Trapping - Cooling - Quantum Control

Summer term 2019 - Lecturer: Tobias Schätz, Leon Karpa

Assignment sheet 4

please hand in your solutions by May 15, 18:00.

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1) π – and $\frac{\pi}{2}$ – pulses

In this exercise we want to study the action of laser pulses of finite length on the state of a two-level atom given by $|\psi\rangle = c_1 |1\rangle + c_2 |2\rangle$. On resonance ($\delta = 0$) the time evolutions for the coefficients c_1 and c_2 can be described by

$$c_1(t) = \cos\left(\frac{\Omega_{12}}{2}t\right), \qquad c_2(t) = -i\sin\left(\frac{\Omega_{12}}{2}t\right),$$

where Ω_{12} denotes the (on-resonance) Rabi frequency. Consider an initial state $|\psi(t=0)\rangle = |1\rangle$, i.e. all the population is in the ground state ($c_1(t) = 1$, $c_2(t) = 0$).

a) Calculate the corresponding density matrix $\rho = |\psi\rangle\langle\psi|$.

(1 Point)

We now want to see what a (resonant) laser pulse of length $t_\pi \equiv \pi/\Omega_{12}$ does to the two-level system.

b) Calculate the density matrix $\rho = |\psi(t_\pi)\rangle\langle\psi(t_\pi)|$.

(1 Point)

c) What is the overall effect of two pulses of length t_π on $|1\rangle$?

(1 Point)

d) Show that a pulse of length $t_{\pi/2} \equiv t_\pi/2$ transfers $\psi(0)$ into $(|1\rangle - i|2\rangle)/\sqrt{2}$.

(1 Point)

e) What is the overall effect of two pulses of length $t_{\pi/2}$?

(1 Point)

f) Assume that the excited state $|2\rangle$ experiences a phase shift of ϕ between the two $\pi/2$-pulses. Derive an expression for the probabilities of ending up in $|1\rangle$ and $|2\rangle$ after the two pulses.

(2 Points)

g) Assume now the accumulated phase ϕ is a linear function of the (waiting) duration t_{wait} between the two pulses, i.e., $\phi \to \phi(t_{\text{wait}}) \propto t_{\text{wait}}$. Sketch the probabilities of ending up in $|1\rangle$ and $|2\rangle$ after the two pulses as a function of the waiting duration t_{wait}.

(2 Points)