

Computational Quantum Physics

Week 8

Due on Week 9

Exercise 1: **Density Matrices**

Given a quantum system formed by N subsystems (spins, atoms, particles etc..) each described by its wave function $\psi_i \in \mathcal{H} = \mathbb{C}^d$, a d -dimensional Hilbert space.

How do you write the total wave function of the system $\Psi(\psi_1, \dots, \psi_N)$? Write a Fortran code to

- (a) describe a N -body non interacting, separable pure state
- (b) a general N -body pure wave function $\Psi \in \mathcal{H}^{\otimes N}$.

Comment on their efficiency. How large N can be? Furthermore, setting $N=2$:

- (a) Write the density matrix of a generic pure state Ψ , $\rho = |\Psi\rangle\langle\Psi|$.
- (b) Given the generic density matrix ρ compute the reduce density matrix of either the left or the right system, e.g. $\rho_1 = \text{Tr}_2 \rho$.
- (c) Test the functions described before (and all others needed) on two-spin one-half (qubits) with different states.