TD1 Quantization

Professor: Marc Antonini (am@i3s.unice.fr)
Lab assistant: Anca Grapa (agrapa@i3s.unice.fr)

INTRODUCTION IN CODING PRINCIPLE

The conventional principle in coding for signal compression is illustrated in Figure 1. A lossy compression method is necessary in the case of high redundancy in the signal. The input signal is first **transformed** in order to be more compressible.There are different kind of transformations such as Discrete Cosine Transform (DCT), wavelets, Gaussian filters, Gabor filters, Laplacian Pyramids, etc. The process of reducing the number of coefficients is called **quantization**. The transform domain is convenient to decide which are the coefficients which are less involved and eliminate them. Last but not least, the quantized signal is sequenced and losslessly packed into the output bit-stream (**entropy coding**). The decoding chain consists of the reverse steps, expect of the quantization process which is irreversible.

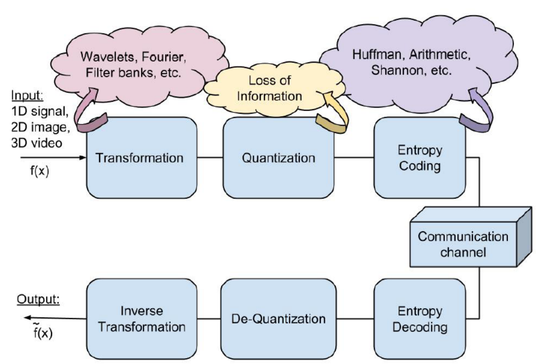

FIGURE 1 – Coding Principle.

Scalar quantization

Scalar quantization is a process that maps a signal with a specified range to another finite set of numerical values, commonly by selecting its nearest neighbour among these.
Regarding the **uniform quantization**, the dynamic range (minimum to maximum values) of a discrete signal is partitioned into L equally sized intervals, each with length Δ, also known as quantization step-size.

- Construct a uniform **scalar quantizer** with L quantization levels, where $L = 2^R$, (R is the number of bits that are required to encode a quantized sample).
- Plot the input/output characteristic function of this quantizer.
- For all the possible values of R quantize the "lena.jpg" image.
- Measure the **distortion** D of quantization by computing the **Mean Square Error (MSE)** between input signal $s(n)$ and quantized signal $\hat{s}(n)$, given by the following formula . Explain.

$$D = \frac{1}{N}\sum_{n=1}^{N}|s(n) - \hat{s}(n)|^2$$

- Plot a graph $D(R)$ with $R = log_2 L$ bits/sample. Explain.

Estimation of the entropy

Generally, the entropy represents the quantity of information provided by an event of the source signal S. In our case, the **histogram** of the image with L gray levels determines the quantity of information as defined by Shannon. The Shannon entropy H of the source signal S with probability distribution $p_X(i)$ has the following formula :

$$H(S) = -\sum_{i=1}^{L} p_S(i)\log_2 p_S(i)$$

- Plot a graph D=function(Total Entropy). Explain.
- Plot a graph $PSNR_{dB}$=function(Total Entropy). Explain. The PSNR (Peak Signal-to-Noise Ratio) is defined in the following way :

$$PSNR_{dB} = 10log_{10}\frac{255^2}{mse}$$