Trapping - Cooling - Quantum Control

Summer term 2019 - Lecturer: Tobias Schätz, Leon Karpa

Assignment sheet 9

please hand in your solutions by June 26th, 18:00.

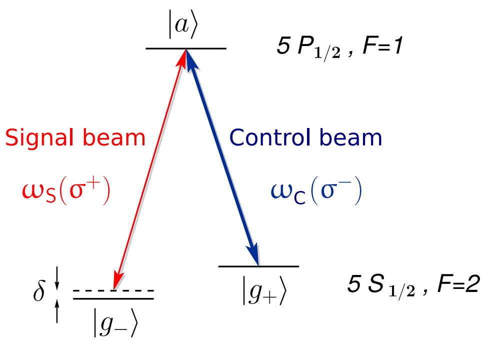
---

1) Electromagnetically Induced Transparency and Slow Light

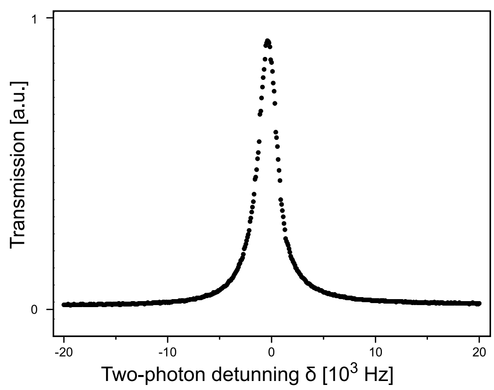
To realize electromagnetically induced transparency (EIT) experimentally, one can use a heated ^{87}Rb spectroscopy cell, filled with Neon as a buffer gas. The signal beam transition $|5\,^2S_{1/2},\, F = 2\rangle \rightarrow |5\,^2P_{1/2},\, F = 1\rangle$ then has a collision- and Doppler-broadened linewidth of $\Delta\nu_{D1} \approx 2\,\mathrm{GHz}$. When the conditions for EIT are sattisfied, a narrow two-photon resonance with a width of $\Delta\nu \approx 2\,\mathrm{kHz}$ appears (see Fig. 1(b)).

a) How does the group velocity v_g depend on the refractive index $n(\omega)$? Estimate v_g for a pulse of light on the signal transition under EIT conditions.
Hint: Assume that $v_g \approx c$ for a signal-light pulse without EIT and that the extremal values of $n(\omega_S)$ are equal under non-EIT and EIT conditions.

(2 Points)



(a) Simplified level scheme of ^{87}Rb, showing the states used for EIT.



(b) Absorption spectrum of a dark state resonance under EIT conditions.

Figure 1: Experimental realization of EIT.

The Hamiltonian of the described system, using the rotating-wave approximation and a transformation to the interaction picture, is given by

$$\hat{H} = \hat{H}_0 + \hat{H}_{\mathrm{int}}\,, \tag{1}$$

where

$$\hat{H}_0 = -\frac{\hbar}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & -2\,\delta & 0 \\ 0 & 0 & -2\,\Delta \end{pmatrix}, \quad \hat{H}_{int} = -\frac{\hbar}{2}\begin{pmatrix} 0 & 0 & \Omega_S \\ 0 & 0 & \Omega_C \\ \Omega_S & \Omega_C & 0 \end{pmatrix}. \tag{2}$$

Here Δ describes the one-photon detuning of the control beam, $\delta = \omega_S - \omega_C - \Delta E_{g_+g_-}/\hbar$ is the two-photon detuning and Ω_i are the respective Rabi frequencies.

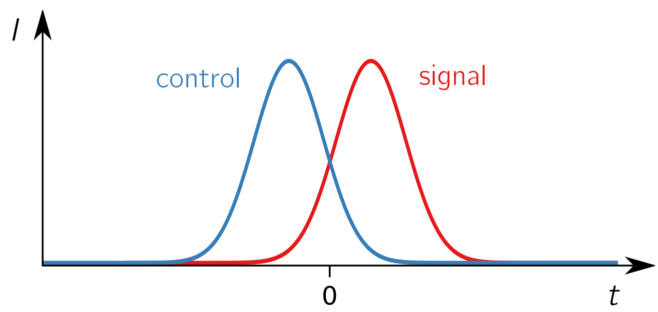


Figure 2: A sequence of two gaussian pulses, as experienced by a moving atom flying through two subsequent laser beams.

b) Calculate the eigenvalues λ_i of $\hat{H}$ for the special case of two-photon resonance $\delta = 0$.

(3 Points)

c) Find the eigenvector $|\Psi_0\rangle$ to the smallest absolute energy eigenvalue λ_0.

(2 Points)

d) Calculate the transition matrix element $\langle a| \hat{H}_{\text{int}} |\Psi_0\rangle$ between $|\Psi_0\rangle$ and the excited state of the unperturbed system $|a\rangle$. How does it depend on the one-photon detuning Δ? What physical properties of $|\Psi_0\rangle$ can you derive from this?

(3 Points)

2) Come to the Dark State

In the lecture, you have shown that a state

$$|\Psi\rangle = a\,|g_-\rangle + b\,|g_+\rangle \tag{3}$$

is a dark state, i.e. its dipole transition matrix element with the excited state $|a\rangle$ vanishes, when

$$b = -a\,\frac{\Omega_S}{\Omega_C}\,. \tag{4}$$

The Rabi frequencies can be controled experimentally, e.g. by varying the intensity of the signal or coupling beam. We will now consider an atom initially in the ground state $|g_-\rangle$ to understand how switching of the beam intensities influences the dynamics of the system. The beams are tuned to resonance, i.e. $\Delta = \delta = 0$.

a) Assume that at $t = 0$ both beams are switched on with equal intensity. Describe qualitatively how the system evolves in time. What role does spontaneous emission play?

(2 Points)

b) Now assume a sequence of two pulses as depicted in Fig. 2, where first the control beam is switched on smoothly, then switched off, while the signal beam is switched on. In which state will the system be at $t = 0$ and after the pulse sequence? What role does spontaneous emission play now?

(2 Points)