

(1) When plane surface is inclined to H.P. :

When a plane surface is inclined to H.P. and a side or a diagonal or a diameter is parallel to H.P. and inclined to V.P., projections of the plane will be obtained in following stages :

Initial position :

Initially plane is assumed to be parallel to H.P. or on the H.P. If a plane is inclined to H.P., it will be either rotated about one of its sides or about one of its corners.

- If a side of the plane is on H.P., it will be kept perpendicular to V.P. (i.e. XY).
- If a corner of the plane is on H.P., it will be kept in such a way that the line joining the corner and centre of the plane must be parallel to V.P. (i.e. XY).

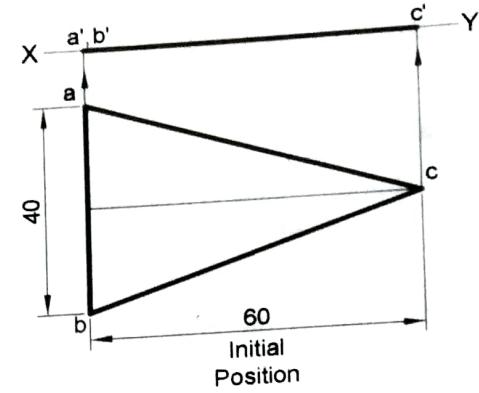


Fig. 4.6

Intermediate position :

The plane is then rotated so as to make the required angle with H.P. Its F.V. in this position will be line view and its T.V. will show the apparent shape.

Final Position :

- In the final position the T.V. of the plane is rotated to the required inclination with V.P. Remember, only the position of the T.V. will change, its shape and size will not be changed.
- In the F.V. of final position of the plane distances of all corners from H.P. (i.e. XY) will remain the same as in the F.V. of intermediate position.

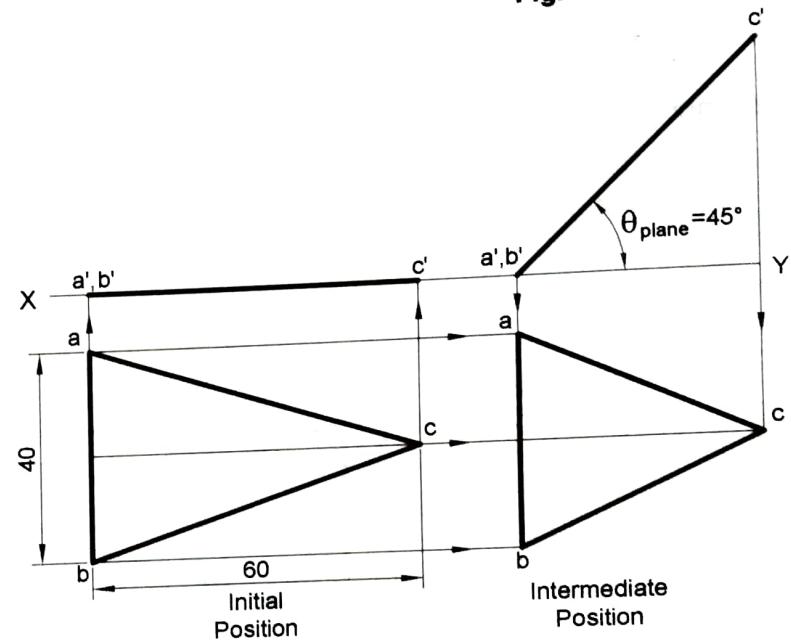


Fig. 4.7

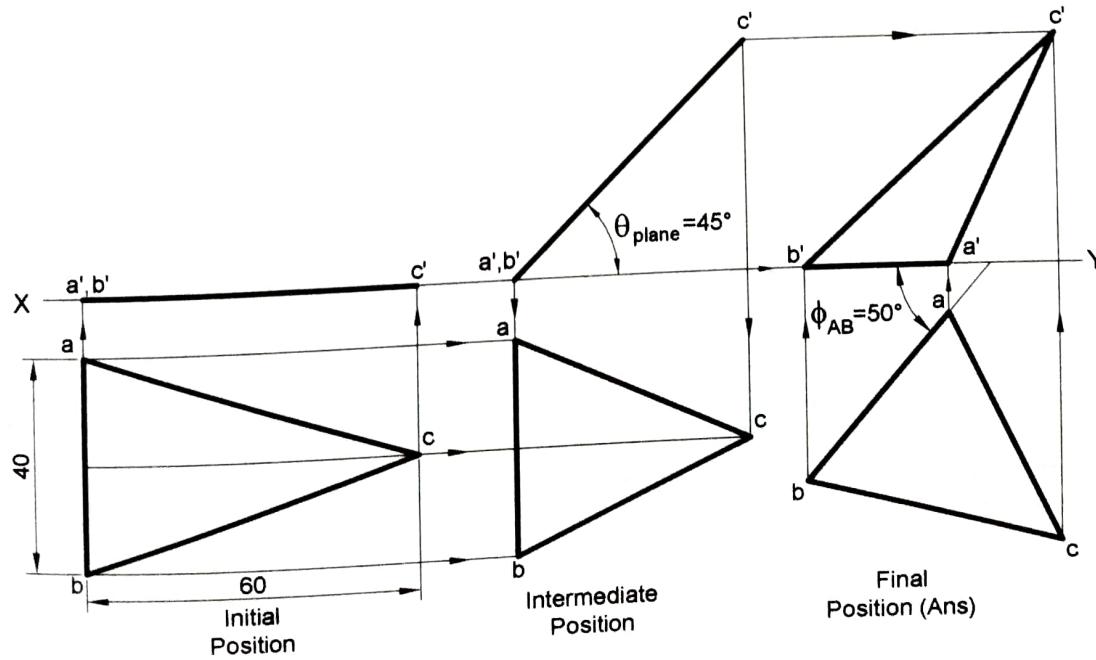


Fig. 4.8

Fig. 4.11

4.6.1 Projection of Triangular Plane

Prob. 39 : An isosceles triangle ABC having its base $AB = 40$ mm and altitude 60 mm is resting on the H.P. on its base AB. Draw the projections of the plane when its surface is inclined to H.P. at an angle of 45° and the base AB which is on the H.P. is making an angle of 50° to the V.P.

Soln. :

Method 1 : Change of position method :

In this method plane is rotated and principal planes do not change their position.

Refer Fig. Prob. 39(a).

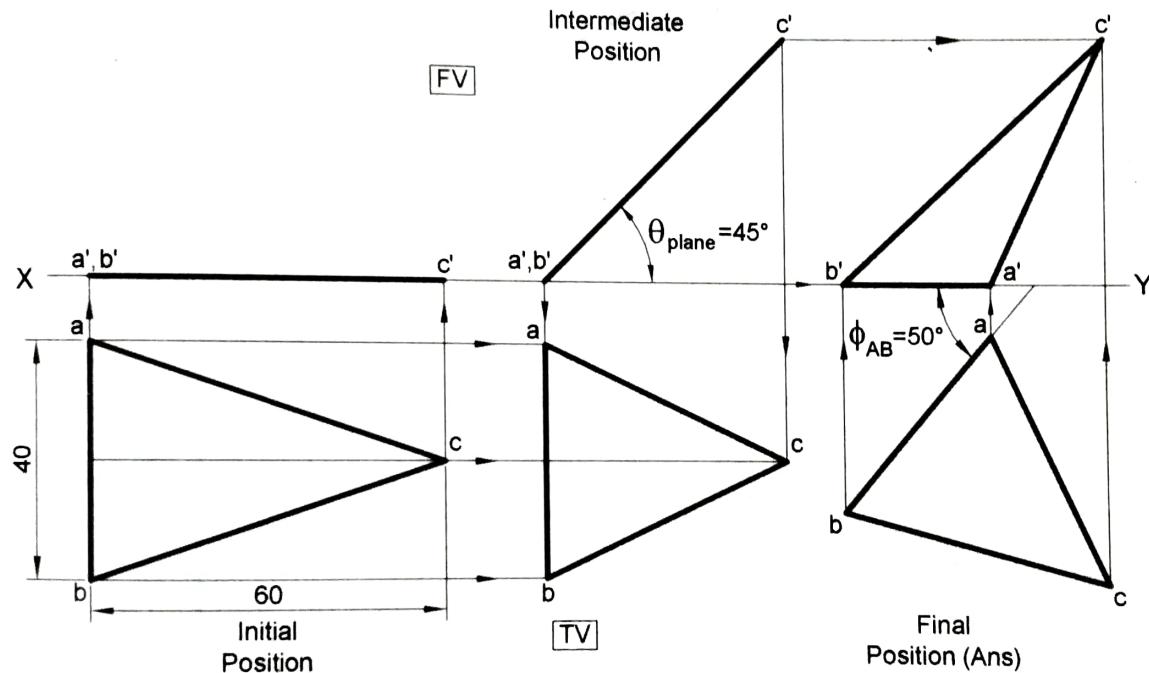


Fig. Prob. 39(a)

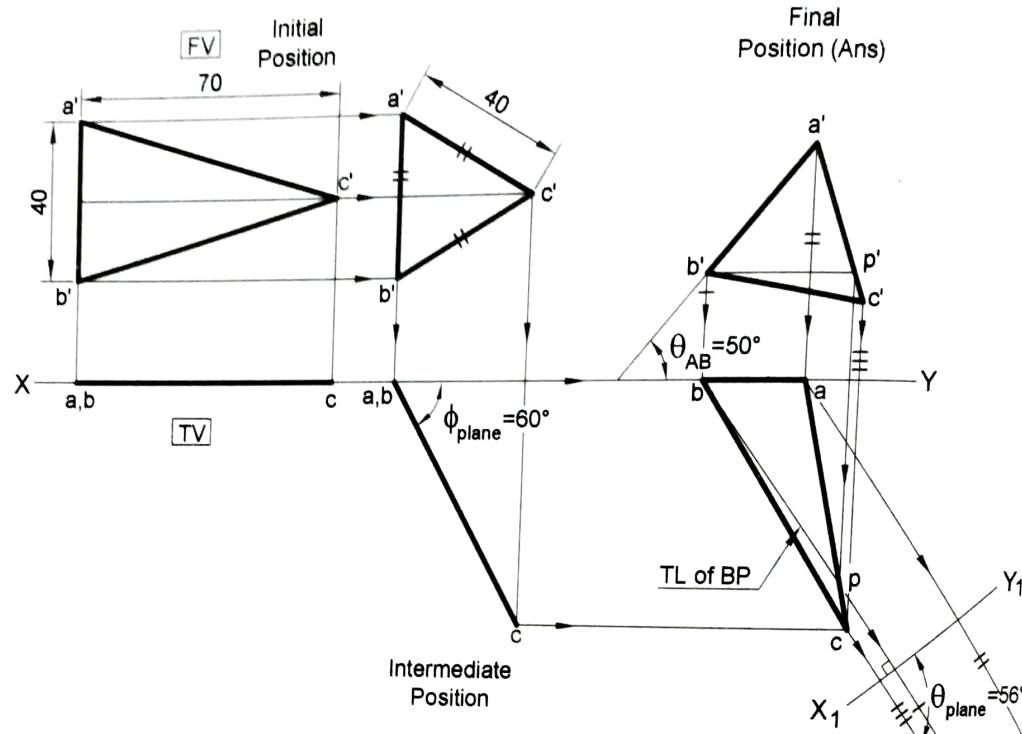


Prob. 45 : Draw the projections of an isosceles triangular lamina of base 40 mm and altitude 70 mm. when it is inclined with V.P. in such a way that its elevation is an equilateral triangle, while the base is inclined at 50° with H.P. and is in V.P. Find the inclination of the lamina with reference planes. Assume the object in the first quadrant.

Soln. : Refer Fig. Prob. 45.

Initial position :

Step 1 : Since elevation of isosceles triangle ABC is an equilateral triangle it is inclined to V.P. with its side AB on the V.P. So initially we will assume the plane to be kept on the V.P. with side AB perpendicular to H.P. (i.e. XY).



Step 2 : Draw the F.V. $a' b' c'$ of given dimensions keeping side $a' b'$

4.2 Projections of Quadrilateral Planes :

Prob. 56 : A rectangular plane ABCD with side AB = 30 mm and BC = 50 mm is resting on the H.P. on its smaller side AB. Draw the projections of the plane when its surface makes an angle of 45° with the H.P. and the side AB which is on the H.P. is inclined at 45° to the V.P. Draw the projections of the plane.

Soln. : Method 1 : Change of position method : Refer Fig. Prob. 56(a)

Initial position :
Step 1 : Since surface of the plane is given to be inclined with H.P. with its side AB on the H.P., initially we will assume the plane to be kept on the H.P. with side AB perpendicular to V.P. (i.e. XY).

Step 2 : As we have kept the plane on H.P., its T.V. will show the true shape. Hence draw the T.V. abcd with side perpendicular to the XY line and project the F.V. $a'b' - c'd'$ which will be line view coinciding with the XY line.

Intermediate position :

Step 1 : When a rectangle ABCD will be inclined to H.P. at an angle of 45° , its F.V. will appear as a line making 45° with the XY line. Hence redraw the F.V. of first stage at an angle of $\theta_{\text{plane}} = 45^\circ$ with the XY line keeping $a'b'$ line.

Step 2 : Project the points a', b', c' and d' from the F.V. vertically down and project the points a, b, c and d from stage T.V. parallel to XY line and to the right to get the same points in the second stage T.V.

Final position :

Step 1 : Since side AB is making angle of 45° with the V.P. redraw the T.V. of the second stage such that it is making an angle of 45° with the XY line.

Step 2 : Project the points of the third stage T.V. (i.e. a, b, c and d) vertically up. Again project the points of stage F.V. (i.e. a', b', c' and d') parallel to the XY line and to the right to intersect vertical projection of stage T.V. and locate points a', b', c' and d' .

Step 3 : Join points a', b', c' and d' in proper sequence to get the F.V. of final position.

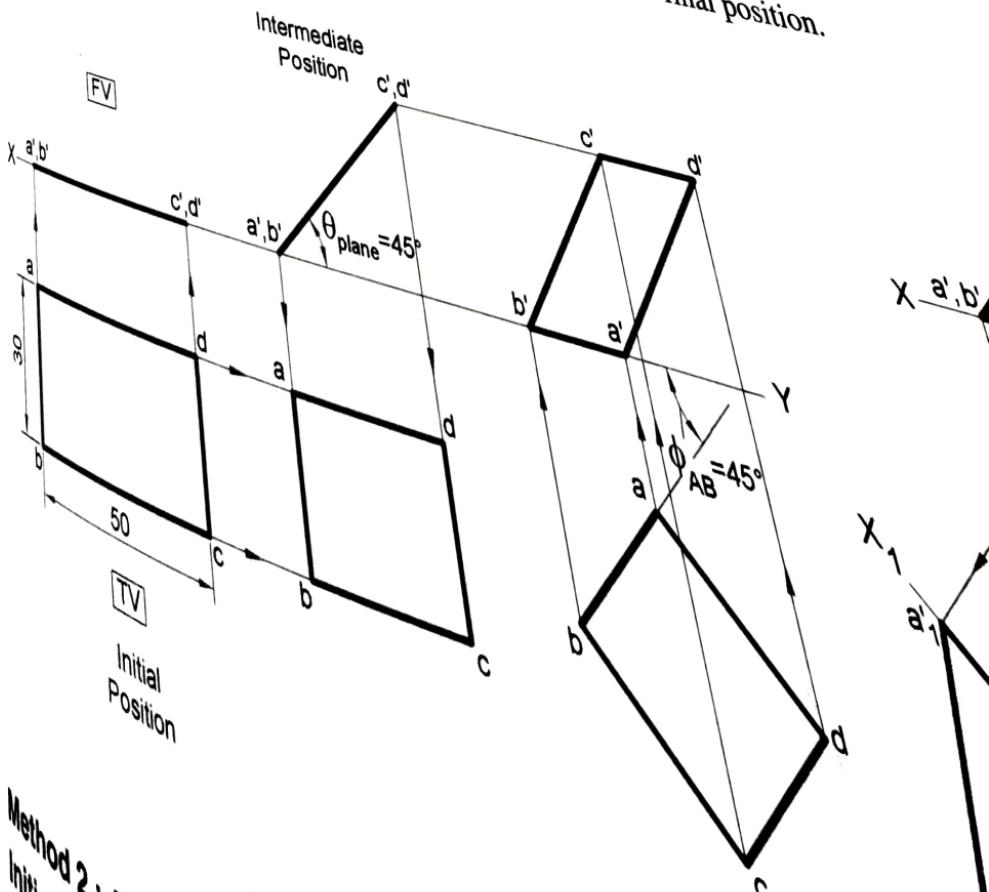


Fig. Prob. 56(a)

Method 2 : Auxiliary plane method : Refer to the Fig. Prob. 56(a)

Initial position : Same as described in Method 1.

Intermediate position : Now rectangle ABCD has been rotated so that corners C and D lie on the V.P. (XY line).

Step 1 : Now rectangle ABCD has been rotated so that corners C and D lie on the V.P. (XY line).

Step 2 : Draw the front view and side view of the plane and mark points a' , b' , c' and d' in advance.

Step 3 : Join points a' , b' , c' and d' in proper sequence which will give us the final position.

Prob. 57 : A rectangular plane $ABCD$ with side $AB = 30 \text{ mm}$ and $BC = 50 \text{ mm}$ is resting on the V.P. on its smaller side AB . Draw the projections of the plane when its surface makes an angle of 45° with the V.P. and the side AB which is on the V.P. is inclined at 45° to the H.P. Draw the projections of the plane.

Soln. : Method 1 : Change of position method- Refer Fig. Prob. 57(a)

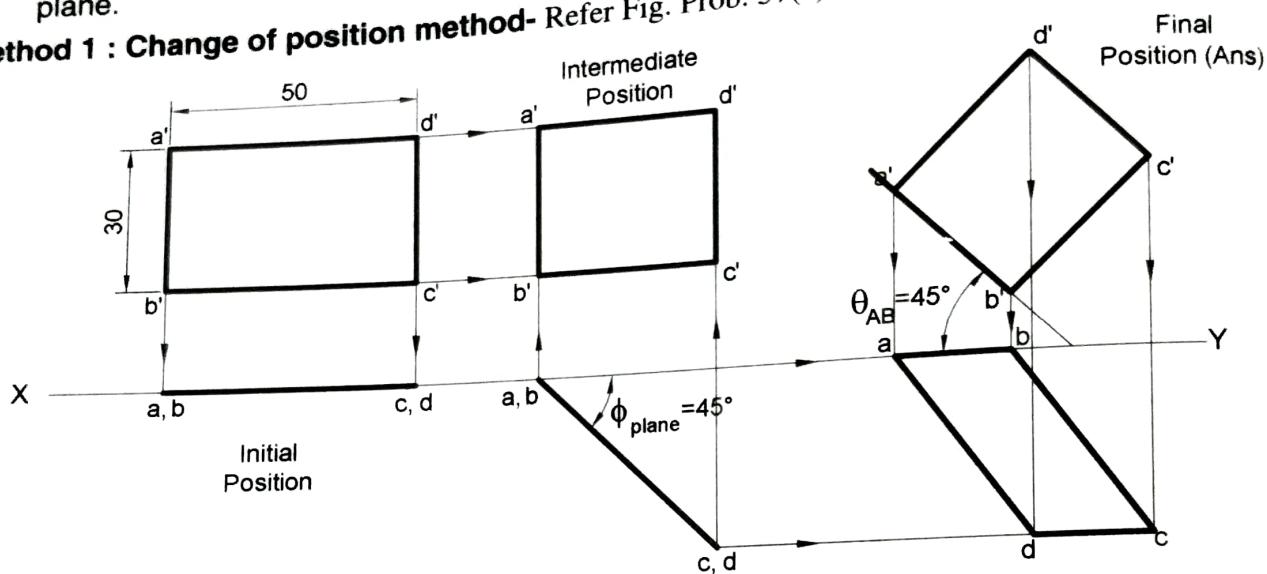


Fig. Prob. 57(a)

Initial position :

Step 1 : Since surface of the plane is given to be inclined with V.P. with its side AB on the V.P., initially we will assume the plane to be kept on the V.P. with side AB perpendicular to H.P. (i.e. XY).

Step 2 : As we have kept the plane on V.P., its F.V. will show the true shape. Hence draw the F.V. $a'b'c'd'$ with side a' perpendicular to the XY line and project the T.V. $ab - cd$ which will be line view coinciding with the XY line.

Intermediate position :

Step 1 : When a rectangle $ABCD$ will be inclined to V.P. at an angle of 45° , its T.V. will appear as a line making 45° to the XY line. Hence redraw the T.V. of first stage at an angle of $\phi_{plane} = 45^\circ$ with the XY line keeping ab on the XY line.

Step 2 : Project the points a , b , c and d from the T.V. vertically up and project the points a' , b' , c' and d' from the first stage F.V. parallel to XY line and to the right to get the same points in the second stage F.V.

Step 3 : Join points a' , b' , c' and d' in a proper sequence in the second stage F.V.

Final position :

Step 1 : Since side AB is making an angle of 45° with the H.P. redraw the F.V. of the second stage such that $a'b'$ is making an angle of 45° with the XY line.

Step 2 : Project the points of the third stage F.V. (i.e. a' , b' , c' and d') vertically down. Again project the points of second stage T.V. (i.e. a , b , c and d) parallel to the XY line and to the right to intersect vertical projectors of third stage F.V. and locate points a , b , c and d .

Step 3 : Join points a , b , c and d in proper sequence to get the T.V. of final position.

with the help
Join points a'_1 , b'_1 , c'_1 and d'_1 in proper sequence which will give us the final T.V.

Ques. 58 : A rhombus $ABCD$ having major diagonal $AC = 90$ mm and minor diagonal $BD = 60$ mm is resting on the H.P. on its corner A. Draw the projections of the rhombus when its surface is inclined to H.P. at an angle of 45° and diagonal AC is contained by an auxiliary vertical plane making an angle of 30° to the V.P.

In. : Refer Fig. Prob. 58

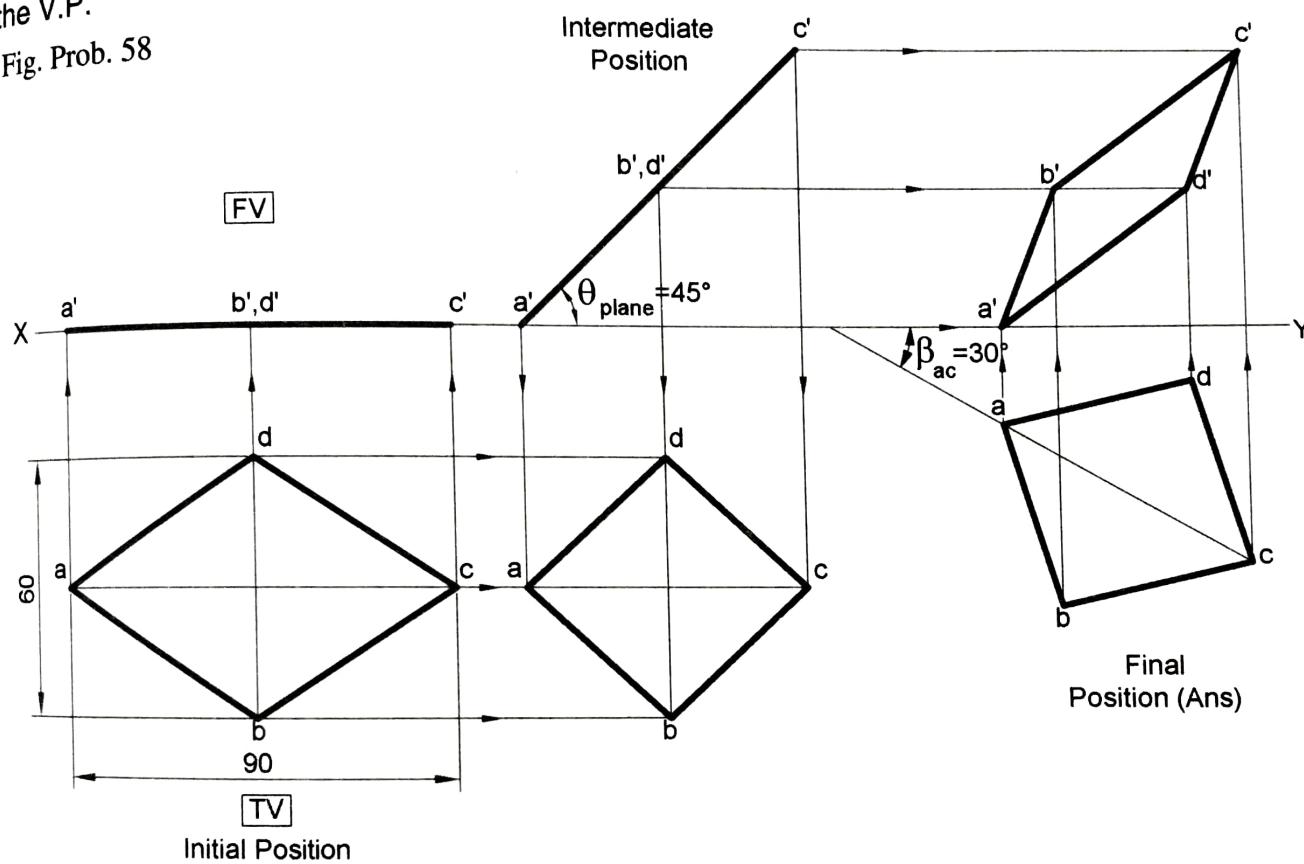


Fig. Prob. 58

stage T.V. and locate points a' , b' , c' and d' in proper sequence to get the F.V. of the object.

Step 3 : Join points a' , b' , c' and d' in proper sequence to get the F.V. on one of its smaller sides in such a way that its surface makes an angle of 45° with the V.P. Draw the projections of the rectangle when one of its longer sides makes an angle of 30° to H.P.

Soln. : Refer Fig. Prob. 61.

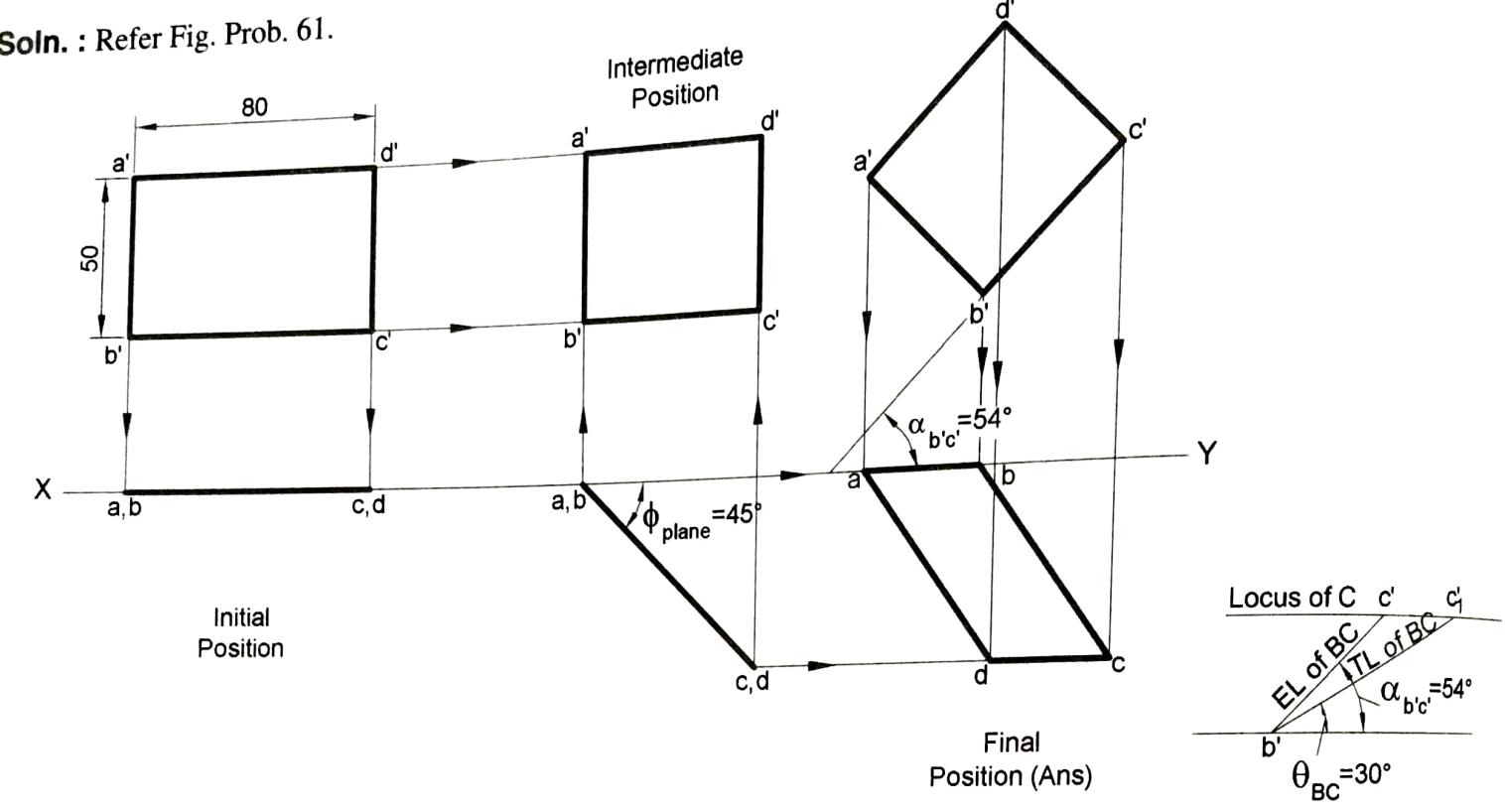


Fig. Prob. 61

Initial position :

Step 1 : Let the rectangular plate be $ABCD$ with AB as one of its smaller sides. Since rectangle is inclined with V.P. with its smaller side AB on the V.P., initially we will assume the rectangle to be kept on the V.P. with side AB perpendicular to H.P. (i.e. XY).

Step 2 : Draw the F.V. $a'b'c'd'$ with side $a'b'$ perpendicular to the XY line and project the T.V. $ab - cd$ which will be line view coinciding with the XY line.

Intermediate position :

Step 1 : Redraw the T.V. of first stage at an angle of $\phi_{plane} = 45^\circ$ with the XY line keeping ab on the XY line.

Step 2 : Project the points a, b, c and d from the T.V. vertically up and project the points a', b', c' and d' from the first stage F.V. parallel to XY line and to the right to get the same points in the second stage F.V.

Step 3 : Join points a', b', c' and d' in a proper sequence in the second stage F.V.

Final position :

Step 1 : Let longer side BC is making an angle of 30° with the H.P. but it is seen in its apparent length in the second stage F.V. Hence we will calculate $\alpha_{b'c'}$ as shown in Fig. Prob. 61. Redraw the F.V. of the second stage such that elevation $b'c'$ is making an angle of $\alpha_{b'c'}$ with the XY line.

Step 2 : Project the points of the third stage F.V. (i.e. a', b', c' and d') vertically down. Again project the points of the second stage T.V. (i.e. a, b, c and d) parallel to the XY line and to the right to intersect vertical projectors from the third stage F.V. and locate points a, b, c and d .

Step 3 : Join points a, b, c and d in proper sequence to get the T.V. of final position.

Prob. 62: A rectangular plate having sides 45 mm and 75 mm is resting on the H.P. on one of its smaller sides and is inclined to H.P. in such a way that its plan appears to be another rectangle of smaller side 45 mm and longer side 65 mm. Draw the projections of the rectangle when one of its longer sides makes an angle of 45° to V.P.

Soln.: Refer Fig. Prob. 62.

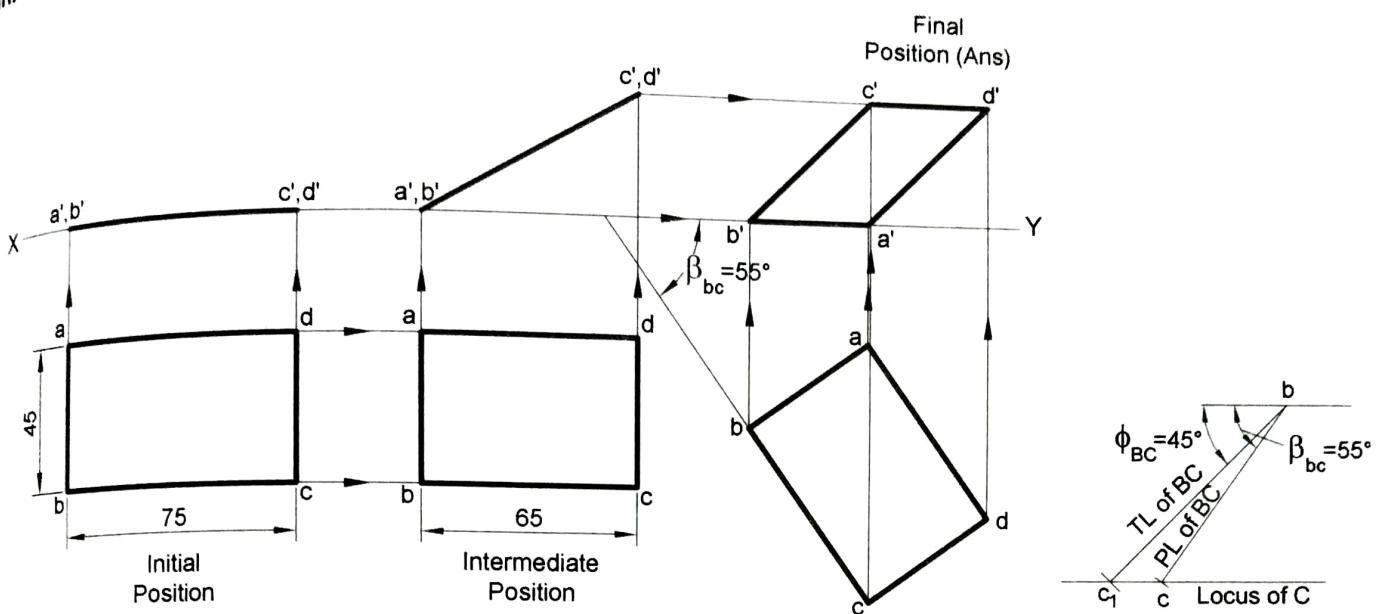


Fig. Prob. 62

Initial position :

Step 1: Let the rectangle be $ABCD$ having AB as one of its smaller sides and BC as one of its longer sides. Since plan of the rectangle is showing its apparent shape, it is inclined to H.P. with its side AB on the H.P. So initially we will assume the plane to be kept on the H.P. with side AB perpendicular to V.P. (i.e. XY).

Step 2: Draw the T.V. $abcd$ of given dimensions keeping side ab perpendicular to the XY line and project the F.V. $a' b' c' d'$ which will be line view coinciding with the XY line.

Intermediate position :

Step 1: Draw projectors parallel to the XY line and to the right, from corners ' a' ', ' b' ', ' c' ' and ' d' ' of the initial stage T.V.

Step 2: Draw line ab perpendicular to XY line keeping ' a' ' and ' b' ' on their respective projectors and construct the rectangle $abcd$ having side $bc = 65$ mm and keeping points ' c' ' and ' d' ' on their projectors.

Step 3: Now draw two projectors through points ' a' ' and ' d' ' in the T.V. and redraw the first stage F.V. keeping ' $a' b'$ ' on the projector of ' a' ' and ' $c' d'$ ' on the projector of ' d' '.

Final position :

Step 1: Since one of the longer sides, say BC is making an angle of 45° with the V.P. but it is seen in its apparent length in the intermediate stage T.V. Hence we will calculate β_{bc} as shown in Fig. Prob. 62. Redraw the T.V. of the intermediate stage such that plan of longer side bc is making an angle of β_{bc} with the XY line.

Step 2: Project the points of the final stage T.V. (i.e. a , b , c and d) vertically up. Again project the points of the intermediate stage F.V. (i.e. a' , b' , c' and d') parallel to the XY line and to the right to intersect vertical projectors from final stage T.V. and locate points a' , b' , c' and d' .

Step 3: Join points a' , b' , c' and d' in proper sequence to get the F.V. of final position.

Prob. 63 : A rectangular plate having sides 45 mm and 75 mm is resting on the V.P. on one of its smaller sides and is inclined to V.P. in such a way that its elevation appears to be another rectangle of smaller side 45 mm and longer side 65 mm. Draw the projections of the rectangle when the smaller side which is in V.P. makes an angle of 45° to H.P.

Soln. : Refer Fig. Prob. 63.

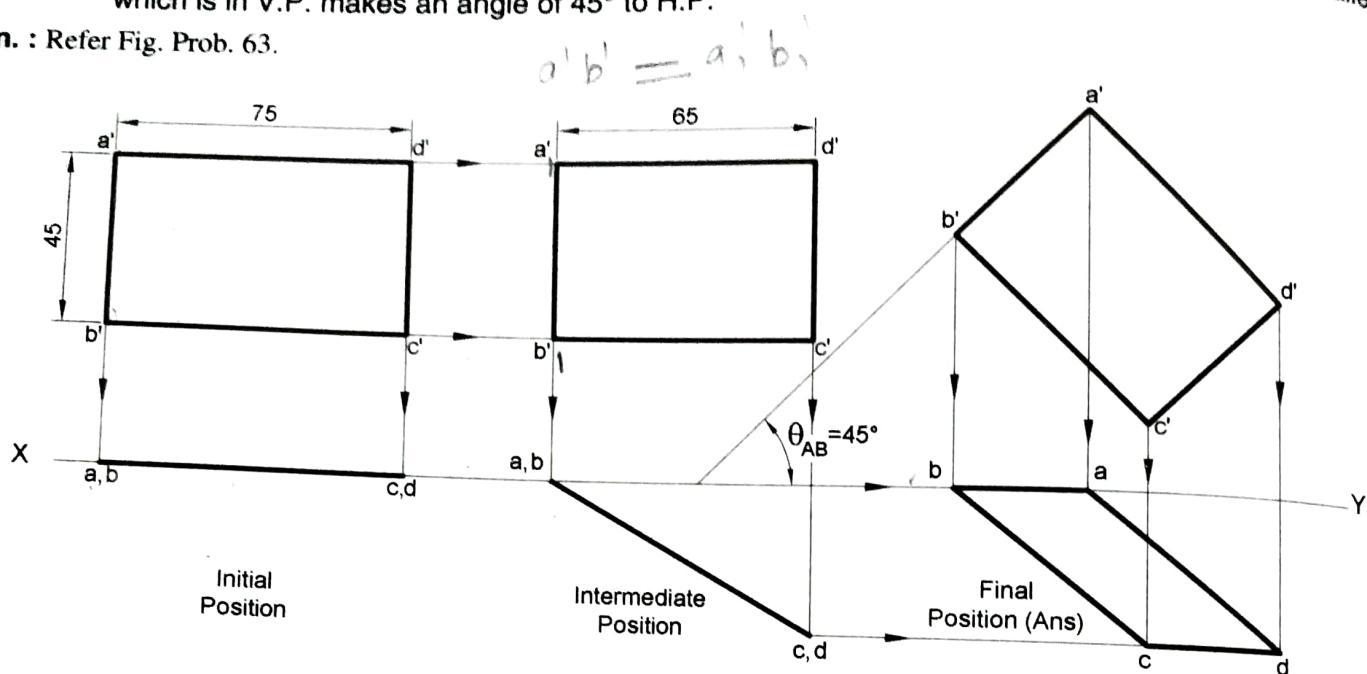


Fig. Prob. 63

Initial position :

Step 1 : Let the rectangle be $ABCD$ having AB as one of its smaller sides and BC as one of its longer sides. Since elevation of the rectangle is showing its apparent shape, it is inclined to V.P. with its side AB on the V.P. So initially we will assume the plane to be kept on the V.P. with side AB perpendicular to H.P. (i.e. XY).

Step 2 : Draw the F.V. $a'b'c'd'$ of given dimensions keeping side $a'b'$ perpendicular to the XY line and project the T.V. $ab - cd$ which will be line view coinciding with the XY line.

Intermediate position :

Step 1 : Draw projectors parallel to the XY line and to the right , from corners a' , b' , c' and d' of the initial stage F.V.
Step 2 : Draw line $a'b'$ perpendicular to XY line keeping a' and b' on their respective projectors and construct the rectangle $a'b'c'd'$ having side $b'c' = 65$ mm keeping points c' and d' on their projectors.
Step 3 : Now draw two projectors through points b' and c' in the F.V. and redraw the initial stage T.V. keeping ab on the projector of b' and cd on the projector of c' .

Final position :

Step 1 : Since smaller side AB is making an angle of 45° with the H.P., redraw the F.V. of the intermediate stage such that side $a'b'$ is making an angle of 45° with the XY line.
Step 2 : Project the points of the final stage F.V. (i.e. a' , b' , c' and d') vertically down. Again project the points of the intermediate stage T.V. (i.e. a , b , c and d) parallel to the XY line and to the right to intersect vertical projectors from final stage F.V. and locate points a , b , c and d .
Step 3 : Join points a , b , c and d in proper sequence to get the T.V. of final position

Prob. 66 : A rectangular plane ABCD with side $AB = 40 \text{ mm}$ and $BC = 60 \text{ mm}$ has its side AB on V.P. and side BC making an angle of 30° with H.P. Draw the projections of the rectangle if its F.V. appears to be square.

Soln. : Refer Fig. Prob. 66.

Initial position :

Step 1: Since elevation of the rectangle is showing its apparent shape, it is inclined to V.P. with its side AB on the V.P. So initially we will assume the plane to be kept on the V.P. with side AB perpendicular to H.P. (i.e. XY).

Step 2: Draw the F.V. $a' b' c' d'$ of given dimensions keeping side $a' b'$ perpendicular to the XY line and project the T.V. $ab - cd$ which will be line view coinciding with the XY line.

Intermediate position :

Step 1: Draw projectors parallel to the XY line and to the right, from corners a', b', c' and d' of the initial stage F.V.

Step 2: Draw line $a'b'$ perpendicular to XY line keeping a' and b' on their respective projectors and construct the square $a'b'c'd'$ keeping points c' and d' on their projectors.

Step 3: Now draw two projectors vertically down through points b' and c' in the F.V. and redraw the first stage T.V. keeping ab on the projector of b' and cd on the projector of c' .

Final position :

Step 1: Since the longer side, BC is making an angle of 30° with the H.P. but it is seen in its apparent length in the intermediate stage F.V. Hence we will calculate α_{bc} as shown in Fig. Prob. 66. Redraw the F.V. of the intermediate stage such that elevation of longer side BC is making an angle of $\alpha_{b'c'}$ with the XY line.

Step 2: Project the points of the final stage F.V. (i.e. a', b', c' and d') vertically down. Again project the points of the intermediate stage T.V. (i.e. a, b, c and d) parallel to the XY line and to the right to intersect vertical projectors from final stage F.V. and locate points a, b, c and d .

Step 3: Join points a, b, c and d in proper sequence to get the T.V. of final position.

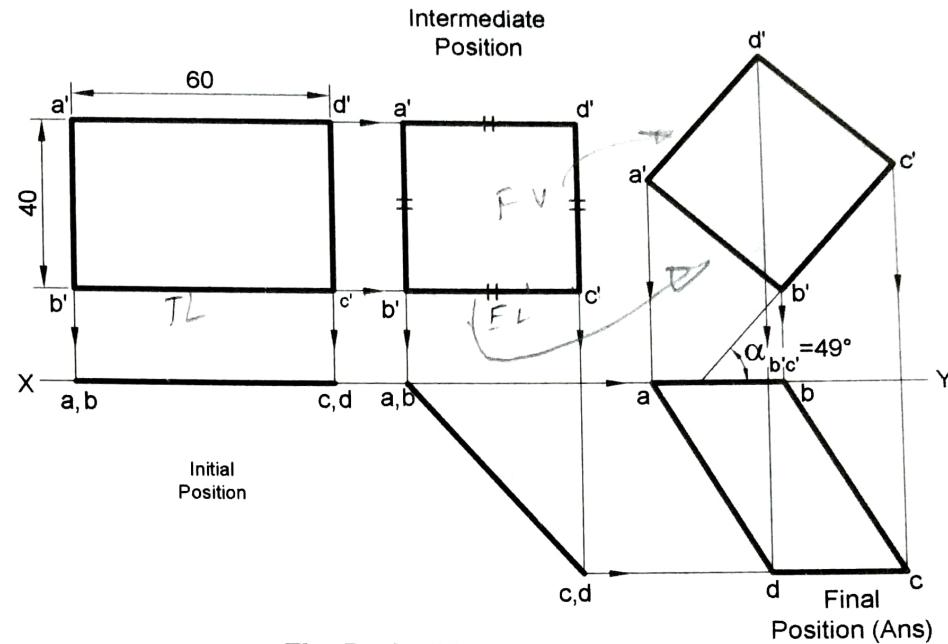
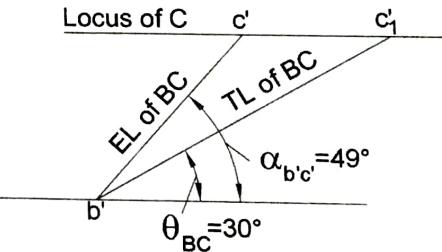


Fig. Prob. 66



4.6.5 Projections of Circular and Semicircular Planes :

Prob. 87 : A circular plate of diameter 60 mm is kept on the H.P. on a point of its circumference. The surface of the circular plate makes an angle of 40° to the H.P. Draw the projections of the circle when diameter passing through the point on H.P. makes an angle of 30° to V.P.

Soln. : Refer Fig. Prob. 87.

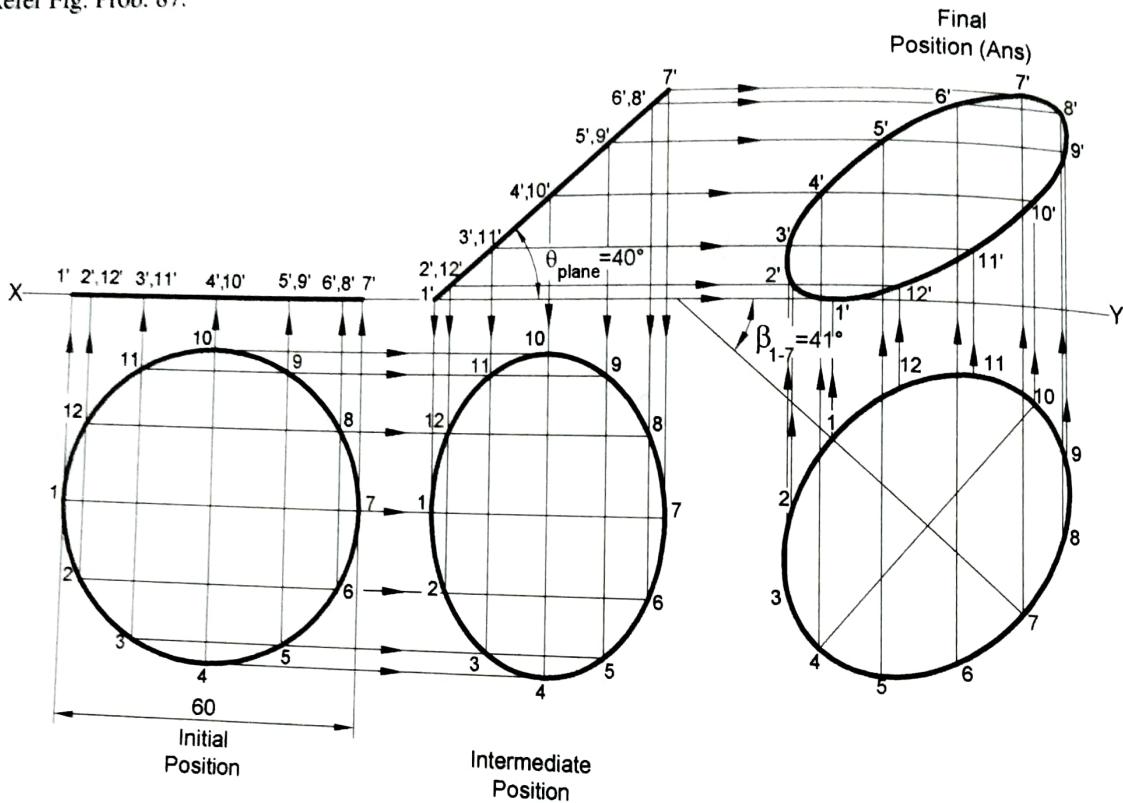


Fig. Prob. 87

Initial position :

Step 1 : Since circular plate is inclined to H.P. we will assume it to be kept on H.P. initially.

Step 2 : Draw a circle of diameter 60 mm in the T.V. Divide this circle into 12 equal parts and name it as 1, 2, 3, ... 12. Project points 1, 2, 3, ... 12 to the F.V. as 1', 2', 3', ... 12'. Keep diameter 1-7 parallel to the XY line in the T.V.

Intermediate position :

Step 1 : Redraw the F.V. of first stage at an angle of $\theta_{plane} = 40^\circ$ with the XY line keeping 1' on the XY line.

Step 2 : Project the points 1', 2', 3', ... 12' from the F.V. vertically down and project the points 1, 2, 3, ... 12 from the first stage T.V. parallel to XY line and to the right to get the same points in the second stage T.V.

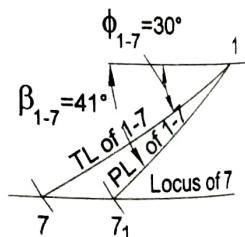
Step 3 : Join points 1, 2, 3, ... 12 in a proper sequence by smooth curve in the second stage T.V.

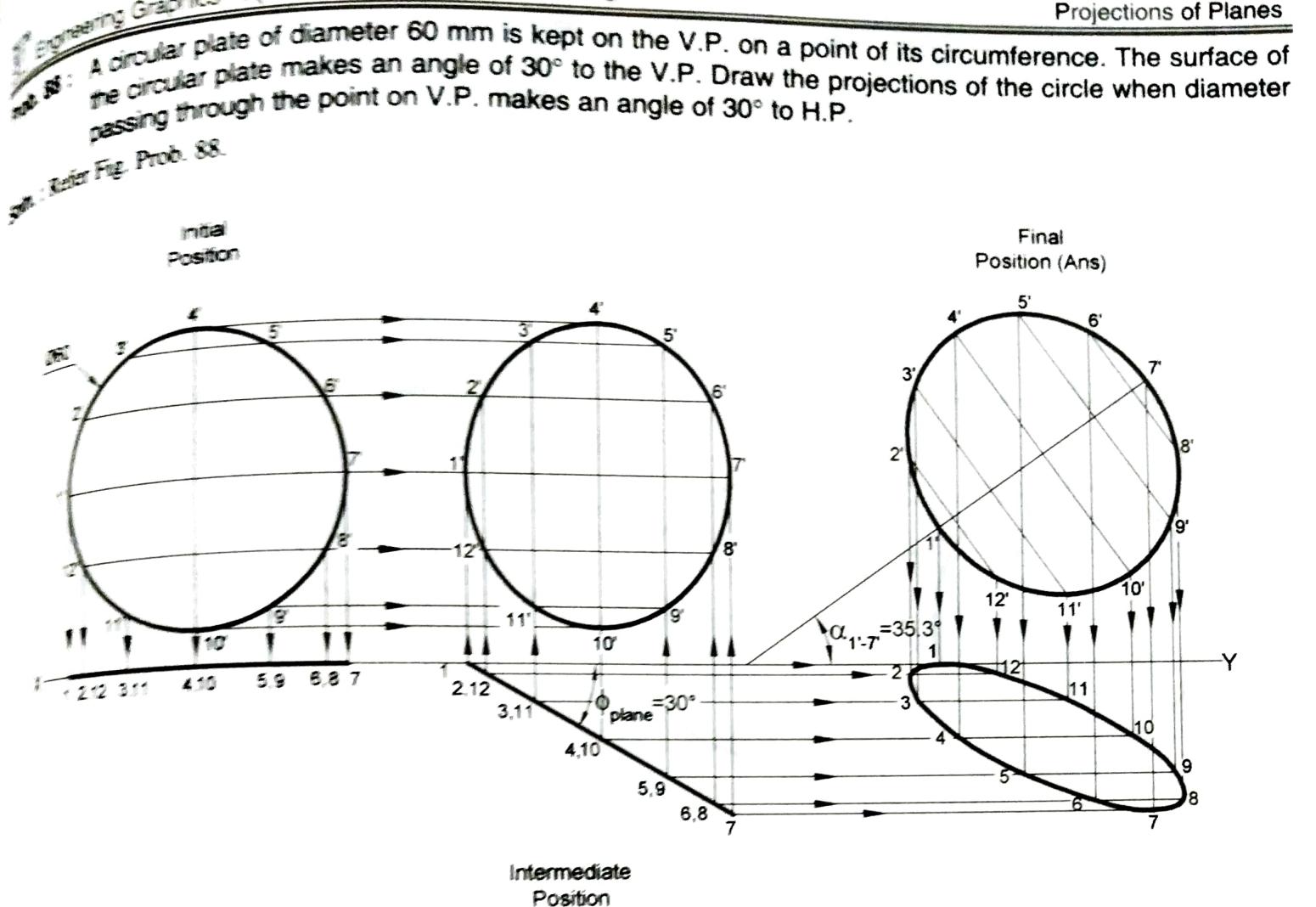
Final position :

Step 1 : Diameter 1-7 is making an angle of 30° with the V.P. but it is seen in its apparent length in the second stage T.V. Hence we will calculate β_{1-7} as shown in Fig. Prob. 87. Redraw the T.V. of the second stage such that plan of diameter 1-7 is making an angle of β_{1-7} with the XY line.

Step 2 : Project the points of the third stage T.V. (i.e. 1, 2, 3, ... 12) vertically up. Again project the points of the second stage F.V. (i.e. 1', 2', 3', ... 12') parallel to the XY line and to the right to intersect vertical projectors from third stage T.V. and locate points 1', 2', 3', ... 12'.

Step 3 : Join points 1', 2', 3', ... 12' in proper sequence by a smooth curve to get the final answer.



**Fig. Prob. 88****Initial position :**

(i) Since circular plate is inclined to V.P., we will assume it to be kept on V.P. initially.

(ii) Draw a circle of diameter 60 mm in the F.V. Divide this circle into 12 equal parts and name it as $1', 2', 3', \dots, 12'$. Project points $1', 2', 3', \dots, 12'$ to the T.V. as $1, 2, 3, \dots, 12$. Keep diameter $1'7'$ parallel to the XY line in the F.V.

Intermediate position :

(i) Redraw the T.V. of first stage at an angle of $\phi_{plane} = 30^\circ$ with the XY line keeping point '1' on the XY line.

(ii) Project the points $1, 2, 3, \dots, 12$ from the T.V. vertically up and project the points $1', 2', 3', \dots, 12'$ from the first stage F.V. parallel to XY line and to the right to get the same points in the second stage F.V.

(iii) Join points $1', 2', 3', \dots, 12'$ in a proper sequence by smooth curve in the second stage F.V.

Final position :

(i) Diameter $1-7$ is making an angle of 30° with the H.P. but it is seen in its apparent length in the second stage F.V. Hence we will calculate α_{1-7} as shown in Fig. Prob. 88. Redraw the F.V. of the second stage such that elevation of diameter $1'7'$ is making an angle of α_{1-7} with the XY line.

(ii) Project the points of the third stage F.V. (i.e. $1', 2', 3', \dots, 12'$) vertically down. Again project the points of the third stage F.V. (i.e. $1', 2', 3', \dots, 12'$) parallel to the XY line and to the right to intersect vertical projectors from

