Eliciting ambiguity

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Outline

Subjective ambiguity

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- Two components: beliefs and preferences,

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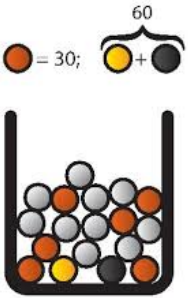
  $\Rightarrow$ Need to measure subjective ambiguity.

From the Ellsberg Urn to real-world applications



$\Longrightarrow$

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- Theory: Mixing bets reveal ambiguity perception and attitude for a single natural event.

- Experiment: Mixing bets are applicable and valid.

- Ambiguity relevant for stock exchange and other participants' behavior.

- Estimation of general model that accounts for stochastic choice, heterogeneity, probability weighting, and hedging.

Related literature

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- ... equivalent ranges of probability (Abdellaoui et al., 2021; Hill, 2023)
- ... exchangeability and Hurwicz Expected Utility (Bleichrodt et al., 2023)
- ... or directly ask for imprecise probabilities (Manski, 2018; Manski and Molinari, 2010; Giustinelli et al., 2021; Bachmann et al., 2020; Henkel, 2022)

Outline

Introduction: Subjective ambiguity

Mixing bets

Experiment

Conclusion and next steps

Mechanism: A mixing bet

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- is endowed with lottery tickets. Each ticket represents a fixed probability to win a monetary reward.

- has to bet each ticket on the event or the complement.

- obtains only the tickets placed on the true realization multiplied by the odds.

Multiple mixing bets

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- Mixing interval := Interval of odds, where agent prefers to mix

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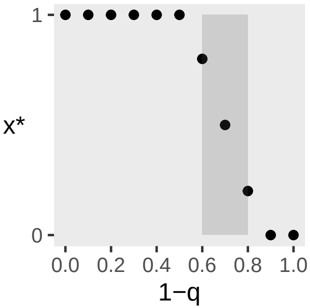


Maxmin

$$\min_{p \in B} \mathbb{E}_p[u(\cdot)]$$

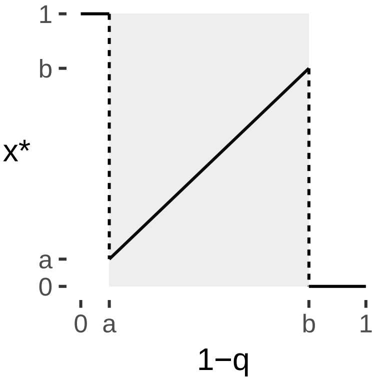
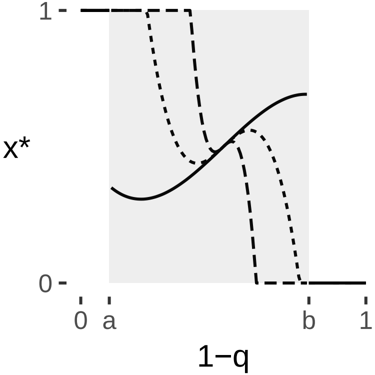


Figure: Optimal ratio of tickets put on event x^* versus odds of complement $1 - q$ for agent with belief interval $[a, b]$.

Variational preferences

$$\min_{p \in B} \mathbb{E}_p[u(\cdot)] + c(p).$$



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Second order preferences

$$\mathbb{E}_{p\sim\mathbb{P}}[\phi(\mathbb{E}_p[u(\cdot)])].$$

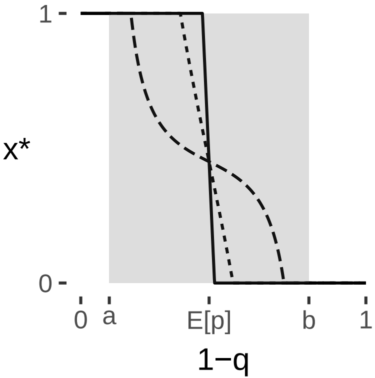

Figure: Optimal ratio of tickets put on event x^* versus odds of complement $1-q$ for agent with belief interval $supp(\mathbb{P}) = [a, b]$.

Summary of theory

- For maxmin preferences, variatonal preferences, and smooth second-order preferences:

$$\text{mixing interval} \subseteq \textit{belief interval}$$

with equality for maxmin and for large utility difference between prizes.

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- For maxmin preferences, variatonal preferences, and smooth second-order preferences:

  $$\text{mixing interval} \subseteq \textit{belief interval}$$

  with equality for maxmin and for large utility difference between prizes.

- The length of the mixing interval quantifies ambiguity perception.

- The shape of the mixing choices classifies ambiguity attitude.

Outline

The Experiment

Elicit mixing bets for the following events

- risky color drawn from urn (risk)
- ambiguous color drawn from urn (ambiguity)
- ambiguous color drawn after information update (updated)
- stock index rising (stock)
- other player cooperating (social)

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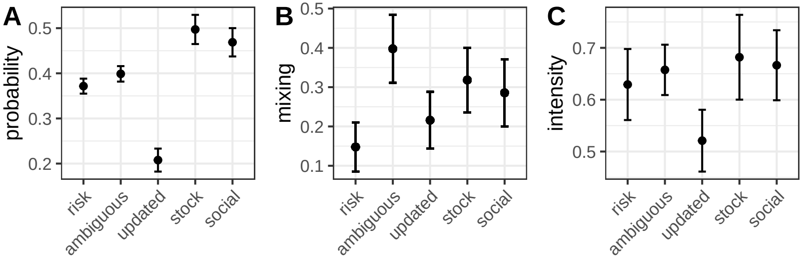
Estimate

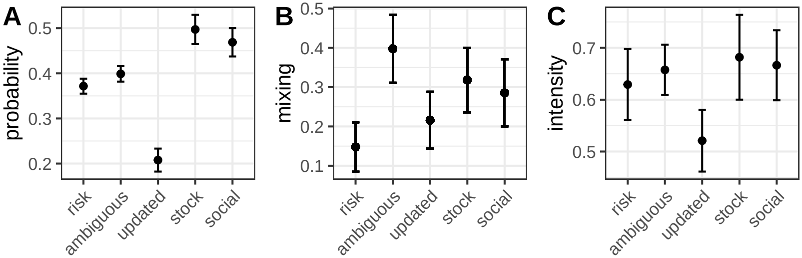
- midpoint of belief interval (probability)
- length of belief interval (ambiguity perception)
- mixing intensity (ambiguity attitude)

Results



- A - probability: Average midpoint of mixing interval

Results



- A - probability: Average midpoint of mixing interval
- B - mixing: Average likelihood of mixing

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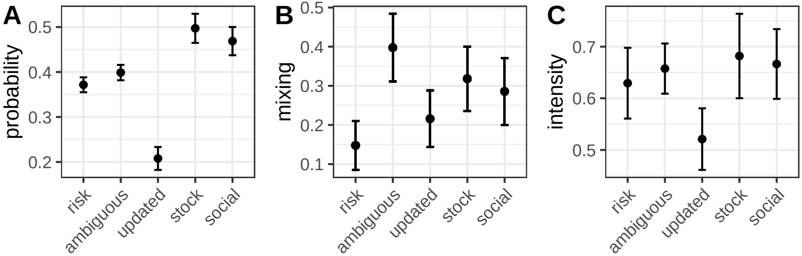

- A - probability: Average midpoint of mixing interval
- B - mixing: Average likelihood of mixing
- C - intensity: Average intensity of mixing

Structural model

Goals:

- estimate individual parameters
- adjust for stochastic choice / noise
- adjust for hedging
- adjust for probability weighting

Solution: Estimate discrete choice model with Bayesian hierarchical structure.

$$U(x, q, B, \theta) = \min_{p \in B} E_\gamma(p, x, q) + c_\theta(p),$$

$$\mathbb{P}(x = k) \sim e^{\sigma U(k,q,B,\theta)+\epsilon}$$

Results: measurement model

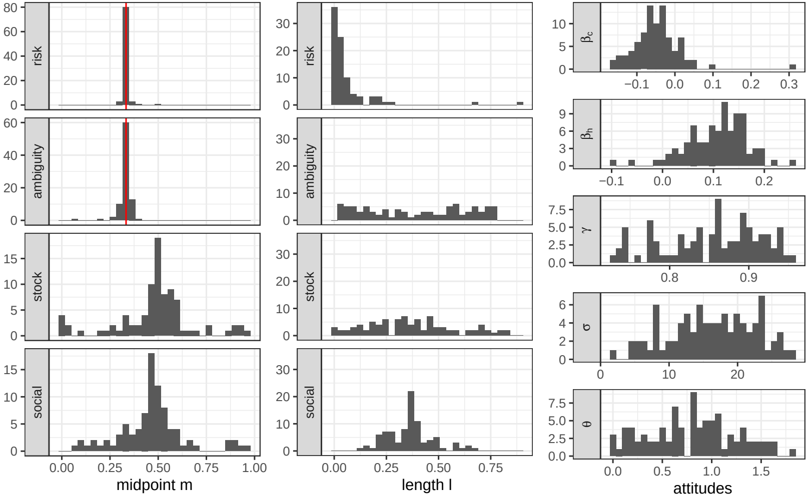

Figure: Posterior means of midpoint of belief interval, length of belief interval, and attitudes.

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- Relatively simple and fast mechanism
- Applicable for whole range of ambiguity averse preferences
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Limitations:

- Multiple measurements prone to agent hedging across reports
- Assumes expected utility for the payout lottery
- Does not reveal attitude for ambiguity seeking preferences

Follow-up

- Experiment shows relevance of ambiguity perception for natural events
  - Open question: Importance for natural decisions?

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- Experiment shows relevance of ambiguity perception for natural events
  - Open question: Importance for natural decisions?
- Experiment shows heterogeneity in student population.
  - Open question: Heterogeneity in representative samples?
- Sever methods allow to measure ambiguity for natural events.
  - Open question: Which method (or combination of methods) is most useful in applied settings?

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A simplified mixing bet

- Let the odds be 1 : 1 and the reward 10 Euros. Will England win the Euro2024?

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- Betting against England: If you put all tickets on "not England", you get 10 Euros if England does not win.
- Mixing: If you divide the tickets equally, you win 10 Euros with a chance of 50%.

An intuitive explanation

Consider the choice between

$[E_q]$ a lottery that pays with probability q if the event E realizes

$[C_q]$ a lottery that pays with probability $1 - q$ if the the event E does not realize, and

$[M_q]$ a lottery that pays with probability $q(1 - q)$.