DiffDrive Notes

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1 Required Objects for the DiffDrive Class

In order to create the DiffDrive class, I will need to define certain attributes about the robot and provide functions to compute the Forward Kinematics, Inverse Kinematics, and Twist of the robot in space.

The wheel radius and wheel track were provided from the .yaml file referenced in the launch file, so these values will be used throughout the calculations as constant variables. By writing methods and equations for Forward Kinematics, Inverse Kinematics, and Integrate twist (over a timestep)

2 Mathmatics for Turtlebot3 Kinematics

To calculate the kinematics of the Turtlebot, we will need to represent the important frames of the turtlebot: The body frame, the wheel frames, and the world frame.
The adjoint matrix is critical to represent the twist in each frame which can be written as:
$Ad_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ y & cos(\theta) & sin(\theta) \\ -x & sin(\theta) & cos(\theta) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ y & 1 & 0 \\ -x & 0 & 1 \end{bmatrix}$ when the frames have no rotation in the transformation between them. The twist V_i can be represented as $V_i = \begin{bmatrix} \dot{\theta}_i \\ \dot{x}_i \\ \dot{y}_i \end{bmatrix} = Ad_{ij}V_j$ and the wheel velocities can be related to the wheel rotations through the equation $\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} r\dot{\phi} \\ 0 \end{bmatrix}$ where $v_x, v_y, r, \dot{\phi}$ are the x-velocity of the wheel frame, the y-velocity of the wheel frame, the wheel radius, and the wheel's rotational velocity about the wheel's axle. The Turtlebot3 can be represented using the figure 1.
The B frame represents the body frame of the robot while the w frames represent the wheels. Using the transformations $T_{w1} = (\omega, x, y) = (0, 0, TW/2)$ and $T_{w2} = (\omega, x, y) = (0, 0, -TW/2)$ where TW refers to the track width of the robot, we can substitute in the equations we have to get:

$$V_{w1} = \begin{bmatrix} \dot{\theta}_{w1} \\ \dot{x}_{w1} \\ \dot{y}_{w1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -TW/2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_B \\ \dot{x}_B \\ \dot{y}_B \end{bmatrix} = \begin{bmatrix} \dot{\theta}_B \\ (-TW/2)\dot{\theta}_B + \dot{x}_B \\ \dot{y}_B \end{bmatrix}$$

$$V_{w2} = \begin{bmatrix} \dot{\theta}_{w2} \\ \dot{x}_{w2} \\ \dot{y}_{w2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ TW/2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_B \\ \dot{x}_B \\ \dot{y}_B \end{bmatrix} = \begin{bmatrix} \dot{\theta}_B \\ (TW/2)\dot{\theta}_B + \dot{x}_B \\ \dot{y}_B \end{bmatrix}$$

Substituting the wheel rotation equations, we get:

$$\begin{bmatrix} \dot{\theta}_B \\ r\dot{\phi}_{w1} \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\theta}_B \\ -(TW/2)\dot{\theta}_B + \dot{x}_B \\ \dot{y}_B \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta}_B \\ r\dot{\phi}_{w2} \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\theta}_B \\ (TW/2)\dot{\theta}_B + \dot{x}_B \\ \dot{y}_B \end{bmatrix}$$

It should be noted that the y-velocity values are 0 because this is a differential drive robot, so if that value is non-zero, we know we have slipping. From this we can get the H matrix $\dot{\phi} = HV_B$

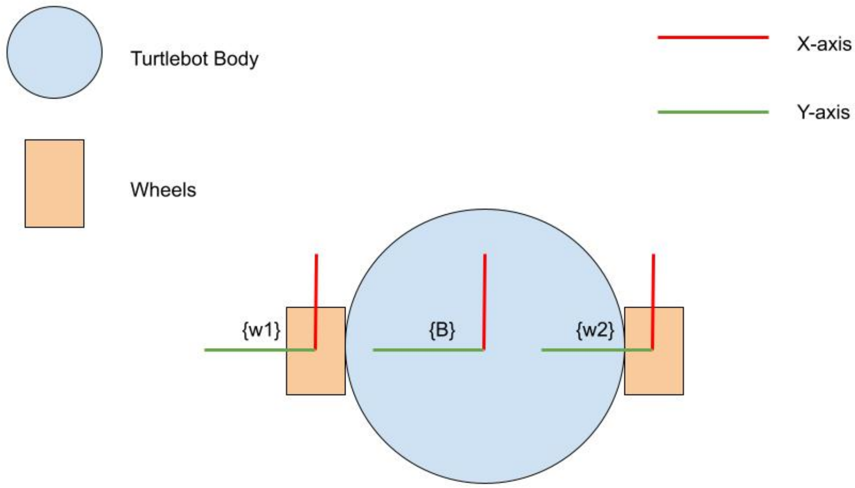

Figure 1: Turtlebot3 Drawing with Axes and Frames

$$\begin{bmatrix} \dot{\phi}_{w1} \\ \dot{\phi}_{w2} \end{bmatrix} = 1/r \begin{bmatrix} -(TW/2) & 1 & 0 \\ (TW/2) & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_B \\ \dot{x}_B \\ \dot{y}_B \end{bmatrix}$$

To then get the twist from the wheel motion, we can use the equation $V_B = H^{\dagger}\dot{\phi}$ [3]

$$V_b = r \begin{bmatrix} -(1/TW) & (1/TW) \\ 1/2 & 1/2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\phi}_{w1} \\ \dot{\phi}_{w2} \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta}_B \\ \dot{x}_B \\ \dot{y}_B \end{bmatrix} = \begin{bmatrix} (r/TW)(-\dot{\phi}_{w1} + \dot{\phi}_{w2}) \\ (r/2)(\dot{\phi}_{w1} + \dot{\phi}_{w2}) \\ 0 \end{bmatrix} \tag{1}$$

In this equation we again see that the y-velocity is 0 due to this robot being a differential drive robot. If we want to get the configuration of the robot over one time step dt, we could multiply the equation by dt to get the following. Because the time step is always one "units", we can do this easily.

$$\begin{bmatrix} \Delta\theta_B \\ \Delta x_B \\ \Delta y_B \end{bmatrix} = \begin{bmatrix} (r/TW)(-\Delta\phi_{w1} + \Delta\phi_{w2}) \\ (r/2)(\Delta\phi_{w1} + \Delta\phi_{w2}) \\ 0 \end{bmatrix} \tag{2}$$

3 Other Conversions

To get a new body frame B' from frame B after a twist motion, we can use the equation

$$T_{BB'} = \text{integrate\_twist}(V_B) \text{ where integrate\_twist() is a function using equation 2.} \tag{3}$$

The body frame B in the world (space) frame is simply the configuration of the robot at that time.

$$T_{SB} = q \tag{4}$$

To get B' a simple use of the subscript rule can yield that transform.

$$T_{SB'} = T_{SB} T_{BB'} \tag{5}$$

Extracting the configuration from this transform yields the new configuration of the robot in the world (space) frame.