Trapping - Cooling - Quantum Control

Summer term 2019 - Lecturer: Tobias Schätz, Leon Karpa

Assignment sheet 6

please hand in your solutions by May 29, 18:00.

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1) General Questions

- Typical potential depths of Paul traps are on the order of 1 eV. What temperatures would this correspond to?
- In which way are optical dipole traps and Paul traps analogous?

2) Single Beam Optical Dipole Trap

The electric field of a laser is described by a gaussian beam

$$\mathbf{E}(r,z) = \mathbf{E}_0 \frac{w_0}{w(z)} \exp\left\{\frac{-r^2}{w(z)^2}\right\} \exp\{-i(kz + \phi(z))\} \ , \tag{1}$$

where w_0 is the waist radius, $r = \sqrt{x^2 + y^2}$ is the *radial* distance from the center of the beam and $\phi(z)$ is an additional phase that will not be considered here. The evolution of the waist along the z-direction (*axial*) is given by

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_\mathrm{R}}\right)^2} \ , \tag{2}$$

in terms of the Rayleigh length

$$z_\mathrm{R} = \frac{\pi w_0^2}{\lambda} \ . \tag{3}$$

a) What is the intensity profile of a gaussian beam? Derive an expression in terms of the total power transmitted by the beam $P_\mathrm{tot} = \frac{\pi}{2} I_0 w_0^2$, where $I_0 = {}^1\!/_2\, \epsilon_0\, c\, E_0^2$. Sketch the intensity profiles along the axial ($r = 0$) and radial ($z = 0$) direction.

(2 Points)

b) Optical dipole traps for ultracold atoms are typically ten times deeper than the thermal energy $k_B T$ of the confined atoms. Consider a dilute gas of lithium-6 (^{6}Li) atoms ($\lambda_0 = 671\,\mathrm{nm}$, $\Gamma = 2\pi \times 6\,\mathrm{MHz}$, $I_\mathrm{sat} = 2.54\,\mathrm{mW/cm^2}$) at a temperature of 300 µK. When using a trapping laser with a wavelength of $\lambda = 1064\,\mathrm{nm}$ focussed to $w_0 = 15\,\mu\mathrm{m}$, what optical power is required to trap the cloud?

(2 Points)

c) Assume the coherence of the atoms in b) is limited only by off-resonant scattering of photons from the trapping beam. What is the coherence time?

(1 Point)

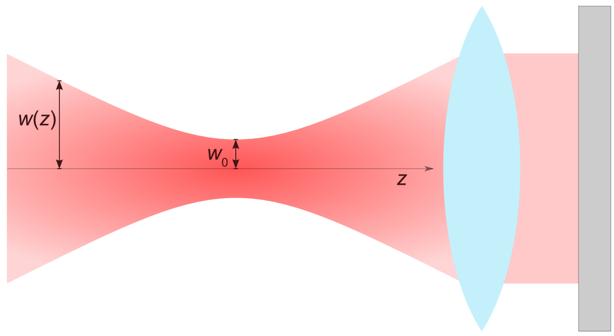

Figure 1: A gaussian beam, collimated and retro-reflected, can be used to increase trap depth and axial confinement with respect to a single beam setup.

d) Approximate the trapping potential in axial and radial direction around $z \approx 0$ as a harmonic oscillator. Find expressions for the axial and radial trapping frequencies ω_{ax} and ω_{rad} in terms of the trap depth U_0. For the above setup, what are the oscillation frequencies $\nu = \frac{\omega}{2\pi}$ of the atoms in the trap?

(3 Points)

3) Optical Lattice

Consider the experimental setup shown in Fig. 1, where the trapping laser is collimated by a lens and then retro-reflected.

a) Using (1) calculate the electric field $\mathbf{E}(z, r)$ of the superimposed beams. You can assume a case in which the waist is much larger than the wavelength, such that $\frac{z}{z_R} \approx 0$ and $\phi(z) \approx 0$. Give an expression for and sketch the intensity along the axial direction ($r = 0$).

(2 Points)

b) The intensity now shows spatially periodic maxima in which atoms can be trapped. What is the maximum trap depth U_0/k_B for ^{6}Li atoms (see 1) when a laser of $\lambda_0 = 1064\,\text{nm}$, $P_{\text{tot}} = 20\,\text{W}$ and $w_0 = 50\,\mu\text{m}$ is used.

(1 Point)

c) Derive an expression for the axial frequency ω_{ax} in terms of the trap depth U_0. Compare to the result of 1d). By what factor is the axial confinement increased?

(2 Points)

d) **BONUS:** A $\lambda/4$-plate is placed into the beam path between the lens and the mirror. Describe how the trapping potential changes.

(2 Points)