Information Theory and Computation
Exercise 10

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Theory

Real Space Renormalization Group

The Real Space Renormalization Group procedure for the Ising chain in a transverse field starts from an Hamiltonian of N spins which we will take equals to:

$$H_N = \lambda \sum_{i=1}^{N} \sigma_i^z + \sum_{i=1}^{N-1} \sigma_i^x \sigma_{i+1}^x$$

At each iteration of the algorithm we want to double the number of spins represented keeping the same dimensionality of the Hamiltonian ($2^N \times 2^N$). At each iteration we have an Hamiltonian $\tilde{H}_N$ (which is, at the first step, the original Hamiltonian) and the interaction Hamiltonians (living in n Hilbert space relative to $N/2$ spins) H_L and H_R for the left sites and right sites of the system; at first we have $H_L = \mathbb{I} \otimes \cdots \otimes \mathbb{I} \otimes \sigma_{N/2}^x$ and $H_R = \sigma_{N/2+1}^x \otimes \mathbb{I} \otimes \cdots \otimes \mathbb{I}$. Given the initial conditions the procedure goes as follows:

- Diagonalize $\tilde{H}_N$ obtaining the projection matrix P
- Project $\tilde{H}_N$ using a reduced version of P, P', relative only to the first $2^{N/2}$ eigenvector hence obtaining a new Hamiltonian $\tilde{H}'_{N/2}$ representing N spins in a $N/2$ spins effective Hamiltonian.
- We restore our previous dimensionality by sticking together two equal $\tilde{H}'_{N/2}$. The interaction $\tilde{H}_{int}$ between the two is obtained by first updating H_L and H_R using the same projector $H_L \leftarrow P'^{\dagger} H_L P'$ and $H_R \leftarrow P'^{\dagger} H_R P'$ and second defining $\tilde{H}_{int} = H_L \otimes H_R$
- The Hamiltonian $\tilde{H}_N$ is updated by $\tilde{H}'_{N/2} \otimes \mathbb{I}^{N/2} + \mathbb{I}^{N/2} \otimes \tilde{H}'_{N/2} + \tilde{H}_{int}$

Since at each iteration we double the number of spins represented hence at the end, if we made N_{itr} iterations, we would have $2^{N_{itr}} N$ spins. The ground state energy density of the system is obtained by diving the smallest eigenvalue of the final $\tilde{H}_N$ by $2^{N_{itr}} N$.

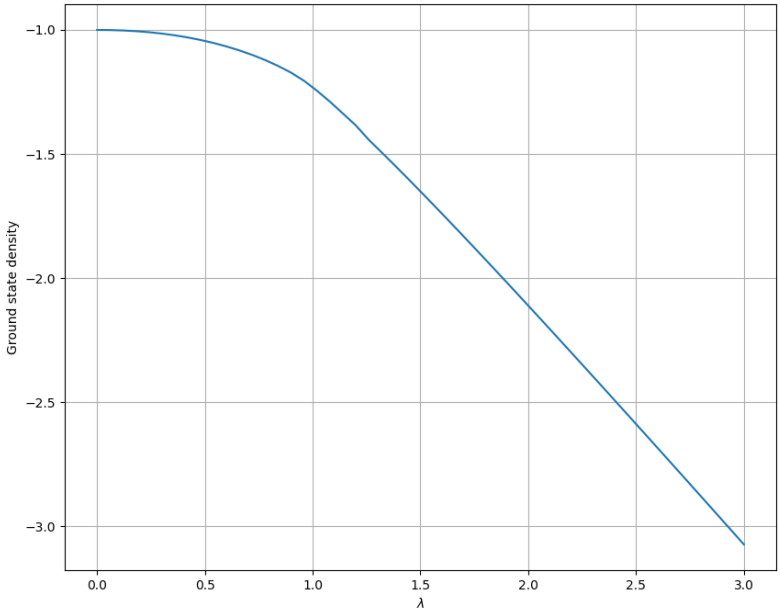
Code Development

The following code block applies the RSRG procedure on a N dimensional Hamiltonian H. The functions *applyIdentitiesLeft* and *applyIdentitiesRight* applies a tensor product of a certain number of identity matrices eitehr on the left or on the right of the given matrix. The function *applyProjection(A,P)* returns the matrix $A' = P^\dagger AP$. The final Hamiltonian is found in the variable H_{new}.

```
NHalf = N/2
sz = 2**N
szHalf = 2**NHalf
allocate(P(sz,sz))
allocate(Hnew(sz,sz))
P=H
Hnew=H
HL=applyIdentitiesLeft(sX, 2, NHalf-1)
HR=applyIdentitiesRight(sX, 2, NHalf-1)
do iter = 1, Niters
        call herm_diag(P,sz,eigs,'V',info)
        deallocate(eigs)
        Ht = applyProjection(Hnew,P(1:sz, 1:szHalf))
        HL = applyIdentitiesLeft(HL, 2, NHalf)
        HR = applyIdentitiesRight(HR, 2, NHalf)
        HL = applyProjection(HL,P(1:sz, 1:szHalf))
        HR = applyProjection(HR,P(1:sz, 1:szHalf))
        P = applyIdentitiesRight(Ht,2,NHalf) + applyIdentitiesLeft(Ht,2,NHalf)
        P = P + tensor_product(HL,HR, szHalf, szHalf)
        Hnew=P
end do
```

Results

Below we show the resulting ground state energy as a function of λ. We used $N = 4$ states as a starting point and $N_{itr} = 1000$ iterations.



For $\lambda = 0$ one expects a theoretical value of -1 and the obtained one (using double precision arithmetic) is -1.0000000000005036. This very small error is also obtained using a smaller number of iterations.

Final considerations

Such implementation of the RSRG algorithm was really simplified by the use of auxiliary functions which took care of the annoying parts of the code allowing the code to show only the mathematical operations performed.