contains the posteriors, we divide the third column by the normalizing constant.

That's it. Simple, right?

Well, you might be bothered by one detail. I write $p(D|H)$ in terms of percentages, not probabilities, which means it is off by a factor of 10,000. But that cancels out when we divide through by the normalizing constant, so it doesn't affect the result.

When the set of hypotheses is mutually exclusive and collectively exhaustive, you can multiply the likelihoods by any factor, if it is convenient, as long as you apply the same factor to the entire column.

1.7 The Monty Hall problem

The Monty Hall problem might be the most contentious question in the history of probability. The scenario is simple, but the correct answer is so counterintuitive that many people just can't accept it, and many smart people have embarrassed themselves not just by getting it wrong but by arguing the wrong side, aggressively, in public.

Monty Hall was the original host of the game show *Let's Make a Deal*. The Monty Hall problem is based on one of the regular games on the show. If you are on the show, here's what happens:

- Monty shows you three closed doors and tells you that there is a prize behind each door: one prize is a car, the other two are less valuable prizes like peanut butter and fake finger nails. The prizes are arranged at random.
- The object of the game is to guess which door has the car. If you guess right, you get to keep the car.
- You pick a door, which we will call Door A. We'll call the other doors B and C.
- Before opening the door you chose, Monty increases the suspense by opening either Door B or C, whichever does not have the car. (If the car is actually behind Door A, Monty can safely open B or C, so he chooses one at random.)
- Then Monty offers you the option to stick with your original choice or switch to the one remaining unopened door.

The question is, should you "stick" or "switch" or does it make no difference?

Most people have the strong intuition that it makes no difference. There are two doors left, they reason, so the chance that the car is behind Door A is 50%.

But that is wrong. In fact, the chance of winning if you stick with Door A is only 1/3; if you switch, your chances are 2/3.

By applying Bayes's theorem, we can break this problem into simple pieces, and maybe convince ourselves that the correct answer is, in fact, correct.

To start, we should make a careful statement of the data. In this case D consists of two parts: Monty chooses Door B *and* there is no car there.

Next we define three hypotheses: A, B, and C represent the hypothesis that the car is behind Door A, Door B, or Door C. Again, let's apply the table method:

	Prior $\mathrm{p}(H)$	Likelihood $\mathrm{p}(D\|H)$	$\mathrm{p}(H)\ \mathrm{p}(D\|H)$	Posterior $\mathrm{p}(H\|D)$
A	1/3	1/2	1/6	1/3
B	1/3	0	0	0
C	1/3	1	1/3	2/3

Filling in the priors is easy because we are told that the prizes are arranged at random, which suggests that the car is equally likely to be behind any door.

Figuring out the likelihoods takes some thought, but with reasonable care we can be confident that we have it right:

- If the car is actually behind A, Monty could safely open Doors B or C. So the probability that he chooses B is 1/2. And since the car is actually behind A, the probability that the car is not behind B is 1.
- If the car is actually behind B, Monty has to open door C, so the probability that he opens door B is 0.
- Finally, if the car is behind Door C, Monty opens B with probability 1 and finds no car there with probability 1.

Now the hard part is over; the rest is just arithmetic. The sum of the third column is 1/2. Dividing through yields $\mathrm{p}(A|D) = 1/3$ and $\mathrm{p}(C|D) = 2/3$. So you are better off switching.