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25/04/22.

Tutorial - 5

Q.1. Orthogonal trajectories of the family of curves.

1. $r^2 = a\sin 2\theta$
2. $e^x + e^{-y} = c$
3. $x^2 + 2y^2 = c^2$
4. $r = \frac{2a}{1+\cos\theta}$

① $r^2 = a\sin 2\theta$

$a = \frac{r^2}{\sin 2\theta}$

differentiate wrt θ,

$0 = \left[2r\cdot\left(\frac{dr}{d\theta}\right)\cdot(\sin 2\theta)\right] - (\cos 2\theta\cdot 2)\cdot r^2$

$\frac{dr}{d\theta} = \frac{2\cos 2\theta\cdot r^{\cancel{2}}}{\cancel{2r}\cdot\sin 2\theta} = \frac{r\cos 2\theta}{\sin 2\theta}$

Replacing $\frac{dr}{d\theta}$ by $\left(-r^2\frac{d\theta}{dr}\right)$

$-r^2\frac{d\theta}{dr} = \frac{\cancel{r}\cdot\cos 2\theta}{\sin 2\theta}$

$\frac{d\theta}{dr} = -\frac{\cos 2\theta}{\sin 2\theta\,(r)}$

$\int \frac{d\theta\cdot\sin 2\theta}{-\cos 2\theta} = \int\frac{dr}{r}$

$-\int \tan 2\theta\, d\theta = \log r + \log c.$

$-\frac{1}{2}\log|\cos 2\theta| = \log r\cdot c$

$\log(\cos 2\theta) = -2\log(rc) = \log(rc)^{-2}$

$\boxed{\cos 2\theta = \frac{1}{(rc)^2}}$

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(2) $e^{x} + e^{-y} = c$

$e^{x} + e^{-y} \cdot \dfrac{-dy}{dx} = 0$ [diff wrt x]

Replacing $\dfrac{dy}{dx}$ by $-\dfrac{dx}{dy}$,

$e^{x} + e^{-y} \cdot \dfrac{dx}{dy} = 0$

$\dfrac{dx}{dy} = \dfrac{-e^{x}}{e^{-y}}$

$\dfrac{dx}{-e^{x}} = \dfrac{dy}{e^{-y}}$

$-\int dx \cdot e^{-x} = \int dy \cdot e^{y}$

$-e^{-x} \cdot (-1) + c = e^{y}$

$e^{-x} + c = e^{y}$

$\boxed{e^{y} - e^{-x} = c}$

(3) $x^2 + 2y^2 = c^2$

Differentiating wrt x,

$2x + 4y \cdot \dfrac{dy}{dx} = 0$

Replacing $\dfrac{dy}{dx}$ with $\dfrac{-dx}{dy}$;

$2x - 4y \dfrac{dx}{dy} = 0$; $x - 2y \cdot \dfrac{dx}{dy} = 0$

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$$\frac{dx}{dx\,dy} = \frac{-x}{-2y} = \frac{x}{2y}$$

~~log x = 1~~

$$\frac{dx}{x} = \frac{dy}{2y}$$

$$\log x + \log c = 2\log y$$

$$\log xc = \log y^2$$

$$\boxed{xc = y^2}$$

(4) $$r = \frac{2a}{1+\cos\theta}$$

$$r = \frac{2a}{2\cos^2\theta/2}$$

$$r = a\cdot\sec^2\theta/2 \quad -(1)$$

Differentiating wrt θ,

$$\frac{dr}{d\theta} = 2a\,\sec\theta/2\cdot\sec\theta/2\cdot\tan\theta/2\cdot 1/2$$

$$= 2a\cdot\sec^2\theta/2\cdot\tan\theta/2\cdot 1/2$$

$$\frac{dr}{d\theta} = \underbrace{a\cdot\sec^2\theta/2}\cdot\tan\theta/2 \quad -(2)$$

From (1), $a\sec^2\theta/2 = r$

~~$a\sec^2\theta/2 = r$~~

So Substituting ① in ②

$$\frac{dr}{d\theta} = r \cdot \tan\frac{\theta}{2}$$

Now replacing $\frac{dr}{d\theta}$ by $-r^2\frac{d\theta}{dr}$

$\therefore$ Differential eqn. of OT is

$$-r^2\frac{d\theta}{dr} = \cancel{r} \cdot \tan\theta/2$$

$$\frac{d\theta}{dr} = \frac{-\tan\theta/2}{r}$$

$$\frac{-1}{\tan\theta/2} \cdot d\theta = \frac{dr}{r}$$

$$\int -\cot\theta/2 \cdot d\theta = \int \frac{dr}{r}$$

$$-2\log|\sin\theta/2| = \log r + \log c$$

$$-\log \sin^{+2}\theta/2 = \cancel{\log rc} \; \log r + \log c$$

$$\log \sin^2\theta/2 + \log r = -\log c$$

$$\log\left(r \cdot \sin^2\theta/2\right) = -\log c$$
$$= \log c_1$$

$$r \cdot \sin^2\theta/2 = c_1$$

$$r = \frac{2c_1}{2\sin^2\theta/2} = \boxed{\frac{2c_1}{1-\cos\theta} = r}$$

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