Two-way split-plot design

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Two-way ANOVA & pairwise comparison post hoc tests in a split-plot design.

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	<pre># (install &) load packages pacman::p_load(conflicted, desplot, emmeans, ggtext, lme4, lmerTest, MetBrewer, multcomp, multcompView, tidyverse)</pre>	
	<pre># handle function conflicts conflicts_prefer(dplyr::filter)</pre>	

```
conflicts_prefer(dplyr::select)
conflicts_prefer(lmerTest::lmer)
```

1 Data

This dataset was originally published in Gomez and Gomez (1984) from a yield (kg/ha) trial with 4 genotypes (G) and 6 nitrogen levels (N), leading to 24 treatment level combinations. The data set here has 3 complete replicates (rep) and is laid out as a split-plot design.

1.1 Import

```
# data is available online:
path <- "https://raw.githubusercontent.com/SchmidtPaul/dsfair_quarto/master/data/Gomez&Gom
dat <- read_csv(path) # use path from above
dat
# A tibble: 72 x 7
   yield
           row
                           mainplot G
                                           N
                 col rep
                                     <chr> <chr>
   <dbl> <dbl> <dbl> <chr> <chr>
 1 4520
                           mp01
             4
                   1 rep1
                                     Simba Goomba
 2 5598
             2
                   2 rep1
                           mp02
                                     Simba Koopa
 3 6192
                                     Simba Toad
             1
                   3 rep1
                           mp03
 4 8542
                                     Simba Peach
             2
                   4 rep1
                           mp04
 5 5806
             2
                   5 rep1
                           mp05
                                     Simba Diddy
 6 7470
                           mp06
                                     Simba Yoshi
             1
                   6 rep1
             2
 7 4034
                   1 rep1
                           mp01
                                     Nala Goomba
 8 6682
                           mp02
                                     Nala Koopa
             4
                   2 rep1
 9
   6869
                   3 rep1
                           mp03
                                     Nala Toad
10 6318
                   4 rep1
                           mp04
                                     Nala Peach
# i 62 more rows
```

1.2 Format

Before anything, the columns rep, N and G should be encoded as factors, since R by default encoded them as character.

```
dat <- dat %>%
  mutate(across(c(rep:N), ~ as.factor(.x)))
```

1.3 Explore

We make use of <code>dlookr::describe()</code> to conveniently obtain descriptive summary tables. Here, we get can summarize per nitrogen level, per genotype and also per nitrogen-genotype-combination.

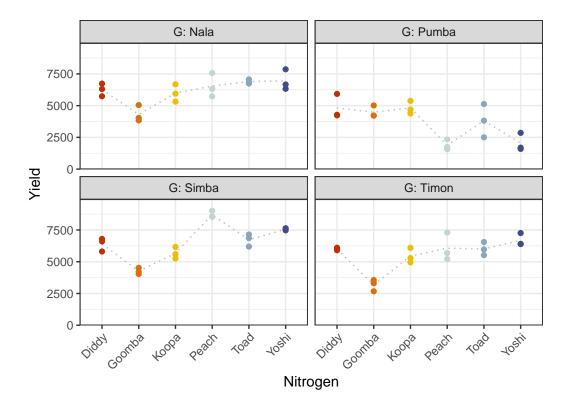
```
dat %>%
  group_by(N) %>%
  dlookr::describe(yield) %>%
  select(2:sd) %>%
  arrange(desc(mean))
# A tibble: 6 x 5
                               sd
 N
             n
                  na mean
  <fct>
        <int> <int> <dbl> <dbl>
                   0 5866. 832.
1 Diddy
            12
2 Toad
            12
                   0 5864. 1434.
3 Yoshi
            12
                   0 5812 2349.
                   0 5797. 2660.
4 Peach
            12
5 Koopa
            12
                   0 5478. 657.
                   0 4054. 672.
6 Goomba
            12
dat %>%
  group_by(G) %>%
  dlookr::describe(yield) %>%
  select(2:sd) %>%
  arrange(desc(mean))
# A tibble: 4 x 5
            n
                 na mean
                              sd
  <fct> <int> <int> <dbl> <dbl>
1 Simba
           18
                  0 6554. 1475.
                  0 6156. 1078.
2 Nala
           18
                  0 5563. 1269.
3 Timon
           18
4 Pumba
           18
                  0 3642. 1434.
```

```
dat %>%
  group_by(N, G) %>%
  dlookr::describe(yield) %>%
  select(2:sd) %>%
  arrange(desc(mean)) %>%
  print(n=Inf)
# A tibble: 24 x 6
          G
                                         sd
                     n
                           na
                              mean
   <fct>
          <fct> <int> <int> <dbl>
                                      <dbl>
                            0 8701.
 1 Peach
          Simba
                     3
                                      270.
 2 Yoshi
                     3
                            0 7563.
                                       86.9
          Simba
 3 Yoshi
                     3
                            0 6951.
          Nala
                                      808.
 4 Toad
          Nala
                     3
                            0 6895
                                      166.
 5 Toad
          Simba
                     3
                            0 6733.
                                      490.
 6 Yoshi
          Timon
                     3
                            0 6687.
                                      496.
 7 Peach
          Nala
                     3
                            0 6540.
                                      936.
 8 Diddy
                     3
                            0 6400
                                      523.
          Simba
 9 Diddy
          Nala
                     3
                            0 6259
                                      499.
10 Peach
          Timon
                     3
                            0 6065.
                                     1097.
11 Toad
          Timon
                      3
                            0 6014
                                      515.
12 Diddy
          Timon
                     3
                            0 5994
                                      101.
13 Koopa
                     3
                            0 5982
          Nala
                                      684.
14 Koopa
          Simba
                     3
                            0 5672
                                      458.
15 Koopa
          Timon
                     3
                            0 5443.
                                      589.
16 Koopa
                     3
                            0 4816
          Pumba
                                      506.
17 Diddy
                     3
                            0 4812
                                      963.
          Pumba
18 Goomba Pumba
                     3
                            0 4481.
                                      463.
19 Goomba Nala
                     3
                            0 4306
                                      646.
20 Goomba Simba
                     3
                            0 4253.
                                      248.
21 Toad
          Pumba
                     3
                            0 3816
                                     1311.
22 Goomba Timon
                     3
                            0 3177.
                                      453.
23 Yoshi
          Pumba
                     3
                            0 2047.
                                      703.
24 Peach
          Pumba
                     3
                            0 1881.
                                      407.
```

Additionally, we can decide to plot our data. One way to deal with the combination of two factors would be to use panels/facets in ggplot2.

Note that we here define a custom set of colors for the Nitrogen levels that will be used throughout this chapter.

```
Ncolors <- met.brewer("VanGogh2", 6) %>%
 as.vector() %>%
 set_names(levels(dat$N))
ggplot(data = dat) +
 aes(y = yield, x = N, color = N) +
 facet_wrap(~G, labeller = label_both) +
 stat_summary(
   fun = mean,
   colour = "grey",
    geom = "line",
   linetype = "dotted",
   group = 1
 geom_point() +
 scale_x_discrete(
   name = "Nitrogen"
 ) +
 scale_y_continuous(
   name = "Yield",
   limits = c(0, NA),
   expand = expansion(mult = c(0, 0.1))
 ) +
 scale_color_manual(
   values = Ncolors,
   guide = "none"
 ) +
 theme_bw() +
 theme(axis.text.x = element_text(
    angle = 45,
   hjust = 1,
   vjust = 1
 ))
```



Finally, since this is an experiment that was laid with a certain experimental design (= a split-plot design) - it makes sense to also get a field plan. This can be done via desplot() from {desplot}.

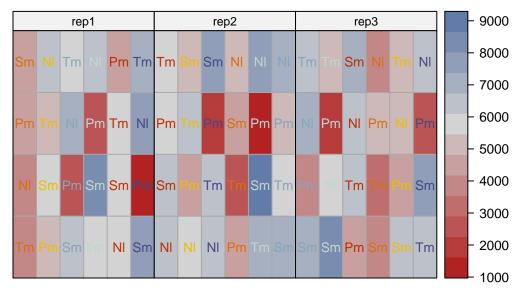
```
desplot(
  data = dat,
  form = rep ~ col + row | rep, # fill color per rep, headers per rep
  col.regions = c("white", "grey95", "grey90"),
  text = G, # genotype names per plot
  cex = 0.8, # genotype names: font size
  shorten = "abb", # genotype names: abbreviate
  col = N, # color of genotype names for each N-level
  col.text = Ncolors, # use custom colors from above
  out1 = col, out1.gpar = list(col = "darkgrey"), # lines between columns
  out2 = row, out2.gpar = list(col = "darkgrey"), # lines between rows
  main = "Field layout", # plot title
  show.key = TRUE, # show legend
  key.cex = 0.7 # legend font size
)
```

Field layout

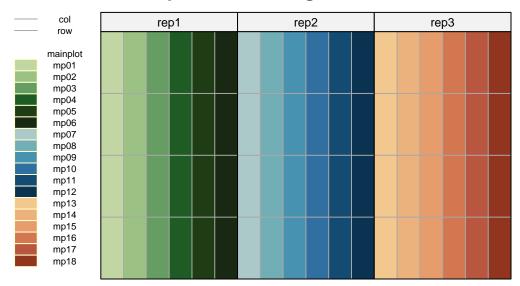
	col	•
	row	rep1 rep2 rep3
	rep rep1 rep2 rep3	Sm NI Tm NI Pm Tm Tm Sm Sm NI NI NI Tm Tm Sm NI Tm NI
•	N Diddy Goomba Koopa	Pm Tm NI Pm Tm NI Pm Tm Pm Sm Pm Pm NI Pm NI Pm NI Pm
•	Peach Toad Yoshi	NI Sm Pm Sm Sm Pm Sm Pm Tm Tm Sm Tm Pm NI Tm Tm Pm Sm
NI Pm Sm Tm	G Nala Pumba Simba Timon	TmPmSmTm NI Sm NI NI NI PmTmSmSmSmPmSmSmTm

```
desplot(
  data = dat,
  form = yield ~ col + row | rep, # fill color per rep, headers per rep
  text = G, # genotype names per plot
  cex = 0.8, # genotype names: font size
  shorten = "abb", # genotype names: abbreviate
  col = N, # color of genotype names for each N-level
  col.text = Ncolors, # use custom colors from above
  out1 = col, out1.gpar = list(col = "darkgrey"), # lines between columns
  out2 = row, out2.gpar = list(col = "darkgrey"), # lines between rows
  main = "Yield per plot", # plot title
  show.key = FALSE # show legend
)
```

Yield per plot







2 Model

Finally, we can decide to fit a linear model with yield as the response variable. In this example it makes sense to mentally group the effects in our model as either design effects or treatment effects. The treatments here are the genotypes G and the nitrogen levels N which we will include in the model as main effects, but also via their interaction effect N:G. Regarding the design, the model needs to contain a block (rep) effect representing the three complete blocks. Additionally, there should also be random effects for the 18 mainplots, since they represent additional randomization units.

```
mod \leftarrow lmer(yield \sim G + N + G:N +
                rep + (1 | rep:mainplot),
              data = dat)
```

⚠ Model assumptions met? (click to show)

It would be at this moment (i.e. after fitting the model and before running the ANOVA), that you should check whether the model assumptions are met. Find out more in the summary article "Model Diagnostics"

3 ANOVA

Based on our model, we can then conduct an ANOVA:

```
ANOVA <- anova(mod)
ANOVA
Type III Analysis of Variance Table with Satterthwaite's method
      Sum Sq Mean Sq NumDF DenDF F value
                                            Pr(>F)
    89885051 29961684
                         3
                              36 85.7416 < 2.2e-16 ***
    19192886
                              10 10.9849 0.0008277 ***
N
             3838577
                         5
      683088
              341544
                         2
                              10 0.9774 0.4095330
rep
G:N 69378044 4625203
                        15
                              36 13.2360 2.078e-10 ***
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

Accordingly, the ANOVA's F-test found the nitrogen-genotype-interaction to be statistically significant (p < .001***).

4 Mean comparison

Besides an ANOVA, one may also want to compare adjusted yield means between cultivars via post hoc tests (t-test, Tukey test etc.). Especially because of the results of this ANOVA, we should compare means for all N:G interactions and **not** for the N and/or G main effects. When doing so, we still have multiple options to choose from. I here decide to compare all genotype means per nitrogen

```
mean_comp <- mod %>%
  emmeans(specs = ~ N|G) %>% # adj. mean per cultivar
  cld(adjust = "Tukey", Letters = letters) # compact letter display (CLD)
mean_comp
G = Nala:
                     df lower.CL upper.CL .group
 N
        emmean SE
          4306 366 41.9
                             3297
                                      5315
 Goomba
                                           a
 Koopa
          5982 366 41.9
                             4973
                                      6991
                                             b
 Diddy
          6259 366 41.9
                             5250
                                      7268
 Peach
          6540 366 41.9
                             5531
                                      7550
                                             b
          6895 366 41.9
                             5886
                                      7904
 Toad
```

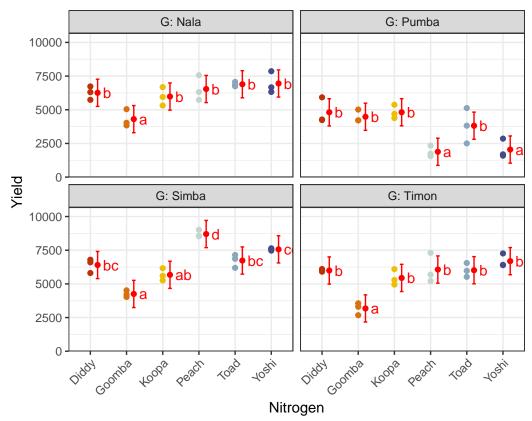
```
Yoshi
          6951 366 41.9
                            5941
                                      7960
G = Pumba:
                     df lower.CL upper.CL .group
N
        emmean SE
Peach
          1881 366 41.9
                              871
                                      2890
Yoshi
          2047 366 41.9
                             1037
                                      3056
                                            a
Toad
          3816 366 41.9
                                      4825
                             2807
                                             b
 Goomba
          4481 366 41.9
                             3472
                                      5491
Diddy
          4812 366 41.9
                             3803
                                      5821
                                             b
Koopa
          4816 366 41.9
                             3807
                                      5825
G = Simba:
                     df lower.CL upper.CL .group
N
        emmean SE
Goomba
          4253 366 41.9
                             3243
                                      5262
                                            а
Koopa
          5672 366 41.9
                             4663
                                      6681
                                            ab
Diddy
          6400 366 41.9
                             5391
                                      7409
                                             bc
 Toad
          6733 366 41.9
                             5723
                                      7742
                                             bc
 Yoshi
          7563 366 41.9
                             6554
                                      8573
                                              cd
 Peach
          8701 366 41.9
                             7691
                                      9710
                                               d
G = Timon:
                     df lower.CL upper.CL .group
        emmean SE
Goomba
          3177 366 41.9
                             2168
                                      4187
Koopa
          5443 366 41.9
                             4433
                                      6452
                                             b
Diddy
          5994 366 41.9
                             4985
                                      7003
 Toad
          6014 366 41.9
                             5005
                                      7023
                                             b
Peach
          6065 366 41.9
                             5056
                                      7075
                                             b
 Yoshi
          6687 366 41.9
                             5678
                                      7697
                                             b
Results are averaged over the levels of: rep
Degrees-of-freedom method: kenward-roger
Confidence level used: 0.95
Conf-level adjustment: sidak method for 6 estimates
P value adjustment: tukey method for comparing a family of 6 estimates
significance level used: alpha = 0.05
NOTE: If two or more means share the same grouping symbol,
      then we cannot show them to be different.
      But we also did not show them to be the same.
```

Note that if you would like to see the underlying individual contrasts/differences between adjusted means, simply add details = TRUE to the cld() statement. Furthermore, check out the Summary Article "Compact Letter Display".

Finally, we can create a plot that displays both the raw data and the results, i.e. the comparisons of the adjusted means that are based on the linear model.

```
my_caption <- "The four facettes represent genotypes Simba, Nala, Timon and Pumba. Black d
ggplot() +
  facet_wrap(~G, labeller = label_both) + # facette per G level
  aes(x = N) +
  # black dots representing the raw data
  geom_point(
    data = dat,
    aes(y = yield, color = N)
  # red dots representing the adjusted means
  geom_point(
    data = mean_comp,
    aes(y = emmean),
    color = "red",
    position = position_nudge(x = 0.2)
  # red error bars representing the confidence limits of the adjusted means
  geom_errorbar(
    data = mean_comp,
    aes(ymin = lower.CL, ymax = upper.CL),
    color = "red",
    width = 0.1,
    position = position_nudge(x = 0.2)
  ) +
  # red letters
  geom_text(
    data = mean_comp,
    aes(y = emmean, label = str_trim(.group)),
    color = "red",
    position = position_nudge(x = 0.35),
    hjust = 0
  ) +
  scale_x_discrete(
    name = "Nitrogen"
  ) +
  scale_y_continuous(
    name = "Yield",
    limits = c(0, NA),
```

```
expand = expansion(mult = c(0, 0.1))
) +
scale_color_manual(
 values = Ncolors,
  guide = "none"
) +
theme_bw() +
labs(caption = my_caption) +
theme(
  plot.caption = element_textbox_simple(margin = margin(t = 5)),
 plot.caption.position = "plot",
  axis.text.x = element_text(
    angle = 45,
   hjust = 1,
   vjust = 1
 )
)
```



The four facettes represent genotypes Simba, Nala, Timon and Pumba. Black dots represent raw data. Red dots and error bars represent adjusted means with 95% confidence limits per cultivar. For each genotype separately, means followed by a common letter are not significantly different according to the Tukey–test.

Gomez, Kwanchai A, and Arturo A Gomez. 1984. Statistical Procedures for Agricultural Research. 2nd ed. An International Rice Research Institute Book. Nashville, TN: John Wiley & Sons.