One-way augmented design

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One-way ANOVA & pairwise comparison post hoc tests in a non-resolvable augmented design.

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	# (install &) load packages	
	pacman::p_load(
	conflicted,	
	desplot,	
	emmeans,	
	ggtext,	
	lme4,	
	<pre>lmerTest,</pre>	
	multcomp,	
	multcompView,	
	tidyverse)	

```
# handle function conflicts
conflicts_prefer(dplyr::filter)
conflicts_prefer(dplyr::select)
conflicts_prefer(lmerTest::lmer)
```

1 Data

This example is taken from Chapter "3.7 Analysis of a non-resolvable augmented design" of the course material "Mixed models for metric data (3402-451)" by Prof. Dr. Hans-Peter Piepho. It considers data published in Petersen (1994) from a yield trial laid out as an augmented design. The genotypes (gen) include 3 standards (st, ci, wa) and 30 new cultivars of interest. The trial was laid out in 6 blocks (block). The 3 standards are tested in each block, while each entry is tested in only one of the blocks. Therefore, the blocks are "incomplete blocks".

1.1 Import

```
# data is available online:
path <- "https://raw.githubusercontent.com/SchmidtPaul/dsfair_quarto/master/data/Petersen1
dat <- read_csv(path) # use path from above</pre>
dat
# A tibble: 48 x 5
         yield block
                         row
                                col
   <chr> <dbl> <chr> <dbl> <dbl> <dbl>
 1 st
           2972 I
                           1
 2 14
           2405 I
                           2
                                  1
                           3
 3 26
           2855 I
                                  1
 4 ci
           2592 I
                           4
                                  1
                           5
 5 17
           2572 I
                                  1
           2608 I
                           6
                                  1
 6 wa
 7 22
           2705 I
                           7
                                  1
           2391 I
                           8
                                  1
 8 13
                           1
                                  2
 9 st
           3122 II
                                  2
10 ci
           3023 II
# i 38 more rows
```

1.2 Format

Before anything, the columns gen and block should be encoded as factors, since R by default encoded them as character.

```
dat <- dat %>%
  mutate(across(c(gen, block), ~ as.factor(.x)))
```

1.3 Explore

We make use of **dlookr::describe()** to conveniently obtain descriptive summary tables. Here, we get can summarize per block and per cultivar.

```
dat %>%
  group_by(gen) %>%
 dlookr::describe(yield) %>%
  select(2:sd) %>%
  arrange(desc(n), desc(mean))
# A tibble: 33 x 5
   gen
                               sd
             n
                   na mean
   <fct> <int> <int> <dbl> <dbl>
             6
                    0 2759.
                             832.
 1 st
 2 ci
             6
                    0 2726.
                             711.
 3 wa
             6
                    0 2678.
                             615.
                    0 3643
 4 19
             1
                              NA
                    0 3380
5 11
             1
                              NA
 6 07
             1
                    0 3265
                              NA
7 03
             1
                    0 3055
                              NA
8 04
             1
                    0 3018
                              NA
9 01
             1
                    0 3013
                              NA
             1
                    0 2955
10 30
                              NA
# i 23 more rows
dat %>%
  group_by(block) %>%
  dlookr::describe(yield) %>%
  select(2:sd) %>%
  arrange(desc(mean))
```

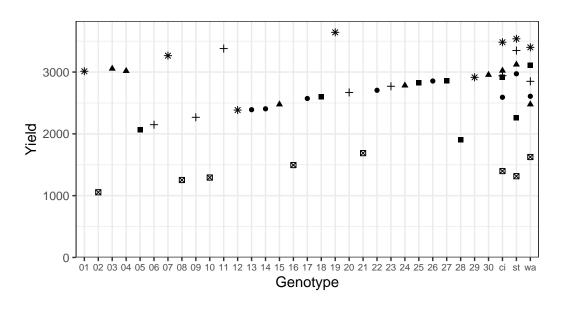
```
# A tibble: 6 x 5
 block
            n
                 na mean
  <fct> <int> <int> <dbl> <dbl>
            8
                  0 3205. 417.
1 VI
                  0 2864.
2 II
            8
                           258.
3 IV
            8
                  0 2797. 445.
4 I
            8
                  0 2638. 202.
                  0 2567. 440.
            8
5 III
6 V
            8
                  0 1390.
                           207.
```

Additionally, we can decide to plot our data. Note that we here define custom colors for the genotypes, where all unreplicated entries get a shade of green and all replicated checks get a shade of red.

```
greens30 <- colorRampPalette(c("#bce2cc", "#00923f"))(30)</pre>
oranges3 <- colorRampPalette(c("#e4572e", "#ad0000"))(3)</pre>
gen_cols <- set_names(c(greens30, oranges3), nm = levels(dat$gen))</pre>
ggplot(data = dat) +
  aes(
    y = yield,
    x = gen,
    shape = block
  ) +
  geom point() +
    scale_x_discrete(
    name = "Genotype"
  scale_y_continuous(
    name = "Yield",
    limits = c(0, NA),
    expand = expansion(mult = c(0, 0.05))
  ) +
  scale_color_manual(
    guide = "none",
    values = gen_cols
  scale_shape_discrete(
    name = "Block"
  ) +
  guides(shape = guide_legend(nrow = 1)) +
```

```
theme_bw() +
theme(
  legend.position = "top",
  axis.text.x = element_text(size = 7)
)
```

Block • I ▲ II ■ III + IV 図 V * VI



Finally, since this is an experiment that was laid with a certain experimental design (= a non-resolvable augmented design) - it makes sense to also get a field plan. This can be done via desplot() from {desplot}.

```
desplot(
  data = dat,
  flip = TRUE, # row 1 on top, not on bottom
  form = gen ~ col + row, # fill color per cultivar
  col.regions = gen_cols, # custom fill colors
  out1 = block, # line between blocks
  text = gen, # cultivar names per plot
  cex = 1, # cultviar names: font size
  shorten = FALSE, # cultivar names: don't abbreviate
  main = "Field layout", # plot title
  show.key = FALSE # hide legend
```

Field layout

st	st	st
14	ci	18
26 ci	04	27 ci
ci	15	ci
17	30	25
wa	03	25 28
22	wa	05
13	24	wa
st	st	st
09	02	29 07
06	21	07
ci	wa	ci
wa	ci	01
20	10	wa
11	08	12 19
23	16	19

2 Model

Finally, we can decide to fit a linear model with yield as the response variable and gen as fixed effects, since our goal is to compare them to each other. Since the trial was laid out in blocks, we also need block effects in the model, but these can be taken either as a fixed or as random effects. Since our goal is to compare genotypes, we will determine which of the two models we prefer by comparing the average standard error of a difference (s.e.d.) for the comparisons between adjusted genotype means - the lower the s.e.d. the better.

```
pull("SE") %>% # extract s.e.d. column
  mean() # get arithmetic mean
[1] 461.3938
# blocks as random (linear mixed model)
mod_rb <- lmer(yield ~ gen + (1 | block),</pre>
               data = dat)
mod_rb %>%
  emmeans(pairwise ~ "gen",
          adjust = "tukey",
          lmer.df = "kenward-roger") %>%
  pluck("contrasts") %>% # extract diffs
  as_tibble() %>% # format to table
  pull("SE") %>% # extract s.e.d. column
  mean() # get arithmetic mean
[1] 462.0431
```

As a result, we find that the model with fixed block effects has the slightly smaller s.e.d. and is therefore more precise in terms of comparing genotypes.



⚠ Model assumptions met? (click to show)

It would be at this moment (i.e. after fitting the model and before running the ANOVA), that you should check whether the model assumptions are met. Find out more in the summary article "Model Diagnostics"

3 ANOVA

Based on our model, we can then conduct an ANOVA:

```
ANOVA <- anova(mod_fb)
ANOVA
Analysis of Variance Table
```

```
Response: yield

Df Sum Sq Mean Sq F value Pr(>F)

gen 32 12626173 394568 4.331 0.0091056 **

block 5 6968486 1393697 15.298 0.0002082 ***

Residuals 10 911027 91103

---

Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

Accordingly, the ANOVA's F-test found the cultivar effects to be statistically significant ($p = 0.009^{**}$). Additionally, the block effects are also statistically significant ($p < .001^{***}$), but this is only of secondary concern for us.

4 Mean comparison

Besides an ANOVA, one may also want to compare adjusted yield means between cultivars via post hoc tests (t-test, Tukey test etc.).

```
mean_comp <- mod_fb %>%
  emmeans(specs = ~ gen) %>% # adj. mean per genotype
  cld(adjust = "Tukey", Letters = letters) # compact letter display (CLD)
mean_comp
 gen emmean SE df lower.CL upper.CL .group
       1632 341 10
                                 3100
 12
                         164
                                       a
 06
       1823 341 10
                                 3291
                         355
                                       a
 28
       1862 341 10
                         394
                                 3330
 09
       1943 341 10
                         475
                                 3411
 05
       2024 341 10
                                 3492 a
                         556
 29
       2162 341 10
                         694
                                 3630
 01
       2260 341 10
                         792
                                 3728
       2324 341 10
 15
                         856
                                 3792
 02
       2330 341 10
                                 3798
                         862
                                      a
 20
       2345 341 10
                         877
                                 3813 a
 13
       2388 341 10
                         920
                                 3856 a
 14
       2402 341 10
                         934
                                 3870
 23
       2445 341 10
                         977
                                 3913
 07
       2512 341 10
                        1044
                                 3980
 80
       2528 341 10
                        1060
                                 3996
 18
       2562 341 10
                        1094
                                 4030 a
```

```
10
      2568 341 10
                                4036
                       1100
17
      2569 341 10
                       1101
                                4037
24
      2630 341 10
                       1162
                                4098
      2678 123 10
พล
                       2148
                                3208
                                      a
      2702 341 10
22
                       1234
                                4170
      2726 123 10
ci
                       2195
                                3256
      2759 123 10
st
                       2229
                                3289
16
      2770 341 10
                       1302
                                4238
25
      2784 341 10
                       1316
                                4252
30
      2802 341 10
                       1334
                                4270
27
      2816 341 10
                                4284
                       1348
      2852 341 10
26
                       1384
                                4320
04
      2865 341 10
                                4333 a
                       1397
19
      2890 341 10
                       1422
                                4358 a
03
      2902 341 10
                       1434
                                4370
21
      2963 341 10
                       1495
                                 4431
      3055 341 10
11
                       1587
                                4523
```

```
Results are averaged over the levels of: block
Confidence level used: 0.95
Conf-level adjustment: sidak method for 33 estimates
P value adjustment: tukey method for comparing a family of 33 estimates
significance level used: alpha = 0.05
NOTE: If two or more means share the same grouping symbol,
then we cannot show them to be different.
But we also did not show them to be the same.
```

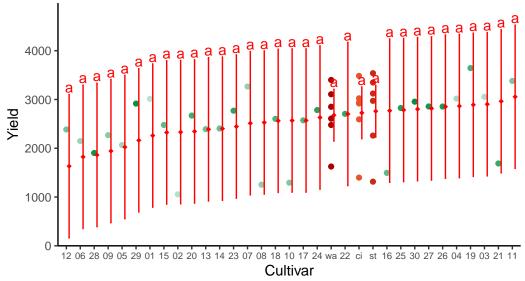
It can be seen that while some genotypes have a higher yield than others, no differences are found to be statistically significant here. Accordingly, notice that e.g. for gen 11, which is the genotype with the highest adjusted yield mean (=3055), its lower confidence limit (=1587) includes gen 12, which is the genotype with the lowest adjusted yield mean (=1632).

Note that if you would like to see the underlying individual contrasts/differences between adjusted means, simply add details = TRUE to the cld() statement. Furthermore, check out the Summary Article "Compact Letter Display".

Finally, we can create a plot that displays both the raw data and the results, i.e. the comparisons of the adjusted means that are based on the linear model.

```
# reorder genotype factor levels according to adjusted mean
my_caption <- "Dots represent raw data. Red diamonds and error bars represent adjusted mea
ggplot() +</pre>
```

```
# green/red dots representing the raw data
geom_point(
 data = dat,
 aes(y = yield, x = gen, color = gen)
# red diamonds representing the adjusted means
geom_point(
  data = mean_comp,
  aes(y = emmean, x = gen),
  shape = 18,
 color = "red",
 position = position_nudge(x = 0.2)
# red error bars representing the confidence limits of the adjusted means
geom_errorbar(
  data = mean_comp,
  aes(ymin = lower.CL, ymax = upper.CL, x = gen),
  color = "red",
 width = 0.1,
  position = position_nudge(x = 0.2)
) +
# red letters
geom_text(
 data = mean_comp,
 aes(y = upper.CL, x = gen, label = str_trim(.group)),
 color = "red",
 vjust = -0.2,
 position = position_nudge(x = 0.2)
) +
scale_color_manual(
  guide = "none",
 values = gen_cols
) +
scale_x_discrete(
 name = "Cultivar",
 limits = as.character(mean_comp$gen)
) +
scale_y_continuous(
 name = "Yield",
 limits = c(0, NA),
  expand = expansion(mult = c(0, 0.1))
```



Dots represent raw data. Red diamonds and error bars represent adjusted means with 95% confidence limits per cultivar. Means followed by a common letter are not significantly different according to the Tukey–test.

5 Bonus

Here are some other things you would maybe want to look at for the analysis of this dataset.

5.1 Variance components

To extract variance components from our models, we unfortunately need different functions per model since only of of them is a mixed model and we used different functions to fit them.

```
# Residual Variance
summary(mod_fb)$sigma^2
```

```
[1] 91102.66
  # Both Variance Components
  as_tibble(VarCorr(mod_rb))
  # A tibble: 2 \times 5
    grp
              var1
                           var2
                                    vcov sdcor
    <chr>
                                    <dbl> <dbl>
              <chr>
                           <chr>
  1 block
              (Intercept) <NA>
                                 434198.
                                           659.
  2 Residual <NA>
                                           302.
                           <NA>
                                   91103.
Petersen, Roger G. 1994. Agricultural Field Experiments. CRC Press. https://doi.org/10.
```

1201/9781482277371.