A] ENGINEERING CURVES

We often come across curves in engineering field. Few laws of nature put on graph, give us such curves. The trajectory of space vehicles is decided by these curves. These curves are very useful in understanding the manufacture of various items, in designing the mechanisms, in analysis of forces in construction of bridges, dams, water tanks etc.

1. Conic Sections

A cone is obtained by rotating a straight line keeping one of its ends at a fixed point and other end in contact with a closed curve. The rotating straight line is called a generator, the fixed point is known as apex or vertex and the closed curve is called a base. The base of solid circular cone is a circle. An imaginary line joining the centre of base circle and the apex is known as axis of the cone. If the axis of circular cone is perpendicular to the base, it is called right circular cone. The sections obtained by cutting the right circular cone at different angles are called conic sections.

Types of Conic Sections

The following are types of conic sections obtained by cutting a right circular cone by section planes A, B, C, D and E w.r.t. cone axis, base or generator. [Refer Figure 4.1 (i to v)]

i. **Circle:** When a right circular cone is cut by a section plane A, perpendicular to the axis of the cone and cutting all its generators, the section obtained is called a circle.

ii. **Ellipse:** When a right circular cone is cut by a section plane B, inclined to the axis of the cone and if it cuts all its generators, the section is called an ellipse. Here section plane is not parallel to any generator of the cone.

iii. **Parabola:** A right circular cone cut by a section plane C, inclined to the axis of the cone and parallel to one of its generators, the section obtained is called a parabola.

iv. **Hyperbola:** If a cone is cut by a plane making an angle with the axis smaller than that of the generator, the conic section obtained is called a hyperbola. Also if the cutting plane is parallel to the axis of the cone, we get a rectangular hyperbola as the section.

v. **Isosceles Triangle:** When a right circular cone is cut by a section plane through the apex of the cone and intersecting the base, the section obtained is an isosceles triangle.

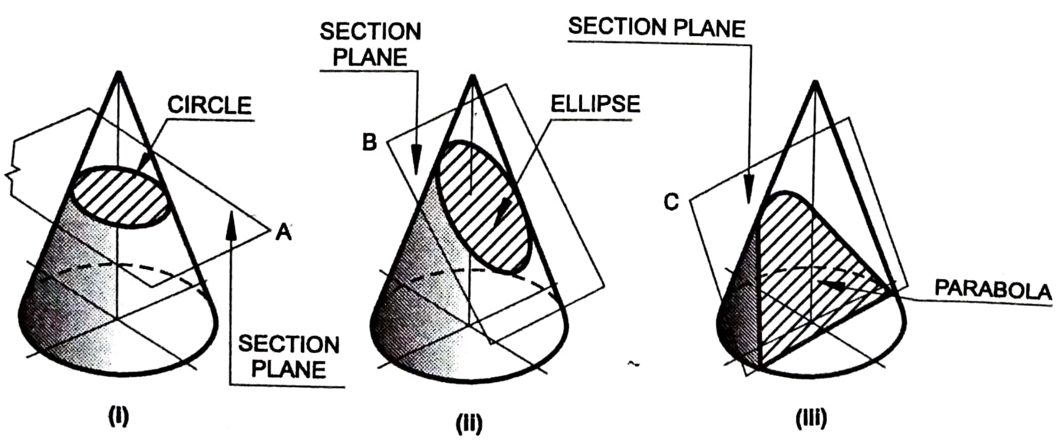

Figure 4.1: (I to iii)

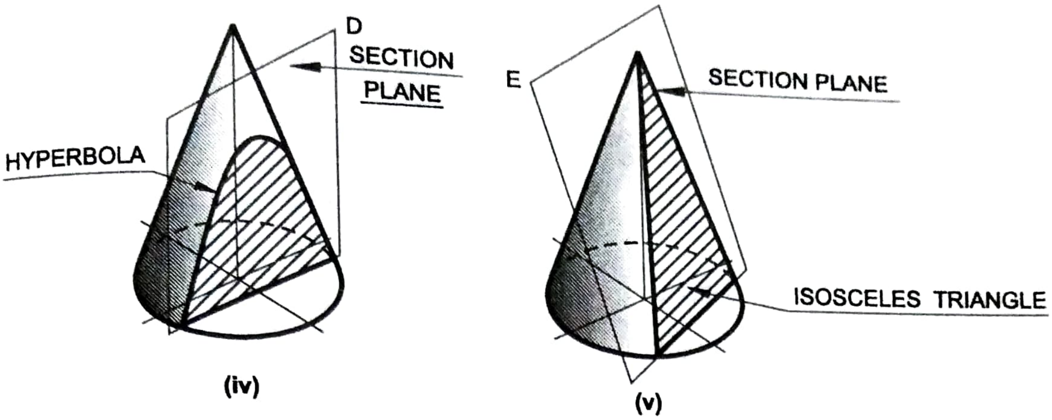

Figure 4.1: (iv to v) Types of conic sections

Note: Two conic sections viz. circle and isosceles triangle are very common. Hence they are not discussed.

Conic Defined as Loci

A conic can be defined as the locus of a point moving in a plane, such that the ratio of its distance from a fixed point and a fixed straight line is a constant. The fixed point is called focus and the fixed straight line is called directrix.

Eccentricity of a conic is the ratio of the distance of the point from the focus divided by its perpendicular distance from the directrix. It is a constant for a conic and is usually denoted by e.

Let P be any point on curve moving in such a way, that the ratio of its distance from a fixed point F and a fixed straight line DD is constant. PF is the distance of the moving point P from the focus F and PQ is the distance of the moving point P from the directrix. (Refer Figure 4.2)

a. If ratio $\frac{P_1F}{P_1Q_1} < 1$, the curve obtained is an ellipse.
(For ellipse, eccentricity e < 1)

b. If ratio $\frac{P_2F}{P_2Q_2} = 1$, the curve is a parabola.
(For parabola, eccentricity e = 1)

c. If ratio $\frac{P_3F}{P_3Q_3} > 1$, the curve obtained is called hyperbola.
(For hyperbola, eccentricity e > 1)

2. Ellipse

Ellipse is the locus of a point, which moves in a plane such that, the ratio of its distance from a fixed point (Focus) and a fixed straight line (Directrix) is a constant and is always less than unity. (Refer Figure 4.2)

An ellipse can also be defined as a curve traced out by a point, moving in a plane such that, the sum of its distances from two fixed points is always a constant. These fixed points are called foci (plural of focus).

The sum of the distances is equal to the major axis of the ellipse. An ellipse has two foci and two directrices.

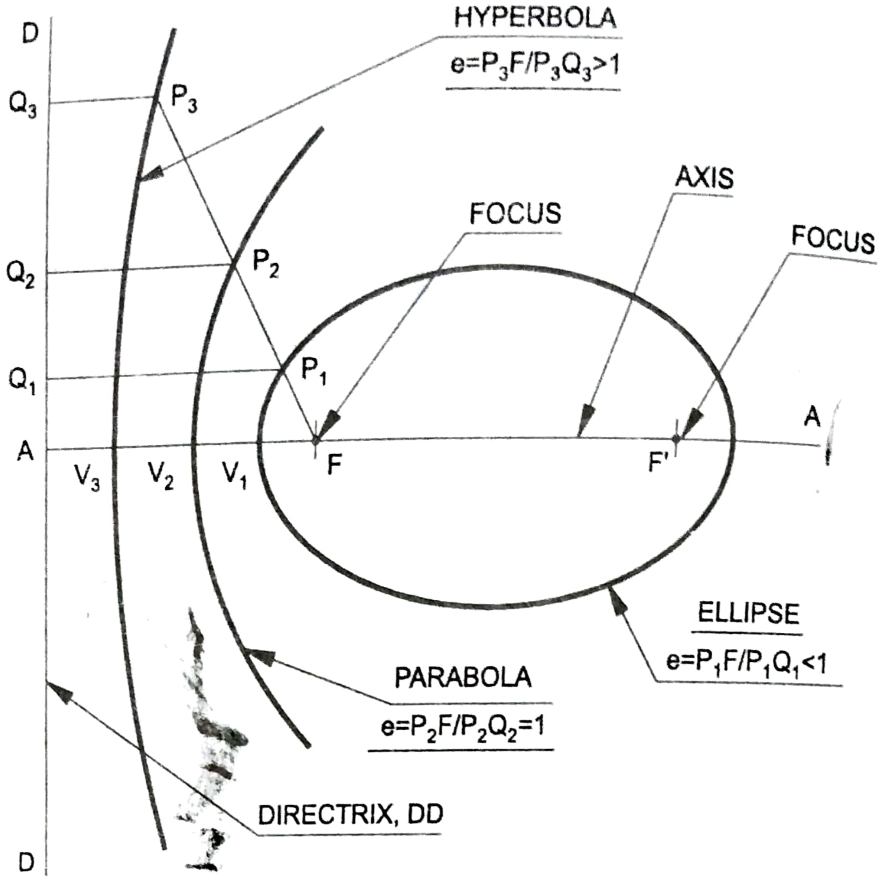

Figure 4.2: Conic curves defined as loci

Uses of Elliptical Shapes:

i. Shape of trays
ii. Monument
iii. Shape of manhole
iv. Shape of tank in garden
v. Path of earth around the sun
vi. Flanges of pipes, glands and stuffing boxes
vii. Shapes used in bridges and arches

Terminology of Ellipse

The important terms related to the ellipse are defined as under - [Refer Figure 4.3 (a)]

a. **Major Axis:** It is a line passing through the foci and terminated by the curve. (VV')

b. **Minor Axis:** It is the line, which passes through the geometrical centre of the ellipse, bisecting the major axis at the right angles and terminated by the ellipse. (KL)

e. **Normal to an Ellipse :** Normal to an ellipse at any point is perpendicular to the tangent at that point [Normal NM as shown in Figure 4.3 (b)]

In Figure 4.3 (b), AB is major axis, CD is minor axis and F_1 and F_2 are the foci. Foci are equidistant from the centre O. The points A, P, C etc. are on the curve.

Therefore as per definition

$(AF_1 + AF_2) = (PF_1 + PF_2) = (CF_1 + CF_2)$ etc.

But $AF_1 + AF_2 = AB$

Therefore $PF_1 + PF_2 = AB$ i.e. major axis.

Therefore, the sum of distances of any point on the curve from the two foci is equal to the major axis.

Also $(CF_1 + CF_2) = AB$

But $CF_1 = CF_2$ Therefore $CF_1 = CF_2 = 1/2\,AB$

Therefore the distances of the ends of the minor axis from the foci are equal to half the major axis.

Let us study different methods of drawing ellipse one by one, with the help of problems. Method No.2 to No.6 are to be used only if the major and minor axes of ellipse are known.

Directrix and Focus Method

Problem: 1

The distance between a fixed point a fixed straight line is 60 mm. Draw the locus of a moving point P, such that its distance from that fixed point (at a particular position) is 24 mm and that from the fixed straight line is 36 m Draw tangent and normal to this curve at any point on the curve. Name the curve, the fixed point and f straight-line.

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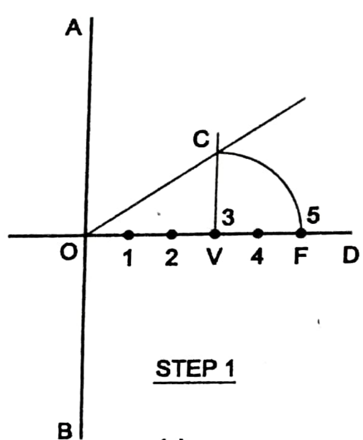
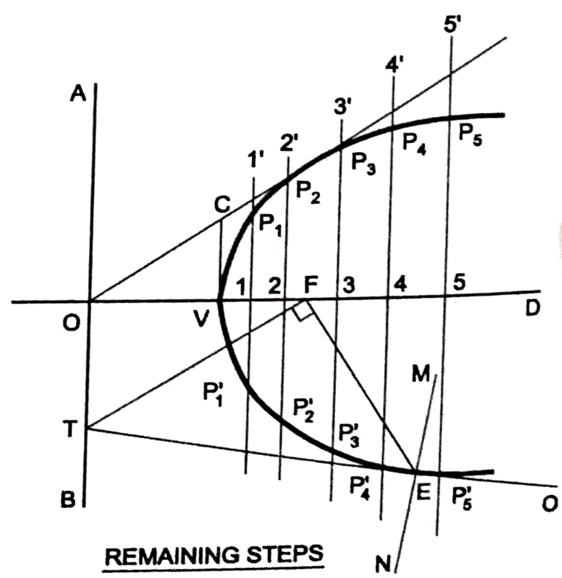
Solution: (Refer Figure 4.4)



(a)



(b)

Figure 4.4 (a & b)

3. Parabola

Parabola can be defined as a curve traced out by a point moving in a plane such that its distance from a fixed point, called focus is always equal to its distance from a fixed line, called directrix.

A parabola has one focus and one directrix.

Use of parabolic curves.

1) Sound reflectors.
2) Construction of bridges and arches.
3) Head lamp reflectors of automobiles.
4) Path described by projectile.
5) Shape of cooling tower.

Let us study methods of drawing parabola with the help of problems.

Directrix and Focus Method

Problem : 1

Draw a line AB of any length, preferably vertical. Mark a point O on it. At O, erect a perpendicular OF, 60 mm long. A point P moves in a plane in such a manner that it is always equidistant from the point F and the line AB. Trace the locus of point P. Name the curve. Draw a tangent and normal at any point on the curve.

Solution: (Refer Fig. 4.9) **P.U.,**

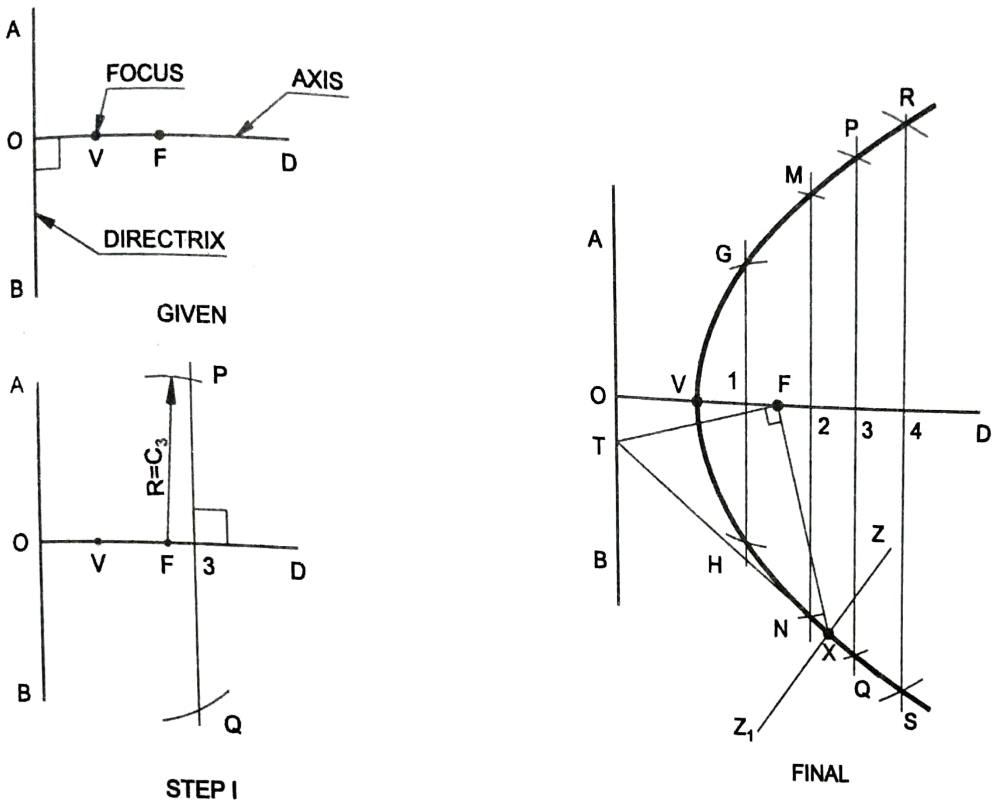


Figure 4.9

As the eccentricity of this curve is one, the curve is parabola. Draw a directrix AB and axis OD perpendicular to AB. Mark focus F such that OF = 60 mm. By definition e = 1 = VF/VO, thus mark point V (vertex) as a midpoint of OF as shown in figure (step I).

Mark point 3 on line OD and erect a perpendicular through it. With centre at F and radius equal to 03, draw an arc to cut the vertical line through point 3 at P and Q as shown in step I. Take number of points 1,2,3,.....V onwards on line OD. Erect perpendiculars through all the points. Now using F as centre and radii equal to 01,02,03.... Etc. cut the respective vertical lines through those points at G-H-M, N-P, Q-R, S etc. All these points are on parabola, hence join all these points by a smooth curve to obtain the required parabola.

For tangent and normal, mark any point X on the curve. Join X to focus F. Plot a perpendicular line FT to this line FX, meeting AB in T. Join TX, which is a tangent to the parabola at point X. Draw a perpendicular line ZZ_1, to the tangent through point X and thus ZZ_1, will be required normal at point X.

Rectangle Method

Problem : 2

A ball is thrown up in the air, reaches a maximum height of 70 metres and travels a horizontal distance of 3 metres. Trace the path of the ball assuming it to be a parabolic. Draw tangent and normal at any point on the curve

P.U., A.U.,

Solution: [Refer Fig. 4.10 (a)]

Rectangular (or Equilateral) Hyperbola

It is a curve traced out by moving a point in such a way that the product of its distances from two fixed lines at right angles to each other is a constant.

The fixed lines are called asymptotes of the rectangular hyperbola. This curve graphically represents the Boyle's Law viz. p x v = a constant

Use of Hyperbolic Curves

i) Nature of graph of Boyle's Law.
ii) Shape of cooling tower.
iii) Shape of over head water tank.

Let us study methods of drawing hyperbola with the help of problems.

1. Directrix and Focus Method

Problem : 1

Construct a conic section by using the following data:

Distance of focus from directrix = 60 mm. Eccentricity = 5/3. Name the curve. Draw a tangent and normal to the curve at any point on the curve.

Solution: (Refer Fig. 4.12) **A.U.,**

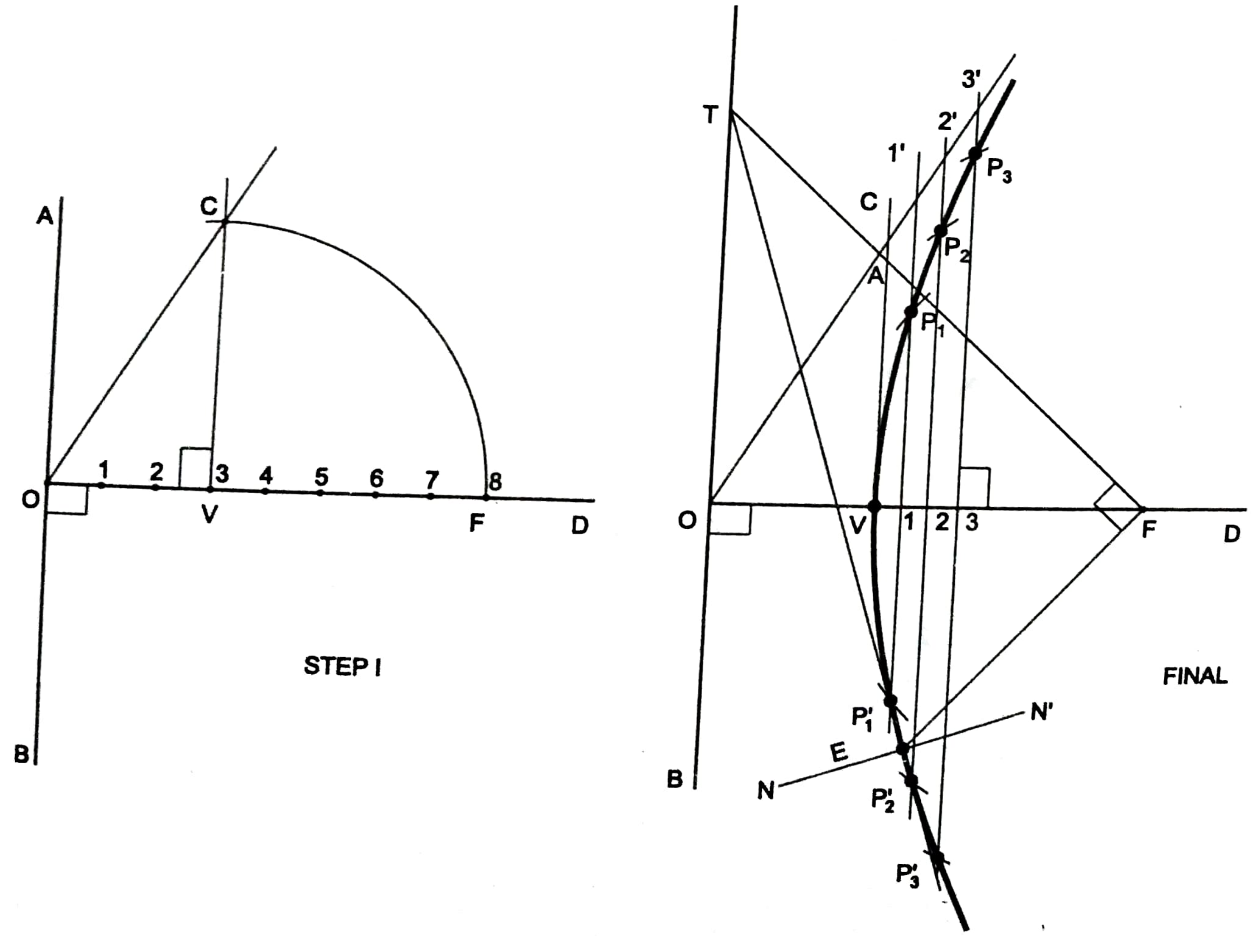


Figure 4.12

...these P_1-P_2...P_6 points by a ... such as

4. Involute

The involute of a circle or polygon is a curve traced out by an end of a piece of inelastic string unwound from the circle or polygon, the string being kept tight.

It may also be defined as a curve traced out by a point in a straight line, which rolls around the circle, without slipping.

The normal to an involute is tangent to the circle. The construction of involute is explained through the following problems. Gear teeth profiles are based on the involute of a circle.

Problem :1

Draw a involute of a circle of 40 mm diameter. Draw also a normal and tangent to it at a point M, 100 mm from the centre of circle.

Solution: (Refer Fig. 4.15)

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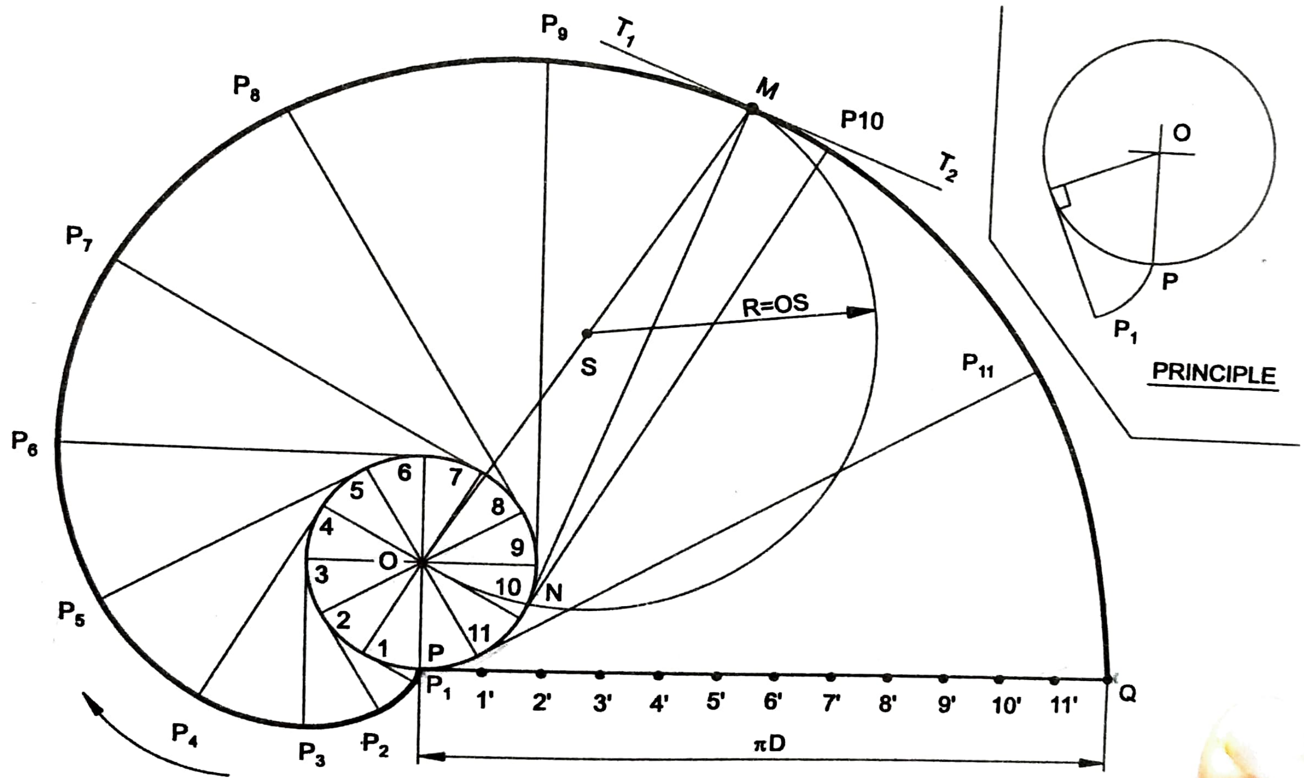

Figure 4.15

5. Cycloid

Cycloid is a curve generated by a point on the circumference of a circle, which rolls along a straight line (without slipping). The rolling circle is called a generating circle.

Cycloidal curves are drawn to represent the profile of gear teeth.

Normal and tangent

In case of all the cycloidal curves, a normal at any point on the cycloidal curve will pass through the point of contact between directing line/circle and that of generating circle which passes through the said point on the curve.

The tangent at any point is perpendicular to the normal at that point.

Problem : 1

A circle of diameter 50 mm rolls horizontally without slipping. If a point P is on the circumference of the circle, draw a locus of a point P for one revolution of 50 mm diameter circle and name the curve.

Draw tangent and normal at any point on the curve

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Solution: [Refer Fig. 4.20 (A & B)]

Draw horizontal line O_0-0_8 equal to the circumference of the generating circle. Hence, length O_0-$O_8 = \pi D = \pi 50$ mm. With centre O_0 and radius equal to 25 mm, draw initial position of circle. Line O_0-O_8 is locus of the centre of rolling circle. Divide circle and line O_0-O_8 into same number of equal parts (say 8). Mark centres of circles as O_1,O_2.....etc. and circle parts as 1,2,...etc. Draw tangent P_0Q to the circle, which will be a directing line and is equal to O_0-O_8.

Let P_0 be the initial position of point P on the circle. When circle completes one resolution, final position of the point P will be at P_8 line. The in between positions of point P are located as under :

Draw horizontal lines through points 1,2,3... etc. on circle. These will be loci lines for these points, during the rolling of the circle. To get the positions of point P, when the circle centre will be at O_3, use center as O_3 and with radius equal to 25 mm mark position of point P as P_3, on the locus line of point 3. In the same way with centre at O_5, draw an arc of radius equal to 25 mm to cut the locus line through point 5 at P_5. (Refer Step-I). Similarly obtain other points like P_2-P_3-P_4...etc. Draw a smooth curve passing through all these points to get the required Cycloid.

To obtain a tangent at any point M on curve, draw an arc (with centre at M) of radius equal to 25 mm and the line O_0-O_8 at Om. Erect a perpendicular through Om to meet P_0 Q line in point N. Join NM. Then line NM will be the required normal to the curve. A line perpendicular to NMN$_1$ will be a tangent. Therefore TMT$_1$ is the required tangent to the curve at point M.

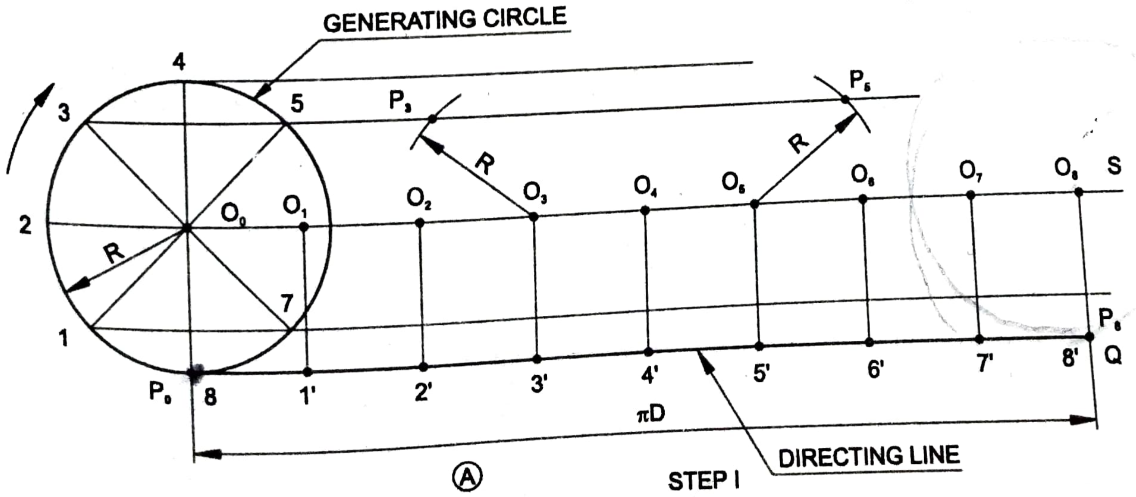

Figure 4.20

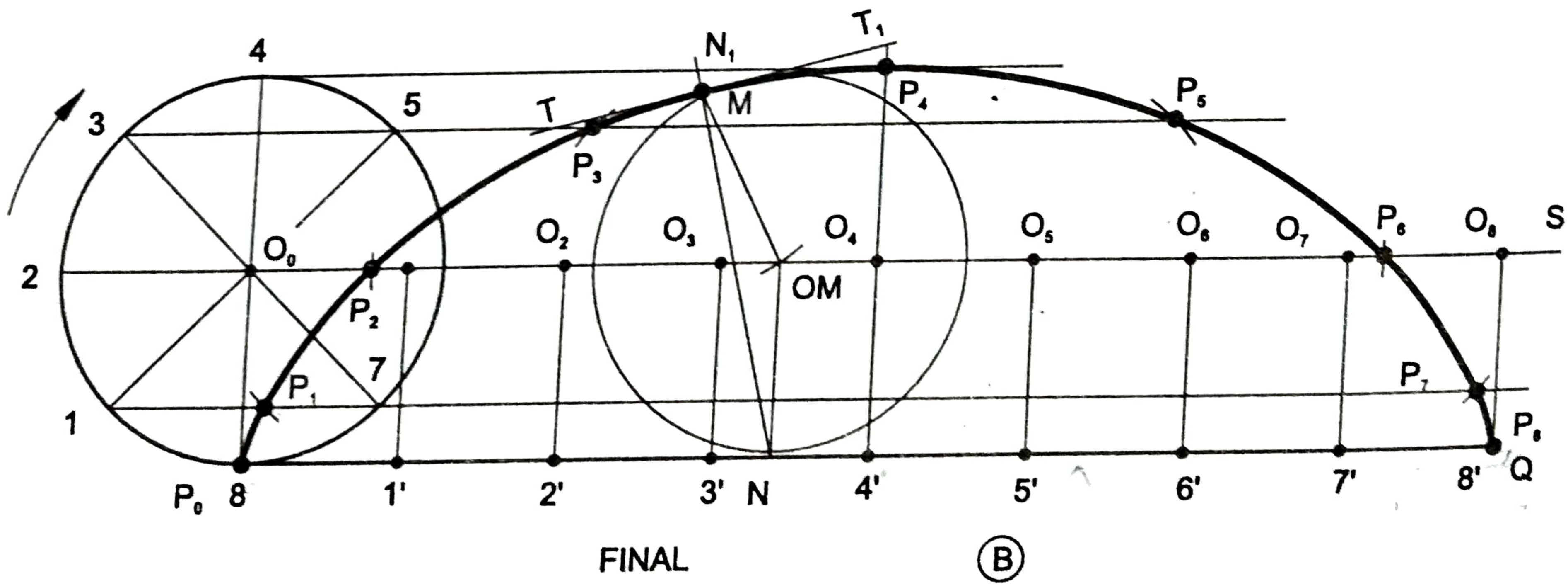

Figure 4.20 (A & B)

n:2

plane about one of its ends, ... moving point is called a spiral. The point about which the line rotates is called a pole.

Radius vector is a line joining any point on the curve to the pole. The angle between the initial position of and radius vector is called a vectorial angle. The moving point will trace one convolution when the line ro... once. A spiral can make any number of convolutions before reaching the pole.

Archimedean Spiral

It is a curve traced out by a point which moves uniformly both about the centre and at the same time away towards the centre.

Applications of Spiral

1. Shape of springs of watches and wall clocks.
2. Profile of cam for automation.
3. Screw threads and worm gears.
4. Scroll plate of self centring chucks for lathes.
5. Sound tracks on gramophone records.
6. Heat and pressure gauges elements.
7. Clamping devices of jigs and fixtures.

The construction of spiral is explained through the following problems.

To draw tangent and normal to the curve at any point K on it.

$$\text{Constant of curve} = \frac{\text{Difference in length of any two radius vectors}}{\text{Angle between the corresponding radius vectors in radians}}$$

$= (OP_6 - OP_4)/(\pi/2)$ where OP_4 = the length of vector OP_4 at certain position

$= (15)/(\pi/2) = 9.55$ mm OP_6 = length of vector OP_6 after moving 90° angle i.e. $\pi/2$ radians

(Refer Fig. 4.28 for construction)

Problem : 1

A point P moves radially outwards from the centre of a circular disc to the periphery when disc completes one revolution.

Radial movement of the point P and the angular motion of the disc are assumed to be uniform. Take diameter of disc as 120 mm. Trace the locus of the point P and name the curve. A.U.,

Solution: (Refer Fig. 4.28)

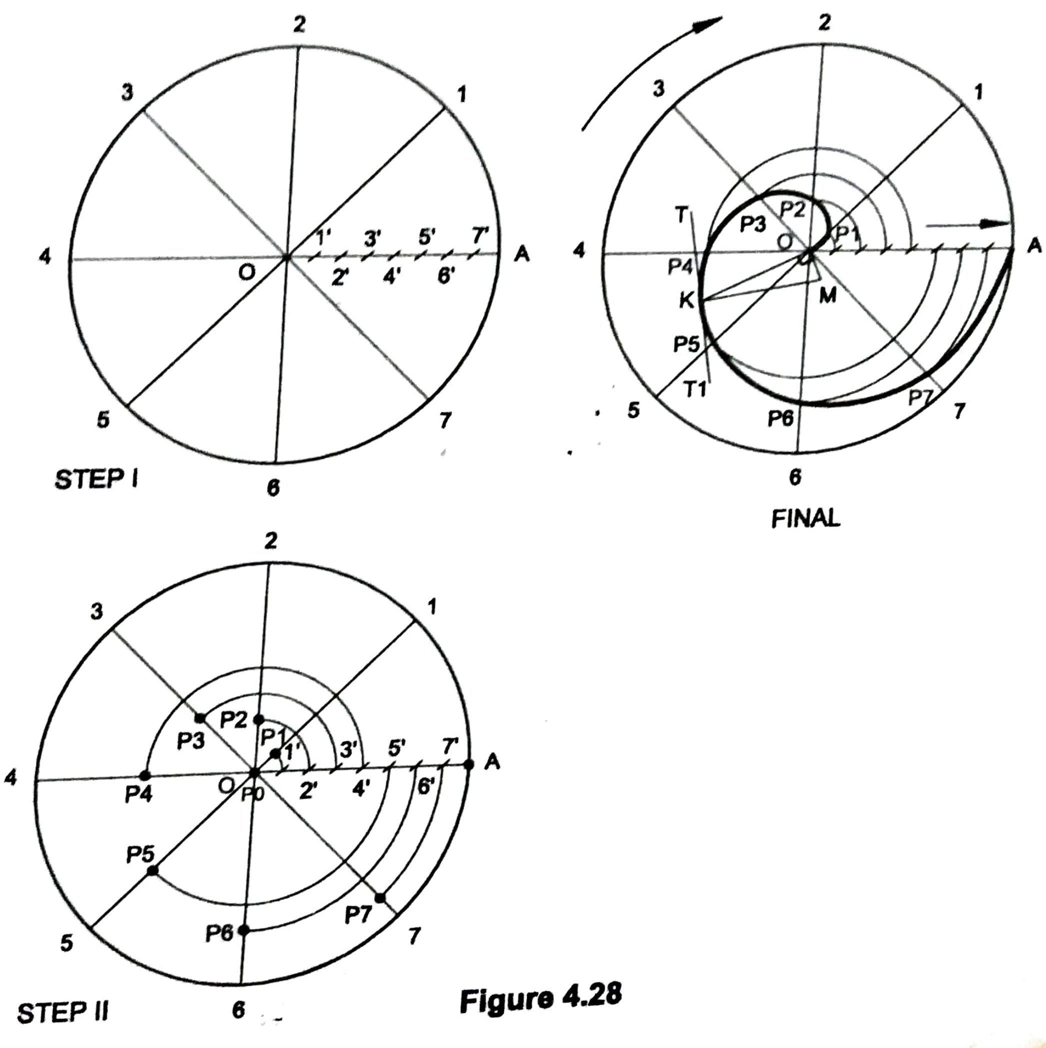

Figure 4.28

... centre O to represent disc. Divide disc circ...

divide the height of the cylinder (equal to pitch) into same number of equal parts and name the parts.

Let P0 be the starting position of point P on the helix. As it moves around the circumference of the cylinder in the anticlockwise direction through an angle as shown by 0 to 1 in the plan (i.e. 1/8 of rotation), it gets an axial advancement from 0' to 1' in the elevation (i.e. 1/8 of pitch) simultaneously. This position of point P is represented by the point P1. This is the point intersection of the vertical line through 1 (in plan) and horizontal line through point 1' in the elevation. Repeat this process to obtain other points as P2-P3-P4.

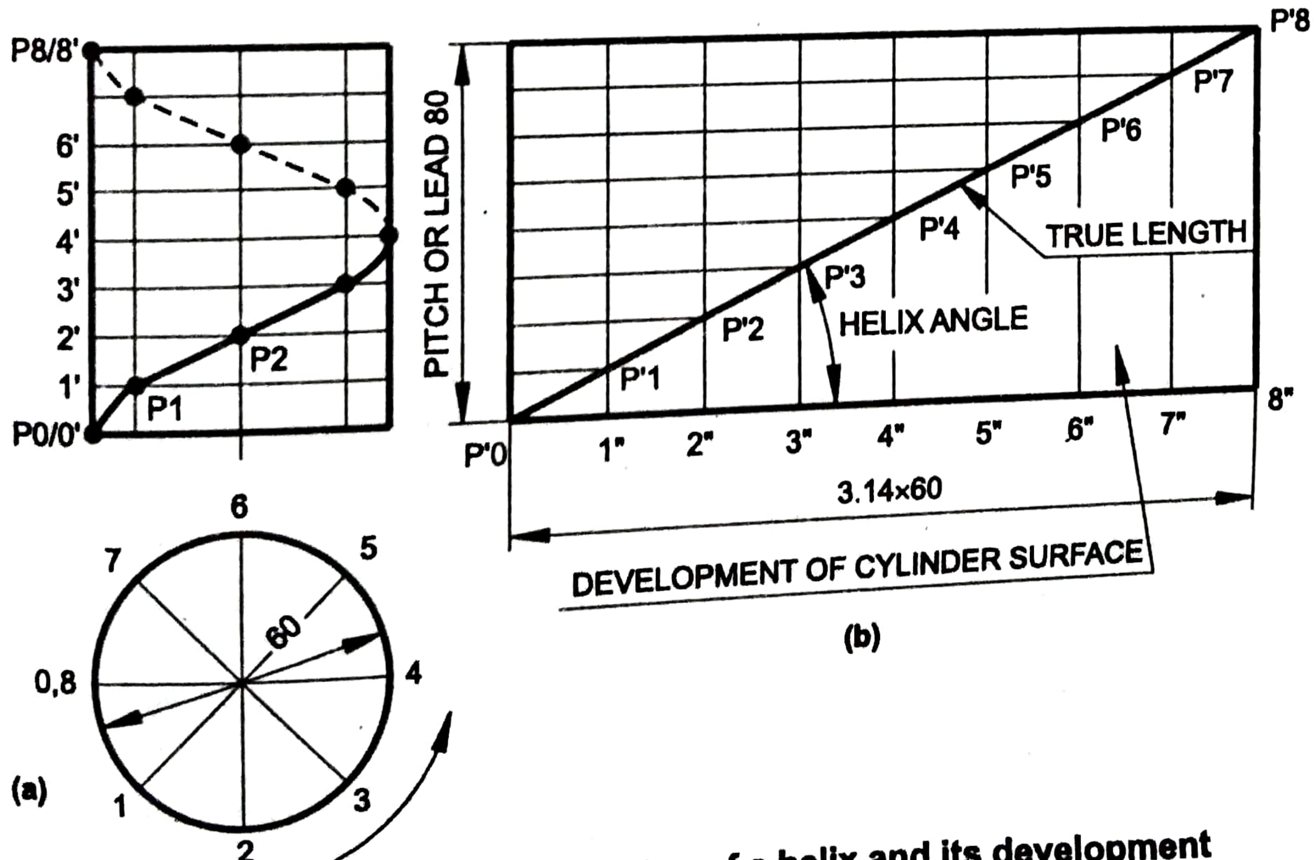

Figure 4.31 (a & b): Construction of a helix and its development

The portion of the curve from P4 to P8 is on the rear side of the cylinder (i.e. invisible) hence it is shown by dotted line. Join points P0-P1-P2-P3.... etc. to obtain the required helical curve for one convolution.

Fig.2.14(b) shows the development of the helix. It is a straight line and is the hypotenuse of a right-angled triangle, having base equal to the circumference of the circle and the vertical side equal to the pitch of the helix. The angle, which it makes with the base, is called a helix-angle.