

MIT WORLD PEACE UNIVERSITY

Engineering Mechanics
First Year B. Tech, Trimester 1

GRAPHICAL REPRESENTATION OF PROBLEMS
INVOLVING RECTILINEAR MOTION AND MOTION
CURVES

EXPERIMENT 10
PRACTICAL REPORT

Prepared By
Krishnaraj Thadesar
Division 9, Roll No. 54
November 21, 2021

Contents

1 Objective	1
2 Theory	1
2.1 Velocity	1
2.2 Acceleration	1
2.3 Equations of Motion	1
2.4 Distance Time Graph	1
2.5 Velocity Time Graph	2
2.6 Acceleration Time Graph	3
3 Procedure	3
4 Analytical Method	4
5 Graphical Method	8
6 Results	12
7 Conclusion	12

1 Objective

To find Graphically and Analytically the resultant of a set of problems involving Rectilinear Motion and Motion Curves, and to compare the results thereby finding the Percentage Error.

2 Theory

The Following laws and concepts have been used in this experiment.

2.1 Velocity

The velocity of an object is the rate of change of its position with respect to a frame of reference, and is a function of time. Velocity is equivalent to a specification of an object's speed and direction of motion (e.g. 60 km/h to the north).

2.2 Acceleration

In mechanics, acceleration is the rate of change of the velocity of an object with respect to time. Accelerations are vector quantities (in that they have magnitude and direction).

2.3 Equations of Motion

In physics, equations of motion are equations that describe the behavior of a physical system in terms of its motion as a function of time. More specifically, the equations of motion describe the behavior of a physical system as a set of mathematical functions in terms of dynamic variables. These variables are usually spatial coordinates and time.

The Equations used here are applicable to uniformly accelerated motion.

$$\begin{aligned}v &= u + at \\s &= ut + \frac{1}{2}at^2 \\v^2 &= u^2 + 2as\end{aligned}\tag{1}$$

2.4 Distance Time Graph

A distance-time graph shows how the distance and speed of an object changes with time. This graph shows the movement of three objects over time. The slope, or steepness, of each line indicates the object's rate of speed. In general, the steeper the slope, the faster the object's speed. A straight line indicates constant speed, whereas changes in slope at different time intervals indicate changing speeds.

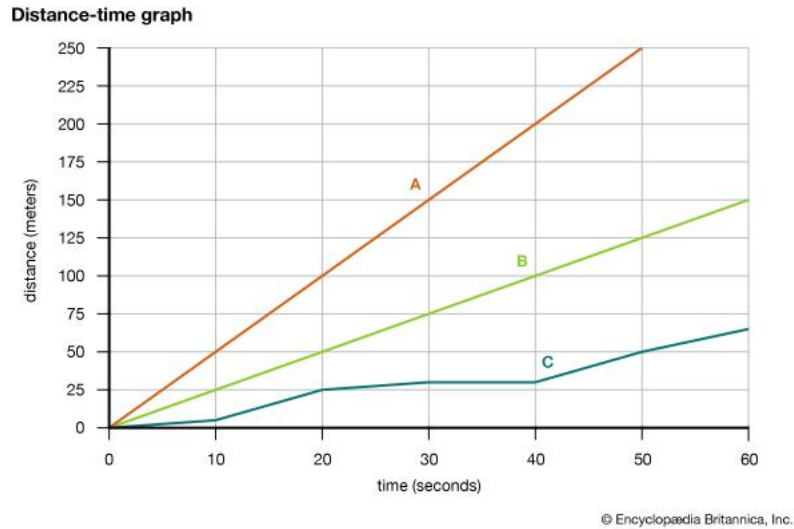


Figure 1: Distance Time Graph

The Slope of a Distance time graph is the velocity.

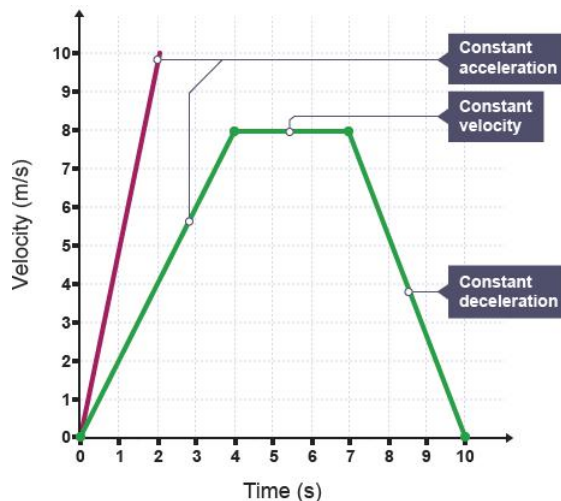
$$\text{Velocity } v = \frac{\text{Distance}}{\text{Time}}$$

Instantaneous Velocity can be found as:

$$v(t) = \lim_{\Delta t \rightarrow 0} \frac{x(t + \Delta t) - x(t)}{\Delta t} = \frac{dx(t)}{dt} \quad (2)$$

2.5 Velocity Time Graph

The velocity of an object is its speed in a particular direction. Two cars travelling at the same speed but in opposite directions have different velocities. A velocity-time graph shows the speed and direction an object travels over a specific period of time. Velocity-time graphs are also called speed-time graphs. The vertical axis of a velocity-time graph is the velocity of the object. The horizontal axis is the time from the start.¹

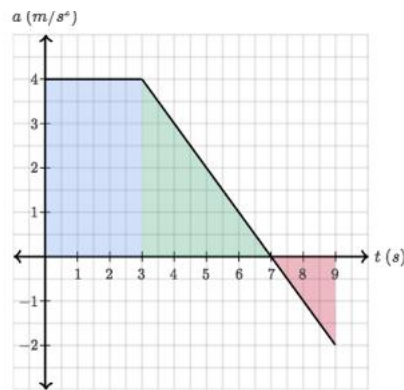


The Slope of the velocity time graph shows the Acceleration.

$$a(t) = \frac{d}{dt}v(t) \quad (3)$$

2.6 Acceleration Time Graph

The acceleration time graph is the graph that is used to determine the change in velocity in the given interval of the time. In the acceleration vs time graph on the x-axis you have the time taken by the object and on the y-axis acceleration of the object, in which the area under the graph gives you the change in velocity of the object over the given period of the time. The acceleration time graph is used to find the change in the velocity of the moving object for the given period of time and this can be determined by finding the area under the curve.



Distance under the Acceleration Time graph gives the velocity

$$v = a \times t$$

3 Procedure

1. Solve the Given Question analytically.
2. Solve the Given Question Graphically.
3. Tabulate the Results
4. Calculate the Error using the Formula

$$\text{Percentage Error } (\eta) = \frac{|\text{Graphical Value} - \text{Analytical Value}|}{\text{Analytical Value}} * 100$$

4 Analytical Method

Q1. A car starting from rest moves along a straight track with an acceleration shown in the Figure. Determine the time t for the car to reach a speed of 50 m/s and construct the v-t diagram that describes the motion until the time 't'.

We know ;

$$u = 0$$

a = increasing in a straight line
till 10s

$$a = 8 \text{ m/s}^2 \text{ after } 10 \text{ s}$$

To find time taken to reach 50 m/s velocity
let it be 't'.

From the graph, slope point form can be applied

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{a - 0}{t - 0} = \frac{8 - 0}{10 - 0}$$

$$\Rightarrow \boxed{a = 0.8 \cdot t} \text{ --- ①}$$

$$\text{But } a = \frac{dv}{dt} ; \boxed{dv = a dt} \text{ --- ②}$$

Integrating ② by substituting ①,

$$\int dv = \int a \cdot dt$$

$$v = \int 0.8 \cdot t \cdot dt = 0.8 \int t \cdot dt$$

$$v = 0.8 \cdot \frac{t^2}{2}$$

So $v = 0.4 t^2$ at 10 s. So Velocity at 10 s,

$$v_{10} = (0.4)(100) = \underline{\underline{40 \text{ m/s}}}$$

From 10 s to time 't', acceleration = 8 m/s^2

$$a = v/t \quad ; \quad v = a \cdot t = 8 \cdot t$$

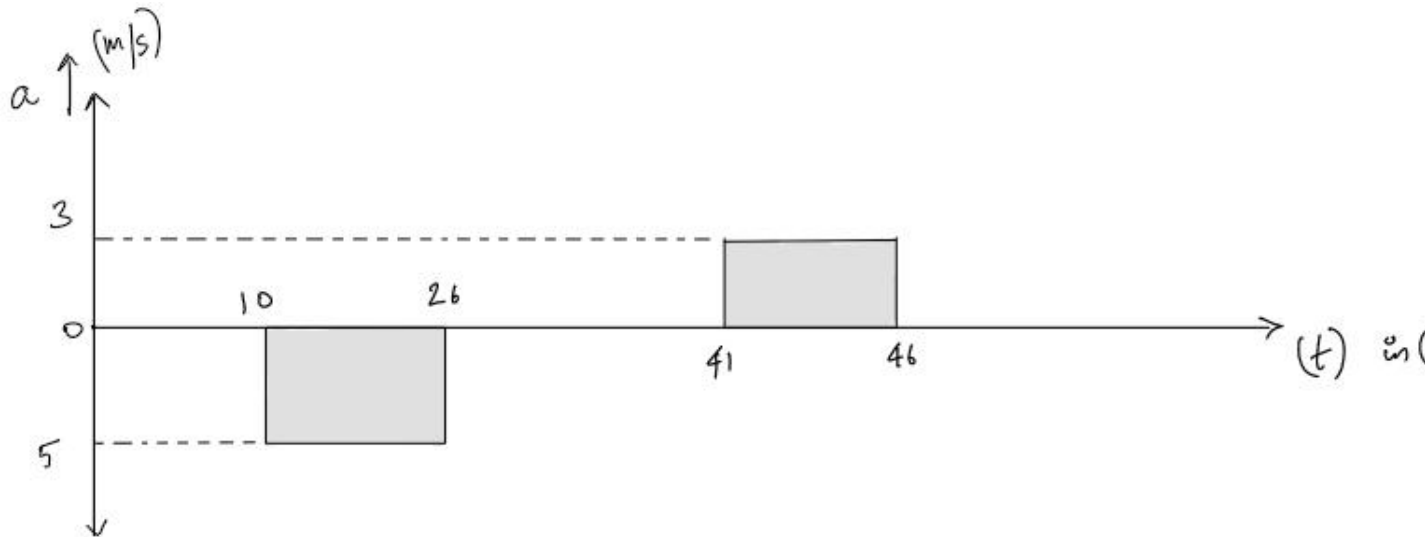
To increase v by 10 m/s ,

$$10 = 8 \cdot t \quad , \quad t = 10/8 = 1.25 \text{ s}$$

$$\begin{aligned} \text{So total velocity} &= 40 + 10 = \boxed{50 \text{ m/s}} \\ \text{total time} &= 10 + 1.25 = \boxed{11.25 \text{ s}} \end{aligned}$$

Engineering Mechanics Experiment 9

Q2. A particle moves in a straight line with the acceleration shown in the figure. If $x = -540\text{m}$ and $v = 60\text{m/s}$ and $t = 0$ then find the total distance travelled by the particle when $t = 50\text{s}$.



Case 1 : Till 10 s

at $t=0$;

$v = 60\text{ m/s}$

$s = -540\text{ m}$

$a = 0$ till 10 s

\Rightarrow so till 10 s,

$$s = v \cdot t$$

$$= 60 \cdot 10$$

$$= \underline{600\text{ m.}}$$

Case 2 : From 10 to 26 s

$$a = -5\text{ m/s}^2$$

$$u = 60\text{ m/s}$$

$$v = ?$$

$$t = 16\text{ s}$$

$$v = u + at$$

$$= 60 - 80 = -20\text{ m/s}$$

so velocity becomes 0 at some point

$$v = 0 \Rightarrow 0 = 60 - 5 \cdot t ; t = 12\text{ s}$$

So at $t = 10 + 12.5 = 22.5\text{ s}$, $v = 0$.

From 10 to 22.5 s

Engineering Mechanics Experiment 9

$$v = 60, v = 0$$

$$s_{21} = \frac{v^2 - v^2}{2a} = \frac{3600}{10} = \underline{\underline{360 \text{ m}}}$$

From 225 to 265,

$$v = -ve$$

$$v = 0$$

$$v = -20$$

$$s_{22} = \frac{v^2 - v^2}{2a} = \frac{400}{-10} = \underline{\underline{40 \text{ m}}}$$

Case 3: $t = 265$ to $t = 415$

$$u = -20 \text{ m/s} \quad ; \quad t = 15 \text{ s}$$

$$a = 0$$

$$s_3 = -v \cdot t = u \cdot t = -15 \times 20 = \underline{\underline{-300 \text{ m}}}$$

Case 4: $t = 41$ to $t = 46 \text{ s}$.

$$u = -20 \text{ m/s} \quad ; \quad t = 5 \text{ s}$$

$$a = 3 \text{ m/s}^2$$

$$s = ut + \frac{1}{2}at^2 = -20 \times 5 + \frac{1}{2} \cdot 3 \times 25$$

$$s_4 = \underline{\underline{-62.5}}$$

Case 5: $t = 46 \rightarrow t = 50$

$$s = u \cdot t$$

$$u = -5 \text{ m/s}$$

$$s = \underline{\underline{-20 \text{ m}}}$$

So to find the total distance,

$$s_{\text{total}} = |s_1| + |s_{21}| + |s_{22}| + |s_3| + |s_4| + |s_5| = \\ 600 + 360 + 40 + 300 + 62.5 + 20 = \underline{\underline{1382.5 \text{ m}}}$$

5 Graphical Method

Q1. A car starting from rest moves along a straight track with an acceleration shown in the Figure. Determine the time t for the car to reach a speed of 50 m/s and construct the v-t diagram that describes the motion until the time 't'.

$$u = 0$$

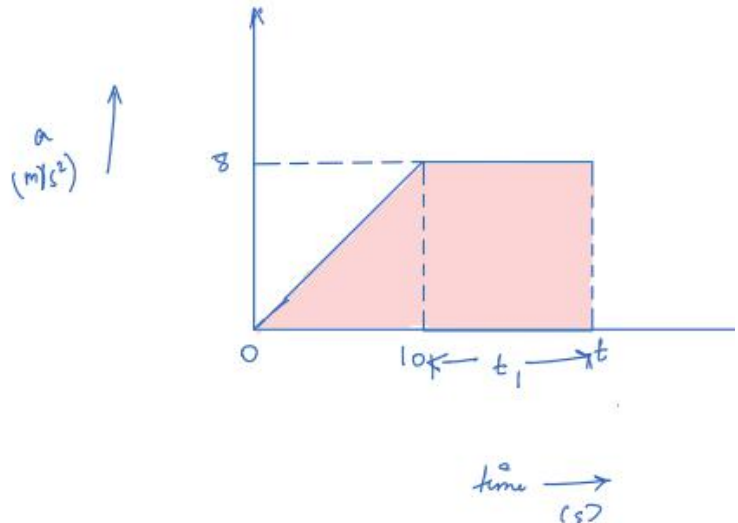
Constant increasing acceleration till 10 s,

$$a = v/t \quad \therefore at = \frac{v}{t} \cdot t$$

To reach 50 m/s velocity, let time taken be $t_1 + (10)$

$$50 = \frac{1}{2} \cdot 8 \times 10 + 8 \times t_1$$

$$t_1 = 1.25 \text{ s}$$



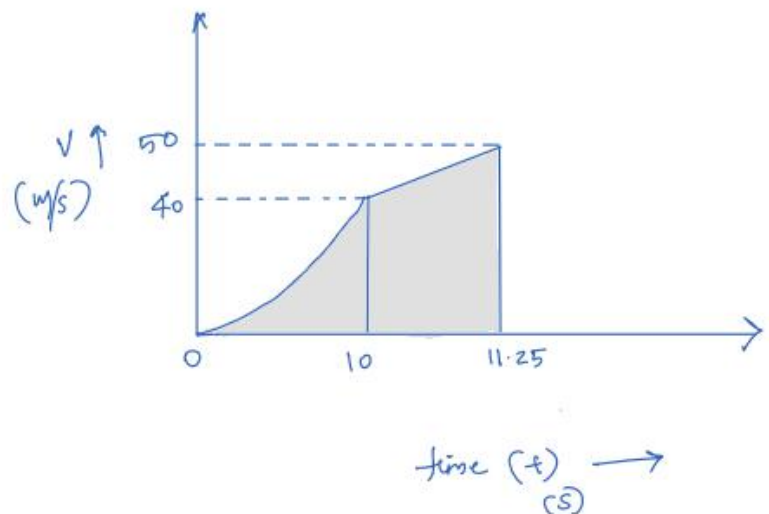
$$\begin{aligned} \text{So total time taken to reach 50 m/s} &= 1.25 \text{ s} + 10 \text{ s} \\ &= 11.25 \text{ s} \end{aligned}$$

As there is constant acceleration till 10 s, let velocity after 10 s be V_{10}

$$V_{10} = \frac{1}{2} \times 8 \times 10 = 40 \text{ m/s}$$

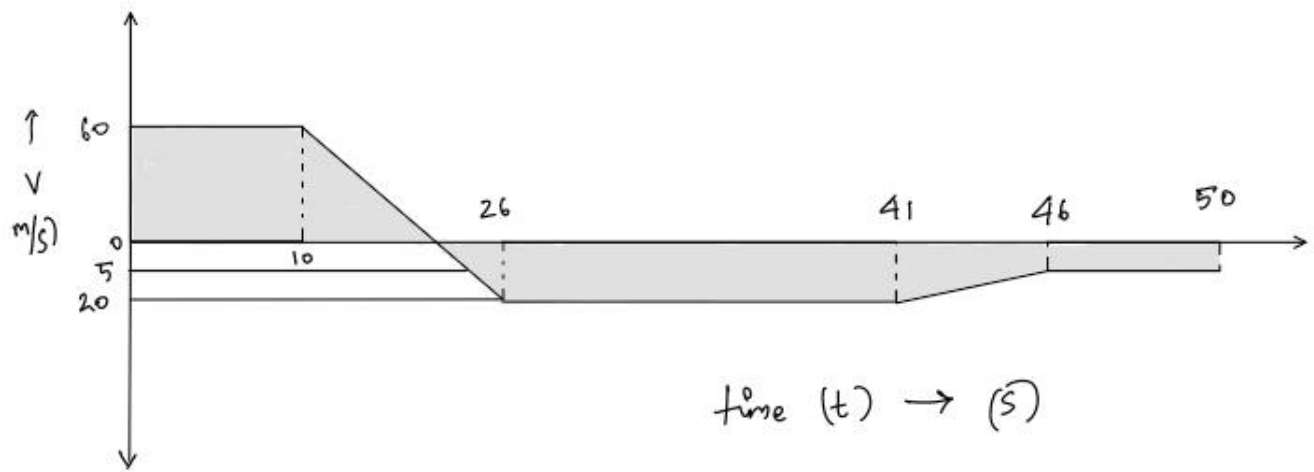
$$V_{11.25} = 50 \text{ m/s.}$$

So velocity time graph would look like



Engineering Mechanics Experiment 9

Q2. A particle moves in a straight line with the acceleration shown in the figure. If $x = -540\text{m}$ and $v = 60\text{m/s}$ and $t = 0$ then find the total distance travelled by the particle when $t = 50\text{s}$.



Area of an $a-t$ diagram gives velocity:

$$V_0 = 60 \text{ m/s}$$

$$V_{10} = 60 \text{ m/s}$$

$$V_{26} = 60 - 16 \times 5 = -20 \text{ m/s}$$

$$V_{41} = -20 \text{ m/s}$$

$$V_{46} = 60 - 16 \times 5 + 5 \times 3 = -5 \text{ m/s}$$

$$V_{50} = -5 \text{ m/s}$$

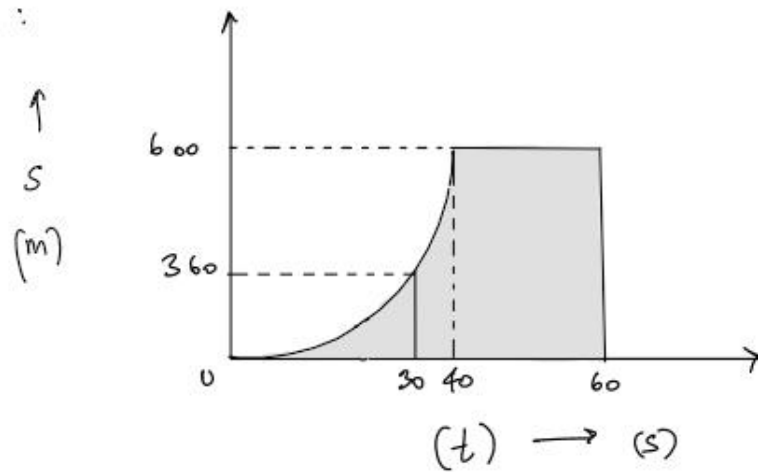
Area of $v-t$ diagram gives us the distance travelled:

$$\begin{aligned} S_{\text{total}} &= 60 \times 10 + \frac{1}{2} \times 12 \times 60 + \frac{1}{2} \times 4 \times 20 + 15 \times 20 \\ &\quad + \left(\frac{20 + 5}{2} \right) \times 5 + 4 \times 5 = \underline{\underline{1382.5 \text{ m}}} \end{aligned}$$

Engineering Mechanics Experiment 9

Q3. The $s-t$ curve for a train travelling along a straight track has been experimentally determined. From the data construct the $v-t$ and $a-t$ curve for the motion from 0 to 60 Seconds. From 0 to 30 seconds the curve is $s = (0.4t^2)m$ where t is in s.

Given Data :



At $t = 30s$, Velocity and acceleration is given as

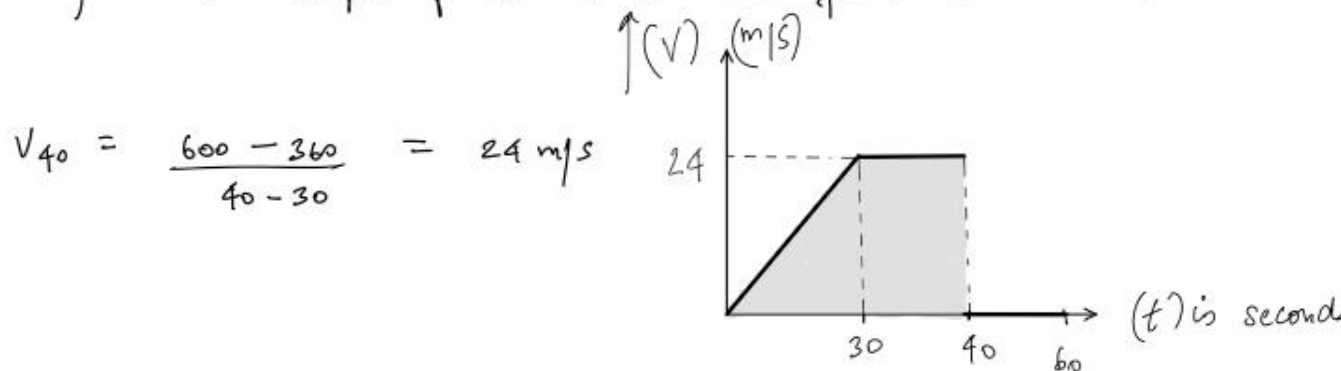
$$s = 0.4t^2$$

$$v = \frac{ds}{dt} = 0.8t$$

$$a = \frac{dv}{dt} = 0.8$$

$$\text{at } t = 30s, v = 24 \text{ m/s}, a = 0.8 \text{ m/s}^2$$

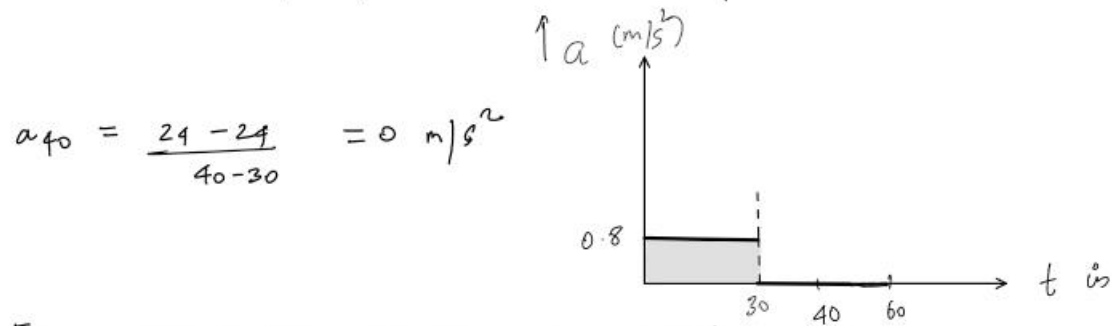
Velocity is the slope of the $s-t$ curve from $t = 30s$ to $40s$



$$v_{40} = \frac{600 - 360}{40 - 30} = 24 \text{ m/s}$$

Engineering Mechanics Experiment 9

Acceleration is slope of the v-t curve from $t = 30\text{ s}$ to 40 s



From 40 s to 60 s , velocity and acceleration are zero.

6 Results

Question Number	Analytical Solution	Graphical Solution	Percent Error
1	$t = 11.25\text{s}$	$t = 11.25\text{s}$	$\eta_t = 0\%$
2	$x = 1382.5\text{m}$	$x = 1382.5$	$\eta_x = 0\%$

7 Conclusion

A set of problems involving Rectilinear Motion and Motion Curves were solved using graphical as well as analytical methods. The Percentage error between the two answers was found.