Matrix Factorization for Recommender Systems

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Galvanize

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Overview

Matrix Factorzation Recommenders

- Setup/Intuition
- Factorization
- Algorithms
- Nuances
- Final Thoughts

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Setup → Sparse Ratings Matrix

	Movie 1	Movie 2	Movie 3	···	Movie m
User 1	4	?	?	...	1
User 2	3	3	2	?	2
User 3	?	3	?	?	?
⋮	...	...	?	?	...
User n	?	5	4	...	5

X - Ratings Matrix

Could be very, very sparse → 99% of entries unknown.

Downfall of Collaborative Filtering

Item-Item: I like action movies $\rightarrow$ rate *Top Gun* and *Mission Impossible* 5s.
$\rightarrow$ I'm recommended *Jerry Maguire* even though I won't like it.

User-User: I like Tom Cruise $\rightarrow$ rate *Top Gun* and *Mission Impossible* 5s.
$\rightarrow$ I'm recommended *Transformers* even though I won't like it.

Movies (and Everything Else) Have Attributes

- Action, Romance, Comedy, etc.
- Tom Criuse, Tom Hanks, Megan Fox, etc.
- Long, Short, Subtitles, Foreign, Happy, Sad, etc.

Could We Use a Linear Regression?

$$
\begin{aligned}
\textit{Rating Prediction} = \beta_0 &+ \beta_1 \times \textit{actionness} + \ldots \\
&+ \beta_i \times \textit{foxiness} + \ldots \\
&+ \beta_j \times \textit{sadness} + \epsilon
\end{aligned}
$$

Could We Use a Linear Regression?

$$\begin{aligned} \textit{Rating Prediction} &= \beta_0 + \beta_1 \times \textit{actionness} + \ldots \\ &+ \beta_i \times \textit{foxiness} + \ldots \\ &+ \beta_j \times \textit{sadness} + \epsilon \end{aligned}$$

Possibly...though we'd have to come up with some measure of actionness, etc. This is both subject to error and rather brittle.

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What About Factorization?

- Factorization could account for something along the lines of these attributes as was our hope in LR.
- All of the factorization models that we know can be interpreted as a linear combination of bases.
- There's a chance, especially with NMF, that those bases, latent features, could correspond with some of these "attributes" that we're looking to describe the movies with.

Factorization Problem

- **Problem:** PCA, SVD and NMF **all** must be computed on a **dense** data matrix, X.
- Potential Solution: inpute missing values, naively, with something like the mean of the known values. *Note: This is what sklearn does when it says it factorizes sparse matrices.*

Factorization Goal

- Create a factorization for a sparse data matrix, X, into $U \cdot V$, such that the reconstruction to $\hat{X}$ serves as a model.
- More formally, for a previously unknown entry in X, $X_{i,j}$ the corresponding entry in $\hat{X}$, $\hat{X}_{i,j}$ serves as a prediction.
- *Note: Since we could easily overfit the known values in X we want to regularize, one way to do this is by reducing the inner dimension in U and V, k.*

Factorization Visual

X (sparse)

	Movie 1	Movie 2	...	Movie m
User 1	4	?	...	1
User 2	3	3	...	2
User 3	?	?	...	?
⋮	...	...	...	...
User n	?	5	...	5

≈

U (dense)

LF 1	...	LF k
0.5	...	6.3
2	...	-1.1
..	...	...
...	...	...
-0.1	...	0.2

•

V (dense)

	Movie 1	Movie 2	...	Movie m
LF 1	1.5	-0.4	...	-2.9
⋮	...	...	...	...
LF k	5.3	-1.1	...	0.2

Reconstruction Visual

LF 1	...	LF k
0.5	...	6.3
2	...	-1.1
..	...	...
...	...	...
-0.1	...	0.2

U

dense

•

	Movie 1	Movie 2	...	Movie m
LF 1	1.5	-0.4	...	-2.9
⋮	...	...	...	...
LF k	5.3	-1.1	...	0.2

V

dense

=

	Movie 1	Movie 2	...	Movie m
User 1	4.23	3.2	...	1.03
User 2	2.97	3.12	...	1.8
User 3	3.78	4.36	...	4.65
⋮	...	...	...	...
User n	2.45	4.89	...	4.77

$\hat{X}$

dense

Difference between CF and MF

- Collaborative Filtering (neighborhood models) $\rightarrow$ **Memory Based**. Just store data so we can query what/whom is most similar when asked to recommend.
- Factorization Techniques $\rightarrow$ **Model Based**. Creates predictions, from which the most suitable can be recommended.

Computing the Factorization

- Similar to what we did to find the factorization in NMF, we're going to minimize a cost function.
- Now, though we can't do it at the level of the entirety of X, since it is sparse.
- However, we can optimize with respect to the data in X that we do have.

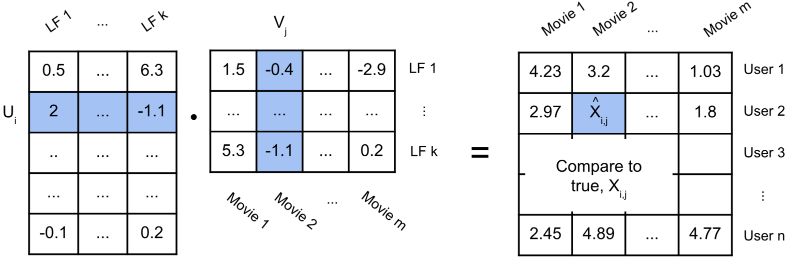
Factorization Plan

For each of the known ratings in $X_{i,j}$ we want to minimize the square error in the prediction that results from $U_i \cdot V_j$, a.k.a.

$$\min_{U,V} \sum_{(i,j)\in K} (X_{i,j} - U_i \cdot V_j)^2.$$

Where U_i is the i^{th} row of U, V_j is the j^{th} column of V, and K is the set of indices in X that have data.

Reconstructing a Single Entry



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Algorithms

- This minimization can be solved with ALS, rotating between fixing the U_is to solve for the V_js and fixing the V_js to solve for the U_is.
- A more popular alternative is a version of gradient descent popularized by Simon Funk during the Netflix prize, know as Funk SVD.

Funk SVD

- Define the error on a particular prediction in X as $e_{i,j} = X_{i,j} - \hat{X}_{i,j}$.
- Then we can update the columns in U and V with:
  - $U_i \leftarrow U_i + \nu(e_{i,j} V_j)$
  - $V_j \leftarrow V_j + \nu(e_{i,j} U_i)$

Funk SVD Algorithm

Initialize U and V with small random values.

While error is decreasing:

- For each user, i:
  - For each item rated by that user, j:
    1. Predict rating, $\hat{X}_{i,j}$.
    2. Calculate $e_{i,j}$.
    3. Update U_i and V_j.

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Baseline Predictors (Biases)

- Much of the observed ratings are associated with a specific user's personality or an item's intrinsic value, not an interaction between the two which is what get captured in the factorization.
- To encapsulate these effects, which do not involve user-item interactions, we introduce **baseline predictors**.
  - μ: Baseline average value in X.
  - b_i: Baseline rating for user i.
  - b_j: Baseline rating for item j.
- From this we can describe our predictions with:

$$\hat{X}_{i,j} = \mu + b_i + b_j + U_i \cdot V_j.$$

Regularization

- Another way to regularize our decomposition to help prevent from overfitting to our sparse data is via a penalty, λ, placed on the magnitude of: b_i, b_j, U_i and V_j. The most common is the L_2 norm.
- Such a penalty changes our cost function to:

$$\min_{b_:,U,V} \sum_{(i,j)\in K} (X_{i,j} - U_i \cdot V_j)^2 + \lambda(b_i^2 + b_j^2 + |U_i|^2 + |V_j|^2)$$

Regularization Update Rules

With these considerations the update rules become:

- $b_i \leftarrow b_i + \nu(e_{i,j} - \lambda b_i)$
- $b_j \leftarrow b_j + \nu(e_{i,j} - \lambda b_j)$
- $U_i \leftarrow U_i + \nu(e_{i,j} V_j - \lambda U_i)$
- $V_j \leftarrow V_j + \nu(e_{i,j} U_i - \lambda V_j)$

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Validation

Validating any recommender is difficult, but it is necessary as we're going to want to tune the hyperparameters that we introduced into our model, ν and λ.

The most frequently used metric is Root Mean Squared Error (RMSE) on the known data:

$$RMSE = \sqrt{\sum_{(i,j) \in K} (X_{i,j} - \hat{X}_{i,j})^2}$$

MF - Pros/Cons

Pros

- Decent with sparsity, so long as we regularize.
- Prediction is fast, only need to do an inner product.
- Can inspect latent features for topical meaning.
- Can be extended to include side information.

Cons

- Need to re-factorize with new data. Very slow.
- Fails in the cold start case.
- Not great open source tools for huge matrices.
- Difficult to tune directly to the type of recommendation you want to make. Tied to the difficulty of measuring success.

Advanced Factorization Methods

- Non-negativity constraint $\rightarrow$ More interpretable latent features.
- SVD++ $\rightarrow$ uses implicit feedback (clicks, likes, etc.) to enhance model.
- Time-aware factor model $\rightarrow$ accounts for temporal information about data.