Chapter 2

KINETICS OF PARTICLES

Kinematics

The relationship between displacement, velocity, acceleration & time are called as Kinematics. It is study of geometry of motion we do not consider the mass of the body & the forces action on the body while studying kinematics

Kinetics

The study of relationship between force, mass & acceleration is called as kinematics but for solving the problems of kinetics, knowledge of kinematics is the pre-requisite. Thus kinetics is the analysis of motion of the body in which displacement, velocity, acceleration, time as well as the force & mass of the body is involved kinetics is based on Newton's 2nd Law of motion.

Newton's 2nd Law of motion is expressed in 3 different ways in kinetics:-

1) D' Alemberts principle
2) Impulse- Momentum Principle
3) Work- Energy Principle

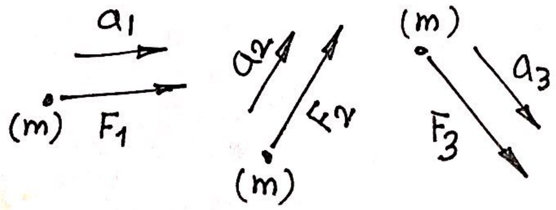
Newton's 2nd Law of Motion:-

If the resultant force acting of the particle is not zero the particle will have an acceleration proportional to the magnitude of resultant force and in the direction of the resultant force. The constant $\frac{f}{a}$ is a characteristic of a particle under consideration and it is denoted by 'm' i.e. mass of the particle 'm' is always a positive scalar quantity.

In this case, $\frac{F_1}{a_1} = \frac{F_2}{a_2} = \frac{F_3}{a_3} = \text{Constant}$

Thus, we get that $\frac{F_1}{a_1} = \frac{F_2}{a_2} = \frac{F_3}{a_3} = m$

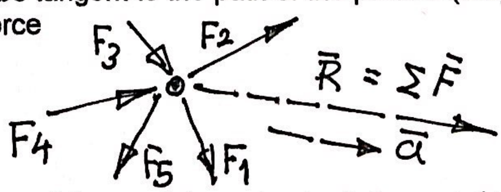
This gives us, $\boxed{\bar{F} = m.\bar{a}}$



The above mathematical relation proves that the force 'F' and acceleration 'a' are always directed along the same line. Thus the magnitudes of the resultant force 'F' and the corresponding acceleration a are proportional to each other but they will not be tangent to the path of the particle (they can be in any direction). For a particle subjected to several force

$$\sum \bar{F} = m\bar{a}$$

$$\boxed{\sum \bar{F} = (m \cdot \bar{a})}$$



Here, 'a' should always the absolute acceleration of the particle and not relative acceleration.

Newtonian frame of Reference:-

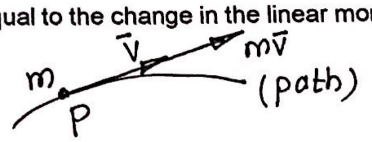
(Inertial frame) For the validity of Newton's 2nd Law, the co-ordinate axes must have constant orientation w.r.t. even the stars and their origin must be attached to the sun or the mass centre of the solar system. This frame of reference is assumed to be absolute stationary. Hence it is called as **Newtonian frame of Reference.** While defining this frame of reference, it was assumed that the solar system is stationary in the universe but later on it has been proved that the solar system is not stationary and because of this even Newtonian frame or reference is not stationary; such a moving frame of reference is called as **relative frame of reference**. But for the analysis of domestic Engg. Problems, we always consider the frame of reference attached to our earth (assuming that, it is stationary).

Newton's 1st Law from Newton's 2nd Law:-

1) If a particle is at rest w.r.t. N. frame ref^n then $\bar{F}=0$ and $\bar{a}=0$ and the particle continues to be in state of rest.
2) If initially, the particle is moving with a constant velocity 'v_o' then also $\bar{F}=0$ and due to that a = 0 and the partied will continue to be in state of uniform rectilinear motion (this is possible only in vacuum)

   Thus, If $\Sigma\ \bar{F}=0$ then the particle will not change its original state, what ever it may be.

   Thus, the resultant force acting on a particle is equal to the change in the linear momentum of the particle.



If $\sum \bar{F}=0,\ L=0,\ L=\text{constant}$

This is called as **principle of conservation of linear momentum** and if the resultant force acting on the particle is zero, the linear momentum remains constant both in magnitude as well as in direction of the motion. This is also called as Newton's 1st law of motion.

In Rectangular Components:-

$$\sum\left(f_x\hat{i}+f_y\hat{j}+f_z\hat{k}\right)=m(a_x\hat{i}+a_y\hat{j}+a_z\hat{k})$$

$$\therefore F_x=m\cdot a_x = m\cdot\ddot{x}$$

$$F_y=m\cdot a_y = m\cdot\ddot{y}$$

$$F_z=m\cdot a_z = m\cdot\ddot{z}$$

In Path Variables

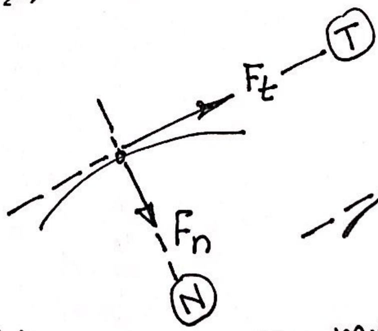
$$F_t=m.a_t = m\cdot dV/dt$$

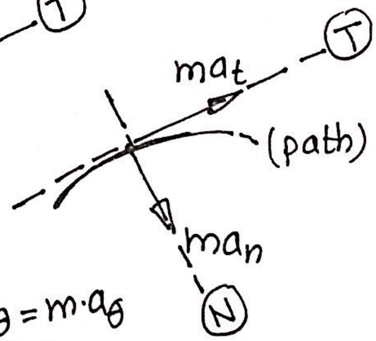
$$F_n=m.a_n = m\cdot V^2/\rho$$

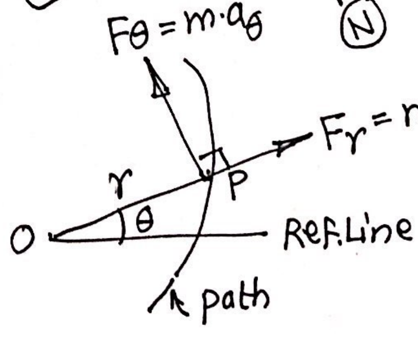
In Polar Co – ordinates:

$$F_r=m.a_r = m\cdot(\ddot{r}-r\dot{\theta}^2)$$

$$F_\theta=m.a_\theta = m\cdot(r\ddot{\theta}+2\dot{r}\dot{\theta})$$







The Concept Of Dynamic Equilibrium

(D'Alembert's Principle)

By Newton's Second Law,

$\bar{F}=m\cdot\bar{a}$

D'Alembert had modified this equation to

$\bar{F}-m\cdot\bar{a}=0$ i.e. $\boxed{\bar{F}+(-m\cdot\bar{a})=0}$

The force $\left(-m\bar{a}\right)$ is called as inertial vector or D Alembert's force.

Thus by the addition of $\left(-m\bar{a}\right)$ force, we are converting the problem of Dynamics into the problem of statics. The force system on LHS of above equation is a balanced system to which we can apply equations of static equilibrium i.e. $\sum Fx=0, \sum Fy=0\ \&\ \sum M=0$ the force.

The force $(-m\bar{a})$ is called as inertial force and it can be experienced in every case of accelerated or decelerated motion. Thus, by the addition of $(-m\bar{a})$ force, we are superimposing the conditions of equilibrium on a body in motion. Because of this the above concept is called as **principle of Dynamic Equilibrium**.

Impulse Momentum Principle:-

By Newtons 2nd Low motion, we have

$$f = m\bar{a} = m\frac{d\bar{v}}{dt}$$

$$\bar{f}dt = md\bar{v}$$

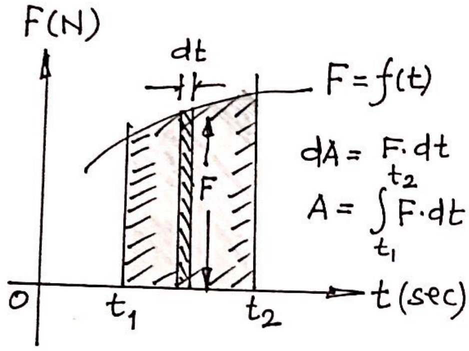
$$\int_{t_1}^{t_2} \bar{f}dt = \int_{u}^{v} mdv$$

$$F(t_2 - t_1) = m(v - u)$$

$$\text{let } t_2 - t_1 = t$$

$$\text{Hence, } Ft = mv - mu$$

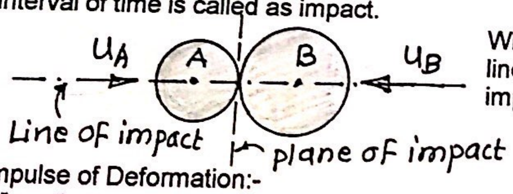
$$\boxed{(F \cdot t) = m \cdot \bar{v} - m \cdot \bar{u}}$$



'F.t' is called as impulse acting on particle for time 't'. Thus, the impulse action on the particle for time 't' is equal to the change in the linear momentum in that time. This is called as **Impulse Momentum Principle**. Here the force acting on the particle is constant for time 't'. If the force acting on the particle is changing and force, $F = f(t)$, then the area under (F – t) diagram is the impulse acting on the particle for time 't'.

Direct Central Impact:-

Collision of two bodies in which each body exerts tremendous pressure on the other for a very short interval of time is called as impact.



When the mass centers of the colliding bodies are lying on the line of impact and their velocities are collinear to the line of impact then it is called are **direct central impact**.

Impulse of Deformation:-

When the two colliding bodies touch each other initially they have a tendency to push the other body. This stage is called as **deformation stage**, the impulse acting on the body is during deformation is called as **impulse of deformation**.

Impulse of Recovery:-

After the deformation stage, the bodies develop the tendency of separating away from each other this is called as **Recovery**. Recovery may be 100% or 0% or partial. The impulse acting on the body during recovery is called as **Impulse of recovery**.

$$\begin{pmatrix}\Delta t \\ \text{time of impact}\end{pmatrix} = \begin{pmatrix}\Delta t_d \\ \text{time of deformation}\end{pmatrix} + \begin{pmatrix}\Delta t_r \\ \text{time of recovery}\end{pmatrix}$$

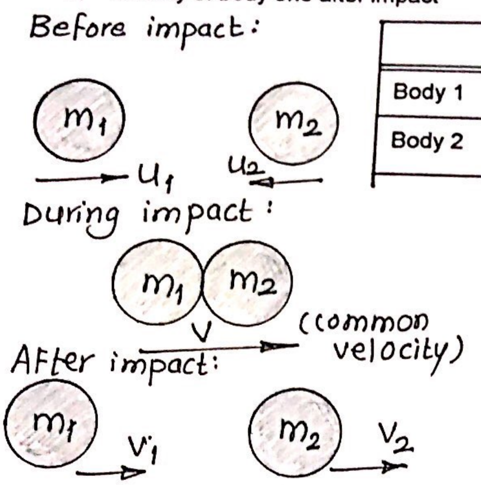
Coefficient of restitution:

The ratio of impulse of recovery to the impulse of deformation is called as **coefficient of restitution** of the two colliding bodies.

Consider two colliding bodies as under ;

Let m_1 = mass of body one,
u_1 = velocity of body one before impact
V_1 = velocity of body one after impact

m_2 = mass of body one,
u_2 = velocity of body one before impact
V_2 = velocity of body one after impact



	Impulse of Deformation	Impulse of recovery
Body 1	$m_1v - m_1u1$	$m1v1 - m1v$
Body 2	$m_2v - m_2u_2$	$m_2v_2 - m_2v$

$$e = \frac{m_1v_1 - m_1v}{m_1v - m_1u_1}$$

$$= \frac{v_1 - v}{v - u_1}$$

$$= \frac{m_2v_2 - m_2v}{m_2v - m_2u_2}$$

$$= \frac{v_2 - v}{v - u_2}$$

i.e. mass will not have any effect on the coefficient of restitution,

$$e = \frac{v_1 - v}{v - u_1} = \frac{v_2 - v}{v - u_2} = \frac{v_1 - v - v_2 + v}{v - u_1 - v + u_2}$$

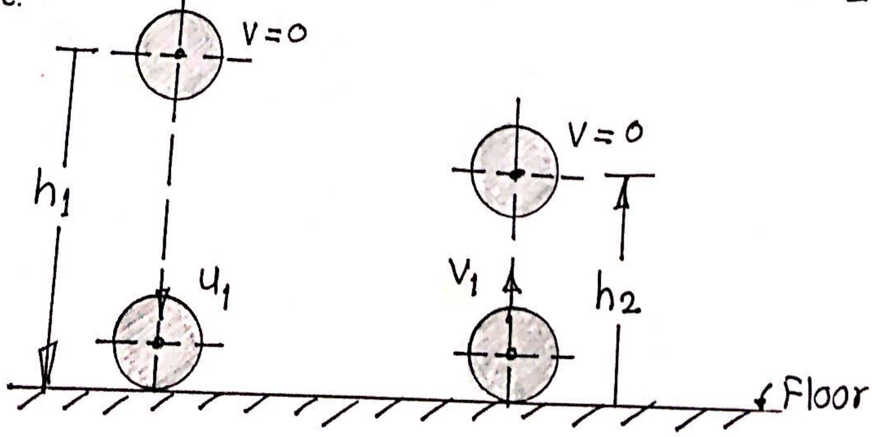
$$e = \frac{v_1 - v_2}{u_2 - u_1} = -\left[\frac{v_1 - v_2}{u_1 - u_2}\right] = -\left[\frac{v_{1/2}}{u_{1/2}}\right] \qquad \boxed{e = \left[\frac{v_1 - v_2}{u_2 - u_1}\right]}$$

$$e = -\left[\frac{\text{relative velocity of (i) w.r.t. (ii) after impact}}{\text{relative velocity of (i) w.r.t. (ii) before impact}}\right]$$

$$e = -\left[\frac{\text{velocity of seperalion}}{\text{velocity of approach}}\right]$$

Collision with a body of ∞ mass

Consider a ball of mass m, released from height h1 on the floor. It rebounces to height h2 after impact as shown in figure.



u_1 = striking velocity $\quad v_1$ = Rebounding velocity

$u_1 = \sqrt{2gh_1}(\downarrow) \quad v_1 = \sqrt{2gh_2}(\uparrow)$

Ball = Body (1) ....(Finite mass)...........(u1 , v1)

Floor = Body (2)..........(Infinite mass) ....(u2 = v2 = 0)

$$\therefore e = \frac{v_1 - v_2}{u_2 - u_1} = \frac{v_1}{-u_1} = \frac{\sqrt{2gh_2}}{+\sqrt{2gh_1}} \quad \boxed{e = \sqrt{\frac{h_2}{h_1}}}$$

Based on coefficient of restitution the phenomenon of impact is classified in to three types:-

1) **Elastic Impact:-**

i) The two bodies separate after the impact.

ii) coefficient of restitution is one. $\boxed{e = 1}$

iii) Linear momentum is conserved ($m_1 u_1 + m_2 m u_2 = m_1 v_1 + m_2 v_2$)

iv) Kinetic energy is conserved.

(K.E. of the system before impact) = (K.E. of the system after impact)

$$T1 = T2$$

$$\left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) = \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2r_2^2\right)$$

(K.E. of the system before impact) = (K.E. of the system after impact)

v) Recovery is 100% and the two bodies regain their original shape and size.

2) **Semi – Elastic Impact:-**

i) The two bodies separate after the impact

ii) Coefficient of restitution is between zero and one. $\boxed{0 < e < 1.}$

iii) Linear momentum is conserved ($m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$)

iv) Kinetic Energy of system is not conserved

(K.E. of the system before impact) > (K.E. of the system after impact)

$$T_1 > T_2$$

$$\left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) > \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)$$

Energy lost in impact = $(T_1 - T_2)$

$$\% \text{ loss in energy} = \left(\frac{T_1 - T_2}{T_1}\right) \times 100$$

v) The Recovery is partial and there is same permanent damage of the bodies

3) Plastic Impact:-

i) The 2 bodies do not separate after the impact but they more with a common velocity 'v

ii) Coefficient of restitution is zero $\boxed{e = 0}$

iii) Linear momentum is conserved ($m_1u_1 + m_2u_2) = (m_1 + m_2)v$

iv) There is a great loss of Kinetic Energy. Hence Kinetic Energy , KE is not conserved

(KE of the system before impact) > (KE of the system after impact)

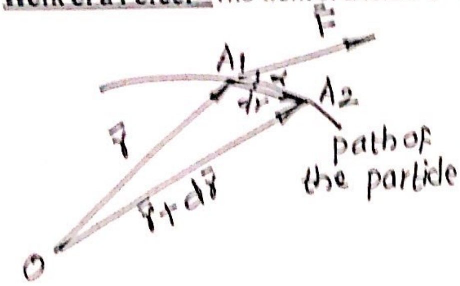
$$\left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) > \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)$$

$$\text{Energy lost in impact} = (T_1 - T_2)$$

$$\text{\% loss in energy} = \left(\frac{T_1 - T_2}{T_1}\right) \times 100$$

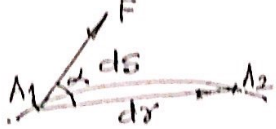
v) The Recovery is not possible there is permanent damage of the two bodies.

<u>Work of a Force:-</u> The work of a force '$\vec{F}$' corresponding to a displacement '$d\vec{r}$' is defines as the quantity



$du = \vec{F}.d\vec{r}$

$du = (F\cos\alpha \times ds)$



where,

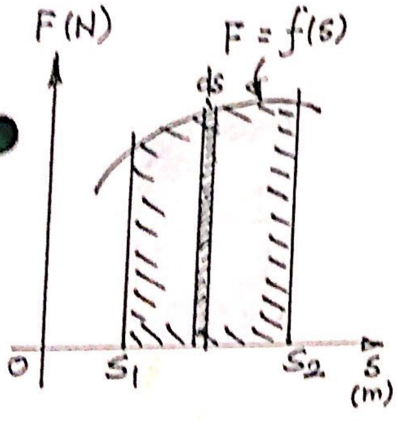
F = magnitude of the Force

ds = magnitude of the Displacement

$\vec{F} = (Fx)\hat{i} + (Fy)\hat{j} + (Fz)\hat{k}$

and $d\vec{r} = (dx)\hat{i} + (dy)\hat{j} + (dz)\hat{k}$

then , $du = (fxdx) + (fydy) + (fzdz)$

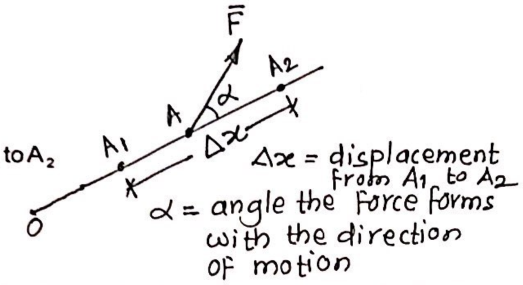


1 Nm = 1 joule work, in the above figure, if the angle 'α' is acute then work 'du' is positive. If angle 'α' is obtuse then work 'du' is negative and if angle $\alpha = 90°$ then work done is zero. If the force and displacement are having same direction, then work done is positive. If the force and displacement are having opposite direction then work done is negative. If the force and the displacement are at right angles to each other, work done is zero. In the above discussion it is assumed that the force 'f' remains constant when the particle travels from A1 to A2.If the force 'F' is changing and F = f (s), then consider the area under force-displacement diagram.

$$A = [\text{ total area under (f-s) curve from } s_1 \text{ to } s_2] = \int_{s_1}^{s_2} F ds = U$$

U = work done in displacing the body from position A1 to A2

Work of A Constant Force:-



A) Work of a constant force in rectilinear motion.

u_{1-2} = work done in displacing the partide form A_1 to A_2

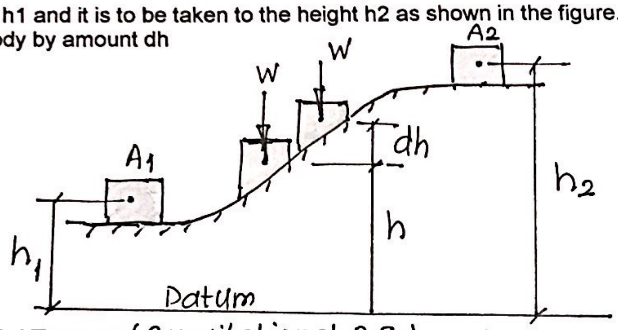
$u_{1-2} = (F\cos\alpha)(\Delta x)$

$= (F.\Delta x)$

B) Work of the force of gravity:-

Consider a block of mass m at a height h1 and it is to be taken to the height h2 as shown in the figure.

dU = very small w.d in displacing the body by amount dh



$$dU = -w.dh$$

$$u_{1-2} = \int_{A_1}^{A_2} du = \int_{h_1}^{h_2} -w.dh$$

$$= w\int_{h_2}^{h_1} dh = wh_1 - wh_2$$

$$u_{1-2} = (mgh_1) - (mgh_2)$$

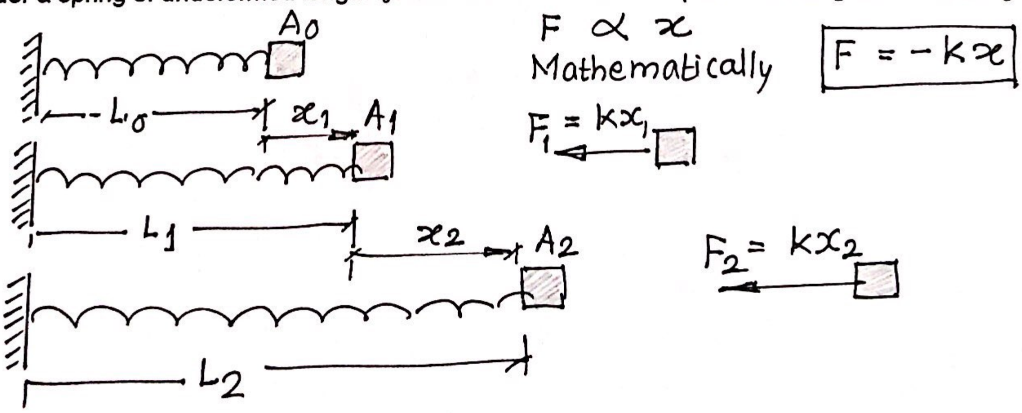
let $v_G = mgh$ = ~~Grairtationl~~ Potential Energy. (Gravitational P.E.)

$$u_{1-2} = (v_G)_1 - (v_G)_2$$

When the block is moving up, work done by force 'W' is negative. When the block is moving down, work done by force 'W' is positive. The work of the force 'W' is independent of the path followed. It depends only upon the initial and final positions. Hence the term mgh is called as **potential energy** of the body w.r.t. the force of gravity. Theoretically, GPE represents the amount of energy required to move the particle of mass 'm' from the centre of the earth and take it away form it at a distance of 'h' form the centre of the earth. But in practice, we consider a convenient datum line and we measure the height 'h' w.r.t. that datum line. If GPE increases then u_{1-2} is negative. If GPE decreases then u_{1-2} is positive. If the body is above the datum level, GPE is positive. If the body is below the datum level, GPE is negative. If the body is at the datum level GPE is zero.

C) <u>Work Done by The resisting Force of spring:-</u>

Consider a spring of undeformed length l_0. it is to be stretched from position l1 to l_2 as shown in figure.



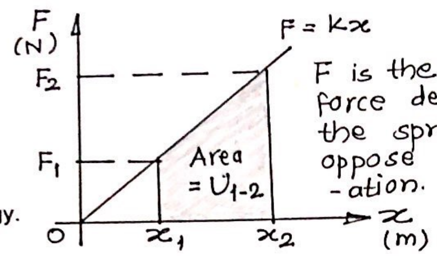
$$u_{1-2} = \left(\frac{1}{2}x_2 f_2 - \frac{1}{2}x_1 F_1\right)$$

$$= \left(\frac{1}{2}x_1^2 k - \frac{1}{2}x_2^2 k\right)$$

$$u_{1-2} = \left(\frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2\right)$$

$$V_E = \frac{1}{2}kx^2 = \text{Elastic Potential Energy.}$$

$$u_{1-2} = (VE)_1 - (VE)_2$$



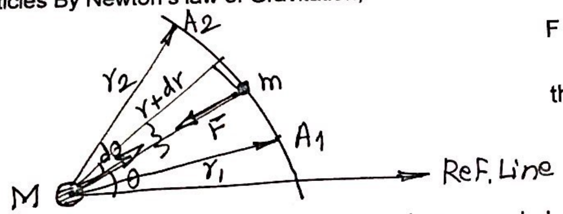
F is the resisting force developed by the spring to oppose the deformation.

When any spring is subjected to deformation 'x' due to external force, then the force exerted by the spring to resist the deformation is proportional to 'x'. When the spring is subjected to deformation, the force of the spring is doing negative work and when the spring is returning back to its original position, the work done by the force 'f' is positive.

D) Work of A Gravitational Force:-

Two particular of masses 'M' and 'm' separated by a distance 'r' exerts a force of attraction on each other, which is represented by 'F' i.e. equal and opposite forces directed along the line joining the particles By Newton's law of Gravitation,

$$F = \frac{GMm}{r^2}$$



$$\text{then } du = -Fdr = -\left(\frac{GMm}{r^2} \times dr\right)$$

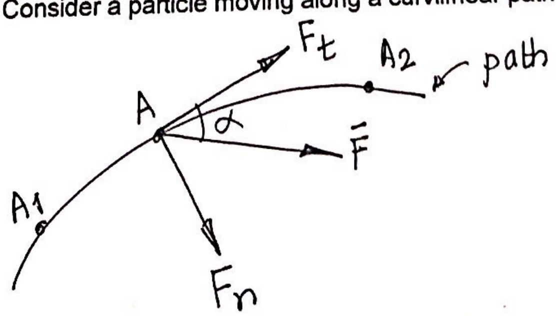
As force F is directed towards 'O' and 'dr' away form o. work done is negative.

$$u_{1-2} = \int_1^2 du = -\int_{r_1}^{r_2} \frac{GM}{r^2} dr$$

$$u_{1-2} = \left[\frac{GMm}{r_1} - \frac{GMm}{r_2}\right]$$

Work Energy principle:-

Consider a particle moving along a curvilinear path and directed by force 'F'



$$f_t = m.a_t$$

$$f_t = m.\frac{dv}{dt}$$

$$f_t = m\frac{dv}{ds}\frac{ds}{dt}$$

$$f_t = m.v.\frac{dv}{ds}$$

$$\int_{s_1}^{s_2} f_t.ds = \int_u^v mvdv$$

$$\boxed{u_{1-2} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2}$$

Work Done = change in Kinetic Energy

Let,

$$T = \frac{1}{2}mv^2 = KE$$

$$\boxed{u_{1-2} = T_2 - T_1}$$

$$T_2 = T_1 + u_{1-2}$$

KE of particle represents the capacity to do work.

Power:- Rate of doing work is called as power

$$P = \frac{du}{dt}$$

$$P = \frac{f.dr}{dt}$$

$$P = F\left(\frac{dr}{dt}\right)$$

$$\boxed{P = F.V}$$

Unit:- 1J/S = 1 watt.

746 w = 1Horse Power

1 watt = 1Nm/s

1 Kilowatt = 1000 Watts

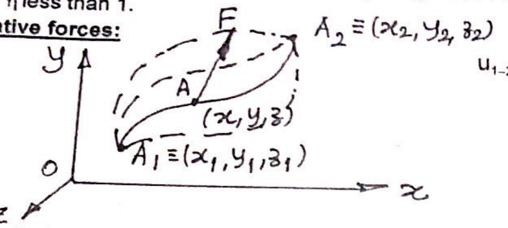
Efficiency:-

$$\eta = \frac{\text{output work}}{\text{input work}}$$

$$\eta = \frac{\text{Power output}}{\text{Power input}}$$

(This def^n is valid only if the rate of doing work is contant)
In practice work done by force of friction is always negative. Hence, work out put is always less than work input and η less than 1.

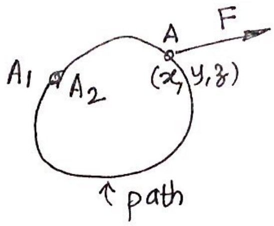
Conservative forces:



$$u_{1-2} = v(x_1, y_1, z_1) - v(x_2, y_2, z_2)$$
$$= v_1 - v_2$$
$$= \text{change in Potential Energy.}$$

V(x,y,z) is called as Potential Function. When work of a force is independent of the path followed by the point of application of that force, then that force is called as conservative force. If the system is having all conservative forces, then it is called as conservative force system. When A1 is coincides with A2 then, $\oint F.dr = 0$. The 0 on the $\int$ sign indicates that the path is closed.

Thus the elementary work of a conservative force is an exact differential.



Thus,

$$dU = Fxdx + Fydy + fzdz$$
$$= -dv(x, y, z)$$
$$= -\left(\frac{dv}{dx}dx + \frac{dv}{dy}dy + \frac{dv}{dz}dz\right)$$
$$fx = -\frac{dv}{dx}, \quad Fy = -\frac{dv}{dy}, \quad Fz = \frac{dv}{dz}$$

But force $F = Fxi + Fyj + Fzk$

$$\text{then} \quad F = -\left(\frac{dv}{dx}\right)\hat{i} - \left(\frac{dv}{dy}\right)\hat{j} - \left(\frac{dv}{dz}\right)\hat{k}$$

$$\boxed{F = -\text{grad } v}$$

As the work done by the force of friction depends upon the path , frictional force is not a conservative force. Hence, in a conservative force system, frictional force is absent. Because of that it is also called as an ideal system. The Law of conservation of mechanical. Energy is application to only conservative force systems.

Conservation of energy:- for a conservative force system.

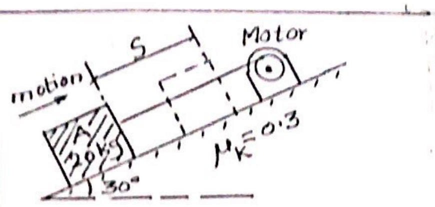
$$T_2 - T_1 = V_1 - V_2$$
$$T_1 + V_1 = T_2 + V_2$$
$$(KE + PE)_1 = (K.E. + P.E.)_2$$
$$(T.M.E.)_1 = (T.M.E.)_2$$
$$T.M.E. = \text{Total Mechanical Energy} = K.E. + P.E.$$

When a particle moves under the action of conservative forces, the sum of the K.E. and P.E. of the particle remains constant at all positions thus T.M.E. = Constant.
This is called as law of conservation of mechanical energy.

Lecture No: **Rectilinear Kinetics (Newton's Second Law of Motion)**

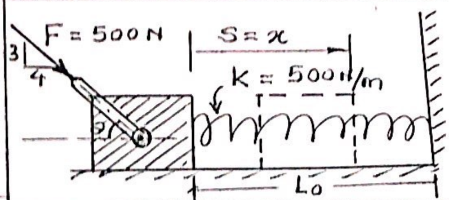
13

1. The motor winds in the cable with a constant acceleration, such that the 20-kg. Crate moves a distance s= 6m in 3 sec., starting from rest. Determine the tension developed in the cable. The coefficient of kinetic friction between the crate and the plane is $\mu_k = 0.3$

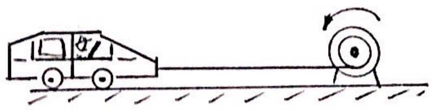


2. If motor M exerts a force of $F = (10t^2 + 100)$ N on the cable, where t is in seconds, determine the velocity of the 25- kg. crate when t = 4 s. The coefficients of static and kinetic friction between the crate and the plane are $\mu_s = 0.3$ and $\mu_k = 0.25$ respectively. The crate is initially at rest.

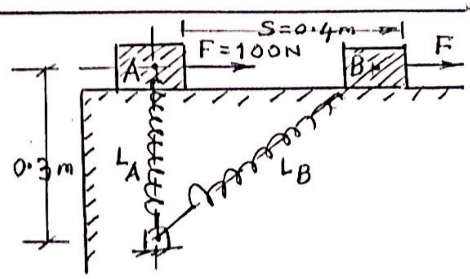
3. A spring of stiffness k = 500 N/m is mounted against the 10 kg block. If the block is subjected to the force of F = 500 N, determine its velocity at s = 0.5m. When s=0, the block is at rest and the spring is uncompressed. The contact surface is smooth.



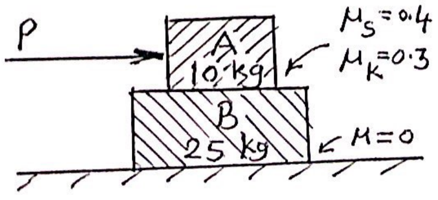
4. The 2 Mg. car is being towed by a winch. If the winch exerts a force of T= 100 (s + 1) N on the cable where s is the displacement of the car in meters, determine the speed of the car when s= 10 m, starting from rest. Neglect rolling resistance of the car.



5. The spring has a stiffness k = 200 N / m and is unstretched when the 25 kg block is at A. Determine the acceleration of the block when s = 0.4 m. The contact surface between the block and the plane is smooth.

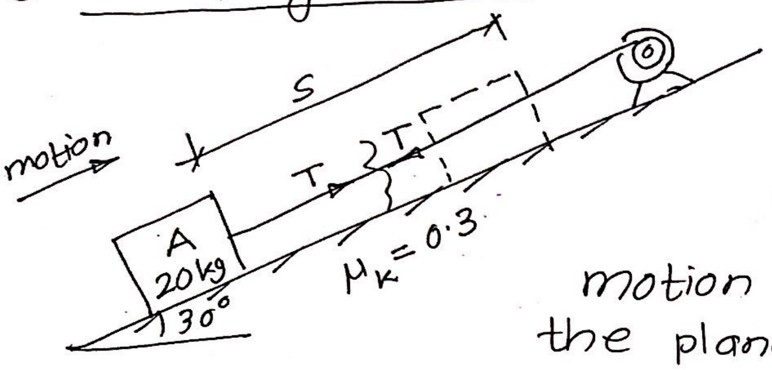


6. Block B rests upon a smooth surface. If the coefficients of static and kinetic friction between A and B are $\mu_s = 0.4$ and $\mu_k = 0.3$, respectively, determine the acceleration of each block if P = 30N.



Lecture No. 9 D'Alembert's Principle / Newton's 2nd law (Rectilinear motion)

(1) F 13.1 / pg. 742 / RCH



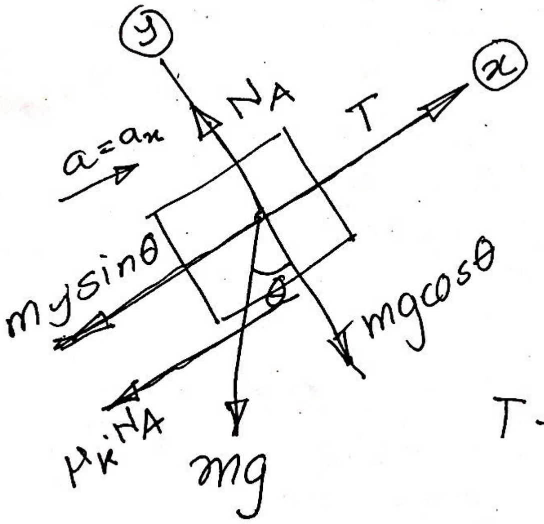
$m = 20 kg$
$u = 0$
a = constant
$s = 6 m$ at $t = 3 s$
$\theta = 30°$

motion of the crate along the plane is uni. acc. motion.

$$s = ut + \frac{1}{2}at^2$$

$$6 = 0 + \frac{1}{2} \times a \times (3)^2$$

$$a = 1.333 \ m/s^2$$



Along 'y' axis, $\sum F_y = m \cdot a_y$

But, $a_y = 0 \quad \therefore \sum F_y = 0$

$\therefore N_A - (20 \times 9.81 \times \cos 30°) = 0$

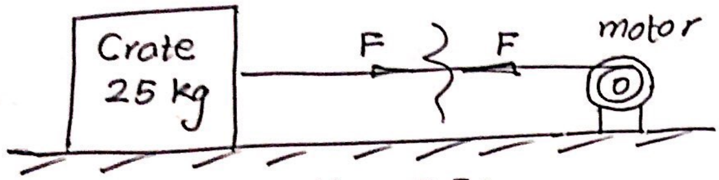
$N_A = 169.914 \ N$

Along 'x' axis, $\sum F_x = m \cdot a_x$

$T - (20 \times 9.81 \times \sin 30°) - (0.3 \times 169.914) = (20 \times 1.333)$

$\therefore \boxed{T = 176 \ N}$

② F13.2/Pg.742/RCH :



$\mu_s = 0.30$

$\mu_k = 0.25$

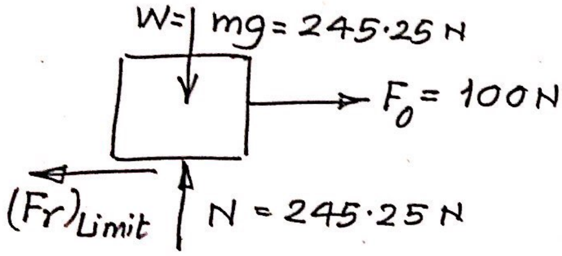
At $t = 0$, $v = 0$

$F = (10 \cdot t^2 + 100)$ N

At $t = 0$, $F_0 = 100$ N

$(Fr)_{max} = (Fr)_{Limit} = \mu_s \cdot N$

$= (0.3 \times 25 \times 9.81) = 73.575$ N



As, $F_0 > (Fr)_{Limit}$ at $t = 0$, the crate will start moving immediately after 'F_0' is applied.

During motion of the crate,

$\Sigma F_y = m \cdot a_y = 0$

$N - 245.25 = 0 \quad \therefore N = 245.25$ N

$\Sigma F_x = m \cdot a_x$

$F - (Fr)_{kinetic} = 25 \cdot a_x$

$(10 \cdot t^2 + 100) - (0.25 \times 245.25) = 25 \cdot a_x$

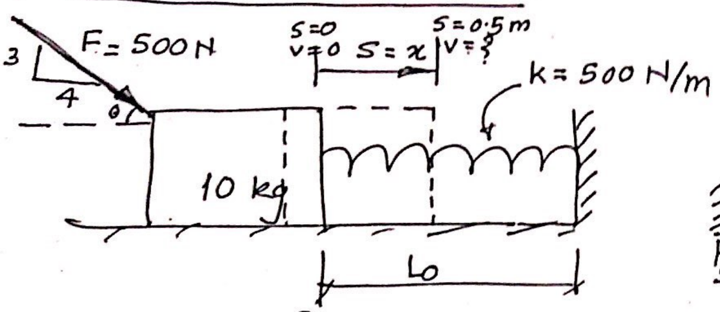
$\therefore \quad a_x = a = [(0.4)t^2 + (1.5475)]$ m/s²

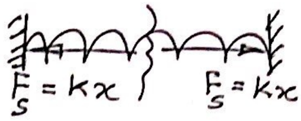
$dV = a \cdot dt$

$$\int_0^V dV = \int_0^4 [(0.4)t^2 + (1.5475)]\,dt$$

$$\therefore \boxed{v_4 = 14.72 \text{ m/s } (\rightarrow)}$$

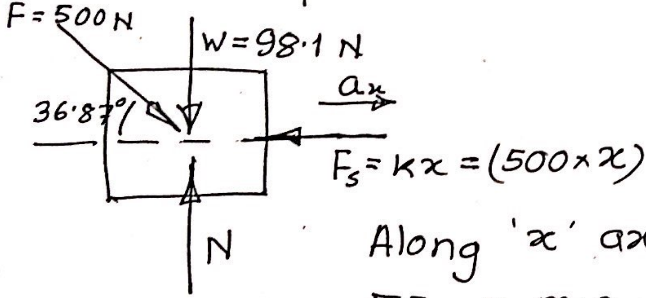
(3) F 13.3/ pg. 742/ RCH :





$\tan\theta = \frac{3}{4}$ $\quad \therefore\ \theta = 36.87^\circ$

x = deformation of the spring in 'm'



Along 'x' axis,

$$\Sigma F_x = m \cdot a_x$$

$$(500)(\cos 36.87^\circ) - (500 \cdot x) = 10 \cdot a_x$$

$$\therefore\ a_x = (40 - 50 \cdot x)\ m/s^2 = a \text{ (say)}$$

But, $\quad a \cdot dx = v \cdot dv$

$$\int_0^{0.5\,m} (40 - 50 \cdot x)\,dx = \int_0^{v} v \cdot dv$$

$$\left(40 \cdot x - 25 \cdot x^2\right)_0^{0.5} = \left[\frac{v^2}{2}\right]_0^{v}$$

$$\therefore\ \boxed{v = 5.24\ m/s\ (\rightarrow)}$$

④ F 13.4 / Pg. 742 / RCH

$m = 2000$ kg

$T = (100)(s+1)$ N

Neglect the rolling resistance

when $s = 0$, $v = 0$

$s = 10$ m, $V = ?$

$\Sigma F_x = m \cdot a_x$

$\therefore \quad T = m \cdot a_x$

$(100)(s+1) = (2000)\, a_x$

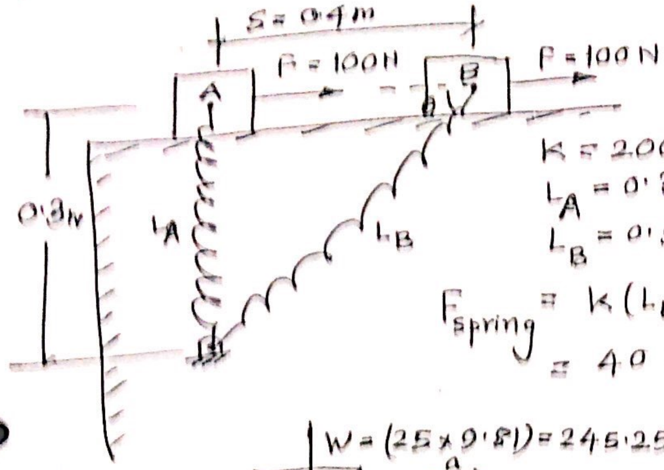
$\therefore \quad a_x = a = [(0.05)x + (0.05)]$ m/s²

Now, $\quad a \cdot dx = v \cdot dv$

$$\int_0^{10} [(0.05)x + (0.05)]\, dx = \int_0^{v} v \cdot dv$$

$\therefore \quad \boxed{v = 2.45 \text{ m/s} (\rightarrow)}$

(E) F 13.5/pg. 742/RCH :



$\tan\theta = \frac{0.3}{0.4}$

$\theta = 36.87^\circ$

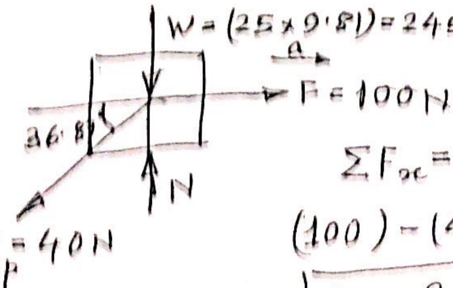
$K = 200\ N/m$

$L_A = 0.3\ m = L_0$

$L_B = 0.5\ m,\ (L_B - L_0) = x_B$

$F_{spring} = K(L_B - L_0) = (200)(0.5 - 0.3)$

$= 40\ N$

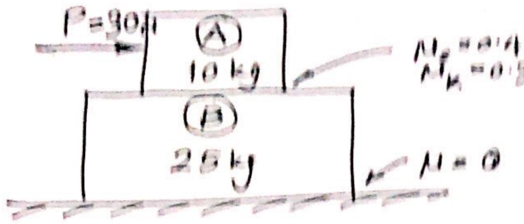


$\Sigma F_x = m \cdot a_x$

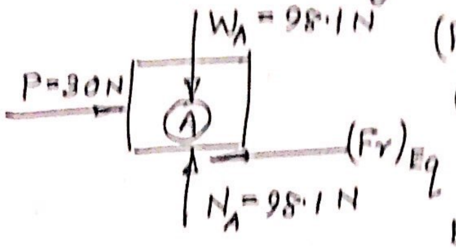
$(100) - (40)(\cos 36.87^\circ) = 25 \cdot a_x$

$\therefore\ a_x = \boxed{a = 2.72\ m/s^2\ (\rightarrow)}$

(6) F 13-6 / pg. 742 / RCH



I) Check, if slipping occurs betn A & B :



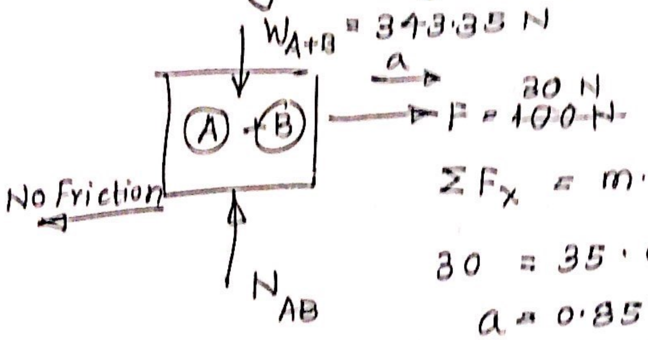
$(Fr)_{Eq} = 30\ N$

$(Fr)_{Limit} = (0.4 \times 98.1) = 39.24\ N$

As, $(Fr)_{Equi.} < (Fr)_{Limit}$

Block A is <u>not sliding</u> at the top of block B.

II) Consider blocks A and B, together as one rigid body :



$\Sigma F_x = m \cdot a$

$30 = 35 \cdot a$

$a = 0.857\ m/s^2$

$\therefore \boxed{a_A = a_B = 0.857\ m/s^2\ (\rightarrow)}$