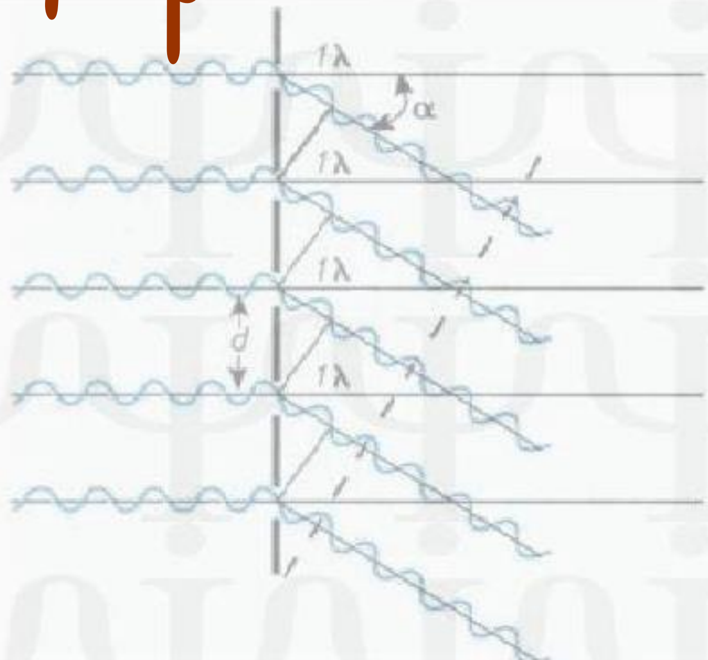
Wave nature of particles



$$E\Psi = -\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\Psi}{\mathrm{d}x^2} + V\Psi$$

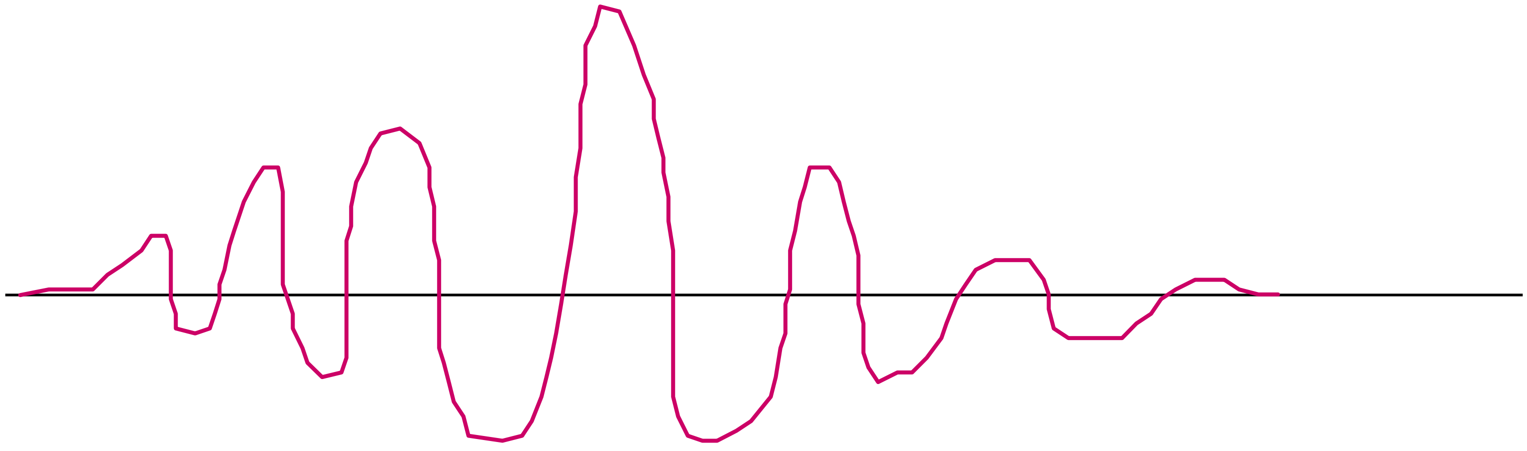
Heisenberg Uncertainty Principle

b) A **particle** does have a definite location at a specific time, but it does not have a frequency or wavelength.

c) **In between case**: a group of sine waves can add together (via Fourier analysis) to give a semi-definite location: a result of Fourier analysis is this: the more the group shows up as a spike, the more waves it takes to make the group.

Heisenberg Uncertainty Principle

A rough drawing of a sample in between case, where the wave is somewhat localized, and made up of several frequencies.



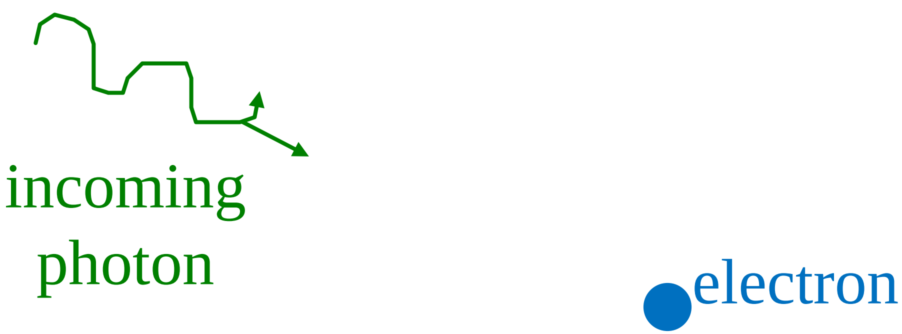
Heisenberg Uncertainty Principle

Let’s look at **how this works in practice**.

Consider trying to locate an electron somewhere in space. You might try to “see” the electron by hitting it with a photon. The next slide will show an idealized diagram, that is, it will show a diagram assuming a definite position for the electron.

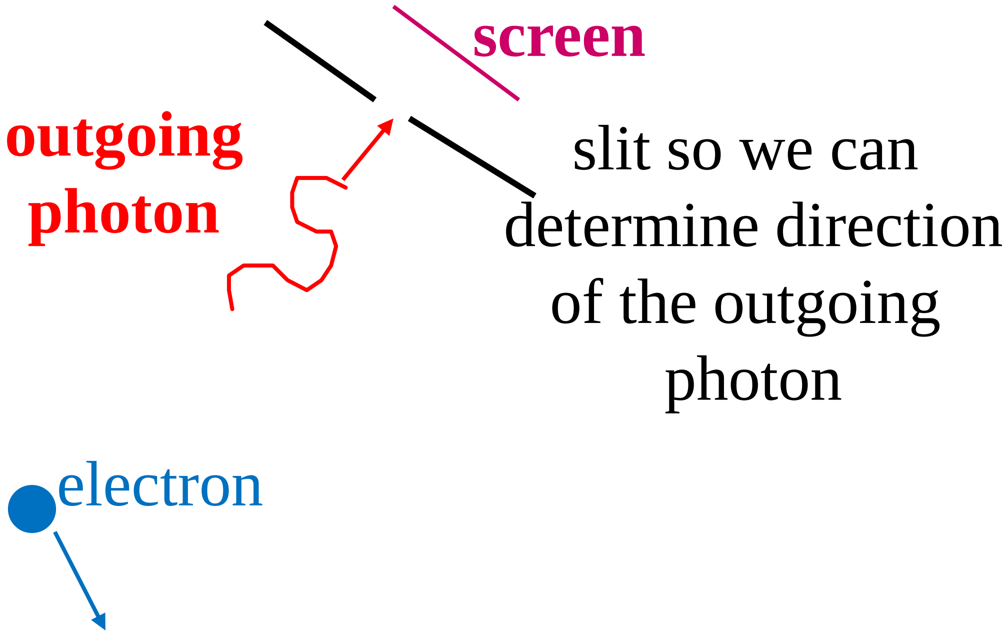
Heisenberg Uncertainty Principle

We fire an incoming photon at the electron, have the photon hit and bounce, then trace the path of the outgoing photon back to see where the electron was.



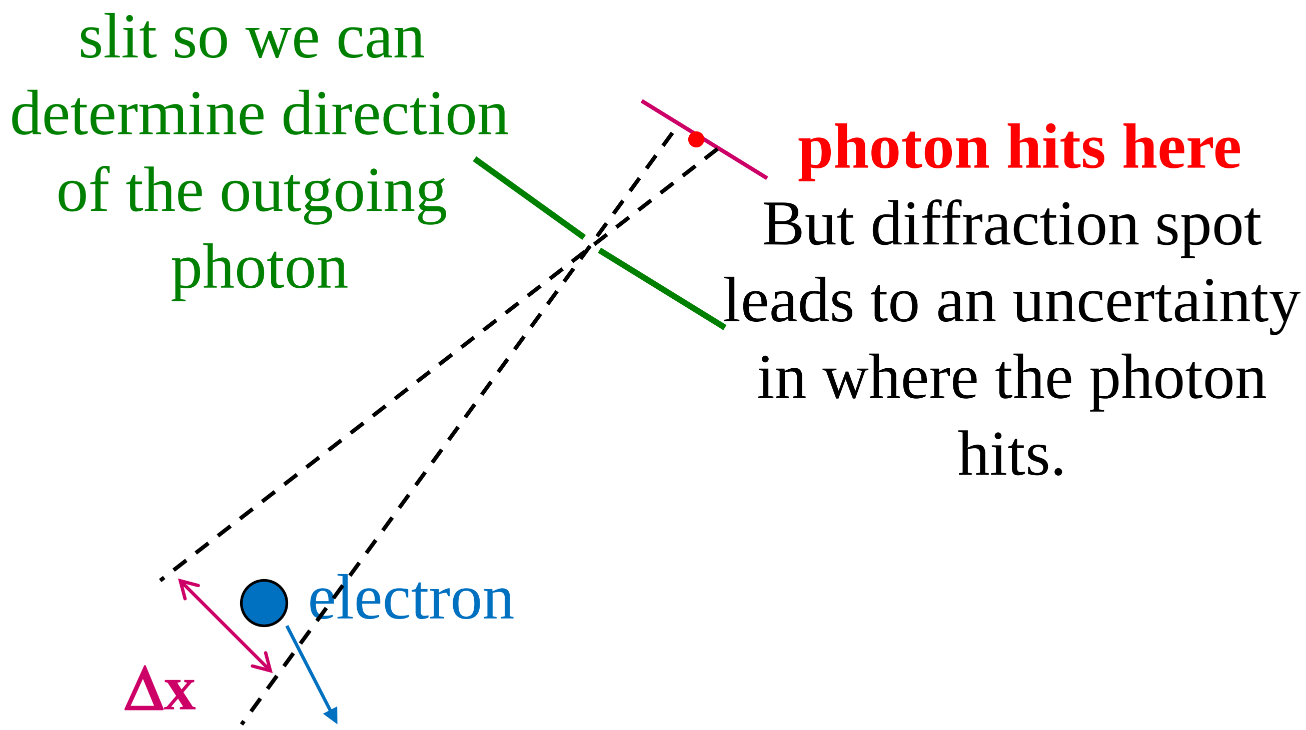
We would have to fire a lot of photons, but only the one photon that hits the electron will matter.

Heisenberg Uncertainty Principle



Heisenberg Uncertainty Principle

Here the wave-particle duality creates a problem in determining where the electron was.



Heisenberg Uncertainty Principle

If we make the slit narrower to better determine the direction of the photon (and hence the location of the electron), the wave nature of light will cause the light to be diffracted more. This bigger diffraction pattern will cause more uncertainty in where the photon actually came from, and hence more uncertainty in where the electron was .

Heisenberg Uncertainty Principle

We can reduce the diffraction angle if we reduce the wavelength (and hence increase the frequency and the energy of the photon).

But if we do increase the energy of the photon, the photon will hit the electron harder and make it move more from its location, which will increase the uncertainty in the momentum of the electron.

Heisenberg Uncertainty Principle

The quantum mechanics being discussed here is essentially a science of motions of subatomic entities.

Mechanics is a discipline which describes the motions of the objects.

The parameters which are typically used for describing the motion of an object are position (x), momentum (p), energy (E) and time (t).

In classical mechanics, which is based on Newton's second law of motion, the method to obtain these parameters is as follows

For determining the motion of any object (i.e. for determining x, p, E and t), the force acting on the object should be known. This force should be substituted in the equation of motion,

Heisenberg Uncertainty Principle

$$F = ma$$

$$a = \frac{F}{m}$$

$$\frac{dv}{dt} = \frac{F}{m}$$

$$v = \int \frac{F}{m} dt$$

$$p = mv$$

$$E = \frac{p^2}{2m}$$

$$x = \int v dt$$

What is mentioned is a simplified method of classical mechanics used for determining the motion of an object.

If the required mathematics is followed precisely, there seems to be no reason why the quantities like x, p, E and t should not be determinable accurately, precisely (or deterministically).

Can similar approach be followed for subatomic particles?

Certainly not!

The fundamental reason behind this is that all <u>subatomic particles behave like waves</u> (previously we have seen that because of extremely small value of the Planck's constant, the wavelike properties of day to day objects are negligible).

Heisenberg Uncertainty Principle

The simultaneous measurement of two conjugate variables (such as the momentum and position or the energy and time for a moving particle) entails a limitation on the precision (standard deviation) of each measurement.

The more precise the measurement of position, the more imprecise the measurement of momentum, and vice versa. In the most extreme case, absolute precision of one variable would entail absolute imprecision regarding the other.

Heisenberg Uncertainty Principle

Consider an electron having

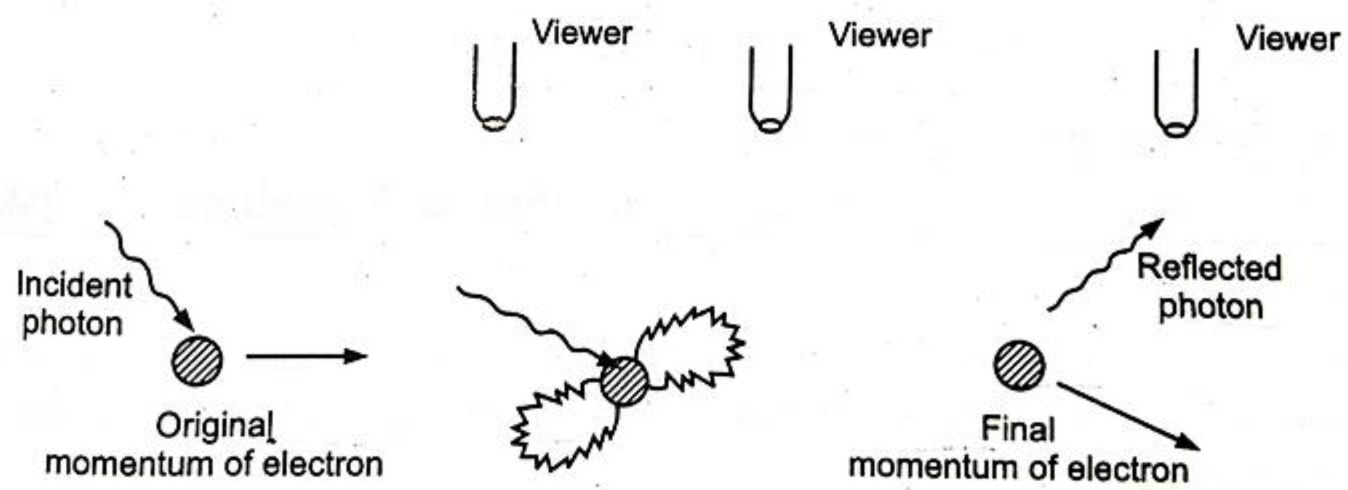
Mass - **m**

Wavelength of associated matter wave – λ **and electro can be found somewhere within this wave**

Uncertainty in its position – $\Delta x = \lambda$ **……….Equation 1**

Illuminate with light (Photon) of wavelength – λ

Photon momentum - $\frac{h}{\lambda}$



Heisenberg Uncertainty Principle

Uncertainty in the momentum = Incident Photon momentum

$$\Delta p = \frac{h}{\lambda}$$ ……….Equation 2

From Equation 1 and 2

$$\Delta x \,.\, \Delta p = \lambda \,.\, \frac{h}{\lambda} = h$$

More correctly

$$\Delta x \,.\, \Delta p \geq \hbar = \frac{h}{2\pi}$$

Heisenberg Uncertainty Principle

Consider an electron having

Mass - **m**

Moving with velocity – **v**

Kinetic Energy of electron as $E = \frac{1}{2} mv^2$ **……….Equation 1**

By differentiating the equation the uncertainty in the energy measurement **Δx** can be found

$$\Delta E = \frac{1}{2} m \cdot 2 v \Delta v$$

$$\Delta E = v (m \cdot \Delta v)$$

$$\Delta E = v \cdot \Delta p$$

$$\Delta E = \frac{\Delta x}{\Delta t} \cdot \Delta p \qquad (\because v = \frac{\Delta x}{\Delta t})$$

$$\Delta E \cdot \Delta t = \Delta x \cdot \Delta p$$

$$\Delta E \,.\, \Delta t \geq \hbar = \frac{h}{2\pi}$$