# Trapping - Cooling - Quantum Control

Summer term 2019 - Lecturer: Tobias Schätz, Leon Karpa

## Assignment sheet 7

please hand in your solutions by June 5th, 18:00.

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**1) Evaporative Cooling**

To study evaporative cooling, we will analyze the effect of a single evaporative cooling step on a cloud of atoms with temperature $T$. The atoms have a Boltzmann energy distribution $N(E) = N_0\,\beta\,e^{-\beta E}$, where $\beta = \frac{1}{k_B T}$. The total energy of the cloud is

$$E_0 = N_0\,\beta \int_0^\infty E\,e^{-\beta E}\,dE = N_0\,k_B T \tag{1}$$

Thus, the mean energy per atom is $\overline{E}_0 = k_B T$. In one step of evaporative cooling we let all atoms with energy greater than $\epsilon$ escape.

a) Calculate the fraction of atoms that escape ${}^{\Delta N}/_{N_0}$, as well as the relative loss in total energy ${}^{\Delta E}/_{E_0}$ in terms of $\beta$ and $\epsilon$.

(2 Points)

b) What is the mean energy per atom $\overline{E}_1 = {}^{E_1}/_{N_1}$ after the first cooling step?

(1 Point)

c) Evaluate the expressions obtained in a) for energy cuts at $\epsilon = 3\,k_B T$ and $\epsilon = 6\,k_B T$. For both cases, what is the relative energy loss per lost atom $\frac{\Delta E/E_0}{\Delta N/N_0}$. What are the implications for evaporative cooling?

(2 Points)

d) Plot the atom number and the temperature of an atomic cloud that is cooled down to $\frac{T}{T_{\text{init}}} = 0.01$ in subsequent evaporation steps for different values of $\epsilon/k_B T = [1, 3, 6, 10]$.

(2 Points)

e) A ramp consisting of many consecutive evaporation steps is limited in its speed by the time that the atoms need to thermalize between the steps. Thermalization is achieved by elastic collisions between the atoms. The collision rate between atoms in the cloud is given by

$$R_{\text{coll}} = n\,\overline{v}\,\sigma\,, \tag{2}$$

where $n$ is the number density, $\overline{v}$ is the mean velocity and $\sigma$ is their collision cross-section. Assuming a constant scattering rate, show that in a harmonic oscillator potential

$$R_{\text{coll}} \propto \frac{N}{E}\,. \tag{3}$$

What is the relative increase of the collision rate for an evaporative cooling step of $\frac{\epsilon}{k_B T} = 3$?

*Hint:* A cloud of thermal atoms in a harmonic oscillator potential with an oscillation frequency $\omega$ has a radius $r$ given by

$$\frac{1}{2}m\omega^2 r^2 = \frac{1}{2}k_B T.$$

(4 Points)

**2) Bose-Einstein Condensation**

Assume a cloud of $4 \times 10^6$ Na atoms trapped in a single-beam optical-dipole trap. They experience a harmonic potential with radial oscillation frequencies $\omega_x = \omega_y = 2\pi \times 250\,\text{Hz}$ and axial oscillation frequency $\omega_z = 2\pi \times 16\,\text{Hz}$.

a) What is the critical temperature $T_C$ at which the transition from a thermal gas to a Bose-Einstein condensate occurs? Express the result as a temperature and in terms of the effective harmonic oscillator level spacing $\hbar\overline{\omega}$.

(2 Points)

b) The Na atoms are evaporatively cooled down to $0.8T_C$. How many atoms are condensed to the ground state?

(2 Points)