With the Standard Model (SM) physicist try to describe the elementary particles and interactions. It is successful in describing most processes in physics. But the SM is still incomplete. It can't describe phenomenons like gravity, neutrino masses, matter-antimatter asymmetry etc. This is where effective field theory (EFT) comes in. Here one looks at the SM as a low energy approximation of an underlying ultraviolet(UV) theory (figure 0.1). An in depth introduction into EFT is shown in [1] and [2].

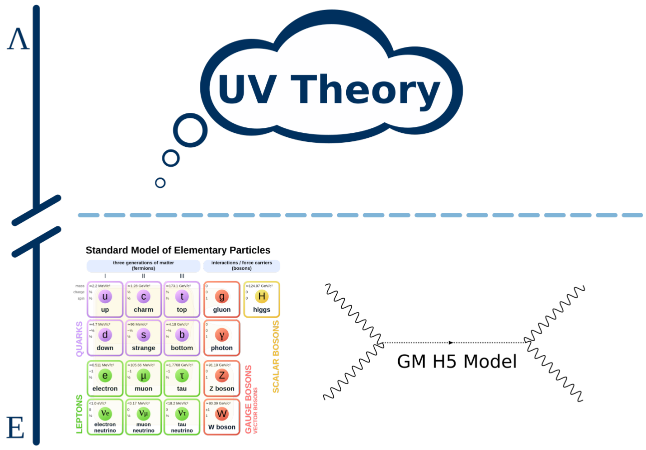


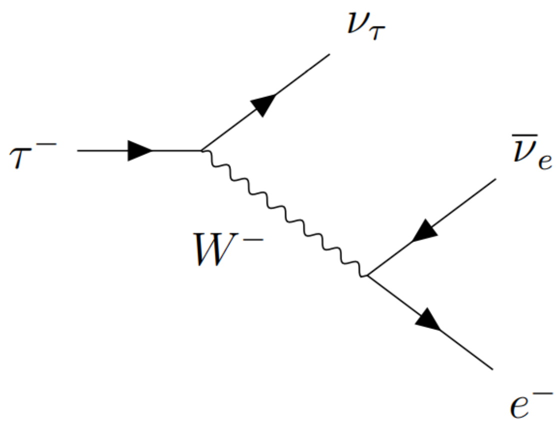
Figure 0.1: Sketch of the Basic EFT concept with the low energy observer having energy E and cutoff at Λ. The SM and BSM theory(here GM H5 Model [3]) are combined in the EFT. [4]

0.0.1 Fermi Theory on the example Tau Decay

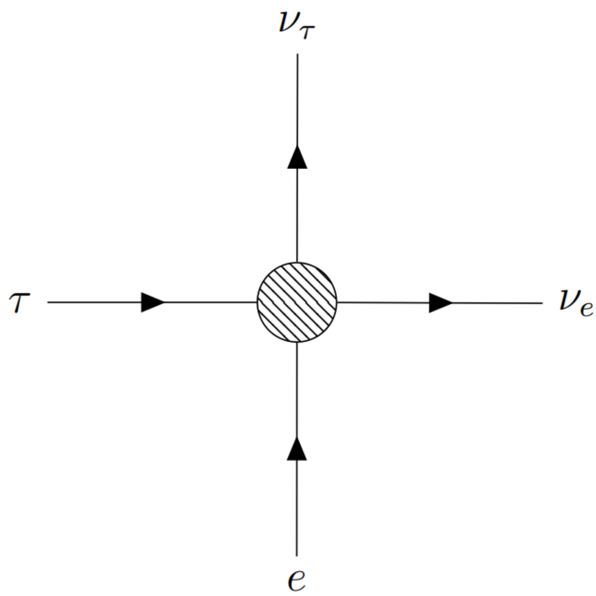
In 1933 Enrico Fermi proposed the Fermi theory in order to describe the β-decay. His theory was able to describe the weak coupling quite well without the former knowledge about the $W^{\pm}$-Boson which was only later theorized in 1968 by Steven Weinberg, Sheldon Glashow and Abdus Salam. The $W^{\pm}$-Boson was finally discovered in 1983. 50 years after Fermi's first successful description of an interaction involving the $W^{\pm}$-Boson. Today one would call the Fermi theory the low-energy EFT of the $W^{\pm}$-Boson. Let's try now to create a different Fermi theory but instead of describing the β decay[5], this theory will look at the τ-decay specifically the decay mode $\tau \to \nu_\tau e^- \nu_e$. Even though this is technically cheating since Fermi didn't have modern knowledge about the tensor structure of the weak interaction in Quantum Field Theory or the knowledge about Feynman diagrams. One can start by drawing the Feynman diagram and writing down the Lorenz invariant matrix element.

$$-i\mathcal{M}_{fi} = \left[\frac{g_W}{\sqrt{2}}\overline{u}(k_{\nu_\tau})\frac{1}{2}\gamma^\mu(1-\gamma^5)u(k_\tau)\right]\frac{g_{\mu\sigma}-\frac{k_\nu k_\sigma}{m_W^2}}{k^2-m_W^2}\left[\frac{g_W}{\sqrt{2}}\overline{u}(k_e)\frac{1}{2}\gamma^\sigma(1-\gamma^5)v(k_{\overline{\nu_e}})\right] \tag{0.1}$$

In most low energy decay processes the momentum of the intermediate W^--Boson is small



compared to its mass. Therefore, one can approximate the propagator (0.2) in orders of the momentum k. But the matrix element in here written in orders of mass since these are more interesting for this thesis. In the Feynman diagram this can be expressed by collapsing the propagator into a single vertex.



$$\frac{-ig_{\mu\sigma} - \frac{k_\nu k_\sigma}{m_W^2}}{k^2 - m_W^2} \xrightarrow{|q|^2 \ll m_W^2} \frac{ig_{\mu\sigma}}{m_W^2}\left(1 + \frac{k^2}{m_w^2} + \frac{k^4}{m_W^6} + \mathcal{O}(k^6)\right) \tag{0.2}$$

$$\Rightarrow i\mathcal{M}_{fi} = \frac{g_W^2}{8m_W^2}\left[\overline{u}(k_{\nu_\tau})\gamma^\mu(1-\gamma^5)u(k_\tau)\right] g_{\mu\sigma}\left[\overline{u}(k_e)\gamma^\sigma(1-\gamma^5)v(k_{\overline{\nu_e}})\right] + \mathcal{O}(m_W^4) \tag{0.3}$$

One can now compare the result with the result Fermi would have gotten if he knew about the parity violation discovered by Wu in 1957.

$$i\mathcal{M}_{fi} = \frac{G_F}{\sqrt{2}} \left[\overline{u}(k_{\nu_\tau})\gamma^\mu(1-\gamma^5)u(k_\tau)\right] g_{\mu\sigma} \left[\overline{u}(k_e)\gamma^\sigma(1-\gamma^5)v(k_{\overline{\nu_e}})\right] \tag{0.4}$$

With $G_F \approx 4.5437957 \times 10^{14} J^{-2}$ being the Fermi constant which is typically measured in the muon decay. The $1/\sqrt{2}$ is added in order to not change the numerical value of G_F while considering the parity violation. From equation 0.3 and 0.4 on gets the following result.

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2} \tag{0.5}$$

Defining the expansion scale $\Lambda = m_W$ and the Wilson coefficient $c = \frac{g_W^2}{8}$ one can see that the matrix element has dimension 2. With this one can also write down a new effective Lagrangian without the $W^\pm$-Boson. But the EFT described has to contain the tau, electron, their respective neutrino fields as well as the interaction Lagrangian.

$$\mathcal{L}_{EFT} = \frac{c}{\Lambda^2} \left(\overline{\nu}_\tau \overline{\gamma}_\rho \tau\right) \left(\overline{e \gamma}_\rho \nu_e\right) + \mathcal{O}(\frac{1}{\Lambda^4}) \tag{0.6}$$

This Fermi theory is only valid for low energies since the scattering amplitude from the SM and the EFT would start to diverge once the energy gets close to the mass of the $W^\pm$-Boson. The m_W defines the validity scale of the EFT in this case the Fermi theory. For energies higher than m_W the Fermi theory is no longer valid. One can increase the validity scale by including therms of higher dimension in Λ. In this example the Matrix element was derived from the SM and then compared the result with the result from Fermi in order to determine $\frac{c}{\Lambda}$. This requires knowledge about the UV theory which is not accessible in low-Energy measurements. As a result one has to make assumptions about the UV theory in order to construct the right EFT (called Matching 0.2)

0.0.2 Standard Model Effective Field Theory (SMEFT)

As the Name suggests The Standard Model effective field theory (SMEFT)[1] aims to extend the SM. Except the g-Factor of the myon the SM has been robust without meaningful deviations from experimental measurements. For this reason using a theory that keeps the $SU(3) \times SU(2) \times U(1)$ symmetry with the Higgs field breaking gauge symmetry is desirable. One can construct the Lagrangian from the gauge invariance of SM fields and allow arbitrarily large mass dimensions. With that any SMEFT Lagrangian can be written as:

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \frac{1}{\Lambda^3}\mathcal{L}_7 + \frac{1}{\Lambda^4}\mathcal{L}_8 + \ldots \text{ with } \mathcal{L}_i = \sum_i c_i^D \mathcal{O}_i^D \tag{0.7}$$

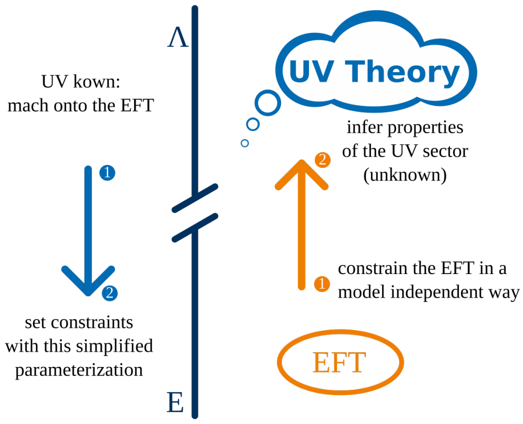


Figure 0.2: Schematic representation for the matching of an EFT to the UV theory using top-down and bottom-up matching method. [6]

The operators $\mathcal{O}_i$ are constructed from the gauge invariance of the SM fields while the Wilson coefficients c_i contain the information on heavy degrees of freedom. Normally the heavy degrees of freedom are integrated out to have a renormalizable theory but are needed to describe high energy particles. The Wilson coefficients are constructed from the operator product expansion in 0.7 as shown in the Fermi theory(0.0.1). For a characteristic heavy scale Λ the operators are ordered by there dimension d_i fixing the dimension of their respective coefficients.

$$[\mathcal{O}_i] = d_i \longrightarrow c_i \sim \frac{1}{\Lambda^{d_i-4}} \tag{0.8}$$

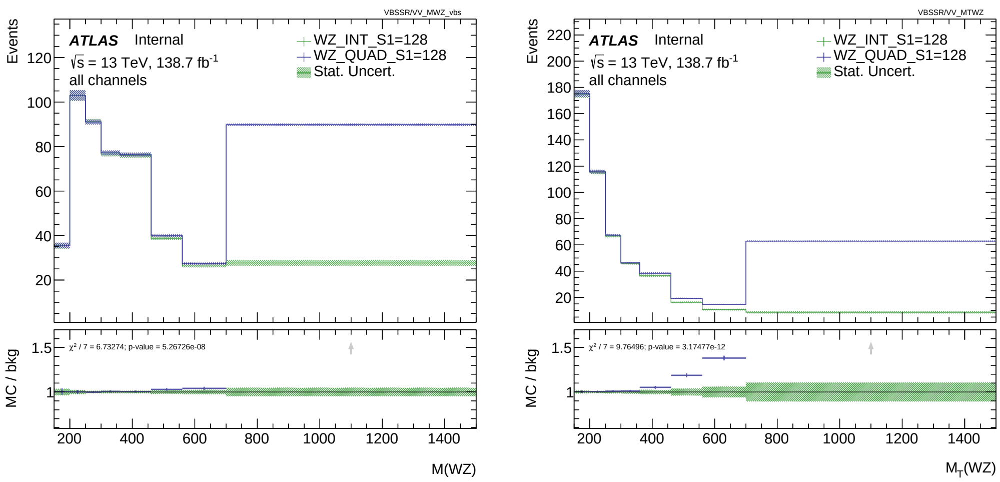
The leading order $D = 4$ (marginal operator) term in 0.7 is the SM Lagrangian while deviations of the SM are described by operators with a dimension $D > 4$ also called irrelevant operators since they are suppressed by the scale E/Λ. Even though they are called irrelevant they often contain important information for processes with high energy. In a Fermi theory these contain information about processes with leading order flavour-change. Relevant operators $D < 4$ become relevant for energies close to the validity scale and only EFT's with relevant and marginal operators are normalizable making the predictions valid up to E/Λ.

Using knowledge about SM fields and known symmetries one can construct an operator basis. All allowed invariant structures are found using the SM fields and their symmetries at a dimension D. Removing all terms from the S-matrix that would result in the same physics provides the basis. The General choice of basis in the search for new physics is the Warsaw basis [6], other basis can be chosen as physics should be basis independent. In vector boson scattering (VBS) processes the Eboli basis[7] is commonly used because the operators can be categorized in longitudinal operators only containing covariant derivatives D^μ and a Higgs doublet fields Φ, transverse operators only containing field strength tensors $\widehat{W}_{\mu\nu}$ and mixed operators containing combinations. Most of the operators can be discarded since they don't contribute to $W^{\pm}Z$ processes leaving only six dimension-8 operators.

$$\begin{aligned}
&\text{Longitudinal:} && \mathcal{O}_{S,1} = [(D_\mu\Phi)^\dagger D_\mu\Phi] \times [(D^\nu\Phi)^\dagger D^\nu\Phi] \\
&\text{Mixed:} && \mathcal{O}_{M,0} = Tr[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}] \times [(D_\beta\Phi)^\dagger D^\beta\Phi] \\
& && \mathcal{O}_{M,1} = Tr[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\beta}] \times [(D_\beta\Phi)^\dagger D^\mu\Phi] \\
&\text{Transverse:} && \mathcal{O}_{T,0} = Tr[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}] \times Tr[\widehat{W}_{\alpha\beta}\widehat{W}^{\alpha\beta}] \\
& && \mathcal{O}_{T,1} = Tr[\widehat{W}_{\alpha\nu}\widehat{W}^{\mu\beta}] \times Tr[\widehat{W}_{\mu\beta}\widehat{W}^{\alpha\nu}] \\
& && \mathcal{O}_{T,2} = Tr[\widehat{W}_{\alpha\mu}\widehat{W}^{\mu\beta}] \times Tr[\widehat{W}_{\beta\nu}\widehat{W}^{\nu\alpha}]
\end{aligned}$$

0.0.3 EFT validity and unitarity violation

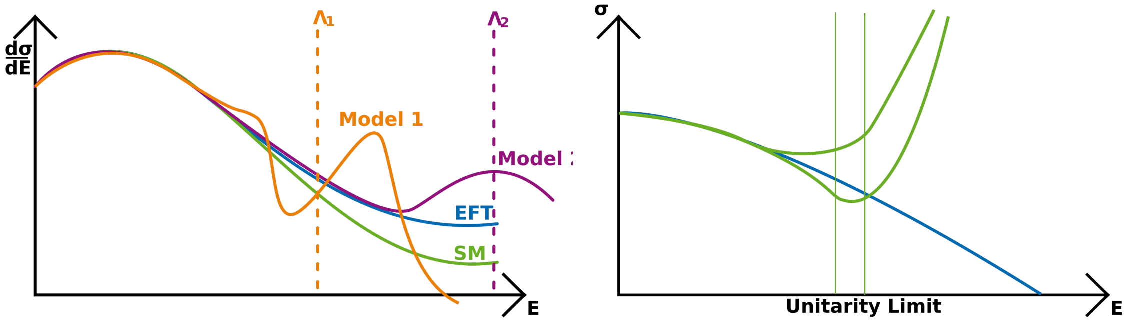
The time evolution of states in quantum mechanics is mathematically described by unitary operators. Time evolution without unitary operators is proposed in new approaches to quantum mechanics like Carl M. Benders talk in 2020 at the TU-Dresden proposing a quantum theory including non Hermitian Hamiltonian [8]. Unitarity was a decisive concept for the inclusion of the Higgs field in the SM as it restored unitarity at tree level. EFT isn't a complete theory by extending the SM, the triple gauge coupling (TGC) and quadratic gauge coupling (QGC) processes are modified leading to unitarity violation. The expected behaviors of a unitary EFT is dim-6 interference $>$ dim-6 quadratic $\sim$ dim-8 interference $>$ dim-8 quadratic. This however is not the case as dim-8 operators cause the scattering amplitude to grow asymptotically $\sim \Lambda^2$. This eventually leads to unitarity violation 0.3.



(a) (b)

Figure 0.3: Comparission of inteferenz term and quadratic term for the S1 parameters created with value S1=128. The quadratic therm is bigger for higher energies since no unitarisation is used. **a)**$M(WZ)$,**b)**$M_T(WZ)$

One can now apply a cutoff at the validity scale $M = \Lambda$ but in practice the value of Λ is unknown and can only be gained by analysing data. This means that EFT predictions are only valid for an operator dependent unitarity limit. It should be noted that unitarity can be restored by using unitarisation. In ATLAS research the relatively simple K-matrix method is commonen tool for unitarisation as shown in [9]. This however does not restore EFT validity and makes connecting EFT results to UV theories harder. In conclusion the search for beyond the standard model(BSM) effects using EFT is only possible in an energy range as light states are not detectable and high Energy states are removed by the unitarity limit 0.4.



(a) (b)

Figure 0.4: a) Not all models can be fitted using EFT only ones with energy scale higher than the SM [4]**b)** Schematic of unitarity limits the energy range changes depending on the cutoff [10]