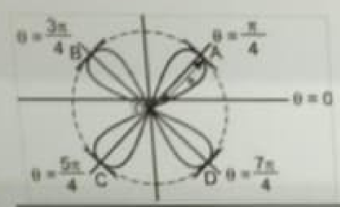
Tag to Revisit

0 of 15 answered

1 Points Reset Next >

The following figure represents the curve whose equation is



- $r = a\cos 3\theta$
- $r = a(1+\cos\theta)$
- $r = a\sin 2\theta$
- $r = a\sin 3\theta$

Tag to Revisit

0 of 15 answered

1 Points Reset Next >

The solution of the differential equation $\frac{dy}{dx} = e^{x+y} + x^2e^y$

- $e^y + e^x + \frac{x^3}{3} + c = 0$
- None of these
- $e^{-y} + e^x + \frac{x^3}{3} + c = 0$
- $e^{-y} + e^x - \frac{x^3}{3} + c = 0$

Tag to Revisit

0 of 15 answered

1 Points Reset Next

The points of intersection with Y & X-axis of the curve $y^2x = a(x^2 - a^2)$

- No point of intersection on both axes
- No point on Y-axis & $(\pm a, o)$
- No point on X-axis & $(0, \pm a)$
- ✓ None of these

Finish Remaining 00:28:58

Tag to Revisit

0 of 15 answered

1 Points Reset Next

If the differential equation of family of curves $r^2 = a \sin 2\theta$ is $\frac{dr}{d\theta} = r \cot 2\theta$ then its orthogonal trajectories is given by (2)

- $r^2 = \log \sec 2\theta + k$
- $r^2 = k \sin 2\theta$
- $r^2 = k \cos 2\theta$
- $\log r = -\frac{1}{2} \sec^2 2\theta + k$

Tag to Revisit

The region of absence for the curve represented by the equation
$a^2x^2 = y^3(2a-y)$

- ☐ $y<0$ and $y<2a$
- ☐ $y>0$ and $y<2a$
- ☐ $y<0$ and $y>2a$
- ☐ $x<0$ and $y<2a$

Tag to Revisit

The curve represented by the equation $x = a(t-\sin t)$, $y = a(1-\cos t)$ is

- ☐ symmetric about x axis and not passing through origin.
- ☐ symmetric about y axis and not passing through origin.
- ☐ symmetric about y axis and passing through origin.
- ☐ symmetric about x axis and passing through origin.

Tag to Revisit

The tangent at the origin to the curve represented by the equation $x = a(t - \sin t)$, $y = a(1 - \cos t)$ is

- x=a
- y=a
- x=0
- y=0

Tag to Revisit

The D.E. $(x + y - 5)dx + (x - y + 4)dy$ is

- Homogeneous
- Linear
- Exact
- Non Exact

Tag to Revisit

1 Points

The equation of tangent to the curve at origin represented by the equation $y^2(4-x)=x(x-2)^2$ is

- ☐ x=2
- ☐ x=0
- ☐ y=0
- ☐ x=4

Tag to Revisit

1 Points Reset Next

The differential equation for the current i in an electric circuit containing resistance 100 ohm and an inductance of 0.5 henry connected in series with battery of 20 volts is (1)

- ○ $0.5\frac{di}{dt}+100i=0$
- ○ $0.5\frac{di}{dt}+100i=20$
- ○ $100\frac{di}{dt}+0.5i=20$
- ○ $100\frac{di}{dt}+0.5R=0$

Tag to Revisit

1 Points Reset

If $I = \frac{E}{R}(1 - e^{\frac{-Rt}{L}})$ & E=500volts R=250Ω L=640 H. Then maximum value of I is

- 0.5
- 2
- 0
- None of these

Finish

Tag to Revisit

1 Points Reset

The region of presence for the curve represented by the equation
$x = t^2,\ y = t - \frac{t^3}{3}$

- $x<0$
- $x>0$
- $y>0$
- $y<0$

Tag to Revisit

The angle between the radius vector & the tangent to the curve

$$r = \frac{a}{2}(1 + cos\theta)$$

- $\frac{\pi}{2} - \frac{\theta}{2}$
- $\pi - \frac{\theta}{2}$
- $\frac{\pi}{2} + \frac{\theta}{2}$
- $\pi + \frac{\theta}{2}$

1 Points Reset Next

Tag to Revisit

The charge Q on the plate of the condenser of capacity C charged through a resistance R by a steady voltage V satisfy the differential equation $R\frac{dQ}{dt} + \frac{Q}{C} = V$.If Q=0 at t=0 then$Q = CV(1 - e^{\frac{-t}{RC}})$. Then maximum current is

- CV
- None of these
- $\frac{V}{R}$
- 0

Tag to Revisit

The G.S. of $\log \frac{dy}{dx} = ax + by$

- $ae^{-by} + be^{-ax} + c = 0$
- $ae^{-by} + be^{ax} + c = 0$
- $ae^{by} + be^{ax} + c = 0$
- NONE

Tag to Revisit

Let P is any point on the curve & if $(\frac{dy}{dx})_p > 0$ then

- Tangent makes obtuse angle with x
- Tangent parallel to y-axis
- Tangent makes acute angle with x-axis
- Tangent parallel to x-axis

Tag to Revisit

1 Points Reset Next

The integrating factor of the D.E. $\frac{dy}{dx} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}$

- $2\sqrt{x}$
- ✓ $e^{2\sqrt{x}}$
- $e^{-2\sqrt{x}}$
- $e^{\frac{1}{\sqrt{x}}}$

Tag to Revisit

The curve $a^2y^2 = a^2x^2 - x^4$ has

- ☐ two asymptotes
- ☐ one asymptote
- ☐ origin is node
- ☐ origin is cusp

Tag to Revisit

1 Points

Reset

The tangents at pole to the polar curve $r = a\cos 2\theta$ are

- [ ] $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots\ldots$
- [ ] $\theta = 0, \pi, 2\pi, 3\pi, \ldots\ldots$
- [ ] $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots\ldots$
- [ ] $\theta = \frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \ldots\ldots$

Tag to Revisit

1 Points

The angle ϕ between radius vector and tangent line using $\tan\phi = r\dfrac{d\theta}{dr}$ for the polar equation $r^2 = a^2\cos 2\theta$ is equal to

- [ ] $\dfrac{\pi}{4} + 2\theta$
- [ ] $\pi + 2\theta$
- [ ] $\dfrac{\pi}{2} + \theta$
- [ ] $\dfrac{\pi}{2} + 2\theta$

A steam pipe 20 cm in diameter is protected with covering 6 cm thick for which thermal conductivity k = 0.0003 in steady state. The inner surface of the pipe is at 200°C and outer surface of the covering is at 30°C and differential equation of conduction of heat is $dT = -\frac{Q}{2\pi k}\frac{dx}{x}$. The amount of heat loss Q is (2)

- $\dfrac{170\,(2\pi k)}{\log(1.6)}$
- $-\dfrac{170\,(2\pi k)}{\log(1.6)}$
- $\dfrac{\log(1.6)}{170\,(2\pi k)}$
- 170

Tag to Revisit

The D.E. of orthogonal trajectory of $r = a\cos\theta$ is $\frac{dr}{r} = \cot\theta d\theta$ then orthogonal trajectory is

- $r = c\sin\theta$
- $r = c\cos\theta$
- $r = \cos\theta$
- None

Untag

1 Points Rese

The differential equation $(x^3 + 3y^2x)dx(y^3 + 3x^2y) = 0$

- Only Exact
- Exact and Homogeneous
- Only Homogeneous
- None

Tag to Revisit

1 Points Reset

The integrating factor of the differential equation $(x^4e^x - 2mxy^2)dx + 2mx^2ydy = 0$

$\int \frac{-4}{x} dx$

e

- $\frac{4}{x}$
- 4x
- $\frac{-4}{x}$
- Exact so factor is 1

1 Points Reset

The equation of tangent to the curve at origin represented by the equation $y = x(x^2 - 1)$ is

- [ ] x=0
- [ ] y=x
- [ ] y=o
- [ ] y= -x

1 Points Res

The charge flowing through the R-C series cct with no applied E.M.F is

- [ ] $Q = e^{\frac{t}{RC}} K$ K=constant
- [ ] None of these
- [ ] $Q = e^{\frac{-t}{RC}} K$ K=constant
- [ ] $Q = e^{-tRC} K$ K=constar

1 Points Reset Next

The linear form of D.E. $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$ by putting $e^y = v$

- $\frac{dv}{dx} + e^x = e^{2x}$
- $\frac{dv}{dx} + ve^x = e^{2x}$
- $\frac{dv}{dx} + ve^x = e^x$
- None

Tag to Revisit

1 Points

The tangent at the origin to the curve represented by the equation $x=t^2,\ y=t-\dfrac{t^3}{3}$ is

- [ ] y=x
- [ ] y=-x
- [ ] y=x
- [ ] y=0

RESET

NEXT

1 Points

Tag to Revisit

The equation of tangents to the curve at origin represented by the equation $x^2(x^2-4a^2)=y^2(x^2-a^2)$, where $a>0$ is

- ☐ $y=2x, y=-2x$
- ☐ $x=a, x=-a$
- ☐ $x=2a, x=-2a$
- ☐ $y=x, y=-x$

Asymptote parallel to X-axis to the curve $x^2(x^2-4a^2)=y^2(x^2-a^2)$

- ☐ $\pm a$
- ☐ No Asymptote
- ☐ a
- ☐ None of these

Tag to Revisit

1 Points

The equation of asymptotes parallel to $y-$axis to the curve represented by the equation $y(1+x^2)=x$ is

- [ ] x=0
- [ ] $x=1,\ x=-1$
- [ ] y=1
- [ ] none of the above

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Tag to Revisit

1 Points Reset

Let I be the current flowing in the circuit containing inductance L & capacitance C in a series without applied e.m.f. E then the differential equation is

- None of these
- $L\frac{dI}{dt} + \frac{q}{C} = E$ ✓
- $L\frac{dI}{dt} + \frac{q}{C} = 0$
- $L\frac{dI}{dt} - \frac{q}{C} = 0$

---

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Tag to Revisit

1 of 15

1 Points Reset

The differential equation of orthogonal trajectories of family of curves $2x^2 + y^2 = cx$ is (1)

- $4x + 2y\frac{dy}{dx} = \frac{2x^2 + y^2}{x}$
- $4x - 2y\frac{dx}{dy} = \frac{2x^2 + y^2}{x}$ ✓
- $4x + 2y\frac{dx}{dy} = \frac{2x^2 + y^2}{x}$
- none of these

---

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Tag to Revisit

1 Points

The solution of D.E. $(x^2 + e^x)dx + ydy = 0$

- $\frac{x^3}{3} + e^x + \frac{y^2}{2} = c$ ✓
- $x^3 + e^x + y^2 = c$
- $x^3 + e^x + \frac{y^2}{2} = c$
- None of these

Tag to Revisit

The differential equation of orthogonal trajectories of family of straight lines $y = mx$ is (1)

- $\frac{dy}{dx} = \frac{y}{x}$
- $\frac{dx}{dy} = -\frac{y}{x}$
- $\frac{dx}{dy} = -\frac{x}{y}$
- $\frac{dy}{dx} = m$

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Tag to Revisit

The integrating factor for the differential equation of R-L series circuit is

- $e^{-\frac{Lt}{R}}$
- $e^{-\frac{Rt}{L}}$
- $e^{\frac{Lt}{R}}$
- $e^{\frac{Rt}{L}}$

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Tag to Revisit

1 Points Reset

The curve $r = acos5\theta$ can be obtained from $r = asin5\theta$ by rotating plane through

- 5π
- 10π
- $\frac{\pi}{5}$
- $\frac{\pi}{10}$

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Tag to Revisit

1 Points

Region of existence of the curve $y^2 = \frac{a^2(a-x)}{x}$

- None of these
- $x < 0, \quad x < a$
- $0 < x < a$
- $x > 0, \ x > a$

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Tag to Revisit

1 Points

$2\frac{dy}{dx} - ysecx = y^3tanx$ linear form of these equation is

- [ ] $\frac{du}{dx} + (secx)u = tanx$
- [ ] $\frac{du}{dx} - (secx)u = tanx$
- [ ] $\frac{du}{dx} + (secx)u = -tanx$
- [ ] $\frac{du}{dx} - (secx)u = -tanx$

Tangent at origin to the curve $r = a\cos 3\theta$

- [ ] $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \ldots$
- [ ] None of these
- [ ] $0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi \ldots$
- [ ] $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6} \ldots$

Tag to Revisit

1 Points Reset Finish

The G.S of D.E $x^3 \frac{dy}{dx} = secy$ is

- $siny + \frac{1}{x^2} = c$
- $cosy + \frac{1}{2x^2} = c$
- $cosy - \frac{1}{2x^2} = c$
- $siny + \frac{1}{2x^2} = c$

Ordinary Differential Equations

1. Form a differential equation whose general solution is

$i)\ y = ae^{-2x} + be^{-3x}$ **(Ans : $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$)**

$ii)\ y = e^{x}(Acosx + Bsinx)$ **(Ans : $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$)**

Solve the differential equations-

1. $\frac{dy}{dx} = \frac{x\ sinx}{2e^{y}sinhy}$ **(Ans: $\frac{e^{2y}}{2} - y + xcosx - sinx = C$)**
2. $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$ **(Ans : log[1 + tan($\frac{x+y}{2}$)] – x = C)**
3. $\frac{dy}{dx} = \frac{x^3-y^3}{yx^2}$ **(Ans: cy = $e^{\frac{-x^3}{3y^3}}$)**
4. (x + y cot $\frac{x}{y}$)dy – y dx =0 (Ans : y cos $\frac{x}{y}$=C)
5. $\frac{dy}{dx} = \frac{\tan y - 2xy - y}{x^2 - x\tan^2 y + \sec^2 y}$ (Ans : x.tany – xy – x^2y – tany = C)
6. $ylogy\ dx + (x - \log y)dy = 0$ **(Ans :2xlogy – (logy)^2= C)**
7. $(2y + y^4)dx + (2y^4 + xy^3 - 4x)dy = 0$ **(Ans : $xy + \frac{x}{y^2} + y^2 = C$)**
8. $xcos\ x\ \frac{dy}{dx} + (\cos x - xsin\ x)y = 1$ **(Ans : $xycosx - x = C$)**
9. $\frac{dy}{dx} - xy = -y^3e^{x^2}$ **(Ans : $\frac{e^{x^2}}{y^2} = 2x + C$)**
10. $(y - 2x^3)dx - x(1 - xy)dx = 0$ **(Ans: $\frac{y}{x} + x^2 - \frac{y^2}{2} = C$)**
11. $(x^2y - 2xy^2)dx = (x^3 - 3x^2y)dy$ **Ans : $\frac{x}{y} - 2logx + 3logy = C$**
12. $ye^{y}dx = (y^3 + 2xe^{y})dy$ **(Ans : $\frac{x}{y^2} + e^{-y} = C$)**
13. $sin\ y\ \frac{dy}{dx} - cosx(2cosy - \sin^2 x)\ y = 0$
(Ans : $4cosy = 2\sin^2 x + 2sinx = 1 = Ce^{-2sinx}$)
14. $\cos y - xsin\ y\ \frac{dy}{dx} = \sec^2 x$ **(Ans: $xcosy = tanx + C$)**
15. $\left(xy^2 - e^{\frac{1}{x^3}}\right)dx + x^2\ y\ dy = 0$ **(Ans : $\frac{3y^2}{x^2} - 2e^{-x^{-3}} = C$)**

APPLICATIONS OF DIFFERENTIAL EQUATIONS

Orthogonal Trajectories

13. Find the orthogonal trajectories of the family of $y^2 = 4ax$. **(Ans : $2x^2 + y^2 = C$)**

14. Find the orthogonal trajectory of the family of ellipses $\frac{1}{2}x^2 + y^2 = C$, where C > 0 **(Ans : $x^2 = ky$)**

[**Ref: Kreyszig, page-36**]

15. Find the orthogonal trajectory of the family of r = a cos θ. (Ans : $r = Csin\theta$)

16. Find the orthogonal trajectory of the family of r = a (1- cos θ). (Ans: r = C(1+cosθ))

17. Find the orthogonal trajectory of the family of $e^x + e^{-y} = c$. (Ans: $e^y - e^{-x} = C$)

Electric Circuits

20. An electric circuit contains an inductance of 0.5 henry and a resistance of 10 ohms in series with electro motive force of 20 volts. Find the current at any time t, it is zero at t=0. (Ans : $\frac{1}{5}(1 - e^{-200t}$)

21. A circuit consist of resistance R ohms and condenser C farads connected to constant electromotive force E, if $\frac{q}{C}$ is the voltage of condenser at time t after closing the circuit. Show that the voltage at time t, is $E\left(1 - e^{-\frac{t}{RC}}\right)$. Also find current flowing into the circuit.

22. The charge Q on the plate of a condenser of capacity $'C'$ charged through a resistance $'R'$ by steady voltage $'V'$ satisfies the differential equation $R\frac{dQ}{dt} + \frac{Q}{C} = V$. If $Q = 0$ at $t = 0$ then show that $Q = CV\left[1 - e^{-t/RC}\right]$. Find the current flowing into the plate. (Ans: $i = \frac{V}{R}\, e^{-\frac{t}{RC}}$)

23. Find the current i in the circuit having resistance R and condenser of capacity C in series with emf E sin ωt.

24. The equation of L-R circuit is given by L$\frac{dI}{dt}$. + RI = 10 sin t .If I=0, at t = 0 , express I as a function of t . (Ans: $I = \frac{10}{\sqrt{R^2+L^2}}[\sin(t - \emptyset) + sin\emptyset\, e^{\frac{-Rt}{L}}$)

Heat Conduction

23. A steam pipe 20 cm in diameter is protected with a covering 6 cm thick for which the coefficient of thermal conductivity is k = 0.0003 cal/cm in steady state. Find the heat lost per hour through a meter length of the pipe, if the inner surface of the pipe is at 200 ℃ and the outer surface of the covering is at 30 ℃. (Ans : q=245443.3861)

24. A long hollow pipe has an inner diameter of 10 cm and outer diameter of 20 cm the inner surface is kept at 200 ℃ and outer surface at 50 ℃. The thermal conductivity is 0.12. How much heat is lost per minute from a portion of the pipe 20 m long and also find temperature at distance x=7.5 cm from the centre of pipe. (Ans: T=113 C)

Tracing of Curve

Trace the following curves

1) $y^2(2a - x) = x^3$
2) $(x^2 + y^2)x = (x^2 - y^2)$
3) $xy^2 = a^2(a - x)$
4) $x^2y^2 = a^2(y^2 - x^2)$
5) $(x^2 + a^2)y^2 = a^2x^2$
6) $(x^2 + 4a^2)y = 8a^3$
7) $x = a(t + sint), y = a(1 - cost)$
8) $x = a(t - sint), y = a(1 - cost)$
9) $x = a(t + sint), y = a(1 + cost)$
10) $r^2 = a^2 cos2\theta$
11) $r = a\,cos2\theta$
12) $r = a\,cos5\theta$
13) $r = a\,(1 - cos\theta)$
14) $r = a\,sin2\theta$
15) $r = 2\,sin5\theta$

Reduction Formulae, Beta and Gamma

1. Evaluate $\int_0^{\pi} x\sin^5 x\cos^8 x\,dx$ **Ans.** $\frac{8\pi}{1287}$
2. Evaluate $\int_0^{2a} x^3(2ax-x^2)^{\frac{3}{2}}\,dx$ **Ans.** $\frac{9\pi a^7}{16}$
3. Find the reduction formula for $\int_0^{\frac{\pi}{3}} \cos^n x\,dx$ and using it evaluate $\int_0^{\frac{\pi}{3}} \cos^6 x\,dx$.

   Ans. $\boldsymbol{I_n = \frac{\sqrt{3}}{n2^n} + \frac{n-1}{n} I_{n-2}\,,\ \frac{3\sqrt{3}}{32} + \frac{5\pi}{48}}$
4. If $I_n = \int_0^{\frac{\pi}{4}} \frac{\sin(2n-1)x}{\sin x}\,dx$ then prove that $n(I_{n+1} - I_n) = \sin\frac{n\pi}{2}$ and hence find I_3.

   Ans. $1 + \frac{\pi}{4}$
5. If $I_n = \int_0^{\infty} e^{-x}\sin^n x\,dx$, Obtain the relation between I_n *and* I_{n-2} .

   Ans. $I_n = \frac{n(n-1)}{n^2+1} I_{n-2}$
6. Evaluate $\int_0^{\infty} x^7 e^{-2x^2}\,dx$ **Ans.** $3/16$
7. Evaluate $\int_0^{\infty} 3^{-4x^2}\,dx$ **Ans.** $\frac{\sqrt{\pi}}{4\sqrt{\log 3}}$
8. Evaluate $\int_0^{\infty} \frac{x^4}{4^x}\,dx$ **Ans.** $\frac{24}{(\log 4)^5}$
9. Evaluate $\int_0^1 \frac{dx}{\sqrt{-\log x}}$ **Ans.** $\sqrt{\pi}$
10. Evaluate $\int_0^1 x^3(\log x)^4\,dx$ **Ans.** $\frac{3}{128}$
11. Show that $\int_0^1 \frac{dx}{\sqrt[3]{1-x^3}} = \frac{2\pi}{3\sqrt{3}}$
12. Show that $\int_0^{\infty} \frac{x^6-x^3}{(1+x^3)^5} x^2\,dx = 0$
13. Evaluate $\int_3^7 (x-3)^{\frac{1}{4}}(7-x)^{\frac{1}{4}}\,dx$ **Ans.** $\frac{2}{3\sqrt{3}}\left(\gamma\left(\frac{1}{4}\right)\right)^2$
14. Prove that $\int_0^{\infty} \frac{dx}{1+x^4} = \frac{\pi}{2\sqrt{2}}$
15. Show that $\int_0^{\frac{\pi}{2}} \sqrt{\tan\theta}\,d\theta \int_0^{\frac{\pi}{2}} \sqrt{\cot\theta}\,d\theta = \frac{\pi^2}{2}$.

F.Y. B. Tech.
Mathematics-II (SCI105A)
Practice Problems

Differentiation Under Integral Sign (DUIS)

1. Show that $\int_0^1 \frac{x^a-1}{logx} = \log(a+1)\,,\ \ a \geq 0$
2. Show that $\int_0^\infty \frac{e^{-x}-e^{-ax}}{xsecx} dx = \frac{1}{2}\log(\frac{a^2+1}{2})$
3. Find $\int_0^\infty \frac{e^{-ax}sinmx}{x} dx$ and hence evaluate $\int_0^\infty \frac{sinx}{x} dx$
4. Prove that $\int_0^\infty \frac{1-cosax}{x^2} dx = \frac{\pi a}{2}$
5. If $y = \int_0^x f(t)sina(x-t)dt$ then show that $\frac{d^2y}{dx^2} + a^2y = af(x)$
6. If $\emptyset(a) = \int_a^{a^2} \frac{sinax}{x} dx$ then find $\frac{d\emptyset}{da}$
7. Verify the DUIS rule for the $\int_a^{a^2} logaxdx$

Error Function

1. Prove that $erfc(-x) + erfc(x) = 2$
2. Show that $\int_a^b e^{-x^2}\, dx = \frac{\sqrt{\pi}}{2}[\mathrm{erf}(b) - \mathrm{erf}(a)]$
3. Find $\frac{d}{dx} erfc(ax^n)$
4. Prove that $\mathrm{erf}(x)$ is an odd function. Deduce that $erfc(-x) - \mathrm{erf}(x) = 1$
5. Show that $\int_0^\infty e^{-st}\, \mathrm{erf}(\sqrt{t})\, dt = \frac{1}{s\sqrt{s+1}}$
6. Show that $\int_0^\infty e^{-(x+a)^2}\, dx = \frac{\sqrt{\pi}}{2}[1 - \mathrm{erf}(a)]$
7. Show that $\frac{d}{dt}\mathrm{erf}(\sqrt{t}) = \frac{e^{-t}}{\sqrt{\pi t}}$ and hence evaluate $\int_0^\infty e^{-t}\, \mathrm{erf}(\sqrt{t})\, dt$.
8. Show that $\int_0^t \mathrm{erf}(ax)\, dx + \int_0^t erfc(ax)dx = t$.

Double Integral and Applications

1. $\int_1^2 \int_{-\sqrt{2-y}}^{\sqrt{2-y}} 2x^2\, y^2 dx\, dy$ (Ans: $\frac{856}{945}$)
2. $\iint \sqrt{4x^2 - y^2}\;\; dx\, dy$ over the area of triangle $y = 0\,, y = x\; \&\; x = 1$

   Ans: $\frac{1}{3}\left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\right)$
3. $\iint_R xy\,\sqrt{1 - x - y}\, dxdy$ over the region $x \geq 0\,, y \geq 0\; \&\; x + y \leq 1$ (Ans: $\frac{16}{945}$)
4. Evaluate $\iint_R x^2 + y^2\, dxdy$ over area of triangle whose vertices are (0,1) (1 ,1) & (1, 2). (Ans: $\frac{7}{6}$)
5. Show that $\int_0^a \int_0^{a-\sqrt{a^2-y^2}} \frac{xyiog(x+a)}{(x-a)^2} dxdy = \frac{a^2}{8}(2loga + 1)$
6. Evaluate by changing the order

   I) $\int_0^1 \int_{x^2}^{2-x} xy\, dxdy$ (Ans: $\frac{3}{8}$)

   II) $\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{e^y}{(e^y+1)\sqrt{1-x^2-y^2}} dxdy$ (Ans : $\frac{\pi}{2}\log\left(\frac{e+1}{2}\right)$
7. Express the following integral as a single integral

   $\int_0^1 \int_0^y f(x,y)dxdy + \int_1^\infty \int_0^{\frac{1}{y}} f(x,y)dxdy$ (Ans: $\int_0^1 \int_x^{\frac{1}{x}} f(x,y)dxdy$)
8. Evaluate

   I) $\int_0^{\frac{a}{\sqrt{2}}} \int_0^{\sqrt{a^2-y^2}} \ln(x^2 + y^2)\, dxdy$ (Ans: $\frac{\pi}{2}\left[\frac{a^2}{2}loga - \frac{a^2}{4}\right]$)

   II) $\int_0^{\frac{a}{\sqrt{2}}} \int_x^{\sqrt{a^2-x^2}} \frac{x}{\sqrt{x^2+y^2}} dxdy$ (Ans: $\frac{a^2}{2}\left(1 - \frac{1}{\sqrt{2}}\right)$)
9. Evaluate over one loop of $r^2 = a^2cos2\theta$ $\iint_R \frac{rdrd\theta}{\sqrt{a^2+r^2}}$ (Ans : 2a$(1 - \frac{\pi}{4})$)
10. Find area bounded by curve $y^2\,(2a - x) = x^3$ & its Asymptote (Ans :$3\pi a^2$)
11. Find area of cardioid $r = a(1 + cos\theta)$ (Ans: $\frac{3\pi a^2}{2}$)
12. Find area bounded by curve $y^2x = 16(4 - x)$ & its Asymptote. (Ans:16π)
13. Find area bounded by curves $y^2 = 4x\;\;\&\;\; 2x - y - 4 = 0$ (Ans: 9)
14. Find area bounded by curves $y^2 = x\;\;\&\;\; x^2 = -8y$ (Ans: $\frac{8}{3}$)
15. Evaluate $\int_0^{\frac{\pi}{2}} \int_0^y cos2y\,\sqrt{1 - a^2sin^2x}\;\; dxdy$ (Ans: $\frac{1}{3a^2}[(1 - a^2)^{\frac{3}{2}} - 1$)

Triple Integral and Applications

1. Evaluate $\iiint xyz\ dx\,dy\,dz$ Over positive octant of a sphere $x^2+y^2+z^2=a^2$.
   Ans: $\frac{a^6}{48}$
2. Evaluate $\int_0^2\int_0^y\int_{x-y}^{x+y}(x+y+z)dzdx\,dy$ Ans: 16
3. Evaluate $\iiint x^2yz\ dxdydz$ throughout the volume bounded by planes $x=0, y=0, z=0$ and $\frac{x}{2}-y+z=1$. Ans: $\frac{8}{2520}$
4. Evaluate $\iiint \frac{z^2dxdydz}{x^2+y^2+z^2}$ over the volume of sphere $x^2+y^2+z^2=2$ Ans: $\frac{8\pi\sqrt{2}}{9}$
5. Evaluate $\iiint z^2dxdydz$ over the volume bounded by the cylinder $x^2+y^2=a^2$ and the paraboloid $x^2+y^2=z$ and the plane $z=0$. Ans: $\frac{\pi a^8}{12}$
6. Evaluate $\int_0^{\pi/2}\int_0^{a\sin\theta}\int_0^{(a^2-r^2)/a} rdzdrd\theta$ Ans: $\frac{5a^3}{64}$
7. Evaluate $\iiint\sqrt{1-\frac{x^2}{4}-\frac{y^2}{9}-\frac{z^2}{64}}\ dxdydz$ throughout the volume of Ellipsoid $\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{64}=1$. Ans: $12\pi^2$
8. Evaluate $\iiint(x^2+y^2+z^2)^{3/2}\ dx\,dy\,dz$ Over the hemisphere defined by $x^2+y^2+z^2=9 \quad z\geq 0$. Ans: 243π
9. Calculate the volume of the solid bounded by the following surfaces $z=0,\ x^2+y^2=1,\ x+y+z=3$. Ans: 3π
10. Find by triple integration the volume of region bounded by paraboloid $az=x^2+y^2$ and the cylinder $x^2+y^2=r^2$. Ans: $\frac{\pi r^4}{2a}$
11. A cylindrical hole of radius 4 is bored through a sphere of radius 6. Find the volume of the remaining solid. Ans: $\frac{4\pi}{3}(20)^{3/2}$
12. Find the volume bounded by coordinate planes and the plane $lx+my+nz=1$. Ans: $\frac{1}{6mln}$
13. Find the volume bounded by the cylinder $x^2+y^2=4$ and the planes $y+z=4$ and $z=0$. Ans: 16π
14. Find the volume bounded by $y^2=x, x^2=y$ and the planes $z=0, x+y+z=1$. Ans: $\frac{1}{30}$
15. Find the volume of the solid bounded by the cylinder $x^2+y^2=4y$, the paraboloid $x^2+y^2=2z$ and the plane $z=0$ Ans: 12π

F.Y. B. Tech.
Mathematics-II (SCI105A)
Practice Problems

Fourier series

Q.1) Find the Fourier series expansion for $f(x) = a(2-x)$ in the interval $0 \le x \le 2$

Q.2) Find Fourier series for $f(x) = x + x^2$ in the interval $-\pi < x < \pi$

and $f(x) = f(x+2\pi)$ hence show that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} \dots$

Q.3) Find Fourier series for $f(x) = \frac{\pi^2}{12} - \frac{x^2}{4} in\ (-\pi, \pi)$.

Q.4) Obtain Fourier series expansion for $f(x) = 2 - \frac{x^2}{2}, 0 \le x \le 2$.

Q.5) Find the Fourier series expansion for $f(x) = \frac{1}{2}(\pi - x)$ in the interval $0 \le x \le 2\pi$.

Q.8) Find the Fourier series of the function $f(x) = \begin{cases} -1; & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases}$

where $(x) = f(x+2\pi)$.

Q.9) Determine the Fourier series for the following function

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ \sin x & 0 < x < \pi \end{cases}$$

Q.10) Expand the function $f(x) = x \sin x$, as a Fourier series

in the interval $-\pi < x < \pi$. Hence deduce that $\frac{\pi - 2}{4} = \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \cdots$

Q.11) Expand the function $f(x) = x \cos x$, as a Fourier series

in the interval $-\pi < x < \pi$.

Harmonic Analysis

Q.12) Determine the first two harmonics of the Fourier series for the following values:

x^0	0	30	60	90	120	150	180	210	240	270	300	330
y	2.34	3.01	3.68	4.15	3.69	2.20	0.83	0.51	0.88	1.09	1.19	1.64

Express T as a Fourier series up to first two harmonics.

Q.15) Obtain the constant term and the coefficient of the first harmonic in the

Fourier series of $f(x)$ as given in the following table

x	0	1	2	3	4	5
f(x)	9	18	24	28	26	20

Q.N.	Question	ANS
1	Fourier coefficient 'a₀' in the Fourier series expansion of $f(x) = e^{-x}$; $0 \le x \le 2\pi$ and $f(x + 2\pi) = f(x)$ is a) $\frac{1}{\pi}(1 - e^{-2\pi})$ b) $\frac{1}{2\pi}(1 - e^{2\pi})$ c) $\frac{2}{\pi}(e^{-2\pi} - 1)$ d) $\frac{1}{\pi}(1 + e^{2\pi})$	A
2	Fourier coefficient a_0 in the Fourier series expansion of $f(x) = \left(\frac{\pi - x}{2}\right)^2$; $0 \le x \le 2\pi$ and $f(x + 2\pi) = f(x)$ is a) $\frac{\pi^2}{3}$ b) $\frac{\pi^2}{6}$ c) 0 d) $\frac{\pi}{6}$	B
3	$f(x) = x, -\pi \le x \le \pi$ *and* period is 2π.the fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$, then fourier coefficient b_1 is a) 2 b) -1 c) 0 d) $\frac{2}{\pi}$	A
4	$f(x) = \begin{cases} 1, & -1 < x < 0 \\ \cos \pi x, & 0 < x < 1 \end{cases}$ *and* period is 2.the fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\pi x + b_n \sin n\pi x)$, then fourier coefficient a_0 is a) 2 b) -1 c) 1 d) $\frac{2}{\pi}$	C
5	$f(x) = x - x^3, \ -2 < x < 2$ *and* period is 4.the fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\pi x + b_n \sin n\pi x)$, then fourier coefficient a_0 is a)1 b)0 c)-2 d)-1	B
6	For the half range cosine series of $f(x) = \sin x, 0 \le x < \pi$ *and* period is 2π.the fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx)$ fourier coefficient a_0 is a) 4 b) 2 c) $\frac{2}{\pi}$ d) $\frac{4}{\pi}$	D
7	The value of b_1 in Harmonic analysis of y for the following tabulated data is: x: 0, 60, 120, 180, 240, 300, 360 y: 1.0, 1.4, 1.9, 1.7, 1.5, 1.2, 1.0 Sin x: 0, 0.866, 0.866, 0, -0.866, -0.866, 0 a) 0.0989 b) 0.3464 c) 0.1732 d) 0.6932	C
8	The value of the constant term in the fourier series of $f(x) = e^{-x}$ in $0 \le x \le 2\pi$, $f(x + 2\pi) = f(x)$ is a) $\frac{1}{\pi}(1 - e^{-2\pi})$ b) $\frac{1}{2\pi}(1 - e^{-2\pi})$ c) $1 - e^{-2\pi}$ d) $2(1 - e^{-2\pi})$	B
9	The value of the constant term in the fourier series of $f(x) = x \sin x$ in $0 \le x \le 2\pi$, is a) -2 b) 2 c) $-\frac{1}{2}$ d) -1	D

10	If $a_n = \frac{2}{n^2-1}$ for $n > 1$, in the fourier series of f(x) = x sin x in $0 \le x \le 2\pi$, then the value of a_1 is a) -2 b) $\frac{2}{n^2-1}$ c) - $\frac{1}{2}$ d) $\frac{1}{2}$	C
11	The value of the constant term in the fourier series of f(x)= $\begin{cases} -\pi & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$, *is* a) $-\frac{\pi}{4}$ b) $-\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{2}$	A
12	The value of a_n in the fourier series of f(x)= $\begin{cases} \cos x\,; & -\pi < x < 0 \\ -cosx\,; & 0 < x < \pi \end{cases}$ a)$\frac{(-1)^n}{n}$ b)$\frac{1}{n}$ c)$\frac{(-1)^n}{n^2-1}$ d)0	D
13	The value of b_n in the fourier series of f(x) = x in $-\pi < x < \pi$, is a) 0 b) $\frac{cosn\pi}{n}$ c) $-\frac{2cosn\pi}{n}$ d) $-\frac{\cos n\pi}{n}$	C
14	The Fourier constant 'a_n' for f (x) = $4 - x^2$ in the interval $0 < x < 2$ is_______ a) $-\frac{4}{\pi^2 n^2}$ b)$\frac{4}{\pi n^2}$ c) $\frac{4}{\pi^2 n^2}$ d)$\frac{2}{\pi^2 n^2}$	A
15	If f(x)=sin ax defined in the interval ($-l$, l) then value of 'a_n'is________ a)$\frac{2}{\pi n^2}$ b)$\frac{1}{n^2}$ c)0 d) $-\frac{1}{n^2}$	C
16	The function $f(x) = \begin{cases} 0 & -2<x<-1 \\ 1+x & -1<x<0 \\ 1-x & 0<x<1 \\ 0 & 1<x<2 \end{cases}$ Is a)an odd function b) an even function c) neither even nor odd function d)cannot be decided	B
17	The Fourier constant 'a_n'for f(x)=x^2 in the interval$-1 \le x \le 1$ is a) $\frac{4(-1)^n}{\pi^2 n^2}$ b)$\frac{4(-1)^{n+1}}{\pi^2 n^2}$ c) $\frac{4}{\pi^2 n^2}$ d)$-\frac{4}{\pi^2 n^2}$	A
18	If $f(x) = \begin{cases} -x, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$, then f(x) is a) Even function b) odd function c)Neither even nor odd d) none of these	A
19	In fourier series for f(x) = x in the interval $-\pi \le x \le \pi$ which of the following is correct a) $a_0 = \pi\,, a_n = \frac{1+(-1)^n}{n}, b_n = 0$ b) $a_0 = 0\,, a_n = 0\,, b_n = \frac{-2(-1)^n}{n}$ c) $a_0 = \frac{\pi}{2}\,, a_n = \frac{1+(-1)^n}{n}, b_n = \frac{-2(-1)^n}{n}$ d) $a_0 = 0\,, a_n = 0\,, b_n = 0$	B

20	The Fourier constant $'b_n'$for f(x)=2 $-\frac{x^2}{2}$ in the interval $0 \le x \le 2$ is a) $\frac{-2}{n\pi}$ b) $\frac{2}{n\pi}$ c) $\frac{2}{\pi^2 n^2}$ d) none of these.	B
21	If $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ is discontinuous at x = 0.Then value of f(0) is a) $\pi/2$ b) 0 c) $-\frac{\pi}{2}$ d) π	C
22	The function $f(x) = \begin{cases} 0 & -2<x<-1 \\ 1+x & -1<x<0 \\ 1-x & 0<x<1 \\ 0 & 1<x<2 \end{cases}$ Find a_0 a)2 b) 1/4 c) 1/2 d)0	D
23	Which of the following is the half range sine series of f(x)in the interval, $0 \le x \le \pi$? a)f(x)=$\sum_{n=1}^{\infty} b_n sin\frac{n\pi x}{L}$ b) f(x)=$\frac{a_0}{2}\sum_{n=1}^{\infty} b_n sinnx$ c) f(x)=$\sum_{n=1}^{\infty} b_n sinnx$ d) none of these.	C
24	For the function $f(x) = \begin{cases} -\pi, & 0 < x < \pi \\ x-\pi, & \pi < x < 2\pi \end{cases}$ what is the value of a_0 a) $\frac{1}{2}$ b) $-\frac{\pi}{2}$ c) $\frac{\pi}{2}$ d) π	B
25	If f(x) = $e^x, -1 \le x \le 1$ then constant term of f(x) in fourier expansion is a) $\frac{e}{2}$ b) $\frac{e-e^{-1}}{2}$ c) $\frac{e+e^{-1}}{2}$ d) $\frac{1+e}{2}$	B
26	If $f(x) = \begin{cases} -\pi, & 0 < x < \pi \\ x, & \pi < x < 2\pi \end{cases}$ is discontinuous at x= 0 then value of f(0) is a) $-\pi$ b) 0 c) $-\frac{\pi}{2}$ d) π	C
27	If $\sum y = 42$,n=6.$\sum ycos\theta = -8.5, \sum ycos2\theta$= -1.5,what arethe values of a_0, a_1, a_2 a)7,-2.8,-2.8 b)14,-2.8,1.5 c) 7,-1.5,-2.8 d)none of these	D
28	If f(x) = x^4 in (-1,1) then the fourier coefficient b_n is a)$\frac{24(-1)^n}{n^3\pi^3}$ b) $6\left[\frac{(-1)^n+1}{n^4\pi^4}\right]$ c)0 d)None of these.	C
29	For the function $f(x) = 2x - x^2, 0 \le x \le 3$ the value of b_n is, a) 0 b) $\frac{-9}{n^2\pi^2}$ c) $\frac{3}{n\pi}$ d) none of these	C
30	In the Fourier expansion of the function $f(x) = \frac{1}{2}(\pi - x)$ in the interval $0 \le x \le 2\pi$ the value of a_n is, a) $\frac{1}{n^2\pi}$ b) $\frac{\pi}{n^2}$ c) 0 d) $\frac{(-1)^n}{n^2\pi}$	C

<table>
<tr><td>31</td><td>If $f(x) = \frac{\pi^2}{2} - \frac{x^2}{4}, -\pi \le x \le \pi$ then values of a_n and b_n are
a) $0, \frac{3}{n\pi}$ b) $0, \frac{(-1)^{n+1}}{n^2}$ c) $\frac{(-1)^n+1}{n^2-1}, 0$ d) $\frac{-(-1)^n}{n^2}, 0$</td><td>D</td></tr>
<tr><td>32</td><td>The following values of y give the displacement in inches of a certain machine part for the rotation x of the flywheel
<table>
<tr><td>x</td><td>0</td><td>$\pi/6$</td><td>$2\pi/6$</td><td>$3\pi/6$</td><td>$4\pi/6$</td><td>$5\pi/6$</td></tr>
<tr><td>Y</td><td>0</td><td>9.2</td><td>14.4</td><td>17.8</td><td>17.3</td><td>11.7</td></tr>
</table>
What is the value of a_0
a)11.733 b)14.4 c)23.466 d) none of these</td><td>C</td></tr>
<tr><td>33</td><td>If $\sin x = \frac{2}{\pi} - \frac{2}{\pi}\sum_{n=2}^{\infty}\frac{1+(-1)^n}{(n-1)(n+1)}\cos nx$ for $0 \le x \le \pi$ then which of the following correct
a) $\frac{1}{2} = \frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \cdots$ b) $\frac{\pi^2}{2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$
c) $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$ d) none of these</td><td>A</td></tr>
<tr><td>34</td><td>If $a_0 = \frac{4\pi^2}{3}, a_n = \frac{4(-1)^{n+1}}{n^2}$ are the fourier coefficient of f(x) in $-\pi \le x \le \pi$ then which of the following correct
a) f(x)= $\frac{2\pi^2}{3} + \sum_{n=1}^{\infty}\frac{4(-1)^{n+1}}{n^2}\cos\frac{n\pi x}{2}$ b) f(x)= $\frac{4\pi^2}{3} + \sum_{n=1}^{\infty}\frac{4(-1)^{n+1}}{n^2}\sin nx$
c) f(x)= $\frac{2\pi^2}{3} - \sum_{n=1}^{\infty}\frac{4(-1)^n}{n^2}\cos nx$ d) none of these</td><td>C</td></tr>
<tr><td>35</td><td>If f(x) = x^2, $0 < x < 2$ then in half range cosine series $\frac{a_0}{2}$ is
a) 4 b) 12 c) $\frac{8}{3}$ d) 8</td><td>C</td></tr>
<tr><td>36</td><td>For the half range cosine series f(x) = $\left(\frac{\pi-x}{2}\right)^2, 0 \le x \le \pi$, if $a_0 = \frac{\pi^2}{6}, a_n = \frac{1}{n^2}, b_n = 0$, then which of the following statement is correct
a) $\frac{\pi^2}{6} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2}$------ b) $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2}$- - -
c) $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}$------ d) $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2}$- - -</td><td>C</td></tr>
</table>

Q.N.	Question	Ans
1	If $\emptyset(a) = \int_0^{\infty}\frac{e^{-x}}{x}(1 - e^{-ax})dx, a > -1$ then by DUIS rule, $\frac{d\emptyset}{da}$ is a) $\frac{e^{-x}}{x}(1 - e^{-ax})$ b) $\int_0^{\infty}\frac{a}{x}\left(e^{-(a+1)x}\right)dx$ c) $\int_0^{\infty}(e^{-ax})dx$ d) $\int_0^{\infty}\left(e^{-(a+1)x}\right)dx$	D
2	If $\emptyset(a) = \int_0^{1}\frac{x^a - 1}{logx}dx, a \ge 0$ then by DUIS rule, $\frac{d\emptyset}{da}$ is a) $\int_0^1 \frac{x^a loga}{logx}dx$ b) $\int_0^1\frac{ax^{a-1}}{logx}dx$ c) $\int_0^1 x^a dx$ d) $\frac{x^a - 1}{logx}$	C

3	If $\emptyset(\alpha) = \int_0^{\infty} \frac{e^{-x} sin\alpha x}{x} dx$ then by DUIS rule, $\frac{d\emptyset}{d\alpha}$ is a) $\int_0^{\infty} e^{-x} sin\alpha x\, dx$ b) $\int_0^{\infty} e^{-x} cos\alpha x\, dx$ c) $\int_0^{\infty} \frac{\alpha e^{-x} sin\alpha x}{x} dx$ d) $\frac{e^{-x} sin\alpha x}{x}$	B
4	If $\emptyset(a) = \int_0^{\frac{\pi}{2}} \frac{\log(1+asin^2x)}{sin^2x} dx, a > 0$, then by DUIS rule, $\frac{d\emptyset}{da}$ is a) $\int_0^{\frac{\pi}{2}} \frac{2sinxcosx}{(1+asin^2x)} dx$ b) $\int_0^{\frac{\pi}{2}} \frac{1}{(1+asin^2x)sin^2x} dx$ c) $\int_0^{\frac{\pi}{2}} \frac{1}{1+asin^2x} dx$ d) $\int_0^{\frac{\pi}{2}} \frac{sin^2x}{(1+asin^2x)} dx$	C
5	If $\emptyset(a) = \int_a^{a^2} \log(ax) dx$, then by DUIS rule II, $\frac{d\emptyset}{da}$ is a) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx + 2aloga^3$ b) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx$ c) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx + 2aloga^3 - 2loga$ d) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx - 2aloga^3 + 2loga$	A
6	If $\emptyset(a) = \int_0^{a^2} tan^{-1}\left(\frac{x}{a}\right) dx$, then by DUIS rule II, $\frac{d\emptyset}{da}$ is a) $\int_0^{a^2} \frac{\partial}{\partial a} tan^{-1}\left(\frac{x}{a}\right) dx + 2atan^{-1}a$ b) $\int_0^{a^2} \frac{\partial}{\partial a} tan^{-1}\left(\frac{x}{a}\right) dx$ c) $\int_0^{a^2} \frac{\partial}{\partial a} tan^{-1}\left(\frac{x}{a}\right) dx + a^2tan^{-1}a$ d) $\int_0^{a^2} \frac{\partial}{\partial a} tan^{-1}\left(\frac{x}{a}\right) dx + a^2tan^{-1}a - tan^{-1}\left(\frac{x}{a}\right)$	A
7	IF $I(a)=\int_a^{a^2} \frac{1}{x+a} dx$, then by DUIS rule II, $\frac{dI}{da}$ is a) $\int_a^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx - \frac{1}{a^2+a}(2a) + \frac{1}{2a}$ b) $\int_a^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx + \frac{1}{a^2+a}(2a) - \frac{1}{2a}$ c) $\int_a^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx - \frac{1}{a^2+a}$ d) $\int_a^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx$	B
8	Using DUIS Rule the value of the integral $\emptyset(a) = \int_0^{\infty} \frac{e^{-x}}{x}(1 - e^{-ax})dx, a > -1$, given $\frac{d\emptyset}{da} = \frac{1}{a+1}$ is a)log(a+1) b) $-\frac{1}{(a+1)^2}$ c)log(a+1) + π d) $-\frac{1}{(a+1)^2} + 1$	A
9	Using DUIS Rule the value of the integral $\emptyset(a) = \int_0^{\frac{\pi}{2}} \frac{\log(1+asin^2x)}{sin^2x} dx$ with $\frac{d\emptyset}{da} = \frac{\pi}{2}\frac{1}{\sqrt{a+1}}$ is a) $\pi\sqrt{a+1}$ b) $\pi\sqrt{a+1} + \pi$ c) $\pi\sqrt{a+1} - \pi$ d) $3\pi(a+1)^{\frac{3}{2}} - \pi$	C
10	Using DUIS Rule the value of the integral $\emptyset(a) = \int_0^{\infty} \frac{1-cosax}{x^2} dx$, with $\frac{d\emptyset}{da} = \frac{\pi}{2}$ is a) $\frac{\pi}{2}$ b) $\frac{\pi a}{2}$ c) πa d) $\frac{\pi a}{2} + \frac{\pi}{2}$	B
11	If I(a)= $\int_0^1 \frac{x^a - x^b}{logx}$ dx then the value of I(a) is a) log$\left(\frac{a}{b}\right)$ b) log$\left(\frac{a+1}{b+1}\right)$ c) log$\left(\frac{b+1}{a+1}\right)$ d) log$\left(\frac{b}{a}\right)$	B

12	If erf($\sqrt{t}$) $=\frac{2}{\sqrt{\pi}}\int_0^{\sqrt{t}} e^{-u^2}\,du$ then $\frac{d}{dt}$ erf($\sqrt{t}$) is a) $\frac{e^{-t}}{2\sqrt{t}}$ b) $\frac{e^{-t^2}}{\sqrt{\pi t}}$ c) $\frac{e^{-t}}{\sqrt{\pi}}$ d) $\frac{e^{-t}}{\sqrt{\pi t}}$	D
13	$\int_0^t \mathrm{erf}(ax)\,dx + \int_0^t \mathrm{erf\,c}(ax)\,dx =?$ a) t b) x c) 0 d) $\frac{t^2}{2}$	A
14	If $\frac{d}{dx}$ erf(ax)$=\frac{2ae^{-a^2x^2}}{\sqrt{\pi}}$, the value of $\int_0^t \mathrm{erf}(ax)\,dx$ is a) t erf(at)$+\frac{1}{a\sqrt{\pi}}e^{-a^2t^2}-\frac{1}{a\sqrt{\pi}}$ b) t erf(at)$-\frac{1}{a\sqrt{\pi}}e^{-a^2t^2}-\frac{1}{a\sqrt{\pi}}$ c) erf(at)$+\frac{1}{a\sqrt{\pi}}e^{-a^2t^2}+\frac{1}{a\sqrt{\pi}}$ d) t erf(at)$-\frac{1}{a\sqrt{\pi}}e^{-a^2t^2}+\frac{1}{a\sqrt{\pi}}$	A
15	The integral for "erf(b)-erf(a)" is, a) $\frac{2}{\sqrt{\pi}}\int_a^b e^{-t^2}\,dt$ b) $\sqrt{\frac{2}{\pi}}\int_a^b e^{-t^2}\,dt$ c) $\int_a^b e^{-t^2}\,dt$ d) none of these	A

PAGE NO.
DATE / /

(1) The differential equation of all circles touching Y-axis at the origin & centres on X-axis, is

(A) $x^2 - y^2 = 2x\left[x + y\frac{dy}{dx}\right]$

(B) $x^2 + y^2 = 2x\left[x + y\frac{dy}{dx}\right]$

(C) $x^2 + y^2 = 2x\left[x - y\frac{dy}{dx}\right]$

(D) None of these

2) Intigrating factor of $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$

(A) $\frac{1}{x^2y^2}$

(B) $\frac{1}{xy}$

(C) $\frac{1}{x^2y}$

(D) $\frac{1}{xy^2}$

3) If $I = \frac{E}{R}\left[1 - e^{-Rt/L}\right]$ then the value of q is

A) $\frac{E}{R}\left[t + \frac{L}{R}e^{-Rt/L}\right]$

B) $\frac{E}{R}\left[t + \frac{R}{L}e^{-Rt/L}\right]$

C) $\frac{E}{R}\left[t - \frac{R}{L}e^{-Rt/L}\right]$

(D) None of these.

(4) The curve $r = ae^{m\theta}$

(A) Not passes through the pole

(B) passes through the pole

(C) symmetry about y-axis

(D) None of these

(5) The curve $a^2y^2 = x^2(2a-x)(x-a)$ is

(A) Symmetry about y-axis

(B) symmetry about $y=x$

(C) symmetry about x-axis

(D) symmetry about $y=-x$.

(6) Tangent at origin to the curve $x^3+y^3 = 3ax$ is

(A) $x=0$

(B) $y=0$

(C) $x=0, y=0$

(D) None of these

Q.1) $\int_0^1 \int_0^1 \frac{dx\,dy}{\sqrt{1-x^2}\sqrt{1-y^2}} = \cdots$

a) $\frac{\pi^2}{3}$ b) $\frac{\pi^2}{4}$ ✓ c) π d) 1

Q.2) $\int_0^{2\pi} \int_{a\sin\theta}^{a} r\,d\theta\,dr = \cdots$

a) $\frac{\pi a^2}{2}$ ✓ b) $\frac{\pi}{2}$ c) $\frac{\pi^2 a}{2}$ d) 5

Q.3) $\int_0^1 \int_0^x e^{x+y}\,dx\,dy = \cdots$

a) $\frac{(e-1)^2}{2}$ ✓ b) $\frac{e-1}{2}$ c) $\frac{e}{2}$ d) e

Q.4) After changing the order of integration $I = \int_0^1 \int_{4y}^{4} e^{x^2}\,dx\,dy$, the new limits of x & y are

✓a) $0 \le x \le 4$, $0 \le y \le \frac{x}{4}$ b) $4 \le x \le 0$, $\frac{x}{4} \le y \le 0$

c) $0 \le x \le \frac{y}{4}$, $0 \le y \le 4$ d) $0 \le x \le 1$, $0 \le y \le 4$

Q.5) After changing the order of integration of $I = \int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dx\,dy$ the new limits of x & y are

✓a) $0 \le x \le y$ $0 \le y < \infty$ b) $0 \le y \le x$, $0 \le x < \infty$

c) $0 \le x \le 1$ $x \le y \le 1$ d) $0 \le x \le 1$, $0 \le y \le 2$

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Q.1) $\int_0^1 \int_0^1 \frac{dx\,dy}{\sqrt{1-x^2}\sqrt{1-y^2}} = \ldots$

a) $\frac{\pi^2}{3}$ b) $\frac{\pi^2}{4}$ c) π d) 1

Q.2) $\int_0^{\pi} \int_{a\sin\theta}^{a} r\,d\theta\,dr = \ldots$

a) $\frac{\pi a^2}{2}$ b) $\frac{\pi}{2}$ c) $\frac{\pi^2 a}{2}$ d) 5

Q.3) $\int_0^1 \int_0^x e^{x+y}\,dx\,dy = \ldots$

a) $\frac{(e-1)^2}{2}$ b) $\frac{e-1}{2}$ c) $\frac{e}{2}$ d) e

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c) $0 \le x \le \frac{y}{4},\ 0 \le y \le 4$ d) $0 \le x \le 1,\ 0 \le y \le 4$

Q.5) After changing the order of integration of $I = \int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dx\,dy$ the new limits of x & y are

a) $0 \le x \le y,\ 0 \le y < \infty$ b) $0 \le y \le x,\ 0 \le x < \infty$

c) $0 \le x \le 1,\ x \le y \le 1$ d) $0 \le x \le 1,\ 0 \le y \le 2$

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