Krishnaraj PT
109054 I3

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CLASS TEST - 1

Q.1.

$$y = ae^{2x} + be^{3x} \quad -(1)$$

$$\frac{dy}{dx} = y' = 2ae^{2x} + 3be^{3x} \quad -(2)$$

Now, eqn (2) $-(2 \times$ eq. (1))

$$y' - 2y = 2ae^{2x} + 3be^{3x} - (2ae^{2x} + 2be^{3x})$$

$$= be^{3x}$$

$$y' - 2y = be^{3x} \quad -(3)$$

Diff wrt x again,

$$y'' - 2y' = 3be^{3x} \quad -(4)$$

eqn. (4) $-$ (3)$\cdot$(3)

$$= y'' - 2y' - 3y' + 6y = 0$$

$$\boxed{y'' - 5y + 6y = 0}$$

Q.2 (1)

$$\sqrt[3]{d^2y/dx^2} = \sqrt{dy/dx}$$

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^{3/2}$$

$$\therefore \left(\frac{d^2y}{dx^2}\right)^2 = \left(\frac{dy}{dx}\right)^3$$

Order = 2

degree = 2

Rainbow

(2) $\left[(1-x^2)\left(\frac{d^2y}{dx^2}\right)\right]^1 + x \cdot \frac{dy}{dx} = x^2 - y$

order $= 2$

degree $= 1$

Q.3 $\frac{dy}{dx} = 1 - x \cdot \tan(x-y)$

let $x - y = u.$

$\therefore \frac{dy}{dx} = 1 - \frac{du}{dx}$

$\therefore 1 - \frac{du}{dx} = 1 - x \cdot \tan u$

$\frac{du}{dx} = x \cdot \tan u$

$\cot u \,(du) = x \cdot dx.$

On Integrating we get,

$\int \cot u \cdot du = \int x \cdot dx + c$

$\log|\sin u| = \frac{x^2}{2} + c$

$\log|\sin(x-y)| = \frac{x^2}{2} + c.$

Q.4. $(xy - x^2) \cdot \frac{dy}{dx} = y^2$

$\frac{dy}{dx} = \frac{y^2}{xy - x^2}$; (homogeneous equation.)

Put $y = vx$

$$\frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{v^2x^2}{x^2v^2 - x^2}$$

$$x\frac{dv}{dx} = \frac{x^2}{x^2}\left[\frac{v^2}{v^2-1}\right] - v$$

$$x \cdot \frac{dv}{dx} = \frac{v^2}{(v-1)} - \left[\frac{v(v-1)}{(v-1)}\right]$$

$$= \frac{v^2 - v^2 + v}{(v-1)}$$

$$x \cdot \frac{dv}{dx} = \frac{v}{v-1}$$

$$\frac{v-1}{v} \cdot dv = \frac{dx}{x}.$$

on Integrating,

$$\int \frac{v-1}{v} dv = \int \frac{dx}{x}$$

$$v - \log v = \log x + c$$

$$\frac{y}{x} = \log(x \cdot y/x) + c$$

$$\boxed{\frac{y}{x} = \log y + c}$$

Q.5. $y \cdot dx = (\sin y - x)\, dy$

$y\,dx + (x - \sin y)\, dy = 0$

$\frac{\partial M}{\partial y} = 1 \quad ; \quad M = y$

$\frac{\partial N}{\partial x} = \cancel{x \sin y} \; \cancel{1} \quad 1 - 0 = 1$

$N = (x - \sin y)$

so as $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

eq. given equation is exact.

so its solution is given by,

$$\underset{(y\,const)}{\int^{x}} M \cdot dx + \underset{\left(\text{terms free of } x\right)}{\int} N\, dy = c$$

$$\int y \cdot dx + \int -\sin y\, dy = c$$

$$\boxed{yx + \cos y = c}$$