Friction Theory

Belt Friction

Wedges

CHAPTER 4

Friction

Introduction to the Frictional Force

Friction forces are the force effects which are produced when bodies whose surfaces are in contact have actual or impending or sliding motion relative to each other. The lines of action of these friction forces lie in the plane which is tangent to the bodies at the point of contact.

Thus the frictional force may be defined as the contact resistance (opposing force) exerted by on body upon a second body in contact when the second body moves or tends to move past the first body. Friction therefore is a retarding force always acting opposite to the motion or tendency to move (impending motion). If the contact surfaces are "perfectly smooth", there is no frictional force at the surface of contact.

Note that frictional force is a "Passive Force" and brought into play when an external force is applied, the intention of the external force being to cause motion of a body over another contact plane frictional forces are either useful or detrimental. In case of brakes or clutches or in case of an automobile tyres in contact with road surface or in case of a person walking along the ground the friction forces are not only useful bat also essential to the desired function in these situations. Whereas in case of bearings or the situation where machine parts slide relative to one another, the effect of frictional forces is detrimental since it results in wear of these parts.

Limiting Friction

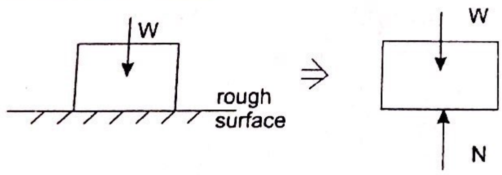


Figure (A)

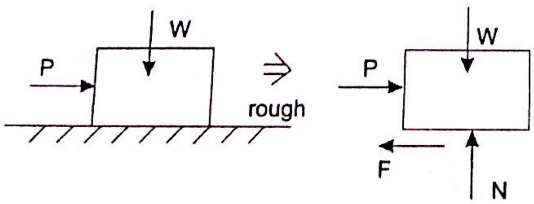


Figure (B)

Figure (A) shows a block of weight W resting on a rough surface. The block is in equilibrium under the action of its weight 'W' and normal reaction of the surface 'N'.

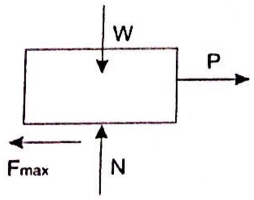
In figure (b) a horizontal force 'P' is applied to the same block. Force 'P' is applied to the same block. Force 'P' is trying to push the block in its direction.

As the block is in equilibrium, we can write

Initially for lower values of 'P' it is observed that even though force P increases, the block continues to be in the state of rest. This proves that there is some resisting force developed at the contact surface opposing the possibility of motion. This resisting force is called as "Frictional Force".

Frictional force is a passive force (i.e. it is developed due to some another force). It is a self adjusting force. It can adjust it magnitude and direction as per the tendency of the motion of the block.

If the force 'P' is gradually increased, a condition will reach where the frictional force on the block is no longer sufficient to prevent the onset of motion of the block. The frictional force for this limiting condition, when motion is impending is the maximum value of 'Frictional force that the surface can export on the block and is called as 'limiting friction'.



If the external force P is increased beyond the value $P = F_{max}$, the block will have a resultant force of magnitude $(P - F_{max})$ acting on it. From Newton's second law, the block will accelerate in the direction of the resultant force and the problem will be then treated in dynamics.

Co-efficient of Friction (μ)

The magnitude of the frictional force is independent of the area of contact surfaces but is proportional to the normal reaction

Thus, $F_{max} = \mu_s N$

From this, we get, $\mu_s = \dfrac{F_{max}}{N}$

The ratio of the maximum frictional force to the normal reaction of the surface is called as the "coefficient of static friction" (μ_s). This coefficient is experimentally determined. It is the property of the two surfaces in contact.

Representative values of the coefficient of static friction

Sr. No.	Surfaces	Dry	Lubricated
1.	Steel on steel	0.8	0.16

2.	Steel on Brass	0.35	0.19
3.	Steel on Graphite	0.1	0.1
4.	Steel on Teflon	0.04	0.04
5.	Aluminium on Aluminium	1.35	0.3
6.	Wood on Wood (dry)	0.2 – 0.6	
7.	Wood on Wood (wet)	0.2	
8.	Leather on Wood	0.3 – 0.4	
9.	Leather on Metal (dry)	0.6	
10.	Leather on Metal (wet)	0.4	

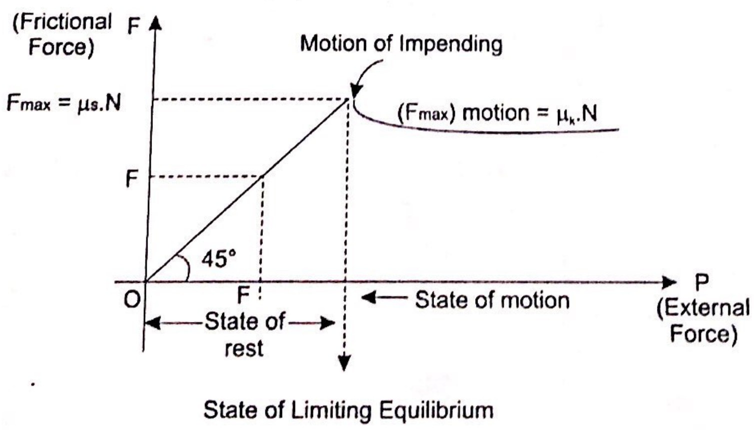
Experimentally it has proved that the coefficient of friction decreases slightly once the sliding motion starts.

Due to this the value of F_{max} also decreases during motion. This is called as Kinetic Friction.

$$\therefore (F_{max})_{motion} = \mu_k N$$

From this, $\mu_k = \frac{(F_{max})_{motion}}{N}$

The ratio of the maximum frictional force during motion to the normal reaction is called as "coefficient of kinetic friction" (μk)



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Considering equilibrium along the plane at this critical stage,

$-W.\sin\theta + F_{max} = 0$

$\therefore -W.\sin\theta + \mu_s W.\cos\theta = 0$

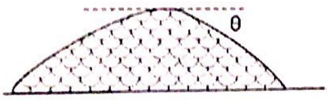
$\therefore$ We get $\mu_s = \tan\theta$, but we also know that

$\mu_s = \tan\phi$

$\therefore \tan\theta = \tan\phi$

$\therefore$ Angle of repose $= \theta = \phi =$ Angle of friction

i) when $\theta < \phi$ then the body remains in equilibrium
ii) when $\theta = \phi$ then the body is in limiting equilibrium or motion is impending
iii) when $\theta > \phi$ then the body slide down the plane



If granular material is poured into a pile on a horizontal surface, it will form a mound which has a approximate shape of a right circular cone then angle θ is the angle of repose. Then $\theta = \phi$ i.e. angle of friction for the loose material. This angle may be used to compute the dimensions of a mound of loose material.

Coulomb's Laws of Dry Friction

1. The frictional force F acts along the "tangent plane" at the surface of contact and acts in a direction opposite to the motion or impending motion of the bodies in contact. The maximum frictional force is proportional to the normal reaction N.

   $\therefore F_{max} = \mu_s N$ where μ_s depends upon the nature of surfaces in contact.

2. The frictional force is independent of the extent of area of contact for a given value of normal reaction.

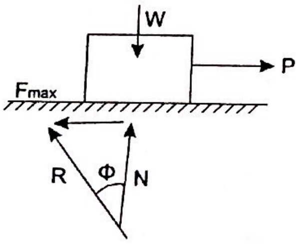
3. The ratio of maximum or limiting frictional force to the normal reaction remains constant for the given surfaces of contact.

4. The maximum value of friction in dynamic conditions is less than that in static condition.

   (F_{max}) motion $= \mu_k N$

As F_{max} > (F_{max}) motion, the force required to start the motion is always greater than the force required to maintain the motion.

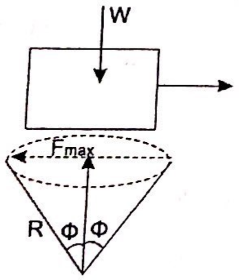
Angle of Friction (ϕ)



In the limiting equilibrium position the resultant of the maximum frictional force 'F_{max}' and the normal reaction 'N' is called as the "resultant reaction" (R). The angle made by the resultant reaction R with the normal to the plane of contact is called as "angle of friction" (ϕ)

$$\tan\varphi = \frac{F_{max}}{N} = \frac{\mu_s N}{N} \qquad \tan\varphi = \mu_s$$

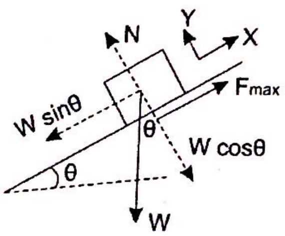
Cone of Friction



Imaginary surface generated by revolving the resultant reaction R about the vertical (as shown in figure – vertical being normal to the surface of contact) called as the "cone of friction". The semi-vertex angle of this cone is ϕ i.e. angle of friction. As long as the resultant reaction lies within this surface, the body will be in equilibrium, no matter what the direction of force 'P' is the horizontal plane is.

Angle of Repose

Consider a body placed on an inclined plane whose inclination can be varied as may be desired. Initially, for low values of angle θ the frictional force is sufficient to prevent the tendency of the body to slide down the plane develop due to (W.sinθ).



As the inclination θ is gradually increased, a stage will reach when the body is just on the verge of sliding down the plane. This limiting angle is called as "angle of repose".

Equilibrium of a Body on Inclined Plane when the angle of inclination is greater than the angle of repose

If the angle of inclination of the plane to the horizontal is less than the angle of repose, the body on the inclined plane with rough surface of contact will remain in equilibrium without any force of external force.

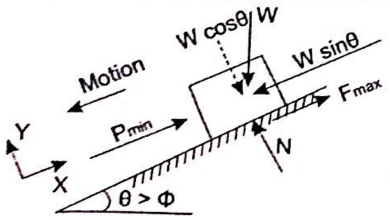
However, if the angle of inclination is greater than the angle of repose. For the body on the inclined plane, an external force is required to keep the body in equilibrium.

Two important cases shall be considered.

Case 1 : The external force 'P' applied parallel to the inclined plane.

Case #2: The external force 'P' applied parallel to the inclined plane.

Case 1 : The External Force 'P' Applied Parallel to the Inclined Plane



Consider a block of weight W placed on an inclined plane with inclination 'θ' greater than the angle of repose 'ϕ'. As $\theta > \phi$ the block is having natural tendency to slide down the plane. This is resisted by the combined effect of the frictional force and an external force 'P', In the figure force P is such that it just prevents the block from sliding down the plane. The frictional force is maximum i.e.

$F_{max} = \mu_s.N = \mu_s.W \cos\theta.$

Then in the limiting equilibrium position

$P_{min} + F_{max} = W \sin\theta$

$\therefore P_{min} + \mu_s.W\cos\theta = W.\sin\theta$

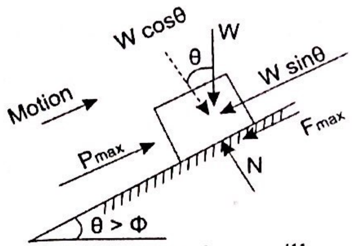
But $\qquad \mu_s = \mathrm{Tan}\varphi = \dfrac{\sin\varphi}{\cos\varphi}$

$$\therefore P_{min} = W(\sin\theta - \mu_s \cos\theta)$$

$$= W\left(\sin\theta - \frac{\sin\varphi \cdot \cos\theta}{\cos\varphi}\right)$$

$$= \frac{W(\sin\theta \cdot \cos\varphi - \cos\theta \cdot \sin\varphi)}{\cos\varphi}$$

$$\therefore P_{min} = \left[\frac{W\sin(\theta-\varphi)}{\cos\varphi}\right] \quad \text{.......................................(i)}$$



Any value of P less than P_{min} will cause the body slide down the plane.

As we increase the values of P from P_{min}, the frictional force starts decreasing from F_{max}. Finally, when P = W.sinθ the value of the frictional force become zero.

As we increase P further gradually ($P > W\sin\theta$) the tendency of the block will now be to move up the plane and the frictional force will now act down the plane. With increase in force P the frictional force also increase (but in downward direction). Finally a stage will reach at which the block is on the verge of sliding up the plane. Then in the limiting equilibrium position.

$P_{max} = W\sin\theta + F_{max}$

$= W.\sin\theta + \mu s.W.\cos\theta$

But $\mu_s = \tan\varphi = \dfrac{\sin\varphi}{\cos\varphi}$

$$\therefore P_{max} = W(\sin\theta + \mu_s \cos\theta)$$

$$= W\sin\theta + \frac{W.\sin\varphi.\sin\theta}{\cos\varphi}$$

$$= \frac{W(\sin\theta \cdot \cos\varphi + \cos\theta \cdot \sin\varphi)}{\cos\varphi}$$

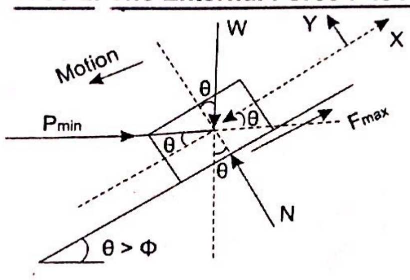
$$\therefore P_{max} = \left[\frac{W\sin(\theta+\varphi)}{\cos\varphi}\right] \quad \text{.......................................(ii)}$$

If we apply a force larger than P_{max}, the equilibrium would be disturbed and the body would start moving up the plane.

Hence, the range of values of 'P' for which the body can be kept in equilibrium is from P_{min} to P_{max}.

For the values of 'P' from Pmin to 'Wsinθ' the frictional force acts up the plane. At P = W.sinθ there is no frictional force developed. For the values of P from W.sinθ to P_{max}, the frictional force F acts down the plane.

Case 2: The External Force P is Applied Horizontally



In this figure force P is applied horizontally to just prevent the block from sliding down the plane. Then in the limiting equilibrium position,

$N = W.\cos\theta + P_{min}.\sin\theta$

And $P_{min}.\cos\theta + F_{max} = W.\sin\theta$

$\therefore P_{min}.\cos\theta + \mu sN = W.\sin\theta$

$\therefore P_{min}.\cos\theta + \mu s.W\cos\theta + \mu s.P_{min}.\sin\theta = W.\sin\theta$

$\therefore P_{min}(\cos\theta + \mu s.\sin\theta) = W(\sin\theta - \mu s.\cos\theta)$

Substituting $\mu_s = \tan\phi$, we get,

$$\therefore P_{min} = \left(\frac{\cos\theta.\cos\varphi + \sin\theta\sin\phi}{\cos\varphi}\right) = \left(\frac{W(\sin\theta.\cos\varphi - \cos\theta.\sin\varphi}{\cos\varphi}\right)$$

$$\therefore P_{min}.\cos(\theta-\varphi) = W.\sin(\theta-\varphi)$$

$$\therefore P_{min} = W.\tan(\theta-\varphi) \quad \text{.........(iii)}$$

Any value of P less than P_{min} will cause the body slide down the plane.

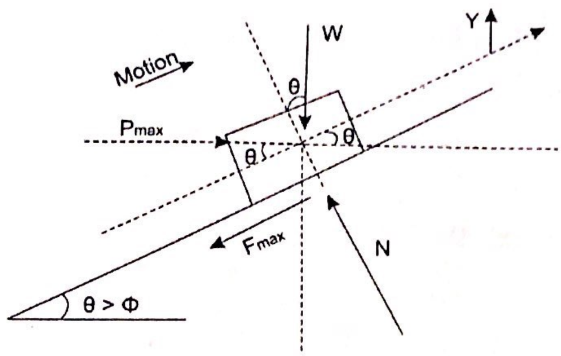
As we increase the values of P from P_{min} the frictional force starts decreasing from F_{max}.

Finally, when Pcosθ = Wsinθ

And P = W.Tanθ, the value of frictional force become zero.

As we increase P, further gradually (P > W.Tanθ) the tendency of the block will now be to move up the plane and frictional force will now act down the plane. With increase in force P the frictional force also increase (but in downward direction). Finally a stage will reach at which the block is on the verge of sliding up the plane.

Then, in the limiting equilibrium position,



$N = W\cos\theta + P_{max}.\sin\theta$

And $P_{max}.\cos\theta = F_{max} + W\sin\theta$

$\therefore\ P_{max}.\cos\theta = \mu_s.N + W\sin\theta$

$\therefore\ P_{max}.\cos\theta = \mu s.W.\cos\theta + \mu_s P_{max}.\sin\theta + W.\sin\theta$

$\therefore\ P_{max}\ (\cos\theta - \mu_s.\sin\theta) = W(\sin\theta + \mu_s.\cos\theta)$

Substituting $\mu_s = \text{Tan}\phi$, we get,

$$\therefore P_{max}.\left(\frac{\cos\theta.\ \cos\varphi\ \ \sin\theta.\sin\varphi}{\cos\varphi}\right) = \left(\frac{W(\sin\theta.\cos\varphi + \cos\theta.\sin\varphi}{\cos\varphi}\right)$$

$$\therefore P_{max}\left[\frac{\cos\theta.\cos\varphi - \sin\theta.\sin\varphi}{\cos\varphi}\right] = \left[\frac{W(\sin\theta.\cos\varphi + \cos\theta\ \sin\phi)}{\cos\varphi}\right]$$

$$\therefore P_{max}.\cos(\theta + \varphi) = W.\sin(\theta + \varphi)$$

$$\therefore P_{max} = W.\text{Tan}(\theta + \varphi) \text{..(iv)}$$

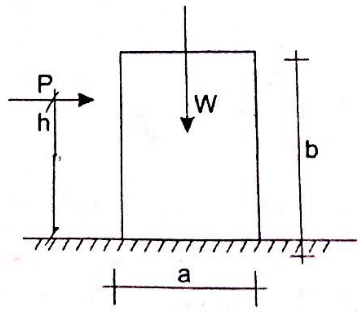
If we apply a force larger than Pmax, the equilibrium would be disturbed and the body would start moving up the plane.

Hence, the range of values of 'P' for which the body can be kept in equilibrium is from P_{min} to P_{max}.

For the values of 'P' from P_{min} to W.Tanθ, the frictional force acts up the plane. At P = W Tanθ there is no frictional force developed. For the values of P from W.Tanθ to P_{max}, the frictional force acts down the plane.

Criteria for Sliding or Tipping

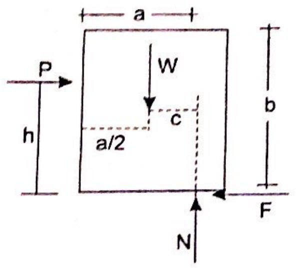
Consider a block of dimension a x b and weight W subjected to force P as shown in figure. Then in the FB.D the



normal reaction N is assumed to act at a distance 'C' to the right of the line of action of the weight 'W'.

Taking moments about the point of application of the normal reaction,

$$-P.h + W.c = 0$$

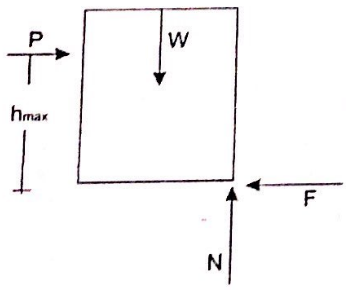


$$\therefore h = \left(\frac{W}{P}\right)c$$

h being necessarily $\leq$ b.

As forces P and W are constants, we can conclude that c = f(h)

Thus, as 'h' increases 'c' also increases. But the limiting case will occur when c = a/2. The body is on the verge of tipping about the right hand corner of the block.



For the moment equilibrium about this corner. For the case of impending tipping,

$$-P.h_{max} + W.\frac{a}{2} = 0$$

$$\therefore h_{max} = \frac{W.a}{2P}$$

If sliding motion of the body is assumed to be I mpending, then

$P = F_{max} = \mu_s.N$

And N = W

$\therefore$ We get $P = \mu_s.W$

If sliding and tipping are assumed to be equally likely to occur then,

$$h_{max} = \frac{W.a}{2P} = \frac{W.a}{2.\mu_s.W}$$

$$\therefore h_{max} = \frac{a}{2\mu_s}$$

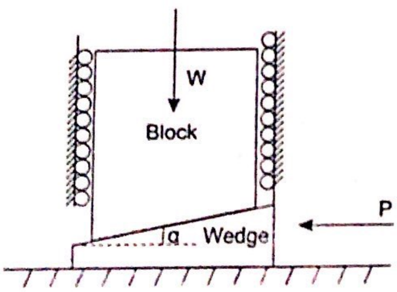
It may be noted that the result in the above equation is independent of the weight 'W' and height 'b' of the block, and of the applied force 'P'. It is a function of the width of the block 'a' and the coefficient of friction'μs.'

From this we get five different conditions as under,

1.	$h < h_{max}$, $p < \mu.W$	Neither sliding, nor tipping will occur. The block remains at rest.
2.	$h < h_{max}$, $P = \mu.W$	The body does not tip and sliding motion is impending
3.	$h < h_{max}$, $P > \mu W$	The body does not tip but sliding motion with increasing velocity will occur
4.	$h = h_{max}$, $P = \mu W$	Both sliding and tipping motion are impending. The occurrence of either situation in equally likely.
5.	$h > h_{max}$ a) If $P \leq \mu W$	The body will tip over. Tipping motion will occur with the tipping edge remaining stationary with respect to the surface.
	b) If $P > \mu W$	Tipping motion will occur with the sliding of the edge of the block along the surface.

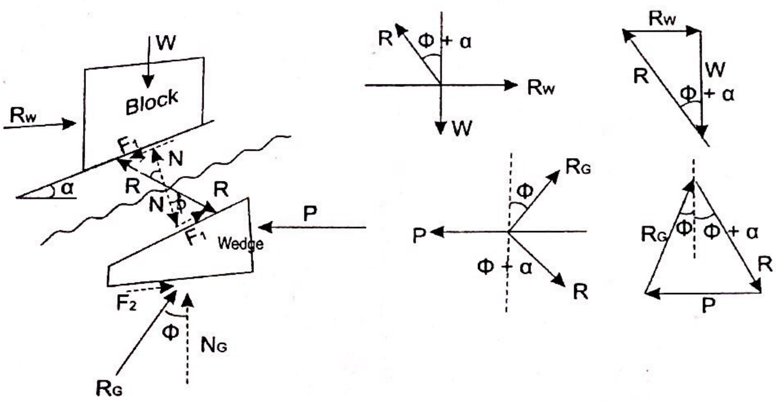
Wedges

The wedge is a simple machine which is intended to transform an applied force into a force at approximately right angles to the directionof the applied force. It is the simplest and most useful of machines and is used as a means of producing small adjustments in the position of the body or as a means of applying large forces. Wedges are dependent on friction.



Consider a block of weight W, whose position in the vertical guide is to be adjusted by a wedge of negligible mass as shown in figure. Then there are two cases to be discussed:

Case 1: When the block is to be raised by ~~pusing~~ pushing the ~~wege~~ wedge in.



In the above figure,

$\phi = \tan^{-1}\mu$ = angle of friction, α = angle of wedge

W = weight of the block

P = external force applied to the wedge to raise the block upwards

N = normal reaction between the block and the wedge

F_1 = limiting frictional force developed between the block and the wedge when sliding of the wedge is impending

R = resultant reaction between the block i.e. resultant of N and F_1

N_G = normal reaction of the ground on the wedge

F_2 = limiting frictional force developed between the wedge and the ground surface when sliding of the wedge is impending.

R_G = resultant reaction of the ground surface on the wedge i.e. resultant of N_G and F_2

Then in the limiting equilibrium position,

For the block,

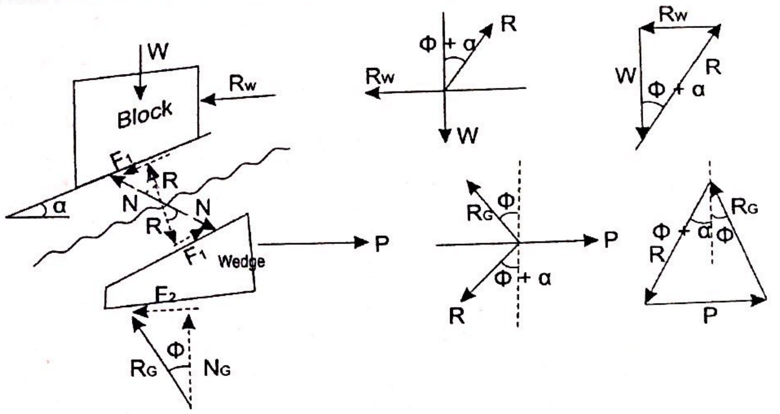
$\overline{W} + \overline{R}_w + \overline{R} = 0$ .... (i)

And for the wedge

$\overline{P} + \overline{R}_G + \overline{R} = 0$ .... (ii)

Solving the above equations graphically or analytically we can get the value of force 'P'.

Case 2: When the block is to be lowered by pulling the wedge out.



If the wedge is self-locking then for lowering the block, it is to be pulled out by applying a pulling force 'P' on to it as shown in the above figure. The direction of frictional forces will now change the direction to oppose the new impending motion.

Then for the limiting equilibrium position,

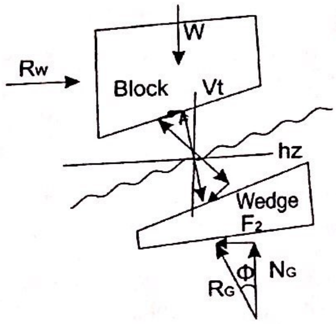
For the block, $\overline{W} + \overline{R}_w + \overline{R} = 0$ .... (i)

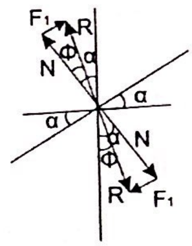
And for the wedge, $\overline{P} + \overline{R}_G + \overline{R} = 0$ .... (ii)

Solving the above equations graphically or analytically we can get the value of force 'P'.

Self-locking Wedge

In the above arrangement if P = 0 and the wedge remains at its place in equilibrium then R and R_G must be collinear.





This is possible only when,

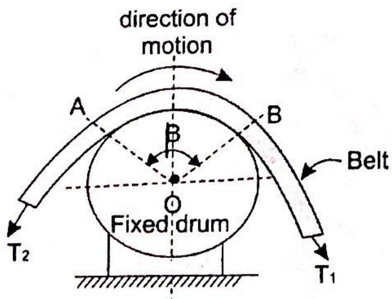
$(\alpha - \phi) = \phi$

$\therefore \phi = (\alpha/2)$

Thus, R and R_G both will make angle $(\alpha/2)$ with respect to the normal to their surfaces. Thus, as long as $(\alpha/2) < \phi$, slipping of the wedge will not occur and the wedge is called as self-locking wedge.

Belt Friction

Belt or chain or rope drives are used to transmit power from one place to the other. It is the frictional force between the belt and the pulley which is responsible for the operation of the belt drive. The impending slippage of the flexible cable or belt or rope over the drums is of importance in the design of belt drives, band brakes and hoisting rigs. If the cross section of the belt is rectangular, it is called as a "Flat Belt", And if the cross section of the belt is a frustum of a cone, it is called as a "V-Belt. V-belts are used for heavy duty machines for transmitting more power.



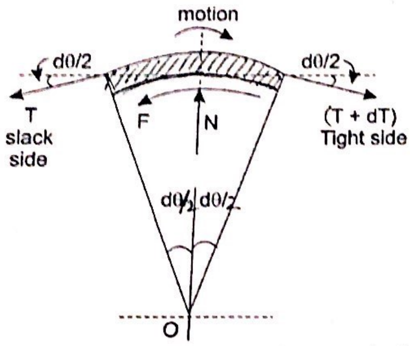
Figures shows a fixed circular drum over which passes a flexible flat belt, rope or cable. Consider that the slipping motion of the belt relative to the drum is impending.

β = m< AOB

= angle of lap

μ = coefficient of friction between the belt and the drum

The tension in the belt in the direction of the sliding motion is called as "tight tension" and is denoted by 'T_1'. The tension in the belt in the direction opposite to the motion is called as "slack tension" and is denoted by 'T_2'. The angle of contact of the belt on the drum is called as "angle of lap" and it is denoted by β. It is expressed in radians.



Consider the free body diagram of the element of the belt for which the central angle is "dθ".

Neglecting the weight of the belt, the forces acting on the element of the belt are shown in the F.B.D.

T = Tension in the belt on slack side

T + dT = tension in the belt on the tight side

N = normal reaction of the drum on the belt

F = Frictional force opposing the impending motion of the belt

Then,

$F = \mu.N$

Applying equations of equilibrium,

$\Sigma F_x = 0$ gives,

$$(T + dT)\cos\frac{d\theta}{2} - T.\cos\frac{d\theta}{2} - \mu.N = 0$$

$$\therefore dT.\cos\frac{d\theta}{2} - \mu N = 0$$

$$\therefore dT = \mu N \quad \text{.......................................} \quad (i)$$

$\Sigma f_y = 0$ gives,

$$N - T.\sin\frac{d\theta}{2} - (T + dT).\sin\frac{d\theta}{2} = 0$$

$$\therefore N - 2.T.\sin\frac{d\theta}{2} - dT.\sin\frac{d\theta}{2} = 0$$

when, $d\theta \to 0, \sin\frac{d\theta}{2} \to \frac{d\theta}{2}$

and $\left(dT \times \frac{d\theta}{2}\right) \to 0$

$$\therefore N - 2.T.\frac{d\theta}{2} = 0$$

$$\therefore N = T.d\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

Substituting in equation (i) we get,

$$dT = \mu.T.d\theta$$

$$\therefore \frac{dT}{T} = \mu.d\theta$$

Integrating the above equation

$$\int_{T_2}^{T_1}\frac{dT}{T} = \mu.\int_{o}^{\beta} d\theta$$

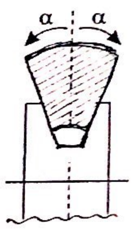
$$\therefore \mathrm{Log}_e T_1 - \mathrm{Log}(T_2) = \mu\beta$$

$$\therefore \mathrm{Log}_e\left(T_1/T_2\right) = \mu\beta$$

$$\boxed{\therefore \frac{T_1}{T_2} = e^{\mu\beta}}$$

This tension ratio is applicable for flat belt.

In case of V-belt on grooved pulleys, a better friction grip than that in flat-belt can be achieved and a larger tension ratio can be obtained. Instead of a V-belt, a rope can be used and the tension ratio will apply the same as in case of V-belt.



$$\boxed{\frac{T_1}{T_2} = e^{\mu\beta/\sin\alpha}}$$

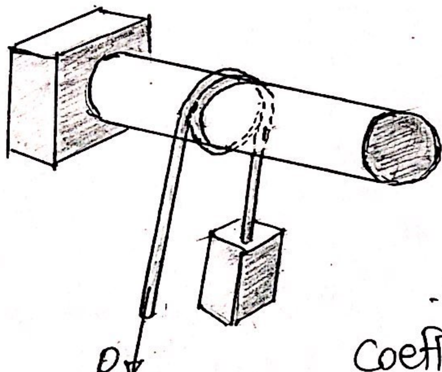
This tension ratio is applicable for V-belt as well as ropes.

The above equation is the fundamental relationship between the belt tensions for the case where slipping motion of the belt on the drum is impending. Since $e^{\mu\beta} > 1$, T_1 is always greater than T_2. It may be also noted that the above relation is independent of the radius of the drum or pulley. Thus, this equation applies to a drum of any contour and β is the total angle of contact in radians.

The value of β is a measure of how far around the circumference of the drum the belt is wrapped.

The above formulae is applicable to the problems involving flat belts passing over fixed cylindrical drums and the problems involving ropes wrapped around a post or capstan. They can also be applied to the problems of band brakes in which the drum which is about to rotate, while the band remains fixed. These formulae can also be applied to the problem involving belt drives. In these problems both pulley and the belt rotate. If the belt is slipping over the pulley i.e. it is moving with respect to the pulley then it is called as slippage and only in that case the above formula is used. If the belt, rope or brake is actually slipping, then in the belt friction expression coefficient of kinetic friction (μk) is to be used. And if the slipping is impending then coefficient of static friction (μs) is used.

② Ex. No. ⑩ A 120 kg block is supported by a rope which is wrapped $1\frac{1}{2}$ times around a horizontal rod. knowing that the coefficient of static friction between the rope and the rod is $\mu_s = 0.15$. Determine the range of values of 'P' for which equilibrium is maintained.



Solution :

1 turn = 360°

$1/2$ turn = 180°

$\therefore 1\frac{1}{2}$ turns $= 540^\circ = 9.425^c$

$\therefore$ Angle of lap, $\beta = 9.425^c$

Coefficient of friction, $\mu_s = 0.15$

By Belt friction Formula,

$$\left[\frac{\text{Tight tension}}{\text{slack tension}}\right] = e^{\mu \cdot \beta} = e^{0.15 \times 9.425} = 4.11134$$

I) For P_{min}: The natural tendency of the 120 kg block is to fall down due to gravity. 'Pmin' is the force, just required to prevent this downward motion of the 120 kg block. Hence, (120 × 9.81 = 1177.2 N) force will be the tight tension and 'P_{min}' will be the slack tension.

By Belt friction formula,

$$\left(\frac{1177.2}{P_{min}}\right) = 4.11134 \quad \therefore \boxed{P_{min} = 286.329 \text{ N}}$$

II) For P_{max}: Now, the external force 'P' will be enough more to pull the 120 kg block in upward direction. Hence, 'P_{max}' will become tight tension and 1177.2 N will be slack tension.

By Belt Friction formula,

$$\therefore \left(\frac{P_{max}}{1177.2}\right) = 4.11134 \quad \therefore \boxed{P_{max} = 4839.874\ N}$$

Ans: The system will remain in equilibrium for all values of 'P' satisfying the condition,

$$286.329\ N \leq P \leq 4839.874\ N$$

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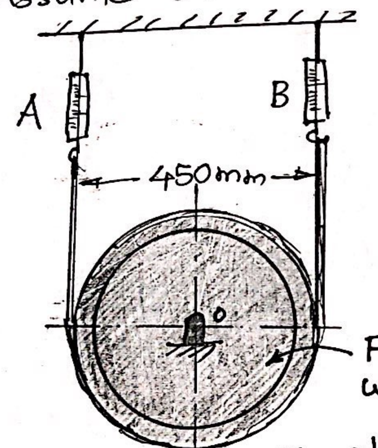
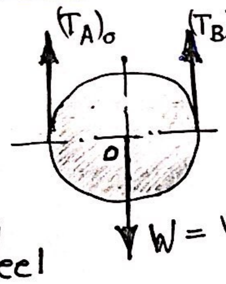
Ex-No.(11) The setup shown is used to measure the output of a small turbine. When the flywheel is at rest, the reading of each spring balance is 70N. If a 12.60 Nm couple must be applied to the flywheel to keep it rotating clockwise at a constant speed determine a) the reading of each balance at that time b) the coefficient of kinetic friction. Assume that the belt is inextensible.



Solution : when the flywheel is at rest,

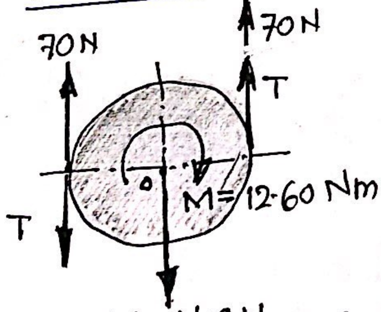


$\therefore (T_A)_0 = (T_B)_0 = 70$ N

$\therefore W = (T_A)_0 + (T_B)_0$
$= 140$ N

W = Weight of the flywheel

when the flywheel is rotating clockwise due to 12.60 Nm couple,



$\begin{bmatrix} \text{External couple acting} \\ \text{on the flywheel} \end{bmatrix} = \begin{pmatrix} \text{frictional} \\ \text{couple} \end{pmatrix}$

$12.60 \text{ Nm} = T \times 0.450$

$\boxed{T = 28 \text{ N}}$

$\therefore$ Tight tension, $T_1 = 70 + 28 = 98 \text{ N} = T_B$

$\therefore$ slack tension, $T_2 = 70 - 28 = 42 \text{ N} = T_A$

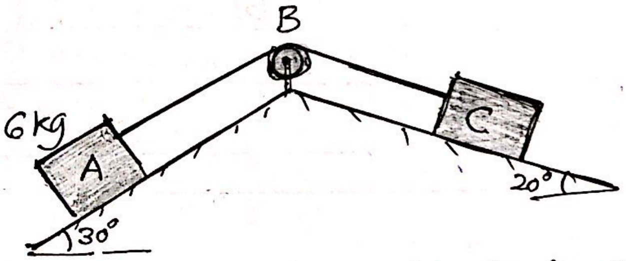
By Belt friction formula,

$\left(\frac{T_1}{T_2}\right) = e^{\mu\beta} \quad \therefore \left(\frac{98}{42}\right) = e^{\mu \cdot \pi} \quad \therefore \boxed{\mu = 0.269}$

Ans: a) $T_A = 42$ N, $T_B = 98$ N

b) $\mu = 0.269$

Ex. No. (12) Blocks 'A' and 'C' are connected by a rope that passes over drum B. Knowing that the drum rotates slowly clockwise and that the coefficients of friction at all surfaces are $\mu_s = 0.30$ and $\mu_k = 0.20$, determine the smallest mass of block C for which block A (a) will remain at rest, (b) will be in impending motion up the incline, (c) will move up the incline at a constant speed.

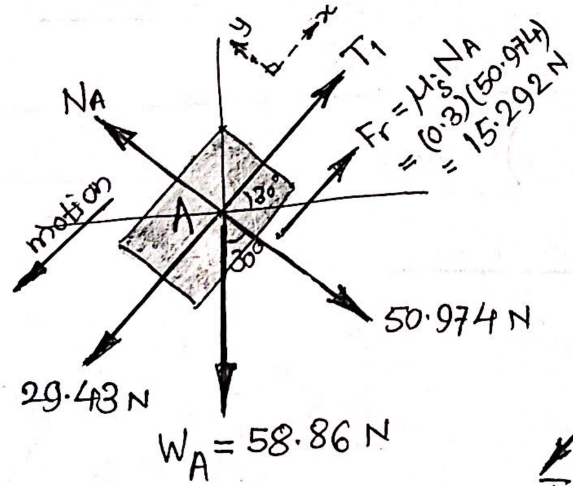


Solution: (a) When block **A** remains at rest:

As, $\mu_s = 0.30 \quad \therefore \phi = 16.7^\circ$

As, $\theta = 30^\circ > 16.7^\circ \therefore$ Block A will have tendency to slide down the plane. This is prevented by the tension in the rope.

consider F.B.D of block A:

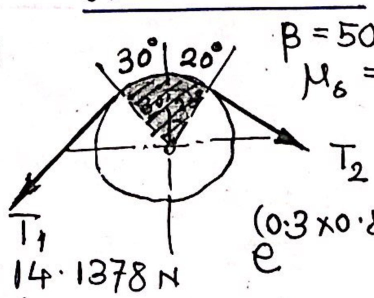


$\Sigma F_x = 0$ gives,

$$T_1 + 15.292 = 29.43$$

$$\therefore \boxed{T_1 = 14.1378 \text{ N}}$$

Now, consider F.B.D. of the drum B:



$\beta = 50^\circ = 0.8726^c$

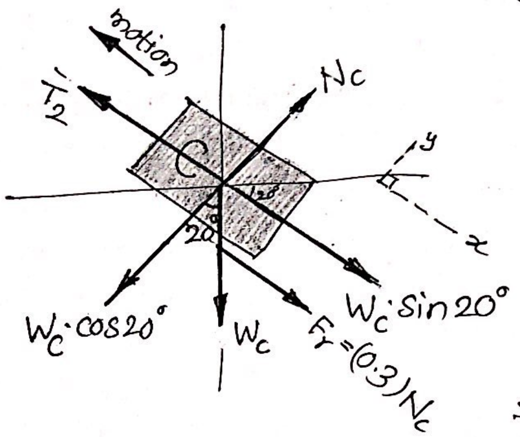
$\mu_s = 0.3$

$e^{(0.3 \times 0.8726)} = 1.2992$

By the Belt Friction formula,

$$\left(\frac{14.1378}{T_2}\right) = 1.2992 \quad \therefore \boxed{T_2 = 10.882 \text{ N}}$$

Now, consider F.B.D. of block C;



$\Sigma F_y = 0$ gives,

$N_c = W_c \cdot \cos 20°$
$= (0.9396) W_c$

$F_r = (0.3) N_c = (0.282) W_c$

$\Sigma F_x = 0$ gives,

$T_2 = (0.342) W_c + (0.282) W_c$

$\therefore T_2 = (0.624) W_c$

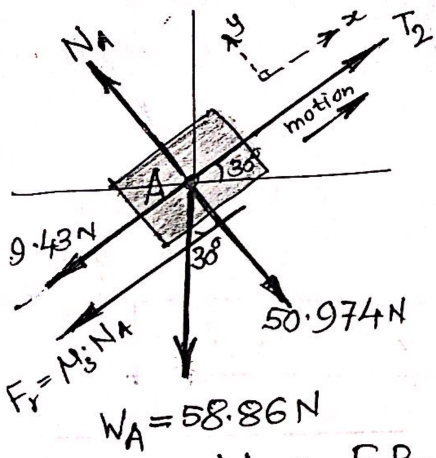
$\boxed{T_2 = 10.882}$

$\therefore W_c = 17.438$ N

$\therefore \boxed{m_c = \left(\frac{17.438}{9.81}\right) = 1.778 \text{ kg}}$

b) when block A is in impending motion up the incline:

Consider F.B.D. of block A:



$\Sigma F_y = 0$ gives,

$N_A = 50.574$ N

$\Sigma F_x = 0$ gives,

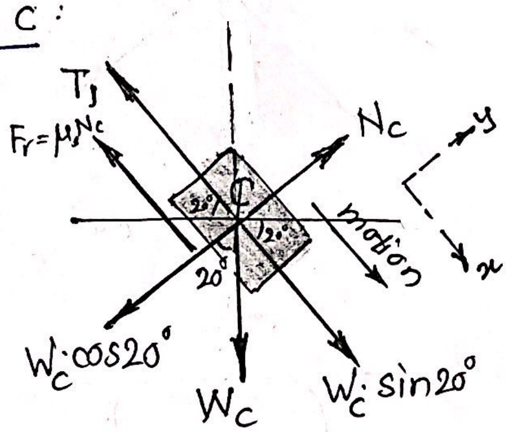
$T_2 = 29.43 + (0.3)(50.574)$

$\boxed{T_2 = 44.602 \text{ N}}$

Now, for the drum B, by Belt friction formula,

$\left(\frac{T_1}{44.602}\right) = 1.2992 \quad \therefore \boxed{T_1 = 57.947 \text{ N}}$

Now, consider F.B.D. of block C:



$\Sigma F_y = 0$ gives,

$N_c = W_c \cdot \cos 20°$

$F_r = \mu_s N_c = (0.3) \cdot W_c (0.9397)$
$= (0.282) W_c$

$\Sigma F_x = 0$ gives,

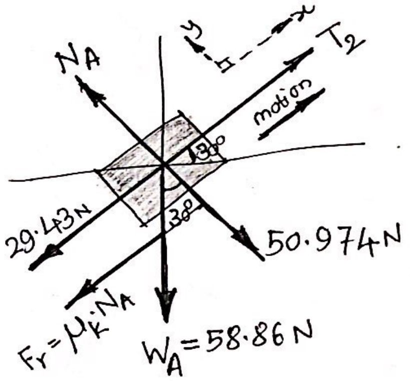
$T_1 + (0.282) W_c = W_c \cdot \sin 20°$

$57.947 = (0.0601) W_c$

$\therefore W_c = 963.978$ N $\therefore \boxed{m_c = 98.265 \text{ kg}}$

(c) when block A will move up the incline at a constant speed.

Consider F.B.D. of block A:



$\Sigma F_y = 0$ gives, $N_A = 50.974$ N

$(F_r)_{kinetic} = \mu_k \cdot N_A = (0.20 \times 50.974)$

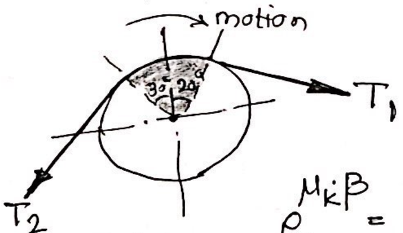
$= 10.1948$ N

$\Sigma F_x = 0$ gives,

$$\boxed{T_2 = 29.43 + 10.1948 = 39.6248\,N}$$

Now, consider F.B.D. of drum B:

$\beta = 50^\circ = 0.8726^c$



$T_2 = 39.6248$ N

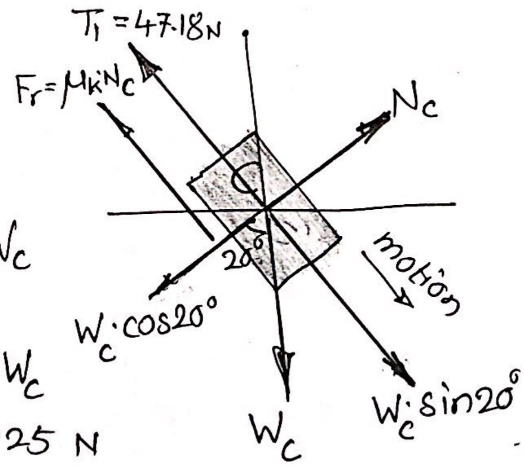
$e^{\mu_k \cdot \beta} = e^{(0.2 \times 0.8726)}$

$= 1.1906$

By Belt friction formula,

$\therefore \left(\dfrac{T_1}{39.6248}\right) = 1.1906 \qquad \therefore \boxed{T_1 = 47.18\,N}$

Now consider F.B.D. of block C:



$\Sigma F_y = 0$ gives,

$N_c = W_c \cdot \cos 20^\circ = (0.9397) W_c$

$\Sigma F_x = 0$ gives,

$W_c \cdot \sin 20^\circ = 47.18 + F_r$

$F_r = \mu_k \cdot N_c = (0.2)(0.9397) W_c$

$= (0.1879) W_c$

$\therefore (0.342) W_c = 47.18 + (0.1879) W_c$

$\therefore W_c = \left(\dfrac{47.18}{0.154}\right) = 306.125$ N

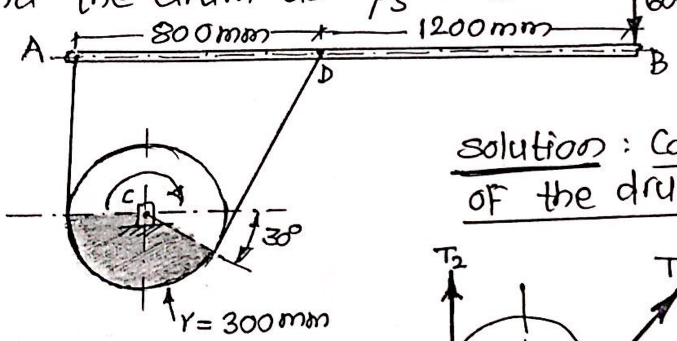
$$\boxed{m_c = 31.205\,kg}$$

Ans: a) $m_c = 1.778$ kg

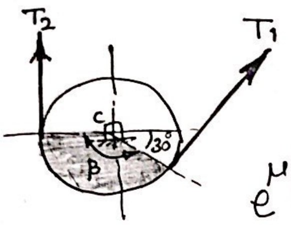
b) $m_c = 98.265$ kg

c) $m_c = 31.205$ kg

Ex. No. 13 Calculate the breaking torque acting on the drum, if the drum is rotating clockwise. Take the coefficient of friction between the belt and the drum as $\mu_s = 0.25$



Solution: Consider the F.B.D. of the drum;



$\beta = 180 - 30$
$= 150^\circ = 2.618^c$

$\mu = 0.25$

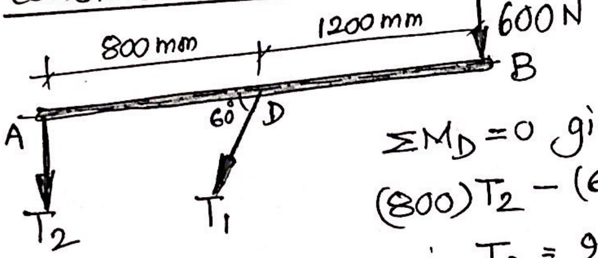
$e^{\mu\beta} = e^{(0.25 \times 2.618)}$
$= 1.9242$

By the Belt Friction Formula,

$$\left(\frac{T_1}{T_2}\right) = e^{\mu\beta} = 1.9242$$

$$\therefore T_1 = (1.9242)\, T_2 \longrightarrow (1)$$

Now, consider the F.B.D. of the lever;



$\Sigma M_D = 0$ gives,

$$(800)T_2 - (600 \times 1200) = 0 \longrightarrow (2)$$

$\therefore T_2 = 900\ N$

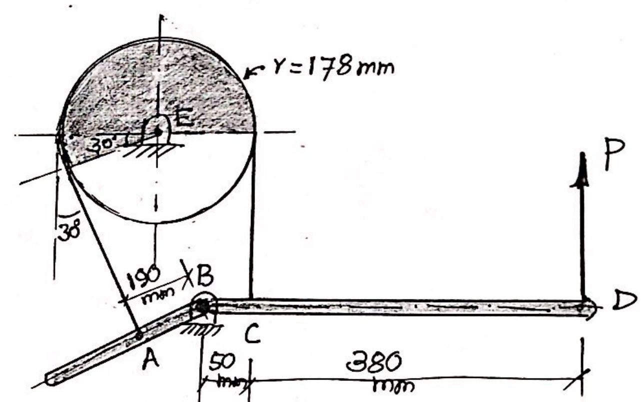
$\therefore T_1 = 1731.78\ N$

$\therefore$ Breaking torque acting on the drum,

$$M = (T_1 - T_2) \cdot r$$
$$= (1731.78 - 900) \times 0.3$$
$$= 249.534\ Nm \text{ (anticlockwise)}$$

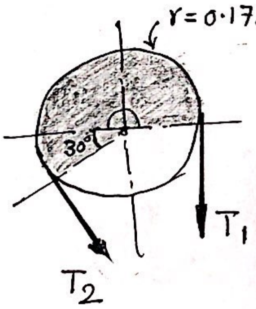
Ans: $\boxed{M = 249.534\ Nm \text{ (anticlockwise)}}$

Ex. No. 14 A differential band brake is used to control the speed of the drum. Determine the minimum value of the coefficient of static friction for which the brake is self-locking when the drum rotates counterclockwise.



Soln:

Consider the F.B.D. of the drum;

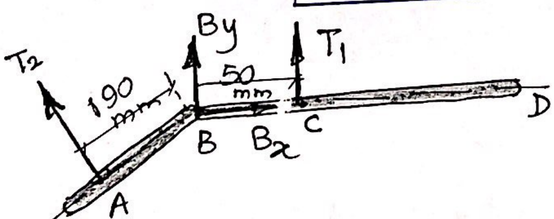


$\beta = 180° + 30° = 210° = 3.6652^c$

By the Belt friction formula,

$$\left(\frac{T_1}{T_2}\right) = e^{\mu\beta} \longrightarrow (1)$$

Now, consider the F.B.D. of the lever. (Note: For self-locking case take $P = 0$)



$\Sigma M_B = 0$ gives,

$$(50)T_1 - (190)T_2 = 0$$

$$\therefore \left(\frac{T_1}{T_2}\right) = \left(\frac{190}{50}\right) = 3.8 \longrightarrow (2)$$

From (1) and (2), we get.

$$\left(\frac{T_1}{T_2}\right) = 3.8 = e^{\mu \times 3.6652}$$

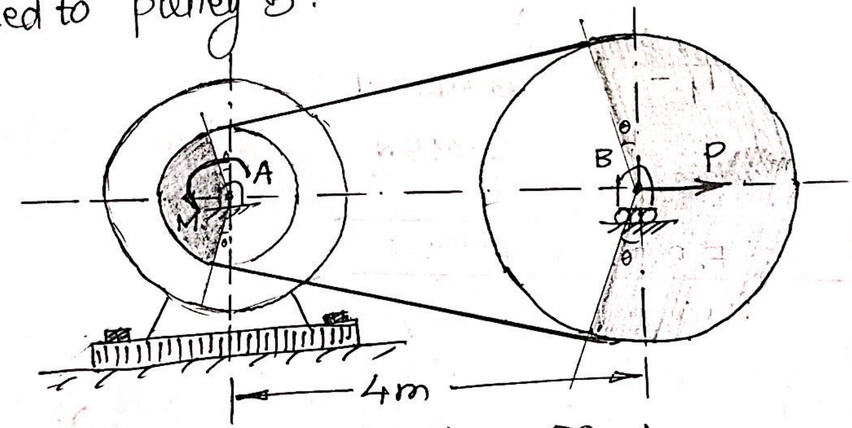
$$\therefore \log_e(3.8) = \mu \times 3.6652$$

$$1.335 = \mu \times 3.6652$$

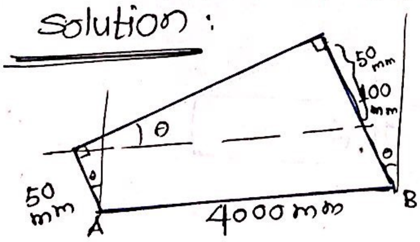
$\therefore$

Ans: $\boxed{\mu = 0.364}$

Ex. No (15) The V pulley's A and B have diameters 100 mm and 200 mm respectively and are connected by a 'V' belt for which $\alpha = 36°$. Pulley A is mounted on the shaft of an electric motor that develops a couple $M = 5$ N.m and the tension in the belt is controlled by a mechanism that applies a horizontal force 'P' to the axle of the pulley B. Knowing that the coefficient of static friction is 0.35, determine the magnitude of 'P' when the maximum couple is transmitted to pulley B.

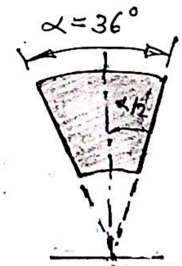


Solution :

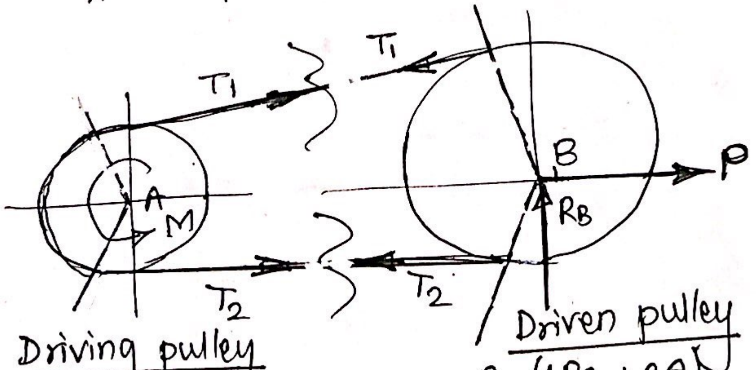


$$\sin\theta = \left(\frac{50}{4000}\right)$$

$$\theta = 0.7162°$$



cross-section of the 'V' belt



Driving pulley

$$\beta = (180 - 2\theta)$$
$$= 178.5676° = 3.1166^c$$

Driven pulley

$$\beta = (180 + 2\theta)$$
$$= 181.4324° = 3.166^c$$

For the driving pulley

$$\left.\begin{aligned}\beta &= 3.1166^c\\ \mu &= 0.35\\ \alpha &= 36^\circ\end{aligned}\right\} \quad e^{\left(\frac{\mu\beta}{\sin\alpha/2}\right)} = e^{\left(\frac{0.35\times 3.1166}{\sin 36/2}\right)} = e^{3.5299}$$

$$= 34.122$$

For the 'V' Belt,

$$\left(\frac{T_1}{T_2}\right) = e^{\left(\frac{\mu\beta}{\sin\alpha/2}\right)} = 34.122$$

$$\therefore T_1 = (34.122)T_2 \longrightarrow (1)$$

Note: $(\alpha/2) = (36/2) = 18^\circ$ is the semi-vertex angle of the 'V' Belt.

Now, $\boxed{M = (T_1 - T_2)\cdot r}$

$$5 \text{ Nm} = (T_1 - T_2)(0.050)$$

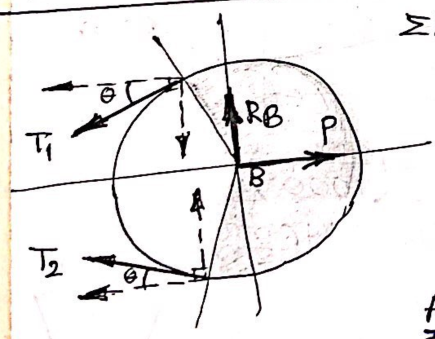
$$\therefore (T_1 - T_2) = 100 \longrightarrow (2)$$

$$T_2(34.122 - 1) = 100$$

$$\therefore T_2 = 3.019 \text{ N}$$

$$\therefore T_1 = 103.019 \text{ N}$$

Consider F.B.D. of pulley B:



$\Sigma F_x = 0$ gives,

$$P = (T_1 + T_2)\cos\theta$$

$$P = (103.019 + 3.019)(\cos 0.7162^\circ)$$

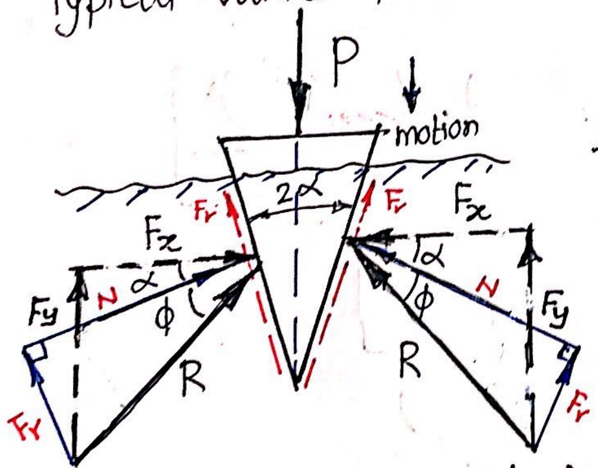
Ans: $\boxed{P = 106.03 \text{ N}}$

(P)

Ex. No. ⑧ a) Define the term 'wedge'.

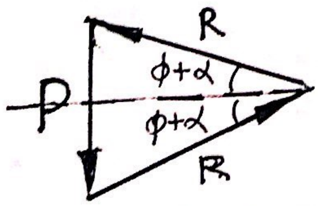
b) Find the relationship between the force applied to a wedge and force produced in the direction perpendicular to the applied force.

Solution : a) The wedge is a simple machine which is intended to transform an applied force, into a force at approximately right angles to the direction of the applied force. A typical example of a wedge is shown in figure..... The wedge angle 2α is usually quite small. Typical values for this angle are 5 to 10°

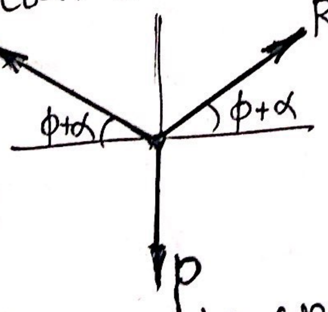


'P' is the force applied to the wedge. Weight of the wedge is assumed to be negligible compared to this force. Motion is assumed to be impending, so that the frictional forces have their maximum values.

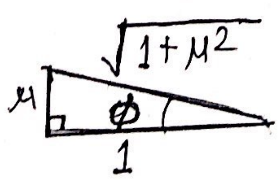
R = Total reaction which is the resultant of the normal reaction and frictional force. (N and Fr)



Force triangle



F.B.D. of the wedge



$\Sigma F_y = 0$ gives,

$$2R \cdot \sin(\phi+\alpha) = P$$

$$\therefore \frac{P}{2} = R\sin(\phi+\alpha)$$

$$R(\sin\phi\cdot\cos\alpha + \cos\phi\cdot\sin\alpha) = \frac{P}{2} \longrightarrow (a)$$

But $\mu = \tan\phi$ ∴ from this we get

$$\sin\phi = \left[\frac{\mu}{\sqrt{1+\mu^2}}\right] \text{ and } \cos\phi = \left[\frac{1}{\sqrt{1+\mu^2}}\right]$$

substituting in eqⁿ (a), we get,

$$R = \left[\frac{P\cdot\sqrt{1+\mu^2}}{2(\sin\alpha + \mu\cos\alpha)}\right] \longrightarrow (b)$$

The force 'F_x' in x direction is given by

$$F_x = R\cdot\cos(\phi+\alpha)$$

$$\therefore F_x = R\left[\cos\phi\cdot\cos\alpha - \sin\phi\cdot\sin\alpha\right]$$

$$\therefore F_x = \left[\frac{P\cdot\sqrt{1+\mu^2}\left[\left(\frac{\cos\alpha}{\sqrt{1+\mu^2}}\right) - \left(\frac{\mu\sin\alpha}{\sqrt{1+\mu^2}}\right)\right]}{2(\sin\alpha + \mu\cos\alpha)}\right]$$

$$\therefore F_x = \left[\frac{P(\cos\alpha - \mu\sin\alpha)}{2(\sin\alpha + \mu\cdot\cos\alpha)}\right]$$

$$\therefore \boxed{\left(\frac{F_x}{P}\right) = \left[\frac{(\cos\alpha - \mu\cdot\sin\alpha)}{2(\sin\alpha + \mu\cdot\cos\alpha)}\right]}$$

The ratio $\left(\frac{F_x}{P}\right)$ may be interpreted as the ratio of the desired force effect 'F_x' and the input force 'P'. It is thus a represen-tation of the force-multiplying effect of the wedge.

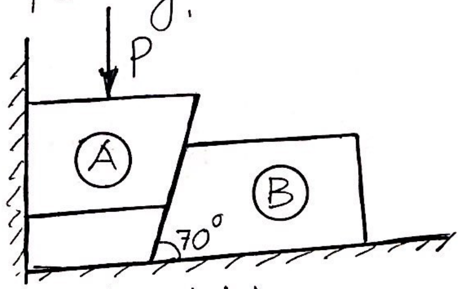
For eg. <u>For $\mu = 0.3$</u>

i) if $\alpha = 5°$, $\left(\frac{F_x}{P}\right) = 1.256$

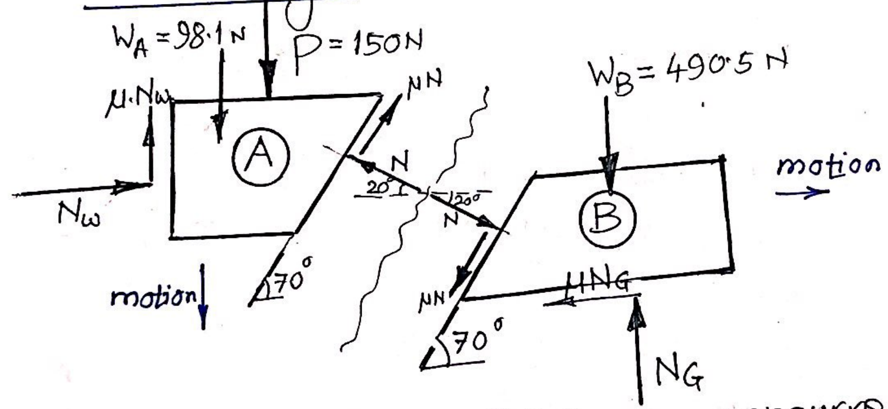
ii) if $\alpha = 4°$, $\left(\frac{F_x}{P}\right) = 1.323$

iii) if $\alpha = 3°$, $\left(\frac{F_x}{P}\right) = 1.396$

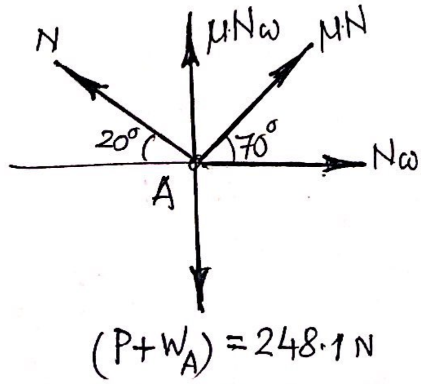
Ex. No. ⑧ The 10 kg block A is resting against a 50 kg block B as shown in Figure... The coefficient of static friction μ_s is the same between all surfaces of contact. If P = 150 N, determine the value of μ_s, for which motion is impending.



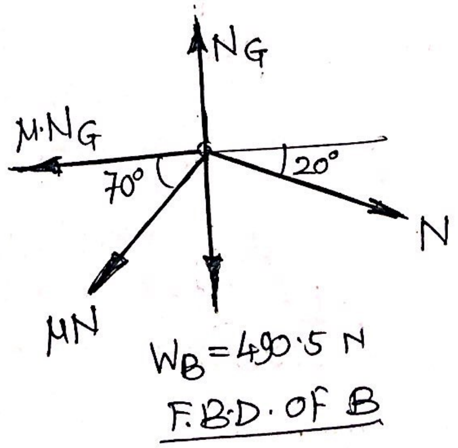
Solution: Consider the F.B.D. of blocks A and B, with motion of A in vertically downward direction and motion of B towards right.



Convert the above F.B.Ds in concurrent coplanar force systems by neglecting the size of the blocks.



F.B.D. OF A



F.B.D. OF B

Applying equations of equilibrium to the F.B.D. of A,

$\Sigma F_x = 0$ gives,

$N_\omega + \mu \cdot N \cdot \cos 70^\circ - N \cdot \cos 20^\circ = 0$

$\therefore N_\omega + \mu \cdot N \cdot (0.342) - (0.94) N = 0 \longrightarrow$ (i)

$\Sigma F_y = 0$ gives,

$\mu \cdot N_\omega + N \cdot \sin 20^\circ + \mu N \cdot \sin 70^\circ - 248.1 = 0$

$\therefore \mu \cdot N_\omega + (0.342) N + (0.94) \mu N - 248.1 = 0 \longrightarrow$ (ii)

Similarly applying equations of equilibrium to the F.B.D. of B,

$\Sigma F_x = 0$ gives,

$N \cdot \cos 20^\circ - \mu \cdot N_G - \mu \cdot N \cdot \cos 70^\circ = 0$

$(0.94) N - \mu \cdot N_G - (0.342) \mu N = 0 \longrightarrow$ (iii)

$\Sigma F_y = 0$ gives,

$N_G - N \cdot \sin 20^\circ - \mu \cdot N \cdot \sin 70^\circ - 490.5 = 0$

$N_G - (0.342) N - (0.94) \mu \cdot N - 490.5 = 0 \longrightarrow$ (iv)

From eq^n (iv) we get,

$N_G = (0.342) N + (0.94) \mu \cdot N + 490.5$

Substituting this in eq^n (iii), we get,

$(0.94) N - 2(0.342) \mu N - (0.94) \mu^2 N - (490.5) \mu = 0 \rightarrow$ (a)

From eq^n (i) we get,

$N_\omega = (0.94) N - (0.342) \cdot \mu \cdot N$

Substituting this ~~eq~~ in eq^n (ii), we get,

$(0.94) \mu N - (0.342) \mu^2 N + (0.342) N - 248.1 = 0 \longrightarrow$ (b)

$\therefore N (1.88 \times \mu - 0.342 \times \mu^2 + 0.342) = 248.1$

$$\therefore N = \left[\frac{248.1}{(0.342 + 1.88 \times \mu - 0.342 \times \mu^2)} \right]$$

Substitute the above the expression for N, in eq^n (a), we get.

$$0 = \left[\frac{233.214}{(0.342 + 1.88\mu - 0.342\mu^2)}\right] - \left[\frac{(169.7)\mu}{(0.342 + 1.88\mu - 0.342\mu^2)}\right]$$

$$- \left[\frac{(233.214)\mu^2}{(0.342 + 1.88\mu - 0.342\mu^2)}\right] - \left[\frac{(490.5)(\mu)(0.342 + 1.88\mu - 0.342\mu^2)}{(0.342 + 1.88\mu - 0.342\mu^2)}\right]$$

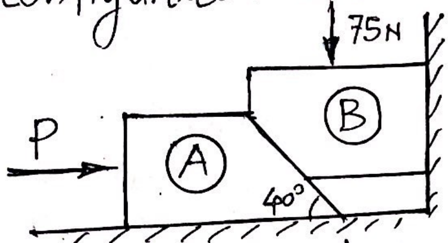
$$\therefore 0 = (233.214) - (169.7)\mu - (233.214)\mu^2 - (167.75)\mu - (922.14)\mu^2 - (167.15)\mu^3$$

$$\therefore (167.15)\mu^3 + (1155.354)\mu^2 + (337.45)\mu - (233.214) = 0$$

Solving the above cubic eqn, we get,

Ans: $\boxed{\mu = 0.332}$

Ex.No. 9 Block A weigh 25N and B weighs 18 N. The coefficient of friction at all surfaces in contact is 0.11. For what range of values of P will the system be in equilibrium in the configuration shown?

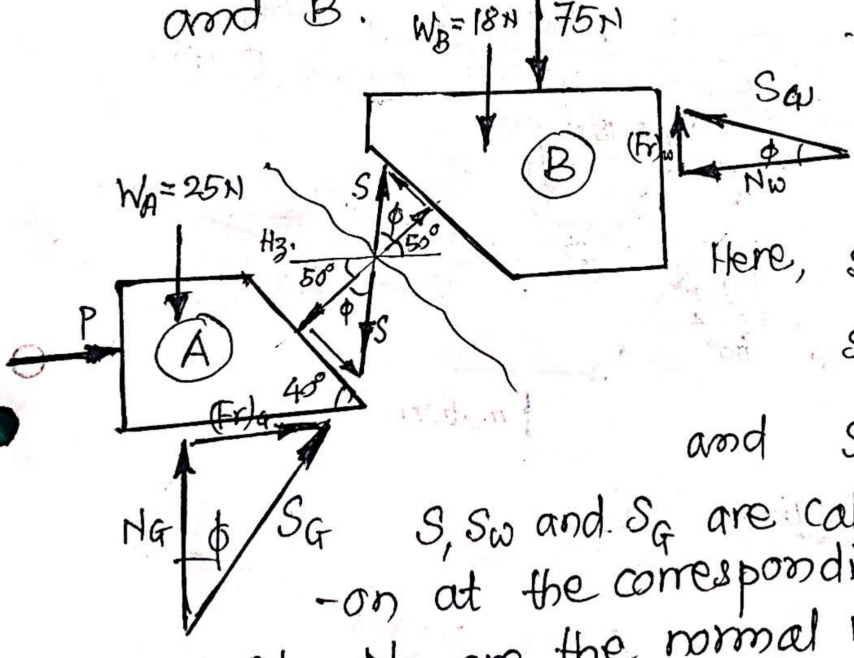


Solution :

A) For Pmin :

Consider the motion of block B in downward direction due to which block A will move to the left.

Now, consider the F.B.D.s of blocks A and B.



$\mu = \tan\phi$

$\therefore \phi = \tan^{-1}\mu$

$\phi = \tan^{-1}\mu = \tan^{-1} 0.11$

$\phi = 6.28°$

Here, $S_w = \sqrt{N_w^2 + (Fr)_w^2}$,

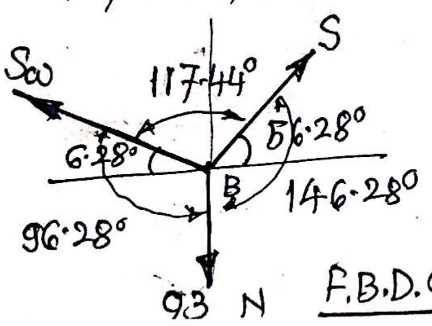
$S_G = \sqrt{N_G^2 + (Fr)_G^2}$

and $S = \sqrt{N^2 + Fr^2}$

S, S_w and S_G are called as total reaction at the corresponding surfaces.

N, N_w, N_G are the normal reactions

Fr, $(Fr)_w$, $(Fr)_G$ are the frictional forces.

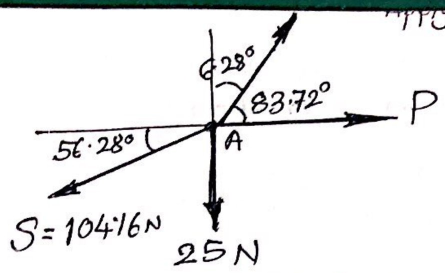


F.B.D. of Block B

Applying Lami's theorem, to the F.B.D. of block B, we get,

$$\left(\frac{93}{\sin 117.44°}\right) = \left(\frac{S}{\sin 96.28°}\right) = \left(\frac{S_w}{\sin 146.28°}\right)$$

$\therefore$ $\boxed{S = 104.16 \text{ N}}$



F.B.D. of block A

Applying ... F.B.D. of block A,

$\Sigma F_x = 0$ gives,

$$P + S_G \cos 83.72^\circ - (104.16 \times \cos 56.28^\circ) = 0$$

$$\therefore P + (0.109) S_G - 57.82 = 0 \longrightarrow (i)$$

$\Sigma F_y = 0$ gives,

$$S_G \sin 83.72^\circ - 25 - (104.16) \sin 56.28 = 0$$

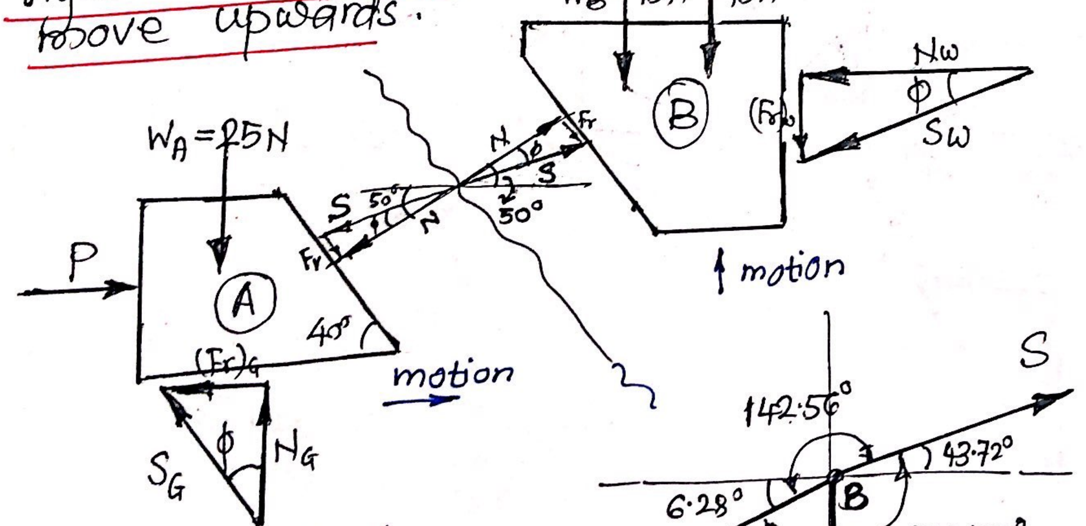
$$(0.99) S_G - 25 - 86.64 = 0 \longrightarrow (ii)$$

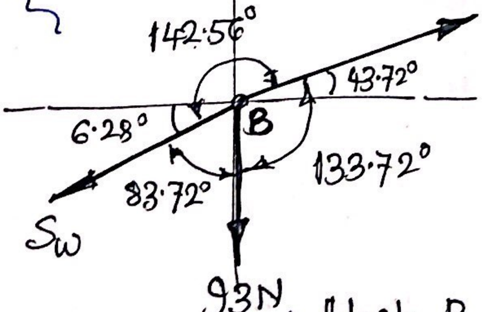
$$\therefore S_G = 112.76 \text{ N}$$

From eq^n (i) we get $\boxed{P_{min} = 45.528 \text{ N}}$

B) For P_{max}:

Consider the motion of block A to the rightword direction due to which block B will move upwards.





F.B.D. of block B

Applying Lami's theorem, to F.B.D. of block B, we get,

$$\left(\frac{93}{\sin 142.56°}\right) = \left(\frac{S}{\sin 83.72°}\right) = \left(\frac{S_W}{\sin 133.72°}\right)$$

$$\therefore \boxed{S = 152.06 \text{ N}}$$

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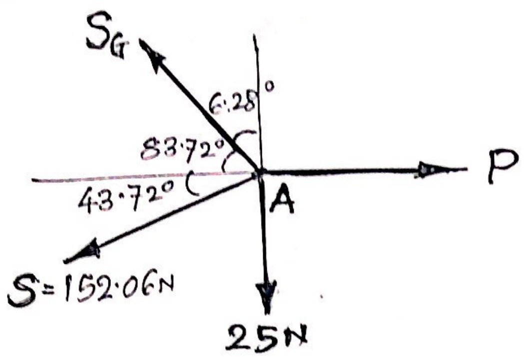

F.B.D. of block A

Applying eq^{ns} of equilibrium to F.B.D. of A,

$\Sigma F_x = 0$ gives,

$$P - S_G \times \cos 83.72° - (152.06)\cos 43.72° = 0$$

$$P - (0.109) S_G - 109.9 = 0 \longrightarrow (i)$$

$\Sigma F_y = 0$ gives,

$$S_G \cdot \sin 83.72° - 25 - (152.06)\sin 43.72° = 0$$

$$(0.99) S_G - 25 - 105.09 = 0 \longrightarrow (ii)$$

$$\therefore \quad S_G = 131.4 \text{ N}$$

From eq^n (i) we get,

$$\boxed{P_{max} = 124.22 \text{ N}}$$

Ans: The range of values of P, for which the system will remain in equilibrium is

$$P_{min} \leq P \leq P_{max}$$

$$\therefore \quad \boxed{45.528 \text{ N} \leq P \leq 124.22 \text{ N}}$$