Information Theory and Computation
Exercise 9bis

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Translational invariant states

Take the translational operator $T = \sum_{i=1}^{N} \sigma_x^i \sigma_x^{i+1}$ (using periodic boundary conditions). Suppose we want to diagonalize this operator: we need to build 2^N states that are traslational invariant. Suppose we find an ensemble of M states $\{t_i\}$ for which

$$Tt_i = t_{i+1} \tag{1}$$

then any Fourier like combination is traslational invariant:

$$Tv_p = T\sum_{q=1}^{M} e^{2\pi i \frac{pq}{M}} t_q = \sum_{q=1}^{M} e^{2\pi i \frac{pq}{M}} t_{q+1} = e^{-2\pi i \frac{p}{M}} v_p \tag{2}$$

We have to build 2^N of these states and we have to do it in a way that exploits the above property. Let's start from the trivial state which is $|0\dots 0\rangle$. Now we proceed by adding one 1 at the previous state i.e. we use as $\{t_i\}$ the followings states:

$$|100\dots00\rangle, |010\dots00\rangle, |001\dots00\rangle, \dots, |000\dots01\rangle \tag{3}$$

There exist N states like this and using the Fourier series we can extract N basis vector. In this way we are block-diagonalizing the Hamiltonian. Let's move to the next kind of state by adding another 1:

$$|1100\dots00\rangle, |0110\dots00\rangle, |0011\dots00\rangle, \dots, |000\dots11\rangle \tag{4}$$

$$|10100\dots000\rangle, |01010\dots000\rangle, |00101\dots000\rangle, \dots, |000\dots101\rangle, |010\dots001\rangle \tag{5}$$

$$|100100\dots000\rangle, |010010\dots000\rangle, |001001\dots000\rangle, \dots, |100000\dots100\rangle, |010000\dots010\rangle, |001000\dots001\rangle \tag{6}$$