Session35:

TRACING OF POLAR CURVES

The following rules will help in tracing a Polar curve.

Rule 1:<u>Symmetry</u>

- ***(a) Symmetry about pole*:** If the equation of the curve remains unchanged by replacing $r\ by\ -r,$ then curve is symmetric to the pole.
- ***(b) Symmetry about initial line*:** If the equation of the curve remains unchanged by replacing $\theta\ \ by -\theta$, then curve is symmetric about the initial line.

(c) *Symmetry about* $\theta = \frac{\pi}{2}$:

1. If the equation of the curve remains unchanged by replacing $\theta\ by -\theta$

and $r\ by - r\ respectively$, then curve is symmetric about the line $\theta = \frac{\pi}{2}$.

2. If the equation of the curve remains unchanged by replacing $\theta\ by\ \ \pi - \theta$

then curve is symmetric about the line $\theta = \frac{\pi}{2}$.

Rule 2: <u>Pole:</u> If for some value of θ, r becomes zero then the pole will lie on the curve.

Rule 3:<u>Tangents:</u> To find tangents at the pole, put r = 0 in the equation, the values of θ gives the tangent at the pole.

Rule 4:<u> Angle between radius vector and tangent $[\boldsymbol{\phi}]$</u> :

Use the formula $\tan\phi = r\frac{d\theta}{dr}$ and find ϕ and also the points where $\phi = 0\ or\ \infty$.

Rule 5:<u> Form the table showing values of r for some values of θ</u>

Rule 6<u>: Find the region of absence of the curve.</u>

Q1. Trace the following curve: $r^2 = a^2\cos 2\theta$

Solution: We check the following points for tracing of the above curve

1. **Limit**: - $|r| \le a$ i. e. the curve lies between $r = -a$ *to* $r = a$.
2. **Symmetry:-**
   (i) About the Pole:-
   If we replace r by $-r$, then the equation of the curve is remains unchanged.
   $\therefore$ The curve is symmetry about pole.
   (ii) About initial line $\theta = 0$:-
   If we replace θ by $-\theta$, then the equation of the curve is remains unchanged.
   $\therefore$ The curve is symmetry about the initial line $\theta = 0$.
   (iii) About the line perpendicular to the initial line at pole or about the line $\theta = \pi/2$:-
   If we replace r by $-r$ and θ by $-\theta$, then the equation of the curve is remains unchanged.
   $\therefore$ The curve is symmetry about the line perpendicular to the initial line at pole or about the line $\theta = \pi/2$.
3. **Pole:-**
   (i) For $\theta = \dfrac{\pi}{4}$, $r = 0$.
   Hence the curve passes through the pole.

   (ii) Tangent at Pole:- If we put $r = 0$, then we get the tangent at pole.
   Putting $\mathrm{r} = 0$ in (1), we have

$$a^2 \cos 2\theta = 0$$

$$\Rightarrow \quad \cos 2\theta = 0 \qquad [\because a \ne 0]$$

$$\Rightarrow \quad 2\theta = \cos^{-1} 0 = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

$$\Rightarrow \quad \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

4. **Tangent:-**

$$\tan\phi = r\frac{d\theta}{dr} = \frac{r}{\frac{dr}{d\theta}} = \frac{r}{-\frac{\cancel{2}a^2 \sin 2\theta}{\cancel{2}r}} = -\frac{r^2}{a^2 \sin 2\theta} = -\frac{\cancel{a^2} \cos 2\theta}{\cancel{a^2} \sin 2\theta} = -\cot 2\theta = \tan\left(\frac{\pi}{2} + 2\theta\right)$$

5. **Asymptotes:-**No asymptotes.
6. **Table values:-**

θ	0	$\pi/4$	$\pi/2$
r	a	0	Imaginary
$r\,d\theta/dr = \tan\phi$	∞ *i.e.* $\phi = \dfrac{\pi}{2}$	$\phi = \pi$	$\phi = 3\pi/2$

It is clear that at $\theta = 0$, $r = a$, and the tangent is perpendicular to the initial line at $(a,\ 0)$ and $(-a,\ 0)$. Again at $\theta = \pi/2$, r is imaginary. Hence there is no part of the curve between $\pi/4$ to $3\pi/4$. Also the curve is symmetry about pole, initial line and the line perpendicular to initial line. Hence the approximate shape of the curve is as follows: