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<tr><td>1</td><td>Fourier coefficient 'a_0 ' in the Fourier series expansion of $f(x) = e^{-x}$; $0 \le x \le 2\pi$ and $f(x + 2\pi) = f(x)$ is
a) $\frac{1}{\pi}(1 - e^{-2\pi})$ b) $\frac{1}{2\pi}(1 - e^{2\pi})$ c) $\frac{2}{\pi}(e^{-2\pi} - 1)$ d) $\frac{1}{\pi}(1 + e^{2\pi})$</td><td>A</td></tr>
<tr><td>2</td><td>Fourier coefficient a_0 in the Fourier series expansion of $f(x) = \left(\frac{\pi - x}{2}\right)^2$; $0 \le x \le 2\pi$ and $f(x + 2\pi) = f(x)$ is
a) $\frac{\pi^2}{3}$ b) $\frac{\pi^2}{6}$ c) 0 d) $\frac{\pi}{6}$</td><td>B</td></tr>
<tr><td>3</td><td>$f(x) = x$, $-\pi \le x \le \pi$ *and* period is 2π the fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$, then fourier coefficient b_1 is
a) 2 b) -1 c) 0 d) $\frac{2}{\pi}$</td><td>A</td></tr>
<tr><td>4</td><td>$f(x) = \begin{cases} 1, & -1 < x < 0 \\ \cos \pi x, & 0 < x < 1 \end{cases}$ *and* period is 2 the fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\pi x + b_n \sin n\pi x)$, then fourier coefficient a_0 is
a) 2 b) -1 c) 1 d) $\frac{2}{\pi}$</td><td>C</td></tr>
<tr><td>5</td><td>$f(x) = x - x^3$, $-2 < x < 2$ *and* period is 4 the fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\pi x + b_n \sin n\pi x)$, then fourier coefficient a_0 is
a)1 b)0 c)-2 d)-1</td><td>B</td></tr>
<tr><td>6</td><td>For the half range cosine series of $f(x) = \sin x$, $0 \le x < \pi$ *and* period is 2π the fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx)$ fourier coefficient a_0 is
a) 4 b) 2 c) $\frac{2}{\pi}$ d) $\frac{4}{\pi}$</td><td>D</td></tr>
<tr><td>7</td><td>The value of b_1 in Harmonic analysis of y for the following tabulated data is:
<table>
<tr><td>x</td><td>0</td><td>60</td><td>120</td><td>180</td><td>240</td><td>300</td><td>360</td></tr>
<tr><td>y</td><td>1.0</td><td>1.4</td><td>1.9</td><td>1.7</td><td>1.5</td><td>1.2</td><td>1.0</td></tr>
<tr><td>Sin x</td><td>0</td><td>0.866</td><td>0.866</td><td>0</td><td>-0.866</td><td>-0.866</td><td>0</td></tr>
</table>
a) 0.0989 b) 0.3464 c) 0.1732 d) 0.6932</td><td>C</td></tr>
<tr><td>8</td><td>The value of the constant term in the fourier series of $f(x) = e^{-x}$ in $0 \le x \le 2\pi$, $f(x + 2\pi) = f(x)$ is
a) $\frac{1}{\pi}(1 - e^{-2\pi})$ b) $\frac{1}{2\pi}(1 - e^{-2\pi})$ c) $1 - e^{-2\pi}$ d) $2(1 - e^{-2\pi})$</td><td>B</td></tr>
<tr><td>9</td><td>The value of the constant term in the fourier series of $f(x) = x \sin x$ in $0 \le x \le 2\pi$, is
a) -2 b) 2 c) $-\frac{1}{2}$ d) -1</td><td>D</td></tr>
</table>

10	If $a_n = \frac{2}{n^2-1}$ for $n > 1$, in the fourier series of f(x) = x sin x in $0 \le x \le 2\pi$, then the value of a_1 is a) -2 b) $\frac{2}{n^2-1}$ c) $-\frac{1}{2}$ d) $\frac{1}{2}$	C
11	The value of the constant term in the fourier series of $f(x)=\begin{cases} -\pi & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$, is a) $-\frac{\pi}{4}$ b) $-\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{2}$	A
12	The value of a_n in the fourier series of $f(x)=\begin{cases} \cos x; & -\pi < x < 0 \\ -\cos x; & 0 < x < \pi \end{cases}$ a) $\frac{(-1)^n}{n}$ b) $\frac{1}{n}$ c) $\frac{(-1)^n}{n^2-1}$ d) 0	D
13	The value of b_n in the fourier series of f(x) = x in $-\pi < x < \pi$, is a) 0 b) $\frac{\cos n\pi}{n}$ c) $-\frac{2\cos n\pi}{n}$ d) $-\frac{\cos n\pi}{n}$	C
14	The Fourier constant 'a_n' for f (x) = $4 - x^2$ in the interval $0 < x < 2$ is______ a) $-\frac{4}{\pi^2 n^2}$ b) $\frac{4}{\pi n^2}$ c) $\frac{4}{\pi^2 n^2}$ d) $\frac{2}{\pi^2 n^2}$	A
15	If f(x)=sin ax defined in the interval ($-l$, l) then value of 'a_n' is______ a) $\frac{2}{\pi n^2}$ b) $\frac{1}{n^2}$ c) 0 d) $-\frac{1}{n^2}$	C
16	The function $f(x) = \begin{cases} 0 & -2<x<-1 \\ 1+x & -1<x<0 \\ 1-x & 0<x<1 \\ 0 & 1<x<2 \end{cases}$ Is a) an odd function b) an even function c) neither even nor odd function d) cannot be decided	B
17	The Fourier constant 'a_n' for f(x)=x^2 in the interval $-1 \le x \le 1$ is a) $\frac{4(-1)^n}{\pi^2 n^2}$ b) $\frac{4(-1)^{n+1}}{\pi^2 n^2}$ c) $\frac{4}{\pi^2 n^2}$ d) $-\frac{4}{\pi^2 n^2}$	A
18	If $f(x) = \begin{cases} -x, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$, then f(x) is a) Even function b) odd function c) Neither even nor odd d) none of these	A
19	In fourier series for f(x) = x in the interval $-\pi \le x \le \pi$ which of the following is correct a) $a_0 = \pi, a_n = \frac{1+(-1)^n}{n}, b_n = 0$ b) $a_0 = 0, a_n = 0, b_n = \frac{-2(-1)^n}{n}$ c) $a_0 = \frac{\pi}{2}, a_n = \frac{1+(-1)^n}{n}, b_n = \frac{-2(-1)^n}{n}$ d) $a_0 = 0, a_n = 0, b_n = 0$	B

20	The Fourier constant $'b_n'$for f(x)=$2-\frac{x^2}{2}$ in the interval $0 \le x \le 2$ is a) $\frac{-2}{n\pi}$ b)$\frac{2}{n\pi}$ c) $\frac{2}{\pi^2 n^2}$ d) none of these.	B
21	If $f(x)=\begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ is discontinuous at x = 0.Then value of f(0) is a) π /2 b) 0 c) $-\frac{\pi}{2}$ d) π	C
22	The function $f(x)=\begin{cases} 0 & -2<x<-1 \\ 1+x & -1<x<0 \\ 1-x & 0<x<1 \\ 0 & 1<x<2 \end{cases}$ Find a_0 a)2 b) 1/4 c) 1/2 d)0	D
23	Which of the following is the half range sine series of f(x)in the interval, $0 \le x \le \pi$? a)f(x)=$\sum_{n=1}^{\infty} b_n sin\frac{n\pi x}{L}$ b) f(x)=$\frac{a_0}{2}\sum_{n=1}^{\infty} b_n sinnx$ c) f(x)=$\sum_{n=1}^{\infty} b_n sinnx$ d) none of these.	C
24	For the function $f(x)=\begin{cases} -\pi, & 0 < x < \pi \\ x-\pi, & \pi < x < 2\pi \end{cases}$ what is the value of a_0 a) $\frac{1}{2}$ b) $-\frac{\pi}{2}$ c) $\frac{\pi}{2}$ d) π	B
25	If f(x) = $e^x, -1 \le x \le 1$ then constant term of f(x) in fourier expansion is a) $\frac{e}{2}$ b) $\frac{e-e^{-1}}{2}$ c) $\frac{e+e^{-1}}{2}$ d) $\frac{1+e}{2}$	B
26	If $f(x)=\begin{cases} -\pi, & 0 < x < \pi \\ x, & \pi < x < 2\pi \end{cases}$ is discontinuous at x= 0 then value of f(0) is a) $-\pi$ b) 0 c) $-\frac{\pi}{2}$ d) π	C
27	If $\sum y = 42$,n=6.$\sum ycos\theta = -8.5, \sum ycos2\theta$= -1.5,what arethe values of a_0, a_1, a_2 a)7,-2.8,-2.8 b)14,-2.8,1.5 c) 7,-1.5,-2.8 d)none of these	D
28	If f(x) = x^4 in (-1,1) then the fourier coefficient b_n is a)$\frac{24(-1)^n}{n^3\pi^3}$ b) $6\left[\frac{(-1)^n+1}{n^4\pi^4}\right]$ c)0 d)None of these.	C
29	For the function $f(x)=2x-x^2, 0 \le x \le 3$ the value of b_n is, a) 0 b) $\frac{-9}{n^2\pi^2}$ c) $\frac{3}{n\pi}$ d) none of these	C
30	In the Fourier expansion of the function $f(x)=\frac{1}{2}(\pi-x)$ in the interval $0 \le x \le 2\pi$ the value of a_n is, a) $\frac{1}{n^2\pi}$ b) $\frac{\pi}{n^2}$ c) 0 d) $\frac{(-1)^n}{n^2\pi}$	C

31	If $f(x)=\frac{\pi^2}{2}-\frac{x^2}{4}, -\pi\le x\le\pi$ then values of a_n and b_n are a) $0,\frac{3}{n\pi}$ b) $0,\frac{(-1)^{n+1}}{n^2}$ c) $\frac{(-1)^n+1}{n^2-1},0$ d) $\frac{-(-1)^n}{n^2},0$	D

32 The following values of y give the displacement in inches of a certain machine part for the rotation x of the flywheel — **C**

x	0	$\pi/6$	$2\pi/6$	$3\pi/6$	$4\pi/6$	$5\pi/6$
Y	0	9.2	14.4	17.8	17.3	11.7

What is the value of a_0

a)11.733 b)14.4 c)23.466 d) none of these

33	If $\sin x=\frac{2}{\pi}-\frac{2}{\pi}\sum_{n=2}^{\infty}\frac{1+(-1)^n}{(n-1)(n+1)}\cos nx$ for $0\le x\le\pi$ then which of the following correct a) $\frac{1}{2}=\frac{1}{1\cdot3}+\frac{1}{3\cdot5}+\frac{1}{5\cdot7}+\cdots$ b) $\frac{\pi^2}{2}=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots$ c) $\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$ d) none of these	A
34	If $a_0=\frac{4\pi^2}{3}, a_n=\frac{4(-1)^{n+1}}{n^2}$ are the fourier coefficient of f(x) in $-\pi\le x\le\pi$ then which of the following correct a) $f(x)=\frac{2\pi^2}{3}+\sum_{n=1}^{\infty}\frac{4(-1)^{n+1}}{n^2}\cos\frac{n\pi x}{2}$ b) $f(x)=\frac{4\pi^2}{3}+\sum_{n=1}^{\infty}\frac{4(-1)^{n+1}}{n^2}\sin nx$ c) $f(x)=\frac{2\pi^2}{3}-\sum_{n=1}^{\infty}\frac{4(-1)^n}{n^2}\cos nx$ d) none of these	C
35	If $f(x)=x^2, 0<x<2$ then in half range cosine series $\frac{a_0}{2}$ is a) 4 b) 12 c) $\frac{8}{3}$ d) 8	C
36	For the half range cosine series $f(x)=\left(\frac{\pi-x}{2}\right)^2, 0\le x\le\pi$, if $a_0=\frac{\pi^2}{6}, a_n=\frac{1}{n^2}, b_n=0$, then which of the following statement is correct a) $\frac{\pi^2}{6}=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}$------ b) $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}---$ c) $\frac{\pi^2}{6}=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}$------ d) $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}---$	C

Q.N.	Question	Ans
1	If $\emptyset(a)=\int_0^{\infty}\frac{e^{-x}}{x}(1-e^{-ax})dx, a>-1$ then by DUIS rule, $\frac{d\emptyset}{da}$ is a) $\frac{e^{-x}}{x}(1-e^{-ax})$ b) $\int_0^{\infty}\frac{a}{x}\left(e^{-(a+1)x}\right)dx$ c) $\int_0^{\infty}(e^{-ax})dx$ d) $\int_0^{\infty}\left(e^{-(a+1)x}\right)dx$	D
2	If $\emptyset(a)=\int_0^1\frac{x^a-1}{\log x}dx, a\ge0$ then by DUIS rule, $\frac{d\emptyset}{da}$ is a) $\int_0^1\frac{x^a\log a}{\log x}dx$ b) $\int_0^1\frac{ax^{a-1}}{\log x}dx$ c) $\int_0^1 x^a dx$ d) $\frac{x^a-1}{\log x}$	C

3	If $\emptyset(\alpha) = \int_0^{\infty} \frac{e^{-x}sinax}{x} dx$ then by DUIS rule, $\frac{d\emptyset}{da}$ is a) $\int_0^{\infty} e^{-x} sinax\, dx$ b) $\int_0^{\infty} e^{-x} cosax\, dx$ c) $\int_0^{\infty} \frac{ae^{-x}sinax}{x} dx$ d) $\frac{e^{-x}sinax}{x}$	B
4	If $\emptyset(a) = \int_0^{\frac{\pi}{2}} \frac{\log(1+asin^2x)}{sin^2x} dx, a > 0$, then by DUIS rule, $\frac{d\emptyset}{da}$ is a) $\int_0^{\frac{\pi}{2}} \frac{2sinxcosx}{(1+asin^2x)} dx$ b) $\int_0^{\frac{\pi}{2}} \frac{1}{(1+asin^2x)sin^2x} dx$ c) $\int_0^{\frac{\pi}{2}} \frac{1}{1+asin^2x} dx$ d) $\int_0^{\frac{\pi}{2}} \frac{sin^2x}{(1+asin^2x)} dx$	C
5	If $\emptyset(a) = \int_a^{a^2} \log(ax) dx$, then by DUIS rule II, $\frac{d\emptyset}{da}$ is a) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx + 2aloga^3$ b) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx$ c) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx + 2aloga^3 - 2loga$ d) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx - 2aloga^3 + 2loga$	A
6	If $\emptyset(a) = \int_0^{a^2} tan^{-1}\left(\frac{x}{a}\right) dx$, then by DUIS rule II, $\frac{d\emptyset}{da}$ is a) $\int_0^{a^2} \frac{\partial}{\partial a} tan^{-1}\left(\frac{x}{a}\right) dx + 2atan^{-1}a$ b) $\int_0^{a^2} \frac{\partial}{\partial a} tan^{-1}\left(\frac{x}{a}\right) dx$ c) $\int_0^{a^2} \frac{\partial}{\partial a} tan^{-1}\left(\frac{x}{a}\right) dx + a^2tan^{-1}a$ d) $\int_0^{a^2} \frac{\partial}{\partial a} tan^{-1}\left(\frac{x}{a}\right) dx + a^2tan^{-1}a - tan^{-1}\left(\frac{x}{a}\right)$	A
7	IF $I(a)=\int_a^{a^2} \frac{1}{x+a} dx$, then by DUIS rule II, $\frac{dI}{da}$ is a) $\int_a^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx - \frac{1}{a^2+a}(2a) + \frac{1}{2a}$ b) $\int_a^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx + \frac{1}{a^2+a}(2a) - \frac{1}{2a}$ c) $\int_a^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx - \frac{1}{a^2+a}$ d) $\int_a^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx$	B
8	Using DUIS Rule the value of the integral $\emptyset(a) = \int_0^{\infty} \frac{e^{-x}}{x}(1 - e^{-ax}) dx, a > -1$, given $\frac{d\emptyset}{da} = \frac{1}{a+1}$ is a) log(a+1) b) $-\frac{1}{(a+1)^2}$ c) log(a+1) + π d) $-\frac{1}{(a+1)^2} + 1$	A
9	Using DUIS Rule the value of the integral $\emptyset(a) = \int_0^{\frac{\pi}{2}} \frac{\log(1+asin^2x)}{sin^2x} dx$ with $\frac{d\emptyset}{da} = \frac{\pi}{2}\frac{1}{\sqrt{a+1}}$ is a) $\pi\sqrt{a+1}$ b) $\pi\sqrt{a+1} + \pi$ c) $\pi\sqrt{a+1} - \pi$ d) $3\pi(a+1)^{\frac{3}{2}} - \pi$	C
10	Using DUIS Rule the value of the integral $\emptyset(a) = \int_0^{\infty} \frac{1-cosax}{x^2} dx$, with $\frac{d\emptyset}{da} = \frac{\pi}{2}$ is a) $\frac{\pi}{2}$ b) $\frac{\pi a}{2}$ c) πa d) $\frac{\pi a}{2} + \frac{\pi}{2}$	B
11	If $I(a)=\int_0^1 \frac{x^a-x^b}{logx}dx$ then the value of I(a) is a) $\log\left(\frac{a}{b}\right)$ b) $\log\left(\frac{a+1}{b+1}\right)$ c) $\log\left(\frac{b+1}{a+1}\right)$ d) $\log\left(\frac{b}{a}\right)$	B

12	If erf($\sqrt{t}$) $=\frac{2}{\sqrt{\pi}}\int_0^{\sqrt{t}} e^{-u^2}\,du$ then $\frac{d}{dt}$ erf($\sqrt{t}$) is a) $\frac{e^{-t}}{2\sqrt{t}}$ b) $\frac{e^{-t^2}}{\sqrt{\pi t}}$ c) $\frac{e^{-t}}{\sqrt{\pi}}$ d) $\frac{e^{-t}}{\sqrt{\pi t}}$	D
13	$\int_0^t \operatorname{erf}(ax)\,dx + \int_0^t \operatorname{erfc}(ax)\,dx = ?$ a) t b) x c) 0 d) $\frac{t^2}{2}$	A
14	If $\frac{d}{dx}$ erf(ax)=$\frac{2ae^{-a^2x^2}}{\sqrt{\pi}}$, the value of $\int_0^t \operatorname{erf}(ax)\,dx$ is a) t erf(at)+$\frac{1}{a\sqrt{\pi}}e^{-a^2t^2} - \frac{1}{a\sqrt{\pi}}$ b) t erf(at)$-\frac{1}{a\sqrt{\pi}}e^{-a^2t^2} - \frac{1}{a\sqrt{\pi}}$ c) erf(at)+$\frac{1}{a\sqrt{\pi}}e^{-a^2t^2} + \frac{1}{a\sqrt{\pi}}$ d) t erf(at)$-\frac{1}{a\sqrt{\pi}}e^{-a^2t^2} + \frac{1}{a\sqrt{\pi}}$	A
15	The integral for "erf(b)-erf(a)" is, a) $\frac{2}{\sqrt{\pi}}\int_a^b e^{-t^2}\,dt$ b) $\sqrt{\frac{2}{\pi}}\int_a^b e^{-t^2}\,dt$ c) $\int_a^b e^{-t^2}\,dt$ d) none of these	A

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1) The differential equation of all circles touching Y-axis at the origin & centres on X-axis, is

(A) $x^2-y^2 = 2x\left[x+y\frac{dy}{dx}\right]$

(B) $x^2+y^2 = 2x\left[x+y\frac{dy}{dx}\right]$

(C) $x^2+y^2 = 2x\left[x-y\frac{dy}{dx}\right]$

(D) None of these

2] Intigrating factor of $(x^2y-2xy^2)dx-(x^2-3x^2y)dy=0$

(A) $\frac{1}{x^2y^2}$

(B) $\frac{1}{xy}$

(C) $\frac{1}{x^2y}$

(D) $\frac{1}{xy^2}$

3] If $I = \frac{E}{R}\left[1-e^{-Rt/L}\right]$ then the value of q is

A) $\frac{E}{R}\left[t+\frac{L}{R}e^{-Rt/L}\right]$

B) $\frac{E}{R}\left[t+\frac{R}{L}e^{-Rt/L}\right]$

C) $\frac{E}{R}\left[t-\frac{R}{L}e^{-Rt/L}\right]$

(D) None of these

④ The curve $r = ae^{m\theta}$

Ⓐ Not passes through the pole

Ⓑ passes through the pole

Ⓒ symmetry about y-axis

Ⓓ None of these

⑤ The curve $a^2y^2 = x^2(2a-x)(x-a)$ is

Ⓐ symmetry about y-axis

Ⓑ symmetry about $y=x$

Ⓒ symmetry about x-axis

Ⓓ symmetry about $y=-x$

⑥ Tangent at origin to the curve $x^3+y^3=3ax$ is

Ⓐ $x=0$

Ⓑ $y=0$

Ⓒ $x=0, y=0$

Ⓓ None of these

Q.1) $\int_0^1 \int_0^1 \frac{dx\,dy}{\sqrt{1-x^2}\,\sqrt{1-y^2}} = \ldots$

a) $\frac{\pi^2}{3}$ ✓ b) $\frac{\pi^2}{4}$ c) π d) 1

Q.2) $\int_0^{2\pi} \int_{a\sin\theta}^{a} r\,d\theta\,dr = \ldots$

✓ a) $\frac{\pi a^2}{2}$ b) $\frac{\pi}{2}$ c) $\frac{\pi^2 a}{2}$ d) 5

Q.3) $\int_0^1 \int_0^x e^{x+y}\,dx\,dy = \ldots$

✓ a) $\frac{(e-1)^2}{2}$ b) $\frac{e-1}{2}$ c) $\frac{e}{2}$ d) e

Q.4) After changing the order of integration $I = \int_0^1 \int_{4y}^{4} e^{x^2}\,dx\,dy$, the new limits of x & y are

✓ a) $0 \le x \le 4,\ 0 \le y \le \frac{x}{4}$ b) $4 \le x \le 0,\ \frac{x}{4} \le y \le 0$

c) $0 \le x \le \frac{y}{4},\ 0 \le y \le 4$ d) $0 \le x \le 1,\ 0 \le y \le 4$

Q.5) After changing the order of integration of $I = \int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dx\,dy$ the new limits of x & y are

✓ a) $0 \le x \le y$ $0 \le y < \infty$ b) $0 \le y \le x,\ 0 \le x < \infty$

c) $0 \le x \le 1$ $x \le y \le 1$ d) $0 \le x \le 1,\ 0 \le y \le 2$

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Q.1) $\int_0^1 \int_0^1 \frac{dx\,dy}{\sqrt{1-x^2}\sqrt{1-y^2}} = \ldots$

a) $\frac{\pi^2}{3}$ b) $\frac{\pi^2}{4}$ c) π d) 1

Q.2) $\int_0^{2\pi} \int_{a\sin\theta}^{a} r\,d\theta\,dr = \ldots$

a) $\frac{\pi a^2}{2}$ b) $\frac{\pi}{2}$ c) $\frac{\pi^2 a}{2}$ d) 5

Q.3) $\int_0^1 \int_0^x e^{x+y}\,dx\,dy = \ldots$

a) $\frac{(e-1)^2}{2}$ b) $\frac{e-1}{2}$ c) $\frac{e}{2}$ d) e

Q.4) After changing the order of integration $I = \int_0^1 \int_{4y}^4 e^{x^2}\,dx\,dy$, the new limits of x & y are

a) $0 \le x \le 4,\ 0 \le y \le \frac{x}{4}$ b) $4 \le x \le 0,\ \frac{x}{4} \le y \le 0$
c) $0 \le x \le \frac{y}{4},\ 0 \le y \le 4$ d) $0 \le x \le 1,\ 0 \le y \le 4$

Q.5) After changing the order of integration of $I = \int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dx\,dy$ the new limits of x & y are

a) $0 \le x \le y\quad 0 \le y < \infty$ b) $0 \le y \le x,\ 0 \le x < \infty$
c) $0 \le x \le 1\quad x \le y \le 1$ d) $0 \le x \le 1,\ 0 \le y \le 2$

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(M-2)

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Engineering Mathematics (M II)

Chapter 03 – Fourier Series

1) A function $f(x)$ is said to be periodic function with a period T, if
   a) $f(x)=f(x+T), \quad for\ all\ x$
   b) $f(T)=f(x+T), \quad for\ all\ x$
   c) $f(x)=-f(x+T), \quad for\ all\ x$
   d) $f(x)=f\left(\frac{x}{T}\right), \quad for\ all\ x$

2) A smallest positive number T satisfying $f(x)=f(x+T)$ is known as
   a) absolute function b) absolute time
   c) periodic time d)primitive period

3) If T is the fundamental period a function $f(x)$, which of the following is incorrect?
   a) $f(x)=f(x+nT), \quad n\in I$
   b) $f(x)=f(x+n+T), \quad n\in I$
   c) $f(x)=f(x-T)$
   d) $f(x)=f(x+T)$

4) If $f(x+nT)=f(x)$ where n is an integer and T is the smallest positive number, the fundamental period of $f(x)$ is
   a) T b) nT c) $2T$ d) $\frac{T}{2}$

5) If $f(x)$ is a periodic function of period T, then for $n\neq 0$, the function $f(nx)$ is a periodic function of period
   a) T b) T^n c) $\frac{T}{n}$ d) nT

6) The fundamental period of $\sin x$ is
   a) π b) 2π c) 3π d) $\frac{\pi}{2}$

7) The fundamental period of $\sin 2x$ is
   a) π b) 2π c) 3π d) $\frac{\pi}{2}$

8) The fundamental period of $\sin 4x$ is
   a) π b) 2π c) 3π d) $\frac{\pi}{2}$

9) The fundamental period of $\cos 3x$ is
   a) π b) $\frac{2\pi}{3}$ c) $\frac{3\pi}{2}$ d) 3π

10) The fundamental period of $\sin(-3x)$ is
   a) -3π b) 3π c) $-\frac{2\pi}{3}$ d) $\frac{2\pi}{3}$

11) The fundamental period of $\sin\left(-\frac{x}{2}\right)$ is
   a) π b) 2π c) 3π d) 4π

12) The fundamental period of $\cos(x+\pi)$ is
   a) π b) 2π c) 3π d) $\frac{\pi}{2}$

13) The fundamental period of $\sin\left(x+\frac{3\pi}{2}\right)$ is
   a) 2π b) $\frac{2\pi}{3}$ c) 3π d) π

14) The fundamental period of $\tan\left(3x+\frac{\pi}{2}\right)$ is
   a) 2π b) π c) 3π d) $\frac{\pi}{3}$

15) The fundamental period of $\sin\left(x+\frac{\pi}{6}\right)$ is
   a) π b) 2π c) 3π d) $\frac{\pi}{3}$

16) The fundamental period of $2\sin x$ is
   a) π b) 2π c) 3π d) 4π

17) The fundamental period of $\sin x\cos x$ is
   a) π b) 2π c) 3π d) 4π

18) The fundamental period of $\tan x$ is
a) 4π b) 3π c) 2π d) π

19) The fundamental period of $\tan 5x$ is
a) $\frac{\pi}{5}$ b) 5π c) 10π d) π

20) The fundamental period of $2\sec(-3x)$ is
a) $-\frac{2\pi}{3}$ b) $\frac{2\pi}{3}$ c) $\frac{3\pi}{2}$ d) $-\frac{3\pi}{2}$

21) The fundamental period of $\operatorname{cosec} 2x$ is
a) π b) 2π c) 3π d) $\frac{\pi}{2}$

22) A function $f(x)$ defined in the interval $[-a,\ a]$ is said to be even function, if
a) $f(-x)=-f(x)$ b) $f(2x)=2f(x)$
c) $f(-x)=f(x)$ d) $f(x)=-f(x)$

23) A function $f(x)$ defined in the interval $[-a,\ a]$ is said to be odd function, if
a) $f(x)=-f(x)$ b) $f(2x)=2f(x)$
c) $f(-x)=f(x)$ d) $f(-x)=-f(x)$

24) Which of the followings is an even function?
a) $\cosh x$ b) $x^3-\cos x$
c) $\tan 3x$ d) $e^x+\tan^2 x$

25) Which of the followings is an even function?
a) $\sin 3x$ b) $\tan x$ c) $\operatorname{cosec}^3 x$ d) $\tan^2 x$

26) Which of the followings is not an even function?
a) $\sin^3 x$ b) $\sin^2 x$ c) $\tan^2 x$ d) $\sec x$

27) Which of the followings is an odd function?
a) e^{-x} b) $\tan\frac{3x}{2}$
c) $\cos^3 x$ d) $\operatorname{cosec} 2x$

28) Which of the followings is an odd function?
a) $-e^x$ b) $-\tan^2 x$
c) $-\sin x$ d) $-\cos x$

29) Which of the followings is not an odd function?
a) $2\tan x$ b) $\tan^2 x$
c) $\tan x$ d) $\sin 3x$

30) Which of the followings is neither even nor an odd function?
a) $\operatorname{cosech} x$ b) $\tanh x$ c) e^x d) $\sinh x$

31) If $f(x)$ is to be constant function w.r.t. x, then $f(x)$ is
a)even function
b) odd function
c) both even and odd
d) neither even nor odd

32) If $f(x)=x^3+2x-\cos x$, the function $f(x)$ is
a) even function
b) odd function
c) both even and odd
d)neither even nor odd

33) If $f(x)=x^2-\sin^4 x\cdot e^{|x|}$, the function $f(x)$ is
a) even function
b) odd function
c) both even and odd
d) neither even nor odd

34) Which of the following statement is incorrect?
a) Product of even and odd function is an odd function.
b) Multiplication of even and odd function is an odd function.
c) Addition of even and odd function is an odd function.
d) Subtraction of two odd functions is an odd function.

35) Fourier series expansion of a function $f(x)$ defined on the interval $[c,\ c+2L]$ and having period $2L$ is given by
a) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{n\pi x}{L}\right)+b_n\sin\left(\frac{n\pi x}{L}\right)$
b) $a_0+\sum_{n=1}^{\infty}a_n\cos\left(\frac{n\pi x}{L}\right)+b_n\sin\left(\frac{n\pi x}{L}\right)$
c) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(n\pi Lx)+b_n\sin(n\pi Lx)$
d) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{nx}{L}\right)+b_n\sin\left(\frac{nx}{L}\right)$

36) Fourier series expansion of a function $f(x)$ defined on the interval $[0,\ 2\pi]$ and satisfying the Dirichlet's conditions is given by

a) $\dfrac{a_0}{2}+\sum\limits_{n=1}^{\infty} a_n \cos\dfrac{nx}{2}+b_n \sin\dfrac{nx}{2}$

b) $\dfrac{a_0}{2}+2\pi\sum\limits_{n=1}^{\infty} a_n \cos nx+b_n \sin nx$

c) $\dfrac{a_0}{2}+\sum\limits_{n=1}^{\infty} a_n \cos nx+b_n \sin nx$

d) $\dfrac{a_0}{2}+a_n \cos nx+b_n \sin nx$

37) If a function $f(x)$ is defined on the interval $[-\pi,\ \pi]$ and satisfying the Dirichlet's conditions, Fourier series expansion is given by

a) $\dfrac{a_0}{2}+\sum\limits_{n=1}^{\infty} a_n \cos\left(\dfrac{n\pi x}{L}\right)+b_n \sin\left(\dfrac{n\pi x}{L}\right)$

b) $\dfrac{a_0}{2}+\sum\limits_{n=1}^{\infty} a_n \cos\left(\dfrac{nx}{L}\right)+b_n \sin\left(\dfrac{nx}{L}\right)$

c) $\dfrac{a_0}{2}+\sum\limits_{n=1}^{\infty} a_n \cos\left(\dfrac{2n\pi x}{L}\right)+b_n \sin\left(\dfrac{2n\pi x}{L}\right)$

d) $\dfrac{a_0}{2}+\sum\limits_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$

38) If a function $f(x)$ is defined on the interval $[0,\ 4]$ with period $T=4$, Fourier series expansion is given by

a) $\dfrac{a_0}{2}+\sum\limits_{n=1}^{\infty} a_n \cos\left(\dfrac{n\pi x}{4}\right)+b_n \sin\left(\dfrac{n\pi x}{4}\right)$

b) $\dfrac{a_0}{2}+a_n \cos\left(\dfrac{n\pi x}{2}\right)+b_n \sin\left(\dfrac{n\pi x}{2}\right)$

c) $\dfrac{a_0}{2}+\sum\limits_{n=1}^{\infty} a_n \cos\left(\dfrac{n\pi x}{2}\right)+b_n \sin\left(\dfrac{n\pi x}{2}\right)$

d) $\dfrac{a_0}{2}+\sum\limits_{n=1}^{\infty} a_n \cos\left(\dfrac{2n\pi x}{L}\right)+b_n \sin\left(\dfrac{2n\pi x}{L}\right)$

39) Fourier series expansion of a function $f(x)$ defined over a period 2π and satisfying the Dirichlet's conditions is given by

a) $\dfrac{a_0}{2}+\sum\limits_{n=1}^{\infty} a_n \sin nx+b_n \cos nx$

b) $\dfrac{a_0}{2}+2\pi\sum\limits_{n=1}^{\infty} a_n \cos nx+b_n \sin nx$

c) $\dfrac{a_0}{2}+\sum\limits_{n=1}^{\infty} a_n \cos nx+b_n \sin nx$

d) $\dfrac{a_0}{2}+a_n \cos nx+b_n \sin nx$

40) If an even function $f(x)$ with period $2l$ is defined over the interval $(-l,\ l)$, its Fourier series expansion is given by

a) $\dfrac{a_0}{2}+\sum\limits_{n=1}^{\infty} a_n \sin\left(\dfrac{n\pi x}{l}\right)$ b) $\dfrac{a_0}{2}+\sum\limits_{n=1}^{\infty} a_n \cos\left(\dfrac{n\pi x}{l}\right)$

c) $\dfrac{a_0}{2}+\sum\limits_{n=1}^{\infty} a_n \cos\left(\dfrac{n\pi x}{2l}\right)$ d) $\dfrac{a_0}{2}+\sum\limits_{n=1}^{\infty} a_n \cos\left(\dfrac{nx}{l}\right)$

41) If an odd function $f(x)$ is defined over the interval $(-\pi,\ \pi)$, its Fourier series expansion is given by

a) $\sum\limits_{n=1}^{\infty} b_n \sin\left(\dfrac{2nx}{l}\right)$ b) $\sum\limits_{n=1}^{\infty} a_n \sin(nx)$

c) $\dfrac{a_0}{2}+\sum\limits_{n=1}^{\infty} a_n \sin(nx)$ d) $\sum\limits_{n=1}^{\infty} b_n \sin(nx)$

42) If an odd function $f(x)$ is of period 2π, its Fourier series expansion is given by

a) $\sum\limits_{n=1}^{\infty} b_n \sin\left(\dfrac{2nx}{l}\right)$ b) $\sum\limits_{n=1}^{\infty} a_n \sin(nx)$

c) $\dfrac{a_0}{2}+\sum\limits_{n=1}^{\infty} a_n \sin(nx)$ d) $\sum\limits_{n=1}^{\infty} b_n \sin(nx)$

43) The Fourier series expansion of an even function $f(x)$ with period 2π is given by

a) $\dfrac{a_0}{2}+\sum\limits_{n=1}^{\infty} a_n \sin(nx)$ b) $\dfrac{a_0}{2}+\sum\limits_{n=1}^{\infty} a_n \cos(nx)$

c) $\dfrac{a_0}{2}+\sum\limits_{n=1}^{\infty} a_n \cos\left(\dfrac{nx}{2}\right)$ d) $\dfrac{a_0}{2}+\sum\limits_{n=1}^{\infty} a_n \cos\left(\dfrac{nx}{l}\right)$

44) If an odd function $f(x)$ with period $2l$ is defined over the interval $(-l,\ l)$, its Fourier series expansion is given by

a) $\sum\limits_{n=1}^{\infty} b_n \sin\left(\dfrac{nx}{l}\right)$ b) $\dfrac{a_0}{2}+\sum\limits_{n=1}^{\infty} b_n \sin\left(\dfrac{n\pi x}{l}\right)$

c) $\sum\limits_{n=1}^{\infty} b_n \sin\left(\dfrac{n\pi x}{l}\right)$ d) $\sum\limits_{n=1}^{\infty} a_n \sin\left(\dfrac{n\pi x}{l}\right)$

45) If $f(x)$ is periodic function with period $2L$ in the interval C to $C+2L$, the Fourier coefficient a_0 is

a) $\frac{1}{L}\int_C^{C+2L} f(x)\cos\frac{nx}{L}dx$ b) $\frac{1}{L}\int_C^{C+2L} f(x)\sin\frac{nx}{L}dx$

c) $\int_C^{C+2L} f(x)dx$ d) $\frac{1}{L}\int_C^{C+2L} f(x)dx$

46) If $f(x)$ is periodic function with period $2L$ in the interval C to $C+2L$, the Fourier coefficient a_n is

a) $\frac{1}{L}\int_C^{C+2L} f(x)\cos\frac{2n\pi x}{L}dx$

b) $\frac{1}{L}\int_C^{C+2L} f(x)\cos\frac{n\pi x}{L}dx$

c) $\frac{1}{L}\int_C^{C+2L} f(x)\cos\frac{nx}{L}dx$

d) $\frac{1}{2L}\int_C^{C+2L} f(x)\cos\frac{nx}{L}dx$

47) If $f(x)$ is periodic function with period $2L$ in the interval C to $C+2L$, the Fourier coefficient b_n is

a) $\frac{1}{L}\int_C^{C+2L} f(x)\sin\frac{\pi x}{L}dx$

b) $\frac{1}{L}\int_C^{C+2L} f(x)\sin\frac{nx}{L}dx$

c) $\frac{1}{L}\int_C^{C+2L} f(x)\sin\frac{n\pi x}{L}dx$

d) $\frac{1}{L}\int_0^{2L} f(x)\sin\frac{n\pi x}{L}dx$

48) If $f(x)$ is an even function defined in the interval $[-L, L]$ and $f(x)=f(x+2L)$, the Fourier coefficient are

a) $a_0=\frac{2}{L}\int_0^L f(x)dx,\ a_n=\frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx,\ b_n=0$

b) $a_0=\frac{1}{L}\int_0^L f(x)dx,\ \ a_n=\frac{1}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx$

c) $a_0=0,\ \ a_n=\frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx$

d) $a_0=\frac{2}{L}\int_0^L f(x)dx,\ \ a_n=\frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$

49) If $f(x)$ is an odd function defined in the interval $[-L, L]$ and $f(x)=f(x+2L)$, the Fourier coefficient are

a) $a_0=0,\ \ a_n=0,\ \ b_n=\frac{1}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$

b) $a_0=0,\ \ a_n=0,\ \ b_n=\frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$

c) $a_0=0,\ \ a_n=0,\ \ b_n=\frac{2}{L}\int_0^{2L} f(x)\sin\frac{n\pi x}{L}dx$

d) $a_0=0,\ \ a_n=0,\ \ b_n=\frac{2}{L}\int_0^L f(x)\sin\frac{nx}{L}dx$

50) If $f(x)$ is an even periodic function defined in the interval $[-\pi, \pi]$, the Fourier coefficient are

a) $a_0=\frac{2}{\pi}\int_0^\pi f(x)dx,\ a_n=\frac{2}{\pi}\int_0^\pi f(x)\cos nx\,dx,\ b_n=0$

b) $a_0=\frac{2}{L}\int_0^\pi f(x)dx,\ a_n=\frac{2}{L}\int_0^\pi f(x)\cos\frac{nx}{L}dx,\ b_n=0$

c) $a_0=\frac{2}{\pi}\int_0^\pi f(x)dx,\ a_n=\frac{2}{\pi}\int_0^\pi f(x)\sin nxdx,\ b_n=0$

d) $a_0=\frac{1}{\pi}\int_0^\pi f(x)dx,\ a_n=\frac{1}{\pi}\int_0^\pi f(x)\cos nx\,dx,\ b_n=0$

51) If $f(x)$ is an odd periodic function defined in the interval $[-\pi, \pi]$, the Fourier coefficient are

a) $a_0=0,\ a_n=\frac{2}{\pi}\int_0^\pi f(x)\cos nx\,dx,\ b_n=0$

b) $a_0=\frac{2}{\pi}\int_0^\pi f(x)dx,\ a_n=\frac{2}{\pi}\int_0^\pi f(x)\cos nx\,dx,\ b_n=0$

c) $a_0=0,\ a_n=0,\ b_n=\frac{2}{\pi}\int_0^\pi f(x)\sin nx\,dx$

d) $a_0 = \frac{1}{\pi}\int_0^{\pi} f(x)\,dx,\ a_n = \frac{1}{\pi}\int_0^{\pi} f(x)\cos nx\,dx,\ b_n = 0$

52) The Fourier coefficient of an even periodic function $f(x)$ defined in the interval $[-2, 2]$ are

a) $a_0 = \int_0^2 f(x)\,dx,\ a_n = \int_0^2 f(x)\cos nx\,dx,\ b_n = 0$

b) $a_0 = \int_0^2 f(x)\,dx,\ a_n = \int_0^2 f(x)\cos\frac{n\pi x}{2}\,dx,\ b_n = 0$

c) $a_0 = \frac{2}{\pi}\int_0^2 f(x)\,dx,\ a_n = \frac{2}{\pi}\int_0^2 f(x)\cos nx\,dx,\ b_n = 0$

d) $a_0 = \int_0^2 f(x)\,dx,\ a_n = \int_0^2 f(x)\cos nx\,dx,\ b_n = 0$

53) The Fourier coefficient of an even periodic function $f(x)$ defined in the interval $[-1, 1]$ are

a) $a_0 = \frac{2}{\pi}\int_0^1 f(x)\,dx,\ a_n = \frac{2}{\pi}\int_0^1 f(x)\cos n\pi x\,dx,\ b_n = 0$

b) $a_0 = 2\int_0^2 f(x)\,dx,\ a_n = 2\int_0^2 f(x)\cos\frac{n\pi x}{2}\,dx,\ b_n = 0$

c) $a_0 = 2\int_0^1 f(x)\,dx,\ a_n = 2\int_0^1 f(x)\cos n\pi x\,dx,\ b_n = 0$

d) $a_0 = \int_0^1 f(x)\,dx,\ a_n = \int_0^1 f(x)\cos n\pi x\,dx,\ b_n = 0$

54) The Fourier coefficient of an odd periodic function $f(x)$ defined in the interval $[-4, 4]$ are

a) $a_0 = 0,\ \ a_n = 0,\ \ b_n = \frac{1}{2}\int_0^4 f(x)\cos\frac{n\pi x}{4}\,dx$

b) $a_0 = 0,\ \ a_n = 0,\ \ b_n = \frac{1}{4}\int_0^L f(x)\sin n\pi x\,dx$

c) $a_0 = 0,\ \ a_n = 0,\ \ b_n = 2\int_0^4 f(x)\sin\frac{n\pi x}{4}\,dx$

d) $a_0 = 0,\ \ a_n = 0,\ \ b_n = \frac{1}{2}\int_0^4 f(x)\sin\frac{n\pi x}{4}\,dx$

55) If the Fourier series expansion of an even function $f(x)$ over an interval $[-l, l]$ is given by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$, the value of a_0 is obtained by

a) $\frac{2}{l}\int_{-l}^{l} f(x)\,dx$ b) $\frac{2}{l}\int_0^{l} f(x)\cos\frac{n\pi x}{l}\,dx$

c) $\frac{1}{2l}\int_0^{l} f(x)\,dx$ d) $\frac{2}{l}\int_0^{l} f(x)\,dx$

56) If the Fourier series expansion of an even function $f(x)$ over an interval $[-l, l]$ is given by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$, the value of a_n is obtained by

a) $\frac{2}{l}\int_0^{l} f(x)\sin\frac{n\pi x}{l}\,dx$ b) $\frac{1}{l}\int_0^{l} f(x)\cos\frac{n\pi x}{l}\,dx$

c) $\frac{2}{l}\int_0^{l} f(x)\cos\frac{n\pi x}{l}\,dx$ d) $\frac{1}{l}\int_0^{l} f(x)\cos\frac{nx}{l}\,dx$

57) If the Fourier series expansion of an even function $f(x)$ over an interval $[-\pi, \pi]$ is given by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$, the value of a_0 is obtained by

a) $\frac{2}{\pi}\int_0^{\pi} f(x)\,dx$ b) $\frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$

c) $\frac{1}{\pi}\int_0^{\pi} f(x)\,dx$ d) $\frac{2}{\pi}\int_0^{\pi} f(x)\cos nx dx$

58) If the Fourier series expansion of an even function $f(x)$ over an interval $[-\pi, \pi]$ is given by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$, the value of a_n is obtained by

a) $\frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$ b) $\frac{2}{\pi}\int_0^{\pi} f(x)\cos nx dx$

c) $\frac{1}{\pi}\int_0^{l} f(x)\cos\frac{n\pi x}{l}\,dx$ d) $\frac{2}{\pi}\int_0^{\pi} f(x)\cos\frac{nx}{\pi}\,dx$

59) If the Fourier series expansion of an odd function $f(x)$ over an interval $[-l, l]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$, the value of b_0 is obtained by

a) $\int_0^l f(x)\cos\frac{n\pi x}{l}dx$ b) $\frac{1}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}dx$

c) $\frac{2}{l}\int_0^l f(x)dx$ d) none of the above

60) If the Fourier series expansion of an odd function $f(x)$ over an interval $[-l, l]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$, the value of b_n is obtained by

a) $\frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}dx$ b) $\frac{1}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}dx$

c) $\frac{1}{l}\int_0^l f(x)\sin\frac{nx}{l}dx$ d) $\int_0^l f(x)\cos\frac{n\pi x}{l}dx$

62) The half range Fourier cosine series for $f(x)$ defined over the interval $[0, L]$ is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{nx}{L}\right)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right)$

c) $a_0+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$ d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$

63) The half range Fourier sine series for $f(x)$ defined over the interval $[0, L]$ is given by

a) $\sum_{n=1}^{\infty} a_n \sin\left(\frac{nx}{L}\right)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right)$

c) $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$ d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$

64) The half range Fourier cosine series for $f(x)$ defined over the interval $[0, \pi]$ is given by

a) $\sum_{n=1}^{\infty} b_n \sin(nx)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$ d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)$

65) The half range Fourier sine series for $f(x)$ defined over the interval $[0, \pi]$ is given by

a) $\sum_{n=1}^{\infty} b_n \sin\left(\frac{nx}{\pi}\right)$ b) $\sum_{n=1}^{\infty} b_n \sin(nx)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right)$ d) $\sum_{n=1}^{\infty} a_n \sin(n\pi x)$

66) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, L]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$ the value of a_0 is given by

a) $\frac{1}{L}\int_0^L f(x)dx$ b) $\frac{1}{L}\int_0^L f(x)dx$

c) $\frac{2}{\pi}\int_0^\pi f(x)dx$ d) $\frac{2}{L}\int_0^L f(x)dx$

67) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, L]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$ the value of a_n is given by

a) $\frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$

b) $\frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

c) $\frac{2}{L}\int_0^L f(x)\cos(nx)dx$

d) $\frac{1}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

68) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, L]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$ the value of b_0 is given by

a) $\frac{1}{L}\int_0^L f(x)\sin\frac{x}{L}dx$ b) $\frac{2}{L}\int_0^L f(x)\sin x\,dx$

c) 0 d) $\frac{2}{L}\int_0^L f(x)dx$

69) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, L]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$ the value of b_n is given by

a) $\frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$

b) $\frac{1}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$

c) $\frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

d) $\frac{2}{L}\int_0^L f(x)\sin(nx)\,dx$

70) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, \pi]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(nx)$ the value of a_0 is given by

a) $\frac{1}{L}\int_0^L f(x)\,dx$ b) $\frac{1}{L}\int_0^L f(x)\,dx$

c) $\frac{2}{\pi}\int_0^\pi f(x)\,dx$ d) $\frac{2}{L}\int_0^L f(x)\,dx$

71) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, \pi]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(nx)$ the value of a_n is given by

a) $\frac{2}{L}\int_0^L f(x)\sin(nx)\,dx$

b) $\frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

c) $\frac{2}{\pi}\int_0^\pi f(x)\cos(nx)\,dx$

d) $\frac{1}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

72) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, \pi]$ is given by $\sum_{n=1}^{\infty}b_n\sin(nx)$ the value of b_0 is given by

a) $\frac{1}{L}\int_0^L f(x)\sin\frac{x}{L}\,dx$ b) $\frac{2}{L}\int_0^L f(x)\sin x\,dx$

c) 0 d) $\frac{2}{L}\int_0^L f(x)\,dx$

73) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, \pi]$ is given by $\sum_{n=1}^{\infty}b_n\sin(nx)$ the value of b_n is given by

a) $\frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$

b) $\frac{1}{L}\int_0^L f(x)\sin(nx)\,dx$

c) $\frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

d) $\frac{2}{\pi}\int_0^\pi f(x)\sin(nx)\,dx$

74) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, 1]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(n\pi x)$ the value of a_0 is given by

a) $\frac{1}{\pi}\int_0^\pi f(x)\,dx$ b) $2\int_0^1 f(x)\,dx$

c) $\frac{2}{\pi}\int_0^\pi f(x)\,dx$ d) $\int_0^1 f(x)\,dx$

75) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, 2]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{n\pi x}{2}\right)$ the value of a_n is given by

a) $\frac{2}{3}\int_0^2 f(x)\sin\left(\frac{n\pi x}{2}\right)dx$

b) $\frac{1}{2}\int_0^2 f(x)\cos\left(\frac{n\pi x}{2}\right)dx$

c) $\frac{2}{L}\int_0^2 f(x)\cos(nx)\,dx$

d) $\int_0^2 f(x)\cos\left(\frac{n\pi x}{2}\right)dx$

76) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, 3]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{3}\right)$ the value of b_0 is given by

a) $\frac{1}{3}\int_0^3 f(x)\sin\frac{x}{3}\,dx$ b) $\frac{2}{3}\int_0^3 f(x)\sin 3x\,dx$

c) 0 d) $\frac{2}{3}\int_0^3 f(x)\,dx$

77) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, 4]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{4}\right)$ the value of b_n is given by

a) $\frac{2}{3}\int_0^2 f(x)\sin\left(\frac{n\pi x}{4}\right)dx$

b) $\frac{1}{2}\int_0^4 f(x)\sin\left(\frac{n\pi x}{4}\right)dx$

c) $\int_0^2 f(x)\cos\left(\frac{n\pi x}{4}\right)dx$

d) $\frac{1}{2}\int_0^4 f(x)\sin(nx)\,dx$

78) In the harmonic analysis for a function defined over a period of 2π, the term $a_1\cos x + b_1\sin x$ is known as

a) amplitude of $f(x)$ b) second harmonic
c) first harmonic d) none of these

79) In the harmonic analysis for a function defined over a period of 2π, the amplitude of the first harmonic is

a) $\sqrt{a_1^2 - b_1^2}$ b) $\sqrt{a_1^2 + b_1^2}$
c) $\sqrt{a_0^2 + a_1^2}$ d) $a_1^2 + b_1^2$

80) In the harmonic analysis for a function defined over a period of 2π, the amplitude of the second harmonic is

a) $\left(a_2^2 + b_2^2\right)^2$ b) $\frac{1}{2}\left(a_2^2 + b_2^2\right)$

c) $2\sqrt{a_2^2 + b_2^2}$ d) $\sqrt{a_2^2 + b_2^2}$

81) In the harmonic analysis for a function defined over a period of 2π, the amplitude of the second harmonic is

a) $\left(a_n^2 + b_n^2\right)^n$ b) $\sqrt{a_n^2 + b_n^2}$

c) $n\sqrt{a_n^2 + b_n^2}$ d) $\frac{1}{n}\sqrt{a_n^2 + b_n^2}$

82) In the harmonic analysis for a function $f(x)$ defined over a period of $2L$, the first harmonic term is given by

a) $b_1\sin\frac{\pi x}{L}$ b) $a_1\cos\frac{\pi x}{L}$

c) $a_1\cos\frac{\pi x}{L} - b_1\sin\frac{\pi x}{L}$ d) $a_1\cos\frac{\pi x}{L} + b_1\sin\frac{\pi x}{L}$

83) In the harmonic analysis for a function $f(x)$ defined over a period of 2, the first harmonic term is given by

a) $a_1\cos\pi x + b_1\sin\pi x$ b) $a_1\cos\frac{\pi x}{2} + b_1\sin\frac{\pi x}{2}$

c) $a_1\cos 2\pi x + b_1\sin 2\pi x$ d) $a_1\cos\frac{\pi x}{2} - b_1\sin\frac{\pi x}{2}$

84) If $f(x) = x\cos x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(nx) + b_n\sin(nx)$, the Fourier coefficient a_0 is given by

a) $-\frac{1}{\pi}$ b) 0 c) $\frac{1}{\pi}$ d) $\frac{2}{\pi}$

85) If $f(x) = x\cos x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(nx) + b_n\sin(nx)$, the Fourier coefficient a_n is given by

a) $\frac{1}{\pi}$ b) $-\frac{1}{\pi}$ c) $\frac{2}{\pi}$ d) 0

86) If $f(x) = x\cos x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(nx) + b_n\sin(nx)$, the Fourier coefficient b_1 is given by

a) $\frac{1}{\pi}$ b) $-\frac{1}{\pi}$ c) $-\frac{1}{2}$ d) 0

87) If $f(x)=\cos x$ where $-\pi\le x\le\pi$ is of periodic function with period 2π, the Fourier coefficient a_1 is given by

a) 1 b) $-\frac{1}{\pi}$ c) $\frac{2}{\pi}$ d) 0

88) If $f(x)=x$ where $-\pi\le x\le\pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(nx)+b_n\sin(nx)$, the Fourier coefficient a_0 is given by

a) $\frac{\pi}{2}$ b) 0 c) 1 d) $\frac{\pi^2}{2}$

89) If $f(x)=x$ where $-\pi\le x\le\pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(nx)+b_n\sin(nx)$, the Fourier coefficient b_1 is given by

a) 2 b) 0 c) π d) $\frac{\pi}{2}$

90) If $f(x)=2$ where $-\pi\le x\le\pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(nx)+b_n\sin(nx)$, the Fourier coefficient a_0 is given by

a) 2 b) 4 c) 3 d) none of these

91) If $f(x)=2$ where $-\pi\le x\le\pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(nx)+b_n\sin(nx)$, the Fourier coefficient b_n is given by

a) $\frac{3\pi}{2}$ b) $-\frac{\pi}{2}$ c) 0 d) $\frac{\pi}{2}$

92) If $f(x)=a$ where $-\pi\le x\le\pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(nx)+b_n\sin(nx)$, the Fourier coefficient a_0 is given by

a) $2a$ b) 0 c) 2π d) $\frac{\pi}{2}$

93) If $f(x)=a$ where $-\pi\le x\le\pi$ is of periodic function with period 2π, the Fourier coefficient a_0 is given by

a) 2π b) $2a$ c) 0 d) $\frac{\pi}{2}$

94) If $f(x)=\sin^2 x$ where $-\pi\le x\le\pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(nx)+b_n\sin(nx)$, the Fourier coefficient b_n is given by

a) $\frac{3\pi}{2}$ b) $-\frac{\pi}{2}$ c) $\frac{\pi}{2}$ d) 0

95) If $f(x)=\cosh x$ where $-\pi\le x\le\pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(nx)+b_n\sin(nx)$, the Fourier coefficient b_n is given by

a) 0 b) $\frac{\pi}{3}$ c) $e^{-\pi}$ d) $e^{-2\pi}$

96) If $f(x)=\begin{cases}-x & -\pi\le x\le 0\\ x & 0\le x\le\pi\end{cases}$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(nx)+b_n\sin(nx)$, the Fourier coefficient a_0 is given by

a) $\frac{\pi}{2}$ b) π c) 0 d) $-\frac{\pi}{2}$

97) If $f(x)=\begin{cases}-x & -\pi\le x\le 0\\ x & 0\le x\le\pi\end{cases}$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(nx)+b_n\sin(nx)$, the Fourier coefficient b_n is given by

a) $\frac{\pi}{2}$ b) π c) 0 d) $-\frac{\pi}{2}$

98) If $f(x)=\begin{cases}1+\frac{2x}{\pi} & -\pi\le x\le 0\\ 1-\frac{2x}{\pi} & 0\le x\le\pi\end{cases}$ is of periodic function with period 2π, the Fourier coefficient b_n is given by

a) $\frac{\pi}{2}$ b) π c) $-\frac{\pi}{2}$ d) 0

99) If $f(x)=x-x^3$ where $-2\le x\le 2$ is of periodic function with period 2 and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(nx)+b_n\sin(nx)$, the Fourier coefficient b_1 is given by

a) 2 b) 0 c) π d) $\frac{\pi}{2}$

100) If $f(x)=x+\frac{x^2}{4}$ where $-\pi\le x\le\pi$ is of periodic function with period 2π, the Fourier coefficient a_0 is given by

a) 0 b) $\frac{\pi}{2}$ c) $-\frac{\pi}{2}$ d) $\frac{\pi^2}{6}$

101) If $f(x)=e^x$ where $-\pi\le x\le\pi$ is of periodic function with period 2π , the Fourier coefficient a_0 is given by

a) 1 b) $\frac{e^{\pi}-e^{-\pi}}{\pi}$ c) $\frac{e^{\pi}+e^{-\pi}}{\pi}$ d) 0

102) If $f(x)=x-x^2$ where $-1\le x\le 1$ is of periodic function with period 2, the Fourier coefficient a_0 is given by

a) 1 b) $-\frac{2}{3}$ c) π d) 0

103) If $f(x)=1-x^2$ where $-1\le x\le 1$ is of periodic function with period 2, the Fourier coefficient a_0 is given by

a) 1 b) $\frac{2\pi}{3}$ c) $\frac{4}{3}$ d) 0

104) If $f(x)=k$ where $-l\le x\le l$ is of periodic function with period 2, the Fourier coefficient a_0 is given by

a) $2k$ b) $\frac{2k\pi}{3}$ c) $2k\pi$ d) 0

105) If $f(x)=\begin{cases}-a & -2\le x\le 0\\ a & 0\le x\le 2\end{cases}$ is of periodic function with period 4, the Fourier coefficient b_n is given by

a) 0 b) 4 c) $-\frac{\pi}{2}$ d) $-\frac{2a}{n\pi}\left[(-1)^n-1\right]$

106) If $f(x)=\begin{cases}0 & -2\le x\le 0\\ 2 & 0\le x\le 2\end{cases}$ is of periodic function with period 4, the Fourier coefficient a_0 is given by

a) 0 b) 4 c) $-\frac{\pi}{2}$ d) 1

107) If $f(x)=\begin{cases}1 & -1\le x\le 0\\ \cos\pi x & 0\le x\le 1\end{cases}$ is of periodic function with period 2, the Fourier coefficient a_0 is given by

a) 0 b) $\frac{\pi}{2}$ c) $-\frac{\pi}{2}$ d) 1

108) If $f(x)=e^{-x}$ defined over $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, the Fourier coefficient a_0 is given by

a) $\frac{1}{2\pi}\left(1-e^{-2\pi}\right)$ b) $\frac{2}{\pi}\left(1-e^{-2\pi}\right)$

c) $\frac{1}{\pi}\left(1+e^{-x}\right)$ d) $\frac{1}{\pi}\left(1-e^{-2\pi}\right)$

109) If $f(x)=x$ defined over $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, the Fourier coefficient a_0 is given by

a) 3π b) $\frac{\pi}{2}$ c) π d) 2π

110) If $f(x)=x$ defined over $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, the Fourier coefficient a_n is given by

a) 0 b) π c) 2π d) 3π

111) If $f(x)=x$ defined over $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, the Fourier coefficient b_n is given by

a) $-\frac{2}{n\pi}$ b) $-\frac{\pi}{n}$ c) $-\frac{1}{n}$ d) $-\frac{2}{n}$

112) If $f(x)=\sqrt{1-\cos x}$ defined over $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, the Fourier coefficient a_0 is given by

a) 0 b) $-\frac{4\sqrt{2}}{\pi}$ c) $\frac{4\sqrt{2}}{\pi}$ d) $\frac{8\sqrt{2}}{\pi}$

113) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=\dfrac{\pi-x}{2}$ defined over the interval $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, is given by

a) $-\dfrac{\pi}{2}$ b) $\dfrac{\pi}{2}$ c) 0 d) $\dfrac{\pi^2}{6}$

114) The Fourier coefficient a_n for the Fourier series expansion of $f(x)=\dfrac{\pi-x}{2}$ defined over the interval $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, is given by

a) 0 b) π c) $\dfrac{\pi}{2}$ d) $-\dfrac{\pi}{2}$

115) The Fourier coefficient b_n for the Fourier series expansion of $f(x)=\dfrac{\pi-x}{2}$ defined over the interval $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, is given by

a) $-\dfrac{1}{n^2}$ b) $\dfrac{1}{n}$ c) $-\dfrac{1}{n}$ d) $\dfrac{\pi}{n}$

116) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=\left(\dfrac{\pi-x}{2}\right)^2$ defined over the interval $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, is given by

a) $\dfrac{\pi^2}{6}$ b) $\dfrac{\pi^2}{12}$ c) $\dfrac{\pi^2}{3}$ d) $\dfrac{\pi^2-1}{6}$

117) Consider $f(x)=x\sin x,\ x\in[0,2\pi]$ and $f(x+2\pi)=f(x)$. Then the Fourier coefficient a_0 for the Fourier series expansion of $f(x)$ is given by

a) -4 b) $-\dfrac{\pi}{2}$ c) -2 d) $\dfrac{\pi}{2}$

118) If $f(x)=\begin{cases} x & 0\le x\le\pi \\ 0 & \pi\le x\le 2\pi \end{cases}$ is periodic over a period 2π, the Fourier coefficient a_0 for the Fourier series expansion of $f(x)$ is given by

a) π b) $\dfrac{\pi}{2}$ c) $-\dfrac{\pi}{2}$ d) $\dfrac{\pi}{4}$

119) If $f(x)=\begin{cases} 0 & 0\le x\le\pi \\ x & \pi\le x\le 2\pi \end{cases}$ is periodic over a period 2π, the Fourier coefficient a_0 for the Fourier series expansion of $f(x)$ is given by

a) $\dfrac{3\pi}{2}$ b) $\dfrac{\pi}{2}$ c) 3π d) $\dfrac{3\pi}{4}$

120) If the function $f(x)=\begin{cases} -\pi & 0\le x\le\pi \\ x-\pi & \pi\le x\le 2\pi \end{cases}$ and $f(x+2\pi)=f(x)$, the Fourier coefficient a_0 is given by

a) 0 b) $-\dfrac{\pi}{2}$ c) $-\dfrac{\pi}{4}$ d) $-\pi$

121) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=x-x^2$ defined over the interval $0\le x\le 2$ and $f(x+2)=f(x)$, is given by

a) 0 b) $-\dfrac{4}{3}$ c) $-\dfrac{2}{3}$ d) $\dfrac{2}{3}$

122) The Fourier coefficient a_n for the Fourier series expansion of $f(x)=x-x^2$ defined over the interval $0\le x\le 2$ and $f(x+2)=f(x)$, is given by

a) 0 b) $-\dfrac{4}{n^2\pi^2}$ c) $\dfrac{4}{n^2\pi^2}$ d) $-\dfrac{1}{n^2\pi^2}$

123) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=x+x^2$ defined over the interval $0\le x\le 3$ and having period 3, is given by

a) 0 b) $-\dfrac{4}{n^2\pi^2}$ c) $\dfrac{4}{n^2\pi^2}$ d) $\dfrac{3}{2}$

124) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=2x-x^2$ defined over the interval $0\le x\le 4$ and $f(x+4)=f(x)$, is given by

a) $-\dfrac{1}{3}$ b) $-\dfrac{2}{3}$ c) $-\dfrac{4}{3}$ d) $-\dfrac{8}{3}$

125) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=2x-x^2$ defined over the interval $0\le x\le 3$ and $f(x+3)=f(x)$, is given by

a) 0 b) $-\frac{2}{3}$ c) $-\frac{4}{3}$ d) $-\frac{8}{3}$

126) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=1-x^2$ defined over the interval $0\le x\le 2$ and $f(x+2)=f(x)$, is given by

a) $-\frac{1}{3}$ b) $-\frac{2}{3}$ c) $-\frac{4}{3}$ d) $-\frac{8}{3}$

127) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=1-x^2$ defined over the interval $0\le x\le 1$ and $f(x+2)=f(x)$, is given by

a) $\frac{2}{3}$ b) $-\frac{2}{3}$ c) $-\frac{1}{3}$ d) $-\frac{4}{3}$

128) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=4-x^2$ defined over the interval $0\le x\le 2$ and $f(x+2)=f(x)$, is given by

a) $-\frac{1}{3}$ b) $\frac{16}{3}$ c) $-\frac{16}{3}$ d) $-\frac{8}{3}$

129) If $f(x)=\begin{cases}\pi x & 0\le x\le 1\\ \pi(2-x) & 1\le x\le 2\end{cases}$ is periodic over a period 2, the Fourier coefficient a_0 for the Fourier series expansion of $f(x)$ is given by

a) $-\frac{\pi}{2}$ b) π c) $-\pi$ d) $\frac{\pi}{2}$

130) If $f(x)=\begin{cases}x & 0\le x\le 1\\ 0 & 1\le x\le 2\end{cases}$ is periodic over a period 2, the Fourier coefficient a_0 for the Fourier series expansion of $f(x)$ is given by

a) 2 b) 0 c) $\frac{1}{2}$ d) 1

131) The Fourier coefficient a_0 in the half range cosine series expansion of function $f(x)=\sin x$ defined over the interval $[0,\ \pi]$ is given by

a) $\frac{2}{\pi}$ b) $\frac{\pi}{2}$ c) $-\frac{\pi}{2}$ d) 0

132) The Fourier coefficient b_1 in the half range cosine series expansion of function $f(x)=\cos x$ defined over the interval $[0,\ \pi]$ is given by

a) $-\frac{\pi}{2}$ b) 0 c) $\frac{1}{2\pi}$ d) $\frac{\pi}{2}$

133) The Fourier coefficient a_0 in the half range cosine series expansion of function $f(x)=\pi x-x^2$ defined over the interval $[0,\ \pi]$ is given by

a) 0 b) $\frac{\pi^2}{6}$ c) $\frac{2\pi^2}{3}$ d) $\frac{\pi^2}{3}$

134) The Fourier coefficient a_0 in the half range sine series expansion of function $f(x)=\cos x$ defined over the interval $[0,\ \pi]$ is given by

a) $\frac{4}{\pi}$ b) $\frac{2}{\pi}$ c) $\frac{\pi}{2}$ d) 0

135) The Fourier coefficient a_0 in the half range cosine series expansion of function $f(x)=\sin x$ defined over the interval $[0,\ \pi]$ is given by

a) $\frac{4}{\pi}$ b) $\frac{2}{\pi}$ c) 0 d) $\frac{\pi}{2}$

136) The Fourier coefficient a_1 in the half range cosine series expansion of function $f(x)=\sin x$ defined over the interval $[0,\ \pi]$ is given by

a) 1 b) $\frac{2}{\pi}$ c) 0 d) $\frac{\pi}{2}$

137) The Fourier coefficient b_1 in the half range cosine series expansion of function $f(x)=x$ defined over the interval $[0,\ 2]$ with period 4 is given by

a) 0 b) $\frac{1}{\pi}$ c) $\frac{2}{\pi}$ d) $\frac{4}{\pi}$

138) The Fourier coefficient a_0 in the half range cosine series expansion of function $f(x)=x-x^2$ defined over the interval $[0,\ 1]$ is given by

a) $\frac{1}{3}$ b) $-\frac{1}{3}$ c) 0 d) $\frac{2}{\pi}$

139) The Fourier coefficient a_0 in the half range cosine series expansion of function $f(x)=lx-x^2$ defined over the interval $[0,\ l]$ with period $2l$ is given by

a) 0 b) $\frac{l^2}{3}$ c) $\frac{\pi}{2}$ d) $\frac{2}{\pi n^2}(\cos n\pi-1)$

140) The Fourier coefficient a_1 in the half range cosine series expansion of function $f(x)=x-x^2$ defined over the interval $[0,\ 1]$ is given by

a) 2 b) $\frac{2}{\pi}$ c) $\frac{\pi}{2}$ d) $\frac{1}{2}$

141) The Fourier coefficient a_n in the half range cosine series expansion of function $f(x)=x-x^2$ defined over the interval $[0,\ 1]$ is given by

a) 0 b) $\frac{2}{\pi}$ c) $\frac{\pi}{2}$ d) $\frac{2}{\pi n^2}(\cos n\pi-1)$

142) The Fourier coefficient a_n in the half range sine series expansion of function $f(x)=2+x$ defined over the interval $[0,\ 1]$ is given by

a) 4 b) 0 c) $-\frac{2}{n\pi}$ d) $-\frac{2\pi}{n}$

143) The Fourier series expansion for the function $f(x)=\left(\frac{\pi-x}{2}\right)^2$ over the interval $0\le x\le 2\pi$ is given by $\left(\frac{\pi-x}{2}\right)^2=\frac{\pi^2}{12}+\sum_{n=1}^{\infty}\frac{1}{n^2}\cos nx$. Then the value of $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots$ is

a) 1 b) $\frac{\pi^2}{6}$ c) $\frac{\pi^2}{12}$ d) $\frac{\pi^2}{3}$

144) The Fourier series expansion for the function $f(x)=\pi^2-x^2$ over the interval $-\pi\le x\le\pi$ is given by $\pi^2-x^2=\frac{2\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}\cos nx$. Then the value of $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-+\ldots$ is

a) 1 b) $\frac{\pi^2}{6}$ c) $\frac{\pi^2}{12}$ d) $\frac{\pi^2}{3}$

145) The Fourier series expansion for the function $f(x)=\pi^2-x^2$ over the interval $-\pi\le x\le\pi$ is given by $\pi^2-x^2=\frac{2\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}\cos nx$. Then the value of $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots$ is

a) $\frac{\pi^2}{6}$ b) $\frac{\pi^2}{12}$ c) $\frac{\pi^2}{3}$ d) 0

146) The Fourier series expansion for the function $f(x)=\begin{cases}1+\frac{2x}{\pi} & -\pi\le x\le 0\\ 1-\frac{2x}{\pi} & 0\le x\le\pi\end{cases}$ is given by $f(x)=\frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos nx$. Then the value of $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots$ is

a) $\frac{\pi^2}{6}$ b) $\frac{\pi^2}{12}$ c) $\frac{\pi^2}{3}$ d) $\frac{\pi^2}{8}$

147) The Fourier series expansion for the function $f(x)=\begin{cases}\pi x & 0\le x\le 1\\ \pi(2-x) & 1\le x\le 2\end{cases}$ is given by $f(x)=\frac{\pi}{2}-\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos(2n-1)\pi x$. Then the value of $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots$ is

a) $\frac{\pi^2}{6}$ b) $\frac{\pi^2}{12}$ c) $\frac{\pi^2}{8}$ d) $\frac{\pi^2}{3}$

148) In the following harmonic analysis of $y=f(x)$, the value of a_0 is given by

x	0	1	2	3	4	5
y	4	8	15	7	5	3

a) 14 b) 7 c) 3.5 d) 6

149) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	1	2	3	4	5
y	9	18	26	26	26	20

a) 25.01 b) 20.83 c) 41.66 d) 40.89

150) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	30	60	90	120	150	180
y	0	9.2	14.4	17.8	17	12	0

a) 10.23 b) 23.46 c) 46.92 d) 11.73

151) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	60	120	180	240	300	360
y	1	1.4	1.9	1.6	1.6	1.2	1

a) 7.2 b) 1.45 c) 5.8 d) 2.9

152) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	60	120	180	240	300	360
y	1.98	2.15	2.7	-0.22	-0.31	1.5	1.98

a) 4.8 b) 2.6 c) 5.2 d) 1.3

153) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	1	1.4	1.9	1.6	1.6	1.2	1

a) 2.9 b) 5.8 c) 1.45 d) 3.8

154) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
y	1.98	1.35	1	1.3	-0.88	-0.25	1.98

a) 1 b) 0.75 c) 1.5 d) 3

155) In the following harmonic analysis of $y = f(x)$, the value of a_1 is given by

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
y	0	9.2	14.4	17.8	17.3	11.7	0

a) 3.73 b) 5.73 c) 7.73 d) -7.73

156) In the following harmonic analysis of $y = f(x)$, the value of b_1 is given by

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
y	0	9.2	14.4	17.8	17.3	11.7	0

a) 4.38 b) 3.48 c) 4.83 d) 8.43

157) In the following harmonic analysis of $y = f(x)$, the value of a_1 is given by

x	0	1	2	3	4	5
y	9	18	24	28	26	20

a) - 8.37 b) 8.73 c) 7.83 d) 3.78

158) In the following harmonic analysis of $y = f(x)$, the value of b_1 is given by

x	0	1	2	3	4	5
y	9	18	24	28	26	20

a) 1.25 b) -6.3 c) – 3.15 d) - 3.50

159) In the following harmonic analysis of $y = f(x)$, the value of b_1 is given by

x	0	1	2	3	4	5	6
y	9	18	24	28	26	20	9
$\cos\left(\frac{\pi}{3}x\right)$	1	0.5	-0.5	-1	-0.5	0.5	1

a) 3.38 b) – 8.33 c) 8.33 d) 5.83

Chapter 04–Reduction Formulae, Beta and Gamma Functions

I) Reduction Formulae

1) For $I_n = \int_0^{\pi/2} \sin^n x\,dx$, we have

a) $I_n = 2\int_0^{\pi} \sin^n x\,dx$ b) $I_n = \int_0^{\pi/2} \sin^{n-2} x \cos^2 x\,dx$

c) $I_n = \int_0^{\pi/2} \cos^n x\,dx$ d) $I_n = \frac{1}{2}\int_0^{\pi/4} \sin^n x\,dx$

2) For $I_n = \int_0^{\pi} \sin^n x\,dx$, we have

a) 0 b) $I_n = 2\int_0^{\pi/2} \sin^n x\,dx$

c) $I_n = 4\int_0^{\pi/2} \sin^n x\,dx$ d) none of these

3) For $I_n = \int_0^{\pi} \cos^n x\,dx$, where n is an even integer, we have

a) $I_n = 4\int_0^{\pi/2} \cos^n x\,dx$ b) $I_n = 2\int_0^{\pi/4} \cos^n x\,dx$

c) $I_n = 2\int_0^{\pi/2} \cos^n x\,dx$ d) 0

4) For $I_n = \int_0^{\pi} \cos^n x\,dx$, where n is an odd integer, we have

a) $I_n = 4\int_0^{\pi/2} \cos^n x\,dx$ b) $I_n = 2\int_0^{\pi/4} \cos^n x\,dx$

c) $I_n = 2\int_0^{\pi/2} \cos^n x\,dx$ d) 0

5) For $I_n = \int_0^{2\pi} \sin^n x\,dx$, where n is an even integer, we have

a) 0 b) $I_n = 4\int_0^{\pi/4} \sin^n x\,dx$

c) $I_n = 2\int_0^{\pi/2} \sin^n x\,dx$ d) $I_n = 4\int_0^{\pi/2} \sin^n x\,dx$

6) For $I_n = \int_0^{2\pi} \sin^n x\,dx$, where n is an odd integer, we have

a) $I_n = 4\int_0^{\pi/2} \sin^n x\,dx$ b) $I_n = 2\int_0^{\pi/2} \sin^n x\,dx$

c) 0 d) $I_n = 4\int_0^{\pi/4} \sin^n x\,dx$

7) For $I_n = \int_0^{2\pi} \cos^n x\,dx$, where n is an odd integer, we have

a) 0 b) $I_n = 4\int_0^{\pi/2} \cos^n x\,dx$

c) $I_n = 2\int_0^{\pi/2} \cos^n x\,dx$ d) $I_n = 4\int_0^{\pi/4} \cos^n x\,dx$

8) For $I_n = \int_0^{2\pi} \cos^n x\,dx$, where n is an even integer, we have

a) 0 b) $I_n = 4\int_0^{\pi/4} \cos^n x\,dx$

c) $I_n = 4\int_0^{\pi/2} \cos^n x\,dx$ d) $I_n = 2\int_0^{\pi/2} \cos^n x\,dx$

9) For $I_{m,n}=\int_0^{\pi}\sin^m x\cos^n x\,dx$, where both m and n are odd integers, we have

a) $I_{m,n}=2\int_0^{\pi/2}\sin^m x\cos^n x\,dx$ b) 0

c) $I_{m,n}=4\int_0^{\pi/2}\sin^m x\cos^n x\,dx$ d) none

10) For $I_{m,n}=\int_0^{\pi}\sin^m x\cos^n x\,dx$, where both m and n are even integers, we have

a) $I_{m,n}=2\int_0^{\pi/2}\sin^m x\cos^n x\,dx$

b) $I_{m,n}=4\int_0^{\pi/2}\sin^m x\cos^n x\,dx$

c) 0 d) none of the above

11) For $I_{m,n}=\int_0^{\pi}\sin^m x\cos^n x\,dx$, where only n is an even integer, we have

a) 0 b) $I_{m,n}=4\int_0^{\pi/2}\sin^m x\cos^n x\,dx$

c) $I_{m,n}=2\int_0^{\pi/2}\sin^m x\cos^n x\,dx$ d) none

12) For $I_{m,n}=\int_0^{\pi}\sin^m x\cos^n x\,dx$, where only n is an odd integer, we have

a) $I_{m,n}=4\int_0^{\pi/2}\sin^m x\cos^n x\,dx$

b) 0

c) $I_{m,n}=2\int_0^{\pi/2}\sin^m x\cos^n x\,dx$

d) none of the above

13) For $I_n=\int_0^{\pi/2}\sin^n x\,dx$, which of the following is the reduction formula?

a) $I_n=\frac{n-1}{n}I_{n-1}$ b) $I_n=\frac{n}{n+1}I_{n-2}$

c) $I_n=\frac{n+1}{n}I_{n-2}$ d) $I_n=\frac{n-1}{n}I_{n-2}$

14) For $I_n=\int_0^{\pi/2}\cos^n x\,dx$, which of the following is the reduction formula?

a) $I_n=\frac{n-1}{n}I_{n-2}$ b) $I_n=\frac{n-1}{n}I_{n-1}$

c) $I_n=\frac{n}{n+1}I_{n-2}$ d) $I_n=\frac{n+1}{n}I_{n-2}$

15) For $I_n=\int_0^{\pi/2}\sin^n x\,dx$, where n is an even natural number, which of the following is the reduced form?

a) $I_n=\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \ldots\ \cdot\frac{1}{2}\cdot\frac{\pi}{2}$

b) $I_n=\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \ldots\ \cdot\frac{2}{3}\cdot 1$

c) $I_n=\frac{n+1}{n}\cdot\frac{n+3}{n+2}\cdot\frac{n+5}{n+4}\cdot\ \ldots\ \cdot\frac{1}{2}\cdot\frac{\pi}{2}$

d) $I_n=\frac{n-2}{n-1}\cdot\frac{n-4}{n-3}\cdot\frac{n-6}{n-5}\cdot\ \ldots\ \cdot\frac{2}{3}\cdot\frac{\pi}{2}$

16) For $I_n=\int_0^{\pi/2}\cos^n x\,dx$, where n is an even natural number, which of the following is the reduced form?

a) $I_n=\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \ldots\ \cdot\frac{2}{3}\cdot 1$

b) $I_n=\frac{n-2}{n-1}\cdot\frac{n-4}{n-3}\cdot\frac{n-6}{n-5}\cdot\ \ldots\ \cdot\frac{2}{3}\cdot\frac{\pi}{2}$

c) $I_n=\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \ldots\ \cdot\frac{1}{2}\cdot\frac{\pi}{2}$

d) $I_n=\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \ldots\ \cdot\frac{1}{2}\cdot\pi$

17) For $I_n = \int_0^{\pi/2} \sin^n x\,dx$, where n is an odd natural number, which of the following is the reduced form?

a) $I_n = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdot \ldots \cdot \frac{2}{3} \cdot 1$

b) $I_n = \frac{n-2}{n-1} \cdot \frac{n-4}{n-3} \cdot \frac{n-6}{n-5} \cdot \ldots \cdot \frac{2}{3} \cdot \frac{\pi}{2}$

c) $I_n = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdot \ldots \cdot \frac{1}{2} \cdot \frac{\pi}{2}$

d) $I_n = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdot \ldots \cdot \frac{1}{2} \cdot \pi$

18) For $I_n = \int_0^{\pi/2} \cos^n x\,dx$, where n is an odd natural number, which of the following is the reduced form?

a) $I_n = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdot \ldots \cdot \frac{1}{2} \cdot \frac{\pi}{2}$

b) $I_n = \frac{n+1}{n} \cdot \frac{n+3}{n+2} \cdot \frac{n+5}{n+4} \cdot \ldots \cdot \frac{2}{3} \cdot 1$

c) $I_n = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdot \ldots \cdot \frac{2}{3} \cdot 1$

d) $I_n = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdot \ldots \cdot \frac{1}{2} \cdot \pi$

19) For $I_n = \int_0^{\pi/2} \sin^n x \cos^n x\,dx$, where n is an odd natural number, which of the following is the reduced form?

a) $I_{(m,\,n)} = \frac{[(m-1)(m-3)\cdot \ldots \cdot 2 or 1][(n-1)(n-3)\cdot \ldots \cdot 2 or 1]}{(m+n)(m+n-2)\cdot \ldots \cdot 2 or 1} \cdot k$

where $k = \begin{cases} \frac{\pi}{2} & both\ m\ \&\ n\ are\ odd \\ 1 & otherwise \end{cases}$

b) $I_{(m,\,n)} = \frac{[(m-1)(m-3)\cdot \ldots \cdot 2 or 1][(n-1)(n-3)\cdot \ldots \cdot 2 or 1]}{(m+n)(m+n-2)\cdot \ldots \cdot 2 or 1} \cdot k$

where $k = \begin{cases} \frac{\pi}{2} & both\ m\ \&\ n\ are\ even \\ 1 & otherwise \end{cases}$

c) $I_{(m,\,n)} = \frac{[(m-1)(m-3)\cdot \ldots \cdot 2 or 1][(n-1)(n-3)\cdot \ldots \cdot 2 or 1]}{(m+n)(m+n-2)\cdot \ldots \cdot 2 or 1} \cdot k$

where $k = \begin{cases} \frac{\pi}{2} & m+n\ is\ even \\ 1 & otherwise \end{cases}$

d) $I_{(m,\,n)} = \frac{(m+n-1)(m+n-3)\cdot \ldots \cdot 2 or 1}{(m+n)(m+n-2)\cdot \ldots \cdot 2 or 1} \cdot k$

where $k = \begin{cases} \frac{\pi}{2} & both\ m\ \&\ n\ are\ odd \\ 1 & otherwise \end{cases}$

20) The value of $\int_0^{\pi/2} \sin^3 x\,dx$ is equal to

a) $\frac{3\pi}{4}$ b) $\frac{3}{4}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$

21) The value of $\int_0^{\pi/2} \sin^4 x\,dx$ is equal to

a) $\frac{3}{8}$ b) $\frac{3}{16}$ c) $\frac{3\pi}{16}$ d) $\frac{3\pi}{18}$

22) The value of $\int_0^{\pi/2} \sin^5 x\,dx$ is equal to

a) $\frac{4\pi}{15}$ b) $\frac{8\pi}{30}$ c) $\frac{8\pi}{15}$ d) $\frac{8}{15}$

23) The value of $\int_0^{\pi/2} \sin^9 x\,dx$ is equal to

a) $\frac{64}{315}$ b) $\frac{128}{315}$ c) $\frac{128}{315}\pi$ d) $\frac{64}{315}\pi$

24) The value of $\int_0^{\pi/2} \cos^3 x\,dx$ is equal to

a) $\frac{3\pi}{4}$ b) $\frac{3}{4}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$

25) The value of $\int_0^{\pi/2} \cos^4 x\,dx$ is equal to

a) $\frac{3}{8}$ b) $\frac{3}{16}$ c) $\frac{3\pi}{16}$ d) $\frac{3\pi}{18}$

26) The value of $\int_0^{\pi/2} \cos^7 x\,dx$ is equal to

a) $\frac{8}{35}$ b) $\frac{16\pi}{35}$ c) $\frac{16\pi}{70}$ d) $\frac{16}{35}$

27) The value of $\int_0^{\pi/2} \cos^{10} x\,dx$ is equal to

a) $\frac{63\pi}{128}$ b) $\frac{63\pi}{512}$ c) $\frac{63\pi}{256}$ d) $\frac{64}{315}\pi$

28) The value of $\int_0^{\pi/2} \sin^3 x \cos^2 x\,dx$ is equal to

a) $\frac{2}{15}$ b) $\frac{\pi}{30}$ c) $\frac{1}{15}$ d) $\frac{\pi}{15}$

29) The value of $\int_0^{\pi/2} \sin^2 x \cos^3 x\,dx$ is equal to

a) $\frac{1}{15}$ b) $\frac{\pi}{30}$ c) $\frac{\pi}{15}$ d) $\frac{2}{15}$

30) The value of $\int_0^{\pi/2} \sin^3 x \cos^4 x\,dx$ is equal to

a) $\frac{1}{35}$ b) $\frac{2}{35}$ c) $\frac{2\pi}{35}$ d) $\frac{2\pi}{70}$

31) The value of $\int_0^{\pi/2} \sin^4 x \cos^6 x\,dx$ is equal to

a) $\frac{3\pi}{512}$ b) $\frac{3}{256}$ c) $\frac{3\pi}{256}$ d) $\frac{3\pi}{128}$

32) The value of $\int_{-\pi/2}^{\pi/2} \sin^3 x \cos^2 x\,dx$ is equal to

a) $2\int_0^{\pi/2} \sin^3 x \cos^2 x\,dx$ b) $4\int_0^{\pi/2} \sin^3 x \cos^2 x\,dx$

c) 0 d) none of the above

33) The value of $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos^3 x\,dx$ is equal to

a) 0 b) $2\int_0^{\pi/2} \sin^2 x \cos^3 x\,dx$

c) $3\int_0^{\pi/3} \sin^2 x \cos^3 x\,dx$ d) none of the above

34) The value of $\int_{-\pi/2}^{\pi/2} \sin^4 x \cos^2 x\,dx$ is equal to

a) $\frac{3}{16}$ b) $\frac{3\pi}{16}$ c) $\frac{\pi}{16}$ d) 0

35) The value of $\int_{-\pi/2}^{\pi/2} \sin^5 x\,dx$ is equal to

a) $\frac{3\pi}{128}$ b) $\frac{3\pi}{15}$ c) $\frac{32}{256}$ d) 0

36) The value of $\int_0^{2\pi} \sin^4 x \cos^6 x\,dx$ is equal to

a) $\frac{3}{64}$ b) $\frac{2\pi}{35}$ c) $\frac{2}{35}$ d) $\frac{3\pi}{128}$

37) The value of $\int_0^{2\pi} \sin^4 x \cos^7 x\,dx$ is equal to

a) $\frac{5}{128}$ b) $\frac{5\pi}{128}$ c) 0 d) $\frac{5\pi}{256}$

38) The value of $\int_{-\pi}^{\pi} \sin^4 x \cos^7 x\,dx$ is equal to

a) 0 b) $\frac{5\pi}{128}$ c) $\frac{5}{128}$ d) $\frac{5\pi}{256}$

39) The value of $\int_0^{\pi} \cos^3 x\,dx$ is equal to

a) $\frac{5\pi}{256}$ b) $\frac{5\pi}{16}$ c) $\frac{5}{128}$ d) 0

40) The value of $\int_0^{\pi} \cos^6 x\,dx$ is equal to

a) 0 b) $\frac{5\pi}{16}$ c) $\frac{5}{8}$ d) $\frac{5\pi}{256}$

41) The value of $\int_0^{\pi} \cos^7 x\,dx$ is equal to

a) $\frac{5\pi}{256}$ b) $\frac{5\pi}{16}$ c) $\frac{5}{128}$ d) 0

42) The value of $\int_0^{\pi} \sin^7 x\,dx$ is equal to

a) $\frac{5\pi}{32}$ b) $\frac{5\pi}{16}$ c) $\frac{32}{35}$ d) 0

43) The value of $\int_0^{\pi} \sin^6 x\,dx$ is equal to

a) $\frac{5\pi}{16}$ b) $\frac{5\pi}{32}$ c) $\frac{3}{4}$ d) 0

44) The value of $\int_0^{2\pi} \sin^6 \theta\,d\theta$ is equal to

a) $\frac{5\pi}{32}$ b) $\frac{5\pi}{16}$ c) $\frac{5\pi}{8}$ d) 0

45) The value of $\int_0^{2\pi} \sin^8 x\,dx$ is equal to

a) $\frac{5\pi}{16}$ b) $\frac{5\pi}{32}$ c) $\frac{32}{35}$ d) $\frac{35\pi}{32}$

46) The value of $\int_0^{2\pi} \cos^5 x\,dx$ is equal to

a) $\frac{5\pi}{32}$ b) $\frac{5\pi}{16}$ c) $\frac{32}{35}$ d) 0

47) The value of $\int_0^{2\pi} \sin^6 x \cos^4 x\,dx$ is equal to

a) $\frac{5\pi}{256}$ b) $\frac{3\pi}{128}$ c) $\frac{35\pi}{256}$ d) 0

48) The value of $\int_0^{2\pi} \sin^7 x \cos^4 x\,dx$ is equal to

a) $\frac{5\pi}{256}$ b) $\frac{3\pi}{128}$ c) $\frac{35\pi}{256}$ d) 0

49) The value of $\int_0^{2\pi} \sin^7 x \cos^5 x\,dx$ is equal to

a) $\frac{5\pi}{256}$ b) 0 c) $\frac{35\pi}{256}$ d) $\frac{3\pi}{128}$

50) The value of $\int_0^{\pi} \sin^5\left(\frac{x}{2}\right)dx$ is equal to

a) $\frac{16\pi}{15}$ b) $\frac{5\pi}{32}$ c) $\frac{16}{15}$ d) $\frac{5\pi}{16}$

51) The value of $\int_0^{\pi/4} \sin^2(2x)\,dx$ is equal to

a) $\frac{\pi}{8}$ b) $\frac{16}{15}$ c) $\frac{3\pi}{8}$ d) 0

52) The value of $\int_0^{\pi/4} \sin^7(2x)\,dx$ is equal to

a) $\frac{16\pi}{15}$ b) $\frac{5\pi}{16}$ c) $\frac{8}{35}$ d) 0

53) The value of $\int_0^{\pi/4} \cos^2(2x)\,dx$ is equal to

a) $\frac{5\pi}{16}$ b) $\frac{\pi}{8}$ c) $\frac{5\pi}{32}$ d) 0

54) The value of $\int_0^{\pi/3} \sin^5(3x)\,dx$ is equal to

a) $\frac{3\pi}{16}$ b) $\frac{8\pi}{15}$ c) $\frac{8\pi}{45}$ d) $\frac{8}{45}$

55) If $I_n = \int_0^{\pi/4} \sin^{2n} x\,dx = -\frac{1}{2^{n+1}n} + \frac{2n-1}{2n} I_{n-1}$, the value of I_2 is equal to

a) $\frac{3\pi+2}{8}$ b) $\frac{3\pi-8}{32}$ c) $-\frac{8+3\pi}{32}$ d) $\frac{3\pi}{32}$

56) If $I_n = \int_0^{\pi/2} x \sin^n x\,dx = \frac{1}{n^2} + \frac{n-1}{n} I_{n-2}$, the value of I_5 is equal to

a) $\frac{149}{25}$ b) $\frac{19}{225}$ c) $\frac{\pi}{2} - \frac{149}{225}$ d) $\frac{149}{225}$

56) If $I_n = \int_0^{\pi/2} \tan^n x\,dx = \frac{1}{n-1} - I_{n-2}$, the value of I_4 is equal to

a) $\frac{\pi}{4} - \frac{2}{3}$ b) $\frac{\pi}{4} + \frac{2}{3}$ c) $\frac{\pi}{2} - \frac{2}{3}$ d) $\frac{\pi}{4} + \frac{4}{3}$

II) Gamma Functions

57) For $n>0$, the gamma function $\overline{|n}$ is defined as

a) $\int_0^\infty e^x x^{n-1}dx$ b) $\int_0^\infty e^{-x} x^{n+1}dx$

c) $\int_0^\infty e^{-x} x^{n}dx$ d) $\int_0^\infty e^{-x} x^{n-1}dx$

58) $\int_0^\infty e^{-x} x^{n}dx$ is equal to

a) $\overline{|n+1}$ b) $\overline{|n}$ c) $\overline{|n-1}$ d) $\overline{|n-2}$

59) $\int_0^\infty e^{-kx} x^{n}dx$ is equal to

a) $k^{n-1}\overline{|n-1}$ b) $\dfrac{\overline{|n-1}}{k^{n-1}}$ c) $\dfrac{\overline{|n+1}}{k^{n+1}}$ d) $\dfrac{\overline{|n}}{k^{n}}$

60) $\int_0^\infty e^{-kx} x^{n-1}dx$ is equal to

a) $k^{n-1}\overline{|n-1}$ b) $\dfrac{\overline{|n-1}}{k^{n-1}}$ c) $\dfrac{\overline{|n+1}}{k^{n+1}}$ d) $\dfrac{\overline{|n}}{k^{n}}$

61) The value of $\overline{|n}$ is equal to

a) $n\overline{|n-1}$ b) $(n+1)\overline{|n+1}$

c) $(n-1)\overline{|n-1}$ d) $n\overline{|n}$

62) If n is a natural number, the value of $\overline{|n}$ is

a) $\dfrac{n!}{n+1}$ b) $(n-1)!$ c) $n!$ d) $(n+1)!$

63) The value of $\overline{|1}$ is

a) 1 b) 2 c) 3 d) 0

64) The value of $\overline{|2}$ is

a) 0 b) 1 c) 2 d) 3

65) The value of $\overline{|7}$ is

a) 3256 b) 5040 c) 120 d) 720

66) The value of $\overline{|1/2}$ is

a) $\dfrac{1}{2}$ b) $\sqrt{\pi}$ c) $\overline{|\pi}$ d) none

67) The value of $\overline{|5/2}$ is

a) $\dfrac{3\sqrt{\pi}}{2}$ b) $\dfrac{3\sqrt{\pi}}{4}$ c) $\dfrac{3\sqrt{\pi}}{8}$ d) 0

68) The value of $\overline{|\tfrac{1}{4}}\cdot\overline{|\tfrac{3}{4}}$ is

a) $\pi\sqrt{2}$ b) $\dfrac{\pi}{\sqrt{2}}$ c) $\dfrac{\sqrt{2}}{\pi}$ d) none

69) The value of $\overline{|p}\cdot\overline{|1-p}$, for $0<p<1$, is given by the formula

a) $\dfrac{\sin p\pi}{\pi}$ b) $\dfrac{\pi}{\sin p\pi}$

c) $\dfrac{\sqrt{\pi}}{\sin p\pi}$ d) $\dfrac{p\pi}{\sin p\pi}$

70) The value of $\int_0^\infty e^{-x} x^{5}dx$

a) 60 b) 720 c) 120 d) 240

71) The value of $\int_0^\infty e^{-2x} x^{5}dx$

a) $\dfrac{125}{32}$ b) $\dfrac{120}{35}$ c) $\dfrac{25}{8}$ d) $\dfrac{15}{8}$

72) The value of $\int_0^\infty e^{-x} x^{1/2}dx$

a) $\dfrac{\sqrt{\pi}}{2}$ b) $\dfrac{\pi}{2}$ c) $\dfrac{\sqrt{\pi}}{3}$ d) $\sqrt{\pi}$

73) The value of $\int_0^\infty e^{-x} x^{-1/2}dx$

a) $\dfrac{\sqrt{\pi}}{2}$ b) $\sqrt{\pi}$ c) $\dfrac{\sqrt{\pi}}{3}$ d) $\dfrac{\pi}{2}$

74) The value of $\int_0^\infty e^{-x} x^{3/2}dx$

a) $\dfrac{\sqrt{\pi}}{4}$ b) $\dfrac{3\sqrt{\pi}}{8}$ c) $\dfrac{3\sqrt{\pi}}{4}$ d) $\dfrac{3\sqrt{\pi}}{2}$

75) The substitution for the integral $\int_0^\infty \sqrt{x}\cdot e^{-\sqrt{x}}dx$ to reduce it into the form of gamma function is

a) $\sqrt{x}=t$ b) $\sqrt{x}=t^2$

c) $\sqrt{x}=\frac{t}{2}$ d) $x=\sin t$

76) The substitution for the integral $\int_0^\infty x^3\cdot e^{-\sqrt{x}}dx$ to reduce it into the form of gamma function is

a) $x^3=\sin^2 t$ b) $x^3=e^{-t}$

c) $x^3=t$ d) $\sqrt{x}=t$

77) The substitution for the integral $\int_0^\infty x^3\cdot 5^{-x}dx$ to reduce it into the form of gamma function is

a) $5^x=e^t$ b) $x^3=e^{-t}$

c) $5^x=x^{-t}$ d) $\log x=5^{-x}$

78) On using substitution $\sqrt{x}=t$, the value of the integration $\int_0^\infty x\cdot e^{-\sqrt{x}}dx$ is given by

a) 1 b) 3 c) 12 d) 16

79) On using substitution $\sqrt{x}=t$, the value of the integration $\int_0^\infty \sqrt{x}\cdot e^{-\sqrt{x}}dx$ is given by

a) 1 b) 2 c) 3 d) 4

80) On using substitution $\sqrt{t}=x$, the value of the integration $\int_0^\infty e^{-x^2}dx$ is given by

a) $\left|\overline{\tfrac{1}{4}}\right.$ b) 16 c) $\frac{\sqrt{\pi}}{2}$ d) $\sqrt{\pi}$

81) On using substitution $x^3=t$, the value of the integration $\int_0^\infty \sqrt{x}\cdot e^{-x^3}dx$ is given by

a) $\frac{\sqrt{\pi}}{2}$ b) $\frac{\sqrt{\pi}}{3}$ c) $\frac{2\sqrt{\pi}}{3}$ d) $\frac{3\sqrt{\pi}}{4}$

82) On using substitution $x^4=t$, the value of the integration $\int_0^\infty e^{-x^4}dx$ is given by

a) $\sqrt{\pi}$ b) π c) $\frac{1}{4}\left|\overline{\tfrac{1}{4}}\right.$ d) $\left|\overline{\tfrac{3}{4}}\right.$

83) On using substitution $x=t^2$, the value of the integration $\int_0^\infty \sqrt[4]{x}\cdot e^{-\sqrt{x}}dx$ is given by

a) $\frac{3\sqrt{\pi}}{2}$ b) $\frac{2\sqrt{\pi}}{3}$ c) $\frac{\sqrt{\pi}}{3}$ d) $2\sqrt{\pi}$

84) On using substitution $2x^2=t$, the value of the integration $\int_0^\infty x^7\cdot e^{-2x^2}dx$ is given by

a) $\left|\overline{\tfrac{3}{4}}\right.$ b) $\frac{3}{8}$ c) $\frac{2\sqrt{\pi}}{3}$ d) $\frac{3}{16}$

85) On using substitution $2x^2=t$, the value of the integration $\int_0^\infty x^9\cdot e^{-2x^2}dx$ is given by

a) $\left|\overline{\tfrac{3}{4}}\right.$ b) $\frac{3}{8}$ c) $\frac{2\sqrt{\pi}}{3}$ d) $\frac{3}{16}$

86) On using substitution $x^2=t$, the value of the integration $\int_0^\infty x^2\cdot e^{-x^2}dx$ is given by

a) $\frac{1}{3}\left|\overline{\tfrac{3}{2}}\right.$ b) $\frac{3}{2}\left|\overline{\tfrac{3}{2}}\right.$ c) $\frac{1}{2}\left|\overline{\tfrac{3}{2}}\right.$ d) $\frac{1}{2}\left|\overline{\tfrac{2}{3}}\right.$

87) On using substitution $x=t^{1/3}$, the value of the integration $\int_0^\infty \sqrt{x}\cdot e^{-x^3}dx$ is given by

a) $\frac{\sqrt{\pi}}{3}$ b) $\frac{2\sqrt{\pi}}{3}$ c) $\frac{1}{2}\left|\overline{\tfrac{2}{3}}\right.$ d) $\frac{1}{3}\left|\overline{\tfrac{3}{2}}\right.$

88) On using substitution $a^{-x}=e^{-t}$, the value of the integration $\int_0^\infty \frac{x^a}{a^x}dx$ is given by

a) $\frac{\left|\overline{a}\right.}{(\log a)^a}$ b) $\frac{\left|\overline{a-1}\right.}{(\log a)^{a-1}}$

c) $\dfrac{\overline{|a+1}}{(\log a)^{a+1}}$ d) $\dfrac{\overline{|a}}{(\log a)^{a+1}}$

89) On using substitution $3^{-x} = e^{-t}$, the value of the integration $\int_0^\infty \frac{x^3}{3^x} dx$ is given by

a) $\dfrac{3}{(\log 3)^4}$ b) $\dfrac{6}{(\log 3)^4}$
c) $\dfrac{36}{(\log 3)^4}$ d) $\dfrac{6}{(\log 3)^3}$

90) On using substitution $\log x = -t$, the value of the integration $\int_0^1 (x\log x)^3 dx$ is given by

a) $-\dfrac{3}{64}$ b) $\dfrac{3}{64}$ c) $\dfrac{3}{128}$ d) $-\dfrac{3}{128}$

91) On using substitution $\log x = -t$, the value of the integration $\int_0^1 \left(\log\frac{1}{x}\right)^{n-1} dx$ is given by

a) $\overline{|n+1}$ b) $\overline{|n}$ c) $\overline{|n-1}$ d) $-\overline{|1+n}$

92) On using substitution $\log x = -t$, the value of the integration $\int_0^1 \frac{1}{\sqrt{x\log\frac{1}{x}}} dx$ is given by

a) $\sqrt{2\pi}$ b) $\pi\sqrt{2}$ c) $2\sqrt{\pi}$ d) 2π

93) On using substitution $\log x = -t$, the value of the integration $\int_0^1 \frac{1}{\sqrt{-\log x}} dx$ is given by

a) $\sqrt{2\pi}$ b) $\pi\sqrt{2}$ c) $\sqrt{\pi}$ d) $2\sqrt{\pi}$

94) On using substitution $h^2x^2 = t$, the value of the integration $\int_0^\infty x^{n-1} e^{-h^2x^2} dx$ is given by

a) $\sqrt{2\pi}$ b) $\dfrac{\overline{|n/2}}{2h^n}$ c) $\dfrac{\overline{|n/2}}{2h^{n+1}}$ d) $\dfrac{\overline{|1+n/2}}{2h^{n+1}}$

II) Beta Functions

95) The value of $\beta(m, n)$ in the integral form is

a) $\int_0^1 x^m (1-x)^{n-1} dx$ b) $\int_0^1 x^m (1-x)^n dx$
c) $\int_0^1 x^{m+1} (1-x)^{n+1} dx$ d) $\int_0^1 x^{m-1} (1-x)^{n-1} dx$

96) The value of $\beta(m, n)$ in terms of gamma function is

a) $\dfrac{\overline{|m}\cdot\overline{|n}}{\overline{|m+n+1}}$ b) $\dfrac{\overline{|m-1}\cdot\overline{|n-1}}{\overline{|m+n}}$
c) $\dfrac{\overline{|m+1}\cdot\overline{|n+1}}{\overline{|m+n+1}}$ d) $\dfrac{\overline{|m}\cdot\overline{|n}}{\overline{|m+n}}$

97) The value of $\beta(m, n)$, when m and n are positive integers is

a) $\dfrac{(m-1)!(n-1)!}{(m+n-1)!}$ b) $\dfrac{(m+1)!(n+1)!}{(m+n+1)!}$
c) $\dfrac{m!n!}{(m+n)!}$ d) $\dfrac{m!n!}{(m+n+1)!}$

98) $\int_0^{\pi/2} \sin^m x\cos^n x dx$ is given by

a) $\beta(m, n)$ b) $\frac{1}{2}\beta\left(\frac{m-1}{2}, \frac{n-1}{2}\right)$
c) $\frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$ d) $\beta\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$

99) $\int_0^{\pi/2} \sin^{2m-1} x\cos^{2n-1} x dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$ b) $\beta\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$
c) $\frac{1}{2}\beta\left(\frac{m-1}{2}, \frac{n-1}{2}\right)$ d) $\beta(m, n)$

100) $\int_0^{\pi/2} \sin^m x\, dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{m-1}{2}, \frac{1}{2}\right)$ b) $\frac{1}{2}\beta\left(m, \frac{1}{2}\right)$
c) $\frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{1}{2}\right)$ d) $\frac{1}{2}\beta\left(\frac{m+1}{2}, 0\right)$

101) $\int_0^{\pi/2} \cos^m x\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{m-1}{2}, \frac{1}{2}\right)$ b) $\frac{1}{2}\beta\left(m, \frac{1}{2}\right)$

c) $\frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{1}{2}\right)$ d) $\frac{1}{2}\beta\left(\frac{m+1}{2}, 0\right)$

102) $\int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}}\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$ b) $\beta(m, n)$

c) $\beta(m+1, n+1)$ d) $\beta(m-1, n-1)$

103) $\beta(3, 5)$ can be represented by

a) $\int_0^{\infty} x^2 (1-x)^4\,dx$ b) $\int_0^1 x^4 (1-x)^6\,dx$

c) $\int_0^1 x^3 (1-x)^5\,dx$ d) $\int_0^1 x^2 (1-x)^4\,dx$

104) What is the exact value of $\beta(5, 3)$?

a) $\frac{2}{35}$ b) $\frac{2}{105}$ c) $\frac{1}{105}$ d) $\frac{1}{35}$

105) What is the exact value of $\beta\left(\frac{1}{4}, \frac{3}{4}\right)$?

a) $\frac{1}{8}$ b) $\pi\sqrt{2}$ c) $2\sqrt{\pi}$ d) $\sqrt{2\pi}$

106) $\int_0^1 \sqrt{x}(1-x)^{5/2}\,dx$ is equal to

a) $\beta\left(\frac{3}{2}, \frac{7}{2}\right)$ b) $\beta\left(\frac{1}{2}, \frac{5}{2}\right)$

c) $\beta\left(\frac{2}{3}, \frac{5}{3}\right)$ d) $\beta(2, 5)$

107) $\int_0^1 x^4 (1-x)^5\,dx$ is equal to

a) $\frac{3}{462}$ b) $\frac{1}{462}$ c) $\frac{1}{501}$ d) $\frac{1}{231}$

108) $2\int_0^{\pi/2} \sin^{\frac{3}{2}} x \cos^5 x\,dx$ is given by

a) $\beta\left(\frac{5}{4}, 3\right)$ b) $\frac{1}{2}\beta\left(\frac{5}{4}, 3\right)$

c) $\beta\left(\frac{5}{4}, \frac{3}{2}\right)$ d) $\beta\left(\frac{5}{4}, \frac{3}{4}\right)$

109) $2\int_0^{\pi/2} \sqrt{\sin x \cos x}\,dx$ is given by

a) $\beta\left(\frac{1}{2}, \frac{1}{2}\right)$ b) $\beta\left(\frac{5}{4}, \frac{5}{4}\right)$

c) $\beta\left(\frac{3}{4}, \frac{3}{4}\right)$ d) $\beta\left(\frac{3}{2}, \frac{3}{2}\right)$

110) $\int_0^{\pi/2} \frac{1}{\sqrt{\sin x}}\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{1}{4}, \frac{3}{2}\right)$ b) $\beta\left(\frac{1}{4}, \frac{1}{2}\right)$

c) $2\beta\left(\frac{1}{4}, \frac{1}{2}\right)$ d) $\frac{1}{2}\beta\left(\frac{1}{4}, \frac{1}{2}\right)$

111) $\int_0^{\pi/2} \frac{1}{\sqrt{\cos x}}\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{1}{4}, \frac{3}{2}\right)$ b) $\frac{1}{2}\beta\left(\frac{1}{4}, \frac{1}{2}\right)$

c) $\beta\left(\frac{1}{4}, \frac{1}{2}\right)$ d) $2\beta\left(\frac{1}{4}, \frac{1}{2}\right)$

112) $\int_0^{\pi/2} \sqrt{\tan x}\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{3}{4}, \frac{5}{4}\right)$ b) $\beta\left(\frac{3}{4}, \frac{1}{4}\right)$

c) $2\beta\left(\frac{3}{4}, \frac{1}{4}\right)$ d) $\frac{1}{2}\beta\left(\frac{3}{4}, \frac{1}{4}\right)$

113) $\int_0^{\pi/2} \sqrt{\cot x}\,dx$ is given by

a) $2\beta\left(\frac{3}{4}, \frac{1}{4}\right)$ b) $\frac{1}{2}\beta\left(\frac{3}{4}, \frac{5}{4}\right)$

c) $\frac{1}{2}\beta\left(\frac{3}{4}, \frac{1}{4}\right)$ d) $\beta\left(\frac{3}{4}, \frac{1}{4}\right)$

114) $\int_0^{\pi/2} \tan^{3/4} x\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{7}{4}, \frac{1}{4}\right)$ b) $\frac{1}{2}\beta\left(\frac{7}{4}, \frac{1}{4}\right)$

c) $\frac{1}{2}\beta\left(\frac{7}{8}, \frac{1}{8}\right)$ d) $\frac{1}{2}\beta\left(\frac{7}{8}, \frac{7}{8}\right)$

115) The value of the integral $\int_0^\infty \frac{x^4}{(1+x)^7} dx$ is

a) $\frac{1}{30}$ b) 30 c) $\frac{1}{15}$ d) $\frac{1}{3}$

116) The value of the integral $\int_0^\infty \frac{x^3+x^2}{(1+x)^7} dx$ is

a) 30 b) $\frac{1}{3}$ c) $\frac{1}{30}$ d) $\frac{1}{15}$

117) The value of the integral $\int_0^\infty \frac{x^8-x^{14}}{(1+x)^{24}} dx$ is

a) 30 b) 0 c) $\frac{1}{30}$ d) $\frac{1}{15}$

118) The value of the integral $\int_0^\infty \frac{x^6(1-x^8)}{(1+x)^{24}} dx$ is

a) 30 b) 0 c) $\frac{1}{30}$ d) $\frac{1}{15}$

119) $\beta(n, n+1)$ is identical with

a) $\frac{(\Gamma n)^2}{\Gamma 2n}$ b) $\frac{\Gamma n}{\Gamma 2n}$ c) $\frac{\Gamma n}{2\Gamma 2n}$ d) $\frac{(\Gamma n)^2}{2\Gamma 2n}$

120) $\beta(m, n+1)+\beta(m+1, n)$ is equal to

a) $\beta(m+1, n+1)$ b) $\beta(m+1, n)$
c) $\beta(m, n)$ d) $\beta(m, n+1)$

121) $\beta(m, n)\cdot\beta(m+n, k)$ is equal to

a) $\frac{\Gamma m\cdot\Gamma n+k}{\Gamma m+n+k}$ b) $\frac{\Gamma m\cdot\Gamma n\cdot\Gamma k}{\Gamma m+n}$
c) $\frac{\Gamma m\cdot\Gamma n}{\Gamma m+n+k}$ d) $\frac{\Gamma m\cdot\Gamma n\cdot\Gamma k}{\Gamma m+n+k}$

122) $\beta(m, n+1)$ is equal to

a) $\frac{m+n}{n}\beta(m, n)$ b) $\frac{n}{m+n}\beta(m, n)$
c) $\frac{m}{m+n}\beta(m, n)$ d) $\frac{m+n}{m}\beta(m, n)$

123) On using substitution $x^3=8t$, the integral $\int_0^2 x(8-x^3)^{1/3} dx$ is equal to

a) $\frac{5}{81}$ b) $\frac{2}{27}$ c) $\frac{2}{81}$ d) $\frac{1}{81}$

124) The value of the integration $\int_0^1 x^3\left(1-x^{1/2}\right)^5 dx$ by substituting $x=t^2$ is given by

a) $2\beta(8, 6)$ b) $\frac{1}{2}\beta(8, 6)$
c) $\beta(8, 6)$ d) $2\beta(9, 7)$

125) The value of the integration $\int_0^1 \left(1-x^{1/n}\right)^m dx$ by substituting $x=t^n$ is given by

a) $n\beta(m, n+1)$ b) $n\beta(m+1, n)$
c) $n\beta(m, n)$ d) $m\beta(m+1, n)$

Chapter 05–Differentiation Under Integral Sign & Error Function

I) Differentiation Under Integral Sign

1) If $I(\alpha)=\int_a^b f(x,\alpha)\,dx$, where α is parameter and a, b are constants, by differentiation under integral sign rule we have

a) $\frac{dI}{d\alpha}=\int_a^b \frac{\partial}{\partial\alpha}\left[f(x,\alpha)\right]\cdot dx$

b) $\frac{dI}{d\alpha}=\int_a^b \frac{\partial}{\partial x}\left[f(x,\alpha)\right]\cdot dx$

c) $\frac{dI}{dx}=\int_a^b \frac{\partial}{\partial x}\left[f(x,\alpha)\right]\cdot dx$

d) $\frac{dI}{dx}=\int_a^b \frac{\partial}{\partial\alpha}\left[f(x,\alpha)\right]\cdot dx$

2) If $I(\alpha)=\int_a^b f(x,\alpha)\,dx$, where α is parameter and a, b are functions of α, by differentiation under integral sign rule we have

a) $\frac{dI}{d\alpha}=\int_a^b \frac{\partial}{\partial\alpha}\left[f(x,\alpha)\right]\cdot dx-f(x,b)\frac{db}{d\alpha}+f(x,a)\frac{da}{d\alpha}$

b) $\frac{dI}{d\alpha}=\int_a^b \frac{\partial}{\partial\alpha}\left[f(x,\alpha)\right]\cdot dx+f(x,b)\frac{db}{dx}-f(x,a)\frac{da}{dx}$

c) $\frac{dI}{d\alpha}=\int_a^b \frac{\partial}{\partial\alpha}\left[f(x,\alpha)\right]\cdot dx+f(x,b)\frac{db}{d\alpha}-f(x,a)\frac{da}{d\alpha}$

d) $\frac{dI}{dx}=\int_a^b \frac{\partial}{\partial x}\left[f(x,\alpha)\right]\cdot dx+f(x,b)\frac{db}{dx}-f(x,a)\frac{da}{dx}$

Note: Henceforth, we abbreviate "differentiation under integral sign" by "DUIS" for simplicity.

3) If $I=\int_0^\infty e^{-bx^2}\cos 2ax\cdot dx$, where $b>0$, by DUIS rule we have

a) $\frac{dI}{dx}=\int_0^\infty \frac{\partial}{\partial x}\left[e^{-bx^2}\cos 2ax\right]\cdot dx$

b) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial a}\left[e^{-bx^2}\cos 2ax\right]\cdot dx$

c) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial b}\left[e^{-bx^2}\cos 2ax\right]\cdot dx$

d) $\frac{dI}{db}=\int_0^\infty \frac{\partial}{\partial a}\left[e^{-bx^2}\cos 2ax\right]\cdot dx$

4) If $I=\int_0^\infty \frac{e^{-ax}}{x}\left(1-e^{-bx}\right)dx$, where $a, b>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial b}\left[\frac{e^{-ax}}{x}\left(1-e^{-bx}\right)\right]dx$

b) $\frac{dI}{dx}=\int_0^\infty \frac{\partial}{\partial x}\left[\frac{e^{-ax}}{x}\left(1-e^{-bx}\right)\right]dx$

c) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial a}\left[\frac{e^{-ax}}{x}\left(1-e^{-bx}\right)\right]dx$

d) $\frac{dI}{db}=\int_0^\infty \frac{\partial}{\partial a}\left[\frac{e^{-ax}}{x}\left(1-e^{-bx}\right)\right]dx$

5) If $I=\int_0^\infty \frac{e^{-ax}}{x}\left(1-e^{-x}\right)dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial a}\left[\frac{\left(1-e^{-x}\right)}{x}\right]\cdot e^{-ax}dx$

b) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial x}\left[e^{-ax}\right]\cdot\frac{\left(1-e^{-x}\right)}{x}dx$

c) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial x}\left[e^{-ax}\frac{\left(1-e^{-x}\right)}{x}\right]\cdot dx$

d) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial a}\left[e^{-ax}\right]\cdot\frac{\left(1-e^{-x}\right)}{x}dx$

6) If $I=\int_0^\infty \frac{e^{-x}}{x}\left(a-\frac{1}{x}+\frac{1}{x^2}e^{-ax}\right)dx$, where $a, b>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty \frac{e^{-x}}{x}\cdot\frac{\partial}{\partial x}\left(a-\frac{1}{x}+\frac{1}{x^2}e^{-ax}\right)dx$

b) $\dfrac{dI}{dx}=\int_0^\infty \dfrac{e^{-x}}{x}\cdot\dfrac{\partial}{\partial a}\left(a-\dfrac{1}{x}+\dfrac{1}{x^2}e^{-ax}\right)dx$

c) $\dfrac{dI}{dx}=\int_0^\infty \dfrac{e^{-x}}{x}\cdot\dfrac{\partial}{\partial x}\left(a-\dfrac{1}{x}+\dfrac{1}{x^2}e^{-ax}\right)dx$

d) $\dfrac{dI}{da}=\int_0^\infty \dfrac{e^{-x}}{x}\cdot\dfrac{\partial}{\partial a}\left(a-\dfrac{1}{x}+\dfrac{1}{x^2}e^{-ax}\right)dx$

7) If $I=\int_0^\infty \dfrac{e^{-ax}}{x}\left(1-e^{-x}\right)dx$, where $a>0$, by DUIS rule we have

a) $\dfrac{dI}{da}=\int_0^\infty e^{(a+1)x}dx$ b) $\dfrac{dI}{da}=\int_0^\infty e^{-ax}dx$

c) $\dfrac{dI}{da}=\int_0^\infty e^{-(a+1)x}dx$ d) $\dfrac{dI}{da}=\int_0^\infty e^{-(a-1)x}dx$

8) If $I=\int_0^\infty \dfrac{e^{-x}}{x}\left(a-\dfrac{1}{x}+\dfrac{1}{x^2}e^{-ax}\right)dx$, where $a,b>0$, by DUIS rule we have

a) $\dfrac{dI}{da}=\int_0^\infty \dfrac{e^{-x}}{x}\left(1-\dfrac{1}{x}e^{-ax}\right)dx$

b) $\dfrac{dI}{da}=\int_0^\infty \dfrac{e^{-x}}{x}\left(1-e^{-ax}\right)dx$

c) $\dfrac{dI}{da}=\int_0^\infty \dfrac{e^{-x}}{x}\left(1-\dfrac{1}{x^2}e^{-ax}\right)dx$

d) $\dfrac{dI}{da}=\int_0^\infty \left(1-\dfrac{1}{x}e^{-ax}\right)dx$

9) If $I=\int_0^\infty \dfrac{e^{-ax}-e^{-bx}}{x}dx$, where $a,b>0$, by DUIS rule we have

a) $\dfrac{dI}{db}=-\int_0^\infty e^{-ax}dx$ b) $\dfrac{dI}{da}=-\int_0^\infty e^{-ax}dx$

c) $\dfrac{dI}{da}=\int_0^\infty e^{-ax}dx$ d) $\dfrac{dI}{da}=\int_0^\infty \left(e^{-ax}-e^{-ax}\right)dx$

10) If $I=\int_0^\infty \dfrac{e^{-ax}-e^{-bx}}{x}dx$, where $a,b>0$, by DUIS rule we have

a) $\dfrac{dI}{db}=\int_0^\infty \left(e^{-ax}-e^{-ax}\right)dx$ b) $\dfrac{dI}{db}=-\int_0^\infty e^{-bx}dx$

c) $\dfrac{dI}{db}=\int_0^\infty e^{-ax}dx$ d) $\dfrac{dI}{db}=\int_0^\infty e^{-bx}dx$

11) If $I=\int_0^\infty \dfrac{e^{-x}-e^{-ax}}{x\sec x}dx$, where $a>0$, by DUIS rule we have

a) $\dfrac{dI}{da}=\int_0^\infty \dfrac{e^{-ax}}{\sec x}dx$ b) $\dfrac{dI}{da}=\int_0^\infty \dfrac{e^{-ax}}{\sec x\tan x}dx$

c) $\dfrac{dI}{da}=-\int_0^\infty \dfrac{e^{-ax}}{\sec x}dx$ d) $\dfrac{dI}{da}=-\int_0^\infty \dfrac{ae^{-ax}}{x\sec x}dx$

12) If $I=\int_0^\infty e^{-a^2}\cos ax\,da$, where $x>0$, by DUIS rule we have

a) $\dfrac{dI}{dx}=-2\int_0^\infty a^2e^{-a^2}\sin ax\,da$

b) $\dfrac{dI}{dx}=2\int_0^\infty ae^{-a^2}\sin ax\,da$

c) $\dfrac{dI}{dx}=-2\int_0^\infty ae^{-a^2}\cos ax\,da$

d) $\dfrac{dI}{dx}=-\int_0^\infty ae^{-a^2}\sin ax\,da$

13) If $I=\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}dx$, where $a>0$, by DUIS rule we have

a) $\dfrac{dI}{da}=\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}\dfrac{1}{x^2}dx$

b) $\dfrac{dI}{da}=a\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}\dfrac{1}{x^2}dx$

c) $\dfrac{dI}{da}=-2a\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}\dfrac{1}{x^2}dx$

d) $\dfrac{dI}{da}=-\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}\dfrac{1}{x^2}dx$

14) If $I=\int_0^{\infty}\frac{e^{-x}\sin ax}{x}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=-a\int_0^{\infty}\cos ax\,dx$ b) $\frac{dI}{da}=\int_0^{\infty}\sin ax\,dx$

c) $\frac{dI}{da}=-\int_0^{\infty}e^{-x}\cos ax\,dx$ d) $\frac{dI}{da}=\int_0^{\infty}e^{-x}\cos ax\,dx$

15) If $I=\int_0^{\pi}\frac{x^a-1}{\log x}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{\infty}\frac{x^a\log a}{\log x}dx$ b) $\frac{dI}{da}=\int_0^{\pi}x^a\,dx$

c) $\frac{dI}{da}=\int_0^{\pi}x^a\log a\,dx$ d) $\frac{dI}{da}=\int_0^{\pi}\frac{x^a\log a}{\log x}dx$

16) If $I=\int_0^{1}\frac{x^a-x^b}{\log x}dx$, where $a,\ b>0$, by DUIS rule we have

a) x^a-x^b b) $\frac{dI}{da}=\int_0^{\pi}\frac{x^a-x^b}{x\log x}dx$

c) $\frac{dI}{da}=\int_0^{1}\frac{x^a\log a}{\log x}dx$ d) $\frac{dI}{da}=\int_0^{1}x^a\,dx$

17) If $I=\int_0^{\pi}\log(1+a\cos x)\,dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{\pi}\frac{-\sin x}{1+a\cos x}dx$ b) $\frac{dI}{da}=\int_0^{\pi}\frac{\cos x}{1+a\cos x}dx$

c) $\frac{dI}{da}=\int_0^{\pi}\frac{a}{1+a\cos x}dx$ d) $\frac{dI}{da}=-\int_0^{\pi}\frac{\cos x}{1+a\cos x}dx$

18) If $I=\int_0^{\pi}\frac{1}{x^2}\log(1+ax^2)\,dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{\pi}\frac{ax^2}{1+ax^2}dx$ b) $\frac{dI}{da}=2\int_0^{\pi}\frac{x}{1+ax^2}dx$

c) $\frac{dI}{da}=\int_0^{\pi}\frac{1}{1+ax^2}dx$ d) $\frac{dI}{da}=\int_0^{\pi}\frac{2ax}{1+ax^2}dx$

19) If $I=\int_0^{\pi}\frac{1}{\sin^2 x}\log(1+a\sin^2 x)\,dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{\pi}\frac{1}{1+a\sin^2 x}dx$ b) $\frac{dI}{da}=\int_0^{\pi}\frac{\sin 2x}{1+a\sin^2 x}dx$

c) $\frac{dI}{da}=\int_0^{\pi}\frac{a}{1+a\sin^2 x}dx$ d) $\frac{dI}{da}=\int_0^{\pi}\frac{\cos x}{1+a\sin^2 x}dx$

20) If $I=\int_0^{\infty}\frac{1-\cos ax}{x^2}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{\infty}\frac{a\sin ax}{x^2}dx$ b) $\frac{dI}{da}=-\int_0^{\infty}\frac{\cos ax}{x}dx$

c) $\frac{dI}{da}=\int_0^{\infty}\frac{\sin ax}{x}dx$ d) $\frac{dI}{da}=-\int_0^{\infty}\frac{\sin ax}{x}dx$

21) If $I=\int_0^{1}\frac{x^a}{\log x}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{1}\frac{x^a\log a}{\log x}dx$ b) $\frac{dI}{da}=\int_0^{1}x^a\,dx$

c) $\frac{dI}{da}=\int_0^{1}x^a\log a\,dx$ d) $\frac{dI}{da}=\int_0^{1}x^{a-1}\,dx$

22) If $I=\int_0^{\pi/2}\log(a^2\cos^2 x+b^2\sin^2 x)\,dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{\pi/2}\frac{1}{a^2+b^2\tan^2 x}dx$

b) $\frac{dI}{da}=\int_0^{\pi/2}\frac{b^2}{a^2+b^2\tan^2 x}dx$

c) $\frac{dI}{da}=\int_0^{\pi/2}\frac{a^2}{a^2+b^2\tan^2 x}dx$

d) $\frac{dI}{da}=\int_0^{\pi/2}\frac{2a}{a^2+b^2\tan^2 x}dx$

23) If $I=\int_0^{\infty}\frac{\sin ax-\sin bx}{x^2}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=-\int_0^{\infty}\frac{\cos bx}{x}dx$ b) $\frac{dI}{da}=\int_0^{\infty}\frac{\cos ax}{x}dx$

c) $\frac{dI}{da}=-\int_0^{\infty}\frac{\cos ax}{x}dx$ d) $\frac{dI}{db}=\int_0^{\infty}\frac{\cos ax}{x}dx$

24) If $I=\int_0^{a^2}\tan^{-1}\left(\frac{x}{a}\right)dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{a^2}\frac{\partial}{\partial a}\tan^{-1}\left(\frac{x}{a}\right)dx$

b) $\frac{dI}{da}=\int_0^{a^2}\frac{\partial}{\partial a}\tan^{-1}\left(\frac{x}{a}\right)dx-2a\tan^{-1}a$

c) $\frac{dI}{da}=\int_0^{a^2}\frac{\partial}{\partial a}\tan^{-1}\left(\frac{x}{a}\right)dx+2a\tan^{-1}x$

d) $\frac{dI}{da}=\int_0^{a^2}\frac{\partial}{\partial a}\tan^{-1}\left(\frac{x}{a}\right)dx+2a\tan^{-1}a$

25) If $I=\int_0^{a}\frac{\log(1+ax)}{1+x^2}dx$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{a}\frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right]dx+\frac{\log(1+a^2)}{1+a^2}$

b) $\frac{dI}{da}=\int_0^{a}\frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right]dx-\frac{\log(1+a^2)}{1+a^2}$

c) $\frac{dI}{da}=\int_0^{a}\frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right]dx$

d) $\frac{dI}{da}=\int_0^{a}\frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right]dx+\frac{\log(1+x^2)}{1+x^2}$

26) If $I=\int_a^{a^2}\log(ax)\,dx$, by DUIS rule we have

a) $\frac{dI}{da}=\int_a^{a^2}\frac{\partial}{\partial a}\log(ax)\,dx$

b) $\frac{dI}{da}=\int_a^{a^2}\frac{\partial}{\partial a}\log(ax)\,dx-(6a-2)\log a$

c) $\frac{dI}{da}=\int_a^{a^2}\frac{\partial}{\partial a}\log(ax)\,dx+(6a+2)\log a$

d) $\frac{dI}{da}=\int_a^{a^2}\frac{\partial}{\partial a}\log(ax)\,dx+(6a-2)\log a$

27) If $I=\int_t^{t^2}e^{tx^2}\,dx$, by DUIS rule we have

a) $\frac{dI}{dt}=\int_t^{t^2}x^2e^{tx^2}\,dx+2te^{t^5}-e^{t^3}$

b) $\frac{dI}{dt}=\int_t^{t^2}x^2e^{tx^2}\,dx-2te^{t^5}+e^{t^3}$

c) $\frac{dI}{dt}=\int_t^{t^2}te^{tx^2}\,dx+2te^{t^5}-e^{t^3}$

d) $\frac{dI}{dt}=\int_t^{t^2}t^3e^{tx^2}\,dx+2te^{t^5}-e^{t^3}$

28) If $I=\int_0^{a^2}\tan^{-1}\left(\frac{x}{a}\right)dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{a^2}\frac{x}{a^2+x^2}dx+2a\tan^{-1}a$

b) $\frac{dI}{da}=-\int_0^{a^2}\frac{a}{a^2+x^2}dx+2a\tan^{-1}a$

c) $\frac{dI}{da}=-\int_0^{a^2}\frac{x}{a^2+x^2}dx+2a\tan^{-1}a$

d) $\frac{dI}{da}=-\int_0^{a^2}\frac{x}{a^2+x^2}dx-2a\tan^{-1}a$

29) If $I=\int_a^{a^2}\log(ax)\,dx$, by DUIS rule we have

a) $\frac{dI}{da}=\int_a^{a^2}\frac{1}{x}dx-(6a-2)\log a$

b) $\frac{dI}{da}=\int_a^{a^2}\frac{1}{a}dx+(6a-2)\log a$

c) $\frac{dI}{da}=\int_a^{a^2}\frac{1}{a}dx-(6a-2)\log a$

d) $\frac{dI}{da}=\int_a^{a^2}\frac{1}{a}dx-(6a-2)\log a$

30) If $I=\int_0^a \frac{\log(1+ax)}{1+x^2}dx$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^a \frac{x}{(1+x^2)(1+ax)}dx+\frac{\log(1+a^2)}{1+a^2}$

b) $\frac{dI}{da}=\int_0^a \frac{1}{(1+x^2)(1+ax)}dx+\frac{\log(1+a^2)}{1+a^2}$

c) $\frac{dI}{da}=\int_0^a \frac{a}{(1+x^2)(1+ax)}dx+\frac{\log(1+a^2)}{1+a^2}$

d) $\frac{dI}{da}=\int_0^a \frac{x}{(1+x^2)(1+ax)}dx-\frac{\log(1+a^2)}{1+a^2}$

31) If $I=\int_{\pi/6a}^{\pi/3a}\frac{\sin ax}{x}dx$, by DUIS rule we have

a) $\frac{dI}{da}=\int_{\pi/6a}^{\pi/3a}\cos ax\,dx+\frac{1}{a}$

b) $\frac{dI}{da}=\int_{\pi/6a}^{\pi/3a}\cos ax\,dx-\frac{1}{2a}$

c) $\frac{dI}{da}=\int_{\pi/6a}^{\pi/3a}\cos ax\,dx-\frac{1}{a}$

d) $\frac{dI}{da}=\int_{\pi/6a}^{\pi/3a}\frac{\cos ax}{x}dx-\frac{1}{a}$

32) If $f(x)=\int_a^x (x-t)^2 G(t)\,dt$, we have

a) $\frac{df}{dx}=\int_a^x \frac{\partial}{\partial x}(x-t)^2 G(t)\,dt+(x-a)^2 G(a)$

b) $\frac{df}{dx}=\int_a^x \frac{\partial}{\partial x}(x-t)^2 G(t)\,dt$

c) $\frac{df}{dx}=\int_a^x \frac{\partial}{\partial x}(x-t)^2 G(t)\,dt-(x-a)^2 G(a)$

d) $\frac{df}{dx}=\int_a^x \frac{\partial}{\partial x}(x-t)^2 G(t)\,dt+a^2 G(a)$

33) If $y=\int_0^x f(t)\sin a(x-t)\,dt$, we have

a) $\frac{dy}{dx}=\int_0^x xf(t)\cos a(x-t)\,dt$

b) $\frac{dy}{dx}=\int_0^x af(t)\cos a(x-t)\,dt+f(x)$

c) $\frac{dy}{dx}=\int_0^x af(t)\cos a(x-t)\,dt-af(x)$

d) $\frac{dy}{dx}=a\int_0^x f(t)\cos a(x-t)\,dt$

34) For the integral $I(a)=\int_0^\infty \frac{e^{-x}}{x}(1-e^{-ax})dx$, we have $\frac{dI}{da}=\frac{1}{a+1}$, then I is

a) $\log(a+1)-1$ b) $\log(a+1)$

c) $\log(a+1)+1$ d) $-\frac{1}{(a+1)^2}$

35) The value of integration $I(a)=\int_0^1 \frac{x^a-1}{\log x}dx$ with $\frac{dI}{da}=\frac{1}{a+1}$ is given by

a) $\log(a+1)$ b) $\log(a+1)-1$

c) $\log(a+1)+1$ d) $-\frac{1}{(a+1)^2}$

36) The value of integration $I(a)=\int_0^1 \frac{e^{-2x}\sin ax}{x}dx$ with $\frac{dI}{da}=\frac{2}{a^2+4}$ is given by

a) $\tan^{-1}\left(\frac{a}{2}\right)+\frac{\pi}{2}$ b) $\tan^{-1}\left(\frac{a}{2}\right)$

c) $\frac{1}{2}\tan^{-1}\left(\frac{a}{2}\right)$ d) $\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$

37) The value of integration $I(a)=\int_0^\infty \frac{e^{-x}-e^{-ax}}{x\sec x}dx$ with $\frac{dI}{da}=\frac{a}{a^2+1}$ is given by

a) $2\log\left(\frac{2}{a^2+1}\right)$ b) $\frac{1}{2}\log\left(\frac{2}{a^2+1}\right)$

c) $\frac{1}{2}\log\left(\frac{a^2+1}{2}\right)$ d) $2\log\left(\frac{a^2+1}{2}\right)$

38) The value of integration $I(a)=\int_0^{\infty}\frac{1-\cos ax}{x^2}dx$ with $\frac{dI}{da}=\frac{\pi}{2}$ is given by

a) $2\pi a$ b) $\frac{\pi a}{3}$ c) $\frac{\pi}{2}$ d) $\frac{\pi a}{2}$

39) The value of integration $I=\int_0^{\infty}\frac{\log(1+ax^2)}{x^2}dx$, with $\frac{dI}{da}=\frac{\pi}{2\sqrt{a}}$ is given by

a) $\pi\sqrt{a}$ b) $2\sqrt{a}$ c) $\pi\sqrt{2}$ d) $a\sqrt{\pi}$

40) The value of integration $I=\int_0^{\pi/2}\frac{\log(1+a\sin^2 x)}{\sin^2 x}dx$, with $\frac{dI}{da}=\frac{\pi}{2\sqrt{a+1}}$ is given by

a) $\pi\sqrt{a+1}+\pi$ b) $\pi\sqrt{a+1}-\pi$

c) $\pi\sqrt{a+1}-\frac{\pi}{a}$ d) $\frac{\pi\sqrt{a+1}-\pi}{a}$

II) Error Functions

41) $erf(x)$ is given by

a) $\frac{1}{2\sqrt{\pi}}\int_0^x e^{-u^2}du$ b) $\frac{\sqrt{\pi}}{2}\int_0^x e^{-u^2}du$

c) $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$ d) $\int_0^x e^{-u^2}du$

42) $erfc(x)$ is given by

a) $\frac{2}{\sqrt{\pi}}\int_x^{\infty} e^{-u^2}du$ b) $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$

c) $\frac{\sqrt{\pi}}{2}\int_0^x e^{-u^2}du$ d) $\frac{\sqrt{\pi}}{2}\int_x^{\infty} e^{-u^2}du$

43) $erf(0)$ is given by

a) $\frac{2}{\sqrt{\pi}}$ b) 1 c) ∞ d) 0

44) $erf(\infty)$ is given by

a) 1 b) 0 c) $\frac{2}{\sqrt{\pi}}$ d) ∞

45) $erfc(0)$ is given by

a) 0 b) $\frac{2}{\sqrt{\pi}}$ c) ∞ d) 1

46) $erf(x)+erfc(x)=?$

a) 2 b) ∞ c) 1 d) 0

47) $erf(-x)=?$

a) $erfc(x)$ b) $-erf(x)$

c) $erf(x)$ d) $-erf(x^2)$

48) Error function is an

a) even function b) neither even nor odd

c) odd function d) none of these

49) $erf(x)+erf(-x)=?$

a) 0 b) 1 c) 2 d) 3

50) $erf(-x)+erfc(-x)=?$

a) 0 b) 3 c) 2 d) 1

51) $erfc(-x)-erf(x)=?$
a) ∞ b) 2 c) 1 d) 0

52) $erfc(x)+erfc(-x)=?$
a) 2 b) 1 c) 0 d) ∞

53) If $erf(ax)=\frac{2}{\sqrt{\pi}}\int_0^{ax} e^{-u^2}du$, then $\frac{d}{dx}[erf(ax)]$ is
a) $\frac{2a}{\sqrt{\pi}}e^{-x^2}$ b) $\frac{a}{2\sqrt{\pi}}e^{-a^2x^2}$
c) $\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$ d) $\frac{2x}{\sqrt{\pi}}e^{-a^2x^2}$

54) If $erfc(ax)=\frac{2}{\sqrt{\pi}}\int_{ax}^{\infty} e^{-u^2}du$, then $\frac{d}{dx}[erfc(ax)]$ is
a) $-\frac{2x}{\sqrt{\pi}}e^{-a^2x^2}$ b) $\frac{2x}{\sqrt{\pi}}e^{-a^2x^2}$
c) $-\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$ d) $\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$

55) If $erf(\sqrt{t})=\frac{2}{\sqrt{\pi}}\int_0^{\sqrt{t}} e^{-u^2}du$, then $\frac{d}{dx}[erf(\sqrt{t})]$ is
a) $\frac{1}{t\sqrt{\pi}}e^{-t}$ b) $\frac{1}{\sqrt{\pi t}}e^{-t}$
c) $\frac{2}{\sqrt{\pi t}}e^{-t}$ d) $\frac{1}{\sqrt{\pi t}}e^{-t^2}$

56) If $erfc(\sqrt{t})=\frac{2}{\sqrt{\pi}}\int_{\sqrt{t}}^{\infty} e^{-u^2}du$, then $\frac{d}{dx}[erfc(\sqrt{t})]$ is
a) $\frac{2}{\sqrt{\pi t}}e^{-t}$ b) $\frac{1}{\sqrt{\pi t}}e^{-t^2}$
c) $\frac{1}{t\sqrt{\pi}}e^{-t}$ d) $-\frac{1}{\sqrt{\pi t}}e^{-t}$

57) $\frac{d}{dx}[erf(x)+erfc(x)]=?$
a) 1 b) 0 c) 2 d) ∞

58) If $\frac{d}{dx}[erf(ax)]=\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$, then $\frac{d}{dx}[erfc(ax)]$ is
a) $-\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$ b) $\frac{2a}{\sqrt{\pi}}e^{a^2x^2}$
c) $\frac{1}{\sqrt{\pi}}e^{-a^2x^2}$ d) $\frac{4a^2}{\sqrt{\pi}}e^{-a^2x^2}$

59) On substitution $x+a=u$ in the integration $\int_0^{\infty} e^{-(x+a)^2}dx$, then the value of integration is
a) $\frac{\sqrt{\pi}}{2}erf(a)$ b) $\frac{2}{\sqrt{\pi}}erf(a)$
c) $\frac{\sqrt{\pi}}{2}erfc(a)$ d) $erfc(a)$

60) $\int_0^t erf(ax)dx+\int_0^t erfc(ax)dx=?$
a) 1 b) ∞ c) 0 d) t

61) If $\frac{dy}{dx}[erf(\sqrt{t})]=\frac{e^{-t}}{\sqrt{\pi t}}$, the integration $\int_0^{\infty} e^{-t}erf(\sqrt{t})dt$ is
a) $\frac{1}{\sqrt{\pi}}\int_0^{\infty} e^{-t}t^{1/2}dt$ b) $\frac{2}{\sqrt{\pi}}\int_0^{\infty} e^{-2t}t^{1/2}dt$
c) $-\frac{1}{\sqrt{\pi}}\int_0^{\infty} e^{-2t}t^{-1/2}dt$ d) $\frac{1}{\sqrt{\pi}}\int_0^{\infty} e^{-2t}t^{-1/2}dt$

62) The power series expansion of $erf(x)$ is
a) $\frac{2}{\sqrt{\pi}}\left[x-\frac{x^3}{3}+\frac{x^5}{10}-\frac{x^7}{42}+-\ldots\right]$
b) $\frac{2}{\sqrt{\pi}}\left[x+\frac{x^3}{3}+\frac{x^5}{10}+\frac{x^7}{42}+\ldots\right]$
c) $\frac{2}{\sqrt{\pi}}\left[x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+-\ldots\right]$
d) $\frac{2}{\sqrt{\pi}}\left[x+\frac{x^3}{3!}+\frac{x^5}{5!}+\frac{x^7}{7!}+\ldots\right]$

Chapter 06 – Curve Tracing & Rectification of Curves

I) Curve Tracing

1) If the portion of the curve lies on the both sides of the point lying above the tangent at that point, the curve is known as
a) concave upward b) concave downward
c) inflexion point d) none of these

2) If the portion of the curve lies on the both sides of the point lying above the tangent at that point, the curve is known as
a) inflexion point b) concave downward
c) inflexion point d) none of these

3) A point through which two branches of the same curve passes is known as
a) double point b) inflexion point
c) multiple point d) conjugate point

4) A point through which many branches of the same curve passes is known as
a) double point b) inflexion point
c) multiple point d) conjugate point

5) A double point through which the branches of the curve passes and the tangent at that point are real and distinct, the point is known as
a) conjugate point b) node
c) point of inflexion d) cusp

6) A double point through which the branches of the curve passes and the tangent at that point are real but the same, the point is known as
a) conjugate point b) point of inflexion
c) cusp d) node

7) A double point is said to be node if the tangents to the curve at that point are
a) imaginary b) perpendicular to each other
c) real but the same d) real and distinct

8) A double point is said to be cusp if the tangents at that point are
a) imaginary b) real and distinct
c) real but the same d) none of these

9) If at a point $\frac{dy}{dx}=0$, the tangent to the curve at that point is
a) parallel to the line $x+y=0$
b) parallel to x-axis
c) perpendicular to x-axis
d) parallel to $y=x$

10) If at a point $\frac{dy}{dx}=\infty$, the tangent to the curve at that point is
a) parallel to the line $x+y=0$
b) parallel to $y=x$
c) parallel to x-axis
d) perpendicular to x-axis

11) The standard equation of x-axis in the Cartesian form is given by
a) $x-y=0$ b) $x+y=0$
c) $y=0$ d) $x=0$

12) The standard equation of y-axis in the Cartesian form is given by
a) $x-y=0$ b) $x+y=0$
c) $y=0$ d) $x=0$

13) If the all the powers of y in the Cartesian form are even, the curve is symmetrical about
a) $y-axis$ b) $x,\ y-axes$
c) $x-axis$ d) the line $y=x$

14) If the all the powers of x in the Cartesian form are even, the curve is symmetrical about
a) $x,\ y-axes$ b) $y-axis$
c) $x-axis$ d) the line $y=x$

15) If the all the powers of x and y in the Cartesian form are even, the curve is symmetrical about
a) the line $y=x$ b) $x-axis$ only
c) $y-axis$ only d) $x,\ y-axes$

16) If in the equation of the Cartesian form by replacing $x\to y$ and $y\to x$, the equation is symmetrical about
a) the line $y=x$ b) $x,\ y-axes$

c) $x-axis$ d) $y-axis$

17) If in the equation of the Cartesian form by replacing $x \to -y$ and $y \to -x$, the equation is symmetrical about
a) the line $y=-x$ b) the line $y=x$
c) $x,\ y-axes$ d) $y-axis$ only

18) If in the equation of the Cartesian form by replacing $x \to -x$ and $y \to -y$, the equation is symmetrical about
a) the line $y=-x$ b) $x,\ y-axes$
c) opposite quadrants d) the line $y=x$

19) The equation of the tangent at origin when the curve is passing through origin is obtained by equating to zero
a) the lowest degree term of the equation
b) the highest degree term of x in equation
c) the highest degree term of y in equation
d) the coefficient of the term xy

20) In the Cartesian form, the asymptote to the curve parallel to x-axis may be obtained by equating to zero
a) the coefficient of lowest degree term in y
b) the coefficient of highest degree term in y
c) the coefficient of highest degree term in x
d) the coefficient of lowest degree term in x

21) In the Cartesian form, the asymptote to the curve parallel to y-axis may be obtained by equating to zero
a) the coefficient of lowest degree term in y
b) the coefficient of highest degree term in y
c) the coefficient of highest degree term in x
d) the coefficient of lowest degree term in x

22) Oblique asymptote are obtained only when the curve is
a) symmetrical about x-axis
b) symmetrical about y-axis
c) symmetrical about both x and y-axis
d) not symmetrical about both x and y-axes

23) In the Cartesian form if the coefficient of the highest degree term in x is constant, the curve has
a) no asymptote parallel to $x=y$
b) no asymptote parallel to y-axis
c) no asymptote parallel to x-axis
d) none of these

24) In the Cartesian form if the coefficient of the highest degree term in y is constant, the curve has
a) no asymptote parallel to $x+y=0$
b) no asymptote parallel to $x=y$
c) no asymptote parallel to x-axis
d) no asymptote parallel to y-axis

25) In the polar form, if the equation of the curve remains unchanged by replacing $\theta \to -\theta$, the curve is symmetrical about
a) the line $\theta=\frac{\pi}{4}$ b) the initial line
c) pole d) the line $\theta=\frac{\pi}{2}$

26) In the polar form, if the equation of the curve remains unchanged by replacing $r \to -r$, the curve is symmetrical about
a) the line $\theta=\frac{\pi}{4}$ b) the initial line
c) pole d) the line $\theta=\frac{\pi}{2}$

27) In the polar form, if the equation of the curve remains unchanged by replacing $\theta \to \pi-\theta$, the curve is symmetrical about
a) the line $\theta=\frac{\pi}{2}$ b) the line $\theta=\frac{\pi}{4}$
c) the initial line d) pole

28) The pole is point of the curve, if for given angle θ, the value of
a) $r=\infty$ b) $r=0$ c) $r<0$ d) $r>0$

29) If a curve is passing through the pole, the tangent to the curve at pole are obtained by solving
a) $r=0$ b) $r=\infty$ c) $\theta=0$ d) $\theta=\pi$

30) In the polar form, the relation between the angle ϕ formed by the radius vector and the tangent to the curve at that point, is given by
a) $\tan\phi=r^2\frac{d\theta}{dr}$ b) $\cot\phi=r\frac{d\theta}{dr}$
c) $\tan\phi=r\frac{d\theta}{dr}$ d) $\tan\phi=r\frac{dr}{d\theta}$

31) In the parametric form $x = f(t)$, $y = g(t)$, the curve is symmetrical about y-axis, if
a) $x = f(t)$ is odd and $y = g(t)$ is even
b) $x = f(t)$ is even and $y = g(t)$ is odd
c) $x = f(t)$ is odd and $y = g(t)$ is odd
d) $x = f(t)$ is even and $y = g(t)$ is even

32) In the parametric form $x = f(t)$, $y = g(t)$, the curve is symmetrical about y-axis, if
a) $x = f(t)$ is odd and $y = g(t)$ is odd
b) $x = f(t)$ is even and $y = g(t)$ is odd
c) $x = f(t)$ is odd and $y = g(t)$ is even
d) $x = f(t)$ is even and $y = g(t)$ is even

33) The curve represented by the equation $x^2y^2 = x^2 + 1$ is symmetrical about
a) the line $y = x$ b) x-axis only
c) y-axis only d) both x and y-axes

34) The curve represented by the equation $x\left(x^2 + y^2\right) = a\left(x^2 - y^2\right)$ is
a) symmetrical about y-axis but not passing through origin
b) symmetrical about y-axis and passing through origin
c) symmetrical about x-axis but not passing through origin
d) symmetrical about x-axis and passing through origin

35) The curve represented by the equation $a^2y^2 = x^2\left(a^2 - x^2\right)$ is
a) symmetrical about both x and y-axis but not passing through origin
b) symmetrical about both x and y-axis and passing through origin
c) symmetrical about x-axis but not passing through origin
d) symmetrical about x-axis only and passing through origin

36) The curve represented by the equation $(2a - x)y^2 = x^3$ is
a) symmetrical about y-axis and passing through origin
b) symmetrical about x-axis but not passing through origin
c) symmetrical about x-axis and passing through origin
d) symmetrical about y-axis not passing through origin

37) The curve represented by the equation $(2a - y)y^3 = a^2x^2$ is
a) symmetrical about y-axis and passing through origin and $(0, 2a)$
b) symmetrical about x-axis but not passing through origin and $(0, 2a)$
c) symmetrical about x-axis and passing through origin and $(0, 2a)$
d) symmetrical about y-axis not passing through origin and $(0, 2a)$

38) The curve represented by the equation $xy^2 = 4a^2(a - x)$ is
a) symmetrical about y-axis and passing through $(a, 0)$
b) symmetrical about x-axis but not passing through $(a, 0)$
c) symmetrical about x-axis and passing through $(a, 0)$
d) symmetrical about y-axis not passing through $(a, 0)$

39) The curve represented by the equation $xy^2 = 4a^2(a - x)$ has at origin
a) node b) cusp c) inflexion d) none

40) The curve represented by the equation $(2a - x)y^2 = x^3$ has the tangent at origin whose equation is
a) $x + y = 0$ b) y-axis c) x-axis d) $y = x$

41) The curve represented by the equation $\left(1 + x^2\right)y = x$ has the tangent at origin whose equation is
a) $y = x$ b) x-axis c) y-axis d) $x + y = 0$

42) The curve represented by the equation $3ay^2 = x(x - a)^2$ has the tangent at origin whose equation is
a) $x + y = 0$ b) $y = x$ c) x-axis d) y-axis

43) The curve represented by the equation $3ay^2 = x(x-a)^2$ has the asymptote parallel to x-axis whose equation is
a) $x+y=0$ b) $y=x$ c) x-axis d) y-axis

44) For the curve given by equation $x^2y = 4a^2(2a-y)$, the asymptote is
a) $y=2a$ b) $y=x$ c) y-axis d) x-axis

45) The curve represented by the equation $y^2(4-x) = x(x-2)^2$ has the asymptote parallel to y-axis whose equation is
a) $x=y$ b) $x=0$ c) $x=2$ d) $x=4$

46) The curve represented by the equation $x^2y^2 = a^2(y^2-x^2)$ has the asymptote parallel to y-axis whose equation is
a) $x=0$ b) $x=\pm a$ c) $x=y$ d) $y=0$

47) For the curve given by equation $x^2y = 4a^2(2a-y)$, the region of absence is
a) $0<y<2a$ b) $y>0,\ \ y>2a$
c) $y<0,\ \ y<2a$ d) $y<0,\ \ y>2a$

48) For the curve given by equation $x^3 = 4y^2(2a-x)$, the region of absence is
a) $0<x<2a$ b) $x<0,\ \ x>2a$
c) $x>0,\ \ x>2a$ d) $x<0,\ \ x<2a$

49) For the curve given by equation $xy^2 = 4a^2(a-x)$, the region of absence is
a) $0<x<a$ b) $x>0,\ \ x>a$
c) $x<0,\ \ x>a$ d) $x<0,\ \ x<a$

50) For the curve given by equation $y^2 = \dfrac{4x^2(a-x)}{x+a}$, the region of absence along x-axis is
a) $[-\infty,\ -a]$ & $[a,\ \infty]$ b) $[-\infty,\ a]$ & $[-a,\ \infty]$
c) $[-\infty,\ -a]$ d) $[-a,\ \infty]$

51) The curve represented by the equation $x^{1/2} + y^{1/2} = a^{1/2}$ is symmetrical about
a) $y=x$ b) x-axis c) y-axis d) $x+y=0$

52) The curve represented by the equation $\sqrt{x}+\sqrt{y}=\sqrt{a}$ has tangent at origin whose equation is
a) x-axis b) no tangent exists
c) y-axis d) $x+y=0$

53) The curve represented by the equation $\sqrt{x}+\sqrt{y}=\sqrt{a}$ has tangent at $(a,\ 0)$ which is
a) the line $x+y=0$ b) the line $y=x$
c) parallel to y-axis d) parallel to x-axis

54) The curve represented by the equation $x=t^2,\ \ y=t-\dfrac{t^3}{3}$ is symmetrical about
a) symmetrical about y-axis but not passing through origin
b) symmetrical about y-axis and passing through origin
c) symmetrical about x-axis but not passing through origin
d) symmetrical about x-axis and passing through origin

55) The curve represented by the equation $x=a(\theta+\sin\theta),\ \ y=a(1+\cos\theta)$ is symmetrical about
a) symmetrical about y-axis but not passing through origin
b) symmetrical about y-axis and passing through origin
c) symmetrical about x-axis but not passing through origin
d) symmetrical about x-axis and passing through origin

56) The curve represented by the equation $r=a(1+\cos\theta)$ is
a) symmetrical about initial line and not passing through the pole
b) symmetrical about initial line and passing through the pole
c) not symmetrical about initial line and passing through the pole
d) not symmetrical about initial line and not passing through the pole

57) The curve represented by the equation $r^2 = a^2 \cos 2\theta$ is

a) symmetrical about initial line as well as pole and not passing through the pole
b) symmetrical about initial line as well as pole and passing through the pole
c) not symmetrical about initial line as well as pole and passing through the pole
d) not symmetrical about initial line as well as pole and not passing through the pole

58) The curve represented by the equation $r^2 = a^2 \sin 2\theta$ is

a) symmetrical about the line $\theta = \frac{\pi}{4}$ and not passing through the pole
b) symmetrical about initial line as well as pole and passing through the pole
c) not symmetrical about the line $\theta = \frac{\pi}{4}$ and passing through the pole
d) not symmetrical about initial line as well as pole and not passing through the pole

59) The curve represented by the equation $r(1+\cos\theta) = 2a^2$ is

a) symmetrical about the line $\theta = \frac{\pi}{4}$ and not passing through the pole
b) symmetrical about initial line and passing through the pole
c) not symmetrical about the line $\theta = \frac{\pi}{4}$ and passing through the pole
d) symmetrical about initial and not passing through the pole

60) The equations of the tangents at pole to the curve $r = a\sin 3\theta$ are given by

a) $\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \ldots$
b) $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{3}, \ldots$
c) $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \ldots$
d) no such tangent exists

61) The equations of the tangents at pole to the curve $r = a\cos 2\theta$ are given by

a) $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \ldots$
b) $\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \ldots$
c) $\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$
d) $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots$

62) For the rose curve $r = a\sin n\theta$, if n is even, the curve is consisting of

a) $2n$ equal loops b) $2n+1$ equal loops
c) n equal loops d) $2n-1$ equal loops

63) For the rose curve $r = a\cos n\theta$, if n is even, the curve is consisting of

a) n equal loops b) $2n+1$ equal loops
c) $2n$ equal loops d) $2n-1$ equal loops

64) For the rose curve $r = a\sin n\theta$, if n is odd, the curve is consisting of

a) $2n$ equal loops b) n equal loops
c) $2n+1$ equal loops d) $2n-1$ equal loops

65) For the rose curve $r = a\cos n\theta$, if n is odd, the curve is consisting of

a) n equal loops b) $2n+1$ equal loops
c) $2n$ equal loops d) $2n-1$ equal loops

I) Rectification of Curve

66) If $A(a_1, b_1)$ $B(a_2, b_2)$ are two points on the curve on xy-plane, the length of arc is given by

a) $\int_{b_1}^{b_2} \sqrt{1-\left(\frac{dy}{dx}\right)^2} \cdot dy$ b) $\int_{b_1}^{b_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2} \cdot dy$

c) $\int_{a_1}^{a_2} \sqrt{1-\left(\frac{dy}{dx}\right)^2} \cdot dx$ d) $\int_{a_1}^{a_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2} \cdot dx$

67) If $A(a_1, b_1)$ $B(a_2, b_2)$ are two points on the curve on xy-plane, the length of arc is given by

a) $\int_{b_1}^{b_2} \sqrt{1+\left(\frac{dx}{dy}\right)^2} \cdot dy$ b) $\int_{a_1}^{a_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2} \cdot dx$

c) $\int_{a_1}^{a_2} \sqrt{1-\left(\frac{dy}{dx}\right)^2} \cdot dx$ d) $\int_{b_1}^{b_2} \sqrt{1-\left(\frac{dx}{dy}\right)^2} \cdot dy$

68) If $A(r_1, \theta_1)$ $B(r_2, \theta_2)$ are two points on the curve on the polar plane, the length of arc is given by

a) $\int_{\theta_1}^{\theta_2} \sqrt{r^2-\left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$ b) $\int_{\theta_1}^{\theta_2} \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

c) $\int_{r_1}^{r_2} \sqrt{1+r^2\left(\frac{d\theta}{dr}\right)^2} \cdot dr$ d) $\int_{r_1}^{r_2} \sqrt{1-r^2\left(\frac{d\theta}{dr}\right)^2} \cdot dr$

69) If $A(r_1, \theta_1)$ $B(r_2, \theta_2)$ are two points on the curve on the polar plane, the length of arc is given by

a) $\int_{\theta_1}^{\theta_2} \sqrt{r^2-\left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$ b) $\int_{\theta_1}^{\theta_2} \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

c) $\int_{r_1}^{r_2} \sqrt{1+r^2\left(\frac{d\theta}{dr}\right)^2} \cdot dr$ d) $\int_{r_1}^{r_2} \sqrt{1-r^2\left(\frac{d\theta}{dr}\right)^2} \cdot dr$

70) If $A(t_1)$ $B(t_2)$ are two points on the curve given by $x = f(t)$, $y = g(t)$ on the xy-plane, the length of arc is given by

a) $\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2} \cdot dt$

b) $\int_{t_1}^{t_2} \sqrt{\left(\frac{dt}{dx}\right)^2+\left(\frac{dt}{dy}\right)^2} \cdot dt$

c) $\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2-\left(\frac{dy}{dt}\right)^2} \cdot dt$

d) $\int_{t_1}^{t_2} \left[\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2\right] \cdot dt$

71) The arc length of the upper part of the loop of the curve $9y^2 = (x+7)(x+4)^2$ is obtained by solving the integration

a) $\int_{0}^{4} \sqrt{1+\left(\frac{dy}{dx}\right)^2} \cdot dx$ b) $\int_{-7}^{0} \sqrt{1+\left(\frac{dy}{dx}\right)^2} \cdot dx$

c) $\int_{-7}^{-4} \sqrt{1+\left(\frac{dy}{dx}\right)^2} \cdot dx$ d) $\int_{7}^{4} \sqrt{1+\left(\frac{dy}{dx}\right)^2} \cdot dx$

72) The arc length of the upper part of the curve $y^2 = 4x$ which is cut by the line $3y = 8x$ is obtained by solving the integration

a) $\int_{1}^{9/16} \sqrt{1+\left(\frac{dy}{dx}\right)^2} \cdot dx$ b) $\int_{0}^{9/16} \sqrt{1+\left(\frac{dy}{dx}\right)^2} \cdot dx$

c) $\int_{0}^{3/4} \sqrt{1+\left(\frac{dy}{dx}\right)^2} \cdot dx$ d) $\int_{3}^{8} \sqrt{1+\left(\frac{dy}{dx}\right)^2} \cdot dx$

73) The points $A(a, 0)$ $B(0, a)$ are two points on the curve $x^2 + y^2 = a^2$ on xy-plane such that $1+\left(\frac{dy}{dx}\right)^2 = \frac{a^2}{a^2 - x^2}$, the length of arc is given by

a) $4a$ b) πa c) $\frac{\pi a}{4}$ d) $\frac{\pi a}{2}$

74) The points $A(0, 0)$ $B(a, b)$ are two points on the curve $y = a\cosh\left(\frac{x}{a}\right)$ on xy-plane such that $1+\left(\frac{dy}{dx}\right)^2 = \cosh^2\left(\frac{x}{a}\right)$, the length of arc is given by

a) $S = a\sinh\left(\frac{x}{a}\right)$ b) $S = a\tanh\left(\frac{x}{a}\right)$

c) $S = \sinh\left(\frac{x}{a}\right)$ d) $S = a\,\mathrm{sech}\left(\frac{x}{a}\right)$

75) The points $A(0,0)$ $B(1,0)$ are two points on the curve $3y^2 = x(x-1)^2$ on xy-plane such that $1+\left(\frac{dy}{dx}\right)^2 = \frac{(3x+1)^2}{12x}$, the length of arc is given by

a) $\frac{3}{\sqrt{3}}$ b) $\frac{1}{2\sqrt{3}}$ c) $\frac{1}{\sqrt{3}}$ d) $\frac{2}{\sqrt{3}}$

76) The total arc length of the part of the curve $r = a(1+\cos\theta)$ which is cut by the circle $r + a\cos\theta = 0$ is obtained by solving the integration

a) $\int_0^{2\pi/3} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

b) $2\int_0^{2\pi/3} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

c) $2\int_0^{2\pi/3} \sqrt{r^2 - \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

d) $\int_0^{2\pi/3} \sqrt{r^2 - \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

77) The total arc length of the upper part of the curve $r^2 = a^2\cos 2\theta$ is obtained by solving the integration

a) $2\int_0^{\pi/2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

b) $\int_0^{\pi/2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

c) $\int_0^{\pi/4} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

d) $\int_0^{\pi/3} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

78) The total length of the arc of the curve $r = ae^{m\theta}$ using $1 + r^2\left(\frac{d\theta}{dr}\right)^2 = 1 + \frac{1}{m^2}$ when r varies from r_1 to r_2 is given by

a) $\frac{\sqrt{1+m^2}}{m}(r_2 - r_1)$ b) $\frac{\sqrt{1+m^2}}{m}(r_2 + r_1)$

c) $\frac{\sqrt{1+m^2}}{m}(r_1 - r_2)$ d) $\frac{\sqrt{1-m^2}}{m}(r_2 - r_1)$

79) The total length of the arc formed by the upper half of the cardioide $r = a(1+\cos\theta)$ using $r^2 + \left(\frac{dr}{d\theta}\right)^2 = 2a^2(1+\cos\theta)$ when θ varies from 0 to π is given by

a) 4π b) 2π c) $4a$ d) $2a$

80) The total arc length of the upper part of the curve $x = t^2,\ y = t - \frac{t^3}{3}$ is obtained by solving the integration

a) $\int_0^2 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$

b) $\int_0^3 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$

c) $\int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$

d) $\int_0^{\sqrt{3}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$

81) The total arc length of the two consecutive cusps lies in the first quadrant of the curve $x = a\cos^3\theta,\ y = a\sin^3\theta$ is obtained by solving the integration

a) $\int_0^{\pi/4} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \cdot d\theta$

b) $\int_0^{\pi/3} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \cdot d\theta$

c) $\int_0^{\pi/2} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \cdot d\theta$

d) $\int_0^{\pi} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \cdot d\theta$

82) The total arc length of the upper part of the curve $x = t^2,\ y = t - \frac{t^3}{3}$ between $t = 0$ to $t = \sqrt{3}$ with $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (1+t^2)^2$ is given by

a) $2\sqrt{3}$ b) $\sqrt{3}$ c) $\frac{2}{\sqrt{3}}$ d) $4\sqrt{3}$

83) The total arc length of the two consecutive cusps lies in the first quadrant of the curve $x = a\cos^3\theta,\ y = a\sin^3\theta$ between $\theta = 0$ to $\theta = \frac{\pi}{2}$ with $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 9a^2\sin^2\theta\cos^2\theta$ is given by

a) $\frac{3a}{4}$ b) $3a$ c) $\frac{3a}{2}$ d) $\frac{2a}{3}$

84) The total arc length of the two cusps between $\theta = -\pi$ to $\theta = \pi$ of the curve

$$x = a(\theta + \sin\theta),\ y = a(1 - \cos\theta)$$

with $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 4a^2\cos^2\left(\frac{\theta}{2}\right)$ is

a) $4a$ b) $8a$ c) $2a$ d) a

85) The total arc length of the two cusps between $\theta = 0$ to $\theta = \frac{\pi}{2}$ of the curve $x = e^{\theta}\cos\theta$, and $y = e^{\theta}\sin\theta$ with $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 2e^{2\theta}$ is

a) $\sqrt{2}\left(1 - e^{\pi/2}\right)$ b) $\sqrt{2}\left(e^{\pi} - 1\right)$

c) $\sqrt{2}\left(e^{\pi/2} + 1\right)$ d) $\sqrt{2}\left(e^{\pi/2} - 1\right)$

Chapter 03) Fourier Series

1	a	41	d	81	b	121	c
2	d	42	d	82	d	122	b
3	b	43	b	83	a	123	d
4	a	44	c	84	b	124	d
5	c	45	d	85	d	125	a
6	d	46	b	86	c	126	b
7	a	47	c	87	a	127	a
8	d	48	a	88	b	128	b
9	b	49	b	89	a	129	b
10	d	50	a	90	b'	130	c
11	d	51	c	91	c	131	a
12	b	52	b	92	a	132	b
13	a	53	c	93	c	133	d
14	d	54	d	94	d	134	d
15	b	55	d	95	a	135	a
16	b	56	c	96	b	136	c
17	a	57	a	97	c	137	d
18	d	58	b	98	d	138	a
19	a	59	d	99	b	139	b
20	b	60	a	100	d	140	a
21	a			101	d	141	d
22	c	62	d	102	b	142	c
23	d	63	c	103	c	143	b
24	a	64	d	104	a	144	c
25	d	65	b	105	d	145	a
26	a	66	d	106	b	146	d
27	d	67	b	107	d	147	c
28	c	68	c	108	d	148	a
29	b	69	a	109	d	149	c
30	c	70	c	110	a	150	b
31	a	71	c	111	d	151	d
32	d	72	c	112	c	152	b
33	a	73	d	113	c	153	a
34	c	74	b	114	a	154	c
35	a	75	d	115	b	155	d
36	c	76	c	116	a	156	d
37	a	77	b	117	c	157	a
38	c	78	c	118	b	158	c
39	c	79	b	119	a	159	b
40	b	80	d	120	b		

Chapter 04) Reduction Formulae & Beta, Gamma Function

1	c	26	d	51	a	76	d	101	c
2	b	27	b	52	c	77	a	102	b
3	c	28	c	53	b	78	c	103	d
4	d	29	a	54	d	79	d	104	c
5	d	30	b	55	b	80	c	105	b
6	c	31	a	56	d	81	b	106	a
7	a	32	c	57	a	82	c	107	b
8	c	33	b	58	d	83	a	108	a
9	b	34	c	59	a	84	d	109	c
10	a	35	d	60	c	85	b	110	d
11	c	36	d	61	d	86	c	111	b
12	b	37	c	62	c	87	a	112	d
13	d	38	a	63	b	88	c	113	c
14	a	39	d	64	a	89	b	114	c
15	a	40	b	65	c	90	d	115	a
16	c	41	d	66	d	91	b	116	c
17	c	42	c	67	b	92	a	117	b
18	c	43	a	68	a	93	c	118	b
19	b	44	b	69	b	94	b	119	d
20	d	45	d	70	c	95	d	120	c
21	c	46	d	71	d	96	d	121	d
22	d	47	b	72	a	97	a	122	b
23	b	48	d	73	b	98	c	123	c
24	d	49	b	74	c	99	d	124	a
25	c	50	c	75	a	100	c	125	b

Chapter 05) Differentiation Under Integral Sign & Error Function

1	a	14	d	27	a	40	b	53	c
2	c	15	b	28	c	41	c	54	c
3	b	16	d	29	d	42	a	55	b
4	c	17	b	30	a	43	d	56	d
5	d	18	c	31	c	44	a	57	b
6	d	19	a	32	b	45	d	58	a
7	c	20	c	33	d	46	c	59	c
8	a	21	b	34	b	47	b	60	d
9	b	22	d	35	a	48	c	61	d
10	d	23	b	36	b	49	a	62	a
11	a	24	d	37	c	50	d		
12	d	25	a	38	d	51	c		
13	c	26	d	39	a	52	a		

Chapter 06) Curve Tracing & Rectification of Curves

1	a		18	c		35	d		52	b		69	c
2	b		19	a		36	c		53	d		70	a
3	a		20	c		37	a		54	d		71	c
4	c		21	b		38	c		55	a		72	b
5	b		22	d		39	b		56	b		73	d
6	c		23	c		40	d		57	b		74	a
7	d		24	d		41	a		58	a		75	d
8	c		25	b		42	d		59	d		76	b
9	b		26	c		43	c		60	a		77	c
10	d		27	a		44	d		61	d		78	a
11	c		28	b		45	d		62	a		79	c
12	d		29	a		46	b		63	c		80	d
13	c		30	c		47	d		64	b		81	c
14	b		31	b		48	b		65	a		82	a
15	d		32	c		49	c		66	d		83	c
16	a		33	d		50	a		67	a		84	b
17	a		34	d		51	a		68	b		85	d

Savitribai Phule Pune University – FE – Sem. II

Engineering Mathematics (M II)

Chapter 01–Ordinary Differential Equations

1) The order of the differential equation is
   a) the order of the highest ordered differential coefficient appearing in the differential equation.
   b) the order of the lowest ordered differential coefficient appearing in the differential equation.
   c) the power of the highest ordered differential coefficient appearing in the differential equation.
   d) the degree of the highest ordered differential coefficient appearing in the differential equation.

2) The degree of the differential equation is
   a) the highest ordered differential coefficient appearing in the differential equation.
   b) the lowest power of the highest ordered differential coefficient appearing in the differential equation.
   c) the highest power of the highest ordered differential coefficient appearing in the differential equation.
   d) the coefficient power of the highest ordered differential coefficient appearing in the differential equation.

3) A solution of a differential equation is a relation between
   a) dependent variables
   b) independent variables
   c) dependent and independent variables not containing any differential coefficient
   d) none of the above

4) In the general solution, the number of arbitrary constants is equal to
   a)order of the differential equation
   b) degree of the differential equation
   c) sum of order and degree of diff. eqn.
   d) difference of order and degree of diff. eqn.

5) The general solution of n^{th} order ordinary differential equation must involve
   a) $n+1$ arbitrary constants
   b) $n-1$ arbitrary constants
   c) n arbitrary constants
   d) none of the above

6) The solution obtained by assigning particular values to arbitrary constants in general solution of differential equation is known as
   a) singular solution b) particular solution
   c) general solution d) none of above

7) The order of differential equation whose general solution is $y=(c_1+c_2x)e^x+x$, where $c_1,\ c_2$ are arbitrary constants, is
   a) 1 b)2 c) 3 d) 0

8) The order of differential equation whose general solution is $y=(c_1+c_2x+c_3x)e^x+\frac{x^2}{12}$, where $c_1,\ c_2,\ c_3$ are arbitrary constants, is
   a) 0 b) 1 c) 2 d)3

9) The order of differential equation whose general solution is $y=(c_1+c_2)e^x+\frac{x^4}{3}$, where $c_1,\ c_2$ are arbitrary constants, is
   a) 0 b)1 c)2 d) 3

10) The order of differential equation whose general solution is $y=cx+c^2$, where c is arbitrary constant, is
   a) 0 b)1 c)2 d) 3

11) The order of differential equation whose general solution is $y=Ax+\frac{B}{x}$, where $A,\ B$ are arbitrary constants, is
   a) 0 b) 1 c)2 d) 3

12) The order of differential equation whose general solution is $y = Ax + \frac{A^2}{x}$, where A, B are arbitrary constants, is
a) 0 b) 1 c) 2 d) 3

13) The order of differential equation whose general solution is $y = \log(x-a) + b$, where a, b are arbitrary constants, is
a)2 b) 1 c) 0 d) none

14) The order of differential equation whose general solution is $x = A\sin(kt + B)$, where A, B are arbitrary constants and k is fixed constant, is
a) 0 b) 1 c)2 d) 3

15) The order of differential equation whose general solution is $x = (A + Bt)e^t$, where A, B are arbitrary constants, is
a) 0 b)2 c) 1 d) 3

16) The order of differential equation whose general solution is $y + \sqrt{x^2 + y^2} = cx + c^3$, where c is arbitrary constant, is
a) 0 b) 2 c) 3 d)1

17) The order of differential equation whose general solution is $y = 4(x - A)^2$, where A is arbitrary constant, is
a)1 b) 2 c) 3 d) none

18) The order of differential equation whose solution is $y = (c_1 + c_2 x)e^x + (c_3 + c_4 x)e^{2x}$, where c_1, c_2, c_3, c_4 are arbitrary constants, is
a) 1 b) 2 c) 3 d)4

19) The order of differential equation whose solution is $y = c_1 x + c_2 e^x + c_3 e^{2x} + c_4 e^{3x}$, where c_1, c_2, c_3, c_4 are arbitrary constants, is
a) 1 b)4 c) 2 d) 3

20) The order of differential equation whose solution is $y = (Ax^2 + Bx + C)e^x$, where A, B, C are arbitrary constants, is
a) 1 b) 2 c)3 d) 4

21) The order of differential equation whose general solution is $y = \sqrt{kx + c}$, where c is the only arbitrary constant, is
a)1 b) 2 c) 3 d) 0

22) The order of differential equation whose general solution is $y = c^2 + \frac{c}{x}$, where c is arbitrary constant, is
a) 0 b) 2 c) 3 d)1

23) The order of differential equation whose general solution is $y = A\cos(x+5)$, where A is arbitrary constant, is
a) 0 b)1 c) 2 d) 3

24) The order and the degree of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is
a)1, 1 b) 1, 2 c) 2, 1 d) 2, 2

25) The order and the degree of the differential equation $\frac{dy}{dx} + y\log x = \sin x$ is
a) 0, 1 b) 1, 0 c) 2, 1 d)1, 1

26) The order and the degree of the differential equation $\frac{dy}{dx} + 2y = \cos x$ is
a) 0, 1 b)1, 1 c) 1, 2 d) 2, 1

27) The order and the degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 5y = \sin 7x$ is
a) 0, 1 b) 1, 1 c) 1, 2 d) 2, 1

28) The order and the degree of the differential equation $1 + \frac{dy}{dx} = \left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}}$ is
a) order 2, degree 1 b) order 1, degree 2
c)order 2, degree 3 d) order 2, degree $\frac{3}{2}$

29) The order and the degree of the differential equation $\sqrt{1 + \frac{dy}{dx}} = \frac{d^2y}{dx^2}$ is
a) order 2, degree 2 b) order 2, degree 1
c) order 1, degree 2 d) order 1, degree 1

30) The order and the degree of the differential equation $\dfrac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}=k$ is

a) order 2, degree 1 b) order 2, degree 2

c) order 2, degree 3 d) order 2, degree $\dfrac{3}{2}$

31) The order and the degree of the differential equation $\sqrt{1+\dfrac{dy}{dx}}=\dfrac{d^2y}{dx^2}$ is

a)order 2, degree 2 b) order 2, degree 1

c) order 1, degree 2 d) order 1, degree 1

32) The order and the degree of the differential equation $\dfrac{d^2y}{dx^2}=\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}$ is

a) order 2, degree 1 b) order 2, degree 2

c) order 1, degree 2 d) order 1, degree 1

33) The order and the degree of the differential equation $x=\dfrac{1}{\sqrt{1+\dfrac{dy}{dx}+\dfrac{d^2y}{dx^2}}}$ is

a) order 2, degree 2 b)order 2, degree 1

c) order 1, degree 2 d) order 1, degree $-\dfrac{1}{2}$

34) The order and the degree of the differential equation $1+\dfrac{dy}{dx}=\dfrac{y}{\dfrac{dy}{dx}}$ is

a) order 1, degree 1 b) order 2, degree 1

c)order 1, degree 2 d) order 2, degree 2

35) The order and the degree of the differential equation $y+\dfrac{d^2y}{dx^2}+\dfrac{x}{\dfrac{dy}{dx}}=1$ is

a) order 1, degree 1 b) order 2, degree 1

c) order 1, degree 2 d) order 2, degree 2

36) The order and the degree of the differential equation $(2x-3y+2)dy+(x-2y+7)dx=0$ is

a)1, 1 b) 1, 2 c) 2, 1 d) none

37) By eliminating the arbitrary constant m, the differential equation for the general solution $y=mx$ is given by

a) $\dfrac{dy}{dx}=\dfrac{y}{x}$ b) $\dfrac{dy}{dx}-xy=0$

c) $\dfrac{dy}{dx}+\dfrac{y}{x}=0$ d) $\dfrac{dy}{dx}-y=0$

38) The differential equation satisfied by the general solution $y+x^3=Ax$ with A is arbitrary constant, is given by

a) $y\dfrac{dy}{dx}+2x-y^3=0$ b) $x\dfrac{dy}{dx}+2x^3-y=0$

c) $\dfrac{dy}{dx}+2x^2-y=0$ d) $x^3\dfrac{dy}{dx}+2(x-y)=0$

39) $y=5+\sqrt{cx}$, where c is the arbitrary constant, is the general solution of

a) $y\dfrac{dy}{dx}=5+2x$ b) $y=2x\dfrac{dy}{dx}$

c) $y=5+2x\dfrac{dy}{dx}$ d) $y=5+2x\sqrt{\dfrac{dy}{dx}}$

40) By eliminating the arbitrary constant c, the differential equation of $y=cx-c^2$ is

a) $\dfrac{dy}{dx}-x\left(\dfrac{dy}{dx}\right)^2+y=0$ b) $\left(\dfrac{dy}{dx}\right)^2-x\dfrac{dy}{dx}+y=0$

c) $\left(\dfrac{dy}{dx}\right)^2+x\dfrac{dy}{dx}-y=0$ d) $\left(\dfrac{dy}{dx}\right)^2-xy=0$

41) The differential equation whose primitive is $y=c^2+\dfrac{c}{x}$, is given by

a) $x^4\left(\dfrac{dy}{dx}\right)^2-xy=0$ b) $\left(\dfrac{dy}{dx}\right)^2+\dfrac{dy}{dx}+y=0$

c) $\left(\dfrac{dy}{dx}\right)^2-x^4\dfrac{dy}{dx}-y=0$ d) $x^4\left(\dfrac{dy}{dx}\right)^2-x\dfrac{dy}{dx}-y=0$

42) By eliminating the arbitrary constant c present in the function $x=cy-y^2$, the differential equation is given by

a) $\left(\dfrac{x+y^2}{y}\right)\dfrac{dy}{dx}-2y\dfrac{dy}{dx}-1=0$

b) $\left(\dfrac{x+y^2}{y}\right)\left(\dfrac{dy}{dx}\right)^2-2y\dfrac{dy}{dx}-1=0$

c) $x\frac{dy}{dx}-2\left(\frac{x+y^2}{y}\right)\frac{dy}{dx}-1=0$

d) $y^2\frac{dy}{dx}-2xy\frac{dy}{dx}+1=0$

43) The differential equation whose solution is $y^2=4ax$ is given by

a) $\left(\frac{dy}{dx}\right)^2-2xy=0$ b) $\frac{dy}{dx}-xy^2=0$

c) $2xy\frac{dy}{dx}-y^2=0$ d) $2xy\frac{dy}{dx}+y^2=0$

44) The differential equation of family of curves $x^2+y^2+xy+x+y=c$ is

a) $\frac{dy}{dx}=-\frac{2x+y+1}{x+2y+1}$ b) $y_2+4y=0$

c) $\frac{dy}{dx}=\frac{2x-y}{x+2y+1}$ d) $x^2y_2-xy_1+y=0$

45) The differential equation whose generalized solution is $xy+y^2-x^2-x-3y=c$, is

a) $\frac{dy}{dx}=-\frac{2x-y+1}{x-2y+3}$ b) $\frac{dy}{dx}=\frac{x-2y-1}{x-2y+3}$

c) $\frac{dy}{dx}=\frac{2x+y+1}{x+2y+3}$ d) $\frac{dy}{dx}=\frac{2x-y+1}{x+2y-3}$

46) The differential equation satisfied by family of circles $x^2+y^2=2Ax$ is given by

a) $\frac{dy}{dx}+x^2+y^2=0$ b) $\frac{dy}{dx}+\frac{y^2-x^2}{xy}=0$

c) $\frac{dy}{dx}+\frac{x^2-y^2}{2xy}=0$ d) $\frac{dy}{dx}-\frac{x^2-y^2}{2xy}=0$

47) The differential equation whose general solution is $x^3+y^3=3Ax$, where A is arbitrary constant, is

a) $y_1=\frac{x^3+y^3-3x^2}{3xy^2}$ b) $x^2y_1+y=3y_1$

c) $xy_1+y^2+x=0$ d) none of these

48) $y^2=x^2-1+Ax$, where A is arbitrary constant, is the general solution of the equation

a) $\frac{dy}{dx}=\frac{x^2+y^2}{2xy}$ b) $y\frac{dy}{dx}+x^2+y^2=0$

c) $2xy\frac{dy}{dx}=x^2+y^2+1$ d) $2xy\frac{dy}{dx}-\left(x^2+y^2\right)=0$

49) The differential equation of $y=4(x-A)^2$, where A is arbitrary constant, is

a) $\frac{dy}{dx}-16y^2=0$ b) $\left(\frac{dy}{dx}\right)^2-16y=0$

c) $\left(\frac{dy}{dx}\right)^2+4y=0$ d) $\left(\frac{dy}{dx}\right)^2+16y=0$

50) $\left(1+x^2\right)=A\left(1+y^2\right)$ is a general solution of the differential equation

a) $\frac{dy}{dx}+\frac{1+x^2}{1-y^2}=0$ b) $\frac{x}{y}\frac{dy}{dx}+\left(\frac{1+x^2}{1-y^2}\right)=0$

c) $\left(\frac{1+x^2}{1-y^2}\right)\frac{dy}{dx}+\frac{x}{y}=0$ d) $\frac{dy}{dx}+\frac{x}{y}\left(\frac{1+x^2}{1-y^2}\right)=0$

51) The differential equation representing the family of loops $y^2=c\left(4+e^{2x}\right)$ is

a) $\left(4+e^{2x}\right)\frac{dy}{dx}+4ye^{2x}=0$ b) $\left(4+e^{2x}\right)\frac{dy}{dx}+y=0$

c) $\frac{dy}{dx}-ye^{2x}=0$ d) $\left(4+e^{2x}\right)\frac{dy}{dx}-ye^{2x}=0$

52) The differential equation whose general solution is $y=\sqrt{3x+c}$, is given by

a) $\frac{dy}{dx}-3y=0$ b) $2y\frac{dy}{dx}+3=0$

c) $2y\frac{dy}{dx}-3=0$ d) $2\frac{dy}{dx}-3y=0$

53) By eliminating the arbitrary constant A from $y=A\cos(x+3)$ the differential equation is

a) $\frac{dy}{dx}+y=0$ b) $\frac{dy}{dx}+y\cot(x+3)=0$

c) $\tan(x+3)\frac{dy}{dx}+y=0$ d) $\cot(x+3)\frac{dy}{dx}+y=0$

54) By eliminating the arbitrary constant c, the differential equation of $\cos(y-x)=ce^{-x}$ is

a) $x^2y_1-xy=4y_1$ b) $\tan(y-x)\left(\frac{dy}{dx}-1\right)-1=0$

c) $xy_1-y+x\sin\left(\frac{y}{x}\right)=0$ d) none of these

55) The differential equation whose generalized solution is $\sin(y-x)=ce^{-\frac{x^2}{2}}$, is given by

a) $\tan(y-x)\left(\frac{dy}{dx}-1\right)+x=0$

b) $\cot(y-x)\left(\frac{dy}{dx}+1\right)+y=0$

c) $\left(\frac{dy}{dx}-1\right)+\frac{x}{\cot(y-x)}=0$

d) $\cot(y-x)\left(\frac{dy}{dx}-1\right)+x=0$

56) The differential equation of the family of curves $y=Ae^{-x^2}$ is given by

a) $y\frac{dy}{dx}-2x^2=0$ b) $\frac{dy}{dx}+2xy=0$

c) $y\frac{dy}{dx}+2\log x=0$ d) $\frac{dy}{dx}-x^2y=0$

57) The differential equation whose general solution is $y=Ae^{\frac{x}{y}}$, is given by

a) $(x+y)y_1-y=0$ b) $(x+y)^2y_1+y=0$

c) $(x-y)y_1+y=0$ d) $xy_1-\frac{y}{x}=0$

58) By eliminating the arbitrary constant c from the function $y=5ce^{\frac{x}{y}}$, the differential equation is

a) $(x+y)\frac{dy}{dx}-y=0$ b) $\frac{dy}{dx}-\frac{y}{x+y}=0$

c) $\left(\frac{x+y}{x}\right)\frac{dy}{dx}-\frac{y}{x}=0$ d) $\frac{dy}{dx}-\frac{y-x}{x+y}=0$

59) The differential equation for the function $\sin\left(\frac{y}{x}\right)=Ax$ is obtained by eliminating A and is given by

a) $\frac{dy}{dx}+\frac{y}{x}=x\tan\left(\frac{y}{x}\right)$ b) $\frac{dy}{dx}+xy=\tan\left(\frac{y}{x}\right)$

c) $x\frac{dy}{dx}-y=x\cot\left(\frac{y}{x}\right)$ d) $x\frac{dy}{dx}-y=x\tan\left(\frac{y}{x}\right)$

60) The differential equation of $\cos\left(\frac{y}{x}\right)=cx$ is

a) $xy_1-y+x\cot\left(\frac{y}{x}\right)=0$

b) $xy_1-y+x\sin\left(\frac{y}{x}\right)=0$

c) $x^2y_1-y+x=0$

d) $x^2y_1-y+x\sin\left(\frac{y}{x}\right)=0$

61) The differential equation for the function $xy=c^2$, where c is arbitrary constant, is

a) $x\frac{dy}{dx}-y=0$ b) $\frac{dy}{dx}+xy=0$

c) $x\frac{dy}{dx}+y=0$ d) $x\left(\frac{dy}{dx}\right)^2+y=0$

62) The differential equation satisfying the general solution $xy=ce^x$ is

a) $x^2y_1-xy+e^x=0$ b) $xy_1+y=e^x$

c) $xy_1+y(1+x)=0$ d) $xy_1+y(1-x)=0$

63) The differential equation whose general solution is $y^2=2c\left(x+\sqrt{c}\right)$, where c is arbitrary constant, is

a) $2\frac{dy}{dx}\left(x+\sqrt{y\frac{dy}{dx}}\right)-y=0$ b) $x+\sqrt{y\frac{dy}{dx}}-y=0$

c) $\frac{dy}{dx}\left(x+\sqrt{y\frac{dy}{dx}}\right)+y=0$ d) $2x\frac{dy}{dx}+y\left(\frac{dy}{dx}\right)^2=0$

64) The differential equation satisfying the function $y=Ax+Bx^2$ is given by

a) $x^2y_2-4xy_1+y=0$ b) $y_2^2+2xy_1+2y=0$

c) $x^2y_2-2xy_1+2y=0$ d) $x^2y_2+xy_1+y=0$

65) By eliminating the arbitrary constants c_1, c_2 from the function $y=\sqrt{4x^2+c_1x+c_2}$ we get the differential equation

a) $y_2+xy_1=0$ b) $yy_2+y_1^2=4$

c) $x^2y_1y_2-y^2=0$ d) $x^2y_2+xy_1+4y=0$

66) $\frac{x^2}{4}-\frac{y^2}{a}=1$ is a general solution of

a) $xy_1-4y=xy$ b) $x^2y_1-4xy_1+16y=0$

c) $x^2y_1-4y_1-xy=0$ d) none of these

67) The differential equation representing the family of ellipse $\frac{x^2}{a^2}+\frac{y^2}{9}=1$, is given by

a) $y\frac{dy}{dx}-x^2y+9=0$ b) $xy\frac{dy}{dx}-y^2+9=0$

c) $xy\frac{dy}{dx}-y^2=0$ d) $xy\frac{dy}{dx}+y^2-9=0$

68) The differential equation whose primitive is $y^2=4A(x-B)$, where A and B are arbitrary constants, is

a) $x^2y_1y_2-y^2=0$ b) $x^2y_2+xy_1+4y=0$

c) $y_2+xy_1=0$ d) $yy_2+{y_1}^2=0$

69) On the elimination of the arbitrary constants A and B as well from $y^2=5A(x-3B)$, the differential equation formed is

a) $\frac{d^2y}{dx^2}+y=0$ b) $y^2\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$

c) $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$ d) $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2-y=0$

70) The differential equation with general solution $x=A\cos(B-5t)$ is given by

a) $\frac{d^2x}{dt^2}+5\frac{dx}{dt}-25t=0$ b) $\frac{d^2x}{dt^2}-\frac{dx}{dt}-xt=0$

c) $\frac{d^2x}{dt^2}-25x=0$ d) $\frac{d^2y}{dx^2}-25y=0$

71) The differential equation whose general solution is $y=\log(Ax+B)$ is

a) $y_2+{y_1}^2=0$ b) $x^2y_2+{y_1}^2=0$

c) $y_2+x{y_1}^2+y=0$ d) $xy_2+{y_1}^2-y=0$

72) $y=A\sin x+B\cos x$ is the solution satisfying the differential equation

a) $\frac{d^2y}{dx^2}+\frac{y}{x}=0$ b) $y^2\frac{d^2y}{dx^2}+xy+x=0$

c) $\frac{d^2y}{dx^2}+xy=0$ d) $\frac{d^2y}{dx^2}+y=0$

73) The differential equation whose general solution is $y=A\sin 3x+B\cos 3x$ where A, B are arbitrary constants, is

a) $x^2y_2-xy-9y_1=0$ b) $xy_2-9y_1+y=0$

c) $y_2-9y=0$ d) $y_2+9y=0$

74) The differential equation whose solution is $y=A\cos\frac{4x}{3}+B\sin\frac{4x}{3}$, where A and B are arbitrary constants, is given by

a) $\frac{d^2y}{dx^2}-\frac{dy}{dx}-\frac{4}{3}y=0$ b) $\frac{d^2y}{dx^2}+\frac{16}{9}y=0$

c) $9\frac{d^2y}{dx^2}-16y=0$ d) $\frac{d^2y}{dx^2}+\frac{dy}{dx}+\frac{16}{9}y=0$

75) The differential equation whose primitive is $y=A\cos\log x+B\sin\log x$, where A and B are arbitrary constants, is given by

a) $x^2y_2+y_1+xy=0$ b) $x^2y_2+xy_1+y=0$

c) $x^2y_2+y_1+y=0$ d) $y_2-x^2y_1-xy=0$

76) The differential equation whose general solution is $y=Ae^{-x}+B$, where A and B are arbitrary constants, is

a) $y=x^2y_2+y_1$ b) $x^2y_2+xy_1+y=0$

c) $y_2+y_1=0$ d) $x{y_2}^2+y_1=0$

77) $y=Ae^{-x}+Be^{-x}$, where A and B both are arbitrary constants, is the solution for the differential equation

a) $x\frac{d^2y}{dx^2}+y=0$ b) $\frac{d^2y}{dx^2}-\frac{dy}{dx}-y=0$

c) $\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=0$ d) $\frac{d^2y}{dx^2}-y=0$

78) By eliminating the arbitrary constants A and B both from the function $xy=Ae^{x}+Be^{-x}$, we get the differential equation

a) $\frac{x}{y}\frac{d^2y}{dx^2}+2\frac{dy}{dx}-\frac{x}{y}=0$ b) $x\frac{d^2y}{dx^2}+2\frac{dy}{dx}-xy=0$

c) $y\frac{d^2y}{dx^2}+2\frac{dy}{dx}-xy=0$ d) $\frac{d^2y}{dx^2}+\frac{dy}{dx}+xy=0$

79) The differential equation, whose solution is given by $y = Ae^{-3x} + Be^{3x}$, is

a) $xy_2{}^2 + y_1 - xy = 0$ b) $x^2y_2 + y_1 + xy = 0$

c) $x^2y_2 - xy_1 + y = 0$ d) $y_2 - 4y = 0$

80) $e^{-t}y = A + Bt$ is a general solution of the differential equation

a) $y_2 - 2y_1 + y = 0$ b) $y_2 + y_1t + yt^2 = 0$

c) $xy_2 + y_1 + y = 0$ d) $4y_2 + 2y_1 + y = 0$

81) The differential equation having generalized solution $e^{-t}x = At - B$ is given by

a) $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = 0$ b) $x\frac{d^2x}{dt^2} + \frac{dx}{dt} + xt = 0$

c) $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + t = 0$ d) $x^2\frac{d^2x}{dt^2} - 2xt + x = 0$

82) The general form of the differential equation of I order and I degree can be expressed as

a) $\frac{dy}{dx} = c$ b) $M(x, y)dx + N(x, y)dy = 0$

c) $\frac{dy}{dx} + y = du$ d) $M(x, y)dx + N(x, y)dy = du$

83) The differential equation of the form $f_1(x)dx + f_2(y)dy = 0$ is known as

a) Linear form b) Non homogeneous form

c) exact form d)variable separable form

84) The differential equation in the form $\frac{dy}{dx} = x^n f\left(\frac{y}{x}\right)$ is known as

a) Linear form b) homogeneous form

c) exact form d) variable separable form

85) The differential equation in the form $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$, where f and g both are homogeneous functions of x and y of the same degree, is known as

a) Linear form b) homogeneous form

c) exact form d) variable separable form

86) The homogenous differential equation of the form $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$ is solved by substitution

a) no substitution, direct solution b) $x^n = v$

c) $xy = v$ d) $\frac{y}{x} = v$

87) The differential equation of the form $M(x, y)dx + N(x, y)dy = 0$ is exact, if

a) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ b) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

c) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ d) $\frac{\partial M}{\partial y} \times \frac{\partial N}{\partial x} = 1$

88) The differential equation $\frac{dy}{dx} = e^{2x+y} + 3x^4e^y$ is of the form

a) Linear form b) Non homogeneous form

c) exact form d) variable separable form

89) The form of the differential equation $(y^3 - 3x^2y)dx + (x^2y + 3x^3)dy = 0$ is

a) Linear form b) homogeneous form

c) exact form d) variable separable form

90) The differential equation is of the form $(x + y)dx + (x - y + 1)dy = 0$

a) Linear form b) non homogeneous form

c) exact form d) variable separable form

91) The differential equation $xy - \frac{dy}{dx} = y^3e^{-x^2}$ is of the form

a) Linear form b) non homogeneous form

c) exact form d) variable separable form

92) The substitution which can be used to solve the equation $(x + y + 7)dx + (3x + 3y - 7)dy = 0$ is

a) $x + y = v$ b) $x - y = v$

c) $xy = v$ d) $\frac{y}{x} = v$

93) The general solution of the differential equation $\frac{3e^x}{1 - e^x}dx + \frac{\sec^2 y}{\tan y}dy = 0$ is

a) $\tan y = c(1 - e^x)^3$ b) $(1 - e^x)^3 \tan y = c$

c) $(1 - e^{-x})^3 \cot y = c$ d) $\cot y = c(1 - e^x)^3$

94) The general solution of the differential equation $\frac{dy}{dx}+y=0$ is

a) $y=ce^{-x}$ b) $y=Ae^{-x}+B$
c) $y=ce^{x}$ d) $x=ce^{-y}$

95) The general solution of the differential equation $\frac{dx}{dy}+x=0$ is

a) $y=ce^{-x}$ b) $y=Ae^{-x}+B$
c) $y=ce^{x}$ d) $x=ce^{-y}$

96) The general solution of the differential equation $\frac{dy}{dx}+x=0$ is

a) $y=ce^{-x}$ b) $y^2+2x=c$
c) $x^2+2y=c$ d) $x=ce^{-y}$

97) The general solution of the differential equation $ydx+xdy=0$ is

a) $x^2+y^2=c$ b) $xy=c$ c) $\frac{y}{x}=c$ d) $\frac{x}{y}=c$

98) The general solution of the differential equation $\frac{dy}{dx}+\tan x=0$ is

a) $y=\log\sin x+c$ b) $y-\log\sec x=c$
c) $y=\log\sec x+c$ d) $y=\log\cos x+c$

99) The general solution of the differential equation $\frac{dy}{dx}+xy=0$ is

a) $\log x+\log y=c$ b) $\frac{x^2}{2}+\log y=c$
c) $x^2+\log y=c$ d) $x^2+y^2=c$

100) The general solution of the differential equation $\frac{dy}{dx}+\frac{1+x}{1+y}=0$ is

a) $x^2+y^2+2x+2y=c$ b) $(x+y)^2+2(x+y)=c$
c) $x^2+y^2+x+y=c$ d) $(1+x)=c(1+y)$

101) The general solution of the differential equation $\frac{dy}{dx}=\frac{1+y}{1+x}$ is

a) $(1+x)=c(1+y)^2$ b) $(1+y)=c(1+x)$
c) $(1+x)=c(1+y)$ d) $x=cy$

102) The general solution of the differential equation $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$ is

a) $\log\left(\frac{1+x^2}{1+y^2}\right)$ b) $\log(1+x^2)+\log(1+y^2)=c$
c) $\tan^{-1}x+\tan^{-1}y=c$ d) $\tan^{-1}x-\tan^{-1}y=c$

103) The general solution of the differential equation $\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$ is

a) $\frac{1}{2}\log\left(\frac{1-y^2}{1-x^2}\right)=c$ b) $\sec^{-1}x+\sec^{-1}y=c$
c) $\tan^{-1}x+\tan^{-1}y=c$ d) $\sin^{-1}x+\sin^{-1}y=c$

104) The general solution of the differential equation $x(1+y^2)dx+y(1+x^2)dy=0$ is

a) $(1+y^2)(1+x^2)=c$ b) $\log\left(\frac{1+y^2}{1+x^2}\right)=c$
c) $(1+y^2)=c(1+x^2)$ d) $\tan^{-1}x+\tan^{-1}y=c$

105) The general solution of the differential equation $\frac{dy}{dx}=(1+x)(1+y^2)$ is

a) $\log(1+y^2)=x+\frac{x^2}{2}+c$ b) $\tan^{-1}y=x+\frac{x^2}{2}+c$
c) $\log(1+x)+\tan^{-1}y=c$ d) $\tan^{-1}y+x+x^2=c$

106) The general solution of the differential equation $(e^x+1)ydy=(y+1)e^x dx$ is

a) $y+\log(y+1)+\log(e^x+1)=c$
b) $x+\log(y+1)=\log(e^x+1)+c$
c) $y-\log(y+1)=\log(e^x+1)+c$
d) $\frac{y^2}{2}+\log(y+1)=\log(e^x+1)+c$

107) The general solution of the differential equation $\frac{dy}{dx}=e^{x+y}+e^{y-x}$ is

a) $e^{-x}-e^{x}-e^{-y}=c$ b) $e^{x}-e^{2x}-e^{-y}=c$
c) $e^{-x}+e^{x}+e^{-y}=c$ d) $e^{x}-e^{-x}-e^{y}=c$

108) The general solution of the differential equation $\frac{dy}{dx}=e^{x-y}+3x^2e^{-y}$ is

a) $\frac{e^x+x^3}{e^y}=c$ b) $e^{x-y}=e^y+x^3+c$

c) $e^y=e^x+x^3+c$ d) $e^y+e^x+x^3=c$

109) The general solution of the differential equation $y(1+\log x)\frac{dx}{dy}-x\log x=0$ is

a) $\frac{x}{\log x}=yc$ b) $\frac{x}{y}\log x=y+c$

c) $x(\log x+1)=yc$ d) $x\log x=yc$

110) The general solution of the differential equation $\sec^2 x\tan y dx+\sec^2 y\tan x dy=0$ is

a) $\tan x\tan y=c$ b) $\tan x=c\tan y$

c) $\tan x+\tan y=c$ d) $\tan y=c\tan x$

111) The general solution of the differential equation $y\sec^2 x+(y-5)\tan x\frac{dy}{dx}=0$ is

a) $y^5-y+\tan x=c$ b) $y+5\log y+\log\sec x=c$

c) $y+5\log\frac{\tan x}{y}=c$ d) $y-5\log y+\log\tan x=c$

112) The general solution of the differential equation $e^x\cos y+(1+e^x)\sin y\frac{dy}{dx}=0$ is

a) $(1+e^x)\tan y=c$ b) $(1+e^x)\sec y=c$

c) $(1+e^x)\cos y=c$ d) $\sec y=c(1+e^x)$

113) The general solution of the differential equation $e^y\cos x dx+(e^y+1)\sin x dy=0$ is

a) $\sec x(e^y+1)=c$ b) $\sin x=c(e^y+1)$

c) $\sin y(1+e^x)=c$ d) $\sin x(e^y+1)=c$

114) The general solution of the differential equation $(4+e^{2x})\frac{dy}{dx}=ye^{2x}$ is

a) $\frac{y^2}{2}=A+(4+e^{2x})$ b) $y^2(4+e^{2x})=A$

c) $y^2=A(4+e^{2x})$ d) $x^2=A(4+e^{2x})$

115) The general solution of the differential equation $y-x\frac{dy}{dx}=2\left(y+\frac{dy}{dx}\right)$ is

a) $(x+2)y=c$ b) $x+2y=c$

c) $y=c(x+2)$ d) $(x+2)^2y=c$

116) The general solution of the differential equation $(x+1)\frac{dy}{dx}+1=2e^{-y}$ is

a) $(x+1)(2+e^{-y})=c$ b) $(2-e^y)=c(x+1)$

c) $(x+1)(2-e^y)=c$ d) $(x+1)=c(2-e^y)$

117) The general solution of the differential equation $x^3\left(x\frac{dy}{dx}+y\right)-\sec(xy)=0$ is

a) $\sin(xy)=2cx^2$ b) $\sin(xy)-\frac{1}{2x^2}=c$

c) $\sec(xy)+\frac{1}{2x^2}=c$ d) $\sin(xy)+\frac{1}{2x^2}=c$

118) The general solution of the differential equation $(y-ay^2)dx=(a+x)dy$ is

a) $\log(a+x)+\frac{1}{2}\log(1-ay)-\frac{1}{3}\log y=c$

b) $\log(a+x)-\frac{1}{a}\log(1-ay)-\log y=c$

c) $\log(a+x)+\log(1-ay)-\log y=c$

d) $\log(a+x)+\frac{\log(1-ay)}{-a}+\log y=c$

119) The necessary and sufficient condition for the equation $M(x,y)dx+N(x,y)dy=0$ to be exact is

a) $\frac{\partial M}{\partial x}=\frac{\partial N}{\partial y};\ My+Nx=0$

b) $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x};\ My+Nx\neq 0$

c) $\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x};\ My+Nx\neq 0$

d) $\frac{\partial M}{\partial y}\times\frac{\partial N}{\partial x}=1;\ My+Nx\neq 0$

120) If the differential equation $Mdx+Ndy=0$ is a homogeneous but not exact, its integrating factor is

a) $\frac{1}{Mx-Ny}$; $My-Nx\neq 0$

b) $\frac{1}{Mx+Ny}$; $Mx+Ny\neq 0$

c) $\frac{1}{My-Nx}$; $My-Nx\neq 0$

d) $\frac{1}{My+Nx}$; $My+Nx\neq 0$

121) If the differential equation $Mdx+Ndy=0$ is not exact but can be expressed in the form $yf_1(xy)dx+xf_2(xy)dy=0$, its integrating factor is

a) $\frac{1}{Mx+Ny}$; $Mx+Ny\neq 0$

b) $\frac{1}{My-Nx}$; $My-Nx\neq 0$

c) $\frac{1}{My+Nx}$; $My+Nx\neq 0$

d) $\frac{1}{Mx+Ny}$; $Mx+Ny=0$

122) If the differential equation $Mdx+Ndy=0$ is not exact and $\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}=f(x)$, its integrating factor is

a) $e^{\int f(x)dx}$
b) $e^{f(x)}$
c) $e^{\int f(y)dy}$
d) $f(x)$

123) If the differential equation $Mdx+Ndy=0$ is not exact and $\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{-M}=f(y)$, its integrating factor is

a) $e^{\int f(x)dx}$
b) $e^{f(x)}$
c) $e^{\int f(y)dy}$
d) $f(x)$

124) The total derivative of $dx+dy$ is

a) $d\left(\frac{x}{y}\right)$
b) $d(x+y)$
c) $d(x-y)$
d) $d(xy)$

125) The total derivative of $dx-dy$ is

a) $d\left(\frac{x}{y}\right)$
b) $d(x+y)$
c) $d(x-y)$
d) $d(xy)$

126) The total derivative of $xdy+ydx$ is

a) $d\left(\frac{x}{y}\right)$
b) $d(x+y)$
c) $d(x-y)$
d) $d(xy)$

127) The total derivative of $xdy-ydx$ with the integrating factor $\frac{1}{x^2}$ is

a) $d(x-y)$
b) $d\left(\frac{x}{y}\right)$
c) $d\left(\frac{y}{x}\right)$
d) $d(xy)$

128) The total derivative of $2(xdx+ydy)$ is

a) $d(x+y)$
b) $d(xy)$
c) $d(xy)^2$
d) $d(x^2+y^2)$

129) The total derivative of $2(xdx-ydy)$ is

a) $d(xy)$
b) $d\left(\frac{x^2}{y^2}\right)$
c) $d(x^2-y^2)$
d) $d(x^2+y^2)$

130) The total derivative of $\frac{ydx-xdy}{y^2}$ is

a) $d\left(\frac{y}{x}\right)$
b) $d\left(\frac{x}{y}\right)$
c) $d\left(\frac{x-y}{y}\right)$
d) $d(x^2-y^2)$

131) The total derivative of $ydx-xdy$ with the integrating factor $\frac{1}{y^2}$ is

a) $d\left(\frac{x}{y}\right)$
b) $d\left(\frac{y}{x}\right)$
c) $d\left(\frac{x-y}{y}\right)$
d) $d(x^2-y^2)$

132) The total derivative of $dx+dy$ with the integrating factor $\frac{1}{x+y}$ is

a) $d\left[\log(x-y)\right]$ b) $d\left[\log(x+y)\right]$

c) $d\left[\log(xy)\right]$ d) $d\left[\log(x^2+y^2)\right]$

133) The total derivative of $dx-dy$ with the integrating factor $\frac{1}{x-y}$ is

a) $d\left[\log(x-y)\right]$ b) $d\left[\log(x+y)\right]$

c) $d\left[\log(xy)\right]$ d) $d\left[\log(x^2+y^2)\right]$

134) The total derivative of $xdy+ydx$ with the integrating factor $\frac{1}{xy}$ is

a) $d\left[\log(x-y)\right]$ b) $d\left[\log(x+y)\right]$

c) $d\left[\log(xy)\right]$ d) $d\left[\log(x^2+y^2)\right]$

135) The total derivative of $xdy-ydx$ with the integrating factor $\frac{1}{xy}$ is

a) $d\left[\log(x-y)\right]$ b) $d\left[\log\left(\frac{x}{y}\right)\right]$

c) $d\left[\log\left(\frac{y}{x}\right)\right]$ d) $d\left[\log(xy)\right]$

136) The total derivative of $2(xdx+ydy)$ with the integrating factor $\frac{1}{x^2+y^2}$ is

a) $d\left[\log(x-y)\right]$ b) $d\left[\log(x+y)\right]$

c) $d\left[\log(xy)\right]$ d) $d\left[\log(x^2+y^2)\right]$

137) The total derivative of $2(xdx-ydy)$ with the integrating factor $\frac{1}{x^2-y^2}$ is

a) $d\left[\log(x^2-y^2)\right]$ b) $d\left[\log(x+y)\right]$

c) $d\left[\log(xy)\right]$ d) $d\left[\log(x^2+y^2)\right]$

138) The total derivative of $xdy-ydx$ with the integrating factor $\frac{1}{x^2+y^2}$ is

a) $d\left[\log(x^2-y^2)\right]$ b) $d\left[\log(x^2+y^2)\right]$

c) $d\left(\tan^{-1}\frac{y}{x}\right)$ d) $d\left(\tan^{-1}\frac{x}{y}\right)$

139) If the integrating factor of $\frac{ydx-xdy}{y^2}$ is $\frac{y}{x}$, its total derivative is

a) $d\left(\tan^{-1}\frac{x}{y}\right)$ b) $d\left(\log(x+y)\right)$

c) $d\left(\log\frac{y}{x}\right)$ d) $d\left(\log\frac{x}{y}\right)$

140) If the integrating factor of $\frac{xdy-ydx}{x^2}$ is $\frac{x}{y}$, its total derivative is

a) $d\left(\tan^{-1}\frac{y}{x}\right)$ b) $d\left(\tan^{-1}\frac{x}{y}\right)$

c) $d\left(\log\frac{x}{y}\right)$ d) $d\left(\log\frac{y}{x}\right)$

141) If the integrating factor of $\frac{ydx-xdy}{y^2}$ is $\frac{y^2}{x^2+y^2}$, its total derivative is

a) $d\left(\log\frac{y}{x}\right)$ b) $d\left(\tan^{-1}\frac{y}{x}\right)$

c) $d\left(\tan^{-1}\frac{x}{y}\right)$ d) $\log(x^2+y^2)$

142) The total derivative of $dx+dy$ with the integrating factor $\frac{1}{1+(x+y)^2}$ is

a) $d\left(\tan^{-1}(x+y)\right)$ b) $d\left(\log\frac{y}{x}\right)$

c) $d\left(\sec^{-1}(x+y)\right)$ d) $\log(x+y)$

143) The equation $(x+y+3)dx+(x-y-7)dy=0$ is of the form

a) variable separable b)exact differential

c) linear differential d) homogeneous

144) Equation $(3x+2y+1)dx+(2x-7y-3)dy=0$ is of the form
a) variable separable b) exact differential
c) linear differential d) homogeneous

145) For what value of λ, the differential equation $(5x+\lambda y-3)dx+(3x-7y+5)dy=0$ is exact?
a) 0 b) 1 c) 2 d)3

146) For what value of a, the differential equation $(xy^2+ax^2y)dx+(x^3+x^2y)dy=0$ is exact?
a)3 b) 2 c) 1 d) 5

147) For what value of a, the differential equation $(\tan y+ax^2y-y)dx+(x\tan^2 y-x^3-\sec^2 y)dy=0$ is exact?
a) 2 b) -2 c) 3 d)-3

148) The differential equation $\frac{dy}{dx}=\frac{ay+1}{(y+2)e^y-x}$ is exact, if the value of a is
a) -2 b) 2 c) -1 d)1

149) Differential equation $\frac{dy}{dx}+\frac{3+ay\cos x}{2\sin x-4y^3}=0$ is exact, if the value of a is
a) -3 b) 3 c)2 d)-2

150) For what values of a and b, the differential equation $(ay^2+x+x^8)dx+(y^2+y-bxy)dy=0$ is an exact differential equation?
a) $2a+b=0$ b) $a=2b$
c) $a-2b=3$ d) $a=1=b$

151) The equation $(1+axy^2)dx+(1+bx^2y)dy=0$ is exact differential equation, if
a) $a+2b=0$ b) $a=1,\ b=-3$
c) $a=b$ d) $a=2,\ b=3$

152) For what values of a and b, differential equation $(axy^4+\sin y)dx+(bx^2y^3+x\cos y)dy=0$ is formed to be exact?
a) $a=3b$ b) $a=2,\ b=4$
c) $a+b=1$ d) $a=3,\ b=-3$

153) The integrating factor for the differential equation $(y^2-2xy)dx+(2x^2+3xy)dy=0$ is
a) $\frac{1}{4xy^2}$ b) $\frac{1}{4x^2y^2}$ c) $\frac{1}{2x^2y}$ d) $\frac{1}{2xy}$

154) The integrating factor for the differential equation $(xy-2y^2)dx-(x^2-3xy)dy=0$ is
a) $\frac{1}{2x^2y^2}$ b) $\frac{1}{x^2y}$ c) $\frac{1}{xy}$ d) $\frac{1}{xy^2}$

155) The integrating factor for the differential equation $(x^2-3xy+2y^2)dx-(2xy-3x^2)dy=0$ is
a) $\frac{1}{x^3}$ b) $\frac{1}{x^3y}$ c) $\frac{1}{y^3}$ d) $\frac{1}{x^2y^2}$

156) The differential equation $(y^3-2x^2y)dx+(2xy^2-x^3)dy=0$ can be reduced to exact if the equation is multiplied by the integrating factor
a) $\frac{1}{xy}$ b) x^2y^2 c) $\frac{1}{x^2y^2}$ d) xy

157) The integrating factor for the differential equation $(x^2y-2xy^2)dx+(3x^2y-x^3)dy=0$ is
a) $\frac{1}{2xy}$ b) $\frac{1}{x^2y}$ c) $\frac{1}{x^2y^2}$ d) $\frac{1}{xy^2}$

158) The integrating factor for the differential equation $(xy+1)ydx-(xy-1)xdy=0$ is
a) $\frac{1}{2x^2y^2}$ b) $\frac{1}{2x^2y}$ c) $\frac{1}{2xy^2}$ d) $\frac{1}{2xy}$

159) The integrating factor for the differential equation $(xy+1)ydx+(x^2y^2+xy+1)xdy=0$ is
a) $\frac{1}{x^3y}$ b) $-\frac{1}{x^3y^3}$ c) $-\frac{1}{x^2y^2}$ d) $\frac{1}{xy^3}$

160) The integrating factor for the equation $(x^2y^2+xy+1)ydx+(x^2y^2-xy+1)xdy=0$ is
a) $\frac{1}{2x^2y^2}$ b) $\frac{1}{2x^2y}$ c) $\frac{1}{2xy^2}$ d) $\frac{1}{2x^3y^3}$

161) The integrating factor for the equation $\left(x^2y^2+5xy+2\right)ydx+\left(x^2y^2+4xy+2\right)xdy=0$ is

a) $\frac{1}{x^2y^3}$ b) $\frac{1}{x^2y}$ c) $\frac{1}{xy^2}$ d) $\frac{1}{x^2y^2}$

162) The differential equation $y\left(xy+2x^2y^2\right)dx+x\left(xy-x^2y^2\right)dy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) $\frac{1}{2x^2y}$ b) $\frac{1}{3x^3y}$ c) $\frac{1}{2x^2y^2}$ d) $\frac{1}{3x^3y^3}$

163) The integrating factor for the differential equation $\left(x^2+y^2+x\right)ydx+xydy=0$ is

a) $\frac{1}{2xy^2}$ b) $\frac{1}{2xy}$ c) x d) $\frac{1}{x}$

164) The integrating factor for the equation $\left(xy\sin xy+\cos xy\right)ydx+\left(xy\sin xy-\cos xy\right)xdx=0$ is

a) $\frac{1}{2xy}$ b) $\frac{1}{2xy\cos xy}$

c) $\frac{1}{2xy\sin xy}$ d) $\frac{1}{2\cos xy}$

165) The integrating factor for the differential equation $\left(y+\frac{y^3}{3}+\frac{x^2}{2}\right)dx+\left(\frac{x+xy^2}{4}\right)dy=0$ is

a) $\frac{1}{x^2}$ b) x^2 c) $\frac{1}{x^3}$ d) x^3

166) The integrating factor for the differential equation $\left(2x\log x-xy\right)dy+2ydx=0$ is

a) x^2 b) $\frac{1}{x}$ c) $\frac{1}{x^2}$ d) $\frac{1}{x^3}$

167) The integrating factor for the differential equation $\left(x^2+y^2+1\right)dx-2xydy=0$ is

a) x^2 b) $\frac{1}{x}$ c) $\frac{1}{x^2}$ d) $\frac{1}{x^3}$

168) The integrating factor for the differential equation $y\left(2xy+e^x\right)dx-e^xdy=0$ is

a) $\frac{1}{y^2}$ b) $\frac{1}{x^2}$ c) $\frac{1}{y^3}$ d) $\frac{1}{x^3}$

169) The integrating factor for the differential equation $y\log ydx+\left(x-\log y\right)dy=0$ is

a) $\frac{1}{y^2}$ b) $\frac{1}{x^2}$ c) $\frac{1}{y}$ d) $\frac{1}{x}$

170) The differential equation $\left(y^4+2y\right)dx+\left(xy^3+2y^4-4x\right)dy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) $\frac{1}{y^2}$ b) $\frac{1}{y^3}$ c) $\frac{1}{x^3}$ d) $\frac{1}{x^2}$

171) The differential equation $\left(2x+e^x\log y\right)ydx+e^xdy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) x^2 b) $\frac{1}{x^3}$ c) $\frac{1}{x}$ d) $\frac{1}{y}$

172) The differential equation $\left(xy^3+y\right)dx+2\left(x^2y^2+x+y^4\right)dy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) x b) y c) $\frac{1}{y}$ d) $\frac{1}{x}$

173) The differential equation $\left(xy^2-e^{\frac{1}{x^3}}\right)dx-x^2ydy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) $\frac{1}{x^4}$ b) $\frac{1}{x^3}$ c) $\frac{1}{y^2}$ d) $\frac{1}{y^3}$

174) $\left(x^2-3xy+2y^2\right)dx-\left(e^x+y^3\right)dy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) $\frac{1}{y^2}$ b) $\frac{1}{y^3}$ c) $\frac{1}{x^3}$ d) $\frac{1}{x^4}$

175) $\left(y^4+2y\right)dx+\left(xy^3+2y^4-4x\right)dy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) $\frac{1}{x^3}$ b) $\frac{1}{y^2}$ c) $\frac{1}{y^3}$ d) $\frac{1}{x^4}$

176) The solution of the exact differential equation $(x+y-2)dx+(x-y+4)dy=0$ is

a) $x^2+y^2+xy+x+y+c=0$

b) $x^2+y^2+2xy+4x+6y+c=0$

c) $x^2+y^2+2xy+4x+8y+c=0$

d) $x^2-y^2+2xy-4x+8y+c=0$

177) The solution of the exact differential equation $\left(y^2e^{xy^2}+4x^3\right)dx+\left(2xye^{xy^2}-3y^2\right)dy=0$ is

a) $\frac{1}{y^2}e^{xy^2}+\frac{x^4}{4}-\frac{y^3}{3}=c$ b) $e^{xy^2}+x^4-y^3=c$

c) $e^{xy^2}+x^4+y^3=c$ d) $e^{xy^2}+\frac{x^4}{4}-\frac{y^3}{3}=c$

178) The solution of the exact differential equation $\left(x^2-4xy-2y^2\right)dx+\left(y^2-4xy-2x^2\right)dy=0$ is

a) $x^3-6x^2y-6xy^2+y^3=c$

b) $\frac{x^3}{3}-6x^2y-6xy^2+\frac{y^3}{3}=c$

c) $x^3+x^2y+xy^2+y^3=c$

d) $x^3+x^2y-3xy^2+2y^3=c$

179) The solution of the exact differential equation $(1+\log xy)dx+\left(1+\frac{x}{y}\right)dy=0$ is

a) $y-x\log x+\log y=c$ b) $y+x\log xy=c$

c) $1+\frac{x}{y}\log xy=c$ d) $\frac{y}{x}+\log xy=c$

180) The solution of the exact differential equation $\left(1+x^2\right)(xdy+ydx)+2x^2ydx=0$ is

a) $x^2+y\left(1+x^2\right)=c$ b) $x+y-\left(1+x^2\right)=c$

c) $xy\left(1+x^2\right)=c$ d) $x+y\left(1+x^2\right)=c$

181) The solution of the exact differential equation $\cos y-x\sin y\frac{dy}{dx}=\sec^2 x$ is

a) $\frac{x}{y}\cos y=c\tan x$ b) $\cot x-x^2\cos y=c$

c) $\tan^2 x-x\sin y=c$ d) $\tan x-x\cos y=c$

182) The solution of the exact differential equation $\frac{dy}{dx}=\frac{1+y^2+3x^2y}{1-2xy-x^3}$ is

a) $x\left(1+y^2\right)+x^3y-y=c$

b) $\frac{1+y^2}{x}+x^2y-y=c$

c) $1+y^2+x^2y-xy=c$

d) $x\left(1+\frac{y^2}{2}\right)-\frac{x^3y}{3}-y=c$

183) The solution of the exact differential equation $\left(x^2y-2xy^2\right)dx+\left(3x^2y-x^3\right)dy=0$ with the integrating factor $\frac{1}{x^2y^2}$ is

a) $\frac{y}{x}-\log x+\log y=c$

b) $\frac{x}{y}-2\log x+3\log y=c$

c) $x+2y\log x+3x\log y=c$

d) $\frac{x^2}{2}-2y\log x+3\log y=c$

184) The solution of the exact differential equation $\left(3xy^2-y^3\right)dx+\left(xy^2-2x^2y\right)dy=0$ with the integrating factor $\frac{1}{x^2y^2}$ is

a) $\frac{y}{x}+3\log x+2\log y=c$

b) $y\log x+3\log x-2\log y=c$

c) $\frac{y}{x}+3\log x-2\log y=c$

d) $\frac{y^2}{x^2}+3x\log x-2y\log y=c$

185) The solution of the exact differential equation $\left(x^2-3xy+2y^2\right)dx+x(3x-2y)dy=0$ with the integrating factor $\frac{1}{x^3}$ is

a) $x^2\log x+3xy-y^2=cx^2$

b) $\log x+3x^2y-y^2=c$

c) $x^3\log x+3x^2y-xy^2=cx^3$

d) $3\log x+3xy+y^2=cx^2$

186) The solution of the exact differential equation $(1+xy)ydx+(1-xy)xdy=0$ with the integrating factor $\frac{1}{2x^2y^2}$ is

a) $3\log\left(\frac{x}{y}\right)+\frac{1}{x^2y^2}=c$ b) $\log\left(\frac{x}{y}\right)+\frac{1}{xy}=c$

c) $3\log\left(\frac{x}{y}\right)-\frac{1}{x^2y}=c$ d) $\log\left(\frac{x}{y}\right)-\frac{1}{xy}=c$

187) The solution of the exact differential equation $(x^2y^2+5xy+2)ydx+(x^2y^2+4xy+2)xdy=0$ with the integrating factor $\frac{1}{x^2y^2}$ is

a) $xy+5\log x-\frac{2}{xy}+4\log y=c$

b) $x^2y+5\log x-\frac{1}{xy}+2\log y=c$

c) $xy+5\log x+\frac{1}{xy}+3\log y=c$

d) $x^2y^2+5\log x+\frac{2}{xy}+4\log y=c$

188) The solution of the exact differential equation $(x^2y^2+xy+1)ydx+(x^2y^2-xy+1)xdy=0$ with the integrating factor $\frac{1}{2x^2y^2}$ is

a) $xy-\frac{1}{xy}+x\log x+y\log y=c$

b) $xy-\frac{1}{xy}+\log x+\log y=c$

c) $\frac{x}{y}-\frac{1}{xy}+\log\left(\frac{x}{y}\right)=c$

d) $xy-\frac{1}{xy}+\log\left(\frac{x}{y}\right)=c$

189) The solution of the exact differential equation $y(xy+2x^2y^2)dx+x(xy-x^2y^2)dy=0$ with the integrating factor $\frac{1}{3x^3y^3}$ is

a) $2\log\left(\frac{x}{y}\right)-\frac{1}{xy}=c$ b) $\frac{1}{2}\log\left(\frac{x}{y}\right)+\frac{1}{xy}=c$

c) $\log\left(\frac{x^2}{y}\right)-\frac{1}{xy}=c$ d) $\log\left(\frac{x}{y^2}\right)+\frac{1}{xy}=c$

190) The solution of the exact differential equation $(x^2+y^2+x)ydx+xydy=0$ with the integrating factor x is

a) $x^4+x^2y^3+x^3=c$ b) $y\left(\frac{x^4}{4}+\frac{x^2y^2}{2}+\frac{x^3}{3}\right)=c$

c) $y(x^4+x^2y^2+x^3)=c$ d) $\frac{x^4}{4}+\frac{x^2y^2}{2}+\frac{x^3}{3}=c$

191) The solution of the exact differential equation $(xy\sin xy+\cos xy)ydx+(xy\sin xy-\cos xy)xdx=0$ with the integrating factor $\frac{1}{2xy\cos xy}$ is

a) $x\log(\sec xy)=cy$ b) $xy\sec xy=c$

c) $x\sec xy=cy$ d) $x\cos xy=cy$

192) The solution of the exact differential equation $(x^2-3xy+2y^2)dx+(3x^2-2xy)dy=0$ with the integrating factor $\frac{1}{x^3}$ is

a) $\log x+\frac{3y}{x}-\left(\frac{y}{x}\right)^2=c$ b) $\log x+3yx-\left(\frac{y}{x}\right)^2=c$

c) $\log x+\frac{y}{x}+\left(\frac{y}{x}\right)^2=c$ d) $3\log x+\frac{y}{x}-\frac{y^2}{x}=c$

193) The solution of the exact differential equation $(xy^3+y)dx+2(x^2y^2+x+y^4)dy=0$ with the integrating factor y is

a) $\frac{3}{4}x^2y^4+\frac{6}{5}xy^2+2y^6=c$

b) $3x^2y^4+6x^2y+2x^6=c$

c) $x^3y^4+3xy^2+5y^6=c$

d) $3x^2y^4+6xy^2+2y^6=c$

194) The solution of the exact differential equation $(y^4+2y)dx+(xy^3-4x+2y^4)dy=0$ with the integrating factor $\frac{1}{y^3}$ is

a) $x(y^3+2)+y^2=c$ b) $x^2(y^3+2)-y^4=cy^2$

c) $x(y^3+2)+y^4=cy^2$ d) $(y^3+2)xy^4=cy^2$

195) The solution of the exact differential equation $\left(3x+2y^2\right)ydx+2x\left(2x+3y^2\right)dy=0$ with the integrating factor xy^3 is

a) $x^3y^4+x^2y^6=c$ b) $x^3y^3+x^4y^3=c$

c) $x^2y^4+xy^6=c$ d) $\frac{1}{3}x^3y^4+\frac{1}{4}x^2y^6=c$

196) The solution of the exact differential equation $\left(x^2y+y^4\right)dx+\left(2x^3+4xy^3\right)dy=0$ with the integrating factor $x^{\frac{5}{2}}y^{10}$ is

a) $\frac{12}{11}x^{\frac{11}{2}}y^{11}+\frac{12}{7}x^{\frac{7}{2}}y^{14}=c$

b) $\frac{2}{11}x^{\frac{11}{12}}y^{11}+\frac{2}{7}x^{\frac{7}{12}}y^{14}=c$

c) $\frac{2}{11}x^{\frac{11}{2}}y^{11}+\frac{2}{7}x^{\frac{7}{2}}y^{14}=c$

d) $\frac{2}{11}x^{\frac{11}{2}}y^{11}-\frac{2}{7}x^{\frac{7}{2}}y^{14}=c$

197) The solution of the exact differential equation $\left(y^2+2x^2y\right)dx+\left(2x^3-xy\right)dy=0$ with the integrating factor $\frac{1}{x^{5/2}y^{1/2}}$ is

a) $4xy-\frac{2}{3}\left(\frac{y}{x}\right)^{\frac{3}{2}}=c$ b) $4\sqrt{xy}-\frac{2}{3}\left(\frac{y}{x}\right)^{\frac{3}{2}}=c$

c) $4\sqrt{xy}-\frac{2}{3}\sqrt{\frac{y}{x}}=c$ d) $\sqrt{xy}+\left(\frac{y}{x}\right)^{\frac{3}{2}}=c$

198) The solution of the exact differential equation $\left(y^4-2x^3y\right)dx+\left(x^4-2xy^3\right)dy=0$ with the integrating factor $\frac{1}{x^2y^2}$ is

a) $\frac{2x^2}{y}+\frac{3y^2}{x}=c$ b) $\frac{x^2}{y}-\frac{y^2}{x}=c$

c) $\frac{x^2}{2y}+\frac{y^2}{3x}=c$ d) $\frac{x^2}{y}+\frac{y^2}{x}=c$

199) The solution of the exact differential equation $\left(y^3-2x^2y\right)dx-\left(x^3-2xy^2\right)dy=0$ with the integrating factor xy is

a) $x^3y^3\left(y^2+x^2\right)=c$ b) $x^2y^2\left(y^2-x^2\right)=c$

c) $x^2y^2\left(y^2+x^2\right)=c$ d) $x^2+y^2\left(y^2-x^2\right)=c$

200) The solution of the exact differential equation $\left(3x^2y^4+2xy\right)dx+\left(2x^3y^3-x^2\right)dy=0$ with the integrating factor $\frac{1}{y^2}$ is

a) $x^3y^2+\frac{x^2}{y}=c$ b) $x^2y^2+\frac{x^2}{y^2}=c$

c) $x^3y^3-\frac{x^2}{y}=c$ d) $x^2y^3-\frac{x^2}{y^3}=c$

201) The solution of the exact differential equation $y\left(x^2y+e^x\right)dx-e^xdy=0$ with the integrating factor $\frac{1}{y^2}$ is

a) $\frac{x^2}{2}+\frac{e^x}{y}=c$ b) $\frac{x^3}{3}-\frac{e^x}{y}=c$

c) $\frac{x^3}{3}+\frac{e^x}{y}=c$ d) $\frac{x^3}{3}+\frac{e^x}{2}=c$

202) The solution of the exact differential equation $\left(2x+e^x\log y\right)ydx+\left(e^x\right)dy=0$ with the integrating factor $\frac{1}{y}$ is

a) $x^2+e^x+\log y=c$ b) $x^2-e^x\log y=c$

c) $\frac{x^2}{2}+e^x\log y=c$ d) $x^2+e^x\log y=c$

203) The solution of $\frac{dy}{dx}\left(x+2y^3\right)=y+2x^3y^2$ with the integrating factor $\frac{1}{y^2}$ is

a) $\frac{x}{y}-\frac{x^4}{y}+y^2=c$ b) $\frac{x}{y}+\frac{x^4}{2}-\frac{y^2}{2}=c$

c) $\frac{x}{3}+\frac{x^4}{2}+y^2=c$ d) $\frac{x}{y}+\frac{x^4}{2}-y^2=c$

204) The solution of the exact differential equation $y\log ydx+\left(x-\log y\right)dy=0$ with the integrating factor $\frac{1}{y}$ is

a) $2x\log y-\left(\log y\right)^2=c$

b) $x^2\log y+\left(\log y\right)^2=c$

c) $2x\log y+\left(\log y\right)^3=c$

d) $\frac{2x}{3}\log y - \log y^2 = c$

205) The solution of the exact differential equation $y\left(2x^2y+e^x\right)dx=\left(e^x+y^3\right)dy$ with the integrating factor $\frac{1}{y^2}$ is

a) $\frac{1}{3}x^3+\frac{e^x}{x}-\frac{1}{2}y^2=c$ b) $\frac{2}{3}x^3+\frac{e^x}{y}+\frac{1}{2}y^3=c$

c) $\frac{2}{3}x^3+\frac{e^x}{y}-\frac{1}{2}y^2=c$ d) $x^3+\frac{e^x}{y}-y^2=c$

206) The solution of the exact differential equation $\left(2x\log x - xy\right)dy+2ydx=0$ with the integrating factor $\frac{1}{x}$ is

a) $2x\log x-\frac{x^2}{2}=c$ b) $2y\log x-\frac{y^2}{2}=c$

c) $\frac{y}{2}\log x-\frac{y^2}{2}=c$ d) $y\log x+\frac{y^2}{2}=c$

207) The solution of the exact differential equation $\left(x^4e^x-2mxy^2\right)dx+2mx^2ydy=0$ with the integrating factor $\frac{1}{x^4}$ is

a) $e^x+\frac{m^2y^2}{x^2}=cm$ b) $e^x-\frac{my^2}{x^2}=c$

c) $\frac{e^x}{y}-\frac{my^2}{x^2}=c$ d) $e^x+\frac{my^2}{x^2}=c$

208) The differential equation which can be expressed in the form $\frac{dy}{dx}+Py=Q$, where P and Q are functions of x only, is known as

a) variable separable equation in x, y
b) homogeneous differential equation in x, y
c) linear differential equation in x w.r.t y
d)linear differential equation in y w.r.t x

209) The differential equation which can be expressed in the form $\frac{dx}{dy}+Px=Q$, where P and Q are functions of y only, is known as

a)linear differential equation in x w.r.t y
b) linear differential equation in y w.r.t x
c) homogeneous differential equation in x, y
d) variable separable equation in x, y

210) The integrating factor of the linear differential equation $\frac{dy}{dx}+Py=Q$, where P and Q are functions of x only, is

a) $e^{\int Pdx}$ b) $e^{\int Qdx}$ c) $e^{\int Pdy}$ d) $e^{\int Qdy}$

211) The integrating factor of the linear differential equation $\frac{dx}{dy}+Px=Q$, where P and Q are functions of y only, is

a) $e^{\int Pdx}$ b) $e^{\int Qdx}$ c) $e^{\int Pdy}$ d) $e^{\int Qdy}$

212) The general solution of the linear differential equation $\frac{dy}{dx}+Py=Q$, where P and Q are functions of x only, is given by

a) $ye^{\int Pdx}=\int Qe^{\int Pdx}dx+c$
b) $xe^{\int Pdx}=\int Qe^{\int Pdx}dx+c$
c) $xe^{\int Pdy}=\int Qe^{\int Pdy}dy+c$
d) $xe^{\int Qdx}=\int Pe^{\int Qdx}dx+c$

213) The general solution of the linear differential equation $\frac{dx}{dy}+Px=Q$, where P and Q are functions of y only, is given by

a) $ye^{\int Pdx}=\int Qe^{\int Pdx}dx+c$
b) $xe^{\int Pdx}=\int Qe^{\int Pdx}dx+c$
c) $xe^{\int Pdy}=\int Qe^{\int Pdy}dy+c$
d) $xe^{\int Qdx}=\int Pe^{\int Qdx}dx+c$

214) A differential equation which can be expressed in the form $\frac{dy}{dx}+Py=Qy^n$, where P and Q are functions of x only, is known as

a) Non-linear differential equation
b)Bernoulli's linear differential equation
c) exact differential equation
d) homogenous differential equation

215) A differential equation which can be expressed in the form $\frac{dx}{dy}+Px=Qx^n$, where P and Q are functions of x only, is known as

a) Non-linear differential equation
b) Bernoulli's linear differential equation
c) exact differential equation
d) homogenous differential equation

216) A differential equation which can be expressed in the form $f'(y)\frac{dy}{dx}+Pf(y)=Q$, where P and Q are functions of x only, can be reduced into the linear form by substituting
a) $P=v$ b) $Q=v$
c) $f(y)=v$ d) $f'(y)=v$

217) A differential equation which can be expressed in the form $\frac{dy}{dx}+Py=Qy^n$, where P and Q are functions of x only, can be reduced into the linear form by substituting
a) $y^n=v$ b) $y^{1-n}=v$
c) $y^{n-1}=v$ d) $y^{n+1}=v$

218) If I_1, I_2 are the integrating factors of the equations $\frac{dx}{dy}+Px=Q$ and $\frac{dx}{dy}-Px=Q$ respectively, the relation between them is
a) $I_1=-I_2$ b) $I_1=I_2$
c) $I_1\cdot I_2=-1$ d) $I_1\cdot I_2=1$

219) The integrating factor of the linear differential equation $\frac{dy}{dx}+xy=x^5$ is
a) $e^{\log\frac{x^2}{2}}$ b) $e^{\frac{x^2}{2}}$ c) e^{x^2} d) x^2

220) The integrating factor of the linear differential equation $\frac{dy}{dx}+2xy=\frac{\tan^{-1}x}{1+x^2}$ is
a) $\frac{x^2}{2}$ b) $e^{\frac{x^2}{2}}$ c) e^{x^2} d) $2x^2$

221) The integrating factor of the linear differential equation $\frac{dx}{dy}+xy=y^5$ is
a) $e^{\frac{y^2}{2}}$ b) $\frac{y^2}{2}$ c) $e^{\frac{x^2}{2}}$ d) e^{x^2}

222) The integrating factor of the linear differential equation $\frac{dy}{dx}+\frac{y}{1+x}=x^3$ is
a) $e^{\frac{(1+x)^2}{2}}$ b) $1+x$ c) $\frac{1}{1+x}$ d) e^{1+x}

223) The integrating factor of the linear differential equation $\frac{dy}{dx}+\frac{y}{1-x}=\sin x$ is
a) $\frac{1}{1-x}$ b) $1-x$ c) e^{1-x} d) $e^{\frac{(1-x)^2}{2}}$

224) The integrating factor of the linear differential equation $\frac{dy}{dx}+\frac{y}{1+x^2}=\sec x\tan x$ is
a) $\frac{(1+x^2)^2}{2}$ b) $1+x^2$ c) $e^{\tan^{-1}x}$ d) e^{1+x^2}

225) The integrating factor of the linear differential equation $\frac{dy}{dx}+\frac{2x}{1+x^2}y=\tan^{-1}x$ is
a) $\frac{(1+x^2)^2}{2}$ b) $1+x^2$ c) $e^{\tan^{-1}x}$ d) e^{1+x^2}

226) The integrating factor of the linear equation $\frac{dy}{dx}+y\tan x=e^x\sin(2x-3)$ is
a) $\sec^2 x$ b) $\cos x$ c) $\sec x$ d) $e^{\sec x}$

227) The integrating factor of the linear differential equation $\tan x\frac{dy}{dx}+y=e^x\sin x$ is
a) e b) $e^{\sin x}$ c) $\log(\sin x)$ d) $\sin x$

228) The integrating factor of the linear equation $(1+x^2)\frac{dy}{dx}+xy=2x^3-3x+5$ is
a) e^{1+x^2} b) $\frac{1}{1+x^2}$ c) $1+x^2$ d) $\sqrt{1+x^2}$

229) The integrating factor of the linear equation $(1+x^2)\frac{dy}{dx}+4xy=\frac{1}{(1+x^2)^3}$ is
a) $1+x^2$ b) e^{1+x^2} c) $(1+x^2)^2$ d) $\frac{1}{1+x^2}$

230) The integrating factor of the linear equation $\left(1+x^2\right)\frac{dy}{dx}+2xy=\frac{1}{\left(1+x^2\right)^3}$ is

a) $1+x^2$ b) e^{1+x^2} c) $\left(1+x^2\right)^2$ d) $\frac{1}{1+x^2}$

231) The integrating factor of the linear equation $\left(1+x^2\right)\frac{dy}{dx}-2xy=\frac{1}{\left(1+x^2\right)^3}$ is

a) $1+x^2$ b) e^{1+x^2} c) $\left(1+x^2\right)^2$ d) $\frac{1}{1+x^2}$

232) The integrating factor of the linear differential equation $\frac{dx}{dy}+\frac{xy}{1+y^2}=\sec y$ is

a) $\sqrt{1+x^2}$ b) $\sqrt{1+y^2}$ c) $\tan^{-1} y$ d) $e^{\tan^{-1} y}$

233) The integrating factor of the linear differential equation $\frac{dy}{dx}+y\cot x=\tan x$ is

a) $\sin x$ b) $e^{\sec x}$
c) $\cos x$ d) $\sec x+\tan x$

234) The integrating factor of the linear differential equation $\cos x\frac{dy}{dx}+y=\tan x$ is

a) $e^{\sec x+\tan x}$ b) $e^{\sec x}$
c) $\cos x$ d) $\sec x+\tan x$

235) The integrating factor of the differential equation $\frac{dy}{dx}+\sqrt{x}\,y=\sin\sqrt{x}\cos\sqrt{x}$ is

a) $\sin\sqrt{x}$ b) $e^{\log\sqrt{x}}$
c) $e^{\frac{2}{3}x\sqrt{x}}$ *or* $e^{\frac{2}{3}x^{\frac{3}{2}}}$ d) $\frac{2}{3}x\sqrt{x}$ *or* $\frac{2}{3}x^{\frac{3}{2}}$

236) The integrating factor of the linear equation $\frac{dy}{dx}+\left(\tan x+\frac{1}{x}\right)y=\frac{1}{x}\sec x$ is

a) $x\sec x$ b) $e^{x\sec x}$ c) $e^{x+\sec x}$ d) $x+\sec x$

237) The integrating factor of the linear differential equation $\left(1-x^2\right)\frac{dy}{dx}=x^3+xy$ is

a) $e^{\tan^{-1} x}$ b) $e^{\sqrt{1-x^2}}$ c) $\frac{1}{\sqrt{1-x^2}}$ d) $\sqrt{1-x^2}$

238) The integrating factor of the linear differential equation $\left(1-x^2\right)\frac{dy}{dx}=x^3-xy$ is

a) $\frac{1}{\sqrt{1-x^2}}$ b) $\sqrt{1-x^2}$ c) $e^{\sqrt{1-x^2}}$ d) $e^{\tan^{-1} x}$

239) The integrating factor of the differential equation $1+y^2+\left(x-e^{\tan^{-1} x}\right)\frac{dy}{dx}=0$ is

a) $\tan^{-1} x$ b) $\tan^{-1} y$ c) $e^{\tan^{-1} x}$ d) $e^{\tan^{-1} y}$

240) The integrating factor of the differential equation $1+x^2+\left(y-e^{\tan^{-1} y}\right)\frac{dx}{dy}=0$ is

a) $\tan^{-1} x$ b) $\tan^{-1} y$ c) $e^{\tan^{-1} x}$ d) $e^{\tan^{-1} y}$

241) The integrating factor of the differential equation $\left(1+y^2\right)dx=\left(e^{\tan^{-1} x}-x\right)dy$ is

a) $\tan^{-1} x$ b) $\tan^{-1} y$ c) $e^{\tan^{-1} x}$ d) $e^{\tan^{-1} y}$

242) The integrating factor of the linear differential equation $y^2+\left(x-\frac{1}{y}\right)\frac{dy}{dx}=0$ is

a) $2\log x$ b) $\log y$ c) $-\frac{1}{y}$ d) $-\frac{1}{y^2}$

243) The integrating factor of the linear differential equation $\sin 2y\,dx=\left(\tan y-x\right)dy$ is

a) $\frac{\tan x}{2}$ b) $\sqrt{\tan y}$ c) $\sqrt{\tan x}$ d) $\frac{\tan y}{2}$

244) The integrating factor of the linear equation $y\log y\,dx+\left(x-\log y\right)dy=0$ is

a) $\left(\log y\right)^2$ b) $x\log y$ c) $\log y$ d) $\log x$

245) The integrating factor of the linear differential equation $y\,dx-\left(y-x\right)dy=0$ is

a) y b) x c) y^2 d) x^2

246) The integrating factor of the linear equation $\sqrt{a^2+x^2}\frac{dy}{dx}+y=\sqrt{a^2+x^2}-x$ is

a) $\frac{1}{2a}\log\left(x+\sqrt{a^2+x^2}\right)$ b) $\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

c) $x+\sqrt{a^2+x^2}$ d) $x-\sqrt{a^2+x^2}$

247) The integrating factor of the linear differential equation $\frac{dy}{dx}=\frac{e^x-2xy}{x^3}$ is

a) $e^{\frac{x^3}{3}}$ b) x^3 c) $\frac{1}{x^3}$ d) e^{x^3}

248) The integrating factor of linear differential equation $\left(x^2+1\right)\frac{dy}{dx}=x^3-2xy+x$ is

a) $\tan^{-1}x$ b) $e^{\tan^{-1}x}$ c) $\frac{1}{x^2+1}$ d) x^2+1

249) The integrating factor of the linear differential equation $x^2\frac{dy}{dx}=3x^2-2xy+1$ is

a) x^2-1 b) x^2 c) x^2+1 d) $\frac{1}{x^2}$

250) The integrating factor of the linear differential equation $\left(e^{-y}\sec^2 y-x\right)dy=dx$ is

a) $e^{\tan y}$ b) $\tan y$ c) e^x d) e^y

251) The differential equation $\frac{dy}{dx}-y\tan x=y^4\sec x$ is reduced into the linear form

a) $\frac{du}{dx}+3u\tan x=-3\sec x;\ u=y^{-3}$

b) $\frac{du}{dx}-3u\tan x=3\sec x;\ u=y^{-3}$

c) $\frac{du}{dx}-3u\tan x=-3\sec x;\ u=y^{-3}$

d) $\frac{du}{dx}+3u\cot x=-3\sec x;\ u=y^{-3}$

252) The differential equation $\frac{dy}{dx}-xy=-y^3e^{-x^2}$ can be reduced to the linear form

a) $\frac{dy}{dx}+xu=-2e^{-x^2};\ u=\frac{1}{y^2}$

b) $\frac{dy}{dx}+xu=e^{-x^2};\ u=\frac{1}{y^2}$

c) $\frac{dy}{dx}-2xu=2e^{-x^2};\ u=\frac{1}{y^2}$

d) $\frac{dy}{dx}+2xu=2e^{-x^2};\ u=\frac{1}{y^2}$

253) The value of k for which e^{ky^2} is an integrating factor of linear differential equation $\frac{dx}{dy}+xy=e^{-\frac{y^2}{2}}$ is

a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) 2 d) -2

254) The general solution of $\frac{dy}{dx}+\frac{y}{1+x}=-x(1-x)$ with the integrating factor $\frac{1}{1-x}$ is

a) $\frac{y}{1-x}=-\frac{x^3}{3}+c$ b) $y=-\frac{x^2}{2}(1-x)+c$

c) $\frac{y}{1-x}=-\frac{x^2}{2}+c$ d) $\frac{y}{1-x}=\frac{x^2}{2}+c$

255) The general solution of $\frac{dy}{dx}+\frac{y}{(1-x)\sqrt{x}}=1-\sqrt{x}$ with the integrating factor $\frac{1+\sqrt{x}}{1-\sqrt{x}}$ is

a) $\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)y^2=x^2+\frac{2}{3}x^{\frac{3}{2}}+c$

b) $\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)y=x+\frac{2}{3}x^{\frac{3}{2}}+c$

c) $\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)y=x+\frac{2}{3}x^{\frac{3}{2}}+c$

d) $\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)y=x-\frac{2}{3}x^{\frac{3}{2}}+c$

256) The general solution of $\frac{dy}{dx}+y\cot x=\sin 2x$ with the integrating factor $\sin x$ is

a) $y\sin x=\frac{2}{3}\sin^3 x+c$ b) $y\sin x=\frac{1}{3}\sin^3 x+c$

c) $x\sin y=\frac{2}{3}\sin^3 x+c$ d) $y\sin x=\sin^3 x+c$

257) The general solution of $\frac{dy}{dx}+\frac{3y}{x}=\frac{e^x}{x^2}$ with the integrating factor x^3 is

a) $x^3y=e^{-x}(x+1)+c$ b) $xy^3=e^x(x-1)+c$

c) $x^3y=e^x(x-1)+c$ d) $x^3y=e^x(x+1)+c$

258) The general solution of $\frac{dy}{dx}+\frac{3y}{x}=x^2$ with the integrating factor x^3 is

a) $x^3y=\frac{x^4}{4}+c$ b) $x^3y=\frac{x^6}{6}+c$

c) $x^3y=\frac{x^2}{2}+c$ d) $xy^3=\frac{x^3}{3}+c$

259) The general solution of $\frac{dy}{dx}+\left(\tan x+\frac{1}{x}\right)y=\frac{1}{x}\sec x$ with the integrating factor $x\sec x$ is

a) $xy\sin x=\tan x+c$ b) $xy\sec x=-\tan x+c$

c) $xy\tan x=\cot x+c$ d) $xy\sec x=\tan x+c$

260) The general solution of $\frac{dy}{dx}+\frac{2y}{x}=\frac{1}{x^3}$ with the integrating factor x^2 is

a) $y=x^2\log x+c$ b) $x^2y=\log x+c$

c) $xy^2=\log x+c$ d) $x^2y=\log\frac{1}{x}+c$

261) The general solution of $\frac{dy}{dx}+(1+2x)y=e^{-x^2}$ with the integrating factor e^{x+x^2} is

a) $ye^{x+x^2}=e^x+c$ b) $ye^{x+x^2}=-e^x+c$

c) $e^{x+x^2}=ye^x+c$ d) $ye^{x-x^2}=e^x+c$

262) The general solution of $\frac{dy}{dx}+\frac{x}{1+y^2}=\frac{e^{-\tan^{-1}y}}{1+y^2}$ with the integrating factor $e^{\tan^{-1}y}$ is

a) $ye^{\tan^{-1}y}=\tan^{-1}x+c$ b) $xe^{\tan^{-1}y}=\tan^{-1}y+c$

c) $xe^{\tan^{-1}y}=\cot^{-1}y+c$ d) $ye^{\tan^{-1}y}=\tan^{-1}y+c$

263) The general solution of $\frac{dy}{dx}+x\sec y=\frac{2y\cos y}{1+\sin y}$ with the integrating factor $\sec y+\tan y$ is

a) $(\sec y+\tan y)x^2=y+c$

b) $(\sec y+\tan y)x=-y^2+c$

c) $(\sec y+\tan y)x=y^2+c$

d) $x=\frac{y^2}{\sec y+\tan y}+c$

Chapter 02 – Applications of Ordinary Differential Equations

1) Two families of curves are said to be orthogonal trajectories of each other, if
   a) Every member of one family cuts every member of other family at right angle.
   b) Every member of one family cuts every member of other family at origin.
   c) Every member of one family cuts every member of other family at common point.
   d) None of the above.

2) In the two dimensional Cartesian form, to find orthogonal trajectories of given family of curves, in its differential equation we replace $\frac{dy}{dx}$ by
   a) $-y\frac{dx}{dy}$ b) $-\frac{dy}{dx}$ c) $-\frac{dx}{dy}$ d) $-x\frac{dx}{dy}$

3) In the two dimensional polar form, to find orthogonal trajectories of given family of curves, in its differential equation we replace $\frac{dr}{d\theta}$ by
   a) $r\frac{d\theta}{dr}$ b) $-r\frac{d\theta}{dr}$ c) $-r^2\frac{d\theta}{dr}$ d) $-\frac{d\theta}{dr}$

4) The differential equation of orthogonal trajectories of family of straight lines $y=mx$ is
   a) $\frac{dx}{dy}+y=0$ b) $\frac{dy}{dx}=-\frac{y}{x}$
   c) $\frac{dx}{dy}=-\frac{y}{x}$ d) $\frac{dx}{dy}=-\frac{x}{y}$

5) For the family of the curves $x^2+y^2=c^2$, the differential equation of orthogonal trajectories is
   a) $x^2+y^2\frac{dx}{dy}=0$ b) $x+y\frac{dy}{dx}=0$
   c) $x+xy\frac{dx}{dy}=0$ d) $x-y\frac{dx}{dy}=0$

6) The differential equation of orthogonal trajectories of family of $x^2+2y^2=c^2$ is
   a) $y-2x\frac{dy}{dc}=0$ b) $x-2y\frac{dx}{dy}=0$
   c) $x+2y\frac{dy}{dx}=0$ d) $x+2y\frac{dx}{dy}=0$

7) For the family of the curves $y^2=4ax$, the differential equation of orthogonal trajectories is
   a) $2y\frac{dy}{dx}=4x$ b) $2y\frac{dy}{dx}=\frac{y}{x^2}$
   c) $-2y\frac{dy}{dx}=\frac{y^2}{x}$ d) $-2y\frac{dx}{dy}=\frac{y^2}{x}$

8) For the family of the curves $y=4ax^2$, the differential equation of orthogonal trajectories is
   a) $y\frac{dy}{dx}=2x$ b) $\frac{dy}{dx}=-\frac{2}{x^2}$
   c) $\frac{dy}{dx}=-\frac{2y}{x}$ d) $-2\frac{dx}{dy}=\frac{1}{xy}$

9) For the family of the curves $xy=c$, the differential equation of orthogonal trajectories is
   a) $x^2\frac{dx}{dy}+2y=0$ b) $-x\frac{dx}{dy}+y=0$
   c) $2x\frac{dx}{dy}-y=0$ d) $x\frac{dy}{dx}-y=0$

10) The differential equation of orthogonal trajectories of family of $2x^2+y^2=cx$ is
   a) $4x-2y\frac{dx}{dy}=\frac{2x^2+y^2}{x}$
   b) $4x+2y\frac{dy}{dx}=\frac{2x^2+y^2}{x}$
   c) $4x^2+2y\frac{dx}{dy}=\frac{2x^2+y^2}{x}$
   d) $4x-2xy\frac{dy}{dx}=\frac{2x^2+y^2}{x}$

11) For the family of the curves $x^2+cy^2=1$, the differential equation of orthogonal trajectories is

a) $x+\left(\frac{1-x^2}{y}\right)\frac{dy}{dx}=0$ b) $x+\left(\frac{1+x^2}{y}\right)\frac{dx}{dy}=0$

c) $x-\left(\frac{1-x^2}{y}\right)\frac{dx}{dy}=0$ d) $x^2+\left(\frac{1-x^2}{y}\right)\frac{dy}{dx}=0$

12) For the family of the curves $e^x+e^{-y}=c$, the differential equation of orthogonal trajectories is

a) $e^{2x}-e^{-2y}\frac{dx}{dy}=0$ b) $e^{-x}+e^{y}\frac{dx}{dy}=0$

c) $e^{x}-e^{-y}\frac{dy}{dx}=0$ d) $e^{x}+e^{-y}\frac{dx}{dy}=0$

13) The differential equation of orthogonal trajectories of family of $r=a\cos\theta$ is

a) $-r\frac{dr}{d\theta}=\cot\theta$ b) $-r\frac{dr}{d\theta}=\tan\theta$

c) $r\frac{d\theta}{dr}=\cot\theta$ d) $r\frac{d\theta}{dr}=\tan\theta$

14) For the family of the curves $r=a\sin\theta$, the differential equation of orthogonal trajectories is

a) $\frac{1}{r}\frac{d\theta}{dr}=-\cot\theta$ b) $r\frac{d\theta}{dr}=-\tan\theta$

c) $r\frac{d\theta}{dr}=-\cot\theta$ d) $-\frac{1}{r}\frac{dr}{d\theta}=\tan\theta$

15) For the family of the curves $r^2=a\sin\theta$, the differential equation of orthogonal trajectories is

a) $2r\frac{d\theta}{dr}=-\cot\theta$ b) $r\frac{d\theta}{dr}=-\frac{\tan\theta}{2}$

c) $r^2\frac{d\theta}{dr}=-\cot\theta$ d) $-\frac{2}{r}\frac{dr}{d\theta}=\tan\theta$

16) For the family of the curves $r=a(1-\cos\theta)$, the differential equation of orthogonal trajectories is

a) $-r\frac{d\theta}{dr}=\frac{1-\cos\theta}{\sin\theta}$ b) $r\frac{dr}{d\theta}=\frac{\sin\theta}{1-\cos\theta}$

c) $-r\frac{d\theta}{dr}=\frac{\sin\theta}{1-\cos\theta}$ d) $r^2\frac{d\theta}{dr}=\frac{\sin\theta}{1-\cos\theta}$

17) For the family of the curves $r^2=a\sin 2\theta$, the differential equation of orthogonal trajectories is

a) $-r^2\frac{dr}{d\theta}=\cot 2\theta$ b) $r\frac{d\theta}{dr}=-\cot 2\theta$

c) $r\frac{d\theta}{dr}=-\tan 2\theta$ d) $-\frac{dr}{d\theta}=\cot 2\theta$

18) For the family of the curves $r^2=a\cos 2\theta$, the differential equation of orthogonal trajectories is

a) $\frac{dr}{d\theta}=r\tan 2\theta$ b) $\frac{1}{r}\frac{d\theta}{dr}=-\tan 2\theta$

c) $r\frac{d\theta}{dr}=\cot 2\theta$ d) $r\frac{d\theta}{dr}=\tan 2\theta$

19) For the family of the curves $r^2=a\cos 2\theta$, the differential equation of orthogonal trajectories is

a) $\frac{dr}{d\theta}=r\tan 2\theta$ b) $\frac{1}{r}\frac{d\theta}{dr}=-\tan 2\theta$

c) $r\cot 2\theta\frac{d\theta}{dr}=1$ d) $r\frac{d\theta}{dr}+\tan 2\theta=0$

20) For the family of the curves $r=a\cos^2\theta$, the differential equation of orthogonal trajectories is

a) $r\frac{d\theta}{dr}=\frac{\sin 2\theta}{\cos^2\theta}$ b) $r^2\frac{d\theta}{dr}=-\frac{\sin 2\theta}{\cos^2\theta}$

c) $\frac{dr}{d\theta}=-\frac{\sin 2\theta}{\cos^2\theta}$ d) $r\frac{d\theta}{dr}=-\frac{\sin 2\theta}{\cos 2\theta}$

21) For the family of the curves $r=a\sec^2\left(\frac{\theta}{2}\right)$, the differential equation of orthogonal trajectories is

a) $r\frac{d\theta}{dr}=-\tan\frac{\theta}{2}$ b) $r\frac{dr}{d\theta}=-\cot\frac{\theta}{2}$

c) $\frac{1}{r}\frac{d\theta}{dr}=-\cot\frac{\theta}{2}$ d) $r\frac{d\theta}{dr}=-\tan 2\theta$

22) The orthogonal trajectories of family of curves having differential equation $y=mx$ is $\frac{dy}{dx}=\frac{y}{x}$, is given by

a) $x^2 - y^2 = c$ b) $\frac{x^2}{2} + y^2 = c$

c) $x^2 + y^2 = c$ d) $x^2 + 2y^2 = c$

23) If the differential equation of family of curves $xy = c$ is $x\frac{dy}{dx} = -y$, then its family of orthogonal trajectories is given by

a) $x^2 - 2y^2 = c$ b) $x^2 + 2y^2 = c$

c) $x^2 - y^2 = c^2$ d) $x^2 + y^2 = c$

24) The orthogonal trajectories of family of curves having differential equation $x^2 + y^2 = k^2$ is $\frac{dy}{dx} = -\frac{x}{y}$, is given by

a) $x^2 = 4ay$ b) $x^2 - y^2 = c$

c) $y^2 = x + c$ d) $y = cx$

25) If the differential equation of family of curves $x^2 - y^2 = c$ is $y\frac{dy}{dx} = x$, then its family of orthogonal trajectories is given by

a) $y = cx$ b) $xy = c$

c) $x^2 = 4ay$ d) $y^2 = x + c$

26) The orthogonal trajectories of family of curves having differential equation $x^2 + 2y^2 = c^2$ is $\frac{dy}{dx} + \frac{x}{2y} = 0$, is given by

a) $x^2 - cx + c^2 = 0$ b) $y = 2cx^2 + x$

c) $x^2 = ky$ d) $y = 2cx^2$

27) The orthogonal trajectories of family of curves having differential equation $x^2 + cy^2 = 1$ is $\frac{dy}{dx} = -\frac{xy}{1-x^2}$, is given by

a) $\log x + x^2 + y^2 = c$ b) $\log x - x^2 - y^2 = c$

c) $\log x - \frac{x^2}{2} - \frac{y^2}{2} = c$ d) $\log x + \frac{x^2}{2} + \frac{y^2}{2} = c$

28) The orthogonal trajectories of family of curves having differential equation $y = 4ax^2$ is $\frac{dy}{dx} = \frac{y}{x}$, is given by

a) $2x^2 = cy^2$ b) $2x^2 - y^2 = c^2$

c) $2x^2 + y^2 = c$ d) $x^2 + 2y^2 = c$

29) If the differential equation of family of curves $y^2 = 4ax$ is $2x\frac{dy}{dx} = y$, then its family of orthogonal trajectories is given by

a) $2x^2 + y^2 = c$ b) $2x^2 - y^2 = c^2$

c) $x^2 + 2y^2 = c$ d) $2x^2 = cy^2$

30) The orthogonal trajectories of family of curves having differential equation $e^x + e^{-y} = e^c$ is $\frac{dy}{dx} = \frac{e^x}{e^{-y}}$, is given by

a) $e^{2x} + e^{-2y} = k$ b) $e^x - e^{-y} = k$

c) $e^x + e^{-y} = e^c$ d) $e^{-x} + e^y = e^c$

31) If the differential equation of family of curves $e^x - e^{-y} = c$ is $\frac{dy}{dx} + \frac{e^{-y}}{e^x} = 0$, then its family of orthogonal trajectories is given by

a) $e^x + e^{-y} = k$ b) $e^{-x} + e^y = e^c$

c) $e^x + e^{-y} = e^c$ d) $e^{2x} + e^{-2y} = k$

32) If the differential equation of family of curves $x^2 = ce^{x^2+y^2}$ is $\frac{dy}{dx} = \frac{1-x^2}{xy}$, then its family of orthogonal trajectories is given by

a) $\log\left(1-x^2\right) + 2\log y = c$

b) $\log\left(1-x^2\right) - 2\log y = c$

c) $2\log\left(1-x^2\right) - 3\log y = c$

d) $\log\left(1-x^2\right) + \log y = c$

33) The orthogonal trajectories of family of curves having differential equation $\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1$ is $x + \left(\frac{a^2-x^2}{y}\right)\frac{dy}{dx} = 0$, where a and b are fixed constants, is given by

a) $\frac{y^2}{2} = \lambda \log x + \frac{x^2}{2} + k$

b) $y^2 - x^2 = a^2 \log x + k$

c) $\frac{y^2}{2} = a^2 \log x - \frac{x^2}{2} + k$

d) $x^2 + y^2 = a^2 \log x + k$

34) If the differential equation of family of curves $r = a(1-\cos\theta)$ is $(1-\cos\theta)\frac{dr}{d\theta} = r$, then its family of orthogonal trajectories is given by

a) $r^2 = A(1+\cos\theta)$ b) $r = A(1+\sin\theta)$

c) $r = A(1-\cos\theta)$ d) $r = A(1+\cos\theta)$

35) If the differential equation of family of curves $r = a(1-\cos\theta)$ is $\frac{dr}{d\theta} = r\cot\frac{\theta}{2}$, then its family of orthogonal trajectories is given by

a) $\log\cos\left(\frac{\theta}{2}\right) = 2\log r + c$

b) $2\log\sin\left(\frac{\theta}{2}\right) = \frac{1}{2}\log r + c$

c) $2\log\cos\left(\frac{\theta}{2}\right) = \log r + c$

d) $\log 2\cos\left(\frac{\theta}{2}\right) = \log r + c$

36) The orthogonal trajectories of family of curves having differential equation $r = a\sin\theta$ is $\frac{dr}{d\theta} = r\cot\theta$, is given by

a) $r = A\cos\theta$ b) $r = A\tan\theta$

c) $r\cos\theta = A$ d) $r^2 = A\cos\theta$

37) The orthogonal trajectories of family of curves having differential equation $r = a\cos\theta$ is $\frac{dr}{d\theta} + r\tan\theta = 0$, is given by

a) $r = C\operatorname{cosec}2\theta$ b) $r^2 = C\sin^2\theta$

c) $r = C\tan\theta$ d) $r = C\sin\theta$

38) If the differential equation of family of curves $r^2 = a^2\cos 2\theta$ is $\frac{dr}{d\theta} + r\tan 2\theta = 0$, then its family of orthogonal trajectories is given by

a) $r^2 = c\sin^2 2\theta$ b) $r = c\sin 2\theta$

c) $r^2 = c^2\sin 2\theta$ d) $r^2 = c^2\cos 2\theta$

39) If the differential equation of family of curves $r^2 = a\sin 2\theta$ is $\frac{dr}{d\theta} = r\cot 2\theta$, then its family of orthogonal trajectories is given by

a) $r^2\cos 2\theta = k$ b) $r^2 = k\cos 2\theta$

c) $2\log r = \log\sec 2\theta + k$ d) $r^2 = k\cot 2\theta$

40) The orthogonal trajectories of family of curves having differential equation $r = a^2\cos^2\theta$ is $\frac{dr}{d\theta} + 2r\tan\theta = 0$, is given by

a) $\log\tan\theta = 2\log r + c$ b) $2\log\sin\theta = \log r + c$

c) $\frac{3}{2}\log\sin\theta = 2\log r + c$ d) $\frac{\log\sin\theta}{2} = \log r + c$

41) If the differential equation of family of curves $r = a\theta$ is $r = \theta\frac{dr}{d\theta}$, then its family of orthogonal trajectories is given by

a) $r = ce^{-\frac{\theta^2}{2}}$ b) $r = ce^{-\theta^2}$

c) $r^2 = ce^{-\frac{\theta^2}{2}}$ d) $r^2 = ce^{\theta^2}$

42) Newton's law of cooling states that

a) The temperature of the body is inversely proportional to the difference between the body temperature and the surrounding temperature.

b) The temperature of the body is proportional to the sum of the body temperature and the surrounding temperature.

c) The temperature of the body is proportional to the difference between the body temperature and the surrounding temperature.

d) The temperature of the body is proportional to the surrounding of the body temperature.

43) For $\theta =$ the temperature of the body and $\theta_0 =$ the temperature of the surrounding, then Newton's law of cooling states the differential equation

a) $\frac{d\theta}{dt} = -k\theta_0$ b) $\frac{d\theta}{dt} = -k\theta + \theta_0$

c) $\frac{d\theta}{dt} = -k(\theta + \theta_0)$ d) $\frac{d\theta}{dt} = -k(\theta - \theta_0)$

44) A body having initially temperature 90°C is kept in surrounding of temperature 26°C. Then the differential equation satisfied by body temperature θ at any time t is given by

a) $\frac{d\theta}{dt} = -k(\theta - 64)$ b) $\frac{d\theta}{dt} = -k(\theta - 26)$

c) $\frac{d\theta}{dt}=-k(\theta+26)$ d) $\frac{d\theta}{dt}=-k(\theta-90)$

45) Consider a substance at initial temperature 32°C is surrounded by room temperature 10°C. According to Newton's law of cooling the differential equation satisfied by its temperature T at time t hour is

a) $\frac{dT}{dt}=-kT(T-10)$ b) $\frac{dT}{dt}=-k(T-32)$
c) $\frac{dT}{dt}=-k(10-32T)$ d) $\frac{dT}{dt}=-k(T-10)$

46) A metallic object is heated up to getting temperature 100°C and the placed in water of temperature 50°C. Then the differential equation of the object temperature θ at time t is given by Newton's law of cooling as

a) $\frac{d\theta}{dt}=-k\theta(\theta-26)$ b) $\frac{d\theta}{dt}=-k(\theta-50)$
c) $\frac{d\theta}{dt}=-k(\theta-150)$ d) $\frac{d\theta}{dt}=-k(\theta+50)$

47) If a body originally at 120°C cools to 35°C in 40 minute in the air of constant temperature 45°C. Then according to Newton's law, its differential equation is given by

a) $\frac{d\theta}{dt}=-k(\theta-120)$ b) $\frac{d\theta}{dt}=-k(\theta-40)$
c) $\frac{d\theta}{dt}=-k(\theta-45)$ d) $\frac{d\theta}{dt}=-k(\theta-35)$

48) Assuming the temperature of the surrounding is being kept constant at 25°C and a body cools from temperature 80°C to 35°C in 45 minute. Then it must satisfy the differential equation

a) $\frac{dT}{dt}=-k(T-25)$ b) $\frac{dT}{dt}=-k(T-80)$
c) $\frac{dT}{dt}=-k(T-35)$ d) $\frac{dT}{dt}=-k(T+25)$

49) The rate of change of temperature of a body is proportional to the difference between the temperature of body and its surrounding nearby. If temperature of the air is 35°C and that of the body is 96°C and cools down to 55°C in just 25 minute. Then we must have

a) $\frac{dT}{dt}=-k(T+25)$ b) $\frac{dT}{dt}=-k(T-55)$
c) $\frac{dT}{dt}=-k(T-35)$ d) $\frac{dT}{dt}=-k(T-25)$

50) A metal ball is placed in the oven till it obtain temperature of 100°C and then at time t = 0, it is then placed in water of temperature 40°C. By Newton's law, if the temperature of the ball is decreased to 70°C in 10 minutes, then it must satisfy the differential equation

a) $\frac{dT}{dt}=-k(T-70)$ b) $\frac{dT}{dt}=-k(T-40)$
c) $\frac{dT}{dt}=-k(T-55)$ d) $\frac{dT}{dt}=-k(T-100)$

51) If a body of temperature T at time t kept in the surrounding of temperature T_0 satisfies the differential equation $\frac{dT}{dt}=-k(T-T_0)$, the relation between T and t is given as

a) $T=T_0-ke^{-kt}$ b) $T=T_0+ke^{-kt}$
c) $T=T_0+ke^{-kt}$ d) $T=-k(T_0-e^{-kt})$

52) A body is heated to a temperature of 100°C and then at time recording t = 0 it is then placed liquid of temperature 40°C. The temperature of the body is then reduced to 60°C in 4 minute. By Newton's law of cooling its differential equation is $\frac{d\theta}{dt}=-\frac{1}{4}(\theta-40)\log 3$. The time required to reduce the temperature of body to 50°C is

a) 5 min 6 sec b) 5.6 min
c) 65 min d) 6.5 min

53) A corpse of temperature 32°C is kept in the mortuary of constant temperature 10°C and the temperature of the corpse decreases to 20°C in 5 minutes. The differential equation of the system is given as $\frac{dT}{dt}=-0.05(T-10)$. Then T is

a) $T=22e^{-0.05t}$ b) $T=22+10e^{0.05t}$
c) $T=10-22e^{-0.05t}$ d) $T=10+22e^{-0.05t}$

54) A thermometer is taken outdoors of temperature 0ºC from a room of temperature 21ºC and the reading on the thermometer drops to 10ºC in 5 minutes and satisfies sufficiently the differential equation $\frac{dT}{dt} = -0.7419T$. What is its primitive?
a) $T = 21e^{-0.7419t}$ b) $T = 21 - 10e^{0.7419t}$
c) $T = 10 + 21e^{0.7419t}$ d) $T = 21e^{0.7419t}$

55) A metal body of mass 5 kg is heated to a temperature upto 100ºC exactly and then, at time considered to be t = 0, it is immersed in oil of temperature 30ºC. In just 3 minutes, the temperature of body drops to 70ºC in 3 minute and satisfies $\frac{d\theta}{dt} = -\frac{\theta - 30}{3}\log\left(\frac{7}{4}\right)$. What is time taken to drop temperature of body to 31ºC.
a) 15.28 min b) 12.78 min
c) 32.78 sec d) 22.78 min

56) If the temperature of body drops down to 70ºC from 100ºC in 15 minute, and satisfying the Newton's law of cooling $\frac{d\theta}{dt} = -k(\theta - 30)$, the value of k is
a) $\frac{1}{15}\log\frac{7}{4}$ b) $-\frac{1}{15}\log\frac{7}{4}$
c) $15\log\frac{7}{4}$ d) $-15\log\frac{7}{4}$

57) A metal ball of temperature 100ºC is placed in air conditioned room of temperature 20ºC. The temperature drops by 40ºC in 5 minute. Its differential equation in accordance with Newton's law of cooling is given by $\frac{dT}{dt} = -\frac{T - 20}{5}\log 2$. The temperature after 8 minute is
a) 6.44 b) 64.4 c) 46.4 d) 44.6

58) A body cools down from 80ºC to 60ºC from 1.00 PM to 1.20 PM in a room of temperature 40ºC and satisfies the differential equation $\frac{d\theta}{dt} = -0.03465(\theta - 40)$. The temperature of body at 1.40 PM is
a) 45 b) 50 c) 55 d) 60

59) The temperature of body cooling down from 100ºC to 60ºC in 60 seconds when it is kept in the air surrounding of constant temperature 20ºC and satisfies the equation $\frac{d\theta}{dt} = -k(\theta - 20)$. The value of k is then
a) $\log 2$ b) $\log 3$ c) $\log 4$ d) $\log 5$

60) A metal ball made by brass of mass 50 gm cools down from 80ºC to 60ºC after a recorded time of 20 minute in air atmosphere of 40ºC. The differential equation is $\frac{d\theta}{dt} = -k(\theta - 40)$. What is the value of k?
a) $-\frac{3}{20}\log_e 2$ b) $-20\log_e 2$
c) $\frac{1}{20}\log 2$ d) $-\frac{1}{20}\log_e 2$

61) A body of temperature 90ºC is placed in water of temperature 30ºC for 6 minute and then its temperature calculated is to be just 50ºC. The Newton's law of cooling is $\frac{d\theta}{dt} = -k(\theta - 40)$. Then what of followings is correct.
a) $k = \frac{1}{6}\log_e\frac{1}{3}$ b) $k = \frac{1}{6}\log_e 3$
c) $k = -\frac{1}{6}\log_e 2$ d) $k = -\frac{1}{6}\log_e\frac{1}{4}$

62) An iron ball is heated for temperature 100ºC is placed in water of temperature 50ºC at t = 0 and at t = 5 minute then its temperature calculated which is read to be 70ºC. The Newton's law of cooling is $\frac{d\theta}{dt} = -k(\theta - 50)$. Then what of followings is correct?
a) $k = -\frac{3}{4}\log_e\frac{2}{5}$ b) $k = \frac{1}{5}\log_e\frac{2}{5}$
c) $k = -\frac{2}{5}\log_e\frac{1}{5}$ d) $k = -\frac{1}{5}\log_e\frac{2}{5}$

63) A circuit consisting of resistance R, inductance L connected in series with voltage of amount E. By Kirchhoff's law, the differential equation for the current i in terms of t is
a) $L\frac{di}{dt} + \frac{i}{R} = E$ b) $L\frac{di}{dt} + Ri = E$

c) $L\frac{di}{dt}+Ri=0$ d) $R\frac{di}{dt}+Li=E$

64) A circuit consisting of resistance R, inductance L connected in series without voltage of amount E. By Kirchhoff's law, the differential equation for the current i in terms of t is

a) $L\frac{di}{dt}+\frac{i}{R}=E$ b) $L\frac{di}{dt}+Ri=E$

c) $L\frac{di}{dt}+Ri=0$ d) $R\frac{di}{dt}+Li=E$

65) An electrical circuit is consisting of inductance L, capacitance C in series with voltage source E. By Kirchhoff's law, we have

a) $L\frac{dq}{dt}+\frac{q}{C}=E$ b) $L\frac{dq}{dt}+\frac{q}{R}=E$

c) $C\frac{di}{dt}+\frac{i}{R}=E$ d) $\frac{di}{dt}+\frac{i}{C}=ER$

66) An electrical circuit is consisting of resistance R, capacitance C in series with voltage source E. By Kirchhoff's law, we have

a) $R\frac{dq}{dt}+\frac{q}{C}=E$ b) $L\frac{dq}{dt}+\frac{q}{R}=E$

c) $C\frac{di}{dt}+\frac{i}{R}=E$ d) $\frac{di}{dt}+\frac{i}{C}=ER$

67) A circuit consisting of resistance R, inductance L connected in series with voltage of amount $E\cos\omega t$. By Kirchhoff's law, the differential equation for the current i in terms of t is

a) $L\frac{di}{dt}+\frac{i}{R}=E\cos\omega t$ b) $L\frac{di}{dt}+Ri=E\cos\omega t$

c) $L\frac{di}{dt}+Ri=0$ d) $R\frac{di}{dt}+Li=E\cos\omega t$

68) The differential equation for the current i in an electrical circuit consisting of inductance L, resistance R in series with electromotive force of Ee^{-at} is given by

a) $\frac{di}{dt}+Ri=\frac{E}{L}e^{-at}$ b) $L\frac{di}{dt}+Ri=Ee^{-at}$

c) $L\frac{di}{dt}+\frac{i}{R}=Ee^{-at}$ d) $R\frac{di}{dt}+Li=Ee^{-at}$

69) The differential equation for the current i in an electrical circuit composing of resistance of 120 ohm and an inductance of 0.7 henry connected in series with battery of 30 volt is

a) $0.7\frac{di}{dt}-120i=30$ b) $120\frac{di}{dt}+0.7i=30$

c) $0.7\frac{di}{dt}+120i=30$ d) $0.7\frac{di}{dt}+\frac{i}{120}=30$

70) The differential equation for the current i in an electrical circuit composing of resistance of 200 ohm and an inductance of 100 henry connected in series with battery of 440 volt is

a) $20\frac{di}{dt}+10i=44$ b) $\frac{di}{dt}+2i=40$

c) $5\frac{di}{dt}+10i=44$ d) $10\frac{di}{dt}+20i=44$

71) A capacitance of 0.03 farad and resistance of 10 ohm in series with electromotive force of 20 volts are in a circuit. If initially the capacitor is totally discharged, the differential equation for the charge q is

a) $10\frac{dq}{dt}+\frac{q}{0.03}=20;\ q(0)=0$

b) $\frac{dq}{dt}+\frac{q}{0.03}=2;\ q(0)=0$

c) $\frac{dq}{dt}+\frac{q}{0.3}=2;\ q(0)=0$

d) $10\frac{dq}{dt}+0.03q=20;\ q(0)=0$

72) In an electrical circuit of R and L in series with steady EMF, the current i satisfies the equation $i=\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$. The time required for the maximum value is

a) 0 b) $\frac{L}{R}\log 10$

c) $-\frac{L}{R}\log 90$ d) $\frac{E}{R}\log 10$

73) In an electrical circuit of R and L in series with steady EMF, the current i satisfies the equation $i=\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$. The time required for the current gets 90% of maximum value is

a) 0 b) $\frac{L}{R}\log 2$

c) $-\frac{L}{R}\log 2$ d) $\frac{E}{R}\log 2$

74) If the differential equation for the current i is $R\frac{di}{dt}+Ri=E$, the current i at time t is

a) $i=\frac{E}{R}+ce^{-\frac{R}{L}t}$ b) $iR=1-cEe^{-\frac{R}{L}t}$

c) $i=\frac{E}{R}+ce^{\frac{R}{L}t}$ d) $i=\frac{E}{R}ce^{-\frac{R}{L}t}$

75) A charge q on the plate of condenser of capacity C through resistance R in series with steady state EMF V volt satisfies the differential equation $R\frac{dq}{dt}+\frac{q}{C}=V$. Then q in terms of t is

a) $q=\frac{C}{V}+ke^{-\frac{t}{RC}}$ b) $q=CV+ke^{\frac{t}{RC}}$

c) $q=CVke^{-\frac{t}{RC}}$ d) $q=CV+ke^{-\frac{t}{RC}}$

76) A charge q on the plate of condenser of capacity C through resistance R in series with steady state EMF V volt satisfies the equation $q=CV\left(1-e^{-\frac{t}{RC}}\right)$. Then i in terms of t is

a) $i=\frac{V}{R}e^{-\frac{t}{RC}}$ b) $i=\frac{V}{R}+e^{-\frac{t}{RC}}$

c) $i=VRe^{-\frac{t}{RC}}$ d) $i=\frac{V}{R}e^{\frac{t}{RC}}$

77) The differential equation for the current i is given to be $0.5\frac{di}{dt}+100i=20$ for an electrical circuit containing resistance R = 100 ohm, inductance L = 0.5 henry in series. Then

a) $i=0.2+Ae^{200t}$ b) $i=20+Ae^{-200t}$

c) $i=0.2Ae^{-200t}$ d) $i=0.2+Ae^{-200t}$

78) If an electrical circuit of R-C in series, charge $q=q(t)$ as function of t is $q=e^{3t}-e^{6t}$, the time required for maximum charge on capacitor is given by

a) $\frac{1}{2}\log 3$ b) $\frac{2}{3}\log 2$

c) $\frac{1}{3}\log 2$ d) $\frac{1}{3}\log\frac{1}{2}$

79) An electrical circuit of resistance R, inductance L in series with an electromotive force of E is satisfying the differential equation for the current i as $L\frac{di}{dt}+Ri=E$. For L = 640 henry, R = 250 ohm, E = 500 volt, the integrating factor of the above equation is

a) $e^{\frac{64}{25}t}$ b) $e^{\frac{25}{64}t}$ c) $e^{-\frac{25}{64}t}$ d) $e^{-\frac{64}{25}t}$

80) In an electrical circuit of L = 640 H, R = 250 Ω and E = 500 with EMF of 20 volts, the differential equation is

a) $\frac{di}{dt}+\frac{64}{25}i=\frac{32}{25}$ b) $\frac{di}{dt}+\frac{64}{25}i=\frac{25}{32}$

c) $\frac{di}{dt}+\frac{25}{64}i=\frac{25}{32}$ d) $\frac{di}{dt}+\frac{25}{64}i=\frac{32}{25}$

81) Rectilinear motion is the motion of body along

a) straight line b) circular motion

c) curvilinear d) parabolic path

82) The algebraic sum of the forces acting on a body along a given direction is equal to

a) mass × total force b) mass × distance

c) mass × velocity d) mass × acceleration

83) A particle moving in a straight line with acceleration $k\left(x+\frac{a^4}{x^3}\right)$ is directed towards origin. Then the equation of motion is

a) $\frac{dv}{dx}=-kv\left(x+\frac{a^4}{x^3}\right)$ b) $v\frac{dv}{dt}=-k\left(x+\frac{a^4}{x^3}\right)$

c) $\frac{d^2x}{dt^2}=-k\left(x+\frac{a^4}{x^3}\right)$ d) $k\frac{d^2x}{dt^2}=\left(x+\frac{a^4}{x^3}\right)$

84) A body of mass m kg moves in straight line with acceleration $\frac{k}{x^3}$ at a distance x and directed towards center. Then

a) $v\frac{dv}{dx}=-\frac{k}{x^3}$ b) $\frac{dv}{dx}=v\frac{k}{x^3}$

c) $v\frac{dv}{dx}=\frac{k}{x^3}$ d) $v\frac{dv}{dt}=-\frac{k}{x^3}$

85) A body of mass m falling freely from rest under gravitational force of attraction and air resistance proportional to square of velocity kv^2. Then

a) $\frac{dv}{dx}=v\left(mg-kv^2\right)$ b) $v\frac{dv}{dx}=m\left(g-kv^2\right)$

c) $mv\frac{dv}{dx}=mg-kv^2$ d) $v\frac{dv}{dx}=g-kv^2$

86) A particle is projected vertically upward with initial velocity v_1 and resistance of air produces retardation kv^2 where v is velocity at time t. Then

a) $mv\frac{dv}{dx}=mg-kv^2$ b) $v\frac{dv}{dx}=-g-kv^2$

c) $v\frac{dv}{dx}=m\left(g-kv^2\right)$ d) $v\frac{dv}{dx}=g-kv^2$

87) A particle starts moving horizontally from rest is opposed by a force cx, resistance per unit mass of value bv^2, where v and x are velocity and displacement of body at time t. Then

a) $v\frac{dv}{dx}=cs+bv^2$ b) $v\frac{dv}{dx}=-cs+bv^2$

c) $v\frac{dv}{dx}=cs-bv^2$ d) $v\frac{dv}{dx}=-cs-bv^2$

88) A body of mass m falls from rest under gravity in a liquid having resistance to motion at time t is mk times velocity. Then

a) $\frac{dv}{dt}=g+kv$ b) $\frac{dv}{dt}=g-kv$

c) $\frac{dv}{dt}=-g-kv$ d) $\frac{dv}{dt}=-g+kv$

89) A particle of mass m is projected vertically upward with velocity v, assuming the air resistance k times its velocity. Then

a) $m\frac{dv}{dt}=-mg-kv$ b) $m\frac{dv}{dt}=-mg+kv$

c) $m\frac{dv}{dt}=mg-kv$ d) $m\frac{dv}{dt}=mg+kv$

90) Assuming that the resistance to movement of a ship through water in the form of $a^2+b^2v^2$, where v is the velocity. Then the differential equation for retardation of the ship moving with engine stopped is

a) $m\frac{dv}{dt}=a^2+b^2v^2$ b) $m\frac{dv}{dt}=-a^2+b^2v^2$

c) $m\frac{dv}{dt}=-a^2-b^2v^2$ d) $m\frac{dv}{dt}=a^2-b^2v^2$

91) The differential equation of motion of particle of mass m falls from rest under gravity in a fluid satisfies the equation $\frac{dv}{dt}=g-kv$, then

a) $t=-k\log\left(\frac{g}{g-kv}\right)$ b) $t=k\log\left(\frac{g}{g-kv}\right)$

c) $t=-\frac{1}{k}\log\left(\frac{g}{g-kv}\right)$ d) $t=\frac{1}{k}\log\left(\frac{g}{g-kv}\right)$

92) A body of mass m falling freely under gravity satisfies the equation $mv\frac{dv}{dx}=k\left(a^2-v^2\right)$ with condition $ka^2=mg$, then

a) $x=\frac{m}{2k}\log\left(a^2-v^2\right)$ b) $x=\frac{m}{2}k\log\left(\frac{a^2}{a^2-v^2}\right)$

c) $x=-\frac{m}{2k}\log\left(\frac{a^2}{a^2-v^2}\right)$ d) $x=\frac{m}{2k}\log\left(\frac{a^2}{a^2-v^2}\right)$

93) A body starts from rest with an acceleration $\frac{dv}{dt}=k\left(1-\frac{t}{T}\right)$. Then its velocity is

a) $v=k\left(t-\frac{t^2}{2T}\right)$ b) $\frac{v^2}{2}=k\left(t-\frac{t^2}{2T}\right)$

c) $v=-k\left(t-\frac{t^2}{2T}\right)$ d) $v=k\left(\frac{t}{2}-\frac{t^2}{T}\right)$

94) A particle of unit mass starts from rest with an acceleration $v\frac{dv}{dr}=-\frac{k}{r^3}$. If initially it was at rest at r = a, then

a) $v^2=-k\left(\frac{1}{r^2}-\frac{1}{a^2}\right)$ b) $v^2=k\left(\frac{1}{r^2}+\frac{1}{a^2}\right)$

c) $v^2=k\left(\frac{1}{r^2}-\frac{1}{a^2}\right)$ d) $v^2=k\left(a^2-r^2\right)$

95) A particle of mass m is subjected projected upward with velocity V with its equation of motion $m\frac{dv}{dt}=-mg-kv$, then the velocity at time t is

a) $t=\log\left(\frac{mg+kv}{mg+kV}\right)$ b) $t=\frac{m}{k}\log\left(\frac{mg+kv}{mg+kV}\right)$

c) $t=-\frac{m}{k}\log\left(\frac{mg+kv}{mg+kV}\right)$ d) $t=\frac{m}{k}\log\left(\frac{mg-kv}{mg-kV}\right)$

96) A particle of mass m falls freely from rest under gravitational force in fluid producing resistance to motion of amount mkv, where k is constant. The differential equation is $\frac{dv}{dt} = g - kv$, then its terminal velocity is

a) $-\frac{g}{k}$ b) gk c) $-gk$ d) $\frac{g}{k}$

97) A bullet is fired into a sand tank and satisfies the differential equation $\frac{dv}{dt} = -k\sqrt{v}$. If v_0 is its initial velocity, we have

a) $2\sqrt{v} = -kt + 2\sqrt{v_0}$ b) $2\sqrt{v} = -\left(kt + 2\sqrt{v_0}\right)$
c) $2\sqrt{v} = kt + 2\sqrt{v_0}$ d) $\sqrt{v} = kt - 2\sqrt{v_0}$

98) A particle is in motion of horizontal straight line with acceleration $k\left(x + \frac{a^4}{x^3}\right)$ directed towards its origin and satisfies the differential equation $v\frac{dv}{dt} = -k\left(x + \frac{a^4}{x^3}\right)$. Assuming that it starts from rest at a distance x = a from origin, we have

a) $v^2 = -k\left(x^2 - \frac{a^4}{x^2}\right)$ b) $v^2 = k\left(x^2 + \frac{a^4}{x^2}\right)$
c) $v^2 = k\left(x^2 - \frac{a^4}{x^2}\right)$ d) $v^2 = -k\left(2x^2 - \frac{a^4}{2x^2}\right)$

99) If a particle moves in a straight line so that the force acting on it is directed towards a fixed point in the line of motion and proportional to its displacement from the point, it is then known as
a) curvilinear motion
b) rectilinear motion
c) Simple harmonic motion
d) circular motion

100) If a particle execute SHM, then its differential equation is given by

a) $\frac{d^2x}{dt^2} = -\omega^2 x$ b) $\frac{d^2x}{dt^2} - \omega^2 x = 0$
c) $\frac{d^2x}{dt^2} = k\omega x^2$ d) $\frac{d^2x}{dt^2} = -\omega x^2$

101) Fourier's law of heat conduction states that, the quantity of heat flow across the area of cross section A is
a) inversely proportional to the product of A with temperature gradient
b) proportional to the difference of A with temperature gradient
c) proportional to the product of A with temperature gradient
d) proportional to the sum of A and temperature gradient

102) If q quantity of heat flow across the cross sectional area A and thickness dx per unit time where the difference between temperatures at the faces is dT, the by Fourier's heat law

a) $q = -k - A\frac{dT}{dx}$ b) $q = -kA\frac{dT}{dx}$
c) $q = kA\frac{dT}{dx}$ d) $q = -kA + \frac{dT}{dx}$

103) The differential equation of steady state heat conduction per unit time from unit length of pipe of uniform radius r_0 carrying steam of temperature T_0 and thermal conductivity k, if the pipe is covered with material in a constant surrounding temperature, is given by

a) $Q = -\frac{2kr}{\pi}\cdot\frac{dT}{dr}$ b) $Q = -kr\frac{dT}{dr}$
c) $Q = 2k\pi r\frac{dT}{dr}$ d) $Q = -2k\pi r\frac{dT}{dr}$

104) The difference equation for steady state heat loss in unit time from a spherical shell of thermal conductivity covered by insulating material and kept in surrounding of constant temperature during heat flow, is

a) $Q = -\frac{4\pi r^2}{k}\cdot\frac{dT}{dr}$ b) $Q = 4k\pi r^2\frac{dT}{dr}$
c) $Q = -4k\pi r^2\frac{dT}{dr}$ d) $Q = -2k\pi r\frac{dT}{dr}$

105) The differential equation for steady state heat loss per unit time from unit length of pipe covered with insulating material which is kept in constant surrounding temperature, is

$Q=-2k\pi r\frac{dT}{dr}$. Then the temperature T is given by

a) $T=-\frac{Q}{k}\log r+c$ b) $T=-\frac{Q}{2\pi k}\log\frac{1}{r}+c$

c) $T=\frac{Q}{2\pi k}\log r+c$ d) $T=-\frac{Q}{2\pi k}\log r+c$

106) The differential equation for heat conductivity in spherical shell is described by $Q=-4k\pi r^2\frac{dT}{dr}$. Then

a) $T=\frac{Q}{kr}+c$ b) $T=\frac{Q}{4\pi kr}+c$

c) $T=\frac{Q}{4\pi k}r+c$ d) $T=-\frac{Q}{4\pi kr}+c$

107) A pipe of 10 cm radius carries steam of 150ºC and covered with insulating material of thickness 5 cm with thermal conductivity k = 0.0025 and it is kept in surrounding of temperature 40ºC. The equation is $dT=-\frac{Q}{2\pi k}\cdot\frac{dx}{x}$. Then the heat loss is

a) $220\pi k\log 1.5$ b) $\frac{220k}{\log 1.5}$

c) $\frac{220\pi k}{\log 1.5}$ d) $\frac{110\pi k}{\log 1.5}$

108) Heat is flowing through a hollow pipe of diameter 10 cm and outer diameter 20 cm and it is covered by insulating material of k = 0.12 and kept in surrounding of 200ºC. The differential equation is being $dT=-\frac{Q}{2\pi k}\cdot\frac{dx}{x}$. Then the heat loss is

a) $\frac{300\pi k}{\log 2}$ b) $\frac{150\pi k}{\log 2}$

c) $-\frac{300\pi k}{\log 2}$ d) $\frac{300\pi k}{\log 0.2}$

109) Steam of temperature 200ºC is set into pipe of 20 cm diameter covered with material of 6 cm thickness in surrounding of 30ºC. The equation is $dT=-\frac{Q}{2\pi k}\cdot\frac{dx}{x}$. The heat loss is

a) $\frac{170\pi k}{\log 16}$ b) $\frac{170(2\pi k)}{\log 1.6}$

c) $\frac{170\pi k}{\log 1.6}$ d) $-\frac{170\pi k}{\log 1.6}$

110) Steam of 100ºC is flowing through pipe of diameter 10 cm covered with asbestos of 5 cm thick and thermal conductivity k = 0.0006. The outer temperature is being 30ºC and the differential equation is $dT=-\frac{Q}{2\pi k}\cdot\frac{dx}{x}$. What is the amount of heat loss?

a) $\frac{140\pi k}{\log 2}$ b) $70\pi k\log 2$

c) $\frac{70\pi k}{\log 2}$ d) $-\frac{70\pi k}{\log 2}$

111) A tank contains 50 liters of fresh water. Brine of 2 gm/liter flows into the tank at the rate of 2 liters/minute and mixed with uniform continuity and the same amount runs out with the same rate. If Q is total amount of salt present at time t in the brine, we have

a) $\frac{dQ}{dt}=-4+\frac{Q}{25}$ b) $\frac{dQ}{dt}=-4-\frac{Q}{25}$

c) $\frac{dQ}{dt}=4-\frac{Q}{25}$ d) $\frac{dQ}{dt}=4+\frac{Q}{25}$

112) A tank contains 10000 liters of Brine of 200 kg dissolve salt. Fresh water flows into the tank at the rate of 100 liters/minute and mixed with uniform continuity and the same amount runs out with the same rate. If Q is total amount of salt present at time t in the brine, we have

a) $\frac{dQ}{dt}=200+\frac{Q}{100}$ b) $\frac{dQ}{dt}=-\frac{Q}{100}$

c) $\frac{dQ}{dt}=200-\frac{Q}{100}$ d) $\frac{dQ}{dt}=\frac{Q}{100}$

113) A tank contains 100 liters of fresh water. Brine of 1 gm/liter flows into the tank at the rate of 2 liters/minute and mixed with uniform continuity and the same amount runs out with the same rate. If Q is total amount of salt present at time t in the brine, we have

a) $\frac{dQ}{dt}=-\frac{Q}{100+t}$ b) $\frac{dQ}{dt}=2+\frac{Q}{100+t}$

c) $\frac{dQ}{dt}=2-\frac{Q}{100+t}$ d) $\frac{dQ}{dt}=2-\frac{Q}{100t}$

114) A tank contains 10000 liters of Brine of 20 kg dissolve salt. Brine of 0.1 kg/liter flows into the tank at the rate of 40 liters/minute and mixed with uniform continuity and the same amount runs out with the rate 30 liters/minute. If Q is total amount of salt present at time t in the brine, we have

a) $\frac{dQ}{dt}=4-\frac{3Q}{1000+10t}$ b) $\frac{dQ}{dt}=4-\frac{30Q}{100+t}$

c) $\frac{dQ}{dt}=-\frac{3Q}{100+t}$ d) $\frac{dQ}{dt}=4-\frac{3Q}{100+t}$

115) A tank contains 5000 liters of fresh water. Brine of 100 gm/liter flows into the tank at the rate of 10 liters/minute and mixed with uniform continuity and the same amount runs out with the same rate. If Q is total amount of salt present at time t in the brine, we have

a) $\frac{dQ}{dt}=\frac{5000-Q}{500}$ b) $\frac{dQ}{dt}=5000-\frac{Q}{500}$

c) $\frac{dQ}{dt}=1000+\frac{Q}{5}$ d) $\frac{dQ}{dt}=1000-\frac{Q}{500}$

116) A tank contains 10000 liters of Brine of 200 kg dissolve salt. Fresh water flows into the tank at the rate of 100 liters/minute and mixed with uniform continuity and the same amount runs out with the same rate. If Q is total amount of salt present at time t in the brine, we have $\frac{dQ}{dt}=-\frac{Q}{100}$. Then

a) $\log Q=-\frac{t}{100}$

b) $\log Q=-\frac{t}{100}-\log 200$

c) $\log Q=-\frac{t}{100}+\log 200$

d) $\log Q=\frac{t}{100}+\log 200$

117) A tank contains 50 liters of fresh water. Brine of 2 gm/liter flows into the tank at the rate of 2 liters/minute and mixed with uniform continuity and the same amount runs out with the same rate. If Q is total amount of salt present at time t, we have $\frac{dQ}{dt}=4-\frac{Q}{25}$. Then

a) $t=50\log 10-25\log(100-Q)$

b) $t=25\log 10-25\log(100-Q)$

c) $t=50\log 10+25\log(100-Q)$

d) $t=25\log 10+25\log(100-Q)$

118) The rate of decay of a substance is directly proportional to the amount of substance present at that time. Hence

a) $\frac{dt}{dx}=-kx$ b) $\frac{dx}{dt}=-kx$

c) $\frac{dx}{dt}=-kx+t$ d) $\frac{dx}{dt}=-kx^2+c$

Unit I : Ordinary Differential Equations

1	A	41	B	81	A	121	B	161	D	201	C	241	D
2	C	42	A	82	B	122	A	162	D	202	D	242	C
3	C	43	C	83	D	123	C	163	C	203	D	243	B
4	A	44	A	84	B	124	B	164	B	204	A	244	C
5	C	45	D	85	B	125	C	165	D	205	C	245	A
6	B	46	C	86	D	126	B	166	B	206	B	246	C
7	A	47	A	87	A	127	C	167	C	207	D	247	B
8	D	48	C	88	D	128	D	168	A	208	D	248	D
9	C	49	B	89	B	129	C	169	C	209	A	249	B
10	B	50	C	90	B	130	B	170	B	210	A	250	D
11	C	51	D	91	A	131	A	171	B	211	C	251	A
12	B	52	C	92	A	132	B	172	B	212	A	252	D
13	A	53	D	93	A	133	A	173	A	213	C	253	A
14	C	54	B	94	A	134	C	174	A	214	B	254	C
15	B	55	D	95	D	135	C	175	C	215	B	255	B
16	D	56	B	96	C	136	D	176	D	216	C	256	A
17	A	57	A	97	B	137	A	177	B	217	B	257	C
18	D	58	A	98	D	138	C	178	A	218	D	258	B
19	B	59	D	99	B	139	D	179	B	219	B	259	D
20	C	60	A	100	A	140	D	180	C	220	C	260	B
21	A	61	C	101	B	141	C	181	D	221	A	261	A
22	D	62	D	102	C	142	A	182	A	222	B	262	B
23	B	63	A	103	D	143	B	183	B	223	A	263	C
24	A	64	C	104	A	144	B	184	C	224	C		
25	D	65	B	105	B	145	D	185	A	225	B		
26	B	66	C	106	C	146	A	186	D	226	C		
27	D	67	B	107	A	147	D	187	A	227	D		
28	C	68	D	108	C	148	D	188	D	228	D		
29	A	69	C	109	D	149	C	189	C	229	C		
30	B	70	C	110	A	150	A	190	B	230	A		
31	A	71	A	111	D	151	C	191	C	231	D		
32	B	72	D	112	B	152	B	192	A	232	B		
33	B	73	D	113	D	153	A	193	D	233	A		
34	C	74	B	114	C	154	D	194	C	234	D		
35	B	75	B	115	A	155	A	195	A	235	C		
36	A	76	C	116	C	156	D	196	C	236	A		
37	A	77	D	117	D	157	C	197	B	237	D		
38	B	78	B	118	C	158	A	198	D	238	A		
39	C	79	D	119	B	159	B	199	B	239	D		
40	B	80	A	120	D	160	A	200	A	240	C		

Unit II : Applications of Ordinary Differential Equations

1	A	18	D	35	C	52	D	69	C	86	B	103	D
2	C	19	C	36	A	53	D	70	D	87	D	104	C
3	B	20	A	37	D	54	A	71	A	88	B	105	D
4	C	21	A	38	C	55	D	72	B	89	A	106	B
5	D	22	C	39	B	56	A	73	B	90	C	107	C
6	B	23	C	40	D	57	C	74	A	91	D	108	A
7	D	24	D	41	A	58	B	75	D	92	D	109	B
8	C	25	B	42	C	59	A	76	A	93	A	110	A
9	B	26	C	43	D	60	C	77	D	94	C	111	C
10	A	27	C	44	B	61	B	78	C	95	B	112	B
11	C	28	D	45	D	62	D	79	B	96	D	113	C
12	D	29	A	46	B	63	B	80	C	97	A	114	D
13	D	30	B	47	C	64	C	81	A	98	A	115	D
14	A	31	A	48	A	65	A	82	D	99	C	116	C
15	A	32	B	49	C	66	A	83	C	100	A	117	A
16	C	33	C	50	B	67	B	84	A	101	C	118	B
17	B	34	D	51	B	68	B	85	C	102	B		

Unit-4
Integral Calculus
Differentiation Under Integral Sign (DUIS)

1. Introduction.

- In addition to variables, additional parameters

$$I(\alpha) = \int_a^b f(x,\alpha)dx$$ $\Rightarrow$ α = Parameter , x = Variable.

Rule 1 : Integrals with constant limits.

$$If\ \ I(\alpha) = \int_a^b f(x,\alpha)dx \quad then \quad \boxed{\frac{d}{d\alpha}\int_a^b f(x,\alpha)dx = \int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)dx}$$

a & b constants $\Rightarrow$ LHS derivative $\Rightarrow$ Partial derivative RHS

LEIBNITZ RULE : Integrals with limits as Functions of the Parameter.

If a and b are functions of parameter α

$$i.e.\ I(\alpha) = \int_{a(\alpha)}^{b(\alpha)} f(x,\alpha)\,dx \ Then$$

$$\frac{d}{d\alpha}\int_a^b f(x,\alpha)\,dx = \int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)\,dx + f(b,\alpha).\frac{db}{d\alpha} - f(a,\alpha).\frac{da}{d\alpha}$$

1)	If $I(a) = \int_0^\infty \frac{e^{-x}}{x}(1 - e^{-ax})dx \quad (a > -1)$ then the value of $\frac{dI(a)}{da}$ is			
	a)	$1/(a+1)$	b)	$-1/(a+1)$
	c)	$\log(a+1)$	d)	0

2)	If $I(a) = \int_0^\infty \frac{\cos \lambda x}{x}(e^{-ax} - e^{-bx})dx; \quad a > 0, b > 0$ then			
	a)	$\frac{dI(a)}{da} + \int_0^\infty e^{-ax} \cos \lambda x \, dx = 0$	b)	$\frac{dI(a)}{da} - \int_0^\infty e^{-ax} \cos \lambda x \, dx = 0$
	c)	$\frac{dI(a)}{da} + \int_0^\infty e^{-ax} \sin \lambda x \, dx = 0$	d)	$\frac{dI(a)}{da} - \int_0^\infty e^{-ax} \sin \lambda x \, dx = 0$

3)	**The value of** $\frac{d}{db}\left[\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x}\,dx\right]$ **where** $a>0, b>0$, **is**			
	a)	$\int\limits_0^\infty \frac{be^{-bx}}{x}\,dx$	b)	$\int\limits_0^\infty \frac{-be^{-bx}}{x}\,dx$
	c)	$\int\limits_0^\infty e^{-ax}\,dx$	d)	$\boxed{\int\limits_0^\infty e^{-bx}\,dx}$

4)	**If** $I(a)=\int_a^{a^2} e^{ax^2}\,dx$ **then** $\frac{dI(a)}{da}=$			
	a)	$\int\limits_a^{a^2} x^2e^{ax^2}\,dx+e^{a^5}-e^{a^3}$	b)	$\int\limits_a^{a^2} 2axe^{ax^2}\,dx+2ae^{a^5}-e^{a^3}$
	c)	$\boxed{\int\limits_a^{a^2} x^2e^{ax^2}\,dx+2ae^{a^5}-e^{a^3}}$	d)	$\int\limits_a^{a^2} e^{ax^2}\,dx+e^{a^5}-2ae^{a^3}$

5)	If $II(aa) = \int_0^\infty \frac{e^{-ax}-e^{-bx}}{x} dx$ and $I'(a) = -\frac{1}{a}$ then the value of $I(a)$ is			
	a)	$\log a$	b)	$\log(a/b)$
	c)	$\log b$	d)	$\log(b/a)$

6)	If $\frac{dI}{da} = \frac{a}{a^2+1}$, then the value of integral $I(a) = \int_0^\infty \frac{e^{-x}-e^{-ax}}{x \sec x} dx$ is			
	a)	$\frac{1}{2}\log\left(\frac{a^2+1}{2}\right)$	b)	$\frac{1}{2}\log\left(\frac{a^2+1}{a}\right)$
	c)	$\log(a^2+1)$	d)	$-\log(a^2+1)$

7)	**If $I(a)=\int_0^{a^2}\tan^{-1}\left(\frac{x}{a}\right)dx$, then $\frac{dI}{da}$ is**			
	a)	$\int_0^{a^2}\frac{\partial}{\partial a}\tan^{-1}\left(\frac{x}{a}\right)dx+2a\tan^{-1}a$	b)	$\int_0^{a^2}\frac{\partial}{\partial a}\tan^{-1}\left(\frac{x}{a}\right)dx+a^2\tan^{-1}a$
	c)	$\int_0^{a^2}\frac{\partial}{\partial x}\tan^{-1}\left(\frac{x}{a}\right)dx+2a\tan^{-1}a$	d)	$\int_0^{a^2}\frac{\partial}{\partial a}\tan^{-1}\left(\frac{x}{a}\right)dx+a\tan^{-1}a^2$
8)	The value of $\frac{d}{da}\left[\int_a^{a^2}\frac{dx}{x+a}\right]$ is			
	a)	$\int_a^{a^2}\frac{dx}{(x+a)^2}+\frac{2}{a+1}+\frac{1}{2a}$	b)	$\int_a^{a^2}-\frac{dx}{(x+a)^2}+\frac{2}{a^2+a}-\frac{1}{2a}$
	c)	$\int_a^{a^2}-\frac{dx}{(x+a)^2}+\frac{2}{a+1}-\frac{1}{2a}$	d)	$\int_a^{a^2}-\frac{dx}{(x+a)^2}+\frac{2}{a^2+a}$

ERROR FUNCTION

Error function Of x or Probability Integral

$$erf\,(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2}\, du \qquad ----------------(1)$$

Complementary Error function Of x

$$erf_c\,(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-u^2}\, du \qquad ----------------(2)$$

Properties

(1) $\text{erf}\,(0) = 0$ Put $x = 0$ in (1) or (3)

(2) $\text{erf}\,(\infty) = 1$ Put $x = \infty$ in (3) and use the property $\Gamma \tfrac{1}{2} = \frac{1}{\sqrt{\pi}} \int_0^\infty e^{-t} t^{-1/2} dt = \sqrt{\pi}$

(3) $\text{erf}(x) + \text{erf}_c(x) = 1$

(4) $erf(x)$ is an odd function

===

1	**The definition of $\mathrm{erf}(\sqrt{t})$ is**			
	a)	$\frac{2}{\sqrt{\pi}}\int_0^t e^{-u^2}du$	b)	$\frac{2}{\sqrt{\pi}}\int_t^\infty e^{-u^2}du$
	c)	$\frac{2}{\sqrt{\pi}}\int_0^{\sqrt{t}} e^{-u^2}du$	d)	$\frac{2}{\sqrt{\pi}}\int_{\sqrt{t}}^\infty e^{-u^2}du$
2	The value of $\mathrm{erf}(\infty)$ is			
	a)	0	b)	1
	c)	∞	d)	-1
3	The value of $erfc(x) + erfc(-x)$ is			
	a)	1	b)	2
	c)	0	d)	-1

4)	**The value of $\mathrm{erf}(-\infty)$ is**			
	a)	0	b)	1
	c)	∞	d)	-1
5)	Error function is			
	a)	Even function	b)	Neither ever nor odd function
	c)	Odd function	d)	Constant function
6)	The value of $erfc(0)$ is			
	a)	0	b)	1
	c)	-1	d)	∞

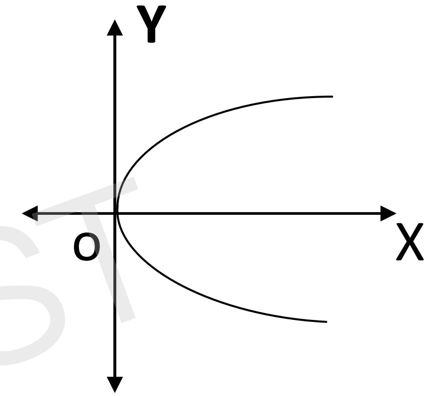
7)	**If $\alpha(x) = \sqrt{\frac{2}{\pi}} \int_0^x e^{-t^2/2} dt$ then the value of $\alpha(x\sqrt{2})$ is**			
	a)	$\operatorname{erf}(x\sqrt{2})$	b)	$-\operatorname{erf}(x)$
	c)	$\operatorname{erf}(2x)$	d)	$\boxed{\operatorname{erf}(x)}$
8)	The value of $\int_0^2 \operatorname{erf} c(x)\, dx + \int_0^2 \operatorname{erfc}(-x)\, dx$ is			
	a)	0	b)	$\boxed{4}$
	c)	2	d)	-2
9)	The value of $\frac{d}{dx} \operatorname{erf}(x)$ is			
	a)	$\boxed{\frac{2}{\sqrt{\pi}} e^{-tx^2}}$	b)	$\frac{2}{\sqrt{\pi}} e^{x^2}$
	c)	0	d)	$\frac{2}{\sqrt{\pi}} e^{-2x}$

10	**The value of $erfcerfc(-\infty)$ is**			
	a)	0	b)	1
	c)	-1	d)	2
11	The value of $erf(\infty) + erfc(-\infty)$ is			
	a)	3	b)	2
	c)	1	d)	0

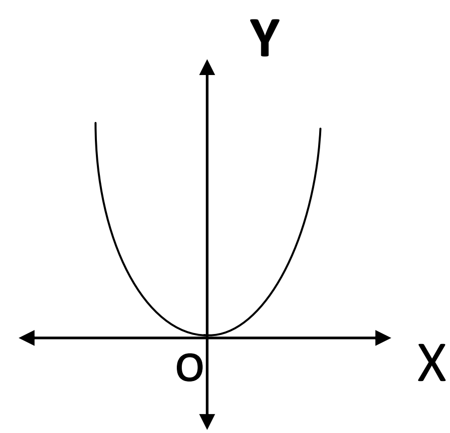
Rules For Tracing Of Cartesian Curves.

Rule 1 : Symmetry :

(a) Symmetry about X- axis: If equation of the curve remains unchanged by changing y to – y or all the powers of y in the equation are even. e.g. $y^2 = 4ax$.

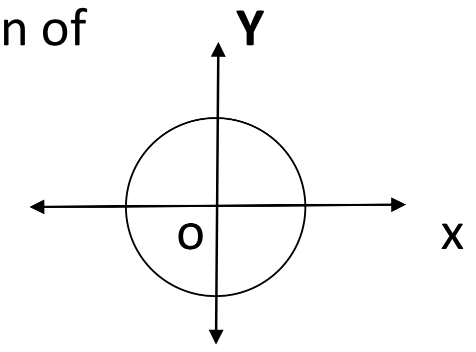


(b) Symmetry about Y- axis: If equation of the curve remains unchanged by changing x to – x **or** all the powers of x in the equation are even. e.g. $x^2 = 4ay$.



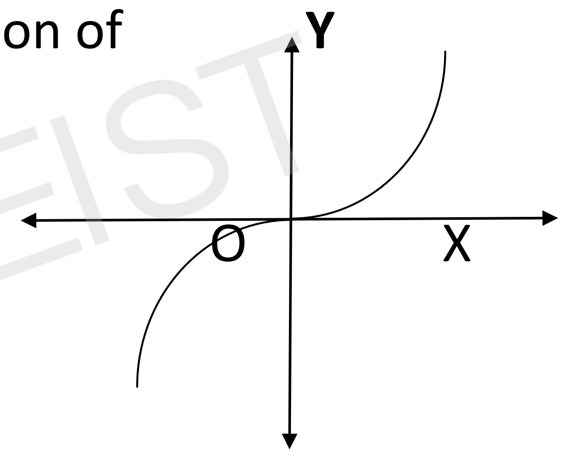
(c) Symmetry about both X and Y axes: If equation of the curve contains all even powers of x and y .

e.g. $x^2 + y^2 = r^2$.



(d) Symmetry in opposite quadrants: If equation of the curve remains unchanged by changing x to – x and y to – y simultaneously.

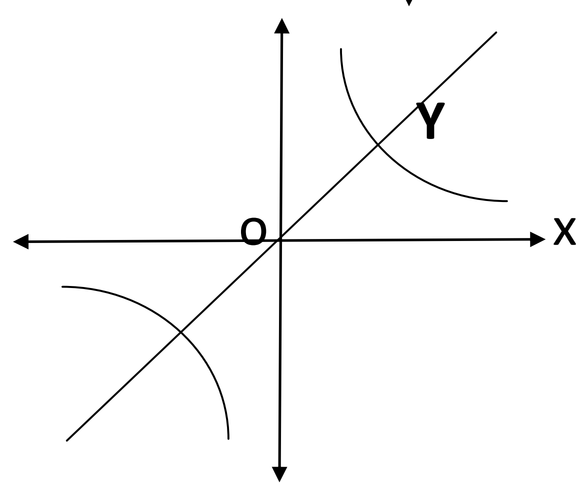
e.g. $y = x^3$



(e) Symmetry about the line y = x:

If equation of the curve remains unchanged by changing x to y and y to x .

e.g. $xy = c^2$

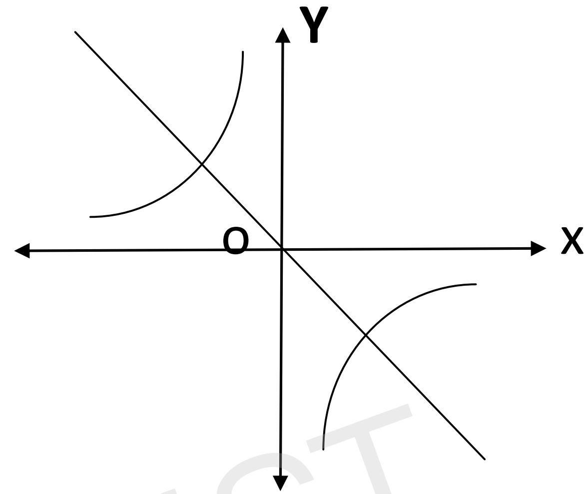


(e) Symmetry about the line y = - x :

If equation of the curve remains unchanged

by changing x to - y and y to - x

e.g. $xy = -c^2$



Rule 2 : Points Of Intersection :

(a) **Origin :** If the equation of the curve does not contain any arbitrary constant then the curve passes through origin.

(b) **Intersections with the co-ordinate axes :** If possible express the equation in the explicit form, y = f(x) or x = f(y).

Intersection with X-axis; put y = 0 and Intersection with Y-axis; put x = 0.

Find the tangents at these points, if necessary and position of the curve relative to these lines

(c) If a curve is symmetric about the line y = x or y = - x find the points of intersections of the curve with these lines and also the tangents at that point because ***tangent leads the curve.***

Rule 3 :Tangents:

(a) Tangents at the origin : If a curve is given by a rational integral algebraic equation and passes through origin : **the equation of the tangent or tangents at origin can be obtained by equating to zero, the lowest degree terms taken together in the equation of the curve.**

(b) Tangents at any other points : To find nature of tangent at any point P find $\frac{dy}{dx}$ at that point.

(i) If $\left(\frac{dy}{dx}\right)_P = 0 \Longrightarrow$ Tangent at P is parallel to X- axis.

(ii) If $\left(\frac{dy}{dx}\right)_P = \infty \Longrightarrow$ Tangent at P is parallel to Y- axis.

Rule 4 : Asymptotes : Asymptotes are tangents at infinity.

(a) Asymptotes parallel to X - axis are obtained by equating to zero the coefficients of highest degree term in x.

(b) Asymptotes parallel to Y - axis are obtained by equating to zero the coefficients of highest degree term in y.

(c) Oblique asymptotes: Asymptotes not parallel to co – ordinate axes are called oblique asymptotes. If curve is not symmetric about X or Y – axis then we check for oblique asymptotes. Equation of oblique asymptote can be obtained by two methods.

(i) **Method 1 :** Let $y = mx + c$ be the asymptote. To find m and c substitute this y in the given equation $f(x, y)$ so we get the

points of intersection with the curve i.e. $f(x, mx + c) = 0$.

Equating to zero two successive highest powers of x we find m and c.

Rule 5 : Region of absence of The curve :

(a) If possible express the equation in the explicit form, $y = f(x)$
And examine how y varies as x varies continuously.

(b) For $y = f(x)$, if y becomes imaginary for some value of $x > a$ (say)
Then no part of the curve exists beyond $x = a$.

(c) For $x = f(y)$, if x becomes imaginary for some value of $y > b$ (say)
Then no part of the curve exists beyond $y = b$.

Some Useful Remarks :

(a) When we have to solve for $y = f(x)$, put $x = 0$ see what is y.
Observe how y varies as x increases from 0 to $+\infty$ with special attention to the values of y for which $y = 0$ or $y \to +\infty$.
Also observe how y behaves as x becomes negative and $x \to -\infty$. with special attention for y becoming zero or $y \to -\infty$.

(b) If $y \to \infty$ as $x \to a$ then $x = a$ must be an asymptote ∥ to Y – axis.
If $x \to \infty$ as $y \to b$ then $y = b$ must be an asymptote ∥ to x – axis.

(c) If $y \to \infty$ as $x \to \infty$ and there is approximately linear relation between x and y for larger values of x, we may expect an oblique asymptote.

(d) If the curve is symmetric about X – axis or in the opposite quadrants then only positive values of y may be considered. We may draw the curve for negative values of y by symmetry.
Similarly, for symmetry about Y – axis only positive values of x may be considered.

Type 3 : Curves Given by Parametric Equations, x = f (t) , y = f(t)

Where t is a parameter.

Rules For Tracing parametric Curves.

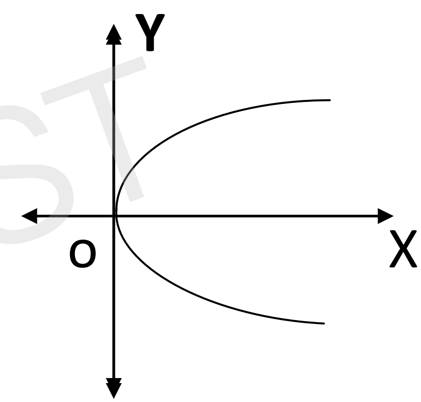
Rule 1 : Symmetry :

(a) Symmetry about X- axis: If equation of

X remains unchanged by changing ‘t’ to ‘ – t’

And y changes the sign then curve will be symmetric

About X – axis. e.g. x = at^2 , y = at i.e. $y^2 = 4ax$.Parabola.

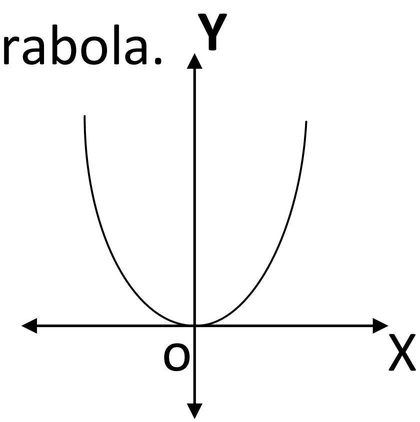


(b) Symmetry about Y- axis: If equation of

y remains unchanged by changing ‘t’ to ‘ – t’

And X changes the sign then curve will be symmetric

About Y – axis. e.g. x = , at y = at^2 i.e. $x^2 = 4ay$.Parabola.

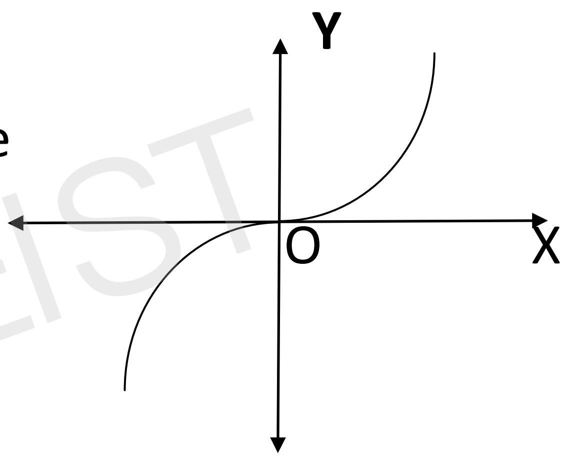


Note : For trigonometric equations if on replacing t to $\pi - t$, y remains unchanged and X changes the sign then also curve will be symmetric about Y – axis.

(c) Symmetry about origin: If on **replacing t by - t** if both x and y change the sign then curve is symmetric about otigin.

i.e. both x(t) and y(t)are odd functions of t.

e.g. x = t , y = t^2.



Rule 2 : Points Of Intersection :

1. If for some value of t both x and y become zero, then the curve passes through origin.
2. Find x and y intercepts if any.

Rule 3: Nature of tangents

1. $\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dx}\right)}$ **2. Form the table of values of t , x, y ,** $\frac{dy}{dx}$

Rule 4: Asymptotes and Region

1. Find asymptotes if any. **2.** Find region of absence.

Rules For Tracing Polar Curves.

Terminology : In polar curves

a) $\theta = 0$, positive $X-$ axix is called as ***Initial line***.

b) Equation of polar curves is often given by $r = f(\theta)$.

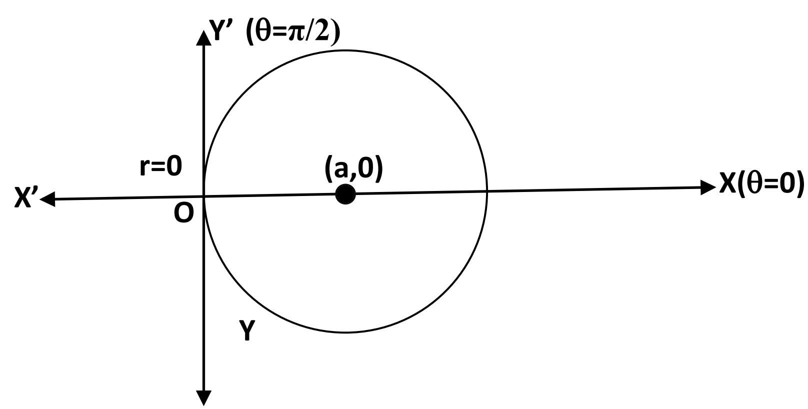
c) Origin O is called as Pole.

d) R is called as radius vector.

Symmetry :

a) If on changing θ to - θ ,equation of the curve remains unchanged then curve is symmetric about initial line (X - axis).

e.g. $x^2 + y^2 = 2ax$, Polar equation $r = 2a\cos\theta$



b)If on changing r to - r ,equation of the curve remains unchanged then curve is symmetric about the pole.

e.g. $(x^2 + y^2) = a^2(x^2 - y^2)$, Polar equation $r^2 = a^2 \cos 2\theta$

d) If on changing r to - r and θ to - θ,equation of the curve remains unchanged then curve is symmetric about Y - axis.

OR

If on changing θ to π- θ, equation of the curve remains unchanged then curve is symmetric about Y - axis.

e.g. (i) $(x^2 + y^2) = 2ay$, Polar equation $r = 2a \sin\theta$

(ii) r = (1+ $\sin\theta$) , First rule fails but second rule gives symmetry about Y – axis.

Pole : If for some values of θ ,r becomes zero then the pole lies on the curve.

e.g. (i) $(x^2 + y^2) = a^2(x^2 - y^2)$, Polar equation $r^2 = a^2 \cos 2\theta$

$$\theta = \frac{\pi}{4}, \quad r = 0 \quad \Longrightarrow \text{curve passes through the pole.}$$

Tangents at Pole : To find tangents at pole, put r = 0 in the equation, the value of θ gives tangent at the pole.

e.g. r = a sin3 θ , r = 0 $\Longrightarrow sin3\theta = 0$

$$3\theta = 0, \pi, 2\pi, 3\pi, \ldots \ldots .\ \theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \ldots \ldots \text{ are all tangents at pole.}$$

- **Prepare a table showing the values of r and .**
- **Find the angle between radius vector and the tangent (∅)**
- $\tan\emptyset = r\,\dfrac{d\theta}{dr} = \dfrac{r}{\left(\frac{dr}{d\theta}\right)}$, find the value of θ ,for which $\emptyset = 0$ *or* ∞
- The values of θ for which

$$\emptyset = 0, tangents\ coincide\ with\ radius\ vector.$$
$$\emptyset = \frac{\pi}{2}, tangents\ are\ perpendicular\ to\ radius\ vector.$$

ROSE CURVES.

If the polar equations are of the type

$$r = a \sin n\theta \qquad OR \qquad r = a \cos n\theta$$

Rules for tracing Rose curves :

1. Symmetry : (i) If $\theta \rightarrow -\theta$ and equation is not changed

$\Longrightarrow$ curve is symmetri about initial line.

(ii) If $\theta \rightarrow -\theta$ and $r \rightarrow -r$, and equation is not changed

$\Longrightarrow$ curve is symmetri aboutthe line $\theta = \frac{\pi}{2}$ through the pole perpedicular to initial line.

2.Since $|sin\ n\theta| \leq 1\ and\ |cos\ n\theta| \leq 1$, the maximum value of r is a

The rose curves lie in a circle of radius a.

3. Find in particular values of θ for which r = 0.

4. If the pole lies on the curve then find the equarion or equations of tangents at pole. Put r = 0 , values of θ give tangents at pole.

5. Since $\sin\theta$ and $\cos\theta$ are periodic functions of period 2π

Values of θ from 0 to 2π should only be considered. The values $\theta > 2\pi$

Do not give any new branch of the curve.

6. The curves r = a $\sin n\theta$ and $r = a\cos n\theta$ consists of

(i) n equal loops if n is odd.

(ii) 2n equal loops if n is even.

7. For drawing the loops of the curve r = a sin$n\theta$

(a) Divide each quadrant into 'n' equal parts.

(b) First loop is drawn along $\theta = \frac{\pi/2}{n}$.

(c) If n is even, draw loops in two sectors consecutively from $\theta = 0$ to $\theta = 2\pi$.

(d) If n is odd, draw loops in two sectors alternatively keeping two sectors between the loops vacant.

8. For drawing the loops of the curve r = a cos$n\theta$

(a) Divide each quadrant into 'n' equal parts.

(b) First loop is drawn along $\theta = 0$.

(c) If n is even, draw loops in two sectors consecutively from $\theta = 0$ to $\theta = 2\pi$.

(d) If n is odd, draw loops in two sectors alternatively keeping two sectors between the loops vacant.

Prepare the table of the values r and θ observe how r varies as θ increase from 0 to 2π

NOTE.

1. $sinn\theta = 0 \; for \; n\theta = 0, \pi, 2\pi, 3\pi, 4\pi \ldots\ldots\ldots\ldots\ldots$

   $$\therefore \; \theta = 0 \; , \; \frac{\pi}{n}, \frac{2\pi}{n}, \frac{3\pi}{n}, \frac{4\pi}{n} \ldots\ldots\ldots\ldots\ldots$$

2. $cosn\theta = 0 \; for \; n\theta = \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots\ldots\ldots\ldots\ldots$

   $$\therefore \; \theta = \frac{-\pi}{2n}, \frac{\pi}{2n}, \frac{3\pi}{2n}, \frac{5\pi}{2n} \ldots\ldots\ldots\ldots\ldots$$

1)	**A double point is Node if**			
	a)	Distinct branches have a common tangent	b)	Distinct branches have distinct tangent
	c)	Tangent at double point is above the curve	d)	Tangent at double point is below the curve
2)	A double point is Cusp if			
	a)	Two branches have distinct tangents	b)	Tangent line cuts the curve unusually
	c)	Two branches have a common tangent	d)	None of the above
3)	If all powers of y are even in the equation then curve is symmetrical about			
	a)	y −axis	b)	line $y = x$
	c)	x −axis	d)	line $y = -x$

4)	**If the equation of curve remains unchanged by replacing y by $-y$, then the curve is symmetric about**			
	a)	y −axis	b)	line $y = x$
	c)	x −axis	d)	line $y = -x$
5)	If all terms of x are of even degree in the equation of curve, then the curve is symmetric about			
	a)	y −axis	b)	line $y = x$
	c)	x −axis	d)	line $y = -x$

6)	**If the equation of curve does not contains any absolute constant term then the curve**			
	a)	Passes through origin	b)	Is increasing
	c)	Does not pass through origin	d)	Is decreasing
7)	If the curve passes through origin then the tangent to the curve at origin is obtained by			
	a)	Equating highest degree terms to zero	b)	Equating odd degree terms to zero
	c)	Equating even degree terms to zero	d)	Equating lowest degree terms to zero

8)	**Asymptotes are the tangents to the curve**			
	a)	At origin parallel to y −axis	b)	At origin not parallel to co-ordinate axis
	c)	At origin parallel to x −axis	d)	At infinity and are of the form $y = mx + c$
9)	Asymptotes parallel to x −axis are obtained by equating			
	a)	Coefficient of highest degree terms of y in the equation to zero	b)	Lowest degree terms to zero
	c)	Highest degree terms to zero	d)	Coefficient of highest degree terms of x in the equation to zero
10)	The parametric curve $x = f(t), y = g(t)$ is symmetric about x −axis if			
	a)	$f(t)$ is even and $g(t)$ is an odd function of t	b)	Both $f(t)$ and $g(t)$ are odd functions of t
	c)	$f(t)$ is an odd and $g(t)$ is even function of t	d)	Both $f(t)$ and $g(t)$ are even functions of t

11)	The curve $xy^2 = a^2(a - x)$ is symmetric about		
a)	x −axis	b)	line $y = x$
c)	y −axis	d)	line $y = -x$

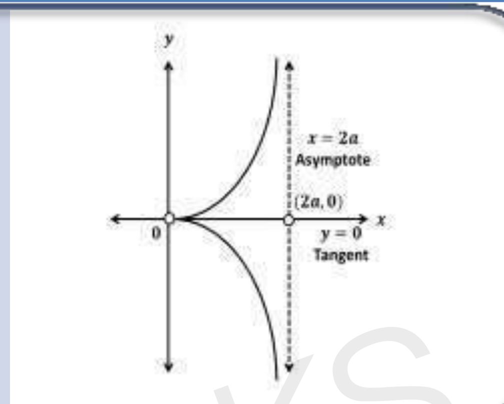
12)	The curve $xy^2 = a^2(a - x)$		
a)	passes through the point $(-a, 0)$	b)	does not pass through origin
c)	passes through the origin	d)	passes through the point (a, a)

13)	For the rose curve if is odd then the curve consist of $r = asinn\theta$ & $r = asinn\theta$		
a)	$2n$ equal loops	b)	$(n - 1)$equal loops
c)	$(n + 1)$equal loops	d)	n equal loops

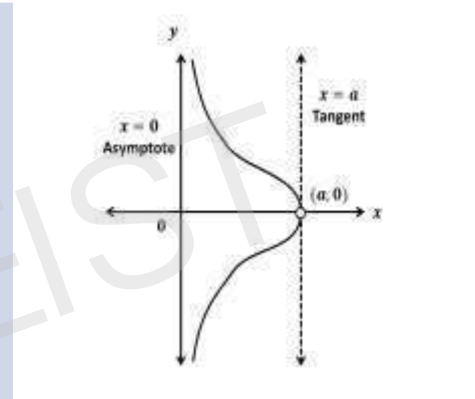
14)	The curve represented by the equation is symmetrical about		
a)	$y = -x$	b)	both x & y $axis$
c)	x $axis$ only	d)	$y = x$

15 The equation $y^2(2a - x) = x^3$ represents the curve

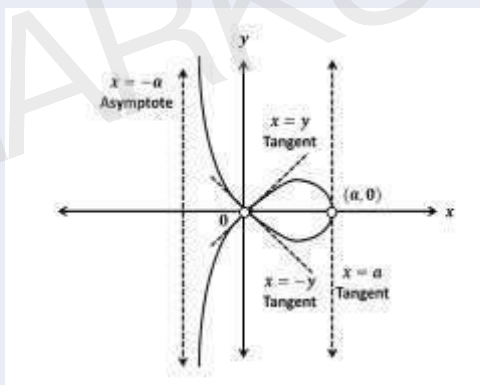
a)



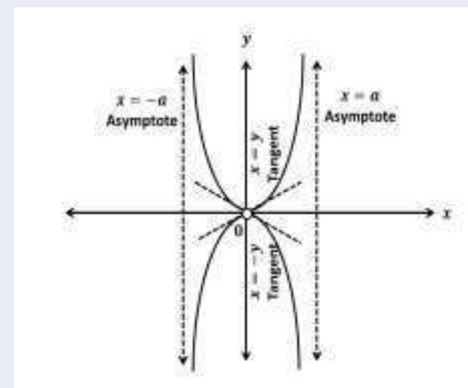
b)

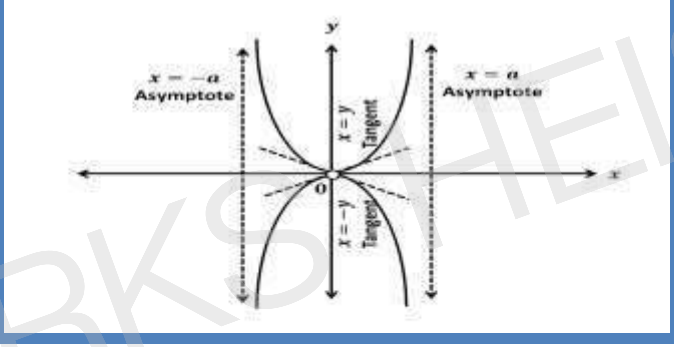


c)



d)



16)	The equation of curve represented in the following figure is 			
	a)	$xy^2 = a^2(a - x)$	b)	$y^2(2a - x) = x^3$
	c)	$x(x^2 + y^2) = a(x^2 - y^2)$	d)	$\boxed{x^2y^2 = a^2(y^2 - x^2)}$

UNIT – IV
Rectification Of Curves.

Definition : The process of determination of lengths of the plane curves whose equations are in Cartesian, Parametric and Polar forms is known as **Rectification of curves.**

If 's' is length of the curve from A to B then rectification formulae are

Equation.	ds	s
y = f (x)	$\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ dx	$\int_{x_1}^{x_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$
x = f (y)	$\sqrt{1+\left(\frac{dx}{dy}\right)^2}$ dy	$\int_{y_1}^{y_2} \sqrt{1+\left(\frac{dx}{dy}\right)^2}$ dy
$x = f_1(t)$ $y = f_2(t)$	$\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}$ dt	$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}$ dt

Equation.	ds	s
r = f (θ)	$\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\ d\theta$	$\int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\ d\theta$
θ = f (r)	$\sqrt{1 + r^2\left(\frac{d\theta}{dr}\right)^2}\ dr$	$\int_{r_1}^{r_2} \sqrt{1 + r^2\left(\frac{d\theta}{dr}\right)^2}\ dr$

-------------------------------- $\mathscr{R}$ --------------------------------

1)	The length of arc of upper part of loop of the curve $3y^2 = x(x-1)^2$ from $(0,0)$ to $(1,0)$ using $1+\left(\frac{dy}{dx}\right)^2 = \frac{(3x+1)^2}{12x}$, is			
	a)	$4/\sqrt{3}$	b)	$1/\sqrt{3}$
	c)	$\sqrt{3}$	d)	$\boxed{2/\sqrt{3}}$

2)	The length of upper half of the cardioide $r = a(1+\cos\theta)$ where θ varies from 0 to π using $r^2 + \left(\frac{dr}{d\theta}\right)^2 = 2a^2(1+\cos\theta)$ is			
	a)	a	b)	$2a$
	c)	$\boxed{4a}$	d)	$8a$

3)	The length of arc of the curve $x = e^{\theta}\cos\theta$, $y = e^{\theta}\sin\theta$, from $\theta = 0$ to $\theta = \pi/2$, using $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 2e^{2\theta}$ is			
	a)	$\sqrt{2}e^{\pi/2}$	b)	$\sqrt{2}(e^{\frac{\pi}{2}} + 1)$
	c)	$\boxed{\sqrt{2}(e^{\frac{\pi}{2}} - 1)}$	d)	$(e^{\frac{\pi}{2}} - 1)$

4)	For the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ the expression for $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2$ is			
	a)	$3a^2\sin^2\theta\cos^2\theta$	b)	$3a\sin^2\theta\cos^2\theta$
	c)	$3\,a\sin\theta\cos\theta$	d)	$\boxed{9a^2\sin^2\theta\cos^2\theta}$

5)	For the curve $ay^2 = x^3$, the expression for $1 + \left(\frac{dy}{dx}\right)^2$ is			
	a)	$9x/4a$	b)	$1 - (9x/4a)$
	c)	$\boxed{1 + (9x/4a)}$	d)	$4a + 9x$

6)	The total length of the loop of the curve $x = t^2,\ y = t\left(1 - \frac{t^2}{3}\right)$ if $ds^2 = (1 + t^2)^2$ and $0 < t < \sqrt{3}$ is			
	a)	4	b)	$\boxed{4\sqrt{3}}$
	c)	$\sqrt{3}$	d)	$4 + \sqrt{3}$

7)	The limits of θ for finding the perimeter of $r = a(1+\cos\theta)$ are		
a)	$0 < \theta < \pi$	b)	$\boxed{0 < \theta < 2\pi}$
c)	$0 < \theta < \pi/2$	d)	$0 < \theta < \pi/4$

Unit I
Differential Equations

<u>Order of a D.E.</u>

It is the highest order derivative appearing in the equation.

<u>Degree of a D. E.</u>

It is the degree of the highest ordered derivative when the derivatives are free from radicals.

Solution of a D.E.

It is a relation between the variables which satisfies the given D. E.

General Solution

It is a solution of a D.E. in which the number of arbitrary constants equals to the order of D.E.

Particular Solution

It is a solution of a D.E. obtained by assigning particular values to the arbitrary constants in general solution of D.E.

Ordinary D.E. of 1^{st} order and 1^{st} degree

It is the D.E. of the form

$$Mdx + Ndy = 0$$

where M and N are functions of x, y or constants

Depending upon the nature of M & N,we have 8 different types of solution of 1^{st} order , 1^{st} degree ordinary differential equation.

Methods of solving O.D.E. of 1^{st} order and 1^{st} degree

1. <u>Variable separable</u>

The given D.E. can be written as

$$f(x)dx = g(y)dy$$

G. S. is obtained by taking integration on both sides

$$\int f(x)dx = \int g(y)dy + C$$

2. D.E. reducible to variable separable by using substitution

Note certain terms in x and y namely $e^{xy}, e^{x/y}, e^{y/x}, \cos y/x, \cos(x+y), \sin(x-y)$ when appear in an equation lead to an identification as reducible to variable separable form.

3. Homogeneous D.E.

A D.E. $M\,dx+N\,dy=0$ is said to be homogeneous D.E. if M and N both are homogeneous functions of x and y of same degree.

Homogeneous D.E. can be reduce to variable separable form by substituting $\boldsymbol{y=ux}$

4. <u>Non-homogeneous D.E.</u>

The D.E. of the form $\dfrac{dy}{dx} = \dfrac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}$

is called non-homogeneous D.E.

If $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2}$ in these case the expression $a_1x + b_1y$ and $a_2x + b_2y$ have a common factor the equation can be reduce to variable separable form.

If $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$ then put $x = X + h$ and $y = Y + k$

Choose h and k such that the equation becomes homogeneous in X and Y.

5. <u>Exact D.E.</u>

A D.E. $M\,dx + N\,dy = 0$ is said to be exact if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

The G.S. of exact D.E. is given by

$$\int_{y=const} M\,dx + \int [terms\ of\ N\ not\ containing\ x]\ dy = C$$

6. D.E. Reducible to Exact Form By Using Integrating Factor.

If $M\ dx + N\ dy = 0$ is not exact then by multiplying the equation by function $k(x,y)$ called as Integrating Factor (I.F.) , the equation can be made exact.

Rules of finding I.F.

- If the given D.E. is homogeneous and $xM + yN \neq 0$ then $I.F. = \frac{1}{xM+yN}$
- If the given D.E. is of the form $yf(xy)dx + xg(xy)dy = 0$ and $xM - yN \neq 0$ then $I.F. = \frac{1}{xM-yN}$.
- If $\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N} = f(x)$ then $I.F. = e^{\int f(x)dx}$
- If $\frac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M} = g(y)$ then $I.F. = e^{\int g(y)dy}$
- If the given D.E. can be written as $x^a y^b(mydx + nxdy) + x^c y^d(pydx + qxdy) = 0$ then $I.F. = x^h y^k$, choose h, k such that condition of exactness is satisfied.

7. Linear D.E.

A D.E. of the form

$$\frac{dy}{dx} + Py = Q$$

where P, Q are functions of x or constants, is called a linear D.E. in y

$$I.F. = e^{\int P\,dx}$$

G.S. of linear D.E. is

$$ye^{\int Pdx} = \int Qe^{\int Pdx}dx + c$$

A D.E. of the form

$$\frac{dx}{dy} + Px = Q$$

where P, Q are functions of y or constants, is called a linear D.E. in x

$$I.F. = e^{\int P\,dy}$$

G.S. of linear D.E. is

$$xe^{\int Pdy} = \int Qe^{\int Pdy} dy + c$$

8. **Equation reducible to linear form**

The D.E. of the form

$$f'(y)\frac{dy}{dx}+P(x)f(y)=Q(x)$$

can be reduce to linear D.E. by substituting

$$f(y)=u, \quad \therefore f'(y)\frac{dy}{dx}=\frac{du}{dx}$$

$$\frac{du}{dx}+P(x)u=Q(x)$$

which is linear D.E. in u

$\therefore$ G.S. is $ue^{\int Pdx}=\int Qe^{\int Pdx}dx+c$

Similarly, the D.E. of the form

$$f'(x)\frac{dx}{dy}+P(y)f(x)=Q(y)$$

can be reduce to linear D.E. by substituting

$$f(x)=u, \quad \therefore f'(x)\frac{dx}{dy}=\frac{du}{dy}$$

$$\frac{du}{dy}+P(y)u=Q(y)$$

which is linear D.E. in u

Bernoulli's D.E.

A D.E. of the form

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

is called Bernoulli's D.E. in y

$Divide\ by\ y^n$

$$y^{-n}\frac{dy}{dx} + P(x)y^{1-n} = Q(x)$$

$Put \quad y^{1-n} = u\ and\ solve$

Similarly, a D.E. of the form

$$\frac{dx}{dy} + P(y)x = Q(y)x^n$$

is called Bernoulli's D.E. in x

Divide by x^n

$$x^{-n}\frac{dx}{dy} + P(y)x^{1-n} = Q(y)$$

Put $x^{1-n} = u$ *and solve*

- Variable Separable
- Reducible to variable separable by Substitution
- Homogeneous D.E.
- Non-homogeneous D.E.
- Exact D.E.
- Reducible to Exact D.E.
- Linear D.E.
- Reducible to Linear D.E.

Variable Separable $f(x)dx = g(y)dy$

Reducible to variable separable by Substitution

Homogeneous D.E. Put $y = ux$

Non-homogeneous D.E. $\frac{dy}{dx} = \frac{a_1 x + b_1 y + c_1}{a_2 x + b_2 y + c_2}$

$$\frac{dy}{dx} + Py = Q$$

Linear D.E.

Reducible to Linear D.E.

$$Mdx + Ndy = 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Exact D.E.

Reducible to Exact D.E.

The order and degree of the D.E $\left(1+\left(\frac{dy}{dx}\right)^3\right)^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ is

a) 2,3

b) 2,2

c) 2,1

d) 3,2

The order and degree of the D.E $\frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^2 + \int y\,dx = sin\mathrm{x}$ is

a) 4,1

b) 4,2

c) 2,2

d) None of these

The order and degree of the D.E $\left(\frac{dr}{dt}\right)^4 + \left(\frac{d^2r}{dt^2}\right)^3 + \left(\frac{d^3r}{dt^3}\right)^2 + \left(\frac{d^4r}{dt^4}\right)$=0

a) 1,4

b) 4,4

c) 4,1

d) 3,2

The order and degree of the D.E $\frac{dy}{dx} = \frac{ax+by+c}{3x+2by+5}\ is$

a) 1,0

b) 0,1

c) 1,1

d) None of these

The number of arbitrary constants in the general solution of ordinary differential equation is equal to

a) The order of D.E

b) The degree of D.E

c) Coefficient of highest order differential coefficient

d) None of these

The order of differential equation whose general solution is $y = \frac{c_1}{c_2} cos(4x + c_3) + c_4 e^{2x - c_5}$, where c_1, c_2, c_3, c_4, c_5 are arbitrary constants, is

a) 2

b) 3

c) 4

d) 5

The order of differential equation whose general solution is $c_1 y = c_2 + c_3 x + c_3 x^2$, where c_1, c_2, c_3 are arbitrary constants, is

a) 1

b) 2

c) 3

d) 4

The order of differential equation whose general solution is $c_1 y e^{x + c_2} = c_3 x e^{4x + c_4}$, where c_1, c_2, c_3, c_4 are arbitrary constants, is

a) 1

b) 2

c) 3

d) 4

The D.E whose general solution is $y = \sqrt{5x + C}$ where C is arbitrary constant, is

a) $2y\frac{dy}{dx} - 1 = 0$

b) $\boxed{2y\frac{dy}{dx} - 5 = 0}$

c) $\frac{dy}{dx} - \frac{5}{2}\frac{1}{\sqrt{5x + C}} = 0$

d) $y\frac{dy}{dx} - 5 = 0$

The D.E whose general solution is $y = Cx - C^2$, where c is arbitrary constant, is

a) $\frac{dy}{dx} = C$

b) $\left(\frac{dy}{dx}\right)^2 + xy = 0$

c) $\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} - y = 0$

d) $\boxed{\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0}$

The D.E whose general solution is $y = C^2 + \frac{C}{x}$, where C is arbitrary constant, is

a) $x^4y_1^2 + xy_1 - y = 0$

b) $\boxed{x^4y_1^2 - xy_1 - y = 0}$

c) $x^2y_1^2 - xy_1 - y = 0$

d) $y_1 = -\frac{c}{x^2}$

The D.E whose general solution is $y = A\cos(x+3)$, where A is arb. constant, is

a) $\boxed{\cot(x+3)y_1 + y = 0}$ b) $\tan(x+3)y_1 + y = 0$

c) $\cot(x+3)y_1 - y = 0$ d) $\tan(x+3)y_1 - y = 0$

The D.E representing the family of curves $y^2 = 2C\left(x+\sqrt{C}\right)$ where C is arbitrary constant, is

a) $2yy_1(x+\sqrt{yy_1}) - y^2 = 1$ b) $\boxed{2y_1(x+\sqrt{yy_1}) - y = 0}$

c) $y = 2y_1\left(x+\sqrt{C}\right)$ d) $y_1(x+\sqrt{yy_1}) - y = 0$

The solution of the D.E. $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ is

a) $\boxed{\tan^{-1}y = \tan^{-1}x + c}$ b) $\tan^{-1}x + \tan^{-1}y = c$

c) $y - x = c$ d) None of these

The D.E. representing family of curves $x^2 + y^2 = 2Ax$, where A is arb. constant, is

a) $$y_1 = \frac{y^2 + x^2}{2xy}$$

b) $$\boxed{y_1 = \frac{y^2 - x^2}{2xy}}$$

c) $$y_1 = \frac{y^2 + x^2}{2y}$$

d) $$y_1 = \frac{2xy}{y^2 + x^2}$$

The D.E. satisfied by G.S. $y = A\cos x + B\sin x$,where A,B are arb. constants, is

a) $$\frac{d^2y}{dx^2} + \frac{dy}{dx} = B\sin x$$

b) $$\frac{d^2y}{dx^2} - y = 0$$

c) $$\frac{d^2y}{dx^2} + \frac{dy}{dx} = A\cos x$$

d) $$\boxed{\frac{d^2y}{dx^2} + y = 0}$$

The D.E. satisfied by general solution

$y = A\cos(\log x) + B\sin(\log x)$, where A,B are arbitrary constants, is

a) $$x\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$$

b) $$\boxed{x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0}$$

c) $$x^2\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$$

d) $$x^2\frac{d^2y}{dx^2} + y = 0$$

The D.E. satisfied by general solution $y = Ae^{x} + Be^{-x}$,where A,B are arb const is

a) $\boxed{y_2 - y = 0}$

b) $y_2 + \mathrm{y} = 0$

c) $y_2 + \mathrm{y} = Ae^{x} + Be^{-x}$

d) $y_2 - y = 2Ae^{x}$

The D.E. satisfied by general solution $y^2 = 4A(x - B)$,where A,B are arb const, is

a) $y_2 + y_1^2 = 0$

b) $yy_2 + y_1 = 0$

c) $yy_2 - y_1^2 = 0$

d) $\boxed{yy_2 + y_1^2 = 0}$

The D.E. of family of circles having their center at $(A, 5)$ and radius 5,where A is arbitrary constant is

a) $(y-5)^2\left\{1 + \frac{dy}{dx}\right\} = 5$

b) $(y-5)^2\left\{1 - \left(\frac{dy}{dx}\right)^2\right\} = 25$

c) $\boxed{(y-5)^2\left\{1 + \left(\frac{dy}{dx}\right)^2\right\} = 25}$

d) None of these

The D.E. $\frac{dy}{dx} = e^{x-y} + 3x^2e^{-y}$ is of the form

a) Variable separable

b) Homogeneous

c) Linear

d) Exact

For solving D.E. $(x + y + 1)dx + (2x + 2y + 4)dy = 0$ appropriate substitution is

a) $x + y = 1$

b) $x + y = u$

c) $x - y = u$

d) None of these

The D.E. $\frac{dy}{dx} = \frac{x^3-3xy^2}{y^3-3x^2y}$ is of the form

a) Variable separable

b) Homogeneous

c) Linear

d) Exact

The D.E. $\frac{dy}{dx} = \frac{x+2y-3}{3x+6y-1}$ is of the form

a) Variable separable

b) Homogeneous

c) $\boxed{\text{Non-homogeneous}}$

d) Exact

The solution of D.E. is $\frac{dy}{dx} = e^{x-y} + 3x^2e^{-y}$

a) $\boxed{e^y = e^x + x^3 + C}$

b) $e^y = e^x + 3x^3 + C$

c) $e^y = e^x + 3x + C$

d) $e^x + e^y = 3x^3 + C$

The solution of D.E. $x^3\left(x\frac{dy}{dx} + y\right) - sec(xy) = 0$ is

a) $tan(xy) + \frac{1}{2x^2} = C$

b) $\boxed{sin(xy) + \frac{1}{2x^2} = C}$

c) $sin(xy) - \frac{1}{2x^2} = C$

d) $sin(xy) - \frac{1}{4x^2} = C$

The solution of D.E. $sec^2 x\, tan\, y\, dx + sec^2 y\, tan\, x\, dy = 0$ is

a) $\sec^2 x \; tan\, y = C$

b) $tan\, x\, sec^2 y = C$

c) $\boxed{tan\, x\, tan\, y = C}$

d) $sec^2 x\, sec^2 y = C$

The solution of D.E. $e^x cos\, y\, dx + (1 + e^x)\, sin\, y \; dy = 0$ is

a) $x(1 + e^x) = C \sec y$

b) $\boxed{(1 + e^x)\, sec\, y = C}$

c) $\frac{sec\, y}{(1+e^x)}$=C

d) $(1 + e^x)\, cos\, y = C$

The solution of D.E. $y(1 + log\, x)\frac{dx}{dy} - x\, log\, x = 0 \;\; is$

a) $log(x\, log\, x) = yC$

b) $\frac{x}{logx} = yC$

c) $y\, log\, x = xC$

d) $\boxed{x\, log\, x = yC}$

The solution of D.E. $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$

a) $tan^{-1} x + cot^{-1} y = C$

b) $\boxed{sin^{-1} x + sin^{-1} y = C}$

c) $sec^{-1} x + cosec^{-1} x = C$

d) $sin^{-1} x - sin^{-1} y = C$

The necessary and sufficient condition that the D.E M(x,y) dx + N(x,y) dy = 0 be exact is

a) $\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x};}$

b) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y};$

c) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x};$

d) $\frac{\partial M}{\partial x} - \frac{\partial N}{\partial y} = 1;$

If the integrating factor of differential equation $(x^2 + y^2 + 1)dx - 2xydy = 0$ is $\frac{1}{x^2}$ then its general solution is

a) $x - y = c$

b) $x^3 + 3y^2 = c$

c) $\boxed{x^2 - y^2 - 1 = cx}$

d) $x^2 + y^2 - 1 = cy$

If the I.F. of $(2x \log x - xy)dy + 2ydx = 0$ is $\frac{1}{x}$ then its general solution is

a) $x^2 \log y - \frac{y}{3} = c$

b) $\boxed{2y \log x - \frac{y^2}{2} = c}$

c) $2x^2 \log x - xy^2 = c$

d) $x \log y - x = c$

If the I.F. of $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)\, dy = 0$ is $\frac{1}{y^3}$ then its general solution is

a) $\boxed{\left(y + \frac{2}{y^2}\right) x + y^2 = c}$

b) $\left(1 + \frac{1}{y^2}\right) x + y = c$

c) $xy^4 - 2xy + x^2y^4 = 0$

d) $y^3 + 2xy - 2x^2 = c$

If the I.F. of $(x^7y^2 + 3y)dx + (3x^8y - x)dy = 0$ is $\frac{y}{x^7}$ then its general solution is

a) $x^3y + x^7y^4 = c$

b) $x^7y^3 - x^2 = cx^5$

c) $\boxed{xy^3 - \frac{y^2}{2x^6} = c}$

d) $xy + \frac{y^2}{x^7} = c$

Integrating factor for the differential equation $\frac{dy}{dx} + \frac{4x}{x^2+1}y = \frac{1}{(x^2+1)^3}$ is

a) x^2

b) $1 + x^2$

c) $(1 + x^2)^{-2}$

d) $(1 + x^2)^2$

If the integrating factor of differential equation $\frac{dx}{dy} + \mathrm{x}\, sec\, y = \frac{2y\, cos\, y}{1 + sin\, y}$ is $sec\, y + tan\, y$ then its general solution is

a) $(sec\, y + tan\, y)x = y^2 + c$

b) $x^2\, sec\, y + tan\, y = c$

c) $sec\, y + tan\, y = xy + c$

d) $sec\, y + x^2\, tan\, y = x^2 + c$

The differential equation $(1 + sin\, y)dx = (2y\, cos\, y - x\, sec\, y - x\, tan\, y)dy$ is

a) Homogeneous

b) Variable separable

c) Linear in x

d) None of these

The substitution for reducing non-homogeneous differential equation $\frac{dy}{dx} = \frac{2x-5y+3}{2x+4y-6}$ to homogeneous differential equation is

a) $x = X + 1, \quad y = Y - 3$

b) $x = X + 2, \quad y = Y + 2$

c) $x = X + 1, \quad y = Y + 1$

d) $x = X - 1, \quad y = Y + 2$

The integrating factor for the linear differential equation $\frac{dy}{dx} + \frac{y}{\sqrt{x}} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$ is

a) $e^{\sqrt{x}}$

b) $e^{\frac{1}{\sqrt{x}}}$

c) $e^{2\sqrt{x}}$

d) $e^{-\sqrt{x}}$

If homogeneous D.E. $\mathrm{M(x,y)\ dx\ +\ N(x,y)\ dy\ =\ 0}$is not exact then the integrating factor is

a) $\frac{1}{My+Nx}$ $My+Nx \neq 0$

b) $\frac{1}{Mx-Ny}$ $Mx-Ny \neq 0$

c) $\boxed{\frac{1}{Mx+Ny} \quad Mx+Ny \neq 0}$

d) $\frac{1}{My-Nx}$ $My-Nx \neq 0$

If the D.E. $\mathrm{M(x,y)\ dx\ +\ N(x,y)\ dy\ =\ 0}$ is not exat and it can be written as $yf_1(xy)dx + xf_2(xy)dy = 0$ then the I.F. is

a) $\frac{1}{My+Nx}$ $My+Nx \neq 0$

b) $\boxed{\frac{1}{Mx-Ny} \quad Mx-Ny \neq 0}$

c) $\frac{1}{Mx+Ny}$ $Mx+Ny \neq 0$

d) $\frac{1}{My-Nx}$ $My-Nx \neq 0$

If the D.E. $\mathrm{M(x,y)\ dx\ +\ N(x,y)\ dy\ =\ 0}$is not exact and $\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{M} = f(x)$ then the I.F. is

a) $e^{f(x)}$

b) $e^{\int f(x)dy}$

c) $f(x)$

d) $\boxed{e^{\int f(x)dx}}$

If the D.E. $\mathrm{M(x,y)\,dx\; + \; N(x,y)\,dy \; = \; 0}$is not exact and $\frac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M} = f(y)$ then the I.F. is

a) $e^{f(y)}$

b) $e^{\int f(y)dx}$

c) $f(y)$

d) $\boxed{e^{\int f(y)dy}}$

The D.E. $(x+y-2)dx+(x-y+4)dy=0$ is of the form

a) Exact

b) Homogeneous

c) Linear

d) None of these

The value of λ for which the D.E. $(xy^2+\lambda x^2y)dx+(x^3+x^2y)dy=0$ is exact is

a) -3

b) 2

c) 3

d) 1

The D.E. $(ay^2+x+x^8)dx+(y^8-y+bxy)dy=0$ is exact if

a) $b \neq 2a$

b) $b=a$

c) $a=1, b=3$

d) $b=2a$

The D.E.$(3 + by\cos x)dx + (2\sin x - 4y^3)dy = 0$ is exact if

a) $b = -2$

b) $b = 3$

c) $b = 0$

d) $b = 2$

I.F. of homogeneous D.E. $(xy - 2y^2)dx + (3xy - x^2)dy = 0$ is

a) $1/xy$

b) $1/x^2y^2$

c) $1/x^2y$

d) $1/x\ y^2$

I.F. of D.E. $(1 + xy)ydx + (x^2y^2 + xy + 1)xdy = 0$ is

a) $1/(x^2y)$

b) $1/x^3y^3$

c) $1/(xy^2)$

d) $1/x^2y^2$

I.F. of D.E. $(x^2 + y^2 + x)dx + (xy)dy = 0$ is

a) $1/x$

b) $1/x^2$

c) $\boxed{x^2}$

d) x

I.F. of D.E. $\left(y + \frac{y^3}{3} + \frac{x^2}{2}\right) dx + \left(\frac{x+xy^2}{4}\right) dy = 0$ is

a) x^2

b) $\boxed{x^3}$

c) $1/x$

d) $1/x^3$

I.F. of D.E. $(2x\, log\, x - xy)dy + (2y)dx = 0$ is

a) $\boxed{1/x}$

b) $1/x^2y^2$

c) $1/x^2$

d) 1/y

I.F. of D.E. $(2xy^2 + ye^x)dx - e^x dy = 0$

a) $1/x$

b) $1/y$

c) $1/x^2$

d) $\boxed{1/y^2}$

I.F. of D.E. $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$

a) $2/y$

b) $1/y$

c) $\boxed{\frac{1}{y^3}}$

d) $\frac{2}{y^2}$

Solution of non—exact D.E.$(x^2 - 3xy + 2y^2)dx + x(3x - 2y)dy = 0$

With I.F.$\frac{1}{x^3}$ is

a) $3\frac{y}{x} - \frac{y^2}{x^2} = C$

b) $log\, x - 3\frac{y}{x} + \frac{y^2}{x^2} = C$

c) $log\, x + 3\frac{y}{x} - 2\frac{y^2}{x^2} = C$

d) $\boxed{log\, x + 3\frac{y}{x} - \frac{y^2}{x^2} = C}$

Solution of non—exact D.E. $(3xy^2 - y^3)dx + (xy^2 - 2x^2y)dy = 0$

With I.F. $\frac{1}{x^2y^2}$ is

a) $3\ log\ x - \frac{2y}{x^3} - 2\ log\ y = C$

b) $3\ log\ x + \frac{y}{x} - 2\ log\ y = C$

c) $3\ log\ x + \frac{y}{x} = C$

d) $log\ x - \frac{y}{x} + 2\ log\ y = C$

Solution of non—exact D.E. $(x^4e^x - 2mxy^2)dx + (2mx^2y)dy = 0$

With I.F. $\frac{1}{x^4}$ is

a) $e^x + \frac{6my^2}{x^4} = C$

b) $e^x + \frac{2my^2}{x^2} = C$

c) $e^x + \frac{y^2}{x^2} = C$

d) $e^x + \frac{my^2}{x^2} = C$

The differential equation $\frac{dy}{dx} + P(x)y = Q(x)y^n$ is

a) Linear equation

b) Non-linear equation

c) Bernoulli's equation

d) None of these

The integrating factor for differential equation

$(1+y^2)\frac{dx}{dy}+x=e^{-\tan^{-1}y}$ is

a) $\frac{1}{1+y^2}$

b) $e^{\tan^{-1}x}$

c) $e^{\tan^{-1}y}$

d) None of these

The substitution for reducing non-homogeneous differential equation $\frac{dy}{dx}=\frac{-x-2y}{y-1}$ to homogeneous differential equation is

a) $x=X-1, \quad y=Y-3$

b) $x=X-2, \quad y=Y+1$

c) $x=X+1, \quad y=Y+1$

d) $x=X-1, \quad y=Y+2$

For what values of a and b, the differential equation $(y + x^3)dx + (ax + by^3)dy = 0$ is exact.

a) $b = 1, for\ all\ values\ of\ b$

b) $a = 2, b = 1$

c) $\boxed{a = 1, for\ all\ values\ of\ b}$

d) $a = -1, b = 3$

For what values of a, the differential equation $(ye^{xy} + ay^3)dx + (xe^{xy} + 12xy^2 - 2y)dy = 0$ is exact.

a) $a = 2$

b) $\boxed{a = 4}$

c) $a = 3$

d) $a = 1$

Unit II
Applications of Differential Equations

Orthogonal Trajectory

Method of finding the orthogonal trajectory of family of curves $F(x, y, c) = 0$ (1)

Obtain D.E. of (1) by eliminating the arbitrary constant c, resulting in

$$\frac{dy}{dx} = f(x, y) \qquad (2)$$

Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$ in (2) we get

$$-\frac{dx}{dy} = f(x, y) \qquad (3)$$

Solving (3) gives $\mathrm{G}(x, y, k) = 0$ which is the required orthogonal trajectory of (1)

Method of finding orthogonal trajectory of family of curves $F(r,\theta,c)=0\quad(1)$

Obtain D.E. of (1) by eliminating arb. const. c.

$$\frac{dr}{d\theta}=f(r,\theta)\qquad(2)$$

Replace $\frac{dr}{d\theta}$ by $-r^2\frac{d\theta}{dr}$ in (2)

$$\therefore -r^2\frac{d\theta}{dr}=f(r,\theta)\qquad(3)$$

Solving (3) gives $G(r,\theta,k)=0$ which is the required orthogonal trajectory.

Newton's law of Cooling

The rate at which the temperature of a body θ changes is proportional to the difference between the temperature of body and the temperature of the surrounding medium θ_0

$$\frac{d\theta}{dt} \propto \theta - \theta_0$$

$$\therefore \frac{d\theta}{dt} = -k(\theta - \theta_0)$$

Simple Electrical Circuits

If $\boldsymbol{q}$ is charge and $\boldsymbol{i = \frac{dq}{dt}}$ the current in a circuit at any time $\boldsymbol{t}$ then

Voltage drop across a **resistor** of resistance R is $\boldsymbol{Ri}$

Voltage drop across a **capacitor** of capacitance C is $\boldsymbol{\frac{q}{C}}$

and

Voltage drop across an **inductor** of inductance L is

$$\boldsymbol{L\frac{di}{dt} = L\frac{d^2q}{dt^2}}$$

Kirchhoff's Voltage law

The algebraic **sum** of all the **voltage drops** across the components of an electrical circuit is **equal to e.m.f.**

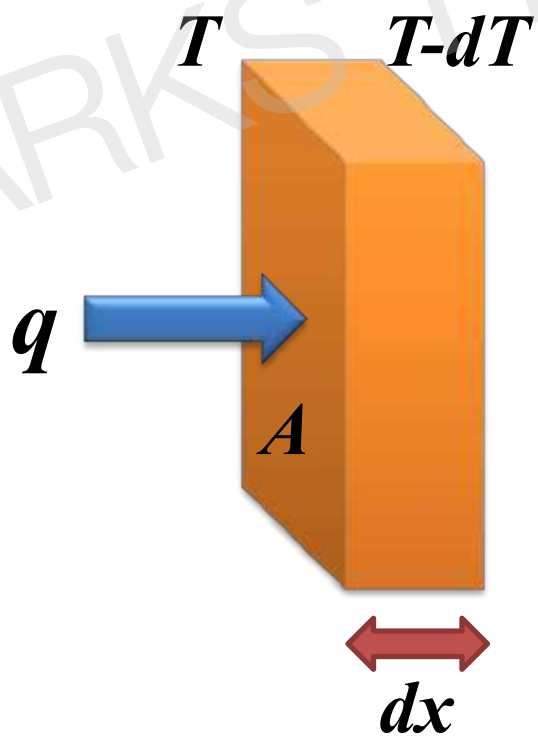
Heat Flow

Fourier's law of Heat conduction

The heat flowing across a surface is proportional to its surface area and to the rate of change of temp w.r.t. its distance normal to the surface.

If q **(cal/sec)** be the quantity of heat that flows across a slab of surface area A **cm²** and thickness dx in 1 sec where the difference of temp at the faces of the slab is dT and k coefficient of thermal conductivity then

$$q = -kA\frac{dT}{dx}$$



Law of natural decay

A rate of decay of a material is proportional to its amount present at that instant.

If m is amount of material at time t then

$$\frac{dm}{dt} = -km$$

Rectilinear Motion

Rectilinear motion (also called as linear motion) is **motion along a straight line.**

If x is displacement of a particle at time t then its

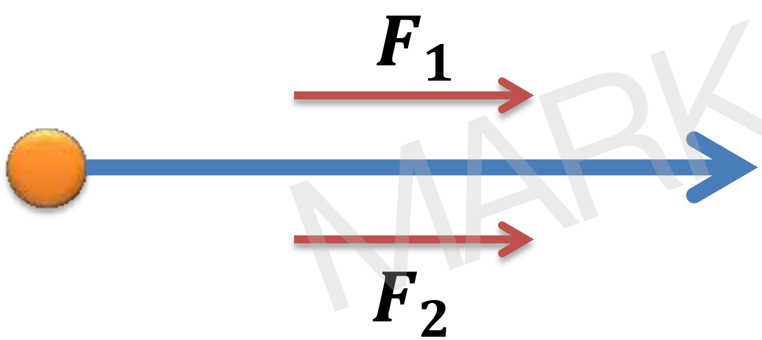
Velocity $v = \dfrac{dx}{dt}$

Acceleration $a = \dfrac{dv}{dt} = \dfrac{d^2x}{dt^2} = v\dfrac{dv}{dx}$

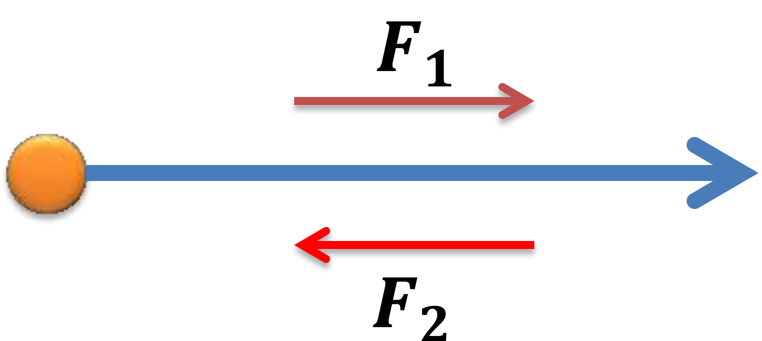
D'Alembert's principle

Algebraic sum of the forces acting on a body along a given direction is equal to the product of mass and acceleration in that direction.

Net force = Mass X Acceleration



Net force = $F_1 + F_2$



Net force = $F_1 - F_2$

S.H.M.

Equation of SHM is

$$Acceleration = \frac{d^2x}{dt^2} = -\omega^2 x$$

$$Period\ T = \frac{2\pi}{\omega}$$

	For finding orthogonal trajectory of $f(x, y, c) = 0$ we replace $\frac{dy}{dx}$ by				[01]
	a)	$-dx/dy$	b)	$-dy/dx$	
	c)	$2dx/dy$	d)	dy/dx	
	The orthogonal trajectory of $y = ax^2$ is				[02]
	a)	$x^2 + y^2 = c^2$	b)	$x^2 + (y^2/2) = c^2$	
	c)	$(x^2/2) + y^2 = c$	d)	None of these	
	The orthogonal trajectory of parabola is				[02]
	a)	Circle	b)	Hyperbola	
	c)	Ellipse	d)	Straight line	

	The orthogonal trajectory of the family of circles with centre at $(0,0)$ is a family of				**[02]**
	a)	Circles	b)	Straight lines through $(0,0)$	
	c)	any straight line	d)	Parabola	
	The DE for the orthogonal trajectory of the family of curves $x^2 + 2y^2 = c^2$ is				[01]
	a)	$x + 2y\dfrac{dy}{dx} = 0$	b)	$2\dfrac{dx}{x} = \dfrac{dy}{y}$	
	c)	$xdx + ydy = 0$	d)	$\dfrac{dx}{x} = \dfrac{dy}{y}$	
	The DE of orthogonal trajectory of the family of curves $r^2 = a\sin 2\theta$ is				[01]
	a)	$\dfrac{dr}{r} = -\tan 2\theta\, d\theta$	b)	$\dfrac{dr}{r} = \tan 2\theta\, d\theta$	
	c)	$dr = \tan 2\theta\, d\theta$	d)	None of these	

	The DE of orthogonal trajectory of the family of curves $r^2 = a^2 \cos 2\theta$ is				[01]
	a)	$r\frac{d\theta}{dr} = \tan 2\theta$	b)	$rdr = \tan 2\theta \, d\theta$	
	c)	$rdr = \cot 2\theta \, d\theta$	d)	$rdr + \tan\theta \, d\theta = 0$	
	If the DE of orthogonal trajectory of a curve is $r\frac{d\theta}{dr} + \cot(\theta/2) = 0$ then its orthogonal trajectory is				[01]
	a)	$r = \cos\theta$	b)	$r = c(1 - \sin\theta)$	
	c)	$r = c(1 - \cos\theta)$	d)	$r = b(1 + \cos\theta)$	
	If temperature of surrounding medium is θ_0 and temperature of body at any time t is θ, then in a process of heating $d\theta/dt$ is				[01]
	a)	$\theta - \theta_0$	b)	$k(\theta - \theta_0); \;\; k > 0$	
	c)	$-k(\theta - \theta_0); \;\; k > 0$	d)	None of these	

In certain data of newton's law of cooling, $-kt = \log\left(\frac{\theta-40}{60}\right)$ and at $t = 4, \theta = 60^0$, then the value of k is					**[02]**
a)	$\log(1/3)$	b)	$-\log(1/3)$		
c)	$4\log(1/3)$	d)	$(1/4)\log 3$		
If the temperature of water initially is 100^0C and $\theta_0 = 20^0C$, and water cools down to 60^0C in first 20 minutes with $k = \frac{1}{20}\log 2$, then during what time will it cool to 30^0C					[02]
a)	60 min	b)	50 min		
c)	1.5 hour	d)	40 min		
If a body originally at 80^0C, with $\theta_0 = 40^0C$ and $k = \frac{1}{20}\log 2$, then the temperature of body after 40 min is					[02]
a)	40^0C	b)	50^0C		
c)	80^0C	d)	30^0C		

If the body at 100^0C is placed in room whose temperature is 10^0C and cools to 60^0C in 5 minutes then the value of k is				**[02]**
a)	$\log 2$	b)	$-\log 2$	
c)	$(1/5)\log 2$ s	d)	$5\log 2$	
The linear form of DE for R-L series circuit with emf E is				[01]
a)	$L\frac{di}{dt} + Ri = E$	b)	$\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}$	
c)	$L\frac{di}{dt} + Ri = 0$	d)	none of these	
The integrating factor for the DE of R-L series circuit with emf E is				[02]
a)	$e^{\int Rdt}$	b)	e^{Rt+c}	
c)	$e^{\int \frac{R}{L}dt}$	d)	$e^{\int idt}$	

	If $i = \frac{E}{R} + ke^{-\frac{Rt}{L}}$ then the maximum value of i is				[01]
	a)	R/L	b)	E/R	
	c)	−E/R	d)	2R/L	
	The linear form of DE for R-C series circuit with emf E is				[01]
	a)	$Ri + \frac{q}{c} = E(t)$	b)	$Ri + \frac{1}{C}\int i\,dt = E$	
	c)	$R\frac{di}{dt} + \frac{i}{C} = \frac{dE}{dt}$	d)	$\frac{di}{dt} + \frac{i}{RC} = \frac{1}{R}\frac{dE}{dt}$	
	The integrating factor for the DE of R-C series circuit with emf E is				[01]
	a)	$e^{\int RC dt}$	b)	$e^{\int \frac{1}{RC} dt}$	
	c)	$e^{\int \frac{1}{R} dt}$	d)	$e^{\int \frac{1}{C} dt}$	

	If $i = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$ then the 50% of maximum current is				[01]
	a)	E/R	b)	E/2R	
	c)	2E/R	d)	2R/E	
	Which one of the following is not correct?				[01]
	a)	$F = ma$	b)	$F = m\frac{dv}{dt}$	
	c)	$F = m\,v\frac{dv}{dx}$	d)	$F = m\,v\frac{dv}{dt}$	
	A motion of a body or particle along straight line is known as				[01]
	a)	rectilinear motion	b)	curvilinear motion	
	c)	Motion	d)	None of these	

If a body of mass m falling from rest is subjected to the force of gravity and an air resistance proportional to the square of velocity kv^2, then the equation of motion is		**[01]**
a) $mv\frac{dv}{dx} = mg + kv^2$	b) $ma = -mg + kv^2$	
c) $ma = mg - kv^2$	d) None of these	

If a body opposed by force per unit mass of value cx and resistance per unit mass of value kv^2 then the equation of motion is		[01]
a) $a = cx - bv^2$	b) $a = bv^2 - cx$	
c) $v\frac{dv}{dx} = -cx - bv^2$	d) $v\frac{dv}{dx} = cx + bv^2$	

	The quantity of heat in a body is proportional to its				[01]
	a)	mass only	b)	temperature only	
	c)	mass and temperature	d)	none of these	

	The motion of a particle moving along a straight line is $\frac{d^2x}{dt^2} + 16x = 0$, then its period is				[01]
	a)	$2\pi/\sqrt{2}$	b)	$\pi/2$	
	c)	2π	d)	π	

	The orthogonal trajectories of the series of hyperbolas $xy = c^2$ is				[02]
	a)	$x^2 + y^2 = c^2$	b)	$x^2y^2 = c^2$	
	c)	$y^2 - x^2 = c^2$	d)	None of these	

	The differential equation of orthogonal trajectories of family of straight lines $y = mx$ is				**[01]**
	a)	$x\,dx - y\,dy = 0$	b)	$ydx - xdy = 0$	
	c)	$x\,dx + ydy = 0$	d)	$ydx + xdy = 0$	
	Let the population of country be decreasing at the rate proportional to its population. If the population has decreased to 25% in 10 years, how long will it take to half.				[02]
	a)	20 years	b)	8.3 years	
	c)	15 years	d)	5 years	
	The orthogonal trajectories of the family of straight lines $y = mx$ is				[01]
	a)	$x^2 - y^2 = c^2$	b)	$x^2 = my^2$	
	c)	$y^2 = m^2x^2$	d)	$x^2 + y^2 = c^2$	

	The set of orthogonal trajectories to a family of curves whose DE is $\phi\left(x, y, \frac{dy}{dx}\right) = 0$ is obtained by DE				**[01]**
	a)	$\phi\left(x, y, x\dfrac{dy}{dx}\right) = 0$	b)	$\phi\left(x, y, \dfrac{-dx}{dy}\right) = 0$	
	c)	$\phi\left(x, y, \dfrac{dy}{dx}\right) = 0$	d)	$\phi\left(x, y, \dfrac{-dy}{dx}\right) = 0$	
	The orthogonal trajectories of the family of curves $r\cos\theta = a$ is				[02]
	a)	$r\sin\theta = c$	b)	$r\tan\theta = c$	
	c)	$\dfrac{r}{\sin\theta} = c$	d)	None of these	
	If 10 grams of some radioactive substance reduces to 8 gm in 60 years, in how many years will 2 gm of it will be left ?				[02]
	a)	120 yrs	b)	378 yrs	
	c)	220 yrs	d)	433 yrs	

	Voltage drop across inductance L is given by				[01]
	a)	Li	b)	$L\frac{di}{dt}$	
	c)	$\frac{dL}{dt}$	d)	None of these	
	A ball at temperature of 32^0C is kept in a room where the temperature is 10^0C. If the ball cools to 27^0C in hour then its temperature is given by				[02]
	a)	$T = 22\ e^{0.205\ t}$	b)	$T = 10\ e^{1.163t}$	
	c)	$T = 10 + 22e^{-0.258t}$	d)	$T = 32 - 10e^{-0.093t}$	

Unit III

Fourier Series, Reduction Formulae, Gamma Functions, Beta Functions

Multiple Choice Questions

Periodic functions

A function $f(x)$ is said to be periodic if it is defined for all real x and if there is some positive number T such that

$$f(x+T)=f(x) \qquad \forall\, x$$

The number T is then called period of $f(x)$.

$\sin x, \cos x$ are periodic functions of period 2π

$\tan x\,, \cot x$ are periodic functions of period π

Fourier Series

If $f(x)$ is a periodic function of period 2π, defined in the interval $c \leq x \leq c + 2\pi$ then

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

this representation of $f(x)$ is called Fourier Series and the coefficients a_0, a_n, b_n are called the Fourier coefficients.

Euler's Formulae

$$a_0 = \frac{1}{\pi} \int_{c}^{c+2\pi} f(x)dx$$

$$a_n = \frac{1}{\pi} \int_{c}^{c+2\pi} f(x) \cos nx \, dx$$

$$b_n = \frac{1}{\pi} \int_{c}^{c+2\pi} f(x) \sin nx \, dx$$

1	If $f(x+nT)=f(x)$ where n is any integer then the fundamental period of $f(x)$ is			
	a)	$2T$	b)	$T/2$
	c)	$\boxed{T}$	d)	3T

2	If $f(x)$ is a periodic function with period T then $f(ax)$, $a \neq 0$ is periodic function with fundamental period			
	a)	T	b)	$\boxed{T/a}$
	c)	aT	d)	π

3	Fundamental period of $\cos 2x$ is		
a)	$\frac{\pi}{4}$	b)	$\frac{\pi}{2}$
c)	$\boxed{\pi}$	d)	2π

4	Fundamental period of $\tan 3x$ is		
a)	$\frac{\pi}{2}$	b)	$\boxed{\frac{\pi}{3}}$
c)	π	d)	$\frac{\pi}{4}$

5	The value of constant terms in the Fourier series of $f(x)=e^{-x}$ in $0\le x\le 2\pi,\ f(x+2\pi)=f(x)$ is				
	a)	$\frac{1}{\pi}(1-e^{-2\pi})$	b)	$\boxed{\frac{1}{2\pi}(1-e^{-2\pi})}$	
	c)	$2(1-e^{-2\pi})$	d)	$(1-e^{-2\pi})$	

6	Fourier coefficient a_0 in the Fourier series expansion of $f(x)=\left(\frac{\pi-x}{2}\right)^2$; $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$			
	a)	$\frac{\pi^2}{3}$	b)	$\boxed{\frac{\pi^2}{6}}$
	c)	0	d)	$\pi/6$

For function defined in the interval $-\pi \leq x \leq \pi$

- If $f(x)$ is even then

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{2}{\pi}\int_{0}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx = \frac{2}{\pi}\int_{0}^{\pi} f(x)\cos nx\, dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx = 0$$

For function defined in the interval $-\pi \leq x \leq \pi$

- If $f(x)$ is odd then

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx = \frac{2}{\pi}\int_{0}^{\pi} f(x)\sin nx\, dx$$

7	**Fourier series representation of periodic function** $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$ $f(x) = \frac{8}{\pi^2}\left[\frac{1}{1^2}\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x + \cdots\right]$ **then value of** $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots =$			
a)	$\frac{\pi^2}{4}$	b)	$\boxed{\frac{\pi^2}{8}}$
c)	$\frac{\pi^2}{16}$	d)	$\frac{8}{\pi^2}$

31 $f(x) = x, -\pi \le x \le \pi$ and period is 2π. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$, Fourier coefficient b_1 is

a) $\boxed{2}$ b) -1

c) 0 d) $2/\pi$

If $f(x)$ is a periodic function of period $2L$, defined in the interval $c \leq x \leq c+2L$ then

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$$

$$a_0 = \frac{1}{L}\int_{c}^{c+2L} f(x)dx \qquad a_n = \frac{1}{L}\int_{c}^{c+2L} f(x)\cos\frac{n\pi x}{L}dx$$

$$b_n = \frac{1}{L}\int_{c}^{c+2L} f(x)\sin\frac{n\pi x}{L}dx$$

If $f(x)$ is a periodic function of period $2L$, defined in the interval $-\mathrm{L} \leq x \leq L$ and

if $f(x)$ is an even function then

$$a_0 = \frac{2}{L}\int_0^L f(x)dx \qquad a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx \qquad b_n = 0$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L}\right)$$

if $f(x)$ is an odd function then

$$a_0 = 0 \qquad a_n = 0 \qquad b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$$

$$f(x) = \sum_{n=1}^{\infty}\left(b_n \sin\frac{n\pi x}{L}\right)$$

Half range expansions

- Half range cosine series: If $f(x)$ is defined in the interval $0 \leq x \leq L$, then the half range cosine series of $f(x)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$$

$$a_0 = \frac{2}{L}\int_0^L f(x)dx \qquad a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx$$

- Half range sine series: If $f(x)$ is defined in the interval $0 \leq x \leq L$, then the half range sine series of $f(x)$ is given by

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$$

$$b_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} \, dx$$

9	The Fourier constant a_n for $f(x) = 4 - x^2$ in the interval $0 < x < 2$ is			
	a)	$\boxed{4/\pi^2 n^2}$	b)	$2/n^2\pi^2$
	c)	$4/n^2\pi$	d)	$2/n\pi^2$

10	For half range sine series of $f(x) = x, 0 \le x \le 2$ and period is 4. Fourier series is represented by $\sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{2}$, then Fourier coefficient b_1 is			
	a)	4	b)	2
	c)	$\frac{2}{\pi}$	d)	$\boxed{\frac{4}{\pi}}$

		1st Harmonic		2nd Harmonic		3rd Harmonic	
x	y	$y\cos\frac{\pi x}{L}$	$y\sin\frac{\pi x}{L}$	$y\cos\frac{2\pi x}{L}$	$y\sin\frac{2\pi x}{L}$	$y\cos\frac{3\pi x}{L}$	$y\sin\frac{3\pi x}{L}$
x_0	y_0						
$\vdots$	$\vdots$						
x_{m-1}	y_{m-1}						
Σ							

$$a_0 = \frac{2}{m}\sum_{i=0}^{m-1} y_i \qquad a_n = \frac{2}{m}\sum_{i=0}^{m-1} y_i \cos\frac{n\pi x_i}{L} \qquad b_n = \frac{2}{m}\sum_{i=0}^{m-1} y_i \sin\frac{n\pi x_i}{L}$$

$$f(x) = \frac{a_0}{2} + a_1\cos\frac{\pi x}{L} + b_1\sin\frac{\pi x}{L} + a_2\cos\frac{2\pi x}{L} + b_2\sin\frac{2\pi x}{L}$$
$$+ a_3\cos\frac{3\pi x}{L} + b_3\sin\frac{3\pi x}{L} + \cdots$$

1. The term $\left[a_1 \cos\left(\frac{\pi x}{L}\right) + b_1 \sin\left(\frac{\pi x}{L}\right)\right]$ is called as **'Fundamental or First harmonic'.**

2. The term $\left[a_2 \cos\left(\frac{2\pi x}{L}\right) + b_2 \sin\left(\frac{2\pi x}{L}\right)\right]$ is called as **' second harmonic'** and so on.

3. The amplitude of n$^{\text{th}}$ harmonic is $+\sqrt{a_n^2 + b_n^2}$.

4. Percentage of n$^{\text{th}}$ harmonic =

$$\frac{\text{amplitude of n}^{\text{th}} \text{ harmonic}}{\text{amplitude of I}^{\text{st}} \text{ harmonic}} \times 100$$

11	**For the certain data if $a_0 = 1.5,\ a_1 = 0.373,\ b_1 = 1.004$ then the amplitude of 1st harmonic is**			
	a)	1.07	b)	2.07
	c)	1.004	d)	1.377

12	**The value of a_0 in harmonic analysis of y for the following tabulated data is**						
x°	0	60	120	180	240	300	360
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0
	a)	1.45	b)	5.8			
	c)	2.9	d)	2.48			

13	The value of a_1 in Harmonic analysis of y for the following tabulated data is :						
x	0	1	2	3	4	5	6
y	4	8	15	7	6	2	4
$\cos\frac{\pi x}{3}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1
	a)	-4.16	b)	-8.32			
	c)	-3.57	d)	$\boxed{-10.98}$			

14	The value of $a_1,\ a_2$ in Fourier cosine series of y for the following tabulated data are			
x	0	$\pi/4$	$\pi/2$	$3\pi/4$
y	0	$\sqrt{2}$	2	$\sqrt{2}$
	a)	$-1/2, 1/2$	b)	$\boxed{-1/2, -1/2}$
	c)	$2, -2$	d)	$-2, 0$

Reduction Formulae

1. $\int_0^{\pi/2} \cos^n x\, dx = \int_0^{\pi/2} \sin^n x\, dx$

$$= \frac{[\,(n-1)\text{subtract } 2 \ldots\ldots\ldots\ldots\ldots\ldots\ 2 \text{ or } 1\,]}{[\,(n)\ \text{subtract } 2 \ldots\ldots\ldots\ldots\ldots\ldots\ 2 \text{ or } 1]} \times \left(\frac{\pi}{2}\right) \text{ if n is even.}$$

$$= \frac{[\,(n-1)\text{subtract } 2 \ldots\ldots\ldots\ldots\ldots\ldots\ 2 \text{ or } 1\,]}{[\,(n)\ \text{subtract } 2 \ldots\ldots\ldots\ldots\ldots\ldots\ 2 \text{ or } 1]} \times 1 \text{ if n is odd.}$$

2.(a) $\int_0^{\pi/2} \sin^m x \cos^n x\, dx$

$$= \left\{\frac{[\,(m-1)\text{ subtract } 2 \ldots\ldots\ 2 \text{ or } 1\,].[\,(n-1)\text{ subtract } 2 \ldots\ldots\ 2 \text{ or } 1\,]}{[\,(m+n)\text{ subtract } 2 \ldots\ldots\ldots\ 2 \text{ or } 1\,]}\right\} \times \left(\frac{\pi}{2}\right)$$

If m and n both are even.

2.(b) $\int_0^{\pi/2} \sin^m x \cos^n x\, dx$

$$= \left\{\frac{[\,(m-1)\text{ subtract } 2 \ldots\ldots\ 2 \text{ or } 1\,].[\,(n-1)\text{ subtract } 2 \ldots\ldots\ 2 \text{ or } 1\,]}{[\,(m+n)\text{ subtract } 2 \ldots\ldots\ldots\ 2 \text{ or } 1\,]}\right\} \times (1)$$

Otherwise .

3] $\int_0^{\pi/2} \sin^m x \cos x \, dx = \int_0^{\pi/2} \cos^m x \sin x \, dx = \frac{1}{m+1}$

Conversion Formulae :

1] $\int_0^{2\pi} sin^m x \, cos^n x \, dx = \begin{cases} = 4\int_0^{\pi/2} sin^m x \, cos^n x \, dx \, , \boldsymbol{if\ m\ ,\ n\ even.} \\ = 0 \ , \qquad Otherwise. \end{cases}$

2] $\int_0^{\pi} sin^m x \, cos^n x \, dx$

$$= \begin{cases} = 2\int_0^{\pi/2} sin^m x \, cos^n x \, dx \, , \boldsymbol{if\ n\ even\ , for\ any\ m.} \\ = 0 \ , \qquad , \boldsymbol{if\ n\ odd\ , for\ any\ m.} \end{cases}$$

3] $\int_0^{2\pi} sin^n x \, dx = \begin{cases} = 4\int_0^{\pi/2} sin^n x \, dx \, , \boldsymbol{if\ n\ is\ even.} \\ = 0 \qquad , \boldsymbol{if\ n\ is\ odd.} \end{cases}$

4] $\int_0^{2\pi} cos^n\, x\, dx = \begin{cases} = 4\int_0^{\pi/2} cos^n\, x\, dx\ , \boldsymbol{if\ n\ is\ even.} \\ = 0 \qquad\qquad\qquad\quad , \boldsymbol{if\ n\ is\ odd.} \end{cases}$

5] $\int_0^{\pi} sin^n\, x\, dx = 2\int_0^{\pi/2} sin^n\, x\, dx\ , \boldsymbol{for\ any\ n}\ .$

6] $\int_0^{\pi} cos^n\, x\, dx = \begin{cases} = 2\int_0^{\pi/2} cos^n\, x\, dx\ , \boldsymbol{if\ n\ is\ even.} \\ = 0 \qquad\qquad\qquad\quad , \boldsymbol{if\ n\ is\ odd.} \end{cases}$

1	The value of the integral $\int_0^{\frac{\pi}{6}} \cos^6 3x \ dx$ is			
	a)	$5\pi/96$	b)	$7/48$
	c)	$5\pi/32$	d)	0

2	The value of $\int_{-\pi/2}^{\pi/2} \sin^4 x \, dx$ is			
	a)	$3\pi/16$	b)	$3\pi/8$
	c)	$3\pi/4$	d)	0

$$\int_0^{\pi/2} cos^n x \, dx = \int_0^{\pi/2} sin^n x \, dx \boxed{= \frac{n-1}{n} I_{n-2}}$$

3	**The value of the integral** $\int_0^{\pi/2} \sin^4 x \cos^3 x \; dx$ **is**			
	a)	$\pi/35$	b)	$\boxed{2/35}$
	c)	0	d)	$53/2$

4	**The value of** $\int_{-2\pi}^{2\pi} \sin^3 x \cos^2 x \, dx$ **is**			
	a)	$\boxed{0}$	b)	$\pi/4$
	c)	$\pi/16$	d)	$\pi/32$

5	The value of the integral $\int_0^{2\pi} \cos^5 x \; dx$ is				
	a)	$\boxed{0}$	b)	$5/16$	
	c)	$5/32$	d)	$5\pi/32$	

6	The value of the integral $\int_0^{\pi} \sin^5 x \; dx$ is				
	a)	$8\pi/15$	b)	$\pi/2$	
	c)	$\boxed{16/15}$	d)	0	

7	If $I_n = \int_{\pi/4}^{\pi/2} \cot^n x \, dx$ and $I_n = \frac{1}{n-1} - I_{n-2}$, then the value of I_6 is			
	a)	$\frac{13}{15}$	b)	$\frac{13}{15} + \frac{\pi}{4}$
	c)	$\boxed{\frac{13}{15} - \frac{\pi}{4}}$	d)	$\frac{13}{15} - \frac{\pi}{2}$
8	If $I_n = \int_0^{\pi/4} \sin^{2n} x \, dx$ and $I_n = \left(1 - \frac{1}{2n}\right) I_{n-1} - \frac{1}{n2^{n+1}}$, then the value of $\int_0^{\pi/4} \sin^4 x \, dx$ is			
	a)	$\frac{3\pi}{32} + \frac{1}{4}$	b)	$\boxed{\frac{3\pi}{32} - \frac{1}{4}}$
	c)	$\frac{\pi}{16} - \frac{1}{4}$	d)	$\frac{3\pi}{16} + \frac{1}{4}$

9	If $I_{m,n} = \int_0^{\pi/2} (\cos^m x)(\sin nx)dx$ and $I_{m,n} = \frac{1+m\, I_{m-1,n-1}}{m+n}$, then the value of $\int_0^{\pi/2} (\cos^2 x)(\sin 4x)dx$ is			
	a)	3	b)	2
	c)	$1/3$	d)	$2/3$

10	If $I_n = \int_0^{\pi/2} x \cdot \sin^n x \cdot dx$ and $I_n = \frac{n-1}{n} I_{n-2} + \frac{1}{n^2}$, then the value of $\int_0^{\pi/2} x \cdot \sin^4 x \cdot dx$			
	a)	$\frac{3\pi^2}{64} + \frac{1}{4}$	b)	$\frac{\pi^2}{64} + \frac{1}{4}$
	c)	$\frac{3\pi^2}{32} - \frac{1}{4}$	d)	$\frac{3\pi^2}{64} - \frac{1}{4}$

1. Gamma Function

Definition: The integral $\int_0^\infty e^{-x}x^{n-1}dx$ is called as Gamma function

and denoted by $\Gamma n = \int_0^\infty e^{-x}x^{n-1}dx$ (n > 0)

Properties :

$\boxed{1.\ \Gamma 1 = 1}$

2. Reduction formula : $\Gamma(n+1) = n\,\Gamma n$

$= n!$, if n is $+ve$ integer

3. $\Gamma 0 = \infty$

4. $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

5. $\Gamma P\ \Gamma(1-P) = \frac{\pi}{\sin p\pi}$

11	The value of the integral $\int_0^{\infty} \frac{x^5}{5^x} dx$ by using substitution $5^x = e^t$ is			
	a)	$120/(\log 5)^6$	b)	$24/(\log 4)^5$
	c)	$120/(\log 5)^5$	d)	$24/(\log 4)^4$

12	The value of the integral $\int_0^{1} \frac{dx}{\sqrt{x \log\left(\frac{1}{x}\right)}}$ by using the substitution $\log\left(\frac{1}{x}\right) = t$ is			
	a)	$\sqrt{\pi}/2$	b)	$\sqrt{2\pi}$
	c)	$\sqrt{\pi}$	d)	$2\sqrt{\pi}$

13	The formula for $\Gamma(n+1)$ is			
	a)	$\int_0^\infty e^{-x}x^{n-1}dx$	b)	$\int_0^\infty e^{-x}x^{n}dx$
	c)	$2\int_0^\infty e^{-x}x^{n-1}dx$	d)	$\int_0^\infty e^{-x}x^{n-2}dx$

14	The value of the integral $\int_0^\infty e^{-4x}x^3\,dx$ is			
	a)	$4!$	b)	$3!$
	c)	$\frac{3!}{64}$	d)	$\frac{3!}{256}$

15	The value of $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)$ is			
	a)	$2\pi/\sqrt{3}$	b)	$\pi/\sqrt{3}$
	c)	2π	d)	$2/\sqrt{3}$

16	The value of $\int_0^1 (\log x)^n \, dx$ is			
	a)	$(-1)^n\Gamma(\mathrm{n}+1)$	b)	$(\log n)\Gamma n$
	c)	Γn	d)	$\Gamma(n+1)$

Beta Function.

Definition : $\boxed{\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx}$; *where* m, n *are* +*ve integers*

Properties Of Beta Function.

1. $\beta(m,n) = \beta(n,m)$

2. $\beta(m,n) = \int_0^{\infty} \frac{y^{m-1}}{(1+y)^{m+n}}\,dy$

3. $\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta\,d\theta$

4. $$\int_0^{\pi/2} \sin^p\theta \cos^q\theta \, d\theta = \frac{1}{2}\beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$$

5. Relation Between Beta and Gamma Function.

$$\boxed{\beta\,(m\,, n) = \frac{\Gamma m \; \Gamma n}{\Gamma m+n}}$$

6. Legendre's duplication formula :

$$\boxed{\Gamma m \;\; \Gamma m + 1/2 \; = \; \frac{\sqrt{\pi}}{2^{2m-1}} \; \Gamma 2m}$$

17	Value of $B\left(\frac{3}{4},\frac{1}{4}\right)$ is			
	a)	2π	b)	$\pi\sqrt{2}$
	c)	$\pi/2$	d)	$\sqrt{2}$

18	Value of $\int_0^{\pi/2}\sqrt{\tan x}\,dx$ is			
	a)	$\frac{1}{2}B\left(\frac{3}{4},\frac{1}{4}\right)$	b)	$\frac{1}{2}B\left(\frac{3}{4},\frac{3}{4}\right)$
	c)	$B\left(\frac{3}{4},\frac{1}{4}\right)$	d)	$B\left(\frac{3}{4},\frac{3}{4}\right)$

19	If $B(n+1,1)=\frac{1}{4}$ and n is a positive integer then value of n is			
	a)	1	b)	2
	c)	3	d)	4

20	Value of $\int_0^{\pi/2}\sqrt{2\sin 2x}\,dx$ is			
	a)	$\frac{1}{2}B\left(\frac{3}{4},\frac{1}{4}\right)$	b)	$\frac{1}{2}B\left(\frac{3}{4},\frac{3}{4}\right)$
	c)	$B\left(\frac{3}{4},\frac{1}{4}\right)$	d)	$B\left(\frac{3}{4},\frac{3}{4}\right)$

21	The value of $\int_0^\infty \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}} dx$ is			
	a)	0	b)	$\frac{B(m,n)}{2}$
	c)	$2B(m,n)$	d)	1

22	By Duplication formula, the value of $\Gamma m \cdot \Gamma(m+\frac{1}{2})$ is			
	a)	$\frac{\sqrt{\pi}}{2^{m-1}}\Gamma(2m)$	b)	$\frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(m)$
	c)	$\frac{\sqrt{\pi}}{2^{m}}\Gamma(2m)$	d)	$\frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$

Unit II
Applications of Differential Equations

Orthogonal Trajectory

Method of finding the orthogonal trajectory of family of curves $F(x, y, c) = 0$ (1)

Obtain D.E. of (1) by eliminating the arbitrary constant c, resulting in

$$\frac{dy}{dx} = f(x, y) \qquad (2)$$

Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$ in (2) we get

$$-\frac{dx}{dy} = f(x, y) \qquad (3)$$

Solving (3) gives $\mathrm{G}(x, y, k) = 0$ which is the required orthogonal trajectory of (1)

Method of finding orthogonal trajectory of family of curves $F(r,\theta,c)=0 \quad (1)$

Obtain D.E. of (1) by eliminating arb. const. c.

$$\frac{dr}{d\theta}=f(r,\theta) \quad (2)$$

Replace $\frac{dr}{d\theta}$ by $-r^2\frac{d\theta}{dr}$ in (2)

$$\therefore -r^2\frac{d\theta}{dr}=f(r,\theta) \quad (3)$$

Solving (3) gives $G(r,\theta,k)=0$ which is the required orthogonal trajectory.

Newton's law of Cooling

The rate at which the temperature of a body θ changes is proportional to the difference between the temperature of body and the temperature of the surrounding medium θ_0

$$\frac{d\theta}{dt} \propto \theta - \theta_0$$

$$\therefore \frac{d\theta}{dt} = -k(\theta - \theta_0)$$

Simple Electrical Circuits

If $\boldsymbol{q}$ is charge and $\boldsymbol{i = \frac{dq}{dt}}$ the current in a circuit at any time $\boldsymbol{t}$ then

Voltage drop across a **resistor** of resistance R is $\boldsymbol{Ri}$

Voltage drop across a **capacitor** of capacitance C is $\boldsymbol{\frac{q}{C}}$

and

Voltage drop across an **inductor** of inductance L is

$$\boldsymbol{L\frac{di}{dt} = L\frac{d^2q}{dt^2}}$$

Kirchhoff's Voltage law

The algebraic **sum** of all the **voltage drops** across the components of an electrical circuit is **equal to e.m.f.**

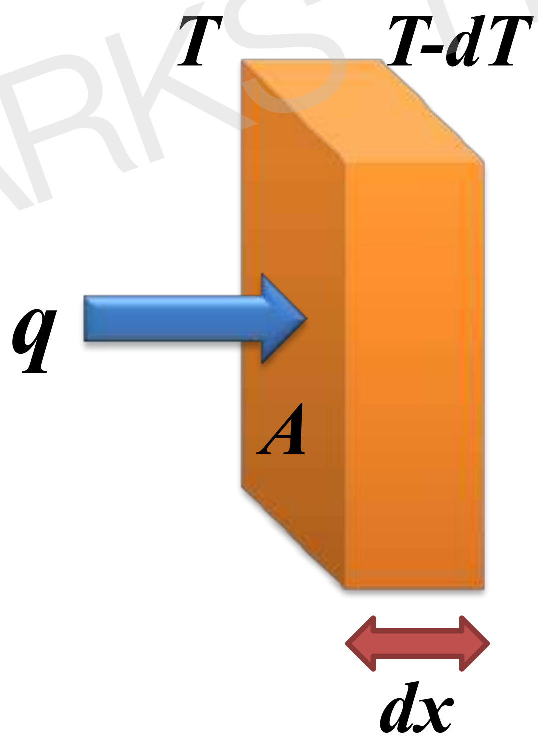
Heat Flow

Fourier's law of Heat conduction

The heat flowing across a surface is proportional to its surface area and to the rate of change of temp w.r.t. its distance normal to the surface.

If q **(cal/sec)** be the quantity of heat that flows across a slab of surface area A **cm²** and thickness dx in 1 sec where the difference of temp at the faces of the slab is dT and k coefficient of thermal conductivity then

$$q = -kA\frac{dT}{dx}$$



Law of natural decay

A rate of decay of a material is proportional to its amount present at that instant.

If m is amount of material at time t then

$$\frac{dm}{dt} = -km$$

Rectilinear Motion

Rectilinear motion (also called as linear motion) is **motion along a straight line.**

If x is displacement of a particle at time t then its

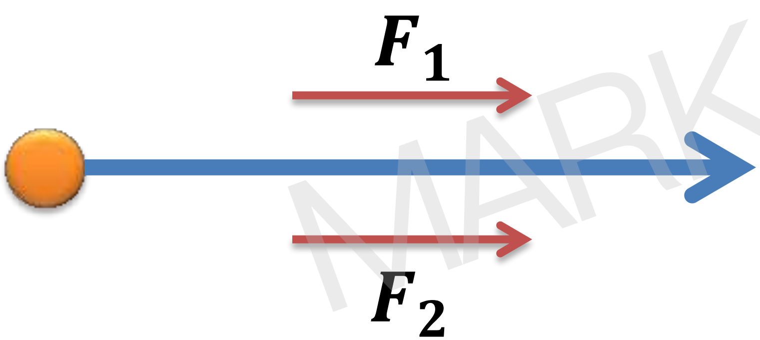
Velocity $v = \frac{dx}{dt}$

Acceleration $a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = v\frac{dv}{dx}$

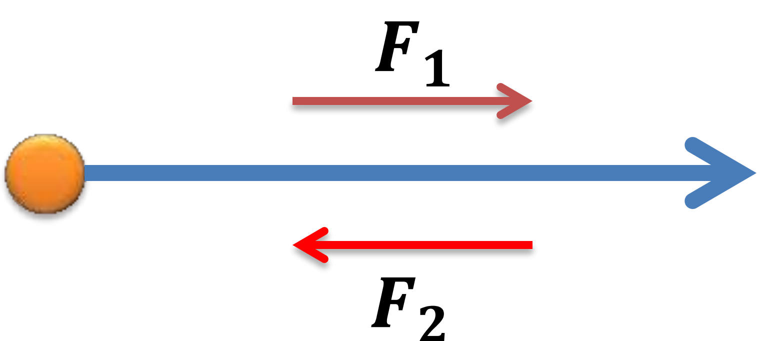
D'Alembert's principle

Algebraic sum of the forces acting on a body along a given direction is equal to the product of mass and acceleration in that direction.

Net force = Mass X Acceleration



Net force = $F_1 + F_2$



Net force = $F_1 - F_2$

S.H.M.

Equation of SHM is

$$Acceleration = \frac{d^2x}{dt^2} = -\omega^2 x$$

$$Period\ T = \frac{2\pi}{\omega}$$

	For finding orthogonal trajectory of $f(x, y, c) = 0$ we replace $\frac{dy}{dx}$ by				**[01]**
	a)	$-dx/dy$	b)	$-dy/dx$	
	c)	$2dx/dy$	d)	dy/dx	
	The orthogonal trajectory of $y = ax^2$ is				[02]
	a)	$x^2 + y^2 = c^2$	b)	$x^2 + (y^2/2) = c^2$	
	c)	$(x^2/2) + y^2 = c$	d)	None of these	
	The orthogonal trajectory of parabola is				[02]
	a)	Circle	b)	Hyperbola	
	c)	Ellipse	d)	Straight line	

	The orthogonal trajectory of the family of circles with centre at $(0,0)$ is a family of				[02]
	a)	Circles	b)	Straight lines through $(0,0)$	
	c)	any straight line	d)	Parabola	
	The DE for the orthogonal trajectory of the family of curves $x^2 + 2y^2 = c^2$ is				[01]
	a)	$x + 2y\frac{dy}{dx} = 0$	b)	$2\frac{dx}{x} = \frac{dy}{y}$	
	c)	$xdx + ydy = 0$	d)	$\frac{dx}{x} = \frac{dy}{y}$	
	The DE of orthogonal trajectory of the family of curves $r^2 = a\sin 2\theta$ is				[01]
	a)	$\frac{dr}{r} = -\tan 2\theta\, d\theta$	b)	$\frac{dr}{r} = \tan 2\theta\, d\theta$	
	c)	$dr = \tan 2\theta\, d\theta$	d)	None of these	

	The DE of orthogonal trajectory of the family of curves $r^2 = a^2 \cos 2\theta$ is				[01]
	a)	$r\frac{d\theta}{dr} = \tan 2\theta$	b)	$rdr = \tan 2\theta \, d\theta$	
	c)	$rdr = \cot 2\theta \, d\theta$	d)	$rdr + \tan\theta \, d\theta = 0$	
	If the DE of orthogonal trajectory of a curve is $r\frac{d\theta}{dr} + \cot(\theta/2) = 0$ then its orthogonal trajectory is				[01]
	a)	$r = \cos\theta$	b)	$r = c(1 - \sin\theta)$	
	c)	$r = c(1 - \cos\theta)$	d)	$r = b(1 + \cos\theta)$	
	If temperature of surrounding medium is θ_0 and temperature of body at any time t is θ, then in a process of heating $d\theta/dt$ is				[01]
	a)	$\theta - \theta_0$	b)	$k(\theta - \theta_0); \;\; k > 0$	
	c)	$-k(\theta - \theta_0); \;\; k > 0$	d)	None of these	

	In certain data of newton's law of cooling, $-kt = \log\left(\frac{\theta-40}{60}\right)$ and at $t = 4, \theta = 60^0$, then the value of k is				**[02]**
	a)	$\log(1/3)$	b)	$-\log(1/3)$	
	c)	$4\log(1/3)$	d)	$(1/4)\log 3$	
	If the temperature of water initially is 100^0C and $\theta_0 = 20^0C$, and water cools down to 60^0C in first 20 minutes with $k = \frac{1}{20}\log 2$, then during what time will it cool to 30^0C				[02]
	a)	60 min	b)	50 min	
	c)	1.5 hour	d)	40 min	
	If a body originally at 80^0C, with $\theta_0 = 40^0C$ and $k = \frac{1}{20}\log 2$, then the temperature of body after 40 min is				[02]
	a)	40^0C	b)	50^0C	
	c)	80^0C	d)	30^0C	

	If the body at 100^0C is placed in room whose temperature is 10^0C and cools to 60^0C in 5 minutes then the value of k is				**[02]**
	a)	$\log 2$	b)	$-\log 2$	
	c)	$(1/5)\log 2$ s	d)	$5\log 2$	
	The linear form of DE for R-L series circuit with emf E is				[01]
	a)	$L\frac{di}{dt}+Ri=E$	b)	$\frac{di}{dt}+\frac{R}{L}i=\frac{E}{L}$	
	c)	$L\frac{di}{dt}+Ri=0$	d)	none of these	
	The integrating factor for the DE of R-L series circuit with emf E is				[02]
	a)	$e^{\int Rdt}$	b)	e^{Rt+c}	
	c)	$e^{\int \frac{R}{L}dt}$	d)	$e^{\int idt}$	

	If $i = \frac{E}{R} + ke^{-\frac{Rt}{L}}$ then the maximum value of i is				[01]
	a)	R/L	b)	E/R	
	c)	−E/R	d)	2R/L	
	The linear form of DE for R-C series circuit with emf E is				[01]
	a)	$Ri + \frac{q}{c} = E(t)$	b)	$Ri + \frac{1}{C}\int i\,dt = E$	
	c)	$R\frac{di}{dt} + \frac{i}{C} = \frac{dE}{dt}$	d)	$\frac{di}{dt} + \frac{i}{RC} = \frac{1}{R}\frac{dE}{dt}$	
	The integrating factor for the DE of R-C series circuit with emf E is				[01]
	a)	$e^{\int RC dt}$	b)	$e^{\int \frac{1}{RC} dt}$	
	c)	$e^{\int \frac{1}{R} dt}$	d)	$e^{\int \frac{1}{C} dt}$	

	If $i = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$ then the 50% of maximum current is				**[01]**
	a)	E/R	b)	E/2R	
	c)	2E/R	d)	2R/E	
	Which one of the following is not correct?				[01]
	a)	$F = ma$	b)	$F = m\frac{dv}{dt}$	
	c)	$F = m\,v\frac{dv}{dx}$	d)	$F = m\,v\frac{dv}{dt}$	
	A motion of a body or particle along straight line is known as				[01]
	a)	rectilinear motion	b)	curvilinear motion	
	c)	Motion	d)	None of these	

If a body of mass m falling from rest is subjected to the force of gravity and an air resistance proportional to the square of velocity kv^2, then the equation of motion is	**[01]**

a)	$$mv\frac{dv}{dx} = mg + kv^2$$	b)	$$ma = -mg + kv^2$$
c)	$$ma = mg - kv^2$$	d)	None of these

If a body opposed by force per unit mass of value cx and resistance per unit mass of value kv^2 then the equation of motion is	[01]

a)	$$a = cx - bv^2$$	b)	$$a = bv^2 - cx$$
c)	$$v\frac{dv}{dx} = -cx - bv^2$$	d)	$$v\frac{dv}{dx} = cx + bv^2$$

	The quantity of heat in a body is proportional to its			[01]
a)	mass only	b)	temperature only	
c)	mass and temperature	d)	none of these	

	The motion of a particle moving along a straight line is $\frac{d^2x}{dt^2} + 16x = 0$, then its period is			[01]
a)	$2\pi/\sqrt{2}$	b)	$\pi/2$	
c)	2π	d)	π	

	The orthogonal trajectories of the series of hyperbolas $xy = c^2$ is			[02]
a)	$x^2 + y^2 = c^2$	b)	$x^2y^2 = c^2$	
c)	$y^2 - x^2 = c^2$	d)	None of these	

	The differential equation of orthogonal trajectories of family of straight lines $y = mx$ is				**[01]**
	a)	$x\,dx - y\,dy = 0$	b)	$ydx - xdy = 0$	
	c)	$x\,dx + ydy = 0$	d)	$ydx + xdy = 0$	
	Let the population of country be decreasing at the rate proportional to its population. If the population has decreased to 25% in 10 years, how long will it take to half.				[02]
	a)	20 years	b)	8.3 years	
	c)	15 years	d)	5 years	
	The orthogonal trajectories of the family of straight lines $y = mx$ is				[01]
	a)	$x^2 - y^2 = c^2$	b)	$x^2 = my^2$	
	c)	$y^2 = m^2x^2$	d)	$x^2 + y^2 = c^2$	

The set of orthogonal trajectories to a family of curves whose DE is $\phi\left(x, y, \frac{dy}{dx}\right) = 0$ is obtained by DE				[01]
a)	$\phi\left(x, y, x\frac{dy}{dx}\right) = 0$	b)	$\phi\left(x, y, \frac{-dx}{dy}\right) = 0$	
c)	$\phi\left(x, y, \frac{dy}{dx}\right) = 0$	d)	$\phi\left(x, y, \frac{-dy}{dx}\right) = 0$	
The orthogonal trajectories of the family of curves $r\cos\theta = a$ is				[02]
a)	$r\sin\theta = c$	b)	$r\tan\theta = c$	
c)	$\frac{r}{\sin\theta} = c$	d)	None of these	
If 10 grams of some radioactive substance reduces to 8 gm in 60 years, in how many years will 2 gm of it will be left ?				[02]
a)	120 yrs	b)	378 yrs	
c)	220 yrs	d)	433 yrs	

	Voltage drop across inductance L is given by				**[01]**
	a)	Li	b)	$L\frac{di}{dt}$	
	c)	$\frac{dL}{dt}$	d)	None of these	
	A ball at temperature of 32^0C is kept in a room where the temperature is 10^0C. If the ball cools to 27^0C in hour then its temperature is given by				[02]
	a)	$T = 22\ e^{0.205\ t}$	b)	$T = 10\ e^{1.163t}$	
	c)	$T = 10 + 22e^{-0.258t}$	d)	$T = 32 - 10e^{-0.093t}$	

Unit III

Fourier Series, Reduction Formulae, Gamma Functions, Beta Functions

Multiple Choice Questions

Periodic functions

A function $f(x)$ is said to be periodic if it is defined for all real x and if there is some positive number T such that

$$f(x+T) = f(x) \qquad \forall\, x$$

The number T is then called period of $f(x)$.

$\sin x, \cos x$ are periodic functions of period 2π

$\tan x\,, \cot x$ are periodic functions of period π

Fourier Series

If $f(x)$ is a periodic function of period 2π, defined in the interval $c \le x \le c + 2\pi$ then

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

this representation of $f(x)$ is called Fourier Series and the coefficients a_0, a_n, b_n are called the Fourier coefficients.

Euler's Formulae

$$a_0 = \frac{1}{\pi} \int_{c}^{c+2\pi} f(x)dx$$

$$a_n = \frac{1}{\pi} \int_{c}^{c+2\pi} f(x) \cos nx \, dx$$

$$b_n = \frac{1}{\pi} \int_{c}^{c+2\pi} f(x) \sin nx \, dx$$

1	If $f(x+nT)=f(x)$ where n is any integer then the fundamental period of $f(x)$ is			
	a)	$2T$	b)	$T/2$
	c)	$\boxed{T}$	d)	3T

2	If $f(x)$ is a periodic function with period T then $f(ax)$, $a \neq 0$ is periodic function with fundamental period			
	a)	T	b)	$\boxed{T/a}$
	c)	aT	d)	π

3	Fundamental period of $\cos 2x$ is			
	a)	$\frac{\pi}{4}$	b)	$\frac{\pi}{2}$
	c)	$\boxed{\pi}$	d)	2π

4	Fundamental period of $\tan 3x$ is			
	a)	$\frac{\pi}{2}$	b)	$\boxed{\frac{\pi}{3}}$
	c)	π	d)	$\frac{\pi}{4}$

5	The value of constant terms in the Fourier series of $f(x) = e^{-x}$ in $0 \leq x \leq 2\pi,\ f(x+2\pi) = f(x)$ is			
	a)	$\frac{1}{\pi}(1 - e^{-2\pi})$	b)	$\boxed{\frac{1}{2\pi}(1 - e^{-2\pi})}$
	c)	$2(1 - e^{-2\pi})$	d)	$(1 - e^{-2\pi})$

6	Fourier coefficient a_0 in the Fourier series expansion of $f(x) = \left(\frac{\pi - x}{2}\right)^2$; $0 \leq x \leq 2\pi$ and $f(x+2\pi) = f(x)$			
	a)	$\frac{\pi^2}{3}$	b)	$\boxed{\frac{\pi^2}{6}}$
	c)	0	d)	$\pi/6$

For function defined in the interval $-\pi \leq x \leq \pi$

- If $f(x)$ is even then

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{2}{\pi}\int_{0}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx = \frac{2}{\pi}\int_{0}^{\pi} f(x)\cos nx\, dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx = 0$$

For function defined in the interval $-\pi \leq x \leq \pi$

- If $f(x)$ is odd then

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{2}{\pi}\int_{0}^{\pi} f(x)\sin nx\,dx$$

7	**Fourier series representation of periodic function** $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$ $f(x) = \frac{8}{\pi^2}\left[\frac{1}{1^2}\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x + \cdots\right]$ **then value of** $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots =$

	a)	$\frac{\pi^2}{4}$	b)	$\boxed{\frac{\pi^2}{8}}$
	c)	$\frac{\pi^2}{16}$	d)	$\frac{8}{\pi^2}$

31 $f(x) = x, -\pi \le x \le \pi$ and period is 2π. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$, Fourier coefficient b_1 is

a) $\boxed{2}$ b) -1

c) 0 d) $2/\pi$

If $f(x)$ is a periodic function of period $2L$, defined in the interval $c \le x \le c + 2L$ then

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$$

$$a_0 = \frac{1}{L}\int_{c}^{c+2L} f(x)dx \qquad a_n = \frac{1}{L}\int_{c}^{c+2L} f(x)\cos\frac{n\pi x}{L}dx$$

$$b_n = \frac{1}{L}\int_{c}^{c+2L} f(x)\sin\frac{n\pi x}{L}dx$$

If $f(x)$ is a periodic function of period $2L$, defined in the interval $-\mathrm{L} \leq x \leq L$ and

if $f(x)$ is an even function then

$$a_0 = \frac{2}{L}\int_0^L f(x)dx \qquad a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx \qquad b_n = 0$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L}\right)$$

if $f(x)$ is an odd function then

$$a_0 = 0 \qquad a_n = 0 \qquad b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$$

$$f(x) = \sum_{n=1}^{\infty}\left(b_n \sin\frac{n\pi x}{L}\right)$$

Half range expansions

- Half range cosine series: If $f(x)$ is defined in the interval $0 \leq x \leq L$, then the half range cosine series of $f(x)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}$$

$$a_0 = \frac{2}{L}\int_0^L f(x)dx \qquad a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx$$

- Half range sine series: If $f(x)$ is defined in the interval $0 \leq x \leq L$, then the half range sine series of $f(x)$ is given by

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$$

$$b_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx$$

9	The Fourier constant a_n for $f(x) = 4 - x^2$ in the interval $0 < x < 2$ is			
	a)	$\boxed{4/\pi^2 n^2}$	b)	$2/n^2\pi^2$
	c)	$4/n^2\pi$	d)	$2/n\pi^2$

10	For half range sine series of $f(x) = x, 0 \leq x \leq 2$ and period is 4.Fourier series is represented by $\sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{2}$, then Fourier coefficient b_1 is			
	a)	4	b)	2
	c)	$\frac{2}{\pi}$	d)	$\boxed{\frac{4}{\pi}}$

		1st Harmonic		2nd Harmonic		3rd Harmonic	
x	y	$y\cos\frac{\pi x}{L}$	$y\sin\frac{\pi x}{L}$	$y\cos\frac{2\pi x}{L}$	$y\sin\frac{2\pi x}{L}$	$y\cos\frac{3\pi x}{L}$	$y\sin\frac{3\pi x}{L}$
x_0	y_0						
$\vdots$	$\vdots$						
x_{m-1}	y_{m-1}						
Σ							

$$a_0 = \frac{2}{m}\sum_{i=0}^{m-1} y_i \qquad a_n = \frac{2}{m}\sum_{i=0}^{m-1} y_i \cos\frac{n\pi x_i}{L} \qquad b_n = \frac{2}{m}\sum_{i=0}^{m-1} y_i \sin\frac{n\pi x_i}{L}$$

$$f(x) = \frac{a_0}{2} + a_1\cos\frac{\pi x}{L} + b_1\sin\frac{\pi x}{L} + a_2\cos\frac{2\pi x}{L} + b_2\sin\frac{2\pi x}{L}$$
$$+ a_3\cos\frac{3\pi x}{L} + b_3\sin\frac{3\pi x}{L} + \cdots$$

1. The term $\left[a_1 \cos\left(\frac{\pi x}{L}\right) + b_1 \sin\left(\frac{\pi x}{L}\right)\right]$ is called as **'Fundamental or First harmonic'.**

2. The term $\left[a_2 \cos\left(\frac{2\pi x}{L}\right) + b_2 \sin\left(\frac{2\pi x}{L}\right)\right]$ is called as **' second harmonic'** and so on.

3. The amplitude of n^{th} harmonic is $+\sqrt{a_n^2 + b_n^2}$.

4. Percentage of n^{th} harmonic =

$$\frac{\text{amplitude of } n^{th} \text{ harmonic}}{\text{amplitude of } I^{st} \text{ harmonic}} \times 100$$

11	For the certain data if $a_0 = 1.5,\ a_1 = 0.373,\ b_1 = 1.004$ then the amplitude of 1st harmonic is		
a)	1.07	b)	2.07
c)	1.004	d)	1.377

12	The value of a_0 in harmonic analysis of y for the following tabulated data is					
x°	0	60	120	180	240	300	360
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0
	a)	1.45	b)	5.8			
	c)	2.9	d)	2.48			

13	The value of a_1 in Harmonic analysis of y for the following tabulated data is :						
x	0	1	2	3	4	5	6
y	4	8	15	7	6	2	4
$\cos\frac{\pi x}{3}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1
	a)	-4.16	b)	-8.32			
	c)	-3.57	d)	$\boxed{-10.98}$			

14	The value of $a_1,\ a_2$ in Fourier cosine series of y for the following tabulated data are			
x	0	$\pi/4$	$\pi/2$	$3\pi/4$
y	0	$\sqrt{2}$	2	$\sqrt{2}$
	a)	$-1/2, 1/2$	b)	$\boxed{-1/2, -1/2}$
	c)	$2, -2$	d)	$-2, 0$

Reduction Formulae

1. $\int_0^{\pi/2} \cos^n x\, dx = \int_0^{\pi/2} \sin^n x\, dx$

$$= \frac{[\,(n-1)\text{subtract } 2 \ldots\ldots\ldots\ldots\ldots\ldots 2 \text{ or } 1\,]}{[\,(n)\ \text{subtract } 2 \ldots\ldots\ldots\ldots\ldots\ldots 2 \text{ or } 1]} \times \left(\frac{\pi}{2}\right) \text{ if n is even.}$$

$$= \frac{[\,(n-1)\text{subtract } 2 \ldots\ldots\ldots\ldots\ldots\ldots 2 \text{ or } 1\,]}{[\,(n)\ \text{subtract } 2 \ldots\ldots\ldots\ldots\ldots\ldots 2 \text{ or } 1]} \times 1 \text{ if n is odd.}$$

2.(a) $\int_0^{\pi/2} \sin^m x \cos^n x\, dx$

$$= \left\{\frac{[\,(m-1)\text{ subtract } 2\ldots\ldots 2 \text{ or } 1\,].[\,(n-1)\text{ subtract } 2\ldots\ldots 2 \text{ or } 1\,]}{[\,(m+n)\text{ subtract } 2\ldots\ldots\ldots 2 \text{ or } 1\,]}\right\} \times \left(\frac{\pi}{2}\right)$$

If m and n both are even.

2.(b) $\int_0^{\pi/2} \sin^m x \cos^n x\, dx$

$$= \left\{\frac{[\,(m-1)\text{ subtract } 2\ldots\ldots 2 \text{ or } 1\,].[\,(n-1)\text{ subtract } 2\ldots\ldots 2 \text{ or } 1\,]}{[\,(m+n)\text{ subtract } 2\ldots\ldots\ldots 2 \text{ or } 1\,]}\right\} \times (1)$$

Otherwise .

3] $\int_0^{\pi/2} \sin^m x \cos x \, dx = \int_0^{\pi/2} \cos^m x \sin x \, dx = \frac{1}{m+1}$

Conversion Formulae :

1] $\int_0^{2\pi} \sin^m x \cos^n x \, dx = \begin{cases} = 4\int_0^{\pi/2} \sin^m x \cos^n x \, dx, \ \boldsymbol{if\ m,\ n\ even}. \\ = 0, \quad Otherwise. \end{cases}$

2] $\int_0^{\pi} \sin^m x \cos^n x \, dx$

$= \begin{cases} = 2\int_0^{\pi/2} \sin^m x \cos^n x \, dx, \ \boldsymbol{if\ n\ even,\ for\ any\ m}. \\ = 0, \quad \boldsymbol{,if\ n\ odd,\ for\ any\ m}. \end{cases}$

3] $\int_0^{2\pi} \sin^n x \, dx = \begin{cases} = 4\int_0^{\pi/2} \sin^n x \, dx, \ \boldsymbol{if\ n\ is\ even}. \\ = 0 \quad \boldsymbol{,if\ n\ is\ odd}. \end{cases}$

4] $\int_0^{2\pi} cos^n\, x\, dx = \begin{cases} = 4\int_0^{\pi/2} cos^n\, x\, dx\ , \boldsymbol{if\ n\ is\ even.} \\ = 0 \qquad\qquad\qquad\qquad\quad , \boldsymbol{if\ n\ is\ odd.} \end{cases}$

5] $\int_0^{\pi} sin^n\, x\, dx = 2\int_0^{\pi/2} sin^n\, x\, dx\ , \boldsymbol{for\ any\ n}$.

6] $\int_0^{\pi} cos^n\, x\, dx = \begin{cases} = 2\int_0^{\pi/2} cos^n\, x\, dx\ , \boldsymbol{if\ n\ is\ even.} \\ = 0 \qquad\qquad\qquad\qquad\quad , \boldsymbol{if\ n\ is\ odd.} \end{cases}$

1	**The value of the integral** $\int_0^{\frac{\pi}{6}} \cos^6 3x \ dx$ **is**			
	a)	$5\pi/96$	b)	$7/48$
	c)	$5\pi/32$	d)	0

2	**The value of** $\int_{-\pi/2}^{\pi/2} \sin^4 x \, dx$ **is**			
	a)	$3\pi/16$	b)	$3\pi/8$
	c)	$3\pi/4$	d)	0

$$\int_0^{\pi/2} cos^n x \, dx = \int_0^{\pi/2} sin^n x \, dx \boxed{= \frac{n-1}{n} I_{n-2}}$$

3	The value of the integral $\int_0^{\pi/2} \sin^4 x \cos^3 x \; dx$ is			
	a)	$\pi/35$	b)	$\boxed{2/35}$
	c)	0	d)	$53/2$

4	The value of $\int_{-2\pi}^{2\pi} \sin^3 x \cos^2 x \, dx$ is			
	a)	$\boxed{0}$	b)	$\pi/4$
	c)	$\pi/16$	d)	$\pi/32$

5	The value of the integral $\int_0^{2\pi} \cos^5 x \, dx$ is			
	a)	$\boxed{0}$	b)	$5/16$
	c)	$5/32$	d)	$5\pi/32$

6	The value of the integral $\int_0^{\pi} \sin^5 x \, dx$ is			
	a)	$8\pi/15$	b)	$\pi/2$
	c)	$\boxed{16/15}$	d)	0

7	**If** $I_n = \int_{\pi/4}^{\pi/2} \cot^n x \, dx$ **and** $I_n = \frac{1}{n-1} - I_{n-2}$, **then the value of** I_6 **is**			
	a)	$\frac{13}{15}$	b)	$\frac{13}{15} + \frac{\pi}{4}$
	c)	$\boxed{\frac{13}{15} - \frac{\pi}{4}}$	d)	$\frac{13}{15} - \frac{\pi}{2}$
8	**If** $I_n = \int_0^{\pi/4} \sin^{2n} x \, dx$ **and** $I_n = \left(1 - \frac{1}{2n}\right) I_{n-1} - \frac{1}{n2^{n+1}}$, **then the value of** $\int_0^{\pi/4} \sin^4 x \, dx$ **is**			
	a)	$\frac{3\pi}{32} + \frac{1}{4}$	b)	$\boxed{\frac{3\pi}{32} - \frac{1}{4}}$
	c)	$\frac{\pi}{16} - \frac{1}{4}$	d)	$\frac{3\pi}{16} + \frac{1}{4}$

9	If $I_{m,n} = \int_0^{\pi/2} (\cos^{\mathrm{m}} x)(\sin nx)dx$ and $I_{m,n} = \frac{1+m\, I_{m-1,n-1}}{m+n}$, then the value of $\int_0^{\pi/2} (\cos^2 x)(\sin 4x)dx$ is			
	a)	3	b)	2
	c)	1/3	d)	2/3

10	If $I_n = \int_0^{\pi/2} x \cdot \sin^n x \cdot dx$ and $I_n = \frac{n-1}{n} I_{n-2} + \frac{1}{n^2}$, then the value of $\int_0^{\pi/2} x \cdot \sin^4 x \cdot dx$			
	a)	$\frac{3\pi^2}{64} + \frac{1}{4}$	b)	$\frac{\pi^2}{64} + \frac{1}{4}$
	c)	$\frac{3\pi^2}{32} - \frac{1}{4}$	d)	$\frac{3\pi^2}{64} - \frac{1}{4}$

1. Gamma Function

Definition: The integral $\int_0^\infty e^{-x}x^{n-1}dx$ is called as Gamma function

and denoted by $\Gamma n = \int_0^\infty e^{-x}x^{n-1}dx$ (n > 0)

Properties :

$$\boxed{1.\ \Gamma 1 = 1}$$

2. Reduction formula : $\Gamma n+1 = n\,\Gamma n$

$= n!$, if n is $+ve$ integer

3. $\Gamma 0 = \infty$

4. $\Gamma \frac{1}{2} = \sqrt{\pi}$

5. $\Gamma P\ \Gamma 1-P = \dfrac{\pi}{\sin p\pi}$

11 The value of the integral $\int_0^\infty \frac{x^5}{5^x}\,dx$ by using substitution $5^x = e^t$ is

a)	$\boxed{120/(\log 5)^6}$	b)	$24/(\log 4)^5$
c)	$120/(\log 5)^5$	d)	$24/(\log 4)^4$

12 The value of the integral $\int_0^1 \frac{dx}{\sqrt{x\log\left(\frac{1}{x}\right)}}$ by using the substitution $\log\left(\frac{1}{x}\right) = t$ is

a)	$\sqrt{\pi}/2$	b)	$\boxed{\sqrt{2\pi}}$
c)	$\sqrt{\pi}$	d)	$2\sqrt{\pi}$

13	The formula for $\Gamma(n+1)$ is			
	a)	$\int_0^\infty e^{-x} x^{n-1} dx$	b)	$\boxed{\int_0^\infty e^{-x} x^{n} dx}$
	c)	$2\int_0^\infty e^{-x} x^{n-1} dx$	d)	$\int_0^\infty e^{-x} x^{n-2} dx$

14	The value of the integral $\int_0^\infty e^{-4x} x^3\, dx$ is			
	a)	$4!$	b)	$3!$
	c)	$\frac{3!}{64}$	d)	$\boxed{\frac{3!}{256}}$

15	The value of $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)$ is			
	a)	$2\pi/\sqrt{3}$	b)	$\pi/\sqrt{3}$
	c)	2π	d)	$2/\sqrt{3}$

16	The value of $\int_0^1 (\log x)^n\, dx$ is			
	a)	$(-1)^n\Gamma(n+1)$	b)	$(\log n)\Gamma n$
	c)	Γn	d)	$\Gamma(n+1)$

Beta Function.

$$Definition : \boxed{\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx} \; ; \; where \;\; m,n \; are \; +ve \; \text{integers}$$

Properties Of Beta Function.

1. $$\beta(m,n) = \beta(n,m)$$

2. $$\beta(m,n) = \int_0^\infty \frac{y^{m-1}}{(1+y)^{m+n}}\,dy$$

3. $$\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \; d\theta$$

4. $$\int_0^{\pi/2} \sin^p \theta \cos^q \theta \, d\theta = \frac{1}{2}\beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$$

5. Relation Between Beta and Gamma Function.

$$\boxed{\beta\,(m\,,n\,) = \frac{\Gamma m \; \Gamma n}{\Gamma m+n}}$$

6. Legendre's duplication formula :

$$\boxed{\Gamma m \;\; \Gamma m + 1/2 \; = \; \frac{\sqrt{\pi}}{2^{2m-1}} \; \Gamma 2m}$$

17	Value of $B\left(\frac{3}{4},\frac{1}{4}\right)$ is			
	a)	2π	b)	$\boxed{\pi\sqrt{2}}$
	c)	$\pi/2$	d)	$\sqrt{2}$

18	Value of $\int_0^{\pi/2}\sqrt{\tan x}\,dx$ is			
	a)	$\boxed{\frac{1}{2}B\left(\frac{3}{4},\frac{1}{4}\right)}$	b)	$\frac{1}{2}B\left(\frac{3}{4},\frac{3}{4}\right)$
	c)	$B\left(\frac{3}{4},\frac{1}{4}\right)$	d)	$B\left(\frac{3}{4},\frac{3}{4}\right)$

19	If $B(n+1,1)=\frac{1}{4}$ and n is a positive integer then value of n is			
	a)	1	b)	2
	c)	$\boxed{3}$	d)	4

20	Value of $\int_0^{\pi/2}\sqrt{2\sin 2x}\,dx$ is			
	a)	$\frac{1}{2}B\left(\frac{3}{4},\frac{1}{4}\right)$	b)	$\frac{1}{2}B\left(\frac{3}{4},\frac{3}{4}\right)$
	c)	$\boxed{B\left(\frac{3}{4},\frac{1}{4}\right)}$	d)	$B\left(\frac{3}{4},\frac{3}{4}\right)$

21	The value of $\int_0^\infty \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx$ is			
	a)	0	b)	$\frac{B(m,n)}{2}$
	c)	$\boxed{2B(m,n)}$	d)	1

22	By Duplication formula, the value of $\Gamma m \cdot \Gamma(m+\frac{1}{2})$ is			
	a)	$\frac{\sqrt{\pi}}{2^{m-1}}\Gamma(2m)$	b)	$\frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(m)$
	c)	$\frac{\sqrt{\pi}}{2^{m}}\Gamma(2m)$	d)	$\boxed{\frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)}$

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First Online Examination

First Year of Engineering

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2015-16

Savitribai Phule Pune University – FE – Sem. II

Engineering Mathematics (M II)

Chapter 01–Ordinary Differential Equations

1) The order of the differential equation is
   a) the order of the highest ordered differential coefficient appearing in the differential equation.
   b) the order of the lowest ordered differential coefficient appearing in the differential equation.
   c) the power of the highest ordered differential coefficient appearing in the differential equation.
   d) the degree of the highest ordered differential coefficient appearing in the differential equation.

2) The degree of the differential equation is
   a) the highest ordered differential coefficient appearing in the differential equation.
   b) the lowest power of the highest ordered differential coefficient appearing in the differential equation.
   c) the highest power of the highest ordered differential coefficient appearing in the differential equation.
   d) the coefficient power of the highest ordered differential coefficient appearing in the differential equation.

3) A solution of a differential equation is a relation between
   a) dependent variables
   b) independent variables
   c) dependent and independent variables not containing any differential coefficient
   d) none of the above

4) In the general solution, the number of arbitrary constants is equal to
   a)order of the differential equation
   b) degree of the differential equation
   c) sum of order and degree of diff. eqn.
   d) difference of order and degree of diff. eqn.

5) The general solution of n^{th} order ordinary differential equation must involve
   a) $n+1$ arbitrary constants
   b) $n-1$ arbitrary constants
   c) n arbitrary constants
   d) none of the above

6) The solution obtained by assigning particular values to arbitrary constants in general solution of differential equation is known as
   a) singular solution b) particular solution
   c) general solution d) none of above

7) The order of differential equation whose general solution is $y=(c_1+c_2x)e^x+x$, where $c_1,\ c_2$ are arbitrary constants, is
   a) 1 b)2 c) 3 d) 0

8) The order of differential equation whose general solution is $y=(c_1+c_2x+c_3x)e^x+\frac{x^2}{12}$, where $c_1,\ c_2,\ c_3$ are arbitrary constants, is
   a) 0 b) 1 c) 2 d)3

9) The order of differential equation whose general solution is $y=(c_1+c_2)e^x+\frac{x^4}{3}$, where $c_1,\ c_2$ are arbitrary constants, is
   a) 0 b)1 c)2 d) 3

10) The order of differential equation whose general solution is $y=cx+c^2$, where c is arbitrary constant, is
   a) 0 b)1 c)2 d) 3

11) The order of differential equation whose general solution is $y=Ax+\frac{B}{x}$, where $A,\ B$ are arbitrary constants, is
   a) 0 b) 1 c)2 d) 3

12) The order of differential equation whose general solution is $y = Ax + \frac{A^2}{x}$, where A, B are arbitrary constants, is
a) 0 b) 1 c) 2 d) 3

13) The order of differential equation whose general solution is $y = \log(x-a) + b$, where a, b are arbitrary constants, is
a)2 b) 1 c) 0 d) none

14) The order of differential equation whose general solution is $x = A\sin(kt + B)$, where A, B are arbitrary constants and k is fixed constant, is
a) 0 b) 1 c)2 d) 3

15) The order of differential equation whose general solution is $x = (A + Bt)e^t$, where A, B are arbitrary constants, is
a) 0 b)2 c) 1 d) 3

16) The order of differential equation whose general solution is $y + \sqrt{x^2 + y^2} = cx + c^3$, where c is arbitrary constant, is
a) 0 b) 2 c) 3 d)1

17) The order of differential equation whose general solution is $y = 4(x - A)^2$, where A is arbitrary constant, is
a)1 b) 2 c) 3 d) none

18) The order of differential equation whose solution is $y = (c_1 + c_2 x)e^x + (c_3 + c_4 x)e^{2x}$, where c_1, c_2, c_3, c_4 are arbitrary constants, is
a) 1 b) 2 c) 3 d)4

19) The order of differential equation whose solution is $y = c_1 x + c_2 e^x + c_3 e^{2x} + c_4 e^{3x}$, where c_1, c_2, c_3, c_4 are arbitrary constants, is
a) 1 b)4 c) 2 d) 3

20) The order of differential equation whose solution is $y = (Ax^2 + Bx + C)e^x$, where A, B, C are arbitrary constants, is
a) 1 b) 2 c)3 d) 4

21) The order of differential equation whose general solution is $y = \sqrt{kx + c}$, where c is the only arbitrary constant, is
a)1 b) 2 c) 3 d) 0

22) The order of differential equation whose general solution is $y = c^2 + \frac{c}{x}$, where c is arbitrary constant, is
a) 0 b) 2 c) 3 d)1

23) The order of differential equation whose general solution is $y = A\cos(x+5)$, where A is arbitrary constant, is
a) 0 b)1 c) 2 d) 3

24) The order and the degree of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is
a)1, 1 b) 1, 2 c) 2, 1 d) 2, 2

25) The order and the degree of the differential equation $\frac{dy}{dx} + y\log x = \sin x$ is
a) 0, 1 b) 1, 0 c) 2, 1 d)1, 1

26) The order and the degree of the differential equation $\frac{dy}{dx} + 2y = \cos x$ is
a) 0, 1 b)1, 1 c) 1, 2 d) 2, 1

27) The order and the degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 5y = \sin 7x$ is
a) 0, 1 b) 1, 1 c) 1, 2 d) 2, 1

28) The order and the degree of the differential equation $1 + \frac{dy}{dx} = \left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}}$ is
a) order 2, degree 1 b) order 1, degree 2
c)order 2, degree 3 d) order 2, degree $\frac{3}{2}$

29) The order and the degree of the differential equation $\sqrt{1 + \frac{dy}{dx}} = \frac{d^2y}{dx^2}$ is
a) order 2, degree 2 b) order 2, degree 1
c) order 1, degree 2 d) order 1, degree 1

30) The order and the degree of the differential equation $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}=k$ is

a) order 2, degree 1 b) order 2, degree 2

c) order 2, degree 3 d) order 2, degree $\frac{3}{2}$

31) The order and the degree of the differential equation $\sqrt{1+\frac{dy}{dx}}=\frac{d^2y}{dx^2}$ is

a)order 2, degree 2 b) order 2, degree 1

c) order 1, degree 2 d) order 1, degree 1

32) The order and the degree of the differential equation $\frac{d^2y}{dx^2}=\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ is

a) order 2, degree 1 b) order 2, degree 2

c) order 1, degree 2 d) order 1, degree 1

33) The order and the degree of the differential equation $x=\frac{1}{\sqrt{1+\frac{dy}{dx}+\frac{d^2y}{dx^2}}}$ is

a) order 2, degree 2 b)order 2, degree 1

c) order 1, degree 2 d) order 1, degree $-\frac{1}{2}$

34) The order and the degree of the differential equation $1+\frac{dy}{dx}=\frac{y}{\frac{dy}{dx}}$ is

a) order 1, degree 1 b) order 2, degree 1

c)order 1, degree 2 d) order 2, degree 2

35) The order and the degree of the differential equation $y+\frac{d^2y}{dx^2}+\frac{x}{\frac{dy}{dx}}=1$ is

a) order 1, degree 1 b) order 2, degree 1

c) order 1, degree 2 d) order 2, degree 2

36) The order and the degree of the differential equation $(2x-3y+2)dy+(x-2y+7)dx=0$ is

a)1, 1 b) 1, 2 c) 2, 1 d) none

37) By eliminating the arbitrary constant m, the differential equation for the general solution $y=mx$ is given by

a) $\frac{dy}{dx}=\frac{y}{x}$ b) $\frac{dy}{dx}-xy=0$

c) $\frac{dy}{dx}+\frac{y}{x}=0$ d) $\frac{dy}{dx}-y=0$

38) The differential equation satisfied by the general solution $y+x^3=Ax$ with A is arbitrary constant, is given by

a) $y\frac{dy}{dx}+2x-y^3=0$ b) $x\frac{dy}{dx}+2x^3-y=0$

c) $\frac{dy}{dx}+2x^2-y=0$ d) $x^3\frac{dy}{dx}+2(x-y)=0$

39) $y=5+\sqrt{cx}$, where c is the arbitrary constant, is the general solution of

a) $y\frac{dy}{dx}=5+2x$ b) $y=2x\frac{dy}{dx}$

c) $y=5+2x\frac{dy}{dx}$ d) $y=5+2x\sqrt{\frac{dy}{dx}}$

40) By eliminating the arbitrary constant c, the differential equation of $y=cx-c^2$ is

a) $\frac{dy}{dx}-x\left(\frac{dy}{dx}\right)^2+y=0$ b) $\left(\frac{dy}{dx}\right)^2-x\frac{dy}{dx}+y=0$

c) $\left(\frac{dy}{dx}\right)^2+x\frac{dy}{dx}-y=0$ d) $\left(\frac{dy}{dx}\right)^2-xy=0$

41) The differential equation whose primitive is $y=c^2+\frac{c}{x}$, is given by

a) $x^4\left(\frac{dy}{dx}\right)^2-xy=0$ b) $\left(\frac{dy}{dx}\right)^2+\frac{dy}{dx}+y=0$

c) $\left(\frac{dy}{dx}\right)^2-x^4\frac{dy}{dx}-y=0$ d) $x^4\left(\frac{dy}{dx}\right)^2-x\frac{dy}{dx}-y=0$

42) By eliminating the arbitrary constant c present in the function $x=cy-y^2$, the differential equation is given by

a) $\left(\frac{x+y^2}{y}\right)\frac{dy}{dx}-2y\frac{dy}{dx}-1=0$

b) $\left(\frac{x+y^2}{y}\right)\left(\frac{dy}{dx}\right)^2-2y\frac{dy}{dx}-1=0$

c) $x\dfrac{dy}{dx}-2\left(\dfrac{x+y^2}{y}\right)\dfrac{dy}{dx}-1=0$

d) $y^2\dfrac{dy}{dx}-2xy\dfrac{dy}{dx}+1=0$

43) The differential equation whose solution is $y^2=4ax$ is given by

a) $\left(\dfrac{dy}{dx}\right)^2-2xy=0$ b) $\dfrac{dy}{dx}-xy^2=0$

c) $2xy\dfrac{dy}{dx}-y^2=0$ d) $2xy\dfrac{dy}{dx}+y^2=0$

44) The differential equation of family of curves $x^2+y^2+xy+x+y=c$ is

a) $\dfrac{dy}{dx}=-\dfrac{2x+y+1}{x+2y+1}$ b) $y_2+4y=0$

c) $\dfrac{dy}{dx}=\dfrac{2x-y}{x+2y+1}$ d) $x^2y_2-xy_1+y=0$

45) The differential equation whose generalized solution is $xy+y^2-x^2-x-3y=c$, is

a) $\dfrac{dy}{dx}=-\dfrac{2x-y+1}{x-2y+3}$ b) $\dfrac{dy}{dx}=\dfrac{x-2y-1}{x-2y+3}$

c) $\dfrac{dy}{dx}=\dfrac{2x+y+1}{x+2y+3}$ d) $\dfrac{dy}{dx}=\dfrac{2x-y+1}{x+2y-3}$

46) The differential equation satisfied by family of circles $x^2+y^2=2Ax$ is given by

a) $\dfrac{dy}{dx}+x^2+y^2=0$ b) $\dfrac{dy}{dx}+\dfrac{y^2-x^2}{xy}=0$

c) $\dfrac{dy}{dx}+\dfrac{x^2-y^2}{2xy}=0$ d) $\dfrac{dy}{dx}-\dfrac{x^2-y^2}{2xy}=0$

47) The differential equation whose general solution is $x^3+y^3=3Ax$, where A is arbitrary constant, is

a) $y_1=\dfrac{x^3+y^3-3x^2}{3xy^2}$ b) $x^2y_1+y=3y_1$

c) $xy_1+y^2+x=0$ d) none of these

48) $y^2=x^2-1+Ax$, where A is arbitrary constant, is the general solution of the equation

a) $\dfrac{dy}{dx}=\dfrac{x^2+y^2}{2xy}$ b) $y\dfrac{dy}{dx}+x^2+y^2=0$

c) $2xy\dfrac{dy}{dx}=x^2+y^2+1$ d) $2xy\dfrac{dy}{dx}-\left(x^2+y^2\right)=0$

49) The differential equation of $y=4(x-A)^2$, where A is arbitrary constant, is

a) $\dfrac{dy}{dx}-16y^2=0$ b) $\left(\dfrac{dy}{dx}\right)^2-16y=0$

c) $\left(\dfrac{dy}{dx}\right)^2+4y=0$ d) $\left(\dfrac{dy}{dx}\right)^2+16y=0$

50) $\left(1+x^2\right)=A\left(1+y^2\right)$ is a general solution of the differential equation

a) $\dfrac{dy}{dx}+\dfrac{1+x^2}{1-y^2}=0$ b) $\dfrac{x}{y}\dfrac{dy}{dx}+\left(\dfrac{1+x^2}{1-y^2}\right)=0$

c) $\left(\dfrac{1+x^2}{1-y^2}\right)\dfrac{dy}{dx}+\dfrac{x}{y}=0$ d) $\dfrac{dy}{dx}+\dfrac{x}{y}\left(\dfrac{1+x^2}{1-y^2}\right)=0$

51) The differential equation representing the family of loops $y^2=c\left(4+e^{2x}\right)$ is

a) $\left(4+e^{2x}\right)\dfrac{dy}{dx}+4ye^{2x}=0$ b) $\left(4+e^{2x}\right)\dfrac{dy}{dx}+y=0$

c) $\dfrac{dy}{dx}-ye^{2x}=0$ d) $\left(4+e^{2x}\right)\dfrac{dy}{dx}-ye^{2x}=0$

52) The differential equation whose general solution is $y=\sqrt{3x+c}$, is given by

a) $\dfrac{dy}{dx}-3y=0$ b) $2y\dfrac{dy}{dx}+3=0$

c) $2y\dfrac{dy}{dx}-3=0$ d) $2\dfrac{dy}{dx}-3y=0$

53) By eliminating the arbitrary constant A from $y=A\cos(x+3)$ the differential equation is

a) $\dfrac{dy}{dx}+y=0$ b) $\dfrac{dy}{dx}+y\cot(x+3)=0$

c) $\tan(x+3)\dfrac{dy}{dx}+y=0$ d) $\cot(x+3)\dfrac{dy}{dx}+y=0$

54) By eliminating the arbitrary constant c, the differential equation of $\cos(y-x)=ce^{-x}$ is

a) $x^2y_1-xy=4y_1$ b) $\tan(y-x)\left(\dfrac{dy}{dx}-1\right)-1=0$

c) $xy_1-y+x\sin\left(\dfrac{y}{x}\right)=0$ d) none of these

55) The differential equation whose generalized solution is $\sin(y-x)=ce^{-\frac{x^2}{2}}$, is given by

a) $\tan(y-x)\left(\frac{dy}{dx}-1\right)+x=0$

b) $\cot(y-x)\left(\frac{dy}{dx}+1\right)+y=0$

c) $\left(\frac{dy}{dx}-1\right)+\frac{x}{\cot(y-x)}=0$

d) $\cot(y-x)\left(\frac{dy}{dx}-1\right)+x=0$

56) The differential equation of the family of curves $y=Ae^{-x^2}$ is given by

a) $y\frac{dy}{dx}-2x^2=0$ b) $\frac{dy}{dx}+2xy=0$

c) $y\frac{dy}{dx}+2\log x=0$ d) $\frac{dy}{dx}-x^2y=0$

57) The differential equation whose general solution is $y=Ae^{\frac{x}{y}}$, is given by

a) $(x+y)y_1-y=0$ b) $(x+y)^2y_1+y=0$

c) $(x-y)y_1+y=0$ d) $xy_1-\frac{y}{x}=0$

58) By eliminating the arbitrary constant c from the function $y=5ce^{\frac{x}{y}}$, the differential equation is

a) $(x+y)\frac{dy}{dx}-y=0$ b) $\frac{dy}{dx}-\frac{y}{x+y}=0$

c) $\left(\frac{x+y}{x}\right)\frac{dy}{dx}-\frac{y}{x}=0$ d) $\frac{dy}{dx}-\frac{y-x}{x+y}=0$

59) The differential equation for the function $\sin\left(\frac{y}{x}\right)=Ax$ is obtained by eliminating A and is given by

a) $\frac{dy}{dx}+\frac{y}{x}=x\tan\left(\frac{y}{x}\right)$ b) $\frac{dy}{dx}+xy=\tan\left(\frac{y}{x}\right)$

c) $x\frac{dy}{dx}-y=x\cot\left(\frac{y}{x}\right)$ d) $x\frac{dy}{dx}-y=x\tan\left(\frac{y}{x}\right)$

60) The differential equation of $\cos\left(\frac{y}{x}\right)=cx$ is

a) $xy_1-y+x\cot\left(\frac{y}{x}\right)=0$

b) $xy_1-y+x\sin\left(\frac{y}{x}\right)=0$

c) $x^2y_1-y+x=0$

d) $x^2y_1-y+x\sin\left(\frac{y}{x}\right)=0$

61) The differential equation for the function $xy=c^2$, where c is arbitrary constant, is

a) $x\frac{dy}{dx}-y=0$ b) $\frac{dy}{dx}+xy=0$

c) $x\frac{dy}{dx}+y=0$ d) $x\left(\frac{dy}{dx}\right)^2+y=0$

62) The differential equation satisfying the general solution $xy=ce^x$ is

a) $x^2y_1-xy+e^x=0$ b) $xy_1+y=e^x$

c) $xy_1+y(1+x)=0$ d) $xy_1+y(1-x)=0$

63) The differential equation whose general solution is $y^2=2c\left(x+\sqrt{c}\right)$, where c is arbitrary constant, is

a) $2\frac{dy}{dx}\left(x+\sqrt{y\frac{dy}{dx}}\right)-y=0$ b) $x+\sqrt{y\frac{dy}{dx}}-y=0$

c) $\frac{dy}{dx}\left(x+\sqrt{y\frac{dy}{dx}}\right)+y=0$ d) $2x\frac{dy}{dx}+y\left(\frac{dy}{dx}\right)^2=0$

64) The differential equation satisfying the function $y=Ax+Bx^2$ is given by

a) $x^2y_2-4xy_1+y=0$ b) $y_2^{\,2}+2xy_1+2y=0$

c) $x^2y_2-2xy_1+2y=0$ d) $x^2y_2+xy_1+y=0$

65) By eliminating the arbitrary constants c_1, c_2 from the function $y=\sqrt{4x^2+c_1x+c_2}$ we get the differential equation

a) $y_2+xy_1=0$ b) $yy_2+y_1^{\,2}=4$

c) $x^2y_1y_2-y^2=0$ d) $x^2y_2+xy_1+4y=0$

66) $\frac{x^2}{4}-\frac{y^2}{a}=1$ is a general solution of

a) $xy_1-4y=xy$ b) $x^2y_1-4xy_1+16y=0$

c) $x^2y_1-4y_1-xy=0$ d) none of these

67) The differential equation representing the family of ellipse $\frac{x^2}{a^2}+\frac{y^2}{9}=1$, is given by

a) $y\frac{dy}{dx}-x^2y+9=0$ b) $xy\frac{dy}{dx}-y^2+9=0$

c) $xy\frac{dy}{dx}-y^2=0$ d) $xy\frac{dy}{dx}+y^2-9=0$

68) The differential equation whose primitive is $y^2=4A(x-B)$, where A and B are arbitrary constants, is

a) $x^2y_1y_2-y^2=0$ b) $x^2y_2+xy_1+4y=0$

c) $y_2+xy_1=0$ d) $yy_2+{y_1}^2=0$

69) On the elimination of the arbitrary constants A and B as well from $y^2=5A(x-3B)$, the differential equation formed is

a) $\frac{d^2y}{dx^2}+y=0$ b) $y^2\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$

c) $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$ d) $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2-y=0$

70) The differential equation with general solution $x=A\cos(B-5t)$ is given by

a) $\frac{d^2x}{dt^2}+5\frac{dx}{dt}-25t=0$ b) $\frac{d^2x}{dt^2}-\frac{dx}{dt}-xt=0$

c) $\frac{d^2x}{dt^2}-25x=0$ d) $\frac{d^2y}{dx^2}-25y=0$

71) The differential equation whose general solution is $y=\log(Ax+B)$ is

a) $y_2+{y_1}^2=0$ b) $x^2y_2+{y_1}^2=0$

c) $y_2+x{y_1}^2+y=0$ d) $xy_2+{y_1}^2-y=0$

72) $y=A\sin x+B\cos x$ is the solution satisfying the differential equation

a) $\frac{d^2y}{dx^2}+\frac{y}{x}=0$ b) $y^2\frac{d^2y}{dx^2}+xy+x=0$

c) $\frac{d^2y}{dx^2}+xy=0$ d) $\frac{d^2y}{dx^2}+y=0$

73) The differential equation whose general solution is $y=A\sin 3x+B\cos 3x$ where A, B are arbitrary constants, is

a) $x^2y_2-xy-9y_1=0$ b) $xy_2-9y_1+y=0$

c) $y_2-9y=0$ d) $y_2+9y=0$

74) The differential equation whose solution is $y=A\cos\frac{4x}{3}+B\sin\frac{4x}{3}$, where A and B are arbitrary constants, is given by

a) $\frac{d^2y}{dx^2}-\frac{dy}{dx}-\frac{4}{3}y=0$ b) $\frac{d^2y}{dx^2}+\frac{16}{9}y=0$

c) $9\frac{d^2y}{dx^2}-16y=0$ d) $\frac{d^2y}{dx^2}+\frac{dy}{dx}+\frac{16}{9}y=0$

75) The differential equation whose primitive is $y=A\cos\log x+B\sin\log x$, where A and B are arbitrary constants, is given by

a) $x^2y_2+y_1+xy=0$ b) $x^2y_2+xy_1+y=0$

c) $x^2y_2+y_1+y=0$ d) $y_2-x^2y_1-xy=0$

76) The differential equation whose general solution is $y=Ae^{-x}+B$, where A and B are arbitrary constants, is

a) $y=x^2y_2+y_1$ b) $x^2y_2+xy_1+y=0$

c) $y_2+y_1=0$ d) $x{y_2}^2+y_1=0$

77) $y=Ae^{-x}+Be^{-x}$, where A and B both are arbitrary constants, is the solution for the differential equation

a) $x\frac{d^2y}{dx^2}+y=0$ b) $\frac{d^2y}{dx^2}-\frac{dy}{dx}-y=0$

c) $\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=0$ d) $\frac{d^2y}{dx^2}-y=0$

78) By eliminating the arbitrary constants A and B both from the function $xy=Ae^{x}+Be^{-x}$, we get the differential equation

a) $\frac{x}{y}\frac{d^2y}{dx^2}+2\frac{dy}{dx}-\frac{x}{y}=0$ b) $x\frac{d^2y}{dx^2}+2\frac{dy}{dx}-xy=0$

c) $y\frac{d^2y}{dx^2}+2\frac{dy}{dx}-xy=0$ d) $\frac{d^2y}{dx^2}+\frac{dy}{dx}+xy=0$

79) The differential equation, whose solution is given by $y = Ae^{-3x} + Be^{3x}$, is

a) $xy_2^{\,2} + y_1 - xy = 0$ b) $x^2 y_2 + y_1 + xy = 0$

c) $x^2 y_2 - xy_1 + y = 0$ d) $y_2 - 4y = 0$

80) $e^{-t} y = A + Bt$ is a general solution of the differential equation

a) $y_2 - 2y_1 + y = 0$ b) $y_2 + y_1 t + yt^2 = 0$

c) $xy_2 + y_1 + y = 0$ d) $4y_2 + 2y_1 + y = 0$

81) The differential equation having generalized solution $e^{-t} x = At - B$ is given by

a) $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = 0$ b) $x\frac{d^2x}{dt^2} + \frac{dx}{dt} + xt = 0$

c) $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + t = 0$ d) $x^2\frac{d^2x}{dt^2} - 2xt + x = 0$

82) The general form of the differential equation of I order and I degree can be expressed as

a) $\frac{dy}{dx} = c$ b) $M(x, y)dx + N(x, y)dy = 0$

c) $\frac{dy}{dx} + y = du$ d) $M(x, y)dx + N(x, y)dy = du$

83) The differential equation of the form $f_1(x)dx + f_2(y)dy = 0$ is known as

a) Linear form b) Non homogeneous form

c) exact form d)variable separable form

84) The differential equation in the form $\frac{dy}{dx} = x^n f\left(\frac{y}{x}\right)$ is known as

a) Linear form b) homogeneous form

c) exact form d) variable separable form

85) The differential equation in the form $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$, where f and g both are homogeneous functions of x and y of the same degree, is known as

a) Linear form b) homogeneous form

c) exact form d) variable separable form

86) The homogenous differential equation of the form $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$ is solved by substitution

a) no substitution, direct solution b) $x^n = v$

c) $xy = v$ d) $\frac{y}{x} = v$

87) The differential equation of the form $M(x, y)dx + N(x, y)dy = 0$ is exact, if

a) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ b) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

c) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ d) $\frac{\partial M}{\partial y} \times \frac{\partial N}{\partial x} = 1$

88) The differential equation $\frac{dy}{dx} = e^{2x+y} + 3x^4 e^y$ is of the form

a) Linear form b) Non homogeneous form

c) exact form d) variable separable form

89) The form of the differential equation $(y^3 - 3x^2 y)dx + (x^2 y + 3x^3)dy = 0$ is

a) Linear form b) homogeneous form

c) exact form d) variable separable form

90) The differential equation is of the form $(x + y)dx + (x - y + 1)dy = 0$

a) Linear form b) non homogeneous form

c) exact form d) variable separable form

91) The differential equation $xy - \frac{dy}{dx} = y^3 e^{-x^2}$ is of the form

a) Linear form b) non homogeneous form

c) exact form d) variable separable form

92) The substitution which can be used to solve the equation $(x + y + 7)dx + (3x + 3y - 7)dy = 0$ is

a) $x + y = v$ b) $x - y = v$

c) $xy = v$ d) $\frac{y}{x} = v$

93) The general solution of the differential equation $\frac{3e^x}{1 - e^x}dx + \frac{\sec^2 y}{\tan y}dy = 0$ is

a) $\tan y = c(1 - e^x)^3$ b) $(1 - e^x)^3 \tan y = c$

c) $(1 - e^{-x})^3 \cot y = c$ d) $\cot y = c(1 - e^x)^3$

94) The general solution of the differential equation $\frac{dy}{dx}+y=0$ is

a) $y=ce^{-x}$ b) $y=Ae^{-x}+B$

c) $y=ce^{x}$ d) $x=ce^{-y}$

95) The general solution of the differential equation $\frac{dx}{dy}+x=0$ is

a) $y=ce^{-x}$ b) $y=Ae^{-x}+B$

c) $y=ce^{x}$ d) $x=ce^{-y}$

96) The general solution of the differential equation $\frac{dy}{dx}+x=0$ is

a) $y=ce^{-x}$ b) $y^2+2x=c$

c) $x^2+2y=c$ d) $x=ce^{-y}$

97) The general solution of the differential equation $ydx+xdy=0$ is

a) $x^2+y^2=c$ b) $xy=c$ c) $\frac{y}{x}=c$ d) $\frac{x}{y}=c$

98) The general solution of the differential equation $\frac{dy}{dx}+\tan x=0$ is

a) $y=\log\sin x+c$ b) $y-\log\sec x=c$

c) $y=\log\sec x+c$ d) $y=\log\cos x+c$

99) The general solution of the differential equation $\frac{dy}{dx}+xy=0$ is

a) $\log x+\log y=c$ b) $\frac{x^2}{2}+\log y=c$

c) $x^2+\log y=c$ d) $x^2+y^2=c$

100) The general solution of the differential equation $\frac{dy}{dx}+\frac{1+x}{1+y}=0$ is

a) $x^2+y^2+2x+2y=c$ b) $(x+y)^2+2(x+y)=c$

c) $x^2+y^2+x+y=c$ d) $(1+x)=c(1+y)$

101) The general solution of the differential equation $\frac{dy}{dx}=\frac{1+y}{1+x}$ is

a) $(1+x)=c(1+y)^2$ b) $(1+y)=c(1+x)$

c) $(1+x)=c(1+y)$ d) $x=cy$

102) The general solution of the differential equation $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$ is

a) $\log\left(\frac{1+x^2}{1+y^2}\right)$ b) $\log(1+x^2)+\log(1+y^2)=c$

c) $\tan^{-1}x+\tan^{-1}y=c$ d) $\tan^{-1}x-\tan^{-1}y=c$

103) The general solution of the differential equation $\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$ is

a) $\frac{1}{2}\log\left(\frac{1-y^2}{1-x^2}\right)=c$ b) $\sec^{-1}x+\sec^{-1}y=c$

c) $\tan^{-1}x+\tan^{-1}y=c$ d) $\sin^{-1}x+\sin^{-1}y=c$

104) The general solution of the differential equation $x(1+y^2)dx+y(1+x^2)dy=0$ is

a) $(1+y^2)(1+x^2)=c$ b) $\log\left(\frac{1+y^2}{1+x^2}\right)=c$

c) $(1+y^2)=c(1+x^2)$ d) $\tan^{-1}x+\tan^{-1}y=c$

105) The general solution of the differential equation $\frac{dy}{dx}=(1+x)(1+y^2)$ is

a) $\log(1+y^2)=x+\frac{x^2}{2}+c$ b) $\tan^{-1}y=x+\frac{x^2}{2}+c$

c) $\log(1+x)+\tan^{-1}y=c$ d) $\tan^{-1}y+x+x^2=c$

106) The general solution of the differential equation $(e^x+1)ydy=(y+1)e^xdx$ is

a) $y+\log(y+1)+\log(e^x+1)=c$

b) $x+\log(y+1)=\log(e^x+1)+c$

c) $y-\log(y+1)=\log(e^x+1)+c$

d) $\frac{y^2}{2}+\log(y+1)=\log(e^x+1)+c$

107) The general solution of the differential equation $\frac{dy}{dx}=e^{x+y}+e^{y-x}$ is

a) $e^{-x}-e^{x}-e^{-y}=c$ b) $e^{x}-e^{2x}-e^{-y}=c$

c) $e^{-x}+e^{x}+e^{-y}=c$ d) $e^{x}-e^{-x}-e^{y}=c$

108) The general solution of the differential equation $\frac{dy}{dx}=e^{x-y}+3x^2e^{-y}$ is

a) $\frac{e^x+x^3}{e^y}=c$ b) $e^{x-y}=e^y+x^3+c$

c) $e^y=e^x+x^3+c$ d) $e^y+e^x+x^3=c$

109) The general solution of the differential equation $y(1+\log x)\frac{dx}{dy}-x\log x=0$ is

a) $\frac{x}{\log x}=yc$ b) $\frac{x}{y}\log x=y+c$

c) $x(\log x+1)=yc$ d) $x\log x=yc$

110) The general solution of the differential equation $\sec^2 x\tan y dx+\sec^2 y\tan x dy=0$ is

a) $\tan x\tan y=c$ b) $\tan x=c\tan y$

c) $\tan x+\tan y=c$ d) $\tan y=c\tan x$

111) The general solution of the differential equation $y\sec^2 x+(y-5)\tan x\frac{dy}{dx}=0$ is

a) $y^5-y+\tan x=c$ b) $y+5\log y+\log\sec x=c$

c) $y+5\log\frac{\tan x}{y}=c$ d) $y-5\log y+\log\tan x=c$

112) The general solution of the differential equation $e^x\cos y+(1+e^x)\sin y\frac{dy}{dx}=0$ is

a) $(1+e^x)\tan y=c$ b) $(1+e^x)\sec y=c$

c) $(1+e^x)\cos y=c$ d) $\sec y=c(1+e^x)$

113) The general solution of the differential equation $e^y\cos x dx+(e^y+1)\sin x dy=0$ is

a) $\sec x(e^y+1)=c$ b) $\sin x=c(e^y+1)$

c) $\sin y(1+e^x)=c$ d) $\sin x(e^y+1)=c$

114) The general solution of the differential equation $(4+e^{2x})\frac{dy}{dx}=ye^{2x}$ is

a) $\frac{y^2}{2}=A+(4+e^{2x})$ b) $y^2(4+e^{2x})=A$

c) $y^2=A(4+e^{2x})$ d) $x^2=A(4+e^{2x})$

115) The general solution of the differential equation $y-x\frac{dy}{dx}=2\left(y+\frac{dy}{dx}\right)$ is

a) $(x+2)y=c$ b) $x+2y=c$

c) $y=c(x+2)$ d) $(x+2)^2y=c$

116) The general solution of the differential equation $(x+1)\frac{dy}{dx}+1=2e^{-y}$ is

a) $(x+1)(2+e^{-y})=c$ b) $(2-e^y)=c(x+1)$

c) $(x+1)(2-e^y)=c$ d) $(x+1)=c(2-e^y)$

117) The general solution of the differential equation $x^3\left(x\frac{dy}{dx}+y\right)-\sec(xy)=0$ is

a) $\sin(xy)=2cx^2$ b) $\sin(xy)-\frac{1}{2x^2}=c$

c) $\sec(xy)+\frac{1}{2x^2}=c$ d) $\sin(xy)+\frac{1}{2x^2}=c$

118) The general solution of the differential equation $(y-ay^2)dx=(a+x)dy$ is

a) $\log(a+x)+\frac{1}{2}\log(1-ay)-\frac{1}{3}\log y=c$

b) $\log(a+x)-\frac{1}{a}\log(1-ay)-\log y=c$

c) $\log(a+x)+\log(1-ay)-\log y=c$

d) $\log(a+x)+\frac{\log(1-ay)}{-a}+\log y=c$

119) The necessary and sufficient condition for the equation $M(x,y)dx+N(x,y)dy=0$ to be exact is

a) $\frac{\partial M}{\partial x}=\frac{\partial N}{\partial y};\ My+Nx=0$

b) $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x};\ My+Nx\neq 0$

c) $\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x};\ My+Nx\neq 0$

d) $\frac{\partial M}{\partial y}\times\frac{\partial N}{\partial x}=1;\ My+Nx\neq 0$

120) If the differential equation $Mdx+Ndy=0$ is a homogeneous but not exact, its integrating factor is

a) $\dfrac{1}{Mx-Ny};\ My-Nx\neq 0$

b) $\dfrac{1}{Mx+Ny};\ Mx+Ny\neq 0$

c) $\dfrac{1}{My-Nx};\ My-Nx\neq 0$

d) $\dfrac{1}{My+Nx};\ My+Nx\neq 0$

121) If the differential equation $Mdx+Ndy=0$ is not exact but can be expressed in the form $yf_1(xy)dx+xf_2(xy)dy=0$, its integrating factor is

a) $\dfrac{1}{Mx+Ny};\ Mx+Ny\neq 0$

b) $\dfrac{1}{My-Nx};\ My-Nx\neq 0$

c) $\dfrac{1}{My+Nx};\ My+Nx\neq 0$

d) $\dfrac{1}{Mx+Ny};\ Mx+Ny=0$

122) If the differential equation $Mdx+Ndy=0$ is not exact and $\dfrac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}=f(x)$, its integrating factor is

a) $e^{\int f(x)dx}$ b) $e^{f(x)}$

c) $e^{\int f(y)dy}$ d) $f(x)$

123) If the differential equation $Mdx+Ndy=0$ is not exact and $\dfrac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{-M}=f(y)$, its integrating factor is

a) $e^{\int f(x)dx}$ b) $e^{f(x)}$

c) $e^{\int f(y)dy}$ d) $f(x)$

124) The total derivative of $dx+dy$ is

a) $d\left(\dfrac{x}{y}\right)$ b) $d(x+y)$

c) $d(x-y)$ d) $d(xy)$

125) The total derivative of $dx-dy$ is

a) $d\left(\dfrac{x}{y}\right)$ b) $d(x+y)$

c) $d(x-y)$ d) $d(xy)$

126) The total derivative of $xdy+ydx$ is

a) $d\left(\dfrac{x}{y}\right)$ b) $d(x+y)$

c) $d(x-y)$ d) $d(xy)$

127) The total derivative of $xdy-ydx$ with the integrating factor $\dfrac{1}{x^2}$ is

a) $d(x-y)$ b) $d\left(\dfrac{x}{y}\right)$

c) $d\left(\dfrac{y}{x}\right)$ d) $d(xy)$

128) The total derivative of $2(xdx+ydy)$ is

a) $d(x+y)$ b) $d(xy)$

c) $d(xy)^2$ d) $d\left(x^2+y^2\right)$

129) The total derivative of $2(xdx-ydy)$ is

a) $d(xy)$ b) $d\left(\dfrac{x^2}{y^2}\right)$

c) $d\left(x^2-y^2\right)$ d) $d\left(x^2+y^2\right)$

130) The total derivative of $\dfrac{ydx-xdy}{y^2}$ is

a) $d\left(\dfrac{y}{x}\right)$ b) $d\left(\dfrac{x}{y}\right)$

c) $d\left(\dfrac{x-y}{y}\right)$ d) $d\left(x^2-y^2\right)$

131) The total derivative of $ydx-xdy$ with the integrating factor $\dfrac{1}{y^2}$ is

a) $d\left(\dfrac{x}{y}\right)$ b) $d\left(\dfrac{y}{x}\right)$

c) $d\left(\dfrac{x-y}{y}\right)$ d) $d\left(x^2-y^2\right)$

132) The total derivative of $dx+dy$ with the integrating factor $\frac{1}{x+y}$ is

a) $d\left[\log(x-y)\right]$ b) $d\left[\log(x+y)\right]$

c) $d\left[\log(xy)\right]$ d) $d\left[\log\left(x^2+y^2\right)\right]$

133) The total derivative of $dx-dy$ with the integrating factor $\frac{1}{x-y}$ is

a) $d\left[\log(x-y)\right]$ b) $d\left[\log(x+y)\right]$

c) $d\left[\log(xy)\right]$ d) $d\left[\log\left(x^2+y^2\right)\right]$

134) The total derivative of $xdy+ydx$ with the integrating factor $\frac{1}{xy}$ is

a) $d\left[\log(x-y)\right]$ b) $d\left[\log(x+y)\right]$

c) $d\left[\log(xy)\right]$ d) $d\left[\log\left(x^2+y^2\right)\right]$

135) The total derivative of $xdy-ydx$ with the integrating factor $\frac{1}{xy}$ is

a) $d\left[\log(x-y)\right]$ b) $d\left[\log\left(\frac{x}{y}\right)\right]$

c) $d\left[\log\left(\frac{y}{x}\right)\right]$ d) $d\left[\log(xy)\right]$

136) The total derivative of $2(xdx+ydy)$ with the integrating factor $\frac{1}{x^2+y^2}$ is

a) $d\left[\log(x-y)\right]$ b) $d\left[\log(x+y)\right]$

c) $d\left[\log(xy)\right]$ d) $d\left[\log\left(x^2+y^2\right)\right]$

137) The total derivative of $2(xdx-ydy)$ with the integrating factor $\frac{1}{x^2-y^2}$ is

a) $d\left[\log\left(x^2-y^2\right)\right]$ b) $d\left[\log(x+y)\right]$

c) $d\left[\log(xy)\right]$ d) $d\left[\log\left(x^2+y^2\right)\right]$

138) The total derivative of $xdy-ydx$ with the integrating factor $\frac{1}{x^2+y^2}$ is

a) $d\left[\log\left(x^2-y^2\right)\right]$ b) $d\left[\log\left(x^2+y^2\right)\right]$

c) $d\left(\tan^{-1}\frac{y}{x}\right)$ d) $d\left(\tan^{-1}\frac{x}{y}\right)$

139) If the integrating factor of $\frac{ydx-xdy}{y^2}$ is $\frac{y}{x}$, its total derivative is

a) $d\left(\tan^{-1}\frac{x}{y}\right)$ b) $d\left(\log(x+y)\right)$

c) $d\left(\log\frac{y}{x}\right)$ d) $d\left(\log\frac{x}{y}\right)$

140) If the integrating factor of $\frac{xdy-ydx}{x^2}$ is $\frac{x}{y}$, its total derivative is

a) $d\left(\tan^{-1}\frac{y}{x}\right)$ b) $d\left(\tan^{-1}\frac{x}{y}\right)$

c) $d\left(\log\frac{x}{y}\right)$ d) $d\left(\log\frac{y}{x}\right)$

141) If the integrating factor of $\frac{ydx-xdy}{y^2}$ is $\frac{y^2}{x^2+y^2}$, its total derivative is

a) $d\left(\log\frac{y}{x}\right)$ b) $d\left(\tan^{-1}\frac{y}{x}\right)$

c) $d\left(\tan^{-1}\frac{x}{y}\right)$ d) $\log\left(x^2+y^2\right)$

142) The total derivative of $dx+dy$ with the integrating factor $\frac{1}{1+(x+y)^2}$ is

a) $d\left(\tan^{-1}(x+y)\right)$ b) $d\left(\log\frac{y}{x}\right)$

c) $d\left(\sec^{-1}(x+y)\right)$ d) $\log(x+y)$

143) The equation $(x+y+3)dx+(x-y-7)dy=0$ is of the form

a) variable separable b)exact differential

c) linear differential d) homogeneous

144) Equation $(3x+2y+1)dx+(2x-7y-3)dy=0$ is of the form
a) variable separable b) exact differential
c) linear differential d) homogeneous

145) For what value of λ, the differential equation $(5x+\lambda y-3)dx+(3x-7y+5)dy=0$ is exact?
a) 0 b) 1 c) 2 d)3

146) For what value of a, the differential equation $(xy^2+ax^2y)dx+(x^3+x^2y)dy=0$ is exact?
a)3 b) 2 c) 1 d) 5

147) For what value of a, the differential equation $(\tan y+ax^2y-y)dx+(x\tan^2 y-x^3-\sec^2 y)dy=0$ is exact?
a) 2 b) -2 c) 3 d)-3

148) The differential equation $\frac{dy}{dx}=\frac{ay+1}{(y+2)e^y-x}$ is exact, if the value of a is
a) -2 b) 2 c) -1 d)1

149) Differential equation $\frac{dy}{dx}+\frac{3+ay\cos x}{2\sin x-4y^3}=0$ is exact, if the value of a is
a) -3 b) 3 c)2 d)-2

150) For what values of a and b, the differential equation $(ay^2+x+x^8)dx+(y^2+y-bxy)dy=0$ is an exact differential equation?
a) $2a+b=0$ b) $a=2b$
c) $a-2b=3$ d) $a=1=b$

151) The equation $(1+axy^2)dx+(1+bx^2y)dy=0$ is exact differential equation, if
a) $a+2b=0$ b) $a=1,\ b=-3$
c) $a=b$ d) $a=2,\ b=3$

152) For what values of a and b, differential equation $(axy^4+\sin y)dx+(bx^2y^3+x\cos y)dy=0$ is formed to be exact?
a) $a=3b$ b) $a=2,\ b=4$
c) $a+b=1$ d) $a=3,\ b=-3$

153) The integrating factor for the differential equation $(y^2-2xy)dx+(2x^2+3xy)dy=0$ is
a) $\frac{1}{4xy^2}$ b) $\frac{1}{4x^2y^2}$ c) $\frac{1}{2x^2y}$ d) $\frac{1}{2xy}$

154) The integrating factor for the differential equation $(xy-2y^2)dx-(x^2-3xy)dy=0$ is
a) $\frac{1}{2x^2y^2}$ b) $\frac{1}{x^2y}$ c) $\frac{1}{xy}$ d) $\frac{1}{xy^2}$

155) The integrating factor for the differential equation $(x^2-3xy+2y^2)dx-(2xy-3x^2)dy=0$ is
a) $\frac{1}{x^3}$ b) $\frac{1}{x^3y}$ c) $\frac{1}{y^3}$ d) $\frac{1}{x^2y^2}$

156) The differential equation $(y^3-2x^2y)dx+(2xy^2-x^3)dy=0$ can be reduced to exact if the equation is multiplied by the integrating factor
a) $\frac{1}{xy}$ b) x^2y^2 c) $\frac{1}{x^2y^2}$ d) xy

157) The integrating factor for the differential equation $(x^2y-2xy^2)dx+(3x^2y-x^3)dy=0$ is
a) $\frac{1}{2xy}$ b) $\frac{1}{x^2y}$ c) $\frac{1}{x^2y^2}$ d) $\frac{1}{xy^2}$

158) The integrating factor for the differential equation $(xy+1)ydx-(xy-1)xdy=0$ is
a) $\frac{1}{2x^2y^2}$ b) $\frac{1}{2x^2y}$ c) $\frac{1}{2xy^2}$ d) $\frac{1}{2xy}$

159) The integrating factor for the differential equation $(xy+1)ydx+(x^2y^2+xy+1)xdy=0$ is
a) $\frac{1}{x^3y}$ b) $-\frac{1}{x^3y^3}$ c) $-\frac{1}{x^2y^2}$ d) $\frac{1}{xy^3}$

160) The integrating factor for the equation $(x^2y^2+xy+1)ydx+(x^2y^2-xy+1)xdy=0$ is
a) $\frac{1}{2x^2y^2}$ b) $\frac{1}{2x^2y}$ c) $\frac{1}{2xy^2}$ d) $\frac{1}{2x^3y^3}$

161) The integrating factor for the equation $\left(x^2y^2+5xy+2\right)ydx+\left(x^2y^2+4xy+2\right)xdy=0$ is

a) $\frac{1}{x^2y^3}$ b) $\frac{1}{x^2y}$ c) $\frac{1}{xy^2}$ d) $\frac{1}{x^2y^2}$

162) The differential equation $y\left(xy+2x^2y^2\right)dx+x\left(xy-x^2y^2\right)dy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) $\frac{1}{2x^2y}$ b) $\frac{1}{3x^3y}$ c) $\frac{1}{2x^2y^2}$ d) $\frac{1}{3x^3y^3}$

163) The integrating factor for the differential equation $\left(x^2+y^2+x\right)ydx+xydy=0$ is

a) $\frac{1}{2xy^2}$ b) $\frac{1}{2xy}$ c) x d) $\frac{1}{x}$

164) The integrating factor for the equation $(xy\sin xy+\cos xy)ydx+(xy\sin xy-\cos xy)xdx=0$ is

a) $\frac{1}{2xy}$ b) $\frac{1}{2xy\cos xy}$

c) $\frac{1}{2xy\sin xy}$ d) $\frac{1}{2\cos xy}$

165) The integrating factor for the differential equation $\left(y+\frac{y^3}{3}+\frac{x^2}{2}\right)dx+\left(\frac{x+xy^2}{4}\right)dy=0$ is

a) $\frac{1}{x^2}$ b) x^2 c) $\frac{1}{x^3}$ d) x^3

166) The integrating factor for the differential equation $(2x\log x-xy)dy+2ydx=0$ is

a) x^2 b) $\frac{1}{x}$ c) $\frac{1}{x^2}$ d) $\frac{1}{x^3}$

167) The integrating factor for the differential equation $\left(x^2+y^2+1\right)dx-2xydy=0$ is

a) x^2 b) $\frac{1}{x}$ c) $\frac{1}{x^2}$ d) $\frac{1}{x^3}$

168) The integrating factor for the differential equation $y\left(2xy+e^x\right)dx-e^xdy=0$ is

a) $\frac{1}{y^2}$ b) $\frac{1}{x^2}$ c) $\frac{1}{y^3}$ d) $\frac{1}{x^3}$

169) The integrating factor for the differential equation $y\log ydx+(x-\log y)dy=0$ is

a) $\frac{1}{y^2}$ b) $\frac{1}{x^2}$ c) $\frac{1}{y}$ d) $\frac{1}{x}$

170) The differential equation $\left(y^4+2y\right)dx+\left(xy^3+2y^4-4x\right)dy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) $\frac{1}{y^2}$ b) $\frac{1}{y^3}$ c) $\frac{1}{x^3}$ d) $\frac{1}{x^2}$

171) The differential equation $\left(2x+e^x\log y\right)ydx+e^xdy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) x^2 b) $\frac{1}{x^3}$ c) $\frac{1}{x}$ d) $\frac{1}{y}$

172) The differential equation $\left(xy^3+y\right)dx+2\left(x^2y^2+x+y^4\right)dy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) x b) y c) $\frac{1}{y}$ d) $\frac{1}{x}$

173) The differential equation $\left(xy^2-e^{\frac{1}{x^3}}\right)dx-x^2ydy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) $\frac{1}{x^4}$ b) $\frac{1}{x^3}$ c) $\frac{1}{y^2}$ d) $\frac{1}{y^3}$

174) $\left(x^2-3xy+2y^2\right)dx-\left(e^x+y^3\right)dy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) $\frac{1}{y^2}$ b) $\frac{1}{y^3}$ c) $\frac{1}{x^3}$ d) $\frac{1}{x^4}$

175) $\left(y^4+2y\right)dx+\left(xy^3+2y^4-4x\right)dy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) $\frac{1}{x^3}$ b) $\frac{1}{y^2}$ c) $\frac{1}{y^3}$ d) $\frac{1}{x^4}$

176) The solution of the exact differential equation $(x+y-2)dx+(x-y+4)dy=0$ is

a) $x^2+y^2+xy+x+y+c=0$

b) $x^2+y^2+2xy+4x+6y+c=0$

c) $x^2+y^2+2xy+4x+8y+c=0$

d) $x^2-y^2+2xy-4x+8y+c=0$

177) The solution of the exact differential equation $\left(y^2e^{xy^2}+4x^3\right)dx+\left(2xye^{xy^2}-3y^2\right)dy=0$ is

a) $\frac{1}{y^2}e^{xy^2}+\frac{x^4}{4}-\frac{y^3}{3}=c$ b) $e^{xy^2}+x^4-y^3=c$

c) $e^{xy^2}+x^4+y^3=c$ d) $e^{xy^2}+\frac{x^4}{4}-\frac{y^3}{3}=c$

178) The solution of the exact differential equation $\left(x^2-4xy-2y^2\right)dx+\left(y^2-4xy-2x^2\right)dy=0$ is

a) $x^3-6x^2y-6xy^2+y^3=c$

b) $\frac{x^3}{3}-6x^2y-6xy^2+\frac{y^3}{3}=c$

c) $x^3+x^2y+xy^2+y^3=c$

d) $x^3+x^2y-3xy^2+2y^3=c$

179) The solution of the exact differential equation $(1+\log xy)dx+\left(1+\frac{x}{y}\right)dy=0$ is

a) $y-x\log x+\log y=c$ b) $y+x\log xy=c$

c) $1+\frac{x}{y}\log xy=c$ d) $\frac{y}{x}+\log xy=c$

180) The solution of the exact differential equation $\left(1+x^2\right)(xdy+ydx)+2x^2ydx=0$ is

a) $x^2+y\left(1+x^2\right)=c$ b) $x+y-\left(1+x^2\right)=c$

c) $xy\left(1+x^2\right)=c$ d) $x+y\left(1+x^2\right)=c$

181) The solution of the exact differential equation $\cos y-x\sin y\frac{dy}{dx}=\sec^2 x$ is

a) $\frac{x}{y}\cos y=c\tan x$ b) $\cot x-x^2\cos y=c$

c) $\tan^2 x-x\sin y=c$ d) $\tan x-x\cos y=c$

182) The solution of the exact differential equation $\frac{dy}{dx}=\frac{1+y^2+3x^2y}{1-2xy-x^3}$ is

a) $x\left(1+y^2\right)+x^3y-y=c$

b) $\frac{1+y^2}{x}+x^2y-y=c$

c) $1+y^2+x^2y-xy=c$

d) $x\left(1+\frac{y^2}{2}\right)-\frac{x^3y}{3}-y=c$

183) The solution of the exact differential equation $\left(x^2y-2xy^2\right)dx+\left(3x^2y-x^3\right)dy=0$ with the integrating factor $\frac{1}{x^2y^2}$ is

a) $\frac{y}{x}-\log x+\log y=c$

b) $\frac{x}{y}-2\log x+3\log y=c$

c) $x+2y\log x+3x\log y=c$

d) $\frac{x^2}{2}-2y\log x+3\log y=c$

184) The solution of the exact differential equation $\left(3xy^2-y^3\right)dx+\left(xy^2-2x^2y\right)dy=0$ with the integrating factor $\frac{1}{x^2y^2}$ is

a) $\frac{y}{x}+3\log x+2\log y=c$

b) $y\log x+3\log x-2\log y=c$

c) $\frac{y}{x}+3\log x-2\log y=c$

d) $\frac{y^2}{x^2}+3x\log x-2y\log y=c$

185) The solution of the exact differential equation $\left(x^2-3xy+2y^2\right)dx+x(3x-2y)dy=0$ with the integrating factor $\frac{1}{x^3}$ is

a) $x^2\log x+3xy-y^2=cx^2$

b) $\log x+3x^2y-y^2=c$

c) $x^3\log x+3x^2y-xy^2=cx^3$

d) $3\log x+3xy+y^2=cx^2$

186) The solution of the exact differential equation $(1+xy)ydx+(1-xy)xdy=0$ with the integrating factor $\frac{1}{2x^2y^2}$ is

a) $3\log\left(\frac{x}{y}\right)+\frac{1}{x^2y^2}=c$ b) $\log\left(\frac{x}{y}\right)+\frac{1}{xy}=c$

c) $3\log\left(\frac{x}{y}\right)-\frac{1}{x^2y}=c$ d) $\log\left(\frac{x}{y}\right)-\frac{1}{xy}=c$

187) The solution of the exact differential equation $(x^2y^2+5xy+2)ydx+(x^2y^2+4xy+2)xdy=0$ with the integrating factor $\frac{1}{x^2y^2}$ is

a) $xy+5\log x-\frac{2}{xy}+4\log y=c$

b) $x^2y+5\log x-\frac{1}{xy}+2\log y=c$

c) $xy+5\log x+\frac{1}{xy}+3\log y=c$

d) $x^2y^2+5\log x+\frac{2}{xy}+4\log y=c$

188) The solution of the exact differential equation $(x^2y^2+xy+1)ydx+(x^2y^2-xy+1)xdy=0$ with the integrating factor $\frac{1}{2x^2y^2}$ is

a) $xy-\frac{1}{xy}+x\log x+y\log y=c$

b) $xy-\frac{1}{xy}+\log x+\log y=c$

c) $\frac{x}{y}-\frac{1}{xy}+\log\left(\frac{x}{y}\right)=c$

d) $xy-\frac{1}{xy}+\log\left(\frac{x}{y}\right)=c$

189) The solution of the exact differential equation $y(xy+2x^2y^2)dx+x(xy-x^2y^2)dy=0$ with the integrating factor $\frac{1}{3x^3y^3}$ is

a) $2\log\left(\frac{x}{y}\right)-\frac{1}{xy}=c$ b) $\frac{1}{2}\log\left(\frac{x}{y}\right)+\frac{1}{xy}=c$

c) $\log\left(\frac{x^2}{y}\right)-\frac{1}{xy}=c$ d) $\log\left(\frac{x}{y^2}\right)+\frac{1}{xy}=c$

190) The solution of the exact differential equation $(x^2+y^2+x)ydx+xydy=0$ with the integrating factor x is

a) $x^4+x^2y^3+x^3=c$ b) $y\left(\frac{x^4}{4}+\frac{x^2y^2}{2}+\frac{x^3}{3}\right)=c$

c) $y(x^4+x^2y^2+x^3)=c$ d) $\frac{x^4}{4}+\frac{x^2y^2}{2}+\frac{x^3}{3}=c$

191) The solution of the exact differential equation $(xy\sin xy+\cos xy)ydx+(xy\sin xy-\cos xy)xdx=0$ with the integrating factor $\frac{1}{2xy\cos xy}$ is

a) $x\log(\sec xy)=cy$ b) $xy\sec xy=c$

c) $x\sec xy=cy$ d) $x\cos xy=cy$

192) The solution of the exact differential equation $(x^2-3xy+2y^2)dx+(3x^2-2xy)dy=0$ with the integrating factor $\frac{1}{x^3}$ is

a) $\log x+\frac{3y}{x}-\left(\frac{y}{x}\right)^2=c$ b) $\log x+3yx-\left(\frac{y}{x}\right)^2=c$

c) $\log x+\frac{y}{x}+\left(\frac{y}{x}\right)^2=c$ d) $3\log x+\frac{y}{x}-\frac{y^2}{x}=c$

193) The solution of the exact differential equation $(xy^3+y)dx+2(x^2y^2+x+y^4)dy=0$ with the integrating factor y is

a) $\frac{3}{4}x^2y^4+\frac{6}{5}xy^2+2y^6=c$

b) $3x^2y^4+6x^2y+2x^6=c$

c) $x^3y^4+3xy^2+5y^6=c$

d) $3x^2y^4+6xy^2+2y^6=c$

194) The solution of the exact differential equation $(y^4+2y)dx+(xy^3-4x+2y^4)dy=0$ with the integrating factor $\frac{1}{y^3}$ is

a) $x(y^3+2)+y^2=c$ b) $x^2(y^3+2)-y^4=cy^2$

c) $x(y^3+2)+y^4=cy^2$ d) $(y^3+2)xy^4=cy^2$

195) The solution of the exact differential equation $\left(3x+2y^2\right)ydx+2x\left(2x+3y^2\right)dy=0$ with the integrating factor xy^3 is

a) $x^3y^4+x^2y^6=c$ b) $x^3y^3+x^4y^3=c$

c) $x^2y^4+xy^6=c$ d) $\frac{1}{3}x^3y^4+\frac{1}{4}x^2y^6=c$

196) The solution of the exact differential equation $\left(x^2y+y^4\right)dx+\left(2x^3+4xy^3\right)dy=0$ with the integrating factor $x^{\frac{5}{2}}y^{10}$ is

a) $\frac{12}{11}x^{\frac{11}{2}}y^{11}+\frac{12}{7}x^{\frac{7}{2}}y^{14}=c$

b) $\frac{2}{11}x^{\frac{11}{12}}y^{11}+\frac{2}{7}x^{\frac{7}{12}}y^{14}=c$

c) $\frac{2}{11}x^{\frac{11}{2}}y^{11}+\frac{2}{7}x^{\frac{7}{2}}y^{14}=c$

d) $\frac{2}{11}x^{\frac{11}{2}}y^{11}-\frac{2}{7}x^{\frac{7}{2}}y^{14}=c$

197) The solution of the exact differential equation $\left(y^2+2x^2y\right)dx+\left(2x^3-xy\right)dy=0$ with the integrating factor $\frac{1}{x^{5/2}y^{1/2}}$ is

a) $4xy-\frac{2}{3}\left(\frac{y}{x}\right)^{\frac{3}{2}}=c$ b) $4\sqrt{xy}-\frac{2}{3}\left(\frac{y}{x}\right)^{\frac{3}{2}}=c$

c) $4\sqrt{xy}-\frac{2}{3}\sqrt{\frac{y}{x}}=c$ d) $\sqrt{xy}+\left(\frac{y}{x}\right)^{\frac{3}{2}}=c$

198) The solution of the exact differential equation $\left(y^4-2x^3y\right)dx+\left(x^4-2xy^3\right)dy=0$ with the integrating factor $\frac{1}{x^2y^2}$ is

a) $\frac{2x^2}{y}+\frac{3y^2}{x}=c$ b) $\frac{x^2}{y}-\frac{y^2}{x}=c$

c) $\frac{x^2}{2y}+\frac{y^2}{3x}=c$ d) $\frac{x^2}{y}+\frac{y^2}{x}=c$

199) The solution of the exact differential equation $\left(y^3-2x^2y\right)dx-\left(x^3-2xy^2\right)dy=0$ with the integrating factor xy is

a) $x^3y^3\left(y^2+x^2\right)=c$ b) $x^2y^2\left(y^2-x^2\right)=c$

c) $x^2y^2\left(y^2+x^2\right)=c$ d) $x^2+y^2\left(y^2-x^2\right)=c$

200) The solution of the exact differential equation $\left(3x^2y^4+2xy\right)dx+\left(2x^3y^3-x^2\right)dy=0$ with the integrating factor $\frac{1}{y^2}$ is

a) $x^3y^2+\frac{x^2}{y}=c$ b) $x^2y^2+\frac{x^2}{y^2}=c$

c) $x^3y^3-\frac{x^2}{y}=c$ d) $x^2y^3-\frac{x^2}{y^3}=c$

201) The solution of the exact differential equation $y\left(x^2y+e^x\right)dx-e^xdy=0$ with the integrating factor $\frac{1}{y^2}$ is

a) $\frac{x^2}{2}+\frac{e^x}{y}=c$ b) $\frac{x^3}{3}-\frac{e^x}{y}=c$

c) $\frac{x^3}{3}+\frac{e^x}{y}=c$ d) $\frac{x^3}{3}+\frac{e^x}{2}=c$

202) The solution of the exact differential equation $\left(2x+e^x\log y\right)ydx+\left(e^x\right)dy=0$ with the integrating factor $\frac{1}{y}$ is

a) $x^2+e^x+\log y=c$ b) $x^2-e^x\log y=c$

c) $\frac{x^2}{2}+e^x\log y=c$ d) $x^2+e^x\log y=c$

203) The solution of $\frac{dy}{dx}\left(x+2y^3\right)=y+2x^3y^2$ with the integrating factor $\frac{1}{y^2}$ is

a) $\frac{x}{y}-\frac{x^4}{y}+y^2=c$ b) $\frac{x}{y}+\frac{x^4}{2}-\frac{y^2}{2}=c$

c) $\frac{x}{3}+\frac{x^4}{2}+y^2=c$ d) $\frac{x}{y}+\frac{x^4}{2}-y^2=c$

204) The solution of the exact differential equation $y\log ydx+\left(x-\log y\right)dy=0$ with the integrating factor $\frac{1}{y}$ is

a) $2x\log y-\left(\log y\right)^2=c$

b) $x^2\log y+\left(\log y\right)^2=c$

c) $2x\log y+\left(\log y\right)^3=c$

d) $\frac{2x}{3}\log y - \log y^2 = c$

205) The solution of the exact differential equation $y\left(2x^2y + e^x\right)dx = \left(e^x + y^3\right)dy$ with the integrating factor $\frac{1}{y^2}$ is

a) $\frac{1}{3}x^3 + \frac{e^x}{x} - \frac{1}{2}y^2 = c$ b) $\frac{2}{3}x^3 + \frac{e^x}{y} + \frac{1}{2}y^3 = c$

c) $\frac{2}{3}x^3 + \frac{e^x}{y} - \frac{1}{2}y^2 = c$ d) $x^3 + \frac{e^x}{y} - y^2 = c$

206) The solution of the exact differential equation $\left(2x\log x - xy\right)dy + 2ydx = 0$ with the integrating factor $\frac{1}{x}$ is

a) $2x\log x - \frac{x^2}{2} = c$ b) $2y\log x - \frac{y^2}{2} = c$

c) $\frac{y}{2}\log x - \frac{y^2}{2} = c$ d) $y\log x + \frac{y^2}{2} = c$

207) The solution of the exact differential equation $\left(x^4e^x - 2mxy^2\right)dx + 2mx^2ydy = 0$ with the integrating factor $\frac{1}{x^4}$ is

a) $e^x + \frac{m^2y^2}{x^2} = cm$ b) $e^x - \frac{my^2}{x^2} = c$

c) $\frac{e^x}{y} - \frac{my^2}{x^2} = c$ d) $e^x + \frac{my^2}{x^2} = c$

208) The differential equation which can be expressed in the form $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x only, is known as

a) variable separable equation in x, y
b) homogeneous differential equation in x, y
c) linear differential equation in x w.r.t y
d)linear differential equation in y w.r.t x

209) The differential equation which can be expressed in the form $\frac{dx}{dy} + Px = Q$, where P and Q are functions of y only, is known as

a)linear differential equation in x w.r.t y
b) linear differential equation in y w.r.t x
c) homogeneous differential equation in x, y
d) variable separable equation in x, y

210) The integrating factor of the linear differential equation $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x only, is

a) $e^{\int Pdx}$ b) $e^{\int Qdx}$ c) $e^{\int Pdy}$ d) $e^{\int Qdy}$

211) The integrating factor of the linear differential equation $\frac{dx}{dy} + Px = Q$, where P and Q are functions of y only, is

a) $e^{\int Pdx}$ b) $e^{\int Qdx}$ c) $e^{\int Pdy}$ d) $e^{\int Qdy}$

212) The general solution of the linear differential equation $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x only, is given by

a) $ye^{\int Pdx} = \int Qe^{\int Pdx}dx + c$
b) $xe^{\int Pdx} = \int Qe^{\int Pdx}dx + c$
c) $xe^{\int Pdy} = \int Qe^{\int Pdy}dy + c$
d) $xe^{\int Qdx} = \int Pe^{\int Qdx}dx + c$

213) The general solution of the linear differential equation $\frac{dx}{dy} + Px = Q$, where P and Q are functions of y only, is given by

a) $ye^{\int Pdx} = \int Qe^{\int Pdx}dx + c$
b) $xe^{\int Pdx} = \int Qe^{\int Pdx}dx + c$
c) $xe^{\int Pdy} = \int Qe^{\int Pdy}dy + c$
d) $xe^{\int Qdx} = \int Pe^{\int Qdx}dx + c$

214) A differential equation which can be expressed in the form $\frac{dy}{dx} + Py = Qy^n$, where P and Q are functions of x only, is known as

a) Non-linear differential equation
b)Bernoulli's linear differential equation
c) exact differential equation
d) homogenous differential equation

215) A differential equation which can be expressed in the form $\frac{dx}{dy} + Px = Qx^n$, where P and Q are functions of x only, is known as

a) Non-linear differential equation
b) Bernoulli's linear differential equation
c) exact differential equation
d) homogenous differential equation

216) A differential equation which can be expressed in the form $f'(y)\frac{dy}{dx}+Pf(y)=Q$, where P and Q are functions of x only, can be reduced into the linear form by substituting
a) $P=v$ b) $Q=v$
c) $f(y)=v$ d) $f'(y)=v$

217) A differential equation which can be expressed in the form $\frac{dy}{dx}+Py=Qy^n$, where P and Q are functions of x only, can be reduced into the linear form by substituting
a) $y^n=v$ b) $y^{1-n}=v$
c) $y^{n-1}=v$ d) $y^{n+1}=v$

218) If I_1, I_2 are the integrating factors of the equations $\frac{dx}{dy}+Px=Q$ and $\frac{dx}{dy}-Px=Q$ respectively, the relation between them is
a) $I_1=-I_2$ b) $I_1=I_2$
c) $I_1\cdot I_2=-1$ d) $I_1\cdot I_2=1$

219) The integrating factor of the linear differential equation $\frac{dy}{dx}+xy=x^5$ is
a) $e^{\log\frac{x^2}{2}}$ b) $e^{\frac{x^2}{2}}$ c) e^{x^2} d) x^2

220) The integrating factor of the linear differential equation $\frac{dy}{dx}+2xy=\frac{\tan^{-1}x}{1+x^2}$ is
a) $\frac{x^2}{2}$ b) $e^{\frac{x^2}{2}}$ c) e^{x^2} d) $2x^2$

221) The integrating factor of the linear differential equation $\frac{dx}{dy}+xy=y^5$ is
a) $e^{\frac{y^2}{2}}$ b) $\frac{y^2}{2}$ c) $e^{\frac{x^2}{2}}$ d) e^{x^2}

222) The integrating factor of the linear differential equation $\frac{dy}{dx}+\frac{y}{1+x}=x^3$ is
a) $e^{\frac{(1+x)^2}{2}}$ b) $1+x$ c) $\frac{1}{1+x}$ d) e^{1+x}

223) The integrating factor of the linear differential equation $\frac{dy}{dx}+\frac{y}{1-x}=\sin x$ is
a) $\frac{1}{1-x}$ b) $1-x$ c) e^{1-x} d) $e^{\frac{(1-x)^2}{2}}$

224) The integrating factor of the linear differential equation $\frac{dy}{dx}+\frac{y}{1+x^2}=\sec x\tan x$ is
a) $\frac{(1+x^2)^2}{2}$ b) $1+x^2$ c) $e^{\tan^{-1}x}$ d) e^{1+x^2}

225) The integrating factor of the linear differential equation $\frac{dy}{dx}+\frac{2x}{1+x^2}y=\tan^{-1}x$ is
a) $\frac{(1+x^2)^2}{2}$ b) $1+x^2$ c) $e^{\tan^{-1}x}$ d) e^{1+x^2}

226) The integrating factor of the linear equation $\frac{dy}{dx}+y\tan x=e^x\sin(2x-3)$ is
a) $\sec^2 x$ b) $\cos x$ c) $\sec x$ d) $e^{\sec x}$

227) The integrating factor of the linear differential equation $\tan x\frac{dy}{dx}+y=e^x\sin x$ is
a) e b) $e^{\sin x}$ c) $\log(\sin x)$ d) $\sin x$

228) The integrating factor of the linear equation $(1+x^2)\frac{dy}{dx}+xy=2x^3-3x+5$ is
a) e^{1+x^2} b) $\frac{1}{1+x^2}$ c) $1+x^2$ d) $\sqrt{1+x^2}$

229) The integrating factor of the linear equation $(1+x^2)\frac{dy}{dx}+4xy=\frac{1}{(1+x^2)^3}$ is
a) $1+x^2$ b) e^{1+x^2} c) $(1+x^2)^2$ d) $\frac{1}{1+x^2}$

230) The integrating factor of the linear equation $\left(1+x^2\right)\frac{dy}{dx}+2xy=\frac{1}{\left(1+x^2\right)^3}$ is

a) $1+x^2$ b) e^{1+x^2} c) $\left(1+x^2\right)^2$ d) $\frac{1}{1+x^2}$

231) The integrating factor of the linear equation $\left(1+x^2\right)\frac{dy}{dx}-2xy=\frac{1}{\left(1+x^2\right)^3}$ is

a) $1+x^2$ b) e^{1+x^2} c) $\left(1+x^2\right)^2$ d) $\frac{1}{1+x^2}$

232) The integrating factor of the linear differential equation $\frac{dx}{dy}+\frac{xy}{1+y^2}=\sec y$ is

a) $\sqrt{1+x^2}$ b) $\sqrt{1+y^2}$ c) $\tan^{-1} y$ d) $e^{\tan^{-1} y}$

233) The integrating factor of the linear differential equation $\frac{dy}{dx}+y\cot x=\tan x$ is

a) $\sin x$ b) $e^{\sec x}$
c) $\cos x$ d) $\sec x+\tan x$

234) The integrating factor of the linear differential equation $\cos x\frac{dy}{dx}+y=\tan x$ is

a) $e^{\sec x+\tan x}$ b) $e^{\sec x}$
c) $\cos x$ d) $\sec x+\tan x$

235) The integrating factor of the differential equation $\frac{dy}{dx}+\sqrt{x}y=\sin\sqrt{x}\cos\sqrt{x}$ is

a) $\sin\sqrt{x}$ b) $e^{\log\sqrt{x}}$
c) $e^{\frac{2}{3}x\sqrt{x}}$ *or* $e^{\frac{2}{3}x^{\frac{3}{2}}}$ d) $\frac{2}{3}x\sqrt{x}$ *or* $\frac{2}{3}x^{\frac{3}{2}}$

236) The integrating factor of the linear equation $\frac{dy}{dx}+\left(\tan x+\frac{1}{x}\right)y=\frac{1}{x}\sec x$ is

a) $x\sec x$ b) $e^{x\sec x}$ c) $e^{x+\sec x}$ d) $x+\sec x$

237) The integrating factor of the linear differential equation $\left(1-x^2\right)\frac{dy}{dx}=x^3+xy$ is

a) $e^{\tan^{-1} x}$ b) $e^{\sqrt{1-x^2}}$ c) $\frac{1}{\sqrt{1-x^2}}$ d) $\sqrt{1-x^2}$

238) The integrating factor of the linear differential equation $\left(1-x^2\right)\frac{dy}{dx}=x^3-xy$ is

a) $\frac{1}{\sqrt{1-x^2}}$ b) $\sqrt{1-x^2}$ c) $e^{\sqrt{1-x^2}}$ d) $e^{\tan^{-1} x}$

239) The integrating factor of the differential equation $1+y^2+\left(x-e^{\tan^{-1} x}\right)\frac{dy}{dx}=0$ is

a) $\tan^{-1} x$ b) $\tan^{-1} y$ c) $e^{\tan^{-1} x}$ d) $e^{\tan^{-1} y}$

240) The integrating factor of the differential equation $1+x^2+\left(y-e^{\tan^{-1} y}\right)\frac{dx}{dy}=0$ is

a) $\tan^{-1} x$ b) $\tan^{-1} y$ c) $e^{\tan^{-1} x}$ d) $e^{\tan^{-1} y}$

241) The integrating factor of the differential equation $\left(1+y^2\right)dx=\left(e^{\tan^{-1} x}-x\right)dy$ is

a) $\tan^{-1} x$ b) $\tan^{-1} y$ c) $e^{\tan^{-1} x}$ d) $e^{\tan^{-1} y}$

242) The integrating factor of the linear differential equation $y^2+\left(x-\frac{1}{y}\right)\frac{dy}{dx}=0$ is

a) $2\log x$ b) $\log y$ c) $-\frac{1}{y}$ d) $-\frac{1}{y^2}$

243) The integrating factor of the linear differential equation $\sin 2y\,dx=\left(\tan y-x\right)dy$ is

a) $\frac{\tan x}{2}$ b) $\sqrt{\tan y}$ c) $\sqrt{\tan x}$ d) $\frac{\tan y}{2}$

244) The integrating factor of the linear equation $y\log y\,dx+\left(x-\log y\right)dy=0$ is

a) $\left(\log y\right)^2$ b) $x\log y$ c) $\log y$ d) $\log x$

245) The integrating factor of the linear differential equation $ydx-\left(y-x\right)dy=0$ is

a) y b) x c) y^2 d) x^2

246) The integrating factor of the linear equation $\sqrt{a^2+x^2}\frac{dy}{dx}+y=\sqrt{a^2+x^2}-x$ is

a) $\frac{1}{2a}\log\left(x+\sqrt{a^2+x^2}\right)$ b) $\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

c) $x+\sqrt{a^2+x^2}$ d) $x-\sqrt{a^2+x^2}$

247) The integrating factor of the linear differential equation $\frac{dy}{dx}=\frac{e^x-2xy}{x^3}$ is

a) $e^{\frac{x^3}{3}}$ b) x^3 c) $\frac{1}{x^3}$ d) e^{x^3}

248) The integrating factor of linear differential equation $\left(x^2+1\right)\frac{dy}{dx}=x^3-2xy+x$ is

a) $\tan^{-1}x$ b) $e^{\tan^{-1}x}$ c) $\frac{1}{x^2+1}$ d) x^2+1

249) The integrating factor of the linear differential equation $x^2\frac{dy}{dx}=3x^2-2xy+1$ is

a) x^2-1 b) x^2 c) x^2+1 d) $\frac{1}{x^2}$

250) The integrating factor of the linear differential equation $\left(e^{-y}\sec^2 y-x\right)dy=dx$ is

a) $e^{\tan y}$ b) $\tan y$ c) e^x d) e^y

251) The differential equation $\frac{dy}{dx}-y\tan x=y^4\sec x$ is reduced into the linear form

a) $\frac{du}{dx}+3u\tan x=-3\sec x;\ u=y^{-3}$

b) $\frac{du}{dx}-3u\tan x=3\sec x;\ u=y^{-3}$

c) $\frac{du}{dx}-3u\tan x=-3\sec x;\ u=y^{-3}$

d) $\frac{du}{dx}+3u\cot x=-3\sec x;\ u=y^{-3}$

252) The differential equation $\frac{dy}{dx}-xy=-y^3e^{-x^2}$ can be reduced to the linear form

a) $\frac{dy}{dx}+xu=-2e^{-x^2};\ u=\frac{1}{y^2}$

b) $\frac{dy}{dx}+xu=e^{-x^2};\ u=\frac{1}{y^2}$

c) $\frac{dy}{dx}-2xu=2e^{-x^2};\ u=\frac{1}{y^2}$

d) $\frac{dy}{dx}+2xu=2e^{-x^2};\ u=\frac{1}{y^2}$

253) The value of k for which e^{ky^2} is an integrating factor of linear differential equation $\frac{dx}{dy}+xy=e^{-\frac{y^2}{2}}$ is

a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) 2 d) -2

254) The general solution of $\frac{dy}{dx}+\frac{y}{1+x}=-x(1-x)$ with the integrating factor $\frac{1}{1-x}$ is

a) $\frac{y}{1-x}=-\frac{x^3}{3}+c$ b) $y=-\frac{x^2}{2}(1-x)+c$

c) $\frac{y}{1-x}=-\frac{x^2}{2}+c$ d) $\frac{y}{1-x}=\frac{x^2}{2}+c$

255) The general solution of $\frac{dy}{dx}+\frac{y}{(1-x)\sqrt{x}}=1-\sqrt{x}$ with the integrating factor $\frac{1+\sqrt{x}}{1-\sqrt{x}}$ is

a) $\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)y^2=x^2+\frac{2}{3}x^{\frac{3}{2}}+c$

b) $\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)y=x+\frac{2}{3}x^{\frac{3}{2}}+c$

c) $\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)y=x+\frac{2}{3}x^{\frac{3}{2}}+c$

d) $\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)y=x-\frac{2}{3}x^{\frac{3}{2}}+c$

256) The general solution of $\frac{dy}{dx}+y\cot x=\sin 2x$ with the integrating factor $\sin x$ is

a) $y\sin x=\frac{2}{3}\sin^3 x+c$ b) $y\sin x=\frac{1}{3}\sin^3 x+c$

c) $x\sin y=\frac{2}{3}\sin^3 x+c$ d) $y\sin x=\sin^3 x+c$

257) The general solution of $\frac{dy}{dx}+\frac{3y}{x}=\frac{e^x}{x^2}$ with the integrating factor x^3 is

a) $x^3y=e^{-x}(x+1)+c$ b) $xy^3=e^x(x-1)+c$

c) $x^3y=e^x(x-1)+c$ d) $x^3y=e^x(x+1)+c$

258) The general solution of $\frac{dy}{dx}+\frac{3y}{x}=x^2$ with the integrating factor x^3 is

a) $x^3y=\frac{x^4}{4}+c$ b) $x^3y=\frac{x^6}{6}+c$

c) $x^3y=\frac{x^2}{2}+c$ d) $xy^3=\frac{x^3}{3}+c$

259) The general solution of $\frac{dy}{dx}+\left(\tan x+\frac{1}{x}\right)y=\frac{1}{x}\sec x$ with the integrating factor $x\sec x$ is

a) $xy\sin x=\tan x+c$ b) $xy\sec x=-\tan x+c$

c) $xy\tan x=\cot x+c$ d) $xy\sec x=\tan x+c$

260) The general solution of $\frac{dy}{dx}+\frac{2y}{x}=\frac{1}{x^3}$ with the integrating factor x^2 is

a) $y=x^2\log x+c$ b) $x^2y=\log x+c$

c) $xy^2=\log x+c$ d) $x^2y=\log\frac{1}{x}+c$

261) The general solution of $\frac{dy}{dx}+(1+2x)y=e^{-x^2}$ with the integrating factor e^{x+x^2} is

a) $ye^{x+x^2}=e^x+c$ b) $ye^{x+x^2}=-e^x+c$

c) $e^{x+x^2}=ye^x+c$ d) $ye^{x-x^2}=e^x+c$

262) The general solution of $\frac{dy}{dx}+\frac{x}{1+y^2}=\frac{e^{-\tan^{-1}y}}{1+y^2}$ with the integrating factor $e^{\tan^{-1}y}$ is

a) $ye^{\tan^{-1}y}=\tan^{-1}x+c$ b) $xe^{\tan^{-1}y}=\tan^{-1}y+c$

c) $xe^{\tan^{-1}y}=\cot^{-1}y+c$ d) $ye^{\tan^{-1}y}=\tan^{-1}y+c$

263) The general solution of $\frac{dy}{dx}+x\sec y=\frac{2y\cos y}{1+\sin y}$ with the integrating factor $\sec y+\tan y$ is

a) $(\sec y+\tan y)x^2=y+c$

b) $(\sec y+\tan y)x=-y^2+c$

c) $(\sec y+\tan y)x=y^2+c$

d) $x=\frac{y^2}{\sec y+\tan y}+c$

Chapter 02 – Applications of Ordinary Differential Equations

1) Two families of curves are said to be orthogonal trajectories of each other, if
   a) Every member of one family cuts every member of other family at right angle.
   b) Every member of one family cuts every member of other family at origin.
   c) Every member of one family cuts every member of other family at common point.
   d) None of the above.

2) In the two dimensional Cartesian form, to find orthogonal trajectories of given family of curves, in its differential equation we replace $\frac{dy}{dx}$ by
   a) $-y\frac{dx}{dy}$ b) $-\frac{dy}{dx}$ c) $-\frac{dx}{dy}$ d) $-x\frac{dx}{dy}$

3) In the two dimensional polar form, to find orthogonal trajectories of given family of curves, in its differential equation we replace $\frac{dr}{d\theta}$ by
   a) $r\frac{d\theta}{dr}$ b) $-r\frac{d\theta}{dr}$ c) $-r^2\frac{d\theta}{dr}$ d) $-\frac{d\theta}{dr}$

4) The differential equation of orthogonal trajectories of family of straight lines $y = mx$ is
   a) $\frac{dx}{dy} + y = 0$ b) $\frac{dy}{dx} = -\frac{y}{x}$
   c) $\frac{dx}{dy} = -\frac{y}{x}$ d) $\frac{dx}{dy} = -\frac{x}{y}$

5) For the family of the curves $x^2 + y^2 = c^2$, the differential equation of orthogonal trajectories is
   a) $x^2 + y^2\frac{dx}{dy} = 0$ b) $x + y\frac{dy}{dx} = 0$
   c) $x + xy\frac{dx}{dy} = 0$ d) $x - y\frac{dx}{dy} = 0$

6) The differential equation of orthogonal trajectories of family of $x^2 + 2y^2 = c^2$ is
   a) $y - 2x\frac{dy}{dc} = 0$ b) $x - 2y\frac{dx}{dy} = 0$
   c) $x + 2y\frac{dy}{dx} = 0$ d) $x + 2y\frac{dx}{dy} = 0$

7) For the family of the curves $y^2 = 4ax$, the differential equation of orthogonal trajectories is
   a) $2y\frac{dy}{dx} = 4x$ b) $2y\frac{dy}{dx} = \frac{y}{x^2}$
   c) $-2y\frac{dy}{dx} = \frac{y^2}{x}$ d) $-2y\frac{dx}{dy} = \frac{y^2}{x}$

8) For the family of the curves $y = 4ax^2$, the differential equation of orthogonal trajectories is
   a) $y\frac{dy}{dx} = 2x$ b) $\frac{dy}{dx} = -\frac{2}{x^2}$
   c) $\frac{dy}{dx} = -\frac{2y}{x}$ d) $-2\frac{dx}{dy} = \frac{1}{xy}$

9) For the family of the curves $xy = c$, the differential equation of orthogonal trajectories is
   a) $x^2\frac{dx}{dy} + 2y = 0$ b) $-x\frac{dx}{dy} + y = 0$
   c) $2x\frac{dx}{dy} - y = 0$ d) $x\frac{dy}{dx} - y = 0$

10) The differential equation of orthogonal trajectories of family of $2x^2 + y^2 = cx$ is
   a) $4x - 2y\frac{dx}{dy} = \frac{2x^2 + y^2}{x}$
   b) $4x + 2y\frac{dy}{dx} = \frac{2x^2 + y^2}{x}$
   c) $4x^2 + 2y\frac{dx}{dy} = \frac{2x^2 + y^2}{x}$
   d) $4x - 2xy\frac{dy}{dx} = \frac{2x^2 + y^2}{x}$

11) For the family of the curves $x^2+cy^2=1$, the differential equation of orthogonal trajectories is

a) $x+\left(\frac{1-x^2}{y}\right)\frac{dy}{dx}=0$ b) $x+\left(\frac{1+x^2}{y}\right)\frac{dx}{dy}=0$

c) $x-\left(\frac{1-x^2}{y}\right)\frac{dx}{dy}=0$ d) $x^2+\left(\frac{1-x^2}{y}\right)\frac{dy}{dx}=0$

12) For the family of the curves $e^x+e^{-y}=c$, the differential equation of orthogonal trajectories is

a) $e^{2x}-e^{-2y}\frac{dx}{dy}=0$ b) $e^{-x}+e^{y}\frac{dx}{dy}=0$

c) $e^{x}-e^{-y}\frac{dy}{dx}=0$ d) $e^{x}+e^{-y}\frac{dx}{dy}=0$

13) The differential equation of orthogonal trajectories of family of $r=a\cos\theta$ is

a) $-r\frac{dr}{d\theta}=\cot\theta$ b) $-r\frac{dr}{d\theta}=\tan\theta$

c) $r\frac{d\theta}{dr}=\cot\theta$ d) $r\frac{d\theta}{dr}=\tan\theta$

14) For the family of the curves $r=a\sin\theta$, the differential equation of orthogonal trajectories is

a) $\frac{1}{r}\frac{d\theta}{dr}=-\cot\theta$ b) $r\frac{d\theta}{dr}=-\tan\theta$

c) $r\frac{d\theta}{dr}=-\cot\theta$ d) $-\frac{1}{r}\frac{dr}{d\theta}=\tan\theta$

15) For the family of the curves $r^2=a\sin\theta$, the differential equation of orthogonal trajectories is

a) $2r\frac{d\theta}{dr}=-\cot\theta$ b) $r\frac{d\theta}{dr}=-\frac{\tan\theta}{2}$

c) $r^2\frac{d\theta}{dr}=-\cot\theta$ d) $-\frac{2}{r}\frac{dr}{d\theta}=\tan\theta$

16) For the family of the curves $r=a(1-\cos\theta)$, the differential equation of orthogonal trajectories is

a) $-r\frac{d\theta}{dr}=\frac{1-\cos\theta}{\sin\theta}$ b) $r\frac{dr}{d\theta}=\frac{\sin\theta}{1-\cos\theta}$

c) $-r\frac{d\theta}{dr}=\frac{\sin\theta}{1-\cos\theta}$ d) $r^2\frac{d\theta}{dr}=\frac{\sin\theta}{1-\cos\theta}$

17) For the family of the curves $r^2=a\sin 2\theta$, the differential equation of orthogonal trajectories is

a) $-r^2\frac{dr}{d\theta}=\cot 2\theta$ b) $r\frac{d\theta}{dr}=-\cot 2\theta$

c) $r\frac{d\theta}{dr}=-\tan 2\theta$ d) $-\frac{dr}{d\theta}=\cot 2\theta$

18) For the family of the curves $r^2=a\cos 2\theta$, the differential equation of orthogonal trajectories is

a) $\frac{dr}{d\theta}=r\tan 2\theta$ b) $\frac{1}{r}\frac{d\theta}{dr}=-\tan 2\theta$

c) $r\frac{d\theta}{dr}=\cot 2\theta$ d) $r\frac{d\theta}{dr}=\tan 2\theta$

19) For the family of the curves $r^2=a\cos 2\theta$, the differential equation of orthogonal trajectories is

a) $\frac{dr}{d\theta}=r\tan 2\theta$ b) $\frac{1}{r}\frac{d\theta}{dr}=-\tan 2\theta$

c) $r\cot 2\theta\frac{d\theta}{dr}=1$ d) $r\frac{d\theta}{dr}+\tan 2\theta=0$

20) For the family of the curves $r=a\cos^2\theta$, the differential equation of orthogonal trajectories is

a) $r\frac{d\theta}{dr}=\frac{\sin 2\theta}{\cos^2\theta}$ b) $r^2\frac{d\theta}{dr}=-\frac{\sin 2\theta}{\cos^2\theta}$

c) $\frac{dr}{d\theta}=-\frac{\sin 2\theta}{\cos^2\theta}$ d) $r\frac{d\theta}{dr}=-\frac{\sin 2\theta}{\cos 2\theta}$

21) For the family of the curves $r=a\sec^2\left(\frac{\theta}{2}\right)$, the differential equation of orthogonal trajectories is

a) $r\frac{d\theta}{dr}=-\tan\frac{\theta}{2}$ b) $r\frac{dr}{d\theta}=-\cot\frac{\theta}{2}$

c) $\frac{1}{r}\frac{d\theta}{dr}=-\cot\frac{\theta}{2}$ d) $r\frac{d\theta}{dr}=-\tan 2\theta$

22) The orthogonal trajectories of family of curves having differential equation $y=mx$ is $\frac{dy}{dx}=\frac{y}{x}$, is given by

a) $x^2-y^2=c$ b) $\frac{x^2}{2}+y^2=c$

c) $x^2+y^2=c$ d) $x^2+2y^2=c$

23) If the differential equation of family of curves $xy=c$ is $x\frac{dy}{dx}=-y$, then its family of orthogonal trajectories is given by

a) $x^2-2y^2=c$ b) $x^2+2y^2=c$

c) $x^2-y^2=c^2$ d) $x^2+y^2=c$

24) The orthogonal trajectories of family of curves having differential equation $x^2+y^2=k^2$ is $\frac{dy}{dx}=-\frac{x}{y}$, is given by

a) $x^2=4ay$ b) $x^2-y^2=c$

c) $y^2=x+c$ d) $y=cx$

25) If the differential equation of family of curves $x^2-y^2=c$ is $y\frac{dy}{dx}=x$, then its family of orthogonal trajectories is given by

a) $y=cx$ b) $xy=c$

c) $x^2=4ay$ d) $y^2=x+c$

26) The orthogonal trajectories of family of curves having differential equation $x^2+2y^2=c^2$ is $\frac{dy}{dx}+\frac{x}{2y}=0$, is given by

a) $x^2-cx+c^2=0$ b) $y=2cx^2+x$

c) $x^2=ky$ d) $y=2cx^2$

27) The orthogonal trajectories of family of curves having differential equation $x^2+cy^2=1$ is $\frac{dy}{dx}=-\frac{xy}{1-x^2}$, is given by

a) $\log x+x^2+y^2=c$ b) $\log x-x^2-y^2=c$

c) $\log x-\frac{x^2}{2}-\frac{y^2}{2}=c$ d) $\log x+\frac{x^2}{2}+\frac{y^2}{2}=c$

28) The orthogonal trajectories of family of curves having differential equation $y=4ax^2$ is $\frac{dy}{dx}=\frac{y}{x}$, is given by

a) $2x^2=cy^2$ b) $2x^2-y^2=c^2$

c) $2x^2+y^2=c$ d) $x^2+2y^2=c$

29) If the differential equation of family of curves $y^2=4ax$ is $2x\frac{dy}{dx}=y$, then its family of orthogonal trajectories is given by

a) $2x^2+y^2=c$ b) $2x^2-y^2=c^2$

c) $x^2+2y^2=c$ d) $2x^2=cy^2$

30) The orthogonal trajectories of family of curves having differential equation $e^x+e^{-y}=e^c$ is $\frac{dy}{dx}=\frac{e^x}{e^{-y}}$, is given by

a) $e^{2x}+e^{-2y}=k$ b) $e^x-e^{-y}=k$

c) $e^x+e^{-y}=e^c$ d) $e^{-x}+e^y=e^c$

31) If the differential equation of family of curves $e^x-e^{-y}=c$ is $\frac{dy}{dx}+\frac{e^{-y}}{e^x}=0$, then its family of orthogonal trajectories is given by

a) $e^x+e^{-y}=k$ b) $e^{-x}+e^y=e^c$

c) $e^x+e^{-y}=e^c$ d) $e^{2x}+e^{-2y}=k$

32) If the differential equation of family of curves $x^2=ce^{x^2+y^2}$ is $\frac{dy}{dx}=\frac{1-x^2}{xy}$, then its family of orthogonal trajectories is given by

a) $\log\left(1-x^2\right)+2\log y=c$

b) $\log\left(1-x^2\right)-2\log y=c$

c) $2\log\left(1-x^2\right)-3\log y=c$

d) $\log\left(1-x^2\right)+\log y=c$

33) The orthogonal trajectories of family of curves having differential equation $\frac{x^2}{a^2}+\frac{y^2}{b^2+\lambda}=1$ is $x+\left(\frac{a^2-x^2}{y}\right)\frac{dy}{dx}=0$, where a and b are fixed constants, is given by

a) $\frac{y^2}{2}=\lambda\log x+\frac{x^2}{2}+k$

b) $y^2-x^2=a^2\log x+k$

c) $\frac{y^2}{2}=a^2\log x-\frac{x^2}{2}+k$

d) $x^2+y^2=a^2\log x+k$

34) If the differential equation of family of curves $r = a(1-\cos\theta)$ is $(1-\cos\theta)\frac{dr}{d\theta} = r$, then its family of orthogonal trajectories is given by
a) $r^2 = A(1+\cos\theta)$ b) $r = A(1+\sin\theta)$
c) $r = A(1-\cos\theta)$ d) $r = A(1+\cos\theta)$

35) If the differential equation of family of curves $r = a(1-\cos\theta)$ is $\frac{dr}{d\theta} = r\cot\frac{\theta}{2}$, then its family of orthogonal trajectories is given by
a) $\log\cos\left(\frac{\theta}{2}\right) = 2\log r + c$
b) $2\log\sin\left(\frac{\theta}{2}\right) = \frac{1}{2}\log r + c$
c) $2\log\cos\left(\frac{\theta}{2}\right) = \log r + c$
d) $\log 2\cos\left(\frac{\theta}{2}\right) = \log r + c$

36) The orthogonal trajectories of family of curves having differential equation $r = a\sin\theta$ is $\frac{dr}{d\theta} = r\cot\theta$, is given by
a) $r = A\cos\theta$ b) $r = A\tan\theta$
c) $r\cos\theta = A$ d) $r^2 = A\cos\theta$

37) The orthogonal trajectories of family of curves having differential equation $r = a\cos\theta$ is $\frac{dr}{d\theta} + r\tan\theta = 0$, is given by
a) $r = C\,\mathrm{cosec}\,2\theta$ b) $r^2 = C\sin^2\theta$
c) $r = C\tan\theta$ d) $r = C\sin\theta$

38) If the differential equation of family of curves $r^2 = a^2\cos 2\theta$ is $\frac{dr}{d\theta} + r\tan 2\theta = 0$, then its family of orthogonal trajectories is given by
a) $r^2 = c\sin^2 2\theta$ b) $r = c\sin 2\theta$
c) $r^2 = c^2\sin 2\theta$ d) $r^2 = c^2\cos 2\theta$

39) If the differential equation of family of curves $r^2 = a\sin 2\theta$ is $\frac{dr}{d\theta} = r\cot 2\theta$, then its family of orthogonal trajectories is given by
a) $r^2\cos 2\theta = k$ b) $r^2 = k\cos 2\theta$
c) $2\log r = \log\sec 2\theta + k$ d) $r^2 = k\cot 2\theta$

40) The orthogonal trajectories of family of curves having differential equation $r = a^2\cos^2\theta$ is $\frac{dr}{d\theta} + 2r\tan\theta = 0$, is given by
a) $\log\tan\theta = 2\log r + c$ b) $2\log\sin\theta = \log r + c$
c) $\frac{3}{2}\log\sin\theta = 2\log r + c$ d) $\frac{\log\sin\theta}{2} = \log r + c$

41) If the differential equation of family of curves $r = a\theta$ is $r = \theta\frac{dr}{d\theta}$, then its family of orthogonal trajectories is given by
a) $r = ce^{-\frac{\theta^2}{2}}$ b) $r = ce^{-\theta^2}$
c) $r^2 = ce^{-\frac{\theta^2}{2}}$ d) $r^2 = ce^{\theta^2}$

42) Newton's law of cooling states that
a) The temperature of the body is inversely proportional to the difference between the body temperature and the surrounding temperature.
b) The temperature of the body is proportional to the sum of the body temperature and the surrounding temperature.
c) The temperature of the body is proportional to the difference between the body temperature and the surrounding temperature.
d) The temperature of the body is proportional to the surrounding of the body temperature.

43) For $\theta =$ the temperature of the body and $\theta_0 =$ the temperature of the surrounding, then Newton's law of cooling states the differential equation
a) $\frac{d\theta}{dt} = -k\theta_0$ b) $\frac{d\theta}{dt} = -k\theta + \theta_0$
c) $\frac{d\theta}{dt} = -k(\theta + \theta_0)$ d) $\frac{d\theta}{dt} = -k(\theta - \theta_0)$

44) A body having initially temperature 90ºC is kept in surrounding of temperature 26ºC. Then the differential equation satisfied by body temperature θ at any time t is given by
a) $\frac{d\theta}{dt} = -k(\theta - 64)$ b) $\frac{d\theta}{dt} = -k(\theta - 26)$

c) $\frac{d\theta}{dt}=-k(\theta+26)$ d) $\frac{d\theta}{dt}=-k(\theta-90)$

45) Consider a substance at initial temperature 32°C is surrounded by room temperature 10°C. According to Newton's law of cooling the differential equation satisfied by its temperature T at time t hour is

a) $\frac{dT}{dt}=-kT(T-10)$ b) $\frac{dT}{dt}=-k(T-32)$

c) $\frac{dT}{dt}=-k(10-32T)$ d) $\frac{dT}{dt}=-k(T-10)$

46) A metallic object is heated up to getting temperature 100°C and the placed in water of temperature 50°C. Then the differential equation of the object temperature θ at time t is given by Newton's law of cooling as

a) $\frac{d\theta}{dt}=-k\theta(\theta-26)$ b) $\frac{d\theta}{dt}=-k(\theta-50)$

c) $\frac{d\theta}{dt}=-k(\theta-150)$ d) $\frac{d\theta}{dt}=-k(\theta+50)$

47) If a body originally at 120°C cools to 35°C in 40 minute in the air of constant temperature 45°C. Then according to Newton's law, its differential equation is given by

a) $\frac{d\theta}{dt}=-k(\theta-120)$ b) $\frac{d\theta}{dt}=-k(\theta-40)$

c) $\frac{d\theta}{dt}=-k(\theta-45)$ d) $\frac{d\theta}{dt}=-k(\theta-35)$

48) Assuming the temperature of the surrounding is being kept constant at 25°C and a body cools from temperature 80°C to 35°C in 45 minute. Then it must satisfy the differential equation

a) $\frac{dT}{dt}=-k(T-25)$ b) $\frac{dT}{dt}=-k(T-80)$

c) $\frac{dT}{dt}=-k(T-35)$ d) $\frac{dT}{dt}=-k(T+25)$

49) The rate of change of temperature of a body is proportional to the difference between the temperature of body and its surrounding nearby. If temperature of the air is 35°C and that of the body is 96°C and cools down to 55°C in just 25 minute. Then we must have

a) $\frac{dT}{dt}=-k(T+25)$ b) $\frac{dT}{dt}=-k(T-55)$

c) $\frac{dT}{dt}=-k(T-35)$ d) $\frac{dT}{dt}=-k(T-25)$

50) A metal ball is placed in the oven till it obtain temperature of 100°C and then at time t = 0, it is then placed in water of temperature 40°C. By Newton's law, if the temperature of the ball is decreased to 70°C in 10 minutes, then it must satisfy the differential equation

a) $\frac{dT}{dt}=-k(T-70)$ b) $\frac{dT}{dt}=-k(T-40)$

c) $\frac{dT}{dt}=-k(T-55)$ d) $\frac{dT}{dt}=-k(T-100)$

51) If a body of temperature T at time t kept in the surrounding of temperature T_0 satisfies the differential equation $\frac{dT}{dt}=-k(T-T_0)$, the relation between T and t is given as

a) $T=T_0-ke^{-kt}$ b) $T=T_0+ke^{-kt}$

c) $T=T_0+ke^{-kt}$ d) $T=-k(T_0-e^{-kt})$

52) A body is heated to a temperature of 100°C and then at time recording t = 0 it is then placed liquid of temperature 40°C. The temperature of the body is then reduced to 60°C in 4 minute. By Newton's law of cooling its differential equation is $\frac{d\theta}{dt}=-\frac{1}{4}(\theta-40)\log 3$. The time required to reduce the temperature of body to 50°C is

a) 5 min 6 sec b) 5.6 min

c) 65 min d) 6.5 min

53) A corpse of temperature 32°C is kept in the mortuary of constant temperature 10°C and the temperature of the corpse decreases to 20°C in 5 minutes. The differential equation of the system is given as $\frac{dT}{dt}=-0.05(T-10)$. Then T is

a) $T=22e^{-0.05t}$ b) $T=22+10e^{0.05t}$

c) $T=10-22e^{-0.05t}$ d) $T=10+22e^{-0.05t}$

54) A thermometer is taken outdoors of temperature 0°C from a room of temperature 21°C and the reading on the thermometer drops to 10°C in 5 minutes and satisfies sufficiently the differential equation $\frac{dT}{dt} = -0.7419T$. What is its primitive?

a) $T = 21e^{-0.7419t}$ b) $T = 21 - 10e^{0.7419t}$
c) $T = 10 + 21e^{0.7419t}$ d) $T = 21e^{0.7419t}$

55) A metal body of mass 5 kg is heated to a temperature upto 100°C exactly and then, at time considered to be t = 0, it is immersed in oil of temperature 30°C. In just 3 minutes, the temperature of body drops to 70°C in 3 minute and satisfies $\frac{d\theta}{dt} = -\frac{\theta - 30}{3}\log\left(\frac{7}{4}\right)$. What is time taken to drop temperature of body to 31°C.

a) 15.28 min b) 12.78 min
c) 32.78 sec d) 22.78 min

56) If the temperature of body drops down to 70°C from 100°C in 15 minute, and satisfying the Newton's law of cooling $\frac{d\theta}{dt} = -k(\theta - 30)$, the value of k is

a) $\frac{1}{15}\log\frac{7}{4}$ b) $-\frac{1}{15}\log\frac{7}{4}$
c) $15\log\frac{7}{4}$ d) $-15\log\frac{7}{4}$

57) A metal ball of temperature 100°C is placed in air conditioned room of temperature 20°C. The temperature drops by 40°C in 5 minute. Its differential equation in accordance with Newton's law of cooling is given by $\frac{dT}{dt} = -\frac{T - 20}{5}\log 2$. The temperature after 8 minute is

a) 6.44 b) 64.4 c) 46.4 d) 44.6

58) A body cools down from 80°C to 60°C from 1.00 PM to 1.20 PM in a room of temperature 40°C and satisfies the differential equation $\frac{d\theta}{dt} = -0.03465(\theta - 40)$. The temperature of body at 1.40 PM is

a) 45 b) 50 c) 55 d) 60

59) The temperature of body cooling down from 100°C to 60°C in 60 seconds when it is kept in the air surrounding of constant temperature 20°C and satisfies the equation $\frac{d\theta}{dt} = -k(\theta - 20)$. The value of k is then

a) $\log 2$ b) $\log 3$ c) $\log 4$ d) $\log 5$

60) A metal ball made by brass of mass 50 gm cools down from 80°C to 60°C after a recorded time of 20 minute in air atmosphere of 40°C. The differential equation is $\frac{d\theta}{dt} = -k(\theta - 40)$. What is the value of k?

a) $-\frac{3}{20}\log_e 2$ b) $-20\log_e 2$
c) $\frac{1}{20}\log 2$ d) $-\frac{1}{20}\log_e 2$

61) A body of temperature 90°C is placed in water of temperature 30°C for 6 minute and then its temperature calculated is to be just 50°C. The Newton's law of cooling is $\frac{d\theta}{dt} = -k(\theta - 40)$. Then what of followings is correct.

a) $k = \frac{1}{6}\log_e\frac{1}{3}$ b) $k = \frac{1}{6}\log_e 3$
c) $k = -\frac{1}{6}\log_e 2$ d) $k = -\frac{1}{6}\log_e\frac{1}{4}$

62) An iron ball is heated for temperature 100°C is placed in water of temperature 50°C at t = 0 and at t = 5 minute then its temperature calculated which is read to be 70°C. The Newton's law of cooling is $\frac{d\theta}{dt} = -k(\theta - 50)$. Then what of followings is correct?

a) $k = -\frac{3}{4}\log_e\frac{2}{5}$ b) $k = \frac{1}{5}\log_e\frac{2}{5}$
c) $k = -\frac{2}{5}\log_e\frac{1}{5}$ d) $k = -\frac{1}{5}\log_e\frac{2}{5}$

63) A circuit consisting of resistance R, inductance L connected in series with voltage of amount E. By Kirchhoff's law, the differential equation for the current i in terms of t is

a) $L\frac{di}{dt} + \frac{i}{R} = E$ b) $L\frac{di}{dt} + Ri = E$

c) $L\frac{di}{dt}+Ri=0$ d) $R\frac{di}{dt}+Li=E$

64) A circuit consisting of resistance R, inductance L connected in series without voltage of amount E. By Kirchhoff's law, the differential equation for the current i in terms of t is

a) $L\frac{di}{dt}+\frac{i}{R}=E$ b) $L\frac{di}{dt}+Ri=E$

c) $L\frac{di}{dt}+Ri=0$ d) $R\frac{di}{dt}+Li=E$

65) An electrical circuit is consisting of inductance L, capacitance C in series with voltage source E. By Kirchhoff's law, we have

a) $L\frac{dq}{dt}+\frac{q}{C}=E$ b) $L\frac{dq}{dt}+\frac{q}{R}=E$

c) $C\frac{di}{dt}+\frac{i}{R}=E$ d) $\frac{di}{dt}+\frac{i}{C}=ER$

66) An electrical circuit is consisting of resistance R, capacitance C in series with voltage source E. By Kirchhoff's law, we have

a) $R\frac{dq}{dt}+\frac{q}{C}=E$ b) $L\frac{dq}{dt}+\frac{q}{R}=E$

c) $C\frac{di}{dt}+\frac{i}{R}=E$ d) $\frac{di}{dt}+\frac{i}{C}=ER$

67) A circuit consisting of resistance R, inductance L connected in series with voltage of amount $E\cos\omega t$. By Kirchhoff's law, the differential equation for the current i in terms of t is

a) $L\frac{di}{dt}+\frac{i}{R}=E\cos\omega t$ b) $L\frac{di}{dt}+Ri=E\cos\omega t$

c) $L\frac{di}{dt}+Ri=0$ d) $R\frac{di}{dt}+Li=E\cos\omega t$

68) The differential equation for the current i in an electrical circuit consisting of inductance L, resistance R in series with electromotive force of Ee^{-at} is given by

a) $\frac{di}{dt}+Ri=\frac{E}{L}e^{-at}$ b) $L\frac{di}{dt}+Ri=Ee^{-at}$

c) $L\frac{di}{dt}+\frac{i}{R}=Ee^{-at}$ d) $R\frac{di}{dt}+Li=Ee^{-at}$

69) The differential equation for the current i in an electrical circuit composing of resistance of 120 ohm and an inductance of 0.7 henry connected in series with battery of 30 volt is

a) $0.7\frac{di}{dt}-120i=30$ b) $120\frac{di}{dt}+0.7i=30$

c) $0.7\frac{di}{dt}+120i=30$ d) $0.7\frac{di}{dt}+\frac{i}{120}=30$

70) The differential equation for the current i in an electrical circuit composing of resistance of 200 ohm and an inductance of 100 henry connected in series with battery of 440 volt is

a) $20\frac{di}{dt}+10i=44$ b) $\frac{di}{dt}+2i=40$

c) $5\frac{di}{dt}+10i=44$ d) $10\frac{di}{dt}+20i=44$

71) A capacitance of 0.03 farad and resistance of 10 ohm in series with electromotive force of 20 volts are in a circuit. If initially the capacitor is totally discharged, the differential equation for the charge q is

a) $10\frac{dq}{dt}+\frac{q}{0.03}=20;\ q(0)=0$

b) $\frac{dq}{dt}+\frac{q}{0.03}=2;\ q(0)=0$

c) $\frac{dq}{dt}+\frac{q}{0.3}=2;\ q(0)=0$

d) $10\frac{dq}{dt}+0.03q=20;\ q(0)=0$

72) In an electrical circuit of R and L in series with steady EMF, the current i satisfies the equation $i=\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$. The time required for the maximum value is

a) 0 b) $\frac{L}{R}\log 10$

c) $-\frac{L}{R}\log 90$ d) $\frac{E}{R}\log 10$

73) In an electrical circuit of R and L in series with steady EMF, the current i satisfies the equation $i=\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$. The time required for the current gets 90% of maximum value is

a) 0 b) $\frac{L}{R}\log 2$

c) $-\frac{L}{R}\log 2$ d) $\frac{E}{R}\log 2$

74) If the differential equation for the current i is $R\frac{di}{dt}+Ri=E$, the current i at time t is

a) $i=\frac{E}{R}+ce^{-\frac{R}{L}t}$ b) $iR=1-cEe^{-\frac{R}{L}t}$

c) $i=\frac{E}{R}+ce^{\frac{R}{L}t}$ d) $i=\frac{E}{R}ce^{-\frac{R}{L}t}$

75) A charge q on the plate of condenser of capacity C through resistance R in series with steady state EMF V volt satisfies the differential equation $R\frac{dq}{dt}+\frac{q}{C}=V$. Then q in terms of t is

a) $q=\frac{C}{V}+ke^{-\frac{t}{RC}}$ b) $q=CV+ke^{\frac{t}{RC}}$

c) $q=CVke^{-\frac{t}{RC}}$ d) $q=CV+ke^{-\frac{t}{RC}}$

76) A charge q on the plate of condenser of capacity C through resistance R in series with steady state EMF V volt satisfies the equation $q=CV\left(1-e^{-\frac{t}{RC}}\right)$. Then i in terms of t is

a) $i=\frac{V}{R}e^{-\frac{t}{RC}}$ b) $i=\frac{V}{R}+e^{-\frac{t}{RC}}$

c) $i=VRe^{-\frac{t}{RC}}$ d) $i=\frac{V}{R}e^{\frac{t}{RC}}$

77) The differential equation for the current i is given to be $0.5\frac{di}{dt}+100i=20$ for an electrical circuit containing resistance R = 100 ohm, inductance L = 0.5 henry in series. Then

a) $i=0.2+Ae^{200t}$ b) $i=20+Ae^{-200t}$

c) $i=0.2Ae^{-200t}$ d) $i=0.2+Ae^{-200t}$

78) If an electrical circuit of R-C in series, charge $q=q(t)$ as function of t is $q=e^{3t}-e^{6t}$, the time required for maximum charge on capacitor is given by

a) $\frac{1}{2}\log 3$ b) $\frac{2}{3}\log 2$

c) $\frac{1}{3}\log 2$ d) $\frac{1}{3}\log\frac{1}{2}$

79) An electrical circuit of resistance R, inductance L in series with an electromotive force of E is satisfying the differential equation for the current i as $L\frac{di}{dt}+Ri=E$. For L = 640 henry, R = 250 ohm, E = 500 volt, the integrating factor of the above equation is

a) $e^{\frac{64}{25}t}$ b) $e^{\frac{25}{64}t}$ c) $e^{-\frac{25}{64}t}$ d) $e^{-\frac{64}{25}t}$

80) In an electrical circuit of L = 640 H, R = 250 Ω and E = 500 with EMF of 20 volts, the differential equation is

a) $\frac{di}{dt}+\frac{64}{25}i=\frac{32}{25}$ b) $\frac{di}{dt}+\frac{64}{25}i=\frac{25}{32}$

c) $\frac{di}{dt}+\frac{25}{64}i=\frac{25}{32}$ d) $\frac{di}{dt}+\frac{25}{64}i=\frac{32}{25}$

81) Rectilinear motion is the motion of body along

a) straight line b) circular motion

c) curvilinear d) parabolic path

82) The algebraic sum of the forces acting on a body along a given direction is equal to

a) mass × total force b) mass × distance

c) mass × velocity d) mass × acceleration

83) A particle moving in a straight line with acceleration $k\left(x+\frac{a^4}{x^3}\right)$ is directed towards origin. Then the equation of motion is

a) $\frac{dv}{dx}=-kv\left(x+\frac{a^4}{x^3}\right)$ b) $v\frac{dv}{dt}=-k\left(x+\frac{a^4}{x^3}\right)$

c) $\frac{d^2x}{dt^2}=-k\left(x+\frac{a^4}{x^3}\right)$ d) $k\frac{d^2x}{dt^2}=\left(x+\frac{a^4}{x^3}\right)$

84) A body of mass m kg moves in straight line with acceleration $\frac{k}{x^3}$ at a distance x and directed towards center. Then

a) $v\frac{dv}{dx}=-\frac{k}{x^3}$ b) $\frac{dv}{dx}=v\frac{k}{x^3}$

c) $v\frac{dv}{dx}=\frac{k}{x^3}$ d) $v\frac{dv}{dt}=-\frac{k}{x^3}$

85) A body of mass m falling freely from rest under gravitational force of attraction and air resistance proportional to square of velocity kv^2. Then

a) $\frac{dv}{dx}=v\left(mg-kv^2\right)$ b) $v\frac{dv}{dx}=m\left(g-kv^2\right)$

c) $mv\frac{dv}{dx}=mg-kv^2$ d) $v\frac{dv}{dx}=g-kv^2$

86) A particle is projected vertically upward with initial velocity v_1 and resistance of air produces retardation kv^2 where v is velocity at time t. Then

a) $mv\frac{dv}{dx}=mg-kv^2$ b) $v\frac{dv}{dx}=-g-kv^2$

c) $v\frac{dv}{dx}=m\left(g-kv^2\right)$ d) $v\frac{dv}{dx}=g-kv^2$

87) A particle starts moving horizontally from rest is opposed by a force cx, resistance per unit mass of value bv^2, where v and x are velocity and displacement of body at time t. Then

a) $v\frac{dv}{dx}=cs+bv^2$ b) $v\frac{dv}{dx}=-cs+bv^2$

c) $v\frac{dv}{dx}=cs-bv^2$ d) $v\frac{dv}{dx}=-cs-bv^2$

88) A body of mass m falls from rest under gravity in a liquid having resistance to motion at time t is mk times velocity. Then

a) $\frac{dv}{dt}=g+kv$ b) $\frac{dv}{dt}=g-kv$

c) $\frac{dv}{dt}=-g-kv$ d) $\frac{dv}{dt}=-g+kv$

89) A particle of mass m is projected vertically upward with velocity v, assuming the air resistance k times its velocity. Then

a) $m\frac{dv}{dt}=-mg-kv$ b) $m\frac{dv}{dt}=-mg+kv$

c) $m\frac{dv}{dt}=mg-kv$ d) $m\frac{dv}{dt}=mg+kv$

90) Assuming that the resistance to movement of a ship through water in the form of $a^2+b^2v^2$, where v is the velocity. Then the differential equation for retardation of the ship moving with engine stopped is

a) $m\frac{dv}{dt}=a^2+b^2v^2$ b) $m\frac{dv}{dt}=-a^2+b^2v^2$

c) $m\frac{dv}{dt}=-a^2-b^2v^2$ d) $m\frac{dv}{dt}=a^2-b^2v^2$

91) The differential equation of motion of particle of mass m falls from rest under gravity in a fluid satisfies the equation $\frac{dv}{dt}=g-kv$, then

a) $t=-k\log\left(\frac{g}{g-kv}\right)$ b) $t=k\log\left(\frac{g}{g-kv}\right)$

c) $t=-\frac{1}{k}\log\left(\frac{g}{g-kv}\right)$ d) $t=\frac{1}{k}\log\left(\frac{g}{g-kv}\right)$

92) A body of mass m falling freely under gravity satisfies the equation $mv\frac{dv}{dx}=k\left(a^2-v^2\right)$ with condition $ka^2=mg$, then

a) $x=\frac{m}{2k}\log\left(a^2-v^2\right)$ b) $x=\frac{m}{2}k\log\left(\frac{a^2}{a^2-v^2}\right)$

c) $x=-\frac{m}{2k}\log\left(\frac{a^2}{a^2-v^2}\right)$ d) $x=\frac{m}{2k}\log\left(\frac{a^2}{a^2-v^2}\right)$

93) A body starts from rest with an acceleration $\frac{dv}{dt}=k\left(1-\frac{t}{T}\right)$. Then its velocity is

a) $v=k\left(t-\frac{t^2}{2T}\right)$ b) $\frac{v^2}{2}=k\left(t-\frac{t^2}{2T}\right)$

c) $v=-k\left(t-\frac{t^2}{2T}\right)$ d) $v=k\left(\frac{t}{2}-\frac{t^2}{T}\right)$

94) A particle of unit mass starts from rest with an acceleration $v\frac{dv}{dr}=-\frac{k}{r^3}$. If initially it was at rest at r = a, then

a) $v^2=-k\left(\frac{1}{r^2}-\frac{1}{a^2}\right)$ b) $v^2=k\left(\frac{1}{r^2}+\frac{1}{a^2}\right)$

c) $v^2=k\left(\frac{1}{r^2}-\frac{1}{a^2}\right)$ d) $v^2=k\left(a^2-r^2\right)$

95) A particle of mass m is subjected projected upward with velocity V with its equation of motion $m\frac{dv}{dt}=-mg-kv$, then the velocity at time t is

a) $t=\log\left(\frac{mg+kv}{mg+kV}\right)$ b) $t=\frac{m}{k}\log\left(\frac{mg+kv}{mg+kV}\right)$

c) $t=-\frac{m}{k}\log\left(\frac{mg+kv}{mg+kV}\right)$ d) $t=\frac{m}{k}\log\left(\frac{mg-kv}{mg-kV}\right)$

96) A particle of mass m falls freely from rest under gravitational force in fluid producing resistance to motion of amount mkv, where k is constant. The differential equation is $\frac{dv}{dt} = g - kv$, then its terminal velocity is

a) $-\frac{g}{k}$ b) gk c) $-gk$ d) $\frac{g}{k}$

97) A bullet is fired into a sand tank and satisfies the differential equation $\frac{dv}{dt} = -k\sqrt{v}$. If v_0 is its initial velocity, we have

a) $2\sqrt{v} = -kt + 2\sqrt{v_0}$ b) $2\sqrt{v} = -\left(kt + 2\sqrt{v_0}\right)$

c) $2\sqrt{v} = kt + 2\sqrt{v_0}$ d) $\sqrt{v} = kt - 2\sqrt{v_0}$

98) A particle is in motion of horizontal straight line with acceleration $k\left(x + \frac{a^4}{x^3}\right)$ directed towards its origin and satisfies the differential equation $v\frac{dv}{dt} = -k\left(x + \frac{a^4}{x^3}\right)$. Assuming that it starts from rest at a distance x = a from origin, we have

a) $v^2 = -k\left(x^2 - \frac{a^4}{x^2}\right)$ b) $v^2 = k\left(x^2 + \frac{a^4}{x^2}\right)$

c) $v^2 = k\left(x^2 - \frac{a^4}{x^2}\right)$ d) $v^2 = -k\left(2x^2 - \frac{a^4}{2x^2}\right)$

99) If a particle moves in a straight line so that the force acting on it is directed towards a fixed point in the line of motion and proportional to its displacement from the point, it is then known as

a) curvilinear motion
b) rectilinear motion
c) Simple harmonic motion
d) circular motion

100) If a particle execute SHM, then its differential equation is given by

a) $\frac{d^2x}{dt^2} = -\omega^2 x$ b) $\frac{d^2x}{dt^2} - \omega^2 x = 0$

c) $\frac{d^2x}{dt^2} = k\omega x^2$ d) $\frac{d^2x}{dt^2} = -\omega x^2$

101) Fourier's law of heat conduction states that, the quantity of heat flow across the area of cross section A is

a) inversely proportional to the product of A with temperature gradient
b) proportional to the difference of A with temperature gradient
c) proportional to the product of A with temperature gradient
d) proportional to the sum of A and temperature gradient

102) If q quantity of heat flow across the cross sectional area A and thickness dx per unit time where the difference between temperatures at the faces is dT, the by Fourier's heat law

a) $q = -k - A\frac{dT}{dx}$ b) $q = -kA\frac{dT}{dx}$

c) $q = kA\frac{dT}{dx}$ d) $q = -kA + \frac{dT}{dx}$

103) The differential equation of steady state heat conduction per unit time from unit length of pipe of uniform radius r_0 carrying steam of temperature T_0 and thermal conductivity k, if the pipe is covered with material in a constant surrounding temperature, is given by

a) $Q = -\frac{2kr}{\pi}\cdot\frac{dT}{dr}$ b) $Q = -kr\frac{dT}{dr}$

c) $Q = 2k\pi r\frac{dT}{dr}$ d) $Q = -2k\pi r\frac{dT}{dr}$

104) The difference equation for steady state heat loss in unit time from a spherical shell of thermal conductivity covered by insulating material and kept in surrounding of constant temperature during heat flow, is

a) $Q = -\frac{4\pi r^2}{k}\cdot\frac{dT}{dr}$ b) $Q = 4k\pi r^2\frac{dT}{dr}$

c) $Q = -4k\pi r^2\frac{dT}{dr}$ d) $Q = -2k\pi r\frac{dT}{dr}$

105) The differential equation for steady state heat loss per unit time from unit length of pipe covered with insulating material which is kept in constant surrounding temperature, is

$Q=-2k\pi r\dfrac{dT}{dr}$. Then the temperature T is given by

a) $T=-\dfrac{Q}{k}\log r+c$ b) $T=-\dfrac{Q}{2\pi k}\log\dfrac{1}{r}+c$

c) $T=\dfrac{Q}{2\pi k}\log r+c$ d) $T=-\dfrac{Q}{2\pi k}\log r+c$

106) The differential equation for heat conductivity in spherical shell is described by $Q=-4k\pi r^2\dfrac{dT}{dr}$. Then

a) $T=\dfrac{Q}{kr}+c$ b) $T=\dfrac{Q}{4\pi kr}+c$

c) $T=\dfrac{Q}{4\pi k}r+c$ d) $T=-\dfrac{Q}{4\pi kr}+c$

107) A pipe of 10 cm radius carries steam of 150ºC and covered with insulating material of thickness 5 cm with thermal conductivity k = 0.0025 and it is kept in surrounding of temperature 40ºC. The equation is $dT=-\dfrac{Q}{2\pi k}\cdot\dfrac{dx}{x}$. Then the heat loss is

a) $220\pi k\log 1.5$ b) $\dfrac{220k}{\log 1.5}$

c) $\dfrac{220\pi k}{\log 1.5}$ d) $\dfrac{110\pi k}{\log 1.5}$

108) Heat is flowing through a hollow pipe of diameter 10 cm and outer diameter 20 cm and it is covered by insulating material of k = 0.12 and kept in surrounding of 200ºC. The differential equation is being $dT=-\dfrac{Q}{2\pi k}\cdot\dfrac{dx}{x}$. Then the heat loss is

a) $\dfrac{300\pi k}{\log 2}$ b) $\dfrac{150\pi k}{\log 2}$

c) $-\dfrac{300\pi k}{\log 2}$ d) $\dfrac{300\pi k}{\log 0.2}$

109) Steam of temperature 200ºC is set into pipe of 20 cm diameter covered with material of 6 cm thickness in surrounding of 30ºC. The equation is $dT=-\dfrac{Q}{2\pi k}\cdot\dfrac{dx}{x}$. The heat loss is

a) $\dfrac{170\pi k}{\log 16}$ b) $\dfrac{170(2\pi k)}{\log 1.6}$

c) $\dfrac{170\pi k}{\log 1.6}$ d) $-\dfrac{170\pi k}{\log 1.6}$

110) Steam of 100ºC is flowing through pipe of diameter 10 cm covered with asbestos of 5 cm thick and thermal conductivity k = 0.0006. The outer temperature is being 30ºC and the differential equation is $dT=-\dfrac{Q}{2\pi k}\cdot\dfrac{dx}{x}$. What is the amount of heat loss?

a) $\dfrac{140\pi k}{\log 2}$ b) $70\pi k\log 2$

c) $\dfrac{70\pi k}{\log 2}$ d) $-\dfrac{70\pi k}{\log 2}$

111) A tank contains 50 liters of fresh water. Brine of 2 gm/liter flows into the tank at the rate of 2 liters/minute and mixed with uniform continuity and the same amount runs out with the same rate. If Q is total amount of salt present at time t in the brine, we have

a) $\dfrac{dQ}{dt}=-4+\dfrac{Q}{25}$ b) $\dfrac{dQ}{dt}=-4-\dfrac{Q}{25}$

c) $\dfrac{dQ}{dt}=4-\dfrac{Q}{25}$ d) $\dfrac{dQ}{dt}=4+\dfrac{Q}{25}$

112) A tank contains 10000 liters of Brine of 200 kg dissolve salt. Fresh water flows into the tank at the rate of 100 liters/minute and mixed with uniform continuity and the same amount runs out with the same rate. If Q is total amount of salt present at time t in the brine, we have

a) $\dfrac{dQ}{dt}=200+\dfrac{Q}{100}$ b) $\dfrac{dQ}{dt}=-\dfrac{Q}{100}$

c) $\dfrac{dQ}{dt}=200-\dfrac{Q}{100}$ d) $\dfrac{dQ}{dt}=\dfrac{Q}{100}$

113) A tank contains 100 liters of fresh water. Brine of 1 gm/liter flows into the tank at the rate of 2 liters/minute and mixed with uniform continuity and the same amount runs out with the same rate. If Q is total amount of salt present at time t in the brine, we have

a) $\dfrac{dQ}{dt}=-\dfrac{Q}{100+t}$ b) $\dfrac{dQ}{dt}=2+\dfrac{Q}{100+t}$

c) $\dfrac{dQ}{dt}=2-\dfrac{Q}{100+t}$ d) $\dfrac{dQ}{dt}=2-\dfrac{Q}{100t}$

114) A tank contains 10000 liters of Brine of 20 kg dissolve salt. Brine of 0.1 kg/liter flows into the tank at the rate of 40 liters/minute and mixed with uniform continuity and the same amount runs out with the rate 30 liters/minute. If Q is total amount of salt present at time t in the brine, we have

a) $\frac{dQ}{dt}=4-\frac{3Q}{1000+10t}$ b) $\frac{dQ}{dt}=4-\frac{30Q}{100+t}$

c) $\frac{dQ}{dt}=-\frac{3Q}{100+t}$ d) $\frac{dQ}{dt}=4-\frac{3Q}{100+t}$

115) A tank contains 5000 liters of fresh water. Brine of 100 gm/liter flows into the tank at the rate of 10 liters/minute and mixed with uniform continuity and the same amount runs out with the same rate. If Q is total amount of salt present at time t in the brine, we have

a) $\frac{dQ}{dt}=\frac{5000-Q}{500}$ b) $\frac{dQ}{dt}=5000-\frac{Q}{500}$

c) $\frac{dQ}{dt}=1000+\frac{Q}{5}$ d) $\frac{dQ}{dt}=1000-\frac{Q}{500}$

116) A tank contains 10000 liters of Brine of 200 kg dissolve salt. Fresh water flows into the tank at the rate of 100 liters/minute and mixed with uniform continuity and the same amount runs out with the same rate. If Q is total amount of salt present at time t in the brine, we have $\frac{dQ}{dt}=-\frac{Q}{100}$. Then

a) $\log Q=-\frac{t}{100}$

b) $\log Q=-\frac{t}{100}-\log 200$

c) $\log Q=-\frac{t}{100}+\log 200$

d) $\log Q=\frac{t}{100}+\log 200$

117) A tank contains 50 liters of fresh water. Brine of 2 gm/liter flows into the tank at the rate of 2 liters/minute and mixed with uniform continuity and the same amount runs out with the same rate. If Q is total amount of salt present at time t, we have $\frac{dQ}{dt}=4-\frac{Q}{25}$. Then

a) $t=50\log 10-25\log(100-Q)$

b) $t=25\log 10-25\log(100-Q)$

c) $t=50\log 10+25\log(100-Q)$

d) $t=25\log 10+25\log(100-Q)$

118) The rate of decay of a substance is directly proportional to the amount of substance present at that time. Hence

a) $\frac{dt}{dx}=-kx$ b) $\frac{dx}{dt}=-kx$

c) $\frac{dx}{dt}=-kx+t$ d) $\frac{dx}{dt}=-kx^2+c$

Unit I : Ordinary Differential Equations

1	A	41	B	81	A	121	B	161	D	201	C	241	D
2	C	42	A	82	B	122	A	162	D	202	D	242	C
3	C	43	C	83	D	123	C	163	C	203	D	243	B
4	A	44	A	84	B	124	B	164	B	204	A	244	C
5	C	45	D	85	B	125	C	165	D	205	C	245	A
6	B	46	C	86	D	126	B	166	B	206	B	246	C
7	A	47	A	87	A	127	C	167	C	207	D	247	B
8	D	48	C	88	D	128	D	168	A	208	D	248	D
9	C	49	B	89	B	129	C	169	C	209	A	249	B
10	B	50	C	90	B	130	B	170	B	210	A	250	D
11	C	51	D	91	A	131	A	171	B	211	C	251	A
12	B	52	C	92	A	132	B	172	B	212	A	252	D
13	A	53	D	93	A	133	A	173	A	213	C	253	A
14	C	54	B	94	A	134	C	174	A	214	B	254	C
15	B	55	D	95	D	135	C	175	C	215	B	255	B
16	D	56	B	96	C	136	D	176	D	216	C	256	A
17	A	57	A	97	B	137	A	177	B	217	B	257	C
18	D	58	A	98	D	138	C	178	A	218	D	258	B
19	B	59	D	99	B	139	D	179	B	219	B	259	D
20	C	60	A	100	A	140	D	180	C	220	C	260	B
21	A	61	C	101	B	141	C	181	D	221	A	261	A
22	D	62	D	102	C	142	A	182	A	222	B	262	B
23	B	63	A	103	D	143	B	183	B	223	A	263	C
24	A	64	C	104	A	144	B	184	C	224	C		
25	D	65	B	105	B	145	D	185	A	225	B		
26	B	66	C	106	C	146	A	186	D	226	C		
27	D	67	B	107	A	147	D	187	A	227	D		
28	C	68	D	108	C	148	D	188	D	228	D		
29	A	69	C	109	D	149	C	189	C	229	C		
30	B	70	C	110	A	150	A	190	B	230	A		
31	A	71	A	111	D	151	C	191	C	231	D		
32	B	72	D	112	B	152	B	192	A	232	B		
33	B	73	D	113	D	153	A	193	D	233	A		
34	C	74	B	114	C	154	D	194	C	234	D		
35	B	75	B	115	A	155	A	195	A	235	C		
36	A	76	C	116	C	156	D	196	C	236	A		
37	A	77	D	117	D	157	C	197	B	237	D		
38	B	78	B	118	C	158	A	198	D	238	A		
39	C	79	D	119	B	159	B	199	B	239	D		
40	B	80	A	120	D	160	A	200	A	240	C		

Unit II : Applications of Ordinary Differential Equations

1	A	18	D	35	C	52	D	69	C	86	B	103	D
2	C	19	C	36	A	53	D	70	D	87	D	104	C
3	B	20	A	37	D	54	A	71	A	88	B	105	D
4	C	21	A	38	C	55	D	72	B	89	A	106	B
5	D	22	C	39	B	56	A	73	B	90	C	107	C
6	B	23	C	40	D	57	C	74	A	91	D	108	A
7	D	24	D	41	A	58	B	75	D	92	D	109	B
8	C	25	B	42	C	59	A	76	A	93	A	110	A
9	B	26	C	43	D	60	C	77	D	94	C	111	C
10	A	27	C	44	B	61	B	78	C	95	B	112	B
11	C	28	D	45	D	62	D	79	B	96	D	113	C
12	D	29	A	46	B	63	B	80	C	97	A	114	D
13	D	30	B	47	C	64	C	81	A	98	A	115	D
14	A	31	A	48	A	65	A	82	D	99	C	116	C
15	A	32	B	49	C	66	A	83	C	100	A	117	A
16	C	33	C	50	B	67	B	84	A	101	C	118	B
17	B	34	D	51	B	68	B	85	C	102	B		

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First Year Degree Course in Engineering – Semester II

Engineering Mathematics (M II)

Savitribai Phule Pune University

Second Online Examination

First Year of Engineering

Dr. Chavan N. S.

2015-16

Savitribai Phule Pune University – FE – Sem. II

Engineering Mathematics (M II)

Chapter 03 – Fourier Series

1) A function $f(x)$ is said to be periodic function with a period T, if
   a) $f(x)=f(x+T), \quad for\ all\ x$
   b) $f(T)=f(x+T), \quad for\ all\ x$
   c) $f(x)=-f(x+T), \quad for\ all\ x$
   d) $f(x)=f\left(\frac{x}{T}\right), \quad for\ all\ x$

2) A smallest positive number T satisfying $f(x)=f(x+T)$ is known as
   a) absolute function b) absolute time
   c) periodic time d)primitive period

3) If T is the fundamental period a function $f(x)$, which of the following is incorrect?
   a) $f(x)=f(x+nT), \quad n\in I$
   b) $f(x)=f(x+n+T), \ n\in I$
   c) $f(x)=f(x-T)$
   d) $f(x)=f(x+T)$

4) If $f(x+nT)=f(x)$ where n is an integer and T is the smallest positive number, the fundamental period of $f(x)$ is
   a) T b) nT c) $2T$ d) $\frac{T}{2}$

5) If $f(x)$ is a periodic function of period T, then for $n\neq 0$, the function $f(nx)$ is a periodic function of period
   a) T b) T^n c) $\frac{T}{n}$ d) nT

6) The fundamental period of $\sin x$ is
   a) π b) 2π c) 3π d) $\frac{\pi}{2}$

7) The fundamental period of $\sin 2x$ is
   a) π b) 2π c) 3π d) $\frac{\pi}{2}$

8) The fundamental period of $\sin 4x$ is
   a) π b) 2π c) 3π d) $\frac{\pi}{2}$

9) The fundamental period of $\cos 3x$ is
   a) π b) $\frac{2\pi}{3}$ c) $\frac{3\pi}{2}$ d) 3π

10) The fundamental period of $\sin(-3x)$ is
   a) -3π b) 3π c) $-\frac{2\pi}{3}$ d) $\frac{2\pi}{3}$

11) The fundamental period of $\sin\left(-\frac{x}{2}\right)$ is
   a) π b) 2π c) 3π d) 4π

12) The fundamental period of $\cos(x+\pi)$ is
   a) π b) 2π c) 3π d) $\frac{\pi}{2}$

13) The fundamental period of $\sin\left(x+\frac{3\pi}{2}\right)$ is
   a) 2π b) $\frac{2\pi}{3}$ c) 3π d) π

14) The fundamental period of $\tan\left(3x+\frac{\pi}{2}\right)$ is
   a) 2π b) π c) 3π d) $\frac{\pi}{3}$

15) The fundamental period of $\sin\left(x+\frac{\pi}{6}\right)$ is
   a) π b) 2π c) 3π d) $\frac{\pi}{3}$

16) The fundamental period of $2\sin x$ is
   a) π b) 2π c) 3π d) 4π

17) The fundamental period of $\sin x\cos x$ is
   a) π b) 2π c) 3π d) 4π

18) The fundamental period of $\tan x$ is
a) 4π b) 3π c) 2π d) π

19) The fundamental period of $\tan 5x$ is
a) $\frac{\pi}{5}$ b) 5π c) 10π d) π

20) The fundamental period of $2\sec(-3x)$ is
a) $-\frac{2\pi}{3}$ b) $\frac{2\pi}{3}$ c) $\frac{3\pi}{2}$ d) $-\frac{3\pi}{2}$

21) The fundamental period of $\operatorname{cosec} 2x$ is
a) π b) 2π c) 3π d) $\frac{\pi}{2}$

22) A function $f(x)$ defined in the interval $[-a,\ a]$ is said to be even function, if
a) $f(-x)=-f(x)$ b) $f(2x)=2f(x)$
c) $f(-x)=f(x)$ d) $f(x)=-f(x)$

23) A function $f(x)$ defined in the interval $[-a,\ a]$ is said to be odd function, if
a) $f(x)=-f(x)$ b) $f(2x)=2f(x)$
c) $f(-x)=f(x)$ d) $f(-x)=-f(x)$

24) Which of the followings is an even function?
a) $\cosh x$ b) $x^3-\cos x$
c) $\tan 3x$ d) $e^x+\tan^2 x$

25) Which of the followings is an even function?
a) $\sin 3x$ b) $\tan x$ c) $\operatorname{cosec}^3 x$ d) $\tan^2 x$

26) Which of the followings is not an even function?
a) $\sin^3 x$ b) $\sin^2 x$ c) $\tan^2 x$ d) $\sec x$

27) Which of the followings is an odd function?
a) e^{-x} b) $\tan\frac{3x}{2}$
c) $\cos^3 x$ d) $\operatorname{cosec} 2x$

28) Which of the followings is an odd function?
a) $-e^x$ b) $-\tan^2 x$
c) $-\sin x$ d) $-\cos x$

29) Which of the followings is not an odd function?
a) $2\tan x$ b) $\tan^2 x$
c) $\tan x$ d) $\sin 3x$

30) Which of the followings is neither even nor an odd function?
a) $\operatorname{cosech} x$ b) $\tanh x$ c) e^x d) $\sinh x$

31) If $f(x)$ is to be constant function w.r.t. x, then $f(x)$ is
a) even function
b) odd function
c) both even and odd
d) neither even nor odd

32) If $f(x)=x^3+2x-\cos x$, the function $f(x)$ is
a) even function
b) odd function
c) both even and odd
d) neither even nor odd

33) If $f(x)=x^2-\sin^4 x\cdot e^{|x|}$, the function $f(x)$ is
a) even function
b) odd function
c) both even and odd
d) neither even nor odd

34) Which of the following statement is incorrect?
a) Product of even and odd function is an odd function.
b) Multiplication of even and odd function is an odd function.
c) Addition of even and odd function is an odd function.
d) Subtraction of two odd functions is an odd function.

35) Fourier series expansion of a function $f(x)$ defined on the interval $[c,\ c+2L]$ and having period $2L$ is given by
a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos\left(\frac{n\pi x}{L}\right)+b_n\sin\left(\frac{n\pi x}{L}\right)$
b) $a_0+\sum_{n=1}^{\infty} a_n\cos\left(\frac{n\pi x}{L}\right)+b_n\sin\left(\frac{n\pi x}{L}\right)$
c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos(n\pi Lx)+b_n\sin(n\pi Lx)$
d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos\left(\frac{nx}{L}\right)+b_n\sin\left(\frac{nx}{L}\right)$

36) Fourier series expansion of a function $f(x)$ defined on the interval $[0,\ 2\pi]$ and satisfying the Dirichlet's conditions is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\frac{nx}{2}+b_n \sin\frac{nx}{2}$

b) $\frac{a_0}{2}+2\pi\sum_{n=1}^{\infty} a_n \cos nx+b_n \sin nx$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nx+b_n \sin nx$

d) $\frac{a_0}{2}+a_n \cos nx+b_n \sin nx$

37) If a function $f(x)$ is defined on the interval $[-\pi,\ \pi]$ and satisfying the Dirichlet's conditions, Fourier series expansion is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)+b_n \sin\left(\frac{n\pi x}{L}\right)$

b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{nx}{L}\right)+b_n \sin\left(\frac{nx}{L}\right)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{2n\pi x}{L}\right)+b_n \sin\left(\frac{2n\pi x}{L}\right)$

d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$

38) If a function $f(x)$ is defined on the interval $[0,\ 4]$ with period $T=4$, Fourier series expansion is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{4}\right)+b_n \sin\left(\frac{n\pi x}{4}\right)$

b) $\frac{a_0}{2}+a_n \cos\left(\frac{n\pi x}{2}\right)+b_n \sin\left(\frac{n\pi x}{2}\right)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{2}\right)+b_n \sin\left(\frac{n\pi x}{2}\right)$

d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{2n\pi x}{L}\right)+b_n \sin\left(\frac{2n\pi x}{L}\right)$

39) Fourier series expansion of a function $f(x)$ defined over a period 2π and satisfying the Dirichlet's conditions is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin nx+b_n \cos nx$

b) $\frac{a_0}{2}+2\pi\sum_{n=1}^{\infty} a_n \cos nx+b_n \sin nx$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nx+b_n \sin nx$

d) $\frac{a_0}{2}+a_n \cos nx+b_n \sin nx$

40) If an even function $f(x)$ with period $2l$ is defined over the interval $(-l,\ l)$, its Fourier series expansion is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{l}\right)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{2l}\right)$ d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{nx}{l}\right)$

41) If an odd function $f(x)$ is defined over the interval $(-\pi,\ \pi)$, its Fourier series expansion is given by

a) $\sum_{n=1}^{\infty} b_n \sin\left(\frac{2nx}{l}\right)$ b) $\sum_{n=1}^{\infty} a_n \sin(nx)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin(nx)$ d) $\sum_{n=1}^{\infty} b_n \sin(nx)$

42) If an odd function $f(x)$ is of period 2π, its Fourier series expansion is given by

a) $\sum_{n=1}^{\infty} b_n \sin\left(\frac{2nx}{l}\right)$ b) $\sum_{n=1}^{\infty} a_n \sin(nx)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin(nx)$ d) $\sum_{n=1}^{\infty} b_n \sin(nx)$

43) The Fourier series expansion of an even function $f(x)$ with period 2π is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin(nx)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{nx}{2}\right)$ d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{nx}{l}\right)$

44) If an odd function $f(x)$ with period $2l$ is defined over the interval $(-l,\ l)$, its Fourier series expansion is given by

a) $\sum_{n=1}^{\infty} b_n \sin\left(\frac{nx}{l}\right)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$

c) $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$ d) $\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{l}\right)$

45) If $f(x)$ is periodic function with period $2L$ in the interval C to $C+2L$, the Fourier coefficient a_0 is

a) $\frac{1}{L}\int_{C}^{C+2L} f(x)\cos\frac{nx}{L}dx$ b) $\frac{1}{L}\int_{C}^{C+2L} f(x)\sin\frac{nx}{L}dx$

c) $\int_{C}^{C+2L} f(x)dx$ d) $\frac{1}{L}\int_{C}^{C+2L} f(x)dx$

46) If $f(x)$ is periodic function with period $2L$ in the interval C to $C+2L$, the Fourier coefficient a_n is

a) $\frac{1}{L}\int_{C}^{C+2L} f(x)\cos\frac{2n\pi x}{L}dx$

b) $\frac{1}{L}\int_{C}^{C+2L} f(x)\cos\frac{n\pi x}{L}dx$

c) $\frac{1}{L}\int_{C}^{C+2L} f(x)\cos\frac{nx}{L}dx$

d) $\frac{1}{2L}\int_{C}^{C+2L} f(x)\cos\frac{nx}{L}dx$

47) If $f(x)$ is periodic function with period $2L$ in the interval C to $C+2L$, the Fourier coefficient b_n is

a) $\frac{1}{L}\int_{C}^{C+2L} f(x)\sin\frac{\pi x}{L}dx$

b) $\frac{1}{L}\int_{C}^{C+2L} f(x)\sin\frac{nx}{L}dx$

c) $\frac{1}{L}\int_{C}^{C+2L} f(x)\sin\frac{n\pi x}{L}dx$

d) $\frac{1}{L}\int_{0}^{2L} f(x)\sin\frac{n\pi x}{L}dx$

48) If $f(x)$ is an even function defined in the interval $[-L, L]$ and $f(x)=f(x+2L)$, the Fourier coefficient are

a) $a_0=\frac{2}{L}\int_0^L f(x)dx,\ a_n=\frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx,\ b_n=0$

b) $a_0=\frac{1}{L}\int_0^L f(x)dx,\quad a_n=\frac{1}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx$

c) $a_0=0,\quad a_n=\frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx$

d) $a_0=\frac{2}{L}\int_0^L f(x)dx,\quad a_n=\frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$

49) If $f(x)$ is an odd function defined in the interval $[-L, L]$ and $f(x)=f(x+2L)$, the Fourier coefficient are

a) $a_0=0,\quad a_n=0,\quad b_n=\frac{1}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$

b) $a_0=0,\quad a_n=0,\quad b_n=\frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$

c) $a_0=0,\quad a_n=0,\quad b_n=\frac{2}{L}\int_0^{2L} f(x)\sin\frac{n\pi x}{L}dx$

d) $a_0=0,\quad a_n=0,\quad b_n=\frac{2}{L}\int_0^L f(x)\sin\frac{nx}{L}dx$

50) If $f(x)$ is an even periodic function defined in the interval $[-\pi, \pi]$, the Fourier coefficient are

a) $a_0=\frac{2}{\pi}\int_0^\pi f(x)dx,\ a_n=\frac{2}{\pi}\int_0^\pi f(x)\cos nx\,dx,\ b_n=0$

b) $a_0=\frac{2}{L}\int_0^\pi f(x)dx,\ a_n=\frac{2}{L}\int_0^\pi f(x)\cos\frac{nx}{L}dx,\ b_n=0$

c) $a_0=\frac{2}{\pi}\int_0^\pi f(x)dx,\ a_n=\frac{2}{\pi}\int_0^\pi f(x)\sin nxdx,\ b_n=0$

d) $a_0=\frac{1}{\pi}\int_0^\pi f(x)dx,\ a_n=\frac{1}{\pi}\int_0^\pi f(x)\cos nx\,dx,\ b_n=0$

51) If $f(x)$ is an odd periodic function defined in the interval $[-\pi, \pi]$, the Fourier coefficient are

a) $a_0=0,\ a_n=\frac{2}{\pi}\int_0^\pi f(x)\cos nx\,dx,\ b_n=0$

b) $a_0=\frac{2}{\pi}\int_0^\pi f(x)dx,\ a_n=\frac{2}{\pi}\int_0^\pi f(x)\cos nx\,dx,\ b_n=0$

c) $a_0=0,\ a_n=0,\ b_n=\frac{2}{\pi}\int_0^\pi f(x)\sin nx\,dx$

d) $a_0 = \frac{1}{\pi}\int_0^{\pi} f(x)\,dx,\ a_n = \frac{1}{\pi}\int_0^{\pi} f(x)\cos nx\,dx,\ b_n = 0$

52) The Fourier coefficient of an even periodic function $f(x)$ defined in the interval $[-2, 2]$ are

a) $a_0 = \int_0^2 f(x)\,dx,\ a_n = \int_0^2 f(x)\cos nx\,dx,\ b_n = 0$

b) $a_0 = \int_0^2 f(x)\,dx,\ a_n = \int_0^2 f(x)\cos\frac{n\pi x}{2}\,dx,\ b_n = 0$

c) $a_0 = \frac{2}{\pi}\int_0^2 f(x)\,dx,\ a_n = \frac{2}{\pi}\int_0^2 f(x)\cos nx\,dx,\ b_n = 0$

d) $a_0 = \int_0^2 f(x)\,dx,\ a_n = \int_0^2 f(x)\cos nx\,dx,\ b_n = 0$

53) The Fourier coefficient of an even periodic function $f(x)$ defined in the interval $[-1, 1]$ are

a) $a_0 = \frac{2}{\pi}\int_0^1 f(x)\,dx,\ a_n = \frac{2}{\pi}\int_0^1 f(x)\cos n\pi x\,dx,\ b_n = 0$

b) $a_0 = 2\int_0^2 f(x)\,dx,\ a_n = 2\int_0^2 f(x)\cos\frac{n\pi x}{2}\,dx,\ b_n = 0$

c) $a_0 = 2\int_0^1 f(x)\,dx,\ a_n = 2\int_0^1 f(x)\cos n\pi x\,dx,\ b_n = 0$

d) $a_0 = \int_0^1 f(x)\,dx,\ a_n = \int_0^1 f(x)\cos n\pi x\,dx,\ b_n = 0$

54) The Fourier coefficient of an odd periodic function $f(x)$ defined in the interval $[-4, 4]$ are

a) $a_0 = 0,\ \ a_n = 0,\ \ b_n = \frac{1}{2}\int_0^4 f(x)\cos\frac{n\pi x}{4}\,dx$

b) $a_0 = 0,\ \ a_n = 0,\ \ b_n = \frac{1}{4}\int_0^L f(x)\sin n\pi x\,dx$

c) $a_0 = 0,\ \ a_n = 0,\ \ b_n = 2\int_0^4 f(x)\sin\frac{n\pi x}{4}\,dx$

d) $a_0 = 0,\ \ a_n = 0,\ \ b_n = \frac{1}{2}\int_0^4 f(x)\sin\frac{n\pi x}{4}\,dx$

55) If the Fourier series expansion of an even function $f(x)$ over an interval $[-l, l]$ is given by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$, the value of a_0 is obtained by

a) $\frac{2}{l}\int_{-l}^{l} f(x)\,dx$ b) $\frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$

c) $\frac{1}{2l}\int_0^l f(x)\,dx$ d) $\frac{2}{l}\int_0^l f(x)\,dx$

56) If the Fourier series expansion of an even function $f(x)$ over an interval $[-l, l]$ is given by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$, the value of a_n is obtained by

a) $\frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}\,dx$ b) $\frac{1}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$

c) $\frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$ d) $\frac{1}{l}\int_0^l f(x)\cos\frac{nx}{l}\,dx$

57) If the Fourier series expansion of an even function $f(x)$ over an interval $[-\pi, \pi]$ is given by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$, the value of a_0 is obtained by

a) $\frac{2}{\pi}\int_0^{\pi} f(x)\,dx$ b) $\frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$

c) $\frac{1}{\pi}\int_0^{\pi} f(x)\,dx$ d) $\frac{2}{\pi}\int_0^{\pi} f(x)\cos nx dx$

58) If the Fourier series expansion of an even function $f(x)$ over an interval $[-\pi, \pi]$ is given by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$, the value of a_n is obtained by

a) $\frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$ b) $\frac{2}{\pi}\int_0^{\pi} f(x)\cos nx dx$

c) $\frac{1}{\pi}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$ d) $\frac{2}{\pi}\int_0^{\pi} f(x)\cos\frac{nx}{\pi}\,dx$

59) If the Fourier series expansion of an odd function $f(x)$ over an interval $[-l, l]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$, the value of b_0 is obtained by

a) $\int_0^l f(x)\cos\frac{n\pi x}{l}dx$ b) $\frac{1}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}dx$

c) $\frac{2}{l}\int_0^l f(x)dx$ d) none of the above

60) If the Fourier series expansion of an odd function $f(x)$ over an interval $[-l, l]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$, the value of b_n is obtained by

a) $\frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}dx$ b) $\frac{1}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}dx$

c) $\frac{1}{l}\int_0^l f(x)\sin\frac{nx}{l}dx$ d) $\int_0^l f(x)\cos\frac{n\pi x}{l}dx$

62) The half range Fourier cosine series for $f(x)$ defined over the interval $[0, L]$ is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{nx}{L}\right)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right)$

c) $a_0+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$ d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$

63) The half range Fourier sine series for $f(x)$ defined over the interval $[0, L]$ is given by

a) $\sum_{n=1}^{\infty} a_n \sin\left(\frac{nx}{L}\right)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right)$

c) $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$ d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$

64) The half range Fourier cosine series for $f(x)$ defined over the interval $[0, \pi]$ is given by

a) $\sum_{n=1}^{\infty} b_n \sin(nx)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$ d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)$

65) The half range Fourier sine series for $f(x)$ defined over the interval $[0, \pi]$ is given by

a) $\sum_{n=1}^{\infty} b_n \sin\left(\frac{nx}{\pi}\right)$ b) $\sum_{n=1}^{\infty} b_n \sin(nx)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right)$ d) $\sum_{n=1}^{\infty} a_n \sin(n\pi x)$

66) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, L]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$ the value of a_0 is given by

a) $\frac{1}{L}\int_0^L f(x)dx$ b) $\frac{1}{L}\int_0^L f(x)dx$

c) $\frac{2}{\pi}\int_0^\pi f(x)dx$ d) $\frac{2}{L}\int_0^L f(x)dx$

67) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, L]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$ the value of a_n is given by

a) $\frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$

b) $\frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

c) $\frac{2}{L}\int_0^L f(x)\cos(nx)dx$

d) $\frac{1}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

68) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, L]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$ the value of b_0 is given by

a) $\frac{1}{L}\int_0^L f(x)\sin\frac{x}{L}dx$ b) $\frac{2}{L}\int_0^L f(x)\sin x\,dx$

c) 0 d) $\frac{2}{L}\int_0^L f(x)dx$

69) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, L]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$ the value of b_n is given by

a) $\frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$

b) $\frac{1}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$

c) $\frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

d) $\frac{2}{L}\int_0^L f(x)\sin(nx)\,dx$

70) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, \pi]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos(nx)$ the value of a_0 is given by

a) $\frac{1}{L}\int_0^L f(x)\,dx$ b) $\frac{1}{L}\int_0^L f(x)\,dx$

c) $\frac{2}{\pi}\int_0^\pi f(x)\,dx$ d) $\frac{2}{L}\int_0^L f(x)\,dx$

71) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, \pi]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos(nx)$ the value of a_n is given by

a) $\frac{2}{L}\int_0^L f(x)\sin(nx)\,dx$

b) $\frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

c) $\frac{2}{\pi}\int_0^\pi f(x)\cos(nx)\,dx$

d) $\frac{1}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

72) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, \pi]$ is given by $\sum_{n=1}^{\infty} b_n\sin(nx)$ the value of b_0 is given by

a) $\frac{1}{L}\int_0^L f(x)\sin\frac{x}{L}\,dx$ b) $\frac{2}{L}\int_0^L f(x)\sin x\,dx$

c) 0 d) $\frac{2}{L}\int_0^L f(x)\,dx$

73) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, \pi]$ is given by $\sum_{n=1}^{\infty} b_n\sin(nx)$ the value of b_n is given by

a) $\frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$

b) $\frac{1}{L}\int_0^L f(x)\sin(nx)\,dx$

c) $\frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

d) $\frac{2}{\pi}\int_0^\pi f(x)\sin(nx)\,dx$

74) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, 1]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos(n\pi x)$ the value of a_0 is given by

a) $\frac{1}{\pi}\int_0^\pi f(x)\,dx$ b) $2\int_0^1 f(x)\,dx$

c) $\frac{2}{\pi}\int_0^\pi f(x)\,dx$ d) $\int_0^1 f(x)\,dx$

75) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, 2]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos\left(\frac{n\pi x}{2}\right)$ the value of a_n is given by

a) $\frac{2}{3}\int_0^2 f(x)\sin\left(\frac{n\pi x}{2}\right)dx$

b) $\frac{1}{2}\int_0^2 f(x)\cos\left(\frac{n\pi x}{2}\right)dx$

c) $\frac{2}{L}\int_0^2 f(x)\cos(nx)\,dx$

d) $\int_0^2 f(x)\cos\left(\frac{n\pi x}{2}\right)dx$

76) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, 3]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{3}\right)$ the value of b_0 is given by

a) $\frac{1}{3}\int_0^3 f(x)\sin\frac{x}{3}dx$ b) $\frac{2}{3}\int_0^3 f(x)\sin 3x\, dx$

c) 0 d) $\frac{2}{3}\int_0^3 f(x)dx$

77) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, 4]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{4}\right)$ the value of b_n is given by

a) $\frac{2}{3}\int_0^2 f(x)\sin\left(\frac{n\pi x}{4}\right)dx$

b) $\frac{1}{2}\int_0^4 f(x)\sin\left(\frac{n\pi x}{4}\right)dx$

c) $\int_0^2 f(x)\cos\left(\frac{n\pi x}{4}\right)dx$

d) $\frac{1}{2}\int_0^4 f(x)\sin(nx)dx$

78) In the harmonic analysis for a function defined over a period of 2π, the term $a_1\cos x + b_1 \sin x$ is known as

a) amplitude of $f(x)$ b) second harmonic

c) first harmonic d) none of these

79) In the harmonic analysis for a function defined over a period of 2π, the amplitude of the first harmonic is

a) $\sqrt{a_1^2 - b_1^2}$ b) $\sqrt{a_1^2 + b_1^2}$

c) $\sqrt{a_0^2 + a_1^2}$ d) $a_1^2 + b_1^2$

80) In the harmonic analysis for a function defined over a period of 2π, the amplitude of the second harmonic is

a) $\left(a_2^2 + b_2^2\right)^2$ b) $\frac{1}{2}\left(a_2^2 + b_2^2\right)$

c) $2\sqrt{a_2^2 + b_2^2}$ d) $\sqrt{a_2^2 + b_2^2}$

81) In the harmonic analysis for a function defined over a period of 2π, the amplitude of the second harmonic is

a) $\left(a_n^2 + b_n^2\right)^n$ b) $\sqrt{a_n^2 + b_n^2}$

c) $n\sqrt{a_n^2 + b_n^2}$ d) $\frac{1}{n}\sqrt{a_n^2 + b_n^2}$

82) In the harmonic analysis for a function $f(x)$ defined over a period of $2L$, the first harmonic term is given by

a) $b_1 \sin\frac{\pi x}{L}$ b) $a_1 \cos\frac{\pi x}{L}$

c) $a_1 \cos\frac{\pi x}{L} - b_1 \sin\frac{\pi x}{L}$ d) $a_1 \cos\frac{\pi x}{L} + b_1 \sin\frac{\pi x}{L}$

83) In the harmonic analysis for a function $f(x)$ defined over a period of 2, the first harmonic term is given by

a) $a_1\cos\pi x + b_1 \sin \pi x$ b) $a_1 \cos\frac{\pi x}{2} + b_1 \sin\frac{\pi x}{2}$

c) $a_1 \cos 2\pi x + b_1 \sin 2\pi x$ d) $a_1 \cos\frac{\pi x}{2} - b_1 \sin\frac{\pi x}{2}$

84) If $f(x) = x\cos x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(nx) + b_n \sin(nx)$, the Fourier coefficient a_0 is given by

a) $-\frac{1}{\pi}$ b) 0 c) $\frac{1}{\pi}$ d) $\frac{2}{\pi}$

85) If $f(x) = x\cos x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(nx) + b_n \sin(nx)$, the Fourier coefficient a_n is given by

a) $\frac{1}{\pi}$ b) $-\frac{1}{\pi}$ c) $\frac{2}{\pi}$ d) 0

86) If $f(x) = x\cos x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(nx) + b_n \sin(nx)$, the Fourier coefficient b_1 is given by

a) $\frac{1}{\pi}$ b) $-\frac{1}{\pi}$ c) $-\frac{1}{2}$ d) 0

87) If $f(x)=\cos x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π, the Fourier coefficient a_1 is given by

a) 1 b) $-\frac{1}{\pi}$ c) $\frac{2}{\pi}$ d) 0

88) If $f(x)=x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient a_0 is given by

a) $\frac{\pi}{2}$ b) 0 c) 1 d) $\frac{\pi^2}{2}$

89) If $f(x)=x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient b_1 is given by

a) 2 b) 0 c) π d) $\frac{\pi}{2}$

90) If $f(x)=2$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient a_0 is given by

a) 2 b) 4 c) 3 d) none of these

91) If $f(x)=2$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient b_n is given by

a) $\frac{3\pi}{2}$ b) $-\frac{\pi}{2}$ c) 0 d) $\frac{\pi}{2}$

92) If $f(x)=a$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient a_0 is given by

a) $2a$ b) 0 c) 2π d) $\frac{\pi}{2}$

93) If $f(x)=a$ where $-\pi \le x \le \pi$ is of periodic function with period 2π, the Fourier coefficient a_0 is given by

a) 2π b) $2a$ c) 0 d) $\frac{\pi}{2}$

94) If $f(x)=\sin^2 x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient b_n is given by

a) $\frac{3\pi}{2}$ b) $-\frac{\pi}{2}$ c) $\frac{\pi}{2}$ d) 0

95) If $f(x)=\cosh x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient b_n is given by

a) 0 b) $\frac{\pi}{3}$ c) $e^{-\pi}$ d) $e^{-2\pi}$

96) If $f(x)=\begin{cases} -x & -\pi \le x \le 0 \\ x & 0 \le x \le \pi \end{cases}$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient a_0 is given by

a) $\frac{\pi}{2}$ b) π c) 0 d) $-\frac{\pi}{2}$

97) If $f(x)=\begin{cases} -x & -\pi \le x \le 0 \\ x & 0 \le x \le \pi \end{cases}$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient b_n is given by

a) $\frac{\pi}{2}$ b) π c) 0 d) $-\frac{\pi}{2}$

98) If $f(x)=\begin{cases} 1+\frac{2x}{\pi} & -\pi \le x \le 0 \\ 1-\frac{2x}{\pi} & 0 \le x \le \pi \end{cases}$ is of periodic function with period 2π, the Fourier coefficient b_n is given by

a) $\frac{\pi}{2}$ b) π c) $-\frac{\pi}{2}$ d) 0

99) If $f(x)=x-x^3$ where $-2\le x\le 2$ is of periodic function with period 2 and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(nx)+b_n\sin(nx)$, the Fourier coefficient b_1 is given by

a) 2 b) 0 c) π d) $\frac{\pi}{2}$

100) If $f(x)=x+\frac{x^2}{4}$ where $-\pi\le x\le\pi$ is of periodic function with period 2π, the Fourier coefficient a_0 is given by

a) 0 b) $\frac{\pi}{2}$ c) $-\frac{\pi}{2}$ d) $\frac{\pi^2}{6}$

101) If $f(x)=e^x$ where $-\pi\le x\le\pi$ is of periodic function with period 2π, the Fourier coefficient a_0 is given by

a) 1 b) $\frac{e^{\pi}-e^{-\pi}}{\pi}$ c) $\frac{e^{\pi}+e^{-\pi}}{\pi}$ d) 0

102) If $f(x)=x-x^2$ where $-1\le x\le 1$ is of periodic function with period 2, the Fourier coefficient a_0 is given by

a) 1 b) $-\frac{2}{3}$ c) π d) 0

103) If $f(x)=1-x^2$ where $-1\le x\le 1$ is of periodic function with period 2, the Fourier coefficient a_0 is given by

a) 1 b) $\frac{2\pi}{3}$ c) $\frac{4}{3}$ d) 0

104) If $f(x)=k$ where $-l\le x\le l$ is of periodic function with period 2, the Fourier coefficient a_0 is given by

a) $2k$ b) $\frac{2k\pi}{3}$ c) $2k\pi$ d) 0

105) If $f(x)=\begin{cases}-a & -2\le x\le 0\\ a & 0\le x\le 2\end{cases}$ is of periodic function with period 4, the Fourier coefficient b_n is given by

a) 0 b) 4 c) $-\frac{\pi}{2}$ d) $-\frac{2a}{n\pi}\left[(-1)^n-1\right]$

106) If $f(x)=\begin{cases}0 & -2\le x\le 0\\ 2 & 0\le x\le 2\end{cases}$ is of periodic function with period 4, the Fourier coefficient a_0 is given by

a) 0 b) 4 c) $-\frac{\pi}{2}$ d) 1

107) If $f(x)=\begin{cases}1 & -1\le x\le 0\\ \cos\pi x & 0\le x\le 1\end{cases}$ is of periodic function with period 2, the Fourier coefficient a_0 is given by

a) 0 b) $\frac{\pi}{2}$ c) $-\frac{\pi}{2}$ d) 1

108) If $f(x)=e^{-x}$ defined over $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, the Fourier coefficient a_0 is given by

a) $\frac{1}{2\pi}\left(1-e^{-2\pi}\right)$ b) $\frac{2}{\pi}\left(1-e^{-2\pi}\right)$

c) $\frac{1}{\pi}\left(1+e^{-x}\right)$ d) $\frac{1}{\pi}\left(1-e^{-2\pi}\right)$

109) If $f(x)=x$ defined over $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, the Fourier coefficient a_0 is given by

a) 3π b) $\frac{\pi}{2}$ c) π d) 2π

110) If $f(x)=x$ defined over $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, the Fourier coefficient a_n is given by

a) 0 b) π c) 2π d) 3π

111) If $f(x)=x$ defined over $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, the Fourier coefficient b_n is given by

a) $-\frac{2}{n\pi}$ b) $-\frac{\pi}{n}$ c) $-\frac{1}{n}$ d) $-\frac{2}{n}$

112) If $f(x)=\sqrt{1-\cos x}$ defined over $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, the Fourier coefficient a_0 is given by

a) 0 b) $-\frac{4\sqrt{2}}{\pi}$ c) $\frac{4\sqrt{2}}{\pi}$ d) $\frac{8\sqrt{2}}{\pi}$

113) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=\dfrac{\pi-x}{2}$ defined over the interval $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, is given by

a) $-\dfrac{\pi}{2}$ b) $\dfrac{\pi}{2}$ c) 0 d) $\dfrac{\pi^2}{6}$

114) The Fourier coefficient a_n for the Fourier series expansion of $f(x)=\dfrac{\pi-x}{2}$ defined over the interval $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, is given by

a) 0 b) π c) $\dfrac{\pi}{2}$ d) $-\dfrac{\pi}{2}$

115) The Fourier coefficient b_n for the Fourier series expansion of $f(x)=\dfrac{\pi-x}{2}$ defined over the interval $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, is given by

a) $-\dfrac{1}{n^2}$ b) $\dfrac{1}{n}$ c) $-\dfrac{1}{n}$ d) $\dfrac{\pi}{n}$

116) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=\left(\dfrac{\pi-x}{2}\right)^2$ defined over the interval $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, is given by

a) $\dfrac{\pi^2}{6}$ b) $\dfrac{\pi^2}{12}$ c) $\dfrac{\pi^2}{3}$ d) $\dfrac{\pi^2-1}{6}$

117) Consider $f(x)=x\sin x,\ x\in[0,2\pi]$ and $f(x+2\pi)=f(x)$. Then the Fourier coefficient a_0 for the Fourier series expansion of $f(x)$ is given by

a) -4 b) $-\dfrac{\pi}{2}$ c) -2 d) $\dfrac{\pi}{2}$

118) If $f(x)=\begin{cases}x & 0\le x\le \pi\\ 0 & \pi\le x\le 2\pi\end{cases}$ is periodic over a period 2π, the Fourier coefficient a_0 for the Fourier series expansion of $f(x)$ is given by

a) π b) $\dfrac{\pi}{2}$ c) $-\dfrac{\pi}{2}$ d) $\dfrac{\pi}{4}$

119) If $f(x)=\begin{cases}0 & 0\le x\le \pi\\ x & \pi\le x\le 2\pi\end{cases}$ is periodic over a period 2π, the Fourier coefficient a_0 for the Fourier series expansion of $f(x)$ is given by

a) $\dfrac{3\pi}{2}$ b) $\dfrac{\pi}{2}$ c) 3π d) $\dfrac{3\pi}{4}$

120) If the function $f(x)=\begin{cases}-\pi & 0\le x\le \pi\\ x-\pi & \pi\le x\le 2\pi\end{cases}$ and $f(x+2\pi)=f(x)$, the Fourier coefficient a_0 is given by

a) 0 b) $-\dfrac{\pi}{2}$ c) $-\dfrac{\pi}{4}$ d) $-\pi$

121) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=x-x^2$ defined over the interval $0\le x\le 2$ and $f(x+2)=f(x)$, is given by

a) 0 b) $-\dfrac{4}{3}$ c) $-\dfrac{2}{3}$ d) $\dfrac{2}{3}$

122) The Fourier coefficient a_n for the Fourier series expansion of $f(x)=x-x^2$ defined over the interval $0\le x\le 2$ and $f(x+2)=f(x)$, is given by

a) 0 b) $-\dfrac{4}{n^2\pi^2}$ c) $\dfrac{4}{n^2\pi^2}$ d) $-\dfrac{1}{n^2\pi^2}$

123) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=x+x^2$ defined over the interval $0\le x\le 3$ and having period 3, is given by

a) 0 b) $-\dfrac{4}{n^2\pi^2}$ c) $\dfrac{4}{n^2\pi^2}$ d) $\dfrac{3}{2}$

124) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=2x-x^2$ defined over the interval $0\le x\le 4$ and $f(x+4)=f(x)$, is given by

a) $-\dfrac{1}{3}$ b) $-\dfrac{2}{3}$ c) $-\dfrac{4}{3}$ d) $-\dfrac{8}{3}$

125) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=2x-x^2$ defined over the interval $0\le x\le 3$ and $f(x+3)=f(x)$, is given by

a) 0 b) $-\frac{2}{3}$ c) $-\frac{4}{3}$ d) $-\frac{8}{3}$

126) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=1-x^2$ defined over the interval $0\le x\le 2$ and $f(x+2)=f(x)$, is given by

a) $-\frac{1}{3}$ b) $-\frac{2}{3}$ c) $-\frac{4}{3}$ d) $-\frac{8}{3}$

127) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=1-x^2$ defined over the interval $0\le x\le 1$ and $f(x+2)=f(x)$, is given by

a) $\frac{2}{3}$ b) $-\frac{2}{3}$ c) $-\frac{1}{3}$ d) $-\frac{4}{3}$

128) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=4-x^2$ defined over the interval $0\le x\le 2$ and $f(x+2)=f(x)$, is given by

a) $-\frac{1}{3}$ b) $\frac{16}{3}$ c) $-\frac{16}{3}$ d) $-\frac{8}{3}$

129) If $f(x)=\begin{cases}\pi x & 0\le x\le 1\\ \pi(2-x) & 1\le x\le 2\end{cases}$ is periodic over a period 2, the Fourier coefficient a_0 for the Fourier series expansion of $f(x)$ is given by

a) $-\frac{\pi}{2}$ b) π c) $-\pi$ d) $\frac{\pi}{2}$

130) If $f(x)=\begin{cases}x & 0\le x\le 1\\ 0 & 1\le x\le 2\end{cases}$ is periodic over a period 2, the Fourier coefficient a_0 for the Fourier series expansion of $f(x)$ is given by

a) 2 b) 0 c) $\frac{1}{2}$ d) 1

131) The Fourier coefficient a_0 in the half range cosine series expansion of function $f(x)=\sin x$ defined over the interval $[0,\ \pi]$ is given by

a) $\frac{2}{\pi}$ b) $\frac{\pi}{2}$ c) $-\frac{\pi}{2}$ d) 0

132) The Fourier coefficient b_1 in the half range cosine series expansion of function $f(x)=\cos x$ defined over the interval $[0,\ \pi]$ is given by

a) $-\frac{\pi}{2}$ b) 0 c) $\frac{1}{2\pi}$ d) $\frac{\pi}{2}$

133) The Fourier coefficient a_0 in the half range cosine series expansion of function $f(x)=\pi x-x^2$ defined over the interval $[0,\ \pi]$ is given by

a) 0 b) $\frac{\pi^2}{6}$ c) $\frac{2\pi^2}{3}$ d) $\frac{\pi^2}{3}$

134) The Fourier coefficient a_0 in the half range sine series expansion of function $f(x)=\cos x$ defined over the interval $[0,\ \pi]$ is given by

a) $\frac{4}{\pi}$ b) $\frac{2}{\pi}$ c) $\frac{\pi}{2}$ d) 0

135) The Fourier coefficient a_0 in the half range cosine series expansion of function $f(x)=\sin x$ defined over the interval $[0,\ \pi]$ is given by

a) $\frac{4}{\pi}$ b) $\frac{2}{\pi}$ c) 0 d) $\frac{\pi}{2}$

136) The Fourier coefficient a_1 in the half range cosine series expansion of function $f(x)=\sin x$ defined over the interval $[0,\ \pi]$ is given by

a) 1 b) $\frac{2}{\pi}$ c) 0 d) $\frac{\pi}{2}$

137) The Fourier coefficient b_1 in the half range cosine series expansion of function $f(x)=x$ defined over the interval $[0,\ 2]$ with period 4 is given by

a) 0 b) $\frac{1}{\pi}$ c) $\frac{2}{\pi}$ d) $\frac{4}{\pi}$

138) The Fourier coefficient a_0 in the half range cosine series expansion of function $f(x)=x-x^2$ defined over the interval $[0,\ 1]$ is given by

a) $\frac{1}{3}$ b) $-\frac{1}{3}$ c) 0 d) $\frac{2}{\pi}$

139) The Fourier coefficient a_0 in the half range cosine series expansion of function $f(x)=lx-x^2$ defined over the interval $[0,\ l]$ with period $2l$ is given by

a) 0 b) $\frac{l^2}{3}$ c) $\frac{\pi}{2}$ d) $\frac{2}{\pi n^2}(\cos n\pi-1)$

140) The Fourier coefficient a_1 in the half range cosine series expansion of function $f(x)=x-x^2$ defined over the interval $[0,\ 1]$ is given by

a) 2 b) $\frac{2}{\pi}$ c) $\frac{\pi}{2}$ d) $\frac{1}{2}$

141) The Fourier coefficient a_n in the half range cosine series expansion of function $f(x)=x-x^2$ defined over the interval $[0,\ 1]$ is given by

a) 0 b) $\frac{2}{\pi}$ c) $\frac{\pi}{2}$ d) $\frac{2}{\pi n^2}(\cos n\pi-1)$

142) The Fourier coefficient a_n in the half range sine series expansion of function $f(x)=2+x$ defined over the interval $[0,\ 1]$ is given by

a) 4 b) 0 c) $-\frac{2}{n\pi}$ d) $-\frac{2\pi}{n}$

143) The Fourier series expansion for the function $f(x)=\left(\frac{\pi-x}{2}\right)^2$ over the interval $0\le x\le 2\pi$ is given by $\left(\frac{\pi-x}{2}\right)^2=\frac{\pi^2}{12}+\sum_{n=1}^{\infty}\frac{1}{n^2}\cos nx$. Then the value of $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots$ is

a) 1 b) $\frac{\pi^2}{6}$ c) $\frac{\pi^2}{12}$ d) $\frac{\pi^2}{3}$

144) The Fourier series expansion for the function $f(x)=\pi^2-x^2$ over the interval $-\pi\le x\le\pi$ is given by $\pi^2-x^2=\frac{2\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}\cos nx$. Then the value of $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-+\ldots$ is

a) 1 b) $\frac{\pi^2}{6}$ c) $\frac{\pi^2}{12}$ d) $\frac{\pi^2}{3}$

145) The Fourier series expansion for the function $f(x)=\pi^2-x^2$ over the interval $-\pi\le x\le\pi$ is given by $\pi^2-x^2=\frac{2\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}\cos nx$. Then the value of $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots$ is

a) $\frac{\pi^2}{6}$ b) $\frac{\pi^2}{12}$ c) $\frac{\pi^2}{3}$ d) 0

146) The Fourier series expansion for the function $f(x)=\begin{cases}1+\frac{2x}{\pi} & -\pi\le x\le 0\\ 1-\frac{2x}{\pi} & 0\le x\le\pi\end{cases}$ is given by $f(x)=\frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos nx$. Then the value of $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots$ is

a) $\frac{\pi^2}{6}$ b) $\frac{\pi^2}{12}$ c) $\frac{\pi^2}{3}$ d) $\frac{\pi^2}{8}$

147) The Fourier series expansion for the function $f(x)=\begin{cases}\pi x & 0\le x\le 1\\ \pi(2-x) & 1\le x\le 2\end{cases}$ is given by $f(x)=\frac{\pi}{2}-\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos(2n-1)\pi x$. Then the value of $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots$ is

a) $\frac{\pi^2}{6}$ b) $\frac{\pi^2}{12}$ c) $\frac{\pi^2}{8}$ d) $\frac{\pi^2}{3}$

148) In the following harmonic analysis of $y=f(x)$, the value of a_0 is given by

x	0	1	2	3	4	5
y	4	8	15	7	5	3

a) 14 b) 7 c) 3.5 d) 6

149) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	1	2	3	4	5
y	9	18	26	26	26	20

a) 25.01 b) 20.83 c) 41.66 d) 40.89

150) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	30	60	90	120	150	180
y	0	9.2	14.4	17.8	17	12	0

a) 10.23 b) 23.46 c) 46.92 d) 11.73

151) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	60	120	180	240	300	360
y	1	1.4	1.9	1.6	1.6	1.2	1

a) 7.2 b) 1.45 c) 5.8 d) 2.9

152) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	60	120	180	240	300	360
y	1.98	2.15	2.7	-0.22	-0.31	1.5	1.98

a) 4.8 b) 2.6 c) 5.2 d) 1.3

153) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	1	1.4	1.9	1.6	1.6	1.2	1

a) 2.9 b) 5.8 c) 1.45 d) 3.8

154) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
y	1.98	1.35	1	1.3	-0.88	-0.25	1.98

a) 1 b) 0.75 c) 1.5 d) 3

155) In the following harmonic analysis of $y = f(x)$, the value of a_1 is given by

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
y	0	9.2	14.4	17.8	17.3	11.7	0

a) 3.73 b) 5.73 c) 7.73 d) -7.73

156) In the following harmonic analysis of $y = f(x)$, the value of b_1 is given by

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
y	0	9.2	14.4	17.8	17.3	11.7	0

a) 4.38 b) 3.48 c) 4.83 d) 8.43

157) In the following harmonic analysis of $y = f(x)$, the value of a_1 is given by

x	0	1	2	3	4	5
y	9	18	24	28	26	20

a) - 8.37 b) 8.73 c) 7.83 d) 3.78

158) In the following harmonic analysis of $y = f(x)$, the value of b_1 is given by

x	0	1	2	3	4	5
y	9	18	24	28	26	20

a) 1.25 b) -6.3 c) – 3.15 d) - 3.50

159) In the following harmonic analysis of $y = f(x)$, the value of b_1 is given by

x	0	1	2	3	4	5	6
y	9	18	24	28	26	20	9
$\cos\left(\frac{\pi}{3}x\right)$	1	0.5	-0.5	-1	-0.5	0.5	1

a) 3.38 b) – 8.33 c) 8.33 d) 5.83

Chapter 04–Reduction Formulae, Beta and Gamma Functions

I) Reduction Formulae

1) For $I_n = \int_0^{\pi/2} \sin^n x\, dx$, we have

a) $I_n = 2\int_0^{\pi} \sin^n x\, dx$ b) $I_n = \int_0^{\pi/2} \sin^{n-2} x \cos^2 x\, dx$

c) $I_n = \int_0^{\pi/2} \cos^n x\, dx$ d) $I_n = \frac{1}{2}\int_0^{\pi/4} \sin^n x\, dx$

2) For $I_n = \int_0^{\pi} \sin^n x\, dx$, we have

a) 0 b) $I_n = 2\int_0^{\pi/2} \sin^n x\, dx$

c) $I_n = 4\int_0^{\pi/2} \sin^n x\, dx$ d) none of these

3) For $I_n = \int_0^{\pi} \cos^n x\, dx$, where n is an even integer, we have

a) $I_n = 4\int_0^{\pi/2} \cos^n x\, dx$ b) $I_n = 2\int_0^{\pi/4} \cos^n x\, dx$

c) $I_n = 2\int_0^{\pi/2} \cos^n x\, dx$ d) 0

4) For $I_n = \int_0^{\pi} \cos^n x\, dx$, where n is an odd integer, we have

a) $I_n = 4\int_0^{\pi/2} \cos^n x\, dx$ b) $I_n = 2\int_0^{\pi/4} \cos^n x\, dx$

c) $I_n = 2\int_0^{\pi/2} \cos^n x\, dx$ d) 0

5) For $I_n = \int_0^{2\pi} \sin^n x\, dx$, where n is an even integer, we have

a) 0 b) $I_n = 4\int_0^{\pi/4} \sin^n x\, dx$

c) $I_n = 2\int_0^{\pi/2} \sin^n x\, dx$ d) $I_n = 4\int_0^{\pi/2} \sin^n x\, dx$

6) For $I_n = \int_0^{2\pi} \sin^n x\, dx$, where n is an odd integer, we have

a) $I_n = 4\int_0^{\pi/2} \sin^n x\, dx$ b) $I_n = 2\int_0^{\pi/2} \sin^n x\, dx$

c) 0 d) $I_n = 4\int_0^{\pi/4} \sin^n x\, dx$

7) For $I_n = \int_0^{2\pi} \cos^n x\, dx$, where n is an odd integer, we have

a) 0 b) $I_n = 4\int_0^{\pi/2} \cos^n x\, dx$

c) $I_n = 2\int_0^{\pi/2} \cos^n x\, dx$ d) $I_n = 4\int_0^{\pi/4} \cos^n x\, dx$

8) For $I_n = \int_0^{2\pi} \cos^n x\, dx$, where n is an even integer, we have

a) 0 b) $I_n = 4\int_0^{\pi/4} \cos^n x\, dx$

c) $I_n = 4\int_0^{\pi/2} \cos^n x\, dx$ d) $I_n = 2\int_0^{\pi/2} \cos^n x\, dx$

9) For $I_{m,n}=\int_0^{\pi}\sin^m x\cos^n x\,dx$, where both m and n are odd integers, we have

a) $I_{m,n}=2\int_0^{\pi/2}\sin^m x\cos^n x\,dx$ b) 0

c) $I_{m,n}=4\int_0^{\pi/2}\sin^m x\cos^n x\,dx$ d) none

10) For $I_{m,n}=\int_0^{\pi}\sin^m x\cos^n x\,dx$, where both m and n are even integers, we have

a) $I_{m,n}=2\int_0^{\pi/2}\sin^m x\cos^n x\,dx$

b) $I_{m,n}=4\int_0^{\pi/2}\sin^m x\cos^n x\,dx$

c) 0 d) none of the above

11) For $I_{m,n}=\int_0^{\pi}\sin^m x\cos^n x\,dx$, where only n is an even integer, we have

a) 0 b) $I_{m,n}=4\int_0^{\pi/2}\sin^m x\cos^n x\,dx$

c) $I_{m,n}=2\int_0^{\pi/2}\sin^m x\cos^n x\,dx$ d) none

12) For $I_{m,n}=\int_0^{\pi}\sin^m x\cos^n x\,dx$, where only n is an odd integer, we have

a) $I_{m,n}=4\int_0^{\pi/2}\sin^m x\cos^n x\,dx$

b) 0

c) $I_{m,n}=2\int_0^{\pi/2}\sin^m x\cos^n x\,dx$

d) none of the above

13) For $I_n=\int_0^{\pi/2}\sin^n x\,dx$, which of the following is the reduction formula?

a) $I_n=\frac{n-1}{n}I_{n-1}$ b) $I_n=\frac{n}{n+1}I_{n-2}$

c) $I_n=\frac{n+1}{n}I_{n-2}$ d) $I_n=\frac{n-1}{n}I_{n-2}$

14) For $I_n=\int_0^{\pi/2}\cos^n x\,dx$, which of the following is the reduction formula?

a) $I_n=\frac{n-1}{n}I_{n-2}$ b) $I_n=\frac{n-1}{n}I_{n-1}$

c) $I_n=\frac{n}{n+1}I_{n-2}$ d) $I_n=\frac{n+1}{n}I_{n-2}$

15) For $I_n=\int_0^{\pi/2}\sin^n x\,dx$, where n is an even natural number, which of the following is the reduced form?

a) $I_n=\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \ldots\ \cdot\frac{1}{2}\cdot\frac{\pi}{2}$

b) $I_n=\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \ldots\ \cdot\frac{2}{3}\cdot 1$

c) $I_n=\frac{n+1}{n}\cdot\frac{n+3}{n+2}\cdot\frac{n+5}{n+4}\cdot\ \ldots\ \cdot\frac{1}{2}\cdot\frac{\pi}{2}$

d) $I_n=\frac{n-2}{n-1}\cdot\frac{n-4}{n-3}\cdot\frac{n-6}{n-5}\cdot\ \ldots\ \cdot\frac{2}{3}\cdot\frac{\pi}{2}$

16) For $I_n=\int_0^{\pi/2}\cos^n x\,dx$, where n is an even natural number, which of the following is the reduced form?

a) $I_n=\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \ldots\ \cdot\frac{2}{3}\cdot 1$

b) $I_n=\frac{n-2}{n-1}\cdot\frac{n-4}{n-3}\cdot\frac{n-6}{n-5}\cdot\ \ldots\ \cdot\frac{2}{3}\cdot\frac{\pi}{2}$

c) $I_n=\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \ldots\ \cdot\frac{1}{2}\cdot\frac{\pi}{2}$

d) $I_n=\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \ldots\ \cdot\frac{1}{2}\cdot\pi$

17) For $I_n = \int_0^{\pi/2} \sin^n x\,dx$, where n is an odd natural number, which of the following is the reduced form?

a) $I_n = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ ...\ \cdot\frac{2}{3}\cdot 1$

b) $I_n = \frac{n-2}{n-1}\cdot\frac{n-4}{n-3}\cdot\frac{n-6}{n-5}\cdot\ ...\ \cdot\frac{2}{3}\cdot\frac{\pi}{2}$

c) $I_n = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ ...\ \cdot\frac{1}{2}\cdot\frac{\pi}{2}$

d) $I_n = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ ...\ \cdot\frac{1}{2}\cdot\pi$

18) For $I_n = \int_0^{\pi/2} \cos^n x\,dx$, where n is an odd natural number, which of the following is the reduced form?

a) $I_n = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ ...\ \cdot\frac{1}{2}\cdot\frac{\pi}{2}$

b) $I_n = \frac{n+1}{n}\cdot\frac{n+3}{n+2}\cdot\frac{n+5}{n+4}\cdot\ ...\ \cdot\frac{2}{3}\cdot 1$

c) $I_n = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ ...\ \cdot\frac{2}{3}\cdot 1$

d) $I_n = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ ...\ \cdot\frac{1}{2}\cdot\pi$

19) For $I_n = \int_0^{\pi/2} \sin^n x\cos^n x\,dx$, where n is an odd natural number, which of the following is the reduced form?

a) $I_{(m,\,n)} = \frac{[(m-1)(m-3)\cdot\ ...\ \cdot 2\,or\,1][(n-1)(n-3)\cdot\ ...\ \cdot 2\,or\,1]}{(m+n)(m+n-2)\cdot\ ...\ \cdot 2\,or\,1}\cdot k$

where $k = \begin{cases} \frac{\pi}{2} & \textit{both } m\ \&\ n \textit{ are odd} \\ 1 & \textit{otherwise} \end{cases}$

b) $I_{(m,\,n)} = \frac{[(m-1)(m-3)\cdot\ ...\ \cdot 2\,or\,1][(n-1)(n-3)\cdot\ ...\ \cdot 2\,or\,1]}{(m+n)(m+n-2)\cdot\ ...\ \cdot 2\,or\,1}\cdot k$

where $k = \begin{cases} \frac{\pi}{2} & \textit{both } m\ \&\ n \textit{ are even} \\ 1 & \textit{otherwise} \end{cases}$

c) $I_{(m,\,n)} = \frac{[(m-1)(m-3)\cdot\ ...\ \cdot 2\,or\,1][(n-1)(n-3)\cdot\ ...\ \cdot 2\,or\,1]}{(m+n)(m+n-2)\cdot\ ...\ \cdot 2\,or\,1}\cdot k$

where $k = \begin{cases} \frac{\pi}{2} & m+n \textit{ is even} \\ 1 & \textit{otherwise} \end{cases}$

d) $I_{(m,\,n)} = \frac{(m+n-1)(m+n-3)\cdot\ ...\ \cdot 2\,or\,1}{(m+n)(m+n-2)\cdot\ ...\ \cdot 2\,or\,1}\cdot k$

where $k = \begin{cases} \frac{\pi}{2} & \textit{both } m\ \&\ n \textit{ are odd} \\ 1 & \textit{otherwise} \end{cases}$

20) The value of $\int_0^{\pi/2} \sin^3 x\,dx$ is equal to

a) $\frac{3\pi}{4}$ b) $\frac{3}{4}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$

21) The value of $\int_0^{\pi/2} \sin^4 x\,dx$ is equal to

a) $\frac{3}{8}$ b) $\frac{3}{16}$ c) $\frac{3\pi}{16}$ d) $\frac{3\pi}{18}$

22) The value of $\int_0^{\pi/2} \sin^5 x\,dx$ is equal to

a) $\frac{4\pi}{15}$ b) $\frac{8\pi}{30}$ c) $\frac{8\pi}{15}$ d) $\frac{8}{15}$

23) The value of $\int_0^{\pi/2} \sin^9 x\,dx$ is equal to

a) $\frac{64}{315}$ b) $\frac{128}{315}$ c) $\frac{128}{315}\pi$ d) $\frac{64}{315}\pi$

24) The value of $\int_0^{\pi/2} \cos^3 x\,dx$ is equal to

a) $\frac{3\pi}{4}$ b) $\frac{3}{4}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$

25) The value of $\int_0^{\pi/2} \cos^4 x\,dx$ is equal to

a) $\frac{3}{8}$ b) $\frac{3}{16}$ c) $\frac{3\pi}{16}$ d) $\frac{3\pi}{18}$

26) The value of $\int_0^{\pi/2} \cos^7 x\,dx$ is equal to

a) $\frac{8}{35}$ b) $\frac{16\pi}{35}$ c) $\frac{16\pi}{70}$ d) $\frac{16}{35}$

27) The value of $\int_0^{\pi/2} \cos^{10} x\,dx$ is equal to

a) $\frac{63\pi}{128}$ b) $\frac{63\pi}{512}$ c) $\frac{63\pi}{256}$ d) $\frac{64}{315}\pi$

28) The value of $\int_0^{\pi/2} \sin^3 x \cos^2 x\,dx$ is equal to

a) $\frac{2}{15}$ b) $\frac{\pi}{30}$ c) $\frac{1}{15}$ d) $\frac{\pi}{15}$

29) The value of $\int_0^{\pi/2} \sin^2 x \cos^3 x\,dx$ is equal to

a) $\frac{1}{15}$ b) $\frac{\pi}{30}$ c) $\frac{\pi}{15}$ d) $\frac{2}{15}$

30) The value of $\int_0^{\pi/2} \sin^3 x \cos^4 x\,dx$ is equal to

a) $\frac{1}{35}$ b) $\frac{2}{35}$ c) $\frac{2\pi}{35}$ d) $\frac{2\pi}{70}$

31) The value of $\int_0^{\pi/2} \sin^4 x \cos^6 x\,dx$ is equal to

a) $\frac{3\pi}{512}$ b) $\frac{3}{256}$ c) $\frac{3\pi}{256}$ d) $\frac{3\pi}{128}$

32) The value of $\int_{-\pi/2}^{\pi/2} \sin^3 x \cos^2 x\,dx$ is equal to

a) $2\int_0^{\pi/2} \sin^3 x \cos^2 x\,dx$ b) $4\int_0^{\pi/2} \sin^3 x \cos^2 x\,dx$

c) 0 d) none of the above

33) The value of $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos^3 x\,dx$ is equal to

a) 0 b) $2\int_0^{\pi/2} \sin^2 x \cos^3 x\,dx$

c) $3\int_0^{\pi/3} \sin^2 x \cos^3 x\,dx$ d) none of the above

34) The value of $\int_{-\pi/2}^{\pi/2} \sin^4 x \cos^2 x\,dx$ is equal to

a) $\frac{3}{16}$ b) $\frac{3\pi}{16}$ c) $\frac{\pi}{16}$ d) 0

35) The value of $\int_{-\pi/2}^{\pi/2} \sin^5 x\,dx$ is equal to

a) $\frac{3\pi}{128}$ b) $\frac{3\pi}{15}$ c) $\frac{32}{256}$ d) 0

36) The value of $\int_0^{2\pi} \sin^4 x \cos^6 x\,dx$ is equal to

a) $\frac{3}{64}$ b) $\frac{2\pi}{35}$ c) $\frac{2}{35}$ d) $\frac{3\pi}{128}$

37) The value of $\int_0^{2\pi} \sin^4 x \cos^7 x\,dx$ is equal to

a) $\frac{5}{128}$ b) $\frac{5\pi}{128}$ c) 0 d) $\frac{5\pi}{256}$

38) The value of $\int_{-\pi}^{\pi} \sin^4 x \cos^7 x\,dx$ is equal to

a) 0 b) $\frac{5\pi}{128}$ c) $\frac{5}{128}$ d) $\frac{5\pi}{256}$

39) The value of $\int_0^{\pi} \cos^3 x\,dx$ is equal to

a) $\frac{5\pi}{256}$ b) $\frac{5\pi}{16}$ c) $\frac{5}{128}$ d) 0

40) The value of $\int_0^{\pi} \cos^6 x\,dx$ is equal to

a) 0 b) $\frac{5\pi}{16}$ c) $\frac{5}{8}$ d) $\frac{5\pi}{256}$

41) The value of $\int_0^{\pi} \cos^7 x\,dx$ is equal to

a) $\frac{5\pi}{256}$ b) $\frac{5\pi}{16}$ c) $\frac{5}{128}$ d) 0

42) The value of $\int_0^{\pi} \sin^7 x\,dx$ is equal to

a) $\frac{5\pi}{32}$ b) $\frac{5\pi}{16}$ c) $\frac{32}{35}$ d) 0

43) The value of $\int_0^{\pi} \sin^6 x\,dx$ is equal to

a) $\frac{5\pi}{16}$ b) $\frac{5\pi}{32}$ c) $\frac{3}{4}$ d) 0

44) The value of $\int_0^{2\pi} \sin^6 \theta\,d\theta$ is equal to

a) $\frac{5\pi}{32}$ b) $\frac{5\pi}{16}$ c) $\frac{5\pi}{8}$ d) 0

45) The value of $\int_0^{2\pi} \sin^8 x\,dx$ is equal to

a) $\frac{5\pi}{16}$ b) $\frac{5\pi}{32}$ c) $\frac{32}{35}$ d) $\frac{35\pi}{32}$

46) The value of $\int_0^{2\pi} \cos^5 x\,dx$ is equal to

a) $\frac{5\pi}{32}$ b) $\frac{5\pi}{16}$ c) $\frac{32}{35}$ d) 0

47) The value of $\int_0^{2\pi} \sin^6 x \cos^4 x\,dx$ is equal to

a) $\frac{5\pi}{256}$ b) $\frac{3\pi}{128}$ c) $\frac{35\pi}{256}$ d) 0

48) The value of $\int_0^{2\pi} \sin^7 x \cos^4 x\,dx$ is equal to

a) $\frac{5\pi}{256}$ b) $\frac{3\pi}{128}$ c) $\frac{35\pi}{256}$ d) 0

49) The value of $\int_0^{2\pi} \sin^7 x \cos^5 x\,dx$ is equal to

a) $\frac{5\pi}{256}$ b) 0 c) $\frac{35\pi}{256}$ d) $\frac{3\pi}{128}$

50) The value of $\int_0^{\pi} \sin^5\left(\frac{x}{2}\right)dx$ is equal to

a) $\frac{16\pi}{15}$ b) $\frac{5\pi}{32}$ c) $\frac{16}{15}$ d) $\frac{5\pi}{16}$

51) The value of $\int_0^{\pi/4} \sin^2(2x)\,dx$ is equal to

a) $\frac{\pi}{8}$ b) $\frac{16}{15}$ c) $\frac{3\pi}{8}$ d) 0

52) The value of $\int_0^{\pi/4} \sin^7(2x)\,dx$ is equal to

a) $\frac{16\pi}{15}$ b) $\frac{5\pi}{16}$ c) $\frac{8}{35}$ d) 0

53) The value of $\int_0^{\pi/4} \cos^2(2x)\,dx$ is equal to

a) $\frac{5\pi}{16}$ b) $\frac{\pi}{8}$ c) $\frac{5\pi}{32}$ d) 0

54) The value of $\int_0^{\pi/3} \sin^5(3x)\,dx$ is equal to

a) $\frac{3\pi}{16}$ b) $\frac{8\pi}{15}$ c) $\frac{8\pi}{45}$ d) $\frac{8}{45}$

55) If $I_n = \int_0^{\pi/4} \sin^{2n} x\,dx = -\frac{1}{2^{n+1}n} + \frac{2n-1}{2n} I_{n-1}$, the value of I_2 is equal to

a) $\frac{3\pi+2}{8}$ b) $\frac{3\pi-8}{32}$ c) $-\frac{8+3\pi}{32}$ d) $\frac{3\pi}{32}$

56) If $I_n = \int_0^{\pi/2} x\sin^n x\,dx = \frac{1}{n^2} + \frac{n-1}{n} I_{n-2}$, the value of I_5 is equal to

a) $\frac{149}{25}$ b) $\frac{19}{225}$ c) $\frac{\pi}{2} - \frac{149}{225}$ d) $\frac{149}{225}$

56) If $I_n = \int_0^{\pi/2} \tan^n x\,dx = \frac{1}{n-1} - I_{n-2}$, the value of I_4 is equal to

a) $\frac{\pi}{4} - \frac{2}{3}$ b) $\frac{\pi}{4} + \frac{2}{3}$ c) $\frac{\pi}{2} - \frac{2}{3}$ d) $\frac{\pi}{4} + \frac{4}{3}$

II) Gamma Functions

57) For $n>0$, the gamma function $\overline{|n}$ is defined as

a) $\int_0^\infty e^{x}x^{n-1}dx$ b) $\int_0^\infty e^{-x}x^{n+1}dx$

c) $\int_0^\infty e^{-x}x^{n}dx$ d) $\int_0^\infty e^{-x}x^{n-1}dx$

58) $\int_0^\infty e^{-x}x^{n}dx$ is equal to

a) $\overline{|n+1}$ b) $\overline{|n}$ c) $\overline{|n-1}$ d) $\overline{|n-2}$

59) $\int_0^\infty e^{-kx}x^{n}dx$ is equal to

a) $k^{n-1}\overline{|n-1}$ b) $\frac{\overline{|n-1}}{k^{n-1}}$ c) $\frac{\overline{|n+1}}{k^{n+1}}$ d) $\frac{\overline{|n}}{k^{n}}$

60) $\int_0^\infty e^{-kx}x^{n-1}dx$ is equal to

a) $k^{n-1}\overline{|n-1}$ b) $\frac{\overline{|n-1}}{k^{n-1}}$ c) $\frac{\overline{|n+1}}{k^{n+1}}$ d) $\frac{\overline{|n}}{k^{n}}$

61) The value of $\overline{|n}$ is equal to

a) $n\overline{|n-1}$ b) $(n+1)\overline{|n+1}$

c) $(n-1)\overline{|n-1}$ d) $n\overline{|n}$

62) If n is a natural number, the value of $\overline{|n}$ is

a) $\frac{n!}{n+1}$ b) $(n-1)!$ c) $n!$ d) $(n+1)!$

63) The value of $\overline{|1}$ is

a) 1 b) 2 c) 3 d) 0

64) The value of $\overline{|2}$ is

a) 0 b) 1 c) 2 d) 3

65) The value of $\overline{|7}$ is

a) 3256 b) 5040 c) 120 d) 720

66) The value of $\overline{|1/2}$ is

a) $\frac{1}{2}$ b) $\sqrt{\pi}$ c) $\overline{|\pi}$ d) none

67) The value of $\overline{|5/2}$ is

a) $\frac{3\sqrt{\pi}}{2}$ b) $\frac{3\sqrt{\pi}}{4}$ c) $\frac{3\sqrt{\pi}}{8}$ d) 0

68) The value of $\overline{\left|\tfrac{1}{4}\right.}\cdot\overline{\left|\tfrac{3}{4}\right.}$ is

a) $\pi\sqrt{2}$ b) $\frac{\pi}{\sqrt{2}}$ c) $\frac{\sqrt{2}}{\pi}$ d) none

69) The value of $\overline{|p}\cdot\overline{|1-p}$, for $0<p<1$, is given by the formula

a) $\frac{\sin p\pi}{\pi}$ b) $\frac{\pi}{\sin p\pi}$

c) $\frac{\sqrt{\pi}}{\sin p\pi}$ d) $\frac{p\pi}{\sin p\pi}$

70) The value of $\int_0^\infty e^{-x}x^{5}dx$

a) 60 b) 720 c) 120 d) 240

71) The value of $\int_0^\infty e^{-2x}x^{5}dx$

a) $\frac{125}{32}$ b) $\frac{120}{35}$ c) $\frac{25}{8}$ d) $\frac{15}{8}$

72) The value of $\int_0^\infty e^{-x}x^{1/2}dx$

a) $\frac{\sqrt{\pi}}{2}$ b) $\frac{\pi}{2}$ c) $\frac{\sqrt{\pi}}{3}$ d) $\sqrt{\pi}$

73) The value of $\int_0^\infty e^{-x}x^{-1/2}dx$

a) $\frac{\sqrt{\pi}}{2}$ b) $\sqrt{\pi}$ c) $\frac{\sqrt{\pi}}{3}$ d) $\frac{\pi}{2}$

74) The value of $\int_0^\infty e^{-x}x^{3/2}dx$

a) $\frac{\sqrt{\pi}}{4}$ b) $\frac{3\sqrt{\pi}}{8}$ c) $\frac{3\sqrt{\pi}}{4}$ d) $\frac{3\sqrt{\pi}}{2}$

75) The substitution for the integral $\int_0^\infty \sqrt{x}\cdot e^{-\sqrt{x}}dx$ to reduce it into the form of gamma function is

a) $\sqrt{x}=t$ b) $\sqrt{x}=t^2$
c) $\sqrt{x}=\frac{t}{2}$ d) $x=\sin t$

76) The substitution for the integral $\int_0^\infty x^3\cdot e^{-\sqrt{x}}dx$ to reduce it into the form of gamma function is

a) $x^3=\sin^2 t$ b) $x^3=e^{-t}$
c) $x^3=t$ d) $\sqrt{x}=t$

77) The substitution for the integral $\int_0^\infty x^3\cdot 5^{-x}dx$ to reduce it into the form of gamma function is

a) $5^x=e^t$ b) $x^3=e^{-t}$
c) $5^x=x^{-t}$ d) $\log x=5^{-x}$

78) On using substitution $\sqrt{x}=t$, the value of the integration $\int_0^\infty x\cdot e^{-\sqrt{x}}dx$ is given by

a) 1 b) 3 c) 12 d) 16

79) On using substitution $\sqrt{x}=t$, the value of the integration $\int_0^\infty \sqrt{x}\cdot e^{-\sqrt{x}}dx$ is given by

a) 1 b) 2 c) 3 d) 4

80) On using substitution $\sqrt{t}=x$, the value of the integration $\int_0^\infty e^{-x^2}dx$ is given by

a) $\left|\frac{1}{4}\right.$ b) 16 c) $\frac{\sqrt{\pi}}{2}$ d) $\sqrt{\pi}$

81) On using substitution $x^3=t$, the value of the integration $\int_0^\infty \sqrt{x}\cdot e^{-x^3}dx$ is given by

a) $\frac{\sqrt{\pi}}{2}$ b) $\frac{\sqrt{\pi}}{3}$ c) $\frac{2\sqrt{\pi}}{3}$ d) $\frac{3\sqrt{\pi}}{4}$

82) On using substitution $x^4=t$, the value of the integration $\int_0^\infty e^{-x^4}dx$ is given by

a) $\sqrt{\pi}$ b) π c) $\frac{1}{4}\left|\frac{1}{4}\right.$ d) $\left|\frac{3}{4}\right.$

83) On using substitution $x=t^2$, the value of the integration $\int_0^\infty \sqrt[4]{x}\cdot e^{-\sqrt{x}}dx$ is given by

a) $\frac{3\sqrt{\pi}}{2}$ b) $\frac{2\sqrt{\pi}}{3}$ c) $\frac{\sqrt{\pi}}{3}$ d) $2\sqrt{\pi}$

84) On using substitution $2x^2=t$, the value of the integration $\int_0^\infty x^7\cdot e^{-2x^2}dx$ is given by

a) $\left|\frac{3}{4}\right.$ b) $\frac{3}{8}$ c) $\frac{2\sqrt{\pi}}{3}$ d) $\frac{3}{16}$

85) On using substitution $2x^2=t$, the value of the integration $\int_0^\infty x^9\cdot e^{-2x^2}dx$ is given by

a) $\left|\frac{3}{4}\right.$ b) $\frac{3}{8}$ c) $\frac{2\sqrt{\pi}}{3}$ d) $\frac{3}{16}$

86) On using substitution $x^2=t$, the value of the integration $\int_0^\infty x^2\cdot e^{-x^2}dx$ is given by

a) $\frac{1}{3}\left|\frac{3}{2}\right.$ b) $\frac{3}{2}\left|\frac{3}{2}\right.$ c) $\frac{1}{2}\left|\frac{3}{2}\right.$ d) $\frac{1}{2}\left|\frac{2}{3}\right.$

87) On using substitution $x=t^{1/3}$, the value of the integration $\int_0^\infty \sqrt{x}\cdot e^{-x^3}dx$ is given by

a) $\frac{\sqrt{\pi}}{3}$ b) $\frac{2\sqrt{\pi}}{3}$ c) $\frac{1}{2}\left|\frac{2}{3}\right.$ d) $\frac{1}{3}\left|\frac{3}{2}\right.$

88) On using substitution $a^{-x}=e^{-t}$, the value of the integration $\int_0^\infty \frac{x^a}{a^x}dx$ is given by

a) $\frac{\lceil a}{(\log a)^a}$ b) $\frac{\lceil a-1}{(\log a)^{a-1}}$

c) $\dfrac{\Gamma(a+1)}{(\log a)^{a+1}}$ d) $\dfrac{\Gamma(a)}{(\log a)^{a+1}}$

89) On using substitution $3^{-x} = e^{-t}$, the value of the integration $\int_0^\infty \frac{x^3}{3^x}dx$ is given by

a) $\dfrac{3}{(\log 3)^4}$ b) $\dfrac{6}{(\log 3)^4}$

c) $\dfrac{36}{(\log 3)^4}$ d) $\dfrac{6}{(\log 3)^3}$

90) On using substitution $\log x = -t$, the value of the integration $\int_0^1 (x\log x)^3 dx$ is given by

a) $-\dfrac{3}{64}$ b) $\dfrac{3}{64}$ c) $\dfrac{3}{128}$ d) $-\dfrac{3}{128}$

91) On using substitution $\log x = -t$, the value of the integration $\int_0^1 \left(\log\frac{1}{x}\right)^{n-1} dx$ is given by

a) $\Gamma(n+1)$ b) $\Gamma(n)$ c) $\Gamma(n-1)$ d) $-\Gamma(1+n)$

92) On using substitution $\log x = -t$, the value of the integration $\int_0^1 \frac{1}{\sqrt{x\log\frac{1}{x}}}dx$ is given by

a) $\sqrt{2\pi}$ b) $\pi\sqrt{2}$ c) $2\sqrt{\pi}$ d) 2π

93) On using substitution $\log x = -t$, the value of the integration $\int_0^1 \frac{1}{\sqrt{-\log x}}dx$ is given by

a) $\sqrt{2\pi}$ b) $\pi\sqrt{2}$ c) $\sqrt{\pi}$ d) $2\sqrt{\pi}$

94) On using substitution $h^2x^2 = t$, the value of the integration $\int_0^\infty x^{n-1}e^{-h^2x^2}dx$ is given by

a) $\sqrt{2\pi}$ b) $\dfrac{\Gamma(n/2)}{2h^n}$ c) $\dfrac{\Gamma(n/2)}{2h^{n+1}}$ d) $\dfrac{\Gamma(1+n/2)}{2h^{n+1}}$

II) Beta Functions

95) The value of $\beta(m, n)$ in the integral form is

a) $\int_0^1 x^m(1-x)^{n-1}dx$ b) $\int_0^1 x^m(1-x)^n dx$

c) $\int_0^1 x^{m+1}(1-x)^{n+1}dx$ d) $\int_0^1 x^{m-1}(1-x)^{n-1}dx$

96) The value of $\beta(m, n)$ in terms of gamma function is

a) $\dfrac{\Gamma(m)\cdot\Gamma(n)}{\Gamma(m+n+1)}$ b) $\dfrac{\Gamma(m-1)\cdot\Gamma(n-1)}{\Gamma(m+n)}$

c) $\dfrac{\Gamma(m+1)\cdot\Gamma(n+1)}{\Gamma(m+n+1)}$ d) $\dfrac{\Gamma(m)\cdot\Gamma(n)}{\Gamma(m+n)}$

97) The value of $\beta(m, n)$, when m and n are positive integers is

a) $\dfrac{(m-1)!(n-1)!}{(m+n-1)!}$ b) $\dfrac{(m+1)!(n+1)!}{(m+n+1)!}$

c) $\dfrac{m!n!}{(m+n)!}$ d) $\dfrac{m!n!}{(m+n+1)!}$

98) $\int_0^{\pi/2} \sin^m x\cos^n x dx$ is given by

a) $\beta(m, n)$ b) $\frac{1}{2}\beta\left(\frac{m-1}{2}, \frac{n-1}{2}\right)$

c) $\frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$ d) $\beta\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$

99) $\int_0^{\pi/2} \sin^{2m-1} x\cos^{2n-1} x dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$ b) $\beta\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$

c) $\frac{1}{2}\beta\left(\frac{m-1}{2}, \frac{n-1}{2}\right)$ d) $\beta(m, n)$

100) $\int_0^{\pi/2} \sin^m x\, dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{m-1}{2}, \frac{1}{2}\right)$ b) $\frac{1}{2}\beta\left(m, \frac{1}{2}\right)$

c) $\frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{1}{2}\right)$ d) $\frac{1}{2}\beta\left(\frac{m+1}{2}, 0\right)$

101) $\int_0^{\pi/2} \cos^m x\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{m-1}{2}, \frac{1}{2}\right)$ b) $\frac{1}{2}\beta\left(m, \frac{1}{2}\right)$

c) $\frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{1}{2}\right)$ d) $\frac{1}{2}\beta\left(\frac{m+1}{2}, 0\right)$

102) $\int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}}\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$ b) $\beta(m, n)$

c) $\beta(m+1, n+1)$ d) $\beta(m-1, n-1)$

103) $\beta(3, 5)$ can be represented by

a) $\int_0^{\infty} x^2(1-x)^4\,dx$ b) $\int_0^1 x^4(1-x)^6\,dx$

c) $\int_0^1 x^3(1-x)^5\,dx$ d) $\int_0^1 x^2(1-x)^4\,dx$

104) What is the exact value of $\beta(5, 3)$?

a) $\frac{2}{35}$ b) $\frac{2}{105}$ c) $\frac{1}{105}$ d) $\frac{1}{35}$

105) What is the exact value of $\beta\left(\frac{1}{4}, \frac{3}{4}\right)$?

a) $\frac{1}{8}$ b) $\pi\sqrt{2}$ c) $2\sqrt{\pi}$ d) $\sqrt{2\pi}$

106) $\int_0^1 \sqrt{x}\,(1-x)^{5/2}\,dx$ is equal to

a) $\beta\left(\frac{3}{2}, \frac{7}{2}\right)$ b) $\beta\left(\frac{1}{2}, \frac{5}{2}\right)$

c) $\beta\left(\frac{2}{3}, \frac{5}{3}\right)$ d) $\beta(2, 5)$

107) $\int_0^1 x^4(1-x)^5\,dx$ is equal to

a) $\frac{3}{462}$ b) $\frac{1}{462}$ c) $\frac{1}{501}$ d) $\frac{1}{231}$

108) $2\int_0^{\pi/2} \sin^{\frac{3}{2}} x \cos^5 x\,dx$ is given by

a) $\beta\left(\frac{5}{4}, 3\right)$ b) $\frac{1}{2}\beta\left(\frac{5}{4}, 3\right)$

c) $\beta\left(\frac{5}{4}, \frac{3}{2}\right)$ d) $\beta\left(\frac{5}{4}, \frac{3}{4}\right)$

109) $2\int_0^{\pi/2} \sqrt{\sin x \cos x}\,dx$ is given by

a) $\beta\left(\frac{1}{2}, \frac{1}{2}\right)$ b) $\beta\left(\frac{5}{4}, \frac{5}{4}\right)$

c) $\beta\left(\frac{3}{4}, \frac{3}{4}\right)$ d) $\beta\left(\frac{3}{2}, \frac{3}{2}\right)$

110) $\int_0^{\pi/2} \frac{1}{\sqrt{\sin x}}\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{1}{4}, \frac{3}{2}\right)$ b) $\beta\left(\frac{1}{4}, \frac{1}{2}\right)$

c) $2\beta\left(\frac{1}{4}, \frac{1}{2}\right)$ d) $\frac{1}{2}\beta\left(\frac{1}{4}, \frac{1}{2}\right)$

111) $\int_0^{\pi/2} \frac{1}{\sqrt{\cos x}}\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{1}{4}, \frac{3}{2}\right)$ b) $\frac{1}{2}\beta\left(\frac{1}{4}, \frac{1}{2}\right)$

c) $\beta\left(\frac{1}{4}, \frac{1}{2}\right)$ d) $2\beta\left(\frac{1}{4}, \frac{1}{2}\right)$

112) $\int_0^{\pi/2} \sqrt{\tan x}\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{3}{4}, \frac{5}{4}\right)$ b) $\beta\left(\frac{3}{4}, \frac{1}{4}\right)$

c) $2\beta\left(\frac{3}{4}, \frac{1}{4}\right)$ d) $\frac{1}{2}\beta\left(\frac{3}{4}, \frac{1}{4}\right)$

113) $\int_0^{\pi/2} \sqrt{\cot x}\,dx$ is given by

a) $2\beta\left(\frac{3}{4}, \frac{1}{4}\right)$ b) $\frac{1}{2}\beta\left(\frac{3}{4}, \frac{5}{4}\right)$

c) $\frac{1}{2}\beta\left(\frac{3}{4}, \frac{1}{4}\right)$ d) $\beta\left(\frac{3}{4}, \frac{1}{4}\right)$

114) $\int_0^{\pi/2} \tan^{3/4} x\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{7}{4}, \frac{1}{4}\right)$ b) $\frac{1}{2}\beta\left(\frac{7}{4}, \frac{1}{4}\right)$

c) $\frac{1}{2}\beta\left(\frac{7}{8}, \frac{1}{8}\right)$ d) $\frac{1}{2}\beta\left(\frac{7}{8}, \frac{7}{8}\right)$

115) The value of the integral $\int_0^\infty \frac{x^4}{(1+x)^7}dx$ is

a) $\frac{1}{30}$ b) 30 c) $\frac{1}{15}$ d) $\frac{1}{3}$

116) The value of the integral $\int_0^\infty \frac{x^3+x^2}{(1+x)^7}dx$ is

a) 30 b) $\frac{1}{3}$ c) $\frac{1}{30}$ d) $\frac{1}{15}$

117) The value of the integral $\int_0^\infty \frac{x^8-x^{14}}{(1+x)^{24}}dx$ is

a) 30 b) 0 c) $\frac{1}{30}$ d) $\frac{1}{15}$

118) The value of the integral $\int_0^\infty \frac{x^6\left(1-x^8\right)}{(1+x)^{24}}dx$ is

a) 30 b) 0 c) $\frac{1}{30}$ d) $\frac{1}{15}$

119) $\beta(n,\ n+1)$ is identical with

a) $\frac{(\Gamma n)^2}{\Gamma 2n}$ b) $\frac{\Gamma n}{\Gamma 2n}$ c) $\frac{\Gamma n}{2\Gamma 2n}$ d) $\frac{(\Gamma n)^2}{2\Gamma 2n}$

120) $\beta(m,\ n+1)+\beta(m+1,\ n)$ is equal to

a) $\beta(m+1,\ n+1)$ b) $\beta(m+1,\ n)$

c) $\beta(m,\ n)$ d) $\beta(m,\ n+1)$

121) $\beta(m,\ n)\cdot\beta(m+n,\ k)$ is equal to

a) $\frac{\Gamma m\cdot\Gamma(n+k)}{\Gamma(m+n+k)}$ b) $\frac{\Gamma m\cdot\Gamma n\cdot\Gamma k}{\Gamma(m+n)}$

c) $\frac{\Gamma m\cdot\Gamma n}{\Gamma(m+n+k)}$ d) $\frac{\Gamma m\cdot\Gamma n\cdot\Gamma k}{\Gamma(m+n+k)}$

122) $\beta(m,\ n+1)$ is equal to

a) $\frac{m+n}{n}\beta(m,\ n)$ b) $\frac{n}{m+n}\beta(m,\ n)$

c) $\frac{m}{m+n}\beta(m,\ n)$ d) $\frac{m+n}{m}\beta(m,\ n)$

123) On using substitution $x^3=8t$, the integral $\int_0^2 x\left(8-x^3\right)^{1/3}dx$ is equal to

a) $\frac{5}{81}$ b) $\frac{2}{27}$ c) $\frac{2}{81}$ d) $\frac{1}{81}$

124) The value of the integration $\int_0^1 x^3\left(1-x^{1/2}\right)^5 dx$ by substituting $x=t^2$ is given by

a) $2\beta(8,\ 6)$ b) $\frac{1}{2}\beta(8,\ 6)$

c) $\beta(8,\ 6)$ d) $2\beta(9,\ 7)$

125) The value of the integration $\int_0^1 \left(1-x^{1/n}\right)^m dx$ by substituting $x=t^n$ is given by

a) $n\beta(m,\ n+1)$ b) $n\beta(m+1,\ n)$

c) $n\beta(m,\ n)$ d) $m\beta(m+1,\ n)$

Chapter 05–Differentiation Under Integral Sign & Error Function

I) Differentiation Under Integral Sign

1) If $I(\alpha)=\int_a^b f(x,\alpha)\,dx$, where α is parameter and a, b are constants, by differentiation under integral sign rule we have

a) $\frac{dI}{d\alpha}=\int_a^b \frac{\partial}{\partial\alpha}\left[f(x,\alpha)\right]\cdot dx$

b) $\frac{dI}{d\alpha}=\int_a^b \frac{\partial}{\partial x}\left[f(x,\alpha)\right]\cdot dx$

c) $\frac{dI}{dx}=\int_a^b \frac{\partial}{\partial x}\left[f(x,\alpha)\right]\cdot dx$

d) $\frac{dI}{dx}=\int_a^b \frac{\partial}{\partial\alpha}\left[f(x,\alpha)\right]\cdot dx$

2) If $I(\alpha)=\int_a^b f(x,\alpha)\,dx$, where α is parameter and a, b are functions of α, by differentiation under integral sign rule we have

a) $\frac{dI}{d\alpha}=\int_a^b \frac{\partial}{\partial\alpha}\left[f(x,\alpha)\right]\cdot dx-f(x,b)\frac{db}{d\alpha}+f(x,a)\frac{da}{d\alpha}$

b) $\frac{dI}{d\alpha}=\int_a^b \frac{\partial}{\partial\alpha}\left[f(x,\alpha)\right]\cdot dx+f(x,b)\frac{db}{dx}-f(x,a)\frac{da}{dx}$

c) $\frac{dI}{d\alpha}=\int_a^b \frac{\partial}{\partial\alpha}\left[f(x,\alpha)\right]\cdot dx+f(x,b)\frac{db}{d\alpha}-f(x,a)\frac{da}{d\alpha}$

d) $\frac{dI}{dx}=\int_a^b \frac{\partial}{\partial x}\left[f(x,\alpha)\right]\cdot dx+f(x,b)\frac{db}{dx}-f(x,a)\frac{da}{dx}$

Note: Henceforth, we abbreviate "differentiation under integral sign" by "DUIS" for simplicity.

3) If $I=\int_0^\infty e^{-bx^2}\cos 2ax\cdot dx$, where $b>0$, by DUIS rule we have

a) $\frac{dI}{dx}=\int_0^\infty \frac{\partial}{\partial x}\left[e^{-bx^2}\cos 2ax\right]\cdot dx$

b) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial a}\left[e^{-bx^2}\cos 2ax\right]\cdot dx$

c) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial b}\left[e^{-bx^2}\cos 2ax\right]\cdot dx$

d) $\frac{dI}{db}=\int_0^\infty \frac{\partial}{\partial a}\left[e^{-bx^2}\cos 2ax\right]\cdot dx$

4) If $I=\int_0^\infty \frac{e^{-ax}}{x}\left(1-e^{-bx}\right)dx$, where $a, b>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial b}\left[\frac{e^{-ax}}{x}\left(1-e^{-bx}\right)\right]dx$

b) $\frac{dI}{dx}=\int_0^\infty \frac{\partial}{\partial x}\left[\frac{e^{-ax}}{x}\left(1-e^{-bx}\right)\right]dx$

c) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial a}\left[\frac{e^{-ax}}{x}\left(1-e^{-bx}\right)\right]dx$

d) $\frac{dI}{db}=\int_0^\infty \frac{\partial}{\partial a}\left[\frac{e^{-ax}}{x}\left(1-e^{-bx}\right)\right]dx$

5) If $I=\int_0^\infty \frac{e^{-ax}}{x}\left(1-e^{-x}\right)dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial a}\left[\frac{\left(1-e^{-x}\right)}{x}\right]\cdot e^{-ax}dx$

b) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial x}\left[e^{-ax}\right]\cdot\frac{\left(1-e^{-x}\right)}{x}dx$

c) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial x}\left[e^{-ax}\frac{\left(1-e^{-x}\right)}{x}\right]\cdot dx$

d) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial a}\left[e^{-ax}\right]\cdot\frac{\left(1-e^{-x}\right)}{x}dx$

6) If $I=\int_0^\infty \frac{e^{-x}}{x}\left(a-\frac{1}{x}+\frac{1}{x^2}e^{-ax}\right)dx$, where $a, b>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty \frac{e^{-x}}{x}\cdot\frac{\partial}{\partial x}\left(a-\frac{1}{x}+\frac{1}{x^2}e^{-ax}\right)dx$

b) $\frac{dI}{dx}=\int_0^\infty \frac{e^{-x}}{x}\cdot\frac{\partial}{\partial a}\left(a-\frac{1}{x}+\frac{1}{x^2}e^{-ax}\right)dx$

c) $\frac{dI}{dx}=\int_0^\infty \frac{e^{-x}}{x}\cdot\frac{\partial}{\partial x}\left(a-\frac{1}{x}+\frac{1}{x^2}e^{-ax}\right)dx$

d) $\frac{dI}{da}=\int_0^\infty \frac{e^{-x}}{x}\cdot\frac{\partial}{\partial a}\left(a-\frac{1}{x}+\frac{1}{x^2}e^{-ax}\right)dx$

7) If $I=\int_0^\infty \frac{e^{-ax}}{x}\left(1-e^{-x}\right)dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty e^{(a+1)x}dx$ b) $\frac{dI}{da}=\int_0^\infty e^{-ax}dx$

c) $\frac{dI}{da}=\int_0^\infty e^{-(a+1)x}dx$ d) $\frac{dI}{da}=\int_0^\infty e^{-(a-1)x}dx$

8) If $I=\int_0^\infty \frac{e^{-x}}{x}\left(a-\frac{1}{x}+\frac{1}{x^2}e^{-ax}\right)dx$, where $a,b>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty \frac{e^{-x}}{x}\left(1-\frac{1}{x}e^{-ax}\right)dx$

b) $\frac{dI}{da}=\int_0^\infty \frac{e^{-x}}{x}\left(1-e^{-ax}\right)dx$

c) $\frac{dI}{da}=\int_0^\infty \frac{e^{-x}}{x}\left(1-\frac{1}{x^2}e^{-ax}\right)dx$

d) $\frac{dI}{da}=\int_0^\infty \left(1-\frac{1}{x}e^{-ax}\right)dx$

9) If $I=\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x}dx$, where $a,b>0$, by DUIS rule we have

a) $\frac{dI}{db}=-\int_0^\infty e^{-ax}dx$ b) $\frac{dI}{da}=-\int_0^\infty e^{-ax}dx$

c) $\frac{dI}{da}=\int_0^\infty e^{-ax}dx$ d) $\frac{dI}{da}=\int_0^\infty \left(e^{-ax}-e^{-ax}\right)dx$

10) If $I=\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x}dx$, where $a,b>0$, by DUIS rule we have

a) $\frac{dI}{db}=\int_0^\infty \left(e^{-ax}-e^{-ax}\right)dx$ b) $\frac{dI}{db}=-\int_0^\infty e^{-bx}dx$

c) $\frac{dI}{db}=\int_0^\infty e^{-ax}dx$ d) $\frac{dI}{db}=\int_0^\infty e^{-bx}dx$

11) If $I=\int_0^\infty \frac{e^{-x}-e^{-ax}}{x\sec x}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty \frac{e^{-ax}}{\sec x}dx$ b) $\frac{dI}{da}=\int_0^\infty \frac{e^{-ax}}{\sec x\tan x}dx$

c) $\frac{dI}{da}=-\int_0^\infty \frac{e^{-ax}}{\sec x}dx$ d) $\frac{dI}{da}=-\int_0^\infty \frac{ae^{-ax}}{x\sec x}dx$

12) If $I=\int_0^\infty e^{-a^2}\cos ax\,da$, where $x>0$, by DUIS rule we have

a) $\frac{dI}{dx}=-2\int_0^\infty a^2e^{-a^2}\sin ax\,da$

b) $\frac{dI}{dx}=2\int_0^\infty ae^{-a^2}\sin ax\,da$

c) $\frac{dI}{dx}=-2\int_0^\infty ae^{-a^2}\cos ax\,da$

d) $\frac{dI}{dx}=-\int_0^\infty ae^{-a^2}\sin ax\,da$

13) If $I=\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}\frac{1}{x^2}dx$

b) $\frac{dI}{da}=a\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}\frac{1}{x^2}dx$

c) $\frac{dI}{da}=-2a\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}\frac{1}{x^2}dx$

d) $\frac{dI}{da}=-\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}\frac{1}{x^2}dx$

14) If $I=\int_0^\infty \frac{e^{-x}\sin ax}{x}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=-a\int_0^\infty \cos ax\,dx$ b) $\frac{dI}{da}=\int_0^\infty \sin ax\,dx$

c) $\frac{dI}{da}=-\int_0^\infty e^{-x}\cos ax\,dx$ d) $\frac{dI}{da}=\int_0^\infty e^{-x}\cos ax\,dx$

15) If $I=\int_0^\pi \frac{x^a-1}{\log x}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty \frac{x^a\log a}{\log x}dx$ b) $\frac{dI}{da}=\int_0^\pi x^a\,dx$

c) $\frac{dI}{da}=\int_0^\pi x^a\log a\,dx$ d) $\frac{dI}{da}=\int_0^\pi \frac{x^a\log a}{\log x}dx$

16) If $I=\int_0^1 \frac{x^a-x^b}{\log x}dx$, where $a,\ b>0$, by DUIS rule we have

a) x^a-x^b b) $\frac{dI}{da}=\int_0^\pi \frac{x^a-x^b}{x\log x}dx$

c) $\frac{dI}{da}=\int_0^1 \frac{x^a\log a}{\log x}dx$ d) $\frac{dI}{da}=\int_0^1 x^a\,dx$

17) If $I=\int_0^\pi \log(1+a\cos x)\,dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\pi \frac{-\sin x}{1+a\cos x}dx$ b) $\frac{dI}{da}=\int_0^\pi \frac{\cos x}{1+a\cos x}dx$

c) $\frac{dI}{da}=\int_0^\pi \frac{a}{1+a\cos x}dx$ d) $\frac{dI}{da}=-\int_0^\pi \frac{\cos x}{1+a\cos x}dx$

18) If $I=\int_0^\pi \frac{1}{x^2}\log(1+ax^2)\,dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\pi \frac{ax^2}{1+ax^2}dx$ b) $\frac{dI}{da}=2\int_0^\pi \frac{x}{1+ax^2}dx$

c) $\frac{dI}{da}=\int_0^\pi \frac{1}{1+ax^2}dx$ d) $\frac{dI}{da}=\int_0^\pi \frac{2ax}{1+ax^2}dx$

19) If $I=\int_0^\pi \frac{1}{\sin^2 x}\log(1+a\sin^2 x)\,dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\pi \frac{1}{1+a\sin^2 x}dx$ b) $\frac{dI}{da}=\int_0^\pi \frac{\sin 2x}{1+a\sin^2 x}dx$

c) $\frac{dI}{da}=\int_0^\pi \frac{a}{1+a\sin^2 x}dx$ d) $\frac{dI}{da}=\int_0^\pi \frac{\cos x}{1+a\sin^2 x}dx$

20) If $I=\int_0^\infty \frac{1-\cos ax}{x^2}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty \frac{a\sin ax}{x^2}dx$ b) $\frac{dI}{da}=-\int_0^\infty \frac{\cos ax}{x}dx$

c) $\frac{dI}{da}=\int_0^\infty \frac{\sin ax}{x}dx$ d) $\frac{dI}{da}=-\int_0^\infty \frac{\sin ax}{x}dx$

21) If $I=\int_0^1 \frac{x^a}{\log x}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^1 \frac{x^a\log a}{\log x}dx$ b) $\frac{dI}{da}=\int_0^1 x^a\,dx$

c) $\frac{dI}{da}=\int_0^1 x^a\log a\,dx$ d) $\frac{dI}{da}=\int_0^1 x^{a-1}\,dx$

22) If $I=\int_0^{\pi/2} \log(a^2\cos^2 x+b^2\sin^2 x)\,dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{\pi/2} \frac{1}{a^2+b^2\tan^2 x}dx$

b) $\frac{dI}{da}=\int_0^{\pi/2} \frac{b^2}{a^2+b^2\tan^2 x}dx$

c) $\frac{dI}{da}=\int_0^{\pi/2} \frac{a^2}{a^2+b^2\tan^2 x}dx$

d) $\frac{dI}{da}=\int_0^{\pi/2} \frac{2a}{a^2+b^2\tan^2 x}dx$

23) If $I=\int_0^{\infty}\frac{\sin ax-\sin bx}{x^2}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=-\int_0^{\infty}\frac{\cos bx}{x}dx$ b) $\frac{dI}{da}=\int_0^{\infty}\frac{\cos ax}{x}dx$

c) $\frac{dI}{da}=-\int_0^{\infty}\frac{\cos ax}{x}dx$ d) $\frac{dI}{db}=\int_0^{\infty}\frac{\cos ax}{x}dx$

24) If $I=\int_0^{a^2}\tan^{-1}\left(\frac{x}{a}\right)dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{a^2}\frac{\partial}{\partial a}\tan^{-1}\left(\frac{x}{a}\right)dx$

b) $\frac{dI}{da}=\int_0^{a^2}\frac{\partial}{\partial a}\tan^{-1}\left(\frac{x}{a}\right)dx-2a\tan^{-1}a$

c) $\frac{dI}{da}=\int_0^{a^2}\frac{\partial}{\partial a}\tan^{-1}\left(\frac{x}{a}\right)dx+2a\tan^{-1}x$

d) $\frac{dI}{da}=\int_0^{a^2}\frac{\partial}{\partial a}\tan^{-1}\left(\frac{x}{a}\right)dx+2a\tan^{-1}a$

25) If $I=\int_0^{a}\frac{\log(1+ax)}{1+x^2}dx$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{a}\frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right]dx+\frac{\log(1+a^2)}{1+a^2}$

b) $\frac{dI}{da}=\int_0^{a}\frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right]dx-\frac{\log(1+a^2)}{1+a^2}$

c) $\frac{dI}{da}=\int_0^{a}\frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right]dx$

d) $\frac{dI}{da}=\int_0^{a}\frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right]dx+\frac{\log(1+x^2)}{1+x^2}$

26) If $I=\int_a^{a^2}\log(ax)\,dx$, by DUIS rule we have

a) $\frac{dI}{da}=\int_a^{a^2}\frac{\partial}{\partial a}\log(ax)\,dx$

b) $\frac{dI}{da}=\int_a^{a^2}\frac{\partial}{\partial a}\log(ax)\,dx-(6a-2)\log a$

c) $\frac{dI}{da}=\int_a^{a^2}\frac{\partial}{\partial a}\log(ax)\,dx+(6a+2)\log a$

d) $\frac{dI}{da}=\int_a^{a^2}\frac{\partial}{\partial a}\log(ax)\,dx+(6a-2)\log a$

27) If $I=\int_t^{t^2}e^{tx^2}\,dx$, by DUIS rule we have

a) $\frac{dI}{dt}=\int_t^{t^2}x^2e^{tx^2}\,dx+2te^{t^5}-e^{t^3}$

b) $\frac{dI}{dt}=\int_t^{t^2}x^2e^{tx^2}\,dx-2te^{t^5}+e^{t^3}$

c) $\frac{dI}{dt}=\int_t^{t^2}te^{tx^2}\,dx+2te^{t^5}-e^{t^3}$

d) $\frac{dI}{dt}=\int_t^{t^2}t^3e^{tx^2}\,dx+2te^{t^5}-e^{t^3}$

28) If $I=\int_0^{a^2}\tan^{-1}\left(\frac{x}{a}\right)dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{a^2}\frac{x}{a^2+x^2}dx+2a\tan^{-1}a$

b) $\frac{dI}{da}=-\int_0^{a^2}\frac{a}{a^2+x^2}dx+2a\tan^{-1}a$

c) $\frac{dI}{da}=-\int_0^{a^2}\frac{x}{a^2+x^2}dx+2a\tan^{-1}a$

d) $\frac{dI}{da}=-\int_0^{a^2}\frac{x}{a^2+x^2}dx-2a\tan^{-1}a$

29) If $I=\int_a^{a^2}\log(ax)\,dx$, by DUIS rule we have

a) $\frac{dI}{da}=\int_a^{a^2}\frac{1}{x}dx-(6a-2)\log a$

b) $\frac{dI}{da}=\int_a^{a^2}\frac{1}{a}dx+(6a-2)\log a$

c) $\frac{dI}{da}=\int_a^{a^2}\frac{1}{a}dx-(6a-2)\log a$

d) $\frac{dI}{da}=\int_a^{a^2}\frac{1}{a}dx-(6a-2)\log a$

30) If $I=\int_0^a \frac{\log(1+ax)}{1+x^2}dx$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^a \frac{x}{(1+x^2)(1+ax)}dx+\frac{\log(1+a^2)}{1+a^2}$

b) $\frac{dI}{da}=\int_0^a \frac{1}{(1+x^2)(1+ax)}dx+\frac{\log(1+a^2)}{1+a^2}$

c) $\frac{dI}{da}=\int_0^a \frac{a}{(1+x^2)(1+ax)}dx+\frac{\log(1+a^2)}{1+a^2}$

d) $\frac{dI}{da}=\int_0^a \frac{x}{(1+x^2)(1+ax)}dx-\frac{\log(1+a^2)}{1+a^2}$

31) If $I=\int_{\pi/6a}^{\pi/3a}\frac{\sin ax}{x}dx$, by DUIS rule we have

a) $\frac{dI}{da}=\int_{\pi/6a}^{\pi/3a}\cos ax\,dx+\frac{1}{a}$

b) $\frac{dI}{da}=\int_{\pi/6a}^{\pi/3a}\cos ax\,dx-\frac{1}{2a}$

c) $\frac{dI}{da}=\int_{\pi/6a}^{\pi/3a}\cos ax\,dx-\frac{1}{a}$

d) $\frac{dI}{da}=\int_{\pi/6a}^{\pi/3a}\frac{\cos ax}{x}dx-\frac{1}{a}$

32) If $f(x)=\int_a^x (x-t)^2 G(t)\,dt$, we have

a) $\frac{df}{dx}=\int_a^x \frac{\partial}{\partial x}(x-t)^2 G(t)\,dt+(x-a)^2 G(a)$

b) $\frac{df}{dx}=\int_a^x \frac{\partial}{\partial x}(x-t)^2 G(t)\,dt$

c) $\frac{df}{dx}=\int_a^x \frac{\partial}{\partial x}(x-t)^2 G(t)\,dt-(x-a)^2 G(a)$

d) $\frac{df}{dx}=\int_a^x \frac{\partial}{\partial x}(x-t)^2 G(t)\,dt+a^2 G(a)$

33) If $y=\int_0^x f(t)\sin a(x-t)\,dt$, we have

a) $\frac{dy}{dx}=\int_0^x xf(t)\cos a(x-t)\,dt$

b) $\frac{dy}{dx}=\int_0^x af(t)\cos a(x-t)\,dt+f(x)$

c) $\frac{dy}{dx}=\int_0^x af(t)\cos a(x-t)\,dt-af(x)$

d) $\frac{dy}{dx}=a\int_0^x f(t)\cos a(x-t)\,dt$

34) For the integral $I(a)=\int_0^\infty \frac{e^{-x}}{x}(1-e^{-ax})dx$, we have $\frac{dI}{da}=\frac{1}{a+1}$, then I is

a) $\log(a+1)-1$ b) $\log(a+1)$

c) $\log(a+1)+1$ d) $-\frac{1}{(a+1)^2}$

35) The value of integration $I(a)=\int_0^1 \frac{x^a-1}{\log x}dx$ with $\frac{dI}{da}=\frac{1}{a+1}$ is given by

a) $\log(a+1)$ b) $\log(a+1)-1$

c) $\log(a+1)+1$ d) $-\frac{1}{(a+1)^2}$

36) The value of integration $I(a)=\int_0^1 \frac{e^{-2x}\sin ax}{x}dx$ with $\frac{dI}{da}=\frac{2}{a^2+4}$ is given by

a) $\tan^{-1}\left(\frac{a}{2}\right)+\frac{\pi}{2}$ b) $\tan^{-1}\left(\frac{a}{2}\right)$

c) $\frac{1}{2}\tan^{-1}\left(\frac{a}{2}\right)$ d) $\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$

37) The value of integration $I(a)=\int_0^\infty \frac{e^{-x}-e^{-ax}}{x\sec x}dx$ with $\frac{dI}{da}=\frac{a}{a^2+1}$ is given by

a) $2\log\left(\frac{2}{a^2+1}\right)$ b) $\frac{1}{2}\log\left(\frac{2}{a^2+1}\right)$

c) $\frac{1}{2}\log\left(\frac{a^2+1}{2}\right)$ d) $2\log\left(\frac{a^2+1}{2}\right)$

38) The value of integration $I(a)=\int_0^\infty \frac{1-\cos ax}{x^2}dx$ with $\frac{dI}{da}=\frac{\pi}{2}$ is given by

a) $2\pi a$ b) $\frac{\pi a}{3}$ c) $\frac{\pi}{2}$ d) $\frac{\pi a}{2}$

39) The value of integration $I=\int_0^\infty \frac{\log(1+ax^2)}{x^2}dx$, with $\frac{dI}{da}=\frac{\pi}{2\sqrt{a}}$ is given by

a) $\pi\sqrt{a}$ b) $2\sqrt{a}$ c) $\pi\sqrt{2}$ d) $a\sqrt{\pi}$

40) The value of integration $I=\int_0^{\pi/2} \frac{\log(1+a\sin^2 x)}{\sin^2 x}dx$, with $\frac{dI}{da}=\frac{\pi}{2\sqrt{a+1}}$ is given by

a) $\pi\sqrt{a+1}+\pi$ b) $\pi\sqrt{a+1}-\pi$

c) $\pi\sqrt{a+1}-\frac{\pi}{a}$ d) $\frac{\pi\sqrt{a+1}-\pi}{a}$

II) Error Functions

41) $erf(x)$ is given by

a) $\frac{1}{2\sqrt{\pi}}\int_0^x e^{-u^2}du$ b) $\frac{\sqrt{\pi}}{2}\int_0^x e^{-u^2}du$

c) $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$ d) $\int_0^x e^{-u^2}du$

42) $erfc(x)$ is given by

a) $\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2}du$ b) $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$

c) $\frac{\sqrt{\pi}}{2}\int_0^x e^{-u^2}du$ d) $\frac{\sqrt{\pi}}{2}\int_x^\infty e^{-u^2}du$

43) $erf(0)$ is given by

a) $\frac{2}{\sqrt{\pi}}$ b) 1 c) ∞ d) 0

44) $erf(\infty)$ is given by

a) 1 b) 0 c) $\frac{2}{\sqrt{\pi}}$ d) ∞

45) $erfc(0)$ is given by

a) 0 b) $\frac{2}{\sqrt{\pi}}$ c) ∞ d) 1

46) $erf(x)+erfc(x)=?$

a) 2 b) ∞ c) 1 d) 0

47) $erf(-x)=?$

a) $erfc(x)$ b) $-erf(x)$

c) $erf(x)$ d) $-erf(x^2)$

48) Error function is an

a) even function b) neither even nor odd

c) odd function d) none of these

49) $erf(x)+erf(-x)=?$

a) 0 b) 1 c) 2 d) 3

50) $erf(-x)+erfc(-x)=?$

a) 0 b) 3 c) 2 d) 1

51) $erfc(-x)-erf(x)=?$
a) ∞ b) 2 c) 1 d) 0

52) $erfc(x)+erfc(-x)=?$
a) 2 b) 1 c) 0 d) ∞

53) If $erf(ax)=\frac{2}{\sqrt{\pi}}\int_0^{ax} e^{-u^2}du$, then $\frac{d}{dx}[erf(ax)]$ is
a) $\frac{2a}{\sqrt{\pi}}e^{-x^2}$ b) $\frac{a}{2\sqrt{\pi}}e^{-a^2x^2}$
c) $\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$ d) $\frac{2x}{\sqrt{\pi}}e^{-a^2x^2}$

54) If $erfc(ax)=\frac{2}{\sqrt{\pi}}\int_{ax}^{\infty} e^{-u^2}du$, then $\frac{d}{dx}[erfc(ax)]$ is
a) $-\frac{2x}{\sqrt{\pi}}e^{-a^2x^2}$ b) $\frac{2x}{\sqrt{\pi}}e^{-a^2x^2}$
c) $-\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$ d) $\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$

55) If $erf(\sqrt{t})=\frac{2}{\sqrt{\pi}}\int_0^{\sqrt{t}} e^{-u^2}du$, then $\frac{d}{dx}[erf(\sqrt{t})]$ is
a) $\frac{1}{t\sqrt{\pi}}e^{-t}$ b) $\frac{1}{\sqrt{\pi t}}e^{-t}$
c) $\frac{2}{\sqrt{\pi t}}e^{-t}$ d) $\frac{1}{\sqrt{\pi t}}e^{-t^2}$

56) If $erfc(\sqrt{t})=\frac{2}{\sqrt{\pi}}\int_{\sqrt{t}}^{\infty} e^{-u^2}du$, then $\frac{d}{dx}[erfc(\sqrt{t})]$ is
a) $\frac{2}{\sqrt{\pi t}}e^{-t}$ b) $\frac{1}{\sqrt{\pi t}}e^{-t^2}$
c) $\frac{1}{t\sqrt{\pi}}e^{-t}$ d) $-\frac{1}{\sqrt{\pi t}}e^{-t}$

57) $\frac{d}{dx}[erf(x)+erfc(x)]=?$
a) 1 b) 0 c) 2 d) ∞

58) If $\frac{d}{dx}[erf(ax)]=\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$, then $\frac{d}{dx}[erfc(ax)]$ is
a) $-\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$ b) $\frac{2a}{\sqrt{\pi}}e^{a^2x^2}$
c) $\frac{1}{\sqrt{\pi}}e^{-a^2x^2}$ d) $\frac{4a^2}{\sqrt{\pi}}e^{-a^2x^2}$

59) On substitution $x+a=u$ in the integration $\int_0^{\infty} e^{-(x+a)^2}dx$, then the value of integration is
a) $\frac{\sqrt{\pi}}{2}erf(a)$ b) $\frac{2}{\sqrt{\pi}}erf(a)$
c) $\frac{\sqrt{\pi}}{2}erfc(a)$ d) $erfc(a)$

60) $\int_0^t erf(ax)dx+\int_0^t erfc(ax)dx=?$
a) 1 b) ∞ c) 0 d) t

61) If $\frac{dy}{dx}[erf(\sqrt{t})]=\frac{e^{-t}}{\sqrt{\pi t}}$, the integration $\int_0^{\infty} e^{-t}erf(\sqrt{t})dt$ is
a) $\frac{1}{\sqrt{\pi}}\int_0^{\infty} e^{-t}t^{1/2}dt$ b) $\frac{2}{\sqrt{\pi}}\int_0^{\infty} e^{-2t}t^{1/2}dt$
c) $-\frac{1}{\sqrt{\pi}}\int_0^{\infty} e^{-2t}t^{-1/2}dt$ d) $\frac{1}{\sqrt{\pi}}\int_0^{\infty} e^{-2t}t^{-1/2}dt$

62) The power series expansion of $erf(x)$ is
a) $\frac{2}{\sqrt{\pi}}\left[x-\frac{x^3}{3}+\frac{x^5}{10}-\frac{x^7}{42}+-\ldots\right]$
b) $\frac{2}{\sqrt{\pi}}\left[x+\frac{x^3}{3}+\frac{x^5}{10}+\frac{x^7}{42}+\ldots\right]$
c) $\frac{2}{\sqrt{\pi}}\left[x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+-\ldots\right]$
d) $\frac{2}{\sqrt{\pi}}\left[x+\frac{x^3}{3!}+\frac{x^5}{5!}+\frac{x^7}{7!}+\ldots\right]$

Chapter 06 – Curve Tracing & Rectification of Curves

I) Curve Tracing

1) If the portion of the curve lies on the both sides of the point lying above the tangent at that point, the curve is known as
a) concave upward b) concave downward
c) inflexion point d) none of these

2) If the portion of the curve lies on the both sides of the point lying above the tangent at that point, the curve is known as
a) inflexion point b) concave downward
c) inflexion point d) none of these

3) A point through which two branches of the same curve passes is known as
a) double point b) inflexion point
c) multiple point d) conjugate point

4) A point through which many branches of the same curve passes is known as
a) double point b) inflexion point
c) multiple point d) conjugate point

5) A double point through which the branches of the curve passes and the tangent at that point are real and distinct, the point is known as
a) conjugate point b) node
c) point of inflexion d) cusp

6) A double point through which the branches of the curve passes and the tangent at that point are real but the same, the point is known as
a) conjugate point b) point of inflexion
c) cusp d) node

7) A double point is said to be node if the tangents to the curve at that point are
a) imaginary b) perpendicular to each other
c) real but the same d) real and distinct

8) A double point is said to be cusp if the tangents at that point are
a) imaginary b) real and distinct
c) real but the same d) none of these

9) If at a point $\frac{dy}{dx}=0$, the tangent to the curve at that point is
a) parallel to the line $x+y=0$
b) parallel to x-axis
c) perpendicular to x-axis
d) parallel to $y=x$

10) If at a point $\frac{dy}{dx}=\infty$, the tangent to the curve at that point is
a) parallel to the line $x+y=0$
b) parallel to $y=x$
c) parallel to x-axis
d) perpendicular to x-axis

11) The standard equation of x-axis in the Cartesian form is given by
a) $x-y=0$ b) $x+y=0$
c) $y=0$ d) $x=0$

12) The standard equation of y-axis in the Cartesian form is given by
a) $x-y=0$ b) $x+y=0$
c) $y=0$ d) $x=0$

13) If the all the powers of y in the Cartesian form are even, the curve is symmetrical about
a) $y-axis$ b) $x,\ y-axes$
c) $x-axis$ d) the line $y=x$

14) If the all the powers of x in the Cartesian form are even, the curve is symmetrical about
a) $x,\ y-axes$ b) $y-axis$
c) $x-axis$ d) the line $y=x$

15) If the all the powers of x and y in the Cartesian form are even, the curve is symmetrical about
a) the line $y=x$ b) $x-axis$ only
c) $y-axis$ only d) $x,\ y-axes$

16) If in the equation of the Cartesian form by replacing $x\to y$ and $y\to x$, the equation is symmetrical about
a) the line $y=x$ b) $x,\ y-axes$

c) $x-axis$ d) $y-axis$

17) If in the equation of the Cartesian form by replacing $x \to -y$ and $y \to -x$, the equation is symmetrical about
a) the line $y=-x$ b) the line $y=x$
c) $x,\ y-axes$ d) $y-axis$ only

18) If in the equation of the Cartesian form by replacing $x \to -x$ and $y \to -y$, the equation is symmetrical about
a) the line $y=-x$ b) $x,\ y-axes$
c) opposite quadrants d) the line $y=x$

19) The equation of the tangent at origin when the curve is passing through origin is obtained by equating to zero
a) the lowest degree term of the equation
b) the highest degree term of x in equation
c) the highest degree term of y in equation
d) the coefficient of the term xy

20) In the Cartesian form, the asymptote to the curve parallel to x-axis may be obtained by equating to zero
a) the coefficient of lowest degree term in y
b) the coefficient of highest degree term in y
c) the coefficient of highest degree term in x
d) the coefficient of lowest degree term in x

21) In the Cartesian form, the asymptote to the curve parallel to y-axis may be obtained by equating to zero
a) the coefficient of lowest degree term in y
b) the coefficient of highest degree term in y
c) the coefficient of highest degree term in x
d) the coefficient of lowest degree term in x

22) Oblique asymptote are obtained only when the curve is
a) symmetrical about x-axis
b) symmetrical about y-axis
c) symmetrical about both x and y-axis
d) not symmetrical about both x and y-axes

23) In the Cartesian form if the coefficient of the highest degree term in x is constant, the curve has
a) no asymptote parallel to $x=y$
b) no asymptote parallel to y-axis
c) no asymptote parallel to x-axis
d) none of these

24) In the Cartesian form if the coefficient of the highest degree term in y is constant, the curve has
a) no asymptote parallel to $x+y=0$
b) no asymptote parallel to $x=y$
c) no asymptote parallel to x-axis
d) no asymptote parallel to y-axis

25) In the polar form, if the equation of the curve remains unchanged by replacing $\theta \to -\theta$, the curve is symmetrical about
a) the line $\theta=\frac{\pi}{4}$ b) the initial line
c) pole d) the line $\theta=\frac{\pi}{2}$

26) In the polar form, if the equation of the curve remains unchanged by replacing $r \to -r$, the curve is symmetrical about
a) the line $\theta=\frac{\pi}{4}$ b) the initial line
c) pole d) the line $\theta=\frac{\pi}{2}$

27) In the polar form, if the equation of the curve remains unchanged by replacing $\theta \to \pi-\theta$, the curve is symmetrical about
a) the line $\theta=\frac{\pi}{2}$ b) the line $\theta=\frac{\pi}{4}$
c) the initial line d) pole

28) The pole is point of the curve, if for given angle θ, the value of
a) $r=\infty$ b) $r=0$ c) $r<0$ d) $r>0$

29) If a curve is passing through the pole, the tangent to the curve at pole are obtained by solving
a) $r=0$ b) $r=\infty$ c) $\theta=0$ d) $\theta=\pi$

30) In the polar form, the relation between the angle ϕ formed by the radius vector and the tangent to the curve at that point, is given by
a) $\tan\phi = r^2\frac{d\theta}{dr}$ b) $\cot\phi = r\frac{d\theta}{dr}$
c) $\tan\phi = r\frac{d\theta}{dr}$ d) $\tan\phi = r\frac{dr}{d\theta}$

31) In the parametric form $x = f(t)$, $y = g(t)$, the curve is symmetrical about y-axis, if
a) $x = f(t)$ is odd and $y = g(t)$ is even
b) $x = f(t)$ is even and $y = g(t)$ is odd
c) $x = f(t)$ is odd and $y = g(t)$ is odd
d) $x = f(t)$ is even and $y = g(t)$ is even

32) In the parametric form $x = f(t)$, $y = g(t)$, the curve is symmetrical about y-axis, if
a) $x = f(t)$ is odd and $y = g(t)$ is odd
b) $x = f(t)$ is even and $y = g(t)$ is odd
c) $x = f(t)$ is odd and $y = g(t)$ is even
d) $x = f(t)$ is even and $y = g(t)$ is even

33) The curve represented by the equation $x^2y^2 = x^2 + 1$ is symmetrical about
a) the line $y = x$ b) x-axis only
c) y-axis only d) both x and y-axes

34) The curve represented by the equation $x(x^2 + y^2) = a(x^2 - y^2)$ is
a) symmetrical about y-axis but not passing through origin
b) symmetrical about y-axis and passing through origin
c) symmetrical about x-axis but not passing through origin
d) symmetrical about x-axis and passing through origin

35) The curve represented by the equation $a^2y^2 = x^2(a^2 - x^2)$ is
a) symmetrical about both x and y-axis but not passing through origin
b) symmetrical about both x and y-axis and passing through origin
c) symmetrical about x-axis but not passing through origin
d) symmetrical about x-axis only and passing through origin

36) The curve represented by the equation $(2a - x)y^2 = x^3$ is
a) symmetrical about y-axis and passing through origin
b) symmetrical about x-axis but not passing through origin
c) symmetrical about x-axis and passing through origin
d) symmetrical about y-axis not passing through origin

37) The curve represented by the equation $(2a - y)y^3 = a^2x^2$ is
a) symmetrical about y-axis and passing through origin and $(0, 2a)$
b) symmetrical about x-axis but not passing through origin and $(0, 2a)$
c) symmetrical about x-axis and passing through origin and $(0, 2a)$
d) symmetrical about y-axis not passing through origin and $(0, 2a)$

38) The curve represented by the equation $xy^2 = 4a^2(a - x)$ is
a) symmetrical about y-axis and passing through $(a, 0)$
b) symmetrical about x-axis but not passing through $(a, 0)$
c) symmetrical about x-axis and passing through $(a, 0)$
d) symmetrical about y-axis not passing through $(a, 0)$

39) The curve represented by the equation $xy^2 = 4a^2(a - x)$ has at origin
a) node b) cusp c) inflexion d) none

40) The curve represented by the equation $(2a - x)y^2 = x^3$ has the tangent at origin whose equation is
a) $x + y = 0$ b) y-axis c) x-axis d) $y = x$

41) The curve represented by the equation $(1 + x^2)y = x$ has the tangent at origin whose equation is
a) $y = x$ b) x-axis c) y-axis d) $x + y = 0$

42) The curve represented by the equation $3ay^2 = x(x - a)^2$ has the tangent at origin whose equation is
a) $x + y = 0$ b) $y = x$ c) x-axis d) y-axis

43) The curve represented by the equation $3ay^2 = x(x-a)^2$ has the asymptote parallel to x-axis whose equation is
a) $x+y=0$ b) $y=x$ c) x-axis d) y-axis

44) For the curve given by equation $x^2y = 4a^2(2a-y)$, the asymptote is
a) $y=2a$ b) $y=x$ c) y-axis d) x-axis

45) The curve represented by the equation $y^2(4-x) = x(x-2)^2$ has the asymptote parallel to y-axis whose equation is
a) $x=y$ b) $x=0$ c) $x=2$ d) $x=4$

46) The curve represented by the equation $x^2y^2 = a^2(y^2-x^2)$ has the asymptote parallel to y-axis whose equation is
a) $x=0$ b) $x=\pm a$ c) $x=y$ d) $y=0$

47) For the curve given by equation $x^2y = 4a^2(2a-y)$, the region of absence is
a) $0<y<2a$ b) $y>0,\ \ y>2a$
c) $y<0,\ \ y<2a$ d) $y<0,\ \ y>2a$

48) For the curve given by equation $x^3 = 4y^2(2a-x)$, the region of absence is
a) $0<x<2a$ b) $x<0,\ \ x>2a$
c) $x>0,\ \ x>2a$ d) $x<0,\ \ x<2a$

49) For the curve given by equation $xy^2 = 4a^2(a-x)$, the region of absence is
a) $0<x<a$ b) $x>0,\ \ x>a$
c) $x<0,\ \ x>a$ d) $x<0,\ \ x<a$

50) For the curve given by equation $y^2 = \dfrac{4x^2(a-x)}{x+a}$, the region of absence along x-axis is
a) $[-\infty,\ -a]$ & $[a,\ \infty]$ b) $[-\infty,\ a]$ & $[-a,\ \infty]$
c) $[-\infty,\ -a]$ d) $[-a,\ \infty]$

51) The curve represented by the equation $x^{1/2} + y^{1/2} = a^{1/2}$ is symmetrical about
a) $y=x$ b) x-axis c) y-axis d) $x+y=0$

52) The curve represented by the equation $\sqrt{x}+\sqrt{y}=\sqrt{a}$ has tangent at origin whose equation is
a) x-axis b) no tangent exists
c) y-axis d) $x+y=0$

53) The curve represented by the equation $\sqrt{x}+\sqrt{y}=\sqrt{a}$ has tangent at $(a,\ 0)$ which is
a) the line $x+y=0$ b) the line $y=x$
c) parallel to y-axis d) parallel to x-axis

54) The curve represented by the equation $x=t^2,\ \ y=t-\dfrac{t^3}{3}$ is symmetrical about
a) symmetrical about y-axis but not passing through origin
b) symmetrical about y-axis and passing through origin
c) symmetrical about x-axis but not passing through origin
d) symmetrical about x-axis and passing through origin

55) The curve represented by the equation $x=a(\theta+\sin\theta),\ \ y=a(1+\cos\theta)$ is symmetrical about
a) symmetrical about y-axis but not passing through origin
b) symmetrical about y-axis and passing through origin
c) symmetrical about x-axis but not passing through origin
d) symmetrical about x-axis and passing through origin

56) The curve represented by the equation $r=a(1+\cos\theta)$ is
a) symmetrical about initial line and not passing through the pole
b) symmetrical about initial line and passing through the pole
c) not symmetrical about initial line and passing through the pole
d) not symmetrical about initial line and not passing through the pole

57) The curve represented by the equation $r^2 = a^2 \cos 2\theta$ is
a) symmetrical about initial line as well as pole and not passing through the pole
b) symmetrical about initial line as well as pole and passing through the pole
c) not symmetrical about initial line as well as pole and passing through the pole
d) not symmetrical about initial line as well as pole and not passing through the pole

58) The curve represented by the equation $r^2 = a^2 \sin 2\theta$ is
a) symmetrical about the line $\theta = \frac{\pi}{4}$ and not passing through the pole
b) symmetrical about initial line as well as pole and passing through the pole
c) not symmetrical about the line $\theta = \frac{\pi}{4}$ and passing through the pole
d) not symmetrical about initial line as well as pole and not passing through the pole

59) The curve represented by the equation $r(1+\cos\theta) = 2a^2$ is
a) symmetrical about the line $\theta = \frac{\pi}{4}$ and not passing through the pole
b) symmetrical about initial line and passing through the pole
c) not symmetrical about the line $\theta = \frac{\pi}{4}$ and passing through the pole
d) symmetrical about initial and not passing through the pole

60) The equations of the tangents at pole to the curve $r = a\sin 3\theta$ are given by
a) $\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \ldots$
b) $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{3}, \ldots$
c) $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \ldots$
d) no such tangent exists

61) The equations of the tangents at pole to the curve $r = a\cos 2\theta$ are given by
a) $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \ldots$
b) $\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \ldots$
c) $\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$
d) $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots$

62) For the rose curve $r = a\sin n\theta$, if n is even, the curve is consisting of
a) $2n$ equal loops b) $2n+1$ equal loops
c) n equal loops d) $2n-1$ equal loops

63) For the rose curve $r = a\cos n\theta$, if n is even, the curve is consisting of
a) n equal loops b) $2n+1$ equal loops
c) $2n$ equal loops d) $2n-1$ equal loops

64) For the rose curve $r = a\sin n\theta$, if n is odd, the curve is consisting of
a) $2n$ equal loops b) n equal loops
c) $2n+1$ equal loops d) $2n-1$ equal loops

65) For the rose curve $r = a\cos n\theta$, if n is odd, the curve is consisting of
a) n equal loops b) $2n+1$ equal loops
c) $2n$ equal loops d) $2n-1$ equal loops

I) Rectification of Curve

66) If $A(a_1, b_1)$ $B(a_2, b_2)$ are two points on the curve on xy-plane, the length of arc is given by

a) $\int_{b_1}^{b_2}\sqrt{1-\left(\frac{dy}{dx}\right)^2}\cdot dy$ b) $\int_{b_1}^{b_2}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dy$

c) $\int_{a_1}^{a_2}\sqrt{1-\left(\frac{dy}{dx}\right)^2}\cdot dx$ d) $\int_{a_1}^{a_2}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$

67) If $A(a_1, b_1)$ $B(a_2, b_2)$ are two points on the curve on xy-plane, the length of arc is given by

a) $\int_{b_1}^{b_2}\sqrt{1+\left(\frac{dx}{dy}\right)^2}\cdot dy$ b) $\int_{a_1}^{a_2}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$

c) $\int_{a_1}^{a_2}\sqrt{1-\left(\frac{dy}{dx}\right)^2}\cdot dx$ d) $\int_{b_1}^{b_2}\sqrt{1-\left(\frac{dx}{dy}\right)^2}\cdot dy$

68) If $A(r_1, \theta_1)$ $B(r_2, \theta_2)$ are two points on the curve on the polar plane, the length of arc is given by

a) $\int_{\theta_1}^{\theta_2}\sqrt{r^2-\left(\frac{dr}{d\theta}\right)^2}\cdot d\theta$ b) $\int_{\theta_1}^{\theta_2}\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\cdot d\theta$

c) $\int_{r_1}^{r_2}\sqrt{1+r^2\left(\frac{d\theta}{dr}\right)^2}\cdot dr$ d) $\int_{r_1}^{r_2}\sqrt{1-r^2\left(\frac{d\theta}{dr}\right)^2}\cdot dr$

69) If $A(r_1, \theta_1)$ $B(r_2, \theta_2)$ are two points on the curve on the polar plane, the length of arc is given by

a) $\int_{\theta_1}^{\theta_2}\sqrt{r^2-\left(\frac{dr}{d\theta}\right)^2}\cdot d\theta$ b) $\int_{\theta_1}^{\theta_2}\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\cdot d\theta$

c) $\int_{r_1}^{r_2}\sqrt{1+r^2\left(\frac{d\theta}{dr}\right)^2}\cdot dr$ d) $\int_{r_1}^{r_2}\sqrt{1-r^2\left(\frac{d\theta}{dr}\right)^2}\cdot dr$

70) If $A(t_1)$ $B(t_2)$ are two points on the curve given by $x = f(t)$, $y = g(t)$ on the xy-plane, the length of arc is given by

a) $\int_{t_1}^{t_2}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\cdot dt$

b) $\int_{t_1}^{t_2}\sqrt{\left(\frac{dt}{dx}\right)^2+\left(\frac{dt}{dy}\right)^2}\cdot dt$

c) $\int_{t_1}^{t_2}\sqrt{\left(\frac{dx}{dt}\right)^2-\left(\frac{dy}{dt}\right)^2}\cdot dt$

d) $\int_{t_1}^{t_2}\left[\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2\right]\cdot dt$

71) The arc length of the upper part of the loop of the curve $9y^2 = (x+7)(x+4)^2$ is obtained by solving the integration

a) $\int_{0}^{4}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$ b) $\int_{-7}^{0}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$

c) $\int_{-7}^{-4}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$ d) $\int_{7}^{4}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$

72) The arc length of the upper part of the curve $y^2 = 4x$ which is cut by the line $3y = 8x$ is obtained by solving the integration

a) $\int_{1}^{9/16}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$ b) $\int_{0}^{9/16}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$

c) $\int_{0}^{3/4}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$ d) $\int_{3}^{8}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$

73) The points $A(a, 0)$ $B(0, a)$ are two points on the curve $x^2 + y^2 = a^2$ on xy-plane such that $1+\left(\frac{dy}{dx}\right)^2 = \frac{a^2}{a^2 - x^2}$, the length of arc is given by

a) $4a$ b) πa c) $\frac{\pi a}{4}$ d) $\frac{\pi a}{2}$

74) The points $A(0, 0)$ $B(a, b)$ are two points on the curve $y = a\cosh\left(\frac{x}{a}\right)$ on xy-plane such that $1+\left(\frac{dy}{dx}\right)^2 = \cosh^2\left(\frac{x}{a}\right)$, the length of arc is given by

a) $S = a\sinh\left(\frac{x}{a}\right)$ b) $S = a\tanh\left(\frac{x}{a}\right)$

c) $S = \sinh\left(\frac{x}{a}\right)$ d) $S = a\,\mathrm{sech}\left(\frac{x}{a}\right)$

75) The points $A(0,0)$ $B(1,0)$ are two points on the curve $3y^2 = x(x-1)^2$ on xy-plane such that $1+\left(\frac{dy}{dx}\right)^2 = \frac{(3x+1)^2}{12x}$, the length of arc is given by

a) $\frac{3}{\sqrt{3}}$ b) $\frac{1}{2\sqrt{3}}$ c) $\frac{1}{\sqrt{3}}$ d) $\frac{2}{\sqrt{3}}$

76) The total arc length of the part of the curve $r = a(1+\cos\theta)$ which is cut by the circle $r + a\cos\theta = 0$ is obtained by solving the integration

a) $\int_0^{2\pi/3} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

b) $2\int_0^{2\pi/3} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

c) $2\int_0^{2\pi/3} \sqrt{r^2 - \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

d) $\int_0^{2\pi/3} \sqrt{r^2 - \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

77) The total arc length of the upper part of the curve $r^2 = a^2\cos 2\theta$ is obtained by solving the integration

a) $2\int_0^{\pi/2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

b) $\int_0^{\pi/2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

c) $\int_0^{\pi/4} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

d) $\int_0^{\pi/3} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

78) The total length of the arc of the curve $r = ae^{m\theta}$ using $1 + r^2\left(\frac{d\theta}{dr}\right)^2 = 1 + \frac{1}{m^2}$ when r varies from r_1 to r_2 is given by

a) $\frac{\sqrt{1+m^2}}{m}(r_2 - r_1)$ b) $\frac{\sqrt{1+m^2}}{m}(r_2 + r_1)$

c) $\frac{\sqrt{1+m^2}}{m}(r_1 - r_2)$ d) $\frac{\sqrt{1-m^2}}{m}(r_2 - r_1)$

79) The total length of the arc formed by the upper half of the cardioide $r = a(1+\cos\theta)$ using $r^2 + \left(\frac{dr}{d\theta}\right)^2 = 2a^2(1+\cos\theta)$ when θ varies from 0 to π is given by

a) 4π b) 2π c) $4a$ d) $2a$

80) The total arc length of the upper part of the curve $x = t^2,\ y = t - \frac{t^3}{3}$ is obtained by solving the integration

a) $\int_0^{2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$

b) $\int_0^{3} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$

c) $\int_0^{1} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$

d) $\int_0^{\sqrt{3}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$

81) The total arc length of the two consecutive cusps lies in the first quadrant of the curve $x = a\cos^3\theta,\ y = a\sin^3\theta$ is obtained by solving the integration

a) $\int_0^{\pi/4} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \cdot d\theta$

b) $\int_0^{\pi/3} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \cdot d\theta$

c) $\int_0^{\pi/2} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \cdot d\theta$

d) $\int_0^{\pi} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \cdot d\theta$

82) The total arc length of the upper part of the curve $x = t^2,\ y = t - \frac{t^3}{3}$ between $t = 0$ to $t = \sqrt{3}$ with $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (1+t^2)^2$ is given by

a) $2\sqrt{3}$ b) $\sqrt{3}$ c) $\dfrac{2}{\sqrt{3}}$ d) $4\sqrt{3}$

83) The total arc length of the two consecutive cusps lies in the first quadrant of the curve $x = a\cos^3\theta,\ y = a\sin^3\theta$ between $\theta = 0$ to $\theta = \dfrac{\pi}{2}$ with $\left(\dfrac{dx}{d\theta}\right)^2 + \left(\dfrac{dy}{d\theta}\right)^2 = 9a^2\sin^2\theta\cos^2\theta$ is given by

a) $\dfrac{3a}{4}$ b) $3a$ c) $\dfrac{3a}{2}$ d) $\dfrac{2a}{3}$

84) The total arc length of the two cusps between $\theta = -\pi$ to $\theta = \pi$ of the curve

$$x = a(\theta + \sin\theta),\ y = a(1 - \cos\theta)$$

with $\left(\dfrac{dx}{d\theta}\right)^2 + \left(\dfrac{dy}{d\theta}\right)^2 = 4a^2\cos^2\left(\dfrac{\theta}{2}\right)$ is

a) $4a$ b) $8a$ c) $2a$ d) a

85) The total arc length of the two cusps between $\theta = 0$ to $\theta = \dfrac{\pi}{2}$ of the curve $x = e^{\theta}\cos\theta$, and $y = e^{\theta}\sin\theta$ with $\left(\dfrac{dx}{d\theta}\right)^2 + \left(\dfrac{dy}{d\theta}\right)^2 = 2e^{2\theta}$ is

a) $\sqrt{2}\left(1 - e^{\pi/2}\right)$ b) $\sqrt{2}\left(e^{\pi} - 1\right)$

c) $\sqrt{2}\left(e^{\pi/2} + 1\right)$ d) $\sqrt{2}\left(e^{\pi/2} - 1\right)$

Chapter 03) Fourier Series

1	a	41	d	81	b	121	c
2	d	42	d	82	d	122	b
3	b	43	b	83	a	123	d
4	a	44	c	84	b	124	d
5	c	45	d	85	d	125	a
6	d	46	b	86	c	126	b
7	a	47	c	87	a	127	a
8	d	48	a	88	b	128	b
9	b	49	b	89	a	129	b
10	d	50	a	90	b'	130	c
11	d	51	c	91	c	131	a
12	b	52	b	92	a	132	b
13	a	53	c	93	c	133	d
14	d	54	d	94	d	134	d
15	b	55	d	95	a	135	a
16	b	56	c	96	b	136	c
17	a	57	a	97	c	137	d
18	d	58	b	98	d	138	a
19	a	59	d	99	b	139	b
20	b	60	a	100	d	140	a
21	a			101	d	141	d
22	c	62	d	102	b	142	c
23	d	63	c	103	c	143	b
24	a	64	d	104	a	144	c
25	d	65	b	105	d	145	a
26	a	66	d	106	b	146	d
27	d	67	b	107	d	147	c
28	c	68	c	108	d	148	a
29	b	69	a	109	d	149	c
30	c	70	c	110	a	150	b
31	a	71	c	111	d	151	d
32	d	72	c	112	c	152	b
33	a	73	d	113	c	153	a
34	c	74	b	114	a	154	c
35	a	75	d	115	b	155	d
36	c	76	c	116	a	156	d
37	a	77	b	117	c	157	a
38	c	78	c	118	b	158	c
39	c	79	b	119	a	159	b
40	b	80	d	120	b		

Chapter 04) Reduction Formulae & Beta, Gamma Function

1	c	26	d	51	a	76	d	101	c
2	b	27	b	52	c	77	a	102	b
3	c	28	c	53	b	78	c	103	d
4	d	29	a	54	d	79	d	104	c
5	d	30	b	55	b	80	c	105	b
6	c	31	a	56	d	81	b	106	a
7	a	32	c	57	a	82	c	107	b
8	c	33	b	58	d	83	a	108	a
9	b	34	c	59	a	84	d	109	c
10	a	35	d	60	c	85	b	110	d
11	c	36	d	61	d	86	c	111	b
12	b	37	c	62	c	87	a	112	d
13	d	38	a	63	b	88	c	113	c
14	a	39	d	64	a	89	b	114	c
15	a	40	b	65	c	90	d	115	a
16	c	41	d	66	d	91	b	116	c
17	c	42	c	67	b	92	a	117	b
18	c	43	a	68	a	93	c	118	b
19	b	44	b	69	b	94	b	119	d
20	d	45	d	70	c	95	d	120	c
21	c	46	d	71	d	96	d	121	d
22	d	47	b	72	a	97	a	122	b
23	b	48	d	73	b	98	c	123	c
24	d	49	b	74	c	99	d	124	a
25	c	50	c	75	a	100	c	125	b

Chapter 05) Differentiation Under Integral Sign & Error Function

1	a	14	d	27	a	40	b	53	c
2	c	15	b	28	c	41	c	54	c
3	b	16	d	29	d	42	a	55	b
4	c	17	b	30	a	43	d	56	d
5	d	18	c	31	c	44	a	57	b
6	d	19	a	32	b	45	d	58	a
7	c	20	c	33	d	46	c	59	c
8	a	21	b	34	b	47	b	60	d
9	b	22	d	35	a	48	c	61	d
10	d	23	b	36	b	49	a	62	a
11	a	24	d	37	c	50	d		
12	d	25	a	38	d	51	c		
13	c	26	d	39	a	52	a		

Chapter 06) Curve Tracing & Rectification of Curves

1	a	18	c	35	d	52	b	69	c
2	b	19	a	36	c	53	d	70	a
3	a	20	c	37	a	54	d	71	c
4	c	21	b	38	c	55	a	72	b
5	b	22	d	39	b	56	b	73	d
6	c	23	c	40	d	57	b	74	a
7	d	24	d	41	a	58	a	75	d
8	c	25	b	42	d	59	d	76	b
9	b	26	c	43	c	60	a	77	c
10	d	27	a	44	d	61	d	78	a
11	c	28	b	45	d	62	a	79	c
12	d	29	a	46	b	63	c	80	d
13	c	30	c	47	d	64	b	81	c
14	b	31	b	48	b	65	a	82	a
15	d	32	c	49	c	66	d	83	c
16	a	33	d	50	a	67	a	84	b
17	a	34	d	51	a	68	b	85	d

MULTIPLE CHOICE QUESTIONS

Order, Degree and Formation of Differential Equation :

1. The differential equation $1+\frac{dy}{dx}-\left(\frac{d^2y}{dx^2}\right)^{3/2}=0$ is of (1)

(A) order 1 and degree 2
(B) order 2 and degree 3
(C) order 3 and degree 6
(D) order 3 and degree 3

2. The differential equation $\sqrt{1+\frac{dy}{dx}}=\frac{d^2y}{dx^2}$ is of (1)

(A) order 2 and degree 2
(B) order 1 and degree 2
(C) order 2 and degree 1
(D) order 1 and degree 1

3. The differential equation $x = \dfrac{1}{\sqrt{1+\dfrac{dy}{dx}+\dfrac{d^2y}{dx^2}}}$ is of (1)

(A) order 2 and degree 2
(B) order 1 and degree 2
(C) order 2 and degree 1
(D) order 1 and degree 1

4. The differential equation $1+\dfrac{dy}{dx}=\dfrac{y}{\dfrac{dy}{dx}}$ is of (1)

(A) order 2 and degree 2
(B) order 1 and degree 1
(C) order 2 and degree 1
(D) order 1 and degree 2

5. The differential equation $(2x - y + 3)\,dx + (y - 2x - 2)\,dy = 0$ is of (1)

(A) order 1 and degree 1
(B) order 1 and degree 2
(C) order 2 and degree 1
(D) order 2 and degree 2

6. The number of arbitrary constants in the general solution of ordinary differential equation is equal to (1)

(A) the order of differential equation
(B) the degree of differential equation
(C) coefficient of highest order differential coefficient
(D) none of these

7. The general solution of n^{th} order ordinary differential equation must involve (1)

(A) $(n + 1)$ arbitrary constants
(B) $(n - 1)$ arbitrary constants
(C) n arbitrary constants
(D) none of these

8. The solution obtained by assigning particular values to arbitrary constants in general solution of ordinary differential equation is called (1)

(A) the general solution
(B) a particular solution
(C) a singular solution
(D) none of these

9. The order of differential equation whose general solution is

$y = C_1 + C_2e^{-2x} + C_3e^{3x} + C_4e^{-3x}$, where C_1, C_2, C_3, C_4 are arbitrary constants, is (1)

(A) 1 (B) 3 (C) 2 (D) 4

10. The order of differential equation whose general solution is

$y = e^x(Ax^2 + Bx + C)$, where A, B, C are arbitrary constants, is (1)

(A) 2 (B) 4 (C) 3 (D) 1

11. The differential equation whose general solution is $y = \sqrt{5x + C}$, where C is arbitrary constant, is (1)

(A) $2y\dfrac{dy}{dx} - 1 = 0$ (B) $2y\dfrac{dy}{dx} - 5 = 0$

(C) $\dfrac{dy}{dx} - \dfrac{5}{2}\dfrac{1}{\sqrt{5x+C}} = 0$ (D) $y\dfrac{dy}{dx} - 5 = 0$

12. $y = Cx - C^2$, where C is arbitrary constant is the general solution of the differential equation (1)

(A) $\dfrac{dy}{dx} = C$ (B) $\left(\dfrac{dy}{dx}\right)^2 + xy = 0$

(C) $\left(\dfrac{dy}{dx}\right)^2 + x\dfrac{dy}{dx} - y = 0$ (D) $\left(\dfrac{dy}{dx}\right)^2 - x\dfrac{dy}{dx} + y = 0$

13. The differential equation whose general solution is $y = C^2 + \dfrac{C}{x}$, where C is arbitrary constant is (1)

(A) $x^4 y_1^2 + xy_1 - y = 0$ (B) $x^4 y_1^2 - xy_1 - y = 0$

(C) $x^2 y_1^2 - xy_1 - y = 0$ (D) $y_1 = -\dfrac{C}{x^2}$

14. By eliminating arbitrary constant A the differential equation whose general solution is $y = 4(x - A)^2$ (1)

(A) $y_1^2 + 16y = 0$ (B) $y_1 - 2y = 0$ (C) $y_1^2 - 16y = 0$ (D) $y_1 - 8(x - A) = 0$

15. The differential equation whose general solution is $y = A\cos(x + 3)$, where A is arbitrary constant, is (1)

(A) $\cot(x + 3)\, y_1 + y = 0$ (B) $\tan(x + 3)\, y_1 + y = 0$

(C) $\cot(x + 3)\, y_1 - y = 0$ (D) $\tan(x + 3)\, y_1 - y = 0$

16. By eliminating arbitrary constant a the differential equation whose general solution is $y^2 = 4ax$ is (1)

(A) $xy\dfrac{dy}{dx} - y^2 = 0$ (B) $2xy\dfrac{dy}{dx} + y^2 = 0$ (C) $2xy\dfrac{dy}{dx} - y^2 = 0$ (D) $8xy\dfrac{dy}{dx} - y^2 = 0$

17. The differential equation whose general solution is $xy = C^2$, where C is arbitrary constant, is (1)

(A) $xy_1 - y = 0$ (B) $xy_2 + y_1 = 0$ (C) $xy_1 = C^2$ (D) $xy_1 + y = 0$

18. The differential equation representing the family of curves $y^2 = 2C(x + \sqrt{C})$, where C is arbitrary constant, is (1)

(A) $2yy_1\left(x + \sqrt{yy_1}\right) - y^2 = 1$
(B) $2y_1\left(x + \sqrt{yy_1}\right) - y = 0$
(C) $y = 2y_1\left(x + \sqrt{C}\right)$
(D) $y_1\left(x + \sqrt{yy_1}\right) - y = 0$

19. By eliminating arbitrary constant A the differential equation whose general solution is $y = Ae^{-x^2}$ is (1)

(A) $\frac{dy}{dx} - 2xy = 0$
(B) $y\frac{dy}{dx} - 2x = 0$
(C) $\frac{dy}{dx} + 2xy = 0$
(D) $y\frac{dy}{dx} + 2x = 0$

20. y = mx where m is arbitrary constant is the general solution of the differential equation is (1)

(A) $\frac{dy}{dx} = \frac{y}{x}$
(B) $\frac{dy}{dx} = \frac{x}{y}$
(C) $\frac{dy}{dx} = m$
(D) $\frac{dy}{dx} = -\frac{y}{x}$

21. The differential equation representing the family of curves $y = 3 + \sqrt{Cx}$, where C is arbitrary constant, is (2)

(A) $y = 3 + 2x\frac{dy}{dx}$
(B) $y = 3 + 2\sqrt{x}\frac{dy}{dx}$
(C) $y = 2x\frac{dy}{dx}$
(D) $\frac{dy}{dx} = \frac{\sqrt{c}}{2\sqrt{x}}$

22. The differential equation satisfied by general solution $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$, where a is arbitrary constant, is (2)

(A) $xyy_1 - y + 4 = 0$
(B) $xyy_1 + y^2 - 4 = 0$
(C) $x^2yy_1 + y^2x - 1 = 0$
(D) $xyy_1 - y^2 + 4 = 0$

23. The differential equation representing the family of curves $x^2 + y^2 = 2Ax$, where A is arbitrary constant, is (2)

(A) $y_1 = \frac{y^2 + x^2}{2xy}$
(B) $y_1 = \frac{y^2 - x^2}{2xy}$
(C) $y_1 = \frac{y^2 - x^2}{2y}$
(D) $y_1 = \frac{2xy}{y^2 - x^2}$

24. $y^2 = C(4 + e^{2x})$ where C is arbitrary constant is the general solution of the differential equation (2)

(A) $(4 + e^{2x})\frac{dy}{dx} - ye^{2x} = 0$
(B) $y\frac{dy}{dx} - e^{2x}(4 + e^{2x}) = 0$
(C) $e^{2x}\frac{dy}{dx} - y^2e^{2x} = 0$
(D) $y(4 + e^{2x})\frac{dy}{dx} - e^{2x} = 0$

25. $\sin(y - x) = Ce^{-\frac{x^2}{2}}$ where C is arbitrary constant is the general solution of the differential equation (2)

(A) $\tan(y-x)\left[\frac{dy}{dx}-1\right]+x=0$

(B) $\cot(y-x)\frac{dy}{dx}-1+x=0$

(C) $\cot(y-x)\left[\frac{dy}{dx}-1\right]+x=0$

(D) $\cot(y-x)\left[\frac{dy}{dx}-1\right]=0$

26. By eliminating arbitrary constant A the differential equation whose general solution is $(1 + x^2) = A(1 - y^2)$ is (2

(A) $\frac{(1+x^2)}{(1-y^2)}\frac{dy}{dx}+\frac{x}{y}=0$

(B) $\frac{(1-y^2)}{(1+x^2)}\frac{dy}{dx}+\frac{x}{y}=0$

(C) $\frac{(1+x^2)}{(1-y^2)}\frac{dy}{dx}+\frac{y}{x}=0$

(D) $\frac{(1-y^2)}{(1+x^2)}\frac{dy}{dx}-\frac{x}{y}=0$

27. The differential equation satisfied by general solution $x = Cy - y^2$, where C is arbitrary constants, is (2)

(A) $\left(\frac{y}{x+y^2}\right)y_1-2yy_1-1=0$

(B) $\left(\frac{x+y^2}{y}\right)y_1-2yy_1-1=0$

(C) $\left(\frac{x+y^2}{y}\right)y_1+2yy_1+1=0$

(D) $\left(\frac{x+y^2}{y}\right)y_1+2yy_1=0$

28. The differential equation satisfied by general solution $y + x^3 = Cx$, where C is arbitrary constants, is (2)

(A) $\frac{dy}{dx}+3x^2=C$

(B) $x\frac{dy}{dx}+2x^2-y=0$

(C) $\frac{dy}{dx}+2x^2-y=0$

(D) $x\frac{dy}{dx}+2x^3-y=0$

29. $xy + y^2 - x^2 - 3y - x = C$, where C is arbitrary constant is the general solution of the differential equation (2)

(A) $\frac{dy}{dx}=\frac{2x-y+1}{x+2y-3}$

(B) $\frac{dy}{dx}=\frac{2x+1}{x+2y-3}$

(C) $\frac{dy}{dx}=\frac{y-2x-1}{x-2y+3}$

(D) $\frac{dy}{dx}=\frac{x+2y-3}{2x-y+1}$

30. $y = Ce^{x/y}$, where C is arbitrary constant is the general solution of the differential equation (2)

(A) $yy_1 + (y - xy_1) = 0$
(B) $yy_1 - (y - xy_1) = 0$
(C) $y^2y_1 - (y - xy_1) = 0$
(D) $\frac{dy}{dx} - 1 = 0$

31. $\sin\left(\frac{y}{x}\right) = Cx$, where C is arbitrary constant is the general solution of the differential equation (2)

(A) $xy_1 + y = x\tan\left(\frac{y}{x}\right)$
(B) $xy_1 - y = x\cot\left(\frac{y}{x}\right)$
(C) $xy_1 - y = x\tan\left(\frac{x}{y}\right)$
(D) $xy_1 - y = x\tan\left(\frac{y}{x}\right)$

32. By eliminating arbitrary constant A the differential equation whose general solution is $y^2 = x^2 - 1 + Ax$ is (2)

(A) $2xy\frac{dy}{dx} = x^2 + y^2 + 1$
(B) $2xy\frac{dy}{dx} = x^2 + x + y^2 + 1$
(C) $2xy\frac{dy}{dx} = y^2 + 1$
(D) $2y\frac{dy}{dx} = 2x + A$

33. The differential equation satisfied by general solution $y = A\cos x + B\sin x$, where A and B are arbitrary constants, is (2)

(A) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = B\sin x$
(B) $\frac{d^2y}{dx^2} - y = 0$
(C) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = A\cos x$
(D) $\frac{d^2y}{dx^2} + y = 0$

34. The differential equation satisfied by general solution $y = A\cos\frac{2x}{3} + B\sin\frac{2x}{3}$, where A and B are arbitrary constants, is

(A) $\frac{d^2y}{dx^2} + \frac{9}{4}y = 0$
(B) $\frac{d^2y}{dx^2} - \frac{4}{9}y = 0$
(C) $\frac{d^2y}{dx^2} + \frac{4}{9}y = 0$
(D) $\frac{d^2y}{dx^2} - \frac{9}{4}y = 0$

35. The differential equation satisfied by general solution $y = A\cos(\log x) + B\sin(\log x)$ where A and B are arbitrary constants, is (2)

(A) $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$
(B) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$
(C) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$
(D) $x^2\frac{d^2y}{dx^2} + y = 0$

36. The differential equation satisfied by general solution $y = Ae^{x} + Be^{-x}$, where A and B are arbitrary constants, is (2)

(A) $y_2 - y = 0$ (B) $y_2 + y = 0$

(C) $y_2 + y = Ae^{x} - Be^{-x}$ (D) $y_2 - y = 2Ae^{x}$

37. The differential equation satisfied by general solution $xy = Ae^{x} + Be^{-x}$, where A and B are arbitrary constants, is (2)

(A) $xy_2 + 2y_1 + xy = 0$ (B) $xy_2 - 2y_1 + xy = 0$

(C) $xy_2 + 2y_1 - xy = 0$ (D) $xy_2 + y_1 - xy = 0$

$x = A\cos(2t+B)$

38. The differential equation satisfied by general solution $x = A\cos(2t + B)$, where A and B are arbitrary constants, is (2)

(A) $\frac{d^2x}{dt^2} + 4x = 0$ (B) $\frac{d^2x}{dt^2} - 2x = 0$ (C) $\frac{d^2x}{dt^2} - 4x = 0$ (D) $\frac{d^2x}{dt^2} + x = 0$

39. By eliminating arbitrary constants A and B the differential equation whose general solution is $e^{-t}x = (A + Bt)$ is (2)

(A) $x_2 + 2x_1 - x = 0$ (B) $x_2 - 2x_1 + x = 0$ (C) $x_2 - x_1 + x = 0$ (D) $x_2 + x = 0$

40. By eliminating arbitrary constants A and B the differential equation whose general solution is $y^2 = 4A(x - B)$ is (2)

(A) $y_2 + y_1^2 = 0$ (B) $yy_2 + y_1 = 0$ (C) $yy_2 - y_1^2 = 0$ (D) $yy_2 + y_1^2 = 0$

41. The differential equation of family of circles having their centres at (A, 5) and radius 5, where A is arbitrary constant is (2)

(A) $(y-5)^2\left\{1 + \frac{dy}{dx}\right\} = 5$ (B) $(y-5)^2\left\{1 - \left(\frac{dy}{dx}\right)^2\right\} = 25$

(C) $(y-5)^2\left\{1 + \left(\frac{dy}{dx}\right)^2\right\} = 25$ (D) none of these

42. The differential equation of family of circles having their centres at origin and radius a where a is arbitrary constant, is (2)

(A) $x - y\frac{dy}{dx} = 0$ (B) $x + y\frac{dy}{dx} = 0$ (C) $x\frac{dy}{dx} + y = 0$ (D) $x + y\frac{dy}{dx} = \frac{a^2}{2}$

43. By eliminating arbitrary constants A and B the differential equation whose general solution is $(x - A)^2 = 4(y - B)$ is (2)

(A) $2\frac{dy}{dx} - (x - A) = 0$ (B) $\frac{d^2y}{dx^2} + \frac{1}{2} = 0$ (C) $\frac{d^2y}{dx^2} - 2 = 0$ (D) $\frac{d^2y}{dx^2} - \frac{1}{2} = 0$

44. The differential equation satisfied by general solution $y = A\cos 4x + B\sin 4x + C$, where A, B and C are arbitrary constants, is (2)

(A) $\frac{d^2y}{dx^2} - 16y = 0$ (B) $\frac{d^3y}{dx^3} - 16\frac{dy}{dx} = 0$ (C) $\frac{d^3y}{dx^3} + 16\frac{dy}{dx} = 0$ (D) $\frac{d^3y}{dx^3} + \frac{dy}{dx} = 0$

45. The differential equation satisfied by general solution $y = Ax^2 + Bx + C$, where A, B and C are arbitrary constants, is (2)

(A) $\frac{d^3y}{dx^3} = 0$ (B) $\frac{d^2y}{dx^2} = 2A$ (C) $\frac{d^3y}{dx^3} = A$ (D) $\frac{d^4y}{dx^4} = 0$

ANSWERS

1. (B)	2. (A)	3. (C)	4. (D)	5. (A)	6. (A)	7. (C)	8. (B)
9. (D)	10. (C)	11. (B)	12. (D)	13. (B)	14. (C)	15. (A)	16. (C)
17. (D)	18. (B)	19. (C)	20. (A)	21. (A)	22. (D)	23. (B)	24. (A)
25. (C)	26. (A)	27. (B)	28. (D)	29. (A)	30. (B)	31. (D)	32. (A)
33. (D)	34. (C)	35. (B)	36. (A)	37. (C)	38. (A)	39. (B)	40. (D)
41. (C)	42. (B)	43. (D)	44. (C)	45. (A)			

MULTIPLE CHOICE QUESTIONS

1. The differential equation $\frac{dy}{dx} = e^{x-y} + 3x^2e^{-y}$ is of the form (1)

(A) variable separable (B) homogeneous (C) linear (D) exact

2. For solving the differential equation $(x + y + 1)\,dx + (2x + 2y + 4)\,dy = 0$ appropriate substitution is (1)

(A) $x + y = 1$ (B) $x + y = u$ (C) $x - y = u$ (D) none of these

3. The differential equation $\frac{dy}{dx} = \frac{x^3 - 3xy^2}{y^3 - 3x^2y}$ is of the form (1)

(A) variable separable (B) homogeneous (C) linear (D) exact

4. The differential equation $\frac{dy}{dx} = \frac{x + 2y - 3}{3x + 6y - 1}$ is of the form (1)

(A) variable separable (B) exact

(C) non-homogeneous (D) homogeneous

5. The solution of differential equation $\frac{dy}{dx} + y = 0$ is (1)

(A) $y = Ae^{-x}$ (B) $y = Ae^{x}$ (C) $x = Ae^{-y}$ (D) $x = Ae^{y}$

6. The solution of differential equation $\frac{dy}{dx} + x = 0$ is (1)

(A) $x + y^2 = C$ (B) $x + y = C$ (C) $x^2 + y = C$ (D) $x^2 + 2y = C$

7. The solution of differential equation $y\,dx + x\,dy = 0$ is (1)

(A) $x^2y = C$ (B) $xy = C$ (C) $xy^2 = C$ (D) $xy + 1 = C$

8. The solution of differential equation $\frac{dy}{dx} + \tan x = 0$ is (1)

(A) $y + \log \sin x = C$ (B) $y + \sec^2 x = C$

(C) $y - \log \cos x = C$ (D) $y + \log \cot x = C$

9. The solution of differential equation $\frac{dy}{dx} = \frac{1 + y}{1 + x}$ is (1)

(A) $(1 + y) = C(1 + x)$ (B) $(1 + x) = \frac{C}{(1 + y)}$

(C) $xy(1 + y) = C$ (D) $(1 + y)^2 = C(1 + x)$

10. The solution of differential equation $\frac{dy}{dx} + \frac{1+y^2}{1+x^2} = 0$ is (2)

(A) $\tan^{-1} y - \tan^{-1} x = C$
(B) $\tan^{-1} y + \tan^{-1} x = C$
(C) $\tan y + \tan x = C$
(D) $\cos y + \cos x = C$

11. The solution of differential equation $(4 + e^{2x}) \frac{dy}{dx} = ye^{2x}$ is (2)

(A) $y^2 = (4 + e^{2x})\, C$ (B) $y = (4 + e^{2x})\, C$ (C) $y\,(4 + e^{2x}) = C$ (D) $y^2\,(4 + e^{2x}) = C$

12. The solution of differential equation $y - x\frac{dy}{dx} = 2\left(y + \frac{dy}{dx}\right)$ is (2)

(A) $y + (x + 2) = C$ (B) $y - (x + 2) = C$ (C) $y = C\,(x + 2)$ (D) $y\,(x + 2) = C$

13. The solution of differential equation $xdy - ydx = 0$ is (2)

(A) $y = x + C$ (B) $x^2 - y^2 = C$ (C) $xy = C$ (D) $y = Cx$

14. The solution of differential equation $\frac{dy}{dx} = e^{x-y} + 3x^2e^{-y}$ is (2)

(A) $e^y = e^x + x^3 + C$
(B) $e^y = e^x + 3x^3 + C$
(C) $e^y = e^x + 3x + C$
(D) $e^x + e^y = 3x^3 + C$

15. The solution of differential equation is $x^3\left(x\frac{dy}{dx} + y\right) - \sec(xy) = 0$ by substitution $xy = u$ is (2)

(A) $\tan(xy) + \frac{1}{2x^2} = C$
(B) $\sin(xy) + \frac{1}{2x^2} = C$
(C) $\sin(xy) - \frac{1}{2x^2} = C$
(D) $\sin(xy) - \frac{1}{4x^4} = C$

16. The solution of differential equation $\sec^2 x \tan y dx + \sec^2 y \tan x\, dy = 0$ is (2)

(A) $\sec^2 x \tan y = C$ (B) $\tan x \sec^2 y = C$ (C) $\tan x \tan y = C$ (D) $\sec^2 x \sec^2 y = C$

17. The solution of differential equation $y \sec^2 x + (y + 7) \tan x \frac{dy}{dx} = 0$ is (2)

(A) $y + 7 \log y = -\log \tan x + C$
(B) $y + \log (7 + y) = -\log \tan x + C$
(C) $y - 7 \log y = \log \tan x + C$
(D) $y + \log y = -\log \tan x + C$

18. The solution of differential equation $e^x \cos y dx + (1 + e^x) \sin y\, dy = 0$ is (2)

(A) $(1 + e^x) = C \sec y$
(B) $(1 + e^x) \sec y = C$
(C) $\frac{\sec y}{(1 + e^x)} = C$
(D) $(1 + e^x) \cos y = C$

19. The solution of differential equation $y(1+\log x)\frac{dx}{dy} - x\log x = 0$ is (2)

(A) $\log(x\log x) = yC$
(B) $\frac{x}{\log x} = yC$
(C) $y(\log x) = xC$
(D) $x(\log x) = yC$

20. The solution of differential equation $3e^x \tan y\, dx + (1+e^x)\sec^2 y\, dy = 0$ is (2)

(A) $3\log(1-e^x) = -\log\tan y + \log C$
(B) $\log(1+e^x) = \log\tan y + \log C$
(C) $3\log(1+e^x) = -\log\tan y + \log C$
(D) $\log(1+e^x) = -\log\sin y + \log C$

21. The solution of differential equation $x(1+y^2)\,dx + y(1+x^2)\,dy = 0$ is (2)

(A) $(1-x^2)(1+y^2) = C$
(B) $\tan^{-1}x + \tan^{-1}y = C$
(C) $(1+x^2) = C(1+y^2)$
(D) $(1+x^2)(1+y^2) = C$

22. The solution of differential equation $\frac{dy}{dx} = (1+x)(1+y^2)$ is (2)

(A) $\tan^{-1}y = x + \frac{x^2}{2} + C$
(B) $\log(1+y^2) = x + \frac{x^2}{2} + C$
(C) $\tan^{-1}x = y + \frac{y^2}{2} + C$
(D) $\frac{1}{2}\log\left(\frac{1+y}{1-y}\right) = x + \frac{x^2}{2} + C$

23. The solution of differential equation $(e^x+1)\,y\,dy = (y+1)\,e^x\,dx$ is (2)

(A) $y - \log(1-y) = \log(e^x-1) + \log C$
(B) $y - \log(1+y) = \log(e^x+1) + \log C$
(C) $y + \log(1-y) = \log(e^x+1) + \log C$
(D) $y - \log(1+y) = \log(e^x-1) + \log C$

24. The solution of differential equation $\frac{dy}{dx} = e^{x+y} + e^{y-x}$ is (2)

(A) $e^{-y} = e^{-x} - C$
(B) $e^{y} = e^{x} - e^{-x} + C$
(C) $-e^{-y} = e^{x} - e^{-x} + C$
(D) $e^{-y} = e^{x} + e^{-x} + C$

25. The solution of differential equation $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$ is (2)

(A) $\tan^{-1}x + \cot^{-1}y = C$
(B) $\sin^{-1}x + \sin^{-1}y = C$
(C) $\sec^{-1}x + \operatorname{cosec}^{-1}y = C$
(D) $\sin^{-1}x - \sin^{-1}y = C$

ANSWERS

1. (A)	2. (B)	3. (B)	4. (C)	5. (A)	6. (D)	7. (B)	8. (C)
9. (A)	10. (B)	11. (A)	12. (D)	13. (D)	14. (A)	15. (B)	16. (C)
17. (A)	18. (B)	19. (D)	20. (C)	21. (D)	22. (A)	23. (B)	24. (C)
25. (B)							

MULTIPLE CHOICE QUESTIONS

Exact Differential Equations and Reducible to Exact Differential Equation :

1. The necessary and sufficient condition that the differential equation $M(x, y)\,dx + N(x, y)\,dy = 0$ be exact is (1)

   (A) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$; $My + Nx \neq 0$
   (B) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$; $Mx - Ny \neq 0$
   (C) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$; $Mx + Ny \neq 0$
   (D) $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 1$; $My - Nx \neq 0$

2. If homogeneous differential equation $M(x, y)\,dx + N(x, y)\,dy = 0$ is not exact then the integrating factor is (1)

   (A) $\frac{1}{My + Nx}$; $My + Nx \neq 0$
   (B) $\frac{1}{Mx - Ny}$; $Mx - Ny \neq 0$
   (C) $\frac{1}{Mx + Ny}$; $Mx + Ny \neq 0$
   (D) $\frac{1}{My - Nx}$; $My - Nx \neq 0$

3. If the differential equation $M(x, y)\,dx + N(x, y)\,dy = 0$ is not exact and it can be written as $yf_1(xy)\,dx + xf_2(xy)\,dy = 0$ then the integrating factor is (1)

   (A) $\frac{1}{My + Nx}$; $My + Nx \neq 0$
   (B) $\frac{1}{Mx - Ny}$; $Mx - Ny \neq 0$
   (C) $\frac{1}{Mx + Ny}$; $Mx + Ny \neq 0$
   (D) $\frac{1}{My - Nx}$; $My - Nx \neq 0$

4. If the differential equation M (x, y) dx + N (x, y) dy = 0 is not exact and $\dfrac{\dfrac{\partial M}{\partial y}-\dfrac{\partial N}{\partial x}}{N} = f(x)$ then the integrating factor is (1)

(A) $e^{f(x)}$ (B) $e^{\int f(x)\,dy}$ (C) $f(x)$ (D) $e^{\int f(x)\,dx}$

5. If the differential equation M(x, y) dx + N(x, y) dy = 0 is not exact and $\dfrac{\dfrac{\partial M}{\partial y}-\dfrac{\partial N}{\partial x}}{-M} = f(y)$ then the integrating factor is (1)

(A) $e^{\int f(y)\,dy}$ (B) $e^{\int f(y)\,dx}$ (C) $f(y)$ (D) $e^{f(y)}$

6. The total derivative of xdy + ydx is (1)

(A) $d\left(\dfrac{y}{x}\right)$ (B) $d\left(\dfrac{x}{y}\right)$ (C) $d(xy)$ (D) $d(x+y)$

7. The total derivative of xdy – ydx with integrating factor $\dfrac{1}{x^2}$ is (1)

(A) $d\left(\dfrac{x}{y}\right)$ (B) $d\left(\dfrac{y}{x}\right)$ (C) $d\left(\log\dfrac{x}{y}\right)$ (D) $d(x-y)$

8. The total derivative of xdy + ydx with integrating factor $\dfrac{1}{xy}$ is (1)

(A) $d\left(\log\dfrac{x}{y}\right)$ (B) $d\left(\log\dfrac{y}{x}\right)$ (C) $d\,[\log(x+y)]$ (D) $d\,(\log xy)$

9. The total derivative of xdy – ydx with integrating factor $\dfrac{1}{xy}$ is (1)

(A) $d\left(\log\dfrac{x}{y}\right)$ (B) $d\left(\log\dfrac{y}{x}\right)$ (C) $d\left[\dfrac{y}{x}\right]$ (D) $d\,(\log xy)$

10. The total derivative of xdy – ydx with integrating factor $\dfrac{1}{x^2+y^2}$ is (1)

(A) $d\left(\tan^{-1}\dfrac{y}{x}\right)$ (B) $d\left(\tan^{-1}\dfrac{x}{y}\right)$ (C) $d\,[\log(x^2+y^2)]$ (D) none of these

11. The total derivative of dx + dy with integrating factor $\dfrac{1}{x+y}$ is (1)

(A) $d\,[\log(x-y)]$ (B) $d\,[\log x^2-y^2)]$ (C) $d\,[\log(x+y)]$ (D) none of these

12. The differential equation $(x+y-2)\,dx + (x-y+4)\,dy = 0$ is of the form (1)

(A) exact (B) homogeneous (C) linear (D) none of these

13. The value of λ for which the differential equation $(xy^2+\lambda x^2y)\,dx + (x^3+x^2y)\,dy = 0$ is exact is (2)

(A) –3 (B) 2 (C) 3 (D) 1

14. The differential equation $(ay^2+x+x^8)\,dx + (y^8-y+bxy)\,dy = 0$ is exact if (2)

(A) $b \neq 2a$ (B) $b = a$ (C) $a = 1, b = 3$ (D) $b = 2a$

15. The differential equation $(3+by\cos x)\,dx + (2\sin x-4y^3)\,dy = 0$ is exact if (2)

(A) $b = -2$ (B) $b = 3$ (C) $b = 0$ (D) $b = 2$

16. The differential equation $(\tan y - ax^2y - y)\,dx + (x\tan^2 y - x^3 - \sec^2 y)\,dy = 0$ is exact if (2)

(A) $a = 2$ (B) $a = 3$ (C) $a = -3$ (D) $a = -2$

17. The differential equation $\left(\frac{2x}{y^3}\right)dx + \left(\frac{y^2 + ax^2}{y^4}\right)dy = 0$ is exact if (2)

(A) $a = -3$ (B) $a = 3$ (C) $a = -2$ (D) $a = 6$

18. Integrating factor of homogeneous differential equation

$(xy - 2y^2)\,dx + (3xy - x^2)\,dy = 0$ is (2)

(A) $\frac{1}{xy}$ (B) $\frac{1}{x^2y^2}$ (C) $\frac{1}{x^2y}$ (D) $\frac{1}{xy^2}$

19. Integrating factor of homogeneous differential equation

$(x^2 - 3xy + 2y^2)\,dx + (3x^2 - 2xy)\,dy = 0$ is (2)

(A) $\frac{1}{xy}$ (B) $\frac{1}{x^3}$ (C) $\frac{1}{x^2y}$ (D) $\frac{1}{x^2}$

20. Integrating factor of homogeneous differential equation

$(y^2 - 2xy)\,dx + (2x^2 + 3xy)\,dy = 0$ is (2)

(A) $\frac{1}{x^2y^2}$ (B) $\frac{1}{x^2y}$ (C) $\frac{1}{4xy^2}$ (D) $\frac{1}{y^2}$

21. Integrating factor of homogeneous differential equation

$(x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy = 0$ is (2)

(A) $\frac{1}{x^2y^2}$ (B) $\frac{1}{xy}$ (C) $\frac{2}{x}$ (D) $\frac{1}{x^2y}$

22. Integrating factor for differential equation

$(x^2y^2 + xy + 1)\,y\,dx + (x^2y^2 - xy + 1)\,x\,dy = 0$ is (2)

(A) $\frac{1}{2x^3y^3}$ (B) $\frac{1}{xy}$ (C) $\frac{1}{2x^2y^2}$ (D) $\frac{1}{x^2y}$

23. Integrating factor for differential equation $(1 + xy)\,y\,dx + (1 - xy)\,x\,dy = 0$ is (2)

(A) $\frac{1}{2x^2y^2}$ (B) $\frac{1}{x^2y}$ (C) $\frac{1}{xy^2}$ (D) $\frac{1}{y}$

24. Integrating factor for differential equation $(1 + xy)\,y\,dx + (x^2y^2 + xy + 1)\,x\,dy = 0$ is (2)

(A) $\frac{1}{x^2y}$ (B) $-\frac{1}{x^3y^3}$ (C) $\frac{1}{xy^2}$ (D) $\frac{1}{x^2y^2}$

25. Integrating factor for differential equation $(x^2 + y^2 + x)\,dx + (xy)\,dy = 0$ is (2)

(A) $\frac{1}{x}$ (B) $\frac{1}{x^2}$ (C) x^2 (D) x

26. Integrating factor for differential equation $\left(y+\frac{y^3}{3}+\frac{x^2}{2}\right)dx+\left(\frac{x+xy^2}{4}\right)dy=0$ is (2)

(A) $\frac{1}{x}$ (B) x^3 (C) x^2 (D) $\frac{1}{x^3}$

27. Integrating factor for differential equation $(2x \log x - xy)\, dy + (2y)\, dx = 0$ is (2)

(A) $\frac{1}{x}$ (B) $\frac{1}{x^2y^2}$ (C) $\frac{1}{x^2}$ (D) $\frac{1}{y}$

28. Integrating factor for differential equation $(x^2 + y^2 + 1)\, dx - 2xydy = 0$ is (2)

(A) $\frac{1}{x}$ (B) $\frac{1}{x^3}$ (C) $\frac{1}{x^2}$ (D) $\frac{1}{xy}$

29. Integrating factor for differential equation $y\,(2xy + e^x)\, dx - e^x\, dy = 0$ is (2)

(A) $\frac{1}{x}$ (B) $\frac{1}{y}$ (C) $\frac{1}{x^2}$ (D) $\frac{1}{y^2}$

30. Integrating factor for differential equation $y \log y\, dx + (x - \log y)\, dy = 0$ is (2)

(A) $\frac{1}{x}$ (B) $\frac{1}{y}$ (C) $\frac{1}{x^2}$ (D) $\frac{1}{y^2}$

31. Integrating factor for differential equation $(y^4 + 2y)\, dx + (xy^3 + 2y^4 - 4x)\, dy = 0$ (2)

(A) $\frac{2}{x}$ (B) $\frac{1}{y}$ (C) $\frac{1}{y^3}$ (D) $\frac{2}{y^2}$

32. Integrating factor for differential equation $(2x + e^x \log y)\, ydx + (e^x)\, dy = 0$ is (2)

(A) $\frac{1}{x}$ (B) $\frac{1}{y^2}$ (C) $\frac{1}{x^2}$ (D) $\frac{1}{y}$

33. Solution of non-exact differential equation $(x^2 - 3xy + 2y^2)\, dx + x\,(3x - 2y)\, dy = 0$ with integrating factor $\frac{1}{x^3}$ is (2)

(A) $3\frac{y}{x}-\frac{y^2}{x^2}=C$

(B) $\log x-3\frac{y}{x}+\frac{y^2}{x^2}=C$

(C) $\log x+3\frac{y}{x}-2\frac{y^2}{x^2}=C$

(D) $\log x+3\frac{y}{x}-\frac{y^2}{x^2}=C$

34. Solution of non-exact differential equation $(3xy^2 - y^3)\, dx + (xy^2 - 2x^2y)\, dy = 0$ with integrating factor $\frac{1}{x^2y^2}$ is (2)

(A) $3\log x-\frac{2y}{x^3}-2\log y=C$

(B) $3\log x+\frac{y}{x}-2\log y=C$

(C) $3\log x+\frac{y}{x}=C$

(D) $\log x-\frac{y}{x}+2\log y=C$

35. Solution of non-exact differential equation $(1 + xy)\, ydx + (1 - xy)\, xdy = 0$ is integating factor $\frac{1}{x^2y^2}$ is (2)

(A) $\frac{2}{xy} - \log\left(\frac{x}{y}\right) = C$ (B) $-\frac{1}{xy} + \log\left(\frac{y}{x}\right) = C$

(C) $-\frac{1}{xy} + \log\left(\frac{x}{y}\right) = C$ (D) $-\frac{2}{x^3y} + \log\left(\frac{x}{y}\right) = C$

36. Solution of non-exact differential equation $(2 + x^2y^2)\, ydx + (2 - 2x^2y^2)\, xdy = 0$ with integrating factor $\frac{1}{x^3y^3}$ is (2)

(A) $\log\left(\frac{x}{y^2}\right) - \frac{1}{x^2y^2} = C$ (B) $\log\left(\frac{x}{y^2}\right) + \frac{1}{x^2y^2} = C$

(C) $\log\left(\frac{y^2}{x}\right) - \frac{1}{x^2y^2} = C$ (D) $\log x - \frac{1}{x^2y^2} = C$

37. Soluton of non-exact differential equation $y\,(2xy + e^x)\, dx - e^x\, dy = 0$ with integating factor $\frac{1}{y^2}$ is (2)

(A) $x^2 + \frac{e^x}{y} - e^x \log y = C$ (B) $x^2 + \frac{e^x}{y} = C$

(C) $x^2 + \frac{2e^x}{y} = C$ (D) $x^2 - \frac{e^x}{y} = C$

38. Solution of non-exact differential equation $(x^4e^x - 2mxy^2)\, dx + (2mx^2y)\, dy = 0$ with integating factor $\frac{1}{x^4}$ is (2)

(A) $e^x + \frac{6my^2}{x^4} = C$ (B) $e^x + \frac{2my^2}{x^2} = C$ (C) $e^x + \frac{y^2}{x^2} = C$ (D) $e^x + \frac{my^2}{x^2} = C$

ANSWERS

1. (A)	2. (C)	3. (B)	4. (D)	5. (A)	6. (C)	7. (B)	8. (D)
9. (B)	10. (A)	11. (C)	12. (A)	13. (C)	14. (D)	15. (D)	16. (B)
17. (A)	18. (D)	19. (B)	20. (C)	21. (A)	22. (C)	23. (A)	24. (B)
25. (D)	26. (B)	27. (A)	28. (C)	29. (D)	30. (B)	31. (C)	32. (D)
33. (D)	34. (B)	35. (C)	36. (A)	37. (B)	38. (D)		

MULTIPLE CHOICE QUESTIONS

Linear Differential Equations and Reducible to Linear Differential Equation :

1. The differential equation of the form $\frac{dy}{dx} + Py = Q$ where P and Q are functions of x or constants, is (1)

   (A) exact differential equation

   (B) linear differential equation in y

   (C) linear differential equation in x

   (D) non-homogeneous differential equation

2. The differential equation of the form $\frac{dx}{dy} + Px = Q$ where P and Q are functions of y or constants, is (1)

   (A) exact differential equation

   (B) linear differential equation in y

   (C) linear differential equation in x

   (D) non-homogeneous differential equation

3. Integrating factor of linear differential equation $\frac{dy}{dx} + Py = Q$ where P and Q are functions of x or constants, is (1)

   (A) $e^{\int P\,dy}$ (B) $e^{\int Q\,dx}$ (C) $e^{\int Q\,dx}$ (D) $e^{\int P\,dx}$

4. Integrating factor of linear differential equation $\frac{dx}{dy} + Px = Q$ where P and Q are functions of y or constants, is (1)

   (A) $e^{\int P\,dy}$ (B) $e^{\int P\,dx}$ (C) $e^{\int Q\,dx}$ (D) $e^{\int Q\,dy}$

5. The general solution of linear differential equation $\frac{dy}{dx} + Py = Q$ where P and Q are functions of x or constants, is (1)

   (A) $xe^{\int P\,dy} = \int Qe^{\int P\,dy}\,dy + C$

   (B) $y = \int Qe^{\int P\,dx}\,dx + C$

   (C) $ye^{\int P\,dx} = \int Q\,dx + C$

   (D) $ye^{\int P\,dx} = \int Qe^{\int P\,dx}\,dx + C$

6. The general solution of linear differential equation $\frac{dx}{dy} + Px = Q$ where P and Q are functions of y or constants, is (1)

   (A) $x = \int Qe^{\int P\,dy}\,dy + C$

   (B) $xe^{\int P\,dx} = \int Qe^{\int P\,dx}\,dx + C$

   (C) $ye^{\int P\,dx} = \int Qe^{\int P\,dx}\,dx + C$

   (D) $xe^{\int P\,dy} = \int Qe^{\int P\,dy}\,dy + C$

7. The differential equation of the form $\frac{dy}{dx} + Py = Qy^n$, $n \neq 1$ where P and Q are functions of x or constants, is (1)

(A) Bernoulli's differential equation (B) exact differential equation
(C) symmetric differential equation (D) linear differential equation

8. The differential equation of the form $\frac{dx}{dy} + Px = Qx^n$, $n \neq 1$ where P and Q are functions of y or constants, is (1)

(A) Bernoulli's differential equation (B) exact differential equation
(C) symmetric differential equation (D) linear differential equation

9. The differential equation of the form $f'(y)\frac{dy}{dx} + P\,f(y) = Q$ where P and Q are functions of x or constants, can be reduced to linear differential equation by the substitution (1)

(A) $f'(y) = u$ (B) $P = u$ (C) $f(y) = u$ (D) $Q = u$

10. The differential equation of the form $f'(x)\frac{dx}{dy} + P\,f(x) = Q$ where P and Q are functions of y or constants, can be reduced to linear differential equation by the substitution (1)

(A) $f'(x) = u$ (B) $f(x) = u$ (C) $P = u$ (D) $Q = u$

11. Integrating factor of linear differential equation $\frac{dy}{dx} + xy = x^3$ is (2)

(A) $e^{\log x}$ (B) e^x (C) x^2 (D) $e^{\frac{x^2}{2}}$

12. Integrating factor of linear differential equation $\frac{dx}{dy} + yx = y^2$ is (2)

(A) $e^{\frac{y^2}{2}}$ (B) $e^{\frac{x^2}{2}}$ (C) y^2 (D) $e^{\log y}$

13. The differential equation $\frac{dy}{dx} + \frac{y}{1+x^2} = x^2$ has integrating factor (2)

(A) $e^{\frac{1}{1+y^2}}$ (B) $e^{\tan^{-1}x}$ (C) $e^{\frac{1}{1+x^2}}$ (D) $e^{\tan^{-1}y}$

14. The differential equation $\frac{dx}{dy} + \frac{x}{1+y^2} = y^2$ has integrating factor (2)

(A) $e^{\frac{1}{1+y^2}}$ (B) $e^{\tan^{-1}x}$ (C) $e^{\frac{1}{1+x^2}}$ (D) $e^{\tan^{-1}y}$

15. The differential equation $\frac{dy}{dx} + \sqrt{x}\,y = x^3$ has integrating factor (2)

(A) $e^{\frac{2}{3}x\sqrt{x}}$ (B) $e^{\frac{1}{3}x\sqrt{x}}$ (C) $e^{\sqrt{x}}$ (D) e^{-x}

16. The linear differential equation $(1 + y^2) + (x - e^{-\tan^{-1} y})\frac{dy}{dx} = 0$ has integrating factor(2)

(A) $e^{\tan^{-1} x}$ (B) $e^{\frac{1}{1+y^2}}$ (C) $e^{\tan^{-1} y}$ (D) e^{2y}

17. The linear differential equation $(1 - x^2)\frac{dy}{dx} = 1 + xy$ has integrating factor (2)

(A) $\sqrt{1-x^2}$ (B) $\frac{1}{\sqrt{1-x^2}}$ (C) $e^{\tan^{-1} x}$ (D) $x\sqrt{1-x^2}$

18. The linear differential equation $(2y + x^2)\, dx = x dy$ has integrating factor (2)

(A) $\frac{1}{x}$ (B) $\frac{1}{x^2}$ (C) x (D) $\frac{1}{y^2}$

19. The linear differential equation $y^2 + \left(x - \frac{1}{y}\right)\frac{dy}{dx} = 0$ has integrating factor (2)

(A) e^x (B) e^y (C) $\frac{1}{y^2}$ (D) $e^{-\frac{1}{y}}$

20. The differential equation $\frac{dy}{dx} + y \cot x = \sin 2x$ has integrating factor (2)

(A) $\cos x$ (B) $e^{\cot x}$ (C) $\sin x$ (D) $\sec x$

21. The differential equation $\cos x \frac{dy}{dx} + y = \sin x$ has integrating factor (2)

(A) $e^{\sec x}$ (B) $(\operatorname{cosec} x - \cot x)$

(C) $(\sec x + \tan x)$ (D) $(\sec x - \tan x)$

22. The differential equation $(x^2 + 1)\frac{dy}{dx} + 4xy = \frac{1}{(x^2+1)^2}$ has integrating factor (2)

(A) $(x^2 + 1)^2$ (B) $(x^2 + 1)$ (C) $e^{\frac{4x}{(x^2+1)}}$ (D) e^{4x}

23. The Bernoulli's differential equation $\frac{dy}{dx} - y \tan x = y^4 \sec x$ reduces to linear differential equation (2)

(A) $\frac{du}{dx} + (3 \tan x)\, u = -3 \sec x$ where $y^{-3} = u$

(B) $\frac{du}{dx} - (3 \tan x)\, u = 3 \sec x$ where $y^{-3} = u$

(C) $\frac{du}{dx} + (\tan x)\, u = -\sec x$ where $y^{-3} = u$

(D) none of these

24. The Bernoulli's differential equation $\frac{dy}{dx} - xy = -y^3 e^{-x^2}$ reduces to linear differential equation (2)

(A) $\frac{du}{dx} + (2x)\,u = 2e^{-x^2}$ where $y^{-2} = u$ (B) $\frac{du}{dx} + (x)\,u = e^{-x^2}$ where $y^{-2} = u$

(C) $\frac{du}{dx} - (2x)\,u = -2e^{-x^2}$ where $y^{-2} = u$ (D) none of these

25. The differential equation $\tan y \frac{dy}{dx} + \tan x = \cos^2 x \cos y$ reduces to linear differential equation (2)

(A) $\frac{du}{dx} - \tan(x)\,u = -\cos^2 x$ where $\sec y = u$

(B) $\frac{du}{dx} + (\tan x)\,u = \cos^2 x$ where $\sec y = u$

(C) $\frac{du}{dx} + (\cot x)\,u = \cos^2 x$ where $\sec y = u$

(D) none of these

26. The differential equation $\sin y \frac{dy}{dx} - 2\cos x \cos y = -\cos x \sin^2 x$ reduces to linear differential equation (2)

(A) $\frac{du}{dx} + (\cos x)\,u = \cos x \sin^2 x$ where $\cos y = u$

(B) $\frac{du}{dx} - (2\cos x)\,u = -\cos x \sin^2 x$ where $\cos y = u$

(C) $\frac{du}{dx} + (2\cos x)\,u = \cos x \sin^2 x$ where $\cos y = u$

(D) none of these

27. The value of α so that '$e^{\alpha y^2}$' is an integrating factor of linear differential equation $\frac{dx}{dy} + xy = e^{-\frac{y^2}{2}}$ is (2)

(A) -1 (B) $-\frac{1}{2}$ (C) 1 (D) $\frac{1}{2}$

28. The value of α so that $e^{\alpha x^2}$ is an integrating factor of linear differential equation $\frac{dy}{dx} - xy = x$ is (2)

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) 1 (D) -2

29. If I_1, I_2 are integrating factors of the equation $x\frac{dy}{dx} + 2y = 1$ and $x\frac{dy}{dx} - 2y = 1$ then true relation is (2)

(A) $I_1 = -I_2$ (B) $I_1I_2 = 1$ (C) $I_1 = x^2I_2$ (D) $I_1I_2 = x^2$

30. The general solution of $\frac{dy}{dx} + \frac{1}{1-x}y = -x(1-x)$ with integrating factor $\frac{1}{1-x}$ is (2)

(A) $y = -\frac{x^2}{2}\left(\frac{1}{1-x}\right) + C$

(B) $y\frac{1}{1-x} = x^2 + C$

(C) $y\frac{1}{1-x} = \frac{x^2}{2} + C$

(D) $y\frac{1}{1-x} = -\frac{x^2}{2} + C$

31. The general solution of $\frac{dy}{dx} + \frac{3}{x}y = \frac{e^x}{x^2}$ with integrating factor x^3 is (2)

(A) $y\,x^3 = (x+1)\,e^x + C$

(B) $y\,x^3 = (x-1)\,e^x + C$

(C) $x\,y^3 = (x-1)\,e^x + C$

(D) none of these

32. The general solution of $\frac{dy}{dx} + (\cot x)\,y = \sin 2x$ with integrating factor $\sin x$ is (2)

(A) $y\sin x = \frac{2}{3}\sin^2 x + C$

(B) $y\sin x = \sin^3 x + C$

(C) $y\sin x = \frac{2}{3}\sin^3 x + C$

(D) none of these

33. The general solution of $\frac{dy}{dx} + \frac{1}{(1-x)\sqrt{x}}y = (1-\sqrt{x})$ with integrating factor $\frac{1+\sqrt{x}}{1-\sqrt{x}}$ is (2)

(A) $y\frac{1+\sqrt{x}}{1-\sqrt{x}} = -x - \frac{2}{3}x^{3/2} + C$

(B) $y\frac{1+\sqrt{x}}{1-\sqrt{x}} = x + \frac{2}{3}x^{3/2} + C$

(C) $y\frac{1+\sqrt{x}}{1-\sqrt{x}} = x + \frac{3}{2}x^{1/2} + C$

(D) none of these

34. The general solution of $\frac{dy}{dx} + \left(\tan x + \frac{1}{x}\right)y = \frac{1}{x}\sec x$ with integrating factor $x\sec x$ is (2)

(A) $y\,(x\sec x) = \tan x + C$

(B) $y\,(x\sec x) = \frac{x^3}{3} + C$

(C) $x\,(y\sec y) = \tan x + C$

(D) none of these

35. The general solution of $\frac{dy}{dx} + \frac{3}{x}y = x^2$ with integrating factor x^3 is (2)

(A) $y\,x^3 = \frac{x^6}{6} + C$ (B) $y\,x^3 = \frac{x^2}{2} + C$ (C) $y\,x^3 = \log x + C$ (D) none of these

36. The general solution of $\dfrac{dy}{dx}+\dfrac{2}{x}y=\dfrac{1}{x^3}$ with integrating factor x^2 is (2)

(A) $y\,x^2=\dfrac{x^2}{2}+C$ (B) $y\,x^2=\log x+C$ (C) $y\,x^2=\dfrac{x^6}{6}+C$ (D) none of these

37. The general solution of $\dfrac{dy}{dx}+(1+2x)\,y=e^{-x^2}$ with integrating factor e^{x+x^2} is (2)

(A) $y\,e^{x+x^2}=\dfrac{e^{x+x^2}}{2}+C$ (B) $y\,e^{x+x^2}=e^{x^2}+C$

(C) $y\,e^{x+x^2}=e^{x}+C$ (D) none of these

38. The general solution of $\dfrac{dx}{dy}+\dfrac{1}{1+y^2}x=\dfrac{e^{-\tan^{-1}y}}{1+y^2}$ with integrating factor $e^{\tan^{-1}y}$ is (2)

(A) $x\,e^{\tan^{-1}y}=\tan^{-1}y+C$ (B) $y\,e^{\tan^{-1}y}=\tan^{-1}y+C$

(C) $e^{\tan^{-1}y}=\tan^{-1}y+C$ (D) none of these

39. The general solution of $\dfrac{dx}{dy}+(\sec y)\,x=\dfrac{2y\cos y}{1+\sin y}$ with integrating factor $(\sec y+\tan y)$ is (2)

(A) $y\,(\sec y+\tan y)=y^2+C$ (B) $x\,(\sec y+\tan y)=\dfrac{y^2}{2}+C$

(C) $x\,(\sec y+\tan y)=y^2+C$ (D) none of these

40. The general solution of $\dfrac{dx}{dy}-\dfrac{2}{y}x=y^2e^{-y}$ with integrating factor $\dfrac{1}{y^2}$ is (2)

(A) $x\dfrac{1}{y^2}=-e^{-y}+C$ (B) $x\dfrac{1}{y^2}=e^{-y}+C$ (C) $y\dfrac{1}{x^2}=-e^{-y}+C$ (D) none of these

ANSWERS

1. (B)	2. (C)	3. (D)	4. (A)	5. (D)	6. (D)	7. (A)	8. (A)
9. (C)	10. (B)	11. (D)	12. (A)	13. (B)	14. (D)	15. (A)	16. (C)
17. (A)	18. (B)	19. (D)	20. (C)	21. (C)	22. (A)	23. (A)	24. (A)
25. (B)	26. (C)	27. (D)	28. (A)	29. (B)	30. (D)	31. (B)	32. (C)
33. (B)	34. (A)	35. (A)	36. (B)	37. (C)	38. (A)	39. (C)	40. (A)

MULTIPLE CHOICE QUESTIONS

Orthogonal Trajectories :

1. The differential equation of orthogonal trajectories of family of straight lines $y = mx$ is (1)

(A) $\frac{dx}{dy} = -\frac{y}{x}$ (B) $\frac{dx}{dy} = -\frac{x}{y}$ (C) $\frac{dy}{dx} = \frac{y}{x}$ (D) $\frac{dy}{dx} = m$

2. If the family of curves is given by $x^2 + 2y^2 = c^2$ then the differential equation of orthogonal trajectories of family is (1)

(A) $x - 2y\frac{dy}{dx} = 0$ (B) $x + 2y\frac{dx}{dy} = 0$ (C) $x + 2y\frac{dy}{dx} = 0$ (D) $x - 2y\frac{dx}{dy} = 0$

3. The differential equation of orthogonal trajectories of family of curves $xy = c$ is (1)

(A) $x\frac{dx}{dy} + y = 0$ (B) $-x\frac{dx}{dy} + y = 0$ (C) $-x\frac{dx}{dy} - y = 0$ (D) $x\frac{dy}{dx} + y = 0$

4. If the family of curves is given by $y^2 = 4ax$ then the differential equation of orthogonal trajectories of family is (1)

(A) $2y\frac{dy}{dx} = 4a$ (B) $2y\frac{dy}{dx} = \frac{y^2}{x}$ (C) $-2y\frac{dx}{dy} = \frac{y^2}{x}$ (D) $2y\frac{dy}{dx} = \frac{x}{y^2}$

5. The differential equation of orthogonal trajectories of family of curves $2x^2 + y^2 = cx$ is (1)

(A) $4x + 2y\frac{dy}{dx} = \frac{2x^2 + y^2}{x}$ (B) $4x - 2y\frac{dx}{dy} = \frac{2x^2 + y^2}{x}$

(C) $4x + 2y\frac{dx}{dy} = \frac{2x^2 + y^2}{x}$ (D) none of these

6. The differential equation of orthogonal trajectories of family of curves $x^2 + cy^2 = 1$ is (1)

(A) $x - \left(\frac{1 - x^2}{y}\right)\frac{dx}{dy} = 0$ (B) $x + \left(\frac{1 - x^2}{y}\right)\frac{dy}{dx} = 0$

(C) $x + \left(\frac{1 - x^2}{y}\right)\frac{dx}{dy} = 0$ (D) none of these

7. The differential equation of orthogonal trajectories of family of curves $e^x + e^{-y} = c$ is (1)

(A) $e^x - e^{-y}\frac{dy}{dx} = 0$ (B) $e^x - e^{-y}\frac{dx}{dy} = 0$ (C) $e^x + e^{-y}\frac{dx}{dy} = 0$ (D) none of these

8. The differential equation of orthogonal trajectories of family of curves $r = a\cos\theta$ is (1)

(A) $r^2\dfrac{d\theta}{dr} = \tan\theta$ (B) $\dfrac{1}{r}\dfrac{d\theta}{dr} = -\tan\theta$ (C) $r\dfrac{d\theta}{dr} = -\tan\theta$ (D) $r\dfrac{d\theta}{dr} = \tan\theta$

9. The differential equation of orthogonal trajectories of family of curves $r = a\sin\theta$ is (1)

(A) $\dfrac{1}{r}\dfrac{d\theta}{dr} = \cot\theta$ (B) $r\dfrac{d\theta}{dr} = -\cot\theta$ (C) $r\dfrac{d\theta}{dr} = -\tan\theta$ (D) $r^2\dfrac{d\theta}{dr} = \tan\theta$

10. The differential equation of orthogonal trajectories of family of curves $r = a(1-\cos\theta)$ is (1)

(A) $-r\dfrac{d\theta}{dr} = \dfrac{\sin\theta}{1-\cos\theta}$ (B) $r\dfrac{d\theta}{dr} = \dfrac{\sin\theta}{1-\cos\theta}$

(C) $-r^2\dfrac{d\theta}{dr} = \dfrac{\sin\theta}{1-\cos\theta}$ (D) $-r\dfrac{d\theta}{dr} = \dfrac{1-\cos\theta}{\sin\theta}$

11. The differential equation of orthogonal trajectories of family of curves $r^2 = a\sin 2\theta$ is (1)

(A) $r\dfrac{d\theta}{dr} = \tan 2\theta$ (B) $r\dfrac{d\theta}{dr} = \cot 2\theta$ (C) $-r\dfrac{d\theta}{dr} = \cot 2\theta$ (D) $\dfrac{dr}{d\theta} = r\cot 2\theta$

12. The differential equation of orthogonal trajectories of family of curves $r^2 = a\cos 2\theta$ is (1)

(A) $-r^2\dfrac{d\theta}{dr} = \tan 2\theta$ (B) $r\dfrac{d\theta}{dr} = \cot 2\theta$ (C) $r\dfrac{d\theta}{dr} = \tan 2\theta$ (D) $\dfrac{dr}{d\theta} = -r\tan 2\theta$

13. The differential equation of orthogonal trajectories of family of curves $r = a\sec^2\dfrac{\theta}{2}$ is (1)

(A) $-r\dfrac{d\theta}{dr} = \tan\dfrac{\theta}{2}$ (B) $r\dfrac{d\theta}{dr} = \tan\dfrac{\theta}{2}$ (C) $-r\dfrac{d\theta}{dr} = \cot\dfrac{\theta}{2}$ (D) $\dfrac{dr}{d\theta} = 2\tan\dfrac{\theta}{2}$

14. The differential equation of orthogonal trajectories of family of curves $r = a\cos^2\theta$ is (1)

(A) $\dfrac{dr}{d\theta} = -\dfrac{r\sin 2\theta}{\cos^2\theta}$ (B) $-r\dfrac{d\theta}{dr} = \dfrac{\sin 2\theta}{\cos^2\theta}$ (C) $r\dfrac{d\theta}{dr} = \dfrac{\cos^2\theta}{\sin 2\theta}$ (D) $r\dfrac{d\theta}{dr} = \dfrac{\sin 2\theta}{\cos^2\theta}$

15. The differential equation of orthogonal trajectories of family of curves $r^2 = a\sin\theta$ is (1)

(A) $2r\dfrac{d\theta}{dr} = \cot\theta$ (B) $2r\dfrac{d\theta}{dr} = -\cot\theta$ (C) $2r\dfrac{d\theta}{dr} = -\tan\theta$ (D) $2\dfrac{d\theta}{dr} = -r^2\cot\theta$

16. If the differential equation of family of straight lines $y = mx$ is $\frac{dy}{dx} = \frac{y}{x}$ then its orthogonal trajectories is (2)

(A) $xy = k$ (B) $x^2 - y^2 = k^2$ (C) $y = kx$ (D) $x^2 + y^2 = k^2$

17. If the differential equation of family of rectangular hyperbola $xy = c$ is $x\frac{dy}{dx} = -y$ then its orthogonal trajectories is (2)

(A) $x^2 - y^2 = k^2$ (B) $x^2 + y^2 = k^2$ (C) $y^2 = kx$ (D) $xy = k_1$

18. Orthogonal trajectories of family of circles $x^2 + y^2 = c^2$ whose differential equation is $\frac{dy}{dx} = -\frac{x}{y}$, is equal to (2)

(A) $x^2 - y^2 = k^2$ (B) $y = kx$ (C) $y^2 = kx$ (D) $x^2 + y^2 = k^2$

19. If the differential equation of family of rectangular hyperbola $x^2 - y^2 = c^2$ is $\frac{dy}{dx} = \frac{x}{y}$, then its orthogonal trajectories is (2)

(A) $y^2 = kx$ (B) $x^2 + y^2 = k^2$ (C) $xy = k$ (D) $y = kx$

20. Orthogonal trajectories of family of curves $x^2 + 2y^2 = c^2$ whose differential equation is $\frac{dy}{dx} = -\frac{x}{2y}$, is (2)

(A) $x^2 = ky$ (B) $x^2 = \frac{k}{y}$ (C) $x^2 + 2y^2 = k^2$ (D) none of these

21. Orthogonal trajectories of family of curves $y^2 = 4ax$, whose differential equation is $\frac{dy}{dx} = \frac{y}{2x}$, is equal to (2)

(A) $x^2 + y^2 = k^2$ (B) $x^2 + 2y^2 = k^2$ (C) $y^2 = 4kx$ (D) $2x^2 + y^2 = k$

22. If the differential equation of family of curves $e^x + e^{-y} = c$ is $\frac{dy}{dx} = \frac{e^x}{e^{-y}}$, then its orthogonal trajectories is (2)

(A) $e^{-x} + e^{-y} = k$ (B) $e^{-x} - e^y = k$ (C) $e^x - e^{-y} = k$ (D) $e^x + e^{-y} = k$

23. If the differential equation of family of curves $e^y - e^{-x} = c$ is $\frac{dy}{dx} = -\frac{e^{-x}}{e^y}$, then its orthogonal trajectories is (2)

(A) $e^{-x} + e^{-y} = k$ (B) $e^x - e^{-y} = k$ (C) $e^x + e^{-y} = k$ (D) $e^y - e^{-x} = k$

24. Orthogonal trajectories of family of curves $x^2 + cy^2 = 1$ whose differential equation is $\frac{dy}{dx} = -\frac{xy}{1-x^2}$, is (2)

(A) $\log x + \frac{x^2}{2} = \frac{y^2}{2} + k$ (B) $\log x - \frac{x^2}{2} = \frac{y^2}{2} + k$ (C) $x^2 + y^2 = k$ (D) $x^2 + ky^2 = 1$

25. Orthogonal trajectories of family of curves $x^2 = ce^{x^2+y^2}$ whose differential equation is $\frac{dy}{dx} = \frac{1-x^2}{xy}$, is (2)

(A) $\log(1-x^2) - 2\log y = \log k$
(B) $2\log(1-x^2) - \log y = \log k$
(C) $\log(1-x^2) + 2\log y = \log k$
(D) $\log(1-x^2) - 2\log y = \log k + k$

26. Orthogonal trajectories of family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1$, λ is an arbitrary constant, whose differential equation is $x + \left(\frac{a^2-x^2}{y}\right)\frac{dy}{dx} = 0$, is (2)

(A) $\frac{y^2}{2} = -a^2\log x + \frac{x^2}{2} + k$
(B) $y^2 = b^2\log x - x^2 + k$
(C) $\frac{y^2}{2} = a^2\log x - \frac{x^2}{2} + k$
(D) $\frac{y^2}{2} = -a^2\log x + x^2 + k$

27. If the differential equation of family of curves $r = a(1-\cos\theta)$ is $\frac{dr}{d\theta} = r\cot\frac{\theta}{2}$ then its orthogonal trajectories is given by (2)

(A) $2\log\sec\frac{\theta}{2} = \log r + \log k$
(B) $2\log\cos\frac{\theta}{2} = \log r + \log k$
(C) $\frac{1}{2}\log\cos\frac{\theta}{2} = \log r + \log k$
(D) $\frac{1}{2}\log\sec\frac{\theta}{2} = \log r + \log k$

28. If the differential equation of family of curves $r = a\sec^2\frac{\theta}{2}$ is $\frac{dr}{d\theta} = r\tan\frac{\theta}{2}$ then its orthogonal trajectories is given by (2)

(A) $-2\log\cos\frac{\theta}{2} = \log r + \log k$
(B) $2\log\sin\frac{\theta}{2} = \log r + \log k$
(C) $-2\log\sin\frac{\theta}{2} = \log r + \log k$
(D) $2\log\cos\frac{\theta}{2} = \log r + \log k$

29. If the differential equation of family of curves $r = a\sin\theta$ is $\frac{dr}{d\theta} = r\cot\theta$ then its orthogonal trajectories is given by (2)

(A) $r = k\cos\theta$
(B) $r = k\sec\theta$
(C) $r = k\sin\theta$
(D) $\log\cos\theta = rk$

30. If the differential equation of family of curves $r = a\cos\theta$ is $\frac{dr}{d\theta} = -r\tan\theta$ then its orthogonal trajectories is given by (2)

(A) $\log r = -\operatorname{cosec}^2\theta + k$
(B) $r = k\cos\theta$
(C) $r = k\operatorname{cosec}\theta$
(D) $r = k\sin\theta$

31. If the differential equation of family of curves $r^2 = a \sin 2\theta$ is $\frac{dr}{d\theta} = r \cot 2\theta$ then its orthogonal trajectories is given by (2)

(A) $r^2 = \log \sec 2\theta + k$ (B) $r^2 = k \sin 2\theta$

(C) $r^2 = k \cos 2\theta$ (D) $\log r = -\frac{1}{2} \sec^2 2\theta + k$

32. If the differential equation of family of curves $r^2 = a \cos 2\theta$ is $\frac{dr}{d\theta} = -r \tan 2\theta$ then its orthogonal trajectories is given by (2)

(A) $\frac{1}{2} \log \cos 2\theta = \log r + \log k$ (B) $\frac{1}{2} \log \sin 2\theta = \log r + \log k$

(C) $\log \sin 2\theta = -r^2 + k$ (D) $\frac{1}{2} \log \sin 2\theta = -\log r + \log k$

33. If the differential equation of family of curves $r = a \cos^2 \theta$ is $\frac{dr}{d\theta} = -2r \tan \theta$ then its orthogonal trajectories is given by (2)

(A) $\frac{1}{2} \log \sin \theta = \log r + \log k$ (B) $\frac{1}{2} \log \sin \theta = -\log r + \log k$

(C) $\log \sin \theta = r + k$ (D) $\log \sec \theta = -\log r + \log k$

ANSWERS

1. (A)	2. (D)	3. (B)	4. (C)	5. (B)	6. (A)	7. (C)	8. (D)
9. (B)	10. (A)	11. (C)	12. (C)	13. (A)	14. (D)	15. (B)	16. (D)
17. (A)	18. (B)	19. (C)	20. (A)	21. (D)	22. (B)	23. (C)	24. (B)
25. (A)	26. (C)	27. (B)	28. (C)	29. (A)	30. (D)	31. (C)	32. (B)
33. (A)							

MULTIPLE CHOICE QUESTIONS

Newton's Law of Cooling :

1. Newton's law of cooling states that (1)

(A) the temperature of a body changes at the rate which is proportional to the temperatures of surrounding medium

(B) the temperature of a body changes at the rate which is inversely proportional to the difference in temperatures between that of surrounding medium and that of body itself

(C) the temperature of a body changes at the rate which is proportional to the sum of temperatures of surrounding medium and that of body itself

(D) the temperature of a body changes at the rate which is proportional to the difference in temperatures between that of surrounding medium and that of body itself

2. A metal ball is heated to a temperature of 100°C and at time $t = 0$ it is placed in water which is maintained at 40°C. By Newton's law of cooling the differential equation satisfied by temperature θ of metal ball at any time t is (1)

(A) $\frac{d\theta}{dt} = -k(\theta - 100)$ (B) $\frac{d\theta}{dt} = -k(\theta - 40)$ (C) $\frac{d\theta}{dt} = -k\theta$ (D) $\frac{d\theta}{dt} = -k\theta(\theta - 40)$

3. According to Newton's law of cooling, the rate at which a substance cools in moving air is proportional to the difference between the temperature of the substance and that of air. A substance initially at temperature 90°C is kept in moving air at temperature 26°C, the differential equation satisfied by temperature θ of substance at any time t is (1)

(A) $\frac{d\theta}{dt} = -k(\theta - 26)$ (B) $\frac{d\theta}{dt} = -k(\theta - 90)$ (C) $\frac{d\theta}{dt} = -k\theta$ (D) $\frac{d\theta}{dt} = -k(\theta - 64)$

4. Suppose a corpse at a temperature of 32°C arrives at mortuary where the temperature is kept at 10°C. Then by Newton's law of cooling the differential equation satisfied by temperature T of corpse t hours later is (1)

(A) $\frac{dT}{dt} = -kT(T - 10)$ (B) $\frac{dT}{dt} = -k(T - 32)$

(C) $\frac{dT}{dt} = -k(T - 10)$ (D) $\frac{dT}{dt} = -kT(T - 32)$

5. A thermometer is taken outdoors where the temperature is 0°C, from a room in which the temperature is 21°C and temperature drops to 10°C in 1 minute. Then by Newton's law of cooling the differential equation satisfied by temperature T at time t is (1)

(A) $\frac{dT}{dt} = -k(T-21)$ (B) $\frac{dT}{dt} = -kT$ (C) $\frac{dT}{dt} = kT$ (D) $\frac{dT}{dt} = -kT(T-21)$

6. If θ_0 is the temperature of the surrounding and θ is temperature of the body at any time t satisfies the differential equation $\frac{d\theta}{dt} = -k(\theta - \theta_0)$ then θ is given by (2)

(A) $\theta = \theta_0 e^{-kt}$ (B) $\theta = \theta_0 + Ae^{kt}$
(C) $\theta = -k(\theta_0 + Ae^{-kt})$ (D) $\theta = \theta_0 + Ae^{-kt}$

7. Suppose a corpse at a temperature of 32°C arrives at mortuary where the temperature is kept at 10°C. If the corpse cools to 27°C in 5 minutes. If the differential equation by Newton's law of cooling is $\frac{dT}{dt} = -(0.05)(T-10)$, then temperature T of corpse at any time t is given by (2)

(A) $T = 22e^{-0.05t}$ (B) $T = 10 + 22e^{0.05t}$
(C) $T = 10 + 22e^{-0.05t}$ (D) $T = 10 - 22e^{-0.05t}$

8. A thermometer is taken outdoors where the temperature is 0°C, from a room in which the temperature is 21°C and temperature drops to 10°C in 1 minute. If the differential equation by Newton's law of cooling is $\frac{dT}{dt} = -(0.7419)T$, then temperature T of thermometer at time t is given by

(A) $T = 21 + 11e^{-0.7419t}$ (B) $T = 21e^{0.7419t}$
(C) $T = 10 + 21e^{-0.7419t}$ (D) $T = 21e^{-0.7419t}$

9. A body originally at 80°C cools down to 60°C in 20 minutes in a room where the temperature is 40°C. The differential equation by Newton's law of cooling is $\frac{d\theta}{dt} = -k(\theta - 40)$, then the value of k is (2)

(A) $-\frac{1}{20}\log_e 2$ (B) $\frac{1}{20}\log_e 2$ (C) $20\log_e 2$ (D) $\log_e 2$

10. If the temperature of the body drops from 100°C to 60°C in 1 minute when the temperature of surrounding is 20°C satisfies the differential equation $\frac{d\theta}{dt} = -k(\theta - 20)$, then the value of k is (2)

(A) $\log_e 2$ (B) $-\log_e 2$ (C) $\log_e 4$ (D) $\log_e 8$

11. The temperature of the air is 30°C and the substance cools from 100°C to 70°C in 15 minutes. If differential equation by Newton's law of cooling is $\frac{d\theta}{dt} = -k(\theta - 30)$, then the value of k is (2)

(A) $\log_e \frac{7}{4}$ (B) $\frac{1}{15}\log_e \frac{4}{7}$ (C) $\frac{1}{15}\log_e \frac{7}{4}$ (D) $15\log_e \frac{7}{4}$

12. By Newton's law of cooling the differential equation of body originally at 80°C cools down to 60°C in 20 minutes in surrounding temperature of 40°C is $\frac{d\theta}{dt} = -(0.03465)(\theta - 40)$. The temperature of the body after 40 minutes is (2)

(A) 60°C (B) 50°C (C) 35°C (D) 85°C

13. A metal ball is heated to a temperature of 100°C and at time t = 0 it is placed in water which is maintained at 40°C. The temperature of the ball reduces to 60°C in 4 minutes. By Newton's law of cooling the differential equation is $\frac{d\theta}{dt} = -\left(\frac{1}{4}\log_e 3\right)$ $(\theta - 40)$. Then the time required to reduce the temperature of ball to 50°C is (2)

(A) 7.5 min (B) 3.5 min (C) 10 min (D) 6.5 min

14. A body at temperature 100°C is placed in a room whose temperature is 20°C and cools to 60°C in 5 minutes. By Newton's law of cooling the differential equation is $\frac{d\theta}{dt} = -\left(\frac{1}{5}\log_e 2\right)(\theta - 20)$. Then the temperature after 8 minutes is (2)

(A) 46.4°C (B) 65.4°C (C) 40.4°C (D) 20°C

15. A copper ball is heated to a temperature of 100°C and at time t = 0 it is placed in water which is maintained at 30°C. The temperature of the ball reduces to 70°C in 3 minutes. The differential equation by Newton's law of cooling is $\frac{d\theta}{dt} = -\left(\frac{1}{3}\log_e \frac{7}{4}\right)(\theta - 30)$. Then the time required to reduce the temperature of ball to 31°C is (2)

(A) 3 min (B) 7.78 min (C) 22.78 min (D) 15.78 min

ANSWERS

1. (D)	2. (B)	3. (A)	4. (C)	5. (B)	6. (D)	7. (C)	8. (D)
9. (B)	10. (A)	11. (C)	12. (B)	13. (D)	14. (A)	15. (C)	

MULTIPLE CHOICE QUESTIONS

Rectilinear Motion :

1. Rectilinear motion is a motion of body along a (1)

(A) straight line (B) circular path (C) parabolic path (D) none of these

2. According to D'Alembert's principle, algebraic sum of forces acting on a body along a given direction is equal to (1)

(A) velocity × acceleration (B) mass × velocity

(C) mass × displacement (D) mass × acceleration

3. A particle moving in a straight line with acceleration $k\left(x+\frac{a^4}{x^3}\right)$ directed towards the origin. The equation of motion is (1)

(A) $\frac{dv}{dx}=-k\left(x+\frac{a^4}{x^3}\right)$ (B) $v\frac{dv}{dx}=k\left(x+\frac{a^4}{x^3}\right)$

(C) $v\frac{dv}{dx}=-k\left(x+\frac{a^4}{x^3}\right)$ (D) $\frac{dv}{dx}=\left(x+\frac{a^4}{x^3}\right)$

4. A particle of mass m moves in a horizontal straight line OA with acceleration $\frac{k}{x^3}$ at a distance x and directed towards the origin O. Then the differential equation of motion is (1)

(A) $v\frac{dv}{dx}=\frac{k}{x^3}$ (B) $v\frac{dv}{dx}=-\frac{k}{x^3}$ (C) $\frac{dv}{dx}=-\frac{k}{x^3}$ (D) $\frac{dv}{dx}=\frac{k}{x^3}$

5. A body of mass m falling from rest is subjected to a force of gravity and air resistance proportional to square of velocity (kv^2). The equation of motion is (1)

(A) $m\frac{dv}{dx}=mg-kv^2$ (B) $mv\frac{dv}{dx}=mg+kv^2$

(C) $mv\frac{dv}{dx}=-kv^2$ (D) $mv\frac{dv}{dx}=mg-kv^2$

6. A particle is projected vertically upward with velocity v_1 and resistance of air produces retardation (kv^2) where v is velocity. The equation of motion is (1)

(A) $v\frac{dv}{dx}=-g-kv^2$ (B) $v\frac{dv}{dx}=-g+kv^2$ (C) $v\frac{dv}{dx}=-kv^2$ (D) $v\frac{dv}{dx}=g-kv^2$

7. A body starts moving from rest is opposed by a force per unit mass of value (cx) resistance per unit mass of value (bv^2) where v and x are velocity and displacement of body at that instant. The differential equation of motion is (1)

(A) $mv\frac{dv}{dx}=-cx-bv^2$ (B) $v\frac{dv}{dx}=cx+bv^2$

(C) $v\frac{dv}{dx}=-cx-bv^2$ (D) $\frac{dv}{dx}=-cx-bv^2$

8. A body of mass m falls from rest under gravity, in a fluid whose resistance to motion at any time t is mk times its velocity where k is constant. The differential equation of motion is (1)

(A) $\frac{dv}{dt}=-g-kv$ (B) $\frac{dv}{dt}=g-kv$ (C) $\frac{dv}{dt}=g+kv$ (D) $\frac{dv}{dt}=mg-mkv$

9. A particle of mass m is projected vertically upward with velocity v, assuming the air resistance k times its velocity where k is constant. The differential equation of motion is (1)

(A) $\frac{dv}{dt}=mg-kv$ (B) $m\frac{dv}{dt}=-mg+kv$

(C) $m\frac{dv}{dt}=-kv$ (D) $m\frac{dv}{dt}=-mg-kv$

10. Assuming that the resistance to movement of a ship through water in the form of $(a^2 + b^2v^2)$ where v is the velocity, a and b are constants. The differential equation for retardation of the ship moving with engine stopped is (1)

(A) $m\frac{dv}{dt} = -(a^2 + b^2v^2)^2$ (B) $m\frac{dv}{dt} = +(a^2 + b^2v^2)$

(C) $m\frac{dv}{dt} = -(a^2 + b^2v^2)$ (D) $m\frac{dv}{dx} = -(a^2 + b^2v^2)$

11. Differential equation of motion of a body of mass m falls from rest under gravity in a fluid whose resistance to motion at any time t is mk times its velocity where k is constant is $\frac{dv}{dt} = g - kv$ then the relation between velocity and time t is (2)

(A) $t = \frac{1}{k}\log\frac{g-kv}{g}$ (B) $t = \frac{1}{k}\log\frac{g}{g-kv}$

(C) $t = \frac{1}{k}\log\frac{g}{g+kv}$ (D) $t = -\frac{1}{k}\log\frac{1}{g-kv}$

12. A body of mass m falling from rest is subjected to the force of gravity and air resistance proportional to square of velocity (kv^2) satisfies the differential equation $mv\frac{dv}{dx} = k(a^2 - v^2)$ where $ka^2 = mg$, then the relation between velocity and displacement is (2)

(A) $\frac{2kx}{m} = \log\frac{a^2}{a^2 - v^2}$ (B) $\frac{2kx}{m} = \log\frac{a^2 - v^2}{a^2}$

(C) $2kx = \log\frac{1}{a^2 - v^2}$ (D) $\frac{x}{m} = \log\frac{a^2}{a^2 - v^2}$

13. A vehicle starts from rest and its acceleration is given by $\frac{dv}{dt} = k\left(1 - \frac{t}{T}\right)$ where k and T are constant. Then the velocity v in terms of time t is given by (2)

(A) $v = k\left(t - \frac{t^2}{2}\right)$ (B) $v = k\left(t - \frac{t^2}{T}\right)$ (C) $v = k\left(\frac{t^2}{2} - \frac{t^3}{3T}\right)$ (D) $v = k\left(t - \frac{t^2}{2T}\right)$

14. A particle of unit mass moves in a horizontal straight line OA with an acceleration $\frac{k}{r^3}$ at a distance r and directed towards O. If initially the particle was at rest at r = a and equation of motion is $v\frac{dv}{dr} = -\frac{k}{r^3}$ then the relation between r, v is (2)

(A) $v^2 = k\left(\frac{1}{r^2} + \frac{1}{a^2}\right)$ (B) $v^2 = k\left(\frac{1}{a^2} - \frac{1}{r^2}\right)$ (C) $v^2 = k\left(\frac{1}{r^2} - \frac{1}{a^2}\right)$ (D) $v^2 = k\left(\frac{1}{r^4} - \frac{1}{a^2}\right)$

15. A particle of mass m is projected upward with velocity V. Assuming the air resistance k times its velocity and equation of motion is $m\frac{dv}{dt} = -mg - kv$ then the relation between velocity v and time t is (2)

(A) $t = \frac{m}{k}\log\left(\frac{mg + kV}{mg + kv}\right)$ (B) $t = \frac{m}{k}\log\left(\frac{mg + kv}{mg + kV}\right)$

(C) $t = m\log\left(\frac{mg + kV}{mg + kv}\right)$ (D) $t = \log\left(\frac{mg + kv}{mg + kV}\right)$

16. A body of mass m falls from rest under gravity in a fluid whose resistance to motion at any instant is mkv where k is constant. The differential equation of motion is $\frac{dv}{dt} = g - kv$ then the terminal velocity is (2)

(A) $\frac{k}{g}$ (B) $\frac{g}{k}$ (C) $-\frac{g}{k}$ (D) none of these

17. A bullet is fired into a sand tank, its retardation is proportional to the square root of its velocity. The differential equation of motion is $\frac{dv}{dt} = -k\sqrt{v}$. If v_0 is initial velocity then the relation between velocity v and time t is (2)

(A) $\sqrt{v} = -t + \sqrt{v_0}$ (B) $2\sqrt{v} = -kt$

(C) $\sqrt{v} = -kt + \sqrt{v_0}$ (D) $2\sqrt{v} = -kt + 2\sqrt{v_0}$

18. A particle moving in a straight line with acceleration $k\left(x + \frac{a^4}{x^3}\right)$ directed towards the origin. the equation of motion is $v\frac{dv}{dx} = -k\left(x + \frac{a^4}{x^3}\right)$. If it starts from rest at a distance x = a from the origin then the relation between velocity v and displacement x is (2)

(A) $\frac{v^2}{2} = k\left(\frac{x^2}{2} + \frac{a^4}{2x^2}\right)$ (B) $\frac{v^2}{2} = -k\left(\frac{x^2}{2} - \frac{a^4}{2x^2}\right)$

(C) $\frac{v^2}{2} = -k\left(\frac{x^2}{2} - \frac{a^4}{2x^2}\right) + \frac{a^2}{2}$ (D) $\frac{v^2}{2} = -k\left(\frac{x^2}{2} - \frac{3a^4}{x^4}\right)$

ANSWERS

1. (A)	2. (D)	3. (C)	4. (B)	5. (D)	6. (A)	7. (C)	8. (B)
9. (D)	10. (C)	11. (B)	12. (A)	13. (D)	14. (C)	15. (A)	16. (B)
17. (D)	18. (B)						

MULTIPLE CHOICE QUESTIONS

Applications to Electrical Circuits :

1. A circuit containing resistance R and inductance L in series with voltage source E. By Kirchhoff's voltage law, differential equation for current i is (1)

   (A) $Li + R\frac{di}{dt} = E$ (B) $L\frac{di}{dt} + Ri = E$ (C) $L\frac{di}{dt} + Ri = 0$ (D) $L\frac{di}{dt} + \frac{q}{C} = E$

2. A circuit containing resistance R and capacitance C in series with voltage source E. By Kirchhoff's voltage law, differential equation for current $i = \frac{dq}{dt}$ is (1)

   (A) $L\frac{di}{dt} + \frac{q}{C} = E$ (B) $R\frac{dq}{dt} + \frac{q}{C} = 0$ (C) $L\frac{di}{dt} + Ri = 0$ (D) $R\frac{dq}{dt} + \frac{q}{C} = E$

3. A circuit containing inductance L, capacitance C in series without applied electromotive force. By Kirchhoff's voltage law, differential equation for current i is (1)

   (A) $L\frac{di}{dt} + \frac{q}{C} = 0$ (C) $L\frac{di}{dt} + Ri = 0$ (C) $L\frac{di}{dt} + Ri = E$ (D) $L\frac{di}{dt} + \frac{q}{C} = E$

4. A circuit containing inductance L, capacitance C in series with applied electromotive force E. By Kirchhoff's voltage law, differential equation for current i is (1)

   (A) $L\frac{di}{dt} + Ri = E$ (B) $L\frac{di}{dt} + Ri = 0$ (C) $L\frac{di}{dt} + \frac{q}{C} = E$ (D) $L\frac{di}{dt} + \frac{q}{C} = 0$

5. The differential equation for the current in an electric circuit containing resistance R and inductance L in series with voltage source E sin ωt is (1)

   (A) $L\frac{di}{dt} + \frac{q}{C} = E$ (B) $Li + R\frac{di}{dt} = E \sin \omega t$

   (C) $L\frac{di}{dt} + Ri = 0$ (D) $L\frac{di}{dt} + Ri = E \sin \omega t$

6. In a circuit containing resistance R and inductance L in series with constant voltage source E, current i is given by $i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$, then maximum current i_{max} is (1)

   (A) $\frac{E}{R}$ (B) $\frac{R}{E}$ (C) ER (D) 0

7. The differential equation for the current i in an electric circuit containing resistance 100 ohm and an inductance of 0.5 henry connected in series with battery of 20 volts is (1)

   (A) $0.5\frac{di}{dt} + 100i = 0$ (B) $0.5\frac{di}{dt} + 100i = 20$

   (C) $100\frac{di}{dt} + 0.5i = 20$ (D) $100\frac{di}{dt} + 0.5R = 0$

8. The differential equation for the current i in an electric circuit containing resistance R = 250 ohm and an inductance of L = 640 henry in series with an electromotive force E = 500 volts is (1)

(A) $640\frac{di}{dt}+250i=0$ (B) $250\frac{di}{dt}+640i=500$

(C) $640\frac{di}{dt}+250i=500$ (D) $250\frac{di}{dt}+640i=0$

9. A capacitor C = 0.01 farad in series with resistor R = 20 ohms is charged from battery E = 10 volts. If initially capacitor is completely discharged then differential equation for charge q(t) is given by (1)

(A) $20\frac{dq}{dt}+\frac{q}{0.01}=0;\ q(0)=0$ (B) $20\frac{dq}{dt}+0.01q=10;\ q(0)=0$

(C) $20\frac{dq}{dt}+\frac{q}{0.01}=10;\ q(0)=0$ (D) $20\frac{dq}{dt}+0.01q=0;\ q(0)=0$

10. In a circuit containing resistance R and inductance L in series with constant e.m.f. E, the current i is given by $i=\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$, then the time required to build current half of its theoretical maximum is (2)

(A) $\frac{L}{R\log 2}$ (B) $\frac{L\log 2}{R}$ (C) $\frac{R\log 2}{L}$ (D) 0

11. In a circuit containing resistance R and inductance L in series with constant e.m.f. E, the current i is given by $i=\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$, then the time required before current reaches its 90% of maximum value is (2)

(A) 0 (B) $\frac{L}{R\log 10}$ (C) $\frac{R\log 10}{L}$ (D) $\frac{L\log 10}{R}$

12. If the differential equation for current in an electric circuit containing resistance R and inductance L in series with constant e.m.f. E, the current i is $L\frac{di}{dt}+Ri=E$, then the current at any time t is given by (2)

(A) $i=\frac{E}{R}-Ae^{-\frac{R}{L}t}$; A is arbitrary constant

(B) $i=\frac{E}{R}+Ae^{-\frac{R}{L}t}$; A is arbitrary constant

(C) $i=\frac{E}{R}+Ae^{\frac{R}{L}t}$; A is arbitrary constant

(D) $i=\frac{E}{R}+e^{-\frac{R}{L}t}$

13. A charge q on the plate of condenser of capacity C charge through a resistance R by steady voltage V satisfies differential equation $R\frac{dq}{dt}+\frac{q}{C}=V$, then charge q at any time t is (2)

(A) $q = CV + Ae^{-\frac{1}{RC}t}$; A is arbitrary constant

(B) $q = CV - Ae^{\frac{1}{RC}t}$; A is arbitrary constant

(C) $q = C + Ae^{\frac{1}{RC}t}$; A is arbitrary constant

(D) $q = CV + e^{\frac{1}{RC}t}$

14. The charge q on the plate of condenser of capacity C charge through a resistance R by steady voltage V is given by $q = CV\left(1-e^{-\frac{1}{RC}t}\right)$ then current flowing through the plate is (2)

(A) $i=\frac{V}{R}e^{-\frac{R}{L}t}$ (B) $i=\frac{V}{R}e^{\frac{1}{RC}t}$

(C) $i=\frac{V}{R}e^{-\frac{1}{RC}t}$ (D) $i = CV\left(1-e^{-\frac{1}{RC}t}\right)$

15. A resistance R = 100 ohms, an inductance L = 0.5 henry are connected in series with a battery of 20 volts. The differential equation for the current i is $0.5\frac{di}{dt}+100i=20$, then current i at any time t is (2)

(A) Ae^{-200t}; A is arbitrary constant (B) $\frac{1}{5}+Ae^{200t}$; A is arbitrary constant

(C) $2+Ae^{-200t}$; A is arbitrary constant (D) $\frac{1}{5}+Ae^{-200t}$; A is arbitrary constant

16. If an R-C circuit, charge q as function of time t is $q = e^{-3t} - e^{-6t}$, then time required for maximum charge on capacitor is (2)

(A) 3 log 2 (B) $-\frac{1}{3}\log 2$ (C) $\frac{1}{3}\log 2$ (D) $\frac{1}{2}\log 3$

17. A circuit containing resistance R and inductance L in series with voltage source E. The differential equation for current i is $L\frac{di}{dt}+Ri=E$. Given L = 640 H, R = 250 Ω and E = 500 volts then integrating factor of differential equation is (2)

(A) $e^{\frac{64}{25}t}$ (B) $e^{\frac{25}{64}t}$ (C) $e^{-\frac{25}{64}t}$ (D) e^{250t}

ANSWERS

1. (B)	2. (D)	3. (A)	4. (C)	5. (D)	6. (A)	7. (B)	8. (C)
9. (C)	10 (B)	11. (D)	12. (B)	13. (A)	14. (C)	15. (D)	16. (C)
17. (B)							

Simple Harmonic Motion :

1. If a particle moves on a straight line so that the force acting on it is always directed towards a fixed point on the line and proportional to its distance from the point then the particle is said to be in (1)

(A) simple harmonic motion
(B) motion under the gravity
(C) periodic motion
(D) circular motion

Ans. (A)

2. A particle executes simple harmonic motion then the differential equation of motion is (1)

(A) $\frac{d^2x}{dt^2} = -\omega^2 x$
(B) $\frac{d^2x}{dt^2} = \omega^2 x$
(C) $\frac{d^2x}{dt^2} = -\frac{\omega^2}{x}$
(D) $\frac{dx}{dt} = -\omega^2 x$

Ans. (A)

MULTIPLE CHOICE QUESTIONS

Chemical Engineering Problems :

1. A tank contains 10,000 litres of brine in which 200 kg salt dissolved. Fresh water runs into the tank at the rate of 100 litres per minute and the mixture kept uniform by stirring, runs out at the same rate. If Q be the total amount of salt at any time t then the governing differential equation is (2)

   (A) $\frac{dQ}{dt}=200-\frac{Q}{100}$ (B) $\frac{dQ}{dt}=-\frac{Q}{10000}$ (C) $\frac{dQ}{dt}=-\frac{Q}{100}$ (D) $\frac{dQ}{dt}=\frac{Q}{100}$

2. A tank initially contains 50 litres of fresh water. Brine containing 2 grams per litre of salt, flows into the tank at the rate of 2 litres per minute and the mixture kept uniform by stirring, runs out at the same rate. If Q be the total amount of salt at any time t then the differential equation in terms of Q and t is (2)

   (A) $\frac{dQ}{dt}=4-\frac{Q}{50}$ (B) $\frac{dQ}{dt}=4-\frac{Q}{25}$ (C) $\frac{dQ}{dt}=2-\frac{Q}{50}$ (D) $\frac{dQ}{dt}=2-\frac{Q}{25}$

3. A tank initially contains 100 litres of fresh water. 2 litres of brine each containing 1 gram of dissolved salt, runs into the tank per minute and the mixture kept uniform by stirring, runs out at the rate of 1 litre per minute. Let Q be the quantity of salt present at any time t then $\frac{dQ}{dt}$ the rate at which salt content changing is (2)

   (A) $\frac{dQ}{dt}=1-\frac{Q}{100+t}$ (B) $\frac{dQ}{dt}=2-\frac{Q}{100}$ (C) $\frac{dQ}{dt}=\frac{Q}{100+t}$ (D) $\frac{dQ}{dt}=2-\frac{Q}{100+t}$

4. A tank contains 1000 litres of brine in which 20 kg salt dissolved. Brine containing 0.1 kg per litre of salt, runs into the tank at the rate of 40 litres per minute and the mixture, assumed to be kept uniform by stirring, runs out at the rate of 30 litres per minute. Assuming that tank is sufficiently large to avoid overflow, the governing differential equation for rate at which the salt content changing $\frac{dQ}{dt}$ at any time t is (2)

   (A) $\frac{dQ}{dt}=4-30\frac{Q}{1000+10t}$ (B) $\frac{dQ}{dt}=4-30\frac{Q}{1000}$

   (C) $\frac{dQ}{dt}=30\frac{Q}{1000+10t}$ (D) $\frac{dQ}{dt}=4-\frac{Q}{1000+10t}$

5. A tank initially contains 5000 litres of fresh water. Salt water containing 100 grams per litre of salt, flows into it at the rate of 10 litres per minute and the mixture kept uniform by stirring, runs out at the same rate. If Q be the total amount of salt at any time t then the differential equation relating Q and t is (2)

   (A) $\frac{dQ}{dt}=100-\frac{Q}{500}$ (B) $\frac{dQ}{dt}=1000-\frac{Q}{500}$

   (C) $\frac{dQ}{dt}=1000-\frac{Q}{5000}$ (D) $\frac{dQ}{dt}=-\frac{Q}{500}$

6. A tank contains 10,000 litres of brine in which 200 kg of salt are dissolved. Fresh water runs into the tank at the rate of 100 litres per minute and the mixture kept uniform by stirring, runs out at the same rate. If governing differential equation is $\frac{dQ}{dt} = -\frac{Q}{100}$, the amount of salt Q at any time t is (2)

(A) $\log Q = -\frac{\log t}{100} + \log 200$
(B) $\log Q = -\frac{t}{100}$
(C) $\log Q = -\frac{t}{100} + \log 200$
(D) $\log Q = -\frac{t}{100} - \log 200$

7. A tank initially contains 50 litres of fresh water. Brine containing 2 grams per litre of salt, flows into the tank at the rate of 2 litres per minute and the mixture kept uniform by stirring, runs out at the same rate. The differential equation in terms of Q and t is $\frac{dQ}{dt} = 4 - \frac{Q}{25}$. The total amount of salt Q at any time t is (2)

(A) $t = -25 \log_e (100 - Q) + 25 \log_e 100$
(B) $t = 25 \log_e (100 - Q) - 25 \log_e 100$
(C) $t = -\log_e (100 - Q) + \log_e 10$
(D) none of these

8. A tank initially contains 500 litres of fresh water. Salt water containing 100 grams per litre of salt, flows into it at the rate of 10 litres per minute and the mixture kept uniform by stirring, runs out at the same rate. The differential equation in terms of Q and t is $\frac{dQ}{dt} = 1000 - \frac{Q}{500}$. The amount of salt Q at any time t is (2)

(A) $t = -\log_e (500000 - Q) + k$
(B) $t = 500 \log_e (500000 - Q) + k$
(C) $t = -500 \log_e (5000 - Q) + k$
(D) $t = -500 \log_e (500000 - Q) + k$

ANSWERS

1. (C)	2. (B)	3. (D)	4. (A)	5. (B)	6. (C)	7. (A)	8. (D)

MULTIPLE CHOICE QUESTIONS

One Dimensional Conduction of Heat :

1. Fourier's law of heat conduction states that, the quantity of heat flow across an area A cm^2 is (2)

(A) proportional to the product of area A and temperature gradient $\frac{dT}{dx}$

(B) inversely proportional to the product of area A and temperature gradient $\frac{dT}{dx}$

(C) equal to sum of area A and temperature gradient $\frac{dT}{dx}$

(D) equal to difference of area A and temperature gradient $\frac{dT}{dx}$

2. If q be the quantity of heat that flows across an area A cm^2 and thickness δx in one second where the difference of temperature at the faces is δT, then by Fourier's law of heat conduction (2)

(A) $q = -k\left(A - \frac{dT}{dx}\right)$, where k is thermal conductivity

(B) $q = kA\frac{dT}{dx}$, where k is thermal conductivity

(C) $q = -k\left(A + \frac{dT}{dx}\right)$, where k is thermal conductivity

(D) $q = -kA\frac{dT}{dx}$, where k is thermal conductivity

3. The differential equation for steady state heat loss per unit time from a unit length of pipe with thermal conductivity k, radius r_0 carrying steam at temperature T_0, if the pipe is covered with insulation of thickness w, the outer surface of which remains at the constant temperature T_1, is (2)

(A) $Q = k(2\pi r)\frac{dT}{dr}$

(B) $Q = -k(2\pi r)\frac{dT}{dr}$

(C) $Q = -k(2\pi r^2)\frac{dT}{dr}$

(D) $Q = -k(\pi r^2)\frac{dT}{dr}$

4. The differential equation for steady state heat loss per unit time from a spherical shell with thermal conductivity k radius r_0 carrying steam at temperature T_0, if the spherical shell is covered with insulation of thickness w, the outer surface of which remains at the constant temperature T_1, is (2)

(A) $Q = -k\,(2\pi r)\dfrac{dT}{dr}$ (B) $Q = k\,(2\pi r)\dfrac{dT}{dr}$

(C) $Q = -k\,(4\pi r^2)\dfrac{dT}{dr}$ (D) $Q = -k\,(\pi r^2)\dfrac{dT}{dr}$

5. The differential equation for steady state heat loss Q per unit time from a unit length of pipe with thermal conductivity k, radius r_0 carrying steam at temperature T_0, if the pipe is covered with insulation of thickness W, the outer surface of which remains at the constant temperature T_1, is $Q = -k\,(2\pi r)\dfrac{dT}{dr}$. Then the temperature T of surface of pipe of radius r is (2)

(A) $T = \dfrac{Q}{2\pi k}\dfrac{1}{r} + C$ (B) $T = \dfrac{Q}{2\pi k}\log r + C$

(C) $T = -\dfrac{Q}{2\pi k}\dfrac{1}{r} + C$ (D) $T = -\dfrac{Q}{2\pi k}\log r + C$

6. The differential equation for steady state heat loss Q per unit time from a spherical shell with thermal conductivity k, radius r_0 carrying steam at temperature T_0, if the spherical shell is covered with insulation of thickness w, the outer surface of which remains at the constant temperature T_1, is $Q = -k\,(4\pi r^2)\dfrac{dT}{dr}$. Then the temperature T of spherical shell of radius r is (2)

(A) $T = -\dfrac{Q}{4\pi k}\dfrac{1}{r^2} + C$ (B) $T = \dfrac{Q}{4\pi k}\dfrac{1}{r} + C$ (C) $T = -\dfrac{Q}{4\pi k}\dfrac{1}{r} + C$ (D) $T = -\dfrac{Q}{2\pi k}\dfrac{1}{r^3} + C$

7. A pipe 20 cm in diameter contains steam at 150°C and is protected with covering 5 cm thick for which k = 0.0025. If the temperature of the outer surface of the covering is 40°C and differential equation of conduction of heat is $dT = -\dfrac{Q}{2\pi k}\dfrac{dx}{x}$. The amount of heat loss Q is (2)

(A) $\dfrac{110\,(2\pi k)}{\log(1.5)}$ (B) $\dfrac{\log(1.5)}{110\,(2\pi k)}$ (C) $-\dfrac{110\,(2\pi k)}{\log(1.5)}$ (D) $\dfrac{110}{\log(1.5)}$

8. A long hollow pipe has an inner diameter of 10 cm and outer diameter of 20 cm. The inner surface is kept at 200°C and outer surface at 50°C. The thermal conductivity k = 0.12. The differential equation of conduction of heat is $dT = -\dfrac{Q}{2\pi k}\dfrac{dx}{x}$. Then the amount of heat loss Q cal/sec is (2)

(A) $-\dfrac{150\,(2\pi k)}{\log 2}$ (B) $\dfrac{\log 2}{150\,(2\pi k)}$ (C) $\dfrac{150\,(2\pi k)}{\log 2}$ (D) $\dfrac{(2\pi k)}{\log 2}$

9. A steam pipe 20 cm in diameter is protected with covering 6 cm thick for which thermal conductivity k = 0.0003 in steady state. The inner surface of the pipe is at 200°C and outer surface of the covering is at 30°C and differential equation of conduction of heat is $dT = -\frac{Q}{2\pi k}\frac{dx}{x}$. The amount of heat loss Q is (2)

(A) $\frac{170\,(2\pi k)}{\log(1.6)}$ (B) $-\frac{170\,(2\pi k)}{\log(1.6)}$ (C) $\frac{\log(1.6)}{170\,(2\pi k)}$ (D) $\frac{170}{\log(1.6)}$

10. A pipe 10 cm in diameter contains steam at 100°C. It is protected with asbestos 5 cm thick for which k = 0.0006 and outer surface is at 30°C. The differential equation of conduction of heat is $dT = -\frac{Q}{2\pi k}\frac{dx}{x}$. The amount of heat loss Q is (2)

(A) $\frac{\log 2}{70\,(2\pi k)}$ (B) $\frac{70\,(2\pi k)}{\log 2}$ (C) $-\frac{70\,(2\pi k)}{\log 2}$ (D) $\frac{(2\pi k)}{\log 2}$

ANSWERS

1. (A)	2. (D)	3. (B)	4. (C)	5. (D)	6. (B)	7. (A)	8. (C)
9. (A)	10. (B)						

Miscellaneous Examples :

1. In a certain culture of bacteria, the rate of increase is proportional to the number present. If y denote the number of bacteria at time t hours then the governing differential equation is (1)

(A) $\frac{dy}{dt} = ky$ (B) $\frac{dy}{dt} = -ky$ (C) $\frac{dy}{dt} = \frac{k}{y}$ (D) $\frac{dy}{dt} = ky^2$

2. The differential equation of the population model for natural growth of bacteria is $\frac{dy}{dt} = ky$. The general solution of the equation is (1)

(A) $y = c \log kt$ (B) $ye^{kt} = ct$ (C) $y = ce^{kt}$ (D) $y = ce^{-kt}$

3. The amount x of a substance present in certain chemical reaction at time t is given by $\frac{dx}{dt} + \frac{1}{10}x = 2 - (1.5)\, e^{-\frac{1}{10}t}$, then the amount x of substance present at time t is (1)

(A) $x = -\frac{3}{2}te^{-\frac{1}{10}t} + Ce^{-\frac{1}{10}t}$ (B) $x = 20 + \frac{3}{2}te^{-\frac{1}{10}t} - Ce^{-\frac{1}{10}t}$

(C) $x = 20 - \frac{3}{2}t + C$ (D) $x = 20 - \frac{3}{2}te^{-\frac{1}{10}t} + Ce^{-\frac{1}{10}t}$

4. Biotransformation of an organic compound having concentration x can be modeled using an ordinary differential equation $\frac{dx}{dt} + kx^2 = 0$ where k is reaction rate constant. If $x = a$ at $t = 0$, the solution of equation is (2)

(A) $x = ae^{-kt}$ (B) $\frac{1}{x} = \frac{1}{a} + kt$ (C) $x = a\,(1 - e^{-kt})$ (D) $x = a + kt$

ANSWERS

1. (A)	2. (C)	3. (D)	4. (B)

MULTIPLE CHOICE QUESTIONS

Fourier Series and Harmonic Analysis :

1. A function f(x) is said to be periodic of period T if (1)

   (A) $f(x+T) = f(x)$ for all x
   (B) $f(x+T) = f(T)$ for all x
   (C) $f(-x) = f(x)$ for all x
   (D) $f(-x) = -f(x)$ for all x

2. If $f(x + nT) = f(x)$ where n is any integer then the fundamental period of f(x) is (1)

   (A) 2T
   (B) $\frac{T}{2}$
   (C) T
   (D) 3T

3. If f(x) is a periodic function with period T then f(ax), $a \neq 0$ is periodic function with fundamental period (1)

   (A) T
   (B) $\frac{T}{a}$
   (C) aT
   (D) π

4. Fundamental period of sin 2x is sin 2(180) (1)

   (A) $\frac{\pi}{4}$
   (B) $\frac{\pi}{2}$
   (C) 2π
   (D) π

5. Fundamental period of cos 2x is (1)

   (A) $\frac{\pi}{4}$
   (B) $\frac{\pi}{2}$
   (C) π
   (D) 2π

6. Fundamental period of tan 3x is (1)

   (A) $\frac{\pi}{2}$
   (B) $\frac{\pi}{3}$
   (C) π
   (D) $\frac{\pi}{4}$

7. Fourier series representation of periodic function f(x) with period 2π which satisfies the Dirichlet's conditions is (1)

(A) $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ (B) $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\pi x + b_n \sin n\pi x)$

(C) $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx)(b_n \sin nx)$ (D) $\frac{a_0}{2} + (a_n \cos nx + b_n \sin nx)$

8. Fourier series representation of periodic function f(x) with period 2L which satisfies the Dirichlet's conditions is (1)

2L

(a) $\frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(n\pi x) + b_n \sin(n\pi x)]$ (B) $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_1 \cos\left(\frac{n\pi x}{L}\right) + b_1 \sin\left(\frac{n\pi x}{L}\right)\right]$

(C) $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{L}\right) \times b_n \sin\left(\frac{n\pi x}{L}\right)\right]$ (D) $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right]$

9. If f(x) is periodic function with period 2L defined in the interval C to C + 2L then Fourier coefficient a_0 is (1)

(A) $\int_C^{C+2L} f(x)\,dx$ (B) $\frac{1}{L}\int_C^{C+2L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx$

(C) $\frac{1}{L}\int_C^{C+2L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx$ (D) $\frac{1}{L}\int_C^{C+2L} f(x)\,dx$

10. If f(x) is periodic function with period 2L defined in the interval C to C + 2L then Fourier coefficient a_n is (1)

(A) $\int_C^{C+2L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx$ (B) $\frac{1}{L}\int_C^{C+2L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx$

(C) $\frac{1}{L}\int_C^{C+2L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx$ (D) $\frac{1}{L}\int_C^{C+2L} f(x)\,dx$

11. If f(x) is periodic function with period 2L defined in the interval C to C + 2L then Fourier coefficient b_n is (1)

(A) $\int_C^{C+2L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx$ (B) $\frac{1}{L}\int_C^{C+2L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx$

(C) $\frac{1}{L}\int_C^{C+2L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx$ (D) $\frac{1}{L}\int_C^{C+2L} f(x)\,dx$

12. A function f(x) is said to be even if (1)

(A) $f(-x) = f(x)$ (B) $f(-x) = -f(x)$ (C) $f(x + 2\pi) = f(x)$ (D) $f(-x) = [f(x)]^2$

13. A function f(x) is said to be odd if (1)

(A) $f(-x) = f(x)$ (B) $f(-x) = -f(x)$ (C) $f(x + 2\pi) = f(x)$ (D) $f(-x) = [f(x)]^2$

14. Which of the following is an odd function ? (1)

(A) $\sin x$ (B) $e^x + e^{-x}$ (C) $e^{|x|}$ (D) $\pi^2 - x^2$

15. Which of the following is an even function ? (1)

(A) $\sin x$ (B) $e^x - e^{-x}$ (C) $x \cos x$ (D) $\cos x$

16. Which of the following is neither an even function nor an odd function ? (1)

(A) $x \sin x$ (B) x^2 (C) e^{-x} (D) $x \cos x$

17. For an even function f(x) defined in the interval $-\pi \le x \le \pi$ and $f(x + 2\pi) = f(x)$ the Fourier series is (1)

(A) $\sum_{n=1}^{\infty} b_n \sin nx$ (B) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}$

(C) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$ (D) $\sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$

18. For an odd function f(x) defined in the interval $-\pi \le x \le \pi$ and $f(x + 2\pi) = f(x)$ the Fourier series is (1)

(A) $\sum_{n=1}^{\infty} b_n \sin nx$ (B) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}$

(C) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$ (D) $\sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$

19. Fourier coefficients for an even function f(x) defined in the interval $-\pi \le x \le \pi$ and $f(x + 2\pi) = f(x)$ are (1)

(A) $a_0 = 0, a_n = 0, \; b_n = \frac{2}{\pi}\int_0^{\pi} f(x) \sin nx dx$

(B) $a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\, dx, \; a_n = \frac{2}{\pi}\int_0^{\pi} f(x) \cos nx\, dx, \; b_n = 0$

(C) $a_0 = 0, a_n = 0, b_n = 0$

(D) $a_0 = 0, \; a_n = 0, \; b_n = \frac{2}{\pi}\int_0^{\pi} f(x) \cos nx\, dx$

20. Fourier coefficients for an odd function f(x) defined in the interval $-\pi \le x \le \pi$ and $f(x + 2\pi) = f(x)$ are (1)

(A) $a_0 = 0,\ a_n = 0,\ b_n = \frac{2}{\pi}\int_0^\pi f(x)\cos nx\,dx$

(B) $a_0 = \frac{2}{\pi}\int_0^\pi f(x)\,dx,\ a_n = \frac{2}{\pi}\int_0^\pi f(x)\cos nx\,dx,\ b_n = 0$

(C) $a_0 = 0, a_n = 0, b_n = 0$

(D) $a_0 = 0,\ a_n = 0,\ b_n = \frac{2}{\pi}\int_0^\pi f(x)\sin nx\,dx$

21. Fourier coefficients for an even function f(x) defined in the interval $-L \le x \le L$ and $f(x + 2L) = f(x)$ are (1)

(A) $a_0 = 0, a_n = 0,\ b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$

(B) $a_0 = \frac{2}{L}\int_0^L f(x)\,dx,\ \ a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx, b_n = 0$

(C) $a_0 = 0, a_n = 0,\ b_n = 0$

(D) $a_0 = 0,\ a_n = 0,\ \ b_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx$

22. Fourier coefficients for an odd function f(x) defined in the interval $-L \le x \le L$ and $f(x + 2L) = f(x)$ are (1)

(A) $a_0 = 0, a_n = 0,\ b_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx$

(B) $a_0 = \frac{2}{L}\int_0^L f(x)\,dx,\ \ a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx,\ \ b_n = 0$

(C) $a_0 = 0,\ a_n = 0, b_n = 0$

(D) $a_0 = 0, a_n = 0,\ b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$

23. Half range Fourier cosine series for f(x) defined in the interval $0 \le x \le L$ is (1)

(A) $\sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$

(B) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{nx}{L}$

(C) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$

(D) $\frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$

24. Half range Fourier sine series for f(x) defined in the interval $0 \le x \le L$ is (1)

(A) $\sum_{n=1}^{\infty} b_n \sin \frac{nx}{L}$ (B) $\sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$

(C) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}$ (D) $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right)$

25. In Harmonic analysis for a function with period 2π, the term $a_1 \cos x + b_1 \sin x$ is called (1)

(A) second harmonic (B) first harmonic

(C) third harmonic (D) none of these

26. In Harmonic analysis for a function with period 2π, the amplitude of first harmonic $a_1 \cos x + b_1 \sin x$ is (1)

(A) $\sqrt{a_1^2 - b_1^2}$ (B) $a_1^2 + b_1^2$ (C) $\sqrt{a_1^2 + b_1^2}$ (D) $\left(a_1^2 + b_1^2\right)^2$

27. The value of a_0 in Fourier series of y with period 6 for the following tabulated data(1)

x	0	1	2	3	4	5
y	9	18	24	28	26	20

125

(A) 17.85 (B) 20.83 (C) 35.71 (D) 41.66

28. The value of a_0 in Fourier series of y with period 180° for the following tabulated data is (1)

x^0	0	30	60	90	120	150
y	0	9.2	14.4	17.8	17.3	11.7

(A) 23.46 (B) 20.11 (C) 11.73 (D) 10.50

29. The values of a_0 in Fourier series of y with period 6 for the following tabulated data is (1)

x	0	1	2	3	4	5
y	4	8	15	7	6	2

(A) 3.5 (B) 14 (C) 6 (D) 7

30. The value of a_0 in Fourier series of y with period 360° for the following tabulated data is (1)

x^0	0	60	120	180	240	300
y	1.0	1.4	1.9	1.7	1.5	1.2

(A) 1.45 (B) 5.8 (C) 2.9 (D) 2.48

31. Fourier coefficient a_0 in the Fourier series expansion of $f(x) = e^{-x}$; $0 \le x \le 2\pi$ and $f(x + 2\pi) = f(x)$ is (2)

(a) $\frac{1}{\pi}(1 - e^{-2\pi})$ (B) $\frac{1}{2\pi}(1 - e^{2\pi})$ (C) $\frac{2}{\pi}(e^{-2\pi} - 1)$ (D) $\frac{1}{\pi}(1 + e^{2\pi})$

32. Fourier coefficient a_0 in the Fourier series expansion of $f(x) = \left(\frac{\pi - x}{2}\right)^2$; $0 \le x \le 2\pi$ and $f(x + 2\pi) = f(x)$ is (2)

(A) $\frac{\pi^2}{3}$ (B) $\frac{\pi^2}{6}$ (C) 0 (D) $\frac{\pi}{6}$

33. Fourier coefficient a_0 in the Fourier series expansion of $f(x) = x \sin x$; $0 \le x \le 2\pi$ and $f(x + 2\pi) = f(x)$ is (2)

(A) +2 (B) 0 (C) −2 (D) −4

34. $f(x) = \begin{cases} x, & 0 \le x \le \pi \\ 0, & \pi < x \le 2\pi \end{cases}$ and $f(x + 2\pi) = f(x)$. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$, then Fourier coefficient a_0 is (2)

(A) 2π (B) $\frac{\pi}{3}$ (C) 0 (D) $\frac{\pi}{2}$

35. $f(x) = \begin{cases} 0, & 0 \le x \le \pi \\ x, & \pi < x \le 2\pi \end{cases}$ and $f(x + 2\pi) = f(x)$. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$, then Fourier coefficient a_0 is (2)

(A) $\frac{3\pi}{2}$ (B) $\frac{3\pi}{8}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

36. $f(x) = 2x - x^2$, $0 \le x \le 3$ and period is 3. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2n\pi x}{3} + b_n \sin \frac{2n\pi x}{3}\right)$, then Fourier coefficient a_0 is (2)

(A) $\frac{3}{2}$ (B) 0 (C) 12 (D) $\frac{3}{4}$

37. $f(x) = 4 - x^2$, $0 \le x \le 2$ and period is 2. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\pi x + b_n \sin n\pi x)$, then Fourier coefficient a_0 is (2)

(A) $\frac{11}{3}$ (B) 0 (C) $\frac{16}{3}$ (D) $\frac{8}{5}$

38. $f(x) = \begin{cases} -x, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ and $f(x + 2\pi) = f(x)$. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$, then Fourier coefficient a_0 is (2)

(A) $\frac{\pi}{3}$ (B) $\frac{2}{\pi}$ (C) $\frac{\pi}{4}$ (D) π

39. $f(x) = x \cos x$, $-\pi \le x \le \pi$ and period is 2π. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$, then Fourier coefficient a_0 is (2)

(A) $-\frac{2}{\pi}$ (B) 0 (C) $\frac{4}{\pi}$ (D) $-\frac{4}{\pi}$

40. $f(x) = 2$, $-\pi \le x \le \pi$ and period is 2π. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$, then Fourier coefficient a_0 is (2)

(A) 4 (B) 2 (C) $\frac{4}{\pi}$ (D) $\frac{2}{\pi}$

41. $f(x) = x$, $-\pi \le x \le \pi$ and period is 2π. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$, then Fourier coefficient b_1 is (2)

(A) 2 (B) -1 (C) 0 (d) $\frac{2}{\pi}$

42. $f(x) = \begin{cases} 1, & -1 < x < 0 \\ \cos \pi x, & 0 < x < 1 \end{cases}$ and period 2. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\pi x + b_n \sin n\pi x)$, then Fourier coefficient a_0 is (2)

(A) 2 (B) 0 (C) 1 (D) -1

43. $f(x) = x - x^3$, $-2 < x < 2$ and period 4. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\pi x + b_n \sin n\pi x)$, then Fourier coefficient a_0 is (2)

(A) 1 (B) 0 (C) -2 (D) -1

44. For half range cosine series of $f(x) = \sin x$, $0 \le x < \pi$ and period is 2π. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$, then Fourier coefficient a_0 is (2)

(A) 4 (B) 2 (C) $\frac{2}{\pi}$ (D) $\frac{4}{\pi}$

45. For half range sine series of $f(x) = \cos x$, $0 \le x \le \pi$ and period is 2π. Fourier series is represented by $\sum_{n=1}^{\infty} b_n \sin nx$, then Fourier coefficient b_1 is (2)

(A) $\frac{1}{\pi}$ (B) 0 (C) $\frac{2}{\pi}$ (D) $\frac{2}{\pi}$

46. For half range cosine series of $f(x) = lx - x^2, 0 \le x \le l$ and period is $2l$. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l}$, then Fourier coefficient a_0 is (2)

(A) $\frac{l^2}{3}$ (B) $\frac{2l^2}{3}$ (C) $\frac{l^2}{6}$ (D) 0

47. For half range sine series of $f(x) = x, 0 \le x \le 2$ and period is 4. Fourier series is represented by $\sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{2}$, then Fourier coefficient b_1 is (2)

(A) 4 (B) 2 (C) $\frac{2}{\pi}$ (D) $\frac{4}{\pi}$

48. Fourier series representation of periodic function $f(x) = \left(\frac{\pi - x}{2}\right)^2, 0 \le x \le 2\pi$ is $\left(\frac{\pi - x}{2}\right)^2 = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{1}{n^2} \cos nx$, then value of $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots =$ (2)

(A) $\frac{\pi^2}{6}$ (B) $\frac{\pi^2}{12}$ (C) $\frac{\pi^2}{3}$ (D) 0

49. Fourier series representation of periodic function $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$ is

$f(x) = \frac{8}{\pi^2}\left[\frac{1}{1^2} \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \dots\right]$, then value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots =$ (2)

(A) $\frac{\pi^2}{4}$ (B) $\frac{\pi^2}{8}$ (C) $\frac{\pi^2}{16}$ (D) $\frac{8}{\pi^2}$

50. Fourier series representation of periodic function $f(x) = \pi^2 - x^2, -\pi \le x \le \pi$ is $\pi^2 - x^2 = \frac{2\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \cos nx$, then value of $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots =$ (2)

(A) $\frac{\pi^2}{3}$ (B) $\frac{\pi^2}{4}$ (C) $\frac{\pi^2}{6}$ (D) $\frac{\pi^2}{12}$

51. Fourier series representation of periodic function $f(x) = \pi^2 - x^2, -\pi \le x \le \pi$ is $\pi^2 - x^2 = \frac{2\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \cos nx$, then value of $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots =$ (2)

(A) $\frac{\pi^2}{3}$ (B) $\frac{\pi^2}{12}$ (C) $\frac{\pi^2}{6}$ (D) 0

52. The value of a_1 in Fourier series of y with period 6 for the following tabulated data is : (2)

x	0	1	2	3	4	5
y	9	18	24	28	26	20
$\cos\frac{\pi x}{3}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	−1	$-\frac{1}{2}$	$\frac{1}{2}$

(A) −8.33 (B) −7.14 (C) −4.16 (D) 0

53. The value of b_1 in Fourier series of y with period π for the following tabulated data is : (2)

x^0	0	30	60	90	120	150
y	0	9.2	14.4	17.8	17.3	11.7
sin 2x	0	0.866	0.866	0	−0.866	−0.866

(A) −3.116 (B) −1.558 (C) −4.16 (D) −1.336

54. The value of a_1 in Fourier series of y with period 6 for the following tabulated data is : (2)

x	0	1	2	3	4	5
y	4	8	15	7	6	2
$\cos\frac{\pi x}{3}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	−1	$-\frac{1}{2}$	$\frac{1}{2}$

(A) −2.83 (B) −8.32 (C) −3.57 (D) −10.98

55. The value of b_1 in Fourier series of y with period 2π for the following tabulated data is : (2)

x^0	0	60	120	180	240	300
y	1.0	1.4	1.9	1.7	1.5	1.2
sin x	0	0.866	0.866	0	−0.866	−0.866

(A) 0.0989 (B) 0.3464 (C) 0.1732 (D) 0.6932

ANSWERS

1. (A)	2. (C)	3. (B)	4. (D)	5. (C)	6. (B)	7. (A)	8. (D)
9. (D)	10. (C)	11. (B)	12. (A)	13. (B)	14. (A)	15. (D)	16. (C)
17. (C)	18. (A)	19. (B)	20. (D)	21. (B)	22. (D)	23. (C)	24. (B)
25. (B)	26. (C)	27. (D)	28. (A)	29. (B)	30. (C)	31. (A)	32. (B)
33. (C)	34. (D)	35. (A)	36. (B)	37. (C)	38. (D)	39. (B)	40. (A)
41. (A)	42. (C)	43. (B)	44. (D)	45. (B)	46. (A)	47. (D)	48. (A)
49. (B)	50. (D)	51. (C)	52. (A)	53. (B)	54. (D)	55. (C)	

MULTIPLE CHOICE QUESTIONS

Reduction Formulae :

1. If $I_n = \int_0^{\pi/2} \sin^n x \, dx$ then which of the following relation is true ? (1)

(A) $I_n = \frac{n-1}{n} I_{n-2}$ (B) $I_n = \frac{n-1}{n} I_{n-1}$ (C) $I_n = \frac{n}{n-1} I_{n-2}$ (D) $I_n = \frac{n-2}{n} I_{n-1}$

2. If $I_n = \int_0^{\pi/2} \cos^n x \, dx$ then which of the following relation is true ? (1)

(A) $I_n = \frac{n-1}{n} I_{n-1}$ (B) $I_n = \frac{n-1}{n} I_{n-2}$ (C) $I_n = \frac{n}{n-1} I_{n-2}$ (D) $I_n = n(n+1) I_{n-1}$

3. If $I_n = \int_0^{\pi/2} \sin^n x \, dx$, n positive even integer then I_n is calculated from (1)

(A) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{3}{4} \cdot \frac{1}{2}$

(B) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{3}{4} \cdot \frac{1}{2} \cdot \pi$

(C) $\left(\frac{n+1}{n} \cdot \frac{n+3}{n+2} \cdot \frac{n+5}{n+4} \cdots\right) \cdot \frac{\pi}{2}$

(D) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}$

4. If $I_n = \int_0^{\pi/2} \sin^n x \, dx$, n positive odd integer then I_n is calculated from (1)

(A) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{4}{5} \cdot \frac{2}{3} \cdot \frac{\pi}{2}$

(B) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}$

(C) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{4}{5} \cdot \frac{2}{3} \cdot 1$

(D) $\frac{n+1}{n} \cdot \frac{n+3}{n+2} \cdot \frac{n+5}{n+4}$

5. If $I_n = \int_0^{\pi/2} \cos^n x \, dx$, n positive even integer then I_n is calculated from (1)

(A) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{3}{4} \cdot \frac{1}{2}$

(B) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}$

(C) $\left(\frac{n+1}{n} \cdot \frac{n+3}{n+2} \cdot \frac{n+5}{n+4} \cdots\right) \cdot \frac{\pi}{2}$

(D) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{3}{4} \cdot \frac{1}{2} \cdot \pi$

6. If $I_n = \int_0^{\pi/2} \cos^n x \, dx$, n positive odd integer then I_n is calculated from (1)

(A) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{4}{5} \cdot \frac{2}{3} \cdot \frac{\pi}{2}$

(B) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}$

(C) $\frac{n+1}{n} \cdot \frac{n+3}{n+2} \cdot \frac{n+5}{n+4}$

(D) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{4}{5} \cdot \frac{2}{3} \cdot 1$

7. The value of integral $I_{m,n} = \int_0^{\pi/2} \sin^m x \cos^n x \, dx$; m, n are positive integers ≥ 2 is (1)

(A) $I_{m,n} = \frac{[(m-1)(m-3)\ldots 2 \text{ or } 1]\cdot[(n-1)(n-3)\ldots 2 \text{ or } 1]}{[(m+n)(m+n-2)\ldots 2 \text{ or } 1]} \times P$

where $P = \frac{\pi}{2}$ if m and n are both even

$= 1$ for all other values of m and n

(B) $I_{m,n} = \frac{[(m-1)(m-3)\ldots 2 \text{ or } 1]\cdot(n-1)(n-3)\ldots 2 \text{ or } 1]}{[(m+n)(m+n-2)\ldots 2 \text{ or } 1}\times P$

where P = 1 if m and are both even

$= \frac{\pi}{2}$ for all other values of m and n.

(C) $I_{m,n} = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdots\frac{4}{5}\cdot\frac{2}{3}\cdot 1$ (D) $I_{m,n} = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdots\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}$

8. $\int_0^{\pi/2} \sin^4 x \, dx$ is equal to (2)

(A) $\frac{\pi}{2}$ (B) $\frac{3\pi}{8}$ (C) $\frac{\pi}{4}$ (D) $\frac{3\pi}{16}$

9. $\int_0^{\pi/2} \sin^5 x \, dx$ is equal to (2)

(A) $\frac{8}{15}\cdot\frac{\pi}{2}$ (B) $\frac{15}{8}$ (C) $\frac{8}{15}$ (D) 0

10. $\int_0^{\pi/2} \cos^3 x \, dx$ is equal to (2)

(A) $\frac{2}{3}$ (B) $\frac{1}{4}$ (C) $\frac{2}{3}\cdot\frac{\pi}{2}$ (D) $\frac{1}{3}$

11. $\int_0^{\pi/2} \cos^6 x \, dx$ is equal to (2)

(A) $\frac{5}{16}$ (B) $\frac{5}{16}\cdot\frac{\pi}{2}$ (C) $\frac{16}{5}\cdot\frac{\pi}{2}$ (D) $\frac{5}{48}\cdot\frac{\pi}{2}$

12. $\int_0^{\pi/4} \sin^2 (2x) \, dx$ is equal to (2)

(A) $\frac{1}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{8}$

13. $\int_0^{\pi/4} \cos^2 (2x) \, dx$ is equal to (2)

(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{1}{4}$

14. $\int_0^{\pi} \sin^5\left(\frac{x}{2}\right) dx$ is equal to (2)

(A) $\frac{8\pi}{15}$ (B) $\frac{32}{15}$ (C) $\frac{16}{15}$ (D) $\frac{8}{15}$

15. $\int_0^{\pi} \cos^3 x \, dx$ is equal to (2)

(A) 1 (B) $\frac{2}{3}$ (C) $\frac{4}{3}$ (D) 0

16. $\int_0^{\pi} \sin^6 t \, dt$ is equal to (2)

(A) $\frac{\pi}{8}$ (B) $\frac{5\pi}{16}$ (C) $\frac{5}{8}$ (D) $\frac{5\pi}{8}$

17. $\int_0^{2\pi} \sin^5 t \, dt$ is equal to (2)

(A) $\frac{5}{4}$ (B) $\frac{5\pi}{32}$ (C) $\frac{5\pi}{8}$ (D) 0

18. $\int_0^{2\pi} \cos^7 t \, dt$ is equal to (2)

(A) $\frac{32}{35}$ (B) $\frac{32\pi}{70}$ (C) 0 (D) $\frac{16}{35}$

19. $\int_0^{\pi/2} \sin^6 x \cos^4 x \, dx$ is equal to (2)

(A) $\frac{3}{256}$ (B) $\frac{3\pi}{512}$ (C) $\frac{3}{128}$ (D) $\frac{512}{3}\pi$

20. $\int_0^{\pi/2} \sin^4 x \cos^5 x \, dx$ is equal to (2)

(A) $\frac{4\pi}{315}$ (B) $\frac{315}{8}$ (C) $\frac{8\pi}{630}$ (D) $\frac{8}{315}$

21. $\int_0^{2\pi} \sin^6 x \cos^4 x \, dx$ is equal to (2)

(A) $\frac{5\pi}{128}$ (B) $\frac{3\pi}{512}$ (C) $\frac{3\pi}{128}$ (D) $\frac{5\pi}{64}$

22. $\int_0^{2\pi} \sin^5 \theta \cos^4 \theta \, d\theta$ is equal to (2)

(A) 0 (B) $\frac{8}{315}$ (C) $\frac{3\pi}{128}$ (D) $\frac{\pi}{128}$

23. $\int_{-\pi/2}^{\pi/2} \sin^4\theta\, d\theta$ is equal to (2)

(A) $\frac{3\pi}{16}$ (B) $\frac{3\pi}{4}$ (C) 0 (D) $\frac{3\pi}{8}$

24. $\int_{-\pi/2}^{\pi/2} \sin^4\theta \cos^2\theta\, d\theta$ is equal to (2)

(A) $\frac{\pi}{32}$ (B) $\frac{\pi}{16}$ (C) $\frac{\pi}{8}$ (D) 0

25. If $I_n = \int_0^{\pi/4} \tan^n x\, dx$ and $I_n = \frac{1}{n-1} - I_{n-2}$ then I_4 is equal to (2)

(A) $-\frac{2}{3}+\frac{\pi}{2}$ (B) $-\frac{2}{3}-\frac{\pi}{4}$ (C) $-\frac{2}{3}+\frac{\pi}{4}$ (D) $-\frac{4}{3}+\frac{\pi}{4}$

26. If $I_n = \int_{\pi/4}^{\pi/2} \cot^n x\, dx$ and $I_n = \frac{1}{n-1} - I_{n-2}$ then I_4 is equal to (2)

(A) $-\frac{4}{3}+\frac{\pi}{4}$ (B) $-\frac{2}{3}+\frac{\pi}{2}$ (C) $-\frac{2}{3}-\frac{\pi}{4}$ (D) $-\frac{2}{3}+\frac{\pi}{4}$

27. If $I_n = \int_0^{\pi/4} \sin^{2n} x\, dx$ and $I_n = -\frac{1}{n2^{n+1}} + \left(\frac{2n-1}{2n}\right) I_{n-1}$ then I_2 is equal to (2)

(A) $-\frac{1}{4}+\frac{3\pi}{32}$ (B) $-\frac{3}{4}+\frac{3\pi}{32}$ (C) $\frac{1}{4}+\frac{\pi}{32}$ (D) $\frac{1}{4}-\frac{\pi}{16}$

28. If $I_n = \int_0^{\pi/4} \cos^{2n} x\, dx$ and $I_n = \frac{1}{n2^{n+1}} + \left(\frac{2n-1}{2n}\right) I_{n-1}$, then I_2 is equal to (2)

(A) $-\frac{1}{4}+\frac{3\pi}{32}$ (B) $\frac{1}{4}+\frac{3\pi}{32}$ (C) $\frac{1}{4}+\frac{3\pi}{16}$ (D) $\frac{1}{4}+\frac{\pi}{16}$

29. If $I_n = \int_0^{\pi/3} \cos^n x\, dx$ and $I_n = \frac{\sqrt{3}}{n2^n} + \left(\frac{n-1}{n}\right) I_{n-2}$ then I_2 is equal to (2)

(A) $\frac{\sqrt{3}}{4}+\frac{\pi}{3}$ (B) $\left(\frac{\sqrt{3}}{8}+\frac{1}{2}\right)\frac{\pi}{6}$ (C) $\frac{\sqrt{3}}{8}+\frac{\pi}{6}$ (D) $\frac{\sqrt{3}}{16}+\frac{\pi}{12}$

30. If $I_n = \int_0^{\infty} e^{-px} \sin^n x\, dx$ and $I_n = \frac{n(n-1)}{n^2+p^2} I_{n-2}$ then value of $\int_0^{\infty} e^{-2x} \sin^2 x\, dx$ is equal to (2)

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{8}$ (D) 2

31. If $I_n = \int_0^{\pi/4} \frac{\sin(2n-1)x}{\sin x} dx$ and $I_{n+1} = \frac{1}{n}\sin\frac{n\pi}{2} + I_n$ then I_3 is equal to (2)

(A) $2+\frac{\pi}{4}$ (B) $2-\frac{\pi}{2}$ (C) $1-\frac{\pi}{4}$ (D) $1+\frac{\pi}{4}$

32. If $I_n = \int (\log x)^n \, dx$ then (2)

(A) $I_n + nI_{n-1} = x(\log x)^n$

(B) $I_n - nI_{n-1} = x(\log x)^n$

(C) $I_n + I_{n-1} = x(\log x)^n$

(D) $I_n + nI_{n-1} = (\log x)$

ANSWERS

1. (A)	2. (B)	3. (D)	4. (C)	5. (B)	6. (D)	7. (A)	8. (D)
9. (C)	10. (A)	11. (B)	12. (D)	13. (A)	14. (C)	15. (D)	16. (B)
17. (A)	18. (C)	19. (B)	20. (D)	21. (C)	22.(A)	23. (D)	24. (B)
25. (C)	26. (D)	27. (A)	28. (B)	29. (C)	30. (C)	31. (D)	32.(A)

MULTIPLE CHOICE QUESTIONS

Gamma Functions :

1. Gamma function of n (n > 0), is defined as (1)

   (A) $\int_0^\infty e^{-x} x^{n-1} dx$ (B) $\int_0^\infty e^{x} x^{n-1} dx$ (C) $\int_0^1 e^{-x} x^{n-1} dx$ (D) $\int_0^\infty e^{-x} x^{1-n} dx$

2. The value of equivalent form of Gamma function $\int_0^\infty e^{-kx} x^{n-1} dx$ is (1)

   (A) $\frac{\overline{|n}}{n^k}$ (B) $\frac{\overline{|n}}{k!}$ (C) $\frac{\overline{|n}}{k^n}$ (D) $\overline{|n+k+1}$

3. Reduction formula for Gamma function is (1)

   (A) $\Gamma(n+1) = (n-1)\Gamma(n-1)$ (B) $\Gamma(n+1) = n\,\Gamma(n)$

   (C) $\Gamma(n+1) = (n-1)\,\Gamma(n)$ (D) $\Gamma(n+1) = n\,\Gamma(n-1)$

4. If n is a positive integer, then $\Gamma(n+1)$ is (1)

   (A) $(n+1)!$ (B) $(n-1)!$ (C) $(n+2)!$ (D) $n!$

5. $\Gamma(1)$ is equal to (1)

(A) 2! (B) 1 (C) 0 (D) $\sqrt{\pi}$

6. $\Gamma\left(\frac{1}{2}\right)$ is equal to (1)

(A) $\sqrt{\pi}$ (B) π (C) $\frac{1}{2}$ (D) 1

7. $\Gamma(7)$ is equal to (1)

(A) 8! (B) 7! (C) 6! (D) 6

8. $\Gamma\left(\frac{5}{2}\right)$ is equal to (1)

(A) $\frac{5}{2}\sqrt{\pi}$ (B) $\frac{15}{8}\sqrt{\pi}$ (C) $\frac{4}{3}\sqrt{\pi}$ (D) $\frac{3}{4}\sqrt{\pi}$

9. By using $\Gamma(p)\,\Gamma(1-p) = \frac{\pi}{\sin p\pi}$, if $0 \le p \le 1$ the value of $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$ is (1)

(A) $\frac{\pi}{\sqrt{2}}$ (B) π (C) $\sqrt{2}\pi$ (D) 2π

10. $\int_0^\infty e^{-t}\, t^{3/2}\, dt$ is equal to (1)

(A) $\frac{3}{4}\sqrt{\pi}$ (B) $\frac{15}{4}\sqrt{\pi}$ (C) $\frac{3}{4}\pi$ (D) $\frac{3}{2}\sqrt{\pi}$

11. $\int_0^\infty e^{-5x}\, x^4\, dx$ is equal to (1)

(A) $\frac{4!}{4^5}$ (B) $\frac{5!}{4^4}$ (C) $\frac{5!}{5^5}$ (D) $\frac{4!}{5^5}$

12. The appropriate substitution to reduce the given integral $\int_0^\infty \sqrt{x}\, e^{-\sqrt{x}}\, dx$ to Gamma function integral (1)

(A) $x^3 = t$ (B) $\sqrt{x} = t$ (C) $-x^3 = t$ (D) $\log x = t$

13. The appropriate substitution to reduce the given integral $\int_0^1 (x \log x)^4\, dx$ to Gamma function integral (1)

(A) $\log x = -t$ (B) $x = -e^t$ (C) $x = t^2$ (D) $\log x = t$

14. The appropriate substitution to reduce the given integral $\int_0^\infty \frac{x^5}{5^x} dx$ to Gamma function integral (1)

(A) $\log x = -t$ (B) $5^x = e^{-t}$ (C) $5^x = e^t$ (D) $5^x = t$

15. The value of integral $\int_0^\infty \sqrt{x}\, e^{-\sqrt{x}} dx$ by using substitution $\sqrt{x} = t$ is (2)

(A) 1 (B) 4 (C) 2 (D) 3

16. The value of integral $\int_0^\infty e^{-x^2} dx$ by using substitution $x^2 = t$ is (2)

(A) $\sqrt{\pi}$ (B) $\frac{\sqrt{\pi}}{2}$ (C) $\frac{\sqrt{\pi}}{3}$ (D) $2\sqrt{\pi}$

17. The value of integral $\int_0^\infty e^{-x^4} dx$ by using substitution $x^4 = t$ is (2)

(A) $\Gamma\frac{5}{4}$ (B) $\Gamma\frac{1}{4}$ (C) $\frac{1}{4}\Gamma\frac{1}{4}$ (D) $\frac{1}{4}\Gamma\frac{5}{4}$

18. The value of integral $\int_0^\infty \sqrt{x}\, e^{-x^3} dx$ by using substitution $x^3 = t$ is (2)

(A) $\frac{\sqrt{\pi}}{6}$ (B) $\frac{\sqrt{\pi}}{2}$ (C) $3\sqrt{\pi}$ (D) $\frac{\sqrt{\pi}}{3}$

19. The value of integral $\int_0^\infty \sqrt[4]{x}\, e^{-\sqrt{x}} dx$ by using substitution $x^2 = t$ is (2)

(A) $\frac{3\sqrt{\pi}}{4}$ (B) $\frac{3\sqrt{\pi}}{2}$ (C) $\frac{15\sqrt{\pi}}{4}$ (D) $\frac{\sqrt{\pi}}{3}$

20. The value of integral $\int_0^\infty x^9 e^{-2x^2} dx$ by using substitution $2x^2 = t$ is (2)

(A) $\frac{\Gamma 5}{64}$ (B) $\frac{\Gamma 6}{64}$ (C) $\frac{\Gamma 5}{32}$ (D) $\frac{\Gamma 6}{32}$

21. The value of integral $\int_0^1 \left[\log\left(\frac{1}{y}\right)\right]^{n-1} dy$ by using substitution $\log\left(\frac{1}{y}\right) = t$ is (2)

(A) Γn (B) $\Gamma(n+1)$ (C) $-\Gamma n$ (D) $\Gamma(n-1)$

22. The value of integral $\int_0^1 \frac{dx}{\sqrt{x \log\left(\frac{1}{x}\right)}}$ by using substitution $\log\left(\frac{1}{x}\right) = t$ is (2)

(A) $\frac{\sqrt{\pi}}{2}$ (B) $\sqrt{\pi}$ (C) $\sqrt{2\pi}$ (D) $2\sqrt{\pi}$

23. The value of integral $\int_0^1 \frac{dx}{\sqrt{-\log x}}$ by using substitution $-\log x = t$ is (2)

(A) $\frac{\sqrt{\pi}}{2}$ (B) $\sqrt{\pi}$ (C) $\sqrt{2\pi}$ (D) $2\sqrt{\pi}$

24. The value of integral $\int_0^\infty \frac{x^4}{4^x}\,dx$ by using substitution $4^x = e^t$ is (2)

(A) $\frac{4}{(\log 4)^4}$ (B) $\frac{24}{(\log 4)^3}$ (C) $\frac{24}{(\log 4)^5}$ (D) $\frac{12}{(\log 4)^4}$

25. The value of integral $\int_0^\infty \frac{x^2}{2^x}\,dx$ by using substitution $2^x = e^t$ is (2)

(A) $\frac{1}{(\log 2)^2}$ (B) $\frac{2}{(\log 2)^2}$ (C) $\frac{2}{(\log 2)^3}$ (D) $\frac{3}{(\log 2)^4}$

ANSWERS

1. (A)	2. (C)	3. (B)	4. (D)	5. (B)	6. (A)	7. (C)	8. (D)
9. (C)	10. (A)	11. (D)	12. (B)	13. (A)	14. (C)	15. (B)	16. (B)
17. (C)	18. (D)	19. (B)	20. (A)	21. (A)	22. (C)	23. (B)	24. (C)
25. (C)							

MULTIPLE CHOICE QUESTIONS

DUIS

1. If $I(\alpha) = \int_a^b f(x, \alpha)\, dx$, where α is parameter and a, b are constants then by DUIS rule, $\frac{dI(\alpha)}{d\alpha}$ is (1)

(A) $\int_a^b \frac{\partial}{\partial \alpha} f(x, \alpha)\, dx$ (B) $\int_a^b \frac{\partial}{\partial x} f(x, \alpha)\, dx$

(C) $f(b, \alpha) - f(a, \alpha)$ (D) $f(x, \alpha)$

2. If $I(\alpha) = \int_a^b f(x, \alpha)\, dx$ where a, b are functions of parameter α then by DUIS rule, $\frac{dI(\alpha)}{d\alpha}$ is (1)

(A) $\int_a^b \frac{\partial}{\partial \alpha} f(x, \alpha)\, dx + f(a, \alpha)\frac{da}{d\alpha} - f(b, \alpha)\frac{db}{d\alpha}$

(B) $\int_a^b \frac{\partial}{\partial \alpha} f(x, \alpha)\, dx + f(b, \alpha)\frac{db}{d\alpha} + f(a, \alpha)\frac{da}{d\alpha}$

(C) $\int_a^b \frac{\partial}{\partial \alpha} f(x, \alpha)\, dx + f(b, \alpha)\frac{db}{d\alpha} - f(a, \alpha)\frac{da}{d\alpha}$

(D) $\int_a^b \frac{\partial}{\partial \alpha} f(x, \alpha)\, dx$

3. If $\phi(a) = \int_0^\infty \frac{e^{-x}}{x}(1 - e^{-ax})\, dx$, $a > -1$ then by DUIS rule, $\frac{d\phi}{da}$ is (1)

(A) $\int_0^\infty \frac{\partial}{\partial a} e^{-x}(1 - e^{-ax})\, dx$

(B) $\int_0^\infty \frac{\partial}{\partial a} \frac{e^{-x}}{x}(1 - e^{-ax})\, dx$

(C) $\int_0^\infty \frac{\partial}{\partial x} \frac{e^{-x}}{x}(1 - e^{-ax})\, dx$

(D) $\frac{e^{-x}}{x}(1 - e^{-ax})$

4. If $\phi(a) = \int_0^\infty e^{-bx^2} \cos(2ax)\, dx$, $b > 0$ then by DUIS rule, $\frac{d\phi}{da}$ is (1)

(A) $\int_0^\infty \frac{\partial}{\partial b}\left[e^{-bx^2}\cos(2ax)\right] dx$

(B) $e^{-bx^2}\cos(2ax)$

(C) $\int_0^\infty \frac{\partial}{\partial x}\left[e^{-bx^2}\cos(2ax)\right] dx$

(D) $\int_0^\infty \frac{\partial}{\partial a}\left[e^{-bx^2}\cos(2ax)\right] dx$

5. If $\phi(a) = \int_0^\infty \frac{e^{-x}}{x}\left(a - \frac{1}{x} + \frac{1}{x}e^{-ax}\right) dx$, then by DUIS rule, $\frac{d\phi}{da}$ is (1)

(A) $\frac{e^{-x}}{x}\left(a - \frac{1}{x} + \frac{1}{x}e^{-ax}\right)$

(B) $\int_0^\infty \frac{\partial}{\partial a}\frac{e^{-x}}{x}\left(a - \frac{1}{x} + \frac{1}{x}e^{-ax}\right) dx$

(C) $\int_0^\infty \frac{\partial}{\partial x}\frac{e^{-x}}{x}\left(a - \frac{1}{x} + \frac{1}{x}e^{-ax}\right) dx$

(D) $\int_0^\infty \frac{\partial}{\partial a} e^{-x}\left(a - \frac{1}{x} + \frac{1}{x}e^{-ax}\right) dx$

6. If $\phi(a) = \int_0^{\infty} \frac{e^{-x}}{x}(1 - e^{-ax})\, dx$, a > –1 then by DUIS rule, $\frac{d\phi}{da}$ is (2)

(A) $\frac{e^{-x}}{x}(1 - e^{-ax})$ (B) $\int_0^{\infty} \frac{a}{x} e^{-(a+1)x}\, dx$ (C) $\int_0^{\infty} e^{-ax}\, dx$ (D) $\int_0^{\infty} e^{-(a+1)x}\, dx$

7. If $\phi(a) = \int_0^1 \frac{x^a - 1}{\log x}\, dx$; $a \geq 0$ then by DUIS rule, $\frac{d\phi}{da}$ is (2)

(A) $\int_0^1 \frac{x^a \log a}{\log x}\, dx$ (B) $\int_0^1 \frac{a x^{a-1}}{\log x}\, dx$ (C) $\int_0^1 x^a\, dx$ (D) $\frac{x^a - 1}{\log x}$

8. If $\phi(\alpha) = \int_0^{\infty} \frac{e^{-x} \sin \alpha x}{x}\, dx$ then by DUIS rule, $\frac{d\phi}{d\alpha}$ is (2)

(A) $\int_0^{\infty} e^{-x} \sin \alpha x\, dx$

(B) $\int_0^{\infty} e^{-x} \cos \alpha x\, dx$

(C) $\int_0^{\infty} \frac{\alpha e^{-x} \sin \alpha x}{x}\, dx$

(D) $\frac{e^{-x} \sin \alpha x}{x}$

9. If $I(a) = \int_0^{\infty} e^{-\left(x^2 + \frac{a^2}{x^2}\right)}\, dx$; a > 0 then by DUIS rule, $\frac{dI}{da}$ is (2)

(A) $\int_0^{\infty} e^{-\left(x^2 + \frac{a^2}{x^2}\right)}\left(-\frac{2a}{x^2}\right) dx$

(B) $\int_0^{\infty} e^{-\left(x^2 + \frac{a^2}{x^2}\right)}\left(\frac{2a}{x^2}\right) dx$

(C) $\int_0^{\infty} e^{-\left(x^2 + \frac{a^2}{x^2}\right)}\left(-2x - \frac{2a^2}{x^3}\right) dx$

(D) $e^{-\left(x^2 + \frac{a^2}{x^2}\right)}$

10. If $I(a) = \int_0^{\pi} \log(1 - a \cos x)\, dx$; $|a| < 1$ then by DUIS rule, $\frac{dI}{da}$ is (2)

(A) $\int_0^{\pi} \frac{-a \sin x}{1 - a \cos x}\, dx$

(B) $\int_0^{\pi} \frac{\cos x}{1 - a \cos x}\, dx$

(C) $\int_0^{\pi} \frac{-\cos x}{1 - a \cos x}\, dx$

(D) $\int_0^{\pi} \frac{1}{1 - a \cos x}\, dx$

11. By DUIS rule $\frac{d}{da}\left[\int_0^\infty \frac{e^{-x}}{x}\left(a-\frac{1}{x}+\frac{1}{x}e^{-ax}\right)dx\right]$, a is parameter, is (2)

(A) $\int_0^\infty \frac{e^{-x}}{x}(1+e^{-ax})\,dx$

(B) $\int_0^\infty \frac{e^{-x}}{x}\left(1-\frac{ae^{-ax}}{x}\right)dx$

(C) $\int_0^\infty e^{-x}(1-e^{-ax})\,dx$

(D) $\int_0^\infty \frac{e^{-x}}{x}(1-e^{-ax})\,dx$

12. If $I(x) = \int_0^\infty e^{-a^2}\cos axda$, x is parameter then by DUIS rule, $\frac{dI}{dx}$ is (2)

(A) $\int_0^\infty xe^{-a^2}\sin(xa)\,da$

(B) $\int_0^\infty ae^{-a^2}\sin(xa)\,da$

(C) $\int_0^\infty -ae^{-a^2}\sin(xa)\,da$

(D) $\int_0^\infty ae^{-a^2}\cos(xa)\,da$

13. If $I(a) = \int_0^\infty \frac{e^{-x}-e^{-ax}}{x\sec x}\,dx$, $a > 0$ then by DUIS rule, $\frac{dI}{da}$ is (2)

(A) $\int_0^\infty \frac{e^{-x}+e^{-ax}x}{x\sec x}\,dx$

(B) $\int_0^\infty \frac{e^{-x}}{\sec x}\,dx$

(C) $\int_0^\infty (e^{-x}-e^{-ax})\,dx$

(D) $\int_0^\infty \frac{e^{-ax}}{\sec x}\,dx$

14. If $\phi(a) = \int_0^1 \frac{x^a-x^b}{\log x}\,dx$; $a > 0, b > 0$ then by DUIS rule, $\frac{d\phi}{da}$ is (2)

(A) $\int_0^1 \frac{x^a\log a}{\log x}\,dx$ (B) $\int_0^1 x^a\,dx$ (C) $\int_0^1 x^b\,dx$ (D) $\int_0^1 (x^a-x^b)\,dx$

15. If $\phi(b) = \int_0^\infty \frac{e^{-ax}-e^{-bx}}{x}\,dx$; $a > 0, b > 0$ then by DUIS rule, $\frac{d\phi}{db}$ is (2)

(A) $\int_0^\infty e^{-bx}\,dx$

(B) $\int_0^\infty \frac{e^{-ax}(-a)-e^{-bx}(-b)}{x}\,dx$

(C) $\int_0^\infty e^{-ax}\,dx$

(D) $\int_0^\infty (e^{-ax}-e^{-bx})\,dx$

16. If $\phi(a) = \int_0^\infty \frac{1}{x^2} \log(1 + ax^2)\, dx;\ a > 0,$ then by DUIS rule, $\frac{d\phi}{da}$ is (2)

(A) $\int_0^\infty \frac{a}{x(1 + ax^2)} dx$ (B) $\int_0^\infty \frac{\log(1 + ax^2)}{x} dx$

(C) $\int_0^\infty \frac{2a}{x(1 + ax^2)} dx$ (D) $\int_0^\infty \frac{1}{1 + ax^2} dx$

17. If $\phi(a) = \int_0^{\pi/2} \frac{\log(1 + a \sin^2 x)}{\sin^2 x} dx;\ a > 0,$ then by DUIS rule, $\frac{d\phi}{da}$ is (2)

(A) $\int_0^{\pi/2} \frac{2 \sin x \cos x}{(1 + a \sin^2 x)} dx$ (B) $\int_0^{\pi/2} \frac{1}{(1 + a \sin^2 x) \sin^2 x} dx$

(C) $\int_0^{\pi/2} \frac{1}{1 + a \sin^2 x} dx$ (D) $\int_0^{\pi/2} \frac{\sin^2 x}{(1 + a \sin^2 x)} dx$

18. If $\phi(a) = \int_a^{a^2} \log(ax)\, dx,$ then by DUIS rule II, $\frac{d\phi}{da}$ is (2)

(A) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx + 2a \log a^3$ (B) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx$

(C) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx + 2a \log a^3 - 2 \log a$ (D) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx - 2a \log a^3 + 2 \log a$

19. If $\phi(a) = \int_0^{a^2} \tan^{-1}\left(\frac{x}{a}\right) dx,$ then by DUIS rule II, $\frac{d\phi}{da}$ is (2)

(A) $\int_0^{a^2} \frac{\partial}{\partial a} \tan^{-1}\left(\frac{x}{a}\right) dx + 2a \tan^{-1} a$

(B) $\int_0^{a^2} \frac{\partial}{\partial a} \tan^{-1}\left(\frac{x}{a}\right) dx$

(C) $\int_0^{a^2} \frac{\partial}{\partial a} \tan^{-1}\left(\frac{x}{a}\right) dx + a^2 \tan^{-1} a$

(D) $\int_0^{a^2} \frac{\partial}{\partial a} \tan^{-1}\left(\frac{x}{a}\right) dx + a^2 \tan^{-1} a - \tan^{-1}\left(\frac{x}{a}\right)$

20. If $I(a) = \int_{a}^{a^2} \frac{1}{x+a} dx$, then by DUIS rule II, $\frac{dI}{da}$ is (2)

(A) $\int_{a}^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx - \frac{1}{a^2+a}(2a) + \frac{1}{2a}$

(B) $\int_{a}^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx + \frac{1}{a^2+a}(2a) - \frac{1}{2a}$

(C) $\int_{a}^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx - \frac{1}{a^2+a}$

(D) $\int_{a}^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx$

21. If $\phi(a) = \int_{0}^{a} \frac{\log(1+ax)}{1+x^2} dx$, then by DUIS rule II, $\frac{d\phi}{da}$ is (2)

(A) $\int_{0}^{a} \frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right] dx$

(B) $\int_{0}^{a} \frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right] dx - \frac{\log(1+a)}{1+a^2}$

(C) $\int_{0}^{a} \frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right] dx - \frac{\log(1+a^2)}{1+a^2}$

(D) $\int_{0}^{a} \frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right] dx + \frac{\log(1+a^2)}{1+a^2}$

22. If $F(t) = \int_{t}^{t^2} e^{tx^2} dx$, then by DUIS rule II, $\frac{dF}{dt}$ is (2)

(A) $\int_{t}^{t^2} \frac{\partial}{\partial t} e^{tx^2} dx + (2t)e^{t^4} - e^{t^2}$

(B) $\int_{t}^{t^2} \frac{\partial}{\partial t} e^{tx^2} dx + e^{t^5} - e^{t^3}$

(C) $\int_{t}^{t^2} \frac{\partial}{\partial t} e^{tx^2} dx + (2t)e^{t^5} - e^{t^3}$

(D) $\int_{t}^{t^2} \frac{\partial}{\partial t} e^{tx^2} dx$

23. If $f(x) = \int_{a}^{x} (x-t)^2 G(t)\, dt$, a is constant and x is parameter then by DUIS rule II, $\frac{df}{dx}$ is (2)

(A) $\int_{a}^{x} \frac{\partial}{\partial x}(x-t)^2 G(t)\, dt + G(x)$

(B) $\int_{a}^{x} \frac{\partial}{\partial x}(x-t)^2 G(t)\, dt$

(C) $\int_{a}^{x} \frac{\partial}{\partial x}(x-t)^2 G(t)\, dt - (x-a)^2 G(a)$

(D) $(x-t)^2 G(t)$

24. If $y = \int_0^x f(t) \sin a(x-t)\, dt$, then by DUIS rule II, $\frac{dy}{dx}$ is (2)

(A) $\int_0^x af(t) \sin a(x-t)\, dt$ (B) $\int_0^x f(t) \cos a(x-t)\, dt$

(C) $\int_0^x af(t) \cos a(x-t)\, dt$ (D) $\int_0^x af(t) \cos a(x-t)\, dt + f(x)$

25. Using DUIS rule the value of integral $\phi(a) = \int_0^\infty \frac{e^{-x}}{x}(1-e^{-ax})\, dx, a > -1$, given $\frac{d\phi}{da} = \frac{1}{a+1}$ is (2)

(A) $\log(a+1)$ (B) $-\frac{1}{(a+1)^2}$ (C) $\log(a+1)+\pi$ (D) $-\frac{1}{(a+1)^2}+1$

26. Using DUIS rule the value of integral $\phi(a) = \int_0^1 \frac{x^a-1}{\log x}\, dx, a \ge 0$, given $\frac{d\phi}{da} = \frac{1}{a+1}$ is (2)

(A) $\log(a+1)$ (B) $-\frac{1}{(a+1)^2}$ (C) $\log(a+1)+\pi$ (D) $-\frac{1}{(a+1)^2}+1$

27. Using DUIS rule the value of integral $\phi(\alpha) = \int_0^\infty \frac{e^{-2x}\sin \alpha x}{x}\, dx$, with $\frac{d\phi}{d\alpha} = \frac{2}{\alpha^2+4}$ is (2)

(a) $2\log(\alpha^2+4)$ (B) $2\tan^{-1}\left(\frac{\alpha}{2}\right)$ (C) $\frac{1}{2}\tan^{-1}\left(\frac{\alpha}{2}\right)$ (D) $\tan^{-1}\left(\frac{\alpha}{2}\right)$

28. Using DUIS rule the value of integral $\phi(\alpha) = \int_0^\infty \frac{e^{-\alpha x}\sin x}{x}\, dx$, with $\frac{d\phi}{d\alpha} = -\frac{1}{\alpha^2+1}$ and assuming $\int_0^\infty \frac{\sin x}{x}\, dx = \frac{\pi}{2}$ is (2)

(A) $\tan^{-1}\alpha + \frac{\pi}{2}$ (B) $-\tan^{-1}\alpha + \frac{\pi}{2}$ (C) $-\tan^{-1}\alpha$ (D) $\log(\alpha^2+1)+\frac{\pi}{2}$

29. Using DUIS rule the value of integral $\phi(a) = \int_0^{\pi/2} \frac{\log(1+a\sin^2 x)}{\sin^2 x}\, dx$, with $\frac{d\phi}{da} = \frac{\pi}{2}\frac{1}{\sqrt{a+1}}$ is (2)

(A) $\pi\sqrt{a+1}$ (B) $\pi\sqrt{a+1}+\pi$

(C) $\pi\sqrt{a+1}-\pi$ (D) $3\pi(a+1)^{3/2}-\pi$

30. Using DUIS rule the value of integral $\phi(a) = \int_0^\infty \frac{\log(1+ax^2)}{x^2}dx$; $a > 0$ with $\frac{d\phi}{da} = \frac{\pi}{2}\frac{1}{\sqrt{a}}$ is (2)

(A) $\pi\sqrt{a}$ (B) $\frac{\pi\sqrt{a}}{4}$ (C) $-\frac{\pi}{4a^{3/2}}$ (D) $\frac{\pi}{4\sqrt{a}}$

31. Using DUIS rule the value of integral $\phi(a) = \int_0^\infty \frac{e^{-x} - e^{-ax}}{x \sec x}dx$, with $\frac{d\phi}{da} = \frac{a}{a^2+1}$ is (2)

(A) $\tan^{-1} a + \frac{\pi}{4}$ (B) $\log\left(\frac{2}{a^2+1}\right)$ (C) $\frac{1}{2}\log(a^2+1)$ (D) $\frac{1}{2}\log\left(\frac{a^2+1}{2}\right)$

32. Using DUIS rule the value of integral $\phi(a) = \int_0^\infty \frac{1-\cos ax}{x^2}dx$, with $\frac{d\phi}{da} = \frac{\pi}{2}$ is (2)

(A) $\frac{\pi}{2}$ (B) $\frac{\pi a}{2}$ (C) πa (D) $\frac{\pi a}{2} + \frac{\pi}{2}$

ANSWERS

1. (A)	2. (C)	3. (B)	4. (D)	5. (B)	6. (D)	7. (C)	8. (B)
9. (A)	10. (C)	11. (D)	12. (C)	13. (D)	14. (B)	15. (A)	16. (D)
17. (C)	18. (C)	19. (A)	20. (B)	21. (D)	22. (C)	23. (B)	24. (C)
25. (A)	26. (A)	27. (D)	28. (B)	29. (C)	30. (A)	31. (D)	32. (B)

MULTIPLE CHOICE QUESTIONS

Error Functions :

1. Error function of x, erf(x) is defined as (1)

(A) $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\,du$ (B) $\frac{2}{\sqrt{\pi}}\int_0^\infty e^{-u^2}\,du$ (C) $\int_0^\infty e^{-x} x^{n-1}\,dx$ (D) $\frac{2}{\sqrt{\pi}}\int_0^\infty e^{-u^2}\,du$

2. Complimentary error function of x, erfc (c) is defined as (1)

(A) $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\,du$ (B) $\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2}\,du$ (C) $\int_0^\infty e^{-x} x^{n-1}\,dx$ (D) $\frac{2}{\sqrt{\pi}}\int_0^\infty e^{-u^2}\,du$

3. The value of erf (∞) is (1)

(A) 0 (B) ∞ (C) 1 (D) $\frac{2}{\sqrt{\pi}}$

4. The value of erf (0) is (1)

(A) –1 (B) ∞ (C) 1 (D) 0

5. The value of erfc(0) is (1)

(A) –1 (B) ∞ (C) 1 (D) 0

6. Which of the following is true? (1)

(A) $\text{erf}(x) - \text{erfc}(x) = 1$ (B) $\text{erf}(x) + \text{erfc}(x) = 1$

(C) $\text{erf}(x) + \text{erfc}(x) = 2$ (D) $\text{erf}(-x) = \text{erf}(x)$

7. Error function is (1)

(A) a periodic function (B) an even function

(C) a harmonic function (D) an odd function

8. $\text{erf}(-x)$ is equal to (1)

(A) $-\text{erf}(x)$ (B) $\text{erf}(x)$ (C) $\text{erfc}(x)$ (D) $\text{erfc}(-x)$

9. The proper substitution to reduce the integral $\int_0^\infty e^{-(x+a)^2}\,dx$ to complimentary error function is (1)

(A) $(x+a)^2 = u$ (B) $-(x+a) = u$ (C) $x + a = u$ (D) $-(x+a)^2 = u$

10. $\text{erf}(x) + \text{erf}(-x) =$ (1)

(A) 2 (B) 1 (C) −1 (D) 0

11. $\text{erf}(-x) + \text{erfc}(-x) =$ (1)

(A) 2 (B) 1 (C) −1 (D) 0

12. $\text{erfc}(-x) - \text{erf}(x) =$ (1)

(A) 2 (B) −1 (C) 1 (D) 0

13. $\text{erfc}(-x) + \text{erfc}(x) =$ (1)

(A) 2 (B) −1 (C) 1 (D) 0

14. If $\text{erf}(ax) = \frac{2}{\sqrt{\pi}}\int_0^{ax} e^{-u^2}\,du$ then $\frac{d}{dx}\text{erf}(ax)$ is (2)

(A) $\frac{2x}{\sqrt{\pi}}e^{-a^2x^2}$ (B) $\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$ (C) $ae^{-a^2x^2}$ (D) $\frac{2a}{\sqrt{\pi}}e^{a^2x^2}$

15. If $\text{erf}(\sqrt{t}) = \frac{2}{\sqrt{\pi}}\int_0^{\sqrt{t}} e^{-u^2}\,du$ then $\frac{d}{dt}\text{erf}(\sqrt{t})$ is (2)

(A) $\frac{e^{-t}}{2\sqrt{t}}$ (B) $\frac{e^{-t^2}}{\sqrt{\pi t}}$ (C) $\frac{e^{-t}}{\sqrt{\pi}}$ (D) $\frac{e^{-t}}{\sqrt{\pi t}}$

16. If $\operatorname{erfc}(ax) = \frac{2}{\sqrt{\pi}} \int_{ax}^{\infty} e^{-u^2} du$ then $\frac{d}{dx} \operatorname{erfc}(ax)$ is (2)

(a) $-\frac{1}{\sqrt{\pi}} e^{-a^2x^2}$ (B) $-\frac{2}{\sqrt{\pi}} e^{-a^2x^2}$ (C) $-\frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$ (D) $\frac{2a}{\sqrt{\pi}} e^{a^2x^2}$

17. If $\operatorname{erfc}\left(\sqrt{t}\right) = \frac{2}{\sqrt{\pi}} \int_{\sqrt{t}}^{\infty} e^{-u^2} du$ then $\frac{d}{dt} \operatorname{erfc}\left(\sqrt{t}\right)$ is (2)

(A) $\frac{e^{-t}}{2\sqrt{t}}$ (B) $-\frac{e^{-t}}{\sqrt{\pi t}}$ (C) $\frac{e^{-t}}{\sqrt{\pi t}}$ (D) $\frac{e^{-t^2}}{\sqrt{\pi t}}$

18. If $\frac{d}{dx} \operatorname{erf}(ax) = \frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$ then $\frac{d}{dx} \operatorname{erfc}(ax)$ is (2)

(A) $\frac{2a}{\sqrt{\pi}} e^{a^2x^2}$ (B) $1 - \frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$ (C) $-\frac{1}{\sqrt{\pi}} e^{-a^2x^2}$ (D) $-\frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$

19. If $\frac{d}{dt} \operatorname{erf}\left(\sqrt{t}\right) = \frac{e^{-t}}{\sqrt{\pi t}}$ then $\frac{d}{dt} \operatorname{erfc}\left(\sqrt{t}\right)$ is (2)

(A) $-\frac{e^{-t}}{\sqrt{\pi t}}$ (B) $1 - \frac{e^{-t}}{\sqrt{\pi t}}$ (C) $-\frac{e^{t}}{\sqrt{\pi t}}$ (D) $\frac{e^{-t}}{\sqrt{\pi t}}$

20. If $\frac{d}{dx} \operatorname{erfc}(ax) = -\frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$ then $\frac{d}{dx} \operatorname{erf}(ax)$ is (2)

(A) $ae^{-a^2x^2}$ (B) $1 - \frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$ (C) $\frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$ (D) $-\frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$

21. If $\frac{d}{da} \operatorname{erfc}(ax) = -\frac{2x}{\sqrt{\pi}} e^{-a^2x^2}$ then $\frac{d}{da} \operatorname{erf}(ax)$ is (2)

(A) $xe^{-a^2x^2}$ (B) $-\frac{2x}{\sqrt{\pi}} e^{-a^2x^2}$ (C) $\frac{1}{\sqrt{\pi}} e^{-a^2x^2}$ (D) $\frac{2x}{\sqrt{\pi}} e^{-a^2x^2}$

22. If $\operatorname{erfc}(a) = \frac{2}{\sqrt{\pi}} \int_{a}^{\infty} e^{-u^2} du$ then by using substitution $x + a = u$, the integral $\int_{0}^{\infty} e^{-(x+a)^2} dx$ in terms of erfc (a) is (2)

(A) $\frac{2}{\sqrt{\pi}} \operatorname{erfc}(a)$ (B) erfc (a) (C) $\frac{\sqrt{\pi}}{2} \operatorname{erfc}(a)$ (D) $\frac{\sqrt{\pi}}{2} \operatorname{erf}(a)$

23. $\int_0^t \text{erf}(ax)\,dx + \int_0^t \text{erfc}(ax)\,dx =$ (2)

(A) t (B) x (C) 0 (D) $\frac{t^2}{2}$

24. The integral $\int_0^\infty e^{-t}\,\text{erf}\left(\sqrt{t}\right) dt$ using $\frac{d}{dt}\text{erf}\left(\sqrt{t}\right) = \frac{e^{-t}}{\sqrt{\pi t}}$ is (2)

(A) $\frac{1}{\sqrt{\pi}}\int_0^\infty e^{2t}\, t^{1/2}\, dt$

(B) $\frac{1}{\sqrt{\pi}}\int_0^\infty e^{-2t}\, t^{-1/2}\, dt$

(C) $\int_0^\infty e^{-2t}\, t^{-1/2}\, dt$

(D) $\frac{1}{\sqrt{\pi}}\int_0^\infty e^{-t}\, t^{1/2}\, dt$

25. Expansion of erf (x) in series is (2)

(A) $\frac{2}{\sqrt{\pi}}\left[x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \ldots\right]$

(B) $\frac{2}{\sqrt{\pi}}\left[x + \frac{x^3}{3} + \frac{x^5}{10} + \frac{x^7}{42} + \ldots\right]$

(C) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$

(D) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

ANSWERS

1. (A)	2. (B)	3. (C)	4. (D)	5. (C)	6. (B)	7. (D)	8. (A)
9. (C)	10. (D)	11. (B)	12. (C)	13. (A)	14. (B)	15. (D)	16. (C)
17. (B)	18. (D)	19. (A)	20. (C)	21. (D)	22. (C)	23. (A)	24. (B)
25. (A)							

Fig. 6.51

MULTIPLE CHOICE QUESTIONS

Curve Tracing :

1. If the powers of y in the cartesian equation are even everywhere then the curve is symmetrical about (1)

(A) x-axis
(B) y-axis
(C) both x and y axes
(D) line $y = x$

2. If the powers of x in the cartesian equation are even everywhere then the curve is symmetrical about (1)

(A) x-axis (B) y-axis

(C) both x and y axes (D) line y = x

3. If the powers of x and y both in the cartesian equation are even everywhere then the curve is symmetrical about (1)

(A) x-axis only (B) y-axis only

(C) both x and y axes (D) line y = x

4. On replacing x and y by –x and –y respectively if the cartesian equation remains unchanged then the curve is symmetrical about (1)

(A) line y = x (B) y-axis

(C) both x and y axes (D) opposite quadrants

5. If x and y are interchanged and cartesian equation remains unchanged then the curve is symmetrical about (1)

(A) both x and y axes (B) line y = –x

(C) line y = x (D) opposite quadrants

6. If x is changed to –y and y to –x and cartesian equation remains unchanged then the curve is symmetrical about (1)

(A) both x and y axes (B) line y = –x line y = -x

(C) line y = x (D) opposite quadrants

7. If the curve passes through origin then tangents at origin to the cartesian curve can be obtained by equating to zero (1)

(A) lowest degree term in the equation

(B) highest degree term in the equation

(C) coefficient of lowest degree term in the equation

(D) coefficient of highest degree term in the equation

8. A double point is called node if the tangents to the curve at the double point are (1)

(A) real and equal (B) imaginary

(C) always perpendicular (D) real and distinct

9. A double point is called cusp if the tangents to the curve at the double point are (1)

(A) real and equal (B) imaginary

(C) always perpendicular (D) real and distinct

10. In cartesian equation the points where $\frac{dy}{dx} = 0$, tangent to the curve at those points will be (1)

(A) parallel to y-axis
(B) parallel to x-axis
(C) parallel to $y = x$
(D) parallel to $y = -x$

11. In cartesian equation the points where $\frac{dy}{dx} = \infty$, tangent to the curve at those points will be (1)

(A) parallel to $y = -x$
(B) parallel to x-axis
(C) parallel to $y = x$
(D) parallel to y-axis

12. The asymptotes to the cartesian curve parallel to x-axis if exists is obtained by equating to zero (1)

(A) coefficient of highest degree term in y
(B) lowest degree term in the equation
(C) coefficient of highest degree term in x
(D) highest degree term in the equation

13. The asymptotes to the cartesian curve parallel to y-axis if exists is obtained by equating to zero (1)

(A) coefficient of highest degree term in y
(B) lowest degree term in the equation
(C) coefficient of highest degree term in x
(D) highest degree term in the equation

14. If the polar equation to the curve remains unchanged by changing θ to $-\theta$ then the curve is symmetrical about (1)

(A) line $\theta = \frac{\pi}{4}$
(B) pole
(C) line $\theta = \frac{\pi}{2}$
(D) initial line $\theta = 0$

15. If the polar equation to the curve remains unchanged by changing r to $-r$ then the curve is symmetrical about (1)

(A) line $\theta = \frac{\pi}{4}$
(B) pole
(C) line $\theta = \frac{\pi}{2}$
(D) initial line $\theta = 0$

16. If the polar equation to the curve remains unchanged by changing θ to $\pi - \theta$ then the curve is symmetrical about (1)

(A) initial line $\theta = 0$
(B) pole
(C) line passing through pole and perpendicular to the initial line
(D) line $\theta = \frac{\pi}{4}$

17. Pole will lie on the curve if for some value of θ (1)

(A) r becomes zero
(B) r becomes infinite
(C) $r > 0$
(D) $r < 0$

18. The tangents to the polar curve at pole if exist can be obtained by putting in the polar (1)

    (A) $\theta = 0$ (B) $\theta = \pi$ (C) $r = 0$ (D) $r = a, a > 0$

19. For the rose curve $r = a \cos n\theta$ and $r = a \sin n\theta$ if n is odd then the curve consist of (1)

    (A) 2n equal loops (B) (n + 1) equal loops
    (C) (n − 1) equal loops (D) n equal loops

20. For the rose curve $r = a \cos n\theta$ and $r = a \sin n\theta$ if n is even then the curve consist of (1)

    (A) (n + 1) equal loops (B) 2n equal loops
    (C) (n − 1) equal loops (D) n equal loops

21. For the polar curve, angle ϕ between radius vector and tangent line is obtained by the formula (1)

    (A) $\cot \phi = r\frac{d\theta}{dr}$ (B) $\tan \phi = r\frac{d\theta}{dr}$ (C) $\tan \phi = r\frac{dr}{d\theta}$ (D) $\sin \phi = r\frac{d\theta}{dr}$

22. The cartesian parametric curve $x = f(t)$, $y = g(t)$ is symmetrical about x-axis if (1)

    (A) f(t) is even and g(t) is odd (B) f(t) is odd and g(t) is even
    (C) f(t) is even and g(t) is even (D) f(t) is odd and g(t) is odd

23. The cartesian parametric curve $x = f(t)$, $y = g(t)$ is symmetrical about y-axis if (1)

    (A) f(t) is even and g(t) is odd (B) f(t) is even and g(t) is even
    (C) f(t) is odd and g(t) is even (D) f(t) is odd and g(t) is odd

24. The curve represented by the equation $x^{1/2} + y^{1/2} = a^{1/2}$ is symmetrical about (1)

    (A) $y = -x$ (B) x-axis
    (C) both x and y axes (D) $y = x$

25. The curve represented by the equation $x^2 y^2 = x^2 + 1$ is symmetrical about (1)

    (A) $y = -x$ (B) x-axis only
    (C) both x and y axes (D) $y = x$

26. The curve represented by the equation $r^2\theta = a^2$ is symmetrical about (1)

    (A) pole (B) initial line $\theta = 0$
    (C) line $\theta = \frac{\pi}{2}$ (D) line $\theta = \frac{\pi}{4}$

27. The curve represented by the equation $r = 2a \sin \theta$ is symmetrical about (1)

    (A) pole (B) initial line $\theta = 0$
    (C) line $\theta = \frac{\pi}{4}$ (D) line $\theta = \frac{\pi}{2}$

28. The curve represented by the equation $x = at^2$, $y = 2at$ is symmetrical about (1)

(A) y-axis (B) x-axis

(C) both x and y axes (D) opposite quadrants

29. The asymptote parallel to y-axis to the curve $xy^2 = a^2(a - x)$ is (1)

(A) $y = 0$ (B) $x = 0$ (C) $x = a$ (D) $x = -a$

30. The number of loops in the rose curve $r = a \cos 2\theta$ are (1)

(A) 4 (B) 2 (C) 3 (D) 8

31. The number of loops in the rose curve $r = a \sin 3\theta$ are (1)

(A) 6 (B) 4 (C) 3 (D) 9

32. The curve represented by the equation $y^2(2a - x) = x^3$ is (2)

(A) symmetrical about y-axis and passing through origin

(B) symmetrical about x-axis and not passing through origin

(C) symmetrical about y-axis and passing through (2a, 0)

(D) symmetrical about x-axis and passing through origin

33. The curve represented by the equation $x(x^2 + y^2) = a(x^2 - y^2)$ is (2)

(A) symmetrical about x-axis and passing through origin

(B) symmetrical about x-axis and not passing through origin

(C) symmetrical about y-axis and passing through (a, 0)

(D) symmetrical about y-axis and passing through origin

34. The curve represented by the equation $a^2x^2 = y^3(2a - y)$ is (2)

(A) symmetrical about x-axis and passing through (2a, 0)

(B) symmetrical about both x-axis and y-axis and passing through origin

(C) symmetrical about y-axis and passing through (0, 2a)

(D) symmetrical about both x-axis and y-axis and passing through (2a, 0)

35. The equation of tangents to the curve at origin, if exist, represented by the equation $y^2(2a - x) = x^3$ is (2)

(A) $y = 0, y = 0$

(B) $x = 0, x = 2a$

(C) $x = 0, x = 0$

(D) $y = x$

36. The equation of tangents to the curve at origin, if exist, represented by the equation $y(1+x^2) = x$ is (2)

(A) $y = x$ (B) $x = 0$

(C) $x = 1$ and $x = -1$ (D) $y = 0$

37. The equation of tangents to the curve at origin, if exists, represented by the equation $3ay^2 = x(x-a)^2$ is (2)

(A) $x = a$ (B) $x = 0$ and $y = 0$

(C) $x = 0$ (D) $y = 0$

38. The equation of asymptotes parallel to x-axis to the curve represented by the equation $y(1+x^2) = x$ is (2)

(A) $x = 1, x = -1$ (B) $x = 0$

(C) $y = x$ (D) $y = 0$

39. The equation of asymptotes parallel to y-axis to the curve represented by the equation $y^2(4-x) = x(x-2)^2$ is (2)

(A) $x = 2$ (B) $x = 4$ (C) $y = 0$ (D) $x = 0$

40. The equation of asymptotes parallel to y-axis to the curve represented by the equation $x^2y^2 = a^2(y^2 - x^2)$ is (2)

(A) $x = a, x = -a$ (B) $y = a, y = -a$ (C) $y = x, y = -x$ (D) $x = 0, y = 0$

41. The region of absence for the curve represented by the equation $x^2 = \frac{4a^2(2a-y)}{y}$ is (2)

(A) $y < 0$ and $y > 2a$ (B) $y > 0$ and $y < 2a$

(C) $y > 0$ and $y > 2a$ (D) $y < 0$ and $y < 2a$

42. The region of absence for the curve represented by the equation $y^2(2a-x) = x^3$ is (2)

(A) $x > 0$ and $x < 2a$ (B) $x < 0$ and $x > 2a$

(C) $x < 0$ and $x < 2a$ (D) $x > 0$ and $x > 2a$

43. The region of absence for the curve represented by the equation $xy^2 = a^2(a-x)$ is (2)

(A) $x > 0$ and $x < a$ (B) $x < 0$ and $x < a$

(C) $x < 0$ and $x > a$ (D) $x > 0$ and $x > a$

44. The region of absence for the curve represented by the equation $y^2 = \frac{x^2(a-x)}{a+x}$ is (2)

(A) $x > a$ and $x > -a$ (B) $x < a$ and $x < -a$

(C) $x < a$ and $x > -a$ (D) $x > a$ and $x < -a$

45. The region of absence for the curve represented by the equation $x^2 = \frac{a^2y^2}{a^2-y^2}$ is (2)

(A) $y < a$ and $y > -a$ (B) $y > a$ and $y < -a$

(C) $y > a$ and $y > -a$ (D) $y < a$ and $y < -a$

46. The curve represented by the equation $r = a(1 + \cos\theta)$ is (2)

(A) symmetrical about initial line and passing through pole

(B) symmetrical about initial line and not passing through pole

(C) symmetrical about $\theta = \frac{\pi}{2}$ and passing through pole

(D) symmetrical about $\theta = \frac{\pi}{4}$ and passing through pole

47. The curve represented by the equation $r^2 = a^2 \cos 2\theta$ is (2)

(A) symmetrical about $\theta = \frac{\pi}{2}$ and not passing through pole

(B) symmetrical about $\theta = \frac{\pi}{4}$ and not passing through pole

(C) symmetrical about initial line and pole

(D) symmetrical about $\theta = \frac{\pi}{4}$ and passing through pole

48. The curve represented by the equation $r^2 = a^2 \sin 2\theta$ is (2)

(A) symmetrical about initial line and passing through pole

(B) symmetrical about initial line and not passing through pole

(C) symmetrical about $\theta = \frac{\pi}{2}$ and passing through pole

(D) symmetrical about $\theta = \frac{\pi}{4}$ and passing through pole

49. The curve represented by the equation $r = \frac{2a}{1+\cos\theta}$ is (2)

(A) symmetrical about initial line and passing through pole

(B) symmetrical about initial line and not passing through pole

(C) symmetrical about $\theta = \frac{\pi}{2}$ and passing through pole

(D) symmetrical about $\theta = \frac{\pi}{4}$ and passing through pole

50. The tangents at pole to the polar curve $r = a \sin 3\theta$ are (2)

(A) $\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, \ldots$ (B) $\theta = \frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \ldots$

(C) $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi \ldots$ (D) $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots$

51. The tangents at pole to the polar curve $r = a \cos 2\theta$ are (2)

(A) $\theta = 0, \pi, 2\pi, 3\pi, \ldots$ (B) $\theta = \frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \ldots$

(C) $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots$ (D) $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots$

52. The curve represented by the equation $x = t^2, y = t - \frac{t^3}{3}$ is (2)

(A) symmetrical about y-axis and passing through origin

(B) symmetrical about x-axis and not passing through origin

(C) symmetrical about y-axis and passing through (3, 0)

(D) symmetrical about x-axis and passing through origin

53. The curve represented by the equation $x = a (t + \sin t), y = a (1 + \cos t)$ is (2)

(A) symmetrical about y-axis and not passing through origin

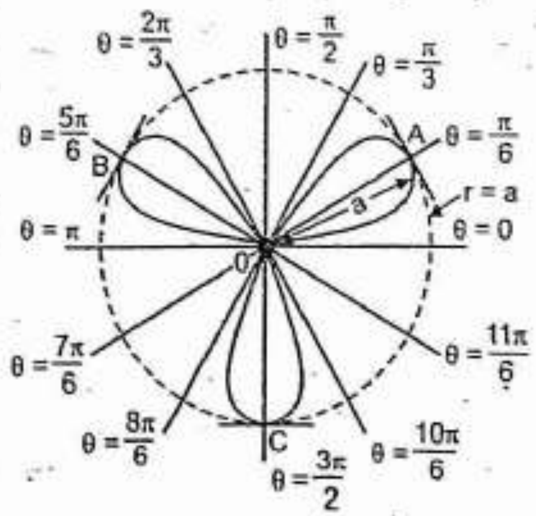
(B) symmetrical about x-axis and not passing through origin

(C) symmetrical about y-axis and passing through origin

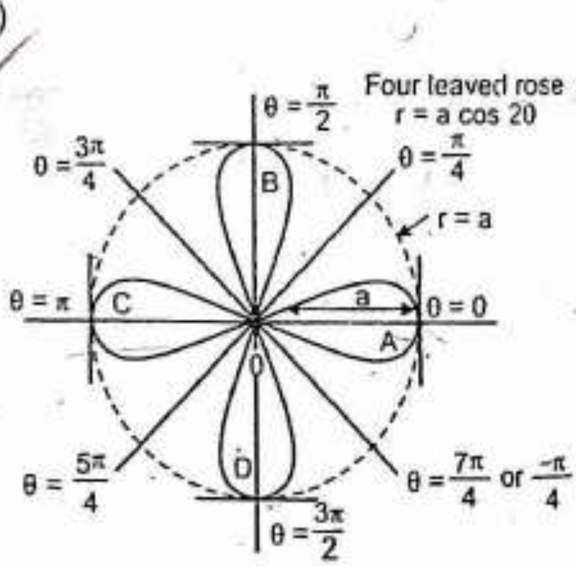
(D) symmetrical about x-axis and passing through origin

54. The equation $r = a \cos 2\theta$ represents the curve (1)

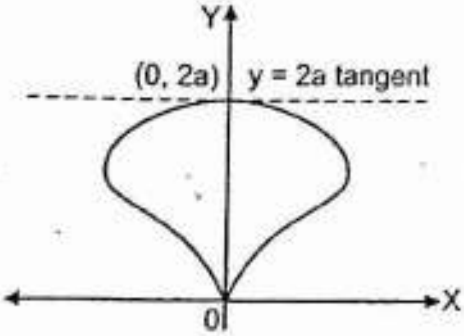
(A)



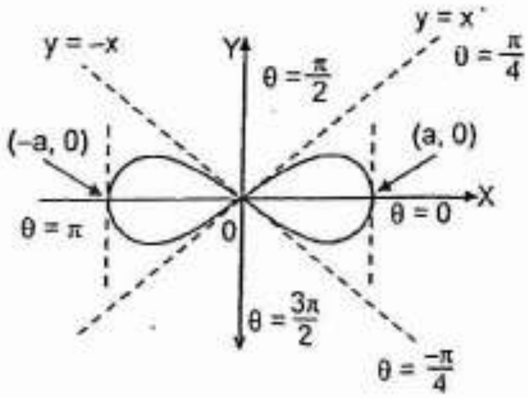
(B)



(C)

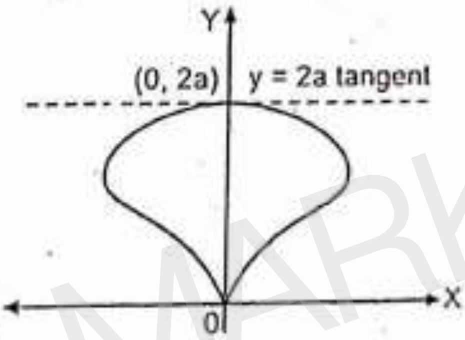


(D)



55. The equation $r = a \sin 3\theta$ represents the curve (1)

(A)



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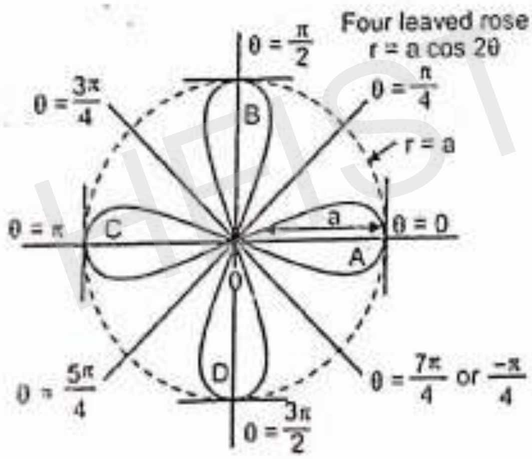
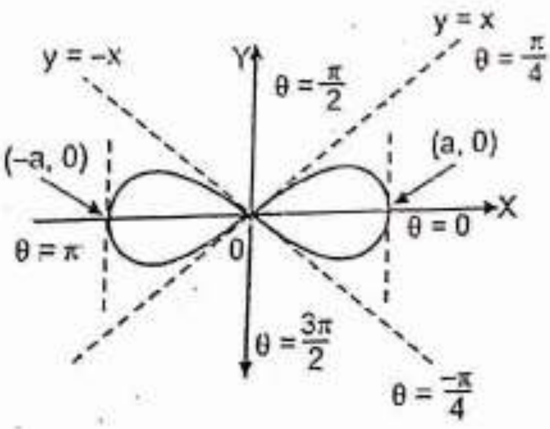
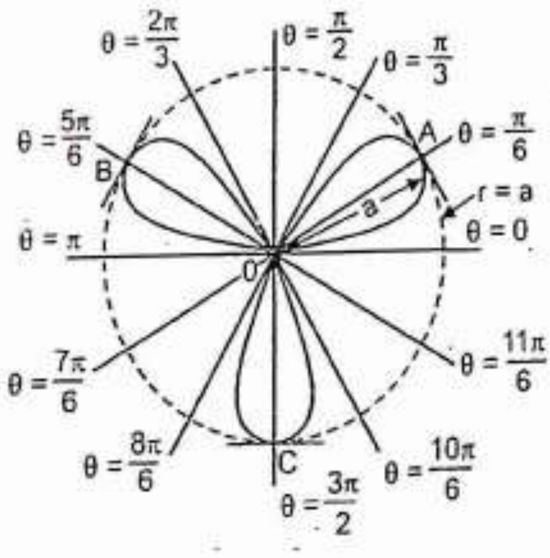


(C)

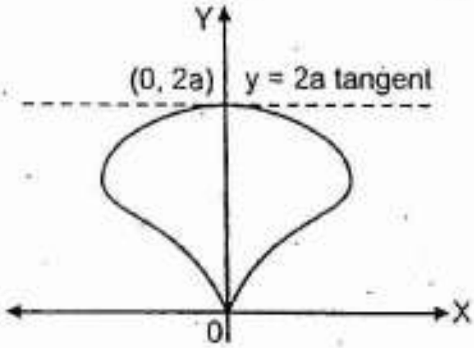


(D)

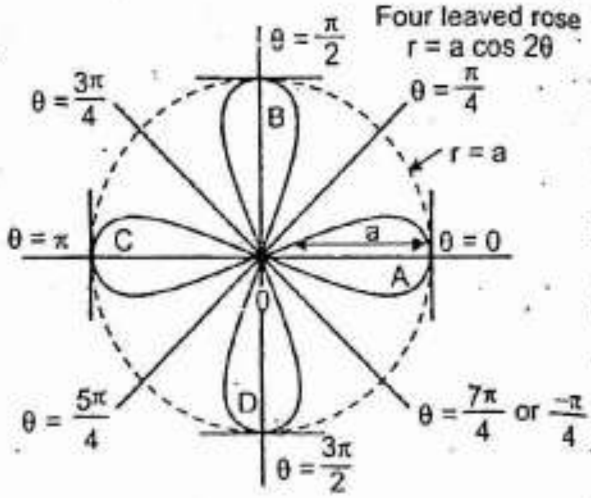


56. The equation $r^2 = a^2 \cos 2\theta$ represents the curve (1)

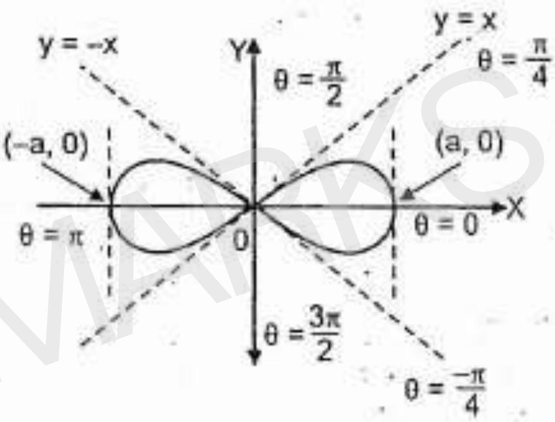
(A)



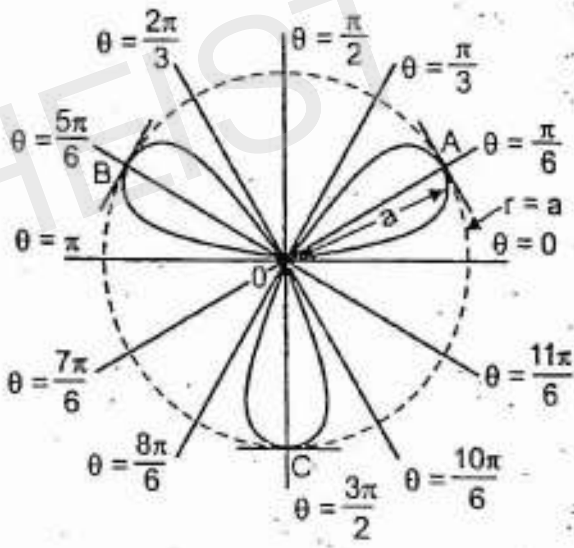
(B)



(C)

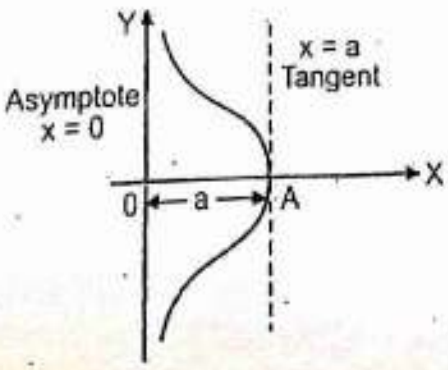


(D)

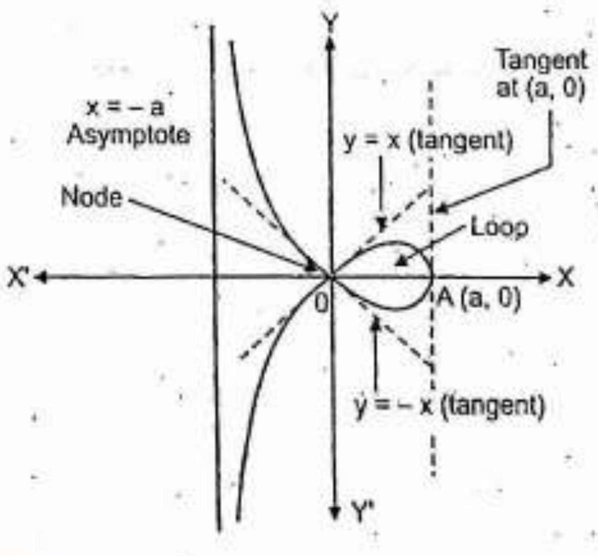


57. The equation $xy^2 = a^2 (a - x)$ represents the curve (2)

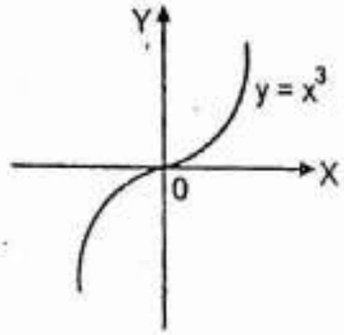
(A)



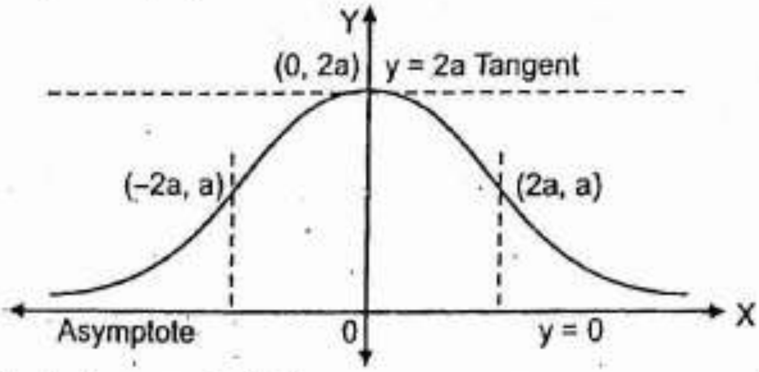
(B)



(C)

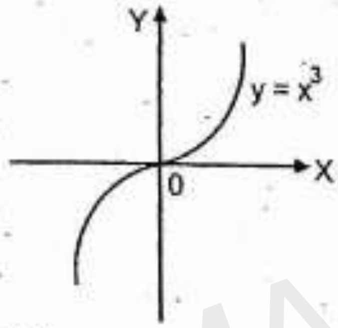


(D)

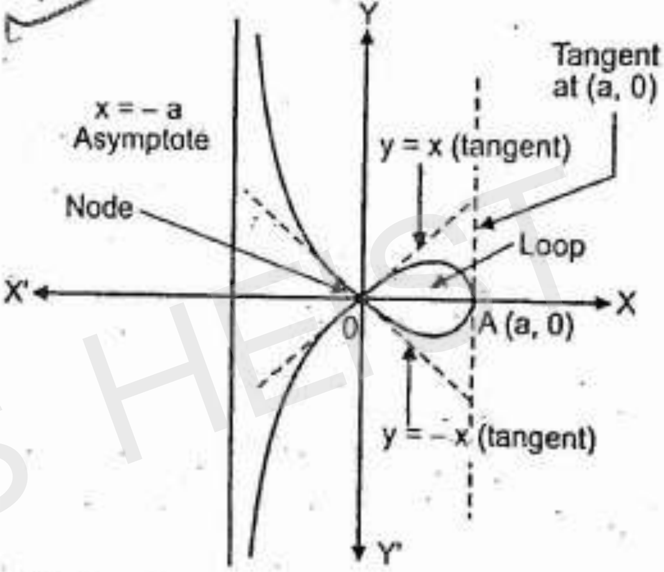


58. The equation $x(x^2 + y^2) = a(x^2 - y^2)$, $a > 0$ represents the curve (2)

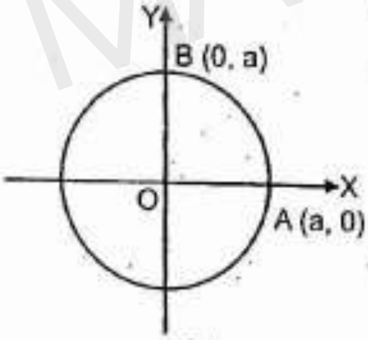
(A)



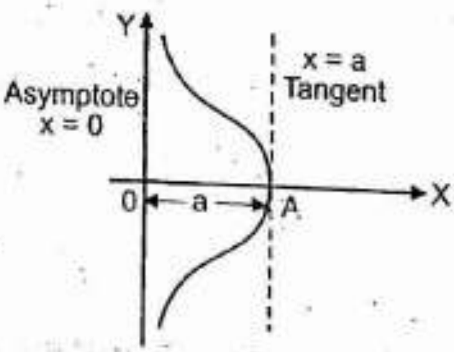
(B)



(C)

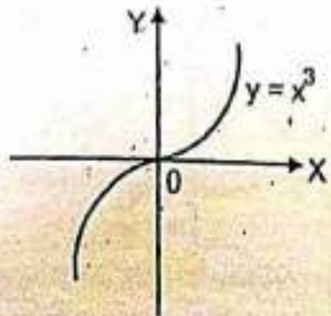


(D)

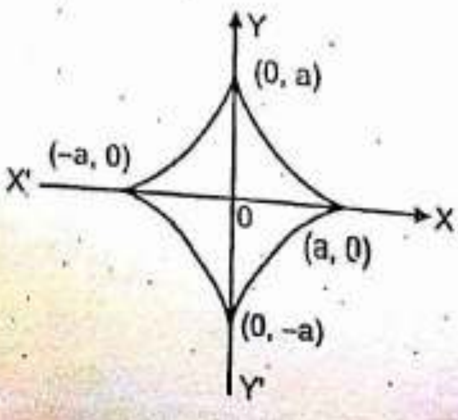


59. The equation $x^{2/3} + y^{2/3} = a^{2/3}$ represents the curve (2)

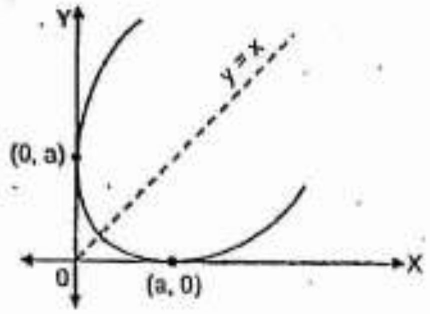
(A)



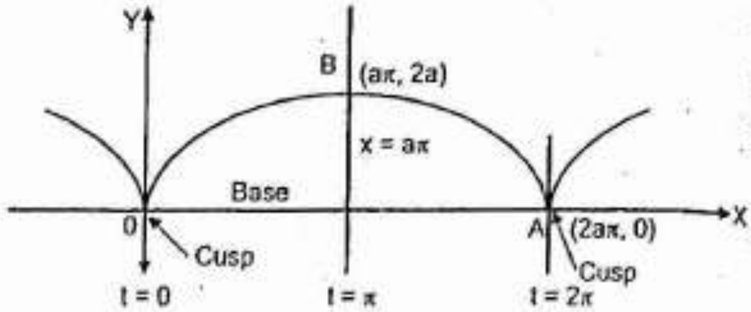
(B)



(C)

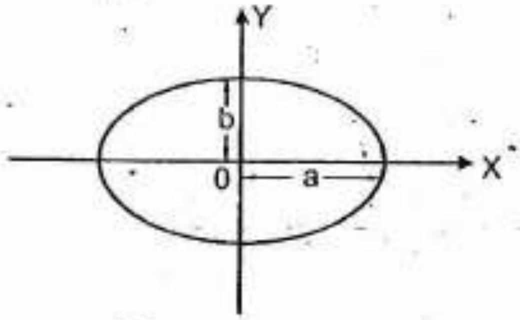


(D)

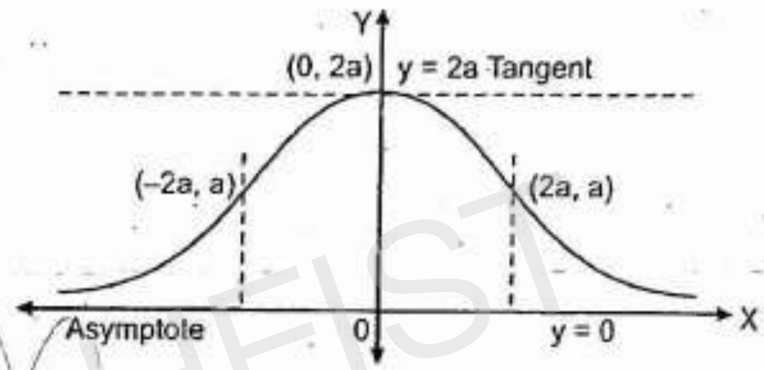


60. The equation $a^2x^2 = y^3 (2a - y)$, $a > 0$ represents the curve (2)

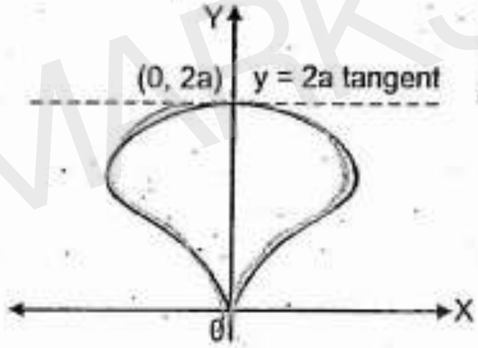
(A)



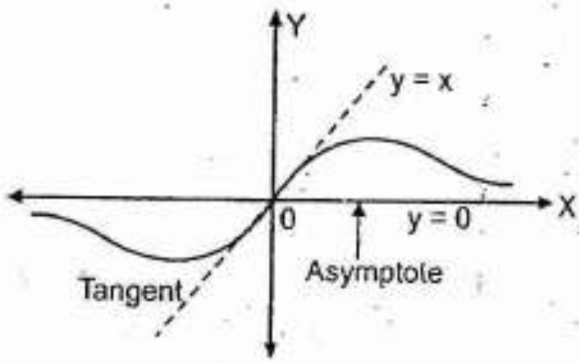
(B)



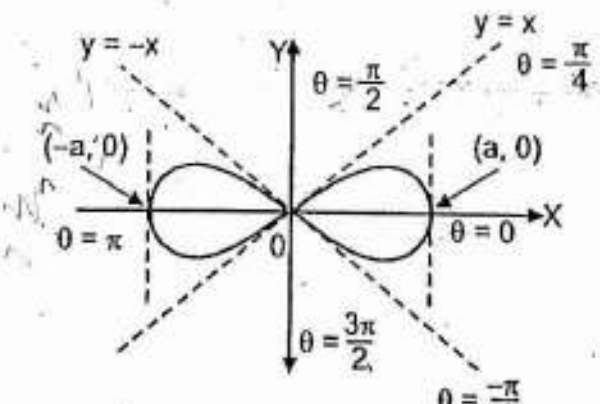
(C)



(D)

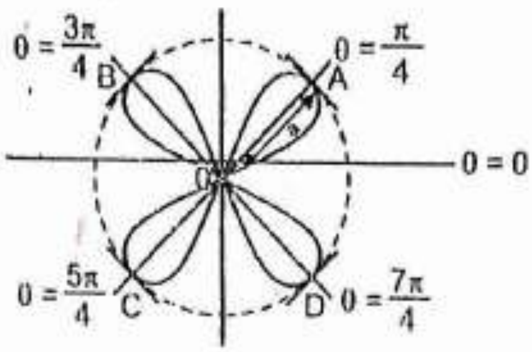


61. The following figure represents the curve whose equation is (2)



(A) $r^2 = a^2 \cos 2\theta$ (B) $r^2 = a^2 \sin 2\theta$ (C) $r = a \cos 2\theta$ (D) $r = a(1 + \cos\theta)$

62. The following figure represents the curve whose equation is (2)



(A) $r = a \cos 3\theta$

(B) $r = a \sin 2\theta$

(C) $r = a \sin 3\theta$

(D) $r = a\,(1 + \cos \theta)$

ANSWERS

1. (A)	2. (B)	3. (C)	4. (D)	5. (C)	6. (B)	7. (A)	8. (D)
9. (A)	10. (B)	11. (D)	12. (C)	13. (A)	14. (D)	15. (B)	16. (C)
17. (A)	18. (C)	19. (D)	20. (B)	21. (B)	22. (A)	23. (C)	24. (D)
25. (C)	26. (A)	27. (D)	28. (B)	29. (B)	30. (A)	31. (C)	32. (D)
33. (A)	34. (C)	35. (D)	36. (A)	37. (C)	38. (D)	39. (B)	40. (A)
41. (A)	42. (B)	43. (C)	44. (D)	45. (B)	46. (A)	47. (C)	48. (D)
49. (B)	50. (A)	51. (C)	52. (D)	53. (A)	54. (B)	55. (D)	56. (C)
57. (A)	58. (B)	59. (B)	60. (C)	61. (A)	62. (B)		

MULTIPLE CHOICE QUESTIONS

Rectification of Curves :

1. Formula for measuring the arc length AB where $A(x_1, y_1)$, $B(x_2, y_2)$ are any two points on the curve $y = f(x)$ is (1)

(A) $\int_{x_1}^{x_2} \sqrt{dx}$

(B) $\int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dx$

(C) $\int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

(D) $\int_{x_1}^{x_2} \sqrt{1 - \left(\frac{dy}{dx}\right)^2}\, dx$

2. Formula for measuring the arc length AB where $A(x_1, y_1)$, $B(x_2, y_2)$ are any two points on the curve $x = g(y)$ is (1)

(A) $\int_{y_1}^{y_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dy$

(B) $\int_{y_1}^{y_2} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

(C) $\int_{0}^{y_2} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

(D) $\int_{y_1}^{y_2} \sqrt{1 - \left(\frac{dx}{dy}\right)^2}\, dy$

3. Formula for measuring the arc length AB where $A(r_1, \theta_1)$, $B(r_2, \theta_2)$ are any two points on the curve $r = f(\theta)$ is (1)

(A) $\int_{\theta_1}^{\theta_2} \sqrt{1 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

(B) $\int_{\theta_1}^{\theta_2} \sqrt{1 + r^2\left(\frac{d\theta}{dr}\right)^2}\, dr$

(C) $\int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

(D) $\int_{\theta_1}^{\theta_2} \sqrt{r^2 + r^2\left(\frac{dr}{d\theta}\right)^2}\, dr$

4. Formula for measuring the arc length AB where $A(r_1, \theta_1)$, $B(r_2, \theta_2)$ are any two points on the curve $\theta = f(r)$ is (1)

(A) $\int_{r_1}^{r_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, dr$

(B) $\int_{r_1}^{r_2} \sqrt{1 + \left(\frac{d\theta}{dr}\right)^2}\, dr$

(C) $\int_{r_1}^{r_2} \sqrt{1 - r^2\left(\frac{d\theta}{dr}\right)^2}\, dr$

(D) $\int_{r_1}^{r_2} \sqrt{1 + r^2\left(\frac{d\theta}{dr}\right)^2}\, dr$

5. Formula for measuring the arc length AB where A, B are any two points on the parametric curve $x = f_1(t)$, $y = f_2(t)$, corresponding to parameters t_1, t_2 respectively is (1)

(A) $\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 - \left(\frac{dy}{dt}\right)^2}\, dt$ (B) $\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dx}\right)^2}\, dt$

(C) $\int_{t_1}^{t_2} \left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right] dt$ (D) $\int_{t_1}^{t_2} \sqrt{x^2(t) + y^2(t)}\, dt$

6. The arc length AB where A(a, 0), B(0, a) are any two points on the circle $x^2 + y^2 = a^2$, using $1 + \left(\frac{dy}{dx}\right)^2 = \frac{a^2}{a^2 - x^2}$, is (2)

(A) $\frac{\pi a}{2}$ (B) a log a (C) $\frac{\pi a}{4}$ (D) a

7. The length of arc from vertex (0, 0) to any point (x, y) of catenary $y = a\cosh\frac{x}{a}$, using $1 + \left(\frac{dy}{dx}\right)^2 = \cosh^2\frac{x}{a}$, is (2)

(A) $a\cosh\frac{x}{a}$ (B) $\sinh\frac{x}{a}$ (C) $a\sinh\frac{x}{a}$ (D) $\cosh\frac{x}{a}$

8. The length of arc of upper part of loop of the curve $3y^2 = x(x-1)^2$ from (0, 0) to (1, 0) using $1 + \left(\frac{dy}{dx}\right)^2 = \frac{(3x+1)^2}{12x}$, is (2)

(A) $\frac{4}{\sqrt{3}}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\sqrt{3}$ (D) $\frac{2}{\sqrt{3}}$

9. Integral for calculating the length of upper arc of loop of the curve $9y^2 = (x+7)(x+4)^2$ is (2)

(A) $\int_{-7}^{-4} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ (B) $\int_{4}^{7} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

(C) $\int_{0}^{-4} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ (D) $\int_{-7}^{0} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

10. Integral for calculating the length of arc of parabola $y^2 = 4x$, cut off by the line $3y = 8x$ is (2)

(A) $\int_{0}^{\frac{16}{9}} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ (B) $\int_{0}^{\frac{9}{16}} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

(C) $\int_{0}^{\frac{8}{3}} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ (D) $\int_{0}^{\frac{3}{8}} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

11. The length of upper half of cardioide $r = a(1 + \cos\theta)$ where θ varies from 0 to π using $r^2 + \left(\frac{dr}{d\theta}\right)^2 = 2a^2(1 + \cos\theta)$, is (2

(A) 2a (B) 8a (C) 4a (D) a

12. The length of the arc of curve $r = ae^{m\theta}$ intercepted between radii r_1 and r_2 using $1 + r^2\left(\frac{d\theta}{dr}\right)^2 = 1 + \frac{1}{m^2}$, is (2

(A) $\frac{m}{\sqrt{1+m^2}}(r_2 - r_1)$ (B) $\frac{\sqrt{1+m^2}}{m} r_2$

(C) $\frac{\sqrt{1+m^2}}{m}(r_2 + r_1)$ (D) $\frac{\sqrt{1+m^2}}{m}(r_2 - r_1)$

13. Integral for calculating the length of cardioide $r = a(1 + \cos\theta)$ which lies outside the circle $r = -a\cos\theta$ is (2

(A) $2\int_0^{\pi/3}\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,dr$ (B) $2\int_0^{\pi}\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,dr$

(C) $2\int_0^{2\pi/3}\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$ (D) $2\int_0^{\pi/2}\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,dr$

14. Integral for calculating the length of upper arc of one loop of Bernoulli's lemniscate $r^2 = a^2\cos 2\theta$ in the first quadrant is (2)

(A) $\int_0^{\pi/4}\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$ (B) $\int_0^{\pi/6}\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,dr$

(C) $\int_0^{\pi/2}\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,dr$ (D) $\int_0^{\pi/3}\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,dr$

15. Integral for calculating the length of upper arc of loop of the curve $x = t^2$, $y = t\left(1 - \frac{t^2}{3}\right)$ is (2)

(A) $\int_0^{9}\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$ (B) $\int_0^{3}\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

(C) $\int_0^{1}\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$ (D) $\int_0^{\sqrt{3}}\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

16. Integral for calculating the length of the arc of Astroid $x = a\cos^3\theta$, $y = a\sin^3\theta$ in the first quadrant between two consecutive cusps, is (2)

(A) $\int_0^{\pi/4}\sqrt{\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2}\,d\theta$ (B) $\int_0^{\pi/2}\sqrt{\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2}\,d\theta$

(C) $\int_0^{\pi/3}\sqrt{\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2}\,d\theta$ (D) $\int_0^{\pi/6}\sqrt{\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2}\,d\theta$

17. The length of arc of upper part of loop of the curve $x = t^2$, $y = t\left(1-\frac{t^2}{3}\right)$ where t varies from 0 to $\sqrt{3}$, using $\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2 = (1+t^2)^2$, is (2)

(A) $2\sqrt{3}$ (B) $4\sqrt{3}$ (D) $\sqrt{3}$ (D) $\frac{2}{\sqrt{3}}$

18. The length of the arc of Astroid $x = a\cos^3\theta$, $y = a\sin^3\theta$ in the first quadrant between two consecutive cusps, where θ varies from 0 to $\frac{\pi}{2}$, using $\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2 = 9a^2\sin^2\theta\cos^2\theta$, is (2)

(A) 3a (B) $\frac{3a}{2}$ (C) $\frac{3a}{4}$ (D) $\frac{3a}{8}$

19. The length of arc of the curve $x = e^\theta\cos\theta$, $y = e^\theta\sin\theta$, from $\theta = 0$ to $\theta = \frac{\pi}{2}$, using $\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2 = 2e^{2\theta}$, is (2)

(A) $\sqrt{2}\,e^{\pi/2}$ (B) $\sqrt{2}\,(e^{\pi/2}+1)$ (C) $\sqrt{2}\,(e^{\pi/2}-1)$ (D) $(e^{\pi/2}+1)$

20. The length of arc of the cycloid $x = a(\theta+\sin\theta)$, $y = a(1-\cos\theta)$, from one cusp $\theta = -\pi$ to another cusp $\theta = \pi$, using $\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2 = 4a^2\cos^2\frac{\theta}{2}$, is (2)

(A) 2a (B) a (C) 4a (D) 8a

ANSWERS

1. (C)	2. (B)	3. (C)	4. (D)	5. (B)	6. (A)	7. (C)	8. (D)
9. (A)	10. (B)	11. (C)	12. (D)	13. (C)	14. (A)	15. (D)	16. (B)
17. (A)	18. (B)	19. (C)	20. (D)				

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Department of Engg. Sciences & Allied Engg.
Engg.Mathematics-II

Unit : IV Curve Tracing

Q. 1	If the portion of the curve on both side of the point lies above the tangent to the curve at that point ,the curve is called
a	Concave upward
b	Concave downward
c	Convex upward
d	None of the above
Q. 2	If the portion of the curve on both side of the point lies below the tangent to the curve at that point ,the curve is called
a	Concave upward
b	Concave downward
c	Convex upward
d	None of the above
Q. 3	A double point where the tangents are real and distinct is called
a	Node
b	Cusp
c	Isolated point

d	Point of inflection
Q. 4	A double point where the tangents are real and equal is called
a	Node
b	Cusp
c	Isolated point
d	Point of inflection
Q. 5	A point through which more than one branches of the curve pass is called
a	Node
b	Cusp
c	Multiple point
d	Point of inflection
Q. 6	The graph of the curve $y = f(x)$ is increasing in the interval $[a, b]$ if
a	$f'(x) > 0$, for all $x \in [a, b]$
b	$f'(x) < 0$, for all $x \in [a, b]$
c	$f'(x) = 0$, for all $x \in [a, b]$
d	None of the above
Q. 7	Asymptote are

a	Tangents to the curve at infinity
b	Tangents to the curve at finite distance
c	Tangents to the curve at origin
d	None of the above
Q. 8	Asymptotes parallel to X –axis is obtained by equating the
a	Coefficient of highest degree term in x to zero
b	Coefficient of highest degree term in y to zero
c	Coefficient of lowest degree term in x to zero
d	Coefficient of lowest degree term in y to zero
Q. 9	Asymptotes parallel to Y –axis is obtained by equating the
a	Coefficient of highest degree term in x to zero
b	Coefficient of highest degree term in y to zero
c	Coefficient of lowest degree term in x to zero
d	Coefficient of lowest degree term in y to zero
Q. 10	Oblique asymptote are
a	Parallel to X –axis only.
b	Parallel to Y –axis only.

c	Parallel to both X -axis and Y –axis.
d	Not parallel to both X -axis and Y –axis.
Q. 11	To obtain the equation of oblique asymptote $y = mx + c$ of the given cartesian curve substitute y in the given curve and then $\boldsymbol{m}$ and $\boldsymbol{c}$ can be obtained
a	By equating two successive highest powers of x to zero
b	By equating two successive highest powers of y to zero.
c	Coefficient of highest degree term in y to zero.
d	Coefficient of highest degree term in x to zero
Q. 12	From the equation $x^3 + m^3\,x^3 + 3\,m^2c\,x^2 + 3mx\,c^2 + c^3 = 3am\,x^2 + 3\,acx$ the equation of oblique asymptote $y = mx + c$ for the cartesian curve $x^3 + y^3 = 3\,a\,xy$, is is obtained which is
a	$x + y + a = 0$
b	$x + y - a = 0$
c	$x + y = 0$
d	$x - y = 0$
Q. 13	For cartesian curve, if $y \to \infty$ as $x \to a$ then $x = a$ is asymptote parallel to
a	X - axis
b	Y - axis
c	The line $y = x$

d	None of the above
Q. 14	For cartesian curve, if $x \to \infty$ as $x \to b$ then $y = b$ is asymptote parallel to
a	X - axis
b	Y - axis
c	The line $y = x$
d	None of the above
Q. 15	For the curve $y^2(a^2 + x^2) = x^2(a^2 - x^2)$, equation of tangent at the origin is
a	$y = 0$
b	$x = 0$
c	$y = \pm x$
d	None of the above
Q. 16	The equation of asymptote parallel to Y-axis for the curve $x^2y^2 = a^2(x^2 + y^2)$ is
a	$y = \pm a$
b	$x = 0$
c	$y = 0$
d	$x = \pm a$
Q. 17	The equation of asymptote for the curve $x^2(2a - y) = y^3$ is
a	$y = 0$
b	$x = 0$
c	$y = 2a$
d	$x = \pm a$
Q. 18	The curve $y^2(x - a) = x^2(x + a)$ is symmetrical about the line
a	$y = a$

b	$x = 0$
c	$y = x$
d	X-axis
Q. 19	For the curve $x^3 + y^3 = 3\,axy$, equation of tangent at the origin is
a	$y = 0\,, x = 1$
b	$x = 0\,,\ y = 1$
c	$x = 1\,,\ y = 1$
d	$x = 0\,,\ y = 0$
Q. 20	The curve represented by the equation $(4 - x)y^2 = x^2$ is
a	Symmetrical about X-axis and having $x = 4$ is the equation of asymptote.
b	Symmetrical about Y-axis and having $x = 4$ is the equation of asymptote
c	Symmetrical about X-axis and having $y = 1$ is the equation of asymptote
d	Symmetrical about Y-axis and having $y = 1$ is the equation of asymptote

Q.no.	Ans	Q.no.	Ans	Q.no.	Ans	Q.no.	Ans
1	a	6	a	11	a	16	d
2	b	7	a	12	a	17	c
3	a	8	a	13	b	18	d
4	b	9	b	14	a	19	d
5	c	10	b	15	c	20	a

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Unit : I Application of Differential Equations **Assignment No: 5**

1	Rectilinear motion is a motion of body along a
A	straight line
B	circular path
C	Parabolic path
D	None of these
Ans	**A**
Marks	**1**
Unit	**IIe**

2	According to D'Alembert's principle algebraic sum of forces acting on a body along a given direction is equal to
A	Velocity $\times$ Acceleration
B	Mass $\times$ Velocity
C	Mass $\times$ displacement
D	Mass $\times$ Acceleration
Ans	**D**
Marks	**1**
Unit	**IIe**

3	A particle moving in straight line with acceleration $k\left(x+\frac{a^4}{x^3}\right)$ directed towards origin. The equation of motion is
A	$\frac{dv}{dx}=-k\left(x+\frac{a^4}{x^3}\right)$
B	$v\frac{dv}{dx}=k\left(x+\frac{a^4}{x^3}\right)$
C	$v\frac{dv}{dx}=-k\left(x+\frac{a^4}{x^3}\right)$
D	$\frac{dv}{dx}=\left(x+\frac{a^4}{x^3}\right)$
Ans	**C**
Marks	**1**
Unit	**IIe**

4	A particle of mass m moves in a horizontal straight line OA with acceleration $\frac{k}{x^3}$ at a distance x and directed towards origin O. Then the differential equation of motion is
A	$v\frac{dv}{dx}=\frac{k}{x^3}$
B	$v\frac{dv}{dx}=-\frac{k}{x^3}$
C	$\frac{dv}{dx}=-\frac{k}{x^3}$
D	$\frac{dv}{dx}=\frac{k}{x^3}$
Ans	**B**
Marks	**1**
Unit	**IIe**

5	A body of mass m falling from rest is subjected to a force of gravity and air resistance proportional to square of velocity $\left(kv^2\right)$. The equation of motion is
A	$m\frac{dv}{dx}=mg-kv^2$
B	$mv\frac{dv}{dx}=mg+kv^2$
C	$mv\frac{dv}{dx}=-kv^2$
D	$mv\frac{dv}{dx}=mg-kv^2$
Ans	**D**
Marks	**1**
Unit	**IIe**

6	A particle is projected vertically upward with velocity v_1 and resistance of air produces retardation $\left(kv^2\right)$ where v is velocity. The equation of motion is
A	$v\frac{dv}{dx}=-g-kv^2$
B	$v\frac{dv}{dx}=-g+kv^2$
C	$v\frac{dv}{dx}=-kv^2$
D	$v\frac{dv}{dx}=g-kv^2$
Ans	**A**
Marks	**1**
Unit	**IIe**

7	A body starts moving from rest is opposed by a force per unit mass of value (cx) resistance per unit mass of value (bv^2) where v and x are velocity and displacement of body at that instant. The differential equation of motion is
A	$mv\frac{dv}{dx} = -cx - bv^2$
B	$v\frac{dv}{dx} = cx + bv^2$
C	$v\frac{dv}{dx} = -cx - bv^2$
D	$\frac{dv}{dx} = -cx - bv^2$
Ans	**C**
Marks	**1**
Unit	**IIe**

8	The body of mass m falls from rest under gravity, in a fluid whose resistance to motion at any time t is mk times its velocity where k is constant. The differential equation of motion is
A	$\frac{dv}{dt} = -g - kv$
B	$\frac{dv}{dt} = g - kv$
C	$\frac{dv}{dt} = g + kv$
D	$\frac{dv}{dt} = mg - mkv$
Ans	**B**
Marks	**1**
Unit	**IIe**

9	A particle of mass m is projected vertically upward with velocity V, assuming the air resistance k times its velocity where k is constant. The differential equation of motion is
A	$\frac{dv}{dt} = mg - kv$
B	$m\frac{dv}{dt} = -mg + kv$
C	$m\frac{dv}{dt} = -kv$
D	$m\frac{dv}{dt} = -mg - kv$
Ans	**D**
Marks	**1**
Unit	**IIe**

10	Assuming that the resistance to movement of a ship through water in the form of $\left(a^2 + b^2v^2\right)$ where v is the velocity, a and b are constants. The differential equation for retardation of the ship moving with engine stopped.
A	$m\frac{dv}{dt} = -\left(a^2 + b^2v^2\right)^2$
B	$m\frac{dv}{dt} = +\left(a^2 + b^2v^2\right)$
C	$m\frac{dv}{dt} = -\left(a^2 + b^2v^2\right)$
D	$m\frac{dv}{dx} = -\left(a^2 + b^2v^2\right)$
Ans	**C**
Marks	**1**
Unit	**IIe**

11	A bullet is fired into sand tank , its retardation proportional to square root of its velocity .The differential equation of motion is
A	$m\frac{dv}{dt}=-mk\sqrt{v}$
B	$m\frac{dv}{dt}=mk\sqrt{v}$
C	$m\frac{dv}{dx}=-mk\sqrt{v}$
D	$m\frac{d^2v}{dt^2}=-mk\sqrt{v}$
Ans	**A**
Marks	**1**
Unit	**IIe**

12	A particle of mass m moves with velocity v along a straight line whose resistance per unit mass is μ rimes cube of velocity .Then the differential equation of motion is
A	$\frac{d^2v}{dt^2}=-\mu v^3$
B	$\frac{dv}{dx}=-\mu v^3$
C	$\frac{dv}{dt}=-\mu v^3$
D	$\frac{dv}{dt}=\mu v^3$
Ans	**C**
Marks	**1**
Unit	**IIe**

13	A particle of unit mass moves in a straight line under the attraction varying inversely as the $\frac{3}{2}th$ power of distance.The differential equation of motion is
A	$\frac{dv}{dt}=k\frac{1}{x^{3/2}}$
B	$\frac{dv}{dt}=-k\frac{1}{x^{3/2}}$
C	$\frac{dv}{dt}=-k\,x^{3/2}$
D	$\frac{dv}{dt}=k\,x^{3/2}$
Ans	**B**
Marks	**1**
Unit	**IIe**

14	The distance x travelled by the particle moving in a straight line at any time t is given by $\cos^{-1}\left(\frac{x^2}{a^2}\right)=2\sqrt{k}t$ where k and a are constant .If the particle will start at a distance a from origin then the particle will arrive at origin in time
A	$\frac{\pi\sqrt{k}}{4}$
B	$\frac{\pi}{\sqrt{k}}$
C	$\frac{\pi}{2\sqrt{k}}$
D	$\frac{\pi}{4\sqrt{k}}$
Ans	**D**
Marks	**1**
Unit	**IIe**

15	The velocity v of a vehicle at any time t is given by $v=k\left(t-\frac{t^2}{2T}\right)$ where k is constant and T is the time taken to attain maximum speed . Then the maximum speed of the vehicle is
A	$\frac{kT}{2}$
B	$\frac{kT}{4}$
C	kT
D	$\frac{kT}{3}$
Ans	**A**
Marks	**1**
Unit	**IIe**

Type: Applications to Electrical Circuits

1	A circuit containing resistance R and inductance L in series with voltage source E. By Kirchhoff's voltage law differential equation for current i is
A	$Li+R\frac{di}{dt}=E$
B	$L\frac{di}{dt}+Ri=E$
C	$L\frac{di}{dt}+Ri=0$
D	$L\frac{di}{dt}+\frac{q}{C}=E$
Ans	**B**
Marks	**1**
Unit	**IIe**

2	A circuit containing resistance R and capacitance C in series with voltage source E. By Kirchhoff's voltage law differential equation for current $i=\frac{dq}{dt}$ is
A	$L\frac{di}{dt}+\frac{q}{C}=E$
B	$R\frac{dq}{dt}+\frac{q}{C}=0$
C	$L\frac{di}{dt}+Ri=0$
D	$R\frac{dq}{dt}+\frac{q}{C}=E$
Ans	**D**
Marks	**1**
Unit	**IIe**

3	A circuit containing inductance L capacitance C in series without applied electromotive force. By Kirchhoff's voltage law differential equation for current i is
A	$L\frac{di}{dt}+\frac{q}{C}=0$
B	$L\frac{di}{dt}+Ri=0$
C	$L\frac{di}{dt}+Ri=E$
D	$L\frac{di}{dt}+\frac{q}{C}=E$
Ans	**A**
Marks	**1**
Unit	**IIe**

4	A circuit containing inductance L capacitance C in series with applied electromotive force E. By Kirchhoff's voltage law differential equation for current i is
A	$L\frac{di}{dt}+Ri=E$
B	$L\frac{di}{dt}+Ri=0$
C	$L\frac{di}{dt}+\frac{q}{C}=E$
D	$L\frac{di}{dt}+\frac{q}{C}=0$
Ans	**C**
Marks	**1**
Unit	**IIe**

5	The differential equation for the current in an electric circuit containing resistance R and inductance L in series with voltage source $E\sin\omega t$ is
A	$L\frac{di}{dt}+\frac{q}{C}=E$
B	$Li+R\frac{di}{dt}=E\sin\omega t$
C	$L\frac{di}{dt}+Ri=0$
D	$L\frac{di}{dt}+Ri=E\sin\omega t$
Ans	**D**
Marks	**1**
Unit	**IIe**

6	In a circuit containing resistance R and inductance L in series with constant voltage source E current i is given by $i=\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$, then maximum current $i_{\max}$ is
A	$\frac{E}{R}$
B	$\frac{R}{E}$
C	ER
D	0
Ans	**A**
Marks	**1**
Unit	**IIe**

7	In a circuit containing resistance R and inductance L in series with constant voltage source E current i is given by $i=\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$ and $L=640H, R=250\Omega, E=500V$, then maximum current $i_{\max}$ is
A	1
B	2
C	$\frac{1}{2}$
D	3
Ans	**B**
Marks	**1**
Unit	**IIe**

8	In a circuit containing resistance R and inductance L in series with constant voltage source E current i is given by $i=\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$ and $L=640H, R=250\Omega, E=500V$, then 50% of maximum current is
A	1
B	2
C	$\frac{1}{2}$
D	3
Ans	**A**
Marks	**1**
Unit	**IIe**

9	The differential equation for the current i in an electric circuit containing resistance 100 ohm and an inductance of 0.5 henery connected in series with battery of 20 volts is
A	$0.5\frac{di}{dt}+100\ \ i=0$
B	$0.5\frac{di}{dt}+100\ i=20$
C	$100\frac{di}{dt}+0.5\ \ i=20$
D	$100\frac{di}{dt}+0.5R=0$
Ans	**B**
Marks	**1**
Unit	**IIe**

10	The differential equation for the current i in an electric circuit containing resistance $R=250$ ohm and an inductance of $L=640$ henery in series with an electromotive force $E=500$ volts is
A	$640\frac{di}{dt}+250\ i=0$
B	$250\frac{di}{dt}+640\,i=500$
C	$640\frac{di}{dt}+250\ i=500$
D	$250\frac{di}{dt}+640\,i=0$
Ans	**C**
Marks	**1**
Unit	**IIe**

11	A capacitor $C = 0.01$ farad in series with resistor $R = 20$ ohms is charged from battery $E = 10$ volts. If initially capacitor is completely discharged then differential equation for charge $q(t)$ is given by
A	$20\frac{dq}{dt} + \frac{q}{0.01} = 0; \qquad q(0) = 0$
B	$20\frac{dq}{dt} + 0.01q = 10; \qquad q(0) = 0$
C	$20\frac{dq}{dt} + \frac{q}{0.01} = 10; \qquad q(0) = 0$
D	$20\frac{dq}{dt} + 0.01q = 0; \qquad q(0) = 0$
Ans	**C**
Marks	**1**
Unit	**IIe**

12	In a circuit containing resistance R and inductance L in series with constant voltage source E current i is given by $i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$, $R = 100\Omega, E = 20V$, $L = 0.5H$ then the value of i at $t = 0$ is
A	$\frac{1}{5}$
B	$-\frac{1}{5}$
C	$\frac{2}{5}$
D	0
Ans	**D**
Marks	**1**
Unit	**IIe**

Type: One Dimensional Conduction of Heat

1	Fourier law of heat conduction states that, the quantity of heat flow across an area A cm^2 is
A	proportional to product of area A and temperature gradient $\frac{dT}{dx}$
B	inversely proportional to product of area A and temperature gradient $\frac{dT}{dx}$
C	equal to sum of area A and temperature gradient $\frac{dT}{dx}$
D	equal to difference of area A and temperature gradient $\frac{dT}{dx}$
Ans	**A**
Marks	**1**
Unit	**IIe**

2	If q be the quantity of heat that flows across an area A cm^2 and thickness δx in one second where the difference of temperature at the faces is δT, then by Fourier's law of heat conduction
A	$q = -k\left(A - \frac{dT}{dx}\right)$, where k is thermal conductivity
B	$q = kA\frac{dT}{dx}$, where k is thermal conductivity
C	$q = -k\left(A + \frac{dT}{dx}\right)$, where k is thermal conductivity
D	$q = -kA\frac{dT}{dx}$, where k is thermal conductivity
Ans	**D**
Marks	**1**
Unit	**IIe**

3	The differential equation for steady state heat loss per unit time from a unit length of pipe with thermal conductivity k radius r_0 carrying steam at temperature T_0, if the pipe is covered with insulation of thickness w, the outer surface of which remains at the constant temperature T_1, is
A	$Q = k(2\pi r)\dfrac{dT}{dr}$
B	$Q = -k(2\pi r)\dfrac{dT}{dr}$
C	$Q = -k(2\pi r^2)\dfrac{dT}{dr}$
D	$Q = -k(\pi r^2)\dfrac{dT}{dr}$
Ans	**B**
Marks	**1**
Unit	**IIe**

4	The differential equation for steady state heat loss per unit time from a spherical shell with thermal conductivity k radius r_0 carrying steam at temperature T_0, if the spherical shell is covered with insulation of thickness w, the outer surface of which remains at the constant temperature T_1, is
A	$Q = -k(2\pi r)\dfrac{dT}{dr}$
B	$Q = k(2\pi r)\dfrac{dT}{dr}$
C	$Q = -k(4\pi r^2)\dfrac{dT}{dr}$
D	$Q = -k(\pi r^2)\dfrac{dT}{dr}$
Ans	**C**
Marks	**1**
Unit	**IIe**

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Unit : I Application of Differential Equations **Assignment No: 1**

Question 1	If the differential equation of family of straight lines $y = mx$ is $\frac{dy}{dx} = \frac{y}{x}$ then its orthogonal trajectories is
a	$xy = k$
b	$x^2 - y^2 = k^2$
c	$y = kx$
d	$x^2 + y^2 = k^2$
Question 2	If the differential equation of family of curves $y = cx^2$ is $\frac{dy}{dx} = \frac{2y}{x}$ then its orthogonal trajectories is
a	$2x^2 - y^2 = k$
b	$\frac{1}{2}\log y = \log x + k$
c	$y = kx^2$
d	$x^2 + 2y^2 = k$
Question 3	If the differential equation of family of curves $y = ce^x$ is $\frac{dy}{dx} = y$ then its orthogonal trajectories is
a	$x^2 + 2y = k$
b	$2x - y^2 = k$
c	$2x + y^2 = k$

d	$2x^2 + y = k$
Question 4	If the differential equation of family of curves $y = cx^3$ is $\frac{dy}{dx} = \frac{3y}{2x}$ then its orthogonal trajectories is
a	$3x^2 + 2y^2 = k$
b	$2x^2 + 3y^2 = k$
c	$2x^2 - 3y^2 = k$
d	$3x^2 - 2y^2 = k$
Question 5	If the differential equation of family of curves $2x^2 + y^2 = c$ is $4x + 2y\frac{dy}{dx} = 0$ then its orthogonal trajectories is
a	$y^2 = kx$
b	$x^2 = ky$
c	$x^2 + ky = 0$
d	$x + ky^2 = 0$
Question 6	If the differential equation of family of curves $y = (x-k)^2$ is $\frac{dy}{dx} = 2\sqrt{y}$ then its orthogonal trajectories is
a	$\frac{4}{3}y^{\frac{3}{2}} = \frac{2}{3}x + k$
b	$\frac{1}{3}y^{\frac{3}{2}} + x = k$
c	$\frac{4}{3}y^{\frac{3}{2}} + \frac{2}{3}x = k$
d	$y^{\frac{1}{2}} = (x-k)$

Question 7	If the differential equation of family of curves $x^{\frac{2}{3}}+y^{\frac{2}{3}}=c^{\frac{2}{3}}$ is $y^{-\frac{1}{3}}\frac{dy}{dx}=-x^{-\frac{1}{3}}$ then its orthogonal trajectories is
a	$x^{\frac{4}{3}}+y^{\frac{4}{3}}=k$
b	$x^{\frac{4}{3}}-y^{\frac{4}{3}}=k$
c	$x^{\frac{2}{3}}+y^{\frac{2}{3}}=k$
d	$x^{\frac{2}{3}}-y^{\frac{2}{3}}=k$
Question 8	The orthogonal trajectories of family of curves $y-2x=c$ is
a	$x-2y=k$
b	$x+y=k$
c	$2x+y=k$
d	$x+2y=k$
Question 9	Orthogonal trajectories of family of curves $x^2+2y^2=c^2$ whose differential equation is $\frac{dy}{dx}=-\frac{x}{2y}$, is
a	$x^2=ky$
b	$x^2=\frac{k}{y}$
c	$x^2+2y^2=k^2$
d	None of these
Question 10	If the differential equation of family of curves $r=ce^{\theta}$ is $\frac{dr}{d\theta}=r$ then its orthogonal trajectories is given by

a	$\frac{r^2}{2}+\theta=k$
b	$r=ke^{-\theta}$
c	$\frac{r^2}{2}-\theta=k$
d	$r=\log\theta+k$
Question 11	If the differential equation of family of curves $r=c(\sec\theta+\tan\theta)$ is $\frac{dr}{d\theta}=r\sec\theta$ then its orthogonal trajectories is given by
a	$\log r-\sin\theta=k$
b	$-\sin\theta=\frac{r^2}{2}+k$
c	$-\frac{r^2}{2}=\log(\sec\theta+\tan\theta)+k$
d	$\log r=-\sin\theta+k$
Question 12	If the differential equation of family of curves $r^n\cos n\theta=a^n$ is $\frac{1}{r}\frac{dr}{d\theta}=\tan n\theta$ then its orthogonal trajectories is given by
a	$r^n\sin n\theta=k^n$
b	$r^n=k^n\sin n\theta$
c	$\frac{1}{r}=\log\sin n\theta+k$
d	$r^2=2k^n\sin n\theta$
Question 13	If the differential equation of family of curves $r^n=b\operatorname{cosec}n\theta$ is $\frac{1}{r}\frac{dr}{d\theta}=-\cot n\theta$ then its orthogonal trajectories is given by
a	$\frac{1}{r}=\log\cos n\theta+k$
b	$r^n=k\cos n\theta$

c	$r^n = k\sec n\theta$
d	$r^2 = 2k^n \cos n\theta$
Question 14	If the differential equation of family of curves $r = \frac{a\theta}{1+\theta}$ is $\frac{dr}{d\theta} = \frac{r}{\theta(1+\theta)}$ then its orthogonal trajectories is given by
a	$r(1+\theta) = k\theta$
b	$\frac{\theta^2}{2} + \log\theta = \log r + k$
c	$\theta^2 + \theta^3 = -6\log r + k$
d	$3\theta^2 + 2\theta^3 = -6\log r + k$
Question 15	If the differential equation of family of curves $r = 2a(\sin\theta + \cos\theta)$ is $\frac{dr}{d\theta} = \frac{r(\cos\theta - \sin\theta)}{(\cos\theta + \sin\theta)}$ then its orthogonal trajectories is given by
a	$r = \frac{k}{\cos\theta - \sin\theta}$
b	$r = k(\cos\theta - \sin\theta)$
c	$r = k(\cos\theta + \sin\theta)$
d	$r + k(\cos\theta + \sin\theta) = 0$
Question 16	If the differential equation of family of curves $r = a\sec^2\frac{\theta}{2}$ is $\frac{dr}{d\theta} = r\tan\frac{\theta}{2}$ then its orthogonal trajectories is given by
a	$-2\log cos\frac{\theta}{2} = \log r + \log k$
b	$2\log\sin\frac{\theta}{2} = \log r + \log k$
c	$-2\log\sin\frac{\theta}{2} = \log r + \log k$

d	$2\log\cos\frac{\theta}{2}=\log r+\log k$
Question 17	If the differential equation of family of curves $r^2=a\cos 2\theta$ is $\frac{dr}{d\theta}=-r\tan 2\theta$ then its orthogonal trajectories is given by
a	$\frac{1}{2}\log\cos 2\theta=\log r+\log k$
b	$\frac{1}{2}\log\sin 2\theta=\log r+\log k$
c	$\log\sin 2\theta=-r^2+k$
d	$\frac{1}{2}\log\sin 2\theta=-\log r+\log k$
Question 18	By Newton's law of cooling the differential equation of body originally at $80^0 C$ cools down to $60^0 C$ in 20 minutes in surrounding temperature of $40^0 C$ is $\frac{d\theta}{dt}=-(0.03465)(\theta-40)$. The temperature of the body after 40 minutes is
a	$60^0 C$
b	$50^0 C$
c	$35^0 C$
d	$85^0 C$
Question 19	A metal ball is heated to a temperature of $100^0 C$ and at time $t=0$ it is placed in water which is maintained at $40^0 C$. The temperature of the ball reduces to $60^0 C$ in 4 minutes. By Newton's law of cooling the differential equation is $\frac{d\theta}{dt}=-\left(\frac{1}{4}\log_e 3\right)(\theta-40)$. Then the time required to reduce the temperature of ball to $50^0 C$ is
a	7.5 min
b	3.5 min

c	10 min
d	6.5 min
Question 20	A body whose temperature is initially $100^\circ C$ is allowed to cool in air , whose temperature remains at a constant temperature $20^\circ C$ .It is given that after 10 minutes, the body has cooled to $40^\circ C$ By Newton's law of cooling the differential equation .
a	$\frac{d\theta}{dt} = -k(\theta - 60)$
b	$\frac{d\theta}{dt} = -k(\theta - 100)$
c	$\frac{d\theta}{dt} = -k(\theta - 40)$
d	$\frac{d\theta}{dt} = -k(\theta - 20)$

Q.no.	Ans	Q.no.	Ans	Q.no.	Ans	Q.no.	Ans
1	d	6	c	11	d	16	c
2	d	7	b	12	a	17	b
3	c	8	d	13	c	18	b
4	d	9	a	14	d	19	d
5	a	10	b	15	a	20	d

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Unit : I Application of Differential Equations **Assignment No: 2**

Question 1	Suppose a corpse at a temperature of 32^0C arrives at mortuary where the temperature is kept at 10^0C. If the corpse cools to 27^0C in 5 minutes. If the differential equation by Newton's law of cooling is $\frac{dT}{dt}=-(0.05)(T-10)$, then temperature T of corpse at any time t is given by
a	$T=22e^{-0.05t}$
b	$T=10+22e^{0.05t}$
c	$T=10+22e^{-0.05t}$
d	$T=10-22e^{-0.05t}$
Question 2	A body originally at 80^0C cools down to 60^0C in 20 minutes in a room where the temperature is 40^0C. The differential equation by Newton's law of cooling is $\frac{d\theta}{dt}=-k(\theta-40)$, then the value of k is
a	$-\frac{1}{20}\log_e 2$
b	$\frac{1}{20}\log_e 2$
c	$20\log_e 2$
d	$\log_e 2$
Question 3	By Newton's law of cooling the differential equation of body originally at 80^0C cools down to 60^0C in 20 minutes in surrounding temperature of 40^0C is $\frac{d\theta}{dt}=-(0.03465)(\theta-40)$. The temperature of the body after 40 minutes is
a	60^0C

b	$50^0 C$
c	$35^0 C$
d	$85^0 C$
Question 4	A metal ball is heated to a temperature of $100^0 C$ and at time $t = 0$ it is placed in water which is maintained at $40^0 C$. The temperature of the ball reduces to $60^0 C$ in 4 minutes. By Newton's law of cooling the differential equation is $\frac{d\theta}{dt} = -\left(\frac{1}{4}\log_e 3\right)(\theta - 40)$. Then the time required to reduce the temperature of ball to $50^0 C$ is
a	7.5 min
b	3.5 min
c	10 min
d	6.5 min
Question 5	Suppose a body at a temperature of 30°C cools down to $27^\circ C$ in 2 minutes, where the temperature of the surrounding is 10°C If the differential equation by Newton's law of cooling is $\frac{dT}{dt} = -(0.08)(T - 10)$, then temperature T of the body at any time t is given by
a	$T = 20e^{-0.08t}$
b	$T = 10 + 20e^{0.08t}$
c	$T = 10 + 20e^{-0.08t}$
d	$T = 10 - 20e^{-0.08t}$
Question 6	A body originally at 100°C cools down to 80 °C in 20 minutes in a room where the temperature is 30°C. The differential equation by Newton's law of cooling is, $\frac{d\theta}{dt} = -k(\theta - 30)$ then the value of k is

a	$-\frac{1}{20} ln\left(\frac{5}{7}\right)$
b	$\frac{1}{20} ln\left(\frac{5}{7}\right)$
c	$20\ ln\left(\frac{5}{7}\right)$
d	$ln\left(\frac{5}{7}\right)$
Question 7	By Newton's law of cooling the differential equation of body originally at $60°C$ cools down to $40°C$ in 20 minutes in surrounding temperature of $20°C$ is $\frac{d\theta}{dt} = -(0.3465)(\theta - 20)$. The temperature of the body after 40 minutes is
a	$20°C$
b	$10°C$
c	$180°C$
d	$30°C$
Question 8	A thermometer is taken outdoors where the temperature of the surrounding is 5°C from the room at temperature 35°C and the reading drops to $15°C$ in 2 min. By Newton's law of cooling the differential equation is $\frac{d\theta}{dt} = -(0.55)(\theta - 5)$.. Then the time required to reduce the temperature 10°C is
a	3.26 min
b	2.5 min
c	10 min
d	0.38 min
Question 9	A thermometer is taken outdoors where the temperature is 0°C , from a room in which the temperature is 21°C and temperature drops to 10°C in 1 min. If the differential equation by Newton's law of cooling is $\frac{dT}{dt} = -(0.7419)T$, then the temperature T of thermometer at time t is given by

a	$T = 21 + 11e^{-0.74198t}$
b	$T = 21e^{0.7419t}$
c	$T = 10 + 21e^{-0.7419t}$
d	$T = 21e^{-0.7419t}$
Question 10	If the temperature of the air is $30°K$ and substance cools from $37°K$ to $34°K$ in 15 min. By Newton's law of cooling the differential equation is $\frac{d\theta}{dt} = -(0.0373)(\theta - 30)$. Then the time at which the temperature will be $31°K$ is
a	18 min
b	52.15 min
c	30 min
d	22.65 min
Question 11	In a circuit containing resistance R and inductance L in series with constant e.m.f. E, the current i is given by $i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$, then the time required to builds current half of its theoretical maximum is
a	$\frac{L}{R \log 2}$
b	$\frac{L \log 2}{R}$
c	$\frac{R \log 2}{L}$
d	0
Question 12	If the differential equation for current in an electric circuit containing resistance R and inductance L in series with constant e.m.f. E, the current i is $L\frac{di}{dt} + Ri = E$, then the current at any time t is given by

a	$i=\frac{E}{R}-Ae^{-\frac{R}{L}t}$; A is arbitrary constant
b	$i=\frac{E}{R}+Ae^{-\frac{R}{L}t}$; A is arbitrary constant
c	$i=\frac{E}{R}+Ae^{\frac{R}{L}t}$; A is arbitrary constant
d	$i=\frac{E}{R}+e^{-\frac{R}{L}t}$
Question 13	The charge q on the plate of condenser of capacity C charge through a resistance R by steady voltage V satisfies differential equation $R\frac{dq}{dt}+\frac{q}{C}=V$, then charge q at any time t is
a	$q=CV+Ae^{-\frac{1}{RC}t}$; A is arbitrary constant
b	$q=CV-Ae^{\frac{1}{RC}t}$; A is arbitrary constant
c	$q=C+Ae^{\frac{1}{RC}t}$; A is arbitrary constant
d	$q=CV+e^{\frac{1}{RC}t}$
Question 14	A resistance $R=100$ ohms, an inductance $L=0.5$ henry are connected in series with a battery of 20 Volts. The differential equation for the current i is $0.5\frac{di}{dt}+100i=20$, then current i at any time t is
a	Ae^{-200t}; A is arbitrary constant
b	$\frac{1}{5}+Ae^{200t}$; A is arbitrary constant
c	$2+Ae^{-200t}$; A is arbitrary constant
d	$\frac{1}{5}+Ae^{-200t}$; A is arbitrary constant
Question 15	Suppose a corpse at a temperature of 32^0C arrives at mortuary where the temperature is kept at 10^0C. If the corpse cools to 27^0C in 5 minutes. If the differential equation by Newton's law of cooling is $\frac{dT}{dt}=-(0.05)(T-10)$, then temperature T of corpse at any time t is given by

a	$T=22e^{-0.05\,t}$
b	$T=10+22e^{0.05\,t}$
c	$T=10+22e^{-0.05\,t}$
d	$T=10-22e^{-0.05\,t}$
Question 16	A body originally at 80^0C cools down to 60^0C in 20 minutes in a room where the temperature is 40^0C. The differential equation by Newton's law of cooling is $\frac{d\theta}{dt}=-k(\theta-40)$, then the value of k is
a	$-\frac{1}{20}\log_e 2$
b	$\frac{1}{20}\log_e 2$
c	$20\log_e 2$
d	$\log_e 2$
Question 17	By Newton's law of cooling the differential equation of body originally at 80^0C cools down to 60^0C in 20 minutes in surrounding temperature of 40^0C is $\frac{d\theta}{dt}=-(0.03465)(\theta-40)$. The temperature of the body after 40 minutes is
a	60^0C
b	50^0C
c	35^0C
d	85^0C
Question 18	A metal ball is heated to a temperature of 100^0C and at time $t=0$ it is placed in water which is maintained at 40^0C. The temperature of the ball reduces to 60^0C in 4 minutes. By Newton's law of cooling the differential equation is

	$\frac{d\theta}{dt} = -\left(\frac{1}{4}\log_e 3\right)(\theta - 40)$. Then the time required to reduce the temperature of ball to $50^0 C$ is
a	7.5 min
b	3.5 min
c	10 min
d	6.5 min
Question 19	Suppose a corpse at a temperature of $32^0 C$ arrives at mortuary where the temperature is kept at $10^0 C$. If the corpse cools to $27^0 C$ in 5 minutes. If the differential equation by Newton's law of cooling is $\frac{dT}{dt} = -(0.05)(T - 10)$, then temperature T of corpse at any time t is given by
a	$T = 22e^{-0.05t}$
b	$T = 10 + 22e^{0.05t}$
c	$T = 10 + 22e^{-0.05t}$
d	$T = 10 - 22e^{-0.05t}$
Question 20	A body originally at $80^0 C$ cools down to $60^0 C$ in 20 minutes in a room where the temperature is $40^0 C$. The differential equation by Newton's law of cooling is $\frac{d\theta}{dt} = -k(\theta - 40)$, then the value of k is
a	$-\frac{1}{20}\log_e 2$
b	$\frac{1}{20}\log_e 2$
c	$20\log_e 2$
d	$\log_e 2$

Q.no.	Ans	Q.no.	Ans	Q.no.	Ans	Q.no.	Ans
1	c	6	a	11	b	16	b
2	b	7	d	12	b	17	b
3	b	8	a	13	a	18	d
4	d	9	d	14	d	19	c
5	c	10	b	15	c	20	b

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Unit : I Application of Differential Equations **Assignment No: 3**

Type: Rectilinear Motion

1	Differential equation of motion of a body of mass m falls from rest under gravity in a fluid whose resistance to motion at any time t is mk times its velocity where k is constant is $\frac{dv}{dt} = g - kv$ then the relation between velocity and time t is
A	$t = \frac{1}{k}\log\frac{g-kv}{g}$
B	$t = \frac{1}{k}\log\frac{g}{g-kv}$
C	$t = \frac{1}{k}\log\frac{g}{g+kv}$
D	$t = -\frac{1}{k}\log\frac{1}{g-kv}$
Ans	**B**
Marks	**2**
Unit	**IIc**

2	A body of mass m falling from rest is subjected to the force of gravity and air resistance proportional to square of velocity $\left(kv^2\right)$ satisfies the differential equation $mv\frac{dv}{dx} = k\left(a^2 - v^2\right)$ where $ka^2 = mg$, then the relation between velocity and displacement is
A	$\frac{2kx}{m} = \log\frac{a^2}{a^2 - v^2}$
B	$\frac{2kx}{m} = \log\frac{a^2 - v^2}{a^2}$
C	$2kx = \log\frac{1}{a^2 - v^2}$
D	$\frac{x}{m} = \log\frac{a^2}{a^2 - v^2}$
Ans	**A**

Marks	**2**
Unit	**IIc**

3	A vehicle starts from rest and its acceleration is given by $\frac{dv}{dt}=k\left(1-\frac{t}{T}\right)$ where k and T are constant then the velocity v in terms of time t is given by
A	$v=k\left(t-\frac{t^2}{2}\right)$
B	$v=k\left(t-\frac{t^2}{T}\right)$
C	$v=k\left(\frac{t^2}{2}-\frac{t^3}{3T}\right)$
D	$v=k\left(t-\frac{t^2}{2T}\right)$
Ans	**D**
Marks	**2**
Unit	**IIc**

4	A particle of mass m is projected upward with velocity V. Assuming the air resistance k times its velocity and equation of motion is $m\frac{dv}{dt}=-mg-kv$ then relation between velocity v and time t is
A	$t=\frac{m}{k}\log\left(\frac{mg+kV}{mg+kv}\right)$
B	$t=\frac{m}{k}\log\left(\frac{mg+kv}{mg+kV}\right)$
C	$t=m\log\left(\frac{mg+kV}{mg+kv}\right)$
D	$t=\log\left(\frac{mg+kv}{mg+kV}\right)$
Ans	**A**
Marks	**2**
Unit	**IIc**

5	A body of mass m falls from rest under gravity and retarding force due to air resistance is proportional to square of velocity (kv^2) satisfies the differential equation $\frac{dv}{dt} = k(a^2 - v^2)$, where $a^2 = \frac{g}{k}$ then the relation between velocity and time t is
A	$t = \frac{1}{2ak}\log\left(\frac{a+v}{a-v}\right)$
B	$t = \log\left(\frac{a+v}{a-v}\right)$
C	$t = \frac{1}{2ak}tan^{-1}\left(\frac{v}{a}\right)$
D	$t = \frac{1}{2ak}\log(a^2 - v^2)$
Ans	**A**
Marks	**2**
Unit	**IIc**

6	A moving body opposed by a force per unit mass of value cx and resistance per unit mass of value bv^2 where x and v are displacement and velocity of the body at that instant, satisfies the differential equation $v\frac{dv}{dx} + bv^2 = -cx$ then the integrating factor is
A	e^{-2bx}
B	$\frac{1}{e^{-bx}}$
C	e^{2bx}
D	$-\frac{1}{e^{-2bx}}$
Ans	**C**
Marks	**2**
Unit	**IIc**

7	A moving body opposed by a force per unit mass of value cx and resistance per unit mass of value bv^2 where x and v are displacement and velocity of the body at that instant, satisfies the differential equation $\frac{du}{dx} + 2bu = -2cx$ with the integrating factor e^{2bx} the relation between velocity and distance is (take $u = v^2$)
A	$v^2 = \frac{c}{2b^2} - \frac{c\,x}{b} - Ae^{2bx}$
B	$v^2 = \frac{c}{2b^2} + \frac{c\,x}{b} + Ae^{-2bx}$
C	$v^2 = \frac{c}{2b^2} + \frac{c\,x}{b} + Ae^{2bx}$, A is constant of integration
D	$v^2 = \frac{c}{2b^2} - \frac{c\,x}{b} + Ae^{-2bx}$, A is constant of integration
Ans	**D**
Marks	**2**
Unit	**IIc**

8	A particle of mass m is projected vertically upward with velocity V_0 and the air resistance produced retardation kv^2, where v is the velocity at any instant satisfies the differential equation $\frac{v\,dv}{g+kv^2} = -dx$,then the relation between distance x and velocity v is
A	$x = \frac{1}{2k}\log\left[\frac{g + k{V_0}^2}{g + kv^2}\right]$
B	$x = \frac{1}{2k}\log\left[\frac{g - k{V_0}^2}{g - kv^2}\right]$
C	$x = \frac{1}{2k}\log\left[\frac{g - kv^2}{g + k{V_0}^2}\right]$
D	$x = \log\left[\frac{g + k{V_0}^2}{g + kv^2}\right]$
Ans	**A**
Marks	**2**
Unit	**IIc**

9	A particle is moving in a straight line with acceleration $k\left(x+\frac{a^4}{x^3}\right)$ directed towards the origin. If it starts from rest at a distance $'a'$ from the origin and the differential equation of motion is $v\frac{dv}{dx}=-k\left(x+\frac{a^4}{x^3}\right)$ then the relation between velocity and distance is
A	$\frac{v^2}{2}=k\left[\frac{x^2}{2}-\frac{a^4}{2x^2}\right]$
B	$\frac{v^2}{2}=-k\left[\frac{x^2}{2}-\frac{a^4}{2x^2}\right]$
C	$\frac{v^2}{2}=-k\left[\frac{x^2}{2}+\frac{a^4}{2x^2}\right]$
D	$\frac{v^2}{2}=k\left[\frac{x^2}{2}+\frac{a^4}{2x^2}\right]$
Ans	**B**
Marks	**2**
Unit	**IIc**

10	A bullet is fired into sand tank with initial velocity V_0 and its retardation is proportional to $\sqrt{v}$. The equation of motion is $\frac{dv}{dt}=-k\sqrt{v}$ then relation between time t and velocity v is
A	$t=\frac{2}{k}\left(\sqrt{V_0-v}\right)$
B	$t=\frac{2}{k}\left(\sqrt{V_0}+\sqrt{v}\right)$
C	$t=\frac{2}{k}\left(\sqrt{V_0}-\sqrt{v}\right)$
D	$t=\left(\sqrt{v}-\sqrt{V_0}\right)$
Ans	**C**
Marks	**2**
Unit	**IIc**

Type: One Dimensional Conduction of Heat

1	Fourier law of heat conduction states that, the quantity of heat flow across an area A cm^2 is
A	proportional to product of area A and temperature gradient $\frac{dT}{dx}$
B	inversely proportional to product of area A and temperature gradient $\frac{dT}{dx}$
C	equal to sum of area A and temperature gradient $\frac{dT}{dx}$
D	equal to difference of area A and temperature gradient $\frac{dT}{dx}$
Ans	**A**
Marks	**1**
Unit	**IIe**

2	If q be the quantity of heat that flows across an area A cm^2 and thickness δx in one second where the difference of temperature at the faces is δT, then by Fourier's law of heat conduction
A	$q = -k\left(A - \frac{dT}{dx}\right)$, where k is thermal conductivity
	$q = kA\frac{dT}{dx}$, where k is thermal conductivity
C	$q = -k\left(A + \frac{dT}{dx}\right)$, where k is thermal conductivity
	$q = -kA\frac{dT}{dx}$, where k is thermal conductivity
Ans	**D**
Marks	**1**
Unit	**IIe**

3	The differential equation for steady state heat loss Q per unit time from a spherical shell with thermal conductivity k radius r_0 carrying steam at temperature T_0, if the spherical shell is covered with insulation of thickness w, the outer surface of which remains at the constant temperature T_1, is $Q=-k\left(4\pi r^2\right)\frac{dT}{dr}$ Then the temperature T of spherical shell of radius r is
A	$T=-\frac{Q}{4\pi k}\frac{1}{r^2}+C$
B	$T=\frac{Q}{4\pi k}\frac{1}{r}+C$
C	$T=-\frac{Q}{4\pi k}\frac{1}{r}+C$
D	$T=-\frac{Q}{2\pi k}\frac{1}{r^3}+C$
Ans	**B**
Marks	**2**
Unit	**IIc**

4	A pipe 20 cm in diameter contains steam at $150^0 C$ and is protected with covering 5cm thick for which k = 0.0025. If the temperature of the outer surface of the covering is $40^0 C$ and differential equation of conduction of heat is $dT=-\frac{Q}{2\pi k}\frac{dx}{x}$. The amount of heat loss Q is
A	$\frac{110(2\pi k)}{\log(1.5)}$
B	$\frac{\log(1.5)}{110(2\pi k)}$
C	$-\frac{110(2\pi k)}{\log(1.5)}$
D	$\frac{110}{\log(1.5)}$
Ans	**A**
Marks	**1**
Unit	**IIc**

5	A pipe 10 cm in diameter contains steam at $100^0 C$. It is protected with asbestos 5 cm thick for which k = 0.0006 and outer surface is at $30^0 C$. The differential equation of conduction of heat is $dT = -\frac{Q}{2\pi k}\frac{dx}{x}$. The amount of heat loss Q is
A	$\frac{\log 2}{70(2\pi k)}$
B	$\frac{70(2\pi k)}{\log 2}$
C	$-\frac{70(2\pi k)}{\log 2}$
D	$\frac{(2\pi k)}{\log 2}$
Ans	**B**
Marks	**1**
Unit	**IIc**

6	A steam pipe 20 cm in diameter is protected with covering 6 cm thick and k = 0.0003 cal/cm in a steady state. If the inner surface of the pipe is at $200\ ^0C$ and the outer surfaceof the covering is at $30^0 C$ then heat loss Q in the pipe using $\frac{Q}{2\pi k}\int_{x_1}^{x_2}\frac{dx}{x} = -\int_{t_1}^{t_2} dT$, where x_1 ,x_2 are inner and outer radii and t_1 ,t_2 are temperatures at inner and outer surfaces is
A	$Q = 0.6815 \quad cal/sec$
B	$Q = 68.15 \quad cal/sec$
C	$Q = -0.6815 \quad cal/sec$
D	$Q = 6815 \quad cal/sec$
Ans	**A**
Marks	**1**
Unit	**IIc**

7	A pipe 20 cm is diameter contains steam at $150\ {}^0C$ and is protected with a covering 5 cm thick for which k = 0.0025. If the temperature of the outer surface of the covering is $40\ {}^0C$ .What is the heat loss (Q) through the covering under steady state conditions. [Use $\frac{Q}{2\pi k}\int_{x_1}^{x_2}\frac{dx}{x} = -\int_{t_1}^{t_2} dT$, where$Q$ is a heat loss.]
A	$Q = \frac{2\,\pi\,k}{log_e 1.5}\quad cal/sec$
B	$Q = \frac{-220\,\pi\,k}{log_e 1.5}\quad cal/sec$
C	$Q = \frac{220\,\pi\,k}{log_e 1.5}\quad cal/sec$
D	None of the above
Ans	**B**
Marks	**1**
Unit	**IIc**

8	A pipe 20 cm is diameter contains steam at $150\ {}^0C$ and is protected with a covering 5 cm thick for which k = 0.0025. If the temperature of the outer surface of the covering is $40\ {}^0C$ and the heat loss $Q = 0.03874\ \ cal/sec$. What is the temperature half-way through the covering under steady state conditions using $\frac{Q}{2\pi k}\int_{x_1}^{x_2}\frac{dx}{x} = -\int_{t_1}^{t_2} dT$.
A	$T = 98.5\ {}^0C$
B	$T = 30.5\ {}^0C$
C	$T = 89.5\ {}^0C$
D	$T = 895\ {}^0C$
Ans	**C**
Marks	**1**
Unit	**IIc**

9	What is heat loss per unit time from a unit length of pipe of radius r_0 carrying steam of temperature T_0 if the pipe is covered with insulation of thickness w. The outer surface of which remains at constant temperature T_1 . In a steady state condition it form a differential equation $Q\ \frac{dr}{r} = -2\ \pi\ k\ dT$, where Q is a heat loss
A	$Q = \frac{2\ \pi\ k\ [T_1 - T_0]}{\log\left(\frac{r_0\ +\ w}{r_0}\right)}$
B	$Q = \frac{2\ \pi\ k\ [T_1 + T_0]}{\log\left(\frac{r_0\ +\ w}{r_0}\right)}$
C	$Q = -\frac{2\ \pi\ k\ [T_1 + T_0]}{\log\left(\frac{r_0\ -\ w}{r}\right)}$
D	$Q = -\frac{2\ \pi\ k\ [T_1 - T_0]}{\log\left(\frac{r_0\ +\ w}{r_0}\right)}$
Ans	**D**
Marks	**1**
Unit	**IIc**

10	A long hollow pipe has an inner diameter of 10 cm and outer diameter of 20 cm.The inner surface is kept at $200\ ^0C$ and outer surface at $50\ ^0C$ for which k = 0.12. What is the heat loss . In a steady state condition it form a differential equation $Q\ \frac{dr}{r} = -2\ \pi\ k\ dT$, where Q is a heat loss. [Use $\frac{Q}{2\pi k}\int_{x_1}^{x_2}\frac{dx}{x} = -\int_{t_1}^{t_2} dT$]
A	$Q = \frac{-\,300\ \pi\ k}{log_e\ 2}\quad cal/sec$
B	$Q = \frac{300\ \pi\ k}{log_e\ 2}\quad cal/sec$
C	$Q = \frac{-\,30\ \pi\ k}{log_e\ 2}\quad cal/sec$
D	$Q = \frac{300\ \pi\ k}{log_e 1.2}\quad cal/sec$
Ans	**B**
Marks	**1**
Unit	**IIc**

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Rectification of Curves

Q. 1 The arc length of the curve $y = f(x)$ from any point $A(x_1, y_1)$ to $B(x_2, y_2)$ is measured by the formula

(a) $s = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ (b) $s = -\int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

(c) $s = \int_{y_1}^{y} \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \cdot dy$ (d) $s = -\int_{y_1}^{y} \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \cdot dy$ **Ans. : (a)**

Explanation : Please refer Section 7.9

Q. 2 The arc length of the curve $x = f(y)$ from any point $A(x_1, y_1)$ to $B(x_2, y_2)$ is measured by the formula

(a) $s = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ (b) $s = -\int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

(c) $s = \int_{y_1}^{y} \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \cdot dy$ (d) $s = -\int_{y_1}^{y_2} \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \cdot dy$ **Ans. : (c)**

Explanation : Please refer Section 7.9

Q. 3 The arc length of the curve $x = f_1(t)$; $y = f_2(t)$,(where t is parameter) from any point t_1 to t_2 is measured by the formula

(a) $s = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ (b) $s = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$

(c) $s = \int_{y_1}^{y} \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \cdot dy$ (d) $s = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 - \left(\frac{dy}{dt}\right)^2} \cdot dt$ **Ans. : (b)**

Explanation : Please refer Section 7.9

Q. 4 The arc length of the curve $r = f(\theta)$ from any point $\theta = \theta_1$ to θ_2 is measured by the formula

(a) $s = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ (b) $s = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$

(c) $\int_{r_1}^{r_2} \sqrt{1 + r^2\left(\frac{d\theta}{dr}\right)^2} \cdot dr$ (d) $s = \int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$ **Ans. : (d)**

Q. 5 The arc length of the curve $y = \log \sec x$ from $x = 0$ to $x = \pi/3$, if $1 + \left(\frac{dy}{dx}\right)^2 = \sec^2 x$ is

(a) $s = \log (2 + \sqrt{3})$ (b) $s = \log \sqrt{2} + 3\sqrt{3})$

(c) $s = \log (\sqrt{2} + 1\sqrt{3})$ (d) $s = \log (1 + \sqrt{3})$ **Ans. : (a)**

Explanation : Please refer Ex. 7.11.3

Q. 6 The arc length of the curve $y = f(x)$ by using $\left(\frac{dy}{dx}\right)^2 = x^2 - 1$ from $x = 0$ to $x = 2$ is

(a) 3 (b) 2 (c) 4 (d) 1 **Ans. : (b)**

Explanation :

$$\text{Given, } \left(\frac{dy}{dx}\right)^2 = x^2 - 1$$

$$1 + \left(\frac{dy}{dx}\right)^2 = x^2$$

$$\text{Arc length (s)} = \int_{x=0}^{2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_0^2 x\, dx = \left(\frac{x^2}{2}\right)_0^2 = 2$$

Q. 7 The arc length of the curve $y = f(x)$ by using $1 + \left(\frac{dy}{dx}\right)^2 = e^{4x}$ between $x = 0$ to $x = 1$ is

(a) e^4 (b) 1 (c) $\frac{e^2 - 1}{2}$ (d) $\frac{e^2 + 1}{2}$ **Ans. : (c)**

Explanation :

$$\text{Given, } 1 + \left(\frac{dy}{dx}\right)^2 = e^{4x}$$

$$\text{Arc length (s)} = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_0^1 e^{2x}\, dx = \left(\frac{e^{2x}}{2}\right)_0^1 = \frac{e^2 - 1}{2}$$

Q. 8 If the slope of the tangent of the curve at any point (x, y) is $(x - 1)$; then the length of arc between $x = 1$ and $x = 4$ is

(a) $\frac{3}{2}\sqrt{10} - \frac{1}{2}\left(3 + \sqrt{10}\right)$ (b) 3

(c) $\frac{1}{2}\sqrt{10}$ (d) $\frac{3}{2}\sqrt{10} + \frac{1}{2}\left(3 + \sqrt{10}\right)$ **Ans. : (d)**

Explanation :

$$\text{Given, } \frac{dy}{dx} = (x - 1) \qquad \text{Hence, } \sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + (x-1)^2}$$

$$\text{Arc length (s)} = \int_{x=1}^{4} \sqrt{1 + (x-1)^2}\, dx$$

$$= \left[\frac{x-1}{2}\sqrt{1+(x-1)^2} + \frac{1}{2}\left[(x-1) + \sqrt{1+(x-1)^2} \right] \right]_1^4$$

$$= \left[\frac{3}{2}\sqrt{10} + \frac{1}{2}\left(3+\sqrt{10}\right) \right]$$

Q. 9 The arc length of the circle $x^2 + y^2 = a^2$ by using $1 + \left(\frac{dy}{dx}\right)^2 = \frac{a^2}{a^2 - x^2}$ from $x = 0$ to $x = a$ is

(a) a (b) $\frac{\pi}{a}$ (c) $\frac{\pi}{2}$ a (d) log (2a) **Ans. : (c)**

Explanation :

$$\text{Arc length } s = \int_0^a \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_0^a \sqrt{\frac{a^2}{a^2 - x^2}}\, dx$$

$$= a\left[\sin^{-1}\left(\frac{x}{a}\right) \right]_0^a = a\left[\frac{\pi}{2} - 0 \right] = \frac{\pi}{2} a$$

Q. 10 The arc length of the catenary. $y = c \cosh\left(\frac{x}{c}\right)$ from the vertex (0, 0) to any point (x, y), by using $\left(\frac{dy}{dx}\right)^2 = \cosh^2 \frac{x}{c} - 1$; is

(a) $\cosh \frac{x}{c}$ (b) $c \sinh \frac{x}{c}$ (c) $\cosh^2 \frac{x}{c}$ (d) $\tanh\left(\frac{x}{c}\right)$ **Ans. : (b)**

Explanation :

$$\text{Given, } \left(\frac{dy}{dx}\right)^2 = \cosh^2 \frac{x}{c} - 1$$

$$1 + \left(\frac{dy}{dx}\right)^2 = \cosh^2\left(\frac{x}{c}\right)$$

$$\text{Arc length, } s = \int_{x=0}^{x} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_0^a \cosh \frac{x}{c}\, dx$$

$$s = \left(\frac{\sinh \frac{x}{c}}{\frac{1}{c}} \right)_0^x = C \cdot \sinh \frac{x}{c}$$

Q. 11 The arc length of the cycloid $x = (\theta - \sin\theta)$; $y = a(1 - \cos\theta)$ from $\theta = 0$ to $\theta = 2\pi$, by using $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 4a^2 \sin^2 \frac{\theta}{2}$, is

(a) 8a (b) 4a (c) $\sqrt{8a}$ (d) 2a **Ans. : (a)**

Explanation :

$$\text{Given, } \left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 4a^2 \sin^2 \frac{\theta}{2},$$

$$\text{Arc length } s = \int_{\theta_1 = 0}^{2\pi} \sqrt{4a^2 \sin^2 \frac{\theta}{2}}\, d\theta = 2a \left(\frac{\cos \frac{\theta}{2}}{\frac{1}{2}} \right)_0^{2\pi}$$

$$s = -4a(-1-1) = 8a$$

Q. 12 The total length of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ by using $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 9a^2 \cos^2\theta \sin^2\theta$, is

(a) 6a (b) 3a (c) 9a (d) $\frac{\pi}{2}$ **Ans. : (a)**

Explanation : Please refer Ex. 7.12.4

Q. 13 The total length of the arc of circle $x^2 + y^2 = 16$ by using $\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \frac{4}{y}$ is

(a) 2π (b) -2π (c) π (d) 16π **Ans. : (a)**

Explanation : Please refer Ex. 7.11.1

Q. 14 The length of the arc of parabola $y^2 = 16x$ from the vertex ((0, 0) to (4, 0)), by using $\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ is equal to

(a) 2π

(b) $\left(\frac{x+4}{x}\right)$

(c) $\left[\sqrt{2} - \log\left(1 + \sqrt{2}\right)\right]$

(d) $\left[\sqrt{2} + \log\left(1 - \sqrt{2}\right)\right]$ **Ans. : (b)**

Explanation : Please refer Ex. 7.11.9.

Q. 15 The length of the arc of the curve $y = \frac{a}{2}\left(e^{x/a} + e^{-x/a}\right)$ between the points (0, a) to (x, y), by using $1 + \left(\frac{dy}{dx}\right)^2 = \frac{1}{4}\left(e^{x/a} + e^{-x/a}\right)$ is

(a) $\left(e^{x+y}\right)$ (b) $\frac{a}{2}\left(e^{x/a} + e^{-x/a}\right)$ (c) $\frac{a}{2}\left(e^{x/a} - e^{-x/a}\right)$ (d) $\left(e^{x/a} - e^{-x/a}\right)$ **Ans. : (c)**

Explanation : Please refer Ex. 7.11.6

Q. 16 The length of the arc of parabola $y^2 = 4a\,x$ cut off by the line $3y = 8x$ from the point (0, 0) to $\left(\frac{9a}{16}, \frac{3a}{2}\right)$, by using $= \frac{1}{2a}\sqrt{4\,a^2 + y^2}$ is

(a) $a\left[\frac{15}{16} - \log 2\right]$

(b) $4\left[\sqrt{2} + \log\left(1 + \sqrt{2}\right)\right]$

(c) $\left[\sqrt{2} - \log\left(1 + \sqrt{2}\right)\right]$

(d) $a\left[\frac{15}{16} + \log 2\right]$ **Ans. : (d)**

Explanation : Please refer Ex. 7.11.9

Q. 17 The length of the arc of the curve $x^2 + y^2 = 16$ between the points x = 0 to 4 using $\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = 4 \cdot \frac{1}{\sqrt{16 - x^2}}$ is

(a) 2π (b) 4π (c) 3π (d) π **Ans. : (a)**

Sinhgad College of Engineering,Vadgaon

Engineering Mathematics – II

Unit 1- First Order Ordinary Differential Equations

1) The differential equation $1+\frac{dy}{dx}-\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}}=0$ is of
   A) order 1 and degree 2
   B) order 2 and degree 3
   C) order 3 and degree 6
   D) order 3 and degree 3

   Ans. B)

2) The differential equation $1+\frac{dy}{dx}=\frac{y}{\frac{dy}{dx}}$ is of
   A) order 2 and degree 2
   B) order 1 and degree 1
   C) order 2 and degree 1
   D) order 1 and degree 2

   Ans. D)

3) The differential equation $(2x-y+3)dx+(y-2x-2)dy=0$ is of
   A) order 1 and degree 1
   B) order 1 and degree 2
   C) order 2 and degree 1
   D) order 2 and degree 2

   Ans. A)

4) The number of arbitrary constants in the general solution of ordinary differential equation is equal to
   A) the order of differential equation
   B) the degree of differential equation
   C) coefficient of highest order differential derivative
   D) none of these

   Ans. A)

5) The order of differential equation whose general solution is $y = C_1 + C_2e^{-2x} + C_3e^{3x} + C_4e^{-3x}$, where C_1, C_2, C_3, C_4 are arbitrary constants is
   A) 1
   B) 3
   C) 2
   D) 4

Ans. D)

6) $y = Cx - C^2$,where C is arbitrary constant is the general solution of the differential equation
   A) $\frac{dy}{dx} = C$
   B) $\left(\frac{dy}{dx}\right)^2 + xy = 0$
   C) $\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} - y = 0$
   D) $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$

Ans. D)

7) By eliminating arbitrary constant A the differential equation whose general solution is $y = 4(x - A)^2$is
   A) ${y_1}^2 + 16y = 0$
   B) $y_1 - 2y = 0$
   C) ${y_1}^2 - 16y = 0$
   D) $y_1 - 8(x - A) = 0$

Ans. C)

8) The differential equation satisfied by general solution $y = A\cos x + B\sin x$ where A and B are arbitrary constants is,
   A) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = B\sin x$
   B) $\frac{d^2y}{dx^2} - y = 0$
   C) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = A\cos x$
   D) $\frac{d^2y}{dx^2} + y = 0$

Ans. D)

9) The differential equation satisfied by general solution $y = Ae^x + Be^{-x}$ where A and B are arbitrary constants,is

A) $y_2 - y = 0$
B) $y_2 + y = 0$
C) $y_2 + y = Ae^x - Be^{-x}$
D) $y_2 - y = 2Ae^x$

Ans. A)

10) The differential equation satisfied by general solution $y = Ax^2 + Bx + C$ where A, B, C are arbitrary constants is

A) $\frac{d^3y}{dx^3} = 0$
B) $\frac{d^2y}{dx^2} = 2A$
C) $\frac{d^3y}{dx^3} = A$
D) $\frac{d^4y}{dx^4} = 0$

Ans. A)

11) The solution of differential equation $\frac{dy}{dx} + y = 0$ is

A) $y = Ae^{-x}$
B) $y = Ae^x$
C) $x = Ae^{-y}$
D) $x = Ae^y$

Ans. A)

12) The solution of differential equation $\frac{dy}{dx} + \frac{1+y^2}{1+x^2} = 0$ is

A) $\tan^{-1} y - \tan^{-1} x = C$
B) $\tan^{-1} y + \tan^{-1} x = C$
C) $\tan y + \tan x = C$
D) $\cos y + \cos x = C$

Ans. B)

13) The solution of differential equation $sec^2 x \tan y\, dx + sec^2 y \tan x\, dy = 0$ is

A) $sec^2 x \tan y = C$
B) $sec^2 y \tan x = C$
C) $\tan x \tan y = C$
D) $sec^2 x sec^2 y = C$

Ans. C)

14) The solution of differential equation $e^x \cos y\, dx + (1 + e^x) \sin y\, dy = 0$ is

A) $(1 + e^x) = C \sec y$
B) $(1 + e^x) \sec y = C$
C) $\frac{\sec y}{(1+e^x)} = C$
D) $(1 + e^x) \cos y = C$

Ans. B)

15) The solution of differential equation $x(1 + y^2)dx + y(1 + x^2)dy = 0$ is

A) $(1 - x^2)(1 + y^2) = C$
B) $\tan^{-1} x + \tan^{-1} y = C$
C) $(1 + x^2) = C(1 + y^2)$
D) $(1 + x^2)(1 + y^2) = C$

Ans. D)

16) The necessary and sufficient condition that the differential equation $M(x, y)dx + N(x, y)dy = 0$ be exact is

A) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$
B) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$
C) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$
D) $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 1$

Ans . A)

17) If homogeneous differential equation $M(x,y)dx + N(x,y)dy = 0$ is not exact then the integrating factor is

A) $\frac{1}{My+Nx}; My + Nx \neq 0$
B) $\frac{1}{Mx-Ny}; Mx - Ny \neq 0$
C) $\frac{1}{Mx+Ny}; Mx + Ny \neq 0$
D) $\frac{1}{My-Nx}; My - Nx \neq 0$

Ans. C)

18) If the differential equation $M(x,y)dx + N(x,y)dy = 0$ is not exact and it can be written as $yf_1(xy)dx + xf_2(xy)dy = 0$ then the integrating factor is

A) $\frac{1}{My+Nx}; My + Nx \neq 0$
B) $\frac{1}{Mx-Ny}; Mx - Ny \neq 0$
C) $\frac{1}{Mx+Ny}; Mx + Ny \neq 0$
D) $\frac{1}{My-Nx}; My - Nx \neq 0$

Ans. B)

19) If the differential equation $M(x,y)dx + N(x,y)dy = 0$ is not exact and $\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x)$ then the integrating factor is

A) $e^{f(x)}$
B) $e^{\int f(x)dy}$
C) $f(x)$
D) $e^{\int f(x)dx}$

Ans. D)

20) If the differential equation $M(x,y)dx + N(x,y)dy = 0$ is not exact and $\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = f(y)$ then the integrating factor is

A) $e^{\int f(y)dy}$
B) $e^{\int f(y)dx}$
C) $f(y)$
D) $e^{f(y)}$

Ans. A)

21) The value of λ for which the differential equation

$(xy^2 + \lambda x^2y)dx + (x^3 + x^2y)dy = 0$ is exact is

A) -3
B) 2
C) 3
D) 1

Ans. C)

22) The differential equation $\left(\frac{2x}{y^3}\right)dx + \left(\frac{y^2+ax^2}{y^4}\right)dy = 0$ is exact if

A) a= -3
B) a= 3
C) a= -2
D) a= 6

Ans. A)

23) Integrating factor of homogeneous differential equation

$(xy - 2y^2)dx + (3xy - x^2)dy = 0$ is

A) $\frac{1}{xy}$
B) $\frac{1}{x^2y^2}$
C) $\frac{1}{x^2y}$
D) $\frac{1}{xy^2}$

Ans. D)

24) Integrating factor for differential equation $(x^2y^2 + xy + 1)ydx + (x^2y^2 - xy + 1)xdy = 0$ is

A) $\frac{1}{2x^3y^3}$
B) $\frac{1}{xy}$
C) $\frac{1}{2x^2y^2}$
D) $\frac{1}{x^2y}$

Ans. C)

25) Integrating factor for differential equation $(x^2 + y^2 + 1)dx - 2xydy = 0$ is

A) $\frac{1}{x}$
B) $\frac{1}{x^3}$
C) $\frac{1}{x^2}$
D) $\frac{1}{xy}$

Ans. C)

26) Integrating factor for differential equation $y \log y\, dx + (x - \log y)dy = 0$ is

A) $\frac{1}{x}$
B) $\frac{1}{y}$
C) $\frac{1}{x^2}$
D) $\frac{1}{y^2}$

Ans. B)

27) Solution of non-exact diff. equation $(3xy^2 - y^3)dx + (xy^2 - 2x^2y)dy = 0$ with integrating factor $\frac{1}{x^2y^2}$ is

A) $3\log x - \frac{2y}{x^3} - 2\log y = C$
B) $3\log x + \frac{y}{x} - 2\log y = C$
C) $3\log x + \frac{y}{x} = C$
D) $\log x - \frac{y}{x} + 2\log y = C$

Ans. B)

28) Solution of non-exact diff. equation $(1+xy)ydx+(1-xy)xdy=0$ with integrating factor $\frac{1}{x^2y^2}$ is

A) $\frac{2}{xy} - \log\left(\frac{x}{y}\right) = C$
B) $-\frac{1}{xy} + \log\left(\frac{y}{x}\right) = C$
C) $-\frac{1}{xy} + \log\left(\frac{x}{y}\right) = C$
D) $-\frac{2}{x^3y} + \log\left(\frac{x}{y}\right) = C$

Ans. C)

29) The differential equation $(3+by\cos x)dx+(2\sin x-4y^3)dy=0$ is exact if

A) b= 4
B) b= 3
C) b= 0
D) b= 2

Ans. D)

30) Integrating factor for differential equation $(x^2+y^2+x)dx+(xy)dy=0$ is

A) $\frac{1}{x}$
B) $\frac{1}{x^2}$
C) x^2
D) x

Ans. D)

31) The differential equation of the form $\frac{dy}{dx}+Py=Q$ where P and Q are functions of x or constants, is

A) Exact differential equation
B) Linear differential equation in y
C) Linear differential equation in x
D) Non-homogeneous differential equation

Ans. B)

32) The differential equation of the form $\frac{dx}{dy} + Px = Q$ where P and Q are functions of y or constants, is

A) Exact differential equation
B) Linear differential equation in y
C) Linear differential equation in x
D) Non-homogeneous differential equation

Ans. C)

33) Integrating factor of linear differential equation $\frac{dy}{dx} + Py = Q$ where P and Q are functions of x or constants, is

A) $e^{\int P dy}$
B) $e^{\int Q dy}$
C) $e^{\int Q dx}$
D) $e^{\int P dx}$

Ans. D)

34) Integrating factor of linear differential equation $\frac{dx}{dy} + Px = Q$ where P and Q are functions of x or constants, is

A) $e^{\int P dy}$
B) $e^{P dx}$
C) $e^{\int Q dx}$
D) $e^{\int Q dy}$

Ans. A)

35) The differential equation of the form $\frac{dy}{dx} + Py = Qy^n, n \neq 1$ where P and Q are functions of x or constants, is

A) Bernoulli's differential equation
B) Exact differential equation
C) Symmetric differential equation
D) Linear differential equation

Ans. A)

36) The general solution of linear differential equation $\frac{dy}{dx}+Py=Q$ where P and Q are functions of x or constants, is

A) $xe^{\int Pdy}=\int Qe^{\int Pdy}\,dy+C$
B) $y=\int Qe^{\int Pdx}\,dx+C$
C) $ye^{\int Pdx}=\int Q\,dx+C$
D) $ye^{\int Pdx}=\int Qe^{\int Pdx}\,dx+C$

Ans. D)

37) The differential equation of the form $f'(x)\frac{dx}{dy}+Pf(x)=Q$ where P and Q are functions of y or constants,can be reduced to linear differential equation by the substitution

A) $f'(x)=u$
B) $f(x)=u$
C) $P=u$
D) $Q=u$

Ans. B)

38) Integrating factor of linear differential equation $\frac{dy}{dx}+xy=x^3$ is

A) $e^{\log x}$
B) e^{x}
C) x^2
D) $e^{\frac{x^2}{2}}$

Ans. D)

39) The differential equation $\frac{dx}{dy}+\frac{x}{1+y^2}=y^2$ has integrating factor

A) $e^{\frac{1}{1+y^2}}$
B) $e^{\tan^{-1}x}$
C) $e^{\frac{1}{1+x^2}}$
D) $e^{\tan^{-1}y}$

Ans. D)

40) The differential equation $\cos x \frac{dy}{dx} + y = \sin x$ has integrating factor

A) $e^{\sec x}$
B) $(cosecx - cotx)$
C) $(\sec x + \tan x)$
D) $(\sec x - \tan x)$

Ans. C)

41) The Bernoulli's differential equation $\frac{dy}{dx} - y \tan x = y^4 \sec x$ reduces to linear differential equation

A) $\frac{du}{dx} + (3 \tan x)u = -3 \sec x \ where\ y^{-3} = u$
B) $\frac{du}{dx} - (3 \tan x)u = 3 \sec x \ where\ y^{-3} = u$
C) $\frac{du}{dx} + (\tan x)u = -\sec x \ where\ y^{-3} = u$
D) None of these

Ans. A)

42) The general solution $\frac{dy}{dx} + \frac{3}{x}y = x^2$ with integrating factor x^3 is...

A) $yx^3 = \frac{x^6}{6} + C$
B) $yx^3 = \frac{x^2}{6} + C$
C) $yx^3 = \log x + C$
D) None of these

Ans. A)

43) The general solution of $\frac{dx}{dy} + \frac{1}{1+y^2}x = \frac{e^{-\tan^{-1} y}}{1+y^2}$ with integrating factor $e^{\tan^{-1} y}$

A) $xe^{\tan^{-1} y} = \tan^{-1} y + C$
B) $ye^{\tan^{-1} y} = \tan^{-1} y + C$
C) $e^{\tan^{-1} y} = \tan^{-1} y + C$
D) $e^{-\tan^{-1} y} = \tan^{-1} y + C$

Ans. A)

44) The differential equation $\sin y \frac{dy}{dx} - 2\cos x \cos y = -\cos x\, sin^2 x$ reduces to linear differential equation...

A) $\frac{du}{dx} + (\cos x)u = \cos x\, sin^2 x\ where\ \cos y = u$
B) $\frac{du}{dx} - (2\cos x)u = \cos x\, sin^2 x\ where\ \cos y = u$
C) $\frac{du}{dx} + (2\cos x)u = \cos x\, sin^2 x\ where\ \cos y = u$
D) $\frac{du}{dx} - (\cos x)u = \cos x\, sin^2 x\ where\ \cos y = u$

Ans. C)

45) The general solution of $\frac{dy}{dx} + \frac{2}{x}y = \frac{1}{x^3}$ with integrating factor x^2 is...

A) $yx^2 = \frac{x^2}{2} + C$
B) $yx^2 = \log x + C$
C) $yx^2 = \frac{x^6}{6} + C$
D) None of these

Ans. B)

46) The differential equation $\frac{dy}{dx} + y\cot x = \sin 2x$ has integrating factor

A) $\cos x$
B) $e^{\cot x}$
C) $\sin x$
D) $\sec x$

Ans. C)

47) The solution of differential equation $xdy - ydx = 0$ is

A) $y = x + c$
B) $x^2 - y^2 = c$
C) $xy = c$
D) $y = cx$

Ans. D)

48) The solution of differential equation $(e^x + 1)ydy = (y + 1)e^x dx$ is

A) $y - \log(1 - y) = \log(e^x - 1) + \log C$
B) $y - \log(1 + y) = \log(e^x + 1) + \log C$
C) $y + \log(1 - y) = \log(e^x + 1) + \log C$
D) $y - \log(1 + y) = \log(e^x - 1) + \log C$

Ans. B)

49) Integrating factor of homogeneous differential equation

$(x^2 - 3xy + 2y^2)dx + (3x^2 - 2xy)dy = 0$ is

A) $\frac{1}{xy}$
B) $\frac{1}{x^3}$
C) $\frac{1}{x^2y}$
D) $\frac{1}{x^2}$

Ans. B)

50) The differential equation $\frac{dy}{dx} + \sqrt{x}y = x^3$ has integrating factor

A) $e^{\frac{2}{3}x\sqrt{x}}$
B) $e^{\frac{1}{3}x\sqrt{x}}$
C) $e^{\sqrt{x}}$
D) e^{-x}

Ans. A)

Sinhgad College of Engineering,Vadgaon

Engineering Mathematics – II

Unit 2- Applications of Differential Equations

Orthogonal Trajectories

1. The differential equation of orthogonal trajectories of family of curves $xy = c$ is
   A. $x\frac{dx}{dy} + y = 0$
   B. $-x\frac{dx}{dy} + y = 0$
   C. $-x\frac{dx}{dy} - y = 0$
   D. $x\frac{dy}{dx} + y = 0$

   ANS (B)

2. If the family of curves is given by $y^2 = 4ax$ then the differential equation of orthogonal trajectories of family is
   A. $2y\frac{dy}{dx} = 4a$
   B. $2y\frac{dy}{dx} = \frac{y^2}{x}$
   C. $-2y\frac{dx}{dy} = \frac{y^2}{x}$
   D. $2y\frac{dy}{dx} = \frac{x}{y^2}$

   ANS (C)

3. The differential equation of orthogonal trajectories of family of curves $2x^2 + y^2 = cx$ is
   A. $4x + 2y\frac{dy}{dx} = \frac{2x^2+y^2}{x}$
   B. $4x - 2y\frac{dx}{dy} = \frac{2x^2+y^2}{x}$
   C. $4x - 2y\frac{dx}{dy} = \frac{2x^2+y^2}{x}$
   D. none of these

ANS (B)

4. The differential equation of orthogonal trajectories of family of curves $x^2 + cy^2 = 1$ is
   A. $x - \left(\frac{1-x^2}{y}\right)\frac{dx}{dy} = 0$
   B. $x + \left(\frac{1-x^2}{y}\right)\frac{dy}{dx} = 0$
   C. $x + \left(\frac{1-x^2}{y}\right)\frac{dx}{dy} = 0$
   D. none of these

ANS (A)

5. The differential equation of orthogonal trajectories of family of curves $e^x + e^{-y} = c$ is
   A. $e^x - e^{-y}\frac{dy}{dx} = 0$
   B. $e^x - e^{-y}\frac{dx}{dy} = 0$
   C. $e^x + e^{-y}\frac{dx}{dy} = 0$
   D. none of these

ANS (C)

6. The differential equation of orthogonal trajectories of family of curves $r = a\sin\theta$ is
   A. $\frac{1}{r}\frac{d\theta}{dr} = \cot\theta$
   B. $r\frac{d\theta}{dr} = -\cot\theta$
   C. $r\frac{d\theta}{dr} = -\tan\theta$
   D. $r^2\frac{d\theta}{dr} = \tan\theta$

ANS (B)

7. Orthogonal trajectories of family of circle $x^2 + y^2 = c^2$ whose differential equation is $\frac{dy}{dx} = -\frac{x}{y}$, is equal to
   A. $x^2 - y^2 = k^2$
   B. $y = kx$
   C. $y^2 = kx$
   D. $x^2 + y^2 = k^2$

ANS (B)

8. If the differential equation of family of rectangular hyperbola $x^2 - y^2 = c^2$ is $\frac{dy}{dx} = \frac{x}{y}$, then its orthogonal trajectories is
   A. $y^2 = kx$
   B. $x^2 + y^2 = k^2$
   C. $yx = k$
   D. $y = kx$

ANS (C)

9. Orthogonal trajectories of family of curves $x^2 + 2y^2 = c^2$ whose differential equation is $\frac{dy}{dx} = -\frac{x}{2y}$, is equal to
   A. $x^2 = ky$
   B. $x^2 = \frac{k}{y}$
   C. $x^2 + 2y^2 = k^2$
   D. none of these

ANS (A)

10. Orthogonal trajectories of family of curves $y^2 = 4ax$ whose differential equation is $\frac{dy}{dx} = \frac{y}{2x}$, is equal to
   A. $x^2 + y^2 = k^2$
   B. $x^2 + 2y^2 = k^2$
   C. $y^2 = 4kx$
   D. $2x^2 + y^2 = k$

ANS (D)

Newton's Law of Cooling:

1. If the temperature of the body drops from $100^0\mathrm{C}$ to $60^0\mathrm{C}$ in 1 minute when the temperature of surrounding is $20^0\mathrm{C}$ satisfies the differential equation $\frac{d\theta}{dt} = -k(\theta - 20)$, then the value of k is
   A. $\log_e 2$
   B. $-\log_e 2$
   C. $\log_e 4$
   D. $\log_e 8$

   ANS (A)

2. The temperature of the air is $30^0\mathrm{C}$ and the substance cools from $100^0\mathrm{C}$ to $70^0\mathrm{C}$ in 15 minutes. If differential equation by Newton's law of cooling is $\frac{d\theta}{dt} = -k(\theta - 30)$, then the value of k is
   A. $\log_e \frac{7}{4}$
   B. $\frac{1}{15}\log_e \frac{4}{7}$
   C. $\frac{1}{15}\log_e \frac{7}{4}$
   D. $15\log_e \frac{7}{4}$

   ANS (C)

3. By Newton's law of cooling the differential equation of body originally at $80^0\mathrm{C}$ cools down to $60^0\mathrm{C}$ in 20 minutes in surrounding temperature of $40^0\mathrm{C}$ is $\frac{d\theta}{dt} = -(0.03465)(\theta - 40)$. The temperature of the body after 40 minutes is
   A. $60^0\mathrm{C}$
   B. $50^0\mathrm{C}$
   C. $35^0\mathrm{C}$
   D. $85^0\mathrm{C}$

   ANS (B)

4. A metal ball is heated to a temperature of 100^0C and at time t = 0 it is placed in water which is maintained at 40^0C.The temperature of the ball reduces to 60^0C in 4 minutes. By Newton's law of cooling the differential equation is $\frac{d\theta}{dt} = -\left(\frac{1}{4}\log_e 3\right)(\theta - 40)$. Then the time required to reduce the a temperature of ball to 50^0C is
   A. 7.5 min
   B. 3.5 min
   C. 10 min
   D. 6.5 min

   ANS (D)

5. A body at a temperature 100^0C and at time t = 0 it is placed in a room whose temperature is 20^0C and cools to 60^0C in 5 minutes. By Newton's law of cooling the differential equation is $\frac{d\theta}{dt} = -\left(\frac{1}{5}\log_e 2\right)(\theta - 20)$. Then the temperature after 8 minutes is
   A. 46.4^0C
   B. 65.4^0C
   C. 40.4^0C
   D. 20^0C

   ANS(A)

Applications to Electrical Circuits:

1. The differential equation for the current in an electric circuit containing resistance 100 ohm and an inductance of 0.5 henry connected in series with battery of 20 volts is
   A. $0.5\frac{di}{dt} + 100i = 0$
   B. $0.5\frac{di}{dt} + 100i = 20$
   C. $100\frac{di}{dt} + 0.5i = 20$
   D. $100\frac{di}{dt} + 0.5R = 0$

   ANS (B)

2. The differential equation for the current in an electric circuit containing resistance R = 250 ohm and an inductance of L = 640 henry in series with an electromotive force E = 500 volts is
   A. $640\frac{di}{dt} + 250i = 0$
   B. $250\frac{di}{dt} + 640i = 500$
   C. $640\frac{di}{dt} + 250i = 500$
   D. $250\frac{di}{dt} + 640i = 0$

   ANS (C)

3. A capacitor C = 0.01 farad in series with resistor R = 20 ohms is charged from battery E = 10 volts. If initially capacitor is completely discharged then differential equation for charge q(t) is given by
   A. $20\frac{dq}{dt} + \frac{q}{0.01} = 0; q(0) = 0$
   B. $20\frac{dq}{dt} + 0.01q = 10; q(0) = 0$
   C. $20\frac{dq}{dt} + \frac{q}{0.01} = 10; q(0) = 0$
   D. $20\frac{dq}{dt} + 0.01q = 0; q(0) = 0$

   ANS (C)

4. In a circuit containing resistance R and inductance L in series with constant e.m.f.E, the current i is given by $i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$, then the time required to build current half of its theoretical maximum is

   A. $\frac{L}{R \log 2}$
   B. $\frac{L \log 2}{R}$
   C. $\frac{R \log 2}{L}$
   D. 0

ANS (B)

5. In a circuit containing resistance R and inductance L in series with constant e.m.f.E, the current i is given by $i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$, then the time required before current reaches its 90% maximum value is

   A. 0
   B. $\frac{L}{R \log 10}$
   C. $\frac{R \log 10}{L}$
   D. $\frac{L \log 10}{R}$

ANS (D)

6. The differential equation for the current in an electric circuit containing resistance R and inductance of L in series with constant e.m.f. E, the current i is $L = \frac{di}{dt} + Ri = E$, then the current at any time t is given by

   A. $i = \frac{E}{R} - Ae^{-\frac{R}{L}t}$; A is arbitrary constant
   B. $i = \frac{E}{R} + Ae^{-\frac{R}{L}t}$; A is arbitrary constant
   C. $i = \frac{E}{R} + Ae^{\frac{R}{L}t}$; A is arbitrary constant
   D. $i = \frac{E}{R} + e^{-\frac{R}{L}t}$

ANS (B)

7. A charge q on the plate of condenser of capacity C charge through a resistance R by steady voltage V satisfies differential equation $R\frac{dq}{dt}+\frac{q}{C}=V$, then charge q at any time t is

   A. $q = CV+Ae^{-\frac{1}{RC}t}$; A is arbitrary constant
   B. $q = CV-Ae^{\frac{1}{RC}t}$; A is arbitrary constant
   C. $q = C+Ae^{\frac{1}{RC}t}$; A is arbitrary constant
   D. $q = CV+e^{\frac{1}{RC}t}$

ANS (A)

8. A charge q on the plate of condenser of capacity C charge through a resistance R by steady voltage V is given by $q = CV\left(1-e^{\frac{1}{-RC}t}\right)$ then current flowing through the plate is

   A. $i = \frac{V}{R}e^{-\frac{R}{L}t}$
   B. $i = \frac{V}{R}e^{\frac{1}{RC}t}$
   C. $i = \frac{V}{R}e^{\frac{1}{-RC}t}$
   D. $i = CV\left(1-e^{\frac{1}{-RC}t}\right)$

ANS (C)

9. A resistance R = 100 ohms, an inductance L = 0.5 henry are connected in series with a battery of 20 volts. The differential equation for the current i is $0.5\frac{di}{dt}+100i=20$ then current i at any time t is

   A. Ae^{-200t}; A is arbitrary constant
   B. $\frac{1}{5}+Ae^{200t}$; A is arbitrary constant
   C. $2+Ae^{-200t}$; A is arbitrary constant
   D. $\frac{1}{5}+Ae^{-200t}$; A is arbitrary constant

ANS (D)

10. A circuit containing resistance R and inductance L in series with voltage source E. The differential equation for the current i is $L\frac{di}{dt} + Ri = E$. Given $L = 640$ H, $R = 250\ \Omega$ and $E = 500$ volts then integrating factor of differential equation is

A. $e^{\frac{64}{25}t}$

B. $e^{\frac{25}{64}t}$

C. $e^{-\frac{25}{64}t}$

D. e^{250t}

Ans. B

Rectilinear Motion,heat flow,S.H.M.

1) Rectilinear motion is a motion of the body along a ------

A) straight line B) circular path

C) parabolic path D) none of these

2) According to D'Alemberts's principle, algebraic sum of forces acting on a body along a given direction is equal to -----

A) velocity X acceleration B) mass x velocity

C) mass X Displacement D) mass X acceleration

3) A particle moving in a straight line with acceleration $k\left(x+\frac{a^4}{x^3}\right)$ directed towards the origin.

The equation of motion is ------

A) $\frac{dv}{dx} = -k\left(x+\frac{a^4}{x^3}\right)$ B) $v.\frac{dv}{dx} = k\left(x+\frac{a^4}{x^3}\right)$

C) $v.\frac{dv}{dx} = -k\left(x+\frac{a^4}{x^3}\right)$ D) $\frac{dv}{dx} = \left(x+\frac{a^4}{x^3}\right)$

4) A particle of mass m moves in a horizontal straight line OA with acceleration $\frac{k}{x^3}$ at a distance x and directed towards the origin O. Then the differential equation of motion is -----

A) $v.\frac{dv}{dx} = \frac{k}{x^3}$ B) $v.\frac{dv}{dx} = -\frac{k}{x^3}$

C) $\frac{dv}{dx} = -\frac{k}{x^3}$ D) $\frac{dv}{dx} = \frac{k}{x^3}$

5) A body of mass m falling from rest is subjected to a force of gravity and air resistance proportional to square of velocity (kv^2). The equation of motion is ----

A) $m\frac{dv}{dx} = mg - kv^2$ B) $mv\frac{dv}{dx} = mg + kv^2$

C) $m\frac{dv}{dx} = -kv^2$ D $mv\frac{dv}{dx} = mg - kv^2$

6) A particle is projected vertically upward with velocity v_1 and resistance of air produces retardation (kv^2). Where v is velocity. The equation of motion is -----

A) $v\frac{dv}{dx} = -g - kv^2$

B) $v\frac{dv}{dx} = -g + kv^2$

C) $v\frac{dv}{dx} = -kv^2$

D) $v\frac{dv}{dx} = g - kv^2$

7) A body starts moving from rest is opposed by a force per unit mass of value (cx) resistance per unit mass of value (bv^2).where v and x are velocity and displacement of body at that instant. The differential equation of motion is ----

A) $mv\frac{dv}{dx} = -cx - bv^2$

B) $v\frac{dv}{dx} = cx + bv^2$

C) $v\frac{dv}{dx} = -cx - bv^2$

D) $\frac{dv}{dx} = -cx - bv^2$

8) A body of mass m falls from rest under gravity, in a fluid whose resistance to motion at any time t is mk times its velocity where k is constant. The differential equation of motion is ----

A) $\frac{dv}{dt} = -g - kv$

B) $\frac{dv}{dt} = g - kv$

C) $\frac{dv}{dt} = g + kv$

D) $\frac{dv}{dt} = mg - mkv$

9) A particle of mass m is projected vertically upward with velocity v, assuming the air resistance k times its velocity where k is constant. The differential equation of motion is -----

A) $\frac{dv}{dt} = mg - kv$

B) $m\frac{dv}{dt} = -mg + kv$

C) $m\frac{dv}{dt} = -kv$

D) $m\frac{dv}{dt} = -mg - kv$

10) Assuming that the resistance to movement of a ship through water in the form of $(a^2 + b^2v^2)$ where v is the velocity, a and b are constants. The differential equation for retardation of the ship moving with engine stopped is -----

A) $m\frac{dv}{dt} = -(a^2 + b^2v^2)^2$

B) $m\frac{dv}{dt} = (a^2 + b^2v^2)$

C) $m\frac{dv}{dt} = -(a^2 + b^2v^2)$

D) $m\frac{dv}{dX} = -(a^2 + b^2v^2)$

11) A bullet is fired into a sand tank; its retardation is proportional to the square root of its velocity. The differential equation of motion is $\frac{dv}{dt} = -k\sqrt{v}$ .If v_0 is initial velocity then the relation between velocity v and the time t is ----

A) $\sqrt{v} = -t + \sqrt{v_0}$

B) $2\sqrt{v} = -kt$

C) $\sqrt{v} = -kt + \sqrt{v_0}$

D) $2\sqrt{v} = -kt + 2\sqrt{v_0}$

12) A vehicle starts from rest and its acceleration is given by $\frac{dv}{dt} = k\left(1 - \frac{t}{T}\right)$ where k and T are constant. Then the velocity v in terms of t is given by ----

A) $v = k\left(t - \frac{t^2}{2}\right)$

B) $v = k\left(t - \frac{t^2}{T}\right)$

C) $v = k\left(\frac{t^2}{2} - \frac{t^3}{3T}\right)$

D) $v = k\left(t - \frac{t^2}{2T}\right)$

13) A body of mass m falls from rest under gravity in a fluid whose resistance to motion at any instant is mkv where k is constant. The differential equation of motion is $\frac{dv}{dt} = g - kv$ then the terminal velocity is -----

A) $\frac{k}{g}$

B) $\frac{g}{k}$

C) $-\frac{g}{k}$

D) None of these

14) A particle of mass m is projected upward with velocity V .Assuming the air resistance k times its velocity and equation of motion is $m\,\frac{dV}{dt} = -mg - kv$ then the relation between velocity v and time t is ----

A) $t = \frac{m}{k} log\left(\frac{mg+kV}{mg+kv}\right)$

B) $t = \frac{m}{k} log\left(\frac{mg+kv}{mg+kV}\right)$

C) $t = m\ log\left(\frac{mg+kV}{mg+kv}\right)$

D) $t = log\left(\frac{mg+kv}{mg+kV}\right)$

15.Fourier law of heat conduction states that, the quantity of heat flow across an area A cm^2 is ...

A) proportional to the product of area A and temperature gradient $\frac{dT}{dX}$

B) inversely proportional to the product of area A and temperature gradient $\frac{dT}{dX}$

C) equal to sum of area A and temperature gradient $\frac{dT}{dX}$

D) equal to the difference of area A and temperature gradient $\frac{dT}{dX}$

16. If q be the quantity of heat that flows across an area A cm^2 and thickness δx in one second where the difference of temperature at the faces is δT,then by Fourier's law of heat conduction.....

A) $q = -k\left(A - \frac{dT}{dX}\right)$, where k is thermal conductivity.

B) $q = kA\frac{dT}{dX}$, where k is thermal conductivity.

C) $q = -k\left(A + \frac{dT}{dX}\right)$, where k is thermal conductivity.

D) $q = -kA\frac{dT}{dX}$, where k is thermal conductivity.

17. The differential equation for steady state heat loss per unit time from a unit length of pipe with thermal conductivity k, radius r_0 carrying steam at temperature T_0,if the pipe is covered with insulation of thickness W, the outer surface of which remains at the constant temperature T_1,is ---

A) $Q = k(2\pi r)\frac{dT}{dr}$

B) $Q = -k(2\pi r)\frac{dT}{dr}$

C) $Q = -k\,(2\pi r^2)\,\frac{dT}{dr}$

D) $Q = -k\,(\pi r^2)\,\frac{dT}{dX}$

18. The differential equation for steady state heat loss per unit time from a spherical shell of pipe with thermal conductivity k, radius r_0 carrying steam at temperature T_0,if the spherical shell is covered with insulation of thickness W, the outer surface of which remains at the constant temperature T_1,is -----

A) $Q = -k(2\pi r)\frac{dT}{dr}$

B) $Q = k(2\pi r)\frac{dT}{dr}$

C) $Q = -k\,(4\pi r^2)\,\frac{dT}{dr}$

D) $Q = -k\,(\pi r^2)\,\frac{dT}{dX}$

19. The differential equation for steady state heat loss Q per unit time from a unit length of pipe with thermal conductivity k, radius r_0 carrying steam at temperature T_0,if the pipe is covered with insulation of thickness W, the outer surface of which remains at the constant temperature T_1, is $Q = -k(2\pi r)\frac{dT}{dr}$ .Then the temperature T of surface of pipe of radius r is ------

A) $T = \frac{Q}{2\pi k}\frac{1}{r} + C$

B) $T = \frac{Q}{2\pi k} logr + C$

C) $T = -\frac{Q}{2\pi k}\frac{1}{r} + C$

D) $T = -\frac{Q}{2\pi k} logr + C$

20. The differential equation for steady state heat loss Q per unit time from a spherical shell with thermal conductivity K, radius r_0 carrying steam at temperature T_0,if the spherical shell is covered with insulation of thickness W, the outer surface of which remains at the constant temperature T_1, is $Q = -k(4\pi r^2)\frac{dT}{dr}$ .Then the temperature T of spherical shell of radius r is ------

A) $T = -\frac{Q}{4\pi k}\frac{1}{r^2}$ +C

B) $T = \frac{Q}{4\pi k}\frac{1}{r} + C$

C) $T = -\frac{Q}{4\pi k}\frac{1}{r} + C$

D) $T = \frac{Q}{2\pi k}\frac{1}{r^3} + C$

21. A pipe 20 cm in diameter contains steam at 150°C and is protected with covering 5cm thick for which k = 0.0025. If the temperature of the outer surface of the covering is 40°C and differential equation of conduction of heat is $dT = -\frac{Q}{2\pi k}\frac{dX}{X}$. The amount of heat loss Q is ------

A) $\frac{110\,(2\pi k)}{log(1.5)}$

B) $\frac{\log(1.5)}{110(2\pi k)}$

C) $-\frac{110\,(2\pi k)}{log(1.5)}$

D) $\frac{110}{log(1.5)}$

22. A long hollow pipe has an inner diameter of 10 cm and outer diameter of 20 cm.The inner surface is kept at 200°C and outer surface at 50°C. The thermal conductivity k = 0.12.The differential equation of conduction of heat is $dT = -\frac{Q}{2\pi k}\frac{dX}{X}$.Then the amount of heat loss Q Cal/sec is .....

A) $-\frac{150(2\pi k)}{\log 2}$

B) $\frac{\log 2}{150(2\pi k)}$

C) $\frac{150(\,2\pi k)}{log2}$

D) $\frac{(2\pi k)}{\log 2}$

23.A steam pipe 20 cm in diameter is protected with covering 6 cm thick for which thermal conductivity k = 0.0003 in steady state. The inner surface of the pipe is at $200^{\circ}C$ and outer surface of the covering is at $30^{\circ}C$ and differential equation of conduction of heat is $dT = -\frac{Q}{2\pi k}\frac{dX}{X}$. The amount of heat loss Q is ......

A) $\frac{170\,(2\pi k)}{log(1.6)}$ B) $-\frac{170\,(2\pi k)}{log(1.6)}$

C) $\frac{\log(1.6)}{170(2\pi k)}$ D) $\frac{170}{log(1.6)}$

24. A pipe 10 cm in diameter contains steam at $100^{\circ}C$ .It is protected with asbestos 5 cm thick for which k = 0.0006 and outer surface is at 30 °C. The differential equation of conduction of heat is $dT = -\frac{Q}{2\pi k}\frac{dX}{X}$. The amount of heat loss Q is ......

A) $\frac{\log 2}{70(2\pi k)}$ B) $\frac{70\,(2\pi k)}{\log 2}$

C) $-\frac{70\,(2\pi k)}{\log 2}$ D) $\frac{(\,2\pi k\,)}{\log 2}$

25.If a particle moves on a straight line so that the force acting on it is always directed towards a fixed point on the line and proportional to its distance from the point then the particle is said to be in ....

A) Simple harmonic motion B) motion under the gravity

V)Periodic motion D) circular motion

26. A particle executes simple harmonic motion then the differential equation of motion is .....

A) $\frac{d^2X}{dt^2} = -\,w^2X$ B) $\frac{d^2X}{dt^2} = \,w^2X$

C) $\frac{d^2X}{dt^2} = -\,\frac{w^2}{X}$ D) $\frac{dX}{dt} = -\,w^2X$

--ANSWERS--

Q.1 - A	Q.2 - D	Q.3 - C	Q.4 - B	Q.5 - D	Q.6 - A
Q.7 - C	Q.8 - B	Q.9 - D	Q.10 - C	Q.11 - D	Q.12 - D
Q.13 - B	Q14 – A	Q.15 - A	Q.16 - D	Q.17 - B	Q.18 - C
Q.19 - D	Q.20 - B	Q.21 - A	Q.22 - C	Q.23 - A	Q.24 - B
Q.25 - A	Q.26 - A				

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Engineering Mathematics – II

Unit 3- Integral Calculus

1) If $I_n = \int_0^{\frac{\pi}{2}} sin^n\, xdx$ then which of the following relation is true ?

   A) $I_{n=} \frac{n-1}{n} I_{n-2}$
   B) $I_{n=} \frac{n-1}{n} I_{n-1}$
   C) $I_{n=} \frac{n}{n-1} I_{n-2}$
   D) $I_{n=} \frac{n-2}{n} I_{n-1}$

   Ans.A

2) If $I_n = \int_0^{\frac{\pi}{2}} cos^n\, xdx$ then which of the following relation is true ?

   A)$I_{n=} \frac{n-1}{n} I_{n-1}$

   $B)I_{n=} \frac{n-1}{n} I_{n-2}$

   $C)I_{n=} \frac{n}{n-1} I_{n-2}$

   $D)I_{n=} n(n+1)\, I_{n-1}$

   Ans. B

3) If $I_n = \int_0^{\frac{\pi}{2}} sin^n\, xdx$, n positive even integer then I_n is calculated from
   A) $\frac{n-1}{n} . \frac{n-3}{n-2} . \frac{n-5}{n-4} \dots \frac{3}{4} . \frac{1}{2}$
   B) $\frac{n-1}{n} . \frac{n-3}{n-2} . \frac{n-5}{n-4} \dots \frac{3}{4} . \frac{1}{2} \pi$
   C) $\left(\frac{n+1}{n} . \frac{n+3}{n+2} . \frac{n+5}{n+4} \dots\right) \frac{\pi}{2}$
   D) $\frac{n-1}{n} . \frac{n-3}{n-2} . \frac{n-5}{n-4} \dots \frac{3}{4} . \frac{1}{2} \frac{\pi}{2}$

   Ans D)

4) If $I_n = \int_0^{\frac{\pi}{2}} sin^n\, xdx$, n positive odd integer then I_n is calculated from

$A)\quad \frac{n-1}{n}.\frac{n-3}{n-2}.\frac{n-5}{n-4}\dots\frac{4}{5}.\frac{2}{3}\frac{\pi}{2}$

B)$\frac{n-1}{n}.\frac{n-3}{n-2}.\frac{n-5}{n-4}\dots\frac{3}{4}.\frac{1}{2}\frac{\pi}{2}$

C)$\frac{n-1}{n}.\frac{n-3}{n-2}.\frac{n-5}{n-4}\dots\frac{4}{5}.\frac{2}{3}.1$

D)$\frac{n+1}{n}.\frac{n+3}{n+2}.\frac{n+5}{n+4}$

Ans C)

5) If $I_n = \int_0^{\frac{\pi}{2}} cos^n\, xdx$, n positive even integer then I_n is calculated from

A) $\frac{n-1}{n}.\frac{n-3}{n-2}.\frac{n-5}{n-4}\dots\frac{3}{4}.\frac{1}{2}$

B) $\frac{n-1}{n}.\frac{n-3}{n-2}.\frac{n-5}{n-4}\dots\frac{3}{4}.\frac{1}{2}\frac{\pi}{2}$

C) $\left(\frac{n+1}{n}.\frac{n+3}{n+2}.\frac{n+5}{n+4}\dots\right)\frac{\pi}{2}$

D) $\frac{n-1}{n}.\frac{n-3}{n-2}.\frac{n-5}{n-4}\dots\frac{3}{4}.\frac{1}{2}\,\pi$

Ans B)

6) If $I_n = \int_0^{\frac{\pi}{2}} cos^n\, xdx$, n positive odd integer then I_n is calculated from

A)$\frac{n-1}{n}.\frac{n-3}{n-2}.\frac{n-5}{n-4}\dots\frac{4}{5}.\frac{2}{3}.\frac{\pi}{2}$

B) $\frac{n-1}{n}.\frac{n-3}{n-2}.\frac{n-5}{n-4}\dots\frac{3}{4}.\frac{1}{2}\frac{\pi}{2}$

C) $\frac{n+1}{n}.\frac{n+3}{n+2}.\frac{n+5}{n+4}$

D) $\frac{n-1}{n}.\frac{n-3}{n-2}.\frac{n-5}{n-4}\dots\frac{4}{5}.\frac{2}{3}.1$

Ans D)

7) $\int_0^{\frac{\pi}{2}} sin^4\, xdx$ is equal to

A) $\frac{\pi}{2}$ B) $\frac{3\pi}{8}$ C) $\frac{\pi}{4}$ D) $\frac{3\pi}{16}$

Ans .D)

8) $\int_0^{\frac{\pi}{2}} sin^5\,xdx$ is equal to

A)$\frac{8}{15}\frac{\pi}{2}$ B)$\frac{15}{8}$ C) $\frac{8}{15}$ D)0

Ans .C)

9) $\int_0^{\frac{\pi}{2}} cos^3\,xdx$ is equal to

A) $\frac{2}{3}$ B) $\frac{1}{4}$ C) $\frac{2}{3}\ \frac{\pi}{2}$ D) $\frac{1}{3}$

ANS . A)

10) $\int_0^{\frac{\pi}{2}} cos^6\,xdx$ is equal to

A)$\frac{5}{16}$ B) $\frac{5}{16}\frac{\pi}{2}$ C) $\frac{16}{5}\frac{\pi}{2}$ D) $\frac{5}{48}\frac{\pi}{2}$

Ans. B

11) $\int_0^{\frac{\pi}{4}} sin^2\,(2x)dx$ is equal to

A) $\frac{1}{4}$ B) $\frac{\pi}{2}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{8}$

Ans.D)

12) $\int_0^{\frac{\pi}{4}} cos^2\,(2x)dx$ is equal to

A) $\frac{\pi}{8}$ B) $\frac{\pi}{2}$ C) $\frac{\pi}{4}$ D) $\frac{1}{4}$

Ans .A)

13) $\int_0^{\pi} sin^5\left(\frac{x}{2}\right)dx$ is equal to

A)$\frac{8\pi}{15}$ B) $\frac{32}{15}$ C) $\frac{16}{15}$ D) $\frac{8}{15}$

Ans.C)

14) $\int_0^{\pi} cos^3\,xdx$ is equal to

A) 1 B) $\frac{2}{3}$ C) $\frac{4}{3}$ D)0

Ans.D)

15) $\int_0^{\pi} sin^6\, tdt$ is equal to

A) $\frac{\pi}{8}$ B) $\frac{5\pi}{16}$ D) $\frac{5}{8}$ C) $\frac{5\pi}{8}$

Ans.B)

16) $\int_0^{2\pi} sin^6\, tdt$ is equal to

A) $\frac{5}{4}$ B) $\frac{5\pi}{32}$ C) $\frac{5\pi}{8}$ D)0

Ans.C)

17) $\int_0^{2\pi} cos^7\, tdt$ is equal to

A) $\frac{32}{35}$ B) $\frac{32\pi}{70}$ C)0 D) $\frac{16}{35}$

Ans.C)

18)) $\int_0^{\frac{\pi}{2}} sin^6\, xcos^4 x\, dx$ is equal to

A)$\frac{3}{256}$ $B)\,\frac{3\pi}{512}$ $C)\,\frac{3}{128}$ $D)\,\frac{512\pi}{3}$

Ans .B)

19) $\int_0^{2\pi} sin^5\, \theta cos^4\theta\, d\theta \quad is\ equal\ to$

A)0 B) $\frac{8}{315}$ $C)\,\frac{3\pi}{128}$ $D)\,\frac{\pi}{128}$

Ans. A)

20) If $\mathrm{I}_n = \int_0^{\frac{\pi}{4}} tan^n xdx \quad and \quad \mathrm{I}_n = \frac{1}{n-1} - I_{n-2}\ then\ I_4\ is\ equal\ to$

A)$-\frac{2}{3}+\frac{\pi}{2}$ B) $-\frac{2}{3}-\frac{\pi}{4}$ C) $-\frac{2}{3}+\frac{\pi}{4}$ D) $-\frac{4}{3}+\frac{\pi}{4}$

Ans c)

21)If $\mathrm{I}_n = \int_0^{\frac{\pi}{4}} sin^{2n} xdx \quad and \quad \mathrm{I}_n = -\frac{1}{n2^{n+1}} + \left(\frac{2n-1}{2n}\right) I_{n-1}\ then\ I_2\ is\ equal\ to$

A)$-\frac{1}{4}+\frac{3\pi}{32}$ B))$-\frac{3}{4}+\frac{3\pi}{32}$ C))$\frac{1}{4}+\frac{\pi}{32}$ D))$\frac{1}{4}-\frac{\pi}{16}$

Ans.A)

22. The value of the integral $\int_0^1 (1 - x^{1/n})^m \, dx$ by using substitution $x^{1/n}$=t is

A) $B(n, m + 1)$

B) $nB(n, m + 1)$

C) $B(m, n + 1)$

D) $mB(m, n + 1)$

Ans. B

23)The value of the integral $\int_0^1 \frac{dx}{\sqrt{xlog\left(\frac{1}{x}\right)}}$dx by using substitution $log\frac{1}{x} = t$ is

A)$\frac{\sqrt{\pi}}{2}$ B) $\sqrt{\pi}$ C)$\sqrt{2\pi}$ D) $2\sqrt{\pi}$

Ans. C

24) $\int_0^\infty e^{-5x} x^4 \, dx$ is equal

A) $\frac{4!}{4^5}$ B) $\frac{5!}{5^5}$

C) $\frac{4!}{5^5}$ D) $\frac{5!}{4^4}$

Ans. C

25) $\int_0^\infty \frac{x^4}{(1+x)^7} dx$ is equal to

A) $\frac{1}{30}$ B) $\frac{1}{15}$

C) $\frac{1}{60}$ D) $\frac{1}{20}$

Ans. A

26). The value of the integral $\int_0^\infty \frac{x^4}{4^x}$dx by is

A)$\frac{4}{(log4)^4}$ B) $\frac{24}{(log4)^3}$ C)$\frac{24}{(log4)^4}$ D)$\frac{24}{(log4)^5}$

Ans. D

27)The value of integral $\int_0^\infty \sqrt{x}e^{-x^3}$ dx using substitution $x^3 = t$

A) $\frac{\sqrt{\pi}}{6}$ B)$\frac{\sqrt{\pi}}{2}$ C)$3\sqrt{\pi}$ D)$\frac{\sqrt{\pi}}{3}$

Ans. D

28) $\int_0^{\pi/2} \sqrt{\cot\theta}\, d\theta$ is equal to

A) $\frac{1}{2}\mathrm{B}\left(-\frac{1}{2},\frac{1}{2}\right)$ B) $\mathrm{B}\left(\frac{1}{4},\frac{3}{4}\right)$

C)$\mathrm{B}\left(\frac{1}{2},-\frac{1}{2}\right)$ D) $\frac{1}{2}\mathrm{B}\left(\frac{1}{4},\frac{3}{4}\right)$

Ans. D

29) The value of $B(m,n+1) + B(m+1,n) is$

A) $B(m,n)$

B) $2B(m+1,n)$

C) $2B\ (m,n+1)$

D) $2B\ (m,n+1)$

Ans. A

30. If $\emptyset(a) = \int_0^\infty \frac{e^{-x}}{x}(1 - e^{-ax})dx, a > -1$ then by DUIS rule, $\frac{d\emptyset}{da}$ is

$A)\int_0^\infty \frac{\partial}{\partial a}\ e^{-x}(1 - e^{-ax})dx$

B) $\int_0^\infty \frac{\partial}{\partial a}\ \frac{e^{-x}}{x}(1 - e^{-ax})dx$

C) $\int_0^\infty \frac{\partial}{\partial x}\ \frac{e^{-x}}{x}(1 - e^{-ax})dx$

$D)\frac{e^{-x}}{x}(1 - e^{-ax})$

Ans. B

31. If $\emptyset(a) = \int_0^\infty \frac{e^{-x}}{x}(\mathrm{a} - \frac{1}{\mathrm{x}} + \frac{1}{\mathrm{x}}e^{-ax})\ dx$ then by DUIS rule,$\frac{d\emptyset}{da}$ is

$A)\frac{e^{-x}}{x}\left(\mathrm{a} - \frac{1}{\mathrm{x}} + \frac{1}{\mathrm{x}}e^{-ax}\right)$

B) $\int_0^\infty \frac{\partial}{\partial a}\ \frac{e^{-x}}{x}(\mathrm{a} - \frac{1}{\mathrm{x}} + \frac{1}{\mathrm{x}}e^{-ax})\ dx$

C) $\int_0^\infty \frac{\partial}{\partial x}\ \frac{e^{-x}}{x}(\mathrm{a} - \frac{1}{\mathrm{x}} + \frac{1}{\mathrm{x}}e^{-ax})\ dx$

D) $\int_0^\infty \frac{\partial}{\partial a}\ e^{-x}(\mathrm{a} - \frac{1}{\mathrm{x}} + \frac{1}{\mathrm{x}}e^{-ax})\ dx$

Ans. B

32. If $I(b) = \int_0^\infty e^{-(x+b)^2}$dx then by DUIS rule,$\frac{dI}{db}$ is

A) $\int_0^\infty \frac{\partial}{\partial b} e^{-(x+b)^2}$dx

B) $\int_0^\infty \frac{\partial}{\partial x} e^{-(x+b)^2}$dx

C) $\int_0^\infty e^{-(x+b)^2}$

D) $e^{-(x+b)^2}$

Ans. A

33. If $\emptyset(a) = \int_0^\infty \frac{e^{-x}}{x}(1 - e^{-ax})dx, a > -1$ then by DUIS rule, $\frac{d\emptyset}{da}$ is

A) $\frac{e^{-x}}{x}(1 - e^{-ax})$

$B)\ \int_0^\infty \frac{a}{x}\left(e^{-(a+1)x}\right)dx$

C) $\int_0^\infty (e^{-ax})dx$

D) $\int_0^\infty \left(e^{-(a+1)x}\right)dx$

Ans. D

34. If $\emptyset(a) = \int_0^1 \frac{x^a - 1}{logx} dx, a \geq 0$ then by DUIS rule, $\frac{d\emptyset}{da}$ is

A) $\int_0^1 \frac{x^a loga}{logx} dx$

B) $\int_0^1 \frac{ax^{a-1}}{logx} dx$

C) $\int_0^1 x^a dx$

D) $\frac{x^a - 1}{logx}$

Ans. C

35. If $\emptyset(\alpha) = \int_0^\infty \frac{e^{-x} sin\alpha x}{x} dx$ then by DUIS rule, $\frac{d\emptyset}{d\alpha}$ is

A) $\int_0^\infty e^{-x} sin\alpha x\, dx$

B) $\int_0^\infty e^{-x} cos\alpha x\, dx$

C) $\int_0^\infty \frac{\alpha e^{-x} sin\alpha x}{x} dx$

D) $\frac{e^{-x} sin\alpha x}{x}$

Ans. B

36. If I(a)= $\int_0^\infty e^{-(x^2 + \frac{a^2}{x^2})} dx, a > 0$ then by DUIS rule, $\frac{dI}{da}$ is

A) $\int_0^\infty e^{-\left(x^2 + \frac{a^2}{x^2}\right)} \left(\frac{-2a}{x^2}\right) dx$

B) $\int_0^\infty e^{-\left(x^2 + \frac{a^2}{x^2}\right)} \left(\frac{2a}{x^2}\right) dx$

C) $\int_0^\infty e^{-\left(x^2 + \frac{a^2}{x^2}\right)} \left(-2x - \frac{2a^2}{x^3}\right) dx$

D) $e^{-(x^2 + \frac{a^2}{x^2})}$

Ans. A

37. If I(a)= $\int_0^\pi \log(1 - acosx)\, dx, |a| < 1$ then by DUIS rule, $\frac{dI}{da}$ is

A) $\int_0^\pi \frac{-asinx}{1 - acosx} dx$

B) $\int_0^\pi \frac{cosx}{1 - acosx} dx$

C) $\int_0^\pi \frac{-cosx}{1 - acosx} dx$

D) $\int_0^\pi \frac{1}{1 - acosx} dx$

Ans. C

38. If I(x)= $\int_0^\infty \mathrm{e}^{-\mathrm{a}^2} cosaxda$, x is parameter then by DUIS rule, $\frac{dI}{dx}$ is

A) $\int_0^\infty x\mathrm{e}^{-\mathrm{a}^2} \sin(xa)\, da$

B) $\int_0^\infty a\mathrm{e}^{-\mathrm{a}^2} \sin(xa)\, da$

C) $\int_0^\infty -a\mathrm{e}^{-\mathrm{a}^2} \sin(xa) da$

D) $\int_0^\infty a\mathrm{e}^{-\mathrm{a}^2} \cos(xa) da$

Ans. C

39. If $I(a) = \int_0^\infty \frac{e^{-x}-e^{-ax}}{xsecx} dx, a > 0$ then by DUIS rule,$\frac{dI}{da}$ is

A) $\int_0^\infty \frac{e^{-x}+e^{-ax}}{xsecx} dx$ B) $\int_0^\infty \frac{e^{-x}}{secx} dx$

C) $\int_0^\infty (e^{-x} - e^{-ax})dx$ D) $\int_0^\infty \frac{e^{-ax}}{secx} dx$

Ans. D

40. If $\emptyset(a) = \int_0^\infty \frac{1}{x^2} \log(1 + ax^2)dx, a > 0$, then by DUIS rule,$\frac{d\emptyset}{da}$ is

A) $\int_0^\infty \frac{a}{x(1+ax^2)} dx$ B) $\int_0^\infty \frac{\log(1+ax^2)}{x} dx$

C) $\int_0^\infty \frac{2a}{x(1+ax^2)} dx$ D) $\int_0^\infty \frac{1}{1+ax^2} dx$

Ans. D

41. If $\emptyset(a) = \int_0^{\frac{\pi}{2}} \frac{\log(1+asin^2x)}{sin^2x} dx, a > 0$, then by DUIS rule,$\frac{d\emptyset}{da}$ is

A) $\int_0^{\frac{\pi}{2}} \frac{2\text{sinxcosx}}{(1+asin^2x)} dx$ B) $\int_0^{\frac{\pi}{2}} \frac{1}{(1+asin^2x)sin^2x} dx$

C) $\int_0^{\frac{\pi}{2}} \frac{1}{1+asin^2x} dx$ D) $\int_0^{\frac{\pi}{2}} \frac{sin^2x}{(1+asin^2x)} dx$

Ans. C

42. If $\emptyset(a) = \int_0^1 \frac{x^a-x^b}{logx} dx, a > 0, b > 0$ then by DUIS rule, $\frac{d\emptyset}{da}$ is

A) $\int_0^1 \frac{x^a loga}{logx} dx$ B) $\int_0^1 x^a dx$

C) $\int_0^1 x^b dx$ D) $\int_0^1 (x^a - x^b)dx$

Ans. B

43. If $\emptyset(b) = \int_0^\infty \frac{e^{-ax}-e^{-bx}}{x} dx, a > 0, b > 0$ then by DUIS rule, $\frac{d\emptyset}{db}$ is

A) $\int_0^\infty e^{-bx}\, dx$

$B)$ $\int_0^\infty \frac{e^{-ax}(-a)-e^{-bx}(-b)}{x} dx$

C) $\int_0^\infty e^{-ax}\, dx$

D) $\int_0^\infty (e^{-ax} - e^{-bx}) dx$

Ans. A

44.By DUIS rule $\frac{d}{da}\int_0^\infty \frac{e^{-x}}{x}(\mathrm{a} - \frac{1}{\mathrm{x}} + \frac{1}{\mathrm{x}} e^{-ax})\, dx$,a is parameter,is

A) $\int_0^\infty \frac{e^{-x}}{x}(1+e^{-ax})dx$ B) $\int_0^\infty \frac{e^{-x}}{x}(1-\frac{xe^{-ax}}{x})dx$

C) $\int_0^\infty e^{-x}(1-e^{-ax})dx$ D) $\int_0^\infty \frac{e^{-x}}{x}(1-e^{-ax})dx$

Ans. D

45. Using DUIS Rule the value of the integral $\emptyset(a) = \int_0^\infty \frac{e^{-x}}{x}(1 - e^{-ax})dx, a > -1$, given $\frac{d\emptyset}{da} = \frac{1}{a+1}$ is

A) log(a+1) B) $-\frac{1}{(a+1)^2}$

C) log(a+1) + π D) $-\frac{1}{(a+1)^2} + 1$

Ans. A

46. Using DUIS Rule the value of the integral $\emptyset(a) = \int_0^\infty \frac{x^a-1}{logx} dx, a \geq 0$, given $\frac{d\emptyset}{da} = \frac{1}{a+1}$ is

A) log(a+1) B) $-\frac{1}{(a+1)^2}$

C) log(a+1) + π D) $-\frac{1}{(a+1)^2} + 1$

Ans. A

47. Using DUIS Rule the value of the integral $\emptyset(\alpha) = \int_0^\infty \frac{e^{-2x} sin\alpha x}{x} dx$, with $\frac{d\emptyset}{d\alpha} = \frac{2}{\alpha^2+4}$ is

A) $2\log(\alpha^2 + 4)$ B) $2tan^{-1}\left(\frac{\alpha}{2}\right)$

C) $\frac{1}{2}tan^{-1}\left(\frac{\alpha}{2}\right)$ D) $tan^{-1}\left(\frac{\alpha}{2}\right)$

Ans. D

48. Using DUIS Rule the value of the integral $\emptyset(\alpha) = \int_0^\infty \frac{e^{-\alpha x} sinx}{x} dx$, with $\frac{d\emptyset}{d\alpha} = -\frac{1}{\alpha^2+1}$ and assuming $\int_0^\infty \frac{sinx}{x} dx = \frac{\pi}{2}$ is

A) $tan^{-1}\alpha + \frac{\pi}{2}$ B) $-tan^{-1}\alpha + \frac{\pi}{2}$

C) $-tan^{-1}\alpha$ D) $\log(\alpha^2 + 1) + \frac{\pi}{2}$

Ans. B

49. If $\phi(a) = \int_a^{a^2} \log(ax)\ dx$ then by DUIS rule II, $\frac{d\phi}{da}$ is...

A) $\int_a^{a^2} \frac{\partial}{\partial a}\{\log(ax)\}dx + 2a\log(a^3)$

B) $\int_a^{a^2} \frac{\partial}{\partial a}\{\log(ax)\}dx$

C) $\int_a^{a^2} \frac{\partial}{\partial a}\{\log(ax)\}dx + 2a\log(a^3) - 2\log(a)$

D) $\int_a^{a^2} \frac{\partial}{\partial a}\{\log(ax)\}dx - 2a\log(a^3) + 2\log(a)$

Ans. C

50. If $(x) = \int_a^x (x-t)^2 G(t)dt$, where a is constant and x is parameter then by DUIS rule II , $\frac{df}{dx}$=…

A) $\int_a^x \frac{\partial}{\partial x}\{(x-t)^2 G(t)\}dt + G(x)$

B) $\int_a^x \frac{\partial}{\partial x}\{(x-t)^2 G(t)\}dt$

C) $\int_a^x \frac{\partial}{\partial x}\{(x-t)^2 G(t)\}dt - \{(x-a)^2 G(a)\}$

D) None of these

Ans. B

51) Error function of x, erf(x) is defined as

A) $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2} du$ B) $\frac{2}{\sqrt{\pi}}\int_0^\infty e^{-u^2} du$ C) $\int_0^\infty e^{-x} x^{n-1} dx$ D) $\frac{2}{\sqrt{\pi}}\int_0^\infty e^{u^2} du$

Ans.A)

52) complimentary error function of of x, erfc (c) is defined as

A) $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2} du$ B) $\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2} du$ C) $\int_0^\infty e^{-x} x^{n-1} dx$ D) $\frac{2}{\sqrt{\pi}}\int_0^\infty e^{u^2} du$

Ans.B)

53) The value of erf(∞) is

A) 0 B) ∞ C) 1 D) $\frac{2}{\sqrt{\pi}}$

Ans.C

54) The value of erf(0) is

A) -1 B) ∞ C) 1 D) 0

Ans.D

55) The value of erfc(0) is

A) -1 B) ∞ C) 1 D) 0

Ans. C

56) Which of the following is true ?

A) erf(x) – erfc(x) =1 B) erf(x) + erfc(x) =1

C) erf(x) + erfc(x) =2 D) erf(-x) = erf(x)

Ans.B

57) Error fuction is

A) a periodic function B) an even function

B) a harmonic function D) an odd function

Ans.D

58) erf(-x) is equal to

A) −erf(x) B) erf(x) C) erfc(x) D) erfc(-x)

Ans. A

59) erf(x) + erf(-x) =

A) 2 B) 1 C) -1 D) 0

Ans.D

60) If erf(ax) = $\frac{2}{\sqrt{\pi}}\int_0^{ax} e^{-u^2} du$ then $\frac{d}{dx}$ $\text{erf}(ax)$ is

A) $\frac{2x}{\sqrt{\pi}}e^{-a^2x^2}$ B) $\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$ C) a$e^{-a^2x^2}$ D) $\frac{2a}{\sqrt{\pi}}e^{a^2x^2}$

Ans.B)

61) If $\frac{d}{dx}$ $\text{erf}(ax)$ = $\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$ then $\frac{d}{dx}$ $\text{erfc}(ax)$ is

A)) $\frac{2a}{\sqrt{\pi}}e^{a^2x^2}$ B) 1- $\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$ C)$-\frac{1}{\sqrt{\pi}}e^{-a^2x^2}$ D)$-\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$

Ans. D

62) If erfc($\sqrt{t}$) = $\frac{2}{\sqrt{\pi}}\int_{\sqrt{t}}^{\infty} e^{-u^2} du$ then $\frac{d}{dt}$ erfc($\sqrt{t}$) is

A) $\frac{e^{-t}}{2\sqrt{t}}$ B) $-\frac{e^{-t}}{\sqrt{\pi t}}$ C) $\frac{e^{-t}}{\sqrt{\pi t}}$ D) $\frac{e^{-t^2}}{\sqrt{\pi t}}$

Ans B)

63) If erfc (a) = $\frac{2}{\sqrt{\pi}}\int_a^{\infty} e^{-u^2} du$ then by using substitution x+a = u ,the

Integral $\int_0^{\infty} e^{-(x+a)^2} dx$ in terms of erfc(a) is

A) $\frac{2}{\sqrt{\pi}}\ erfc(a)$ B) erfc(a) C) $\frac{\sqrt{\pi}}{2}\ erfc(a)$ D) $\frac{\sqrt{\pi}}{2}\ erf(a)$

Ans. C

Sinhgad College of Engineering,Vadgaon

Engineering Mathematics – II

Unit 4- Curve Tracing

1. If the powers of x in the Cartesian equation are even everywhere then the curve is symmetrical about

A) x-axis B) y-axis C) both x and y axes D) line $y = x$

Ans.-B

2. If the powers of x and y both in the Cartesian equation are even everywhere then the curve is symmetrical about

A) x-axis only B) y-axis only C) both x and y axes D) line $y = x$

Ans.- C

3. On replacing $x\ and\ y$ by $-x\ and -y$ respectively if the Cartesian equation remains unchanged then the curve is symmetrical about

A) line $y = x$ B) y-axis C) both x and y axes D) opposite quadrants

Ans.- D

4. If x and y are interchanged and Cartesian equation remains unchanged then the curve is symmetrical about

A) both x and y axes B) line $y = -x$ C) line $y = x$ D) opposite quadrants

Ans. - C

5. If the curve passes through origin then tangents at origin to the Cartesian curve can be obtained by equating to zero

A) lowest degree term in the equation

B) highest degree term in the equation

C) coefficient of lowest degree term in the equation

D) coefficient of highest degree term in the equation

Ans.- A

6. In Cartesian equation the points where $\frac{dy}{dx} = 0$, tangent to the curve at those points will be

A) Parallel to y-axis B) Parallel to x-axis

C) Parallel to $y = x$ D) Parallel to $y = -x$

Ans.- B

7. In Cartesian equation the points where $\frac{dy}{dx} = \infty$,tangent to the curve at those points will be

A) Parallel to $y = -x$ B) Parallel to x-axis

C) Parallel to y-axis D) Parallel to $y = x$

Ans.- C

8. If the powers of y in the Cartesian equation are even everywhere then the curve is symmetrical about

A) x-axis B) y-axis C) both $x$and y axes D) line $y = x$

Ans. - A

9. The asymptotes to the Cartesian curve parallel to x-axis if exists is obtained by equating to zero

A) Coefficient of highest degree term in y B) Lowest degree term in the equation

C) Coefficient of highest degree term in x D) Highest degree term in the equation

Ans. - C

10. The asymptotes to the Cartesian curve parallel to y-axis if exists is obtained by equating to zero

A) Coefficient of highest degree term in y B) Lowest degree term in the equation

C) Coefficient of highest degree term in x D) Highest degree term in the equation

Ans. – A

11. The curve represented by the equation $x^2y^2 = x^2 + 1$ is symmetrical about

A) $y = -x$ B) $x - axis\ only$ C) both x and y axes D) $y = x$

Ans. -C

12. The asymptote parallel to y-axis to the curve $xy^2 = a^2(a - x)$ is

A) $y = 0$ B) $x = 0$ C) $x = a$ D) $x = -a$

Ans. - B

13. The curve represented by the equation $y^2(2a - x) = x^3$ is

A) Symmetrical about y-axis and passing through origin

B) Symmetrical about x-axis and not passing through origin

C) Symmetrical about y-axis and passing through $(2a, 0)$

D) Symmetrical about x-axis and passing through origin

Ans. – D

14. The equation of tangents to the curve at origin if exist, represented by the equation

$3ay^2 = x(x-a)^2$ is

A) $x = a$ B) $x = 0\ and\ y = 0$ C) $x = 0$ D) $y = 0$

Ans. - C

15. The equation of asymptotes parallel to y-axis to the curve represented by the equation

$x^2y^2 = a^2(y^2 - x^2)$ is

A) $x = a\ , x = -a$ B) $y = a\ , y = -a$ C) $y = x\ , y = -x$ D) $x = 0\ , y = 0$

Ans. – A

16. The region of absence for the curve represented by the equation $x^2 = \frac{4a^2(2a-y)}{y}$ is

A) $y < 0\ and\ y > 2a$ B) $y > 0\ and\ y < 2a$

C) $y > 0\ and\ y > 2a$ D) $y < 0\ and\ y < 2a$

Ans. - A

17. The region of absence for the curve represented by the equation $xy^2 = a^2(a-x)$is

A) $x > 0\ and\ x < a$ B) $x < 0\ and\ x < a$ C) $x < 0\ and\ x > a$ D) $x > 0\ and\ x > a$

Ans. - C

18. The curve represented by the equation $r^2 = a^2cos2\theta$ is…

A) symmetric about θ =$\frac{\pi}{2}$ and not passing through pole

B) symmetric about θ =$\frac{\pi}{4}$ and not passing through pole

C) symmetric about initial line and pole

D) symmetric about θ =$\frac{\pi}{4}$ and passing through pole

Ans.-C

19. The curve represented by the equation $r^2 = a^2sin2\theta$ is…

A) symmetric about initial line and passing through pole
B) symmetric about initial line and pole
C) symmetric about θ =$\frac{\pi}{4}$ and not passing through pole
D) symmetric about θ =$\frac{\pi}{4}$ and passing through pole

Ans.- D

20. The curve represented by the equation $r = \frac{2a}{1+cos\theta}$ is

A) symmetric about initial line and passing through pole
B) symmetric about initial line and not passing through pole
C) symmetric about θ =$\frac{\pi}{4}$ and not passing through pole
D) symmetric about θ =$\frac{\pi}{4}$ and passing through pole

Ans.- B

21. If the polar equation to the curve remains unchanged by changing θ to – θ then the curve is symmetric about …

A) line θ = $\frac{\pi}{4}$ B) pole C) line θ = $\frac{\pi}{2}$ D) initial line θ = 0

Ans.- D

22. If the polar equation to the curve remains unchanged by changing θ to π– θ then the curve is symmetric about …

A) initial line θ = 0 B) pole
C) line passing through pole and perpendicular to the initial line
D) line θ = $\frac{\pi}{4}$

Ans. -C

23. Pole will lie on the curve if for some value of θ

A) r becomes zero B) r becomes infinite

C) r $>$ 0 D) r $<$ 0

Ans.- A

24. The tangents to the polar curve at pole if exist can be obtained by putting in the polar

A) θ = 0 B) θ = π C) r = 0 D) r = a, a$>$ 0

Ans.- C

25. If the polar equation to the curve remains unchanged by changing $r\ to - r$ then the curve is symmetrical about

A) line $\theta = \frac{\pi}{4}$ B) pole C) line $\theta = \frac{\pi}{2}$ D) Initial line $\theta = 0$

Ans. - B

26. For the polar curve, angle φ between radius vector and tangent line is obtained by the formula

A) $cos\varphi = r\frac{d\theta}{dr}$ B) $tan\varphi = r\frac{d\theta}{dr}$ C) $tan\varphi = r\frac{dr}{d\theta}$ D) $sin\varphi = r\frac{d\theta}{dr}$

Ans. -B

27. The curve represented by equation $r = 2asin\theta$ is symmetrical about

A) Pole B) Initial line $\theta = 0$ C) Line $\theta = \frac{\pi}{4}$ D) Line $\theta = \frac{\pi}{2}$

Ans. – D

28. The curve represented by equation $r = a(1 + cos\theta)$ is symmetrical about

A) Initial line and passing through pole B) Initial line and not passing through pole

C) $\theta = \frac{\pi}{2}$ and passing through pole D) $\theta = \frac{\pi}{4}$ and passing through pole

Ans. – A

29. For the rose curve $r = a\cos n\theta$ and $r = a\sin n\theta$ if n is odd then the curve consists of

A) $2n$ equal loops B) $(n+1)$ equal loops c) $(n-1)$ equal loops D) n equal loops

Ans.- D

30. For the rose curve $r = a\cos n\theta$ and $r = a\sin n\theta$ if n is even then the curve consists of

A) $2n$ equal loops B) $(n+1)$ equal loops c) $(n-1)$ equal loops D) n equal loops

Ans.- A

31. The tangents at pole to the curve $r = a\sin 3\theta$ are

A) $\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3} \ldots$ B) $\theta = \frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6} \ldots$

C) $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \ldots$ D) $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots$

Ans.- A

32. The tangents at pole to the curve $r = a\cos 2\theta$ are

A) $\theta = 0, \pi, 2\pi, 3\pi \ldots$ B) $\theta = \frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6} \ldots$

C) $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \ldots$ D) $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots$

Ans. – C

33. The Cartesian parametric curve $x = f(t),\ y = g(t)$ is symmetrical about X-axis if……..

A) f(t) is even and g(t) is odd B) f(t) is odd and g(t) is even

C) f(t) is even and g(t) is even D) f(t) is odd and g(t) is odd

Ans.- A

34. The Cartesian parametric curve $x = f(t),\ y = g(t)$ is symmetrical about Y-axis if……..

A) f(t) is even and g(t) is odd B) f(t) is odd and g(t) is even

C) f(t) is even and g(t) is even D) f(t) is odd and g(t) is odd

Ans.- B

35. The curve represented by the equation $x = at^2, y = 2at$ is symmetrical about …..

A) Y-axis B) X-axis C) both X and Y axis D) opposite quadrants

Ans.- B

36. The Cartesian parametric curve $x = f\ (t),\ y = g(t)$ is symmetrical in opposite quadrants if……..

A) f(t) is even and g(t) is odd B) f(t) is odd and g(t) is even

C) f(t) is even and g(t) is even D) f(t) is odd and g(t) is odd

Ans. -D

37. The curve represented by the equation $x = t, y = t^3$ is symmetrical about …..

A) Y-axis B) X-axis C) both X and Y axis D) opposite quadrants

Ans.- D

38. The curve represented by the equation $x = a(\theta + sin\theta), y = a(1 + cos\theta)$ where θ is parameter is symmetrical about …..

A) Y-axis B) X-axis C) both X and Y axis D) opposite quadrants

Ans.-A

39. Formula for measuring the arc length AB where A(x_1, y_1), B(x_2, y_2) are any two points on the curve $y = f(x)$ is

A) $\int_{x_1}^{x_2} \sqrt{dx}$ B) $\int_{x_1}^{x_2} \sqrt{1 + (\frac{dy}{dx})^2}\, dx$ C) $\int_{x_1}^{x_2} \sqrt{1 + (\frac{dx}{dy})^2}\, dx$ D) $\int_{x_1}^{x_2} \sqrt{1 - (\frac{dy}{dx})^2}\, dx$

Ans.- B

40. Formula for measuring the arc length AB where A(x_1, y_1), B(x_2, y_2) are any two points on the curve $x = g(y)$ is

A) $\int_{y_1}^{y_2} \sqrt{dx}$ B) $\int_{y_1}^{y_2} \sqrt{1 + (\frac{dy}{dx})^2}\, dy$ C) $\int_{y_1}^{y_2} \sqrt{1 + (\frac{dx}{dy})^2}\, dy$ D) $\int_{y_1}^{y_2} \sqrt{1 - (\frac{dy}{dx})^2}\, dy$

Ans.- C

41. Formula for measuring the arc length AB where A(r_1, θ_1), B(r_2, θ_2) are any two points on the curve $r = f(\theta)$ is

A) $\int_{\theta_1}^{\theta_2} \sqrt{1 + (\frac{dr}{d\theta})^2}d\theta$ B) $\int_{\theta_1}^{\theta_2} \sqrt{1 + r^2(\frac{d\theta}{dr})^2}d\theta$

C) $\int_{\theta_1}^{\theta_2} \sqrt{r^2 + (\frac{dr}{d\theta})^2}d\theta$ D) $\int_{\theta_1}^{\theta_2} \sqrt{r^2 + r^2(\frac{dr}{d\theta})^2}dr$

Ans.- C

42. Formula for measuring the arc length AB where A(r_1, θ_1), B(r_2, θ_2) are any two points on the curve θ= $f(r)$ is θ

A) $\int_{r_1}^{r_2} \sqrt{r^2 + (\frac{dr}{d\theta})^2} dr$ B) $\int_{r_1}^{r_2} \sqrt{1 + (\frac{d\theta}{dr})^2} dr$

C) $\int_{r_1}^{r_2} \sqrt{1 - r^2(\frac{d\theta}{dr})^2} dr$ D) $\int_{r_1}^{r_2} \sqrt{1 + r^2(\frac{d\theta}{dr})^2} dr$

Ans.- D

43. Formula for measuring the arc length AB where A, B are any two points on the parametric curve $x = f_1(t), y = f_2(t)$, corresponding to parameters t_1, t_2 respectively is…….

A) $\int_{t_1}^{t_2} \sqrt{(\frac{dx}{dt})^2 - (\frac{dy}{dt})^2} dt$ B) $\int_{t_1}^{t_2} \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} dt$

C) $\int_{t_1}^{t_2} \left[(\frac{dx}{dt})^2 - \left(\frac{dy}{dt}\right)^2 \right] dt$ D) $\int_{t_1}^{t_2} \sqrt{(x^2(t) - y^2(t)} dt$

Ans. - B

44. The arc length AB where A(a, 0), B(0, a) are any two points on the circle $x^2 + y^2 = a^2$ using $1 + (\frac{dy}{dx})^2 = \frac{a^2}{a^2 - x^2}$ is …

A) $\frac{\pi a}{2}$ B) $a \; loga$ C) $\frac{\pi a}{4}$ D) a

Ans.- A

45. The length of arc of the curve $x = e^{\theta} cos\theta$, $y = e^{\theta} sin\theta$

from θ = 0 to θ =$\frac{\pi}{2}$ using $(\frac{dx}{d\theta})^2 + (\frac{dy}{d\theta})^2$ = $2e^{2\theta}$ is……

A) $\sqrt{2}e^{\frac{\pi}{2}}$

B) $\sqrt{2}(e^{\frac{\pi}{2}} + 1)$

C) $\sqrt{2}(e^{\frac{\pi}{2}} - 1)$

D) $(e^{\frac{\pi}{2}} + 1)$

Ans.- C

46. Integral for calculating the length of upper arc of loop of the curve $9y^2 = (x + 7)(x + 4)^2$ is………

A) $\int_{-7}^{-4} \sqrt{1 + (\frac{dy}{dx})^2} \, dx$

B) $\int_{7}^{4} \sqrt{1 + (\frac{dy}{dx})^2} \, dx$

C) $\int_{-7}^{0} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$

D) $\int_{0}^{-4} \sqrt{1 + (\frac{dy}{dx})^2} \, dx$

Ans.- A

47. Integral for calculating the length of arc of parabola $y^2 = 4x$ cut off by the line $3y = 8x$ is

A. $\int_0^{\frac{16}{9}} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

B. $\int_0^{\frac{9}{16}} \sqrt{1+(\frac{dy}{dx})^2}\, dx$

C. $\int_0^{\frac{8}{3}} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

D. $\int_0^{\frac{3}{8}} \sqrt{1+(\frac{dy}{dx})^2}\, dx$

Ans. - B

48. The length of arc of upper part of loop of the curve

$3y^2 = x(x-1)^2$ from (0,0) to (1 , 0) using

$1+(\frac{dy}{dx})^2 = \frac{(3x+1)^2}{12x}$ is....

A) $\frac{4}{\sqrt{3}}$

B) $\frac{1}{\sqrt{3}}$

C) $\frac{2}{\sqrt{3}}$

D) $\sqrt{3}$

Ans. - C

49. Integral for calculating the length of the upper arc of the loop of the curve $x = t^2, y = t(1-\frac{t^2}{3})$ is.......

A) $\int_0^9 \sqrt{(\frac{dx}{dt})^2 - (\frac{dy}{dt})^2}dt$

B) $\int_0^{\sqrt{3}} \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}dt$

C) $\int_0^1 [\,(\frac{dx}{dt})^2 - \left(\frac{dy}{dt}\right)^2\Big]\, dt$

D) $\int_0^{\sqrt{3}} \sqrt{(x^2(t) - y^2(t)}dt$

Ans. - B

50. Integral for calculating the length of arc of Astroid $x = acos^3\theta, x = asin^3\theta$ in the first quadrant between two consecutive cusps is …….

A) $\int_0^{\frac{\pi}{4}} \sqrt{(\frac{dx}{d\theta})^2 + (\frac{dy}{d\theta})^2} d\theta$

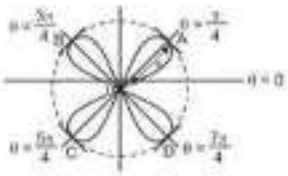
B) $\int_0^{\frac{\pi}{2}} \sqrt{(\frac{dx}{d\theta})^2 + (\frac{dy}{d\theta})^2} d\theta$

C) $\int_0^{\frac{\pi}{3}} \sqrt{(\frac{dx}{d\theta})^2 + (\frac{dy}{d\theta})^2} d\theta$

D) $\int_0^{\frac{\pi}{6}} \sqrt{(\frac{dx}{d\theta})^2 + (\frac{dy}{d\theta})^2} d\theta$

Ans. – B

The following figure represents the curve whose equation is .... (2)



(A) $r = a\cos 3\theta$

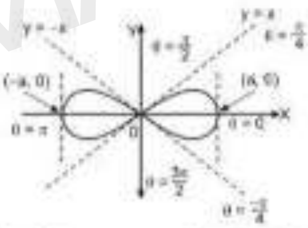
(B) $r = a\sin 2\theta$

(C) $r = a\sin 3\theta$

(D) $r = a(1 + \cos\theta)$

Ans.- B

The following figure represents the curve whose equation is .... (2)



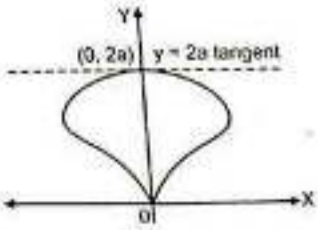
(A) $r^2 = a^2\cos 2\theta$ (B) $r^2 = a^2\sin 2\theta$

(C) $r = a\cos 2\theta$ (D) $r = a(1 + \cos\theta)$

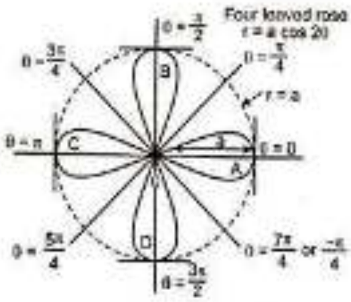
Ans.- A

The equation $r^2 = a^2 \cos 2\theta$ represents the curve ...

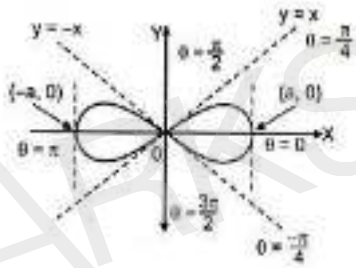
(A)



(B)



(C)



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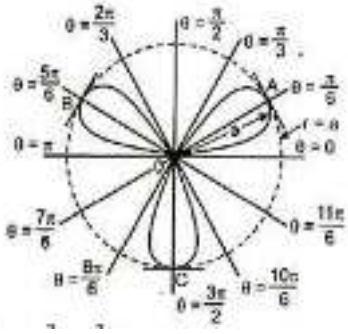
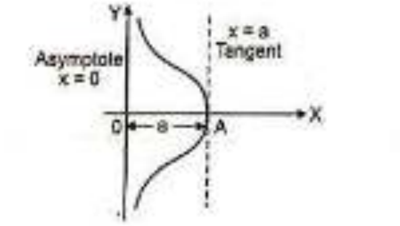


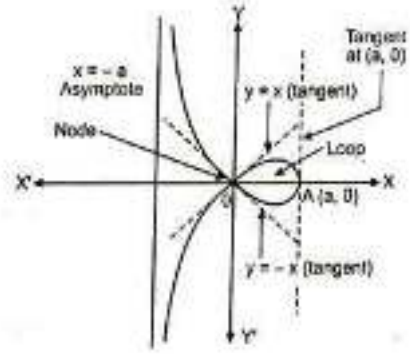
Ans.- C

The equation $xy^2 = a^2 (a - x)$ represents the curve ...

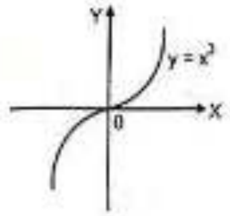
(A)

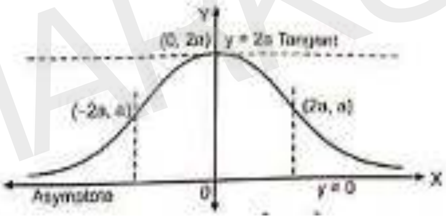


(B)



C)

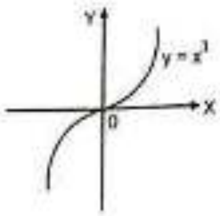




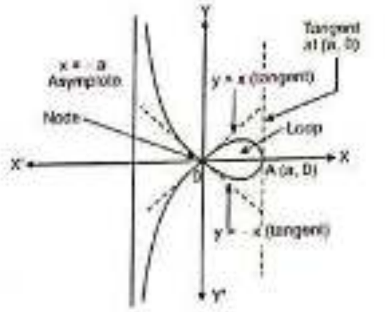
Ans.- A

The equation $x(x^2 + y^2) = a(x^2 - y^2)$, $a > 0$ represents the curve ... (2)

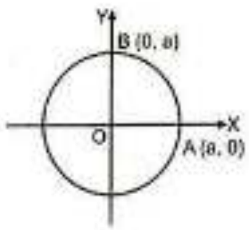
(A)



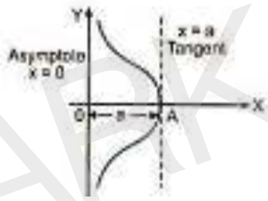
(B)



(C)



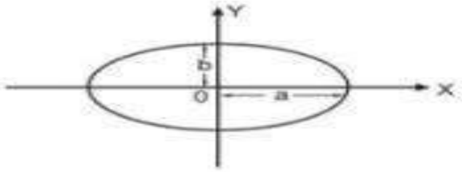
(D)



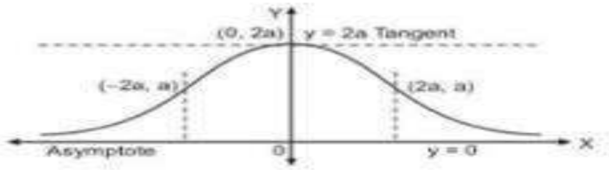
Ans.- B

The equation $a^2x^2 = y^3(2a - y)$, $a > 0$ represents the curv
... (2

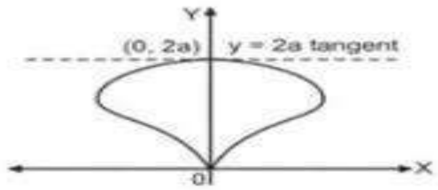
(A)



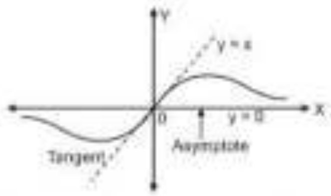
(B)



(C)



(D)



Ans.- C

Sinhgad College of Engineering,Vadgaon

Engineering Mathematics – II

Unit 5- Solid Geometry

1) Equation of sphere whose Centre at (2, -3,1) and radius is 5 is

A) $x^2 + y^2 + z^2 - 4x + 6y - 2z - 11 = 0$

B) $x^2 + y^2 + z^2 - 8x + 12y - 4z - 11 = 0$

C) $x^2 + y^2 + z^2 - 4x + 4y - 6z - 13 = 0$

D) $x^2 + y^2 + z^2 - 8x + 12y - 4z - 13 = 0$

Ans- A

2)Equation of spere with Centre at (2, -2,3) and passing through (7,-3,5) is

A) $x^2 + y^2 + z^2 - 4x + 6y - 2z - 11 = 0$

B) $x^2 + y^2 + z^2 - 8x + 12y - 4z - 11 = 0$

C) $x^2 + y^2 + z^2 - 4x + 4y - 6z - 13 = 0$

D) $x^2 + y^2 + z^2 - 8x + 12y - 4z - 13 = 0$

Ans-C

3)Equation of sphere passing through origin and making equal intercepts of unit length on axes is

A) $x^2 + y^2 + z^2 - x - y - z = 0$

B) $x^2 + y^2 + z^2 = 0$

C) $x^2 + y^2 + z^2 - x + y - z = 0$

D) $x^2 + y^2 + z^2 - x - y - z - 1 = 0$

Ans-A

4)equation of spere passing through (1,3,1), (2,-1,1),(1,2,0),(1,-1,1) is

A) $x^2+y^2+z^2-4x+2y-2z+8=0$

B) $x^2+y^2+z^2-4x+2y-2z-8=0$

C) $x^2+y^2+z^2+4x+2y-2z+8=0$

D) $x^2+y^2+z^2-4x+2y+2z+8=0$

Ans-B

5)spherical coordinates of a point (3,4,5) are-

A)3,4,5 B)$5\sqrt{2}, 45^0, 53.13^0$

C) $5, 45^0, 53^0$ D) $\sqrt{3}, 126.26^0, 135^0$

Ans-B

6) spherical coordinates of a point (-1,1, -1) are-

A) $5\sqrt{3}, 126.26^0, 135^0$ B)$5\sqrt{2}, 45^0, 53.13^0$

C) $5, 45^0, 53^0$ D) $\sqrt{3}, 126.26^0, 135^0$

Ans-D

7) $x^2+y^2+z^2-4x+6y-2z-11=0$ is equation of sphere then Centre and radius are

A) (-2, 3, -1),5 B) (2, -3,1),25 C) (-2, 3, -1),25 D) (2, -3,1),5

Ans-D

8) two spheres $x^2+y^2+z^2-2x+4y-4z=0$ and

$x^2+y^2+z^2+10x+2z+10=0$ touch each other then the point of contact is

A) $\left(\frac{-11}{7}, \frac{-8}{7}, \frac{-5}{7}\right)$ B) $\left(\frac{11}{7}, \frac{8}{7}, \frac{5}{7}\right)$ C) $\left(\frac{-11}{7}, \frac{-8}{7}, \frac{5}{7}\right)$ D) $\left(\frac{-11}{7}, \frac{8}{7}, \frac{5}{7}\right)$

Ans-c

9) the equation of the sphere tangential to the plane $x - 2y - 2z = 7$ at $(3, -1, -1)$ and passing through the point $(1,1,-3)$ is

A) $x^2 + y^2 + z^2 - 10y - 10z - 31 = 0$

B) $x^2 + y^2 + z^2 - 10y - 10z + 31 = 0$

C) $x^2 + y^2 + z^2 + 10y - 10z - 31 = 0$

D) $x^2 + y^2 + z^2 + 10y + 10z + 31 = 0$

Ans-A

10) the equation of the sphere which passed through $(3,1,2)$ and meets XOY plane in a circle of radius 3 units with the center at $(1, -2,0)$ is

A) $x^2 + y^2 + z^2 + 2x + 4y - 4z - 4 = 0$

B) $x^2 + y^2 + z^2 - 2x + 4y - 4z - 4 = 0$

C) $x^2 + y^2 + z^2 - 2x - 4y - 4z - 4 = 0$

D) $x^2 + y^2 + z^2 - 2x + 4y + 4z + 4 = 0$

Ans-B

11) Find the equation of the sphere passing through the circle $x^2+y^2 + z^2 = 9,\ 2x + 3y + 4z = 5$, and point $(1,2,3)$ is

A) $(x^2 + y^2 + z^2) - 2x - 3y - 4z - 22 = 0$

B) $(x^2 + y^2 + z^2) - 2x - 3y - 4z - 22 = 0$

C) $3(x^2 + y^2 + z^2) - 2x - 3y - 4z - 22 = 0$

D) $(x^2 + y^2 + z^2) - 2x - 3y - 4z - 22 = 0$

Ans-C

12) the equation of the sphere through the circle

$x^2+y^2+z^2-x+9y-5z-5=0,\ x^2+y^2+z^2+10y-4z-8=0$ as a great circle is

A) $x^2+y^2+z^2+4x+6y-8z+4=0$

B) $x^2+y^2+z^2-4x+6y-8z-4=0$

C) $x^2+y^2+z^2-4x+6y+8z+4=0$

D) $x^2+y^2+z^2-4x+6y-8z+4=0$

Ans-D

13) **t**he equation of the sphere passing through the circle $3x-4y+5z-15=0,$ and $x^2+y^2+z^2-2x+3y-4z+6=0,$ and cuts the sphere $x^2+y^2+z^2+2x+4y-6z+11=0$ orthogonally is

A) $5(x^2+y^2+z^2)-13x+19y-25z+45=0$

B) $5(x^2+y^2+z^2)+13x+19y-25z+45=0$

C) $5(x^2+y^2+z^2)-13x-19y-25z+45=0$

D) $5(x^2+y^2+z^2)-13x+19y-25z-45=0$

Ans-A

14) Consider two sphere

$S_1 \equiv x^2+y^2+z^2+2u_1x+2v_1y+2w_1z+d_1=0$

$S_2 \equiv x^2+y^2+z^2+2u_2x+2v_2y+2w_2z+d_2=0$

Then condition for orthogonality is

A) $2u_1u_2+2v_1v_2+2w_1w_2=d_1-d_2$
B) $2u_1u_2-2v_1v_2+2w_1w_2=d_1+d_2$
C) $2u_1u_2-2v_1v_2-2w_1w_2=d_1-d_2$
D) $2u_1u_2+2v_1v_2+2w_1w_2=d_1+d_2$
Ans-D

15) The center of the circle, which is an intersection of the sphere,

$x^2+y^2+z^2-2y-4z-11=0$ by the plane $x+2\mathrm{y}+2\mathrm{z}=15$ is

A) (1,3,4) B) (1,2,3) C) (0,0,0) D) (1,3,2)

Ans-A

16)The Equation of Right circular cone whose vertex is at (α,β,γ), semi-vertical angle θ and the line Axis $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$ is

A) $cos\theta=\frac{l(x+\alpha)+m(y+\beta)+n(z+\gamma)}{\sqrt{l^2+m^2+n^2}\sqrt{(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2}}$

B) $cos\theta=\frac{l(x-\alpha)-m(y-\beta)-n(z-\gamma)}{\sqrt{l^2+m^2+n^2}\sqrt{(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2}}$

C) $cos\theta=\frac{l(x-\alpha)+m(y-\beta)+n(z-\gamma)}{\sqrt{l^2-m^2-n^2}\sqrt{(x-\alpha)^2-(y-\beta)^2-(z-\gamma)^2}}$

D) $cos\theta=\frac{l(x-\alpha)+m(y-\beta)+n(z-\gamma)}{\sqrt{l^2+m^2+n^2}\sqrt{(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2}}$

Ans. D

17) Equation of Right Circular cone with vertex at (1,2,3) and axis of cone $\frac{x-1}{2}=\frac{y-2}{-1}=\frac{z-3}{4}$ and semi-Vertical angle 60^0 is given by

A) $cos60=\frac{2(x-1)-1(y-2)+4(z-3)}{\sqrt{2^2+(-1)^2+4^2}\sqrt{(x-1)^2+(y-2)^2+(z-3)^2}}$

B) $cos60=\frac{2(x-1)+1(y-2)+4(z-3)}{\sqrt{2^2+(-1)^2+4^2}\sqrt{(x-1)^2+(x-2)^2+(x-3)^2}}$

C) $cos60=\frac{2(x-1)-1(y-2)-4(z-3)}{\sqrt{2^2+(-1)^2+4^2}\sqrt{(x-1)^2+(x-2)^2+(x-3)^2}}$

D) $cos60=\frac{2(x-1)-1(y-2)+4(z-3)}{\sqrt{2^2-(1)^2+4^2}\sqrt{(x-1)^2+(x-2)^2+(x-3)^2}}$

Ans. A

18) Equation of Right Circular cone with vertex at (-1,-2,-3) and axis of cone

$\frac{x+1}{3} = \frac{y+2}{1} = \frac{z+3}{4}$ and semi-Vertical angle 30° is given by

A) $cos30 = \frac{3(x+1)-1(y+2)-4(z+3)}{\sqrt{3^2+1^2+4^2}\sqrt{(x+1)^2+(x+2)^2+(x+3)^2}}$

B) $cos30 = \frac{3(x+1)+1(y+2)+4(z+3)}{\sqrt{3^2+1^2+4^2}\sqrt{(x+1)^2+(y+2)^2+(z+3)^2}}$

C) $cos30 = \frac{3(x-1)-1(y-2)-4(z-3)}{\sqrt{3^2+1^2+4^2}\sqrt{(x-1)^2+(y-2)^2+(z-3)^2}}$

D) $cos30 = \frac{3(x-1)+1(y-2)+4(z-3)}{\sqrt{3^2+1^2+4^2}\sqrt{(x-1)^2+(x-2)^2+(x-3)^2}}$

Ans. B

19) Equation of Right Circular cone with vertex at (1,2,3) and axis of cone

$\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{4}$ and semi-Vertical angle 60^0 is given by

A) $21[(x-1)^2 + (y-2)^2 + (z-3)^2] = 4[2x - y + 4z - 12]^2$

B) $21[(x+1)^2 + (y+2)^2 + (z+3)^2] = 4[2x - y + 4z - 12]^2$

C) $4[(x-1)^2 + (y-2)^2 + (z-3)^2] = 25[2x - y + 4z - 12]^2$

D) $4[(x+1)^2 + (y+2)^2 + (z+3)^2] = 25[2x - y + 4z - 12]^2$

Ans. A

20) Equation of Right Circular cone with vertex at (1,1,1) and axis of cone

$\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{3}$ and semi-Vertical angle $\frac{\pi}{4}$ is given by

A) $2[(x-1)^2 + (y-1)^2 + (z-1)^2] = 14[x + 2y + 3z - 6]^2$

B) $14[(x-1)^2 + (y-1)^2 + (z-1)^2] = 2[x + 2y + 3z + 6]^2$

C) $14[(x-1)^2 + (y-1)^2 + (z-1)^2] = 2[x + 2y + 3z - 6]^2$

D) $2[(x-1)^2 + (y-1)^2 + (z-1)^2] = 14[x + 2y + 3z + 6]^2$

Ans. C

21) Semi- vertical angle of right circular cone which passes through the point (2,1,3) with vertex at $(1,1,2)$ and axis parallel to the line $\frac{x-2}{2} = \frac{y-1}{-4} = \frac{z+2}{3}$ is...

A) $cos\theta = \frac{5}{\sqrt{29}}$ B) $cos\theta = \frac{5}{\sqrt{58}}$ C) $cos\theta = \frac{-5}{\sqrt{58}}$ D) $cos\theta = \frac{-5}{\sqrt{29}}$

Ans. B

22) Semi- vertical angle of right circular cone with vertex at (0,0,2) direction ratio of generator are 0, 3, -2 and axis is z-axis is given by

A) $cos\theta = \frac{-4}{\sqrt{23}}$ B) $cos\theta = \frac{4}{\sqrt{23}}$ C) $cos\theta = \frac{2}{\sqrt{13}}$ D) $cos\theta = \frac{-2}{\sqrt{13}}$

Ans. D

23) Semi vertical angle of right circular cone having its vertex at $(0,0,0)$ and which passes through the point (2,-2,1) and axis parallel to the line

$\frac{x-2}{5} = \frac{y-1}{1} = \frac{z+2}{1}$

A) $cos\theta = \frac{9}{\sqrt{9}\sqrt{27}}$ B) $cos\theta = \frac{13}{\sqrt{9}\sqrt{27}}$ C) $cos\theta = \frac{-9}{\sqrt{9}\sqrt{27}}$ D) $cos\theta = \frac{-13}{\sqrt{9}\sqrt{27}}$

Ans. A

24) Semi vertical angle of right circular cone having its vertex at $(0,0,0)$ and direction ratio of one of the generator of cone are $1, -2,\ 2$ and axis makes equal angles with co-ordinate axes is given by

A) $cos\theta = \frac{-1}{3\sqrt{3}}$ B) $cos\theta = \frac{-3}{3\sqrt{3}}$ C) $cos\theta = \frac{1}{3\sqrt{3}}$ D) $cos\theta = \frac{3}{3\sqrt{3}}$

Ans. C

25) Semi vertical angle of Right Circular cone which passes through the point $(1,1,2)$ and has its axis as Line $6x = -3y = 4z$ and vertex origin is …

A) $cos\theta = \frac{-4}{\sqrt{174}}$ B) $cos\theta = \frac{12}{\sqrt{23}}$ C) $cos\theta = \frac{-12}{\sqrt{13}}$ D) $cos\theta = \frac{4}{\sqrt{174}}$

Ans. D

26) The Equation of Right circular cone whose vertex is at (0,0,0) semi-vertical angle θ and the line Axis $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ is

A) $cos\theta=\frac{l(x)+m(y)+n(z)}{\sqrt{l^2+m^2+n^2}\sqrt{(x)^2+(y)^2+(z)^2}}$ B) $cos\theta=\frac{l(x)-m(y)-n(z)}{\sqrt{l^2+m^2+n^2}\sqrt{(x)^2+(y)^2+(z)^2}}$

C) $cos\theta=\frac{l(x)+m(y)+n(z)}{\sqrt{l^2-m^2-n^2}\sqrt{(x)^2-(y)^2-(z)^2}}$ D) $cos\theta=\frac{l(x)+m(y)+n(z)}{\sqrt{l^2+m^2+n^2}\sqrt{(x)^2-(y)^2-(z)^2}}$

Ans. A

27) Equation of Right Circular cone with vertex at (0,0,0) and axis of cone $\frac{x}{2}=\frac{y}{-1}=\frac{z}{2}$ and semi vertical angle $\frac{\pi}{4}$ is given by

A) $cos\frac{\pi}{4}=\frac{2(x)-1(y)+2(z)}{\sqrt{(2)^2-(1)^2-(2)^2}\sqrt{(x)^2-(y)^2-(z)^2}}$

B) $cos\frac{\pi}{4}=\frac{2(x)-1(y)+2(z)}{\sqrt{(2)^2+(-1)^2+(2)^2}\sqrt{(x)^2-(y)^2-(z)^2}}$

C) $cos\frac{\pi}{4}=\frac{2(x)-1(y)+2(z)}{\sqrt{(2)^2+(-1)^2+(2)^2}\sqrt{(x)^2+(y)^2+(z)^2}}$

D) $cos\frac{\pi}{4}=\frac{2(x)+1(y)+2(z)}{\sqrt{(2)^2-(-1)^2+(2)^2}\sqrt{(x)^2+(y)^2+(z)^2}}$

Ans. C

28) Equation of Right Circular cone with vertex at (0,0,0) and axis of cone $\frac{x}{2}=\frac{y}{-1}=\frac{z}{2}$ and semi vertical angle $\frac{\pi}{4}$ is given by

A) $x^2+y^2+z^2+8xy-16xz-8yz=0$

B) $x^2+7y^2+z^2+8xy-16xz+8yz=0$

C) $x^2+7y^2+z^2+8xy-16xz-8yz=0$

D) $x^2+8y^2+z^2+8xy-16xz+8yz=0$

Ans. B

29) Equation of Right Circular cone with vertex at (0,0,0) and axis of cone $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and semi vertical angle $\frac{\pi}{6}$ is given by

A) $\frac{1}{2} \quad = \frac{x+2y+2z}{\sqrt{14}\sqrt{(x)^2+(y)^2+(z)^2}}$ B) $\frac{\sqrt{3}}{2} \quad = \frac{x-2y-3z}{\sqrt{14}\sqrt{(x)^2+(y)^2+(z)^2}}$

C) $\frac{1}{2} \quad = \frac{x+2y+3z}{\sqrt{14}\sqrt{(x)^2-(y)^2-(z)^2}}$ D) $\frac{\sqrt{3}}{2} \quad = \frac{x+2y+3z}{\sqrt{14}\sqrt{(x)^2+(y)^2+(z)^2}}$

Ans. D

30) Equation of Right circular cone with vertex at (0,0,0) and direction ratio of axis of cone are 5, 1, 1 and $cos\theta = \frac{9}{\sqrt{9}\sqrt{27}}$ is given by

A) $\frac{9}{\sqrt{9}\sqrt{27}} \quad = \frac{5x+y+z}{\sqrt{(5)^2-(1)^2-(1)^2}\sqrt{(x)^2+(y)^2+(z)^2}}$ B) $\frac{9}{\sqrt{9}\sqrt{27}} \quad = \frac{5x+y+z}{\sqrt{(5)^2+(1)^2+(1)^2}\sqrt{(x)^2+(y)^2+(z)^2}}$

C) $\frac{9}{\sqrt{9}\sqrt{27}} \quad = \frac{5x-y-z}{\sqrt{(5)^2-(1)^2-(1)^2}\sqrt{(x)^2+(y)^2+(z)^2}}$ D) $\frac{9}{\sqrt{9}\sqrt{27}} \quad = \frac{5x-y-z}{\sqrt{(5)^2+(1)^2+(1)^2}\sqrt{(x)^2+(y)^2+(z)^2}}$

Ans. B

31) Equation of Right circular cone with vertex at (0,0,0) and has its axis as Line

$6x = -3y = 4z$ and $cos\theta = \frac{4}{\sqrt{174}}$ is given by

A) $\frac{4}{\sqrt{174}} \quad = \frac{2x+4y+3z}{\sqrt{29}\sqrt{(x)^2+(y)^2+(z)^2}}$

B) $\frac{4}{\sqrt{174}} = \frac{2x-4y+3z}{\sqrt{29}\sqrt{(x)^2+(y)^2+(z)^2}}$

C) $\frac{4}{\sqrt{174}} = \frac{6x-3y+4z}{\sqrt{29}\sqrt{(x)^2+(y)^2+(z)^2}}$

D) $\frac{4}{\sqrt{174}} \quad = \frac{6x+3y+4z}{\sqrt{29}\sqrt{(x)^2+(y)^2+(z)^2}}$

Ans. B

32) Equation of Right circular cone with vertex at (0,0,0) and direction ratio of axis of cone are 5, 1, 1 and $cos\theta = \frac{9}{\sqrt{9}\sqrt{27}}$ is given by

A) $8x^2 - 4y^2 - 4z^2 + 5xy + yz + 5xz = 0$

B) $8x^2 + 4y^2 - 8z^2 + 5xy - yz - 5xz = 0$

C) $8x^2 + 4y^2 - 8z^2 + 5xy - yz + 5xz = 0$

D) $8x^2 + 4y^2 - 4z^2 + 5xy - yz - 5xz = 0$

Ans. A

33) Equation of Right circular cone with vertex at (0,0,0) axis is the y axis and semi vertical angle 30^0 is given by

A) $\frac{\sqrt{3}}{2} = \frac{y}{\sqrt{(x)^2+(y)^2+(z)^2}}$ B) $\frac{1}{2} = \frac{y}{\sqrt{(x)^2+(y)^2+(z)^2}}$

C) $\frac{\sqrt{3}}{2} = \frac{x+z}{\sqrt{(x)^2+(y)^2+(z)^2}}$ D) $\frac{1}{2} = \frac{x+z}{\sqrt{(x)^2+(y)^2+(z)^2}}$

Ans. A

34) Equation of Right circular cone with vertex at (0,0,0) axis is the y axis and semi vertical angle 30^0 is given by

A) $3x^2 + y^2 + 3z^2 = 0$ B) $3x^2 - y^2 + 3z^2 = 0$

C) $x^2 - y^2 + 3z^2 = 0$ D) $3x^2 - y^2 - 3z^2 = 0$

Ans. B

35) Equation of Right circular cone with vertex at $(0, 0, 0)$ axis is the z axis and semi vertical angle 45^0 is

$A) \frac{1}{\sqrt{2}} = \frac{-z}{\sqrt{(x)^2+(y)^2+(z)^2}}$

$B) \frac{1}{\sqrt{2}} = \frac{z}{\sqrt{(x)^2+(y)^2+(z)^2}}$

$C) \frac{1}{\sqrt{2}} = \frac{x-y}{\sqrt{(x)^2+(y)^2+(z)^2}}$

$D) \frac{1}{\sqrt{2}} = \frac{x+y}{\sqrt{(x)^2+(y)^2+(z)^2}}$

Ans. B

36) Direction ratio of axis of Right Circular cone which passes through the point (1,1,2) and has its axis as Line $6x = -3y = 4z$ and vertex origin is given by

A) $6, -3, 4$ B) $2, -4, 3$ C) $2, 4, -3$ D) $-6, 3, -4$

Ans. B

37) Direction ratio of axis of Right Circular cone which passes through the point (1,1,2) and has its axis as Line $2x = -y = 4z$ and vertex origin is given by

A) $-1, -2, -2$ B) $-2, 1, -4$ C) $1, -2, 2$ D) $2, -4, 1$

Ans. D

38)The equation of the right circular cylinder whose radius r and axis is the line $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$ is...

A) $(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2-\left\{\frac{l(x-\alpha)+m(y-\beta)+n(z-\gamma)}{\sqrt{l^2+m^2+n^2}}\right\}^2=r^2$

B) $(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2+\left\{\frac{l(x-\alpha)+m(y-\beta)+n(z-\gamma)}{\sqrt{l^2+m^2+n^2}}\right\}^2=r^2$

C) $(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2-\left\{\frac{(x-\alpha)+(y-\beta)+(z-\gamma)}{\sqrt{l^2+m^2+n^2}}\right\}^2=r^2$

D) $(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2-\left\{\frac{l(x+\alpha)+m(y+\beta)+n(z+\gamma)}{\sqrt{l^2+m^2+n^2}}\right\}^2=r^2$

Ans. A

39) The equation of the right circular cylinder whose radius 2 and axis is the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{2}$ is...

A) $(x-1)^2+(y-2)^2+(z-3)^2-\left\{\frac{2(x-1)+1(y-2)+1(z-3)}{3}\right\}^2=4$

B) $(x-1)^2+(y-2)^2+(z-3)^2+\left\{\frac{2(x-1)+1(y-2)+1(z-3)}{3}\right\}^2=4$

C) $(x-1)^2+(y-2)^2+(z-3)^2-\left\{\frac{(x-1)+(y-2)+(z-3)}{3}\right\}^2=4$

D) $(x-1)^2+(y-2)^2+(z-3)^2-\left\{\frac{2(x+1)+1(y+2)+1(z+3)}{3}\right\}^2=4$

Ans. A

40) The radius of right circular cylinder whose axis is $\frac{x-2}{2}=\frac{y-1}{1}=\frac{z}{3}$ and which passes through the point (0,0,3) is…

A) $\sqrt{\frac{10}{7}}$ B) $\frac{90}{7}$ C) $\sqrt{\frac{90}{7}}$ D) $\frac{10}{7}$

Ans. C

41) Equation of right circular cylinder of radius a, whose axis passes through the origin and makes equal angles with co-ordinate axes is..

A) $2(x^2+y^2+z^2+xy+yz+zx)=a^2$

B) $2(x^2+y^2+z^2+xy+yz+zx)=3a^2$

C) $2(x^2-y^2-z^2+xy+yz+zx)=a^2$

D) $2(x^2+y^2+z^2-xy-yz-zx)=3a^2$

Ans. D

42) The equation of the right circular cylinder whose axis is $x=2y=-z$ and radius 4 is…

A) $5x^2-8y^2+5z^2-4yz+8zx-4xy+144=0$

B) $5x^2-8y^2+5z^2-4yz+8zx-4xy-144=0$

C) $5x^2-8y^2-5z^2-4yz+8zx-4xy+144=0$

D) $5x^2+8y^2+5z^2+4yz+8zx-4xy-144=0$

Ans. D

43) The radius of right circular cylinder whose axis is $\frac{x}{2}=\frac{y}{1}=\frac{z}{2}$ and which passes through the point (1,2,1) is…

A) 2 B) $2\sqrt{2}$ C) $\sqrt{2}$ D) $\frac{1}{2}$

Ans. C

44) The equation of the right circular cylinder whose radius 5 and axis is the line $\frac{x-2}{2}=\frac{y-3}{1}=\frac{z+1}{1}$ is…

A) $(x-2)^2+(y-3)^2+(z+1)^2-\left\{\frac{2(x-2)+1(y-3)+1(z+1)}{\sqrt{6}}\right\}^2=25$

B) $(x+2)^2+(y+3)^2+(z+1)^2+\left\{\frac{2(x-1)+1(y-2)+1(z-3)}{3}\right\}^2=25$

C) $(x-2)^2+(y-3)^2+(z+1)^2+\left\{\frac{(x-1)+(y-2)+(z-3)}{3}\right\}^2=25$

D) None of these

Ans. A

45) The equation of right circular cylinder whose axis is $\frac{x-2}{2}=\frac{y-1}{1}=\frac{z}{3}$ and radius $\sqrt{\frac{90}{7}}$ is…

A) $10x^2+13y^2+5z^2-4xy-6yz-12zx-36x-18y+30z-135=0$

B) $10x^2+13y^2+5z^2+4xy+6yz+12zx-36x-18y+30z-135=0$

C) $10x^2+13y^2+5z^2-4xy-6yz-12zx-36x-18y+30z+135=0$

D) None of these

Ans. A

46) The axis of right circular cylinder has direction cosines proportional to 2,3,6. The direction ratios of axis are..

A) $\frac{-2}{7},\frac{-3}{7},\frac{-6}{7}$ B) $\frac{2}{7},\frac{3}{7},\frac{6}{7}$ C) $\frac{-2}{49},\frac{-3}{49},\frac{-6}{49}$ D) $\frac{-2}{7},\frac{3}{7},\frac{-6}{7}$

Ans. B

47) The equation of right circular cylinder of radius 2 whose axis passes through (1,2,3) and has direction cosines proportional to 2,-3,6 is…

A) $(x-1)^2+(y-2)^2+(z+1)^2-\left\{\frac{2(x-2)+1(y-3)+1(z+1)}{\sqrt{6}}\right\}^2=4$

B) $(x-2)^2+(y-3)^2+(z+1)^2+\left\{\frac{2(x-2)+1(y-3)+1(z+1)}{\sqrt{6}}\right\}^2=4$

C) $(x-1)^2+(y-2)^2+(z-3)^2-\left\{\frac{2x-3y+6z-14}{7}\right\}^2=4$

D) $(x-1)^2+(y-2)^2+(z-3)^2+\left\{\frac{2x-3y+6z-14}{7}\right\}^2=4$

Ans. C

Sinhgad College of Engineering,Vadgaon

Engineering Mathematics – II

Unit 6-Multiple integrals and their applications

Q.1) The value of $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \sin x \, dx \, dy$ is

A)0

B)1

C)$\frac{\pi}{2}$

D)π

Ans-C

Q.2) The value of $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \cos x \, dx \, dy$ is

A)$\frac{\pi}{2}$

B)1

C)0

D)π

Ans-A

Q.3) The value of $\int_0^1 \int_0^y x\, dx\, dy$ is

A)$\frac{1}{2}$

B)$\frac{1}{3}$

C)$\frac{1}{8}$

D)$\frac{1}{6}$

Ans-D

Q.4) The value of $\int_0^1 \int_0^x e^y\, dx\, dy$ is

A)e^2

B)$e - 2$

C)e

D)$\frac{1}{2}(e^2 - 1)$

Ans: B

Q.5)Using polar transformation $x = r\cos\theta$, $y = r\sin\theta$ the Cartesian double integral $\iint_R f(x,y)dxdy$ becomes

A) $\iint_R f(r,\theta)drd\theta$

B) $\iint_R f(r,\theta)rdrd\theta$

C) $\iint_R f(r,\theta)r^2drd\theta$

D) $\iint_R f(r,\theta)\theta drd\theta$

Ans:B

Q.6) On changing the order of integration of $\int_0^1 \int_0^x f(x.y)dxdy$ becomes

A) $\int_0^1 \int_0^1 f(x.y)dxdy$

B) $\int_0^1 \int_0^y f(x.y)dxdy$

C) $\int_0^1 \int_1^y f(x.y)dxdy$

D) $\int_0^1 \int_y^1 f(x.y)dxdy$

Ans: D

Q.7) On changing the order of integration of $\int_0^1 \int_{x^2}^1 f(x.y)dxdy$ becomes

A) $\int_0^1 \int_0^{\sqrt{y}} f(x.y)dxdy$

B) $\int_0^1 \int_0^{-\sqrt{y}} f(x.y)dxdy$

C) $\int_0^1 \int_0^{\sqrt{x}} f(x.y)dxdy$

D) $\int_0^1 \int_0^{-\sqrt{x}} f(x.y)dxdy$

Ans: A

Q.8) on transforming into the polar co-ordinates the double integration $\int_0^1 \int_0^{\sqrt{1-x^2}} f(x,y)dxdy$ becomes

A) $\int_0^{\pi} \left\{ \int_0^1 f(r,\theta) r dr \right\} d\theta$

B) $\int_0^{\frac{\pi}{2}} \left\{ \int_0^1 f(r,\theta) r d\theta \right\} dr$

C) $\int_0^{\frac{\pi}{2}} \left\{ \int_0^1 f(r,\theta) r dr \right\} d\theta$

D) $\int_0^{2\pi} \left\{ \int_0^1 f(r,\theta) r dr \right\} d\theta$

Ans: C

Q.11) on transforming into the polar co-ordinates the double integration $\int_0^a \int_0^{\sqrt{a^2-y^2}} e^{-x^2-y^2} dxdy$ becomes

A) $\int_0^{\frac{\pi}{2}} \int_0^a e^{-r^2} dr d\theta$

B) $\int_0^{\frac{\pi}{2}} \int_0^a e^{-r^2} r^2 dr d\theta$

C) $\int_0^{\frac{\pi}{2}} \int_0^a e^{-r^2} r dr d\theta$

D) $\int_0^{\frac{\pi}{2}} \int_0^a e^{-r} r dr d\theta$

Ans. C

Q.12) To find the area of upper half of a cardioid $r = a(1+\cos\theta)$ the double integral becomes

A) $\int_0^{\pi} \int_0^{a(1+\cos\theta)} r dr d\theta$

B) $\int_0^{\pi} \int_0^{a(1+\cos\theta)} dr d\theta$

C) $\int_0^{2\pi} \int_0^{a(1+\cos\theta)} r^2 dr d\theta$

D) $\int_0^{2\pi} \int_0^{a(1+\cos\theta)} r dr d\theta$

Ans: A

13. The value of $\int_0^2 \int_0^x e^{x+y}\, dydx$ is

A) $\frac{1}{2}(e^2-1)$ B) $\frac{1}{2}(e^2-e)$ C) $\frac{1}{2}(e^2-1)^2$ D) None of these

Ans. C

14. On changing the order of integration for $\int_0^\infty \int_x^\infty f(x,y)\,dydx$,the integral becomes

A) $\int_0^\infty \int_0^\infty f(x,y)\,dydx$ B) $\int_0^\infty \int_y^\infty f(x,y)\,dxdy$

C) $\int_0^\infty \int_0^y f(x,y)\,dxdy$ D) $\int_0^\infty \int_0^x f(x,y)\,dydx$

Ans. C

15. On changing the order of integration for $\int_0^a \int_{\frac{y^2}{a}}^{y} f(x,y)dxdy$,the integral becomes

A) $\int_0^a \int_0^x f(x,y)\,dxdy$ B) $\int_0^a \int_x^{\sqrt{ax}} f(x,y)\,dydx$

C) $\int_0^a \int_0^a f(x,y)\,dydx$ D) $\int_0^\infty \int_x^\infty f(x,y)\,dxdy$

Ans. B

16.Find the value of $\int_0^1 \int_0^{1-x} (x+y)dxdy$
A) $^1/_2$ B) $^1/_3$ C) $^1/_4$ D) $^1/_5$

Ans B

17. Evaluate $\int_0^1 \int_0^y xydxdy$

A) $^1/_2$ B) $^1/_3$ C) $^1/_4$ D) $^1/_8$

Ans D

18. Evaluate $\int_0^1 \int_0^1 \frac{dxdy}{(1+x^2)(1+y^2)}$

A) $\frac{\pi^2}{16}$ B) $\frac{5}{16}$ C) $^1/_{16}$ D) $^1/_8$

Ans A

19.Find the value of $\iint xye^{x+y}$ dxdy.
A) $ye^y(xe^x-e^x)$ B) $(ye^y-e^y)(xe^x-e^x)$
C) $(ye^y-e^y)xe^x$ D) $(ye^y-e^y)(xe^x+e^x)$

Ans B

20. using double integration and the strip parallel to X-axis the area enclosed between the curves $y^2 = 4x$ and $x^2 = 4y$ using points of cutting (0,0) and (4,4) is

A) $\int_0^4 \int_0^{4x} dxdy$ B) $\int_0^4 \int_0^4 dxdy$ C) $\int_0^{4y} \int_0^{4x} dxdy$ D) $\int_{y=0}^{y=4} \int_{\frac{y^2}{4}}^{2\sqrt{y}} dxdy$

Ans. D

21. The area enclosed between the straight line y=x and parabola $y = x^2$ in the XOY plane using double integration is

A) $\int_{x=0}^{x=1} \int_{y=x}^{y=x^2} dxdy$ B) $\int_{x=0}^{x=\infty} \int_{y=x}^{y=\infty} dxdy$

C) $\int_{x=0}^{x=1} \int_{y=0}^{y=1} dxdy$ D) $\int_{x=1}^{x=\infty} \int_{y=1}^{y=\infty} dxdy$

Ans. A

22. The area bounded by $y^2 = 4x$ and $2x - y - 4 = 0$ is

A) 2 B) 1 C) 8 D) 9

Ans. D

23. Area bounded by $y^2 = x$ and $x^2 = -8y$ is

A) $\frac{2}{3}$ B) $\frac{8}{3}$ C) $\frac{7}{3}$ D) $\frac{1}{3}$

Ans. B

24. Area bounded by $y^2 = 4ax$ and $x^2 = 4ay$ is

A) $\frac{a}{3}$ B) $\frac{a^2}{3}$ C) $\frac{16a^2}{3}$ D) $\frac{16}{3}$

Ans. C

25. Area bounded by $x^2 = 4y$ and $x - 2y + 4 = 0$ is

A) 9 B) 4 C) 16 D) 5

Ans. A

26. If $(\bar{x}, \bar{y})$ is centre of gravity of arc AB of the curve $y = f(x)$, then $(\bar{x}, \bar{y})$=

A) $\bar{x} = \frac{\int x \rho \, ds}{\int \rho \, ds}$; $\bar{y} = \frac{\int y \, \rho \, ds}{\int \rho \, ds}$.

B) $\bar{x} = \frac{\int y \rho \, ds}{\int \rho \, ds}$; $\bar{y} = \frac{\int x \rho \, ds}{\int \rho \, ds}$.

C) $\bar{x} = \frac{\int x^2 \rho \, ds}{\int \rho \, ds}$; $\bar{y} = \frac{\int y^2 \rho \, ds}{\int \rho \, ds}$.

D) $\bar{x} = \frac{\int xy \rho \, ds}{\int \rho \, ds}$; $\bar{y} = \frac{\int y \, \rho \, ds}{\int \rho \, ds}$.

Ans. A

27. If $(\bar{x}, \bar{y})$ are corrdinates of centre of gravity of region R plane lamina bonded by curve C and ρ (density) is constant then $(\bar{x}, \bar{y})$=……….

A) $\bar{x} = \frac{\iint_R x \, dx \, dy}{\iint_R dx \, dy}$; $\bar{y} = \frac{\iint_R y \, dx \, dy}{\iint_R dx \, dy}$

B) $\bar{x} = \frac{\iint_R x^2 \, dx \, dy}{\iint_R dx \, dy}$; $\bar{y} = \frac{\iint_R y \, dx \, dy}{\iint_R dx \, dy}$

C) $\bar{x} = \frac{\iint_R x \, dx \, dy}{\iint_R dx \, dy}$; $\bar{y} = \frac{\iint_R y^2 \, dx \, dy}{\iint_R dx \, dy}$

D) $\bar{x} = \frac{\iint_R x^2 \, dx \, dy}{\iint_R dx \, dy}$; $\bar{y} = \frac{\iint_R y^2 \, dx \, dy}{\iint_R dx \, dy}$

Ans. A

28. The centroid of the loop of the curve $y^2 = \frac{x^2(a-x)}{(a+x)}$ will lie on …………

A) Y-axis

B) X- axis

C) origin

D) None of the above

Ans. B

29. The centroid of the area bounded by

$y^2(2a - x) = x^3$ and its asymptote is

A) $(\bar{x}, \bar{y}) = \left(\frac{5a}{3}, 2a\right)$

B) $(\bar{x}, \bar{y}) = \left(\frac{5a}{3}, a\right)$

C) $(\bar{x}, \bar{y}) = \left(\frac{5a}{3}, 0\right)$

D) $(\bar{x}, \bar{y}) = \left(\frac{5a}{3}, a\right)$

Ans. C

30. The centroid of the loop of the curve $x^2 = \frac{y^2(a-y)}{(a+y)}$ will lie on …………

A) Y-axis

B) X- axis

C) origin

D) None of the above

Ans. A

31. If ($\bar{x}$, $\bar{y}$, $\bar{z}$) be coordinates of centre of gravity of the solid which encloses volume V. then $\bar{x}$ =……

A) $\bar{x} = \frac{\iiint_V x \, \rho \, dx \, dy \, dz}{\iiint_V \rho \, dx \, dy \, dz}$;

B) $\bar{x} = \frac{\iiint_V y \, \rho \, dx \, dy \, dz}{\iiint_V \rho \, dx \, dy \, dz}$;

C) $\bar{x} = \frac{\iiint_V z \, \rho \, dx \, dy \, dz}{\iiint_V \rho \, dx \, dy \, dz}$z

D) $None\ of\ the\ above$

Ans.A

32. The Moment of inertia of a plane lamina R bounded by the curve C about X-axis is………… .

A) $\iint_R \rho \, y^2 \, dx \, dy$ B) $\iint_R \rho \, x^2 \, dx \, dy$

C) $\iint_R \rho \, (x+y)^2 \, dx \, dy$ D) $\iint_R \rho \, x \, dx \, dy$

Ans.A

33. The Moment of inertia of a plane lamina R bounded by the curve C about Y-axis is……….. .

A) $\iint_R \rho \, y^2 \, dx \, dy$ B) $\iint_R \rho \, x^2 \, dx \, dy$

C) $\iint_R \rho \, (x+y)^2 \, dx \, dy$ D) $\iint_R \rho \, x \, dx \, dy$

Ans.B

34. The Moment of inertia of a plane lamina R bounded by the curve C in polar coordinates is

A) M.I. $= \iint_R \rho \, p^2 r \, d\theta \, dr$ B) M.I. $= \iint_R \rho \, p^2 \, d\theta \, dr$

C) M.I. $= \iint_R \rho \, p \, r \, d\theta \, dr$ D) M.I. $= \iint_R \rho \, p^2 \theta \, d\theta \, dr$

Ans. A

35. The Moment of inertia of solid which is at a distance p from the axis is ……

A) $\iiint_V \rho \, p \, dx \, dy \, dz$ B) $\iiint_V \rho \, p^2 \, dx \, dy \, dz$

C) $\iiint_V \rho \, dx \, dy \, dz$ D) $\iiint_V \rho \, p^3 \, dx \, dy \, dz$

Ans.B

36. The Moment of inertia of solid about X-axis is ......

A) $\iiint_V \rho(y^2 + z^2)dx\, dy\, dz$

B) $\iiint_V \rho(x^2 + z^2)dx\, dy\, dz$

C) $\iiint_V \rho(y^2 + x^2)dx\, dy\, dz$

D) $\iiint_V \rho\, x^2 dx\, dy\, dz$

Ans.A

37. The Moment of inertia of solid about Y -axis is ......

A) $\iiint_V \rho(y^2 + z^2)dx\, dy\, dz$

B) $\iiint_V \rho(x^2 + z^2)dx\, dy\, dz$

C) $\iiint_V \rho(y^2 + x^2)dx\, dy\, dz$

D) $\iiint_V \rho\, x^2 dx\, dy\, dz$

Ans.B

38. The Moment of inertia of solid about Z-axis is ......

A) $\iiint_V \rho(y^2 + z^2)dx\, dy\, dz$ s

B) $\iiint_V \rho(x^2 + z^2)dx\, dy\, dz$

C) $\iiint_V \rho(y^2 + x^2)dx\, dy\, dz$

D) $\iiint_V \rho\, x^2 dx\, dy\, dz$

Ans.C

39. The Moment of inertia about the line $\theta = \frac{\pi}{2}$ of the area enclosed by $r = a(1 + cos\theta)$

A) M.I. = $\iint r^2 sin^2\theta\, r\, dr\, d\theta$

B) M.I. = $\iint r^2 cos^2\theta\, r\, dr\, d\theta$

C) M.I. = $\iint r^2 cos^2\theta\, dr\, d\theta$

D) M.I. = $\iint cos^2\theta\, r\, dr\, d\theta$

Ans.B

40. The Moment of Inertia about the X-axis of the area enclosed by the lines $x = 0, \frac{x}{a} + \frac{y}{b} = 1$ is

A) $\rho \int_{y=0}^{b} \cdot \int_{0}^{\frac{a}{b}(b-y)} x^2 dx\, dy$

B) $\rho \int_{y=0}^{b} \cdot \int_{0}^{\frac{a}{b}(b-y)} x\, dx\, dy$

C) $\rho \int_{y=0}^{b} \cdot \int_{0}^{\frac{a}{b}(b-y)} y^2 dx\, dy$

D) $\rho \int_{x=0}^{b} \cdot \int_{y=0}^{\frac{a}{b}(b-y)} y^2 dy\, dx$

Ans.C

41. The Moment of Inertia about the Y-axis of the area enclosed by the lines $x = 0, \frac{x}{a} + \frac{y}{b} = 1$ is

A) $\rho \int_{y=0}^{b} \cdot \int_{0}^{\frac{a}{b}(b-y)} x^2 dx\, dy$

B) $\rho \int_{y=0}^{b} \cdot \int_{0}^{\frac{a}{b}(b-y)} x\, dx\, dy$

C) $\rho \int_{y=0}^{b} \cdot \int_{0}^{\frac{a}{b}(b-y)} y^2 dx\, dy$

D) $\rho \int_{x=0}^{b} \cdot \int_{y=0}^{\frac{a}{b}(b-y)} y^2 dy\, dx$

Ans.A

42. Transformation of triple integration to spherical polar coordinates is

A) $\iiint_V F(r,\theta,\emptyset)\, r^2 sin\theta\, d\theta\, d\emptyset\, dr$

B) $\iiint_V F(r,\theta,z)\, r^2 sin\theta\, d\theta\, d\emptyset\, dr$

C) $\iiint_V F(r,\theta,\emptyset)\, sin\theta\, d\theta\, d\emptyset\, dr$

D) $\iiint_V F(r,\theta,\emptyset)\, r^2\, d\theta\, d\emptyset\, dr$

Answer : A

43. $\int_{-1}^{1} \int_{0}^{z} \int_{x-z}^{x+z} (x + y + z) dxdydz =$

A) 1 B) 0 C) -1 D) none of these

Answer : B

44. $\int_0^1 dy \int_{y^2}^{1} dx \int_0^{1-x} xdz$

A) $-\frac{4}{35}$ B) $\frac{4}{35}$ C) $\frac{2}{35}$ D) $\frac{-2}{35}$

Answer : B

45. $\iiint (x^2 y^2 + y^2 z^2 + z^2 x^2) dxdydz$ throughout the volume of the sphere

$x^2 + y^2 + z^2 = a^2$ is

A) $\frac{4}{35} a^7$ B) $\frac{-4}{35} a^7$ C) $\frac{4}{35} a^7 \pi$ D) $a^7 \pi$

Answer : C

46. $\iiint\limits_V \sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}}\, dxdydz$ throughout the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is

A) $\frac{\pi^2 abc}{4}$ B) $\frac{\pi abc}{4}$ C) $\frac{abc}{4}$ D) $\frac{\pi^2}{4}$

Answer : A

47. $\int_0^a \int_0^x \int_0^{\sqrt{x+y}} z\, dxdydz =$

A) $\frac{-a^2}{4}$ B) $\frac{a}{4}$ C) $\frac{a^3}{4}$ D) $\frac{a^2}{4}$

Answer : C

48. $\iiint (x+y+z)\, dxdydz$ over the positive octant of the sphere $x^2+y^2+z^2=a^2$ is

A) $\frac{-\pi a^4}{16}$ B) $\frac{3\pi a^4}{16}$ C) $\frac{3\pi a^2}{16}$ D) $\frac{\pi a^4}{6}$

Answer : B

49. The Dirichlet's theorem for 3 variables x, y, z is $\iiint x^{a-1}\, y^{b-1} z^{c-1} dxdydz$ =

A) $\frac{\Gamma a\, \Gamma b\, \Gamma c}{\Gamma 1+a-b+c}$ B) $\frac{\Gamma a\, \Gamma b\, \Gamma c}{\Gamma 1-a+b+c}$ C) $\frac{\Gamma a\, \Gamma b\, \Gamma c}{\Gamma 1+a+b+c}$ D) $\frac{\Gamma a\, \Gamma b\, \Gamma c}{\Gamma 1+a+b-c}$

Answer : C

50. The volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is given by

$V = 8abc \int_0^{\pi/2} \int_0^{\pi/2} \int_0^1 r^2 sin\theta\, d\theta\, d\emptyset\, dr =$

A) $\frac{2abc}{3}$ B) $\frac{abc\pi}{3}$ C) $\frac{2abc}{3}$ D) $\frac{4abc}{3}$

Answer : D

51. The volume of the tetrahedron bounded by the co-ordinate planes and the

$\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=1$ is

A) 2 B) 3 C) 4 D) 1

Answer : C

52. The volume enclosed by the cone $x^2+y^2=z^2$ and the paraboloid

$x^2+y^2=z$ given by $V=4\int_0^{\pi/2}\int_0^1(r-r^2)rd\theta dr=$

A) $\frac{\pi}{4}$ B) $\frac{\pi}{6}$ C) $\frac{\pi}{2}$ D) $-\frac{\pi}{4}$

Answer : B

53. The volume of the cylinder $x^2+y^2=2ax$ intercepted between paraboloid

$x^2+y^2=2az$ and XY plane is given by $V=\frac{1}{2a}2\int_0^{\pi/2}\int_0^{2acos\theta}r^2rd\theta dr=$

A) $\frac{3\pi}{4}$ B) $\frac{3\pi a^3}{4}$ c) $\frac{a^3\pi}{4}$ D) $\frac{3a^3}{4}$

Answer : B

Sinhgad College of Engineering,Vadgaon

Engineering Mathematics – II

Unit 1- First Order Ordinary Differential Equations

1) The differential equation $1+\frac{dy}{dx}-\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}}=0$ is of
   A) order 1 and degree 2
   B) order 2 and degree 3
   C) order 3 and degree 6
   D) order 3 and degree 3

   Ans. B)

2) The differential equation $1+\frac{dy}{dx}=\frac{y}{\frac{dy}{dx}}$ is of
   A) order 2 and degree 2
   B) order 1 and degree 1
   C) order 2 and degree 1
   D) order 1 and degree 2

   Ans. D)

3) The differential equation $(2x-y+3)dx+(y-2x-2)dy=0$ is of
   A) order 1 and degree 1
   B) order 1 and degree 2
   C) order 2 and degree 1
   D) order 2 and degree 2

   Ans. A)

4) The number of arbitrary constants in the general solution of ordinary differential equation is equal to
   A) the order of differential equation
   B) the degree of differential equation
   C) coefficient of highest order differential derivative
   D) none of these

   Ans. A)

5) The order of differential equation whose general solution is $y = C_1 + C_2 e^{-2x} + C_3 e^{3x} + C_4 e^{-3x}$, where C_1, C_2, C_3, C_4 are arbitrary constants is

A) 1
B) 3
C) 2
D) 4

Ans. D)

6) $y = Cx - C^2$,where C is arbitrary constant is the general solution of the differential equation

A) $\frac{dy}{dx} = C$
B) $\left(\frac{dy}{dx}\right)^2 + xy = 0$
C) $\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} - y = 0$
D) $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$

Ans. D)

7) By eliminating arbitrary constant A the differential equation whose general solution is $y = 4(x - A)^2$is

A) ${y_1}^2 + 16y = 0$
B) $y_1 - 2y = 0$
C) ${y_1}^2 - 16y = 0$
D) $y_1 - 8(x - A) = 0$

Ans. C)

8) The differential equation satisfied by general solution $y = A\cos x + B\sin x$ where A and B are arbitrary constants is,

A) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = B\sin x$
B) $\frac{d^2y}{dx^2} - y = 0$
C) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = A\cos x$
D) $\frac{d^2y}{dx^2} + y = 0$

Ans. D)

9) The differential equation satisfied by general solution $y = Ae^{x} + Be^{-x}$ where A and B are arbitrary constants,is

A) $y_2 - y = 0$
B) $y_2 + y = 0$
C) $y_2 + y = Ae^{x} - Be^{-x}$
D) $y_2 - y = 2Ae^{x}$

Ans. A)

10) The differential equation satisfied by general solution $y = Ax^2 + Bx + C$ where A, B, C are arbitrary constants is

A) $\frac{d^3y}{dx^3} = 0$
B) $\frac{d^2y}{dx^2} = 2A$
C) $\frac{d^3y}{dx^3} = A$
D) $\frac{d^4y}{dx^4} = 0$

Ans. A)

11) The solution of differential equation $\frac{dy}{dx} + y = 0$ is

A) $y = Ae^{-x}$
B) $y = Ae^{x}$
C) $x = Ae^{-y}$
D) $x = Ae^{y}$

Ans. A)

12) The solution of differential equation $\frac{dy}{dx} + \frac{1+y^2}{1+x^2} = 0$ is

A) $\tan^{-1} y - \tan^{-1} x = C$
B) $\tan^{-1} y + \tan^{-1} x = C$
C) $\tan y + \tan x = C$
D) $\cos y + \cos x = C$

Ans. B)

13) The solution of differential equation $sec^2x \tan y\, dx + sec^2y \tan x\, dy = 0$ is

A) $sec^2x \tan y = C$
B) $sec^2y \tan x = C$
C) $\tan x \tan y = C$
D) $sec^2xsec^2y = C$

Ans. C)

14) The solution of differential equation $e^x \cos y\, dx + (1 + e^x) \sin y\, dy = 0$ is

A) $(1 + e^x) = C \sec y$
B) $(1 + e^x) \sec y = C$
C) $\frac{\sec y}{(1+e^x)} = C$
D) $(1 + e^x) \cos y = C$

Ans. B)

15) The solution of differential equation $x(1 + y^2)dx + y(1 + x^2)dy = 0$ is

A) $(1 - x^2)(1 + y^2) = C$
B) $\tan^{-1} x + \tan^{-1} y = C$
C) $(1 + x^2) = C(1 + y^2)$
D) $(1 + x^2)(1 + y^2) = C$

Ans. D)

16) The necessary and sufficient condition that the differential equation $M(x, y)dx + N(x, y)dy = 0$ be exact is

A) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$
B) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$
C) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$
D) $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 1$

Ans . A)

17) If homogeneous differential equation $M(x,y)dx + N(x,y)dy = 0$ is not exact then the integrating factor is

A) $\frac{1}{My+Nx}; My + Nx \neq 0$
B) $\frac{1}{Mx-Ny}; Mx - Ny \neq 0$
C) $\frac{1}{Mx+Ny}; Mx + Ny \neq 0$
D) $\frac{1}{My-Nx}; My - Nx \neq 0$

Ans. C)

18) If the differential equation $M(x,y)dx + N(x,y)dy = 0$ is not exact and it can be written as $yf_1(xy)dx + xf_2(xy)dy = 0$ then the integrating factor is

A) $\frac{1}{My+Nx}; My + Nx \neq 0$
B) $\frac{1}{Mx-Ny}; Mx - Ny \neq 0$
C) $\frac{1}{Mx+Ny}; Mx + Ny \neq 0$
D) $\frac{1}{My-Nx}; My - Nx \neq 0$

Ans. B)

19) If the differential equation $M(x,y)dx + N(x,y)dy = 0$ is not exact and $\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x)$ then the integrating factor is

A) $e^{f(x)}$
B) $e^{\int f(x)dy}$
C) $f(x)$
D) $e^{\int f(x)dx}$

Ans. D)

20) If the differential equation $M(x,y)dx + N(x,y)dy = 0$ is not exact and $\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = f(y)$ then the integrating factor is

A) $e^{\int f(y)dy}$
B) $e^{\int f(y)dx}$
C) $f(y)$
D) $e^{f(y)}$

Ans. A)

21) The value of λ for which the differential equation $(xy^2 + \lambda x^2 y)dx + (x^3 + x^2 y)dy = 0$ is exact is

A) -3
B) 2
C) 3
D) 1

Ans. C)

22) The differential equation $\left(\frac{2x}{y^3}\right)dx + \left(\frac{y^2+ax^2}{y^4}\right)dy = 0$ is exact if

A) a= -3
B) a= 3
C) a= -2
D) a= 6

Ans. A)

23) Integrating factor of homogeneous differential equation $(xy - 2y^2)dx + (3xy - x^2)dy = 0$ is

A) $\frac{1}{xy}$
B) $\frac{1}{x^2y^2}$
C) $\frac{1}{x^2y}$
D) $\frac{1}{xy^2}$

Ans. D)

24) Integrating factor for differential equation $(x^2y^2 + xy + 1)ydx + (x^2y^2 - xy + 1)xdy = 0$ is

A) $\frac{1}{2x^3y^3}$
B) $\frac{1}{xy}$
C) $\frac{1}{2x^2y^2}$
D) $\frac{1}{x^2y}$

Ans. C)

25) Integrating factor for differential equation $(x^2 + y^2 + 1)dx - 2xydy = 0$ is

A) $\frac{1}{x}$
B) $\frac{1}{x^3}$
C) $\frac{1}{x^2}$
D) $\frac{1}{xy}$

Ans. C)

26) Integrating factor for differential equation $y \log y \, dx + (x - \log y)dy = 0$ is

A) $\frac{1}{x}$
B) $\frac{1}{y}$
C) $\frac{1}{x^2}$
D) $\frac{1}{y^2}$

Ans. B)

27) Solution of non-exact diff. equation $(3xy^2 - y^3)dx + (xy^2 - 2x^2y)dy = 0$ with integrating factor $\frac{1}{x^2y^2}$ is

A) $3\log x - \frac{2y}{x^3} - 2\log y = C$
B) $3\log x + \frac{y}{x} - 2\log y = C$
C) $3\log x + \frac{y}{x} = C$
D) $\log x - \frac{y}{x} + 2\log y = C$

Ans. B)

28) Solution of non-exact diff. equation $(1+xy)ydx+(1-xy)xdy=0$ with integrating factor $\frac{1}{x^2y^2}$ is

A) $\frac{2}{xy} - \log\left(\frac{x}{y}\right) = C$
B) $-\frac{1}{xy} + \log\left(\frac{y}{x}\right) = C$
C) $-\frac{1}{xy} + \log\left(\frac{x}{y}\right) = C$
D) $-\frac{2}{x^3y} + \log\left(\frac{x}{y}\right) = C$

Ans. C)

29) The differential equation $(3+by\cos x)dx+(2\sin x-4y^3)dy=0$ is exact if

A) b= 4
B) b= 3
C) b= 0
D) b= 2

Ans. D)

30) Integrating factor for differential equation $(x^2+y^2+x)dx+(xy)dy=0$ is

A) $\frac{1}{x}$
B) $\frac{1}{x^2}$
C) x^2
D) x

Ans. D)

31) The differential equation of the form $\frac{dy}{dx}+Py=Q$ where P and Q are functions of x or constants, is

A) Exact differential equation
B) Linear differential equation in y
C) Linear differential equation in x
D) Non-homogeneous differential equation

Ans. B)

32) The differential equation of the form $\frac{dx}{dy} + Px = Q$ where P and Q are functions of y or constants, is

A) Exact differential equation
B) Linear differential equation in y
C) Linear differential equation in x
D) Non-homogeneous differential equation

Ans. C)

33) Integrating factor of linear differential equation $\frac{dy}{dx} + Py = Q$ where P and Q are functions of x or constants, is

A) $e^{\int Pdy}$
B) $e^{\int Qdy}$
C) $e^{\int Qdx}$
D) $e^{\int Pdx}$

Ans. D)

34) Integrating factor of linear differential equation $\frac{dx}{dy} + Px = Q$ where P and Q are functions of x or constants, is

A) $e^{\int Pdy}$
B) e^{Pdx}
C) $e^{\int Qdx}$
D) $e^{\int Qdy}$

Ans. A)

35) The differential equation of the form $\frac{dy}{dx} + Py = Qy^n, n \neq 1$ where P and Q are functions of x or constants, is

A) Bernoulli's differential equation
B) Exact differential equation
C) Symmetric differential equation
D) Linear differential equation

Ans. A)

36) The general solution of linear differential equation $\frac{dy}{dx} + Py = Q$ where P and Q are functions of x or constants, is

A) $xe^{\int Pdy} = \int Qe^{\int Pdy}\, dy + C$
B) $y = \int Qe^{\int Pdx}\, dx + C$
C) $ye^{\int Pdx} = \int Q\, dx + C$
D) $ye^{\int Pdx} = \int Qe^{\int Pdx}\, dx + C$

Ans. D)

37) The differential equation of the form $f'(x)\frac{dx}{dy} + Pf(x) = Q$ where P and Q are functions of y or constants,can be reduced to linear differential equation by the substitution

A) $f'(x) = u$
B) $f(x) = u$
C) $P = u$
D) $Q = u$

Ans. B)

38) Integrating factor of linear differential equation $\frac{dy}{dx} + xy = x^3$ is

A) $e^{\log x}$
B) e^x
C) x^2
D) $e^{\frac{x^2}{2}}$

Ans. D)

39) The differential equation $\frac{dx}{dy} + \frac{x}{1+y^2} = y^2$ has integrating factor

A) $e^{\frac{1}{1+y^2}}$
B) $e^{\tan^{-1} x}$
C) $e^{\frac{1}{1+x^2}}$
D) $e^{\tan^{-1} y}$

Ans. D)

40) The differential equation $\cos x \frac{dy}{dx} + y = \sin x$ has integrating factor

A) $e^{\sec x}$
B) $(cosecx - cotx)$
C) $(\sec x + \tan x)$
D) $(\sec x - \tan x)$

Ans. C)

41) The Bernoulli's differential equation $\frac{dy}{dx} - y \tan x = y^4 \sec x$ reduces to linear differential equation

A) $\frac{du}{dx} + (3 \tan x)u = -3 \sec x \ where \ y^{-3} = u$
B) $\frac{du}{dx} - (3 \tan x)u = 3 \sec x \ where \ y^{-3} = u$
C) $\frac{du}{dx} + (\tan x)u = -\sec x \ where \ y^{-3} = u$
D) None of these

Ans. A)

42) The general solution $\frac{dy}{dx} + \frac{3}{x}y = x^2$ with integrating factor x^3 is...

A) $yx^3 = \frac{x^6}{6} + C$
B) $yx^3 = \frac{x^2}{6} + C$
C) $yx^3 = \log x + C$
D) None of these

Ans. A)

43) The general solution of $\frac{dx}{dy} + \frac{1}{1+y^2}x = \frac{e^{-\tan^{-1} y}}{1+y^2}$ with integrating factor $e^{\tan^{-1} y}$

A) $xe^{\tan^{-1} y} = \tan^{-1} y + C$
B) $ye^{\tan^{-1} y} = \tan^{-1} y + C$
C) $e^{\tan^{-1} y} = \tan^{-1} y + C$
D) $e^{-\tan^{-1} y} = \tan^{-1} y + C$

Ans. A)

44) The differential equation $\sin y \frac{dy}{dx} - 2\cos x \cos y = -\cos x\, sin^2 x$ reduces to linear differential equation...

A) $\frac{du}{dx} + (\cos x)u = \cos x\, sin^2 x\ where\ \cos y = u$
B) $\frac{du}{dx} - (2\cos x)u = \cos x\, sin^2 x\ where\ \cos y = u$
C) $\frac{du}{dx} + (2\cos x)u = \cos x\, sin^2 x\ where\ \cos y = u$
D) $\frac{du}{dx} - (\cos x)u = \cos x\, sin^2 x\ where\ \cos y = u$

Ans. C)

45) The general solution of $\frac{dy}{dx} + \frac{2}{x}y = \frac{1}{x^3}$ with integrating factor x^2 is...

A) $yx^2 = \frac{x^2}{2} + C$
B) $yx^2 = \log x + C$
C) $yx^2 = \frac{x^6}{6} + C$
D) None of these

Ans. B)

46) The differential equation $\frac{dy}{dx} + y\cot x = \sin 2x$ has integrating factor

A) $\cos x$
B) $e^{\cot x}$
C) $\sin x$
D) $\sec x$

Ans. C)

47) The solution of differential equation $xdy - ydx = 0$ is

A) $y = x + c$
B) $x^2 - y^2 = c$
C) $xy = c$
D) $y = cx$

Ans. D)

48) The solution of differential equation $(e^x + 1)ydy = (y + 1)e^x dx$ is

A) $y - \log(1 - y) = \log(e^x - 1) + \log C$
B) $y - \log(1 + y) = \log(e^x + 1) + \log C$
C) $y + \log(1 - y) = \log(e^x + 1) + \log C$
D) $y - \log(1 + y) = \log(e^x - 1) + \log C$

Ans. B)

49) Integrating factor of homogeneous differential equation

$(x^2 - 3xy + 2y^2)dx + (3x^2 - 2xy)dy = 0$ is

A) $\frac{1}{xy}$
B) $\frac{1}{x^3}$
C) $\frac{1}{x^2 y}$
D) $\frac{1}{x^2}$

Ans. B)

50) The differential equation $\frac{dy}{dx} + \sqrt{x}y = x^3$ has integrating factor

A) $e^{\frac{2}{3}x\sqrt{x}}$
B) $e^{\frac{1}{3}x\sqrt{x}}$
C) $e^{\sqrt{x}}$
D) e^{-x}

Ans. A)

Sinhgad College of Engineering,Vadgaon

Engineering Mathematics – II

Unit 4- Curve Tracing

1. If the powers of x in the Cartesian equation are even everywhere then the curve is symmetrical about

   A) x-axis B) y-axis C) both x and y axes D) line $y = x$

Ans.-B

2. If the powers of x and y both in the Cartesian equation are even everywhere then the curve is symmetrical about

   A) x-axis only B) y-axis only C) both x and y axes D) line $y = x$

Ans.- C

3. On replacing $x \, and \, y$ by $-x \, and - y$ respectively if the Cartesian equation remains unchanged then the curve is symmetrical about

   A) line $y = x$ B) y-axis C) both x and y axes D) opposite quadrants

Ans.- D

4. If x and y are interchanged and Cartesian equation remains unchanged then the curve is symmetrical about

   A) both x and y axes B) line $y = -x$ C) line $y = x$ D) opposite quadrants

Ans. - C

5. If the curve passes through origin then tangents at origin to the Cartesian curve can be obtained by equating to zero

   A) lowest degree term in the equation

   B) highest degree term in the equation

   C) coefficient of lowest degree term in the equation

   D) coefficient of highest degree term in the equation

Ans.- A

6. In Cartesian equation the points where $\frac{dy}{dx} = 0$, tangent to the curve at those points will be

   A) Parallel to y-axis B) Parallel to x-axis

   C) Parallel to $y = x$ D) Parallel to $y = -x$

Ans.- B

7. In Cartesian equation the points where $\frac{dy}{dx} = \infty$,tangent to the curve at those points will be

A) Parallel to $y = -x$ B) Parallel to x-axis

C) Parallel to y-axis D) Parallel to $y = x$

Ans.- C

8. If the powers of y in the Cartesian equation are even everywhere then the curve is symmetrical about

A) x-axis B) y-axis C) both $x$and y axes D) line $y = x$

Ans. - A

9. The asymptotes to the Cartesian curve parallel to x-axis if exists is obtained by equating to zero

A) Coefficient of highest degree term in y B) Lowest degree term in the equation

C) Coefficient of highest degree term in x D) Highest degree term in the equation

Ans. - C

10. The asymptotes to the Cartesian curve parallel to y-axis if exists is obtained by equating to zero

A) Coefficient of highest degree term in y B) Lowest degree term in the equation

C) Coefficient of highest degree term in x D) Highest degree term in the equation

Ans. – A

11. The curve represented by the equation $x^2y^2 = x^2 + 1$ is symmetrical about

A) $y = -x$ B) $x - axis\ only$ C) both x and y axes D) $y = x$

Ans. -C

12. The asymptote parallel to y-axis to the curve $xy^2 = a^2(a - x)$ is

A) $y = 0$ B) $x = 0$ C) $x = a$ D) $x = -a$

Ans. - B

13. The curve represented by the equation $y^2(2a - x) = x^3$ is

A) Symmetrical about y-axis and passing through origin

B) Symmetrical about x-axis and not passing through origin

C) Symmetrical about y-axis and passing through $(2a, 0)$

D) Symmetrical about x-axis and passing through origin

Ans. – D

14. The equation of tangents to the curve at origin if exist, represented by the equation

$3ay^2 = x(x-a)^2$ is

A) $x = a$ B) $x = 0\ and\ y = 0$ C) $x = 0$ D) $y = 0$

Ans. - C

15. The equation of asymptotes parallel to y-axis to the curve represented by the equation

$x^2y^2 = a^2(y^2 - x^2)$ is

A) $x = a\,, x = -a$ B) $y = a\,, y = -a$ C) $y = x\,, y = -x$ D) $x = 0\,, y = 0$

Ans. – A

16. The region of absence for the curve represented by the equation $x^2 = \frac{4a^2(2a-y)}{y}$ is

A) $y < 0\ and\ y > 2a$ B) $y > 0\ and\ y < 2a$

C) $y > 0\ and\ y > 2a$ D) $y < 0\ and\ y < 2a$

Ans. - A

17. The region of absence for the curve represented by the equation $xy^2 = a^2(a-x)$is

A) $x > 0\ and\ x < a$ B) $x < 0\ and\ x < a$ C) $x < 0\ and\ x > a$ D) $x > 0\ and\ x > a$

Ans. - C

18. The curve represented by the equation $r^2 = a^2cos2\theta$ is…

A) symmetric about θ =$\frac{\pi}{2}$ and not passing through pole

B) symmetric about θ =$\frac{\pi}{4}$ and not passing through pole

C) symmetric about initial line and pole

D) symmetric about θ =$\frac{\pi}{4}$ and passing through pole

Ans.-C

19. The curve represented by the equation $r^2 = a^2sin2\theta$ is…

A) symmetric about initial line and passing through pole
B) symmetric about initial line and pole
C) symmetric about θ =$\frac{\pi}{4}$ and not passing through pole
D) symmetric about θ =$\frac{\pi}{4}$ and passing through pole

Ans.- D

20. The curve represented by the equation $r = \frac{2a}{1+cos\theta}$ is

A) symmetric about initial line and passing through pole
B) symmetric about initial line and not passing through pole
C) symmetric about θ =$\frac{\pi}{4}$ and not passing through pole
D) symmetric about θ =$\frac{\pi}{4}$ and passing through pole

Ans.- B

21. If the polar equation to the curve remains unchanged by changing θ to – θ then the curve is symmetric about …

A) line θ = $\frac{\pi}{4}$ B) pole C) line θ = $\frac{\pi}{2}$ D) initial line θ = 0

Ans.- D

22. If the polar equation to the curve remains unchanged by changing θ to π– θ then the curve is symmetric about …

A) initial line θ = 0 B) pole
C) line passing through pole and perpendicular to the initial line
D) line θ = $\frac{\pi}{4}$

Ans. -C

23. Pole will lie on the curve if for some value of θ

A) r becomes zero B) r becomes infinite

C) r $>$ 0 D) r $<$ 0

Ans.- A

24. The tangents to the polar curve at pole if exist can be obtained by putting in the polar

A) θ = 0 B) θ = π C) r = 0 D) r = a, a$>$ 0

Ans.- C

25. If the polar equation to the curve remains unchanged by changing $r\ to -r$ then the curve is symmetrical about

A) line $\theta = \frac{\pi}{4}$ B) pole C) line $\theta = \frac{\pi}{2}$ D) Initial line $\theta = 0$

Ans. - B

26. For the polar curve, angle φ between radius vector and tangent line is obtained by the formula

A) $cos\varphi = r\frac{d\theta}{dr}$ B) $tan\varphi = r\frac{d\theta}{dr}$ C) $tan\varphi = r\frac{dr}{d\theta}$ D) $sin\varphi = r\frac{d\theta}{dr}$

Ans. -B

27. The curve represented by equation $r = 2asin\theta$ is symmetrical about

A) Pole B) Initial line $\theta = 0$ C) Line $\theta = \frac{\pi}{4}$ D) Line $\theta = \frac{\pi}{2}$

Ans. – D

28. The curve represented by equation $r = a(1 + cos\theta)$ is symmetrical about

A) Initial line and passing through pole B) Initial line and not passing through pole

C) $\theta = \frac{\pi}{2}$ and passing through pole D) $\theta = \frac{\pi}{4}$ and passing through pole

Ans. – A

29. For the rose curve $r = a\cos n\theta$ and $r = a\sin n\theta$ if n is odd then the curve consists of

A) $2n$ equal loops B) $(n+1)$ equal loops c) $(n-1)$ equal loops D) n equal loops

Ans.- D

30. For the rose curve $r = a\cos n\theta$ and $r = a\sin n\theta$ if n is even then the curve consists of

A) $2n$ equal loops B) $(n+1)$ equal loops c) $(n-1)$ equal loops D) n equal loops

Ans.- A

31. The tangents at pole to the curve $r = a\sin 3\theta$ are

A) $\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3} \ldots$ B) $\theta = \frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6} \ldots$

C) $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \ldots$ D) $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots$

Ans.- A

32. The tangents at pole to the curve $r = a\cos 2\theta$ are

A) $\theta = 0, \pi, 2\pi, 3\pi \ldots$ B) $\theta = \frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6} \ldots$

C) $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \ldots$ D) $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots$

Ans. – C

33. The Cartesian parametric curve $x = f\,(t),\ y = g(t)$ is symmetrical about X-axis if……..

A) f(t) is even and g(t) is odd B) f(t) is odd and g(t) is even

C) f(t) is even and g(t) is even D) f(t) is odd and g(t) is odd

Ans.- A

34. The Cartesian parametric curve $x = f\,(t),\ y = g(t)$ is symmetrical about Y-axis if……..

A) f(t) is even and g(t) is odd B) f(t) is odd and g(t) is even

C) f(t) is even and g(t) is even D) f(t) is odd and g(t) is odd

Ans.- B

35. The curve represented by the equation $x = at^2, y = 2at$ is symmetrical about …..

A) Y-axis B) X-axis C) both X and Y axis D) opposite quadrants

Ans.- B

36. The Cartesian parametric curve $x = f\ (t),\ y = g(t)$ is symmetrical in opposite quadrants if……..

A) f(t) is even and g(t) is odd B) f(t) is odd and g(t) is even

C) f(t) is even and g(t) is even D) f(t) is odd and g(t) is odd

Ans. -D

37. The curve represented by the equation $x = t, y = t^3$ is symmetrical about …..

A) Y-axis B) X-axis C) both X and Y axis D) opposite quadrants

Ans.- D

38. The curve represented by the equation $x = a(\theta + sin\theta), y = a(1 + cos\theta)$ where θ is parameter is symmetrical about …..

A) Y-axis B) X-axis C) both X and Y axis D) opposite quadrants

Ans.-A

39. Formula for measuring the arc length AB where A(x_1, y_1), B(x_2, y_2) are any two points on the curve $y = f(x)$ is

A) $\int_{x_1}^{x_2} \sqrt{dx}$ B) $\int_{x_1}^{x_2} \sqrt{1 + (\frac{dy}{dx})^2}\, dx$ C) $\int_{x_1}^{x_2} \sqrt{1 + (\frac{dx}{dy})^2}\, dx$ D) $\int_{x_1}^{x_2} \sqrt{1 - (\frac{dy}{dx})^2}\, dx$

Ans.- B

40. Formula for measuring the arc length AB where A(x_1, y_1), B(x_2, y_2) are any two points on the curve $x = g(y)$ is

A) $\int_{y_1}^{y_2} \sqrt{dx}$ B) $\int_{y_1}^{y_2} \sqrt{1 + (\frac{dy}{dx})^2}\, dy$ C) $\int_{y_1}^{y_2} \sqrt{1 + (\frac{dx}{dy})^2}\, dy$ D) $\int_{y_1}^{y_2} \sqrt{1 - (\frac{dy}{dx})^2}\, dy$

Ans.- C

41. Formula for measuring the arc length AB where A(r_1, θ_1), B(r_2, θ_2) are any two points on the curve $r = f(\theta)$ is

A) $\int_{\theta_1}^{\theta_2} \sqrt{1 + (\frac{dr}{d\theta})^2} d\theta$ B) $\int_{\theta_1}^{\theta_2} \sqrt{1 + r^2(\frac{d\theta}{dr})^2} d\theta$

C) $\int_{\theta_1}^{\theta_2} \sqrt{r^2 + (\frac{dr}{d\theta})^2} d\theta$ D) $\int_{\theta_1}^{\theta_2} \sqrt{r^2 + r^2(\frac{dr}{d\theta})^2} dr$

Ans.- C

42. Formula for measuring the arc length AB where A(r_1, θ_1), B(r_2, θ_2) are any two points on the curve θ= $f(r)$ is θ

A)$\int_{r_1}^{r_2} \sqrt{r^2 + (\frac{dr}{d\theta})^2} dr$ B) $\int_{r_1}^{r_2} \sqrt{1 + (\frac{d\theta}{dr})^2} dr$

$C)$ $\int_{r_1}^{r_2} \sqrt{1 - r^2(\frac{d\theta}{dr})^2} dr$ D) $\int_{r_1}^{r_2} \sqrt{1 + r^2(\frac{d\theta}{dr})^2} dr$

Ans.- D

43. Formula for measuring the arc length AB where A, B are any two points on the parametric curve $x = f_1(t), y = f_2(t)$, corresponding to parameters t_1, t_2 respectively is…….

A) $\int_{t_1}^{t_2} \sqrt{(\frac{dx}{dt})^2 - (\frac{dy}{dt})^2} dt$ B) $\int_{t_1}^{t_2} \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} dt$

C) $\int_{t_1}^{t_2} \left[(\frac{dx}{dt})^2 - \left(\frac{dy}{dt}\right)^2 \right] dt$ $D)$ $\int_{t_1}^{t_2} \sqrt{(x^2(t) - y^2(t)} dt$

Ans. - B

44. The arc length AB where A(a, 0), B(0, a) are any two points on the circle $x^2 + y^2 = a^2$ using $1 + (\frac{dy}{dx})^2 = \frac{a^2}{a^2 - x^2}$ is …

A) $\frac{\pi a}{2}$ B) $a \, loga$ C) $\frac{\pi a}{4}$ D) a

Ans.- A

45. The length of arc of the curve $x = e^{\theta} cos\theta$, $y = e^{\theta} sin\theta$

from θ = 0 to θ =$\frac{\pi}{2}$ using $(\frac{dx}{d\theta})^2 + (\frac{dy}{d\theta})^2$ = $2e^{2\theta}$ is……

A) $\sqrt{2} e^{\frac{\pi}{2}}$

B) $\sqrt{2}(e^{\frac{\pi}{2}} + 1)$

C) $\sqrt{2}(e^{\frac{\pi}{2}} - 1)$

D) $(e^{\frac{\pi}{2}} + 1)$

Ans.- C

46. Integral for calculating the length of upper arc of loop of the curve $9y^2 = (x + 7)(x + 4)^2$ is………

A) $\int_{-7}^{-4} \sqrt{1 + (\frac{dy}{dx})^2} \, dx$

B) $\int_{7}^{4} \sqrt{1 + (\frac{dy}{dx})^2} \, dx$

C) $\int_{-7}^{0} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$

D) $\int_{0}^{-4} \sqrt{1 + (\frac{dy}{dx})^2} \, dx$

Ans.- A

47. Integral for calculating the length of arc of parabola $y^2 = 4x$ cut off by the line $3y = 8x$ is

A. $\int_0^{\frac{16}{9}} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

B. $\int_0^{\frac{9}{16}} \sqrt{1 + (\frac{dy}{dx})^2}\, dx$

C. $\int_0^{\frac{8}{3}} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

D. $\int_0^{\frac{3}{8}} \sqrt{1 + (\frac{dy}{dx})^2}\, dx$

Ans. - B

48. The length of arc of upper part of loop of the curve

$3y^2 = x(x-1)^2$ from (0,0) to (1 , 0) using

$1 + (\frac{dy}{dx})^2 = \frac{(3x+1)^2}{12x}$ is....

A) $\frac{4}{\sqrt{3}}$

B) $\frac{1}{\sqrt{3}}$

C) $\frac{2}{\sqrt{3}}$

D) $\sqrt{3}$

Ans. - C

49. Integral for calculating the length of the upper arc of the loop of the curve $x = t^2, y = t(1 - \frac{t^2}{3})$ is.......

A) $\int_0^9 \sqrt{(\frac{dx}{dt})^2 - (\frac{dy}{dt})^2} dt$

B) $\int_0^{\sqrt{3}} \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} dt$

C) $\int_0^1 \left[(\frac{dx}{dt})^2 - \left(\frac{dy}{dt}\right)^2 \right] dt$

D) $\int_0^{\sqrt{3}} \sqrt{(x^2(t) - y^2(t)} dt$

Ans. - B

50. Integral for calculating the length of arc of Astroid $x = acos^3\theta, x = asin^3\theta$ in the first quadrant between two consecutive cusps is …….

A) $\int_0^{\frac{\pi}{4}} \sqrt{(\frac{dx}{d\theta})^2 + (\frac{dy}{d\theta})^2} d\theta$

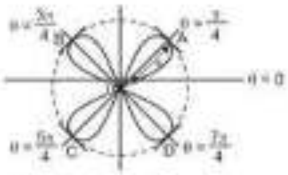
B) $\int_0^{\frac{\pi}{2}} \sqrt{(\frac{dx}{d\theta})^2 + (\frac{dy}{d\theta})^2} d\theta$

C) $\int_0^{\frac{\pi}{3}} \sqrt{(\frac{dx}{d\theta})^2 + (\frac{dy}{d\theta})^2} d\theta$

D) $\int_0^{\frac{\pi}{6}} \sqrt{(\frac{dx}{d\theta})^2 + (\frac{dy}{d\theta})^2} d\theta$

Ans. – B

The following figure represents the curve whose equation is …. (2)



(A) $r = a \cos 3\theta$

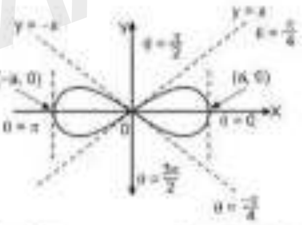
(B) $r = a \sin 2\theta$

(C) $r = a \sin 3\theta$

(D) $r = a (1 + \cos \theta)$

Ans.- B

. The following figure represents the curve whose equation is …. (2)



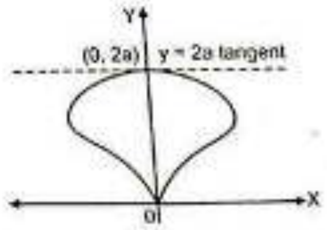
(A) $r^2 = a^2 \cos 2\theta$ (B) $r^2 = a^2 \sin 2\theta$

(C) $r = a \cos 2\theta$ (D) $r = a (1 + \cos \theta)$

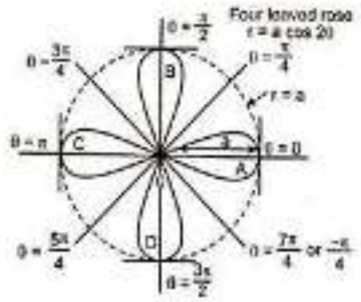
Ans.- A

The equation $r^2 = a^2 \cos 2\theta$ represents the curve ...

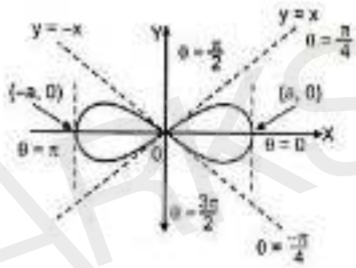
(A)



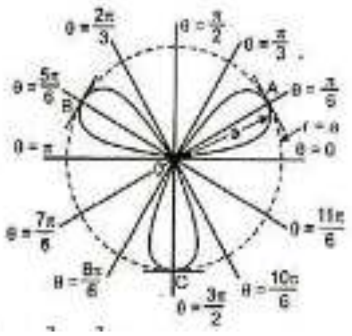
(B)



(C)



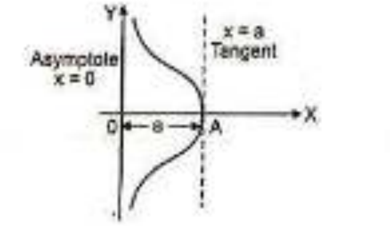
(D)



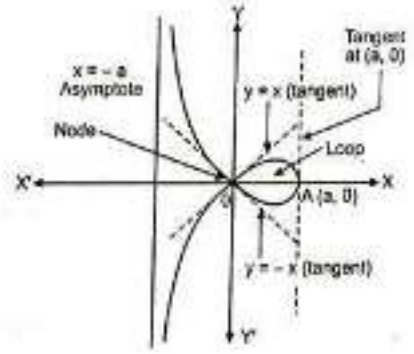
Ans.- C

The equation $xy^2 = a^2 (a - x)$ represents the curve ...

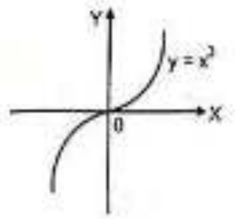
(A)

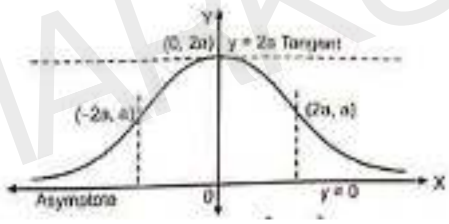


(B)



C)

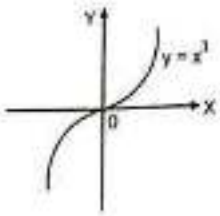




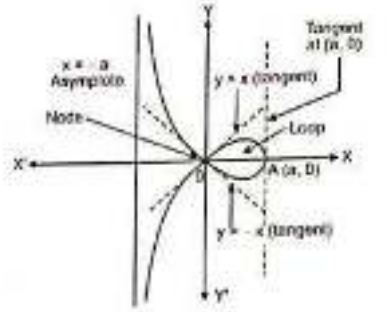
Ans.- A

The equation $x\,(x^2 + y^2) = a\,(x^2 - y^2)$, $a > 0$ represents the curve ... (2)

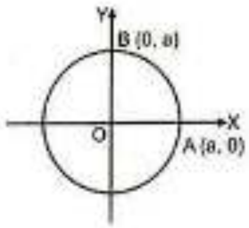
(A)



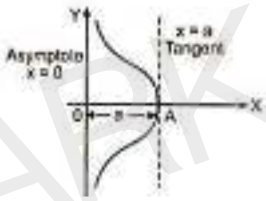
(B)



(C)



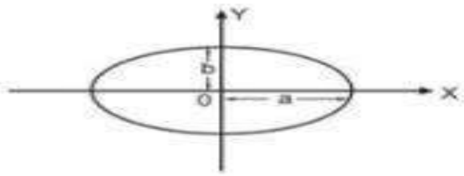
(D)



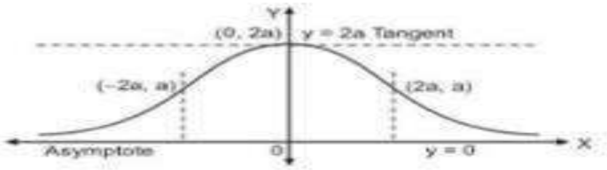
Ans.- B

The equation $a^2x^2 = y^3(2a - y)$, $a > 0$ represents the curve ... (2

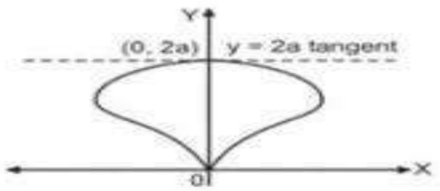
(A)



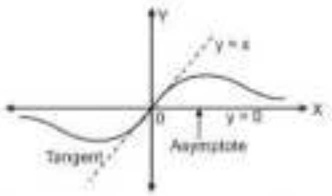
(B)



(C)



(D)



Ans.- C

Sinhgad College of Engineering,Vadgaon

Engineering Mathematics – II

Unit 5- Solid Geometry

1) Equation of sphere whose Centre at (2, -3,1) and radius is 5 is

A) $x^2 + y^2 + z^2 - 4x + 6y - 2z - 11 = 0$

B) $x^2 + y^2 + z^2 - 8x + 12y - 4z - 11 = 0$

C) $x^2 + y^2 + z^2 - 4x + 4y - 6z - 13 = 0$

D) $x^2 + y^2 + z^2 - 8x + 12y - 4z - 13 = 0$

Ans- A

2)Equation of spere with Centre at (2, -2,3) and passing through (7,-3,5) is

A) $x^2 + y^2 + z^2 - 4x + 6y - 2z - 11 = 0$

B) $x^2 + y^2 + z^2 - 8x + 12y - 4z - 11 = 0$

C) $x^2 + y^2 + z^2 - 4x + 4y - 6z - 13 = 0$

D) $x^2 + y^2 + z^2 - 8x + 12y - 4z - 13 = 0$

Ans-C

3)Equation of sphere passing through origin and making equal intercepts of unit length on axes is

A) $x^2 + y^2 + z^2 - x - y - z = 0$

B) $x^2 + y^2 + z^2 = 0$

C) $x^2 + y^2 + z^2 - x + y - z = 0$

D) $x^2 + y^2 + z^2 - x - y - z - 1 = 0$

Ans-A

4)equation of spere passing through (1,3,1), (2,-1,1),(1,2,0),(1,-1,1) is

A) $x^2+y^2+z^2-4x+2y-2z+8=0$

B) $x^2+y^2+z^2-4x+2y-2z-8=0$

C) $x^2+y^2+z^2+4x+2y-2z+8=0$

D) $x^2+y^2+z^2-4x+2y+2z+8=0$

Ans-B

5)spherical coordinates of a point (3,4,5) are-

A)3,4,5 B)$5\sqrt{2}, 45^0, 53.13^0$

C) $5, 45^0, 53^0$ D) $\sqrt{3}, 126.26^0, 135^0$

Ans-B

6) spherical coordinates of a point (-1,1, -1) are-

A) $5\sqrt{3}, 126.26^0, 135^0$ B)$5\sqrt{2}, 45^0, 53.13^0$

C) $5, 45^0, 53^0$ D) $\sqrt{3}, 126.26^0, 135^0$

Ans-D

7) $x^2+y^2+z^2-4x+6y-2z-11=0$ is equation of sphere then Centre and radius are

A) (-2, 3, -1),5 B) (2, -3,1),25 C) (-2, 3, -1),25 D) (2, -3,1),5

Ans-D

8) two spheres $x^2+y^2+z^2-2x+4y-4z=0$ and

$x^2+y^2+z^2+10x+2z+10=0$ touch each other then the point of contact is

A) $\left(\frac{-11}{7},\frac{-8}{7},\frac{-5}{7}\right)$ B) $\left(\frac{11}{7},\frac{8}{7},\frac{5}{7}\right)$ C) $\left(\frac{-11}{7},\frac{-8}{7},\frac{5}{7}\right)$ D) $\left(\frac{-11}{7},\frac{8}{7},\frac{5}{7}\right)$

Ans-c

9) the equation of the sphere tangential to the plane $x - 2y - 2z = 7$ at $(3, -1, -1)$ and passing through the point $(1,1,-3)$ is

A) $x^2 + y^2 + z^2 - 10y - 10z - 31 = 0$

B) $x^2 + y^2 + z^2 - 10y - 10z + 31 = 0$

C) $x^2 + y^2 + z^2 + 10y - 10z - 31 = 0$

D) $x^2 + y^2 + z^2 + 10y + 10z + 31 = 0$

Ans-A

10) the equation of the sphere which passed through $(3,1,2)$ and meets XOY plane in a circle of radius 3 units with the center at $(1, -2,0)$ is

A) $x^2 + y^2 + z^2 + 2x + 4y - 4z - 4 = 0$

B) $x^2 + y^2 + z^2 - 2x + 4y - 4z - 4 = 0$

C) $x^2 + y^2 + z^2 - 2x - 4y - 4z - 4 = 0$

D) $x^2 + y^2 + z^2 - 2x + 4y + 4z + 4 = 0$

Ans-B

11) Find the equation of the sphere passing through the circle $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$, and point $(1,2,3)$ is

A) $(x^2 + y^2 + z^2) - 2x - 3y - 4z - 22 = 0$

B) $(x^2 + y^2 + z^2) - 2x - 3y - 4z - 22 = 0$

C) $3(x^2 + y^2 + z^2) - 2x - 3y - 4z - 22 = 0$

D) $(x^2 + y^2 + z^2) - 2x - 3y - 4z - 22 = 0$

Ans-C

12) the equation of the sphere through the circle

$x^2+y^2+z^2-x+9y-5z-5=0,\ x^2+y^2+z^2+10y-4z-8=0$ as a great circle is

A) $x^2+y^2+z^2+4x+6y-8z+4=0$

B) $x^2+y^2+z^2-4x+6y-8z-4=0$

C) $x^2+y^2+z^2-4x+6y+8z+4=0$

D) $x^2+y^2+z^2-4x+6y-8z+4=0$

Ans-D

13) the equation of the sphere passing through the circle $3x-4y+5z-15=0$, and $x^2+y^2+z^2-2x+3y-4z+6=0$, and cuts the sphere $x^2+y^2+z^2+2x+4y-6z+11=0$ orthogonally is

A) $5(x^2+y^2+z^2)-13x+19y-25z+45=0$

B) $5(x^2+y^2+z^2)+13x+19y-25z+45=0$

C) $5(x^2+y^2+z^2)-13x-19y-25z+45=0$

D) $5(x^2+y^2+z^2)-13x+19y-25z-45=0$

Ans-A

14) Consider two sphere

$S_1 \equiv x^2+y^2+z^2+2u_1x+2v_1y+2w_1z+d_1=0$

$S_2 \equiv x^2+y^2+z^2+2u_2x+2v_2y+2w_2z+d_2=0$

Then condition for orthogonality is

A) $2u_1u_2+2v_1v_2+2w_1w_2=d_1-d_2$
B) $2u_1u_2-2v_1v_2+2w_1w_2=d_1+d_2$
C) $2u_1u_2-2v_1v_2-2w_1w_2=d_1-d_2$
D) $2u_1u_2+2v_1v_2+2w_1w_2=d_1+d_2$
Ans-D

15) The center of the circle, which is an intersection of the sphere,

$x^2+y^2+z^2-2y-4z-11=0$ by the plane $x+2\mathrm{y}+2\mathrm{z}=15$ is

A) (1,3,4) B) (1,2,3) C) (0,0,0) D) (1,3,2)

Ans-A

16)The Equation of Right circular cone whose vertex is at (α,β,γ), semi-vertical angle θ and the line Axis $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$ is

A) $cos\theta=\frac{l(x+\alpha)+m(y+\beta)+n(z+\gamma)}{\sqrt{l^2+m^2+n^2}\sqrt{(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2}}$

B) $cos\theta=\frac{l(x-\alpha)-m(y-\beta)-n(z-\gamma)}{\sqrt{l^2+m^2+n^2}\sqrt{(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2}}$

C) $cos\theta=\frac{l(x-\alpha)+m(y-\beta)+n(z-\gamma)}{\sqrt{l^2-m^2-n^2}\sqrt{(x-\alpha)^2-(y-\beta)^2-(z-\gamma)^2}}$

D) $cos\theta=\frac{l(x-\alpha)+m(y-\beta)+n(z-\gamma)}{\sqrt{l^2+m^2+n^2}\sqrt{(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2}}$

Ans. D

17) Equation of Right Circular cone with vertex at (1,2,3) and axis of cone $\frac{x-1}{2}=\frac{y-2}{-1}=\frac{z-3}{4}$ and semi-Vertical angle 60^0 is given by

A) $cos60=\frac{2(x-1)-1(y-2)+4(z-3)}{\sqrt{2^2+(-1)^2+4^2}\sqrt{(x-1)^2+(y-2)^2+(z-3)^2}}$

B) $cos60=\frac{2(x-1)+1(y-2)+4(z-3)}{\sqrt{2^2+(-1)^2+4^2}\sqrt{(x-1)^2+(x-2)^2+(x-3)^2}}$

C) $cos60=\frac{2(x-1)-1(y-2)-4(z-3)}{\sqrt{2^2+(-1)^2+4^2}\sqrt{(x-1)^2+(x-2)^2+(x-3)^2}}$

D) $cos60=\frac{2(x-1)-1(y-2)+4(z-3)}{\sqrt{2^2-(1)^2+4^2}\sqrt{(x-1)^2+(x-2)^2+(x-3)^2}}$

Ans. A

18) Equation of Right Circular cone with vertex at (-1,-2,-3) and axis of cone

$\frac{x+1}{3} = \frac{y+2}{1} = \frac{z+3}{4}$ and semi-Vertical angle 30° is given by

A) $cos30 = \frac{3(x+1)-1(y+2)-4(z+3)}{\sqrt{3^2+1^2+4^2}\sqrt{(x+1)^2+(x+2)^2+(x+3)^2}}$

B) $cos30 = \frac{3(x+1)+1(y+2)+4(z+3)}{\sqrt{3^2+1^2+4^2}\sqrt{(x+1)^2+(y+2)^2+(z+3)^2}}$

C) $cos30 = \frac{3(x-1)-1(y-2)-4(z-3)}{\sqrt{3^2+1^2+4^2}\sqrt{(x-1)^2+(y-2)^2+(z-3)^2}}$

D) $cos30 = \frac{3(x-1)+1(y-2)+4(z-3)}{\sqrt{3^2+1^2+4^2}\sqrt{(x-1)^2+(x-2)^2+(x-3)^2}}$

Ans. B

19) Equation of Right Circular cone with vertex at (1,2,3) and axis of cone

$\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{4}$ and semi-Vertical angle 60^0 is given by

A) $21[(x-1)^2 + (y-2)^2 + (z-3)^2] = 4[2x - y + 4z - 12]^2$

B) $21[(x+1)^2 + (y+2)^2 + (z+3)^2] = 4[2x - y + 4z - 12]^2$

C) $4[(x-1)^2 + (y-2)^2 + (z-3)^2] = 25[2x - y + 4z - 12]^2$

D) $4[(x+1)^2 + (y+2)^2 + (z+3)^2] = 25[2x - y + 4z - 12]^2$

Ans. A

20) Equation of Right Circular cone with vertex at (1,1,1) and axis of cone

$\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{3}$ and semi-Vertical angle $\frac{\pi}{4}$ is given by

A) $2[(x-1)^2 + (y-1)^2 + (z-1)^2] = 14[x + 2y + 3z - 6]^2$

B) $14[(x-1)^2 + (y-1)^2 + (z-1)^2] = 2[x + 2y + 3z + 6]^2$

C) $14[(x-1)^2 + (y-1)^2 + (z-1)^2] = 2[x + 2y + 3z - 6]^2$

D) $2[(x-1)^2 + (y-1)^2 + (z-1)^2] = 14[x + 2y + 3z + 6]^2$

Ans. C

21) Semi- vertical angle of right circular cone which passes through the point (2,1,3) with vertex at $(1,1,2)$ and axis parallel to the line $\frac{x-2}{2}=\frac{y-1}{-4}=\frac{z+2}{3}$ is...

A) $cos\theta=\frac{5}{\sqrt{29}}$ B) $cos\theta=\frac{5}{\sqrt{58}}$ C) $cos\theta=\frac{-5}{\sqrt{58}}$ D) $cos\theta=\frac{-5}{\sqrt{29}}$

Ans. B

22) Semi- vertical angle of right circular cone with vertex at (0,0,2) direction ratio of generator are 0, 3, -2 and axis is z-axis is given by

A) $cos\theta=\frac{-4}{\sqrt{23}}$ B) $cos\theta=\frac{4}{\sqrt{23}}$ C) $cos\theta=\frac{2}{\sqrt{13}}$ D) $cos\theta=\frac{-2}{\sqrt{13}}$

Ans. D

23) Semi vertical angle of right circular cone having its vertex at $(0,0,0)$ and which passes through the point (2,-2,1) and axis parallel to the line $\frac{x-2}{5}=\frac{y-1}{1}=\frac{z+2}{1}$

A) $cos\theta=\frac{9}{\sqrt{9}\sqrt{27}}$ B) $cos\theta=\frac{13}{\sqrt{9}\sqrt{27}}$ C) $cos\theta=\frac{-9}{\sqrt{9}\sqrt{27}}$ D) $cos\theta=\frac{-13}{\sqrt{9}\sqrt{27}}$

Ans. A

24) Semi vertical angle of right circular cone having its vertex at $(0,0,0)$ and direction ratio of one of the generator of cone are $1,-2,\ 2$ and axis makes equal angles with co-ordinate axes is given by

A) $cos\theta=\frac{-1}{3\sqrt{3}}$ B) $cos\theta=\frac{-3}{3\sqrt{3}}$ C) $cos\theta=\frac{1}{3\sqrt{3}}$ D) $cos\theta=\frac{3}{3\sqrt{3}}$

Ans. C

25) Semi vertical angle of Right Circular cone which passes through the point $(1,1,2)$ and has its axis as Line $6x=-3y=4z$ and vertex origin is …

A) $cos\theta=\frac{-4}{\sqrt{174}}$ B) $cos\theta=\frac{12}{\sqrt{23}}$ C) $cos\theta=\frac{-12}{\sqrt{13}}$ D) $cos\theta=\frac{4}{\sqrt{174}}$

Ans. D

26) The Equation of Right circular cone whose vertex is at (0,0,0) semi-vertical angle θ and the line Axis $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ is

A) $cos\theta = \frac{l(x)+m(y)+n(z)}{\sqrt{l^2+m^2+n^2}\sqrt{(x)^2+(y)^2+(z)^2}}$ B) $cos\theta = \frac{l(x)-m(y)-n(z)}{\sqrt{l^2+m^2+n^2}\sqrt{(x)^2+(y)^2+(z)^2}}$

C) $cos\theta = \frac{l(x)+m(y)+n(z)}{\sqrt{l^2-m^2-n^2}\sqrt{(x)^2-(y)^2-(z)^2}}$ D) $cos\theta = \frac{l(x)+m(y)+n(z)}{\sqrt{l^2+m^2+n^2}\sqrt{(x)^2-(y)^2-(z)^2}}$

Ans. A

27) Equation of Right Circular cone with vertex at (0,0,0) and axis of cone $\frac{x}{2} = \frac{y}{-1} = \frac{z}{2}$ and semi vertical angle $\frac{\pi}{4}$ is given by

A) $cos\frac{\pi}{4} = \frac{2(x)-1(y)+2(z)}{\sqrt{(2)^2-(1)^2-(2)^2}\sqrt{(x)^2-(y)^2-(z)^2}}$

B) $cos\frac{\pi}{4} = \frac{2(x)-1(y)+2(z)}{\sqrt{(2)^2+(-1)^2+(2)^2}\sqrt{(x)^2-(y)^2-(z)^2}}$

C) $cos\frac{\pi}{4} = \frac{2(x)-1(y)+2(z)}{\sqrt{(2)^2+(-1)^2+(2)^2}\sqrt{(x)^2+(y)^2+(z)^2}}$

D) $cos\frac{\pi}{4} = \frac{2(x)+1(y)+2(z)}{\sqrt{(2)^2-(-1)^2+(2)^2}\sqrt{(x)^2+(y)^2+(z)^2}}$

Ans. C

28) Equation of Right Circular cone with vertex at (0,0,0) and axis of cone $\frac{x}{2} = \frac{y}{-1} = \frac{z}{2}$ and semi vertical angle $\frac{\pi}{4}$ is given by

A) $x^2 + y^2 + z^2 + 8xy - 16xz - 8yz = 0$

B) $x^2 + 7y^2 + z^2 + 8xy - 16xz + 8yz = 0$

C) $x^2 + 7y^2 + z^2 + 8xy - 16xz - 8yz = 0$

D) $x^2 + 8y^2 + z^2 + 8xy - 16xz + 8yz = 0$

Ans. B

29) Equation of Right Circular cone with vertex at (0,0,0) and axis of cone $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and semi vertical angle $\frac{\pi}{6}$ is given by

A) $\frac{1}{2} = \frac{x+2y+2z}{\sqrt{14}\sqrt{(x)^2+(y)^2+(z)^2}}$ B) $\frac{\sqrt{3}}{2} = \frac{x-2y-3z}{\sqrt{14}\sqrt{(x)^2+(y)^2+(z)^2}}$

C) $\frac{1}{2} = \frac{x+2y+3z}{\sqrt{14}\sqrt{(x)^2-(y)^2-(z)^2}}$ D) $\frac{\sqrt{3}}{2} = \frac{x+2y+3z}{\sqrt{14}\sqrt{(x)^2+(y)^2+(z)^2}}$

Ans. D

30) Equation of Right circular cone with vertex at (0,0,0) and direction ratio of axis of cone are 5, 1, 1 and $cos\theta = \frac{9}{\sqrt{9}\sqrt{27}}$ is given by

A) $\frac{9}{\sqrt{9}\sqrt{27}} = \frac{5x+y+z}{\sqrt{(5)^2-(1)^2-(1)^2}\sqrt{(x)^2+(y)^2+(z)^2}}$ B) $\frac{9}{\sqrt{9}\sqrt{27}} = \frac{5x+y+z}{\sqrt{(5)^2+(1)^2+(1)^2}\sqrt{(x)^2+(y)^2+(z)^2}}$

C) $\frac{9}{\sqrt{9}\sqrt{27}} = \frac{5x-y-z}{\sqrt{(5)^2-(1)^2-(1)^2}\sqrt{(x)^2+(y)^2+(z)^2}}$ D) $\frac{9}{\sqrt{9}\sqrt{27}} = \frac{5x-y-z}{\sqrt{(5)^2+(1)^2+(1)^2}\sqrt{(x)^2+(y)^2+(z)^2}}$

Ans. B

31) Equation of Right circular cone with vertex at (0,0,0) and has its axis as Line

$6x = -3y = 4z$ and $cos\theta = \frac{4}{\sqrt{174}}$ is given by

A) $\frac{4}{\sqrt{174}} = \frac{2x+4y+3z}{\sqrt{29}\sqrt{(x)^2+(y)^2+(z)^2}}$

B) $\frac{4}{\sqrt{174}} = \frac{2x-4y+3z}{\sqrt{29}\sqrt{(x)^2+(y)^2+(z)^2}}$

C) $\frac{4}{\sqrt{174}} = \frac{6x-3y+4z}{\sqrt{29}\sqrt{(x)^2+(y)^2+(z)^2}}$

D) $\frac{4}{\sqrt{174}} = \frac{6x+3y+4z}{\sqrt{29}\sqrt{(x)^2+(y)^2+(z)^2}}$

Ans. B

32) Equation of Right circular cone with vertex at (0,0,0) and direction ratio of axis of cone are 5, 1, 1 and $cos\theta = \frac{9}{\sqrt{9}\sqrt{27}}$ is given by

A) $8x^2 - 4y^2 - 4z^2 + 5xy + yz + 5xz = 0$

B) $8x^2 + 4y^2 - 8z^2 + 5xy - yz - 5xz = 0$

C) $8x^2 + 4y^2 - 8z^2 + 5xy - yz + 5xz = 0$

D) $8x^2 + 4y^2 - 4z^2 + 5xy - yz - 5xz = 0$

Ans. A

33) Equation of Right circular cone with vertex at (0,0,0) axis is the y axis and semi vertical angle 30^0 is given by

A) $\frac{\sqrt{3}}{2} = \frac{y}{\sqrt{(x)^2+(y)^2+(z)^2}}$ B) $\frac{1}{2} = \frac{y}{\sqrt{(x)^2+(y)^2+(z)^2}}$

C) $\frac{\sqrt{3}}{2} = \frac{x+z}{\sqrt{(x)^2+(y)^2+(z)^2}}$ D) $\frac{1}{2} = \frac{x+z}{\sqrt{(x)^2+(y)^2+(z)^2}}$

Ans. A

34) Equation of Right circular cone with vertex at (0,0,0) axis is the y axis and semi vertical angle 30^0 is given by

A) $3x^2 + y^2 + 3z^2 = 0$ B) $3x^2 - y^2 + 3z^2 = 0$

C) $x^2 - y^2 + 3z^2 = 0$ D) $3x^2 - y^2 - 3z^2 = 0$

Ans. B

35) Equation of Right circular cone with vertex at $(0, 0, 0)$ axis is the z axis and semi vertical angle 45^0 is

$A)\ \frac{1}{\sqrt{2}} = \frac{-z}{\sqrt{(x)^2+(y)^2+(z)^2}}$

$B)\ \frac{1}{\sqrt{2}} = \frac{z}{\sqrt{(x)^2+(y)^2+(z)^2}}$

$C)\ \frac{1}{\sqrt{2}} = \frac{x-y}{\sqrt{(x)^2+(y)^2+(z)^2}}$

$D)\ \frac{1}{\sqrt{2}} = \frac{x+y}{\sqrt{(x)^2+(y)^2+(z)^2}}$

Ans. B

36) Direction ratio of axis of Right Circular cone which passes through the point $(1,1,2)$ and has its axis as Line $6x = -3y = 4z$ and vertex origin is given by

A) $6, -3, 4$ B) $2, -4, 3$ C) $2, 4, -3$ D) $-6, 3, -4$

Ans. B

37) Direction ratio of axis of Right Circular cone which passes through the point (1,1,2) and has its axis as Line $2x = -y = 4z$ and vertex origin is given by

A) $-1, -2, -2$ B) $-2, 1, -4$ C) $1, -2, 2$ D) $2, -4, 1$

Ans. D

38)The equation of the right circular cylinder whose radius r and axis is the line $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$ is...

A) $(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2-\left\{\frac{l(x-\alpha)+m(y-\beta)+n(z-\gamma)}{\sqrt{l^2+m^2+n^2}}\right\}^2=r^2$

B) $(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2+\left\{\frac{l(x-\alpha)+m(y-\beta)+n(z-\gamma)}{\sqrt{l^2+m^2+n^2}}\right\}^2=r^2$

C) $(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2-\left\{\frac{(x-\alpha)+(y-\beta)+(z-\gamma)}{\sqrt{l^2+m^2+n^2}}\right\}^2=r^2$

D) $(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2-\left\{\frac{l(x+\alpha)+m(y+\beta)+n(z+\gamma)}{\sqrt{l^2+m^2+n^2}}\right\}^2=r^2$

Ans. A

39) The equation of the right circular cylinder whose radius 2 and axis is the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{2}$ is...

A) $(x-1)^2+(y-2)^2+(z-3)^2-\left\{\frac{2(x-1)+1(y-2)+1(z-3)}{3}\right\}^2=4$

B) $(x-1)^2+(y-2)^2+(z-3)^2+\left\{\frac{2(x-1)+1(y-2)+1(z-3)}{3}\right\}^2=4$

C) $(x-1)^2+(y-2)^2+(z-3)^2-\left\{\frac{(x-1)+(y-2)+(z-3)}{3}\right\}^2=4$

D) $(x-1)^2+(y-2)^2+(z-3)^2-\left\{\frac{2(x+1)+1(y+2)+1(z+3)}{3}\right\}^2=4$

Ans. A

40) The radius of right circular cylinder whose axis is $\frac{x-2}{2}=\frac{y-1}{1}=\frac{z}{3}$ and which passes through the point (0,0,3) is…

A) $\sqrt{\frac{10}{7}}$ B) $\frac{90}{7}$ C) $\sqrt{\frac{90}{7}}$ D) $\frac{10}{7}$

Ans. C

41) Equation of right circular cylinder of radius a, whose axis passes through the origin and makes equal angles with co-ordinate axes is..

A) $2(x^2+y^2+z^2+xy+yz+zx)=a^2$

B) $2(x^2+y^2+z^2+xy+yz+zx)=3a^2$

C) $2(x^2-y^2-z^2+xy+yz+zx)=a^2$

D) $2(x^2+y^2+z^2-xy-yz-zx)=3a^2$

Ans. D

42) The equation of the right circular cylinder whose axis is $x=2y=-z$ and radius 4 is…

A) $5x^2-8y^2+5z^2-4yz+8zx-4xy+144=0$

B) $5x^2-8y^2+5z^2-4yz+8zx-4xy-144=0$

C) $5x^2-8y^2-5z^2-4yz+8zx-4xy+144=0$

D) $5x^2+8y^2+5z^2+4yz+8zx-4xy-144=0$

Ans. D

43) The radius of right circular cylinder whose axis is $\frac{x}{2}=\frac{y}{1}=\frac{z}{2}$ and which passes through the point (1,2,1) is…

A) 2 B) $2\sqrt{2}$ C) $\sqrt{2}$ D) $\frac{1}{2}$

Ans. C

44) The equation of the right circular cylinder whose radius 5 and axis is the line $\frac{x-2}{2}=\frac{y-3}{1}=\frac{z+1}{1}$ is…

A) $(x-2)^2+(y-3)^2+(z+1)^2-\left\{\frac{2(x-2)+1(y-3)+1(z+1)}{\sqrt{6}}\right\}^2=25$

B) $(x+2)^2+(y+3)^2+(z+1)^2+\left\{\frac{2(x-1)+1(y-2)+1(z-3)}{3}\right\}^2=25$

C) $(x-2)^2+(y-3)^2+(z+1)^2+\left\{\frac{(x-1)+(y-2)+(z-3)}{3}\right\}^2=25$

D) None of these

Ans. A

45) The equation of right circular cylinder whose axis is $\frac{x-2}{2}=\frac{y-1}{1}=\frac{z}{3}$ and radius $\sqrt{\frac{90}{7}}$ is…

A) $10x^2+13y^2+5z^2-4xy-6yz-12zx-36x-18y+30z-135=0$

B) $10x^2+13y^2+5z^2+4xy+6yz+12zx-36x-18y+30z-135=0$

C) $10x^2+13y^2+5z^2-4xy-6yz-12zx-36x-18y+30z+135=0$

D) None of these

Ans. A

46) The axis of right circular cylinder has direction cosines proportional to 2,3,6. The direction ratios of axis are..

A) $\frac{-2}{7},\frac{-3}{7},\frac{-6}{7}$ B) $\frac{2}{7},\frac{3}{7},\frac{6}{7}$ C) $\frac{-2}{49},\frac{-3}{49},\frac{-6}{49}$ D) $\frac{-2}{7},\frac{3}{7},\frac{-6}{7}$

Ans. B

47) The equation of right circular cylinder of radius 2 whose axis passes through (1,2,3) and has direction cosines proportional to 2,-3,6 is…

A) $(x-1)^2+(y-2)^2+(z+1)^2-\left\{\frac{2(x-2)+1(y-3)+1(z+1)}{\sqrt{6}}\right\}^2=4$

B) $(x-2)^2+(y-3)^2+(z+1)^2+\left\{\frac{2(x-2)+1(y-3)+1(z+1)}{\sqrt{6}}\right\}^2=4$

C) $(x-1)^2+(y-2)^2+(z-3)^2-\left\{\frac{2x-3y+6z-14}{7}\right\}^2=4$

D) $(x-1)^2+(y-2)^2+(z-3)^2+\left\{\frac{2x-3y+6z-14}{7}\right\}^2=4$

Ans. C

Sinhgad College of Engineering,Vadgaon

Engineering Mathematics – II

Unit 6-Multiple integrals and their applications

Q.1) The value of $\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \sin x \, dx \, dy$ is

A)0

B)1

C)$\frac{\pi}{2}$

D)π

Ans-C

Q.2) The value of $\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \cos x \, dx \, dy$ is

A)$\frac{\pi}{2}$

B)1

C)0

D)π

Ans-A

Q.3) The value of $\int_0^1 \int_0^y x\,dx\,dy$ is

A)$\frac{1}{2}$

B)$\frac{1}{3}$

C)$\frac{1}{8}$

D)$\frac{1}{6}$

Ans-D

Q.4) The value of $\int_0^1 \int_0^x e^y\,dx\,dy$ is

A)e^2

B)$e-2$

C)e

D)$\frac{1}{2}(e^2-1)$

Ans: B

Q.5)Using polar transformation $x = r\cos\theta$, $y = r\sin\theta$ the Cartesian double integral $\iint_R f(x,y)dxdy$ becomes

A) $\iint_R f(r,\theta)drd\theta$

B) $\iint_R f(r,\theta)rdrd\theta$

C) $\iint_R f(r,\theta)r^2drd\theta$

D) $\iint_R f(r,\theta)\theta drd\theta$

Ans:B

Q.6) On changing the order of integration of $\int_0^1\int_0^x f(x.y)dxdy$ becomes

A) $\int_0^1\int_0^1 f(x.y)dxdy$

B) $\int_0^1\int_0^y f(x.y)dxdy$

C) $\int_0^1\int_1^y f(x.y)dxdy$

D) $\int_0^1\int_y^1 f(x.y)dxdy$

Ans: D

Q.7) On changing the order of integration of $\int_0^1\int_{x^2}^1 f(x.y)dxdy$ becomes

A) $\int_0^1\int_0^{\sqrt{y}} f(x.y)dxdy$

B) $\int_0^1\int_0^{-\sqrt{y}} f(x.y)dxdy$

C) $\int_0^1\int_0^{\sqrt{x}} f(x.y)dxdy$

D) $\int_0^1\int_0^{-\sqrt{x}} f(x.y)dxdy$

Ans: A

Q.8) on transforming into the polar co-ordinates the double integration $\int_0^1 \int_0^{\sqrt{1-x^2}} f(x,y)dxdy$ becomes

A) $\int_0^{\pi} \left\{ \int_0^1 f(r,\theta) r dr \right\} d\theta$

B) $\int_0^{\frac{\pi}{2}} \left\{ \int_0^1 f(r,\theta) r d\theta \right\} dr$

C) $\int_0^{\frac{\pi}{2}} \left\{ \int_0^1 f(r,\theta) r dr \right\} d\theta$

D) $\int_0^{2\pi} \left\{ \int_0^1 f(r,\theta) r dr \right\} d\theta$

Ans: C

Q.11) on transforming into the polar co-ordinates the double integration $\int_0^a \int_0^{\sqrt{a^2-y^2}} e^{-x^2-y^2} dxdy$ becomes

A) $\int_0^{\frac{\pi}{2}} \int_0^a e^{-r^2} dr d\theta$

B) $\int_0^{\frac{\pi}{2}} \int_0^a e^{-r^2} r^2 dr d\theta$

C) $\int_0^{\frac{\pi}{2}} \int_0^a e^{-r^2} r dr d\theta$

D) $\int_0^{\frac{\pi}{2}} \int_0^a e^{-r} r dr d\theta$

Ans. C

Q.12)To find the area of upper half of a cardioid $r = a(1+\cos\theta)$ the double integral becomes

A) $\int_0^{\pi} \int_0^{a(1+\cos\theta)} r dr d\theta$

B) $\int_0^{\pi} \int_0^{a(1+\cos\theta)} dr d\theta$

C) $\int_0^{2\pi} \int_0^{a(1+\cos\theta)} r^2 dr d\theta$

D) $\int_0^{2\pi} \int_0^{a(1+\cos\theta)} r dr d\theta$

Ans: A

13. The value of $\int_0^2 \int_0^x e^{x+y}\, dydx$ is

A) $\frac{1}{2}(e^2-1)$ B) $\frac{1}{2}(e^2-e)$ C) $\frac{1}{2}(e^2-1)^2$ D) None of these

Ans. C

14. On changing the order of integration for $\int_0^\infty \int_x^\infty f(x,y)\,dydx$,the integral becomes

A) $\int_0^\infty \int_0^\infty f(x,y)\,dydx$ B) $\int_0^\infty \int_y^\infty f(x,y)\,dxdy$

C) $\int_0^\infty \int_0^y f(x,y)\,dxdy$ D) $\int_0^\infty \int_0^x f(x,y)\,dydx$

Ans. C

15. On changing the order of integration for $\int_0^a \int_{\frac{y^2}{a}}^{y} f(x,y)dxdy$,the integral becomes

A) $\int_0^a \int_0^x f(x,y)\,dxdy$ B) $\int_0^a \int_x^{\sqrt{ax}} f(x,y)\,dydx$

C) $\int_0^a \int_0^a f(x,y)\,dydx$ D) $\int_0^\infty \int_x^\infty f(x,y)\,dxdy$

Ans. B

16.Find the value of $\int_0^1 \int_0^{1-x} (x+y)dxdy$

A) $1/2$ B) $1/3$ C) $1/4$ D) $1/5$

Ans B

17. Evaluate $\int_0^1 \int_0^y xydxdy$

A) $1/2$ B) $1/3$ C) $1/4$ D) $1/8$

Ans D

18. Evaluate $\int_0^1 \int_0^1 \frac{dxdy}{(1+x^2)(1+y^2)}$

A) $\frac{\pi^2}{16}$ B) $\frac{5}{16}$ C) $1/16$ D) $1/8$

Ans A

19.Find the value of ∫∫xye^{x+y} dxdy.

A) $ye^y(xe^x-e^x)$ B) $(ye^y-e^y)(xe^x-e^x)$

C) $(ye^y-e^y)xe^x$ D) $(ye^y-e^y)(xe^x+e^x)$

Ans B

20. using double integration and the strip parallel to X-axis the area enclosed between the curves $y^2 = 4x$ and $x^2 = 4y$ using points of cutting (0,0) and (4,4) is

A) $\int_0^4 \int_0^{4x} dxdy$ B) $\int_0^4 \int_0^4 dxdy$ C) $\int_0^{4y} \int_0^{4x} dxdy$ D) $\int_{y=0}^{y=4} \int_{\frac{y^2}{4}}^{2\sqrt{y}} dxdy$

Ans. D

21. The area enclosed between the straight line y=x and parabola $y = x^2$ in the XOY plane using double integration is

A) $\int_{x=0}^{x=1} \int_{y=x}^{y=x^2} dxdy$ B) $\int_{x=0}^{x=\infty} \int_{y=x}^{y=\infty} dxdy$

C) $\int_{x=0}^{x=1} \int_{y=0}^{y=1} dxdy$ D) $\int_{x=1}^{x=\infty} \int_{y=1}^{y=\infty} dxdy$

Ans. A

22. The area bounded by $y^2 = 4x$ and $2x - y - 4 = 0$ is

A) 2 B) 1 C) 8 D) 9

Ans. D

23. Area bounded by $y^2 = x$ and $x^2 = -8y$ is

A) $\frac{2}{3}$ B) $\frac{8}{3}$ C) $\frac{7}{3}$ D) $\frac{1}{3}$

Ans. B

24. Area bounded by $y^2 = 4ax$ and $x^2 = 4ay$ is

A) $\frac{a}{3}$ B) $\frac{a^2}{3}$ C) $\frac{16a^2}{3}$ D) $\frac{16}{3}$

Ans. C

25. Area bounded by $x^2 = 4y$ and $x - 2y + 4 = 0$ is

A) 9 B) 4 C) 16 D) 5

Ans. A

26. If $(\bar{x}, \bar{y})$ is centre of gravity of arc AB of the curve $y = f(x)$, then $(\bar{x}, \bar{y})$=

A) $\bar{x} = \frac{\int x\,\rho\,ds}{\int \rho\,ds}$; $\bar{y} = \frac{\int y\ \rho\,ds}{\int \rho\,ds}$.

B) $\bar{x} = \frac{\int y\,\rho\,ds}{\int \rho\,ds}$; $\bar{y} = \frac{\int x\,\rho\,ds}{\int \rho\,ds}$.

C) $\bar{x} = \frac{\int x^2\,\rho\,ds}{\int \rho\,ds}$; $\bar{y} = \frac{\int y^2 \rho\,ds}{\int \rho\,ds}$.

D) $\bar{x} = \frac{\int xy\,\rho\,ds}{\int \rho\,ds}$; $\bar{y} = \frac{\int y\ \rho\,ds}{\int \rho\,ds}$.

Ans. A

27. If $(\bar{x}, \bar{y})$ are corrdinates of centre of gravity of region R plane lamina bonded by curve C and ρ (density) is constant then $(\bar{x}, \bar{y})$=……….

A) $\bar{x} = \frac{\iint_R x\,dx\,dy}{\iint_R dx\,dy}$; $\bar{y} = \frac{\iint_R y\,dx\,dy}{\iint_R dx\,dy}$

B) $\bar{x} = \frac{\iint_R x^2\,dx\,dy}{\iint_R dx\,dy}$; $\bar{y} = \frac{\iint_R y\,dx\,dy}{\iint_R dx\,dy}$

C) $\bar{x} = \frac{\iint_R x\,dx\,dy}{\iint_R dx\,dy}$; $\bar{y} = \frac{\iint_R y^2\,dx\,dy}{\iint_R dx\,dy}$

D) $\bar{x} = \frac{\iint_R x^2\,dx\,dy}{\iint_R dx\,dy}$; $\bar{y} = \frac{\iint_R y^2\,dx\,dy}{\iint_R dx\,dy}$

Ans. A

28. The centroid of the loop of the curve $y^2 = \frac{x^2(a-x)}{(a+x)}$ will lie on …………

A) Y-axis

B) X- axis

C) origin

D) None of the above

Ans. B

29. The centroid of the area bounded by

$y^2(2a - x) = x^3$ and its asymptote is

A) $(\bar{x}, \bar{y}) = (\frac{5a}{3}, 2a)$

B) $(\bar{x}, \bar{y}) = (\frac{5a}{3}, a)$

C) $(\bar{x}, \bar{y}) = (\frac{5a}{3}, 0)$

D) $(\bar{x}, \bar{y}) = (\frac{5a}{3}, a)$

Ans. C

30. The centroid of the loop of the curve $x^2 = \frac{y^2(a-y)}{(a+y)}$ will lie on …………

A) Y-axis

B) X- axis

C) origin

D) None of the above

Ans. A

31. If ($\bar{x}$, $\bar{y}$, $\bar{z}$) be coordinates of centre of gravity of the solid which encloses volume V. then $\bar{x}$ =……

A) $\bar{x} = \frac{\iiint_V x\ \rho\ dx\ dy\ dz}{\iiint_V \rho\ dx\ dy\ dz}$;

B) $\bar{x} = \frac{\iiint_V y\ \rho\ dx\ dy\ dz}{\iiint_V \rho\ dx\ dy\ dz}$;

C) $\bar{x} = \frac{\iiint_V z\ \rho\ dx\ dy\ dz}{\iiint_V \rho\ dx\ dy\ dz}$z

D) $None\ of the\ above$

Ans.A

32. The Moment of inertia of a plane lamina R bounded by the curve C about X-axis is………… .

A) $\iint_R \rho\ y^2\ dx\ dy$ B) $\iint_R \rho\ x^2\ dx\ dy$

C) $\iint_R \rho\ (x+y)^2\ dx\ dy$ D) $\iint_R \rho\ x\ dx\ dy$

Ans.A

33. The Moment of inertia of a plane lamina R bounded by the curve C about Y-axis is……….. .

A) $\iint_R \rho\ y^2\ dx\ dy$ B) $\iint_R \rho\ x^2\ dx\ dy$

C) $\iint_R \rho\ (x+y)^2\ dx\ dy$ D) $\iint_R \rho\ x\ dx\ dy$

Ans.B

34. The Moment of inertia of a plane lamina R bounded by the curve C in polar coordinates is

A) M.I. $=\iint_R \rho\ p^2 r\ d\theta\ dr$ B) M.I. $=\iint_R \rho\ p^2\ d\theta\ dr$

C) M.I. $=\iint_R \rho\ p\ r\ d\theta\ dr$ D) M.I. $=\iint_R \rho\ p^2\theta\ d\theta\ dr$

Ans. A

35. The Moment of inertia of solid which is at a distance p from the axis is ……

A) $\iiint_V \rho\ p\ dx\ dy\ dz$ B) $\iiint_V \rho\ p^2\ dx\ dy\ dz$

C) $\iiint_V \rho\ \ dx\ dy\ dz$ D) $\iiint_V \rho\ p^3\ dx\ dy\ dz$

Ans.B

36. The Moment of inertia of solid about X-axis is ……

A) $\iiint_V \rho(y^2 + z^2)dx\,dy\,dz$

B) $\iiint_V \rho(x^2 + z^2)dx\,dy\,dz$

C) $\iiint_V \rho(y^2 + x^2)dx\,dy\,dz$

D) $\iiint_V \rho\, x^2 dx\,dy\,dz$

Ans.A

37. The Moment of inertia of solid about Y -axis is ……

A) $\iiint_V \rho(y^2 + z^2)dx\,dy\,dz$

B) $\iiint_V \rho(x^2 + z^2)dx\,dy\,dz$

C) $\iiint_V \rho(y^2 + x^2)dx\,dy\,dz$

D) $\iiint_V \rho\, x^2 dx\,dy\,dz$

Ans.B

38. The Moment of inertia of solid about Z-axis is ……

A) $\iiint_V \rho(y^2 + z^2)dx\,dy\,dz$ s

B) $\iiint_V \rho(x^2 + z^2)dx\,dy\,dz$

C) $\iiint_V \rho(y^2 + x^2)dx\,dy\,dz$

D) $\iiint_V \rho\, x^2 dx\,dy\,dz$

Ans.C

39. The Moment of inertia about the line $\theta = \frac{\pi}{2}$ of the area enclosed by $r = a(1 + cos\theta)$

A) M.I. = $\iint r^2 sin^2\theta\, r\, dr\, d\theta$

B) M.I. = $\iint r^2 cos^2\theta\, r\, dr\, d\theta$

C) M.I. = $\iint r^2 cos^2\theta\, dr\, d\theta$

D) M.I. = $\iint cos^2\theta\, r\, dr\, d\theta$

Ans.B

40. The Moment of Inertia about the X-axis of the area enclosed by the lines $x = 0, \frac{x}{a} + \frac{y}{b} = 1$ is

A) $\rho \int_{y=0}^{b} \int_0^{\frac{a}{b}(b-y)} x^2 dx\,dy$

B) $\rho \int_{y=0}^{b} \int_0^{\frac{a}{b}(b-y)} x\, dx\,dy$

C) $\rho \int_{y=0}^{b} \int_0^{\frac{a}{b}(b-y)} y^2 dx\,dy$

D) $\rho \int_{x=0}^{b} \int_{y=0}^{\frac{a}{b}(b-y)} y^2 dy\,dx$

Ans.C

41. The Moment of Inertia about the Y-axis of the area enclosed by the lines $x = 0, \frac{x}{a} + \frac{y}{b} = 1$ is

A) $\rho \int_{y=0}^{b} \int_{0}^{\frac{a}{b}(b-y)} x^2 dx\, dy$ B) $\rho \int_{y=0}^{b} \int_{0}^{\frac{a}{b}(b-y)} x\, dx\, dy$

C) $\rho \int_{y=0}^{b} \int_{0}^{\frac{a}{b}(b-y)} y^2 dx\, dy$ D) $\rho \int_{x=0}^{b} \int_{y=0}^{\frac{a}{b}(b-y)} y^2 dy\, dx$

Ans.A

42. Transformation of triple integration to spherical polar coordinates is

A) $\iiint_V F(r,\theta,\emptyset)\, r^2 sin\theta\; d\theta\; d\emptyset\; dr$

B) $\iiint_V F(r,\theta,z)\, r^2 sin\theta\; d\theta\; d\emptyset\; dr$

C) $\iiint_V F(r,\theta,\emptyset)\, sin\theta\; d\theta\; d\emptyset\; dr$

D) $\iiint_V F(r,\theta,\emptyset)\, r^2\; d\theta\; d\emptyset\; dr$

Answer : A

43. $\int_{-1}^{1} \int_{0}^{z} \int_{x-z}^{x+z} (x + y + z) dxdydz =$

A) 1 B) 0 C) -1 D) none of these

Answer : B

44. $\int_{0}^{1} dy \int_{y^2}^{1} dx \int_{0}^{1-x} xdz$

A) $-\frac{4}{35}$ B) $\frac{4}{35}$ C) $\frac{2}{35}$ D) $\frac{-2}{35}$

Answer : B

45. $\iiint (x^2\, y^2 + y^2 z^2 + z^2 x^2) dxdydz$ throughout the volume of the sphere

$x^2 + y^2 + z^2 = a^2$ is

A) $\frac{4}{35} a^7$ B) $\frac{-4}{35} a^7$ C) $\frac{4}{35} a^7 \pi$ D) $a^7 \pi$

Answer : C

46. $\iiint\limits_V \sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}}\ dxdydz$ throughout the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is

A) $\frac{\pi^2 abc}{4}$ B) $\frac{\pi abc}{4}$ C) $\frac{abc}{4}$ D) $\frac{\pi^2}{4}$

Answer : A

47. $\int_0^a \int_0^x \int_0^{\sqrt{x+y}} z\ dxdydz =$

A) $\frac{-a^2}{4}$ B) $\frac{a}{4}$ C) $\frac{a^3}{4}$ D) $\frac{a^2}{4}$

Answer : C

48. $\iiint (x+y+z)\ dxdydz$ over the positive octant of the sphere $x^2+y^2+z^2=a^2$ is

A) $\frac{-\pi a^4}{16}$ B) $\frac{3\pi a^4}{16}$ C) $\frac{3\pi a^2}{16}$ D) $\frac{\pi a^4}{6}$

Answer : B

49. The Dirichlet's theorem for 3 variables x, y, z is $\iiint x^{a-1}\, y^{b-1} z^{c-1} dxdydz$ =

A) $\frac{\Gamma a\, \Gamma b\, \Gamma c}{\Gamma 1+a-b+c}$ B) $\frac{\Gamma a\, \Gamma b\, \Gamma c}{\Gamma 1-a+b+c}$ C) $\frac{\Gamma a\, \Gamma b\, \Gamma c}{\Gamma 1+a+b+c}$ D) $\frac{\Gamma a\, \Gamma b\, \Gamma c}{\Gamma 1+a+b-c}$

Answer : C

50. The volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is given by

$V = 8abc \int_0^{\pi/2} \int_0^{\pi/2} \int_0^1 r^2 sin\theta\ d\theta\ d\emptyset\ dr =$

A) $\frac{2abc}{3}$ B) $\frac{abc\pi}{3}$ C) $\frac{2abc}{3}$ D) $\frac{4abc}{3}$

Answer : D

51. The volume of the tetrahedron bounded by the co-ordinate planes and the

$\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=1$ is

A) 2 B) 3 C) 4 D) 1

Answer : C

52. The volume enclosed by the cone $x^2+y^2=z^2$ and the paraboloid

$x^2+y^2=z$ given by $V=4\int_0^{\pi/2}\int_0^1(r-r^2)rd\theta dr=$

A) $\frac{\pi}{4}$ B) $\frac{\pi}{6}$ C) $\frac{\pi}{2}$ D) $-\frac{\pi}{4}$

Answer : B

53. The volume of the cylinder $x^2+y^2=2ax$ intercepted between paraboloid

$x^2+y^2=2az$ and XY plane is given by $V=\frac{1}{2a}2\int_0^{\pi/2}\int_0^{2acos\theta}r^2rd\theta dr=$

A) $\frac{3\pi}{4}$ B) $\frac{3\pi a^3}{4}$ c) $\frac{a^3\pi}{4}$ D) $\frac{3a^3}{4}$

Answer : B

Seat No -

Total number of questions : 60

1000505 T1 ENGINEERING MATHEMATICS II

Time : 1hr

Max Marks : 50

N.B

1) All questions are Multiple Choice Questions having single correct option.

2) Attempt any 50 questions out of 60.

3) Use of calculator is allowed.

4) Each question carries 1 Mark.

5) Specially abled students are allowed 20 minutes extra for examination.

6) Do not use pencils to darken answer.

7) Use only black/blue ball point pen to darken the appropriate circle.

8) No change will be allowed once the answer is marked on OMR Sheet.

9) Rough work shall not be done on OMR sheet or on question paper.

10) Darken ONLY ONE CIRCLE for each answer.

Q.no 1. To change Cartesian coordinates (x, y, z) to spherical polar coordinate (r, θ, ϕ);$dxdydz$ is replaced by

A : To change Cartesian coordinates (x, y, z) to spherical polar coordinate (r, θ, ϕ);$dxdydz$ is replaced by

B : $r^2 \cos\theta dr d\theta d\phi$

C : $r^2 dr d\theta d\phi$

D : $r dr d\theta d\phi$

Q.no 2. Moment of inertia of the lamina A about the x axis is equal to

A : $\int\int_A \rho x^2 dxdy$

B : $\int \int_A \rho y^2 dxdy$

C : $\int \int_A \rho(x^2 + y^2) dxdy$

D : $\int \int_A \rho x^2 y^2 dxdy$

Q.no 3. Rectilinear motion is a motion of body along a

A : Straight line

B : Circular path

C : Parabolic path

D : Hyperbolic path

Q.no 4. If $\phi(a) = \int_0^\infty e^{-bx^2} \cos(2ax)\, dx,\ b > 0$ then by DUIS rule $\frac{d\phi}{da}$ is

A : $\int_0^\infty \frac{\partial}{\partial b}\left[e^{-bx^2} \cos(2ax)\right] dx$

B : $e^{-bx^2} \cos(2ax)$

C : $\int_0^\infty \frac{\partial}{\partial x}\left[e^{-bx^2} \cos(2ax)\right] dx$

D : $\int_0^\infty \frac{\partial}{\partial a}\left[e^{-bx^2} \cos(2ax)\right] dx$

Q.no 5. The differential equation for the current i in an electric circuit containing resistance 100 ohm and an inductance of 0.5 henery connected in series with battery of 20 volts is

A : $0.5\frac{di}{dt} + 100i = 0$

B : $0.5\frac{di}{dt} + 100i = 20$

C : $100\frac{di}{dt}+0.5i=0$

D : $100\frac{di}{dt}+0.5R=0$

Q.no 6. The formula to find the length of the curve $y=f(x)$ from $x=a$ to $x=b$ is $s=$

A : $\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

B : $\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dy$

C : $\int_a^b \sqrt{a^2+\left(\frac{dy}{dx}\right)^2}\,dx$

D : $\int_a^b \sqrt{a^2+b^2\left(\frac{dy}{dx}\right)^2}\,dy$

Q.no 7.
Condition for the two spheres $x^2+y^2+z^2+2u_1x+2v_1y+2w_1z+d_1=0$ and $x^2+y^2+z^2+2u_2x+2v_2y+2w_2z+d_2=0$ to be orthogonal is ---

A : $S+\lambda U=0$

B : $S_1-S_2=0$

C : $2u_1u_2+2v_1v_2+2w_1w_2=d_1+d_2$

D : $u_1u_2+v_1v_2+w_1w_2=d_1+d_2$

Q.no 8. The general solution of Linear Differential equation $\frac{dx}{dy}+Px=Q$ where P and Q are functions of y or constants, is

A : $x\,e^{\int p\,dx}=\int Q\,e^{\int p\,dx}dx+c$

B : $x\, e^{\int p dy} = \int Q\, e^{\int p dy} dy + c$

C : $y\, e^{\int p dx} = \int Q\, e^{\int p dx} dx + c$

D : $y\, e^{\int p dx} = \int Q\, e^{\int p dx} dy + c$

Q.no 9. The necessary and sufficient condition that the Differential equation $Mdx + Ndy = 0$ is exact is

A : $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

B : $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

C : $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

D : $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial x}$

Q.no 10. Fourier law of heat conduction states that, the quantity of heat flow across an area A cm2 is

A : proportional to product of area A and temperature gradient dT/dx

B : inversely proportional to product of area A and temperature gradient dT/dx

C : equal to sum of area A and temperature gradient

D : equal to difference of area A and temperature gradient dT/dx

Q.no 11.

Fourier series representation of periodic function $f(x)$ with period 2π which satisfies the Dirichlet's conditions is

A : $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$

B : $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\pi x + b_n \sin n\pi x)$

C : $\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n \cos nx)(b_n \sin nx)$

D : $\frac{a_0}{2}+(a_n \cos nx + b_n \sin nx)$

Q.no 12. The integrating factor of $(y^4+2y)dx+(xy^3+2y^4-4x)dy=0$ using non exact rule $\frac{\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)}{M}$ is

A : $\frac{1}{y^3}$

B : 1/x

C : y^3

D : $\frac{1}{x^3}$

Q.no 13. A circuit containing inductance L capacitance C in series without applied electromotive force. By Kirchhoff's voltage law differential equation for current i is

A : $L\frac{di}{dt}+\frac{q}{c}=0$

B : $L\frac{di}{dt}+Ri=0$

C : $L\frac{di}{dt}+Ri=E$

D : $L\frac{di}{dt}+\frac{q}{c}=E$

Q.no 14. $\int_0^{\pi/2} sin^p\theta cos^q\theta \, d\theta$ is equal to

A : $B\left(\frac{p-1}{2},\frac{q-1}{2}\right)$

B : $\frac{1}{2}B\left(\frac{p+1}{2},\frac{q+1}{2}\right)$

C : $B\left(\frac{p+1}{2},\frac{q+1}{2}\right)$

D : $B(p,q)$

Q.no 15. The integrating factor of $\frac{dy}{dx}+Py=Q$ is

A : $e^{\int p dx}$

B : $e^{\int p dy}$

C : $e^{\int Q dx}$

D : $e^{\int Q dy}$

Q.no 16. Error function of x, $erf(x)$ is defined as

A : $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$

B : $\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2}du$

C : $\int_0^\infty e^{-x}x^{n-1}dx$

D : $\frac{2}{\sqrt{\pi}}\int_0^\infty e^{-u^2}du$

Q.no 17. The differential equation for the current in an electric circuit containing resistance R and inductance L in series with voltage source E sin(wt) is

A : $L\frac{di}{dt}+\frac{q}{C}=E$

B : $Li + R\frac{di}{dt} = Esin\omega t$

C : $L\frac{di}{dt} + Ri = 0$

D : $L\frac{di}{dt} + Ri = 0$

Q.no 18. Moment of inertia of the lamina A about the y axis is equal to

A : $\int \int_A \rho x^2 dxdy$

B : $\int \int_A \rho y^2 dxdy$

C : $\int \int_A \rho (x^2 + y^2) dxdy$

D : $\int \int_A \rho x^2 y^2 dxdy$

Q.no 19. The solution of the DE $\frac{dy}{dx} = \frac{x^2}{y^3}$ is

A : $y^3 - x^3 = c$

B : $3y^4 - 4x^3 = c$

C : $y^2 - x^3 = c$

D : $x^2 - y^2 = c$

Q.no 20. If the differential equation $M(x,y)dx + N(x,y)dy = 0$ is not exact and it can be written as $yf_1(x,y)dx + xf_2(x,y)dy = 0$ then integrating factor is

A : $\frac{1}{My + Nx}; My + Nx \neq 0$

B : $\frac{1}{Mx+Ny}$; $Mx+Ny \neq 0$

C : $\frac{1}{Mx-Ny}$; $Mx-Ny \neq 0$

D : $\frac{1}{My-Nx}$; $My-Nx \neq 0$

Q.no 21. The integrating factor of the DE $cosx\frac{dy}{dx}+ysinx=1$ is

A : sec x

B : cot x

C : tan x

D : sin x

Q.no 22. If double integral in Cartesian coordinate is given by $\int\int_R f(x,y)\,dxdy$ then the value of same integral in polar form is

A : $\int\int_R f(r\cos\theta, r\sin\theta)\,rdrd\theta$

B : $\int\int_R f(r\cos\theta, r\sin\theta)\,rdrd\theta$

C : $\int\int_R f(r\cos\theta, r\sin\theta)\,r^2drd\theta$

D : $\int\int_R f(r\sin\theta, r\cos\theta)\,rdrd\theta$

Q.no 23. A particle moving in straight line with acceleration $k(x+\frac{a^4}{x^3})$ directed towards origin. The equation of motion is

A : $\frac{dv}{dx}=-k(x+\frac{a^4}{x^3})$

B : $v\frac{dv}{dx}=k(x+\frac{a^4}{x^3})$

C : $v\dfrac{dv}{dx} = -k(x + \dfrac{a^4}{x^3})$

D : $\dfrac{dv}{dx} = (x + \dfrac{a^4}{x^3})$

Q.no 24. Area enclosed by the polar curve $r = f_1(\theta)$ and $r = f_2(\theta)$ and the line $\theta = \alpha, \theta = \beta \alpha < \beta$ is given by

A : $Area = \int_{\alpha}^{\beta} \int_{f_1(\theta)}^{f_2(\theta)} r dr d\theta$

B : $Area = \int_{\alpha}^{\beta} \int_{f_1(\theta)}^{f_2(\theta)} dr d\theta$

C : $Area = \int_{\alpha}^{\beta} \int_{f_1(\theta)}^{f_2(\theta)} r^2 \sin\theta dr d\theta$

D : $Area = \int_{\alpha}^{\beta} \int_{f_1(\theta)}^{f_2(\theta)} r^2 \cos\theta dr d\theta$

Q.no 25.

The equation of asymptotes parallel to y-axis to the curve represented by the equation

$y^2(4-x) = x(x-2)^2$ is

A : x=2

B : y=0

C : x=4

D : x=0

Q.no 26. The value of integration $\int_0^a \int_0^b \int_0^c x^2 y^2 z^2 dx dy dz$ is equal to

A : $\frac{abc}{3}$

B : $\frac{a^2b^2c^2}{3}$

C : $\frac{a^3b^3c^3}{27}$

D : $\frac{a^2b^2c^2}{9}$

Q.no 27. The center and radius of the sphere $3(x^2 + y^2 + z^2) - 30x + 12y - 18z + 89 = 0$ is

A : $(10,-4,6)$, $\frac{5}{\sqrt{3}}$

B : $(15,-6,9)$, $\frac{\sqrt{203}}{\sqrt{3}}$

C : $(5,-2,3)$, $\frac{5}{\sqrt{3}}$

D : $(-5,2,-3)$, $\frac{5}{3}$

Q.no 28.
Find semi-vertical angle for a right circular cone with vertex at the point (1,0,1)which passes through the point (1,1,1) and axis of cone has direction ratios 1,1,1

A : $\sin\alpha = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

B : $\cos\alpha = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

C : $\cos\alpha = a_1a_2 + b_1b_2 + c_1c_2$

D : $\sin\alpha = a_1a_2 + b_1b_2 + c_1c_2$

Q.no 29. $erf(x) - erfc(x) = 1 \quad erfc(x) = ?$

A : $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\,du$

B : $\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2}\,du$

C : $\frac{2}{\sqrt{\pi}}\int_{0}^{x^2} e^{-t}t^{-1/2}dt$

D : 0

Q.no 30. The integrating factor of the DE $\frac{dy}{dx} + ycosx = \frac{sin2x}{2}$ is

A : e^{cosx}

B : e^{sinx}

C : sin x

D : cos x

Q.no 31. According to D'Alembert's principle algebraic sum of forces acting on a body along a given direction is equal to

A : Velocity x Acceleration

B : Mass x Velocity

C : Mass x displacement

D : Mass x Acceleration

Q.no 32. If $I_n = \int_0^{\pi/2} sin^n x\, dx$, where n is positive even integer then I_n is calculated from

A : $\frac{n-1}{n}.\frac{n-3}{n-2.}.\frac{n-5}{n-4}......\frac{3}{4}.\frac{1}{2}$

B : $\frac{n-1}{n}.\frac{n-3}{n-2.}.\frac{n-5}{n-4}......\frac{3}{4}.\frac{1}{2}.\pi$

C : $\left(\frac{n+1}{n}.\frac{n.+3}{n+2.}.\frac{n+5}{n+4}\right)......\frac{\pi}{2}$

D : $\frac{n-1}{n}.\frac{n-3}{n-2.}.\frac{n-5}{n-4}......\frac{3}{4}.\frac{1}{2}.\frac{\pi}{2}$

Q.no 33. The value of λ for which the differential equation $(xy^2 + \lambda x^2 y)dx + (x+y)x^2 dy = 0$ is exact

A : 2

B : -3

C : 3

D : -2

Q.no 34.

If $(\bar{x}, \bar{y})$ the coordinates of centroid of the plane area is given by

A : $\bar{x} = \frac{\int\int x\rho dxdy}{\int\int \rho dxdy}, \bar{y} = \frac{\int\int y\rho dxdy}{\int\int \rho dxdy}$

B : $\bar{x} = \frac{\int\int x^2\rho dxdy}{\int\int \rho dxdy}, \bar{y} = \frac{\int\int y^2\rho dxdy}{\int\int \rho dxdy}$

C : $\bar{x} = \frac{\int\int \rho dxdy}{\int\int x\rho dxdy}, \bar{y} = \frac{\int\int \rho dxdy}{\int\int y\rho dxdy}$

D : None of these

Q.no 35. The integrating factor of $(x^2 + y^2 + x)dx + xydy = 0$ using non exact rule $\frac{\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)}{N}$ is

A : x

B : 1/x

C : x2

D : xy

Q.no 36. The formula to find the length of the curve $r = f(\theta)$ from $\theta = \theta_1$ to $\theta = \theta_2$ is

A : $\int_{\theta_1}^{\theta_2} \sqrt{1 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

B : $\int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

C : $\int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, dr$

D : $\int_{\theta_2}^{\theta_1} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

Q.no 37. The value of $erf\,(3) + erf_c\,(3)$ is

A : 3

B : 2

C : 1

D : 0

Q.no 38. If the equation of curve remains unchanged by replacing y by -y, then the curve is symmetric about

A : y-axis

B : line y=x

C : x-axis

D : line y=-x

Q.no 39. If the sphere $x^2 + y^2 + z^2 = 25$,

$$x^2 + y^2 + z^2 - 18x - 24y - 40z + 225 = 0$$

touches each other externally then distance between their centers is:

A : 20

B : 5

C : 15

D : 25

Q.no 40. The integrating factor of the DE $\frac{dx}{dy} + \frac{3}{y}x = \frac{1}{y^2}$

A : $-y^3$

B : y^3

C : x^2y

D : x^3

Q.no 41. The integrating factor of $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$ using non exact rule $\frac{\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)}{M}$ is

A : y^2

B : $\frac{1}{y^2}$

C : y

D : 1/y

Q.no 42. The integrating factor for the DE $(1 + x^2)\frac{dy}{dx} + xy = 0$ is

A : e^{1+x^2}

B : $\log(x^2 + 1)$

C : $\sqrt{x^2 + 1}$

D : $1 + x^2$

Q.no 43.

The two spheres $x^2 + y^2 + z^2 - 12x - 18y - 24z + 29 = 0$ and $x^2 + y^2 + z^2 + 3x - 3y + 3z - 56 = 0$

A : touches each other internally

B : touches each other externally

C : touches each other Orthogonally

D : Intersecting each other

Q.no 44. Radius of right circular cylinder is 5and whose axis is the line $\frac{x-2}{3}=\frac{y-3}{1}=\frac{z+1}{1}$ Then the equation of right circular cylinder is

A : $(x+2)^2+(y+3)^2+(z+1)^2=5^2+\left(\frac{3x+y+z-8}{\sqrt{11}}\right)^2$

B : $(x-2)^2+(y-3)^2+(z-1)^2=5^2+\left(\frac{3x+y+z-8}{\sqrt{11}}\right)^2$

C : $(x+2)^2+(y-3)^2+(z+1)^2=5^2+\left(\frac{3x+y+z-8}{\sqrt{11}}\right)^2$

D : $(x-2)^2+(y-3)^2+(z+1)^2=5^2+\left(\frac{3x+y+z-8}{\sqrt{11}}\right)^2$

Q.no 45. q56.jpg

A : 1

B : 0

C : q56_3.jpg

D : q56_4.jpg

Q.no 46.

Area bounded by the parabola $y^2=4x$ and the line $y=x$ is calculated using

A : $\int_{y=0}^{4}\int_{x=y^2/4}^{y} dxdy$

B : $\int_{y=0}^{4}\int_{x=0}^{4} dxdy$

C : $\int_{y=0}^{4}\int_{x=0}^{y} dxdy$

D : $\int_{x=0}^{4}\int_{y=0}^{x} dxdy$

Q.no 47. The value of integration $\int_0^1\int_0^x (x^2+y^2)\,dxdy$ is equal to

A : 4/3

B : 5/3

C : 2/3

D : 1

Q.no 48. The value of integration $\int_0^1\int_0^1\int_0^1 xyzdxdydz$ is equal to

A : 0.125

B : 0.25

C : -0.125

D : 1

Q.no 49. The value of integration $\int_0^1\int_0^{x^2} xe^y dxdy$ is equal to

A : e/2

B : e-1

C : 1-e

D : (e/2)-1

Q.no 50. $B\left(\frac{1}{4},\frac{3}{4}\right)$ is equal to

A : $\frac{2\pi}{\sqrt{3}}$

B : $\frac{2\pi}{\sqrt{3}}$

C : 0

D : $\pi\sqrt{2}$

Q.no 51. A capacitor C=0.01 farad in series with resistor R=20 ohms is charged from battery E=10 volts. If initially capacitor is completely discharged then differential equation for charge q(t) is given by

A : $20\frac{dq}{dt}+\frac{q}{0.01}=0\ ;\ q(0)=0$

B : $20\frac{dq}{dt}+0.01q=10\ ;\ q(0)=0$

C : $20\frac{dq}{dt}+\frac{q}{0.01}=10\ ;\ q(0)=0$

D : $20\frac{dq}{dt}+0.01q=0\ ;\ q(0)=0$

Q.no 52. If $I(\alpha)=\int_0^1 \frac{x^\alpha-1}{\log x}dx$, $\alpha \geq 0$ then by DUIS $\frac{dI}{d\alpha}$ is

A : $\int_0^1 \log x\, dx$

B : $\int_0^1 x^\alpha \log x\, dx$

C : $\int_0^1 x^\alpha dx$

D : $\int_0^1 x^{-\alpha} dx$

Q.no 53.

The equation of sphere which passes through the point (4, 6, 3) and passes through the circle $(x-1)^2+(y-2)^2=25$, $z=0$ is

A : $x^2+y^2+z^2-2x-4y-3z-20=0$

B : $x^2 + y^2 + z^2 + 3x - 4y + 3z - 16 = 0$

C : $2x^2 + 2y^2 + 2z^2 + 6x - 3y + 2z = 0$

D : $x^2 + y^2 + z^2 + 4x - 4y + 3z - 20 = 0$

Q.no 54. The solution of exact DE $(x + 2y - 2)dx + (2x - y + 3)dy = 0$ is

A : $x^2 + 4xy - 4x + 6y = c$

B : $x^2 + 4xy - 4x - y^2 + 6y = c$

C : $x^2 + 8xy + y^2 = c$

D : $x^2 + 4xy - \frac{y^2}{2} + y = c$

Q.no 55. If $I_n = \int_0^{\pi/2} \cos^n x \, dx$ then which of the following relation is true?

A : $\frac{n-1}{n} \cdot \frac{n-3}{n-2.} \cdot \frac{n-5}{n-4} \ldots \ldots \frac{3}{4} \cdot \frac{1}{2}$

B : $I_n = \frac{n}{n-1} I_{n-2}$

C : $I_n = \frac{n-1}{n} I_{n-2}$

D : $I_n = n(n+1) I_{n-1}$

Q.no 56. In spherical co-ordinates volume is given by

A : $V = \int \int \int_V dr d\theta d\phi$

B : $V = \int \int \int_V r dr d\theta d\phi$

C : $V = \int\int\int_V r^2 \cos\theta dr d\theta d\phi$

D : $V = \int\int\int_V r^2 \sin\theta dr d\theta d\phi$

Q.no 57.

The radius of the circle of intersection of the sphere $x^2 + y^2 + z^2 - 2x + 4y + 2z - 6 = 0$ by the plane $x + 2y + 2z - 4 = 0$ is:

[Centre of the circle is(2,0,1)]

A : 9

B : 3

C : 12

D : $\sqrt{3}$

Q.no 58.

The value of a_0 in harmonic analysis of y for the following tabulated data is

x	0	1	2	3	4	5	6
y	9	18	24	28	26	20	9

A : 17.85

B : 20.83

C : 35.71

D : 41.66

Q.no 59. $\frac{5\pi}{128}$

A : $\frac{5\pi}{128}$

B : $\frac{3\pi}{128}$

C : $\frac{3\pi}{512}$

D : $\frac{5\pi}{64}$

Q.no 60. The value of integration $\int_0^2 \int_0^{x^2} e^{\frac{y}{x}} dx dy$ is equal to

A : $e^2 - 2$

B : $2e^2 - 1$

C : $e^2 - 1$

D : $e^2 + 1$

Answer for Question No 1. is a

Answer for Question No 2. is b

Answer for Question No 3. is a

Answer for Question No 4. is d

Answer for Question No 5. is b

Answer for Question No 6. is a

Answer for Question No 7. is c

Answer for Question No 8. is b

Answer for Question No 9. is a

Answer for Question No 10. is a

Answer for Question No 11. is b

Answer for Question No 12. is a

Answer for Question No 13. is a

Answer for Question No 14. is b

Answer for Question No 15. is a

Answer for Question No 16. is a

Answer for Question No 17. is d

Answer for Question No 18. is a

Answer for Question No 19. is b

Answer for Question No 20. is c

Answer for Question No 21. is a

Answer for Question No 22. is a

Answer for Question No 23. is c

Answer for Question No 24. is a

Answer for Question No 25. is c

Answer for Question No 26. is c

Answer for Question No 27. is c

Answer for Question No 28. is b

Answer for Question No 29. is b

Answer for Question No 30. is b

Answer for Question No 31. is d

Answer for Question No 32. is d

Answer for Question No 33. is c

Answer for Question No 34. is a

Answer for Question No 35. is a

Answer for Question No 36. is b

Answer for Question No 37. is c

Answer for Question No 38. is c

Answer for Question No 39. is d

Answer for Question No 40. is b

Answer for Question No 41. is b

Answer for Question No 42. is b

Answer for Question No 43. is c

Answer for Question No 44. is d

Answer for Question No 45. is c

Answer for Question No 46. is a

Answer for Question No 47. is c

Answer for Question No 48. is a

Answer for Question No 49. is b

Answer for Question No 50. is d

Answer for Question No 51. is c

Answer for Question No 52. is c

Answer for Question No 53. is a

Answer for Question No 54. is b

Answer for Question No 55. is c

Answer for Question No 56. is d

Answer for Question No 57. is d

Answer for Question No 58. is d

Answer for Question No 59. is b

Answer for Question No 60. is c

Seat No -

Total number of questions : 60

1000505 T1 ENGINEERING MATHEMATICS II

Time : 1hr

Max Marks : 50

N.B

1) All questions are Multiple Choice Questions having single correct option.

2) Attempt any 50 questions out of 60.

3) Use of calculator is allowed.

4) Each question carries 1 Mark.

5) Specially abled students are allowed 20 minutes extra for examination.

6) Do not use pencils to darken answer.

7) Use only black/blue ball point pen to darken the appropriate circle.

8) No change will be allowed once the answer is marked on OMR Sheet.

9) Rough work shall not be done on OMR sheet or on question paper.

10) Darken ONLY ONE CIRCLE for each answer.

In a D.E. $L\frac{di}{dt}+Ri=E$. E means

Q.no 1.

A : Voltage

B : Current

C : Resistance

D : Electromotive force

$\int_0^{\pi/2} sin^p\theta cos^q\theta \, d\theta$ is equal to

Q.no 2.

A : $B\left(\frac{p-1}{2},\frac{q-1}{2}\right)$

B : $\frac{1}{2}B\left(\frac{p+1}{2},\frac{q+1}{2}\right)$

C : $B\left(\frac{p+1}{2},\frac{q+1}{2}\right)$

D : $B(p,q)$

Q.no 3. If q be the quantity of heat that flows across an area A cm2 and thickness δx in one second where the difference of temperature at the faces is δT, then by Fourier's law of heat conduction

A : $q=-k\left(A-\frac{dt}{dx}\right)$, where k is thermal conductivity

B : $q=kA\frac{dT}{dx}$, where k is thermal conductivity

C : $q=-k\left(A+\frac{dt}{dx}\right)$, where k is thermal conductivity

D : $q=-kA\frac{dT}{dx}$, , where k is thermal conductivity

Q.no 4. Moment of inertia of the lamina A about the x axis is equal to

A : $\int\int_A \rho x^2 dxdy$

B : $\int\int_A \rho y^2 dxdy$

C : $\int\int_A \rho(x^2+y^2)dxdy$

D : $\int\int_A \rho x^2y^2 dxdy$

Q.no 5. According to D'Alembert's principle algebraic sum of forces acting on a body along a given direction is equal to

A : Velocity x Acceleration

B : Mass x Velocity

C : Mass x displacement

D : Mass x Acceleration

Q.no 6. The amount of heat Q flowing through the area per unit time is

A : Q = thermal conductivity x Area x Rate of change of temp. across an area

B : Q = thermal conductivity x Area of slab

C : Q = thermal conductivity x Area x temp. gradient

D : Q = thermal conductivity x Area + Rate of flow of heat

Q.no 7. Fourier law of heat conduction states that, the quantity of heat flow across an area A cm2 is

A : proportional to product of area A and temperature gradient dT/dx

B : inversely proportional to product of area A and temperature gradient dT/dx

C : equal to sum of area A and temperature gradient

D : equal to difference of area A and temperature gradient dT/dx

Q.no 8. If the equation of curve remains unchanged by replacing y by -y, then the curve is symmetric about

A : y-axis

B : line y=x

C : x-axis

D : line y=-x

Q.no 9. Equation of right circular cylinder whose radius is 'r' and axis is the line $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ is

A : $x^2+y^2+z^2=r^2+\left(\frac{lx+my+nz}{\sqrt{l^2+m^2+n^2}}\right)^2$

B : $\alpha=\cos^{-1}1$

C : $x^2+y^2+z^2=r^2+\frac{lx+my+nz}{\sqrt{l^2+m^2+n^2}}$

D : $(x+y+z)=r^2+\left(\frac{lx+my+nz}{\sqrt{l^2+m^2+n^2}}\right)^2$

Q.no 10.

The equation of tangent to the curve at origin represented by the equation $y(1+x^2)=x$ is

A : y=x

B : x=0

C : x=1

D : y=0

Q.no 11. Rectilinear motion is a motion of body along a

A : Straight line

B : Circular path

C : Parabolic path

D : Hyperbolic path

Q.no 12. The general solution of Linear Differential equation $\frac{dx}{dy} + Px = Q$ where P and Q are functions of y or constants, is

A : $x\, e^{\int p dx} = \int Q\, e^{\int p dx} dx + c$

B : $x\, e^{\int p dy} = \int Q\, e^{\int p dy} dy + c$

C : $y\, e^{\int p dx} = \int Q\, e^{\int p dx} dx + c$

D : $y\, e^{\int p dx} = \int Q\, e^{\int p dx} dy + c$

Q.no 13. The solution of the DE $\frac{dy}{dx} = \frac{x^2}{y^3}$ is

A : $y^3 - x^3 = c$

B : $3y^4 - 4x^3 = c$

C : $y^2 - x^3 = c$

D : $x^2 - y^2 = c$

Q.no 14. Half range Fourier cosine series for $f(x)$ defined in the interval $0 \le x \le L$ is

A : Half range Fourier cosine series for $f(x)$ defined in the interval $0 \le x \le L$ is

B : $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\frac{nx}{L}$

C : $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$

D : $\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\, cos\frac{n\pi x}{L}+b_n \sin\frac{n\pi x}{L}\right)$

Q.no 15. Gamma function of n > 0 is defined as

A : $\int_0^{\infty} e^{x} x^{n-1}\, dx$

B : $\int_0^{\infty} e^{x} x^{n-1}\, dx$

C : $\int_0^{\infty} e^{-x} x^{n-1}\, dx$

D : $\int_0^{\infty} e^{-x} x^{1-n}\, dx$

Q.no 16. The value of $erf\,(3)+erf_c\,(3)$ is

A : 3

B : 2

C : 1

D : 0

Q.no 17. If $I(\alpha)=\int_a^b f(x,\alpha)dx$ where α is parameter and a, b are constants then by DUIS rule $\frac{dI(\alpha)}{d\alpha}$ is

A : $\int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)dx$

B : $\int_a^b \frac{\partial}{\partial x} f(x,\alpha)dx$

C : $f(b,\alpha)-f(a,\alpha)$

D : $f(x,\alpha)$

Q.no 18. For RC circuit the charge q satisfies the linear D.E.

A : $R+\frac{dq}{dt}=E$

B : $Ri+q=0$

C : $A=\frac{1}{3}\pi r^2 l$

D : $R\frac{dq}{dt}+\frac{q}{C}=E$

Q.no 19. The integrating factor of $(x^2+y^2+x)dx+xydy=0$ using non exact rule $\frac{\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)}{N}$ is

A : x

B : 1/x

C : x2

D : xy

Q.no 20. To change Cartesian coordinates (x,y,z) to spherical polar coordinate (r,θ,ϕ);$dxdydz$ is replaced by

A : To change Cartesian coordinates (x,y,z) to spherical polar coordinate (r,θ,ϕ);$dxdydz$ is replaced by

B : $r^2\cos\theta dr d\theta d\phi$

C : $r^2 dr d\theta d\phi$

D : $rdrd\theta d\phi$

Q.no 21. Voltage drop across inductance L is given by

A : Li

B : L di/dt

C : dL/dt

D : L dL/dt

Q.no 22. $erf(x) - erfc(x) = 1 \quad erfc(x) = ?$

A : $\frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$

B : $\frac{2}{\sqrt{\pi}} \int_x^\infty e^{-u^2} du$

C : $\frac{2}{\sqrt{\pi}} \int_0^{x^2} e^{-t} t^{-1/2} dt$

D : 0

Q.no 23. Reduction formula for gamma function is

A : $\Gamma(n+1) = (n-1)\Gamma(n-1)$

B : $\Gamma(n+1) = n\Gamma(n)$

C : $\Gamma(n+1) = (n-1)\Gamma(n)$

D : $\Gamma(n+1) = n \quad \Gamma(n-1)$

Q.no 24.

Find semi-vertical angle for a right circular cone with vertex at the point (1,0,1)which passes through the point (1,1,1) and axis of cone has direction ratios 1,1,1

A : $\sin\alpha = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

B : $\cos\alpha = \dfrac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\,\sqrt{a_2^2+b_2^2+c_2^2}}$

C : $\cos\alpha = a_1a_2+b_1b_2+c_1c_2$

D : $\sin\alpha = a_1a_2+b_1b_2+c_1c_2$

Q.no 25. The value of equivalent form of gamma function $\int_0^\infty e^{-kx}x^{n-1}\,dx$ is

A : $\dfrac{\Gamma n}{n^k}$

B : $\dfrac{\Gamma n}{k^n}$

C : $\dfrac{\Gamma n}{k!}$

D : $\Gamma(n+k+1)$

Q.no 26. The value of the integral $\int_0^b e^{-u^2}\,du$ is

A : $\dfrac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\,du$

B : $\dfrac{\sqrt{\pi}}{2}\,erf\,(b)$

C : $\dfrac{2}{\sqrt{\pi}}\,erf_C\,(b)$

D : $erf\,(b)$

Q.no 27. The formula to find the length of the curve $r=f(\theta)$ from $\theta=\theta_1$ to $\theta=\theta_2$ is

A : $\int_{\theta_1}^{\theta_2} \sqrt{1+\left(\frac{dr}{d\theta}\right)^2}\, d\theta$

B : $\int_{\theta_1}^{\theta_2} \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\, d\theta$

C : $\int_{\theta_1}^{\theta_2} \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\, dr$

D : $\int_{\theta_2}^{\theta_1} \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\, d\theta$

Q.no 28. If $I_n=\int_0^{\pi/2} sin^n x\, dx$, where n is positive even integer then I_n is calculated from

A : $\frac{n-1}{n}.\frac{n-3}{n-2.}.\frac{n-5}{n-4}\dots\dots\frac{3}{4}.\frac{1}{2}$

B : $\frac{n-1}{n}.\frac{n-3}{n-2.}.\frac{n-5}{n-4}\dots\dots\frac{3}{4}.\frac{1}{2}.\pi$

C : $\left(\frac{n+1}{n}.\frac{n.+3}{n+2.}.\frac{n+5}{n+4}\right)\dots\dots\frac{\pi}{2}$

D : $\frac{n-1}{n}.\frac{n-3}{n-2.}.\frac{n-5}{n-4}\dots\dots\frac{3}{4}.\frac{1}{2}.\frac{\pi}{2}$

Q.no 29. The value of λ for which the differential equation $(xy^2+\lambda x^2y)dx + (x+y)x^2dy = 0$ is exact

A : 2

B : -3

C : 3

D : -2

Q.no 30. If the sphere $x^2 + y^2 + z^2 = 25$,

$$x^2 + y^2 + z^2 - 18x - 24y - 40z + 225 = 0$$

touches each other externally then distance between their centers is:

A : 20

B : 5

C : 15

D : 25

Q.no 31. Which of the following is true?

A : $erf(x) - erfc(x) = 1$

B : $erf(x) + erfc(x) = 1$

C : $erf(x) + erfc(x) = 2$

D : $erf(-x) = erf(x)$

Q.no 32. The integrating factor for the DE $(1 + x^2)\frac{dy}{dx} + xy = 0$ is

A : e^{1+x^2}

B : $\log(x^2 + 1)$

C : $\sqrt{x^2 + 1}$

D : $1 + x^2$

Q.no 33. A particle moving in straight line with acceleration $k(x + \frac{a^4}{x^3})$ directed towards origin. The equation of motion is

A : $\frac{dv}{dx} = -k(x + \frac{a^4}{x^3})$

B : $v\frac{dv}{dx} = k(x + \frac{a^4}{x^3})$

C : $v\frac{dv}{dx} = -k(x + \frac{a^4}{x^3})$

D : $\frac{dv}{dx} = (x + \frac{a^4}{x^3})$

Q.no 34.

The equation of asymptotes parallel to y-axis to the curve represented by the equation

$y^2(4 - x) = x(x - 2)^2$ is

A : x=2

B : y=0

C : x=4

D : x=0

Q.no 35. The differential equation for the current in an electric circuit containing resistance R and inductance L in series with voltage source E sin(wt) is

A : $L\frac{di}{dt} + \frac{q}{C} = E$

B : $Li + R\frac{di}{dt} = Esin\omega t$

C : $L\frac{di}{dt} + Ri = 0$

D : $L\frac{di}{dt} + Ri = 0$

Q.no 36. The curve $xy^2 = a^2(a - x)$

A : passes through the point (-a,0)

B : does not pass through origin

C : passes through the origin

D : passes through the point (a,a)

Q.no 37.

Fourier series representation of periodic function $f(x)$ with period 2π which satisfies the Dirichlet's conditions is

A : $\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$

B : $\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n \cos n\pi x + b_n \sin n\pi x)$

C : $\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n \cos nx)(b_n \sin nx)$

D : $\frac{a_0}{2}+(a_n \cos nx + b_n \sin nx)$

Q.no 38. The center and radius of the sphere $3(x^2+\ y^2+\ z^2)-30x+12y-18z+89=0$ is

A : (10,-4,6) , $\frac{5}{\sqrt{3}}$

B : (15,-6,9) , $\frac{\sqrt{203}}{\sqrt{3}}$

C : (5,-2,3) , $\frac{5}{\sqrt{3}}$

D : (-5,2,-3) , $\frac{5}{3}$

Q.no 39. If n is a positive integer, then $\Gamma(n+1)$ is

A : (n+1)!

B : (n+2)!

C : (n-1)!

D : n!

Q.no 40. The formula to find the length of the curve $y = f(x)$ from $x = a$ to $x = b$ is $s =$

A : $\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

B : $\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dy$

C : $\int_a^b \sqrt{a^2 + \left(\frac{dy}{dx}\right)^2}\, dx$

D : $\int_a^b \sqrt{a^2 + b^2\left(\frac{dy}{dx}\right)^2}\, dy$

Q.no 41. The differential equation for the current i in an electric circuit containing resistance 100 ohm and an inductance of 0.5 henery connected in series with battery of 20 volts is

A : $0.5\frac{di}{dt} + 100i = 0$

B : $0.5\frac{di}{dt} + 100i = 20$

C : $100\frac{di}{dt} + 0.5i = 0$

D : $100\frac{di}{dt} + 0.5R = 0$

Q.no 42. The integrating factor of the DE $\frac{dx}{dy} + \frac{3}{y}x = \frac{1}{y^2}$

A : $-y^3$

B : y^3

C : x^2y

D : x^3

Q.no 43.

Axis of the right circular cylinder is $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-2}{5}$ what are direction cosines of axis

A : $\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

B : $\left(\frac{2}{\sqrt{30}}, \frac{-1}{\sqrt{30}}, \frac{5}{\sqrt{30}}\right)$

C : $\left(\frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}\right)$

D : $\left(\frac{2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{5}{\sqrt{30}}\right)$

Q.no 44.

The equation of sphere which passes through the point (4, 6, 3) and passes through the circle $(x-1)^2 + (y-2)^2 = 25,\ z = 0$ is

A : $x^2 + y^2 + z^2 - 2x - 4y - 3z - 20 = 0$

B : $x^2 + y^2 + z^2 + 3x - 4y + 3z - 16 = 0$

C : $2x^2 + 2y^2 + 2z^2 + 6x - 3y + 2z = 0$

D : $x^2 + y^2 + z^2 + 4x - 4y + 3z - 20 = 0$

Q.no 45. The value of $erf(0) + erf(\infty)$ is

A : 1

B : -1

C : 0

D : ∞

Q.no 46. If $I(\alpha) = \int_0^1 \frac{x^\alpha - 1}{\log x} dx$, $\alpha \geq 0$ then by DUIS $\frac{dI}{d\alpha}$ is

A : $\int_0^1 \log x \, dx$

B : $\int_0^1 x^{\alpha} \log x \, dx$

C : $\int_0^1 x^{\alpha} dx$

D : $\int_0^1 x^{-\alpha} dx$

Q.no 47. In a circuit containing resistance R and inductance L in series with constant voltage source E current i is given by $i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$, then maximum current $i_{\max}$ is

A : E/R

B : R/E

C : ER

D : 0

Q.no 48. A capacitor C=0.01 farad in series with resistor R=20 ohms is charged from battery E=10 volts. If initially capacitor is completely discharged then differential equation for charge q(t) is given by

A : $20\frac{dq}{dt} + \frac{q}{0.01} = 0 \; ; \; q(0) = 0$

B : $20\frac{dq}{dt} + 0.01q = 10 \; ; \; q(0) = 0$

C : $20\frac{dq}{dt} + \frac{q}{0.01} = 10 \; ; \; q(0) = 0$

D : $20\frac{dq}{dt} + 0.01q = 0 \; ; \; q(0) = 0$

Q.no 49. Radius of right circular cylinder is 2 and direction ratios 2,-1,5 and fixed point on axis of the line (1,-3,2)then the equation of right circular cylinder is

A : $(x+1)^2+(y+3)^2+(z-2)^2=2^2+\left(\frac{2x-y+5z-15}{\sqrt{30}}\right)^2$

B : $(x+1)^2+(y+3)^2+(z-2)^2=2^2+\left(\frac{2x-y+5z-15}{\sqrt{30}}\right)^2$

C : $(x-1)^2+(y+3)^2-(z-2)^2=2^2+\left(\frac{2x-y-5z-15}{\sqrt{30}}\right)^2$

D : $(x-1)^2+(y+3)^2+(z-2)^2=2^2+\left(\frac{2x-y+5z-15}{\sqrt{30}}\right)^2$

Q.no 50. B(4,5)=

A : 1/280

B : 4/105

C : 1/70

D : 1/210

Q.no 51. If $I(\alpha)=\int_a^b f(x,\alpha)dx$ where $a,\ b$ are function of parameter α then by DUIS rule $\frac{dI(\alpha)}{d\alpha}$ is

A : $\int_a^b \frac{\partial}{\partial\alpha}f(x,\alpha)dx+f(a,\alpha)\frac{da}{d\alpha}-f(b,\alpha)\frac{db}{d\alpha}$

B : $\int_a^b \frac{\partial}{\partial\alpha}f(x,\alpha)dx+f(b,\alpha)\frac{db}{d\alpha}-f(a,\alpha)\frac{da}{d\alpha}$

C : $\int_a^b \frac{\partial}{\partial\alpha}f(x,\alpha)dx+f(b,\alpha)\frac{db}{d\alpha}-f(a,\alpha)\frac{da}{d\alpha}$

D : $\int_a^b \frac{\partial}{\partial\alpha}f(x,\alpha)dx$

Q.no 52. Radius of right circular cylinder is 5and whose axis is the line $\frac{x-2}{3} = \frac{y-3}{1} = \frac{z+1}{1}$ Then the equation of right circular cylinder is

A : $(x+2)^2 + (y+3)^2 + (z+1)^2 = 5^2 + \left(\frac{3x+y+z-8}{\sqrt{11}}\right)^2$

B : $(x-2)^2 + (y-3)^2 + (z-1)^2 = 5^2 + \left(\frac{3x+y+z-8}{\sqrt{11}}\right)^2$

C : $(x+2)^2 + (y-3)^2 + (z+1)^2 = 5^2 + \left(\frac{3x+y+z-8}{\sqrt{11}}\right)^2$

D : $(x-2)^2 + (y-3)^2 + (z+1)^2 = 5^2 + \left(\frac{3x+y+z-8}{\sqrt{11}}\right)^2$

Q.no 53. The solution of the DE $\frac{dy}{dx} + y = e^{-x}$ is

A : $ye^{-x} + c = x$

B : $ye^{x} + x = c$

C : $ye^{x} = x + c$

D : $y = x + c$

Q.no 54. The value of integration $\int_0^1 \int_x^{\sqrt{x}} xy\,dx\,dy$ is equal to

A : 0

B : -1/24

C : 1/24

D : 24

Q.no 55.

Find the equation of right circular cone whose vertex is at origin, whose axis is the line $\frac{x}{2} = \frac{y}{-1} = \frac{z}{2}$ and semi-vertical angle $\frac{\pi}{4}$

A : $3(x^2 + y^2 + z^2) = 4\,(2x - y + 2z)^2$

B : $9(x^2+y^2+z^2)=2\,(2x-y+2z)^2$

C : $6(x^2+y^2+z^2)=2\,(2x-y+2z)^2$

D : $(x^2+y^2+z^2)=\,(2x-y+2z)^2$

Q.no 56. $B\left(\frac{1}{4},\frac{3}{4}\right)$ is equal to

A : $\frac{2\pi}{\sqrt{3}}$

B : $\frac{2\pi}{\sqrt{3}}$

C : 0

D : $\pi\sqrt{2}$

Q.no 57.

The two spheres $x^2+\ y^2+\ z^2-\ 12x-18y-24z+29=0$ and $x^2+\ y^2+\ z^2+\ 3x-3y+3z-56=0$

A : touches each other internally

B : touches each other externally

C : touches each other Orthogonally

D : Intersecting each other

Q.no 58. The value of integration $\int_0^1 \int_{x^2}^{x} xy(x+y)dxdy$ is equal to

A : 3/15

B : 2/15

C : 4/15

D : 1/15

Q.no 59. Fourier coefficient a_0 in the Fourier series expansion of

$f(x) = x\sin x\,;\, 0 \le x \le 2\pi$ and $f(x+2\pi) = f(x)$

A : 2

B : 0

C : -2

D : -4

Q.no 60. The value of integration $\int_0^2 \int_0^{x^2} e^{\frac{y}{x}} dxdy$ is equal to

A : $e^2 - 2$

B : $2e^2 - 1$

C : $e^2 - 1$

D : $e^2 + 1$

Answer for Question No 1. is d

Answer for Question No 2. is b

Answer for Question No 3. is d

Answer for Question No 4. is b

Answer for Question No 5. is d

Answer for Question No 6. is c

Answer for Question No 7. is a

Answer for Question No 8. is c

Answer for Question No 9. is a

Answer for Question No 10. is a

Answer for Question No 11. is a

Answer for Question No 12. is b

Answer for Question No 13. is b

Answer for Question No 14. is c

Answer for Question No 15. is c

Answer for Question No 16. is c

Answer for Question No 17. is a

Answer for Question No 18. is d

Answer for Question No 19. is a

Answer for Question No 20. is a

Answer for Question No 21. is b

Answer for Question No 22. is b

Answer for Question No 23. is b

Answer for Question No 24. is b

Answer for Question No 25. is b

Answer for Question No 26. is b

Answer for Question No 27. is b

Answer for Question No 28. is d

Answer for Question No 29. is c

Answer for Question No 30. is d

Answer for Question No 31. is b

Answer for Question No 32. is b

Answer for Question No 33. is c

Answer for Question No 34. is c

Answer for Question No 35. is d

Answer for Question No 36. is b

Answer for Question No 37. is b

Answer for Question No 38. is c

Answer for Question No 39. is d

Answer for Question No 40. is a

Answer for Question No 41. is b

Answer for Question No 42. is b

Answer for Question No 43. is b

Answer for Question No 44. is a

Answer for Question No 45. is a

Answer for Question No 46. is c

Answer for Question No 47. is a

Answer for Question No 48. is c

Answer for Question No 49. is d

Answer for Question No 50. is a

Answer for Question No 51. is c

Answer for Question No 52. is d

Answer for Question No 53. is c

Answer for Question No 54. is c

Answer for Question No 55. is b

Answer for Question No 56. is d

Answer for Question No 57. is c

Answer for Question No 58. is b

Answer for Question No 59. is c

Answer for Question No 60. is c

Seat No -

Total number of questions : 60

1000505 T1 ENGINEERING MATHEMATICS II

Time : 1hr

Max Marks : 50

N.B

1) All questions are Multiple Choice Questions having single correct option.

2) Attempt any 50 questions out of 60.

3) Use of calculator is allowed.

4) Each question carries 1 Mark.

5) Specially abled students are allowed 20 minutes extra for examination.

6) Do not use pencils to darken answer.

7) Use only black/blue ball point pen to darken the appropriate circle.

8) No change will be allowed once the answer is marked on OMR Sheet.

9) Rough work shall not be done on OMR sheet or on question paper.

10) Darken ONLY ONE CIRCLE for each answer.

Q.no 1. The curve $xy^2 = a^2(a-x)$

A : passes through the point (-a,0)

B : does not pass through origin

C : passes through the origin

D : passes through the point (a,a)

Q.no 2.

Find semi-vertical angle for a right circular cone with vertex at the point $(1,0,1)$ which passes through the point $(1,1,1)$ and axis of cone has direction ratios 1,1,1

A : $\sin\alpha = \dfrac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\,\sqrt{a_2^2+b_2^2+c_2^2}}$

B : $\cos\alpha = \dfrac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\,\sqrt{a_2^2+b_2^2+c_2^2}}$

C : $\cos\alpha = a_1a_2 + b_1b_2 + c_1c_2$

D : $\sin\alpha = a_1a_2 + b_1b_2 + c_1c_2$

Q.no 3. Fourier law of heat conduction states that, the quantity of heat flow across an area A cm2 is

A : proportional to product of area A and temperature gradient dT/dx

B : inversely proportional to product of area A and temperature gradient dT/dx

C : equal to sum of area A and temperature gradient

D : equal to difference of area A and temperature gradient dT/dx

Q.no 4. If the sphere $x^2 + y^2 + z^2 = 25$,

$$x^2 + y^2 + z^2 - 18x - 24y - 40z + 225 = 0$$

touches each other externally then distance between their centers is:

A : 20

B : 5

C : 15

D : 25

Q.no 5. The solution of the DE $\frac{dy}{dx} = \frac{x^2}{y^3}$ is

A : $y^3 - x^3 = c$

B : $3y^4 - 4x^3 = c$

C : $y^2 - x^3 = c$

D : $x^2 - y^2 = c$

Q.no 6. If the differential equation $M(x,y)dx + N(x,y)dy = 0$ is not exact and it can be written as $yf_1(x,y)dx + xf_2(x,y)dy = 0$ then integrating factor is

A : $\frac{1}{My+Nx}; My+Nx \neq 0$

B : $\frac{1}{Mx+Ny}; Mx+Ny \neq 0$

C : $\frac{1}{Mx-Ny}; Mx-Ny \neq 0$

D : $\frac{1}{My-Nx}; My-Nx \neq 0$

Q.no 7. The general solution of Linear Differential equation $\frac{dx}{dy} + Px = Q$ where P and Q are functions of y or constants, is

A : $x\, e^{\int p dx} = \int Q\, e^{\int p dx} dx + c$

B : $x\, e^{\int p dy} = \int Q\, e^{\int p dy} dy + c$

C : $y\, e^{\int p dx} = \int Q\, e^{\int p dx} dx + c$

D : $y\, e^{\int p dx} = \int Q\, e^{\int p dx} dy + c$

Q.no 8. Gamma function of n > 0 is defined as

A : $\int_0^\infty e^x x^{n-1}\, dx$

B : $\int_0^\infty e^x x^{n-1}\, dx$

C : $\int_0^\infty e^{-x} x^{n-1}\, dx$

D : $\int_0^\infty e^{-x} x^{1-n}\, dx$

The value of the integral $\int_0^b e^{-u^2}\,du$ is

Q.no 9.

A : $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\,du$

B : $\frac{\sqrt{\pi}}{2} erf\ (b)$

C : $\frac{2}{\sqrt{\pi}} erf_c\,(b)$

D : $erf\ (b)$

Q.no 10. A circuit containing inductance L capacitance C in series without applied electromotive force. By Kirchhoff's voltage law differential equation for current i is

A : $L\frac{di}{dt}+\frac{q}{c}=0$

B : $L\frac{di}{dt}+Ri=0$

C : $L\frac{di}{dt}+Ri=E$

D : $L\frac{di}{dt}+\frac{q}{c}=E$

In a D.E. $L\frac{di}{dt}+Ri=E$, E means

Q.no 11.

A : Voltage

B : Current

C : Resistance

D : Electromotive force

$\int_0^{\pi/2} sin^p\theta\, cos^q\theta\ d\theta$ is equal to

Q.no 12.

A : $B\left(\frac{p-1}{2},\frac{q-1}{2}\right)$

B : $\frac{1}{2}B\left(\frac{p+1}{2},\frac{q+1}{2}\right)$

C : $B\left(\frac{p+1}{2},\frac{q+1}{2}\right)$

D : $B(p,q)$

Q.no 13. If $I_n=\int_0^{\pi/2} sin^n x\, dx$, where n is positive even integer then I_n is calculated from

A : $\frac{n-1}{n}.\frac{n-3}{n-2.}.\frac{n-5}{n-4}\dots\dots\frac{3}{4}.\frac{1}{2}$

B : $\frac{n-1}{n}.\frac{n-3}{n-2.}.\frac{n-5}{n-4}\dots\dots\frac{3}{4}.\frac{1}{2}.\pi$

C : $\left(\frac{n+1}{n}.\frac{n.+3}{n+2.}.\frac{n+5}{n+4}\right)\dots\dots\frac{\pi}{2}$

D : $\frac{n-1}{n}.\frac{n-3}{n-2.}.\frac{n-5}{n-4}\dots\dots\frac{3}{4}.\frac{1}{2}.\frac{\pi}{2}$

Q.no 14. The integrating factor for the DE $(1+x^2)\frac{dy}{dx}+xy=0$ is

A : e^{1+x^2}

B : $\log(x^2+1)$

C : $\sqrt{x^2+1}$

D : $1+x^2$

Q.no 15. The integrating factor for the DE $\frac{dy}{dx}+\frac{x}{1+x}y=1+x$ is

A : $e^{x(1+x)}$

B : $e^{x(1+x)}$

C : $e^{x/(1+x)}$

D : $1+x^2$

Q.no 16. The differential equation for the current i in an electric circuit containing resistance 100 ohm and an inductance of 0.5 henery connected in series with battery of 20 volts is

A : $0.5\frac{di}{dt}+100i=0$

B : $0.5\frac{di}{dt}+100i=20$

C : $100\frac{di}{dt}+0.5i=0$

D : $100\frac{di}{dt}+0.5R=0$

Q.no 17. The integrating factor of $(x^2+y^2+x)dx+xydy=0$ using non exact rule $\frac{\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)}{N}$ is

A : x

B : 1/x

C : x2

D : xy

Q.no 18. A pipe contains steam and is protected with a covering then in the formula $q = kA\,\frac{dT}{dt}$

A : $A=2\pi rl$

B : $A=\frac{1}{3}\pi r^2 l$

C : $A = \frac{1}{3}\pi r^2 l$

D : $A = rl$

Q.no 19. The integrating factor of the DE $\frac{dy}{dx} + y\cos x = \frac{\sin 2x}{2}$ is

A : $e^{\cos x}$

B : $e^{\sin x}$

C : sin x

D : cos x

Q.no 20. Reduction formula for gamma function is

A : $\Gamma(n+1) = (n-1)\Gamma(n-1)$

B : $\Gamma(n+1) = n\Gamma(n)$

C : $\Gamma(n+1) = (n-1)\Gamma(n)$

D : $\Gamma(n+1) = n \quad \Gamma(n-1)$

Q.no 21. The integrating factor of the DE $\frac{dx}{dy} + \frac{3}{y}x = \frac{1}{y^2}$

A : $-y^3$

B : y^3

C : x^2y

D : x^3

Q.no 22. According to D'Alembert's principle algebraic sum of forces acting on a body along a given direction is equal to

A : Velocity x Acceleration

B : Mass x Velocity

C : Mass x displacement

D : Mass x Acceleration

Q.no 23. $erf(x) - erfc(x) = 1 \quad erfc(x) = ?$

A : $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\,du$

B : $\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2}\,du$

C : $\frac{2}{\sqrt{\pi}}\int_0^{x^2} e^{-t}t^{-1/2}\,dt$

D : 0

Q.no 24. If the equation of curve remains unchanged by replacing y by -y, then the curve is symmetric about

A : y-axis

B : line y=x

C : x-axis

D : line y=-x

Q.no 25. Moment of inertia of the lamina A about the y axis is equal to

A : $\int\int_A \rho x^2 dxdy$

B : $\int\int_A \rho y^2 dxdy$

C : $\int\int_A \rho(x^2 + y^2) dxdy$

D : $\int\int_A \rho x^2 y^2 dxdy$

Q.no 26. The integrating factor of $\frac{dy}{dx} + Py = Q$ is

A : $e^{\int p dx}$

B : $e^{\int pdy}$

C : $e^{\int Qdx}$

D : $e^{\int Qdy}$

Q.no 27. If $\phi(a) = \int_0^\infty e^{-bx^2} \cos(2ax)\,dx,\ b > 0$ then by DUIS rule $\frac{d\phi}{da}$ is

A : $\int_0^\infty \frac{\partial}{\partial b}\left[e^{-bx^2} \cos(2ax)\right] dx$

B : $e^{-bx^2} \cos(2ax)$

C : $\int_0^\infty \frac{\partial}{\partial x}\left[e^{-bx^2} \cos(2ax)\right] dx$

D : $\int_0^\infty \frac{\partial}{\partial a}\left[e^{-bx^2} \cos(2ax)\right] dx$

Q.no 28. If double integral in Cartesian coordinate is given by $\int\int_R f(x, y)\, dxdy$ then the value of same integral in polar form is

A : $\int\int_R f(r\cos\theta, r\sin\theta)\, rdrd\theta$

B : $\int\int_R f(r\cos\theta, r\sin\theta)\, rdrd\theta$

C : $\int\int_R f(r\cos\theta, r\sin\theta)\, r^2drd\theta$

D : $\int\int_R f(r\sin\theta, r\cos\theta)\, rdrd\theta$

Q.no 29.

Fourier series representation of periodic function $f(x)$ with period 2π which satisfies the Dirichlet's conditions is

A : $\dfrac{a_0}{2}+\sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$

B : $\dfrac{a_0}{2}+\sum_{n=1}^{\infty}(a_n \cos n\pi x + b_n \sin n\pi x)$

C : $\dfrac{a_0}{2}+\sum_{n=1}^{\infty}(a_n \cos nx)(b_n \sin nx)$

D : $\dfrac{a_0}{2}+(a_n \cos nx + b_n \sin nx)$

Q.no 30. B(m,n) is equal to

A : $\int_0^1 x^{m-1}(1-x)^{n-1}dx$

B : $\int_0^{\infty} x^{m-1}(1-x)^{n-1}dx$

C : $\int_0^1 x^{m}(1-x)^{n}dx$

D : $\int_0^1 e^{-x}x^{n-1}dx$

Q.no 31. The integrating factor of $(y^4+2y)dx+(xy^3+2y^4-4x)dy=0$ using non exact rule $\dfrac{\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)}{M}$ is

A : $\dfrac{1}{y^3}$

B : 1/x

C : y^3

D : $\frac{1}{x^3}$

Q.no 32. Moment of inertia of the lamina A about the x axis is equal to

A : $\int\int_A \rho x^2 dxdy$

B : $\int\int_A \rho y^2 dxdy$

C : $\int\int_A \rho(x^2+y^2) dxdy$

D : $\int\int_A \rho x^2 y^2 dxdy$

Q.no 33.

Equation of right circular cylinder whose radius is 'r' and axis is the line $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ is

A : $x^2+y^2+z^2=r^2+\left(\frac{lx+my+nz}{\sqrt{l^2+m^2+n^2}}\right)^2$

B : $\alpha=\cos^{-1}1$

C : $x^2+y^2+z^2=r^2+\frac{lx+my+nz}{\sqrt{l^2+m^2+n^2}}$

D : $(x+y+z)=r^2+\left(\frac{lx+my+nz}{\sqrt{l^2+m^2+n^2}}\right)^2$

Q.no 34. The integrating factor of the DE $cosx\frac{dy}{dx}+ysinx=1$ is

A : sec x

B : cot x

C : tan x

D : sin x

Q.no 35. The formula to find the length of the curve $r=f(\theta)$ from $\theta=\theta_1$ to $\theta=\theta_2$ is

A : $\int_{\theta_1}^{\theta_2} \sqrt{1 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

B : $\int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

C : $\int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, dr$

D : $\int_{\theta_2}^{\theta_1} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

Q.no 36. Error function of x, $erf(x)$ is defined as

A : $\frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2}\, du$

B : $\frac{2}{\sqrt{\pi}} \int_x^\infty e^{-u^2}\, du$

C : $\int_0^\infty e^{-x} x^{n-1}\, dx$

D : $\frac{2}{\sqrt{\pi}} \int_0^\infty e^{-u^2}\, du$

Q.no 37. For RC circuit the charge q satisfies the linear D.E.

A : $R + \frac{dq}{dt} = E$

B : $Ri + q = 0$

C : $A = \frac{1}{3}\pi r^2 l$

D : $R\frac{dq}{dt}+\frac{q}{C}=E$

Q.no 38. If the DE $Mdx+Ndy=0$ can be written as $x^a y^b(mydx+nxdy)+x^r y^s(pydx+qxdy)=0$ then integrating factor is

A : $x^h y^k$

B : x^h

C : xy

D : $\frac{1}{x^h y^k}$

Q.no 39. Rectilinear motion is a motion of body along a

A : Straight line

B : Circular path

C : Parabolic path

D : Hyperbolic path

Q.no 40. Half range Fourier cosine series for $f(x)$ defined in the interval $0\le x\le L$ is

A : Half range Fourier cosine series for $f(x)$ defined in the interval $0\le x\le L$ is

B : $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos\frac{nx}{L}$

C : $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos\frac{n\pi x}{L}$

D : $\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\, cos\frac{n\pi x}{L}+b_n\sin\frac{n\pi x}{L}\right)$

Q.no 41. If $I(\alpha)=\int_{a}^{b} f(x,\alpha)dx$ where α is parameter and a, b are constants then by DUIS rule $\frac{dI(\alpha)}{d\alpha}$ is

A : $\int_{a}^{b} \frac{\partial}{\partial\alpha} f(x,\alpha)dx$

B : $\int_{a}^{b} \frac{\partial}{\partial x} f(x,\alpha)dx$

C : $f(b,\alpha)-f(a,\alpha)$

D : $f(x,\alpha)$

Q.no 42.

The equation of asymptotes parallel to y-axis to the curve represented by the equation $y^2(4-x)=x(x-2)^2$ is

A : x=2

B : y=0

C : x=4

D : x=0

Q.no 43. If $I(\alpha)=\int_0^1 \frac{x^{\alpha}-1}{\log x}dx$, $\alpha\geq 0$ then by DUIS $\frac{dI}{d\alpha}$ is

A : $\int_0^1 \log x\, dx$

B : $\int_0^1 x^{\alpha}\log x\, dx$

C : $\int_0^1 x^{\alpha} dx$

D : $\int_0^1 x^{-\alpha} dx$

Q.no 44. In spherical co-ordinates volume is given by

A : $V = \int \int \int_V dr d\theta d\phi$

B : $V = \int \int \int_V r dr d\theta d\phi$

C : $V = \int \int \int_V r^2 \cos\theta dr d\theta d\phi$

D : $V = \int \int \int_V r^2 \sin\theta dr d\theta d\phi$

Q.no 45. A capacitor C=0.01 farad in series with resistor R=20 ohms is charged from battery E=10 volts. If initially capacitor is completely discharged then differential equation for charge q(t) is given by

A : $20\frac{dq}{dt} + \frac{q}{0.01} = 0 \ ; \ q(0) = 0$

B : $20\frac{dq}{dt} + 0.01q = 10 \ ; \ q(0) = 0$

C : $20\frac{dq}{dt} + \frac{q}{0.01} = 10 \ ; \ q(0) = 0$

D : $20\frac{dq}{dt} + 0.01q = 0 \ ; \ q(0) = 0$

Q.no 46. Radius of right circular cylinder is 2 and direction ratios 2,-1,5 and fixed point on axis of the line (1,-3,2)then the equation of right circular cylinder is

A : $(x+1)^2 + (y+3)^2 + (z-2)^2 = 2^2 + \left(\frac{2x - y + 5z - 15}{\sqrt{30}}\right)^2$

B : $(x+1)^2 + (y+3)^2 + (z-2)^2 = 2^2 + \left(\frac{2x - y + 5z - 15}{\sqrt{30}}\right)^2$

C : $(x-1)^2 + (y+3)^2 - (z-2)^2 = 2^2 + \left(\frac{2x-y-5z-15}{\sqrt{30}}\right)^2$

D : $(x-1)^2 + (y+3)^2 + (z-2)^2 = 2^2 + \left(\frac{2x-y+5z-15}{\sqrt{30}}\right)^2$

Q.no 47.

The equation of sphere which passes through the point (4, 6, 3) and passes through the circle $(x-1)^2 + (y-2)^2 = 25$, $z = 0$ is

A : $x^2 + y^2 + z^2 - 2x - 4y - 3z - 20 = 0$

B : $x^2 + y^2 + z^2 + 3x - 4y + 3z - 16 = 0$

C : $2x^2 + 2y^2 + 2z^2 + 6x - 3y + 2z = 0$

D : $x^2 + y^2 + z^2 + 4x - 4y + 3z - 20 = 0$

Q.no 48. If $I(\alpha) = \int_0^1 \frac{x^\alpha - 1}{\log x} dx$, $\alpha \geq 0$ then by DUIS $\frac{dI}{d\alpha}$ is

A : $\log(\alpha + 1)$

B : $\log(\alpha - 1)$

C : $\log \alpha$

D : 0

Q.no 49. The solution of the DE $\frac{dy}{dx} + y = e^{-x}$ is

A : $ye^{-x} + c = x$

B : $ye^{x} + x = c$

C : $ye^{x} = x + c$

D : $y = x + c$

Q.no 50. B(4,5)=

A : 1/280

B : 4/105

C : 1/70

D : 1/210

Q.no 51. If $I(\alpha)=\int_a^b f(x,\alpha)dx$ where a, b are function of parameter α then by DUIS rule $\frac{dI(\alpha)}{d\alpha}$ is

A : $\int_a^b \frac{\partial}{\partial\alpha}f(x,\alpha)dx+f(a,\alpha)\frac{da}{d\alpha}-f(b,\alpha)\frac{db}{d\alpha}$

B : $\int_a^b \frac{\partial}{\partial\alpha}f(x,\alpha)dx+f(b,\alpha)\frac{db}{d\alpha}-f(a,\alpha)\frac{da}{d\alpha}$

C : $\int_a^b \frac{\partial}{\partial\alpha}f(x,\alpha)dx+f(b,\alpha)\frac{db}{d\alpha}-f(a,\alpha)\frac{da}{d\alpha}$

D : $\int_a^b \frac{\partial}{\partial\alpha}f(x,\alpha)dx$

Q.no 52. The value of integration $\int_0^1 \int_0^x (x^2+y^2)\,dxdy$ is equal to

A : 4/3

B : 5/3

C : 2/3

D : 1

Q.no 53. Radius of right circular cylinder is 5and whose axis is the line $\frac{x-2}{3}=\frac{y-3}{1}=\frac{z+1}{1}$ Then the equation of right circular cylinder is

A : $(x+2)^2+(y+3)^2+(z+1)^2=5^2+\left(\frac{3x+y+z-8}{\sqrt{11}}\right)^2$

B : $(x-2)^2 + (y-3)^2 + (z-1)^2 = 5^2 + \left(\frac{3x+y+z-8}{\sqrt{11}}\right)^2$

C : $(x+2)^2 + (y-3)^2 + (z+1)^2 = 5^2 + \left(\frac{3x+y+z-8}{\sqrt{11}}\right)^2$

D : $(x-2)^2 + (y-3)^2 + (z+1)^2 = 5^2 + \left(\frac{3x+y+z-8}{\sqrt{11}}\right)^2$

Q.no 54. The solution of exact DE $(x+2y-2)dx + (2x-y+3)dy = 0$ is

A : $x^2 + 4xy - 4x + 6y = c$

B : $x^2 + 4xy - 4x - y^2 + 6y = c$

C : $x^2 + 8xy + y^2 = c$

D : $x^2 + 4xy - \frac{y^2}{2} + y = c$

Q.no 55. The value of $erf(0) + erf(\infty)$ is

A : 1

B : -1

C : 0

D : ∞

Q.no 56.

Area bounded by the parabola $y^2 = 4x$ and the line $y = x$ is calculated using

A : $\int_{y=0}^{4} \int_{x=y^2/4}^{y} dxdy$

B : $\int_{y=0}^{4} \int_{x=0}^{4} dxdy$

C : $\int_{y=0}^{4}\int_{x=0}^{y} dxdy$

D : $\int_{x=0}^{4}\int_{y=0}^{x} dxdy$

Q.no 57. In a circuit containing resistance R and inductance L in series with constant voltage source E current i is given by $i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$, then maximum current $i_{\max}$ is

A : E/R

B : R/E

C : ER

D : 0

Q.no 58. $\int_0^{\pi/2} cos^6 x\, dx$ is equal to

A : $\frac{5}{16}$

B : $\frac{16}{5}.\frac{\pi}{2}$

C : $\frac{5}{16}.\frac{\pi}{2}$

D : $\frac{5}{48}.\frac{\pi}{2}$

Q.no 59. $\frac{5\pi}{128}$

A : $\frac{5\pi}{128}$

B : $\frac{3\pi}{128}$

C : $\frac{3\pi}{512}$

D : $\frac{5\pi}{64}$

Q.no 60.
If the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ then the point of contact is given by—

A : (1, 4, 3)

B : (-1, 4, -2)

C : (0, 1,2)

D : (1,2,-1)

Answer for Question No 1. is b

Answer for Question No 2. is b

Answer for Question No 3. is a

Answer for Question No 4. is d

Answer for Question No 5. is b

Answer for Question No 6. is c

Answer for Question No 7. is b

Answer for Question No 8. is c

Answer for Question No 9. is b

Answer for Question No 10. is a

Answer for Question No 11. is d

Answer for Question No 12. is b

Answer for Question No 13. is d

Answer for Question No 14. is b

Answer for Question No 15. is c

Answer for Question No 16. is b

Answer for Question No 17. is a

Answer for Question No 18. is a

Answer for Question No 19. is b

Answer for Question No 20. is b

Answer for Question No 21. is b

Answer for Question No 22. is d

Answer for Question No 23. is b

Answer for Question No 24. is c

Answer for Question No 25. is a

Answer for Question No 26. is a

Answer for Question No 27. is d

Answer for Question No 28. is a

Answer for Question No 29. is b

Answer for Question No 30. is a

Answer for Question No 31. is a

Answer for Question No 32. is b

Answer for Question No 33. is a

Answer for Question No 34. is a

Answer for Question No 35. is b

Answer for Question No 36. is a

Answer for Question No 37. is d

Answer for Question No 38. is a

Answer for Question No 39. is a

Answer for Question No 40. is c

Answer for Question No 41. is a

Answer for Question No 42. is c

Answer for Question No 43. is c

Answer for Question No 44. is d

Answer for Question No 45. is c

Answer for Question No 46. is d

Answer for Question No 47. is a

Answer for Question No 48. is a

Answer for Question No 49. is c

Answer for Question No 50. is a

Answer for Question No 51. is c

Answer for Question No 52. is c

Answer for Question No 53. is d

Answer for Question No 54. is b

Answer for Question No 55. is a

Answer for Question No 56. is a

Answer for Question No 57. is a

Answer for Question No 58. is c

Answer for Question No 59. is b

Answer for Question No 60. is b

Seat No -

Total number of questions : 60

1000505 T1 ENGINEERING MATHEMATICS II

Time : 1hr

Max Marks : 50

N.B

1) All questions are Multiple Choice Questions having single correct option.

2) Attempt any 50 questions out of 60.

3) Use of calculator is allowed.

4) Each question carries 1 Mark.

5) Specially abled students are allowed 20 minutes extra for examination.

6) Do not use pencils to darken answer.

7) Use only black/blue ball point pen to darken the appropriate circle.

8) No change will be allowed once the answer is marked on OMR Sheet.

9) Rough work shall not be done on OMR sheet or on question paper.

10) Darken ONLY ONE CIRCLE for each answer.

Q.no 1. The value of integration $\int_0^a \int_0^b \int_0^c x^2y^2z^2dxdydz$ is equal to

A : $\frac{abc}{3}$

B : $\frac{a^2b^2c^2}{3}$

C : $\frac{a^3b^3c^3}{27}$

D : $\frac{a^2b^2c^2}{9}$

Q.no 2. The integrating factor of the DE $cosx\frac{dy}{dx} + ysinx = 1$ is

A : sec x

B : cot x

C : tan x

D : sin x

Q.no 3. The value of equivalent form of gamma function $\int_0^\infty e^{-kx} x^{n-1}\, dx$ is

A : $\dfrac{\Gamma n}{n^k}$

B : $\dfrac{\Gamma n}{k^n}$

C : $\dfrac{\Gamma n}{k!}$

D : $\Gamma(n+k+1)$

Q.no 4. A circuit containing inductance L capacitance C in series with applied electromotive force E. By Kirchhoff's voltage law differential equation for current i is

A : $L\frac{di}{dt} + Ri = E$

B : $L\frac{di}{dt} + Ri = 0$

C : $L\frac{di}{dt} + \frac{q}{C} = E$

D : $L\frac{di}{dt} + \frac{q}{C} = 0$

Q.no 5. The formula to find the length of the curve $r = f(\theta)$ from $\theta = \theta_1$ to $\theta = \theta_2$ is

A : $\int_{\theta_1}^{\theta_2} \sqrt{1 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

B : $\int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\ d\theta$

C : $\int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\ dr$

D : $\int_{\theta_2}^{\theta_1} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\ d\theta$

Q.no 6.

If $(\bar{x}, \bar{y})$ the coordinates of centroid of the plane area is given by

A : $\bar{x} = \frac{\int\int x\rho dxdy}{\int\int \rho dxdy}, \bar{y} = \frac{\int\int y\rho dxdy}{\int\int \rho dxdy}$

B : $\bar{x} = \frac{\int\int x^2\rho dxdy}{\int\int \rho dxdy}, \bar{y} = \frac{\int\int y^2\rho dxdy}{\int\int \rho dxdy}$

C : $\bar{x} = \frac{\int\int \rho dxdy}{\int\int x\rho dxdy}, \bar{y} = \frac{\int\int \rho dxdy}{\int\int y\rho dxdy}$

D : None of these

Q.no 7. The integrating factor of $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$ using non exact rule $\frac{\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)}{M}$ is

A : $\frac{1}{y^3}$

B : 1/x

C : y^3

D : $\frac{1}{x^3}$

Q.no 8. The differential equation for the current in an electric circuit containing resistance R and inductance L in series with voltage source E sin(wt) is

A : $L\frac{di}{dt}+\frac{q}{C}=E$

B : $Li+R\frac{di}{dt}=Esin\omega t$

C : $L\frac{di}{dt}+Ri=0$

D : $L\frac{di}{dt}+Ri=0$

Q.no 9.

Find semi-vertical angle for a right circular cone with vertex at the point $(1,0,1)$ which passes through the point $(1,1,1)$ and axis of cone has direction ratios 1,1,1

A : $\sin\alpha=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

B : $\cos\alpha=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

C : $\cos\alpha=a_1a_2+b_1b_2+c_1c_2$

D : $\sin\alpha=a_1a_2+b_1b_2+c_1c_2$

Q.no 10.

The equation of asymptotes parallel to y-axis to the curve represented by the equation

$y^2(4-x)=x(x-2)^2$ is

A : x=2

B : y=0

C : x=4

D : x=0

Q.no 11. The integrating factor for the DE $(1+x^2)\frac{dy}{dx}+xy=0$ is

A : e^{1+x^2}

B : $\log(x^2+1)$

C : $\sqrt{x^2+1}$

D : $1+x^2$

Q.no 12. A particle moving in straight line with acceleration $k(x+\frac{a^4}{x^3})$ directed towards origin. The equation of motion is

A : $\frac{dv}{dx}=-k(x+\frac{a^4}{x^3})$

B : $v\frac{dv}{dx}=k(x+\frac{a^4}{x^3})$

C : $v\frac{dv}{dx}=-k(x+\frac{a^4}{x^3})$

D : $\frac{dv}{dx}=(x+\frac{a^4}{x^3})$

Q.no 13. The differential equation for the current i in an electric circuit containing resistance R=250 ohm and an inductance of L=640 henery in series with an electromotive force E=500 volts is

A : $640\frac{di}{dt}+250i=0$

B : $250\frac{di}{dt}+640i=500$

C : $640\frac{di}{dt}+250i=500$

D : $250\frac{di}{dt}+640i=0$

Q.no 14.

Condition for the two spheres $x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$ and $x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 = 0$ to be orthogonal is ---

A : $S + \lambda U = 0$

B : $S_1 - S_2 = 0$

C : $2u_1u_2 + 2v_1v_2 + 2w_1w_2 = d_1 + d_2$

D : $u_1u_2 + v_1v_2 + w_1w_2 = d_1 + d_2$

Q.no 15. If the equation of curve remains unchanged by replacing y by -y, then the curve is symmetric about

A : y-axis

B : line y=x

C : x-axis

D : line y=-x

Q.no 16. If $I(\alpha)=\int_a^b f(x,\alpha)dx$ where α is parameter and a, b are constants then by DUIS rule $\frac{dI(\alpha)}{d\alpha}$ is

A : $\int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)dx$

B : $\int_a^b \frac{\partial}{\partial x} f(x,\alpha)dx$

C : $f(b,\alpha)-f(a,\alpha)$

D : $f(x,\alpha)$

Q.no 17. The center and radius of the sphere $3(x^2 + y^2 + z^2) - 30x + 12y - 18z + 89 = 0$ is

A : $(10,-4,6)$, $\frac{5}{\sqrt{3}}$

B : $(15,-6,9)$, $\frac{\sqrt{203}}{\sqrt{3}}$

C : $(5,-2,3)$, $\frac{5}{\sqrt{3}}$

D : $(-5,2,-3)$, $\frac{5}{3}$

Q.no 18.

Find semi-vertical angle for a right circular cone with vertex at the point $(1,0,1)$which passes through the point $(1,1,1)$ and axis of cone has direction ratios 1,1,1

A : $\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

B : $\alpha = \cos^{-1} 1$

C : $\alpha = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

D : $\alpha = \cos^{-1}(\sqrt{3})$

Q.no 19. The value of $\int\int\int_V dxdydz$ where V is the volume bounded by $x^2 + y^2 + z^2 = 1$

A : $\frac{4}{3}\pi$

B : 4π

C : π

D : $\frac{1}{3}\pi$

Q.no 20. If the sphere $x^2 + y^2 + z^2 = 25$,

$$x^2 + y^2 + z^2 - 18x - 24y - 40z + 225 = 0$$

touches each other externally then distance between their centers is:

A : 20

B : 5

C : 15

D : 25

Q.no 21. The integrating factor of the DE $\frac{dy}{dx} + ycosx = \frac{sin2x}{2}$ is

A : e^{cosx}

B : e^{sinx}

C : sin x

D : cos x

Q.no 22.

The equation of tangent to the curve at origin represented by the equation $y(1 + x^2) = x$ is

A : y=x

B : x=0

C : x=1

D : y=0

Q.no 23. $\int_0^{\pi/2} sin^p\theta cos^q\theta \, d\theta$ is equal to

A : $B\left(\frac{p-1}{2}, \frac{q-1}{2}\right)$

B : $\frac{1}{2}B\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$

C : $B\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$

D : $B(p,q)$

Q.no 24. Fourier law of heat conduction states that, the quantity of heat flow across an area A cm2 is

A : proportional to product of area A and temperature gradient dT/dx

B : inversely proportional to product of area A and temperature gradient dT/dx

C : equal to sum of area A and temperature gradient

D : equal to difference of area A and temperature gradient dT/dx

Q.no 25. The integrating factor of the DE $\frac{dx}{dy} + \frac{3}{y}x = \frac{1}{y^2}$

A : $-y^3$

B : y^3

C : x^2y

D : x^3

Q.no 26. The integrating factor for the DE $\frac{dy}{dx} + \frac{x}{1+x}y = 1 + x$ is

A : $e^{x(1+x)}$

B : $e^{x(1+x)}$

C : $e^{x/(1+x)}$

D : $1 + x^2$

Q.no 27. For RC circuit the charge q satisfies the linear D.E.

A : $R + \frac{dq}{dt} = E$

B : $Ri + q = 0$

C : $A = \frac{1}{3}\pi r^2 l$

D : $R\frac{dq}{dt} + \frac{q}{C} = E$

Q.no 28. Reduction formula for gamma function is

A : $\Gamma(n+1) = (n-1)\Gamma(n-1)$

B : $\Gamma(n+1) = n\Gamma(n)$

C : $\Gamma(n+1) = (n-1)\Gamma(n)$

D : $\Gamma(n+1) = n \quad \Gamma(n-1)$

Q.no 29. $erf(x) - erfc(x) = 1 \quad erfc\,(x) = ?$

A : $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\,du$

B : $\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2}\,du$

C : $\frac{2}{\sqrt{\pi}}\int_0^{x^2} e^{-t}t^{-1/2}\,dt$

D : 0

Q.no 30. If q be the quantity of heat that flows across an area A cm2 and thickness δx in one second where the difference of temperature at the faces is δT, then by Fourier's law of heat conduction

A : $q = -k\left(A - \frac{dt}{dx}\right)$, where k is thermal conductivity

B : $q = kA\frac{dT}{dx}$, where k is thermal conductivity

C : $q = -k\left(A + \frac{dt}{dx}\right)$, where k is thermal conductivity

D : $q = -kA\frac{dT}{dx}$, , where k is thermal conductivity

Q.no 31. The value of λ for which the differential equation $(xy^2 + \lambda x^2 y)dx + (x+y)x^2 dy = 0$ is exact

A : 2

B : -3

C : 3

D : -2

Q.no 32. Rectilinear motion is a motion of body along a

A : Straight line

B : Circular path

C : Parabolic path

D : Hyperbolic path

Q.no 33. To change Cartesian coordinates (x, y, z) to spherical polar coordinate (r, θ, ϕ);$dxdydz$ is replaced by

A : To change Cartesian coordinates (x, y, z) to spherical polar coordinate (r, θ, ϕ);$dxdydz$ is replaced by

B : $r^2 \cos\theta dr d\theta d\phi$

C : $r^2 dr d\theta d\phi$

D : $r dr d\theta d\phi$

Q.no 34. The integrating factor of $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$ using non exact rule $\frac{\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)}{M}$ is

A : y^2

B : $\frac{1}{y^2}$

C : y

D : 1/y

In a D.E. $L\frac{di}{dt} + Ri = E$, E means

Q.no 35.

A : Voltage

B : Current

C : Resistance

D : Electromotive force

Area enclosed by the polar curve $r = f_1(\theta)$ and $r = f_2(\theta)$ and the line $\theta = \alpha, \theta = \beta\alpha < \beta$ is given by

Q.no 36.

A : $Area = \int_\alpha^\beta \int_{f_1(\theta)}^{f_2(\theta)} r dr d\theta$

B : $Area = \int_\alpha^\beta \int_{f_1(\theta)}^{f_2(\theta)} dr d\theta$

C : $Area = \int_\alpha^\beta \int_{f_1(\theta)}^{f_2(\theta)} r^2 \sin\theta dr d\theta$

D : $Area = \int_\alpha^\beta \int_{f_1(\theta)}^{f_2(\theta)} r^2 \cos\theta dr d\theta$

A pipe contains steam and is protected with a covering then in the formula $q = kA\,\frac{dT}{dt}$

Q.no 37.

A : $A = 2\pi rl$

B : $A = \frac{1}{3}\pi r^2 l$

C : $A = \frac{1}{3}\pi r^2 l$

D : $A = rl$

Q.no 38. If $I_n = \int_0^{\pi/2} sin^n x\, dx$, where n is positive even integer then I_n is calculated from

A : $\frac{n-1}{n} \cdot \frac{n-3}{n-2.} \cdot \frac{n-5}{n-4} \ldots\ldots \frac{3}{4} \cdot \frac{1}{2}$

B : $\frac{n-1}{n} \cdot \frac{n-3}{n-2.} \cdot \frac{n-5}{n-4} \ldots\ldots \frac{3}{4} \cdot \frac{1}{2} \cdot \pi$

C : $\left(\frac{n+1}{n} \cdot \frac{n.+3}{n+2.} \cdot \frac{n+5}{n+4}\right) \ldots\ldots \frac{\pi}{2}$

D : $\frac{n-1}{n} \cdot \frac{n-3}{n-2.} \cdot \frac{n-5}{n-4} \ldots\ldots \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}$

Q.no 39.

Equation of right circular cylinder whose radius is ‘r’ and axis is the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ is

A : $x^2 + y^2 + z^2 = r^2 + \left(\frac{lx + my + nz}{\sqrt{l^2 + m^2 + n^2}}\right)^2$

B : $\alpha = \cos^{-1} 1$

C : $x^2 + y^2 + z^2 = r^2 + \frac{lx + my + nz}{\sqrt{l^2 + m^2 + n^2}}$

D : $(x + y + z) = r^2 + \left(\frac{lx + my + nz}{\sqrt{l^2 + m^2 + n^2}}\right)^2$

Q.no 40. The amount of heat Q flowing through the area per unit time is

A : Q = thermal conductivity x Area x Rate of change of temp. across an area

B : Q = thermal conductivity x Area of slab

C : Q = thermal conductivity x Area x temp. gradient

D : Q = thermal conductivity x Area + Rate of flow of heat

Q.no 41. If double integral in Cartesian coordinate is given by $\int\int_R f(x,y)\,dxdy$ then the value of same integral in polar form is

A : $\int\int_R f(r\cos\theta, r\sin\theta)\,rdrd\theta$

B : $\int\int_R f(r\cos\theta, r\sin\theta)\,rdrd\theta$

C : $\int\int_R f(r\cos\theta, r\sin\theta)\,r^2drd\theta$

D : $\int\int_R f(r\sin\theta, r\cos\theta)\,rdrd\theta$

Q.no 42. Error function of x, $erf(x)$ is defined as

A : $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$

B : $\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2}du$

C : $\int_0^\infty e^{-x}x^{n-1}dx$

D : $\frac{2}{\sqrt{\pi}}\int_0^\infty e^{-u^2}du$

Q.no 43. B(4,5)=

A : 1/280

B : 4/105

C : 1/70

D : 1/210

Q.no 44.

If the plane $4x - 3y + 6z - 35 = 0$ is tangential to the sphere

$x^2 + y^2 + z^2 - y - 2z - 14 = 0$ then the length of perpendicular from any point P on the plane is---

A : $\frac{61}{2}$

B : $\frac{61}{2}$

C : 30

D : $-\frac{\sqrt{61}}{2}$

Q.no 45.

A particle of mass m moves in a horizontal straight line OA with acceleration $\frac{k}{x^3}$ at a distance x and directed towards origin O. Then the differential equation of motion is

A : $v\frac{dv}{dx} = \frac{k}{x^3}$

B : $v\frac{dv}{dx} = -\frac{k}{x^3}$

C : $\frac{dv}{dx} = -\frac{k}{x^3}$

D : $\frac{dv}{dx} = \frac{k}{x^3}$

Q.no 46.

Area bounded by the parabola $y^2 = 4x$ and the line $y = x$ is calculated using

A : $\int_{y=0}^{4} \int_{x=y^2/4}^{y} dxdy$

B : $\int_{y=0}^{4} \int_{x=0}^{4} dxdy$

C : $\int_{y=0}^{4} \int_{x=0}^{y} dxdy$

D : $\int_{x=0}^{4} \int_{y=0}^{x} dxdy$

Q.no 47. If $I(\alpha) = \int_0^1 \frac{x^\alpha - 1}{\log x} dx$, $\alpha \geq 0$ then by DUIS $\frac{dI}{d\alpha}$ is

A : $\log(\alpha + 1)$

B : $\log(\alpha - 1)$

C : $\log \alpha$

D : 0

Q.no 48. In spherical co-ordinates volume is given by

A : $V = \int \int \int_V drd\theta d\phi$

B : $V = \int \int \int_V rdrd\theta d\phi$

C : $V = \int \int \int_V r^2 \cos\theta drd\theta d\phi$

D : $V = \int \int \int_V r^2 \sin\theta drd\theta d\phi$

Q.no 49.

Axis of the right circular cylinder is $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-2}{5}$ what are direction cosines of axis

A : $\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

B : $\left(\frac{2}{\sqrt{30}}, \frac{-1}{\sqrt{30}}, \frac{5}{\sqrt{30}}\right)$

C : $\left(\frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}\right)$

D : $\left(\frac{2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{5}{\sqrt{30}}\right)$

Q.no 50.

If $I(\alpha) = \int_a^b f(x,\alpha)dx$ where a, b are function of parameter α then by DUIS rule $\frac{dI(\alpha)}{d\alpha}$ is

A : $\int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)dx + f(a,\alpha)\frac{da}{d\alpha} - f(b,\alpha)\frac{db}{d\alpha}$

B : $\int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)dx + f(b,\alpha)\frac{db}{d\alpha} - f(a,\alpha)\frac{da}{d\alpha}$

C : $\int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)dx + f(b,\alpha)\frac{db}{d\alpha} - f(a,\alpha)\frac{da}{d\alpha}$

D : $\int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)dx$

Q.no 51.

Find the equation of right circular cone whose vertex is at origin, whose axis is the line $\frac{x}{2} = \frac{y}{-1} = \frac{z}{2}$ and semi-vertical angle $\frac{\pi}{4}$

A : $3(x^2 + y^2 + z^2) = 4\,(2x - y + 2z)^2$

B : $9(x^2+y^2+z^2) = 2\,(2x-y+2z)^2$

C : $6(x^2+y^2+z^2) = 2\,(2x-y+2z)^2$

D : $(x^2+y^2+z^2) = (2x-y+2z)^2$

Q.no 52. The value of integration $\int_0^1 \int_x^{\sqrt{x}} xydxdy$ is equal to

A : 0

B : -1/24

C : 1/24

D : 24

Q.no 53. The solution of exact DE $(x+2y-2)dx + (2x-y+3)dy = 0$ is

A : $x^2+4xy-4x+6y=c$

B : $x^2+4xy-4x-y^2+6y=c$

C : $x^2+8xy+y^2=c$

D : $x^2+4xy-\frac{y^2}{2}+y=c$

Q.no 54. $B\left(\frac{1}{4},\frac{3}{4}\right)$ is equal to

A : $\frac{2\pi}{\sqrt{3}}$

B : $\frac{2\pi}{\sqrt{3}}$

C : 0

D : $\pi\sqrt{2}$

Q.no 55.

The value of integration $\int_0^1 \int_0^x (x^2 + y^2)\, dx dy$ is equal to

A : 4/3

B : 5/3

C : 2/3

D : 1

Q.no 56. q56.jpg

A : 1

B : 0

C : q56_3.jpg

D : q56_4.jpg

In a circuit containing resistance R and inductance L in series with constant voltage source E current i is given by $i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$, then maximum current $i_{\max}$ is

Q.no 57.

A : E/R

B : R/E

C : ER

D : 0

Q.no 58.

If the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ then the point of contact is given by—

A : (1, 4, 3)

B : (-1, 4, -2)

C : (0, 1,2)

D : (1,2,-1)

Q.no 59. The value of integration $\int_0^2 \int_0^{x^2} e^{\frac{y}{x}} dxdy$ is equal to

A : $e^2 - 2$

B : $2e^2 - 1$

C : $e^2 - 1$

D : $e^2 + 1$

Q.no 60.

The value of integration $\int_0^1 \int_{x^2}^{x} xy(x+y) dxdy$ is equal to

A : 3/15

B : 2/15

C : 4/15

D : 1/15

Answer for Question No 1. is c

Answer for Question No 2. is a

Answer for Question No 3. is b

Answer for Question No 4. is c

Answer for Question No 5. is b

Answer for Question No 6. is a

Answer for Question No 7. is a

Answer for Question No 8. is d

Answer for Question No 9. is b

Answer for Question No 10. is c

Answer for Question No 11. is b

Answer for Question No 12. is c

Answer for Question No 13. is c

Answer for Question No 14. is c

Answer for Question No 15. is c

Answer for Question No 16. is a

Answer for Question No 17. is c

Answer for Question No 18. is a

Answer for Question No 19. is a

Answer for Question No 20. is d

Answer for Question No 21. is b

Answer for Question No 22. is a

Answer for Question No 23. is b

Answer for Question No 24. is a

Answer for Question No 25. is b

Answer for Question No 26. is c

Answer for Question No 27. is d

Answer for Question No 28. is b

Answer for Question No 29. is b

Answer for Question No 30. is d

Answer for Question No 31. is c

Answer for Question No 32. is a

Answer for Question No 33. is a

Answer for Question No 34. is b

Answer for Question No 35. is d

Answer for Question No 36. is a

Answer for Question No 37. is a

Answer for Question No 38. is d

Answer for Question No 39. is a

Answer for Question No 40. is c

Answer for Question No 41. is a

Answer for Question No 42. is a

Answer for Question No 43. is a

Answer for Question No 44. is a

Answer for Question No 45. is b

Answer for Question No 46. is a

Answer for Question No 47. is a

Answer for Question No 48. is d

Answer for Question No 49. is b

Answer for Question No 50. is c

Answer for Question No 51. is b

Answer for Question No 52. is c

Answer for Question No 53. is b

Answer for Question No 54. is d

Answer for Question No 55. is c

Answer for Question No 56. is c

Answer for Question No 57. is a

Answer for Question No 58. is b

Answer for Question No 59. is c

Answer for Question No 60. is b

Seat No -

Total number of questions : 60

1000505 T1 ENGINEERING MATHEMATICS II

Time : 1hr

Max Marks : 50

N.B

1) All questions are Multiple Choice Questions having single correct option.

2) Attempt any 50 questions out of 60.

3) Use of calculator is allowed.

4) Each question carries 1 Mark.

5) Specially abled students are allowed 20 minutes extra for examination.

6) Do not use pencils to darken answer.

7) Use only black/blue ball point pen to darken the appropriate circle.

8) No change will be allowed once the answer is marked on OMR Sheet.

9) Rough work shall not be done on OMR sheet or on question paper.

10) Darken ONLY ONE CIRCLE for each answer.

Q.no 1. To change Cartesian coordinates (x, y, z) to spherical polar coordinate (r, θ, ϕ);$dxdydz$ is replaced by

A : To change Cartesian coordinates (x, y, z) to spherical polar coordinate (r, θ, ϕ);$dxdydz$ is replaced by

B : $r^2 \cos\theta dr d\theta d\phi$

C : $r^2 dr d\theta d\phi$

D : $r dr d\theta d\phi$

Q.no 2. If n is a positive integer, then $\Gamma(n+1)$ is

A : (n+1)!

B : (n+2)!

C : (n-1)!

D : n!

Q.no 3.

Fourier series representation of periodic function $f(x)$ with period 2π which satisfies the Dirichlet's conditions is

A : $\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$

B : $\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n \cos n\pi x + b_n \sin n\pi x)$

C : $\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n \cos nx)(b_n \sin nx)$

D : $\frac{a_0}{2}+(a_n \cos nx + b_n \sin nx)$

Q.no 4. A circuit containing inductance L capacitance C in series without applied electromotive force. By Kirchhoff's voltage law differential equation for current i is

A : $L\frac{di}{dt}+\frac{q}{c}=0$

B : $L\frac{di}{dt}+Ri=0$

C : $L\frac{di}{dt}+Ri=E$

D : $L\frac{di}{dt}+\frac{q}{c}=E$

Q.no 5. $erf(x)-erfc(x)=1 \quad erfc(x)=?$

A : $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\,du$

B : $\frac{2}{\sqrt{\pi}}\int_{x}^{\infty} e^{-u^2}\,du$

C : $\frac{2}{\sqrt{\pi}}\int_{0}^{x^2} e^{-t}t^{-1/2}\,dt$

D : 0

Q.no 6. The value of $\int\int\int_V dxdydz$ where V is the volume bounded by $x^2+y^2+z^2=1$

A : $\frac{4}{3}\pi$

B : 4π

C : π

D : $\frac{1}{3}\pi$

Q.no 7. Reduction formula for gamma function is

A : $\Gamma(n+1)=(n-1)\Gamma(n-1)$

B : $\Gamma(n+1)=n\Gamma(n)$

C : $\Gamma(n+1)=(n-1)\Gamma(n)$

D : $\Gamma(n+1)=n \quad \Gamma(n-1)$

Q.no 8. If $I_n=\int_0^{\pi/2} sin^n x\, dx$, where n is positive even integer then I_n is calculated from

A : $\frac{n-1}{n}.\frac{n-3}{n-2.}.\frac{n-5}{n-4}\dots\dots\frac{3}{4}.\frac{1}{2}$

B : $\frac{n-1}{n}.\frac{n-3}{n-2.}.\frac{n-5}{n-4}\dots\dots\frac{3}{4}.\frac{1}{2}.\pi$

C : $\left(\frac{n+1}{n}.\frac{n.+3}{n+2.}.\frac{n+5}{n+4}\right)\dots\dots\frac{\pi}{2}$

D : $\frac{n-1}{n}.\frac{n-3}{n-2.}.\frac{n-5}{n-4}\dots\dots\frac{3}{4}.\frac{1}{2}.\frac{\pi}{2}$

Q.no 9. Error function of x, $erf(x)$ is defined as

A : $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$

B : $\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2}du$

C : $\int_0^\infty e^{-x}x^{n-1}dx$

D : $\frac{2}{\sqrt{\pi}}\int_0^\infty e^{-u^2}du$

Q.no 10. The necessary and sufficient condition that the Differential equation $Mdx + Ndy = 0$ is exact is

A : $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$

B : $\frac{\partial M}{\partial x}=\frac{\partial N}{\partial y}$

C : $\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}$

D : $\frac{\partial M}{\partial x}=\frac{\partial N}{\partial x}$

Q.no 11. If double integral in Cartesian coordinate is given by $\int\int_R f(x,y)\,dxdy$ then the value of same integral in polar form is

A : $\int\int_R f(r\cos\theta, r\sin\theta)\, r dr d\theta$

B : $\int\int_R f(r\cos\theta, r\sin\theta)\, r dr d\theta$

C : $\int\int_R f(r\cos\theta, r\sin\theta)\, r^2 dr d\theta$

D : $\int\int_R f(r\sin\theta, r\cos\theta)\, r dr d\theta$

Q.no 12.

The equation of tangent to the curve at origin represented by the equation $y(1+x^2) = x$ is

A : y=x

B : x=0

C : x=1

D : y=0

Q.no 13. The integrating factor for the DE $\frac{dy}{dx} + \frac{x}{1+x} y = 1 + x$ is

A : $e^{x(1+x)}$

B : $e^{x(1+x)}$

C : $e^{x/(1+x)}$

D : $1 + x^2$

Q.no 14. B(m,n) is equal to

A : $\int_0^1 x^{m-1}(1-x)^{n-1} dx$

B : $\int_0^\infty x^{m-1}(1-x)^{n-1} dx$

C : $\int_0^1 x^m (1-x)^n dx$

D : $\int_0^1 e^{-x} x^{n-1} dx$

Q.no 15. Rectilinear motion is a motion of body along a

A : Straight line

B : Circular path

C : Parabolic path

D : Hyperbolic path

Q.no 16. Moment of inertia of the lamina A about the x axis is equal to

A : $\int \int_A \rho x^2 dxdy$

B : $\int \int_A \rho y^2 dxdy$

C : $\int \int_A \rho (x^2 + y^2) dxdy$

D : $\int \int_A \rho x^2 y^2 dxdy$

Q.no 17. The value of λ for which the differential equation $(xy^2 + \lambda x^2 y)dx + (x+y)x^2 dy = 0$ is exact

A : 2

B : -3

C : 3

D : -2

Q.no 18. According to D'Alembert's principle algebraic sum of forces acting on a body along a given direction is equal to

A : Velocity x Acceleration

B : Mass x Velocity

C : Mass x displacement

D : Mass x Acceleration

Q.no 19. The integrating factor of $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$ using non exact rule $\frac{\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)}{M}$ is

A : $\frac{1}{y^3}$

B : 1/x

C : y^3

D : $\frac{1}{x^3}$

Q.no 20. If the DE $Mdx + Ndy = 0$ can be written as $x^a y^b(mydx + nxdy) + x^r y^s(pydx + qxdy) = 0$ then integrating factor is

A : $x^h y^k$

B : x^h

C : xy

D : $\frac{1}{x^h y^k}$

Q.no 21. The solution of the DE $\frac{dy}{dx} = \frac{x^2}{y^3}$ is

A : $y^3 - x^3 = c$

B : $3y^4 - 4x^3 = c$

C : $y^2 - x^3 = c$

D : $x^2 - y^2 = c$

Q.no 22. Which of the following is true?

A : $erf(x) - erfc(x) = 1$

B : $erf(x) + erfc(x) = 1$

C : $erf(x) + erfc(x) = 2$

D : $erf(-x) = erf(x)$

Q.no 23. The integrating factor of the DE $\frac{dy}{dx} + ycosx = \frac{sin2x}{2}$ is

A : e^{cosx}

B : e^{sinx}

C : sin x

D : cos x

Q.no 24. A particle moving in straight line with acceleration $k(x + \frac{a^4}{x^3})$ directed towards origin. The equation of motion is

A : $\frac{dv}{dx} = -k(x + \frac{a^4}{x^3})$

B : $v\frac{dv}{dx} = k(x + \frac{a^4}{x^3})$

C : $v\frac{dv}{dx} = -k(x + \frac{a^4}{x^3})$

D : $\frac{dv}{dx} = (x + \frac{a^4}{x^3})$

Q.no 25. The integrating factor of $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$ using non exact rule $\frac{\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)}{M}$ is

A : y^2

B : $\frac{1}{y^2}$

C : y

D : 1/y

Q.no 26. Gamma function of n > 0 is defined as

A : $\int_0^\infty e^x x^{n-1}\, dx$

B : $\int_0^\infty e^x x^{n-1}\, dx$

C : $\int_0^\infty e^{-x} x^{n-1}\, dx$

D : $\int_0^\infty e^{-x} x^{1-n}\, dx$

Q.no 27. The general solution of Linear Differential equation $\frac{dx}{dy} + Px = Q$ where P and Q are functions of y or constants, is

A : $x\, e^{\int p dx} = \int Q\, e^{\int p dx} dx + c$

B : $x\, e^{\int p dy} = \int Q\, e^{\int p dy} dy + c$

C : $y\, e^{\int p dx} = \int Q\, e^{\int p dx} dx + c$

D : $y\, e^{\int p dx} = \int Q\, e^{\int p dx} dy + c$

Q.no 28.
Condition for the two spheres $x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$ and $x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 = 0$ to be orthogonal is ---

A : $S + \lambda U = 0$

B : $S_1 - S_2 = 0$

C : $2u_1u_2 + 2v_1v_2 + 2w_1w_2 = d_1 + d_2$

D : $u_1u_2 + v_1v_2 + w_1w_2 = d_1 + d_2$

Q.no 29. The integrating factor for the DE $(1+x^2)\frac{dy}{dx} + xy = 0$ is

A : e^{1+x^2}

B : $\log(x^2+1)$

C : $\sqrt{x^2+1}$

D : $1+x^2$

Q.no 30.

Equation of right circular cylinder whose radius is ‘r’ and axis is the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ is

A : $x^2+y^2+z^2 = r^2 + \left(\frac{lx+my+nz}{\sqrt{l^2+m^2+n^2}}\right)^2$

B : $\alpha = \cos^{-1} 1$

C : $x^2+y^2+z^2 = r^2 + \frac{lx+my+nz}{\sqrt{l^2+m^2+n^2}}$

D : $(x+y+z) = r^2 + \left(\frac{lx+my+nz}{\sqrt{l^2+m^2+n^2}}\right)^2$

Q.no 31. The formula to find the length of the curve $r = f(\theta)$ from $\theta = \theta_1$ to $\theta = \theta_2$ is

A : $\int_{\theta_1}^{\theta_2} \sqrt{1 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

B : $\int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

C : $\int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, dr$

D : $\int_{\theta_2}^{\theta_1} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

Q.no 32. Half range Fourier cosine series for $f(x)$ defined in the interval $0 \leq x \leq L$ is

A : Half range Fourier cosine series for $f(x)$ defined in the interval $0 \leq x \leq L$ is

B : $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{nx}{L}$

C : $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$

D : $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$

Q.no 33. For RC circuit the charge q satisfies the linear D.E.

A : $R + \frac{dq}{dt} = E$

B : $Ri + q = 0$

C : $A = \frac{1}{3}\pi r^2 l$

D : $R\frac{dq}{dt} + \frac{q}{C} = E$

Q.no 34. The differential equation for the current i in an electric circuit containing resistance 100 ohm and an inductance of 0.5 henery connected in series with battery of 20 volts is

A : $0.5\frac{di}{dt} + 100i = 0$

B : $0.5\frac{di}{dt} + 100i = 20$

C : $100\frac{di}{dt} + 0.5i = 0$

D : $100\frac{di}{dt} + 0.5R = 0$

Q.no 35. The differential equation for the current i in an electric circuit containing resistance R=250 ohm and an inductance of L=640 henery in series with an electromotive force E=500 volts is

A : $640\frac{di}{dt} + 250i = 0$

B : $250\frac{di}{dt} + 640i = 500$

C : $640\frac{di}{dt} + 250i = 500$

D : $250\frac{di}{dt} + 640i = 0$

Q.no 36. The integrating factor of the DE $\frac{dx}{dy} + \frac{3}{y}x = \frac{1}{y^2}$

A : $-y^3$

B : y^3

C : x^2y

D : x^3

Q.no 37.

If $(\bar{x}, \bar{y})$ the coordinates of centroid of the plane area is given by

A : $\bar{x} = \frac{\int\int x\rho dxdy}{\int\int \rho dxdy}, \bar{y} = \frac{\int\int y\rho dxdy}{\int\int \rho dxdy}$

B : $\bar{x} = \frac{\int\int x^2 \rho dxdy}{\int\int \rho dxdy}, \bar{y} = \frac{\int\int y^2 \rho dxdy}{\int\int \rho dxdy}$

C : $\bar{x} = \frac{\int\int \rho dxdy}{\int\int x\rho dxdy}, \bar{y} = \frac{\int\int \rho dxdy}{\int\int y\rho dxdy}$

D : None of these

Q.no 38. If the equation of curve remains unchanged by replacing y by -y, then the curve is symmetric about

A : y-axis

B : line y=x

C : x-axis

D : line y=-x

Q.no 39. If $\phi(a) = \int_0^\infty e^{-bx^2} \cos(2ax)\, dx,\ b > 0$ then by DUIS rule $\frac{d\phi}{da}$ is

A : $\int_0^\infty \frac{\partial}{\partial b}\left[e^{-bx^2} \cos(2ax)\right] dx$

B : $e^{-bx^2} \cos(2ax)$

C : $\int_0^\infty \frac{\partial}{\partial x}\left[e^{-bx^2} \cos(2ax)\right] dx$

D : $\int_0^\infty \frac{\partial}{\partial a}\left[e^{-bx^2} \cos(2ax)\right] dx$

Q.no 40. Fourier law of heat conduction states that, the quantity of heat flow across an area A cm2 is

A : proportional to product of area A and temperature gradient dT/dx

B : inversely proportional to product of area A and temperature gradient dT/dx

C : equal to sum of area A and temperature gradient

D : equal to difference of area A and temperature gradient dT/dx

Q.no 41. Moment of inertia of the lamina A about the y axis is equal to

A : $\int \int_A \rho x^2 dxdy$

B : $\int \int_A \rho y^2 dxdy$

C : $\int \int_A \rho(x^2 + y^2) dxdy$

D : $\int \int_A \rho x^2 y^2 dxdy$

Q.no 42. The value of the integral $\int_0^b e^{-u^2} du$ is

A : $\frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$

B : $\frac{\sqrt{\pi}}{2} erf\ (b)$

C : $\frac{2}{\sqrt{\pi}} erf_C\ (b)$

D : $erf\ (b)$

Q.no 43.

The two spheres $x^2 + y^2 + z^2 - 12x - 18y - 24z + 29 = 0$ and $x^2 + y^2 + z^2 + 3x - 3y + 3z - 56 = 0$

A : touches each other internally

B : touches each other externally

C : touches each other Orthogonally

D : Intersecting each other

Q.no 44. The value of integration $\int_0^1 \int_0^{x^2} xe^y dxdy$ is equal to

A : e/2

B : e-1

C : 1-e

D : (e/2)-1

Q.no 45. q56.jpg

A : 1

B : 0

C : q56_3.jpg

D : q56_4.jpg

Q.no 46.

Axis of the right circular cylinder is $\frac{x-1}{2}=\frac{y+3}{-1}=\frac{z-2}{5}$ what are direction cosines of axis

A : $\alpha=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

B : $\left(\frac{2}{\sqrt{30}},\frac{-1}{\sqrt{30}},\frac{5}{\sqrt{30}}\right)$

C : $\left(\frac{-2}{\sqrt{30}},\frac{1}{\sqrt{30}},\frac{-5}{\sqrt{30}}\right)$

D : $\left(\frac{2}{\sqrt{30}},\frac{1}{\sqrt{30}},\frac{5}{\sqrt{30}}\right)$

Q.no 47. In a circuit containing resistance R and inductance L in series with constant voltage source E current i is given by $i=\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$, then maximum current $i_{\max}$ is

A : E/R

B : R/E

C : ER

D : 0

Q.no 48. The solution of exact DE $(x+2y-2)dx+(2x-y+3)dy=0$ is

A : $x^2+4xy-4x+6y=c$

B : $x^2+4xy-4x-y^2+6y=c$

C : $x^2+8xy+y^2=c$

D : $x^2+4xy-\frac{y^2}{2}+y=c$

Q.no 49. The value of integration $\int_0^1\int_0^x (x^2+y^2)\,dxdy$ is equal to

A : 4/3

B : 5/3

C : 2/3

D : 1

Q.no 50.

The equation of sphere which passes through the point (4, 6, 3) and passes through the circle $(x-1)^2+(y-2)^2=25$, $z=0$ is

A : $x^2+y^2+z^2-2x-4y-3z-20=0$

B : $x^2+y^2+z^2+3x-4y+3z-16=0$

C : $2x^2+2y^2+2z^2+6x-3y+2z=0$

D : $x^2+y^2+z^2+4x-4y+3z-20=0$

Q.no 51. $B\left(\frac{1}{4},\frac{3}{4}\right)$ is equal to

A : $\frac{2\pi}{\sqrt{3}}$

B : $\frac{2\pi}{\sqrt{3}}$

C : 0

D : $\pi\sqrt{2}$

Q.no 52. The value of $erf(0)+erf(\infty)$ is

A : 1

B : -1

C : 0

D : ∞

Q.no 53.
If the plane $4x-3y+6z-35=0$ is tangential to the sphere

$x^2+y^2+z^2-y-2z-14=0$ then the length of perpendicular from any point P on the plane is---

A : $\frac{61}{2}$

B : $\frac{61}{2}$

C : 30

D : $-\frac{\sqrt{61}}{2}$

Q.no 54. Radius of right circular cylinder is 5and whose axis is the line $\frac{x-2}{3}=\frac{y-3}{1}=\frac{z+1}{1}$ Then the equation of right circular cylinder is

A : $(x+2)^2+(y+3)^2+(z+1)^2=5^2+\left(\frac{3x+y+z-8}{\sqrt{11}}\right)^2$

B : $(x-2)^2+(y-3)^2+(z-1)^2=5^2+\left(\frac{3x+y+z-8}{\sqrt{11}}\right)^2$

C : $(x+2)^2 + (y-3)^2 + (z+1)^2 = 5^2 + \left(\frac{3x+y+z-8}{\sqrt{11}}\right)^2$

D : $(x-2)^2 + (y-3)^2 + (z+1)^2 = 5^2 + \left(\frac{3x+y+z-8}{\sqrt{11}}\right)^2$

Q.no 55. If $I(\alpha) = \int_0^1 \frac{x^\alpha - 1}{\log x} dx,\ \alpha \geq 0$ then by DUIS $\frac{dI}{d\alpha}$ is

A : $\log(\alpha+1)$

B : $\log(\alpha-1)$

C : $\log\alpha$

D : 0

Q.no 56. The value of integration $\int_0^1 \int_x^{\sqrt{x}} xydxdy$ is equal to

A : 0

B : -1/24

C : 1/24

D : 24

Q.no 57. In spherical co-ordinates volume is given by

A : $V = \int\int\int_V drd\theta d\phi$

B : $V = \int\int\int_V rdrd\theta d\phi$

C : $V = \int\int\int_V r^2 \cos\theta drd\theta d\phi$

D : $V = \int\int\int_V r^2 \sin\theta drd\theta d\phi$

Q.no 58. $\int_0^{\pi/2} cos^6 x\, dx$ is equal to

A : $\frac{5}{16}$

B : $\frac{16}{5} . \frac{\pi}{2}$

C : $\frac{5}{16} . \frac{\pi}{2}$

D : $\frac{5}{48} . \frac{\pi}{2}$

Q.no 59. The value of integration $\int_0^2 \int_0^{x^2} e^{\frac{y}{x}} dxdy$ is equal to

A : $e^2 - 2$

B : $2e^2 - 1$

C : $e^2 - 1$

D : $e^2 + 1$

Q.no 60. The value of a_0 in harmonic analysis of y for the following tabulated data is

x	0	1	2	3	4	5	6
y	9	18	24	28	26	20	9

A : 17.85

B : 20.83

C : 35.71

D : 41.66

Answer for Question No 1. is a

Answer for Question No 2. is d

Answer for Question No 3. is b

Answer for Question No 4. is a

Answer for Question No 5. is b

Answer for Question No 6. is a

Answer for Question No 7. is b

Answer for Question No 8. is d

Answer for Question No 9. is a

Answer for Question No 10. is a

Answer for Question No 11. is a

Answer for Question No 12. is a

Answer for Question No 13. is c

Answer for Question No 14. is a

Answer for Question No 15. is a

Answer for Question No 16. is b

Answer for Question No 17. is c

Answer for Question No 18. is d

Answer for Question No 19. is a

Answer for Question No 20. is a

Answer for Question No 21. is b

Answer for Question No 22. is b

Answer for Question No 23. is b

Answer for Question No 24. is c

Answer for Question No 25. is b

Answer for Question No 26. is c

Answer for Question No 27. is b

Answer for Question No 28. is c

Answer for Question No 29. is b

Answer for Question No 30. is a

Answer for Question No 31. is b

Answer for Question No 32. is c

Answer for Question No 33. is d

Answer for Question No 34. is b

Answer for Question No 35. is c

Answer for Question No 36. is b

Answer for Question No 37. is a

Answer for Question No 38. is c

Answer for Question No 39. is d

Answer for Question No 40. is a

Answer for Question No 41. is a

Answer for Question No 42. is b

Answer for Question No 43. is c

Answer for Question No 44. is b

Answer for Question No 45. is c

Answer for Question No 46. is b

Answer for Question No 47. is a

Answer for Question No 48. is b

Answer for Question No 49. is c

Answer for Question No 50. is a

Answer for Question No 51. is d

Answer for Question No 52. is a

Answer for Question No 53. is a

Answer for Question No 54. is d

Answer for Question No 55. is a

Answer for Question No 56. is c

Answer for Question No 57. is d

Answer for Question No 58. is c

Answer for Question No 59. is c

Answer for Question No 60. is d

Seat No -

Total number of questions : 60

1000505 T1 ENGINEERING MATHEMATICS II

Time : 1hr

Max Marks : 50

N.B

1) All questions are Multiple Choice Questions having single correct option.

2) Attempt any 50 questions out of 60.

3) Use of calculator is allowed.

4) Each question carries 1 Mark.

5) Specially abled students are allowed 20 minutes extra for examination.

6) Do not use pencils to darken answer.

7) Use only black/blue ball point pen to darken the appropriate circle.

8) No change will be allowed once the answer is marked on OMR Sheet.

9) Rough work shall not be done on OMR sheet or on question paper.

10) Darken ONLY ONE CIRCLE for each answer.

Q.no 1. To change Cartesian coordinates (x, y, z) to spherical polar coordinate (r, θ, ϕ);$dxdydz$ is replaced by

A : To change Cartesian coordinates (x, y, z) to spherical polar coordinate (r, θ, ϕ);$dxdydz$ is replaced by

B : $r^2 \cos\theta dr d\theta d\phi$

C : $r^2 dr d\theta d\phi$

D : $r dr d\theta d\phi$

Q.no 2. Which of the following is true?

A : $erf(x) - erfc(x) = 1$

B : $erf(x) + erfc(x) = 1$

C : $erf(x) + erfc(x) = 2$

D : $erf(-x) = erf(x)$

Q.no 3. A circuit containing inductance L capacitance C in series with applied electromotive force E. By Kirchhoff's voltage law differential equation for current i is

A : $L\frac{di}{dt} + Ri = E$

B : $L\frac{di}{dt} + Ri = 0$

C : $L\frac{di}{dt} + \frac{q}{C} = E$

D : $L\frac{di}{dt} + \frac{q}{C} = 0$

Q.no 4. A pipe contains steam and is protected with a covering then in the formula $q = kA\,\frac{dT}{dt}$

A : $A = 2\pi rl$

B : $A = \frac{1}{3}\pi r^2 l$

C : $A = \frac{1}{3}\pi r^2 l$

D : $A = rl$

Q.no 5. Moment of inertia of the lamina A about the y axis is equal to

A : $\int\int_A \rho x^2\,dx\,dy$

B : $\int\int_A \rho y^2 dxdy$

C : $\int\int_A \rho(x^2+y^2)dxdy$

D : $\int\int_A \rho x^2y^2 dxdy$

Q.no 6. The formula to find the length of the curve $r = f(\theta)$ from $\theta = \theta_1$ to $\theta = \theta_2$ is

A : $\int_{\theta_1}^{\theta_2} \sqrt{1 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

B : $\int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

C : $\int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, dr$

D : $\int_{\theta_2}^{\theta_1} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

Q.no 7. The amount of heat Q flowing through the area per unit time is

A : Q = thermal conductivity x Area x Rate of change of temp. across an area

B : Q = thermal conductivity x Area of slab

C : Q = thermal conductivity x Area x temp. gradient

D : Q = thermal conductivity x Area + Rate of flow of heat

Q.no 8. The integrating factor of $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$ using non exact rule $\frac{\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)}{M}$ is

A : $\frac{1}{y^3}$

B : 1/x

C : y^3

D : $\frac{1}{x^3}$

Q.no 9.

Find semi-vertical angle for a right circular cone with vertex at the point (1,0,1)which passes through the point (1,1,1) and axis of cone has direction ratios 1,1,1

A : $\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

B : $\alpha = \cos^{-1} 1$

C : $\alpha = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

D : $\alpha = \cos^{-1}(\sqrt{3})$

In a D.E. $L\frac{di}{dt} + Ri = E$, E means

Q.no 10.

A : Voltage

B : Current

C : Resistance

D : Electromotive force

The center and radius of the sphere $3(x^2 + y^2 + z^2) - 30x + 12y - 18z + 89 = 0$ is

Q.no 11.

A : (10,-4,6) , $\frac{5}{\sqrt{3}}$

B : $(15,-6,9)$, $\frac{\sqrt{203}}{\sqrt{3}}$

C : $(5,-2,3)$, $\frac{5}{\sqrt{3}}$

D : $(-5,2,-3)$, $\frac{5}{3}$

Q.no 12. The value of $erf\,(3) + erf_c\,(3)$ is

A : 3

B : 2

C : 1

D : 0

Q.no 13. If double integral in Cartesian coordinate is given by $\int\int_R f(x,y)\,dxdy$ then the value of same integral in polar form is

A : $\int\int_R f(r\cos\theta, r\sin\theta)\, rdrd\theta$

B : $\int\int_R f(r\cos\theta, r\sin\theta)\, rdrd\theta$

C : $\int\int_R f(r\cos\theta, r\sin\theta)\, r^2drd\theta$

D : $\int\int_R f(r\sin\theta, r\cos\theta)\, rdrd\theta$

Q.no 14.

The formula to find the length of the curve $y = f(x)$ from $x = a$ to $x = b$ is $s =$

A : $\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

B : $\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dy$

C : $\int_a^b \sqrt{a^2 + \left(\frac{dy}{dx}\right)^2}\, dx$

D : $\int_a^b \sqrt{a^2 + b^2\left(\frac{dy}{dx}\right)^2}\, dy$

Q.no 15. $\int_0^{\pi/2} sin^p\theta cos^q\theta\, d\theta$ is equal to

A : $B\left(\frac{p-1}{2}, \frac{q-1}{2}\right)$

B : $\frac{1}{2}B\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$

C : $B\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$

D : $B(p,q)$

Q.no 16. The value of λ for which the differential equation $(xy^2 + \lambda x^2 y)dx + (x+y)x^2 dy = 0$ is exact

A : 2

B : -3

C : 3

D : -2

Q.no 17. The value of integration $\int_0^a \int_0^b \int_0^c x^2y^2z^2 dxdydz$ is equal to

A : $\frac{abc}{3}$

B : $\frac{a^2b^2c^2}{3}$

C : $\frac{a^3b^3c^3}{27}$

D : $\frac{a^2b^2c^2}{9}$

Q.no 18. If the sphere $x^2 + y^2 + z^2 = 25$,

$$x^2 + y^2 + z^2 - 18x - 24y - 40z + 225 = 0$$

touches each other externally then distance between their centers is:

A : 20

B : 5

C : 15

D : 25

Q.no 19. The integrating factor of the DE $\frac{dy}{dx} + ycosx = \frac{sin2x}{2}$ is

A : e^{cosx}

B : e^{sinx}

C : sin x

D : cos x

Q.no 20. Error function of x, $erf(x)$ is defined as

A : $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\,du$

B : $\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2}\,du$

C : $\int_0^\infty e^{-x}x^{n-1}dx$

D : $\frac{2}{\sqrt{\pi}}\int_0^{\infty} e^{-u^2}du$

Q.no 21. The differential equation for the current i in an electric circuit containing resistance R=250 ohm and an inductance of L=640 henery in series with an electromotive force E=500 volts is

A : $640\frac{di}{dt}+250i=0$

B : $250\frac{di}{dt}+640i=500$

C : $640\frac{di}{dt}+250i=500$

D : $250\frac{di}{dt}+640i=0$

Q.no 22. Rectilinear motion is a motion of body along a

A : Straight line

B : Circular path

C : Parabolic path

D : Hyperbolic path

Q.no 23. The integrating factor of $(x^2+y^2+x)dx+xydy=0$ using non exact rule $\frac{\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)}{N}$ is

A : x

B : 1/x

C : x2

D : xy

Q.no 24. If n is a positive integer, then $\Gamma(n+1)$ is

A : (n+1)!

B : (n+2)!

C : (n-1)!

D : n!

Q.no 25. Voltage drop across inductance L is given by

A : Li

B : L di/dt

C : dL/dt

D : L dL/dt

Q.no 26. Fourier law of heat conduction states that, the quantity of heat flow across an area A cm2 is

A : proportional to product of area A and temperature gradient dT/dx

B : inversely proportional to product of area A and temperature gradient dT/dx

C : equal to sum of area A and temperature gradient

D : equal to difference of area A and temperature gradient dT/dx

Q.no 27. A circuit containing inductance L capacitance C in series without applied electromotive force. By Kirchhoff's voltage law differential equation for current i is

A : $L\frac{di}{dt}+\frac{q}{c}=0$

B : $L\frac{di}{dt}+Ri=0$

C : $L\frac{di}{dt}+Ri=E$

D : $L\frac{di}{dt}+\frac{q}{c}=E$

Q.no 28. If the DE $Mdx+Ndy=0$ can be written as $x^a y^b(mydx+nxdy)+x^r y^s(pydx+qxdy)=0$ then integrating factor is

A : $x^h y^k$

B : x^h

C : xy

D : $\frac{1}{x^h y^k}$

Q.no 29. The general solution of Linear Differential equation $\frac{dx}{dy} + Px = Q$ where P and Q are functions of y or constants, is

A : $x\, e^{\int p dx} = \int Q\, e^{\int p dx} dx + c$

B : $x\, e^{\int p dy} = \int Q\, e^{\int p dy} dy + c$

C : $y\, e^{\int p dx} = \int Q\, e^{\int p dx} dx + c$

D : $y\, e^{\int p dx} = \int Q\, e^{\int p dx} dy + c$

Q.no 30. The integrating factor of the DE $\frac{dx}{dy} + \frac{3}{y} x = \frac{1}{y^2}$

A : $-y^3$

B : y^3

C : $x^2 y$

D : x^3

Q.no 31.

If $(\bar{x}, \bar{y})$ the coordinates of centroid of the plane area is given by

A : $\bar{x} = \frac{\int\int x\rho dxdy}{\int\int \rho dxdy}, \bar{y} = \frac{\int\int y\rho dxdy}{\int\int \rho dxdy}$

B : $\bar{x} = \frac{\int\int x^2\rho dxdy}{\int\int \rho dxdy}, \bar{y} = \frac{\int\int y^2\rho dxdy}{\int\int \rho dxdy}$

C : $\bar{x} = \frac{\int\int \rho dxdy}{\int\int x\rho dxdy}, \bar{y} = \frac{\int\int \rho dxdy}{\int\int y\rho dxdy}$

D : None of these

Q.no 32. Half range Fourier cosine series for $f(x)$ defined in the interval $0 \le x \le L$ is

A : Half range Fourier cosine series for $f(x)$ defined in the interval $0 \le x \le L$ is

B : $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{nx}{L}$

C : $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$

D : $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n\, cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$

Q.no 33. If $I_n = \int_0^{\pi/2} sin^n x\, dx$, where n is positive even integer then I_n is calculated from

A : $\frac{n-1}{n} . \frac{n-3}{n-2.} . \frac{n-5}{n-4} \dots\dots \frac{3}{4} . \frac{1}{2}$

B : $\frac{n-1}{n} . \frac{n-3}{n-2.} . \frac{n-5}{n-4} \dots\dots \frac{3}{4} . \frac{1}{2} . \pi$

C : $\left(\frac{n+1}{n} . \frac{n.+3}{n+2.} . \frac{n+5}{n+4}\right) \dots\dots \frac{\pi}{2}$

D : $\frac{n-1}{n} . \frac{n-3}{n-2.} . \frac{n-5}{n-4} \dots\dots \frac{3}{4} . \frac{1}{2} . \frac{\pi}{2}$

Q.no 34. The value of $\int\int\int_V dxdydz$ where V is the volume bounded by $x^2 + y^2 + z^2 = 1$

A : $\frac{4}{3}\pi$

B : 4π

C : π

D : $\frac{1}{3}\pi$

Q.no 35. If the equation of curve remains unchanged by replacing y by -y, then the curve is symmetric about

A : y-axis

B : line y=x

C : x-axis

D : line y=-x

Q.no 36. If the differential equation $M(x,y)dx + N(x,y)dy = 0$ is not exact and it can be written as $yf_1(x,y)dx + xf_2(x,y)dy = 0$ then integrating factor is

A : $\frac{1}{My+Nx}; My+Nx \neq 0$

B : $\frac{1}{Mx+Ny}; Mx+Ny \neq 0$

C : $\frac{1}{Mx-Ny}; Mx-Ny \neq 0$

D : $\frac{1}{My-Nx}; My-Nx \neq 0$

Q.no 37.

The equation of tangent to the curve at origin represented by the equation $y(1+x^2) = x$ is

A : y=x

B : x=0

C : x=1

D : y=0

Q.no 38. If $\phi(a)=\int_0^\infty e^{-bx^2}\cos(2ax)\,dx,\ b>0$ then by DUIS rule $\frac{d\phi}{da}$ is

A : $\int_0^\infty \frac{\partial}{\partial b}\left[e^{-bx^2}\cos(2ax)\right]dx$

B : $e^{-bx^2}\cos(2ax)$

C : $\int_0^\infty \frac{\partial}{\partial x}\left[e^{-bx^2}\cos(2ax)\right]dx$

D : $\int_0^\infty \frac{\partial}{\partial a}\left[e^{-bx^2}\cos(2ax)\right]dx$

Q.no 39. If $I(\alpha)=\int_a^b f(x,\alpha)\,dx$ where α is parameter and a, b are constants then by DUIS rule $\frac{dI(\alpha)}{d\alpha}$ is

A : $\int_a^b \frac{\partial}{\partial \alpha} f(x,\alpha)dx$

B : $\int_a^b \frac{\partial}{\partial x} f(x,\alpha)dx$

C : $f(b,\alpha)-f(a,\alpha)$

D : $f(x,\alpha)$

Q.no 40. Reduction formula for gamma function is

A : $\Gamma(n+1)=(n-1)\Gamma(n-1)$

B : $\Gamma(n+1) = n\Gamma(n)$

C : $\Gamma(n+1) = (n-1)\Gamma(n)$

D : $\Gamma(n+1) = n \quad \Gamma(n-1)$

Q.no 41. Area enclosed by the polar curve $r = f_1(\theta)$ and $r = f_2(\theta)$ and the line $\theta = \alpha, \theta = \beta \alpha < \beta$ is given by

A : $Area = \int_\alpha^\beta \int_{f_1(\theta)}^{f_2(\theta)} r dr d\theta$

B : $Area = \int_\alpha^\beta \int_{f_1(\theta)}^{f_2(\theta)} dr d\theta$

C : $Area = \int_\alpha^\beta \int_{f_1(\theta)}^{f_2(\theta)} r^2 \sin\theta dr d\theta$

D : $Area = \int_\alpha^\beta \int_{f_1(\theta)}^{f_2(\theta)} r^2 \cos\theta dr d\theta$

Q.no 42. The differential equation for the current in an electric circuit containing resistance R and inductance L in series with voltage source E sin(wt) is

A : $L\frac{di}{dt} + \frac{q}{C} = E$

B : $Li + R\frac{di}{dt} = Esin\omega t$

C : $L\frac{di}{dt} + Ri = 0$

D : $L\frac{di}{dt} + Ri = 0$

Q.no 43. If $I(\alpha) = \int_0^1 \frac{x^\alpha - 1}{\log x} dx$, $\alpha \geq 0$ then by DUIS $\frac{dI}{d\alpha}$ is

A : $\log(\alpha + 1)$

B : $\log(\alpha - 1)$

C : $\log \alpha$

D : 0

Q.no 44.
The radius of the circle of intersection of the sphere $x^2 + y^2 + z^2 - 2x + 4y + 2z - 6 = 0$ by the plane $x + 2y + 2z - 4 = 0$ is:

[Centre of the circle is(2,0,1)]

A : 9

B : 3

C : 12

D : $\sqrt{3}$

Q.no 45. The solution of exact DE $(x + 2y - 2)dx + (2x - y + 3)dy = 0$ is

A : $x^2 + 4xy - 4x + 6y = c$

B : $x^2 + 4xy - 4x - y^2 + 6y = c$

C : $x^2 + 8xy + y^2 = c$

D : $x^2 + 4xy - \frac{y^2}{2} + y = c$

Q.no 46. B(4,5)=

A : 1/280

B : 4/105

C : 1/70

D : 1/210

Q.no 47. In a circuit containing resistance R and inductance L in series with constant voltage source E current i is given by $i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$, then maximum current $i_{\max}$ is

A : E/R

B : R/E

C : ER

D : 0

Q.no 48. If $I(\alpha) = \int_0^1 \frac{x^\alpha - 1}{\log x} dx$, $\alpha \geq 0$ then by DUIS $\frac{dI}{d\alpha}$ is

A : $\int_0^1 \log x \, dx$

B : $\int_0^1 x^\alpha \log x \, dx$

C : $\int_0^1 x^\alpha dx$

D : $\int_0^1 x^{-\alpha} dx$

Q.no 49. Radius of right circular cylinder is 2 and direction ratios 2,-1,5 and fixed point on axis of the line (1,-3,2)then the equation of right circular cylinder is

A : $(x+1)^2 + (y+3)^2 + (z-2)^2 = 2^2 + \left(\frac{2x - y + 5z - 15}{\sqrt{30}}\right)^2$

B : $(x+1)^2 + (y+3)^2 + (z-2)^2 = 2^2 + \left(\frac{2x - y + 5z - 15}{\sqrt{30}}\right)^2$

C : $(x-1)^2 + (y+3)^2 - (z-2)^2 = 2^2 + \left(\frac{2x-y-5z-15}{\sqrt{30}}\right)^2$

D : $(x-1)^2 + (y+3)^2 + (z-2)^2 = 2^2 + \left(\frac{2x-y+5z-15}{\sqrt{30}}\right)^2$

Q.no 50.

The value of integration $\int_0^1 \int_0^1 \int_0^1 xyzdxdydz$ is equal to

A : 0.125

B : 0.25

C : -0.125

D : 1

Q.no 51. The value of integration $\int_0^1 \int_0^{x^2} xe^y dxdy$ is equal to

A : e/2

B : e-1

C : 1-e

D : (e/2)-1

Q.no 52. If $I_n = \int_0^{\pi/2} cos^n x\, dx$ then which of the following relation is true?

A : $\frac{n-1}{n} \cdot \frac{n-3}{n-2.} \cdot \frac{n-5}{n-4} \ldots\ldots \frac{3}{4} \cdot \frac{1}{2}$

B : $I_n = \frac{n}{n-1} I_{n-2}$

C : $I_n = \frac{n-1}{n} I_{n-2}$

D : $I_n = n(n+1) I_{n-1}$

Q.no 53.

Axis of the right circular cylinder is $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-2}{5}$ what are direction cosines of axis

A : $\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

B : $\left(\frac{2}{\sqrt{30}}, \frac{-1}{\sqrt{30}}, \frac{5}{\sqrt{30}}\right)$

C : $\left(\frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}\right)$

D : $\left(\frac{2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{5}{\sqrt{30}}\right)$

Q.no 54.

If $I(\alpha) = \int_a^b f(x,\alpha)dx$ where a, b are function of parameter α then by DUIS rule $\frac{dI(\alpha)}{d\alpha}$ is

A : $\int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)dx + f(a,\alpha)\frac{da}{d\alpha} - f(b,\alpha)\frac{db}{d\alpha}$

B : $\int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)dx + f(b,\alpha)\frac{db}{d\alpha} - f(a,\alpha)\frac{da}{d\alpha}$

C : $\int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)dx + f(b,\alpha)\frac{db}{d\alpha} - f(a,\alpha)\frac{da}{d\alpha}$

D : $\int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)dx$

Q.no 55.

A particle of mass m moves in a horizontal straight line OA with acceleration $\frac{k}{x^3}$ at a distance x and directed towards origin O. Then the differential equation of motion is

A : $v\frac{dv}{dx} = \frac{k}{x^3}$

B : $v\frac{dv}{dx} = -\frac{k}{x^3}$

C : $\frac{dv}{dx} = -\frac{k}{x^3}$

D : $\frac{dv}{dx} = \frac{k}{x^3}$

Q.no 56. The value of $erf(0) + erf(\infty)$ is

A : 1

B : -1

C : 0

D : ∞

Q.no 57.

Area bounded by the parabola $y^2 = 4x$ and the line $y = x$ is calculated using

A : $\int_{y=0}^{4} \int_{x=y^2/4}^{y} dxdy$

B : $\int_{y=0}^{4} \int_{x=0}^{4} dxdy$

C : $\int_{y=0}^{4} \int_{x=0}^{y} dxdy$

D : $\int_{x=0}^{4} \int_{y=0}^{x} dxdy$

Q.no 58. Fourier coefficient a_0 in the Fourier series expansion of

$f(x) = x\sin x\,;\, 0 \le x \le 2\pi$ and $f(x+2\pi) = f(x)$

A : 2

B : 0

C : -2

D : -4

Q.no 59. The value of a_0 in harmonic analysis of y for the following tabulated data is

x	0	1	2	3	4	5	6
y	9	18	24	28	26	20	9

A : 17.85

B : 20.83

C : 35.71

D : 41.66

Q.no 60. The value of integration $\int_0^2 \int_0^{x^2} e^{\frac{y}{x}} dx dy$ is equal to

A : $e^2 - 2$

B : $2e^2 - 1$

C : $e^2 - 1$

D : $e^2 + 1$

Answer for Question No 1. is a

Answer for Question No 2. is b

Answer for Question No 3. is c

Answer for Question No 4. is a

Answer for Question No 5. is a

Answer for Question No 6. is b

Answer for Question No 7. is c

Answer for Question No 8. is a

Answer for Question No 9. is a

Answer for Question No 10. is d

Answer for Question No 11. is c

Answer for Question No 12. is c

Answer for Question No 13. is a

Answer for Question No 14. is a

Answer for Question No 15. is b

Answer for Question No 16. is c

Answer for Question No 17. is c

Answer for Question No 18. is d

Answer for Question No 19. is b

Answer for Question No 20. is a

Answer for Question No 21. is c

Answer for Question No 22. is a

Answer for Question No 23. is a

Answer for Question No 24. is d

Answer for Question No 25. is b

Answer for Question No 26. is a

Answer for Question No 27. is a

Answer for Question No 28. is a

Answer for Question No 29. is b

Answer for Question No 30. is b

Answer for Question No 31. is a

Answer for Question No 32. is c

Answer for Question No 33. is d

Answer for Question No 34. is a

Answer for Question No 35. is c

Answer for Question No 36. is c

Answer for Question No 37. is a

Answer for Question No 38. is d

Answer for Question No 39. is a

Answer for Question No 40. is b

Answer for Question No 41. is a

Answer for Question No 42. is d

Answer for Question No 43. is a

Answer for Question No 44. is d

Answer for Question No 45. is b

Answer for Question No 46. is a

Answer for Question No 47. is a

Answer for Question No 48. is c

Answer for Question No 49. is d

Answer for Question No 50. is a

Answer for Question No 51. is b

Answer for Question No 52. is c

Answer for Question No 53. is b

Answer for Question No 54. is c

Answer for Question No 55. is b

Answer for Question No 56. is a

Answer for Question No 57. is a

Answer for Question No 58. is c

Answer for Question No 59. is d

Answer for Question No 60. is c

Seat No -

Total number of questions : 60

1000505 T1 ENGINEERING MATHEMATICS II

Time : 1hr

Max Marks : 50

N.B

1) All questions are Multiple Choice Questions having single correct option.

2) Attempt any 50 questions out of 60.

3) Use of calculator is allowed.

4) Each question carries 1 Mark.

5) Specially abled students are allowed 20 minutes extra for examination.

6) Do not use pencils to darken answer.

7) Use only black/blue ball point pen to darken the appropriate circle.

8) No change will be allowed once the answer is marked on OMR Sheet.

9) Rough work shall not be done on OMR sheet or on question paper.

10) Darken ONLY ONE CIRCLE for each answer.

Q.no 1. The differential equation for the current i in an electric circuit containing resistance R=250 ohm and an inductance of L=640 henery in series with an electromotive force E=500 volts is

A : $640\frac{di}{dt} + 250i = 0$

B : $250\frac{di}{dt} + 640i = 500$

C : $640\frac{di}{dt} + 250i = 500$

D : $250\frac{di}{dt} + 640i = 0$

Q.no 2. The general solution of Linear Differential equation $\frac{dx}{dy} + Px = Q$ where P and Q are functions of y or constants, is

A : $x\, e^{\int p dx} = \int Q\, e^{\int p dx} dx + c$

B : $x\, e^{\int p dy} = \int Q\, e^{\int p dy} dy + c$

C : $y\, e^{\int p dx} = \int Q\, e^{\int p dx} dx + c$

D : $y\, e^{\int p dx} = \int Q\, e^{\int p dx} dy + c$

Q.no 3.

The equation of asymptotes parallel to y-axis to the curve represented by the equation $y^2(4-x) = x(x-2)^2$ is

A : x=2

B : y=0

C : x=4

D : x=0

Q.no 4. The integrating factor of $\frac{dy}{dx} + Py = Q$ is

A : $e^{\int p dx}$

B : $e^{\int p dy}$

C : $e^{\int Q dx}$

D : $e^{\int Q dy}$

Q.no 5. The value of $erf\,(3) + erf_c\,(3)$ is

A : 3

B : 2

C : 1

D : 0

Q.no 6. A circuit containing inductance L capacitance C in series without applied electromotive force. By Kirchhoff's voltage law differential equation for current i is

A : $L\frac{di}{dt} + \frac{q}{c} = 0$

B : $L\frac{di}{dt} + Ri = 0$

C : $L\frac{di}{dt} + Ri = E$

D : $L\frac{di}{dt} + \frac{q}{c} = E$

Q.no 7. The necessary and sufficient condition that the Differential equation $Mdx + Ndy = 0$ is exact is

A : $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

B : $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

C : $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

D : $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial x}$

Q.no 8. The value of λ for which the differential equation $(xy^2 + \lambda x^2 y)dx + (x+y)x^2 dy = 0$ is exact

A : 2

B : -3

C : 3

D : -2

Q.no 9.

Find semi-vertical angle for a right circular cone with vertex at the point $(1,0,1)$ which passes through the point $(1,1,1)$ and axis of cone has direction ratios 1,1,1

A : $\sin\alpha = \dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

B : $\cos\alpha = \dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

C : $\cos\alpha = a_1a_2 + b_1b_2 + c_1c_2$

D : $\sin\alpha = a_1a_2 + b_1b_2 + c_1c_2$

Q.no 10. The formula to find the length of the curve $r = f(\theta)$ from $\theta = \theta_1$ to $\theta = \theta_2$ is

A : $\int_{\theta_1}^{\theta_2} \sqrt{1 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

B : $\int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

C : $\int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, dr$

D : $\int_{\theta_2}^{\theta_1} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

Q.no 11. If $\phi(a) = \int_0^\infty e^{-bx^2}\cos(2ax)\,dx,\ b > 0$ then by DUIS rule $\frac{d\phi}{da}$ is

A : $\int_0^\infty \frac{\partial}{\partial b}\left[e^{-bx^2}\cos(2ax)\right]dx$

B : $e^{-bx^2}\cos(2ax)$

C : $\int_0^\infty \frac{\partial}{\partial x}\left[e^{-bx^2}\cos(2ax)\right]dx$

D : $\int_0^\infty \frac{\partial}{\partial a}\left[e^{-bx^2}\cos(2ax)\right]dx$

Q.no 12.

The formula to find the length of the curve $y = f(x)$ from $x = a$ to $x = b$ is $s =$

A : $\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

B : $\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dy$

C : $\int_a^b \sqrt{a^2+\left(\frac{dy}{dx}\right)^2}\, dx$

D : $\int_a^b \sqrt{a^2+b^2\left(\frac{dy}{dx}\right)^2}\, dy$

Q.no 13. Moment of inertia of the lamina A about the y axis is equal to

A : $\int\int_A \rho x^2 dxdy$

B : $\int\int_A \rho y^2 dxdy$

C : $\int\int_A \rho(x^2+y^2)dxdy$

D : $\int\int_A \rho x^2y^2 dxdy$

Q.no 14. Which of the following is true?

A : $erf(x) - erfc(x) = 1$

B : $erf(x) + erfc(x) = 1$

C : $erf(x) + erfc(x) = 2$

D : $erf(-x) = erf(x)$

Q.no 15.

Find semi-vertical angle for a right circular cone with vertex at the point $(1,0,1)$ which passes through the point $(1,1,1)$ and axis of cone has direction ratios 1,1,1

A : $\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

B : $\alpha = \cos^{-1} 1$

C : $\alpha = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

D : $\alpha = \cos^{-1}(\sqrt{3})$

Q.no 16.

The equation of tangent to the curve at origin represented by the equation $y(1+x^2) = x$ is

A : y=x

B : x=0

C : x=1

D : y=0

Q.no 17.

A particle moving in straight line with acceleration $k(x + \frac{a^4}{x^3})$ directed towards origin. The equation of motion is

A : $\frac{dv}{dx} = -k(x + \frac{a^4}{x^3})$

B : $v\frac{dv}{dx} = k(x + \frac{a^4}{x^3})$

C : $v\frac{dv}{dx} = -k(x + \frac{a^4}{x^3})$

D : $\frac{dv}{dx} = (x + \frac{a^4}{x^3})$

Q.no 18. Half range Fourier cosine series for $f(x)$ defined in the interval $0 \le x \le L$ is

A : Half range Fourier cosine series for $f(x)$ defined in the interval $0 \le x \le L$ is

B : $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{nx}{L}$

C : $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$

D : $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n\, cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$

Q.no 19. The integrating factor of the DE $cosx\frac{dy}{dx} + ysinx = 1$ is

A : sec x

B : cot x

C : tan x

D : sin x

Q.no 20. Fourier law of heat conduction states that, the quantity of heat flow across an area A cm2 is

A : proportional to product of area A and temperature gradient dT/dx

B : inversely proportional to product of area A and temperature gradient dT/dx

C : equal to sum of area A and temperature gradient

D : equal to difference of area A and temperature gradient dT/dx

Q.no 21. The amount of heat Q flowing through the area per unit time is

A : Q = thermal conductivity x Area x Rate of change of temp. across an area

B : Q = thermal conductivity x Area of slab

C : Q = thermal conductivity x Area x temp. gradient

D : Q = thermal conductivity x Area + Rate of flow of heat

Q.no 22. $\int_0^{\pi/2} sin^p\theta cos^q\theta \, d\theta$ is equal to

A : $B\left(\frac{p-1}{2}, \frac{q-1}{2}\right)$

B : $\frac{1}{2}B\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$

C : $B\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$

D : $B(p,q)$

Q.no 23. The curve $xy^2 = a^2(a-x)$

A : passes through the point (-a,0)

B : does not pass through origin

C : passes through the origin

D : passes through the point (a,a)

Q.no 24. If n is a positive integer, then $\Gamma(n+1)$ is

A : (n+1)!

B : (n+2)!

C : (n-1)!

D : n!

Q.no 25. If the differential equation $M(x,y)dx + N(x,y)dy = 0$ is not exact and it can be written as $yf_1(x,y)dx + xf_2(x,y)dy = 0$ then integrating factor is

A : $\frac{1}{My + Nx}; My + Nx \neq 0$

B : $\frac{1}{Mx + Ny}; Mx + Ny \neq 0$

C : $\frac{1}{Mx - Ny}; Mx - Ny \neq 0$

D : $\frac{1}{My - Nx}; My - Nx \neq 0$

Q.no 26. Error function of x, $erf(x)$ is defined as

A : $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$

B : $\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2}du$

C : $\int_0^\infty e^{-x}x^{n-1}dx$

D : $\frac{2}{\sqrt{\pi}}\int_0^\infty e^{-u^2}du$

Q.no 27. $erf(x) - erfc(x) = 1$ $erfc(x) = ?$

A : $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$

B : $\frac{2}{\sqrt{\pi}}\int_{x}^{\infty} e^{-u^2}\,du$

C : $\frac{2}{\sqrt{\pi}}\int_{0}^{x^2} e^{-t}t^{-1/2}\,dt$

D : 0

Q.no 28. Rectilinear motion is a motion of body along a

A : Straight line

B : Circular path

C : Parabolic path

D : Hyperbolic path

Q.no 29. Moment of inertia of the lamina A about the x axis is equal to

A : $\int\int_A \rho x^2 dxdy$

B : $\int\int_A \rho y^2 dxdy$

C : $\int\int_A \rho(x^2+y^2) dxdy$

D : $\int\int_A \rho x^2 y^2 dxdy$

Q.no 30. The value of $\int\int\int_V dxdydz$ where V is the volume bounded by $x^2+y^2+z^2=1$

A : $\frac{4}{3}\pi$

B : 4π

C : π

D : $\frac{1}{3}\pi$

Q.no 31. A circuit containing inductance L capacitance C in series with applied electromotive force E. By Kirchhoff's voltage law differential equation for current i is

A : $L\frac{di}{dt}+Ri=E$

B : $L\frac{di}{dt}+Ri=0$

C : $L\frac{di}{dt}+\frac{q}{C}=E$

D : $L\frac{di}{dt}+\frac{q}{C}=0$

Q.no 32. The integrating factor of the DE $\frac{dy}{dx}+ycosx=\frac{sin2x}{2}$ is

A : e^{cosx}

B : e^{sinx}

C : sin x

D : cos x

Q.no 33. If $I(\alpha)=\int_a^b f(x,\alpha)dx$ where α is parameter and a, b are constants then by DUIS rule $\frac{dI(\alpha)}{d\alpha}$ is

A : $\int_a^b \frac{\partial}{\partial\alpha}f(x,\alpha)dx$

B : $\int_a^b \frac{\partial}{\partial x}f(x,\alpha)dx$

C : $f(b,\alpha)-f(a,\alpha)$

D : $f(x, \alpha)$

Q.no 34. For RC circuit the charge q satisfies the linear D.E.

A : $R + \frac{dq}{dt} = E$

B : $Ri + q = 0$

C : $A = \frac{1}{3}\pi r^2 l$

D : $R\frac{dq}{dt} + \frac{q}{C} = E$

Q.no 35. The integrating factor of $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$ using non exact rule $\frac{\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)}{M}$ is

A : y^2

B : $\frac{1}{y^2}$

C : y

D : 1/y

Q.no 36. The integrating factor for the DE $\frac{dy}{dx} + \frac{x}{1+x}y = 1 + x$ is

A : $e^{x(1+x)}$

B : $e^{x(1+x)}$

C : $e^{x/(1+x)}$

D : $1 + x^2$

Q.no 37. The solution of the DE $\frac{dy}{dx} = \frac{x^2}{y^3}$ is

A : $y^3 - x^3 = c$

B : $3y^4 - 4x^3 = c$

C : $y^2 - x^3 = c$

D : $x^2 - y^2 = c$

Q.no 38. If double integral in Cartesian coordinate is given by $\int\int_R f(x, y)\, dxdy$ then the value of same integral in polar form is

A : $\int\int_R f(r\cos\theta, r\sin\theta)\, rdrd\theta$

B : $\int\int_R f(r\cos\theta, r\sin\theta)\, rdrd\theta$

C : $\int\int_R f(r\cos\theta, r\sin\theta)\, r^2drd\theta$

D : $\int\int_R f(r\sin\theta, r\cos\theta)\, rdrd\theta$

Q.no 39.

Equation of right circular cylinder whose radius is 'r' and axis is the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ is

A : $x^2 + y^2 + z^2 = r^2 + \left(\frac{lx + my + nz}{\sqrt{l^2 + m^2 + n^2}}\right)^2$

B : $\alpha = \cos^{-1} 1$

C : $x^2 + y^2 + z^2 = r^2 + \frac{lx + my + nz}{\sqrt{l^2 + m^2 + n^2}}$

D : $(x + y + z) = r^2 + \left(\frac{lx + my + nz}{\sqrt{l^2 + m^2 + n^2}}\right)^2$

Q.no 40. Voltage drop across inductance L is given by

A : Li

B : L di/dt

C : dL/dt

D : L dL/dt

Q.no 41. According to D'Alembert's principle algebraic sum of forces acting on a body along a given direction is equal to

A : Velocity x Acceleration

B : Mass x Velocity

C : Mass x displacement

D : Mass x Acceleration

Q.no 42. If q be the quantity of heat that flows across an area A cm2 and thickness δx in one second where the difference of temperature at the faces is δT, then by Fourier's law of heat conduction

A : $q = -k\left(A - \frac{dt}{dx}\right)$, where k is thermal conductivity

B : $q = kA\frac{dT}{dx}$, where k is thermal conductivity

C : $q = -k\left(A + \frac{dt}{dx}\right)$, where k is thermal conductivity

D : $q = -kA\frac{dT}{dx}$, , where k is thermal conductivity

Q.no 43. q56.jpg

A : 1

B : 0

C : q56_3.jpg

D : q56_4.jpg

Q.no 44. Radius of right circular cylinder is 2 and direction ratios 2,-1,5 and fixed point on axis of the line (1,-3,2)then the equation of right circular cylinder is

A : $(x+1)^2 + (y+3)^2 + (z-2)^2 = 2^2 + \left(\frac{2x-y+5z-15}{\sqrt{30}}\right)^2$

B : $(x+1)^2 + (y+3)^2 + (z-2)^2 = 2^2 + \left(\frac{2x-y+5z-15}{\sqrt{30}}\right)^2$

C : $(x-1)^2 + (y+3)^2 - (z-2)^2 = 2^2 + \left(\frac{2x-y-5z-15}{\sqrt{30}}\right)^2$

D : $(x-1)^2 + (y+3)^2 + (z-2)^2 = 2^2 + \left(\frac{2x-y+5z-15}{\sqrt{30}}\right)^2$

Q.no 45.

The value of integration $\int_0^1 \int_0^x (x^2+y^2)\,dxdy$ is equal to

A : 4/3

B : 5/3

C : 2/3

D : 1

Q.no 46. B(4,5)=

A : 1/280

B : 4/105

C : 1/70

D : 1/210

Q.no 47.

The radius of the circle of intersection of the sphere $x^2 + y^2 + z^2 - 2x + 4y + 2z - 6 = 0$ by the plane $x + 2y + 2z - 4 = 0$ is:

[Centre of the circle is(2,0,1)]

A : 9

B : 3

C : 12

D : $\sqrt{3}$

Q.no 48. The value of $erf(0)+erf(\infty)$ is

A : 1

B : -1

C : 0

D : ∞

Q.no 49.

Axis of the right circular cylinder is $\frac{x-1}{2}=\frac{y+3}{-1}=\frac{z-2}{5}$ what are direction cosines of axis

A : $\alpha=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

B : $\left(\frac{2}{\sqrt{30}},\frac{-1}{\sqrt{30}},\frac{5}{\sqrt{30}}\right)$

C : $\left(\frac{-2}{\sqrt{30}},\frac{1}{\sqrt{30}},\frac{-5}{\sqrt{30}}\right)$

D : $\left(\frac{2}{\sqrt{30}},\frac{1}{\sqrt{30}},\frac{5}{\sqrt{30}}\right)$

Q.no 50.

The equation of sphere which passes through the point (4, 6, 3) and passes through the circle $(x-1)^2+(y-2)^2=25,\ z=0$ is

A : $x^2+y^2+z^2-2x-4y-3z-20=0$

B : $x^2+y^2+z^2+3x-4y+3z-16=0$

C : $2x^2+2y^2+2z^2+6x-3y+2z=0$

D : $x^2+y^2+z^2+4x-4y+3z-20=0$

Q.no 51.

If the plane $4x - 3y + 6z - 35 = 0$ is tangential to the sphere

$x^2 + y^2 + z^2 - y - 2z - 14 = 0$ then the length of perpendicular from any point P on the plane is---

A : $\frac{61}{2}$

B : $\frac{61}{2}$

C : 30

D : $-\frac{\sqrt{61}}{2}$

Q.no 52. Radius of right circular cylinder is 5and whose axis is the line $\frac{x-2}{3} = \frac{y-3}{1} = \frac{z+1}{1}$ Then the equation of right circular cylinder is

A : $(x+2)^2 + (y+3)^2 + (z+1)^2 = 5^2 + \left(\frac{3x+y+z-8}{\sqrt{11}}\right)^2$

B : $(x-2)^2 + (y-3)^2 + (z-1)^2 = 5^2 + \left(\frac{3x+y+z-8}{\sqrt{11}}\right)^2$

C : $(x+2)^2 + (y-3)^2 + (z+1)^2 = 5^2 + \left(\frac{3x+y+z-8}{\sqrt{11}}\right)^2$

D : $(x-2)^2 + (y-3)^2 + (z+1)^2 = 5^2 + \left(\frac{3x+y+z-8}{\sqrt{11}}\right)^2$

Q.no 53.

The two spheres $x^2 + y^2 + z^2 - 12x - 18y - 24z + 29 = 0$ and $x^2 + y^2 + z^2 + 3x - 3y + 3z - 56 = 0$

A : touches each other internally

B : touches each other externally

C : touches each other Orthogonally

D : Intersecting each other

Q.no 54. A particle of mass m moves in a horizontal straight line OA with acceleration $\frac{k}{x^3}$ at a distance x and directed towards origin O. Then the differential equation of motion is

A : $v\frac{dv}{dx} = \frac{k}{x^3}$

B : $v\frac{dv}{dx} = -\frac{k}{x^3}$

C : $\frac{dv}{dx} = -\frac{k}{x^3}$

D : $\frac{dv}{dx} = \frac{k}{x^3}$

Q.no 55. The value of integration $\int_0^1 \int_x^{\sqrt{x}} xy\,dx\,dy$ is equal to

A : 0

B : -1/24

C : 1/24

D : 24

Q.no 56. The solution of the DE $\frac{dy}{dx} + y = e^{-x}$ is

A : $ye^{-x} + c = x$

B : $ye^{x} + x = c$

C : $ye^{x} = x + c$

D : $y = x + c$

Q.no 57. If $I_n = \int_0^{\pi/2} \cos^n x \, dx$ then which of the following relation is true?

A : $\frac{n-1}{n} \cdot \frac{n-3}{n-2.} \cdot \frac{n-5}{n-4} \cdots \cdots \frac{3}{4} \cdot \frac{1}{2}$

B : $\mathrm{I_n} = \frac{n}{n-1} I_{n-2}$

C : $\mathrm{I_n} = \frac{n-1}{n} I_{n-2}$

D : $\mathrm{I_n} = n(n+1) I_{n-1}$

Q.no 58. $\frac{5\pi}{128}$

A : $\frac{5\pi}{128}$

B : $\frac{3\pi}{128}$

C : $\frac{3\pi}{512}$

D : $\frac{5\pi}{64}$

Q.no 59. The value of a_0 in harmonic analysis of y for the following tabulated data is

x	0	1	2	3	4	5	6
y	9	18	24	28	26	20	9

A : 17.85

B : 20.83

C : 35.71

D : 41.66

Q.no 60. The value of integration $\int_0^2 \int_0^{x^2} e^{\frac{y}{x}} dxdy$ is equal to

A : $e^2 - 2$

B : $2e^2 - 1$

C : $e^2 - 1$

D : $e^2 + 1$

Answer for Question No 1. is c

Answer for Question No 2. is b

Answer for Question No 3. is c

Answer for Question No 4. is a

Answer for Question No 5. is c

Answer for Question No 6. is a

Answer for Question No 7. is a

Answer for Question No 8. is c

Answer for Question No 9. is b

Answer for Question No 10. is b

Answer for Question No 11. is d

Answer for Question No 12. is a

Answer for Question No 13. is a

Answer for Question No 14. is b

Answer for Question No 15. is a

Answer for Question No 16. is a

Answer for Question No 17. is c

Answer for Question No 18. is c

Answer for Question No 19. is a

Answer for Question No 20. is a

Answer for Question No 21. is c

Answer for Question No 22. is b

Answer for Question No 23. is b

Answer for Question No 24. is d

Answer for Question No 25. is c

Answer for Question No 26. is a

Answer for Question No 27. is b

Answer for Question No 28. is a

Answer for Question No 29. is b

Answer for Question No 30. is a

Answer for Question No 31. is c

Answer for Question No 32. is b

Answer for Question No 33. is a

Answer for Question No 34. is d

Answer for Question No 35. is b

Answer for Question No 36. is c

Answer for Question No 37. is b

Answer for Question No 38. is a

Answer for Question No 39. is a

Answer for Question No 40. is b

Answer for Question No 41. is d

Answer for Question No 42. is d

Answer for Question No 43. is c

Answer for Question No 44. is d

Answer for Question No 45. is c

Answer for Question No 46. is a

Answer for Question No 47. is d

Answer for Question No 48. is a

Answer for Question No 49. is b

Answer for Question No 50. is a

Answer for Question No 51. is a

Answer for Question No 52. is d

Answer for Question No 53. is c

Answer for Question No 54. is b

Answer for Question No 55. is c

Answer for Question No 56. is c

Answer for Question No 57. is c

Answer for Question No 58. is b

Answer for Question No 59. is d

Answer for Question No 60. is c

Seat No -

Total number of questions : 60

1000505 T1 ENGINEERING MATHEMATICS II

Time : 1hr

Max Marks : 50

N.B

1) All questions are Multiple Choice Questions having single correct option.

2) Attempt any 50 questions out of 60.

3) Use of calculator is allowed.

4) Each question carries 1 Mark.

5) Specially abled students are allowed 20 minutes extra for examination.

6) Do not use pencils to darken answer.

7) Use only black/blue ball point pen to darken the appropriate circle.

8) No change will be allowed once the answer is marked on OMR Sheet.

9) Rough work shall not be done on OMR sheet or on question paper.

10) Darken ONLY ONE CIRCLE for each answer.

Q.no 1. According to D'Alembert's principle algebraic sum of forces acting on a body along a given direction is equal to

A : Velocity x Acceleration

B : Mass x Velocity

C : Mass x displacement

D : Mass x Acceleration

Q.no 2. The amount of heat Q flowing through the area per unit time is

A : Q = thermal conductivity x Area x Rate of change of temp. across an area

B : Q = thermal conductivity x Area of slab

C : Q = thermal conductivity x Area x temp. gradient

D : Q = thermal conductivity x Area + Rate of flow of heat

Q.no 3. Half range Fourier cosine series for $f(x)$ defined in the interval $0 \leq x \leq L$ is

A : Half range Fourier cosine series for $f(x)$ defined in the interval $0 \le x \le L$ is

B : $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{nx}{L}$

C : $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$

D : $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n\, cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$

Q.no 4. A pipe contains steam and is protected with a covering then in the formula $q = kA\,\frac{dT}{dt}$

A : $A = 2\pi rl$

B : $A = \frac{1}{3}\pi r^2 l$

C : $A = \frac{1}{3}\pi r^2 l$

D : $A = rl$

Q.no 5. B(m,n) is equal to

A : $\int_0^1 x^{m-1}(1-x)^{n-1}dx$

B : $\int_0^{\infty} x^{m-1}(1-x)^{n-1}dx$

C : $\int_0^1 x^m(1-x)^n dx$

D : $\int_0^1 e^{-x}x^{n-1}dx$

Q.no 6. The formula to find the length of the curve $r = f(\theta)$ from $\theta = \theta_1$ to $\theta = \theta_2$ is

A : $\int_{\theta_1}^{\theta_2} \sqrt{1 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

B : $\int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

C : $\int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, dr$

D : $\int_{\theta_2}^{\theta_1} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

Q.no 7. If the DE $Mdx + Ndy = 0$ can be written as $x^a y^b(mydx + nxdy) + x^r y^s(pydx + qxdy) = 0$ then integrating factor is

A : $x^h y^k$

B : x^h

C : xy

D : $\frac{1}{x^h y^k}$

Q.no 8. The value of λ for which the differential equation $(xy^2 + \lambda x^2 y)dx + (x + y)x^2 dy = 0$ is exact

A : 2

B : -3

C : 3

D : -2

Q.no 9. A circuit containing inductance L capacitance C in series with applied electromotive force E. By Kirchhoff's voltage law differential equation for current i is

A : $L\frac{di}{dt} + Ri = E$

B : $L\frac{di}{dt} + Ri = 0$

C : $L\frac{di}{dt} + \frac{q}{C} = E$

D : $L\frac{di}{dt} + \frac{q}{C} = 0$

Q.no 10.

The formula to find the length of the curve $y = f(x)$ from $x = a$ to $x = b$ is $s =$

A : $\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

B : $\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dy$

C : $\int_a^b \sqrt{a^2 + \left(\frac{dy}{dx}\right)^2}\, dx$

D : $\int_a^b \sqrt{a^2 + b^2\left(\frac{dy}{dx}\right)^2}\, dy$

Q.no 11. The differential equation for the current in an electric circuit containing resistance R and inductance L in series with voltage source E sin(wt) is

A : $L\frac{di}{dt} + \frac{q}{C} = E$

B : $Li + R\frac{di}{dt} = Esin\omega t$

C : $L\frac{di}{dt} + Ri = 0$

D : $L\frac{di}{dt} + Ri = 0$

Q.no 12.

Find semi-vertical angle for a right circular cone with vertex at the point $(1,0,1)$which passes through the point $(1,1,1)$ and axis of cone has direction ratios $1,1,1$

A : $\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

B : $\alpha = \cos^{-1} 1$

C : $\alpha = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

D : $\alpha = \cos^{-1}(\sqrt{3})$

Q.no 13. For RC circuit the charge q satisfies the linear D.E.

A : $R + \frac{dq}{dt} = E$

B : $Ri + q = 0$

C : $A = \frac{1}{3}\pi r^2 l$

D : $R\frac{dq}{dt} + \frac{q}{C} = E$

Q.no 14. The value of integration $\int_0^a \int_0^b \int_0^c x^2y^2z^2dxdydz$ is equal to

A : $\frac{abc}{3}$

B : $\frac{a^2b^2c^2}{3}$

C : $\frac{a^3b^3c^3}{27}$

D : $\frac{a^2b^2c^2}{9}$

Q.no 15. If double integral in Cartesian coordinate is given by $\int\int_R f(x,y)\,dxdy$ then the value of same integral in polar form is

A : $\int\int_R f(r\cos\theta, r\sin\theta)\,rdrd\theta$

B : $\int\int_R f(r\cos\theta, r\sin\theta)\,rdrd\theta$

C : $\int\int_R f(r\cos\theta, r\sin\theta)\,r^2drd\theta$

D : $\int\int_R f(r\sin\theta, r\cos\theta)\,rdrd\theta$

Q.no 16. The integrating factor for the DE $(1+x^2)\frac{dy}{dx}+xy=0$ is

A : e^{1+x^2}

B : $\log(x^2+1)$

C : $\sqrt{x^2+1}$

D : $1+x^2$

Q.no 17. If n is a positive integer, then $\Gamma(n+1)$ is

A : (n+1)!

B : (n+2)!

C : (n-1)!

D : n!

Q.no 18. The integrating factor of the DE $cosx\frac{dy}{dx} + ysinx = 1$ is

A : sec x

B : cot x

C : tan x

D : sin x

Q.no 19. The value of $\int\int\int_V dxdydz$ where V is the volume bounded by $x^2 + y^2 + z^2 = 1$

A : $\frac{4}{3}\pi$

B : 4π

C : π

D : $\frac{1}{3}\pi$

Q.no 20. If the equation of curve remains unchanged by replacing y by -y, then the curve is symmetric about

A : y-axis

B : line y=x

C : x-axis

D : line y=-x

Q.no 21. If the differential equation $M(x,y)dx + N(x,y)dy = 0$ is not exact and it can be written as $yf_1(x,y)dx + xf_2(x,y)dy = 0$ then integrating factor is

A : $\frac{1}{My+Nx}; My + Nx \neq 0$

B : $\frac{1}{Mx+Ny}; Mx + Ny \neq 0$

C : $\frac{1}{Mx - Ny}; Mx - Ny \neq 0$

D : $\frac{1}{My - Nx}; My - Nx \neq 0$

Q.no 22. Area enclosed by the polar curve $r = f_1(\theta)$ and $r = f_2(\theta)$ and the line $\theta = \alpha, \theta = \beta \alpha < \beta$ is given by

A : $Area = \int_{\alpha}^{\beta} \int_{f_1(\theta)}^{f_2(\theta)} r dr d\theta$

B : $Area = \int_{\alpha}^{\beta} \int_{f_1(\theta)}^{f_2(\theta)} dr d\theta$

C : $Area = \int_{\alpha}^{\beta} \int_{f_1(\theta)}^{f_2(\theta)} r^2 \sin\theta dr d\theta$

D : $Area = \int_{\alpha}^{\beta} \int_{f_1(\theta)}^{f_2(\theta)} r^2 \cos\theta dr d\theta$

Q.no 23.
Find semi-vertical angle for a right circular cone with vertex at the point $(1,0,1)$which passes through the point $(1,1,1)$ and axis of cone has direction ratios 1,1,1

A : $\sin\alpha = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

B : $\cos\alpha = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

C : $\cos\alpha = a_1a_2 + b_1b_2 + c_1c_2$

D : $\sin\alpha = a_1a_2 + b_1b_2 + c_1c_2$

Q.no 24. The integrating factor of the DE $\frac{dx}{dy} + \frac{3}{y}x = \frac{1}{y^2}$

A : $-y^3$

B : y^3

C : x^2y

D : x^3

Q.no 25. A particle moving in straight line with acceleration $k(x+\frac{a^4}{x^3})$ directed towards origin. The equation of motion is

A : $\frac{dv}{dx}=-k(x+\frac{a^4}{x^3})$

B : $v\frac{dv}{dx}=k(x+\frac{a^4}{x^3})$

C : $v\frac{dv}{dx}=-k(x+\frac{a^4}{x^3})$

D : $\frac{dv}{dx}=(x+\frac{a^4}{x^3})$

Q.no 26. The differential equation for the current i in an electric circuit containing resistance R=250 ohm and an inductance of L=640 henery in series with an electromotive force E=500 volts is

A : $640\frac{di}{dt}+250i=0$

B : $250\frac{di}{dt}+640i=500$

C : $640\frac{di}{dt}+250i=500$

D : $250\frac{di}{dt}+640i=0$

Q.no 27. Rectilinear motion is a motion of body along a

A : Straight line

B : Circular path

C : Parabolic path

D : Hyperbolic path

Q.no 28. The integrating factor of the DE $\frac{dy}{dx} + ycosx = \frac{sin2x}{2}$ is

A : e^{cosx}

B : e^{sinx}

C : sin x

D : cos x

Q.no 29. To change Cartesian coordinates (x, y, z) to spherical polar coordinate (r, θ, ϕ);$dxdydz$ is replaced by

A : To change Cartesian coordinates (x, y, z) to spherical polar coordinate (r, θ, ϕ);$dxdydz$ is replaced by

B : $r^2 \cos\theta dr d\theta d\phi$

C : $r^2 dr d\theta d\phi$

D : $r dr d\theta d\phi$

Q.no 30. If $I(\alpha) = \int_a^b f(x,\alpha)dx$ where α is parameter and a, b are constants then by DUIS rule $\frac{dI(\alpha)}{d\alpha}$ is

A : $\int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)dx$

B : $\int_a^b \frac{\partial}{\partial x} f(x,\alpha)dx$

C : $f(b, \alpha) - f(a, \alpha)$

D : $f(x, \alpha)$

Q.no 31. Fourier law of heat conduction states that, the quantity of heat flow across an area A cm2 is

A : proportional to product of area A and temperature gradient dT/dx

B : inversely proportional to product of area A and temperature gradient dT/dx

C : equal to sum of area A and temperature gradient

D : equal to difference of area A and temperature gradient dT/dx

Q.no 32. The necessary and sufficient condition that the Differential equation $Mdx + Ndy = 0$ is exact is

A : $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

B : $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

C : $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

D : $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial x}$

Q.no 33. Moment of inertia of the lamina A about the y axis is equal to

A : $\int \int_A \rho x^2 dx dy$

B : $\int \int_A \rho y^2 dx dy$

C : $\int \int_A \rho (x^2 + y^2) dx dy$

D : $\int \int_A \rho x^2 y^2 dx dy$

Q.no 34. If $\phi(a)=\int_0^\infty e^{-bx^2}\cos(2ax)\,dx,\ b>0$ then by DUIS rule $\frac{d\phi}{da}$ is

A : $\int_0^\infty \frac{\partial}{\partial b}\left[e^{-bx^2}\cos(2ax)\right]dx$

B : $e^{-bx^2}\cos(2ax)$

C : $\int_0^\infty \frac{\partial}{\partial x}\left[e^{-bx^2}\cos(2ax)\right]dx$

D : $\int_0^\infty \frac{\partial}{\partial a}\left[e^{-bx^2}\cos(2ax)\right]dx$

Q.no 35. $erf(x)-erfc(x)=1 \quad erfc(x)=?$

A : $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$

B : $\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2}du$

C : $\frac{2}{\sqrt{\pi}}\int_0^{x^2} e^{-t}t^{-1/2}dt$

D : 0

Q.no 36. The integrating factor of $(x^2+y^2+x)dx+xydy=0$ using non exact rule $\frac{\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)}{N}$ is

A : x

B : 1/x

C : x2

D : xy

Q.no 37. $\int_0^{\pi/2} sin^p\theta cos^q\theta\, d\theta$ is equal to

A : $B\left(\frac{p-1}{2},\frac{q-1}{2}\right)$

B : $\frac{1}{2}B\left(\frac{p+1}{2},\frac{q+1}{2}\right)$

C : $B\left(\frac{p+1}{2},\frac{q+1}{2}\right)$

D : $B(p,q)$

Q.no 38.

Condition for the two spheres $x^2+ y^2+ z^2+ 2u_1x+2v_1y+2w_1z+d_1=0$ and $x^2+ y^2+ z^2+ 2u_2x+2v_2y+2w_2z+d_2=0$ to be orthogonal is ---

A : $S+\lambda U=0$

B : $S_1- S_2=0$

C : $2u_1u_2+ 2v_1v_2+2w_1w_2= d_1+ d_2$

D : $u_1u_2+ v_1v_2+w_1w_2= d_1+ d_2$

Q.no 39. The integrating factor of $(3x^2y^4+2xy)dx+(2x^3y^3-x^2)dy=0$ using non exact rule $\frac{\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)}{M}$ is

A : y^2

B : $\frac{1}{y^2}$

C : y

D : 1/y

Q.no 40.

The equation of tangent to the curve at origin represented by the equation $y(1+x^2)=x$ is

A : y=x

B : x=0

C : x=1

D : y=0

Q.no 41. Error function of x, $erf\left(x\right)$ is defined as

A : $\frac{2}{\sqrt{\pi}}\int_{0}^{x} e^{-u^2} du$

B : $\frac{2}{\sqrt{\pi}}\int_{x}^{\infty} e^{-u^2} du$

C : $\int_{0}^{\infty} e^{-x}x^{n-1}dx$

D : $\frac{2}{\sqrt{\pi}}\int_{0}^{\infty} e^{-u^2} du$

Q.no 42. The value of the integral $\int_{0}^{b} e^{-u^2} du$ is

A : $\frac{2}{\sqrt{\pi}}\int_{0}^{x} e^{-u^2} du$

B : $\frac{\sqrt{\pi}}{2} erf\left(b\right)$

C : $\frac{2}{\sqrt{\pi}} erf_{C}\left(b\right)$

D : $erf\left(b\right)$

Q.no 43. The solution of the DE $\frac{dy}{dx} + y = e^{-x}$ is

A : $ye^{-x} + c = x$

B : $ye^{x} + x = c$

C : $ye^{x} = x + c$

D : $y = x + c$

Q.no 44. q56.jpg

A : 1

B : 0

C : q56_3.jpg

D : q56_4.jpg

Q.no 45. A capacitor C=0.01 farad in series with resistor R=20 ohms is charged from battery E=10 volts. If initially capacitor is completely discharged then differential equation for charge q(t) is given by

A : $20\frac{dq}{dt} + \frac{q}{0.01} = 0 \;;\; q(0) = 0$

B : $20\frac{dq}{dt} + 0.01q = 10 \;;\; q(0) = 0$

C : $20\frac{dq}{dt} + \frac{q}{0.01} = 10 \;;\; q(0) = 0$

D : $20\frac{dq}{dt} + 0.01q = 0 \;;\; q(0) = 0$

Q.no 46. The value of integration $\int_0^1 \int_x^{\sqrt{x}} xy\,dx\,dy$ is equal to

A : 0

B : -1/24

C : 1/24

D : 24

Q.no 47. The value of integration $\int_0^1 \int_0^{x^2} xe^y dxdy$ is equal to

A : e/2

B : e-1

C : 1-e

D : (e/2)-1

Q.no 48. If $I(\alpha) = \int_0^1 \frac{x^\alpha - 1}{\log x} dx$, $\alpha \geq 0$ then by DUIS $\frac{dI}{d\alpha}$ is

A : $\int_0^1 \log x \, dx$

B : $\int_0^1 x^\alpha \log x \, dx$

C : $\int_0^1 x^\alpha dx$

D : $\int_0^1 x^{-\alpha} dx$

Q.no 49. If $I(\alpha) = \int_0^1 \frac{x^\alpha - 1}{\log x} dx$, $\alpha \geq 0$ then by DUIS $\frac{dI}{d\alpha}$ is

A : $\log(\alpha + 1)$

B : $\log(\alpha - 1)$

C : $\log \alpha$

D : 0

Q.no 50.

Find the equation of right circular cone whose vertex is at origin, whose axis is the line $\frac{x}{2} = \frac{y}{-1} = \frac{z}{2}$ and semi-vertical angle $\frac{\pi}{4}$

A : $3(x^2 + y^2 + z^2) = 4\,(2x - y + 2z)^2$

B : $9(x^2 + y^2 + z^2) = 2\,(2x - y + 2z)^2$

C : $6(x^2 + y^2 + z^2) = 2\,(2x - y + 2z)^2$

D : $(x^2 + y^2 + z^2) = (2x - y + 2z)^2$

Q.no 51. In spherical co-ordinates volume is given by

A : $V = \int\int\int_V dr d\theta d\phi$

B : $V = \int\int\int_V r dr d\theta d\phi$

C : $V = \int\int\int_V r^2 \cos\theta dr d\theta d\phi$

D : $V = \int\int\int_V r^2 \sin\theta dr d\theta d\phi$

Q.no 52.

The value of integration $\int_0^1 \int_0^x (x^2 + y^2)\, dx dy$ is equal to

A : 4/3

B : 5/3

C : 2/3

D : 1

Q.no 53.

If $I(\alpha)=\int_a^b f(x,\alpha)\,dx$ where $a,\ b$ are function of parameter α then by DUIS rule $\frac{dI(\alpha)}{d\alpha}$ is

A : $\int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)dx + f(a,\alpha)\frac{da}{d\alpha} - f(b,\alpha)\frac{db}{d\alpha}$

B : $\int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)dx + f(b,\alpha)\frac{db}{d\alpha} - f(a,\alpha)\frac{da}{d\alpha}$

C : $\int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)dx + f(b,\alpha)\frac{db}{d\alpha} - f(a,\alpha)\frac{da}{d\alpha}$

D : $\int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)dx$

Q.no 54. A particle of mass m moves in a horizontal straight line OA with acceleration $\frac{k}{x^3}$ at a distance x and directed towards origin O. Then the differential equation of motion is

A : $v\frac{dv}{dx}=\frac{k}{x^3}$

B : $v\frac{dv}{dx}=-\frac{k}{x^3}$

C : $\frac{dv}{dx}=-\frac{k}{x^3}$

D : $\frac{dv}{dx}=\frac{k}{x^3}$

Q.no 55. B(4,5)=

A : 1/280

B : 4/105

C : 1/70

D : 1/210

Q.no 56. Radius of right circular cylinder is 2 and direction ratios 2,-1,5 and fixed point on axis of the line (1,-3,2)then the equation of right circular cylinder is

A : $(x+1)^2 + (y+3)^2 + (z-2)^2 = 2^2 + \left(\frac{2x-y+5z-15}{\sqrt{30}}\right)^2$

B : $(x+1)^2 + (y+3)^2 + (z-2)^2 = 2^2 + \left(\frac{2x-y+5z-15}{\sqrt{30}}\right)^2$

C : $(x-1)^2 + (y+3)^2 - (z-2)^2 = 2^2 + \left(\frac{2x-y-5z-15}{\sqrt{30}}\right)^2$

D : $(x-1)^2 + (y+3)^2 + (z-2)^2 = 2^2 + \left(\frac{2x-y+5z-15}{\sqrt{30}}\right)^2$

Q.no 57.

The value of integration $\int_0^1 \int_0^1 \int_0^1 xyz\,dxdydz$ is equal to

A : 0.125

B : 0.25

C : -0.125

D : 1

Q.no 58. The value of integration $\int_0^2 \int_0^{x^2} e^{\frac{y}{x}} dxdy$ is equal to

A : $e^2 - 2$

B : $2e^2 - 1$

C : $e^2 - 1$

D : $e^2 + 1$

Q.no 59. Fourier coefficient a_0 in the Fourier series expansion of

$f(x) = x\sin x\,;\, 0 \le x \le 2\pi$ and $f(x+2\pi) = f(x)$

A : 2

B : 0

C : -2

D : -4

Q.no 60. The value of a_0 in harmonic analysis of y for the following tabulated data is

x	0	1	2	3	4	5	6
y	9	18	24	28	26	20	9

A : 17.85

B : 20.83

C : 35.71

D : 41.66

Answer for Question No 1. is d

Answer for Question No 2. is c

Answer for Question No 3. is c

Answer for Question No 4. is a

Answer for Question No 5. is a

Answer for Question No 6. is b

Answer for Question No 7. is a

Answer for Question No 8. is c

Answer for Question No 9. is c

Answer for Question No 10. is a

Answer for Question No 11. is d

Answer for Question No 12. is a

Answer for Question No 13. is d

Answer for Question No 14. is c

Answer for Question No 15. is a

Answer for Question No 16. is b

Answer for Question No 17. is d

Answer for Question No 18. is a

Answer for Question No 19. is a

Answer for Question No 20. is c

Answer for Question No 21. is c

Answer for Question No 22. is a

Answer for Question No 23. is b

Answer for Question No 24. is b

Answer for Question No 25. is c

Answer for Question No 26. is c

Answer for Question No 27. is a

Answer for Question No 28. is b

Answer for Question No 29. is a

Answer for Question No 30. is a

Answer for Question No 31. is a

Answer for Question No 32. is a

Answer for Question No 33. is a

Answer for Question No 34. is d

Answer for Question No 35. is b

Answer for Question No 36. is a

Answer for Question No 37. is b

Answer for Question No 38. is c

Answer for Question No 39. is b

Answer for Question No 40. is a

Answer for Question No 41. is a

Answer for Question No 42. is b

Answer for Question No 43. is c

Answer for Question No 44. is c

Answer for Question No 45. is c

Answer for Question No 46. is c

Answer for Question No 47. is b

Answer for Question No 48. is c

Answer for Question No 49. is a

Answer for Question No 50. is b

Answer for Question No 51. is d

Answer for Question No 52. is c

Answer for Question No 53. is c

Answer for Question No 54. is b

Answer for Question No 55. is a

Answer for Question No 56. is d

Answer for Question No 57. is a

Answer for Question No 58. is c

Answer for Question No 59. is c

Answer for Question No 60. is d

Seat No -

Total number of questions : 60

1000505 T1 ENGINEERING MATHEMATICS II

Time : 1hr

Max Marks : 50

N.B

1) All questions are Multiple Choice Questions having single correct option.

2) Attempt any 50 questions out of 60.

3) Use of calculator is allowed.

4) Each question carries 1 Mark.

5) Specially abled students are allowed 20 minutes extra for examination.

6) Do not use pencils to darken answer.

7) Use only black/blue ball point pen to darken the appropriate circle.

8) No change will be allowed once the answer is marked on OMR Sheet.

9) Rough work shall not be done on OMR sheet or on question paper.

10) Darken ONLY ONE CIRCLE for each answer.

Q.no 1. $erf(x) - erfc(x) = 1 \quad erfc(x) = ?$

A : $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\,du$

B : $\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2}\,du$

C : $\frac{2}{\sqrt{\pi}}\int_0^{x^2} e^{-t}t^{-1/2}\,dt$

D : 0

Q.no 2.

The equation of tangent to the curve at origin represented by the equation $y(1+x^2) = x$ is

A : y=x

B : x=0

C : x=1

D : y=0

Q.no 3. According to D'Alembert's principle algebraic sum of forces acting on a body along a given direction is equal to

A : Velocity x Acceleration

B : Mass x Velocity

C : Mass x displacement

D : Mass x Acceleration

Q.no 4. A particle moving in straight line with acceleration $k(x+\frac{a^4}{x^3})$ directed towards origin. The equation of motion is

A : $\frac{dv}{dx}=-k(x+\frac{a^4}{x^3})$

B : $v\frac{dv}{dx}=k(x+\frac{a^4}{x^3})$

C : $v\frac{dv}{dx}=-k(x+\frac{a^4}{x^3})$

D : $\frac{dv}{dx}=(x+\frac{a^4}{x^3})$

Q.no 5. If the DE $Mdx+Ndy=0$ can be written as $x^a y^b(mydx+nxdy)+x^r y^s(pydx+qxdy)=0$ then integrating factor is

A : $x^h y^k$

B : x^h

C : xy

D : $\frac{1}{x^h y^k}$

Q.no 6. The integrating factor for the DE $(1+x^2)\frac{dy}{dx}+xy=0$ is

A : e^{1+x^2}

B : $\log(x^2+1)$

C : $\sqrt{x^2+1}$

D : $1+x^2$

Q.no 7. Voltage drop across inductance L is given by

A : Li

B : L di/dt

C : dL/dt

D : L dL/dt

Q.no 8. For RC circuit the charge q satisfies the linear D.E.

A : $R+\frac{dq}{dt}=E$

B : $Ri+q=0$

C : $A=\frac{1}{3}\pi r^2 l$

D : $R\frac{dq}{dt}+\frac{q}{C}=E$

Q.no 9. The value of $\int\int\int_V dxdydz$ where V is the volume bounded by $x^2+y^2+z^2=1$

A : $\frac{4}{3}\pi$

B : 4π

C : π

D : $\frac{1}{3}\pi$

Q.no 10.

Find semi-vertical angle for a right circular cone with vertex at the point (1,0,1)which passes through the point (1,1,1) and axis of cone has direction ratios 1,1,1

A : $\sin\alpha = \dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\,\sqrt{a_2^2 + b_2^2 + c_2^2}}$

B : $\cos\alpha = \dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\,\sqrt{a_2^2 + b_2^2 + c_2^2}}$

C : $\cos\alpha = a_1a_2 + b_1b_2 + c_1c_2$

D : $\sin\alpha = a_1a_2 + b_1b_2 + c_1c_2$

Q.no 11. Moment of inertia of the lamina A about the y axis is equal to

A : $\int\int_A \rho x^2 dxdy$

B : $\int\int_A \rho y^2 dxdy$

C : $\int\int_A \rho(x^2 + y^2) dxdy$

D : $\int\int_A \rho x^2y^2 dxdy$

Q.no 12. B(m,n) is equal to

A : $\int_0^1 x^{m-1}(1-x)^{n-1}dx$

B : $\int_0^\infty x^{m-1}(1-x)^{n-1}dx$

C : $\int_0^1 x^m (1-x)^n dx$

D : $\int_0^1 e^{-x} x^{n-1} dx$

Q.no 13. The value of equivalent form of gamma function $\int_0^\infty e^{-kx} x^{n-1}\, dx$ is

A : $\frac{\Gamma n}{n^k}$

B : $\frac{\Gamma n}{k^n}$

C : $\frac{\Gamma n}{k!}$

D : $\Gamma(n+k+1)$

Q.no 14. Rectilinear motion is a motion of body along a

A : Straight line

B : Circular path

C : Parabolic path

D : Hyperbolic path

Q.no 15. In a D.E. $L\frac{di}{dt} + Ri = E$, E means

A : Voltage

B : Current

C : Resistance

D : Electromotive force

Q.no 16. The value of integration $\int_0^a \int_0^b \int_0^c x^2 y^2 z^2 dxdydz$ is equal to

A : $\frac{abc}{3}$

B : $\frac{a^2b^2c^2}{3}$

C : $\frac{a^3b^3c^3}{27}$

D : $\frac{a^2b^2c^2}{9}$

Q.no 17. The value of $erf\left(3\right)+erf_c\left(3\right)$ is

A : 3

B : 2

C : 1

D : 0

Q.no 18. If double integral in Cartesian coordinate is given by $\int\int_R f(x,y)\,dxdy$ then the value of same integral in polar form is

A : $\int\int_R f(r\cos\theta, r\sin\theta)\, rdrd\theta$

B : $\int\int_R f(r\cos\theta, r\sin\theta)\, rdrd\theta$

C : $\int\int_R f(r\cos\theta, r\sin\theta)\, r^2drd\theta$

D : $\int\int_R f(r\sin\theta, r\cos\theta)\, rdrd\theta$

Q.no 19. If $I_n=\int_0^{\pi/2} sin^n x\, dx$, where n is positive even integer then I_n is calculated from

A : $\frac{n-1}{n}.\frac{n-3}{n-2.}.\frac{n-5}{n-4}\cdots\cdots\frac{3}{4}.\frac{1}{2}$

B : $\frac{n-1}{n}.\frac{n-3}{n-2.}.\frac{n-5}{n-4}\ldots\ldots\frac{3}{4}.\frac{1}{2}.\pi$

C : $\left(\frac{n+1}{n}.\frac{n.+3}{n+2.}.\frac{n+5}{n+4}\right)\ldots\ldots\frac{\pi}{2}$

D : $\frac{n-1}{n}.\frac{n-3}{n-2.}.\frac{n-5}{n-4}\ldots\ldots\frac{3}{4}.\frac{1}{2}.\frac{\pi}{2}$

Q.no 20. The integrating factor of the DE $\frac{dy}{dx} + ycosx = \frac{sin2x}{2}$ is

A : e^{cosx}

B : e^{sinx}

C : sin x

D : cos x

Q.no 21. To change Cartesian coordinates (x, y, z) to spherical polar coordinate (r, θ, ϕ);$dxdydz$ is replaced by

A : To change Cartesian coordinates (x, y, z) to spherical polar coordinate (r, θ, ϕ);$dxdydz$ is replaced by

B : $r^2 \cos\theta dr d\theta d\phi$

C : $r^2 dr d\theta d\phi$

D : $r dr d\theta d\phi$

Q.no 22. The differential equation for the current i in an electric circuit containing resistance 100 ohm and an inductance of 0.5 henery connected in series with battery of 20 volts is

A : $0.5\frac{di}{dt} + 100i = 0$

B : $0.5\frac{di}{dt} + 100i = 20$

C : $100\frac{di}{dt} + 0.5i = 0$

D : $100\frac{di}{dt} + 0.5R = 0$

Q.no 23.

If $(\bar{x}, \bar{y})$ the coordinates of centroid of the plane area is given by

A : $\bar{x} = \frac{\int\int x\rho dxdy}{\int\int \rho dxdy}, \bar{y} = \frac{\int\int y\rho dxdy}{\int\int \rho dxdy}$

B : $\bar{x} = \frac{\int\int x^2\rho dxdy}{\int\int \rho dxdy}, \bar{y} = \frac{\int\int y^2\rho dxdy}{\int\int \rho dxdy}$

C : $\bar{x} = \frac{\int\int \rho dxdy}{\int\int x\rho dxdy}, \bar{y} = \frac{\int\int \rho dxdy}{\int\int y\rho dxdy}$

D : None of these

Q.no 24. The integrating factor of $(x^2 + y^2 + x)dx + xydy = 0$ using non exact rule $\frac{\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)}{N}$ is

A : x

B : 1/x

C : x2

D : xy

Q.no 25. If the equation of curve remains unchanged by replacing y by -y, then the curve is symmetric about

A : y-axis

B : line y=x

C : x-axis

D : line y=-x

Q.no 26. If q be the quantity of heat that flows across an area A cm2 and thickness δx in one second where the difference of temperature at the faces is δT, then by Fourier's law of heat conduction

A : $q = -k\left(A - \frac{dt}{dx}\right)$, where k is thermal conductivity

B : $q = kA\frac{dT}{dx}$, where k is thermal conductivity

C : $q = -k\left(A + \frac{dt}{dx}\right)$, where k is thermal conductivity

D : $q = -kA\frac{dT}{dx}$, , where k is thermal conductivity

Q.no 27. The integrating factor of $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$ using non exact rule $\frac{\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)}{M}$ is

A : y^2

B : $\frac{1}{y^2}$

C : y

D : 1/y

Q.no 28. The solution of the DE $\frac{dy}{dx} = \frac{x^2}{y^3}$ is

A : $y^3 - x^3 = c$

B : $3y^4 - 4x^3 = c$

C : $y^2 - x^3 = c$

D : $x^2 - y^2 = c$

Q.no 29. A circuit containing inductance L capacitance C in series with applied electromotive force E. By Kirchhoff's voltage law differential equation for current i is

A : $L\frac{di}{dt} + Ri = E$

B : $L\frac{di}{dt} + Ri = 0$

C : $L\frac{di}{dt} + \frac{q}{C} = E$

D : $L\frac{di}{dt} + \frac{q}{C} = 0$

Q.no 30. Error function of x, $erf(x)$ is defined as

A : $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2} du$

B : $\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2} du$

C : $\int_0^\infty e^{-x} x^{n-1} dx$

D : $\frac{2}{\sqrt{\pi}}\int_0^\infty e^{-u^2} du$

Q.no 31. The integrating factor of $\frac{dy}{dx} + Py = Q$ is

A : $e^{\int pdx}$

B : $e^{\int pdy}$

C : $e^{\int Qdx}$

D : $e^{\int Qdy}$

Q.no 32. Half range Fourier cosine series for $f(x)$ defined in the interval $0 \leq x \leq L$ is

A : Half range Fourier cosine series for $f(x)$ defined in the interval $0 \leq x \leq L$ is

B : $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{nx}{L}$

C : $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$

D : $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n\, cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$

Q.no 33. The necessary and sufficient condition that the Differential equation $Mdx + Ndy = 0$ is exact is

A : $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

B : $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

C : $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

D : $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial x}$

Q.no 34. The general solution of Linear Differential equation $\frac{dx}{dy} + Px = Q$ where P and Q are functions of y or constants, is

A : $x\, e^{\int pdx} = \int Q\, e^{\int pdx} dx + c$

B : $x\, e^{\int p dy} = \int Q\, e^{\int p dy} dy + c$

C : $y\, e^{\int p dx} = \int Q\, e^{\int p dx} dx + c$

D : $y\, e^{\int p dx} = \int Q\, e^{\int p dx} dy + c$

Q.no 35.

The equation of asymptotes parallel to y-axis to the curve represented by the equation $y^2(4-x) = x(x-2)^2$ is

A : x=2

B : y=0

C : x=4

D : x=0

Q.no 36. If the differential equation $M(x,y)dx + N(x,y)dy = 0$ is not exact and it can be written as $yf_1(x,y)dx + xf_2(x,y)dy = 0$ then integrating factor is

A : $\frac{1}{My+Nx}; My + Nx \neq 0$

B : $\frac{1}{Mx+Ny}; Mx + Ny \neq 0$

C : $\frac{1}{Mx-Ny}; Mx - Ny \neq 0$

D : $\frac{1}{My-Nx}; My - Nx \neq 0$

Q.no 37. Reduction formula for gamma function is

A : $\Gamma(n+1) = (n-1)\Gamma(n-1)$

B : $\Gamma(n+1) = n\Gamma(n)$

C : $\Gamma(n+1) = (n-1)\Gamma(n)$

D : $\Gamma(n+1) = n \quad \Gamma(n-1)$

Q.no 38. A pipe contains steam and is protected with a covering then in the formula $q = kA\,\frac{dT}{dt}$

A : $A = 2\pi r l$

B : $A = \frac{1}{3}\pi r^2 l$

C : $A = \frac{1}{3}\pi r^2 l$

D : $A = rl$

Q.no 39. If $\phi(a) = \int_0^\infty e^{-bx^2}\cos(2ax)\,dx,\ b>0$ then by DUIS rule $\frac{d\phi}{da}$ is

A : $\int_0^\infty \frac{\partial}{\partial b}\left[e^{-bx^2}\cos(2ax)\right]dx$

B : $e^{-bx^2}\cos(2ax)$

C : $\int_0^\infty \frac{\partial}{\partial x}\left[e^{-bx^2}\cos(2ax)\right]dx$

D : $\int_0^\infty \frac{\partial}{\partial a}\left[e^{-bx^2}\cos(2ax)\right]dx$

Q.no 40. The center and radius of the sphere $3(x^2 + y^2 + z^2) - 30x + 12y - 18z + 89 = 0$ is

A : (10,-4,6) , $\frac{5}{\sqrt{3}}$

B : $(15,-6,9)$, $\frac{\sqrt{203}}{\sqrt{3}}$

C : $(5,-2,3)$, $\frac{5}{\sqrt{3}}$

D : $(-5,2,-3)$, $\frac{5}{3}$

Q.no 41. Which of the following is true?

A : $erf(x) - erfc(x) = 1$

B : $erf(x) + erfc(x) = 1$

C : $erf(x) + erfc(x) = 2$

D : $erf(-x) = erf(x)$

Q.no 42. Area enclosed by the polar curve $r = f_1(\theta)$ and $r = f_2(\theta)$ and the line $\theta = \alpha, \theta = \beta \alpha < \beta$ is given by

A : $Area = \int_\alpha^\beta \int_{f_1(\theta)}^{f_2(\theta)} r dr d\theta$

B : $Area = \int_\alpha^\beta \int_{f_1(\theta)}^{f_2(\theta)} dr d\theta$

C : $Area = \int_\alpha^\beta \int_{f_1(\theta)}^{f_2(\theta)} r^2 \sin\theta dr d\theta$

D : $Area = \int_\alpha^\beta \int_{f_1(\theta)}^{f_2(\theta)} r^2 \cos\theta dr d\theta$

Q.no 43. The value of integration $\int_0^1 \int_x^{\sqrt{x}} xy dx dy$ is equal to

A : 0

B : -1/24

C : 1/24

D : 24

Q.no 44. The solution of exact DE $(x+2y-2)dx+(2x-y+3)dy=0$ is

A : $x^2+4xy-4x+6y=c$

B : $x^2+4xy-4x-y^2+6y=c$

C : $x^2+8xy+y^2=c$

D : $x^2+4xy-\frac{y^2}{2}+y=c$

Q.no 45. If $I(\alpha)=\int_0^1 \frac{x^\alpha-1}{\log x}dx$, $\alpha\geq 0$ then by DUIS $\frac{dI}{d\alpha}$ is

A : $\log(\alpha+1)$

B : $\log(\alpha-1)$

C : $\log\alpha$

D : 0

Q.no 46. A capacitor C=0.01 farad in series with resistor R=20 ohms is charged from battery E=10 volts. If initially capacitor is completely discharged then differential equation for charge q(t) is given by

A : $20\frac{dq}{dt}+\frac{q}{0.01}=0\ ;\ q(0)=0$

B : $20\frac{dq}{dt}+0.01q=10\ ;\ q(0)=0$

C : $20\frac{dq}{dt}+\frac{q}{0.01}=10\ ;\ q(0)=0$

D : $20\frac{dq}{dt}+0.01q=0\ ;\ q(0)=0$

Q.no 47. The value of $erf(0)+erf(\infty)$ is

A : 1

B : -1

C : 0

D : ∞

Q.no 48.

The equation of sphere which passes through the point (4, 6, 3) and passes through the circle $(x-1)^2+(y-2)^2=25, z=0$ is

A : $x^2+y^2+z^2-2x-4y-3z-20=0$

B : $x^2+y^2+z^2+3x-4y+3z-16=0$

C : $2x^2+2y^2+2z^2+6x-3y+2z=0$

D : $x^2+y^2+z^2+4x-4y+3z-20=0$

Q.no 49. In spherical co-ordinates volume is given by

A : $V=\int\int\int_V drd\theta d\phi$

B : $V=\int\int\int_V rdrd\theta d\phi$

C : $V=\int\int\int_V r^2\cos\theta drd\theta d\phi$

D : $V=\int\int\int_V r^2\sin\theta drd\theta d\phi$

Q.no 50. q56.jpg

A : 1

B : 0

C : q56_3.jpg

D : q56_4.jpg

Q.no 51.

The radius of the circle of intersection of the sphere $x^2 + y^2 + z^2 - 2x + 4y + 2z - 6 = 0$ by the plane $x + 2y + 2z - 4 = 0$ is:

[Centre of the circle is(2,0,1)]

A : 9

B : 3

C : 12

D : $\sqrt{3}$

Q.no 52. B(4,5)=

A : 1/280

B : 4/105

C : 1/70

D : 1/210

Q.no 53. Radius of right circular cylinder is 5and whose axis is the line $\frac{x-2}{3} = \frac{y-3}{1} = \frac{z+1}{1}$ Then the equation of right circular cylinder is

A : $(x+2)^2 + (y+3)^2 + (z+1)^2 = 5^2 + \left(\frac{3x+y+z-8}{\sqrt{11}}\right)^2$

B : $(x-2)^2 + (y-3)^2 + (z-1)^2 = 5^2 + \left(\frac{3x+y+z-8}{\sqrt{11}}\right)^2$

C : $(x+2)^2 + (y-3)^2 + (z+1)^2 = 5^2 + \left(\frac{3x+y+z-8}{\sqrt{11}}\right)^2$

D : $(x-2)^2 + (y-3)^2 + (z+1)^2 = 5^2 + \left(\frac{3x+y+z-8}{\sqrt{11}}\right)^2$

Q.no 54. In a circuit containing resistance R and inductance L in series with constant voltage source E current i is given by $i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$, then maximum current $i_{\max}$ is

A : E/R

B : R/E

C : ER

D : 0

Q.no 55. The value of integration $\int_0^1 \int_0^{x^2} xe^y dxdy$ is equal to

A : e/2

B : e-1

C : 1-e

D : (e/2)-1

Q.no 56.

The value of integration $\int_0^1 \int_0^1 \int_0^1 xyzdxdydz$ is equal to

A : 0.125

B : 0.25

C : -0.125

D : 1

Q.no 57.

Area bounded by the parabola $y^2 = 4x$ and the line $y = x$ is calculated using

A : $\int_{y=0}^{4} \int_{x=y^2/4}^{y} dxdy$

B : $\int_{y=0}^{4}\int_{x=0}^{4} dxdy$

C : $\int_{y=0}^{4}\int_{x=0}^{y} dxdy$

D : $\int_{x=0}^{4}\int_{y=0}^{x} dxdy$

Q.no 58. Fourier coefficient a_0 in the Fourier series expansion of $f(x) = x\sin x\,;\, 0 \le x \le 2\pi$ and $f(x+2\pi) = f(x)$

A : 2

B : 0

C : -2

D : -4

Q.no 59. $\frac{5\pi}{128}$

A : $\frac{5\pi}{128}$

B : $\frac{3\pi}{128}$

C : $\frac{3\pi}{512}$

D : $\frac{5\pi}{64}$

Q.no 60. $\int_0^{\pi/2} cos^6 x\, dx$ is equal to

A : $\frac{5}{16}$

B : $\frac{16}{5}.\frac{\pi}{2}$

C : $\frac{5}{16}.\frac{\pi}{2}$

D : $\frac{5}{48}.\frac{\pi}{2}$

Answer for Question No 1. is b

Answer for Question No 2. is a

Answer for Question No 3. is d

Answer for Question No 4. is c

Answer for Question No 5. is a

Answer for Question No 6. is b

Answer for Question No 7. is b

Answer for Question No 8. is d

Answer for Question No 9. is a

Answer for Question No 10. is b

Answer for Question No 11. is a

Answer for Question No 12. is a

Answer for Question No 13. is b

Answer for Question No 14. is a

Answer for Question No 15. is d

Answer for Question No 16. is c

Answer for Question No 17. is c

Answer for Question No 18. is a

Answer for Question No 19. is d

Answer for Question No 20. is b

Answer for Question No 21. is a

Answer for Question No 22. is b

Answer for Question No 23. is a

Answer for Question No 24. is a

Answer for Question No 25. is c

Answer for Question No 26. is d

Answer for Question No 27. is b

Answer for Question No 28. is b

Answer for Question No 29. is c

Answer for Question No 30. is a

Answer for Question No 31. is a

Answer for Question No 32. is c

Answer for Question No 33. is a

Answer for Question No 34. is b

Answer for Question No 35. is c

Answer for Question No 36. is c

Answer for Question No 37. is b

Answer for Question No 38. is a

Answer for Question No 39. is d

Answer for Question No 40. is c

Answer for Question No 41. is b

Answer for Question No 42. is a

Answer for Question No 43. is c

Answer for Question No 44. is b

Answer for Question No 45. is a

Answer for Question No 46. is c

Answer for Question No 47. is a

Answer for Question No 48. is a

Answer for Question No 49. is d

Answer for Question No 50. is c

Answer for Question No 51. is d

Answer for Question No 52. is a

Answer for Question No 53. is d

Answer for Question No 54. is a

Answer for Question No 55. is b

Answer for Question No 56. is a

Answer for Question No 57. is a

Answer for Question No 58. is c

Answer for Question No 59. is b

Answer for Question No 60. is c

Seat No -

Total number of questions : 60

1000505 T1 ENGINEERING MATHEMATICS II

Time : 1hr

Max Marks : 50

N.B

1) All questions are Multiple Choice Questions having single correct option.

2) Attempt any 50 questions out of 60.

3) Use of calculator is allowed.

4) Each question carries 1 Mark.

5) Specially abled students are allowed 20 minutes extra for examination.

6) Do not use pencils to darken answer.

7) Use only black/blue ball point pen to darken the appropriate circle.

8) No change will be allowed once the answer is marked on OMR Sheet.

9) Rough work shall not be done on OMR sheet or on question paper.

10) Darken ONLY ONE CIRCLE for each answer.

Q.no 1. The general solution of Linear Differential equation $\frac{dx}{dy} + Px = Q$ where P and Q are functions of y or constants, is

A : $x\, e^{\int p dx} = \int Q\, e^{\int p dx} dx + c$

B : $x\, e^{\int p dy} = \int Q\, e^{\int p dy} dy + c$

C : $y\, e^{\int p dx} = \int Q\, e^{\int p dx} dx + c$

D : $y\, e^{\int p dx} = \int Q\, e^{\int p dx} dy + c$

Q.no 2. Area enclosed by the polar curve $r = f_1(\theta)$ and $r = f_2(\theta)$ and the line $\theta = \alpha, \theta = \beta \alpha < \beta$ is given by

A : $Area = \int_{\alpha}^{\beta} \int_{f_1(\theta)}^{f_2(\theta)} r dr d\theta$

B : $Area = \int_{\alpha}^{\beta} \int_{f_1(\theta)}^{f_2(\theta)} dr d\theta$

C : $Area = \int_{\alpha}^{\beta} \int_{f_1(\theta)}^{f_2(\theta)} r^2 \sin\theta dr d\theta$

D : $Area = \int_{\alpha}^{\beta} \int_{f_1(\theta)}^{f_2(\theta)} r^2 \cos\theta dr d\theta$

Q.no 3. If $I(\alpha) = \int_a^b f(x,\alpha)dx$ where α is parameter and a, b are constants then by DUIS rule $\frac{dI(\alpha)}{d\alpha}$ is

A : $\int_a^b \frac{\partial}{\partial \alpha} f(x,\alpha)dx$

B : $\int_a^b \frac{\partial}{\partial x} f(x,\alpha)dx$

C : $f(b, \alpha) - f(a, \alpha)$

D : $f(x, \alpha)$

Q.no 4. If $\phi(a) = \int_0^{\infty} e^{-bx^2} \cos(2ax)dx,\ b > 0$ then by DUIS rule $\frac{d\phi}{da}$ is

A : $\int_0^{\infty} \frac{\partial}{\partial b}\left[e^{-bx^2} \cos(2ax)\right]dx$

B : $e^{-bx^2} \cos(2ax)$

C : $\int_0^{\infty} \frac{\partial}{\partial x}\left[e^{-bx^2} \cos(2ax)\right]dx$

D : $\int_0^\infty \frac{\partial}{\partial a}\left[e^{-bx^2}\cos(2ax)\right]dx$

Q.no 5. The integrating factor of the DE $cosx\frac{dy}{dx} + ysinx = 1$ is

A : sec x

B : cot x

C : tan x

D : sin x

Q.no 6. The differential equation for the current i in an electric circuit containing resistance R=250 ohm and an inductance of L=640 henery in series with an electromotive force E=500 volts is

A : $640\frac{di}{dt} + 250i = 0$

B : $250\frac{di}{dt} + 640i = 500$

C : $640\frac{di}{dt} + 250i = 500$

D : $250\frac{di}{dt} + 640i = 0$

Q.no 7. If n is a positive integer, then $\Gamma(n+1)$ is

A : (n+1)!

B : (n+2)!

C : (n-1)!

D : n!

Q.no 8. Moment of inertia of the lamina A about the x axis is equal to

A : $\int\int_A \rho x^2 dxdy$

B : $\int\int_A \rho y^2 dxdy$

C : $\int\int_A \rho(x^2+y^2)dxdy$

D : $\int\int_A \rho x^2y^2 dxdy$

Q.no 9. The integrating factor of $(y^4+2y)dx+(xy^3+2y^4-4x)dy=0$ using non exact rule $\frac{\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)}{M}$ is

A : $\frac{1}{y^3}$

B : 1/x

C : y^3

D : $\frac{1}{x^3}$

Q.no 10.

The equation of tangent to the curve at origin represented by the equation $y(1+x^2)=x$ is

A : y=x

B : x=0

C : x=1

D : y=0

Q.no 11. The solution of the DE $\frac{dy}{dx}=\frac{x^2}{y^3}$ is

A : $y^3-x^3=c$

B : $3y^4-4x^3=c$

C : $y^2-x^3=c$

D : $x^2 - y^2 = c$

Q.no 12. If $I_n = \int_0^{\pi/2} \sin^n x \, dx$, where n is positive even integer then I_n is calculated from

A : $\frac{n-1}{n} \cdot \frac{n-3}{n-2.} \cdot \frac{n-5}{n-4} \cdots\cdots \frac{3}{4} \cdot \frac{1}{2}$

B : $\frac{n-1}{n} \cdot \frac{n-3}{n-2.} \cdot \frac{n-5}{n-4} \cdots\cdots \frac{3}{4} \cdot \frac{1}{2} \cdot \pi$

C : $\left(\frac{n+1}{n} \cdot \frac{n.+3}{n+2.} \cdot \frac{n+5}{n+4}\right) \cdots\cdots \frac{\pi}{2}$

D : $\frac{n-1}{n} \cdot \frac{n-3}{n-2.} \cdot \frac{n-5}{n-4} \cdots\cdots \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}$

Q.no 13.

Find semi-vertical angle for a right circular cone with vertex at the point (1,0,1)which passes through the point (1,1,1) and axis of cone has direction ratios 1,1,1

A : $\sin\alpha = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

B : $\cos\alpha = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

C : $\cos\alpha = a_1 a_2 + b_1 b_2 + c_1 c_2$

D : $\sin\alpha = a_1 a_2 + b_1 b_2 + c_1 c_2$

Q.no 14. The center and radius of the sphere $3(x^2 + y^2 + z^2) - 30x + 12y - 18z + 89 = 0$ is

A : (10,-4,6) , $\frac{5}{\sqrt{3}}$

B : $(15,-6,9)$, $\frac{\sqrt{203}}{\sqrt{3}}$

C : $(5,-2,3)$, $\frac{5}{\sqrt{3}}$

D : $(-5,2,-3)$, $\frac{5}{3}$

Q.no 15. The amount of heat Q flowing through the area per unit time is

A : Q = thermal conductivity x Area x Rate of change of temp. across an area

B : Q = thermal conductivity x Area of slab

C : Q = thermal conductivity x Area x temp. gradient

D : Q = thermal conductivity x Area + Rate of flow of heat

Q.no 16. Half range Fourier cosine series for $f(x)$ defined in the interval $0 \le x \le L$ is

A : Half range Fourier cosine series for $f(x)$ defined in the interval $0 \le x \le L$ is

B : $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{nx}{L}$

C : $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$

D : $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n\, cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$

Q.no 17. The integrating factor of $(x^2 + y^2 + x)dx + xydy = 0$ using non exact rule $\frac{\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)}{N}$ is

A : x

B : 1/x

C : x2

D : xy

Q.no 18. Error function of x, $erf(x)$ is defined as

A : $\frac{2}{\sqrt{\pi}}\int_{0}^{x} e^{-u^2} du$

B : $\frac{2}{\sqrt{\pi}}\int_{x}^{\infty} e^{-u^2} du$

C : $\int_{0}^{\infty} e^{-x}x^{n-1} dx$

D : $\frac{2}{\sqrt{\pi}}\int_{0}^{\infty} e^{-u^2} du$

Q.no 19. Rectilinear motion is a motion of body along a

A : Straight line

B : Circular path

C : Parabolic path

D : Hyperbolic path

Q.no 20. If the sphere $x^2 + y^2 + z^2 = 25$,

$x^2 + y^2 + z^2 - 18x - 24y - 40z + 225 = 0$

touches each other externally then distance between their centers is:

A : 20

B : 5

C : 15

D : 25

Q.no 21. Reduction formula for gamma function is

A : $\Gamma(n+1) = (n-1)\Gamma(n-1)$

B : $\Gamma(n+1) = n\Gamma(n)$

C : $\Gamma(n+1) = (n-1)\Gamma(n)$

D : $\Gamma(n+1) = n \quad \Gamma(n-1)$

Q.no 22. A pipe contains steam and is protected with a covering then in the formula $q = kA\,\frac{dT}{dt}$

A : $A = 2\pi r l$

B : $A = \frac{1}{3}\pi r^2 l$

C : $A = \frac{1}{3}\pi r^2 l$

D : $A = rl$

Q.no 23. The curve $xy^2 = a^2(a-x)$

A : passes through the point (-a,0)

B : does not pass through origin

C : passes through the origin

D : passes through the point (a,a)

Q.no 24. The value of the integral $\int_0^b e^{-u^2}\,du$ is

A : $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\,du$

B : $\frac{\sqrt{\pi}}{2} erf\,(b)$

C : $\frac{2}{\sqrt{\pi}} erf_C(b)$

D : $erf\,(b)$

Q.no 25.

Condition for the two spheres $x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$ and $x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 = 0$ to be orthogonal is ---

A : $S + \lambda U = 0$

B : $S_1 - S_2 = 0$

C : $2u_1u_2 + 2v_1v_2 + 2w_1w_2 = d_1 + d_2$

D : $u_1u_2 + v_1v_2 + w_1w_2 = d_1 + d_2$

Q.no 26.

If $(\bar{x}, \bar{y})$ the coordinates of centroid of the plane area is given by

A : $\bar{x} = \frac{\int\int x\rho dxdy}{\int\int \rho dxdy}, \bar{y} = \frac{\int\int y\rho dxdy}{\int\int \rho dxdy}$

B : $\bar{x} = \frac{\int\int x^2\rho dxdy}{\int\int \rho dxdy}, \bar{y} = \frac{\int\int y^2\rho dxdy}{\int\int \rho dxdy}$

C : $\bar{x} = \frac{\int\int \rho dxdy}{\int\int x\rho dxdy}, \bar{y} = \frac{\int\int \rho dxdy}{\int\int y\rho dxdy}$

D : None of these

Q.no 27. Gamma function of n > 0 is defined as

A : $\int_0^\infty e^x x^{n-1}\, dx$

B : $\int_0^\infty e^x x^{n-1}\, dx$

C : $\int_0^\infty e^{-x} x^{n-1}\,dx$

D : $\int_0^\infty e^{-x} x^{1-n}\,dx$

Q.no 28. A circuit containing inductance L capacitance C in series without applied electromotive force. By Kirchhoff's voltage law differential equation for current i is

A : $L\frac{di}{dt}+\frac{q}{c}=0$

B : $L\frac{di}{dt}+Ri=0$

C : $L\frac{di}{dt}+Ri=E$

D : $L\frac{di}{dt}+\frac{q}{c}=E$

Q.no 29. According to D'Alembert's principle algebraic sum of forces acting on a body along a given direction is equal to

A : Velocity x Acceleration

B : Mass x Velocity

C : Mass x displacement

D : Mass x Acceleration

Q.no 30. If the differential equation $M(x,y)dx+N(x,y)dy=0$ is not exact and it can be written as $yf_1(x,y)dx+xf_2(x,y)dy=0$ then integrating factor is

A : $\frac{1}{My+Nx}; My+Nx\neq 0$

B : $\frac{1}{Mx+Ny}; Mx+Ny\neq 0$

C : $\frac{1}{Mx - Ny}\ ; Mx - Ny \neq 0$

D : $\frac{1}{My - Nx}; My - Nx \neq 0$

Q.no 31. The integrating factor for the DE $\frac{dy}{dx} + \frac{x}{1+x} y = 1 + x$ is

A : $e^{x(1+x)}$

B : $e^{x(1+x)}$

C : $e^{x/(1+x)}$

D : $1 + x^2$

Q.no 32. The integrating factor of the DE $\frac{dy}{dx} + ycosx = \frac{sin2x}{2}$ is

A : e^{cosx}

B : e^{sinx}

C : sin x

D : cos x

Q.no 33. For RC circuit the charge q satisfies the linear D.E.

A : $R + \frac{dq}{dt} = E$

B : $Ri + q = 0$

C : $A = \frac{1}{3}\pi r^2 l$

D : $R\frac{dq}{dt} + \frac{q}{C} = E$

Q.no 34. Moment of inertia of the lamina A about the y axis is equal to

A : $\int \int_A \rho x^2 dxdy$

B : $\int \int_A \rho y^2 dxdy$

C : $\int \int_A \rho(x^2 + y^2) dxdy$

D : $\int \int_A \rho x^2 y^2 dxdy$

Q.no 35. The integrating factor of $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$ using non exact rule $\frac{\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)}{M}$ is

A : y^2

B : $\frac{1}{y^2}$

C : y

D : 1/y

Q.no 36. A particle moving in straight line with acceleration $k(x + \frac{a^4}{x^3})$ directed towards origin. The equation of motion is

A : $\frac{dv}{dx} = -k(x + \frac{a^4}{x^3})$

B : $v\frac{dv}{dx} = k(x + \frac{a^4}{x^3})$

C : $v\frac{dv}{dx} = -k(x + \frac{a^4}{x^3})$

D : $\frac{dv}{dx} = (x + \frac{a^4}{x^3})$

Q.no 37. A circuit containing inductance L capacitance C in series with applied electromotive force E. By Kirchhoff's voltage law differential equation for current i is

A : $L\frac{di}{dt} + Ri = E$

B : $L\frac{di}{dt} + Ri = 0$

C : $L\frac{di}{dt} + \frac{q}{C} = E$

D : $L\frac{di}{dt} + \frac{q}{C} = 0$

Q.no 38. The integrating factor for the DE $(1 + x^2)\frac{dy}{dx} + xy = 0$ is

A : e^{1+x^2}

B : $\log(x^2 + 1)$

C : $\sqrt{x^2 + 1}$

D : $1 + x^2$

Q.no 39. The value of integration $\int_0^a \int_0^b \int_0^c x^2y^2z^2 dxdydz$ is equal to

A : $\frac{abc}{3}$

B : $\frac{a^2b^2c^2}{3}$

C : $\frac{a^3b^3c^3}{27}$

D : $\frac{a^2b^2c^2}{9}$

Q.no 40. The value of equivalent form of gamma function $\int_0^\infty e^{-kx} x^{n-1}\, dx$ is

A : $\frac{\Gamma n}{n^k}$

B : $\frac{\Gamma n}{k^n}$

C : $\frac{\Gamma n}{k!}$

D : $\Gamma(n+k+1)$

Q.no 41. The value of $\int\int\int_V dxdydz$ where V is the volume bounded by $x^2+y^2+z^2=1$

A : $\frac{4}{3}\pi$

B : 4π

C : π

D : $\frac{1}{3}\pi$

Q.no 42. If double integral in Cartesian coordinate is given by $\int\int_R f(x,y)\,dxdy$ then the value of same integral in polar form is

A : $\int\int_R f(r\cos\theta, r\sin\theta)\,rdrd\theta$

B : $\int\int_R f(r\cos\theta, r\sin\theta)\,rdrd\theta$

C : $\int\int_R f(r\cos\theta, r\sin\theta)\,r^2drd\theta$

D : $\int\int_R f(r\sin\theta, r\cos\theta)\,rdrd\theta$

Q.no 43. The solution of the DE $\frac{dy}{dx}+y=e^{-x}$ is

A : $ye^{-x} + c = x$

B : $ye^{x} + x = c$

C : $ye^{x} = x + c$

D : $y = x + c$

Q.no 44.

The two spheres $x^2 + y^2 + z^2 - 12x - 18y - 24z + 29 = 0$ and $x^2 + y^2 + z^2 + 3x - 3y + 3z - 56 = 0$

A : touches each other internally

B : touches each other externally

C : touches each other Orthogonally

D : Intersecting each other

Q.no 45.

If $I(\alpha) = \int_a^b f(x,\alpha)dx$ where a, b are function of parameter α then by DUIS rule $\frac{dI(\alpha)}{d\alpha}$ is

A : $\int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)dx + f(a,\alpha)\frac{da}{d\alpha} - f(b,\alpha)\frac{db}{d\alpha}$

B : $\int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)dx + f(b,\alpha)\frac{db}{d\alpha} - f(a,\alpha)\frac{da}{d\alpha}$

C : $\int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)dx + f(b,\alpha)\frac{db}{d\alpha} - f(a,\alpha)\frac{da}{d\alpha}$

D : $\int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)dx$

Q.no 46. In spherical co-ordinates volume is given by

A : $V = \int \int \int_V drd\theta d\phi$

B : $V = \int \int \int_V rdrd\theta d\phi$

C : $V = \int \int \int_V r^2 \cos\theta drd\theta d\phi$

D : $V = \int \int \int_V r^2 \sin\theta drd\theta d\phi$

Q.no 47. B(4,5)=

A : 1/280

B : 4/105

C : 1/70

D : 1/210

Q.no 48. A particle of mass m moves in a horizontal straight line OA with acceleration $\frac{k}{x^3}$ at a distance x and directed towards origin O. Then the differential equation of motion is

A : $v\frac{dv}{dx} = \frac{k}{x^3}$

B : $v\frac{dv}{dx} = -\frac{k}{x^3}$

C : $\frac{dv}{dx} = -\frac{k}{x^3}$

D : $\frac{dv}{dx} = \frac{k}{x^3}$

Q.no 49.

If the plane $4x - 3y + 6z - 35 = 0$ is tangential to the sphere

$x^2 + y^2 + z^2 - y - 2z - 14 = 0$ then the length of perpendicular from any point P on the plane is---

A : $\frac{61}{2}$

B : $\frac{61}{2}$

C : 30

D : $-\frac{\sqrt{61}}{2}$

Q.no 50.

Area bounded by the parabola $y^2 = 4x$ and the line $y = x$ is calculated using

A : $\int_{y=0}^{4} \int_{x=y^2/4}^{y} dxdy$

B : $\int_{y=0}^{4} \int_{x=0}^{4} dxdy$

C : $\int_{y=0}^{4} \int_{x=0}^{y} dxdy$

D : $\int_{x=0}^{4} \int_{y=0}^{x} dxdy$

Q.no 51.

The value of integration $\int_0^1 \int_0^x (x^2 + y^2)\, dxdy$ is equal to

A : 4/3

B : 5/3

C : 2/3

D : 1

Q.no 52. q56.jpg

A : 1

B : 0

C : q56_3.jpg

D : q56_4.jpg

Q.no 53. If $I(\alpha)=\int_0^1 \frac{x^\alpha-1}{\log x}dx$, $\alpha\geq 0$ then by DUIS $\frac{dI}{d\alpha}$ is

A : $\log(\alpha+1)$

B : $\log(\alpha-1)$

C : $\log\alpha$

D : 0

Q.no 54. $B\left(\frac{1}{4},\frac{3}{4}\right)$ is equal to

A : $\frac{2\pi}{\sqrt{3}}$

B : $\frac{2\pi}{\sqrt{3}}$

C : 0

D : $\pi\sqrt{2}$

Q.no 55. If $I(\alpha)=\int_0^1 \frac{x^\alpha-1}{\log x}dx$, $\alpha\geq 0$ then by DUIS $\frac{dI}{d\alpha}$ is

A : $\int_0^1 \log x \, dx$

B : $\int_0^1 x^{\alpha} \log x \, dx$

C : $\int_0^1 x^{\alpha} dx$

D : $\int_0^1 x^{-\alpha} dx$

Q.no 56. If $I_n=\int_0^{\pi/2} cos^n x \, dx$ then which of the following relation is true?

A : $\frac{n-1}{n} \cdot \frac{n-3}{n-2.} \cdot \frac{n-5}{n-4} \cdots\cdots \frac{3}{4} \cdot \frac{1}{2}$

B : $I_n = \frac{n}{n-1} I_{n-2}$

C : $I_n = \frac{n-1}{n} I_{n-2}$

D : $I_n = n(n+1) I_{n-1}$

Q.no 57. A capacitor C=0.01 farad in series with resistor R=20 ohms is charged from battery E=10 volts. If initially capacitor is completely discharged then differential equation for charge q(t) is given by

A : $20\frac{dq}{dt} + \frac{q}{0.01} = 0 \; ; \; q(0) = 0$

B : $20\frac{dq}{dt} + 0.01q = 10 \; ; \; q(0) = 0$

C : $20\frac{dq}{dt} + \frac{q}{0.01} = 10 \; ; \; q(0) = 0$

D : $20\frac{dq}{dt} + 0.01q = 0$; $q(0) = 0$

Q.no 58. Fourier coefficient a_0 in the Fourier series expansion of

$f(x) = x\sin x$; $0 \le x \le 2\pi$ and $f(x+2\pi) = f(x)$

A : 2

B : 0

C : -2

D : -4

Q.no 59. The value of a_0 in harmonic analysis of y for the following tabulated data is

x	0	1	2	3	4	5	6
y	9	18	24	28	26	20	9

A : 17.85

B : 20.83

C : 35.71

D : 41.66

Q.no 60.

The value of integration $\int_0^1 \int_{x^2}^{x} xy(x+y)dxdy$ is equal to

A : 3/15

B : 2/15

C : 4/15

D : 1/15

Answer for Question No 1. is b

Answer for Question No 2. is a

Answer for Question No 3. is a

Answer for Question No 4. is d

Answer for Question No 5. is a

Answer for Question No 6. is c

Answer for Question No 7. is d

Answer for Question No 8. is b

Answer for Question No 9. is a

Answer for Question No 10. is a

Answer for Question No 11. is b

Answer for Question No 12. is d

Answer for Question No 13. is b

Answer for Question No 14. is c

Answer for Question No 15. is c

Answer for Question No 16. is c

Answer for Question No 17. is a

Answer for Question No 18. is a

Answer for Question No 19. is a

Answer for Question No 20. is d

Answer for Question No 21. is b

Answer for Question No 22. is a

Answer for Question No 23. is b

Answer for Question No 24. is b

Answer for Question No 25. is c

Answer for Question No 26. is a

Answer for Question No 27. is c

Answer for Question No 28. is a

Answer for Question No 29. is d

Answer for Question No 30. is c

Answer for Question No 31. is c

Answer for Question No 32. is b

Answer for Question No 33. is d

Answer for Question No 34. is a

Answer for Question No 35. is b

Answer for Question No 36. is c

Answer for Question No 37. is c

Answer for Question No 38. is b

Answer for Question No 39. is c

Answer for Question No 40. is b

Answer for Question No 41. is a

Answer for Question No 42. is a

Answer for Question No 43. is c

Answer for Question No 44. is c

Answer for Question No 45. is c

Answer for Question No 46. is d

Answer for Question No 47. is a

Answer for Question No 48. is b

Answer for Question No 49. is a

Answer for Question No 50. is a

Answer for Question No 51. is c

Answer for Question No 52. is c

Answer for Question No 53. is a

Answer for Question No 54. is d

Answer for Question No 55. is c

Answer for Question No 56. is c

Answer for Question No 57. is c

Answer for Question No 58. is c

Answer for Question No 59. is d

Answer for Question No 60. is b

MULTIPLE CHOICE QUESTIONS

Order, Degree and Formation of Differential Equation :

1. The differential equation $1+\frac{dy}{dx}-\left(\frac{d^2y}{dx^2}\right)^{3/2}=0$ is of (1)

(A) order 1 and degree 2

(B) order 2 and degree 3

(C) order 3 and degree 6

(D) order 3 and degree 3

2. The differential equation $\sqrt{1+\frac{dy}{dx}}=\frac{d^2y}{dx^2}$ is of (1)

(A) order 2 and degree 2

(B) order 1 and degree 2

(C) order 2 and degree 1

(D) order 1 and degree 1

3. The differential equation $x = \frac{1}{\sqrt{1 + \frac{dy}{dx} + \frac{d^2y}{dx^2}}}$ is of (1)

(A) order 2 and degree 2
(B) order 1 and degree 2
(C) order 2 and degree 1
(D) order 1 and degree 1

4. The differential equation $1 + \frac{dy}{dx} = \frac{y}{\frac{dy}{dx}}$ is of (1)

(A) order 2 and degree 2
(B) order 1 and degree 1
(C) order 2 and degree 1
(D) order 1 and degree 2

5. The differential equation $(2x - y + 3)\,dx + (y - 2x - 2)\,dy = 0$ is of (1)

(A) order 1 and degree 1
(B) order 1 and degree 2
(C) order 2 and degree 1
(D) order 2 and degree 2

6. The number of arbitrary constants in the general solution of ordinary differential equation is equal to (1)

(A) the order of differential equation
(B) the degree of differential equation
(C) coefficient of highest order differential coefficient
(D) none of these

7. The general solution of n^{th} order ordinary differential equation must involve (1)

(A) $(n + 1)$ arbitrary constants
(B) $(n - 1)$ arbitrary constants
(C) n arbitrary constants
(D) none of these

8. The solution obtained by assigning particular values to arbitrary constants in general solution of ordinary differential equation is called (1)

(A) the general solution
(B) a particular solution
(C) a singular solution
(D) none of these

9. The order of differential equation whose general solution is

$y = C_1 + C_2e^{-2x} + C_3e^{3x} + C_4e^{-3x}$, where C_1, C_2, C_3, C_4 are arbitrary constants, is (1)

(A) 1
(B) 3
(C) 2
(D) 4

10. The order of differential equation whose general solution is

$y = e^x(Ax^2 + Bx + C)$, where A, B, C are arbitrary constants, is (1)

(A) 2
(B) 4
(C) 3
(D) 1

11. The differential equation whose general solution is $y = \sqrt{5x + C}$, where C is arbitrary constant, is (1)

(A) $2y\dfrac{dy}{dx} - 1 = 0$ (B) $2y\dfrac{dy}{dx} - 5 = 0$

(C) $\dfrac{dy}{dx} - \dfrac{5}{2}\dfrac{1}{\sqrt{5x+C}} = 0$ (D) $y\dfrac{dy}{dx} - 5 = 0$

12. $y = Cx - C^2$, where C is arbitrary constant is the general solution of the differential equation (1)

(A) $\dfrac{dy}{dx} = C$ (B) $\left(\dfrac{dy}{dx}\right)^2 + xy = 0$

(C) $\left(\dfrac{dy}{dx}\right)^2 + x\dfrac{dy}{dx} - y = 0$ (D) $\left(\dfrac{dy}{dx}\right)^2 - x\dfrac{dy}{dx} + y = 0$

13. The differential equation whose general solution is $y = C^2 + \dfrac{C}{x}$, where C is arbitrary constant is (1)

(A) $x^4 y_1^2 + xy_1 - y = 0$ (B) $x^4 y_1^2 - xy_1 - y = 0$

(C) $x^2 y_1^2 - xy_1 - y = 0$ (D) $y_1 = -\dfrac{C}{x^2}$

14. By eliminating arbitrary constant A the differential equation whose general solution is $y = 4(x - A)^2$ (1)

(A) $y_1^2 + 16y = 0$ (B) $y_1 - 2y = 0$ (C) $y_1^2 - 16y = 0$ (D) $y_1 - 8(x - A) = 0$

15. The differential equation whose general solution is $y = A\cos(x + 3)$, where A is arbitrary constant, is (1)

(A) $\cot(x + 3)\, y_1 + y = 0$ (B) $\tan(x + 3)\, y_1 + y = 0$

(C) $\cot(x + 3)\, y_1 - y = 0$ (D) $\tan(x + 3)\, y_1 - y = 0$

16. By eliminating arbitrary constant a the differential equation whose general solution is $y^2 = 4ax$ is (1)

(A) $xy\dfrac{dy}{dx} - y^2 = 0$ (B) $2xy\dfrac{dy}{dx} + y^2 = 0$ (C) $2xy\dfrac{dy}{dx} - y^2 = 0$ (D) $8xy\dfrac{dy}{dx} - y^2 = 0$

17. The differential equation whose general solution is $xy = C^2$, where C is arbitrary constant, is (1)

(A) $xy_1 - y = 0$ (B) $xy_2 + y_1 = 0$ (C) $xy_1 = C^2$ (D) $xy_1 + y = 0$

18. The differential equation representing the family of curves $y^2 = 2C(x + \sqrt{C})$, where C is arbitrary constant, is (1)

(A) $2yy_1\left(x + \sqrt{yy_1}\right) - y^2 = 1$
(B) $2y_1\left(x + \sqrt{yy_1}\right) - y = 0$
(C) $y = 2y_1\left(x + \sqrt{C}\right)$
(D) $y_1\left(x + \sqrt{yy_1}\right) - y = 0$

19. By eliminating arbitrary constant A the differential equation whose general solution is $y = Ae^{-x^2}$ is (1)

(A) $\frac{dy}{dx} - 2xy = 0$ (B) $y\frac{dy}{dx} - 2x = 0$ (C) $\frac{dy}{dx} + 2xy = 0$ (D) $y\frac{dy}{dx} + 2x = 0$

20. y = mx where m is arbitrary constant is the general solution of the differential equation is (1)

(A) $\frac{dy}{dx} = \frac{y}{x}$ (B) $\frac{dy}{dx} = \frac{x}{y}$ (C) $\frac{dy}{dx} = m$ (D) $\frac{dy}{dx} = -\frac{y}{x}$

21. The differential equation representing the family of curves $y = 3 + \sqrt{Cx}$, where C is arbitrary constant, is (2)

(A) $y = 3 + 2x\frac{dy}{dx}$ (B) $y = 3 + 2\sqrt{x}\frac{dy}{dx}$ (C) $y = 2x\frac{dy}{dx}$ (D) $\frac{dy}{dx} = \frac{\sqrt{c}}{2\sqrt{x}}$

22. The differential equation satisfied by general solution $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$, where a is arbitrary constant, is (2)

(A) $xyy_1 - y + 4 = 0$
(B) $xyy_1 + y^2 - 4 = 0$
(C) $x^2yy_1 + y^2x - 1 = 0$
(D) $xyy_1 - y^2 + 4 = 0$

23. The differential equation representing the family of curves $x^2 + y^2 = 2Ax$, where A is arbitrary constant, is (2)

(A) $y_1 = \frac{y^2 + x^2}{2xy}$ (B) $y_1 = \frac{y^2 - x^2}{2xy}$ (C) $y_1 = \frac{y^2 - x^2}{2y}$ (D) $y_1 = \frac{2xy}{y^2 - x^2}$

24. $y^2 = C(4 + e^{2x})$ where C is arbitrary constant is the general solution of the differential equation (2)

(A) $(4 + e^{2x})\frac{dy}{dx} - ye^{2x} = 0$
(B) $y\frac{dy}{dx} - e^{2x}(4 + e^{2x}) = 0$
(C) $e^{2x}\frac{dy}{dx} - y^2e^{2x} = 0$
(D) $y(4 + e^{2x})\frac{dy}{dx} - e^{2x} = 0$

25. $\sin(y - x) = Ce^{-\frac{x^2}{2}}$ where C is arbitrary constant is the general solution of the differential equation (2)

(A) $\tan(y-x)\left[\frac{dy}{dx}-1\right]+x=0$

(B) $\cot(y-x)\frac{dy}{dx}-1+x=0$

(C) $\cot(y-x)\left[\frac{dy}{dx}-1\right]+x=0$

(D) $\cot(y-x)\left[\frac{dy}{dx}-1\right]=0$

26. By eliminating arbitrary constant A the differential equation whose general solution is $(1+x^2) = A(1-y^2)$ is (2)

(A) $\frac{(1+x^2)}{(1-y^2)}\frac{dy}{dx}+\frac{x}{y}=0$

(B) $\frac{(1-y^2)}{(1+x^2)}\frac{dy}{dx}+\frac{x}{y}=0$

(C) $\frac{(1+x^2)}{(1-y^2)}\frac{dy}{dx}+\frac{y}{x}=0$

(D) $\frac{(1-y^2)}{(1+x^2)}\frac{dy}{dx}-\frac{x}{y}=0$

27. The differential equation satisfied by general solution $x = Cy - y^2$, where C is arbitrary constants, is (2)

(A) $\left(\frac{y}{x+y^2}\right)y_1-2yy_1-1=0$

(B) $\left(\frac{x+y^2}{y}\right)y_1-2yy_1-1=0$

(C) $\left(\frac{x+y^2}{y}\right)y_1+2yy_1+1=0$

(D) $\left(\frac{x+y^2}{y}\right)y_1+2yy_1=0$

28. The differential equation satisfied by general solution $y + x^3 = Cx$, where C is arbitrary constants, is (2)

(A) $\frac{dy}{dx}+3x^2=C$

(B) $x\frac{dy}{dx}+2x^2-y=0$

(C) $\frac{dy}{dx}+2x^2-y=0$

(D) $x\frac{dy}{dx}+2x^3-y=0$

29. $xy + y^2 - x^2 - 3y - x = C$, where C is arbitrary constant is the general solution of the differential equation (2)

(A) $\frac{dy}{dx}=\frac{2x-y+1}{x+2y-3}$

(B) $\frac{dy}{dx}=\frac{2x+1}{x+2y-3}$

(C) $\frac{dy}{dx}=\frac{y-2x-1}{x-2y+3}$

(D) $\frac{dy}{dx}=\frac{x+2y-3}{2x-y+1}$

30. $y = Ce^{x/y}$, where C is arbitrary constant is the general solution of the differential equation (2)

(A) $yy_1 + (y - xy_1) = 0$
(B) $yy_1 - (y - xy_1) = 0$
(C) $y^2y_1 - (y - xy_1) = 0$
(D) $\frac{dy}{dx} - 1 = 0$

31. $\sin\left(\frac{y}{x}\right) = Cx$, where C is arbitrary constant is the general solution of the differential equation (2)

(A) $xy_1 + y = x\tan\left(\frac{y}{x}\right)$
(B) $xy_1 - y = x\cot\left(\frac{y}{x}\right)$
(C) $xy_1 - y = x\tan\left(\frac{x}{y}\right)$
(D) $xy_1 - y = x\tan\left(\frac{y}{x}\right)$

32. By eliminating arbitrary constant A the differential equation whose general solution is $y^2 = x^2 - 1 + Ax$ is (2)

(A) $2xy\frac{dy}{dx} = x^2 + y^2 + 1$
(B) $2xy\frac{dy}{dx} = x^2 + x + y^2 + 1$
(C) $2xy\frac{dy}{dx} = y^2 + 1$
(D) $2y\frac{dy}{dx} = 2x + A$

33. The differential equation satisfied by general solution $y = A\cos x + B\sin x$, where A and B are arbitrary constants, is (2)

(A) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = B\sin x$
(B) $\frac{d^2y}{dx^2} - y = 0$
(C) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = A\cos x$
(D) $\frac{d^2y}{dx^2} + y = 0$

34. The differential equation satisfied by general solution $y = A\cos\frac{2x}{3} + B\sin\frac{2x}{3}$, where A and B are arbitrary constants, is

(A) $\frac{d^2y}{dx^2} + \frac{9}{4}y = 0$
(B) $\frac{d^2y}{dx^2} - \frac{4}{9}y = 0$
(C) $\frac{d^2y}{dx^2} + \frac{4}{9}y = 0$
(D) $\frac{d^2y}{dx^2} - \frac{9}{4}y = 0$

35. The differential equation satisfied by general solution $y = A\cos(\log x) + B\sin(\log x)$ where A and B are arbitrary constants, is (2)

(A) $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$
(B) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$
(C) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$
(D) $x^2\frac{d^2y}{dx^2} + y = 0$

36. The differential equation satisfied by general solution $y = Ae^{x} + Be^{-x}$, where A and B are arbitrary constants, is (2)

(A) $y_2 - y = 0$ (B) $y_2 + y = 0$

(C) $y_2 + y = Ae^{x} - Be^{-x}$ (D) $y_2 - y = 2Ae^{x}$

37. The differential equation satisfied by general solution $xy = Ae^{x} + Be^{-x}$, where A and B are arbitrary constants, is (2)

(A) $xy_2 + 2y_1 + xy = 0$ (B) $xy_2 - 2y_1 + xy = 0$

(C) $xy_2 + 2y_1 - xy = 0$ (D) $xy_2 + y_1 - xy = 0$

$x = A\cos(2t + B$
$x = A\cos(2t$

38. The differential equation satisfied by general solution $x = A\cos(2t + B)$, where A and B are arbitrary constants, is (2)

(A) $\frac{d^2x}{dt^2} + 4x = 0$ (B) $\frac{d^2x}{dt^2} - 2x = 0$ (C) $\frac{d^2x}{dt^2} - 4x = 0$ (D) $\frac{d^2x}{dt^2} + x = 0$

39. By eliminating arbitrary constants A and B the differential equation whose general solution is $e^{-t}x = (A + Bt)$ is (2)

(A) $x_2 + 2x_1 - x = 0$ (B) $x_2 - 2x_1 + x = 0$ (C) $x_2 - x_1 + x = 0$ (D) $x_2 + x = 0$

40. By eliminating arbitrary constants A and B the differential equation whose general solution is $y^2 = 4A(x - B)$ is (2)

(A) $y_2 + y_1^2 = 0$ (B) $yy_2 + y_1 = 0$ (C) $yy_2 - y_1^2 = 0$ (D) $yy_2 + y_1^2 = 0$

41. The differential equation of family of circles having their centres at (A, 5) and radius 5, where A is arbitrary constant is (2)

(A) $(y-5)^2\left\{1 + \frac{dy}{dx}\right\} = 5$ (B) $(y-5)^2\left\{1 - \left(\frac{dy}{dx}\right)^2\right\} = 25$

(C) $(y-5)^2\left\{1 + \left(\frac{dy}{dx}\right)^2\right\} = 25$ (D) none of these

42. The differential equation of family of circles having their centres at origin and radius a where a is arbitrary constant, is (2)

(A) $x - y\frac{dy}{dx} = 0$ (B) $x + y\frac{dy}{dx} = 0$ (C) $x\frac{dy}{dx} + y = 0$ (D) $x + y\frac{dy}{dx} = \frac{a^2}{2}$

43. By eliminating arbitrary constants A and B the differential equation whose general solution is $(x - A)^2 = 4(y - B)$ is (2)

(A) $2\frac{dy}{dx} - (x - A) = 0$ (B) $\frac{d^2y}{dx^2} + \frac{1}{2} = 0$ (C) $\frac{d^2y}{dx^2} - 2 = 0$ (D) $\frac{d^2y}{dx^2} - \frac{1}{2} = 0$

44. The differential equation satisfied by general solution $y = A\cos 4x + B\sin 4x + C$, where A, B and C are arbitrary constants, is (2)

(A) $\frac{d^2y}{dx^2} - 16y = 0$ (B) $\frac{d^3y}{dx^3} - 16\frac{dy}{dx} = 0$ (C) $\frac{d^3y}{dx^3} + 16\frac{dy}{dx} = 0$ (D) $\frac{d^3y}{dx^3} + \frac{dy}{dx} = 0$

45. The differential equation satisfied by general solution $y = Ax^2 + Bx + C$, where A, B and C are arbitrary constants, is (2)

(A) $\frac{d^3y}{dx^3} = 0$ (B) $\frac{d^2y}{dx^2} = 2A$ (C) $\frac{d^3y}{dx^3} = A$ (D) $\frac{d^4y}{dx^4} = 0$

ANSWERS

1. (B)	2. (A)	3. (C)	4. (D)	5. (A)	6. (A)	7. (C)	8. (B)
9. (D)	10. (C)	11. (B)	12. (D)	13. (B)	14. (C)	15. (A)	16. (C)
17. (D)	18. (B)	19. (C)	20. (A)	21. (A)	22. (D)	23. (B)	24. (A)
25. (C)	26. (A)	27. (B)	28. (D)	29. (A)	30. (B)	31. (D)	32. (A)
33. (D)	34. (C)	35. (B)	36. (A)	37. (C)	38. (A)	39. (B)	40. (D)
41. (C)	42. (B)	43. (D)	44. (C)	45. (A)			

MULTIPLE CHOICE QUESTIONS

1. The differential equation $\frac{dy}{dx} = e^{x-y} + 3x^2e^{-y}$ is of the form (1)

(A) variable separable (B) homogeneous (C) linear (D) exact

2. For solving the differential equation $(x + y + 1)\,dx + (2x + 2y + 4)\,dy = 0$ appropriate substitution is (1)

(A) $x + y = 1$ (B) $x + y = u$ (C) $x - y = u$ (D) none of these

3. The differential equation $\frac{dy}{dx} = \frac{x^3 - 3xy^2}{y^3 - 3x^2y}$ is of the form (1)

(A) variable separable (B) homogeneous (C) linear (D) exact

4. The differential equation $\frac{dy}{dx} = \frac{x + 2y - 3}{3x + 6y - 1}$ is of the form (1)

(A) variable separable (B) exact

(C) non-homogeneous (D) homogeneous

5. The solution of differential equation $\frac{dy}{dx} + y = 0$ is (1)

(A) $y = Ae^{-x}$ (B) $y = Ae^{x}$ (C) $x = Ae^{-y}$ (D) $x = Ae^{y}$

6. The solution of differential equation $\frac{dy}{dx} + x = 0$ is (1)

(A) $x + y^2 = C$ (B) $x + y = C$ (C) $x^2 + y = C$ (D) $x^2 + 2y = C$

7. The solution of differential equation $ydx + xdy = 0$ is (1)

(A) $x^2y = C$ (B) $xy = C$ (C) $xy^2 = C$ (D) $xy + 1 = C$

8. The solution of differential equation $\frac{dy}{dx} + \tan x = 0$ is (1)

(A) $y + \log \sin x = C$ (B) $y + \sec^2 x = C$

(C) $y - \log \cos x = C$ (D) $y + \log \cot x = C$

9. The solution of differential equation $\frac{dy}{dx} = \frac{1 + y}{1 + x}$ is (1)

(A) $(1 + y) = C\,(1 + x)$ (B) $(1 + x) = \frac{C}{(1 + y)}$

(C) $xy\,(1 + y) = C$ (D) $(1 + y)^2 = C\,(1 + x)$

10. The solution of differential equation $\frac{dy}{dx} + \frac{1+y^2}{1+x^2} = 0$ is (2)

(A) $\tan^{-1} y - \tan^{-1} x = C$
(B) $\tan^{-1} y + \tan^{-1} x = C$
(C) $\tan y + \tan x = C$
(D) $\cos y + \cos x = C$

11. The solution of differential equation $(4 + e^{2x}) \frac{dy}{dx} = ye^{2x}$ is (2)

(A) $y^2 = (4 + e^{2x}) C$ (B) $y = (4 + e^{2x}) C$ (C) $y (4 + e^{2x}) = C$ (D) $y^2 (4 + e^{2x}) = C$

12. The solution of differential equation $y - x\frac{dy}{dx} = 2\left(y + \frac{dy}{dx}\right)$ is (2)

(A) $y + (x + 2) = C$ (B) $y - (x + 2) = C$ (C) $y = C (x + 2)$ (D) $y (x + 2) = C$

13. The solution of differential equation $xdy - ydx = 0$ is (2)

(A) $y = x + C$ (B) $x^2 - y^2 = C$ (C) $xy = C$ (D) $y = Cx$

14. The solution of differential equation $\frac{dy}{dx} = e^{x-y} + 3x^2e^{-y}$ is (2)

(A) $e^y = e^x + x^3 + C$
(B) $e^y = e^x + 3x^3 + C$
(C) $e^y = e^x + 3x + C$
(D) $e^x + e^y = 3x^3 + C$

15. The solution of differential equation is $x^3\left(x\frac{dy}{dx} + y\right) - \sec(xy) = 0$ by substitution $xy = u$ is (2)

(A) $\tan(xy) + \frac{1}{2x^2} = C$
(B) $\sin(xy) + \frac{1}{2x^2} = C$
(C) $\sin(xy) - \frac{1}{2x^2} = C$
(D) $\sin(xy) - \frac{1}{4x^4} = C$

16. The solution of differential equation $\sec^2 x \tan y dx + \sec^2 y \tan x\, dy = 0$ is (2)

(A) $\sec^2 x \tan y = C$ (B) $\tan x \sec^2 y = C$ (C) $\tan x \tan y = C$ (D) $\sec^2 x \sec^2 y = C$

17. The solution of differential equation $y \sec^2 x + (y + 7) \tan x \frac{dy}{dx} = 0$ is (2)

(A) $y + 7 \log y = -\log \tan x + C$
(B) $y + \log (7 + y) = -\log \tan x + C$
(C) $y - 7 \log y = \log \tan x + C$
(D) $y + \log y = -\log \tan x + C$

18. The solution of differential equation $e^x \cos y dx + (1 + e^x) \sin y\, dy = 0$ is (2)

(A) $(1 + e^x) = C \sec y$
(B) $(1 + e^x) \sec y = C$
(C) $\frac{\sec y}{(1 + e^x)} = C$
(D) $(1 + e^x) \cos y = C$

19. The solution of differential equation $y(1 + \log x)\frac{dx}{dy} - x \log x = 0$ is (2)

(A) $\log(x \log x) = yC$ (B) $\frac{x}{\log x} = yC$

(C) $y(\log x) = xC$ (D) $x(\log x) = yC$

20. The solution of differential equation $3e^x \tan y\, dx + (1 + e^x) \sec^2 y dy = 0$ is (2)

(A) $3 \log(1 - e^x) = -\log \tan y + \log C$ (B) $\log(1 + e^x) = \log \tan y + \log C$

(C) $3 \log(1 + e^x) = -\log \tan y + \log C$ (D) $\log(1 + e^x) = -\log \sin y + \log C$

21. The solution of differential equation $x(1 + y^2) dx + y(1 + x^2) dy = 0$ is (2)

(A) $(1 - x^2)(1 + y^2) = C$ (B) $\tan^{-1} x + \tan^{-1} y = C$

(C) $(1 + x^2) = C(1 + y^2)$ (D) $(1 + x^2)(1 + y^2) = C$

22. The solution of differential equation $\frac{dy}{dx} = (1 + x)(1 + y^2)$ is (2)

(A) $\tan^{-1} y = x + \frac{x^2}{2} + C$ (B) $\log(1 + y^2) = x + \frac{x^2}{2} + C$

(C) $\tan^{-1} x = y + \frac{y^2}{2} + C$ (D) $\frac{1}{2}\log\left(\frac{1+y}{1-y}\right) = x + \frac{x^2}{2} + C$

23. The solution of differential equation $(e^x + 1) ydy = (y + 1) e^x dx$ is (2)

(A) $y - \log(1 - y) = \log(e^x - 1) + \log C$ (B) $y - \log(1 + y) = \log(e^x + 1) + \log C$

(C) $y + \log(1 - y) = \log(e^x + 1) + \log C$ (D) $y - \log(1 + y) = \log(e^x - 1) + \log C$

24. The solution of differential equation $\frac{dy}{dx} = e^{x+y} + e^{y-x}$ is (2)

(A) $e^{-y} = e^{-x} - C$ (B) $e^y = e^x - e^{-x} + C$

(C) $-e^{-y} = e^x - e^{-x} + C$ (D) $e^{-y} = e^x + e^{-x} + C$

25. The solution of differential equation $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$ is (2)

(A) $\tan^{-1} x + \cot^{-1} y = C$ (B) $\sin^{-1} x + \sin^{-1} y = C$

(C) $\sec^{-1} x + \operatorname{cosec}^{-1} y = C$ (D) $\sin^{-1} x - \sin^{-1} y = C$

ANSWERS

1. (A)	2. (B)	3. (B)	4. (C)	5. (A)	6. (D)	7. (B)	8. (C)
9. (A)	10. (B)	11. (A)	12. (D)	13. (D)	14. (A)	15. (B)	16. (C)
17. (A)	18. (B)	19. (D)	20. (C)	21. (D)	22. (A)	23. (B)	24. (C)
25. (B)							

MULTIPLE CHOICE QUESTIONS

Exact Differential Equations and Reducible to Exact Differential Equation :

1. The necessary and sufficient condition that the differential equation $M(x, y)\,dx + N(x, y)\,dy = 0$ be exact is (1)

   (A) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$; $My + Nx \neq 0$ (B) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$; $Mx - Ny \neq 0$

   (C) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$; $Mx + Ny \neq 0$ (D) $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 1$; $My - Nx \neq 0$

2. If homogeneous differential equation $M(x, y)\,dx + N(x, y)\,dy = 0$ is not exact then the integrating factor is (1)

   (A) $\frac{1}{My + Nx}$; $My + Nx \neq 0$ (B) $\frac{1}{Mx - Ny}$; $Mx - Ny \neq 0$

   (C) $\frac{1}{Mx + Ny}$; $Mx + Ny \neq 0$ (D) $\frac{1}{My - Nx}$; $My - Nx \neq 0$

3. If the differential equation $M(x, y)\,dx + N(x, y)\,dy = 0$ is not exact and it can be written as $yf_1(xy)\,dx + xf_2(xy)\,dy = 0$ then the integrating factor is (1)

   (A) $\frac{1}{My + Nx}$; $My + Nx \neq 0$ (B) $\frac{1}{Mx - Ny}$; $Mx - Ny \neq 0$

   (C) $\frac{1}{Mx + Ny}$; $Mx + Ny \neq 0$ (D) $\frac{1}{My - Nx}$; $My - Nx \neq 0$

4. If the differential equation M (x, y) dx + N (x, y) dy = 0 is not exact and $\dfrac{\dfrac{\partial M}{\partial y}-\dfrac{\partial N}{\partial x}}{N}=f(x)$ then the integrating factor is (1)

(A) $e^{f(x)}$ (B) $e^{\int f(x)\,dy}$ (C) f(x) (D) $e^{\int f(x)\,dx}$

5. If the differential equation M(x, y) dx + N(x, y) dy = 0 is not exact and $\dfrac{\dfrac{\partial M}{\partial y}-\dfrac{\partial N}{\partial x}}{-M}=f(y)$ then the integrating factor is (1)

(A) $e^{\int f(y)\,dy}$ (B) $e^{\int f(y)\,dx}$ (C) f(y) (D) $e^{f(y)}$

6. The total derivative of xdy + ydx is (1)

(A) $d\left(\dfrac{y}{x}\right)$ (B) $d\left(\dfrac{x}{y}\right)$ (C) d(xy) (D) d(x + y)

7. The total derivative of xdy − ydx with integrating factor $\dfrac{1}{x^2}$ is (1)

(A) $d\left(\dfrac{x}{y}\right)$ (B) $d\left(\dfrac{y}{x}\right)$ (C) $d\left(\log\dfrac{x}{y}\right)$ (D) d(x − y)

8. The total derivative of xdy + ydx with integrating factor $\dfrac{1}{xy}$ is (1)

(A) $d\left(\log\dfrac{x}{y}\right)$ (B) $d\left(\log\dfrac{y}{x}\right)$ (C) d [log (x + y)] (D) d (log xy)

9. The total derivative of xdy − ydx with integrating factor $\dfrac{1}{xy}$ is (1)

(A) $d\left(\log\dfrac{x}{y}\right)$ (B) $d\left(\log\dfrac{y}{x}\right)$ (C) $d\left[\dfrac{y}{x}\right]$ (D) d (log xy)

10. The total derivative of xdy − ydx with integrating factor $\dfrac{1}{x^2+y^2}$ is (1)

(A) $d\left(\tan^{-1}\dfrac{y}{x}\right)$ (B) $d\left(\tan^{-1}\dfrac{x}{y}\right)$ (C) d [log (x² + y²)] (D) none of these

11. The total derivative of dx + dy with integrating factor $\dfrac{1}{x+y}$ is (1)

(A) d [log (x − y)] (B) d [log x² − y²)] (C) d [log (x + y)] (D) none of these

12. The differential equation (x + y − 2) dx + (x − y + 4) dy = 0 is of the form (1)

(A) exact (B) homogeneous (C) linear (D) none of these

13. The value of λ for which the differential equation $(xy^2+\lambda x^2y)\,dx+(x^3+x^2y)\,dy=0$ is exact is (2)

(A) −3 (B) 2 (C) 3 (D) 1

14. The differential equation $(ay^2+x+x^8)\,dx+(y^8-y+bxy)\,dy=0$ is exact if (2)

(A) b ≠ 2a (B) b = a (C) a = 1, b = 3 (D) b = 2a

15. The differential equation $(3+by\cos x)\,dx+(2\sin x-4y^3)\,dy=0$ is exact if (2)

(A) b = −2 (B) b = 3 (C) b = 0 (D) b = 2

16. The differential equation $(\tan y - ax^2y - y)\,dx + (x\tan^2 y - x^3 - \sec^2 y)\,dy = 0$ is exact if (2)

(A) $a = 2$ (B) $a = 3$ (C) $a = -3$ (D) $a = -2$

17. The differential equation $\left(\frac{2x}{y^3}\right)dx + \left(\frac{y^2 + ax^2}{y^4}\right)dy = 0$ is exact if (2)

(A) $a = -3$ (B) $a = 3$ (C) $a = -2$ (D) $a = 6$

18. Integrating factor of homogeneous differential equation

$(xy - 2y^2)\,dx + (3xy - x^2)\,dy = 0$ is (2)

(A) $\frac{1}{xy}$ (B) $\frac{1}{x^2y^2}$ (C) $\frac{1}{x^2y}$ (D) $\frac{1}{xy^2}$

19. Integrating factor of homogeneous differential equation

$(x^2 - 3xy + 2y^2)\,dx + (3x^2 - 2xy)\,dy = 0$ is (2)

(A) $\frac{1}{xy}$ (B) $\frac{1}{x^3}$ (C) $\frac{1}{x^2y}$ (D) $\frac{1}{x^2}$

20. Integrating factor of homogeneous differential equation

$(y^2 - 2xy)\,dx + (2x^2 + 3xy)\,dy = 0$ is (2)

(A) $\frac{1}{x^2y^2}$ (B) $\frac{1}{x^2y}$ (C) $\frac{1}{4xy^2}$ (D) $\frac{1}{y^2}$

21. Integrating factor of homogeneous differential equation

$(x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy = 0$ is (2)

(A) $\frac{1}{x^2y^2}$ (B) $\frac{1}{xy}$ (C) $\frac{2}{x}$ (D) $\frac{1}{x^2y}$

22. Integrating factor for differential equation

$(x^2y^2 + xy + 1)\,ydx + (x^2y^2 - xy + 1)\,xdy = 0$ is (2)

(A) $\frac{1}{2x^3y^3}$ (B) $\frac{1}{xy}$ (C) $\frac{1}{2x^2y^2}$ (D) $\frac{1}{x^2y}$

23. Integrating factor for differential equation $(1 + xy)\,ydx + (1 - xy)\,xdy = 0$ is (2)

(A) $\frac{1}{2x^2y^2}$ (B) $\frac{1}{x^2y}$ (C) $\frac{1}{xy^2}$ (D) $\frac{1}{y}$

24. Integrating factor for differential equation $(1 + xy)\,ydx + (x^2y^2 + xy + 1)\,xdy = 0$ is (2)

(A) $\frac{1}{x^2y}$ (B) $-\frac{1}{x^3y^3}$ (C) $\frac{1}{xy^2}$ (D) $\frac{1}{x^2y^2}$

25. Integrating factor for differential equation $(x^2 + y^2 + x)\,dx + (xy)\,dy = 0$ is (2)

(A) $\frac{1}{x}$ (B) $\frac{1}{x^2}$ (C) x^2 (D) x

26. Integrating factor for differential equation $\left(y+\frac{y^3}{3}+\frac{x^2}{2}\right)dx+\left(\frac{x+xy^2}{4}\right)dy=0$ is (2)

(A) $\frac{1}{x}$ (B) x^3 (C) x^2 (D) $\frac{1}{x^3}$

27. Integrating factor for differential equation $(2x\log x-xy)\,dy+(2y)\,dx=0$ is (2)

(A) $\frac{1}{x}$ (B) $\frac{1}{x^2y^2}$ (C) $\frac{1}{x^2}$ (D) $\frac{1}{y}$

28. Integrating factor for differential equation $(x^2+y^2+1)\,dx-2xy\,dy=0$ is (2)

(A) $\frac{1}{x}$ (B) $\frac{1}{x^3}$ (C) $\frac{1}{x^2}$ (D) $\frac{1}{xy}$

29. Integrating factor for differential equation $y\,(2xy+e^x)\,dx-e^x\,dy=0$ is (2)

(A) $\frac{1}{x}$ (B) $\frac{1}{y}$ (C) $\frac{1}{x^2}$ (D) $\frac{1}{y^2}$

30. Integrating factor for differential equation $y\log y\,dx+(x-\log y)\,dy=0$ is (2)

(A) $\frac{1}{x}$ (B) $\frac{1}{y}$ (C) $\frac{1}{x^2}$ (D) $\frac{1}{y^2}$

31. Integrating factor for differential equation $(y^4+2y)\,dx+(xy^3+2y^4-4x)\,dy=0$ (2)

(A) $\frac{2}{x}$ (B) $\frac{1}{y}$ (C) $\frac{1}{y^3}$ (D) $\frac{2}{y^2}$

32. Integrating factor for differential equation $(2x+e^x\log y)\,y\,dx+(e^x)\,dy=0$ is (2)

(A) $\frac{1}{x}$ (B) $\frac{1}{y^2}$ (C) $\frac{1}{x^2}$ (D) $\frac{1}{y}$

33. Solution of non-exact differential equation $(x^2-3xy+2y^2)\,dx+x\,(3x-2y)\,dy=0$ with integrating factor $\frac{1}{x^3}$ is (2)

(A) $3\frac{y}{x}-\frac{y^2}{x^2}=C$ (B) $\log x-3\frac{y}{x}+\frac{y^2}{x^2}=C$

(C) $\log x+3\frac{y}{x}-2\frac{y^2}{x^2}=C$ (D) $\log x+3\frac{y}{x}-\frac{y^2}{x^2}=C$

34. Solution of non-exact differential equation $(3xy^2-y^3)\,dx+(xy^2-2x^2y)\,dy=0$ with integrating factor $\frac{1}{x^2y^2}$ is (2)

(A) $3\log x-\frac{2y}{x^3}-2\log y=C$ (B) $3\log x+\frac{y}{x}-2\log y=C$

(C) $3\log x+\frac{y}{x}=C$ (D) $\log x-\frac{y}{x}+2\log y=C$

35. Solution of non-exact differential equation $(1 + xy)\, ydx + (1 - xy)\, xdy = 0$ is integating factor $\frac{1}{x^2y^2}$ is (2)

(A) $\frac{2}{xy} - \log\left(\frac{x}{y}\right) = C$ (B) $-\frac{1}{xy} + \log\left(\frac{y}{x}\right) = C$

(C) $-\frac{1}{xy} + \log\left(\frac{x}{y}\right) = C$ (D) $-\frac{2}{x^3y} + \log\left(\frac{x}{y}\right) = C$

36. Solution of non-exact differential equation $(2 + x^2y^2)\, ydx + (2 - 2x^2y^2)\, xdy = 0$ with integrating factor $\frac{1}{x^3y^3}$ is (2)

(A) $\log\left(\frac{x}{y^2}\right) - \frac{1}{x^2y^2} = C$ (B) $\log\left(\frac{x}{y^2}\right) + \frac{1}{x^2y^2} = C$

(C) $\log\left(\frac{y^2}{x}\right) - \frac{1}{x^2y^2} = C$ (D) $\log x - \frac{1}{x^2y^2} = C$

37. Soluton of non-exact differential equation $y\,(2xy + e^x)\, dx - e^x\, dy = 0$ with integating factor $\frac{1}{y^2}$ is (2)

(A) $x^2 + \frac{e^x}{y} - e^x \log y = C$ (B) $x^2 + \frac{e^x}{y} = C$

(C) $x^2 + \frac{2e^x}{y} = C$ (D) $x^2 - \frac{e^x}{y} = C$

38. Solution of non-exact differential equation $(x^4e^x - 2mxy^2)\, dx + (2mx^2y)\, dy = 0$ with integating factor $\frac{1}{x^4}$ is (2)

(A) $e^x + \frac{6my^2}{x^4} = C$ (B) $e^x + \frac{2my^2}{x^2} = C$ (C) $e^x + \frac{y^2}{x^2} = C$ (D) $e^x + \frac{my^2}{x^2} = C$

ANSWERS

1. (A)	2. (C)	3. (B)	4. (D)	5. (A)	6. (C)	7. (B)	8. (D)
9. (B)	10. (A)	11. (C)	12. (A)	13. (C)	14. (D)	15. (D)	16. (B)
17. (A)	18. (D)	19. (B)	20. (C)	21. (A)	22. (C)	23. (A)	24. (B)
25. (D)	26. (B)	27. (A)	28. (C)	29. (D)	30. (B)	31. (C)	32. (D)
33. (D)	34. (B)	35. (C)	36. (A)	37. (B)	38. (D)		

MULTIPLE CHOICE QUESTIONS

Linear Differential Equations and Reducible to Linear Differential Equation :

1. The differential equation of the form $\frac{dy}{dx} + Py = Q$ where P and Q are functions of x or constants, is (1)

(A) exact differential equation

(B) linear differential equation in y

(C) linear differential equation in x

(D) non-homogeneous differential equation

2. The differential equation of the form $\frac{dx}{dy} + Px = Q$ where P and Q are functions of y or constants, is (1)

(A) exact differential equation

(B) linear differential equation in y

(C) linear differential equation in x

(D) non-homogeneous differential equation

3. Integrating factor of linear differential equation $\frac{dy}{dx} + Py = Q$ where P and Q are functions of x or constants, is (1)

(A) $e^{\int P\,dy}$ (B) $e^{\int Q\,dx}$ (C) $e^{\int Q\,dx}$ (D) $e^{\int P\,dx}$

4. Integrating factor of linear differential equation $\frac{dx}{dy} + Px = Q$ where P and Q are functions of y or constants, is (1)

(A) $e^{\int P\,dy}$ (B) $e^{\int P\,dx}$ (C) $e^{\int Q\,dx}$ (D) $e^{\int Q\,dy}$

5. The general solution of linear differential equation $\frac{dy}{dx} + Py = Q$ where P and Q are functions of x or constants, is (1)

(A) $xe^{\int P\,dy} = \int Qe^{\int P\,dy}\,dy + C$ (B) $y = \int Qe^{\int P\,dx}\,dx + C$

(C) $ye^{\int P\,dx} = \int Q\,dx + C$ (D) $ye^{\int P\,dx} = \int Qe^{\int P\,dx}\,dx + C$

6. The general solution of linear differential equation $\frac{dx}{dy} + Px = Q$ where P and Q are functions of y or constants, is (1)

(A) $x = \int Qe^{\int P\,dy}\,dy + C$ (B) $xe^{\int P\,dx} = \int Qe^{\int P\,dx}\,dx + C$

(C) $ye^{\int P\,dx} = \int Qe^{\int P\,dx}\,dx + C$ (D) $xe^{\int P\,dy} = \int Qe^{\int P\,dy}\,dy + C$

7. The differential equation of the form $\frac{dy}{dx} + Py = Qy^n$, $n \neq 1$ where P and Q are functions of x or constants, is (1)

(A) Bernoulli's differential equation (B) exact differential equation
(C) symmetric differential equation (D) linear differential equation

8. The differential equation of the form $\frac{dx}{dy} + Px = Qx^n$, $n \neq 1$ where P and Q are functions of y or constants, is (1)

(A) Bernoulli's differential equation (B) exact differential equation
(C) symmetric differential equation (D) linear differential equation

9. The differential equation of the form $f'(y)\frac{dy}{dx} + P\,f(y) = Q$ where P and Q are functions of x or constants, can be reduced to linear differential equation by the substitution (1)

(A) $f'(y) = u$ (B) $P = u$ (C) $f(y) = u$ (D) $Q = u$

10. The differential equation of the form $f'(x)\frac{dx}{dy} + P\,f(x) = Q$ where P and Q are functions of y or constants, can be reduced to linear differential equation by the substitution. (1)

(A) $f'(x) = u$ (B) $f(x) = u$ (C) $P = u$ (D) $Q = u$

11. Integrating factor of linear differential equation $\frac{dy}{dx} + xy = x^3$ is (2)

(A) $e^{\log x}$ (B) e^x (C) x^2 (D) $e^{\frac{x^2}{2}}$

12. Integrating factor of linear differential equation $\frac{dx}{dy} + yx = y^2$ is (2)

(A) $e^{\frac{y^2}{2}}$ (B) $e^{\frac{x^2}{2}}$ (C) y^2 (D) $e^{\log y}$

13. The differential equation $\frac{dy}{dx} + \frac{y}{1+x^2} = x^2$ has integrating factor (2)

(A) $e^{\frac{1}{1+y^2}}$ (B) $e^{\tan^{-1} x}$ (C) $e^{\frac{1}{1+x^2}}$ (D) $e^{\tan^{-1} y}$

14. The differential equation $\frac{dx}{dy} + \frac{x}{1+y^2} = y^2$ has integrating factor (2)

(A) $e^{\frac{1}{1+y^2}}$ (B) $e^{\tan^{-1} x}$ (C) $e^{\frac{1}{1+x^2}}$ (D) $e^{\tan^{-1} y}$

15. The differential equation $\frac{dy}{dx} + \sqrt{x}\,y = x^3$ has integrating factor (2)

(A) $e^{\frac{2}{3} \times \sqrt{x}}$ (B) $e^{\frac{1}{3} \times \sqrt{x}}$ (C) $e^{\sqrt{x}}$ (D) e^{-x}

16. The linear differential equation $(1 + y^2) + (x - e^{-\tan^{-1} y})\dfrac{dy}{dx} = 0$ has integrating factor (2)

(A) $e^{\tan^{-1} x}$ (B) $e^{\frac{1}{1+y^2}}$ (C) $e^{\tan^{-1} y}$ (D) e^{2y}

17. The linear differential equation $(1 - x^2)\dfrac{dy}{dx} = 1 + xy$ has integrating factor (2)

(A) $\sqrt{1-x^2}$ (B) $\dfrac{1}{\sqrt{1-x^2}}$ (C) $e^{\tan^{-1} x}$ (D) $x\sqrt{1-x^2}$

18. The linear differential equation $(2y + x^2)\,dx = x\,dy$ has integrating factor (2)

(A) $\dfrac{1}{x}$ (B) $\dfrac{1}{x^2}$ (C) x (D) $\dfrac{1}{y^2}$

19. The linear differential equation $y^2 + \left(x - \dfrac{1}{y}\right)\dfrac{dy}{dx} = 0$ has integrating factor (2)

(A) e^x (B) e^y (C) $\dfrac{1}{y^2}$ (D) $e^{-\frac{1}{y}}$

20. The differential equation $\dfrac{dy}{dx} + y\cot x = \sin 2x$ has integrating factor (2)

(A) $\cos x$ (B) $e^{\cot x}$ (C) $\sin x$ (D) $\sec x$

21. The differential equation $\cos x\dfrac{dy}{dx} + y = \sin x$ has integrating factor (2)

(A) $e^{\sec x}$

(B) $(\text{cosec } x - \cot x)$

(C) $(\sec x + \tan x)$

(D) $(\sec x - \tan x)$

22. The differential equation $(x^2 + 1)\dfrac{dy}{dx} + 4xy = \dfrac{1}{(x^2+1)^2}$ has integrating factor (2)

(A) $(x^2 + 1)^2$ (B) $(x^2 + 1)$ (C) $e^{\frac{4x}{(x^2+1)}}$ (D) e^{4x}

23. The Bernoulli's differential equation $\dfrac{dy}{dx} - y\tan x = y^4 \sec x$ reduces to linear differential equation (2)

(A) $\dfrac{du}{dx} + (3\tan x)\,u = -3\sec x$ where $y^{-3} = u$

(B) $\dfrac{du}{dx} - (3\tan x)\,u = 3\sec x$ where $y^{-3} = u$

(C) $\dfrac{du}{dx} + (\tan x)\,u = -\sec x$ where $y^{-3} = u$

(D) none of these

24. The Bernoulli's differential equation $\frac{dy}{dx} - xy = -y^3 e^{-x^2}$ reduces to linear differential equation (2)

(A) $\frac{du}{dx} + (2x)\,u = 2e^{-x^2}$ where $y^{-2} = u$ (B) $\frac{du}{dx} + (x)\,u = e^{-x^2}$ where $y^{-2} = u$

(C) $\frac{du}{dx} - (2x)\,u = -2e^{-x^2}$ where $y^{-2} = u$ (D) none of these

25. The differential equation $\tan y \frac{dy}{dx} + \tan x = \cos^2 x \cos y$ reduces to linear differential equation (2)

(A) $\frac{du}{dx} - \tan(x)\,u = -\cos^2 x$ where $\sec y = u$

(B) $\frac{du}{dx} + (\tan x)\,u = \cos^2 x$ where $\sec y = u$

(C) $\frac{du}{dx} + (\cot x)\,u = \cos^2 x$ where $\sec y = u$

(D) none of these

26. The differential equation $\sin y \frac{dy}{dx} - 2\cos x \cos y = -\cos x \sin^2 x$ reduces to linear differential equation (2)

(A) $\frac{du}{dx} + (\cos x)\,u = \cos x \sin^2 x$ where $\cos y = u$

(B) $\frac{du}{dx} - (2\cos x)\,u = -\cos x \sin^2 x$ where $\cos y = u$

(C) $\frac{du}{dx} + (2\cos x)\,u = \cos x \sin^2 x$ where $\cos y = u$

(D) none of these

27. The value of α so that $e^{\alpha y^2}$ is an integrating factor of linear differential equation $\frac{dx}{dy} + xy = e^{-\frac{y^2}{2}}$ is (2)

(A) -1 (B) $-\frac{1}{2}$ (C) 1 (D) $\frac{1}{2}$

28. The value of α so that $e^{\alpha x^2}$ is an integrating factor of linear differential equation $\frac{dy}{dx} - xy = x$ is (2)

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) 1 (D) -2

29. If I_1, I_2 are integrating factors of the equation $x\frac{dy}{dx} + 2y = 1$ and $x\frac{dy}{dx} - 2y = 1$ then true relation is (2)

(A) $I_1 = -I_2$ (B) $I_1 I_2 = 1$ (C) $I_1 = x^2 I_2$ (D) $I_1 I_2 = x^2$

30. The general solution of $\frac{dy}{dx} + \frac{1}{1-x}y = -x(1-x)$ with integrating factor $\frac{1}{1-x}$ is (2)

(A) $y = -\frac{x^2}{2}\left(\frac{1}{1-x}\right) + C$

(B) $y\frac{1}{1-x} = x^2 + C$

(C) $y\frac{1}{1-x} = \frac{x^2}{2} + C$

(D) $y\frac{1}{1-x} = -\frac{x^2}{2} + C$

31. The general solution of $\frac{dy}{dx} + \frac{3}{x}y = \frac{e^x}{x^2}$ with integrating factor x^3 is (2)

(A) $y\,x^3 = (x+1)\,e^x + C$

(B) $y\,x^3 = (x-1)\,e^x + C$

(C) $x\,y^3 = (x-1)\,e^x + C$

(D) none of these

32. The general solution of $\frac{dy}{dx} + (\cot x)\,y = \sin 2x$ with integrating factor $\sin x$ is (2)

(A) $y\sin x = \frac{2}{3}\sin^2 x + C$

(B) $y\sin x = \sin^3 x + C$

(C) $y\sin x = \frac{2}{3}\sin^3 x + C$

(D) none of these

33. The general solution of $\frac{dy}{dx} + \frac{1}{(1-x)\sqrt{x}}y = (1-\sqrt{x})$ with integrating factor $\frac{1+\sqrt{x}}{1-\sqrt{x}}$ is (2)

(A) $y\frac{1+\sqrt{x}}{1-\sqrt{x}} = -x - \frac{2}{3}x^{3/2} + C$

(B) $y\frac{1+\sqrt{x}}{1-\sqrt{x}} = x + \frac{2}{3}x^{3/2} + C$

(C) $y\frac{1+\sqrt{x}}{1-\sqrt{x}} = x + \frac{3}{2}x^{1/2} + C$

(D) none of these

34. The general solution of $\frac{dy}{dx} + \left(\tan x + \frac{1}{x}\right)y = \frac{1}{x}\sec x$ with integrating factor $x\sec x$ is (2)

(A) $y\,(x\sec x) = \tan x + C$

(B) $y\,(x\sec x) = \frac{x^3}{3} + C$

(C) $x\,(y\sec y) = \tan x + C$

(D) none of these

35. The general solution of $\frac{dy}{dx} + \frac{3}{x}y = x^2$ with integrating factor x^3 is (2)

(A) $y\,x^3 = \frac{x^6}{6} + C$ (B) $y\,x^3 = \frac{x^2}{2} + C$ (C) $y\,x^3 = \log x + C$ (D) none of these

36. The general solution of $\frac{dy}{dx} + \frac{2}{x} y = \frac{1}{x^3}$ with integrating factor x^2 is (2)

(A) $y\,x^2 = \frac{x^2}{2} + C$ (B) $y\,x^2 = \log x + C$ (C) $y\,x^2 = \frac{x^6}{6} + C$ (D) none of these

37. The general solution of $\frac{dy}{dx} + (1 + 2x)\,y = e^{-x^2}$ with integrating factor $e^{x + x^2}$ is (2)

(A) $y\,e^{x + x^2} = \frac{e^{x + x^2}}{2} + C$ (B) $y\,e^{x + x^2} = e^{x^2} + C$

(C) $y\,e^{x + x^2} = e^x + C$ (D) none of these

38. The general solution of $\frac{dx}{dy} + \frac{1}{1 + y^2} x = \frac{e^{-\tan^{-1} y}}{1 + y^2}$ with integrating factor $e^{\tan^{-1} y}$ is (2)

(A) $x\,e^{\tan^{-1} y} = \tan^{-1} y + C$ (B) $y\,e^{\tan^{-1} y} = \tan^{-1} y + C$

(C) $e^{\tan^{-1} y} = \tan^{-1} y + C$ (D) none of these

39. The general solution of $\frac{dx}{dy} + (\sec y)\,x = \frac{2y \cos y}{1 + \sin y}$ with integrating factor $(\sec y + \tan y)$ is (2)

(A) $y\,(\sec y + \tan y) = y^2 + C$ (B) $x\,(\sec y + \tan y) = \frac{y^2}{2} + C$

(C) $x\,(\sec y + \tan y) = y^2 + C$ (D) none of these

40. The general solution of $\frac{dx}{dy} - \frac{2}{y} x = y^2 e^{-y}$ with integrating factor $\frac{1}{y^2}$ is (2)

(A) $x \frac{1}{y^2} = -e^{-y} + C$ (B) $x \frac{1}{y^2} = e^{-y} + C$ (C) $y \frac{1}{x^2} = -e^{-y} + C$ (D) none of these

ANSWERS

1. (B)	2. (C)	3. (D)	4. (A)	5. (D)	6. (D)	7. (A)	8. (A)
9. (C)	10. (B)	11. (D)	12. (A)	13. (B)	14. (D)	15. (A)	16. (C)
17. (A)	18. (B)	19. (D)	20. (C)	21. (C)	22. (A)	23. (A)	24. (A)
25. (B)	26. (C)	27. (D)	28. (A)	29. (B)	30. (D)	31. (B)	32. (C)
33. (B)	34. (A)	35. (A)	36. (B)	37. (C)	38. (A)	39. (C)	40. (A)

MULTIPLE CHOICE QUESTIONS

Orthogonal Trajectories :

1. The differential equation of orthogonal trajectories of family of straight lines $y = mx$ is (1)

(A) $\frac{dx}{dy} = -\frac{y}{x}$ (B) $\frac{dx}{dy} = -\frac{x}{y}$ (C) $\frac{dy}{dx} = \frac{y}{x}$ (D) $\frac{dy}{dx} = m$

2. If the family of curves is given by $x^2 + 2y^2 = c^2$ then the differential equation of orthogonal trajectories of family is (1)

(A) $x - 2y\frac{dy}{dx} = 0$ (B) $x + 2y\frac{dx}{dy} = 0$ (C) $x + 2y\frac{dy}{dx} = 0$ (D) $x - 2y\frac{dx}{dy} = 0$

3. The differential equation of orthogonal trajectories of family of curves $xy = c$ is (1)

(A) $x\frac{dx}{dy} + y = 0$ (B) $-x\frac{dx}{dy} + y = 0$ (C) $-x\frac{dx}{dy} - y = 0$ (D) $x\frac{dy}{dx} + y = 0$

4. If the family of curves is given by $y^2 = 4ax$ then the differential equation of orthogonal trajectories of family is (1)

(A) $2y\frac{dy}{dx} = 4a$ (B) $2y\frac{dy}{dx} = \frac{y^2}{x}$ (C) $-2y\frac{dx}{dy} = \frac{y^2}{x}$ (D) $2y\frac{dy}{dx} = \frac{x}{y^2}$

5. The differential equation of orthogonal trajectories of family of curves $2x^2 + y^2 = cx$ is (1)

(A) $4x + 2y\frac{dy}{dx} = \frac{2x^2 + y^2}{x}$ (B) $4x - 2y\frac{dx}{dy} = \frac{2x^2 + y^2}{x}$

(C) $4x + 2y\frac{dx}{dy} = \frac{2x^2 + y^2}{x}$ (D) none of these

6. The differential equation of orthogonal trajectories of family of curves $x^2 + cy^2 = 1$ is (1)

(A) $x - \left(\frac{1 - x^2}{y}\right)\frac{dx}{dy} = 0$ (B) $x + \left(\frac{1 - x^2}{y}\right)\frac{dy}{dx} = 0$

(C) $x + \left(\frac{1 - x^2}{y}\right)\frac{dx}{dy} = 0$ (D) none of these

7. The differential equation of orthogonal trajectories of family of curves $e^x + e^{-y} = c$ is (1)

(A) $e^x - e^{-y}\frac{dy}{dx} = 0$ (B) $e^x - e^{-y}\frac{dx}{dy} = 0$ (C) $e^x + e^{-y}\frac{dx}{dy} = 0$ (D) none of these

8. The differential equation of orthogonal trajectories of family of curves $r = a\cos\theta$ is (1)

(A) $r^2\dfrac{d\theta}{dr} = \tan\theta$ (B) $\dfrac{1}{r}\dfrac{d\theta}{dr} = -\tan\theta$ (C) $r\dfrac{d\theta}{dr} = -\tan\theta$ (D) $r\dfrac{d\theta}{dr} = \tan\theta$

9. The differential equation of orthogonal trajectories of family of curves $r = a\sin\theta$ is (1)

(A) $\dfrac{1}{r}\dfrac{d\theta}{dr} = \cot\theta$ (B) $r\dfrac{d\theta}{dr} = -\cot\theta$ (C) $r\dfrac{d\theta}{dr} = -\tan\theta$ (D) $r^2\dfrac{d\theta}{dr} = \tan\theta$

10. The differential equation of orthogonal trajectories of family of curves $r = a(1-\cos\theta)$ is (1)

(A) $-r\dfrac{d\theta}{dr} = \dfrac{\sin\theta}{1-\cos\theta}$ (B) $r\dfrac{d\theta}{dr} = \dfrac{\sin\theta}{1-\cos\theta}$

(C) $-r^2\dfrac{d\theta}{dr} = \dfrac{\sin\theta}{1-\cos\theta}$ (D) $-r\dfrac{d\theta}{dr} = \dfrac{1-\cos\theta}{\sin\theta}$

11. The differential equation of orthogonal trajectories of family of curves $r^2 = a\sin 2\theta$ is (1)

(A) $r\dfrac{d\theta}{dr} = \tan 2\theta$ (B) $r\dfrac{d\theta}{dr} = \cot 2\theta$ (C) $-r\dfrac{d\theta}{dr} = \cot 2\theta$ (D) $\dfrac{dr}{d\theta} = r\cot 2\theta$

12. The differential equation of orthogonal trajectories of family of curves $r^2 = a\cos 2\theta$ is (1)

(A) $-r^2\dfrac{d\theta}{dr} = \tan 2\theta$ (B) $r\dfrac{d\theta}{dr} = \cot 2\theta$ (C) $r\dfrac{d\theta}{dr} = \tan 2\theta$ (D) $\dfrac{dr}{d\theta} = -r\tan 2\theta$

13. The differential equation of orthogonal trajectories of family of curves $r = a\sec^2\dfrac{\theta}{2}$ is (1)

(A) $-r\dfrac{d\theta}{dr} = \tan\dfrac{\theta}{2}$ (B) $r\dfrac{d\theta}{dr} = \tan\dfrac{\theta}{2}$ (C) $-r\dfrac{d\theta}{dr} = \cot\dfrac{\theta}{2}$ (D) $\dfrac{dr}{d\theta} = 2\tan\dfrac{\theta}{2}$

14. The differential equation of orthogonal trajectories of family of curves $r = a\cos^2\theta$ is (1)

(A) $\dfrac{dr}{d\theta} = -\dfrac{r\sin 2\theta}{\cos^2\theta}$ (B) $-r\dfrac{d\theta}{dr} = \dfrac{\sin 2\theta}{\cos^2\theta}$ (C) $r\dfrac{d\theta}{dr} = \dfrac{\cos^2\theta}{\sin 2\theta}$ (D) $r\dfrac{d\theta}{dr} = \dfrac{\sin 2\theta}{\cos^2\theta}$

15. The differential equation of orthogonal trajectories of family of curves $r^2 = a\sin\theta$ is (1)

(A) $2r\dfrac{d\theta}{dr} = \cot\theta$ (B) $2r\dfrac{d\theta}{dr} = -\cot\theta$ (C) $2r\dfrac{d\theta}{dr} = -\tan\theta$ (D) $2\dfrac{d\theta}{dr} = -r^2\cot\theta$

16. If the differential equation of family of straight lines $y = mx$ is $\frac{dy}{dx} = \frac{y}{x}$ then its orthogonal trajectories is (2)

(A) $xy = k$ (B) $x^2 - y^2 = k^2$ (C) $y = kx$ (D) $x^2 + y^2 = k^2$

17. If the differential equation of family of rectangular hyperbola $xy = c$ is $x\frac{dy}{dx} = -y$ then its orthogonal trajectories is (2)

(A) $x^2 - y^2 = k^2$ (B) $x^2 + y^2 = k^2$ (C) $y^2 = kx$ (D) $xy = k_1$

18. Orthogonal trajectories of family of circles $x^2 + y^2 = c^2$ whose differential equation is $\frac{dy}{dx} = -\frac{x}{y}$, is equal to (2)

(A) $x^2 - y^2 = k^2$ (B) $y = kx$ (C) $y^2 = kx$ (D) $x^2 + y^2 = k^2$

19. If the differential equation of family of rectangular hyperbola $x^2 - y^2 = c^2$ is $\frac{dy}{dx} = \frac{x}{y}$, then its orthogonal trajectories is (2)

(A) $y^2 = kx$ (B) $x^2 + y^2 = k^2$ (C) $xy = k$ (D) $y = kx$

20. Orthogonal trajectories of family of curves $x^2 + 2y^2 = c^2$ whose differential equation is $\frac{dy}{dx} = -\frac{x}{2y}$, is (2)

(A) $x^2 = ky$ (B) $x^2 = \frac{k}{y}$ (C) $x^2 + 2y^2 = k^2$ (D) none of these

21. Orthogonal trajectories of family of curves $y^2 = 4ax$, whose differential equation is $\frac{dy}{dx} = \frac{y}{2x}$, is equal to (2)

(A) $x^2 + y^2 = k^2$ (B) $x^2 + 2y^2 = k^2$ (C) $y^2 = 4kx$ (D) $2x^2 + y^2 = k$

22. If the differential equation of family of curves $e^x + e^{-y} = c$ is $\frac{dy}{dx} = \frac{e^x}{e^{-y}}$, then its orthogonal trajectories is (2)

(A) $e^{-x} + e^{-y} = k$ (B) $e^{-x} - e^y = k$ (C) $e^x - e^{-y} = k$ (D) $e^x + e^{-y} = k$

23. If the differential equation of family of curves $e^y - e^{-x} = c$ is $\frac{dy}{dx} = -\frac{e^{-x}}{e^y}$, then its orthogonal trajectories is (2)

(A) $e^{-x} + e^{-y} = k$ (B) $e^x - e^{-y} = k$ (C) $e^x + e^{-y} = k$ (D) $e^y - e^{-x} = k$

24. Orthogonal trajectories of family of curves $x^2 + cy^2 = 1$ whose differential equation is $\frac{dy}{dx} = -\frac{xy}{1 - x^2}$, is (2)

(A) $\log x + \frac{x^2}{2} = \frac{y^2}{2} + k$ (B) $\log x - \frac{x^2}{2} = \frac{y^2}{2} + k$ (C) $x^2 + y^2 = k$ (D) $x^2 + ky^2 = 1$

25. Orthogonal trajectories of family of curves $x^2 = ce^{x^2+y^2}$ whose differential equation is $\frac{dy}{dx} = \frac{1-x^2}{xy}$, is (2)

(A) $\log(1-x^2) - 2\log y = \log k$
(B) $2\log(1-x^2) - \log y = \log k$
(C) $\log(1-x^2) + 2\log y = \log k$
(D) $\log(1-x^2) - 2\log y = \log k + k$

26. Orthogonal trajectories of family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1$, λ is an arbitrary constant, whose differential equation is $x + \left(\frac{a^2-x^2}{y}\right)\frac{dy}{dx} = 0$, is (2)

(A) $\frac{y^2}{2} = -a^2\log x + \frac{x^2}{2} + k$
(B) $y^2 = b^2\log x - x^2 + k$
(C) $\frac{y^2}{2} = a^2\log x - \frac{x^2}{2} + k$
(D) $\frac{y^2}{2} = -a^2\log x + x^2 + k$

27. If the differential equation of family of curves $r = a(1-\cos\theta)$ is $\frac{dr}{d\theta} = r\cot\frac{\theta}{2}$ then its orthogonal trajectories is given by (2)

(A) $2\log\sec\frac{\theta}{2} = \log r + \log k$
(B) $2\log\cos\frac{\theta}{2} = \log r + \log k$
(C) $\frac{1}{2}\log\cos\frac{\theta}{2} = \log r + \log k$
(D) $\frac{1}{2}\log\sec\frac{\theta}{2} = \log r + \log k$

28. If the differential equation of family of curves $r = a\sec^2\frac{\theta}{2}$ is $\frac{dr}{d\theta} = r\tan\frac{\theta}{2}$ then its orthogonal trajectories is given by (2)

(A) $-2\log\cos\frac{\theta}{2} = \log r + \log k$
(B) $2\log\sin\frac{\theta}{2} = \log r + \log k$
(C) $-2\log\sin\frac{\theta}{2} = \log r + \log k$
(D) $2\log\cos\frac{\theta}{2} = \log r + \log k$

29. If the differential equation of family of curves $r = a\sin\theta$ is $\frac{dr}{d\theta} = r\cot\theta$ then its orthogonal trajectories is given by (2)

(A) $r = k\cos\theta$
(B) $r = k\sec\theta$
(C) $r = k\sin\theta$
(D) $\log\cos\theta = rk$

30. If the differential equation of family of curves $r = a\cos\theta$ is $\frac{dr}{d\theta} = -r\tan\theta$ then its orthogonal trajectories is given by (2)

(A) $\log r = -\operatorname{cosec}^2\theta + k$
(B) $r = k\cos\theta$
(C) $r = k\operatorname{cosec}\theta$
(D) $r = k\sin\theta$

31. If the differential equation of family of curves $r^2 = a \sin 2\theta$ is $\frac{dr}{d\theta} = r \cot 2\theta$ then its orthogonal trajectories is given by (2)

(A) $r^2 = \log \sec 2\theta + k$
(B) $r^2 = k \sin 2\theta$
(C) $r^2 = k \cos 2\theta$
(D) $\log r = -\frac{1}{2} \sec^2 2\theta + k$

32. If the differential equation of family of curves $r^2 = a \cos 2\theta$ is $\frac{dr}{d\theta} = -r \tan 2\theta$ then its orthogonal trajectories is given by (2)

(A) $\frac{1}{2} \log \cos 2\theta = \log r + \log k$
(B) $\frac{1}{2} \log \sin 2\theta = \log r + \log k$
(C) $\log \sin 2\theta = -r^2 + k$
(D) $\frac{1}{2} \log \sin 2\theta = -\log r + \log k$

33. If the differential equation of family of curves $r = a \cos^2 \theta$ is $\frac{dr}{d\theta} = -2r \tan \theta$ then its orthogonal trajectories is given by (2)

(A) $\frac{1}{2} \log \sin \theta = \log r + \log k$
(B) $\frac{1}{2} \log \sin \theta = -\log r + \log k$
(C) $\log \sin \theta = r + k$
(D) $\log \sec \theta = -\log r + \log k$

ANSWERS

1. (A)	2. (D)	3. (B)	4. (C)	5. (B)	6. (A)	7. (C)	8. (D)
9. (B)	10. (A)	11. (C)	12. (C)	13. (A)	14. (D)	15. (B)	16. (D)
17. (A)	18. (B)	19. (C)	20. (A)	21. (D)	22. (B)	23. (C)	24. (B)
25. (A)	26. (C)	27. (B)	28. (C)	29. (A)	30. (D)	31. (C)	32. (B)
33. (A)							

MULTIPLE CHOICE QUESTIONS

Newton's Law of Cooling :

1. Newton's law of cooling states that (1)
   (A) the temperature of a body changes at the rate which is proportional to the temperatures of surrounding medium
   (B) the temperature of a body changes at the rate which is inversely proportional to the difference in temperatures between that of surrounding medium and that of body itself
   (C) the temperature of a body changes at the rate which is proportional to the sum of temperatures of surrounding medium and that of body itself
   (D) the temperature of a body changes at the rate which is proportional to the difference in temperatures between that of surrounding medium and that of body itself

2. A metal ball is heated to a temperature of 100°C and at time t = 0 it is placed in water which is maintained at 40°C. By Newton's law of cooling the differential equation satisfied by temperature θ of metal ball at any time t is (1)
   (A) $\frac{d\theta}{dt} = -k(\theta - 100)$ (B) $\frac{d\theta}{dt} = -k(\theta - 40)$ (C) $\frac{d\theta}{dt} = -k\theta$ (D) $\frac{d\theta}{dt} = -k\theta(\theta - 40)$

3. According to Newton's law of cooling, the rate at which a substance cools in moving air is proportional to the difference between the temperature of the substance and that of air. A substance initially at temperature 90°C is kept in moving air at temperature 26°C, the differential equation satisfied by temperature θ of substance at any time t is (1)
   (A) $\frac{d\theta}{dt} = -k(\theta - 26)$ (B) $\frac{d\theta}{dt} = -k(\theta - 90)$ (C) $\frac{d\theta}{dt} = -k\theta$ (D) $\frac{d\theta}{dt} = -k(\theta - 64)$

4. Suppose a corpse at a temperature of 32°C arrives at mortuary where the temperature is kept at 10°C. Then by Newton's law of cooling the differential equation satisfied by temperature T of corpse t hours later is (1)
   (A) $\frac{dT}{dt} = -kT(T - 10)$
   (B) $\frac{dT}{dt} = -k(T - 32)$
   (C) $\frac{dT}{dt} = -k(T - 10)$
   (D) $\frac{dT}{dt} = -kT(T - 32)$

5. A thermometer is taken outdoors where the temperature is 0°C, from a room in which the temperature is 21°C and temperature drops to 10°C in 1 minute. Then by Newton's law of cooling the differential equation satisfied by temperature T at time t is (1)

(A) $\frac{dT}{dt} = -k(T-21)$ (B) $\frac{dT}{dt} = -kT$ (C) $\frac{dT}{dt} = kT$ (D) $\frac{dT}{dt} = -kT(T-21)$

6. If θ_0 is the temperature of the surrounding and θ is temperature of the body at any time t satisfies the differential equation $\frac{d\theta}{dt} = -k(\theta - \theta_0)$ then θ is given by (2)

(A) $\theta = \theta_0 e^{-kt}$ (B) $\theta = \theta_0 + Ae^{kt}$

(C) $\theta = -k(\theta_0 + Ae^{-kt})$ (D) $\theta = \theta_0 + Ae^{-kt}$

7. Suppose a corpse at a temperature of 32°C arrives at mortuary where the temperature is kept at 10°C. If the corpse cools to 27°C in 5 minutes. If the differential equation by Newton's law of cooling is $\frac{dT}{dt} = -(0.05)(T-10)$, then temperature T of corpse at any time t is given by (2)

(A) $T = 22e^{-0.05t}$ (B) $T = 10 + 22e^{0.05t}$

(C) $T = 10 + 22e^{-0.05t}$ (D) $T = 10 - 22e^{-0.05t}$

8. A thermometer is taken outdoors where the temperature is 0°C, from a room in which the temperature is 21°C and temperature drops to 10°C in 1 minute. If the differential equation by Newton's law of cooling is $\frac{dT}{dt} = -(0.7419)T$, then temperature T of thermometer at time t is given by

(A) $T = 21 + 11e^{-0.7419t}$ (B) $T = 21e^{0.7419t}$

(C) $T = 10 + 21e^{-0.7419t}$ (D) $T = 21e^{-0.7419t}$

9. A body originally at 80°C cools down to 60°C in 20 minutes in a room where the temperature is 40°C. The differential equation by Newton's law of cooling is $\frac{d\theta}{dt} = -k(\theta - 40)$, then the value of k is (2)

(A) $-\frac{1}{20}\log_e 2$ (B) $\frac{1}{20}\log_e 2$ (C) $20\log_e 2$ (D) $\log_e 2$

10. If the temperature of the body drops from 100°C to 60°C in 1 minute when the temperature of surrounding is 20°C satisfies the differential equation $\frac{d\theta}{dt} = -k(\theta - 20)$, then the value of k is (2)

(A) $\log_e 2$ (B) $-\log_e 2$ (C) $\log_e 4$ (D) $\log_e 8$

11. The temperature of the air is 30°C and the substance cools from 100°C to 70°C in 15 minutes. If differential equation by Newton's law of cooling is $\frac{d\theta}{dt} = -k(\theta - 30)$, then the value of k is (2)

(A) $\log_e \frac{7}{4}$ (B) $\frac{1}{15}\log_e \frac{4}{7}$ (C) $\frac{1}{15}\log_e \frac{7}{4}$ (D) $15\log_e \frac{7}{4}$

12. By Newton's law of cooling the differential equation of body originally at 80°C cools down to 60°C in 20 minutes in surrounding temperature of 40°C is $\frac{d\theta}{dt} = -(0.03465)(\theta - 40)$. The temperature of the body after 40 minutes is (2)

(A) 60°C (B) 50°C (C) 35°C (D) 85°C

13. A metal ball is heated to a temperature of 100°C and at time t = 0 it is placed in water which is maintained at 40°C. The temperature of the ball reduces to 60°C in 4 minutes. By Newton's law of cooling the differential equation is $\frac{d\theta}{dt} = -\left(\frac{1}{4}\log_e 3\right)$ $(\theta - 40)$. Then the time required to reduce the temperature of ball to 50°C is (2)

(A) 7.5 min (B) 3.5 min (C) 10 min (D) 6.5 min

14. A body at temperature 100°C is placed in a room whose temperature is 20°C and cools to 60°C in 5 minutes. By Newton's law of cooling the differential equation is $\frac{d\theta}{dt} = -\left(\frac{1}{5}\log_e 2\right)(\theta - 20)$. Then the temperature after 8 minutes is (2)

(A) 46.4°C (B) 65.4°C (C) 40.4°C (D) 20°C

15. A copper ball is heated to a temperature of 100°C and at time t = 0 it is placed in water which is maintained at 30°C. The temperature of the ball reduces to 70°C in 3 minutes. The differential equation by Newton's law of cooling is $\frac{d\theta}{dt} = -\left(\frac{1}{3}\log_e \frac{7}{4}\right)(\theta - 30)$. Then the time required to reduce the temperature of ball to 31°C is (2)

(A) 3 min (B) 7.78 min (C) 22.78 min (D) 15.78 min

ANSWERS

1. (D)	2. (B)	3. (A)	4. (C)	5. (B)	6. (D)	7. (C)	8. (D)
9. (B)	10. (A)	11. (C)	12. (B)	13. (D)	14. (A)	15. (C)	

MULTIPLE CHOICE QUESTIONS

Rectilinear Motion :

1. Rectilinear motion is a motion of body along a (1)

   (A) straight line (B) circular path (C) parabolic path (D) none of these

2. According to D'Alembert's principle, algebraic sum of forces acting on a body along a given direction is equal to (1)

   (A) velocity × acceleration (B) mass × velocity

   (C) mass × displacement (D) mass × acceleration

3. A particle moving in a straight line with acceleration $k\left(x+\frac{a^4}{x^3}\right)$ directed towards the origin. The equation of motion is (1)

(A) $\frac{dv}{dx}=-k\left(x+\frac{a^4}{x^3}\right)$ (B) $v\frac{dv}{dx}=k\left(x+\frac{a^4}{x^3}\right)$

(C) $v\frac{dv}{dx}=-k\left(x+\frac{a^4}{x^3}\right)$ (D) $\frac{dv}{dx}=\left(x+\frac{a^4}{x^3}\right)$

4. A particle of mass m moves in a horizontal straight line OA with acceleration $\frac{k}{x^3}$ at a distance x and directed towards the origin O. Then the differential equation of motion is (1)

(A) $v\frac{dv}{dx}=\frac{k}{x^3}$ (B) $v\frac{dv}{dx}=-\frac{k}{x^3}$ (C) $\frac{dv}{dx}=-\frac{k}{x^3}$ (D) $\frac{dv}{dx}=\frac{k}{x^3}$

5. A body of mass m falling from rest is subjected to a force of gravity and air resistance proportional to square of velocity (kv^2). The equation of motion is (1)

(A) $m\frac{dv}{dx}=mg-kv^2$ (B) $mv\frac{dv}{dx}=mg+kv^2$

(C) $mv\frac{dv}{dx}=-kv^2$ (D) $mv\frac{dv}{dx}=mg-kv^2$

6. A particle is projected vertically upward with velocity v_1 and resistance of air produces retardation (kv^2) where v is velocity. The equation of motion is (1)

(A) $v\frac{dv}{dx}=-g-kv^2$ (B) $v\frac{dv}{dx}=-g+kv^2$ (C) $v\frac{dv}{dx}=-kv^2$ (D) $v\frac{dv}{dx}=g-kv^2$

7. A body starts moving from rest is opposed by a force per unit mass of value (cx) resistance per unit mass of value (bv^2) where v and x are velocity and displacement of body at that instant. The differential equation of motion is (1)

(A) $mv\frac{dv}{dx}=-cx-bv^2$ (B) $v\frac{dv}{dx}=cx+bv^2$

(C) $v\frac{dv}{dx}=-cx-bv^2$ (D) $\frac{dv}{dx}=-cx-bv^2$

8. A body of mass m falls from rest under gravity, in a fluid whose resistance to motion at any time t is mk times its velocity where k is constant. The differential equation of motion is (1)

(A) $\frac{dv}{dt}=-g-kv$ (B) $\frac{dv}{dt}=g-kv$ (C) $\frac{dv}{dt}=g+kv$ (D) $\frac{dv}{dt}=mg-mkv$

9. A particle of mass m is projected vertically upward with velocity v, assuming the air resistance k times its velocity where k is constant. The differential equation of motion is (1)

(A) $\frac{dv}{dt}=mg-kv$ (B) $m\frac{dv}{dt}=-mg+kv$

(C) $m\frac{dv}{dt}=-kv$ (D) $m\frac{dv}{dt}=-mg-kv$

10. Assuming that the resistance to movement of a ship through water in the form of $(a^2 + b^2v^2)$ where v is the velocity, a and b are constants. The differential equation for retardation of the ship moving with engine stopped is (1)

(A) $m\frac{dv}{dt} = -(a^2 + b^2v^2)^2$ (B) $m\frac{dv}{dt} = +(a^2 + b^2v^2)$

(C) $m\frac{dv}{dt} = -(a^2 + b^2v^2)$ (D) $m\frac{dv}{dx} = -(a^2 + b^2v^2)$

11. Differential equation of motion of a body of mass m falls from rest under gravity in a fluid whose resistance to motion at any time t is mk times its velocity where k is constant is $\frac{dv}{dt} = g - kv$ then the relation between velocity and time t is (2)

(A) $t = \frac{1}{k}\log\frac{g-kv}{g}$ (B) $t = \frac{1}{k}\log\frac{g}{g-kv}$

(C) $t = \frac{1}{k}\log\frac{g}{g+kv}$ (D) $t = -\frac{1}{k}\log\frac{1}{g-kv}$

12. A body of mass m falling from rest is subjected to the force of gravity and air resistance proportional to square of velocity (kv^2) satisfies the differential equation $mv\frac{dv}{dx} = k(a^2 - v^2)$ where $ka^2 = mg$, then the relation between velocity and displacement is (2)

(A) $\frac{2kx}{m} = \log\frac{a^2}{a^2-v^2}$ (B) $\frac{2kx}{m} = \log\frac{a^2-v^2}{a^2}$

(C) $2kx = \log\frac{1}{a^2-v^2}$ (D) $\frac{x}{m} = \log\frac{a^2}{a^2-v^2}$

13. A vehicle starts from rest and its acceleration is given by $\frac{dv}{dt} = k\left(1-\frac{t}{T}\right)$ where k and T are constant. Then the velocity v in terms of time t is given by (2)

(A) $v = k\left(t-\frac{t^2}{2}\right)$ (B) $v = k\left(t-\frac{t^2}{T}\right)$ (C) $v = k\left(\frac{t^2}{2}-\frac{t^3}{3T}\right)$ (D) $v = k\left(t-\frac{t^2}{2T}\right)$

14. A particle of unit mass moves in a horizontal straight line OA with an acceleration $\frac{k}{r^3}$ at a distance r and directed towards O. If initially the particle was at rest at r = a and equation of motion is $v\frac{dv}{dr} = -\frac{k}{r^3}$ then the relation between r, v is (2)

(A) $v^2 = k\left(\frac{1}{r^2}+\frac{1}{a^2}\right)$ (B) $v^2 = k\left(\frac{1}{a^2}-\frac{1}{r^2}\right)$ (C) $v^2 = k\left(\frac{1}{r^2}-\frac{1}{a^2}\right)$ (D) $v^2 = k\left(\frac{1}{r^4}-\frac{1}{a^2}\right)$

15. A particle of mass m is projected upward with velocity V. Assuming the air resistance k times its velocity and equation of motion is $m\frac{dv}{dt} = -mg - kv$ then the relation between velocity v and time t is (2)

(A) $t = \frac{m}{k}\log\left(\frac{mg + kV}{mg + kv}\right)$ (B) $t = \frac{m}{k}\log\left(\frac{mg + kv}{mg + kV}\right)$

(C) $t = m\log\left(\frac{mg + kV}{mg + kv}\right)$ (D) $t = \log\left(\frac{mg + kv}{mg + kV}\right)$

16. A body of mass m falls from rest under gravity in a fluid whose resistance to motion at any instant is mkv where k is constant. The differential equation of motion is $\frac{dv}{dt} = g - kv$ then the terminal velocity is (2)

(A) $\frac{k}{g}$ (B) $\frac{g}{k}$ (C) $-\frac{g}{k}$ (D) none of these

17. A bullet is fired into a sand tank, its retardation is proportional to the square root of its velocity. The differential equation of motion is $\frac{dv}{dt} = -k\sqrt{v}$. If v_0 is initial velocity then the relation between velocity v and time t is (2)

(A) $\sqrt{v} = -t + \sqrt{v_0}$ (B) $2\sqrt{v} = -kt$

(C) $\sqrt{v} = -kt + \sqrt{v_0}$ (D) $2\sqrt{v} = -kt + 2\sqrt{v_0}$

18. A particle moving in a straight line with acceleration $k\left(x + \frac{a^4}{x^3}\right)$ directed towards the origin. the equation of motion is $v\frac{dv}{dx} = -k\left(x + \frac{a^4}{x^3}\right)$. If it starts from rest at a distance x = a from the origin then the relation between velocity v and displacement x is (2)

(A) $\frac{v^2}{2} = k\left(\frac{x^2}{2} + \frac{a^4}{2x^2}\right)$ (B) $\frac{v^2}{2} = -k\left(\frac{x^2}{2} - \frac{a^4}{2x^2}\right)$

(C) $\frac{v^2}{2} = -k\left(\frac{x^2}{2} - \frac{a^4}{2x^2}\right) + \frac{a^2}{2}$ (D) $\frac{v^2}{2} = -k\left(\frac{x^2}{2} - \frac{3a^4}{x^4}\right)$

ANSWERS

1. (A)	2. (D)	3. (C)	4. (B)	5. (D)	6. (A)	7. (C)	8. (B)
9. (D)	10. (C)	11. (B)	12. (A)	13. (D)	14. (C)	15. (A)	16. (B)
17. (D)	18. (B)						

MULTIPLE CHOICE QUESTIONS

Applications to Electrical Circuits :

1. A circuit containing resistance R and inductance L in series with voltage source E. By Kirchhoff's voltage law, differential equation for current i is (1)

(A) $Li + R\frac{di}{dt} = E$ (B) $L\frac{di}{dt} + Ri = E$ (C) $L\frac{di}{dt} + Ri = 0$ (D) $L\frac{di}{dt} + \frac{q}{C} = E$

2. A circuit containing resistance R and capacitance C in series with voltage source E. By Kirchhoff's voltage law, differential equation for current $i = \frac{dq}{dt}$ is (1)

(A) $L\frac{di}{dt} + \frac{q}{C} = E$ (B) $R\frac{dq}{dt} + \frac{q}{C} = 0$ (C) $L\frac{di}{dt} + Ri = 0$ (D) $R\frac{dq}{dt} + \frac{q}{C} = E$

3. A circuit containing inductance L, capacitance C in series without applied electromotive force. By Kirchhoff's voltage law, differential equation for current i is (1)

(A) $L\frac{di}{dt} + \frac{q}{C} = 0$ (C) $L\frac{di}{dt} + Ri = 0$ (C) $L\frac{di}{dt} + Ri = E$ (D) $L\frac{di}{dt} + \frac{q}{C} = E$

4. A circuit containing inductance L, capacitance C in series with applied electromotive force E. By Kirchhoff's voltage law, differential equation for current i is (1)

(A) $L\frac{di}{dt} + Ri = E$ (B) $L\frac{di}{dt} + Ri = 0$ (C) $L\frac{di}{dt} + \frac{q}{C} = E$ (D) $L\frac{di}{dt} + \frac{q}{C} = 0$

5. The differential equation for the current in an electric circuit containing resistance R and inductance L in series with voltage source E sin ωt is (1)

(A) $L\frac{di}{dt} + \frac{q}{C} = E$ (B) $Li + R\frac{di}{dt} = E \sin \omega t$

(C) $L\frac{di}{dt} + Ri = 0$ (D) $L\frac{di}{dt} + Ri = E \sin \omega t$

6. In a circuit containing resistance R and inductance L in series with constant voltage source E, current i is given by $i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$, then maximum current i_{max} is (1)

(A) $\frac{E}{R}$ (B) $\frac{R}{E}$ (C) ER (D) 0

7. The differential equation for the current i in an electric circuit containing resistance 100 ohm and an inductance of 0.5 henry connected in series with battery of 20 volts is (1)

(A) $0.5\frac{di}{dt} + 100i = 0$ (B) $0.5\frac{di}{dt} + 100i = 20$

(C) $100\frac{di}{dt} + 0.5i = 20$ (D) $100\frac{di}{dt} + 0.5R = 0$

8. The differential equation for the current i in an electric circuit containing resistance R = 250 ohm and an inductance of L = 640 henry in series with an electromotive force E = 500 volts is (1)

(A) $640\frac{di}{dt} + 250i = 0$ (B) $250\frac{di}{dt} + 640i = 500$

(C) $640\frac{di}{dt} + 250i = 500$ (D) $250\frac{di}{dt} + 640i = 0$

9. A capacitor C = 0.01 farad in series with resistor R = 20 ohms is charged from battery E = 10 volts. If initially capacitor is completely discharged then differential equation for charge q(t) is given by (1)

(A) $20\frac{dq}{dt} + \frac{q}{0.01} = 0;\ q(0) = 0$ (B) $20\frac{dq}{dt} + 0.01q = 10;\ q(0) = 0$

(C) $20\frac{dq}{dt} + \frac{q}{0.01} = 10;\ q(0) = 0$ (D) $20\frac{dq}{dt} + 0.01q = 0;\ q(0) = 0$

10. In a circuit containing resistance R and inductance L in series with constant e.m.f. E, the current i is given by $i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$, then the time required to build current half of its theoretical maximum is (2)

(A) $\frac{L}{R\log 2}$ (B) $\frac{L\log 2}{R}$ (C) $\frac{R\log 2}{L}$ (D) 0

11. In a circuit containing resistance R and inductance L in series with constant e.m.f. E, the current i is given by $i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$, then the time required before current reaches its 90% of maximum value is (2)

(A) 0 (B) $\frac{L}{R\log 10}$ (C) $\frac{R\log 10}{L}$ (D) $\frac{L\log 10}{R}$

12. If the differential equation for current in an electric circuit containing resistance R and inductance L in series with constant e.m.f. E, the current i is $L\frac{di}{dt} + Ri = E$, then the current at any time t is given by (2)

(A) $i = \frac{E}{R} - Ae^{-\frac{R}{L}t}$; A is arbitrary constant

(B) $i = \frac{E}{R} + Ae^{-\frac{R}{L}t}$; A is arbitrary constant

(C) $i = \frac{E}{R} + Ae^{\frac{R}{L}t}$; A is arbitrary constant

(D) $i = \frac{E}{R} + e^{-\frac{R}{L}t}$

13. A charge q on the plate of condenser of capacity C charge through a resistance R by steady voltage V satisfies differential equation $R\frac{dq}{dt}+\frac{q}{C}=V$, then charge q at any time t is (2)

(A) $q = CV + Ae^{-\frac{1}{RC}t}$; A is arbitrary constant

(B) $q = CV - Ae^{\frac{1}{RC}t}$; A is arbitrary constant

(C) $q = C + Ae^{\frac{1}{RC}t}$; A is arbitrary constant

(D) $q = CV + e^{\frac{1}{RC}t}$

14. The charge q on the plate of condenser of capacity C charge through a resistance R by steady voltage V is given by $q = CV\left(1-e^{-\frac{1}{RC}t}\right)$ then current flowing through the plate is (2)

(A) $i = \frac{V}{R}e^{-\frac{R}{L}t}$ (B) $i = \frac{V}{R}e^{\frac{1}{RC}t}$

(C) $i = \frac{V}{R}e^{-\frac{1}{RC}t}$ (D) $i = CV\left(1-e^{-\frac{1}{RC}t}\right)$

15. A resistance R = 100 ohms, an inductance L = 0.5 henry are connected in series with a battery of 20 volts. The differential equation for the current i is $0.5\frac{di}{dt}+100i = 20$, then current i at any time t is (2)

(A) Ae^{-200t}; A is arbitrary constant (B) $\frac{1}{5}+Ae^{200t}$; A is arbitrary constant

(C) $2+Ae^{-200t}$; A is arbitrary constant (D) $\frac{1}{5}+Ae^{-200t}$; A is arbitrary constant

16. If an R-C circuit, charge q as function of time t is $q = e^{-3t}-e^{-6t}$, then time required for maximum charge on capacitor is (2)

(A) $3\log 2$ (B) $-\frac{1}{3}\log 2$ (C) $\frac{1}{3}\log 2$ (D) $\frac{1}{2}\log 3$

17. A circuit containing resistance R and inductance L in series with voltage source E. The differential equation for current i is $L\frac{di}{dt}+Ri = E$. Given L = 640 H, R = 250 Ω and E = 500 volts then integrating factor of differential equation is (2)

(A) $e^{\frac{64}{25}t}$ (B) $e^{\frac{25}{64}t}$ (C) $e^{-\frac{25}{64}t}$ (D) e^{250t}

ANSWERS

1. (B)	2. (D)	3. (A)	4. (C)	5. (D)	6. (A)	7. (B)	8. (C)
9. (C)	10 (B)	11. (D)	12. (B)	13. (A)	14. (C)	15. (D)	16. (C)
17. (B)							

Simple Harmonic Motion :

1. If a particle moves on a straight line so that the force acting on it is always directed towards a fixed point on the line and proportional to its distance from the point then the particle is said to be in (1)

(A) simple harmonic motion (B) motion under the gravity

(C) periodic motion (D) circular motion

Ans. (A)

2. A particle executes simple harmonic motion then the differential equation of motion is (1)

(A) $\frac{d^2x}{dt^2} = -\omega^2 x$ (B) $\frac{d^2x}{dt^2} = \omega^2 x$ (C) $\frac{d^2x}{dt^2} = -\frac{\omega^2}{x}$ (D) $\frac{dx}{dt} = -\omega^2 x$

Ans. (A)

MULTIPLE CHOICE QUESTIONS

Chemical Engineering Problems :

1. A tank contains 10,000 litres of brine in which 200 kg salt dissolved. Fresh water runs into the tank at the rate of 100 litres per minute and the mixture kept uniform by stirring, runs out at the same rate. If Q be the total amount of salt at any time t then the governing differential equation is (2)

(A) $\frac{dQ}{dt} = 200 - \frac{Q}{100}$ (B) $\frac{dQ}{dt} = -\frac{Q}{10000}$ (C) $\frac{dQ}{dt} = -\frac{Q}{100}$ (D) $\frac{dQ}{dt} = \frac{Q}{100}$

2. A tank initially contains 50 litres of fresh water. Brine containing 2 grams per litre of salt, flows into the tank at the rate of 2 litres per minute and the mixture kept uniform by stirring, runs out at the same rate. If Q be the total amount of salt at any time t then the differential equation in terms of Q and t is (2)

(A) $\frac{dQ}{dt} = 4 - \frac{Q}{50}$ (B) $\frac{dQ}{dt} = 4 - \frac{Q}{25}$ (C) $\frac{dQ}{dt} = 2 - \frac{Q}{50}$ (D) $\frac{dQ}{dt} = 2 - \frac{Q}{25}$

3. A tank initially contains 100 litres of fresh water. 2 litres of brine each containing 1 gram of dissolved salt, runs into the tank per minute and the mixture kept uniform by stirring, runs out at the rate of 1 litre per minute. Let Q be the quantity of salt present at any time t then $\frac{dQ}{dt}$ the rate at which salt content changing is (2)

(A) $\frac{dQ}{dt} = 1 - \frac{Q}{100+t}$ (B) $\frac{dQ}{dt} = 2 - \frac{Q}{100}$ (C) $\frac{dQ}{dt} = \frac{Q}{100+t}$ (D) $\frac{dQ}{dt} = 2 - \frac{Q}{100+t}$

4. A tank contains 1000 litres of brine in which 20 kg salt dissolved. Brine containing 0.1 kg per litre of salt, runs into the tank at the rate of 40 litres per minute and the mixture, assumed to be kept uniform by stirring, runs out at the rate of 30 litres per minute. Assuming that tank is sufficiently large to avoid overflow, the governing differential equation for rate at which the salt content changing $\frac{dQ}{dt}$ at any time t is (2)

(A) $\frac{dQ}{dt} = 4 - 30\frac{Q}{1000+10t}$ (B) $\frac{dQ}{dt} = 4 - 30\frac{Q}{1000}$

(C) $\frac{dQ}{dt} = 30\frac{Q}{1000+10t}$ (D) $\frac{dQ}{dt} = 4 - \frac{Q}{1000+10t}$

5. A tank initially contains 5000 litres of fresh water. Salt water containing 100 grams per litre of salt, flows into it at the rate of 10 litres per minute and the mixture kept uniform by stirring, runs out at the same rate. If Q be the total amount of salt at any time t then the differential equation relating Q and t is (2)

(A) $\frac{dQ}{dt} = 100 - \frac{Q}{500}$ (B) $\frac{dQ}{dt} = 1000 - \frac{Q}{500}$

(C) $\frac{dQ}{dt} = 1000 - \frac{Q}{5000}$ (D) $\frac{dQ}{dt} = -\frac{Q}{500}$

6. A tank contains 10,000 litres of brine in which 200 kg of salt are dissolved. Fresh water runs into the tank at the rate of 100 litres per minute and the mixture kept uniform by stirring, runs out at the same rate. If governing differential equation is $\frac{dQ}{dt} = -\frac{Q}{100}$, the amount of salt Q at any time t is (2)

(A) $\log Q = -\frac{\log t}{100} + \log 200$ (B) $\log Q = -\frac{t}{100}$

(C) $\log Q = -\frac{t}{100} + \log 200$ (D) $\log Q = -\frac{t}{100} - \log 200$

7. A tank initially contains 50 litres of fresh water. Brine containing 2 grams per litre of salt, flows into the tank at the rate of 2 litres per minute and the mixture kept uniform by stirring, runs out at the same rate. The differential equation in terms of Q and t is $\frac{dQ}{dt} = 4 - \frac{Q}{25}$. The total amount of salt Q at any time t is (2)

(A) $t = -25 \log_e (100 - Q) + 25 \log_e 100$ (B) $t = 25 \log_e (100 - Q) - 25 \log_e 100$

(C) $t = -\log_e (100 - Q) + \log_e 10$ (D) none of these

8. A tank initially contains 500 litres of fresh water. Salt water containing 100 grams per litre of salt, flows into it at the rate of 10 litres per minute and the mixture kept uniform by stirring, runs out at the same rate. The differential equation in terms of Q and t is $\frac{dQ}{dt} = 1000 - \frac{Q}{500}$. The amount of salt Q at any time t is (2)

(A) $t = -\log_e (500000 - Q) + k$ (B) $t = 500 \log_e (500000 - Q) + k$

(C) $t = -500 \log_e (5000 - Q) + k$ (D) $t = -500 \log_e (500000 - Q) + k$

ANSWERS

1. (C)	2. (B)	3. (D)	4. (A)	5. (B)	6. (C)	7. (A)	8. (D)

MULTIPLE CHOICE QUESTIONS

One Dimensional Conduction of Heat :

1. Fourier's law of heat conduction states that, the quantity of heat flow across an area A cm^2 is (2)

(A) proportional to the product of area A and temperature gradient $\frac{dT}{dx}$

(B) inversely proportional to the product of area A and temperature gradient $\frac{dT}{dx}$

(C) equal to sum of area A and temperature gradient $\frac{dT}{dx}$

(D) equal to difference of area A and temperature gradient $\frac{dT}{dx}$

2. If q be the quantity of heat that flows across an area A cm^2 and thickness δx in one second where the difference of temperature at the faces is δT, then by Fourier's law of heat conduction (2)

(A) $q = -k\left(A - \frac{dT}{dx}\right)$, where k is thermal conductivity

(B) $q = kA\frac{dT}{dx}$, where k is thermal conductivity

(C) $q = -k\left(A + \frac{dT}{dx}\right)$, where k is thermal conductivity

(D) $q = -kA\frac{dT}{dx}$, where k is thermal conductivity

3. The differential equation for steady state heat loss per unit time from a unit length of pipe with thermal conductivity k, radius r_0 carrying steam at temperature T_0, if the pipe is covered with insulation of thickness w, the outer surface of which remains at the constant temperature T_1, is (2)

(A) $Q = k\,(2\pi r)\frac{dT}{dr}$

(B) $Q = -k\,(2\pi r)\frac{dT}{dr}$

(C) $Q = -k\,(2\pi r^2)\frac{dT}{dr}$

(D) $Q = -k\,(\pi r^2)\frac{dT}{dr}$

4. The differential equation for steady state heat loss per unit time from a spherical shell with thermal conductivity k radius r_0 carrying steam at temperature T_0, if the spherical shell is covered with insulation of thickness w, the outer surface of which remains at the constant temperature T_1, is (2)

(A) $Q = -k\,(2\pi r)\dfrac{dT}{dr}$ (B) $Q = k\,(2\pi r)\dfrac{dT}{dr}$

(C) $Q = -k\,(4\pi r^2)\dfrac{dT}{dr}$ (D) $Q = -k\,(\pi r^2)\dfrac{dT}{dr}$

5. The differential equation for steady state heat loss Q per unit time from a unit length of pipe with thermal conductivity k, radius r_0 carrying steam at temperature T_0, if the pipe is covered with insulation of thickness W, the outer surface of which remains at the constant temperature T_1, is $Q = -k\,(2\pi r)\dfrac{dT}{dr}$. Then the temperature T of surface of pipe of radius r is (2)

(A) $T = \dfrac{Q}{2\pi k}\dfrac{1}{r} + C$ (B) $T = \dfrac{Q}{2\pi k}\log r + C$

(C) $T = -\dfrac{Q}{2\pi k}\dfrac{1}{r} + C$ (D) $T = -\dfrac{Q}{2\pi k}\log r + C$

6. The differential equation for steady state heat loss Q per unit time from a spherical shell with thermal conductivity k, radius r_0 carrying steam at temperature T_0, if the spherical shell is covered with insulation of thickness w, the outer surface of which remains at the constant temperature T_1, is $Q = -k\,(4\pi r^2)\dfrac{dT}{dr}$. Then the temperature T of spherical shell of radius r is (2)

(A) $T = -\dfrac{Q}{4\pi k}\dfrac{1}{r^2} + C$ (B) $T = \dfrac{Q}{4\pi k}\dfrac{1}{r} + C$ (C) $T = -\dfrac{Q}{4\pi k}\dfrac{1}{r} + C$ (D) $T = -\dfrac{Q}{2\pi k}\dfrac{1}{r^3} + C$

7. A pipe 20 cm in diameter contains steam at 150°C and is protected with covering 5 cm thick for which k = 0.0025. If the temperature of the outer surface of the covering is 40°C and differential equation of conduction of heat is $dT = -\dfrac{Q}{2\pi k}\dfrac{dx}{x}$. The amount of heat loss Q is (2)

(A) $\dfrac{110\,(2\pi k)}{\log(1.5)}$ (B) $\dfrac{\log(1.5)}{110\,(2\pi k)}$ (C) $-\dfrac{110\,(2\pi k)}{\log(1.5)}$ (D) $\dfrac{110}{\log(1.5)}$

8. A long hollow pipe has an inner diameter of 10 cm and outer diameter of 20 cm. The inner surface is kept at 200°C and outer surface at 50°C. The thermal conductivity k = 0.12. The differential equation of conduction of heat is $dT = -\dfrac{Q}{2\pi k}\dfrac{dx}{x}$. Then the amount of heat loss Q cal/sec is (2)

(A) $-\dfrac{150\,(2\pi k)}{\log 2}$ (B) $\dfrac{\log 2}{150\,(2\pi k)}$ (C) $\dfrac{150\,(2\pi k)}{\log 2}$ (D) $\dfrac{(2\pi k)}{\log 2}$

9. A steam pipe 20 cm in diameter is protected with covering 6 cm thick for which thermal conductivity k = 0.0003 in steady state. The inner surface of the pipe is at 200°C and outer surface of the covering is at 30°C and differential equation of conduction of heat is $dT = -\frac{Q}{2\pi k}\frac{dx}{x}$. The amount of heat loss Q is (2)

(A) $\frac{170\,(2\pi k)}{\log(1.6)}$ (B) $-\frac{170\,(2\pi k)}{\log(1.6)}$ (C) $\frac{\log(1.6)}{170\,(2\pi k)}$ (D) $\frac{170}{\log(1.6)}$

10. A pipe 10 cm in diameter contains steam at 100°C. It is protected with asbestos 5 cm thick for which k = 0.0006 and outer surface is at 30°C. The differential equation of conduction of heat is $dT = -\frac{Q}{2\pi k}\frac{dx}{x}$. The amount of heat loss Q is (2)

(A) $\frac{\log 2}{70\,(2\pi k)}$ (B) $\frac{70\,(2\pi k)}{\log 2}$ (C) $-\frac{70\,(2\pi k)}{\log 2}$ (D) $\frac{(2\pi k)}{\log 2}$

ANSWERS

1. (A)	2. (D)	3. (B)	4. (C)	5. (D)	6. (B)	7. (A)	8. (C)
9. (A)	10. (B)						

Miscellaneous Examples :

1. In a certain culture of bacteria, the rate of increase is proportional to the number present. If y denote the number of bacteria at time t hours then the governing differential equation is (1)

(A) $\frac{dy}{dt} = ky$ (B) $\frac{dy}{dt} = -ky$ (C) $\frac{dy}{dt} = \frac{k}{y}$ (D) $\frac{dy}{dt} = ky^2$

2. The differential equation of the population model for natural growth of bacteria is $\frac{dy}{dt} = ky$. The general solution of the equation is (1)

(A) $y = c \log kt$ (B) $ye^{kt} = ct$ (C) $y = ce^{kt}$ (D) $y = ce^{-kt}$

3. The amount x of a substance present in certain chemical reaction at time t is given by $\frac{dx}{dt} + \frac{1}{10}x = 2 - (1.5)\,e^{-\frac{1}{10}t}$, then the amount x of substance present at time t is (1)

(A) $x = -\frac{3}{2}te^{-\frac{1}{10}t} + Ce^{-\frac{1}{10}t}$ (B) $x = 20 + \frac{3}{2}te^{-\frac{1}{10}t} - Ce^{-\frac{1}{10}t}$

(C) $x = 20 - \frac{3}{2}t + C$ (D) $x = 20 - \frac{3}{2}te^{-\frac{1}{10}t} + Ce^{-\frac{1}{10}t}$

4. Biotransformation of an organic compound having concentration x can be modeled using an ordinary differential equation $\frac{dx}{dt} + kx^2 = 0$ where k is reaction rate constant. If $x = a$ at $t = 0$, the solution of equation is (2)

(A) $x = ae^{-kt}$ (B) $\frac{1}{x} = \frac{1}{a} + kt$ (C) $x = a(1 - e^{-kt})$ (D) $x = a + kt$

ANSWERS

1. (A)	2. (C)	3. (D)	4. (B)

MULTIPLE CHOICE QUESTIONS

Fourier Series and Harmonic Analysis :

1. A function f(x) is said to be periodic of period T if (1)

   (A) $f(x+T) = f(x)$ for all x (B) $f(x+T) = f(T)$ for all x

   (C) $f(-x) = f(x)$ for all x (D) $f(-x) = -f(x)$ for all x

2. If $f(x + nT) = f(x)$ where n is any integer then the fundamental period of f(x) is (1)

   (A) 2T (B) $\frac{T}{2}$ (C) T (D) 3T

3. If f(x) is a periodic function with period T then f(ax), $a \neq 0$ is periodic function with fundamental period (1)

   (A) T (B) $\frac{T}{a}$ (C) aT (D) π

4. Fundamental period of sin 2x is sin 2(180) (1)

   (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) 2π (D) π

5. Fundamental period of cos 2x is (1)

   (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) π (D) 2π

6. Fundamental period of tan 3x is (1)

   (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) π (D) $\frac{\pi}{4}$

7. Fourier series representation of periodic function f(x) with period 2π which satisfies the Dirichlet's conditions is (1)

(A) $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ (B) $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\pi x + b_n \sin n\pi x)$

(C) $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx)(b_n \sin nx)$ (D) $\frac{a_0}{2} + (a_n \cos nx + b_n \sin nx)$

8. Fourier series representation of periodic function f(x) with period 2L which satisfies the Dirichlet's conditions is (1)

2L

(a) $\frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos (n\pi x) + b_n \sin (n\pi x)]$ (B) $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_1 \cos\left(\frac{n\pi x}{L}\right) + b_1 \sin\left(\frac{n\pi x}{L}\right)\right]$

(C) $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{L}\right) \times b_n \sin\left(\frac{n\pi x}{L}\right)\right]$ (D) $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right]$

9. If f(x) is periodic function with period 2L defined in the interval C to C + 2L then Fourier coefficient a_0 is (1)

(A) $\int_C^{C+2L} f(x)\,dx$ (B) $\frac{1}{L}\int_C^{C+2L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx$

(C) $\frac{1}{L}\int_C^{C+2L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx$ (D) $\frac{1}{L}\int_C^{C+2L} f(x)\,dx$

10. If f(x) is periodic function with period 2L defined in the interval C to C + 2L then Fourier coefficient a_n is (1)

(A) $\int_C^{C+2L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx$ (B) $\frac{1}{L}\int_C^{C+2L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx$

(C) $\frac{1}{L}\int_C^{C+2L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx$ (D) $\frac{1}{L}\int_C^{C+2L} f(x)\,dx$

11. If f(x) is periodic function with period 2L defined in the interval C to C + 2L then Fourier coefficient b_n is (1)

(A) $\int_C^{C+2L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx$ (B) $\frac{1}{L}\int_C^{C+2L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx$

(C) $\frac{1}{L}\int_C^{C+2L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx$ (D) $\frac{1}{L}\int_C^{C+2L} f(x)\,dx$

12. A function f(x) is said to be even if (1)

(A) $f(-x) = f(x)$ (B) $f(-x) = -f(x)$ (C) $f(x + 2\pi) = f(x)$ (D) $f(-x) = [f(x)]^2$

13. A function f(x) is said to be odd if (1)

(A) $f(-x) = f(x)$ (B) $f(-x) = -f(x)$ (C) $f(x + 2\pi) = f(x)$ (D) $f(-x) = [f(x)]^2$

14. Which of the following is an odd function ? (1)

(A) $\sin x$ (B) $e^x + e^{-x}$ (C) $e^{|x|}$ (D) $\pi^2 - x^2$

15. Which of the following is an even function ? (1)

(A) $\sin x$ (B) $e^x - e^{-x}$ (C) $x \cos x$ (D) $\cos x$

16. Which of the following is neither an even function nor an odd function ? (1)

(A) $x \sin x$ (B) x^2 (C) e^{-x} (D) $x \cos x$

17. For an even function f(x) defined in the interval $-\pi \le x \le \pi$ and $f(x + 2\pi) = f(x)$ the Fourier series is (1)

(A) $\sum\limits_{n=1}^{\infty} b_n \sin nx$ (B) $\frac{a_0}{2} + \sum\limits_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}$

(C) $\frac{a_0}{2} + \sum\limits_{n=1}^{\infty} a_n \cos nx$ (D) $\sum\limits_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$

18. For an odd function f(x) defined in the interval $-\pi \le x \le \pi$ and $f(x + 2\pi) = f(x)$ the Fourier series is (1)

(A) $\sum\limits_{n=1}^{\infty} b_n \sin nx$ (B) $\frac{a_0}{2} + \sum\limits_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}$

(C) $\frac{a_0}{2} + \sum\limits_{n=1}^{\infty} a_n \cos nx$ (D) $\sum\limits_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$

19. Fourier coefficients for an even function f(x) defined in the interval $-\pi \le x \le \pi$ and $f(x + 2\pi) = f(x)$ are (1)

(A) $a_0 = 0, a_n = 0, \; b_n = \frac{2}{\pi} \int\limits_0^{\pi} f(x) \sin nx dx$

(B) $a_0 = \frac{2}{\pi} \int\limits_0^{\pi} f(x)\, dx, \; a_n = \frac{2}{\pi} \int\limits_0^{\pi} f(x) \cos nx\, dx, \; b_n = 0$

(C) $a_0 = 0, a_n = 0, b_n = 0$

(D) $a_0 = 0, \; a_n = 0, \; b_n = \frac{2}{\pi} \int\limits_0^{\pi} f(x) \cos nx\, dx$

20. Fourier coefficients for an odd function f(x) defined in the interval $-\pi \le x \le \pi$ and $f(x+2\pi) = f(x)$ are (1)

(A) $a_0 = 0,\ a_n = 0,\ b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$

(B) $a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx,\ a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx,\ b_n = 0$

(C) $a_0 = 0, a_n = 0, b_n = 0$

(D) $a_0 = 0,\ a_n = 0,\ b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$

21. Fourier coefficients for an even function f(x) defined in the interval $-L \le x \le L$ and $f(x+2L) = f(x)$ are (1)

(A) $a_0 = 0, a_n = 0,\ b_n = \frac{2}{L}\int_0^{L} f(x)\sin\frac{n\pi x}{L}dx$

(B) $a_0 = \frac{2}{L}\int_0^{L} f(x)\,dx,\ \ a_n = \frac{2}{L}\int_0^{L} f(x)\cos\frac{n\pi x}{L}dx, b_n = 0$

(C) $a_0 = 0, a_n = 0,\ b_n = 0$

(D) $a_0 = 0,\ a_n = 0,\ \ b_n = \frac{2}{L}\int_0^{L} f(x)\cos\frac{n\pi x}{L}dx$

22. Fourier coefficients for an odd function f(x) defined in the interval $-L \le x \le L$ and $f(x+2L) = f(x)$ are (1)

(A) $a_0 = 0, a_n = 0,\ b_n = \frac{2}{L}\int_0^{L} f(x)\cos\frac{n\pi x}{L}dx$

(B) $a_0 = \frac{2}{L}\int_0^{L} f(x)\,dx,\ \ a_n = \frac{2}{L}\int_0^{L} f(x)\cos\frac{n\pi x}{L}dx,\ \ b_n = 0$

(C) $a_0 = 0,\ a_n = 0, b_n = 0$

(D) $a_0 = 0, a_n = 0,\ b_n = \frac{2}{L}\int_0^{L} f(x)\sin\frac{n\pi x}{L}dx$

23. Half range Fourier cosine series for f(x) defined in the interval $0 \le x \le L$ is (1)

(A) $\sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$

(B) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{nx}{L}$

(C) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$

(D) $\frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$

24. Half range Fourier sine series for f(x) defined in the interval $0 \le x \le L$ is (1)

(A) $\sum_{n=1}^{\infty} b_n \sin \frac{nx}{L}$ (B) $\sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$

(C) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}$ (D) $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L}\right)$

25. In Harmonic analysis for a function with period 2π, the term $a_1 \cos x + b_1 \sin x$ is called (1)

(A) second harmonic (B) first harmonic

(C) third harmonic (D) none of these

26. In Harmonic analysis for a function with period 2π, the amplitude of first harmonic $a_1 \cos x + b_1 \sin x$ is (1)

(A) $\sqrt{a_1^2 - b_1^2}$ (B) $a_1^2 + b_1^2$ (C) $\sqrt{a_1^2 + b_1^2}$ (D) $\left(a_1^2 + b_1^2\right)^2$

27. The value of a_0 in Fourier series of y with period 6 for the following tabulated data (1)

x	0	1	2	3	4	5
y	9	18	24	28	26	20

(A) 17.85 (B) 20.83 (C) 35.71 (D) 41.66

28. The value of a_0 in Fourier series of y with period 180° for the following tabulated data is (1)

x^0	0	30	60	90	120	150
y	0	9.2	14.4	17.8	17.3	11.7

(A) 23.46 (B) 20.11 (C) 11.73 (D) 10.50

29. The values of a_0 in Fourier series of y with period 6 for the following tabulated data is (1)

x	0	1	2	3	4	5
y	4	8	15	7	6	2

(A) 3.5 (B) 14 (C) 6 (D) 7

30. The value of a_0 in Fourier series of y with period 360° for the following tabulated data is (1)

x^0	0	60	120	180	240	300
y	1.0	1.4	1.9	1.7	1.5	1.2

(A) 1.45 (B) 5.8 (C) 2.9 (D) 2.48

31. Fourier coefficient a_0 in the Fourier series expansion of $f(x) = e^{-x}$; $0 \le x \le 2\pi$ and $f(x + 2\pi) = f(x)$ is (2)

(a) $\frac{1}{\pi}(1 - e^{-2\pi})$ (B) $\frac{1}{2\pi}(1 - e^{2\pi})$ (C) $\frac{2}{\pi}(e^{-2\pi} - 1)$ (D) $\frac{1}{\pi}(1 + e^{2\pi})$

32. Fourier coefficient a_0 in the Fourier series expansion of $f(x) = \left(\frac{\pi - x}{2}\right)^2$; $0 \le x \le 2\pi$ and $f(x + 2\pi) = f(x)$ is (2)

(A) $\frac{\pi^2}{3}$ (B) $\frac{\pi^2}{6}$ (C) 0 (D) $\frac{\pi}{6}$

33. Fourier coefficient a_0 in the Fourier series expansion of $f(x) = x \sin x$; $0 \le x \le 2\pi$ and $f(x + 2\pi) = f(x)$ is (2)

(A) +2 (B) 0 (C) –2 (D) –4

34. $f(x) = \begin{cases} x, & 0 \le x \le \pi \\ 0, & \pi < x \le 2\pi \end{cases}$ and $f(x + 2\pi) = f(x)$. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$, then Fourier coefficient a_0 is (2)

(A) 2π (B) $\frac{\pi}{3}$ (C) 0 (D) $\frac{\pi}{2}$

35. $f(x) = \begin{cases} 0, & 0 \le x \le \pi \\ x, & \pi < x \le 2\pi \end{cases}$ and $f(x + 2\pi) = f(x)$. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$, then Fourier coefficient a_0 is (2)

(A) $\frac{3\pi}{2}$ (B) $\frac{3\pi}{8}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

36. $f(x) = 2x - x^2$, $0 \le x \le 3$ and period is 3. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2n\pi x}{3} + b_n \sin \frac{2n\pi x}{3}\right)$, then Fourier coefficient a_0 is (2)

(A) $\frac{3}{2}$ (B) 0 (C) 12 (D) $\frac{3}{4}$

37. $f(x) = 4 - x^2$, $0 \le x \le 2$ and period is 2. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\pi x + b_n \sin n\pi x)$, then Fourier coefficient a_0 is (2)

(A) $\frac{11}{3}$ (B) 0 (C) $\frac{16}{3}$ (D) $\frac{8}{5}$

38. $f(x) = \begin{cases} -x, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ and $f(x + 2\pi) = f(x)$. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$, then Fourier coefficient a_0 is (2)

(A) $\frac{\pi}{3}$ (B) $\frac{2}{\pi}$ (C) $\frac{\pi}{4}$ (D) π

39. $f(x) = x \cos x$, $-\pi \le x \le \pi$ and period is 2π. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$, then Fourier coefficient a_0 is (2)

(A) $-\frac{2}{\pi}$ (B) 0 (C) $\frac{4}{\pi}$ (D) $-\frac{4}{\pi}$

40. $f(x) = 2$, $-\pi \le x \le \pi$ and period is 2π. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$, then Fourier coefficient a_0 is (2)

(A) 4 (B) 2 (C) $\frac{4}{\pi}$ (D) $\frac{2}{\pi}$

41. $f(x) = x$, $-\pi \le x \le \pi$ and period is 2π. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$, then Fourier coefficient b_1 is (2)

(A) 2 (B) -1 (C) 0 (d) $\frac{2}{\pi}$

42. $f(x) = \begin{cases} 1, & -1 < x < 0 \\ \cos \pi x, & 0 < x < 1 \end{cases}$ and period 2. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\pi x + b_n \sin n\pi x)$, then Fourier coefficient a_0 is (2)

(A) 2 (B) 0 (C) 1 (D) -1

43. $f(x) = x - x^3$, $-2 < x < 2$ and period 4. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\pi x + b_n \sin n\pi x)$, then Fourier coefficient a_0 is (2)

(A) 1 (B) 0 (C) -2 (D) -1

44. For half range cosine series of $f(x) = \sin x$, $0 \le x < \pi$ and period is 2π. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$, then Fourier coefficient a_0 is (2)

(A) 4 (B) 2 (C) $\frac{2}{\pi}$ (D) $\frac{4}{\pi}$

45. For half range sine series of $f(x) = \cos x$, $0 \le x \le \pi$ and period is 2π. Fourier series is represented by $\sum_{n=1}^{\infty} b_n \sin nx$, then Fourier coefficient b_1 is (2)

(A) $\frac{1}{\pi}$ (B) 0 (C) $\frac{2}{\pi}$ (D) $-\frac{2}{\pi}$

46. For half range cosine series of $f(x) = lx - x^2, 0 \le x \le l$ and period is $2l$. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}$, then Fourier coefficient a_0 is (2)

(A) $\frac{l^2}{3}$ (B) $\frac{2l^2}{3}$ (C) $\frac{l^2}{6}$ (D) 0

47. For half range sine series of $f(x) = x, 0 \le x \le 2$ and period is 4. Fourier series is represented by $\sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{2}$, then Fourier coefficient b_1 is (2)

(A) 4 (B) 2 (C) $\frac{2}{\pi}$ (D) $\frac{4}{\pi}$

48. Fourier series representation of periodic function $f(x) = \left(\frac{\pi - x}{2}\right)^2, 0 \le x \le 2\pi$ is $\left(\frac{\pi - x}{2}\right)^2 = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{1}{n^2}\cos nx$, then value of $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots =$ (2)

(A) $\frac{\pi^2}{6}$ (B) $\frac{\pi^2}{12}$ (C) $\frac{\pi^2}{3}$ (D) 0

49. Fourier series representation of periodic function $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$ is $f(x) = \frac{8}{\pi^2}\left[\frac{1}{1^2}\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x + \ldots\right]$, then value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots =$ (2)

(A) $\frac{\pi^2}{4}$ (B) $\frac{\pi^2}{8}$ (C) $\frac{\pi^2}{16}$ (D) $\frac{8}{\pi^2}$

50. Fourier series representation of periodic function $f(x) = \pi^2 - x^2, -\pi \le x \le \pi$ is $\pi^2 - x^2 = \frac{2\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}\cos nx$, then value of $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots =$ (2)

(A) $\frac{\pi^2}{3}$ (B) $\frac{\pi^2}{4}$ (C) $\frac{\pi^2}{6}$ (D) $\frac{\pi^2}{12}$

51. Fourier series representation of periodic function $f(x) = \pi^2 - x^2, -\pi \le x \le \pi$ is $\pi^2 - x^2 = \frac{2\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}\cos nx$, then value of $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots =$ (2)

(A) $\frac{\pi^2}{3}$ (B) $\frac{\pi^2}{12}$ (C) $\frac{\pi^2}{6}$ (D) 0

52. The value of a_1 in Fourier series of y with period 6 for the following tabulated data is : (2)

x	0	1	2	3	4	5
y	9	18	24	28	26	20
$\cos\frac{\pi x}{3}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	–1	$-\frac{1}{2}$	$\frac{1}{2}$

(A) –8.33 (B) –7.14 (C) –4.16 (D) 0

53. The value of b_1 in Fourier series of y with period π for the following tabulated data is : (2)

x^0	0	30	60	90	120	150
y	0	9.2	14.4	17.8	17.3	11.7
sin 2x	0	0.866	0.866	0	–0.866	–0.866

(A) –3.116 (B) –1.558 (C) –4.16 (D) –1.336

54. The value of a_1 in Fourier series of y with period 6 for the following tabulated data is : (2)

x	0	1	2	3	4	5
y	4	8	15	7	6	2
$\cos\frac{\pi x}{3}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	–1	$-\frac{1}{2}$	$\frac{1}{2}$

(A) –2.83 (B) –8.32 (C) –3.57 (D) –10.98

55. The value of b_1 in Fourier series of y with period 2π for the following tabulated data is : (2)

x^0	0	60	120	180	240	300
y	1.0	1.4	1.9	1.7	1.5	1.2
sin x	0	0.866	0.866	0	–0.866	–0.866

(A) 0.0989 (B) 0.3464 (C) 0.1732 (D) 0.6932

ANSWERS

1. (A)	2. (C)	3. (B)	4. (D)	5. (C)	6. (B)	7. (A)	8. (D)
9. (D)	10. (C)	11. (B)	12. (A)	13. (B)	14. (A)	15. (D)	16. (C)
17. (C)	18. (A)	19. (B)	20. (D)	21. (B)	22. (D)	23. (C)	24. (B)
25. (B)	26. (C)	27. (D)	28. (A)	29. (B)	30. (C)	31. (A)	32. (B)
33. (C)	34. (D)	35. (A)	36. (B)	37. (C)	38. (D)	39. (B)	40. (A)
41. (A)	42. (C)	43. (B)	44. (D)	45. (B)	46. (A)	47. (D)	48. (A)
49. (B)	50. (D)	51. (C)	52. (A)	53. (B)	54. (D)	55. (C)	

MULTIPLE CHOICE QUESTIONS

Reduction Formulae :

1. If $I_n = \int_0^{\pi/2} \sin^n x \, dx$ then which of the following relation is true ? (1)

(A) $I_n = \frac{n-1}{n} I_{n-2}$ (B) $I_n = \frac{n-1}{n} I_{n-1}$ (C) $I_n = \frac{n}{n-1} I_{n-2}$ (D) $I_n = \frac{n-2}{n} I_{n-1}$

2. If $I_n = \int_0^{\pi/2} \cos^n x \, dx$ then which of the following relation is true ? (1)

(A) $I_n = \frac{n-1}{n} I_{n-1}$ (B) $I_n = \frac{n-1}{n} I_{n-2}$ (C) $I_n = \frac{n}{n-1} I_{n-2}$ (D) $I_n = n(n+1) I_{n-1}$

3. If $I_n = \int_0^{\pi/2} \sin^n x \, dx$, n positive even integer then I_n is calculated from (1)

(A) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{3}{4} \cdot \frac{1}{2}$

(B) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{3}{4} \cdot \frac{1}{2} \cdot \pi$

(C) $\left(\frac{n+1}{n} \cdot \frac{n+3}{n+2} \cdot \frac{n+5}{n+4} \cdots\right) \cdot \frac{\pi}{2}$

(D) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}$

4. If $I_n = \int_0^{\pi/2} \sin^n x \, dx$, n positive odd integer then I_n is calculated from (1)

(A) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{4}{5} \cdot \frac{2}{3} \cdot \frac{\pi}{2}$

(B) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}$

(C) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{4}{5} \cdot \frac{2}{3} \cdot 1$

(D) $\frac{n+1}{n} \cdot \frac{n+3}{n+2} \cdot \frac{n+5}{n+4}$

5. If $I_n = \int_0^{\pi/2} \cos^n x \, dx$, n positive even integer then I_n is calculated from (1)

(A) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{3}{4} \cdot \frac{1}{2}$

(B) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}$

(C) $\left(\frac{n+1}{n} \cdot \frac{n+3}{n+2} \cdot \frac{n+5}{n+4} \cdots\right) \cdot \frac{\pi}{2}$

(D) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{3}{4} \cdot \frac{1}{2} \cdot \pi$

6. If $I_n = \int_0^{\pi/2} \cos^n x \, dx$, n positive odd integer then I_n is calculated from (1)

(A) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{4}{5} \cdot \frac{2}{3} \cdot \frac{\pi}{2}$

(B) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}$

(C) $\frac{n+1}{n} \cdot \frac{n+3}{n+2} \cdot \frac{n+5}{n+4}$

(D) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{4}{5} \cdot \frac{2}{3} \cdot 1$

7. The value of integral $I_{m,n} = \int_0^{\pi/2} \sin^m x \cos^n x\, dx$; m, n are positive integers ≥ 2 is (1)

(A) $I_{m,n} = \frac{[(m-1)(m-3)\ldots 2 \text{ or } 1]\cdot[(n-1)(n-3)\ldots 2 \text{ or } 1]}{[(m+n)(m+n-2)\ldots 2 \text{ or } 1]} \times P$

where $P = \frac{\pi}{2}$ if m and n are both even

$= 1$ for all other values of m and n

(B) $I_{m,n} = \frac{[(m-1)(m-3)\ldots 2 \text{ or } 1]\cdot(n-1)(n-3)\ldots 2 \text{ or } 1]}{[(m+n)(m+n-2)\ldots 2 \text{ or } 1}\times P$

where $P = 1$ if m and are both even

$= \frac{\pi}{2}$ for all other values of m and n.

(C) $I_{m,n} = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdots\frac{4}{5}\cdot\frac{2}{3}\cdot 1$ (D) $I_{m,n} = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdots\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}$

8. $\int_0^{\pi/2} \sin^4 x\, dx$ is equal to (2)

(A) $\frac{\pi}{2}$ (B) $\frac{3\pi}{8}$ (C) $\frac{\pi}{4}$ (D) $\frac{3\pi}{16}$

9. $\int_0^{\pi/2} \sin^5 x\, dx$ is equal to (2)

(A) $\frac{8}{15}\cdot\frac{\pi}{2}$ (B) $\frac{15}{8}$ (C) $\frac{8}{15}$ (D) 0

10. $\int_0^{\pi/2} \cos^3 x\, dx$ is equal to (2)

(A) $\frac{2}{3}$ (B) $\frac{1}{4}$ (C) $\frac{2}{3}\cdot\frac{\pi}{2}$ (D) $\frac{1}{3}$

11. $\int_0^{\pi/2} \cos^6 x\, dx$ is equal to (2)

(A) $\frac{5}{16}$ (B) $\frac{5}{16}\cdot\frac{\pi}{2}$ (C) $\frac{16}{5}\cdot\frac{\pi}{2}$ (D) $\frac{5}{48}\cdot\frac{\pi}{2}$

12. $\int_0^{\pi/4} \sin^2(2x)\, dx$ is equal to (2)

(A) $\frac{1}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{8}$

13. $\int_0^{\pi/4} \cos^2(2x)\, dx$ is equal to (2)

(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{1}{4}$

14. $\int_0^{\pi} \sin^5\left(\frac{x}{2}\right) dx$ is equal to (2)

(A) $\frac{8\pi}{15}$ (B) $\frac{32}{15}$ (C) $\frac{16}{15}$ (D) $\frac{8}{15}$

15. $\int_0^{\pi} \cos^3 x \, dx$ is equal to (2)

(A) 1 (B) $\frac{2}{3}$ (C) $\frac{4}{3}$ (D) 0

16. $\int_0^{\pi} \sin^6 t \, dt$ is equal to (2)

(A) $\frac{\pi}{8}$ (B) $\frac{5\pi}{16}$ (C) $\frac{5}{8}$ (D) $\frac{5\pi}{8}$

17. $\int_0^{2\pi} \sin^6 t \, dt$ is equal to (2)

(A) $\frac{5}{4}$ (B) $\frac{5\pi}{32}$ (C) $\frac{5\pi}{8}$ (D) 0

18. $\int_0^{2\pi} \cos^7 t \, dt$ is equal to (2)

(A) $\frac{32}{35}$ (B) $\frac{32\pi}{70}$ (C) 0 (D) $\frac{16}{35}$

19. $\int_0^{\pi/2} \sin^6 x \cos^4 x \, dx$ is equal to (2)

(A) $\frac{3}{256}$ (B) $\frac{3\pi}{512}$ (C) $\frac{3}{128}$ (D) $\frac{512}{3}\pi$

20. $\int_0^{\pi/2} \sin^4 x \cos^5 x \, dx$ is equal to (2)

(A) $\frac{4\pi}{315}$ (B) $\frac{315}{8}$ (C) $\frac{8\pi}{630}$ (D) $\frac{8}{315}$

21. $\int_0^{2\pi} \sin^6 x \cos^4 x \, dx$ is equal to (2)

(A) $\frac{5\pi}{128}$ (B) $\frac{3\pi}{512}$ (C) $\frac{3\pi}{128}$ (D) $\frac{5\pi}{64}$

22. $\int_0^{2\pi} \sin^5 \theta \cos^4 \theta \, d\theta$ is equal to (2)

(A) 0 (B) $\frac{8}{315}$ (C) $\frac{3\pi}{128}$ (D) $\frac{\pi}{128}$

23. $\int_{-\pi/2}^{\pi/2} \sin^4\theta \, d\theta$ is equal to (2)

(A) $\frac{3\pi}{16}$ (B) $\frac{3\pi}{4}$ (C) 0 (D) $\frac{3\pi}{8}$

24. $\int_{-\pi/2}^{\pi/2} \sin^4\theta \cos^2\theta \, d\theta$ is equal to (2)

(A) $\frac{\pi}{32}$ (B) $\frac{\pi}{16}$ (C) $\frac{\pi}{8}$ (D) 0

25. If $I_n = \int_0^{\pi/4} \tan^n x \, dx$ and $I_n = \frac{1}{n-1} - I_{n-2}$ then I_4 is equal to (2)

(A) $-\frac{2}{3}+\frac{\pi}{2}$ (B) $-\frac{2}{3}-\frac{\pi}{4}$ (C) $-\frac{2}{3}+\frac{\pi}{4}$ (D) $-\frac{4}{3}+\frac{\pi}{4}$

26. If $I_n = \int_{\pi/4}^{\pi/2} \cot^n x \, dx$ and $I_n = \frac{1}{n-1} - I_{n-2}$ then I_4 is equal to (2)

(A) $-\frac{4}{3}+\frac{\pi}{4}$ (B) $-\frac{2}{3}+\frac{\pi}{2}$ (C) $-\frac{2}{3}-\frac{\pi}{4}$ (D) $-\frac{2}{3}+\frac{\pi}{4}$

27. If $I_n = \int_0^{\pi/4} \sin^{2n} x \, dx$ and $I_n = -\frac{1}{n2^{n+1}} + \left(\frac{2n-1}{2n}\right) I_{n-1}$ then I_2 is equal to (2)

(A) $-\frac{1}{4}+\frac{3\pi}{32}$ (B) $-\frac{3}{4}+\frac{3\pi}{32}$ (C) $\frac{1}{4}+\frac{\pi}{32}$ (D) $\frac{1}{4}-\frac{\pi}{16}$

28. If $I_n = \int_0^{\pi/4} \cos^{2n} x \, dx$ and $I_n = \frac{1}{n2^{n+1}} + \left(\frac{2n-1}{2n}\right) I_{n-1}$ then I_2 is equal to (2)

(A) $-\frac{1}{4}+\frac{3\pi}{32}$ (B) $\frac{1}{4}+\frac{3\pi}{32}$ (C) $\frac{1}{4}+\frac{3\pi}{16}$ (D) $\frac{1}{4}+\frac{\pi}{16}$

29. If $I_n = \int_0^{\pi/3} \cos^n x \, dx$ and $I_n = \frac{\sqrt{3}}{n2^n} + \left(\frac{n-1}{n}\right) I_{n-2}$ then I_2 is equal to (2)

(A) $\frac{\sqrt{3}}{4}+\frac{\pi}{3}$ (B) $\left(\frac{\sqrt{3}}{8}+\frac{1}{2}\right)\frac{\pi}{6}$ (C) $\frac{\sqrt{3}}{8}+\frac{\pi}{6}$ (D) $\frac{\sqrt{3}}{16}+\frac{\pi}{12}$

30. If $I_n = \int_0^{\infty} e^{-px} \sin^n x \, dx$ and $I_n = \frac{n(n-1)}{n^2+p^2} I_{n-2}$ then value of $\int_0^{\infty} e^{-2x} \sin^2 x \, dx$ is equal to (2)

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{8}$ (D) 2

31. If $I_n = \int_0^{\pi/4} \frac{\sin(2n-1)x}{\sin x} dx$ and $I_{n+1} = \frac{1}{n}\sin\frac{n\pi}{2} + I_n$ then I_3 is equal to (2)

(A) $2+\frac{\pi}{4}$ (B) $2-\frac{\pi}{2}$ (C) $1-\frac{\pi}{4}$ (D) $1+\frac{\pi}{4}$

32. If $I_n = \int (\log x)^n \, dx$ then (2)

(A) $I_n + nI_{n-1} = x(\log x)^n$
(B) $I_n - nI_{n-1} = x(\log x)^n$
(C) $I_n + I_{n-1} = x(\log x)^n$
(D) $I_n + nI_{n-1} = (\log x)$

ANSWERS

1. (A)	2. (B)	3. (D)	4. (C)	5. (B)	6. (D)	7. (A)	8. (D)
9. (C)	10. (A)	11. (B)	12. (D)	13. (A)	14. (C)	15. (D)	16. (B)
17. (A)	18. (C)	19. (B)	20. (D)	21. (C)	22.(A)	23. (D)	24. (B)
25. (C)	26. (D)	27. (A)	28. (B)	29. (C)	30. (C)	31. (D)	32.(A)

MULTIPLE CHOICE QUESTIONS

Gamma Functions :

1. Gamma function of n (n > 0), is defined as (1)

(A) $\int_0^\infty e^{-x} x^{n-1}\, dx$ (B) $\int_0^\infty e^{x} x^{n-1}\, dx$ (C) $\int_0^1 e^{-x} x^{n-1}\, dx$ (D) $\int_0^\infty e^{-x} x^{1-n}\, dx$

2. The value of equivalent form of Gamma function $\int_0^\infty e^{-kx} x^{n-1}\, dx$ is (1)

(A) $\frac{\overline{|n}}{n^k}$ (B) $\frac{\overline{|n}}{k!}$ (C) $\frac{\overline{|n}}{k^n}$ (D) $\overline{|n+k+1}$

3. Reduction formula for Gamma function is (1)

(A) $\Gamma(n+1) = (n-1)\,\Gamma(n-1)$ (B) $\Gamma(n+1) = n\,\Gamma(n)$

(C) $\Gamma(n+1) = (n-1)\,\Gamma(n)$ (D) $\Gamma(n+1) = n\,\Gamma(n-1)$

4. If n is a positive integer, then $\Gamma(n+1)$ is (1)

(A) $(n+1)!$ (B) $(n-1)!$ (C) $(n+2)!$ (D) $n!$

5. $\Gamma(1)$ is equal to (1)

(A) 2! (B) 1 (C) 0 (D) $\sqrt{\pi}$

6. $\Gamma\left(\frac{1}{2}\right)$ is equal to (1)

(A) $\sqrt{\pi}$ (B) π (C) $\frac{1}{2}$ (D) 1

7. $\Gamma(7)$ is equal to (1)

(A) 8! (B) 7! (C) 6! (D) 6

8. $\Gamma\left(\frac{5}{2}\right)$ is equal to (1)

(A) $\frac{5}{2}\sqrt{\pi}$ (B) $\frac{15}{8}\sqrt{\pi}$ (C) $\frac{4}{3}\sqrt{\pi}$ (D) $\frac{3}{4}\sqrt{\pi}$

9. By using $\Gamma(p)\,\Gamma(1-p) = \frac{\pi}{\sin p\pi}$, if $0 \le p \le 1$ the value of $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$ is (1)

(A) $\frac{\pi}{\sqrt{2}}$ (B) π (C) $\sqrt{2}\pi$ (D) 2π

10. $\int_0^\infty e^{-t}\, t^{3/2}\, dt$ is equal to (1)

(A) $\frac{3}{4}\sqrt{\pi}$ (B) $\frac{15}{4}\sqrt{\pi}$ (C) $\frac{3}{4}\pi$ (D) $\frac{3}{2}\sqrt{\pi}$

11. $\int_0^\infty e^{-5x}\, x^4\, dx$ is equal to (1)

(A) $\frac{4!}{4^5}$ (B) $\frac{5!}{4^4}$ (C) $\frac{5!}{5^5}$ (D) $\frac{4!}{5^5}$

12. The appropriate substitution to reduce the given integral $\int_0^\infty \sqrt{x}\, e^{-\sqrt{x}}\, dx$ to Gamma function integral (1)

(A) $x^3 = t$ (B) $\sqrt{x} = t$ (C) $-x^3 = t$ (D) $\log x = t$

13. The appropriate substitution to reduce the given integral $\int_0^1 (x \log x)^4\, dx$ to Gamma function integral (1)

(A) $\log x = -t$ (B) $x = -e^t$ (C) $x = t^2$ (D) $\log x = t$

14. The appropriate substitution to reduce the given integral $\int_0^{\infty} \frac{x^5}{5^x} dx$ to Gamma function integral (1)

(A) $\log x = -t$ (B) $5^x = e^{-t}$ (C) $5^x = e^t$ (D) $5^x = t$

15. The value of integral $\int_0^{\infty} \sqrt{x}\, e^{-\sqrt{x}} dx$ by using substitution $\sqrt{x} = t$ is (2)

(A) 1 (B) 4 (C) 2 (D) 3

16. The value of integral $\int_0^{\infty} e^{-x^2} dx$ by using substitution $x^2 = t$ is (2)

(A) $\sqrt{\pi}$ (B) $\frac{\sqrt{\pi}}{2}$ (C) $\frac{\sqrt{\pi}}{3}$ (D) $2\sqrt{\pi}$

17. The value of integral $\int_0^{\infty} e^{-x^4} dx$ by using substitution $x^4 = t$ is (2)

(A) $\left\lceil \frac{5}{4} \right.$ (B) $\left\lceil \frac{1}{4} \right.$ (C) $\frac{1}{4}\left\lceil \frac{1}{4} \right.$ (D) $\frac{1}{4}\left\lceil \frac{5}{4} \right.$

18. The value of integral $\int_0^{\infty} \sqrt{x}\, e^{-x^3} dx$ by using substitution $x^3 = t$ is (2)

(A) $\frac{\sqrt{\pi}}{6}$ (B) $\frac{\sqrt{\pi}}{2}$ (C) $3\sqrt{\pi}$ (D) $\frac{\sqrt{\pi}}{3}$

19. The value of integral $\int_0^{\infty} \sqrt[4]{x}\, e^{-\sqrt{x}} dx$ by using substitution $x^2 = t$ is (2)

(A) $\frac{3\sqrt{\pi}}{4}$ (B) $\frac{3\sqrt{\pi}}{2}$ (C) $\frac{15\sqrt{\pi}}{4}$ (D) $\frac{\sqrt{\pi}}{3}$

20. The value of integral $\int_0^{\infty} x^9 e^{-2x^2} dx$ by using substitution $2x^2 = t$ is (2)

(A) $\frac{\lceil 5}{64}$ (B) $\frac{\lceil 6}{64}$ (C) $\frac{\lceil 5}{32}$ (D) $\frac{\lceil 6}{32}$

21. The value of integral $\int_0^{1} \left[\log\left(\frac{1}{y}\right)\right]^{n-1} dy$ by using substitution $\log\left(\frac{1}{y}\right) = t$ is (2)

(A) $\lceil n$ (B) $\lceil n+1$ (C) $-\lceil n$ (D) $\lceil n-1$

22. The value of integral $\int_0^1 \frac{dx}{\sqrt{x \log\left(\frac{1}{x}\right)}}$ by using substitution $\log\left(\frac{1}{x}\right) = t$ is (2)

(A) $\frac{\sqrt{\pi}}{2}$ (B) $\sqrt{\pi}$ (C) $\sqrt{2\pi}$ (D) $2\sqrt{\pi}$

23. The value of integral $\int_0^1 \frac{dx}{\sqrt{-\log x}}$ by using substitution $-\log x = t$ is (2)

(A) $\frac{\sqrt{\pi}}{2}$ (B) $\sqrt{\pi}$ (C) $\sqrt{2\pi}$ (D) $2\sqrt{\pi}$

24. The value of integral $\int_0^\infty \frac{x^4}{4^x}\, dx$ by using substitution $4^x = e^t$ is (2)

(A) $\frac{4}{(\log 4)^4}$ (B) $\frac{24}{(\log 4)^3}$ (C) $\frac{24}{(\log 4)^5}$ (D) $\frac{12}{(\log 4)^4}$

25. The value of integral $\int_0^\infty \frac{x^2}{2^x}\, dx$ by using substitution $2^x = e^t$ is (2)

(A) $\frac{1}{(\log 2)^2}$ (B) $\frac{2}{(\log 2)^2}$ (C) $\frac{2}{(\log 2)^3}$ (D) $\frac{3}{(\log 2)^4}$

ANSWERS

1. (A)	2. (C)	3. (B)	4. (D)	5. (B)	6. (A)	7. (C)	8. (D)
9. (C)	10. (A)	11. (D)	12. (B)	13. (A)	14. (C)	15. (B)	16. (B)
17. (C)	18. (D)	19. (B)	20. (A)	21. (A)	22. (C)	23. (B)	24. (C)
25. (C)							

MULTIPLE CHOICE QUESTIONS

DUIS

1. If $I(\alpha) = \int_a^b f(x, \alpha)\, dx$, where α is parameter and a, b are constants then by DUIS rule, $\frac{dI(\alpha)}{d\alpha}$ is (1)

~~(A)~~ $\int_a^b \frac{\partial}{\partial \alpha} f(x, \alpha)\, dx$

(B) $\int_a^b \frac{\partial}{\partial x} f(x, \alpha)\, dx$

(C) $f(b, \alpha) - f(a, \alpha)$

(D) $f(x, \alpha)$

2. If $I(\alpha) = \int_a^b f(x, \alpha)\, dx$ where a, b are functions of parameter α then by DUIS rule, $\frac{dI(\alpha)}{d\alpha}$ is (1)

(A) $\int_a^b \frac{\partial}{\partial \alpha} f(x, \alpha)\, dx + f(a, \alpha)\frac{da}{d\alpha} - f(b, \alpha)\frac{db}{d\alpha}$

(B) $\int_a^b \frac{\partial}{\partial \alpha} f(x, \alpha)\, dx + f(b, \alpha)\frac{db}{d\alpha} + f(a, \alpha)\frac{da}{d\alpha}$

(C) $\int_a^b \frac{\partial}{\partial \alpha} f(x, \alpha)\, dx + f(b, \alpha)\frac{db}{d\alpha} - f(a, \alpha)\frac{da}{d\alpha}$

(D) $\int_a^b \frac{\partial}{\partial \alpha} f(x, \alpha)\, dx$

3. If $\phi(a) = \int_0^\infty \frac{e^{-x}}{x}(1 - e^{-ax})\, dx$, $a > -1$ then by DUIS rule, $\frac{d\phi}{da}$ is (1)

(A) $\int_0^\infty \frac{\partial}{\partial a} e^{-x}(1 - e^{-ax})\, dx$

(B) $\int_0^\infty \frac{\partial}{\partial a} \frac{e^{-x}}{x}(1 - e^{-ax})\, dx$

(C) $\int_0^\infty \frac{\partial}{\partial x} \frac{e^{-x}}{x}(1 - e^{-ax})\, dx$

(D) $\frac{e^{-x}}{x}(1 - e^{-ax})$

4. If $\phi(a) = \int_0^\infty e^{-bx^2}\cos(2ax)\, dx$, $b > 0$ then by DUIS rule, $\frac{d\phi}{da}$ is (1)

(A) $\int_0^\infty \frac{\partial}{\partial b}\left[e^{-bx^2}\cos(2ax)\right] dx$

(B) $e^{-bx^2}\cos(2ax)$

(C) $\int_0^\infty \frac{\partial}{\partial x}\left[e^{-bx^2}\cos(2ax)\right] dx$

(D) $\int_0^\infty \frac{\partial}{\partial a}\left[e^{-bx^2}\cos(2ax)\right] dx$

5. If $\phi(a) = \int_0^\infty \frac{e^{-x}}{x}\left(a - \frac{1}{x} + \frac{1}{x}e^{-ax}\right) dx$, then by DUIS rule, $\frac{d\phi}{da}$ is (1)

(A) $\frac{e^{-x}}{x}\left(a - \frac{1}{x} + \frac{1}{x}e^{-ax}\right)$

(B) $\int_0^\infty \frac{\partial}{\partial a}\frac{e^{-x}}{x}\left(a - \frac{1}{x} + \frac{1}{x}e^{-ax}\right) dx$

(C) $\int_0^\infty \frac{\partial}{\partial x}\frac{e^{-x}}{x}\left(a - \frac{1}{x} + \frac{1}{x}e^{-ax}\right) dx$

(D) $\int_0^\infty \frac{\partial}{\partial a} e^{-x}\left(a - \frac{1}{x} + \frac{1}{x}e^{-ax}\right) dx$

6. If $\phi(a) = \int_0^\infty \frac{e^{-x}}{x}(1 - e^{-ax})\, dx$, $a > -1$ then by DUIS rule, $\frac{d\phi}{da}$ is (2)

(A) $\frac{e^{-x}}{x}(1 - e^{-ax})$ (B) $\int_0^\infty \frac{a}{x} e^{-(a+1)x}\, dx$ (C) $\int_0^\infty e^{-ax}\, dx$ (D) $\int_0^\infty e^{-(a+1)x}\, dx$

7. If $\phi(a) = \int_0^1 \frac{x^a - 1}{\log x}\, dx$; $a \geq 0$ then by DUIS rule, $\frac{d\phi}{da}$ is (2)

(A) $\int_0^1 \frac{x^a \log a}{\log x}\, dx$ (B) $\int_0^1 \frac{ax^{a-1}}{\log x}\, dx$ (C) $\int_0^1 x^a\, dx$ (D) $\frac{x^a - 1}{\log x}$

8. If $\phi(\alpha) = \int_0^\infty \frac{e^{-x} \sin \alpha x}{x}\, dx$ then by DUIS rule, $\frac{d\phi}{d\alpha}$ is (2)

(A) $\int_0^\infty e^{-x} \sin \alpha x\, dx$ (B) $\int_0^\infty e^{-x} \cos \alpha x\, dx$

(C) $\int_0^\infty \frac{\alpha e^{-x} \sin \alpha x}{x}\, dx$ (D) $\frac{e^{-x} \sin \alpha x}{x}$

9. If $I(a) = \int_0^\infty e^{-\left(x^2 + \frac{a^2}{x^2}\right)}\, dx$; $a > 0$ then by DUIS rule, $\frac{dI}{da}$ is (2)

(A) $\int_0^\infty e^{-\left(x^2 + \frac{a^2}{x^2}\right)} \left(-\frac{2a}{x^2}\right) dx$ (B) $\int_0^\infty e^{-\left(x^2 + \frac{a^2}{x^2}\right)} \left(\frac{2a}{x^2}\right) dx$

(C) $\int_0^\infty e^{-\left(x^2 + \frac{a^2}{x^2}\right)} \left(-2x - \frac{2a^2}{x^3}\right) dx$ (D) $e^{-\left(x^2 + \frac{a^2}{x^2}\right)}$

10. If $I(a) = \int_0^\pi \log(1 - a \cos x)\, dx$; $|a| < 1$ then by DUIS rule, $\frac{dI}{da}$ is (2)

(A) $\int_0^\pi \frac{-a \sin x}{1 - a \cos x}\, dx$ (B) $\int_0^\pi \frac{\cos x}{1 - a \cos x}\, dx$

(C) $\int_0^\pi \frac{-\cos x}{1 - a \cos x}\, dx$ (D) $\int_0^\pi \frac{1}{1 - a \cos x}\, dx$

11. By DUIS rule $\frac{d}{da}\left[\int_0^\infty \frac{e^{-x}}{x}\left(a-\frac{1}{x}+\frac{1}{x}e^{-ax}\right)dx\right]$, a is parameter, is (2)

(A) $\int_0^\infty \frac{e^{-x}}{x}(1+e^{-ax})\,dx$ (B) $\int_0^\infty \frac{e^{-x}}{x}\left(1-\frac{ae^{-ax}}{x}\right)dx$

(C) $\int_0^\infty e^{-x}(1-e^{-ax})\,dx$ (D) $\int_0^\infty \frac{e^{-x}}{x}(1-e^{-ax})\,dx$

12. If $I(x)=\int_0^\infty e^{-a^2}\cos ax\,da$, x is parameter then by DUIS rule, $\frac{dI}{dx}$ is (2)

(A) $\int_0^\infty xe^{-a^2}\sin(xa)\,da$ (B) $\int_0^\infty ae^{-a^2}\sin(xa)\,da$

(C) $\int_0^\infty -ae^{-a^2}\sin(xa)\,da$ (D) $\int_0^\infty ae^{-a^2}\cos(xa)\,da$

13. If $I(a)=\int_0^\infty \frac{e^{-x}-e^{-ax}}{x\sec x}dx$, $a>0$ then by DUIS rule, $\frac{dI}{da}$ is (2)

(A) $\int_0^\infty \frac{e^{-x}+e^{-ax}x}{x\sec x}dx$ (B) $\int_0^\infty \frac{e^{-x}}{\sec x}dx$

(C) $\int_0^\infty (e^{-x}-e^{-ax})\,dx$ (D) $\int_0^\infty \frac{e^{-ax}}{\sec x}dx$

14. If $\phi(a)=\int_0^1 \frac{x^a-x^b}{\log x}dx$; $a>0, b>0$ then by DUIS rule, $\frac{d\phi}{da}$ is (2)

(A) $\int_0^1 \frac{x^a\log a}{\log x}dx$ (B) $\int_0^1 x^a\,dx$ (C) $\int_0^1 x^b\,dx$ (D) $\int_0^1 (x^a-x^b)\,dx$

15. If $\phi(b)=\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x}dx$; $a>0, b>0$ then by DUIS rule, $\frac{d\phi}{db}$ is (2)

(A) $\int_0^\infty e^{-bx}\,dx$ (B) $\int_0^\infty \frac{e^{-ax}(-a)-e^{-bx}(-b)}{x}dx$

(C) $\int_0^\infty e^{-ax}\,dx$ (D) $\int_0^\infty (e^{-ax}-e^{-bx})\,dx$

16. If $\phi(a) = \int_0^{\infty} \frac{1}{x^2} \log(1 + ax^2)\, dx;\ a > 0,$ then by DUIS rule, $\frac{d\phi}{da}$ is (2)

(A) $\int_0^{\infty} \frac{a}{x(1 + ax^2)} dx$

(B) $\int_0^{\infty} \frac{\log(1 + ax^2)}{x} dx$

(C) $\int_0^{\infty} \frac{2a}{x(1 + ax^2)} dx$

(D) $\int_0^{\infty} \frac{1}{1 + ax^2} dx$

17. If $\phi(a) = \int_0^{\pi/2} \frac{\log(1 + a\sin^2 x)}{\sin^2 x} dx;\ a > 0,$ then by DUIS rule, $\frac{d\phi}{da}$ is (2)

(A) $\int_0^{\pi/2} \frac{2\sin x \cos x}{(1 + a\sin^2 x)} dx$

(B) $\int_0^{\pi/2} \frac{1}{(1 + a\sin^2 x)\sin^2 x} dx$

(C) $\int_0^{\pi/2} \frac{1}{1 + a\sin^2 x} dx$

(D) $\int_0^{\pi/2} \frac{\sin^2 x}{(1 + a\sin^2 x)} dx$

18. If $\phi(a) = \int_a^{a^2} \log(ax)\, dx,$ then by DUIS rule II, $\frac{d\phi}{da}$ is (2)

(A) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx + 2a \log a^3$

(B) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx$

(C) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx + 2a \log a^3 - 2\log a$

(D) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx - 2a \log a^3 + 2\log a$

19. If $\phi(a) = \int_0^{a^2} \tan^{-1}\left(\frac{x}{a}\right) dx,$ then by DUIS rule II, $\frac{d\phi}{da}$ is (2)

(A) $\int_0^{a^2} \frac{\partial}{\partial a} \tan^{-1}\left(\frac{x}{a}\right) dx + 2a \tan^{-1} a$

(B) $\int_0^{a^2} \frac{\partial}{\partial a} \tan^{-1}\left(\frac{x}{a}\right) dx$

(C) $\int_0^{a^2} \frac{\partial}{\partial a} \tan^{-1}\left(\frac{x}{a}\right) dx + a^2 \tan^{-1} a$

(D) $\int_0^{a^2} \frac{\partial}{\partial a} \tan^{-1}\left(\frac{x}{a}\right) dx + a^2 \tan^{-1} a - \tan^{-1}\left(\frac{x}{a}\right)$

20. If $I(a) = \int_{a}^{a^2} \frac{1}{x+a} dx$, then by DUIS rule II, $\frac{dI}{da}$ is (2)

(A) $\int_{a}^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx - \frac{1}{a^2+a}(2a) + \frac{1}{2a}$

(B) $\int_{a}^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx + \frac{1}{a^2+a}(2a) - \frac{1}{2a}$

(C) $\int_{a}^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx - \frac{1}{a^2+a}$

(D) $\int_{a}^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx$

21. If $\phi(a) = \int_{0}^{a} \frac{\log(1+ax)}{1+x^2} dx$, then by DUIS rule II, $\frac{d\phi}{da}$ is (2)

(A) $\int_{0}^{a} \frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right] dx$

(B) $\int_{0}^{a} \frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right] dx - \frac{\log(1+a)}{1+a^2}$

(C) $\int_{0}^{a} \frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right] dx - \frac{\log(1+a^2)}{1+a^2}$

(D) $\int_{0}^{a} \frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right] dx + \frac{\log(1+a^2)}{1+a^2}$

22. If $F(t) = \int_{t}^{t^2} e^{tx^2} dx$, then by DUIS rule II, $\frac{dF}{dt}$ is (2)

(A) $\int_{t}^{t^2} \frac{\partial}{\partial t} e^{tx^2} dx + (2t)\, e^{t^4} - e^{t^2}$

(B) $\int_{t}^{t^2} \frac{\partial}{\partial t} e^{tx^2} dx + e^{t^5} - e^{t^3}$

(C) $\int_{t}^{t^2} \frac{\partial}{\partial t} e^{tx^2} dx + (2t)\, e^{t^5} - e^{t^3}$

(D) $\int_{t}^{t^2} \frac{\partial}{\partial t} e^{tx^2} dx$

23. If $f(x) = \int_{a}^{x} (x-t)^2 G(t)\, dt$, a is constant and x is parameter then by DUIS rule II, $\frac{df}{dx}$ is

(2)

(A) $\int_{a}^{x} \frac{\partial}{\partial x} (x-t)^2 G(t)\, dt + G(x)$

(B) $\int_{a}^{x} \frac{\partial}{\partial x} (x-t)^2 G(t)\, dt$

(C) $\int_{a}^{x} \frac{\partial}{\partial x} (x-t)^2 G(t)\, dt - (x-a)^2 G(a)$

(D) $(x-t)^2 G(t)$

24. If $y = \int_0^x f(t) \sin a(x-t)\, dt$, then by DUIS rule II, $\frac{dy}{dx}$ is (2)

(A) $\int_0^x af(t) \sin a(x-t)\, dt$

(B) $\int_0^x f(t) \cos a(x-t)\, dt$

(C) $\int_0^x af(t) \cos a(x-t)\, dt$

(D) $\int_0^x af(t) \cos a(x-t)\, dt + f(x)$

25. Using DUIS rule the value of integral $\phi(a) = \int_0^\infty \frac{e^{-x}}{x}(1-e^{-ax})\, dx, a > -1$, given $\frac{d\phi}{da} = \frac{1}{a+1}$ is (2)

(A) $\log(a+1)$ (B) $-\frac{1}{(a+1)^2}$ (C) $\log(a+1)+\pi$ (D) $-\frac{1}{(a+1)^2}+1$

26. Using DUIS rule the value of integral $\phi(a) = \int_0^1 \frac{x^a-1}{\log x}\, dx, a \geq 0$, given $\frac{d\phi}{da} = \frac{1}{a+1}$ is (2)

(A) $\log(a+1)$ (B) $-\frac{1}{(a+1)^2}$ (C) $\log(a+1)+\pi$ (D) $-\frac{1}{(a+1)^2}+1$

27. Using DUIS rule the value of integral $\phi(\alpha) = \int_0^\infty \frac{e^{-2x}\sin \alpha x}{x}\, dx$, with $\frac{d\phi}{d\alpha} = \frac{2}{\alpha^2+4}$ is (2)

(a) $2\log(\alpha^2+4)$ (B) $2\tan^{-1}\left(\frac{\alpha}{2}\right)$ (C) $\frac{1}{2}\tan^{-1}\left(\frac{\alpha}{2}\right)$ (D) $\tan^{-1}\left(\frac{\alpha}{2}\right)$

28. Using DUIS rule the value of integral $\phi(\alpha) = \int_0^\infty \frac{e^{-\alpha x}\sin x}{x}\, dx$, with $\frac{d\phi}{d\alpha} = -\frac{1}{\alpha^2+1}$ and assuming $\int_0^\infty \frac{\sin x}{x}\, dx = \frac{\pi}{2}$ is (2)

(A) $\tan^{-1}\alpha + \frac{\pi}{2}$ (B) $-\tan^{-1}\alpha + \frac{\pi}{2}$ (C) $-\tan^{-1}\alpha$ (D) $\log(\alpha^2+1)+\frac{\pi}{2}$

29. Using DUIS rule the value of integral $\phi(a) = \int_0^{\pi/2} \frac{\log(1+a.\sin^2 x)}{\sin^2 x}\, dx$, with $\frac{d\phi}{da} = \frac{\pi}{2}\frac{1}{\sqrt{a+1}}$ is (2)

(A) $\pi\sqrt{a+1}$

(B) $\pi\sqrt{a+1}+\pi$

(C) $\pi\sqrt{a+1}-\pi$

(D) $3\pi(a+1)^{3/2}-\pi$

30. Using DUIS rule the value of integral $\phi(a) = \int_0^\infty \frac{\log(1+ax^2)}{x^2}\,dx;\ a > 0$ with $\frac{d\phi}{da} = \frac{\pi}{2}\frac{1}{\sqrt{a}}$ is (2)

(A) $\pi\sqrt{a}$ (B) $\frac{\pi\sqrt{a}}{4}$ (C) $-\frac{\pi}{4a^{3/2}}$ (D) $\frac{\pi}{4\sqrt{a}}$

31. Using DUIS rule the value of integral $\phi(a) = \int_0^\infty \frac{e^{-x} - e^{-ax}}{x \sec x}\,dx$, with $\frac{d\phi}{da} = \frac{a}{a^2+1}$ is (2)

(A) $\tan^{-1} a + \frac{\pi}{4}$ (B) $\log\left(\frac{2}{a^2+1}\right)$ (C) $\frac{1}{2}\log(a^2+1)$ (D) $\frac{1}{2}\log\left(\frac{a^2+1}{2}\right)$

32. Using DUIS rule the value of integral $\phi(a) = \int_0^\infty \frac{1-\cos ax}{x^2}\,dx$, with $\frac{d\phi}{da} = \frac{\pi}{2}$ is (2)

(A) $\frac{\pi}{2}$ (B) $\frac{\pi a}{2}$ (C) πa (D) $\frac{\pi a}{2} + \frac{\pi}{2}$

ANSWERS

1. (A)	2. (C)	3. (B)	4. (D)	5. (B)	6. (D)	7. (C)	8. (B)
9. (A)	10. (C)	11. (D)	12. (C)	13. (D)	14. (B)	15. (A)	16. (D)
17. (C)	18. (C)	19. (A)	20. (B)	21. (D)	22. (C)	23. (B)	24. (C)
25. (A)	26. (A)	27. (D)	28. (B)	29. (C)	30. (A)	31. (D)	32. (B)

MULTIPLE CHOICE QUESTIONS

Error Functions :

1. Error function of x, erf(x) is defined as (1)

(A) $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\,du$ (B) $\frac{2}{\sqrt{\pi}}\int_0^\infty e^{-u^2}\,du$ (C) $\int_0^\infty e^{-x} x^{n-1}\,dx$ (D) $\frac{2}{\sqrt{\pi}}\int_0^\infty e^{-u^2}\,du$

2. Complimentary error function of x, erfc (c) is defined as (1)

(A) $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\,du$ (B) $\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2}\,du$ (C) $\int_0^\infty e^{-x} x^{n-1}\,dx$ (D) $\frac{2}{\sqrt{\pi}}\int_0^\infty e^{-u^2}\,du$

3. The value of erf (∞) is (1)

(A) 0 (B) ∞ (C) 1 (D) $\frac{2}{\sqrt{\pi}}$

4. The value of erf (0) is (1)

(A) –1 (B) ∞ (C) 1 (D) 0

5. The value of erfc(0) is (1)

(A) –1 (B) ∞ (C) 1 (D) 0

6. Which of the following is true ? (1)

(A) $\text{erf}(x) - \text{erfc}(x) = 1$ (B) $\text{erf}(x) + \text{erfc}(x) = 1$

(C) $\text{erf}(x) + \text{erfc}(x) = 2$ (D) $\text{erf}(-x) = \text{erf}(x)$

7. Error function is (1)

(A) a periodic function (B) an even function

(C) a harmonic function (D) an odd function

8. $\text{erf}(-x)$ is equal to (1)

(A) $-\text{erf}(x)$ (B) $\text{erf}(x)$ (C) $\text{erfc}(x)$ (D) $\text{erfc}(-x)$

9. The proper substitution to reduce the integral $\int_0^{\infty} e^{-(x+a)^2}\,dx$ to complimentary error function is (1)

(A) $(x+a)^2 = u$ (B) $-(x+a) = u$ (C) $x + a = u$ (D) $-(x+a)^2 = u$

10. $\text{erf}(x) + \text{erf}(-x) =$ (1)

(A) 2 (B) 1 (C) -1 (D) 0

11. $\text{erf}(-x) + \text{erfc}(-x) =$ (1)

(A) 2 (B) 1 (C) -1 (D) 0

12. $\text{erfc}(-x) - \text{erf}(x) =$ (1)

(A) 2 (B) -1 (C) 1 (D) 0

13. $\text{erfc}(-x) + \text{erfc}(x) =$ (1)

(A) 2 (B) -1 (C) 1 (D) 0

14. If $\text{erf}(ax) = \frac{2}{\sqrt{\pi}} \int_0^{ax} e^{-u^2}\,du$ then $\frac{d}{dx}\text{erf}(ax)$ is (2)

(A) $\frac{2x}{\sqrt{\pi}} e^{-a^2x^2}$ (B) $\frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$ (C) $a e^{-a^2x^2}$ (D) $\frac{2a}{\sqrt{\pi}} e^{a^2x^2}$

15. If $\text{erf}(\sqrt{t}) = \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{t}} e^{-u^2}\,du$ then $\frac{d}{dt}\text{erf}(\sqrt{t})$ is (2)

(A) $\frac{e^{-t}}{2\sqrt{t}}$ (B) $\frac{e^{-t^2}}{\sqrt{\pi t}}$ (C) $\frac{e^{-t}}{\sqrt{\pi}}$ (D) $\frac{e^{-t}}{\sqrt{\pi t}}$

16. If $\text{erfc}(ax) = \frac{2}{\sqrt{\pi}}\int_{ax}^{\infty} e^{-u^2}\,du$ then $\frac{d}{dx}\text{erfc}(ax)$ is (2)

(a) $-\frac{1}{\sqrt{\pi}}e^{-a^2x^2}$ (B) $-\frac{2}{\sqrt{\pi}}e^{-a^2x^2}$ (C) $-\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$ (D) $\frac{2a}{\sqrt{\pi}}e^{a^2x^2}$

17. If $\text{erfc}\left(\sqrt{t}\right) = \frac{2}{\sqrt{\pi}}\int_{\sqrt{t}}^{\infty} e^{-u^2}\,du$ then $\frac{d}{dt}\text{erfc}\left(\sqrt{t}\right)$ is (2)

(A) $\frac{e^{-t}}{2\sqrt{t}}$ (B) $-\frac{e^{-t}}{\sqrt{\pi t}}$ (C) $\frac{e^{-t}}{\sqrt{\pi t}}$ (D) $\frac{e^{-t^2}}{\sqrt{\pi t}}$

18. If $\frac{d}{dx}\text{erf}(ax) = \frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$ then $\frac{d}{dx}\text{erfc}(ax)$ is (2)

(A) $\frac{2a}{\sqrt{\pi}}e^{a^2x^2}$ (B) $1-\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$ (C) $-\frac{1}{\sqrt{\pi}}e^{-a^2x^2}$ (D) $-\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$

19. If $\frac{d}{dt}\text{erf}\left(\sqrt{t}\right) = \frac{e^{-t}}{\sqrt{\pi t}}$ then $\frac{d}{dt}\text{erfc}\left(\sqrt{t}\right)$ is (2)

(A) $-\frac{e^{-t}}{\sqrt{\pi t}}$ (B) $1-\frac{e^{-t}}{\sqrt{\pi t}}$ (C) $-\frac{e^{t}}{\sqrt{\pi t}}$ (D) $\frac{e^{-t}}{\sqrt{\pi t}}$

20. If $\frac{d}{dx}\text{erfc}(ax) = -\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$ then $\frac{d}{dx}\text{erf}(ax)$ is (2)

(A) $ae^{-a^2x^2}$ (B) $1-\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$ (C) $\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$ (D) $-\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$

21. If $\frac{d}{da}\text{erfc}(ax) = -\frac{2x}{\sqrt{\pi}}e^{-a^2x^2}$ then $\frac{d}{da}\text{erf}(ax)$ is (2)

(A) $xe^{-a^2x^2}$ (B) $-\frac{2x}{\sqrt{\pi}}e^{-a^2x^2}$ (C) $\frac{1}{\sqrt{\pi}}e^{-a^2x^2}$ (D) $\frac{2x}{\sqrt{\pi}}e^{-a^2x^2}$

22. If $\text{erfc}(a) = \frac{2}{\sqrt{\pi}}\int_{a}^{\infty} e^{-u^2}\,du$ then by using substitution $x + a = u$, the integral $\int_{0}^{\infty} e^{-(x+a)^2}\,dx$ in terms of erfc (a) is (2)

(A) $\frac{2}{\sqrt{\pi}}\text{erfc}(a)$ (B) erfc (a) (C) $\frac{\sqrt{\pi}}{2}\text{erfc}(a)$ (D) $\frac{\sqrt{\pi}}{2}\text{erf}(a)$

23. $\int_0^t \text{erf}(ax)\,dx + \int_0^t \text{erfc}(ax)\,dx =$ (2)

(A) t (B) x (C) 0 (D) $\frac{t^2}{2}$

24. The integral $\int_0^\infty e^{-t}\,\text{erf}\left(\sqrt{t}\right) dt$ using $\frac{d}{dt}\text{erf}\left(\sqrt{t}\right) = \frac{e^{-t}}{\sqrt{\pi t}}$ is (2)

(A) $\frac{1}{\sqrt{\pi}}\int_0^\infty e^{2t}\,t^{1/2}\,dt$

(B) $\frac{1}{\sqrt{\pi}}\int_0^\infty e^{-2t}\,t^{-1/2}\,dt$

(C) $\int_0^\infty e^{-2t}\,t^{-1/2}\,dt$

(D) $\frac{1}{\sqrt{\pi}}\int_0^\infty e^{-t}\,t^{1/2}\,dt$

25. Expansion of erf (x) in series is (2)

(A) $\frac{2}{\sqrt{\pi}}\left[x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \ldots\right]$

(B) $\frac{2}{\sqrt{\pi}}\left[x + \frac{x^3}{3} + \frac{x^5}{10} + \frac{x^7}{42} + \ldots\right]$

(C) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$

(D) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

ANSWERS

1. (A)	2. (B)	3. (C)	4. (D)	5. (C)	6. (B)	7. (D)	8. (A)
9. (C)	10. (D)	11. (B)	12. (C)	13. (A)	14. (B)	15. (D)	16. (C)
17. (B)	18. (D)	19. (A)	20. (C)	21. (D)	22. (C)	23. (A)	24. (B)
25. (A)							

Fig. 6.51

MULTIPLE CHOICE QUESTIONS

Curve Tracing :

1. If the powers of y in the cartesian equation are even everywhere then the curve is symmetrical about (1)

(A) x-axis

(B) y-axis

(C) both x and y axes

(D) line y = x

2. If the powers of x in the cartesian equation are even everywhere then the curve is symmetrical about (1)

   (A) x-axis (B) y-axis

   (C) both x and y axes (D) line y = x

3. If the powers of x and y both in the cartesian equation are even everywhere then the curve is symmetrical about (1)

   (A) x-axis only (B) y-axis only

   (C) both x and y axes (D) line y = x

4. On replacing x and y by –x and –y respectively if the cartesian equation remains unchanged then the curve is symmetrical about (1)

   (A) line y = x (B) y-axis

   (C) both x and y axes (D) opposite quadrants

5. If x and y are interchanged and cartesian equation remains unchanged then the curve is symmetrical about (1)

   (A) both x and y axes (B) line y = –x

   (C) line y = x (D) opposite quadrants

6. If x is changed to –y and y to –x and cartesian equation remains unchanged then the curve is symmetrical about (1)

   (A) both x and y axes (B) line y = –x line y = –x

   (C) line y = x (D) opposite quadrants

7. If the curve passes through origin then tangents at origin to the cartesian curve can be obtained by equating to zero (1)

   (A) lowest degree term in the equation

   (B) highest degree term in the equation

   (C) coefficient of lowest degree term in the equation

   (D) coefficient of highest degree term in the equation

8. A double point is called node if the tangents to the curve at the double point are (1)

   (A) real and equal (B) imaginary

   (C) always perpendicular (D) real and distinct

9. A double point is called cusp if the tangents to the curve at the double point are (1)

   (A) real and equal (B) imaginary

   (C) always perpendicular (D) real and distinct

10. In cartesian equation the points where $\frac{dy}{dx} = 0$, tangent to the curve at those points will be (1)

(A) parallel to y-axis (B) parallel to x-axis

(C) parallel to $y = x$ (D) parallel to $y = -x$

11. In cartesian equation the points where $\frac{dy}{dx} = \infty$, tangent to the curve at those points will be (1)

(A) parallel to $y = -x$ (B) parallel to x-axis

(C) parallel to $y = x$ (D) parallel to y-axis

12. The asymptotes to the cartesian curve parallel to x-axis if exists is obtained by equating to zero (1)

(A) coefficient of highest degree term in y (B) lowest degree term in the equation

(C) coefficient of highest degree term in x (D) highest degree term in the equation

13. The asymptotes to the cartesian curve parallel to y-axis if exists is obtained by equating to zero (1)

(A) coefficient of highest degree term in y (B) lowest degree term in the equation

(C) coefficient of highest degree term in x (D) highest degree term in the equation

14. If the polar equation to the curve remains unchanged by changing θ to $-\theta$ then the curve is symmetrical about (1)

(A) line $\theta = \frac{\pi}{4}$ (B) pole (C) line $\theta = \frac{\pi}{2}$ (D) initial line $\theta = 0$

15. If the polar equation to the curve remains unchanged by changing r to $-r$ then the curve is symmetrical about (1)

(A) line $\theta = \frac{\pi}{4}$ (B) pole (C) line $\theta = \frac{\pi}{2}$ (D) initial line $\theta = 0$

16. If the polar equation to the curve remains unchanged by changing θ to $\pi - \theta$ then the curve is symmetrical about (1)

(A) initial line $\theta = 0$

(B) pole

(C) line passing through pole and perpendicular to the initial line

(D) line $\theta = \frac{\pi}{4}$

17. Pole will lie on the curve if for some value of θ (1)

(A) r becomes zero (B) r becomes infinite

(C) $r > 0$ (D) $r < 0$

18. The tangents to the polar curve at pole if exist can be obtained by putting in the polar (1)

(A) $\theta = 0$ (B) $\theta = \pi$ (C) $r = 0$ (D) $r = a, a > 0$

19. For the rose curve $r = a \cos n\theta$ and $r = a \sin n\theta$ if n is odd then the curve consist of (1)

(A) 2n equal loops (B) (n + 1) equal loops

(C) (n – 1) equal loops (D) n equal loops

20. For the rose curve $r = a \cos n\theta$ and $r = a \sin n\theta$ if n is even then the curve consist of (1)

(A) (n + 1) equal loops (B) 2n equal loops

(C) (n – 1) equal loops (D) n equal loops

21. For the polar curve, angle ϕ between radius vector and tangent line is obtained by the formula (1)

(A) $\cot \phi = r \dfrac{d\theta}{dr}$ (B) $\tan \phi = r \dfrac{d\theta}{dr}$ (C) $\tan \phi = r \dfrac{dr}{d\theta}$ (D) $\sin \phi = r \dfrac{d\theta}{dr}$

22. The cartesian parametric curve $x = f(t)$, $y = g(t)$ is symmetrical about x-axis if (1)

(A) f(t) is even and g(t) is odd (B) f(t) is odd and g(t) is even

(C) f(t) is even and g(t) is even (D) f(t) is odd and g(t) is odd

23. The cartesian parametric curve $x = f(t)$, $y = g(t)$ is symmetrical about y-axis if (1)

(A) f(t) is even and g(t) is odd (B) f(t) is even and g(t) is even

(C) f(t) is odd and g(t) is even (D) f(t) is odd and g(t) is odd

24. The curve represented by the equation $x^{1/2} + y^{1/2} = a^{1/2}$ is symmetrical about (1)

(A) $y = -x$ (B) x-axis

(C) both x and y axes (D) $y = x$

25. The curve represented by the equation $x^2 y^2 = x^2 + 1$ is symmetrical about (1)

(A) $y = -x$ (B) x-axis only

(C) both x and y axes (D) $y = x$

26. The curve represented by the equation $r^2\theta = a^2$ is symmetrical about (1)

(A) pole (B) initial line $\theta = 0$

(C) line $\theta = \dfrac{\pi}{2}$ (D) line $\theta = \dfrac{\pi}{4}$

27. The curve represented by the equation $r = 2a \sin \theta$ is symmetrical about (1)

(A) pole (B) initial line $\theta = 0$

(C) line $\theta = \dfrac{\pi}{4}$ (D) line $\theta = \dfrac{\pi}{2}$

28. The curve represented by the equation $x = at^2$, $y = 2at$ is symmetrical about (1)

(A) y-axis (B) x-axis

(C) both x and y axes (D) opposite quadrants

29. The asymptote parallel to y-axis to the curve $xy^2 = a^2(a - x)$ is (1)

(A) $y = 0$ (B) $x = 0$ (C) $x = a$ (D) $x = -a$

30. The number of loops in the rose curve $r = a \cos 2\theta$ are (1)

(A) 4 (B) 2 (C) 3 (D) 8

31. The number of loops in the rose curve $r = a \sin 3\theta$ are (1)

(A) 6 (B) 4 (C) 3 (D) 9

32. The curve represented by the equation $y^2(2a - x) = x^3$ is (2)

(A) symmetrical about y-axis and passing through origin

(B) symmetrical about x-axis and not passing through origin

(C) symmetrical about y-axis and passing through (2a, 0)

(D) symmetrical about x-axis and passing through origin

33. The curve represented by the equation $x(x^2 + y^2) = a(x^2 - y^2)$ is (2)

(A) symmetrical about x-axis and passing through origin

(B) symmetrical about x-axis and not passing through origin

(C) symmetrical about y-axis and passing through (a, 0)

(D) symmetrical about y-axis and passing through origin

34. The curve represented by the equation $a^2x^2 = y^3(2a - y)$ is (2)

(A) symmetrical about x-axis and passing through (2a, 0)

(B) symmetrical about both x-axis and y-axis and passing through origin

(C) symmetrical about y-axis and passing through (0, 2a)

(D) symmetrical about both x-axis and y-axis and passing through (2a, 0)

35. The equation of tangents to the curve at origin, if exist, represented by the equation $y^2(2a - x) = x^3$ is (2)

(A) $y = 0, y = 0$

(B) $x = 0, x = 2a$

(C) $x = 0, x = 0$

(D) $y = x$

36. The equation of tangents to the curve at origin, if exist, represented by the equation $y(1+x^2)=x$ is (2)

(A) $y=x$ (B) $x=0$

(C) $x=1$ and $x=-1$ (D) $y=0$

37. The equation of tangents to the curve at origin, if exists, represented by the equation $3ay^2=x(x-a)^2$ is (2)

(A) $x=a$ (B) $x=0$ and $y=0$

(C) $x=0$ (D) $y=0$

38. The equation of asymptotes parallel to x-axis to the curve represented by the equation $y(1+x^2)=x$ is (2)

(A) $x=1, x=-1$ (B) $x=0$

(C) $y=x$ (D) $y=0$

39. The equation of asymptotes parallel to y-axis to the curve represented by the equation $y^2(4-x)=x(x-2)^2$ is (2)

(A) $x=2$ (B) $x=4$ (C) $y=0$ (D) $x=0$

40. The equation of asymptotes parallel to y-axis to the curve represented by the equation $x^2y^2=a^2(y^2-x^2)$ is (2)

(A) $x=a, x=-a$ (B) $y=a, y=-a$ (C) $y=x, y=-x$ (D) $x=0, y=0$

41. The region of absence for the curve represented by the equation $x^2=\frac{4a^2(2a-y)}{y}$ is (2)

(A) $y<0$ and $y>2a$ (B) $y>0$ and $y<2a$

(C) $y>0$ and $y>2a$ (D) $y<0$ and $y<2a$

42. The region of absence for the curve represented by the equation $y^2(2a-x)=x^3$ is (2)

(A) $x>0$ and $x<2a$ (B) $x<0$ and $x>2a$

(C) $x<0$ and $x<2a$ (D) $x>0$ and $x>2a$

43. The region of absence for the curve represented by the equation $xy^2=a^2(a-x)$ is (2)

(A) $x>0$ and $x<a$ (B) $x<0$ and $x<a$

(C) $x<0$ and $x>a$ (D) $x>0$ and $x>a$

44. The region of absence for the curve represented by the equation $y^2 = \frac{x^2(a-x)}{a+x}$ is (2)

(A) $x > a$ and $x > -a$
(B) $x < a$ and $x < -a$
(C) $x < a$ and $x > -a$
(D) $x > a$ and $x < -a$

45. The region of absence for the curve represented by the equation $x^2 = \frac{a^2 y^2}{a^2 - y^2}$ is (2)

(A) $y < a$ and $y > -a$
(B) $y > a$ and $y < -a$
(C) $y > a$ and $y > -a$
(D) $y < a$ and $y < -a$

46. The curve represented by the equation $r = a(1 + \cos\theta)$ is (2)

(A) symmetrical about initial line and passing through pole
(B) symmetrical about initial line and not passing through pole
(C) symmetrical about $\theta = \frac{\pi}{2}$ and passing through pole
(D) symmetrical about $\theta = \frac{\pi}{4}$ and passing through pole

47. The curve represented by the equation $r^2 = a^2 \cos 2\theta$ is (2)

(A) symmetrical about $\theta = \frac{\pi}{2}$ and not passing through pole
(B) symmetrical about $\theta = \frac{\pi}{4}$ and not passing through pole
(C) symmetrical about initial line and pole
(D) symmetrical about $\theta = \frac{\pi}{4}$ and passing through pole

48. The curve represented by the equation $r^2 = a^2 \sin 2\theta$ is (2)

(A) symmetrical about initial line and passing through pole
(B) symmetrical about initial line and not passing through pole
(C) symmetrical about $\theta = \frac{\pi}{2}$ and passing through pole
(D) symmetrical about $\theta = \frac{\pi}{4}$ and passing through pole

49. The curve represented by the equation $r = \frac{2a}{1 + \cos\theta}$ is (2)

(A) symmetrical about initial line and passing through pole
(B) symmetrical about initial line and not passing through pole
(C) symmetrical about $\theta = \frac{\pi}{2}$ and passing through pole
(D) symmetrical about $\theta = \frac{\pi}{4}$ and passing through pole

50. The tangents at pole to the polar curve $r = a \sin 3\theta$ are (2)

(A) $\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, \ldots$

(B) $\theta = \frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \ldots$

(C) $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi \ldots$

(D) $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots$

51. The tangents at pole to the polar curve $r = a \cos 2\theta$ are (2)

(A) $\theta = 0, \pi, 2\pi, 3\pi, \ldots$

(B) $\theta = \frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \ldots$

(C) $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots$

(D) $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots$

52. The curve represented by the equation $x = t^2, y = t - \frac{t^3}{3}$ is (2)

(A) symmetrical about y-axis and passing through origin

(B) symmetrical about x-axis and not passing through origin

(C) symmetrical about y-axis and passing through (3, 0)

(D) symmetrical about x-axis and passing through origin

53. The curve represented by the equation $x = a(t + \sin t), y = a(1 + \cos t)$ is (2)

(A) symmetrical about y-axis and not passing through origin

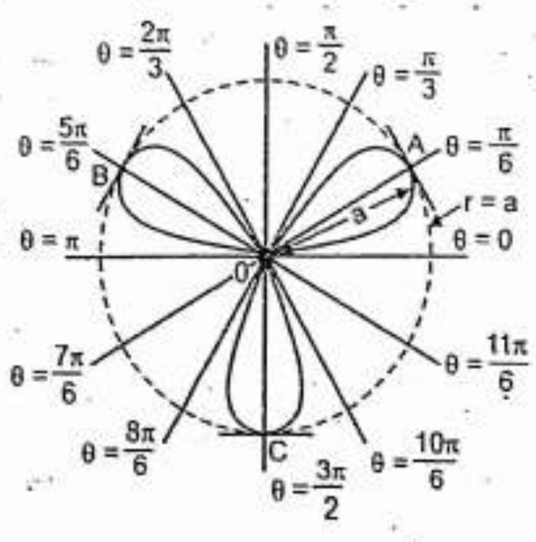
(B) symmetrical about x-axis and not passing through origin

(C) symmetrical about y-axis and passing through origin

(D) symmetrical about x-axis and passing through origin

54. The equation $r = a \cos 2\theta$ represents the curve (1)

(A)



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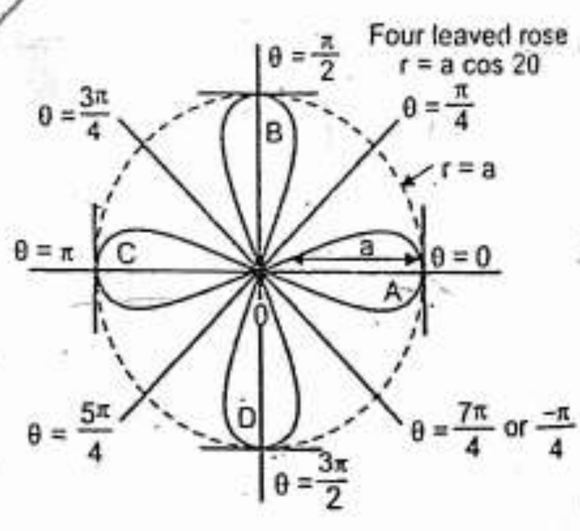

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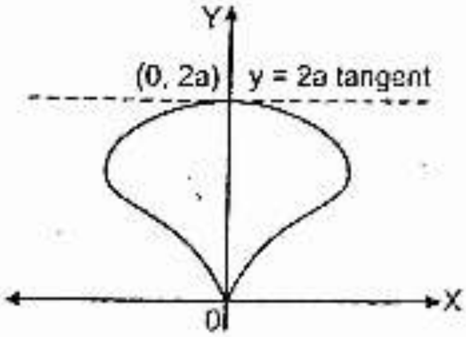

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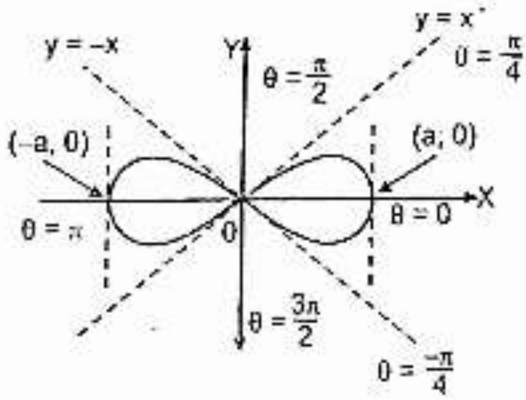
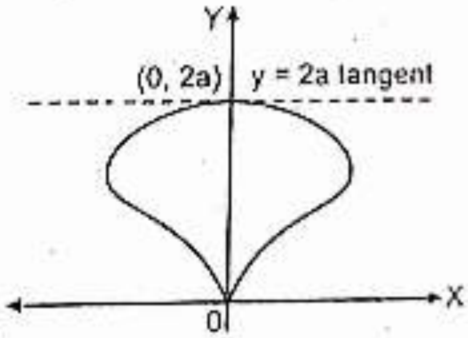


55. The equation r = a sin 3θ represents the curve (1)

(A)



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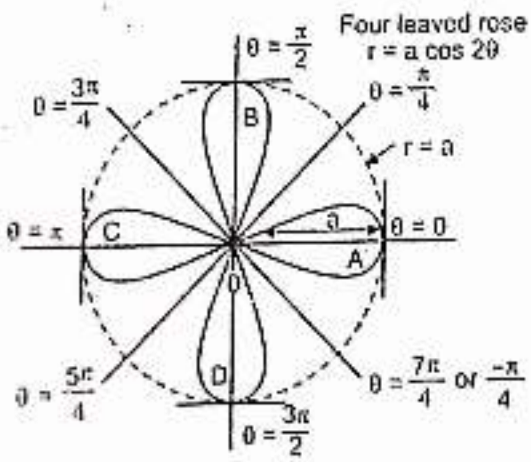

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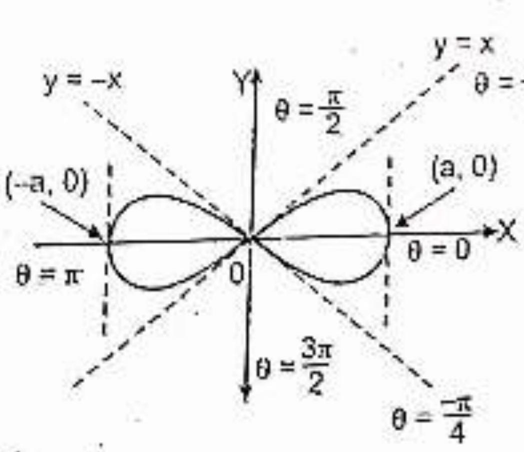
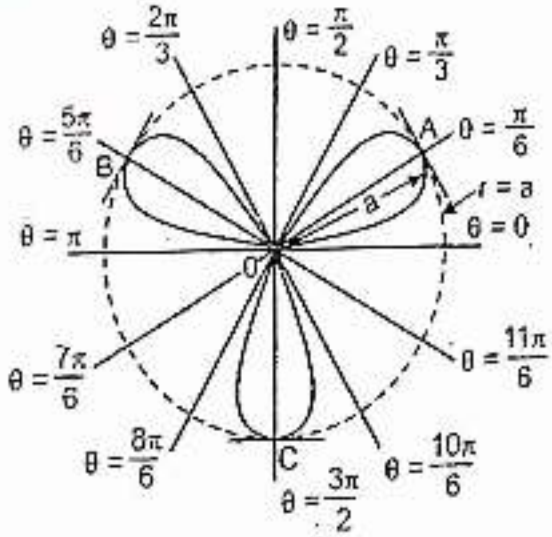
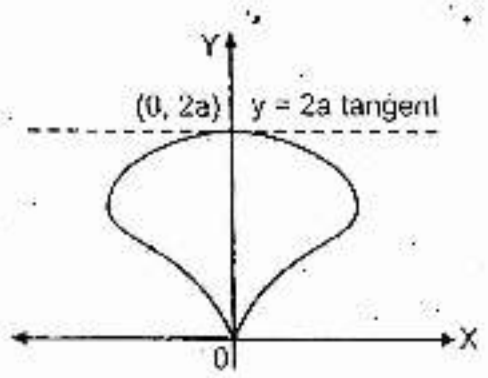


(D)

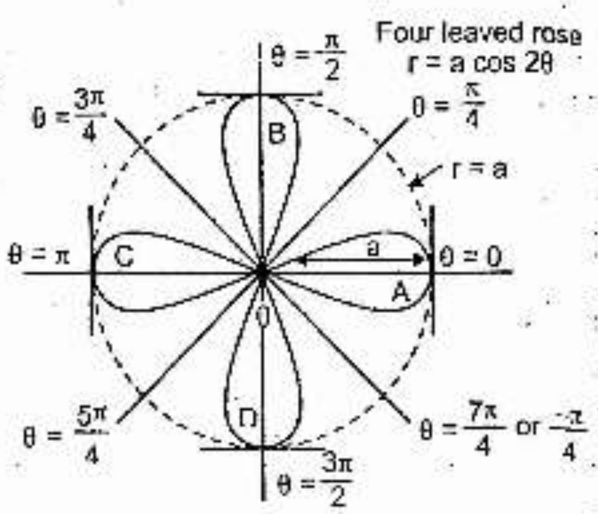


56. The equation $r^2 = a^2 \cos 2\theta$ represents the curve (1)

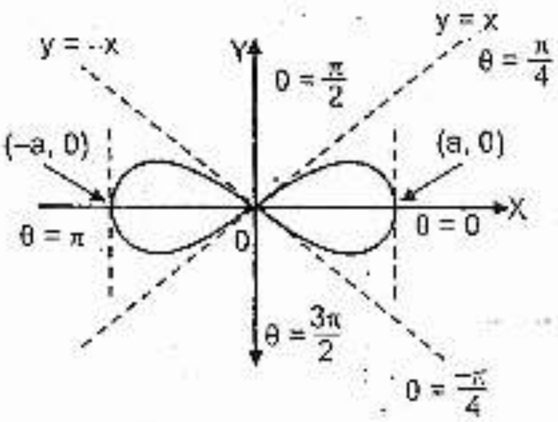
(A)



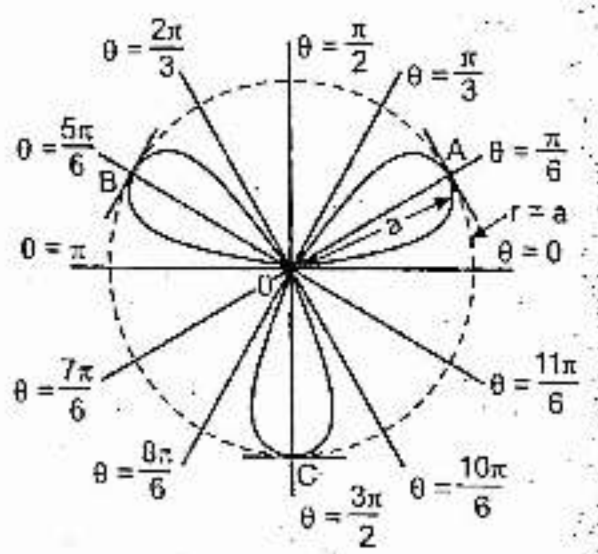
(B)



(C)

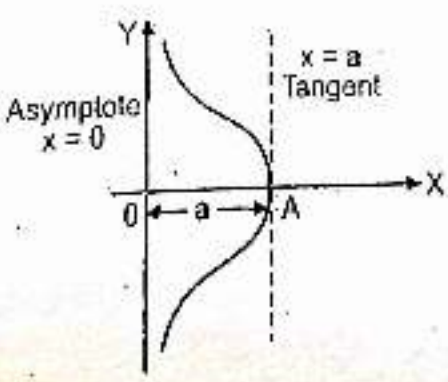


(D)

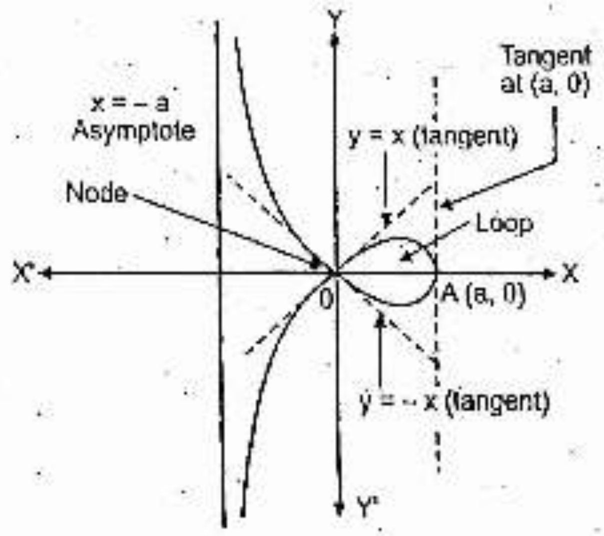


57. The equation $xy^2 = a^2 (a - x)$ represents the curve (2)

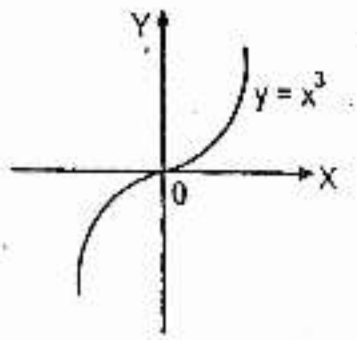
(A)



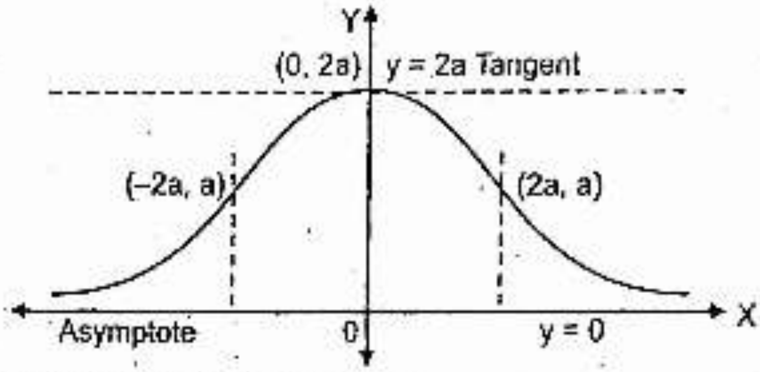
(B)



(C)

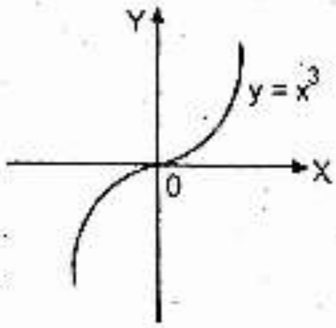


(D)

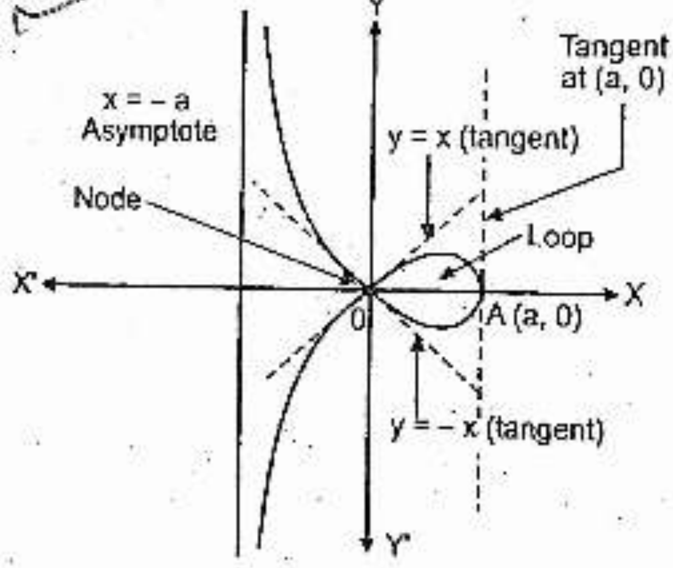


58. The equation $x(x^2 + y^2) = a(x^2 - y^2)$, $a > 0$ represents the curve (2)

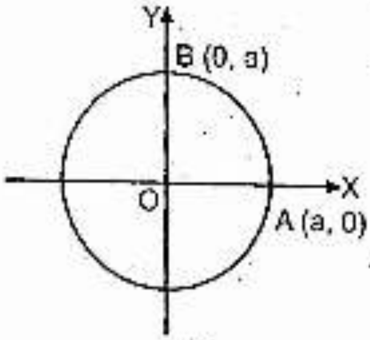
(A)



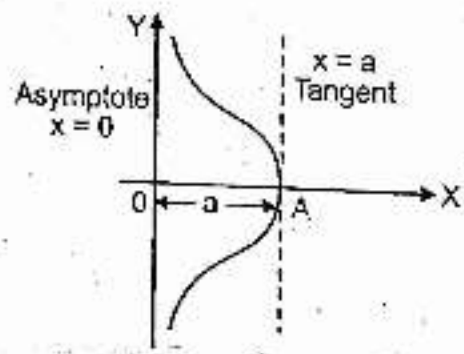
(B)



(C)

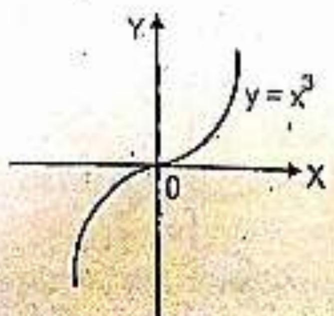


(D)

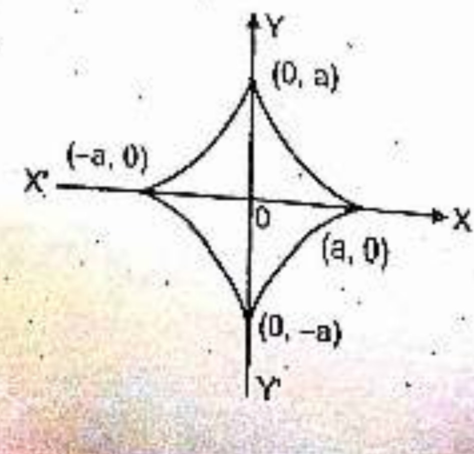


59. The equation $x^{2/3} + y^{2/3} = a^{2/3}$ represents the curve (2)

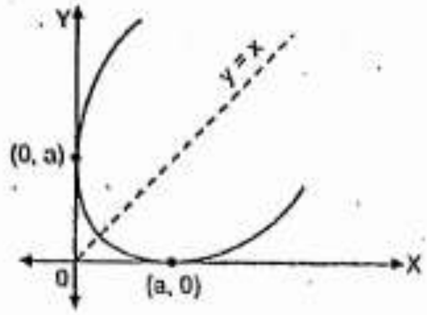
(A)



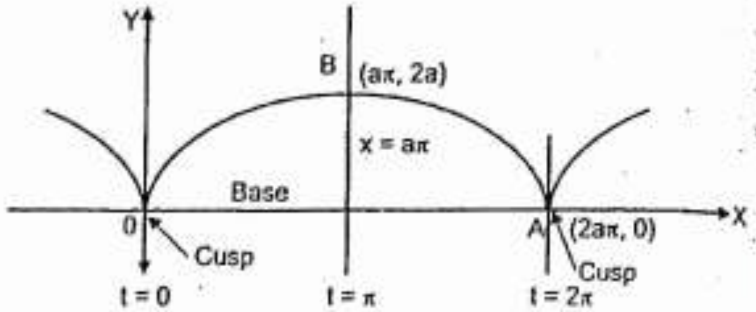
(B)



(C)

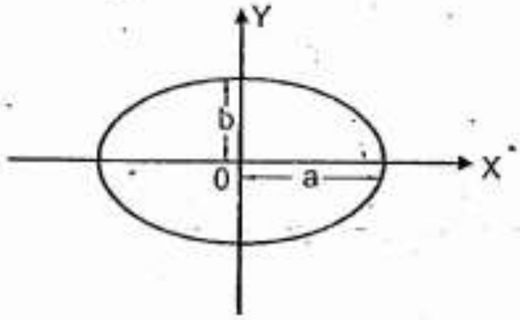


(D)

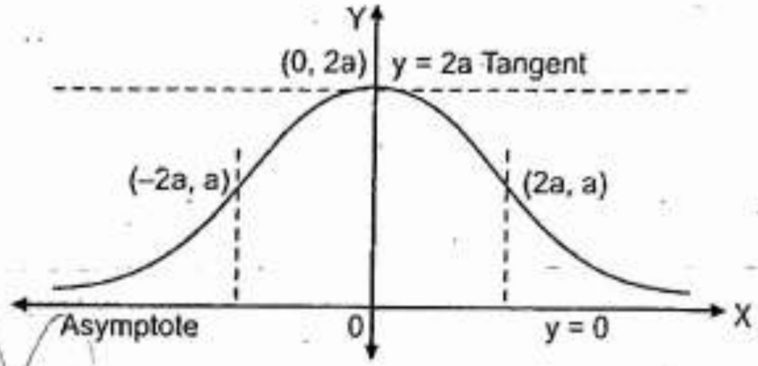


60. The equation $a^2x^2 = y^3 (2a - y)$, $a > 0$ represents the curve (2)

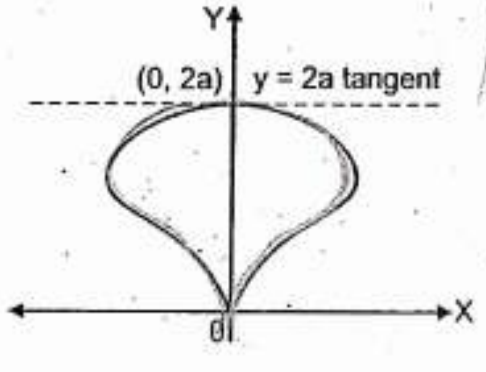
(A)



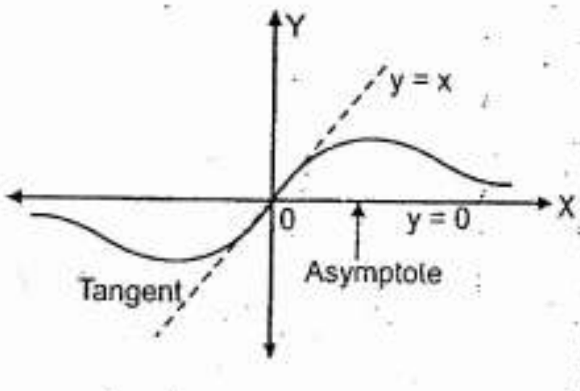
(B)



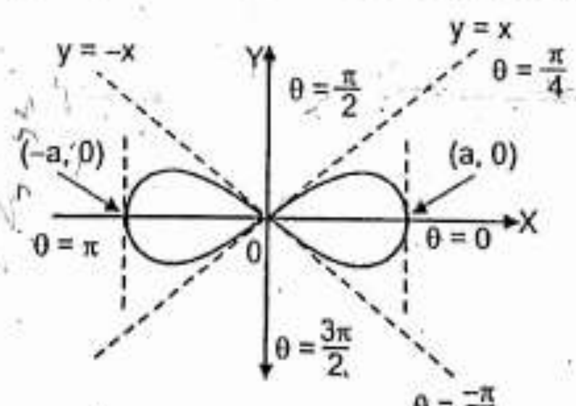
(C)



(D)

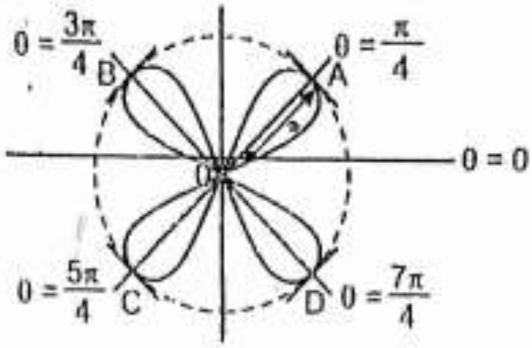


61. The following figure represents the curve whose equation is (2)



(A) $r^2 = a^2 \cos 2\theta$ (B) $r^2 = a^2 \sin 2\theta$ (C) $r = a \cos 2\theta$ (D) $r = a (1 + \cos \theta)$

62. The following figure represents the curve whose equation is (2)



(A) $r = a \cos 3\theta$ (B) $r = a \sin 2\theta$

(C) $r = a \sin 3\theta$ (D) $r = a (1 + \cos \theta)$

ANSWERS

1. (A)	2. (B)	3. (C)	4. (D)	5. (C)	6. (B)	7. (A)	8. (D)
9. (A)	10. (B)	11. (D)	12. (C)	13. (A)	14. (D)	15. (B)	16. (C)
17. (A)	18. (C)	19. (D)	20. (B)	21. (B)	22. (A)	23. (C)	24. (D)
25. (C)	26. (A)	27. (D)	28. (B)	29. (B)	30. (A)	31. (C)	32. (D)
33. (A)	34. (C)	35. (D)	36. (A)	37. (C)	38. (D)	39. (B)	40. (A)
41. (A)	42. (B)	43. (C)	44. (D)	45. (B)	46. (A)	47. (C)	48. (D)
49. (B)	50. (A)	51. (C)	52. (D)	53. (A)	54. (B)	55. (D)	56. (C)
57. (A)	58. (B)	59. (B)	60. (C)	61. (A)	62. (B)		

MULTIPLE CHOICE QUESTIONS

Rectification of Curves :

1. Formula for measuring the arc length AB where $A(x_1, y_1)$, $B(x_2, y_2)$ are any two points on the curve $y = f(x)$ is (1)

(A) $\int_{x_1}^{x_2} \sqrt{dx}$

(B) $\int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dx$

(C) $\int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

(D) $\int_{x_1}^{x_2} \sqrt{1 - \left(\frac{dy}{dx}\right)^2}\, dx$

2. Formula for measuring the arc length AB where $A(x_1, y_1)$, $B(x_2, y_2)$ are any two points on the curve $x = g(y)$ is (1)

(A) $\int_{y_1}^{y_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dy$

(B) $\int_{y_1}^{y_2} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

(C) $\int_{0}^{y_2} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

(D) $\int_{y_1}^{y_2} \sqrt{1 - \left(\frac{dx}{dy}\right)^2}\, dy$

3. Formula for measuring the arc length AB where $A(r_1, \theta_1)$, $B(r_2, \theta_2)$ are any two points on the curve $r = f(\theta)$ is (1)

(A) $\int_{\theta_1}^{\theta_2} \sqrt{1 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

(B) $\int_{\theta_1}^{\theta_2} \sqrt{1 + r^2\left(\frac{d\theta}{dr}\right)^2}\, dr$

(C) $\int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

(D) $\int_{\theta_1}^{\theta_2} \sqrt{r^2 + r^2\left(\frac{dr}{d\theta}\right)^2}\, dr$

4. Formula for measuring the arc length AB where $A(r_1, \theta_1)$, $B(r_2, \theta_2)$ are any two points on the curve $\theta = f(r)$ is (1)

(A) $\int_{r_1}^{r_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, dr$

(B) $\int_{r_1}^{r_2} \sqrt{1 + \left(\frac{d\theta}{dr}\right)^2}\, dr$

(C) $\int_{r_1}^{r_2} \sqrt{1 - r^2\left(\frac{d\theta}{dr}\right)^2}\, dr$

(D) $\int_{r_1}^{r_2} \sqrt{1 + r^2\left(\frac{d\theta}{dr}\right)^2}\, dr$

5. Formula for measuring the arc length AB where A, B are any two points on the parametric curve $x = f_1(t)$, $y = f_2(t)$, corresponding to parameters t_1, t_2 respectively is (1)

(A) $\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 - \left(\frac{dy}{dt}\right)^2}\, dt$ (B) $\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dx}\right)^2}\, dt$

(C) $\int_{t_1}^{t_2} \left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right] dt$ (D) $\int_{t_1}^{t_2} \sqrt{x^2(t) + y^2(t)}\, dt$

6. The arc length AB where A(a, 0), B(0, a) are any two points on the circle $x^2 + y^2 = a^2$, using $1 + \left(\frac{dy}{dx}\right)^2 = \frac{a^2}{a^2 - x^2}$, is (2)

(A) $\frac{\pi a}{2}$ (B) a log a (C) $\frac{\pi a}{4}$ (D) a

7. The length of arc from vertex (0, 0) to any point (x, y) of catenary $y = a \cosh \frac{x}{a}$, using $1 + \left(\frac{dy}{dx}\right)^2 = \cosh^2 \frac{x}{a}$, is (2)

(A) $a \cosh \frac{x}{a}$ (B) $\sinh \frac{x}{a}$ (C) $a \sinh \frac{x}{a}$ (D) $\cosh \frac{x}{a}$

8. The length of arc of upper part of loop of the curve $3y^2 = x(x-1)^2$ from (0, 0) to (1, 0) using $1 + \left(\frac{dy}{dx}\right)^2 = \frac{(3x+1)^2}{12x}$, is (2)

(A) $\frac{4}{\sqrt{3}}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\sqrt{3}$ (D) $\frac{2}{\sqrt{3}}$

9. Integral for calculating the length of upper arc of loop of the curve $9y^2 = (x+7)(x+4)^2$ is (2)

(A) $\int_{-7}^{-4} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ (B) $\int_{4}^{7} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

(C) $\int_{0}^{-4} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ (D) $\int_{-7}^{0} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

10. Integral for calculating the length of arc of parabola $y^2 = 4x$, cut off by the line $3y = 8x$ is (2)

(A) $\int_{0}^{\frac{16}{9}} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ (B) $\int_{0}^{\frac{9}{16}} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

(C) $\int_{0}^{\frac{8}{3}} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ (D) $\int_{0}^{\frac{3}{8}} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

11. The length of upper half of cardioide $r = a(1 + \cos\theta)$ where θ varies from 0 to π using $r^2 + \left(\frac{dr}{d\theta}\right)^2 = 2a^2(1 + \cos\theta)$, is (2

(A) 2a (B) 8a (C) 4a (D) a

12. The length of the arc of curve $r = ae^{m\theta}$ intercepted between radii r_1 and r_2 using $1 + r^2\left(\frac{d\theta}{dr}\right)^2 = 1 + \frac{1}{m^2}$, is (2

(A) $\frac{m}{\sqrt{1+m^2}}(r_2 - r_1)$ (B) $\frac{\sqrt{1+m^2}}{m} r_2$

(C) $\frac{\sqrt{1+m^2}}{m}(r_2 + r_1)$ (D) $\frac{\sqrt{1+m^2}}{m}(r_2 - r_1)$

13. Integral for calculating the length of cardioide $r = a(1 + \cos\theta)$ which lies outside the circle $r = -a\cos\theta$ is (2

(A) $2\int_0^{\pi/3}\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, dr$ (B) $2\int_0^{\pi}\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, dr$

(C) $2\int_0^{2\pi/3}\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$ (D) $2\int_0^{\pi/2}\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, dr$

14. Integral for calculating the length of upper arc of one loop of Bernoulli's lemniscate $r^2 = a^2\cos 2\theta$ in the first quadrant is (2)

(A) $\int_0^{\pi/4}\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$ (B) $\int_0^{\pi/6}\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, dr$

(C) $\int_0^{\pi/2}\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, dr$ (D) $\int_0^{\pi/3}\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, dr$

15. Integral for calculating the length of upper arc of loop of the curve $x = t^2$, $y = t\left(1 - \frac{t^2}{3}\right)$ is (2)

(A) $\int_0^{9}\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ (B) $\int_0^{3}\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

(C) $\int_0^{1}\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ (D) $\int_0^{\sqrt{3}}\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

16. Integral for calculating the length of the arc of Astroid $x = a\cos^3\theta$, $y = a\sin^3\theta$ in the first quadrant between two consecutive cusps, is (2)

(A) $\int_0^{\pi/4}\sqrt{\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2}\,d\theta$ (B) $\int_0^{\pi/2}\sqrt{\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2}\,d\theta$

(C) $\int_0^{\pi/3}\sqrt{\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2}\,d\theta$ (D) $\int_0^{\pi/6}\sqrt{\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2}\,d\theta$

17. The length of arc of upper part of loop of the curve $x = t^2$, $y = t\left(1-\frac{t^2}{3}\right)$ where t varies from 0 to $\sqrt{3}$, using $\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2 = (1+t^2)^2$, is (2)

(A) $2\sqrt{3}$ (B) $4\sqrt{3}$ (D) $\sqrt{3}$ (D) $\frac{2}{\sqrt{3}}$

18. The length of the arc of Astroid $x = a\cos^3\theta$, $y = a\sin^3\theta$ in the first quadrant between two consecutive cusps, where θ varies from 0 to $\frac{\pi}{2}$, using $\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2 = 9a^2\sin^2\theta\cos^2\theta$, is (2)

(A) $3a$ (B) $\frac{3a}{2}$ (C) $\frac{3a}{4}$ (D) $\frac{3a}{8}$

19. The length of arc of the curve $x = e^\theta\cos\theta$, $y = e^\theta\sin\theta$, from $\theta = 0$ to $\theta = \frac{\pi}{2}$, using $\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2 = 2e^{2\theta}$, is (2)

(A) $\sqrt{2}\,e^{\pi/2}$ (B) $\sqrt{2}\left(e^{\pi/2}+1\right)$ (C) $\sqrt{2}\left(e^{\pi/2}-1\right)$ (D) $\left(e^{\pi/2}+1\right)$

20. The length of arc of the cycloid $x = a(\theta+\sin\theta)$, $y = a(1-\cos\theta)$, from one cusp $\theta = -\pi$ to another cusp $\theta = \pi$, using $\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2 = 4a^2\cos^2\frac{\theta}{2}$, is (2)

(A) 2a (B) a (C) 4a (D) 8a

ANSWERS

1. (C)	2. (B)	3. (C)	4. (D)	5. (B)	6. (A)	7. (C)	8. (D)
9. (A)	10. (B)	11. (C)	12. (D)	13. (C)	14. (A)	15. (D)	16. (B)
17. (A)	18. (B)	19. (C)	20. (D)				

Sinhgad College of Engineering, Vadgoan-Ambegaon (Bk.), Pune – 411041.

First Year Degree Course in Engineering – Semester II

Engineering Mathematics (M II)

Savitribai Phule Pune University

Second Online Examination

First Year of Engineering

Dr. Chavan N. S.

2015-16

Savitribai Phule Pune University – FE – Sem. II

Engineering Mathematics (M II)

Chapter 03 – Fourier Series

1) A function $f(x)$ is said to be periodic function with a period T, if
   a) $f(x)=f(x+T), \quad \textit{for all } x$
   b) $f(T)=f(x+T), \quad \textit{for all } x$
   c) $f(x)=-f(x+T), \quad \textit{for all } x$
   d) $f(x)=f\left(\frac{x}{T}\right), \quad \textit{for all } x$

2) A smallest positive number T satisfying $f(x)=f(x+T)$ is known as
   a) absolute function b) absolute time
   c) periodic time d)primitive period

3) If T is the fundamental period a function $f(x)$, which of the following is incorrect?
   a) $f(x)=f(x+nT), \quad n\in I$
   b) $f(x)=f(x+n+T), \quad n\in I$
   c) $f(x)=f(x-T)$
   d) $f(x)=f(x+T)$

4) If $f(x+nT)=f(x)$ where n is an integer and T is the smallest positive number, the fundamental period of $f(x)$ is
   a) T b) nT c) $2T$ d) $\frac{T}{2}$

5) If $f(x)$ is a periodic function of period T, then for $n\neq 0$, the function $f(nx)$ is a periodic function of period
   a) T b) T^n c) $\frac{T}{n}$ d) nT

6) The fundamental period of $\sin x$ is
   a) π b) 2π c) 3π d) $\frac{\pi}{2}$

7) The fundamental period of $\sin 2x$ is
   a) π b) 2π c) 3π d) $\frac{\pi}{2}$

8) The fundamental period of $\sin 4x$ is
   a) π b) 2π c) 3π d) $\frac{\pi}{2}$

9) The fundamental period of $\cos 3x$ is
   a) π b) $\frac{2\pi}{3}$ c) $\frac{3\pi}{2}$ d) 3π

10) The fundamental period of $\sin(-3x)$ is
   a) -3π b) 3π c) $-\frac{2\pi}{3}$ d) $\frac{2\pi}{3}$

11) The fundamental period of $\sin\left(-\frac{x}{2}\right)$ is
   a) π b) 2π c) 3π d) 4π

12) The fundamental period of $\cos(x+\pi)$ is
   a) π b) 2π c) 3π d) $\frac{\pi}{2}$

13) The fundamental period of $\sin\left(x+\frac{3\pi}{2}\right)$ is
   a) 2π b) $\frac{2\pi}{3}$ c) 3π d) π

14) The fundamental period of $\tan\left(3x+\frac{\pi}{2}\right)$ is
   a) 2π b) π c) 3π d) $\frac{\pi}{3}$

15) The fundamental period of $\sin\left(x+\frac{\pi}{6}\right)$ is
   a) π b) 2π c) 3π d) $\frac{\pi}{3}$

16) The fundamental period of $2\sin x$ is
   a) π b) 2π c) 3π d) 4π

17) The fundamental period of $\sin x\cos x$ is
   a) π b) 2π c) 3π d) 4π

18) The fundamental period of $\tan x$ is
a) 4π b) 3π c) 2π d) π

19) The fundamental period of $\tan 5x$ is
a) $\frac{\pi}{5}$ b) 5π c) 10π d) π

20) The fundamental period of $2\sec(-3x)$ is
a) $-\frac{2\pi}{3}$ b) $\frac{2\pi}{3}$ c) $\frac{3\pi}{2}$ d) $-\frac{3\pi}{2}$

21) The fundamental period of $\operatorname{cosec} 2x$ is
a) π b) 2π c) 3π d) $\frac{\pi}{2}$

22) A function $f(x)$ defined in the interval $[-a,\ a]$ is said to be even function, if
a) $f(-x)=-f(x)$ b) $f(2x)=2f(x)$
c) $f(-x)=f(x)$ d) $f(x)=-f(x)$

23) A function $f(x)$ defined in the interval $[-a,\ a]$ is said to be odd function, if
a) $f(x)=-f(x)$ b) $f(2x)=2f(x)$
c) $f(-x)=f(x)$ d) $f(-x)=-f(x)$

24) Which of the followings is an even function?
a) $\cosh x$ b) $x^3-\cos x$
c) $\tan 3x$ d) $e^x+\tan^2 x$

25) Which of the followings is an even function?
a) $\sin 3x$ b) $\tan x$ c) $\operatorname{cosec}^3 x$ d) $\tan^2 x$

26) Which of the followings is not an even function?
a) $\sin^3 x$ b) $\sin^2 x$ c) $\tan^2 x$ d) $\sec x$

27) Which of the followings is an odd function?
a) e^{-x} b) $\tan\frac{3x}{2}$
c) $\cos^3 x$ d) $\operatorname{cosec} 2x$

28) Which of the followings is an odd function?
a) $-e^x$ b) $-\tan^2 x$
c) $-\sin x$ d) $-\cos x$

29) Which of the followings is not an odd function?
a) $2\tan x$ b) $\tan^2 x$
c) $\tan x$ d) $\sin 3x$

30) Which of the followings is neither even nor an odd function?
a) $\operatorname{cosech} x$ b) $\tanh x$ c) e^x d) $\sinh x$

31) If $f(x)$ is to be constant function w.r.t. x, then $f(x)$ is
a)even function
b) odd function
c) both even and odd
d) neither even nor odd

32) If $f(x)=x^3+2x-\cos x$, the function $f(x)$ is
a) even function
b) odd function
c) both even and odd
d)neither even nor odd

33) If $f(x)=x^2-\sin^4 x\cdot e^{|x|}$, the function $f(x)$ is
a) even function
b) odd function
c) both even and odd
d) neither even nor odd

34) Which of the following statement is incorrect?
a) Product of even and odd function is an odd function.
b) Multiplication of even and odd function is an odd function.
c) Addition of even and odd function is an odd function.
d) Subtraction of two odd functions is an odd function.

35) Fourier series expansion of a function $f(x)$ defined on the interval $[c,\ c+2L]$ and having period $2L$ is given by
a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos\left(\frac{n\pi x}{L}\right)+b_n\sin\left(\frac{n\pi x}{L}\right)$
b) $a_0+\sum_{n=1}^{\infty} a_n\cos\left(\frac{n\pi x}{L}\right)+b_n\sin\left(\frac{n\pi x}{L}\right)$
c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos(n\pi Lx)+b_n\sin(n\pi Lx)$
d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos\left(\frac{nx}{L}\right)+b_n\sin\left(\frac{nx}{L}\right)$

36) Fourier series expansion of a function $f(x)$ defined on the interval $[0,\ 2\pi]$ and satisfying the Dirichlet's conditions is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\frac{nx}{2}+b_n \sin\frac{nx}{2}$

b) $\frac{a_0}{2}+2\pi\sum_{n=1}^{\infty} a_n \cos nx+b_n \sin nx$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nx+b_n \sin nx$

d) $\frac{a_0}{2}+a_n \cos nx+b_n \sin nx$

37) If a function $f(x)$ is defined on the interval $[-\pi,\ \pi]$ and satisfying the Dirichlet's conditions, Fourier series expansion is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)+b_n \sin\left(\frac{n\pi x}{L}\right)$

b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{nx}{L}\right)+b_n \sin\left(\frac{nx}{L}\right)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{2n\pi x}{L}\right)+b_n \sin\left(\frac{2n\pi x}{L}\right)$

d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$

38) If a function $f(x)$ is defined on the interval $[0,\ 4]$ with period $T=4$, Fourier series expansion is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{4}\right)+b_n \sin\left(\frac{n\pi x}{4}\right)$

b) $\frac{a_0}{2}+a_n \cos\left(\frac{n\pi x}{2}\right)+b_n \sin\left(\frac{n\pi x}{2}\right)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{2}\right)+b_n \sin\left(\frac{n\pi x}{2}\right)$

d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{2n\pi x}{L}\right)+b_n \sin\left(\frac{2n\pi x}{L}\right)$

39) Fourier series expansion of a function $f(x)$ defined over a period 2π and satisfying the Dirichlet's conditions is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin nx+b_n \cos nx$

b) $\frac{a_0}{2}+2\pi\sum_{n=1}^{\infty} a_n \cos nx+b_n \sin nx$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nx+b_n \sin nx$

d) $\frac{a_0}{2}+a_n \cos nx+b_n \sin nx$

40) If an even function $f(x)$ with period $2l$ is defined over the interval $(-l,\ l)$, its Fourier series expansion is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{l}\right)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{2l}\right)$ d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{nx}{l}\right)$

41) If an odd function $f(x)$ is defined over the interval $(-\pi,\ \pi)$, its Fourier series expansion is given by

a) $\sum_{n=1}^{\infty} b_n \sin\left(\frac{2nx}{l}\right)$ b) $\sum_{n=1}^{\infty} a_n \sin(nx)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin(nx)$ d) $\sum_{n=1}^{\infty} b_n \sin(nx)$

42) If an odd function $f(x)$ is of period 2π, its Fourier series expansion is given by

a) $\sum_{n=1}^{\infty} b_n \sin\left(\frac{2nx}{l}\right)$ b) $\sum_{n=1}^{\infty} a_n \sin(nx)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin(nx)$ d) $\sum_{n=1}^{\infty} b_n \sin(nx)$

43) The Fourier series expansion of an even function $f(x)$ with period 2π is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin(nx)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{nx}{2}\right)$ d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{nx}{l}\right)$

44) If an odd function $f(x)$ with period $2l$ is defined over the interval $(-l,\ l)$, its Fourier series expansion is given by

a) $\sum_{n=1}^{\infty} b_n \sin\left(\frac{nx}{l}\right)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$

c) $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$ d) $\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{l}\right)$

45) If $f(x)$ is periodic function with period $2L$ in the interval C to $C+2L$, the Fourier coefficient a_0 is

a) $\frac{1}{L}\int_C^{C+2L} f(x)\cos\frac{nx}{L}dx$ b) $\frac{1}{L}\int_C^{C+2L} f(x)\sin\frac{nx}{L}dx$

c) $\int_C^{C+2L} f(x)dx$ d) $\frac{1}{L}\int_C^{C+2L} f(x)dx$

46) If $f(x)$ is periodic function with period $2L$ in the interval C to $C+2L$, the Fourier coefficient a_n is

a) $\frac{1}{L}\int_C^{C+2L} f(x)\cos\frac{2n\pi x}{L}dx$

b) $\frac{1}{L}\int_C^{C+2L} f(x)\cos\frac{n\pi x}{L}dx$

c) $\frac{1}{L}\int_C^{C+2L} f(x)\cos\frac{nx}{L}dx$

d) $\frac{1}{2L}\int_C^{C+2L} f(x)\cos\frac{nx}{L}dx$

47) If $f(x)$ is periodic function with period $2L$ in the interval C to $C+2L$, the Fourier coefficient b_n is

a) $\frac{1}{L}\int_C^{C+2L} f(x)\sin\frac{\pi x}{L}dx$

b) $\frac{1}{L}\int_C^{C+2L} f(x)\sin\frac{nx}{L}dx$

c) $\frac{1}{L}\int_C^{C+2L} f(x)\sin\frac{n\pi x}{L}dx$

d) $\frac{1}{L}\int_0^{2L} f(x)\sin\frac{n\pi x}{L}dx$

48) If $f(x)$ is an even function defined in the interval $[-L, L]$ and $f(x)=f(x+2L)$, the Fourier coefficient are

a) $a_0=\frac{2}{L}\int_0^L f(x)dx,\ a_n=\frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx,\ b_n=0$

b) $a_0=\frac{1}{L}\int_0^L f(x)dx,\ \ a_n=\frac{1}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx$

c) $a_0=0,\ \ a_n=\frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx$

d) $a_0=\frac{2}{L}\int_0^L f(x)dx,\ \ a_n=\frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$

49) If $f(x)$ is an odd function defined in the interval $[-L, L]$ and $f(x)=f(x+2L)$, the Fourier coefficient are

a) $a_0=0,\ \ a_n=0,\ \ b_n=\frac{1}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$

b) $a_0=0,\ \ a_n=0,\ \ b_n=\frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$

c) $a_0=0,\ \ a_n=0,\ \ b_n=\frac{2}{L}\int_0^{2L} f(x)\sin\frac{n\pi x}{L}dx$

d) $a_0=0,\ \ a_n=0,\ \ b_n=\frac{2}{L}\int_0^L f(x)\sin\frac{nx}{L}dx$

50) If $f(x)$ is an even periodic function defined in the interval $[-\pi, \pi]$, the Fourier coefficient are

a) $a_0=\frac{2}{\pi}\int_0^\pi f(x)dx,\ a_n=\frac{2}{\pi}\int_0^\pi f(x)\cos nx\,dx,\ b_n=0$

b) $a_0=\frac{2}{L}\int_0^\pi f(x)dx,\ a_n=\frac{2}{L}\int_0^\pi f(x)\cos\frac{nx}{L}dx,\ b_n=0$

c) $a_0=\frac{2}{\pi}\int_0^\pi f(x)dx,\ a_n=\frac{2}{\pi}\int_0^\pi f(x)\sin nxdx,\ b_n=0$

d) $a_0=\frac{1}{\pi}\int_0^\pi f(x)dx,\ a_n=\frac{1}{\pi}\int_0^\pi f(x)\cos nx\,dx,\ b_n=0$

51) If $f(x)$ is an odd periodic function defined in the interval $[-\pi, \pi]$, the Fourier coefficient are

a) $a_0=0,\ a_n=\frac{2}{\pi}\int_0^\pi f(x)\cos nx\,dx,\ b_n=0$

b) $a_0=\frac{2}{\pi}\int_0^\pi f(x)dx,\ a_n=\frac{2}{\pi}\int_0^\pi f(x)\cos nx\,dx,\ b_n=0$

c) $a_0=0,\ a_n=0,\ b_n=\frac{2}{\pi}\int_0^\pi f(x)\sin nx\,dx$

d) $a_0 = \frac{1}{\pi}\int_0^{\pi} f(x)\,dx,\ a_n = \frac{1}{\pi}\int_0^{\pi} f(x)\cos nx\,dx,\ b_n = 0$

52) The Fourier coefficient of an even periodic function $f(x)$ defined in the interval $[-2, 2]$ are

a) $a_0 = \int_0^2 f(x)\,dx,\ a_n = \int_0^2 f(x)\cos nx\,dx,\ b_n = 0$

b) $a_0 = \int_0^2 f(x)\,dx,\ a_n = \int_0^2 f(x)\cos\frac{n\pi x}{2}\,dx,\ b_n = 0$

c) $a_0 = \frac{2}{\pi}\int_0^2 f(x)\,dx,\ a_n = \frac{2}{\pi}\int_0^2 f(x)\cos nx\,dx,\ b_n = 0$

d) $a_0 = \int_0^2 f(x)\,dx,\ a_n = \int_0^2 f(x)\cos nx\,dx,\ b_n = 0$

53) The Fourier coefficient of an even periodic function $f(x)$ defined in the interval $[-1, 1]$ are

a) $a_0 = \frac{2}{\pi}\int_0^1 f(x)\,dx,\ a_n = \frac{2}{\pi}\int_0^1 f(x)\cos n\pi x\,dx,\ b_n = 0$

b) $a_0 = 2\int_0^2 f(x)\,dx,\ a_n = 2\int_0^2 f(x)\cos\frac{n\pi x}{2}\,dx,\ b_n = 0$

c) $a_0 = 2\int_0^1 f(x)\,dx,\ a_n = 2\int_0^1 f(x)\cos n\pi x\,dx,\ b_n = 0$

d) $a_0 = \int_0^1 f(x)\,dx,\ a_n = \int_0^1 f(x)\cos n\pi x\,dx,\ b_n = 0$

54) The Fourier coefficient of an odd periodic function $f(x)$ defined in the interval $[-4, 4]$ are

a) $a_0 = 0,\ \ a_n = 0,\ \ b_n = \frac{1}{2}\int_0^4 f(x)\cos\frac{n\pi x}{4}\,dx$

b) $a_0 = 0,\ \ a_n = 0,\ \ b_n = \frac{1}{4}\int_0^L f(x)\sin n\pi x\,dx$

c) $a_0 = 0,\ \ a_n = 0,\ \ b_n = 2\int_0^4 f(x)\sin\frac{n\pi x}{4}\,dx$

d) $a_0 = 0,\ \ a_n = 0,\ \ b_n = \frac{1}{2}\int_0^4 f(x)\sin\frac{n\pi x}{4}\,dx$

55) If the Fourier series expansion of an even function $f(x)$ over an interval $[-l, l]$ is given by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$, the value of a_0 is obtained by

a) $\frac{2}{l}\int_{-l}^{l} f(x)\,dx$ b) $\frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$

c) $\frac{1}{2l}\int_0^l f(x)\,dx$ d) $\frac{2}{l}\int_0^l f(x)\,dx$

56) If the Fourier series expansion of an even function $f(x)$ over an interval $[-l, l]$ is given by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$, the value of a_n is obtained by

a) $\frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}\,dx$ b) $\frac{1}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$

c) $\frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$ d) $\frac{1}{l}\int_0^l f(x)\cos\frac{nx}{l}\,dx$

57) If the Fourier series expansion of an even function $f(x)$ over an interval $[-\pi, \pi]$ is given by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$, the value of a_0 is obtained by

a) $\frac{2}{\pi}\int_0^{\pi} f(x)\,dx$ b) $\frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$

c) $\frac{1}{\pi}\int_0^{\pi} f(x)\,dx$ d) $\frac{2}{\pi}\int_0^{\pi} f(x)\cos nx dx$

58) If the Fourier series expansion of an even function $f(x)$ over an interval $[-\pi, \pi]$ is given by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$, the value of a_n is obtained by

a) $\frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$ b) $\frac{2}{\pi}\int_0^{\pi} f(x)\cos nx dx$

c) $\frac{1}{\pi}\int_0^{l} f(x)\cos\frac{n\pi x}{l}\,dx$ d) $\frac{2}{\pi}\int_0^{\pi} f(x)\cos\frac{nx}{\pi}\,dx$

59) If the Fourier series expansion of an odd function $f(x)$ over an interval $[-l, l]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$, the value of b_0 is obtained by

a) $\int_0^l f(x)\cos\frac{n\pi x}{l}dx$ b) $\frac{1}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}dx$

c) $\frac{2}{l}\int_0^l f(x)dx$ d) none of the above

60) If the Fourier series expansion of an odd function $f(x)$ over an interval $[-l, l]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$, the value of b_n is obtained by

a) $\frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}dx$ b) $\frac{1}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}dx$

c) $\frac{1}{l}\int_0^l f(x)\sin\frac{nx}{l}dx$ d) $\int_0^l f(x)\cos\frac{n\pi x}{l}dx$

62) The half range Fourier cosine series for $f(x)$ defined over the interval $[0, L]$ is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{nx}{L}\right)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right)$

c) $a_0+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$ d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$

63) The half range Fourier sine series for $f(x)$ defined over the interval $[0, L]$ is given by

a) $\sum_{n=1}^{\infty} a_n \sin\left(\frac{nx}{L}\right)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right)$

c) $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$ d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$

64) The half range Fourier cosine series for $f(x)$ defined over the interval $[0, \pi]$ is given by

a) $\sum_{n=1}^{\infty} b_n \sin(nx)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$ d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)$

65) The half range Fourier sine series for $f(x)$ defined over the interval $[0, \pi]$ is given by

a) $\sum_{n=1}^{\infty} b_n \sin\left(\frac{nx}{\pi}\right)$ b) $\sum_{n=1}^{\infty} b_n \sin(nx)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right)$ d) $\sum_{n=1}^{\infty} a_n \sin(n\pi x)$

66) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, L]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$ the value of a_0 is given by

a) $\frac{1}{L}\int_0^L f(x)dx$ b) $\frac{1}{L}\int_0^L f(x)dx$

c) $\frac{2}{\pi}\int_0^{\pi} f(x)dx$ d) $\frac{2}{L}\int_0^L f(x)dx$

67) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, L]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$ the value of a_n is given by

a) $\frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$

b) $\frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

c) $\frac{2}{L}\int_0^L f(x)\cos(nx)dx$

d) $\frac{1}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

68) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, L]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$ the value of b_0 is given by

a) $\frac{1}{L}\int_0^L f(x)\sin\frac{x}{L}dx$ b) $\frac{2}{L}\int_0^L f(x)\sin x\,dx$

c) 0 d) $\frac{2}{L}\int_0^L f(x)dx$

69) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, L]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$ the value of b_n is given by

a) $\frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$

b) $\frac{1}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$

c) $\frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

d) $\frac{2}{L}\int_0^L f(x)\sin(nx)dx$

70) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, \pi]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(nx)$ the value of a_0 is given by

a) $\frac{1}{L}\int_0^L f(x)dx$ b) $\frac{1}{L}\int_0^L f(x)dx$

c) $\frac{2}{\pi}\int_0^\pi f(x)dx$ d) $\frac{2}{L}\int_0^L f(x)dx$

71) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, \pi]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(nx)$ the value of a_n is given by

a) $\frac{2}{L}\int_0^L f(x)\sin(nx)dx$

b) $\frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

c) $\frac{2}{\pi}\int_0^\pi f(x)\cos(nx)dx$

d) $\frac{1}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

72) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, \pi]$ is given by $\sum_{n=1}^{\infty}b_n\sin(nx)$ the value of b_0 is given by

a) $\frac{1}{L}\int_0^L f(x)\sin\frac{x}{L}dx$ b) $\frac{2}{L}\int_0^L f(x)\sin x\,dx$

c) 0 d) $\frac{2}{L}\int_0^L f(x)dx$

73) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, \pi]$ is given by $\sum_{n=1}^{\infty}b_n\sin(nx)$ the value of b_n is given by

a) $\frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$

b) $\frac{1}{L}\int_0^L f(x)\sin(nx)dx$

c) $\frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

d) $\frac{2}{\pi}\int_0^\pi f(x)\sin(nx)dx$

74) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, 1]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(n\pi x)$ the value of a_0 is given by

a) $\frac{1}{\pi}\int_0^\pi f(x)dx$ b) $2\int_0^1 f(x)dx$

c) $\frac{2}{\pi}\int_0^\pi f(x)dx$ d) $\int_0^1 f(x)dx$

75) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, 2]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{n\pi x}{2}\right)$ the value of a_n is given by

a) $\frac{2}{3}\int_0^2 f(x)\sin\left(\frac{n\pi x}{2}\right)dx$

b) $\frac{1}{2}\int_0^2 f(x)\cos\left(\frac{n\pi x}{2}\right)dx$

c) $\frac{2}{L}\int_0^2 f(x)\cos(nx)dx$

d) $\int_0^2 f(x)\cos\left(\frac{n\pi x}{2}\right)dx$

76) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, 3]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{3}\right)$ the value of b_0 is given by

a) $\frac{1}{3}\int_0^3 f(x)\sin\frac{x}{3}dx$ b) $\frac{2}{3}\int_0^3 f(x)\sin 3x\,dx$

c) 0 d) $\frac{2}{3}\int_0^3 f(x)dx$

77) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, 4]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{4}\right)$ the value of b_n is given by

a) $\frac{2}{3}\int_0^2 f(x)\sin\left(\frac{n\pi x}{4}\right)dx$

b) $\frac{1}{2}\int_0^4 f(x)\sin\left(\frac{n\pi x}{4}\right)dx$

c) $\int_0^2 f(x)\cos\left(\frac{n\pi x}{4}\right)dx$

d) $\frac{1}{2}\int_0^4 f(x)\sin(nx)dx$

78) In the harmonic analysis for a function defined over a period of 2π, the term $a_1\cos x + b_1\sin x$ is known as

a) amplitude of $f(x)$ b) second harmonic

c) first harmonic d) none of these

79) In the harmonic analysis for a function defined over a period of 2π, the amplitude of the first harmonic is

a) $\sqrt{a_1^2 - b_1^2}$ b) $\sqrt{a_1^2 + b_1^2}$

c) $\sqrt{a_0^2 + a_1^2}$ d) $a_1^2 + b_1^2$

80) In the harmonic analysis for a function defined over a period of 2π, the amplitude of the second harmonic is

a) $\left(a_2^2 + b_2^2\right)^2$ b) $\frac{1}{2}\left(a_2^2 + b_2^2\right)$

c) $2\sqrt{a_2^2 + b_2^2}$ d) $\sqrt{a_2^2 + b_2^2}$

81) In the harmonic analysis for a function defined over a period of 2π, the amplitude of the second harmonic is

a) $\left(a_n^2 + b_n^2\right)^n$ b) $\sqrt{a_n^2 + b_n^2}$

c) $n\sqrt{a_n^2 + b_n^2}$ d) $\frac{1}{n}\sqrt{a_n^2 + b_n^2}$

82) In the harmonic analysis for a function $f(x)$ defined over a period of $2L$, the first harmonic term is given by

a) $b_1\sin\frac{\pi x}{L}$ b) $a_1\cos\frac{\pi x}{L}$

c) $a_1\cos\frac{\pi x}{L} - b_1\sin\frac{\pi x}{L}$ d) $a_1\cos\frac{\pi x}{L} + b_1\sin\frac{\pi x}{L}$

83) In the harmonic analysis for a function $f(x)$ defined over a period of 2, the first harmonic term is given by

a) $a_1\cos\pi x + b_1\sin\pi x$ b) $a_1\cos\frac{\pi x}{2} + b_1\sin\frac{\pi x}{2}$

c) $a_1\cos 2\pi x + b_1\sin 2\pi x$ d) $a_1\cos\frac{\pi x}{2} - b_1\sin\frac{\pi x}{2}$

84) If $f(x) = x\cos x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(nx) + b_n\sin(nx)$, the Fourier coefficient a_0 is given by

a) $-\frac{1}{\pi}$ b) 0 c) $\frac{1}{\pi}$ d) $\frac{2}{\pi}$

85) If $f(x) = x\cos x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(nx) + b_n\sin(nx)$, the Fourier coefficient a_n is given by

a) $\frac{1}{\pi}$ b) $-\frac{1}{\pi}$ c) $\frac{2}{\pi}$ d) 0

86) If $f(x) = x\cos x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(nx) + b_n\sin(nx)$, the Fourier coefficient b_1 is given by

a) $\frac{1}{\pi}$ b) $-\frac{1}{\pi}$ c) $-\frac{1}{2}$ d) 0

87) If $f(x)=\cos x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π, the Fourier coefficient a_1 is given by

a) 1 b) $-\frac{1}{\pi}$ c) $\frac{2}{\pi}$ d) 0

88) If $f(x)=x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient a_0 is given by

a) $\frac{\pi}{2}$ b) 0 c) 1 d) $\frac{\pi^2}{2}$

89) If $f(x)=x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient b_1 is given by

a) 2 b) 0 c) π d) $\frac{\pi}{2}$

90) If $f(x)=2$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient a_0 is given by

a) 2 b) 4 c) 3 d) none of these

91) If $f(x)=2$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient b_n is given by

a) $\frac{3\pi}{2}$ b) $-\frac{\pi}{2}$ c) 0 d) $\frac{\pi}{2}$

92) If $f(x)=a$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient a_0 is given by

a) $2a$ b) 0 c) 2π d) $\frac{\pi}{2}$

93) If $f(x)=a$ where $-\pi \le x \le \pi$ is of periodic function with period 2π, the Fourier coefficient a_0 is given by

a) 2π b) $2a$ c) 0 d) $\frac{\pi}{2}$

94) If $f(x)=\sin^2 x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient b_n is given by

a) $\frac{3\pi}{2}$ b) $-\frac{\pi}{2}$ c) $\frac{\pi}{2}$ d) 0

95) If $f(x)=\cosh x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient b_n is given by

a) 0 b) $\frac{\pi}{3}$ c) $e^{-\pi}$ d) $e^{-2\pi}$

96) If $f(x)=\begin{cases} -x & -\pi \le x \le 0 \\ x & 0 \le x \le \pi \end{cases}$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient a_0 is given by

a) $\frac{\pi}{2}$ b) π c) 0 d) $-\frac{\pi}{2}$

97) If $f(x)=\begin{cases} -x & -\pi \le x \le 0 \\ x & 0 \le x \le \pi \end{cases}$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient b_n is given by

a) $\frac{\pi}{2}$ b) π c) 0 d) $-\frac{\pi}{2}$

98) If $f(x)=\begin{cases} 1+\frac{2x}{\pi} & -\pi \le x \le 0 \\ 1-\frac{2x}{\pi} & 0 \le x \le \pi \end{cases}$ is of periodic function with period 2π, the Fourier coefficient b_n is given by

a) $\frac{\pi}{2}$ b) π c) $-\frac{\pi}{2}$ d) 0

99) If $f(x)=x-x^3$ where $-2\le x\le 2$ is of periodic function with period 2 and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(nx)+b_n\sin(nx)$, the Fourier coefficient b_1 is given by

a) 2 b) 0 c) π d) $\frac{\pi}{2}$

100) If $f(x)=x+\frac{x^2}{4}$ where $-\pi\le x\le\pi$ is of periodic function with period 2π, the Fourier coefficient a_0 is given by

a) 0 b) $\frac{\pi}{2}$ c) $-\frac{\pi}{2}$ d) $\frac{\pi^2}{6}$

101) If $f(x)=e^x$ where $-\pi\le x\le\pi$ is of periodic function with period 2π , the Fourier coefficient a_0 is given by

a) 1 b) $\frac{e^{\pi}-e^{-\pi}}{\pi}$ c) $\frac{e^{\pi}+e^{-\pi}}{\pi}$ d) 0

102) If $f(x)=x-x^2$ where $-1\le x\le 1$ is of periodic function with period 2, the Fourier coefficient a_0 is given by

a) 1 b) $-\frac{2}{3}$ c) π d) 0

103) If $f(x)=1-x^2$ where $-1\le x\le 1$ is of periodic function with period 2, the Fourier coefficient a_0 is given by

a) 1 b) $\frac{2\pi}{3}$ c) $\frac{4}{3}$ d) 0

104) If $f(x)=k$ where $-l\le x\le l$ is of periodic function with period 2, the Fourier coefficient a_0 is given by

a) $2k$ b) $\frac{2k\pi}{3}$ c) $2k\pi$ d) 0

105) If $f(x)=\begin{cases}-a & -2\le x\le 0\\ a & 0\le x\le 2\end{cases}$ is of periodic function with period 4, the Fourier coefficient b_n is given by

a) 0 b) 4 c) $-\frac{\pi}{2}$ d) $-\frac{2a}{n\pi}\left[(-1)^n-1\right]$

106) If $f(x)=\begin{cases}0 & -2\le x\le 0\\ 2 & 0\le x\le 2\end{cases}$ is of periodic function with period 4, the Fourier coefficient a_0 is given by

a) 0 b) 4 c) $-\frac{\pi}{2}$ d) 1

107) If $f(x)=\begin{cases}1 & -1\le x\le 0\\ \cos\pi x & 0\le x\le 1\end{cases}$ is of periodic function with period 2, the Fourier coefficient a_0 is given by

a) 0 b) $\frac{\pi}{2}$ c) $-\frac{\pi}{2}$ d) 1

108) If $f(x)=e^{-x}$ defined over $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, the Fourier coefficient a_0 is given by

a) $\frac{1}{2\pi}\left(1-e^{-2\pi}\right)$ b) $\frac{2}{\pi}\left(1-e^{-2\pi}\right)$
c) $\frac{1}{\pi}\left(1+e^{-x}\right)$ d) $\frac{1}{\pi}\left(1-e^{-2\pi}\right)$

109) If $f(x)=x$ defined over $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, the Fourier coefficient a_0 is given by

a) 3π b) $\frac{\pi}{2}$ c) π d) 2π

110) If $f(x)=x$ defined over $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, the Fourier coefficient a_n is given by
a) 0 b) π c) 2π d) 3π

111) If $f(x)=x$ defined over $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, the Fourier coefficient b_n is given by

a) $-\frac{2}{n\pi}$ b) $-\frac{\pi}{n}$ c) $-\frac{1}{n}$ d) $-\frac{2}{n}$

112) If $f(x)=\sqrt{1-\cos x}$ defined over $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, the Fourier coefficient a_0 is given by

a) 0 b) $-\frac{4\sqrt{2}}{\pi}$ c) $\frac{4\sqrt{2}}{\pi}$ d) $\frac{8\sqrt{2}}{\pi}$

113) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=\frac{\pi-x}{2}$ defined over the interval $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, is given by

a) $-\frac{\pi}{2}$ b) $\frac{\pi}{2}$ c) 0 d) $\frac{\pi^2}{6}$

114) The Fourier coefficient a_n for the Fourier series expansion of $f(x)=\frac{\pi-x}{2}$ defined over the interval $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, is given by

a) 0 b) π c) $\frac{\pi}{2}$ d) $-\frac{\pi}{2}$

115) The Fourier coefficient b_n for the Fourier series expansion of $f(x)=\frac{\pi-x}{2}$ defined over the interval $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, is given by

a) $-\frac{1}{n^2}$ b) $\frac{1}{n}$ c) $-\frac{1}{n}$ d) $\frac{\pi}{n}$

116) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=\left(\frac{\pi-x}{2}\right)^2$ defined over the interval $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, is given by

a) $\frac{\pi^2}{6}$ b) $\frac{\pi^2}{12}$ c) $\frac{\pi^2}{3}$ d) $\frac{\pi^2-1}{6}$

117) Consider $f(x)=x\sin x,\ x\in[0,2\pi]$ and $f(x+2\pi)=f(x)$. Then the Fourier coefficient a_0 for the Fourier series expansion of $f(x)$ is given by

a) -4 b) $-\frac{\pi}{2}$ c) -2 d) $\frac{\pi}{2}$

118) If $f(x)=\begin{cases} x & 0\le x\le\pi \\ 0 & \pi\le x\le 2\pi \end{cases}$ is periodic over a period 2π, the Fourier coefficient a_0 for the Fourier series expansion of $f(x)$ is given by

a) π b) $\frac{\pi}{2}$ c) $-\frac{\pi}{2}$ d) $\frac{\pi}{4}$

119) If $f(x)=\begin{cases} 0 & 0\le x\le\pi \\ x & \pi\le x\le 2\pi \end{cases}$ is periodic over a period 2π, the Fourier coefficient a_0 for the Fourier series expansion of $f(x)$ is given by

a) $\frac{3\pi}{2}$ b) $\frac{\pi}{2}$ c) 3π d) $\frac{3\pi}{4}$

120) If the function $f(x)=\begin{cases} -\pi & 0\le x\le\pi \\ x-\pi & \pi\le x\le 2\pi \end{cases}$ and $f(x+2\pi)=f(x)$, the Fourier coefficient a_0 is given by

a) 0 b) $-\frac{\pi}{2}$ c) $-\frac{\pi}{4}$ d) $-\pi$

121) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=x-x^2$ defined over the interval $0\le x\le 2$ and $f(x+2)=f(x)$, is given by

a) 0 b) $-\frac{4}{3}$ c) $-\frac{2}{3}$ d) $\frac{2}{3}$

122) The Fourier coefficient a_n for the Fourier series expansion of $f(x)=x-x^2$ defined over the interval $0\le x\le 2$ and $f(x+2)=f(x)$, is given by

a) 0 b) $-\frac{4}{n^2\pi^2}$ c) $\frac{4}{n^2\pi^2}$ d) $-\frac{1}{n^2\pi^2}$

123) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=x+x^2$ defined over the interval $0\le x\le 3$ and having period 3, is given by

a) 0 b) $-\frac{4}{n^2\pi^2}$ c) $\frac{4}{n^2\pi^2}$ d) $\frac{3}{2}$

124) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=2x-x^2$ defined over the interval $0\le x\le 4$ and $f(x+4)=f(x)$, is given by

a) $-\frac{1}{3}$ b) $-\frac{2}{3}$ c) $-\frac{4}{3}$ d) $-\frac{8}{3}$

125) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=2x-x^2$ defined over the interval $0\le x\le 3$ and $f(x+3)=f(x)$, is given by

a) 0 b) $-\frac{2}{3}$ c) $-\frac{4}{3}$ d) $-\frac{8}{3}$

126) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=1-x^2$ defined over the interval $0\le x\le 2$ and $f(x+2)=f(x)$, is given by

a) $-\frac{1}{3}$ b) $-\frac{2}{3}$ c) $-\frac{4}{3}$ d) $-\frac{8}{3}$

127) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=1-x^2$ defined over the interval $0\le x\le 1$ and $f(x+2)=f(x)$, is given by

a) $\frac{2}{3}$ b) $-\frac{2}{3}$ c) $-\frac{1}{3}$ d) $-\frac{4}{3}$

128) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=4-x^2$ defined over the interval $0\le x\le 2$ and $f(x+2)=f(x)$, is given by

a) $-\frac{1}{3}$ b) $\frac{16}{3}$ c) $-\frac{16}{3}$ d) $-\frac{8}{3}$

129) If $f(x)=\begin{cases}\pi x & 0\le x\le 1\\ \pi(2-x) & 1\le x\le 2\end{cases}$ is periodic over a period 2, the Fourier coefficient a_0 for the Fourier series expansion of $f(x)$ is given by

a) $-\frac{\pi}{2}$ b) π c) $-\pi$ d) $\frac{\pi}{2}$

130) If $f(x)=\begin{cases}x & 0\le x\le 1\\ 0 & 1\le x\le 2\end{cases}$ is periodic over a period 2, the Fourier coefficient a_0 for the Fourier series expansion of $f(x)$ is given by

a) 2 b) 0 c) $\frac{1}{2}$ d) 1

131) The Fourier coefficient a_0 in the half range cosine series expansion of function $f(x)=\sin x$ defined over the interval $[0,\ \pi]$ is given by

a) $\frac{2}{\pi}$ b) $\frac{\pi}{2}$ c) $-\frac{\pi}{2}$ d) 0

132) The Fourier coefficient b_1 in the half range cosine series expansion of function $f(x)=\cos x$ defined over the interval $[0,\ \pi]$ is given by

a) $-\frac{\pi}{2}$ b) 0 c) $\frac{1}{2\pi}$ d) $\frac{\pi}{2}$

133) The Fourier coefficient a_0 in the half range cosine series expansion of function $f(x)=\pi x-x^2$ defined over the interval $[0,\ \pi]$ is given by

a) 0 b) $\frac{\pi^2}{6}$ c) $\frac{2\pi^2}{3}$ d) $\frac{\pi^2}{3}$

134) The Fourier coefficient a_0 in the half range sine series expansion of function $f(x)=\cos x$ defined over the interval $[0,\ \pi]$ is given by

a) $\frac{4}{\pi}$ b) $\frac{2}{\pi}$ c) $\frac{\pi}{2}$ d) 0

135) The Fourier coefficient a_0 in the half range cosine series expansion of function $f(x)=\sin x$ defined over the interval $[0,\ \pi]$ is given by

a) $\frac{4}{\pi}$ b) $\frac{2}{\pi}$ c) 0 d) $\frac{\pi}{2}$

136) The Fourier coefficient a_1 in the half range cosine series expansion of function $f(x)=\sin x$ defined over the interval $[0,\ \pi]$ is given by

a) 1 b) $\frac{2}{\pi}$ c) 0 d) $\frac{\pi}{2}$

137) The Fourier coefficient b_1 in the half range cosine series expansion of function $f(x)=x$ defined over the interval $[0,\ 2]$ with period 4 is given by

a) 0 b) $\frac{1}{\pi}$ c) $\frac{2}{\pi}$ d) $\frac{4}{\pi}$

138) The Fourier coefficient a_0 in the half range cosine series expansion of function $f(x)=x-x^2$ defined over the interval $[0,\ 1]$ is given by

a) $\frac{1}{3}$ b) $-\frac{1}{3}$ c) 0 d) $\frac{2}{\pi}$

139) The Fourier coefficient a_0 in the half range cosine series expansion of function $f(x)=lx-x^2$ defined over the interval $[0,\ l]$ with period $2l$ is given by

a) 0 b) $\frac{l^2}{3}$ c) $\frac{\pi}{2}$ d) $\frac{2}{\pi n^2}(\cos n\pi-1)$

140) The Fourier coefficient a_1 in the half range cosine series expansion of function $f(x)=x-x^2$ defined over the interval $[0,\ 1]$ is given by

a) 2 b) $\frac{2}{\pi}$ c) $\frac{\pi}{2}$ d) $\frac{1}{2}$

141) The Fourier coefficient a_n in the half range cosine series expansion of function $f(x)=x-x^2$ defined over the interval $[0,\ 1]$ is given by

a) 0 b) $\frac{2}{\pi}$ c) $\frac{\pi}{2}$ d) $\frac{2}{\pi n^2}(\cos n\pi-1)$

142) The Fourier coefficient a_n in the half range sine series expansion of function $f(x)=2+x$ defined over the interval $[0,\ 1]$ is given by

a) 4 b) 0 c) $-\frac{2}{n\pi}$ d) $-\frac{2\pi}{n}$

143) The Fourier series expansion for the function $f(x)=\left(\frac{\pi-x}{2}\right)^2$ over the interval $0\le x\le 2\pi$ is given by $\left(\frac{\pi-x}{2}\right)^2=\frac{\pi^2}{12}+\sum_{n=1}^{\infty}\frac{1}{n^2}\cos nx$. Then the value of $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots$ is

a) 1 b) $\frac{\pi^2}{6}$ c) $\frac{\pi^2}{12}$ d) $\frac{\pi^2}{3}$

144) The Fourier series expansion for the function $f(x)=\pi^2-x^2$ over the interval $-\pi\le x\le\pi$ is given by $\pi^2-x^2=\frac{2\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}\cos nx$. Then the value of $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-+\ldots$ is

a) 1 b) $\frac{\pi^2}{6}$ c) $\frac{\pi^2}{12}$ d) $\frac{\pi^2}{3}$

145) The Fourier series expansion for the function $f(x)=\pi^2-x^2$ over the interval $-\pi\le x\le\pi$ is given by $\pi^2-x^2=\frac{2\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}\cos nx$. Then the value of $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots$ is

a) $\frac{\pi^2}{6}$ b) $\frac{\pi^2}{12}$ c) $\frac{\pi^2}{3}$ d) 0

146) The Fourier series expansion for the function $f(x)=\begin{cases}1+\frac{2x}{\pi} & -\pi\le x\le 0\\ 1-\frac{2x}{\pi} & 0\le x\le\pi\end{cases}$ is given by $f(x)=\frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos nx$. Then the value of $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots$ is

a) $\frac{\pi^2}{6}$ b) $\frac{\pi^2}{12}$ c) $\frac{\pi^2}{3}$ d) $\frac{\pi^2}{8}$

147) The Fourier series expansion for the function $f(x)=\begin{cases}\pi x & 0\le x\le 1\\ \pi(2-x) & 1\le x\le 2\end{cases}$ is given by $f(x)=\frac{\pi}{2}-\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos(2n-1)\pi x$. Then the value of $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots$ is

a) $\frac{\pi^2}{6}$ b) $\frac{\pi^2}{12}$ c) $\frac{\pi^2}{8}$ d) $\frac{\pi^2}{3}$

148) In the following harmonic analysis of $y=f(x)$, the value of a_0 is given by

x	0	1	2	3	4	5
y	4	8	15	7	5	3

a) 14 b) 7 c) 3.5 d) 6

149) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	1	2	3	4	5
y	9	18	26	26	26	20

a) 25.01 b) 20.83 c) 41.66 d) 40.89

150) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	30	60	90	120	150	180
y	0	9.2	14.4	17.8	17	12	0

a) 10.23 b) 23.46 c) 46.92 d) 11.73

151) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	60	120	180	240	300	360
y	1	1.4	1.9	1.6	1.6	1.2	1

a) 7.2 b) 1.45 c) 5.8 d) 2.9

152) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	60	120	180	240	300	360
y	1.98	2.15	2.7	-0.22	-0.31	1.5	1.98

a) 4.8 b) 2.6 c) 5.2 d) 1.3

153) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	1	1.4	1.9	1.6	1.6	1.2	1

a) 2.9 b) 5.8 c) 1.45 d) 3.8

154) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
y	1.98	1.35	1	1.3	-0.88	-0.25	1.98

a) 1 b) 0.75 c) 1.5 d) 3

155) In the following harmonic analysis of $y = f(x)$, the value of a_1 is given by

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
y	0	9.2	14.4	17.8	17.3	11.7	0

a) 3.73 b) 5.73 c) 7.73 d) -7.73

156) In the following harmonic analysis of $y = f(x)$, the value of b_1 is given by

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
y	0	9.2	14.4	17.8	17.3	11.7	0

a) 4.38 b) 3.48 c) 4.83 d) 8.43

157) In the following harmonic analysis of $y = f(x)$, the value of a_1 is given by

x	0	1	2	3	4	5
y	9	18	24	28	26	20

a) - 8.37 b) 8.73 c) 7.83 d) 3.78

158) In the following harmonic analysis of $y = f(x)$, the value of b_1 is given by

x	0	1	2	3	4	5
y	9	18	24	28	26	20

a) 1.25 b) -6.3 c) – 3.15 d) - 3.50

159) In the following harmonic analysis of $y = f(x)$, the value of b_1 is given by

x	0	1	2	3	4	5	6
y	9	18	24	28	26	20	9
$\cos\left(\frac{\pi}{3}x\right)$	1	0.5	-0.5	-1	-0.5	0.5	1

a) 3.38 b) – 8.33 c) 8.33 d) 5.83

Chapter 04–Reduction Formulae, Beta and Gamma Functions

I) Reduction Formulae

1) For $I_n = \int_0^{\pi/2} \sin^n x\,dx$, we have

a) $I_n = 2\int_0^{\pi} \sin^n x\,dx$ b) $I_n = \int_0^{\pi/2} \sin^{n-2} x \cos^2 x\,dx$

c) $I_n = \int_0^{\pi/2} \cos^n x\,dx$ d) $I_n = \frac{1}{2}\int_0^{\pi/4} \sin^n x\,dx$

2) For $I_n = \int_0^{\pi} \sin^n x\,dx$, we have

a) 0 b) $I_n = 2\int_0^{\pi/2} \sin^n x\,dx$

c) $I_n = 4\int_0^{\pi/2} \sin^n x\,dx$ d) none of these

3) For $I_n = \int_0^{\pi} \cos^n x\,dx$, where n is an even integer, we have

a) $I_n = 4\int_0^{\pi/2} \cos^n x\,dx$ b) $I_n = 2\int_0^{\pi/4} \cos^n x\,dx$

c) $I_n = 2\int_0^{\pi/2} \cos^n x\,dx$ d) 0

4) For $I_n = \int_0^{\pi} \cos^n x\,dx$, where n is an odd integer, we have

a) $I_n = 4\int_0^{\pi/2} \cos^n x\,dx$ b) $I_n = 2\int_0^{\pi/4} \cos^n x\,dx$

c) $I_n = 2\int_0^{\pi/2} \cos^n x\,dx$ d) 0

5) For $I_n = \int_0^{2\pi} \sin^n x\,dx$, where n is an even integer, we have

a) 0 b) $I_n = 4\int_0^{\pi/4} \sin^n x\,dx$

c) $I_n = 2\int_0^{\pi/2} \sin^n x\,dx$ d) $I_n = 4\int_0^{\pi/2} \sin^n x\,dx$

6) For $I_n = \int_0^{2\pi} \sin^n x\,dx$, where n is an odd integer, we have

a) $I_n = 4\int_0^{\pi/2} \sin^n x\,dx$ b) $I_n = 2\int_0^{\pi/2} \sin^n x\,dx$

c) 0 d) $I_n = 4\int_0^{\pi/4} \sin^n x\,dx$

7) For $I_n = \int_0^{2\pi} \cos^n x\,dx$, where n is an odd integer, we have

a) 0 b) $I_n = 4\int_0^{\pi/2} \cos^n x\,dx$

c) $I_n = 2\int_0^{\pi/2} \cos^n x\,dx$ d) $I_n = 4\int_0^{\pi/4} \cos^n x\,dx$

8) For $I_n = \int_0^{2\pi} \cos^n x\,dx$, where n is an even integer, we have

a) 0 b) $I_n = 4\int_0^{\pi/4} \cos^n x\,dx$

c) $I_n = 4\int_0^{\pi/2} \cos^n x\,dx$ d) $I_n = 2\int_0^{\pi/2} \cos^n x\,dx$

9) For $I_{m,n} = \int_0^{\pi} \sin^m x \cos^n x\, dx$, where both m and n are odd integers, we have

a) $I_{m,n} = 2\int_0^{\pi/2} \sin^m x \cos^n x\, dx$ b) 0

c) $I_{m,n} = 4\int_0^{\pi/2} \sin^m x \cos^n x\, dx$ d) none

10) For $I_{m,n} = \int_0^{\pi} \sin^m x \cos^n x\, dx$, where both m and n are even integers, we have

a) $I_{m,n} = 2\int_0^{\pi/2} \sin^m x \cos^n x\, dx$

b) $I_{m,n} = 4\int_0^{\pi/2} \sin^m x \cos^n x\, dx$

c) 0 d) none of the above

11) For $I_{m,n} = \int_0^{\pi} \sin^m x \cos^n x\, dx$, where only n is an even integer, we have

a) 0 b) $I_{m,n} = 4\int_0^{\pi/2} \sin^m x \cos^n x\, dx$

c) $I_{m,n} = 2\int_0^{\pi/2} \sin^m x \cos^n x\, dx$ d) none

12) For $I_{m,n} = \int_0^{\pi} \sin^m x \cos^n x\, dx$, where only n is an odd integer, we have

a) $I_{m,n} = 4\int_0^{\pi/2} \sin^m x \cos^n x\, dx$

b) 0

c) $I_{m,n} = 2\int_0^{\pi/2} \sin^m x \cos^n x\, dx$

d) none of the above

13) For $I_n = \int_0^{\pi/2} \sin^n x\, dx$, which of the following is the reduction formula?

a) $I_n = \frac{n-1}{n} I_{n-1}$ b) $I_n = \frac{n}{n+1} I_{n-2}$

c) $I_n = \frac{n+1}{n} I_{n-2}$ d) $I_n = \frac{n-1}{n} I_{n-2}$

14) For $I_n = \int_0^{\pi/2} \cos^n x\, dx$, which of the following is the reduction formula?

a) $I_n = \frac{n-1}{n} I_{n-2}$ b) $I_n = \frac{n-1}{n} I_{n-1}$

c) $I_n = \frac{n}{n+1} I_{n-2}$ d) $I_n = \frac{n+1}{n} I_{n-2}$

15) For $I_n = \int_0^{\pi/2} \sin^n x\, dx$, where n is an even natural number, which of the following is the reduced form?

a) $I_n = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot \ldots \cdot\frac{1}{2}\cdot\frac{\pi}{2}$

b) $I_n = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot \ldots \cdot\frac{2}{3}\cdot 1$

c) $I_n = \frac{n+1}{n}\cdot\frac{n+3}{n+2}\cdot\frac{n+5}{n+4}\cdot \ldots \cdot\frac{1}{2}\cdot\frac{\pi}{2}$

d) $I_n = \frac{n-2}{n-1}\cdot\frac{n-4}{n-3}\cdot\frac{n-6}{n-5}\cdot \ldots \cdot\frac{2}{3}\cdot\frac{\pi}{2}$

16) For $I_n = \int_0^{\pi/2} \cos^n x\, dx$, where n is an even natural number, which of the following is the reduced form?

a) $I_n = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot \ldots \cdot\frac{2}{3}\cdot 1$

b) $I_n = \frac{n-2}{n-1}\cdot\frac{n-4}{n-3}\cdot\frac{n-6}{n-5}\cdot \ldots \cdot\frac{2}{3}\cdot\frac{\pi}{2}$

c) $I_n = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot \ldots \cdot\frac{1}{2}\cdot\frac{\pi}{2}$

d) $I_n = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot \ldots \cdot\frac{1}{2}\cdot\pi$

17) For $I_n = \int_0^{\pi/2} \sin^n x\, dx$, where n is an odd natural number, which of the following is the reduced form?

a) $I_n = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdot \ldots \cdot \frac{2}{3} \cdot 1$

b) $I_n = \frac{n-2}{n-1} \cdot \frac{n-4}{n-3} \cdot \frac{n-6}{n-5} \cdot \ldots \cdot \frac{2}{3} \cdot \frac{\pi}{2}$

c) $I_n = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdot \ldots \cdot \frac{1}{2} \cdot \frac{\pi}{2}$

d) $I_n = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdot \ldots \cdot \frac{1}{2} \cdot \pi$

18) For $I_n = \int_0^{\pi/2} \cos^n x\, dx$, where n is an odd natural number, which of the following is the reduced form?

a) $I_n = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdot \ldots \cdot \frac{1}{2} \cdot \frac{\pi}{2}$

b) $I_n = \frac{n+1}{n} \cdot \frac{n+3}{n+2} \cdot \frac{n+5}{n+4} \cdot \ldots \cdot \frac{2}{3} \cdot 1$

c) $I_n = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdot \ldots \cdot \frac{2}{3} \cdot 1$

d) $I_n = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdot \ldots \cdot \frac{1}{2} \cdot \pi$

19) For $I_n = \int_0^{\pi/2} \sin^n x \cos^n x\, dx$, where n is an odd natural number, which of the following is the reduced form?

a) $I_{(m,\,n)} = \frac{[(m-1)(m-3)\cdot \ldots \cdot 2 or 1][(n-1)(n-3)\cdot \ldots \cdot 2 or 1]}{(m+n)(m+n-2)\cdot \ldots \cdot 2 or 1} \cdot k$

where $k = \begin{cases} \frac{\pi}{2} & both\ m\ \&\ n\ are\ odd \\ 1 & otherwise \end{cases}$

b) $I_{(m,\,n)} = \frac{[(m-1)(m-3)\cdot \ldots \cdot 2 or 1][(n-1)(n-3)\cdot \ldots \cdot 2 or 1]}{(m+n)(m+n-2)\cdot \ldots \cdot 2 or 1} \cdot k$

where $k = \begin{cases} \frac{\pi}{2} & both\ m\ \&\ n\ are\ even \\ 1 & otherwise \end{cases}$

c) $I_{(m,\,n)} = \frac{[(m-1)(m-3)\cdot \ldots \cdot 2 or 1][(n-1)(n-3)\cdot \ldots \cdot 2 or 1]}{(m+n)(m+n-2)\cdot \ldots \cdot 2 or 1} \cdot k$

where $k = \begin{cases} \frac{\pi}{2} & m+n\ is\ even \\ 1 & otherwise \end{cases}$

d) $I_{(m,\,n)} = \frac{(m+n-1)(m+n-3)\cdot \ldots \cdot 2 or 1}{(m+n)(m+n-2)\cdot \ldots \cdot 2 or 1} \cdot k$

where $k = \begin{cases} \frac{\pi}{2} & both\ m\ \&\ n\ are\ odd \\ 1 & otherwise \end{cases}$

20) The value of $\int_0^{\pi/2} \sin^3 x\, dx$ is equal to

a) $\frac{3\pi}{4}$ b) $\frac{3}{4}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$

21) The value of $\int_0^{\pi/2} \sin^4 x\, dx$ is equal to

a) $\frac{3}{8}$ b) $\frac{3}{16}$ c) $\frac{3\pi}{16}$ d) $\frac{3\pi}{18}$

22) The value of $\int_0^{\pi/2} \sin^5 x\, dx$ is equal to

a) $\frac{4\pi}{15}$ b) $\frac{8\pi}{30}$ c) $\frac{8\pi}{15}$ d) $\frac{8}{15}$

23) The value of $\int_0^{\pi/2} \sin^9 x\, dx$ is equal to

a) $\frac{64}{315}$ b) $\frac{128}{315}$ c) $\frac{128}{315}\pi$ d) $\frac{64}{315}\pi$

24) The value of $\int_0^{\pi/2} \cos^3 x\, dx$ is equal to

a) $\frac{3\pi}{4}$ b) $\frac{3}{4}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$

25) The value of $\int_0^{\pi/2} \cos^4 x\, dx$ is equal to

a) $\frac{3}{8}$ b) $\frac{3}{16}$ c) $\frac{3\pi}{16}$ d) $\frac{3\pi}{18}$

26) The value of $\int_0^{\pi/2} \cos^7 x\, dx$ is equal to

a) $\frac{8}{35}$ b) $\frac{16\pi}{35}$ c) $\frac{16\pi}{70}$ d) $\frac{16}{35}$

27) The value of $\int_0^{\pi/2} \cos^{10} x\,dx$ is equal to

a) $\frac{63\pi}{128}$ b) $\frac{63\pi}{512}$ c) $\frac{63\pi}{256}$ d) $\frac{64}{315}\pi$

28) The value of $\int_0^{\pi/2} \sin^3 x \cos^2 x\,dx$ is equal to

a) $\frac{2}{15}$ b) $\frac{\pi}{30}$ c) $\frac{1}{15}$ d) $\frac{\pi}{15}$

29) The value of $\int_0^{\pi/2} \sin^2 x \cos^3 x\,dx$ is equal to

a) $\frac{1}{15}$ b) $\frac{\pi}{30}$ c) $\frac{\pi}{15}$ d) $\frac{2}{15}$

30) The value of $\int_0^{\pi/2} \sin^3 x \cos^4 x\,dx$ is equal to

a) $\frac{1}{35}$ b) $\frac{2}{35}$ c) $\frac{2\pi}{35}$ d) $\frac{2\pi}{70}$

31) The value of $\int_0^{\pi/2} \sin^4 x \cos^6 x\,dx$ is equal to

a) $\frac{3\pi}{512}$ b) $\frac{3}{256}$ c) $\frac{3\pi}{256}$ d) $\frac{3\pi}{128}$

32) The value of $\int_{-\pi/2}^{\pi/2} \sin^3 x \cos^2 x\,dx$ is equal to

a) $2\int_0^{\pi/2} \sin^3 x \cos^2 x\,dx$ b) $4\int_0^{\pi/2} \sin^3 x \cos^2 x\,dx$

c) 0 d) none of the above

33) The value of $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos^3 x\,dx$ is equal to

a) 0 b) $2\int_0^{\pi/2} \sin^2 x \cos^3 x\,dx$

c) $3\int_0^{\pi/3} \sin^2 x \cos^3 x\,dx$ d) none of the above

34) The value of $\int_{-\pi/2}^{\pi/2} \sin^4 x \cos^2 x\,dx$ is equal to

a) $\frac{3}{16}$ b) $\frac{3\pi}{16}$ c) $\frac{\pi}{16}$ d) 0

35) The value of $\int_{-\pi/2}^{\pi/2} \sin^5 x\,dx$ is equal to

a) $\frac{3\pi}{128}$ b) $\frac{3\pi}{15}$ c) $\frac{32}{256}$ d) 0

36) The value of $\int_0^{2\pi} \sin^4 x \cos^6 x\,dx$ is equal to

a) $\frac{3}{64}$ b) $\frac{2\pi}{35}$ c) $\frac{2}{35}$ d) $\frac{3\pi}{128}$

37) The value of $\int_0^{2\pi} \sin^4 x \cos^7 x\,dx$ is equal to

a) $\frac{5}{128}$ b) $\frac{5\pi}{128}$ c) 0 d) $\frac{5\pi}{256}$

38) The value of $\int_{-\pi}^{\pi} \sin^4 x \cos^7 x\,dx$ is equal to

a) 0 b) $\frac{5\pi}{128}$ c) $\frac{5}{128}$ d) $\frac{5\pi}{256}$

39) The value of $\int_0^{\pi} \cos^3 x\,dx$ is equal to

a) $\frac{5\pi}{256}$ b) $\frac{5\pi}{16}$ c) $\frac{5}{128}$ d) 0

40) The value of $\int_0^{\pi} \cos^6 x\,dx$ is equal to

a) 0 b) $\frac{5\pi}{16}$ c) $\frac{5}{8}$ d) $\frac{5\pi}{256}$

41) The value of $\int_0^{\pi} \cos^7 x\,dx$ is equal to

a) $\frac{5\pi}{256}$ b) $\frac{5\pi}{16}$ c) $\frac{5}{128}$ d) 0

42) The value of $\int_0^{\pi} \sin^7 x\,dx$ is equal to

a) $\frac{5\pi}{32}$ b) $\frac{5\pi}{16}$ c) $\frac{32}{35}$ d) 0

43) The value of $\int_0^{\pi} \sin^6 x\,dx$ is equal to

a) $\frac{5\pi}{16}$ b) $\frac{5\pi}{32}$ c) $\frac{3}{4}$ d) 0

44) The value of $\int_0^{2\pi} \sin^6 \theta\,d\theta$ is equal to

a) $\frac{5\pi}{32}$ b) $\frac{5\pi}{16}$ c) $\frac{5\pi}{8}$ d) 0

45) The value of $\int_0^{2\pi} \sin^8 x\,dx$ is equal to

a) $\frac{5\pi}{16}$ b) $\frac{5\pi}{32}$ c) $\frac{32}{35}$ d) $\frac{35\pi}{32}$

46) The value of $\int_0^{2\pi} \cos^5 x\,dx$ is equal to

a) $\frac{5\pi}{32}$ b) $\frac{5\pi}{16}$ c) $\frac{32}{35}$ d) 0

47) The value of $\int_0^{2\pi} \sin^6 x \cos^4 x\,dx$ is equal to

a) $\frac{5\pi}{256}$ b) $\frac{3\pi}{128}$ c) $\frac{35\pi}{256}$ d) 0

48) The value of $\int_0^{2\pi} \sin^7 x \cos^4 x\,dx$ is equal to

a) $\frac{5\pi}{256}$ b) $\frac{3\pi}{128}$ c) $\frac{35\pi}{256}$ d) 0

49) The value of $\int_0^{2\pi} \sin^7 x \cos^5 x\,dx$ is equal to

a) $\frac{5\pi}{256}$ b) 0 c) $\frac{35\pi}{256}$ d) $\frac{3\pi}{128}$

50) The value of $\int_0^{\pi} \sin^5\left(\frac{x}{2}\right)dx$ is equal to

a) $\frac{16\pi}{15}$ b) $\frac{5\pi}{32}$ c) $\frac{16}{15}$ d) $\frac{5\pi}{16}$

51) The value of $\int_0^{\pi/4} \sin^2(2x)\,dx$ is equal to

a) $\frac{\pi}{8}$ b) $\frac{16}{15}$ c) $\frac{3\pi}{8}$ d) 0

52) The value of $\int_0^{\pi/4} \sin^7(2x)\,dx$ is equal to

a) $\frac{16\pi}{15}$ b) $\frac{5\pi}{16}$ c) $\frac{8}{35}$ d) 0

53) The value of $\int_0^{\pi/4} \cos^2(2x)\,dx$ is equal to

a) $\frac{5\pi}{16}$ b) $\frac{\pi}{8}$ c) $\frac{5\pi}{32}$ d) 0

54) The value of $\int_0^{\pi/3} \sin^5(3x)\,dx$ is equal to

a) $\frac{3\pi}{16}$ b) $\frac{8\pi}{15}$ c) $\frac{8\pi}{45}$ d) $\frac{8}{45}$

55) If $I_n = \int_0^{\pi/4} \sin^{2n} x\,dx = -\frac{1}{2^{n+1}n} + \frac{2n-1}{2n} I_{n-1}$, the value of I_2 is equal to

a) $\frac{3\pi+2}{8}$ b) $\frac{3\pi-8}{32}$ c) $-\frac{8+3\pi}{32}$ d) $\frac{3\pi}{32}$

56) If $I_n = \int_0^{\pi/2} x \sin^n x\,dx = \frac{1}{n^2} + \frac{n-1}{n} I_{n-2}$, the value of I_5 is equal to

a) $\frac{149}{25}$ b) $\frac{19}{225}$ c) $\frac{\pi}{2} - \frac{149}{225}$ d) $\frac{149}{225}$

56) If $I_n = \int_0^{\pi/2} \tan^n x\,dx = \frac{1}{n-1} - I_{n-2}$, the value of I_4 is equal to

a) $\frac{\pi}{4} - \frac{2}{3}$ b) $\frac{\pi}{4} + \frac{2}{3}$ c) $\frac{\pi}{2} - \frac{2}{3}$ d) $\frac{\pi}{4} + \frac{4}{3}$

II) Gamma Functions

57) For $n>0$, the gamma function Γn is defined as

a) $\int_0^\infty e^{x}x^{n-1}dx$ b) $\int_0^\infty e^{-x}x^{n+1}dx$

c) $\int_0^\infty e^{-x}x^{n}dx$ d) $\int_0^\infty e^{-x}x^{n-1}dx$

58) $\int_0^\infty e^{-x}x^{n}dx$ is equal to

a) $\Gamma(n+1)$ b) Γn c) $\Gamma(n-1)$ d) $\Gamma(n-2)$

59) $\int_0^\infty e^{-kx}x^{n}dx$ is equal to

a) $k^{n-1}\Gamma(n-1)$ b) $\frac{\Gamma(n-1)}{k^{n-1}}$ c) $\frac{\Gamma(n+1)}{k^{n+1}}$ d) $\frac{\Gamma n}{k^{n}}$

60) $\int_0^\infty e^{-kx}x^{n-1}dx$ is equal to

a) $k^{n-1}\Gamma(n-1)$ b) $\frac{\Gamma(n-1)}{k^{n-1}}$ c) $\frac{\Gamma(n+1)}{k^{n+1}}$ d) $\frac{\Gamma n}{k^{n}}$

61) The value of Γn is equal to

a) $n\Gamma(n-1)$ b) $(n+1)\Gamma(n+1)$

c) $(n-1)\Gamma(n-1)$ d) $n\Gamma n$

62) If n is a natural number, the value of Γn is

a) $\frac{n!}{n+1}$ b) $(n-1)!$ c) $n!$ d) $(n+1)!$

63) The value of $\Gamma 1$ is

a) 1 b) 2 c) 3 d) 0

64) The value of $\Gamma 2$ is

a) 0 b) 1 c) 2 d) 3

65) The value of $\Gamma 7$ is

a) 3256 b) 5040 c) 120 d) 720

66) The value of $\Gamma\left(\frac{1}{2}\right)$ is

a) $\frac{1}{2}$ b) $\sqrt{\pi}$ c) $\Gamma\pi$ d) none

67) The value of $\Gamma\left(\frac{5}{2}\right)$ is

a) $\frac{3\sqrt{\pi}}{2}$ b) $\frac{3\sqrt{\pi}}{4}$ c) $\frac{3\sqrt{\pi}}{8}$ d) 0

68) The value of $\Gamma\left(\frac{1}{4}\right)\cdot\Gamma\left(\frac{3}{4}\right)$ is

a) $\pi\sqrt{2}$ b) $\frac{\pi}{\sqrt{2}}$ c) $\frac{\sqrt{2}}{\pi}$ d) none

69) The value of $\Gamma p\cdot\Gamma(1-p)$, for $0<p<1$, is given by the formula

a) $\frac{\sin p\pi}{\pi}$ b) $\frac{\pi}{\sin p\pi}$

c) $\frac{\sqrt{\pi}}{\sin p\pi}$ d) $\frac{p\pi}{\sin p\pi}$

70) The value of $\int_0^\infty e^{-x}x^{5}dx$

a) 60 b) 720 c) 120 d) 240

71) The value of $\int_0^\infty e^{-2x}x^{5}dx$

a) $\frac{125}{32}$ b) $\frac{120}{35}$ c) $\frac{25}{8}$ d) $\frac{15}{8}$

72) The value of $\int_0^\infty e^{-x}x^{1/2}dx$

a) $\frac{\sqrt{\pi}}{2}$ b) $\frac{\pi}{2}$ c) $\frac{\sqrt{\pi}}{3}$ d) $\sqrt{\pi}$

73) The value of $\int_0^\infty e^{-x}x^{-1/2}dx$

a) $\frac{\sqrt{\pi}}{2}$ b) $\sqrt{\pi}$ c) $\frac{\sqrt{\pi}}{3}$ d) $\frac{\pi}{2}$

74) The value of $\int_0^\infty e^{-x}x^{3/2}dx$

a) $\frac{\sqrt{\pi}}{4}$ b) $\frac{3\sqrt{\pi}}{8}$ c) $\frac{3\sqrt{\pi}}{4}$ d) $\frac{3\sqrt{\pi}}{2}$

75) The substitution for the integral $\int_0^\infty \sqrt{x}\cdot e^{-\sqrt{x}}dx$ to reduce it into the form of gamma function is

a) $\sqrt{x}=t$ b) $\sqrt{x}=t^2$

c) $\sqrt{x}=\frac{t}{2}$ d) $x=\sin t$

76) The substitution for the integral $\int_0^\infty x^3\cdot e^{-\sqrt{x}}dx$ to reduce it into the form of gamma function is

a) $x^3=\sin^2 t$ b) $x^3=e^{-t}$

c) $x^3=t$ d) $\sqrt{x}=t$

77) The substitution for the integral $\int_0^\infty x^3\cdot 5^{-x}dx$ to reduce it into the form of gamma function is

a) $5^x=e^t$ b) $x^3=e^{-t}$

c) $5^x=x^{-t}$ d) $\log x=5^{-x}$

78) On using substitution $\sqrt{x}=t$, the value of the integration $\int_0^\infty x\cdot e^{-\sqrt{x}}dx$ is given by

a) 1 b) 3 c) 12 d) 16

79) On using substitution $\sqrt{x}=t$, the value of the integration $\int_0^\infty \sqrt{x}\cdot e^{-\sqrt{x}}dx$ is given by

a) 1 b) 2 c) 3 d) 4

80) On using substitution $\sqrt{t}=x$, the value of the integration $\int_0^\infty e^{-x^2}dx$ is given by

a) $\left|\frac{1}{4}\right.$ b) 16 c) $\frac{\sqrt{\pi}}{2}$ d) $\sqrt{\pi}$

81) On using substitution $x^3=t$, the value of the integration $\int_0^\infty \sqrt{x}\cdot e^{-x^3}dx$ is given by

a) $\frac{\sqrt{\pi}}{2}$ b) $\frac{\sqrt{\pi}}{3}$ c) $\frac{2\sqrt{\pi}}{3}$ d) $\frac{3\sqrt{\pi}}{4}$

82) On using substitution $x^4=t$, the value of the integration $\int_0^\infty e^{-x^4}dx$ is given by

a) $\sqrt{\pi}$ b) π c) $\frac{1}{4}\left|\frac{1}{4}\right.$ d) $\left|\frac{3}{4}\right.$

83) On using substitution $x=t^2$, the value of the integration $\int_0^\infty \sqrt[4]{x}\cdot e^{-\sqrt{x}}dx$ is given by

a) $\frac{3\sqrt{\pi}}{2}$ b) $\frac{2\sqrt{\pi}}{3}$ c) $\frac{\sqrt{\pi}}{3}$ d) $2\sqrt{\pi}$

84) On using substitution $2x^2=t$, the value of the integration $\int_0^\infty x^7\cdot e^{-2x^2}dx$ is given by

a) $\left|\frac{3}{4}\right.$ b) $\frac{3}{8}$ c) $\frac{2\sqrt{\pi}}{3}$ d) $\frac{3}{16}$

85) On using substitution $2x^2=t$, the value of the integration $\int_0^\infty x^9\cdot e^{-2x^2}dx$ is given by

a) $\left|\frac{3}{4}\right.$ b) $\frac{3}{8}$ c) $\frac{2\sqrt{\pi}}{3}$ d) $\frac{3}{16}$

86) On using substitution $x^2=t$, the value of the integration $\int_0^\infty x^2\cdot e^{-x^2}dx$ is given by

a) $\frac{1}{3}\left|\frac{3}{2}\right.$ b) $\frac{3}{2}\left|\frac{3}{2}\right.$ c) $\frac{1}{2}\left|\frac{3}{2}\right.$ d) $\frac{1}{2}\left|\frac{2}{3}\right.$

87) On using substitution $x=t^{1/3}$, the value of the integration $\int_0^\infty \sqrt{x}\cdot e^{-x^3}dx$ is given by

a) $\frac{\sqrt{\pi}}{3}$ b) $\frac{2\sqrt{\pi}}{3}$ c) $\frac{1}{2}\left|\frac{2}{3}\right.$ d) $\frac{1}{3}\left|\frac{3}{2}\right.$

88) On using substitution $a^{-x}=e^{-t}$, the value of the integration $\int_0^\infty \frac{x^a}{a^x}dx$ is given by

a) $\frac{\left|a\right.}{(\log a)^a}$ b) $\frac{\left|a-1\right.}{(\log a)^{a-1}}$

c) $\dfrac{\overline{|a+1}}{(\log a)^{a+1}}$ d) $\dfrac{\overline{|a}}{(\log a)^{a+1}}$

89) On using substitution $3^{-x} = e^{-t}$, the value of the integration $\int_0^\infty \frac{x^3}{3^x} dx$ is given by

a) $\dfrac{3}{(\log 3)^4}$ b) $\dfrac{6}{(\log 3)^4}$

c) $\dfrac{36}{(\log 3)^4}$ d) $\dfrac{6}{(\log 3)^3}$

90) On using substitution $\log x = -t$, the value of the integration $\int_0^1 (x\log x)^3 dx$ is given by

a) $-\dfrac{3}{64}$ b) $\dfrac{3}{64}$ c) $\dfrac{3}{128}$ d) $-\dfrac{3}{128}$

91) On using substitution $\log x = -t$, the value of the integration $\int_0^1 \left(\log\frac{1}{x}\right)^{n-1} dx$ is given by

a) $\overline{|n+1}$ b) $\overline{|n}$ c) $\overline{|n-1}$ d) $-\overline{|1+n}$

92) On using substitution $\log x = -t$, the value of the integration $\int_0^1 \frac{1}{\sqrt{x\log\frac{1}{x}}} dx$ is given by

a) $\sqrt{2\pi}$ b) $\pi\sqrt{2}$ c) $2\sqrt{\pi}$ d) 2π

93) On using substitution $\log x = -t$, the value of the integration $\int_0^1 \frac{1}{\sqrt{-\log x}} dx$ is given by

a) $\sqrt{2\pi}$ b) $\pi\sqrt{2}$ c) $\sqrt{\pi}$ d) $2\sqrt{\pi}$

94) On using substitution $h^2x^2 = t$, the value of the integration $\int_0^\infty x^{n-1}e^{-h^2x^2} dx$ is given by

a) $\sqrt{2\pi}$ b) $\dfrac{\overline{|n/2}}{2h^n}$ c) $\dfrac{\overline{|n/2}}{2h^{n+1}}$ d) $\dfrac{\overline{|1+n/2}}{2h^{n+1}}$

II) Beta Functions

95) The value of $\beta(m, n)$ in the integral form is

a) $\int_0^1 x^m(1-x)^{n-1} dx$ b) $\int_0^1 x^m(1-x)^n dx$

c) $\int_0^1 x^{m+1}(1-x)^{n+1} dx$ d) $\int_0^1 x^{m-1}(1-x)^{n-1} dx$

96) The value of $\beta(m, n)$ in terms of gamma function is

a) $\dfrac{\overline{|m}\cdot\overline{|n}}{\overline{|m+n+1}}$ b) $\dfrac{\overline{|m-1}\cdot\overline{|n-1}}{\overline{|m+n}}$

c) $\dfrac{\overline{|m+1}\cdot\overline{|n+1}}{\overline{|m+n+1}}$ d) $\dfrac{\overline{|m}\cdot\overline{|n}}{\overline{|m+n}}$

97) The value of $\beta(m, n)$, when m and n are positive integers is

a) $\dfrac{(m-1)!(n-1)!}{(m+n-1)!}$ b) $\dfrac{(m+1)!(n+1)!}{(m+n+1)!}$

c) $\dfrac{m!n!}{(m+n)!}$ d) $\dfrac{m!n!}{(m+n+1)!}$

98) $\int_0^{\pi/2} \sin^m x\cos^n x dx$ is given by

a) $\beta(m, n)$ b) $\frac{1}{2}\beta\left(\frac{m-1}{2}, \frac{n-1}{2}\right)$

c) $\frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$ d) $\beta\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$

99) $\int_0^{\pi/2} \sin^{2m-1} x\cos^{2n-1} x dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$ b) $\beta\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$

c) $\frac{1}{2}\beta\left(\frac{m-1}{2}, \frac{n-1}{2}\right)$ d) $\beta(m, n)$

100) $\int_0^{\pi/2} \sin^m x\, dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{m-1}{2}, \frac{1}{2}\right)$ b) $\frac{1}{2}\beta\left(m, \frac{1}{2}\right)$

c) $\frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{1}{2}\right)$ d) $\frac{1}{2}\beta\left(\frac{m+1}{2}, 0\right)$

101) $\int_0^{\pi/2} \cos^m x\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{m-1}{2}, \frac{1}{2}\right)$ b) $\frac{1}{2}\beta\left(m, \frac{1}{2}\right)$

c) $\frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{1}{2}\right)$ d) $\frac{1}{2}\beta\left(\frac{m+1}{2}, 0\right)$

102) $\int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}}\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$ b) $\beta(m, n)$

c) $\beta(m+1, n+1)$ d) $\beta(m-1, n-1)$

103) $\beta(3, 5)$ can be represented by

a) $\int_0^{\infty} x^2(1-x)^4\,dx$ b) $\int_0^1 x^4(1-x)^6\,dx$

c) $\int_0^1 x^3(1-x)^5\,dx$ d) $\int_0^1 x^2(1-x)^4\,dx$

104) What is the exact value of $\beta(5, 3)$?

a) $\frac{2}{35}$ b) $\frac{2}{105}$ c) $\frac{1}{105}$ d) $\frac{1}{35}$

105) What is the exact value of $\beta\left(\frac{1}{4}, \frac{3}{4}\right)$?

a) $\frac{1}{8}$ b) $\pi\sqrt{2}$ c) $2\sqrt{\pi}$ d) $\sqrt{2\pi}$

106) $\int_0^1 \sqrt{x}(1-x)^{5/2}\,dx$ is equal to

a) $\beta\left(\frac{3}{2}, \frac{7}{2}\right)$ b) $\beta\left(\frac{1}{2}, \frac{5}{2}\right)$

c) $\beta\left(\frac{2}{3}, \frac{5}{3}\right)$ d) $\beta(2, 5)$

107) $\int_0^1 x^4(1-x)^5\,dx$ is equal to

a) $\frac{3}{462}$ b) $\frac{1}{462}$ c) $\frac{1}{501}$ d) $\frac{1}{231}$

108) $2\int_0^{\pi/2} \sin^{\frac{3}{2}} x\cos^5 x\,dx$ is given by

a) $\beta\left(\frac{5}{4}, 3\right)$ b) $\frac{1}{2}\beta\left(\frac{5}{4}, 3\right)$

c) $\beta\left(\frac{5}{4}, \frac{3}{2}\right)$ d) $\beta\left(\frac{5}{4}, \frac{3}{4}\right)$

109) $2\int_0^{\pi/2} \sqrt{\sin x\cos x}\,dx$ is given by

a) $\beta\left(\frac{1}{2}, \frac{1}{2}\right)$ b) $\beta\left(\frac{5}{4}, \frac{5}{4}\right)$

c) $\beta\left(\frac{3}{4}, \frac{3}{4}\right)$ d) $\beta\left(\frac{3}{2}, \frac{3}{2}\right)$

110) $\int_0^{\pi/2} \frac{1}{\sqrt{\sin x}}\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{1}{4}, \frac{3}{2}\right)$ b) $\beta\left(\frac{1}{4}, \frac{1}{2}\right)$

c) $2\beta\left(\frac{1}{4}, \frac{1}{2}\right)$ d) $\frac{1}{2}\beta\left(\frac{1}{4}, \frac{1}{2}\right)$

111) $\int_0^{\pi/2} \frac{1}{\sqrt{\cos x}}\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{1}{4}, \frac{3}{2}\right)$ b) $\frac{1}{2}\beta\left(\frac{1}{4}, \frac{1}{2}\right)$

c) $\beta\left(\frac{1}{4}, \frac{1}{2}\right)$ d) $2\beta\left(\frac{1}{4}, \frac{1}{2}\right)$

112) $\int_0^{\pi/2} \sqrt{\tan x}\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{3}{4}, \frac{5}{4}\right)$ b) $\beta\left(\frac{3}{4}, \frac{1}{4}\right)$

c) $2\beta\left(\frac{3}{4}, \frac{1}{4}\right)$ d) $\frac{1}{2}\beta\left(\frac{3}{4}, \frac{1}{4}\right)$

113) $\int_0^{\pi/2} \sqrt{\cot x}\,dx$ is given by

a) $2\beta\left(\frac{3}{4}, \frac{1}{4}\right)$ b) $\frac{1}{2}\beta\left(\frac{3}{4}, \frac{5}{4}\right)$

c) $\frac{1}{2}\beta\left(\frac{3}{4}, \frac{1}{4}\right)$ d) $\beta\left(\frac{3}{4}, \frac{1}{4}\right)$

114) $\int_0^{\pi/2} \tan^{3/4} x\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{7}{4}, \frac{1}{4}\right)$ b) $\frac{1}{2}\beta\left(\frac{7}{4}, \frac{1}{4}\right)$

c) $\frac{1}{2}\beta\left(\frac{7}{8}, \frac{1}{8}\right)$ d) $\frac{1}{2}\beta\left(\frac{7}{8}, \frac{7}{8}\right)$

115) The value of the integral $\int_0^\infty \frac{x^4}{(1+x)^7}dx$ is

a) $\frac{1}{30}$ b) 30 c) $\frac{1}{15}$ d) $\frac{1}{3}$

116) The value of the integral $\int_0^\infty \frac{x^3+x^2}{(1+x)^7}dx$ is

a) 30 b) $\frac{1}{3}$ c) $\frac{1}{30}$ d) $\frac{1}{15}$

117) The value of the integral $\int_0^\infty \frac{x^8-x^{14}}{(1+x)^{24}}dx$ is

a) 30 b) 0 c) $\frac{1}{30}$ d) $\frac{1}{15}$

118) The value of the integral $\int_0^\infty \frac{x^6(1-x^8)}{(1+x)^{24}}dx$ is

a) 30 b) 0 c) $\frac{1}{30}$ d) $\frac{1}{15}$

119) $\beta(n, n+1)$ is identical with

a) $\frac{(\Gamma n)^2}{\Gamma 2n}$ b) $\frac{\Gamma n}{\Gamma 2n}$ c) $\frac{\Gamma n}{2\Gamma 2n}$ d) $\frac{(\Gamma n)^2}{2\Gamma 2n}$

120) $\beta(m, n+1)+\beta(m+1, n)$ is equal to

a) $\beta(m+1, n+1)$ b) $\beta(m+1, n)$
c) $\beta(m, n)$ d) $\beta(m, n+1)$

121) $\beta(m, n)\cdot\beta(m+n, k)$ is equal to

a) $\frac{\Gamma m\cdot\Gamma n+k}{\Gamma m+n+k}$ b) $\frac{\Gamma m\cdot\Gamma n\cdot\Gamma k}{\Gamma m+n}$
c) $\frac{\Gamma m\cdot\Gamma n}{\Gamma m+n+k}$ d) $\frac{\Gamma m\cdot\Gamma n\cdot\Gamma k}{\Gamma m+n+k}$

122) $\beta(m, n+1)$ is equal to

a) $\frac{m+n}{n}\beta(m, n)$ b) $\frac{n}{m+n}\beta(m, n)$
c) $\frac{m}{m+n}\beta(m, n)$ d) $\frac{m+n}{m}\beta(m, n)$

123) On using substitution $x^3=8t$, the integral $\int_0^2 x(8-x^3)^{1/3}dx$ is equal to

a) $\frac{5}{81}$ b) $\frac{2}{27}$ c) $\frac{2}{81}$ d) $\frac{1}{81}$

124) The value of the integration $\int_0^1 x^3\left(1-x^{1/2}\right)^5 dx$ by substituting $x=t^2$ is given by

a) $2\beta(8, 6)$ b) $\frac{1}{2}\beta(8, 6)$
c) $\beta(8, 6)$ d) $2\beta(9, 7)$

125) The value of the integration $\int_0^1\left(1-x^{1/n}\right)^m dx$ by substituting $x=t^n$ is given by

a) $n\beta(m, n+1)$ b) $n\beta(m+1, n)$
c) $n\beta(m, n)$ d) $m\beta(m+1, n)$

Chapter 05–Differentiation Under Integral Sign & Error Function

I) Differentiation Under Integral Sign

1) If $I(\alpha)=\int_a^b f(x,\alpha)\,dx$, where α is parameter and a, b are constants, by differentiation under integral sign rule we have

a) $\frac{dI}{d\alpha}=\int_a^b \frac{\partial}{\partial\alpha}\left[f(x,\alpha)\right]\cdot dx$

b) $\frac{dI}{d\alpha}=\int_a^b \frac{\partial}{\partial x}\left[f(x,\alpha)\right]\cdot dx$

c) $\frac{dI}{dx}=\int_a^b \frac{\partial}{\partial x}\left[f(x,\alpha)\right]\cdot dx$

d) $\frac{dI}{dx}=\int_a^b \frac{\partial}{\partial\alpha}\left[f(x,\alpha)\right]\cdot dx$

2) If $I(\alpha)=\int_a^b f(x,\alpha)\,dx$, where α is parameter and a, b are functions of α, by differentiation under integral sign rule we have

a) $\frac{dI}{d\alpha}=\int_a^b \frac{\partial}{\partial\alpha}\left[f(x,\alpha)\right]\cdot dx-f(x,b)\frac{db}{d\alpha}+f(x,a)\frac{da}{d\alpha}$

b) $\frac{dI}{d\alpha}=\int_a^b \frac{\partial}{\partial\alpha}\left[f(x,\alpha)\right]\cdot dx+f(x,b)\frac{db}{dx}-f(x,a)\frac{da}{dx}$

c) $\frac{dI}{d\alpha}=\int_a^b \frac{\partial}{\partial\alpha}\left[f(x,\alpha)\right]\cdot dx+f(x,b)\frac{db}{d\alpha}-f(x,a)\frac{da}{d\alpha}$

d) $\frac{dI}{dx}=\int_a^b \frac{\partial}{\partial x}\left[f(x,\alpha)\right]\cdot dx+f(x,b)\frac{db}{dx}-f(x,a)\frac{da}{dx}$

Note: Henceforth, we abbreviate "differentiation under integral sign" by "DUIS" for simplicity.

3) If $I=\int_0^\infty e^{-bx^2}\cos 2ax\cdot dx$, where $b>0$, by DUIS rule we have

a) $\frac{dI}{dx}=\int_0^\infty \frac{\partial}{\partial x}\left[e^{-bx^2}\cos 2ax\right]\cdot dx$

b) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial a}\left[e^{-bx^2}\cos 2ax\right]\cdot dx$

c) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial b}\left[e^{-bx^2}\cos 2ax\right]\cdot dx$

d) $\frac{dI}{db}=\int_0^\infty \frac{\partial}{\partial a}\left[e^{-bx^2}\cos 2ax\right]\cdot dx$

4) If $I=\int_0^\infty \frac{e^{-ax}}{x}\left(1-e^{-bx}\right)dx$, where $a,b>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial b}\left[\frac{e^{-ax}}{x}\left(1-e^{-bx}\right)\right]dx$

b) $\frac{dI}{dx}=\int_0^\infty \frac{\partial}{\partial x}\left[\frac{e^{-ax}}{x}\left(1-e^{-bx}\right)\right]dx$

c) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial a}\left[\frac{e^{-ax}}{x}\left(1-e^{-bx}\right)\right]dx$

d) $\frac{dI}{db}=\int_0^\infty \frac{\partial}{\partial a}\left[\frac{e^{-ax}}{x}\left(1-e^{-bx}\right)\right]dx$

5) If $I=\int_0^\infty \frac{e^{-ax}}{x}\left(1-e^{-x}\right)dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial a}\left[\frac{\left(1-e^{-x}\right)}{x}\right]\cdot e^{-ax}dx$

b) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial x}\left[e^{-ax}\right]\cdot\frac{\left(1-e^{-x}\right)}{x}dx$

c) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial x}\left[e^{-ax}\frac{\left(1-e^{-x}\right)}{x}\right]\cdot dx$

d) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial a}\left[e^{-ax}\right]\cdot\frac{\left(1-e^{-x}\right)}{x}dx$

6) If $I=\int_0^\infty \frac{e^{-x}}{x}\left(a-\frac{1}{x}+\frac{1}{x^2}e^{-ax}\right)dx$, where $a,b>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty \frac{e^{-x}}{x}\cdot\frac{\partial}{\partial x}\left(a-\frac{1}{x}+\frac{1}{x^2}e^{-ax}\right)dx$

b) $\frac{dI}{dx}=\int_0^\infty \frac{e^{-x}}{x}\cdot\frac{\partial}{\partial a}\left(a-\frac{1}{x}+\frac{1}{x^2}e^{-ax}\right)dx$

c) $\frac{dI}{dx}=\int_0^\infty \frac{e^{-x}}{x}\cdot\frac{\partial}{\partial x}\left(a-\frac{1}{x}+\frac{1}{x^2}e^{-ax}\right)dx$

d) $\frac{dI}{da}=\int_0^\infty \frac{e^{-x}}{x}\cdot\frac{\partial}{\partial a}\left(a-\frac{1}{x}+\frac{1}{x^2}e^{-ax}\right)dx$

7) If $I=\int_0^\infty \frac{e^{-ax}}{x}\left(1-e^{-x}\right)dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty e^{(a+1)x}dx$ b) $\frac{dI}{da}=\int_0^\infty e^{-ax}dx$

c) $\frac{dI}{da}=\int_0^\infty e^{-(a+1)x}dx$ d) $\frac{dI}{da}=\int_0^\infty e^{-(a-1)x}dx$

8) If $I=\int_0^\infty \frac{e^{-x}}{x}\left(a-\frac{1}{x}+\frac{1}{x^2}e^{-ax}\right)dx$, where $a,b>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty \frac{e^{-x}}{x}\left(1-\frac{1}{x}e^{-ax}\right)dx$

b) $\frac{dI}{da}=\int_0^\infty \frac{e^{-x}}{x}\left(1-e^{-ax}\right)dx$

c) $\frac{dI}{da}=\int_0^\infty \frac{e^{-x}}{x}\left(1-\frac{1}{x^2}e^{-ax}\right)dx$

d) $\frac{dI}{da}=\int_0^\infty \left(1-\frac{1}{x}e^{-ax}\right)dx$

9) If $I=\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x}dx$, where $a,b>0$, by DUIS rule we have

a) $\frac{dI}{db}=-\int_0^\infty e^{-ax}dx$ b) $\frac{dI}{da}=-\int_0^\infty e^{-ax}dx$

c) $\frac{dI}{da}=\int_0^\infty e^{-ax}dx$ d) $\frac{dI}{da}=\int_0^\infty \left(e^{-ax}-e^{-ax}\right)dx$

10) If $I=\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x}dx$, where $a,b>0$, by DUIS rule we have

a) $\frac{dI}{db}=\int_0^\infty \left(e^{-ax}-e^{-ax}\right)dx$ b) $\frac{dI}{db}=-\int_0^\infty e^{-bx}dx$

c) $\frac{dI}{db}=\int_0^\infty e^{-ax}dx$ d) $\frac{dI}{db}=\int_0^\infty e^{-bx}dx$

11) If $I=\int_0^\infty \frac{e^{-x}-e^{-ax}}{x\sec x}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty \frac{e^{-ax}}{\sec x}dx$ b) $\frac{dI}{da}=\int_0^\infty \frac{e^{-ax}}{\sec x\tan x}dx$

c) $\frac{dI}{da}=-\int_0^\infty \frac{e^{-ax}}{\sec x}dx$ d) $\frac{dI}{da}=-\int_0^\infty \frac{ae^{-ax}}{x\sec x}dx$

12) If $I=\int_0^\infty e^{-a^2}\cos ax\,da$, where $x>0$, by DUIS rule we have

a) $\frac{dI}{dx}=-2\int_0^\infty a^2e^{-a^2}\sin ax\,da$

b) $\frac{dI}{dx}=2\int_0^\infty ae^{-a^2}\sin ax\,da$

c) $\frac{dI}{dx}=-2\int_0^\infty ae^{-a^2}\cos ax\,da$

d) $\frac{dI}{dx}=-\int_0^\infty ae^{-a^2}\sin ax\,da$

13) If $I=\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}\frac{1}{x^2}dx$

b) $\frac{dI}{da}=a\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}\frac{1}{x^2}dx$

c) $\frac{dI}{da}=-2a\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}\frac{1}{x^2}dx$

d) $\frac{dI}{da}=-\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}\frac{1}{x^2}dx$

14) If $I=\int_0^\infty \frac{e^{-x}\sin ax}{x}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=-a\int_0^\infty \cos ax\,dx$ b) $\frac{dI}{da}=\int_0^\infty \sin ax\,dx$

c) $\frac{dI}{da}=-\int_0^\infty e^{-x}\cos ax\,dx$ d) $\frac{dI}{da}=\int_0^\infty e^{-x}\cos ax\,dx$

15) If $I=\int_0^\pi \frac{x^a-1}{\log x}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty \frac{x^a\log a}{\log x}dx$ b) $\frac{dI}{da}=\int_0^\pi x^a\,dx$

c) $\frac{dI}{da}=\int_0^\pi x^a\log a\,dx$ d) $\frac{dI}{da}=\int_0^\pi \frac{x^a\log a}{\log x}dx$

16) If $I=\int_0^1 \frac{x^a-x^b}{\log x}dx$, where $a,\ b>0$, by DUIS rule we have

a) x^a-x^b b) $\frac{dI}{da}=\int_0^\pi \frac{x^a-x^b}{x\log x}dx$

c) $\frac{dI}{da}=\int_0^1 \frac{x^a\log a}{\log x}dx$ d) $\frac{dI}{da}=\int_0^1 x^a\,dx$

17) If $I=\int_0^\pi \log(1+a\cos x)\,dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\pi \frac{-\sin x}{1+a\cos x}dx$ b) $\frac{dI}{da}=\int_0^\pi \frac{\cos x}{1+a\cos x}dx$

c) $\frac{dI}{da}=\int_0^\pi \frac{a}{1+a\cos x}dx$ d) $\frac{dI}{da}=-\int_0^\pi \frac{\cos x}{1+a\cos x}dx$

18) If $I=\int_0^\pi \frac{1}{x^2}\log(1+ax^2)\,dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\pi \frac{ax^2}{1+ax^2}dx$ b) $\frac{dI}{da}=2\int_0^\pi \frac{x}{1+ax^2}dx$

c) $\frac{dI}{da}=\int_0^\pi \frac{1}{1+ax^2}dx$ d) $\frac{dI}{da}=\int_0^\pi \frac{2ax}{1+ax^2}dx$

19) If $I=\int_0^\pi \frac{1}{\sin^2 x}\log(1+a\sin^2 x)\,dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\pi \frac{1}{1+a\sin^2 x}dx$ b) $\frac{dI}{da}=\int_0^\pi \frac{\sin 2x}{1+a\sin^2 x}dx$

c) $\frac{dI}{da}=\int_0^\pi \frac{a}{1+a\sin^2 x}dx$ d) $\frac{dI}{da}=\int_0^\pi \frac{\cos x}{1+a\sin^2 x}dx$

20) If $I=\int_0^\infty \frac{1-\cos ax}{x^2}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty \frac{a\sin ax}{x^2}dx$ b) $\frac{dI}{da}=-\int_0^\infty \frac{\cos ax}{x}dx$

c) $\frac{dI}{da}=\int_0^\infty \frac{\sin ax}{x}dx$ d) $\frac{dI}{da}=-\int_0^\infty \frac{\sin ax}{x}dx$

21) If $I=\int_0^1 \frac{x^a}{\log x}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^1 \frac{x^a\log a}{\log x}dx$ b) $\frac{dI}{da}=\int_0^1 x^a\,dx$

c) $\frac{dI}{da}=\int_0^1 x^a\log a\,dx$ d) $\frac{dI}{da}=\int_0^1 x^{a-1}\,dx$

22) If $I=\int_0^{\pi/2} \log(a^2\cos^2 x+b^2\sin^2 x)\,dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{\pi/2} \frac{1}{a^2+b^2\tan^2 x}dx$

b) $\frac{dI}{da}=\int_0^{\pi/2} \frac{b^2}{a^2+b^2\tan^2 x}dx$

c) $\frac{dI}{da}=\int_0^{\pi/2} \frac{a^2}{a^2+b^2\tan^2 x}dx$

d) $\frac{dI}{da}=\int_0^{\pi/2} \frac{2a}{a^2+b^2\tan^2 x}dx$

23) If $I=\int_0^{\infty}\frac{\sin ax-\sin bx}{x^2}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=-\int_0^{\infty}\frac{\cos bx}{x}dx$ b) $\frac{dI}{da}=\int_0^{\infty}\frac{\cos ax}{x}dx$

c) $\frac{dI}{da}=-\int_0^{\infty}\frac{\cos ax}{x}dx$ d) $\frac{dI}{db}=\int_0^{\infty}\frac{\cos ax}{x}dx$

24) If $I=\int_0^{a^2}\tan^{-1}\left(\frac{x}{a}\right)dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{a^2}\frac{\partial}{\partial a}\tan^{-1}\left(\frac{x}{a}\right)dx$

b) $\frac{dI}{da}=\int_0^{a^2}\frac{\partial}{\partial a}\tan^{-1}\left(\frac{x}{a}\right)dx-2a\tan^{-1}a$

c) $\frac{dI}{da}=\int_0^{a^2}\frac{\partial}{\partial a}\tan^{-1}\left(\frac{x}{a}\right)dx+2a\tan^{-1}x$

d) $\frac{dI}{da}=\int_0^{a^2}\frac{\partial}{\partial a}\tan^{-1}\left(\frac{x}{a}\right)dx+2a\tan^{-1}a$

25) If $I=\int_0^{a}\frac{\log(1+ax)}{1+x^2}dx$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{a}\frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right]dx+\frac{\log(1+a^2)}{1+a^2}$

b) $\frac{dI}{da}=\int_0^{a}\frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right]dx-\frac{\log(1+a^2)}{1+a^2}$

c) $\frac{dI}{da}=\int_0^{a}\frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right]dx$

d) $\frac{dI}{da}=\int_0^{a}\frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right]dx+\frac{\log(1+x^2)}{1+x^2}$

26) If $I=\int_a^{a^2}\log(ax)dx$, by DUIS rule we have

a) $\frac{dI}{da}=\int_a^{a^2}\frac{\partial}{\partial a}\log(ax)dx$

b) $\frac{dI}{da}=\int_a^{a^2}\frac{\partial}{\partial a}\log(ax)dx-(6a-2)\log a$

c) $\frac{dI}{da}=\int_a^{a^2}\frac{\partial}{\partial a}\log(ax)dx+(6a+2)\log a$

d) $\frac{dI}{da}=\int_a^{a^2}\frac{\partial}{\partial a}\log(ax)dx+(6a-2)\log a$

27) If $I=\int_t^{t^2}e^{tx^2}\,dx$, by DUIS rule we have

a) $\frac{dI}{dt}=\int_t^{t^2}x^2e^{tx^2}\,dx+2te^{t^5}-e^{t^3}$

b) $\frac{dI}{dt}=\int_t^{t^2}x^2e^{tx^2}\,dx-2te^{t^5}+e^{t^3}$

c) $\frac{dI}{dt}=\int_t^{t^2}te^{tx^2}\,dx+2te^{t^5}-e^{t^3}$

d) $\frac{dI}{dt}=\int_t^{t^2}t^3e^{tx^2}\,dx+2te^{t^5}-e^{t^3}$

28) If $I=\int_0^{a^2}\tan^{-1}\left(\frac{x}{a}\right)dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{a^2}\frac{x}{a^2+x^2}dx+2a\tan^{-1}a$

b) $\frac{dI}{da}=-\int_0^{a^2}\frac{a}{a^2+x^2}dx+2a\tan^{-1}a$

c) $\frac{dI}{da}=-\int_0^{a^2}\frac{x}{a^2+x^2}dx+2a\tan^{-1}a$

d) $\frac{dI}{da}=-\int_0^{a^2}\frac{x}{a^2+x^2}dx-2a\tan^{-1}a$

29) If $I=\int_a^{a^2}\log(ax)dx$, by DUIS rule we have

a) $\frac{dI}{da}=\int_a^{a^2}\frac{1}{x}dx-(6a-2)\log a$

b) $\frac{dI}{da}=\int_a^{a^2}\frac{1}{a}dx+(6a-2)\log a$

c) $\frac{dI}{da}=\int_a^{a^2}\frac{1}{a}dx-(6a-2)\log a$

d) $\dfrac{dI}{da}=\displaystyle\int_a^{a^2}\frac{1}{a}dx-(6a-2)\log a$

30) If $I=\displaystyle\int_0^a\frac{\log(1+ax)}{1+x^2}dx$, by DUIS rule we have

a) $\dfrac{dI}{da}=\displaystyle\int_0^a\frac{x}{(1+x^2)(1+ax)}dx+\frac{\log(1+a^2)}{1+a^2}$

b) $\dfrac{dI}{da}=\displaystyle\int_0^a\frac{1}{(1+x^2)(1+ax)}dx+\frac{\log(1+a^2)}{1+a^2}$

c) $\dfrac{dI}{da}=\displaystyle\int_0^a\frac{a}{(1+x^2)(1+ax)}dx+\frac{\log(1+a^2)}{1+a^2}$

d) $\dfrac{dI}{da}=\displaystyle\int_0^a\frac{x}{(1+x^2)(1+ax)}dx-\frac{\log(1+a^2)}{1+a^2}$

31) If $I=\displaystyle\int_{\pi/6a}^{\pi/3a}\frac{\sin ax}{x}dx$, by DUIS rule we have

a) $\dfrac{dI}{da}=\displaystyle\int_{\pi/6a}^{\pi/3a}\cos ax\,dx+\frac{1}{a}$

b) $\dfrac{dI}{da}=\displaystyle\int_{\pi/6a}^{\pi/3a}\cos ax\,dx-\frac{1}{2a}$

c) $\dfrac{dI}{da}=\displaystyle\int_{\pi/6a}^{\pi/3a}\cos ax\,dx-\frac{1}{a}$

d) $\dfrac{dI}{da}=\displaystyle\int_{\pi/6a}^{\pi/3a}\frac{\cos ax}{x}dx-\frac{1}{a}$

32) If $f(x)=\displaystyle\int_a^x(x-t)^2G(t)\,dt$, we have

a) $\dfrac{df}{dx}=\displaystyle\int_a^x\frac{\partial}{\partial x}(x-t)^2G(t)\,dt+(x-a)^2G(a)$

b) $\dfrac{df}{dx}=\displaystyle\int_a^x\frac{\partial}{\partial x}(x-t)^2G(t)\,dt$

c) $\dfrac{df}{dx}=\displaystyle\int_a^x\frac{\partial}{\partial x}(x-t)^2G(t)\,dt-(x-a)^2G(a)$

d) $\dfrac{df}{dx}=\displaystyle\int_a^x\frac{\partial}{\partial x}(x-t)^2G(t)\,dt+a^2G(a)$

33) If $y=\displaystyle\int_0^x f(t)\sin a(x-t)\,dt$, we have

a) $\dfrac{dy}{dx}=\displaystyle\int_0^x xf(t)\cos a(x-t)\,dt$

b) $\dfrac{dy}{dx}=\displaystyle\int_0^x af(t)\cos a(x-t)\,dt+f(x)$

c) $\dfrac{dy}{dx}=\displaystyle\int_0^x af(t)\cos a(x-t)\,dt-af(x)$

d) $\dfrac{dy}{dx}=a\displaystyle\int_0^x f(t)\cos a(x-t)\,dt$

34) For the integral $I(a)=\displaystyle\int_0^\infty\frac{e^{-x}}{x}(1-e^{-ax})dx$, we have $\dfrac{dI}{da}=\dfrac{1}{a+1}$, then I is

a) $\log(a+1)-1$ b) $\log(a+1)$

c) $\log(a+1)+1$ d) $-\dfrac{1}{(a+1)^2}$

35) The value of integration $I(a)=\displaystyle\int_0^1\frac{x^a-1}{\log x}dx$ with $\dfrac{dI}{da}=\dfrac{1}{a+1}$ is given by

a) $\log(a+1)$ b) $\log(a+1)-1$

c) $\log(a+1)+1$ d) $-\dfrac{1}{(a+1)^2}$

36) The value of integration $I(a)=\displaystyle\int_0^1\frac{e^{-2x}\sin ax}{x}dx$ with $\dfrac{dI}{da}=\dfrac{2}{a^2+4}$ is given by

a) $\tan^{-1}\left(\dfrac{a}{2}\right)+\dfrac{\pi}{2}$ b) $\tan^{-1}\left(\dfrac{a}{2}\right)$

c) $\dfrac{1}{2}\tan^{-1}\left(\dfrac{a}{2}\right)$ d) $\dfrac{1}{2}\tan^{-1}\left(\dfrac{x}{2}\right)$

37) The value of integration $I(a)=\displaystyle\int_0^\infty\frac{e^{-x}-e^{-ax}}{x\sec x}dx$ with $\dfrac{dI}{da}=\dfrac{a}{a^2+1}$ is given by

a) $2\log\left(\dfrac{2}{a^2+1}\right)$ b) $\dfrac{1}{2}\log\left(\dfrac{2}{a^2+1}\right)$

c) $\frac{1}{2}\log\left(\frac{a^2+1}{2}\right)$ d) $2\log\left(\frac{a^2+1}{2}\right)$

38) The value of integration $I(a)=\int_0^{\infty}\frac{1-\cos ax}{x^2}dx$ with $\frac{dI}{da}=\frac{\pi}{2}$ is given by

a) $2\pi a$ b) $\frac{\pi a}{3}$ c) $\frac{\pi}{2}$ d) $\frac{\pi a}{2}$

39) The value of integration $I=\int_0^{\infty}\frac{\log(1+ax^2)}{x^2}dx$, with $\frac{dI}{da}=\frac{\pi}{2\sqrt{a}}$ is given by

a) $\pi\sqrt{a}$ b) $2\sqrt{a}$ c) $\pi\sqrt{2}$ d) $a\sqrt{\pi}$

40) The value of integration $I=\int_0^{\pi/2}\frac{\log(1+a\sin^2 x)}{\sin^2 x}dx$, with $\frac{dI}{da}=\frac{\pi}{2\sqrt{a+1}}$ is given by

a) $\pi\sqrt{a+1}+\pi$ b) $\pi\sqrt{a+1}-\pi$

c) $\pi\sqrt{a+1}-\frac{\pi}{a}$ d) $\frac{\pi\sqrt{a+1}-\pi}{a}$

II) Error Functions

41) $erf(x)$ is given by

a) $\frac{1}{2\sqrt{\pi}}\int_0^x e^{-u^2}du$ b) $\frac{\sqrt{\pi}}{2}\int_0^x e^{-u^2}du$

c) $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$ d) $\int_0^x e^{-u^2}du$

42) $erfc(x)$ is given by

a) $\frac{2}{\sqrt{\pi}}\int_x^{\infty} e^{-u^2}du$ b) $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$

c) $\frac{\sqrt{\pi}}{2}\int_0^x e^{-u^2}du$ d) $\frac{\sqrt{\pi}}{2}\int_x^{\infty} e^{-u^2}du$

43) $erf(0)$ is given by

a) $\frac{2}{\sqrt{\pi}}$ b) 1 c) ∞ d) 0

44) $erf(\infty)$ is given by

a) 1 b) 0 c) $\frac{2}{\sqrt{\pi}}$ d) ∞

45) $erfc(0)$ is given by

a) 0 b) $\frac{2}{\sqrt{\pi}}$ c) ∞ d) 1

46) $erf(x)+erfc(x)=?$

a) 2 b) ∞ c) 1 d) 0

47) $erf(-x)=?$

a) $erfc(x)$ b) $-erf(x)$

c) $erf(x)$ d) $-erf(x^2)$

48) Error function is an

a) even function b) neither even nor odd

c) odd function d) none of these

49) $erf(x)+erf(-x)=?$

a) 0 b) 1 c) 2 d) 3

50) $erf(-x)+erfc(-x)=?$

a) 0 b) 3 c) 2 d) 1

51) $erfc(-x) - erf(x) = ?$

a) ∞ b) 2 c) 1 d) 0

52) $erfc(x) + erfc(-x) = ?$

a) 2 b) 1 c) 0 d) ∞

53) If $erf(ax) = \frac{2}{\sqrt{\pi}} \int_0^{ax} e^{-u^2} du$, then $\frac{d}{dx}[erf(ax)]$ is

a) $\frac{2a}{\sqrt{\pi}} e^{-x^2}$ b) $\frac{a}{2\sqrt{\pi}} e^{-a^2x^2}$

c) $\frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$ d) $\frac{2x}{\sqrt{\pi}} e^{-a^2x^2}$

54) If $erfc(ax) = \frac{2}{\sqrt{\pi}} \int_{ax}^{\infty} e^{-u^2} du$, then $\frac{d}{dx}[erfc(ax)]$ is

a) $-\frac{2x}{\sqrt{\pi}} e^{-a^2x^2}$ b) $\frac{2x}{\sqrt{\pi}} e^{-a^2x^2}$

c) $-\frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$ d) $\frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$

55) If $erf(\sqrt{t}) = \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{t}} e^{-u^2} du$, then $\frac{d}{dx}[erf(\sqrt{t})]$ is

a) $\frac{1}{t\sqrt{\pi}} e^{-t}$ b) $\frac{1}{\sqrt{\pi t}} e^{-t}$

c) $\frac{2}{\sqrt{\pi t}} e^{-t}$ d) $\frac{1}{\sqrt{\pi t}} e^{-t^2}$

56) If $erfc(\sqrt{t}) = \frac{2}{\sqrt{\pi}} \int_{\sqrt{t}}^{\infty} e^{-u^2} du$, then $\frac{d}{dx}[erfc(\sqrt{t})]$ is

a) $\frac{2}{\sqrt{\pi t}} e^{-t}$ b) $\frac{1}{\sqrt{\pi t}} e^{-t^2}$

c) $\frac{1}{t\sqrt{\pi}} e^{-t}$ d) $-\frac{1}{\sqrt{\pi t}} e^{-t}$

57) $\frac{d}{dx}[erf(x) + erfc(x)] = ?$

a) 1 b) 0 c) 2 d) ∞

58) If $\frac{d}{dx}[erf(ax)] = \frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$, then $\frac{d}{dx}[erfc(ax)]$ is

a) $-\frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$ b) $\frac{2a}{\sqrt{\pi}} e^{a^2x^2}$

c) $\frac{1}{\sqrt{\pi}} e^{-a^2x^2}$ d) $\frac{4a^2}{\sqrt{\pi}} e^{-a^2x^2}$

59) On substitution $x + a = u$ in the integration $\int_0^{\infty} e^{-(x+a)^2} dx$, then the value of integration is

a) $\frac{\sqrt{\pi}}{2} erf(a)$ b) $\frac{2}{\sqrt{\pi}} erf(a)$

c) $\frac{\sqrt{\pi}}{2} erfc(a)$ d) $erfc(a)$

60) $\int_0^t erf(ax)\,dx + \int_0^t erfc(ax)\,dx = ?$

a) 1 b) ∞ c) 0 d) t

61) If $\frac{dy}{dx}[erf(\sqrt{t})] = \frac{e^{-t}}{\sqrt{\pi t}}$, the integration $\int_0^{\infty} e^{-t} erf(\sqrt{t})\,dt$ is

a) $\frac{1}{\sqrt{\pi}} \int_0^{\infty} e^{-t} t^{1/2}\,dt$ b) $\frac{2}{\sqrt{\pi}} \int_0^{\infty} e^{-2t} t^{1/2}\,dt$

c) $-\frac{1}{\sqrt{\pi}} \int_0^{\infty} e^{-2t} t^{-1/2}\,dt$ d) $\frac{1}{\sqrt{\pi}} \int_0^{\infty} e^{-2t} t^{-1/2}\,dt$

62) The power series expansion of $erf(x)$ is

a) $\frac{2}{\sqrt{\pi}}\left[x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + - \dots\right]$

b) $\frac{2}{\sqrt{\pi}}\left[x + \frac{x^3}{3} + \frac{x^5}{10} + \frac{x^7}{42} + \dots\right]$

c) $\frac{2}{\sqrt{\pi}}\left[x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + - \dots\right]$

d) $\frac{2}{\sqrt{\pi}}\left[x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots\right]$

Chapter 06 – Curve Tracing & Rectification of Curves

I) Curve Tracing

1) If the portion of the curve lies on the both sides of the point lying above the tangent at that point, the curve is known as
a) concave upward b) concave downward
c) inflexion point d) none of these

2) If the portion of the curve lies on the both sides of the point lying above the tangent at that point, the curve is known as
a) inflexion point b) concave downward
c) inflexion point d) none of these

3) A point through which two branches of the same curve passes is known as
a) double point b) inflexion point
c) multiple point d) conjugate point

4) A point through which many branches of the same curve passes is known as
a) double point b) inflexion point
c) multiple point d) conjugate point

5) A double point through which the branches of the curve passes and the tangent at that point are real and distinct, the point is known as
a) conjugate point b) node
c) point of inflexion d) cusp

6) A double point through which the branches of the curve passes and the tangent at that point are real but the same, the point is known as
a) conjugate point b) point of inflexion
c) cusp d) node

7) A double point is said to be node if the tangents to the curve at that point are
a) imaginary b) perpendicular to each other
c) real but the same d) real and distinct

8) A double point is said to be cusp if the tangents at that point are
a) imaginary b) real and distinct
c) real but the same d) none of these

9) If at a point $\frac{dy}{dx}=0$, the tangent to the curve at that point is
a) parallel to the line $x+y=0$
b) parallel to x-axis
c) perpendicular to x-axis
d) parallel to $y=x$

10) If at a point $\frac{dy}{dx}=\infty$, the tangent to the curve at that point is
a) parallel to the line $x+y=0$
b) parallel to $y=x$
c) parallel to x-axis
d) perpendicular to x-axis

11) The standard equation of x-axis in the Cartesian form is given by
a) $x-y=0$ b) $x+y=0$
c) $y=0$ d) $x=0$

12) The standard equation of y-axis in the Cartesian form is given by
a) $x-y=0$ b) $x+y=0$
c) $y=0$ d) $x=0$

13) If the all the powers of y in the Cartesian form are even, the curve is symmetrical about
a) $y-axis$ b) $x,\ y-axes$
c) $x-axis$ d) the line $y=x$

14) If the all the powers of x in the Cartesian form are even, the curve is symmetrical about
a) $x,\ y-axes$ b) $y-axis$
c) $x-axis$ d) the line $y=x$

15) If the all the powers of x and y in the Cartesian form are even, the curve is symmetrical about
a) the line $y=x$ b) $x-axis$ only
c) $y-axis$ only d) $x,\ y-axes$

16) If in the equation of the Cartesian form by replacing $x\to y$ and $y\to x$, the equation is symmetrical about
a) the line $y=x$ b) $x,\ y-axes$

c) $x-axis$ d) $y-axis$

17) If in the equation of the Cartesian form by replacing $x \to -y$ and $y \to -x$, the equation is symmetrical about
a) the line $y=-x$ b) the line $y=x$
c) $x,\ y-axes$ d) $y-axis$ only

18) If in the equation of the Cartesian form by replacing $x \to -x$ and $y \to -y$, the equation is symmetrical about
a) the line $y=-x$ b) $x,\ y-axes$
c) opposite quadrants d) the line $y=x$

19) The equation of the tangent at origin when the curve is passing through origin is obtained by equating to zero
a) the lowest degree term of the equation
b) the highest degree term of x in equation
c) the highest degree term of y in equation
d) the coefficient of the term xy

20) In the Cartesian form, the asymptote to the curve parallel to x-axis may be obtained by equating to zero
a) the coefficient of lowest degree term in y
b) the coefficient of highest degree term in y
c) the coefficient of highest degree term in x
d) the coefficient of lowest degree term in x

21) In the Cartesian form, the asymptote to the curve parallel to y-axis may be obtained by equating to zero
a) the coefficient of lowest degree term in y
b) the coefficient of highest degree term in y
c) the coefficient of highest degree term in x
d) the coefficient of lowest degree term in x

22) Oblique asymptote are obtained only when the curve is
a) symmetrical about x-axis
b) symmetrical about y-axis
c) symmetrical about both x and y-axis
d) not symmetrical about both x and y-axes

23) In the Cartesian form if the coefficient of the highest degree term in x is constant, the curve has
a) no asymptote parallel to $x=y$
b) no asymptote parallel to y-axis
c) no asymptote parallel to x-axis
d) none of these

24) In the Cartesian form if the coefficient of the highest degree term in y is constant, the curve has
a) no asymptote parallel to $x+y=0$
b) no asymptote parallel to $x=y$
c) no asymptote parallel to x-axis
d) no asymptote parallel to y-axis

25) In the polar form, if the equation of the curve remains unchanged by replacing $\theta \to -\theta$, the curve is symmetrical about
a) the line $\theta=\dfrac{\pi}{4}$ b) the initial line
c) pole d) the line $\theta=\dfrac{\pi}{2}$

26) In the polar form, if the equation of the curve remains unchanged by replacing $r \to -r$, the curve is symmetrical about
a) the line $\theta=\dfrac{\pi}{4}$ b) the initial line
c) pole d) the line $\theta=\dfrac{\pi}{2}$

27) In the polar form, if the equation of the curve remains unchanged by replacing $\theta \to \pi-\theta$, the curve is symmetrical about
a) the line $\theta=\dfrac{\pi}{2}$ b) the line $\theta=\dfrac{\pi}{4}$
c) the initial line d) pole

28) The pole is point of the curve, if for given angle θ, the value of
a) $r=\infty$ b) $r=0$ c) $r<0$ d) $r>0$

29) If a curve is passing through the pole, the tangent to the curve at pole are obtained by solving
a) $r=0$ b) $r=\infty$ c) $\theta=0$ d) $\theta=\pi$

30) In the polar form, the relation between the angle ϕ formed by the radius vector and the tangent to the curve at that point, is given by
a) $\tan\phi=r^2\dfrac{d\theta}{dr}$ b) $\cot\phi=r\dfrac{d\theta}{dr}$
c) $\tan\phi=r\dfrac{d\theta}{dr}$ d) $\tan\phi=r\dfrac{dr}{d\theta}$

31) In the parametric form $x = f(t)$, $y = g(t)$, the curve is symmetrical about y-axis, if
   a) $x = f(t)$ is odd and $y = g(t)$ is even
   b) $x = f(t)$ is even and $y = g(t)$ is odd
   c) $x = f(t)$ is odd and $y = g(t)$ is odd
   d) $x = f(t)$ is even and $y = g(t)$ is even

32) In the parametric form $x = f(t)$, $y = g(t)$, the curve is symmetrical about y-axis, if
   a) $x = f(t)$ is odd and $y = g(t)$ is odd
   b) $x = f(t)$ is even and $y = g(t)$ is odd
   c) $x = f(t)$ is odd and $y = g(t)$ is even
   d) $x = f(t)$ is even and $y = g(t)$ is even

33) The curve represented by the equation $x^2y^2 = x^2 + 1$ is symmetrical about
   a) the line $y = x$ b) x-axis only
   c) y-axis only d) both x and y-axes

34) The curve represented by the equation $x(x^2 + y^2) = a(x^2 - y^2)$ is
   a) symmetrical about y-axis but not passing through origin
   b) symmetrical about y-axis and passing through origin
   c) symmetrical about x-axis but not passing through origin
   d) symmetrical about x-axis and passing through origin

35) The curve represented by the equation $a^2y^2 = x^2(a^2 - x^2)$ is
   a) symmetrical about both x and y-axis but not passing through origin
   b) symmetrical about both x and y-axis and passing through origin
   c) symmetrical about x-axis but not passing through origin
   d) symmetrical about x-axis only and passing through origin

36) The curve represented by the equation $(2a - x)y^2 = x^3$ is
   a) symmetrical about y-axis and passing through origin
   b) symmetrical about x-axis but not passing through origin
   c) symmetrical about x-axis and passing through origin
   d) symmetrical about y-axis not passing through origin

37) The curve represented by the equation $(2a - y)y^3 = a^2x^2$ is
   a) symmetrical about y-axis and passing through origin and $(0, 2a)$
   b) symmetrical about x-axis but not passing through origin and $(0, 2a)$
   c) symmetrical about x-axis and passing through origin and $(0, 2a)$
   d) symmetrical about y-axis not passing through origin and $(0, 2a)$

38) The curve represented by the equation $xy^2 = 4a^2(a - x)$ is
   a) symmetrical about y-axis and passing through $(a, 0)$
   b) symmetrical about x-axis but not passing through $(a, 0)$
   c) symmetrical about x-axis and passing through $(a, 0)$
   d) symmetrical about y-axis not passing through $(a, 0)$

39) The curve represented by the equation $xy^2 = 4a^2(a - x)$ has at origin
   a) node b) cusp c) inflexion d) none

40) The curve represented by the equation $(2a - x)y^2 = x^3$ has the tangent at origin whose equation is
   a) $x + y = 0$ b) y-axis c) x-axis d) $y = x$

41) The curve represented by the equation $(1 + x^2)y = x$ has the tangent at origin whose equation is
   a) $y = x$ b) x-axis c) y-axis d) $x + y = 0$

42) The curve represented by the equation $3ay^2 = x(x - a)^2$ has the tangent at origin whose equation is
   a) $x + y = 0$ b) $y = x$ c) x-axis d) y-axis

43) The curve represented by the equation $3ay^2 = x(x-a)^2$ has the asymptote parallel to x-axis whose equation is
a) $x+y=0$ b) $y=x$ c) x-axis d) y-axis

44) For the curve given by equation $x^2y = 4a^2(2a-y)$, the asymptote is
a) $y=2a$ b) $y=x$ c) y-axis d) x-axis

45) The curve represented by the equation $y^2(4-x) = x(x-2)^2$ has the asymptote parallel to y-axis whose equation is
a) $x=y$ b) $x=0$ c) $x=2$ d) $x=4$

46) The curve represented by the equation $x^2y^2 = a^2(y^2-x^2)$ has the asymptote parallel to y-axis whose equation is
a) $x=0$ b) $x=\pm a$ c) $x=y$ d) $y=0$

47) For the curve given by equation $x^2y = 4a^2(2a-y)$, the region of absence is
a) $0<y<2a$ b) $y>0,\ y>2a$
c) $y<0,\ y<2a$ d) $y<0,\ y>2a$

48) For the curve given by equation $x^3 = 4y^2(2a-x)$, the region of absence is
a) $0<x<2a$ b) $x<0,\ x>2a$
c) $x>0,\ x>2a$ d) $x<0,\ x<2a$

49) For the curve given by equation $xy^2 = 4a^2(a-x)$, the region of absence is
a) $0<x<a$ b) $x>0,\ x>a$
c) $x<0,\ x>a$ d) $x<0,\ x<a$

50) For the curve given by equation $y^2 = \frac{4x^2(a-x)}{x+a}$, the region of absence along x-axis is
a) $[-\infty,\ -a]$ & $[a,\ \infty]$ b) $[-\infty,\ a]$ & $[-a,\ \infty]$
c) $[-\infty,\ -a]$ d) $[-a,\ \infty]$

51) The curve represented by the equation $x^{1/2} + y^{1/2} = a^{1/2}$ is symmetrical about
a) $y=x$ b) x-axis c) y-axis d) $x+y=0$

52) The curve represented by the equation $\sqrt{x}+\sqrt{y}=\sqrt{a}$ has tangent at origin whose equation is
a) x-axis b) no tangent exists
c) y-axis d) $x+y=0$

53) The curve represented by the equation $\sqrt{x}+\sqrt{y}=\sqrt{a}$ has tangent at $(a,\ 0)$ which is
a) the line $x+y=0$ b) the line $y=x$
c) parallel to y-axis d) parallel to x-axis

54) The curve represented by the equation $x=t^2,\ y=t-\frac{t^3}{3}$ is symmetrical about
a) symmetrical about y-axis but not passing through origin
b) symmetrical about y-axis and passing through origin
c) symmetrical about x-axis but not passing through origin
d) symmetrical about x-axis and passing through origin

55) The curve represented by the equation $x=a(\theta+\sin\theta),\ y=a(1+\cos\theta)$ is symmetrical about
a) symmetrical about y-axis but not passing through origin
b) symmetrical about y-axis and passing through origin
c) symmetrical about x-axis but not passing through origin
d) symmetrical about x-axis and passing through origin

56) The curve represented by the equation $r=a(1+\cos\theta)$ is
a) symmetrical about initial line and not passing through the pole
b) symmetrical about initial line and passing through the pole
c) not symmetrical about initial line and passing through the pole
d) not symmetrical about initial line and not passing through the pole

57) The curve represented by the equation $r^2 = a^2 \cos 2\theta$ is
a) symmetrical about initial line as well as pole and not passing through the pole
b) symmetrical about initial line as well as pole and passing through the pole
c) not symmetrical about initial line as well as pole and passing through the pole
d) not symmetrical about initial line as well as pole and not passing through the pole

58) The curve represented by the equation $r^2 = a^2 \sin 2\theta$ is
a) symmetrical about the line $\theta = \frac{\pi}{4}$ and not passing through the pole
b) symmetrical about initial line as well as pole and passing through the pole
c) not symmetrical about the line $\theta = \frac{\pi}{4}$ and passing through the pole
d) not symmetrical about initial line as well as pole and not passing through the pole

59) The curve represented by the equation $r(1+\cos\theta) = 2a^2$ is
a) symmetrical about the line $\theta = \frac{\pi}{4}$ and not passing through the pole
b) symmetrical about initial line and passing through the pole
c) not symmetrical about the line $\theta = \frac{\pi}{4}$ and passing through the pole
d) symmetrical about initial and not passing through the pole

60) The equations of the tangents at pole to the curve $r = a\sin 3\theta$ are given by
a) $\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \ldots$
b) $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{3}, \ldots$
c) $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \ldots$
d) no such tangent exists

61) The equations of the tangents at pole to the curve $r = a\cos 2\theta$ are given by
a) $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \ldots$
b) $\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \ldots$
c) $\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$
d) $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots$

62) For the rose curve $r = a\sin n\theta$, if n is even, the curve is consisting of
a) $2n$ equal loops b) $2n+1$ equal loops
c) n equal loops d) $2n-1$ equal loops

63) For the rose curve $r = a\cos n\theta$, if n is even, the curve is consisting of
a) n equal loops b) $2n+1$ equal loops
c) $2n$ equal loops d) $2n-1$ equal loops

64) For the rose curve $r = a\sin n\theta$, if n is odd, the curve is consisting of
a) $2n$ equal loops b) n equal loops
c) $2n+1$ equal loops d) $2n-1$ equal loops

65) For the rose curve $r = a\cos n\theta$, if n is odd, the curve is consisting of
a) n equal loops b) $2n+1$ equal loops
c) $2n$ equal loops d) $2n-1$ equal loops

I) Rectification of Curve

66) If $A(a_1, b_1)$ $B(a_2, b_2)$ are two points on the curve on xy-plane, the length of arc is given by

a) $\int_{b_1}^{b_2}\sqrt{1-\left(\frac{dy}{dx}\right)^2}\cdot dy$ b) $\int_{b_1}^{b_2}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dy$

c) $\int_{a_1}^{a_2}\sqrt{1-\left(\frac{dy}{dx}\right)^2}\cdot dx$ d) $\int_{a_1}^{a_2}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$

67) If $A(a_1, b_1)$ $B(a_2, b_2)$ are two points on the curve on xy-plane, the length of arc is given by

a) $\int_{b_1}^{b_2}\sqrt{1+\left(\frac{dx}{dy}\right)^2}\cdot dy$ b) $\int_{a_1}^{a_2}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$

c) $\int_{a_1}^{a_2}\sqrt{1-\left(\frac{dy}{dx}\right)^2}\cdot dx$ d) $\int_{b_1}^{b_2}\sqrt{1-\left(\frac{dx}{dy}\right)^2}\cdot dy$

68) If $A(r_1, \theta_1)$ $B(r_2, \theta_2)$ are two points on the curve on the polar plane, the length of arc is given by

a) $\int_{\theta_1}^{\theta_2}\sqrt{r^2-\left(\frac{dr}{d\theta}\right)^2}\cdot d\theta$ b) $\int_{\theta_1}^{\theta_2}\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\cdot d\theta$

c) $\int_{r_1}^{r_2}\sqrt{1+r^2\left(\frac{d\theta}{dr}\right)^2}\cdot dr$ d) $\int_{r_1}^{r_2}\sqrt{1-r^2\left(\frac{d\theta}{dr}\right)^2}\cdot dr$

69) If $A(r_1, \theta_1)$ $B(r_2, \theta_2)$ are two points on the curve on the polar plane, the length of arc is given by

a) $\int_{\theta_1}^{\theta_2}\sqrt{r^2-\left(\frac{dr}{d\theta}\right)^2}\cdot d\theta$ b) $\int_{\theta_1}^{\theta_2}\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\cdot d\theta$

c) $\int_{r_1}^{r_2}\sqrt{1+r^2\left(\frac{d\theta}{dr}\right)^2}\cdot dr$ d) $\int_{r_1}^{r_2}\sqrt{1-r^2\left(\frac{d\theta}{dr}\right)^2}\cdot dr$

70) If $A(t_1)$ $B(t_2)$ are two points on the curve given by $x=f(t)$, $y=g(t)$ on the xy-plane, the length of arc is given by

a) $\int_{t_1}^{t_2}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\cdot dt$

b) $\int_{t_1}^{t_2}\sqrt{\left(\frac{dt}{dx}\right)^2+\left(\frac{dt}{dy}\right)^2}\cdot dt$

c) $\int_{t_1}^{t_2}\sqrt{\left(\frac{dx}{dt}\right)^2-\left(\frac{dy}{dt}\right)^2}\cdot dt$

d) $\int_{t_1}^{t_2}\left[\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2\right]\cdot dt$

71) The arc length of the upper part of the loop of the curve $9y^2=(x+7)(x+4)^2$ is obtained by solving the integration

a) $\int_{0}^{4}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$ b) $\int_{-7}^{0}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$

c) $\int_{-7}^{-4}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$ d) $\int_{7}^{4}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$

72) The arc length of the upper part of the curve $y^2=4x$ which is cut by the line $3y=8x$ is obtained by solving the integration

a) $\int_{1}^{9/16}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$ b) $\int_{0}^{9/16}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$

c) $\int_{0}^{3/4}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$ d) $\int_{3}^{8}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$

73) The points $A(a, 0)$ $B(0, a)$ are two points on the curve $x^2+y^2=a^2$ on xy-plane such that $1+\left(\frac{dy}{dx}\right)^2=\frac{a^2}{a^2-x^2}$, the length of arc is given by

a) $4a$ b) πa c) $\frac{\pi a}{4}$ d) $\frac{\pi a}{2}$

74) The points $A(0, 0)$ $B(a, b)$ are two points on the curve $y=a\cosh\left(\frac{x}{a}\right)$ on xy-plane such that $1+\left(\frac{dy}{dx}\right)^2=\cosh^2\left(\frac{x}{a}\right)$, the length of arc is given by

a) $S=a\sinh\left(\frac{x}{a}\right)$ b) $S=a\tanh\left(\frac{x}{a}\right)$

c) $S=\sinh\left(\frac{x}{a}\right)$ d) $S=a\,\mathrm{sech}\left(\frac{x}{a}\right)$

75) The points $A(0,0)$ $B(1,0)$ are two points on the curve $3y^2 = x(x-1)^2$ on xy-plane such that $1+\left(\frac{dy}{dx}\right)^2 = \frac{(3x+1)^2}{12x}$, the length of arc is given by

a) $\frac{3}{\sqrt{3}}$ b) $\frac{1}{2\sqrt{3}}$ c) $\frac{1}{\sqrt{3}}$ d) $\frac{2}{\sqrt{3}}$

76) The total arc length of the part of the curve $r = a(1+\cos\theta)$ which is cut by the circle $r + a\cos\theta = 0$ is obtained by solving the integration

a) $\int_0^{2\pi/3} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

b) $2\int_0^{2\pi/3} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

c) $2\int_0^{2\pi/3} \sqrt{r^2 - \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

d) $\int_0^{2\pi/3} \sqrt{r^2 - \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

77) The total arc length of the upper part of the curve $r^2 = a^2\cos 2\theta$ is obtained by solving the integration

a) $2\int_0^{\pi/2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

b) $\int_0^{\pi/2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

c) $\int_0^{\pi/4} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

d) $\int_0^{\pi/3} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

78) The total length of the arc of the curve $r = ae^{m\theta}$ using $1 + r^2\left(\frac{d\theta}{dr}\right)^2 = 1 + \frac{1}{m^2}$ when r varies from r_1 to r_2 is given by

a) $\frac{\sqrt{1+m^2}}{m}(r_2 - r_1)$ b) $\frac{\sqrt{1+m^2}}{m}(r_2 + r_1)$

c) $\frac{\sqrt{1+m^2}}{m}(r_1 - r_2)$ d) $\frac{\sqrt{1-m^2}}{m}(r_2 - r_1)$

79) The total length of the arc formed by the upper half of the cardioide $r = a(1+\cos\theta)$ using $r^2 + \left(\frac{dr}{d\theta}\right)^2 = 2a^2(1+\cos\theta)$ when θ varies from 0 to π is given by

a) 4π b) 2π c) $4a$ d) $2a$

80) The total arc length of the upper part of the curve $x = t^2,\ y = t - \frac{t^3}{3}$ is obtained by solving the integration

a) $\int_0^{2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$

b) $\int_0^{3} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$

c) $\int_0^{1} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$

d) $\int_0^{\sqrt{3}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$

81) The total arc length of the two consecutive cusps lies in the first quadrant of the curve $x = a\cos^3\theta,\ y = a\sin^3\theta$ is obtained by solving the integration

a) $\int_0^{\pi/4} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \cdot d\theta$

b) $\int_0^{\pi/3} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \cdot d\theta$

c) $\int_0^{\pi/2} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \cdot d\theta$

d) $\int_0^{\pi} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \cdot d\theta$

82) The total arc length of the upper part of the curve $x = t^2,\ y = t - \frac{t^3}{3}$ between $t = 0$ to $t = \sqrt{3}$ with $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (1+t^2)^2$ is given by

a) $2\sqrt{3}$ b) $\sqrt{3}$ c) $\frac{2}{\sqrt{3}}$ d) $4\sqrt{3}$

83) The total arc length of the two consecutive cusps lies in the first quadrant of the curve $x = a\cos^3\theta,\ y = a\sin^3\theta$ between $\theta = 0$ to $\theta = \frac{\pi}{2}$ with $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 9a^2\sin^2\theta\cos^2\theta$ is given by

a) $\frac{3a}{4}$ b) $3a$ c) $\frac{3a}{2}$ d) $\frac{2a}{3}$

84) The total arc length of the two cusps between $\theta = -\pi$ to $\theta = \pi$ of the curve

$$x = a(\theta + \sin\theta),\ y = a(1-\cos\theta)$$

with $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 4a^2\cos^2\left(\frac{\theta}{2}\right)$ is

a) $4a$ b) $8a$ c) $2a$ d) a

85) The total arc length of the two cusps between $\theta = 0$ to $\theta = \frac{\pi}{2}$ of the curve $x = e^{\theta}\cos\theta$, and $y = e^{\theta}\sin\theta$ with $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 2e^{2\theta}$ is

a) $\sqrt{2}\left(1-e^{\pi/2}\right)$ b) $\sqrt{2}\left(e^{\pi}-1\right)$

c) $\sqrt{2}\left(e^{\pi/2}+1\right)$ d) $\sqrt{2}\left(e^{\pi/2}-1\right)$

Chapter 03) Fourier Series

1	a	41	d	81	b	121	c
2	d	42	d	82	d	122	b
3	b	43	b	83	a	123	d
4	a	44	c	84	b	124	d
5	c	45	d	85	d	125	a
6	d	46	b	86	c	126	b
7	a	47	c	87	a	127	a
8	d	48	a	88	b	128	b
9	b	49	b	89	a	129	b
10	d	50	a	90	b'	130	c
11	d	51	c	91	c	131	a
12	b	52	b	92	a	132	b
13	a	53	c	93	c	133	d
14	d	54	d	94	d	134	d
15	b	55	d	95	a	135	a
16	b	56	c	96	b	136	c
17	a	57	a	97	c	137	d
18	d	58	b	98	d	138	a
19	a	59	d	99	b	139	b
20	b	60	a	100	d	140	a
21	a			101	d	141	d
22	c	62	d	102	b	142	c
23	d	63	c	103	c	143	b
24	a	64	d	104	a	144	c
25	d	65	b	105	d	145	a
26	a	66	d	106	b	146	d
27	d	67	b	107	d	147	c
28	c	68	c	108	d	148	a
29	b	69	a	109	d	149	c
30	c	70	c	110	a	150	b
31	a	71	c	111	d	151	d
32	d	72	c	112	c	152	b
33	a	73	d	113	c	153	a
34	c	74	b	114	a	154	c
35	a	75	d	115	b	155	d
36	c	76	c	116	a	156	d
37	a	77	b	117	c	157	a
38	c	78	c	118	b	158	c
39	c	79	b	119	a	159	b
40	b	80	d	120	b		

Chapter 04) Reduction Formulae & Beta, Gamma Function

1	c	26	d	51	a	76	d	101	c
2	b	27	b	52	c	77	a	102	b
3	c	28	c	53	b	78	c	103	d
4	d	29	a	54	d	79	d	104	c
5	d	30	b	55	b	80	c	105	b
6	c	31	a	56	d	81	b	106	a
7	a	32	c	57	a	82	c	107	b
8	c	33	b	58	d	83	a	108	a
9	b	34	c	59	a	84	d	109	c
10	a	35	d	60	c	85	b	110	d
11	c	36	d	61	d	86	c	111	b
12	b	37	c	62	c	87	a	112	d
13	d	38	a	63	b	88	c	113	c
14	a	39	d	64	a	89	b	114	c
15	a	40	b	65	c	90	d	115	a
16	c	41	d	66	d	91	b	116	c
17	c	42	c	67	b	92	a	117	b
18	c	43	a	68	a	93	c	118	b
19	b	44	b	69	b	94	b	119	d
20	d	45	d	70	c	95	d	120	c
21	c	46	d	71	d	96	d	121	d
22	d	47	b	72	a	97	a	122	b
23	b	48	d	73	b	98	c	123	c
24	d	49	b	74	c	99	d	124	a
25	c	50	c	75	a	100	c	125	b

Chapter 05) Differentiation Under Integral Sign & Error Function

1	a	14	d	27	a	40	b	53	c
2	c	15	b	28	c	41	c	54	c
3	b	16	d	29	d	42	a	55	b
4	c	17	b	30	a	43	d	56	d
5	d	18	c	31	c	44	a	57	b
6	d	19	a	32	b	45	d	58	a
7	c	20	c	33	d	46	c	59	c
8	a	21	b	34	b	47	b	60	d
9	b	22	d	35	a	48	c	61	d
10	d	23	b	36	b	49	a	62	a
11	a	24	d	37	c	50	d		
12	d	25	a	38	d	51	c		
13	c	26	d	39	a	52	a		

Chapter 06) Curve Tracing & Rectification of Curves

1	a		18	c		35	d		52	b		69	c
2	b		19	a		36	c		53	d		70	a
3	a		20	c		37	a		54	d		71	c
4	c		21	b		38	c		55	a		72	b
5	b		22	d		39	b		56	b		73	d
6	c		23	c		40	d		57	b		74	a
7	d		24	d		41	a		58	a		75	d
8	c		25	b		42	d		59	d		76	b
9	b		26	c		43	c		60	a		77	c
10	d		27	a		44	d		61	d		78	a
11	c		28	b		45	d		62	a		79	c
12	d		29	a		46	b		63	c		80	d
13	c		30	c		47	d		64	b		81	c
14	b		31	b		48	b		65	a		82	a
15	d		32	c		49	c		66	d		83	c
16	a		33	d		50	a		67	a		84	b
17	a		34	d		51	a		68	b		85	d

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First Year Degree Course in Engineering – Semester II

Engineering Mathematics (M II)

Savitribai Phule Pune University

First Online Examination

First Year of Engineering

Dr. Chavan N. S.

2015-16

Savitribai Phule Pune University – FE – Sem. II

Engineering Mathematics (M II)

Chapter 01–Ordinary Differential Equations

1) The order of the differential equation is
   a) the order of the highest ordered differential coefficient appearing in the differential equation.
   b) the order of the lowest ordered differential coefficient appearing in the differential equation.
   c) the power of the highest ordered differential coefficient appearing in the differential equation.
   d) the degree of the highest ordered differential coefficient appearing in the differential equation.

2) The degree of the differential equation is
   a) the highest ordered differential coefficient appearing in the differential equation.
   b) the lowest power of the highest ordered differential coefficient appearing in the differential equation.
   c) the highest power of the highest ordered differential coefficient appearing in the differential equation.
   d) the coefficient power of the highest ordered differential coefficient appearing in the differential equation.

3) A solution of a differential equation is a relation between
   a) dependent variables
   b) independent variables
   c) dependent and independent variables not containing any differential coefficient
   d) none of the above

4) In the general solution, the number of arbitrary constants is equal to
   a)order of the differential equation
   b) degree of the differential equation
   c) sum of order and degree of diff. eqn.
   d) difference of order and degree of diff. eqn.

5) The general solution of n^{th} order ordinary differential equation must involve
   a) $n+1$ arbitrary constants
   b) $n-1$ arbitrary constants
   c) n arbitrary constants
   d) none of the above

6) The solution obtained by assigning particular values to arbitrary constants in general solution of differential equation is known as
   a) singular solution b) particular solution
   c) general solution d) none of above

7) The order of differential equation whose general solution is $y=(c_1+c_2x)e^x+x$, where $c_1,\ c_2$ are arbitrary constants, is
   a) 1 b)2 c) 3 d) 0

8) The order of differential equation whose general solution is $y=\left(c_1+c_2x+c_3x\right)e^x+\frac{x^2}{12}$, where $c_1,\ c_2,\ c_3$ are arbitrary constants, is
   a) 0 b) 1 c) 2 d)3

9) The order of differential equation whose general solution is $y=\left(c_1+c_2\right)e^x+\frac{x^4}{3}$, where $c_1,\ c_2$ are arbitrary constants, is
   a) 0 b)1 c)2 d) 3

10) The order of differential equation whose general solution is $y=cx+c^2$, where c is arbitrary constant, is
   a) 0 b)1 c)2 d) 3

11) The order of differential equation whose general solution is $y=Ax+\frac{B}{x}$, where $A,\ B$ are arbitrary constants, is
   a) 0 b) 1 c)2 d) 3

12) The order of differential equation whose general solution is $y = Ax + \frac{A^2}{x}$, where A, B are arbitrary constants, is
a) 0 b) 1 c) 2 d) 3

13) The order of differential equation whose general solution is $y = \log(x-a)+b$, where a, b are arbitrary constants, is
a)2 b) 1 c) 0 d) none

14) The order of differential equation whose general solution is $x = A\sin(kt+B)$, where A, B are arbitrary constants and k is fixed constant, is
a) 0 b) 1 c)2 d) 3

15) The order of differential equation whose general solution is $x = (A+Bt)e^t$, where A, B are arbitrary constants, is
a) 0 b)2 c) 1 d) 3

16) The order of differential equation whose general solution is $y + \sqrt{x^2+y^2} = cx + c^3$, where c is arbitrary constant, is
a) 0 b) 2 c) 3 d)1

17) The order of differential equation whose general solution is $y = 4(x-A)^2$, where A is arbitrary constant, is
a)1 b) 2 c) 3 d) none

18) The order of differential equation whose solution is $y = (c_1 + c_2 x)e^x + (c_3 + c_4 x)e^{2x}$, where c_1, c_2, c_3, c_4 are arbitrary constants, is
a) 1 b) 2 c) 3 d)4

19) The order of differential equation whose solution is $y = c_1 x + c_2 e^x + c_3 e^{2x} + c_4 e^{3x}$, where c_1, c_2, c_3, c_4 are arbitrary constants, is
a) 1 b)4 c) 2 d) 3

20) The order of differential equation whose solution is $y = (Ax^2 + Bx + C)e^x$, where A, B, C are arbitrary constants, is
a) 1 b) 2 c)3 d) 4

21) The order of differential equation whose general solution is $y = \sqrt{kx+c}$, where c is the only arbitrary constant, is
a)1 b) 2 c) 3 d) 0

22) The order of differential equation whose general solution is $y = c^2 + \frac{c}{x}$, where c is arbitrary constant, is
a) 0 b) 2 c) 3 d)1

23) The order of differential equation whose general solution is $y = A\cos(x+5)$, where A is arbitrary constant, is
a) 0 b)1 c) 2 d) 3

24) The order and the degree of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is
a)1, 1 b) 1, 2 c) 2, 1 d) 2, 2

25) The order and the degree of the differential equation $\frac{dy}{dx} + y\log x = \sin x$ is
a) 0, 1 b) 1, 0 c) 2, 1 d)1, 1

26) The order and the degree of the differential equation $\frac{dy}{dx} + 2y = \cos x$ is
a) 0, 1 b)1, 1 c) 1, 2 d) 2, 1

27) The order and the degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 5y = \sin 7x$ is
a) 0, 1 b) 1, 1 c) 1, 2 d) 2, 1

28) The order and the degree of the differential equation $1 + \frac{dy}{dx} = \left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}}$ is
a) order 2, degree 1 b) order 1, degree 2
c)order 2, degree 3 d) order 2, degree $\frac{3}{2}$

29) The order and the degree of the differential equation $\sqrt{1+\frac{dy}{dx}} = \frac{d^2y}{dx^2}$ is
a) order 2, degree 2 b) order 2, degree 1
c) order 1, degree 2 d) order 1, degree 1

30) The order and the degree of the differential equation $\dfrac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}=k$ is

a) order 2, degree 1 b) order 2, degree 2

c) order 2, degree 3 d) order 2, degree $\dfrac{3}{2}$

31) The order and the degree of the differential equation $\sqrt{1+\dfrac{dy}{dx}}=\dfrac{d^2y}{dx^2}$ is

a)order 2, degree 2 b) order 2, degree 1
c) order 1, degree 2 d) order 1, degree 1

32) The order and the degree of the differential equation $\dfrac{d^2y}{dx^2}=\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}$ is

a) order 2, degree 1 b) order 2, degree 2
c) order 1, degree 2 d) order 1, degree 1

33) The order and the degree of the differential equation $x=\dfrac{1}{\sqrt{1+\dfrac{dy}{dx}+\dfrac{d^2y}{dx^2}}}$ is

a) order 2, degree 2 b)order 2, degree 1

c) order 1, degree 2 d) order 1, degree $-\dfrac{1}{2}$

34) The order and the degree of the differential equation $1+\dfrac{dy}{dx}=\dfrac{y}{\dfrac{dy}{dx}}$ is

a) order 1, degree 1 b) order 2, degree 1
c)order 1, degree 2 d) order 2, degree 2

35) The order and the degree of the differential equation $y+\dfrac{d^2y}{dx^2}+\dfrac{x}{\dfrac{dy}{dx}}=1$ is

a) order 1, degree 1 b) order 2, degree 1
c) order 1, degree 2 d) order 2, degree 2

36) The order and the degree of the differential equation $(2x-3y+2)dy+(x-2y+7)dx=0$ is

a)1, 1 b) 1, 2 c) 2, 1 d) none

37) By eliminating the arbitrary constant m, the differential equation for the general solution $y=mx$ is given by

a) $\dfrac{dy}{dx}=\dfrac{y}{x}$ b) $\dfrac{dy}{dx}-xy=0$

c) $\dfrac{dy}{dx}+\dfrac{y}{x}=0$ d) $\dfrac{dy}{dx}-y=0$

38) The differential equation satisfied by the general solution $y+x^3=Ax$ with A is arbitrary constant, is given by

a) $y\dfrac{dy}{dx}+2x-y^3=0$ b) $x\dfrac{dy}{dx}+2x^3-y=0$

c) $\dfrac{dy}{dx}+2x^2-y=0$ d) $x^3\dfrac{dy}{dx}+2(x-y)=0$

39) $y=5+\sqrt{cx}$, where c is the arbitrary constant, is the general solution of

a) $y\dfrac{dy}{dx}=5+2x$ b) $y=2x\dfrac{dy}{dx}$

c) $y=5+2x\dfrac{dy}{dx}$ d) $y=5+2x\sqrt{\dfrac{dy}{dx}}$

40) By eliminating the arbitrary constant c, the differential equation of $y=cx-c^2$ is

a) $\dfrac{dy}{dx}-x\left(\dfrac{dy}{dx}\right)^2+y=0$ b) $\left(\dfrac{dy}{dx}\right)^2-x\dfrac{dy}{dx}+y=0$

c) $\left(\dfrac{dy}{dx}\right)^2+x\dfrac{dy}{dx}-y=0$ d) $\left(\dfrac{dy}{dx}\right)^2-xy=0$

41) The differential equation whose primitive is $y=c^2+\dfrac{c}{x}$, is given by

a) $x^4\left(\dfrac{dy}{dx}\right)^2-xy=0$ b) $\left(\dfrac{dy}{dx}\right)^2+\dfrac{dy}{dx}+y=0$

c) $\left(\dfrac{dy}{dx}\right)^2-x^4\dfrac{dy}{dx}-y=0$ d) $x^4\left(\dfrac{dy}{dx}\right)^2-x\dfrac{dy}{dx}-y=0$

42) By eliminating the arbitrary constant c present in the function $x=cy-y^2$, the differential equation is given by

a) $\left(\dfrac{x+y^2}{y}\right)\dfrac{dy}{dx}-2y\dfrac{dy}{dx}-1=0$

b) $\left(\dfrac{x+y^2}{y}\right)\left(\dfrac{dy}{dx}\right)^2-2y\dfrac{dy}{dx}-1=0$

c) $x\frac{dy}{dx}-2\left(\frac{x+y^2}{y}\right)\frac{dy}{dx}-1=0$

d) $y^2\frac{dy}{dx}-2xy\frac{dy}{dx}+1=0$

43) The differential equation whose solution is $y^2=4ax$ is given by

a) $\left(\frac{dy}{dx}\right)^2-2xy=0$ b) $\frac{dy}{dx}-xy^2=0$

c) $2xy\frac{dy}{dx}-y^2=0$ d) $2xy\frac{dy}{dx}+y^2=0$

44) The differential equation of family of curves $x^2+y^2+xy+x+y=c$ is

a) $\frac{dy}{dx}=-\frac{2x+y+1}{x+2y+1}$ b) $y_2+4y=0$

c) $\frac{dy}{dx}=\frac{2x-y}{x+2y+1}$ d) $x^2y_2-xy_1+y=0$

45) The differential equation whose generalized solution is $xy+y^2-x^2-x-3y=c$, is

a) $\frac{dy}{dx}=-\frac{2x-y+1}{x-2y+3}$ b) $\frac{dy}{dx}=\frac{x-2y-1}{x-2y+3}$

c) $\frac{dy}{dx}=\frac{2x+y+1}{x+2y+3}$ d) $\frac{dy}{dx}=\frac{2x-y+1}{x+2y-3}$

46) The differential equation satisfied by family of circles $x^2+y^2=2Ax$ is given by

a) $\frac{dy}{dx}+x^2+y^2=0$ b) $\frac{dy}{dx}+\frac{y^2-x^2}{xy}=0$

c) $\frac{dy}{dx}+\frac{x^2-y^2}{2xy}=0$ d) $\frac{dy}{dx}-\frac{x^2-y^2}{2xy}=0$

47) The differential equation whose general solution is $x^3+y^3=3Ax$, where A is arbitrary constant, is

a) $y_1=\frac{x^3+y^3-3x^2}{3xy^2}$ b) $x^2y_1+y=3y_1$

c) $xy_1+y^2+x=0$ d) none of these

48) $y^2=x^2-1+Ax$, where A is arbitrary constant, is the general solution of the equation

a) $\frac{dy}{dx}=\frac{x^2+y^2}{2xy}$ b) $y\frac{dy}{dx}+x^2+y^2=0$

c) $2xy\frac{dy}{dx}=x^2+y^2+1$ d) $2xy\frac{dy}{dx}-\left(x^2+y^2\right)=0$

49) The differential equation of $y=4(x-A)^2$, where A is arbitrary constant, is

a) $\frac{dy}{dx}-16y^2=0$ b) $\left(\frac{dy}{dx}\right)^2-16y=0$

c) $\left(\frac{dy}{dx}\right)^2+4y=0$ d) $\left(\frac{dy}{dx}\right)^2+16y=0$

50) $\left(1+x^2\right)=A\left(1+y^2\right)$ is a general solution of the differential equation

a) $\frac{dy}{dx}+\frac{1+x^2}{1-y^2}=0$ b) $\frac{x}{y}\frac{dy}{dx}+\left(\frac{1+x^2}{1-y^2}\right)=0$

c) $\left(\frac{1+x^2}{1-y^2}\right)\frac{dy}{dx}+\frac{x}{y}=0$ d) $\frac{dy}{dx}+\frac{x}{y}\left(\frac{1+x^2}{1-y^2}\right)=0$

51) The differential equation representing the family of loops $y^2=c\left(4+e^{2x}\right)$ is

a) $\left(4+e^{2x}\right)\frac{dy}{dx}+4ye^{2x}=0$ b) $\left(4+e^{2x}\right)\frac{dy}{dx}+y=0$

c) $\frac{dy}{dx}-ye^{2x}=0$ d) $\left(4+e^{2x}\right)\frac{dy}{dx}-ye^{2x}=0$

52) The differential equation whose general solution is $y=\sqrt{3x+c}$, is given by

a) $\frac{dy}{dx}-3y=0$ b) $2y\frac{dy}{dx}+3=0$

c) $2y\frac{dy}{dx}-3=0$ d) $2\frac{dy}{dx}-3y=0$

53) By eliminating the arbitrary constant A from $y=A\cos(x+3)$ the differential equation is

a) $\frac{dy}{dx}+y=0$ b) $\frac{dy}{dx}+y\cot(x+3)=0$

c) $\tan(x+3)\frac{dy}{dx}+y=0$ d) $\cot(x+3)\frac{dy}{dx}+y=0$

54) By eliminating the arbitrary constant c, the differential equation of $\cos(y-x)=ce^{-x}$ is

a) $x^2y_1-xy=4y_1$ b) $\tan(y-x)\left(\frac{dy}{dx}-1\right)-1=0$

c) $xy_1-y+x\sin\left(\frac{y}{x}\right)=0$ d) none of these

55) The differential equation whose generalized solution is $\sin(y-x)=ce^{-\frac{x^2}{2}}$, is given by

a) $\tan(y-x)\left(\frac{dy}{dx}-1\right)+x=0$

b) $\cot(y-x)\left(\frac{dy}{dx}+1\right)+y=0$

c) $\left(\frac{dy}{dx}-1\right)+\frac{x}{\cot(y-x)}=0$

d) $\cot(y-x)\left(\frac{dy}{dx}-1\right)+x=0$

56) The differential equation of the family of curves $y=Ae^{-x^2}$ is given by

a) $y\frac{dy}{dx}-2x^2=0$ b) $\frac{dy}{dx}+2xy=0$

c) $y\frac{dy}{dx}+2\log x=0$ d) $\frac{dy}{dx}-x^2y=0$

57) The differential equation whose general solution is $y=Ae^{\frac{x}{y}}$, is given by

a) $(x+y)y_1-y=0$ b) $(x+y)^2y_1+y=0$

c) $(x-y)y_1+y=0$ d) $xy_1-\frac{y}{x}=0$

58) By eliminating the arbitrary constant c from the function $y=5ce^{\frac{x}{y}}$, the differential equation is

a) $(x+y)\frac{dy}{dx}-y=0$ b) $\frac{dy}{dx}-\frac{y}{x+y}=0$

c) $\left(\frac{x+y}{x}\right)\frac{dy}{dx}-\frac{y}{x}=0$ d) $\frac{dy}{dx}-\frac{y-x}{x+y}=0$

59) The differential equation for the function $\sin\left(\frac{y}{x}\right)=Ax$ is obtained by eliminating A and is given by

a) $\frac{dy}{dx}+\frac{y}{x}=x\tan\left(\frac{y}{x}\right)$ b) $\frac{dy}{dx}+xy=\tan\left(\frac{y}{x}\right)$

c) $x\frac{dy}{dx}-y=x\cot\left(\frac{y}{x}\right)$ d) $x\frac{dy}{dx}-y=x\tan\left(\frac{y}{x}\right)$

60) The differential equation of $\cos\left(\frac{y}{x}\right)=cx$ is

a) $xy_1-y+x\cot\left(\frac{y}{x}\right)=0$

b) $xy_1-y+x\sin\left(\frac{y}{x}\right)=0$

c) $x^2y_1-y+x=0$

d) $x^2y_1-y+x\sin\left(\frac{y}{x}\right)=0$

61) The differential equation for the function $xy=c^2$, where c is arbitrary constant, is

a) $x\frac{dy}{dx}-y=0$ b) $\frac{dy}{dx}+xy=0$

c) $x\frac{dy}{dx}+y=0$ d) $x\left(\frac{dy}{dx}\right)^2+y=0$

62) The differential equation satisfying the general solution $xy=ce^x$ is

a) $x^2y_1-xy+e^x=0$ b) $xy_1+y=e^x$

c) $xy_1+y(1+x)=0$ d) $xy_1+y(1-x)=0$

63) The differential equation whose general solution is $y^2=2c\left(x+\sqrt{c}\right)$, where c is arbitrary constant, is

a) $2\frac{dy}{dx}\left(x+\sqrt{y\frac{dy}{dx}}\right)-y=0$ b) $x+\sqrt{y\frac{dy}{dx}}-y=0$

c) $\frac{dy}{dx}\left(x+\sqrt{y\frac{dy}{dx}}\right)+y=0$ d) $2x\frac{dy}{dx}+y\left(\frac{dy}{dx}\right)^2=0$

64) The differential equation satisfying the function $y=Ax+Bx^2$ is given by

a) $x^2y_2-4xy_1+y=0$ b) $y_2^{\,2}+2xy_1+2y=0$

c) $x^2y_2-2xy_1+2y=0$ d) $x^2y_2+xy_1+y=0$

65) By eliminating the arbitrary constants c_1, c_2 from the function $y=\sqrt{4x^2+c_1x+c_2}$ we get the differential equation

a) $y_2+xy_1=0$ b) $yy_2+y_1^{\,2}=4$

c) $x^2y_1y_2-y^2=0$ d) $x^2y_2+xy_1+4y=0$

66) $\frac{x^2}{4}-\frac{y^2}{a}=1$ is a general solution of

a) $xy_1-4y=xy$ b) $x^2y_1-4xy_1+16y=0$

c) $x^2y_1-4y_1-xy=0$ d) none of these

67) The differential equation representing the family of ellipse $\frac{x^2}{a^2}+\frac{y^2}{9}=1$, is given by

a) $y\frac{dy}{dx}-x^2y+9=0$ b) $xy\frac{dy}{dx}-y^2+9=0$

c) $xy\frac{dy}{dx}-y^2=0$ d) $xy\frac{dy}{dx}+y^2-9=0$

68) The differential equation whose primitive is $y^2=4A(x-B)$, where A and B are arbitrary constants, is

a) $x^2y_1y_2-y^2=0$ b) $x^2y_2+xy_1+4y=0$

c) $y_2+xy_1=0$ d) $yy_2+y_1^2=0$

69) On the elimination of the arbitrary constants A and B as well from $y^2=5A(x-3B)$, the differential equation formed is

a) $\frac{d^2y}{dx^2}+y=0$ b) $y^2\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$

c) $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$ d) $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2-y=0$

70) The differential equation with general solution $x=A\cos(B-5t)$ is given by

a) $\frac{d^2x}{dt^2}+5\frac{dx}{dt}-25t=0$ b) $\frac{d^2x}{dt^2}-\frac{dx}{dt}-xt=0$

c) $\frac{d^2x}{dt^2}-25x=0$ d) $\frac{d^2y}{dx^2}-25y=0$

71) The differential equation whose general solution is $y=\log(Ax+B)$ is

a) $y_2+y_1^2=0$ b) $x^2y_2+y_1^2=0$

c) $y_2+xy_1^2+y=0$ d) $xy_2+y_1^2-y=0$

72) $y=A\sin x+B\cos x$ is the solution satisfying the differential equation

a) $\frac{d^2y}{dx^2}+\frac{y}{x}=0$ b) $y^2\frac{d^2y}{dx^2}+xy+x=0$

c) $\frac{d^2y}{dx^2}+xy=0$ d) $\frac{d^2y}{dx^2}+y=0$

73) The differential equation whose general solution is $y=A\sin 3x+B\cos 3x$ where A, B are arbitrary constants, is

a) $x^2y_2-xy-9y_1=0$ b) $xy_2-9y_1+y=0$

c) $y_2-9y=0$ d) $y_2+9y=0$

74) The differential equation whose solution is $y=A\cos\frac{4x}{3}+B\sin\frac{4x}{3}$, where A and B are arbitrary constants, is given by

a) $\frac{d^2y}{dx^2}-\frac{dy}{dx}-\frac{4}{3}y=0$ b) $\frac{d^2y}{dx^2}+\frac{16}{9}y=0$

c) $9\frac{d^2y}{dx^2}-16y=0$ d) $\frac{d^2y}{dx^2}+\frac{dy}{dx}+\frac{16}{9}y=0$

75) The differential equation whose primitive is $y=A\cos\log x+B\sin\log x$, where A and B are arbitrary constants, is given by

a) $x^2y_2+y_1+xy=0$ b) $x^2y_2+xy_1+y=0$

c) $x^2y_2+y_1+y=0$ d) $y_2-x^2y_1-xy=0$

76) The differential equation whose general solution is $y=Ae^{-x}+B$, where A and B are arbitrary constants, is

a) $y=x^2y_2+y_1$ b) $x^2y_2+xy_1+y=0$

c) $y_2+y_1=0$ d) $xy_2^2+y_1=0$

77) $y=Ae^{-x}+Be^{-x}$, where A and B both are arbitrary constants, is the solution for the differential equation

a) $x\frac{d^2y}{dx^2}+y=0$ b) $\frac{d^2y}{dx^2}-\frac{dy}{dx}-y=0$

c) $\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=0$ d) $\frac{d^2y}{dx^2}-y=0$

78) By eliminating the arbitrary constants A and B both from the function $xy=Ae^{x}+Be^{-x}$, we get the differential equation

a) $\frac{x}{y}\frac{d^2y}{dx^2}+2\frac{dy}{dx}-\frac{x}{y}=0$ b) $x\frac{d^2y}{dx^2}+2\frac{dy}{dx}-xy=0$

c) $y\frac{d^2y}{dx^2}+2\frac{dy}{dx}-xy=0$ d) $\frac{d^2y}{dx^2}+\frac{dy}{dx}+xy=0$

79) The differential equation, whose solution is given by $y = Ae^{-3x} + Be^{3x}$, is
a) $xy_2^{\,2} + y_1 - xy = 0$ b) $x^2 y_2 + y_1 + xy = 0$
c) $x^2 y_2 - xy_1 + y = 0$ d) $y_2 - 4y = 0$

80) $e^{-t} y = A + Bt$ is a general solution of the differential equation
a) $y_2 - 2y_1 + y = 0$ b) $y_2 + y_1 t + yt^2 = 0$
c) $xy_2 + y_1 + y = 0$ d) $4y_2 + 2y_1 + y = 0$

81) The differential equation having generalized solution $e^{-t} x = At - B$ is given by
a) $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = 0$ b) $x\frac{d^2x}{dt^2} + \frac{dx}{dt} + xt = 0$
c) $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + t = 0$ d) $x^2\frac{d^2x}{dt^2} - 2xt + x = 0$

82) The general form of the differential equation of I order and I degree can be expressed as
a) $\frac{dy}{dx} = c$ b) $M(x, y)dx + N(x, y)dy = 0$
c) $\frac{dy}{dx} + y = du$ d) $M(x, y)dx + N(x, y)dy = du$

83) The differential equation of the form $f_1(x)dx + f_2(y)dy = 0$ is known as
a) Linear form b) Non homogeneous form
c) exact form d)variable separable form

84) The differential equation in the form $\frac{dy}{dx} = x^n f\left(\frac{y}{x}\right)$ is known as
a) Linear form b) homogeneous form
c) exact form d) variable separable form

85) The differential equation in the form $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$, where f and g both are homogeneous functions of x and y of the same degree, is known as
a) Linear form b) homogeneous form
c) exact form d) variable separable form

86) The homogenous differential equation of the form $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$ is solved by substitution
a) no substitution, direct solution b) $x^n = v$
c) $xy = v$ d) $\frac{y}{x} = v$

87) The differential equation of the form $M(x, y)dx + N(x, y)dy = 0$ is exact, if
a) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ b) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$
c) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ d) $\frac{\partial M}{\partial y} \times \frac{\partial N}{\partial x} = 1$

88) The differential equation $\frac{dy}{dx} = e^{2x+y} + 3x^4 e^y$ is of the form
a) Linear form b) Non homogeneous form
c) exact form d) variable separable form

89) The form of the differential equation $\left(y^3 - 3x^2 y\right)dx + \left(x^2 y + 3x^3\right)dy = 0$ is
a) Linear form b) homogeneous form
c) exact form d) variable separable form

90) The differential equation is of the form $(x + y)dx + (x - y + 1)dy = 0$
a) Linear form b) non homogeneous form
c) exact form d) variable separable form

91) The differential equation $xy - \frac{dy}{dx} = y^3 e^{-x^2}$ is of the form
a) Linear form b) non homogeneous form
c) exact form d) variable separable form

92) The substitution which can be used to solve the equation $(x + y + 7)dx + (3x + 3y - 7)dy = 0$ is
a) $x + y = v$ b) $x - y = v$
c) $xy = v$ d) $\frac{y}{x} = v$

93) The general solution of the differential equation $\frac{3e^x}{1 - e^x}dx + \frac{\sec^2 y}{\tan y}dy = 0$ is
a) $\tan y = c\left(1 - e^x\right)^3$ b) $\left(1 - e^x\right)^3 \tan y = c$
c) $\left(1 - e^{-x}\right)^3 \cot y = c$ d) $\cot y = c\left(1 - e^x\right)^3$

94) The general solution of the differential equation $\frac{dy}{dx}+y=0$ is

a) $y=ce^{-x}$ b) $y=Ae^{-x}+B$
c) $y=ce^{x}$ d) $x=ce^{-y}$

95) The general solution of the differential equation $\frac{dx}{dy}+x=0$ is

a) $y=ce^{-x}$ b) $y=Ae^{-x}+B$
c) $y=ce^{x}$ d) $x=ce^{-y}$

96) The general solution of the differential equation $\frac{dy}{dx}+x=0$ is

a) $y=ce^{-x}$ b) $y^2+2x=c$
c) $x^2+2y=c$ d) $x=ce^{-y}$

97) The general solution of the differential equation $ydx+xdy=0$ is

a) $x^2+y^2=c$ b) $xy=c$ c) $\frac{y}{x}=c$ d) $\frac{x}{y}=c$

98) The general solution of the differential equation $\frac{dy}{dx}+\tan x=0$ is

a) $y=\log\sin x+c$ b) $y-\log\sec x=c$
c) $y=\log\sec x+c$ d) $y=\log\cos x+c$

99) The general solution of the differential equation $\frac{dy}{dx}+xy=0$ is

a) $\log x+\log y=c$ b) $\frac{x^2}{2}+\log y=c$
c) $x^2+\log y=c$ d) $x^2+y^2=c$

100) The general solution of the differential equation $\frac{dy}{dx}+\frac{1+x}{1+y}=0$ is

a) $x^2+y^2+2x+2y=c$ b) $(x+y)^2+2(x+y)=c$
c) $x^2+y^2+x+y=c$ d) $(1+x)=c(1+y)$

101) The general solution of the differential equation $\frac{dy}{dx}=\frac{1+y}{1+x}$ is

a) $(1+x)=c(1+y)^2$ b) $(1+y)=c(1+x)$
c) $(1+x)=c(1+y)$ d) $x=cy$

102) The general solution of the differential equation $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$ is

a) $\log\left(\frac{1+x^2}{1+y^2}\right)$ b) $\log(1+x^2)+\log(1+y^2)=c$
c) $\tan^{-1}x+\tan^{-1}y=c$ d) $\tan^{-1}x-\tan^{-1}y=c$

103) The general solution of the differential equation $\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$ is

a) $\frac{1}{2}\log\left(\frac{1-y^2}{1-x^2}\right)=c$ b) $\sec^{-1}x+\sec^{-1}y=c$
c) $\tan^{-1}x+\tan^{-1}y=c$ d) $\sin^{-1}x+\sin^{-1}y=c$

104) The general solution of the differential equation $x(1+y^2)dx+y(1+x^2)dy=0$ is

a) $(1+y^2)(1+x^2)=c$ b) $\log\left(\frac{1+y^2}{1+x^2}\right)=c$
c) $(1+y^2)=c(1+x^2)$ d) $\tan^{-1}x+\tan^{-1}y=c$

105) The general solution of the differential equation $\frac{dy}{dx}=(1+x)(1+y^2)$ is

a) $\log(1+y^2)=x+\frac{x^2}{2}+c$ b) $\tan^{-1}y=x+\frac{x^2}{2}+c$
c) $\log(1+x)+\tan^{-1}y=c$ d) $\tan^{-1}y+x+x^2=c$

106) The general solution of the differential equation $(e^x+1)ydy=(y+1)e^xdx$ is

a) $y+\log(y+1)+\log(e^x+1)=c$
b) $x+\log(y+1)=\log(e^x+1)+c$
c) $y-\log(y+1)=\log(e^x+1)+c$
d) $\frac{y^2}{2}+\log(y+1)=\log(e^x+1)+c$

107) The general solution of the differential equation $\frac{dy}{dx}=e^{x+y}+e^{y-x}$ is

a) $e^{-x}-e^{x}-e^{-y}=c$ b) $e^{x}-e^{2x}-e^{-y}=c$
c) $e^{-x}+e^{x}+e^{-y}=c$ d) $e^{x}-e^{-x}-e^{y}=c$

108) The general solution of the differential equation $\frac{dy}{dx}=e^{x-y}+3x^2e^{-y}$ is

a) $\frac{e^x+x^3}{e^y}=c$ b) $e^{x-y}=e^y+x^3+c$

c) $e^y=e^x+x^3+c$ d) $e^y+e^x+x^3=c$

109) The general solution of the differential equation $y(1+\log x)\frac{dx}{dy}-x\log x=0$ is

a) $\frac{x}{\log x}=yc$ b) $\frac{x}{y}\log x=y+c$

c) $x(\log x+1)=yc$ d) $x\log x=yc$

110) The general solution of the differential equation $\sec^2 x\tan y dx+\sec^2 y\tan x dy=0$ is

a) $\tan x\tan y=c$ b) $\tan x=c\tan y$

c) $\tan x+\tan y=c$ d) $\tan y=c\tan x$

111) The general solution of the differential equation $y\sec^2 x+(y-5)\tan x\frac{dy}{dx}=0$ is

a) $y^5-y+\tan x=c$ b) $y+5\log y+\log\sec x=c$

c) $y+5\log\frac{\tan x}{y}=c$ d) $y-5\log y+\log\tan x=c$

112) The general solution of the differential equation $e^x\cos y+(1+e^x)\sin y\frac{dy}{dx}=0$ is

a) $(1+e^x)\tan y=c$ b) $(1+e^x)\sec y=c$

c) $(1+e^x)\cos y=c$ d) $\sec y=c(1+e^x)$

113) The general solution of the differential equation $e^y\cos x dx+(e^y+1)\sin x dy=0$ is

a) $\sec x(e^y+1)=c$ b) $\sin x=c(e^y+1)$

c) $\sin y(1+e^x)=c$ d) $\sin x(e^y+1)=c$

114) The general solution of the differential equation $(4+e^{2x})\frac{dy}{dx}=ye^{2x}$ is

a) $\frac{y^2}{2}=A+(4+e^{2x})$ b) $y^2(4+e^{2x})=A$

c) $y^2=A(4+e^{2x})$ d) $x^2=A(4+e^{2x})$

115) The general solution of the differential equation $y-x\frac{dy}{dx}=2\left(y+\frac{dy}{dx}\right)$ is

a) $(x+2)y=c$ b) $x+2y=c$

c) $y=c(x+2)$ d) $(x+2)^2y=c$

116) The general solution of the differential equation $(x+1)\frac{dy}{dx}+1=2e^{-y}$ is

a) $(x+1)(2+e^{-y})=c$ b) $(2-e^y)=c(x+1)$

c) $(x+1)(2-e^y)=c$ d) $(x+1)=c(2-e^y)$

117) The general solution of the differential equation $x^3\left(x\frac{dy}{dx}+y\right)-\sec(xy)=0$ is

a) $\sin(xy)=2cx^2$ b) $\sin(xy)-\frac{1}{2x^2}=c$

c) $\sec(xy)+\frac{1}{2x^2}=c$ d) $\sin(xy)+\frac{1}{2x^2}=c$

118) The general solution of the differential equation $(y-ay^2)dx=(a+x)dy$ is

a) $\log(a+x)+\frac{1}{2}\log(1-ay)-\frac{1}{3}\log y=c$

b) $\log(a+x)-\frac{1}{a}\log(1-ay)-\log y=c$

c) $\log(a+x)+\log(1-ay)-\log y=c$

d) $\log(a+x)+\frac{\log(1-ay)}{-a}+\log y=c$

119) The necessary and sufficient condition for the equation $M(x,y)dx+N(x,y)dy=0$ to be exact is

a) $\frac{\partial M}{\partial x}=\frac{\partial N}{\partial y}$; $My+Nx=0$

b) $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$; $My+Nx\neq 0$

c) $\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}$; $My+Nx\neq 0$

d) $\frac{\partial M}{\partial y}\times\frac{\partial N}{\partial x}=1$; $My+Nx\neq 0$

120) If the differential equation $Mdx+Ndy=0$ is a homogeneous but not exact, its integrating factor is

a) $\dfrac{1}{Mx-Ny}$; $My-Nx\neq 0$

b) $\dfrac{1}{Mx+Ny}$; $Mx+Ny\neq 0$

c) $\dfrac{1}{My-Nx}$; $My-Nx\neq 0$

d) $\dfrac{1}{My+Nx}$; $My+Nx\neq 0$

121) If the differential equation $Mdx+Ndy=0$ is not exact but can be expressed in the form $yf_1(xy)dx+xf_2(xy)dy=0$, its integrating factor is

a) $\dfrac{1}{Mx+Ny}$; $Mx+Ny\neq 0$

b) $\dfrac{1}{My-Nx}$; $My-Nx\neq 0$

c) $\dfrac{1}{My+Nx}$; $My+Nx\neq 0$

d) $\dfrac{1}{Mx+Ny}$; $Mx+Ny=0$

122) If the differential equation $Mdx+Ndy=0$ is not exact and $\dfrac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}=f(x)$, its integrating factor is

a) $e^{\int f(x)dx}$ b) $e^{f(x)}$

c) $e^{\int f(y)dy}$ d) $f(x)$

123) If the differential equation $Mdx+Ndy=0$ is not exact and $\dfrac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{-M}=f(y)$, its integrating factor is

a) $e^{\int f(x)dx}$ b) $e^{f(x)}$

c) $e^{\int f(y)dy}$ d) $f(x)$

124) The total derivative of $dx+dy$ is

a) $d\left(\dfrac{x}{y}\right)$ b) $d(x+y)$

c) $d(x-y)$ d) $d(xy)$

125) The total derivative of $dx-dy$ is

a) $d\left(\dfrac{x}{y}\right)$ b) $d(x+y)$

c) $d(x-y)$ d) $d(xy)$

126) The total derivative of $xdy+ydx$ is

a) $d\left(\dfrac{x}{y}\right)$ b) $d(x+y)$

c) $d(x-y)$ d) $d(xy)$

127) The total derivative of $xdy-ydx$ with the integrating factor $\dfrac{1}{x^2}$ is

a) $d(x-y)$ b) $d\left(\dfrac{x}{y}\right)$

c) $d\left(\dfrac{y}{x}\right)$ d) $d(xy)$

128) The total derivative of $2(xdx+ydy)$ is

a) $d(x+y)$ b) $d(xy)$

c) $d(xy)^2$ d) $d\left(x^2+y^2\right)$

129) The total derivative of $2(xdx-ydy)$ is

a) $d(xy)$ b) $d\left(\dfrac{x^2}{y^2}\right)$

c) $d\left(x^2-y^2\right)$ d) $d\left(x^2+y^2\right)$

130) The total derivative of $\dfrac{ydx-xdy}{y^2}$ is

a) $d\left(\dfrac{y}{x}\right)$ b) $d\left(\dfrac{x}{y}\right)$

c) $d\left(\dfrac{x-y}{y}\right)$ d) $d\left(x^2-y^2\right)$

131) The total derivative of $ydx-xdy$ with the integrating factor $\dfrac{1}{y^2}$ is

a) $d\left(\dfrac{x}{y}\right)$ b) $d\left(\dfrac{y}{x}\right)$

c) $d\left(\dfrac{x-y}{y}\right)$ d) $d\left(x^2-y^2\right)$

132) The total derivative of $dx+dy$ with the integrating factor $\frac{1}{x+y}$ is

a) $d\left[\log(x-y)\right]$ b) $d\left[\log(x+y)\right]$
c) $d\left[\log(xy)\right]$ d) $d\left[\log\left(x^2+y^2\right)\right]$

133) The total derivative of $dx-dy$ with the integrating factor $\frac{1}{x-y}$ is

a) $d\left[\log(x-y)\right]$ b) $d\left[\log(x+y)\right]$
c) $d\left[\log(xy)\right]$ d) $d\left[\log\left(x^2+y^2\right)\right]$

134) The total derivative of $xdy+ydx$ with the integrating factor $\frac{1}{xy}$ is

a) $d\left[\log(x-y)\right]$ b) $d\left[\log(x+y)\right]$
c) $d\left[\log(xy)\right]$ d) $d\left[\log\left(x^2+y^2\right)\right]$

135) The total derivative of $xdy-ydx$ with the integrating factor $\frac{1}{xy}$ is

a) $d\left[\log(x-y)\right]$ b) $d\left[\log\left(\frac{x}{y}\right)\right]$
c) $d\left[\log\left(\frac{y}{x}\right)\right]$ d) $d\left[\log(xy)\right]$

136) The total derivative of $2(xdx+ydy)$ with the integrating factor $\frac{1}{x^2+y^2}$ is

a) $d\left[\log(x-y)\right]$ b) $d\left[\log(x+y)\right]$
c) $d\left[\log(xy)\right]$ d) $d\left[\log\left(x^2+y^2\right)\right]$

137) The total derivative of $2(xdx-ydy)$ with the integrating factor $\frac{1}{x^2-y^2}$ is

a) $d\left[\log\left(x^2-y^2\right)\right]$ b) $d\left[\log(x+y)\right]$
c) $d\left[\log(xy)\right]$ d) $d\left[\log\left(x^2+y^2\right)\right]$

138) The total derivative of $xdy-ydx$ with the integrating factor $\frac{1}{x^2+y^2}$ is

a) $d\left[\log\left(x^2-y^2\right)\right]$ b) $d\left[\log\left(x^2+y^2\right)\right]$
c) $d\left(\tan^{-1}\frac{y}{x}\right)$ d) $d\left(\tan^{-1}\frac{x}{y}\right)$

139) If the integrating factor of $\frac{ydx-xdy}{y^2}$ is $\frac{y}{x}$, its total derivative is

a) $d\left(\tan^{-1}\frac{x}{y}\right)$ b) $d\left(\log(x+y)\right)$
c) $d\left(\log\frac{y}{x}\right)$ d) $d\left(\log\frac{x}{y}\right)$

140) If the integrating factor of $\frac{xdy-ydx}{x^2}$ is $\frac{x}{y}$, its total derivative is

a) $d\left(\tan^{-1}\frac{y}{x}\right)$ b) $d\left(\tan^{-1}\frac{x}{y}\right)$
c) $d\left(\log\frac{x}{y}\right)$ d) $d\left(\log\frac{y}{x}\right)$

141) If the integrating factor of $\frac{ydx-xdy}{y^2}$ is $\frac{y^2}{x^2+y^2}$, its total derivative is

a) $d\left(\log\frac{y}{x}\right)$ b) $d\left(\tan^{-1}\frac{y}{x}\right)$
c) $d\left(\tan^{-1}\frac{x}{y}\right)$ d) $\log\left(x^2+y^2\right)$

142) The total derivative of $dx+dy$ with the integrating factor $\frac{1}{1+(x+y)^2}$ is

a) $d\left(\tan^{-1}(x+y)\right)$ b) $d\left(\log\frac{y}{x}\right)$
c) $d\left(\sec^{-1}(x+y)\right)$ d) $\log(x+y)$

143) The equation $(x+y+3)dx+(x-y-7)dy=0$ is of the form

a) variable separable b)exact differential
c) linear differential d) homogeneous

144) Equation $(3x+2y+1)dx+(2x-7y-3)dy=0$ is of the form
a) variable separable b) exact differential
c) linear differential d) homogeneous

145) For what value of λ, the differential equation $(5x+\lambda y-3)dx+(3x-7y+5)dy=0$ is exact?
a) 0 b) 1 c) 2 d)3

146) For what value of a, the differential equation $(xy^2+ax^2y)dx+(x^3+x^2y)dy=0$ is exact?
a)3 b) 2 c) 1 d) 5

147) For what value of a, the differential equation $(\tan y+ax^2y-y)dx+(x\tan^2 y-x^3-\sec^2 y)dy=0$ is exact?
a) 2 b) -2 c) 3 d)-3

148) The differential equation $\frac{dy}{dx}=\frac{ay+1}{(y+2)e^y-x}$ is exact, if the value of a is
a) -2 b) 2 c) -1 d)1

149) Differential equation $\frac{dy}{dx}+\frac{3+ay\cos x}{2\sin x-4y^3}=0$ is exact, if the value of a is
a) -3 b) 3 c)2 d)-2

150) For what values of a and b, the differential equation $(ay^2+x+x^8)dx+(y^2+y-bxy)dy=0$ is an exact differential equation?
a) $2a+b=0$ b) $a=2b$
c) $a-2b=3$ d) $a=1=b$

151) The equation $(1+axy^2)dx+(1+bx^2y)dy=0$ is exact differential equation, if
a) $a+2b=0$ b) $a=1,\ b=-3$
c) $a=b$ d) $a=2,\ b=3$

152) For what values of a and b, differential equation $(axy^4+\sin y)dx+(bx^2y^3+x\cos y)dy=0$ is formed to be exact?
a) $a=3b$ b) $a=2,\ b=4$
c) $a+b=1$ d) $a=3,\ b=-3$

153) The integrating factor for the differential equation $(y^2-2xy)dx+(2x^2+3xy)dy=0$ is
a) $\frac{1}{4xy^2}$ b) $\frac{1}{4x^2y^2}$ c) $\frac{1}{2x^2y}$ d) $\frac{1}{2xy}$

154) The integrating factor for the differential equation $(xy-2y^2)dx-(x^2-3xy)dy=0$ is
a) $\frac{1}{2x^2y^2}$ b) $\frac{1}{x^2y}$ c) $\frac{1}{xy}$ d) $\frac{1}{xy^2}$

155) The integrating factor for the differential equation $(x^2-3xy+2y^2)dx-(2xy-3x^2)dy=0$ is
a) $\frac{1}{x^3}$ b) $\frac{1}{x^3y}$ c) $\frac{1}{y^3}$ d) $\frac{1}{x^2y^2}$

156) The differential equation $(y^3-2x^2y)dx+(2xy^2-x^3)dy=0$ can be reduced to exact if the equation is multiplied by the integrating factor
a) $\frac{1}{xy}$ b) x^2y^2 c) $\frac{1}{x^2y^2}$ d) xy

157) The integrating factor for the differential equation $(x^2y-2xy^2)dx+(3x^2y-x^3)dy=0$ is
a) $\frac{1}{2xy}$ b) $\frac{1}{x^2y}$ c) $\frac{1}{x^2y^2}$ d) $\frac{1}{xy^2}$

158) The integrating factor for the differential equation $(xy+1)ydx-(xy-1)xdy=0$ is
a) $\frac{1}{2x^2y^2}$ b) $\frac{1}{2x^2y}$ c) $\frac{1}{2xy^2}$ d) $\frac{1}{2xy}$

159) The integrating factor for the differential equation $(xy+1)ydx+(x^2y^2+xy+1)xdy=0$ is
a) $\frac{1}{x^3y}$ b) $-\frac{1}{x^3y^3}$ c) $-\frac{1}{x^2y^2}$ d) $\frac{1}{xy^3}$

160) The integrating factor for the equation $(x^2y^2+xy+1)ydx+(x^2y^2-xy+1)xdy=0$ is
a) $\frac{1}{2x^2y^2}$ b) $\frac{1}{2x^2y}$ c) $\frac{1}{2xy^2}$ d) $\frac{1}{2x^3y^3}$

161) The integrating factor for the equation $\left(x^2y^2+5xy+2\right)ydx+\left(x^2y^2+4xy+2\right)xdy=0$ is

a) $\frac{1}{x^2y^3}$ b) $\frac{1}{x^2y}$ c) $\frac{1}{xy^2}$ d) $\frac{1}{x^2y^2}$

162) The differential equation $y\left(xy+2x^2y^2\right)dx+x\left(xy-x^2y^2\right)dy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) $\frac{1}{2x^2y}$ b) $\frac{1}{3x^3y}$ c) $\frac{1}{2x^2y^2}$ d) $\frac{1}{3x^3y^3}$

163) The integrating factor for the differential equation $\left(x^2+y^2+x\right)ydx+xydy=0$ is

a) $\frac{1}{2xy^2}$ b) $\frac{1}{2xy}$ c) x d) $\frac{1}{x}$

164) The integrating factor for the equation $\left(xy\sin xy+\cos xy\right)ydx+\left(xy\sin xy-\cos xy\right)xdx=0$ is

a) $\frac{1}{2xy}$ b) $\frac{1}{2xy\cos xy}$

c) $\frac{1}{2xy\sin xy}$ d) $\frac{1}{2\cos xy}$

165) The integrating factor for the differential equation $\left(y+\frac{y^3}{3}+\frac{x^2}{2}\right)dx+\left(\frac{x+xy^2}{4}\right)dy=0$ is

a) $\frac{1}{x^2}$ b) x^2 c) $\frac{1}{x^3}$ d) x^3

166) The integrating factor for the differential equation $\left(2x\log x-xy\right)dy+2ydx=0$ is

a) x^2 b) $\frac{1}{x}$ c) $\frac{1}{x^2}$ d) $\frac{1}{x^3}$

167) The integrating factor for the differential equation $\left(x^2+y^2+1\right)dx-2xydy=0$ is

a) x^2 b) $\frac{1}{x}$ c) $\frac{1}{x^2}$ d) $\frac{1}{x^3}$

168) The integrating factor for the differential equation $y\left(2xy+e^x\right)dx-e^xdy=0$ is

a) $\frac{1}{y^2}$ b) $\frac{1}{x^2}$ c) $\frac{1}{y^3}$ d) $\frac{1}{x^3}$

169) The integrating factor for the differential equation $y\log ydx+\left(x-\log y\right)dy=0$ is

a) $\frac{1}{y^2}$ b) $\frac{1}{x^2}$ c) $\frac{1}{y}$ d) $\frac{1}{x}$

170) The differential equation $\left(y^4+2y\right)dx+\left(xy^3+2y^4-4x\right)dy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) $\frac{1}{y^2}$ b) $\frac{1}{y^3}$ c) $\frac{1}{x^3}$ d) $\frac{1}{x^2}$

171) The differential equation $\left(2x+e^x\log y\right)ydx+e^xdy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) x^2 b) $\frac{1}{x^3}$ c) $\frac{1}{x}$ d) $\frac{1}{y}$

172) The differential equation $\left(xy^3+y\right)dx+2\left(x^2y^2+x+y^4\right)dy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) x b) y c) $\frac{1}{y}$ d) $\frac{1}{x}$

173) The differential equation $\left(xy^2-e^{\frac{1}{x^3}}\right)dx-x^2ydy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) $\frac{1}{x^4}$ b) $\frac{1}{x^3}$ c) $\frac{1}{y^2}$ d) $\frac{1}{y^3}$

174) $\left(x^2-3xy+2y^2\right)dx-\left(e^x+y^3\right)dy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) $\frac{1}{y^2}$ b) $\frac{1}{y^3}$ c) $\frac{1}{x^3}$ d) $\frac{1}{x^4}$

175) $\left(y^4+2y\right)dx+\left(xy^3+2y^4-4x\right)dy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) $\frac{1}{x^3}$ b) $\frac{1}{y^2}$ c) $\frac{1}{y^3}$ d) $\frac{1}{x^4}$

176) The solution of the exact differential equation $(x+y-2)dx+(x-y+4)dy=0$ is

a) $x^2+y^2+xy+x+y+c=0$

b) $x^2+y^2+2xy+4x+6y+c=0$

c) $x^2+y^2+2xy+4x+8y+c=0$

d) $x^2-y^2+2xy-4x+8y+c=0$

177) The solution of the exact differential equation $\left(y^2e^{xy^2}+4x^3\right)dx+\left(2xye^{xy^2}-3y^2\right)dy=0$ is

a) $\frac{1}{y^2}e^{xy^2}+\frac{x^4}{4}-\frac{y^3}{3}=c$ b) $e^{xy^2}+x^4-y^3=c$

c) $e^{xy^2}+x^4+y^3=c$ d) $e^{xy^2}+\frac{x^4}{4}-\frac{y^3}{3}=c$

178) The solution of the exact differential equation $\left(x^2-4xy-2y^2\right)dx+\left(y^2-4xy-2x^2\right)dy=0$ is

a) $x^3-6x^2y-6xy^2+y^3=c$

b) $\frac{x^3}{3}-6x^2y-6xy^2+\frac{y^3}{3}=c$

c) $x^3+x^2y+xy^2+y^3=c$

d) $x^3+x^2y-3xy^2+2y^3=c$

179) The solution of the exact differential equation $(1+\log xy)dx+\left(1+\frac{x}{y}\right)dy=0$ is

a) $y-x\log x+\log y=c$ b) $y+x\log xy=c$

c) $1+\frac{x}{y}\log xy=c$ d) $\frac{y}{x}+\log xy=c$

180) The solution of the exact differential equation $\left(1+x^2\right)(xdy+ydx)+2x^2ydx=0$ is

a) $x^2+y\left(1+x^2\right)=c$ b) $x+y-\left(1+x^2\right)=c$

c) $xy\left(1+x^2\right)=c$ d) $x+y\left(1+x^2\right)=c$

181) The solution of the exact differential equation $\cos y-x\sin y\frac{dy}{dx}=\sec^2 x$ is

a) $\frac{x}{y}\cos y=c\tan x$ b) $\cot x-x^2\cos y=c$

c) $\tan^2 x-x\sin y=c$ d) $\tan x-x\cos y=c$

182) The solution of the exact differential equation $\frac{dy}{dx}=\frac{1+y^2+3x^2y}{1-2xy-x^3}$ is

a) $x\left(1+y^2\right)+x^3y-y=c$

b) $\frac{1+y^2}{x}+x^2y-y=c$

c) $1+y^2+x^2y-xy=c$

d) $x\left(1+\frac{y^2}{2}\right)-\frac{x^3y}{3}-y=c$

183) The solution of the exact differential equation $\left(x^2y-2xy^2\right)dx+\left(3x^2y-x^3\right)dy=0$ with the integrating factor $\frac{1}{x^2y^2}$ is

a) $\frac{y}{x}-\log x+\log y=c$

b) $\frac{x}{y}-2\log x+3\log y=c$

c) $x+2y\log x+3x\log y=c$

d) $\frac{x^2}{2}-2y\log x+3\log y=c$

184) The solution of the exact differential equation $\left(3xy^2-y^3\right)dx+\left(xy^2-2x^2y\right)dy=0$ with the integrating factor $\frac{1}{x^2y^2}$ is

a) $\frac{y}{x}+3\log x+2\log y=c$

b) $y\log x+3\log x-2\log y=c$

c) $\frac{y}{x}+3\log x-2\log y=c$

d) $\frac{y^2}{x^2}+3x\log x-2y\log y=c$

185) The solution of the exact differential equation $\left(x^2-3xy+2y^2\right)dx+x(3x-2y)dy=0$ with the integrating factor $\frac{1}{x^3}$ is

a) $x^2\log x+3xy-y^2=cx^2$

b) $\log x+3x^2y-y^2=c$

c) $x^3\log x+3x^2y-xy^2=cx^3$

d) $3\log x+3xy+y^2=cx^2$

186) The solution of the exact differential equation $(1+xy)ydx+(1-xy)xdy=0$ with the integrating factor $\frac{1}{2x^2y^2}$ is

a) $3\log\left(\frac{x}{y}\right)+\frac{1}{x^2y^2}=c$ b) $\log\left(\frac{x}{y}\right)+\frac{1}{xy}=c$

c) $3\log\left(\frac{x}{y}\right)-\frac{1}{x^2y}=c$ d) $\log\left(\frac{x}{y}\right)-\frac{1}{xy}=c$

187) The solution of the exact differential equation $(x^2y^2+5xy+2)ydx+(x^2y^2+4xy+2)xdy=0$ with the integrating factor $\frac{1}{x^2y^2}$ is

a) $xy+5\log x-\frac{2}{xy}+4\log y=c$

b) $x^2y+5\log x-\frac{1}{xy}+2\log y=c$

c) $xy+5\log x+\frac{1}{xy}+3\log y=c$

d) $x^2y^2+5\log x+\frac{2}{xy}+4\log y=c$

188) The solution of the exact differential equation $(x^2y^2+xy+1)ydx+(x^2y^2-xy+1)xdy=0$ with the integrating factor $\frac{1}{2x^2y^2}$ is

a) $xy-\frac{1}{xy}+x\log x+y\log y=c$

b) $xy-\frac{1}{xy}+\log x+\log y=c$

c) $\frac{x}{y}-\frac{1}{xy}+\log\left(\frac{x}{y}\right)=c$

d) $xy-\frac{1}{xy}+\log\left(\frac{x}{y}\right)=c$

189) The solution of the exact differential equation $y(xy+2x^2y^2)dx+x(xy-x^2y^2)dy=0$ with the integrating factor $\frac{1}{3x^3y^3}$ is

a) $2\log\left(\frac{x}{y}\right)-\frac{1}{xy}=c$ b) $\frac{1}{2}\log\left(\frac{x}{y}\right)+\frac{1}{xy}=c$

c) $\log\left(\frac{x^2}{y}\right)-\frac{1}{xy}=c$ d) $\log\left(\frac{x}{y^2}\right)+\frac{1}{xy}=c$

190) The solution of the exact differential equation $(x^2+y^2+x)ydx+xydy=0$ with the integrating factor x is

a) $x^4+x^2y^3+x^3=c$ b) $y\left(\frac{x^4}{4}+\frac{x^2y^2}{2}+\frac{x^3}{3}\right)=c$

c) $y(x^4+x^2y^2+x^3)=c$ d) $\frac{x^4}{4}+\frac{x^2y^2}{2}+\frac{x^3}{3}=c$

191) The solution of the exact differential equation $(xy\sin xy+\cos xy)ydx+(xy\sin xy-\cos xy)xdx=0$ with the integrating factor $\frac{1}{2xy\cos xy}$ is

a) $x\log(\sec xy)=cy$ b) $xy\sec xy=c$

c) $x\sec xy=cy$ d) $x\cos xy=cy$

192) The solution of the exact differential equation $(x^2-3xy+2y^2)dx+(3x^2-2xy)dy=0$ with the integrating factor $\frac{1}{x^3}$ is

a) $\log x+\frac{3y}{x}-\left(\frac{y}{x}\right)^2=c$ b) $\log x+3yx-\left(\frac{y}{x}\right)^2=c$

c) $\log x+\frac{y}{x}+\left(\frac{y}{x}\right)^2=c$ d) $3\log x+\frac{y}{x}-\frac{y^2}{x}=c$

193) The solution of the exact differential equation $(xy^3+y)dx+2(x^2y^2+x+y^4)dy=0$ with the integrating factor y is

a) $\frac{3}{4}x^2y^4+\frac{6}{5}xy^2+2y^6=c$

b) $3x^2y^4+6x^2y+2x^6=c$

c) $x^3y^4+3xy^2+5y^6=c$

d) $3x^2y^4+6xy^2+2y^6=c$

194) The solution of the exact differential equation $(y^4+2y)dx+(xy^3-4x+2y^4)dy=0$ with the integrating factor $\frac{1}{y^3}$ is

a) $x(y^3+2)+y^2=c$ b) $x^2(y^3+2)-y^4=cy^2$

c) $x(y^3+2)+y^4=cy^2$ d) $(y^3+2)xy^4=cy^2$

195) The solution of the exact differential equation $\left(3x+2y^2\right)ydx+2x\left(2x+3y^2\right)dy=0$ with the integrating factor xy^3 is

a) $x^3y^4+x^2y^6=c$ b) $x^3y^3+x^4y^3=c$

c) $x^2y^4+xy^6=c$ d) $\frac{1}{3}x^3y^4+\frac{1}{4}x^2y^6=c$

196) The solution of the exact differential equation $\left(x^2y+y^4\right)dx+\left(2x^3+4xy^3\right)dy=0$ with the integrating factor $x^{\frac{5}{2}}y^{10}$ is

a) $\frac{12}{11}x^{\frac{11}{2}}y^{11}+\frac{12}{7}x^{\frac{7}{2}}y^{14}=c$

b) $\frac{2}{11}x^{\frac{11}{12}}y^{11}+\frac{2}{7}x^{\frac{7}{12}}y^{14}=c$

c) $\frac{2}{11}x^{\frac{11}{2}}y^{11}+\frac{2}{7}x^{\frac{7}{2}}y^{14}=c$

d) $\frac{2}{11}x^{\frac{11}{2}}y^{11}-\frac{2}{7}x^{\frac{7}{2}}y^{14}=c$

197) The solution of the exact differential equation $\left(y^2+2x^2y\right)dx+\left(2x^3-xy\right)dy=0$ with the integrating factor $\frac{1}{x^{5/2}y^{1/2}}$ is

a) $4xy-\frac{2}{3}\left(\frac{y}{x}\right)^{\frac{3}{2}}=c$ b) $4\sqrt{xy}-\frac{2}{3}\left(\frac{y}{x}\right)^{\frac{3}{2}}=c$

c) $4\sqrt{xy}-\frac{2}{3}\sqrt{\frac{y}{x}}=c$ d) $\sqrt{xy}+\left(\frac{y}{x}\right)^{\frac{3}{2}}=c$

198) The solution of the exact differential equation $\left(y^4-2x^3y\right)dx+\left(x^4-2xy^3\right)dy=0$ with the integrating factor $\frac{1}{x^2y^2}$ is

a) $\frac{2x^2}{y}+\frac{3y^2}{x}=c$ b) $\frac{x^2}{y}-\frac{y^2}{x}=c$

c) $\frac{x^2}{2y}+\frac{y^2}{3x}=c$ d) $\frac{x^2}{y}+\frac{y^2}{x}=c$

199) The solution of the exact differential equation $\left(y^3-2x^2y\right)dx-\left(x^3-2xy^2\right)dy=0$ with the integrating factor xy is

a) $x^3y^3\left(y^2+x^2\right)=c$ b) $x^2y^2\left(y^2-x^2\right)=c$

c) $x^2y^2\left(y^2+x^2\right)=c$ d) $x^2+y^2\left(y^2-x^2\right)=c$

200) The solution of the exact differential equation $\left(3x^2y^4+2xy\right)dx+\left(2x^3y^3-x^2\right)dy=0$ with the integrating factor $\frac{1}{y^2}$ is

a) $x^3y^2+\frac{x^2}{y}=c$ b) $x^2y^2+\frac{x^2}{y^2}=c$

c) $x^3y^3-\frac{x^2}{y}=c$ d) $x^2y^3-\frac{x^2}{y^3}=c$

201) The solution of the exact differential equation $y\left(x^2y+e^x\right)dx-e^xdy=0$ with the integrating factor $\frac{1}{y^2}$ is

a) $\frac{x^2}{2}+\frac{e^x}{y}=c$ b) $\frac{x^3}{3}-\frac{e^x}{y}=c$

c) $\frac{x^3}{3}+\frac{e^x}{y}=c$ d) $\frac{x^3}{3}+\frac{e^x}{2}=c$

202) The solution of the exact differential equation $\left(2x+e^x\log y\right)ydx+\left(e^x\right)dy=0$ with the integrating factor $\frac{1}{y}$ is

a) $x^2+e^x+\log y=c$ b) $x^2-e^x\log y=c$

c) $\frac{x^2}{2}+e^x\log y=c$ d) $x^2+e^x\log y=c$

203) The solution of $\frac{dy}{dx}\left(x+2y^3\right)=y+2x^3y^2$ with the integrating factor $\frac{1}{y^2}$ is

a) $\frac{x}{y}-\frac{x^4}{y}+y^2=c$ b) $\frac{x}{y}+\frac{x^4}{2}-\frac{y^2}{2}=c$

c) $\frac{x}{3}+\frac{x^4}{2}+y^2=c$ d) $\frac{x}{y}+\frac{x^4}{2}-y^2=c$

204) The solution of the exact differential equation $y\log ydx+\left(x-\log y\right)dy=0$ with the integrating factor $\frac{1}{y}$ is

a) $2x\log y-\left(\log y\right)^2=c$

b) $x^2\log y+\left(\log y\right)^2=c$

c) $2x\log y+\left(\log y\right)^3=c$

d) $\frac{2x}{3}\log y-\log y^2=c$

205) The solution of the exact differential equation $y\left(2x^2y+e^x\right)dx=\left(e^x+y^3\right)dy$ with the integrating factor $\frac{1}{y^2}$ is

a) $\frac{1}{3}x^3+\frac{e^x}{x}-\frac{1}{2}y^2=c$ b) $\frac{2}{3}x^3+\frac{e^x}{y}+\frac{1}{2}y^3=c$

c) $\frac{2}{3}x^3+\frac{e^x}{y}-\frac{1}{2}y^2=c$ d) $x^3+\frac{e^x}{y}-y^2=c$

206) The solution of the exact differential equation $\left(2x\log x-xy\right)dy+2ydx=0$ with the integrating factor $\frac{1}{x}$ is

a) $2x\log x-\frac{x^2}{2}=c$ b) $2y\log x-\frac{y^2}{2}=c$

c) $\frac{y}{2}\log x-\frac{y^2}{2}=c$ d) $y\log x+\frac{y^2}{2}=c$

207) The solution of the exact differential equation $\left(x^4e^x-2mxy^2\right)dx+2mx^2ydy=0$ with the integrating factor $\frac{1}{x^4}$ is

a) $e^x+\frac{m^2y^2}{x^2}=cm$ b) $e^x-\frac{my^2}{x^2}=c$

c) $\frac{e^x}{y}-\frac{my^2}{x^2}=c$ d) $e^x+\frac{my^2}{x^2}=c$

208) The differential equation which can be expressed in the form $\frac{dy}{dx}+Py=Q$, where P and Q are functions of x only, is known as

a) variable separable equation in x, y
b) homogeneous differential equation in x, y
c) linear differential equation in x w.r.t y
d)linear differential equation in y w.r.t x

209) The differential equation which can be expressed in the form $\frac{dx}{dy}+Px=Q$, where P and Q are functions of y only, is known as

a)linear differential equation in x w.r.t y
b) linear differential equation in y w.r.t x
c) homogeneous differential equation in x, y
d) variable separable equation in x, y

210) The integrating factor of the linear differential equation $\frac{dy}{dx}+Py=Q$, where P and Q are functions of x only, is

a) $e^{\int Pdx}$ b) $e^{\int Qdx}$ c) $e^{\int Pdy}$ d) $e^{\int Qdy}$

211) The integrating factor of the linear differential equation $\frac{dx}{dy}+Px=Q$, where P and Q are functions of y only, is

a) $e^{\int Pdx}$ b) $e^{\int Qdx}$ c) $e^{\int Pdy}$ d) $e^{\int Qdy}$

212) The general solution of the linear differential equation $\frac{dy}{dx}+Py=Q$, where P and Q are functions of x only, is given by

a) $ye^{\int Pdx}=\int Qe^{\int Pdx}dx+c$

b) $xe^{\int Pdx}=\int Qe^{\int Pdx}dx+c$

c) $xe^{\int Pdy}=\int Qe^{\int Pdy}dy+c$

d) $xe^{\int Qdx}=\int Pe^{\int Qdx}dx+c$

213) The general solution of the linear differential equation $\frac{dx}{dy}+Px=Q$, where P and Q are functions of y only, is given by

a) $ye^{\int Pdx}=\int Qe^{\int Pdx}dx+c$

b) $xe^{\int Pdx}=\int Qe^{\int Pdx}dx+c$

c) $xe^{\int Pdy}=\int Qe^{\int Pdy}dy+c$

d) $xe^{\int Qdx}=\int Pe^{\int Qdx}dx+c$

214) A differential equation which can be expressed in the form $\frac{dy}{dx}+Py=Qy^n$, where P and Q are functions of x only, is known as

a) Non-linear differential equation
b)Bernoulli's linear differential equation
c) exact differential equation
d) homogenous differential equation

215) A differential equation which can be expressed in the form $\frac{dx}{dy}+Px=Qx^n$, where P and Q are functions of x only, is known as

a) Non-linear differential equation
b) Bernoulli's linear differential equation
c) exact differential equation
d) homogenous differential equation

216) A differential equation which can be expressed in the form $f'(y)\frac{dy}{dx}+Pf(y)=Q$, where P and Q are functions of x only, can be reduced into the linear form by substituting
a) $P=v$ b) $Q=v$
c) $f(y)=v$ d) $f'(y)=v$

217) A differential equation which can be expressed in the form $\frac{dy}{dx}+Py=Qy^n$, where P and Q are functions of x only, can be reduced into the linear form by substituting
a) $y^n=v$ b) $y^{1-n}=v$
c) $y^{n-1}=v$ d) $y^{n+1}=v$

218) If I_1, I_2 are the integrating factors of the equations $\frac{dx}{dy}+Px=Q$ and $\frac{dx}{dy}-Px=Q$ respectively, the relation between them is
a) $I_1=-I_2$ b) $I_1=I_2$
c) $I_1\cdot I_2=-1$ d) $I_1\cdot I_2=1$

219) The integrating factor of the linear differential equation $\frac{dy}{dx}+xy=x^5$ is
a) $e^{\log\frac{x^2}{2}}$ b) $e^{\frac{x^2}{2}}$ c) e^{x^2} d) x^2

220) The integrating factor of the linear differential equation $\frac{dy}{dx}+2xy=\frac{\tan^{-1}x}{1+x^2}$ is
a) $\frac{x^2}{2}$ b) $e^{\frac{x^2}{2}}$ c) e^{x^2} d) $2x^2$

221) The integrating factor of the linear differential equation $\frac{dx}{dy}+xy=y^5$ is
a) $e^{\frac{y^2}{2}}$ b) $\frac{y^2}{2}$ c) $e^{\frac{x^2}{2}}$ d) e^{x^2}

222) The integrating factor of the linear differential equation $\frac{dy}{dx}+\frac{y}{1+x}=x^3$ is
a) $e^{\frac{(1+x)^2}{2}}$ b) $1+x$ c) $\frac{1}{1+x}$ d) e^{1+x}

223) The integrating factor of the linear differential equation $\frac{dy}{dx}+\frac{y}{1-x}=\sin x$ is
a) $\frac{1}{1-x}$ b) $1-x$ c) e^{1-x} d) $e^{\frac{(1-x)^2}{2}}$

224) The integrating factor of the linear differential equation $\frac{dy}{dx}+\frac{y}{1+x^2}=\sec x\tan x$ is
a) $\frac{(1+x^2)^2}{2}$ b) $1+x^2$ c) $e^{\tan^{-1}x}$ d) e^{1+x^2}

225) The integrating factor of the linear differential equation $\frac{dy}{dx}+\frac{2x}{1+x^2}y=\tan^{-1}x$ is
a) $\frac{(1+x^2)^2}{2}$ b) $1+x^2$ c) $e^{\tan^{-1}x}$ d) e^{1+x^2}

226) The integrating factor of the linear equation $\frac{dy}{dx}+y\tan x=e^x\sin(2x-3)$ is
a) $\sec^2 x$ b) $\cos x$ c) $\sec x$ d) $e^{\sec x}$

227) The integrating factor of the linear differential equation $\tan x\frac{dy}{dx}+y=e^x\sin x$ is
a) e b) $e^{\sin x}$ c) $\log(\sin x)$ d) $\sin x$

228) The integrating factor of the linear equation $(1+x^2)\frac{dy}{dx}+xy=2x^3-3x+5$ is
a) e^{1+x^2} b) $\frac{1}{1+x^2}$ c) $1+x^2$ d) $\sqrt{1+x^2}$

229) The integrating factor of the linear equation $(1+x^2)\frac{dy}{dx}+4xy=\frac{1}{(1+x^2)^3}$ is
a) $1+x^2$ b) e^{1+x^2} c) $(1+x^2)^2$ d) $\frac{1}{1+x^2}$

230) The integrating factor of the linear equation $\left(1+x^2\right)\frac{dy}{dx}+2xy=\frac{1}{\left(1+x^2\right)^3}$ is

a) $1+x^2$ b) e^{1+x^2} c) $\left(1+x^2\right)^2$ d) $\frac{1}{1+x^2}$

231) The integrating factor of the linear equation $\left(1+x^2\right)\frac{dy}{dx}-2xy=\frac{1}{\left(1+x^2\right)^3}$ is

a) $1+x^2$ b) e^{1+x^2} c) $\left(1+x^2\right)^2$ d) $\frac{1}{1+x^2}$

232) The integrating factor of the linear differential equation $\frac{dx}{dy}+\frac{xy}{1+y^2}=\sec y$ is

a) $\sqrt{1+x^2}$ b) $\sqrt{1+y^2}$ c) $\tan^{-1} y$ d) $e^{\tan^{-1} y}$

233) The integrating factor of the linear differential equation $\frac{dy}{dx}+y\cot x=\tan x$ is

a) $\sin x$ b) $e^{\sec x}$
c) $\cos x$ d) $\sec x+\tan x$

234) The integrating factor of the linear differential equation $\cos x\frac{dy}{dx}+y=\tan x$ is

a) $e^{\sec x+\tan x}$ b) $e^{\sec x}$
c) $\cos x$ d) $\sec x+\tan x$

235) The integrating factor of the differential equation $\frac{dy}{dx}+\sqrt{x}y=\sin\sqrt{x}\cos\sqrt{x}$ is

a) $\sin\sqrt{x}$ b) $e^{\log\sqrt{x}}$
c) $e^{\frac{2}{3}x\sqrt{x}}$ *or* $e^{\frac{2}{3}x^{\frac{3}{2}}}$ d) $\frac{2}{3}x\sqrt{x}$ *or* $\frac{2}{3}x^{\frac{3}{2}}$

236) The integrating factor of the linear equation $\frac{dy}{dx}+\left(\tan x+\frac{1}{x}\right)y=\frac{1}{x}\sec x$ is

a) $x\sec x$ b) $e^{x\sec x}$ c) $e^{x+\sec x}$ d) $x+\sec x$

237) The integrating factor of the linear differential equation $\left(1-x^2\right)\frac{dy}{dx}=x^3+xy$ is

a) $e^{\tan^{-1} x}$ b) $e^{\sqrt{1-x^2}}$ c) $\frac{1}{\sqrt{1-x^2}}$ d) $\sqrt{1-x^2}$

238) The integrating factor of the linear differential equation $\left(1-x^2\right)\frac{dy}{dx}=x^3-xy$ is

a) $\frac{1}{\sqrt{1-x^2}}$ b) $\sqrt{1-x^2}$ c) $e^{\sqrt{1-x^2}}$ d) $e^{\tan^{-1} x}$

239) The integrating factor of the differential equation $1+y^2+\left(x-e^{\tan^{-1} x}\right)\frac{dy}{dx}=0$ is

a) $\tan^{-1} x$ b) $\tan^{-1} y$ c) $e^{\tan^{-1} x}$ d) $e^{\tan^{-1} y}$

240) The integrating factor of the differential equation $1+x^2+\left(y-e^{\tan^{-1} y}\right)\frac{dx}{dy}=0$ is

a) $\tan^{-1} x$ b) $\tan^{-1} y$ c) $e^{\tan^{-1} x}$ d) $e^{\tan^{-1} y}$

241) The integrating factor of the differential equation $\left(1+y^2\right)dx=\left(e^{\tan^{-1} x}-x\right)dy$ is

a) $\tan^{-1} x$ b) $\tan^{-1} y$ c) $e^{\tan^{-1} x}$ d) $e^{\tan^{-1} y}$

242) The integrating factor of the linear differential equation $y^2+\left(x-\frac{1}{y}\right)\frac{dy}{dx}=0$ is

a) $2\log x$ b) $\log y$ c) $-\frac{1}{y}$ d) $-\frac{1}{y^2}$

243) The integrating factor of the linear differential equation $\sin 2y dx=\left(\tan y-x\right)dy$ is

a) $\frac{\tan x}{2}$ b) $\sqrt{\tan y}$ c) $\sqrt{\tan x}$ d) $\frac{\tan y}{2}$

244) The integrating factor of the linear equation $y\log y dx+\left(x-\log y\right)dy=0$ is

a) $\left(\log y\right)^2$ b) $x\log y$ c) $\log y$ d) $\log x$

245) The integrating factor of the linear differential equation $ydx-\left(y-x\right)dy=0$ is

a) y b) x c) y^2 d) x^2

246) The integrating factor of the linear equation $\sqrt{a^2+x^2}\frac{dy}{dx}+y=\sqrt{a^2+x^2}-x$ is

a) $\frac{1}{2a}\log\left(x+\sqrt{a^2+x^2}\right)$ b) $\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

c) $x+\sqrt{a^2+x^2}$ d) $x-\sqrt{a^2+x^2}$

247) The integrating factor of the linear differential equation $\frac{dy}{dx}=\frac{e^x-2xy}{x^3}$ is

a) $e^{\frac{x^3}{3}}$ b) x^3 c) $\frac{1}{x^3}$ d) e^{x^3}

248) The integrating factor of linear differential equation $\left(x^2+1\right)\frac{dy}{dx}=x^3-2xy+x$ is

a) $\tan^{-1}x$ b) $e^{\tan^{-1}x}$ c) $\frac{1}{x^2+1}$ d) x^2+1

249) The integrating factor of the linear differential equation $x^2\frac{dy}{dx}=3x^2-2xy+1$ is

a) x^2-1 b) x^2 c) x^2+1 d) $\frac{1}{x^2}$

250) The integrating factor of the linear differential equation $\left(e^{-y}\sec^2 y-x\right)dy=dx$ is

a) $e^{\tan y}$ b) $\tan y$ c) e^x d) e^y

251) The differential equation $\frac{dy}{dx}-y\tan x=y^4\sec x$ is reduced into the linear form

a) $\frac{du}{dx}+3u\tan x=-3\sec x;\ u=y^{-3}$

b) $\frac{du}{dx}-3u\tan x=3\sec x;\ u=y^{-3}$

c) $\frac{du}{dx}-3u\tan x=-3\sec x;\ u=y^{-3}$

d) $\frac{du}{dx}+3u\cot x=-3\sec x;\ u=y^{-3}$

252) The differential equation $\frac{dy}{dx}-xy=-y^3e^{-x^2}$ can be reduced to the linear form

a) $\frac{dy}{dx}+xu=-2e^{-x^2};\ u=\frac{1}{y^2}$

b) $\frac{dy}{dx}+xu=e^{-x^2};\ u=\frac{1}{y^2}$

c) $\frac{dy}{dx}-2xu=2e^{-x^2};\ u=\frac{1}{y^2}$

d) $\frac{dy}{dx}+2xu=2e^{-x^2};\ u=\frac{1}{y^2}$

253) The value of k for which e^{ky^2} is an integrating factor of linear differential equation $\frac{dx}{dy}+xy=e^{-\frac{y^2}{2}}$ is

a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) 2 d) -2

254) The general solution of $\frac{dy}{dx}+\frac{y}{1+x}=-x(1-x)$ with the integrating factor $\frac{1}{1-x}$ is

a) $\frac{y}{1-x}=-\frac{x^3}{3}+c$ b) $y=-\frac{x^2}{2}(1-x)+c$

c) $\frac{y}{1-x}=-\frac{x^2}{2}+c$ d) $\frac{y}{1-x}=\frac{x^2}{2}+c$

255) The general solution of $\frac{dy}{dx}+\frac{y}{(1-x)\sqrt{x}}=1-\sqrt{x}$ with the integrating factor $\frac{1+\sqrt{x}}{1-\sqrt{x}}$ is

a) $\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)y^2=x^2+\frac{2}{3}x^{\frac{3}{2}}+c$

b) $\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)y=x+\frac{2}{3}x^{\frac{3}{2}}+c$

c) $\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)y=x+\frac{2}{3}x^{\frac{3}{2}}+c$

d) $\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)y=x-\frac{2}{3}x^{\frac{3}{2}}+c$

256) The general solution of $\frac{dy}{dx}+y\cot x=\sin 2x$ with the integrating factor $\sin x$ is

a) $y\sin x=\frac{2}{3}\sin^3 x+c$ b) $y\sin x=\frac{1}{3}\sin^3 x+c$

c) $x\sin y=\frac{2}{3}\sin^3 x+c$ d) $y\sin x=\sin^3 x+c$

257) The general solution of $\frac{dy}{dx}+\frac{3y}{x}=\frac{e^x}{x^2}$ with the integrating factor x^3 is

a) $x^3y=e^{-x}(x+1)+c$ b) $xy^3=e^x(x-1)+c$

c) $x^3y=e^x(x-1)+c$ d) $x^3y=e^x(x+1)+c$

258) The general solution of $\frac{dy}{dx}+\frac{3y}{x}=x^2$ with the integrating factor x^3 is

a) $x^3y=\frac{x^4}{4}+c$ b) $x^3y=\frac{x^6}{6}+c$

c) $x^3y=\frac{x^2}{2}+c$ d) $xy^3=\frac{x^3}{3}+c$

259) The general solution of $\frac{dy}{dx}+\left(\tan x+\frac{1}{x}\right)y=\frac{1}{x}\sec x$ with the integrating factor $x\sec x$ is

a) $xy\sin x=\tan x+c$ b) $xy\sec x=-\tan x+c$

c) $xy\tan x=\cot x+c$ d) $xy\sec x=\tan x+c$

260) The general solution of $\frac{dy}{dx}+\frac{2y}{x}=\frac{1}{x^3}$ with the integrating factor x^2 is

a) $y=x^2\log x+c$ b) $x^2y=\log x+c$

c) $xy^2=\log x+c$ d) $x^2y=\log\frac{1}{x}+c$

261) The general solution of $\frac{dy}{dx}+(1+2x)y=e^{-x^2}$ with the integrating factor e^{x+x^2} is

a) $ye^{x+x^2}=e^x+c$ b) $ye^{x+x^2}=-e^x+c$

c) $e^{x+x^2}=ye^x+c$ d) $ye^{x-x^2}=e^x+c$

262) The general solution of $\frac{dy}{dx}+\frac{x}{1+y^2}=\frac{e^{-\tan^{-1}y}}{1+y^2}$ with the integrating factor $e^{\tan^{-1}y}$ is

a) $ye^{\tan^{-1}y}=\tan^{-1}x+c$ b) $xe^{\tan^{-1}y}=\tan^{-1}y+c$

c) $xe^{\tan^{-1}y}=\cot^{-1}y+c$ d) $ye^{\tan^{-1}y}=\tan^{-1}y+c$

263) The general solution of $\frac{dy}{dx}+x\sec y=\frac{2y\cos y}{1+\sin y}$ with the integrating factor $\sec y+\tan y$ is

a) $(\sec y+\tan y)x^2=y+c$

b) $(\sec y+\tan y)x=-y^2+c$

c) $(\sec y+\tan y)x=y^2+c$

d) $x=\frac{y^2}{\sec y+\tan y}+c$

Chapter 02 – Applications of Ordinary Differential Equations

1) Two families of curves are said to be orthogonal trajectories of each other, if
   a) Every member of one family cuts every member of other family at right angle.
   b) Every member of one family cuts every member of other family at origin.
   c) Every member of one family cuts every member of other family at common point.
   d) None of the above.

2) In the two dimensional Cartesian form, to find orthogonal trajectories of given family of curves, in its differential equation we replace $\frac{dy}{dx}$ by

   a) $-y\frac{dx}{dy}$ b) $-\frac{dy}{dx}$ c) $-\frac{dx}{dy}$ d) $-x\frac{dx}{dy}$

3) In the two dimensional polar form, to find orthogonal trajectories of given family of curves, in its differential equation we replace $\frac{dr}{d\theta}$ by

   a) $r\frac{d\theta}{dr}$ b) $-r\frac{d\theta}{dr}$ c) $-r^2\frac{d\theta}{dr}$ d) $-\frac{d\theta}{dr}$

4) The differential equation of orthogonal trajectories of family of straight lines $y = mx$ is

   a) $\frac{dx}{dy} + y = 0$ b) $\frac{dy}{dx} = -\frac{y}{x}$

   c) $\frac{dx}{dy} = -\frac{y}{x}$ d) $\frac{dx}{dy} = -\frac{x}{y}$

5) For the family of the curves $x^2 + y^2 = c^2$, the differential equation of orthogonal trajectories is

   a) $x^2 + y^2\frac{dx}{dy} = 0$ b) $x + y\frac{dy}{dx} = 0$

   c) $x + xy\frac{dx}{dy} = 0$ d) $x - y\frac{dx}{dy} = 0$

6) The differential equation of orthogonal trajectories of family of $x^2 + 2y^2 = c^2$ is

   a) $y - 2x\frac{dy}{dc} = 0$ b) $x - 2y\frac{dx}{dy} = 0$

   c) $x + 2y\frac{dy}{dx} = 0$ d) $x + 2y\frac{dx}{dy} = 0$

7) For the family of the curves $y^2 = 4ax$, the differential equation of orthogonal trajectories is

   a) $2y\frac{dy}{dx} = 4x$ b) $2y\frac{dy}{dx} = \frac{y}{x^2}$

   c) $-2y\frac{dy}{dx} = \frac{y^2}{x}$ d) $-2y\frac{dx}{dy} = \frac{y^2}{x}$

8) For the family of the curves $y = 4ax^2$, the differential equation of orthogonal trajectories is

   a) $y\frac{dy}{dx} = 2x$ b) $\frac{dy}{dx} = -\frac{2}{x^2}$

   c) $\frac{dy}{dx} = -\frac{2y}{x}$ d) $-2\frac{dx}{dy} = \frac{1}{xy}$

9) For the family of the curves $xy = c$, the differential equation of orthogonal trajectories is

   a) $x^2\frac{dx}{dy} + 2y = 0$ b) $-x\frac{dx}{dy} + y = 0$

   c) $2x\frac{dx}{dy} - y = 0$ d) $x\frac{dy}{dx} - y = 0$

10) The differential equation of orthogonal trajectories of family of $2x^2 + y^2 = cx$ is

   a) $4x - 2y\frac{dx}{dy} = \frac{2x^2 + y^2}{x}$

   b) $4x + 2y\frac{dy}{dx} = \frac{2x^2 + y^2}{x}$

   c) $4x^2 + 2y\frac{dx}{dy} = \frac{2x^2 + y^2}{x}$

   d) $4x - 2xy\frac{dy}{dx} = \frac{2x^2 + y^2}{x}$

11) For the family of the curves $x^2+cy^2=1$, the differential equation of orthogonal trajectories is

a) $x+\left(\frac{1-x^2}{y}\right)\frac{dy}{dx}=0$ b) $x+\left(\frac{1+x^2}{y}\right)\frac{dx}{dy}=0$

c) $x-\left(\frac{1-x^2}{y}\right)\frac{dx}{dy}=0$ d) $x^2+\left(\frac{1-x^2}{y}\right)\frac{dy}{dx}=0$

12) For the family of the curves $e^x+e^{-y}=c$, the differential equation of orthogonal trajectories is

a) $e^{2x}-e^{-2y}\frac{dx}{dy}=0$ b) $e^{-x}+e^{y}\frac{dx}{dy}=0$

c) $e^{x}-e^{-y}\frac{dy}{dx}=0$ d) $e^{x}+e^{-y}\frac{dx}{dy}=0$

13) The differential equation of orthogonal trajectories of family of $r=a\cos\theta$ is

a) $-r\frac{dr}{d\theta}=\cot\theta$ b) $-r\frac{dr}{d\theta}=\tan\theta$

c) $r\frac{d\theta}{dr}=\cot\theta$ d) $r\frac{d\theta}{dr}=\tan\theta$

14) For the family of the curves $r=a\sin\theta$, the differential equation of orthogonal trajectories is

a) $\frac{1}{r}\frac{d\theta}{dr}=-\cot\theta$ b) $r\frac{d\theta}{dr}=-\tan\theta$

c) $r\frac{d\theta}{dr}=-\cot\theta$ d) $-\frac{1}{r}\frac{dr}{d\theta}=\tan\theta$

15) For the family of the curves $r^2=a\sin\theta$, the differential equation of orthogonal trajectories is

a) $2r\frac{d\theta}{dr}=-\cot\theta$ b) $r\frac{d\theta}{dr}=-\frac{\tan\theta}{2}$

c) $r^2\frac{d\theta}{dr}=-\cot\theta$ d) $-\frac{2}{r}\frac{dr}{d\theta}=\tan\theta$

16) For the family of the curves $r=a\left(1-\cos\theta\right)$, the differential equation of orthogonal trajectories is

a) $-r\frac{d\theta}{dr}=\frac{1-\cos\theta}{\sin\theta}$ b) $r\frac{dr}{d\theta}=\frac{\sin\theta}{1-\cos\theta}$

c) $-r\frac{d\theta}{dr}=\frac{\sin\theta}{1-\cos\theta}$ d) $r^2\frac{d\theta}{dr}=\frac{\sin\theta}{1-\cos\theta}$

17) For the family of the curves $r^2=a\sin 2\theta$, the differential equation of orthogonal trajectories is

a) $-r^2\frac{dr}{d\theta}=\cot 2\theta$ b) $r\frac{d\theta}{dr}=-\cot 2\theta$

c) $r\frac{d\theta}{dr}=-\tan 2\theta$ d) $-\frac{dr}{d\theta}=\cot 2\theta$

18) For the family of the curves $r^2=a\cos 2\theta$, the differential equation of orthogonal trajectories is

a) $\frac{dr}{d\theta}=r\tan 2\theta$ b) $\frac{1}{r}\frac{d\theta}{dr}=-\tan 2\theta$

c) $r\frac{d\theta}{dr}=\cot 2\theta$ d) $r\frac{d\theta}{dr}=\tan 2\theta$

19) For the family of the curves $r^2=a\cos 2\theta$, the differential equation of orthogonal trajectories is

a) $\frac{dr}{d\theta}=r\tan 2\theta$ b) $\frac{1}{r}\frac{d\theta}{dr}=-\tan 2\theta$

c) $r\cot 2\theta\frac{d\theta}{dr}=1$ d) $r\frac{d\theta}{dr}+\tan 2\theta=0$

20) For the family of the curves $r=a\cos^2\theta$, the differential equation of orthogonal trajectories is

a) $r\frac{d\theta}{dr}=\frac{\sin 2\theta}{\cos^2\theta}$ b) $r^2\frac{d\theta}{dr}=-\frac{\sin 2\theta}{\cos^2\theta}$

c) $\frac{dr}{d\theta}=-\frac{\sin 2\theta}{\cos^2\theta}$ d) $r\frac{d\theta}{dr}=-\frac{\sin 2\theta}{\cos 2\theta}$

21) For the family of the curves $r=a\sec^2\left(\frac{\theta}{2}\right)$, the differential equation of orthogonal trajectories is

a) $r\frac{d\theta}{dr}=-\tan\frac{\theta}{2}$ b) $r\frac{dr}{d\theta}=-\cot\frac{\theta}{2}$

c) $\frac{1}{r}\frac{d\theta}{dr}=-\cot\frac{\theta}{2}$ d) $r\frac{d\theta}{dr}=-\tan 2\theta$

22) The orthogonal trajectories of family of curves having differential equation $y=mx$ is $\frac{dy}{dx}=\frac{y}{x}$, is given by

a) $x^2-y^2=c$ b) $\frac{x^2}{2}+y^2=c$
c) $x^2+y^2=c$ d) $x^2+2y^2=c$

23) If the differential equation of family of curves $xy=c$ is $x\frac{dy}{dx}=-y$, then its family of orthogonal trajectories is given by
a) $x^2-2y^2=c$ b) $x^2+2y^2=c$
c) $x^2-y^2=c^2$ d) $x^2+y^2=c$

24) The orthogonal trajectories of family of curves having differential equation $x^2+y^2=k^2$ is $\frac{dy}{dx}=-\frac{x}{y}$, is given by
a) $x^2=4ay$ b) $x^2-y^2=c$
c) $y^2=x+c$ d) $y=cx$

25) If the differential equation of family of curves $x^2-y^2=c$ is $y\frac{dy}{dx}=x$, then its family of orthogonal trajectories is given by
a) $y=cx$ b) $xy=c$
c) $x^2=4ay$ d) $y^2=x+c$

26) The orthogonal trajectories of family of curves having differential equation $x^2+2y^2=c^2$ is $\frac{dy}{dx}+\frac{x}{2y}=0$, is given by
a) $x^2-cx+c^2=0$ b) $y=2cx^2+x$
c) $x^2=ky$ d) $y=2cx^2$

27) The orthogonal trajectories of family of curves having differential equation $x^2+cy^2=1$ is $\frac{dy}{dx}=-\frac{xy}{1-x^2}$, is given by
a) $\log x+x^2+y^2=c$ b) $\log x-x^2-y^2=c$
c) $\log x-\frac{x^2}{2}-\frac{y^2}{2}=c$ d) $\log x+\frac{x^2}{2}+\frac{y^2}{2}=c$

28) The orthogonal trajectories of family of curves having differential equation $y=4ax^2$ is $\frac{dy}{dx}=\frac{y}{x}$, is given by
a) $2x^2=cy^2$ b) $2x^2-y^2=c^2$
c) $2x^2+y^2=c$ d) $x^2+2y^2=c$

29) If the differential equation of family of curves $y^2=4ax$ is $2x\frac{dy}{dx}=y$, then its family of orthogonal trajectories is given by
a) $2x^2+y^2=c$ b) $2x^2-y^2=c^2$
c) $x^2+2y^2=c$ d) $2x^2=cy^2$

30) The orthogonal trajectories of family of curves having differential equation $e^x+e^{-y}=e^c$ is $\frac{dy}{dx}=\frac{e^x}{e^{-y}}$, is given by
a) $e^{2x}+e^{-2y}=k$ b) $e^x-e^{-y}=k$
c) $e^x+e^{-y}=e^c$ d) $e^{-x}+e^y=e^c$

31) If the differential equation of family of curves $e^x-e^{-y}=c$ is $\frac{dy}{dx}+\frac{e^{-y}}{e^x}=0$, then its family of orthogonal trajectories is given by
a) $e^x+e^{-y}=k$ b) $e^{-x}+e^y=e^c$
c) $e^x+e^{-y}=e^c$ d) $e^{2x}+e^{-2y}=k$

32) If the differential equation of family of curves $x^2=ce^{x^2+y^2}$ is $\frac{dy}{dx}=\frac{1-x^2}{xy}$, then its family of orthogonal trajectories is given by
a) $\log\left(1-x^2\right)+2\log y=c$
b) $\log\left(1-x^2\right)-2\log y=c$
c) $2\log\left(1-x^2\right)-3\log y=c$
d) $\log\left(1-x^2\right)+\log y=c$

33) The orthogonal trajectories of family of curves having differential equation $\frac{x^2}{a^2}+\frac{y^2}{b^2+\lambda}=1$ is $x+\left(\frac{a^2-x^2}{y}\right)\frac{dy}{dx}=0$, where a and b are fixed constants, is given by
a) $\frac{y^2}{2}=\lambda\log x+\frac{x^2}{2}+k$
b) $y^2-x^2=a^2\log x+k$
c) $\frac{y^2}{2}=a^2\log x-\frac{x^2}{2}+k$
d) $x^2+y^2=a^2\log x+k$

34) If the differential equation of family of curves $r = a(1-\cos\theta)$ is $(1-\cos\theta)\frac{dr}{d\theta} = r$, then its family of orthogonal trajectories is given by

a) $r^2 = A(1+\cos\theta)$ b) $r = A(1+\sin\theta)$
c) $r = A(1-\cos\theta)$ d) $r = A(1+\cos\theta)$

35) If the differential equation of family of curves $r = a(1-\cos\theta)$ is $\frac{dr}{d\theta} = r\cot\frac{\theta}{2}$, then its family of orthogonal trajectories is given by

a) $\log\cos\left(\frac{\theta}{2}\right) = 2\log r + c$
b) $2\log\sin\left(\frac{\theta}{2}\right) = \frac{1}{2}\log r + c$
c) $2\log\cos\left(\frac{\theta}{2}\right) = \log r + c$
d) $\log 2\cos\left(\frac{\theta}{2}\right) = \log r + c$

36) The orthogonal trajectories of family of curves having differential equation $r = a\sin\theta$ is $\frac{dr}{d\theta} = r\cot\theta$, is given by

a) $r = A\cos\theta$ b) $r = A\tan\theta$
c) $r\cos\theta = A$ d) $r^2 = A\cos\theta$

37) The orthogonal trajectories of family of curves having differential equation $r = a\cos\theta$ is $\frac{dr}{d\theta} + r\tan\theta = 0$, is given by

a) $r = C\operatorname{cosec}2\theta$ b) $r^2 = C\sin^2\theta$
c) $r = C\tan\theta$ d) $r = C\sin\theta$

38) If the differential equation of family of curves $r^2 = a^2\cos 2\theta$ is $\frac{dr}{d\theta} + r\tan 2\theta = 0$, then its family of orthogonal trajectories is given by

a) $r^2 = c\sin^2 2\theta$ b) $r = c\sin 2\theta$
c) $r^2 = c^2\sin 2\theta$ d) $r^2 = c^2\cos 2\theta$

39) If the differential equation of family of curves $r^2 = a\sin 2\theta$ is $\frac{dr}{d\theta} = r\cot 2\theta$, then its family of orthogonal trajectories is given by

a) $r^2\cos 2\theta = k$ b) $r^2 = k\cos 2\theta$
c) $2\log r = \log\sec 2\theta + k$ d) $r^2 = k\cot 2\theta$

40) The orthogonal trajectories of family of curves having differential equation $r = a^2\cos^2\theta$ is $\frac{dr}{d\theta} + 2r\tan\theta = 0$, is given by

a) $\log\tan\theta = 2\log r + c$ b) $2\log\sin\theta = \log r + c$
c) $\frac{3}{2}\log\sin\theta = 2\log r + c$ d) $\frac{\log\sin\theta}{2} = \log r + c$

41) If the differential equation of family of curves $r = a\theta$ is $r = \theta\frac{dr}{d\theta}$, then its family of orthogonal trajectories is given by

a) $r = ce^{-\frac{\theta^2}{2}}$ b) $r = ce^{-\theta^2}$
c) $r^2 = ce^{-\frac{\theta^2}{2}}$ d) $r^2 = ce^{\theta^2}$

42) Newton's law of cooling states that

a) The temperature of the body is inversely proportional to the difference between the body temperature and the surrounding temperature.
b) The temperature of the body is proportional to the sum of the body temperature and the surrounding temperature.
c) The temperature of the body is proportional to the difference between the body temperature and the surrounding temperature.
d) The temperature of the body is proportional to the surrounding of the body temperature.

43) For $\theta =$ the temperature of the body and $\theta_0 =$ the temperature of the surrounding, then Newton's law of cooling states the differential equation

a) $\frac{d\theta}{dt} = -k\theta_0$ b) $\frac{d\theta}{dt} = -k\theta + \theta_0$
c) $\frac{d\theta}{dt} = -k(\theta+\theta_0)$ d) $\frac{d\theta}{dt} = -k(\theta-\theta_0)$

44) A body having initially temperature 90ºC is kept in surrounding of temperature 26ºC. Then the differential equation satisfied by body temperature θ at any time t is given by

a) $\frac{d\theta}{dt} = -k(\theta-64)$ b) $\frac{d\theta}{dt} = -k(\theta-26)$

c) $\frac{d\theta}{dt} = -k(\theta + 26)$ d) $\frac{d\theta}{dt} = -k(\theta - 90)$

45) Consider a substance at initial temperature 32°C is surrounded by room temperature 10°C. According to Newton's law of cooling the differential equation satisfied by its temperature T at time t hour is

a) $\frac{dT}{dt} = -kT(T-10)$ b) $\frac{dT}{dt} = -k(T-32)$

c) $\frac{dT}{dt} = -k(10-32T)$ d) $\frac{dT}{dt} = -k(T-10)$

46) A metallic object is heated up to getting temperature 100°C and the placed in water of temperature 50°C. Then the differential equation of the object temperature θ at time t is given by Newton's law of cooling as

a) $\frac{d\theta}{dt} = -k\theta(\theta - 26)$ b) $\frac{d\theta}{dt} = -k(\theta - 50)$

c) $\frac{d\theta}{dt} = -k(\theta - 150)$ d) $\frac{d\theta}{dt} = -k(\theta + 50)$

47) If a body originally at 120°C cools to 35°C in 40 minute in the air of constant temperature 45°C. Then according to Newton's law, its differential equation is given by

a) $\frac{d\theta}{dt} = -k(\theta - 120)$ b) $\frac{d\theta}{dt} = -k(\theta - 40)$

c) $\frac{d\theta}{dt} = -k(\theta - 45)$ d) $\frac{d\theta}{dt} = -k(\theta - 35)$

48) Assuming the temperature of the surrounding is being kept constant at 25°C and a body cools from temperature 80°C to 35°C in 45 minute. Then it must satisfy the differential equation

a) $\frac{dT}{dt} = -k(T-25)$ b) $\frac{dT}{dt} = -k(T-80)$

c) $\frac{dT}{dt} = -k(T-35)$ d) $\frac{dT}{dt} = -k(T+25)$

49) The rate of change of temperature of a body is proportional to the difference between the temperature of body and its surrounding nearby. If temperature of the air is 35°C and that of the body is 96°C and cools down to 55°C in just 25 minute. Then we must have

a) $\frac{dT}{dt} = -k(T+25)$ b) $\frac{dT}{dt} = -k(T-55)$

c) $\frac{dT}{dt} = -k(T-35)$ d) $\frac{dT}{dt} = -k(T-25)$

50) A metal ball is placed in the oven till it obtain temperature of 100°C and then at time t = 0, it is then placed in water of temperature 40°C. By Newton's law, if the temperature of the ball is decreased to 70°C in 10 minutes, then it must satisfy the differential equation

a) $\frac{dT}{dt} = -k(T-70)$ b) $\frac{dT}{dt} = -k(T-40)$

c) $\frac{dT}{dt} = -k(T-55)$ d) $\frac{dT}{dt} = -k(T-100)$

51) If a body of temperature T at time t kept in the surrounding of temperature T_0 satisfies the differential equation $\frac{dT}{dt} = -k(T - T_0)$, the relation between T and t is given as

a) $T = T_0 - ke^{-kt}$ b) $T = T_0 + ke^{-kt}$

c) $T = T_0 + ke^{-kt}$ d) $T = -k(T_0 - e^{-kt})$

52) A body is heated to a temperature of 100°C and then at time recording t = 0 it is then placed liquid of temperature 40°C. The temperature of the body is then reduced to 60°C in 4 minute. By Newton's law of cooling its differential equation is $\frac{d\theta}{dt} = -\frac{1}{4}(\theta - 40)\log 3$. The time required to reduce the temperature of body to 50°C is

a) 5 min 6 sec b) 5.6 min

c) 65 min d) 6.5 min

53) A corpse of temperature 32°C is kept in the mortuary of constant temperature 10°C and the temperature of the corpse decreases to 20°C in 5 minutes. The differential equation of the system is given as $\frac{dT}{dt} = -0.05(T-10)$. Then T is

a) $T = 22e^{-0.05t}$ b) $T = 22 + 10e^{0.05t}$

c) $T = 10 - 22e^{-0.05t}$ d) $T = 10 + 22e^{-0.05t}$

54) A thermometer is taken outdoors of temperature 0°C from a room of temperature 21°C and the reading on the thermometer drops to 10°C in 5 minutes and satisfies sufficiently the differential equation $\frac{dT}{dt} = -0.7419T$. What is its primitive?
a) $T = 21e^{-0.7419t}$ b) $T = 21 - 10e^{0.7419t}$
c) $T = 10 + 21e^{0.7419t}$ d) $T = 21e^{0.7419t}$

55) A metal body of mass 5 kg is heated to a temperature upto 100°C exactly and then, at time considered to be t = 0, it is immersed in oil of temperature 30°C. In just 3 minutes, the temperature of body drops to 70°C in 3 minute and satisfies $\frac{d\theta}{dt} = -\frac{\theta - 30}{3}\log\left(\frac{7}{4}\right)$. What is time taken to drop temperature of body to 31°C.
a) 15.28 min b) 12.78 min
c) 32.78 sec d) 22.78 min

56) If the temperature of body drops down to 70°C from 100°C in 15 minute, and satisfying the Newton's law of cooling $\frac{d\theta}{dt} = -k(\theta - 30)$, the value of k is
a) $\frac{1}{15}\log\frac{7}{4}$ b) $-\frac{1}{15}\log\frac{7}{4}$
c) $15\log\frac{7}{4}$ d) $-15\log\frac{7}{4}$

57) A metal ball of temperature 100°C is placed in air conditioned room of temperature 20°C. The temperature drops by 40°C in 5 minute. Its differential equation in accordance with Newton's law of cooling is given by $\frac{dT}{dt} = -\frac{T - 20}{5}\log 2$. The temperature after 8 minute is
a) 6.44 b) 64.4 c) 46.4 d) 44.6

58) A body cools down from 80°C to 60°C from 1.00 PM to 1.20 PM in a room of temperature 40°C and satisfies the differential equation $\frac{d\theta}{dt} = -0.03465(\theta - 40)$. The temperature of body at 1.40 PM is
a) 45 b) 50 c) 55 d) 60

59) The temperature of body cooling down from 100°C to 60°C in 60 seconds when it is kept in the air surrounding of constant temperature 20°C and satisfies the equation $\frac{d\theta}{dt} = -k(\theta - 20)$. The value of k is then
a) $\log 2$ b) $\log 3$ c) $\log 4$ d) $\log 5$

60) A metal ball made by brass of mass 50 gm cools down from 80°C to 60°C after a recorded time of 20 minute in air atmosphere of 40°C. The differential equation is $\frac{d\theta}{dt} = -k(\theta - 40)$. What is the value of k?
a) $-\frac{3}{20}\log_e 2$ b) $-20\log_e 2$
c) $\frac{1}{20}\log 2$ d) $-\frac{1}{20}\log_e 2$

61) A body of temperature 90°C is placed in water of temperature 30°C for 6 minute and then its temperature calculated is to be just 50°C. The Newton's law of cooling is $\frac{d\theta}{dt} = -k(\theta - 40)$. Then what of followings is correct.
a) $k = \frac{1}{6}\log_e\frac{1}{3}$ b) $k = \frac{1}{6}\log_e 3$
c) $k = -\frac{1}{6}\log_e 2$ d) $k = -\frac{1}{6}\log_e\frac{1}{4}$

62) An iron ball is heated for temperature 100°C is placed in water of temperature 50°C at t = 0 and at t = 5 minute then its temperature calculated which is read to be 70°C. The Newton's law of cooling is $\frac{d\theta}{dt} = -k(\theta - 50)$. Then what of followings is correct?
a) $k = -\frac{3}{4}\log_e\frac{2}{5}$ b) $k = \frac{1}{5}\log_e\frac{2}{5}$
c) $k = -\frac{2}{5}\log_e\frac{1}{5}$ d) $k = -\frac{1}{5}\log_e\frac{2}{5}$

63) A circuit consisting of resistance R, inductance L connected in series with voltage of amount E. By Kirchhoff's law, the differential equation for the current i in terms of t is
a) $L\frac{di}{dt} + \frac{i}{R} = E$ b) $L\frac{di}{dt} + Ri = E$

c) $L\frac{di}{dt}+Ri=0$ d) $R\frac{di}{dt}+Li=E$

64) A circuit consisting of resistance R, inductance L connected in series without voltage of amount E. By Kirchhoff's law, the differential equation for the current i in terms of t is

a) $L\frac{di}{dt}+\frac{i}{R}=E$ b) $L\frac{di}{dt}+Ri=E$

c) $L\frac{di}{dt}+Ri=0$ d) $R\frac{di}{dt}+Li=E$

65) An electrical circuit is consisting of inductance L, capacitance C in series with voltage source E. By Kirchhoff's law, we have

a) $L\frac{dq}{dt}+\frac{q}{C}=E$ b) $L\frac{dq}{dt}+\frac{q}{R}=E$

c) $C\frac{di}{dt}+\frac{i}{R}=E$ d) $\frac{di}{dt}+\frac{i}{C}=ER$

66) An electrical circuit is consisting of resistance R, capacitance C in series with voltage source E. By Kirchhoff's law, we have

a) $R\frac{dq}{dt}+\frac{q}{C}=E$ b) $L\frac{dq}{dt}+\frac{q}{R}=E$

c) $C\frac{di}{dt}+\frac{i}{R}=E$ d) $\frac{di}{dt}+\frac{i}{C}=ER$

67) A circuit consisting of resistance R, inductance L connected in series with voltage of amount $E\cos\omega t$. By Kirchhoff's law, the differential equation for the current i in terms of t is

a) $L\frac{di}{dt}+\frac{i}{R}=E\cos\omega t$ b) $L\frac{di}{dt}+Ri=E\cos\omega t$

c) $L\frac{di}{dt}+Ri=0$ d) $R\frac{di}{dt}+Li=E\cos\omega t$

68) The differential equation for the current i in an electrical circuit consisting of inductance L, resistance R in series with electromotive force of Ee^{-at} is given by

a) $\frac{di}{dt}+Ri=\frac{E}{L}e^{-at}$ b) $L\frac{di}{dt}+Ri=Ee^{-at}$

c) $L\frac{di}{dt}+\frac{i}{R}=Ee^{-at}$ d) $R\frac{di}{dt}+Li=Ee^{-at}$

69) The differential equation for the current i in an electrical circuit composing of resistance of 120 ohm and an inductance of 0.7 henry connected in series with battery of 30 volt is

a) $0.7\frac{di}{dt}-120i=30$ b) $120\frac{di}{dt}+0.7i=30$

c) $0.7\frac{di}{dt}+120i=30$ d) $0.7\frac{di}{dt}+\frac{i}{120}=30$

70) The differential equation for the current i in an electrical circuit composing of resistance of 200 ohm and an inductance of 100 henry connected in series with battery of 440 volt is

a) $20\frac{di}{dt}+10i=44$ b) $\frac{di}{dt}+2i=40$

c) $5\frac{di}{dt}+10i=44$ d) $10\frac{di}{dt}+20i=44$

71) A capacitance of 0.03 farad and resistance of 10 ohm in series with electromotive force of 20 volts are in a circuit. If initially the capacitor is totally discharged, the differential equation for the charge q is

a) $10\frac{dq}{dt}+\frac{q}{0.03}=20;\ q(0)=0$

b) $\frac{dq}{dt}+\frac{q}{0.03}=2;\ q(0)=0$

c) $\frac{dq}{dt}+\frac{q}{0.3}=2;\ q(0)=0$

d) $10\frac{dq}{dt}+0.03q=20;\ q(0)=0$

72) In an electrical circuit of R and L in series with steady EMF, the current i satisfies the equation $i=\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$. The time required for the maximum value is

a) 0 b) $\frac{L}{R}\log 10$

c) $-\frac{L}{R}\log 90$ d) $\frac{E}{R}\log 10$

73) In an electrical circuit of R and L in series with steady EMF, the current i satisfies the equation $i=\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$. The time required for the current gets 90% of maximum value is

a) 0 b) $\frac{L}{R}\log 2$

c) $-\frac{L}{R}\log 2$ d) $\frac{E}{R}\log 2$

74) If the differential equation for the current i is $R\frac{di}{dt}+Ri=E$, the current i at time t is

a) $i=\frac{E}{R}+ce^{-\frac{R}{L}t}$ b) $iR=1-cEe^{-\frac{R}{L}t}$

c) $i=\frac{E}{R}+ce^{\frac{R}{L}t}$ d) $i=\frac{E}{R}ce^{-\frac{R}{L}t}$

75) A charge q on the plate of condenser of capacity C through resistance R in series with steady state EMF V volt satisfies the differential equation $R\frac{dq}{dt}+\frac{q}{C}=V$. Then q in terms of t is

a) $q=\frac{C}{V}+ke^{-\frac{t}{RC}}$ b) $q=CV+ke^{\frac{t}{RC}}$

c) $q=CVke^{-\frac{t}{RC}}$ d) $q=CV+ke^{-\frac{t}{RC}}$

76) A charge q on the plate of condenser of capacity C through resistance R in series with steady state EMF V volt satisfies the equation $q=CV\left(1-e^{-\frac{t}{RC}}\right)$. Then i in terms of t is

a) $i=\frac{V}{R}e^{-\frac{t}{RC}}$ b) $i=\frac{V}{R}+e^{-\frac{t}{RC}}$

c) $i=VRe^{-\frac{t}{RC}}$ d) $i=\frac{V}{R}e^{\frac{t}{RC}}$

77) The differential equation for the current i is given to be $0.5\frac{di}{dt}+100i=20$ for an electrical circuit containing resistance R = 100 ohm, inductance L = 0.5 henry in series. Then

a) $i=0.2+Ae^{200t}$ b) $i=20+Ae^{-200t}$

c) $i=0.2Ae^{-200t}$ d) $i=0.2+Ae^{-200t}$

78) If an electrical circuit of R-C in series, charge $q=q(t)$ as function of t is $q=e^{3t}-e^{6t}$, the time required for maximum charge on capacitor is given by

a) $\frac{1}{2}\log 3$ b) $\frac{2}{3}\log 2$

c) $\frac{1}{3}\log 2$ d) $\frac{1}{3}\log\frac{1}{2}$

79) An electrical circuit of resistance R, inductance L in series with an electromotive force of E is satisfying the differential equation for the current i as $L\frac{di}{dt}+Ri=E$. For L = 640 henry, R = 250 ohm, E = 500 volt, the integrating factor of the above equation is

a) $e^{\frac{64}{25}t}$ b) $e^{\frac{25}{64}t}$ c) $e^{-\frac{25}{64}t}$ d) $e^{-\frac{64}{25}t}$

80) In an electrical circuit of L = 640 H, R = 250 Ω and E = 500 with EMF of 20 volts, the differential equation is

a) $\frac{di}{dt}+\frac{64}{25}i=\frac{32}{25}$ b) $\frac{di}{dt}+\frac{64}{25}i=\frac{25}{32}$

c) $\frac{di}{dt}+\frac{25}{64}i=\frac{25}{32}$ d) $\frac{di}{dt}+\frac{25}{64}i=\frac{32}{25}$

81) Rectilinear motion is the motion of body along

a) straight line b) circular motion

c) curvilinear d) parabolic path

82) The algebraic sum of the forces acting on a body along a given direction is equal to

a) mass × total force b) mass × distance

c) mass × velocity d) mass × acceleration

83) A particle moving in a straight line with acceleration $k\left(x+\frac{a^4}{x^3}\right)$ is directed towards origin. Then the equation of motion is

a) $\frac{dv}{dx}=-kv\left(x+\frac{a^4}{x^3}\right)$ b) $v\frac{dv}{dt}=-k\left(x+\frac{a^4}{x^3}\right)$

c) $\frac{d^2x}{dt^2}=-k\left(x+\frac{a^4}{x^3}\right)$ d) $k\frac{d^2x}{dt^2}=\left(x+\frac{a^4}{x^3}\right)$

84) A body of mass m kg moves in straight line with acceleration $\frac{k}{x^3}$ at a distance x and directed towards center. Then

a) $v\frac{dv}{dx}=-\frac{k}{x^3}$ b) $\frac{dv}{dx}=v\frac{k}{x^3}$

c) $v\frac{dv}{dx}=\frac{k}{x^3}$ d) $v\frac{dv}{dt}=-\frac{k}{x^3}$

85) A body of mass m falling freely from rest under gravitational force of attraction and air resistance proportional to square of velocity kv^2. Then

a) $\frac{dv}{dx}=v\left(mg-kv^2\right)$ b) $v\frac{dv}{dx}=m\left(g-kv^2\right)$

c) $mv\frac{dv}{dx}=mg-kv^2$ d) $v\frac{dv}{dx}=g-kv^2$

86) A particle is projected vertically upward with initial velocity v_1 and resistance of air produces retardation kv^2 where v is velocity at time t. Then

a) $mv\frac{dv}{dx}=mg-kv^2$ b) $v\frac{dv}{dx}=-g-kv^2$

c) $v\frac{dv}{dx}=m\left(g-kv^2\right)$ d) $v\frac{dv}{dx}=g-kv^2$

87) A particle starts moving horizontally from rest is opposed by a force cx, resistance per unit mass of value bv^2, where v and x are velocity and displacement of body at time t. Then

a) $v\frac{dv}{dx}=cs+bv^2$ b) $v\frac{dv}{dx}=-cs+bv^2$

c) $v\frac{dv}{dx}=cs-bv^2$ d) $v\frac{dv}{dx}=-cs-bv^2$

88) A body of mass m falls from rest under gravity in a liquid having resistance to motion at time t is mk times velocity. Then

a) $\frac{dv}{dt}=g+kv$ b) $\frac{dv}{dt}=g-kv$

c) $\frac{dv}{dt}=-g-kv$ d) $\frac{dv}{dt}=-g+kv$

89) A particle of mass m is projected vertically upward with velocity v, assuming the air resistance k times its velocity. Then

a) $m\frac{dv}{dt}=-mg-kv$ b) $m\frac{dv}{dt}=-mg+kv$

c) $m\frac{dv}{dt}=mg-kv$ d) $m\frac{dv}{dt}=mg+kv$

90) Assuming that the resistance to movement of a ship through water in the form of $a^2+b^2v^2$, where v is the velocity. Then the differential equation for retardation of the ship moving with engine stopped is

a) $m\frac{dv}{dt}=a^2+b^2v^2$ b) $m\frac{dv}{dt}=-a^2+b^2v^2$

c) $m\frac{dv}{dt}=-a^2-b^2v^2$ d) $m\frac{dv}{dt}=a^2-b^2v^2$

91) The differential equation of motion of particle of mass m falls from rest under gravity in a fluid satisfies the equation $\frac{dv}{dt}=g-kv$, then

a) $t=-k\log\left(\frac{g}{g-kv}\right)$ b) $t=k\log\left(\frac{g}{g-kv}\right)$

c) $t=-\frac{1}{k}\log\left(\frac{g}{g-kv}\right)$ d) $t=\frac{1}{k}\log\left(\frac{g}{g-kv}\right)$

92) A body of mass m falling freely under gravity satisfies the equation $mv\frac{dv}{dx}=k\left(a^2-v^2\right)$ with condition $ka^2=mg$, then

a) $x=\frac{m}{2k}\log\left(a^2-v^2\right)$ b) $x=\frac{m}{2}k\log\left(\frac{a^2}{a^2-v^2}\right)$

c) $x=-\frac{m}{2k}\log\left(\frac{a^2}{a^2-v^2}\right)$ d) $x=\frac{m}{2k}\log\left(\frac{a^2}{a^2-v^2}\right)$

93) A body starts from rest with an acceleration $\frac{dv}{dt}=k\left(1-\frac{t}{T}\right)$. Then its velocity is

a) $v=k\left(t-\frac{t^2}{2T}\right)$ b) $\frac{v^2}{2}=k\left(t-\frac{t^2}{2T}\right)$

c) $v=-k\left(t-\frac{t^2}{2T}\right)$ d) $v=k\left(\frac{t}{2}-\frac{t^2}{T}\right)$

94) A particle of unit mass starts from rest with an acceleration $v\frac{dv}{dr}=-\frac{k}{r^3}$. If initially it was at rest at r = a, then

a) $v^2=-k\left(\frac{1}{r^2}-\frac{1}{a^2}\right)$ b) $v^2=k\left(\frac{1}{r^2}+\frac{1}{a^2}\right)$

c) $v^2=k\left(\frac{1}{r^2}-\frac{1}{a^2}\right)$ d) $v^2=k\left(a^2-r^2\right)$

95) A particle of mass m is subjected projected upward with velocity V with its equation of motion $m\frac{dv}{dt}=-mg-kv$, then the velocity at time t is

a) $t=\log\left(\frac{mg+kv}{mg+kV}\right)$ b) $t=\frac{m}{k}\log\left(\frac{mg+kv}{mg+kV}\right)$

c) $t=-\frac{m}{k}\log\left(\frac{mg+kv}{mg+kV}\right)$ d) $t=\frac{m}{k}\log\left(\frac{mg-kv}{mg-kV}\right)$

96) A particle of mass m falls freely from rest under gravitational force in fluid producing resistance to motion of amount mkv, where k is constant. The differential equation is $\frac{dv}{dt} = g - kv$, then its terminal velocity is

a) $-\frac{g}{k}$ b) gk c) $-gk$ d) $\frac{g}{k}$

97) A bullet is fired into a sand tank and satisfies the differential equation $\frac{dv}{dt} = -k\sqrt{v}$. If v_0 is its initial velocity, we have

a) $2\sqrt{v} = -kt + 2\sqrt{v_0}$ b) $2\sqrt{v} = -\left(kt + 2\sqrt{v_0}\right)$
c) $2\sqrt{v} = kt + 2\sqrt{v_0}$ d) $\sqrt{v} = kt - 2\sqrt{v_0}$

98) A particle is in motion of horizontal straight line with acceleration $k\left(x + \frac{a^4}{x^3}\right)$ directed towards its origin and satisfies the differential equation $v\frac{dv}{dt} = -k\left(x + \frac{a^4}{x^3}\right)$. Assuming that it starts from rest at a distance x = a from origin, we have

a) $v^2 = -k\left(x^2 - \frac{a^4}{x^2}\right)$ b) $v^2 = k\left(x^2 + \frac{a^4}{x^2}\right)$
c) $v^2 = k\left(x^2 - \frac{a^4}{x^2}\right)$ d) $v^2 = -k\left(2x^2 - \frac{a^4}{2x^2}\right)$

99) If a particle moves in a straight line so that the force acting on it is directed towards a fixed point in the line of motion and proportional to its displacement from the point, it is then known as
a) curvilinear motion
b) rectilinear motion
c) Simple harmonic motion
d) circular motion

100) If a particle execute SHM, then its differential equation is given by

a) $\frac{d^2x}{dt^2} = -\omega^2 x$ b) $\frac{d^2x}{dt^2} - \omega^2 x = 0$
c) $\frac{d^2x}{dt^2} = k\omega x^2$ d) $\frac{d^2x}{dt^2} = -\omega x^2$

101) Fourier's law of heat conduction states that, the quantity of heat flow across the area of cross section A is
a) inversely proportional to the product of A with temperature gradient
b) proportional to the difference of A with temperature gradient
c) proportional to the product of A with temperature gradient
d) proportional to the sum of A and temperature gradient

102) If q quantity of heat flow across the cross sectional area A and thickness dx per unit time where the difference between temperatures at the faces is dT, the by Fourier's heat law

a) $q = -k - A\frac{dT}{dx}$ b) $q = -kA\frac{dT}{dx}$
c) $q = kA\frac{dT}{dx}$ d) $q = -kA + \frac{dT}{dx}$

103) The differential equation of steady state heat conduction per unit time from unit length of pipe of uniform radius r_0 carrying steam of temperature T_0 and thermal conductivity k, if the pipe is covered with material in a constant surrounding temperature, is given by

a) $Q = -\frac{2kr}{\pi} \cdot \frac{dT}{dr}$ b) $Q = -kr\frac{dT}{dr}$
c) $Q = 2k\pi r\frac{dT}{dr}$ d) $Q = -2k\pi r\frac{dT}{dr}$

104) The difference equation for steady state heat loss in unit time from a spherical shell of thermal conductivity covered by insulating material and kept in surrounding of constant temperature during heat flow, is

a) $Q = -\frac{4\pi r^2}{k} \cdot \frac{dT}{dr}$ b) $Q = 4k\pi r^2\frac{dT}{dr}$
c) $Q = -4k\pi r^2\frac{dT}{dr}$ d) $Q = -2k\pi r\frac{dT}{dr}$

105) The differential equation for steady state heat loss per unit time from unit length of pipe covered with insulating material which is kept in constant surrounding temperature, is

$Q = -2k\pi r\dfrac{dT}{dr}$. Then the temperature T is given by

a) $T = -\dfrac{Q}{k}\log r + c$ b) $T = -\dfrac{Q}{2\pi k}\log\dfrac{1}{r} + c$

c) $T = \dfrac{Q}{2\pi k}\log r + c$ d) $T = -\dfrac{Q}{2\pi k}\log r + c$

106) The differential equation for heat conductivity in spherical shell is described by $Q = -4k\pi r^2\dfrac{dT}{dr}$. Then

a) $T = \dfrac{Q}{kr} + c$ b) $T = \dfrac{Q}{4\pi kr} + c$

c) $T = \dfrac{Q}{4\pi k}r + c$ d) $T = -\dfrac{Q}{4\pi kr} + c$

107) A pipe of 10 cm radius carries steam of 150ºC and covered with insulating material of thickness 5 cm with thermal conductivity k = 0.0025 and it is kept in surrounding of temperature 40ºC. The equation is $dT = -\dfrac{Q}{2\pi k}\cdot\dfrac{dx}{x}$. Then the heat loss is

a) $220\pi k\log 1.5$ b) $\dfrac{220k}{\log 1.5}$

c) $\dfrac{220\pi k}{\log 1.5}$ d) $\dfrac{110\pi k}{\log 1.5}$

108) Heat is flowing through a hollow pipe of diameter 10 cm and outer diameter 20 cm and it is covered by insulating material of k = 0.12 and kept in surrounding of 200ºC. The differential equation is being $dT = -\dfrac{Q}{2\pi k}\cdot\dfrac{dx}{x}$. Then the heat loss is

a) $\dfrac{300\pi k}{\log 2}$ b) $\dfrac{150\pi k}{\log 2}$

c) $-\dfrac{300\pi k}{\log 2}$ d) $\dfrac{300\pi k}{\log 0.2}$

109) Steam of temperature 200ºC is set into pipe of 20 cm diameter covered with material of 6 cm thickness in surrounding of 30ºC. The equation is $dT = -\dfrac{Q}{2\pi k}\cdot\dfrac{dx}{x}$. The heat loss is

a) $\dfrac{170\pi k}{\log 16}$ b) $\dfrac{170(2\pi k)}{\log 1.6}$

c) $\dfrac{170\pi k}{\log 1.6}$ d) $-\dfrac{170\pi k}{\log 1.6}$

110) Steam of 100ºC is flowing through pipe of diameter 10 cm covered with asbestos of 5 cm thick and thermal conductivity k = 0.0006. The outer temperature is being 30ºC and the differential equation is $dT = -\dfrac{Q}{2\pi k}\cdot\dfrac{dx}{x}$. What is the amount of heat loss?

a) $\dfrac{140\pi k}{\log 2}$ b) $70\pi k\log 2$

c) $\dfrac{70\pi k}{\log 2}$ d) $-\dfrac{70\pi k}{\log 2}$

111) A tank contains 50 liters of fresh water. Brine of 2 gm/liter flows into the tank at the rate of 2 liters/minute and mixed with uniform continuity and the same amount runs out with the same rate. If Q is total amount of salt present at time t in the brine, we have

a) $\dfrac{dQ}{dt} = -4 + \dfrac{Q}{25}$ b) $\dfrac{dQ}{dt} = -4 - \dfrac{Q}{25}$

c) $\dfrac{dQ}{dt} = 4 - \dfrac{Q}{25}$ d) $\dfrac{dQ}{dt} = 4 + \dfrac{Q}{25}$

112) A tank contains 10000 liters of Brine of 200 kg dissolve salt. Fresh water flows into the tank at the rate of 100 liters/minute and mixed with uniform continuity and the same amount runs out with the same rate. If Q is total amount of salt present at time t in the brine, we have

a) $\dfrac{dQ}{dt} = 200 + \dfrac{Q}{100}$ b) $\dfrac{dQ}{dt} = -\dfrac{Q}{100}$

c) $\dfrac{dQ}{dt} = 200 - \dfrac{Q}{100}$ d) $\dfrac{dQ}{dt} = \dfrac{Q}{100}$

113) A tank contains 100 liters of fresh water. Brine of 1 gm/liter flows into the tank at the rate of 2 liters/minute and mixed with uniform continuity and the same amount runs out with the same rate. If Q is total amount of salt present at time t in the brine, we have

a) $\dfrac{dQ}{dt} = -\dfrac{Q}{100+t}$ b) $\dfrac{dQ}{dt} = 2 + \dfrac{Q}{100+t}$

c) $\dfrac{dQ}{dt} = 2 - \dfrac{Q}{100+t}$ d) $\dfrac{dQ}{dt} = 2 - \dfrac{Q}{100t}$

114) A tank contains 10000 liters of Brine of 20 kg dissolve salt. Brine of 0.1 kg/liter flows into the tank at the rate of 40 liters/minute and mixed with uniform continuity and the same amount runs out with the rate 30 liters/minute. If Q is total amount of salt present at time t in the brine, we have

a) $\frac{dQ}{dt}=4-\frac{3Q}{1000+10t}$ b) $\frac{dQ}{dt}=4-\frac{30Q}{100+t}$

c) $\frac{dQ}{dt}=-\frac{3Q}{100+t}$ d) $\frac{dQ}{dt}=4-\frac{3Q}{100+t}$

115) A tank contains 5000 liters of fresh water. Brine of 100 gm/liter flows into the tank at the rate of 10 liters/minute and mixed with uniform continuity and the same amount runs out with the same rate. If Q is total amount of salt present at time t in the brine, we have

a) $\frac{dQ}{dt}=\frac{5000-Q}{500}$ b) $\frac{dQ}{dt}=5000-\frac{Q}{500}$

c) $\frac{dQ}{dt}=1000+\frac{Q}{5}$ d) $\frac{dQ}{dt}=1000-\frac{Q}{500}$

116) A tank contains 10000 liters of Brine of 200 kg dissolve salt. Fresh water flows into the tank at the rate of 100 liters/minute and mixed with uniform continuity and the same amount runs out with the same rate. If Q is total amount of salt present at time t in the brine, we have $\frac{dQ}{dt}=-\frac{Q}{100}$. Then

a) $\log Q=-\frac{t}{100}$

b) $\log Q=-\frac{t}{100}-\log 200$

c) $\log Q=-\frac{t}{100}+\log 200$

d) $\log Q=\frac{t}{100}+\log 200$

117) A tank contains 50 liters of fresh water. Brine of 2 gm/liter flows into the tank at the rate of 2 liters/minute and mixed with uniform continuity and the same amount runs out with the same rate. If Q is total amount of salt present at time t, we have $\frac{dQ}{dt}=4-\frac{Q}{25}$. Then

a) $t=50\log 10-25\log(100-Q)$

b) $t=25\log 10-25\log(100-Q)$

c) $t=50\log 10+25\log(100-Q)$

d) $t=25\log 10+25\log(100-Q)$

118) The rate of decay of a substance is directly proportional to the amount of substance present at that time. Hence

a) $\frac{dt}{dx}=-kx$ b) $\frac{dx}{dt}=-kx$

c) $\frac{dx}{dt}=-kx+t$ d) $\frac{dx}{dt}=-kx^2+c$

Unit I : Ordinary Differential Equations

1	A	41	B	81	A	121	B	161	D	201	C	241	D
2	C	42	A	82	B	122	A	162	D	202	D	242	C
3	C	43	C	83	D	123	C	163	C	203	D	243	B
4	A	44	A	84	B	124	B	164	B	204	A	244	C
5	C	45	D	85	B	125	C	165	D	205	C	245	A
6	B	46	C	86	D	126	B	166	B	206	B	246	C
7	A	47	A	87	A	127	C	167	C	207	D	247	B
8	D	48	C	88	D	128	D	168	A	208	D	248	D
9	C	49	B	89	B	129	C	169	C	209	A	249	B
10	B	50	C	90	B	130	B	170	B	210	A	250	D
11	C	51	D	91	A	131	A	171	B	211	C	251	A
12	B	52	C	92	A	132	B	172	B	212	A	252	D
13	A	53	D	93	A	133	A	173	A	213	C	253	A
14	C	54	B	94	A	134	C	174	A	214	B	254	C
15	B	55	D	95	D	135	C	175	C	215	B	255	B
16	D	56	B	96	C	136	D	176	D	216	C	256	A
17	A	57	A	97	B	137	A	177	B	217	B	257	C
18	D	58	A	98	D	138	C	178	A	218	D	258	B
19	B	59	D	99	B	139	D	179	B	219	B	259	D
20	C	60	A	100	A	140	D	180	C	220	C	260	B
21	A	61	C	101	B	141	C	181	D	221	A	261	A
22	D	62	D	102	C	142	A	182	A	222	B	262	B
23	B	63	A	103	D	143	B	183	B	223	A	263	C
24	A	64	C	104	A	144	B	184	C	224	C		
25	D	65	B	105	B	145	D	185	A	225	B		
26	B	66	C	106	C	146	A	186	D	226	C		
27	D	67	B	107	A	147	D	187	A	227	D		
28	C	68	D	108	C	148	D	188	D	228	D		
29	A	69	C	109	D	149	C	189	C	229	C		
30	B	70	C	110	A	150	A	190	B	230	A		
31	A	71	A	111	D	151	C	191	C	231	D		
32	B	72	D	112	B	152	B	192	A	232	B		
33	B	73	D	113	D	153	A	193	D	233	A		
34	C	74	B	114	C	154	D	194	C	234	D		
35	B	75	B	115	A	155	A	195	A	235	C		
36	A	76	C	116	C	156	D	196	C	236	A		
37	A	77	D	117	D	157	C	197	B	237	D		
38	B	78	B	118	C	158	A	198	D	238	A		
39	C	79	D	119	B	159	B	199	B	239	D		
40	B	80	A	120	D	160	A	200	A	240	C		

Unit II : Applications of Ordinary Differential Equations

1	A	18	D	35	C	52	D	69	C	86	B	103	D
2	C	19	C	36	A	53	D	70	D	87	D	104	C
3	B	20	A	37	D	54	A	71	A	88	B	105	D
4	C	21	A	38	C	55	D	72	B	89	A	106	B
5	D	22	C	39	B	56	A	73	B	90	C	107	C
6	B	23	C	40	D	57	C	74	A	91	D	108	A
7	D	24	D	41	A	58	B	75	D	92	D	109	B
8	C	25	B	42	C	59	A	76	A	93	A	110	A
9	B	26	C	43	D	60	C	77	D	94	C	111	C
10	A	27	C	44	B	61	B	78	C	95	B	112	B
11	C	28	D	45	D	62	D	79	B	96	D	113	C
12	D	29	A	46	B	63	B	80	C	97	A	114	D
13	D	30	B	47	C	64	C	81	A	98	A	115	D
14	A	31	A	48	A	65	A	82	D	99	C	116	C
15	A	32	B	49	C	66	A	83	C	100	A	117	A
16	C	33	C	50	B	67	B	84	A	101	C	118	B
17	B	34	D	51	B	68	B	85	C	102	B		

Unit IV

Integral Calculus , Tracing of curves.

(Cartesian, Polar and Parametric.)

Unit-4

Integral Calculus

Differentiation Under Integral Sign (DUIS)

1. Introduction.

- In addition to variables, additional parameters

$$I(\alpha)=\int_a^b f(x,\alpha)dx \quad \Rightarrow \quad \alpha = \text{Parameter}, \ x = \text{Variable.}$$

Rule 1 : Integrals with constant limits.

$$If \ \ I(\alpha)=\int_a^b f(x,\alpha)dx \quad then \quad \boxed{\frac{d}{d\alpha}\int_a^b f(x,\alpha)dx=\int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)dx}$$

a & b constants ⇒ LHS derivative ⇒ Partial derivative RHS

LEIBNITZ RULE : Integrals with limits as Functions of the Parameter.

If a and b are functions of parameter α

$$i.e.\ I(\alpha) = \int_{a(\alpha)}^{b(\alpha)} f(x,\alpha)\, dx \ Then$$

$$\frac{d}{d\alpha}\int_a^b f(x,\alpha)\, dx = \int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)\, dx + f(b,\alpha).\frac{db}{d\alpha} - f(a,\alpha).\frac{da}{d\alpha}$$

1)	**If** $I(a)=\int_0^\infty \frac{e^{-x}}{x}(1-e^{-ax})dx\quad (a>-1)$ **then the value of** $\frac{dI(a)}{da}$ **is**				
[redacted]			b)	$-1/(a+1)$	
	c)	$\log(a+1)$	d)	0	

2)	**If** $I(a)=\int_0^\infty \frac{\cos\lambda x}{x}\left(e^{-ax}-e^{-bx}\right)dx;\quad a>0, b>0$ **then**				
[redacted]			b)	$\frac{dI(a)}{da}-\int_0^\infty e^{-ax}\cos\lambda x\,dx=0$	
	c)	$\frac{dI(a)}{da}+\int_0^\infty e^{-ax}\sin\lambda x\,dx=0$	d)	$\frac{dI(a)}{da}-\int_0^\infty e^{-ax}\sin\lambda x\,dx=0$	

3)	**The value of** $\frac{d}{db}\left[\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x}dx\right]$ **where** $a > 0, b > 0$, **is**			
	a)	$\int_0^\infty \frac{be^{-bx}}{x}dx$	b)	$\int_0^\infty \frac{-be^{-bx}}{x}dx$
	c)	$\int_0^\infty e^{-ax}dx$	[redacted]	

4)	**If** $I(a) = \int_a^{a^2} e^{ax^2}dx$ **then** $\frac{dI(a)}{da} =$			
	a)	$\int_a^{a^2} x^2e^{ax^2}dx + e^{a^5} - e^{a^3}$	b)	$\int_a^{a^2} 2axe^{ax^2}dx + 2ae^{a^5} - e^{a^3}$
	[redacted]		d)	$\int_a^{a^2} e^{ax^2}dx + e^{a^5} - 2ae^{a^3}$

5)	If $II(aa) = \int_0^\infty \frac{e^{-ax}-e^{-bx}}{x} dx$ and $I'(a) = -\frac{1}{a}$ then the value of $I(a)$ is			
	a)	$\log a$	b)	$\log(a/b)$
	c)	$\log b$	[redacted]	

6)	If $\frac{dI}{da} = \frac{a}{a^2+1}$, then the value of integral $I(a) = \int_0^\infty \frac{e^{-x}-e^{-ax}}{x \sec x} dx$ is			
	[redacted]		b)	$\frac{1}{2}\log\left(\frac{a^2+1}{a}\right)$
	c)	$\log(a^2+1)$	d)	$-\log(a^2+1)$

7)	**If $I(a)=\int_0^{a^2}\tan^{-1}\left(\frac{x}{a}\right)dx$, then $\frac{dI}{da}$ is**			
[redacted]			b)	$\int_0^{a^2}\frac{\partial}{\partial a}\tan^{-1}\left(\frac{x}{a}\right)dx+a^2\tan^{-1}a$
	c)	$\int_0^{a^2}\frac{\partial}{\partial x}\tan^{-1}\left(\frac{x}{a}\right)dx+2a\tan^{-1}a$	d)	$\int_0^{a^2}\frac{\partial}{\partial a}\tan^{-1}\left(\frac{x}{a}\right)dx+a\tan^{-1}a^2$
8)	The value of $\frac{d}{da}\left[\int_a^{a^2}\frac{dx}{x+a}\right]$ is			
	a)	$\int_a^{a^2}\frac{dx}{(x+a)^2}+\frac{2}{a+1}+\frac{1}{2a}$	b)	$\int_a^{a^2}-\frac{dx}{(x+a)^2}+\frac{2}{a^2+a}-\frac{1}{2a}$
[redacted]			d)	$\int_a^{a^2}-\frac{dx}{(x+a)^2}+\frac{2}{a^2+a}$

ERROR FUNCTION

Error function Of x or Probability Integral

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2}\, du \qquad ----------------(1)$$

Complementary Error function Of x

$$erf_c(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-u^2}\, du \qquad ----------------(2)$$

Properties

(1) $\text{erf}\,(0) = 0$ Put $x = 0$ in (1) or (3)

(2) $\text{erf}\,(\infty) = 1$ Put $x = \infty$ in (3) and use the property $\Gamma \tfrac{1}{2} = \frac{1}{\sqrt{\pi}} \int_0^\infty e^{-t} t^{-1/2} dt = \sqrt{\pi}$

(3) $\text{erf}(x) + \text{erf}_c(x) = 1$

(4) $erf(x)$ is an odd function

==

1	**The definition of $\operatorname{erf}(\sqrt{t})$ is**			
	a)	$\frac{2}{\sqrt{\pi}}\int_{0}^{t} e^{-u^2}du$	b)	$\frac{2}{\sqrt{\pi}}\int_{t}^{\infty} e^{-u^2}du$
	[redacted]		d)	$\frac{2}{\sqrt{\pi}}\int_{\sqrt{t}}^{\infty} e^{-u^2}du$
2	The value of $\operatorname{erf}(\infty)$ is			
	a)	0	[redacted]	
	c)	∞	d)	-1
3	The value of $erfc(x) + erfc(-x)$ is			
	a)	1	[redacted]	
	c)	0	d)	-1

4)	**The value of** $\mathrm{erf}(-\infty)$ **is**			
	a)	0	b)	1
	c)	∞	[redacted]	
5)	Error function is			
	a)	Even function	b)	Neither ever nor odd function
	[redacted]		d)	Constant function
6)	The value of $erfc(0)$ is			
	a)	0	[redacted]	
	c)	-1	d)	∞

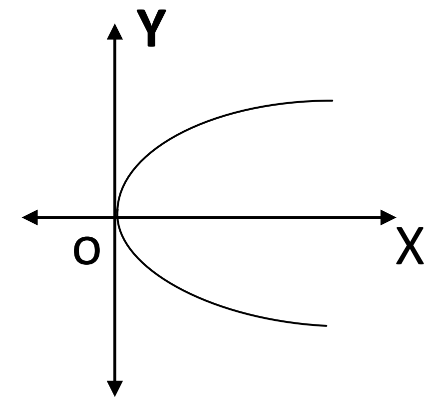
7)	**If $\alpha(x) = \sqrt{\frac{2}{\pi}} \int_0^x e^{-t^2/2} dt$ then the value of $\alpha(x\sqrt{2})$ is**			
	a)	$\mathrm{erf}(x\sqrt{2})$	b)	$-\mathrm{erf}(x)$
	c)	$\mathrm{erf}(2x)$	[redacted]	
8)	The value of $\int_0^2 \mathrm{erf\,c}(x)\, dx + \int_0^2 \mathrm{erfc}(-x)\, dx$ is			
	a)	0	[redacted]	
	c)	2		
9)	The value of $\frac{d}{dx}\mathrm{erf}(x)$ is			
	[redacted]		b)	$\frac{2}{\sqrt{\pi}} e^{x^2}$
	c)	0	d)	$\frac{2}{\sqrt{\pi}} e^{-2x}$

10	**The value of $erfcerfc(-\infty)$ is**			
	a)	0	b)	1
	c)	-1	[redacted]	
11	The value of $erf(\infty) + erfc(-\infty)$ is			
	[redacted]		b)	2
	c)	1	d)	0

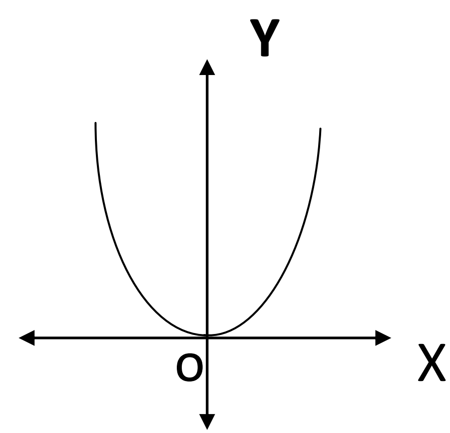
Rules For Tracing Of Cartesian Curves.

Rule 1 : Symmetry :

(a) Symmetry about X- axis: If equation of the curve remains unchanged by changing y to – y or all the powers of y in the equation are even. e.g. $y^2 = 4ax$.

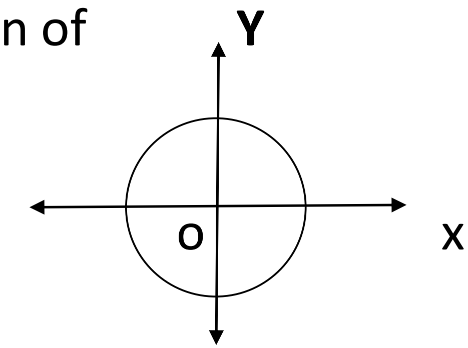


(b) Symmetry about Y- axis: If equation of the curve remains unchanged by changing x to – x **or** all the powers of x in the equation are even. e.g. $x^2 = 4ay$.



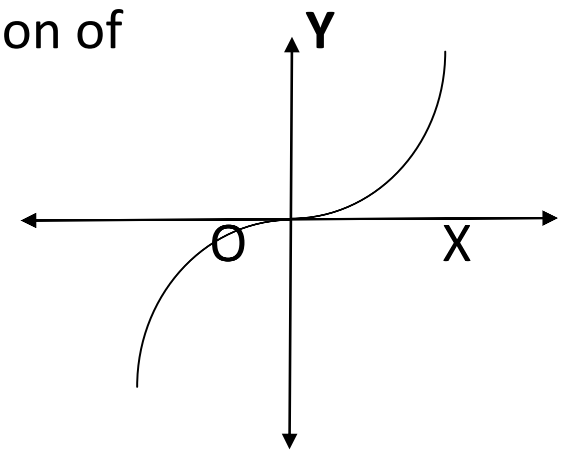
(c) Symmetry about both X and Y axes: If equation of the curve contains all even powers of x and y .

e.g. $x^2 + y^2 = r^2$.



(d) Symmetry in opposite quadrants: If equation of the curve remains unchanged by changing x to – x and y to – y simultaneously.

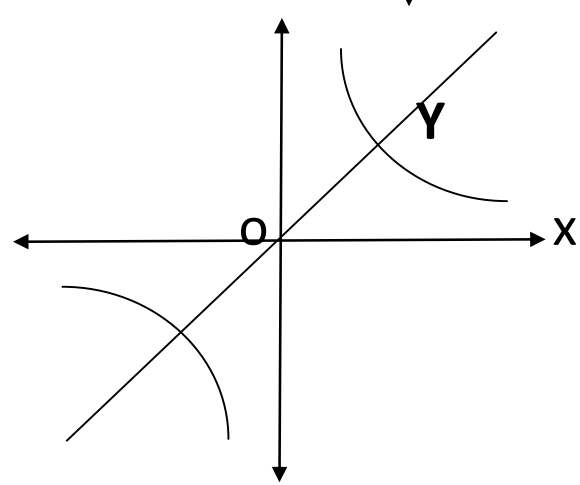
e.g. $y = x^3$



(e) Symmetry about the line y = x:

If equation of the curve remains unchanged by changing x to y and y to x .

e.g. $xy = c^2$

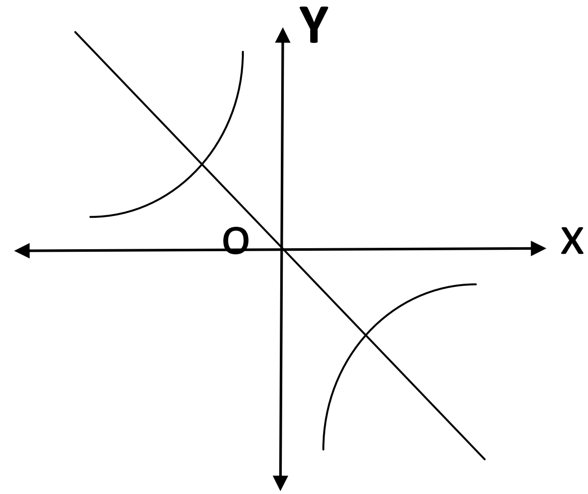


(e) Symmetry about the line y = - x :

If equation of the curve remains unchanged

by changing x to - y and y to - x

e.g. $xy = -c^2$



Rule 2 : Points Of Intersection :

(a) **Origin :** If the equation of the curve does not contain any arbitrary constant then the curve passes through origin.

(b) **Intersections with the co-ordinate axes :** If possible express the equation in the explicit form, $y = f(x)$ or $x = f(y)$.
Intersection with X-axis; put $y = 0$ and Intersection with Y-axis; put $x = 0$.
Find the tangents at these points, if necessary and position of the curve relative to these lines

(c) If a curve is symmetric about the line $y = x$ or $y = -x$ find the points of intersections of the curve with these lines and also the tangents at that point because ***tangent leads the curve.***

Rule 3 :Tangents:

(a) Tangents at the origin : If a curve is given by a rational integral algebraic equation and passes through origin : **the equation of the tangent or tangents at origin can be obtained by equating to zero, the lowest degree terms taken together in the equation of the curve.**

(b) Tangents at any other points : To find nature of tangent at any point P find $\frac{dy}{dx}$ at that point.

(i) If $\left(\frac{dy}{dx}\right)_P = 0 \Longrightarrow$ Tangent at P is parallel to X- axis.

(ii) If $\left(\frac{dy}{dx}\right)_P = \infty \Longrightarrow$ Tangent at P is parallel to Y- axis.

Rule 4 : Asymptotes : Asymptotes are tangents at infinity.

(a) **Asymptotes parallel to X - axis** are obtained by equating to zero the coefficients of highest degree term in x.

(b) **Asymptotes parallel to Y - axis** are obtained by equating to zero the coefficients of highest degree term in y.

(c) **Oblique asymptotes:** Asymptotes not parallel to co – ordinate axes are called oblique asymptotes. If curve is not symmetric about X or Y – axis then we check for oblique asymptotes. Equation of oblique asymptote can be obtained by two methods.

(i) **Method 1 :** Let y = mx + c be the asymptote. To find m and c substitute this y in the given equation f (x , y) so we get the

points of intersection with the curve i.e. f (x , mx + c) = 0.

Equating to zero two successive highest powers of x we find m and c.

Rule 5 : Region of absence of The curve :

(a) If possible express the equation in the explicit form, $y = f(x)$
And examine how y varies as x varies continuously.

(b) For $y = f(x)$, if y becomes imaginary for some value of $x > a$ (say)
Then no part of the curve exists beyond $x = a$.

(c) For $x = f(y)$, if x becomes imaginary for some value of $y > b$ (say)
Then no part of the curve exists beyond $y = b$.

Some Useful Remarks :

(a) When we have to solve for $y = f(x)$, put $x = 0$ see what is y.
Observe how y varies as x increases from 0 to $+\infty$ with special attention to the values of y for which $y = 0$ or $y \to +\infty$.
Also observe how y behaves as x becomes negative and $x \to -\infty$. with special attention for y becoming zero or $y \to -\infty$.

(b) If $y \to \infty$ as $x \to a$ then $x = a$ must be an asymptote ∥ to Y – axis.
If $x \to \infty$ as $y \to b$ then $y = b$ must be an asymptote ∥ to x – axis.

(c) If $y \to \infty$ as $x \to \infty$ and there is approximately linear relation between x and y for larger values of x, we may expect an oblique asymptote.

(d) If the curve is symmetric about X – axis or in the opposite quadrants then only positive values of y may be considered. We may draw the curve for negative values of y by symmetry.
Similarly, for symmetry about Y – axis only positive values of x may be considered.

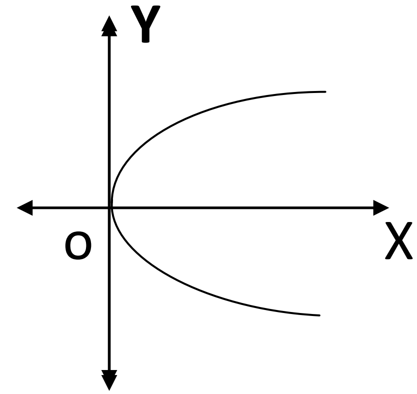
Type 3 : Curves Given by Parametric Equations, x = f (t) , y = f(t)

Where t is a parameter.

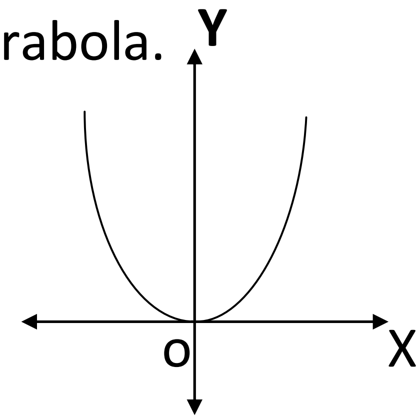
Rules For Tracing parametric Curves.

Rule 1 : Symmetry :

(a) Symmetry about X- axis: If equation of X remains unchanged by changing 't' to '– t' And y changes the sign then curve will be symmetric About X – axis. e.g. x = at^2 , y = at i.e. $y^2 = 4ax$.Parabola.

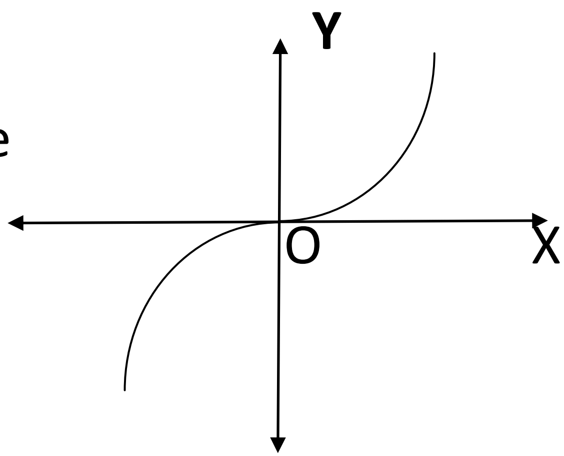


(b) Symmetry about Y- axis: If equation of y remains unchanged by changing 't' to '– t' And X changes the sign then curve will be symmetric About Y – axis. e.g. x = , at y = at^2 i.e. $x^2 = 4ay$.Parabola.



Note : For trigonometric equations if on replacing t to $\pi - t$, y remains unchanged and X changes the sign then also curve will be symmetric about Y – axis.

(c) Symmetry about origin: If on **replacing t by - t** if both x and y change the sign then curve is symmetric about otigin.



i.e. both x(t) and y(t)are odd functions of t.

e.g. x = t , y = t^2.

Rule 2 : Points Of Intersection :

1. If for some value of t both x and y become zero, then the curve passes through origin.
2. Find x and y intercepts if any.

Rule 3: Nature of tangents

1. $\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dx}\right)}$ **2. Form the table of values of t , x, y ,** $\frac{dy}{dx}$

Rule 4: Asymptotes and Region

1. Find asymptotes if any. **2.** Find region of absence.

Rules For Tracing Polar Curves.

Terminology : In polar curves

a) $\theta = 0$, positive $X - $ axix is called as ***Initial line***.

b) Equation of polar curves is often given by r = f **(**θ).

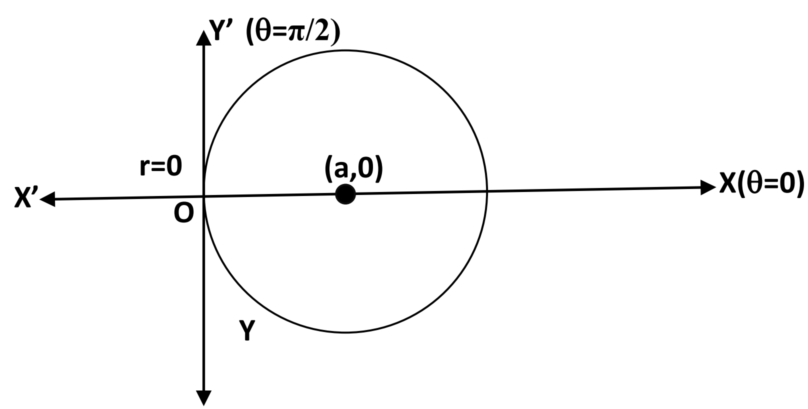
c) Origin O is called as Pole.

d) R is called as radius vector.

Symmetry :

a) If on changing θ to - θ ,equation of the curve remains unchanged then curve is symmetric about initial line (X - axis).

e.g. $x^2 + y^2 = 2ax$, Polar equation $r = 2a\cos\theta$



b)If on changing r to - r ,equation of the curve remains unchanged then curve is symmetric about the pole.

e.g. $(x^2 + y^2) = a^2(x^2 - y^2)$, Polar equation $r^2 = a^2 \cos 2\theta$

d) If on changing r to - r and θ to - θ,equation of the curve remains unchanged then curve is symmetric about Y - axis.

OR

If on changing θ to π- θ, equation of the curve remains unchanged then curve is symmetric about Y - axis.

e.g. (i) $(x^2 + y^2) = 2ay$, Polar equation $r = 2a \sin\theta$

(ii) r = (1+ $\sin\theta$) , First rule fails but second rule gives symmetry about Y – axis.

Pole : If for some values of θ ,r becomes zero then the pole lies on the curve.

e.g. (i) $(x^2 + y^2) = a^2(x^2 - y^2)$, Polar equation $r^2 = a^2 \cos 2\theta$

$$\theta = \frac{\pi}{4}, \quad r = 0 \quad \Longrightarrow \text{curve passes through the pole.}$$

Tangents at Pole : To find tangents at pole, put r = 0 in the equation, the value of θ gives tangent at the pole.

e.g. r = a sin3 θ , r = 0 $\Longrightarrow sin3\theta = 0$

$$3\theta = 0, \pi, 2\pi, 3\pi, \ldots \ldots . \theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \ldots \ldots \text{are all tangents at pole.}$$

- **Prepare a table showing the values of r and .**
- **Find the angle between radius vector and the tangent (∅)**
- $\tan\emptyset = r\,\frac{d\theta}{dr} = \frac{r}{\left(\frac{dr}{d\theta}\right)}$, find the value of θ ,for which $\emptyset = 0\ or\ \infty$
- The values of θ for which

$$\emptyset = 0, tangents\ coincide\ with\ radius\ vector.$$
$$\emptyset = \frac{\pi}{2}, tangents\ are\ perpendicular\ to\ radius\ vector.$$

ROSE CURVES.

If the polar equations are of the type

$$r = a \sin n\theta \qquad OR \qquad r = a \cos n\theta$$

Rules for tracing Rose curves :

1. Symmetry : (i) If $\theta \to -\theta$ and equation is not changed

$\Longrightarrow$ curve is symmetri about initial line.

(ii) If $\theta \to -\theta$ and $r \to -r$, and equation is not changed

$\Longrightarrow$ curve is symmetri aboutthe line $\theta = \frac{\pi}{2}$ through the pole perpedicular to initial line.

2.Since $|sin\ n\theta| \leq 1\ and\ |cos\ n\theta| \leq 1$, the maximum value of r is a

The rose curves lie in a circle of radius a.

3. Find in particular values of θ for which r = 0.

4. If the pole lies on the curve then find the equarion or equations of tangents at pole. Put r = 0 , values of θ give tangents at pole.

5. Since $\sin\theta$ and $\cos\theta$ are periodic functions of period 2π

Values of θ from 0 to 2π should only be considered. The values $\theta > 2\pi$

Do not give any new branch of the curve.

6. The curves r = a $\sin n\theta$ and $r = a \cos n\theta$ consists of

(i) n equal loops if n is odd.

(ii) 2n equal loops if n is even.

7. For drawing the loops of the curve $r = a \sin n\theta$

(a) Divide each quadrant into 'n' equal parts.

(b) First loop is drawn along $\theta = \frac{\pi/2}{n}$.

(c) If n is even, draw loops in two sectors consecutively from $\theta = 0$ to $\theta = 2\pi$.

(d) If n is odd, draw loops in two sectors alternatively keeping two sectors between the loops vacant.

8. For drawing the loops of the curve $r = a \cos n\theta$

(a) Divide each quadrant into 'n' equal parts.

(b) First loop is drawn along $\theta = 0$.

(c) If n is even, draw loops in two sectors consecutively from $\theta = 0$ to $\theta = 2\pi$.

(d) If n is odd, draw loops in two sectors alternatively keeping two sectors between the loops vacant.

Prepare the table of the values r and θ observe how r varies as θ increase from 0 to 2π

NOTE.

1. $\sin n\theta = 0 \; for \; n\theta = 0\,, \pi\,, 2\pi\,, 3\pi\,, 4\pi \ldots \ldots \ldots \ldots \ldots$

$$\therefore \; \theta = 0\,, \; \frac{\pi}{n}, \; \frac{2\pi}{n}, \; \frac{3\pi}{n}, \; \frac{4\pi}{n} \ldots \ldots \ldots \ldots \ldots$$

2. $\cos n\theta = 0 \; for \; n\theta = \frac{-\pi}{2}, \; \frac{\pi}{2}, \; \frac{3\pi}{2}, \; \frac{5\pi}{2} \ldots \ldots \ldots \ldots \ldots$

$$\therefore \; \theta = \frac{-\pi}{2n}, \; \frac{\pi}{2n}, \; \frac{3\pi}{2n}, \; \frac{5\pi}{2n} \ldots \ldots \ldots \ldots \ldots$$

1)	**A double point is Node if**			
	a)	Distinct branches have a common tangent	[redacted]	
	c)	Tangent at double point is above the curve	d)	Tangent at double point is below the curve
2)	A double point is Cusp if			
	a)	Two branches have distinct tangents	b)	Tangent line cuts the curve unusually
	[redacted]		d)	None of the above
3)	If all powers of y are even in the equation then curve is symmetrical about			
	a)	y −axis	b)	line $y = x$
	[redacted]		d)	line $y = -x$

4)	**If the equation of curve remains unchanged by replacing y by $-y$, then the curve is symmetric about**			
	a)	y −axis	b)	line $y = x$
	[redacted]		d)	line $y = -x$
5)	If all terms of x are of even degree in the equation of curve, then the curve is symmetric about			
	[redacted]		b)	line $y = x$
	c)	x −axis	d)	line $y = -x$

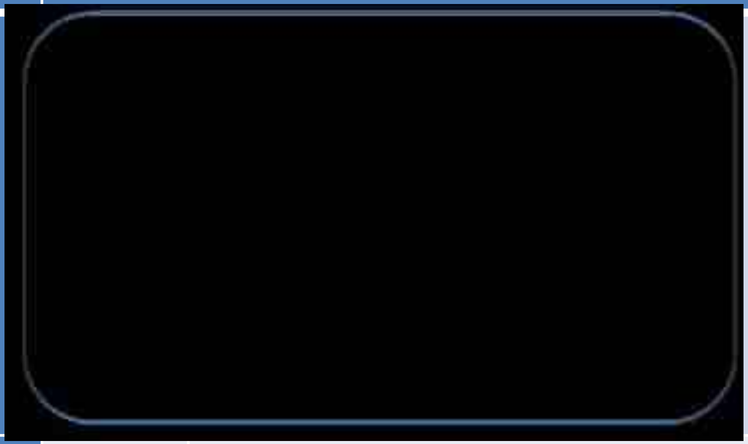
6)	**If the equation of curve does not contains any absolute constant term then the curve**			
	[redacted]		b)	Is increasing
	c)	Does not pass through origin	d)	Is decreasing
7)	If the curve passes through origin then the tangent to the curve at origin is obtained by			
	a)	Equating highest degree terms to zero	b)	Equating odd degree terms to zero
	c)	Equating even degree terms to zero	[redacted]	

8)	**Asymptotes are the tangents to the curve**			
	a)	At origin parallel to y −axis	b)	At origin not parallel to co-ordinate axis
	c)	At origin parallel to x −axis	[redacted]	
9)	Asymptotes parallel to x −axis are obtained by equating			
	a)	Coefficient of highest degree terms of y in the equation to zero	b)	Lowest degree terms to zero
	c)	Highest degree terms to zero	[redacted]	
10)	The parametric curve $x = f(t), y = g(t)$ is symmetric about x −axis if			
	[redacted]		b)	Both $f(t)$ and $g(t)$ are odd functions of t
	c)	$f(t)$ is an odd and $g(t)$ is even function of t	d)	Both $f(t)$ and $g(t)$ are even functions of t

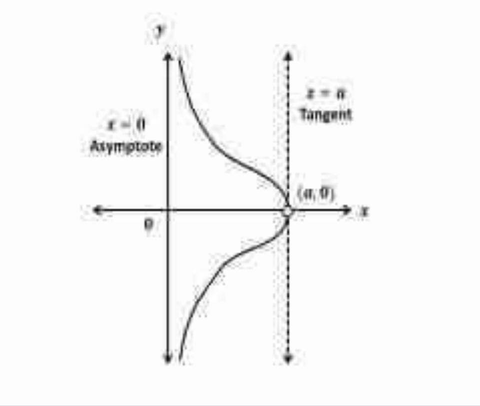
11)	**The curve $xy^2 = a^2(a-x)$ is symmetric about**		
	[redacted]	b)	line $y = x$
	c) y −axis	d)	line $y = -x$
12)	The curve $xy^2 = a^2(a-x)$		
	a) passes through the point $(-a, 0)$	[redacted]	
	c) passes through the origin	d)	passes through the point (a, a)

13)	**For the rose curve if is odd then the curve consist of $r = asinn\theta$ & $r = asinn\theta$**				
	a)	$2n$ equal loops	b)	$(n-1)$equal loops	
	c)	$(n+1)$equal loops	[redacted]		
14)	The curve represented by the equation is symmetrical about				
	a)	$y = -x$	[redacted]		
	c)	$x\ axis$ only	d)	$y = x$	

15 **The equation** $y^2(2a - x) = x^3$ **represents the curve**



b)



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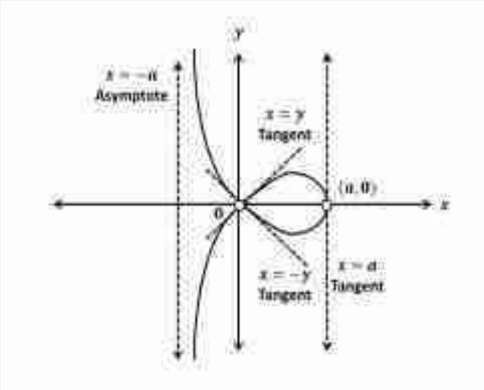

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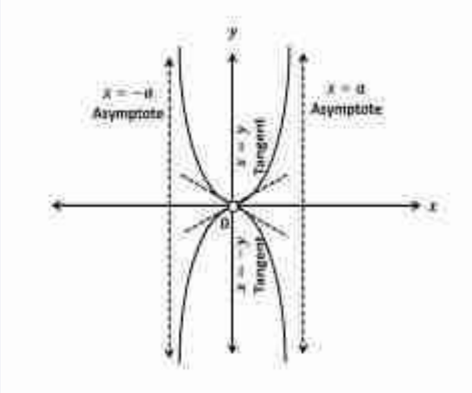
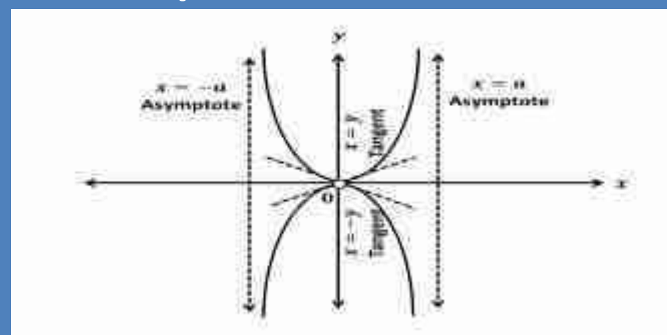


16) **The equation of curve represented in the following figure is**



a)	$xy^2 = a^2(a-x)$	b)	$y^2(2a-x) = x^3$
c)	$x(x^2+y^2) = a(x^2-y^2)$	[redacted]	

UNIT – IV
Rectification Of Curves.

Definition : The process of determination of lengths of the plane curves whose equations are in Cartesian, Parametric and Polar forms is known as **Rectification of curves.**

If 's' is length of the curve from A to B then rectification formulae are

Equation.	ds	s
y = f (x)	$\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ dx	$\int_{x_1}^{x_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$
x = f (y)	$\sqrt{1+\left(\frac{dx}{dy}\right)^2}$ dy	$\int_{y_1}^{y_2} \sqrt{1+\left(\frac{dx}{dy}\right)^2}$ dy
$x = f_1(t)$ $y = f_2(t)$	$\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}$ dt	$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}$ dt

Equation.	ds	s
r = f (θ)	$\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\ d\theta$	$\int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\ d\theta$
θ = f (r)	$\sqrt{1 + r^2\left(\frac{d\theta}{dr}\right)^2}\ dr$	$\int_{r_1}^{r_2} \sqrt{1 + r^2\left(\frac{d\theta}{dr}\right)^2}\ dr$

-- $\mathscr{R}$ --

1)	The length of arc of upper part of loop of the curve $3y^2 = x(x-1)^2$ from $(0,0)$ to $(1,0)$ using $1 + \left(\frac{dy}{dx}\right)^2 = \frac{(3x+1)^2}{12x}$, is			
	a)	$4/\sqrt{3}$	b)	$1/\sqrt{3}$
	c)	$\sqrt{3}$	[redacted]	

2)	The length of upper half of the cardioide $r = a(1+\cos\theta)$ where θ varies from 0 to π using $r^2 + \left(\frac{dr}{d\theta}\right)^2 = 2a^2(1+\cos\theta)$ is			
	a)	a	b)	$2a$
[redacted]			d)	$8a$

3)	**The length of arc of the curve** $x = e^{\theta} \cos\theta$, $y = e^{\theta} \sin\theta$, **from** $\theta = 0$ **to** $\theta = \pi/2$, **using** $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 2e^{2\theta}$ **is**			
	a)	$\sqrt{2}e^{\pi/2}$	b)	$\sqrt{2}(e^{\frac{\pi}{2}} + 1)$
	[redacted]		d)	$(e^{\frac{\pi}{2}} - 1)$

4)	**For the astroid** $x^{2/3} + y^{2/3} = a^{2/3}$ **the expression for** $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2$ **is**			
	a)	$3a^2 \sin^2\theta \cos^2\theta$	b)	$3a \sin^2\theta \cos^2\theta$
	c)	$3\,a \sin\theta \cos\theta$	[redacted]	

5)	For the curve $ay^2 = x^3$, the expression for $1 + \left(\frac{dy}{dx}\right)^2$ is			
	a)	$9x/4a$	b)	$1 - (9x/4a)$
	[redacted]		d)	$4a + 9x$

6)	The total length of the loop of the curve $x = t^2,\ y = t\left(1 - \frac{t^2}{3}\right)$ if $ds^2 = (1 + t^2)^2$ and $0 < t < \sqrt{3}$ is			
	a)	4	[redacted]	
	c)	$\sqrt{3}$	d)	$4 + \sqrt{3}$

7)	The limits of θ for finding the perimeter of $r = a(1 + \cos\theta)$ are			
	a)	$0 < \theta < \pi$	[redacted]	
	c)	$0 < \theta < \pi/2$	d)	$0 < \theta < \pi/4$

Unit I
Differential Equations

Order of a D.E.

It is the highest order derivative appearing in the equation.

Degree of a D. E.

It is the degree of the highest ordered derivative when the derivatives are free from radicals.

Solution of a D.E.

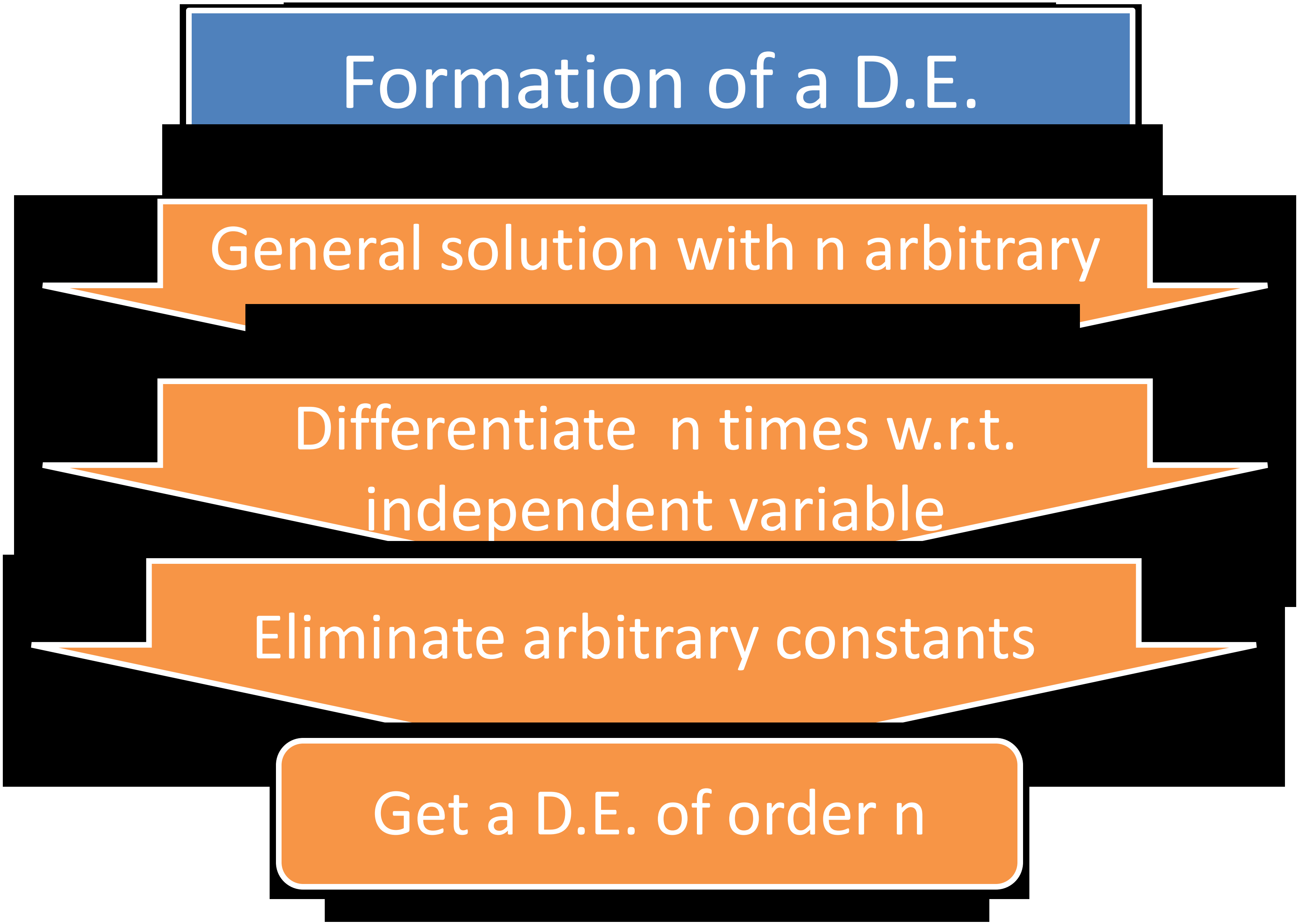
It is a relation between the variables which satisfies the given D. E.

General Solution

It is a solution of a D.E. in which the number of arbitrary constants equals to the order of D.E.

Particular Solution

It is a solution of a D.E. obtained by assigning particular values to the arbitrary constants in general solution of D.E.


Formation of a D.E.
General solution with n arbitrary
Differentiate n times w.r.t. independent variable
Eliminate arbitrary constants
Get a D.E. of order n

Ordinary D.E. of 1st order and 1st degree

It is the D.E. of the form

$$Mdx + Ndy = 0$$

where M and N are functions of x, y or constants

Depending upon the nature of M & N,we have 8 different types of solution of 1^{st} order , 1^{st} degree ordinary differential equation.

Methods of solving O.D.E. of 1st order and 1st degree

1. **Variable separable**

The given D.E. can be written as

$$f(x)dx = g(y)dy$$

G. S. is obtained by taking integration on both sides

$$\int f(x)dx = \int g(y)dy + C$$

2. D.E. reducible to variable separable by using substitution

Note certain terms in x and y namely $e^{xy}, e^{x/y}, e^{y/x}, \cos y/x, \cos(x+y), \sin(x-y)$ when appear in an equation lead to an identification as reducible to variable separable form.

3. Homogeneous D.E.

A D.E. $M\,dx+N\,dy=0$ is said to be homogeneous D.E. if M and N both are homogeneous functions of x and y of same degree.

Homogeneous D.E. can be reduce to variable separable form by substituting $\boldsymbol{y=ux}$

4. Non-homogeneous D.E.

The D.E. of the form $\frac{dy}{dx} = \frac{a_1 x + b_1 y + c_1}{a_2 x + b_2 y + c_2}$

is called non-homogeneous D.E.

If $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ in these case the expression $a_1 x + b_1 y$ and $a_2 x + b_2 y$ have a common factor the equation can be reduce to variable separable form.

If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ then put $x = X + h$ and $y = Y + k$

Choose h and k such that the equation becomes homogeneous in X and Y.

5. Exact D.E.

A D.E. $M\,dx + N\,dy = 0$ is said to be exact if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

The G.S. of exact D.E. is given by

$$\int_{y=const} M\,dx + \int [terms\ of\ N\ not\ containing\ x]\ dy = C$$

6. D.E. Reducible to Exact Form By Using Integrating Factor.

If $M\ dx + N\ dy = 0$ is not exact then by multiplying the equation by function $k(x,y)$ called as Integrating Factor (I.F.) , the equation can be made exact.

Rules of finding I.F.

- If the given D.E. is homogeneous and $xM + yN \neq 0$ then $I.F. = \frac{1}{xM+yN}$
- If the given D.E. is of the form $yf(xy)dx + xg(xy)dy = 0$ and $xM - yN \neq 0$ then $I.F. = \frac{1}{xM-yN}$.
- If $\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x)$ then $I.F. = e^{\int f(x)dx}$
- If $\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = g(y)$ then $I.F. = e^{\int g(y)dy}$
- If the given D.E. can be written as $x^a y^b(mydx + nxdy) + x^c y^d(pydx + qxdy) = 0$ then $I.F. = x^h y^k$, choose h, k such that condition of exactness is satisfied.

7. Linear D.E.

A D.E. of the form

$$\frac{dy}{dx} + Py = Q$$

where P, Q are functions of x or constants, is called a linear D.E. in y

$$I.F. = e^{\int P\,dx}$$

G.S. of linear D.E. is

$$ye^{\int Pdx} = \int Qe^{\int Pdx}dx + c$$

A D.E. of the form

$$\frac{dx}{dy} + Px = Q$$

where P, Q are functions of y or constants, is called a linear D.E. in x

$$I.F. = e^{\int P\,dy}$$

G.S. of linear D.E. is

$$xe^{\int Pdy} = \int Qe^{\int Pdy}\,dy + c$$

8. **Equation reducible to linear form**

The D.E. of the form

$$f'(y)\frac{dy}{dx}+P(x)f(y)=Q(x)$$

can be reduce to linear D.E. by substituting

$$f(y)=u, \quad \therefore f'(y)\frac{dy}{dx}=\frac{du}{dx}$$

$$\frac{du}{dx}+P(x)u=Q(x)$$

which is linear D.E. in u

$\therefore$ G.S. is $ue^{\int Pdx}=\int Qe^{\int Pdx}dx+c$

Similarly, the D.E. of the form

$$f'(x)\frac{dx}{dy}+P(y)f(x)=Q(y)$$

can be reduce to linear D.E. by substituting

$$f(x)=u, \quad \therefore f'(x)\frac{dx}{dy}=\frac{du}{dy}$$

$$\frac{du}{dy}+P(y)u=Q(y)$$

which is linear D.E. in u

Bernoulli's D.E.

A D.E. of the form

$$\frac{dy}{dx}+P(x)y=Q(x)y^{n}$$

is called Bernoulli's D.E. in y

Divide by y^n

$$y^{-n}\frac{dy}{dx}+P(x)y^{1-n}=Q(x)$$

Put $y^{1-n}=u$ *and solve*

Similarly, a D.E. of the form

$$\frac{dx}{dy} + P(y)x = Q(y)x^n$$

is called Bernoulli's D.E. in x

Divide by x^n

$$x^{-n}\frac{dx}{dy} + P(y)x^{1-n} = Q(y)$$

Put $x^{1-n} = u$ *and solve*

- Variable Separable
- Reducible to variable separable by Substitution
- Homogeneous D.E.
- Non-homogeneous D.E.
- Exact D.E.
- Reducible to Exact D.E.
- Linear D.E.
- Reducible to Linear D.E.

Variable Separable $f(x)dx = g(y)dy$

Reducible to variable separable by Substitution

Homogeneous D.E. Put $y = ux$

Non-homogeneous D.E. $\frac{dy}{dx} = \frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}$

$$\frac{dy}{dx} + Py = Q$$

Linear D.E.

Reducible to Linear D.E.

$$Mdx + Ndy = 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Exact D.E.

Reducible to Exact D.E.

The order and degree of the D.E $\left(1+\left(\frac{dy}{dx}\right)^3\right)^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ is

a) 2,3

b) 2,2

c) 2,1

d) 3,2

The order and degree of the D.E $\frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^2 + \int y\,dx = sin$x is

a) 4,1

b) 4,2

c) 2,2

d) None of these

The order and degree of the D.E $\left(\frac{dr}{dt}\right)^4 + \left(\frac{d^2r}{dt^2}\right)^3 + \left(\frac{d^3r}{dt^3}\right)^2 + \left(\frac{d^4r}{dt^4}\right)$=0

a) 1,4

b) 4,4

c) 4,1

d) 3,2

The order and degree of the D.E $\frac{dy}{dx} = \frac{ax+by+c}{3x+2by+5}$ is

a) 1,0

b) 0,1

c) 1,1

d) None of these

The number of arbitrary constants in the general solution of ordinary differential equation is equal to

a) The order of D.E

b) The degree of D.E

c) Coefficient of highest order differential coefficient

d) None of these

The order of differential equation whose general solution is $y = \frac{c_1}{c_2} cos(4x + c_3) + c_4 e^{2x - c_5}$, where c_1, c_2, c_3, c_4, c_5 are arbitrary constants, is

a) 2

b) 3

c) 4

d) 5

The order of differential equation whose general solution is $c_1 y = c_2 + c_3 x + c_3 x^2$, where c_1, c_2, c_3 are arbitrary constants, is

a) 1

b) 2

c) 3

d) 4

The order of differential equation whose general solution is $c_1 y e^{x + c_2} = c_3 x e^{4x + c_4}$, where c_1, c_2, c_3, c_4 are arbitrary constants, is

a) 1

b) 2

c) 3

d) 4

The D.E whose general solution is $y = \sqrt{5x + C}$ where C is arbitrary constant, is

a) $2y\dfrac{dy}{dx} - 1 = 0$

b) $\boxed{2y\dfrac{dy}{dx} - 5 = 0}$

c) $\dfrac{dy}{dx} - \dfrac{5}{2}\dfrac{1}{\sqrt{5x + C}} = 0$

d) $y\dfrac{dy}{dx} - 5 = 0$

The D.E whose general solution is $y = Cx - C^2$, where c is arbitrary constant, is

a) $\dfrac{dy}{dx} = C$

b) $\left(\dfrac{dy}{dx}\right)^2 + xy = 0$

c) $\left(\dfrac{dy}{dx}\right)^2 + x\dfrac{dy}{dx} - y = 0$

d) $\boxed{\left(\dfrac{dy}{dx}\right)^2 - x\dfrac{dy}{dx} + y = 0}$

The D.E whose general solution is $y = C^2 + \frac{C}{x}$, where C is arbitrary constant, is

a) $x^4 y_1^2 + xy_1 - y = 0$

b) $\boxed{x^4 y_1^2 - xy_1 - y = 0}$

c) $x^2 y_1^2 - xy_1 - y = 0$

d) $y_1 = -\dfrac{c}{x^2}$

The D.E whose general solution is $y = Acos(x+3)$, where A is arb. constant, is

a) $\boxed{cot(x+3)y_1 + y = 0}$ b) $tan(x+3)y_1 + y = 0$

c) $cot(x+3)y_1 - y = 0$ d) $tan(x+3)y_1 - y = 0$

The D.E representing the family of curves $y^2 = 2C\left(x+\sqrt{C}\right)$ where C is arbitrary constant, is

a) $2yy_1(x+\sqrt{yy_1}) - y^2 = 1$ b) $\boxed{2y_1(x+\sqrt{yy_1}) - y = 0}$

c) $y = 2y_1\left(x+\sqrt{C}\right)$ d) $y_1(x+\sqrt{yy_1}) - y = 0$

The solution of the D.E. $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ is

a) $\boxed{tan^{-1} y = tan^{-1} x + c}$ b) $tan^{-1} x + tan^{-1} y = c$

c) $y - x = c$ d) None of these

The D.E. representing family of curves $x^2 + y^2 = 2Ax$, where A is arb. constant, is

a) $$y_1 = \frac{y^2 + x^2}{2xy}$$

b) $$\boxed{y_1 = \frac{y^2 - x^2}{2xy}}$$

c) $$y_1 = \frac{y^2 + x^2}{2y}$$

d) $$y_1 = \frac{2xy}{y^2 + x^2}$$

The D.E. satisfied by G.S. $y = A\,cos\,x + B\,sin\,x$,where A,B are arb. constants, is

a) $$\frac{d^2y}{dx^2} + \frac{dy}{dx} = B\,sin\,x$$

b) $$\frac{d^2y}{dx^2} - y = 0$$

c) $$\frac{d^2y}{dx^2} + \frac{dy}{dx} = A\,cos\,x$$

d) $$\boxed{\frac{d^2y}{dx^2} + y = 0}$$

The D.E. satisfied by general solution

$y = A\,cos(log\,x) + B\,sin(log\,x)$, where A,B are arbitrary constants, is

a) $$x\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$$

b) $$\boxed{x^2\frac{d^2y}{dx^2} + \text{x}\frac{dy}{dx} + y = 0}$$

c) $$x^2\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$$

d) $$x^2\frac{d^2y}{dx^2} + y = 0$$

The D.E. satisfied by general solution $y = Ae^{x} + Be^{-x}$,where A,B are arb const is

a) $\boxed{y_2 - y = 0}$

b) $y_2 + \text{y} = 0$

c) $y_2 + \text{y} = Ae^{x} + Be^{-x}$

d) $y_2 - y = 2Ae^{x}$

The D.E. satisfied by general solution $y^2 = 4A(x - B)$,where A,B are arb const, is

a) $y_2 + y_1^2 = 0$

b) $yy_2 + y_1 = 0$

c) $yy_2 - y_1^2 = 0$

d) $\boxed{yy_2 + y_1^2 = 0}$

The D.E. of family of circles having their center at $(A, 5)$ and radius 5,where A is arbitrary constant is

a) $(y - 5)^2 \left\{1 + \frac{dy}{dx}\right\} = 5$

b) $(y - 5)^2 \left\{1 - \left(\frac{dy}{dx}\right)^2\right\} = 25$

c) $\boxed{(y - 5)^2 \left\{1 + \left(\frac{dy}{dx}\right)^2\right\} = 25}$

d) None of these

The D.E. $\frac{dy}{dx} = e^{x-y} + 3x^2 e^{-y}$ is of the form

a) $\boxed{\text{Variable separable}}$

b) Homogeneous

c) Linear

d) Exact

For solving D.E. $(x + y + 1)dx + (2x + 2y + 4)dy = 0$ appropriate substitution is

a) $x + y = 1$

b) $\boxed{x + y = u}$

c) $x - y = u$

d) None of these

The D.E. $\frac{dy}{dx} = \frac{x^3 - 3xy^2}{y^3 - 3x^2 y}$ is of the form

a) Variable separable

b) $\boxed{\text{Homogeneous}}$

c) Linear

d) Exact

The D.E. $\frac{dy}{dx} = \frac{x+2y-3}{3x+6y-1}$ is of the form

a) Variable separable

b) Homogeneous

c) $\boxed{\text{Non-homogeneous}}$

d) Exact

The solution of D.E. is $\frac{dy}{dx} = e^{x-y} + 3x^2e^{-y}$

a) $\boxed{e^y = e^x + x^3 + C}$

b) $e^y = e^x + 3x^3 + C$

c) $e^y = e^x + 3x + C$

d) $e^x + e^y = 3x^3 + C$

The solution of D.E. $x^3\left(x\frac{dy}{dx} + y\right) - sec(xy) = 0$ is

a) $tan(xy) + \frac{1}{2x^2} = C$

b) $\boxed{sin(xy) + \frac{1}{2x^2} = C}$

c) $sin(xy) - \frac{1}{2x^2} = C$

d) $sin(xy) - \frac{1}{4x^2} = C$

The solution of D.E. $sec^2 x\, tan\, y\, dx + sec^2 y\, tan\, x\, dy = 0$ is

a) $\sec^2 x\ \ tan\, y = C$

b) $tan\, x\, sec^2 y = C$

c) $\boxed{tan\, x\, tan\, y = C}$

d) $sec^2 x\, sec^2 y = C$

The solution of D.E. $e^x cos\, y\, dx + (1 + e^x)\, sin\, y\ \ dy = 0$ is

a) $x(1 + e^x) = C \sec \text{y}$

b) $\boxed{(1 + e^x)\, sec\, y = C}$

c) $\frac{sec\, y}{(1+e^x)}$=C

d) $(1 + e^x)\, cos\, y = C$

The solution of D.E. $y(1 + log\, x)\frac{dx}{dy} - x\, log\, x = 0\ \ is$

a) $log(x\, log\, x) = yC$

b) $\frac{x}{logx} = yC$

c) $y\, log\, x = xC$

d) $\boxed{x\, log\, x = yC}$

The solution of D.E. $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$

a) $tan^{-1} x + cot^{-1} y = C$

b) $\boxed{sin^{-1} x + sin^{-1} y = C}$

c) $sec^{-1} x + cosec^{-1} x = C$

d) $sin^{-1} x - sin^{-1} y = C$

The necessary and sufficient condition that the D.E M(x,y) dx + N(x,y) dy = 0 be exact is

a) $\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x};}$

b) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y};$

c) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x};$

d) $\frac{\partial M}{\partial x} - \frac{\partial N}{\partial y} = 1;$

If the integrating factor of differential equation $(x^2 + y^2 + 1)dx - 2xydy = 0$ is $\frac{1}{x^2}$ then its general solution is

a) $x - y = c$

b) $x^3 + 3y^2 = c$

c) $\boxed{x^2 - y^2 - 1 = cx}$

d) $x^2 + y^2 - 1 = cy$

If the I.F. of $(2x\ log\ x - xy)dy + 2ydx = 0$ is $\frac{1}{x}$ then its general solution is

a) $x^2\ log\ y - \frac{y}{3} = c$

b) $\boxed{2y\ log\ x - \frac{y^2}{2} = c}$

c) $2x^2\ log\ x - xy^2 = c$

d) $x\ log\ y - x = c$

If the I.F. of $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)\ dy = 0$ is $\frac{1}{y^3}$ then its general solution is

a) $\boxed{\left(y + \frac{2}{y^2}\right)x + y^2 = c}$

b) $\left(1 + \frac{1}{y^2}\right)x + y = c$

c) $xy^4 - 2xy + x^2y^4 = 0$

d) $y^3 + 2xy - 2x^2 = c$

If the I.F. of $(x^7y^2 + 3y)dx + (3x^8y - x)dy = 0$ is $\frac{y}{x^7}$ then its general solution is

a) $x^3y + x^7y^4 = c$

b) $x^7y^3 - x^2 = cx^5$

c) $\boxed{xy^3 - \frac{y^2}{2x^6} = c}$

d) $xy + \frac{y^2}{x^7} = c$

Integrating factor for the differential equation $\frac{dy}{dx} + \frac{4x}{x^2+1}y = \frac{1}{(x^2+1)^3}$ is

a) x^2

b) $1 + x^2$

c) $(1 + x^2)^{-2}$

d) $\boxed{(1 + x^2)^2}$

If the integrating factor of differential equation $\frac{dx}{dy} + \mathrm{x}\, sec\, y = \frac{2y\, cos\, y}{1+sin\, y}$ is $sec\, y + tan\, y$ then its general solution is

a) $\boxed{(sec\, y + tan\, y)x = y^2 + c}$

b) $x^2\, sec\, y + tan\, y = c$

c) $sec\, y + tan\, y = xy + c$

d) $sec\, y + x^2\, tan\, y = x^2 + c$

The differential equation $(1 + sin\, y)dx = (2y\, cos\, y - x\, sec\, y - x\, tan\, y)dy$ is

a) Homogeneous

b) Variable separable

c) $\boxed{\text{Linear in } x}$

d) None of these

The substitution for reducing non-homogeneous differential equation $\frac{dy}{dx} = \frac{2x-5y+3}{2x+4y-6}$ to homogeneous differential equation is

a) $x = X + 1, \quad y = Y - 3$

b) $x = X + 2, \quad y = Y + 2$

c) $x = X + 1, \quad y = Y + 1$

d) $x = X - 1, \quad y = Y + 2$

The integrating factor for the linear differential equation $\frac{dy}{dx} + \frac{y}{\sqrt{x}} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$ is

a) $e^{\sqrt{x}}$

b) $e^{\frac{1}{\sqrt{x}}}$

c) $e^{2\sqrt{x}}$

d) $e^{-\sqrt{x}}$

If homogeneous D.E. $\mathrm{M(x,y)\ dx\ +\ N(x,y)\ dy\ =\ 0}$is not exact then the integrating factor is

a) $\dfrac{1}{My+Nx}$ $My+Nx \neq 0$

b) $\dfrac{1}{Mx-Ny}$ $Mx-Ny \neq 0$

c) $\boxed{\dfrac{1}{Mx+Ny} \quad Mx+Ny \neq 0}$

d) $\dfrac{1}{My-Nx}$ $My-Nx \neq 0$

If the D.E. $\mathrm{M(x,y)\ dx\ +\ N(x,y)\ dy\ =\ 0}$ is not exat and it can be written as $yf_1(xy)dx + xf_2(xy)dy = 0$ then the I.F. is

a) $\dfrac{1}{My+Nx}$ $My+Nx \neq 0$

b) $\boxed{\dfrac{1}{Mx-Ny} \quad Mx-Ny \neq 0}$

c) $\dfrac{1}{Mx+Ny}$ $Mx+Ny \neq 0$

d) $\frac{1}{My-Nx}$ $My-Nx \neq 0$

If the D.E. $\mathrm{M(x,y)\ dx\ +\ N(x,y)\ dy\ =\ 0}$is not exact and $\dfrac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{M} = f(x)$ then the I.F. is

a) $e^{f(x)}$

b) $e^{\int f(x)dy}$

c) $f(x)$

d) $\boxed{e^{\int f(x)dx}}$

If the D.E. $\mathrm{M}(\mathrm{x},\mathrm{y})\,\mathrm{dx} + \mathrm{N}(\mathrm{x},\mathrm{y})\,\mathrm{dy} = 0$ is not exact and $\frac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M} = f(y)$ then the I.F. is

a) $e^{f(y)}$

b) $e^{\int f(y)dx}$

c) $f(y)$

d) $\boxed{e^{\int f(y)dy}}$

The D.E. $(x + y - 2)dx + (x - y + 4)dy = 0$ is of the form

a) $\boxed{\text{Exact}}$

b) Homogeneous

c) Linear

d) None of these

The value of λ for which the D.E.

$(xy^2 + \lambda x^2 y)dx + (x^3 + x^2 y)dy = 0$ is exact is

a) -3

b) 2

c) $\boxed{3}$

d) 1

The D.E.$(ay^2 + x + x^8)dx + (y^8 - y + bxy)dy = 0$ is exact if

a) $b \neq 2a$

b) $b = a$

c) $a = 1, b = 3$

d) $\boxed{b = 2a}$

The D.E. $(3 + by\cos x)dx + (2\sin x - 4y^3)dy = 0$ is exact if

a) $b = -2$

b) $b = 3$

c) $b = 0$

d) $\boxed{b = 2}$

I.F. of homogeneous D.E. $(xy - 2y^2)dx + (3xy - x^2)dy = 0$ is

a) $1/xy$

b) $1/x^2y^2$

c) $\boxed{1/x^2y}$

d) $1/x\ y^2$

I.F. of D.E. $(1 + xy)ydx + (x^2y^2 + xy + 1)xdy = 0$ is

a) $1/(x^2y)$

b) $\boxed{1/x^3y^3}$

c) $1/(xy^2)$

d) $1/x^2y^2$

I.F. of D.E. $(x^2 + y^2 + x)dx + (xy)dy = 0$ is

a) $1/x$

b) $1/x^2$

c) x^2

d) x

I.F. of D.E. $\left(y + \frac{y^3}{3} + \frac{x^2}{2}\right) dx + \left(\frac{x+xy^2}{4}\right) dy = 0$ is

a) x^2

b) x^3

c) $1/x$

d) $1/x^3$

I.F. of D.E. $(2x\, log\, x - xy)dy + (2y)dx = 0$ is

a) $1/x$

b) $1/x^2y^2$

c) $1/x^2$

d) 1/y

I.F. of D.E. $(2xy^2 + ye^x)dx - e^x dy = 0$

a) $1/x$

b) $1/y$

c) $1/x^2$

d) $1/y^2$

I.F. of D.E. $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$

a) $2/y$

b) $1/y$

c) $\frac{1}{y^3}$

d) $\frac{2}{y^2}$

Solution of non—exact D.E.$(x^2 - 3xy + 2y^2)dx + x(3x - 2y)dy = 0$

With I.F.$\frac{1}{x^3}$ is

a) $3\frac{y}{x} - \frac{y^2}{x^2} = C$

b) $log\, x - 3\frac{y}{x} + \frac{y^2}{x^2} = C$

c) $log\, x + 3\frac{y}{x} - 2\frac{y^2}{x^2} = C$

d) $log\, x + 3\frac{y}{x} - \frac{y^2}{x^2} = C$

Solution of non—exact D.E. $(3xy^2 - y^3)dx + (xy^2 - 2x^2y)dy = 0$

With I.F. $\frac{1}{x^2y^2}$ is

a) $3\,log\,x - \frac{2y}{x^3} - 2\,log\,y = C$

b) $\boxed{3\,log\,x + \frac{y}{x} - 2\,log\,y = C}$

c) $3\,log\,x + \frac{y}{x} = C$

d) $log\,x - \frac{y}{x} + 2\,log\,y = C$

Solution of non—exact D.E. $(x^4e^x - 2mxy^2)dx + (2mx^2y)dy = 0$

With I.F. $\frac{1}{x^4}$ is

a) $e^x + \frac{6my^2}{x^4} = C$

b) $e^x + \frac{2my^2}{x^2} = C$

c) $e^x + \frac{y^2}{x^2} = C$

d) $\boxed{e^x + \frac{my^2}{x^2} = C}$

The differential equation $\frac{dy}{dx} + P(x)y = Q(x)y^n$ is

a) Linear equation

b) Non-linear equation

c) $\boxed{\text{Bernoulli's equation}}$

d) None of these

The integrating factor for differential equation

$(1+y^2)\frac{dx}{dy}+x=e^{-\tan^{-1}y}$ is

a) $\frac{1}{1+y^2}$

b) $e^{\tan^{-1}x}$

c) $\boxed{e^{\tan^{-1}y}}$

d) None of these

The substitution for reducing non-homogeneous differential equation $\frac{dy}{dx}=\frac{-x-2y}{y-1}$ to homogeneous differential equation is

a) $x=X-1, \quad y=Y-3$

b) $x=X-2, \quad y=Y+1$

c) $x=X+1, \quad y=Y+1$

d) $\boxed{x=X-1, \quad y=Y+2}$

For what values of a and b, the differential equation $(y + x^3)dx + (ax + by^3)dy = 0$ is exact.

a) $b = 1, for\ all\ values\ of\ \ b$

b) $a = 2, b = 1$

c) $\boxed{a = 1, for\ all\ \ values\ of\ b}$

d) $a = -1, b = 3$

For what values of a, the differential equation $(ye^{xy} + ay^3)dx + (xe^{xy} + 12xy^2 - 2y)dy = 0$ is exact.

a) $a = 2$

b) $\boxed{a = 4}$

c) $a = 3$

d) $a = 1$

Unit II
Applications of Differential Equations

Orthogonal Trajectory

Method of finding the orthogonal trajectory of family of curves $F(x, y, c) = 0 \quad (1)$

Obtain D.E. of (1) by eliminating the arbitrary constant c, resulting in

$$\frac{dy}{dx} = f(x, y) \qquad (2)$$

Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$ in (2) we get

$$-\frac{dx}{dy} = f(x, y) \quad (3)$$

Solving (3) gives $\mathrm{G}(x, y, k) = 0$ which is the required orthogonal trajectory of (1)

Method of finding orthogonal trajectory of family of curves $F(r, \theta, c) = 0 \quad (1)$

Obtain D.E. of (1) by eliminating arb. const. c.

$$\frac{dr}{d\theta} = f(r, \theta) \quad (2)$$

Replace $\frac{dr}{d\theta}$ by $-r^2 \frac{d\theta}{dr}$ in (2)

$$\therefore -r^2 \frac{d\theta}{dr} = f(r, \theta) \quad (3)$$

Solving (3) gives $G(r, \theta, k) = 0$ which is the required orthogonal trajectory.

Newton's law of Cooling

The rate at which the temperature of a body θ changes is proportional to the difference between the temperature of body and the temperature of the surrounding medium θ_0

$$\frac{d\theta}{dt} \propto \theta - \theta_0$$

$$\therefore \frac{d\theta}{dt} = -k(\theta - \theta_0)$$

Simple Electrical Circuits

If $\boldsymbol{q}$ is charge and $\boldsymbol{i = \frac{dq}{dt}}$ the current in a circuit at any time $\boldsymbol{t}$ then

Voltage drop across a **resistor** of resistance R is $\boldsymbol{Ri}$

Voltage drop across a **capacitor** of capacitance C is $\boldsymbol{\frac{q}{C}}$

and

Voltage drop across an **inductor** of inductance L is

$$\boldsymbol{L\frac{di}{dt} = L\frac{d^2q}{dt^2}}$$

Kirchhoff's Voltage law

The algebraic **sum** of all the **voltage drops** across the components of an electrical circuit is **equal to e.m.f.**

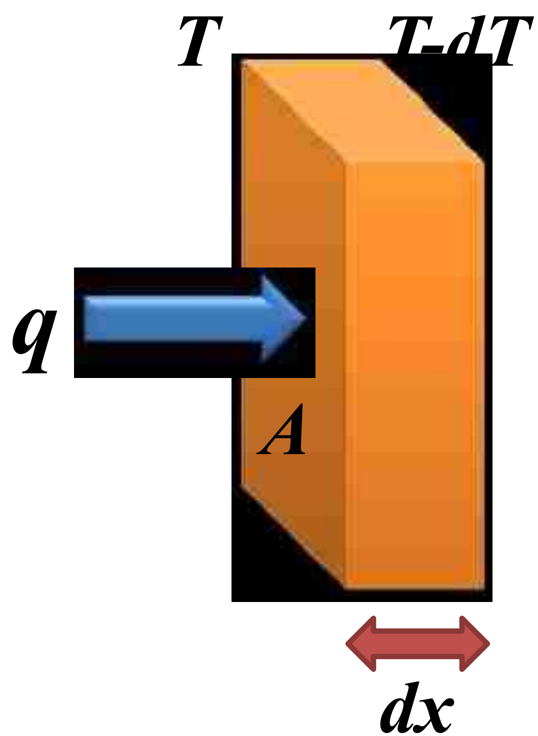
Heat Flow

Fourier’s law of Heat conduction

The heat flowing across a surface is proportional to its surface area and to the rate of change of temp w.r.t. its distance normal to the surface.

If q **(cal/sec)** be the quantity of heat that flows across a slab of surface area A **cm²** and thickness dx in 1 sec where the difference of temp at the faces of the slab is dT and k coefficient of thermal conductivity then

$$q = -kA\frac{dT}{dx}$$



Law of natural decay

A rate of decay of a material is proportional to its amount present at that instant.

If m is amount of material at time t then

$$\frac{dm}{dt} = -km$$

Rectilinear Motion

Rectilinear motion (also called as linear motion) is **motion along a straight line.**

If x is displacement of a particle at time t then its

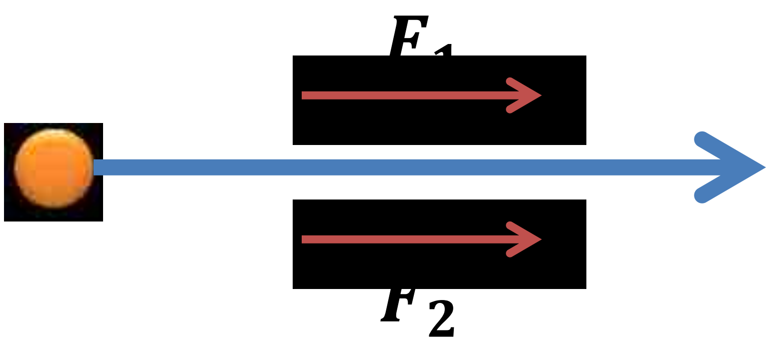
Velocity $$v = \frac{dx}{dt}$$

Acceleration $$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = v\frac{dv}{dx}$$

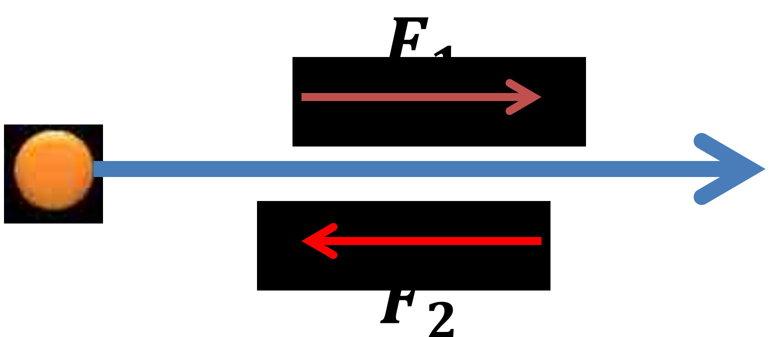
D'Alembert's principle

Algebraic sum of the forces acting on a body along a given direction is equal to the product of mass and acceleration in that direction.

Net force = Mass X Acceleration



Net force = $F_1 + F_2$



Net force = $F_1 - F_2$

S.H.M.

Equation of SHM is

$$Acceleration = \frac{d^2x}{dt^2} = -\omega^2 x$$

$$Period\ T = \frac{2\pi}{\omega}$$

	For finding orthogonal trajectory of $f(x,y,c)=0$ we replace $\frac{dy}{dx}$ by				**[01]**
	a)	$-dx/dy$	b)	$-dy/dx$	
	c)	$2dx/dy$	d)	dy/dx	
	The orthogonal trajectory of $y=ax^2$ is				[02]
	a)	$x^2+y^2=c^2$	b)	$x^2+(y^2/2)=c^2$	
	c)	$(x^2/2)+y^2=c$	d)	None of these	
	The orthogonal trajectory of parabola is				[02]
	a)	Circle	b)	Hyperbola	
	c)	Ellipse	d)	Straight line	

	The orthogonal trajectory of the family of circles with centre at $(0,0)$ is a family of				**[02]**
	a)	Circles	b)	Straight lines through $(0,0)$	
	c)	any straight line	d)	Parabola	
	The DE for the orthogonal trajectory of the family of curves $x^2 + 2y^2 = c^2$ is				[01]
	a)	$x + 2y\dfrac{dy}{dx} = 0$	b)	$2\dfrac{dx}{x} = \dfrac{dy}{y}$	
	c)	$xdx + ydy = 0$	d)	$\dfrac{dx}{x} = \dfrac{dy}{y}$	
	The DE of orthogonal trajectory of the family of curves $r^2 = a\sin 2\theta$ is				[01]
	a)	$\dfrac{dr}{r} = -\tan 2\theta\, d\theta$	b)	$\dfrac{dr}{r} = \tan 2\theta\, d\theta$	
	c)	$dr = \tan 2\theta\, d\theta$	d)	None of these	

The DE of orthogonal trajectory of the family of curves $r^2 = a^2 \cos 2\theta$ is				[01]
a)	$r\frac{d\theta}{dr} = \tan 2\theta$	b)	$rdr = \tan 2\theta\, d\theta$	
c)	$rdr = \cot 2\theta\, d\theta$	d)	$rdr + \tan\theta\, d\theta = 0$	
If the DE of orthogonal trajectory of a curve is $r\frac{d\theta}{dr} + \cot(\theta/2) = 0$ then its orthogonal trajectory is				[01]
a)	$r = \cos\theta$	b)	$r = c(1 - \sin\theta)$	
c)	$r = c(1 - \cos\theta)$	d)	$r = b(1 + \cos\theta)$	
If temperature of surrounding medium is θ_0 and temperature of body at any time t is θ, then in a process of heating $d\theta/dt$ is				[01]
a)	$\theta - \theta_0$	b)	$k(\theta - \theta_0);\ \ k > 0$	
c)	$-k(\theta - \theta_0);\ \ k > 0$	d)	None of these	

	In certain data of newton's law of cooling, $-kt = \log\left(\frac{\theta-40}{60}\right)$ and at $t = 4, \theta = 60^0$, then the value of k is				**[02]**
	a)	$\log(1/3)$	b)	$-\log(1/3)$	
	c)	$4\log(1/3)$	d)	$(1/4)\log 3$	
	If the temperature of water initially is 100^0C and $\theta_0 = 20^0C$, and water cools down to 60^0C in first 20 minutes with $k = \frac{1}{20}\log 2$, then during what time will it cool to 30^0C				[02]
	a)	60 min	b)	50 min	
	c)	1.5 hour	d)	40 min	
	If a body originally at 80^0C, with $\theta_0 = 40^0C$ and $k = \frac{1}{20}\log 2$, then the temperature of body after 40 min is				[02]
	a)	40^0C	b)	50^0C	
	c)	80^0C	d)	30^0C	

	If the body at 100^0C is placed in room whose temperature is 10^0C and cools to 60^0C in 5 minutes then the value of k is				[02]
	a)	$\log 2$	b)	$-\log 2$	
	c)	$(1/5)\log 2$ s	d)	$5\log 2$	
	The linear form of DE for R-L series circuit with emf E is				[01]
	a)	$L\frac{di}{dt}+Ri=E$	b)	$\frac{di}{dt}+\frac{R}{L}i=\frac{E}{L}$	
	c)	$L\frac{di}{dt}+Ri=0$	d)	none of these	
	The integrating factor for the DE of R-L series circuit with emf E is				[02]
	a)	$e^{\int Rdt}$	b)	e^{Rt+c}	
	c)	$e^{\int \frac{R}{L}dt}$	d)	$e^{\int idt}$	

If $i = \frac{E}{R} + ke^{-\frac{Rt}{L}}$ then the maximum value of i is					[01]
a)	R/L	b)	E/R		
c)	−E/R	d)	2R/L		
The linear form of DE for R-C series circuit with emf E is					[01]
a)	$Ri + \frac{q}{c} = E(t)$	b)	$Ri + \frac{1}{C}\int i\,dt = E$		
c)	$R\frac{di}{dt} + \frac{i}{C} = \frac{dE}{dt}$	d)	$\frac{di}{dt} + \frac{i}{RC} = \frac{1}{R}\frac{dE}{dt}$		
The integrating factor for the DE of R-C series circuit with emf E is					[01]
a)	$e^{\int RC dt}$	b)	$e^{\int \frac{1}{RC} dt}$		
c)	$e^{\int \frac{1}{R} dt}$	d)	$e^{\int \frac{1}{C} dt}$		

	If $i = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$ then the 50% of maximum current is				**[01]**
	a)	E/R	b)	E/2R	
	c)	2E/R	d)	2R/E	
	Which one of the following is not correct?				[01]
	a)	$F = ma$	b)	$F = m\frac{dv}{dt}$	
	c)	$F = m\,v\frac{dv}{dx}$	d)	$F = m\,v\frac{dv}{dt}$	
	A motion of a body or particle along straight line is known as				[01]
	a)	rectilinear motion	b)	curvilinear motion	
	c)	Motion	d)	None of these	

If a body of mass m falling from rest is subjected to the force of gravity and an air resistance proportional to the square of velocity kv^2, then the equation of motion is		**[01]**

a)	$mv\dfrac{dv}{dx} = mg + kv^2$	b)	$ma = -mg + kv^2$
c)	$ma = mg - kv^2$	d)	None of these

If a body opposed by force per unit mass of value cx and resistance per unit mass of value kv^2 then the equation of motion is	[01]

a)	$a = cx - bv^2$	b)	$a = bv^2 - cx$
c)	$v\dfrac{dv}{dx} = -cx - bv^2$	d)	$v\dfrac{dv}{dx} = cx + bv^2$

	The quantity of heat in a body is proportional to its				**[01]**
	a)	mass only	b)	temperature only	
	c)	mass and temperature	d)	none of these	

	The motion of a particle moving along a straight line is $\frac{d^2x}{dt^2} + 16x = 0$, then its period is				**[01]**
	a)	$2\pi/\sqrt{2}$	b)	$\pi/2$	
	c)	2π	d)	π	

	The orthogonal trajectories of the series of hyperbolas $xy = c^2$ is				**[02]**
	a)	$x^2 + y^2 = c^2$	b)	$x^2y^2 = c^2$	
	c)	$y^2 - x^2 = c^2$	d)	None of these	

	The differential equation of orthogonal trajectories of family of straight lines $y = mx$ is				**[01]**
	a)	$x\,dx - y\,dy = 0$	b)	$ydx - xdy = 0$	
	c)	$x\,dx + ydy = 0$	d)	$ydx + xdy = 0$	
	Let the population of country be decreasing at the rate proportional to its population. If the population has decreased to 25% in 10 years, how long will it take to half.				[02]
	a)	20 years	b)	8.3 years	
	c)	15 years	d)	5 years	
	The orthogonal trajectories of the family of straight lines $y = mx$ is				[01]
	a)	$x^2 - y^2 = c^2$	b)	$x^2 = my^2$	
	c)	$y^2 = m^2x^2$	d)	$x^2 + y^2 = c^2$	

The set of orthogonal trajectories to a family of curves whose DE is $\phi\left(x, y, \frac{dy}{dx}\right) = 0$ is obtained by DE				**[01]**
a)	$\phi\left(x, y, x\dfrac{dy}{dx}\right) = 0$	b)	$\phi\left(x, y, \dfrac{-dx}{dy}\right) = 0$	
c)	$\phi\left(x, y, \dfrac{dy}{dx}\right) = 0$	d)	$\phi\left(x, y, \dfrac{-dy}{dx}\right) = 0$	
The orthogonal trajectories of the family of curves $r\cos\theta = a$ is				[02]
a)	$r\sin\theta = c$	b)	$r\tan\theta = c$	
c)	$\dfrac{r}{\sin\theta} = c$	d)	None of these	
If 10 grams of some radioactive substance reduces to 8 gm in 60 years, in how many years will 2 gm of it will be left ?				[02]
a)	120 yrs	b)	378 yrs	
c)	220 yrs	d)	433 yrs	

	Voltage drop across inductance L is given by				**[01]**
	a)	Li	b)	$L\frac{di}{dt}$	
	c)	$\frac{dL}{dt}$	d)	None of these	
	A ball at temperature of 32^0C is kept in a room where the temperature is 10^0C. If the ball cools to 27^0C in hour then its temperature is given by				[02]
	a)	$T = 22\, e^{0.205\, t}$	b)	$T = 10\, e^{1.163t}$	
	c)	$T = 10 + 22e^{-0.258t}$	d)	$T = 32 - 10e^{-0.093t}$	

Unit III

Fourier Series, Reduction Formulae, Gamma Functions, Beta Functions

Multiple Choice Questions

Periodic functions

A function $f(x)$ is said to be periodic if it is defined for all real x and if there is some positive number T such that

$$f(x+T) = f(x) \qquad \forall\, x$$

The number T is then called period of $f(x)$.

$\sin x, \cos x$ are periodic functions of period 2π

$\tan x\,, \cot x$ are periodic functions of period π

Fourier Series

If $f(x)$ is a periodic function of period 2π, defined in the interval $c \leq x \leq c + 2\pi$ then

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

this representation of $f(x)$ is called Fourier Series and the coefficients a_0, a_n, b_n are called the Fourier coefficients.

Euler's Formulae

$$a_0 = \frac{1}{\pi} \int_{c}^{c+2\pi} f(x)dx$$

$$a_n = \frac{1}{\pi} \int_{c}^{c+2\pi} f(x) \cos nx \, dx$$

$$b_n = \frac{1}{\pi} \int_{c}^{c+2\pi} f(x) \sin nx \, dx$$

1	If $f(x+nT)=f(x)$ where n is any integer then the fundamental period of $f(x)$ is			
	a)	$2T$	b)	$T/2$
	c)	$\boxed{T}$	d)	3T

2	If $f(x)$ is a periodic function with period T then $f(ax)$, $a \neq 0$ is periodic function with fundamental period			
	a)	T	b)	$\boxed{T/a}$
	c)	aT	d)	π

3	**Fundamental period of** $\cos 2x$ **is**			
	a)	$\frac{\pi}{4}$	b)	$\frac{\pi}{2}$
	c)	$\boxed{\pi}$	d)	2π

4	**Fundamental period of** $\tan 3x$ **is**			
	a)	$\frac{\pi}{2}$	b)	$\boxed{\frac{\pi}{3}}$
	c)	π	d)	$\frac{\pi}{4}$

5	The value of constant terms in the Fourier series of $f(x) = e^{-x}$ in $0 \le x \le 2\pi,\ f(x+2\pi) = f(x)$ is			
	a)	$\frac{1}{\pi}(1-e^{-2\pi})$	b)	$\boxed{\frac{1}{2\pi}(1-e^{-2\pi})}$
	c)	$2(1-e^{-2\pi})$	d)	$(1-e^{-2\pi})$

6	Fourier coefficient a_0 in the Fourier series expansion of $f(x) = \left(\frac{\pi-x}{2}\right)^2; 0 \le x \le 2\pi$ and $f(x+2\pi) = f(x)$			
	a)	$\frac{\pi^2}{3}$	b)	$\boxed{\frac{\pi^2}{6}}$
	c)	0	d)	$\pi/6$

For function defined in the interval $-\pi \leq x \leq \pi$

- If $f(x)$ is even then

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{2}{\pi}\int_{0}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx = \frac{2}{\pi}\int_{0}^{\pi} f(x)\cos nx\, dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx = 0$$

For function defined in the interval $-\pi \leq x \leq \pi$

- If $f(x)$ is odd then

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = 0$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx = 0$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin nx \, dx$$

7	**Fourier series representation of periodic function** $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$ $f(x) = \frac{8}{\pi^2}\left[\frac{1}{1^2}\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x + \cdots\right]$ **then value of** $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots =$

	a)	$\frac{\pi^2}{4}$	b)	$\boxed{\frac{\pi^2}{8}}$
	c)	$\frac{\pi^2}{16}$	d)	$\frac{8}{\pi^2}$

31 $f(x) = x, -\pi \le x \le \pi$ and period is 2π. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$, Fourier coefficient b_1 is

a) $\boxed{2}$ b) -1

c) 0 d) $2/\pi$

If $f(x)$ is a periodic function of period $2L$, defined in the interval $c \leq x \leq c+2L$ then

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$$

$$a_0 = \frac{1}{L}\int_{c}^{c+2L} f(x)dx \qquad a_n = \frac{1}{L}\int_{c}^{c+2L} f(x)\cos\frac{n\pi x}{L}dx$$

$$b_n = \frac{1}{L}\int_{c}^{c+2L} f(x)\sin\frac{n\pi x}{L}dx$$

If $f(x)$ is a periodic function of period $2L$, defined in the interval $-\mathrm{L} \le x \le L$ and

if $f(x)$ is an even function then

$$a_0 = \frac{2}{L}\int_0^L f(x)dx \qquad a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx \qquad b_n = 0$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L}\right)$$

if $f(x)$ is an odd function then

$$a_0 = 0 \qquad a_n = 0 \qquad b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$$

$$f(x) = \sum_{n=1}^{\infty}\left(b_n \sin\frac{n\pi x}{L}\right)$$

Half range expansions

- Half range cosine series: If $f(x)$ is defined in the interval $0 \leq x \leq L$, then the half range cosine series of $f(x)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$$

$$a_0 = \frac{2}{L}\int_0^L f(x)dx \qquad a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx$$

- Half range sine series: If $f(x)$ is defined in the interval $0 \leq x \leq L$, then the half range sine series of $f(x)$ is given by

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$$

$$b_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx$$

9	The Fourier constant a_n for $f(x) = 4 - x^2$ in the interval $0 < x < 2$ is			
	a)	$\boxed{4/\pi^2 n^2}$	b)	$2/n^2\pi^2$
	c)	$4/n^2\pi$	d)	$2/n\pi^2$

10	For half range sine series of $f(x) = x, 0 \leq x \leq 2$ and period is 4.Fourier series is represented by $\sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{2}$, then Fourier coefficient b_1 is			
	a)	4	b)	2
	c)	$\frac{2}{\pi}$	d)	$\boxed{\frac{4}{\pi}}$

		1st Harmonic		2nd Harmonic		3rd Harmonic	
x	y	$y\cos\frac{\pi x}{L}$	$y\sin\frac{\pi x}{L}$	$y\cos\frac{2\pi x}{L}$	$y\sin\frac{2\pi x}{L}$	$y\cos\frac{3\pi x}{L}$	$y\sin\frac{3\pi x}{L}$
x_0	y_0						
$\vdots$	$\vdots$						
x_{m-1}	y_{m-1}						
Σ							

$$a_0 = \frac{2}{m}\sum_{i=0}^{m-1} y_i \qquad a_n = \frac{2}{m}\sum_{i=0}^{m-1} y_i \cos\frac{n\pi x_i}{L} \qquad b_n = \frac{2}{m}\sum_{i=0}^{m-1} y_i \sin\frac{n\pi x_i}{L}$$

$$f(x) = \frac{a_0}{2} + a_1 \cos\frac{\pi x}{L} + b_1 \sin\frac{\pi x}{L} + a_2 \cos\frac{2\pi x}{L} + b_2 \sin\frac{2\pi x}{L}$$
$$+ a_3 \cos\frac{3\pi x}{L} + b_3 \sin\frac{3\pi x}{L} + \cdots$$

1. The term $\left[a_1 \cos\left(\frac{\pi x}{L}\right) + b_1 \sin\left(\frac{\pi x}{L}\right)\right]$ is called as **'Fundamental or First harmonic'.**

2. The term $\left[a_2 \cos\left(\frac{2\pi x}{L}\right) + b_2 \sin\left(\frac{2\pi x}{L}\right)\right]$ is called as ' **second harmonic'** and so on.

3. The amplitude of n^{th} harmonic is $+\sqrt{a_n^2 + b_n^2}$.

4. Percentage of n^{th} harmonic =

$$\frac{\text{amplitude of } n^{\text{th}} \text{ harmonic}}{\text{amplitude of } I^{\text{st}} \text{ harmonic}} \times 100$$

11	For the certain data if $a_0 = 1.5,\ a_1 = 0.373,\ b_1 = 1.004$ then the amplitude of 1st harmonic is				
	a)	$\boxed{1.07}$	b)	2.07
	c)	1.004	d)	1.377

12	The value of a_0 in harmonic analysis of y for the following tabulated data is						
x°	0	60	120	180	240	300	360
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0
	a)	1.45	b)	5.8			
	c)	$\boxed{2.9}$	d)	2.48			

13	The value of a_1 in Harmonic analysis of y for the following tabulated data is :						
x	0	1	2	3	4	5	6
y	4	8	15	7	6	2	4
$\cos\frac{\pi x}{3}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1
	a)	-4.16	b)	-8.32			
	c)	-3.57	d)	$\boxed{-10.98}$			

14	The value of $a_1,\ a_2$ in Fourier cosine series of y for the following tabulated data are			
x	0	$\pi/4$	$\pi/2$	$3\pi/4$
y	0	$\sqrt{2}$	2	$\sqrt{2}$
	a)	$-1/2, 1/2$	b)	$\boxed{-1/2, -1/2}$
	c)	$2, -2$	d)	$-2, 0$

Reduction Formulae

1. $\int_0^{\pi/2} \cos^n x\, dx = \int_0^{\pi/2} \sin^n x\, dx$

$$= \frac{[\,(n-1)\text{subtract } 2 \ldots\ldots\ldots\ldots\ldots\ldots\ 2 \text{ or } 1\,]}{[\,(n)\ \text{ subtract } 2 \ldots\ldots\ldots\ldots\ldots\ldots\ 2 \text{ or } 1]} \times \left(\frac{\pi}{2}\right) \text{ if n is even.}$$

$$= \frac{[\,(n-1)\text{subtract } 2 \ldots\ldots\ldots\ldots\ldots\ldots\ 2 \text{ or } 1\,]}{[\,(n)\ \text{ subtract } 2 \ldots\ldots\ldots\ldots\ldots\ldots\ 2 \text{ or } 1]} \times 1 \text{ if n is odd.}$$

2.(a) $\int_0^{\pi/2} \sin^m x \cos^n x\, dx$

$$= \left\{\frac{[\,(m-1)\text{ subtract } \ 2 \ldots\ldots\ 2 \text{ or } 1\,].[\,(n-1)\text{ subtract } \ 2 \ldots\ldots\ 2 \text{ or } 1\,]}{[\,(m+n)\text{ subtract } \ 2 \ldots\ldots\ldots\ 2 \text{ or } 1\,]}\right\} \times \left(\frac{\pi}{2}\right)$$

If m and n both are even.

2.(b) $\int_0^{\pi/2} \sin^m x \cos^n x\, dx$

$$= \left\{\frac{[\,(m-1)\text{ subtract } \ 2 \ldots\ldots\ 2 \text{ or } 1\,].[\,(n-1)\text{ subtract } \ 2 \ldots\ldots\ 2 \text{ or } 1\,]}{[\,(m+n)\text{ subtract } \ 2 \ldots\ldots\ldots\ 2 \text{ or } 1\,]}\right\} \times (\,1\,)$$

Otherwise .

3] $\int_0^{\pi/2} \sin^m x \cos x \, dx = \int_0^{\pi/2} \cos^m x \sin x \, dx = \frac{1}{m+1}$

Conversion Formulae :

1] $\int_0^{2\pi} \sin^m x \cos^n x \, dx = \begin{cases} = 4\int_0^{\pi/2} \sin^m x \cos^n x \, dx \, , \textbf{\textit{if m , n even}}. \\ = 0 \, , \qquad\qquad\qquad Otherwise. \end{cases}$

2] $\int_0^{\pi} \sin^m x \cos^n x \, dx$

$$= \begin{cases} = 2\int_0^{\pi/2} \sin^m x \cos^n x \, dx \, , \textbf{\textit{if n even , for any m}}. \\ = 0 \, , \qquad\qquad\qquad , \textbf{\textit{if n odd , for any m}}. \end{cases}$$

3] $\int_0^{2\pi} \sin^n x \, dx = \begin{cases} = 4\int_0^{\pi/2} \sin^n x \, dx \, , \textbf{\textit{if n is even}}. \\ = 0 \qquad\qquad , \textbf{\textit{if n is odd}}. \end{cases}$

4] $\int_0^{2\pi} \cos^n x \, dx = \begin{cases} = 4\int_0^{\pi/2} \cos^n x \, dx \;, \boldsymbol{if\ n\ is\ even}. \\ = 0 \qquad\qquad\qquad\qquad\;\;\;\; , \boldsymbol{if\ n\ is\ odd}. \end{cases}$

5] $\int_0^{\pi} \sin^n x \, dx = 2\int_0^{\pi/2} \sin^n x \, dx \;, \boldsymbol{for\ any\ n}$.

6] $\int_0^{\pi} \cos^n x \, dx = \begin{cases} = 2\int_0^{\pi/2} \cos^n x \, dx \;, \boldsymbol{if\ n\ is\ even}. \\ = 0 \qquad\qquad\qquad\qquad\;\;\;\; , \boldsymbol{if\ n\ is\ odd}. \end{cases}$

1	The value of the integral $\int_0^{\frac{\pi}{6}} \cos^6 3x \; dx$ is			
	a)	$\boxed{5\pi/96}$	b)	$7/48$
	c)	$5\pi/32$	d)	0

2	The value of $\int_{-\pi/2}^{\pi/2} \sin^4 x \, dx$ is			
	a)	$3\pi/16$	b)	$\boxed{3\pi/8}$
	c)	$3\pi/4$	d)	0

$$\int_0^{\pi/2} cos^n x \, dx = \int_0^{\pi/2} sin^n x \, dx \boxed{= \frac{n-1}{n} I_{n-2}}$$

3	**The value of the integral** $\int_0^{\pi/2} \sin^4 x \cos^3 x \; dx$ **is**			
	a)	$\pi/35$	b)	$\boxed{2/35}$
	c)	0	d)	$53/2$

4	**The value of** $\int_{-2\pi}^{2\pi} \sin^3 x \cos^2 x \, dx$ **is**			
	a)	$\boxed{0}$	b)	$\pi/4$
	c)	$\pi/16$	d)	$\pi/32$

5	The value of the integral $\int_0^{2\pi} \cos^5 x \; dx$ is			
	a)	$\boxed{0}$	b)	$5/16$
	c)	$5/32$	d)	$5\pi/32$

6	The value of the integral $\int_0^{\pi} \sin^5 x \; dx$ is			
	a)	$8\pi/15$	b)	$\pi/2$
	c)	$\boxed{16/15}$	d)	0

7	If $I_n = \int_{\pi/4}^{\pi/2} \cot^n x\, dx$ and $I_n = \frac{1}{n-1} - I_{n-2}$, then the value of I_6 is			
	a)	$\frac{13}{15}$	b)	$\frac{13}{15} + \frac{\pi}{4}$
	c)	$\boxed{\frac{13}{15} - \frac{\pi}{4}}$	d)	$\frac{13}{15} - \frac{\pi}{2}$

8	If $I_n = \int_0^{\pi/4} \sin^{2n} x\ dx$ and $I_n = \left(1 - \frac{1}{2n}\right) I_{n-1} - \frac{1}{n2^{n+1}}$, then the value of $\int_0^{\pi/4} \sin^4 x\ dx$ is			
	a)	$\frac{3\pi}{32} + \frac{1}{4}$	b)	$\boxed{\frac{3\pi}{32} - \frac{1}{4}}$
	c)	$\frac{\pi}{16} - \frac{1}{4}$	d)	$\frac{3\pi}{16} + \frac{1}{4}$

9	If $I_{m,n} = \int_0^{\pi/2} (\cos^{\mathrm{m}} x)(\sin nx)dx$ and $I_{m,n} = \frac{1+m\, I_{m-1,n-1}}{m+n}$, then the value of $\int_0^{\pi/2} (\cos^2 x)(\sin 4x)dx$ is			
	a)	3	b)	2
	c)	$1/3$	d)	$2/3$

10	If $I_n = \int_0^{\pi/2} x \cdot \sin^n x \cdot dx$ and $I_n = \frac{n-1}{n} I_{n-2} + \frac{1}{n^2}$, then the value of $\int_0^{\pi/2} x \cdot \sin^4 x \cdot dx$			
	a)	$\frac{3\pi^2}{64} + \frac{1}{4}$	b)	$\frac{\pi^2}{64} + \frac{1}{4}$
	c)	$\frac{3\pi^2}{32} - \frac{1}{4}$	d)	$\frac{3\pi^2}{64} - \frac{1}{4}$

1. Gamma Function

Definition: The integral $\int_0^\infty e^{-x} x^{n-1} dx$ is called as Gamma function

and denoted by $\Gamma n = \int_0^\infty e^{-x} x^{n-1} dx$ (n > 0)

Properties :

$\boxed{1.\ \Gamma 1 = 1}$

2. Reduction formula : $\Gamma n+1 = n\,\Gamma n$

$= n!\,,\text{if } n \text{ is } +ve \text{ integer}$

3. $\Gamma 0 = \infty$

4. $\Gamma \dfrac{1}{2} = \sqrt{\pi}$

5. $\Gamma P \ \Gamma 1-P = \dfrac{\pi}{\sin p\pi}$

11	The value of the integral $\int_0^\infty \frac{x^5}{5^x} dx$ by using substitution $5^x = e^t$ is			
	a)	$120/(\log 5)^6$	b)	$24/(\log 4)^5$
	c)	$120/(\log 5)^5$	d)	$24/(\log 4)^4$

12	The value of the integral $\int_0^1 \frac{dx}{\sqrt{x \log\left(\frac{1}{x}\right)}}$ by using the substitution $\log\left(\frac{1}{x}\right) = t$ is			
	a)	$\sqrt{\pi}/2$	b)	$\sqrt{2\pi}$
	c)	$\sqrt{\pi}$	d)	$2\sqrt{\pi}$

13	The formula for $\Gamma(n+1)$ is			
	a)	$$\int_0^\infty e^{-x}x^{n-1}dx$$	b)	$$\int_0^\infty e^{-x}x^{n}dx$$
	c)	$$2\int_0^\infty e^{-x}x^{n-1}dx$$	d)	$$\int_0^\infty e^{-x}x^{n-2}dx$$

14	The value of the integral $\int_0^\infty e^{-4x}x^3\,dx$ is			
	a)	$4!$	b)	$3!$
	c)	$\frac{3!}{64}$	d)	$\frac{3!}{256}$

15	The value of $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)$ is			
	a)	$\boxed{2\pi/\sqrt{3}}$	b)	$\pi/\sqrt{3}$
	c)	2π	d)	$2/\sqrt{3}$

16	The value of $\int_0^1 (\log x)^n\,dx$ is			
	a)	$\boxed{(-1)^n\Gamma(\mathrm{n}+1)}$	b)	$(\log n)\Gamma n$
	c)	Γn	d)	$\Gamma(n+1)$

Beta Function.

$$\text{Definition}: \boxed{\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx}\ ;\ \text{where } m,n \text{ are } +ve \text{ integers}$$

Properties Of Beta Function.

1. $\beta(m,n) = \beta(n,m)$

2. $\beta(m,n) = \int_0^\infty \frac{y^{m-1}}{(1+y)^{m+n}}\,dy$

3. $\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta\, d\theta$

4. $$\int_0^{\pi/2} \sin^p \theta \cos^q \theta \, d\theta = \frac{1}{2}\beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$$

5. Relation Between Beta and Gamma Function.

$$\boxed{\beta\,(m\,,n\,) = \frac{\Gamma m \;\; \Gamma n}{\Gamma(m+n)}}$$

6. Legendre's duplication formula :

$$\boxed{\Gamma m \;\; \Gamma(m + 1/2) = \frac{\sqrt{\pi}}{2^{2m-1}} \;\; \Gamma 2m}$$

17	**Value of $B\left(\frac{3}{4},\frac{1}{4}\right)$ is**			
	a)	2π	b)	$\boxed{\pi\sqrt{2}}$
	c)	$\pi/2$	d)	$\sqrt{2}$

18	**Value of $\int_0^{\pi/2}\sqrt{\tan x}\,dx$ is**			
	a)	$\boxed{\frac{1}{2}B\left(\frac{3}{4},\frac{1}{4}\right)}$	b)	$\frac{1}{2}B\left(\frac{3}{4},\frac{3}{4}\right)$
	c)	$B\left(\frac{3}{4},\frac{1}{4}\right)$	d)	$B\left(\frac{3}{4},\frac{3}{4}\right)$

19	If $B(n+1,1)=\frac{1}{4}$ and n is a positive integer then value of n is			
	a)	1	b)	2
	c)	3	d)	4

20	Value of $\int_0^{\pi/2}\sqrt{2\sin 2x}\,dx$ is			
	a)	$\frac{1}{2}B\left(\frac{3}{4},\frac{1}{4}\right)$	b)	$\frac{1}{2}B\left(\frac{3}{4},\frac{3}{4}\right)$
	c)	$B\left(\frac{3}{4},\frac{1}{4}\right)$	d)	$B\left(\frac{3}{4},\frac{3}{4}\right)$

21	The value of $\int_0^\infty \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\,dx$ is			
	a)	0	b)	$\frac{B(m,n)}{2}$
	c)	$\boxed{2B(m,n)}$	d)	1

22	By Duplication formula, the value of $\Gamma m \cdot \Gamma(m+\frac{1}{2})$ is			
	a)	$\frac{\sqrt{\pi}}{2^{m-1}}\Gamma(2m)$	b)	$\frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(m)$
	c)	$\frac{\sqrt{\pi}}{2^{m}}\Gamma(2m)$	d)	$\boxed{\frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)}$

Unit II
Applications of Differential Equations

Orthogonal Trajectory

Method of finding the orthogonal trajectory of family of curves $F(x, y, c) = 0 \quad (1)$

Obtain D.E. of (1) by eliminating the arbitrary constant c, resulting in

$$\frac{dy}{dx} = f(x, y) \qquad (2)$$

Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$ in (2) we get

$$-\frac{dx}{dy} = f(x, y) \quad (3)$$

Solving (3) gives $\mathrm{G}(x, y, k) = 0$ which is the required orthogonal trajectory of (1)

Method of finding orthogonal trajectory of family of curves $F(r, \theta, c) = 0 \quad (1)$

Obtain D.E. of (1) by eliminating arb. const. c.

$$\frac{dr}{d\theta} = f(r, \theta) \quad (2)$$

Replace $\frac{dr}{d\theta}$ by $-r^2 \frac{d\theta}{dr}$ in (2)

$$\therefore -r^2 \frac{d\theta}{dr} = f(r, \theta) \quad (3)$$

Solving (3) gives $G(r, \theta, k) = 0$ which is the required orthogonal trajectory.

Newton's law of Cooling

The rate at which the temperature of a body θ changes is proportional to the difference between the temperature of body and the temperature of the surrounding medium θ_0

$$\frac{d\theta}{dt} \propto \theta - \theta_0$$

$$\therefore \frac{d\theta}{dt} = -k(\theta - \theta_0)$$

Simple Electrical Circuits

If $\boldsymbol{q}$ is charge and $\boldsymbol{i = \frac{dq}{dt}}$ the current in a circuit at any time $\boldsymbol{t}$ then

Voltage drop across a **resistor** of resistance R is $\boldsymbol{Ri}$

Voltage drop across a **capacitor** of capacitance C is $\boldsymbol{\frac{q}{C}}$

and

Voltage drop across an **inductor** of inductance L is

$$\boldsymbol{L\frac{di}{dt} = L\frac{d^2q}{dt^2}}$$

Kirchhoff's Voltage law

The algebraic **sum** of all the **voltage drops** across the components of an electrical circuit is **equal to e.m.f.**

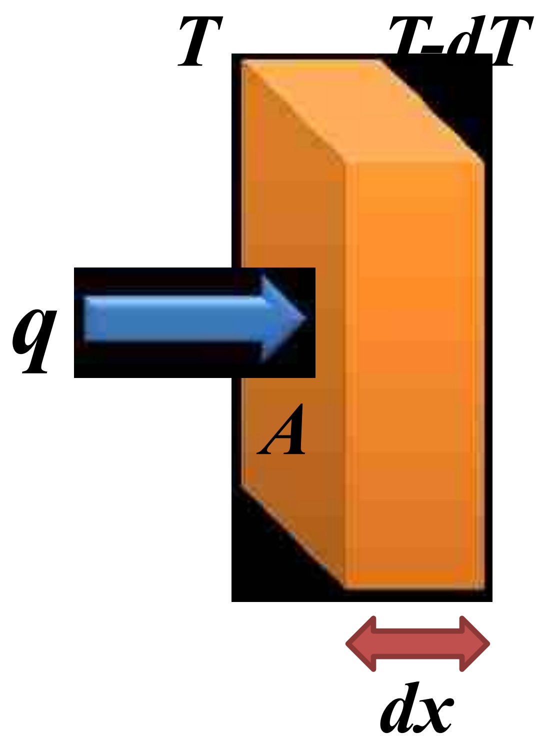
Heat Flow

Fourier's law of Heat conduction

The heat flowing across a surface is proportional to its surface area and to the rate of change of temp w.r.t. its distance normal to the surface.

If q **(cal/sec)** be the quantity of heat that flows across a slab of surface area A **cm²** and thickness dx in 1 sec where the difference of temp at the faces of the slab is dT and k coefficient of thermal conductivity then

$$q = -kA\frac{dT}{dx}$$



Law of natural decay

A rate of decay of a material is proportional to its amount present at that instant.

If m is amount of material at time t then

$$\frac{dm}{dt} = -km$$

Rectilinear Motion

Rectilinear motion (also called as linear motion) is **motion along a straight line.**

If x is displacement of a particle at time t then its

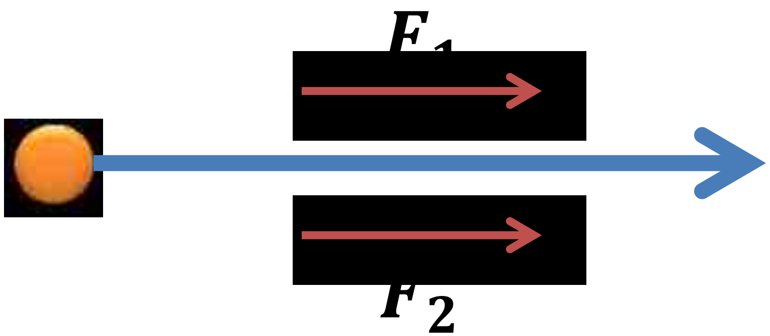
Velocity $v = \dfrac{dx}{dt}$

Acceleration $a = \dfrac{dv}{dt} = \dfrac{d^2x}{dt^2} = v\dfrac{dv}{dx}$

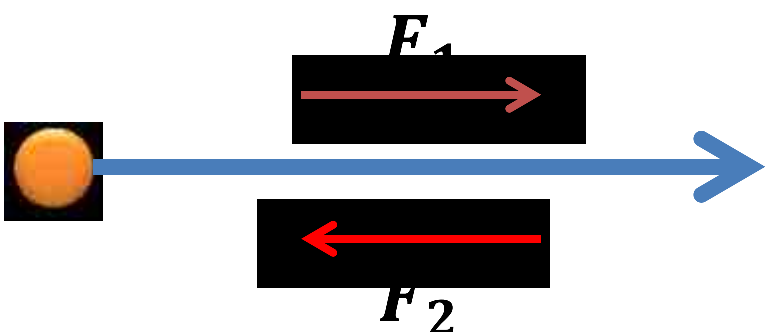
D'Alembert's principle

Algebraic sum of the forces acting on a body along a given direction is equal to the product of mass and acceleration in that direction.

Net force = Mass X Acceleration



Net force = $\boldsymbol{F_1 + F_2}$



Net force = $\boldsymbol{F_1 - F_2}$

S.H.M.

Equation of SHM is

$$Acceleration = \frac{d^2x}{dt^2} = -\omega^2 x$$

$$Period\ T = \frac{2\pi}{\omega}$$

For finding orthogonal trajectory of $f(x, y, c) = 0$ we replace $\frac{dy}{dx}$ by				**[01]**
a)	$-dx/dy$	b)	$-dy/dx$	
c)	$2dx/dy$	d)	dy/dx	
The orthogonal trajectory of $y = ax^2$ is				[02]
a)	$x^2 + y^2 = c^2$	b)	$x^2 + (y^2/2) = c^2$	
c)	$(x^2/2) + y^2 = c$	d)	None of these	
The orthogonal trajectory of parabola is				[02]
a)	Circle	b)	Hyperbola	
c)	Ellipse	d)	Straight line	

	The orthogonal trajectory of the family of circles with centre at $(0,0)$ is a family of				**[02]**
	a)	Circles	b)	Straight lines through $(0,0)$	
	c)	any straight line	d)	Parabola	
	The DE for the orthogonal trajectory of the family of curves $x^2 + 2y^2 = c^2$ is				[01]
	a)	$x + 2y\frac{dy}{dx} = 0$	b)	$2\frac{dx}{x} = \frac{dy}{y}$	
	c)	$xdx + ydy = 0$	d)	$\frac{dx}{x} = \frac{dy}{y}$	
	The DE of orthogonal trajectory of the family of curves $r^2 = a\sin 2\theta$ is				[01]
	a)	$\frac{dr}{r} = -\tan 2\theta\, d\theta$	b)	$\frac{dr}{r} = \tan 2\theta\, d\theta$	
	c)	$dr = \tan 2\theta\, d\theta$	d)	None of these	

	The DE of orthogonal trajectory of the family of curves $r^2 = a^2 \cos 2\theta$ is				**[01]**
	a)	$r\dfrac{d\theta}{dr} = \tan 2\theta$	b)	$rdr = \tan 2\theta\, d\theta$	
	c)	$rdr = \cot 2\theta\, d\theta$	d)	$rdr + \tan\theta\, d\theta = 0$	
	If the DE of orthogonal trajectory of a curve is $r\dfrac{d\theta}{dr} + \cot(\theta/2) = 0$ then its orthogonal trajectory is				[01]
	a)	$r = \cos\theta$	b)	$r = c(1 - \sin\theta)$	
	c)	$r = c(1 - \cos\theta)$	d)	$r = b(1 + \cos\theta)$	
	If temperature of surrounding medium is θ_0 and temperature of body at any time t is θ, then in a process of heating $d\theta/dt$ is				[01]
	a)	$\theta - \theta_0$	b)	$k(\theta - \theta_0);\ \ k > 0$	
	c)	$-k(\theta - \theta_0);\ \ k > 0$	d)	None of these	

	In certain data of newton's law of cooling, $-kt = \log\left(\frac{\theta - 40}{60}\right)$ and at $t = 4, \theta = 60^0$, then the value of k is				**[02]**
	a)	$\log(1/3)$	b)	$-\log(1/3)$	
	c)	$4\log(1/3)$	d)	$(1/4)\log 3$	
	If the temperature of water initially is 100^0C and $\theta_0 = 20^0C$, and water cools down to 60^0C in first 20 minutes with $k = \frac{1}{20}\log 2$, then during what time will it cool to 30^0C				[02]
	a)	60 min	b)	50 min	
	c)	1.5 hour	d)	40 min	
	If a body originally at 80^0C, with $\theta_0 = 40^0C$ and $k = \frac{1}{20}\log 2$, then the temperature of body after 40 min is				[02]
	a)	40^0C	b)	50^0C	
	c)	80^0C	d)	30^0C	

	If the body at 100^0C is placed in room whose temperature is 10^0C and cools to 60^0C in 5 minutes then the value of k is				**[02]**
	a)	$\log 2$	b)	$-\log 2$	
	c)	$(1/5)\log 2$ s	d)	$5\log 2$	
	The linear form of DE for R-L series circuit with emf E is				[01]
	a)	$L\frac{di}{dt}+Ri=E$	b)	$\frac{di}{dt}+\frac{R}{L}i=\frac{E}{L}$	
	c)	$L\frac{di}{dt}+Ri=0$	d)	none of these	
	The integrating factor for the DE of R-L series circuit with emf E is				[02]
	a)	$e^{\int Rdt}$	b)	e^{Rt+c}	
	c)	$e^{\int \frac{R}{L}dt}$	d)	$e^{\int idt}$	

	If $i = \frac{E}{R} + ke^{-\frac{Rt}{L}}$ then the maximum value of i is				[01]
	a)	R/L	b)	E/R	
	c)	−E/R	d)	2R/L	
	The linear form of DE for R-C series circuit with emf E is				[01]
	a)	$Ri + \frac{q}{c} = E(t)$	b)	$Ri + \frac{1}{C}\int i\,dt = E$	
	c)	$R\frac{di}{dt} + \frac{i}{C} = \frac{dE}{dt}$	d)	$\frac{di}{dt} + \frac{i}{RC} = \frac{1}{R}\frac{dE}{dt}$	
	The integrating factor for the DE of R-C series circuit with emf E is				[01]
	a)	$e^{\int RC dt}$	b)	$e^{\int \frac{1}{RC} dt}$	
	c)	$e^{\int \frac{1}{R} dt}$	d)	$e^{\int \frac{1}{C} dt}$	

	If $i = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$ then the 50% of maximum current is				**[01]**
	a)	E/R	b)	E/2R	
	c)	2E/R	d)	2R/E	
	Which one of the following is not correct?				[01]
	a)	$F = ma$	b)	$F = m\frac{dv}{dt}$	
	c)	$F = m\, v\frac{dv}{dx}$	d)	$F = m\, v\frac{dv}{dt}$	
	A motion of a body or particle along straight line is known as				[01]
	a)	rectilinear motion	b)	curvilinear motion	
	c)	Motion	d)	None of these	

If a body of mass m falling from rest is subjected to the force of gravity and an air resistance proportional to the square of velocity kv^2, then the equation of motion is				**[01]**
a)	$mv\dfrac{dv}{dx} = mg + kv^2$	b)	$ma = -mg + kv^2$	
c)	$ma = mg - kv^2$	d)	None of these	
If a body opposed by force per unit mass of value cx and resistance per unit mass of value kv^2 then the equation of motion is				[01]
a)	$a = cx - bv^2$	b)	$a = bv^2 - cx$	
c)	$v\dfrac{dv}{dx} = -cx - bv^2$	d)	$v\dfrac{dv}{dx} = cx + bv^2$	

	The quantity of heat in a body is proportional to its			[01]
a)	mass only	b)	temperature only	
c)	mass and temperature	d)	none of these	

	The motion of a particle moving along a straight line is $\frac{d^2x}{dt^2} + 16x = 0$, then its period is			[01]
a)	$2\pi/\sqrt{2}$	b)	$\pi/2$	
c)	2π	d)	π	

	The orthogonal trajectories of the series of hyperbolas $xy = c^2$ is			[02]
a)	$x^2 + y^2 = c^2$	b)	$x^2y^2 = c^2$	
c)	$y^2 - x^2 = c^2$	d)	None of these	

	The differential equation of orthogonal trajectories of family of straight lines $y = mx$ is				**[01]**
	a)	$x\,dx - y\,dy = 0$	b)	$ydx - xdy = 0$	
	c)	$x\,dx + ydy = 0$	d)	$ydx + xdy = 0$	
	Let the population of country be decreasing at the rate proportional to its population. If the population has decreased to 25% in 10 years, how long will it take to half.				[02]
	a)	20 years	b)	8.3 years	
	c)	15 years	d)	5 years	
	The orthogonal trajectories of the family of straight lines $y = mx$ is				[01]
	a)	$x^2 - y^2 = c^2$	b)	$x^2 = my^2$	
	c)	$y^2 = m^2x^2$	d)	$x^2 + y^2 = c^2$	

	The set of orthogonal trajectories to a family of curves whose DE is $\phi\left(x, y, \frac{dy}{dx}\right) = 0$ is obtained by DE				**[01]**
	a)	$\phi\left(x, y, x\frac{dy}{dx}\right) = 0$	b)	$\phi\left(x, y, \frac{-dx}{dy}\right) = 0$	
	c)	$\phi\left(x, y, \frac{dy}{dx}\right) = 0$	d)	$\phi\left(x, y, \frac{-dy}{dx}\right) = 0$	
	The orthogonal trajectories of the family of curves $r\cos\theta = a$ is				[02]
	a)	$r\sin\theta = c$	b)	$r\tan\theta = c$	
	c)	$\frac{r}{\sin\theta} = c$	d)	None of these	
	If 10 grams of some radioactive substance reduces to 8 gm in 60 years, in how many years will 2 gm of it will be left ?				[02]
	a)	120 yrs	b)	378 yrs	
	c)	220 yrs	d)	433 yrs	

	Voltage drop across inductance L is given by				**[01]**
	a)	Li	b)	$L\frac{di}{dt}$	
	c)	$\frac{dL}{dt}$	d)	None of these	
	A ball at temperature of 32^0C is kept in a room where the temperature is 10^0C. If the ball cools to 27^0C in hour then its temperature is given by				[02]
	a)	$T = 22\ e^{0.205\ t}$	b)	$T = 10\ e^{1.163t}$	
	c)	$T = 10 + 22e^{-0.258t}$	d)	$T = 32 - 10e^{-0.093t}$	

<u>Unit III</u>

Fourier Series, Reduction Formulae, Gamma Functions, Beta Functions

Multiple Choice Questions

Periodic functions

A function $f(x)$ is said to be periodic if it is defined for all real x and if there is some positive number T such that

$$f(x+T) = f(x) \qquad \forall\, x$$

The number T is then called period of $f(x)$.

$\sin x, \cos x$ are periodic functions of period 2π

$\tan x\,, \cot x$ are periodic functions of period π

Fourier Series

If $f(x)$ is a periodic function of period 2π, defined in the interval $c \leq x \leq c + 2\pi$ then

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

this representation of $f(x)$ is called Fourier Series and the coefficients a_0, a_n, b_n are called the Fourier coefficients.

Euler's Formulae

$$a_0 = \frac{1}{\pi} \int_{c}^{c+2\pi} f(x)dx$$

$$a_n = \frac{1}{\pi} \int_{c}^{c+2\pi} f(x) \cos nx \, dx$$

$$b_n = \frac{1}{\pi} \int_{c}^{c+2\pi} f(x) \sin nx \, dx$$

1	**If $f(x+nT)=f(x)$ where n is any integer then the fundamental period of $f(x)$ is**			
	a)	$2T$	b)	$T/2$
	c)	$\boxed{T}$	d)	3T

2	**If $f(x)$ is a periodic function with period T then $f(ax)$, $a \neq 0$ is periodic function with fundamental period**			
	a)	T	b)	$\boxed{T/a}$
	c)	aT	d)	π

3	**Fundamental period of $\cos 2x$ is**			
	a)	$\frac{\pi}{4}$	b)	$\frac{\pi}{2}$
	c)	$\boxed{\pi}$	d)	2π

4	**Fundamental period of $\tan 3x$ is**			
	a)	$\frac{\pi}{2}$	b)	$\boxed{\frac{\pi}{3}}$
	c)	π	d)	$\frac{\pi}{4}$

5	The value of constant terms in the Fourier series of $f(x) = e^{-x}$ in $0 \le x \le 2\pi,\ f(x+2\pi) = f(x)$ is				
	a)	$\frac{1}{\pi}(1-e^{-2\pi})$	b)	$\frac{1}{2\pi}(1-e^{-2\pi})$	
	c)	$2(1-e^{-2\pi})$	d)	$(1-e^{-2\pi})$	

6	Fourier coefficient a_0 in the Fourier series expansion of $f(x) = \left(\frac{\pi-x}{2}\right)^2; 0 \le x \le 2\pi$ and $f(x+2\pi) = f(x)$				
	a)	$\frac{\pi^2}{3}$	b)	$\frac{\pi^2}{6}$	
	c)	0	d)	$\pi/6$	

For function defined in the interval $-\pi \leq x \leq \pi$

- If $f(x)$ is even then

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx \, dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx = 0$$

For function defined in the interval $-\pi \leq x \leq \pi$

- If $f(x)$ is odd then

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx = \frac{2}{\pi}\int_{0}^{\pi} f(x)\sin nx\, dx$$

7	**Fourier series representation of periodic function** $f(x) = \begin{cases} 1+\frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1-\frac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$ $f(x) = \frac{8}{\pi^2}\left[\frac{1}{1^2}\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x + \cdots\right]$ **then value of** $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots =$			
a)	$\frac{\pi^2}{4}$	b)	$\boxed{\frac{\pi^2}{8}}$
c)	$\frac{\pi^2}{16}$	d)	$\frac{8}{\pi^2}$

31 $f(x) = x, -\pi \le x \le \pi$ and period is 2π. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$, Fourier coefficient b_1 is

a) $\boxed{2}$ b) -1

c) 0 d) $2/\pi$

If $f(x)$ is a periodic function of period $2L$, defined in the interval $c \leq x \leq c+2L$ then

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$$

$$a_0 = \frac{1}{L}\int_{c}^{c+2L} f(x)dx \qquad a_n = \frac{1}{L}\int_{c}^{c+2L} f(x)\cos\frac{n\pi x}{L}dx$$

$$b_n = \frac{1}{L}\int_{c}^{c+2L} f(x)\sin\frac{n\pi x}{L}dx$$

If $f(x)$ is a periodic function of period $2L$, defined in the interval $-\mathrm{L} \leq x \leq L$ and

if $f(x)$ is an even function then

$$a_0 = \frac{2}{L}\int_0^L f(x)dx \qquad a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx \qquad b_n = 0$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L}\right)$$

if $f(x)$ is an odd function then

$$a_0 = 0 \qquad a_n = 0 \qquad b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$$

$$f(x) = \sum_{n=1}^{\infty}\left(b_n \sin\frac{n\pi x}{L}\right)$$

Half range expansions

- Half range cosine series: If $f(x)$ is defined in the interval $0 \leq x \leq L$, then the half range cosine series of $f(x)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}$$

$$a_0 = \frac{2}{L} \int_0^L f(x) dx \qquad a_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi x}{L} dx$$

- Half range sine series: If $f(x)$ is defined in the interval $0 \leq x \leq L$, then the half range sine series of $f(x)$ is given by

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$$

$$b_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx$$

9	The Fourier constant a_n for $f(x) = 4 - x^2$ in the interval $0 < x < 2$ is			
	a)	$\boxed{4/\pi^2 n^2}$	b)	$2/n^2\pi^2$
	c)	$4/n^2\pi$	d)	$2/n\pi^2$

10	For half range sine series of $f(x) = x, 0 \leq x \leq 2$ and period is 4.Fourier series is represented by $\sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{2}$, then Fourier coefficient b_1 is			
	a)	4	b)	2
	c)	$\frac{2}{\pi}$	d)	$\boxed{\frac{4}{\pi}}$

		1st Harmonic		2nd Harmonic		3rd Harmonic	
x	y	$y\cos\frac{\pi x}{L}$	$y\sin\frac{\pi x}{L}$	$y\cos\frac{2\pi x}{L}$	$y\sin\frac{2\pi x}{L}$	$y\cos\frac{3\pi x}{L}$	$y\sin\frac{3\pi x}{L}$
x_0	y_0						
$\vdots$	$\vdots$						
x_{m-1}	y_{m-1}						
$\sum$							

$$a_0 = \frac{2}{m}\sum_{i=0}^{m-1} y_i \qquad a_n = \frac{2}{m}\sum_{i=0}^{m-1} y_i \cos\frac{n\pi x_i}{L} \qquad b_n = \frac{2}{m}\sum_{i=0}^{m-1} y_i \sin\frac{n\pi x_i}{L}$$

$$f(x) = \frac{a_0}{2} + a_1 \cos\frac{\pi x}{L} + b_1 \sin\frac{\pi x}{L} + a_2 \cos\frac{2\pi x}{L} + b_2 \sin\frac{2\pi x}{L}$$
$$+ a_3 \cos\frac{3\pi x}{L} + \mathrm{b}_3 \sin\frac{3\pi x}{L} + \cdots$$

1. The term $\left[a_1 \cos\left(\frac{\pi x}{L}\right) + b_1 \sin\left(\frac{\pi x}{L}\right)\right]$ is called as **‘Fundamental or First harmonic’.**

2. The term $\left[a_2 \cos\left(\frac{2\pi x}{L}\right) + b_2 \sin\left(\frac{2\pi x}{L}\right)\right]$ is called as ‘ **second harmonic’** and so on.

3. The amplitude of n$^{\text{th}}$ harmonic is $+\sqrt{a_n^2 + b_n^2}$.

4. Percentage of n$^{\text{th}}$ harmonic =

$$\frac{\text{amplitude of n}^{\text{th}}\text{ harmonic}}{\text{amplitude of I}^{\text{st}}\text{ harmonic}} \times 100$$

11	For the certain data if $a_0 = 1.5,\ a_1 = 0.373,\ b_1 = 1.004$ then the amplitude of 1st harmonic is			
	a)	1.07	b)	2.07
	c)	1.004	d)	1.377

12	The value of a_0 in harmonic analysis of y for the following tabulated data is						
x°	0	60	120	180	240	300	360
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0
	a)	1.45	b)	5.8			
	c)	2.9	d)	2.48			

13	The value of a_1 in Harmonic analysis of y for the following tabulated data is :						
x	0	1	2	3	4	5	6
y	4	8	15	7	6	2	4
$\cos\frac{\pi x}{3}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1
	a)	-4.16	b)	-8.32			
	c)	-3.57	d)	$\boxed{-10.98}$			

14	The value of $a_1,\ a_2$ in Fourier cosine series of y for the following tabulated data are			
x	0	$\pi/4$	$\pi/2$	$3\pi/4$
y	0	$\sqrt{2}$	2	$\sqrt{2}$
	a)	$-1/2, 1/2$	b)	$\boxed{-1/2, -1/2}$
	c)	$2, -2$	d)	$-2, 0$

Reduction Formulae

1. $\int_0^{\pi/2} \cos^n x\, dx = \int_0^{\pi/2} \sin^n x\, dx$

$$= \frac{[\,(n-1)\text{subtract } 2 \ldots\ldots\ldots\ldots\ldots\ldots 2 \text{ or } 1\,]}{[\,(n)\ \text{subtract } 2 \ldots\ldots\ldots\ldots\ldots\ldots 2 \text{ or } 1]} \times \left(\frac{\pi}{2}\right) \text{ if n is even.}$$

$$= \frac{[\,(n-1)\text{subtract } 2 \ldots\ldots\ldots\ldots\ldots\ldots 2 \text{ or } 1\,]}{[\,(n)\ \text{subtract } 2 \ldots\ldots\ldots\ldots\ldots\ldots 2 \text{ or } 1]} \times 1 \text{ if n is odd.}$$

2.(a) $\int_0^{\pi/2} \sin^m x \cos^n x\, dx$

$$= \left\{\frac{[\,(m-1)\text{ subtract } 2\ldots\ldots 2 \text{ or } 1\,].[\,(n-1)\text{ subtract } 2\ldots\ldots 2 \text{ or } 1\,]}{[\,(m+n)\text{ subtract } 2\ldots\ldots\ldots 2 \text{ or } 1\,]}\right\} \times \left(\frac{\pi}{2}\right)$$

If m and n both are even.

2.(b) $\int_0^{\pi/2} \sin^m x \cos^n x\, dx$

$$= \left\{\frac{[\,(m-1)\text{ subtract } 2\ldots\ldots 2 \text{ or } 1\,].[\,(n-1)\text{ subtract } 2\ldots\ldots 2 \text{ or } 1\,]}{[\,(m+n)\text{ subtract } 2\ldots\ldots\ldots 2 \text{ or } 1\,]}\right\} \times (1)$$

Otherwise .

3] $\int_0^{\pi/2} \sin^m x \cos x \, dx = \int_0^{\pi/2} \cos^m x \sin x \, dx = \frac{1}{m+1}$

Conversion Formulae :

1] $\int_0^{2\pi} \sin^m x \cos^n x \, dx = \begin{cases} = 4 \int_0^{\pi/2} \sin^m x \cos^n x \, dx \,, \boldsymbol{if\ m\,,\ n\ even}. \\ = 0 \,, \qquad\qquad Otherwise. \end{cases}$

2] $\int_0^{\pi} \sin^m x \cos^n x \, dx$

$$= \begin{cases} = 2 \int_0^{\pi/2} \sin^m x \cos^n x \, dx \,, \boldsymbol{if\ n\ even\,, for\ any\ m}. \\ = 0 \,, \qquad\qquad \boldsymbol{, if\ n\ odd\,, for\ any\ m}. \end{cases}$$

3] $\int_0^{2\pi} \sin^n x \, dx = \begin{cases} = 4 \int_0^{\pi/2} \sin^n x \, dx \,, \boldsymbol{if\ n\ is\ even}. \\ = 0 \qquad\qquad \boldsymbol{, if\ n\ is\ odd}. \end{cases}$

4] $\int_0^{2\pi} cos^n\, x\, dx = \begin{cases} = 4\int_0^{\pi/2} cos^n\, x\, dx \text{ , } \boldsymbol{if\ n\ is\ even.} \\ = 0 \text{ , } \boldsymbol{if\ n\ is\ odd.} \end{cases}$

5] $\int_0^{\pi} sin^n\, x\, dx = 2\int_0^{\pi/2} sin^n\, x\, dx \text{ , } \boldsymbol{for\ any\ n}$.

6] $\int_0^{\pi} cos^n\, x\, dx = \begin{cases} = 2\int_0^{\pi/2} cos^n\, x\, dx \text{ , } \boldsymbol{if\ n\ is\ even.} \\ = 0 \text{ , } \boldsymbol{if\ n\ is\ odd.} \end{cases}$

1	**The value of the integral** $\int_0^{\frac{\pi}{6}} \cos^6 3x \ dx$ **is**			
	a)	$5\pi/96$	b)	$7/48$
	c)	$5\pi/32$	d)	0

2	**The value of** $\int_{-\pi/2}^{\pi/2} \sin^4 x\, dx$ **is**			
	a)	$3\pi/16$	b)	$3\pi/8$
	c)	$3\pi/4$	d)	0

$$\int_0^{\pi/2} cos^n x\, dx = \int_0^{\pi/2} sin^n x\, dx \boxed{= \frac{n-1}{n} I_{n-2}}$$

3	**The value of the integral** $\int_0^{\pi/2} \sin^4 x \cos^3 x \ dx$ **is**			
	a)	$\pi/35$	b)	$\boxed{2/35}$
	c)	0	d)	$53/2$

4	**The value of** $\int_{-2\pi}^{2\pi} \sin^3 x \cos^2 x \, dx$ **is**			
	a)	$\boxed{0}$	b)	$\pi/4$
	c)	$\pi/16$	d)	$\pi/32$

5	The value of the integral $\int_0^{2\pi} \cos^5 x \; dx$ is			
	a)	$\boxed{0}$	b)	$5/16$
	c)	$5/32$	d)	$5\pi/32$

6	The value of the integral $\int_0^{\pi} \sin^5 x \; dx$ is			
	a)	$8\pi/15$	b)	$\pi/2$
	c)	$\boxed{16/15}$	d)	0

7	**If $I_n = \int_{\pi/4}^{\pi/2} \cot^n x\, dx$ and $I_n = \frac{1}{n-1} - I_{n-2}$, then the value of I_6 is**			
	a)	$\frac{13}{15}$	b)	$\frac{13}{15} + \frac{\pi}{4}$
	c)	$\frac{13}{15} - \frac{\pi}{4}$	d)	$\frac{13}{15} - \frac{\pi}{2}$

8	**If $I_n = \int_0^{\pi/4} \sin^{2n} x\ dx$ and $I_n = \left(1 - \frac{1}{2n}\right) I_{n-1} - \frac{1}{n2^{n+1}}$, then the value of $\int_0^{\pi/4} \sin^4 x\ dx$ is**			
	a)	$\frac{3\pi}{32} + \frac{1}{4}$	b)	$\frac{3\pi}{32} - \frac{1}{4}$
	c)	$\frac{\pi}{16} - \frac{1}{4}$	d)	$\frac{3\pi}{16} + \frac{1}{4}$

9	If $I_{m,n} = \int_0^{\pi/2} (\cos^{m} x)(\sin nx)dx$ and $I_{m,n} = \frac{1+m\, I_{m-1,n-1}}{m+n}$, then the value of $\int_0^{\pi/2} (\cos^2 x)(\sin 4x)dx$ is			
	a)	3	b)	2
	c)	$\boxed{1/3}$	d)	$2/3$

10	If $I_n = \int_0^{\pi/2} x \cdot \sin^n x \cdot dx$ and $I_n = \frac{n-1}{n} I_{n-2} + \frac{1}{n^2}$, then the value of $\int_0^{\pi/2} x \cdot \sin^4 x \cdot dx$			
	a)	$\boxed{\frac{3\pi^2}{64} + \frac{1}{4}}$	b)	$\frac{\pi^2}{64} + \frac{1}{4}$
	c)	$\frac{3\pi^2}{32} - \frac{1}{4}$	d)	$\frac{3\pi^2}{64} - \frac{1}{4}$

1. Gamma Function

Definition: The integral $\int_0^\infty e^{-x} x^{n-1} dx$ is called as Gamma function

and denoted by $\Gamma n = \int_0^\infty e^{-x} x^{n-1} dx$ (n > 0)

Properties :

$$\boxed{1.\ \Gamma 1 = 1}$$

$$2.\ \text{Reduction formula}: \Gamma n+1 = n\,\Gamma n$$

$$= n!\ ,\text{if } n \text{ is } +ve \text{ integer}$$

$$3.\ \Gamma 0 = \infty$$

$$4.\ \Gamma \frac{1}{2} = \sqrt{\pi}$$

$$5.\ \Gamma P\ \Gamma 1-P = \frac{\pi}{\sin p\pi}$$

11	The value of the integral $\int_0^\infty \frac{x^5}{5^x}\,dx$ by using substitution $5^x = e^t$ is			
	a)	$\boxed{120/(\log 5)^6}$	b)	$24/(\log 4)^5$
	c)	$120/(\log 5)^5$	d)	$24/(\log 4)^4$

12	The value of the integral $\int_0^1 \frac{dx}{\sqrt{x \log\left(\frac{1}{x}\right)}}$ by using the substitution $\log\left(\frac{1}{x}\right) = t$ is			
	a)	$\sqrt{\pi}/2$	b)	$\boxed{\sqrt{2\pi}}$
	c)	$\sqrt{\pi}$	d)	$2\sqrt{\pi}$

13	The formula for $\Gamma(\mathrm{n}+1)$ is			
	a)	$\int_0^\infty e^{-x} x^{n-1} dx$	b)	$\boxed{\int_0^\infty e^{-x} x^{n} dx}$
	c)	$2\int_0^\infty e^{-x} x^{n-1} dx$	d)	$\int_0^\infty e^{-x} x^{n-2} dx$

14	The value of the integral $\int_0^\infty e^{-4x} x^3\, dx$ is			
	a)	$4!$	b)	$3!$
	c)	$\frac{3!}{64}$	d)	$\boxed{\frac{3!}{256}}$

15	**The value of** $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)$ **is**			
	a)	$2\pi/\sqrt{3}$	b)	$\pi/\sqrt{3}$
	c)	2π	d)	$2/\sqrt{3}$

16	**The value of** $\int_0^1 (\log x)^n \, dx$ **is**			
	a)	$(-1)^n\Gamma(\mathrm{n}+1)$	b)	$(\log n)\Gamma n$
	c)	Γn	d)	$\Gamma(n+1)$

Beta Function.

$$Definition : \boxed{\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx} \; ; \; where \;\; m,n \; are \; +ve \; \text{integers}$$

Properties Of Beta Function.

1. $\beta\,(m\,,n\,) = \beta\,(n\;\,,m\,)$

2. $\beta(m,n) = \displaystyle\int_0^\infty \frac{y^{m-1}}{(1+y)^{m+n}}\,dy$

3. $\beta(m,n) = 2\displaystyle\int_0^{\pi/2} \sin^{2m-1}\theta\cos^{2n-1}\theta\;d\theta$

4. $$\int_0^{\pi/2} \sin^p \theta \cos^q \theta \, d\theta = \frac{1}{2}\beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$$

5. Relation Between Beta and Gamma Function.

$$\boxed{\beta(m,n) = \frac{\Gamma m \, \Gamma n}{\Gamma (m+n)}}$$

6. Legendre’s duplication formula :

$$\boxed{\Gamma m \, \Gamma (m + 1/2) = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma 2m}$$

17	**Value of** $B\left(\frac{3}{4},\frac{1}{4}\right)$ **is**			
	a)	2π	b)	$\boxed{\pi\sqrt{2}}$
	c)	$\pi/2$	d)	$\sqrt{2}$

18	**Value of** $\int_0^{\pi/2}\sqrt{\tan x}\,dx$ **is**			
	a)	$\boxed{\frac{1}{2}B\left(\frac{3}{4},\frac{1}{4}\right)}$	b)	$\frac{1}{2}B\left(\frac{3}{4},\frac{3}{4}\right)$
	c)	$B\left(\frac{3}{4},\frac{1}{4}\right)$	d)	$B\left(\frac{3}{4},\frac{3}{4}\right)$

19	If $B(n+1,1)=\frac{1}{4}$ and n is a positive integer then value of n is			
	a)	1	b)	2
	c)	$\boxed{3}$	d)	4

20	Value of $\int_0^{\pi/2}\sqrt{2\sin 2x}\,dx$ is			
	a)	$\frac{1}{2}B\left(\frac{3}{4},\frac{1}{4}\right)$	b)	$\frac{1}{2}B\left(\frac{3}{4},\frac{3}{4}\right)$
	c)	$\boxed{B\left(\frac{3}{4},\frac{1}{4}\right)}$	d)	$B\left(\frac{3}{4},\frac{3}{4}\right)$

21	**The value of** $\int_0^\infty \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}} dx$ **is**			
	a)	0	b)	$\frac{B(m,n)}{2}$
	c)	$\boxed{2B(m,n)}$	d)	1

22	**By Duplication formula, the value of** $\Gamma m \cdot \Gamma(m+\frac{1}{2})$ **is**			
	a)	$\frac{\sqrt{\pi}}{2^{m-1}}\Gamma(2m)$	b)	$\frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(m)$
	c)	$\frac{\sqrt{\pi}}{2^{m}}\Gamma(2m)$	d)	$\boxed{\frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)}$

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First Year Degree Course in Engineering – Semester II

Engineering Mathematics (M II)

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First Online Examination

First Year of Engineering

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2015-16

Savitribai Phule Pune University – FE – Sem. II

Engineering Mathematics (M II)

Chapter 01–Ordinary Differential Equations

1) The order of the differential equation is
   a) the order of the highest ordered differential coefficient appearing in the differential equation.
   b) the order of the lowest ordered differential coefficient appearing in the differential equation.
   c) the power of the highest ordered differential coefficient appearing in the differential equation.
   d) the degree of the highest ordered differential coefficient appearing in the differential equation.

2) The degree of the differential equation is
   a) the highest ordered differential coefficient appearing in the differential equation.
   b) the lowest power of the highest ordered differential coefficient appearing in the differential equation.
   c) the highest power of the highest ordered differential coefficient appearing in the differential equation.
   d) the coefficient power of the highest ordered differential coefficient appearing in the differential equation.

3) A solution of a differential equation is a relation between
   a) dependent variables
   b) independent variables
   c) dependent and independent variables not containing any differential coefficient
   d) none of the above

4) In the general solution, the number of arbitrary constants is equal to
   a)order of the differential equation
   b) degree of the differential equation
   c) sum of order and degree of diff. eqn.
   d) difference of order and degree of diff. eqn.

5) The general solution of n^{th} order ordinary differential equation must involve
   a) $n+1$ arbitrary constants
   b) $n-1$ arbitrary constants
   c) n arbitrary constants
   d) none of the above

6) The solution obtained by assigning particular values to arbitrary constants in general solution of differential equation is known as
   a) singular solution b) particular solution
   c) general solution d) none of above

7) The order of differential equation whose general solution is $y=(c_1+c_2x)e^x+x$, where $c_1,\ c_2$ are arbitrary constants, is
   a) 1 b)2 c) 3 d) 0

8) The order of differential equation whose general solution is $y=(c_1+c_2x+c_3x)e^x+\frac{x^2}{12}$, where $c_1,\ c_2,\ c_3$ are arbitrary constants, is
   a) 0 b) 1 c) 2 d)3

9) The order of differential equation whose general solution is $y=(c_1+c_2)e^x+\frac{x^4}{3}$, where $c_1,\ c_2$ are arbitrary constants, is
   a) 0 b)1 c)2 d) 3

10) The order of differential equation whose general solution is $y=cx+c^2$, where c is arbitrary constant, is
   a) 0 b)1 c)2 d) 3

11) The order of differential equation whose general solution is $y=Ax+\frac{B}{x}$, where $A,\ B$ are arbitrary constants, is
   a) 0 b) 1 c)2 d) 3

12) The order of differential equation whose general solution is $y = Ax + \frac{A^2}{x}$, where A, B are arbitrary constants, is
a) 0 b) 1 c) 2 d) 3

13) The order of differential equation whose general solution is $y = \log(x-a)+b$, where a, b are arbitrary constants, is
a)2 b) 1 c) 0 d) none

14) The order of differential equation whose general solution is $x = A\sin(kt+B)$, where A, B are arbitrary constants and k is fixed constant, is
a) 0 b) 1 c)2 d) 3

15) The order of differential equation whose general solution is $x = (A+Bt)e^t$, where A, B are arbitrary constants, is
a) 0 b)2 c) 1 d) 3

16) The order of differential equation whose general solution is $y + \sqrt{x^2+y^2} = cx + c^3$, where c is arbitrary constant, is
a) 0 b) 2 c) 3 d)1

17) The order of differential equation whose general solution is $y = 4(x-A)^2$, where A is arbitrary constant, is
a)1 b) 2 c) 3 d) none

18) The order of differential equation whose solution is $y = (c_1 + c_2 x)e^x + (c_3 + c_4 x)e^{2x}$, where c_1, c_2, c_3, c_4 are arbitrary constants, is
a) 1 b) 2 c) 3 d)4

19) The order of differential equation whose solution is $y = c_1 x + c_2 e^x + c_3 e^{2x} + c_4 e^{3x}$, where c_1, c_2, c_3, c_4 are arbitrary constants, is
a) 1 b)4 c) 2 d) 3

20) The order of differential equation whose solution is $y = (Ax^2 + Bx + C)e^x$, where A, B, C are arbitrary constants, is
a) 1 b) 2 c)3 d) 4

21) The order of differential equation whose general solution is $y = \sqrt{kx+c}$, where c is the only arbitrary constant, is
a)1 b) 2 c) 3 d) 0

22) The order of differential equation whose general solution is $y = c^2 + \frac{c}{x}$, where c is arbitrary constant, is
a) 0 b) 2 c) 3 d)1

23) The order of differential equation whose general solution is $y = A\cos(x+5)$, where A is arbitrary constant, is
a) 0 b)1 c) 2 d) 3

24) The order and the degree of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is
a)1, 1 b) 1, 2 c) 2, 1 d) 2, 2

25) The order and the degree of the differential equation $\frac{dy}{dx} + y\log x = \sin x$ is
a) 0, 1 b) 1, 0 c) 2, 1 d)1, 1

26) The order and the degree of the differential equation $\frac{dy}{dx} + 2y = \cos x$ is
a) 0, 1 b)1, 1 c) 1, 2 d) 2, 1

27) The order and the degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 5y = \sin 7x$ is
a) 0, 1 b) 1, 1 c) 1, 2 d) 2, 1

28) The order and the degree of the differential equation $1 + \frac{dy}{dx} = \left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}}$ is
a) order 2, degree 1 b) order 1, degree 2
c)order 2, degree 3 d) order 2, degree $\frac{3}{2}$

29) The order and the degree of the differential equation $\sqrt{1 + \frac{dy}{dx}} = \frac{d^2y}{dx^2}$ is
a) order 2, degree 2 b) order 2, degree 1
c) order 1, degree 2 d) order 1, degree 1

30) The order and the degree of the differential equation $\dfrac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}=k$ is

a) order 2, degree 1 b) order 2, degree 2
c) order 2, degree 3 d) order 2, degree $\dfrac{3}{2}$

31) The order and the degree of the differential equation $\sqrt{1+\dfrac{dy}{dx}}=\dfrac{d^2y}{dx^2}$ is

a)order 2, degree 2 b) order 2, degree 1
c) order 1, degree 2 d) order 1, degree 1

32) The order and the degree of the differential equation $\dfrac{d^2y}{dx^2}=\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}$ is

a) order 2, degree 1 b) order 2, degree 2
c) order 1, degree 2 d) order 1, degree 1

33) The order and the degree of the differential equation $x=\dfrac{1}{\sqrt{1+\dfrac{dy}{dx}+\dfrac{d^2y}{dx^2}}}$ is

a) order 2, degree 2 b)order 2, degree 1
c) order 1, degree 2 d) order 1, degree $-\dfrac{1}{2}$

34) The order and the degree of the differential equation $1+\dfrac{dy}{dx}=\dfrac{y}{\dfrac{dy}{dx}}$ is

a) order 1, degree 1 b) order 2, degree 1
c)order 1, degree 2 d) order 2, degree 2

35) The order and the degree of the differential equation $y+\dfrac{d^2y}{dx^2}+\dfrac{x}{\dfrac{dy}{dx}}=1$ is

a) order 1, degree 1 b) order 2, degree 1
c) order 1, degree 2 d) order 2, degree 2

36) The order and the degree of the differential equation $(2x-3y+2)dy+(x-2y+7)dx=0$ is

a)1, 1 b) 1, 2 c) 2, 1 d) none

37) By eliminating the arbitrary constant m, the differential equation for the general solution $y=mx$ is given by

a) $\dfrac{dy}{dx}=\dfrac{y}{x}$ b) $\dfrac{dy}{dx}-xy=0$
c) $\dfrac{dy}{dx}+\dfrac{y}{x}=0$ d) $\dfrac{dy}{dx}-y=0$

38) The differential equation satisfied by the general solution $y+x^3=Ax$ with A is arbitrary constant, is given by

a) $y\dfrac{dy}{dx}+2x-y^3=0$ b) $x\dfrac{dy}{dx}+2x^3-y=0$
c) $\dfrac{dy}{dx}+2x^2-y=0$ d) $x^3\dfrac{dy}{dx}+2(x-y)=0$

39) $y=5+\sqrt{cx}$, where c is the arbitrary constant, is the general solution of

a) $y\dfrac{dy}{dx}=5+2x$ b) $y=2x\dfrac{dy}{dx}$
c) $y=5+2x\dfrac{dy}{dx}$ d) $y=5+2x\sqrt{\dfrac{dy}{dx}}$

40) By eliminating the arbitrary constant c, the differential equation of $y=cx-c^2$ is

a) $\dfrac{dy}{dx}-x\left(\dfrac{dy}{dx}\right)^2+y=0$ b) $\left(\dfrac{dy}{dx}\right)^2-x\dfrac{dy}{dx}+y=0$
c) $\left(\dfrac{dy}{dx}\right)^2+x\dfrac{dy}{dx}-y=0$ d) $\left(\dfrac{dy}{dx}\right)^2-xy=0$

41) The differential equation whose primitive is $y=c^2+\dfrac{c}{x}$, is given by

a) $x^4\left(\dfrac{dy}{dx}\right)^2-xy=0$ b) $\left(\dfrac{dy}{dx}\right)^2+\dfrac{dy}{dx}+y=0$
c) $\left(\dfrac{dy}{dx}\right)^2-x^4\dfrac{dy}{dx}-y=0$ d) $x^4\left(\dfrac{dy}{dx}\right)^2-x\dfrac{dy}{dx}-y=0$

42) By eliminating the arbitrary constant c present in the function $x=cy-y^2$, the differential equation is given by

a) $\left(\dfrac{x+y^2}{y}\right)\dfrac{dy}{dx}-2y\dfrac{dy}{dx}-1=0$

b) $\left(\dfrac{x+y^2}{y}\right)\left(\dfrac{dy}{dx}\right)^2-2y\dfrac{dy}{dx}-1=0$

c) $x\frac{dy}{dx}-2\left(\frac{x+y^2}{y}\right)\frac{dy}{dx}-1=0$

d) $y^2\frac{dy}{dx}-2xy\frac{dy}{dx}+1=0$

43) The differential equation whose solution is $y^2=4ax$ is given by

a) $\left(\frac{dy}{dx}\right)^2-2xy=0$ b) $\frac{dy}{dx}-xy^2=0$

c) $2xy\frac{dy}{dx}-y^2=0$ d) $2xy\frac{dy}{dx}+y^2=0$

44) The differential equation of family of curves $x^2+y^2+xy+x+y=c$ is

a) $\frac{dy}{dx}=-\frac{2x+y+1}{x+2y+1}$ b) $y_2+4y=0$

c) $\frac{dy}{dx}=\frac{2x-y}{x+2y+1}$ d) $x^2y_2-xy_1+y=0$

45) The differential equation whose generalized solution is $xy+y^2-x^2-x-3y=c$, is

a) $\frac{dy}{dx}=-\frac{2x-y+1}{x-2y+3}$ b) $\frac{dy}{dx}=\frac{x-2y-1}{x-2y+3}$

c) $\frac{dy}{dx}=\frac{2x+y+1}{x+2y+3}$ d) $\frac{dy}{dx}=\frac{2x-y+1}{x+2y-3}$

46) The differential equation satisfied by family of circles $x^2+y^2=2Ax$ is given by

a) $\frac{dy}{dx}+x^2+y^2=0$ b) $\frac{dy}{dx}+\frac{y^2-x^2}{xy}=0$

c) $\frac{dy}{dx}+\frac{x^2-y^2}{2xy}=0$ d) $\frac{dy}{dx}-\frac{x^2-y^2}{2xy}=0$

47) The differential equation whose general solution is $x^3+y^3=3Ax$, where A is arbitrary constant, is

a) $y_1=\frac{x^3+y^3-3x^2}{3xy^2}$ b) $x^2y_1+y=3y_1$

c) $xy_1+y^2+x=0$ d) none of these

48) $y^2=x^2-1+Ax$, where A is arbitrary constant, is the general solution of the equation

a) $\frac{dy}{dx}=\frac{x^2+y^2}{2xy}$ b) $y\frac{dy}{dx}+x^2+y^2=0$

c) $2xy\frac{dy}{dx}=x^2+y^2+1$ d) $2xy\frac{dy}{dx}-\left(x^2+y^2\right)=0$

49) The differential equation of $y=4\left(x-A\right)^2$, where A is arbitrary constant, is

a) $\frac{dy}{dx}-16y^2=0$ b) $\left(\frac{dy}{dx}\right)^2-16y=0$

c) $\left(\frac{dy}{dx}\right)^2+4y=0$ d) $\left(\frac{dy}{dx}\right)^2+16y=0$

50) $\left(1+x^2\right)=A\left(1+y^2\right)$ is a general solution of the differential equation

a) $\frac{dy}{dx}+\frac{1+x^2}{1-y^2}=0$ b) $\frac{x}{y}\frac{dy}{dx}+\left(\frac{1+x^2}{1-y^2}\right)=0$

c) $\left(\frac{1+x^2}{1-y^2}\right)\frac{dy}{dx}+\frac{x}{y}=0$ d) $\frac{dy}{dx}+\frac{x}{y}\left(\frac{1+x^2}{1-y^2}\right)=0$

51) The differential equation representing the family of loops $y^2=c\left(4+e^{2x}\right)$ is

a) $\left(4+e^{2x}\right)\frac{dy}{dx}+4ye^{2x}=0$ b) $\left(4+e^{2x}\right)\frac{dy}{dx}+y=0$

c) $\frac{dy}{dx}-ye^{2x}=0$ d) $\left(4+e^{2x}\right)\frac{dy}{dx}-ye^{2x}=0$

52) The differential equation whose general solution is $y=\sqrt{3x+c}$, is given by

a) $\frac{dy}{dx}-3y=0$ b) $2y\frac{dy}{dx}+3=0$

c) $2y\frac{dy}{dx}-3=0$ d) $2\frac{dy}{dx}-3y=0$

53) By eliminating the arbitrary constant A from $y=A\cos\left(x+3\right)$ the differential equation is

a) $\frac{dy}{dx}+y=0$ b) $\frac{dy}{dx}+y\cot\left(x+3\right)=0$

c) $\tan\left(x+3\right)\frac{dy}{dx}+y=0$ d) $\cot\left(x+3\right)\frac{dy}{dx}+y=0$

54) By eliminating the arbitrary constant c, the differential equation of $\cos\left(y-x\right)=ce^{-x}$ is

a) $x^2y_1-xy=4y_1$ b) $\tan\left(y-x\right)\left(\frac{dy}{dx}-1\right)-1=0$

c) $xy_1-y+x\sin\left(\frac{y}{x}\right)=0$ d) none of these

55) The differential equation whose generalized solution is $\sin(y-x) = ce^{-\frac{x^2}{2}}$, is given by

a) $\tan(y-x)\left(\frac{dy}{dx}-1\right)+x=0$

b) $\cot(y-x)\left(\frac{dy}{dx}+1\right)+y=0$

c) $\left(\frac{dy}{dx}-1\right)+\frac{x}{\cot(y-x)}=0$

d) $\cot(y-x)\left(\frac{dy}{dx}-1\right)+x=0$

56) The differential equation of the family of curves $y = Ae^{-x^2}$ is given by

a) $y\frac{dy}{dx}-2x^2=0$ b) $\frac{dy}{dx}+2xy=0$

c) $y\frac{dy}{dx}+2\log x=0$ d) $\frac{dy}{dx}-x^2y=0$

57) The differential equation whose general solution is $y = Ae^{\frac{x}{y}}$, is given by

a) $(x+y)y_1-y=0$ b) $(x+y)^2 y_1+y=0$

c) $(x-y)y_1+y=0$ d) $xy_1-\frac{y}{x}=0$

58) By eliminating the arbitrary constant c from the function $y = 5ce^{\frac{x}{y}}$, the differential equation is

a) $(x+y)\frac{dy}{dx}-y=0$ b) $\frac{dy}{dx}-\frac{y}{x+y}=0$

c) $\left(\frac{x+y}{x}\right)\frac{dy}{dx}-\frac{y}{x}=0$ d) $\frac{dy}{dx}-\frac{y-x}{x+y}=0$

59) The differential equation for the function $\sin\left(\frac{y}{x}\right)=Ax$ is obtained by eliminating A and is given by

a) $\frac{dy}{dx}+\frac{y}{x}=x\tan\left(\frac{y}{x}\right)$ b) $\frac{dy}{dx}+xy=\tan\left(\frac{y}{x}\right)$

c) $x\frac{dy}{dx}-y=x\cot\left(\frac{y}{x}\right)$ d) $x\frac{dy}{dx}-y=x\tan\left(\frac{y}{x}\right)$

60) The differential equation of $\cos\left(\frac{y}{x}\right)=cx$ is

a) $xy_1-y+x\cot\left(\frac{y}{x}\right)=0$

b) $xy_1-y+x\sin\left(\frac{y}{x}\right)=0$

c) $x^2y_1-y+x=0$

d) $x^2y_1-y+x\sin\left(\frac{y}{x}\right)=0$

61) The differential equation for the function $xy=c^2$, where c is arbitrary constant, is

a) $x\frac{dy}{dx}-y=0$ b) $\frac{dy}{dx}+xy=0$

c) $x\frac{dy}{dx}+y=0$ d) $x\left(\frac{dy}{dx}\right)^2+y=0$

62) The differential equation satisfying the general solution $xy=ce^x$ is

a) $x^2y_1-xy+e^x=0$ b) $xy_1+y=e^x$

c) $xy_1+y(1+x)=0$ d) $xy_1+y(1-x)=0$

63) The differential equation whose general solution is $y^2=2c\left(x+\sqrt{c}\right)$, where c is arbitrary constant, is

a) $2\frac{dy}{dx}\left(x+\sqrt{y\frac{dy}{dx}}\right)-y=0$ b) $x+\sqrt{y\frac{dy}{dx}}-y=0$

c) $\frac{dy}{dx}\left(x+\sqrt{y\frac{dy}{dx}}\right)+y=0$ d) $2x\frac{dy}{dx}+y\left(\frac{dy}{dx}\right)^2=0$

64) The differential equation satisfying the function $y=Ax+Bx^2$ is given by

a) $x^2y_2-4xy_1+y=0$ b) $y_2^2+2xy_1+2y=0$

c) $x^2y_2-2xy_1+2y=0$ d) $x^2y_2+xy_1+y=0$

65) By eliminating the arbitrary constants c_1, c_2 from the function $y=\sqrt{4x^2+c_1x+c_2}$ we get the differential equation

a) $y_2+xy_1=0$ b) $yy_2+y_1^2=4$

c) $x^2y_1y_2-y^2=0$ d) $x^2y_2+xy_1+4y=0$

66) $\frac{x^2}{4}-\frac{y^2}{a}=1$ is a general solution of

a) $xy_1-4y=xy$ b) $x^2y_1-4xy_1+16y=0$

c) $x^2y_1-4y_1-xy=0$ d) none of these

67) The differential equation representing the family of ellipse $\frac{x^2}{a^2}+\frac{y^2}{9}=1$, is given by

a) $y\frac{dy}{dx}-x^2y+9=0$ b) $xy\frac{dy}{dx}-y^2+9=0$

c) $xy\frac{dy}{dx}-y^2=0$ d) $xy\frac{dy}{dx}+y^2-9=0$

68) The differential equation whose primitive is $y^2=4A(x-B)$, where A and B are arbitrary constants, is

a) $x^2y_1y_2-y^2=0$ b) $x^2y_2+xy_1+4y=0$

c) $y_2+xy_1=0$ d) $yy_2+y_1{}^2=0$

69) On the elimination of the arbitrary constants A and B as well from $y^2=5A(x-3B)$, the differential equation formed is

a) $\frac{d^2y}{dx^2}+y=0$ b) $y^2\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$

c) $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$ d) $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2-y=0$

70) The differential equation with general solution $x=A\cos(B-5t)$ is given by

a) $\frac{d^2x}{dt^2}+5\frac{dx}{dt}-25t=0$ b) $\frac{d^2x}{dt^2}-\frac{dx}{dt}-xt=0$

c) $\frac{d^2x}{dt^2}-25x=0$ d) $\frac{d^2y}{dx^2}-25y=0$

71) The differential equation whose general solution is $y=\log(Ax+B)$ is

a) $y_2+y_1{}^2=0$ b) $x^2y_2+y_1{}^2=0$

c) $y_2+xy_1{}^2+y=0$ d) $xy_2+y_1{}^2-y=0$

72) $y=A\sin x+B\cos x$ is the solution satisfying the differential equation

a) $\frac{d^2y}{dx^2}+\frac{y}{x}=0$ b) $y^2\frac{d^2y}{dx^2}+xy+x=0$

c) $\frac{d^2y}{dx^2}+xy=0$ d) $\frac{d^2y}{dx^2}+y=0$

73) The differential equation whose general solution is $y=A\sin 3x+B\cos 3x$ where A, B are arbitrary constants, is

a) $x^2y_2-xy-9y_1=0$ b) $xy_2-9y_1+y=0$

c) $y_2-9y=0$ d) $y_2+9y=0$

74) The differential equation whose solution is $y=A\cos\frac{4x}{3}+B\sin\frac{4x}{3}$, where A and B are arbitrary constants, is given by

a) $\frac{d^2y}{dx^2}-\frac{dy}{dx}-\frac{4}{3}y=0$ b) $\frac{d^2y}{dx^2}+\frac{16}{9}y=0$

c) $9\frac{d^2y}{dx^2}-16y=0$ d) $\frac{d^2y}{dx^2}+\frac{dy}{dx}+\frac{16}{9}y=0$

75) The differential equation whose primitive is $y=A\cos\log x+B\sin\log x$, where A and B are arbitrary constants, is given by

a) $x^2y_2+y_1+xy=0$ b) $x^2y_2+xy_1+y=0$

c) $x^2y_2+y_1+y=0$ d) $y_2-x^2y_1-xy=0$

76) The differential equation whose general solution is $y=Ae^{-x}+B$, where A and B are arbitrary constants, is

a) $y=x^2y_2+y_1$ b) $x^2y_2+xy_1+y=0$

c) $y_2+y_1=0$ d) $xy_2{}^2+y_1=0$

77) $y=Ae^{-x}+Be^{-x}$, where A and B both are arbitrary constants, is the solution for the differential equation

a) $x\frac{d^2y}{dx^2}+y=0$ b) $\frac{d^2y}{dx^2}-\frac{dy}{dx}-y=0$

c) $\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=0$ d) $\frac{d^2y}{dx^2}-y=0$

78) By eliminating the arbitrary constants A and B both from the function $xy=Ae^{x}+Be^{-x}$, we get the differential equation

a) $\frac{x}{y}\frac{d^2y}{dx^2}+2\frac{dy}{dx}-\frac{x}{y}=0$ b) $x\frac{d^2y}{dx^2}+2\frac{dy}{dx}-xy=0$

c) $y\frac{d^2y}{dx^2}+2\frac{dy}{dx}-xy=0$ d) $\frac{d^2y}{dx^2}+\frac{dy}{dx}+xy=0$

79) The differential equation, whose solution is given by $y = Ae^{-3x} + Be^{3x}$, is
a) $xy_2^2 + y_1 - xy = 0$ b) $x^2 y_2 + y_1 + xy = 0$
c) $x^2 y_2 - xy_1 + y = 0$ d) $y_2 - 4y = 0$

80) $e^{-t} y = A + Bt$ is a general solution of the differential equation
a) $y_2 - 2y_1 + y = 0$ b) $y_2 + y_1 t + yt^2 = 0$
c) $xy_2 + y_1 + y = 0$ d) $4y_2 + 2y_1 + y = 0$

81) The differential equation having generalized solution $e^{-t} x = At - B$ is given by
a) $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = 0$ b) $x\frac{d^2x}{dt^2} + \frac{dx}{dt} + xt = 0$
c) $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + t = 0$ d) $x^2\frac{d^2x}{dt^2} - 2xt + x = 0$

82) The general form of the differential equation of I order and I degree can be expressed as
a) $\frac{dy}{dx} = c$ b) $M(x, y)dx + N(x, y)dy = 0$
c) $\frac{dy}{dx} + y = du$ d) $M(x, y)dx + N(x, y)dy = du$

83) The differential equation of the form $f_1(x)dx + f_2(y)dy = 0$ is known as
a) Linear form b) Non homogeneous form
c) exact form d)variable separable form

84) The differential equation in the form $\frac{dy}{dx} = x^n f\left(\frac{y}{x}\right)$ is known as
a) Linear form b) homogeneous form
c) exact form d) variable separable form

85) The differential equation in the form $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$, where f and g both are homogeneous functions of x and y of the same degree, is known as
a) Linear form b) homogeneous form
c) exact form d) variable separable form

86) The homogenous differential equation of the form $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$ is solved by substitution
a) no substitution, direct solution b) $x^n = v$
c) $xy = v$ d) $\frac{y}{x} = v$

87) The differential equation of the form $M(x, y)dx + N(x, y)dy = 0$ is exact, if
a) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ b) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$
c) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ d) $\frac{\partial M}{\partial y} \times \frac{\partial N}{\partial x} = 1$

88) The differential equation $\frac{dy}{dx} = e^{2x+y} + 3x^4 e^y$ is of the form
a) Linear form b) Non homogeneous form
c) exact form d) variable separable form

89) The form of the differential equation $(y^3 - 3x^2 y)dx + (x^2 y + 3x^3)dy = 0$ is
a) Linear form b) homogeneous form
c) exact form d) variable separable form

90) The differential equation is of the form $(x + y)dx + (x - y + 1)dy = 0$
a) Linear form b) non homogeneous form
c) exact form d) variable separable form

91) The differential equation $xy - \frac{dy}{dx} = y^3 e^{-x^2}$ is of the form
a) Linear form b) non homogeneous form
c) exact form d) variable separable form

92) The substitution which can be used to solve the equation $(x + y + 7)dx + (3x + 3y - 7)dy = 0$ is
a) $x + y = v$ b) $x - y = v$
c) $xy = v$ d) $\frac{y}{x} = v$

93) The general solution of the differential equation $\frac{3e^x}{1 - e^x}dx + \frac{\sec^2 y}{\tan y}dy = 0$ is
a) $\tan y = c(1 - e^x)^3$ b) $(1 - e^x)^3 \tan y = c$
c) $(1 - e^{-x})^3 \cot y = c$ d) $\cot y = c(1 - e^x)^3$

94) The general solution of the differential equation $\frac{dy}{dx}+y=0$ is

a) $y=ce^{-x}$ b) $y=Ae^{-x}+B$
c) $y=ce^{x}$ d) $x=ce^{-y}$

95) The general solution of the differential equation $\frac{dx}{dy}+x=0$ is

a) $y=ce^{-x}$ b) $y=Ae^{-x}+B$
c) $y=ce^{x}$ d) $x=ce^{-y}$

96) The general solution of the differential equation $\frac{dy}{dx}+x=0$ is

a) $y=ce^{-x}$ b) $y^2+2x=c$
c) $x^2+2y=c$ d) $x=ce^{-y}$

97) The general solution of the differential equation $ydx+xdy=0$ is

a) $x^2+y^2=c$ b) $xy=c$ c) $\frac{y}{x}=c$ d) $\frac{x}{y}=c$

98) The general solution of the differential equation $\frac{dy}{dx}+\tan x=0$ is

a) $y=\log\sin x+c$ b) $y-\log\sec x=c$
c) $y=\log\sec x+c$ d) $y=\log\cos x+c$

99) The general solution of the differential equation $\frac{dy}{dx}+xy=0$ is

a) $\log x+\log y=c$ b) $\frac{x^2}{2}+\log y=c$
c) $x^2+\log y=c$ d) $x^2+y^2=c$

100) The general solution of the differential equation $\frac{dy}{dx}+\frac{1+x}{1+y}=0$ is

a) $x^2+y^2+2x+2y=c$ b) $(x+y)^2+2(x+y)=c$
c) $x^2+y^2+x+y=c$ d) $(1+x)=c(1+y)$

101) The general solution of the differential equation $\frac{dy}{dx}=\frac{1+y}{1+x}$ is

a) $(1+x)=c(1+y)^2$ b) $(1+y)=c(1+x)$
c) $(1+x)=c(1+y)$ d) $x=cy$

102) The general solution of the differential equation $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$ is

a) $\log\left(\frac{1+x^2}{1+y^2}\right)$ b) $\log(1+x^2)+\log(1+y^2)=c$
c) $\tan^{-1}x+\tan^{-1}y=c$ d) $\tan^{-1}x-\tan^{-1}y=c$

103) The general solution of the differential equation $\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$ is

a) $\frac{1}{2}\log\left(\frac{1-y^2}{1-x^2}\right)=c$ b) $\sec^{-1}x+\sec^{-1}y=c$
c) $\tan^{-1}x+\tan^{-1}y=c$ d) $\sin^{-1}x+\sin^{-1}y=c$

104) The general solution of the differential equation $x(1+y^2)dx+y(1+x^2)dy=0$ is

a) $(1+y^2)(1+x^2)=c$ b) $\log\left(\frac{1+y^2}{1+x^2}\right)=c$
c) $(1+y^2)=c(1+x^2)$ d) $\tan^{-1}x+\tan^{-1}y=c$

105) The general solution of the differential equation $\frac{dy}{dx}=(1+x)(1+y^2)$ is

a) $\log(1+y^2)=x+\frac{x^2}{2}+c$ b) $\tan^{-1}y=x+\frac{x^2}{2}+c$
c) $\log(1+x)+\tan^{-1}y=c$ d) $\tan^{-1}y+x+x^2=c$

106) The general solution of the differential equation $(e^x+1)ydy=(y+1)e^xdx$ is

a) $y+\log(y+1)+\log(e^x+1)=c$
b) $x+\log(y+1)=\log(e^x+1)+c$
c) $y-\log(y+1)=\log(e^x+1)+c$
d) $\frac{y^2}{2}+\log(y+1)=\log(e^x+1)+c$

107) The general solution of the differential equation $\frac{dy}{dx}=e^{x+y}+e^{y-x}$ is

a) $e^{-x}-e^{x}-e^{-y}=c$ b) $e^{x}-e^{2x}-e^{-y}=c$
c) $e^{-x}+e^{x}+e^{-y}=c$ d) $e^{x}-e^{-x}-e^{y}=c$

108) The general solution of the differential equation $\frac{dy}{dx}=e^{x-y}+3x^2e^{-y}$ is

a) $\frac{e^x+x^3}{e^y}=c$ b) $e^{x-y}=e^y+x^3+c$

c) $e^y=e^x+x^3+c$ d) $e^y+e^x+x^3=c$

109) The general solution of the differential equation $y(1+\log x)\frac{dx}{dy}-x\log x=0$ is

a) $\frac{x}{\log x}=yc$ b) $\frac{x}{y}\log x=y+c$

c) $x(\log x+1)=yc$ d) $x\log x=yc$

110) The general solution of the differential equation $\sec^2 x\tan y dx+\sec^2 y\tan x dy=0$ is

a) $\tan x\tan y=c$ b) $\tan x=c\tan y$

c) $\tan x+\tan y=c$ d) $\tan y=c\tan x$

111) The general solution of the differential equation $y\sec^2 x+(y-5)\tan x\frac{dy}{dx}=0$ is

a) $y^5-y+\tan x=c$ b) $y+5\log y+\log\sec x=c$

c) $y+5\log\frac{\tan x}{y}=c$ d) $y-5\log y+\log\tan x=c$

112) The general solution of the differential equation $e^x\cos y+(1+e^x)\sin y\frac{dy}{dx}=0$ is

a) $(1+e^x)\tan y=c$ b) $(1+e^x)\sec y=c$

c) $(1+e^x)\cos y=c$ d) $\sec y=c(1+e^x)$

113) The general solution of the differential equation $e^y\cos x dx+(e^y+1)\sin x dy=0$ is

a) $\sec x(e^y+1)=c$ b) $\sin x=c(e^y+1)$

c) $\sin y(1+e^x)=c$ d) $\sin x(e^y+1)=c$

114) The general solution of the differential equation $(4+e^{2x})\frac{dy}{dx}=ye^{2x}$ is

a) $\frac{y^2}{2}=A+(4+e^{2x})$ b) $y^2(4+e^{2x})=A$

c) $y^2=A(4+e^{2x})$ d) $x^2=A(4+e^{2x})$

115) The general solution of the differential equation $y-x\frac{dy}{dx}=2\left(y+\frac{dy}{dx}\right)$ is

a) $(x+2)y=c$ b) $x+2y=c$

c) $y=c(x+2)$ d) $(x+2)^2y=c$

116) The general solution of the differential equation $(x+1)\frac{dy}{dx}+1=2e^{-y}$ is

a) $(x+1)(2+e^{-y})=c$ b) $(2-e^y)=c(x+1)$

c) $(x+1)(2-e^y)=c$ d) $(x+1)=c(2-e^y)$

117) The general solution of the differential equation $x^3\left(x\frac{dy}{dx}+y\right)-\sec(xy)=0$ is

a) $\sin(xy)=2cx^2$ b) $\sin(xy)-\frac{1}{2x^2}=c$

c) $\sec(xy)+\frac{1}{2x^2}=c$ d) $\sin(xy)+\frac{1}{2x^2}=c$

118) The general solution of the differential equation $(y-ay^2)dx=(a+x)dy$ is

a) $\log(a+x)+\frac{1}{2}\log(1-ay)-\frac{1}{3}\log y=c$

b) $\log(a+x)-\frac{1}{a}\log(1-ay)-\log y=c$

c) $\log(a+x)+\log(1-ay)-\log y=c$

d) $\log(a+x)+\frac{\log(1-ay)}{-a}+\log y=c$

119) The necessary and sufficient condition for the equation $M(x,y)dx+N(x,y)dy=0$ to be exact is

a) $\frac{\partial M}{\partial x}=\frac{\partial N}{\partial y}$; $My+Nx=0$

b) $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$; $My+Nx\neq 0$

c) $\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}$; $My+Nx\neq 0$

d) $\frac{\partial M}{\partial y}\times\frac{\partial N}{\partial x}=1$; $My+Nx\neq 0$

120) If the differential equation $Mdx+Ndy=0$ is a homogeneous but not exact, its integrating factor is

a) $\frac{1}{Mx-Ny}$; $My-Nx\neq 0$
b) $\frac{1}{Mx+Ny}$; $Mx+Ny\neq 0$
c) $\frac{1}{My-Nx}$; $My-Nx\neq 0$
d) $\frac{1}{My+Nx}$; $My+Nx\neq 0$

121) If the differential equation $Mdx+Ndy=0$ is not exact but can be expressed in the form $yf_1(xy)dx+xf_2(xy)dy=0$, its integrating factor is
a) $\frac{1}{Mx+Ny}$; $Mx+Ny\neq 0$
b) $\frac{1}{My-Nx}$; $My-Nx\neq 0$
c) $\frac{1}{My+Nx}$; $My+Nx\neq 0$
d) $\frac{1}{Mx+Ny}$; $Mx+Ny=0$

122) If the differential equation $Mdx+Ndy=0$ is not exact and $\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}=f(x)$, its integrating factor is
a) $e^{\int f(x)dx}$ b) $e^{f(x)}$
c) $e^{\int f(y)dy}$ d) $f(x)$

123) If the differential equation $Mdx+Ndy=0$ is not exact and $\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{-M}=f(y)$, its integrating factor is
a) $e^{\int f(x)dx}$ b) $e^{f(x)}$
c) $e^{\int f(y)dy}$ d) $f(x)$

124) The total derivative of $dx+dy$ is
a) $d\left(\frac{x}{y}\right)$ b) $d(x+y)$
c) $d(x-y)$ d) $d(xy)$

125) The total derivative of $dx-dy$ is
a) $d\left(\frac{x}{y}\right)$ b) $d(x+y)$
c) $d(x-y)$ d) $d(xy)$

126) The total derivative of $xdy+ydx$ is
a) $d\left(\frac{x}{y}\right)$ b) $d(x+y)$
c) $d(x-y)$ d) $d(xy)$

127) The total derivative of $xdy-ydx$ with the integrating factor $\frac{1}{x^2}$ is
a) $d(x-y)$ b) $d\left(\frac{x}{y}\right)$
c) $d\left(\frac{y}{x}\right)$ d) $d(xy)$

128) The total derivative of $2(xdx+ydy)$ is
a) $d(x+y)$ b) $d(xy)$
c) $d(xy)^2$ d) $d(x^2+y^2)$

129) The total derivative of $2(xdx-ydy)$ is
a) $d(xy)$ b) $d\left(\frac{x^2}{y^2}\right)$
c) $d(x^2-y^2)$ d) $d(x^2+y^2)$

130) The total derivative of $\frac{ydx-xdy}{y^2}$ is
a) $d\left(\frac{y}{x}\right)$ b) $d\left(\frac{x}{y}\right)$
c) $d\left(\frac{x-y}{y}\right)$ d) $d(x^2-y^2)$

131) The total derivative of $ydx-xdy$ with the integrating factor $\frac{1}{y^2}$ is
a) $d\left(\frac{x}{y}\right)$ b) $d\left(\frac{y}{x}\right)$
c) $d\left(\frac{x-y}{y}\right)$ d) $d(x^2-y^2)$

132) The total derivative of $dx+dy$ with the integrating factor $\frac{1}{x+y}$ is

a) $d\left[\log(x-y)\right]$ b) $d\left[\log(x+y)\right]$
c) $d\left[\log(xy)\right]$ d) $d\left[\log\left(x^2+y^2\right)\right]$

133) The total derivative of $dx-dy$ with the integrating factor $\frac{1}{x-y}$ is

a) $d\left[\log(x-y)\right]$ b) $d\left[\log(x+y)\right]$
c) $d\left[\log(xy)\right]$ d) $d\left[\log\left(x^2+y^2\right)\right]$

134) The total derivative of $xdy+ydx$ with the integrating factor $\frac{1}{xy}$ is

a) $d\left[\log(x-y)\right]$ b) $d\left[\log(x+y)\right]$
c) $d\left[\log(xy)\right]$ d) $d\left[\log\left(x^2+y^2\right)\right]$

135) The total derivative of $xdy-ydx$ with the integrating factor $\frac{1}{xy}$ is

a) $d\left[\log(x-y)\right]$ b) $d\left[\log\left(\frac{x}{y}\right)\right]$
c) $d\left[\log\left(\frac{y}{x}\right)\right]$ d) $d\left[\log(xy)\right]$

136) The total derivative of $2(xdx+ydy)$ with the integrating factor $\frac{1}{x^2+y^2}$ is

a) $d\left[\log(x-y)\right]$ b) $d\left[\log(x+y)\right]$
c) $d\left[\log(xy)\right]$ d) $d\left[\log\left(x^2+y^2\right)\right]$

137) The total derivative of $2(xdx-ydy)$ with the integrating factor $\frac{1}{x^2-y^2}$ is

a) $d\left[\log\left(x^2-y^2\right)\right]$ b) $d\left[\log(x+y)\right]$
c) $d\left[\log(xy)\right]$ d) $d\left[\log\left(x^2+y^2\right)\right]$

138) The total derivative of $xdy-ydx$ with the integrating factor $\frac{1}{x^2+y^2}$ is

a) $d\left[\log\left(x^2-y^2\right)\right]$ b) $d\left[\log\left(x^2+y^2\right)\right]$
c) $d\left(\tan^{-1}\frac{y}{x}\right)$ d) $d\left(\tan^{-1}\frac{x}{y}\right)$

139) If the integrating factor of $\frac{ydx-xdy}{y^2}$ is $\frac{y}{x}$, its total derivative is

a) $d\left(\tan^{-1}\frac{x}{y}\right)$ b) $d\left(\log(x+y)\right)$
c) $d\left(\log\frac{y}{x}\right)$ d) $d\left(\log\frac{x}{y}\right)$

140) If the integrating factor of $\frac{xdy-ydx}{x^2}$ is $\frac{x}{y}$, its total derivative is

a) $d\left(\tan^{-1}\frac{y}{x}\right)$ b) $d\left(\tan^{-1}\frac{x}{y}\right)$
c) $d\left(\log\frac{x}{y}\right)$ d) $d\left(\log\frac{y}{x}\right)$

141) If the integrating factor of $\frac{ydx-xdy}{y^2}$ is $\frac{y^2}{x^2+y^2}$, its total derivative is

a) $d\left(\log\frac{y}{x}\right)$ b) $d\left(\tan^{-1}\frac{y}{x}\right)$
c) $d\left(\tan^{-1}\frac{x}{y}\right)$ d) $\log\left(x^2+y^2\right)$

142) The total derivative of $dx+dy$ with the integrating factor $\frac{1}{1+(x+y)^2}$ is

a) $d\left(\tan^{-1}(x+y)\right)$ b) $d\left(\log\frac{y}{x}\right)$
c) $d\left(\sec^{-1}(x+y)\right)$ d) $\log(x+y)$

143) The equation $(x+y+3)dx+(x-y-7)dy=0$ is of the form

a) variable separable b)exact differential
c) linear differential d) homogeneous

144) Equation $(3x+2y+1)dx+(2x-7y-3)dy=0$ is of the form
a) variable separable b) exact differential
c) linear differential d) homogeneous

145) For what value of λ, the differential equation $(5x+\lambda y-3)dx+(3x-7y+5)dy=0$ is exact?
a) 0 b) 1 c) 2 d)3

146) For what value of a, the differential equation $(xy^2+ax^2y)dx+(x^3+x^2y)dy=0$ is exact?
a)3 b) 2 c) 1 d) 5

147) For what value of a, the differential equation $(\tan y+ax^2y-y)dx+(x\tan^2 y-x^3-\sec^2 y)dy=0$ is exact?
a) 2 b) -2 c) 3 d)-3

148) The differential equation $\frac{dy}{dx}=\frac{ay+1}{(y+2)e^y-x}$ is exact, if the value of a is
a) -2 b) 2 c) -1 d)1

149) Differential equation $\frac{dy}{dx}+\frac{3+ay\cos x}{2\sin x-4y^3}=0$ is exact, if the value of a is
a) -3 b) 3 c)2 d)-2

150) For what values of a and b, the differential equation $(ay^2+x+x^8)dx+(y^2+y-bxy)dy=0$ is an exact differential equation?
a) $2a+b=0$ b) $a=2b$
c) $a-2b=3$ d) $a=1=b$

151) The equation $(1+axy^2)dx+(1+bx^2y)dy=0$ is exact differential equation, if
a) $a+2b=0$ b) $a=1,\ b=-3$
c) $a=b$ d) $a=2,\ b=3$

152) For what values of a and b, differential equation $(axy^4+\sin y)dx+(bx^2y^3+x\cos y)dy=0$ is formed to be exact?
a) $a=3b$ b) $a=2,\ b=4$
c) $a+b=1$ d) $a=3,\ b=-3$

153) The integrating factor for the differential equation $(y^2-2xy)dx+(2x^2+3xy)dy=0$ is
a) $\frac{1}{4xy^2}$ b) $\frac{1}{4x^2y^2}$ c) $\frac{1}{2x^2y}$ d) $\frac{1}{2xy}$

154) The integrating factor for the differential equation $(xy-2y^2)dx-(x^2-3xy)dy=0$ is
a) $\frac{1}{2x^2y^2}$ b) $\frac{1}{x^2y}$ c) $\frac{1}{xy}$ d) $\frac{1}{xy^2}$

155) The integrating factor for the differential equation $(x^2-3xy+2y^2)dx-(2xy-3x^2)dy=0$ is
a) $\frac{1}{x^3}$ b) $\frac{1}{x^3y}$ c) $\frac{1}{y^3}$ d) $\frac{1}{x^2y^2}$

156) The differential equation $(y^3-2x^2y)dx+(2xy^2-x^3)dy=0$ can be reduced to exact if the equation is multiplied by the integrating factor
a) $\frac{1}{xy}$ b) x^2y^2 c) $\frac{1}{x^2y^2}$ d) xy

157) The integrating factor for the differential equation $(x^2y-2xy^2)dx+(3x^2y-x^3)dy=0$ is
a) $\frac{1}{2xy}$ b) $\frac{1}{x^2y}$ c) $\frac{1}{x^2y^2}$ d) $\frac{1}{xy^2}$

158) The integrating factor for the differential equation $(xy+1)ydx-(xy-1)xdy=0$ is
a) $\frac{1}{2x^2y^2}$ b) $\frac{1}{2x^2y}$ c) $\frac{1}{2xy^2}$ d) $\frac{1}{2xy}$

159) The integrating factor for the differential equation $(xy+1)ydx+(x^2y^2+xy+1)xdy=0$ is
a) $\frac{1}{x^3y}$ b) $-\frac{1}{x^3y^3}$ c) $-\frac{1}{x^2y^2}$ d) $\frac{1}{xy^3}$

160) The integrating factor for the equation $(x^2y^2+xy+1)ydx+(x^2y^2-xy+1)xdy=0$ is
a) $\frac{1}{2x^2y^2}$ b) $\frac{1}{2x^2y}$ c) $\frac{1}{2xy^2}$ d) $\frac{1}{2x^3y^3}$

161) The integrating factor for the equation $\left(x^2y^2+5xy+2\right)ydx+\left(x^2y^2+4xy+2\right)xdy=0$ is

a) $\frac{1}{x^2y^3}$ b) $\frac{1}{x^2y}$ c) $\frac{1}{xy^2}$ d) $\frac{1}{x^2y^2}$

162) The differential equation $y\left(xy+2x^2y^2\right)dx+x\left(xy-x^2y^2\right)dy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) $\frac{1}{2x^2y}$ b) $\frac{1}{3x^3y}$ c) $\frac{1}{2x^2y^2}$ d) $\frac{1}{3x^3y^3}$

163) The integrating factor for the differential equation $\left(x^2+y^2+x\right)ydx+xydy=0$ is

a) $\frac{1}{2xy^2}$ b) $\frac{1}{2xy}$ c) x d) $\frac{1}{x}$

164) The integrating factor for the equation $(xy\sin xy+\cos xy)\,ydx+(xy\sin xy-\cos xy)\,xdx=0$ is

a) $\frac{1}{2xy}$ b) $\frac{1}{2xy\cos xy}$

c) $\frac{1}{2xy\sin xy}$ d) $\frac{1}{2\cos xy}$

165) The integrating factor for the differential equation $\left(y+\frac{y^3}{3}+\frac{x^2}{2}\right)dx+\left(\frac{x+xy^2}{4}\right)dy=0$ is

a) $\frac{1}{x^2}$ b) x^2 c) $\frac{1}{x^3}$ d) x^3

166) The integrating factor for the differential equation $(2x\log x-xy)\,dy+2ydx=0$ is

a) x^2 b) $\frac{1}{x}$ c) $\frac{1}{x^2}$ d) $\frac{1}{x^3}$

167) The integrating factor for the differential equation $\left(x^2+y^2+1\right)dx-2xydy=0$ is

a) x^2 b) $\frac{1}{x}$ c) $\frac{1}{x^2}$ d) $\frac{1}{x^3}$

168) The integrating factor for the differential equation $y\left(2xy+e^x\right)dx-e^xdy=0$ is

a) $\frac{1}{y^2}$ b) $\frac{1}{x^2}$ c) $\frac{1}{y^3}$ d) $\frac{1}{x^3}$

169) The integrating factor for the differential equation $y\log ydx+\left(x-\log y\right)dy=0$ is

a) $\frac{1}{y^2}$ b) $\frac{1}{x^2}$ c) $\frac{1}{y}$ d) $\frac{1}{x}$

170) The differential equation $\left(y^4+2y\right)dx+\left(xy^3+2y^4-4x\right)dy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) $\frac{1}{y^2}$ b) $\frac{1}{y^3}$ c) $\frac{1}{x^3}$ d) $\frac{1}{x^2}$

171) The differential equation $\left(2x+e^x\log y\right)ydx+e^xdy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) x^2 b) $\frac{1}{x^3}$ c) $\frac{1}{x}$ d) $\frac{1}{y}$

172) The differential equation $\left(xy^3+y\right)dx+2\left(x^2y^2+x+y^4\right)dy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) x b) y c) $\frac{1}{y}$ d) $\frac{1}{x}$

173) The differential equation $\left(xy^2-e^{\frac{1}{x^3}}\right)dx-x^2ydy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) $\frac{1}{x^4}$ b) $\frac{1}{x^3}$ c) $\frac{1}{y^2}$ d) $\frac{1}{y^3}$

174) $\left(x^2-3xy+2y^2\right)dx-\left(e^x+y^3\right)dy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) $\frac{1}{y^2}$ b) $\frac{1}{y^3}$ c) $\frac{1}{x^3}$ d) $\frac{1}{x^4}$

175) $\left(y^4+2y\right)dx+\left(xy^3+2y^4-4x\right)dy=0$ can be reduced to exact if the equation is multiplied by the integrating factor

a) $\frac{1}{x^3}$ b) $\frac{1}{y^2}$ c) $\frac{1}{y^3}$ d) $\frac{1}{x^4}$

176) The solution of the exact differential equation $\left(x+y-2\right)dx+\left(x-y+4\right)dy=0$ is

a) $x^2+y^2+xy+x+y+c=0$
b) $x^2+y^2+2xy+4x+6y+c=0$
c) $x^2+y^2+2xy+4x+8y+c=0$
d) $x^2-y^2+2xy-4x+8y+c=0$

177) The solution of the exact differential equation $\left(y^2e^{xy^2}+4x^3\right)dx+\left(2xye^{xy^2}-3y^2\right)dy=0$ is

a) $\frac{1}{y^2}e^{xy^2}+\frac{x^4}{4}-\frac{y^3}{3}=c$ b) $e^{xy^2}+x^4-y^3=c$
c) $e^{xy^2}+x^4+y^3=c$ d) $e^{xy^2}+\frac{x^4}{4}-\frac{y^3}{3}=c$

178) The solution of the exact differential equation $\left(x^2-4xy-2y^2\right)dx+\left(y^2-4xy-2x^2\right)dy=0$ is

a) $x^3-6x^2y-6xy^2+y^3=c$
b) $\frac{x^3}{3}-6x^2y-6xy^2+\frac{y^3}{3}=c$
c) $x^3+x^2y+xy^2+y^3=c$
d) $x^3+x^2y-3xy^2+2y^3=c$

179) The solution of the exact differential equation $\left(1+\log xy\right)dx+\left(1+\frac{x}{y}\right)dy=0$ is

a) $y-x\log x+\log y=c$ b) $y+x\log xy=c$
c) $1+\frac{x}{y}\log xy=c$ d) $\frac{y}{x}+\log xy=c$

180) The solution of the exact differential equation $\left(1+x^2\right)\left(xdy+ydx\right)+2x^2ydx=0$ is

a) $x^2+y\left(1+x^2\right)=c$ b) $x+y-\left(1+x^2\right)=c$
c) $xy\left(1+x^2\right)=c$ d) $x+y\left(1+x^2\right)=c$

181) The solution of the exact differential equation $\cos y-x\sin y\frac{dy}{dx}=\sec^2 x$ is

a) $\frac{x}{y}\cos y=c\tan x$ b) $\cot x-x^2\cos y=c$
c) $\tan^2 x-x\sin y=c$ d) $\tan x-x\cos y=c$

182) The solution of the exact differential equation $\frac{dy}{dx}=\frac{1+y^2+3x^2y}{1-2xy-x^3}$ is

a) $x\left(1+y^2\right)+x^3y-y=c$
b) $\frac{1+y^2}{x}+x^2y-y=c$
c) $1+y^2+x^2y-xy=c$
d) $x\left(1+\frac{y^2}{2}\right)-\frac{x^3y}{3}-y=c$

183) The solution of the exact differential equation $\left(x^2y-2xy^2\right)dx+\left(3x^2y-x^3\right)dy=0$ with the integrating factor $\frac{1}{x^2y^2}$ is

a) $\frac{y}{x}-\log x+\log y=c$
b) $\frac{x}{y}-2\log x+3\log y=c$
c) $x+2y\log x+3x\log y=c$
d) $\frac{x^2}{2}-2y\log x+3\log y=c$

184) The solution of the exact differential equation $\left(3xy^2-y^3\right)dx+\left(xy^2-2x^2y\right)dy=0$ with the integrating factor $\frac{1}{x^2y^2}$ is

a) $\frac{y}{x}+3\log x+2\log y=c$
b) $y\log x+3\log x-2\log y=c$
c) $\frac{y}{x}+3\log x-2\log y=c$
d) $\frac{y^2}{x^2}+3x\log x-2y\log y=c$

185) The solution of the exact differential equation $\left(x^2-3xy+2y^2\right)dx+x\left(3x-2y\right)dy=0$ with the integrating factor $\frac{1}{x^3}$ is

a) $x^2\log x+3xy-y^2=cx^2$
b) $\log x+3x^2y-y^2=c$
c) $x^3\log x+3x^2y-xy^2=cx^3$
d) $3\log x+3xy+y^2=cx^2$

186) The solution of the exact differential equation $(1+xy)ydx+(1-xy)xdy=0$ with the integrating factor $\frac{1}{2x^2y^2}$ is

a) $3\log\left(\frac{x}{y}\right)+\frac{1}{x^2y^2}=c$ b) $\log\left(\frac{x}{y}\right)+\frac{1}{xy}=c$

c) $3\log\left(\frac{x}{y}\right)-\frac{1}{x^2y}=c$ d) $\log\left(\frac{x}{y}\right)-\frac{1}{xy}=c$

187) The solution of the exact differential equation $(x^2y^2+5xy+2)ydx+(x^2y^2+4xy+2)xdy=0$ with the integrating factor $\frac{1}{x^2y^2}$ is

a) $xy+5\log x-\frac{2}{xy}+4\log y=c$

b) $x^2y+5\log x-\frac{1}{xy}+2\log y=c$

c) $xy+5\log x+\frac{1}{xy}+3\log y=c$

d) $x^2y^2+5\log x+\frac{2}{xy}+4\log y=c$

188) The solution of the exact differential equation $(x^2y^2+xy+1)ydx+(x^2y^2-xy+1)xdy=0$ with the integrating factor $\frac{1}{2x^2y^2}$ is

a) $xy-\frac{1}{xy}+x\log x+y\log y=c$

b) $xy-\frac{1}{xy}+\log x+\log y=c$

c) $\frac{x}{y}-\frac{1}{xy}+\log\left(\frac{x}{y}\right)=c$

d) $xy-\frac{1}{xy}+\log\left(\frac{x}{y}\right)=c$

189) The solution of the exact differential equation $y(xy+2x^2y^2)dx+x(xy-x^2y^2)dy=0$ with the integrating factor $\frac{1}{3x^3y^3}$ is

a) $2\log\left(\frac{x}{y}\right)-\frac{1}{xy}=c$ b) $\frac{1}{2}\log\left(\frac{x}{y}\right)+\frac{1}{xy}=c$

c) $\log\left(\frac{x^2}{y}\right)-\frac{1}{xy}=c$ d) $\log\left(\frac{x}{y^2}\right)+\frac{1}{xy}=c$

190) The solution of the exact differential equation $(x^2+y^2+x)ydx+xydy=0$ with the integrating factor x is

a) $x^4+x^2y^3+x^3=c$ b) $y\left(\frac{x^4}{4}+\frac{x^2y^2}{2}+\frac{x^3}{3}\right)=c$

c) $y(x^4+x^2y^2+x^3)=c$ d) $\frac{x^4}{4}+\frac{x^2y^2}{2}+\frac{x^3}{3}=c$

191) The solution of the exact differential equation $(xy\sin xy+\cos xy)ydx+(xy\sin xy-\cos xy)xdx=0$ with the integrating factor $\frac{1}{2xy\cos xy}$ is

a) $x\log(\sec xy)=cy$ b) $xy\sec xy=c$

c) $x\sec xy=cy$ d) $x\cos xy=cy$

192) The solution of the exact differential equation $(x^2-3xy+2y^2)dx+(3x^2-2xy)dy=0$ with the integrating factor $\frac{1}{x^3}$ is

a) $\log x+\frac{3y}{x}-\left(\frac{y}{x}\right)^2=c$ b) $\log x+3yx-\left(\frac{y}{x}\right)^2=c$

c) $\log x+\frac{y}{x}+\left(\frac{y}{x}\right)^2=c$ d) $3\log x+\frac{y}{x}-\frac{y^2}{x}=c$

193) The solution of the exact differential equation $(xy^3+y)dx+2(x^2y^2+x+y^4)dy=0$ with the integrating factor y is

a) $\frac{3}{4}x^2y^4+\frac{6}{5}xy^2+2y^6=c$

b) $3x^2y^4+6x^2y+2x^6=c$

c) $x^3y^4+3xy^2+5y^6=c$

d) $3x^2y^4+6xy^2+2y^6=c$

194) The solution of the exact differential equation $(y^4+2y)dx+(xy^3-4x+2y^4)dy=0$ with the integrating factor $\frac{1}{y^3}$ is

a) $x(y^3+2)+y^2=c$ b) $x^2(y^3+2)-y^4=cy^2$

c) $x(y^3+2)+y^4=cy^2$ d) $(y^3+2)xy^4=cy^2$

195) The solution of the exact differential equation $(3x+2y^2)ydx+2x(2x+3y^2)dy=0$ with the integrating factor xy^3 is

a) $x^3y^4+x^2y^6=c$ b) $x^3y^3+x^4y^3=c$

c) $x^2y^4+xy^6=c$ d) $\frac{1}{3}x^3y^4+\frac{1}{4}x^2y^6=c$

196) The solution of the exact differential equation $(x^2y+y^4)dx+(2x^3+4xy^3)dy=0$ with the integrating factor $x^{\frac{5}{2}}y^{10}$ is

a) $\frac{12}{11}x^{\frac{11}{2}}y^{11}+\frac{12}{7}x^{\frac{7}{2}}y^{14}=c$

b) $\frac{2}{11}x^{\frac{11}{12}}y^{11}+\frac{2}{7}x^{\frac{7}{12}}y^{14}=c$

c) $\frac{2}{11}x^{\frac{11}{2}}y^{11}+\frac{2}{7}x^{\frac{7}{2}}y^{14}=c$

d) $\frac{2}{11}x^{\frac{11}{2}}y^{11}-\frac{2}{7}x^{\frac{7}{2}}y^{14}=c$

197) The solution of the exact differential equation $(y^2+2x^2y)dx+(2x^3-xy)dy=0$ with the integrating factor $\frac{1}{x^{5/2}y^{1/2}}$ is

a) $4xy-\frac{2}{3}\left(\frac{y}{x}\right)^{\frac{3}{2}}=c$ b) $4\sqrt{xy}-\frac{2}{3}\left(\frac{y}{x}\right)^{\frac{3}{2}}=c$

c) $4\sqrt{xy}-\frac{2}{3}\sqrt{\frac{y}{x}}=c$ d) $\sqrt{xy}+\left(\frac{y}{x}\right)^{\frac{3}{2}}=c$

198) The solution of the exact differential equation $(y^4-2x^3y)dx+(x^4-2xy^3)dy=0$ with the integrating factor $\frac{1}{x^2y^2}$ is

a) $\frac{2x^2}{y}+\frac{3y^2}{x}=c$ b) $\frac{x^2}{y}-\frac{y^2}{x}=c$

c) $\frac{x^2}{2y}+\frac{y^2}{3x}=c$ d) $\frac{x^2}{y}+\frac{y^2}{x}=c$

199) The solution of the exact differential equation $(y^3-2x^2y)dx-(x^3-2xy^2)dy=0$ with the integrating factor xy is

a) $x^3y^3(y^2+x^2)=c$ b) $x^2y^2(y^2-x^2)=c$

c) $x^2y^2(y^2+x^2)=c$ d) $x^2+y^2(y^2-x^2)=c$

200) The solution of the exact differential equation $(3x^2y^4+2xy)dx+(2x^3y^3-x^2)dy=0$ with the integrating factor $\frac{1}{y^2}$ is

a) $x^3y^2+\frac{x^2}{y}=c$ b) $x^2y^2+\frac{x^2}{y^2}=c$

c) $x^3y^3-\frac{x^2}{y}=c$ d) $x^2y^3-\frac{x^2}{y^3}=c$

201) The solution of the exact differential equation $y(x^2y+e^x)dx-e^xdy=0$ with the integrating factor $\frac{1}{y^2}$ is

a) $\frac{x^2}{2}+\frac{e^x}{y}=c$ b) $\frac{x^3}{3}-\frac{e^x}{y}=c$

c) $\frac{x^3}{3}+\frac{e^x}{y}=c$ d) $\frac{x^3}{3}+\frac{e^x}{2}=c$

202) The solution of the exact differential equation $(2x+e^x\log y)ydx+(e^x)dy=0$ with the integrating factor $\frac{1}{y}$ is

a) $x^2+e^x+\log y=c$ b) $x^2-e^x\log y=c$

c) $\frac{x^2}{2}+e^x\log y=c$ d) $x^2+e^x\log y=c$

203) The solution of $\frac{dy}{dx}(x+2y^3)=y+2x^3y^2$ with the integrating factor $\frac{1}{y^2}$ is

a) $\frac{x}{y}-\frac{x^4}{y}+y^2=c$ b) $\frac{x}{y}+\frac{x^4}{2}-\frac{y^2}{2}=c$

c) $\frac{x}{3}+\frac{x^4}{2}+y^2=c$ d) $\frac{x}{y}+\frac{x^4}{2}-y^2=c$

204) The solution of the exact differential equation $y\log ydx+(x-\log y)dy=0$ with the integrating factor $\frac{1}{y}$ is

a) $2x\log y-(\log y)^2=c$

b) $x^2\log y+(\log y)^2=c$

c) $2x\log y+(\log y)^3=c$

d) $\frac{2x}{3}\log y-\log y^2=c$

205) The solution of the exact differential equation $y\left(2x^2y+e^x\right)dx=\left(e^x+y^3\right)dy$ with the integrating factor $\frac{1}{y^2}$ is

a) $\frac{1}{3}x^3+\frac{e^x}{x}-\frac{1}{2}y^2=c$ b) $\frac{2}{3}x^3+\frac{e^x}{y}+\frac{1}{2}y^3=c$

c) $\frac{2}{3}x^3+\frac{e^x}{y}-\frac{1}{2}y^2=c$ d) $x^3+\frac{e^x}{y}-y^2=c$

206) The solution of the exact differential equation $\left(2x\log x-xy\right)dy+2ydx=0$ with the integrating factor $\frac{1}{x}$ is

a) $2x\log x-\frac{x^2}{2}=c$ b) $2y\log x-\frac{y^2}{2}=c$

c) $\frac{y}{2}\log x-\frac{y^2}{2}=c$ d) $y\log x+\frac{y^2}{2}=c$

207) The solution of the exact differential equation $\left(x^4e^x-2mxy^2\right)dx+2mx^2ydy=0$ with the integrating factor $\frac{1}{x^4}$ is

a) $e^x+\frac{m^2y^2}{x^2}=cm$ b) $e^x-\frac{my^2}{x^2}=c$

c) $\frac{e^x}{y}-\frac{my^2}{x^2}=c$ d) $e^x+\frac{my^2}{x^2}=c$

208) The differential equation which can be expressed in the form $\frac{dy}{dx}+Py=Q$, where P and Q are functions of x only, is known as
a) variable separable equation in x, y
b) homogeneous differential equation in x, y
c) linear differential equation in x w.r.t y
d)linear differential equation in y w.r.t x

209) The differential equation which can be expressed in the form $\frac{dx}{dy}+Px=Q$, where P and Q are functions of y only, is known as
a)linear differential equation in x w.r.t y
b) linear differential equation in y w.r.t x
c) homogeneous differential equation in x, y
d) variable separable equation in x, y

210) The integrating factor of the linear differential equation $\frac{dy}{dx}+Py=Q$, where P and Q are functions of x only, is

a) $e^{\int Pdx}$ b) $e^{\int Qdx}$ c) $e^{\int Pdy}$ d) $e^{\int Qdy}$

211) The integrating factor of the linear differential equation $\frac{dx}{dy}+Px=Q$, where P and Q are functions of y only, is

a) $e^{\int Pdx}$ b) $e^{\int Qdx}$ c) $e^{\int Pdy}$ d) $e^{\int Qdy}$

212) The general solution of the linear differential equation $\frac{dy}{dx}+Py=Q$, where P and Q are functions of x only, is given by

a) $ye^{\int Pdx}=\int Qe^{\int Pdx}dx+c$

b) $xe^{\int Pdx}=\int Qe^{\int Pdx}dx+c$

c) $xe^{\int Pdy}=\int Qe^{\int Pdy}dy+c$

d) $xe^{\int Qdx}=\int Pe^{\int Qdx}dx+c$

213) The general solution of the linear differential equation $\frac{dx}{dy}+Px=Q$, where P and Q are functions of y only, is given by

a) $ye^{\int Pdx}=\int Qe^{\int Pdx}dx+c$

b) $xe^{\int Pdx}=\int Qe^{\int Pdx}dx+c$

c) $xe^{\int Pdy}=\int Qe^{\int Pdy}dy+c$

d) $xe^{\int Qdx}=\int Pe^{\int Qdx}dx+c$

214) A differential equation which can be expressed in the form $\frac{dy}{dx}+Py=Qy^n$, where P and Q are functions of x only, is known as
a) Non-linear differential equation
b)Bernoulli's linear differential equation
c) exact differential equation
d) homogenous differential equation

215) A differential equation which can be expressed in the form $\frac{dx}{dy}+Px=Qx^n$, where P and Q are functions of x only, is known as

a) Non-linear differential equation
b) Bernoulli's linear differential equation
c) exact differential equation
d) homogenous differential equation

216) A differential equation which can be expressed in the form $f'(y)\frac{dy}{dx}+Pf(y)=Q$, where P and Q are functions of x only, can be reduced into the linear form by substituting
a) $P=v$ b) $Q=v$
c) $f(y)=v$ d) $f'(y)=v$

217) A differential equation which can be expressed in the form $\frac{dy}{dx}+Py=Qy^n$, where P and Q are functions of x only, can be reduced into the linear form by substituting
a) $y^n=v$ b) $y^{1-n}=v$
c) $y^{n-1}=v$ d) $y^{n+1}=v$

218) If I_1, I_2 are the integrating factors of the equations $\frac{dx}{dy}+Px=Q$ and $\frac{dx}{dy}-Px=Q$ respectively, the relation between them is
a) $I_1=-I_2$ b) $I_1=I_2$
c) $I_1\cdot I_2=-1$ d) $I_1\cdot I_2=1$

219) The integrating factor of the linear differential equation $\frac{dy}{dx}+xy=x^5$ is
a) $e^{\log\frac{x^2}{2}}$ b) $e^{\frac{x^2}{2}}$ c) e^{x^2} d) x^2

220) The integrating factor of the linear differential equation $\frac{dy}{dx}+2xy=\frac{\tan^{-1}x}{1+x^2}$ is
a) $\frac{x^2}{2}$ b) $e^{\frac{x^2}{2}}$ c) e^{x^2} d) $2x^2$

221) The integrating factor of the linear differential equation $\frac{dx}{dy}+xy=y^5$ is
a) $e^{\frac{y^2}{2}}$ b) $\frac{y^2}{2}$ c) $e^{\frac{x^2}{2}}$ d) e^{x^2}

222) The integrating factor of the linear differential equation $\frac{dy}{dx}+\frac{y}{1+x}=x^3$ is
a) $e^{\frac{(1+x)^2}{2}}$ b) $1+x$ c) $\frac{1}{1+x}$ d) e^{1+x}

223) The integrating factor of the linear differential equation $\frac{dy}{dx}+\frac{y}{1-x}=\sin x$ is
a) $\frac{1}{1-x}$ b) $1-x$ c) e^{1-x} d) $e^{\frac{(1-x)^2}{2}}$

224) The integrating factor of the linear differential equation $\frac{dy}{dx}+\frac{y}{1+x^2}=\sec x\tan x$ is
a) $\frac{(1+x^2)^2}{2}$ b) $1+x^2$ c) $e^{\tan^{-1}x}$ d) e^{1+x^2}

225) The integrating factor of the linear differential equation $\frac{dy}{dx}+\frac{2x}{1+x^2}y=\tan^{-1}x$ is
a) $\frac{(1+x^2)^2}{2}$ b) $1+x^2$ c) $e^{\tan^{-1}x}$ d) e^{1+x^2}

226) The integrating factor of the linear equation $\frac{dy}{dx}+y\tan x=e^x\sin(2x-3)$ is
a) $\sec^2 x$ b) $\cos x$ c) $\sec x$ d) $e^{\sec x}$

227) The integrating factor of the linear differential equation $\tan x\frac{dy}{dx}+y=e^x\sin x$ is
a) e b) $e^{\sin x}$ c) $\log(\sin x)$ d) $\sin x$

228) The integrating factor of the linear equation $(1+x^2)\frac{dy}{dx}+xy=2x^3-3x+5$ is
a) e^{1+x^2} b) $\frac{1}{1+x^2}$ c) $1+x^2$ d) $\sqrt{1+x^2}$

229) The integrating factor of the linear equation $(1+x^2)\frac{dy}{dx}+4xy=\frac{1}{(1+x^2)^3}$ is
a) $1+x^2$ b) e^{1+x^2} c) $(1+x^2)^2$ d) $\frac{1}{1+x^2}$

230) The integrating factor of the linear equation $\left(1+x^2\right)\frac{dy}{dx}+2xy=\frac{1}{\left(1+x^2\right)^3}$ is

a) $1+x^2$ b) e^{1+x^2} c) $\left(1+x^2\right)^2$ d) $\frac{1}{1+x^2}$

231) The integrating factor of the linear equation $\left(1+x^2\right)\frac{dy}{dx}-2xy=\frac{1}{\left(1+x^2\right)^3}$ is

a) $1+x^2$ b) e^{1+x^2} c) $\left(1+x^2\right)^2$ d) $\frac{1}{1+x^2}$

232) The integrating factor of the linear differential equation $\frac{dx}{dy}+\frac{xy}{1+y^2}=\sec y$ is

a) $\sqrt{1+x^2}$ b) $\sqrt{1+y^2}$ c) $\tan^{-1} y$ d) $e^{\tan^{-1} y}$

233) The integrating factor of the linear differential equation $\frac{dy}{dx}+y\cot x=\tan x$ is

a) $\sin x$ b) $e^{\sec x}$
c) $\cos x$ d) $\sec x+\tan x$

234) The integrating factor of the linear differential equation $\cos x\frac{dy}{dx}+y=\tan x$ is

a) $e^{\sec x+\tan x}$ b) $e^{\sec x}$
c) $\cos x$ d) $\sec x+\tan x$

235) The integrating factor of the differential equation $\frac{dy}{dx}+\sqrt{x}y=\sin\sqrt{x}\cos\sqrt{x}$ is

a) $\sin\sqrt{x}$ b) $e^{\log\sqrt{x}}$
c) $e^{\frac{2}{3}x\sqrt{x}}$ *or* $e^{\frac{2}{3}x^{\frac{3}{2}}}$ d) $\frac{2}{3}x\sqrt{x}$ *or* $\frac{2}{3}x^{\frac{3}{2}}$

236) The integrating factor of the linear equation $\frac{dy}{dx}+\left(\tan x+\frac{1}{x}\right)y=\frac{1}{x}\sec x$ is

a) $x\sec x$ b) $e^{x\sec x}$ c) $e^{x+\sec x}$ d) $x+\sec x$

237) The integrating factor of the linear differential equation $\left(1-x^2\right)\frac{dy}{dx}=x^3+xy$ is

a) $e^{\tan^{-1} x}$ b) $e^{\sqrt{1-x^2}}$ c) $\frac{1}{\sqrt{1-x^2}}$ d) $\sqrt{1-x^2}$

238) The integrating factor of the linear differential equation $\left(1-x^2\right)\frac{dy}{dx}=x^3-xy$ is

a) $\frac{1}{\sqrt{1-x^2}}$ b) $\sqrt{1-x^2}$ c) $e^{\sqrt{1-x^2}}$ d) $e^{\tan^{-1} x}$

239) The integrating factor of the differential equation $1+y^2+\left(x-e^{\tan^{-1} x}\right)\frac{dy}{dx}=0$ is

a) $\tan^{-1} x$ b) $\tan^{-1} y$ c) $e^{\tan^{-1} x}$ d) $e^{\tan^{-1} y}$

240) The integrating factor of the differential equation $1+x^2+\left(y-e^{\tan^{-1} y}\right)\frac{dx}{dy}=0$ is

a) $\tan^{-1} x$ b) $\tan^{-1} y$ c) $e^{\tan^{-1} x}$ d) $e^{\tan^{-1} y}$

241) The integrating factor of the differential equation $\left(1+y^2\right)dx=\left(e^{\tan^{-1} x}-x\right)dy$ is

a) $\tan^{-1} x$ b) $\tan^{-1} y$ c) $e^{\tan^{-1} x}$ d) $e^{\tan^{-1} y}$

242) The integrating factor of the linear differential equation $y^2+\left(x-\frac{1}{y}\right)\frac{dy}{dx}=0$ is

a) $2\log x$ b) $\log y$ c) $-\frac{1}{y}$ d) $-\frac{1}{y^2}$

243) The integrating factor of the linear differential equation $\sin 2y dx=\left(\tan y-x\right)dy$ is

a) $\frac{\tan x}{2}$ b) $\sqrt{\tan y}$ c) $\sqrt{\tan x}$ d) $\frac{\tan y}{2}$

244) The integrating factor of the linear equation $y\log y dx+\left(x-\log y\right)dy=0$ is

a) $\left(\log y\right)^2$ b) $x\log y$ c) $\log y$ d) $\log x$

245) The integrating factor of the linear differential equation $ydx-\left(y-x\right)dy=0$ is

a) y b) x c) y^2 d) x^2

246) The integrating factor of the linear equation $\sqrt{a^2+x^2}\frac{dy}{dx}+y=\sqrt{a^2+x^2}-x$ is

a) $\frac{1}{2a}\log\left(x+\sqrt{a^2+x^2}\right)$ b) $\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

c) $x+\sqrt{a^2+x^2}$ d) $x-\sqrt{a^2+x^2}$

247) The integrating factor of the linear differential equation $\frac{dy}{dx}=\frac{e^x-2xy}{x^3}$ is

a) $e^{\frac{x^3}{3}}$ b) x^3 c) $\frac{1}{x^3}$ d) e^{x^3}

248) The integrating factor of linear differential equation $\left(x^2+1\right)\frac{dy}{dx}=x^3-2xy+x$ is

a) $\tan^{-1}x$ b) $e^{\tan^{-1}x}$ c) $\frac{1}{x^2+1}$ d) x^2+1

249) The integrating factor of the linear differential equation $x^2\frac{dy}{dx}=3x^2-2xy+1$ is

a) x^2-1 b) x^2 c) x^2+1 d) $\frac{1}{x^2}$

250) The integrating factor of the linear differential equation $\left(e^{-y}\sec^2 y-x\right)dy=dx$ is

a) $e^{\tan y}$ b) $\tan y$ c) e^x d) e^y

251) The differential equation $\frac{dy}{dx}-y\tan x=y^4\sec x$ is reduced into the linear form

a) $\frac{du}{dx}+3u\tan x=-3\sec x;\ u=y^{-3}$

b) $\frac{du}{dx}-3u\tan x=3\sec x;\ u=y^{-3}$

c) $\frac{du}{dx}-3u\tan x=-3\sec x;\ u=y^{-3}$

d) $\frac{du}{dx}+3u\cot x=-3\sec x;\ u=y^{-3}$

252) The differential equation $\frac{dy}{dx}-xy=-y^3e^{-x^2}$ can be reduced to the linear form

a) $\frac{dy}{dx}+xu=-2e^{-x^2};\ u=\frac{1}{y^2}$

b) $\frac{dy}{dx}+xu=e^{-x^2};\ u=\frac{1}{y^2}$

c) $\frac{dy}{dx}-2xu=2e^{-x^2};\ u=\frac{1}{y^2}$

d) $\frac{dy}{dx}+2xu=2e^{-x^2};\ u=\frac{1}{y^2}$

253) The value of k for which e^{ky^2} is an integrating factor of linear differential equation $\frac{dx}{dy}+xy=e^{-\frac{y^2}{2}}$ is

a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) 2 d) -2

254) The general solution of $\frac{dy}{dx}+\frac{y}{1+x}=-x(1-x)$ with the integrating factor $\frac{1}{1-x}$ is

a) $\frac{y}{1-x}=-\frac{x^3}{3}+c$ b) $y=-\frac{x^2}{2}(1-x)+c$

c) $\frac{y}{1-x}=-\frac{x^2}{2}+c$ d) $\frac{y}{1-x}=\frac{x^2}{2}+c$

255) The general solution of $\frac{dy}{dx}+\frac{y}{(1-x)\sqrt{x}}=1-\sqrt{x}$ with the integrating factor $\frac{1+\sqrt{x}}{1-\sqrt{x}}$ is

a) $\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)y^2=x^2+\frac{2}{3}x^{\frac{3}{2}}+c$

b) $\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)y=x+\frac{2}{3}x^{\frac{3}{2}}+c$

c) $\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)y=x+\frac{2}{3}x^{\frac{3}{2}}+c$

d) $\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)y=x-\frac{2}{3}x^{\frac{3}{2}}+c$

256) The general solution of $\frac{dy}{dx}+y\cot x=\sin 2x$ with the integrating factor $\sin x$ is

a) $y\sin x=\frac{2}{3}\sin^3 x+c$ b) $y\sin x=\frac{1}{3}\sin^3 x+c$

c) $x\sin y=\frac{2}{3}\sin^3 x+c$ d) $y\sin x=\sin^3 x+c$

257) The general solution of $\frac{dy}{dx}+\frac{3y}{x}=\frac{e^x}{x^2}$ with the integrating factor x^3 is

a) $x^3y=e^{-x}(x+1)+c$ b) $xy^3=e^x(x-1)+c$

c) $x^3y=e^x(x-1)+c$ d) $x^3y=e^x(x+1)+c$

258) The general solution of $\frac{dy}{dx}+\frac{3y}{x}=x^2$ with the integrating factor x^3 is

a) $x^3y=\frac{x^4}{4}+c$ b) $x^3y=\frac{x^6}{6}+c$

c) $x^3y=\frac{x^2}{2}+c$ d) $xy^3=\frac{x^3}{3}+c$

259) The general solution of $\frac{dy}{dx}+\left(\tan x+\frac{1}{x}\right)y=\frac{1}{x}\sec x$ with the integrating factor $x\sec x$ is

a) $xy\sin x=\tan x+c$ b) $xy\sec x=-\tan x+c$

c) $xy\tan x=\cot x+c$ d) $xy\sec x=\tan x+c$

260) The general solution of $\frac{dy}{dx}+\frac{2y}{x}=\frac{1}{x^3}$ with the integrating factor x^2 is

a) $y=x^2\log x+c$ b) $x^2y=\log x+c$

c) $xy^2=\log x+c$ d) $x^2y=\log\frac{1}{x}+c$

261) The general solution of $\frac{dy}{dx}+(1+2x)y=e^{-x^2}$ with the integrating factor e^{x+x^2} is

a) $ye^{x+x^2}=e^x+c$ b) $ye^{x+x^2}=-e^x+c$

c) $e^{x+x^2}=ye^x+c$ d) $ye^{x-x^2}=e^x+c$

262) The general solution of $\frac{dy}{dx}+\frac{x}{1+y^2}=\frac{e^{-\tan^{-1}y}}{1+y^2}$ with the integrating factor $e^{\tan^{-1}y}$ is

a) $ye^{\tan^{-1}y}=\tan^{-1}x+c$ b) $xe^{\tan^{-1}y}=\tan^{-1}y+c$

c) $xe^{\tan^{-1}y}=\cot^{-1}y+c$ d) $ye^{\tan^{-1}y}=\tan^{-1}y+c$

263) The general solution of $\frac{dy}{dx}+x\sec y=\frac{2y\cos y}{1+\sin y}$ with the integrating factor $\sec y+\tan y$ is

a) $(\sec y+\tan y)x^2=y+c$

b) $(\sec y+\tan y)x=-y^2+c$

c) $(\sec y+\tan y)x=y^2+c$

d) $x=\frac{y^2}{\sec y+\tan y}+c$

Chapter 02 – Applications of Ordinary Differential Equations

1) Two families of curves are said to be orthogonal trajectories of each other, if
   a) Every member of one family cuts every member of other family at right angle.
   b) Every member of one family cuts every member of other family at origin.
   c) Every member of one family cuts every member of other family at common point.
   d) None of the above.

2) In the two dimensional Cartesian form, to find orthogonal trajectories of given family of curves, in its differential equation we replace $\frac{dy}{dx}$ by
   a) $-y\frac{dx}{dy}$ b) $-\frac{dy}{dx}$ c) $-\frac{dx}{dy}$ d) $-x\frac{dx}{dy}$

3) In the two dimensional polar form, to find orthogonal trajectories of given family of curves, in its differential equation we replace $\frac{dr}{d\theta}$ by
   a) $r\frac{d\theta}{dr}$ b) $-r\frac{d\theta}{dr}$ c) $-r^2\frac{d\theta}{dr}$ d) $-\frac{d\theta}{dr}$

4) The differential equation of orthogonal trajectories of family of straight lines $y = mx$ is
   a) $\frac{dx}{dy} + y = 0$ b) $\frac{dy}{dx} = -\frac{y}{x}$
   c) $\frac{dx}{dy} = -\frac{y}{x}$ d) $\frac{dx}{dy} = -\frac{x}{y}$

5) For the family of the curves $x^2 + y^2 = c^2$, the differential equation of orthogonal trajectories is
   a) $x^2 + y^2\frac{dx}{dy} = 0$ b) $x + y\frac{dy}{dx} = 0$
   c) $x + xy\frac{dx}{dy} = 0$ d) $x - y\frac{dx}{dy} = 0$

6) The differential equation of orthogonal trajectories of family of $x^2 + 2y^2 = c^2$ is
   a) $y - 2x\frac{dy}{dc} = 0$ b) $x - 2y\frac{dx}{dy} = 0$
   c) $x + 2y\frac{dy}{dx} = 0$ d) $x + 2y\frac{dx}{dy} = 0$

7) For the family of the curves $y^2 = 4ax$, the differential equation of orthogonal trajectories is
   a) $2y\frac{dy}{dx} = 4x$ b) $2y\frac{dy}{dx} = \frac{y}{x^2}$
   c) $-2y\frac{dy}{dx} = \frac{y^2}{x}$ d) $-2y\frac{dx}{dy} = \frac{y^2}{x}$

8) For the family of the curves $y = 4ax^2$, the differential equation of orthogonal trajectories is
   a) $y\frac{dy}{dx} = 2x$ b) $\frac{dy}{dx} = -\frac{2}{x^2}$
   c) $\frac{dy}{dx} = -\frac{2y}{x}$ d) $-2\frac{dx}{dy} = \frac{1}{xy}$

9) For the family of the curves $xy = c$, the differential equation of orthogonal trajectories is
   a) $x^2\frac{dx}{dy} + 2y = 0$ b) $-x\frac{dx}{dy} + y = 0$
   c) $2x\frac{dx}{dy} - y = 0$ d) $x\frac{dy}{dx} - y = 0$

10) The differential equation of orthogonal trajectories of family of $2x^2 + y^2 = cx$ is
   a) $4x - 2y\frac{dx}{dy} = \frac{2x^2 + y^2}{x}$
   b) $4x + 2y\frac{dy}{dx} = \frac{2x^2 + y^2}{x}$
   c) $4x^2 + 2y\frac{dx}{dy} = \frac{2x^2 + y^2}{x}$
   d) $4x - 2xy\frac{dy}{dx} = \frac{2x^2 + y^2}{x}$

11) For the family of the curves $x^2+cy^2=1$, the differential equation of orthogonal trajectories is

a) $x+\left(\frac{1-x^2}{y}\right)\frac{dy}{dx}=0$ b) $x+\left(\frac{1+x^2}{y}\right)\frac{dx}{dy}=0$

c) $x-\left(\frac{1-x^2}{y}\right)\frac{dx}{dy}=0$ d) $x^2+\left(\frac{1-x^2}{y}\right)\frac{dy}{dx}=0$

12) For the family of the curves $e^x+e^{-y}=c$, the differential equation of orthogonal trajectories is

a) $e^{2x}-e^{-2y}\frac{dx}{dy}=0$ b) $e^{-x}+e^{y}\frac{dx}{dy}=0$

c) $e^{x}-e^{-y}\frac{dy}{dx}=0$ d) $e^{x}+e^{-y}\frac{dx}{dy}=0$

13) The differential equation of orthogonal trajectories of family of $r=a\cos\theta$ is

a) $-r\frac{dr}{d\theta}=\cot\theta$ b) $-r\frac{dr}{d\theta}=\tan\theta$

c) $r\frac{d\theta}{dr}=\cot\theta$ d) $r\frac{d\theta}{dr}=\tan\theta$

14) For the family of the curves $r=a\sin\theta$, the differential equation of orthogonal trajectories is

a) $\frac{1}{r}\frac{d\theta}{dr}=-\cot\theta$ b) $r\frac{d\theta}{dr}=-\tan\theta$

c) $r\frac{d\theta}{dr}=-\cot\theta$ d) $-\frac{1}{r}\frac{dr}{d\theta}=\tan\theta$

15) For the family of the curves $r^2=a\sin\theta$, the differential equation of orthogonal trajectories is

a) $2r\frac{d\theta}{dr}=-\cot\theta$ b) $r\frac{d\theta}{dr}=-\frac{\tan\theta}{2}$

c) $r^2\frac{d\theta}{dr}=-\cot\theta$ d) $-\frac{2}{r}\frac{dr}{d\theta}=\tan\theta$

16) For the family of the curves $r=a(1-\cos\theta)$, the differential equation of orthogonal trajectories is

a) $-r\frac{d\theta}{dr}=\frac{1-\cos\theta}{\sin\theta}$ b) $r\frac{dr}{d\theta}=\frac{\sin\theta}{1-\cos\theta}$

c) $-r\frac{d\theta}{dr}=\frac{\sin\theta}{1-\cos\theta}$ d) $r^2\frac{d\theta}{dr}=\frac{\sin\theta}{1-\cos\theta}$

17) For the family of the curves $r^2=a\sin 2\theta$, the differential equation of orthogonal trajectories is

a) $-r^2\frac{dr}{d\theta}=\cot 2\theta$ b) $r\frac{d\theta}{dr}=-\cot 2\theta$

c) $r\frac{d\theta}{dr}=-\tan 2\theta$ d) $-\frac{dr}{d\theta}=\cot 2\theta$

18) For the family of the curves $r^2=a\cos 2\theta$, the differential equation of orthogonal trajectories is

a) $\frac{dr}{d\theta}=r\tan 2\theta$ b) $\frac{1}{r}\frac{d\theta}{dr}=-\tan 2\theta$

c) $r\frac{d\theta}{dr}=\cot 2\theta$ d) $r\frac{d\theta}{dr}=\tan 2\theta$

19) For the family of the curves $r^2=a\cos 2\theta$, the differential equation of orthogonal trajectories is

a) $\frac{dr}{d\theta}=r\tan 2\theta$ b) $\frac{1}{r}\frac{d\theta}{dr}=-\tan 2\theta$

c) $r\cot 2\theta\frac{d\theta}{dr}=1$ d) $r\frac{d\theta}{dr}+\tan 2\theta=0$

20) For the family of the curves $r=a\cos^2\theta$, the differential equation of orthogonal trajectories is

a) $r\frac{d\theta}{dr}=\frac{\sin 2\theta}{\cos^2\theta}$ b) $r^2\frac{d\theta}{dr}=-\frac{\sin 2\theta}{\cos^2\theta}$

c) $\frac{dr}{d\theta}=-\frac{\sin 2\theta}{\cos^2\theta}$ d) $r\frac{d\theta}{dr}=-\frac{\sin 2\theta}{\cos 2\theta}$

21) For the family of the curves $r=a\sec^2\left(\frac{\theta}{2}\right)$, the differential equation of orthogonal trajectories is

a) $r\frac{d\theta}{dr}=-\tan\frac{\theta}{2}$ b) $r\frac{dr}{d\theta}=-\cot\frac{\theta}{2}$

c) $\frac{1}{r}\frac{d\theta}{dr}=-\cot\frac{\theta}{2}$ d) $r\frac{d\theta}{dr}=-\tan 2\theta$

22) The orthogonal trajectories of family of curves having differential equation $y=mx$ is $\frac{dy}{dx}=\frac{y}{x}$, is given by

a) $x^2 - y^2 = c$ b) $\frac{x^2}{2} + y^2 = c$
c) $x^2 + y^2 = c$ d) $x^2 + 2y^2 = c$

23) If the differential equation of family of curves $xy = c$ is $x\frac{dy}{dx} = -y$, then its family of orthogonal trajectories is given by
a) $x^2 - 2y^2 = c$ b) $x^2 + 2y^2 = c$
c) $x^2 - y^2 = c^2$ d) $x^2 + y^2 = c$

24) The orthogonal trajectories of family of curves having differential equation $x^2 + y^2 = k^2$ is $\frac{dy}{dx} = -\frac{x}{y}$, is given by
a) $x^2 = 4ay$ b) $x^2 - y^2 = c$
c) $y^2 = x + c$ d) $y = cx$

25) If the differential equation of family of curves $x^2 - y^2 = c$ is $y\frac{dy}{dx} = x$, then its family of orthogonal trajectories is given by
a) $y = cx$ b) $xy = c$
c) $x^2 = 4ay$ d) $y^2 = x + c$

26) The orthogonal trajectories of family of curves having differential equation $x^2 + 2y^2 = c^2$ is $\frac{dy}{dx} + \frac{x}{2y} = 0$, is given by
a) $x^2 - cx + c^2 = 0$ b) $y = 2cx^2 + x$
c) $x^2 = ky$ d) $y = 2cx^2$

27) The orthogonal trajectories of family of curves having differential equation $x^2 + cy^2 = 1$ is $\frac{dy}{dx} = -\frac{xy}{1-x^2}$, is given by
a) $\log x + x^2 + y^2 = c$ b) $\log x - x^2 - y^2 = c$
c) $\log x - \frac{x^2}{2} - \frac{y^2}{2} = c$ d) $\log x + \frac{x^2}{2} + \frac{y^2}{2} = c$

28) The orthogonal trajectories of family of curves having differential equation $y = 4ax^2$ is $\frac{dy}{dx} = \frac{y}{x}$, is given by
a) $2x^2 = cy^2$ b) $2x^2 - y^2 = c^2$
c) $2x^2 + y^2 = c$ d) $x^2 + 2y^2 = c$

29) If the differential equation of family of curves $y^2 = 4ax$ is $2x\frac{dy}{dx} = y$, then its family of orthogonal trajectories is given by
a) $2x^2 + y^2 = c$ b) $2x^2 - y^2 = c^2$
c) $x^2 + 2y^2 = c$ d) $2x^2 = cy^2$

30) The orthogonal trajectories of family of curves having differential equation $e^x + e^{-y} = e^c$ is $\frac{dy}{dx} = \frac{e^x}{e^{-y}}$, is given by
a) $e^{2x} + e^{-2y} = k$ b) $e^x - e^{-y} = k$
c) $e^x + e^{-y} = e^c$ d) $e^{-x} + e^y = e^c$

31) If the differential equation of family of curves $e^x - e^{-y} = c$ is $\frac{dy}{dx} + \frac{e^{-y}}{e^x} = 0$, then its family of orthogonal trajectories is given by
a) $e^x + e^{-y} = k$ b) $e^{-x} + e^y = e^c$
c) $e^x + e^{-y} = e^c$ d) $e^{2x} + e^{-2y} = k$

32) If the differential equation of family of curves $x^2 = ce^{x^2+y^2}$ is $\frac{dy}{dx} = \frac{1-x^2}{xy}$, then its family of orthogonal trajectories is given by
a) $\log(1-x^2) + 2\log y = c$
b) $\log(1-x^2) - 2\log y = c$
c) $2\log(1-x^2) - 3\log y = c$
d) $\log(1-x^2) + \log y = c$

33) The orthogonal trajectories of family of curves having differential equation $\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1$ is $x + \left(\frac{a^2 - x^2}{y}\right)\frac{dy}{dx} = 0$, where a and b are fixed constants, is given by
a) $\frac{y^2}{2} = \lambda \log x + \frac{x^2}{2} + k$
b) $y^2 - x^2 = a^2 \log x + k$
c) $\frac{y^2}{2} = a^2 \log x - \frac{x^2}{2} + k$
d) $x^2 + y^2 = a^2 \log x + k$

34) If the differential equation of family of curves $r = a(1-\cos\theta)$ is $(1-\cos\theta)\frac{dr}{d\theta} = r$, then its family of orthogonal trajectories is given by
a) $r^2 = A(1+\cos\theta)$ b) $r = A(1+\sin\theta)$
c) $r = A(1-\cos\theta)$ d) $r = A(1+\cos\theta)$

35) If the differential equation of family of curves $r = a(1-\cos\theta)$ is $\frac{dr}{d\theta} = r\cot\frac{\theta}{2}$, then its family of orthogonal trajectories is given by
a) $\log\cos\left(\frac{\theta}{2}\right) = 2\log r + c$
b) $2\log\sin\left(\frac{\theta}{2}\right) = \frac{1}{2}\log r + c$
c) $2\log\cos\left(\frac{\theta}{2}\right) = \log r + c$
d) $\log 2\cos\left(\frac{\theta}{2}\right) = \log r + c$

36) The orthogonal trajectories of family of curves having differential equation $r = a\sin\theta$ is $\frac{dr}{d\theta} = r\cot\theta$, is given by
a) $r = A\cos\theta$ b) $r = A\tan\theta$
c) $r\cos\theta = A$ d) $r^2 = A\cos\theta$

37) The orthogonal trajectories of family of curves having differential equation $r = a\cos\theta$ is $\frac{dr}{d\theta} + r\tan\theta = 0$, is given by
a) $r = C\cos ec2\theta$ b) $r^2 = C\sin^2\theta$
c) $r = C\tan\theta$ d) $r = C\sin\theta$

38) If the differential equation of family of curves $r^2 = a^2\cos 2\theta$ is $\frac{dr}{d\theta} + r\tan 2\theta = 0$, then its family of orthogonal trajectories is given by
a) $r^2 = c\sin^2 2\theta$ b) $r = c\sin 2\theta$
c) $r^2 = c^2\sin 2\theta$ d) $r^2 = c^2\cos 2\theta$

39) If the differential equation of family of curves $r^2 = a\sin 2\theta$ is $\frac{dr}{d\theta} = r\cot 2\theta$, then its family of orthogonal trajectories is given by
a) $r^2\cos 2\theta = k$ b) $r^2 = k\cos 2\theta$
c) $2\log r = \log\sec 2\theta + k$ d) $r^2 = k\cot 2\theta$

40) The orthogonal trajectories of family of curves having differential equation $r = a^2\cos^2\theta$ is $\frac{dr}{d\theta} + 2r\tan\theta = 0$, is given by
a) $\log\tan\theta = 2\log r + c$ b) $2\log\sin\theta = \log r + c$
c) $\frac{3}{2}\log\sin\theta = 2\log r + c$ d) $\frac{\log\sin\theta}{2} = \log r + c$

41) If the differential equation of family of curves $r = a\theta$ is $r = \theta\frac{dr}{d\theta}$, then its family of orthogonal trajectories is given by
a) $r = ce^{-\frac{\theta^2}{2}}$ b) $r = ce^{-\theta^2}$
c) $r^2 = ce^{-\frac{\theta^2}{2}}$ d) $r^2 = ce^{\theta^2}$

42) Newton's law of cooling states that
a) The temperature of the body is inversely proportional to the difference between the body temperature and the surrounding temperature.
b) The temperature of the body is proportional to the sum of the body temperature and the surrounding temperature.
c) The temperature of the body is proportional to the difference between the body temperature and the surrounding temperature.
d) The temperature of the body is proportional to the surrounding of the body temperature.

43) For $\theta =$ the temperature of the body and $\theta_0 =$ the temperature of the surrounding, then Newton's law of cooling states the differential equation
a) $\frac{d\theta}{dt} = -k\theta_0$ b) $\frac{d\theta}{dt} = -k\theta + \theta_0$
c) $\frac{d\theta}{dt} = -k(\theta + \theta_0)$ d) $\frac{d\theta}{dt} = -k(\theta - \theta_0)$

44) A body having initially temperature 90ºC is kept in surrounding of temperature 26ºC. Then the differential equation satisfied by body temperature θ at any time t is given by
a) $\frac{d\theta}{dt} = -k(\theta - 64)$ b) $\frac{d\theta}{dt} = -k(\theta - 26)$

c) $\frac{d\theta}{dt} = -k(\theta + 26)$ d) $\frac{d\theta}{dt} = -k(\theta - 90)$

45) Consider a substance at initial temperature 32°C is surrounded by room temperature 10°C. According to Newton's law of cooling the differential equation satisfied by its temperature T at time t hour is

a) $\frac{dT}{dt} = -kT(T - 10)$ b) $\frac{dT}{dt} = -k(T - 32)$

c) $\frac{dT}{dt} = -k(10 - 32T)$ d) $\frac{dT}{dt} = -k(T - 10)$

46) A metallic object is heated up to getting temperature 100°C and the placed in water of temperature 50°C. Then the differential equation of the object temperature θ at time t is given by Newton's law of cooling as

a) $\frac{d\theta}{dt} = -k\theta(\theta - 26)$ b) $\frac{d\theta}{dt} = -k(\theta - 50)$

c) $\frac{d\theta}{dt} = -k(\theta - 150)$ d) $\frac{d\theta}{dt} = -k(\theta + 50)$

47) If a body originally at 120°C cools to 35°C in 40 minute in the air of constant temperature 45°C. Then according to Newton's law, its differential equation is given by

a) $\frac{d\theta}{dt} = -k(\theta - 120)$ b) $\frac{d\theta}{dt} = -k(\theta - 40)$

c) $\frac{d\theta}{dt} = -k(\theta - 45)$ d) $\frac{d\theta}{dt} = -k(\theta - 35)$

48) Assuming the temperature of the surrounding is being kept constant at 25°C and a body cools from temperature 80°C to 35°C in 45 minute. Then it must satisfy the differential equation

a) $\frac{dT}{dt} = -k(T - 25)$ b) $\frac{dT}{dt} = -k(T - 80)$

c) $\frac{dT}{dt} = -k(T - 35)$ d) $\frac{dT}{dt} = -k(T + 25)$

49) The rate of change of temperature of a body is proportional to the difference between the temperature of body and its surrounding nearby. If temperature of the air is 35°C and that of the body is 96°C and cools down to 55°C in just 25 minute. Then we must have

a) $\frac{dT}{dt} = -k(T + 25)$ b) $\frac{dT}{dt} = -k(T - 55)$

c) $\frac{dT}{dt} = -k(T - 35)$ d) $\frac{dT}{dt} = -k(T - 25)$

50) A metal ball is placed in the oven till it obtain temperature of 100°C and then at time t = 0, it is then placed in water of temperature 40°C. By Newton's law, if the temperature of the ball is decreased to 70°C in 10 minutes, then it must satisfy the differential equation

a) $\frac{dT}{dt} = -k(T - 70)$ b) $\frac{dT}{dt} = -k(T - 40)$

c) $\frac{dT}{dt} = -k(T - 55)$ d) $\frac{dT}{dt} = -k(T - 100)$

51) If a body of temperature T at time t kept in the surrounding of temperature T_0 satisfies the differential equation $\frac{dT}{dt} = -k(T - T_0)$, the relation between T and t is given as

a) $T = T_0 - ke^{-kt}$ b) $T = T_0 + ke^{-kt}$

c) $T = T_0 + ke^{-kt}$ d) $T = -k(T_0 - e^{-kt})$

52) A body is heated to a temperature of 100°C and then at time recording t = 0 it is then placed liquid of temperature 40°C. The temperature of the body is then reduced to 60°C in 4 minute. By Newton's law of cooling its differential equation is $\frac{d\theta}{dt} = -\frac{1}{4}(\theta - 40)\log 3$. The time required to reduce the temperature of body to 50°C is

a) 5 min 6 sec b) 5.6 min

c) 65 min d) 6.5 min

53) A corpse of temperature 32°C is kept in the mortuary of constant temperature 10°C and the temperature of the corpse decreases to 20°C in 5 minutes. The differential equation of the system is given as $\frac{dT}{dt} = -0.05(T - 10)$. Then T is

a) $T = 22e^{-0.05t}$ b) $T = 22 + 10e^{0.05t}$

c) $T = 10 - 22e^{-0.05t}$ d) $T = 10 + 22e^{-0.05t}$

54) A thermometer is taken outdoors of temperature 0°C from a room of temperature 21°C and the reading on the thermometer drops to 10°C in 5 minutes and satisfies sufficiently the differential equation $\frac{dT}{dt} = -0.7419T$. What is its primitive?

a) $T = 21e^{-0.7419t}$ b) $T = 21 - 10e^{0.7419t}$
c) $T = 10 + 21e^{0.7419t}$ d) $T = 21e^{0.7419t}$

55) A metal body of mass 5 kg is heated to a temperature upto 100°C exactly and then, at time considered to be t = 0, it is immersed in oil of temperature 30°C. In just 3 minutes, the temperature of body drops to 70°C in 3 minute and satisfies $\frac{d\theta}{dt} = -\frac{\theta - 30}{3}\log\left(\frac{7}{4}\right)$. What is time taken to drop temperature of body to 31°C.

a) 15.28 min b) 12.78 min
c) 32.78 sec d) 22.78 min

56) If the temperature of body drops down to 70°C from 100°C in 15 minute, and satisfying the Newton's law of cooling $\frac{d\theta}{dt} = -k(\theta - 30)$, the value of k is

a) $\frac{1}{15}\log\frac{7}{4}$ b) $-\frac{1}{15}\log\frac{7}{4}$
c) $15\log\frac{7}{4}$ d) $-15\log\frac{7}{4}$

57) A metal ball of temperature 100°C is placed in air conditioned room of temperature 20°C. The temperature drops by 40°C in 5 minute. Its differential equation in accordance with Newton's law of cooling is given by $\frac{dT}{dt} = -\frac{T-20}{5}\log 2$. The temperature after 8 minute is

a) 6.44 b) 64.4 c) 46.4 d) 44.6

58) A body cools down from 80°C to 60°C from 1.00 PM to 1.20 PM in a room of temperature 40°C and satisfies the differential equation $\frac{d\theta}{dt} = -0.03465(\theta - 40)$. The temperature of body at 1.40 PM is

a) 45 b) 50 c) 55 d) 60

59) The temperature of body cooling down from 100°C to 60°C in 60 seconds when it is kept in the air surrounding of constant temperature 20°C and satisfies the equation $\frac{d\theta}{dt} = -k(\theta - 20)$. The value of k is then

a) $\log 2$ b) $\log 3$ c) $\log 4$ d) $\log 5$

60) A metal ball made by brass of mass 50 gm cools down from 80°C to 60°C after a recorded time of 20 minute in air atmosphere of 40°C. The differential equation is $\frac{d\theta}{dt} = -k(\theta - 40)$. What is the value of k?

a) $-\frac{3}{20}\log_e 2$ b) $-20\log_e 2$
c) $\frac{1}{20}\log 2$ d) $-\frac{1}{20}\log_e 2$

61) A body of temperature 90°C is placed in water of temperature 30°C for 6 minute and then its temperature calculated is to be just 50°C. The Newton's law of cooling is $\frac{d\theta}{dt} = -k(\theta - 40)$. Then what of followings is correct.

a) $k = \frac{1}{6}\log_e\frac{1}{3}$ b) $k = \frac{1}{6}\log_e 3$
c) $k = -\frac{1}{6}\log_e 2$ d) $k = -\frac{1}{6}\log_e\frac{1}{4}$

62) An iron ball is heated for temperature 100°C is placed in water of temperature 50°C at t = 0 and at t = 5 minute then its temperature calculated which is read to be 70°C. The Newton's law of cooling is $\frac{d\theta}{dt} = -k(\theta - 50)$. Then what of followings is correct?

a) $k = -\frac{3}{4}\log_e\frac{2}{5}$ b) $k = \frac{1}{5}\log_e\frac{2}{5}$
c) $k = -\frac{2}{5}\log_e\frac{1}{5}$ d) $k = -\frac{1}{5}\log_e\frac{2}{5}$

63) A circuit consisting of resistance R, inductance L connected in series with voltage of amount E. By Kirchhoff's law, the differential equation for the current i in terms of t is

a) $L\frac{di}{dt} + \frac{i}{R} = E$ b) $L\frac{di}{dt} + Ri = E$

c) $L\frac{di}{dt}+Ri=0$ d) $R\frac{di}{dt}+Li=E$

64) A circuit consisting of resistance R, inductance L connected in series without voltage of amount E. By Kirchhoff's law, the differential equation for the current i in terms of t is

a) $L\frac{di}{dt}+\frac{i}{R}=E$ b) $L\frac{di}{dt}+Ri=E$

c) $L\frac{di}{dt}+Ri=0$ d) $R\frac{di}{dt}+Li=E$

65) An electrical circuit is consisting of inductance L, capacitance C in series with voltage source E. By Kirchhoff's law, we have

a) $L\frac{dq}{dt}+\frac{q}{C}=E$ b) $L\frac{dq}{dt}+\frac{q}{R}=E$

c) $C\frac{di}{dt}+\frac{i}{R}=E$ d) $\frac{di}{dt}+\frac{i}{C}=ER$

66) An electrical circuit is consisting of resistance R, capacitance C in series with voltage source E. By Kirchhoff's law, we have

a) $R\frac{dq}{dt}+\frac{q}{C}=E$ b) $L\frac{dq}{dt}+\frac{q}{R}=E$

c) $C\frac{di}{dt}+\frac{i}{R}=E$ d) $\frac{di}{dt}+\frac{i}{C}=ER$

67) A circuit consisting of resistance R, inductance L connected in series with voltage of amount $E\cos\omega t$. By Kirchhoff's law, the differential equation for the current i in terms of t is

a) $L\frac{di}{dt}+\frac{i}{R}=E\cos\omega t$ b) $L\frac{di}{dt}+Ri=E\cos\omega t$

c) $L\frac{di}{dt}+Ri=0$ d) $R\frac{di}{dt}+Li=E\cos\omega t$

68) The differential equation for the current i in an electrical circuit consisting of inductance L, resistance R in series with electromotive force of Ee^{-at} is given by

a) $\frac{di}{dt}+Ri=\frac{E}{L}e^{-at}$ b) $L\frac{di}{dt}+Ri=Ee^{-at}$

c) $L\frac{di}{dt}+\frac{i}{R}=Ee^{-at}$ d) $R\frac{di}{dt}+Li=Ee^{-at}$

69) The differential equation for the current i in an electrical circuit composing of resistance of 120 ohm and an inductance of 0.7 henry connected in series with battery of 30 volt is

a) $0.7\frac{di}{dt}-120i=30$ b) $120\frac{di}{dt}+0.7i=30$

c) $0.7\frac{di}{dt}+120i=30$ d) $0.7\frac{di}{dt}+\frac{i}{120}=30$

70) The differential equation for the current i in an electrical circuit composing of resistance of 200 ohm and an inductance of 100 henry connected in series with battery of 440 volt is

a) $20\frac{di}{dt}+10i=44$ b) $\frac{di}{dt}+2i=40$

c) $5\frac{di}{dt}+10i=44$ d) $10\frac{di}{dt}+20i=44$

71) A capacitance of 0.03 farad and resistance of 10 ohm in series with electromotive force of 20 volts are in a circuit. If initially the capacitor is totally discharged, the differential equation for the charge q is

a) $10\frac{dq}{dt}+\frac{q}{0.03}=20;\ \ q(0)=0$

b) $\frac{dq}{dt}+\frac{q}{0.03}=2;\ \ q(0)=0$

c) $\frac{dq}{dt}+\frac{q}{0.3}=2;\ \ q(0)=0$

d) $10\frac{dq}{dt}+0.03q=20;\ \ q(0)=0$

72) In an electrical circuit of R and L in series with steady EMF, the current i satisfies the equation $i=\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$. The time required for the maximum value is

a) 0 b) $\frac{L}{R}\log 10$

c) $-\frac{L}{R}\log 90$ d) $\frac{E}{R}\log 10$

73) In an electrical circuit of R and L in series with steady EMF, the current i satisfies the equation $i=\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$. The time required for the current gets 90% of maximum value is

a) 0 b) $\frac{L}{R}\log 2$

c) $-\frac{L}{R}\log 2$ d) $\frac{E}{R}\log 2$

74) If the differential equation for the current i is $R\frac{di}{dt}+Ri=E$, the current i at time t is

a) $i=\frac{E}{R}+ce^{-\frac{R}{L}t}$ b) $iR=1-cEe^{-\frac{R}{L}t}$

c) $i=\frac{E}{R}+ce^{\frac{R}{L}t}$ d) $i=\frac{E}{R}ce^{-\frac{R}{L}t}$

75) A charge q on the plate of condenser of capacity C through resistance R in series with steady state EMF V volt satisfies the differential equation $R\frac{dq}{dt}+\frac{q}{C}=V$. Then q in terms of t is

a) $q=\frac{C}{V}+ke^{-\frac{t}{RC}}$ b) $q=CV+ke^{\frac{t}{RC}}$

c) $q=CVke^{-\frac{t}{RC}}$ d) $q=CV+ke^{-\frac{t}{RC}}$

76) A charge q on the plate of condenser of capacity C through resistance R in series with steady state EMF V volt satisfies the equation $q=CV\left(1-e^{-\frac{t}{RC}}\right)$. Then i in terms of t is

a) $i=\frac{V}{R}e^{-\frac{t}{RC}}$ b) $i=\frac{V}{R}+e^{-\frac{t}{RC}}$

c) $i=VRe^{-\frac{t}{RC}}$ d) $i=\frac{V}{R}e^{\frac{t}{RC}}$

77) The differential equation for the current i is given to be $0.5\frac{di}{dt}+100i=20$ for an electrical circuit containing resistance R = 100 ohm, inductance L = 0.5 henry in series. Then

a) $i=0.2+Ae^{200t}$ b) $i=20+Ae^{-200t}$

c) $i=0.2Ae^{-200t}$ d) $i=0.2+Ae^{-200t}$

78) If an electrical circuit of R-C in series, charge $q=q(t)$ as function of t is $q=e^{3t}-e^{6t}$, the time required for maximum charge on capacitor is given by

a) $\frac{1}{2}\log 3$ b) $\frac{2}{3}\log 2$

c) $\frac{1}{3}\log 2$ d) $\frac{1}{3}\log\frac{1}{2}$

79) An electrical circuit of resistance R, inductance L in series with an electromotive force of E is satisfying the differential equation for the current i as $L\frac{di}{dt}+Ri=E$. For L = 640 henry, R = 250 ohm, E = 500 volt, the integrating factor of the above equation is

a) $e^{\frac{64}{25}t}$ b) $e^{\frac{25}{64}t}$ c) $e^{-\frac{25}{64}t}$ d) $e^{-\frac{64}{25}t}$

80) In an electrical circuit of L = 640 H, R = 250 Ω and E = 500 with EMF of 20 volts, the differential equation is

a) $\frac{di}{dt}+\frac{64}{25}i=\frac{32}{25}$ b) $\frac{di}{dt}+\frac{64}{25}i=\frac{25}{32}$

c) $\frac{di}{dt}+\frac{25}{64}i=\frac{25}{32}$ d) $\frac{di}{dt}+\frac{25}{64}i=\frac{32}{25}$

81) Rectilinear motion is the motion of body along

a) straight line b) circular motion

c) curvilinear d) parabolic path

82) The algebraic sum of the forces acting on a body along a given direction is equal to

a) mass × total force b) mass × distance

c) mass × velocity d) mass × acceleration

83) A particle moving in a straight line with acceleration $k\left(x+\frac{a^4}{x^3}\right)$ is directed towards origin. Then the equation of motion is

a) $\frac{dv}{dx}=-kv\left(x+\frac{a^4}{x^3}\right)$ b) $v\frac{dv}{dt}=-k\left(x+\frac{a^4}{x^3}\right)$

c) $\frac{d^2x}{dt^2}=-k\left(x+\frac{a^4}{x^3}\right)$ d) $k\frac{d^2x}{dt^2}=\left(x+\frac{a^4}{x^3}\right)$

84) A body of mass m kg moves in straight line with acceleration $\frac{k}{x^3}$ at a distance x and directed towards center. Then

a) $v\frac{dv}{dx}=-\frac{k}{x^3}$ b) $\frac{dv}{dx}=v\frac{k}{x^3}$

c) $v\frac{dv}{dx}=\frac{k}{x^3}$ d) $v\frac{dv}{dt}=-\frac{k}{x^3}$

85) A body of mass m falling freely from rest under gravitational force of attraction and air resistance proportional to square of velocity kv^2. Then

a) $\frac{dv}{dx}=v\left(mg-kv^2\right)$ b) $v\frac{dv}{dx}=m\left(g-kv^2\right)$

c) $mv\frac{dv}{dx}=mg-kv^2$ d) $v\frac{dv}{dx}=g-kv^2$

86) A particle is projected vertically upward with initial velocity v_1 and resistance of air produces retardation kv^2 where v is velocity at time t. Then

a) $mv\frac{dv}{dx}=mg-kv^2$ b) $v\frac{dv}{dx}=-g-kv^2$

c) $v\frac{dv}{dx}=m\left(g-kv^2\right)$ d) $v\frac{dv}{dx}=g-kv^2$

87) A particle starts moving horizontally from rest is opposed by a force cx, resistance per unit mass of value bv^2, where v and x are velocity and displacement of body at time t. Then

a) $v\frac{dv}{dx}=cs+bv^2$ b) $v\frac{dv}{dx}=-cs+bv^2$

c) $v\frac{dv}{dx}=cs-bv^2$ d) $v\frac{dv}{dx}=-cs-bv^2$

88) A body of mass m falls from rest under gravity in a liquid having resistance to motion at time t is mk times velocity. Then

a) $\frac{dv}{dt}=g+kv$ b) $\frac{dv}{dt}=g-kv$

c) $\frac{dv}{dt}=-g-kv$ d) $\frac{dv}{dt}=-g+kv$

89) A particle of mass m is projected vertically upward with velocity v, assuming the air resistance k times its velocity. Then

a) $m\frac{dv}{dt}=-mg-kv$ b) $m\frac{dv}{dt}=-mg+kv$

c) $m\frac{dv}{dt}=mg-kv$ d) $m\frac{dv}{dt}=mg+kv$

90) Assuming that the resistance to movement of a ship through water in the form of $a^2+b^2v^2$, where v is the velocity. Then the differential equation for retardation of the ship moving with engine stopped is

a) $m\frac{dv}{dt}=a^2+b^2v^2$ b) $m\frac{dv}{dt}=-a^2+b^2v^2$

c) $m\frac{dv}{dt}=-a^2-b^2v^2$ d) $m\frac{dv}{dt}=a^2-b^2v^2$

91) The differential equation of motion of particle of mass m falls from rest under gravity in a fluid satisfies the equation $\frac{dv}{dt}=g-kv$, then

a) $t=-k\log\left(\frac{g}{g-kv}\right)$ b) $t=k\log\left(\frac{g}{g-kv}\right)$

c) $t=-\frac{1}{k}\log\left(\frac{g}{g-kv}\right)$ d) $t=\frac{1}{k}\log\left(\frac{g}{g-kv}\right)$

92) A body of mass m falling freely under gravity satisfies the equation $mv\frac{dv}{dx}=k\left(a^2-v^2\right)$ with condition $ka^2=mg$, then

a) $x=\frac{m}{2k}\log\left(a^2-v^2\right)$ b) $x=\frac{m}{2}k\log\left(\frac{a^2}{a^2-v^2}\right)$

c) $x=-\frac{m}{2k}\log\left(\frac{a^2}{a^2-v^2}\right)$ d) $x=\frac{m}{2k}\log\left(\frac{a^2}{a^2-v^2}\right)$

93) A body starts from rest with an acceleration $\frac{dv}{dt}=k\left(1-\frac{t}{T}\right)$. Then its velocity is

a) $v=k\left(t-\frac{t^2}{2T}\right)$ b) $\frac{v^2}{2}=k\left(t-\frac{t^2}{2T}\right)$

c) $v=-k\left(t-\frac{t^2}{2T}\right)$ d) $v=k\left(\frac{t}{2}-\frac{t^2}{T}\right)$

94) A particle of unit mass starts from rest with an acceleration $v\frac{dv}{dr}=-\frac{k}{r^3}$. If initially it was at rest at r = a, then

a) $v^2=-k\left(\frac{1}{r^2}-\frac{1}{a^2}\right)$ b) $v^2=k\left(\frac{1}{r^2}+\frac{1}{a^2}\right)$

c) $v^2=k\left(\frac{1}{r^2}-\frac{1}{a^2}\right)$ d) $v^2=k\left(a^2-r^2\right)$

95) A particle of mass m is subjected projected upward with velocity V with its equation of motion $m\frac{dv}{dt}=-mg-kv$, then the velocity at time t is

a) $t=\log\left(\frac{mg+kv}{mg+kV}\right)$ b) $t=\frac{m}{k}\log\left(\frac{mg+kv}{mg+kV}\right)$

c) $t=-\frac{m}{k}\log\left(\frac{mg+kv}{mg+kV}\right)$ d) $t=\frac{m}{k}\log\left(\frac{mg-kv}{mg-kV}\right)$

96) A particle of mass m falls freely from rest under gravitational force in fluid producing resistance to motion of amount mkv, where k is constant. The differential equation is $\frac{dv}{dt} = g - kv$, then its terminal velocity is

a) $-\frac{g}{k}$ b) gk c) $-gk$ d) $\frac{g}{k}$

97) A bullet is fired into a sand tank and satisfies the differential equation $\frac{dv}{dt} = -k\sqrt{v}$. If v_0 is its initial velocity, we have

a) $2\sqrt{v} = -kt + 2\sqrt{v_0}$ b) $2\sqrt{v} = -\left(kt + 2\sqrt{v_0}\right)$

c) $2\sqrt{v} = kt + 2\sqrt{v_0}$ d) $\sqrt{v} = kt - 2\sqrt{v_0}$

98) A particle is in motion of horizontal straight line with acceleration $k\left(x + \frac{a^4}{x^3}\right)$ directed towards its origin and satisfies the differential equation $v\frac{dv}{dt} = -k\left(x + \frac{a^4}{x^3}\right)$. Assuming that it starts from rest at a distance x = a from origin, we have

a) $v^2 = -k\left(x^2 - \frac{a^4}{x^2}\right)$ b) $v^2 = k\left(x^2 + \frac{a^4}{x^2}\right)$

c) $v^2 = k\left(x^2 - \frac{a^4}{x^2}\right)$ d) $v^2 = -k\left(2x^2 - \frac{a^4}{2x^2}\right)$

99) If a particle moves in a straight line so that the force acting on it is directed towards a fixed point in the line of motion and proportional to its displacement from the point, it is then known as

a) curvilinear motion
b) rectilinear motion
c) Simple harmonic motion
d) circular motion

100) If a particle execute SHM, then its differential equation is given by

a) $\frac{d^2x}{dt^2} = -\omega^2 x$ b) $\frac{d^2x}{dt^2} - \omega^2 x = 0$

c) $\frac{d^2x}{dt^2} = k\omega x^2$ d) $\frac{d^2x}{dt^2} = -\omega x^2$

101) Fourier's law of heat conduction states that, the quantity of heat flow across the area of cross section A is

a) inversely proportional to the product of A with temperature gradient
b) proportional to the difference of A with temperature gradient
c) proportional to the product of A with temperature gradient
d) proportional to the sum of A and temperature gradient

102) If q quantity of heat flow across the cross sectional area A and thickness dx per unit time where the difference between temperatures at the faces is dT, the by Fourier's heat law

a) $q = -k - A\frac{dT}{dx}$ b) $q = -kA\frac{dT}{dx}$

c) $q = kA\frac{dT}{dx}$ d) $q = -kA + \frac{dT}{dx}$

103) The differential equation of steady state heat conduction per unit time from unit length of pipe of uniform radius r_0 carrying steam of temperature T_0 and thermal conductivity k, if the pipe is covered with material in a constant surrounding temperature, is given by

a) $Q = -\frac{2kr}{\pi} \cdot \frac{dT}{dr}$ b) $Q = -kr\frac{dT}{dr}$

c) $Q = 2k\pi r\frac{dT}{dr}$ d) $Q = -2k\pi r\frac{dT}{dr}$

104) The difference equation for steady state heat loss in unit time from a spherical shell of thermal conductivity covered by insulating material and kept in surrounding of constant temperature during heat flow, is

a) $Q = -\frac{4\pi r^2}{k} \cdot \frac{dT}{dr}$ b) $Q = 4k\pi r^2\frac{dT}{dr}$

c) $Q = -4k\pi r^2\frac{dT}{dr}$ d) $Q = -2k\pi r\frac{dT}{dr}$

105) The differential equation for steady state heat loss per unit time from unit length of pipe covered with insulating material which is kept in constant surrounding temperature, is

$Q=-2k\pi r\dfrac{dT}{dr}$. Then the temperature T is given by

a) $T=-\dfrac{Q}{k}\log r+c$ b) $T=-\dfrac{Q}{2\pi k}\log\dfrac{1}{r}+c$

c) $T=\dfrac{Q}{2\pi k}\log r+c$ d) $T=-\dfrac{Q}{2\pi k}\log r+c$

106) The differential equation for heat conductivity in spherical shell is described by $Q=-4k\pi r^{2}\dfrac{dT}{dr}$. Then

a) $T=\dfrac{Q}{kr}+c$ b) $T=\dfrac{Q}{4\pi kr}+c$

c) $T=\dfrac{Q}{4\pi k}r+c$ d) $T=-\dfrac{Q}{4\pi kr}+c$

107) A pipe of 10 cm radius carries steam of 150ºC and covered with insulating material of thickness 5 cm with thermal conductivity k = 0.0025 and it is kept in surrounding of temperature 40ºC. The equation is $dT=-\dfrac{Q}{2\pi k}\cdot\dfrac{dx}{x}$. Then the heat loss is

a) $220\pi k\log 1.5$ b) $\dfrac{220k}{\log 1.5}$

c) $\dfrac{220\pi k}{\log 1.5}$ d) $\dfrac{110\pi k}{\log 1.5}$

108) Heat is flowing through a hollow pipe of diameter 10 cm and outer diameter 20 cm and it is covered by insulating material of k = 0.12 and kept in surrounding of 200ºC. The differential equation is being $dT=-\dfrac{Q}{2\pi k}\cdot\dfrac{dx}{x}$. Then the heat loss is

a) $\dfrac{300\pi k}{\log 2}$ b) $\dfrac{150\pi k}{\log 2}$

c) $-\dfrac{300\pi k}{\log 2}$ d) $\dfrac{300\pi k}{\log 0.2}$

109) Steam of temperature 200ºC is set into pipe of 20 cm diameter covered with material of 6 cm thickness in surrounding of 30ºC. The equation is $dT=-\dfrac{Q}{2\pi k}\cdot\dfrac{dx}{x}$. The heat loss is

a) $\dfrac{170\pi k}{\log 16}$ b) $\dfrac{170(2\pi k)}{\log 1.6}$

c) $\dfrac{170\pi k}{\log 1.6}$ d) $-\dfrac{170\pi k}{\log 1.6}$

110) Steam of 100ºC is flowing through pipe of diameter 10 cm covered with asbestos of 5 cm thick and thermal conductivity k = 0.0006. The outer temperature is being 30ºC and the differential equation is $dT=-\dfrac{Q}{2\pi k}\cdot\dfrac{dx}{x}$. What is the amount of heat loss?

a) $\dfrac{140\pi k}{\log 2}$ b) $70\pi k\log 2$

c) $\dfrac{70\pi k}{\log 2}$ d) $-\dfrac{70\pi k}{\log 2}$

111) A tank contains 50 liters of fresh water. Brine of 2 gm/liter flows into the tank at the rate of 2 liters/minute and mixed with uniform continuity and the same amount runs out with the same rate. If Q is total amount of salt present at time t in the brine, we have

a) $\dfrac{dQ}{dt}=-4+\dfrac{Q}{25}$ b) $\dfrac{dQ}{dt}=-4-\dfrac{Q}{25}$

c) $\dfrac{dQ}{dt}=4-\dfrac{Q}{25}$ d) $\dfrac{dQ}{dt}=4+\dfrac{Q}{25}$

112) A tank contains 10000 liters of Brine of 200 kg dissolve salt. Fresh water flows into the tank at the rate of 100 liters/minute and mixed with uniform continuity and the same amount runs out with the same rate. If Q is total amount of salt present at time t in the brine, we have

a) $\dfrac{dQ}{dt}=200+\dfrac{Q}{100}$ b) $\dfrac{dQ}{dt}=-\dfrac{Q}{100}$

c) $\dfrac{dQ}{dt}=200-\dfrac{Q}{100}$ d) $\dfrac{dQ}{dt}=\dfrac{Q}{100}$

113) A tank contains 100 liters of fresh water. Brine of 1 gm/liter flows into the tank at the rate of 2 liters/minute and mixed with uniform continuity and the same amount runs out with the same rate. If Q is total amount of salt present at time t in the brine, we have

a) $\dfrac{dQ}{dt}=-\dfrac{Q}{100+t}$ b) $\dfrac{dQ}{dt}=2+\dfrac{Q}{100+t}$

c) $\dfrac{dQ}{dt}=2-\dfrac{Q}{100+t}$ d) $\dfrac{dQ}{dt}=2-\dfrac{Q}{100t}$

114) A tank contains 10000 liters of Brine of 20 kg dissolve salt. Brine of 0.1 kg/liter flows into the tank at the rate of 40 liters/minute and mixed with uniform continuity and the same amount runs out with the rate 30 liters/minute. If Q is total amount of salt present at time t in the brine, we have

a) $\frac{dQ}{dt}=4-\frac{3Q}{1000+10t}$ b) $\frac{dQ}{dt}=4-\frac{30Q}{100+t}$

c) $\frac{dQ}{dt}=-\frac{3Q}{100+t}$ d) $\frac{dQ}{dt}=4-\frac{3Q}{100+t}$

115) A tank contains 5000 liters of fresh water. Brine of 100 gm/liter flows into the tank at the rate of 10 liters/minute and mixed with uniform continuity and the same amount runs out with the same rate. If Q is total amount of salt present at time t in the brine, we have

a) $\frac{dQ}{dt}=\frac{5000-Q}{500}$ b) $\frac{dQ}{dt}=5000-\frac{Q}{500}$

c) $\frac{dQ}{dt}=1000+\frac{Q}{5}$ d) $\frac{dQ}{dt}=1000-\frac{Q}{500}$

116) A tank contains 10000 liters of Brine of 200 kg dissolve salt. Fresh water flows into the tank at the rate of 100 liters/minute and mixed with uniform continuity and the same amount runs out with the same rate. If Q is total amount of salt present at time t in the brine, we have $\frac{dQ}{dt}=-\frac{Q}{100}$. Then

a) $\log Q=-\frac{t}{100}$

b) $\log Q=-\frac{t}{100}-\log 200$

c) $\log Q=-\frac{t}{100}+\log 200$

d) $\log Q=\frac{t}{100}+\log 200$

117) A tank contains 50 liters of fresh water. Brine of 2 gm/liter flows into the tank at the rate of 2 liters/minute and mixed with uniform continuity and the same amount runs out with the same rate. If Q is total amount of salt present at time t, we have $\frac{dQ}{dt}=4-\frac{Q}{25}$. Then

a) $t=50\log 10-25\log(100-Q)$

b) $t=25\log 10-25\log(100-Q)$

c) $t=50\log 10+25\log(100-Q)$

d) $t=25\log 10+25\log(100-Q)$

118) The rate of decay of a substance is directly proportional to the amount of substance present at that time. Hence

a) $\frac{dt}{dx}=-kx$ b) $\frac{dx}{dt}=-kx$

c) $\frac{dx}{dt}=-kx+t$ d) $\frac{dx}{dt}=-kx^2+c$

Unit I : Ordinary Differential Equations

1	A	41	B	81	A	121	B	161	D	201	C	241	D
2	C	42	A	82	B	122	A	162	D	202	D	242	C
3	C	43	C	83	D	123	C	163	C	203	D	243	B
4	A	44	A	84	B	124	B	164	B	204	A	244	C
5	C	45	D	85	B	125	C	165	D	205	C	245	A
6	B	46	C	86	D	126	B	166	B	206	B	246	C
7	A	47	A	87	A	127	C	167	C	207	D	247	B
8	D	48	C	88	D	128	D	168	A	208	D	248	D
9	C	49	B	89	B	129	C	169	C	209	A	249	B
10	B	50	C	90	B	130	B	170	B	210	A	250	D
11	C	51	D	91	A	131	A	171	B	211	C	251	A
12	B	52	C	92	A	132	B	172	B	212	A	252	D
13	A	53	D	93	A	133	A	173	A	213	C	253	A
14	C	54	B	94	A	134	C	174	A	214	B	254	C
15	B	55	D	95	D	135	C	175	C	215	B	255	B
16	D	56	B	96	C	136	D	176	D	216	C	256	A
17	A	57	A	97	B	137	A	177	B	217	B	257	C
18	D	58	A	98	D	138	C	178	A	218	D	258	B
19	B	59	D	99	B	139	D	179	B	219	B	259	D
20	C	60	A	100	A	140	D	180	C	220	C	260	B
21	A	61	C	101	B	141	C	181	D	221	A	261	A
22	D	62	D	102	C	142	A	182	A	222	B	262	B
23	B	63	A	103	D	143	B	183	B	223	A	263	C
24	A	64	C	104	A	144	B	184	C	224	C		
25	D	65	B	105	B	145	D	185	A	225	B		
26	B	66	C	106	C	146	A	186	D	226	C		
27	D	67	B	107	A	147	D	187	A	227	D		
28	C	68	D	108	C	148	D	188	D	228	D		
29	A	69	C	109	D	149	C	189	C	229	C		
30	B	70	C	110	A	150	A	190	B	230	A		
31	A	71	A	111	D	151	C	191	C	231	D		
32	B	72	D	112	B	152	B	192	A	232	B		
33	B	73	D	113	D	153	A	193	D	233	A		
34	C	74	B	114	C	154	D	194	C	234	D		
35	B	75	B	115	A	155	A	195	A	235	C		
36	A	76	C	116	C	156	D	196	C	236	A		
37	A	77	D	117	D	157	C	197	B	237	D		
38	B	78	B	118	C	158	A	198	D	238	A		
39	C	79	D	119	B	159	B	199	B	239	D		
40	B	80	A	120	D	160	A	200	A	240	C		

Unit II : Applications of Ordinary Differential Equations

1	A	18	D	35	C	52	D	69	C	86	B	103	D
2	C	19	C	36	A	53	D	70	D	87	D	104	C
3	B	20	A	37	D	54	A	71	A	88	B	105	D
4	C	21	A	38	C	55	D	72	B	89	A	106	B
5	D	22	C	39	B	56	A	73	B	90	C	107	C
6	B	23	C	40	D	57	C	74	A	91	D	108	A
7	D	24	D	41	A	58	B	75	D	92	D	109	B
8	C	25	B	42	C	59	A	76	A	93	A	110	A
9	B	26	C	43	D	60	C	77	D	94	C	111	C
10	A	27	C	44	B	61	B	78	C	95	B	112	B
11	C	28	D	45	D	62	D	79	B	96	D	113	C
12	D	29	A	46	B	63	B	80	C	97	A	114	D
13	D	30	B	47	C	64	C	81	A	98	A	115	D
14	A	31	A	48	A	65	A	82	D	99	C	116	C
15	A	32	B	49	C	66	A	83	C	100	A	117	A
16	C	33	C	50	B	67	B	84	A	101	C	118	B
17	B	34	D	51	B	68	B	85	C	102	B		

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First Year Degree Course in Engineering – Semester II

Engineering Mathematics (M II)

Savitribai Phule Pune University

Second Online Examination

First Year of Engineering

Dr. Chavan N. S.

2015-16

Savitribai Phule Pune University – FE – Sem. II

Engineering Mathematics (M II)

Chapter 03 – Fourier Series

1) A function $f(x)$ is said to be periodic function with a period T, if
   a) $f(x)=f(x+T), \quad \textit{for all } x$
   b) $f(T)=f(x+T), \quad \textit{for all } x$
   c) $f(x)=-f(x+T), \quad \textit{for all } x$
   d) $f(x)=f\left(\frac{x}{T}\right), \quad \textit{for all } x$

2) A smallest positive number T satisfying $f(x)=f(x+T)$ is known as
   a) absolute function b) absolute time
   c) periodic time d)primitive period

3) If T is the fundamental period a function $f(x)$, which of the following is incorrect?
   a) $f(x)=f(x+nT), \quad n\in I$
   b) $f(x)=f(x+n+T), \quad n\in I$
   c) $f(x)=f(x-T)$
   d) $f(x)=f(x+T)$

4) If $f(x+nT)=f(x)$ where n is an integer and T is the smallest positive number, the fundamental period of $f(x)$ is
   a) T b) nT c) $2T$ d) $\frac{T}{2}$

5) If $f(x)$ is a periodic function of period T, then for $n\neq 0$, the function $f(nx)$ is a periodic function of period
   a) T b) T^n c) $\frac{T}{n}$ d) nT

6) The fundamental period of $\sin x$ is
   a) π b) 2π c) 3π d) $\frac{\pi}{2}$

7) The fundamental period of $\sin 2x$ is
   a) π b) 2π c) 3π d) $\frac{\pi}{2}$

8) The fundamental period of $\sin 4x$ is
   a) π b) 2π c) 3π d) $\frac{\pi}{2}$

9) The fundamental period of $\cos 3x$ is
   a) π b) $\frac{2\pi}{3}$ c) $\frac{3\pi}{2}$ d) 3π

10) The fundamental period of $\sin(-3x)$ is
   a) -3π b) 3π c) $-\frac{2\pi}{3}$ d) $\frac{2\pi}{3}$

11) The fundamental period of $\sin\left(-\frac{x}{2}\right)$ is
   a) π b) 2π c) 3π d) 4π

12) The fundamental period of $\cos(x+\pi)$ is
   a) π b) 2π c) 3π d) $\frac{\pi}{2}$

13) The fundamental period of $\sin\left(x+\frac{3\pi}{2}\right)$ is
   a) 2π b) $\frac{2\pi}{3}$ c) 3π d) π

14) The fundamental period of $\tan\left(3x+\frac{\pi}{2}\right)$ is
   a) 2π b) π c) 3π d) $\frac{\pi}{3}$

15) The fundamental period of $\sin\left(x+\frac{\pi}{6}\right)$ is
   a) π b) 2π c) 3π d) $\frac{\pi}{3}$

16) The fundamental period of $2\sin x$ is
   a) π b) 2π c) 3π d) 4π

17) The fundamental period of $\sin x\cos x$ is
   a) π b) 2π c) 3π d) 4π

18) The fundamental period of $\tan x$ is
a) 4π b) 3π c) 2π d) π

19) The fundamental period of $\tan 5x$ is
a) $\frac{\pi}{5}$ b) 5π c) 10π d) π

20) The fundamental period of $2\sec(-3x)$ is
a) $-\frac{2\pi}{3}$ b) $\frac{2\pi}{3}$ c) $\frac{3\pi}{2}$ d) $-\frac{3\pi}{2}$

21) The fundamental period of $\operatorname{cosec} 2x$ is
a) π b) 2π c) 3π d) $\frac{\pi}{2}$

22) A function $f(x)$ defined in the interval $[-a,\ a]$ is said to be even function, if
a) $f(-x)=-f(x)$ b) $f(2x)=2f(x)$
c) $f(-x)=f(x)$ d) $f(x)=-f(x)$

23) A function $f(x)$ defined in the interval $[-a,\ a]$ is said to be odd function, if
a) $f(x)=-f(x)$ b) $f(2x)=2f(x)$
c) $f(-x)=f(x)$ d) $f(-x)=-f(x)$

24) Which of the followings is an even function?
a) $\cosh x$ b) $x^3-\cos x$
c) $\tan 3x$ d) $e^x+\tan^2 x$

25) Which of the followings is an even function?
a) $\sin 3x$ b) $\tan x$ c) $\operatorname{cosec}^3 x$ d) $\tan^2 x$

26) Which of the followings is not an even function?
a) $\sin^3 x$ b) $\sin^2 x$ c) $\tan^2 x$ d) $\sec x$

27) Which of the followings is an odd function?
a) e^{-x} b) $\tan\frac{3x}{2}$
c) $\cos^3 x$ d) $\operatorname{cosec} 2x$

28) Which of the followings is an odd function?
a) $-e^x$ b) $-\tan^2 x$
c) $-\sin x$ d) $-\cos x$

29) Which of the followings is not an odd function?
a) $2\tan x$ b) $\tan^2 x$
c) $\tan x$ d) $\sin 3x$

30) Which of the followings is neither even nor an odd function?
a) $\operatorname{cosech} x$ b) $\tanh x$ c) e^x d) $\sinh x$

31) If $f(x)$ is to be constant function w.r.t. x, then $f(x)$ is
a)even function
b) odd function
c) both even and odd
d) neither even nor odd

32) If $f(x)=x^3+2x-\cos x$, the function $f(x)$ is
a) even function
b) odd function
c) both even and odd
d)neither even nor odd

33) If $f(x)=x^2-\sin^4 x\cdot e^{|x|}$, the function $f(x)$ is
a) even function
b) odd function
c) both even and odd
d) neither even nor odd

34) Which of the following statement is incorrect?
a) Product of even and odd function is an odd function.
b) Multiplication of even and odd function is an odd function.
c) Addition of even and odd function is an odd function.
d) Subtraction of two odd functions is an odd function.

35) Fourier series expansion of a function $f(x)$ defined on the interval $[c,\ c+2L]$ and having period $2L$ is given by
a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos\left(\frac{n\pi x}{L}\right)+b_n\sin\left(\frac{n\pi x}{L}\right)$
b) $a_0+\sum_{n=1}^{\infty} a_n\cos\left(\frac{n\pi x}{L}\right)+b_n\sin\left(\frac{n\pi x}{L}\right)$
c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos(n\pi Lx)+b_n\sin(n\pi Lx)$
d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos\left(\frac{nx}{L}\right)+b_n\sin\left(\frac{nx}{L}\right)$

36) Fourier series expansion of a function $f(x)$ defined on the interval $[0,\ 2\pi]$ and satisfying the Dirichlet's conditions is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\frac{nx}{2}+b_n\sin\frac{nx}{2}$

b) $\frac{a_0}{2}+2\pi\sum_{n=1}^{\infty}a_n\cos nx+b_n\sin nx$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx+b_n\sin nx$

d) $\frac{a_0}{2}+a_n\cos nx+b_n\sin nx$

37) If a function $f(x)$ is defined on the interval $[-\pi,\ \pi]$ and satisfying the Dirichlet's conditions, Fourier series expansion is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{n\pi x}{L}\right)+b_n\sin\left(\frac{n\pi x}{L}\right)$

b) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{nx}{L}\right)+b_n\sin\left(\frac{nx}{L}\right)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{2n\pi x}{L}\right)+b_n\sin\left(\frac{2n\pi x}{L}\right)$

d) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(nx)+b_n\sin(nx)$

38) If a function $f(x)$ is defined on the interval $[0,\ 4]$ with period $T=4$, Fourier series expansion is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{n\pi x}{4}\right)+b_n\sin\left(\frac{n\pi x}{4}\right)$

b) $\frac{a_0}{2}+a_n\cos\left(\frac{n\pi x}{2}\right)+b_n\sin\left(\frac{n\pi x}{2}\right)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{n\pi x}{2}\right)+b_n\sin\left(\frac{n\pi x}{2}\right)$

d) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{2n\pi x}{L}\right)+b_n\sin\left(\frac{2n\pi x}{L}\right)$

39) Fourier series expansion of a function $f(x)$ defined over a period 2π and satisfying the Dirichlet's conditions is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\sin nx+b_n\cos nx$

b) $\frac{a_0}{2}+2\pi\sum_{n=1}^{\infty}a_n\cos nx+b_n\sin nx$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx+b_n\sin nx$

d) $\frac{a_0}{2}+a_n\cos nx+b_n\sin nx$

40) If an even function $f(x)$ with period $2l$ is defined over the interval $(-l,\ l)$, its Fourier series expansion is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\sin\left(\frac{n\pi x}{l}\right)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{n\pi x}{l}\right)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{n\pi x}{2l}\right)$ d) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{nx}{l}\right)$

41) If an odd function $f(x)$ is defined over the interval $(-\pi,\ \pi)$, its Fourier series expansion is given by

a) $\sum_{n=1}^{\infty}b_n\sin\left(\frac{2nx}{l}\right)$ b) $\sum_{n=1}^{\infty}a_n\sin(nx)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\sin(nx)$ d) $\sum_{n=1}^{\infty}b_n\sin(nx)$

42) If an odd function $f(x)$ is of period 2π, its Fourier series expansion is given by

a) $\sum_{n=1}^{\infty}b_n\sin\left(\frac{2nx}{l}\right)$ b) $\sum_{n=1}^{\infty}a_n\sin(nx)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\sin(nx)$ d) $\sum_{n=1}^{\infty}b_n\sin(nx)$

43) The Fourier series expansion of an even function $f(x)$ with period 2π is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\sin(nx)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(nx)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{nx}{2}\right)$ d) $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{nx}{l}\right)$

44) If an odd function $f(x)$ with period $2l$ is defined over the interval $(-l,\ l)$, its Fourier series expansion is given by

a) $\sum_{n=1}^{\infty}b_n\sin\left(\frac{nx}{l}\right)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty}b_n\sin\left(\frac{n\pi x}{l}\right)$

c) $\sum_{n=1}^{\infty}b_n\sin\left(\frac{n\pi x}{l}\right)$ d) $\sum_{n=1}^{\infty}a_n\sin\left(\frac{n\pi x}{l}\right)$

45) If $f(x)$ is periodic function with period $2L$ in the interval C to $C+2L$, the Fourier coefficient a_0 is

a) $\frac{1}{L}\int_{C}^{C+2L} f(x)\cos\frac{nx}{L}dx$ b) $\frac{1}{L}\int_{C}^{C+2L} f(x)\sin\frac{nx}{L}dx$

c) $\int_{C}^{C+2L} f(x)dx$ d) $\frac{1}{L}\int_{C}^{C+2L} f(x)dx$

46) If $f(x)$ is periodic function with period $2L$ in the interval C to $C+2L$, the Fourier coefficient a_n is

a) $\frac{1}{L}\int_{C}^{C+2L} f(x)\cos\frac{2n\pi x}{L}dx$

b) $\frac{1}{L}\int_{C}^{C+2L} f(x)\cos\frac{n\pi x}{L}dx$

c) $\frac{1}{L}\int_{C}^{C+2L} f(x)\cos\frac{nx}{L}dx$

d) $\frac{1}{2L}\int_{C}^{C+2L} f(x)\cos\frac{nx}{L}dx$

47) If $f(x)$ is periodic function with period $2L$ in the interval C to $C+2L$, the Fourier coefficient b_n is

a) $\frac{1}{L}\int_{C}^{C+2L} f(x)\sin\frac{\pi x}{L}dx$

b) $\frac{1}{L}\int_{C}^{C+2L} f(x)\sin\frac{nx}{L}dx$

c) $\frac{1}{L}\int_{C}^{C+2L} f(x)\sin\frac{n\pi x}{L}dx$

d) $\frac{1}{L}\int_{0}^{2L} f(x)\sin\frac{n\pi x}{L}dx$

48) If $f(x)$ is an even function defined in the interval $[-L, L]$ and $f(x)=f(x+2L)$, the Fourier coefficient are

a) $a_0=\frac{2}{L}\int_{0}^{L} f(x)dx,\ a_n=\frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}dx,\ b_n=0$

b) $a_0=\frac{1}{L}\int_{0}^{L} f(x)dx,\quad a_n=\frac{1}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}dx$

c) $a_0=0,\quad a_n=\frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}dx$

d) $a_0=\frac{2}{L}\int_{0}^{L} f(x)dx,\quad a_n=\frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L}dx$

49) If $f(x)$ is an odd function defined in the interval $[-L, L]$ and $f(x)=f(x+2L)$, the Fourier coefficient are

a) $a_0=0,\quad a_n=0,\quad b_n=\frac{1}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L}dx$

b) $a_0=0,\quad a_n=0,\quad b_n=\frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L}dx$

c) $a_0=0,\quad a_n=0,\quad b_n=\frac{2}{L}\int_{0}^{2L} f(x)\sin\frac{n\pi x}{L}dx$

d) $a_0=0,\quad a_n=0,\quad b_n=\frac{2}{L}\int_{0}^{L} f(x)\sin\frac{nx}{L}dx$

50) If $f(x)$ is an even periodic function defined in the interval $[-\pi, \pi]$, the Fourier coefficient are

a) $a_0=\frac{2}{\pi}\int_{0}^{\pi} f(x)dx,\ a_n=\frac{2}{\pi}\int_{0}^{\pi} f(x)\cos nx\,dx,\ b_n=0$

b) $a_0=\frac{2}{L}\int_{0}^{\pi} f(x)dx,\ a_n=\frac{2}{L}\int_{0}^{\pi} f(x)\cos\frac{nx}{L}dx,\ b_n=0$

c) $a_0=\frac{2}{\pi}\int_{0}^{\pi} f(x)dx,\ a_n=\frac{2}{\pi}\int_{0}^{\pi} f(x)\sin nxdx,\ b_n=0$

d) $a_0=\frac{1}{\pi}\int_{0}^{\pi} f(x)dx,\ a_n=\frac{1}{\pi}\int_{0}^{\pi} f(x)\cos nx\,dx,\ b_n=0$

51) If $f(x)$ is an odd periodic function defined in the interval $[-\pi, \pi]$, the Fourier coefficient are

a) $a_0=0,\ a_n=\frac{2}{\pi}\int_{0}^{\pi} f(x)\cos nx\,dx,\ b_n=0$

b) $a_0=\frac{2}{\pi}\int_{0}^{\pi} f(x)dx,\ a_n=\frac{2}{\pi}\int_{0}^{\pi} f(x)\cos nx\,dx,\ b_n=0$

c) $a_0=0,\ a_n=0,\ b_n=\frac{2}{\pi}\int_{0}^{\pi} f(x)\sin nx\,dx$

d) $a_0 = \frac{1}{\pi}\int_0^{\pi} f(x)\,dx,\ a_n = \frac{1}{\pi}\int_0^{\pi} f(x)\cos nx\,dx,\ b_n = 0$

52) The Fourier coefficient of an even periodic function $f(x)$ defined in the interval $[-2, 2]$ are

a) $a_0 = \int_0^2 f(x)\,dx,\ a_n = \int_0^2 f(x)\cos nx\,dx,\ b_n = 0$

b) $a_0 = \int_0^2 f(x)\,dx,\ a_n = \int_0^2 f(x)\cos\frac{n\pi x}{2}\,dx,\ b_n = 0$

c) $a_0 = \frac{2}{\pi}\int_0^2 f(x)\,dx,\ a_n = \frac{2}{\pi}\int_0^2 f(x)\cos nx\,dx,\ b_n = 0$

d) $a_0 = \int_0^2 f(x)\,dx,\ a_n = \int_0^2 f(x)\cos nx\,dx,\ b_n = 0$

53) The Fourier coefficient of an even periodic function $f(x)$ defined in the interval $[-1, 1]$ are

a) $a_0 = \frac{2}{\pi}\int_0^1 f(x)\,dx,\ a_n = \frac{2}{\pi}\int_0^1 f(x)\cos n\pi x\,dx,\ b_n = 0$

b) $a_0 = 2\int_0^2 f(x)\,dx,\ a_n = 2\int_0^2 f(x)\cos\frac{n\pi x}{2}\,dx,\ b_n = 0$

c) $a_0 = 2\int_0^1 f(x)\,dx,\ a_n = 2\int_0^1 f(x)\cos n\pi x\,dx,\ b_n = 0$

d) $a_0 = \int_0^1 f(x)\,dx,\ a_n = \int_0^1 f(x)\cos n\pi x\,dx,\ b_n = 0$

54) The Fourier coefficient of an odd periodic function $f(x)$ defined in the interval $[-4, 4]$ are

a) $a_0 = 0,\ \ a_n = 0,\ \ b_n = \frac{1}{2}\int_0^4 f(x)\cos\frac{n\pi x}{4}\,dx$

b) $a_0 = 0,\ \ a_n = 0,\ \ b_n = \frac{1}{4}\int_0^L f(x)\sin n\pi x\,dx$

c) $a_0 = 0,\ \ a_n = 0,\ \ b_n = 2\int_0^4 f(x)\sin\frac{n\pi x}{4}\,dx$

d) $a_0 = 0,\ \ a_n = 0,\ \ b_n = \frac{1}{2}\int_0^4 f(x)\sin\frac{n\pi x}{4}\,dx$

55) If the Fourier series expansion of an even function $f(x)$ over an interval $[-l, l]$ is given by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$, the value of a_0 is obtained by

a) $\frac{2}{l}\int_{-l}^{l} f(x)\,dx$ b) $\frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$

c) $\frac{1}{2l}\int_0^l f(x)\,dx$ d) $\frac{2}{l}\int_0^l f(x)\,dx$

56) If the Fourier series expansion of an even function $f(x)$ over an interval $[-l, l]$ is given by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$, the value of a_n is obtained by

a) $\frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}\,dx$ b) $\frac{1}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$

c) $\frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$ d) $\frac{1}{l}\int_0^l f(x)\cos\frac{nx}{l}\,dx$

57) If the Fourier series expansion of an even function $f(x)$ over an interval $[-\pi, \pi]$ is given by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$, the value of a_0 is obtained by

a) $\frac{2}{\pi}\int_0^{\pi} f(x)\,dx$ b) $\frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$

c) $\frac{1}{\pi}\int_0^{\pi} f(x)\,dx$ d) $\frac{2}{\pi}\int_0^{\pi} f(x)\cos nx dx$

58) If the Fourier series expansion of an even function $f(x)$ over an interval $[-\pi, \pi]$ is given by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$, the value of a_n is obtained by

a) $\frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$ b) $\frac{2}{\pi}\int_0^{\pi} f(x)\cos nx dx$

c) $\frac{1}{\pi}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$ d) $\frac{2}{\pi}\int_0^{\pi} f(x)\cos\frac{nx}{\pi}\,dx$

59) If the Fourier series expansion of an odd function $f(x)$ over an interval $[-l, l]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$, the value of b_0 is obtained by

a) $\int_0^l f(x)\cos\frac{n\pi x}{l}dx$ b) $\frac{1}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}dx$

c) $\frac{2}{l}\int_0^l f(x)dx$ d) none of the above

60) If the Fourier series expansion of an odd function $f(x)$ over an interval $[-l, l]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$, the value of b_n is obtained by

a) $\frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}dx$ b) $\frac{1}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}dx$

c) $\frac{1}{l}\int_0^l f(x)\sin\frac{nx}{l}dx$ d) $\int_0^l f(x)\cos\frac{n\pi x}{l}dx$

62) The half range Fourier cosine series for $f(x)$ defined over the interval $[0,\ L]$ is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{nx}{L}\right)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right)$

c) $a_0+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$ d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$

63) The half range Fourier sine series for $f(x)$ defined over the interval $[0,\ L]$ is given by

a) $\sum_{n=1}^{\infty} a_n \sin\left(\frac{nx}{L}\right)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right)$

c) $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$ d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$

64) The half range Fourier cosine series for $f(x)$ defined over the interval $[0,\ \pi]$ is given by

a) $\sum_{n=1}^{\infty} b_n \sin(nx)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$ d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)$

65) The half range Fourier sine series for $f(x)$ defined over the interval $[0,\ \pi]$ is given by

a) $\sum_{n=1}^{\infty} b_n \sin\left(\frac{nx}{\pi}\right)$ b) $\sum_{n=1}^{\infty} b_n \sin(nx)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right)$ d) $\sum_{n=1}^{\infty} a_n \sin(n\pi x)$

66) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, L]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$ the value of a_0 is given by

a) $\frac{1}{L}\int_0^L f(x)dx$ b) $\frac{1}{L}\int_0^L f(x)dx$

c) $\frac{2}{\pi}\int_0^\pi f(x)dx$ d) $\frac{2}{L}\int_0^L f(x)dx$

67) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, L]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$ the value of a_n is given by

a) $\frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$

b) $\frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

c) $\frac{2}{L}\int_0^L f(x)\cos(nx)dx$

d) $\frac{1}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

68) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, L]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$ the value of b_0 is given by

a) $\frac{1}{L}\int_0^L f(x)\sin\frac{x}{L}dx$ b) $\frac{2}{L}\int_0^L f(x)\sin x\,dx$

c) 0 d) $\frac{2}{L}\int_0^L f(x)dx$

69) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, L]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$ the value of b_n is given by

a) $\frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$

b) $\frac{1}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$

c) $\frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

d) $\frac{2}{L}\int_0^L f(x)\sin(nx)\,dx$

70) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, \pi]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)$ the value of a_0 is given by

a) $\frac{1}{L}\int_0^L f(x)\,dx$ b) $\frac{1}{L}\int_0^L f(x)\,dx$

c) $\frac{2}{\pi}\int_0^\pi f(x)\,dx$ d) $\frac{2}{L}\int_0^L f(x)\,dx$

71) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, \pi]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)$ the value of a_n is given by

a) $\frac{2}{L}\int_0^L f(x)\sin(nx)\,dx$

b) $\frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

c) $\frac{2}{\pi}\int_0^\pi f(x)\cos(nx)\,dx$

d) $\frac{1}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

72) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, \pi]$ is given by $\sum_{n=1}^{\infty} b_n \sin(nx)$ the value of b_0 is given by

a) $\frac{1}{L}\int_0^L f(x)\sin\frac{x}{L}\,dx$ b) $\frac{2}{L}\int_0^L f(x)\sin x\,dx$

c) 0 d) $\frac{2}{L}\int_0^L f(x)\,dx$

73) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, \pi]$ is given by $\sum_{n=1}^{\infty} b_n \sin(nx)$ the value of b_n is given by

a) $\frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$

b) $\frac{1}{L}\int_0^L f(x)\sin(nx)\,dx$

c) $\frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

d) $\frac{2}{\pi}\int_0^\pi f(x)\sin(nx)\,dx$

74) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, 1]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(n\pi x)$ the value of a_0 is given by

a) $\frac{1}{\pi}\int_0^\pi f(x)\,dx$ b) $2\int_0^1 f(x)\,dx$

c) $\frac{2}{\pi}\int_0^\pi f(x)\,dx$ d) $\int_0^1 f(x)\,dx$

75) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, 2]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{2}\right)$ the value of a_n is given by

a) $\frac{2}{3}\int_0^2 f(x)\sin\left(\frac{n\pi x}{2}\right)dx$

b) $\frac{1}{2}\int_0^2 f(x)\cos\left(\frac{n\pi x}{2}\right)dx$

c) $\frac{2}{L}\int_0^2 f(x)\cos(nx)\,dx$

d) $\int_0^2 f(x)\cos\left(\frac{n\pi x}{2}\right)dx$

76) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, 3]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{3}\right)$ the value of b_0 is given by

a) $\frac{1}{3}\int_0^3 f(x)\sin\frac{x}{3}dx$ b) $\frac{2}{3}\int_0^3 f(x)\sin 3x\,dx$

c) 0 d) $\frac{2}{3}\int_0^3 f(x)dx$

77) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, 4]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{4}\right)$ the value of b_n is given by

a) $\frac{2}{3}\int_0^2 f(x)\sin\left(\frac{n\pi x}{4}\right)dx$

b) $\frac{1}{2}\int_0^4 f(x)\sin\left(\frac{n\pi x}{4}\right)dx$

c) $\int_0^2 f(x)\cos\left(\frac{n\pi x}{4}\right)dx$

d) $\frac{1}{2}\int_0^4 f(x)\sin(nx)dx$

78) In the harmonic analysis for a function defined over a period of 2π, the term $a_1\cos x + b_1\sin x$ is known as

a) amplitude of $f(x)$ b) second harmonic
c) first harmonic d) none of these

79) In the harmonic analysis for a function defined over a period of 2π, the amplitude of the first harmonic is

a) $\sqrt{a_1^2 - b_1^2}$ b) $\sqrt{a_1^2 + b_1^2}$
c) $\sqrt{a_0^2 + a_1^2}$ d) $a_1^2 + b_1^2$

80) In the harmonic analysis for a function defined over a period of 2π, the amplitude of the second harmonic is

a) $\left(a_2^2 + b_2^2\right)^2$ b) $\frac{1}{2}\left(a_2^2 + b_2^2\right)$
c) $2\sqrt{a_2^2 + b_2^2}$ d) $\sqrt{a_2^2 + b_2^2}$

81) In the harmonic analysis for a function defined over a period of 2π, the amplitude of the second harmonic is

a) $\left(a_n^2 + b_n^2\right)^n$ b) $\sqrt{a_n^2 + b_n^2}$
c) $n\sqrt{a_n^2 + b_n^2}$ d) $\frac{1}{n}\sqrt{a_n^2 + b_n^2}$

82) In the harmonic analysis for a function $f(x)$ defined over a period of $2L$, the first harmonic term is given by

a) $b_1\sin\frac{\pi x}{L}$ b) $a_1\cos\frac{\pi x}{L}$
c) $a_1\cos\frac{\pi x}{L} - b_1\sin\frac{\pi x}{L}$ d) $a_1\cos\frac{\pi x}{L} + b_1\sin\frac{\pi x}{L}$

83) In the harmonic analysis for a function $f(x)$ defined over a period of 2, the first harmonic term is given by

a) $a_1\cos\pi x + b_1\sin\pi x$ b) $a_1\cos\frac{\pi x}{2} + b_1\sin\frac{\pi x}{2}$
c) $a_1\cos 2\pi x + b_1\sin 2\pi x$ d) $a_1\cos\frac{\pi x}{2} - b_1\sin\frac{\pi x}{2}$

84) If $f(x) = x\cos x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(nx) + b_n\sin(nx)$, the Fourier coefficient a_0 is given by

a) $-\frac{1}{\pi}$ b) 0 c) $\frac{1}{\pi}$ d) $\frac{2}{\pi}$

85) If $f(x) = x\cos x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(nx) + b_n\sin(nx)$, the Fourier coefficient a_n is given by

a) $\frac{1}{\pi}$ b) $-\frac{1}{\pi}$ c) $\frac{2}{\pi}$ d) 0

86) If $f(x) = x\cos x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(nx) + b_n\sin(nx)$, the Fourier coefficient b_1 is given by

a) $\frac{1}{\pi}$ b) $-\frac{1}{\pi}$ c) $-\frac{1}{2}$ d) 0

87) If $f(x)=\cos x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π, the Fourier coefficient a_1 is given by

a) 1 b) $-\frac{1}{\pi}$ c) $\frac{2}{\pi}$ d) 0

88) If $f(x)=x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient a_0 is given by

a) $\frac{\pi}{2}$ b) 0 c) 1 d) $\frac{\pi^2}{2}$

89) If $f(x)=x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient b_1 is given by

a) 2 b) 0 c) π d) $\frac{\pi}{2}$

90) If $f(x)=2$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient a_0 is given by

a) 2 b) 4 c) 3 d) none of these

91) If $f(x)=2$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient b_n is given by

a) $\frac{3\pi}{2}$ b) $-\frac{\pi}{2}$ c) 0 d) $\frac{\pi}{2}$

92) If $f(x)=a$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient a_0 is given by

a) $2a$ b) 0 c) 2π d) $\frac{\pi}{2}$

93) If $f(x)=a$ where $-\pi \le x \le \pi$ is of periodic function with period 2π, the Fourier coefficient a_0 is given by

a) 2π b) $2a$ c) 0 d) $\frac{\pi}{2}$

94) If $f(x)=\sin^2 x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient b_n is given by

a) $\frac{3\pi}{2}$ b) $-\frac{\pi}{2}$ c) $\frac{\pi}{2}$ d) 0

95) If $f(x)=\cosh x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient b_n is given by

a) 0 b) $\frac{\pi}{3}$ c) $e^{-\pi}$ d) $e^{-2\pi}$

96) If $f(x)=\begin{cases} -x & -\pi \le x \le 0 \\ x & 0 \le x \le \pi \end{cases}$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient a_0 is given by

a) $\frac{\pi}{2}$ b) π c) 0 d) $-\frac{\pi}{2}$

97) If $f(x)=\begin{cases} -x & -\pi \le x \le 0 \\ x & 0 \le x \le \pi \end{cases}$ is of periodic function with period 2π and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient b_n is given by

a) $\frac{\pi}{2}$ b) π c) 0 d) $-\frac{\pi}{2}$

98) If $f(x)=\begin{cases} 1+\frac{2x}{\pi} & -\pi \le x \le 0 \\ 1-\frac{2x}{\pi} & 0 \le x \le \pi \end{cases}$ is of periodic function with period 2π, the Fourier coefficient b_n is given by

a) $\frac{\pi}{2}$ b) π c) $-\frac{\pi}{2}$ d) 0

99) If $f(x)=x-x^3$ where $-2\le x\le 2$ is of periodic function with period 2 and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(nx)+b_n\sin(nx)$, the Fourier coefficient b_1 is given by

a) 2 b) 0 c) π d) $\frac{\pi}{2}$

100) If $f(x)=x+\frac{x^2}{4}$ where $-\pi\le x\le\pi$ is of periodic function with period 2π, the Fourier coefficient a_0 is given by

a) 0 b) $\frac{\pi}{2}$ c) $-\frac{\pi}{2}$ d) $\frac{\pi^2}{6}$

101) If $f(x)=e^x$ where $-\pi\le x\le\pi$ is of periodic function with period 2π , the Fourier coefficient a_0 is given by

a) 1 b) $\frac{e^{\pi}-e^{-\pi}}{\pi}$ c) $\frac{e^{\pi}+e^{-\pi}}{\pi}$ d) 0

102) If $f(x)=x-x^2$ where $-1\le x\le 1$ is of periodic function with period 2, the Fourier coefficient a_0 is given by

a) 1 b) $-\frac{2}{3}$ c) π d) 0

103) If $f(x)=1-x^2$ where $-1\le x\le 1$ is of periodic function with period 2, the Fourier coefficient a_0 is given by

a) 1 b) $\frac{2\pi}{3}$ c) $\frac{4}{3}$ d) 0

104) If $f(x)=k$ where $-l\le x\le l$ is of periodic function with period 2, the Fourier coefficient a_0 is given by

a) $2k$ b) $\frac{2k\pi}{3}$ c) $2k\pi$ d) 0

105) If $f(x)=\begin{cases}-a & -2\le x\le 0\\ a & 0\le x\le 2\end{cases}$ is of periodic function with period 4, the Fourier coefficient b_n is given by

a) 0 b) 4 c) $-\frac{\pi}{2}$ d) $-\frac{2a}{n\pi}\left[(-1)^n-1\right]$

106) If $f(x)=\begin{cases}0 & -2\le x\le 0\\ 2 & 0\le x\le 2\end{cases}$ is of periodic function with period 4, the Fourier coefficient a_0 is given by

a) 0 b) 4 c) $-\frac{\pi}{2}$ d) 1

107) If $f(x)=\begin{cases}1 & -1\le x\le 0\\ \cos\pi x & 0\le x\le 1\end{cases}$ is of periodic function with period 2, the Fourier coefficient a_0 is given by

a) 0 b) $\frac{\pi}{2}$ c) $-\frac{\pi}{2}$ d) 1

108) If $f(x)=e^{-x}$ defined over $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, the Fourier coefficient a_0 is given by

a) $\frac{1}{2\pi}\left(1-e^{-2\pi}\right)$ b) $\frac{2}{\pi}\left(1-e^{-2\pi}\right)$

c) $\frac{1}{\pi}\left(1+e^{-x}\right)$ d) $\frac{1}{\pi}\left(1-e^{-2\pi}\right)$

109) If $f(x)=x$ defined over $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, the Fourier coefficient a_0 is given by

a) 3π b) $\frac{\pi}{2}$ c) π d) 2π

110) If $f(x)=x$ defined over $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, the Fourier coefficient a_n is given by

a) 0 b) π c) 2π d) 3π

111) If $f(x)=x$ defined over $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, the Fourier coefficient b_n is given by

a) $-\frac{2}{n\pi}$ b) $-\frac{\pi}{n}$ c) $-\frac{1}{n}$ d) $-\frac{2}{n}$

112) If $f(x)=\sqrt{1-\cos x}$ defined over $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, the Fourier coefficient a_0 is given by

a) 0 b) $-\frac{4\sqrt{2}}{\pi}$ c) $\frac{4\sqrt{2}}{\pi}$ d) $\frac{8\sqrt{2}}{\pi}$

113) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=\frac{\pi-x}{2}$ defined over the interval $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, is given by

a) $-\frac{\pi}{2}$ b) $\frac{\pi}{2}$ c) 0 d) $\frac{\pi^2}{6}$

114) The Fourier coefficient a_n for the Fourier series expansion of $f(x)=\frac{\pi-x}{2}$ defined over the interval $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, is given by

a) 0 b) π c) $\frac{\pi}{2}$ d) $-\frac{\pi}{2}$

115) The Fourier coefficient b_n for the Fourier series expansion of $f(x)=\frac{\pi-x}{2}$ defined over the interval $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, is given by

a) $-\frac{1}{n^2}$ b) $\frac{1}{n}$ c) $-\frac{1}{n}$ d) $\frac{\pi}{n}$

116) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=\left(\frac{\pi-x}{2}\right)^2$ defined over the interval $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, is given by

a) $\frac{\pi^2}{6}$ b) $\frac{\pi^2}{12}$ c) $\frac{\pi^2}{3}$ d) $\frac{\pi^2-1}{6}$

117) Consider $f(x)=x\sin x,\ x\in[0,2\pi]$ and $f(x+2\pi)=f(x)$. Then the Fourier coefficient a_0 for the Fourier series expansion of $f(x)$ is given by

a) -4 b) $-\frac{\pi}{2}$ c) -2 d) $\frac{\pi}{2}$

118) If $f(x)=\begin{cases} x & 0\le x\le \pi \\ 0 & \pi\le x\le 2\pi \end{cases}$ is periodic over a period 2π, the Fourier coefficient a_0 for the Fourier series expansion of $f(x)$ is given by

a) π b) $\frac{\pi}{2}$ c) $-\frac{\pi}{2}$ d) $\frac{\pi}{4}$

119) If $f(x)=\begin{cases} 0 & 0\le x\le \pi \\ x & \pi\le x\le 2\pi \end{cases}$ is periodic over a period 2π, the Fourier coefficient a_0 for the Fourier series expansion of $f(x)$ is given by

a) $\frac{3\pi}{2}$ b) $\frac{\pi}{2}$ c) 3π d) $\frac{3\pi}{4}$

120) If the function $f(x)=\begin{cases} -\pi & 0\le x\le \pi \\ x-\pi & \pi\le x\le 2\pi \end{cases}$ and $f(x+2\pi)=f(x)$, the Fourier coefficient a_0 is given by

a) 0 b) $-\frac{\pi}{2}$ c) $-\frac{\pi}{4}$ d) $-\pi$

121) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=x-x^2$ defined over the interval $0\le x\le 2$ and $f(x+2)=f(x)$, is given by

a) 0 b) $-\frac{4}{3}$ c) $-\frac{2}{3}$ d) $\frac{2}{3}$

122) The Fourier coefficient a_n for the Fourier series expansion of $f(x)=x-x^2$ defined over the interval $0\le x\le 2$ and $f(x+2)=f(x)$, is given by

a) 0 b) $-\frac{4}{n^2\pi^2}$ c) $\frac{4}{n^2\pi^2}$ d) $-\frac{1}{n^2\pi^2}$

123) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=x+x^2$ defined over the interval $0\le x\le 3$ and having period 3, is given by

a) 0 b) $-\frac{4}{n^2\pi^2}$ c) $\frac{4}{n^2\pi^2}$ d) $\frac{3}{2}$

124) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=2x-x^2$ defined over the interval $0\le x\le 4$ and $f(x+4)=f(x)$, is given by

a) $-\frac{1}{3}$ b) $-\frac{2}{3}$ c) $-\frac{4}{3}$ d) $-\frac{8}{3}$

125) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=2x-x^2$ defined over the interval $0\le x\le 3$ and $f(x+3)=f(x)$, is given by

a) 0 b) $-\frac{2}{3}$ c) $-\frac{4}{3}$ d) $-\frac{8}{3}$

126) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=1-x^2$ defined over the interval $0\le x\le 2$ and $f(x+2)=f(x)$, is given by

a) $-\frac{1}{3}$ b) $-\frac{2}{3}$ c) $-\frac{4}{3}$ d) $-\frac{8}{3}$

127) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=1-x^2$ defined over the interval $0\le x\le 1$ and $f(x+2)=f(x)$, is given by

a) $\frac{2}{3}$ b) $-\frac{2}{3}$ c) $-\frac{1}{3}$ d) $-\frac{4}{3}$

128) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=4-x^2$ defined over the interval $0\le x\le 2$ and $f(x+2)=f(x)$, is given by

a) $-\frac{1}{3}$ b) $\frac{16}{3}$ c) $-\frac{16}{3}$ d) $-\frac{8}{3}$

129) If $f(x)=\begin{cases}\pi x & 0\le x\le 1\\ \pi(2-x) & 1\le x\le 2\end{cases}$ is periodic over a period 2, the Fourier coefficient a_0 for the Fourier series expansion of $f(x)$ is given by

a) $-\frac{\pi}{2}$ b) π c) $-\pi$ d) $\frac{\pi}{2}$

130) If $f(x)=\begin{cases}x & 0\le x\le 1\\ 0 & 1\le x\le 2\end{cases}$ is periodic over a period 2, the Fourier coefficient a_0 for the Fourier series expansion of $f(x)$ is given by

a) 2 b) 0 c) $\frac{1}{2}$ d) 1

131) The Fourier coefficient a_0 in the half range cosine series expansion of function $f(x)=\sin x$ defined over the interval $[0,\ \pi]$ is given by

a) $\frac{2}{\pi}$ b) $\frac{\pi}{2}$ c) $-\frac{\pi}{2}$ d) 0

132) The Fourier coefficient b_1 in the half range cosine series expansion of function $f(x)=\cos x$ defined over the interval $[0,\ \pi]$ is given by

a) $-\frac{\pi}{2}$ b) 0 c) $\frac{1}{2\pi}$ d) $\frac{\pi}{2}$

133) The Fourier coefficient a_0 in the half range cosine series expansion of function $f(x)=\pi x-x^2$ defined over the interval $[0,\ \pi]$ is given by

a) 0 b) $\frac{\pi^2}{6}$ c) $\frac{2\pi^2}{3}$ d) $\frac{\pi^2}{3}$

134) The Fourier coefficient a_0 in the half range sine series expansion of function $f(x)=\cos x$ defined over the interval $[0,\ \pi]$ is given by

a) $\frac{4}{\pi}$ b) $\frac{2}{\pi}$ c) $\frac{\pi}{2}$ d) 0

135) The Fourier coefficient a_0 in the half range cosine series expansion of function $f(x)=\sin x$ defined over the interval $[0,\ \pi]$ is given by

a) $\frac{4}{\pi}$ b) $\frac{2}{\pi}$ c) 0 d) $\frac{\pi}{2}$

136) The Fourier coefficient a_1 in the half range cosine series expansion of function $f(x)=\sin x$ defined over the interval $[0,\ \pi]$ is given by

a) 1 b) $\frac{2}{\pi}$ c) 0 d) $\frac{\pi}{2}$

137) The Fourier coefficient b_1 in the half range cosine series expansion of function $f(x)=x$ defined over the interval $[0,\ 2]$ with period 4 is given by

a) 0 b) $\frac{1}{\pi}$ c) $\frac{2}{\pi}$ d) $\frac{4}{\pi}$

138) The Fourier coefficient a_0 in the half range cosine series expansion of function $f(x)=x-x^2$ defined over the interval $[0,\ 1]$ is given by

a) $\frac{1}{3}$ b) $-\frac{1}{3}$ c) 0 d) $\frac{2}{\pi}$

139) The Fourier coefficient a_0 in the half range cosine series expansion of function $f(x)=lx-x^2$ defined over the interval $[0,\ l]$ with period $2l$ is given by

a) 0 b) $\frac{l^2}{3}$ c) $\frac{\pi}{2}$ d) $\frac{2}{\pi n^2}(\cos n\pi-1)$

140) The Fourier coefficient a_1 in the half range cosine series expansion of function $f(x)=x-x^2$ defined over the interval $[0,\ 1]$ is given by

a) 2 b) $\frac{2}{\pi}$ c) $\frac{\pi}{2}$ d) $\frac{1}{2}$

141) The Fourier coefficient a_n in the half range cosine series expansion of function $f(x)=x-x^2$ defined over the interval $[0,\ 1]$ is given by

a) 0 b) $\frac{2}{\pi}$ c) $\frac{\pi}{2}$ d) $\frac{2}{\pi n^2}(\cos n\pi-1)$

142) The Fourier coefficient a_n in the half range sine series expansion of function $f(x)=2+x$ defined over the interval $[0,\ 1]$ is given by

a) 4 b) 0 c) $-\frac{2}{n\pi}$ d) $-\frac{2\pi}{n}$

143) The Fourier series expansion for the function $f(x)=\left(\frac{\pi-x}{2}\right)^2$ over the interval $0\le x\le 2\pi$ is given by $\left(\frac{\pi-x}{2}\right)^2=\frac{\pi^2}{12}+\sum_{n=1}^{\infty}\frac{1}{n^2}\cos nx$. Then the value of $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots$ is

a) 1 b) $\frac{\pi^2}{6}$ c) $\frac{\pi^2}{12}$ d) $\frac{\pi^2}{3}$

144) The Fourier series expansion for the function $f(x)=\pi^2-x^2$ over the interval $-\pi\le x\le\pi$ is given by $\pi^2-x^2=\frac{2\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}\cos nx$. Then the value of $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-+\ldots$ is

a) 1 b) $\frac{\pi^2}{6}$ c) $\frac{\pi^2}{12}$ d) $\frac{\pi^2}{3}$

145) The Fourier series expansion for the function $f(x)=\pi^2-x^2$ over the interval $-\pi\le x\le\pi$ is given by $\pi^2-x^2=\frac{2\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}\cos nx$. Then the value of $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots$ is

a) $\frac{\pi^2}{6}$ b) $\frac{\pi^2}{12}$ c) $\frac{\pi^2}{3}$ d) 0

146) The Fourier series expansion for the function $f(x)=\begin{cases}1+\frac{2x}{\pi} & -\pi\le x\le 0\\ 1-\frac{2x}{\pi} & 0\le x\le\pi\end{cases}$ is given by $f(x)=\frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos nx$. Then the value of $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots$ is

a) $\frac{\pi^2}{6}$ b) $\frac{\pi^2}{12}$ c) $\frac{\pi^2}{3}$ d) $\frac{\pi^2}{8}$

147) The Fourier series expansion for the function $f(x)=\begin{cases}\pi x & 0\le x\le 1\\ \pi(2-x) & 1\le x\le 2\end{cases}$ is given by $f(x)=\frac{\pi}{2}-\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos(2n-1)\pi x$. Then the value of $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots$ is

a) $\frac{\pi^2}{6}$ b) $\frac{\pi^2}{12}$ c) $\frac{\pi^2}{8}$ d) $\frac{\pi^2}{3}$

148) In the following harmonic analysis of $y=f(x)$, the value of a_0 is given by

x	0	1	2	3	4	5
y	4	8	15	7	5	3

a) 14 b) 7 c) 3.5 d) 6

149) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	1	2	3	4	5
y	9	18	26	26	26	20

a) 25.01 b) 20.83 c) 41.66 d) 40.89

150) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	30	60	90	120	150	180
y	0	9.2	14.4	17.8	17	12	0

a) 10.23 b) 23.46 c) 46.92 d) 11.73

151) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	60	120	180	240	300	360
y	1	1.4	1.9	1.6	1.6	1.2	1

a) 7.2 b) 1.45 c) 5.8 d) 2.9

152) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	60	120	180	240	300	360
y	1.98	2.15	2.7	-0.22	-0.31	1.5	1.98

a) 4.8 b) 2.6 c) 5.2 d) 1.3

153) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	1	1.4	1.9	1.6	1.6	1.2	1

a) 2.9 b) 5.8 c) 1.45 d) 3.8

154) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
y	1.98	1.35	1	1.3	-0.88	-0.25	1.98

a) 1 b) 0.75 c) 1.5 d) 3

155) In the following harmonic analysis of $y = f(x)$, the value of a_1 is given by

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
y	0	9.2	14.4	17.8	17.3	11.7	0

a) 3.73 b) 5.73 c) 7.73 d) -7.73

156) In the following harmonic analysis of $y = f(x)$, the value of b_1 is given by

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
y	0	9.2	14.4	17.8	17.3	11.7	0

a) 4.38 b) 3.48 c) 4.83 d) 8.43

157) In the following harmonic analysis of $y = f(x)$, the value of a_1 is given by

x	0	1	2	3	4	5
y	9	18	24	28	26	20

a) - 8.37 b) 8.73 c) 7.83 d) 3.78

158) In the following harmonic analysis of $y = f(x)$, the value of b_1 is given by

x	0	1	2	3	4	5
y	9	18	24	28	26	20

a) 1.25 b) -6.3 c) – 3.15 d) - 3.50

159) In the following harmonic analysis of $y = f(x)$, the value of b_1 is given by

x	0	1	2	3	4	5	6
y	9	18	24	28	26	20	9
$\cos\left(\frac{\pi}{3}x\right)$	1	0.5	-0.5	-1	-0.5	0.5	1

a) 3.38 b) – 8.33 c) 8.33 d) 5.83

Chapter 04–Reduction Formulae, Beta and Gamma Functions

I) Reduction Formulae

1) For $I_n = \int_0^{\pi/2} \sin^n x\,dx$, we have

a) $I_n = 2\int_0^{\pi} \sin^n x\,dx$ b) $I_n = \int_0^{\pi/2} \sin^{n-2} x\cos^2 x\,dx$

c) $I_n = \int_0^{\pi/2} \cos^n x\,dx$ d) $I_n = \frac{1}{2}\int_0^{\pi/4} \sin^n x\,dx$

2) For $I_n = \int_0^{\pi} \sin^n x\,dx$, we have

a) 0 b) $I_n = 2\int_0^{\pi/2} \sin^n x\,dx$

c) $I_n = 4\int_0^{\pi/2} \sin^n x\,dx$ d) none of these

3) For $I_n = \int_0^{\pi} \cos^n x\,dx$, where n is an even integer, we have

a) $I_n = 4\int_0^{\pi/2} \cos^n x\,dx$ b) $I_n = 2\int_0^{\pi/4} \cos^n x\,dx$

c) $I_n = 2\int_0^{\pi/2} \cos^n x\,dx$ d) 0

4) For $I_n = \int_0^{\pi} \cos^n x\,dx$, where n is an odd integer, we have

a) $I_n = 4\int_0^{\pi/2} \cos^n x\,dx$ b) $I_n = 2\int_0^{\pi/4} \cos^n x\,dx$

c) $I_n = 2\int_0^{\pi/2} \cos^n x\,dx$ d) 0

5) For $I_n = \int_0^{2\pi} \sin^n x\,dx$, where n is an even integer, we have

a) 0 b) $I_n = 4\int_0^{\pi/4} \sin^n x\,dx$

c) $I_n = 2\int_0^{\pi/2} \sin^n x\,dx$ d) $I_n = 4\int_0^{\pi/2} \sin^n x\,dx$

6) For $I_n = \int_0^{2\pi} \sin^n x\,dx$, where n is an odd integer, we have

a) $I_n = 4\int_0^{\pi/2} \sin^n x\,dx$ b) $I_n = 2\int_0^{\pi/2} \sin^n x\,dx$

c) 0 d) $I_n = 4\int_0^{\pi/4} \sin^n x\,dx$

7) For $I_n = \int_0^{2\pi} \cos^n x\,dx$, where n is an odd integer, we have

a) 0 b) $I_n = 4\int_0^{\pi/2} \cos^n x\,dx$

c) $I_n = 2\int_0^{\pi/2} \cos^n x\,dx$ d) $I_n = 4\int_0^{\pi/4} \cos^n x\,dx$

8) For $I_n = \int_0^{2\pi} \cos^n x\,dx$, where n is an even integer, we have

a) 0 b) $I_n = 4\int_0^{\pi/4} \cos^n x\,dx$

c) $I_n = 4\int_0^{\pi/2} \cos^n x\,dx$ d) $I_n = 2\int_0^{\pi/2} \cos^n x\,dx$

9) For $I_{m,n}=\int_0^{\pi}\sin^m x\cos^n x\,dx$, where both m and n are odd integers, we have

a) $I_{m,n}=2\int_0^{\pi/2}\sin^m x\cos^n x\,dx$ b) 0

c) $I_{m,n}=4\int_0^{\pi/2}\sin^m x\cos^n x\,dx$ d) none

10) For $I_{m,n}=\int_0^{\pi}\sin^m x\cos^n x\,dx$, where both m and n are even integers, we have

a) $I_{m,n}=2\int_0^{\pi/2}\sin^m x\cos^n x\,dx$

b) $I_{m,n}=4\int_0^{\pi/2}\sin^m x\cos^n x\,dx$

c) 0 d) none of the above

11) For $I_{m,n}=\int_0^{\pi}\sin^m x\cos^n x\,dx$, where only n is an even integer, we have

a) 0 b) $I_{m,n}=4\int_0^{\pi/2}\sin^m x\cos^n x\,dx$

c) $I_{m,n}=2\int_0^{\pi/2}\sin^m x\cos^n x\,dx$ d) none

12) For $I_{m,n}=\int_0^{\pi}\sin^m x\cos^n x\,dx$, where only n is an odd integer, we have

a) $I_{m,n}=4\int_0^{\pi/2}\sin^m x\cos^n x\,dx$

b) 0

c) $I_{m,n}=2\int_0^{\pi/2}\sin^m x\cos^n x\,dx$

d) none of the above

13) For $I_n=\int_0^{\pi/2}\sin^n x\,dx$, which of the following is the reduction formula?

a) $I_n=\frac{n-1}{n}I_{n-1}$ b) $I_n=\frac{n}{n+1}I_{n-2}$

c) $I_n=\frac{n+1}{n}I_{n-2}$ d) $I_n=\frac{n-1}{n}I_{n-2}$

14) For $I_n=\int_0^{\pi/2}\cos^n x\,dx$, which of the following is the reduction formula?

a) $I_n=\frac{n-1}{n}I_{n-2}$ b) $I_n=\frac{n-1}{n}I_{n-1}$

c) $I_n=\frac{n}{n+1}I_{n-2}$ d) $I_n=\frac{n+1}{n}I_{n-2}$

15) For $I_n=\int_0^{\pi/2}\sin^n x\,dx$, where n is an even natural number, which of the following is the reduced form?

a) $I_n=\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \dots\ \cdot\frac{1}{2}\cdot\frac{\pi}{2}$

b) $I_n=\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \dots\ \cdot\frac{2}{3}\cdot 1$

c) $I_n=\frac{n+1}{n}\cdot\frac{n+3}{n+2}\cdot\frac{n+5}{n+4}\cdot\ \dots\ \cdot\frac{1}{2}\cdot\frac{\pi}{2}$

d) $I_n=\frac{n-2}{n-1}\cdot\frac{n-4}{n-3}\cdot\frac{n-6}{n-5}\cdot\ \dots\ \cdot\frac{2}{3}\cdot\frac{\pi}{2}$

16) For $I_n=\int_0^{\pi/2}\cos^n x\,dx$, where n is an even natural number, which of the following is the reduced form?

a) $I_n=\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \dots\ \cdot\frac{2}{3}\cdot 1$

b) $I_n=\frac{n-2}{n-1}\cdot\frac{n-4}{n-3}\cdot\frac{n-6}{n-5}\cdot\ \dots\ \cdot\frac{2}{3}\cdot\frac{\pi}{2}$

c) $I_n=\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \dots\ \cdot\frac{1}{2}\cdot\frac{\pi}{2}$

d) $I_n=\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \dots\ \cdot\frac{1}{2}\cdot\pi$

17) For $I_n = \int_0^{\pi/2} \sin^n x\,dx$, where n is an odd natural number, which of the following is the reduced form?

a) $I_n = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \dots\ \cdot\frac{2}{3}\cdot 1$

b) $I_n = \frac{n-2}{n-1}\cdot\frac{n-4}{n-3}\cdot\frac{n-6}{n-5}\cdot\ \dots\ \cdot\frac{2}{3}\cdot\frac{\pi}{2}$

c) $I_n = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \dots\ \cdot\frac{1}{2}\cdot\frac{\pi}{2}$

d) $I_n = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \dots\ \cdot\frac{1}{2}\cdot\pi$

18) For $I_n = \int_0^{\pi/2} \cos^n x\,dx$, where n is an odd natural number, which of the following is the reduced form?

a) $I_n = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \dots\ \cdot\frac{1}{2}\cdot\frac{\pi}{2}$

b) $I_n = \frac{n+1}{n}\cdot\frac{n+3}{n+2}\cdot\frac{n+5}{n+4}\cdot\ \dots\ \cdot\frac{2}{3}\cdot 1$

c) $I_n = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \dots\ \cdot\frac{2}{3}\cdot 1$

d) $I_n = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \dots\ \cdot\frac{1}{2}\cdot\pi$

19) For $I_n = \int_0^{\pi/2} \sin^n x \cos^n x\,dx$, where n is an odd natural number, which of the following is the reduced form?

a) $I_{(m,\,n)} = \frac{[(m-1)(m-3)\cdot\ \dots\ \cdot 2 or 1][(n-1)(n-3)\cdot\ \dots\ \cdot 2 or 1]}{(m+n)(m+n-2)\cdot\ \dots\ \cdot 2 or 1}\cdot k$

where $k = \begin{cases} \frac{\pi}{2} & both\ m\ \&\ n\ are\ odd \\ 1 & otherwise \end{cases}$

b) $I_{(m,\,n)} = \frac{[(m-1)(m-3)\cdot\ \dots\ \cdot 2 or 1][(n-1)(n-3)\cdot\ \dots\ \cdot 2 or 1]}{(m+n)(m+n-2)\cdot\ \dots\ \cdot 2 or 1}\cdot k$

where $k = \begin{cases} \frac{\pi}{2} & both\ m\ \&\ n\ are\ even \\ 1 & otherwise \end{cases}$

c) $I_{(m,\,n)} = \frac{[(m-1)(m-3)\cdot\ \dots\ \cdot 2 or 1][(n-1)(n-3)\cdot\ \dots\ \cdot 2 or 1]}{(m+n)(m+n-2)\cdot\ \dots\ \cdot 2 or 1}\cdot k$

where $k = \begin{cases} \frac{\pi}{2} & m+n\ is\ even \\ 1 & otherwise \end{cases}$

d) $I_{(m,\,n)} = \frac{(m+n-1)(m+n-3)\cdot\ \dots\ \cdot 2 or 1}{(m+n)(m+n-2)\cdot\ \dots\ \cdot 2 or 1}\cdot k$

where $k = \begin{cases} \frac{\pi}{2} & both\ m\ \&\ n\ are\ odd \\ 1 & otherwise \end{cases}$

20) The value of $\int_0^{\pi/2} \sin^3 x\,dx$ is equal to

a) $\frac{3\pi}{4}$ b) $\frac{3}{4}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$

21) The value of $\int_0^{\pi/2} \sin^4 x\,dx$ is equal to

a) $\frac{3}{8}$ b) $\frac{3}{16}$ c) $\frac{3\pi}{16}$ d) $\frac{3\pi}{18}$

22) The value of $\int_0^{\pi/2} \sin^5 x\,dx$ is equal to

a) $\frac{4\pi}{15}$ b) $\frac{8\pi}{30}$ c) $\frac{8\pi}{15}$ d) $\frac{8}{15}$

23) The value of $\int_0^{\pi/2} \sin^9 x\,dx$ is equal to

a) $\frac{64}{315}$ b) $\frac{128}{315}$ c) $\frac{128}{315}\pi$ d) $\frac{64}{315}\pi$

24) The value of $\int_0^{\pi/2} \cos^3 x\,dx$ is equal to

a) $\frac{3\pi}{4}$ b) $\frac{3}{4}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$

25) The value of $\int_0^{\pi/2} \cos^4 x\,dx$ is equal to

a) $\frac{3}{8}$ b) $\frac{3}{16}$ c) $\frac{3\pi}{16}$ d) $\frac{3\pi}{18}$

26) The value of $\int_0^{\pi/2} \cos^7 x\,dx$ is equal to

a) $\frac{8}{35}$ b) $\frac{16\pi}{35}$ c) $\frac{16\pi}{70}$ d) $\frac{16}{35}$

27) The value of $\int_0^{\pi/2} \cos^{10} x\,dx$ is equal to

a) $\frac{63\pi}{128}$ b) $\frac{63\pi}{512}$ c) $\frac{63\pi}{256}$ d) $\frac{64}{315}\pi$

28) The value of $\int_0^{\pi/2} \sin^3 x \cos^2 x\,dx$ is equal to

a) $\frac{2}{15}$ b) $\frac{\pi}{30}$ c) $\frac{1}{15}$ d) $\frac{\pi}{15}$

29) The value of $\int_0^{\pi/2} \sin^2 x \cos^3 x\,dx$ is equal to

a) $\frac{1}{15}$ b) $\frac{\pi}{30}$ c) $\frac{\pi}{15}$ d) $\frac{2}{15}$

30) The value of $\int_0^{\pi/2} \sin^3 x \cos^4 x\,dx$ is equal to

a) $\frac{1}{35}$ b) $\frac{2}{35}$ c) $\frac{2\pi}{35}$ d) $\frac{2\pi}{70}$

31) The value of $\int_0^{\pi/2} \sin^4 x \cos^6 x\,dx$ is equal to

a) $\frac{3\pi}{512}$ b) $\frac{3}{256}$ c) $\frac{3\pi}{256}$ d) $\frac{3\pi}{128}$

32) The value of $\int_{-\pi/2}^{\pi/2} \sin^3 x \cos^2 x\,dx$ is equal to

a) $2\int_0^{\pi/2} \sin^3 x \cos^2 x\,dx$ b) $4\int_0^{\pi/2} \sin^3 x \cos^2 x\,dx$

c) 0 d) none of the above

33) The value of $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos^3 x\,dx$ is equal to

a) 0 b) $2\int_0^{\pi/2} \sin^2 x \cos^3 x\,dx$

c) $3\int_0^{\pi/3} \sin^2 x \cos^3 x\,dx$ d) none of the above

34) The value of $\int_{-\pi/2}^{\pi/2} \sin^4 x \cos^2 x\,dx$ is equal to

a) $\frac{3}{16}$ b) $\frac{3\pi}{16}$ c) $\frac{\pi}{16}$ d) 0

35) The value of $\int_{-\pi/2}^{\pi/2} \sin^5 x\,dx$ is equal to

a) $\frac{3\pi}{128}$ b) $\frac{3\pi}{15}$ c) $\frac{32}{256}$ d) 0

36) The value of $\int_0^{2\pi} \sin^4 x \cos^6 x\,dx$ is equal to

a) $\frac{3}{64}$ b) $\frac{2\pi}{35}$ c) $\frac{2}{35}$ d) $\frac{3\pi}{128}$

37) The value of $\int_0^{2\pi} \sin^4 x \cos^7 x\,dx$ is equal to

a) $\frac{5}{128}$ b) $\frac{5\pi}{128}$ c) 0 d) $\frac{5\pi}{256}$

38) The value of $\int_{-\pi}^{\pi} \sin^4 x \cos^7 x\,dx$ is equal to

a) 0 b) $\frac{5\pi}{128}$ c) $\frac{5}{128}$ d) $\frac{5\pi}{256}$

39) The value of $\int_0^{\pi} \cos^3 x\,dx$ is equal to

a) $\frac{5\pi}{256}$ b) $\frac{5\pi}{16}$ c) $\frac{5}{128}$ d) 0

40) The value of $\int_0^{\pi} \cos^6 x\,dx$ is equal to

a) 0 b) $\frac{5\pi}{16}$ c) $\frac{5}{8}$ d) $\frac{5\pi}{256}$

41) The value of $\int_0^{\pi} \cos^7 x\,dx$ is equal to

a) $\frac{5\pi}{256}$ b) $\frac{5\pi}{16}$ c) $\frac{5}{128}$ d) 0

42) The value of $\int_0^{\pi} \sin^7 x\,dx$ is equal to

a) $\frac{5\pi}{32}$ b) $\frac{5\pi}{16}$ c) $\frac{32}{35}$ d) 0

43) The value of $\int_0^{\pi} \sin^6 x\,dx$ is equal to

a) $\frac{5\pi}{16}$ b) $\frac{5\pi}{32}$ c) $\frac{3}{4}$ d) 0

44) The value of $\int_0^{2\pi} \sin^6 \theta\,d\theta$ is equal to

a) $\frac{5\pi}{32}$ b) $\frac{5\pi}{16}$ c) $\frac{5\pi}{8}$ d) 0

45) The value of $\int_0^{2\pi} \sin^8 x\,dx$ is equal to

a) $\frac{5\pi}{16}$ b) $\frac{5\pi}{32}$ c) $\frac{32}{35}$ d) $\frac{35\pi}{32}$

46) The value of $\int_0^{2\pi} \cos^5 x\,dx$ is equal to

a) $\frac{5\pi}{32}$ b) $\frac{5\pi}{16}$ c) $\frac{32}{35}$ d) 0

47) The value of $\int_0^{2\pi} \sin^6 x\cos^4 x\,dx$ is equal to

a) $\frac{5\pi}{256}$ b) $\frac{3\pi}{128}$ c) $\frac{35\pi}{256}$ d) 0

48) The value of $\int_0^{2\pi} \sin^7 x\cos^4 x\,dx$ is equal to

a) $\frac{5\pi}{256}$ b) $\frac{3\pi}{128}$ c) $\frac{35\pi}{256}$ d) 0

49) The value of $\int_0^{2\pi} \sin^7 x\cos^5 x\,dx$ is equal to

a) $\frac{5\pi}{256}$ b) 0 c) $\frac{35\pi}{256}$ d) $\frac{3\pi}{128}$

50) The value of $\int_0^{\pi} \sin^5\left(\frac{x}{2}\right)dx$ is equal to

a) $\frac{16\pi}{15}$ b) $\frac{5\pi}{32}$ c) $\frac{16}{15}$ d) $\frac{5\pi}{16}$

51) The value of $\int_0^{\pi/4} \sin^2(2x)\,dx$ is equal to

a) $\frac{\pi}{8}$ b) $\frac{16}{15}$ c) $\frac{3\pi}{8}$ d) 0

52) The value of $\int_0^{\pi/4} \sin^7(2x)\,dx$ is equal to

a) $\frac{16\pi}{15}$ b) $\frac{5\pi}{16}$ c) $\frac{8}{35}$ d) 0

53) The value of $\int_0^{\pi/4} \cos^2(2x)\,dx$ is equal to

a) $\frac{5\pi}{16}$ b) $\frac{\pi}{8}$ c) $\frac{5\pi}{32}$ d) 0

54) The value of $\int_0^{\pi/3} \sin^5(3x)\,dx$ is equal to

a) $\frac{3\pi}{16}$ b) $\frac{8\pi}{15}$ c) $\frac{8\pi}{45}$ d) $\frac{8}{45}$

55) If $I_n = \int_0^{\pi/4} \sin^{2n} x\,dx = -\frac{1}{2^{n+1}n} + \frac{2n-1}{2n} I_{n-1}$, the value of I_2 is equal to

a) $\frac{3\pi+2}{8}$ b) $\frac{3\pi-8}{32}$ c) $-\frac{8+3\pi}{32}$ d) $\frac{3\pi}{32}$

56) If $I_n = \int_0^{\pi/2} x\sin^n x\,dx = \frac{1}{n^2} + \frac{n-1}{n} I_{n-2}$, the value of I_5 is equal to

a) $\frac{149}{25}$ b) $\frac{19}{225}$ c) $\frac{\pi}{2} - \frac{149}{225}$ d) $\frac{149}{225}$

56) If $I_n = \int_0^{\pi/2} \tan^n x\,dx = \frac{1}{n-1} - I_{n-2}$, the value of I_4 is equal to

a) $\frac{\pi}{4} - \frac{2}{3}$ b) $\frac{\pi}{4} + \frac{2}{3}$ c) $\frac{\pi}{2} - \frac{2}{3}$ d) $\frac{\pi}{4} + \frac{4}{3}$

II) Gamma Functions

57) For $n>0$, the gamma function Γn is defined as

a) $\int_0^\infty e^{x}x^{n-1}dx$ b) $\int_0^\infty e^{-x}x^{n+1}dx$

c) $\int_0^\infty e^{-x}x^{n}dx$ d) $\int_0^\infty e^{-x}x^{n-1}dx$

58) $\int_0^\infty e^{-x}x^{n}dx$ is equal to

a) $\Gamma(n+1)$ b) Γn c) $\Gamma(n-1)$ d) $\Gamma(n-2)$

59) $\int_0^\infty e^{-kx}x^{n}dx$ is equal to

a) $k^{n-1}\Gamma(n-1)$ b) $\frac{\Gamma(n-1)}{k^{n-1}}$ c) $\frac{\Gamma(n+1)}{k^{n+1}}$ d) $\frac{\Gamma n}{k^{n}}$

60) $\int_0^\infty e^{-kx}x^{n-1}dx$ is equal to

a) $k^{n-1}\Gamma(n-1)$ b) $\frac{\Gamma(n-1)}{k^{n-1}}$ c) $\frac{\Gamma(n+1)}{k^{n+1}}$ d) $\frac{\Gamma n}{k^{n}}$

61) The value of Γn is equal to

a) $n\Gamma(n-1)$ b) $(n+1)\Gamma(n+1)$

c) $(n-1)\Gamma(n-1)$ d) $n\Gamma n$

62) If n is a natural number, the value of Γn is

a) $\frac{n!}{n+1}$ b) $(n-1)!$ c) $n!$ d) $(n+1)!$

63) The value of $\Gamma 1$ is

a) 1 b) 2 c) 3 d) 0

64) The value of $\Gamma 2$ is

a) 0 b) 1 c) 2 d) 3

65) The value of $\Gamma 7$ is

a) 3256 b) 5040 c) 120 d) 720

66) The value of $\Gamma\left(\frac{1}{2}\right)$ is

a) $\frac{1}{2}$ b) $\sqrt{\pi}$ c) $\Gamma\pi$ d) none

67) The value of $\Gamma\left(\frac{5}{2}\right)$ is

a) $\frac{3\sqrt{\pi}}{2}$ b) $\frac{3\sqrt{\pi}}{4}$ c) $\frac{3\sqrt{\pi}}{8}$ d) 0

68) The value of $\Gamma\left(\frac{1}{4}\right)\cdot\Gamma\left(\frac{3}{4}\right)$ is

a) $\pi\sqrt{2}$ b) $\frac{\pi}{\sqrt{2}}$ c) $\frac{\sqrt{2}}{\pi}$ d) none

69) The value of $\Gamma p\cdot\Gamma(1-p)$, for $0<p<1$, is given by the formula

a) $\frac{\sin p\pi}{\pi}$ b) $\frac{\pi}{\sin p\pi}$

c) $\frac{\sqrt{\pi}}{\sin p\pi}$ d) $\frac{p\pi}{\sin p\pi}$

70) The value of $\int_0^\infty e^{-x}x^{5}dx$

a) 60 b) 720 c) 120 d) 240

71) The value of $\int_0^\infty e^{-2x}x^{5}dx$

a) $\frac{125}{32}$ b) $\frac{120}{35}$ c) $\frac{25}{8}$ d) $\frac{15}{8}$

72) The value of $\int_0^\infty e^{-x}x^{1/2}dx$

a) $\frac{\sqrt{\pi}}{2}$ b) $\frac{\pi}{2}$ c) $\frac{\sqrt{\pi}}{3}$ d) $\sqrt{\pi}$

73) The value of $\int_0^\infty e^{-x}x^{-1/2}dx$

a) $\frac{\sqrt{\pi}}{2}$ b) $\sqrt{\pi}$ c) $\frac{\sqrt{\pi}}{3}$ d) $\frac{\pi}{2}$

74) The value of $\int_0^\infty e^{-x}x^{3/2}dx$

a) $\frac{\sqrt{\pi}}{4}$ b) $\frac{3\sqrt{\pi}}{8}$ c) $\frac{3\sqrt{\pi}}{4}$ d) $\frac{3\sqrt{\pi}}{2}$

75) The substitution for the integral $\int_0^\infty \sqrt{x}\cdot e^{-\sqrt{x}}dx$ to reduce it into the form of gamma function is

a) $\sqrt{x}=t$ b) $\sqrt{x}=t^2$

c) $\sqrt{x}=\frac{t}{2}$ d) $x=\sin t$

76) The substitution for the integral $\int_0^\infty x^3\cdot e^{-\sqrt{x}}dx$ to reduce it into the form of gamma function is

a) $x^3=\sin^2 t$ b) $x^3=e^{-t}$

c) $x^3=t$ d) $\sqrt{x}=t$

77) The substitution for the integral $\int_0^\infty x^3\cdot 5^{-x}dx$ to reduce it into the form of gamma function is

a) $5^x=e^t$ b) $x^3=e^{-t}$

c) $5^x=x^{-t}$ d) $\log x=5^{-x}$

78) On using substitution $\sqrt{x}=t$, the value of the integration $\int_0^\infty x\cdot e^{-\sqrt{x}}dx$ is given by

a) 1 b) 3 c) 12 d) 16

79) On using substitution $\sqrt{x}=t$, the value of the integration $\int_0^\infty \sqrt{x}\cdot e^{-\sqrt{x}}dx$ is given by

a) 1 b) 2 c) 3 d) 4

80) On using substitution $\sqrt{t}=x$, the value of the integration $\int_0^\infty e^{-x^2}dx$ is given by

a) $\left\lceil \frac{1}{4}\right.$ b) 16 c) $\frac{\sqrt{\pi}}{2}$ d) $\sqrt{\pi}$

81) On using substitution $x^3=t$, the value of the integration $\int_0^\infty \sqrt{x}\cdot e^{-x^3}dx$ is given by

a) $\frac{\sqrt{\pi}}{2}$ b) $\frac{\sqrt{\pi}}{3}$ c) $\frac{2\sqrt{\pi}}{3}$ d) $\frac{3\sqrt{\pi}}{4}$

82) On using substitution $x^4=t$, the value of the integration $\int_0^\infty e^{-x^4}dx$ is given by

a) $\sqrt{\pi}$ b) π c) $\frac{1}{4}\left\lceil\frac{1}{4}\right.$ d) $\left\lceil\frac{3}{4}\right.$

83) On using substitution $x=t^2$, the value of the integration $\int_0^\infty \sqrt[4]{x}\cdot e^{-\sqrt{x}}dx$ is given by

a) $\frac{3\sqrt{\pi}}{2}$ b) $\frac{2\sqrt{\pi}}{3}$ c) $\frac{\sqrt{\pi}}{3}$ d) $2\sqrt{\pi}$

84) On using substitution $2x^2=t$, the value of the integration $\int_0^\infty x^7\cdot e^{-2x^2}dx$ is given by

a) $\left\lceil\frac{3}{4}\right.$ b) $\frac{3}{8}$ c) $\frac{2\sqrt{\pi}}{3}$ d) $\frac{3}{16}$

85) On using substitution $2x^2=t$, the value of the integration $\int_0^\infty x^9\cdot e^{-2x^2}dx$ is given by

a) $\left\lceil\frac{3}{4}\right.$ b) $\frac{3}{8}$ c) $\frac{2\sqrt{\pi}}{3}$ d) $\frac{3}{16}$

86) On using substitution $x^2=t$, the value of the integration $\int_0^\infty x^2\cdot e^{-x^2}dx$ is given by

a) $\frac{1}{3}\left\lceil\frac{3}{2}\right.$ b) $\frac{3}{2}\left\lceil\frac{3}{2}\right.$ c) $\frac{1}{2}\left\lceil\frac{3}{2}\right.$ d) $\frac{1}{2}\left\lceil\frac{2}{3}\right.$

87) On using substitution $x=t^{1/3}$, the value of the integration $\int_0^\infty \sqrt{x}\cdot e^{-x^3}dx$ is given by

a) $\frac{\sqrt{\pi}}{3}$ b) $\frac{2\sqrt{\pi}}{3}$ c) $\frac{1}{2}\left\lceil\frac{2}{3}\right.$ d) $\frac{1}{3}\left\lceil\frac{3}{2}\right.$

88) On using substitution $a^{-x}=e^{-t}$, the value of the integration $\int_0^\infty \frac{x^a}{a^x}dx$ is given by

a) $\frac{\left\lceil a\right.}{(\log a)^a}$ b) $\frac{\left\lceil a-1\right.}{(\log a)^{a-1}}$

c) $\dfrac{\overline{|a+1}}{(\log a)^{a+1}}$ d) $\dfrac{\overline{|a}}{(\log a)^{a+1}}$

89) On using substitution $3^{-x}=e^{-t}$, the value of the integration $\int_0^\infty \frac{x^3}{3^x}dx$ is given by

a) $\dfrac{3}{(\log 3)^4}$ b) $\dfrac{6}{(\log 3)^4}$

c) $\dfrac{36}{(\log 3)^4}$ d) $\dfrac{6}{(\log 3)^3}$

90) On using substitution $\log x=-t$, the value of the integration $\int_0^1 (x\log x)^3 dx$ is given by

a) $-\dfrac{3}{64}$ b) $\dfrac{3}{64}$ c) $\dfrac{3}{128}$ d) $-\dfrac{3}{128}$

91) On using substitution $\log x=-t$, the value of the integration $\int_0^1 \left(\log\frac{1}{x}\right)^{n-1} dx$ is given by

a) $\overline{|n+1}$ b) $\overline{|n}$ c) $\overline{|n-1}$ d) $-\overline{|1+n}$

92) On using substitution $\log x=-t$, the value of the integration $\int_0^1 \frac{1}{\sqrt{x\log\frac{1}{x}}}dx$ is given by

a) $\sqrt{2\pi}$ b) $\pi\sqrt{2}$ c) $2\sqrt{\pi}$ d) 2π

93) On using substitution $\log x=-t$, the value of the integration $\int_0^1 \frac{1}{\sqrt{-\log x}}dx$ is given by

a) $\sqrt{2\pi}$ b) $\pi\sqrt{2}$ c) $\sqrt{\pi}$ d) $2\sqrt{\pi}$

94) On using substitution $h^2x^2=t$, the value of the integration $\int_0^\infty x^{n-1}e^{-h^2x^2}dx$ is given by

a) $\sqrt{2\pi}$ b) $\dfrac{\overline{|n/2}}{2h^n}$ c) $\dfrac{\overline{|n/2}}{2h^{n+1}}$ d) $\dfrac{\overline{|1+n/2}}{2h^{n+1}}$

II) Beta Functions

95) The value of $\beta(m,n)$ in the integral form is

a) $\int_0^1 x^m(1-x)^{n-1}dx$ b) $\int_0^1 x^m(1-x)^n dx$

c) $\int_0^1 x^{m+1}(1-x)^{n+1}dx$ d) $\int_0^1 x^{m-1}(1-x)^{n-1}dx$

96) The value of $\beta(m,n)$ in terms of gamma function is

a) $\dfrac{\overline{|m}\cdot\overline{|n}}{\overline{|m+n+1}}$ b) $\dfrac{\overline{|m-1}\cdot\overline{|n-1}}{\overline{|m+n}}$

c) $\dfrac{\overline{|m+1}\cdot\overline{|n+1}}{\overline{|m+n+1}}$ d) $\dfrac{\overline{|m}\cdot\overline{|n}}{\overline{|m+n}}$

97) The value of $\beta(m,n)$, when m and n are positive integers is

a) $\dfrac{(m-1)!(n-1)!}{(m+n-1)!}$ b) $\dfrac{(m+1)!(n+1)!}{(m+n+1)!}$

c) $\dfrac{m!n!}{(m+n)!}$ d) $\dfrac{m!n!}{(m+n+1)!}$

98) $\int_0^{\pi/2} \sin^m x\cos^n x dx$ is given by

a) $\beta(m,n)$ b) $\frac{1}{2}\beta\left(\frac{m-1}{2},\frac{n-1}{2}\right)$

c) $\frac{1}{2}\beta\left(\frac{m+1}{2},\frac{n+1}{2}\right)$ d) $\beta\left(\frac{m+1}{2},\frac{n+1}{2}\right)$

99) $\int_0^{\pi/2} \sin^{2m-1} x\cos^{2n-1} x dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{m+1}{2},\frac{n+1}{2}\right)$ b) $\beta\left(\frac{m+1}{2},\frac{n+1}{2}\right)$

c) $\frac{1}{2}\beta\left(\frac{m-1}{2},\frac{n-1}{2}\right)$ d) $\beta(m,n)$

100) $\int_0^{\pi/2} \sin^m x\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{m-1}{2},\frac{1}{2}\right)$ b) $\frac{1}{2}\beta\left(m,\frac{1}{2}\right)$

c) $\frac{1}{2}\beta\left(\frac{m+1}{2},\frac{1}{2}\right)$ d) $\frac{1}{2}\beta\left(\frac{m+1}{2},0\right)$

101) $\int_0^{\pi/2} \cos^m x\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{m-1}{2}, \frac{1}{2}\right)$ b) $\frac{1}{2}\beta\left(m, \frac{1}{2}\right)$

c) $\frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{1}{2}\right)$ d) $\frac{1}{2}\beta\left(\frac{m+1}{2}, 0\right)$

102) $\int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}}dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$ b) $\beta(m, n)$

c) $\beta(m+1, n+1)$ d) $\beta(m-1, n-1)$

103) $\beta(3, 5)$ can be represented by

a) $\int_0^{\infty} x^2(1-x)^4\,dx$ b) $\int_0^1 x^4(1-x)^6\,dx$

c) $\int_0^1 x^3(1-x)^5\,dx$ d) $\int_0^1 x^2(1-x)^4\,dx$

104) What is the exact value of $\beta(5, 3)$?

a) $\frac{2}{35}$ b) $\frac{2}{105}$ c) $\frac{1}{105}$ d) $\frac{1}{35}$

105) What is the exact value of $\beta\left(\frac{1}{4}, \frac{3}{4}\right)$?

a) $\frac{1}{8}$ b) $\pi\sqrt{2}$ c) $2\sqrt{\pi}$ d) $\sqrt{2\pi}$

106) $\int_0^1 \sqrt{x}(1-x)^{5/2}\,dx$ is equal to

a) $\beta\left(\frac{3}{2}, \frac{7}{2}\right)$ b) $\beta\left(\frac{1}{2}, \frac{5}{2}\right)$

c) $\beta\left(\frac{2}{3}, \frac{5}{3}\right)$ d) $\beta(2, 5)$

107) $\int_0^1 x^4(1-x)^5\,dx$ is equal to

a) $\frac{3}{462}$ b) $\frac{1}{462}$ c) $\frac{1}{501}$ d) $\frac{1}{231}$

108) $2\int_0^{\pi/2} \sin^{\frac{3}{2}} x\cos^5 x\,dx$ is given by

a) $\beta\left(\frac{5}{4}, 3\right)$ b) $\frac{1}{2}\beta\left(\frac{5}{4}, 3\right)$

c) $\beta\left(\frac{5}{4}, \frac{3}{2}\right)$ d) $\beta\left(\frac{5}{4}, \frac{3}{4}\right)$

109) $2\int_0^{\pi/2} \sqrt{\sin x\cos x}\,dx$ is given by

a) $\beta\left(\frac{1}{2}, \frac{1}{2}\right)$ b) $\beta\left(\frac{5}{4}, \frac{5}{4}\right)$

c) $\beta\left(\frac{3}{4}, \frac{3}{4}\right)$ d) $\beta\left(\frac{3}{2}, \frac{3}{2}\right)$

110) $\int_0^{\pi/2} \frac{1}{\sqrt{\sin x}}dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{1}{4}, \frac{3}{2}\right)$ b) $\beta\left(\frac{1}{4}, \frac{1}{2}\right)$

c) $2\beta\left(\frac{1}{4}, \frac{1}{2}\right)$ d) $\frac{1}{2}\beta\left(\frac{1}{4}, \frac{1}{2}\right)$

111) $\int_0^{\pi/2} \frac{1}{\sqrt{\cos x}}dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{1}{4}, \frac{3}{2}\right)$ b) $\frac{1}{2}\beta\left(\frac{1}{4}, \frac{1}{2}\right)$

c) $\beta\left(\frac{1}{4}, \frac{1}{2}\right)$ d) $2\beta\left(\frac{1}{4}, \frac{1}{2}\right)$

112) $\int_0^{\pi/2} \sqrt{\tan x}\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{3}{4}, \frac{5}{4}\right)$ b) $\beta\left(\frac{3}{4}, \frac{1}{4}\right)$

c) $2\beta\left(\frac{3}{4}, \frac{1}{4}\right)$ d) $\frac{1}{2}\beta\left(\frac{3}{4}, \frac{1}{4}\right)$

113) $\int_0^{\pi/2} \sqrt{\cot x}\,dx$ is given by

a) $2\beta\left(\frac{3}{4}, \frac{1}{4}\right)$ b) $\frac{1}{2}\beta\left(\frac{3}{4}, \frac{5}{4}\right)$

c) $\frac{1}{2}\beta\left(\frac{3}{4}, \frac{1}{4}\right)$ d) $\beta\left(\frac{3}{4}, \frac{1}{4}\right)$

114) $\int_0^{\pi/2} \tan^{3/4} x\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{7}{4}, \frac{1}{4}\right)$ b) $\frac{1}{2}\beta\left(\frac{7}{4}, \frac{1}{4}\right)$

c) $\frac{1}{2}\beta\left(\frac{7}{8}, \frac{1}{8}\right)$　　d) $\frac{1}{2}\beta\left(\frac{7}{8}, \frac{7}{8}\right)$

115) The value of the integral $\int_0^\infty \frac{x^4}{(1+x)^7}dx$ is

a) $\frac{1}{30}$　b) 30　c) $\frac{1}{15}$　d) $\frac{1}{3}$

116) The value of the integral $\int_0^\infty \frac{x^3+x^2}{(1+x)^7}dx$ is

a) 30　b) $\frac{1}{3}$　c) $\frac{1}{30}$　d) $\frac{1}{15}$

117) The value of the integral $\int_0^\infty \frac{x^8-x^{14}}{(1+x)^{24}}dx$ is

a) 30　b) 0　c) $\frac{1}{30}$　d) $\frac{1}{15}$

118) The value of the integral $\int_0^\infty \frac{x^6(1-x^8)}{(1+x)^{24}}dx$ is

a) 30　b) 0　c) $\frac{1}{30}$　d) $\frac{1}{15}$

119) $\beta(n, n+1)$ is identical with

a) $\frac{(\Gamma n)^2}{\Gamma 2n}$　b) $\frac{\Gamma n}{\Gamma 2n}$　c) $\frac{\Gamma n}{2\Gamma 2n}$　d) $\frac{(\Gamma n)^2}{2\Gamma 2n}$

120) $\beta(m, n+1)+\beta(m+1, n)$ is equal to

a) $\beta(m+1, n+1)$　b) $\beta(m+1, n)$

c) $\beta(m, n)$　d) $\beta(m, n+1)$

121) $\beta(m, n)\cdot\beta(m+n, k)$ is equal to

a) $\frac{\Gamma m\cdot\Gamma n+k}{\Gamma m+n+k}$　b) $\frac{\Gamma m\cdot\Gamma n\cdot\Gamma k}{\Gamma m+n}$

c) $\frac{\Gamma m\cdot\Gamma n}{\Gamma m+n+k}$　d) $\frac{\Gamma m\cdot\Gamma n\cdot\Gamma k}{\Gamma m+n+k}$

122) $\beta(m, n+1)$ is equal to

a) $\frac{m+n}{n}\beta(m, n)$　b) $\frac{n}{m+n}\beta(m, n)$

c) $\frac{m}{m+n}\beta(m, n)$　d) $\frac{m+n}{m}\beta(m, n)$

123) On using substitution $x^3=8t$, the integral $\int_0^2 x(8-x^3)^{1/3}dx$ is equal to

a) $\frac{5}{81}$　b) $\frac{2}{27}$　c) $\frac{2}{81}$　d) $\frac{1}{81}$

124) The value of the integration $\int_0^1 x^3\left(1-x^{1/2}\right)^5 dx$ by substituting $x=t^2$ is given by

a) $2\beta(8, 6)$　b) $\frac{1}{2}\beta(8, 6)$

c) $\beta(8, 6)$　d) $2\beta(9, 7)$

125) The value of the integration $\int_0^1\left(1-x^{1/n}\right)^m dx$ by substituting $x=t^n$ is given by

a) $n\beta(m, n+1)$　b) $n\beta(m+1, n)$

c) $n\beta(m, n)$　d) $m\beta(m+1, n)$

Chapter 05–Differentiation Under Integral Sign & Error Function

I) Differentiation Under Integral Sign

1) If $I(\alpha)=\int_a^b f(x,\alpha)\,dx$, where α is parameter and a, b are constants, by differentiation under integral sign rule we have

a) $\frac{dI}{d\alpha}=\int_a^b \frac{\partial}{\partial\alpha}\left[f(x,\alpha)\right]\cdot dx$

b) $\frac{dI}{d\alpha}=\int_a^b \frac{\partial}{\partial x}\left[f(x,\alpha)\right]\cdot dx$

c) $\frac{dI}{dx}=\int_a^b \frac{\partial}{\partial x}\left[f(x,\alpha)\right]\cdot dx$

d) $\frac{dI}{dx}=\int_a^b \frac{\partial}{\partial\alpha}\left[f(x,\alpha)\right]\cdot dx$

2) If $I(\alpha)=\int_a^b f(x,\alpha)\,dx$, where α is parameter and a, b are functions of α, by differentiation under integral sign rule we have

a) $\frac{dI}{d\alpha}=\int_a^b \frac{\partial}{\partial\alpha}\left[f(x,\alpha)\right]\cdot dx-f(x,b)\frac{db}{d\alpha}+f(x,a)\frac{da}{d\alpha}$

b) $\frac{dI}{d\alpha}=\int_a^b \frac{\partial}{\partial\alpha}\left[f(x,\alpha)\right]\cdot dx+f(x,b)\frac{db}{dx}-f(x,a)\frac{da}{dx}$

c) $\frac{dI}{d\alpha}=\int_a^b \frac{\partial}{\partial\alpha}\left[f(x,\alpha)\right]\cdot dx+f(x,b)\frac{db}{d\alpha}-f(x,a)\frac{da}{d\alpha}$

d) $\frac{dI}{dx}=\int_a^b \frac{\partial}{\partial x}\left[f(x,\alpha)\right]\cdot dx+f(x,b)\frac{db}{dx}-f(x,a)\frac{da}{dx}$

Note: Henceforth, we abbreviate "differentiation under integral sign" by "DUIS" for simplicity.

3) If $I=\int_0^\infty e^{-bx^2}\cos 2ax\cdot dx$, where $b>0$, by DUIS rule we have

a) $\frac{dI}{dx}=\int_0^\infty \frac{\partial}{\partial x}\left[e^{-bx^2}\cos 2ax\right]\cdot dx$

b) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial a}\left[e^{-bx^2}\cos 2ax\right]\cdot dx$

c) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial b}\left[e^{-bx^2}\cos 2ax\right]\cdot dx$

d) $\frac{dI}{db}=\int_0^\infty \frac{\partial}{\partial a}\left[e^{-bx^2}\cos 2ax\right]\cdot dx$

4) If $I=\int_0^\infty \frac{e^{-ax}}{x}\left(1-e^{-bx}\right)dx$, where $a,b>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial b}\left[\frac{e^{-ax}}{x}\left(1-e^{-bx}\right)\right]dx$

b) $\frac{dI}{dx}=\int_0^\infty \frac{\partial}{\partial x}\left[\frac{e^{-ax}}{x}\left(1-e^{-bx}\right)\right]dx$

c) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial a}\left[\frac{e^{-ax}}{x}\left(1-e^{-bx}\right)\right]dx$

d) $\frac{dI}{db}=\int_0^\infty \frac{\partial}{\partial a}\left[\frac{e^{-ax}}{x}\left(1-e^{-bx}\right)\right]dx$

5) If $I=\int_0^\infty \frac{e^{-ax}}{x}\left(1-e^{-x}\right)dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial a}\left[\frac{\left(1-e^{-x}\right)}{x}\right]\cdot e^{-ax}dx$

b) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial x}\left[e^{-ax}\right]\cdot\frac{\left(1-e^{-x}\right)}{x}dx$

c) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial x}\left[e^{-ax}\frac{\left(1-e^{-x}\right)}{x}\right]\cdot dx$

d) $\frac{dI}{da}=\int_0^\infty \frac{\partial}{\partial a}\left[e^{-ax}\right]\cdot\frac{\left(1-e^{-x}\right)}{x}dx$

6) If $I=\int_0^\infty \frac{e^{-x}}{x}\left(a-\frac{1}{x}+\frac{1}{x^2}e^{-ax}\right)dx$, where $a,b>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty \frac{e^{-x}}{x}\cdot\frac{\partial}{\partial x}\left(a-\frac{1}{x}+\frac{1}{x^2}e^{-ax}\right)dx$

b) $\frac{dI}{dx}=\int_0^\infty \frac{e^{-x}}{x}\cdot\frac{\partial}{\partial a}\left(a-\frac{1}{x}+\frac{1}{x^2}e^{-ax}\right)dx$

c) $\frac{dI}{dx}=\int_0^\infty \frac{e^{-x}}{x}\cdot\frac{\partial}{\partial x}\left(a-\frac{1}{x}+\frac{1}{x^2}e^{-ax}\right)dx$

d) $\frac{dI}{da}=\int_0^\infty \frac{e^{-x}}{x}\cdot\frac{\partial}{\partial a}\left(a-\frac{1}{x}+\frac{1}{x^2}e^{-ax}\right)dx$

7) If $I=\int_0^\infty \frac{e^{-ax}}{x}\left(1-e^{-x}\right)dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty e^{(a+1)x}dx$ b) $\frac{dI}{da}=\int_0^\infty e^{-ax}dx$

c) $\frac{dI}{da}=\int_0^\infty e^{-(a+1)x}dx$ d) $\frac{dI}{da}=\int_0^\infty e^{-(a-1)x}dx$

8) If $I=\int_0^\infty \frac{e^{-x}}{x}\left(a-\frac{1}{x}+\frac{1}{x^2}e^{-ax}\right)dx$, where $a,b>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty \frac{e^{-x}}{x}\left(1-\frac{1}{x}e^{-ax}\right)dx$

b) $\frac{dI}{da}=\int_0^\infty \frac{e^{-x}}{x}\left(1-e^{-ax}\right)dx$

c) $\frac{dI}{da}=\int_0^\infty \frac{e^{-x}}{x}\left(1-\frac{1}{x^2}e^{-ax}\right)dx$

d) $\frac{dI}{da}=\int_0^\infty \left(1-\frac{1}{x}e^{-ax}\right)dx$

9) If $I=\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x}dx$, where $a,b>0$, by DUIS rule we have

a) $\frac{dI}{db}=-\int_0^\infty e^{-ax}dx$ b) $\frac{dI}{da}=-\int_0^\infty e^{-ax}dx$

c) $\frac{dI}{da}=\int_0^\infty e^{-ax}dx$ d) $\frac{dI}{da}=\int_0^\infty \left(e^{-ax}-e^{-ax}\right)dx$

10) If $I=\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x}dx$, where $a,b>0$, by DUIS rule we have

a) $\frac{dI}{db}=\int_0^\infty \left(e^{-ax}-e^{-ax}\right)dx$ b) $\frac{dI}{db}=-\int_0^\infty e^{-bx}dx$

c) $\frac{dI}{db}=\int_0^\infty e^{-ax}dx$ d) $\frac{dI}{db}=\int_0^\infty e^{-bx}dx$

11) If $I=\int_0^\infty \frac{e^{-x}-e^{-ax}}{x\sec x}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty \frac{e^{-ax}}{\sec x}dx$ b) $\frac{dI}{da}=\int_0^\infty \frac{e^{-ax}}{\sec x\tan x}dx$

c) $\frac{dI}{da}=-\int_0^\infty \frac{e^{-ax}}{\sec x}dx$ d) $\frac{dI}{da}=-\int_0^\infty \frac{ae^{-ax}}{x\sec x}dx$

12) If $I=\int_0^\infty e^{-a^2}\cos ax\,da$, where $x>0$, by DUIS rule we have

a) $\frac{dI}{dx}=-2\int_0^\infty a^2e^{-a^2}\sin ax\,da$

b) $\frac{dI}{dx}=2\int_0^\infty ae^{-a^2}\sin ax\,da$

c) $\frac{dI}{dx}=-2\int_0^\infty ae^{-a^2}\cos ax\,da$

d) $\frac{dI}{dx}=-\int_0^\infty ae^{-a^2}\sin ax\,da$

13) If $I=\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}\frac{1}{x^2}dx$

b) $\frac{dI}{da}=a\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}\frac{1}{x^2}dx$

c) $\frac{dI}{da}=-2a\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}\frac{1}{x^2}dx$

d) $\frac{dI}{da}=-\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}\frac{1}{x^2}dx$

14) If $I=\int_0^\infty \frac{e^{-x}\sin ax}{x}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=-a\int_0^\infty \cos ax\,dx$ b) $\frac{dI}{da}=\int_0^\infty \sin ax\,dx$

c) $\frac{dI}{da}=-\int_0^\infty e^{-x}\cos ax\,dx$ d) $\frac{dI}{da}=\int_0^\infty e^{-x}\cos ax\,dx$

15) If $I=\int_0^\pi \frac{x^a-1}{\log x}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty \frac{x^a\log a}{\log x}dx$ b) $\frac{dI}{da}=\int_0^\pi x^a\,dx$

c) $\frac{dI}{da}=\int_0^\pi x^a\log a\,dx$ d) $\frac{dI}{da}=\int_0^\pi \frac{x^a\log a}{\log x}dx$

16) If $I=\int_0^1 \frac{x^a-x^b}{\log x}dx$, where $a,\ b>0$, by DUIS rule we have

a) x^a-x^b b) $\frac{dI}{da}=\int_0^\pi \frac{x^a-x^b}{x\log x}dx$

c) $\frac{dI}{da}=\int_0^1 \frac{x^a\log a}{\log x}dx$ d) $\frac{dI}{da}=\int_0^1 x^a\,dx$

17) If $I=\int_0^\pi \log(1+a\cos x)\,dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\pi \frac{-\sin x}{1+a\cos x}dx$ b) $\frac{dI}{da}=\int_0^\pi \frac{\cos x}{1+a\cos x}dx$

c) $\frac{dI}{da}=\int_0^\pi \frac{a}{1+a\cos x}dx$ d) $\frac{dI}{da}=-\int_0^\pi \frac{\cos x}{1+a\cos x}dx$

18) If $I=\int_0^\pi \frac{1}{x^2}\log(1+ax^2)\,dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\pi \frac{ax^2}{1+ax^2}dx$ b) $\frac{dI}{da}=2\int_0^\pi \frac{x}{1+ax^2}dx$

c) $\frac{dI}{da}=\int_0^\pi \frac{1}{1+ax^2}dx$ d) $\frac{dI}{da}=\int_0^\pi \frac{2ax}{1+ax^2}dx$

19) If $I=\int_0^\pi \frac{1}{\sin^2 x}\log(1+a\sin^2 x)\,dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\pi \frac{1}{1+a\sin^2 x}dx$ b) $\frac{dI}{da}=\int_0^\pi \frac{\sin 2x}{1+a\sin^2 x}dx$

c) $\frac{dI}{da}=\int_0^\pi \frac{a}{1+a\sin^2 x}dx$ d) $\frac{dI}{da}=\int_0^\pi \frac{\cos x}{1+a\sin^2 x}dx$

20) If $I=\int_0^\infty \frac{1-\cos ax}{x^2}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^\infty \frac{a\sin ax}{x^2}dx$ b) $\frac{dI}{da}=-\int_0^\infty \frac{\cos ax}{x}dx$

c) $\frac{dI}{da}=\int_0^\infty \frac{\sin ax}{x}dx$ d) $\frac{dI}{da}=-\int_0^\infty \frac{\sin ax}{x}dx$

21) If $I=\int_0^1 \frac{x^a}{\log x}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^1 \frac{x^a\log a}{\log x}dx$ b) $\frac{dI}{da}=\int_0^1 x^a\,dx$

c) $\frac{dI}{da}=\int_0^1 x^a\log a\,dx$ d) $\frac{dI}{da}=\int_0^1 x^{a-1}\,dx$

22) If $I=\int_0^{\pi/2} \log(a^2\cos^2 x+b^2\sin^2 x)\,dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{\pi/2} \frac{1}{a^2+b^2\tan^2 x}dx$

b) $\frac{dI}{da}=\int_0^{\pi/2} \frac{b^2}{a^2+b^2\tan^2 x}dx$

c) $\frac{dI}{da}=\int_0^{\pi/2} \frac{a^2}{a^2+b^2\tan^2 x}dx$

d) $\frac{dI}{da}=\int_0^{\pi/2} \frac{2a}{a^2+b^2\tan^2 x}dx$

23) If $I=\int_0^\infty \frac{\sin ax-\sin bx}{x^2}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=-\int_0^\infty \frac{\cos bx}{x}dx$ b) $\frac{dI}{da}=\int_0^\infty \frac{\cos ax}{x}dx$

c) $\frac{dI}{da}=-\int_0^\infty \frac{\cos ax}{x}dx$ d) $\frac{dI}{db}=\int_0^\infty \frac{\cos ax}{x}dx$

24) If $I=\int_0^{a^2} \tan^{-1}\left(\frac{x}{a}\right)dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{a^2} \frac{\partial}{\partial a}\tan^{-1}\left(\frac{x}{a}\right)dx$

b) $\frac{dI}{da}=\int_0^{a^2} \frac{\partial}{\partial a}\tan^{-1}\left(\frac{x}{a}\right)dx-2a\tan^{-1}a$

c) $\frac{dI}{da}=\int_0^{a^2} \frac{\partial}{\partial a}\tan^{-1}\left(\frac{x}{a}\right)dx+2a\tan^{-1}x$

d) $\frac{dI}{da}=\int_0^{a^2} \frac{\partial}{\partial a}\tan^{-1}\left(\frac{x}{a}\right)dx+2a\tan^{-1}a$

25) If $I=\int_0^a \frac{\log(1+ax)}{1+x^2}dx$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^a \frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right]dx+\frac{\log(1+a^2)}{1+a^2}$

b) $\frac{dI}{da}=\int_0^a \frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right]dx-\frac{\log(1+a^2)}{1+a^2}$

c) $\frac{dI}{da}=\int_0^a \frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right]dx$

d) $\frac{dI}{da}=\int_0^a \frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right]dx+\frac{\log(1+x^2)}{1+x^2}$

26) If $I=\int_a^{a^2} \log(ax)\,dx$, by DUIS rule we have

a) $\frac{dI}{da}=\int_a^{a^2} \frac{\partial}{\partial a}\log(ax)\,dx$

b) $\frac{dI}{da}=\int_a^{a^2} \frac{\partial}{\partial a}\log(ax)\,dx-(6a-2)\log a$

c) $\frac{dI}{da}=\int_a^{a^2} \frac{\partial}{\partial a}\log(ax)\,dx+(6a+2)\log a$

d) $\frac{dI}{da}=\int_a^{a^2} \frac{\partial}{\partial a}\log(ax)\,dx+(6a-2)\log a$

27) If $I=\int_t^{t^2} e^{tx^2}\,dx$, by DUIS rule we have

a) $\frac{dI}{dt}=\int_t^{t^2} x^2e^{tx^2}\,dx+2te^{t^5}-e^{t^3}$

b) $\frac{dI}{dt}=\int_t^{t^2} x^2e^{tx^2}\,dx-2te^{t^5}+e^{t^3}$

c) $\frac{dI}{dt}=\int_t^{t^2} te^{tx^2}\,dx+2te^{t^5}-e^{t^3}$

d) $\frac{dI}{dt}=\int_t^{t^2} t^3e^{tx^2}\,dx+2te^{t^5}-e^{t^3}$

28) If $I=\int_0^{a^2} \tan^{-1}\left(\frac{x}{a}\right)dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{a^2} \frac{x}{a^2+x^2}dx+2a\tan^{-1}a$

b) $\frac{dI}{da}=-\int_0^{a^2} \frac{a}{a^2+x^2}dx+2a\tan^{-1}a$

c) $\frac{dI}{da}=-\int_0^{a^2} \frac{x}{a^2+x^2}dx+2a\tan^{-1}a$

d) $\frac{dI}{da}=-\int_0^{a^2} \frac{x}{a^2+x^2}dx-2a\tan^{-1}a$

29) If $I=\int_a^{a^2} \log(ax)\,dx$, by DUIS rule we have

a) $\frac{dI}{da}=\int_a^{a^2} \frac{1}{x}dx-(6a-2)\log a$

b) $\frac{dI}{da}=\int_a^{a^2} \frac{1}{a}dx+(6a-2)\log a$

c) $\frac{dI}{da}=\int_a^{a^2} \frac{1}{a}dx-(6a-2)\log a$

d) $\frac{dI}{da}=\int_a^{a^2}\frac{1}{a}dx-(6a-2)\log a$

30) If $I=\int_0^a\frac{\log(1+ax)}{1+x^2}dx$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^a\frac{x}{(1+x^2)(1+ax)}dx+\frac{\log(1+a^2)}{1+a^2}$

b) $\frac{dI}{da}=\int_0^a\frac{1}{(1+x^2)(1+ax)}dx+\frac{\log(1+a^2)}{1+a^2}$

c) $\frac{dI}{da}=\int_0^a\frac{a}{(1+x^2)(1+ax)}dx+\frac{\log(1+a^2)}{1+a^2}$

d) $\frac{dI}{da}=\int_0^a\frac{x}{(1+x^2)(1+ax)}dx-\frac{\log(1+a^2)}{1+a^2}$

31) If $I=\int_{\pi/6a}^{\pi/3a}\frac{\sin ax}{x}dx$, by DUIS rule we have

a) $\frac{dI}{da}=\int_{\pi/6a}^{\pi/3a}\cos ax\,dx+\frac{1}{a}$

b) $\frac{dI}{da}=\int_{\pi/6a}^{\pi/3a}\cos ax\,dx-\frac{1}{2a}$

c) $\frac{dI}{da}=\int_{\pi/6a}^{\pi/3a}\cos ax\,dx-\frac{1}{a}$

d) $\frac{dI}{da}=\int_{\pi/6a}^{\pi/3a}\frac{\cos ax}{x}dx-\frac{1}{a}$

32) If $f(x)=\int_a^x(x-t)^2G(t)\,dt$, we have

a) $\frac{df}{dx}=\int_a^x\frac{\partial}{\partial x}(x-t)^2G(t)\,dt+(x-a)^2G(a)$

b) $\frac{df}{dx}=\int_a^x\frac{\partial}{\partial x}(x-t)^2G(t)\,dt$

c) $\frac{df}{dx}=\int_a^x\frac{\partial}{\partial x}(x-t)^2G(t)\,dt-(x-a)^2G(a)$

d) $\frac{df}{dx}=\int_a^x\frac{\partial}{\partial x}(x-t)^2G(t)\,dt+a^2G(a)$

33) If $y=\int_0^x f(t)\sin a(x-t)\,dt$, we have

a) $\frac{dy}{dx}=\int_0^x xf(t)\cos a(x-t)\,dt$

b) $\frac{dy}{dx}=\int_0^x af(t)\cos a(x-t)\,dt+f(x)$

c) $\frac{dy}{dx}=\int_0^x af(t)\cos a(x-t)\,dt-af(x)$

d) $\frac{dy}{dx}=a\int_0^x f(t)\cos a(x-t)\,dt$

34) For the integral $I(a)=\int_0^\infty\frac{e^{-x}}{x}(1-e^{-ax})dx$, we have $\frac{dI}{da}=\frac{1}{a+1}$, then I is

a) $\log(a+1)-1$ b) $\log(a+1)$

c) $\log(a+1)+1$ d) $-\frac{1}{(a+1)^2}$

35) The value of integration $I(a)=\int_0^1\frac{x^a-1}{\log x}dx$ with $\frac{dI}{da}=\frac{1}{a+1}$ is given by

a) $\log(a+1)$ b) $\log(a+1)-1$

c) $\log(a+1)+1$ d) $-\frac{1}{(a+1)^2}$

36) The value of integration $I(a)=\int_0^1\frac{e^{-2x}\sin ax}{x}dx$ with $\frac{dI}{da}=\frac{2}{a^2+4}$ is given by

a) $\tan^{-1}\left(\frac{a}{2}\right)+\frac{\pi}{2}$ b) $\tan^{-1}\left(\frac{a}{2}\right)$

c) $\frac{1}{2}\tan^{-1}\left(\frac{a}{2}\right)$ d) $\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$

37) The value of integration $I(a)=\int_0^\infty\frac{e^{-x}-e^{-ax}}{x\sec x}dx$ with $\frac{dI}{da}=\frac{a}{a^2+1}$ is given by

a) $2\log\left(\frac{2}{a^2+1}\right)$ b) $\frac{1}{2}\log\left(\frac{2}{a^2+1}\right)$

c) $\frac{1}{2}\log\left(\frac{a^2+1}{2}\right)$ d) $2\log\left(\frac{a^2+1}{2}\right)$

38) The value of integration $I(a)=\int_0^\infty \frac{1-\cos ax}{x^2}dx$ with $\frac{dI}{da}=\frac{\pi}{2}$ is given by

a) $2\pi a$ b) $\frac{\pi a}{3}$ c) $\frac{\pi}{2}$ d) $\frac{\pi a}{2}$

39) The value of integration $I=\int_0^\infty \frac{\log(1+ax^2)}{x^2}dx$, with $\frac{dI}{da}=\frac{\pi}{2\sqrt{a}}$ is given by

a) $\pi\sqrt{a}$ b) $2\sqrt{a}$ c) $\pi\sqrt{2}$ d) $a\sqrt{\pi}$

40) The value of integration $I=\int_0^{\pi/2} \frac{\log(1+a\sin^2 x)}{\sin^2 x}dx$, with $\frac{dI}{da}=\frac{\pi}{2\sqrt{a+1}}$ is given by

a) $\pi\sqrt{a+1}+\pi$ b) $\pi\sqrt{a+1}-\pi$

c) $\pi\sqrt{a+1}-\frac{\pi}{a}$ d) $\frac{\pi\sqrt{a+1}-\pi}{a}$

II) Error Functions

41) $erf(x)$ is given by

a) $\frac{1}{2\sqrt{\pi}}\int_0^x e^{-u^2}du$ b) $\frac{\sqrt{\pi}}{2}\int_0^x e^{-u^2}du$

c) $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$ d) $\int_0^x e^{-u^2}du$

42) $erfc(x)$ is given by

a) $\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2}du$ b) $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$

c) $\frac{\sqrt{\pi}}{2}\int_0^x e^{-u^2}du$ d) $\frac{\sqrt{\pi}}{2}\int_x^\infty e^{-u^2}du$

43) $erf(0)$ is given by

a) $\frac{2}{\sqrt{\pi}}$ b) 1 c) ∞ d) 0

44) $erf(\infty)$ is given by

a) 1 b) 0 c) $\frac{2}{\sqrt{\pi}}$ d) ∞

45) $erfc(0)$ is given by

a) 0 b) $\frac{2}{\sqrt{\pi}}$ c) ∞ d) 1

46) $erf(x)+erfc(x)=?$

a) 2 b) ∞ c) 1 d) 0

47) $erf(-x)=?$

a) $erfc(x)$ b) $-erf(x)$

c) $erf(x)$ d) $-erf(x^2)$

48) Error function is an

a) even function b) neither even nor odd

c) odd function d) none of these

49) $erf(x)+erf(-x)=?$

a) 0 b) 1 c) 2 d) 3

50) $erf(-x)+erfc(-x)=?$

a) 0 b) 3 c) 2 d) 1

51) $erfc(-x)-erf(x)=?$
a) ∞ b) 2 c) 1 d) 0

52) $erfc(x)+erfc(-x)=?$
a) 2 b) 1 c) 0 d) ∞

53) If $erf(ax)=\frac{2}{\sqrt{\pi}}\int_0^{ax}e^{-u^2}du$, then $\frac{d}{dx}[erf(ax)]$ is
a) $\frac{2a}{\sqrt{\pi}}e^{-x^2}$ b) $\frac{a}{2\sqrt{\pi}}e^{-a^2x^2}$
c) $\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$ d) $\frac{2x}{\sqrt{\pi}}e^{-a^2x^2}$

54) If $erfc(ax)=\frac{2}{\sqrt{\pi}}\int_{ax}^{\infty}e^{-u^2}du$, then $\frac{d}{dx}[erfc(ax)]$ is
a) $-\frac{2x}{\sqrt{\pi}}e^{-a^2x^2}$ b) $\frac{2x}{\sqrt{\pi}}e^{-a^2x^2}$
c) $-\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$ d) $\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$

55) If $erf(\sqrt{t})=\frac{2}{\sqrt{\pi}}\int_0^{\sqrt{t}}e^{-u^2}du$, then $\frac{d}{dx}[erf(\sqrt{t})]$ is
a) $\frac{1}{t\sqrt{\pi}}e^{-t}$ b) $\frac{1}{\sqrt{\pi t}}e^{-t}$
c) $\frac{2}{\sqrt{\pi t}}e^{-t}$ d) $\frac{1}{\sqrt{\pi t}}e^{-t^2}$

56) If $erfc(\sqrt{t})=\frac{2}{\sqrt{\pi}}\int_{\sqrt{t}}^{\infty}e^{-u^2}du$, then $\frac{d}{dx}[erfc(\sqrt{t})]$ is
a) $\frac{2}{\sqrt{\pi t}}e^{-t}$ b) $\frac{1}{\sqrt{\pi t}}e^{-t^2}$
c) $\frac{1}{t\sqrt{\pi}}e^{-t}$ d) $-\frac{1}{\sqrt{\pi t}}e^{-t}$

57) $\frac{d}{dx}[erf(x)+erfc(x)]=?$
a) 1 b) 0 c) 2 d) ∞

58) If $\frac{d}{dx}[erf(ax)]=\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$, then $\frac{d}{dx}[erfc(ax)]$ is
a) $-\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$ b) $\frac{2a}{\sqrt{\pi}}e^{a^2x^2}$
c) $\frac{1}{\sqrt{\pi}}e^{-a^2x^2}$ d) $\frac{4a^2}{\sqrt{\pi}}e^{-a^2x^2}$

59) On substitution $x+a=u$ in the integration $\int_0^{\infty}e^{-(x+a)^2}dx$, then the value of integration is
a) $\frac{\sqrt{\pi}}{2}erf(a)$ b) $\frac{2}{\sqrt{\pi}}erf(a)$
c) $\frac{\sqrt{\pi}}{2}erfc(a)$ d) $erfc(a)$

60) $\int_0^t erf(ax)dx+\int_0^t erfc(ax)dx=?$
a) 1 b) ∞ c) 0 d) t

61) If $\frac{dy}{dx}[erf(\sqrt{t})]=\frac{e^{-t}}{\sqrt{\pi t}}$, the integration $\int_0^{\infty}e^{-t}erf(\sqrt{t})dt$ is
a) $\frac{1}{\sqrt{\pi}}\int_0^{\infty}e^{-t}t^{1/2}dt$ b) $\frac{2}{\sqrt{\pi}}\int_0^{\infty}e^{-2t}t^{1/2}dt$
c) $-\frac{1}{\sqrt{\pi}}\int_0^{\infty}e^{-2t}t^{-1/2}dt$ d) $\frac{1}{\sqrt{\pi}}\int_0^{\infty}e^{-2t}t^{-1/2}dt$

62) The power series expansion of $erf(x)$ is
a) $\frac{2}{\sqrt{\pi}}\left[x-\frac{x^3}{3}+\frac{x^5}{10}-\frac{x^7}{42}+-\ldots\right]$
b) $\frac{2}{\sqrt{\pi}}\left[x+\frac{x^3}{3}+\frac{x^5}{10}+\frac{x^7}{42}+\ldots\right]$
c) $\frac{2}{\sqrt{\pi}}\left[x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+-\ldots\right]$
d) $\frac{2}{\sqrt{\pi}}\left[x+\frac{x^3}{3!}+\frac{x^5}{5!}+\frac{x^7}{7!}+\ldots\right]$

Chapter 06 – Curve Tracing & Rectification of Curves

I) Curve Tracing

1) If the portion of the curve lies on the both sides of the point lying above the tangent at that point, the curve is known as
a) concave upward b) concave downward
c) inflexion point d) none of these

2) If the portion of the curve lies on the both sides of the point lying above the tangent at that point, the curve is known as
a) inflexion point b) concave downward
c) inflexion point d) none of these

3) A point through which two branches of the same curve passes is known as
a) double point b) inflexion point
c) multiple point d) conjugate point

4) A point through which many branches of the same curve passes is known as
a) double point b) inflexion point
c) multiple point d) conjugate point

5) A double point through which the branches of the curve passes and the tangent at that point are real and distinct, the point is known as
a) conjugate point b) node
c) point of inflexion d) cusp

6) A double point through which the branches of the curve passes and the tangent at that point are real but the same, the point is known as
a) conjugate point b) point of inflexion
c) cusp d) node

7) A double point is said to be node if the tangents to the curve at that point are
a) imaginary b) perpendicular to each other
c) real but the same d) real and distinct

8) A double point is said to be cusp if the tangents at that point are
a) imaginary b) real and distinct
c) real but the same d) none of these

9) If at a point $\frac{dy}{dx}=0$, the tangent to the curve at that point is
a) parallel to the line $x+y=0$
b) parallel to x-axis
c) perpendicular to x-axis
d) parallel to $y=x$

10) If at a point $\frac{dy}{dx}=\infty$, the tangent to the curve at that point is
a) parallel to the line $x+y=0$
b) parallel to $y=x$
c) parallel to x-axis
d) perpendicular to x-axis

11) The standard equation of x-axis in the Cartesian form is given by
a) $x-y=0$ b) $x+y=0$
c) $y=0$ d) $x=0$

12) The standard equation of y-axis in the Cartesian form is given by
a) $x-y=0$ b) $x+y=0$
c) $y=0$ d) $x=0$

13) If the all the powers of y in the Cartesian form are even, the curve is symmetrical about
a) $y-axis$ b) $x,\ y-axes$
c) $x-axis$ d) the line $y=x$

14) If the all the powers of x in the Cartesian form are even, the curve is symmetrical about
a) $x,\ y-axes$ b) $y-axis$
c) $x-axis$ d) the line $y=x$

15) If the all the powers of x and y in the Cartesian form are even, the curve is symmetrical about
a) the line $y=x$ b) $x-axis$ only
c) $y-axis$ only d) $x,\ y-axes$

16) If in the equation of the Cartesian form by replacing $x\to y$ and $y\to x$, the equation is symmetrical about
a) the line $y=x$ b) $x,\ y-axes$

c) $x-axis$ d) $y-axis$

17) If in the equation of the Cartesian form by replacing $x \to -y$ and $y \to -x$, the equation is symmetrical about
a) the line $y=-x$ b) the line $y=x$
c) $x,\ y-axes$ d) $y-axis$ only

18) If in the equation of the Cartesian form by replacing $x \to -x$ and $y \to -y$, the equation is symmetrical about
a) the line $y=-x$ b) $x,\ y-axes$
c) opposite quadrants d) the line $y=x$

19) The equation of the tangent at origin when the curve is passing through origin is obtained by equating to zero
a) the lowest degree term of the equation
b) the highest degree term of x in equation
c) the highest degree term of y in equation
d) the coefficient of the term xy

20) In the Cartesian form, the asymptote to the curve parallel to x-axis may be obtained by equating to zero
a) the coefficient of lowest degree term in y
b) the coefficient of highest degree term in y
c) the coefficient of highest degree term in x
d) the coefficient of lowest degree term in x

21) In the Cartesian form, the asymptote to the curve parallel to y-axis may be obtained by equating to zero
a) the coefficient of lowest degree term in y
b) the coefficient of highest degree term in y
c) the coefficient of highest degree term in x
d) the coefficient of lowest degree term in x

22) Oblique asymptote are obtained only when the curve is
a) symmetrical about x-axis
b) symmetrical about y-axis
c) symmetrical about both x and y-axis
d) not symmetrical about both x and y-axes

23) In the Cartesian form if the coefficient of the highest degree term in x is constant, the curve has
a) no asymptote parallel to $x=y$
b) no asymptote parallel to y-axis
c) no asymptote parallel to x-axis
d) none of these

24) In the Cartesian form if the coefficient of the highest degree term in y is constant, the curve has
a) no asymptote parallel to $x+y=0$
b) no asymptote parallel to $x=y$
c) no asymptote parallel to x-axis
d) no asymptote parallel to y-axis

25) In the polar form, if the equation of the curve remains unchanged by replacing $\theta \to -\theta$, the curve is symmetrical about
a) the line $\theta=\frac{\pi}{4}$ b) the initial line
c) pole d) the line $\theta=\frac{\pi}{2}$

26) In the polar form, if the equation of the curve remains unchanged by replacing $r \to -r$, the curve is symmetrical about
a) the line $\theta=\frac{\pi}{4}$ b) the initial line
c) pole d) the line $\theta=\frac{\pi}{2}$

27) In the polar form, if the equation of the curve remains unchanged by replacing $\theta \to \pi-\theta$, the curve is symmetrical about
a) the line $\theta=\frac{\pi}{2}$ b) the line $\theta=\frac{\pi}{4}$
c) the initial line d) pole

28) The pole is point of the curve, if for given angle θ, the value of
a) $r=\infty$ b) $r=0$ c) $r<0$ d) $r>0$

29) If a curve is passing through the pole, the tangent to the curve at pole are obtained by solving
a) $r=0$ b) $r=\infty$ c) $\theta=0$ d) $\theta=\pi$

30) In the polar form, the relation between the angle ϕ formed by the radius vector and the tangent to the curve at that point, is given by
a) $\tan\phi=r^2\frac{d\theta}{dr}$ b) $\cot\phi=r\frac{d\theta}{dr}$
c) $\tan\phi=r\frac{d\theta}{dr}$ d) $\tan\phi=r\frac{dr}{d\theta}$

31) In the parametric form $x = f(t)$, $y = g(t)$, the curve is symmetrical about y-axis, if
a) $x = f(t)$ is odd and $y = g(t)$ is even
b) $x = f(t)$ is even and $y = g(t)$ is odd
c) $x = f(t)$ is odd and $y = g(t)$ is odd
d) $x = f(t)$ is even and $y = g(t)$ is even

32) In the parametric form $x = f(t)$, $y = g(t)$, the curve is symmetrical about y-axis, if
a) $x = f(t)$ is odd and $y = g(t)$ is odd
b) $x = f(t)$ is even and $y = g(t)$ is odd
c) $x = f(t)$ is odd and $y = g(t)$ is even
d) $x = f(t)$ is even and $y = g(t)$ is even

33) The curve represented by the equation $x^2y^2 = x^2 + 1$ is symmetrical about
a) the line $y = x$ b) x-axis only
c) y-axis only d) both x and y-axes

34) The curve represented by the equation $x\left(x^2 + y^2\right) = a\left(x^2 - y^2\right)$ is
a) symmetrical about y-axis but not passing through origin
b) symmetrical about y-axis and passing through origin
c) symmetrical about x-axis but not passing through origin
d) symmetrical about x-axis and passing through origin

35) The curve represented by the equation $a^2y^2 = x^2\left(a^2 - x^2\right)$ is
a) symmetrical about both x and y-axis but not passing through origin
b) symmetrical about both x and y-axis and passing through origin
c) symmetrical about x-axis but not passing through origin
d) symmetrical about x-axis only and passing through origin

36) The curve represented by the equation $(2a - x)y^2 = x^3$ is
a) symmetrical about y-axis and passing through origin
b) symmetrical about x-axis but not passing through origin
c) symmetrical about x-axis and passing through origin
d) symmetrical about y-axis not passing through origin

37) The curve represented by the equation $(2a - y)y^3 = a^2x^2$ is
a) symmetrical about y-axis and passing through origin and $(0, 2a)$
b) symmetrical about x-axis but not passing through origin and $(0, 2a)$
c) symmetrical about x-axis and passing through origin and $(0, 2a)$
d) symmetrical about y-axis not passing through origin and $(0, 2a)$

38) The curve represented by the equation $xy^2 = 4a^2(a - x)$ is
a) symmetrical about y-axis and passing through $(a, 0)$
b) symmetrical about x-axis but not passing through $(a, 0)$
c) symmetrical about x-axis and passing through $(a, 0)$
d) symmetrical about y-axis not passing through $(a, 0)$

39) The curve represented by the equation $xy^2 = 4a^2(a - x)$ has at origin
a) node b) cusp c) inflexion d) none

40) The curve represented by the equation $(2a - x)y^2 = x^3$ has the tangent at origin whose equation is
a) $x + y = 0$ b) y-axis c) x-axis d) $y = x$

41) The curve represented by the equation $\left(1 + x^2\right)y = x$ has the tangent at origin whose equation is
a) $y = x$ b) x-axis c) y-axis d) $x + y = 0$

42) The curve represented by the equation $3ay^2 = x(x - a)^2$ has the tangent at origin whose equation is
a) $x + y = 0$ b) $y = x$ c) x-axis d) y-axis

43) The curve represented by the equation $3ay^2 = x(x-a)^2$ has the asymptote parallel to x-axis whose equation is
a) $x+y=0$ b) $y=x$ c) x-axis d) y-axis

44) For the curve given by equation $x^2y = 4a^2(2a-y)$, the asymptote is
a) $y=2a$ b) $y=x$ c) y-axis d) x-axis

45) The curve represented by the equation $y^2(4-x) = x(x-2)^2$ has the asymptote parallel to y-axis whose equation is
a) $x=y$ b) $x=0$ c) $x=2$ d) $x=4$

46) The curve represented by the equation $x^2y^2 = a^2(y^2-x^2)$ has the asymptote parallel to y-axis whose equation is
a) $x=0$ b) $x=\pm a$ c) $x=y$ d) $y=0$

47) For the curve given by equation $x^2y = 4a^2(2a-y)$, the region of absence is
a) $0<y<2a$ b) $y>0,\ y>2a$
c) $y<0,\ y<2a$ d) $y<0,\ y>2a$

48) For the curve given by equation $x^3 = 4y^2(2a-x)$, the region of absence is
a) $0<x<2a$ b) $x<0,\ x>2a$
c) $x>0,\ x>2a$ d) $x<0,\ x<2a$

49) For the curve given by equation $xy^2 = 4a^2(a-x)$, the region of absence is
a) $0<x<a$ b) $x>0,\ x>a$
c) $x<0,\ x>a$ d) $x<0,\ x<a$

50) For the curve given by equation $y^2 = \dfrac{4x^2(a-x)}{x+a}$, the region of absence along x-axis is
a) $[-\infty,\ -a]$ & $[a,\ \infty]$ b) $[-\infty,\ a]$ & $[-a,\ \infty]$
c) $[-\infty,\ -a]$ d) $[-a,\ \infty]$

51) The curve represented by the equation $x^{1/2} + y^{1/2} = a^{1/2}$ is symmetrical about
a) $y=x$ b) x-axis c) y-axis d) $x+y=0$

52) The curve represented by the equation $\sqrt{x}+\sqrt{y}=\sqrt{a}$ has tangent at origin whose equation is
a) x-axis b) no tangent exists
c) y-axis d) $x+y=0$

53) The curve represented by the equation $\sqrt{x}+\sqrt{y}=\sqrt{a}$ has tangent at $(a,\ 0)$ which is
a) the line $x+y=0$ b) the line $y=x$
c) parallel to y-axis d) parallel to x-axis

54) The curve represented by the equation $x=t^2,\ y=t-\dfrac{t^3}{3}$ is symmetrical about
a) symmetrical about y-axis but not passing through origin
b) symmetrical about y-axis and passing through origin
c) symmetrical about x-axis but not passing through origin
d) symmetrical about x-axis and passing through origin

55) The curve represented by the equation $x=a(\theta+\sin\theta),\ y=a(1+\cos\theta)$ is symmetrical about
a) symmetrical about y-axis but not passing through origin
b) symmetrical about y-axis and passing through origin
c) symmetrical about x-axis but not passing through origin
d) symmetrical about x-axis and passing through origin

56) The curve represented by the equation $r=a(1+\cos\theta)$ is
a) symmetrical about initial line and not passing through the pole
b) symmetrical about initial line and passing through the pole
c) not symmetrical about initial line and passing through the pole
d) not symmetrical about initial line and not passing through the pole

57) The curve represented by the equation $r^2 = a^2 \cos 2\theta$ is
a) symmetrical about initial line as well as pole and not passing through the pole
b) symmetrical about initial line as well as pole and passing through the pole
c) not symmetrical about initial line as well as pole and passing through the pole
d) not symmetrical about initial line as well as pole and not passing through the pole

58) The curve represented by the equation $r^2 = a^2 \sin 2\theta$ is
a) symmetrical about the line $\theta = \frac{\pi}{4}$ and not passing through the pole
b) symmetrical about initial line as well as pole and passing through the pole
c) not symmetrical about the line $\theta = \frac{\pi}{4}$ and passing through the pole
d) not symmetrical about initial line as well as pole and not passing through the pole

59) The curve represented by the equation $r(1+\cos\theta) = 2a^2$ is
a) symmetrical about the line $\theta = \frac{\pi}{4}$ and not passing through the pole
b) symmetrical about initial line and passing through the pole
c) not symmetrical about the line $\theta = \frac{\pi}{4}$ and passing through the pole
d) symmetrical about initial and not passing through the pole

60) The equations of the tangents at pole to the curve $r = a\sin 3\theta$ are given by
a) $\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \ldots$
b) $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{3}, \ldots$
c) $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \ldots$
d) no such tangent exists

61) The equations of the tangents at pole to the curve $r = a\cos 2\theta$ are given by
a) $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \ldots$
b) $\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \ldots$
c) $\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$
d) $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots$

62) For the rose curve $r = a\sin n\theta$, if n is even, the curve is consisting of
a) $2n$ equal loops b) $2n+1$ equal loops
c) n equal loops d) $2n-1$ equal loops

63) For the rose curve $r = a\cos n\theta$, if n is even, the curve is consisting of
a) n equal loops b) $2n+1$ equal loops
c) $2n$ equal loops d) $2n-1$ equal loops

64) For the rose curve $r = a\sin n\theta$, if n is odd, the curve is consisting of
a) $2n$ equal loops b) n equal loops
c) $2n+1$ equal loops d) $2n-1$ equal loops

65) For the rose curve $r = a\cos n\theta$, if n is odd, the curve is consisting of
a) n equal loops b) $2n+1$ equal loops
c) $2n$ equal loops d) $2n-1$ equal loops

I) Rectification of Curve

66) If $A(a_1, b_1)$ $B(a_2, b_2)$ are two points on the curve on xy-plane, the length of arc is given by

a) $\int_{b_1}^{b_2} \sqrt{1-\left(\frac{dy}{dx}\right)^2} \cdot dy$ b) $\int_{b_1}^{b_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2} \cdot dy$

c) $\int_{a_1}^{a_2} \sqrt{1-\left(\frac{dy}{dx}\right)^2} \cdot dx$ d) $\int_{a_1}^{a_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2} \cdot dx$

67) If $A(a_1, b_1)$ $B(a_2, b_2)$ are two points on the curve on xy-plane, the length of arc is given by

a) $\int_{b_1}^{b_2} \sqrt{1+\left(\frac{dx}{dy}\right)^2} \cdot dy$ b) $\int_{a_1}^{a_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2} \cdot dx$

c) $\int_{a_1}^{a_2} \sqrt{1-\left(\frac{dy}{dx}\right)^2} \cdot dx$ d) $\int_{b_1}^{b_2} \sqrt{1-\left(\frac{dx}{dy}\right)^2} \cdot dy$

68) If $A(r_1, \theta_1)$ $B(r_2, \theta_2)$ are two points on the curve on the polar plane, the length of arc is given by

a) $\int_{\theta_1}^{\theta_2} \sqrt{r^2-\left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$ b) $\int_{\theta_1}^{\theta_2} \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

c) $\int_{r_1}^{r_2} \sqrt{1+r^2\left(\frac{d\theta}{dr}\right)^2} \cdot dr$ d) $\int_{r_1}^{r_2} \sqrt{1-r^2\left(\frac{d\theta}{dr}\right)^2} \cdot dr$

69) If $A(r_1, \theta_1)$ $B(r_2, \theta_2)$ are two points on the curve on the polar plane, the length of arc is given by

a) $\int_{\theta_1}^{\theta_2} \sqrt{r^2-\left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$ b) $\int_{\theta_1}^{\theta_2} \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

c) $\int_{r_1}^{r_2} \sqrt{1+r^2\left(\frac{d\theta}{dr}\right)^2} \cdot dr$ d) $\int_{r_1}^{r_2} \sqrt{1-r^2\left(\frac{d\theta}{dr}\right)^2} \cdot dr$

70) If $A(t_1)$ $B(t_2)$ are two points on the curve given by $x = f(t)$, $y = g(t)$ on the xy-plane, the length of arc is given by

a) $\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2} \cdot dt$

b) $\int_{t_1}^{t_2} \sqrt{\left(\frac{dt}{dx}\right)^2+\left(\frac{dt}{dy}\right)^2} \cdot dt$

c) $\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2-\left(\frac{dy}{dt}\right)^2} \cdot dt$

d) $\int_{t_1}^{t_2} \left[\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2\right] \cdot dt$

71) The arc length of the upper part of the loop of the curve $9y^2 = (x+7)(x+4)^2$ is obtained by solving the integration

a) $\int_{0}^{4} \sqrt{1+\left(\frac{dy}{dx}\right)^2} \cdot dx$ b) $\int_{-7}^{0} \sqrt{1+\left(\frac{dy}{dx}\right)^2} \cdot dx$

c) $\int_{-7}^{-4} \sqrt{1+\left(\frac{dy}{dx}\right)^2} \cdot dx$ d) $\int_{7}^{4} \sqrt{1+\left(\frac{dy}{dx}\right)^2} \cdot dx$

72) The arc length of the upper part of the curve $y^2 = 4x$ which is cut by the line $3y = 8x$ is obtained by solving the integration

a) $\int_{1}^{9/16} \sqrt{1+\left(\frac{dy}{dx}\right)^2} \cdot dx$ b) $\int_{0}^{9/16} \sqrt{1+\left(\frac{dy}{dx}\right)^2} \cdot dx$

c) $\int_{0}^{3/4} \sqrt{1+\left(\frac{dy}{dx}\right)^2} \cdot dx$ d) $\int_{3}^{8} \sqrt{1+\left(\frac{dy}{dx}\right)^2} \cdot dx$

73) The points $A(a, 0)$ $B(0, a)$ are two points on the curve $x^2 + y^2 = a^2$ on xy-plane such that $1+\left(\frac{dy}{dx}\right)^2 = \frac{a^2}{a^2-x^2}$, the length of arc is given by

a) $4a$ b) πa c) $\frac{\pi a}{4}$ d) $\frac{\pi a}{2}$

74) The points $A(0, 0)$ $B(a, b)$ are two points on the curve $y = a\cosh\left(\frac{x}{a}\right)$ on xy-plane such that $1+\left(\frac{dy}{dx}\right)^2 = \cosh^2\left(\frac{x}{a}\right)$, the length of arc is given by

a) $S = a\sinh\left(\frac{x}{a}\right)$ b) $S = a\tanh\left(\frac{x}{a}\right)$

c) $S = \sinh\left(\frac{x}{a}\right)$ d) $S = a\,\mathrm{sech}\left(\frac{x}{a}\right)$

75) The points $A(0, 0)$ $B(1, 0)$ are two points on the curve $3y^2 = x(x-1)^2$ on xy-plane such that $1+\left(\frac{dy}{dx}\right)^2 = \frac{(3x+1)^2}{12x}$, the length of arc is given by

a) $\frac{3}{\sqrt{3}}$ b) $\frac{1}{2\sqrt{3}}$ c) $\frac{1}{\sqrt{3}}$ d) $\frac{2}{\sqrt{3}}$

76) The total arc length of the part of the curve $r = a(1+\cos\theta)$ which is cut by the circle $r + a\cos\theta = 0$ is obtained by solving the integration

a) $\int_0^{2\pi/3} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

b) $2\int_0^{2\pi/3} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

c) $2\int_0^{2\pi/3} \sqrt{r^2 - \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

d) $\int_0^{2\pi/3} \sqrt{r^2 - \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

77) The total arc length of the upper part of the curve $r^2 = a^2\cos 2\theta$ is obtained by solving the integration

a) $2\int_0^{\pi/2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

b) $\int_0^{\pi/2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

c) $\int_0^{\pi/4} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

d) $\int_0^{\pi/3} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

78) The total length of the arc of the curve $r = ae^{m\theta}$ using $1 + r^2\left(\frac{d\theta}{dr}\right)^2 = 1 + \frac{1}{m^2}$ when r varies from r_1 to r_2 is given by

a) $\frac{\sqrt{1+m^2}}{m}(r_2 - r_1)$ b) $\frac{\sqrt{1+m^2}}{m}(r_2 + r_1)$

c) $\frac{\sqrt{1+m^2}}{m}(r_1 - r_2)$ d) $\frac{\sqrt{1-m^2}}{m}(r_2 - r_1)$

79) The total length of the arc formed by the upper half of the cardioide $r = a(1+\cos\theta)$ using $r^2 + \left(\frac{dr}{d\theta}\right)^2 = 2a^2(1+\cos\theta)$ when θ varies from 0 to π is given by

a) 4π b) 2π c) $4a$ d) $2a$

80) The total arc length of the upper part of the curve $x = t^2,\ y = t - \frac{t^3}{3}$ is obtained by solving the integration

a) $\int_0^{2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$

b) $\int_0^{3} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$

c) $\int_0^{1} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$

d) $\int_0^{\sqrt{3}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$

81) The total arc length of the two consecutive cusps lies in the first quadrant of the curve $x = a\cos^3\theta,\ y = a\sin^3\theta$ is obtained by solving the integration

a) $\int_0^{\pi/4} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \cdot d\theta$

b) $\int_0^{\pi/3} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \cdot d\theta$

c) $\int_0^{\pi/2} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \cdot d\theta$

d) $\int_0^{\pi} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \cdot d\theta$

82) The total arc length of the upper part of the curve $x = t^2,\ y = t - \frac{t^3}{3}$ between $t = 0$ to $t = \sqrt{3}$ with $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (1+t^2)^2$ is given by

a) $2\sqrt{3}$ b) $\sqrt{3}$ c) $\frac{2}{\sqrt{3}}$ d) $4\sqrt{3}$

83) The total arc length of the two consecutive cusps lies in the first quadrant of the curve $x = a\cos^3\theta,\ y = a\sin^3\theta$ between $\theta = 0$ to $\theta = \frac{\pi}{2}$ with $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 9a^2\sin^2\theta\cos^2\theta$ is given by

a) $\frac{3a}{4}$ b) $3a$ c) $\frac{3a}{2}$ d) $\frac{2a}{3}$

84) The total arc length of the two cusps between $\theta = -\pi$ to $\theta = \pi$ of the curve

$$x = a(\theta + \sin\theta),\ y = a(1 - \cos\theta)$$

with $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 4a^2\cos^2\left(\frac{\theta}{2}\right)$ is

a) $4a$ b) $8a$ c) $2a$ d) a

85) The total arc length of the two cusps between $\theta = 0$ to $\theta = \frac{\pi}{2}$ of the curve $x = e^\theta\cos\theta$, and $y = e^\theta\sin\theta$ with $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 2e^{2\theta}$ is

a) $\sqrt{2}\left(1 - e^{\pi/2}\right)$ b) $\sqrt{2}\left(e^{\pi} - 1\right)$

c) $\sqrt{2}\left(e^{\pi/2} + 1\right)$ d) $\sqrt{2}\left(e^{\pi/2} - 1\right)$

Chapter 03) Fourier Series

1	a		41	d		81	b		121	c
2	d		42	d		82	d		122	b
3	b		43	b		83	a		123	d
4	a		44	c		84	b		124	d
5	c		45	d		85	d		125	a
6	d		46	b		86	c		126	b
7	a		47	c		87	a		127	a
8	d		48	a		88	b		128	b
9	b		49	b		89	a		129	b
10	d		50	a		90	b'		130	c
11	d		51	c		91	c		131	a
12	b		52	b		92	a		132	b
13	a		53	c		93	c		133	d
14	d		54	d		94	d		134	d
15	b		55	d		95	a		135	a
16	b		56	c		96	b		136	c
17	a		57	a		97	c		137	d
18	d		58	b		98	d		138	a
19	a		59	d		99	b		139	b
20	b		60	a		100	d		140	a
21	a					101	d		141	d
22	c		62	d		102	b		142	c
23	d		63	c		103	c		143	b
24	a		64	d		104	a		144	c
25	d		65	b		105	d		145	a
26	a		66	d		106	b		146	d
27	d		67	b		107	d		147	c
28	c		68	c		108	d		148	a
29	b		69	a		109	d		149	c
30	c		70	c		110	a		150	b
31	a		71	c		111	d		151	d
32	d		72	c		112	c		152	b
33	a		73	d		113	c		153	a
34	c		74	b		114	a		154	c
35	a		75	d		115	b		155	d
36	c		76	c		116	a		156	d
37	a		77	b		117	c		157	a
38	c		78	c		118	b		158	c
39	c		79	b		119	a		159	b
40	b		80	d		120	b			

Chapter 04) Reduction Formulae & Beta, Gamma Function

1	c	26	d	51	a	76	d	101	c
2	b	27	b	52	c	77	a	102	b
3	c	28	c	53	b	78	c	103	d
4	d	29	a	54	d	79	d	104	c
5	d	30	b	55	b	80	c	105	b
6	c	31	a	56	d	81	b	106	a
7	a	32	c	57	a	82	c	107	b
8	c	33	b	58	d	83	a	108	a
9	b	34	c	59	a	84	d	109	c
10	a	35	d	60	c	85	b	110	d
11	c	36	d	61	d	86	c	111	b
12	b	37	c	62	c	87	a	112	d
13	d	38	a	63	b	88	c	113	c
14	a	39	d	64	a	89	b	114	c
15	a	40	b	65	c	90	d	115	a
16	c	41	d	66	d	91	b	116	c
17	c	42	c	67	b	92	a	117	b
18	c	43	a	68	a	93	c	118	b
19	b	44	b	69	b	94	b	119	d
20	d	45	d	70	c	95	d	120	c
21	c	46	d	71	d	96	d	121	d
22	d	47	b	72	a	97	a	122	b
23	b	48	d	73	b	98	c	123	c
24	d	49	b	74	c	99	d	124	a
25	c	50	c	75	a	100	c	125	b

Chapter 05) Differentiation Under Integral Sign & Error Function

1	a	14	d	27	a	40	b	53	c
2	c	15	b	28	c	41	c	54	c
3	b	16	d	29	d	42	a	55	b
4	c	17	b	30	a	43	d	56	d
5	d	18	c	31	c	44	a	57	b
6	d	19	a	32	b	45	d	58	a
7	c	20	c	33	d	46	c	59	c
8	a	21	b	34	b	47	b	60	d
9	b	22	d	35	a	48	c	61	d
10	d	23	b	36	b	49	a	62	a
11	a	24	d	37	c	50	d		
12	d	25	a	38	d	51	c		
13	c	26	d	39	a	52	a		

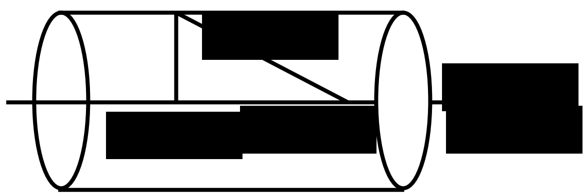
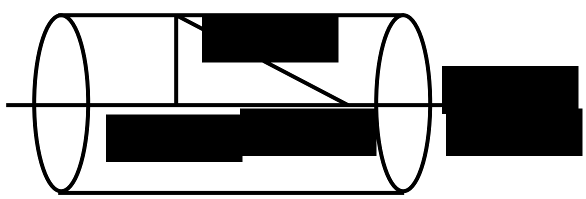
Chapter 06) Curve Tracing & Rectification of Curves

1	a		18	c		35	d		52	b		69	c
2	b		19	a		36	c		53	d		70	a
3	a		20	c		37	a		54	d		71	c
4	c		21	b		38	c		55	a		72	b
5	b		22	d		39	b		56	b		73	d
6	c		23	c		40	d		57	b		74	a
7	d		24	d		41	a		58	a		75	d
8	c		25	b		42	d		59	d		76	b
9	b		26	c		43	c		60	a		77	c
10	d		27	a		44	d		61	d		78	a
11	c		28	b		45	d		62	a		79	c
12	d		29	a		46	b		63	c		80	d
13	c		30	c		47	d		64	b		81	c
14	b		31	b		48	b		65	a		82	a
15	d		32	c		49	c		66	d		83	c
16	a		33	d		50	a		67	a		84	b
17	a		34	d		51	a		68	b		85	d

Cone and cylinder

Q. 1	Let L be any line making an angle $\alpha, \ \beta, \ \gamma$ with $x, \ y$ and z axis respectively. Then direction cosines (dc's) of L are				[01]
	A)	$l=\sin\alpha,\ m=\sin\beta\ ,\ n=\sin\gamma$	C)	$l=\sec\alpha,\ m=\sec\beta\ ,\ n=\sec\gamma$	
	B)	$l=\tan\alpha,\ m=\tan\beta\ ,\ n=\tan\gamma$	D)	$l=\cos\alpha,\ m=\cos\beta\ ,\ n=\cos\gamma$	
Ans.	D				
Q. 2	Let L be any line with l,m,n are direction cosines (dc's) of L. And a,b,c are direction ratios (dr's) of L Then l,m,n are.				[01]
	A)	$l=-\frac{a}{\sqrt{a^2+b^2+c^2}},m=\frac{b}{\sqrt{a^2+b^2+c^2}},n=\frac{c}{\sqrt{a^2+b^2+c^2}}$	C)	$l=-\frac{a}{\sqrt{a^2+b^2+c^2}},m=-\frac{b}{\sqrt{a^2+b^2+c^2}},n=\frac{c}{\sqrt{a^2+b^2+c^2}}$	
	B)	$l=\frac{a}{\sqrt{a^2+b^2+c^2}},m=\frac{b}{\sqrt{a^2+b^2+c^2}},n=\frac{c}{\sqrt{a^2+b^2+c^2}}$	D)	$l=-\frac{a}{\sqrt{a^2+b^2+c^2}},m=-\frac{b}{\sqrt{a^2+b^2+c^2}},n=-\frac{c}{\sqrt{a^2+b^2+c^2}}$	
Ans.	C				
Q. 3	Equation of straight line passing through $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$ is				[01]
	A)	$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$	C)	$\frac{z_2-z_1}{x_2-x_1}=\frac{z_2-z_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$	
	B)	$\frac{y_2-y_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{y_2-y_1}{z_2-z_1}$	D)	$\frac{x-x_1}{x_2-x_1}=\frac{x_2-x_1}{y_2-y_1}=\frac{x_2-x_1}{z_2-z_1}$	
Ans.	A				

Q. 4	Equation of straight line passing through $P(x_1, y_1, z_1)$ and having dcs l, m, n is				[01]
	A)	$\frac{x+x_1}{l}=\frac{y+y_1}{m}=\frac{z+z_1}{n}=r$	C)	$\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}=r$	
	B)	$\frac{x-x_1}{l}=\frac{y+y_1}{m}=\frac{z-z_1}{n}=r$	D)	$\frac{x+x_1}{l}=\frac{y-y_1}{m}=\frac{z+z_1}{n}=r$	
Ans.	C				
Q. 5	Equation of straight line passing through $P(x_1, y_1, z_1)$ and having drs a, b, c is				[01]
	A)	$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}=k$	C)	$\frac{x-x_1}{a}=\frac{y+y_1}{b}=\frac{z+z_1}{c}=k$	
	B)	$\frac{x+x_1}{a}=\frac{y+y_1}{b}=\frac{z+z_1}{c}=k$	D)	$\frac{x+x_1}{a}=\frac{y-y_1}{b}=\frac{z+z_1}{c}=k$	
Ans.	A				
Q. 6	Perpendicular distance of a point $P(x_1, y_1, z_1)$ from a plane $ax+by+cz+d=0$ is given by				[01]
	A)	$\left\lvert\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right\rvert$	C)	$\left\lvert\frac{ax_1+by_1+cz_1+d}{\sqrt{x_1^2+y_1^2+z_1^2}}\right\rvert$	
	B)	$\left\lvert\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2+d^2}}\right\rvert$	D)	None of these	
Ans.	C				
Q. 7	The general equation of cone is				[01]
	A)	$ax^2+by^2+cz^2-2hxy-2fyz-2gzx+2ux+2vy+2wz+d=0$	C)	$ax^2+by^2+cz^2+2hxy+2fyz+2gzx-2ux-2vy-2wz-d=0$	
	B)	$ax^2+by^2+cz^2+2hxy+2fyz+2gzx+2ux+2vy+2wz+d=0$	D)	None of these.	
Ans.	B				
Q. 8	The equation of cone with vertex at origin is				[01]
	A)	$ax^2+by^2+cz^2+2hxy+2fyz+2gzx=0$	C)	$ax^2+by^2+cz^2=0$	
	B)	$ax^2+by^2+cz^2-2hxy-2fyz-2gzx=0$	D)	$ax^2-by^2-cz^2+2hxy+2fyz+2gzx=0$	

Ans.	A				
Q. 9	The equation of right circular cone is				[01]
	A)	$\cos\boldsymbol{\theta} = \dfrac{\boldsymbol{l(x+\alpha)+m(y+\beta)+n(z+\gamma)}}{\sqrt{\boldsymbol{l^2+m^2+n^2}}\sqrt{\boldsymbol{(x+\alpha)^2+(y+\beta)^2+(z+\gamma)^2}}}$	C)	$\cos\boldsymbol{\theta} = \dfrac{\boldsymbol{l(x-\alpha)-m(y-\beta)-n(z-\gamma)}}{\sqrt{\boldsymbol{l^2-m^2-n^2}}\sqrt{\boldsymbol{(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2}}}$	
	B)	$\cos\boldsymbol{\theta} = \dfrac{\boldsymbol{l(x-\alpha)+m(y-\beta)+n(z-\gamma)}}{\sqrt{\boldsymbol{l^2+m^2+n^2}}\sqrt{\boldsymbol{(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2}}}$	D)	$\cos\boldsymbol{\theta} = \dfrac{\boldsymbol{l(x-\alpha)+m(y+\beta)+n(z-\gamma)}}{\sqrt{\boldsymbol{l^2+m^2+n^2}}\sqrt{\boldsymbol{(x-\alpha)^2+(y+\beta)^2+(z-\gamma)^2}}}$	
Ans.	B				
Q.10	The equation of right circular cylinder whose radius is r and axis is the line $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$. 				[01]
	A)	$PA^2+PM^2=AM^2$	C)	$PA^2=-PM^2-AM^2$	
	B)	$PA^2=PM^2-AM^2$	D)	$PA^2=PM^2+AM^2$	
Ans.	D				
Q.11	The equation of right circular cylinder whose radius is r and axis is the line $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$. Is $PA^2=PM^2+AM^2$,$AM = Projection\ of\ PA\ on\ axis$ is given by 				
	A)	$\dfrac{l(x+\alpha)+m(y+\beta)+n(z+\gamma)}{\sqrt{l^2+m^2+n^2}}$	C)	$\dfrac{l(x-\alpha)+m(y-\beta)+n(z-\gamma)}{\sqrt{l^2-m^2-n^2}}$	
	B)	$\dfrac{l(x-\alpha)+m(y-\beta)+n(z-\gamma)}{\sqrt{l^2+m^2+n^2}}$	D)	$\dfrac{l(x-\alpha)-m(y-\beta)-n(z-\gamma)}{\sqrt{l^2-m^2-n^2}}$	
Ans.	B				

Q.12	The right circular cone which passes through the point $(2,-2,1)$ with vertex at the origin and axis parallel to the line $\frac{x-2}{5}=\frac{y-1}{1}=\frac{z+2}{1}$ Then the value of semi-vertical angle θ is				[01]
	A)	$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$	C)	$\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$	
	B)	$-\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$	D)	$\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$	
Ans.	A				
Q.13	The equation of right circular cylinder of radius 2, whose axis is the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{2}$ is $PA^2=PM^2+AM^2$ Then $AM=Proj^n\ of\ PA\ on\ axis$ is given by				[01]
	A)	$AM=\frac{2(x+1)+1(y+2)+2(z+3)}{\sqrt{2^2+1^2+2^2}}$	C)	$AM=\frac{2(x-1)+1(y-2)+2(z-3)}{\sqrt{2^2+1^2+2^2}}$	
	B)	$AM=\frac{2(x-1)+1(y-2)+2(z-3)}{\sqrt{2^2-1^2-2^2}}$	D)	$AM=\frac{2(x-1)-1(y-2)+2(z-3)}{\sqrt{2^2-1^2+2^2}}$	
Ans.	C				

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2015-16

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Chapter 03 – Fourier Series

1) A function $f(x)$ is said to be periodic function with a period T, if
   a) $f(x)=f(x+T)$, *for all x*
   b) $f(T)=f(x+T)$, *for all x*
   c) $f(x)=-f(x+T)$, *for all x*
   d) $f(x)=f\left(\frac{x}{T}\right)$, *for all x*

2) A smallest positive number T satisfying $f(x)=f(x+T)$ is known as
   a) absolute function b) absolute time
   c) periodic time d)primitive period

3) If T is the fundamental period a function $f(x)$, which of the following is incorrect?
   a) $f(x)=f(x+nT)$, $n\in I$
   b) $f(x)=f(x+n+T)$, $n\in I$
   c) $f(x)=f(x-T)$
   d) $f(x)=f(x+T)$

4) If $f(x+nT)=f(x)$ where n is an integer and T is the smallest positive number, the fundamental period of $f(x)$ is
   a) T b) nT c) $2T$ d) $\frac{T}{2}$

5) If $f(x)$ is a periodic function of period T, then for $n\neq 0$, the function $f(nx)$ is a periodic function of period
   a) T b) T^n c) $\frac{T}{n}$ d) nT

6) The fundamental period of $\sin x$ is
   a) π b) 2π c) 3π d) $\frac{\pi}{2}$

7) The fundamental period of $\sin 2x$ is
   a) π b) 2π c) 3π d) $\frac{\pi}{2}$

8) The fundamental period of $\sin 4x$ is
   a) π b) 2π c) 3π d) $\frac{\pi}{2}$

9) The fundamental period of $\cos 3x$ is
   a) π b) $\frac{2\pi}{3}$ c) $\frac{3\pi}{2}$ d) 3π

10) The fundamental period of $\sin(-3x)$ is
   a) -3π b) 3π c) $-\frac{2\pi}{3}$ d) $\frac{2\pi}{3}$

11) The fundamental period of $\sin\left(-\frac{x}{2}\right)$ is
   a) π b) 2π c) 3π d) 4π

12) The fundamental period of $\cos(x+\pi)$ is
   a) π b) 2π c) 3π d) $\frac{\pi}{2}$

13) The fundamental period of $\sin\left(x+\frac{3\pi}{2}\right)$ is
   a) 2π b) $\frac{2\pi}{3}$ c) 3π d) π

14) The fundamental period of $\tan\left(3x+\frac{\pi}{2}\right)$ is
   a) 2π b) π c) 3π d) $\frac{\pi}{3}$

15) The fundamental period of $\sin\left(x+\frac{\pi}{6}\right)$ is
   a) π b) 2π c) 3π d) $\frac{\pi}{3}$

16) The fundamental period of $2\sin x$ is
   a) π b) 2π c) 3π d) 4π

17) The fundamental period of $\sin x\cos x$ is
   a) π b) 2π c) 3π d) 4π

18) The fundamental period of $\tan x$ is
a) 4π b) 3π c) 2π d) π

19) The fundamental period of $\tan 5x$ is
a) $\frac{\pi}{5}$ b) 5π c) 10π d) π

20) The fundamental period of $2\sec(-3x)$ is
a) $-\frac{2\pi}{3}$ b) $\frac{2\pi}{3}$ c) $\frac{3\pi}{2}$ d) $-\frac{3\pi}{2}$

21) The fundamental period of $\operatorname{cosec} 2x$ is
a) π b) 2π c) 3π d) $\frac{\pi}{2}$

22) A function $f(x)$ defined in the interval $[-a,\ a]$ is said to be even function, if
a) $f(-x)=-f(x)$ b) $f(2x)=2f(x)$
c) $f(-x)=f(x)$ d) $f(x)=-f(x)$

23) A function $f(x)$ defined in the interval $[-a,\ a]$ is said to be odd function, if
a) $f(x)=-f(x)$ b) $f(2x)=2f(x)$
c) $f(-x)=f(x)$ d) $f(-x)=-f(x)$

24) Which of the followings is an even function?
a) $\cosh x$ b) $x^3-\cos x$
c) $\tan 3x$ d) $e^x+\tan^2 x$

25) Which of the followings is an even function?
a) $\sin 3x$ b) $\tan x$ c) $\operatorname{cosec}^3 x$ d) $\tan^2 x$

26) Which of the followings is not an even function?
a) $\sin^3 x$ b) $\sin^2 x$ c) $\tan^2 x$ d) $\sec x$

27) Which of the followings is an odd function?
a) e^{-x} b) $\tan\frac{3x}{2}$
c) $\cos^3 x$ d) $\operatorname{cosec} 2x$

28) Which of the followings is an odd function?
a) $-e^x$ b) $-\tan^2 x$
c) $-\sin x$ d) $-\cos x$

29) Which of the followings is not an odd function?
a) $2\tan x$ b) $\tan^2 x$
c) $\tan x$ d) $\sin 3x$

30) Which of the followings is neither even nor an odd function?
a) $\operatorname{cosech} x$ b) $\tanh x$ c) e^x d) $\sinh x$

31) If $f(x)$ is to be constant function w.r.t. x, then $f(x)$ is
a) even function
b) odd function
c) both even and odd
d) neither even nor odd

32) If $f(x)=x^3+2x-\cos x$, the function $f(x)$ is
a) even function
b) odd function
c) both even and odd
d) neither even nor odd

33) If $f(x)=x^2-\sin^4 x\cdot e^{|x|}$, the function $f(x)$ is
a) even function
b) odd function
c) both even and odd
d) neither even nor odd

34) Which of the following statement is incorrect?
a) Product of even and odd function is an odd function.
b) Multiplication of even and odd function is an odd function.
c) Addition of even and odd function is an odd function.
d) Subtraction of two odd functions is an odd function.

35) Fourier series expansion of a function $f(x)$ defined on the interval $[c,\ c+2L]$ and having period $2L$ is given by
a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos\left(\frac{n\pi x}{L}\right)+b_n\sin\left(\frac{n\pi x}{L}\right)$
b) $a_0+\sum_{n=1}^{\infty} a_n\cos\left(\frac{n\pi x}{L}\right)+b_n\sin\left(\frac{n\pi x}{L}\right)$
c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos(n\pi Lx)+b_n\sin(n\pi Lx)$
d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos\left(\frac{nx}{L}\right)+b_n\sin\left(\frac{nx}{L}\right)$

36) Fourier series expansion of a function $f(x)$ defined on the interval $[0,\ 2\pi]$ and satisfying the Dirichlet's conditions is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\frac{nx}{2}+b_n \sin\frac{nx}{2}$

b) $\frac{a_0}{2}+2\pi\sum_{n=1}^{\infty} a_n \cos nx+b_n \sin nx$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nx+b_n \sin nx$

d) $\frac{a_0}{2}+a_n \cos nx+b_n \sin nx$

37) If a function $f(x)$ is defined on the interval $[-\pi,\ \pi]$ and satisfying the Dirichlet's conditions, Fourier series expansion is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)+b_n \sin\left(\frac{n\pi x}{L}\right)$

b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{nx}{L}\right)+b_n \sin\left(\frac{nx}{L}\right)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{2n\pi x}{L}\right)+b_n \sin\left(\frac{2n\pi x}{L}\right)$

d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$

38) If a function $f(x)$ is defined on the interval $[0,\ 4]$ with period $T=4$, Fourier series expansion is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{4}\right)+b_n \sin\left(\frac{n\pi x}{4}\right)$

b) $\frac{a_0}{2}+a_n \cos\left(\frac{n\pi x}{2}\right)+b_n \sin\left(\frac{n\pi x}{2}\right)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{2}\right)+b_n \sin\left(\frac{n\pi x}{2}\right)$

d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{2n\pi x}{L}\right)+b_n \sin\left(\frac{2n\pi x}{L}\right)$

39) Fourier series expansion of a function $f(x)$ defined over a period 2π and satisfying the Dirichlet's conditions is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin nx+b_n \cos nx$

b) $\frac{a_0}{2}+2\pi\sum_{n=1}^{\infty} a_n \cos nx+b_n \sin nx$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nx+b_n \sin nx$

d) $\frac{a_0}{2}+a_n \cos nx+b_n \sin nx$

40) If an even function $f(x)$ with period $2l$ is defined over the interval $(-l,\ l)$, its Fourier series expansion is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{l}\right)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{2l}\right)$ d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{nx}{l}\right)$

41) If an odd function $f(x)$ is defined over the interval $(-\pi,\ \pi)$, its Fourier series expansion is given by

a) $\sum_{n=1}^{\infty} b_n \sin\left(\frac{2nx}{l}\right)$ b) $\sum_{n=1}^{\infty} a_n \sin(nx)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin(nx)$ d) $\sum_{n=1}^{\infty} b_n \sin(nx)$

42) If an odd function $f(x)$ is of period 2π, its Fourier series expansion is given by

a) $\sum_{n=1}^{\infty} b_n \sin\left(\frac{2nx}{l}\right)$ b) $\sum_{n=1}^{\infty} a_n \sin(nx)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin(nx)$ d) $\sum_{n=1}^{\infty} b_n \sin(nx)$

43) The Fourier series expansion of an even function $f(x)$ with period 2π is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin(nx)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{nx}{2}\right)$ d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{nx}{l}\right)$

44) If an odd function $f(x)$ with period $2l$ is defined over the interval $(-l,\ l)$, its Fourier series expansion is given by

a) $\sum_{n=1}^{\infty} b_n \sin\left(\frac{nx}{l}\right)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$

c) $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$ d) $\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{l}\right)$

45) If $f(x)$ is periodic function with period $2L$ in the interval C to $C+2L$, the Fourier coefficient a_0 is

a) $\frac{1}{L}\int_{C}^{C+2L} f(x)\cos\frac{nx}{L}dx$ b) $\frac{1}{L}\int_{C}^{C+2L} f(x)\sin\frac{nx}{L}dx$

c) $\int_{C}^{C+2L} f(x)dx$ d) $\frac{1}{L}\int_{C}^{C+2L} f(x)dx$

46) If $f(x)$ is periodic function with period $2L$ in the interval C to $C+2L$, the Fourier coefficient a_n is

a) $\frac{1}{L}\int_{C}^{C+2L} f(x)\cos\frac{2n\pi x}{L}dx$

b) $\frac{1}{L}\int_{C}^{C+2L} f(x)\cos\frac{n\pi x}{L}dx$

c) $\frac{1}{L}\int_{C}^{C+2L} f(x)\cos\frac{nx}{L}dx$

d) $\frac{1}{2L}\int_{C}^{C+2L} f(x)\cos\frac{nx}{L}dx$

47) If $f(x)$ is periodic function with period $2L$ in the interval C to $C+2L$, the Fourier coefficient b_n is

a) $\frac{1}{L}\int_{C}^{C+2L} f(x)\sin\frac{\pi x}{L}dx$

b) $\frac{1}{L}\int_{C}^{C+2L} f(x)\sin\frac{nx}{L}dx$

c) $\frac{1}{L}\int_{C}^{C+2L} f(x)\sin\frac{n\pi x}{L}dx$

d) $\frac{1}{L}\int_{0}^{2L} f(x)\sin\frac{n\pi x}{L}dx$

48) If $f(x)$ is an even function defined in the interval $[-L, L]$ and $f(x)=f(x+2L)$, the Fourier coefficient are

a) $a_0=\frac{2}{L}\int_0^L f(x)dx,\ a_n=\frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx,\ b_n=0$

b) $a_0=\frac{1}{L}\int_0^L f(x)dx,\ \ a_n=\frac{1}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx$

c) $a_0=0,\ \ a_n=\frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx$

d) $a_0=\frac{2}{L}\int_0^L f(x)dx,\ \ a_n=\frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$

49) If $f(x)$ is an odd function defined in the interval $[-L, L]$ and $f(x)=f(x+2L)$, the Fourier coefficient are

a) $a_0=0,\ \ a_n=0,\ \ b_n=\frac{1}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$

b) $a_0=0,\ \ a_n=0,\ \ b_n=\frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$

c) $a_0=0,\ \ a_n=0,\ \ b_n=\frac{2}{L}\int_0^{2L} f(x)\sin\frac{n\pi x}{L}dx$

d) $a_0=0,\ \ a_n=0,\ \ b_n=\frac{2}{L}\int_0^L f(x)\sin\frac{nx}{L}dx$

50) If $f(x)$ is an even periodic function defined in the interval $[-\pi, \pi]$, the Fourier coefficient are

a) $a_0=\frac{2}{\pi}\int_0^\pi f(x)dx,\ a_n=\frac{2}{\pi}\int_0^\pi f(x)\cos nx\,dx,\ b_n=0$

b) $a_0=\frac{2}{L}\int_0^\pi f(x)dx,\ a_n=\frac{2}{L}\int_0^\pi f(x)\cos\frac{nx}{L}dx,\ b_n=0$

c) $a_0=\frac{2}{\pi}\int_0^\pi f(x)dx,\ a_n=\frac{2}{\pi}\int_0^\pi f(x)\sin nxdx,\ b_n=0$

d) $a_0=\frac{1}{\pi}\int_0^\pi f(x)dx,\ a_n=\frac{1}{\pi}\int_0^\pi f(x)\cos nx\,dx,\ b_n=0$

51) If $f(x)$ is an odd periodic function defined in the interval $[-\pi, \pi]$, the Fourier coefficient are

a) $a_0=0,\ a_n=\frac{2}{\pi}\int_0^\pi f(x)\cos nx\,dx,\ b_n=0$

b) $a_0=\frac{2}{\pi}\int_0^\pi f(x)dx,\ a_n=\frac{2}{\pi}\int_0^\pi f(x)\cos nx\,dx,\ b_n=0$

c) $a_0=0,\ a_n=0,\ b_n=\frac{2}{\pi}\int_0^\pi f(x)\sin nx\,dx$

d) $a_0 = \frac{1}{\pi}\int_0^{\pi} f(x)\,dx,\ a_n = \frac{1}{\pi}\int_0^{\pi} f(x)\cos nx\,dx,\ b_n = 0$

52) The Fourier coefficient of an even periodic function $f(x)$ defined in the interval $[-2, 2]$ are

a) $a_0 = \int_0^2 f(x)\,dx,\ a_n = \int_0^2 f(x)\cos nx\,dx,\ b_n = 0$

b) $a_0 = \int_0^2 f(x)\,dx,\ a_n = \int_0^2 f(x)\cos\frac{n\pi x}{2}\,dx,\ b_n = 0$

c) $a_0 = \frac{2}{\pi}\int_0^2 f(x)\,dx,\ a_n = \frac{2}{\pi}\int_0^2 f(x)\cos nx\,dx,\ b_n = 0$

d) $a_0 = \int_0^2 f(x)\,dx,\ a_n = \int_0^2 f(x)\cos nx\,dx,\ b_n = 0$

53) The Fourier coefficient of an even periodic function $f(x)$ defined in the interval $[-1, 1]$ are

a) $a_0 = \frac{2}{\pi}\int_0^1 f(x)\,dx,\ a_n = \frac{2}{\pi}\int_0^1 f(x)\cos n\pi x\,dx,\ b_n = 0$

b) $a_0 = 2\int_0^2 f(x)\,dx,\ a_n = 2\int_0^2 f(x)\cos\frac{n\pi x}{2}\,dx,\ b_n = 0$

c) $a_0 = 2\int_0^1 f(x)\,dx,\ a_n = 2\int_0^1 f(x)\cos n\pi x\,dx,\ b_n = 0$

d) $a_0 = \int_0^1 f(x)\,dx,\ a_n = \int_0^1 f(x)\cos n\pi x\,dx,\ b_n = 0$

54) The Fourier coefficient of an odd periodic function $f(x)$ defined in the interval $[-4, 4]$ are

a) $a_0 = 0,\ \ a_n = 0,\ \ b_n = \frac{1}{2}\int_0^4 f(x)\cos\frac{n\pi x}{4}\,dx$

b) $a_0 = 0,\ \ a_n = 0,\ \ b_n = \frac{1}{4}\int_0^L f(x)\sin n\pi x\,dx$

c) $a_0 = 0,\ \ a_n = 0,\ \ b_n = 2\int_0^4 f(x)\sin\frac{n\pi x}{4}\,dx$

d) $a_0 = 0,\ \ a_n = 0,\ \ b_n = \frac{1}{2}\int_0^4 f(x)\sin\frac{n\pi x}{4}\,dx$

55) If the Fourier series expansion of an even function $f(x)$ over an interval $[-l, l]$ is given by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$, the value of a_0 is obtained by

a) $\frac{2}{l}\int_{-l}^{l} f(x)\,dx$ b) $\frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$

c) $\frac{1}{2l}\int_0^l f(x)\,dx$ d) $\frac{2}{l}\int_0^l f(x)\,dx$

56) If the Fourier series expansion of an even function $f(x)$ over an interval $[-l, l]$ is given by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$, the value of a_n is obtained by

a) $\frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}\,dx$ b) $\frac{1}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$

c) $\frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$ d) $\frac{1}{l}\int_0^l f(x)\cos\frac{nx}{l}\,dx$

57) If the Fourier series expansion of an even function $f(x)$ over an interval $[-\pi, \pi]$ is given by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$, the value of a_0 is obtained by

a) $\frac{2}{\pi}\int_0^{\pi} f(x)\,dx$ b) $\frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$

c) $\frac{1}{\pi}\int_0^{\pi} f(x)\,dx$ d) $\frac{2}{\pi}\int_0^{\pi} f(x)\cos nxdx$

58) If the Fourier series expansion of an even function $f(x)$ over an interval $[-\pi, \pi]$ is given by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$, the value of a_n is obtained by

a) $\frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$ b) $\frac{2}{\pi}\int_0^{\pi} f(x)\cos nxdx$

c) $\frac{1}{\pi}\int_0^{l} f(x)\cos\frac{n\pi x}{l}\,dx$ d) $\frac{2}{\pi}\int_0^{\pi} f(x)\cos\frac{nx}{\pi}\,dx$

59) If the Fourier series expansion of an odd function $f(x)$ over an interval $[-l, l]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$, the value of b_0 is obtained by

a) $\int_0^l f(x)\cos\frac{n\pi x}{l}dx$ b) $\frac{1}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}dx$

c) $\frac{2}{l}\int_0^l f(x)dx$ d) none of the above

60) If the Fourier series expansion of an odd function $f(x)$ over an interval $[-l, l]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$, the value of b_n is obtained by

a) $\frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}dx$ b) $\frac{1}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}dx$

c) $\frac{1}{l}\int_0^l f(x)\sin\frac{nx}{l}dx$ d) $\int_0^l f(x)\cos\frac{n\pi x}{l}dx$

62) The half range Fourier cosine series for $f(x)$ defined over the interval $[0, L]$ is given by

a) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{nx}{L}\right)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right)$

c) $a_0+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$ d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$

63) The half range Fourier sine series for $f(x)$ defined over the interval $[0, L]$ is given by

a) $\sum_{n=1}^{\infty} a_n \sin\left(\frac{nx}{L}\right)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right)$

c) $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$ d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$

64) The half range Fourier cosine series for $f(x)$ defined over the interval $[0, \pi]$ is given by

a) $\sum_{n=1}^{\infty} b_n \sin(nx)$ b) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$ d) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)$

65) The half range Fourier sine series for $f(x)$ defined over the interval $[0, \pi]$ is given by

a) $\sum_{n=1}^{\infty} b_n \sin\left(\frac{nx}{\pi}\right)$ b) $\sum_{n=1}^{\infty} b_n \sin(nx)$

c) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right)$ d) $\sum_{n=1}^{\infty} a_n \sin(n\pi x)$

66) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, L]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$ the value of a_0 is given by

a) $\frac{1}{L}\int_0^L f(x)dx$ b) $\frac{1}{L}\int_0^L f(x)dx$

c) $\frac{2}{\pi}\int_0^{\pi} f(x)dx$ d) $\frac{2}{L}\int_0^L f(x)dx$

67) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, L]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$ the value of a_n is given by

a) $\frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$

b) $\frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

c) $\frac{2}{L}\int_0^L f(x)\cos(nx)dx$

d) $\frac{1}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

68) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, L]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$ the value of b_0 is given by

a) $\frac{1}{L}\int_0^L f(x)\sin\frac{x}{L}dx$ b) $\frac{2}{L}\int_0^L f(x)\sin x\,dx$

c) 0 d) $\frac{2}{L}\int_0^L f(x)dx$

69) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, L]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$ the value of b_n is given by

a) $\frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$

b) $\frac{1}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$

c) $\frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

d) $\frac{2}{L}\int_0^L f(x)\sin(nx)dx$

70) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, \pi]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(nx)$ the value of a_0 is given by

a) $\frac{1}{L}\int_0^L f(x)dx$ b) $\frac{1}{L}\int_0^L f(x)dx$

c) $\frac{2}{\pi}\int_0^{\pi} f(x)dx$ d) $\frac{2}{L}\int_0^L f(x)dx$

71) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, \pi]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(nx)$ the value of a_n is given by

a) $\frac{2}{L}\int_0^L f(x)\sin(nx)dx$

b) $\frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

c) $\frac{2}{\pi}\int_0^{\pi} f(x)\cos(nx)dx$

d) $\frac{1}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

72) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, \pi]$ is given by $\sum_{n=1}^{\infty}b_n\sin(nx)$ the value of b_0 is given by

a) $\frac{1}{L}\int_0^L f(x)\sin\frac{x}{L}dx$ b) $\frac{2}{L}\int_0^L f(x)\sin x\,dx$

c) 0 d) $\frac{2}{L}\int_0^L f(x)dx$

73) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, \pi]$ is given by $\sum_{n=1}^{\infty}b_n\sin(nx)$ the value of b_n is given by

a) $\frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$

b) $\frac{1}{L}\int_0^L f(x)\sin(nx)dx$

c) $\frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

d) $\frac{2}{\pi}\int_0^{\pi} f(x)\sin(nx)dx$

74) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, 1]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(n\pi x)$ the value of a_0 is given by

a) $\frac{1}{\pi}\int_0^{\pi} f(x)dx$ b) $2\int_0^1 f(x)dx$

c) $\frac{2}{\pi}\int_0^{\pi} f(x)dx$ d) $\int_0^1 f(x)dx$

75) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, 2]$ is given by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{n\pi x}{2}\right)$ the value of a_n is given by

a) $\frac{2}{3}\int_0^2 f(x)\sin\left(\frac{n\pi x}{2}\right)dx$

b) $\frac{1}{2}\int_0^2 f(x)\cos\left(\frac{n\pi x}{2}\right)dx$

c) $\frac{2}{L}\int_0^2 f(x)\cos(nx)dx$

d) $\int_0^2 f(x)\cos\left(\frac{n\pi x}{2}\right)dx$

76) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, 3]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{3}\right)$ the value of b_0 is given by

a) $\frac{1}{3}\int_0^3 f(x)\sin\frac{x}{3}dx$ b) $\frac{2}{3}\int_0^3 f(x)\sin 3x\,dx$

c) 0 d) $\frac{2}{3}\int_0^3 f(x)dx$

77) If the half range Fourier series expansion of a function $f(x)$ defined over the interval $[0, 4]$ is given by $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{4}\right)$ the value of b_n is given by

a) $\frac{2}{3}\int_0^2 f(x)\sin\left(\frac{n\pi x}{4}\right)dx$

b) $\frac{1}{2}\int_0^4 f(x)\sin\left(\frac{n\pi x}{4}\right)dx$

c) $\int_0^2 f(x)\cos\left(\frac{n\pi x}{4}\right)dx$

d) $\frac{1}{2}\int_0^4 f(x)\sin(nx)dx$

78) In the harmonic analysis for a function defined over a period of 2π, the term $a_1\cos x + b_1 \sin x$ is known as

a) amplitude of $f(x)$ b) second harmonic
c) first harmonic d) none of these

79) In the harmonic analysis for a function defined over a period of 2π, the amplitude of the first harmonic is

a) $\sqrt{a_1^2 - b_1^2}$ b) $\sqrt{a_1^2 + b_1^2}$
c) $\sqrt{a_0^2 + a_1^2}$ d) $a_1^2 + b_1^2$

80) In the harmonic analysis for a function defined over a period of 2π, the amplitude of the second harmonic is

a) $\left(a_2^2 + b_2^2\right)^2$ b) $\frac{1}{2}\left(a_2^2 + b_2^2\right)$
c) $2\sqrt{a_2^2 + b_2^2}$ d) $\sqrt{a_2^2 + b_2^2}$

81) In the harmonic analysis for a function defined over a period of 2π, the amplitude of the second harmonic is

a) $\left(a_n^2 + b_n^2\right)^n$ b) $\sqrt{a_n^2 + b_n^2}$
c) $n\sqrt{a_n^2 + b_n^2}$ d) $\frac{1}{n}\sqrt{a_n^2 + b_n^2}$

82) In the harmonic analysis for a function $f(x)$ defined over a period of $2L$, the first harmonic term is given by

a) $b_1 \sin\frac{\pi x}{L}$ b) $a_1 \cos\frac{\pi x}{L}$
c) $a_1 \cos\frac{\pi x}{L} - b_1 \sin\frac{\pi x}{L}$ d) $a_1 \cos\frac{\pi x}{L} + b_1 \sin\frac{\pi x}{L}$

83) In the harmonic analysis for a function $f(x)$ defined over a period of 2, the first harmonic term is given by

a) $a_1 \cos \pi x + b_1 \sin \pi x$ b) $a_1 \cos\frac{\pi x}{2} + b_1 \sin\frac{\pi x}{2}$
c) $a_1 \cos 2\pi x + b_1 \sin 2\pi x$ d) $a_1 \cos\frac{\pi x}{2} - b_1 \sin\frac{\pi x}{2}$

84) If $f(x) = x\cos x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$, the Fourier coefficient a_0 is given by

a) $-\frac{1}{\pi}$ b) 0 c) $\frac{1}{\pi}$ d) $\frac{2}{\pi}$

85) If $f(x) = x\cos x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$, the Fourier coefficient a_n is given by

a) $\frac{1}{\pi}$ b) $-\frac{1}{\pi}$ c) $\frac{2}{\pi}$ d) 0

86) If $f(x) = x\cos x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$, the Fourier coefficient b_1 is given by

a) $\frac{1}{\pi}$ b) $-\frac{1}{\pi}$ c) $-\frac{1}{2}$ d) 0

87) If $f(x)=\cos x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π, the Fourier coefficient a_1 is given by

a) 1 b) $-\dfrac{1}{\pi}$ c) $\dfrac{2}{\pi}$ d) 0

88) If $f(x)=x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\dfrac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient a_0 is given by

a) $\dfrac{\pi}{2}$ b) 0 c) 1 d) $\dfrac{\pi^2}{2}$

89) If $f(x)=x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\dfrac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient b_1 is given by

a) 2 b) 0 c) π d) $\dfrac{\pi}{2}$

90) If $f(x)=2$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\dfrac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient a_0 is given by

a) 2 b) 4 c) 3 d) none of these

91) If $f(x)=2$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\dfrac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient b_n is given by

a) $\dfrac{3\pi}{2}$ b) $-\dfrac{\pi}{2}$ c) 0 d) $\dfrac{\pi}{2}$

92) If $f(x)=a$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\dfrac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient a_0 is given by

a) $2a$ b) 0 c) 2π d) $\dfrac{\pi}{2}$

93) If $f(x)=a$ where $-\pi \le x \le \pi$ is of periodic function with period 2π, the Fourier coefficient a_0 is given by

a) 2π b) $2a$ c) 0 d) $\dfrac{\pi}{2}$

94) If $f(x)=\sin^2 x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\dfrac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient b_n is given by

a) $\dfrac{3\pi}{2}$ b) $-\dfrac{\pi}{2}$ c) $\dfrac{\pi}{2}$ d) 0

95) If $f(x)=\cosh x$ where $-\pi \le x \le \pi$ is of periodic function with period 2π and is represented by $\dfrac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient b_n is given by

a) 0 b) $\dfrac{\pi}{3}$ c) $e^{-\pi}$ d) $e^{-2\pi}$

96) If $f(x)=\begin{cases} -x & -\pi \le x \le 0 \\ x & 0 \le x \le \pi \end{cases}$ is of periodic function with period 2π and is represented by $\dfrac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient a_0 is given by

a) $\dfrac{\pi}{2}$ b) π c) 0 d) $-\dfrac{\pi}{2}$

97) If $f(x)=\begin{cases} -x & -\pi \le x \le 0 \\ x & 0 \le x \le \pi \end{cases}$ is of periodic function with period 2π and is represented by $\dfrac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$, the Fourier coefficient b_n is given by

a) $\dfrac{\pi}{2}$ b) π c) 0 d) $-\dfrac{\pi}{2}$

98) If $f(x)=\begin{cases} 1+\dfrac{2x}{\pi} & -\pi \le x \le 0 \\ 1-\dfrac{2x}{\pi} & 0 \le x \le \pi \end{cases}$ is of periodic function with period 2π, the Fourier coefficient b_n is given by

a) $\dfrac{\pi}{2}$ b) π c) $-\dfrac{\pi}{2}$ d) 0

99) If $f(x)=x-x^3$ where $-2\le x\le 2$ is of periodic function with period 2 and is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(nx)+b_n\sin(nx)$, the Fourier coefficient b_1 is given by

a) 2 b) 0 c) π d) $\frac{\pi}{2}$

100) If $f(x)=x+\frac{x^2}{4}$ where $-\pi\le x\le\pi$ is of periodic function with period 2π, the Fourier coefficient a_0 is given by

a) 0 b) $\frac{\pi}{2}$ c) $-\frac{\pi}{2}$ d) $\frac{\pi^2}{6}$

101) If $f(x)=e^x$ where $-\pi\le x\le\pi$ is of periodic function with period 2π, the Fourier coefficient a_0 is given by

a) 1 b) $\frac{e^{\pi}-e^{-\pi}}{\pi}$ c) $\frac{e^{\pi}+e^{-\pi}}{\pi}$ d) 0

102) If $f(x)=x-x^2$ where $-1\le x\le 1$ is of periodic function with period 2, the Fourier coefficient a_0 is given by

a) 1 b) $-\frac{2}{3}$ c) π d) 0

103) If $f(x)=1-x^2$ where $-1\le x\le 1$ is of periodic function with period 2, the Fourier coefficient a_0 is given by

a) 1 b) $\frac{2\pi}{3}$ c) $\frac{4}{3}$ d) 0

104) If $f(x)=k$ where $-l\le x\le l$ is of periodic function with period 2, the Fourier coefficient a_0 is given by

a) $2k$ b) $\frac{2k\pi}{3}$ c) $2k\pi$ d) 0

105) If $f(x)=\begin{cases}-a & -2\le x\le 0\\ a & 0\le x\le 2\end{cases}$ is of periodic function with period 4, the Fourier coefficient b_n is given by

a) 0 b) 4 c) $-\frac{\pi}{2}$ d) $-\frac{2a}{n\pi}\left[(-1)^n-1\right]$

106) If $f(x)=\begin{cases}0 & -2\le x\le 0\\ 2 & 0\le x\le 2\end{cases}$ is of periodic function with period 4, the Fourier coefficient a_0 is given by

a) 0 b) 4 c) $-\frac{\pi}{2}$ d) 1

107) If $f(x)=\begin{cases}1 & -1\le x\le 0\\ \cos\pi x & 0\le x\le 1\end{cases}$ is of periodic function with period 2, the Fourier coefficient a_0 is given by

a) 0 b) $\frac{\pi}{2}$ c) $-\frac{\pi}{2}$ d) 1

108) If $f(x)=e^{-x}$ defined over $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, the Fourier coefficient a_0 is given by

a) $\frac{1}{2\pi}(1-e^{-2\pi})$ b) $\frac{2}{\pi}(1-e^{-2\pi})$
c) $\frac{1}{\pi}(1+e^{-x})$ d) $\frac{1}{\pi}(1-e^{-2\pi})$

109) If $f(x)=x$ defined over $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, the Fourier coefficient a_0 is given by

a) 3π b) $\frac{\pi}{2}$ c) π d) 2π

110) If $f(x)=x$ defined over $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, the Fourier coefficient a_n is given by
a) 0 b) π c) 2π d) 3π

111) If $f(x)=x$ defined over $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, the Fourier coefficient b_n is given by

a) $-\frac{2}{n\pi}$ b) $-\frac{\pi}{n}$ c) $-\frac{1}{n}$ d) $-\frac{2}{n}$

112) If $f(x)=\sqrt{1-\cos x}$ defined over $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, the Fourier coefficient a_0 is given by

a) 0 b) $-\frac{4\sqrt{2}}{\pi}$ c) $\frac{4\sqrt{2}}{\pi}$ d) $\frac{8\sqrt{2}}{\pi}$

113) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=\dfrac{\pi-x}{2}$ defined over the interval $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, is given by

a) $-\dfrac{\pi}{2}$ b) $\dfrac{\pi}{2}$ c) 0 d) $\dfrac{\pi^2}{6}$

114) The Fourier coefficient a_n for the Fourier series expansion of $f(x)=\dfrac{\pi-x}{2}$ defined over the interval $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, is given by

a) 0 b) π c) $\dfrac{\pi}{2}$ d) $-\dfrac{\pi}{2}$

115) The Fourier coefficient b_n for the Fourier series expansion of $f(x)=\dfrac{\pi-x}{2}$ defined over the interval $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, is given by

a) $-\dfrac{1}{n^2}$ b) $\dfrac{1}{n}$ c) $-\dfrac{1}{n}$ d) $\dfrac{\pi}{n}$

116) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=\left(\dfrac{\pi-x}{2}\right)^2$ defined over the interval $0\le x\le 2\pi$ and $f(x+2\pi)=f(x)$, is given by

a) $\dfrac{\pi^2}{6}$ b) $\dfrac{\pi^2}{12}$ c) $\dfrac{\pi^2}{3}$ d) $\dfrac{\pi^2-1}{6}$

117) Consider $f(x)=x\sin x,\ x\in[0,\,2\pi]$ and $f(x+2\pi)=f(x)$. Then the Fourier coefficient a_0 for the Fourier series expansion of $f(x)$ is given by

a) -4 b) $-\dfrac{\pi}{2}$ c) -2 d) $\dfrac{\pi}{2}$

118) If $f(x)=\begin{cases} x & 0\le x\le \pi \\ 0 & \pi\le x\le 2\pi \end{cases}$ is periodic over a period 2π, the Fourier coefficient a_0 for the Fourier series expansion of $f(x)$ is given by

a) π b) $\dfrac{\pi}{2}$ c) $-\dfrac{\pi}{2}$ d) $\dfrac{\pi}{4}$

119) If $f(x)=\begin{cases} 0 & 0\le x\le \pi \\ x & \pi\le x\le 2\pi \end{cases}$ is periodic over a period 2π, the Fourier coefficient a_0 for the Fourier series expansion of $f(x)$ is given by

a) $\dfrac{3\pi}{2}$ b) $\dfrac{\pi}{2}$ c) 3π d) $\dfrac{3\pi}{4}$

120) If the function $f(x)=\begin{cases} -\pi & 0\le x\le \pi \\ x-\pi & \pi\le x\le 2\pi \end{cases}$ and $f(x+2\pi)=f(x)$, the Fourier coefficient a_0 is given by

a) 0 b) $-\dfrac{\pi}{2}$ c) $-\dfrac{\pi}{4}$ d) $-\pi$

121) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=x-x^2$ defined over the interval $0\le x\le 2$ and $f(x+2)=f(x)$, is given by

a) 0 b) $-\dfrac{4}{3}$ c) $-\dfrac{2}{3}$ d) $\dfrac{2}{3}$

122) The Fourier coefficient a_n for the Fourier series expansion of $f(x)=x-x^2$ defined over the interval $0\le x\le 2$ and $f(x+2)=f(x)$, is given by

a) 0 b) $-\dfrac{4}{n^2\pi^2}$ c) $\dfrac{4}{n^2\pi^2}$ d) $-\dfrac{1}{n^2\pi^2}$

123) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=x+x^2$ defined over the interval $0\le x\le 3$ and having period 3, is given by

a) 0 b) $-\dfrac{4}{n^2\pi^2}$ c) $\dfrac{4}{n^2\pi^2}$ d) $\dfrac{3}{2}$

124) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=2x-x^2$ defined over the interval $0\le x\le 4$ and $f(x+4)=f(x)$, is given by

a) $-\dfrac{1}{3}$ b) $-\dfrac{2}{3}$ c) $-\dfrac{4}{3}$ d) $-\dfrac{8}{3}$

125) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=2x-x^2$ defined over the interval $0\le x\le 3$ and $f(x+3)=f(x)$, is given by

a) 0 b) $-\dfrac{2}{3}$ c) $-\dfrac{4}{3}$ d) $-\dfrac{8}{3}$

126) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=1-x^2$ defined over the interval $0\le x\le 2$ and $f(x+2)=f(x)$, is given by

a) $-\dfrac{1}{3}$ b) $-\dfrac{2}{3}$ c) $-\dfrac{4}{3}$ d) $-\dfrac{8}{3}$

127) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=1-x^2$ defined over the interval $0\le x\le 1$ and $f(x+2)=f(x)$, is given by

a) $\dfrac{2}{3}$ b) $-\dfrac{2}{3}$ c) $-\dfrac{1}{3}$ d) $-\dfrac{4}{3}$

128) The Fourier coefficient a_0 for the Fourier series expansion of $f(x)=4-x^2$ defined over the interval $0\le x\le 2$ and $f(x+2)=f(x)$, is given by

a) $-\dfrac{1}{3}$ b) $\dfrac{16}{3}$ c) $-\dfrac{16}{3}$ d) $-\dfrac{8}{3}$

129) If $f(x)=\begin{cases}\pi x & 0\le x\le 1\\ \pi(2-x) & 1\le x\le 2\end{cases}$ is periodic over a period 2, the Fourier coefficient a_0 for the Fourier series expansion of $f(x)$ is given by

a) $-\dfrac{\pi}{2}$ b) π c) $-\pi$ d) $\dfrac{\pi}{2}$

130) If $f(x)=\begin{cases}x & 0\le x\le 1\\ 0 & 1\le x\le 2\end{cases}$ is periodic over a period 2, the Fourier coefficient a_0 for the Fourier series expansion of $f(x)$ is given by

a) 2 b) 0 c) $\dfrac{1}{2}$ d) 1

131) The Fourier coefficient a_0 in the half range cosine series expansion of function $f(x)=\sin x$ defined over the interval $[0,\ \pi]$ is given by

a) $\dfrac{2}{\pi}$ b) $\dfrac{\pi}{2}$ c) $-\dfrac{\pi}{2}$ d) 0

132) The Fourier coefficient b_1 in the half range cosine series expansion of function $f(x)=\cos x$ defined over the interval $[0,\ \pi]$ is given by

a) $-\dfrac{\pi}{2}$ b) 0 c) $\dfrac{1}{2\pi}$ d) $\dfrac{\pi}{2}$

133) The Fourier coefficient a_0 in the half range cosine series expansion of function $f(x)=\pi x-x^2$ defined over the interval $[0,\ \pi]$ is given by

a) 0 b) $\dfrac{\pi^2}{6}$ c) $\dfrac{2\pi^2}{3}$ d) $\dfrac{\pi^2}{3}$

134) The Fourier coefficient a_0 in the half range sine series expansion of function $f(x)=\cos x$ defined over the interval $[0,\ \pi]$ is given by

a) $\dfrac{4}{\pi}$ b) $\dfrac{2}{\pi}$ c) $\dfrac{\pi}{2}$ d) 0

135) The Fourier coefficient a_0 in the half range cosine series expansion of function $f(x)=\sin x$ defined over the interval $[0,\ \pi]$ is given by

a) $\dfrac{4}{\pi}$ b) $\dfrac{2}{\pi}$ c) 0 d) $\dfrac{\pi}{2}$

136) The Fourier coefficient a_1 in the half range cosine series expansion of function $f(x)=\sin x$ defined over the interval $[0,\ \pi]$ is given by

a) 1 b) $\dfrac{2}{\pi}$ c) 0 d) $\dfrac{\pi}{2}$

137) The Fourier coefficient b_1 in the half range cosine series expansion of function $f(x)=x$ defined over the interval $[0,\ 2]$ with period 4 is given by

a) 0 b) $\dfrac{1}{\pi}$ c) $\dfrac{2}{\pi}$ d) $\dfrac{4}{\pi}$

138) The Fourier coefficient a_0 in the half range cosine series expansion of function $f(x)=x-x^2$ defined over the interval $[0,\ 1]$ is given by

a) $\frac{1}{3}$ b) $-\frac{1}{3}$ c) 0 d) $\frac{2}{\pi}$

139) The Fourier coefficient a_0 in the half range cosine series expansion of function $f(x)=lx-x^2$ defined over the interval $[0,\ l]$ with period $2l$ is given by

a) 0 b) $\frac{l^2}{3}$ c) $\frac{\pi}{2}$ d) $\frac{2}{\pi n^2}(\cos n\pi-1)$

140) The Fourier coefficient a_1 in the half range cosine series expansion of function $f(x)=x-x^2$ defined over the interval $[0,\ 1]$ is given by

a) 2 b) $\frac{2}{\pi}$ c) $\frac{\pi}{2}$ d) $\frac{1}{2}$

141) The Fourier coefficient a_n in the half range cosine series expansion of function $f(x)=x-x^2$ defined over the interval $[0,\ 1]$ is given by

a) 0 b) $\frac{2}{\pi}$ c) $\frac{\pi}{2}$ d) $\frac{2}{\pi n^2}(\cos n\pi-1)$

142) The Fourier coefficient a_n in the half range sine series expansion of function $f(x)=2+x$ defined over the interval $[0,\ 1]$ is given by

a) 4 b) 0 c) $-\frac{2}{n\pi}$ d) $-\frac{2\pi}{n}$

143) The Fourier series expansion for the function $f(x)=\left(\frac{\pi-x}{2}\right)^2$ over the interval $0\le x\le 2\pi$ is given by $\left(\frac{\pi-x}{2}\right)^2=\frac{\pi^2}{12}+\sum_{n=1}^{\infty}\frac{1}{n^2}\cos nx$. Then the value of $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots$ is

a) 1 b) $\frac{\pi^2}{6}$ c) $\frac{\pi^2}{12}$ d) $\frac{\pi^2}{3}$

144) The Fourier series expansion for the function $f(x)=\pi^2-x^2$ over the interval $-\pi\le x\le\pi$ is given by $\pi^2-x^2=\frac{2\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}\cos nx$. Then the value of $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-+\ldots$ is

a) 1 b) $\frac{\pi^2}{6}$ c) $\frac{\pi^2}{12}$ d) $\frac{\pi^2}{3}$

145) The Fourier series expansion for the function $f(x)=\pi^2-x^2$ over the interval $-\pi\le x\le\pi$ is given by $\pi^2-x^2=\frac{2\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}\cos nx$. Then the value of $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots$ is

a) $\frac{\pi^2}{6}$ b) $\frac{\pi^2}{12}$ c) $\frac{\pi^2}{3}$ d) 0

146) The Fourier series expansion for the function $f(x)=\begin{cases}1+\frac{2x}{\pi} & -\pi\le x\le 0\\ 1-\frac{2x}{\pi} & 0\le x\le\pi\end{cases}$ is given by $f(x)=\frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos nx$. Then the value of $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots$ is

a) $\frac{\pi^2}{6}$ b) $\frac{\pi^2}{12}$ c) $\frac{\pi^2}{3}$ d) $\frac{\pi^2}{8}$

147) The Fourier series expansion for the function $f(x)=\begin{cases}\pi x & 0\le x\le 1\\ \pi(2-x) & 1\le x\le 2\end{cases}$ is given by $f(x)=\frac{\pi}{2}-\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos(2n-1)\pi x$. Then the value of $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots$ is

a) $\frac{\pi^2}{6}$ b) $\frac{\pi^2}{12}$ c) $\frac{\pi^2}{8}$ d) $\frac{\pi^2}{3}$

148) In the following harmonic analysis of $y=f(x)$, the value of a_0 is given by

x	0	1	2	3	4	5
y	4	8	15	7	5	3

a) 14 b) 7 c) 3.5 d) 6

149) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	1	2	3	4	5
y	9	18	26	26	26	20

a) 25.01 b) 20.83 c) 41.66 d) 40.89

150) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	30	60	90	120	150	180
y	0	9.2	14.4	17.8	17	12	0

a) 10.23 b) 23.46 c) 46.92 d) 11.73

151) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	60	120	180	240	300	360
y	1	1.4	1.9	1.6	1.6	1.2	1

a) 7.2 b) 1.45 c) 5.8 d) 2.9

152) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	60	120	180	240	300	360
y	1.98	2.15	2.7	-0.22	-0.31	1.5	1.98

a) 4.8 b) 2.6 c) 5.2 d) 1.3

153) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	1	1.4	1.9	1.6	1.6	1.2	1

a) 2.9 b) 5.8 c) 1.45 d) 3.8

154) In the following harmonic analysis of $y = f(x)$, the value of a_0 is given by

x	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
y	1.98	1.35	1	1.3	-0.88	-0.25	1.98

a) 1 b) 0.75 c) 1.5 d) 3

155) In the following harmonic analysis of $y = f(x)$, the value of a_1 is given by

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
y	0	9.2	14.4	17.8	17.3	11.7	0

a) 3.73 b) 5.73 c) 7.73 d) -7.73

156) In the following harmonic analysis of $y = f(x)$, the value of b_1 is given by

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
y	0	9.2	14.4	17.8	17.3	11.7	0

a) 4.38 b) 3.48 c) 4.83 d) 8.43

157) In the following harmonic analysis of $y = f(x)$, the value of a_1 is given by

x	0	1	2	3	4	5
y	9	18	24	28	26	20

a) - 8.37 b) 8.73 c) 7.83 d) 3.78

158) In the following harmonic analysis of $y = f(x)$, the value of b_1 is given by

x	0	1	2	3	4	5
y	9	18	24	28	26	20

a) 1.25 b) -6.3 c) – 3.15 d) - 3.50

159) In the following harmonic analysis of $y = f(x)$, the value of b_1 is given by

x	0	1	2	3	4	5	6
y	9	18	24	28	26	20	9
$\cos\left(\frac{\pi}{3}x\right)$	1	0.5	-0.5	-1	-0.5	0.5	1

a) 3.38 b) – 8.33 c) 8.33 d) 5.83

Chapter 04–Reduction Formulae, Beta and Gamma Functions

I) Reduction Formulae

1) For $I_n = \int_0^{\pi/2} \sin^n x\,dx$, we have

a) $I_n = 2\int_0^{\pi} \sin^n x\,dx$ b) $I_n = \int_0^{\pi/2} \sin^{n-2} x \cos^2 x\,dx$

c) $I_n = \int_0^{\pi/2} \cos^n x\,dx$ d) $I_n = \frac{1}{2}\int_0^{\pi/4} \sin^n x\,dx$

2) For $I_n = \int_0^{\pi} \sin^n x\,dx$, we have

a) 0 b) $I_n = 2\int_0^{\pi/2} \sin^n x\,dx$

c) $I_n = 4\int_0^{\pi/2} \sin^n x\,dx$ d) none of these

3) For $I_n = \int_0^{\pi} \cos^n x\,dx$, where n is an even integer, we have

a) $I_n = 4\int_0^{\pi/2} \cos^n x\,dx$ b) $I_n = 2\int_0^{\pi/4} \cos^n x\,dx$

c) $I_n = 2\int_0^{\pi/2} \cos^n x\,dx$ d) 0

4) For $I_n = \int_0^{\pi} \cos^n x\,dx$, where n is an odd integer, we have

a) $I_n = 4\int_0^{\pi/2} \cos^n x\,dx$ b) $I_n = 2\int_0^{\pi/4} \cos^n x\,dx$

c) $I_n = 2\int_0^{\pi/2} \cos^n x\,dx$ d) 0

5) For $I_n = \int_0^{2\pi} \sin^n x\,dx$, where n is an even integer, we have

a) 0 b) $I_n = 4\int_0^{\pi/4} \sin^n x\,dx$

c) $I_n = 2\int_0^{\pi/2} \sin^n x\,dx$ d) $I_n = 4\int_0^{\pi/2} \sin^n x\,dx$

6) For $I_n = \int_0^{2\pi} \sin^n x\,dx$, where n is an odd integer, we have

a) $I_n = 4\int_0^{\pi/2} \sin^n x\,dx$ b) $I_n = 2\int_0^{\pi/2} \sin^n x\,dx$

c) 0 d) $I_n = 4\int_0^{\pi/4} \sin^n x\,dx$

7) For $I_n = \int_0^{2\pi} \cos^n x\,dx$, where n is an odd integer, we have

a) 0 b) $I_n = 4\int_0^{\pi/2} \cos^n x\,dx$

c) $I_n = 2\int_0^{\pi/2} \cos^n x\,dx$ d) $I_n = 4\int_0^{\pi/4} \cos^n x\,dx$

8) For $I_n = \int_0^{2\pi} \cos^n x\,dx$, where n is an even integer, we have

a) 0 b) $I_n = 4\int_0^{\pi/4} \cos^n x\,dx$

c) $I_n = 4\int_0^{\pi/2} \cos^n x\,dx$ d) $I_n = 2\int_0^{\pi/2} \cos^n x\,dx$

9) For $I_{m,n}=\int_0^{\pi}\sin^m x\cos^n x\,dx$, where both m and n are odd integers, we have

a) $I_{m,n}=2\int_0^{\pi/2}\sin^m x\cos^n x\,dx$ b) 0

c) $I_{m,n}=4\int_0^{\pi/2}\sin^m x\cos^n x\,dx$ d) none

10) For $I_{m,n}=\int_0^{\pi}\sin^m x\cos^n x\,dx$, where both m and n are even integers, we have

a) $I_{m,n}=2\int_0^{\pi/2}\sin^m x\cos^n x\,dx$

b) $I_{m,n}=4\int_0^{\pi/2}\sin^m x\cos^n x\,dx$

c) 0 d) none of the above

11) For $I_{m,n}=\int_0^{\pi}\sin^m x\cos^n x\,dx$, where only n is an even integer, we have

a) 0 b) $I_{m,n}=4\int_0^{\pi/2}\sin^m x\cos^n x\,dx$

c) $I_{m,n}=2\int_0^{\pi/2}\sin^m x\cos^n x\,dx$ d) none

12) For $I_{m,n}=\int_0^{\pi}\sin^m x\cos^n x\,dx$, where only n is an odd integer, we have

a) $I_{m,n}=4\int_0^{\pi/2}\sin^m x\cos^n x\,dx$

b) 0

c) $I_{m,n}=2\int_0^{\pi/2}\sin^m x\cos^n x\,dx$

d) none of the above

13) For $I_n=\int_0^{\pi/2}\sin^n x\,dx$, which of the following is the reduction formula?

a) $I_n=\frac{n-1}{n}I_{n-1}$ b) $I_n=\frac{n}{n+1}I_{n-2}$

c) $I_n=\frac{n+1}{n}I_{n-2}$ d) $I_n=\frac{n-1}{n}I_{n-2}$

14) For $I_n=\int_0^{\pi/2}\cos^n x\,dx$, which of the following is the reduction formula?

a) $I_n=\frac{n-1}{n}I_{n-2}$ b) $I_n=\frac{n-1}{n}I_{n-1}$

c) $I_n=\frac{n}{n+1}I_{n-2}$ d) $I_n=\frac{n+1}{n}I_{n-2}$

15) For $I_n=\int_0^{\pi/2}\sin^n x\,dx$, where n is an even natural number, which of the following is the reduced form?

a) $I_n=\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \ldots\ \cdot\frac{1}{2}\cdot\frac{\pi}{2}$

b) $I_n=\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \ldots\ \cdot\frac{2}{3}\cdot 1$

c) $I_n=\frac{n+1}{n}\cdot\frac{n+3}{n+2}\cdot\frac{n+5}{n+4}\cdot\ \ldots\ \cdot\frac{1}{2}\cdot\frac{\pi}{2}$

d) $I_n=\frac{n-2}{n-1}\cdot\frac{n-4}{n-3}\cdot\frac{n-6}{n-5}\cdot\ \ldots\ \cdot\frac{2}{3}\cdot\frac{\pi}{2}$

16) For $I_n=\int_0^{\pi/2}\cos^n x\,dx$, where n is an even natural number, which of the following is the reduced form?

a) $I_n=\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \ldots\ \cdot\frac{2}{3}\cdot 1$

b) $I_n=\frac{n-2}{n-1}\cdot\frac{n-4}{n-3}\cdot\frac{n-6}{n-5}\cdot\ \ldots\ \cdot\frac{2}{3}\cdot\frac{\pi}{2}$

c) $I_n=\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \ldots\ \cdot\frac{1}{2}\cdot\frac{\pi}{2}$

d) $I_n=\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \ldots\ \cdot\frac{1}{2}\cdot\pi$

17) For $I_n = \int_0^{\pi/2} \sin^n x\, dx$, where n is an odd natural number, which of the following is the reduced form?

a) $I_n = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \ldots\ \cdot\frac{2}{3}\cdot 1$

b) $I_n = \frac{n-2}{n-1}\cdot\frac{n-4}{n-3}\cdot\frac{n-6}{n-5}\cdot\ \ldots\ \cdot\frac{2}{3}\cdot\frac{\pi}{2}$

c) $I_n = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \ldots\ \cdot\frac{1}{2}\cdot\frac{\pi}{2}$

d) $I_n = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \ldots\ \cdot\frac{1}{2}\cdot\pi$

18) For $I_n = \int_0^{\pi/2} \cos^n x\, dx$, where n is an odd natural number, which of the following is the reduced form?

a) $I_n = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \ldots\ \cdot\frac{1}{2}\cdot\frac{\pi}{2}$

b) $I_n = \frac{n+1}{n}\cdot\frac{n+3}{n+2}\cdot\frac{n+5}{n+4}\cdot\ \ldots\ \cdot\frac{2}{3}\cdot 1$

c) $I_n = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \ldots\ \cdot\frac{2}{3}\cdot 1$

d) $I_n = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdot\ \ldots\ \cdot\frac{1}{2}\cdot\pi$

19) For $I_n = \int_0^{\pi/2} \sin^n x \cos^n x\, dx$, where n is an odd natural number, which of the following is the reduced form?

a) $I_{(m,\,n)} = \frac{[(m-1)(m-3)\cdot\ \ldots\ \cdot 2 or 1][(n-1)(n-3)\cdot\ \ldots\ \cdot 2 or 1]}{(m+n)(m+n-2)\cdot\ \ldots\ \cdot 2 or 1}\cdot k$

where $k = \begin{cases} \frac{\pi}{2} & \textit{both m \& n are odd} \\ 1 & \textit{otherwise} \end{cases}$

b) $I_{(m,\,n)} = \frac{[(m-1)(m-3)\cdot\ \ldots\ \cdot 2 or 1][(n-1)(n-3)\cdot\ \ldots\ \cdot 2 or 1]}{(m+n)(m+n-2)\cdot\ \ldots\ \cdot 2 or 1}\cdot k$

where $k = \begin{cases} \frac{\pi}{2} & \textit{both m \& n are even} \\ 1 & \textit{otherwise} \end{cases}$

c) $I_{(m,\,n)} = \frac{[(m-1)(m-3)\cdot\ \ldots\ \cdot 2 or 1][(n-1)(n-3)\cdot\ \ldots\ \cdot 2 or 1]}{(m+n)(m+n-2)\cdot\ \ldots\ \cdot 2 or 1}\cdot k$

where $k = \begin{cases} \frac{\pi}{2} & \textit{m + n is even} \\ 1 & \textit{otherwise} \end{cases}$

d) $I_{(m,\,n)} = \frac{(m+n-1)(m+n-3)\cdot\ \ldots\ \cdot 2 or 1}{(m+n)(m+n-2)\cdot\ \ldots\ \cdot 2 or 1}\cdot k$

where $k = \begin{cases} \frac{\pi}{2} & \textit{both m \& n are odd} \\ 1 & \textit{otherwise} \end{cases}$

20) The value of $\int_0^{\pi/2} \sin^3 x\, dx$ is equal to

a) $\frac{3\pi}{4}$ b) $\frac{3}{4}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$

21) The value of $\int_0^{\pi/2} \sin^4 x\, dx$ is equal to

a) $\frac{3}{8}$ b) $\frac{3}{16}$ c) $\frac{3\pi}{16}$ d) $\frac{3\pi}{18}$

22) The value of $\int_0^{\pi/2} \sin^5 x\, dx$ is equal to

a) $\frac{4\pi}{15}$ b) $\frac{8\pi}{30}$ c) $\frac{8\pi}{15}$ d) $\frac{8}{15}$

23) The value of $\int_0^{\pi/2} \sin^9 x\, dx$ is equal to

a) $\frac{64}{315}$ b) $\frac{128}{315}$ c) $\frac{128}{315}\pi$ d) $\frac{64}{315}\pi$

24) The value of $\int_0^{\pi/2} \cos^3 x\, dx$ is equal to

a) $\frac{3\pi}{4}$ b) $\frac{3}{4}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$

25) The value of $\int_0^{\pi/2} \cos^4 x\, dx$ is equal to

a) $\frac{3}{8}$ b) $\frac{3}{16}$ c) $\frac{3\pi}{16}$ d) $\frac{3\pi}{18}$

26) The value of $\int_0^{\pi/2} \cos^7 x\, dx$ is equal to

a) $\frac{8}{35}$ b) $\frac{16\pi}{35}$ c) $\frac{16\pi}{70}$ d) $\frac{16}{35}$

27) The value of $\int_0^{\pi/2} \cos^{10} x\,dx$ is equal to

a) $\frac{63\pi}{128}$ b) $\frac{63\pi}{512}$ c) $\frac{63\pi}{256}$ d) $\frac{64}{315}\pi$

28) The value of $\int_0^{\pi/2} \sin^3 x\cos^2 x\,dx$ is equal to

a) $\frac{2}{15}$ b) $\frac{\pi}{30}$ c) $\frac{1}{15}$ d) $\frac{\pi}{15}$

29) The value of $\int_0^{\pi/2} \sin^2 x\cos^3 x\,dx$ is equal to

a) $\frac{1}{15}$ b) $\frac{\pi}{30}$ c) $\frac{\pi}{15}$ d) $\frac{2}{15}$

30) The value of $\int_0^{\pi/2} \sin^3 x\cos^4 x\,dx$ is equal to

a) $\frac{1}{35}$ b) $\frac{2}{35}$ c) $\frac{2\pi}{35}$ d) $\frac{2\pi}{70}$

31) The value of $\int_0^{\pi/2} \sin^4 x\cos^6 x\,dx$ is equal to

a) $\frac{3\pi}{512}$ b) $\frac{3}{256}$ c) $\frac{3\pi}{256}$ d) $\frac{3\pi}{128}$

32) The value of $\int_{-\pi/2}^{\pi/2} \sin^3 x\cos^2 x\,dx$ is equal to

a) $2\int_0^{\pi/2} \sin^3 x\cos^2 x\,dx$ b) $4\int_0^{\pi/2} \sin^3 x\cos^2 x\,dx$

c) 0 d) none of the above

33) The value of $\int_{-\pi/2}^{\pi/2} \sin^2 x\cos^3 x\,dx$ is equal to

a) 0 b) $2\int_0^{\pi/2} \sin^2 x\cos^3 x\,dx$

c) $3\int_0^{\pi/3} \sin^2 x\cos^3 x\,dx$ d) none of the above

34) The value of $\int_{-\pi/2}^{\pi/2} \sin^4 x\cos^2 x\,dx$ is equal to

a) $\frac{3}{16}$ b) $\frac{3\pi}{16}$ c) $\frac{\pi}{16}$ d) 0

35) The value of $\int_{-\pi/2}^{\pi/2} \sin^5 x\,dx$ is equal to

a) $\frac{3\pi}{128}$ b) $\frac{3\pi}{15}$ c) $\frac{32}{256}$ d) 0

36) The value of $\int_0^{2\pi} \sin^4 x\cos^6 x\,dx$ is equal to

a) $\frac{3}{64}$ b) $\frac{2\pi}{35}$ c) $\frac{2}{35}$ d) $\frac{3\pi}{128}$

37) The value of $\int_0^{2\pi} \sin^4 x\cos^7 x\,dx$ is equal to

a) $\frac{5}{128}$ b) $\frac{5\pi}{128}$ c) 0 d) $\frac{5\pi}{256}$

38) The value of $\int_{-\pi}^{\pi} \sin^4 x\cos^7 x\,dx$ is equal to

a) 0 b) $\frac{5\pi}{128}$ c) $\frac{5}{128}$ d) $\frac{5\pi}{256}$

39) The value of $\int_0^{\pi} \cos^3 x\,dx$ is equal to

a) $\frac{5\pi}{256}$ b) $\frac{5\pi}{16}$ c) $\frac{5}{128}$ d) 0

40) The value of $\int_0^{\pi} \cos^6 x\,dx$ is equal to

a) 0 b) $\frac{5\pi}{16}$ c) $\frac{5}{8}$ d) $\frac{5\pi}{256}$

41) The value of $\int_0^{\pi} \cos^7 x\,dx$ is equal to

a) $\frac{5\pi}{256}$ b) $\frac{5\pi}{16}$ c) $\frac{5}{128}$ d) 0

42) The value of $\int_0^{\pi} \sin^7 x\,dx$ is equal to

a) $\frac{5\pi}{32}$ b) $\frac{5\pi}{16}$ c) $\frac{32}{35}$ d) 0

43) The value of $\int_0^{\pi} \sin^6 x\,dx$ is equal to

a) $\frac{5\pi}{16}$ b) $\frac{5\pi}{32}$ c) $\frac{3}{4}$ d) 0

44) The value of $\int_0^{2\pi} \sin^6 \theta\,d\theta$ is equal to

a) $\frac{5\pi}{32}$ b) $\frac{5\pi}{16}$ c) $\frac{5\pi}{8}$ d) 0

45) The value of $\int_0^{2\pi} \sin^8 x\,dx$ is equal to

a) $\frac{5\pi}{16}$ b) $\frac{5\pi}{32}$ c) $\frac{32}{35}$ d) $\frac{35\pi}{32}$

46) The value of $\int_0^{2\pi} \cos^5 x\,dx$ is equal to

a) $\frac{5\pi}{32}$ b) $\frac{5\pi}{16}$ c) $\frac{32}{35}$ d) 0

47) The value of $\int_0^{2\pi} \sin^6 x\cos^4 x\,dx$ is equal to

a) $\frac{5\pi}{256}$ b) $\frac{3\pi}{128}$ c) $\frac{35\pi}{256}$ d) 0

48) The value of $\int_0^{2\pi} \sin^7 x\cos^4 x\,dx$ is equal to

a) $\frac{5\pi}{256}$ b) $\frac{3\pi}{128}$ c) $\frac{35\pi}{256}$ d) 0

49) The value of $\int_0^{2\pi} \sin^7 x\cos^5 x\,dx$ is equal to

a) $\frac{5\pi}{256}$ b) 0 c) $\frac{35\pi}{256}$ d) $\frac{3\pi}{128}$

50) The value of $\int_0^{\pi} \sin^5\left(\frac{x}{2}\right)dx$ is equal to

a) $\frac{16\pi}{15}$ b) $\frac{5\pi}{32}$ c) $\frac{16}{15}$ d) $\frac{5\pi}{16}$

51) The value of $\int_0^{\pi/4} \sin^2(2x)\,dx$ is equal to

a) $\frac{\pi}{8}$ b) $\frac{16}{15}$ c) $\frac{3\pi}{8}$ d) 0

52) The value of $\int_0^{\pi/4} \sin^7(2x)\,dx$ is equal to

a) $\frac{16\pi}{15}$ b) $\frac{5\pi}{16}$ c) $\frac{8}{35}$ d) 0

53) The value of $\int_0^{\pi/4} \cos^2(2x)\,dx$ is equal to

a) $\frac{5\pi}{16}$ b) $\frac{\pi}{8}$ c) $\frac{5\pi}{32}$ d) 0

54) The value of $\int_0^{\pi/3} \sin^5(3x)\,dx$ is equal to

a) $\frac{3\pi}{16}$ b) $\frac{8\pi}{15}$ c) $\frac{8\pi}{45}$ d) $\frac{8}{45}$

55) If $I_n = \int_0^{\pi/4} \sin^{2n} x\,dx = -\frac{1}{2^{n+1}n} + \frac{2n-1}{2n} I_{n-1}$, the value of I_2 is equal to

a) $\frac{3\pi+2}{8}$ b) $\frac{3\pi-8}{32}$ c) $-\frac{8+3\pi}{32}$ d) $\frac{3\pi}{32}$

56) If $I_n = \int_0^{\pi/2} x\sin^n x\,dx = \frac{1}{n^2} + \frac{n-1}{n} I_{n-2}$, the value of I_5 is equal to

a) $\frac{149}{25}$ b) $\frac{19}{225}$ c) $\frac{\pi}{2} - \frac{149}{225}$ d) $\frac{149}{225}$

56) If $I_n = \int_0^{\pi/2} \tan^n x\,dx = \frac{1}{n-1} - I_{n-2}$, the value of I_4 is equal to

a) $\frac{\pi}{4} - \frac{2}{3}$ b) $\frac{\pi}{4} + \frac{2}{3}$ c) $\frac{\pi}{2} - \frac{2}{3}$ d) $\frac{\pi}{4} + \frac{4}{3}$

II) Gamma Functions

57) For $n>0$, the gamma function Γn is defined as

a) $\int_0^\infty e^{x}x^{n-1}dx$ b) $\int_0^\infty e^{-x}x^{n+1}dx$

c) $\int_0^\infty e^{-x}x^{n}dx$ d) $\int_0^\infty e^{-x}x^{n-1}dx$

58) $\int_0^\infty e^{-x}x^{n}dx$ is equal to

a) $\Gamma(n+1)$ b) Γn c) $\Gamma(n-1)$ d) $\Gamma(n-2)$

59) $\int_0^\infty e^{-kx}x^{n}dx$ is equal to

a) $k^{n-1}\Gamma(n-1)$ b) $\frac{\Gamma(n-1)}{k^{n-1}}$ c) $\frac{\Gamma(n+1)}{k^{n+1}}$ d) $\frac{\Gamma n}{k^{n}}$

60) $\int_0^\infty e^{-kx}x^{n-1}dx$ is equal to

a) $k^{n-1}\Gamma(n-1)$ b) $\frac{\Gamma(n-1)}{k^{n-1}}$ c) $\frac{\Gamma(n+1)}{k^{n+1}}$ d) $\frac{\Gamma n}{k^{n}}$

61) The value of Γn is equal to

a) $n\Gamma(n-1)$ b) $(n+1)\Gamma(n+1)$

c) $(n-1)\Gamma(n-1)$ d) $n\Gamma n$

62) If n is a natural number, the value of Γn is

a) $\frac{n!}{n+1}$ b) $(n-1)!$ c) $n!$ d) $(n+1)!$

63) The value of $\Gamma 1$ is

a) 1 b) 2 c) 3 d) 0

64) The value of $\Gamma 2$ is

a) 0 b) 1 c) 2 d) 3

65) The value of $\Gamma 7$ is

a) 3256 b) 5040 c) 120 d) 720

66) The value of $\Gamma(1/2)$ is

a) $\frac{1}{2}$ b) $\sqrt{\pi}$ c) $\Gamma\pi$ d) none

67) The value of $\Gamma(5/2)$ is

a) $\frac{3\sqrt{\pi}}{2}$ b) $\frac{3\sqrt{\pi}}{4}$ c) $\frac{3\sqrt{\pi}}{8}$ d) 0

68) The value of $\Gamma\frac{1}{4}\cdot\Gamma\frac{3}{4}$ is

a) $\pi\sqrt{2}$ b) $\frac{\pi}{\sqrt{2}}$ c) $\frac{\sqrt{2}}{\pi}$ d) none

69) The value of $\Gamma p\cdot\Gamma(1-p)$, for $0<p<1$, is given by the formula

a) $\frac{\sin p\pi}{\pi}$ b) $\frac{\pi}{\sin p\pi}$

c) $\frac{\sqrt{\pi}}{\sin p\pi}$ d) $\frac{p\pi}{\sin p\pi}$

70) The value of $\int_0^\infty e^{-x}x^{5}dx$

a) 60 b) 720 c) 120 d) 240

71) The value of $\int_0^\infty e^{-2x}x^{5}dx$

a) $\frac{125}{32}$ b) $\frac{120}{35}$ c) $\frac{25}{8}$ d) $\frac{15}{8}$

72) The value of $\int_0^\infty e^{-x}x^{1/2}dx$

a) $\frac{\sqrt{\pi}}{2}$ b) $\frac{\pi}{2}$ c) $\frac{\sqrt{\pi}}{3}$ d) $\sqrt{\pi}$

73) The value of $\int_0^\infty e^{-x}x^{-1/2}dx$

a) $\frac{\sqrt{\pi}}{2}$ b) $\sqrt{\pi}$ c) $\frac{\sqrt{\pi}}{3}$ d) $\frac{\pi}{2}$

74) The value of $\int_0^\infty e^{-x}x^{3/2}dx$

a) $\frac{\sqrt{\pi}}{4}$ b) $\frac{3\sqrt{\pi}}{8}$ c) $\frac{3\sqrt{\pi}}{4}$ d) $\frac{3\sqrt{\pi}}{2}$

75) The substitution for the integral $\int_0^\infty \sqrt{x}\cdot e^{-\sqrt{x}}dx$ to reduce it into the form of gamma function is

a) $\sqrt{x}=t$ b) $\sqrt{x}=t^2$

c) $\sqrt{x}=\frac{t}{2}$ d) $x=\sin t$

76) The substitution for the integral $\int_0^\infty x^3\cdot e^{-\sqrt{x}}dx$ to reduce it into the form of gamma function is

a) $x^3=\sin^2 t$ b) $x^3=e^{-t}$

c) $x^3=t$ d) $\sqrt{x}=t$

77) The substitution for the integral $\int_0^\infty x^3\cdot 5^{-x}dx$ to reduce it into the form of gamma function is

a) $5^x=e^t$ b) $x^3=e^{-t}$

c) $5^x=x^{-t}$ d) $\log x=5^{-x}$

78) On using substitution $\sqrt{x}=t$, the value of the integration $\int_0^\infty x\cdot e^{-\sqrt{x}}dx$ is given by

a) 1 b) 3 c) 12 d) 16

79) On using substitution $\sqrt{x}=t$, the value of the integration $\int_0^\infty \sqrt{x}\cdot e^{-\sqrt{x}}dx$ is given by

a) 1 b) 2 c) 3 d) 4

80) On using substitution $\sqrt{t}=x$, the value of the integration $\int_0^\infty e^{-x^2}dx$ is given by

a) $\left\lceil\frac{1}{4}\right.$ b) 16 c) $\frac{\sqrt{\pi}}{2}$ d) $\sqrt{\pi}$

81) On using substitution $x^3=t$, the value of the integration $\int_0^\infty \sqrt{x}\cdot e^{-x^3}dx$ is given by

a) $\frac{\sqrt{\pi}}{2}$ b) $\frac{\sqrt{\pi}}{3}$ c) $\frac{2\sqrt{\pi}}{3}$ d) $\frac{3\sqrt{\pi}}{4}$

82) On using substitution $x^4=t$, the value of the integration $\int_0^\infty e^{-x^4}dx$ is given by

a) $\sqrt{\pi}$ b) π c) $\frac{1}{4}\left\lceil\frac{1}{4}\right.$ d) $\left\lceil\frac{3}{4}\right.$

83) On using substitution $x=t^2$, the value of the integration $\int_0^\infty \sqrt[4]{x}\cdot e^{-\sqrt{x}}dx$ is given by

a) $\frac{3\sqrt{\pi}}{2}$ b) $\frac{2\sqrt{\pi}}{3}$ c) $\frac{\sqrt{\pi}}{3}$ d) $2\sqrt{\pi}$

84) On using substitution $2x^2=t$, the value of the integration $\int_0^\infty x^7\cdot e^{-2x^2}dx$ is given by

a) $\left\lceil\frac{3}{4}\right.$ b) $\frac{3}{8}$ c) $\frac{2\sqrt{\pi}}{3}$ d) $\frac{3}{16}$

85) On using substitution $2x^2=t$, the value of the integration $\int_0^\infty x^9\cdot e^{-2x^2}dx$ is given by

a) $\left\lceil\frac{3}{4}\right.$ b) $\frac{3}{8}$ c) $\frac{2\sqrt{\pi}}{3}$ d) $\frac{3}{16}$

86) On using substitution $x^2=t$, the value of the integration $\int_0^\infty x^2\cdot e^{-x^2}dx$ is given by

a) $\frac{1}{3}\left\lceil\frac{3}{2}\right.$ b) $\frac{3}{2}\left\lceil\frac{3}{2}\right.$ c) $\frac{1}{2}\left\lceil\frac{3}{2}\right.$ d) $\frac{1}{2}\left\lceil\frac{2}{3}\right.$

87) On using substitution $x=t^{1/3}$, the value of the integration $\int_0^\infty \sqrt{x}\cdot e^{-x^3}dx$ is given by

a) $\frac{\sqrt{\pi}}{3}$ b) $\frac{2\sqrt{\pi}}{3}$ c) $\frac{1}{2}\left\lceil\frac{2}{3}\right.$ d) $\frac{1}{3}\left\lceil\frac{3}{2}\right.$

88) On using substitution $a^{-x}=e^{-t}$, the value of the integration $\int_0^\infty \frac{x^a}{a^x}dx$ is given by

a) $\frac{\lceil a}{(\log a)^a}$ b) $\frac{\lceil a-1}{(\log a)^{a-1}}$

c) $\dfrac{\Gamma(a+1)}{(\log a)^{a+1}}$ d) $\dfrac{\Gamma(a)}{(\log a)^{a+1}}$

89) On using substitution $3^{-x} = e^{-t}$, the value of the integration $\int_0^\infty \frac{x^3}{3^x}\,dx$ is given by

a) $\dfrac{3}{(\log 3)^4}$ b) $\dfrac{6}{(\log 3)^4}$

c) $\dfrac{36}{(\log 3)^4}$ d) $\dfrac{6}{(\log 3)^3}$

90) On using substitution $\log x = -t$, the value of the integration $\int_0^1 (x\log x)^3\,dx$ is given by

a) $-\dfrac{3}{64}$ b) $\dfrac{3}{64}$ c) $\dfrac{3}{128}$ d) $-\dfrac{3}{128}$

91) On using substitution $\log x = -t$, the value of the integration $\int_0^1 \left(\log\frac{1}{x}\right)^{n-1} dx$ is given by

a) $\Gamma(n+1)$ b) $\Gamma(n)$ c) $\Gamma(n-1)$ d) $-\Gamma(1+n)$

92) On using substitution $\log x = -t$, the value of the integration $\int_0^1 \frac{1}{\sqrt{x\log\frac{1}{x}}}\,dx$ is given by

a) $\sqrt{2\pi}$ b) $\pi\sqrt{2}$ c) $2\sqrt{\pi}$ d) 2π

93) On using substitution $\log x = -t$, the value of the integration $\int_0^1 \frac{1}{\sqrt{-\log x}}\,dx$ is given by

a) $\sqrt{2\pi}$ b) $\pi\sqrt{2}$ c) $\sqrt{\pi}$ d) $2\sqrt{\pi}$

94) On using substitution $h^2x^2 = t$, the value of the integration $\int_0^\infty x^{n-1}e^{-h^2x^2}\,dx$ is given by

a) $\sqrt{2\pi}$ b) $\dfrac{\Gamma(n/2)}{2h^n}$ c) $\dfrac{\Gamma(n/2)}{2h^{n+1}}$ d) $\dfrac{\Gamma(1+n/2)}{2h^{n+1}}$

II) Beta Functions

95) The value of $\beta(m, n)$ in the integral form is

a) $\int_0^1 x^m(1-x)^{n-1}\,dx$ b) $\int_0^1 x^m(1-x)^n\,dx$

c) $\int_0^1 x^{m+1}(1-x)^{n+1}\,dx$ d) $\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$

96) The value of $\beta(m, n)$ in terms of gamma function is

a) $\dfrac{\Gamma(m)\cdot\Gamma(n)}{\Gamma(m+n+1)}$ b) $\dfrac{\Gamma(m-1)\cdot\Gamma(n-1)}{\Gamma(m+n)}$

c) $\dfrac{\Gamma(m+1)\cdot\Gamma(n+1)}{\Gamma(m+n+1)}$ d) $\dfrac{\Gamma(m)\cdot\Gamma(n)}{\Gamma(m+n)}$

97) The value of $\beta(m, n)$, when m and n are positive integers is

a) $\dfrac{(m-1)!(n-1)!}{(m+n-1)!}$ b) $\dfrac{(m+1)!(n+1)!}{(m+n+1)!}$

c) $\dfrac{m!n!}{(m+n)!}$ d) $\dfrac{m!n!}{(m+n+1)!}$

98) $\int_0^{\pi/2} \sin^m x\cos^n x\,dx$ is given by

a) $\beta(m, n)$ b) $\frac{1}{2}\beta\left(\frac{m-1}{2}, \frac{n-1}{2}\right)$

c) $\frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$ d) $\beta\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$

99) $\int_0^{\pi/2} \sin^{2m-1} x\cos^{2n-1} x\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$ b) $\beta\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$

c) $\frac{1}{2}\beta\left(\frac{m-1}{2}, \frac{n-1}{2}\right)$ d) $\beta(m, n)$

100) $\int_0^{\pi/2} \sin^m x\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{m-1}{2}, \frac{1}{2}\right)$ b) $\frac{1}{2}\beta\left(m, \frac{1}{2}\right)$

c) $\frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{1}{2}\right)$ d) $\frac{1}{2}\beta\left(\frac{m+1}{2}, 0\right)$

101) $\int_0^{\pi/2} \cos^m x\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{m-1}{2}, \frac{1}{2}\right)$ b) $\frac{1}{2}\beta\left(m, \frac{1}{2}\right)$

c) $\frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{1}{2}\right)$ d) $\frac{1}{2}\beta\left(\frac{m+1}{2}, 0\right)$

102) $\int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}}dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$ b) $\beta(m, n)$

c) $\beta(m+1, n+1)$ d) $\beta(m-1, n-1)$

103) $\beta(3, 5)$ can be represented by

a) $\int_0^{\infty} x^2(1-x)^4\,dx$ b) $\int_0^1 x^4(1-x)^6\,dx$

c) $\int_0^1 x^3(1-x)^5\,dx$ d) $\int_0^1 x^2(1-x)^4\,dx$

104) What is the exact value of $\beta(5, 3)$?

a) $\frac{2}{35}$ b) $\frac{2}{105}$ c) $\frac{1}{105}$ d) $\frac{1}{35}$

105) What is the exact value of $\beta\left(\frac{1}{4}, \frac{3}{4}\right)$?

a) $\frac{1}{8}$ b) $\pi\sqrt{2}$ c) $2\sqrt{\pi}$ d) $\sqrt{2\pi}$

106) $\int_0^1 \sqrt{x}(1-x)^{5/2}\,dx$ is equal to

a) $\beta\left(\frac{3}{2}, \frac{7}{2}\right)$ b) $\beta\left(\frac{1}{2}, \frac{5}{2}\right)$

c) $\beta\left(\frac{2}{3}, \frac{5}{3}\right)$ d) $\beta(2, 5)$

107) $\int_0^1 x^4(1-x)^5\,dx$ is equal to

a) $\frac{3}{462}$ b) $\frac{1}{462}$ c) $\frac{1}{501}$ d) $\frac{1}{231}$

108) $2\int_0^{\pi/2} \sin^{\frac{3}{2}} x\cos^5 x dx$ is given by

a) $\beta\left(\frac{5}{4}, 3\right)$ b) $\frac{1}{2}\beta\left(\frac{5}{4}, 3\right)$

c) $\beta\left(\frac{5}{4}, \frac{3}{2}\right)$ d) $\beta\left(\frac{5}{4}, \frac{3}{4}\right)$

109) $2\int_0^{\pi/2} \sqrt{\sin x\cos x}\,dx$ is given by

a) $\beta\left(\frac{1}{2}, \frac{1}{2}\right)$ b) $\beta\left(\frac{5}{4}, \frac{5}{4}\right)$

c) $\beta\left(\frac{3}{4}, \frac{3}{4}\right)$ d) $\beta\left(\frac{3}{2}, \frac{3}{2}\right)$

110) $\int_0^{\pi/2} \frac{1}{\sqrt{\sin x}}dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{1}{4}, \frac{3}{2}\right)$ b) $\beta\left(\frac{1}{4}, \frac{1}{2}\right)$

c) $2\beta\left(\frac{1}{4}, \frac{1}{2}\right)$ d) $\frac{1}{2}\beta\left(\frac{1}{4}, \frac{1}{2}\right)$

111) $\int_0^{\pi/2} \frac{1}{\sqrt{\cos x}}dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{1}{4}, \frac{3}{2}\right)$ b) $\frac{1}{2}\beta\left(\frac{1}{4}, \frac{1}{2}\right)$

c) $\beta\left(\frac{1}{4}, \frac{1}{2}\right)$ d) $2\beta\left(\frac{1}{4}, \frac{1}{2}\right)$

112) $\int_0^{\pi/2} \sqrt{\tan x}\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{3}{4}, \frac{5}{4}\right)$ b) $\beta\left(\frac{3}{4}, \frac{1}{4}\right)$

c) $2\beta\left(\frac{3}{4}, \frac{1}{4}\right)$ d) $\frac{1}{2}\beta\left(\frac{3}{4}, \frac{1}{4}\right)$

113) $\int_0^{\pi/2} \sqrt{\cot x}\,dx$ is given by

a) $2\beta\left(\frac{3}{4}, \frac{1}{4}\right)$ b) $\frac{1}{2}\beta\left(\frac{3}{4}, \frac{5}{4}\right)$

c) $\frac{1}{2}\beta\left(\frac{3}{4}, \frac{1}{4}\right)$ d) $\beta\left(\frac{3}{4}, \frac{1}{4}\right)$

114) $\int_0^{\pi/2} \tan^{3/4} x\,dx$ is given by

a) $\frac{1}{2}\beta\left(\frac{7}{4}, \frac{1}{4}\right)$ b) $\frac{1}{2}\beta\left(\frac{7}{4}, \frac{1}{4}\right)$

c) $\frac{1}{2}\beta\left(\frac{7}{8}, \frac{1}{8}\right)$ d) $\frac{1}{2}\beta\left(\frac{7}{8}, \frac{7}{8}\right)$

115) The value of the integral $\int_0^\infty \frac{x^4}{(1+x)^7}dx$ is

a) $\frac{1}{30}$ b) 30 c) $\frac{1}{15}$ d) $\frac{1}{3}$

116) The value of the integral $\int_0^\infty \frac{x^3+x^2}{(1+x)^7}dx$ is

a) 30 b) $\frac{1}{3}$ c) $\frac{1}{30}$ d) $\frac{1}{15}$

117) The value of the integral $\int_0^\infty \frac{x^8-x^{14}}{(1+x)^{24}}dx$ is

a) 30 b) 0 c) $\frac{1}{30}$ d) $\frac{1}{15}$

118) The value of the integral $\int_0^\infty \frac{x^6(1-x^8)}{(1+x)^{24}}dx$ is

a) 30 b) 0 c) $\frac{1}{30}$ d) $\frac{1}{15}$

119) $\beta(n, n+1)$ is identical with

a) $\frac{(\Gamma n)^2}{\Gamma 2n}$ b) $\frac{\Gamma n}{\Gamma 2n}$ c) $\frac{\Gamma n}{2\Gamma 2n}$ d) $\frac{(\Gamma n)^2}{2\Gamma 2n}$

120) $\beta(m, n+1)+\beta(m+1, n)$ is equal to

a) $\beta(m+1, n+1)$ b) $\beta(m+1, n)$

c) $\beta(m, n)$ d) $\beta(m, n+1)$

121) $\beta(m, n)\cdot\beta(m+n, k)$ is equal to

a) $\frac{\Gamma m\cdot\Gamma n+k}{\Gamma m+n+k}$ b) $\frac{\Gamma m\cdot\Gamma n\cdot\Gamma k}{\Gamma m+n}$

c) $\frac{\Gamma m\cdot\Gamma n}{\Gamma m+n+k}$ d) $\frac{\Gamma m\cdot\Gamma n\cdot\Gamma k}{\Gamma m+n+k}$

122) $\beta(m, n+1)$ is equal to

a) $\frac{m+n}{n}\beta(m, n)$ b) $\frac{n}{m+n}\beta(m, n)$

c) $\frac{m}{m+n}\beta(m, n)$ d) $\frac{m+n}{m}\beta(m, n)$

123) On using substitution $x^3=8t$, the integral $\int_0^2 x(8-x^3)^{1/3}dx$ is equal to

a) $\frac{5}{81}$ b) $\frac{2}{27}$ c) $\frac{2}{81}$ d) $\frac{1}{81}$

124) The value of the integration $\int_0^1 x^3\left(1-x^{1/2}\right)^5 dx$ by substituting $x=t^2$ is given by

a) $2\beta(8, 6)$ b) $\frac{1}{2}\beta(8, 6)$

c) $\beta(8, 6)$ d) $2\beta(9, 7)$

125) The value of the integration $\int_0^1\left(1-x^{1/n}\right)^m dx$ by substituting $x=t^n$ is given by

a) $n\beta(m, n+1)$ b) $n\beta(m+1, n)$

c) $n\beta(m, n)$ d) $m\beta(m+1, n)$

Chapter 05–Differentiation Under Integral Sign & Error Function

I) Differentiation Under Integral Sign

1) If $I(\alpha)=\int_a^b f(x,\alpha)\,dx$, where α is parameter and a, b are constants, by differentiation under integral sign rule we have

a) $\dfrac{dI}{d\alpha}=\int_a^b \dfrac{\partial}{\partial\alpha}\left[f(x,\alpha)\right]\cdot dx$

b) $\dfrac{dI}{d\alpha}=\int_a^b \dfrac{\partial}{\partial x}\left[f(x,\alpha)\right]\cdot dx$

c) $\dfrac{dI}{dx}=\int_a^b \dfrac{\partial}{\partial x}\left[f(x,\alpha)\right]\cdot dx$

d) $\dfrac{dI}{dx}=\int_a^b \dfrac{\partial}{\partial\alpha}\left[f(x,\alpha)\right]\cdot dx$

2) If $I(\alpha)=\int_a^b f(x,\alpha)\,dx$, where α is parameter and a, b are functions of α, by differentiation under integral sign rule we have

a) $\dfrac{dI}{d\alpha}=\int_a^b \dfrac{\partial}{\partial\alpha}\left[f(x,\alpha)\right]\cdot dx-f(x,b)\dfrac{db}{d\alpha}+f(x,a)\dfrac{da}{d\alpha}$

b) $\dfrac{dI}{d\alpha}=\int_a^b \dfrac{\partial}{\partial\alpha}\left[f(x,\alpha)\right]\cdot dx+f(x,b)\dfrac{db}{dx}-f(x,a)\dfrac{da}{dx}$

c) $\dfrac{dI}{d\alpha}=\int_a^b \dfrac{\partial}{\partial\alpha}\left[f(x,\alpha)\right]\cdot dx+f(x,b)\dfrac{db}{d\alpha}-f(x,a)\dfrac{da}{d\alpha}$

d) $\dfrac{dI}{dx}=\int_a^b \dfrac{\partial}{\partial x}\left[f(x,\alpha)\right]\cdot dx+f(x,b)\dfrac{db}{dx}-f(x,a)\dfrac{da}{dx}$

Note: Henceforth, we abbreviate "differentiation under integral sign" by "DUIS" for simplicity.

3) If $I=\int_0^\infty e^{-bx^2}\cos 2ax\cdot dx$, where $b>0$, by DUIS rule we have

a) $\dfrac{dI}{dx}=\int_0^\infty \dfrac{\partial}{\partial x}\left[e^{-bx^2}\cos 2ax\right]\cdot dx$

b) $\dfrac{dI}{da}=\int_0^\infty \dfrac{\partial}{\partial a}\left[e^{-bx^2}\cos 2ax\right]\cdot dx$

c) $\dfrac{dI}{da}=\int_0^\infty \dfrac{\partial}{\partial b}\left[e^{-bx^2}\cos 2ax\right]\cdot dx$

d) $\dfrac{dI}{db}=\int_0^\infty \dfrac{\partial}{\partial a}\left[e^{-bx^2}\cos 2ax\right]\cdot dx$

4) If $I=\int_0^\infty \dfrac{e^{-ax}}{x}\left(1-e^{-bx}\right)dx$, where $a,b>0$, by DUIS rule we have

a) $\dfrac{dI}{da}=\int_0^\infty \dfrac{\partial}{\partial b}\left[\dfrac{e^{-ax}}{x}\left(1-e^{-bx}\right)\right]dx$

b) $\dfrac{dI}{dx}=\int_0^\infty \dfrac{\partial}{\partial x}\left[\dfrac{e^{-ax}}{x}\left(1-e^{-bx}\right)\right]dx$

c) $\dfrac{dI}{da}=\int_0^\infty \dfrac{\partial}{\partial a}\left[\dfrac{e^{-ax}}{x}\left(1-e^{-bx}\right)\right]dx$

d) $\dfrac{dI}{db}=\int_0^\infty \dfrac{\partial}{\partial a}\left[\dfrac{e^{-ax}}{x}\left(1-e^{-bx}\right)\right]dx$

5) If $I=\int_0^\infty \dfrac{e^{-ax}}{x}\left(1-e^{-x}\right)dx$, where $a>0$, by DUIS rule we have

a) $\dfrac{dI}{da}=\int_0^\infty \dfrac{\partial}{\partial a}\left[\dfrac{\left(1-e^{-x}\right)}{x}\right]\cdot e^{-ax}dx$

b) $\dfrac{dI}{da}=\int_0^\infty \dfrac{\partial}{\partial x}\left[e^{-ax}\right]\cdot\dfrac{\left(1-e^{-x}\right)}{x}dx$

c) $\dfrac{dI}{da}=\int_0^\infty \dfrac{\partial}{\partial x}\left[e^{-ax}\dfrac{\left(1-e^{-x}\right)}{x}\right]\cdot dx$

d) $\dfrac{dI}{da}=\int_0^\infty \dfrac{\partial}{\partial a}\left[e^{-ax}\right]\cdot\dfrac{\left(1-e^{-x}\right)}{x}dx$

6) If $I=\int_0^\infty \dfrac{e^{-x}}{x}\left(a-\dfrac{1}{x}+\dfrac{1}{x^2}e^{-ax}\right)dx$, where $a,b>0$, by DUIS rule we have

a) $\dfrac{dI}{da}=\int_0^\infty \dfrac{e^{-x}}{x}\cdot\dfrac{\partial}{\partial x}\left(a-\dfrac{1}{x}+\dfrac{1}{x^2}e^{-ax}\right)dx$

b) $\dfrac{dI}{dx}=\int_0^\infty \dfrac{e^{-x}}{x}\cdot\dfrac{\partial}{\partial a}\left(a-\dfrac{1}{x}+\dfrac{1}{x^2}e^{-ax}\right)dx$

c) $\dfrac{dI}{dx}=\int_0^\infty \dfrac{e^{-x}}{x}\cdot\dfrac{\partial}{\partial x}\left(a-\dfrac{1}{x}+\dfrac{1}{x^2}e^{-ax}\right)dx$

d) $\dfrac{dI}{da}=\int_0^\infty \dfrac{e^{-x}}{x}\cdot\dfrac{\partial}{\partial a}\left(a-\dfrac{1}{x}+\dfrac{1}{x^2}e^{-ax}\right)dx$

7) If $I=\int_0^\infty \dfrac{e^{-ax}}{x}\left(1-e^{-x}\right)dx$, where $a>0$, by DUIS rule we have

a) $\dfrac{dI}{da}=\int_0^\infty e^{(a+1)x}dx$ b) $\dfrac{dI}{da}=\int_0^\infty e^{-ax}dx$

c) $\dfrac{dI}{da}=\int_0^\infty e^{-(a+1)x}dx$ d) $\dfrac{dI}{da}=\int_0^\infty e^{-(a-1)x}dx$

8) If $I=\int_0^\infty \dfrac{e^{-x}}{x}\left(a-\dfrac{1}{x}+\dfrac{1}{x^2}e^{-ax}\right)dx$, where $a,b>0$, by DUIS rule we have

a) $\dfrac{dI}{da}=\int_0^\infty \dfrac{e^{-x}}{x}\left(1-\dfrac{1}{x}e^{-ax}\right)dx$

b) $\dfrac{dI}{da}=\int_0^\infty \dfrac{e^{-x}}{x}\left(1-e^{-ax}\right)dx$

c) $\dfrac{dI}{da}=\int_0^\infty \dfrac{e^{-x}}{x}\left(1-\dfrac{1}{x^2}e^{-ax}\right)dx$

d) $\dfrac{dI}{da}=\int_0^\infty \left(1-\dfrac{1}{x}e^{-ax}\right)dx$

9) If $I=\int_0^\infty \dfrac{e^{-ax}-e^{-bx}}{x}dx$, where $a,b>0$, by DUIS rule we have

a) $\dfrac{dI}{db}=-\int_0^\infty e^{-ax}dx$ b) $\dfrac{dI}{da}=-\int_0^\infty e^{-ax}dx$

c) $\dfrac{dI}{da}=\int_0^\infty e^{-ax}dx$ d) $\dfrac{dI}{da}=\int_0^\infty \left(e^{-ax}-e^{-ax}\right)dx$

10) If $I=\int_0^\infty \dfrac{e^{-ax}-e^{-bx}}{x}dx$, where $a,b>0$, by DUIS rule we have

a) $\dfrac{dI}{db}=\int_0^\infty \left(e^{-ax}-e^{-ax}\right)dx$ b) $\dfrac{dI}{db}=-\int_0^\infty e^{-bx}dx$

c) $\dfrac{dI}{db}=\int_0^\infty e^{-ax}dx$ d) $\dfrac{dI}{db}=\int_0^\infty e^{-bx}dx$

11) If $I=\int_0^\infty \dfrac{e^{-x}-e^{-ax}}{x\sec x}dx$, where $a>0$, by DUIS rule we have

a) $\dfrac{dI}{da}=\int_0^\infty \dfrac{e^{-ax}}{\sec x}dx$ b) $\dfrac{dI}{da}=\int_0^\infty \dfrac{e^{-ax}}{\sec x\tan x}dx$

c) $\dfrac{dI}{da}=-\int_0^\infty \dfrac{e^{-ax}}{\sec x}dx$ d) $\dfrac{dI}{da}=-\int_0^\infty \dfrac{ae^{-ax}}{x\sec x}dx$

12) If $I=\int_0^\infty e^{-a^2}\cos ax\,da$, where $x>0$, by DUIS rule we have

a) $\dfrac{dI}{dx}=-2\int_0^\infty a^2e^{-a^2}\sin ax\,da$

b) $\dfrac{dI}{dx}=2\int_0^\infty ae^{-a^2}\sin ax\,da$

c) $\dfrac{dI}{dx}=-2\int_0^\infty ae^{-a^2}\cos ax\,da$

d) $\dfrac{dI}{dx}=-\int_0^\infty ae^{-a^2}\sin ax\,da$

13) If $I=\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}dx$, where $a>0$, by DUIS rule we have

a) $\dfrac{dI}{da}=\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}\dfrac{1}{x^2}dx$

b) $\dfrac{dI}{da}=a\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}\dfrac{1}{x^2}dx$

c) $\dfrac{dI}{da}=-2a\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}\dfrac{1}{x^2}dx$

d) $\dfrac{dI}{da}=-\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)}\dfrac{1}{x^2}dx$

14) If $I=\int_0^{\infty}\frac{e^{-x}\sin ax}{x}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=-a\int_0^{\infty}\cos ax\,dx$ b) $\frac{dI}{da}=\int_0^{\infty}\sin ax\,dx$

c) $\frac{dI}{da}=-\int_0^{\infty}e^{-x}\cos ax\,dx$ d) $\frac{dI}{da}=\int_0^{\infty}e^{-x}\cos ax\,dx$

15) If $I=\int_0^{\pi}\frac{x^a-1}{\log x}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{\infty}\frac{x^a\log a}{\log x}dx$ b) $\frac{dI}{da}=\int_0^{\pi}x^a\,dx$

c) $\frac{dI}{da}=\int_0^{\pi}x^a\log a\,dx$ d) $\frac{dI}{da}=\int_0^{\pi}\frac{x^a\log a}{\log x}dx$

16) If $I=\int_0^{1}\frac{x^a-x^b}{\log x}dx$, where $a,\ b>0$, by DUIS rule we have

a) x^a-x^b b) $\frac{dI}{da}=\int_0^{\pi}\frac{x^a-x^b}{x\log x}dx$

c) $\frac{dI}{da}=\int_0^{1}\frac{x^a\log a}{\log x}dx$ d) $\frac{dI}{da}=\int_0^{1}x^a\,dx$

17) If $I=\int_0^{\pi}\log(1+a\cos x)\,dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{\pi}\frac{-\sin x}{1+a\cos x}dx$ b) $\frac{dI}{da}=\int_0^{\pi}\frac{\cos x}{1+a\cos x}dx$

c) $\frac{dI}{da}=\int_0^{\pi}\frac{a}{1+a\cos x}dx$ d) $\frac{dI}{da}=-\int_0^{\pi}\frac{\cos x}{1+a\cos x}dx$

18) If $I=\int_0^{\pi}\frac{1}{x^2}\log(1+ax^2)\,dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{\pi}\frac{ax^2}{1+ax^2}dx$ b) $\frac{dI}{da}=2\int_0^{\pi}\frac{x}{1+ax^2}dx$

c) $\frac{dI}{da}=\int_0^{\pi}\frac{1}{1+ax^2}dx$ d) $\frac{dI}{da}=\int_0^{\pi}\frac{2ax}{1+ax^2}dx$

19) If $I=\int_0^{\pi}\frac{1}{\sin^2 x}\log(1+a\sin^2 x)\,dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{\pi}\frac{1}{1+a\sin^2 x}dx$ b) $\frac{dI}{da}=\int_0^{\pi}\frac{\sin 2x}{1+a\sin^2 x}dx$

c) $\frac{dI}{da}=\int_0^{\pi}\frac{a}{1+a\sin^2 x}dx$ d) $\frac{dI}{da}=\int_0^{\pi}\frac{\cos x}{1+a\sin^2 x}dx$

20) If $I=\int_0^{\infty}\frac{1-\cos ax}{x^2}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{\infty}\frac{a\sin ax}{x^2}dx$ b) $\frac{dI}{da}=-\int_0^{\infty}\frac{\cos ax}{x}dx$

c) $\frac{dI}{da}=\int_0^{\infty}\frac{\sin ax}{x}dx$ d) $\frac{dI}{da}=-\int_0^{\infty}\frac{\sin ax}{x}dx$

21) If $I=\int_0^{1}\frac{x^a}{\log x}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{1}\frac{x^a\log a}{\log x}dx$ b) $\frac{dI}{da}=\int_0^{1}x^a\,dx$

c) $\frac{dI}{da}=\int_0^{1}x^a\log a\,dx$ d) $\frac{dI}{da}=\int_0^{1}x^{a-1}\,dx$

22) If $I=\int_0^{\pi/2}\log(a^2\cos^2 x+b^2\sin^2 x)\,dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{\pi/2}\frac{1}{a^2+b^2\tan^2 x}dx$

b) $\frac{dI}{da}=\int_0^{\pi/2}\frac{b^2}{a^2+b^2\tan^2 x}dx$

c) $\frac{dI}{da}=\int_0^{\pi/2}\frac{a^2}{a^2+b^2\tan^2 x}dx$

d) $\frac{dI}{da}=\int_0^{\pi/2}\frac{2a}{a^2+b^2\tan^2 x}dx$

23) If $I=\int_0^\infty \frac{\sin ax-\sin bx}{x^2}dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=-\int_0^\infty \frac{\cos bx}{x}dx$ b) $\frac{dI}{da}=\int_0^\infty \frac{\cos ax}{x}dx$

c) $\frac{dI}{da}=-\int_0^\infty \frac{\cos ax}{x}dx$ d) $\frac{dI}{db}=\int_0^\infty \frac{\cos ax}{x}dx$

24) If $I=\int_0^{a^2} \tan^{-1}\left(\frac{x}{a}\right)dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{a^2} \frac{\partial}{\partial a}\tan^{-1}\left(\frac{x}{a}\right)dx$

b) $\frac{dI}{da}=\int_0^{a^2} \frac{\partial}{\partial a}\tan^{-1}\left(\frac{x}{a}\right)dx-2a\tan^{-1}a$

c) $\frac{dI}{da}=\int_0^{a^2} \frac{\partial}{\partial a}\tan^{-1}\left(\frac{x}{a}\right)dx+2a\tan^{-1}x$

d) $\frac{dI}{da}=\int_0^{a^2} \frac{\partial}{\partial a}\tan^{-1}\left(\frac{x}{a}\right)dx+2a\tan^{-1}a$

25) If $I=\int_0^a \frac{\log(1+ax)}{1+x^2}dx$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^a \frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right]dx+\frac{\log(1+a^2)}{1+a^2}$

b) $\frac{dI}{da}=\int_0^a \frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right]dx-\frac{\log(1+a^2)}{1+a^2}$

c) $\frac{dI}{da}=\int_0^a \frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right]dx$

d) $\frac{dI}{da}=\int_0^a \frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right]dx+\frac{\log(1+x^2)}{1+x^2}$

26) If $I=\int_a^{a^2} \log(ax)\,dx$, by DUIS rule we have

a) $\frac{dI}{da}=\int_a^{a^2} \frac{\partial}{\partial a}\log(ax)\,dx$

b) $\frac{dI}{da}=\int_a^{a^2} \frac{\partial}{\partial a}\log(ax)\,dx-(6a-2)\log a$

c) $\frac{dI}{da}=\int_a^{a^2} \frac{\partial}{\partial a}\log(ax)\,dx+(6a+2)\log a$

d) $\frac{dI}{da}=\int_a^{a^2} \frac{\partial}{\partial a}\log(ax)\,dx+(6a-2)\log a$

27) If $I=\int_t^{t^2} e^{tx^2}\,dx$, by DUIS rule we have

a) $\frac{dI}{dt}=\int_t^{t^2} x^2e^{tx^2}\,dx+2te^{t^5}-e^{t^3}$

b) $\frac{dI}{dt}=\int_t^{t^2} x^2e^{tx^2}\,dx-2te^{t^5}+e^{t^3}$

c) $\frac{dI}{dt}=\int_t^{t^2} te^{tx^2}\,dx+2te^{t^5}-e^{t^3}$

d) $\frac{dI}{dt}=\int_t^{t^2} t^3e^{tx^2}\,dx+2te^{t^5}-e^{t^3}$

28) If $I=\int_0^{a^2} \tan^{-1}\left(\frac{x}{a}\right)dx$, where $a>0$, by DUIS rule we have

a) $\frac{dI}{da}=\int_0^{a^2} \frac{x}{a^2+x^2}dx+2a\tan^{-1}a$

b) $\frac{dI}{da}=-\int_0^{a^2} \frac{a}{a^2+x^2}dx+2a\tan^{-1}a$

c) $\frac{dI}{da}=-\int_0^{a^2} \frac{x}{a^2+x^2}dx+2a\tan^{-1}a$

d) $\frac{dI}{da}=-\int_0^{a^2} \frac{x}{a^2+x^2}dx-2a\tan^{-1}a$

29) If $I=\int_a^{a^2} \log(ax)\,dx$, by DUIS rule we have

a) $\frac{dI}{da}=\int_a^{a^2} \frac{1}{x}dx-(6a-2)\log a$

b) $\frac{dI}{da}=\int_a^{a^2} \frac{1}{a}dx+(6a-2)\log a$

c) $\frac{dI}{da}=\int_a^{a^2} \frac{1}{a}dx-(6a-2)\log a$

d) $\dfrac{dI}{da}=\int_{a}^{a^2}\dfrac{1}{a}dx-(6a-2)\log a$

30) If $I=\int_0^a \dfrac{\log(1+ax)}{1+x^2}dx$, by DUIS rule we have

a) $\dfrac{dI}{da}=\int_0^a \dfrac{x}{(1+x^2)(1+ax)}dx+\dfrac{\log(1+a^2)}{1+a^2}$

b) $\dfrac{dI}{da}=\int_0^a \dfrac{1}{(1+x^2)(1+ax)}dx+\dfrac{\log(1+a^2)}{1+a^2}$

c) $\dfrac{dI}{da}=\int_0^a \dfrac{a}{(1+x^2)(1+ax)}dx+\dfrac{\log(1+a^2)}{1+a^2}$

d) $\dfrac{dI}{da}=\int_0^a \dfrac{x}{(1+x^2)(1+ax)}dx-\dfrac{\log(1+a^2)}{1+a^2}$

31) If $I=\int_{\pi/6a}^{\pi/3a}\dfrac{\sin ax}{x}dx$, by DUIS rule we have

a) $\dfrac{dI}{da}=\int_{\pi/6a}^{\pi/3a}\cos ax\,dx+\dfrac{1}{a}$

b) $\dfrac{dI}{da}=\int_{\pi/6a}^{\pi/3a}\cos ax\,dx-\dfrac{1}{2a}$

c) $\dfrac{dI}{da}=\int_{\pi/6a}^{\pi/3a}\cos ax\,dx-\dfrac{1}{a}$

d) $\dfrac{dI}{da}=\int_{\pi/6a}^{\pi/3a}\dfrac{\cos ax}{x}dx-\dfrac{1}{a}$

32) If $f(x)=\int_a^x (x-t)^2\,G(t)\,dt$, we have

a) $\dfrac{df}{dx}=\int_a^x \dfrac{\partial}{\partial x}(x-t)^2\,G(t)\,dt+(x-a)^2\,G(a)$

b) $\dfrac{df}{dx}=\int_a^x \dfrac{\partial}{\partial x}(x-t)^2\,G(t)\,dt$

c) $\dfrac{df}{dx}=\int_a^x \dfrac{\partial}{\partial x}(x-t)^2\,G(t)\,dt-(x-a)^2\,G(a)$

d) $\dfrac{df}{dx}=\int_a^x \dfrac{\partial}{\partial x}(x-t)^2\,G(t)\,dt+a^2G(a)$

33) If $y=\int_0^x f(t)\sin a(x-t)\,dt$, we have

a) $\dfrac{dy}{dx}=\int_0^x xf(t)\cos a(x-t)\,dt$

b) $\dfrac{dy}{dx}=\int_0^x af(t)\cos a(x-t)\,dt+f(x)$

c) $\dfrac{dy}{dx}=\int_0^x af(t)\cos a(x-t)\,dt-af(x)$

d) $\dfrac{dy}{dx}=a\int_0^x f(t)\cos a(x-t)\,dt$

34) For the integral $I(a)=\int_0^\infty \dfrac{e^{-x}}{x}(1-e^{-ax})dx$, we have $\dfrac{dI}{da}=\dfrac{1}{a+1}$, then I is

a) $\log(a+1)-1$ b) $\log(a+1)$

c) $\log(a+1)+1$ d) $-\dfrac{1}{(a+1)^2}$

35) The value of integration $I(a)=\int_0^1 \dfrac{x^a-1}{\log x}dx$ with $\dfrac{dI}{da}=\dfrac{1}{a+1}$ is given by

a) $\log(a+1)$ b) $\log(a+1)-1$

c) $\log(a+1)+1$ d) $-\dfrac{1}{(a+1)^2}$

36) The value of integration $I(a)=\int_0^1 \dfrac{e^{-2x}\sin ax}{x}dx$ with $\dfrac{dI}{da}=\dfrac{2}{a^2+4}$ is given by

a) $\tan^{-1}\left(\dfrac{a}{2}\right)+\dfrac{\pi}{2}$ b) $\tan^{-1}\left(\dfrac{a}{2}\right)$

c) $\dfrac{1}{2}\tan^{-1}\left(\dfrac{a}{2}\right)$ d) $\dfrac{1}{2}\tan^{-1}\left(\dfrac{x}{2}\right)$

37) The value of integration $I(a)=\int_0^\infty \dfrac{e^{-x}-e^{-ax}}{x\sec x}dx$ with $\dfrac{dI}{da}=\dfrac{a}{a^2+1}$ is given by

a) $2\log\left(\dfrac{2}{a^2+1}\right)$ b) $\dfrac{1}{2}\log\left(\dfrac{2}{a^2+1}\right)$

c) $\frac{1}{2}\log\left(\frac{a^2+1}{2}\right)$ d) $2\log\left(\frac{a^2+1}{2}\right)$

38) The value of integration $I(a)=\int_0^\infty \frac{1-\cos ax}{x^2}dx$ with $\frac{dI}{da}=\frac{\pi}{2}$ is given by

a) $2\pi a$ b) $\frac{\pi a}{3}$ c) $\frac{\pi}{2}$ d) $\frac{\pi a}{2}$

39) The value of integration $I=\int_0^\infty \frac{\log(1+ax^2)}{x^2}dx$, with $\frac{dI}{da}=\frac{\pi}{2\sqrt{a}}$ is given by

a) $\pi\sqrt{a}$ b) $2\sqrt{a}$ c) $\pi\sqrt{2}$ d) $a\sqrt{\pi}$

40) The value of integration $I=\int_0^{\pi/2} \frac{\log(1+a\sin^2 x)}{\sin^2 x}dx$, with $\frac{dI}{da}=\frac{\pi}{2\sqrt{a+1}}$ is given by

a) $\pi\sqrt{a+1}+\pi$ b) $\pi\sqrt{a+1}-\pi$

c) $\pi\sqrt{a+1}-\frac{\pi}{a}$ d) $\frac{\pi\sqrt{a+1}-\pi}{a}$

II) Error Functions

41) $erf(x)$ is given by

a) $\frac{1}{2\sqrt{\pi}}\int_0^x e^{-u^2}du$ b) $\frac{\sqrt{\pi}}{2}\int_0^x e^{-u^2}du$

c) $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$ d) $\int_0^x e^{-u^2}du$

42) $erfc(x)$ is given by

a) $\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2}du$ b) $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$

c) $\frac{\sqrt{\pi}}{2}\int_0^x e^{-u^2}du$ d) $\frac{\sqrt{\pi}}{2}\int_x^\infty e^{-u^2}du$

43) $erf(0)$ is given by

a) $\frac{2}{\sqrt{\pi}}$ b) 1   c) ∞ d) 0

44) $erf(\infty)$ is given by

a) 1   b) 0   c) $\frac{2}{\sqrt{\pi}}$ d) ∞

45) $erfc(0)$ is given by

a) 0   b) $\frac{2}{\sqrt{\pi}}$ c) ∞ d) 1

46) $erf(x)+erfc(x)=?$

a) 2   b) ∞ c) 1   d) 0

47) $erf(-x)=?$

a) $erfc(x)$ b) $-erf(x)$

c) $erf(x)$ d) $-erf(x^2)$

48) Error function is an

a) even function   b) neither even nor odd

c) odd function   d) none of these

49) $erf(x)+erf(-x)=?$

a) 0   b) 1   c) 2   d) 3

50) $erf(-x)+erfc(-x)=?$

a) 0   b) 3   c) 2   d) 1

51) $erfc(-x)-erf(x)=?$
a) ∞ b) 2 c) 1 d) 0

52) $erfc(x)+erfc(-x)=?$
a) 2 b) 1 c) 0 d) ∞

53) If $erf(ax)=\frac{2}{\sqrt{\pi}}\int_0^{ax} e^{-u^2}du$, then $\frac{d}{dx}[erf(ax)]$ is
a) $\frac{2a}{\sqrt{\pi}}e^{-x^2}$ b) $\frac{a}{2\sqrt{\pi}}e^{-a^2x^2}$
c) $\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$ d) $\frac{2x}{\sqrt{\pi}}e^{-a^2x^2}$

54) If $erfc(ax)=\frac{2}{\sqrt{\pi}}\int_{ax}^{\infty} e^{-u^2}du$, then $\frac{d}{dx}[erfc(ax)]$ is
a) $-\frac{2x}{\sqrt{\pi}}e^{-a^2x^2}$ b) $\frac{2x}{\sqrt{\pi}}e^{-a^2x^2}$
c) $-\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$ d) $\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$

55) If $erf(\sqrt{t})=\frac{2}{\sqrt{\pi}}\int_0^{\sqrt{t}} e^{-u^2}du$, then $\frac{d}{dx}[erf(\sqrt{t})]$ is
a) $\frac{1}{t\sqrt{\pi}}e^{-t}$ b) $\frac{1}{\sqrt{\pi t}}e^{-t}$
c) $\frac{2}{\sqrt{\pi t}}e^{-t}$ d) $\frac{1}{\sqrt{\pi t}}e^{-t^2}$

56) If $erfc(\sqrt{t})=\frac{2}{\sqrt{\pi}}\int_{\sqrt{t}}^{\infty} e^{-u^2}du$, then $\frac{d}{dx}[erfc(\sqrt{t})]$ is
a) $\frac{2}{\sqrt{\pi t}}e^{-t}$ b) $\frac{1}{\sqrt{\pi t}}e^{-t^2}$
c) $\frac{1}{t\sqrt{\pi}}e^{-t}$ d) $-\frac{1}{\sqrt{\pi t}}e^{-t}$

57) $\frac{d}{dx}[erf(x)+erfc(x)]=?$
a) 1 b) 0 c) 2 d) ∞

58) If $\frac{d}{dx}[erf(ax)]=\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$, then $\frac{d}{dx}[erfc(ax)]$ is
a) $-\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$ b) $\frac{2a}{\sqrt{\pi}}e^{a^2x^2}$
c) $\frac{1}{\sqrt{\pi}}e^{-a^2x^2}$ d) $\frac{4a^2}{\sqrt{\pi}}e^{-a^2x^2}$

59) On substitution $x+a=u$ in the integration $\int_0^{\infty} e^{-(x+a)^2}dx$, then the value of integration is
a) $\frac{\sqrt{\pi}}{2}erf(a)$ b) $\frac{2}{\sqrt{\pi}}erf(a)$
c) $\frac{\sqrt{\pi}}{2}erfc(a)$ d) $erfc(a)$

60) $\int_0^t erf(ax)dx+\int_0^t erfc(ax)dx=?$
a) 1 b) ∞ c) 0 d) t

61) If $\frac{dy}{dx}[erf(\sqrt{t})]=\frac{e^{-t}}{\sqrt{\pi t}}$, the integration $\int_0^{\infty} e^{-t}erf(\sqrt{t})dt$ is
a) $\frac{1}{\sqrt{\pi}}\int_0^{\infty} e^{-t}t^{1/2}dt$ b) $\frac{2}{\sqrt{\pi}}\int_0^{\infty} e^{-2t}t^{1/2}dt$
c) $-\frac{1}{\sqrt{\pi}}\int_0^{\infty} e^{-2t}t^{-1/2}dt$ d) $\frac{1}{\sqrt{\pi}}\int_0^{\infty} e^{-2t}t^{-1/2}dt$

62) The power series expansion of $erf(x)$ is
a) $\frac{2}{\sqrt{\pi}}\left[x-\frac{x^3}{3}+\frac{x^5}{10}-\frac{x^7}{42}+-\ldots\right]$
b) $\frac{2}{\sqrt{\pi}}\left[x+\frac{x^3}{3}+\frac{x^5}{10}+\frac{x^7}{42}+\ldots\right]$
c) $\frac{2}{\sqrt{\pi}}\left[x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+-\ldots\right]$
d) $\frac{2}{\sqrt{\pi}}\left[x+\frac{x^3}{3!}+\frac{x^5}{5!}+\frac{x^7}{7!}+\ldots\right]$

Chapter 06 – Curve Tracing & Rectification of Curves

I) Curve Tracing

1) If the portion of the curve lies on the both sides of the point lying above the tangent at that point, the curve is known as
a) concave upward b) concave downward
c) inflexion point d) none of these

2) If the portion of the curve lies on the both sides of the point lying above the tangent at that point, the curve is known as
a) inflexion point b) concave downward
c) inflexion point d) none of these

3) A point through which two branches of the same curve passes is known as
a) double point b) inflexion point
c) multiple point d) conjugate point

4) A point through which many branches of the same curve passes is known as
a) double point b) inflexion point
c) multiple point d) conjugate point

5) A double point through which the branches of the curve passes and the tangent at that point are real and distinct, the point is known as
a) conjugate point b) node
c) point of inflexion d) cusp

6) A double point through which the branches of the curve passes and the tangent at that point are real but the same, the point is known as
a) conjugate point b) point of inflexion
c) cusp d) node

7) A double point is said to be node if the tangents to the curve at that point are
a) imaginary b) perpendicular to each other
c) real but the same d) real and distinct

8) A double point is said to be cusp if the tangents at that point are
a) imaginary b) real and distinct
c) real but the same d) none of these

9) If at a point $\frac{dy}{dx}=0$, the tangent to the curve at that point is
a) parallel to the line $x+y=0$
b) parallel to x-axis
c) perpendicular to x-axis
d) parallel to $y=x$

10) If at a point $\frac{dy}{dx}=\infty$, the tangent to the curve at that point is
a) parallel to the line $x+y=0$
b) parallel to $y=x$
c) parallel to x-axis
d) perpendicular to x-axis

11) The standard equation of x-axis in the Cartesian form is given by
a) $x-y=0$ b) $x+y=0$
c) $y=0$ d) $x=0$

12) The standard equation of y-axis in the Cartesian form is given by
a) $x-y=0$ b) $x+y=0$
c) $y=0$ d) $x=0$

13) If the all the powers of y in the Cartesian form are even, the curve is symmetrical about
a) $y-axis$ b) $x,\ y-axes$
c) $x-axis$ d) the line $y=x$

14) If the all the powers of x in the Cartesian form are even, the curve is symmetrical about
a) $x,\ y-axes$ b) $y-axis$
c) $x-axis$ d) the line $y=x$

15) If the all the powers of x and y in the Cartesian form are even, the curve is symmetrical about
a) the line $y=x$ b) $x-axis$ only
c) $y-axis$ only d) $x,\ y-axes$

16) If in the equation of the Cartesian form by replacing $x\rightarrow y$ and $y\rightarrow x$, the equation is symmetrical about
a) the line $y=x$ b) $x,\ y-axes$

c) $x-axis$ d) $y-axis$

17) If in the equation of the Cartesian form by replacing $x \to -y$ and $y \to -x$, the equation is symmetrical about
a) the line $y=-x$ b) the line $y=x$
c) $x,\ y-axes$ d) $y-axis$ only

18) If in the equation of the Cartesian form by replacing $x \to -x$ and $y \to -y$, the equation is symmetrical about
a) the line $y=-x$ b) $x,\ y-axes$
c) opposite quadrants d) the line $y=x$

19) The equation of the tangent at origin when the curve is passing through origin is obtained by equating to zero
a) the lowest degree term of the equation
b) the highest degree term of x in equation
c) the highest degree term of y in equation
d) the coefficient of the term xy

20) In the Cartesian form, the asymptote to the curve parallel to x-axis may be obtained by equating to zero
a) the coefficient of lowest degree term in y
b) the coefficient of highest degree term in y
c) the coefficient of highest degree term in x
d) the coefficient of lowest degree term in x

21) In the Cartesian form, the asymptote to the curve parallel to y-axis may be obtained by equating to zero
a) the coefficient of lowest degree term in y
b) the coefficient of highest degree term in y
c) the coefficient of highest degree term in x
d) the coefficient of lowest degree term in x

22) Oblique asymptote are obtained only when the curve is
a) symmetrical about x-axis
b) symmetrical about y-axis
c) symmetrical about both x and y-axis
d) not symmetrical about both x and y-axes

23) In the Cartesian form if the coefficient of the highest degree term in x is constant, the curve has
a) no asymptote parallel to $x=y$
b) no asymptote parallel to y-axis
c) no asymptote parallel to x-axis
d) none of these

24) In the Cartesian form if the coefficient of the highest degree term in y is constant, the curve has
a) no asymptote parallel to $x+y=0$
b) no asymptote parallel to $x=y$
c) no asymptote parallel to x-axis
d) no asymptote parallel to y-axis

25) In the polar form, if the equation of the curve remains unchanged by replacing $\theta \to -\theta$, the curve is symmetrical about
a) the line $\theta=\frac{\pi}{4}$ b) the initial line
c) pole d) the line $\theta=\frac{\pi}{2}$

26) In the polar form, if the equation of the curve remains unchanged by replacing $r \to -r$, the curve is symmetrical about
a) the line $\theta=\frac{\pi}{4}$ b) the initial line
c) pole d) the line $\theta=\frac{\pi}{2}$

27) In the polar form, if the equation of the curve remains unchanged by replacing $\theta \to \pi-\theta$, the curve is symmetrical about
a) the line $\theta=\frac{\pi}{2}$ b) the line $\theta=\frac{\pi}{4}$
c) the initial line d) pole

28) The pole is point of the curve, if for given angle θ, the value of
a) $r=\infty$ b) $r=0$ c) $r<0$ d) $r>0$

29) If a curve is passing through the pole, the tangent to the curve at pole are obtained by solving
a) $r=0$ b) $r=\infty$ c) $\theta=0$ d) $\theta=\pi$

30) In the polar form, the relation between the angle ϕ formed by the radius vector and the tangent to the curve at that point, is given by
a) $\tan\phi=r^2\frac{d\theta}{dr}$ b) $\cot\phi=r\frac{d\theta}{dr}$
c) $\tan\phi=r\frac{d\theta}{dr}$ d) $\tan\phi=r\frac{dr}{d\theta}$

31) In the parametric form $x = f(t)$, $y = g(t)$, the curve is symmetrical about y-axis, if
a) $x = f(t)$ is odd and $y = g(t)$ is even
b) $x = f(t)$ is even and $y = g(t)$ is odd
c) $x = f(t)$ is odd and $y = g(t)$ is odd
d) $x = f(t)$ is even and $y = g(t)$ is even

32) In the parametric form $x = f(t)$, $y = g(t)$, the curve is symmetrical about y-axis, if
a) $x = f(t)$ is odd and $y = g(t)$ is odd
b) $x = f(t)$ is even and $y = g(t)$ is odd
c) $x = f(t)$ is odd and $y = g(t)$ is even
d) $x = f(t)$ is even and $y = g(t)$ is even

33) The curve represented by the equation $x^2y^2 = x^2 + 1$ is symmetrical about
a) the line $y = x$ b) x-axis only
c) y-axis only d) both x and y-axes

34) The curve represented by the equation $x(x^2 + y^2) = a(x^2 - y^2)$ is
a) symmetrical about y-axis but not passing through origin
b) symmetrical about y-axis and passing through origin
c) symmetrical about x-axis but not passing through origin
d) symmetrical about x-axis and passing through origin

35) The curve represented by the equation $a^2y^2 = x^2(a^2 - x^2)$ is
a) symmetrical about both x and y-axis but not passing through origin
b) symmetrical about both x and y-axis and passing through origin
c) symmetrical about x-axis but not passing through origin
d) symmetrical about x-axis only and passing through origin

36) The curve represented by the equation $(2a - x)y^2 = x^3$ is
a) symmetrical about y-axis and passing through origin
b) symmetrical about x-axis but not passing through origin
c) symmetrical about x-axis and passing through origin
d) symmetrical about y-axis not passing through origin

37) The curve represented by the equation $(2a - y)y^3 = a^2x^2$ is
a) symmetrical about y-axis and passing through origin and $(0, 2a)$
b) symmetrical about x-axis but not passing through origin and $(0, 2a)$
c) symmetrical about x-axis and passing through origin and $(0, 2a)$
d) symmetrical about y-axis not passing through origin and $(0, 2a)$

38) The curve represented by the equation $xy^2 = 4a^2(a - x)$ is
a) symmetrical about y-axis and passing through $(a, 0)$
b) symmetrical about x-axis but not passing through $(a, 0)$
c) symmetrical about x-axis and passing through $(a, 0)$
d) symmetrical about y-axis not passing through $(a, 0)$

39) The curve represented by the equation $xy^2 = 4a^2(a - x)$ has at origin
a) node b) cusp c) inflexion d) none

40) The curve represented by the equation $(2a - x)y^2 = x^3$ has the tangent at origin whose equation is
a) $x + y = 0$ b) y-axis c) x-axis d) $y = x$

41) The curve represented by the equation $(1 + x^2)y = x$ has the tangent at origin whose equation is
a) $y = x$ b) x-axis c) y-axis d) $x + y = 0$

42) The curve represented by the equation $3ay^2 = x(x - a)^2$ has the tangent at origin whose equation is
a) $x + y = 0$ b) $y = x$ c) x-axis d) y-axis

43) The curve represented by the equation $3ay^2 = x(x-a)^2$ has the asymptote parallel to x-axis whose equation is
a) $x+y=0$ b) $y=x$ c) x-axis d) y-axis

44) For the curve given by equation $x^2y = 4a^2(2a-y)$, the asymptote is
a) $y=2a$ b) $y=x$ c) y-axis d) x-axis

45) The curve represented by the equation $y^2(4-x) = x(x-2)^2$ has the asymptote parallel to y-axis whose equation is
a) $x=y$ b) $x=0$ c) $x=2$ d) $x=4$

46) The curve represented by the equation $x^2y^2 = a^2(y^2-x^2)$ has the asymptote parallel to y-axis whose equation is
a) $x=0$ b) $x=\pm a$ c) $x=y$ d) $y=0$

47) For the curve given by equation $x^2y = 4a^2(2a-y)$, the region of absence is
a) $0<y<2a$ b) $y>0,\ y>2a$
c) $y<0,\ y<2a$ d) $y<0,\ y>2a$

48) For the curve given by equation $x^3 = 4y^2(2a-x)$, the region of absence is
a) $0<x<2a$ b) $x<0,\ x>2a$
c) $x>0,\ x>2a$ d) $x<0,\ x<2a$

49) For the curve given by equation $xy^2 = 4a^2(a-x)$, the region of absence is
a) $0<x<a$ b) $x>0,\ x>a$
c) $x<0,\ x>a$ d) $x<0,\ x<a$

50) For the curve given by equation $y^2 = \frac{4x^2(a-x)}{x+a}$, the region of absence along x-axis is
a) $[-\infty,\ -a]$ & $[a,\ \infty]$ b) $[-\infty,\ a]$ & $[-a,\ \infty]$
c) $[-\infty,\ -a]$ d) $[-a,\ \infty]$

51) The curve represented by the equation $x^{1/2} + y^{1/2} = a^{1/2}$ is symmetrical about
a) $y=x$ b) x-axis c) y-axis d) $x+y=0$

52) The curve represented by the equation $\sqrt{x}+\sqrt{y}=\sqrt{a}$ has tangent at origin whose equation is
a) x-axis b) no tangent exists
c) y-axis d) $x+y=0$

53) The curve represented by the equation $\sqrt{x}+\sqrt{y}=\sqrt{a}$ has tangent at $(a,\ 0)$ which is
a) the line $x+y=0$ b) the line $y=x$
c) parallel to y-axis d) parallel to x-axis

54) The curve represented by the equation $x=t^2,\ y=t-\frac{t^3}{3}$ is symmetrical about
a) symmetrical about y-axis but not passing through origin
b) symmetrical about y-axis and passing through origin
c) symmetrical about x-axis but not passing through origin
d) symmetrical about x-axis and passing through origin

55) The curve represented by the equation $x=a(\theta+\sin\theta),\ y=a(1+\cos\theta)$ is symmetrical about
a) symmetrical about y-axis but not passing through origin
b) symmetrical about y-axis and passing through origin
c) symmetrical about x-axis but not passing through origin
d) symmetrical about x-axis and passing through origin

56) The curve represented by the equation $r=a(1+\cos\theta)$ is
a) symmetrical about initial line and not passing through the pole
b) symmetrical about initial line and passing through the pole
c) not symmetrical about initial line and passing through the pole
d) not symmetrical about initial line and not passing through the pole

57) The curve represented by the equation $r^2 = a^2 \cos 2\theta$ is
a) symmetrical about initial line as well as pole and not passing through the pole
b) symmetrical about initial line as well as pole and passing through the pole
c) not symmetrical about initial line as well as pole and passing through the pole
d) not symmetrical about initial line as well as pole and not passing through the pole

58) The curve represented by the equation $r^2 = a^2 \sin 2\theta$ is
a) symmetrical about the line $\theta = \frac{\pi}{4}$ and not passing through the pole
b) symmetrical about initial line as well as pole and passing through the pole
c) not symmetrical about the line $\theta = \frac{\pi}{4}$ and passing through the pole
d) not symmetrical about initial line as well as pole and not passing through the pole

59) The curve represented by the equation $r(1+\cos\theta) = 2a^2$ is
a) symmetrical about the line $\theta = \frac{\pi}{4}$ and not passing through the pole
b) symmetrical about initial line and passing through the pole
c) not symmetrical about the line $\theta = \frac{\pi}{4}$ and passing through the pole
d) symmetrical about initial and not passing through the pole

60) The equations of the tangents at pole to the curve $r = a\sin 3\theta$ are given by
a) $\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \ldots$
b) $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{3}, \ldots$
c) $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \ldots$
d) no such tangent exists

61) The equations of the tangents at pole to the curve $r = a\cos 2\theta$ are given by
a) $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \ldots$
b) $\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \ldots$
c) $\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$
d) $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots$

62) For the rose curve $r = a\sin n\theta$, if n is even, the curve is consisting of
a) $2n$ equal loops b) $2n+1$ equal loops
c) n equal loops d) $2n-1$ equal loops

63) For the rose curve $r = a\cos n\theta$, if n is even, the curve is consisting of
a) n equal loops b) $2n+1$ equal loops
c) $2n$ equal loops d) $2n-1$ equal loops

64) For the rose curve $r = a\sin n\theta$, if n is odd, the curve is consisting of
a) $2n$ equal loops b) n equal loops
c) $2n+1$ equal loops d) $2n-1$ equal loops

65) For the rose curve $r = a\cos n\theta$, if n is odd, the curve is consisting of
a) n equal loops b) $2n+1$ equal loops
c) $2n$ equal loops d) $2n-1$ equal loops

I) Rectification of Curve

66) If $A(a_1, b_1)$ $B(a_2, b_2)$ are two points on the curve on xy-plane, the length of arc is given by

a) $\int_{b_1}^{b_2}\sqrt{1-\left(\frac{dy}{dx}\right)^2}\cdot dy$ b) $\int_{b_1}^{b_2}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dy$

c) $\int_{a_1}^{a_2}\sqrt{1-\left(\frac{dy}{dx}\right)^2}\cdot dx$ d) $\int_{a_1}^{a_2}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$

67) If $A(a_1, b_1)$ $B(a_2, b_2)$ are two points on the curve on xy-plane, the length of arc is given by

a) $\int_{b_1}^{b_2}\sqrt{1+\left(\frac{dx}{dy}\right)^2}\cdot dy$ b) $\int_{a_1}^{a_2}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$

c) $\int_{a_1}^{a_2}\sqrt{1-\left(\frac{dy}{dx}\right)^2}\cdot dx$ d) $\int_{b_1}^{b_2}\sqrt{1-\left(\frac{dx}{dy}\right)^2}\cdot dy$

68) If $A(r_1, \theta_1)$ $B(r_2, \theta_2)$ are two points on the curve on the polar plane, the length of arc is given by

a) $\int_{\theta_1}^{\theta_2}\sqrt{r^2-\left(\frac{dr}{d\theta}\right)^2}\cdot d\theta$ b) $\int_{\theta_1}^{\theta_2}\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\cdot d\theta$

c) $\int_{r_1}^{r_2}\sqrt{1+r^2\left(\frac{d\theta}{dr}\right)^2}\cdot dr$ d) $\int_{r_1}^{r_2}\sqrt{1-r^2\left(\frac{d\theta}{dr}\right)^2}\cdot dr$

69) If $A(r_1, \theta_1)$ $B(r_2, \theta_2)$ are two points on the curve on the polar plane, the length of arc is given by

a) $\int_{\theta_1}^{\theta_2}\sqrt{r^2-\left(\frac{dr}{d\theta}\right)^2}\cdot d\theta$ b) $\int_{\theta_1}^{\theta_2}\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\cdot d\theta$

c) $\int_{r_1}^{r_2}\sqrt{1+r^2\left(\frac{d\theta}{dr}\right)^2}\cdot dr$ d) $\int_{r_1}^{r_2}\sqrt{1-r^2\left(\frac{d\theta}{dr}\right)^2}\cdot dr$

70) If $A(t_1)$ $B(t_2)$ are two points on the curve given by $x = f(t)$, $y = g(t)$ on the xy-plane, the length of arc is given by

a) $\int_{t_1}^{t_2}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\cdot dt$

b) $\int_{t_1}^{t_2}\sqrt{\left(\frac{dt}{dx}\right)^2+\left(\frac{dt}{dy}\right)^2}\cdot dt$

c) $\int_{t_1}^{t_2}\sqrt{\left(\frac{dx}{dt}\right)^2-\left(\frac{dy}{dt}\right)^2}\cdot dt$

d) $\int_{t_1}^{t_2}\left[\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2\right]\cdot dt$

71) The arc length of the upper part of the loop of the curve $9y^2 = (x+7)(x+4)^2$ is obtained by solving the integration

a) $\int_{0}^{4}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$ b) $\int_{-7}^{0}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$

c) $\int_{-7}^{-4}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$ d) $\int_{7}^{4}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$

72) The arc length of the upper part of the curve $y^2 = 4x$ which is cut by the line $3y = 8x$ is obtained by solving the integration

a) $\int_{1}^{9/16}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$ b) $\int_{0}^{9/16}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$

c) $\int_{0}^{3/4}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$ d) $\int_{3}^{8}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$

73) The points $A(a, 0)$ $B(0, a)$ are two points on the curve $x^2 + y^2 = a^2$ on xy-plane such that $1+\left(\frac{dy}{dx}\right)^2 = \frac{a^2}{a^2-x^2}$, the length of arc is given by

a) $4a$ b) πa c) $\frac{\pi a}{4}$ d) $\frac{\pi a}{2}$

74) The points $A(0, 0)$ $B(a, b)$ are two points on the curve $y = a\cosh\left(\frac{x}{a}\right)$ on xy-plane such that $1+\left(\frac{dy}{dx}\right)^2 = \cosh^2\left(\frac{x}{a}\right)$, the length of arc is given by

a) $S = a\sinh\left(\frac{x}{a}\right)$ b) $S = a\tanh\left(\frac{x}{a}\right)$

c) $S = \sinh\left(\frac{x}{a}\right)$ d) $S = a\,\mathrm{sech}\left(\frac{x}{a}\right)$

75) The points $A(0, 0)$ $B(1, 0)$ are two points on the curve $3y^2 = x(x-1)^2$ on xy-plane such that $1+\left(\frac{dy}{dx}\right)^2 = \frac{(3x+1)^2}{12x}$, the length of arc is given by

a) $\frac{3}{\sqrt{3}}$ b) $\frac{1}{2\sqrt{3}}$ c) $\frac{1}{\sqrt{3}}$ d) $\frac{2}{\sqrt{3}}$

76) The total arc length of the part of the curve $r = a(1+\cos\theta)$ which is cut by the circle $r + a\cos\theta = 0$ is obtained by solving the integration

a) $\int_0^{2\pi/3} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

b) $2\int_0^{2\pi/3} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

c) $2\int_0^{2\pi/3} \sqrt{r^2 - \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

d) $\int_0^{2\pi/3} \sqrt{r^2 - \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

77) The total arc length of the upper part of the curve $r^2 = a^2\cos 2\theta$ is obtained by solving the integration

a) $2\int_0^{\pi/2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

b) $\int_0^{\pi/2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

c) $\int_0^{\pi/4} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

d) $\int_0^{\pi/3} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

78) The total length of the arc of the curve $r = ae^{m\theta}$ using $1 + r^2\left(\frac{d\theta}{dr}\right)^2 = 1 + \frac{1}{m^2}$ when r varies from r_1 to r_2 is given by

a) $\frac{\sqrt{1+m^2}}{m}(r_2 - r_1)$ b) $\frac{\sqrt{1+m^2}}{m}(r_2 + r_1)$

c) $\frac{\sqrt{1+m^2}}{m}(r_1 - r_2)$ d) $\frac{\sqrt{1-m^2}}{m}(r_2 - r_1)$

79) The total length of the arc formed by the upper half of the cardioide $r = a(1+\cos\theta)$ using $r^2 + \left(\frac{dr}{d\theta}\right)^2 = 2a^2(1+\cos\theta)$ when θ varies from 0 to π is given by

a) 4π b) 2π c) $4a$ d) $2a$

80) The total arc length of the upper part of the curve $x = t^2,\ y = t - \frac{t^3}{3}$ is obtained by solving the integration

a) $\int_0^{2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$

b) $\int_0^{3} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$

c) $\int_0^{1} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$

d) $\int_0^{\sqrt{3}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$

81) The total arc length of the two consecutive cusps lies in the first quadrant of the curve $x = a\cos^3\theta,\ y = a\sin^3\theta$ is obtained by solving the integration

a) $\int_0^{\pi/4} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \cdot d\theta$

b) $\int_0^{\pi/3} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \cdot d\theta$

c) $\int_0^{\pi/2} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \cdot d\theta$

d) $\int_0^{\pi} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \cdot d\theta$

82) The total arc length of the upper part of the curve $x = t^2,\ y = t - \frac{t^3}{3}$ between $t = 0$ to $t = \sqrt{3}$ with $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (1+t^2)^2$ is given by

a) $2\sqrt{3}$ b) $\sqrt{3}$ c) $\frac{2}{\sqrt{3}}$ d) $4\sqrt{3}$

83) The total arc length of the two consecutive cusps lies in the first quadrant of the curve $x = a\cos^3\theta,\ y = a\sin^3\theta$ between $\theta = 0$ to $\theta = \frac{\pi}{2}$ with $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 9a^2\sin^2\theta\cos^2\theta$ is given by

a) $\frac{3a}{4}$ b) $3a$ c) $\frac{3a}{2}$ d) $\frac{2a}{3}$

84) The total arc length of the two cusps between $\theta = -\pi$ to $\theta = \pi$ of the curve
$x = a(\theta + \sin\theta),\ y = a(1-\cos\theta)$
with $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 4a^2\cos^2\left(\frac{\theta}{2}\right)$ is

a) $4a$ b) $8a$ c) $2a$ d) a

85) The total arc length of the two cusps between $\theta = 0$ to $\theta = \frac{\pi}{2}$ of the curve $x = e^{\theta}\cos\theta$, and $y = e^{\theta}\sin\theta$ with $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 2e^{2\theta}$ is

a) $\sqrt{2}\left(1 - e^{\pi/2}\right)$ b) $\sqrt{2}\left(e^{\pi} - 1\right)$
c) $\sqrt{2}\left(e^{\pi/2} + 1\right)$ d) $\sqrt{2}\left(e^{\pi/2} - 1\right)$

Chapter 03) Fourier Series

1	a		41	d		81	b		121	c
2	d		42	d		82	d		122	b
3	b		43	b		83	a		123	d
4	a		44	c		84	b		124	d
5	c		45	d		85	d		125	a
6	d		46	b		86	c		126	b
7	a		47	c		87	a		127	a
8	d		48	a		88	b		128	b
9	b		49	b		89	a		129	b
10	d		50	a		90	b'		130	c
11	d		51	c		91	c		131	a
12	b		52	b		92	a		132	b
13	a		53	c		93	c		133	d
14	d		54	d		94	d		134	d
15	b		55	d		95	a		135	a
16	b		56	c		96	b		136	c
17	a		57	a		97	c		137	d
18	d		58	b		98	d		138	a
19	a		59	d		99	b		139	b
20	b		60	a		100	d		140	a
21	a					101	d		141	d
22	c		62	d		102	b		142	c
23	d		63	c		103	c		143	b
24	a		64	d		104	a		144	c
25	d		65	b		105	d		145	a
26	a		66	d		106	b		146	d
27	d		67	b		107	d		147	c
28	c		68	c		108	d		148	a
29	b		69	a		109	d		149	c
30	c		70	c		110	a		150	b
31	a		71	c		111	d		151	d
32	d		72	c		112	c		152	b
33	a		73	d		113	c		153	a
34	c		74	b		114	a		154	c
35	a		75	d		115	b		155	d
36	c		76	c		116	a		156	d
37	a		77	b		117	c		157	a
38	c		78	c		118	b		158	c
39	c		79	b		119	a		159	b
40	b		80	d		120	b			

Chapter 04) Reduction Formulae & Beta, Gamma Function

1	c	26	d	51	a	76	d	101	c
2	b	27	b	52	c	77	a	102	b
3	c	28	c	53	b	78	c	103	d
4	d	29	a	54	d	79	d	104	c
5	d	30	b	55	b	80	c	105	b
6	c	31	a	56	d	81	b	106	a
7	a	32	c	57	a	82	c	107	b
8	c	33	b	58	d	83	a	108	a
9	b	34	c	59	a	84	d	109	c
10	a	35	d	60	c	85	b	110	d
11	c	36	d	61	d	86	c	111	b
12	b	37	c	62	c	87	a	112	d
13	d	38	a	63	b	88	c	113	c
14	a	39	d	64	a	89	b	114	c
15	a	40	b	65	c	90	d	115	a
16	c	41	d	66	d	91	b	116	c
17	c	42	c	67	b	92	a	117	b
18	c	43	a	68	a	93	c	118	b
19	b	44	b	69	b	94	b	119	d
20	d	45	d	70	c	95	d	120	c
21	c	46	d	71	d	96	d	121	d
22	d	47	b	72	a	97	a	122	b
23	b	48	d	73	b	98	c	123	c
24	d	49	b	74	c	99	d	124	a
25	c	50	c	75	a	100	c	125	b

Chapter 05) Differentiation Under Integral Sign & Error Function

1	a	14	d	27	a	40	b	53	c
2	c	15	b	28	c	41	c	54	c
3	b	16	d	29	d	42	a	55	b
4	c	17	b	30	a	43	d	56	d
5	d	18	c	31	c	44	a	57	b
6	d	19	a	32	b	45	d	58	a
7	c	20	c	33	d	46	c	59	c
8	a	21	b	34	b	47	b	60	d
9	b	22	d	35	a	48	c	61	d
10	d	23	b	36	b	49	a	62	a
11	a	24	d	37	c	50	d		
12	d	25	a	38	d	51	c		
13	c	26	d	39	a	52	a		

Chapter 06) Curve Tracing & Rectification of Curves

1	a		18	c		35	d		52	b		69	c
2	b		19	a		36	c		53	d		70	a
3	a		20	c		37	a		54	d		71	c
4	c		21	b		38	c		55	a		72	b
5	b		22	d		39	b		56	b		73	d
6	c		23	c		40	d		57	b		74	a
7	d		24	d		41	a		58	a		75	d
8	c		25	b		42	d		59	d		76	b
9	b		26	c		43	c		60	a		77	c
10	d		27	a		44	d		61	d		78	a
11	c		28	b		45	d		62	a		79	c
12	d		29	a		46	b		63	c		80	d
13	c		30	c		47	d		64	b		81	c
14	b		31	b		48	b		65	a		82	a
15	d		32	c		49	c		66	d		83	c
16	a		33	d		50	a		67	a		84	b
17	a		34	d		51	a		68	b		85	d

Engineering Mathematics-2 2015 course Unit-6 Double Integration

Q.1) The value of $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \sin x \, dx \, dy$ is

A)0

B)1

C)$\frac{\pi}{2}$

D)π

Ans-C

Q.2) The value of $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \cos x \, dx \, dy$ is

A)$\frac{\pi}{2}$

B)1

C)0

D)π

Ans-A

Q.3) The value of $\int_0^1 \int_0^y x \, dx \, dy$ is

A)$\frac{1}{2}$

B)$\frac{1}{3}$

C)$\frac{1}{8}$

D)$\frac{1}{6}$

Ans-D

Q.4) The value of $\int_0^1 \int_0^x e^y \, dx \, dy$ is

A)e^2

B)$e - 2$

C)e

D)$\frac{1}{2}(e^2 - 1)$

Ans: B

Q.5)Using polar transformation $x = r\cos\theta, y = r\sin\theta$ the Cartesian double integral $\iint_R f(x,y)dxdy$ becomes

A) $\iint_R f(r,\theta)drd\theta$

B) $\iint_R f(r,\theta) r dr d\theta$

C) $\iint_R f(r,\theta) r^2 dr d\theta$

D) $\iint_R f(r,\theta) \theta dr d\theta$

Ans:B

Q.6) On changing the order of integration of $\int_0^1 \int_0^x f(x.y)dxdy$ becomes

A) $\int_0^1 \int_0^1 f(x.y)dxdy$

B) $\int_0^1 \int_0^y f(x.y)dxdy$

C) $\int_0^1 \int_1^y f(x.y)dxdy$

D) $\int_0^1 \int_y^1 f(x.y)dxdy$

Ans: D

Q.7) On changing the order of integration of $\int_0^1 \int_{x^2}^1 f(x.y)dxdy$ becomes

A) $\int_0^1 \int_0^{\sqrt{y}} f(x.y)dxdy$

B) $\int_0^1 \int_0^{-\sqrt{y}} f(x.y)dxdy$

C) $\int_0^1 \int_0^{\sqrt{x}} f(x.y)dxdy$

D) $\int_0^1 \int_0^{-\sqrt{x}} f(x.y)dxdy$

Ans: A

Q.8) on transforming into the polar co-ordinates the double integration $\int_0^1 \int_0^{\sqrt{1-x^2}} f(x.y)dxdy$ becomes

A) $\int_0^{\pi} \left\{\int_0^1 f(r,\theta) r dr\right\} d\theta$

B) $\int_0^{\frac{\pi}{2}} \left\{\int_0^1 f(r,\theta) r d\theta\right\} dr$

C) $\int_0^{\frac{\pi}{2}} \left\{\int_0^1 f(r,\theta) r dr\right\} d\theta$

D) $\int_0^{2\pi} \left\{\int_0^1 f(r,\theta) r dr\right\} d\theta$

Ans: C

Q.9) By considering the strip parallel to Y-axis the integration $\iint_R f(x,y)dxdy$ over the area of triangle whose vertices are $(0,1), (1,1)$ and $(1,2)$ becomes

A) $\int_0^1 \int_1^{x-1} f(x,y)dxdy$

B)$\int_0^1 \int_1^{x+1} f(x,y)dxdy$

C) $\int_0^1 \int_0^{x+1} f(x,y)dxdy$

D) $\int_0^1 \int_1^{x-1} f(x,y)dxdy$

Ans: B

Q.10) By considering the strip parallel to X-axis the integration $\iint_R f(x,y)dxdy$ where R is the region bounded by $y = x^2$ and $y^2 = -x$ becomes

A) $\int_0^1 \int_{\sqrt{y}}^{-y^2} (x,y)dxdy$

B) $\int_0^1 \int_{-\sqrt{y}}^{y^2} (x,y)dxdy$

C)$\int_0^1 \int_{-\sqrt{y}}^{-y^2} (x,y)dxdy$

D) $\int_0^1 \int_{\sqrt{y}}^{y^2} (x,y)dxdy$

Ans: C

Q.11) on transforming into the polar co-ordinates the double integration $\int_0^a \int_0^{\sqrt{a^2-y^2}} e^{-x^2-y^2} dxdy$ becomes

A) $\int_0^{\frac{\pi}{2}} \int_0^a e^{-r^2} drd\theta$

B)$\int_0^{\frac{\pi}{2}} \int_0^a e^{-r^2} r^2 drd\theta$

C) $\int_0^{\frac{\pi}{2}} \int_0^a e^{-r^2} rdrd\theta$

D) $\int_0^{\frac{\pi}{2}} \int_0^a e^{-r} rdrd\theta$

Ans: B

Q.12)To find the area of upper half of a cardioid $r = a(1 + \cos\theta)$ the double integral becomes

A)$\int_0^{\pi} \int_0^{a(1+\cos\theta)} rdrd\theta$

B) $\int_0^{\pi} \int_0^{a(1+\cos\theta)} drd\theta$

C) $\int_0^{2\pi} \int_0^{a(1+\cos\theta)} r^2 drd\theta$

D) $\int_0^{2\pi} \int_0^{a(1+\cos\theta)} rdrd\theta$

Ans: A

Q.13) To find the area of a complete circle $x^2 + y^2 = a^2$ the double integral becomes

A) $2\int_0^{\frac{\pi}{2}} \int_0^a rdrd\theta$

B) $4\int_0^{\frac{\pi}{2}}\int_0^a drd\theta$

C) $2\int_0^{\frac{\pi}{2}}\int_0^a r^2drd\theta$

D)$4\int_0^{\frac{\pi}{2}}\int_0^a rdrd\theta$

Ans: D

• धड्याचे नाव :

Curve Tracing

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● NOTE:

① Curve Tracing:

1) A mathematical function can be better described or understand by its plot or graphical representation.

2) Method to Trace curve depends upon the representation of its equation in the cartesian or parametric form.

3) We are well known about the curve like parabola, hyperbola, Ellipse, cone, circle and straight line etc.

② NOTE:

अ): Linear equation always represent a straight line or plane.

e.g: $ax + by + c = 0$ or $ax + by + cz + d = 0$

Linear eq^n having power 1 always.

आ): Curve has always power 2 or greater than 2.

e.g: circle $x^2 + y^2 = r^2$,

parabola $y^2 = 4ax$

③ Types of CURVES:

1) Cartesian Curve
$f(x, y) = 0$
a) Cartesian Curve Explicit relation
b) Cartesian curve Implicit relation

2) Polar Curve
$f(r, \theta)$
Special Case:
Rose Curve
$r = a\cos n\theta$
or
$r = a\sin n\theta$

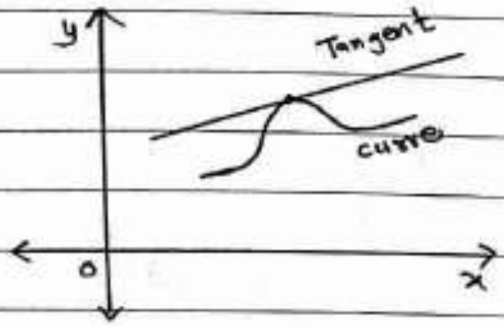
3) Parametric Curve
$x = f(t)$, $y = g(t)$
or
$x = f(\theta)$, $y = g(\theta)$

भाग-3e

(4) Some Important Basic Concepts:

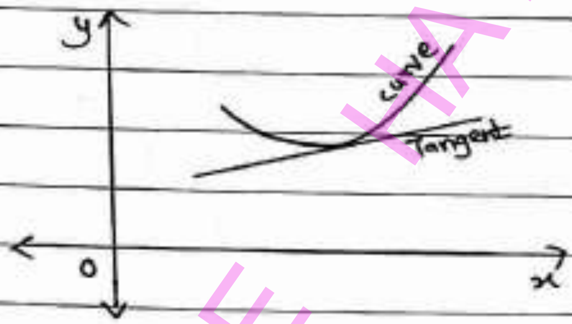
1) Convex Upward:

Portion of curve below the tangent, then curve known as convex upward.



2) Convex Downward:

Portion of curve above the tangent, then curve known as convex downward.



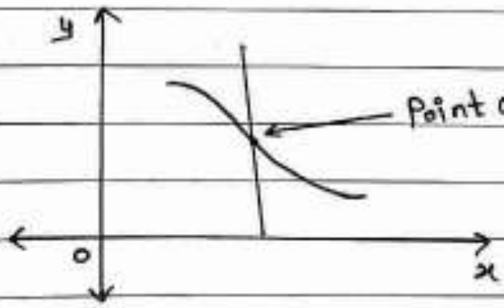
3) Singular Point:

An unusual point on a curve is called a singular point.

viz Point of Inflexion
Double Point
Multiple Point
Node
Conjugate Point

i) Point of Inflexion:

The point that seperates the convex part of a continous curve from the concave part called as point of inflexion.

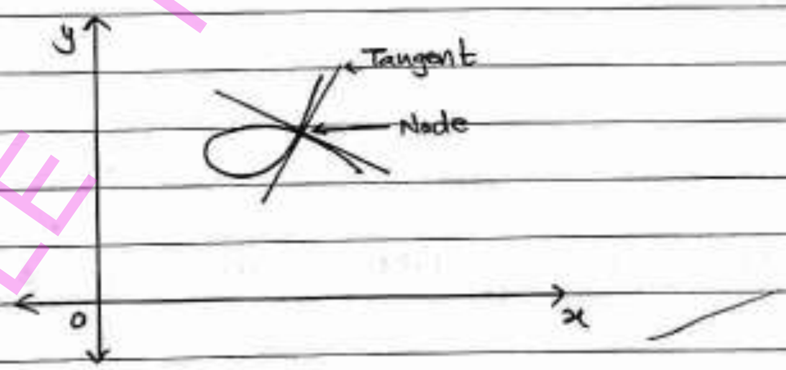


ii) Double Point:

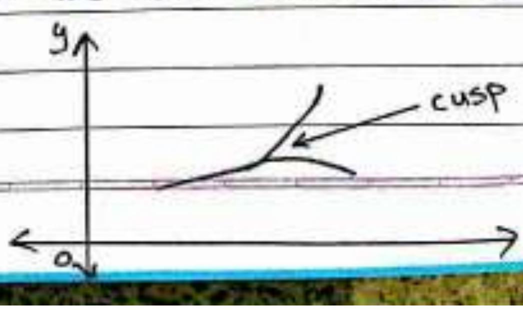
A point on a curve is called double point of two branches of the curve pass through it.

a): Node: A double point is called as node if the branches of curve passing through it & tangents at the common point of intersection are real & different.

e.g: $y = +x$, $y = -x$



b): Cups: A double point is called as cusp if the tangents at it to the two branches of the curve real & same.



iii) Multiple Point:

A point through which omore than one branches of curve passes is called as Multiple points.

iv) Conjugate Point:

A point outside of the curve is called conjugate point or Isolated point.

• HOW TO DRAW CURVES •

(अ):

* Type 01: Cartesian Curve:

* Steps to draw Cartesian Curve:

Step 01: Symmetry

Step 02: Point of Intersection

Step 03: Origine

Step 04: Tangents at Origin

Step 05: Intersection with co-ordinate axes

Step 06: Asymptotes

Step 07: Region of absence of curve

Step 08: Draw the curve from above given Information.

• **Detail Explaination About 8 steps:**

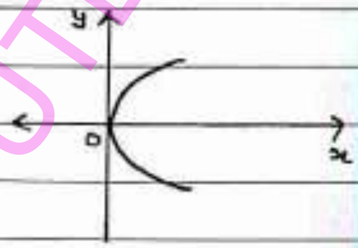
Step 01: Symmetry :-

i) x-axis :

Power of y even everywhere.

Curve symmetric to x-axis.

e.g : $y^2 = 4ax$

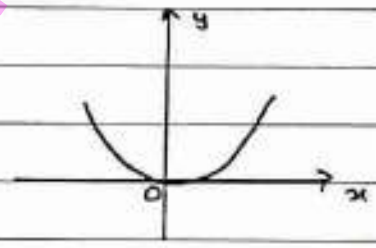


ii) y-axis :

Power of x even everywhere.

Curve symmetric to y-axis.

eg : $x^2 = 4ay$

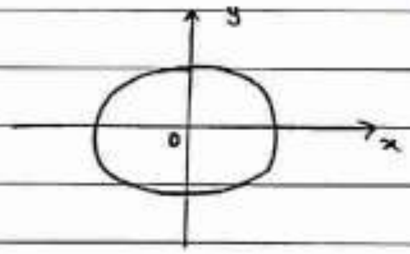


iii) x & y-axis :

Power of x & y both even.

Curve symmetric to x & y axis.

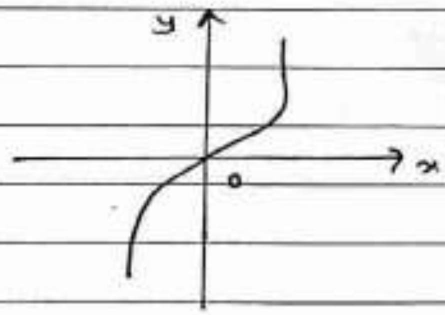
e.g : $x^2 + y^2 = r^2$



iv) Opposite quadrant :

x & y remain unchanged when we replace $x \to -x$ and $y \to -y$ simultaneously.

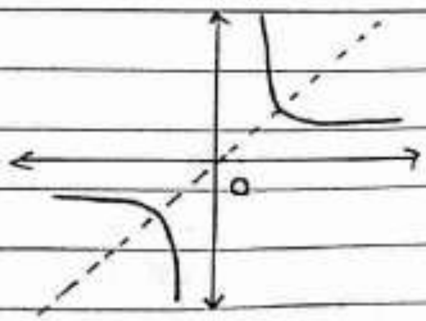
e.g : $y = x^3$



v) line $y = x$:

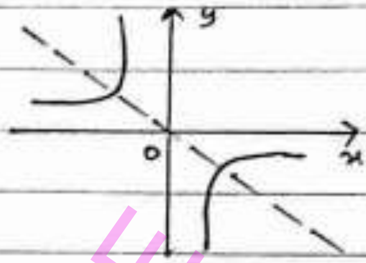
Equation of curve remain unchanged when we replace $x \to y$ & $y \to x$ simultaneously.

e.g : $xy = c^2$



vi) line $y=-x$:

Equation of curve remain unchanged when we replace $x \to -y$ & $y \to -x$ simultaneously.



Step 02: Point of Intersection:

There are two important point of intersection in curve which help us to draw the curve.

Step 03: Origine:

If the equation of the curve does not contains any absolute constant, then it passes through origine.

- Constants Types ⇒

  i) Absolute Constant: e.g: a, b, c, d, ..., -3, -2, -1, 0, 1, 2, 3, ..., 4/5, ...

  ii) Arbitary Constant: e.g: $2x^2$, ax^3

Step 04: Tangent at Origine:

Tangent at origine can be obtained by equating to zero, the lowest degree term taken together in the equation of the curve.

Step 05: Intersection with co-ordinate axes:

Equation of the curve : $y=f(x)$ or $x=f(y)$

i) To find intersection with x-axis : put $y=0$

ii) To find intersection with y-axis : put $x=0$

Step 06: Asymptotes:

Asymptotes are the tangents to the curve at infinity.

1) Asymptotes parellel to x-axis equate to zero, the coefficient of highest degree terms in x.

2) Asymptotes parellel to y-axis equate to zero, the coefficient of highest degree terms in y.

3) Oblique Asymptotes:

Asymptotes which are not parellel to co-ordinate axes are called as oblique asymptotes.

Step 07: Region of Absence of Curve:

1) Express the equation in the explicit form $y = f(x)$ & check how y varies as x varies continously.

2) $y = f(x)$; if y become imaginary for same value of $x > a$ then no part of curve exist beyond $x = a$.

3) $x = f(y)$, if x become imaginary for same value of $y > a$ then no part of curve exist beyond $y = a$.

Step 08: Draw the curve:

Above seven point information conclusion its our curves.

आ:

* Type 02: Polar Curve $f(r, \theta)$:

* Steps to draw Polar Curve:

Step 01: Symmetry

Step 02: Pole & Line

Step 03: Tangent at Pole

Step 04: Draw table such that:

θ	0	....
r	.....	....

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पान-४५

Step 05: Angle between Radius vector & Tangent (ϕ)
Step 06: Assymptotes : Find if it is possible
Step 07: Region of absence of curve
Step 08: Draw the curve using above Information.

• Detail Explaination About 8 steps:

Step 01: Symmetry:

1) Equation of curve remains unchanged when we replace $\theta \to -\theta$, then curve is symmetric to initial line ($\theta = 0$).

2) Equation of curve changed when we replace $\theta \to -\theta$, then curve is symmetric to perpendicular line ($\theta = \pi/2$).

3) Equation of curve remain unchanged by changing $r \to -r$

$\therefore$ Curve symmetric to pole.

e.g: $r^2 = a^2 \cos 2\theta$

4) Equation of the curve remain unchanged when we replace $r \to -r$ and $\theta \to -\theta$ then curve symmetric to pole and perpendicular to initial line.

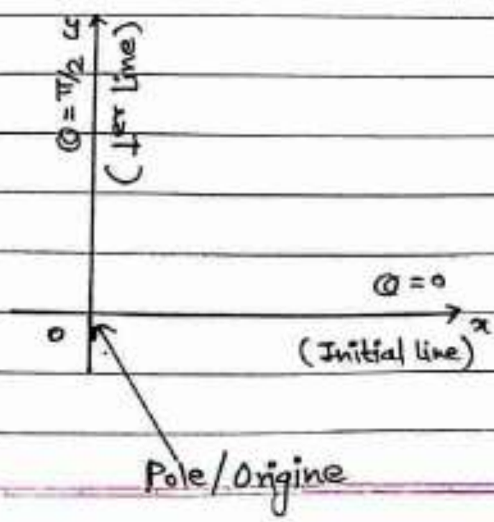
Step 02: Pole & line:

1) The fixed point 'O' is called as Pole / Origine.

2) $\theta = 0$ is initial line.

$\theta = \pi/2$ is perpendicular line.

3) The Pole will lie on the curve if for some values of θ, $r = 0$.



Step 03: Tangent at Pole:

Put $r=0$ then the value of θ gives the tangent at the pole.

Step 04: r and θ Table:

Value of r different for value of θ which is useful to plotting the polar curve.

Table Format:

θ	......	......	....				
r	...	...	......				

Step 05: Angle between Radius (r) and Tangent (ϕ):

$$\tan(\phi) = r \cdot \frac{d\theta}{dr}$$

Find $\phi = 0$ or ∞

By putting value of r and θ appropriate.

Step 06: Assymptotes:

If possible find it.

Assymptotes are tangents at Infinity.

Step 07: Region of Absence of Curve:

1) Values of θ, r^2 becomes negative then curve doesn't exist between these values.

2) In polar equation, only periodic function $\sin\theta$, $\cos\theta$ occures, therefore the value of θ is $0 \rightarrow 2\pi$.

Step 08: Draw the curve:

Using above all information, draw the curve.

* Type 03: Rose Curve: [$r = a\sin n\theta$, $r = a\cdot\cos n\theta$]

* Steps to draw Rose Curve:

Step 01: Symmetry

Step 02: Values of θ which gives $r=0$

Step 03: Put $r=0$, find $\theta = ?$

Step 04: Maximum value of r i.e. $r=a$

Step 05: Periodic function and Period $= 2\pi$

Step 06: No. of loops for $r = a\cdot\sin n\theta$, $r = a\cdot\cos n\theta$

i) n equal loops if $n =$ odd

ii) $2n$ equal loops if $n =$ even

Step 07: First loop:

$r = a\cdot\cos n\theta$ — First Loop $\theta = 0$

$r = a\cdot\sin n\theta$ — First Loop $\theta = \frac{\pi/2}{n}$

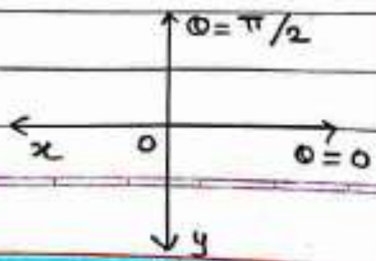
Step 08: Draw Table:

θ	0	$\pi/2n$	......	2π
r				

$r=a$ for $\theta=0$ — $\cos n\theta$ curve

$r=a$ for $\theta=\pi/2n$ — $\sin n\theta$ curve

Step 09: Using all information, draw the curve.



Step 10: Name the curve as:

n leaved Rose.

पान-४८

(इ):

* Type: 04: Parametric Curve: [$x=f(t)$, $y=g(t)$]

* Steps to draw Parametric Curve:

Step 01: Limitation of the curve

Step 02: Symmetry

Step 03: Origin

Step 04: Region of Absence of curve

Table t, x, y, $\frac{dx}{dt}$ and $\frac{dy}{dt}$

Step 05: Draw the curve

Step 06: Cycloid

• Detail Explaination About 6 Steps:

Step 01: Limitation of the curves:

Parametric curve $x=f(t)$, $y=g(t)$.

Find the greatest and least values of x and y for a proper values of t.

Step 02: Symmetry:

1) $f(t)$ = Even function of t

$g(t)$ = odd function of t

$\therefore$ Curve Symmetric about x-axis.

पान-re

2) $f(t)$ = Odd function of t

$g(t)$ = Even function of t

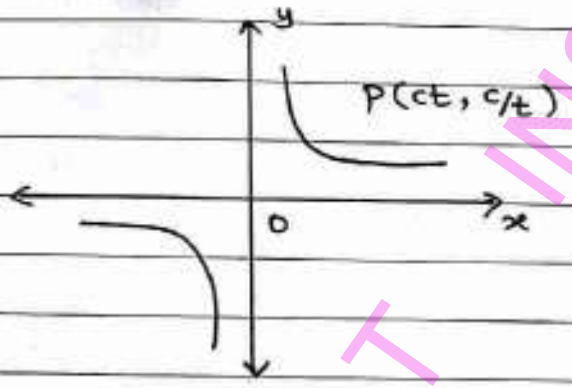
∴ Curve symmetric About y-axis.

3) $f(t)$ = Odd function of t

$g(t)$ = Odd function of t

∴ Curve symmetric in opposite quadrants.

e.g.: $y = \frac{c}{t}$, $x = ct$



Step 03: Origine:

If putting $x = 0$, obtain $y = 0$ for some value of t, then curve passes through the origine.

Step 04: Region of Absence of curve:

1) Find those regions where curve does not exist.

2) Make a table of values t, x, y.

Step 05: Draw the curve:

By using above information draw the curve.

Step 06: Cycloid:

1) When a circle rolls in a plane along given straight line, the locus traced out by a fixed point on the circumference of rolling circle is called as Cycloid.

पान-५०

2> Stretching of cycloid from it's equation depends on the values of x, y, t :

3) Table Type :

t	-2π	$-\pi$	0	π	2π
$x = f(t)$					
$y = g(t)$					

* Note:

Special curve "Spirals" : Equation of the curve $r = a\theta^m$.

SET: 03

पान-५९

* Type: Curve Tracing:

① If the power of y in the cartesian equation are even everywhere then the curve is symmetrical about

A) x-axis B) y-axis c) both x & y D) None

Sol^n ⇒ $\boxed{\text{x-axis}}$

....(बेसिक NOTE पढे।)

....(पान क्र. ३८ - ५० तक पढे।)

② If the power of x in the cartesian equation are even everywhere then the curve is symmetrical about

Sol^n ⇒ $\boxed{\text{y-axis}}$

....(शुरवाती बेसिक NOTE पढे।)

....(पान क्र- ३८ से ५० पढे।)

③ If the power of x and y both in the cartesian equation are even everywhere then the curve is symmetrical about

Sol^n ⇒ $\boxed{\text{Both x \& y axis}}$

....(NOTE: पान क्र. ३८ - ५० पढे।)

④ On replacing x and y by -x and -y resp. if the cartesian equation remains unchanged then the curve is symmetrical about

Sol^n ⇒ $\boxed{\text{Opposite Quadrant}}$

....(NOTE: पान क्र. ३८-५० पढे।)

पान-५२

(5) If x and y are interchanged and cartesian equation remains unchanged then the curve is symmetrical about

Sol^n ⇒ $\boxed{\text{line } y = x}$

....(NOTE: पान क्र. ३८-५० पढ़े।)

(6) If x is changed to $-y$ and y to $-x$ and cartesian equation remains unchanged then the curve is symmetrical about

Sol^n ⇒ $\boxed{\text{line } y = -x}$

....(NOTE: पान क्र. ३८-५० पढ़े।)

(7) If the curve passes through origin then tangents at origine to the cartesian curve can be obtained by equating to zero

Sol^n ⇒ Lowest Degree Term In the Equation.

....(NOTE: पान क्र. ३८-५० पढ़े।)

(8) A double point is called node if the tangents to the curve at the double points are

Sol^n ⇒ Real and Distinct

....(NOTE: पान क्र. ३८-५० पढ़े।)

(9) A double point is called cusp if the tangents to the curve at the double point are

Sol^n ⇒ Real and Same

....(NOTE: पान क्र. ३८-५० पढ़े।)

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पान - ५२

(10) In the cartesian equation the points where $dy/dx = 0$, tangents to the curve at those points will be

Sol^n ⇒

If: $\frac{dy}{dx} = 0$, Then: Tangents $\|^{el}$ to x-axis $(\theta = 0°)$

....(Std Result)

(11) In the cartesian equation the points where $dy/dx = \infty$, tangents to the curve at those points will be

Sol^n ⇒

If: $\frac{dy}{dx} = \infty$, Then: Tangents $\|^{el}$ to y-axis $(\theta = \pi/2^c)$

....(Std Result)

(12) The asymptotes to the cartesian curve parellel to x-axis if exists is obtained by equating to zero

Sol^n ⇒

Coefficient of Highest Degree Terms in x

....(NOTE: पान क्र ३८-५० पहे।)

(13) The asymptotes to the cartesian curve parellel to y-axis if obtained by equating to zero

Sol^n ⇒

Coefficient of Highest Degree Terms in y

.....(NOTE: पान क्र. ३८-५० पहे।)

(14) If the polar equation to the curve remains unchanged by changing θ to $-\theta$ then the curve is symmetrical about

Sol^n ⇒

Initial Line $\theta = 0$

....(NOTE: पान क्र. ३८-५० पहे।)

पान-५४

(15) If the polar equation to the curve remains unchanged by changing r to $-r$ then the curve is symmetrical about

Sol^n ⇒ $\boxed{\text{Pole}}$

....(NOTE: पान क्र. ३८ से ५० पढे।)

(16) If the polar equation to the curve remains unchanged by changing θ to $\pi-\theta$ then the curve is symmetrical about

Sol^n ⇒ $\boxed{\text{line passing through pole and Perpendicular to the initial line}}$

....(NOTE: पान क्र. ३८-५० पढे।)

(17) Pole will lie on the curve if for some value of θ

Sol^n ⇒ $\boxed{\text{'}r\text{' becomes zero, for value of } \theta}$

....(NOTE: पान क्र ३८-५० पढे।)

(18) The tangents to the polar curve at pole if exist can be obtained by putting in the polar

Sol^n ⇒ $\boxed{r=0}$

....(For finding Tangents, $r=0$ is standard substitution.)

(19) for the rose curve $r=a\cdot\cos n\theta$, $r=a\cdot\sin n\theta$ if n is odd then the curve consist of

Sol^n ⇒ $\boxed{\text{'}n\text{' equal loops if } n=\text{odd}}$

....(NOTE: पान क्र. ३८-५० पढे।)

पान-४४

(20) For the rose curve $r = a\cdot\cos n\theta$, $r = a\cdot\sin n\theta$ if n even then the curve consist of

Sol^n ⇒ $\boxed{\text{'2n' equal loops if } n = \text{even}}$

....(NOTE: पान क्र. ३८-४० पढे।)

(21) For the polar curve, angle ϕ between radius vector and tangent line is obtainined by the formula

Sol^n ⇒ $\boxed{\tan\phi = r\frac{d\theta}{dr}}$ ....(यह स्टँडर्ड है।)

(22) The cartesian parametric curve $x = f(t)$, $y = g(t)$ is symmetrical about x-axis if

Sol^n ⇒

f(t) is even and
g(t) is odd function

....(यह स्टँडर्ड है।)

(23) The cartesian parametric curve $x = f(t)$, $y = g(t)$ is symmetrical about y-axis if

Sol^n ⇒

f(t) is odd and
g(t) is even function

....(यह स्टँडर्ड है।)

(24) The curve represented by equation $x^{1/2} + y^{1/2} = a^{1/2}$ is symmetrical about

A) $y = -x$ B) x-axis

C) Both x & y D) $y = x$

Sol^n ⇒ Ans: D)

सि): $x^{1/2} + y^{1/2} = a^{1/2}$

Replace $x \to y$ and $y \to x$

$\therefore \; y^{1/2} + x^{1/2} = a^{1/2}$ (It's Unchanged)

$\therefore$ curve symmetrical to line $y = x$

(25) The curve represented by equation $x^2y^2 = x^2+1$ is symmetrical about

A) $y = -x$ B) x-axis

✓C) Both x & y D) $y = x$

Sol^n ⇒ Ans: C)

सि): $x^2y^2 = x^2+1$

Here, power of x and y is even (i.e. 2) everywhere in the equation.

$\therefore$ curve is symmetrical to Both x & y axis.

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पार्ट-४७

(26) The curve represented by the equation $r^2\theta = a^2$ is Symmetrical about

A) pole B) Initial line $\theta = 0$ C) line $\theta = \pi/2$ D) line $\theta = \pi/4$

Sol^n ⇒

१): $r^2\theta = a^2$

when we replace $r \to -r$, then equation of curved remains unchanged.

$\boxed{\therefore \text{ curve is symmetric about pole.}}$

२): $r^2\theta = a^2$

Put : $r \to -r$

$(-r)^2\theta = a^2$

$+r^2\theta = a^2$

Unchanged in Eq^n : $\therefore$ Ans: A)

(27) The curve represented by equation $r = 2a\cdot\sin\theta$ is symmetrical about

A) Pole B) Initial line $\theta = 0$ C) line $\theta = \pi/2$ D) line $\theta = \pi/4$

Sol^n ⇒ Ans: C)

१): $r = 2a\sin\theta$

Replace $\theta \to -\theta$

$r = 2a\sin(-\theta)$

$r = -2a\cdot\sin\theta$ $\{\because \sin(-\theta) = -\sin\theta\}$

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Sol): By replacing $\theta \to -\theta$

Eq^n show changes

$\therefore$ $\boxed{\text{curve symmetrical about line } \theta = \pi/2}$

(28) The curve represented by the equation $x = at^2$, $y = 2at$ is symmetrical about

A) y B) x C) both x & y D) Opposite quadrants

$Sol^n \Rightarrow$ Ans: B)

Sol):

$x = at^2$	$y = 2at$
Replace $t \to -t$	Replace $t \to -t$
$x = a(-t)^2$	$y = 2a(-t)$
$x = at^2$	$y = -2at$
Even Function	Odd Function

[$\because$ Eq^n of y show changes]

$\therefore$ $\boxed{\text{curve is symmetrical about x-axis.}}$

(29) The asymptote parallel to y-axis to the curve $xy^2 = a^2(a-x)$ is

A) $y=0$ B) $x=0$ C) $x=a$ D) $x=-a$

$Sol^n \Rightarrow$ Ans: B)

Sol): $xy^2 = a^2(a-x)$

$xy^2 = a^3 - a^2x$

पान-ye

Here, power of y even Everywhere in the equation

$\therefore$ curve is symmetrical to x-axis.

3): Compare highest degree coefficient with zero.

$$\therefore \boxed{x=0}$$

(30) The no of loops in the rose curve $r = a\cos 2\theta$ are

A) 4 B) 2 C) 3 D) 8

Sol^n ⇒ Ans: A)

1): $r = a\cdot\cos 2\theta$

compare to ⇒

$r = a\cdot\cos n\theta$ .....(std)

$\therefore$ $n=2$ (Even)

2): No. of Loops $= 2n$

$= 2(2)$

$= \boxed{4}$

(31) The no. of lops in the rose curve $r = a\cdot\sin 3\theta$ are

A) 6 B) 4 C) 3 D) 9

Sol^n ⇒ Ans: C)

1): $r = a\cdot\sin 3\theta$

compare to ⇒

$r = a\cdot\sin n\theta$ .....(std)

पान - ६० (60)

$\therefore n = 3$ (odd)

३) : No. of loops $= n$

$= \boxed{3}$

(32) The curve represented by equation $y^2(2a-x) = x^3$ is

Sol^n ⇒

१) : $y^2(2a-x) = x^3$

$2ay^2 - xy^2 = x^3$

२) : Observations:

1) Power of y (i.e. 2) is even everywhere in eq^n

$\therefore$ curve is symmetrical about x-axis

2) Curve doesn't have any absolute constant

$\therefore$ curve passing through origine.

३) : $\boxed{\text{Symmetrical about x-axis and Passing through origine.}}$

(33) The curve represented by the equation $x(x^2+y^2) = a^2(x^2-y^2)$ is

Sol^n ⇒

१) : $x(x^2+y^2) = a^2(x^2-y^2)$

$x^3 + xy^2 = a^2x^2 - a^2y^2$

२) : Observations:

1) Power of y even (i.e. 2) everywhere

$\therefore$ curve is symmetric about x-axis

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2) Curve dosn't have any Absolute Constant

$\therefore$ Curve passing through origine.

ग):

$\boxed{\text{Curve Symmetrical about x-axis and Passing thr origine.}}$

(34) The curve represented by equation $a^2x^2 = y^3(2a-y)$ is

Sol^n $\Rightarrow$

क):
$$a^2x^2 = y^3(2a-y)$$
$$a^2x^2 = 2ay^3 - y^4$$

ख): Observation:

1) Power of x is even (ie. 2) everywhere

$\therefore$ curve symmetrical about y-axis.

2) Eq^n of curve dosn't have any Absolute constant

$\therefore$ curve passing through origine.

ग):

$\boxed{\text{Curve symmetrical about y-axis and Passing through (0, 2a)}}$

घ):
$$a^2x^2 = 2ay^3 - y^4$$
Put: $x = 0$
$$0 = 2ay^3 - y^4$$
$$y^{\cancel{4}} = 2a\cancel{y^3}$$
$$y = 2a$$

$\therefore$ Points are: $(x, y) = (0, 2a)$.

पान-६२

(35) The equation of tangents to the curve at origin, if exist, represented by equation $y^2(2a-x) = x^3$ is

A) $y=0, y=0$ B) $x=0, x=2a$

C) $x=0, x=0$ ✓D) $y=x$

Sol^n ⇒ Ans: D)

१): $$y^2(2a-x) = x^3$$
$$2ay^2 - y^2x = x^3$$

२): Tangent At Origine ----(given)

Means compare lowest degree term with zero.

$$2ay^2 - y^2x = x^3$$

Degree: 2 2+1 3

✓

(lowest)

३): $$2ay^2 = 0$$
$$y^2 = 0/2a$$
$$y^2 = 0$$
$$y = 0, 0$$

$\therefore$ X-axis is Tangent at Origine.

$\therefore y = x$

(36) The equation of tangents to the curve at origin, if exist, represented by the equation $y(1+x^2) = x$ is

✓A) $y=x$ B) $x=0$ C) $y=0$ D) $x=1, x=-1$

Sol^n ⇒ Ans: A)

पार्ट-६३

5):

$$y(1+x^2)=x$$

$$y+yx^2=x$$

2): Tangent At Origine ---(given)

means compaire Lowest degree term with zero.

$y=0$ OR $x=0$

means

$$\boxed{y=x}$$

(37) The equation of tangents to the curve at origin, if exist, represented by equation $3ay^2=x(x-a)^2$ is

A) $x=a$ B) $x=0, y=0$ C) $x=0$ D) $y=0$

Sol^n → Ans: c)

1):

$$3ay^2=x(x-a)^2$$

$$3ay^2=x[(x)^2-2(a)(x)+(a)^2] \quad \{\because (a-b)^2=a^2-2ab+b^2\}$$

$$3ay^2=x^3-2ax^2+a^2x$$

2): Tangent at origine ....given)

means compares lowest degree terms with zero

$$a^2x=0$$

$$x=0/a^2$$

$$\boxed{x=0}$$

(38) The eq^n of asymptotes parellel to x-axis to the curve represented by equation $y(1+x^2)=x$ is

A) $x=1$, $x=-1$ B) $x=0$

C) $y=x$ D) $y=0$

पान-९४

Sol^n => Ans: D)

क): $y(1+x^2) = x$

$y + yx^2 = x$

(Highest degree power in x term)

ख): Asymptotes $\parallel^{el}$ to x-axis ....(given)

means coefficient of Highest Degree Power of x,

compaire to zero.

$\therefore$ $\boxed{y = 0}$

(39) The eq^n of asymptotes parellel to y-axis to the curve represented by the equation $y^2(4-x) = x(x-2)^2$ is

A) $x = 2$ ✓B) $x = 4$ C) $y = 0$ D) $x = 0$

Sol^n => Ans: B)

क): $y^2(4-x) = x(x-2)^2$

$4y^2 - y^2x = x[x^2 - 2x \times 2 + 2^2]$ $\{\because (a-b)^2 = a^2 - 2ab + b^2\}$

$4y^2 - y^2x = x^3 - 4x^2 + 4x$

(Highest degree power in y term)

ख): Asymptotes $\parallel^{el}$ to y-axis ....(given)

means coefficient of Highest Degree power of y,

compare to zero.

$\therefore$ $4 = 0$ OR $x = 0$

Here RHS are same

$\therefore$ we say LHS also same

$\therefore$ $4 = x$

$\boxed{x = 4}$

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पाठ - ६४

(41) The region of absence for the curve represented by the equation $x^2 = 4a^2(2a-y)/y$ is

A) $y<0$, $y>2a$ c) $y>0$, $y<2a$

B) $y>0$, $y>2a$ D) $y<0$, $y<2a$

Sol^n ⇒ Ans: A)

क):

$$x^2 = \frac{4a^2(2a-y)}{y}$$

$$x = \sqrt{\frac{4a^2(2a-y)}{y}}$$

ख): Region of Absence of curve

$y<0$ ⇒ term -ve in square root
i.e. $x \rightarrow$ is imaginary
∴ curve absent that region.

$y>2a$ ⇒ term -ve in square root
i.e. $x \rightarrow$ is imaginary
∴ curve absent that region.

$$\boxed{y<0\ ,\ y>2a}$$

(42) The region of absence for the curve represented by the equation $y^2(2a-x) = x^3$ is

A) $x>0$, $x<2a$ B) $x<0$, $x>2a$

c) $x<0$, $x<2a$ D) $x>0$, $x>2a$

$\boxed{\text{पान- ६६}}$

Sol^n ⇒ Ans: B)

5): $y^2(2a-x) = x^3$

$$y^2 = \frac{x^3}{2a-x}$$

$$y = \sqrt{\frac{x^3}{2a-x}}$$

2): Region of Absence of curve

$x < 0$ ⇒ y is -ve in square root

$x > 2a$ ⇒ y is -ve in square root

∴ curve absent of -ve terms in square roots.

$$\boxed{x < 0,\ x > 2a}$$

(43) The region of absence for the curve represented by the equation $xy^2 = a^2(a-x)$ is

A) $x > 0,\ x < a$ B) $x < 0,\ x < a$

✓C) $x < 0,\ x > a$ D) $x > 0,\ x > a$

Sol^n ⇒ Ans: C)

5): $xy^2 = a^2(a-x)$

$$y^2 = \frac{a^2(a-x)}{x}$$

$$y = \sqrt{\frac{a^2(a-x)}{x}}$$

पाना - ६७

ख): Region of Absence of curve.

$x < 0 \Rightarrow y$ is -ve in square root

$x > a \Rightarrow y$ is -ve in square root

$\therefore$ curve is absence in this region.

$$\boxed{x < 0,\ x > a}$$

(45) The region of absence for the curve represented by the equation $x^2 = \frac{a^2y^2}{a^2-y^2}$ is

A) $y < a, y > -a$ ✓B) $y > a, y < -a$

C) $y > a, y > -a$ D) $y < a, y < -a$

Sol$^n \Rightarrow$ Ans: B)

क):

$$x^2 = \frac{a^2y^2}{a^2-y^2}$$

$$x = \sqrt{\frac{a^2y^2}{a^2-y^2}}$$

ख): Region of Absence of curve

$y < -a \Rightarrow x$ is -ve in square root

$y > a \Rightarrow x$ is +ve in square root

$\therefore$

$$\boxed{y < -a,\ y > a}$$

पान-६८

(46) The curve represented by the equation $r = a(1+\cos\theta)$ is

✓A) symmetrical about initial line and passing through pole

B) symmetrical about initial line and not passing through pole

c) symmetrical about $\theta = \pi/2$

D) Symmetrical about $\theta = \pi/4$ and passing through pole

Sol^n ⇒ Ans: A)

१): $r = a[1+\cos\theta]$

$r = a + a\cdot\cos\theta$

२रा): 1) Replace $\theta \to -\theta$

$r = a + a\cdot\cos(-\theta)$ $\{\because \cos(-\theta) = \cos\theta\}$

$r = a + a\cdot\cos\theta$

Eq^n remains unchanged

$\therefore$ Curve symmetrical to initial line

३): Put $\theta = \pi$

$r = a + a\cos\pi$ $\{\because \cos\pi = -1\}$

$r = a + a(-1)$

$r = a - a$

$r = 0$

$\therefore$ curve passing through pole.

(47) The curve represented by eq^n $r^2 = a^2 \cdot \cos 2\theta$ is

A) Symmetrical about $\theta = \pi/2$

B) Symmetrical about $\theta = \pi/4$

✓c) Symmetrical about Initial line and Pole

D) Passing through pole

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पान- ६८

Sol$^n \Rightarrow$ Ans: c)

स): $r^2 = a^2 . \cos 2\theta$

Replace $\theta \to -\theta$

$r^2 = a^2 . \cos 2(-\theta)$ $\{\because \cos a(-\theta) = \cos a\theta\}$

$r^2 = a^2 . \cos 2\theta$

Eq^n remains unchanged.

$\therefore$ Curve Symmetrical to Initial line.

अ): Put: $\theta = \pi/4$

$r^2 = a^2 \cos[2 \times \pi/4]$

$r^2 = a^2 \cos(\pi/2)$ $\{\because \cos(\pi/2) = 0\}$

$r^2 = 0$

$r = 0, 0$

$\therefore$ curve passing through pole.

48 The curve represented by $r^2 = a^2 . \sin 2\theta$ is

A) Symmetrical to initial line & passes through pole

B) symmetrical to initial line & not passes through pole

C) symmetrical to $\theta = \pi/4$ & passing through pole

D) not passing through pole

Sol$^n \Rightarrow$ Ans: c)

स): $r^2 = a^2 . \sin 2\theta$ $\{\because$ compare to: $r = a . \sin n\theta$ ---(std) $\therefore n = 2\}$

Replace $\theta \to -\theta$

$r^2 = a^2 . \sin 2(-\theta)$ $\{\because \sin a(-\theta) = -\sin a\theta\}$

$r^2 = -a^2 . \sin 2\theta$

$\therefore$ Eq^n of curve changed

$\therefore$ Curve Symmetrical to perpendicular line i.e. $\theta = \pi/2n$

$\theta = \pi/2(2)$

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$\therefore \quad \theta = \frac{\pi}{4}$

ध): Put $\theta = 0$

$r^2 = a^2 \cdot \sin 2(0)$

$r^2 = a^2 \sin 0 \qquad \{\because \sin 0 = 0\}$

$r^2 = 0$

$r = 0, 0$

$\therefore$ curve passing through pole.

(49) The curve represented by $r = \frac{2a}{1+\cos\theta}$ is

A) Symmetrical to initial line & passing thr origine

B) Symmetrical to initial line & not passing thr origine

c) Symmetrical to $\theta = \pi/2$

D) Symmetrical to $\theta = \pi/4$ & passing thr pole

Sol^n ⇒ Ans: B)

S): $r = \frac{2a}{1+\cos\theta}$

Replace $\theta \to -\theta$

$r = \frac{2a}{1+\cos(-\theta)} \qquad \{\because \cos(-\theta) = \cos\theta\}$

$r = \frac{2a}{1+\cos\theta}$

Eq^n remains unchanged

$\therefore$ curve symmetrical to initial line

ध): For all value of θ, $r \neq 0$

$\therefore$ curve not passes thr the origine.

पान - ६९

(50) The tangents at pole to the polar curve $r = a \cdot \sin 3\theta$ are

✓A) $\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, \ldots$ B) $\theta = \frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \ldots$

C) $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \ldots$ D) $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots$

Sol^n ⇒ Ans: A)

स्प):

$$r = a \cdot \sin 3\theta$$

Put : $r = 0$

$$0 = a \cdot \sin 3\theta$$

$$a \cdot \sin 3\theta = 0$$

$$\sin 3\theta = 0/a$$

$$\sin 3\theta = 0$$

$$3\theta = \sin^{-1}(0)$$

$$3\theta = 0, \pi, 2\pi, 3\pi, \ldots$$

....devide by 3 to each term:

$$\boxed{\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \ldots}$$

(51) The tangents at pole to the polar curve $r = a \cdot \cos 2\theta$ are

A) $\theta = 0, \pi, 2\pi, 3\pi, \ldots$ B) $\theta = \frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \ldots$

✓C) $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots$ D) $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots$

Sol^n ⇒ Ans: c)

स्प):

$$r = a \cdot \cos 2\theta$$

Put : $r = 0$

पान-६२

$$a \cdot \cos 2\theta = 0$$

$$\cos 2\theta = 0/a$$

$$\cos 2\theta = 0$$

$$2\theta = \cos^{-1}(0)$$

$$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

...devide by 2 to each term.

$$\boxed{\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \ldots}$$

52) The curve represented by the equation $x = t^2$, $y = t - t^3/3$ is

A) symmetrical to y axis & pass thr origine

B) symmetrical to x axis & not pass thr origine

C) symmetrical to y axis & pass thr (3,0)

D) symmetrical to x axis & pass thr origine.

Soln => Ans: D)

1): $x = t^2$ | $y = t - t^3/3$

Replace $t \to -t$ | Replace $t \to -t$

$x = (-t)^2$ | $y = (-t) - (-t)^3/3$

$x = t^2$ | $y = -t + t^3/3$

[Eqn of y shows change]

$\therefore$ curve symmetrical to x-axis.

2): $x = t^2$ | $y = t - t^3/3$

Put: $t = 0$

$x = 0^2$ | $y = 0 - 0^3/3$

पान-63

$x = 0$ $y = 0$

By putting $t=0$, we got $x=y=0$

$\therefore$ curve passes through origine.

(53) The cure is $x = a[t+\sin t]$, $y = a[1+\cos t]$ is

A) Symmetrical to y-axis & not pass thr origin

B) Symmetrical to y axis & pass thr origin

c) symmetrical to x axis & pass thr origin

D) None

Sol^n ⇒ Ans: A)

i): $x = a[t+\sin t]$ $y = a[1+\cos t]$

Replace $t \to -t$

$x = a[-t+\sin(-t)]$ $y = a[1+\cos(-t)]$

$x = a[-t-\sin t]$ $y = a[1+\cos t]$

$\{\because \sin(-t) = -\sin t\}$ $\{\because \cos(-t) = \cos t\}$

[Eq^n of x - show change]

$\therefore$ curve is symmetrical to y-axis.

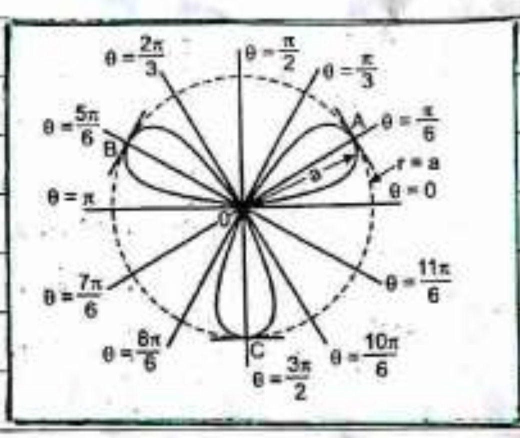
ii): For not any value of t,

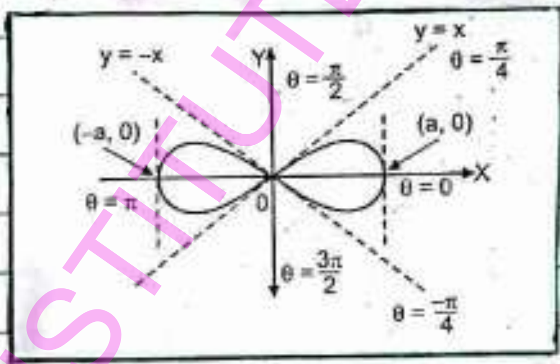
x and y zero simultenously

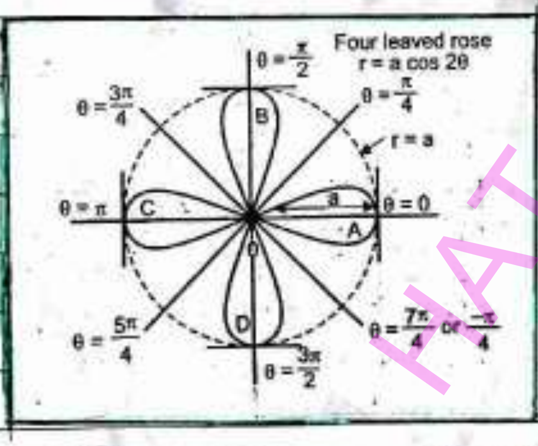
$\therefore$ curve does not passes through origine.

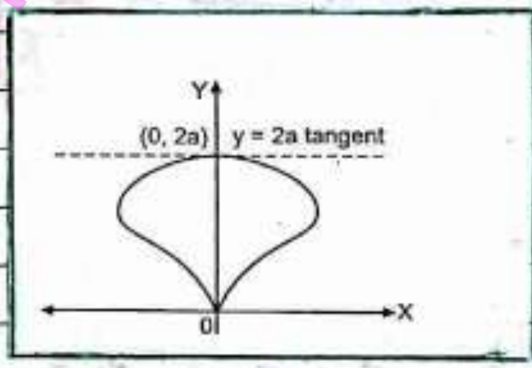
पान-७४

(54) The equation of the curve $r = a\cos 2\theta$ represents curve.

A) 

B) 

✓C) 

D) 

→ Sol^n : C) $r = a\cos 2\theta$

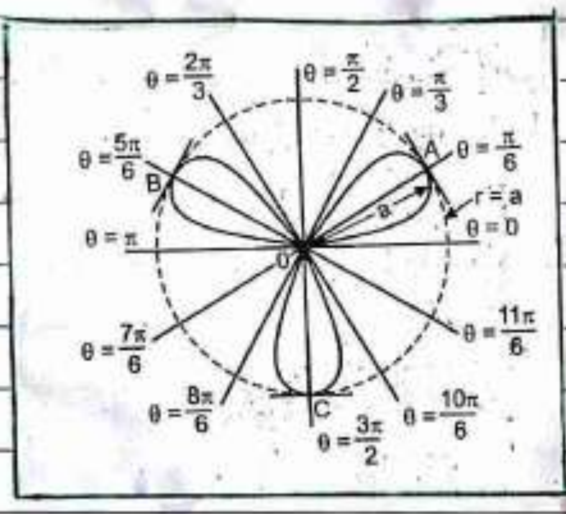
क) it is type of rose cueve,
∴ Eliminate othee options.

ख) $n = 2$ Even value,
∴ No. of loops $= 2n$
$= 2 \times 2 = 4$

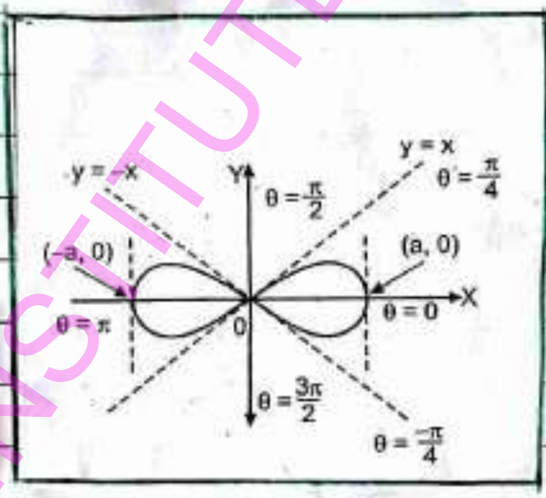
पाठ - ६४

(55) The equation of cure $r = a\sin 3\theta$ represents the curve.

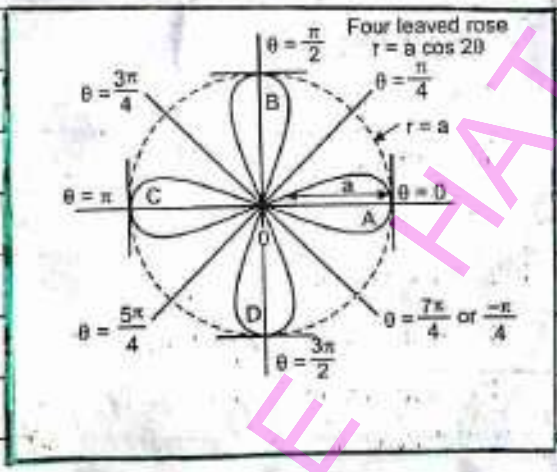
A)



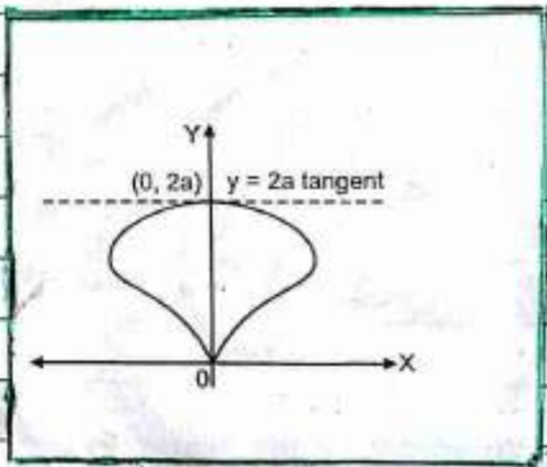
B)



C)



D)



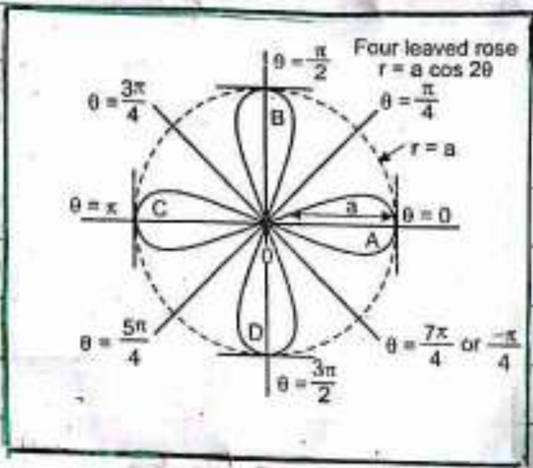
Soln : A)

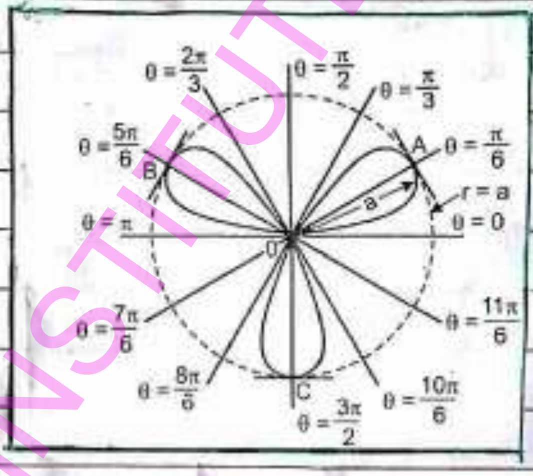
1) It is type of rose curve,
∴ Eliminate other options.

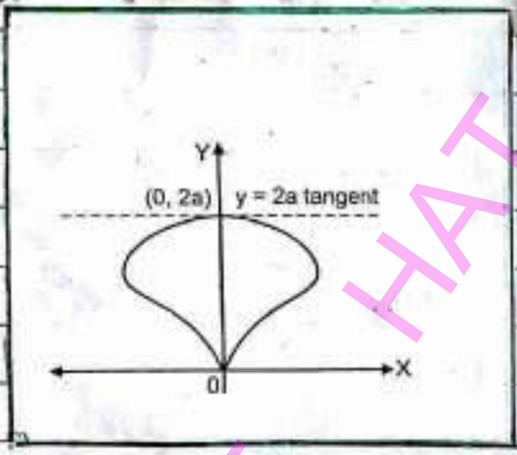
2) $n = 3$ odd value,
∴ No. of loops $= n$
$= 3$

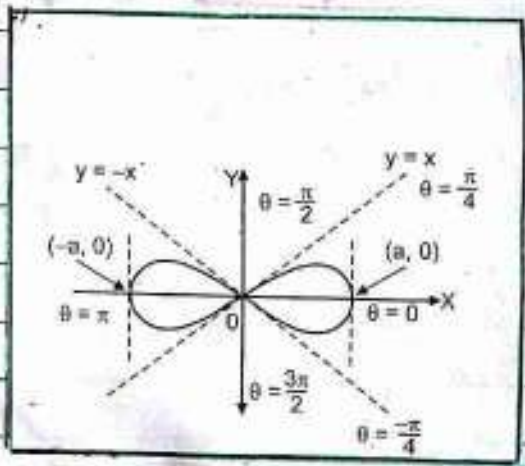
पान-७६

(56) The equation of curve $r^2 = a^2\cos 2\theta$ represents the curve

A) 

B) 

C) 

D) 

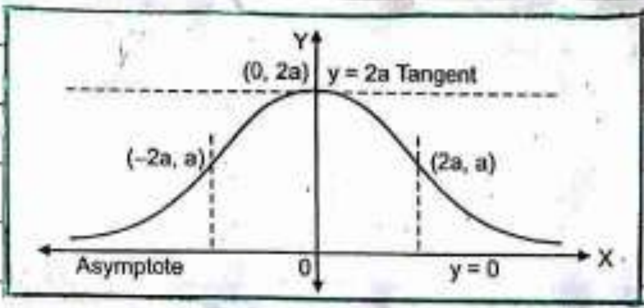
→ Sol^n : D)

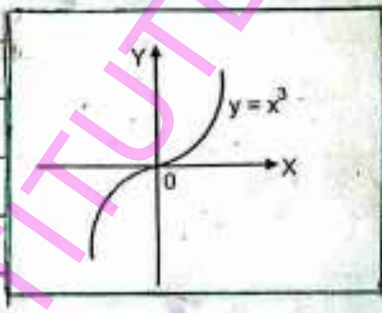
क) $r^2 = a^2\cos 2\theta$ is type of polar curve, $r = f(\theta)$ [Not Rose curve]
∴ Eliminate other options.

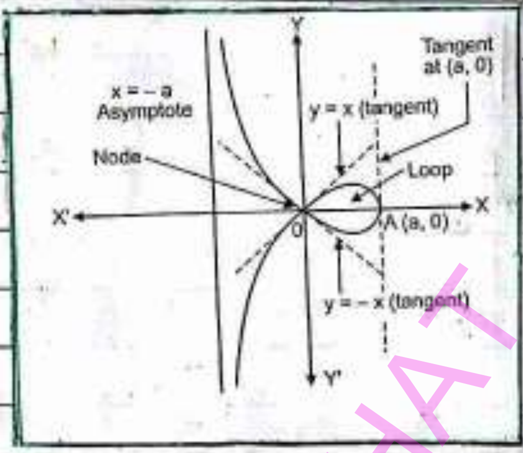
ख) when we replace $r \to -r$ & $\theta \to -\theta$
Equation of curve unaltered,
∴ it is symmetrical to initial line & passes through the origin.

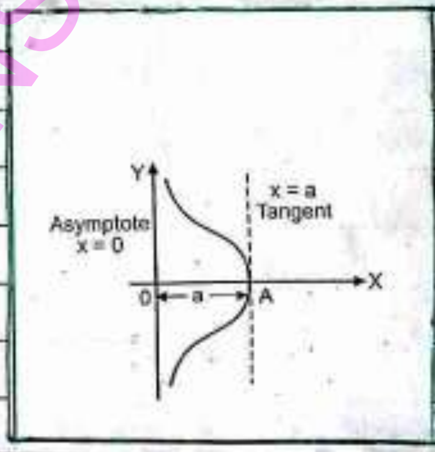
पान-66

(57) The equation of the cueve $xy^2 = a^2(a-x)$ represents the cueve.

A) 

B) 

C) 

D) 

Sol^n : D)

क) $xy^2 = a^2(a-x)$

$= a^3 - a^2x$

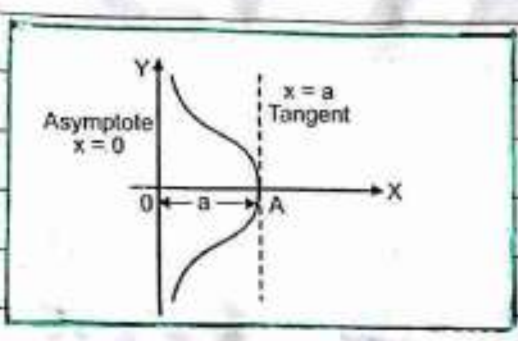
ख) Eq^n of cueve contains even poweɛ of y everywhere,
∴ cueve symmetrical to x-axis.
∴ Eliminate other options.

ग) Eq^n of cueve contains absolute const. i.e a^3
∴ cueve does not passes through origin.
Eliminate otheɛ remaining options.

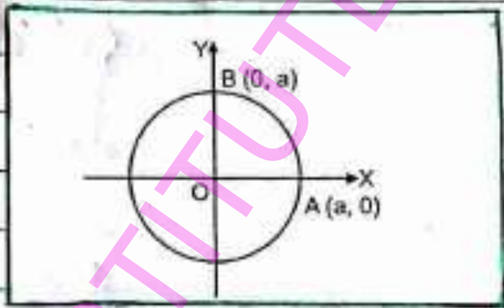
पान - ७८

(58) The equation of the curve $x(x^2+y^2)=a(x^2-y^2)$, $a>0$ represents the curve

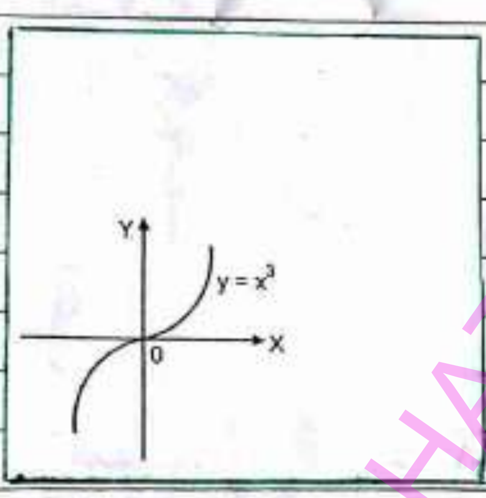
A)



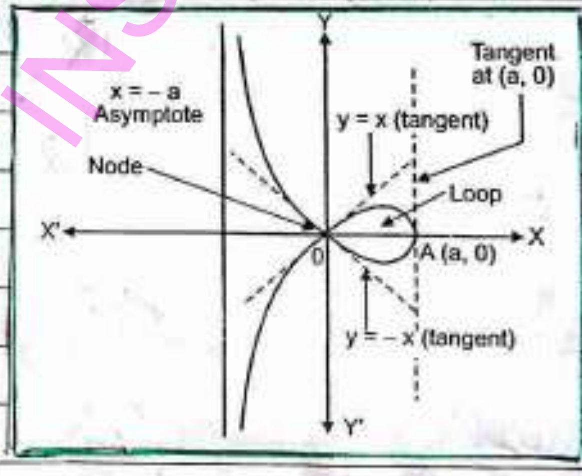
B)



C)



D)



→ Sol^n : D)

क) $$x(x^2+y^2) = a(x^2-y^2) \quad , \quad a>0$$
$$x^3 + xy^2 = ax^2 - ay^2$$

ख) Power of Y - even everywhere,
∴ curve symmetrical to x-axis.
∴ Eliminate other options.

ग) Eq^n of curve does not contains any absolute constant.
∴ it is passes through the origin.
Eliminate other remaining options.

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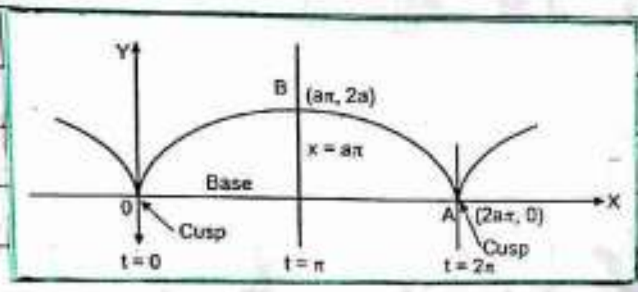
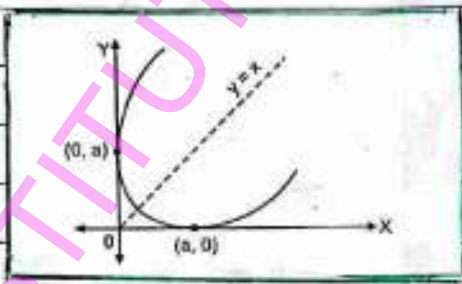
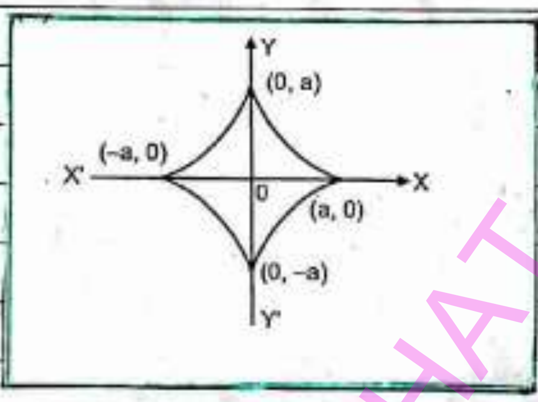
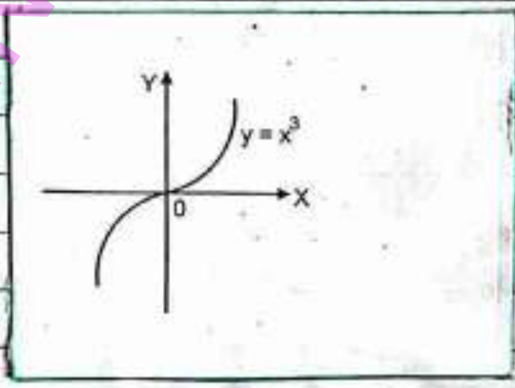
पानी-6e

(59) The equation of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ represents the curve

A) 

B) 

C) 

D) 

→ Sol^n : C)

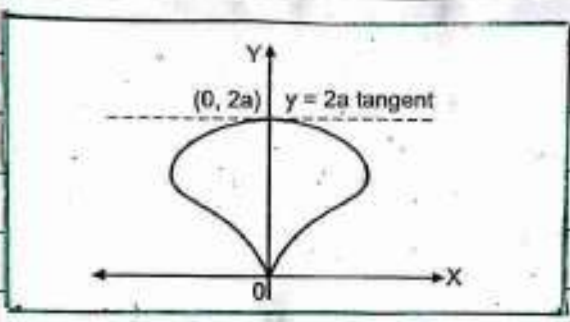
$$x^{2/3} + y^{2/3} = a^{2/3}$$

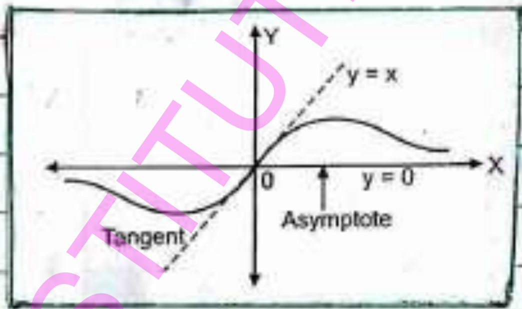
क) Eq^n of curve unaltered when we replace
$x \to y$, $y \to x$
∴ curve symmetrical to line $y = x$ or $y = -x$
∴ Eliminate other options.

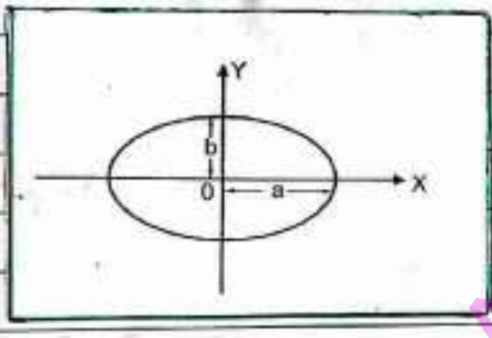
ख) Eq^n of curve contains absolute constant
i.e. $a^{2/3}$
∴ it is not passes through origin.
Eliminate remaining other options.

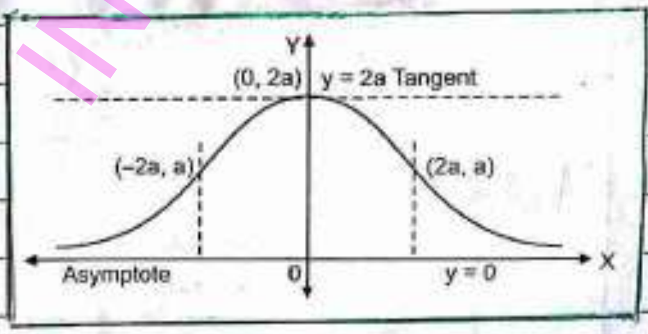
पान-८०

(60) The equation of the curve $a^2x^2 = y^3(2a-y)$, $a>0$ represents the curve

A) 

B) 

C) 

D) 

→ Sol^n: A)

क) $a^2x^2 = y^3(2a-y)$, $a>0$

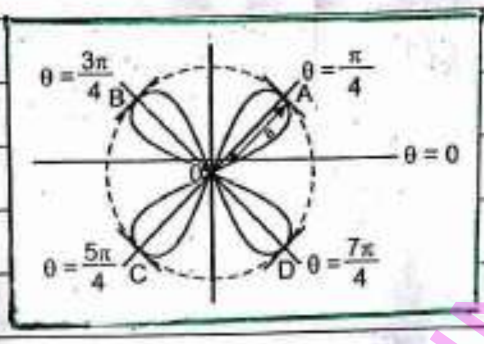
$a^2x^2 = 2ay^3 - y^4$

ख) Power of x even Everywhere,
∴ curve symmetrical to Y-axis
∴ Eliminate other options.

ग) Eq^n of curve does not contains any absolute constant.
∴ curve passes through the origin.
Eliminate remaining other options

पाठ – 29

(61) The following figure represents the curve whose equation is



A) $r = a\sin 2\theta$ ✓ B) $r = a\sin 3\theta$

C) $r = a\cos 3\theta$ D) $r = a(1 + a\cos\theta)$

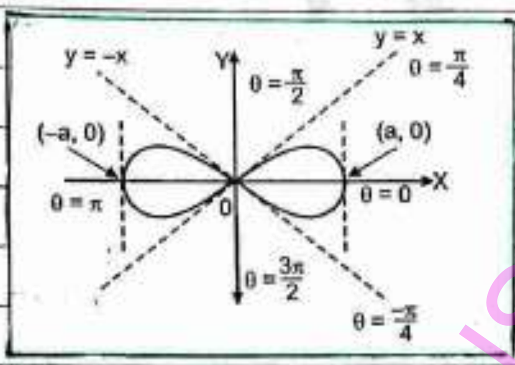
→ Sol^n : A)

अ) By common observation, this curve is Rose curve ($r = a\cos n\theta$ or $r = a\sin n\theta$)

ब) No. of loops = 4
∴ In Equation only Half value of loops present for even loops (i.e. $\frac{n}{2}$)

$\boxed{\therefore\ r = a\sin 2\theta}$

(63) The following figure represents the curve whose equation is



A) $r = a\cos 2\theta$ B) $r^2 = a^2\cos 2\theta$

C) $r = a(1+\cos\theta)$ D) $r^2 = a^2\sin 2\theta$

→ Sol^n : B)

क) By common observation, this curve is polar curve (Not Rose curve)
$\therefore$ Eliminate other options.

ख) curve unchanged when we replace $r \to -r$ and $\theta \to -\theta$,
$\therefore$ curve symmetrical to initial line & passes through the pole.
Eliminate other remaining options

$$\boxed{\therefore\ r^2 = a^2\cos 2\theta}$$

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पान - ३

धड्याचे नाव :

Differential Equation

पाठ-४

• NOTE:

① **Differential Equation (DE):**
Any equation which contains derivatives is called DE.

② **Types:**

D.E.
- Ordinary DE — e.g.: $\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz} \ldots$
- Partial DE — e.g.: $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \ldots$

③ **Order:**
Highest derivatives that appear in equation is called Order.

④ **Degree:**
Power of highest derivative which is free from radical / fraction (ie. a/b) term is called Degree.

⑤ **Formation of DE:**
1) If general solution contains n arbitary constant then differential given equation w.r.t. independant variable n times.
2) Formed DE which is free from arbitary constant which is required formation.
3) Arbitary constant removed from previous equation.

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पत्र - ७

* Type: Order / Degree / Formation of DE:

① The differential equation $1+\frac{dy}{dx}-\left(\frac{d^2y}{dx^2}\right)^{3/2}=0$ is of

A) Order 1 & Degree 2 ✓B) Order 2 & Degree 3

C) Order 3 & Degree 6 D) Order 3 & Degree 3

Sol^n → Ans: B)

B):

$$1+\frac{dy}{dx}-\left(\frac{d^2y}{dx^2}\right)^{3/2}=0$$

$$\left[1+\frac{dy}{dx}\right]=\left(\frac{d^2y}{dx^2}\right)^{3/2}$$

squaring both sides:

$$\left[1+\frac{dy}{dx}\right]^2=\left[\frac{d^2y}{dx^2}\right]^{\frac{3}{\cancel{2}}\times\cancel{2}}$$

$$\left[1+\frac{dy}{dx}\right]^2=\left[\frac{d^2y}{dx^2}\right]^3$$

$\therefore$ $\boxed{\text{Order}=2,\quad \text{Degree}=3}$

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पार्ट-2

(2) The differential equation $\sqrt{1+\frac{dy}{dx}} = \frac{d^2y}{dx^2}$ is of

✓A) Order 2 & Degree 2 B) Order 1 & Degree 2

C) Order 2 & Degree 1 D) Order 1 & Degree 1

Sol^n ⇒ Ans: A)

Q): $\sqrt{1+\frac{dy}{dx}} = \frac{d^2y}{dx^2}$

on squaring both sides,

$$\left[\sqrt{1+\frac{dy}{dx}}\right]^{\cancel{2}} = \left(\frac{d^2y}{dx^2}\right)^2$$

$$\left[1+\frac{dy}{dx}\right] = \left(\frac{d^2y}{dx^2}\right)^2$$

∴ Order = 2
Degree = 2

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पान-e

(3) The differential equation $x = \frac{1}{\sqrt{1+\frac{dy}{dx}+\frac{d^2y}{dx^2}}}$ is of

A) Order 2 & Degree 2 B) Order 1 & Degree 2

✓ C) Order 2 & Degree 1 D) Order 1 & Degree 1

Sol^n → Ans: c)

Ġ):

$$x = \frac{1}{\sqrt{1+\frac{dy}{dx}+\frac{d^2y}{dx^2}}}$$

$$x \cdot \sqrt{1+\frac{dy}{dx}+\frac{d^2y}{dx^2}} = 1$$

on squaring,

$$\left[x\sqrt{1+\frac{dy}{dx}+\frac{d^2y}{dx^2}}\right]^2 = (1)^2$$

$$\left[x^2\left(1+\frac{dy}{dx}+\frac{d^2y}{dx^2}\right)\right] = 1$$

$\therefore$ Order = 2
Degree = 1

पाठ - 90

(4) The differential equation $1+\frac{dy}{dx} = \frac{y}{\frac{dy}{dx}}$ is of

A) Order 2 & Degree 2 B) Order 1 & Degree 1
c) Order 2 & Degree 1 ✓D) Order 1 & Degree 2

Sol^n ⇒ ∴ Ans: D)

D):

$$1+\frac{dy}{dx} = \frac{y}{\frac{dy}{dx}}$$

$$\left(\frac{dy}{dx}\right)\left(1+\frac{dy}{dx}\right) = y$$

$$\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2 = y$$

∴ $\boxed{\begin{array}{l}\text{Order} = 1 \\ \text{Degree} = 2\end{array}}$

(5) The differential equation $(2x-y+3)dx + (y-2x-2)dy = 0$ is of

✓A) Order 1 & Degree 1 B) Order 1 & Degree 2
c) Order 2 & Degree 1 D) Order 2 & Degree 2

Sol^n ⇒ Ans: A)

$$(2x-y+3)dx + (y-2x-2)dy = 0$$

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पान -99

$$(2x-y+3)\,dx = -(y-2x-2)\,dy$$

$$\frac{(2x-y+3)}{-(y-2x-2)} = \frac{dy}{dx}$$

$\therefore$ Order = 1
Degree = 1

(6) The number of arbitary constants in the general solution of ordinary differential equation is equal to

✓ A) Order of differential equation
B) Degree of differential equation
C) Coeffient of highest order differential coefficient
D) None of these

Sol^n ⇒ Ans: A)

Order of differential equation

.... (स्टँडर्ड आहे, पाठांतर करा.)

(7) The general solution of n^{th} order ordinary differential equation must involve

A) (n+1) arbitary constant
B) (n-1) arbitary constant
✓ C) n arbitary constant
D) None of these

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$Sol^n \Rightarrow$ पान - ९२

Ans: c)

$\boxed{\text{n arbitory constants}}$

......(स्टँडर्ड आहे, पाठांतर करा.)

⑧ The solution obtained by assigning particular value to arbitory constants in general solution of ordinary differential equation is called as

A) General Solution ✓B) Perticular Solution
C) Singular Solution D) None of all

$Sol^n \Rightarrow$ Ans: B)

$\boxed{\text{Perticular Solution}}$

......(स्टँडर्ड आहे, पाठांतर करा.)

⑨ The order of differential equation whose general solution is

$y = c_1 + c_2 e^{-2x} + c_3 e^{3x} + c_4 e^{-3x}$

where, c_1, c_2, c_3, c_4 are arbitory constant is

A) 1 B) 3 C) 2 ✓D) 4

$Sol^n \Rightarrow$ Ans: D)

$\left(\text{No of Arbitary Constants}\right) = \left(\text{Order of DE}\right)$ ...... (सूत्र आहे.)

$\therefore \boxed{4}$

(10) The order of differential equation whose general solution is

$y = e^x\,[Ax^2 + Bx + C]$, where A, B, C are arbitary constants is

A) 2 B) 4 ✓C) 3 D) 1

Sol^n ⇒

Ans: C)

$$\begin{bmatrix}\text{No. of Arbitary} \\ \text{Constants}\end{bmatrix} = \begin{bmatrix}\text{Order of} \\ \text{DE}\end{bmatrix}$$ ......(सूत्र)

$$\therefore \boxed{3}$$

(11) The differential equation whose general solution is

$y = \sqrt{5x+C}$, where C is arbitary constant, is

A) $2y\dfrac{dy}{dx} - 1 = 0$ ✓B) $2y\dfrac{dy}{dx} - 5 = 0$

C) $\dfrac{dy}{dx} - \dfrac{5}{2}\,\dfrac{1}{\sqrt{5x+C}} = 0$ D) $y\dfrac{dy}{dx} - 5 = 0$

Sol^n ⇒ Ans: B)

B):

$$y = \sqrt{5x+c}$$

$$(y)^2 = \left[\sqrt{5x+c}\right]^2$$

पान-98

$$y^2 = 5x + C$$

सा): Taking derivative:

$$2y \cdot {dy}/{dx} = 5(1) + 0$$

$$\boxed{2y \frac{dy}{dx} - 5 = 0}$$

(12) $y = Cx - C^2$,where C is arbitary constants is the general solution of the DE

A) $\frac{dy}{dx} = C$ B) $\left(\frac{dy}{dx}\right)^2 + xy = 0$

C) $\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} - y = 0$ ✓D) $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$

Sol^n ⇒ Ans: D)

हे): $y = Cx - C^2$ .... अ

सा): Taking derivative:

$$\frac{dy}{dx} = C(1) - 0$$

$$C = \frac{dy}{dx}$$

Put value of C in eq^n अ:

पान-९५

ग): $y = cx - c^2$

$$y = \left(\frac{dy}{dx}\right)x - \left(\frac{dy}{dx}\right)^2$$

$$\boxed{\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right) + y = 0}$$

(13) The differential equation whose general solution is $y = c^2 + \frac{c}{x}$, where C is arbitary constant is

A) $x^4 y_1^2 + xy_1 - y = 0$ ✓B) $x^4 y_1^2 - xy_1 - y = 0$

c) $x^2 y_1^2 - xy_1 - y = 0$ D) $y_1 = -\frac{c}{x^2}$

Solⁿ → Ans: B)

ग): $y = c^2 + \frac{c}{x}$ ......(1)

Taking derivative:

$$\frac{dy}{dx} = 0 + c\left[\frac{-1}{x^2}\right]$$

$$\left(\frac{x^2}{-1}\right)\left(\frac{dy}{dx}\right) = c$$

$c = -x^2 \frac{dy}{dx}$, put in eqⁿ (1):

पान-९६

ख):

$$y = c^2 + c/x$$

$$y = \left[-x^2 \frac{dy}{dx}\right]^2 + \frac{\left[-x^2 \frac{dy}{dx}\right]}{x}$$

$$y = (x^4)\left(\frac{dy}{dx}\right)^2 - x \cdot \frac{dy}{dx}$$

$$x^4\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right) - y = 0$$

ग):

But : $\frac{dy}{dx} = y_1$, $\frac{d^2y}{dx^2} = y_2$, $\frac{d^3y}{dx^3} = y_3$

$$\therefore \quad x^4 (y_1)^2 - x(y_1) - y = 0$$

$$\therefore \quad \boxed{x^4 y_1^2 - x y_1 - y = 0}$$

(14) By eliminating arbitary constant A the differential equation whose general solution is $y = 4(x-A)^2$

A) $y_1^2 + 16y = 0$ B) $y_1 - 2y = 0$

✓C) $y_1^2 - 16y = 0$ D) $y_1 - 8(x-A) = 0$

Sol^n →

Ans: c)

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पान-१७

क):

$$y = 4(x-A)^2 \quad \ldots\ldots (1)$$

Taking derivative :

$$\frac{dy}{dx} = 4\left[2(x-A)^1 \times (1-0)\right]$$

$$\frac{dy}{dx} = 8(x-A)$$

$$\frac{dy}{dx} \times \frac{1}{8} = (x-A)$$

$$(x-A) = \frac{1}{8} \times \frac{dy}{dx}$$

ख): Put in eq^n (1):

$$y = 4(x-A)^2$$

$$y = 4\left[\frac{1}{8} \times \frac{dy}{dx}\right]^2$$

$$y = \cancel{4}^{1}\left[\frac{1}{\cancel{64}_{16}}\left(\frac{dy}{dx}\right)^2\right]$$

$$y = \frac{1}{16}\left(\frac{dy}{dx}\right)^2$$

$$16y = \left(\frac{dy}{dx}\right)^2$$

But : $\frac{dy}{dx} = y_1$, $\frac{d^2y}{dx^2} = y_2$

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पान-९८

$$16y = (y_1)^2$$

$$y_1^2 - 16y = 0$$

$$\boxed{y_1^2 - 16y = 0}$$

(16) By eliminating arbitary constant 'a' the differential equation whose general solution is $y^2 = 4ax$ is

A) $xy\frac{dy}{dx} - y^2 = 0$ B) $2xy\frac{dy}{dx} + y^2 = 0$

✓ C) $2xy\frac{dy}{dx} - y^2 = 0$ D) $8xy\frac{dy}{dx} - y^2 = 0$

Soln ⇒ Ans : C)

S): $y^2 = 4ax$ ......(1)

Taking derivative :

$$2y\frac{dy}{dx} = 4a(1)$$

$$4a = 2y\frac{dy}{dx}$$

S): Put in eqn (1):

$$y^2 = 4ax$$

$$\cancel{y^2 = \left[2y \cdot \frac{dy}{dx}\right] x}$$

पान-9e

$$y^2 = \left[2y\frac{dy}{dx}\right]\cdot x$$

$$y^2 = 2xy\cdot\frac{dy}{dx}$$

$$\boxed{2xy\frac{dy}{dx} - y^2 = 0}$$

(17) DE whose general solution is $xy = C^2$, where C is arbitary constant, is

A) $xy_1 - y = 0$ B) $xy_2 + y_1 = 0$

C) $xy_1 = c^2$ ✓D) $xy_1 + y = 0$

Sol^n ⇒ Ans: D)

D):

$$xy = c^2$$

Taking derivative:

$$\underbrace{x}_{u}\cdot\underbrace{y}_{v} = c^2 \quad \{\because uv = u'v + uv'\}$$

$$(1)(y) + (x)\left(\frac{dy}{dx}\right) = 0$$

$$x\frac{dy}{dx} + y = 0 \quad \text{But: } dy/dx = y_1$$

$$\boxed{xy_1 + y = 0}$$

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पान- 20

18 The differential equation representing the family of curves $y^2 = 2c[x + \sqrt{c}]$,

where C is constant is

A) $2yy_1[x + \sqrt{yy_1}] - y^2 = 1$

B) $2y_1[x + \sqrt{yy_1}] - y = 0$ ✓

C) $y = 2y_1[x + \sqrt{c}]$

D) $y_1[x + \sqrt{yy_1}] - y = 0$

Solⁿ ⇒

Ans: B)

$$y^2 = 2c(x + \sqrt{c}) \quad \text{.......(1)}$$

Taking derivative:

$$\cancel{2}y\frac{dy}{dx} = \cancel{2}c[1+0]$$

$$c = y\frac{dy}{dx}$$

But: $\frac{dy}{dx} = y_1$

$$c = yy_1$$

put in eqⁿ (1):

$$y^2 = 2c(x + \sqrt{c})$$

$$y^2 = 2(yy_1)[x + \sqrt{yy_1}]$$

पान - 29

$$2yy_1\left(x+\sqrt{yy_1}\right)-y^2=0$$

ग): $$y\left[2y_1\left(x+\sqrt{yy_1}\right)-y\right]=0$$

$$\left[2y_1\left(x+\sqrt{yy_1}\right)-y\right]=\frac{0}{y}$$

$$\boxed{2y_1\left[x+\sqrt{yy_1}\right]-y=0}$$

(19) By eliminating arbitary constant A the differential equation whose general solution is $y = A e^{-x^2}$ is

A) $\frac{dy}{dx}-2xy=0$ B) $y\frac{dy}{dx}-2x=0$

✓C) $\frac{dy}{dx}+2xy=0$ D) $y\frac{dy}{dx}+2x=0$

Sol^n ⇒ Ans: c)

ग): $$y = A e^{-x^2}$$

$$y = \frac{A}{e^{+x^2}}$$

$$ye^{x^2}=A$$

पाठ - २२

29): ~~Taking derivatives:~~

~~$y[e^{x^2}](2x)+$~~

29): Taking derivative:

$$\underset{u}{y}\cdot\underset{v}{e^{x^2}} = A$$

$\{\because uv = u'v + uv'\}$

$$\left(\frac{dy}{dx}\right)\left(e^{x^2}\right) + (y)\left(e^{x^2}\times 2x\right) = 0$$

$$\left(e^{x^2}\right)\left[\ dy/dx + 2xy\ \right] = 0$$

$$\frac{dy}{dx} + 2xy = \frac{0}{e^{x^2}}$$

$$\boxed{2xy + \frac{dy}{dx} = 0}$$

(20) $y = mx$ where m is arbitary constant is the general solution of DE is

✓A) $\frac{dy}{dx} = \frac{y}{x}$ C) $\frac{dy}{dx} = m$

B) $\frac{dy}{dx} = \frac{x}{y}$ D) $\frac{dy}{dx} = -\frac{y}{x}$

Solⁿ ⇒ Ans: A)

पान - २३

अ):

$$y = mx \quad \ldots\ldots (1)$$

Taking derivative :

$$\frac{dy}{dx} = m(1)$$

$$m = \frac{dy}{dx} \quad , \text{ put in } (1):$$

आ) :

$$y = mx$$

$$y = \frac{dy}{dx} \cdot x$$

$$\frac{y}{x} = \frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{y}{x}}$$

(21) DE representing family of curve $y = 3 + \sqrt{cx}$, where c is arbitary constant is,

A) $y = 3 + 2x \frac{dy}{dx}$ ✓ B) $y = 3 + 2\sqrt{x} \frac{dy}{dx}$

C) $y = 2x \frac{dy}{dx}$ D) $\frac{dy}{dx} = \frac{\sqrt{c}}{2\sqrt{x}}$

$Sol^n \Rightarrow$ Ans : A)

क):

$$y = 3 + \sqrt{cx} \quad \ldots\ldots (1)$$

Taking derivative :

$$y = 3 + \sqrt{c} \cdot \sqrt{x}$$

पान-२४

$$\frac{dy}{dx} = 0 + \sqrt{c} \cdot \frac{1}{2\sqrt{x}}$$

$$2\sqrt{x} \cdot \frac{dy}{dx} = \sqrt{c}$$

put in eq^n ①:

अ):

$$y = 3 + \sqrt{c} \cdot \sqrt{x}$$

$$y = 3 + 2\sqrt{x} \cdot {}^{dy}/_{dx} \cdot \sqrt{x}$$

$$y = 3 + 2\sqrt{x} \times \sqrt{x} \cdot \frac{dy}{dx} \qquad \{\because \sqrt{a} \times \sqrt{a} = a\}$$

$$\boxed{y = 3 + 2x \cdot {}^{dy}/_{dx}}$$

(22) Differential eq^n satisfied by general solution $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$, where a is arbitary constant is

A) $xyy_1 - y + 4 = 0$
B) $xyy_1 + y^2 - 4 = 0$
C) $x^2yy_1 + y^2x - 1 = 0$
✓D) $xyy_1 - y^2 + 4 = 0$

Sol^n ⇒ Ans: D)

D):

$$\frac{x^2}{a^2} + \frac{y^2}{4} = 1 \quad \text{.....①}$$

Taking derivative:

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$$\frac{2x}{a^2} + \frac{2y\, dy/dx}{4} = 0$$

But : $\frac{dy}{dx} = y_1$

$$\frac{2x}{a^2} + \frac{2yy_1}{4} = 0$$

$$\frac{2x}{a^2} = \frac{-2yy_1}{4}$$

$$\cancel{2}x \times \frac{4}{-\cancel{2}yy_1} = a^2$$

$$a^2 = \frac{-4x}{yy_1}$$

४): put a^2 in ①:

$$\frac{x^2}{a^2} + \frac{y^2}{4} = 1$$

$$\frac{x^2}{\left[\frac{-4x}{yy_1}\right]} + \frac{y^2}{4} = 1$$

$$x^{\cancel{2}} \times \frac{yy_1}{-4\cancel{x}} + \frac{y^2}{4} = 1$$

$$\frac{xyy_1}{-4} + \frac{y^2}{4} = 1$$

$$\frac{1}{4}\left[\frac{xyy_1}{-1} + \frac{y^2}{1}\right] = 1$$

$$-xyy_1 + y^2 = 1 \times \frac{4}{1}$$

$$-xyy_1 + y^2 = 4$$

$$-xyy_1 + y^2 - 4 = 0$$

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पान - २६

ग): changing sign:

$$\boxed{xyy_1 - y^2 + 4 = 0}$$

(23) The differential equation representing the family of curves $x^2+y^2 = 2Ax$, where A is arbitary constant is

A) $y_1 = \frac{y^2+x^2}{2xy}$ ✓ B) $y_1 = \frac{y^2-x^2}{2xy}$

C) $y_1 = \frac{y^2-x^2}{2y}$ D) $y_1 = \frac{2xy}{y^2-x^2}$

Sol^n ⇒ Ans: B)

क): $x^2+y^2 = 2Ax$ ....①

Taking derivative:

$2x + 2y \cdot y_1 = 2A(1)$ $\{\because \frac{dy}{dx} = y_1\}$

$2x + 2y \cdot y_1 = 2A$

ख): put 2A in ①:

$x^2+y^2 = 2Ax$

$x^2+y^2 = [2x + 2y \cdot y_1]x$

$x^2+y^2 = 2x^2 + 2xyy_1$

$x^2+y^2 - 2x^2 = 2xyy_1$

$-x^2+y^2 = 2xyy_1$

$y^2-x^2 = 2xyy_1$

$$\boxed{\frac{y^2-x^2}{2xy} = y_1}$$

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पान-२७

(25) $\sin(y-x) = c e^{-x^2/2}$ where c is arbitary constant is the general solution of DE

A) $\tan(y-x)\left[\frac{dy}{dx}-1\right]+x=0$ B) $\cot(y-x)\left[\frac{dy}{dx}-1\right]+x=0$

✓ C) $\cot(y-x)\left[\frac{dy}{dx}-1\right]+x=0$ D) $\cot(y-x)\left[\frac{dy}{dx}-1\right]=0$

Sol^n ⇒ Ans: c)

१): $\sin(y-x) = c e^{-x^2/2}$

$$\sin(y-x) = \frac{c}{e^{+x^2/2}}$$

$$\sin(y-x)\cdot e^{x^2/2} = c$$

$$\underset{u}{\uparrow}\qquad\underset{v}{\uparrow}$$

२): Taking derivative: $\{\because uv = u'v + uv'\}$

$$\left[\cos(y-x)(y_1-1)\right]\left[e^{x^2/2}\right] + \left[\sin(y-x)\right]\left[e^{x^2/2} \times \frac{\cancel{2}x}{\cancel{2}}\right] = 0$$

$$\left(e^{x^2/2}\right)\left[\cos(y-x)(y_1-1) + x\cdot\sin(y-x)\right] = 0$$

$$\cos(y-x)(y_1-1) + x\cdot\sin(y-x) = 0/e^{x^2/2}$$

$$\cos(y-x)(y_1-1) + x\cdot\sin(y-x) = 0$$

३): .....devide by $\sin(y-x)$:

$$\frac{\cos(y-x)(y_1-1)}{\sin(y-x)} + \frac{x\cdot\cancel{\sin(y-x)}}{\cancel{\sin(y-x)}} = \frac{0}{\sin(y-x)}$$

$\left\{\frac{\cos}{\sin} = \cot\right\}$

$$\cot(y-x)(y_1-1) + x = 0$$

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पान - २८

डी): But : $y_1 = dy/dx$

$$\boxed{\cot(y-x)\left[\frac{dy}{dx}-1\right]+x=0}$$

(26) By eliminating arbitary constant A the differential equation whose general solution is $(1+x^2) = A(1-y^2)$ is

✓A) $\frac{(1+x^2)}{(1-y^2)}\frac{dy}{dx}+\frac{x}{y}=0$ B) $\frac{(1-y^2)}{(1+x^2)}\frac{dy}{dx}+\frac{x}{y}=0$

C) $\frac{(1+x^2)}{(1-y^2)}\frac{dy}{dx}+\frac{y}{x}=0$ D) $\frac{(1-y^2)}{(1+x^2)}\frac{dy}{dx}-\frac{x}{y}=0$

Sol^n ⇒ Ans: A)

पा): $(1+x^2) = A(1-y^2)$ .....(1)

Taking derivative:

$(0+2x) = A\left[0-2y\cdot\frac{dy}{dx}\right]$,But : $\frac{dy}{dx}=y_1$

$$\cancel{2}x \times \frac{dx}{dy(-\cancel{2}y)} = A$$

$$A = \frac{-x}{y}\cdot\frac{dx}{dy}$$

वा): put A in (1):

$$(1+x^2) = A(1-y^2)$$

$$(1+x^2) = \left(\frac{-x}{y}\cdot\frac{dx}{dy}\right)(1-y^2)$$

$$\frac{(1+x^2)}{(1-y^2)} = \frac{-x}{y}\cdot\frac{dx}{dy}$$

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$$\frac{(1+x^2)}{(1-y^2)} \cdot \frac{dy}{dx} = \frac{-x}{y}$$

$$\boxed{\frac{(1+x^2)}{(1-y^2)} \frac{dy}{dx} + \frac{x}{y} = 0}$$

(27) The differential equation satisfied by general solution $x = cy - y^2$, where c is arbitary constant is

A) $\left(\frac{y}{x+y^2}\right) y_1 - 2yy_1 - 1 = 0$ ✓B) $\left(\frac{x+y^2}{y}\right) y_1 - 2yy_1 - 1 = 0$

C) $\left(\frac{x+y^2}{y}\right) y_1 + 2yy_1 + 1 = 0$ D) $\left(\frac{x+y^2}{y}\right) y_1 + 2yy_1 = 0$

Solⁿ ⇒ Ans: B)

5): $x = cy - y^2$ .....(1)

$x + y^2 = cy$

Taking derivatives:

$1 + 2y \cdot \frac{dy}{dx} = c \cdot \frac{dy}{dx}$, But: $\frac{dy}{dx} = y_1$

$1 + 2y \cdot y_1 = c \cdot y_1$

$c = \frac{1+2yy_1}{y_1}$

4): put c in (1):

$x = cy - y^2$

$x = \left[\frac{1+2yy_1}{y_1}\right] y - y^2$

$x + y^2 = \left[\frac{1+2yy_1}{y_1}\right] y$

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$\boxed{\text{पाठ - 30}}$

$$(x+y^2) \times \frac{y_1}{y} = 1 + 2yy_1$$

$$\frac{(x+y^2)}{y} \cdot y_1 - 1 - 2yy_1 = 0$$

$$\boxed{\left[\frac{x+y^2}{y}\right] y_1 - 2yy_1 - 1 = 0}$$

(28) The differential equation satisfied by general solution $y + x^3 = Cx$, where C is arbitary constants, is

A) $\frac{dy}{dx} + 3x^2 = C$ B) $x\frac{dy}{dx} + 2x^2 - y = 0$

C) $\frac{dy}{dx} + 2x^2 - y = 0$ ✓D) $x\frac{dy}{dx} + 2x^3 - y = 0$

Sol^n ⇒ Ans: D)

D): $y + x^3 = Cx$ .....(1)

Taking derivatives:

$y_1 + 3x^2 = C$, put C in (1):

D):

$$y + x^3 = Cx$$
$$y + x^3 = (y_1 + 3x^2)x$$
$$y + x^3 = y_1 x + 3x^3$$
$$xy_1 + 3x^3 - y - x^3 = 0$$
$$xy_1 + 2x^3 - y = 0$$

$$\boxed{x\frac{dy}{dx} + 2x^3 - y = 0}$$

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(29) $xy + y^2 - x^2 - 3y - x = c$, where c is arbitary constant is the general solution of the differential eq^n

✓A) $\frac{dy}{dx} = \frac{2x-y+1}{x+2y-3}$ B) $\frac{dy}{dx} = \frac{2x+1}{x+2y-3}$

C) $\frac{dy}{dx} = \frac{y-2x-1}{x-2y+3}$ D) $\frac{dy}{dx} = \frac{x+2y-3}{2x-y+1}$

Sol^n ⇒ Ans: A)

5) : $xy + y^2 - x^2 - 3y - x = c$

$\{ \because uv = u'v + uv' \}$

Taking derivative :

$(1)(y) + (x)(y_1) + 2yy_1 - 2x - 3y_1 - 1 = 0$

$xy_1 + 2yy_1 - 3y_1 = -y + 2x + 1$

$y_1 (x + 2y - 3) = 2x - y + 1$

च) : $y_1 = \frac{2x-y+1}{x+2y-3}$, But : $\frac{dy}{dx} = y_1$

$$\boxed{\frac{dy}{dx} = \frac{2x-y+1}{x+2y-3}}$$

(30) $y = Ce^{x/y}$, where C is arbitary constant is the general solution of DE

A) $yy_1 + (y - xy_1) = 0$ ✓B) $yy_1 - (y - xy_1) = 0$

C) $y^2y_1 - (y - xy_1) = 0$ D) $y_1 - 1 = 0$

Sol^n ⇒ Ans: B)

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पान-३२

क):

$$y = ce^{x/y}$$

$$\frac{y}{e^{x/y}} = c$$

$$y \cdot e^{-x/y} = c$$

$$\underset{u}{y} \cdot \underset{v}{e^{-x/y}}$$

$\{\because uv = u'v + uv'\}$

ख): Taking derivative:

$$(y_1)(e^{-x/y}) + (y)\left[e^{-x/y} \times \frac{d}{dx}\left(\frac{-x}{y}\right)\right] = 0 \quad \left\{\because \frac{u}{v} = \frac{u'v - uv'}{v^2}\right.$$

$$y_1 \cdot e^{-x/y} + \cancel{y} \cdot e^{-x/y}\left[\frac{(-1)(y) - (-x)(y_1)}{y^{\cancel{2}}}\right] = 0$$

$$y_1 \cdot e^{-x/y} + e^{-x/y}\frac{(-y + xy_1)}{y} = 0$$

$$e^{-x/y}\left[y_1 + \frac{xy_1 - y}{y}\right] = 0$$

$$e^{-x/y}\left[\frac{yy_1 + xy_1 - y}{y}\right] = 0$$

ग): $e^{-x/y}\left[yy_1 + xy_1 - y\right] = 0$ (Taking -ve sign common)

$$e^{-x/y}\left[yy_1 - (y - xy_1)\right] = 0$$

$$\left[yy_1 - (y - xy_1)\right] = 0/e^{-x/y}$$

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$$\boxed{yy_1 - (y - xy_1) = 0}$$

(32) By eliminating arbitary constant A the differential eq^n whose general solution is $y^2 = x^2 - 1 + Ax$ is

✓A) ~~$2xy\frac{dy}{dx} = x^2 + y^2 + 1$~~ B) ~~$2xy\frac{dy}{dx} = x^2 + x + y^2 + 1$~~

C) ~~$2xy\frac{dy}{dx} = y^2 + 1$~~ D) ~~$2y\frac{dy}{dx} = 2x + A$~~

Sol^n ⇒ Ans : A)

क): $y^2 = x^2 - 1 + Ax$ ....(1)

Taking derivative:

$2yy_1 = 2x - 0 + A$

$2yy_1 - 2x = A$

ख): put A in (1):

$y^2 = x^2 - 1 + (2yy_1 - 2x)x$

$y^2 = x^2 - 1 + 2xyy_1 - 2x^2$

ग): $y^2 = x^2 - 1 + 2xyy_1 - 2x^2$

$y^2 = -x^2 - 1 + 2xyy_1$

$y^2 + x^2 + 1 = 2xyy_1$

$2xy \cdot y_1 = x^2 + y^2 + 1$

But, $y_1 = \frac{dy}{dx}$

$$\boxed{2xy \cdot \frac{dy}{dx} = x^2 + y^2 + 1}$$

(33) The differential equation satisfied by general solution $y = A\cos x + B\sin x$, where A, B are arbitary constant is

A) ~~$\frac{d^2y}{dx^2} + \frac{dy}{dx} = B\sin x$~~ B) ~~$\frac{d^2y}{dx^2} - y = 0$~~

C) $d^2y/dx^2 + dy/dx = A\cos x$ ✓D) $d^2y/dx^2 + y = 0$

Sol^n ⇒

Ans : D)

पु): $y = A\cos x + B\sin x$ ..... ①

Taking derivatives:

$y_1 = A(-\sin x) + B(\cos x)$

२ा): Again derivative:

$y_2 = A(-\cos x) + B(-\sin x)$

$y_2 = -[A\cos x + B\sin x]$

$y_2 = -[y]$ {∵ from ①}

$y_2 + y = 0$, But : $y_1 = \frac{dy}{dx}$, $y_2 = \frac{d^2y}{dx^2}$

$$\boxed{\frac{d^2y}{dx^2} + y = 0}$$

(35) The differential equation satisfied by general solution $y = A\cos(\log x) + B\sin(\log x)$ where A, B constants is

A) $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$ ✓B) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

C) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ D) $x^2\frac{d^2y}{dx^2} + y = 0$

Sol^n ⇒ Ans : B)

पु): $y = A\cos(\log x) + B\sin(\log x)$ ..... ①

Taking derivative:

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$$\frac{dy}{dx} = A\left[-\sin(\log x) \times \frac{1}{x}\right] + B\left[\cos(\log x) \times \frac{1}{x}\right]$$

$$y_1 = \frac{1}{x}\left[-A\sin(\log x) + B\cos(\log x)\right]$$

$\left\{\because \text{But}, \frac{dy}{dx} = y_1\right\}$

$$\underset{u}{x}\,\underset{v}{y_1} = -A\sin(\log x) + B\cos(\log x)$$

स्या): $\{\because uv = u'v + uv'\}$

Again derivative :

$$(1)(y_1) + (x)(y_2) = -A\left[\cos(\log x) \times \frac{1}{x}\right] + B\left[-\sin(\log x) \times \frac{1}{x}\right]$$

$$y_1 + xy_2 = \frac{-1}{x}\left[A\cos(\log x) + B\sin(\log x)\right]$$

$$xy_1 + x^2 y_2 = -1\,[y] \quad \ldots \{\because \text{from } (1)\}$$

$$x^2 y_2 + xy_1 + y = 0$$

But $y_1 = \frac{dy}{dx}$, $y_2 = \frac{d^2y}{dx^2}$

$$\boxed{x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0}$$

(36) The differential equation satisfied by general solution $y = Ae^{x} + Be^{-x}$, where A, B are constant is

✓A) ~~$y_2 - y = 0$~~ B) ~~$y_2 + y = 0$~~

C) $y_2 + y = Ae^{x} + Be^{-x}$ D) $y_2 - y = 2Ae^{x}$

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पान-३६

Sol^n ⇒ Ans : A)

क): $y = Ae^{x} + Be^{-x}$ .....①

Taking derivative:

$y_1 = A(e^{x}) + B(-e^{-x})$

ख): Again derivative:

$y_2 = A(e^{x}) + B(--e^{-x})$

$y_2 = Ae^{x} + Be^{-x}$

$y_2 = y$ ....{∵ from ①}

$\boxed{y_2 - y = 0}$

(37) DE satisfied by solution $xy = Ae^{x} + Be^{-x}$, where A, B are constant is

A) $xy_2 + 2y_1 + xy = 0$ B) $xy_2 - 2y_1 + xy = 0$

✓ C) $xy_2 + 2y_1 - xy = 0$ D) $xy_2 + y_1 - xy = 0$

Sol^n ⇒ Ans: C)

क): $\underset{u}{x}\underset{v}{y} = Ae^{x} + Be^{-x}$ ....①

{∵ $uv = u'v + uv'$ }

Taking derivative:

$(1)(y) + (x)(y_1) = A(e^{x}) + B(-e^{-x})$

$y + \underset{u}{x}\underset{v}{y_1} = Ae^{x} - Be^{-x}$

Again derivative: {∵ $uv = u'v + uv'$ }

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स्प): $y_1 + [(1)(y_1) + (x)(y_2)] = A(e^x) - B(-e^{-x})$

$y_1 + y_1 + xy_2 = Ae^x + Be^{-x}$

$2y_1 + xy_2 = xy$ ..... $\{\because$ from ①$\}$

$xy_2 + 2y_1 - xy = 0$

$$\boxed{xy_2 + 2y_1 - xy = 0}$$

(38) DE by solution $x = A\cos(2t+B)$, where A, B are constant is

✓A) $\frac{d^2x}{dt^2} + 4x = 0$ B) $\frac{d^2x}{dt^2} - 2x = 0$

C) $\frac{d^2x}{dt^2} - 4x = 0$ D) $\frac{d^2x}{dt^2} + x = 0$

Solⁿ→ Ans: A)

दि): $x = A\cos(2t+B)$ .....①

Taking derivative:

$x_1 = A[-\sin(2t+B) \times (2(1)+0)]$ $\{\because x_1 = \frac{dx}{dt}\}$

स्प): Again derivative:

$x_2 = -2A[-\cos(2t+B) \times (2(1)+0)]$

$x_2 = -4A\cos(2t+B)$

$x_2 = -4 \cdot [A\cos(2t+B)]$

$x_2 = -4[x]$ ..... ($\because$ from ①)

$x_2 + 4x = 0$

But: $x_1 = \frac{dx}{dt}$, $x_2 = \frac{d^2x}{dt^2}$

$$\boxed{\frac{d^2x}{dt^2} + 4x = 0}$$

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पाठ - 3L

(39) By eliminating arbitary constants A, B the DE whose general solution is $y^2 = 4A(x-B)$ is

A) $y_2 + y_1^2 = 0$ B) $yy_2 + y_1 = 0$

c) $yy_2 - y_1^2 = 0$ ✓D) $yy_2 + y_1^2 = 0$

Sol^n ⇒ Ans: D)

$y^2 = 4A(x-B)$

Taking derivative:

$2y \cdot y_1 = 4A[1-0]$

$2\underset{u}{y}\underset{v}{y_1} = 4A$ $\{\because uv = u'v + uv'\}$

Again derivative:

$(2y_1)(y_1) + (2y)(y_2) = 0$

$2[y_1^2 + yy_2] = 0$

$y_1^2 + yy_2 = 0/2$

$y_1^2 + yy_2 = 0$

$$\boxed{yy_2 + y_1^2 = 0}$$

(40) By eliminating arbitary constants A, B the DE whose general solution is $e^{-t}x = (A+Bt)$ is

A) $x_2 + 2x_1 + x = 0$ ✓B) $x_2 - 2x_1 + x = 0$

c) $x_2 - x_1 + x = 0$ D) $x_2 + x = 0$

Sol^n ⇒ Ans: B)

$\underset{u}{e^{-t}}\underset{v}{x} = (A+Bt)$ $\{\because uv = u'v + uv'\}$

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पाल-3e

Taking derivative:

$(-e^{-t})(x) + (e^{-t})(x_1) = [0 + B(1)]$ $\{\because x_1 = \frac{dx}{dt}\}$

$-e^{-t}.x + e^{-t}.x_1 = B$

$\underset{u}{\uparrow}\ \underset{v}{\uparrow}\ \ \underset{u}{\uparrow}\ \underset{v}{\uparrow}$ $\{\because uv = u'v + uv'\}$

ब): Again derivative:

$[(--e^{-t})(x) + (-e^{-t})(x_1)] + [(-e^{-t})(x_1) + (e^{-t})(x_2)] = 0$

$e^{-t}x - e^{-t}x_1 - e^{-t}x_1 + e^{-t}x_2 = 0$

$e^{-t}[x - x_1 - x_1 + x_2] = 0$

$e^{-t}[x - 2x_1 + x_2] = 0$

$(x - 2x_1 + x_2) = \frac{0}{e^{-t}}$

$(x_2 - 2x_1 + x) = 0$

$\boxed{x_2 - 2x_1 + x = 0}$

(41) The DE of family of circles having their centeres at (A, 5) and radius 5, where A is arbitary constant is

A) $(y-5)^2\{1 + \frac{dy}{dx}\} = 5$ B) $(y-5)^2\{1 - (\frac{dy}{dx})^2\} = 25$

✓ c) $(y-5)^2\{1 + (\frac{dy}{dx})^2\} = 25$ D) None of one

Sol^n → Ans: c)

ब): Given: center = (A, 5)

radius = 5

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ब): Using eqⁿ of circle:

$$(x-A)^2 + (y-5)^2 = 25 \quad \ldots\ldots (1)$$

Taking derivative:

$$2(x-A)(1-0) + 2(y-5)(y_1 - 0) = 0$$

$$\cancel{2}(x-A) = -\cancel{2}(y-5)y_1$$

$$(x-A) = -(y-5)y_1$$

ड): put $(x-A)$ in eq^n (1) of circle:

$$(x-A)^2 + (y-5)^2 = 25$$

$$[-(y-5).y_1]^2 + (y-5)^2 = 25$$

$$+(y-5)^2 . y_1^2 + (y-5)^2 = 25$$

$$(y-5)^2 [y_1^2 + 1] = 25$$

But: $y_1 = \frac{dy}{dx}$

$$(y-5)^2 \left[\left(\frac{dy}{dx}\right)^2 + 1\right] = 25$$

$$\boxed{(y-5)^2 \left\{1 + (dy/dx)^2\right\} = 25}$$

(42) The differential eq^n of family of circle having their centeres at origin and radius a where a is constant is

A) $x - y\frac{dy}{dx} = 0$ ✓B) $x + y\frac{dy}{dx} = 0$

C) $x\frac{dy}{dx} + y = 0$ D) $x + y\frac{dy}{dx} = \frac{a^2}{2}$

Slⁿ → Ans: B)

अ): Given: center = Origin = (0,0)

radius = a

३): Using eqⁿ of circle:

$$(x-0)^2 + (y-0)^2 = a^2$$

$$x^2 + y^2 = a^2$$

४): Taking derivative:

$$2x + 2y \cdot y_1 = 0$$

$$2[x + yy_1] = 0$$

$$(x + yy_1) = 0/2$$

$$x + yy_1 = 0 \qquad \text{But: } y_1 = \frac{dy}{dx}$$

$$\boxed{x + y\frac{dy}{dx} = 0}$$

43) By eliminating arbitary constants A, B the DE whose general solution is $(x-A)^2 = 4(y-B)$ is

A) $2\frac{dy}{dx} - (x-A) = 0$ B) $\frac{d^2y}{dx^2} + \frac{1}{2} = 0$

C) $\frac{d^2y}{dx^2} - 2 = 0$ ✓D) $\frac{d^2y}{dx^2} - \frac{1}{2} = 0$

Solⁿ → Ans: D)

५): $(x-A)^2 = 4(y-B)$

Taking derivative:

$$2(x-A)(1-0) = 4(y_1 - 0)$$

$$2x - 2A = 4y_1$$

६): Again derivative:

$$2(1) - 0 = 4y_2$$

$$2 = 4y_2$$

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पान-४२

ग):
$$\frac{2}{4} = y_2$$

$$\frac{1}{2} = y_2$$

$$y_2 - \frac{1}{2} = 0 \quad , \text{ But: } y_1 = \frac{dy}{dx} , y_2 = \frac{d^2y}{dx^2}$$

$$\boxed{\frac{d^2y}{dx^2} - \frac{1}{2} = 0}$$

(44) The differential eq^n satisfied by general solution $y = A\cos 4x + B\sin 4x + C$, where A, B, C are arbitary constant is

A) $\frac{d^2y}{dx^2} - 16y = 0$ B) $\frac{d^3y}{dx^3} - 16\frac{dy}{dx} = 0$

✓C) $\frac{d^3y}{dx^3} + 16\frac{dy}{dx} = 0$ D) $\frac{d^3y}{dx^3} + \frac{dy}{dx} = 0$

$Sol^n \rightarrow$

Ans: C)

क):
$$y = A\cos 4x + B\sin 4x + C$$

~~Taking derivative~~

$$\therefore y - C = A\cos 4x + B\sin 4x \quad \ldots\ldots (1)$$

ख): Taking derivative:

$$y_1 - 0 = A[-\sin 4x \times 4(1)] + B[\cos 4x \times 4(1)]$$

$$y_1 = -4A\sin 4x + 4B\cos 4x$$

ग): Again derivative:

$$y_2 = -4A[\cos 4x \times 4(1)] + 4B[-\sin 4x \times 4(1)]$$

$$y_2 = -16A\cos 4x - 16B\sin 4x$$

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पान - ४३

ब): $y_2 = -16[A\cos 4x + B\sin 4x]$

$y_2 = -16[y - c]$ .... ($\because$ from ①)

$y_2 = -16y + 16c$

३): Again derivative:

$y_3 = -16y_1 + 0$

$y_3 + 16y_1 = 0$

$\left\{ \because \text{But: } y_1 = \frac{dy}{dx},\ y_2 = \frac{d^2y}{dx^2},\ y_3 = \frac{d^3y}{dx^3} \right\}$

$$\boxed{\frac{d^3y}{dx^3} + 16\frac{dy}{dx} = 0}$$

(45) The differential equation satisfied by general solⁿ $y = Ax^2 + Bx + C$ where A, B, C are constant is

✓ A) $\frac{d^3y}{dx^3} = 0$ B) $\frac{d^2y}{dx^2} = 0$ C) $\frac{d^3y}{dx^3} = A$ D) $\frac{d^4y}{dx^4} = 0$

Solⁿ ⇒ Ans: A)

१): $y = Ax^2 + Bx + C$

Taking derivative:

$y_1 = 2Ax + B + 0$

२): Taking Again Derivative:

$y_2 = 2A + 0 + 0$

३): Taking Again Again Derivative:

$y_3 = 0 + 0 + 0$

$\therefore y_3 = 0$ But: $y_3 = \frac{d^3y}{dx^3},\ y_2 = \frac{d^2y}{dx^2},\ y_1 = \frac{dy}{dx}$

$$\boxed{\frac{d^3y}{dx^3} = 0}$$

$\therefore$ Ans: A)

• NOTE:

① Variable Seperable Form:

$$M(x)\cdot dx + N(y)\cdot dy = 0$$

OR

$$\frac{1}{M(x)}\cdot dx + \frac{1}{N(y)}\cdot dy = 0$$

$M(x)$ = only function of x

$N(y)$ = only function of y

obtained General Solution by Integrating both sides:

$$\int M(x)\cdot dx + \int N(y)\cdot dy = c$$

OR

$$\int \frac{1}{M(x)}\cdot dx + \int \frac{1}{N(y)}\cdot dy = c$$

② Variable Seperable form by using substitution:

i) Form: $\frac{dy}{dx} = f(ax+by+c)$

(Linear Function)

→ Put: (Linear Function) = u and it's derivative.

→ After that use Variable Seperable method & solve it.

2) Form: $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ OR $\frac{dy}{dx} = f(xy)$

→ Put: $y/x = u$ OR $xy = u$

→ Also put it's derivative.

→ Use V.S. form & solve it.

③ Homogeneous DE:

i) Form: $M(x,y) \cdot dx + N(x,y) \cdot dy = 0$

$M(x,y) = M$ is function of x, y.

$N(x,y) = N$ is function of x, y.

and

Degree of whole equation has throughout same.

→ Put: $y = ux$

→ Derivative: $\frac{dy}{dx} = \left(\frac{du}{dx}\right)(x) + (u)(1)$

→ Use V.S. form & solve it.

④ Non-Homogeneous DE:

i) Form: $\frac{dy}{dx} = \frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}$

• NOTE: Degree not same throughout eq^n.

अ): case 1: $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

→ Put: common term $= u$

→ Reduce into V.S. form and solve it.

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आ): case 2:

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

→ Put: $x = X + h$, $y = Y + h$, $\frac{dY}{dX} = \frac{dy}{dx}$

→ convert into homogeneous form.

→ use homogeneous form method for solving problem.

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पान- रe

✱ Type : General Solution of 1st Degree and 1st Order DE :

① The differential equation $\frac{dy}{dx} = e^{x-y} + 3x^2 e^{-y}$ is of the form

✓A) Variable seperable B) Homogeneous

c) Linear D) Exact

Sol^n ⇒

Ans: A)

5):

$$\frac{dy}{dx} = e^{x-y} + 3x^2 e^{-y}$$

$$y_1 = e^x \cdot e^{-y} + 3x^2 e^{-y} \qquad \left\{ \because \frac{dy}{dx} = y_1 \right\}$$

$$y_1 = e^{-y}[e^x + 3x^2]$$

$$\frac{y_1}{e^{-y}} = [e^x + 3x^2]$$

$$e^y \cdot y_1 = e^x + 3x^2$$

श):

$$e^y \left(\frac{dy}{dx} \right) = e^x + 3x^2 \qquad \left\{ \because y_1 = \frac{dy}{dx} \right\}$$

$$e^y \cdot dy = (e^x + 3x^2) \cdot dx$$

......(this is Variable Seperable Form.)

② For solving differential equation $(x+y+1)dx + (2x+2y+4)dy = 0$ appropriate substitution is

A) $x+y=1$ C) $x-y=4$

✓B) $x+y=4$ D) None

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पान - ५०

Sol^n ⇒ Ans: B)

क): $(x+y+1)dx + (2x+2y+4)dy = 0$

$(x+y+1)dx + [2(x+y)+4]dy = 0$

ख): Common term is $= x+y$

$\therefore$ Put: $x+y = u$

$$\boxed{x+y=u}$$

③ The differential equation $\frac{dy}{dx} = \frac{x^3 - 3xy^2}{y^3 - 3x^2y}$ is of the form

A) Variable seperable ✓B) Homogeneous
C) Linear D) Exact

Sol^n ⇒ Ans: B)

क): Check all terms → All term having same Degree (ie: 3)

$\therefore$ It is Homogeneous DE.

ख): : Term: : Degree: (Power Addition):

$x^3 \Rightarrow 3$

$3xy^2 \Rightarrow 3x^1y^2 \Rightarrow 1+2 = 3$

$y^3 \Rightarrow 3$

$3x^2y \Rightarrow 3x^2y^1 \Rightarrow 2+1 = 3$

ग): • All term having same degree $= 3$

$\therefore$ It is Homogeneous.

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पान - ४९

④ The differential equation $\frac{dy}{dx} = \frac{x+2y-3}{3x+6y-1}$ is of form

A) Variable Seperable B) Exact

✓c) Non-homogeneous D) Homogeneous

Sol^n →

Ans: c)

इ): Degree of all term is not same

∴ It is Non-Homogeneous.

⑤ The solution of differential eq^n $\frac{dy}{dx} + y = 0$ is

✓A) $y = Ae^{-x}$ B) $y = Ae^{x}$ c) $x = Ae^{-y}$ D) $x = Ae^{y}$

Sol^n →

Ans: A)

इ):

$$\frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx} = -y$$

$$\frac{1}{y}\,dy = -dx$$

२इ): Take Integration:

$$\int \frac{1}{y}\,dy = -\int dx$$

$$\log y = -x + \log C$$

$$\log y - \log C = -x$$

$$\log\left(\frac{y}{c}\right) = -x$$

{∵ Formula: Taking Antilog both side.

$$\frac{y}{c} = e^{-x}$$

$$y = c.e^{-x}$$

Replace $C = A$, both constant.

$$\therefore \boxed{y = Ae^{-x}}$$

⑥ The solution of DE $dy/dx + x = 0$ is

A) $x + y^2 = c$ B) $x + y = c$ C) $x^2 + y = c$ ✓D) $x^2 + 2y = c$

Solⁿ ⇒ Ans: D)

१): $\frac{dy}{dx} + x = 0$

$\frac{dy}{dx} = -x$

$dy = -x\,dx$

२): Taking Integration:

$\int dy = -\int x\,dx$

$y = -\frac{x^2}{2} + c$

$y + \frac{x^2}{2} = c$

$\frac{2y + x^2}{2} = c$

$2y + x^2 = c$

$\therefore \boxed{x^2 + 2y = c}$

⑦ The solution of DE $y\,dx + x\,dy = 0$ is

A) $x^2y = c$ ✓B) $xy = c$ C) $xy^2 = c$ D) $xy + 1 = c$

Solⁿ ⇒ Ans: B)

१): $y\,dx + x\,dy = 0$

make adjustment:

$d(yx) + d(xy) = 0$

$d(xy) + d(xy) = 0$

२): Take derivative of only one term only:

$\therefore d(xy) = 0$

म): Taking Integration:

$\int \cancel{d}(xy) = 0$ $\{\because (\text{Integration}) \times (\text{Derivative}) = 1\}$

$(xy) = c$ $\{\because c = \text{constant of Integration}\}$

$\therefore \boxed{xy = c}$

(8) The solution of DE $\frac{dy}{dx} + \tan x = 0$ is

A) $y + \log\cdot\sin x = c$

B) $y + \sec^2 x = c$

✓ C) $y - \log\cdot\cos x = c$

D) $y + \log\cdot\cot x = c$

Sol^n ⇒ Ans: c)

क):

$\frac{dy}{dx} + \tan x = 0$

$\frac{dy}{dx} = -\tan x$

$dy = -\tan x \cdot dx$

ख): Take Integration:

$\int dy = -\int \tan x \, dx$

$y = -[-\log\cdot\cos x] + c$

$y - \log\cdot\cos x + c$

$\boxed{y = -\log\cdot\cos x + c}$

$\{\because \text{Formula: } \int \tan x \, dx = -\log\cos x\}$

(9) The solution of DE $\frac{dy}{dx} = \frac{1+y}{1+x}$ is

A) $(1+y) = c(1+x)$

B) $(1+x) = \frac{c}{(1+y)}$

C) $xy(1+y) = c$

D) $(1+y)^2 = c(1+x)$

पान- ५४

Sol^n ⇒ Ans: A)

पद्धत:
$$\frac{dy}{dx} = \frac{1+y}{1+x}$$

use VS form:
$$\frac{1}{1+y}\cdot dy = \frac{1}{1+x}\cdot \cancel{dy}\, dx$$

२या): Taking Integration:
$$\int \frac{1}{1+y}\, dy = \int \frac{1}{1+x}\, dx$$
$$\log(1+y) = \log(1+x) + \log C$$
$$\cancel{\log}(1+y) = \cancel{\log}[(1+x)\cdot C]$$
$$(1+y) = C\cdot(1+x)$$
$$\boxed{(1+y) = C\cdot(1+x)}$$

10) The solution of DE $\frac{dy}{dx} + \frac{1+y^2}{1+x^2} = 0$ is

A) $\tan^{-1}y - \tan^{-1}x = C$ ✓B) $\tan^{-1}y + \tan^{-1}x = C$
c) $\tan y + \tan x = C$ D) $\cos y + \cos x = C$

Sol^n ⇒ Ans: B)

पद्धत:
$$\frac{dy}{dx} = \frac{-1+y^2}{1+x^2}$$
$$\frac{1}{1+y^2}\, dy = \frac{-1}{1+x^2}\, dx$$

२या): Take Integration:
$$\int \frac{1}{1+y^2}\, dy = -\int \frac{1}{1+x^2}\, dx$$
$$\tan^{-1}y = -\tan^{-1}x + C$$
$$\boxed{\tan^{-1}y + \tan^{-1}x = C}$$

$\left\{ \because \text{formula: } \int \frac{1}{1+x^2}\, dx = \tan^{-1}x \right\}$

पान- ४४

(11) The solution of DE $(4+e^{2x})\frac{dy}{dx} = ye^{2x}$ is

✓A) $y^2 = (4+e^{2x})c$ B) $y = (4+e^{2x})c$

C) $y(4+e^{2x}) = c$ D) $y^2(4+e^{2x}) = c$

Sol^n⇒ Ans: A)

क): $(4+e^{2x})\frac{dy}{dx} = ye^{2x}$

use VS form:

$$\frac{1}{y}dy = \frac{e^{2x}}{(4+e^{2x})}dx$$

make adjustment:

$$\frac{1}{y}dy = \frac{1}{2}\times\frac{2e^{2x}}{(4+e^{2x})}dx$$

ख): Taking Integration:

$$\int\frac{1}{y}dy = \frac{1}{2}\int\frac{2e^{2x}\,dx}{(4+e^{2x})}$$

$$\log y = \frac{1}{2}\log(4+e^{2x}) + \log c$$

$$2\log y = \log(4+e^{2x}) + 2\log c$$

$$\cancel{\log} y^2 = \cancel{\log}[(4+e^{2x})\cdot c^2]$$

$$y^2 = (4+e^{2x})\cdot c^2$$

$\{\because$ Put: $c^2 = c\}$

$$\boxed{y^2 = (4+e^{2x})\cdot c}$$

(12) The solution of DE $y - x\frac{dy}{dx} = 2\left[y + \frac{dy}{dx}\right]$ is

A) $y + (x+2) = c$ B) $y - (x+2) = c$

C) $y = c(x+2)$ ✓D) $y(x+2) = c$

Sol^n⇒ Ans: D)

क): $y - x\frac{dy}{dx} = 2\left[y + \frac{dy}{dx}\right]$

$$y = 2y + 2\frac{dy}{dx} + x\frac{dy}{dx}$$

$$y - 2y = (2+x)\frac{dy}{dx}$$

$$-y = (2+x)\,{}^{dy}/_{dx}$$

२): use VS form:

$$-y\,dx = (2+x)\,dy$$

$$\frac{1}{2+x}\cdot dx = \frac{1}{-y}\cdot dy$$

३): Taking Integration:

$$\int \frac{1}{2+x}\,dx = \int \frac{-1}{y}\,dy$$

$$\log(2+x) = -\log y + \log c$$

$$\log(2+x) = \log y^{-1} + \log c$$

$$-\log y^{-1} = -\log(2+x) + \log c$$

$$\log y^{-1} = \log(2+x) - \log c \qquad \{\because \text{But: } -\log c = +\log c = \text{constant}\}$$

$$\log y^{-1} = \log(2+x) + \log c$$

४): $\cancel{\log} y^{-1} = \cancel{\log}[(2+x)\cdot c]$ $\{\because$ *NOTE: Constant doesn't matter for anything. Everything is possible for constant. $\}$

$$y^{-1} = (2+x)\cdot c$$

$$\frac{1}{c} = \frac{(2+x)}{y^{-1}}$$

$$\frac{1}{c} = (2+x)\cdot y^{+1} \qquad \{\because \text{But: } \frac{1}{c} = c = \text{constant}\}$$

$$c = (2+x)y$$

$$\boxed{y(x+2) = c}$$

(13) The solution of differential equation $x\,dy - y\,dx = 0$ is

A) $y = x + c$ B) $x^2 - y^2 = c$

C) $xy = c$ ✓D) $y = cx$

पान-५७

$Sol^n \Rightarrow$ Ans: D)

क): $x \cdot dy - y \cdot dx = 0$

*NOTE: if two term having all same term
(In this problem x, y, d all are same at both like thi
then → 1) if middle sign +ve,
then multiply xy.
2) if middle sign -ve,
then devide x/y.

ख): $x\,dy - y\,dx = 0$

$d\left(y/x\right) = 0$ (∵ middle sign is -ve)

ग): Take Integration:

$\int \cancel{d}\left(\frac{y}{x}\right) = 0$

$\boxed{\frac{y}{x} = c}$ (∵ c = constant of Integration)

{∵ (Integration) × (derivative) = 1}

(14) The solution of DE $\frac{dy}{dx} = e^{x-y} + 3x^2 e^{-y}$ is

A) $e^y = e^x + x^3 + c$ B) $e^y = e^x + 3x^3 + c$

C) $e^y = e^x + 3x + c$ D) $e^x + e^y = 3x^3 + c$

$Sol^n \Rightarrow$ Ans: A)

क): $\frac{dy}{dx} = e^{x-y} + 3x^2 e^{-y}$

$\frac{dy}{dx} = e^x \cdot e^{-y} + 3x^2 e^{-y}$

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पान- ५८

$$\frac{dy}{dx} = e^{-y}\left[e^{x} + 3x^2\right]$$

$$y_1 = e^{-y}(e^{x} + 3x^2) \qquad \left\{\because \text{But}: \ y_1 = \frac{dy}{dx}\right\}$$

$$\frac{y_1}{e^{-y}} = (e^{x} + 3x^2)$$

$$y_1 \cdot e^{y} = e^{x} + 3x^2$$

ब):

$$\left(\frac{dy}{dx}\right) \cdot e^{y} = e^{x} + 3x^2 \qquad \left\{\because \text{Put}: \ y_1 = \frac{dy}{dx}\right\}$$

$$e^{y} \cdot dy = (e^{x} + 3x^2) \cdot dx$$

$$e^{y} \cdot dy = e^{x} \cdot dx + 3x^2 \cdot dx$$

क): Taking Integration:

$$\int e^{y} \cdot dy = \int e^{x} \cdot dx + 3\int x^2 \cdot dx \qquad \left\{\because \int e^{x}\,dx = e^{x}, \ \int x^n\,dx = \frac{x^{n+1}}{n+1}\right\}$$

$$e^{y} = e^{x} + \cancel{3}\left(\frac{x^3}{\cancel{3}}\right) + c$$

$$\boxed{e^{y} = e^{x} + x^3 + c}$$

(16) The solution of DE $\sec^2 x \cdot \tan y \cdot dx + \sec^2 y \cdot \tan x \cdot dy = 0$ is

A) $\sec^2 x \cdot \tan y = c$ B) $\tan x \cdot \sec^2 y = c$

✓ C) $\tan x \cdot \tan y = c$ D) $\sec^2 x \cdot \sec^2 y = c$

Solⁿ ⇒ Ans: C)

3): use 'drop' method:

$$(\sec^2 x \cdot \tan y)\, dx + (\sec^2 y \cdot \cancel{\tan x})\, dy = 0$$

drop = x

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पाठ-५e

बा): Taking Integration:

$$\tan y \int \sec^2 x \, dx = 0$$

$$\tan y \cdot \tan x = c$$

$\{\because \int \sec^2 x \, dx = \tan x,$ $c =$ constant of Integration$\}$

$$\boxed{\tan x \cdot \tan y = c}$$

(20) The solution of DE $3e^x \cdot \tan y \cdot dx + (1+e^x) \cdot \sec^2 y \cdot dy = 0$ is

A) $3 \log(1-e^x) = -\log \tan y + \log c$

B) $\log(1+e^x) = \log \tan y + \log c$

✓ C) $3 \log(1+e^x) = -\log \tan y + \log c$

D) $\log(1+e^x) = -\log \sin y + \log c$

Sol^n ⇒ Ans: c)

प्र): $3e^x \cdot \tan y \cdot dx + (1+e^x) \cdot \sec^2 y \cdot dy = 0$

$$3e^x \cdot \tan y \cdot dx = -(1+e^x) \cdot \sec^2 y \cdot dy$$

२बा): use VS form:

$$dx \cdot \frac{3e^x}{1+e^x} = -\frac{\sec^2 y}{\tan y} \cdot dy$$

$$3\int \frac{e^x}{1+e^x} dx = -\int \frac{\sec^2 y}{\tan y} dy$$

$\{\because$ Formula used: $\int \frac{f'(x)}{f(x)} dx = \log f(x)\}$

$$\boxed{3 \log(1+e^x) = -\log(\tan y) + \log c}$$

$\{\because \frac{\partial}{\partial x}(1+e^x) = 0 + e^x = e^x\}$

$\{\because \frac{\partial}{\partial x}(\tan x) = \sec^2 x\}$

पान- ६०

(21) The solution of DE $x(1+y^2)\,dx + y(1+x^2)\,dy = 0$

A) $(1-x^2)(1+y^2) = c$

B) $\tan^{-1}x + \tan^{-1}y = c$

C) $(1+x^2) = c(1+y^2)$

D) $(1+x^2)(1+y^2) = c$

Sol^n ⇒ Ans: D)

क):

$$x(1+y^2)\,dx + y(1+x^2)\,dy = 0$$

use VS form:

$$x(1+y^2)\,dx = -y(1+x^2)\,dy$$

$$\frac{x}{1+x^2}\,dx = \frac{-y}{1+y^2}\,dy$$

ख):

$$\int \frac{x}{1+x^2}\,dx = -\int \frac{y}{1+y^2}\,dy$$

adjustment of 2:

$$\frac{1}{2}\int \frac{2x}{1+x^2}\,dx = -\frac{1}{2}\int \frac{2y}{1+y^2}$$

$$\cancel{\tfrac{1}{2}}\log(1+x^2) = -\cancel{\tfrac{1}{2}}\log(1+y^2) + \log c$$

$\left\{ \because \text{formula used: } \int \frac{f'(x)}{f(x)}\,dx = \log f(x) \right\}$

$$\log(1+x^2) + \log(1+y^2) = \log c$$

$$\cancel{\log}\,[(1+x^2)\cdot(1+y^2)] = \cancel{\log}\,c$$

$$\boxed{(1+x^2)(1+y^2) = c}$$

(22) The solution of DE $\frac{dy}{dx} = (1+x)(1+y^2)$ is

A) $\tan^{-1}y = x + \frac{x^2}{2} + c$

B) $\log(1+y^2) = x + \frac{x^2}{2} + c$

C) $\tan^{-1}x = y + \frac{y^2}{2} + c$

D) $\frac{1}{2}\log\left(\frac{1+y}{1-y}\right) = x + \frac{x^2}{2} + c$

पान-69

Sol$^n \Rightarrow$ Ans: A)

पा):
$$\frac{dy}{dx} = (1+x)(1+y^2)$$

use VS form:

$$\frac{1}{1+y^2}\,dy = (1+x)\,dx$$

• Formula used:

$$\left\{\because \int \frac{1}{1+x^2}dx = \tan^{-1}x,\ \int dx = x,\ \int x^n dx = \frac{x^{n+1}}{n+1}\right\}$$

ग्रा): Taking Integration:

$$\int \frac{1}{1+y^2}dy = \int (1+x)\,dx$$

$$\tan^{-1}y = \int(dx + x\,dx)$$

$$\tan^{-1}y = \int dx + \int x\,dx$$

$$\therefore \tan^{-1}y = x + \frac{x^2}{2} + c$$

(23) The solution of DE $(e^x+1)y\cdot dy = (y+1)e^x\cdot dx$ is

A) $y - \log(1-y) = \log(e^x-1) + \log c$

✓ B) $y - \log(1+y) = \log(e^x+1) + \log c$

C) $y + \log(1-y) = \log(e^x+1) + \log c$

D) $y - \log(1+y) = \log(e^x-1) + \log c$

Sol$^n \Rightarrow$ Ans: B)

पा): $(e^x+1)y\cdot dy = (y+1)e^x\cdot dx$

use VS form:

$$\frac{y}{(y+1)}dy = \frac{e^x}{(e^x+1)}dx$$

adjustment of 1:

$$\frac{y+1-1}{y+1}dy = \frac{e^x}{e^x+1}dx$$

Adjustment of Sr:

$$\left(\frac{\cancel{y+1}}{\cancel{y+1}} - \frac{1}{y+1}\right)dy = \frac{e^x}{e^x+1}dx$$

$$\left[1 - \frac{1}{y+1}\right]dy = \frac{e^x}{e^x+1}dx$$

ग्रा): $\left[dy - \frac{1}{y+1}dy\right] = \frac{e^x}{e^x+1}dx$

Taking Integration:

$$\int dy - \int \frac{1}{y+1}dy = \int \frac{e^x}{e^x+1}dx$$

$$\therefore y - \log(1+y) = \log(e^x+1) + \log c$$

$$\left\{\because \text{formula: } \int dy = y,\ \int \frac{1}{x+1}dx = \log(x+1),\ \int \frac{f'(x)}{f(x)}dx = \log f(x)\right\}$$

पान - ६२

(24) The solution of DE $\frac{dy}{dx} = e^{x+y} + e^{y-x}$ is

A) $e^{-y} = e^{-x} - c$ B) $e^{y} = e^{x} - e^{-x} + c$

✓ c) $-e^{-y} = e^{x} - e^{-x} + c$ D) $e^{-y} = e^{x} + e^{-x} + c$

Sol^n ⇒ Ans: c)

स्प):

$$\frac{dy}{dx} = e^{x+y} + e^{y-x}$$

$$\frac{dy}{dx} = e^{x} \cdot \underline{e^{y}} + \underline{e^{y}} \cdot e^{-x}$$

$$\frac{dy}{dx} = e^{y}\left[e^{x} + e^{-x}\right]$$

$$\frac{1}{e^{y}} dy = (e^{x} + e^{-x})\, dx$$

$$e^{-y}\, dy = e^{x}\, dx + e^{-x}\, dx$$

स्प): Taking Integration:

$$\int e^{-y}\, dy = \int e^{x}\, dx + \int e^{-x}\, dx$$

$$\frac{e^{-y}}{-1} = e^{x} + \frac{e^{-x}}{-1} + c$$

$$\therefore \boxed{-e^{-y} = e^{x} - e^{-x} + c}$$

$\left\{\because \text{formula: } \int e^{ax}\, dx = \frac{e^{ax}}{a}\right\}$

(25) The solution of DE $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$ is

A) $\tan^{-1}x + \cot^{-1}y = c$ ✓ B) $\sin^{-1}x + \sin^{-1}y = c$

c) $\sec^{-1}x + \operatorname{cosec}^{-1}y = c$ D) $\sin^{-1}x - \sin^{-1}y = c$

Sol^n ⇒ Ans: B)

स्प):

$$\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1+x^2}} = 0$$

$$\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1+x^2}} \quad \therefore \frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1+x^2}}$$

use VS form:

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ख):

$$\frac{1}{\sqrt{1-y^2}}\,dy = \frac{-1}{\sqrt{1-x^2}}\,dx$$

Taking Integration:

$$\int \frac{1}{\sqrt{1-y^2}}\,dy = -\int \frac{1}{\sqrt{1-x^2}}\,dx$$

formula:
$$\left\{ \because \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x \right\}$$

$$\sin^{-1}y = -\sin^{-1}x + c$$

$$\sin^{-1}y + \sin^{-1}x = c$$

$$\boxed{\sin^{-1}x + \sin^{-1}y = c}$$

$\therefore$ Ans: B)

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• NOTE:

① Beta Function:

1) $\beta(m,n) = \int_0^1 x^{m-1} (1-x)^{n-1}\, dx, \quad m>0, n>0$

Corrollary:

i) $\beta(m+1, n+1) = \int_0^1 x^m (1-x)^n \, dx, \quad m>0, n>0$

ii) $\beta(m,n) = \beta(n,m)$

2) $\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cdot \cos^{2n-1}\theta \, d\theta$

② Standard Formulae:

$$\int_0^{\pi/2} \sin^p\theta \cdot \cos^q\theta \cdot d\theta = \frac{1}{2}\beta\left[\frac{p+1}{2}, \frac{q+1}{2}\right]$$

3) $\int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}} = \beta(m,n)$ .....(alternative defⁿ)

4) $\beta(m,n) = \frac{\Gamma m \cdot \Gamma n}{\Gamma m+n} = \frac{(m-1)!\,(n-1)!}{(m+n-1)!}$

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③ Standard Substitution to convert problems in Beta function:

i) $1 - f(x)$ $\Rightarrow$ $\frac{1}{1} \times f(x) = t$

$\therefore f(x) = t$

ii) $a - f(x)$ $\Rightarrow$ $\frac{1}{a} \times f(x) = t$

iii) $(x-a)^m (b-x)^n$ $\Rightarrow$ $(x-a) = (b-a)t$

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* Type: Beta Function :

① $\beta(m,n)$ is equal to

✓A) $\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$ B) $\int_0^\infty x^{m-1}(1-x)^{n-1}\,dx$

c) $\int_0^1 x^m(1-x)^n\,dx$ c) $\int_0^\infty e^{-x}x^{n-1}\,dx$

Sol^n ⇒ Ans: A)

$$\boxed{\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx}$$

....(यह स्टँडर्ड है, रटलो।)

② $\int_0^{\pi/2} \sin^p\theta \cdot \cos^q\theta\,d\theta$ is equal to

A) $B\left[\frac{p-1}{2}, \frac{q-1}{2}\right]$ B) $B\left[\frac{p+1}{2}, \frac{q+1}{2}\right]$

c) $B[p,q]$ ✓D) $\frac{1}{2}B\left[\frac{p+1}{2}, \frac{q+1}{2}\right]$

Sol^n ⇒ Ans: D)

$$\boxed{\frac{1}{2}B\left[\frac{p+1}{2}, \frac{q+1}{2}\right]}$$

....(यह स्टँडर्ड है, रटलो।)

③ $\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx$ is equal to

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A) $\beta\left(\frac{m-1}{2}, \frac{n-1}{2}\right)$ ✓B) $\beta(m,n)$

C) $\beta(m+1, n+1)$ D) $\beta(m,n)$

Solⁿ ⇒ Ans: B)

$$\boxed{\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} = \beta(m,n)}$$

....(यह स्टँडर्ड है, रखलो।)

(4) $\beta(m,n)$ is equal to

A) $\frac{m\Gamma n}{\Gamma m+n}$ B) $\frac{n\Gamma m}{\Gamma m+n}$ ✓C) $\frac{\Gamma m \cdot \Gamma n}{\Gamma m+n}$ D) $\frac{\Gamma m-1 \cdot \Gamma n-1}{\Gamma m+n}$

Solⁿ ⇒ Ans: C)

$$\boxed{\beta(m,n) = \frac{\Gamma m \cdot \Gamma n}{\Gamma m+n}}$$

....(यह स्टँडर्ड है, रखलो।)

(5) Duplication formula for Gamma function is

A) $\Gamma m \cdot \Gamma m+\frac{1}{2} = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma 2m-1$ B) $\Gamma m \, \Gamma m+\frac{1}{2} = \frac{\sqrt{\pi}}{2^2} \Gamma 2m$

✓C) $\Gamma m \, \Gamma m+\frac{1}{2} = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma 2m$ D) $\Gamma m \, \Gamma m+\frac{1}{2} = \frac{\pi}{2^{2m-1}} \Gamma 2m$

Solⁿ ⇒ Ans: C)

$$\boxed{\Gamma m \cdot \Gamma m+\frac{1}{2} = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma 2m}$$

....(यह स्टँडर्ड है, रखलो।)

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⑥ $B(4,5)$ is represented by

A) $\int_0^{\pi/2} \sin^3 x \cdot \cos^4 x \, dx$ ✓B) $\int_0^1 x^3 (1-x)^4 \, dx$

C) $\int_0^1 x^2 (1-x)^3 \, dx$ D) $\int_0^2 x^3 (1-x)^4 \, dx$

Sol^n ⇒ Ans: B)

$$\beta(m,n) = \int_0^1 x^{m-1} (1-x)^{n-1} \, dx \quad \text{....(std)}$$

$$\beta(4,5) = ? \quad \text{....(given)}$$

$$m = 4, \quad n = 5$$

$$\beta(4,5) = \int_0^1 x^{4-1} (1-x)^{5-1} \, dx$$

$$= \boxed{\int_0^1 x^3 (1-x)^4 \, dx}$$

⑦ $\int_0^1 x^{1/2} (1-x)^{5/2} \, dx$ is equal to

✓A) $B\left(\frac{3}{2}, \frac{7}{2}\right)$ B) $\left(\frac{-1}{2}, \frac{1}{2}\right)$ C) $\frac{1}{2} B\left(\frac{1}{2}, \frac{3}{2}\right)$ D) $B\left(\frac{1}{2}, \frac{3}{2}\right)$

Sol^n ⇒ Ans: A)

$$\int_0^1 x^m (1-x)^n \, dx = \beta(m+1, n+1) \quad \text{....(std)}$$

$$\int_0^1 x^{1/2} (1-x)^{5/2} \, dx = \beta\left[\frac{1}{2}+1, \frac{5}{2}+1\right]$$

$$\boxed{\beta = [3/2, 7/2]}$$

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(8) $\int_0^{\pi/2} \sin^{3/2}\theta \cdot \cos^4\theta \, d\theta$ is equal to

✓A) $\frac{1}{2} B\left[\frac{5}{4}, \frac{5}{2}\right]$ B) $B\left[\frac{5}{2}, \frac{3}{2}\right]$

C) $\frac{1}{2} B\left[\frac{3}{2}, \frac{1}{2}\right]$ D) $B\left[\frac{5}{4}, \frac{5}{2}\right]$

Sol^n ⇒ Ans: A)

$\int_0^{\pi/2} \sin^p\theta \cdot \cos^q\theta \cdot d\theta = \frac{1}{2} B\left[\frac{p+1}{2}, \frac{q+1}{2}\right]$ ....(std)

$\int_0^{\pi/2} \sin^{3/2}\theta \cdot \cos^4\theta \cdot d\theta = \frac{1}{2} B\left[\frac{3/2+1}{2}, \frac{4+1}{2}\right]$

$p = 3/2, \; q = 4$

$= \frac{1}{2} B\left[\frac{5/2}{2}, \frac{5}{2}\right]$

$= \frac{1}{2} B\left[\frac{5}{2} \times \frac{1}{2}, \frac{5}{2}\right]$

$= \boxed{\frac{1}{2} B\left[\frac{5}{4}, \frac{5}{2}\right]}$

(9) $B(3,5)$ is equal to

✓A) $1/105$ B) $2/205$ C) $4/105$ D) $1/21$

Sol^n ⇒ Ans: A)

$B(m,n) = \frac{(m-1)!\,(n-1)!}{(m+n-1)!}$ ....(std)

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$$\beta(3,5) = \frac{(3-1)!\,(5-1)!}{(3+5-1)!}$$

$$= \frac{2!\ 4!}{7!}$$

$$= \frac{2! \times 4!}{7!} \qquad \{\because 2! \times 4! = \text{Calcy: Shift } x^{-1}\}$$

$$= \boxed{\frac{1}{105}}$$

(10) $\int_0^\infty \frac{x^4}{(1+x)^7}\,dx$ is equal to

✓A) $1/30$ B) $1/20$ C) $1/15$ D) $1/60$

Sol^n → Ans: A)

क): $\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx = \beta(m,n)$

$\therefore \int_0^\infty \frac{x^{5-1}}{(1+x)^{5+2}}\,dx = \beta(5,2)$

ख): $\beta(m,n) = \frac{(m-1)!\,(n-1)!}{(m+n-1)!}$

$$\beta(5,2) = \frac{(5-1)!\,(2-1)!}{(5+2-1)!}$$

$$= \frac{4! \times 1!}{6!}$$

$$= \boxed{1/30}$$

(11) $\Gamma p \cdot \Gamma 1-p$ =

A) $\frac{1}{2} \frac{\pi}{\sin p\pi}$ ✓B) $\frac{\pi}{\sin p\pi}$ c) $\frac{1}{\sin p\pi}$ D) $\frac{\pi/2}{\sin p\pi/2}$

Sol^n ⇒ Ans: B)

क): $\boxed{\Gamma p \cdot \Gamma 1-p = \frac{\pi}{\sin p\pi}}$ .....(यह स्टैंडर्ड है, रटलो।)

(12) The appropriate substitution to reduce the given integral $\int_0^2 x \sqrt[3]{8-x^3}\, dx$ to Beta function integral

A) $x = t^3$ B) $x^3 = t$ C) $x = 8t^3$ ✓D) $x^3 = 8t$

Sol^n ⇒ Ans: D)

क): substitution : $a - f(x) = \frac{1}{a} f(x) = t$ ...(plz see NOTE.)

$$8 - x^3 = \frac{1}{8} x^3 = t$$

$$\therefore \boxed{x^3 = 8t}$$

(13) The appropriate substitution to reduce the given integral $\int_0^n x^n (n-x)^m\, dx$ to Beta function integral

A) $x = t^n$ B) $x = mt$ C) $x = t$ ✓D) $x = nt$

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$Sol^n \Rightarrow$ Ans: D)

5): Substitution: $a - f(x) \equiv \frac{1}{a} f(x) = t$ ....(Plz ref. NOTE)

$n - x \equiv \frac{1}{n} x = t$

$\therefore \boxed{x = nt}$

(14) The appropriate substitution to reduce the given integral $\int_a^b (x-a)^m (b-x)^n \, dx$ to Beta function integral

A) $x = (b-a)t$ ✓B) $(x-a) = (b-a)t$

c) $(x+a) = (b-a)t$ D) $(x-a) = (b+a)t$

$Sol^n \Rightarrow$ Ans: B)

$\boxed{(x-a) = (b-a)t}$ .....(यह स्टँडर्ड है, रटलो।)

(15) The appropriate substitution to reduce the given integral $\int_5^9 (x-5)^{1/4} (9-x)^{1/4} \, dx$ is to beta function integral

✓A) $x = 4t+5$ B) $x = 4t$ c) $x = 4t-5$ D) $x = 14t+5$

$Sol^n \Rightarrow$ Ans: A)

5): $\int_5^9 (x-5)^{1/4} (9-x)^{1/4} \, dx$

std substitution:

$(x-a) = (b-a)t$

$(x-5) = (9-5)t$

$x - 5 = 4t$

$\boxed{x = 4t+5}$

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(16) The value of integral $\int_0^1 x^{m-1}(1-x^2)^{n-1}\,dx$ by using substitution $x=\sqrt{t}$ is

A) $\frac{1}{2}B\left[\frac{m-1}{2}, n-1\right]$ ✓ C) $\frac{1}{2}B\left[\frac{m}{2}, n\right]$

B) $B\left[\frac{m}{2}, n\right]$ D) $B\left[\frac{m-1}{2}, n-1\right]$

sol^n ⇒ Ans: C)

क):

Put: $x=\sqrt{t}$

$dx = \frac{1}{2\sqrt{t}}\,dt$

limit:	x	0	1
	t	0	1

$\therefore$ use limit of $t = 0 \to 1$

ख):

$$I = \int_0^1 x^{m-1}(1-x^2)^{n-1}\,dx$$

$$= \int_0^1 (\sqrt{t})^{m-1}\left[1-(\sqrt{t})^2\right]^{n-1}\left(\frac{1}{2\sqrt{t}}\,dt\right)$$

$$= \int_0^1 (t^{1/2})^{m-1}\,[1-t]^{n-1}\left[\frac{1}{2t^{1/2}}\,dt\right]$$

$$= \frac{1}{2}\int_0^1 t^{\frac{1}{2}m-\frac{1}{2}}\cdot t^{-1/2}\,[1-t]^{n-1}\,dt$$

$\{\because a^m \times a^n = a^{m+n}\}$

$$= \frac{1}{2}\int_0^1 t^{\frac{1}{2}m-\frac{1}{2}-\frac{1}{2}}\,(1-t)^{n-1}\,dt$$

$\{\because -\frac{1}{2}-\frac{1}{2} = -1 : \text{calcy}\}$

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$$= \frac{1}{2}\int_0^1 t^{\frac{1}{2}m-1}\,(1-t)^{n-1}\,dt$$

$$= \boxed{\frac{1}{2} B\left[\frac{m}{2}, n\right]}$$

$\left\{\because \text{Formula used: } \int_0^1 t^{m-1}\cdot(1-t)^{n-1}\,dt = B(m,n)\right\}$

(17) The value of integral $\int_0^1 x^3 (1-\sqrt{x})^5 dx$ by using substitution $\sqrt{x} = t$.

A) $2B(8,6)$ B) $B(8,6)$ C) $2B(7,6)$ D) $2B(6,4)$

Sol^n ⇒ Ans: A)

(i): Put: $\sqrt{x} = t$

$x = t^2$

$dx = 2t\,dt$

Limit:

x	0	1
t	0	1

⇒ use limit of $t = 0 \to 1$

(ii): $I = \int_0^1 x^3 [1-\sqrt{x}]^5\,dx$

$$\cancel{= \int_0^1 [\sqrt{x}]^3 [1-t]^5 (2t\,dt)}$$

$$I = \int_0^1 (t^2)^3 [1-t]^5 (2t\,dt)$$

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$$I = 2\int_0^1 t^6 \cdot (1-t)^5 \cdot t \, dt$$

$$= 2\int_0^1 t^7 \, (1-t)^5 \, dt$$

Here: $m = 7$, $n = 5$

$$= 2\,B[7+1, 5+1]$$

$$= \boxed{2\,B[8, 6]}$$

$\left\{ \because \text{Formula used: } \int_0^1 x^m \cdot (1-x)^n \, dx = \beta(m+1, n+1) \right\}$

(18) The value of integral $\int_0^1 (1 - x^{1/n})^m \, dx$ by using substitution $x^{1/n} = t$ is

A) $B(n, m+1)$ B) $nB(n, m+1)$ C) $B(m, n+1)$ D) $mB(m, n+1)$

Sol^n → Ans: B)

(1): Put: $x^{1/n} = t$

$x = t^n$

$dx = n t^{n-1} \, dt$

Limit:		
x	0	1
t	0	1

⇒ use limit of $t = 0 \to 1$

(2): $$I = \int_0^1 [1 - x^{1/n}]^m \, dx$$

$$I = \int_0^1 [1 - (t^{\cancel{n}})^{1/\cancel{n}}]^m \, (n t^{n-1} \, dt)$$

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$$I = \int_0^1 [1-t]^m \cdot n \cdot t^{n-1} \cdot dt$$

$$I = n \int_0^1 (1-t)^m \cdot t^{n-1}\, dt$$

$$\boxed{I = n\,\beta(n, m+1)}$$

$$\left\{ \because\ n\int_0^1 (1-t)^m \cdot t^{n-1}\, dt = n\beta(n, m+1) \ \ldots (\text{std}) \right\}$$

(19) The value of integral $\int_0^1 (1-x^{1/n})^m dx$ by using substition $x^{1/n} = t$ is

A) $B(n, m+1)$ ✓B) $nB(n, m+1)$ C) $B(m, n+1)$ D) $mB(m, n+1)$

Sol^n ⇒ Ans: B) ($\cdots$ Refer Q. 18)

(20) The integral $\int_3^7 \sqrt{(x-3)(7-x)}\, dx$ by using substitution $x = 4t+3$ transform to

A) $\int_0^1 t^{1/2}(1-t)^{1/2} dt$ B) $4\int_0^1 t^{1/2}(1-t)^{1/2} dt$

✓C) $16\int_0^1 t^{1/2}(1-t)^{1/2} dt$ D) $16\int_0^1 t^{1/4}(1-t)^{1/4} dt$

Sol^n ⇒ Ans: C)

ग): Put: $x = 4t+3$

$dx = 4\,dt + 0$

limit:

x	3	7
t	0	1

⇒ use limit of $t = 0 \rightarrow 1$

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Q):

$$I = \int_3^7 \sqrt{(x-3)(7-x)}\, dx$$

$$I = \int_0^1 \sqrt{[(4t+3)-3][7-(4t+3)]}\ (4dt)$$

$$= \int_0^1 \sqrt{(4t)(4-4t)}\ (4dt)$$

$$= \int_0^1 (4t)^{1/2} [4-4t]^{1/2} (4dt)$$

$$= \int_0^1 4^{1/2}.\ t^{1/2} [4(1-t)]^{1/2} (4dt)$$

$\{\because 4^{1/2} = \sqrt{4} = 2\}$

$$= \int_0^1 (2)\ t^{1/2} [4]^{1/2} (1-t)^{1/2} (4dt)$$

$$= \int_0^1 8\ t^{1/2}\ (2)\ (1-t)^{1/2}\ dt$$

$$\boxed{I = 16 \int_0^1 t^{1/2} (1-t)^{1/2}\ dt}$$

21) The value of $\int_0^\infty \frac{x^8}{(1+x)^{24}} dx - \int_0^\infty \frac{x^{14}}{(1+x)^{24}} dx$ is

A) $B(8,14) - B(9,15)$ B) $B(9,15) - B(8,14)$

C) 0 D) None

Sol^n ⇒ Ans: C)

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क): $$= \int_0^\infty \frac{x^8}{(1+x)^{24}}\,dx - \int_0^\infty \frac{x^{14}}{(1+x)^{24}}\,dx$$

$$= \int_0^\infty \frac{x^{9-1}}{(1+x)^{9+15}}\,dx - \int_0^\infty \frac{x^{15-1}}{(1+x)^{15+9}}\,dx$$

$m=9,\ n=15$ $\qquad$ $m=15,\ n=9$

$$= B(9,15) - B(15,9)$$

$$= \boxed{0}$$

$\left\{ \because \text{Formula used: } \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx = B(m,n) \right\}$

22. $\int_0^{\pi/2} \sqrt{\tan\theta}\,d\theta$ is equal to

✓A) $\frac{1}{2}B\left[\frac{3}{4},\frac{1}{4}\right]$ $\quad$ B) $B\left[\frac{3}{4},\frac{1}{4}\right]$ $\quad$ C) $B\left[\frac{1}{2},\frac{-1}{2}\right]$ $\quad$ D) $\frac{1}{2}B\left[\frac{1}{2},\frac{-1}{2}\right]$

Solⁿ→ Ans: A)

क): $$I = \int_0^{\pi/2} \sqrt{\tan\theta}\,d\theta$$

$\{\because \sqrt{x} = x^{1/2}\}$

$$= \int_0^{\pi/2} (\tan\theta)^{1/2}\,d\theta$$

$\left\{\because \tan = \frac{\sin}{\cos}\right\}$

$$= \int_0^{\pi/2} \frac{(\sin\theta)^{1/2}}{(\cos\theta)^{1/2}}\,d\theta$$

$$= \int_0^{\pi/2} \sin^{1/2}\theta \cdot \cos^{-1/2}\theta\,d\theta$$

ख): Formula: $\int_0^{\pi/2} \sin^p\theta \cdot \cos^q\theta\,d\theta = \frac{1}{2}B\left[\frac{p+1}{2}, \frac{q+1}{2}\right]$

$$p = 1/2,\quad q = -1/2$$

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$$= \frac{1}{2} B\left[\frac{1/2+1}{2}, \frac{-1/2+1}{2}\right]$$

$$= \frac{1}{2} B\left[\frac{3/2}{2}, \frac{1/2}{2}\right]$$

$$= \boxed{\frac{1}{2} B\left[\frac{3}{4}, \frac{1}{4}\right]} \qquad \left\{\because \frac{3/2}{2} = \frac{3}{4} : \text{calcy}\right\}$$

(23) $\int_0^{\pi/2} \sqrt{\cot\theta}\, d\theta$ is equal to

A) $\frac{1}{2} B\left[\frac{-1}{2}, \frac{1}{2}\right]$ B) $B\left[\frac{1}{4}, \frac{3}{4}\right]$ C) $B\left[\frac{1}{2}, \frac{-1}{2}\right]$ D) $\frac{1}{2} B\left[\frac{1}{4}, \frac{3}{4}\right]$

Sol^n ⇒ Ans: D)

D): $I = \int_0^{\pi/2} \sqrt{\cot\theta}\, d\theta \qquad \left\{\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right\}$

$$= \int_0^{\pi/2} (\cot\theta)^{1/2} d\theta \qquad \left\{\because \sqrt{x} = x^{1/2}\right\}$$

$$= \int_0^{\pi/2} \frac{(\cos\theta)^{1/2}}{(\sin\theta)^{1/2}} d\theta$$

$$= \int_0^{\pi/2} \sin^{-1/2}\theta \cdot \cos^{1/2}\theta\, d\theta$$

D): Formula: $\int_0^{\pi/2} \sin^p\theta \cdot \cos^q\theta\, d\theta = \frac{1}{2} B\left[\frac{p+1}{2}, \frac{q+1}{2}\right]$

$$p = -1/2, \quad q = 1/2$$

$$= \frac{1}{2} B\left[\frac{-1/2+1}{2}, \frac{1/2+1}{2}\right]$$

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$$= \frac{1}{2} B\left[\frac{1/2}{2}, \frac{3/2}{2}\right] \qquad \left\{\because -\frac{1}{2}+1 = \frac{1}{2} : \text{calcy}\right\}$$

$$= \boxed{\frac{1}{2} B\left[1/4, 3/4\right]} \qquad \left\{\because \frac{1/2}{2} = \frac{1}{4} : \text{calcy}\right\}$$

(24) $\int_0^{\pi/2} 1/\sqrt{\sin\theta}\, d\theta$ is equal is

A) $B\left(\frac{1}{4}, \frac{1}{2}\right)$ B) $\frac{1}{2} B\left(\frac{1}{4}, \frac{1}{2}\right)$ C) $\frac{1}{2} B\left(-\frac{1}{2}, 0\right)$ D) $B\left(-\frac{1}{2}, 0\right)$

Sol^n ⇒ Ans : B)

1): $I = \int_0^{\pi/2} 1/\sqrt{\sin\theta}\, d\theta$

$$I = \int_0^{\pi/2} \frac{1}{(\sin\theta)^{1/2}} d\theta \qquad \left\{\because \sqrt{x} = x^{1/2}\right\}$$

$$= \int_0^{\pi/2} \sin^{-1/2}\theta\, d\theta$$

$$= \int_0^{\pi/2} \sin^{-1/2}\theta \cdot \cos^0\theta\, d\theta \qquad \{\because \cos^0\theta = 1 = \text{Adjustment of } 1\}$$

2): Formula: $\int_0^{\pi/2} \sin^p\theta \cdot \cos^q\theta\, d\theta = \frac{1}{2} B\left[\frac{p+1}{2}, \frac{q+1}{2}\right]$

$p = -1/2,\ q = 0$

$$= \frac{1}{2} \beta\left[\frac{-1/2+1}{2}, \frac{0+1}{2}\right]$$

calcy: $\left\{\because -\frac{1}{2}+1 = \frac{1}{2}, \frac{\frac{1}{2}}{2} = \frac{1}{4}\right\}$

$$= \boxed{\frac{1}{2} B\left[1/4, 1/2\right]}$$

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25 $\int_0^1 \frac{x^3+x^2}{(1+x)^7}\,dx$ is equal to

A) $\frac{1}{240}$ B) $\frac{1}{30}$ C) $\frac{1}{60}$ D) $\frac{1}{120}$

Sol^n ⇒ Ans: C)

ख):

$$I = \int_0^1 \frac{x^3+x^2}{(1+x)^7}\,dx = \boxed{\frac{1}{60}}$$

..... (solve in calcy - ES-calcy)

*Note: ES::

$$\int_0^1 \frac{(x)^3+(x)^2}{(1+x)^7}\,dx = \frac{1}{60}$$

X = ALPHA)

26 $B\left(\frac{1}{4}, \frac{3}{4}\right)$ is equal to

A) $\frac{2\pi}{\sqrt{3}}$ B) 0 C) 2π D) $\pi\sqrt{2}$

Sol^n ⇒ Ans: D)

ख): Use Formula: $B(m,n) = \frac{\Gamma m \cdot \Gamma n}{\Gamma m+n}$

$$B\left(\frac{1}{4}, \frac{3}{4}\right) = \frac{\Gamma 1/4 \cdot \Gamma 3/4}{\Gamma \frac{1}{4}+\frac{3}{4}}$$

$\left\{\because \frac{1}{4}+\frac{3}{4} = 1 : \text{calcy}\right\}$

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$\boxed{\text{पान-902}}$

ख):

$$\beta\left(\tfrac{1}{4},\tfrac{3}{4}\right) = \frac{\Gamma 1/4 \, \Gamma 3/4}{\Gamma 1} \qquad \{\because \Gamma 1 = 1\}$$

$$= \Gamma 1/4 \cdot \Gamma 3/4$$

ग): Use Formula: $\Gamma p \cdot \Gamma 1-p = \dfrac{\pi}{\sin p\pi}$

$$\Gamma 1/4 \cdot \Gamma 1-1/4 = \frac{\pi}{\sin \frac{1}{4}\pi} = \frac{\pi}{\sin \pi/4}$$

$(\because p = 1/4)$

$$= \frac{\pi}{1/\sqrt{2}}$$

$\{\because \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \text{ES calcy : Radian Mode}\}$

$$= \sqrt{2}/1 \times \pi$$

$\{\because \frac{3}{4} = 1 - \frac{1}{4} \text{ .... Adjustment}\}$

$$= \boxed{\pi\sqrt{2}}$$

(27) $\beta(5/4, 5/4)$ is equal to

A) $\frac{2}{3}[\Gamma 1/4]^2$ ✓B) $\frac{1}{12\sqrt{\pi}}[\Gamma 1/4]^2$

c) $\frac{2}{3\sqrt{\pi}}[\Gamma 1/4]$ D) $\frac{1}{\sqrt{\pi}}[\Gamma 1/4]^2$

Sol^n) Ans: B)

क): Use Formula: $\beta(m,n) = \dfrac{\Gamma m \cdot \Gamma n}{\Gamma m+n}$

$$\beta(5/4, 5/4) = \frac{\Gamma 5/4 \, \Gamma 5/4}{\Gamma 5/4 + 5/4}$$

$$= \frac{\left(\frac{5}{4}-1\right)\Gamma\left(\frac{5}{4}-1\right) \cdot \left(\frac{5}{4}-1\right)\Gamma\left(\frac{5}{4}-1\right)}{\Gamma\left(\frac{10}{4}\right)}$$

ख):

{∵ Formula used: $\Gamma n = (n-1)\,\Gamma(n-1)$ }

$$= \frac{\frac{1}{4}\Gamma\frac{1}{4} \cdot \frac{1}{4}\Gamma\frac{1}{4}}{\Gamma\frac{5}{2}} \qquad \left\{\because \frac{5}{4}-1=\frac{1}{4} : \text{calcy}\right\}$$

$$= \frac{\left(\frac{1}{4}\right)^2\left[\Gamma\frac{1}{4}\right]^2}{\Gamma\frac{5}{2}} \quad \text{....} (1)$$

ग): $D^r \rightarrow$ 1) $\Gamma\frac{5}{2} = \left(\frac{5}{2}-1\right)\Gamma\left(\frac{5}{2}-1\right)$ $\quad \{\because \Gamma n = (n-1)\,\Gamma n-1\}$

$$= \frac{3}{2}\Gamma\frac{3}{2} \quad \text{....} (2)$$

2) $\Gamma\frac{3}{2} = \left(\frac{3}{2}-1\right)\Gamma\left(\frac{3}{2}-1\right)$ $\quad \{\because \Gamma n = (n-1)\,\Gamma n-1\}$

$$= \frac{1}{2}\Gamma\frac{1}{2}$$ put in eq^n (2):

घ): $D^r \Rightarrow \Gamma\frac{5}{2} = \frac{3}{2}\left[\frac{1}{2}\Gamma\frac{1}{2}\right]$

$$= \frac{3}{4}\Gamma\frac{1}{2}$$

$$\Gamma\frac{5}{2} = \frac{3}{4}\sqrt{\pi} \qquad \left\{\because \Gamma\frac{1}{2} = \sqrt{\pi}\right\}$$

put in eq^n (1)

पान-१०४

च):

$$= \frac{(1/4)^2 [\Gamma 1/4]^2}{\Gamma 5/2}$$

$$= \frac{(1/4)^2 [\Gamma 1/4]^2}{3/4 \sqrt{\pi}}$$

$$= \frac{1/16 [\Gamma 1/4]^2}{\frac{3}{4} \sqrt{\pi}}$$

$$= \frac{1}{16} \times \frac{4}{3} \cdot \frac{1}{\sqrt{\pi}} \times [\Gamma 1/4]^2$$

$$= \boxed{\frac{1}{12\sqrt{\pi}} (\Gamma 1/4)^2}$$

28 $\beta(n, n+1)$ is equal to

A) $\frac{1}{2} \frac{(\Gamma n)^2}{\Gamma 2n}$ B) $\frac{(\Gamma n)^2}{\Gamma 2n}$ C) $\frac{1}{2} \frac{\Gamma n \, \Gamma n-1}{\Gamma 2n}$ D) $\frac{1}{2} \frac{(\Gamma n+1)^2}{\Gamma 2n}$

Sol^n ⇒ Ans: A)

क): Formula: $\beta(m,n) = \dfrac{\Gamma m \cdot \Gamma n}{\Gamma m+n}$

$\beta(n, n+1)$ ....(given)

∴ Here: $m = n$ & $n = n+1$ put in above formula:

ख): $$\beta(n, n+1) = \frac{\Gamma n \cdot \Gamma n+1}{\Gamma n+n+1}$$

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$\boxed{\text{पान-१०४}}$

$$\beta(m,n) = \frac{\Gamma n \cdot \Gamma n+1}{\Gamma 2n+1}$$

$$= \frac{\Gamma n \cdot \cancel{n}\,\Gamma n}{2\cancel{n}\,\Gamma 2n}$$

Rule:
$\{\because \Gamma n+1 = n\,\Gamma n$ ....(std)
simillary: $\Gamma 2n+1 = 2n\,\Gamma 2n \}$

$$= \boxed{\frac{1}{2}\,\frac{(\Gamma n)^2}{2n}}$$

(29) The value of $\beta(m,n+1) + \beta(m+1,n)$ is

A) $\beta(m,n)$ B) $2\beta(m+1,n)$ C) $2\beta(m,n+1)$ D) $\beta(m-1,n-1)$

Sol^n ⇒ Ans: A)

ब): Formula: $\beta(m,n) = \dfrac{\Gamma m \cdot \Gamma n}{\Gamma m+n}$

अ): $\beta(\underset{m}{\uparrow\, m}, \underset{n}{\uparrow\, n+1}) + \beta(\underset{m}{\uparrow\, m+1}, \underset{n}{\uparrow\, n})$

put in above formula:

$$= \frac{\Gamma m \cdot \Gamma n+1}{\underset{=}{\Gamma m+n+1}} + \frac{\Gamma m+1\ \Gamma n}{\underset{=}{\Gamma m+1+n}}$$

$$= \frac{\Gamma m \cdot n\,\Gamma n + m\,\Gamma m \cdot \Gamma n}{\underset{=}{\Gamma m+n+1}} \qquad \{\because \Gamma n+1 = n\,\Gamma n\}$$

$$= \frac{(\Gamma m \cdot \Gamma n)[\cancel{n+m}]}{(\cancel{m+n})\,\Gamma m+n} \qquad \text{.....}(\Gamma n+1 = n\,\Gamma n \text{ Rule used Here})$$

$$= \frac{\Gamma m \cdot \Gamma n}{\Gamma m+n}$$

$$= \boxed{\beta(m,n)}$$ ....(Plz refer step क))

(30) $\beta(m,n) \times \beta(m+n,p)$ is equal to

A) $\frac{\Gamma m\ \Gamma p}{\Gamma m+n+p}$ B) $\frac{\Gamma m\ \Gamma n\ \Gamma p}{\Gamma m+n+p}$ C) $\frac{\Gamma m\ \Gamma n}{\Gamma m+n+p}$ D) $\frac{\Gamma m\ \Gamma n\ \Gamma p}{\Gamma m-n-p}$

Sol^n ⇒ Ans: B)

क): Formula: $\beta(m,n) = \frac{\Gamma m \cdot \Gamma n}{\Gamma m+n}$

ख): $= \beta(\underset{m}{\uparrow m}, \underset{n}{\uparrow n}) \times \beta(\underset{m}{\uparrow m+n}, \underset{n}{\uparrow p})$

= put this values in above formula:

$$= \frac{\Gamma m \cdot \Gamma n}{\cancel{\Gamma m+n}} \times \frac{\cancel{\Gamma m+n} \cdot \Gamma p}{\Gamma m+n+p}$$

$$= \boxed{\frac{\Gamma m \cdot \Gamma n \cdot \Gamma p}{\Gamma m+n+p}}$$

∴ Ans: B):

पान - ०३

• धड्याचे नाव :

• Application of Differential Equation.

● NOTE:

① Orthogonal Trajectories Types:

Rectangular Trajectory

$f(x, y, a) = 0$

$a \to$ parameter

1) write given DE
i.e. $f(x, y, a) = 0$

2) differentiate it wrt x

3) Eliminate parameter a from eq^n ①

4) Replace: $\frac{dy}{dx} \to \frac{-dx}{dy}$

5) use VS form both side

Polar Trajectories

$f(r, \theta, a) = 0$

$a \to$ parameter

1) write given DE
i.e. $f(r, \theta, a) = 0$

2) differentiate it wrt θ

3) Eliminate parameter a from eq^n ①

4) Replace: $\frac{dr}{d\theta} \to -r^2 \frac{d\theta}{dr}$

5) use VS form both side.

*** NOTE:**

1) $\log e^{f(x)} = f(x) \cdot \log e = f(x)$

2) $e^{\log f(x)} = f(x)$

3) $y^x = a \; = \; \log_a y = x$

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पान - 04

* Type : Orthogonal Trajectories:

(1) The differential equation of orthogonal trajectories of family of straight lines $y = mx$ is

A) $\frac{dx}{dy} = -\frac{y}{x}$ B) $\frac{dx}{dy} = -\frac{x}{y}$ ✓C) $\frac{dy}{dx} = \frac{y}{x}$ D) $\frac{dy}{dx} = m$

Sol^n ⇒ Ans : c)

E): $y = mx$ ⇒ $m = y/x$

Taking derivative:

$\frac{dy}{dx} = m(1)$

$$\boxed{\frac{dy}{dx} = \frac{y}{x}}$$

(2) If the family of curves is given by $x^2 + 2y^2 = c^2$ then the DE of orthogonal trajectories of family is

A) $x - 2y\frac{dy}{dx} = 0$ B) $x + 2y\frac{dx}{dy} = 0$

✓C) $x + 2y\frac{dy}{dx} = 0$ D) $x\frac{dy}{dx} + y = 0$

Sol^n ⇒ Ans : c)

E): $x^2 + 2y^2 = c^2$

Take derivatives:

$2x + 2(2y)\frac{dy}{dx} = 0$

पान - ०६

ब) : $2x + 4y\frac{dy}{dx} = 0$

$2\left[x + 2y\frac{dy}{dx}\right] = 0$

$x + 2y\frac{dy}{dx} = \frac{0}{2}$

$$\boxed{x + 2y\frac{dy}{dx} = 0}$$

③ The differential equation of orthogonal trajectories of family of curves $xy = c$ is

A) $x\frac{dx}{dy} + y = 0$ ✓B) $-x\frac{dx}{dy} + y = 0$

C) $-x\frac{dx}{dy} - y = 0$ D) $x\frac{dy}{dx} + y = 0$

Sol^n ⇒ Ans: B)

ड): $\underset{u}{x}\,\underset{v}{y} = c$ $\{\because uv = u'v + uv'\}$

ब): Take derivatives:

$(1)(y) + (x)(dy/dx) = 0$

$y + x\frac{dy}{dx} = 0$

Replace: $\frac{dy}{dx} = -\frac{dx}{dy}$ ....(By cond^n of Trajectory)

$$\boxed{y - x\,dx/dy = 0}$$

पान-०७

(4) If the family of curves is given by $y^2 = 4ax$ then the DE of orthogonal trajectories of family is

A) $2y\frac{dy}{dx} = 4a$ B) $2y\frac{dy}{dx} = \frac{y^2}{x}$

✓ C) $-2y\frac{dx}{dy} = \frac{y^2}{x}$ D) $2y\frac{dy}{dx} = \frac{x}{y^2}$

Sol^n ⇒ Ans: C)

अ): $y^2 = 4ax \Rightarrow 4a = \frac{y^2}{x}$

Take derivative:

$2y\frac{dy}{dx} = 4a(1)$

ब): $2y\frac{dy}{dx} = \left[\frac{y^2}{x}\right](1)$

क): Replace $\frac{dy}{dx} = -\frac{dx}{dy}$ .... (By trajectory cond^n)

$2y\left(\frac{-dx}{dy}\right) = \frac{y^2}{x}$

$$\boxed{-2y\frac{dx}{dy} = \frac{y^2}{x}}$$

(5) The differential equation of orthogonal trajectories of family of curves $2x^2 + y^2 = cx$ is

A) $4x + 2y\frac{dy}{dx} = \frac{2x^2+y^2}{x}$ ✓ B) $4x + 2y\frac{dx}{dy} = \frac{2x^2+y^2}{x}$

C) $4x + 2y\frac{dx}{dy} = \frac{2x^2+y^2}{x}$ D) None

पान -02

Sol^n ⇒ Ans: B)

क): $2x^2+y^2 = cx \quad \cdots ①$

Take derivative:

$4x + 2y\frac{dy}{dx} = c(1)$

ख): put c in ①:

$2x^2+y^2 = \left[4x+2y\frac{dy}{dx}\right](x)$

$\frac{2x^2+y^2}{x} = 4x+2y\frac{dy}{dx}$

ग): Replace ⇒ $\frac{dy}{dx} = \frac{-dx}{dy}$ ....(By Trajectory cond^n)

$\frac{2x^2+y^2}{x} = 4x + 2y\left(\frac{-dx}{dy}\right)$

$$\boxed{\frac{2x^2+y^2}{x} = 4x - 2y\frac{dx}{dy}}$$

⑥ The differential equation of orthogonal trajectories of family of curves $x^2+cy^2=1$ is

A) $x-\left[\frac{1-x^2}{y}\right]\frac{dx}{dy}=0$ B) $x+\left[\frac{1-x^2}{y}\right]\frac{dy}{dx}=0$

C) $x+\left[\frac{1-x^2}{y}\right]\frac{dx}{dy}=0$ D) None

Sol^n ⇒ Ans: A)

क): $x^2+cy^2=1 \Rightarrow cy^2=1-x^2$

$c = \frac{1-x^2}{y^2}$

पाठ-0e

ख): $x^2+cy^2=1$

Take derivative:

$$2x+2cy\cdot\frac{dy}{dx}=0$$

But: $c=\frac{1-x^2}{y^2}$

$$2x+2\left[\frac{1-x^2}{y^{\cancel{2}}}\right]\cancel{y}\cdot\frac{dy}{dx}=0$$

$$2\left[x+\left(\frac{1-x^2}{y}\right)\frac{dy}{dx}\right]=0$$

ग): By Trajectory condn : Replace: $\frac{dy}{dx}=-\frac{dx}{dy}$

$$2\left[x+\left(\frac{1-x^2}{y}\right)\left(-\frac{dx}{dy}\right)\right]=0$$

$$\left[x+\left(\frac{1-x^2}{y}\right)\left(-\frac{dx}{dy}\right)\right]=0$$

$$\boxed{\left[x-\frac{(1-x^2)}{y}\cdot\frac{dx}{dy}\right]=0}$$

(7) The differential equation of orthogonal trajectories of family of curves $e^x+e^{-x}=c$ is

A) $e^x-e^{-y}\frac{dy}{dx}=0$ B) $e^x-e^{-y}\frac{dx}{dy}=0$

C) $e^x+e^{-y}\frac{dx}{dy}=0$ D) None

Soln ⇒ Ans: C)

पान-90

क): $e^x + e^{-y} = c$

Take derivative:

$e^x - e^{-y}\frac{dy}{dx} = 0$

ख): Replace $\frac{dy}{dx} = \frac{-dx}{dy}$

$e^x - e^{-y}\left(-\frac{dx}{dy}\right) = 0$

$$\boxed{e^x + e^{-y}\frac{dx}{dy} = 0}$$

8. The differential equation of orthogonal trajectories of family of curves $r = a\cos\theta$ is

A) $r^2\frac{d\theta}{dr} = \tan\theta$ B) $\frac{1}{r}\frac{d\theta}{dr} = -\tan\theta$

C) $r\frac{d\theta}{dr} = -\tan\theta$ ✓D) $r\frac{d\theta}{dr} = \tan\theta$

Sol^n ⇒ Ans: D)

क): $r = a\cos\theta$ ....① ⇒ $a = \frac{r}{\cos\theta}$

Take derivative:

$\frac{dr}{d\theta} = a(-\sin\theta)$

ख) $\frac{dr}{d\theta} = \frac{r}{\cos\theta}(-\sin\theta)$ $\left\{\because \frac{\sin}{\cos} = \tan\right\}$

$\frac{dr}{d\theta} = -r\tan\theta$

पाना-99

ग): Replace: $\frac{dr}{d\theta} = -r^2 \frac{d\theta}{dr}$ ....(By Trajectories condⁿ)

$\therefore \cancel{+}r^{\cancel{2}} \frac{d\theta}{dr} = \cancel{+}\cancel{r} \tan\theta$

$$\boxed{r \frac{d\theta}{dr} = \tan\theta}$$

(9) The DE of orthogonal trajectories of family of curves $r = a\sin\theta$ is

A) $\frac{1}{r}\frac{d\theta}{dr} = \cot\theta$ B) $r\frac{d\theta}{dr} = -\cot\theta$

C) $r\frac{d\theta}{dr} = -\tan\theta$ D) $r^2\frac{d\theta}{dr} = \tan\theta$

Solⁿ ⇒ Ans: B)

क): $r = a\sin\theta$ ....① ⇒ $a = \frac{r}{\sin\theta}$

Take derivative:

$\frac{dr}{d\theta} = a\cos\theta$

$\frac{dr}{d\theta} = \left[\frac{r}{\sin\theta}\right]\cos\theta$ $\left\{\because \frac{\cos}{\sin} = \cot\right\}$

$\frac{dr}{d\theta} = r\cot\theta$

ख) Replace: $\frac{dr}{d\theta} = -r^2\frac{d\theta}{dr}$ ....(By Trajectories condⁿ)

$-r^{\cancel{2}}\frac{d\theta}{dr} = \cancel{r}\cot\theta$

(∵ shift -ve sign)

$$\boxed{r\frac{d\theta}{dr} = -\cot\theta}$$

पान-९२

(10) The DE of orthogonal trajectories of family of curves $r = a(1-\cos\theta)$ is

A) $-r\frac{d\theta}{dr} = \frac{\sin\theta}{1-\cos\theta}$ B) $r\frac{d\theta}{dr} = \frac{\sin\theta}{1-\cos\theta}$

C) $-r^2\frac{d\theta}{dr} = \frac{\sin\theta}{1-\cos\theta}$ D) $-r\frac{d\theta}{dr} = \frac{1-\cos\theta}{\sin\theta}$

Sol^n ⇒ Ans: A)

१): $r = a(1-\cos\theta) \Rightarrow a = \frac{r}{1-\cos\theta}$

Take derivative:

$$\frac{dr}{d\theta} = a[0 - -\sin\theta]$$

$$\frac{dr}{d\theta} = a\sin\theta$$

२):

$$\frac{dr}{d\theta} = \left[\frac{r}{1-\cos\theta}\right]\sin\theta$$

३) Replace: $\frac{dr}{d\theta} = -r^2\frac{d\theta}{dr}$

$$-\cancel{r^2}\frac{d\theta}{dr} = \left[\frac{\cancel{r}}{1-\cos\theta}\right]\sin\theta$$

$$\boxed{-r\frac{d\theta}{dr} = \frac{\sin\theta}{1-\cos\theta}}$$

(11), (12), (13), (14), (15) ⇒ Solve all problems like this problems. All are simple & use same method. You will get answers easily.

(15) The differential equation of orthogonal trajectories of family of curves $r^2 = a\sin\theta$ is

A) $2r\dfrac{d\theta}{dr} = \cot\theta$ ✓B) $2r\dfrac{d\theta}{dr} = -\cot\theta$

C) $2r\dfrac{d\theta}{dr} = -\tan\theta$ D) $2\dfrac{d\theta}{dr} = -r^2\cot\theta$

Sol^n ⇒ Ans: B)

अ): $r^2 = a\sin\theta \Rightarrow a = \dfrac{r^2}{\sin\theta}$

Take derivative:

$$2r\frac{dr}{d\theta} = a\cos\theta$$

ब): $2r\dfrac{dr}{d\theta} = \left[\dfrac{r^2}{\sin\theta}\right]\cos\theta$

क): Replace: $\dfrac{dr}{d\theta} = -r^2\dfrac{d\theta}{dr}$

$$2r\left[-\cancel{r^2}\frac{d\theta}{dr}\right] = \left(\frac{\cancel{r^2}}{\sin\theta}\right)\cos\theta$$

$\left\{ \because \dfrac{\cos}{\sin} = \cot \right\}$

$$-2r\frac{d\theta}{dr} = \cot\theta$$

$$\boxed{2r\frac{d\theta}{dr} = -\cot\theta}$$

(16) If the DE of family of straight lines $y = mx$ is $\dfrac{dy}{dx} = \dfrac{y}{x}$ then its orthogonal trajectories is

A) $xy = k$ B) $x^2 - y^2 = k^2$ C) $y = kx$ ✓D) $x^2 + y^2 = k^2$

Sol^n ⇒ Ans: D)

पान-98

क): $y = mx \Rightarrow m = y/x$

Take derivative:

$$\frac{dy}{dx} = m(1)$$

ख): Replace : $\frac{dy}{dx} = -\frac{dx}{dy}$

$$-\frac{dx}{dy} = m$$

ग): use VS form :

$$-\frac{dx}{dy} = \frac{y}{x}$$

$$-x\,dx = y\,dy$$

$$-x\,dx - y\,dy = 0$$

घ): Take Integration:

$$-\int x\,dx - \int y\,dy = 0$$

$$-\frac{x^2}{2} - \frac{y^2}{2} = 0 + C$$

च): multiply by 2 to each term

$$-x^2 - y^2 = 0 + 2C$$

change sign

छ): Put: $\{\because 2c = k\} = \text{constant}$

$$x^2 + y^2 = k$$

(17) If the differential equation of family of rectangular hyperbola $xy = c$ is $x\frac{dy}{dx} = -y$ then its orthogonal trajectories is

A) $x^2 - y^2 = k^2$ B) $x^2 + y^2 = k^2$ C) $y^2 = kx$ D) $xy = k_1$

Sol^n $\Rightarrow$ Ans: A)

क):

$$xy = c$$

Take derivative:

$$\underset{u}{x}\,\underset{v}{y} = c$$

$\{\because uv = u'v + uv'\}$

$$(1)(y) + (x)\left(\frac{dy}{dx}\right) = 0$$

ख): Replace: $\frac{dy}{dx} = -\frac{dx}{dy}$

$$y + x\left(-\frac{dx}{dy}\right) = 0$$

$$y - x\frac{dx}{dy} = 0$$

$$y = x\frac{dx}{dy}$$

ग): use VS form:

$$y\,dy = x\,dx$$

घ): Take Integration:

$$\int y\,dy = \int x\,dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + c$$

$$y^2 = x^2 + 2c$$ Put: $2c = k$ = constant

$$y^2 = x^2 + k$$

$$y^2 - x^2 = k$$

change sign:

$$x^2 - y^2 = -k$$ Put: $-k = k$ = constant

$$\boxed{x^2 - y^2 = k}$$

पाठ - 9 Eg

(18) Orthogonal Trajectories of family of circle $x^2+y^2=c^2$ whose DE is $dy/dx = -x/y$ is equal to

A) $x^2-y^2=k^2$ ✓B) $x=ky$ C) $y^2=kx$ D) $x^2+y^2=k^2$

Solⁿ ⇒ Solution : B):

I): $x^2+y^2=c^2$

$$\frac{dy}{dx} = \frac{-x}{y} \quad \text{....(given)}$$

Replace $\frac{dy}{dx} = -\frac{dx}{dy}$

$$\cancel{-}\frac{dx}{dy} = \cancel{-}\frac{x}{y}$$

II): use VS form:

$$\frac{1}{x}\,dx = \frac{1}{y}\,dy$$

III): Take Integration:

$$\int \frac{1}{x}\,dx = \int \frac{1}{y}\,dy$$

$$\log x = \log y + \log c$$

$$\log x - \log y = \log c$$

$$\cancel{\log}\left(\frac{x}{y}\right) = \cancel{\log} c$$

$$\frac{x}{y} = c \quad \{\because \text{Put}: c=k\}$$

$$x/y = k$$

$$\boxed{x = ky}$$

(19) If DE of family of rectangular hyperbola $x^2-y^2=c^2$ is $\frac{dy}{dx} = \frac{x}{y}$, then its orthogonal trajectories is

A) $y^2=kx$ B) $x^2+y^2=k^2$ ✓C) $xy=k$ D) $y=kx$

Solⁿ ⇒ Ans : C)

I): $\frac{dy}{dx} = \frac{x}{y}$ ....(given)

Replace: $\frac{dy}{dx} = -\frac{dx}{dy}$

$$-\frac{dx}{dy} = \frac{x}{y}$$

पान-96

२): use VS form:

$$\frac{-1}{x}\,dx = \frac{1}{y}\,dy$$

3): Take Integration:

$$-\int \frac{1}{x}\,dx = \int \frac{1}{y}\,dy$$

$$-\log x = \log y + \log k$$

$$-\log x - \log y = \log k$$

4): change sign:

$$\log x + \log y = -\log k$$

$$\log(xy) = \log k \qquad \text{Put: } \{\because -\log k = \log k\}$$

$$xy = k$$

$$\boxed{xy = k}$$

(20) Orthogonal Trajectories of family of curves $x^2+2y^2=c^2$ whose differential equation is $dy/dx = -x/2y$ is

A) $x^2 = ky$ B) $x^2 = k/y$ C) $x^2+2y^2 = k^2$ D) No

Solⁿ ⇒ Ans: A)

1):

$$\frac{dy}{dx} = \frac{-x}{2y} \quad \text{....(given)}$$

Replace: $\frac{dy}{dx} = -\frac{dx}{dy}$

$$\frac{-dx}{dy} = \frac{-x}{2y}$$

2): use VS form:

$$\frac{1}{x}\,dx = \frac{1}{2y}\,dy$$

3): Take Integration:

$$\int \frac{1}{x}\,dx = \frac{1}{2}\int \frac{1}{y}\,dy$$

$$\log x = \frac{1}{2}\log y + \log k$$

4): multiply by 2:

$$2\log x = \log y + 2\log k$$

$$\log x^2 = \log y + \log k \qquad \{\because \text{Put: } 2\log k = \log k\}$$

$$\log x^2 - \log y = \log k$$

$$\log\left(\frac{x^2}{y}\right) = \log k$$

5):

$$\frac{x^2}{y} = k$$

$$\boxed{x^2 = ky}$$

(21) If the DE of family of curves $e^y - e^x = c$ is $\frac{dy}{dx} = -\frac{e^x}{e^y}$, then its orthogonal trajectories is

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पाठ - 9L

A) $e^{-x} + e^{-y} = k$ B) $e^{x} - e^{-y} = k$

✓C) $e^{x} + e^{-y} = k$ D) $e^{y} - e^{-x} = k$

Solⁿ ⇒ Ans : C)

क) : $\frac{dy}{dx} = -\frac{e^{-x}}{e^{y}}$ ....(given)

Replace: $\frac{dy}{dx} = \frac{-dx}{dy}$

$\cancel{+}\frac{dx}{dy} = \cancel{+}\frac{e^{-x}}{e^{y}}$

ख) : use VS form:

$\frac{1}{e^{-x}}\, dx = \frac{1}{e^{y}}\, dy$

$e^{x}\, dx = e^{-y}\, dy$

ग) : Take Integration:

$\int e^{x} dx = \int e^{-y} dy$

$e^{x} = \frac{e^{-y}}{-1} + k$

$e^{x} = -e^{-y} + k$

$\boxed{e^{x} + e^{-y} = k}$

(22) Orthogonal trajectories of family of curves $x^2 + cy^2 = 1$ whose DE is $\frac{dy}{dx} = -\frac{xy}{1-x^2}$ is

A) $\log x + \frac{x^2}{2} = \frac{y^2}{2} + k$ ✓B) $\log x - \frac{x^2}{2} = \frac{y^2}{2} + k$

C) $x^2 + y^2 = k$ D) $x^2 + ky^2 = 1$

Solⁿ ⇒ Ans : B)

क) : $\frac{dy}{dx} = \frac{-xy}{1-x^2}$ ....(given)

Replace: $\frac{dy}{dx} = -\frac{dx}{dy}$

$\cancel{+}\frac{dx}{dy} = \frac{\cancel{+}xy}{1-x^2}$

पान-9e

ख): Use VS form:

$$\frac{(1-x^2)}{x}\,dx = y\,dy$$

$$\left[\frac{1}{x} - \frac{x^2}{x}\right] dx = y\,dy$$

$$\left[\frac{1}{x} - x\right] dx = y\,dy$$

$$\frac{1}{x}\,dx - x\,dx = y\,dy$$

ग): Take integration:

$$\int \frac{1}{x}\,dx - \int x\,dx = \int y\,dy$$

$$\log x - x^2/2 = y^2/2 + K$$

$$\boxed{\log x - \frac{x^2}{2} = \frac{y^2}{2} + K}$$

(23) Orthogal Trajectories of family of curves $x^2 = c\,e^{x^2+y^2}$ whose differential equation is $\frac{dy}{dx} = \frac{1-x^2}{xy}$ is

A) $\log(1-x^2) - 2\log y = \log k$ ✓
B) $2\log(1-x^2) - \log y = \log k$
C) $\log(1-x^2) + 2\log y = \log k$
D) $\log(1-x^2) - 2\log y = \log k + k$

Sol^n ⇒
Ans: A)

क):

$$\frac{dy}{dx} = \frac{1-x^2}{xy} \quad \text{----- (given)}$$

Replace: $\frac{dy}{dx} = \frac{-dx}{dy}$

$$\frac{-dx}{dy} = \frac{1-x^2}{xy}$$

ख): use VS form:

$$\frac{-x\,dx}{1-x^2} = \frac{1}{y}\,dy$$

$$\frac{x\,dx}{1-x^2} + \frac{1}{y}\,dy = 0$$

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पार्ट - २०

ग): Take Integration:

$$\int \frac{x}{1-x^2}\,dx + \int \frac{1}{y}\,dy = 0$$

$$-\frac{1}{2}\int \frac{-2x}{1-x^2}\,dx + \int \frac{1}{y}\,dy = 0 \qquad (\because \text{adjustment of } -2)$$

$$-\frac{1}{2}\log(1-x^2) + \log y = \log k$$

घ): multiply by -2:

$$\log(1-x^2) - 2\log y = -2\log k$$

$\{\because$ Put: $-2\log k = \log k =$ constant$\}$

$\{\because$ Constant doesn't matter$\}$

$$\boxed{\log(1-x^2) - 2\log y = \log k}$$

(24) Orthogonal trajectories of family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1$, λ is arbitrary constant whose DE is $x + \left[\frac{a^2-x^2}{y}\right]\frac{dy}{dx} = 0$ is

A) $\frac{y^2}{2} = -a^2\log x + \frac{x^2}{2} + k$ B) $y^2 = b^2\log x - x^2 + k$

✓ C) $\frac{y^2}{2} = a^2\log x - \frac{x^2}{2} + k$ D) $\frac{y^2}{2} = -a^2\log x + x^2 + k$

Sol^n ⇒ Ans: C)

क): $x + \left(\frac{a^2-x^2}{y}\right)\frac{dy}{dx} = 0$ ....(given)

Replace: $\frac{dy}{dx} = -\frac{dx}{dy}$

$$x + \left(\frac{a^2-x^2}{y}\right)\left(-\frac{dx}{dy}\right) = 0$$

$$x = \frac{a^2-x^2}{y}\,\frac{dx}{dy}$$

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ख): Use VS form:

$y\,dy = \frac{a^2 - x^2}{x}\,dx \quad \therefore\ y\,dy = \frac{a^2}{x}\,dx - \frac{x^2}{x}\,dx$

ग): Take Integration:

$\int y\,dy = a^2\int \frac{1}{x}\,dx - \int x\,dx$

$$\boxed{\frac{y^2}{2} = a^2 \log x - \frac{x^2}{2} + k}$$

(25) If the DE of family of curves $r = a(1-\cos\theta)$ is $\frac{dr}{d\theta} = r\cot\frac{\theta}{2}$ then its orthogonal trajectory is gives by

A) $2\log\sec\frac{\theta}{2} = \log r + \log k$ ✓B) $2\log\cos\frac{\theta}{2} = \log r + \log k$

C) $\frac{1}{2}\log\cos\frac{\theta}{2} = \log r + \log k$ D) $\frac{1}{2}\log\sec\frac{\theta}{2} = \log r + \log k$

Sol^n ⇒ Ans: B)

क): $\frac{dr}{d\theta} = r\cot\frac{\theta}{2}$ ....(given)

Replace: $\frac{dr}{d\theta} = -r^2\frac{d\theta}{dr}$

$-r^{\cancel{2}}\frac{d\theta}{dr} = \cancel{r}\cot\frac{\theta}{2}$

ख): use VS form:

$-\frac{1}{\cot\theta/2}\,d\theta = \frac{1}{r}\,dr$

$-\tan\frac{\theta}{2}\,d\theta = \frac{1}{r}\,dr$

$\{\because 1/\cot = \tan\}$

ग): Take Integration:

$-\int \tan\theta/2\,d\theta = \int 1/r\,dr$

$-\frac{[-\log\cos\theta/2]}{1/2} = \log r + \log k$

$\boxed{+2\cdot\log\cos\theta/2 = \log r + \log k}$

$\left\{\because \int \tan ax\,dx = \frac{-\log\cos ax}{a}\right\}$

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(26) If DE of family of curves $r = a\sec^2 \theta/2$ is $\frac{dr}{d\theta} = r \tan \theta/2$ then its orthogonal trajectories is given by

A) $-2\log\cos\frac{\theta}{2} = \log r + \log k$ B) $2\log\sin\frac{\theta}{2} = \log r + \log k$

✓ C) $-2\log\sin\frac{\theta}{2} = \log r + \log k$ D) $2\log\cos\frac{\theta}{2} = \log r + \log k$

Sol^n ⇒ Ans: c)

S): $\frac{dr}{d\theta} = r\tan\frac{\theta}{2}$ ....(given)

Replace: $\frac{dr}{d\theta} = -r^2\frac{d\theta}{dr}$

$-r^2\frac{d\theta}{dr} = r\tan\frac{\theta}{2}$

S): use VS form:

$-\frac{1}{\tan\theta/2}d\theta = \frac{1}{r}dr$

$-\cot\theta/2\, d\theta = 1/r\, dr$

$\left\{\because \frac{1}{\tan} = \cot\right\}$

S): Take Integration:

$-\int\cot\theta/2\, d\theta = \int 1/r\, dr$

$-\frac{\log\sin\theta/2}{1/2} = \log r + \log k$

$\left\{\because \int\cot ax\, dx = \frac{\log\sin ax}{a}\right\}$

$\{\because$ Here: $a = 1/2\}$

$$\boxed{-2\log\sin\theta/2 = \log r + \log k}$$

(27) If the DE of family of curves $r = a\cos\theta$ is $\frac{dr}{d\theta} = -r\tan\theta$ then its orthogonal trajectories is

A) $\log r = -\operatorname{cosec}^2\theta + k$ B) $r = k\cos\theta$

C) $r = k\operatorname{cosec}\theta$ ✓ D) $r = k\sin\theta$

Sol^n ⇒ Ans: D)

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इ): $\frac{dr}{d\theta} = -r\tan\theta$ ....(given)

Replace: $\frac{dr}{d\theta} = -r^2\frac{d\theta}{dr}$

$\cancel{-}r^{\cancel{2}}\frac{d\theta}{dr} = \cancel{-}\cancel{r}\tan\theta$

ब): use VS form:

$\frac{1}{\tan\theta}\,d\theta = \frac{1}{r}\,dr$

$\cot\theta\,d\theta = 1/r\,dr$

$\{\because 1/\tan = \cot\}$

अ): Take Integration:

$\int \cot\theta\,d\theta = \int 1/r\,dr$

$\log\sin\theta = \log r + \log k$

$\log\sin\theta - \log k = \log r$

$\log\sin\theta + \log k = \log r$

$\{\because \text{Put: } -\log k = \log k\}$

$\cancel{\log}[\sin\theta\cdot k] = \cancel{\log}\, r$

$k\sin\theta = r$

$\boxed{r = k\cdot\sin\theta}$

(28) If the DE of family of curves $r^2 = a\sin 2\theta$ is $\frac{dr}{d\theta} = r\cot 2\theta$ then its orthogonal trajectories is

A) $r^2 = \log\sec 2\theta + k$
B) $r^2 = k\sin 2\theta$
✓C) $r^2 = k\cos 2\theta$
D) $\log r = -1/2\,\sec^2 2\theta + k$

Solⁿ ⇒ Ans: C)

इ): $\frac{dr}{d\theta} = r\cot 2\theta$ ....(given)

Replace: $\frac{dr}{d\theta} = -r^2\frac{d\theta}{dr}$

$-r^{\cancel{2}}\frac{d\theta}{dr} = \cancel{r}\cot 2\theta$

$-\frac{1}{\cot 2\theta}\,d\theta = \frac{1}{r}\,dr$

$-\tan 2\theta\,d\theta = 1/r\,dr$

$\{\because 1/\cot = \tan\}$

ब): Take Integration:

$-\int \tan 2\theta\,d\theta = \int 1/r\,dr$

$-[-\log\cos 2\theta]/2 = \log r + \log k$

multiply by 2:

$+\log\cos 2\theta - 2\log k = 2\log r$

$\log\cos 2\theta + \log k = \log r^2$

$\cancel{\log}[\cos 2\theta\cdot k] = \cancel{\log}\, r^2$

$k\cdot\cos 2\theta = r^2$

Put: $\{\because -2\log k = \log k\}$

$\boxed{r^2 = k\cdot\cos 2\theta}$

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(29) If the DE of family of curves $r = a\cos^2\theta$ is $dr/d\theta = -2r\tan\theta$ then it's orthogonal trajectory is

A) $\frac{1}{2}\log\sin\theta = \log r + \log k$ B) $\frac{1}{2}\log\sin\theta = -\log r + \log k$

c) $\log\sin\theta = r + k$ D) $\log\sec\theta = -\log r + \log k$

Sol^n ⇒ Ans: A)

क):
$$\frac{dr}{d\theta} = -2r\tan\theta$$
......(given)

Replace: $\frac{dr}{d\theta} = -r^2\frac{d\theta}{dr}$

$$\cancel{+}r^{\cancel{2}}\frac{d\theta}{dr} = \cancel{+}2\cancel{r}\tan\theta$$

ख): use VS form:

$$\frac{1}{2\tan\theta}\,d\theta = \frac{1}{r}\,dr$$

$$\frac{1}{2}\cot\theta\,d\theta = \frac{1}{r}dr$$

$\{\because 1/\tan = \cot\}$

ग): Taking Integration:

$$\frac{1}{2}\int\cot\theta\,d\theta = \int\frac{1}{r}\,dr$$

$$1/2\cdot\log\sin\theta = \log r + \log k$$

$$\boxed{\frac{1}{2}\log\sin\theta = \log r + \log k}$$

Formula:
$\{\because \int\cot\theta = \log\sin\theta\}$
$\{\because \int 1/x\,dx = \log x\}$

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पान- ३

• धड्याचे नाव :

Fourier Series and
Gamma & Beta
Functions

पत्र-४

● NOTE:

• Fourier series •

① Fourier Series:

Fourier series is the basic mathematical representation of periodic function.

Fourier series is an infinite series of sines and cosines of multiples of x, satisfying certain conditions known as the Dirichlet's condition.

② Periodic Function:

A function $f(x)$ is said to be periodic if it is defined for all real and x if there is same +ve number T.

1) $f(x+T) = f(x) \quad , \forall x$

where T = period of $f(x)$

2) For n is any integer

$f(x+nT) = f(x)$

where nT = period of $f(x)$

3) If $f(x)$ is periodic function of period T, then $f(ax)$, $a \neq 0$ is also periodic &

Fundamental period = T/a

4) $\sin x$, $\cos x$, $\sec x$, $\operatorname{cosec} x$ are periodic function and

Fundamental period = 2π

5) $\tan x$, $\cot x$ are also periodic function &

Fundamental period = π

6) Constant function i.e. $f(x) = k$ is periodic function.

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③ Dirichlet's condition:

Let $f(x)$ is periodic function of period 2π & defined in the interval $c \leq x \leq c+2\pi$.

• Condition:-

1) $f(x)$ is finite, single valued & its integral exist in the interval.

2) $f(x)$ has at most finite number of finite discontinuities In the interval.

3) $f(x)$ has at most finite number of maxima & minima in the interval.

4) Fourier Trigonometric Series:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cdot \cos nx + b_n \cdot \sin nx]$$

where, a_0, a_n, b_n are coefficient of fourier series and determined by Euler's formula.

• NOTE:-

1) $\sin^{-1}x$, $\cos^{-1}x$ or trigonometric inverse function is not single value function.

2) $\tan x, \cot x, \sec x, \operatorname{cosec} x$ can not have fourier series expansion in $(0, 2\pi)$.

$\tan x$	infinite at $x = \pi/2$
$\cot x$	infinite at $x = 0$
$\sec x$	infinite at $x = \pi/2$
$\operatorname{cosec} x$	infinite at $x = 0$

3) $f(x) = \frac{1}{x-a}$ infinite discontinuity at $x = a$

$\therefore$ It is not fourier series expansion.

4) $f(x) = e^{ax}$ can be expanded as FS in any interval because it satisfy all dirichlet's condition.

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④ Even Function & Odd Function:

i) Even Function:

A function $f(x)$ is said to be even if

$$f(-x) = f(x)$$

e.g.:

1) $f(x) = x^2$

Replace $x \to -x$

$f(-x) = (-x)^2$

$f(-x) = x^2$

$\therefore f(-x) = f(x)$

2) $\cos x,\ x\sin x,\ e^x + e^{-x},\ x^4 + x^2$ etc

ii) Odd Function:

A function $f(x)$ is said to be odd if

$$f(-x) = -f(x)$$

e.g.:

1) $f(x) = \sin x$

Replace $x \to -x$

$f(-x) = \sin(-x)$

$f(-x) = -\sin x$

$\therefore f(-x) = -f(x)$

2) $x,\ x\cos x,\ e^x - e^{-x},\ x^3$ etc

⑤ Revision of Some Imp Formuli:

1) $2\sin A \cdot \cos B = \sin(A+B) + \sin(A-B)$

$2\cos A \cdot \sin B = \sin(A+B) - \sin(A-B)$

$2\cos A \cdot \cos B = \cos(A+B) + \cos(A-B)$

$2\sin A \cdot \sin B = \cos(A-B) - \cos(A+B)$

2) $\sin 2A = 2\sin A \cdot \cos A$, $\cos 2A = 1 - 2\sin^2 A$

$\cos 2A = \cos^2 A - \sin^2 A$, $\cos 2A = 2\cos^2 A - 1$

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3) $\sin(A \pm B) = \sin A \cdot \cos B \pm \cos A \cdot \sin B$

$\cos(A \pm B) = \cos A \cdot \cos B \mp \sin A \cdot \sin B$

4) For any integer n,

i) $\sin n\pi = 0$, $\sin 2\pi n = 0$

ii) $\cos n\pi = (-1)^n$, $\cos 2\pi n = 1$

iii) $\sin(n \pm 1)\pi = 0$, $\cos(n \pm 1)\pi = -\cos n\pi$

5) $$\int e^{ax} \cdot \sin bx \, dx = \frac{e^{ax}}{a^2+b^2}(a \sin bx - b \cos bx)$$

$$\int e^{ax} \cdot \cos bx \, dx = \frac{e^{ax}}{a^2+b^2}(a \cos bx + b \sin bx)$$

6) Advance UV Rule (Integration by part) :

$$\int uv \, dx = u\int v - u' \iint v + u'' \iiint v - \dots\dots$$

find u & v functions by using 'LIATE' Rule.

L = Logarithmic = $\log x$, $\log z$

I = Inverse = $\sin^{-1} x$, $\cos^{-1} x$

A = Algebric = $x, x^2, t^3, 2t^2+1, 4, 3x-5$

T = Trigonometric = $\sin x$, $\cos x$

E = Exponential = e^x, a^x, 3^x, e^{4x}

e.g. $$\int \underset{\uparrow \, u}{(\pi^2 - x^2)} \cdot \underset{\uparrow \, v}{\cos nx} \, dx$$

$$= (\pi^2 - x^2)\int \cos nx \, dx - (\pi^2 - x^2)' \iint \cos nx \, dx + (\pi^2 - x^2)'' \iiint \cos nx \, dx$$

$$= (\pi^2 - x^2)\left(\frac{\sin nx}{n}\right) - (0 - 2x)\left(\frac{-\cos nx}{n}\right) + (-2)\left(\frac{-\sin nx}{n^3}\right)$$

$$= (\pi^2 - x^2)\left(\sin nx/n\right) - (2x)\left(\cos nx/n\right) + 2\left(\sin nx/n^3\right)$$

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पाठ- L

7) $\int_{-a}^{a} f(x)\,dx = \int_{0}^{2a} f(x)\,dx$ .....(for f(x) even function)

$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$ ......(for f(x) even function)

$\int_{-a}^{a} f(x)\,dx = 0$ ......(for f(x) odd function)

(6) Euler's Formula:

1) f(x) is periodic function of interval $c \le x \le c+2\pi$
(Period is Radian)

Fourier Series:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx]$$

a) $a_0 = \frac{1}{\pi} \int_{c}^{c+2\pi} f(x)\,dx$

b) $a_n = \frac{1}{\pi} \int_{c}^{c+2\pi} f(x) \cdot \cos nx\,dx$

c) $b_n = \frac{1}{\pi} \int_{c}^{c+2\pi} f(x) \cdot \sin nx\,dx$

2) f(x) is period $c \le x \le c+2\pi$

If $c=0$, period: $0 \le x \le 2\pi$

$$a_0 = \frac{1}{\pi} \int_{0}^{2\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi} \int_{0}^{2\pi} f(x) \cdot \cos nx\,dx$$

$$b_n = \frac{1}{\pi} \int_{0}^{2\pi} f(x) \cdot \sin nx\,dx$$

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पान-e

3) f(x) is periodic $c \leq x \leq c+2\pi$

If $c = -\pi$, period : $-\pi \leq x \leq \pi$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot \cos nx\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot \sin nx\,dx$$

4) Even & Odd Function :

Interval : $-\pi \leq x \leq \pi$

i) Even Function: $a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cdot \cos nx\,dx$$

$$b_n = 0$$

ii) Odd Function:

$a_0 = 0$,

$a_n = 0$,

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cdot \sin nx\,dx$$

5) Half Range Expansion:

f(x) is periodic function ; period = 2π ,

Half period : $0 \leq x \leq \pi$

i) Half Range cosine:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cdot \cos nx)$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cdot \cos nx\,dx$$

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ii) Half Range Sine:

$$f(x) = \sum_{n=1}^{\infty} bn \cdot \sin nx$$

$$bn = \frac{2}{\pi} \int_0^{\pi} f(x) \cdot \sin nx \, dx$$

(8) Half Range

6) If $f(x)$ is periodic function & interval $c \le x \le c+2L$
(∵ Period is Arbitary)

Fourier Series:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nz + b_n \sin nz]$$

where, $z = \frac{\pi x}{L}$ $\{ \because L = (\text{Total no. of Observation})/2 \}$

i) $a_0 = \frac{1}{L} \int_c^{c+2L} f(x) \, dx$

ii) $a_n = \frac{1}{L} \int_c^{c+2L} f(x) \cdot \cos nz \, dx$

iii) $b_n = \frac{1}{L} \int_c^{c+2L} f(x) \cdot \sin nz \, dx$

7) If $f(x)$ is periodic, $c \le x \le c+2L$
if $c = 0$, period: $0 \le x \le 2L$

$$a_0 = \frac{1}{L} \int_0^{2L} f(x) \, dx$$

$$a_n = \frac{1}{L} \int_0^{2L} f(x) \cdot \cos nz \, dx$$

$$b_n = \frac{1}{L} \int_0^{2L} f(x) \cdot \sin nz \, dx \qquad \{ \because z = \frac{\pi x}{L} \}$$

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8) f(x) is period ; $c \leq x \leq c+2L$

If $c=-L$; period : $-L \leq x \leq L$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x).\cos nz\,dx$$

$$\left\{\because z = \frac{\pi x}{L}\right\}$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x).\sin nz\,dx$$

9) Even & odd Function

Interval : $-L \leq x \leq L$

i) Even function:

$$a_0 = \frac{2}{L}\int_{0}^{L} f(x)\,dx$$

$$a_n = \frac{2}{L}\int_{0}^{L} f(x).\cos nz\,dx \qquad \left\{\because z = \frac{\pi x}{L}\right\}$$

$$b_n = 0$$

ii) Odd function:

$$a_0 = 0$$

$$a_n = 0$$

$$b_n = \frac{2}{L}\int_{0}^{L} f(x).\sin nz\,dx$$

10) Half Range Expansion :

f(x) is periodic function ; period = 2L, Half period = $0 \leq x \leq l$

i) Half range cosine :

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi x}{L}\right]$$

$$a_0 = \frac{2}{L}\int_{0}^{L} f(x)\,dx$$

$$a_n = \frac{2}{L}\int_{0}^{L} f(x).\cos n\pi x/L\,dx$$

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ii) Half range sign:

$$f(x) = \sum_{n=1}^{\infty} bn \cdot \sin \frac{n\pi x}{L}$$

$$bn = \frac{2}{L} \int_0^L f(x) \cdot \sin \frac{n\pi x}{L} \, dx$$

(7) Practical Harmonic Analysis:
[Fourier Table Solving]

$$y = f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [an \cdot \cos nx + bn \cdot \sin nx]$$

$$a_0 = 2 \times \frac{\Sigma y}{(\text{No. of observation } (m))}$$

$$an = 2 \times \frac{\Sigma y \cdot \cos nx}{(\text{No. of observation } (m))}$$

$$bn = 2 \times \frac{\Sigma y \cdot \sin nx}{(\text{No. of observation } (m))}$$

i) For n=1, first harmonic / fundamental harmonic:

$$y = f(x) = \frac{a_0}{2} + [a_1 \cos x + b_1 \sin x]$$

$$a_0 = 2 \times \frac{\Sigma y}{m}$$

$$a_1 = 2 \times \frac{\Sigma y \cdot \cos x}{m}$$

$$b_1 = 2 \times \frac{\Sigma y \cdot \sin x}{m}$$

ii) For n=2, second harmonic:

$$y = f(x) = \frac{a_0}{2} + [a_1 \cos x + b_1 \sin x] + [a_2 \cos 2x + b_2 \sin 2x]$$

$$a_0 = 2 \times \frac{\Sigma y}{m}$$

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पान-93

$$a_1 = 2 \times \frac{\Sigma y \cdot \cos 2x}{m}$$

$$a_2 = 2 \times \frac{\Sigma y \cdot \cos 2x}{m}$$

$$b_1 = 2 \times \frac{\Sigma y \cdot \sin x}{m}$$

$$b_2 = 2 \times \frac{\Sigma y \cdot \sin 2x}{m}$$

and simillarly solving for $n = 3, 4, 5, \ldots\ldots$

- NOTE: use 'ES calculator' for solving problems.

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पान-९४

* Type: Fourier Series and Harmonic Analysis:

① A function $f(x)$ is said to be periodic of period T if

✓A) $f(x+T) = f(x)$ for all x B) $f(x+T) = f(T)$ for all x
C) $f(-x) = f(x)$ for all x D) $f(-x) = -f(x)$ for all x

Sol^n ⇒ Ans: A)

$$f(x+T) = f(x)$$

......(यह स्टैंडर्ड है, रटलो।)

② If $f(x+nT) = f(x)$ where n is any integer then fundamental period of $f(x)$ is

A) $2T$ B) $T/2$ ✓C) T D) $3T$

Sol^n ⇒ Ans: C)

T = period of function $f(x)$

......(यह स्टैंडर्ड है, रटलो।)

③ If $f(x)$ is a periodic function with period T then $f(ax) \neq 0$ is period with fundamental period

A) T ✓B) T/a C) aT D) π

Sol^n ⇒ Ans: B) $\frac{T}{a}$ = Fundamental period

......(यह स्टैंडर्ड है, रटलो।)

④ Fundamental period of $\sin 2x$ is

A) $\pi/4$ B) $\pi/2$ C) 2π ✓D) π

पान - ९४

Sol^n ⇒ Ans : D)

पा): T= Period of $\sin x = 2\pi$

$f(ax) = \sin 2x$ ....(given)

$\therefore a = 2$

क्षा): Fundamental period $= \frac{T}{a} = \frac{\cancel{2}\pi}{\cancel{2}} = \boxed{\pi}$

(5) Fundamental period of $\cos 2x$ is

A) $\pi/4$ B) $\pi/2$ ✓C) π D) 2π

Sol^n ⇒ Ans: C)

पा): T= Period of $\cos 2x = 2\pi$

$f(ax) = \cos 2x$

$\therefore a = 2$

क्षा): Fundamental period $= \frac{T}{a}$

$= \frac{\cancel{2}\pi}{\cancel{2}}$

$= \boxed{\pi}$

(6) Fundamental period of $\tan 3x$ is

A) $\frac{\pi}{2}$ ✓B) $\frac{\pi}{3}$ C) π D) $\frac{\pi}{4}$

Sol^n ⇒ Ans: B)

पा): T= Period of $\tan x = \pi$

$f(ax) = \tan 3x$

$\therefore a = 3$

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पान-९६

उत्तर):

Fundamental Period $= \frac{T}{a}$

$= \boxed{\frac{\pi}{3}}$

(7) Fourier serier representation of periodic function $f(x)$ with period 2π which satisfies the derichlet's conditions is

A) $\frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx]$ B) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$

C) $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx)(b_n \sin nx)$ D) $\frac{a_0}{2} + a_n \cos nx$

Sol^n ⇒ Ans: A)

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx]$$

.....(यह स्टँडर्ड है, रटलो।)

(8) Fourier series representation of periodic function $f(x)$ with period $2L$, which satisfies Dirichlet's conditions is

A) $\frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx]$ B) $\frac{a_0}{2} + \sum_{n=1}^{\infty} [a_1 \cos \frac{n\pi x}{L} + b_1 \sin \frac{n\pi x}{L}]$

C) $\frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos \frac{n\pi x}{L} \times b_n \sin \frac{n\pi x}{L}]$ D) $\frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(\frac{n\pi x}{L}) + b_n \sin(\frac{n\pi x}{L})]$

Sol^n ⇒ Ans: D)

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right]$$

.....(यह स्टँडर्ड है, रटलो।)

पान -९७

(9) If $f(x)$ is periodic function with period 2L. defined in the interval c to c+2L then fourier coefficient a_0 is

A) $\int_c^{c+2L} f(x)\, dx$ B) $\frac{1}{L}\int_c^{c+2L} f(x).\sin\left(\frac{n\pi x}{L}\right) dx$

C) $\frac{1}{L}\int_c^{c+2L} f(x).\cos\left(\frac{n\pi x}{L}\right) dx$ ✓D) $\frac{1}{L}\int_c^{c+2L} f(x)\, dx$

Solⁿ ⇒ Ans: D)

$f(x)$ is periodic function, period: $c \le x \le c+2L$

$$\boxed{a_0 = \frac{1}{L}\int_c^{c+2L} f(x)\cdot dx}$$

....(यह रट्टा है, रटलो।)

(10) If $f(x)$ is periodic function with period 2L defined in the interval c to c+2L then fourier coefficient a_n is

A) $\int_c^{c+2L} f(x).\cos\left(\frac{n\pi x}{L}\right) dx$ B) $\frac{1}{L}\int_c^{c+2L} f(x).\sin\left(\frac{n\pi x}{L}\right) dx$

✓C) $\frac{1}{L}\int_c^{c+2L} f(x).\cos\left(\frac{n\pi x}{L}\right) dx$ D) $\frac{1}{L}\int_c^{c+2L} f(x)\, dx$

Solⁿ ⇒ Ans: c)

$f(x)$ is periodic function, period: $c \le x \le c+2L$

$$\boxed{a_n = \frac{1}{L}\int_c^{c+2L} f(x).\cos\frac{n\pi x}{L}\, dx}$$

(11) If $f(x)$ is periodic function with period 2L defined in the interval c to c+2L then fourier coefficient b_n is

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पान-98

A) $\int_{c}^{c+2L} f(x).\sin\left(\frac{n\pi x}{L}\right)dx$ B) $\frac{1}{L}\int_{c}^{c+2L} f(x).\sin\left(\frac{n\pi x}{L}\right)dx$

C) $\frac{1}{L}\int_{c}^{c+2L} f(x).\cos\left(\frac{n\pi x}{L}\right)dx$ D) $\frac{1}{L}\int_{c}^{c+2L} f(x)\,dx$

Ans: B)

क): $f(x)$ is periodic function, period: $c \le x \le c+2L$

$$\boxed{b_n = \frac{1}{L}\int_{c}^{c+2L} f(x).\sin(n\pi x/L)\,dx}$$

(12) A function $f(x)$ is said to be even if

A) $f(-x) = f(x)$ B) $f(-x) = -f(x)$

C) $f(x+2\pi) = f(x)$ D) $f(-x) = [f(x)]^n$

Sol^n ⇒ Ans: A)

$$\boxed{f(-x) = f(x)}$$

....(यह स्टँडर्ड है, रखो।)

(13) A function $f(x)$ is said to be odd function if

A) $f(-x) = f(x)$ B) $f(-x) = -f(x)$

C) $f(x+2\pi) = f(x)$ D) $f(-x) = [f(x)]^2$

Sol^n ⇒ Ans: B)

$$\boxed{f(-x) = -f(x)}$$

....(यह स्टँडर्ड है, रखो।)

(14) Which of the following in an odd function ?

A) $\sin x$ B) $e^{x} + e^{-x}$

C) $e^{|x|}$ D) $\pi^2 - x^2$

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पाठ-9e

Sol^n ⇒ Ans: A)

स्प):

$f(x)$:	$f(-x)$:
$\sin x$	$\sin(-x) = -\sin x$
$e^{x} + e^{-x}$	$e^{-x} + e^{x}$
$e^{\lvert x\rvert}$	$e^{\lvert x\rvert}$
$\pi^2 - x^2$	$\pi^2 - (-x)^2 = \pi^2 - x^2$

अत): ∵ only $\sin x$ having sign change

∴ $\boxed{\sin x \text{ is an odd function.}}$

(15) Which is of the following is an even function ?

A) $\sin x$ B) $e^{x} - e^{-x}$ C) $x\cos x$ D) $\cos x$ ✓

Sol^n ⇒ Ans: D)

स्प):

$f(x)$:	$f(-x)$:
$\sin x$	$\sin(-x) = -\sin x$
$e^{x} - e^{-x}$	$e^{-x} - e^{x} = -(e^{x} - e^{-x})$
$x\cos x$	$-x\cos(-x) = -x\cos x$
$\cos x$	$\cos(-x) = \cos x$

अत): only $\cos x$ doesnt having sign change

∴ $\boxed{\cos x \text{ is even function.}}$

(16) Which of the following is neither an even function nor an odd function ?

A) $x\sin x$ B) x^2 C) e^{-x} ✓ D) $x\cos x$

Sol^n ⇒ Ans: C)

स्प):

$f(x)$:	$f(-x)$:
e^{-x}	$e^{-(-x)} = e^{+x}$

(Std Result)...... ∴ $\boxed{e^{-x} \text{ is neither even nor odd function}}$

पाठ-२०

(17) For an even function $f(x)$ defined in interval $-\pi \leq x \leq \pi$ and $f(x+2\pi) = f(x)$ the fourier series is

A) $\sum_{n=1}^{\infty} b_n \sin nx$ B) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$

✓C) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$ D) $\sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$

Sol$^n \rightarrow$ Ans: c)

5): For even function $f(x)$, interval: $-\pi \leq x \leq \pi$

$$\therefore \boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cdot \cos nx)}$$

......(यह स्टैंडर्ड है, रटलो।)

(18) For an odd function $f(x)$ defined in interval $-\pi \leq x \leq \pi$ and $f(x+2\pi) = f(x)$ the fourier series is

✓A) $\sum_{n=1}^{\infty} b_n \sin nx$ B) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cdot \cos\frac{n\pi x}{L}$

c) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cdot \cos nx$ D) $\sum_{n=1}^{\infty} b_n \cdot \sin\frac{n\pi x}{L}$

Sol$^n \Rightarrow$ Ans: A)

5): For an odd function $f(x)$, interval: $-\pi \leq x \leq \pi$

$$\boxed{f(x) = \sum_{n=1}^{\infty} b_n \cdot \sin nx}$$

....(यह स्टैंडर्ड है, रटलो।)

(19) Fourier coefficient for an even function $f(x)$ defined in the interval $-\pi \leq x \leq \pi$ and $f(x+2\pi) = f(x)$ are

A) $a_0 = 0, a_n = 0, b_n = \frac{2}{\pi}\int_0^{\pi} f(x) \cdot \sin nx \, dx$

✓B) $a_0 = 2/\pi \int_0^{\pi} f(x) dx$, $a_n = 2/\pi \int_0^{\pi} f(x) \cos nx \, dx$, $b_n = 0$

Ans: B)

Ans: for an even function $f(x)$, interval: $-\pi \leq x \leq \pi$

$$\boxed{\begin{aligned} a_0 &= \frac{2}{\pi}\int_0^{\pi} f(x)\,dx, \\ a_n &= \frac{2}{\pi}\int_0^{\pi} f(x)\cdot\cos nx\,dx, \\ b_n &= 0 \end{aligned}}$$

..... (यह स्टैंडर्ड है, रटलो।)

२० Fourier coefficient for an odd function $f(x)$ defined in the interval $-\pi \leq x \leq \pi$ and $f(x+2\pi) = f(x)$ are

Sol^n ⇒

For odd function $f(x)$, interval: $-\pi \leq x \leq \pi$

$$\boxed{\begin{aligned} a_0 &= 0 \\ a_n &= 0 \\ b_n &= \frac{2}{\pi}\int_0^{\pi} f(x)\cdot\sin nx\,dx \end{aligned}}$$

..... (यह स्टैंडर्ड है, रटलो।)

२१ Fourier coefficient for an even function $f(x)$ defined in the interval $-L \leq x \leq L$ and $f(x+2L) = f(x)$ are

Sol^n ⇒ For even function $f(x)$, interval: $-L \leq x \leq L$

$$\boxed{\begin{aligned} a_0 &= \frac{2}{L}\int_0^{L} f(x)\,dx, \\ a_n &= \frac{2}{L}\int_0^{L} f(x)\cdot\cos\frac{n\pi x}{L}\,dx, \\ b_n &= 0 \end{aligned}}$$

..... (यहभी स्टैंडर्ड है, रटलो।)

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पान - २२

(22) Fourier coefficient for an odd function $f(x)$ defined in the interval $-L \leq x \leq L$ and $f(x+2L) = f(x)$ are

Sol^n =>

For odd function $f(x)$, interval : $-L \leq x \leq L$

$a_0 = 0$,

$a_n = 0$,

$$b_n = \frac{2}{L} \int_0^L f(x) \cdot \sin \frac{n\pi x}{L} \, dx$$

.....(यह स्टँडर्ड है, रटलो।)

(23) Half range Fourier cosine series for $f(x)$ defined in the interval $0 \leq x \leq L$ is

Sol^n =>

Half range cosine : $(0 \leq x \leq L)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cdot \cos \frac{n\pi x}{L}$$

.....(यह स्टँडर्ड है, रटलो।)

(24) Half range fourier sine series for $f(x)$ defined in the interval $0 \leq x \leq L$ is

Sol^n =>

Half range sin : $(0 \leq x \leq L)$

$$f(x) = \sum_{n=1}^{\infty} b_n \cdot \sin n\pi x/L$$

.....(यह स्टँडर्ड है, रटलो।)

(25) In harmonic analysis for a function with period 2π, the term $a_1 \cos x + b_1 \sin x$ is called :

Sol^n => First Harmonic / Fundamental Harmonic. ....(std Result)

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पान - २२

(26) In the harmonic analysis for a function with period 2π, the amplitude of first harmonic $a_1\cos x + b_1 \sin x$ is :

Sol^n ⇒ $\boxed{\text{Amplitude} = \sqrt{a_1^2 + b_1^2}}$

.....(यह स्टैंडर्ड है, रटलो।)

(27) The value of a_0 in Fourier series of y with period 6 for the following tabulated data

x	0	1	2	3	4	5
y	9	18	24	28	26	20

A) 17.85 B) 20.83 C) 35.71 ✓D) 41.66

Sol^n ⇒ Ans: D)

१): Given: Period $= 2L = 6$ $\therefore L = 6/2 = 3$

m = No. of Observation = 6

२): $a_0 = 2 \times \frac{\Sigma y}{m}$ .....(formula: Plz Refer NOTE)

$= 2 \times [9+18+14+28+26+20]/6$

$= 2 \times 125/6$

$\boxed{a_0 = 41.66}$

(28) The value of a_0 in fourier series of y with period $180°$ for the following data is

$x°$	0	30	60	90	120	150
y	0	9.2	14.4	17.8	17.3	11.7

✓A) 23.46 B) 20.11 C) 11.73 D) 10.50

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पान-२४

Sol^n ⇒ Ans: A)

पं): Given: Period $= 180^\circ = \pi^c$

No. of observation $= m = 6$

२ं):

$$a_0 = 2 \times \frac{\Sigma y}{m}$$

$$= 2 \times [0 + 9.2 + 14.4 + 17.8 + 17.3 + 11.7] / 6$$

$$= 2 \times 70/6$$

$$\boxed{a_0 = 23.33}$$

(29) The values of a_0 in Fourier series of y with period 6 for data:

x	0	1	2	3	4	5
y	4	8	15	7	6	2

A) 3.5 B) 14 C) 6 D) 7

Sol^n ⇒ Ans: B)

पं): Given: Period $= 2L = 6$

$m =$ No. of observations $= 6$

२ं):

$$a_0 = 2 \times \frac{\Sigma y}{m}$$

$$a_0 = 2 \times [4 + 8 + 15 + 7 + 6 + 2] / 6$$

$$= 2 \times 42/6$$

$$\boxed{a_0 = 14}$$

पान-२४

(31) Fourier coefficient a_0 in fourier series expansion of $f(x) = e^{-x}$; $0 \le x \le 2\pi$ and $f(x+2\pi) = f(x)$ is

~~A) $\frac{1}{\pi}[1-e^{-2\pi}]$ B) $\frac{1}{2\pi}[1-e^{2\pi}]$ C) $\frac{2}{\pi}[e^{-2\pi}-1]$ D) $\frac{1}{\pi}(1+e^{2\pi})$~~

Sol^n ⇒ Ans: A)

क): $f(x) = e^{-x}$, $0 \le x \le 2\pi$ ....(given)

ख): $a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\, dx$ .....(सूत्र)

$= \frac{1}{\pi}\int_0^{2\pi} e^{-x}\, dx$

....(solve in Radian Mode of Calcy)

$a_0 = 0.3177$

ग): check option: A): $\boxed{\frac{1}{\pi}[1-e^{-2\pi}] = 0.3177}$

• NOTE: कॅल्सीमें कैसे डाले ?

1) ES: $\frac{1}{\pi}\int_0^{2\pi} e^{-x}\, dx = 0.3177$

2) MS: $(1 \div \pi)\int(e\wedge(-x), 0, 2\pi) = 0.3177$

x ⇒ ALPHA) , π ⇒ Shift $\times 10^x$ or Shift EXP

e ⇒ Shift In / ALPHA In

3) Radian Mode ⇒ ES: Shift-Mode-4

MS: Mode Mode Mode Mode → Rad → 2

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पान- २६

(32) Fourier coefficient a_0 in fourier series expansion of

$f(x) = \left[\frac{\pi - x}{2}\right]^2$, $0 \le x \le 2\pi$ and $f(x+2\pi) = f(x)$ is

A) $\frac{\pi^2}{3}$ ✓B) $\frac{\pi^2}{6}$ C) 0 D) $\frac{\pi}{6}$

Sol^n ⇒ Ans: B)

क): $f(x) = (\pi - x/2)^2$, $0 \le x \le 2\pi$ .....(given)

ख): $$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi}\left[\frac{\pi - x}{2}\right]^2 dx$$

.....(solve in Radian mode of calcy)

.....(Plz refer NOTE in problem No. (31))

.....(अच्छे रिजल्ट के लिए ES- कॅलसी इस्तेमाल किजिए।)

$a_0 = 1.64$

ग): check option: B): $\boxed{\frac{\pi^2}{6} = 1.64}$

(33) Fourier coefficient a_0 in the fourier series expansion of

$f(x) = x \sin x$, $0 \le x \le 2\pi$ and $f(x+2\pi) = f(x)$ is

A) +2 B) 0 ✓C) -2 D) -4

Sol^n ⇒ Ans: C)

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पान- २८

फ): $f(x) = x\sin x$; $0 \le x \le 2\pi$ .....(given)

ख):
$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\, dx$$
$$= \frac{1}{\pi}\int_0^{2\pi} x\sin x\, dx$$

..... (solve in Radian Mode of ES calcy)

...... (Plz refer NOTE in Q.31)

$$\boxed{a_0 = -2}$$

(34) $f(x) = \begin{cases} x, & 0 \le x \le \pi \\ 0, & \pi \le x \le 2\pi \end{cases}$ and $f(x+2\pi) = f(x)$,

fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n \cos nx + b_n \sin nx]$, then fourier coefficient a_0 is given as

A) $\frac{3\pi}{2}$ B) $\frac{\pi}{3}$ C) 0 D) $\frac{\pi}{2}$

Sol^n ⇒ Ans: D)

• IMP CONCEPT •

फ): $f(x) = \begin{cases} x, & 0 \le x \le \pi \\ 0, & \pi \le x \le 2\pi \end{cases}$ $\left\{ \begin{array}{l} \text{Limit: } 0 \to 2\pi \\ \because \text{split as: } 0 \to \pi, \\ \pi \to 2\pi \end{array} \right\}$

ख):
$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\, dx$$
$$= \frac{1}{\pi}\left[\int_0^{\pi} f(x)\, dx + \int_{\pi}^{2\pi} f(x)\, dx\right]$$
$$= \frac{1}{\pi}\left[\int_0^{\pi} x\, dx + \int_{\pi}^{2\pi} 0\, dx\right]$$

..... (solve in Radian Mode of ES calcy)

.... (Plz refer NOTE in Q.31)

$$\boxed{a_0 = \pi/2}$$

पाठ-२८

(35)

$$f(x) = \begin{cases} 0, & 0 \le x \le \pi \\ x, & \pi \le x \le 2\pi \end{cases} \quad \text{and } f(x+2\pi) = f(x).$$

Fourier series is represented by

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx],$$

then fourier coefficient a_0 is

A) $\frac{3\pi}{2}$ B) $\frac{3\pi}{8}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{2}$

Sol^n ⇒ Ans: A)

क): $$f(x) = \begin{cases} 0, & 0 \le x \le \pi \\ x, & \pi \le x \le 2\pi \end{cases}$$

ख): Split limit :: $0 \to 2\pi$ AS: $0 \to \pi$, $\pi \to 2\pi$

ग):

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x)\, dx$$

$$a_0 = \frac{1}{\pi} \left[\int_0^{\pi} f(x)\, dx + \int_{\pi}^{2\pi} f(x)\, dx \right]$$

$$= \frac{1}{\pi} \left[\int_0^{\pi} 0\, dx + \int_{\pi}^{2\pi} x\, dx \right]$$

----- (solve in Radian Mode of Calcy)

$$\boxed{a_0 = 3\pi/2}$$

(36)

$f(x) = 2x - x^2$, $0 \le x \le 3$ and period is 3. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos \frac{2n\pi x}{3} + b_n \sin \frac{2n\pi x}{3}\right]$,

then fourier coefficient a_0 is

A) $\frac{3}{2}$ B) 0 C) 12 D) $3/4$

पान-२९

Sol^n ⇒ Ans: B)

S): $f(x) = 2x - x^2$, $0 \le x \le 3$

$\therefore 2L = 3 \quad \therefore L = 3/2$, $0 \le x \le 2L$

म): $$a_0 = \frac{1}{L} \int_0^{2L} f(x)\,dx$$

$$= \frac{1}{(3/2)} \int_0^{2L} (2x - x^2)\,dx$$

.....(solve in Radian Mode of ES calcy)

.....(Plz refer NOTE in Q.31)

$\boxed{a_0 = 0}$

(37) $f(x) = 4 - x^2$, $0 \le x \le 2$ and period is 2. Fourier series is $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos n\pi x + b_n \sin n\pi x \right]$, then fourier coefficient a_0 is

A) $\frac{11}{3}$ B) 0 ✓C) $\frac{16}{3}$ D) $\frac{8}{5}$

Sol^n ⇒ Ans: C)

S): $f(x) = 4 - x^2$, $0 \le x \le 2$ ....(given)

Period $= 2L = 2 \quad \therefore L = 2/2 = 1$

म): $$a_0 = \frac{1}{L} \int_0^{2L} f(x)\,dx = \frac{1}{(1)} \int_0^{2L} (4 - x^2)\,dx$$

.....(solve in calcy ES)

$\boxed{a_0 = 16/3}$

पान-30

(38) $f(x) = \begin{cases} -x, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ and $f(x+2\pi) = f(x)$.

Fourier series is represented by

$\frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx]$, then fourier coefficient is

A) $\pi/3$ B) $2/\pi$ C) $\pi/4$ D) π

Sol^n ⇒ Ans: D)

ड): $f(x) = \begin{cases} -x, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ ..... (given)

क्ष): Split limit:: $-\pi \to \pi$ As: $-\pi \to 0$, $0 \to \pi$

ज्ञ):

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$a_0 = \frac{1}{\pi} \left[\int_{-\pi}^{0} f(x)\, dx + \int_{0}^{\pi} f(x)\, dx \right]$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^{0} (-x)\, dx + \int_{0}^{\pi} (x)\, dx \right]$$

.....(solve in Radian Mode of ES calcy & refer NOTE in Q. 31)

$$\boxed{a_0 = \pi}$$

(39) $f(x) = x \cos x$, $-\pi \le x \le \pi$ and period is 2π. Fourier series given as $\frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx]$, the a_0 is

A) $-2/\pi$ B) 0 C) $4/\pi$ D) $-\pi/4$

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पान - ३९

Sol^n ⇒ Ans: B)

पा): $f(x) = x \cdot \cos x \quad , \quad -\pi < x < \pi$ ....(given)

Period $= 2\pi$

२):
$$a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x)\, dx$$
$$= \frac{1}{\pi} \int_0^{2\pi} (x \cdot \cos x)\, dx$$

..... (put in Radian Mode of ES calcy)

$$\boxed{a_0 = 0}$$

(40) $f(x) = 2 \, , \, -\pi < x < \pi$ and period 2π. Fourier series is $\frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx]$, then fourier coeff a_0 is

A) 4 B) 2 C) $4/\pi$ D) $2/\pi$

Sol^n ⇒ Ans: A)

पा): $f(x) = 2 \quad , \quad -\pi < x < \pi$ .....(given)

Period $= 2\pi$

२):
$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$$
$$= \frac{1}{\pi} \int_{-\pi}^{\pi} 2\, dx$$

..... (solve in Radian Mode of ES calcy)

$$\boxed{a_0 = 4}$$

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पान- ३२

(41) $f(x) = x, \ -\pi \leq x \leq \pi$ and period is 2π. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx]$, then b_1 is

A) 2 B) -1 C) 0 D) $2/\pi$

Sol^n ⇒ Ans: A)

Ⓢ): $f(x) = x, \ -\pi \leq x \leq \pi$ ....(given)

Period = 2π

स्व):

$$b_1 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin x \, dx \quad \text{.....(formula)}$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} x \cdot \sin x \, dx$$

....(put in Radian Mode of Calcy)

$$\boxed{b_1 = 2}$$

(42) $f(x) = \begin{cases} 1, & -1 < x < 0 \\ \cos \pi x, & 0 < x < 1 \end{cases}$ and period 2. Fourier series is

$\frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos n\pi x + b_n \sin n\pi x]$, then a_0 is

A) 2 B) 0 C) 1 D) -1

Ans: C)

Ⓢ): $f(x) = \begin{cases} 1, & -1 < x < 0 \\ \cos \pi x, & 0 < x < 1 \end{cases}$ .....(given)

Period ⇒ $2L = 2 \quad \therefore L = 2/2 = 1$

पान- 33

ख):
$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx \quad \text{.... (formula)}$$

ग): Split Limit:: $-1 \to 1$ As $-1 \to 0$, $0 \to 1$

घ):
$$a_0 = \frac{1}{(1)}\left[\int_{-1}^{0} 1\,dx + \int_{0}^{1} \cos\pi x\,dx\right]$$
..... (solve in Radian Mode in Calcy)

$$\boxed{a_0 = 1}$$

(43) $f(x) = x - x^3$, $-2 < x < 2$ & period 4. Fourier series is $\frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos n\pi x + b_n \sin n\pi x]$, then a_0 is

A) 1 B) 0 C) 1 D) -1

Solⁿ⇒ Ans: B)

क): $f(x) = x - x^3$, $-2 < x < 2$ .... (given)

Period ⇒ $2L = 4$ $\therefore L = 4/2 = 2$

ख):
$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$
$$= \frac{1}{(2)}\int_{-2}^{2} (x - x^3)\,dx$$
..... (solve in calcy)

$$\boxed{a_0 = 0}$$

पान - ३४

(44) For half range cosine series of $f(x) = \sin x, 0 \leq x \leq \pi$ and period is 2π. FS is $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$ then a_0 is

A) 4   B) 2   c) $2/\pi$ D) $4/\pi$

Sol^n ⇒ Ans: D)

इ): $f(x) = \sin x, \quad 0 \leq x \leq \pi$ ....(given)

Period ⇒ 2π

ब): Half Range Cosine :

$$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x)\, dx$$

$$= \frac{2}{\pi} \int_0^{\pi} (\sin x)\, dx$$

.....(solve in Radian Mode of Calcy)

$$\boxed{a_0 = 4/\pi}$$

(45) For half range sine series of $f(x) = \cos x, \ 0 \leq x \leq \pi$ and period is $\sum_{n=1}^{\infty} b_n \cdot \sin nx$, then fourier coefficient b_1 is

A) $\frac{1}{\pi}$ B) 0   c) $\frac{2}{\pi}$ D) $-\frac{2}{\pi}$

Sol^n ⇒ Ans: B)

इ): $f(x) = \cos x, \quad 0 \leq x \leq \pi$ ----(given)

Period = 2π

ब): Half Rane Sine :

$$b_1 = \frac{2}{\pi} \int_0^{\pi} f(x) \cdot \sin x \, dx$$

$$b_1 = \frac{2}{\pi}\int_0^{\pi} \cos x \cdot \sin x \, dx$$

....(solve in Radian Mode of Calcy)

$$\boxed{b_1 = 0}$$

(46) For half range cosine series of $f(x) = lx - x^2$, $0 \le x \le l$ and period $2l$. FS is $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cdot \cos\frac{n\pi x}{l}$, then a_0 is

A) $l^2/3$ B) $2l^2/3$ C) $l^2/6$ D) 0

Sol^n ⇒ Ans: A)

क):

$f(x) = lx - x^2$, $0 \le x \le l$ .....(given)

Period ⇒ $2l$ ∴ $2L = 2l$ ∴ $L = 2l/2 = l$

ख):

$$a_0 = \frac{2}{L}\int_0^{L} f(x)\,dx$$

$$= \frac{2}{(l)}\int_0^{l} (lx - x^2)\,dx$$

(∵ Put $l = 1$ = Assuming value)

$$= \frac{2}{1}\int_0^{1} (1x - x^2)\,dx$$

$$= 2\int_0^{1} (x - x^2)\,dx$$

.....(solve in calcy)

$a_0 = 1/3$

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पान-369

ग):

check option :: ⇒ ($\because$ Put: $l=1$ everywhere)

A) $\frac{l^2}{3}$ B) $\frac{2l^2}{3}$ C) $\frac{l^2}{6}$ D) 0

$= \frac{(1)^2}{3}$ $= \frac{2(1)^2}{3}$ $= \frac{(1)^2}{6}$

$= \frac{1}{3}$ $= \frac{2}{3}$ $= \frac{1}{6}$ $= 0$

✓ × × ×

(Matched)

घ): $\therefore$ $\boxed{a_0 = 1/3 = l^2/3}$

(47) For half range sine series of $f(x) = x$, $0 \le x \le 2$ and period is 4. fourier series is $\sum_{n=1}^{\infty} b_n \cdot \sin \frac{n\pi x}{2}$, b_1 is

A) 4 B) 2 C) $2/\pi$ D) $4/\pi$

Sol^n ⇒ Ans: D)

क): $f(x) = x$, $0 \le x \le 2$ .....(given)

Period ⇒ $2L = 4$ $\therefore L = 4/2 = 2$

ख): $b_1 = \frac{2}{L} \int_0^L f(x) \cdot \sin \frac{\pi x}{L} \, dx$

$= \frac{2}{L} \int_0^L x \cdot \sin \frac{\pi x}{2} \, dx$

$= \frac{2}{2} \int_0^2 x \cdot \sin \frac{\pi x}{2} \, dx$

..... (solve in Radian Mode in ES)

$\boxed{b_1 = 4/\pi}$

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पान - ३७

(48) Fourier series representation of periodic function $f(x) = \left[\frac{\pi - x}{2}\right]^2,\ 0 \le x \le 2\pi$ is $\left(\frac{\pi - x}{2}\right)^2 = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$, then value of $1/1^2 + 1/2^2 + 1/3^2 + \cdots =$

A) $\frac{\pi^2}{6}$ B) $\frac{\pi^2}{12}$ C) $\frac{\pi^2}{3}$ D) 0

Sol^n ⇒ Ans: A)

क): $f(x) = \left[\frac{\pi - x}{2}\right]^2,\ 0 \le x \le 2\pi$

and

$$\left[\frac{\pi - x}{2}\right]^2 = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{\cos nx}{n^2} \quad \ldots (1)$$

ख): Put $x=0$ in (1):

$$\left[\frac{\pi - 0}{2}\right]^2 = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{\cos n0}{n^2} \qquad \{\because \cos n0 = \cos 0 = 1\}$$

$$\frac{\pi^2}{4} = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\frac{\pi^2}{4} - \frac{\pi^2}{12} = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

(solve in ES calcy:)

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

solve Σ summation now: ($\because$ Put: $n = 1, 2, 3, \ldots$)

$$\frac{\pi^2}{6} = \frac{1}{(1)^2} + \frac{1}{(2)^2} + \frac{1}{(3)^2} + \cdots$$

$\therefore$ Answer is: $\boxed{\pi/6}$

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पाली - 38

(49) Fourier series representation of periodic function

$$f(x) = \begin{cases} 1 + 2x/\pi , & -\pi \le x \le 0 \\ 1 - 2x/\pi , & 0 \le x \le \pi \end{cases}$$ is

$$f(x) = \frac{8}{\pi^2}\left[\frac{1}{1^2}\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x + \cdots\right]$$

then value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ is = ?

A) $\frac{\pi^2}{4}$ B) $\frac{\pi^2}{8}$ C) $\frac{\pi^2}{16}$ D) $\frac{8}{\pi^2}$

Solⁿ ⇒ Ans: A)

डी>: $f(x) = \frac{8}{\pi^2}\left[\frac{1}{1^2}\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x + \cdots\right]$

Put: $x = 0$ & $f(x) = 1 + \frac{2x}{\pi}$ & $f(x) = 1 - \frac{2x}{\pi}$.

ऊ>: $$\left[1 + \frac{2x}{\pi}\right] + \left[1 - \frac{2x}{\pi}\right] = \frac{8}{\pi^2}\left[\frac{1}{1^2}\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x + \cdots\right]$$

Put: $x = 0$

$$\left[1 + \frac{2\times 0}{\pi}\right] + \left[1 - \frac{2\times 0}{\pi}\right] = \frac{8}{\pi^2}\left[\frac{1}{1^2}\cos 0 + \frac{1}{3^2}\cos 3\times 0 + \frac{1}{5^2}\cos 5\times 0 + \cdots\right]$$

$\{\because \cos 0 = 1\}$

$$[1+0] + [1+0] = \frac{8}{\pi^2}\left[\frac{1}{1^2}(1) + \frac{1}{3^2}(1) + \frac{1}{5^2}(1) + \cdots\right]$$

$$\cancel{2} \times \frac{\pi^2}{\cancel{8}\,4} = \left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right]$$

$$\frac{\pi^2}{4} = \left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right]$$

∴ Answer is $\boxed{\pi^2/4}$

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पान- ३८

(50) Fourier series representation of periodic function $f(x) = \pi^2 - x^2$, $-\pi \leq x \leq \pi$ is $[\pi^2 - x^2] = \frac{2\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \cos nx$,

then the value of $1/1^2 - 1/2^2 + 1/3^2 - 1/4^2 + \cdots =$

A) $\frac{\pi^2}{3}$ B) $\frac{\pi^2}{4}$ C) $\frac{\pi^2}{6}$ D) $\frac{\pi^2}{12}$

Sol^n ⇒ Ans: D)

क):
$$[\pi^2 - x^2] = \frac{2\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \cos nx$$

Put: $x = 0$

ख):
$$[\pi^2 - 0^2] = \frac{2\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \cos n \times 0$$

$\{\because \cos 0 = 1\}$

$$\pi^2 = \frac{2\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} (1)$$

$$\pi^2 - \frac{2\pi^2}{3} = 4\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$$

(∵ Solve on ES calcy)

$$\frac{\pi^2}{3} = 4\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$$

$$\frac{\pi^2}{3 \times 4} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$$

ग):

Put: $n = 1, 2, 3, 4, \cdots$ (∵ solve Σ)

$$\frac{\pi^2}{12} = \frac{(-1)^{1+1}}{(1)^2} + \frac{(-1)^{2+1}}{(2)^2} + \frac{(-1)^{3+1}}{(3)^2} + \frac{(-1)^{4+1}}{(4)^2} + \cdots$$

$$\frac{\pi^2}{12} = \frac{1}{1^2} + \frac{-1}{2^2} + \frac{1}{3^2} + \frac{-1}{4^2}$$

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2}$$

∴ Answer is $\boxed{\pi^2/12}$

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पाठ-80

(52) Fourier series the value of a_1 in fourier series of y with period 6 for the tabulated data is:

x	0	1	2	3	4	5
y	9	18	24	28	26	20
$\cos \pi x/3$	1	1/2	-1/2	-1	-1/2	1/2

A) -8.33 B) -7.14 C) -4.16 D) 0

Sol^n ⇒ Ans: A)

चरण 1: Given:

Period ⇒ $2L = 6$ ∴ $L = 6/2 = 3$

No. of observations (m) = 6

चरण 2:
$$a_1 = 2 \times \frac{\Sigma y \cdot \cos \pi x/L}{m} \quad \text{.....(formula)}$$

$$a_1 = 2 \times \frac{[9\times1 + 18\times 1/2 + 24\times -1/2 + 28\times -1 + 26\times -1/2 + 20\times 1/2]}{6}$$

$$\boxed{a_1 = -8.33}$$

(53) The value of b1 in Fourier series of y with period π for the following tabulated data is:

x°	0	30	60	90	120	150
y	0	9.2	14.4	17.8	17.3	11.7
$\sin x$	0	0.866	0.866	0	-0.866	-0.866

A) -3.116 B) -1.558 C) -4.16 D) -1.336

Sol^n ⇒ Ans: B)

चरण 1: Given: Period ⇒ π

No. of observations (m) = 6

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पान - ८९

ब):

$$b_1 = 2 \times \frac{\Sigma y \cdot \sin x}{m}$$

$$b_1 = 2 \times [0\times0 + 9.2\times0.866 + 14.4\times0.866 + 17.8\times0 + 17.3\times-0.866 + 11.7\times-0.866] / 6$$

$$\boxed{b_1 = -1.558}$$

(54) The value of a_1 in fourier series of y with period 6 for :

x	0	1	2	3	4	5
y	4	8	15	7	6	2
$\cos\frac{\pi x}{3}$	1	1/2	-1/2	-1	-1/2	1/2

A) = -2.83 B) = -8.32 C) = -3.57 D) = -10.98

Sol^n ⇒ Ans: D)

D): Given:

Period ⇒ $2L = 6 \quad \therefore L = 6/2 = 3$

No. of observations $= 6 = m$

ब):

$$a_1 = 2 \times \frac{\Sigma y \cdot \cos \pi x / L}{m}$$

$$a_1 = 2 \times \frac{[4\times1 + 8\times\frac{1}{2} + 15\times-\frac{1}{2} + 7\times-1 + 6\times-\frac{1}{2} + 2\times\frac{1}{2}]}{6}$$

$$\boxed{a_1 = -10.98}$$

(55) The value of b_1 in fourier series of y with period 2π for the following tabulated data is:

x°	0	60	120	180	240	300
y	1.0	1.4	1.9	1.7	1.5	1.2
$\sin x$	0	0.866	0.866	0	-0.866	-0.866

A) 0.0989 B) 0.3464 C) 0.1732 D) 0.6932

Sol^n ⇒ Ans: c)

क): Given:

Period = 2π

No. of observation $(m) = 6$

ख):

$$b_1 = 2 \times \frac{\Sigma y \cdot \sin x}{m}$$

$$= 2 \times \frac{[1.0 \times 0 + 1.4 \times 0.866 + 1.9 \times 0.866 + 1.7 \times 0 + 1.5 \times -0.866 + 1.2 \times -0.866]}{6}$$

$$\boxed{b_1 = 0.1732}$$

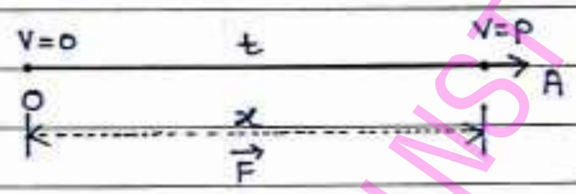
∴ Ans: c):

● NOTE:

① Rectillinear Motion:

It is a motion of a body along a straight line.

let, m-be the mass of body



i) velocity $(v) = \frac{dx}{dt}$

ii) Acceleration $(a) = \frac{dv}{dt}$

OR

Acceleration $(a) = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$

OR

Acceleration $(a) = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = v\frac{dv}{dx}$

Acceleration $(a) = \frac{dv}{dt}$ or $v\frac{dv}{dx}$ or $\frac{d^2x}{dt^2}$

iii) $F = ma = m\frac{dv}{dt}$

iv) $F = mv\frac{dv}{dx}$

v) Net force = Mass x Acceleration

Net force = Sum of all forces

{ D' Alembert's Principal }

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② Sign Convention:

FORCE:	FORCE SIGN:
i) Body fall vertically downward	⇒ +ve
ii) Body grow vertically upward	⇒ -ve
iii) Air resistance on body	⇒ -ve
iv) Opposite force on body	⇒ -ve
v) Body moves towards origin	⇒ -ve
vi) Body alway from origine	⇒ +ve

vii) $e^{-\infty} = 0$ and $e^{\infty} = \infty$

viii) $\left(\frac{\text{Constant}}{\text{Variable}}\right)^0 = 1$

ix) $(\text{Constant})^0 = 1$ x) $(\text{Variable})^0 = 1$

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पान-re

* Type: Rectilinear Motion / Straight line Motion:

① Rectilinear motion is a motion of body along a

A) straight line B) circular path
C) parabolic path D) None of these

Solⁿ ⇒ Ans: A)

straight line

....(यह स्टैंडर्ड है, रटलो।)

② According to D'Alembert's principle algebraic sum of forces acting on a body along a given direction is equal to

A) velocity × acceleration B) mass × velocity
C) mass × displacement D) mass × acceleration

Solⁿ ⇒ Ans: D)

Force = Mass × Acceleration

......(यह स्टैंडर्ड है, रटलो।)

③ A particle moving in a straight line with acceleration $k\left(x+\frac{a^4}{x^3}\right)$ directed towards the origin. The equation of motion is

A) $\frac{dv}{dx} = -k\left(x+\frac{a^4}{x^3}\right)$ B) $v\frac{dv}{dx} = k\left(x+\frac{a^4}{x^3}\right)$

C) $v\frac{dv}{dx} = -k\left(x+\frac{a^4}{x^3}\right)$ D) $\frac{dv}{dx} = \left(x+\frac{a^4}{x^3}\right)$

Solⁿ ⇒ Ans: C)

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पान-40

क): Given: $a = -k\left[x + \frac{a^4}{x^3}\right]$ .....(Refer NOTE for Sign convention)

ख): $F = F_1$ .....(D Alemberts principle)

$ma = ma$ $\{\because F = ma\}$

$\cancel{m}\left[v\frac{dv}{dx}\right] = -k\left[x + \frac{a^4}{x^3}\right]\cancel{m}$ $\{\because a = v\frac{dv}{dx}\}$

$$\boxed{v\frac{dv}{dx} = -k\left(x + \frac{a^4}{x^3}\right)}$$

(4) A particle of mass m moves in a horizontal straight line OA with acceleration k/x^3 at a distance x and directed towards the origine O. Then the differential equation of motion is

A) $v\frac{dv}{dx} = \frac{k}{x^3}$ ✓B) $v\frac{dv}{dx} = \frac{-k}{x^3}$

c) $\frac{dv}{dx} = \frac{-k}{x^3}$ D) $\frac{dv}{dx} = \frac{k}{x^3}$

Solⁿ ⇒ Ans: B)

क) $a = -k/x^3$ .....(Refer NOTE for sign changes)

ख): $F = F_1$ ....(D Alemberts Principal)

$ma = ma$ $\{\because F = ma\}$

$\cancel{m}\left(v\frac{dv}{dx}\right) = \cancel{m}\left(\frac{-k}{x^3}\right)$ $\{\because a = v\frac{dv}{dx}\}$

$$\boxed{v\,dv/dx = -k/x^3}$$

(5) A body of mass पान-४१ m falling from rest is subjected to a force of gravity and air resistance proportional to square of velocity (kv^2).
The equation of motion is

A) $m\frac{dv}{dx} = mg - kv^2$ B) $mv\frac{dv}{dx} = mg + kv^2$

C) $mv\frac{dv}{dx} = -kv^2$ ✓D) $mv\frac{dv}{dx} = mg - kv^2$

Sol^n ⇒ Ans: D)

क) Given: $F_1 = mg$ = Force of Gravity
$F_2 = -kv^2$ ....(Plz refer NOTE for sign changes)

ज्ञ) $F = F_1 + F_2$ .....(D Alemberts Principle)
$F = mg - kv^2$
$ma = mg - kv^2$ $\{\because F = ma \;,\; a = v\frac{dv}{dx}\}$

$$\boxed{m\left[v\,dv/dx\right] = mg - kv^2}$$

(6) A particle is projected vertically upward with velocity V_1 and resistance of air produces retardation (kv^2) where V is velocity. The equation of motion is

✓A) $v\frac{dv}{dx} = -g - kv^2$ B) $v\frac{dv}{dx} = -g + kv^2$

C) $v\frac{dv}{dx} = -kv^2$ D) $v\frac{dv}{dx} = g - kv^2$

Sol^n ⇒ Ans: A)

~~क): Given: m = 1 unit~~
~~$F_1 = cx$, $F_2 = -bv^2$~~

पान - ५२

दि): Given: $F_1 = -mg$ = Force of gravity ..... (Plz refer NOTE for sign convention)

$F_2 = -mkv^2$

उपा):

$F = F_1 + F_2$ .....(D-Alemberts Principle)

$F = -mg - mkv^2$

$\cancel{m}a = \cancel{m}(-g - kv^2)$ $\{\because F = ma,\ a = v\frac{dv}{dx}\}$

$v\frac{dv}{dx} = -g - kv^2$

$$\boxed{v\frac{dv}{dx} = -g - kv^2}$$

(7) A body starts moving from rest is opposed by a force per unit mass of value (cx) resistance per unit mass of value (bv^2) where v & x are velocity & displacement of body at that instant. The differential equation of motion is

A) $mv\frac{dv}{dx} = -cx - bv^2$ B) $v\frac{dv}{dx} = cx + bv^2$

✓C) $v\frac{dv}{dx} = -cx - bv^2$ D) $\frac{dx}{dv} = -cx - bv^2$

Sol^n ⇒ Ans: C)

दि): Given: $m = 1$ unit

$F_1 = -cx$ .....(Plz refer NOTE sign changes)

$F_2 = -bv^2$

सा):

$F = F_1 + F_2$

$ma = -cx - bv^2$

$m\left[v\frac{dv}{dx}\right] = -cx - bv^2$ $\{\because$ Put: $m = 1\}$

उ):

(1) $v \frac{dv}{dx} = -cx - bv^2$

$$\boxed{v \frac{dv}{dx} = -cx - bv^2}$$

(8) A body of mass m falls from rest under gravity, in a fluid whose resistance to motion at any time t is mk times its velocity where k is constant. The DE of motion is

A) $\frac{dv}{dt} = -g - kv$ ✓B) $\frac{dv}{dt} = g - kv$

C) $\frac{dv}{dt} = g + kv$ D) $\frac{dv}{dt} = mg - mkv$

Sol^n ⇒ Ans: B)

दि): Given: $F_1 = mg$ .... (plz refer NOTE for sign change)

$F_2 = -mkv$

उ):

$F = F_1 + F_2$

$ma = mg - mkv$

$\cancel{m}\left[v \frac{dv}{dx}\right] = \cancel{m}(g - kv)$

$v \frac{dv}{dx} = g - kv$

But: $v \frac{dv}{dx} = \frac{dv}{dt}$

$$\boxed{\frac{dv}{dt} = g - kv}$$

(9) A particle of mass m is projected vertically upward with velocity v, assuming the air resistance k times its velocity where k is constant. The differential eqⁿ of motion is

A) $\frac{dv}{dt} = mg - kv$ B) $m\frac{dv}{dt} = -mg + kv$

C) $m\frac{dv}{dt} = -kv$ ✓D) $m\frac{dv}{dt} = -mg - kv$

Solⁿ ⇒ Ans: D)

फ): $F_1 = -mg$ .... (plz refer NOTE for sign change)
  $F_2 = -kv$

ख): $F = F_1 + F_2$
  $ma = -mg - kv$
  $m\left[v\frac{dv}{dx}\right] = -mg - kv$

$$\boxed{m\frac{dv}{dt} = -mg - kv} \qquad \left\{\because v\frac{dv}{dx} = \frac{dv}{dt}\right\}$$

(10) Assuming that the resistance to movement of a ship through water in the form of $(a^2 + b^2v^2)$ where v is the velocity, a, b are constants. The differential eqⁿ for retardation of the ship moving with engine stopped is

A) $m\frac{dv}{dt} = -(a^2 + bv^2)^2$ B) $m\frac{dv}{dt} = (a^2 + b^2v^2)$

✓C) $m\frac{dv}{dt} = -(a^2 + b^2v^2)$ D) $m\frac{dv}{dx} = -(a^2 + b^2v^2)$

Sol^n ⇒ Ans : c)

क): Given : $F_1 = -(a^2+b^2v^2)$ .... (plz refer NOTE for sign change

ख):

$$F = F_1$$

$$ma = -(a^2+b^2v^2)$$

$$m\frac{dv}{dt} = -(a^2+b^2v^2) \qquad \left\{\because a = \frac{dv}{dt}\right\}$$

$$\boxed{m\frac{dv}{dt} = -(a^2+b^2v^2)}$$

(11) Differential equation of motion of a body of mass m falls from rest under gravity in a fluid whose resistance to motion at any time t is mk times its velocity where k is constant is $dv/dt = g-kv$, then the relation between velocity and time t is

A) $t = \frac{1}{k}\log\frac{g-kv}{g}$ ✓B) $t = \frac{1}{k}\log\frac{g}{g-kv}$

c) $t = \frac{1}{k}\log\frac{g}{g+kv}$ D) $t = -\frac{1}{k}\log\frac{1}{g-kv}$

Sol^n ⇒ Ans : B)

क): $\frac{dv}{dt} = g-kv$

use VS form:

ख):

$$\frac{1}{g-kv}\,dv = dt$$

Take Integration:

$$\int\frac{1}{g-kv}\,dv = \int dt$$

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$$\frac{1}{-k}\int \frac{-k}{g-kv}\,dv = \int dt \qquad \text{.....(adjustment of } -k)$$

$$\frac{-1}{k}\log(g-kv) = t + c \qquad \text{.....①}$$

ग) : Put : Initial condition : $t=0,\ v=0$

$$\frac{-1}{k}\log(g-k\times 0) = 0 + c$$

$$c = \frac{-1}{k}\log g = \frac{1}{k}\log g \qquad \left\{\because \frac{-1}{k} = \frac{1}{k} = \text{constant doesnt matter}\right\}$$

put in ①:

घ) : $$\frac{-1}{k}\log(g-kv) = t + \frac{1}{k}\log g$$

$$\frac{-1}{k}\log(g-kv) - \frac{1}{k}\log g = t$$

$$\frac{+1}{k}\left[-\log(g-kv)\right] - \left[\frac{1}{k}\log g\right] = t$$

But: $\frac{-1}{k}\log g = +\frac{1}{k}\log g = \text{constant}$.

Taking common

$$\frac{1}{k}\left[-\underset{\uparrow B}{\log(g-kv)} + \underset{\uparrow A}{\log g}\right] = t$$

$$\frac{1}{k}\left[\log\left[\frac{g}{g-kv}\right]\right] = t$$

$\{\because \log A - \log B = \log A/B\}$

$$\therefore \boxed{t = \frac{1}{k}\log\left(\frac{g}{g-kv}\right)}$$

(12) A body of mass m falling from rest is subjected to the force of gravity and air resistance proportional to square

पान-५७

of velocity (kv^2) satisfies the DE $mv\frac{dv}{dx} = k(a^2-v^2)$ where $ka^2 = mg$, then the relation between velocity and displacement is

✓A) $\frac{2kx}{m} = \log\frac{a^2}{a^2-v^2}$ B) $\frac{2kx}{m} = \log\frac{a^2-x^2}{a^2}$

B) $2kx = \log\frac{1}{a^2-v^2}$ D) $\frac{x}{m} = \log\frac{a^2}{a^2-v^2}$

Sol^n ⇒ Ans : A)

क): Given: $mv\frac{dv}{dx} = k(a^2-v^2)$

use VS form:

$$\frac{mv\,dv}{(a^2-v^2)} = k\,dx$$

ख): Take Integration:

$$m\int\frac{v\,dv}{a^2-v^2} = k\int dx$$

$$\frac{m}{-2}\int\frac{-2v}{a^2-v^2}\,dv = k\int dx \quad \text{.....(adjustment of -2)}$$

$$\frac{m}{-2}\log(a^2-v^2) + c = kx \quad \text{.....(1)}$$

ग): Put: Initial cond^n: $x=0,\ t=0,\ v=0$:

$$-\frac{m}{2}\log(a^2-0^2) + c = k(0)$$

$$-\frac{m}{2}\log a^2 + c = 0$$

$$c = \frac{m}{2}\log a^2$$

$\boxed{\text{पान - ४८}}$

६): put c in ①:

$$\frac{m}{-2}\log(a^2-v^2)+\frac{m}{2}\log a^2 = kx$$

$$\frac{m}{2}\left[-\log(a^2-v^2)+\log a^2\right] = kx$$

$$\underset{B}{-\log(a^2-v^2)}+\underset{A}{\log a^2} = kx \times \frac{2}{m}$$

$$\{\because \log A - \log B = \log A/B\}$$

$$\boxed{\log\left[a^2/(a^2-v^2)\right] = \frac{2kx}{m}}$$

13 A vehicle starts from rest and its acceleration is given by $dv/dt = k(1-t/T)$ where k and T are constant. Then the velocity v in term of time t is given by

A) $v=k\left(t-t^2/2\right)$ B) $v=k\left(t-\frac{t^2}{T}\right)$

C) $v=k\left(t^2/2-t^3/3T\right)$ ✓D) $v=k\left(t-t^2/2T\right)$

Ans: D)

५): $\frac{dv}{dT}=k\left(1-\frac{t}{T}\right)$

use VS form:

$dv = k(1-t/T)\,dt$ $\{\because dT=dt\}$

९): Take Integration:

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$$\int dv = k\int\left(1-\frac{t}{T}\right)dt$$

$$\int dv = k\int 1\,dt - k\frac{1}{T}\int t\,dt \qquad \left\{\because \begin{array}{l} t = \text{variable } (dt) \\ T = \text{constant} \end{array}\right\}$$

$$v = k(t) - k/_T\,(t^2/_2) + c \qquad \ldots\text{(1)}$$

अब: Put: initial condⁿ: $v=0,\ x=0,\ t=0$

$$0 = k(0) - k/_T\,(0^2/_2) + c$$

$$0 = 0 - 0 + c$$

$$c = 0$$

put $c=0$ in (1):

अब:
$$v = k(t) - \frac{k}{T}(t^2/_2) + 0$$

$$v = kt - \frac{kt^2}{2T}$$

$$\boxed{v = k\left[t - \frac{t^2}{2T}\right]}$$

(14) A particle of unit mass moves in a horizontal straight line OA with an acceleration k/r^3 at a distance r and directed towards O. If initially the particle was at rest at $r=a$ and eqⁿ of motion is $v\,dv/dr = -k/r^3$ then the relation between r, v is

A) $v^2 = k\left[\frac{1}{r^2} + \frac{1}{a^2}\right]$

B) $v^2 = k\left[\frac{1}{a^2} - \frac{1}{r^2}\right]$

C) $v^2 = k\left[\frac{1}{r^2} - \frac{1}{a^2}\right]$ ✓

D) $v^2 = k\left[\frac{1}{r^4} - \frac{1}{a^2}\right]$

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Sol^n ⇒ Ans: c)

क):

$$v\frac{dv}{dr} = \frac{-k}{r^3}$$

use VS form:

$$v\,dv = -k\frac{1}{r^3}\,dr$$

ख): Take Integration:

$$\int v\,dv = -k\int \frac{1}{r^3}\,dr$$

$$\int v\,dv = -k\int r^{-3}\,dr$$

$$\frac{v^2}{2} = -k\left(\frac{r^{-2}}{-2}\right) + c$$

$$\frac{v^2}{2} = \frac{k}{2}\frac{1}{r^2} + c \quad \cdots (1)$$

ग): Put: initial cond^n:

$v = 0,\ r = a$

$$\frac{0^2}{2} = \frac{k}{2}\frac{1}{a^2} + c$$

$$0 = \frac{k}{2a^2} + c$$

$$c = \frac{-k}{2a^2}$$

put c in (1):

घ):

$$\frac{v^2}{2} = \frac{k}{2r^2} + c$$

$$\frac{v^2}{\cancel{2}} = \frac{k}{\cancel{2}r^2} - \frac{k}{\cancel{2}a^2}$$

$$\boxed{v^2 = k\left[\frac{1}{r^2} - \frac{1}{a^2}\right]}$$

(15) A body of mass m falls from rest under gravity in a fluid whose resistance to motion at any instant is mkv where k is constant. The differential eq^n of motion is $dv/dt = g - kv$ then terminal velocity is

A) $\frac{k}{g}$ ✓B) $\frac{g}{k}$ C) $-\frac{g}{k}$ D) None

Sol^n ⇒ Ans: B)

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क): $\frac{dv}{dt} = g - kv$

ख): For Terminal velocity = $dv/dt = 0$

$0 = g - kv$

$kv = g$

$$\boxed{v = \frac{g}{k}}$$

(16) A bullet is fired into a sand tank, its retardation is proportional to the square root of its velocity. The DE of motion is $dv/dt = -k\sqrt{v}$. If V_0 is initial velocity then the relation between velocity v and time t is

A) $\sqrt{v} = -t + \sqrt{V_0}$ B) $2\sqrt{v} = -kt$

C) $\sqrt{v} = -kt + \sqrt{V_0}$ ✓D) $2\sqrt{v} = -kt + 2\sqrt{V_0}$

उत्तर⇒ Ans: D)

क): $\frac{dv}{dt} = -k\sqrt{v}$

Use VS form:

$\frac{1}{\sqrt{v}}\, dv = -k\, dt$

ख) Take Integration:

$\int \frac{1}{\sqrt{v}}\, dv = -k \int dt$ $\left\{ \because \int \frac{1}{\sqrt{x}}\, dx = 2\sqrt{x} \right\}$

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$$2\sqrt{v} = -k(t) + c \quad \ldots\ldots (1)$$

ग): Put: initial value $t=0$, $v=v_0$

$$2\sqrt{v_0} = -k \times 0 + c$$

$$2\sqrt{v_0} = c$$

put c in eq^n (1):

घ): $\boxed{2\sqrt{v} = -kt + 2\sqrt{v_0}}$

(17) A particle moving in a straight line with acceleration $k\left(x + \frac{a^4}{x^3}\right)$ directed towards the origin. the eq^n of motion is $v\, dv/dx = -k\left(x + a^4/x^3\right)$. If it starts from rest at a distance $x=a$ from the origin then the relation between velocity v and displacement x is

A) $\frac{v^2}{2} = k\left(\frac{x^2}{2} + \frac{a^4}{2x^2}\right)$ B) $\frac{v^2}{2} = -k\left(\frac{x^2}{2} - \frac{a^4}{2x^2}\right)$

C) $\frac{v^2}{2} = -k\left(\frac{x^2}{2} - \frac{a^4}{2x^2}\right) + \frac{a^2}{2}$ D) $\frac{v^2}{2} = -k\left(\frac{x^2}{2} - \frac{3a^4}{x^4}\right)$

Sol^n ⇒ Ans: B)

क): $v\frac{dv}{dx} = -k\left(x + \frac{a^4}{x^3}\right)$

use VS form:

$$v\, dv = -k\left(x + \frac{a^4}{x^3}\right) dx$$

ख): Taking Integrating:

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$$\int v\,dv = -k\int x\,dx - k\int \frac{a^4}{x^3}\,dx$$

$$v^2/2 = -k\left(x^2/2\right) - k a^4 \int x^{-3}\,dx$$

$$\frac{v^2}{2} = -\frac{k x^2}{2} - k a^4\left(\frac{x^{-2}}{-2}\right) + c \quad \cdots\cdots (1)$$

ग) : Put : initial values $x=0,\ t=0,\ v=0$:

$$\frac{0^2}{2} = -\frac{k\,0^2}{2} - k a^4\left(\frac{0^{-2}}{-2}\right) + c$$

$$0 = 0 - 0 + c$$

$$c = 0$$

put : $c=0$ in (1)

घ) : $$\frac{v^2}{2} = -\frac{k x^2}{2} - k a^4\left(\frac{x^{-2}}{-2}\right) + 0$$

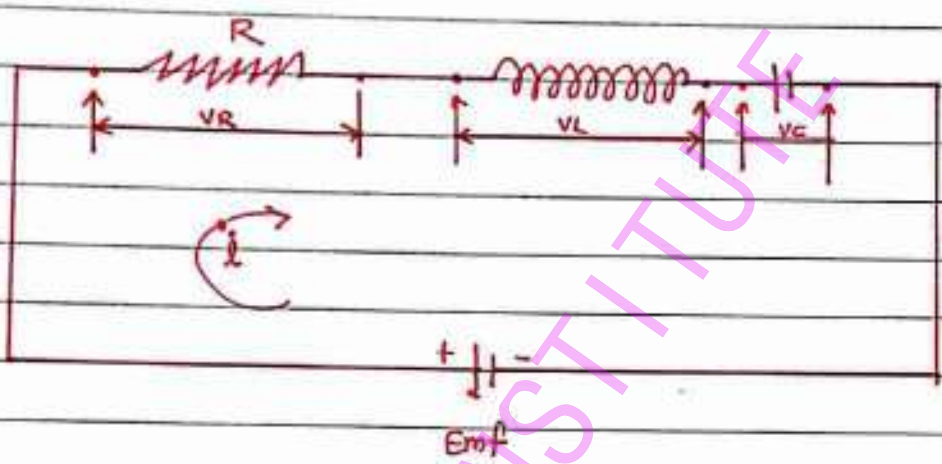
$$\frac{v^2}{2} = -\frac{k x^2}{2} + \frac{k a^4}{2}\left(\frac{1}{x^2}\right)$$

$$\boxed{\frac{v^2}{2} = -k\left[\frac{x^2}{2} - \frac{a^4}{2x^2}\right]}$$

$\therefore$ Ans : B)

• NOTE:

* ELECTRICAL CKT:-



• Basic Terminology:

अ): $i = \frac{dq}{dt}$ or $q = \int i \cdot dt$

आ): voltage drop across resistance $V_R = iR$

इ): voltage drop across inductance $V_L = L\frac{di}{dt}$

ई): voltage drop across capacitance $V_C = q/c$

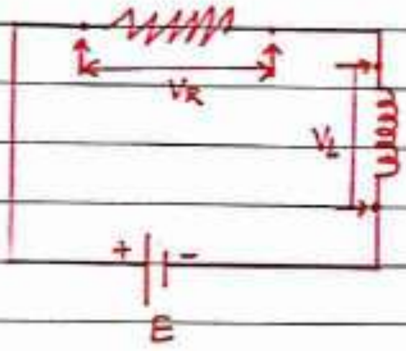
उ): Kirchoff's Voltage Law: KVL:
The algebric sum of the voltage drop around any closed ckt is equal to the resistance electromotive force/ battery in the ckt.

KVL for above ckt is :

$$V_R + V_L + V_C = Emf$$

$$iR + L\frac{di}{dt} + \frac{q}{c} = Emf$$

5): L-R ckt Terminology:



By KVL: $V_R + V_L = E$

$$iR + L\frac{di}{dt} = E$$

$$\therefore \frac{di}{dt} + \frac{R}{L}i = \frac{E}{L} \quad \text{....(LDEq}^n\text{)}$$

$$IF := e^{\int R/L\, dt} = e^{R/L\, t}$$

GS: =

$$i\, e^{R/L\, t} = \int \frac{E}{L} e^{R/L\, t}\, dt + A$$

$$i\, e^{R/L\, t} = \frac{E}{L} \times \frac{L}{R} e^{R/L\, t} + A$$

$$\therefore i = \frac{E}{R} + Ae^{-R/L\, t} \quad \text{.....(Result 01)}$$

Put: initial cond^n : $t=0, v=0, i=0$

$$\therefore A = -\frac{E}{R}$$

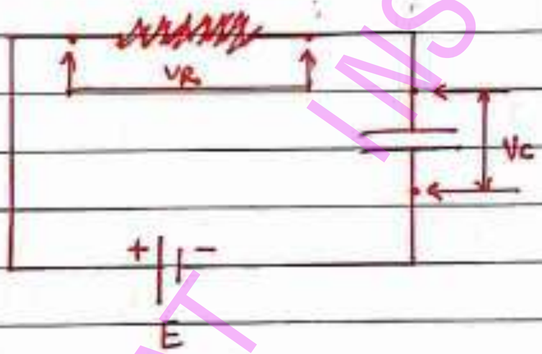
$$\therefore i = \frac{E}{R}\left[1 - e^{-R/L\, t}\right]$$

max current i, when $t = \infty$

$$\therefore i_{max} = \frac{E}{R} \quad \text{....... (Result 02)}$$

$$\{\because e^{-\infty} = 0\}$$

5): R-C ckt Terminology:



By KVL:

$$V_R + V_C = E$$

$$iR + \frac{q}{c} = E$$

$$R\frac{dq}{dt} + \frac{q}{c} = E,$$

$$\frac{dq}{dt} + \frac{1}{RC}q = \frac{E}{R} \quad \text{.... (LDEq}^n\text{)}$$

$$IF := e^{\int 1/RC\, dt} = e^{1/RC\, t}$$

GS:

$$q e^{1/RC\, t} = \int \frac{E}{R} e^{1/RC\, t}\, dt + A$$

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$$q e^{\frac{1}{RC}t} = EC\, e^{\frac{1}{RC}t} + A$$

$$q = EC + A e^{-\frac{1}{RC}t} \quad \text{.........(Result 01)}$$

Put: Initial condⁿ:

$t=0,\ i=0,\ q=q_0$

$\therefore\ q_0 = EC + A \quad$ OR $\quad A = q_0 - EC$

$$q = EC + (q_0 - EC)\, e^{-\frac{1}{RC}t}$$

$$i = \frac{dq}{dt} = \frac{d}{dt}\left\{ EC + (q_0 - EC)\, e^{-\frac{1}{RC}t} \right\}$$

......(Result 02)

ओ): Formula:

1) $V_R = iR$, $V_R = R\frac{dq}{dt}$

2) $V_C = \frac{q}{C}$

3) $V_L = L\frac{di}{dt}$

4) Emf $= E$

5) Inductance $= L$

6) Resistance $= R$

7) Capacitor $= C$

SET: 05

पान - ६८

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* Type: Application of Electrical ckt:

① A circuit containing resistance R and inductance L in the series with voltage source E, by kirchoff's voltage law, differential equation (DE) for current i is

A) $Li + R\frac{di}{dt} = E$ ✓B) $L\frac{di}{dt} + Ri = E$

C) $L\frac{di}{dt} + Ri = 0$ D) $L\frac{di}{dt} + \frac{q}{C} = E$

Sol^n ⇒ Ans: B)

Given:

Resistance = R, $V_R = iR$,

Inductance = L, $V_L = L\frac{di}{dt}$.

Emf = E,

According KVL: $V_R + V_L = E$

$$iR + L\frac{di}{dt} = E$$

$$\boxed{L\frac{di}{dt} + iR = E}$$

② A circuit containing resistance R and capacitance C in series with voltage source E, By kirchoff's voltage law, DE for current $i = dq/dt$ is

A) $L\frac{di}{dt} + \frac{q}{C} = E$ B) $R\frac{dq}{dt} + \frac{q}{C} = 0$

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c) $L\frac{di}{dt}+Ri=0$ ✓D) $R\frac{dq}{dt}+\frac{q}{c}=E$

Sol^n ⇒ Ans : D

5): Given:

Resistance = R ,

capacitance = C ,

Emf = E ,

$V_R = iR = R\frac{dq}{dt}$

$V_c = q/c$

२९): By KVL: $V_R + V_c = E$

$$\boxed{R\frac{dq}{dt}+\frac{q}{c}=E}$$

③ A circuit containing inductance L, capacitance C in series without applied electromotive force. By kirchoff's law, DE for current i is

✓A) $L\frac{di}{dt}+\frac{q}{c}=0$ B) $L\frac{di}{dt}+Ri=0$

c) $L\frac{di}{dt}+Ri=E$ D) $L\frac{di}{dt}+\frac{q}{c}=E$

Sol^n ⇒ Ans: A)

5): Given:

Inductance = L

Capacitance = C

E = Emf = 0

$V_L = L\frac{di}{dt}$

$V_c = \frac{q}{c}$

२९): By KVL: $V_L+V_c=E$ ∴ $\boxed{L\frac{di}{dt}+\frac{q}{c}=0}$

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(4) A circuit containing inductance L, capacitance C in series with applied electromotive force E. By kirchoff's voltage law, DE for current i is

A) $L\frac{di}{dt}+Ri=E$ B) $L\frac{di}{dt}+Ri=0$

✓C) $L\frac{di}{dt}+\frac{q}{c}=E$ D) $L\frac{di}{dt}+\frac{q}{c}=0$

Sol^n ⇒ Ans: C)

1): Given:

Inductance = L $V_L = L\frac{di}{dt}$

Capacitance = C

Emf = E $V_c = \frac{q}{c}$

2): By KVL: $V_L + V_c = E$

$$\boxed{L\frac{di}{dt}+\frac{q}{c}=E}$$

(5) The DE for the current in an electric circuit containing resistance R and inductance L in series with voltage $E\sin\omega t$ is

A) $L\frac{di}{dt}+\frac{q}{c}=E$ B) $Li+R\frac{di}{dt}=E\sin\omega t$

C) $L\frac{di}{dt}+Ri=0$ ✓D) $L\frac{di}{dt}+Ri=E\sin\omega t$

Ans: D)

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क): Given:

Resistance = R $\quad V_R = iR$

Inductance = L $\quad V_L = L\, di/dt$

$\mathcal{E}$ = Emf = $\mathcal{E} \sin\omega t$

ख): By KVL: $V_R + V_L = \mathcal{E}$

$$iR + L\frac{di}{dt} = \mathcal{E}\sin\omega t$$

$$\boxed{L\frac{di}{dt} + Ri = \mathcal{E}\sin\omega t}$$

6. In a circuit contains resistance R and inductance L in series with constant voltage E, current I is given by $i = \frac{E}{R}\left[1 - e^{-R/L\,t}\right]$, then maximum current i_{max} is

✓A) $\frac{E}{R}$ B) $\frac{R}{E}$ C) ER D) 0

Solⁿ ⇒ Ans: A)

क): Given: $i = \frac{E}{R}\left[1 - e^{-R/L\,t}\right]$

i_{max}, when $t = \infty$ and $e^{-\infty} = 0$

$\therefore i = \frac{E}{R}\left[1 - e^{-R/L\cdot\infty}\right] \quad \left\{\because -\frac{R}{L}\times\infty = \infty\right\}$

$\therefore i = \frac{E}{R}\left[1 - e^{-\infty}\right]$

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$$\therefore i = \frac{E}{R}[1-0] \qquad \{\because e^{-\infty}=0\}$$

$$\boxed{i = \frac{E}{R} \quad \text{OR} \quad i_{max} = \frac{E}{R}}$$

(7) The DE for the current i in an electric circuit contains resistance 100 ohm and an inductance of 0.5 henry connected in series with battery of 20 volts is

A) $0.5\frac{di}{dt}+100i=0$ ✓B) $0.5\frac{di}{dt}+100i=20$

C) $100\frac{di}{dt}+0.5i=20$ D) $100\frac{di}{dt}+0.5R=0$

Sol^n ⇒ Ans: B)

$): Given:

Resistance $R = 100\Omega$ $\{\because \text{ohm} = \Omega\}$

Inductance $L = 0.5H$ $V_R = iR, V_L = L\frac{di}{dt}$

emf $= E = 20$ volt

∴): By KVL:

$$V_R + V_L = E$$

$$iR + L\frac{di}{dt} = E$$

$$i(100) + 0.5\frac{di}{dt} = 20$$

$$\boxed{0.5\frac{di}{dt} + 100i = 20}$$

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पान-63

(8) The DE for the current i in an electric circuit containing resistance $R = 250\ \Omega$, inductance $L = 640\ H$ in series with an electromotive force $E = 500$ volts is

A) $640 \frac{di}{dt} + 250i = 0$ B) $250 \frac{di}{dt} + 640i = 500$

✓ C) $640 \frac{di}{dt} + 250i = 500$ D) $250 \frac{di}{dt} + 640i = 0$

Sol^n ⇒ Ans: C)

क): Given:

Resistance $= R = 250$

Inductance $= L = 640$

Emf $= E = 500$

$V_L = L\frac{di}{dt}$, $V_R = iR$

ख): By KVL:

$$V_R + V_L = E$$

$$iR + L\frac{di}{dt} = E$$

$$i(250) + 640\frac{di}{dt} = 500$$

$$\boxed{640\frac{di}{dt} + 250i = 500}$$

(9) A capacitor $C = 0.01$ farad in series with resistor $R = 20\,\Omega$ is charged from battery $E = 10V$. If initially capacitor is completely discharged from then DE for charge $q(t)$ is

A) $20\frac{dq}{dt} + \frac{q}{0.01} = 0,\ q(0) = 0$ C) $20\frac{dq}{dt} + 0.01q = 10,\ q(0) = 0$

✓ B) $20\frac{dq}{dt} + \frac{q}{0.01} = 10,\ q(0) = 0$ D) $20\frac{dq}{dt} + 0.01q = 0,\ q(0) = 0$

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पान-१०८

Sol^n ⇒ Ans: c)

स्प): Given: Capacitor $= C = 0.01$, $V_R = iR = R\frac{dq}{dt}$

Resistance $= R = 20$

Emf $= E = 10$ $V_C = q/c$

उत्तर): By KVL:

$$V_R + V_c = E$$

$$R\frac{dq}{dt} + \frac{q}{c} = E$$

$$\boxed{20\frac{dq}{dt} + \frac{q}{0.01} = 10}$$

(10) In a circuit containing resistance R and inductance L in series with constant emf E, the current i is given by $i = \frac{E}{R}\left[1 - e^{-R/L\,t}\right]$, then the time required to build current half of its theoretical maximum is given as

A) $\frac{L}{R\log 2}$ ✓B) $\frac{L\log 2}{R}$ C) $\frac{R\log 2}{L}$ D) $=0$

Sol^n ⇒ Ans: B)

स्प): $i = \frac{E}{R}\left[1 - e^{-R/L\,t}\right] = i_{max}$

use condition : $i_{max} = \frac{1}{2}\frac{E}{R}$

$$\cancel{\frac{E}{R}}\left[1 - e^{-R/L\,t}\right] = \frac{1}{2}\cancel{\frac{E}{R}}$$

$\boxed{\text{याद-log}}$

$$1-e^{-R/L\,t}=\frac{1}{2}$$

$$1-\frac{1}{2}=e^{-R/L\,t}$$

$$\frac{1}{2}=e^{-R/L\,t}$$

अब): Taking Log:

$$\log\frac{1}{2}=\log e^{-R/L\,t}$$

$$\log 2^{-1}=\log e^{-R/L\,t}$$

$\{\because \log e^{m}=m\log e,$

$\log e^{m}=m\ \}$

Rule:

$$-\log 2=-\frac{R}{L}t\,\cancel{\log e}$$

$$\log 2=\frac{R}{L}t$$

$$\frac{L}{R}\log 2=t$$

$$\boxed{t=\frac{L}{R}\log 2}$$

(11) In a circuit containing resistance R and inductance L in series with constant emf E, the current i is given by $i=\frac{E}{R}\left[1-e^{-R/L\,t}\right]$, then the time required before current reaches its 90% of maximum value is

A) 0 B) $\frac{L}{R\log 10}$ C) $\frac{R\log 10}{L}$ ✓D) $\frac{L\log 10}{R}$

Sol^n ⇒ Ans: D)

पार्ट - ७Eg

5):

$$i = \frac{E}{R}\left[1 - e^{-R/L\,t}\right]$$

we know that $i_{max} = \frac{E}{R}$

current i is 90% of i_{max}

$$\frac{90}{100}\,\cancel{\frac{E}{R}} = \cancel{\frac{E}{R}}\left[1 - e^{-R/L\,t}\right]$$

$$\frac{9}{10} = \left[1 - e^{-R/L\,t}\right]$$

$$e^{-R/L\,t} = 1 - 9/10 \qquad \left\{\because 1 - \frac{9}{10} = \frac{1}{10} = \overset{\text{use}}{\text{Calcy}}\right\}$$

$$e^{-R/L\,t} = 1/10$$

4): Use Log:

$$\log e^{-R/L\,t} = \log\left[1/10\right] \qquad \left\{\because \log e^{m} = m\right\}$$

$$-\frac{R}{L}t = \log 10^{-1}$$

$$-\frac{R}{L}t = -\log 10$$

$$\frac{R}{L}t = \log 10$$

$$\boxed{t = \frac{L\log 10}{R}}$$

(12) If the DE for current in a electric circuit containing resistance R and inductance L in series with constant emf E, the current i is $L\,di/dt + Ri = E$, then the current at any time t is

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पान-100

A) $i=\frac{E}{R}-Ae^{-R/L\,t}$ ✓B) $i=\frac{E}{R}+Ae^{-R/L\,t}$

C) $i=\frac{E}{R}+Ae^{R/L\,t}$ D) $i=\frac{E}{R}+e^{-R/L\,t}$

Solⁿ ⇒ Ans: B)

क): $\boxed{i=\frac{E}{R}+Ae^{-R/L\,t}}$ .....(यह स्टैंडर्ड है, रटलो।)

(• और जानकारी के लिए शुरुवातमें दी गई NOTE पढ़े।)

(13) A charge q on the plate of condenser of capacity c charge through a resistance R by steady voltage V satisfies DE $R\,dq/dt+q/c=V$, then charge q at any time is

✓A) $q=CV+Ae^{-1/RC\,t}$ B) $q=CV-Ae^{1/RC\,t}$

C) $q=C+Ae^{1/RC\,t}$ D) $q=CV+e^{1/RC\,t}$

Solⁿ ⇒ Ans: A)

क): Given: $R\frac{dq}{dt}+\frac{q}{c}=V$ $\{\because V=E\}$

By C-R ckt:

$$\boxed{q=VC+A+e^{-1/RC\,t}}$$

(कृपया, NOTE पढ़े।)

पान-60

(14) The charge q on the plate of condenser of capacity C charge through a resistance R by steady voltage V is given by $q = CV\left[1 - e^{-\frac{1}{RC}t}\right]$ then current flowing through the plate is

A) $i = \frac{V}{R} e^{-\frac{R}{L}t}$ B) $i = \frac{V}{R} e^{\frac{1}{RC}t}$

✓C) $i = \frac{V}{R} e^{-\frac{1}{RC}t}$ D) $i = CV\left[1 - e^{-\frac{1}{RC}t}\right]$

Sol^n ⇒ Ans: C)

C): Given: $q = CV\left[1 - e^{-\frac{1}{RC}t}\right]$

$$\therefore i = \frac{dq}{dt} = \frac{d}{dt}\left\{CV\left[1 - e^{-\frac{1}{RC}t}\right]\right\}$$

$$= CV\left\{0 - e^{-\frac{1}{RC}t} \times \frac{1}{RC}(1)\right\}$$

$$= \frac{\cancel{C}V}{R\cancel{C}}\left[e^{-\frac{1}{RC}t}\right]$$

$$\boxed{i = \frac{V}{R} e^{-\frac{1}{RC}t}}$$

(15) A resistance R = 100 ohm, Inductance L = 0·5 H, are connected in series with battery of 20V. The DE for the current i is $0.5\frac{di}{dt} + 100i = 20$, then current i at any time t is

A) Ae^{-200t} B) $\frac{1}{5} + Ae^{200t}$

C) $2 + Ae^{-200t}$ ✓D) $\frac{1}{5} + Ae^{-200t}$

पान-७८

Sol^n ⇒ Ans: D)

क):

$$0.5\frac{di}{dt} + 100i = 20$$

By L-R eq^n: ....(Refer NOTE)

$$i = \frac{E}{R} + Ae^{-R/L\, t}$$

ख):

$$i = \frac{20}{100} + Ae^{-100/0.5\, t}$$

$$\boxed{i = \frac{1}{5} + Ae^{-200t}} \qquad \left\{\because \frac{100}{0.5} = 200\right\}$$

(16) A circuit containing resistance R and inductance L in series with voltage source E. The DE for current i is

$L\frac{di}{dt} + Ri = E$, Given $L = 640$ H, $R = 250\ \Omega$, $E = 500$ V

then integrating factor of DE is

A) $e^{64/25\, t}$ ✓B) $e^{25/64\, t}$ C) $e^{-25/64\, t}$ D) e^{250t}

Sol^n ⇒ Ans: B)

क): Given: $L = 640$

$R = 250$

$E = 500$

ख):

$$\frac{di}{dt} + \frac{R}{L}i = E$$

$\frac{R}{L}$ ↑ P E ↑ Q

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पान - 20

ग):

$$IF = e^{\int p\,dt}$$

$$= e^{\int R/L\,dt}$$

$$= e^{R/L \int dt}$$

$$IF = e^{R/L\,t}$$

घ): $R = 250$, $L = 640$ ....(given)

$$IF = e^{250/640\,t}$$

$$\boxed{IF = e^{25/64\,t}}$$

$\therefore$ Ans: B)

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पान-L9

(17) If an R-C circuit charge q as function of time t is $q = e^{-3t} - e^{-6t}$, then time required for maximum charge on capacitor c is

A) $3 \log 2$ B) $-\frac{1}{3} \log 2$ C) $\frac{1}{3} \log 2$ D) $\frac{1}{2} \log 3$

Solⁿ → Ans: c)

क): By RC ckt:

$$q = e^{-3t} - e^{-6t}$$

$$\frac{dq}{dt} = -3e^{-3t} + 6e^{-6t}$$

ख): for maximum charge $dq/dt = 0$

$$0 = -3e^{-3t} + 6e^{-6t}$$

$$3e^{-3t} = 6e^{-6t}$$

$$\frac{e^{-3t}}{e^{-6t}} = \frac{\cancel{6}^{2}}{\cancel{3}_{1}}$$

$$e^{-3t} \cdot e^{6t} = 2 \qquad \{\because e^{a} \cdot e^{b} = e^{a+b}\}$$

$$e^{3t} = 2$$

ग): Taking log:

$$\log e^{3t} = \log 2 \qquad \{\because \log e^{m} = m\}$$

$$3t = \log 2$$

$$\boxed{t = \frac{1}{3} \log 2}$$

Ans: c) ::

पान - ६४

● NOTE:

① Exact DE:

$$M(x,y)\cdot dx + N(x,y)\cdot dy = 0$$

For Exact: $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

GS: i) $\int_{y=\text{constant}} M\cdot dx + \int [N \text{ free from } x]\cdot dy = c$

ii) $\int_{x=\text{constant}} N\cdot dy + \int [M \text{ free from } y]\cdot dx = c$

② Non- Exact DE:

$$M(x,y)\cdot dx + N(x,y)\cdot dy = 0$$

For Non Exact: $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

Now, convert it into Exact by multiplying integrating factors.

③ Rules : For Finding Integrating Factors:

$$M\,dx + N\,dy = 0$$

Rule 1:

$x\cdot M + y\cdot N \neq 0$ and given DE is Homogeneous then

$$I.F. = \frac{1}{x\cdot M + y\cdot N}$$

Rule 2:

$xM + yN \neq 0$ and
given DE is Non-homogeneous then

$$I.F. = \frac{1}{xM - yN}$$

Rule 3:

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x)$$

then

$$I.F. = e^{\int F(x)\, dx}$$

Rule 4:

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = g(y)$$

then

$$I.F. = e^{\int g(y)\, dy}$$

Rule 5:

$M\,dx + N\,dy = 0$ and eq^n of form of :

$$x^a y^b [My\,dx + Nx\,dy] + x^r y^s [Py\,dx + Qx\,dy] = 0$$

then

$$I.F. = x^h y^k$$

SET : 03

पान - EE

* Type: Exact DE and Reducible to Exact DE:

① The necessary and sufficient condition that the DE $M(x,y)\,dx + N(x,y)\,dy = 0$ be exact is

A) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, $My + Nx \neq 0$ B) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$, $Mx - Ny \neq 0$

C) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$, $Mx + Ny \neq 0$ D) $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 1$, $My - Nx \neq 0$

Sol^n ⇒

Ans: A) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, $My + Nx \neq 0$

......(स्टँडर्ड आहे, पाठांतर करा.)

② If homogeneous DE $M(x,y) + N(x,y)\frac{dy}{dx} = 0$ is not exact then the integrating factor is

A) $\frac{1}{My + Nx}$, $My + Nx \neq 0$ B) $\frac{1}{Mx - Ny}$, $Mx - Ny \neq 0$

C) $\frac{1}{Mx + Ny}$, $Mx + Ny \neq 0$ D) $\frac{1}{My - Nx}$, $My - Nx \neq 0$

Sol^n ⇒ Ans. C) $\frac{1}{Mx + Ny}$, $Mx + Ny \neq 0$

.....(स्टँडर्ड आहे, पाठांतर करा.)

③ If DE $M(x,y)\,dx + N(x,y)\,dy = 0$ is not exact and it can be written as $y f_1(xy)\,dx + x f_2(xy)\,dy = 0$ then the integrating factor is

पान - ६७

Sol^n/Q

A) $\frac{1}{My+Nx}$, $My+Nx \neq 0$ ✓B) $\frac{1}{Mx-Ny}$, $Mx-Ny \neq 0$

C) $\frac{1}{Mx+Ny}$, $Mx+Ny \neq 0$ D) $\frac{1}{My-Nx}$, $My-Nx \neq 0$

Sol^n →

Ans: B) $\frac{1}{Mx-Ny}$, $Mx-Ny \neq 0$

.....(स्टँडर्ड आहे, पाठांतर करा.)

④ If the DE $M(x,y)\,dx + N(x,y)\,dy = 0$ is not exact and

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x)$$ then the integrating factor is

A) $e^{f(x)}$ B) $e^{\int f(x)\,dy}$ C) $f(x)$ ✓D) $e^{\int f(x)\,dx}$

Sol^n →

Ans: D) $e^{\int f(x)\,dx}$

.....(स्टँडर्ड आहे, पाठांतर करा.)

⑤ If the DE $M(x,y)\,dx + N(x,y)\,dy = 0$ is not exact and

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{-M} = f(y)$$ then the integrating factor is

✓A) $e^{\int f(y)\,dy}$ B) $e^{\int f(y)\,dx}$ C) $f(y)$ D) $e^{f(y)}$

Sol^n →

Ans: A) $e^{\int f(y)\,dy}$

....(स्टँडर्ड आहे, पाठांतर करा.)

प्रश्न-8L

⑥ The total derivative of $x\,dy + y\,dx$ is

A) $d\left(\frac{y}{x}\right)$ B) $d\left(\frac{x}{y}\right)$ C) $d(xy)$ D) $d(x+y)$

Sol^n → Ans: c)

क): $x\,dy + y\,dx = 0$

$d(xy) + d(xy) = 0$

common term, so integrate only one term.

[illegible]

ख): Integrate x wrt dy: OR Integrate y wrt dx:

ie: $x \cdot dy$ ie: $y \cdot dx$

$d(x,y)$ $d(xy)$

$\therefore d(x,y) = d(xy)$

⑦ The total derivative of $x\,dy - y\,dx$ with integrating factor $1/x^2$ is

A) $d(x/y)$ B) $d(y/x)$ C) $d\left[\log x/y\right]$ D) $d(x-y)$

Sol^n → Ans: B)

क): $x\,dy - y\,dx = 0$ & $IF = 1/x^2$ .... (given)

ख): multiply both term by IF:

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पान - ६e

$$\frac{1}{x^2}\cdot x\,dy - \frac{1}{x^2}\cdot y\,dx = 0$$

$$\frac{dy}{x} - \frac{y\,dx}{x^2} = 0$$

$$\frac{x}{x}\left(\frac{dy}{x}\right) - \frac{y\cdot dx}{x^2} = 0 \quad (\because \text{adjustment of } x \text{ \& make same D}^r)$$

$$\frac{x\cdot dy}{x^2} - \frac{y\cdot dx}{x^2} = 0$$

$$\frac{x\,dy - y\,dx}{x^2} = 0$$

ग): This term is opposite of derivative of u/v Rule: $\left\{\because \frac{u}{v} = \frac{vu' - uv'}{v^2}\right.$

(Here: $u = u$ & $v = x$)

then

or $\left. d\left(\frac{u}{v}\right) = \frac{v(du) - u(dv)}{v^2}\right\}$

$\therefore$ we write as:

$$\boxed{d\left(\frac{y}{x}\right)}$$

⑧ The total derivative of $x\,dy + y\,dx$ with integrating factor $1/xy$ is

A) $d[\log x/y]$ B) $d[\log y/x]$

C) $d[\log(x+y)]$ ✓D) $d[\log xy]$

Sol^n ⇒ Ans: D)

क): $x\,dy + y\,dx = 0$ & $IF = 1/xy$ ....(given)

ख): multiply by IF:

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पान - ६०

$$\cancel{x}\, dy \times \frac{1}{\cancel{x}y} + \cancel{y}\, dx \times \frac{1}{x\cancel{y}} = 0$$

$$\frac{1}{y} dy + \frac{1}{x} dx = 0$$

2a): Taking Integrating:

$$\int \frac{1}{y} dy + \int \frac{1}{x} dx = 0$$

$$\log y + \log x = \log c$$

$$\cancel{\log}[yx] = \cancel{\log} c$$

$$yx = c$$

(But this is not in option.)

OR $\log[yx] = \log c$

Taking derivative

$$d[\log(xy)] = d[\log c]$$

$$\therefore d[\log(xy)] = 0$$

(this is Answer)

$\{ \because d(\log c) = 0$
$\because \log c = \text{constant} \}$

$$\therefore \boxed{d(\log(xy)) = 0}$$

(9) The total derivative of $x\,dy - y\,dx$ with integrating factor $1/xy$ is

A) $d[\log x/y]$ ✓B) $d[\log y/x]$

C) $d\left[\frac{y}{x}\right]$ D) $d[\log xy]$

Sol^n ⇒ Ans: B)

....(प्रॉब्लम नंबर (8) सारखाच सोडवा. सोपा आहे.)

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पान - ७९

(10) The total derivative of $x\,dy - y\,dx$ with integrating factor $1/x^2+y^2$ is

A) $d(\tan^{-1} y/x)$ B) $d(\tan^{-1} x/y)$

c) $d[\log(x^2+y^2)]$ D) None of these

Sol^n ⇒ Ans: A)

१): $x\,dy - y\,dx = 0$ & $IF = \frac{1}{x^2+y^2}$ .... (given)

२): multiply by IF:

$$x\,dy \times \frac{1}{x^2+y^2} - y\,dx \times \frac{1}{x^2+y^2} = 0$$

$$\frac{x}{x^2+y^2}\,dy - \frac{y}{x^2+y^2}\,dx = 0$$

३): Take Integration:

~~Aboth~~ Both term having same total no. of variable so integrate only one term.

४):

$$\int \frac{x}{x^2+y^2}\,dy = 0$$

$$x\int \frac{1}{(x)^2+(y^2)}\,dy = 0$$

{∵ Formula use: $\int \frac{1}{(a)^2+(x^2)}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$ }

Here: $a = x$ & $x = y$

$$\therefore \cancel{x}\left[\frac{1}{\cancel{x}}\tan^{-1}\left(\frac{y}{x}\right)\right] = \log c$$ (∵ adjustment of integration of zero is $\log c$)

$$\tan^{-1}\left(\frac{y}{x}\right) = \log c$$

च): Taking derivative:

$$d\left[\tan^{-1}(y/x)\right] = d[\log c] \qquad \left(\because \log c = \text{constant } \& \ \frac{d}{dx}(k) = 0\right)$$

$$\boxed{d\left[\tan^{-1}(y/x)\right] = 0}$$

(11) The total derivative of $dx+dy$ with integrating factor $1/(x+y)$ is

A) $d[\log(x-y)]$ B) $d[\log(x^2-y^2)]$
✓C) $d[\log(x+y)]$ D) None

Sol^n ⇒ Ans: c)

क): $dx + dy = 0$ & $IF = 1/(x+y)$ ---- (given)

ख): multiply by IF:

$$\frac{1}{x+y}dx + \frac{1}{x+y}dy = 0$$

ग): Both term having same & equal no. of variables/letters, so make integration of 1st term only.

$$\int \frac{1}{x+y}dx + \int \frac{1}{x+y}dy = 0$$

Integrate only one term:

$$\int \frac{1}{x+y}dx = 0$$

$$\log(x+y) = \log c$$

पान - ७३

a): Take derivative:

$$d[\log(x+y)] = d[\log c]$$

$$\boxed{d[\log(x+y)] = 0}$$

{∵ derivative of constant is zero & $\log c = \text{const.}$}

(12) The differential equation $(x+y-2)\,dx + (x-y+4)\,dy = 0$ is of the form

✓A) Exact B) Homogeneous C) Linear D) None

Sol^n ⇒ Ans: A)

D): $(x+y-2)\,dx + (x-y+4)\,dy = 0$

$$\underset{M}{\uparrow} \qquad \underset{N}{\uparrow}$$

$$\frac{\partial M}{\partial y} = (0+1-0) \qquad \frac{\partial N}{\partial x} = (1-0+0)$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 1$$

∴ It is Exact.

• Exact means ⇒ $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ both having same value.

(13) The value of λ for which the DE

$(xy^2 + \lambda x^2 y)\,dx + (x^3 + x^2 y)\,dy = 0$ is exact is

A) -3 B) 2 ✓C) 3 D) 1

Sol^n ⇒ Ans: C)

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$\boxed{\text{पान - ७४}}$

क): $(xy^2 + \lambda x^2 y)\,dx + (x^3 + x^2 y)\,dy = 0$

$\uparrow M \qquad \uparrow N$

$\frac{\partial M}{\partial y} = x(2y) + \lambda x^2 (1) = 2xy + \lambda x^2$

$\frac{\partial N}{\partial x} = 3x^2 + 2xy$

ख): Exact $\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$2\cancel{xy} + \lambda x^2 = 3x^2 + 2\cancel{xy}$

$\lambda \cancel{x^2} = 3\cancel{x^2}$

$\boxed{\lambda = 3}$

(14) The differential equation $(ay^2 + x + x^8)\,dx + (y^8 - y + bxy)\,dy = 0$ is exact if

A) $b \neq 2a$ B) $b = a$ C) $a = 1, b = 3$ D) $b = 2a$

Solⁿ ⇒ Ans: D)

क): $(ay^2 + x + x^8)\,dx + (y^8 - y + bxy)\,dy = 0$

$\uparrow M \qquad \uparrow N$

$\frac{\partial M}{\partial y} = a\,2y + 0 + 0 = 2ay$, $\frac{\partial N}{\partial x} = 0 - 0 + b(1)y = by$

ख): Exact $\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$2a\cancel{y} = b\cancel{y}$

$2a = b$

$\boxed{b = 2a}$

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पान-७४

(15) DE $(3+by\cos x)\,dx + (2\sin x - 4y^3)\,dy = 0$ is exact if

A) $b=-2$ B) $b=3$ C) $b=0$ D) $b=2$

Solⁿ⇒ Ans: D)

$$\underbrace{(3+by\cos x)}_{M}dx + \underbrace{(2\sin x - 4y^3)}_{N}dy = 0$$

$$\frac{\partial M}{\partial y} = 0 + b(1)\cos x = b\cos x \quad , \quad \frac{\partial N}{\partial x} = 2\cos x - 0 = 2\cos x$$

Exact $\Rightarrow \dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$

$$b\cancel{\cos x} = 2\cancel{\cos x}$$

$$\boxed{b=2}$$

(18) Integrating factor of homogeneous DE
$(xy-2y^2)\,dx + (3xy - x^2)\,dy = 0$ is

A) $\dfrac{1}{xy}$ B) $\dfrac{1}{x^2y^2}$ C) $\dfrac{1}{x^2y}$ D) $\dfrac{1}{xy^2}$

Solⁿ⇒ Ans: D)

IF for Homogeneous:

$$IF = \frac{1}{xM + yN}$$

पान - ७६

सो): $(xy-2y^2)dx + (3xy-x^2)dy = 0$

$\uparrow$ M $\qquad$ $\uparrow$ N

$$\therefore IF = \frac{1}{x\cdot M + y\cdot N}$$

$$= \frac{1}{x(xy-2y^2)+y(3xy-x^2)}$$

$$= \frac{1}{\cancel{x^2y} - 2xy^2 + 3xy^2 - \cancel{x^2y}} \qquad (\because -2+3=+1)$$

$$= \frac{1}{+1\,xy^2}$$

$$\boxed{IF = \frac{1}{xy^2}}$$

(19) IF of homogeneous DE
$(x^2-3xy+2y^2)dx + (3x^2-2xy)dy = 0$ is

A) $\frac{1}{xy}$ $\quad$ B) $\frac{1}{x^3}$ $\quad$ C) $\frac{1}{x^2y}$ $\quad$ D) $\frac{1}{x^2}$

Sol^n → Ans : B)

प्र): $(x^2-3xy+2y^2)dx + (3x^2-2xy)dy = 0$

$\uparrow$ M $\qquad$ $\uparrow$ N

सो): IF for Homogeneous :

$$IF = \frac{1}{x\cdot M + y\cdot N}$$

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पान - ७७

$$IF = \frac{1}{x(x^2-3xy+2y^2)+y(3x^2-2xy)}$$

$$= \frac{1}{x^3-\cancel{3x^2y}+\cancel{2xy^2}+\cancel{3x^2y}-\cancel{2xy^2}}$$

$$\boxed{IF = \frac{1}{x^3}}$$

(20) Integrating factor of homogeneous DE

$(y^2-2xy)dx + (2x^2+3xy)dy = 0$

A) $\frac{1}{x^2y^2}$ B) $\frac{1}{x^2y}$ C) $\frac{1}{4xy^2}$ D) $\frac{1}{y^2}$

Sol^n ⇒ Ans: c)

5): $\underbrace{(y^2-2xy)}_{M}dx + \underbrace{(2x^2+3xy)}_{N}dy = 0$

- Each term having degree (addition of power) is same ie. = 2

∴ It's Homogeneous.

20):

$$IF = \frac{1}{x\cdot M + y\cdot N}$$

$$= \frac{1}{x(y^2-2xy)+y(2x^2+3xy)}$$

$$= \frac{1}{xy^2-\cancel{2x^2y}+\cancel{2yx^2}+3xy^2}$$

$$\boxed{IF = \frac{1}{4xy^2}}$$

पान-७८

22 Integrating Factor for DE

$(x^2y^2+xy+1)y\,dx + (x^2y^2-xy+1)x\,dy = 0$ is

A) $\frac{1}{2x^3y^3}$ B) $\frac{1}{xy}$ C) ✓ $\frac{1}{2x^2y^2}$ D) $\frac{1}{x^2y}$

Sol^n ⇒ Ans: C)

१): $\underset{\substack{\uparrow \\ M}}{(x^2y^2+xy+1)y}\,dx + \underset{\substack{\uparrow \\ N}}{(x^2y^2-xy+1)x}\,dy = 0$

- Each term having degree (addition of power) is not same
  ∴ It's Non-Homogeneous.

२): IF for Non-Homogeneous:

$$IF = \frac{1}{x\cdot M - y\cdot N}$$

$$= \frac{1}{x(x^2y^2+xy+1)y - y(x^2y^2-xy+1)x}$$

$$= \frac{1}{\cancel{x^3y^3} + x^2y^2 + \cancel{xy} - \cancel{x^3y^3} + x^2y^2 - \cancel{xy}}$$

$$\boxed{IF = \frac{1}{2x^2y^2}}$$

23 IF for DE $(1+xy)y\,dx + (1-xy)x\,dy = 0$ is

A) ✓ $\frac{1}{2x^2y^2}$ B) $\frac{1}{x^2y}$ C) $\frac{1}{xy^2}$ D) $\frac{1}{y}$

Sol^n ⇒ Ans: A)

पान-09

क): $(1+xy)y\,dx + (1-xy)x\,dy = 0$

$\underset{M}{\uparrow}$ $\qquad$ $\underset{N}{\uparrow}$

- Degree of each term is not same

$\therefore$ It's Non-Homogeneous.

ख): IF for Non-Homogeous:

$$IF = \frac{1}{x \cdot M - y \cdot N}$$

$$= \frac{1}{x(1+xy)y - y(1-xy)x}$$

$$= \frac{1}{\cancel{xy} + x^2y^2 - \cancel{yx} + x^2y^2}$$

$$\boxed{IF = \frac{1}{2x^2y^2}}$$

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पान - ८०

(24) Integrating Factor for DE $(1+xy)y\,dx + (x^2y^2+xy+1)x\,dy = 0$ is

A) $\frac{1}{x^2y}$ B) $\frac{-1}{x^3y^3}$ C) $\frac{1}{xy^2}$ D) $\frac{1}{x^2y^2}$

Sol^n → Ans: B)

क): $\underset{M}{(1+xy)y}\,dx + \underset{N}{(x^2y^2+xy+1)x}\,dy = 0$

- Each term having different degree

∴ It's not -Homogeneous.

ख):

$$IF = \frac{1}{x\cdot M - y\cdot N}$$

$$= \frac{1}{x(1+xy)y - y(x^2y^2+xy+1)x}$$

$$= \frac{1}{\cancel{xy} + \cancel{x^2y^2} - x^3y^3 - \cancel{x^2y^2} - \cancel{xy}}$$

$$\boxed{IF = \frac{-1}{x^3y^3}}$$

(25) Integrating factor for DE $(x^2+y^2+x)dx + (xy)dy = 0$ is

A) $\frac{1}{x}$ B) $\frac{1}{x^2}$ C) x^2 D) x

Sol^n ⇒ Ans: D)

क): $\underset{M}{(x^2+y^2+x)}\,dx + \underset{N}{(xy)}\,dy = 0$

पान - L9

ब):
$$\frac{\partial M}{\partial y} = 0 + 2y + 0 = 2y \quad , \quad \frac{\partial N}{\partial x} = (1)y = y$$

क):
$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2y - y}{xy} = \frac{y}{xy} = \frac{1}{x}$$

ड):
$$IF = e^{\int \frac{1}{x} dx}$$
$$= e^{\log x} \qquad \left\{ \because \int \tfrac{1}{x} dx = \log a \;,\; e^{\log m} = m \right\}$$
$$= x$$

$$\boxed{IF = x}$$

(26) Integrating factor for DE $\left(y + \frac{y^3}{3} + \frac{x^2}{2}\right)dx + \left(\frac{x + xy^2}{4}\right)dy = 0$ is

A) $\frac{1}{x}$ B) x^3 C) x^2 D) $\frac{1}{x^3}$

Sol^n → Ans : B)

अ):
$$\underbrace{\left(y + \frac{y^3}{3} + \frac{x^2}{2}\right)}_{M} dx + \underbrace{\left(\frac{x + xy^2}{4}\right)}_{N} dy = 0$$

ब):
$$\frac{\partial M}{\partial y} = 1 + \frac{\cancel{3}y^2}{\cancel{3}} + \frac{0}{2} = 1 + y^2$$
$$\frac{\partial N}{\partial x} = \frac{1 + (1)y^2}{4} = \frac{1 + y^2}{4}$$

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ग):

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{(1+y^2) - \left(\frac{1+y^2}{4}\right)}{\left(\frac{x+xy^2}{4}\right)}$$

$$= \frac{\frac{4(1+y^2) - (1+y^2)}{4}}{\frac{(x+xy^2)}{4}}$$

$$= \frac{4+4y^2-1-y^2}{x+xy^2}$$

$$= \frac{3+3y^2}{x+xy^2}$$

$$= \frac{3\cancel{(1+y^2)}}{x\cancel{(1+y^2)}}$$

$$= \frac{3}{x}$$

घ):

$$IF = e^{\int \frac{3}{x} dx} = e^{3\int \frac{1}{x} dx}$$

$$= e^{3\log x}$$

Rule: $\{\because a \log m = \log m^a\}$

$$= e^{\log x^3}$$

$$\boxed{IF = x^3}$$

(30) Integrating factor for differential equation $y \log y \, dx + (x - \log y) dy = 0$ is

A) $\frac{1}{x}$ B) $\frac{1}{y}$ C) $\frac{1}{x^2}$ D) $\frac{1}{y^2}$

Solⁿ ⇒ Ans: B)

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पान- ८३

क): $y\log y\, dx + (x - \log y)\,dy = 0$

$\uparrow M \qquad \uparrow N$

$$\frac{\partial M}{\partial y} = \underset{u}{y}\cdot\underset{v}{\log y} \qquad \{\because uv = u'v + uv'\}$$

$$\frac{\partial M}{\partial y} = (1)(\log y) + (\cancel{y})\left(\frac{1}{\cancel{y}}\right) = \log y + 1$$

and:

$$\frac{\partial N}{\partial x} = 1 - 0 = 1$$

ख):

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{1 - [\log y + 1]}{y\log y} = \frac{1 - \log y - 1}{y\log y}$$

$$= \frac{-\log y}{y\log y}$$

$$= \frac{-1}{y}$$

ग):

$$IF = e^{\int -\frac{1}{y}dy} = e^{-\int (y)^{-1} dy} \qquad \left\{\because \int \frac{1}{x}dx = \log x\right\}$$

$$= e^{-\int \frac{1}{y}dy}$$

$$= e^{-\log y} \qquad \{\because -a\cdot\log m = \log m^{-a} : \text{Rule}\}$$

$$= \cancel{e}^{\cancel{\log} y^{-1}}$$

$$= y^{-1}$$

$$\boxed{IF = \frac{1}{y}}$$

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पाठ - LX

(31) Integrating factor for DE

$(y^4+2y)\,dx + (xy^3+2y^4-4x)\,dy = 0$

A) $2/x$ B) $1/y$ C) $1/y^3$ D) $2/y^2$

Sol$^n \Rightarrow$ Ans: c)

क): $(y^4+2y)\,dx + (xy^3+2y^4-4x)\,dy = 0$

$$\underset{M}{\uparrow} \qquad \underset{N}{\uparrow}$$

$$\frac{\partial M}{\partial y} = 4y^3 + 2(1) = 4y^3+2$$

$$\frac{\partial N}{\partial x} = (1)y^3 + 0 - 4(1) = y^3 - 4$$

ख): $$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{y^3-4-(4y^3+2)}{(y^4+2y)} = \frac{y^3-4-4y^3-2}{y^4+2y}$$

$$= \frac{-3y^3-6}{y^4+2y}$$

$$= \frac{-3y^3-6}{y(y^3+2)}$$

$$= \frac{-3\cancel{(y^3+2)}}{y\cancel{(y^3+2)}}$$

$$= \frac{-3}{y}$$

ग): $$IF = e^{\int \frac{-3}{y}\,dy}$$

$$= e^{-3\int \frac{1}{y}\,dy} = e^{-3\log y} = e^{\log y^{-3}} = y^{-3}$$

$$= \boxed{\frac{1}{y^3}}$$

32 Integrating factor for DE $(2x + e^{x}\log y)y \cdot dx + e^{x} \cdot dy = 0$ is

A) $\frac{1}{x}$ B) $\frac{1}{y^2}$ C) $\frac{1}{x^2}$ ✓D) $\frac{1}{y}$

Sol^n ⇒ Ans: D)

क): $(2x + e^{x} \cdot \log y) y$ and e^{x}

$(\underset{M}{2xy} + e^{x} \cdot \underset{u}{y} \cdot \underset{v}{\log y})$ $\underset{N}{e^{x}}$

$\{\because uv = u'v + uv'\}$

ख):- ··· (मागच्या गणितासारखाच सोडवा. नक्की जमेल.)

33 Solution of non-exact differential eq^n $(x^2 - 3xy + 2y^2)\, dx + x(3x - 2y)\, dy = 0$ with integrating factor $1/x^3$ is

A) $3\frac{y}{x} - \frac{y^2}{x^2} = c$ B) $\log x - 3\frac{y}{x} + \frac{y^2}{x^2} = c$

C) $\log x + 3\frac{y}{x} - 2\frac{y^2}{x^2} = c$ ✓D) $\log x + 3\frac{y}{x} - \frac{y^2}{x^2} = c$

Sol^n ⇒ Ans: D)

क): $\underset{M}{(x^2 - 3xy + 2y^2)}\, dx + \underset{N}{x(3x - 2y)}\, dy = 0$

Multiply both side by $1/x^3$:

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ख): $(x^2 - 3xy + 2y^2) \cdot dx \times 1/x^3 + (3x^2 - 2xy)\, dy \times 1/x^3 = 0$

$$\left[\frac{x^2}{x^3} - \frac{3xy}{x^3} + \frac{2y^2}{x^3}\right]dx + \left[\frac{3x^2}{x^3} - \frac{2xy}{x^3}\right]dy = 0$$

$$\left[\frac{1}{x} - \frac{3y}{x^2} + \frac{2y^2}{x^3}\right]dx + \left[\frac{3}{x} - \frac{2y}{x^2}\right]dy = 0$$

ग): use method: $\underset{y\text{-constant}}{\int M\, dx} + \int N$ - free from $x = C$

OR

also we can write as: 'DROP' method.

$$\int M \cdot dx + \int \underset{\substack{\uparrow \\ \text{drop}=x}}{N}\, dy = C$$

घ): $\int\left(\frac{1}{x} - \frac{3y}{x^2} + \frac{2y^2}{x^3}\right)dx + \left[\cancel{\frac{3}{x}} - \cancel{\frac{2y}{x^2}}\right]dy = 0$

$\uparrow$ drop $= x$

च): Take integration:

$$\int \frac{1}{x}\, dx - 3y\int \frac{1}{x^2}\, dx + 2y^2\int \frac{1}{x^3}\, dx = C$$

$$\log x - 3y \cdot \left(\frac{-1}{x}\right) + 2y^2 \int x^{-3}\, dx = C$$

$$\log x + \frac{3y}{x} + \cancel{2}y^2\left(\frac{x^{-2}}{-\cancel{2}}\right) = C$$

$\left\{ \because \text{formula used: } \int \frac{1}{x}\, dx = \log x,\ \int \frac{1}{x^2}\, dx = \frac{-1}{x},\ \int x^n\, dx = \frac{x^{n+1}}{n+1} \right\}$

$$\therefore \boxed{\log x + 3y/x - y^2/x^2 = C}$$

पान- ८७

(34) Solution of non-exact DE $(3xy^2-y^3)dx+(xy^2-2x^2y)dy=0$ with integrating factor $\frac{1}{x^2y^2}$ is

A) $3\log x - \frac{2y}{x^3} - 2\log y = c$ B) $3\log x + \frac{y}{x} - 2\log y = c$

C) $3\log x + \frac{y}{x} = c$ D) $\log x - \frac{y}{x} + 2\log y = c$

Sol$^n \Rightarrow$ Ans: B)

१): $(3xy^2-y^3)\,dx + (xy^2-2x^2y)\,dy = 0$

multiply by $\frac{1}{x^2y^2}$:

२): $\left[\frac{3xy^2}{x^2y^2} - \frac{y^3}{x^2y^2}\right]dx + \left[\frac{xy^2}{x^2y^2} - \frac{2x^2y}{x^2y^2}\right]dy = 0$

$\left[\frac{3}{x} - \frac{y}{x^2}\right]dx + \left[\frac{1}{x} - \frac{2}{y}\right]dy = 0$

३): use drop method:

$\left[\frac{3}{x} - \frac{y}{x^2}\right]dx + \left[\cancel{\frac{1}{x}} - \frac{2}{y}\right]dy = 0$

$\uparrow$ drop $= x$

४): Taking Integration:

$3\int\frac{1}{x}dx - y\int\frac{1}{x^2}dx - 2\int\frac{1}{y}dy = c.$

$3\log x - y\left(\frac{-1}{x}\right) - 2\log y = c$

$\therefore$ $\boxed{3\log x + y/x - 2\log y = c}$

(35) Solution of non-exact DE $(1+xy)y\,dx + (1-xy)x\,dy = 0$ with integrating factor $1/(x^2y^2)$ is

A) $\frac{2}{xy} - \log\left(\frac{x}{y}\right) = c$ B) $\frac{-1}{xy} + \log\left(\frac{y}{x}\right) = c$

✓C) $\frac{-1}{xy} + \log\left(\frac{x}{y}\right) = c$ D) $\frac{-2}{x^3y} + \log\left(\frac{x}{y}\right) = c$

Sol^n → Ans : C)

क):

$$(1+xy)y\cdot dx + (1-xy)x\cdot dy = 0$$

$$(y+xy^2)\cdot dx + (x-x^2y)\cdot dy = 0$$

multiply by $\frac{1}{x^2y^2}$:

ख):

$$\left[\frac{\cancel{y}}{x^2y^{\cancel{2}}} + \frac{\cancel{x}y^2}{x^{\cancel{2}}y^2}\right]dx + \left[\frac{x}{x^2y^2} - \frac{\cancel{x^2y}}{x^2y^{\cancel{2}}}\right]dy = 0$$

$$\left[\frac{1}{yx^2} + \frac{1}{x}\right]dx + \left[\frac{1}{xy^2} - \frac{1}{y}\right]dy = 0$$

ग): use drop method:

$$\left[\frac{1}{yx^2} + \frac{1}{x}\right]dx + \left[\cancel{\frac{1}{xy^2}} - \frac{1}{y}\right]dy = 0$$

drop $= x$

घ): Taking Integration:

$$\frac{1}{y}\int\frac{1}{x^2}\,dx + \int\frac{1}{x}\,dx - \int\frac{1}{y}\,dy = c$$

$$\frac{1}{y}\left(\frac{-1}{x}\right) + \log \underset{A}{x} - \log \underset{B}{y} = c$$

Rule: $\{\because \log A - \log B = \log(A/B)\}$

$$\therefore \boxed{\frac{-1}{xy} + \log\left(\frac{x}{y}\right) = c}$$

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(36) Solution of non-exact DE
$(2+x^2y^2)y\,dx + (2-2x^2y^2)x\,dy = 0$ with
integrating factors $\frac{1}{x^3y^3}$ is

A) $\log\left(\frac{x}{y^2}\right) - \frac{1}{x^2y^2} = c$ B) $\log\left(\frac{x}{y^2}\right) + \frac{1}{x^2y^2} = c$

C) $\log\left(\frac{y^2}{x}\right) - \frac{1}{x^2y^2} = c$ D) $\log x - \frac{1}{x^2y^2} = c$

Sol^n $\Rightarrow$ Ans: A)

क): $(2+x^2y^2)y\cdot dx + (2-2x^2y^2)x\cdot dy = 0$

$(2y + x^2y^3)\,dx + (2x - 2x^3y^2)\,dy = 0$

multiply by $\frac{1}{x^3y^3}$:

ख): $$\left[\frac{2y}{x^3y^3} + \frac{x^2y^3}{x^3y^3}\right]dx + \left[\frac{2x}{x^3y^3} - \frac{2x^3y^2}{x^3y^3}\right]dy = 0$$

$$\left[\frac{2}{x^3y^2} + \frac{1}{x}\right]dx + \left[\frac{2}{x^2y^3} - \frac{2}{y}\right]dy = 0$$

ग): use drop method:

$$\left[\frac{2}{x^3y^2} + \frac{1}{x}\right]dx + \left[\frac{2}{x^2y^3} - \frac{2}{y}\right]dy = 0$$

$\uparrow$ drop = x

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ध): Take Integration:

$$\frac{2}{y^2}\int \frac{1}{x^3}dx + \int \frac{1}{x}dx - 2\int \frac{1}{y}dy = c$$

$$\frac{2}{y^2}\int x^{-3}dx + \log x - 2\log y = c$$

$$\frac{\cancel{2}}{y^2}\left[\frac{x^{-2}}{-\cancel{2}}\right] + \log x - \log y^2 = c$$

$$-\frac{1}{y^2}\left[\frac{1}{x^2}\right] + \log \underset{A}{x} - \log \underset{B}{y^2} = c$$

$\{\because \log A - \log B = \log(A/B)\}$

$$\boxed{\frac{-1}{x^2y^2} + \log\left[\frac{x}{y^2}\right] = c}$$

(38) Solution of non-exact DE $(x^4e^x - 2mxy^2)dx + (2mx^2y)dy = 0$ with Integrating factor $\frac{1}{x^4}$ is

A) $e^x + \frac{6my^2}{x^4} = c$ B) $e^x + \frac{2my^2}{x^2} = c$

C) $e^x + \frac{y^2}{x^2} = c$ ✓D) $e^x + \frac{my^2}{x^2} = c$

Solⁿ ⇒ Ans: D)

श): $(x^4e^x - 2mxy^2)dx + (2mx^2y)dy = 0$

multiply by $1/x^4$:

ध): $\left[\frac{x^4e^x}{x^4} - \frac{2mxy^2}{x^4}\right]dx + \left[\frac{2mx^2y}{x^4}\right]dy = 0$

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$$\left[e^x - \frac{2my^2}{x^3}\right] dx + \left[\frac{2my}{x^2}\right] dy = 0$$

अ): use drop method:

$$\left[e^x - \frac{2my^2}{x^3}\right] dx + \left[\frac{2m\cancel{y}}{\cancel{x}^2}\right] dy = 0$$

$\uparrow$ drop = x

ब): Take Integration:

$$\int e^x\,dx - 2my^2\int \frac{1}{x^3}\,dx = c$$

Formula:

$\left\{\because \int e^x dx = e^x\right\}$

$\left\{\because \int x^n dx = \frac{x^{n+1}}{n+1}\right\}$

$$e^x - 2my^2\int x^{-3}\,dx = c$$

$$e^x - 2my^2\left[\frac{x^{-3+1}}{-3+1}\right] = c$$

$$e^x - \cancel{2}my^2\left[\frac{x^{-2}}{-\cancel{2}}\right] = c$$

$$e^x + my^2\left(\frac{1}{x^2}\right) = c$$

$$\boxed{e^x + \frac{my^2}{x^2} = c}$$

$\therefore$ Ans: D):

पान - ३९

● NOTE :

① Newton's Law of cooling :

• Statement ⇒

The temperature of body changes at a rate which is proportional to the difference in temperature between that of the surrounding medium and that of body itself.

Let :

θ = body temperature,

θ_0 = surrounding temperature

t = time

then

ब) : $$\frac{d\theta}{dt} \propto (\theta - \theta_0)$$

$$\frac{d\theta}{dt} = -k(\theta - \theta_0) \quad \text{..... [Result : 01]}$$

{∵ k = Constant of Proportionality,
-ve sign means temperature decreases.}

अ) : Now, Use VS form :

$$\frac{1}{(\theta - \theta_0)} d\theta = -k\,dt$$

$$\int \frac{1}{(\theta - \theta_0)} d\theta = -k \int dt$$

$$\log(\theta - \theta_0) + \log c = -kt$$

$$\log[c \cdot (\theta - \theta_0)] = -kt \quad \text{..... [Result : 02]}$$

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ग): Taking Antilog:

$$c \cdot (\theta - \theta_0) = e^{-kt}$$

$$(\theta - \theta_0) = \frac{1}{c} e^{-kt}$$

{∵ Put $1/c = c_1$ }

$$\theta - \theta_0 = c_1 e^{-kt}$$

$$\theta = \theta_0 + c_1 e^{-kt}$$ .....(Result: 03)

SET : 03

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* Type : Newton's Law of Cooling:

① Newton's law of cooling [Aryabhatta law] is states that

Ans : D) :
the temperature of body changes at the rate which is inversaly proportional to the difference in temperature between that of surrounding medium and that of body itself.
....(करेक्ट आहे, पाठांतर करा.)

② A metal ~~rod~~ ball is heated to a temperature of 100°c and at time t=0 it is placed in water which is maintained at 40°c. By Newton's law of cooling the differential eq^n satisfied by temperature θ of metal ball at any time t is

A) $\frac{d\theta}{dt} = -k(\theta - 100)$ ✓B) $\frac{d\theta}{dt} = -k(\theta - 40)$

C) $\frac{d\theta}{dt} = -k\theta$ D) $\frac{d\theta}{dt} = -k\theta(\theta - 40)$

Sol^n ⇒ Ans: B)

३) : Given: $\theta = 100$
$\theta_0 = 40$
$t = 0$

२) : By Result 01: $\frac{d\theta}{dt} = -k(\theta - \theta_0)$

$$\boxed{\frac{d\theta}{dt} = -k(\theta - 40)}$$

③ According to Newtons law of cooling, rate at which a substance cools in moving air is proportional to the difference between the temperature of substance and air. A substance initially at temperature 90°c is kept in moving air at temperature 26°c, the differential equation satisfied by temperature θ of substance at any time t is

✓A) $\frac{d\theta}{dt} = -k(\theta - 26)$ B) $\frac{d\theta}{dt} = -k(\theta - 90)$

C) $\frac{d\theta}{dt} = -k\theta$ D) $\frac{d\theta}{dt} = -k(\theta - 64)$

Sol^n ⇒ Ans: A)

१): Given: $\theta = 90$, $t = 0$

$\theta_0 = 26$

२): By Result: 01:

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

$$\boxed{\frac{d\theta}{dt} = -k(\theta - 26)}$$

④ Suppose a corpse at a temperature of 32°c arrives at mortuary where the temperature is kept at 10°c. Then by Newton's law of cooling the DE satisfied by temperature T of corpse t hours later is

A) $\frac{dT}{dt} = -kT(T-10)$ B) $\frac{dT}{dt} = -k(T-32)$

✓C) $\frac{dT}{dt} = -k(T-10)$ D) $\frac{dT}{dt} = -kT(T-32)$

Sol^n ⇒ Ans: C)

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पान-३४

5): Given: $\theta = 32$, $t = 0$

$\theta_0 = 10$

2): By Result 01: $\frac{d\theta}{dt} = -k(\theta - \theta_0)$

$$\boxed{\frac{d\theta}{dt} = -k(\theta - 10)}$$

(6) If θ_0 is the temperature of surrounding and θ is temperature of body at time t satisfies the DE $d\theta/dt = -k(\theta - \theta_0)$ then θ is given by

A) $\theta = \theta_0 e^{-kt}$ B) $\theta = \theta_0 + Ae^{kt}$

C) $\theta = -k(\theta_0 + Ae^{-kt})$ ✓D) $\theta = \theta_0 + Ae^{-kt}$

Solⁿ ⇒ Ans: D)

5): $\frac{d\theta}{dt} = -k(\theta - \theta_0)$

By Result 03:

$$\theta = \theta_0 + C_1 e^{-kt}$$

Put: $C_1 = A$ = constant dosn't matter anything.

$$\boxed{\theta = \theta_0 + Ae^{-kt}}$$

(7) Suppose a corpse at a temperature of 32°c arrives at mortuary where the temperature is kept at 10°c. If the corpse cools to 27°c in 5 minutes. If the DE by Newton's law of cooling is $dT/dt = -0.05(T - 10)$, then

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$\boxed{\text{पाल - ३६९}}$

temperature T of corpse at any time t is given by

A) $T = 22 e^{-0.09t}$ B) $T = 10 + 22 e^{0.05t}$

C) $T = 10 + 22 e^{-0.05t}$ D) $T = 10 - 22 e^{-0.05t}$

Sol^n =)

Ans: c)

क): Given: $T = 32 = \theta$

$T_0 = 10 = \theta_0$

$k = 0.05$

$$\frac{dT}{dt} = -0.05(T-10) \text{ .....(given)}$$

$\Rightarrow$ comparing:

$$\frac{d\theta}{dt} = -k(\theta - \theta_0) \text{ ......(std)}$$

..... [Result: 01]

ख): Formula: $c_1 = (T - T_0)$

$c_1 = (32 - 10)$

$c_1 = 22$

ग): By Result 03:

$$\theta = \theta_0 + c_1 e^{-kt}$$

using given values:

$$\boxed{T = 10 + 22 e^{-0.05t}}$$

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पान - 36

(8) A teermpometer is taken outdoors where temperature is 0°c, from a room in which temperature is 21°c and temperature drops to 10°c in 1 minute. If DE by newton is $dT/dt = -(0.7419)T$, then temperature T of the thermometer at time t is

A) $T = 21 + 11e^{-0.7419t}$ C) $T = 21e^{0.7419t}$

B) $T = 10 + 21e^{-0.7419t}$ ✓D) $T = 21\ e^{-0.7419t}$

Sol^n ⇒ Ans: D)

क): Given:

$$\frac{dT}{dt} = -(0.7419)[T] \text{ ----- (given)}$$

⇒ compare:

$$\frac{d\theta}{dt} = -k[T - T_0] \text{ ..... [Result 01]}$$

$\therefore$ $\theta = T$, $k = 0.7419$, $T = T$, $T_0 = 0 = \theta_0$

ख): By Result 03:

$$\theta = \theta_0 + c_1 e^{-kt}$$

ग): $c_1 = T - T_0$ {∵ T = Initial Temperature = 21}

$= 21 - 0$

$c_1 = 21$

घ): $\theta = \theta_0 + c_1 e^{-kt}$

$T = 0 + 21\ e^{-0.7419t}$ $\therefore$ $\boxed{T = 21 \cdot e^{-0.7419t}}$

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पान-32

⑨ A body originally at 80°c cools down to 60°c in 20 min in room where the temperature is 40°c. DE by newton is $d\theta/dt = -k(\theta - 40)$, the value of k is

A) $-\frac{1}{20}\log_e 2$ ✓B) $\frac{1}{20}\log_e 2$ C) $20\log_e 2$ D) $\log_e 2$

Ans : B)

Sol^n ⇒

●NOTE:	• Trick : To find 'k' :
	$k = \frac{-1}{t}\left[\log\left(\frac{\theta_2 - \theta_0}{\theta_1 - \theta_0}\right)\right]$
	where :
	θ_2 = changed temperature of body
	θ_1 = initial temperature of body
	t = time

1): Given: $\theta_1 = 80$

$\theta_2 = 60$

$\theta_0 = 40$

$t = 20$

2): $k = \frac{-1}{t}\left[\log\left[\frac{\theta_2 - \theta_0}{\theta_1 - \theta_0}\right]\right]$

$k = \frac{-1}{20}\left[\log\left(\frac{60-40}{80-40}\right)\right]$

$k = \frac{-1}{20}\left[\log\left(\frac{20}{40}\right)\right]$

$k = \frac{-1}{20}\log\left(\frac{1}{2}\right)$

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पान- ३e

गी):

$$k = \frac{-1}{20}\log(2^{-1})$$

Rule:

$\{\because \log a^m = m \cdot \log a\}$

$$k = \frac{-1}{20}(-1)\log 2$$

$$\boxed{k = \frac{1}{20}\log 2}$$

(10) If the temperature of body drops from 100°c to 60°c in 1 minute when the temperature of surrounding is 20°c satisfies the DE $\frac{d\theta}{dt} = -k(\theta - 20)$, then value of k is

✓A) $\log_e 2$ B) $-\log_e 2$ c) $\log_e 4$ D) $\log_e 8$

Sol^n ⇒ Ans: A)

वी): Given: $\theta_0 = 20$, $\theta_1 = 100$, $\theta_2 = 60$, $t = 1$

वी):

$$k = \frac{-1}{t}\log\left[\frac{\theta_2 - \theta_0}{\theta_1 - \theta_0}\right]$$

$$= \frac{-1}{1}\log\left[\frac{60-20}{100-20}\right]$$

$$= -1\log\left[\frac{40}{80}\right]$$

$$= -1\log\left[\frac{1}{2}\right]$$

$$= -1\log 2^{-1}$$

$\{\because \log a^m = m\log a : \text{Rule}\}$

$$= -1 \times -1 \times \log 2 = \boxed{\log 2}$$

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पान - ४०

(11) The temperature of air is 30°c and the substance cools from 100°c to 70°c in 15 minutes. If DE by newton's law of cooling $\frac{d\theta}{dt} = -k(\theta - 30)$, then value of K is

A) $\log_e \frac{7}{4}$ B) $\frac{1}{15}\log_e \frac{4}{7}$ ✓C) $\frac{1}{15}\log_e \frac{7}{4}$ D) $15\log_e \frac{7}{4}$

Sol^n => Ans: c)

क): Given: $\theta_0 = 30$, $\theta_1 = 100$, $\theta_2 = 70$, $t = 15$

स्प):

$$K = \frac{-1}{t}\log\left[\frac{\theta_2 - \theta_0}{\theta_1 - \theta_0}\right]$$

$$= \frac{-1}{15}\log\left[\frac{70-30}{100-30}\right]$$

$$= \frac{-1}{15}\log\left[\frac{40}{70}\right]$$

$$= \frac{-1}{15}\log\left(\frac{4}{7}\right)$$

Rule:

$$= \frac{-1}{15}\left[-\log\left[\frac{7}{4}\right]\right] \quad \left\{\because \log\left(\frac{A}{B}\right) = -\log\left(\frac{B}{A}\right)\right.$$

$$= \boxed{\frac{1}{15}\log 7/4}$$

(12) By newton's law of cooling the DE of body originally at 80°c cools down to 60°c in 20 minutes in surrounding temperature of 40°c is $\frac{d\theta}{dt} = -0.0365(\theta - 40)$.

The temperature of body after 40 minutes is

A) 60°c ✓B) 50°c C) 35°c D) 85°c

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पान-४१

$Sol^n \Rightarrow$ Ans : B)

• NOTE : 1) $\theta = \theta_0 + c_1 e^{-kt}$

OR

$T = T_0 + A e^{-kt}$

2) $c_1 = $ (Initial Temp) − (Surrounding Temp)

$c_1 = T - T_0 = \theta - \theta_0$

क) : $\frac{d\theta}{dt} = -(0.03465)(\theta - 40)$ ---- (given)

$\Rightarrow$ compare :

$\frac{d\theta}{dt} = -k(\theta - \theta_0)$ .... (Result 01)

Given : $k = 0.03465$, $t = 40$ min

$\theta_0 = 40$, $\theta_1 = 80$ (Initial Temp)

ख) : $c_1 = T - T_0 = \theta_1 - \theta_0$

$= 80 - 40$

$c_1 = 40$

ग) : Formula :

$\theta = \theta_0 + c_1 e^{-kt}$

$\theta = 40 + c_1 e^{-0.03465t}$

$\theta = 40 + 40 \cdot e^{-0.03465t}$

$\theta = 40 + 40 \cdot e^{-0.03465 \times 40}$

solve in calcy : :CALCY:

$\{\because e \wedge (-0.03465 \times 40) = 0.25\}$

$\theta = 40 + 40 \times 0.25$

$$\boxed{\theta = 50}$$

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पान-४२

(13) A metal ball is heated to a temperature of 100°c and at time $t=0$ it is placed in water which is maintained at 40°c. The temperature of the ball reduces to 60°c in 4 minutes. By newton's law of cooling the DE is given as $\frac{d\theta}{dt} = -\left[\frac{1}{4}\log_e 3\right](\theta-40)$. Then the time required to reduce temperature of ball to 50°c is

A) 7.5 min B) 3.5 min C) 10 min D) 6.5 min

Sol^n ⇒

Ans : D)

क): $\frac{d\theta}{dt} = \left[-\frac{1}{4}\log 3\right](\theta-40)$ ----(given)

compare ⇒

$\frac{d\theta}{dt} = -k(\theta-\theta_0)$ ----(Result 01)

ख): Given: $\theta_0 = 40$

$k = +1/4 \log 3$

$\theta_1 = 100$ (Initial Temp)

$\theta = 50$

$t = ?$

$C_1 = \theta_1 - \theta_0 = 100 - 40 = 60$

ग): $\theta = \theta_0 + C_1 e^{-kt}$

$50 = 40 + 60\, e^{-1/4 \log 3 t}$

solve on calcy full eq^n at once :

$\boxed{t = 6.52}$

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पान - ४३

*** NOTE:**

- कॅल्सी पे कैसे 't' की कीमत निकाले ?

$\Rightarrow 50 = 40 + 60\, e^{(-1/4 \log 3)t} \Leftarrow$

$\Downarrow$

① MS कॅल्सी $\Rightarrow$

50 = 40 + 60 e ∧ ((−1 ÷ 4)(In 3) x)

Shift CALC

Shift CALC

$\therefore 6.52$

= : ALPHA CALC

e : ALPHA In

x : ALPHA)

② ES कॅल्सी $\Rightarrow$

$50 = 40 + 60\, e^{-\frac{1}{4} \ln(3) x}$

Shift CALC =

$\therefore 6.52$

= : ALPHA CALC

e : Shift In

(14) A body at temperature 100°c is placed in a room whose temperature is 20°c and cools to 60°c in 5 minutes. By newton's law of cooling the differential equation is $\frac{d\theta}{dt} = -k\left[\frac{1}{5}\log_e 2\right](\theta - 20)$. Then the temperature after 8 minute is

✓A) 46.4°c B) 65.4°c C) 40.4°c D) 20°c

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पान-४४

Sol^n ⇒

Ans : A)

क) :

$$\frac{d\theta}{dt} = -\left(\frac{1}{5}\log_e 2\right)(\theta - 20) \quad \text{.....(given)}$$

compare ⇒

$$\frac{d\theta}{dt} = -k(\theta - \theta_0) \quad \text{.....(Result 01)}$$

ख) : Given: $\theta_0 = 20$

$k = 1/5 \log 2$

$\theta_1 = 100$ (Initial Temp)

$\theta = ?$

$t = 8$

$$c_1 = \theta_1 - \theta_0 = 100 - 20 = 80$$

ग) : $\theta = \theta_0 + c_1 e^{-kt}$

$$\theta = 20 + 80\, e^{(-1/5 \log 2)[8]}$$

solving on calcy : पुरी eq^n कॅलसीपे एकसाथ डाले ।

$\theta = 46.3$

$$\boxed{\theta = 46.3}$$

• यह कॅलक्युलेशनमें दिक्कत आती है, तो गणित क्र. 13 में दी NOTE पढ़े ।

ES ⇒ $20 + 80\, e^{-\frac{1}{5} \ln(2) \times 8}$ ⇒ 46.3

कॅलसी :

MS ⇒ 20+80 e^((-1÷5)(ln 2)×8) ⇒ 46.3

• इसप्रकार कॅलसीमे डाले, आपको उत्तर / जवाब मिल जाएगा ।

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पाठ-४५

(15) A copper ball is heated to a temperature of $100^\circ c$ and at time $t=0$ it is placed in water which is maintained at $30^\circ c$. The temperature of the ball reduces to $70^\circ c$ in 3 minutes. The differential equation by newton's law of cooling is $\frac{d\theta}{dt} = -\left(\frac{1}{3}\log_e\frac{7}{4}\right)(\theta-30)$.

Then the time required to reduce the temperature of ball to $31^\circ c$ is

A) 3 min B) 7.78 min ✓C) 22.78 min D) 15.78 min

Solⁿ ⇒ Ans: C)

क): $$\frac{d\theta}{dt} = -\left(\frac{1}{3}\log_e\frac{7}{4}\right)(\theta-30) \quad \text{.....(given)}$$

⇒ compare:

$$\frac{d\theta}{dt} = -k(\theta-\theta_0) \quad \text{-----(Result 01)}$$

ख): Given: $\theta_0 = 30$

$k = \frac{1}{3}\log_e 7/4$

$\theta_1 = 100$ (Initial Temp)

$\theta = 31$

$t = ?$

$$c_1 = \theta_1 - \theta_0 = 100 - 30 = 70$$

ग): $$\theta = \theta_0 + c_1 e^{-kt}$$

OR

$$\theta = \theta_0 + c_1 e^{-k\times t}$$

: Adjustment on Calcy:

$\{\because -kt = -k\times t\}$

Both are same

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पान-४६

$$31 = 30 + 70\, e^{\left(-\frac{1}{3}\log 7/4\right)t}$$

solving on calcy :

$$\boxed{t = 22.78 \text{ min}}$$

- यह कॅलसीमे सॉल्व करनेसे पहिले गणित क्र. 13 में दी हुई NOTE पहिले पढे। तो यह कॅलसीमें कैसे सॉल्व करते है आप समझ जाओगे।

∴ Ans : c)

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पान - 920

● NOTE :

① Polar Transformation :

If DE not solved by any of previous type, then conversion into polar method is usefull.

Put : 1) $x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

2) $x\,dx + y\,dy = r\,dr$

3) $x\,dy - y\,dx = r^2\,d\theta$

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पान - 929

* Type: Transformation to Polar :

① The differential equation $x^2(x\,dx + y\,dy) + y(x\,dy - y\,dx) = 0$ by using $x = r\cos\theta$ and $y = r\sin\theta$ transformed into

A) $r\,dr + \sin\theta\sec^2\theta\,d\theta = 0$ B) $dr + r\cos\theta\tan^2\theta\,d\theta = 0$

C) $r\,dr + r\sin\theta\cos^2\theta\,d\theta = 0$ ✓D) $r\,dr + r\sin\theta\sec^2\theta\,d\theta = 0$

Sol^n ⇒ Ans: D)

क): $x^2(x\,dx + y\,dy) + y(x\,dy - y\,dx) = 0$

Put: $x = r\cos\theta$, $x\,dx + y\,dy = r\,dr$

$y = r\sin\theta$, $x\,dy - y\,dx = r^2 d\theta$

ख): $(r\cos\theta)^2[r\,dr] + (r\sin\theta)(r^2 d\theta) = 0$

$(r^2\cos^2\theta)(r\,dr) + r^3\sin\theta\,d\theta = 0$

ग): ....devide by $r^2\cos^2\theta$:

$$\frac{(\cancel{r^2\cos^2\theta})(r\,dr)}{\cancel{r^2\cos^2\theta}} + \frac{r^3\sin\theta\,d\theta}{r^2\cos^2\theta} = 0$$

$$r\,dr + r\sin\theta\left[\frac{1}{\cos^2\theta}\right]d\theta = 0 \qquad \left\{\because \frac{1}{\cos x} = \sec x\right\}$$

$$\boxed{r\,dr + r\sin\theta\cdot\sec^2\theta\,d\theta = 0}$$

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पान-१२२

(2) The solution of differential equation

$(x\,dy - y\,dx) + \frac{1}{x^2+y^2}(x\,dx + y\,dy) = 0$ by

using substitution $x = r\cos\theta$, $y = r\sin\theta$ is

A) $\tan^{-1}\frac{y}{x} = \frac{1}{\sqrt{x^2+y^2}} + c$ B) $\tan^{-1}\frac{y}{x} = \frac{1}{2(x^2-y^2)+c}$

✓ C) $\tan^{-1}\frac{y}{x} = \frac{1}{2(x^2+y^2)} + c$ D) $\tan^{-1}\frac{y}{x} = \frac{1}{(x^2+y^2)} + c$

Sol^n → Ans: c)

क) $(x\,dy - y\,dx) + \frac{1}{x^2+y^2}(x\,dx + y\,dy) = 0$

Put: $x = r\cos\theta$, $x^2+y^2 = r^2$

$y = r\sin\theta$, $x\,dx + y\,dy = r\,dr$, $x\,dy - y\,dx = r^2 d\theta$

ख): $(r^2 d\theta) + \frac{1}{r^{\cancel{2}}}(\cancel{r}\,dr) = 0$

$r^2 d\theta = -\frac{1}{r}dr$

$d\theta = -\frac{1}{r}\cdot\frac{1}{r^2}dr$

$d\theta = \frac{-1}{r^3}dr$

ग): Take integration:

$\int d\theta = -\int \frac{1}{r^3}dr$

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पान - १२२

$\theta = -\int r^{-3} dr$ $\left\{ \because \int d\theta = \theta,\ \int x^n dx = \frac{x^{n+1}}{n+1} \right\}$

$\theta = -\frac{r^{-3+1}}{-3+1} + c$

$\theta = -\frac{r^{-2}}{-2} + c$

$\theta = -\frac{1}{-2r^2} + c$

$\theta = \frac{1}{2r^2} + c$

ग): Reput : $r^2 = x^2 + y^2$ and $\theta = \tan^{-1}(y/x)$

$\therefore \boxed{\tan^{-1}(y/x) = \frac{1}{2(x^2+y^2)} + c}$

$\therefore$ Ans: c) :

॥ श्रीगुरवे नमः ॥

A Purple Hat
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पाठ - २४

● NOTE:

*

① Rate of Decay / DisIntegration / Decomposition:

In this, materials take constant of proportionality always -ve.

⇒ Statement:

Disintegration at any instant is proportional to the amount of material at time t is,

$$\frac{du}{dt} \propto u$$

$$\therefore \frac{du}{dt} = -ku$$

{∵ k = Constant of Proportionality, -ve sign means disintegration}

② Rate of Growth / Integration / Composition:

In this, materials take constant of proportionality always +ve.

⇒ Statement:

Integration at any instant is proportional to the growing material present.

$$\frac{dy}{dt} \propto y$$

$$\therefore \frac{dy}{dt} = +ky$$

{∵ k = Constant of Proportionality, +ve sign means growth}

{∵ y = no. of bacteria, t = time}

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पान-२६

① Radium decomposes at the rate proportional to the amount present. The differential equation for the rate of decay of radium is

A) $\frac{du}{dt} = \frac{-k}{u}$ B) $\frac{du}{dt} = ku$ C) $\frac{du}{dt} = -ku$ D) $\frac{du}{dt} = -k$

Ans: C) $\frac{du}{dt} = -ku$ .....(std Result)

② In a certain culture of bacteria, the rate of increase is proportional to the number present. If y denotes the number of bacteria at time t hours then the governing differential equation is

A) $\frac{dy}{dt} = ky$ B) $\frac{dy}{dt} = -ky$ C) $\frac{dy}{dt} = \frac{k}{y}$ D) $\frac{dy}{dt} = ky^2$

Ans: A) $\frac{dy}{dt} = ky$ ....(std Result)

③ The differential equation of the population model for natural growth of bacteria is $dy/dt = ky$. The general solution of equation is

A) $y = c\log kt$ B) $ye^{kt} = ct$ C) $y = ce^{kt}$ D) $y = ce^{-kt}$

Ans: C)

S): $\frac{dy}{dt} = ky$ ....(given)

use VS form:

$\frac{1}{y}dy = k\,dt$

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ड): Take Integration:

$$\int \frac{1}{y}\, dy = k\int dt$$

$$\log y = kt + \log c$$

$$\log y - \log c = kt$$

$$\log y + \log c = kt \qquad \{\text{Put: } -\log c = \log c\}$$

$$\log(yc) = kt$$

ढ): Take Antilog:

$$yc = e^{kt}$$

$$y = \frac{1}{c} e^{kt}$$

$$y = c^{-1} e^{kt} \qquad \{\because \text{ Put: } c^{-1} = c\}$$

$$\boxed{y = ce^{kt}}$$

④ The amount x of substance present in certain chemical reaction at time t is given by

$\frac{dx}{dt} + \frac{1}{10}x = 2 - (1.5)e^{-1/10\, t}$, then the amount x of substance present at time t is

A) $x = -\frac{3}{2}te^{-1/10 t} + ce^{-1/10 t}$ B) $x = 20 + \frac{3}{2}te^{-1/10 t} - ce^{-1/10 t}$

C) $x = 20 - \frac{3}{2}t + c$ ✓D) $x = 20 - \frac{3}{2}te^{-1/10 t} + ce^{-1/10 t}$

Ans: D)

ड): $$\underset{}{\frac{dx}{dt}} + \underset{\uparrow \atop P}{\frac{1}{10}}x = \underset{\uparrow \atop Q}{\left[2 - (1.5)e^{-1/10 t}\right]}$$ ..... (Linear Differential Eqⁿ)

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$\boxed{\text{पान- 28}}$

ब):
$$IF = e^{\int P\,dt} = e^{\int 1/10\,dt}$$
$$= e^{1/10\int dt}$$
$$IF = e^{1/10\,t}$$

ड):
GS:
$$x(IF) = \int \phi(IF)\,dt + c$$
$$x[e^{1/10t}] = \int [2 - 1.5e^{-1/10t}]\cdot e^{1/10t}\cdot dt + c$$
$$= \int 2e^{1/10t}\,dt - \int 1.5\,\cancel{e^{-1/10t}}\cdot\cancel{e^{1/10t}}\,dt + c$$
$$= 2\int e^{1/10t}\,dt - 1.5\int dt + c$$
$$= \frac{2e^{1/10t}}{(1/10)} - 1.5t + c \qquad \left\{\because \int e^{ax}\,dx = \frac{e^{ax}}{a}\right\}$$
$$= \frac{2\times 10}{1}e^{1/10t} - 1.5t + c$$
$$x[e^{1/10t}] = 20e^{1/10t} - 1.5t + c$$

ब): ...devide by $e^{1/10t}$:
$$\frac{x[\cancel{e^{1/10t}}]}{\cancel{e^{1/10t}}} = \frac{20\cancel{e^{1/10t}}}{\cancel{e^{1/10t}}} - \frac{1.5t}{e^{1/10t}} + \frac{c}{e^{1/10t}}$$
$$x = 20 - 1.5t\,e^{-1/10t} + c\cdot e^{-1/10t}$$
{∵ Put 1 dd $e^{1/10t}$ 4 ¢}
$$\left\{\because 1.5 = \frac{3}{2}\right\}$$
$$\boxed{x = 20 - \frac{3}{2}t\,e^{-1/10t} + ce^{-1/10t}}$$

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पान - २८

(5) Biotransformation of an organic compound having concentration of x can be modeled using an ordinary differential equation $\frac{dx}{dt} + k^2 x = 0$, where k is reaction rate constant.

If $x = a$ at $t = 0$, the solution of equation is

A) $x = ae^{-kt}$ B) $\frac{1}{x} = \frac{1}{a} + kt$ C) $x = a(1 - e^{kt})$

D) $x = a + kt$

Ans : B)

अ):

$$\frac{dx}{dt} + kx^2 = 0 \quad \text{....(given)}$$

$$\frac{dx}{dt} = -kx^2$$

ब): Use VS form:

$$-\frac{1}{x^2}\,dx = k\,dt$$

क): Take Integration:

$$-\int \frac{1}{x^2}\,dx = k\int dt$$

$$-\left(\frac{-1}{x}\right) = kt + c$$

$$\frac{1}{x} = kt + c \quad \text{....(1)}$$

ड): Put $x = a$ & $t = 0$ in (1):

$$\therefore \frac{1}{a} = k(0) + c$$

$$\therefore c = \frac{1}{a}$$

इ): Put $c = 1/a$ in (1):

$$\boxed{\frac{1}{x} = kt + \frac{1}{a}}$$

(6) If a particle moves on a straight line so that the force acting on it is always directed towards a fixed point on the line and proportional to its distance from the point then the particle is said to be in

पान - 30

A) Simple Harmonic Motion B) Motion Under the Gravity
C) Periodic Motion D) Circular Motion

Ans: A)

Simple Harmonic Motion .....(std Result)

(7) A particle executes simple harmonic motion then the differential equation of motion is

A) $\frac{d^2x}{dt^2} = -\omega^2 x$ B) $\frac{d^2x}{dt^2} = \omega^2 x$

C) $\frac{d^2x}{dt^2} = -\frac{\omega^2}{x}$ D) $\frac{dx}{dt} = -\omega^2 x$

Ans: A)

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

.....(std Result)

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पाठ - ६४

● NOTE:

① Gamma Function:

$$\sqrt{n} = \int_0^\infty e^{-x} x^{n-1}\,dx \quad , n>0$$

(∵ read gamma of n)

OR

$$\sqrt{n} = 2\int_0^\infty e^{-x^2} x^{2n-1}\,dx$$

② Imp Results:

i) $\sqrt{1} = 1$

ii) $\sqrt{0} = \infty$

iii) $\sqrt{1/2} = \sqrt{\pi}$

iv) $\sqrt{n+1} = n! = n\sqrt{n}$

v) $\sqrt{n} = (n-1)\sqrt{(n-1)}$

vi) $\sqrt{n} = (n-1)!$

vii) $\sqrt{p}\cdot\sqrt{1-p} = \dfrac{\pi}{\sin p\pi}$ (if $0<p<1$)

* Note:

i) $\displaystyle\int_0^\infty e^{-ky} y^{n-1}\,dy = \frac{\sqrt{n}}{k^n}$

ii) $\displaystyle\int_0^\infty e^{-ky} y^{n}\,dy = \frac{\sqrt{n+1}}{k^{n+1}}$

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पान-६४

③ Some IMP Putting : (Transform into Gamma Function)

	Problem Type :	Putting :	Transform:
1)	$e^{-f(x)}$	$f(x) = t$	e^{-t}
2)	a^{x}	$a^{x} = e^{t}$	$\frac{1}{a^x} = \frac{1}{e^t} = e^{-t}$
3)	$\log x$	$\log x = t$	$x = e^{-t}$
4)	$\log(1/x)$	$\log(1/x) = t$ $\log x^{-1} = t$ $-\log x = t$ $\log x = -t$	$t = e^{-t}$

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पार्ट - E1E2

* Type: Gamma Function:

(1) Gamma Function of n (n>0), is defined as

Sol^n ⇒ $\boxed{\Gamma n = \int_0^\infty e^{-x} x^{n-1}\, dx}$

....(यह स्टैंडर्ड है, रटलो।)

(2) The value of equivalent form of Gamma Function $\int_0^\infty e^{-kx} \cdot x^{n-1}\, dx$ is

Sol^n ⇒ $\boxed{\int_0^\infty e^{-kx} \cdot x^{n-1}\, dx = \frac{\Gamma n}{k^n}}$

....(यह स्टैंडर्ड है, रटलो।)

(3) Reduction formula for Gamma function is

Sol^n ⇒ $\boxed{\Gamma n+1 = n\,\Gamma n}$

....(यह स्टैंडर्ड है, रटलो।)

(4) If n is a positive integer, then $\Gamma n+1$ is

Sol^n ⇒ $\boxed{\Gamma n+1 = n!}$

...(यह स्टैंडर्ड है, रटलो।)

(5) $\Gamma 1$ is equal to

Sol^n ⇒ $\boxed{\Gamma 1 = 1}$

....(यह स्टैंडर्ड है, रटलो।)

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पान - ६७

⑥ $\Gamma\frac{1}{2}$ is equal to

Sol^n ⇒

$$\boxed{\Gamma\frac{1}{2} = \sqrt{\pi}}$$

....(यह स्टँडर्ड है, रटलो।)

⑦ $\Gamma 7$ is equal to

Sol^n ⇒

Formula:

$$\Gamma n = (n-1)!$$

$$\therefore \boxed{\Gamma 7 = (7-1)! = 6!}$$

⑧ $\Gamma\frac{5}{2}$ is equal to

Sol^n ⇒

Formula:

$$\Gamma n = (n-1)\,\Gamma(n-1)$$

$$\therefore \Gamma\frac{5}{2} = \left(\frac{5}{2}-1\right)\Gamma\left(\frac{5}{2}-1\right)$$

$$= \frac{3}{2}\,\Gamma\frac{3}{2}$$ again use formula:

$$= \frac{3}{2}\left[\left(\frac{3}{2}-1\right)\Gamma\frac{3}{2}-1\right]$$

$$= \frac{3}{2}\times\frac{1}{2}\,\Gamma\frac{1}{2}$$

$$= \frac{3}{4}\times\Gamma\frac{1}{2}$$

$\left\{\because \Gamma\frac{1}{2} = \sqrt{\pi}\right\}$

$$= \frac{3}{4}\sqrt{\pi}$$

$$\boxed{\Gamma\frac{5}{2} = \frac{3}{4}\sqrt{\pi}}$$

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पान-६८

(9) By using $\Gamma p \cdot \Gamma 1-p = \pi/\sin p\pi$,

If $0 \leq p \leq 1$ the value of $\Gamma 1/4 \cdot \Gamma 3/4$ is

A) $\frac{\pi}{\sqrt{2}}$ B) π ✓C) $\sqrt{2}\,\pi$ D) 2π

Sol^n ⇒ Ans: c)

ष्टे): $\Gamma 1/4 \cdot \Gamma 3/4 = \Gamma 1/4 \cdot \Gamma 1-1/4$ $\{\because 3/4 = 1-1/4\}$

compare to ⇒

$\Gamma p \cdot \Gamma 1-p = \pi/\sin p\pi$

$\therefore p = 1/4$

ष्>:

$$= \frac{\pi}{\sin \frac{1\pi}{4}}$$

ES: calcy

$$= \frac{\pi}{\sin \pi/4}$$ $\{\because \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}\}$

: Radian Mode

$$= \frac{\pi}{1/\sqrt{2}}$$

$$= \boxed{\sqrt{2}\,\pi}$$

(10) $\int_0^\infty e^{-t} t^{3/2}\, dt$ is equal to

✓A) $3/4\sqrt{\pi}$ B) $15/4\sqrt{\pi}$ C) $3/4\,\pi$ D) $3/2\sqrt{\pi}$

Sol^n ⇒ Ans: A)

ष्टे): $\int_0^\infty e^{-t} t^{3/2}\, dt$ compare to std result:

अ): $\Gamma n = \int_0^\infty e^{-t}\, t^{n-1}\, dt$

OR

$\Gamma (n+1) = \int_0^\infty e^{-t}\, t^{n}\, dt$

$\therefore \int_0^\infty e^{-t}\, t^{3/2}\, dt = \Gamma (3/2+1) = \Gamma (5/2)$

ड): $\Gamma (5/2) = \boxed{\frac{3}{4}\sqrt{\pi}}$ .....(Refer Q. 8)

(11) $\int_0^\infty e^{-5x}\, x^4\, dx$ is equal to

A) $4!/4^5$ B) $5!/4^4$ C) $5!/5^5$ D) $4!/5^5$

Soln ⇒ Ans: D)

क): $\int_0^\infty e^{-5x}\, x^4\, dx$

compare with ⇒

$$\int_0^\infty e^{-kx} \cdot x^n\, dx = \frac{\Gamma (n+1)}{k^{n+1}} \quad \text{....(std)}$$

Here $k=5$, $n=4$

ख): $$= \frac{\Gamma (4+1)}{5^{4+1}} \qquad \{\because \Gamma (n+1) = n! \therefore \Gamma (4+1) = 4!\}$$

$$= \frac{4!}{5^5}$$

$$= \boxed{\frac{4!}{5^5}}$$

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पान-60

(12) The appropriate substitution to reduce the given integral $\int_0^\infty \sqrt{x} \cdot e^{-\sqrt{x}} dx$ to Gamma function itegral is

Sol^n ⇒ $\boxed{\sqrt{x} = t}$

....(यह स्टैंडर्ड है, रटलो।)

(13) The appropriate substitution to reduce the given integral $\int_0^1 (x \log x)^4 dx$ to Gamma function integral is

Sol^n ⇒ $\boxed{\log x = -t}$

....(यह स्टैंडर्ड है, रटलो।)

(14) The appropriate substitution to reduce given integral $\int_0^\infty \frac{x^5}{5^x} dx$ to Gamma function is

Sol^n ⇒

$a^x = e^t$ ....(std substitution, refer NOTE)

Here $a = 5$

$\boxed{5^x = e^t}$

(15) The value of Integral $\int_0^\infty \sqrt{x} \cdot e^{-\sqrt{x}} dx$ by using substitution $\sqrt{x} = t$ is

A) 1 B) 2 C) 3 D) 4

Sol^n ⇒ Ans: D)

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पान - 109

क):

$$\int_0^\infty e^{-\sqrt{x}} \cdot \sqrt{x}\ dx$$

Put : $\sqrt{x} = t$, squaring both sides

$(\sqrt{x})^2 = (t)^2$

$x = t^2$

$dx = 2t\,dt$ ....(Derivative)

ख):

$$\int_0^\infty e^{-t}\, t\,(2t\,dt)$$

$$= 2\int_0^\infty e^{-t}\, t^2\, dt$$ $\{\because$ compare to std $\Rightarrow \int_0^\infty e^{-ky} \cdot y^n\, dy = \frac{\lceil n+1}{k^{n+1}}$

$\therefore$ Here $k=1$, $n=2$

ग):

$$= 2\left[\frac{\lceil 2+1}{(1)^{2+1}}\right]$$ $\{\because 1^3 = 1\}$

$$= \frac{2\lceil 2+1}{1}$$

$= 2\lceil 3$ $\{\because \lceil n = (n-1)!\ \therefore \lceil 3 = (3-1)!\}$

$= 2 \times 2!$

$= 2 \times 2$ $\{\because 2! = 2 \Rightarrow$ calcy: shift $x^{-1}\}$

$= \boxed{4}$

(16) The value of Integral $\int_0^\infty e^{-x^2} dx$ by substitution $x^2 = t$ is

A) $\sqrt{\pi}$ B) $\sqrt{\pi}/2$ C) $\sqrt{\pi}/3$ D) $2\sqrt{\pi}$

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Sol^n ⇒

Ans: B)

क): $\int_0^\infty e^{-x^2}\,dx$

Put: $x^2 = t$, Taking square-roots:

$$\sqrt{x^2} = \sqrt{t}$$

$$x = \sqrt{t}$$

$$dx = \frac{1}{2\sqrt{t}}\,dt$$

ख): $\int_0^\infty e^{-t}\left[\frac{1}{2\sqrt{t}}\right]dt = \frac{1}{2}\int_0^\infty e^{-t}\frac{1}{t^{1/2}}\,dt$

$$= \frac{1}{2}\int_0^\infty e^{-t}\,t^{-1/2}\,dt$$

ग): compare to std ⇒

$$\left\{\because \int_0^\infty e^{-y}y^n\,dy = \Gamma{n+1}\right\}$$

Here: $n = -1/2$

$$= \frac{1}{2}\Gamma{\frac{-1}{2}+1} \qquad \left\{\because 1-1/2 = 1/2\right\}$$

$$= \frac{1}{2}\Gamma{\frac{1}{2}}$$

$$= \boxed{\frac{1}{2}\sqrt{\pi}} \qquad \left\{\because \Gamma{1/2} = \sqrt{\pi}\right\}$$

17. The value of integral $\int_0^\infty e^{-x^4}\,dx$ by substitution $x^4 = t$ is

A) $\Gamma{5/4}$ B) $\Gamma{1/4}$ C) $\frac{1}{4}\Gamma{1/4}$ D) $\frac{1}{4}\Gamma{5/4}$

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$Sol^n \Rightarrow$ Ans: c)

5): $\int_0^{\infty} e^{-x^4}\, dx$

Put: $x^4 = t$

$x = t^{1/4}$ ......(understand this adjustment)

$dx = \frac{1}{4} t^{1/4 - 1}\, dt$ .....(derivative)

$dx = \frac{1}{4} t^{-3/4}\, dt$

अ): $\int_0^{\infty} e^{-x^4}\, dx$

$= \int_0^{\infty} e^{-t} \left(\frac{1}{4} t^{-3/4}\, dt\right)$

compare to std $\Rightarrow \left\{\because \int_0^{\infty} e^{-y} y^n\, dy = \sqrt{n+1}\right\}$

Here: $n = -3/4$

∴): $= \frac{1}{4}\sqrt{\frac{-3}{4}+1}$ $\{\because 1 - 3/4 = 1/4 : \text{calcy}\}$

$= \boxed{\frac{1}{4}\sqrt{1/4}}$

(18) The value of integral $\int_0^{\infty} \sqrt{x} \cdot e^{-\sqrt{x}}\, dx$ by putting $x^3 = t$ is

A) $\sqrt{\pi}/6$ B) $\sqrt{\pi}/2$ C) $3\sqrt{\pi}$ D) $\sqrt{\pi}/3$

$Sol^n \Rightarrow$ Ans: D)

Put: $x^3 = t$

$x = t^{1/3}$ .....(understand ts adjustment)

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$$dx = \frac{1}{3} t^{1/3 - 1}\, dt \quad \text{....(derivative)}$$

$$dx = \frac{1}{3} t^{-2/3}\, dt$$

आ):

$$\int_0^\infty \sqrt{x} \cdot e^{-x^3}\, dx$$

$$= \int_0^\infty \sqrt{t^{1/3}} \cdot e^{-t} \left(\frac{1}{3} t^{-2/3}\, dt\right)$$

$$= \int_0^\infty (t^{1/3})^{1/2}\, e^{-t}\, \frac{1}{3} t^{-2/3}\, dt$$

$$= \frac{1}{3} \int_0^\infty t^{1/6}\, e^{-t}\, t^{-2/3}\, dt$$

$$= \frac{1}{3} \int_0^\infty e^{-t}\, t^{1/6 - 2/3}\, dt \qquad \left\{\because \frac{1}{6} - \frac{2}{3} = \frac{-1}{2} : \text{calcy}\right\}$$

compare to std: ⇒

$$\left\{\because \int_0^\infty e^{-y} y^n\, dy = \Gamma(n+1)\right\}$$

$$= \frac{1}{3} \Gamma\left(-\frac{1}{2} + 1\right)$$

$$= \frac{1}{3} \Gamma\left(\frac{1}{2}\right) \qquad \left\{\because -\frac{1}{2} + 1 = \frac{1}{2} : \text{calcy}\right\}$$

$$= \boxed{\frac{1}{3}\sqrt{\pi}} \qquad \left\{\because \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}\right\}$$

20. The value of integral $\int_0^\infty x^9 \cdot e^{-2x^2}\, dx$ by using substitution $2x^2 = t$ is

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पान-64

A) $\frac{\sqrt{5}}{64}$ B) $\frac{\sqrt{6}}{64}$ C) $\frac{\sqrt{5}}{32}$ D) $\frac{\sqrt{6}}{32}$

Sol^n ⇒ Ans: A)

1): Put: $2x^2 = t$

$x^2 = \frac{1}{2} t$

$x = \frac{1}{\sqrt{2}} \sqrt{t}$

$dx = \frac{1}{\sqrt{2}} \left[\frac{1}{2\sqrt{t}} \right] dt$ .....(derivative)

2): $\int_0^\infty x^9 \cdot e^{-2x^2} dx$ $\left\{ \because x = \frac{1}{\sqrt{2}}\sqrt{t} = \frac{\sqrt{t}}{\sqrt{2}} = \left(\frac{t}{2}\right)^{1/2} \right\}$

$= \int_0^\infty \left[\left(t/2\right)^{1/2} \right]^9 e^{-t} \left[\frac{1}{\sqrt{2}} \cdot \frac{1}{2\sqrt{t}} dt \right]$

$= \int_0^\infty \left(\frac{t}{2}\right)^{9/2} e^{-t} \left[\frac{1}{2^{1/2}} \cdot \frac{1}{2^1} \cdot \frac{1}{t^{1/2}} \right] dt$

$\{\because a^m \times a^n = a^{m+n}\}$:calcy: $\{\because \frac{1}{2} + 1 = \frac{3}{2}\}$

$= \int_0^\infty \left(t/2\right)^{9/2} e^{-t} \frac{1}{2^{3/2}} \cdot t^{-1/2} dt$

$= \int_0^\infty \frac{t^{9/2}}{2^{9/2}} \cdot e^{-t} \frac{1}{2^{3/2}} \cdot t^{-1/2} dt$ $\left\{ \because a^m \times a^n = a^{m+n},\ \frac{9}{2} + \frac{3}{2} = 6 \text{ : calcy} \right.$

$= \frac{1}{2^6} \int_0^\infty e^{-t}\, t^{9/2 - 1/2} dt$

$\left\{ \because \frac{9}{2} - \frac{1}{2} = 4 \text{ : calcy} \right\}$

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$$= \int_0^\infty e^{-t}\, t^4\, dt \;\times\; {}^1/_{2^6}$$

ग): compare to std $\Rightarrow$ $\left\{ \because \int_0^\infty e^{-y}\, y^n\, dy = \boxed{\Gamma(n+1)} \right\}$

Here : $n = 4$

$$= \frac{1}{2^6}\, \Gamma(4+1)$$

$$= \boxed{\frac{\Gamma(5)}{2^6}}$$

(21) The value of Integral $\int_0^1 \left[\log\left(\frac{1}{y}\right)\right]^{n-1} dy$ by using substitution $\log\left(\frac{1}{y}\right) = t$ is

A) $\Gamma(n)$ B) $\Gamma(n+1)$ C) $-\Gamma(n)$ D) $\Gamma(n-1)$

Sol^n $\Rightarrow$ Ans: A)

उ): Put: $\log {}^1/_y = t$

$y = e^{-t}$ .... (Plz refer NOTE)

$dy = -e^{-t}\, dt$

limit : $0 \rightarrow 1$ changes to: $\infty \rightarrow 0$

ग): $\int_0^1 \left[\log\left(\frac{1}{y}\right)\right]^{n-1} dy$

$$= \int_\infty^0 [t]^{n-1} \left[-e^{-t}\, dt\right]$$

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$$= -\int_{\infty}^{0} e^{-t}\, t^{n-1}\, dt$$

∴): $= +\int_{0}^{\infty} \bar{e}^{t}\, t^{n-1}\, dt$ {∵ limit changes then sign also changes

$$= \boxed{\lceil n}$$

$\left\{ \because \int_{0}^{\infty} \bar{e}^{t}\, t^{n-1}\, dt = \lceil n \right.$ .... (std Result) }

(22) The value of integral $\int_{0}^{1} \frac{dx}{\sqrt{x \cdot \log(1/x)}}$ by using substitution $\log(1/x) = t$ is

A) $\sqrt{\pi}/2$ B) $\sqrt{\pi}$ ✓C) $\sqrt{2\pi}$ D) $2\sqrt{\pi}$

Sol^n ⇒ Ans: C)

ऽ): Put: $\log(1/x) = t$

$x = \bar{e}^{t}$ .... (Plz refer NOTE)

$dx = -\bar{e}^{t}$ .... (derivative)

Limit $0 \to 1$ changes to: $\infty \to 0$

∴): $I = \int_{0}^{1} \frac{dx}{\sqrt{x \cdot \log(1/x)}}$

$$I = \int_{\infty}^{0} \frac{-\bar{e}^{t}\, dt}{\sqrt{\bar{e}^{t} \cdot t}}$$

पाठ-७L

$$= \int_{\infty}^{0} \frac{-e^{-t}\, dt}{(e^{-t})^{1/2} \cdot (t)^{1/2}}$$ $\{\because (e^{-t})^{1/2} = e^{-1/2 t}\}$

$$= \int_{\infty}^{0} -e^{-t} \cdot (e^{t})^{1/2}\, t^{-1/2}\, dt$$ $\{\because a^m \times a^n = a^{m+n}\}$

$$= -\int_{\infty}^{0} e^{-t+1/2 t} \cdot t^{-1/2}\, dt$$ $\{\because (-1+\frac{1}{2})t = (\frac{-1}{2})t\}$

$$= -\int_{\infty}^{0} e^{-1/2 t} \cdot t^{-1/2}\, dt$$

या): {∵ Limit changes then sign also changes}

$$= +\int_{0}^{\infty} e^{-1/2 t} \cdot t^{-1/2}\, dt$$

compare to std ⇒ $\left\{\because \int_{0}^{\infty} e^{-ky} \cdot t^{n}\, dy = \frac{\Gamma n+1}{k^{n+1}}\right\}$

Here: $k = 1/2$, $n = -1/2$

या):

$$= \frac{\Gamma -1/2+1}{(\frac{1}{2})^{-1/2+1}}$$ $\{\because -\frac{1}{2}+1 = \frac{1}{2} : \text{calcy}\}$

$$= \frac{\Gamma 1/2}{(1/2)^{1/2}} = \frac{\sqrt{\pi}}{\frac{1}{\sqrt{2}}}$$ $\{\because \Gamma 1/2 = \sqrt{\pi}\}$ $\{\because \sqrt{x} = x^{1/2}\}$

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पान-५e

$$= \frac{\sqrt{\pi}}{1/\sqrt{2}}$$

$$= \sqrt{\pi} \times \frac{\sqrt{2}}{1}$$

$$= \sqrt{2} \cdot \sqrt{\pi}$$

$$= \boxed{\sqrt{2\pi}}$$

(23) The value of integral $\int_0^1 \frac{dx}{\sqrt{-\log x}}$ by using substitution $-\log x = t$ is

A) $\frac{\sqrt{\pi}}{2}$ ✓B) $\sqrt{\pi}$ C) $\sqrt{2\pi}$ D) $2\sqrt{\pi}$

Sol^n ⇒ Ans: B)

$\$$): Put: $-\log x = t$

$x = e^{-t}$ ....(Plz see NOTE)

$dx = -e^{-t}$ ....(derivative)

limit $0 \to 1$ changes to: $\infty \to 0$

ख): $\int_0^1 \frac{dx}{\sqrt{-\log x}} = \int_\infty^0 \frac{-e^{-t}\,dt}{\sqrt{+t}}$ $\{\because -\log x = t\}$

$= \int_\infty^0 \frac{-e^{-t}\,dt}{t^{1/2}}$ $\{\because \sqrt{x} = x^{1/2}\}$

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$$= \int_{\infty}^{0} -e^{-t}\, t^{-1/2}\, dt$$

$$= -\int_{\infty}^{0} e^{-t}\, t^{-1/2}\, dt$$

जी): Limit changes to sign changes.

$$= +\int_{0}^{\infty} e^{-t}\, t^{-1/2}\, dt$$

compare to std $\Rightarrow$ $\left\{ \because \int_{0}^{\infty} e^{-y}\, y^{n}\, dy = \Gamma(n+1) \right\}$

Here: $n = -1/2$

$$= \Gamma(-1/2+1) \qquad \left\{ \because \text{calcy: } -\frac{1}{2}+1 = \frac{1}{2} \right\}$$

$$= \Gamma(1/2) \qquad \left\{ \because \Gamma(1/2) = \sqrt{\pi} \quad \dots \text{(std)} \right\}$$

$$= \boxed{\sqrt{\pi}}$$

(24) The value of integral $\int_{0}^{\infty} \frac{x^4}{4^x}\, dx$ by using substitution $4^x = e^t$ is

A) $\frac{4}{(\log 4)^4}$ B) $\frac{24}{(\log 4)^3}$ C) $\frac{24}{(\log 4)^5}$ D) $\frac{12}{(\log 4)^4}$

Soln $\Rightarrow$

Ans: C)

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पान-19

क): Put: $4^x = e^t$

Taking Log:

$\log 4^x = \log e^t$

$x \log 4 = t$

$x = \dfrac{t}{\log 4}$

$dx = \dfrac{dt}{\log 4}$ ... (derivative)

:Rule:

$\{\because \log a^m = m \log a\}$

$\{\because \log e^m = m\}$

ख):

$$I = \int_0^\infty \frac{x^4}{4^x}\,dx$$

$$= \int_0^\infty \frac{(t/\log 4)^4}{e^t}\left(\frac{dt}{\log 4}\right)$$

$$= \int_0^\infty \frac{t^4}{(\log 4)^4} \times \frac{1}{e^t} \times \frac{dt}{(\log 4)}$$

$$= \int_0^\infty t^4 e^{-t}\,dt \times \frac{1}{(\log 4)^5}$$

$$= \frac{1}{(\log 4)^5}\int_0^\infty e^{-t}\cdot t^4\,dt$$

ग): compare to std $\Rightarrow$ $\left\{\because \int_0^\infty e^{-y} y^n\,dy = \overline{|n+1}\right\}$

Here: $n = 4$

$$= \frac{1}{(\log 4)^5}\overline{|4+1}$$

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$$= \frac{4!}{(\log 4)^5} \qquad \left\{ \because \Gamma{n+1} = n! \right\}$$

$$= \boxed{\frac{24}{(\log 4)^5}} \qquad \left\{ \because 4! = 24 : \text{calcy: shift } x^{-1} \right\}$$

(25) The value of integral $\int_0^\infty \frac{x^2}{2^x} dx$ by substitution $2^x = e^t$ is

A) $\frac{1}{(\log 2)^2}$ B) $\frac{2}{(\log 2)^2}$ ✓C) $\frac{2}{(\log 2)^3}$ D) $\frac{3}{(\log 2)^4}$

Solⁿ ⇒ Ans: C)

S): Put: $2^x = e^t$

$$x = \frac{t}{\log 2} \quad (\because \text{refer Q. 24})$$

$$dx = \frac{dt}{\log 2} \quad \text{.....(derivative)}$$

S): $\int_0^\infty \frac{x^2}{2^x} dx = \int_0^\infty \frac{(t/\log 2)^2}{e^t} \left(\frac{dt}{\log 2} \right)$

$$= \int_0^\infty \frac{t^2}{(\log 2)^2} \times \frac{1}{e^t} \times \frac{dt}{(\log 2)}$$

$$= \int_0^\infty e^{-t} \cdot t^2 \, dt \times \frac{1}{(\log 2)^3}$$

$$= \frac{1}{(\log 2)^3} \int_0^\infty e^{-t} t^2 \, dt$$

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ब): compare to std $\Rightarrow$ $\left\{ \because \int_0^\infty e^{-y} y^n \, dy = \sqrt{n+1} \right\}$

Here : $n = 2$

$$= \frac{1}{(\log 2)^3} \sqrt{2+1}$$

$$= \frac{1}{(\log 2)^3} (2!) \qquad \left\{ \because \sqrt{n+1} = n! \right\}$$

$$= \frac{1}{(\log 2)^3} \times (2) \qquad \left\{ \because 2! = 2 : \text{calcy: shift } x^{-1} \right\}$$

$$= \boxed{\frac{2}{(\log 2)^3}}$$

$\therefore$ Ans: c) :

• घड्याचे नाव :

DVIS & Error Function.

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पाठ-०४

● NOTE:

① DUIS:-

Differentiation Under Integral Sign:-

It is used to evaluation of real definite integral.

अ) Rule 01:

Integral with limits (a, b) as constants.

Let

$$I(\alpha) = \int_a^b f(x, \alpha)\, dx$$

where $a, b \to$ constants

$\alpha \to$ variable

Then

$$\frac{dI}{d\alpha} = \int_a^b \frac{\partial}{\partial \alpha} f(x, \alpha)\, dx$$

ब) Rule 02:

Integral with limits as function of parameter.

Let

$$I(\alpha) = \int_{a(\alpha)}^{b(\alpha)} f(x, \alpha)\, dx$$

where

$a(\alpha), b(\alpha) \to$ function of α

$\alpha \to$ variable

Then

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha} \int_{a(\alpha)}^{b(\alpha)} f(x, \alpha)\, dx$$

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and also

$$\frac{dI}{d\alpha} = \int_{a(\alpha)}^{b(\alpha)} \frac{\partial}{\partial \alpha} f(x,\alpha)\, dx + f(b,\alpha)\frac{db}{d\alpha} - f(a,\alpha)\frac{da}{d\alpha}$$

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पान-06

* Type: DUIS:

① If $I(\alpha) = \int_a^b f(x,\alpha)\, dx$, where α is parameter and a, b are constant's then by DUIS rule $\frac{dI(\alpha)}{d\alpha}$ is

Sol^n ⇒

$$\boxed{\frac{d}{d\alpha} I(\alpha) = \int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)\, dx}$$

...(यह स्टँडर्ड है, रटलो।)

② If $I(\alpha) = \int_a^b f(x,\alpha)\, dx$ where a, b are functions of the parameter α then by DUIS rule, $\frac{dI(\alpha)}{d\alpha}$ is

Sol^n ⇒

$$\boxed{\frac{d}{d\alpha} I(\alpha) = \int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)\, dx + f(b,\alpha)\frac{db}{d\alpha} - f(a,\alpha)\frac{da}{d\alpha}}$$

....(यह स्टँडर्ड है, रटलो।)

③ If $\phi(a) = \int_0^\infty \frac{e^{-x}}{x}(1-e^{-ax})\, dx$, $a > -1$ then by DUIS rule $\frac{d\phi}{da}$ is

Sol^n ⇒

$$\boxed{\frac{\partial}{\partial a}\phi(a) = \int_0^\infty \frac{\partial}{\partial a}\left[\frac{e^{-x}}{x}\left[1-e^{-ax}\right]\right] dx}$$

...(यह स्टँडर्ड है, रटलो।)

④ If $\phi(a) = \int_0^\infty e^{-bx^2}\cos(2ax)\, dx$, $b > 0$ then by DUIS, $\frac{d\phi}{da}$ is

Sol^n ⇒ $$\boxed{\frac{d}{da}\phi(a) = \int_0^\infty \frac{\partial}{\partial a}\left[e^{-bx^2}\cdot\cos(2ax)\right] dx}$$ ...(यह स्टँडर्ड है, रटलो।)

(5) If $\phi(a) = \int_0^\infty \frac{e^{-x}}{x} [a - 1/x + 1/x \, e^{-ax}] \, dx$, then $\frac{d\phi}{da}$ is

Sol^n ⇒

$$\boxed{\frac{d}{da}\phi(a) = \int_0^\infty \frac{\partial}{\partial a} \frac{e^{-x}}{x} \left[a - \frac{1}{x} + \frac{1}{x} e^{-ax}\right] dx}$$

......(यह भी स्टॅंडर्ड है, रखो।)

(6) If $\phi(a) = \int_0^\infty \frac{e^{-x}}{x} [1 - e^{-ax}] \, dx$, $a > -1$ then DUIS Rule $d\phi/da$ is

A) $\frac{e^{-x}}{x}(1 - e^{-ax})$ B) $\int_0^\infty \frac{a}{x} (e^{-(a+1)x}) \, dx$

c) $\int_0^\infty e^{-ax} \, dx$ ✓D) $\int_0^\infty e^{-(a+1)x} \, dx$

Sol^n ⇒ Ans: D)

S): $\phi(a) = \int_0^\infty \frac{e^{-x}}{x} [1 - e^{-ax}] \, dx$

$$\frac{\partial\phi}{\partial a} = \int_0^\infty \frac{\partial}{\partial a} \frac{e^{-x}}{x} [1 - e^{-ax}] \, dx$$

स): $= \int_0^\infty \left(\frac{e^{-x}}{x}\right) \left[\frac{\partial}{\partial a}(1) - \frac{\partial}{\partial a} e^{-ax}\right] dx$

$$= \int_0^\infty \left(\frac{e^{-x}}{x}\right) [0 - (-x) e^{-ax}] \, dx$$

$$= \int_0^\infty \left(\frac{e^{-x}}{\cancel{x}}\right) [\cancel{x} e^{-ax}] \, dx \quad \{\because a^m \times a^n = a^{m+n}\}$$

$$= \int_0^\infty e^{-x-ax} \, dx$$

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$$= \boxed{\int_0^\infty e^{-x(1+a)}\, dx}$$

⑦ If $\phi(a) = \int_0^1 \frac{x^a - 1}{\log x} dx$; $a \geq 0$ then by DUIS rule $\frac{d\phi}{da}$ is

A) $\int_0^1 \frac{x^a \log a}{\log x} dx$ B) $\int_0^1 \frac{a x^{a-1}}{\log x} dx$ C) $\int_0^1 x^a dx$ D) $\frac{x^a - 1}{\log x}$

Sol^n ⇒ Ans: c)

प्र):

$$\phi(a) = \int_0^1 \frac{x^a - 1}{\log x} dx$$

$$\frac{\partial \phi}{\partial a} = \int_0^1 \frac{\partial}{\partial a}\left[\frac{x^a - 1}{\log x}\right] dx$$

उ):

$$\frac{\partial \phi}{\partial a} = \int_0^1 \frac{1}{\log x} \frac{\partial}{\partial a}[x^a - 1]\, dx$$

$$= \int_0^1 \frac{1}{\log x}[x^a \log x - 0]\, dx$$

$$= \int_0^1 \frac{1}{\cancel{\log x}}[x^a \cancel{\log x}]\, dx$$

$$\boxed{\frac{\partial \phi}{\partial a} = \int_0^1 x^a \, dx}$$

⑧ If $\phi(\alpha) = \int_0^\infty \frac{e^{-x} \cdot \sin \alpha x}{x} dx$ then by DUIS rule $\frac{d\phi}{d\alpha}$ is

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A) $\int_0^\infty e^{-x} \sin \alpha x \, dx$ B) $\int_0^\infty e^{-x} \cos \alpha x \, dx$

C) $\int_0^\infty \frac{\alpha e^{-x} \sin \alpha x}{x} dx$ D) $e^{-x} \sin \alpha x / x$

Sol^n ⇒ Ans: B)

स्प):

$$\phi(\alpha) = \int_0^\infty \frac{e^{-x} \sin \alpha x}{x} dx$$

$$\frac{d\phi}{d\alpha} = \int_0^\infty \frac{\partial}{\partial \alpha}\left[\frac{e^{-x}\sin \alpha x}{x}\right] dx$$

सा):

$$= \int_0^\infty \left(\frac{e^{-x}}{x}\right) \frac{\partial}{\partial \alpha}(\sin \alpha x) \, dx$$

$$= \int_0^\infty \frac{e^{-x}}{\cancel{x}} \cos \alpha x \, (\cancel{x}) \, dx$$

$$\boxed{\frac{d\phi}{d\alpha} = \int_0^\infty e^{-x} \cos \alpha x \, dx}$$

9) If $I(a) = \int_0^\infty e^{-[x^2 + a^2/x^2]} dx$; $a > 0$ then by DUIS $\frac{dI}{da}$ is

A) $\int_0^\infty e^{-[x^2+a^2/x^2]} \left(-\frac{2a}{x^2}\right) dx$ B) $\int_0^\infty e^{-[x^2+a^2/x^2]} dx \left(\frac{2a}{x^2}\right)$

C) $\int_0^\infty e^{-[x^2+a^2/x^2]} \left(-2x - \frac{2a^2}{x^3}\right) dx$ D) $e^{-[x^2+a^2/x^2]}$

Sol^n ⇒ Ans: A)

स्प):

$$I(a) = \int_0^\infty e^{-[x^2+a^2/x^2]} \, dx$$

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$$\frac{dI}{da} = \int_0^\infty \frac{\partial}{\partial a}\left[e^{-(x^2+a^2/x^2)} \, dx \right]$$

अ):

$$\frac{dI}{da} = \int_0^\infty \left[e^{-(x^2+a^2/x^2)} \cdot -\left(0 + \frac{1}{x^2}(2a)\right)\right] dx$$

$$= \boxed{\int_0^\infty e^{-[x^2+a^2/x^2]} \left[-\frac{2a}{x^2}\right] dx}$$

(10) If $I(a) = \int_0^\pi \log(1 - a\cos x)\, dx$; $|a|<1$ then $\frac{dI}{da}$ is

A) $\int_0^\pi \frac{-a\sin x}{1-a\cos x}\, dx$ B) $\int_0^\pi \frac{\cos x}{1-a\cos x}\, dx$

✓C) $\int_0^\pi \frac{-\cos x}{1-a\cos x}\, dx$ D) $\int_0^\pi \frac{1}{1-a\cos x}\, dx$

Sol^n ⇒ Ans: C)

उ): $I(a) = \int_0^\pi \log(1-a\cos x)\, dx$

$$\frac{dI}{da} = \int_0^\pi \frac{\partial}{\partial a} \log(1-a\cos x)\, dx$$

$$= \int_0^\pi \frac{1}{(1-a\cos x)} \times [0 - (1)\cos x]\, dx$$

$$= \int_0^\pi \frac{1}{(1-a\cos x)} (-\cos x)\, dx$$

$$= \int_0^\pi \boxed{\frac{-\cos x}{1-a\cos x}\, dx}$$

(11) By DUIS rule $\frac{d}{da}\left[\int_0^\infty \frac{e^{-x}}{x}\left[a-\frac{1}{x}+\frac{1}{x}e^{-ax}\right]dx\right]$, a is parameter is

A) $\int_0^\infty \frac{e^{-x}}{x}(1+e^{-ax})\,dx$ B) $\int_0^\infty \frac{e^{-x}}{x}\left[1-\frac{ae^{-ax}}{x}\right]dx$

C) $\int_0^\infty e^{-x}(1-e^{-ax})\,dx$ ✓D) $\int_0^\infty \frac{e^{-x}}{x}\left[1-e^{-ax}\right]dx$

Sol^n ⇒ Ans: D>

D>:

$$\frac{d}{da}\left[\int_0^\infty \frac{e^{-x}}{x}\left[a-\frac{1}{x}+\frac{1}{x}e^{-ax}\right]dx\right]$$

a is parameter :

D>:

$$\int_0^\infty \frac{\partial}{\partial a}\left[\frac{e^{-x}}{x}\left[a-\frac{1}{x}+\frac{1}{x}e^{-ax}\right]\right]dx$$

$$=\int_0^\infty \frac{e^{-x}}{x}\left[\frac{\partial}{\partial a}(a)-\frac{\partial}{\partial a}\left(\frac{1}{x}\right)+\frac{\partial}{\partial a}\frac{1}{x}e^{-ax}\right]dx$$

$$=\int_0^\infty \frac{e^{-x}}{x}\left[1-0+\frac{1}{\cancel{x}}(-\cancel{x})e^{-ax}\right]dx$$

$$=\boxed{\int_0^\infty \frac{e^{-x}}{x}\left[1-e^{-ax}\right]dx}$$

(12) If $f(x)=\int_0^\infty e^{-a^2}.\cos ax\,da$, x is parameter then by DUIS rule $\frac{dI}{dx}$ is

A) $\int_0^\infty x e^{-a^2}\sin(xa)\,da$ B) $\int_0^\infty a e^{-a^2}\sin(xa)\,da$

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c) $\int_0^{\infty} -a e^{-a^2} \sin(xa)\, da$ D) $\int_0^{\infty} a e^{-a^2} \cos(xa)\, da$

Sol^n ⇒ Ans: c)

क) :

$$I(x) = \int_0^{\infty} e^{-a^2} \cos ax \, da$$

$x \rightarrow$ parameter

ख):

$$\frac{dI}{dx} = \int_0^{\infty} \frac{\partial}{\partial x} e^{-a^2} \cos ax \, da$$

$$= \int_0^{\infty} e^{-a^2} (-\sin ax)(a)\, da$$

$$= \boxed{\int_0^{\infty} -a e^{-a^2} \sin(xa)\, da}$$

(13) If $I(a) = \int_0^{\infty} \frac{e^{-x} - e^{-ax}}{x \sec x} dx$, $a > 0$ then by DUIS $\frac{dI}{da}$ is

A) $\int_0^{\infty} \frac{e^{-x} + e^{-ax} x}{x \sec x} dx$ B) $\int_0^{\infty} \frac{e^{-x}}{\sec x} dx$

c) $\int_0^{\infty} (e^{-x} - e^{-ax})\, dx$ D) $\int_0^{\infty} \frac{e^{-ax}}{\sec x} dx$

Sol^n ⇒ Ans: D)

क):

$$\int_0^{\infty} \frac{e^{-x} - e^{-ax}}{x \sec x} dx$$

$$\frac{dI}{da} = \int_0^{\infty} \frac{\partial}{\partial a} \left[\frac{e^{-x} - e^{-ax}}{x \sec x} \right] dx$$

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$$\frac{dI}{da} = \int_0^\infty \frac{1}{x\sec x}\left[\frac{\partial}{\partial a}e^{-x} - \frac{\partial}{\partial a}e^{-ax}\right]dx$$

$$= \int_0^\infty \frac{1}{\cancel{x}\sec x}\left[0 - (-\cancel{x})\,e^{-ax}\right]dx$$

$$= \boxed{\int_0^\infty \frac{+e^{-ax}}{\sec x}\,dx}$$

(14) If $\phi(a) = \int_0^1 \frac{x^a - x^b}{\log x}\,dx$, $a>0$, $b>0$ then by DUIS $\frac{d\phi}{da}$ is

A) $\int_0^1 \frac{x^a \log a}{\log x}\,dx$ ✓B) $\int_0^1 x^a\,dx$ C) $\int_0^1 x^b\,dx$ D) $\int_0^1 (x^a - x^b)\,dx$

Sol^n → Ans: B)

ड): $\phi(a) = \int_0^1 \frac{x^a - x^b}{\log x}\,dx$

$$\frac{d\phi}{da} = \int_0^1 \frac{\partial}{\partial a}\left[\frac{x^a - x^b}{\log x}\right]dx$$

$$= \int_0^1 \frac{1}{\log x}\left[\frac{\partial}{\partial a}x^a - \frac{\partial}{\partial a}x^b\right]dx$$

$$= \int_0^1 \frac{1}{\cancel{\log x}}\left[x^a \cancel{\log x} - 0\right]dx$$

$$= \boxed{\int_0^1 x^a\,dx}$$

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(15) If $\phi(b) = \int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} dx$, $a>0, b>0$ then by DUIS rule $\frac{d\phi}{db}$ is

A) $\int_0^\infty e^{-bx} dx$ B) $\int_0^\infty \frac{e^{-ax}(-a) - e^{-bx}(-b)}{x} dx$ C) $\int_0^\infty e^{-ax} dx$ D) None

Sol^n ⇒ Ans: A)

S):

$$\phi(b) = \int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} dx$$

$$\frac{d\phi}{db} = \int_0^\infty \frac{\partial}{\partial b}\left[\frac{e^{-ax} - e^{-bx}}{x}\right] dx$$

$$= \int_0^\infty \frac{1}{x}\left[\frac{\partial}{\partial b} e^{-ax} - \frac{\partial}{\partial b} e^{-bx}\right] dx$$

$$= \int_0^\infty \frac{1}{\cancel{x}}\left[0 - (-\cancel{x}) e^{-bx}\right] dx$$

$$= \boxed{\int_0^\infty + e^{-bx} \, dx}$$

(16) If $\phi(a) = \int_0^\infty \frac{1}{x^2} \log(1+ax^2) dx$, $a>0$ then by DUIS $\frac{d\phi}{da}$ is

A) $\int_0^\infty \frac{a}{x(1+ax^2)} dx$ B) $\int_0^\infty \frac{\log(1+ax^2)}{x} dx$

C) $\int_0^\infty \frac{2a}{x(1+ax^2)} dx$ D) $\int_0^\infty \frac{1}{1+ax^2} dx$

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Sol$^n \to$ Ans: D)

क): $\phi(a) = \int_0^\infty \frac{1}{x^2} \log(1+ax^2)\, dx$

$$\frac{d\phi}{da} = \int_0^\infty \frac{\partial}{\partial a}\left[\frac{1}{x^2}\log(1+ax^2)\right]dx$$

$$= \int_0^\infty \frac{1}{x^2}\frac{\partial}{\partial a}\left[\log(1+ax^2)\right] dx$$

$$= \int_0^\infty \frac{1}{\cancel{x^2}}\left[\frac{1}{(1+ax^2)}(0+(1)\cancel{x^2})\right] dx$$

$$\boxed{= \int_0^\infty \frac{1}{1+ax^2}\, dx}$$

(17) If $\phi(a) = \int_0^{\pi/2} \frac{\log(1+a\sin^2 x)}{\sin^2 x}\, dx$, $a>0$ by DUIS $d\phi/da$ is

A) $\int_0^{\pi/2} \frac{2\sin x \cos x}{(1+a\sin^2 x)}\, dx$ B) $\int_0^{\pi/2} \frac{1}{(1+a\sin^2 x)\sin^2 x}\, dx$

C) $\int_0^{\pi/2} \frac{1}{1+a\sin^2 x}\, dx$ D) $\int_0^{\pi/2} \frac{\sin^2 x}{(1+a\sin^2 x)}\, dx$

Sol$^n \Rightarrow$ Ans: C)

क): $\phi(a) = \int_0^{\pi/2} \frac{\log(1+a\sin^2 x)}{\sin^2 x}\, dx$

$$\frac{d\phi}{da} = \int_0^{\pi/2} \frac{\partial}{\partial a}\left[\frac{\log(1+a\sin^2 x)}{\sin^2 x}\right] dx$$

$$= \int_0^{\pi/2} \frac{1}{\cancel{\sin^2 x}}\left[\frac{1}{(1+a\sin^2 x)} \times (0+(1)\cancel{\sin^2 x})\right] dx$$

$$\boxed{= \int_0^{\pi/2} 1/(1+a\sin^2 x)\, dx}$$

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पान-९६

(18) If $\phi(a) = \int_a^{a^2} \log(ax)\, dx$, then by DUIS rule II, $\frac{d\phi}{da}$ is

A) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx + 2a \log a^3$ B) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx$

✓ C) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)dx + 2a\log a^3 - 2\log a$ D) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)dx - 2a\log a^3 + 2\log a$

Sol^n ⇒ Ans: c)

S): $\phi(a) = \int_a^{a^2} \log(ax)\, dx$

• DUIS Rule 02:

$$\boxed{\frac{dI}{d\alpha} = \int_{a(\alpha)}^{b(\alpha)} \frac{\partial}{\partial \alpha} f(x,\alpha)\, dx + (b,\alpha)\frac{\partial b}{\partial \alpha} - (a,\alpha)\frac{\partial a}{\partial \alpha}}$$

$$\frac{d\phi}{da} = \int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx + \log(a \times a^2)\frac{\partial}{\partial a} a^2 - \log(a \times a)\frac{\partial}{\partial a}(a)$$

$$= \int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx + \log a^3 (2a) - \log a^2 (1)$$

$\{\because \log a^m = m \cdot \log a$: Rule $\}$

$$\boxed{\frac{d\phi}{da} = \int_a^{a^2} \frac{\partial}{\partial a} \log(ax)dx + 2a\log a^3 - 2\log a}$$

(19) If $\phi(a) = \int_0^{a^2} \tan^{-1}(x/a)dx$, then by DUIS rule II, $d\phi/da$ is

✓ A) $\int_0^{a^2} \frac{\partial}{\partial a} \tan^{-1}\left(\frac{x}{a}\right) dx + 2a\tan^{-1}a$ B) $\int_0^{a^2} \frac{\partial}{\partial a} \tan^{-1}\left(\frac{x}{a}\right) \partial x$

C) $\int_0^{a^2} \frac{\partial}{\partial a} \tan^{-1}\left(\frac{x}{a}\right) dx + a^2 \tan^{-1}a$ D) $\int_0^{a^2} \frac{\partial}{\partial a} \tan^{-1}\left(\frac{x}{a}\right)dx + a^2 \tan^{-1}a - \tan^{-1}\left(\frac{x}{a}\right)$

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$Sol^n \Rightarrow$ Ans: A)

बो):

$$\phi(a) = \int_0^{a^2} \tan^{-1}\left(\frac{x}{a}\right) dx$$

- DUIS Rule II:

$$\frac{d\phi}{da} = \int_0^{a^2} \frac{\partial}{\partial a} \tan^{-1}\left(\frac{x}{a}\right) dx + \tan^{-1}\left(\frac{a^2}{a}\right) \frac{\partial}{\partial a} a^2 - \tan^{-1}\left(\frac{0}{a}\right) \frac{\partial}{\partial a}(0)$$

$$= \int_0^{a^2} \frac{\partial}{\partial a} \tan^{-1}\left(\frac{x}{a}\right) dx + \tan^{-1}(a)(2a) - 0$$

$$= \boxed{\int_0^{a^2} \frac{\partial}{\partial a} \tan^{-1}\left(\frac{x}{a}\right) dx + 2a \tan^{-1} a}$$

20. If $I(a) = \int_a^{a^2} \frac{1}{x+a} dx$, then by DUIS rule II, $\frac{dI}{da}$ is

A) $\int_a^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx - \frac{1}{a^2+a}(2a) + \frac{1}{2a}$

B) $\int_a^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx + \frac{1}{a^2+a}(2a) - \frac{1}{2a}$

C) $\int_a^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx - \frac{1}{a^2+a}$

D) $\int_a^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx$

$Sol^n \Rightarrow$ Ans: B)

बो): $I(a) = \int_a^{a^2} \frac{1}{x+a} dx$, By DUIS Rule II:

$$\frac{dI}{da} = \int_a^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx + \frac{1}{a^2+a} \frac{\partial}{\partial a}(a^2) - \frac{1}{a+a} \frac{\partial}{\partial a}(a)$$

$$\boxed{\frac{dI}{da} = \int_a^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx + \frac{1}{a^2+a}(2a) - \frac{(1)}{2a}(1)}$$

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(21) If $\phi(a) = \int_0^a \left[\frac{\log(1+ax)}{1+x^2}\right] dx$ by DUIS rule II, $\frac{d\phi}{da}$ is

A) $\int_0^a \frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right] dx$

B) $\int_0^a \frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right] dx - \frac{\log(1+a)}{1+a^2}$

C) $\int_0^a \frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right] dx - \frac{\log(1+a^2)}{1+a^2}$

✓D) $\int_0^a \frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right] dx + \frac{\log(1+a^2)}{1+a^2}$

Sol^n ⇒ Ans: D)

$\because$: $\phi(a) = \int_0^a \left[\frac{\log(1+ax)}{1+x^2}\right] dx$

$\therefore$: $$\frac{\partial\phi}{\partial a} = \int_0^a \frac{\partial}{\partial a}\frac{\log(1+ax)}{1+x^2}\, dx + \frac{\log(1+a\times a)}{1+a^2}\frac{\partial}{\partial a}(a)$$

$$\boxed{\frac{\partial\phi}{\partial a} = \int_0^a \frac{\partial}{\partial a}\left[\frac{\log(1+ax)}{1+x^2}\right] dx + \frac{\log(1+a^2)}{1+a^2} \quad (1)}$$

(22) If $F(t) = \int_t^{t^2} e^{tx^2} dx$, then by DUIS rule II, $\frac{dF}{dt}$ is

A) $\int_t^{t^2} \frac{\partial}{\partial t} e^{tx^2} dx + (2t)e^{t^4} - e^{t^2}$

B) $\int_t^{t^2} \frac{\partial}{\partial t} e^{tx^2} dx + e^{t^5} - e^{t^3}$

✓C) $\int_t^{t^2} \frac{\partial}{\partial t} e^{tx^2} dx + (2t)e^{t^5} - e^{t^3}$

D) $\int_t^{t^2} \frac{\partial}{\partial t} e^{tx^2} dx$

Sol^n ⇒ Ans: C)

$\because$: $$\frac{dF}{dt} = \int_t^{t^2} \frac{\partial}{\partial t} e^{tx^2} dx + e^{t\times(t^2)^2}\frac{\partial}{\partial t}t^2 - e^{t\times t^2}\frac{\partial}{\partial t}(t)$$

$$\boxed{\frac{dF}{dt} = \int_t^{t^2} \frac{\partial}{\partial t} e^{tx^2} dx + (2t)e^{t^5} - e^{t^3}}$$

पन्ना-9e

(25) Using DUIS rule the value of integral $\phi(a) = \int_0^\infty \frac{e^{-x}}{x}(1-e^{-ax})\,dx, a>-1$ given $\frac{d\phi}{da} = \frac{1}{a+1}$ is.

A) $\log(a+1)$ B) $-\frac{1}{(a+1)^3}$ C) $\log(a+1)+\pi$ D) $-\frac{1}{(a+1)^2}+1$

Sol^n ⇒ Ans: A)

क): $\phi(a) = \int_0^\infty e^{-x}/x\,(1-e^{-ax})\,dx$

$$\frac{d\phi}{da} = \frac{1}{a+1}$$

ख): On Integrating both side w.r.t a:

$$\int \frac{d\phi}{da}\,da = \int \frac{1}{1+a}\,da$$

$$\phi = \log(1+a) + c \quad \ldots (1)$$

ग): Put: $a=0$

$$\phi = \log(1+0)+c$$
$$\phi = \log 1 + c \qquad \{\because \log 1 = 0\}$$
$$\phi = 0 + c$$
$$\phi = c$$

घ): $\phi(a) = \int_0^\infty e^{-x}/x\,(1-e^{-ax})\,dx$

Put: $a=0$

$$\phi(0) = \int_0^\infty e^{-x}/x\,(1-e^{0})\,dx$$
$$= 0$$
$$\therefore c = 0$$

$$\boxed{\phi(a) = \log(1+a)}$$

पत्र-20

(26) Using DUIS rule the value of integral $\phi(a) = \int_0^1 \frac{x^a - 1}{\log x} dx, a \geq 0$ given $d\phi/da = 1/a+1$ is

A) $\log(a+1)$ B) $-\frac{1}{(a+1)^2}$ C) $\log(a+1)+\pi$ D) $-\frac{1}{(a+1)^2}+1$

Sol^n ⇒ Ans: A)

$$\phi(a) = \int_0^1 \frac{x^a - 1}{\log a} dx, \quad a \geq 0$$

$$\frac{d\phi}{da} = \frac{1}{1+a}$$

$$d\phi = \frac{1}{1+a} da$$

$$\int d\phi = \int \frac{1}{1+a} da$$

$$\phi(a) = \log(1+a) + c \quad \cdots (1)$$

Put: $a = 0$

$$\phi(0) = \log(1+0) + c \qquad \{\because \log 1 = 0\}$$

$$\phi(0) = \int_0^1 \frac{x^0 - 1}{\log x} dx = 0 \quad \therefore c = 0$$

$$\boxed{\phi(a) = \log(1+a)}$$

(27) $\phi(\alpha) = \int_0^\infty \frac{e^{2x} \sin \alpha x}{x} dx$ then integral is when $\frac{d\phi}{d\alpha} = \frac{2}{\alpha^2+4}$

A) $2\log(\alpha^2+4)$ B) $2\tan^{-1}(\alpha/2)$ C) $\frac{1}{2}\tan^{-1}(\alpha/2)$ D) $\tan^{-1}(\alpha/2)$

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Sol^n ⇒ Ans: D)

क):

$$\phi(\alpha) = \int_0^\infty \frac{e^{-2x} \sin \alpha x}{x} \, dx$$

$$\frac{d\phi}{d\alpha} = \frac{2}{\alpha^2+4}$$

$$d\phi = \frac{2}{\alpha^2+4} \, d\alpha$$

Formula:

$$\int d\phi = 2\int \frac{1}{\alpha^2+2^2} \, d\alpha \qquad \left\{ \because \int \frac{1}{x^2+a^2} \, dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) \right\}$$

$$\phi(\alpha) = 2 \times \frac{1}{2} \tan^{-1}\left(\frac{\alpha}{2}\right) + c$$

ख):

Put $\alpha = 0$

$$\phi(0) = \tan^{-1}(0) + c$$

$$\phi(0) = 0 + c$$

ग):

$$\phi(0) = \int_0^\infty \frac{e^{-2x} \sin(0)}{x} \, dx$$

$$= 0$$

$$\therefore c = 0$$

$$\therefore \boxed{\phi(\alpha) = \tan^{-1}(\alpha/2)}$$

(28) Using DUIS rule the value of $\phi(\alpha) = \int_0^\infty \frac{e^{-\alpha x} \sin x}{x} \, dx$ with $\frac{d\phi}{d\alpha} = \frac{-1}{\alpha^2+1}$ and assuming $\int_0^\infty \frac{\sin x}{x} \, dx = \pi/2$ is

A) $\tan^{-1}\alpha + \frac{\pi}{2}$ ✓B) $-\tan^{-1}\alpha + \frac{\pi}{2}$ C) $-\tan^{-1}\alpha$ D) $\log(\alpha^2+1) + \frac{\pi}{2}$

Sol^n ⇒

Ans: B)

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पाना-22

क):

$$\phi(\alpha) = \int_0^\infty \frac{e^{-\alpha x}\sin x}{x}\,dx$$

$$\frac{d\phi}{d\alpha} = \frac{-1}{\alpha^2+1} \quad \text{and} \quad \int_0^\infty \frac{\sin x}{x}\,dx = \frac{\pi}{2}$$

$$\therefore \ d\phi = \frac{-1}{\alpha^2+1}\,d\alpha$$

$$\int d\phi = \int \frac{-1}{\alpha^2+1}\,d\alpha$$

ख):

$$\phi(\alpha) = -\tan^{-1}\alpha + c \quad \ldots (1)$$

Put $\alpha = 0$

$$\phi(0) = 0 + c$$

ग):

$$\phi(0) = \int_0^\infty e^0 \frac{\sin x}{x}\,dx$$

$$= \int_0^\infty \frac{\sin x}{x}\,dx$$

$$= \frac{\pi}{2}$$

$$\therefore \ c = \pi/2$$

$$\therefore \ \boxed{\phi(\alpha) = -\tan^{-1}\alpha + \pi/2}$$

(29) Using DUIS rule the value of integral $\phi(a) = \int_0^{\pi/2} \frac{\log(1+a\sin^2 x)}{\sin^2 x}\,dx$, with $\frac{d\phi}{da} = \frac{\pi}{2}\frac{1}{\sqrt{a+1}}$ is

A) $\pi\sqrt{a+1}$ B) $\pi\sqrt{a+1} + \pi$

C) $\pi\sqrt{a+1} - \pi$ D) $3\pi(a+1)^{3/2} - \pi$

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प्रश्न-२३

Sol^n ⇒ Ans: c)

क): $\phi(a) = \int_0^{\pi/2} \frac{\log(1+a\sin^2 x)}{\sin^2 x}\,dx$

$$\frac{d\phi}{da} = \frac{\pi}{2}\frac{1}{\sqrt{a+1}}$$

$$d\phi = \frac{\pi}{2}\frac{1}{\sqrt{a+1}}\,da$$

$$\int d\phi = \pi\int \frac{1}{2\sqrt{a+1}}\,da \qquad \left\{\because \int \frac{1}{2\sqrt{a}} = \sqrt{a}\right\}$$

$$\phi(a) = \pi(\sqrt{a+1}) + C \quad \cdots (1)$$

ख): Put $a=0$

$$\therefore \phi(0) = \pi\sqrt{0+1} + C$$

ग): $\phi(0) = \int_0^{\pi/2} \frac{\log(1+0\cdot\sin^2 x)}{\sin^2 x}\,dx$

$$= \int_0^{\pi/2} \frac{\log 1}{\sin^2 x}\,dx = 0 \qquad \{\because \log 1 = 0\}$$

$$0 = \pi + C \qquad \therefore C = -\pi$$

$$\boxed{\phi(a) = \pi\sqrt{a+1} - \pi}$$

30. Using DUIS rule the value of integral $\phi(a) = \int_0^{\infty} \frac{\log(1+ax^2)}{x^2}\,dx$, $a>0$ with $\frac{d\phi}{da} = \frac{\pi}{2}\frac{1}{\sqrt{a}}$ is

A) $\pi\sqrt{a}$ B) $\frac{\pi\sqrt{a}}{4}$ C) $\frac{-\pi}{4a^{3/2}}$ D) $\frac{\pi}{4\sqrt{a}}$

Sol^n ⇒ Ans: A)

पाठ-२४

क): $\phi(a) = \int_0^\infty \frac{\log(1+ax^2)}{x^2} dx$, $a>0$

$$\frac{d\phi}{da} = \frac{\pi}{2} \frac{1}{\sqrt{a}}$$

$$d\phi = \frac{\pi}{2} \frac{1}{\sqrt{a}} da$$

$$\int d\phi = \pi \int \frac{1}{2\sqrt{a}} da \qquad \left\{ \because \int \frac{1}{2\sqrt{x}} dx = \sqrt{x} \right\}$$

$$\phi(a) = \pi(\sqrt{a}) + c \quad \ldots (1)$$

ख): Put $a = 0$

$$\phi(0) = \pi\sqrt{0} + c$$

ग): $\phi(0) = \int_0^\infty \frac{\log(1+0)}{x^2} dx = 0 \qquad \{ \because \log 1 = 0 \}$

$$\boxed{\phi(a) = \pi\sqrt{a}}$$

31) Using DUIS rule the value of integral $\phi(a) = \int_0^\infty \frac{e^{-x} - e^{-ax}}{x \cdot \sec x} dx$, with $\frac{d\phi}{da} = \frac{a}{a^2+1}$ is

A) $\tan^{-1} a + \frac{\pi}{4}$ B) $\log\left(\frac{2}{a^2+1}\right)$ C) $\frac{1}{2}\log(a^2+1)$ D) $\frac{1}{2}\log\left(\frac{a^2+1}{2}\right)$

Solⁿ ⇒ Ans: D)

क): $\phi(a) = \int_0^\infty \frac{e^{-x} - e^{-ax}}{x \cdot \sec x} dx$

$$\frac{d\phi}{da} = \frac{a}{a^2+1}$$

$$d\phi = \frac{a}{a^2+1}\, da$$

$$\int d\phi = \int \frac{a}{a^2+1}\, da$$

$$\phi(a) = \frac{1}{2}\log(a^2+1) + c \quad \cdots\cdots (1)$$

ब): Put : $a = 1$

$$\phi(1) = \frac{1}{2}\log(1^2+1) + c$$

$$\phi(1) = \frac{1}{2}\log 2 + c \quad \cdots\cdots (2)$$

ग): $\phi(1) = \int_0^\infty \frac{e^{x} - e^{-x}}{x\sec x}\, dx = 0$

घ): Eq^n (2) rewrite again :

$$0 = \frac{1}{2}\log 2 + c$$

$$c = -\frac{1}{2}\log 2$$

Eq^n (1) becomes:

$$\phi(a) = \frac{1}{2}\cdot\log(a^2+1) - \frac{1}{2}\cdot\log 2$$

$$= \frac{1}{2}\left[\log(a^2+1) - \log 2\right]$$

$$\boxed{\phi(a) = \frac{1}{2}\log\left[\frac{a^2+1}{2}\right]}$$

(32) Using DUIS rule the value of integral $\phi(a) = \int_0^\infty \frac{1-\cos ax}{x^2}\, dx$, with $\frac{d\phi}{da} = \frac{\pi}{2}$ is

A) $\frac{\pi}{2}$ ✓B) $\frac{\pi a}{2}$ C) πa D) $\frac{\pi a}{2} + \frac{\pi}{2}$

पान - २६९

Solⁿ ⇒ Ans: B>

क>: $\phi(a) = \int_0^a \frac{1-\cos ax}{x^2}\,dx$ and

$$\frac{d\phi}{da} = \frac{\pi}{2}$$

$$d\phi = \frac{\pi}{2}\,da$$

$$\int d\phi = \frac{\pi}{2}\int da$$

$$\phi(a) = \frac{\pi}{2}a + c \quad \ldots (1)$$

ख): Put: $a = 0$

$$\phi(0) = 0 + c$$

ग>: $\phi(0) = \int_0^\infty \frac{1-\cos 0}{x^2}\,dx = 0$ $\{\because \cos 0 = 1\}$

∴ Eq^n (1) becomes:

$$\boxed{\phi(a) = \frac{\pi}{2}a}$$

∴ Ans: B>:

● NOTE :

① Error Function Defination:

i) $erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$

↑

(∵ Read: Error function of x)

ii) $erf(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-u^2} du$

↑

(∵ Read: complementory error function of x)

iii) Aternative Def^n :

$$erf(x) = \frac{1}{\sqrt{\pi}} \int_0^{x^2} e^{-t} t^{-1/2} dt$$

* NOTE:

i) variable of integration in definite Integral is immaterial (or doesn't matter)

ii) Adjustment the proper variable & compair with std formula.

② Formulae:

1) $erf(\infty) = 1$

2) $erf(0) = 0$

3) $erf(x) + erf(x) = 1$

4) $erf(x) = 1 - erf(x)$

5) $erf(x) = 1 - erf(x)$

6) $erfc(x) = 1 - erfc(x)$

7) $erf(-x) = -erf(x)$

∴ $erf(x)$ is an odd function

8) $erf(x) = \frac{2}{\sqrt{\pi}} \left[x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} \right] + \ldots$

पान-२८

9) Differentiation of erf(x):

$$\frac{d}{dx} erf(x) = \frac{2a\, e^{-a^2x^2}}{\sqrt{\pi}}$$

10) integration of error function:

$$\int_0^t erf(ax)\, dx = t \cdot erf(at) + \frac{1}{a\sqrt{\pi}} e^{-a^2t^2} - \frac{1}{a\sqrt{\pi}}$$

SET: 02

मान-२e

* Type: Error Function:

① Error function of x, erf(x) is defined as

✓A) $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$ B) $\frac{2}{\sqrt{\pi}}\int_0^\infty e^{-u^2}du$ C) $\int_0^\infty e^{-x}x^{n-1}dx$ D) $\frac{2}{\sqrt{\pi}}\int_0^\infty e^{-u^2}du$

Ans: A)

$$\boxed{erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du}$$ ....(स्टैंडर्ड रिजल्ट है।)

② Complementary error function of x, erfc (c) is defined as

Solⁿ ⇒

$$\boxed{erfc(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2}du}$$ ....(यह स्टैंडर्ड है।)

③ The value of erf(∞) is

Solⁿ ⇒ $\boxed{erf(\infty) = 1}$ ....(स्टैंडर्ड रिजल्ट है।)

④ The value of erf (0) is

Solⁿ ⇒ $\boxed{erf(0) = 0}$ ...(स्टैंडर्ड रिजल्ट है।)

⑤ The value of erf c(0) is

$\boxed{erfc(0) = 1}$ $\{\because erfc(x) = 1 - erf(x)$ $= 1 - 0$ $= 1\}$

....(स्टैंडर्ड रिजल्ट है।)

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(6) Which of the following is true?

Sol^n ⇒ $\boxed{erf(x) + erfc(x) = 1}$

....(यह स्टैंडर्ड रिजल्ट है।)

(7) Error function is

Sol^n ⇒ Error function is: An Odd Function

$$erf(-x) = -erf(x)$$

...(स्टैंडर्ड रिजल्ट है।)

(8) erf(-x) is equal to

Sol^n ⇒ $\boxed{erf(-x) = -erf(x) \equiv \text{odd function}}$

....(स्टैंडर्ड रिजल्ट है।)

(9) The proper substition for integral $\int_0^\infty e^{-(x+a)^2} dx$ to complementory error function is

A) $(x+a)^2 = u$ B) $-(x+a) = u$ C) $x+a = u$ D) $-(x+a)^2 = u$

Sol^n ⇒

$\boxed{\text{substition is: } x+a = u}$

....(स्टैंडर्ड रिजल्ट है।)

(10) $erf(x) + erf(-x) =$

A) 2 B) 1 C) -11 D) 0

Sol^n ⇒ Ans: D)

$$\begin{aligned} \Rightarrow\ &= erf(x) + erf(-x) \\ &= \cancel{erf(x)} - \cancel{erf(x)} \\ &= \boxed{0} \end{aligned}$$

{∵ $erf(-x) = -erf(x)$ ... Result}

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(11) $\text{erf}(-x) + \text{erfc}(-x) =$

A) 2 B) 1 C) -1 D) 0

$\text{Sol}^n \Rightarrow$ Ans: B)

$\text{प्र} :- = \text{erf}(-x) + \text{erfc}(-x)$

Put: $-x = t$

$= \text{erf}(t) + \text{erfc}(t)$ $\{\because \text{Result: } \text{erf}(x) + \text{erfC}(x) = 1\}$

$= \boxed{1}$

(12) $\text{erfc}(-x) - \text{erf}(x) =$

A) 2 B) -1 C) 1 D) 0

$\text{Sol}^n \Rightarrow$ Ans: C)

$\text{प्र} :- = \text{erfc}(-x) - \text{erf}(x)$

$= \text{erfc}(-x) + \text{erf}(-x)$ $\{\because -\text{erf}(x) = +\text{erf}(-x) \text{ ... Result}\}$

Put: $-x = t$

$= \text{erfc}(t) + \text{erf}(t)$

$= \boxed{1}$ $\{\because \text{Result: } \text{erf}(x) + \text{erfc}(x) = 1\}$

(13) $\text{erfc}(-x) + \text{erfc}(x) =$

A) 2 B) -1 C) 1 D) 0

$\text{Sol}^n \Rightarrow$ Ans: A)

$\text{प्र} :- = \text{erfc}(-x) + \text{erfc}(x)$

$=$

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पान - ३२

(14) If $erf(ax) = \frac{2}{\sqrt{\pi}} \int_0^{ax} e^{-u^2} du$ then $\frac{d}{dx} erf(ax)$ is

Sol^n ⇒

$$\boxed{\frac{d}{dx} erf(ax) = \frac{2a}{\sqrt{\pi}} \cdot e^{-a^2x^2}}$$

....(स्टँडर्ड रिजल्ट है।)

(15) If $erf(\sqrt{t}) = \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{t}} e^{-u^2} du$ then $d/dt \; erf(\sqrt{t})$ is

A) $\frac{e^{-t}}{2\sqrt{t}}$ B) $\frac{e^{-t^2}}{\sqrt{\pi t}}$ C) $\frac{e^{-t}}{\sqrt{\pi}}$ D) $\frac{e^{-t}}{\sqrt{\pi t}}$

Sol^n ⇒ Ans: D)

D): $erf(\sqrt{t}) = \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{t}} e^{-u^2} du$

$$\frac{d}{dt} erf(\sqrt{t}) = \frac{2}{\sqrt{\pi}} \frac{d}{dt} \int_0^{\sqrt{t}} e^{-u^2} du$$

D): By DUIS rule II:

$$\frac{d}{dt} erf(\sqrt{t}) = \frac{2}{\sqrt{\pi}} \left[\int_0^{\sqrt{t}} \frac{\partial}{\partial t} e^{-u^2} du + e^{-(\sqrt{t})^2} \frac{\partial}{\partial t} \sqrt{t} - 0 \right]$$

$$= \frac{2}{\sqrt{\pi}} \left[0 + e^{-t} \times \frac{1}{2\sqrt{t}} - 0 \right]$$

$$= \frac{2}{2\sqrt{\pi}} \frac{e^{-t}}{\sqrt{t}}$$

$$= \boxed{\frac{e^{-t}}{\sqrt{\pi t}}}$$

पान-३२

(16) If $erfc(ax) = \frac{2}{\sqrt{\pi}} \int_{ax}^{\infty} e^{-u^2} du$ then $\frac{d}{dx} erfc(ax)$ is

A) $\frac{-1}{\sqrt{\pi}} e^{-a^2x^2}$ B) $-\frac{2}{\sqrt{\pi}} e^{-a^2x^2}$ ✓C) $-\frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$ D) $\frac{2a}{\sqrt{\pi}} e^{a^2x^2}$

Sol^n ⇒ Ans: C)

क): $erfc(ax) = \frac{2}{\sqrt{\pi}} \int_{ax}^{\infty} e^{-u^2} du$

$$\frac{d}{dx} erfc(ax) = \frac{2}{\sqrt{\pi}} \frac{d}{dx} \int_{ax}^{\infty} e^{-u^2} du$$

ख): By DUIS Rule II:

$$\frac{d}{dx} erfc(ax) = \frac{2}{\sqrt{\pi}} \left[\int_{ax}^{\infty} \frac{\partial}{\partial x} e^{-u^2} du + e^{-\infty} \frac{\partial \infty}{\partial x} - e^{-a^2x^2} \frac{\partial}{\partial x} ax \right]$$

$$= \frac{2}{\sqrt{\pi}} \left[0 + 0 - e^{-a^2x^2} (a) \right]$$

$\left\{ \because e^{-\infty} = 0, \ \frac{\partial}{\partial x} e^{-u^2} = 0 \right\}$

$$= \boxed{\frac{-2a \, e^{-a^2x^2}}{\sqrt{\pi}}}$$

(17) If $\frac{d}{dx} erf(ax) = \frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$ then $\frac{d}{dx} erfc(ax)$ is

A) $\frac{2a}{\sqrt{\pi}} e^{a^2x^2}$ B) $1 - \frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$ C) $\frac{-1}{\sqrt{\pi}} e^{-a^2x^2}$ ✓D) $-\frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$

Sol^n ⇒ Ans: D)

D): $\frac{d}{dx} erf(ax) = \frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$

$$\frac{d}{dx} erfc(ax) = ?$$

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पान-३४

Sol):
$$\mathrm{erf}(ax) + \mathrm{erf\,c}(ax) = 1$$

Differentiate both sides with respect to x:

$$\frac{d}{dx}\mathrm{erf}(ax) + \frac{d}{dx}\mathrm{erf\,c}(ax) = \frac{d}{dx}(1)$$

$$\frac{d}{dx}\mathrm{erf}(ax) + \frac{d}{dx}\mathrm{erf\,c}(ax) = 0$$

$$\frac{d}{dx}\mathrm{erf}(ax) = -\frac{d}{dx}\mathrm{erf\,c}(ax)$$

$$= \boxed{-\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}}$$

($\because$ Note: Derivative of erf(x) and its complimentory erf c(x) is opposite in sign only).

(19) If $\frac{d}{dt}\mathrm{erf}(\sqrt{t}) = \frac{e^{-t}}{\sqrt{\pi t}}$ then $\frac{d}{dt}\mathrm{erfc}(\sqrt{t})$ is

✓A) $-\frac{e^{-t}}{\sqrt{\pi t}}$ B) $1-\frac{e^{-t}}{\sqrt{\pi t}}$ C) $-\frac{e^{t}}{\sqrt{\pi t}}$ D) $\frac{e^{-t}}{\sqrt{\pi t}}$

Sol^n => Ans: c)

D): $\frac{d}{dt}\mathrm{erf}(\sqrt{t}) = \frac{e^{-t}}{\sqrt{\pi t}}$

It's complimentory opposite in only sign:

$$\boxed{\therefore \frac{d}{dt}\mathrm{erf\,c}(\sqrt{t}) = -\frac{e^{-t}}{\sqrt{\pi t}}}$$

(20) If $\frac{d}{dx}\mathrm{erf\,c}(ax) = -2a/\sqrt{\pi}\cdot e^{-a^2x^2}$ then $d/dx\ \mathrm{erf}(ax)$ is

A) $ae^{-a^2x^2}$ B) $1-\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$ ✓C) $\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$ D) $-\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$

Sol:

$$\frac{d}{dx}\, erfc(ax) = -\frac{2a}{\sqrt{\pi}}\, e^{-a^2x^2}$$

It's erf(ax) is opposite in only sign.

$$\boxed{\frac{d}{dx}\, erf(ax) = +\frac{2a}{\sqrt{\pi}}\, e^{-a^2x^2}}$$

(21) If $d/da\ erfc(ax) = \frac{-2x}{\sqrt{\pi}}\, e^{-a^2x^2}$ then $d/da\ erf(ax)$ is

A) $x e^{-a^2x^2}$ B) $-2x/\sqrt{\pi} \cdot e^{-a^2x^2}$ C) $1/\sqrt{\pi}\, e^{-a^2x^2}$ D) $2x/\sqrt{\pi} \cdot e^{-a^2x^2}$

Sol^n ⇒ Ans: D)

Sol:

$$\frac{d}{da}\, erfc(ax) = \frac{-2x}{\sqrt{\pi}} \cdot e^{-a^2x^2}$$

It's derivative is opposite in sign only.

$$\boxed{\frac{d}{dx}\, erf(ax) = + \frac{2x}{\sqrt{\pi}}\, e^{-a^2x^2}}$$

(22) If $erfc(a) = \frac{2}{\sqrt{\pi}} \int_a^{\infty} e^{-u^2} du$ then by using substitution $x+a=u$, the integral $\int_0^{\infty} e^{-(x+a)^2} dx$ in terms of erfc(a) is

A) $\frac{2}{\sqrt{\pi}}\, erfc(a)$ B) $erfc(a)$ C) $\frac{\sqrt{\pi}}{2}\, erfc(a)$ D) $\frac{\sqrt{\pi}}{2}\, erf(a)$

Sol^n ⇒ Ans: D)

Sol:

$$\int_a^{\infty} e^{-u^2} du = \frac{\sqrt{\pi}}{2} [erf(a)]$$

Put : $x+a=u$

$dx+0=du$

$dx=du$

Limit:

u	a	∞
x	0	∞

⇒ use limit of x: $0 \to \infty$

$$\therefore \int_0^\infty e^{-(x+a)^2}\,dx = \frac{\sqrt{\pi}}{2}\,\text{erf}(a)$$

(23) $\int_0^t \text{erf}(ax)\,dx + \int_0^t \text{erfc}(ax)\,dx =$

A) t B) x C) 0 D) $\frac{t^2}{2}$

Sol^n ⇒ Ans: A)

S):

$$= \int_0^t \text{erf}(ax)\,dx + \int_0^t \text{erfc}(ax)\,dx$$

$$= \int_0^t [\text{erf}(ax) + \text{erfc}(ax)]\,dx$$

$\{\because \text{erf}(t) + \text{erfc}(t) = 1$ ... Result $\}$

$$= \int_0^t 1\,dx$$

$$= [x]_0^t$$

$$= [t - 0]$$

$$= \boxed{t}$$

(24) The integral $\int_0^\infty e^{-t}\,\text{erf}(\sqrt{t})\,dt$ using $\frac{d}{dt}\text{erf}(\sqrt{t}) = e^{-t}/\sqrt{\pi t}$ is

A) $\frac{1}{\sqrt{\pi}}\int_0^\infty e^{2t}\cdot t^{1/2}\,dt$ B) $\frac{1}{\sqrt{\pi}}\int_0^\infty e^{-2t}\,t^{-1/2}\,dt$

C) $\int_0^\infty e^{-2t}\,t^{-1/2}\,dt$ D) $\frac{1}{\sqrt{\pi}}\int_0^\infty e^{-t}\,t^{1/2}\,dt$

Sol^n ⇒ Ans: B)

पाठ - ३७

क):

$$\frac{d}{dt}\,\mathrm{erf}(\sqrt{t}) = \frac{e^{-t}}{\sqrt{\pi t}}$$

Integrating both sides:

$$\int \frac{d}{dt}\,\mathrm{erf}(\sqrt{t})\,dt = \int \frac{e^{-t}}{\sqrt{\pi t}}\,dt \qquad \{\because \sqrt{t} = t^{1/2}\}$$

$$\mathrm{erf}(\sqrt{t}) = \int \frac{e^{-t}}{\sqrt{\pi}}\,t^{-1/2}\,dt$$

ख): Multiply both sides by e^{-t}:

$$e^{-t}.\,\mathrm{erf}(\sqrt{t}) = \int \frac{e^{-t}.e^{-t}}{\sqrt{\pi}}\,t^{-1/2}\,dt$$

$$\boxed{e^{-t}.\,\mathrm{erf}\sqrt{t} = \int_0^\infty \frac{e^{-2t}}{\sqrt{\pi}}\,t^{-1/2}\,dt}$$

25 Expansion of erf (x) in the series is

Sol^n →

$$\boxed{\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\left[x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \cdots\right]}$$

..... (यह स्टैंडर्ड रिजल्ट है।)

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पाठ - 82

● NOTE:

① Linear Differential Equation:

This equation contains dependent variable and it's derivatives appears only in first degree.

~~Form~~:

$$\frac{dy}{dx} + Py = Q \quad \boxed{OR} \quad \frac{dx}{dy} + Px = Q$$

$P, Q \rightarrow$ free from y $\qquad$ $P, Q \rightarrow$ free from x

• $IF = e^{\int P \cdot dx}$ $\qquad$ • $IF = e^{\int P \cdot dy}$

• GS: $\qquad$ • GS:

$$y \cdot (IF) = \int Q \times IF \cdot dx + c \qquad x \cdot (IF) = \int Q \times IF \cdot dy + c$$

$$y\left[e^{\int P dx}\right] = \int Q \left[e^{\int P dx}\right] dx + c \qquad x\left[e^{\int P dy}\right] = \int Q \cdot \left[e^{\int P dy}\right] dy + c$$

• Note: coeffient of $\frac{dy}{dx}$ or $\frac{dx}{dy}$ must be equal to 1.

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पार्ट-e3

② Equation Reducible into LDE:

आ): Bernoullis DE:

1) Form:

$$\frac{dy}{dx} + Py = Q y^n$$

$P, Q \rightarrow$ free from y

devide above eq^n by y^n:

$$\therefore y^{-n}\frac{dy}{dx} + P y^{1-n} = Q \quad \text{.....} ①$$

Now,

Put : $y^{1-n} = u$

Taking derivative :

$$(1-n) y^{-n} \frac{dy}{dx} = \frac{du}{dx}$$

$$y^{-n}\frac{dy}{dx} = \frac{1}{(1-n)}\frac{du}{dx}$$

put in ①:

$$\therefore \frac{1}{(1-n)}\frac{du}{dx} + P y^{1-n} = Q$$

$$\frac{du}{dx} + (1-n)Pu = (1-n)Q \qquad \{\because y^{1-n} = u\}$$

now solve by LDE method.

2) Form:

$$\frac{dx}{dy} + Px = Q x^n$$

similary solve this by putting $x^{1-n} = u$.
solve like form 1.

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पान - ६४

3) Form:

$$f'(y) \cdot \frac{dy}{dx} + p \cdot f(y) = q \quad \ldots\ldots (1)$$

Put: $f(y) = u$

Take derivative:

$$f'(y) \cdot \frac{dy}{dx} = \frac{du}{dx}$$

put in (1):

$$\therefore \quad \frac{du}{dx} + pu = q$$

now solving using LDE method.

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पान - ६४

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*** Type: Linear Differential Equation and Reducible to LDE:**

① The differential equation of the form $dy/dx + Py = Q$ where P and Q are functions of x or constants, is

Sol^n ⇒ Ans: B) Linear Differential Equation in y ....(std Result)

② The differential equation of the form $dx/dy + Px = Q$ where P and Q are functions of y or constants, is

Sol^n ⇒ Ans: C) Linear Differential Equation in x ......(std Result)

③ Integrating factor of linear differential equation $\frac{dx}{dy} + Px = Q$ where P & Q are functions of y or constants, is

Sol^n ⇒ Ans: A) $IF = e^{\int P\,dy}$ .....(std Result)

④ Integrating factor of linear differential equation $\frac{dy}{dx} + Py = Q$ where P & Q are functions of x or constants, is

Sol^n ⇒ Ans: D):

$IF = e^{\int P\,dx}$ .....(std Result)

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पान-८६

(5) The general solution of linear Differential Equation

$\frac{dy}{dx} + Py = Q$ where P & Q are functions of x or constants, is

Sol^n ⇒ Ans: D)

GS: $y e^{\int P dx} = \int Q \cdot e^{\int P dx} dx + C$ .....(std Result)

(6) The general solution of linear differential equation

$\frac{dx}{dy} + Px = Q$ where P & Q are functions of y or constants, is

Sol^n ⇒ Ans: D)

GS: $x e^{\int P dy} = \int Q \cdot e^{\int P dy} dy + C$ .....(std Result)

(7) The differential equation of the form

$\frac{dy}{dx} + Py = Qy^n$, $n \neq 1$ where P & Q are functions of x or constants, is

Sol^n ⇒ Ans: A)

Bernoullis Differential Equation ....(std Result)

(8) The differential equation of the form

$\frac{dx}{dy} + Px = Qx^n$, $n \neq 1$ where P & Q are functions of y or constants, is

पान-८७

Sol^n ⇒ Ans: A)

Bernoullis Differential Equation ..... (std Result)

(9) The differential equation of the form $f'(y)\frac{dy}{dx} + Pf(y) = Q$ where P & Q are functions of x or constants, can be reduced to linear differential equation by the substitution

Sol^n ⇒ Ans: c)

$$f(y) = u \quad \text{.....(std Result)}$$

(10) The differential equation of the form $f'(x)\frac{dx}{dy} + Pf(x) = Q$ where P & Q are functions of y or constants, can be reduced to linear differential equation by the substitution

Sol^n ⇒ Ans: B)

$$f(x) = u \quad \text{.....(std Result)}$$

(11) Integrating factor of linear differential equation $\frac{dy}{dx} + xy = x^3$ is

A) $e^{\log x}$ B) e^x C) x^2 ✓D) $e^{x^2/2}$

Sol^n ⇒ Ans: D)

$$\text{D)}: \quad \frac{dy}{dx} + \underset{\substack{\uparrow \\ P}}{x}y = \underset{\substack{\uparrow \\ Q}}{x^3}$$

पान-eL

ख): $IF = e^{\int p \cdot dx} = e^{\int x\, dx} = \boxed{e^{x^2/2}}$

$\left\{ \because \int x^n dx = \frac{x^{n+1}}{n+1} \quad \therefore \int x^1 dx = \frac{x^{1+1}}{1+1} \right\}$

(12) Integrating factor of linear differential equation $\frac{dx}{dy} + yx = y^2$ is

A) $e^{y^2/2}$ B) $e^{x^2/2}$ C) y^2 D) $e^{\log y}$

Sol^n ⇒ Ans: A)

क): $\frac{dx}{dy} + \underset{P}{y}\,x = \underset{Q}{y^2}$

ख): $IF = e^{\int P\,dy}$

$= e^{\int y\,dy}$ $\left\{ \because \int x^n dx = \frac{x^{n+1}}{n+1} \right\}$

$= \boxed{e^{y^2/2}}$

(13) The differential equation $\frac{dy}{dx} + \frac{y}{1+x^2} = x^2$ has integrating factor

A) $e^{1/(1+y^2)}$ B) $e^{\tan^{-1}x}$ C) $e^{1/(1+x^2)}$ D) $e^{\tan^{-1}y}$

$\boxed{\text{पाठ-ee}}$

Sol^n ⇒ Ans: B)

क): $\dfrac{dy}{dx} + \left(\dfrac{1}{1+x^2}\right)y = x^2$

$\uparrow$ P $\qquad$ $\uparrow$ Q

ख): $IF = e^{\int P\,dx}$

$$= e^{\int \frac{1}{(1+x^2)}dx} \qquad \left\{\because \int \frac{1}{1+x^2}dx = \tan^{-1}x\right\}$$

$$= \boxed{e^{\tan^{-1}x}}$$

(14) The differential equation $\dfrac{dx}{dy} + \dfrac{x}{1+y^2} = y^2$ has integrating factor

A) $e^{1/(1+y^2)}$ B) $e^{\tan^{-1}x}$ C) $e^{1/(1+x^2)}$ D) $e^{\tan^{-1}y}$

Sol^n ⇒ Ans: D)

क): $\dfrac{dx}{dy} + \left(\dfrac{1}{1+y^2}\right)x = y^2$

$\uparrow$ P $\qquad$ $\uparrow$ Q

ख): $IF = e^{\int P\,dy} = e^{\int \frac{1}{1+y^2}dy}$

$$= \boxed{e^{\tan^{-1}y}}$$

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पान - 900

(15) The differential equation $\frac{dy}{dx} + \sqrt{x}\, y = x^3$ has integrating factor

A) $e^{\frac{2}{3} x\sqrt{x}}$ B) $e^{\frac{1}{3} x\sqrt{x}}$ C) $e^{\sqrt{x}}$ D) e^{-x}

Solⁿ ⇒ Solution : A)

क):

$$\frac{dy}{dx} + \sqrt{x}\, y = x^3$$

$\sqrt{x} \uparrow P$, $x^3 \uparrow Q$

ख):

$$IF = e^{\int P\,dx} = e^{\int \sqrt{x}\,dx}$$

$$= e^{\int x^{1/2} dx} \qquad \left\{\because \sqrt{a} = a^{1/2}\right\}$$

$$= e^{\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}} \qquad \left\{\because \int x^n dx = \frac{x^{n+1}}{n+1}\right\}$$

$$= e^{\frac{x^{\frac{3}{2}}}{\frac{3}{2}}} \qquad \left\{\because \frac{1}{2}+1 = \frac{3}{2}\right\}$$

$$= e^{\frac{2}{3} x^{3/2}}$$

$$= e^{\frac{2}{3} \cdot x^1 \cdot x^{\frac{1}{2}}} \qquad \left\{\because a^{m+n} = a^m \cdot a^n\right\}$$

$$= \boxed{e^{\frac{2}{3} x\sqrt{x}}} \qquad \left\{\because a^{1/2} = \sqrt{a}\right\}$$

पान - 909

(16) The linear differential equation $(1+y^2)+(x-e^{-\tan^{-1}y})\frac{dy}{dx}=0$ has integrating factor

A) $e^{\tan^{-1}x}$ B) $e^{1/(1+y^2)}$ ✓C) $e^{\tan^{-1}y}$ D) e^{2y}

Sol^n ⇒ Ans: c)

S):
$$(1+y^2)+(x-e^{-\tan^{-1}y})\frac{dy}{dx}=0$$

$$(1+y^2)=-(x-e^{-\tan^{-1}y})\frac{dy}{dx}$$

$$(1+y^2)\frac{dx}{dy}=-x+e^{-\tan^{-1}y}$$

$$\frac{dx}{dy}=\frac{-x}{1+y^2}+\frac{e^{-\tan^{-1}y}}{1+y^2}$$

$$\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{e^{-\tan^{-1}y}}{1+y^2}$$

$$\frac{dx}{dy}+\underbrace{\left(\frac{1}{1+y^2}\right)}_{P}x=\underbrace{\frac{e^{-\tan^{-1}y}}{1+y^2}}_{Q}$$

2S):
$$IF=e^{\int P\,dy}=e^{\int\frac{1}{1+y^2}dy}$$

$$=\boxed{e^{\tan^{-1}y}}$$

17. The linear differential equation $(1-x^2)\dfrac{dy}{dx} = 1+xy$ has integrating factor

A) $\sqrt{1-x^2}$ B) $\dfrac{1}{\sqrt{1-x^2}}$ C) $e^{\tan^{-1}x}$ D) $x\sqrt{1-x^2}$

Solⁿ ⇒ Ans: A)

१):

$$(1-x^2)\frac{dy}{dx} = (1+xy)$$

$$(1-x^2)\frac{dy}{dx} - xy = 1$$

..... devide by $(1-x^2)$

$$\frac{\cancel{(1-x^2)}}{\cancel{(1-x^2)}}\frac{dy}{dx} - \frac{xy}{(1-x^2)} = \frac{1}{(1-x^2)}$$

$$\frac{dy}{dx} - \underbrace{\frac{x}{(1-x^2)}}_{P}y = \underbrace{\frac{1}{1-x^2}}_{Q}$$

२):

$$IF = e^{\int P\,dx} = e^{\int \frac{-x}{1-x^2}dx}$$

$$= e^{\frac{1}{2}\int \frac{-2x}{1-x^2}dx}$$

$$= e^{\frac{1}{2}\log(1-x^2)} \qquad \left\{\because \int \frac{f'(x)}{f(x)}dx = \log f(x)\right\}$$

$$= \cancel{e}^{\cancel{\log}(1-x^2)^{1/2}}$$

$$= (1-x^2)^{1/2} \qquad \left\{\because a^{1/2} = \sqrt{a}\right\}$$

$$= \boxed{\sqrt{1-x^2}}$$

(18) The linear differential equation $(2y+x^2)\,dx = x\,dy$ has integrating factor

A) $\frac{1}{x}$ ✓B) $\frac{1}{x^2}$ C) x D) $\frac{1}{y^2}$

Sol^n ⇒ Ans: B)

क): $(2y+x^2)\,dx = x\,dy$

$$\frac{2y+x^2}{x} = \frac{dy}{dx}$$

$$\frac{2y}{x} + \frac{\cancel{x^2}}{\cancel{x}} = \frac{dy}{dx}$$

$$\frac{2y}{x} + x = \frac{dy}{dx}$$

$$\frac{dy}{dx} - \frac{2y}{x} = x$$

$$\frac{dy}{dx} \underbrace{-\left(\frac{2}{x}\right)}_{P} y = \underbrace{x}_{Q}$$

ख):

$$IF = e^{\int P\,dx} = e^{\int -\frac{2}{x}\,dx}$$

$$= e^{-2\int \frac{1}{x}\,dx}$$

$$= e^{-2\log x}$$

$$= \cancel{e}^{\cancel{\log}\, x^{-2}}$$

$$= x^{-2}$$

$$= \boxed{1/x^2}$$

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पान-१०४

(19) The linear differential equation $y^2 + \left(x - \frac{1}{y}\right)\frac{dy}{dx} = 0$ has integrating factor

A) e^x B) e^y C) $1/y^2$ D) $e^{-1/y}$

Sol^n ⇒

Ans: D)

$$y^2 + \left(x - \frac{1}{y}\right)\frac{dy}{dx} = 0$$

$$y^2 = -\left(x - \frac{1}{y}\right)\frac{dy}{dx}$$

$$y^2 \frac{dx}{dy} = -x + \frac{1}{y}$$

$$y^2 \frac{dx}{dy} + x = \frac{1}{y}$$

.....devide by y^2:

$$\frac{dx}{dy} + \underbrace{\frac{1}{y^2}}_{P} x = \underbrace{\frac{1}{y^3}}_{Q}$$

$$IF = e^{\int P\,dy} = e^{\int \frac{1}{y^2} dy}$$

$$= \boxed{e^{-\frac{1}{y}}}$$

$$\left\{ \because \int \frac{1}{x^2} dx = \frac{-1}{x} \right\}$$

(20) The differential equation $\frac{dy}{dx} + y\cot x = \sin 2x$ has integrating factor

A) $\cos x$ B) $e^{\cot x}$ C) $\sin x$ D) $\sec x$

Sol^n ⇒ Ans: c)

१): $\frac{dy}{dx} + y \underset{\uparrow\, P}{\cot x} = \underset{\uparrow\, Q}{\sin 2x}$

२): $IF = e^{\int P\,dx} = e^{\int \cot x\, dx}$

$= e^{\log \sin x}$

$= \boxed{\sin x}$

Formula: $\{\because \int \cot x\, dx = \log \sin x\}$

(21) The differential equation $\cos x \cdot \frac{dy}{dx} + y = \sin x$ has integrating factor

A) $e^{\sec x}$ B) $\operatorname{cosec} x - \cot x$

✓C) $\sec x + \tan x$ D) $\sec x - \tan x$

Sol^n ⇒ Ans: c)

१): $\cos x \cdot \frac{dy}{dx} + y = \sin x$

.....devide by $\cos x$:

$\frac{\cos x}{\cos x} \cdot \frac{dy}{dx} + \frac{y}{\cos x} = \frac{\sin x}{\cos x}$

$\frac{dy}{dx} + \frac{1}{\cos x} y = \tan x$ $\{\because \sec x = \frac{1}{\cos x}\}$

$\frac{dy}{dx} + \underset{\uparrow\, P}{\sec x} \cdot y = \underset{\uparrow\, Q}{\tan x}$

२) $IF = e^{\int P\,dx} = e^{\int \sec x\, dx}$

$= e^{\log(\sec x + \tan x)}$

$= \boxed{\sec x + \tan x}$

Formula: $\{\because \int \sec x\, dx = \log(\sec x + \tan x)\}$

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पाठ -१०६

(25) The differential equation $\tan y \cdot \frac{dy}{dx} + \tan x = \cos^2 x \cdot \cos y$ reduces to linear differential equation

A) $\frac{du}{dx} - \tan(x)u = -\cos^2 x$ where $\sec y = u$

✓B) $\frac{du}{dx} + (\tan x)u = \cos^2 x$ where $\sec y = u$

C) $\frac{du}{dx} + (\cot x)u = \cos^2 x$ where $\sec y = u$

D) None

Solⁿ ⇒ Ans: B)

क): $\tan y \cdot \frac{dy}{dx} + \tan x = \cos^2 x \cdot \cos y$

....devide by cosy:

$$\tan y \cdot \frac{1}{\cos y} \frac{dy}{dx} + \tan x \cdot \frac{1}{\cos y} = \cos^2 x \cdot \frac{\cancel{\cos y}}{\cancel{\cos y}}$$

$$\tan y \cdot \sec y \frac{dy}{dx} + \tan x \cdot \sec y = \cos^2 x \quad \ldots\ldots (1)$$

ख): Put: $\sec y = u$

Taking derivative:

$$\sec y \cdot \tan y \cdot \frac{dy}{dx} = \frac{du}{dx}$$

put in eqⁿ (1):

ग): $$\boxed{\frac{du}{dx} + \tan x \cdot u = \cos^2 x}$$

{∴ Formula: $\frac{1}{\cos x} = \sec x$, $\frac{\partial}{\partial x}(\sec x) = \sec x \cdot \tan x$ }

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पान -१०७

(26) The differential equation $\sin y \frac{dy}{dx} - 2\cos x \cdot \cos y$
$= \cos x \cdot \sin^2 x$
reduces to linear differential equation

A) $\frac{du}{dx} + (\cos x)u = \cos x \cdot \sin^2 x$, where $\cos y = u$

B) $\frac{du}{dx} - (2\cos x)u = -\cos x \cdot \sin^2 x$, where $\cos y = u$

✓C) $\frac{du}{dx} + (2\cos x)u = \cos x \cdot \sin^2 x$, where $\cos y = u$

D) None of these

Sol^n ⇒ Ans: (C)

क):

$$\sin y \frac{dy}{dx} - 2\cos x \cdot \cos y = -\cos x \cdot \sin^2 x \quad \cdots\cdots (1)$$

Put: $\cos y = u$

Taking derivative:

$$-\sin y \frac{dy}{dx} = \frac{du}{dx}$$

$$\sin y \frac{dy}{dx} = -\frac{du}{dx}$$

put in (1):

ख):

$$-\frac{du}{dx} - 2u \cdot \cos x = -\cos x \cdot \sin^2 x$$

$$-\frac{du}{dx} - (2\cos x)u = -\cos x \cdot \sin^2 x$$

changing sign:

ग):

$$\boxed{\frac{du}{dx} + (2\cos x)u = \cos x \cdot \sin^2 x}$$

(27) The value of α so that $e^{\alpha y^2}$ is an integrating factor of linear differential equation

$$\frac{dx}{dy} + xy = e^{-y^2/2}$$ is

A) -1 B) $-1/2$ C) 1 ✓D) $1/2$

Sol^n $\Rightarrow$ Ans: D)

क): $IF = e^{\alpha y^2}$ ......(given) ....(1)

ख): $\frac{dx}{dy} + \underset{\substack{\uparrow \\ P}}{(y)}\, x = \underset{\substack{\uparrow \\ Q}}{e^{-y^2/2}}$

$$IF = e^{\int P\,dy}$$

$$= e^{\int y\,dy} \qquad \left\{ \because \int x^n dx = \frac{x^{n+1}}{n+1} \right\}$$

$$IF = e^{y^2/2} \quad \text{....(2)}$$

ग): $IF = IF$

$$\cancel{e}^{\alpha y^2} = \cancel{e}^{y^2/2}$$

$$\alpha \cancel{y^2} = \cancel{y^2}/2$$

$$\boxed{\alpha = 1/2}$$

पाद-90e

(28) The value of α so that $e^{\alpha x^2}$ is an integrating factor of linear differential equation $\frac{dy}{dx} - xy = x$ is

A) $-\frac{1}{2}$ B) $\frac{1}{2}$ C) 1 D) -2

Sol^n ⇒ Ans: A)

क): $IF = e^{\alpha x^2}$ ....(given) ....①

ख):

$$\frac{dy}{dx} - (x)y = x$$

$\uparrow$ P $\uparrow$ Q

$$IF = e^{\int P\,dx}$$

$$= e^{\int -x\,dx} \qquad \left\{\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right\}$$

$$IF = e^{-x^2/2} \quad ....②$$

ग) $IF = IF$

$$\cancel{e}^{\alpha x^2} = \cancel{e}^{-\frac{x^2}{2}}$$

$$\alpha \cancel{x^2} = -\cancel{x^2}/2$$

$$\boxed{\alpha = \frac{-1}{2}}$$

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पान - 990

(29) If I_1, I_2 are integrating factors of the equation $x\frac{dy}{dx}+2y=1$ and $x\frac{dy}{dx}-2y=1$ then true relation is

A) $I_1=-I_2$ ✓B) $I_1I_2=1$

C) $I_1=x^2I_2$ D) $I_1I_2=x^2$

Soln ⇒ Ans: B)

क): $x\frac{dy}{dx}+2y=1$

$$\frac{dy}{dx}+\underbrace{\frac{2}{x}}_{P}\cdot y=\underbrace{\frac{1}{x}}_{Q}$$

$$IF = e^{\int P.dx}$$
$$= e^{\int 2/x\, dx}$$
$$= e^{2\int 1/x\, dx}$$
$$= e^{2\log x}$$
$$= \cancel{e}^{\cancel{\log} x^2}$$

$\therefore I_1 = IF_1 = x^2$

ख): $x\frac{dy}{dx}-2y=1$

$$\frac{dy}{dx}-\underbrace{\frac{2}{x}}_{P}\cdot y=\underbrace{\frac{1}{x}}_{Q}$$

$$IF = e^{\int P.dx}$$
$$= e^{\int -2/x\, dx}$$
$$= e^{-2\int 1/x\, dx}$$
$$= e^{-2\log x}$$
$$= \cancel{e}^{\cancel{\log} x^{-2}}$$

$\therefore I_2 = IF_2 = x^{-2}$

ग): Now, $(IF_1)\times(IF_2) = x^2\cdot x^{-2}$
$$= x^{2-2}$$
$$= x^0 = \boxed{1}$$

पाठ - 999

(30) The general solution of $\frac{dy}{dx} + \frac{1}{1-x} y = -x(1-x)$ with integrating factors $\frac{1}{1-x}$ is

A) $y = -\frac{x^2}{2}\left(\frac{1}{1-x}\right) + c$ B) $y \frac{1}{1-x} = x^2 + c$

C) $y \frac{1}{1-x} = \frac{x^2}{2} + c$ ✓D) $y \frac{1}{1-x} = -\frac{x^2}{2} + c$

Sol^n ⇒ Ans: D)

ऊ): $\frac{dy}{dx} + \frac{1}{1-x} y = -x(1-x)$ and $IF = \frac{1}{1-x}$

$\uparrow P$ $\uparrow Q$

२व): GS: $y(IF) = \int Q(IF)\, dx + c$

$$y\left[\frac{1}{1-x}\right] = \int -x(\cancel{1-x}) \times \frac{1}{\cancel{1-x}}\, dx + c$$

$$y \frac{1}{1-x} = -\int x\, dx + c \qquad \left\{\because \int x^n dx = \frac{x^{n+1}}{n+1}\right\}$$

$$\boxed{y \frac{1}{1-x} = -\frac{x^2}{2} + c}$$

(31) The general solution of $\frac{dy}{dx} + \frac{3}{x} y = \frac{e^x}{x^2}$ with integrating factor x^3 is

A) $yx^3 = (x+1)e^x + c$ ✓B) $yx^3 = (x-1)e^x + c$

C) $xy^3 = (x-1)e^x + c$ D) None

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पाठ - 992

Sol^n ⇒

Ans: B)

क): $\frac{dy}{dx} + \frac{3}{x} y = \frac{e^x}{x^2}$ and $IF = x^3$

$\uparrow P$ $\uparrow Q$

ख): GS:

$$y(IF) = \int Q(IF)\,dx + c$$

$$yx^3 = \int \frac{e^x}{\cancel{x^2}} (x^{\cancel{3}})\,dx + c$$

$$yx^3 = \int \underset{u}{x}\,\underset{v}{e^x}\,dx + c$$

LIATE

$\underset{u}{x}$ $\underset{v}{e^x}$

$$\left\{ \because \int uv = u\int v - \int \left[\int v \cdot u' \right] \right\}$$

ग): $yx^3 = x\int e^x dx - \int \left[\int e^x dx \times \frac{d}{dx}(x) \right] + c$

$$yx^3 = xe^x - \int [e^x (1)]\,dx + c$$

$$yx^3 = xe^x - \int e^x dx + c$$

$$yx^3 = xe^x - e^x + c$$

$$yx^3 = e^x(x-1) + c$$

$$\boxed{yx^3 = (x-1)e^x + c}$$

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पाठ - 193

(32) The general solution of $\frac{dy}{dx} + (\cot x)y = \sin 2x$ with integrating factor $\sin x$ is

A) $y \sin x = \frac{2}{3}\sin^2 x + c$ B) $y \sin x = \sin^2 x + c$

✓C) $y \sin x = \frac{2}{3}\sin^3 x + c$ D) None

Solⁿ ⇒ Ans: c)

क): $\frac{dy}{dx} + \underset{\substack{\uparrow \\ P}}{(\cot x)}y = \underset{\substack{\uparrow \\ Q}}{\sin 2x}$ and IF = $\sin x$

ख): GS: $y(IF) = \int Q\,(IF)\,dx + C$

$y(\sin x) = \int \sin 2x\,(\sin x)\,dx + C$

$y(\sin x) = \int 2\cdot\sin x\cdot\cos x\cdot(\sin x)\,dx + c$

{∵ Formula: $\sin 2\theta = 2\sin\theta\cos\theta$ }

$y(\sin x) = \int 2\sin^2 x\cdot\cos x\,dx + c$

ग): Put: $\sin x = t$

$\cos x\,dx = dt$ ....(derivative)

$y(\sin x) = \int 2t^2\,dt + c$

$= 2\frac{t^3}{3} + c$ $\left\{\because \int x^n = \frac{x^{n+1}}{n+1}\right\}$

Reput: $t = \sin x$

पाठ - ११४

$$y(\sin x) = \frac{2(\sin x)^3}{3} + c$$

$$\boxed{(y\sin x) = \frac{2}{3}\sin^3 x + c}$$

(34) The general solution of $\frac{dy}{dx} + \left(\tan x + \frac{1}{x}\right)y = \frac{1}{x}\sec x$ with integrating factor $x\cdot\sec x$ is

✓A) $y(x\sec x) = \tan x + c$ B) $y(x\sec x) = x^3/3 + c$
C) $x(y\sec y) = \tan x + c$ D) None

Sol^n ⇒ Ans: A)

क): $\frac{dy}{dx} + \left[\tan x + \frac{1}{x}\right]y = \frac{1}{x}\sec x$ and $IF = x\sec x$
P Q

ख): GS: $y(IF) = \int Q(IF)\,dx + c$

$$y(x\sec x) = \int\left[\frac{1}{\cancel{x}}\sec x\right]\left[\cancel{x}\sec x\right]dx + c$$

$$xy\cdot\sec x = \int \sec^2 x\,dx + c$$

$$xy\sec x = \tan x + c$$

$$\boxed{y(x\sec x) = \tan x + c}$$

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पान-११५

(35) The general solution of $\frac{dy}{dx} + \frac{3}{x}y = x^2$ with IF is x^3 is

✓A) $yx^3 = \frac{x^6}{6} + c$ B) $yx^3 = \frac{x^2}{2} + c$

C) $yx^3 = \log x + c$ D) None

Sol^n ⇒ Ans: A)

क): $\frac{dy}{dx} + \frac{3}{x}y = x^2$ and $IF = x^2$

$\frac{3}{x} \downarrow P$, $x^2 \downarrow Q$

ख): पिछले गणित के तरहाही कीजिए। आसान है।

(36) The general solution of $\frac{dy}{dx} + \frac{2}{x}y = \frac{1}{x^3}$ with IF x^2 is

A) $yx^2 = x^2/2 + c$ ✓B) $yx^2 = \log x + c$

C) $yx^2 = x^6/6 + c$ D) None

Sol^n ⇒ Ans: B)

क): गणित क्रमांक 34 की तरहही किजिएगा।

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पाठ - 996

(37) The general solution of $\frac{dy}{dx} + (1+2x)y = e^{-x^2}$ with $IF = e^{x+x^2}$ is

A) $ye^{x+x^2} = \frac{e^{x+x^2}}{2} + c$ B) $ye^{x+x^2} = e^{x^2} + c$

✓C) $ye^{x+x^2} = e^{x} + c$ D) None

Sol^n ⇒ Ans : c)

क): $\frac{dy}{dx} + \underset{\uparrow P}{(1+2x)}y = \underset{\uparrow Q}{e^{-x^2}}$ and $IF = e^{x+x^2}$

ख): GS: $y(IF) = \int Q\,(IF)\,dx + c$

$$y(e^{x+x^2}) = \int e^{-x^2}(e^{x+x^2})\,dx + c$$

$$= \int \cancel{e^{-x^2}} \cdot e^{x} \cdot \cancel{e^{x^2}}\,dx + c$$

$\{\because a^{m+n} = a^m \cdot a^n\}$

$$= \int e^{x}\,dx + c \qquad \{\because \int e^x dx = e^x\}$$

$$\boxed{y(e^{x+x^2}) = e^{x} + c}$$

(38) The general solution of $\frac{dx}{dy} + \frac{1}{1+y^2}x = \frac{e^{-\tan^{-1}y}}{1+y^2}$ with $IF = e^{\tan^{-1}y}$ is

✓A) $x\,e^{\tan^{-1}y} = \tan^{-1}y + c$ B) $y\,e^{\tan^{-1}y} = \tan^{-1}y + c$

C) $e^{\tan^{-1}y} = \tan^{-1}y + c$ D) No one

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पान-१९६

Sol^n ⇒

Ans: B)

क):
$$\frac{dx}{dy} + \underset{\uparrow P}{\frac{1}{1+y^2}}\, x = \underset{\uparrow Q}{\frac{e^{-\tan^{-1}y}}{1+y^2}} \quad \text{and} \quad IF = e^{\tan^{-1}y}$$

ख): GS:
$$x(IF) = \int Q(IF)\,dy + c$$

$$x\left(e^{\tan^{-1}y}\right) = \int \frac{e^{-\tan^{-1}y}}{1+y^2}\left[e^{\tan^{-1}y}\right] dy + c$$

$$= \int \frac{1}{1+y^2}\, dy + c$$

$$x \cdot e^{\tan^{-1}y} = \tan^{-1}y + c$$

$$\therefore \boxed{x \cdot e^{\tan^{-1}y} = \tan^{-1}y + c}$$

(39) The general solution of $\frac{dx}{dy} + (\sec y)x = \frac{2y\cos y}{1+\sin y}$ with integrating factor $(\sec y + \tan y)$ is

A) $y(\sec y + \tan y) = y^2 + c^2$ B) $x(\sec y + \tan y) = y^2/2 + c$

✓C) $x(\sec y + \tan y) = y^2 + c$ D) None

Sol^n ⇒ Ans: C)

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पाठ-99L

क):

$$\frac{dx}{dy} + \underset{\uparrow P}{(\sec y)}x = \underset{\uparrow Q}{\frac{2y\cos y}{1+\sin y}} \quad \text{and } IF = \sec y + \tan y$$

ख):

GS:

$$x(IF) = \int Q(IF)\,dy + C$$

$$x(\sec y + \tan y) = \int \frac{2y\cos y}{1+\sin y}(\sec y + \tan y)\,dy + C$$

devide by cosy:

$$= \int \frac{\frac{2y\cancel{\cos y}}{\cancel{\cos y}}}{\frac{1+\sin y}{\cos y}}(\sec y + \tan y)\,dy + C$$

$\left\{\because \frac{1}{\cos x} = \sec x\right\}$

$\left\{\because \frac{\sin x}{\cos x} = \tan x\right\}$

$$= \int \frac{2y}{\left[\frac{1}{\cos y} + \frac{\sin y}{\cos y}\right]}(\sec y + \tan y)\,dy + C$$

$$= \int \frac{2y}{\cancel{(\sec y + \tan y)}}\cancel{(\sec y + \tan y)}\,dy + C$$

$$= 2\int y\,dy + C \qquad \left\{\because \int x^n dx = \frac{x^{n+1}}{n+1}\right\}$$

$$= \cancel{2}\,y^2/\cancel{2} + C$$

$$\boxed{x(\sec y + \tan y) = y^2 + C}$$

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पाठ - 19e

(40) The general solution of $\frac{dx}{dy} - \frac{2}{y}x = y^2 e^{-y}$ with integrating factor $\frac{1}{y^2}$ is

A) $x \frac{1}{y^2} = -e^{-y} + c$ B) $x \frac{1}{y^2} = e^{-y} + c$

c) $y \frac{1}{x^2} = -e^{-y} + c$ D) Not any

Solⁿ ⇒ Ans: A)

S): $\frac{dx}{dy} - \frac{2}{y}x = y^2 e^{-y}$ and IF $= \frac{1}{y^2}$

$\frac{2}{y}$ ↑ P, $y^2 e^{-y}$ ↑ Q

S₂): General Solution:

$$x(IF) = \int Q(IF)\,dy + c$$

$$x\left(\frac{1}{y^2}\right) = \int \cancel{y^2} e^{-y} \left(\frac{1}{\cancel{y^2}}\right) dy + c$$

$$x \frac{1}{y^2} = \int e^{-y}\,dy + c \qquad \left\{\because \int e^{-ax}dx = \frac{e^{-ax}}{-a}\right\}$$

$$= \frac{e^{-y}}{-1} + c$$

$$= -e^{-y} + c$$

$$\boxed{x \frac{1}{y^2} = -e^{-y} + c}$$

∴ Ans: A):

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F.Y. B. Tech.
Mathematics-II (SCI105A)
Practice Problems

Ordinary Differential Equations

1. Form a differential equation whose general solution is

$i)\ y = ae^{-2x} + be^{-3x}$ **(Ans : $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$)**

$ii)\ y = e^{x}(Acosx + Bsinx)$ **(Ans : $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$)**

Solve the differential equations-

1. $\frac{dy}{dx} = \frac{x\,sinx}{2e^{y}sinhy}$ **(Ans: $\frac{e^{2y}}{2} - y + xcosx - sinx = C$)**
2. $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$ **(Ans : $\log[1 + \tan(\frac{x+y}{2})] - x = C$)**
3. $\frac{dy}{dx} = \frac{x^3-y^3}{yx^2}$ **(Ans: $cy = e^{\frac{-x^3}{3y^3}}$)**
4. $(x + y\cot\frac{x}{y})dy - y\,dx = 0$ (Ans : $y\cos\frac{x}{y} = C$)
5. $\frac{dy}{dx} = \frac{\tan y - 2xy - y}{x^2 - x\tan^2 y + \sec^2 y}$ (Ans : $x.\tan y - xy - x^2y - \tan y = C$)
6. $ylogy\,dx + (x - \log y)dy = 0$ **(Ans : $2x\log y - (\log y)^2 = C$)**
7. $(2y + y^4)dx + (2y^4 + xy^3 - 4x)dy = 0$ **(Ans : $xy + \frac{x}{y^2} + y^2 = C$)**
8. $xcos\,x\frac{dy}{dx} + (\cos x - xsin\,x)y = 1$ **(Ans : $xycosx - x = C$)**
9. $\frac{dy}{dx} - xy = -y^3e^{x^2}$ **(Ans : $\frac{e^{x^2}}{y^2} = 2x + C$)**
10. $(y - 2x^3)dx - x(1 - xy)dx = 0$ **(Ans: $\frac{y}{x} + x^2 - \frac{y^2}{2} = C$)**
11. $(x^2y - 2xy^2)dx = (x^3 - 3x^2y)dy$ **Ans : $\frac{x}{y} - 2logx + 3logy = C$**
12. $ye^{y}dx = (y^3 + 2xe^{y})dy$ **(Ans : $\frac{x}{y^2} + e^{-y} = C$)**
13. $sin\,y\frac{dy}{dx} - cosx(2cosy - \sin^2 x)\,y = 0$
    (Ans : $4cosy = 2\sin^2 x + 2sinx = 1 = Ce^{-2sinx}$)
14. $\cos y - xsin\,y\frac{dy}{dx} = \sec^2 x$ **(Ans: $xcosy = tanx + C$)**
15. $\left(xy^2 - e^{\frac{1}{x^3}}\right)dx + x^2\,y\,dy = 0$ **(Ans : $\frac{3y^2}{x^2} - 2e^{-x^{-3}} = C$)**

APPLICATIONS OF DIFFERENTIAL EQUATIONS

Orthogonal Trajectories

13. Find the orthogonal trajectories of the family of $y^2 = 4ax$. **(Ans : $2x^2 + y^2 = C$)**

14. Find the orthogonal trajectory of the family of ellipses $\frac{1}{2}x^2 + y^2 = C$, where C > 0

(Ans : $x^2 = ky$)

[Ref: Kreyszig, page-36]

15. Find the orthogonal trajectory of the family of r = a cosθ. (Ans : $r = Csin\theta$)

16. Find the orthogonal trajectory of the family of r = a (1- cosθ). (Ans: r = C(1+cosθ))

17. Find the orthogonal trajectory of the family of $e^x + e^{-y} = c$. (Ans: $e^y - e^{-x} = C$)

Electric Circuits

20. An electric circuit contains an inductance of 0.5 henry and a resistance of 10 ohms in series with electro motive force of 20 volts. Find the current at any time t, it is zero at t=0. (Ans : $\frac{1}{5}(1 - e^{-200t})$)

21. A circuit consist of resistance R ohms and condenser C farads connected to constant electromotive force E, if $\frac{q}{C}$ is the voltage of condenser at time t after closing the circuit. Show that the voltage at time t, is $E\left(1 - e^{-\frac{t}{RC}}\right)$. Also find current flowing into the circuit.

22. The charge Q on the plate of a condenser of capacity $'C'$ charged through a resistance $'R'$ by steady voltage $'V'$ satisfies the differential equation $R\frac{dQ}{dt} + \frac{Q}{C} = V$. If $Q = 0$ at $t = 0$ then show that $Q = CV\left[1 - e^{-t/RC}\right]$. Find the current flowing into the plate. (Ans: $i = \frac{V}{R}\, e^{-\frac{t}{RC}}$)

23.Find the current i in the circuit having resistance R and condenser of capacity C in series with emf E sin ωt.

24.The equation of L-R circuit is given by L$\frac{dI}{dt}$. + RI = 10 sin t .If I=0, at t = 0 , express I as a function of t . (Ans: $I = \frac{10}{\sqrt{R^2+L^2}}[\sin(t - \emptyset) + sin\emptyset\, e^{\frac{-Rt}{L}}$)

Heat Conduction

23. A steam pipe 20 cm in diameter is protected with a covering 6 cm thick for which the coefficient of thermal conductivity is k = 0.0003 cal/cm in steady state. Find the heat lost per hour through a meter length of the pipe, if the inner surface of the pipe is at 200 °C and the outer surface of the covering is at 30 °C. (Ans : q=245443.3861)

24. A long hollow pipe has an inner diameter of 10 cm and outer diameter of 20 cm the inner surface is kept at 200 °C and outer surface at 50 °C. The thermal conductivity is 0.12. How much heat is lost per minute from a portion of the pipe 20 m long and also find temperature at distance x=7.5 cm from the centre of pipe. (Ans: T=113 C)

Tracing of Curve

Trace the following curves

1) $y^2(2a - x) = x^3$
2) $(x^2 + y^2)x = (x^2 - y^2)$
3) $xy^2 = a^2(a - x)$
4) $x^2y^2 = a^2(y^2 - x^2)$
5) $(x^2 + a^2)y^2 = a^2x^2$
6) $(x^2 + 4a^2)y = 8a^3$
7) $x = a(t + sint)\,, y = a(1 - cost)$
8) $x = a(t - sint)\,, y = a(1 - cost)$
9) $x = a(t + sint)\,, y = a(1 + cost)$
10) $r^2 = a^2cos2\theta$
11) $r = a\,cos2\theta$
12) $r = a\,cos5\theta$
13) $r = a\,(1 - cos\theta)$
14) $r = a\,sin2\theta$
15) $r = 2\,sin5\theta$

Reduction Formulae, Beta and Gamma

1. Evaluate $\int_0^{\pi} x\sin^5 x\cos^8 x\,dx$ **Ans.** $\frac{8\pi}{1287}$
2. Evaluate $\int_0^{2a} x^3(2ax-x^2)^{\frac{3}{2}}\,dx$ **Ans.** $\frac{9\pi a^7}{16}$
3. Find the reduction formula for $\int_0^{\frac{\pi}{3}} cos^n x\,dx$ and using it evaluate $\int_0^{\frac{\pi}{3}} \cos^6 x\,dx$.

   Ans. $I_n = \frac{\sqrt{3}}{n2^n} + \frac{n-1}{n} I_{n-2}\,, \frac{3\sqrt{3}}{32} + \frac{5\pi}{48}$
4. If $I_n = \int_0^{\frac{\pi}{4}} \frac{\sin(2n-1)x}{\sin x}\,dx$ then prove that $n(I_{n+1}-I_n) = \sin\frac{n\pi}{2}$ and hence find I_3.

   Ans. $1+\frac{\pi}{4}$
5. If $I_n = \int_0^{\infty} e^{-x}\sin^n x\,dx,$ Obtain the relation between $I_n\ and\ I_{n-2}$.

   Ans. $I_n = \frac{n(n-1)}{n^2+1} I_{n-2}$
6. Evaluate $\int_0^{\infty} x^7 e^{-2x^2} dx$ **Ans.** $3/16$
7. Evaluate $\int_0^{\infty} 3^{-4x^2} dx$ **Ans.** $\frac{\sqrt{\pi}}{4\sqrt{\log 3}}$
8. Evaluate $\int_0^{\infty} \frac{x^4}{4^x}\,dx$ **Ans.** $\frac{24}{(\log 4)^5}$
9. Evaluate $\int_0^1 \frac{dx}{\sqrt{-\log x}}$ **Ans.** $\sqrt{\pi}$
10. Evaluate $\int_0^1 x^3(\log x)^4 dx$ **Ans.** $\frac{3}{128}$
11. Show that $\int_0^1 \frac{dx}{\sqrt[3]{1-x^3}} = \frac{2\pi}{3\sqrt{3}}$
12. Show that $\int_0^{\infty} \frac{x^6-x^3}{(1+x^3)^5} x^2 dx = 0$
13. Evaluate $\int_3^7 (x-3)^{\frac{1}{4}}(7-x)^{\frac{1}{4}}\ dx$ **Ans.** $\frac{2}{3\sqrt{3}}\left(\gamma\left(\frac{1}{4}\right)\right)^2$
14. Prove that $\int_0^{\infty} \frac{dx}{1+x^4} = \frac{\pi}{2\sqrt{2}}$
15. Show that $\int_0^{\frac{\pi}{2}} \sqrt{\tan\theta}\,d\theta \int_0^{\frac{\pi}{2}} \sqrt{\cot\theta}\,d\theta = \frac{\pi^2}{2}$.

Differentiation Under Integral Sign (DUIS)

1. Show that $\int_0^1 \frac{x^a-1}{logx} = \log(a+1)$, $a \geq 0$
2. Show that $\int_0^\infty \frac{e^{-x}-e^{-ax}}{xsecx} dx = \frac{1}{2}\log(\frac{a^2+1}{2})$
3. Find $\int_0^\infty \frac{e^{-ax}sinmx}{x} dx$ and hence evaluate $\int_0^\infty \frac{sinx}{x} dx$
4. Prove that $\int_0^\infty \frac{1-cosax}{x^2} dx = \frac{\pi a}{2}$
5. If $y = \int_0^x f(t)sina(x-t)dt$ then show that $\frac{d^2y}{dx^2} + a^2y = af(x)$
6. If $\emptyset(a) = \int_a^{a^2} \frac{sinax}{x} dx$ then find $\frac{d\emptyset}{da}$
7. Verify the DUIS rule for the $\int_a^{a^2} logaxdx$

Error Function

1. Prove that $erfc(-x) + erfc(x) = 2$
2. Show that $\int_a^b e^{-x^2} dx = \frac{\sqrt{\pi}}{2}[\text{erf}(b) - \text{erf}(a)]$
3. Find $\frac{d}{dx} erfc(ax^n)$
4. Prove that $\text{erf}(x)$ is an odd function. Deduce that $erfc(-x) - \text{erf}(x) = 1$
5. Show that $\int_0^\infty e^{-st}\, \text{erf}(\sqrt{t})\, dt = \frac{1}{s\sqrt{s+1}}$
6. Show that $\int_0^\infty e^{-(x+a)^2} dx = \frac{\sqrt{\pi}}{2}[1 - \text{erf}(a)]$
7. Show that $\frac{d}{dt}\text{erf}(\sqrt{t}) = \frac{e^{-t}}{\sqrt{\pi t}}$ and hence evaluate $\int_0^\infty e^{-t}\, \text{erf}(\sqrt{t})\, dt$.
8. Show that $\int_0^t \text{erf}(ax)\, dx + \int_0^t erfc(ax)dx = t$.

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Mathematics-II (SCI105A)
Practice Problems

Double Integral and Applications

1. $\int_1^2 \int_{-\sqrt{2-y}}^{\sqrt{2-y}} 2x^2\, y^2 dx\, dy$ (Ans: $\frac{856}{945}$)
2. $\iint \sqrt{4x^2 - y^2}\ \ dx\, dy$ over the area of triangle $y = 0\, , y = x\ \&\ x = 1$
   Ans: $\frac{1}{3}\left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\right)$
3. $\iint_R xy\, \sqrt{1 - x - y}\, dxdy$ over the region $x \geq 0\, , y \geq 0\ \&\ x + y \leq 1$ (Ans: $\frac{16}{945}$)
4. Evaluate $\iint_R x^2 + y^2\ dxdy$ over area of triangle whose vertices are (0,1) (1 ,1) & (1, 2). (Ans: $\frac{7}{6}$)
5. Show that $\int_0^a \int_0^{a-\sqrt{a^2-y^2}} \frac{xyiog(x+a)}{(x-a)^2} dxdy = \frac{a^2}{8}(2loga + 1)$
6. Evaluate by changing the order
   I) $\int_0^1 \int_{x^2}^{2-x} xy\, dxdy$ (Ans: $\frac{3}{8}$)
   II) $\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{e^y}{(e^y+1)\sqrt{1-x^2-y^2}} dxdy$ (Ans : $\frac{\pi}{2}\log\left(\frac{e+1}{2}\right)$
7. Express the following integral as a single integral
   $\int_0^1 \int_0^y f(x, y)dxdy + \int_1^\infty \int_0^{\frac{1}{y}} f(x, y)dxdy$ (Ans:$\int_0^1 \int_x^{\frac{1}{x}} f(x, y)dxdy$)
8. Evaluate
   I) $\int_0^{\frac{a}{\sqrt{2}}} \int_0^{\sqrt{a^2-y^2}} \ln(\,x^2 + y^2)\ dxdy$ (Ans: $\frac{\pi}{2}\left[\frac{a^2}{2}loga - \frac{a^2}{4}\right]$)
   II) $\int_0^{\frac{a}{\sqrt{2}}} \int_x^{\sqrt{a^2-x^2}} \frac{x}{\sqrt{x^2+y^2}} dxdy$ (Ans:$\frac{a^2}{2}\left(1 - \frac{1}{\sqrt{2}}\right)$)
9. Evaluate over one loop of $r^2 = \ a^2cos2\theta\ \ \iint_R \frac{rdrd\theta}{\sqrt{a^2+r^2}}$ (Ans : 2a$(1 - \frac{\pi}{4})$)
10. Find area bounded by curve $y^2\ (2a - x) = x^3$ & its Asymptote (Ans :$3\pi a^2$)
11. Find area of cardioid $r = a(1 + cos\theta)$ (Ans: $\frac{3\pi a^2}{2}$)
12. Find area bounded by curve $y^2x = 16(4 - x)$ & its Asymptote. (Ans:16π)
13. Find area bounded by curves $y^2 = 4x\ \ \&\ \ 2x - y - 4 = 0$ (Ans: 9)
14. Find area bounded by curves $y^2 = x\ \ \&\ \ x^2 = -8y$ (Ans: $\frac{8}{3}$)
15. Evaluate $\int_0^{\frac{\pi}{2}} \int_0^y cos2y\, \sqrt{1 - a^2sin^2x}\ dxdy$ (Ans: $\frac{1}{3a^2}[(1 - a^2)^{\frac{3}{2}} - 1$)

Triple Integral and Applications

1. Evaluate $\iiint xyz\ dx\,dy\,dz$ Over positive octant of a sphere $x^2+y^2+z^2=a^2$.
   Ans: $\frac{a^6}{48}$
2. Evaluate $\int_0^2\int_0^y\int_{x-y}^{x+y}(x+y+z)dzdx\,dy$ Ans: 16
3. Evaluate $\iiint x^2yz\ dxdydz$ throughout the volume bounded by planes $x=0, y=0, z=0$ and $\frac{x}{2}-y+z=1$. Ans: $\frac{8}{2520}$
4. Evaluate $\iiint \frac{z^2dxdydz}{x^2+y^2+z^2}$ over the volume of sphere $x^2+y^2+z^2=2$ Ans: $\frac{8\pi\sqrt{2}}{9}$
5. Evaluate $\iiint z^2dxdydz$ over the volume bounded by the cylinder $x^2+y^2=a^2$ and the paraboloid $x^2+y^2=z$ and the plane $z=0$. Ans: $\frac{\pi a^8}{12}$
6. Evaluate $\int_0^{\pi/2}\int_0^{a\sin\theta}\int_0^{(a^2-r^2)/a} rdzdrd\theta$ Ans: $\frac{5a^3}{64}$
7. Evaluate $\iiint\sqrt{1-\frac{x^2}{4}-\frac{y^2}{9}-\frac{z^2}{64}}\ dxdydz$ throughout the volume of Ellipsoid $\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{64}=1$. Ans: $12\pi^2$
8. Evaluate $\iiint(x^2+y^2+z^2)^{3/2}\ dx\,dy\,dz$ Over the hemisphere defined by $x^2+y^2+z^2=9 \quad z\geq 0$. Ans: 243π
9. Calculate the volume of the solid bounded by the following surfaces $z=0,\ x^2+y^2=1,\ x+y+z=3$. Ans: 3π
10. Find by triple integration the volume of region bounded by paraboloid $az=x^2+y^2$ and the cylinder $x^2+y^2=r^2$. Ans: $\frac{\pi r^4}{2a}$
11. A cylindrical hole of radius 4 is bored through a sphere of radius 6. Find the volume of the remaining solid. Ans: $\frac{4\pi}{3}(20)^{3/2}$
12. Find the volume bounded by coordinate planes and the plane $lx+my+nz=1$. Ans: $\frac{1}{6mln}$
13. Find the volume bounded by the cylinder $x^2+y^2=4$ and the planes $y+z=4$ and $z=0$. Ans: 16π
14. Find the volume bounded by $y^2=x, x^2=y$ and the planes $z=0, x+y+z=1$. Ans: $\frac{1}{30}$
15. Find the volume of the solid bounded by the cylinder $x^2+y^2=4y$, the paraboloid $x^2+y^2=2z$ and the plane $z=0$ Ans: 12π

Fourier series

Q.1) Find the Fourier series expansion for $f(x) = a(2 - x)$ in the interval $0 \leq x \leq 2$

Q.2) Find Fourier series for $f(x) = x + x^2$ in the interval $-\pi < x < \pi$

and $f(x) = f(x + 2\pi)$ hence show that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} \dots$

Q.3) Find Fourier series for $f(x) = \frac{\pi^2}{12} - \frac{x^2}{4} in\ (-\pi, \pi)$.

Q.4) Obtain Fourier series expansion for $f(x) = 2 - \frac{x^2}{2}, 0 \leq x \leq 2.$

Q.5) Find the Fourier series expansion for $f(x) = \frac{1}{2}(\pi - x)$ in the interval $0 \leq x \leq 2\pi$.

Q.8) Find the Fourier series of the function $f(x) = \begin{cases} -1; & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases}$

where $(x) = f(x + 2\pi)$.

Q.9) Determine the Fourier series for the following function

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ \sin x & 0 < x < \pi \end{cases}$$

Q.10) Expand the function $f(x) = x \sin x$, as a Fourier series

in the interval $-\pi < x < \pi$. Hence deduce that $\frac{\pi - 2}{4} = \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \cdots$

Q.11) Expand the function $f(x) = x \cos x$, as a Fourier series

in the interval $-\pi < x < \pi$.

Harmonic Analysis

Q.12) Determine the first two harmonics of the Fourier series for the following values:

x^0	0	30	60	90	120	150	180	210	240	270	300	330
y	2.34	3.01	3.68	4.15	3.69	2.20	0.83	0.51	0.88	1.09	1.19	1.64

Express T as a Fourier series up to first two harmonics.

Q.15) Obtain the constant term and the coefficient of the first harmonic in the Fourier series of $f(x)$ as given in the following table

x	0	1	2	3	4	5
f(x)	9	18	24	28	26	20

1) Error function of x , erf(x) is defined as

a) $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\ du$

b) $\frac{2}{\sqrt{\pi}}\int_0^\infty e^{-x^2}\ dx$

c) $\frac{2}{\sqrt{\pi}}\int_0^x e^{x^2}\ dx$

d) $\frac{2}{\sqrt{\pi}}\int_0^u e^{-x^2}\ dx$

Ans.a) $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\ du$

2) The value of erf(∞) is

a) 0

b) ∞

c) 1

d) $2/\sqrt{\pi}$

Ans c)1

3) The value of erf(0) is

a) -1

b) ∞

c) 1

d) 0

Ans.d)0

4) The value of erfc(0) is

a) -1

b) ∞

c) 1

d) 0

Ans .c)1

5) erf(x)+erf(-x) =?

a) 2

b) 1

c) -1

d) 0

Ans.d)0

6) erf(-x)+ erfc(-x) =?

a) 2

b) 1

c) -1

d) 0

Ans.b)1

7) If erf(ax) = $\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\ du$ then $\frac{d}{dx}\text{erf}(ax) = ?$

a) $\frac{2x}{\sqrt{\pi}}e^{-a^2x^2}$
b) $\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$
c) $\frac{2x}{\sqrt{\pi}}e^{a^2x^2}$
d) $ae^{-a^2x^2}$
Ans.b) $\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$

8) If erf($\sqrt{t}$) = $\frac{2}{\sqrt{\pi}}\int_0^{\sqrt{t}} e^{-u^2}\ du \quad then\ \frac{d}{dx}\text{erf}(\sqrt{t}) = ?$
a) $\frac{e^{-t}}{2\sqrt{t}}$
b) $\frac{e^{-t^2}}{\sqrt{\pi t}}$
c) $\frac{e^{-t}}{\sqrt{\pi}}$
d) $\frac{e^{-t}}{\sqrt{\pi t}}$
Ans.d) $\frac{e^{-t}}{\sqrt{\pi t}}$

9) Error function is
a) A periodic function
b) An even function
c) A harmonic function
d) An odd function

Ans. d) An odd function

10) $\int_0^t \text{erf}(ax)\,dx + \int_0^t \text{erf}(ax)\,dx =$
a) t
b) x
c) 0
d) $\frac{t^2}{2}$

Ans.a)t

11) Erf($-x$) is equals to
a) $-\text{erf}(x)$
b) $\text{erf}(x)$
c) $-\text{erfc}(x)$
d) $\text{erfc}(x)$
ans.a) $-\text{erf}(x)$

The value of $\int_{-1}^{1}\int_{-2}^{2}\int_{-3}^{3} dx\, dy\, dz$ A. 48 B. 32 C. 12 D. 0	1	A
The value of $\int_0^1 dx \int_0^2 dy \int_1^2 x^2\, yz dz$ A. 0 B. 1 C. 3 D. 4	1	B
The value of $\int_0^1\int_0^1\int_0^1 (x^2+y^2+z^2)dxdydz$ A. 6 B. 12 C. 5	1	A

The area bounded by the parabola $y^2 = 4ax\ and\ x = a$ is A. $\frac{8a^2}{3}$ B. $\frac{8a^2}{2}$ C. 3 D. $8a^2$	1	A

f(x) = $x - x^3$, $-2 < x < 2$ *and* period is 4.the fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos n\pi x + b_n \sin n\pi x)$, then fourier coefficient a_0 is a)1 b)0 c)-2 d)-1	B
For the half range cosine series of f(x) = sinx, $0 \le x < \pi$ *and* period is 2π.the fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos nx)$ fourier coefficient a_0 is a) 4 b) 2 c) $\frac{2}{\pi}$ d) $\frac{4}{\pi}$	D
The value of b_1 in Harmonic analysis of y for the following tabulated data is: (see table below) a) 0.0989 b) 0.3464 c) 0.1732 d) 0.6932	C
The value of the constant term in the fourier series of f(x) = e^{-x} in $0 \le x \le 2\pi$, f(x + 2 π) = f(x) is a) $\frac{1}{\pi}(1 - e^{-2\pi})$ b) $\frac{1}{2\pi}(1 - e^{-2\pi})$ c) $1 - e^{-2\pi}$ d) $2(1 - e^{-2\pi})$	B
The value of the constant term in the fourier series of f(x) = x sinx in $0 \le x \le 2\pi$, is a) -2 b) 2 c) - $\frac{1}{2}$ d) -1	D

x	0	60	120	180	240	300	360
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0
Sin x	0	0.866	0.866	0	-0.866	-0.866	0

If f(x) = $e^x, -1 \le x \le 1$ then constant term of f(x) in fourier expansion is

a) $\frac{e}{2}$ b) $\frac{e-e^{-1}}{2}$ c) $\frac{e+e^{-1}}{2}$ d) $\frac{1+e}{2}$

If $f(x) = \begin{cases} -\pi, & 0 < x < \pi \\ x, & \pi < x < 2\pi \end{cases}$ is discontinuous at x= 0 then value of f(0) is

a) $-\pi$ b) 0 c) $-\frac{\pi}{2}$ d) π

If $\sum y = 42$,n=6.$\sum y\cos\theta = -8.5$, $\sum y\cos 2\theta$= -1.5,what arethe values of a_0, a_1, a_2

a)7,-2.8,-2.8 b)14,-2.8,1.5 c) 7,-1.5,-2.8 d)none of these

If f(x) = x^4 in (-1,1) then the fourier coefficient b_n is

a)$\frac{24(-1)^n}{n^3\pi^3}$ b) $6\left[\frac{(-1)^n+1}{n^4\pi^4}\right]$ c)0 d)None of these.

For the function $f(x) = 2x - x^2, 0 \le x \le 3$ the value of b_n is,

a) 0 b) $\frac{-9}{n^2\pi^2}$ c) $\frac{3}{n\pi}$ d) none of these

If $\emptyset(a) = \int_0^1 \frac{x^a-1}{logx} dx, a \geq 0$ then by DUIS rule, $\frac{d\emptyset}{da}$ is a) $\int_0^1 \frac{x^a loga}{logx} dx$ b) $\int_0^1 \frac{ax^{a-1}}{logx} dx$ c) $\int_0^1 x^a dx$ d) $\frac{x^a-1}{logx}$	C

If $\emptyset(\alpha) = \int_0^\infty \frac{e^{-x}sin\alpha x}{x} dx$ then by DUIS rule, $\frac{d\emptyset}{d\alpha}$ is a) $\int_0^\infty e^{-x} sin\alpha x \, dx$ b) $\int_0^\infty e^{-x} cos\alpha x \, dx$ c) $\int_0^\infty \frac{\alpha e^{-x} sin\alpha x}{x} dx$ d) $\frac{e^{-x} sin\alpha x}{x}$	B
If $\emptyset(a) = \int_0^{\frac{\pi}{2}} \frac{\log(1+asin^2x)}{sin^2x} dx, a > 0$, then by DUIS rule, $\frac{d\emptyset}{da}$ is a) $\int_0^{\frac{\pi}{2}} \frac{2\text{sinxcosx}}{(1+asin^2x)} dx$ b) $\int_0^{\frac{\pi}{2}} \frac{1}{(1+asin^2x)sin^2x} dx$ c) $\int_0^{\frac{\pi}{2}} \frac{1}{1+asin^2x} dx$ d) $\int_0^{\frac{\pi}{2}} \frac{sin^2x}{(1+asin^2x)} dx$	C
If $\emptyset(a) = \int_a^{a^2} \log(ax) dx$, then by DUIS rule II, $\frac{d\emptyset}{da}$ is a) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx + 2aloga^3$ b) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx$ c) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx + 2aloga^3 - 2loga$ d) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx - 2aloga^3 + 2loga$	A

27) The value of $\int_{r=0}^{a}\int_{\theta=0}^{\pi}\int_{\emptyset=0}^{2\pi} r^2\, sin\theta d\theta\; d\emptyset\; dr$

A.$8a^3\frac{\pi}{3}$ $\quad B.\;\; 4a^3\frac{\pi}{3}$ $\quad c.\; 2a^3\frac{\pi}{3}$ $\quad D.\;\; a^3\frac{\pi}{3}$

Ans. B

28. $Volume = \int_{r=0}^{a}\int_{\theta=0}^{2\pi}\int_{z=0}^{h} rd\theta drdz$

A $\pi a^2 h$ $\quad B.\;\; \frac{\pi a^2 h}{3}$ $\quad C.\quad 2\,\frac{\pi a^2 h}{3}$ $\quad D.4\frac{\pi a^2 h}{3}$

Ans. A

29) The value of $\int_0^a\int_0^a\int_0^a dx\; dy\; dz$

A. $\frac{a^3}{8}$ $\quad B.\;\; a^2 \quad C.\;\; a^3 \quad D.\;\; \frac{a^2}{4}$

Ans. C

30) The value of $\iiint x^{\frac{1}{2}}y^{\frac{1}{2}}z^{-\frac{1}{2}}dxdydz\,,\qquad where\; x+y+z \le 1$

A.

$\int_0^{\pi/2} sin^4 x dx \quad =$

A. $\frac{\pi}{2}$

B. $\frac{3\pi}{8}$

C. $\frac{\pi}{4}$

D. $\frac{3\pi}{16}$

$\int_0^{\pi/2} sin^5 x dx \quad =$

A. $\frac{8}{15}.\frac{\pi}{2}$

B. $\frac{15}{8}$

C. $\frac{8}{15}$

D. 0

$\int_0^{\pi} sin^5\left(\frac{x}{2}\right) dx\ =$ A. $\frac{8\pi}{15}$ B. $\frac{32}{15}$ C. $\frac{16}{15}$ D. $\frac{8}{15}$	1
$\int_0^{\pi/2} sin^4 x\ cos^5 x\ dx\ \ =$ A. $\frac{4\pi}{315}$ B. $\frac{315}{8}$ C. $\frac{8\pi}{630}$ D. $\frac{8}{315}$	1

If $I_n = \int_{\pi/4}^{\pi/2} cot^n\, xdx$ and $I_n = \frac{1}{n-1} - I_{n-2}$ then I_4 is equal to A. $\frac{-4}{3} + \frac{\pi}{4}$ B. $\frac{-2}{3} + \frac{\pi}{2}$ C. $\frac{-2}{3} - \frac{\pi}{4}$ **D.** $\boldsymbol{\frac{-2}{3} + \frac{\pi}{4}}$	1
If $I_n = \int_0^{\pi/4} tan^n\, xdx$ and $I_n = \frac{1}{n-1} - I_{n-2}$ then I_4 is equal to A. $\frac{-2}{3} + \frac{\pi}{2}$ B. $\frac{-2}{3} - \frac{\pi}{4}$ **C.** $\boldsymbol{\frac{-2}{3} + \frac{\pi}{4}}$	1

The appropriate substitution to reduce the integral $\int_0^\infty \sqrt{x}e^{-\sqrt{x}}\,dx$ to Gamma function integral A. x^3= t **B. $\sqrt{x} = t$** C. $-x^3 = t$ D. log x = t	1
The appropriate substitution to reduce the integral $\int_0^1 (x\log x)^4\,dx$ to Gamma function integral **A. log x = - t** B. $x = -e^t$ C. $x = t^2$ D. log x = t	1
The appropriate substitution to reduce the integral $\int_0^\infty \frac{x^5}{5^x}\,dx$ to Gamma function integral A. log x = - t B. $5x = e^t$ **C. $5^x = e^t$** D. $5^x = t$	1

<table>
<tr><th>Q.N.</th><th>Question</th><th>ANS</th></tr>
<tr><td>1</td><td>Fourier coefficient 'a_0 ' in the Fourier series expansion of $f(x) = e^{-x}$; $0 \le x \le 2\pi$ and $f(x + 2\pi) = f(x)$ is
a) $\frac{1}{\pi}(1 - e^{-2\pi})$ b) $\frac{1}{2\pi}(1 - e^{2\pi})$ c) $\frac{2}{\pi}(e^{-2\pi} - 1)$ d) $\frac{1}{\pi}(1 + e^{2\pi})$</td><td>A</td></tr>
<tr><td>2</td><td>Fourier coefficient a_0 in the Fourier series expansion of $f(x) = \left(\frac{\pi - x}{2}\right)^2$; $0 \le x \le 2\pi$ and $f(x + 2\pi) = f(x)$ is
a) $\frac{\pi^2}{3}$ b) $\frac{\pi^2}{6}$ c) 0 d) $\frac{\pi}{6}$</td><td>B</td></tr>
<tr><td>3</td><td>$f(x) = x$, $-\pi \le x \le \pi$ *and* period is 2π the fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$, then fourier coefficient b_1 is
a) 2 b) -1 c) 0 d) $\frac{2}{\pi}$</td><td>A</td></tr>
<tr><td>4</td><td>$f(x) = \begin{cases} 1, & -1 < x < 0 \\ \cos \pi x, & 0 < x < 1 \end{cases}$ *and* period is 2 the fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\pi x + b_n \sin n\pi x)$, then fourier coefficient a_0 is
a) 2 b) -1 c) 1 d) $\frac{2}{\pi}$</td><td>C</td></tr>
<tr><td>5</td><td>$f(x) = x - x^3$, $-2 < x < 2$ *and* period is 4 the fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\pi x + b_n \sin n\pi x)$, then fourier coefficient a_0 is
a)1 b)0 c)-2 d)-1</td><td>B</td></tr>
<tr><td>6</td><td>For the half range cosine series of $f(x) = \sin x$, $0 \le x < \pi$ *and* period is 2π the fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx)$ fourier coefficient a_0 is
a) 4 b) 2 c) $\frac{2}{\pi}$ d) $\frac{4}{\pi}$</td><td>D</td></tr>
<tr><td>7</td><td>The value of b_1 in Harmonic analysis of y for the following tabulated data is:
<table>
<tr><td>x</td><td>0</td><td>60</td><td>120</td><td>180</td><td>240</td><td>300</td><td>360</td></tr>
<tr><td>y</td><td>1.0</td><td>1.4</td><td>1.9</td><td>1.7</td><td>1.5</td><td>1.2</td><td>1.0</td></tr>
<tr><td>Sin x</td><td>0</td><td>0.866</td><td>0.866</td><td>0</td><td>-0.866</td><td>-0.866</td><td>0</td></tr>
</table>
a) 0.0989 b) 0.3464 c) 0.1732 d) 0.6932</td><td>C</td></tr>
<tr><td>8</td><td>The value of the constant term in the fourier series of $f(x) = e^{-x}$ in $0 \le x \le 2\pi$, $f(x + 2\pi) = f(x)$ is
a) $\frac{1}{\pi}(1 - e^{-2\pi})$ b) $\frac{1}{2\pi}(1 - e^{-2\pi})$ c) $1 - e^{-2\pi}$ d) $2(1 - e^{-2\pi})$</td><td>B</td></tr>
<tr><td>9</td><td>The value of the constant term in the fourier series of $f(x) = x \sin x$ in $0 \le x \le 2\pi$, is
a) -2 b) 2 c) $-\frac{1}{2}$ d) -1</td><td>D</td></tr>
</table>

10	If $a_n = \frac{2}{n^2-1}$ for $n > 1$, in the fourier series of f(x) = x sin x in $0 \le x \le 2\pi$, then the value of a_1 is a) -2 b) $\frac{2}{n^2-1}$ c) $-\frac{1}{2}$ d) $\frac{1}{2}$	C
11	The value of the constant term in the fourier series of $f(x)=\begin{cases} -\pi & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$, *is* a) $-\frac{\pi}{4}$ b) $-\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{2}$	A
12	The value of a_n in the fourier series of $f(x)=\begin{cases} \cos x; & -\pi < x < 0 \\ -\cos x; & 0 < x < \pi \end{cases}$ a)$\frac{(-1)^n}{n}$ b)$\frac{1}{n}$ c)$\frac{(-1)^n}{n^2-1}$ d)0	D
13	The value of b_n in the fourier series of f(x) = x in $-\pi < x < \pi$, is a) 0 b) $\frac{\cos n\pi}{n}$ c) $-\frac{2\cos n\pi}{n}$ d) $-\frac{\cos n\pi}{n}$	C
14	The Fourier constant 'a_n' for f (x) = $4 - x^2$ in the interval $0 < x < 2$ is________ a) $-\frac{4}{\pi^2 n^2}$ b)$\frac{4}{\pi n^2}$ c)$\frac{4}{\pi^2 n^2}$ d)$\frac{2}{\pi^2 n^2}$	A
15	If f(x)=sin ax defined in the interval $(-l, l)$ then value of 'a_n'is________ a)$\frac{2}{\pi n^2}$ b)$\frac{1}{n^2}$ c)0 d) $-\frac{1}{n^2}$	C
16	The function $f(x) = \begin{cases} 0 & -2<x<-1 \\ 1+x & -1<x<0 \\ 1-x & 0<x<1 \\ 0 & 1<x<2 \end{cases}$ Is a)an odd function b) an even function c) neither even nor odd function d)cannot be decided	B
17	The Fourier constant 'a_n'for f(x)=x^2 in the interval $-1 \le x \le 1$ is a) $\frac{4(-1)^n}{\pi^2 n^2}$ b)$\frac{4(-1)^{n+1}}{\pi^2 n^2}$ c) $\frac{4}{\pi^2 n^2}$ d)$-\frac{4}{\pi^2 n^2}$	A
18	If $f(x) = \begin{cases} -x, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$, then f(x) is a) Even function b) odd function c)Neither even nor odd d) none of these	A
19	In fourier series for f(x) = x in the interval $-\pi \le x \le \pi$ which of the following is correct a) $a_0 = \pi, a_n = \frac{1+(-1)^n}{n}, b_n = 0$ b) $a_0 = 0, a_n = 0, b_n = \frac{-2(-1)^n}{n}$ c) $a_0 = \frac{\pi}{2}, a_n = \frac{1+(-1)^n}{n}, b_n = \frac{-2(-1)^n}{n}$ d) $a_0 = 0, a_n = 0, b_n = 0$	B

20	The Fourier constant $'b_n'$for f(x)=$2-\frac{x^2}{2}$ in the interval $0 \le x \le 2$ is a) $\frac{-2}{n\pi}$ b) $\frac{2}{n\pi}$ c) $\frac{2}{\pi^2 n^2}$ d) none of these.	B
21	If $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ is discontinuous at x = 0.Then value of f(0) is a) $\pi/2$ b) 0 c) $-\frac{\pi}{2}$ d) π	C
22	The function $f(x) = \begin{cases} 0 & -2<x<-1 \\ 1+x & -1<x<0 \\ 1-x & 0<x<1 \\ 0 & 1<x<2 \end{cases}$ Find a_0 a)2 b) 1/4 c) 1/2 d)0	D
23	Which of the following is the half range sine series of f(x)in the interval, $0 \le x \le \pi$? a)f(x)=$\sum_{n=1}^{\infty} b_n sin\frac{n\pi x}{L}$ b) f(x)=$\frac{a_0}{2}\sum_{n=1}^{\infty} b_n sinnx$ c) f(x)=$\sum_{n=1}^{\infty} b_n sinnx$ d) none of these.	C
24	For the function $f(x) = \begin{cases} -\pi, & 0 < x < \pi \\ x-\pi, & \pi < x < 2\pi \end{cases}$ what is the value of a_0 a) $\frac{1}{2}$ b) $-\frac{\pi}{2}$ c) $\frac{\pi}{2}$ d) π	B
25	If f(x) = $e^x, -1 \le x \le 1$ then constant term of f(x) in fourier expansion is a) $\frac{e}{2}$ b) $\frac{e-e^{-1}}{2}$ c) $\frac{e+e^{-1}}{2}$ d) $\frac{1+e}{2}$	B
26	If $f(x) = \begin{cases} -\pi, & 0 < x < \pi \\ x, & \pi < x < 2\pi \end{cases}$ is discontinuous at x= 0 then value of f(0) is a) $-\pi$ b) 0 c) $-\frac{\pi}{2}$ d) π	C
27	If $\sum y = 42$,n=6.$\sum ycos\theta = -8.5, \sum ycos2\theta$= -1.5,what arethe values of a_0, a_1, a_2 a)7,-2.8,-2.8 b)14,-2.8,1.5 c) 7,-1.5,-2.8 d)none of these	D
28	If f(x) = x^4 in (-1,1) then the fourier coefficient b_n is a)$\frac{24(-1)^n}{n^3\pi^3}$ b) $6\left[\frac{(-1)^n+1}{n^4\pi^4}\right]$ c)0 d)None of these.	C
29	For the function $f(x) = 2x - x^2, 0 \le x \le 3$ the value of b_n is, a) 0 b) $\frac{-9}{n^2\pi^2}$ c) $\frac{3}{n\pi}$ d) none of these	C
30	In the Fourier expansion of the function $f(x) = \frac{1}{2}(\pi - x)$ in the interval $0 \le x \le 2\pi$ the value of a_n is, a) $\frac{1}{n^2\pi}$ b) $\frac{\pi}{n^2}$ c) 0 d) $\frac{(-1)^n}{n^2\pi}$	C

<table>
<tr><td>31</td><td>If $f(x)=\frac{\pi^2}{2}-\frac{x^2}{4}, -\pi \le x \le \pi$ then values of a_n and b_n are
a) $0, \frac{3}{n\pi}$ b) $0, \frac{(-1)^{n+1}}{n^2}$ c) $\frac{(-1)^n+1}{n^2-1}, 0$ d) $\frac{-(-1)^n}{n^2}, 0$</td><td>D</td></tr>
<tr><td>32</td><td>The following values of y give the displacement in inches of a certain machine part for the rotation x of the flywheel
<table><tr><td>x</td><td>0</td><td>$\pi/6$</td><td>$2\pi/6$</td><td>$3\pi/6$</td><td>$4\pi/6$</td><td>$5\pi/6$</td></tr><tr><td>Y</td><td>0</td><td>9.2</td><td>14.4</td><td>17.8</td><td>17.3</td><td>11.7</td></tr></table>What is the value of a_0
a)11.733 b)14.4 c)23.466 d) none of these</td><td>C</td></tr>
<tr><td>33</td><td>If $\sin x = \frac{2}{\pi}-\frac{2}{\pi}\sum_{n=2}^{\infty}\frac{1+(-1)^n}{(n-1)(n+1)}\cos nx$ for $0 \le x \le \pi$ then which of the following correct
a) $\frac{1}{2}= \frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdots$ b) $\frac{\pi^2}{2}=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots$
c) $\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$ d) none of these</td><td>A</td></tr>
<tr><td>34</td><td>If $a_0=\frac{4\pi^2}{3}, a_n=\frac{4(-1)^{n+1}}{n^2}$ are the fourier coefficient of f(x) in $-\pi \le x \le \pi$ then which of the following correct
a) f(x)= $\frac{2\pi^2}{3}+\sum_{n=1}^{\infty}\frac{4(-1)^{n+1}}{n^2}\cos\frac{n\pi x}{2}$ b) f(x)= $\frac{4\pi^2}{3}+\sum_{n=1}^{\infty}\frac{4(-1)^{n+1}}{n^2}\sin nx$
c) f(x)= $\frac{2\pi^2}{3}-\sum_{n=1}^{\infty}\frac{4(-1)^{n}}{n^2}\cos nx$ d) none of these</td><td>C</td></tr>
<tr><td>35</td><td>If f(x) = x^2, $0 < x < 2$ then in half range cosine series $\frac{a_0}{2}$ is
a) 4 b) 12 c) $\frac{8}{3}$ d) 8</td><td>C</td></tr>
<tr><td>36</td><td>For the half range cosine series f(x) = $\left(\frac{\pi-x}{2}\right)^2, 0 \le x \le \pi$, if $a_0=\frac{\pi^2}{6}, a_n=\frac{1}{n^2}, b_n=0$, then which of the following statement is correct
a) $\frac{\pi^2}{6}=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}$------ b) $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}$- - -
c) $\frac{\pi^2}{6}=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}$------ d) $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}$- - -</td><td>C</td></tr>
</table>

Q.N.	Question	Ans
1	If $\emptyset(a)=\int_0^{\infty}\frac{e^{-x}}{x}(1-e^{-ax})dx, a>-1$ then by DUIS rule, $\frac{d\emptyset}{da}$ is a) $\frac{e^{-x}}{x}(1-e^{-ax})$ b) $\int_0^{\infty}\frac{a}{x}\left(e^{-(a+1)x}\right)dx$ c) $\int_0^{\infty}(e^{-ax})dx$ d) $\int_0^{\infty}\left(e^{-(a+1)x}\right)dx$	D
2	If $\emptyset(a)=\int_0^1\frac{x^a-1}{logx}dx, a \ge 0$ then by DUIS rule, $\frac{d\emptyset}{da}$ is a) $\int_0^1\frac{x^a loga}{logx}dx$ b) $\int_0^1\frac{ax^{a-1}}{logx}dx$ c) $\int_0^1 x^a dx$ d) $\frac{x^a-1}{logx}$	C

3	If $\emptyset(\alpha) = \int_0^{\infty} \frac{e^{-x} sin\alpha x}{x} dx$ then by DUIS rule, $\frac{d\emptyset}{d\alpha}$ is a) $\int_0^{\infty} e^{-x} sin\alpha x\, dx$ b) $\int_0^{\infty} e^{-x} cos\alpha x\, dx$ c) $\int_0^{\infty} \frac{\alpha e^{-x} sin\alpha x}{x} dx$ d) $\frac{e^{-x} sin\alpha x}{x}$	B
4	If $\emptyset(a) = \int_0^{\frac{\pi}{2}} \frac{\log(1+a sin^2 x)}{sin^2 x} dx, a > 0$, then by DUIS rule, $\frac{d\emptyset}{da}$ is a) $\int_0^{\frac{\pi}{2}} \frac{2 sinx cosx}{(1+a sin^2 x)} dx$ b) $\int_0^{\frac{\pi}{2}} \frac{1}{(1+a sin^2 x) sin^2 x} dx$ c) $\int_0^{\frac{\pi}{2}} \frac{1}{1+a sin^2 x} dx$ d) $\int_0^{\frac{\pi}{2}} \frac{sin^2 x}{(1+a sin^2 x)} dx$	C
5	If $\emptyset(a) = \int_a^{a^2} \log(ax) dx$, then by DUIS rule II, $\frac{d\emptyset}{da}$ is a) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx + 2a loga^3$ b) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx$ c) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx + 2a loga^3 - 2 loga$ d) $\int_a^{a^2} \frac{\partial}{\partial a} \log(ax)\, dx - 2a loga^3 + 2 loga$	A
6	If $\emptyset(a) = \int_0^{a^2} tan^{-1}\left(\frac{x}{a}\right) dx$, then by DUIS rule II, $\frac{d\emptyset}{da}$ is a) $\int_0^{a^2} \frac{\partial}{\partial a} tan^{-1}\left(\frac{x}{a}\right) dx + 2a tan^{-1} a$ b) $\int_0^{a^2} \frac{\partial}{\partial a} tan^{-1}\left(\frac{x}{a}\right) dx$ c) $\int_0^{a^2} \frac{\partial}{\partial a} tan^{-1}\left(\frac{x}{a}\right) dx + a^2 tan^{-1} a$ d) $\int_0^{a^2} \frac{\partial}{\partial a} tan^{-1}\left(\frac{x}{a}\right) dx + a^2 tan^{-1} a - tan^{-1}\left(\frac{x}{a}\right)$	A
7	IF $I(a)$=$\int_a^{a^2} \frac{1}{x+a} dx$, then by DUIS rule II, $\frac{dI}{da}$ is a) $\int_a^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx - \frac{1}{a^2+a}(2a) + \frac{1}{2a}$ b) $\int_a^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx + \frac{1}{a^2+a}(2a) - \frac{1}{2a}$ c) $\int_a^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx - \frac{1}{a^2+a}$ d) $\int_a^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx$	B
8	Using DUIS Rule the value of the integral $\emptyset(a) = \int_0^{\infty} \frac{e^{-x}}{x}(1 - e^{-ax}) dx, a > -1$, given $\frac{d\emptyset}{da} = \frac{1}{a+1}$ is a)log(a+1) b)$-\frac{1}{(a+1)^2}$ c)log(a+1) + π d) $-\frac{1}{(a+1)^2} + 1$	A
9	Using DUIS Rule the value of the integral $\emptyset(a) = \int_0^{\frac{\pi}{2}} \frac{\log(1+a sin^2 x)}{sin^2 x} dx$ with $\frac{d\emptyset}{da} = \frac{\pi}{2} \frac{1}{\sqrt{a+1}}$ is a)$\pi\sqrt{a+1}$ b) $\pi\sqrt{a+1} + \pi$ c) $\pi\sqrt{a+1} - \pi$ d) $3\pi(a+1)^{\frac{3}{2}} - \pi$	C
10	Using DUIS Rule the value of the integral $\emptyset(a) = \int_0^{\infty} \frac{1-\cos ax}{x^2} dx$, with $\frac{d\emptyset}{da} = \frac{\pi}{2}$ is a) $\frac{\pi}{2}$ b) $\frac{\pi a}{2}$ c) πa d) $\frac{\pi a}{2} + \frac{\pi}{2}$	B
11	If I(a)=$\int_0^1 \frac{x^a - x^b}{log x}$dx then the value of I(a) is a) $\log\left(\frac{a}{b}\right)$ b) $\log\left(\frac{a+1}{b+1}\right)$ c) $\log\left(\frac{b+1}{a+1}\right)$ d) $\log\left(\frac{b}{a}\right)$	B

12	If erf($\sqrt{t}$) =$\frac{2}{\sqrt{\pi}}\int_0^{\sqrt{t}} e^{-u^2}\,du$ then $\frac{d}{dt}$ erf($\sqrt{t}$) is a)$\frac{e^{-t}}{2\sqrt{t}}$ b) $\frac{e^{-t^2}}{\sqrt{\pi t}}$ c) $\frac{e^{-t}}{\sqrt{\pi}}$ d) $\frac{e^{-t}}{\sqrt{\pi t}}$	D
13	$\int_0^t \mathrm{erf}(ax)\,dx + \int_0^t \mathrm{erf\,c}(ax)\,dx = ?$ a) t b) x c) 0 d) $\frac{t^2}{2}$	A
14	If $\frac{d}{dx}$ erf(ax)=$\frac{2ae^{-a^2x^2}}{\sqrt{\pi}}$, the value of $\int_0^t \mathrm{erf}(ax)\,dx$ is a) t erf(at)+$\frac{1}{a\sqrt{\pi}}e^{-a^2t^2}-\frac{1}{a\sqrt{\pi}}$ b) t erf(at)$-\frac{1}{a\sqrt{\pi}}e^{-a^2t^2}-\frac{1}{a\sqrt{\pi}}$ c) erf(at)+$\frac{1}{a\sqrt{\pi}}e^{-a^2t^2}+\frac{1}{a\sqrt{\pi}}$ d) t erf(at)$-\frac{1}{a\sqrt{\pi}}e^{-a^2t^2}+\frac{1}{a\sqrt{\pi}}$	A
15	The integral for "erf(b)-erf(a)" is, a) $\frac{2}{\sqrt{\pi}}\int_a^b e^{-t^2}\,dt$ b) $\sqrt{\frac{2}{\pi}}\int_a^b e^{-t^2}\,dt$ c) $\int_a^b e^{-t^2}\,dt$ d) none of these	A

(1) The differential equation of all circles touching Y-axis at the origin & centres on X-axis, is

(A) $x^2 - y^2 = 2x\left[x + y\frac{dy}{dx}\right]$

(B) $x^2 + y^2 = 2x\left[x + y\frac{dy}{dx}\right]$

(C) $x^2 + y^2 = 2x\left[x - y\frac{dy}{dx}\right]$

(D) None of these

2] Integrating factor of $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$

(A) $\frac{1}{x^2y^2}$

(B) $\frac{1}{xy}$

(C) $\frac{1}{x^2y}$

(D) $\frac{1}{xy^2}$

3] If $I = \frac{E}{R}\left[1 - e^{-Rt/L}\right]$ then the value of q is

A) $\frac{E}{R}\left[t + \frac{L}{R}e^{-Rt/L}\right]$

B) $\frac{E}{R}\left[t + \frac{R}{L}e^{-Rt/L}\right]$

c) $\frac{E}{R}\left[t - \frac{R}{L}e^{-Rt/L}\right]$

(D) None of these

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(4) The curve $r = a e^{m\theta}$

(A) Not passes through the pole

(B) passes through the pole

(C) symmetry about y-axis

(D) None of these

(5) The curve $a^2y^2 = x^2(2a-x)(x-a)$ is

(A) symmetry about y-axis

(B) symmetry about $y=x$

(C) symmetry about x-axis

(D) symmetry about $y=-x$

(6) Tangent at origin to the curve $x^3+y^3=3ax$ is

(A) $x=0$

(B) $y=0$

(C) $x=0$, $y=0$

(D) None of these

Q.1) $\int_0^1 \int_0^1 \frac{dx\,dy}{\sqrt{1-x^2}\,\sqrt{1-y^2}} = \cdots$

a) $\frac{\pi^2}{3}$ b) $\frac{\pi^2}{4}$ c) π d) 1

Q.2) $\int_0^{2\pi} \int_{a\sin\theta}^{a} r\,d\theta\,dr = \cdots$

a) $\frac{\pi a^2}{2}$ b) $\frac{\pi}{2}$ c) $\frac{\pi^2 a}{2}$ d) 5

Q.3) $\int_0^1 \int_0^x e^{x+y}\,dx\,dy = \cdots$

a) $\frac{(e-1)^2}{2}$ b) $\frac{e-1}{2}$ c) $\frac{e}{2}$ d) e

Q.4) After changing the order of integration $I = \int_0^1 \int_{4y}^{4} e^{x^2}\,dx\,dy$, the new limits of x & y are

a) $0 \le x \le 4,\ 0 \le y \le \frac{x}{4}$ b) $4 \le x \le 0,\ \frac{x}{4} \le y \le 0$

c) $0 \le x \le \frac{y}{4},\ 0 \le y \le 4$ d) $0 \le x \le 1,\ 0 \le y \le 4$

Q.5) After changing the order of integration of $I = \int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dx\,dy$ the new limits of x & y are

a) $0 \le x \le y$ $0 \le y < \infty$ b) $0 \le y \le x,\ 0 \le x < \infty$

c) $0 \le x \le 1$ $x \le y \le 1$ d) $0 \le x \le 1,\ 0 \le y \le 2$

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Q.1) $\int_0^1 \int_0^1 \frac{dx\,dy}{\sqrt{1-x^2}\sqrt{1-y^2}} = \ldots$

a) $\frac{\pi^2}{3}$ b) $\frac{\pi^2}{4}$ ✓ c) π d) 1

Q.2) $\int_0^{2\pi} \int_{a\sin\theta}^{a} r\,d\theta\,dr = \ldots$

a) $\frac{\pi a^2}{2}$ ✓ b) $\frac{\pi}{2}$ c) $\frac{\pi^2 a}{2}$ d) 5

Q.3) $\int_0^1 \int_0^x e^{x+y}\,dx\,dy = \ldots$

a) $\frac{(e-1)^2}{2}$ ✓ b) $\frac{e-1}{2}$ c) $\frac{e}{2}$ d) e

Q.4) After changing the order of integration $I = \int_0^1 \int_{4y}^{4} e^{x^2}\,dx\,dy$, the new limits of x & y are

✓ a) $0 \le x \le 4,\ 0 \le y \le \frac{x}{4}$ b) $4 \le x \le 0,\ \frac{x}{4} \le y \le 0$

c) $0 \le x \le \frac{y}{4},\ 0 \le y \le 4$ d) $0 \le x \le 1,\ 0 \le y \le 4$

Q.5) After changing the order of integration of $I = \int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dx\,dy$ the new limits of x & y are

✓ a) $0 \le x \le y\ \ 0 \le y < \infty$ b) $0 \le y \le x,\ 0 \le x < \infty$

c) $0 \le x \le 1\ \ x \le y \le 1$ d) $0 \le x \le 1,\ 0 \le y \le 2$

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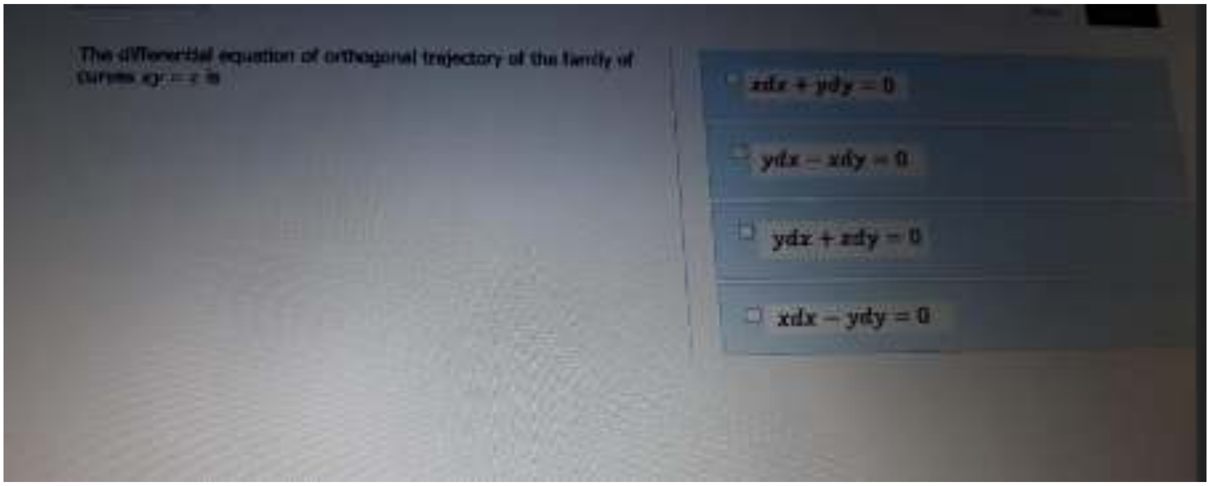
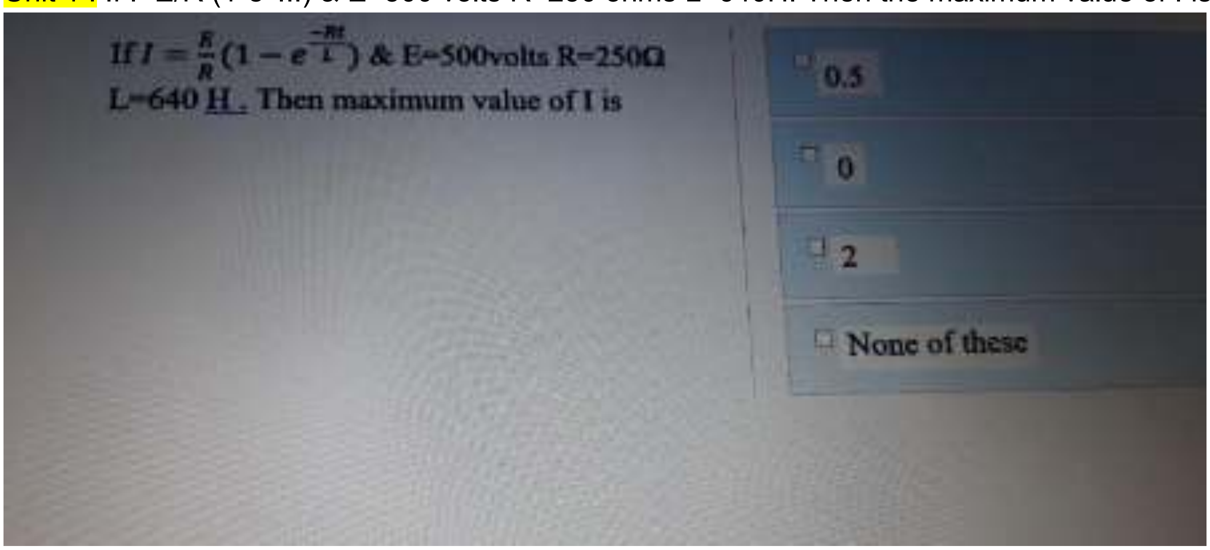
Unit: 1 (34 Questions)

Unit:1 The differential equation of orthogonal trajectory of family of curves xy=c is



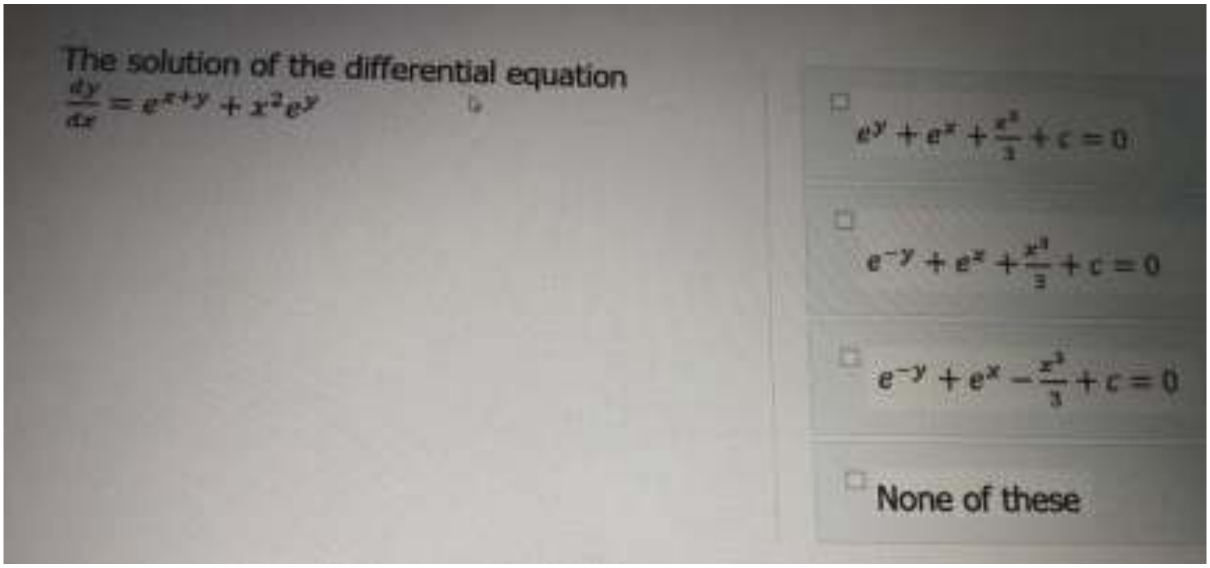
Ans:D (xdx-ydy=0)

Unit 1 : If I=E/R (1-e^...) & E=500 volts R=250 ohms L=640H. Then the maximum value of I is



Ans:c (=2)

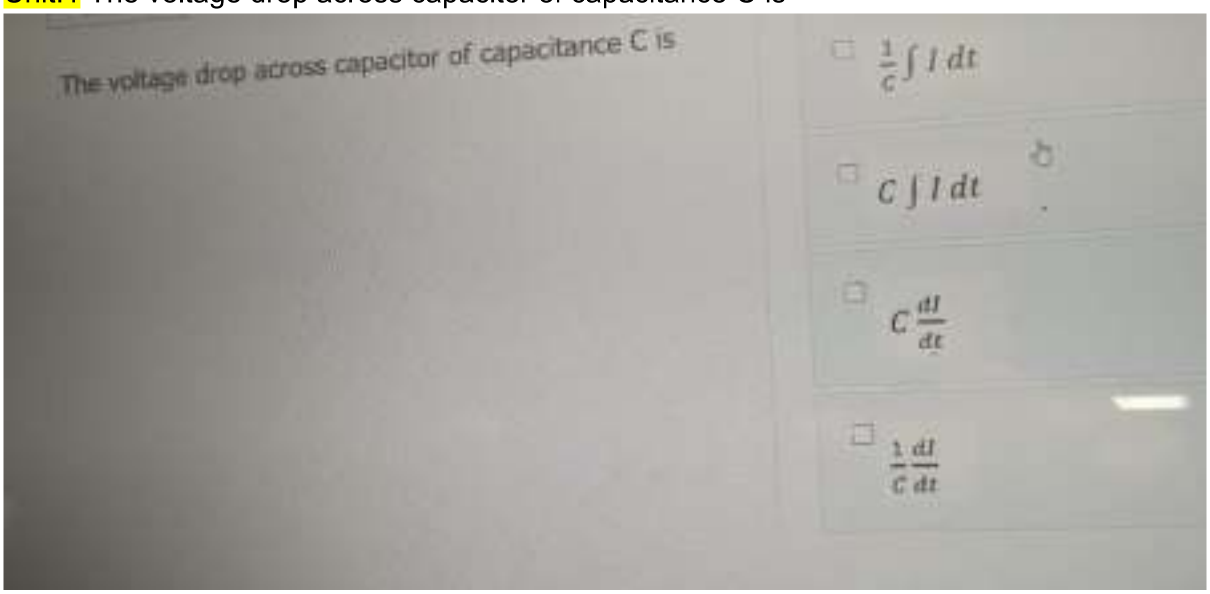
Unit 1: The solution of the differential equation: dy/dx=e^{x+y}+x^2e^y


The solution of the differential equation $\frac{dy}{dx} = e^{x+y} + x^2e^y$

- $e^y + e^x + \frac{x^3}{3} + c = 0$
- $e^{-y} + e^x + \frac{x^3}{3} + c = 0$
- $e^{-y} + e^x - \frac{x^3}{3} + c = 0$
- None of these

Ans:B

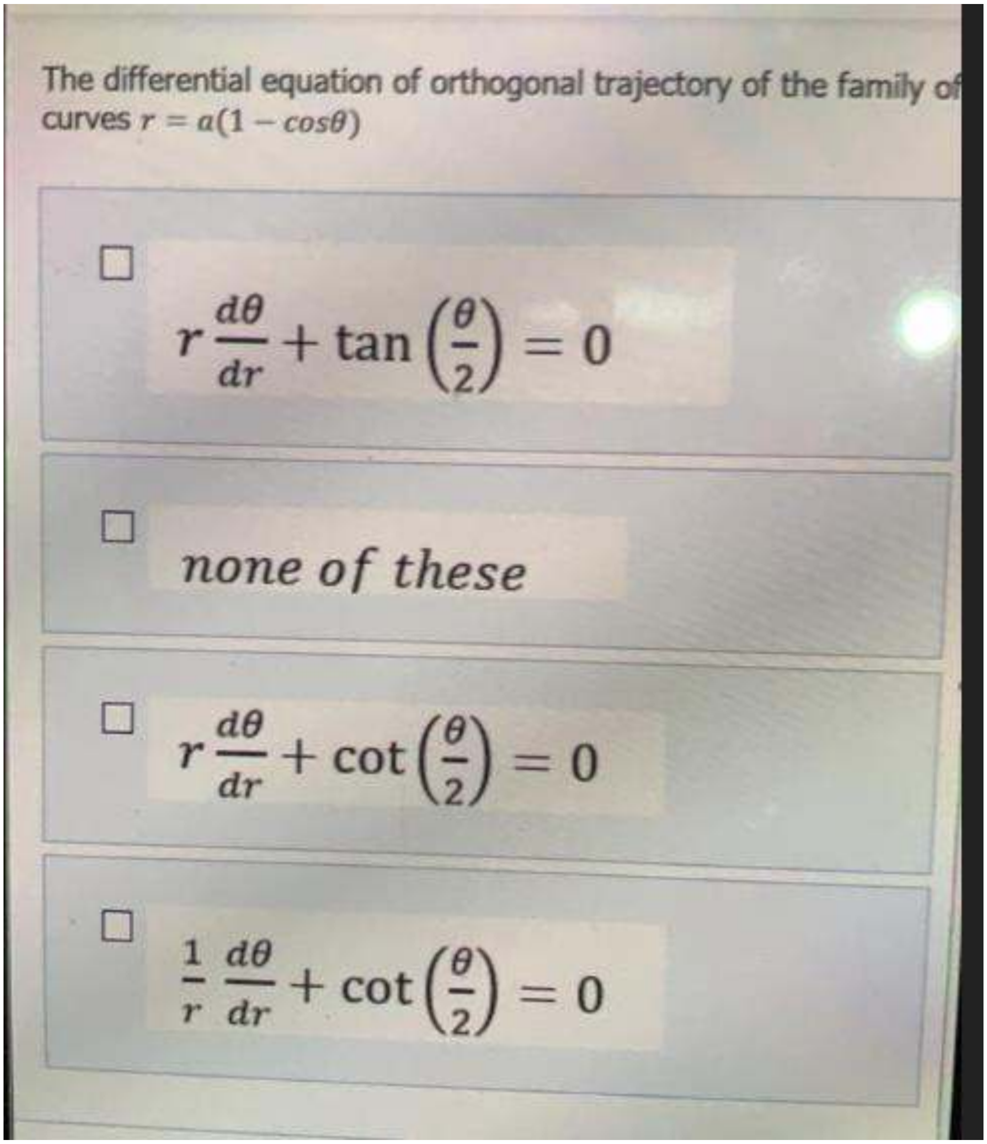
Unit:1 The voltage drop across capacitor of capacitance C is


The voltage drop across capacitor of capacitance C is

- $\frac{1}{C}\int I\,dt$
- $C\int I\,dt$
- $C\frac{dI}{dt}$
- $\frac{1}{C}\frac{dI}{dt}$

Ans: A [1/C integral(I dt)]

Unit:1 The differential equation of orthogonal trajectory of the family of curves r=a(1-cos theta)

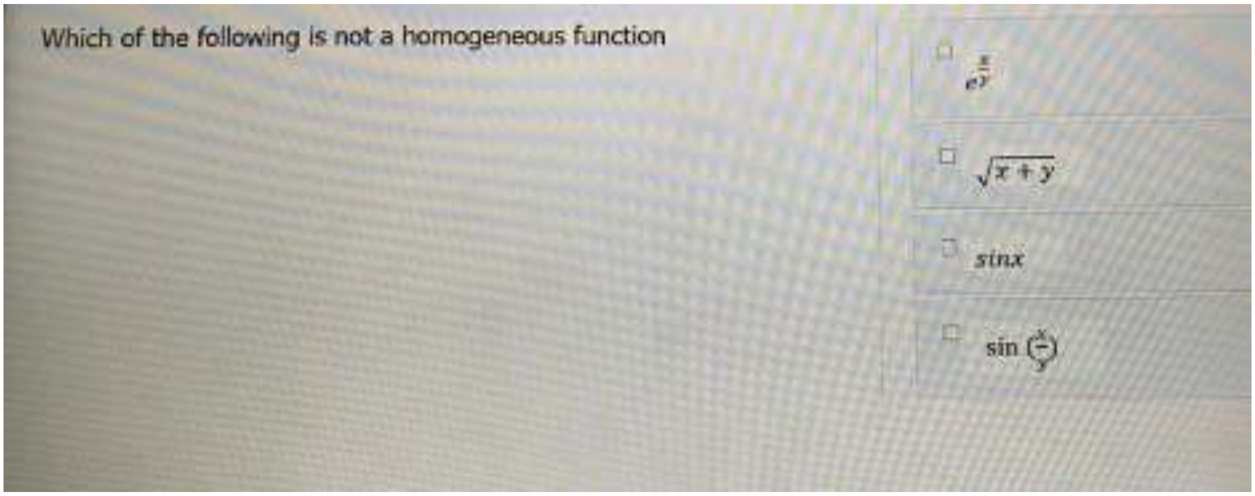


The differential equation of orthogonal trajectory of the family of curves $r = a(1 - cos\theta)$

- [ ] $r\dfrac{d\theta}{dr} + \tan\left(\dfrac{\theta}{2}\right) = 0$
- [ ] *none of these*
- [ ] $r\dfrac{d\theta}{dr} + \cot\left(\dfrac{\theta}{2}\right) = 0$
- [ ] $\dfrac{1}{r}\dfrac{d\theta}{dr} + \cot\left(\dfrac{\theta}{2}\right) = 0$

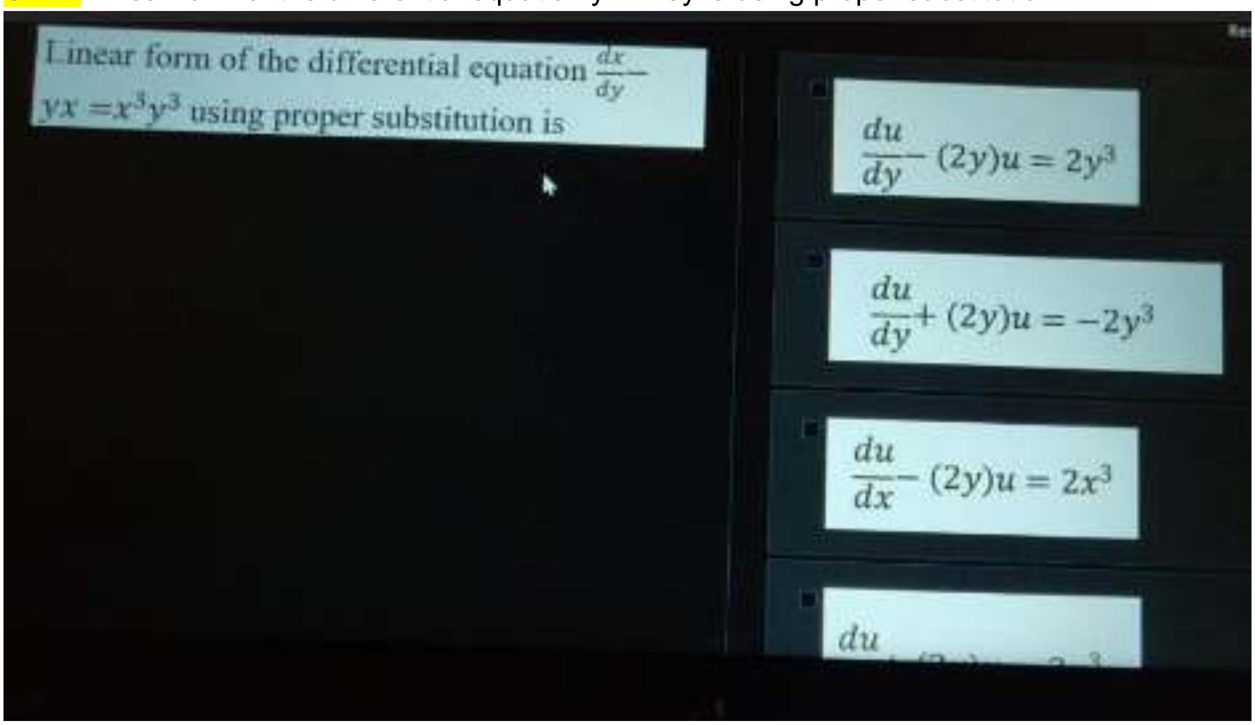
Ans: option C : r(dtheta/dr) + cot(theta/2)

Unit 1: Which of these is not a homogeneous function



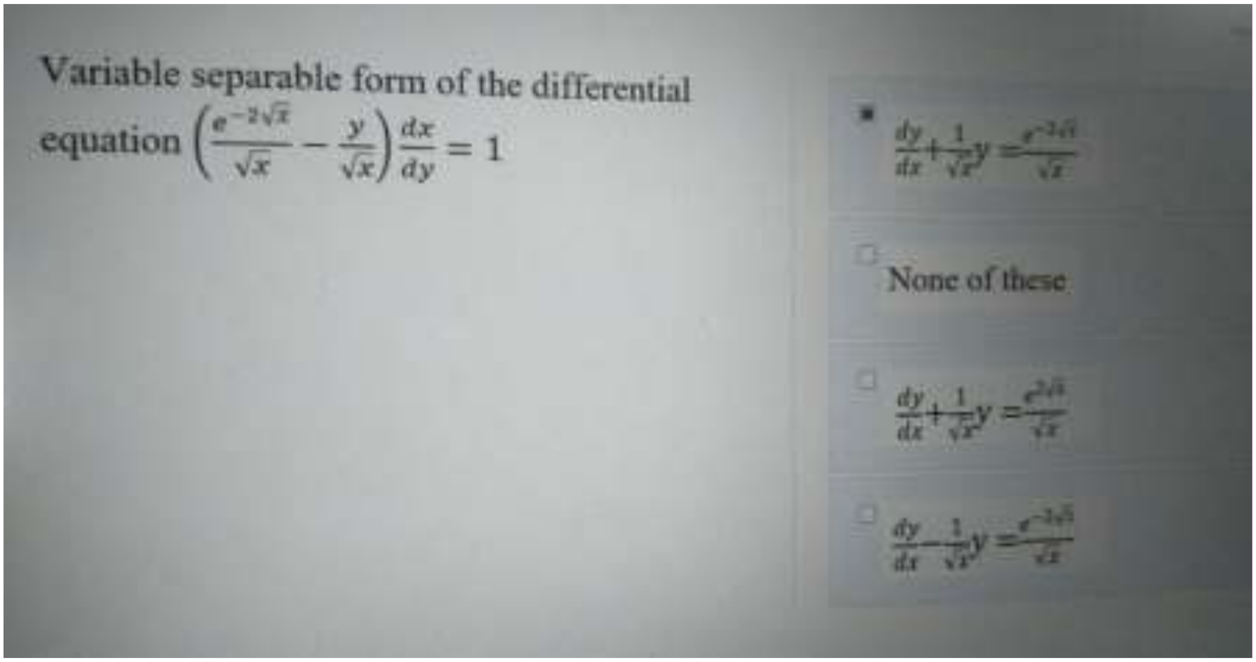
Ans: Sin X

Unit:1 Linear form of the differential equation yx=x^3y^3 using proper substitution



Ans:B du/dy + (2y)u=-2Y^3

UNIT 1:Variable separable form of the differential equation

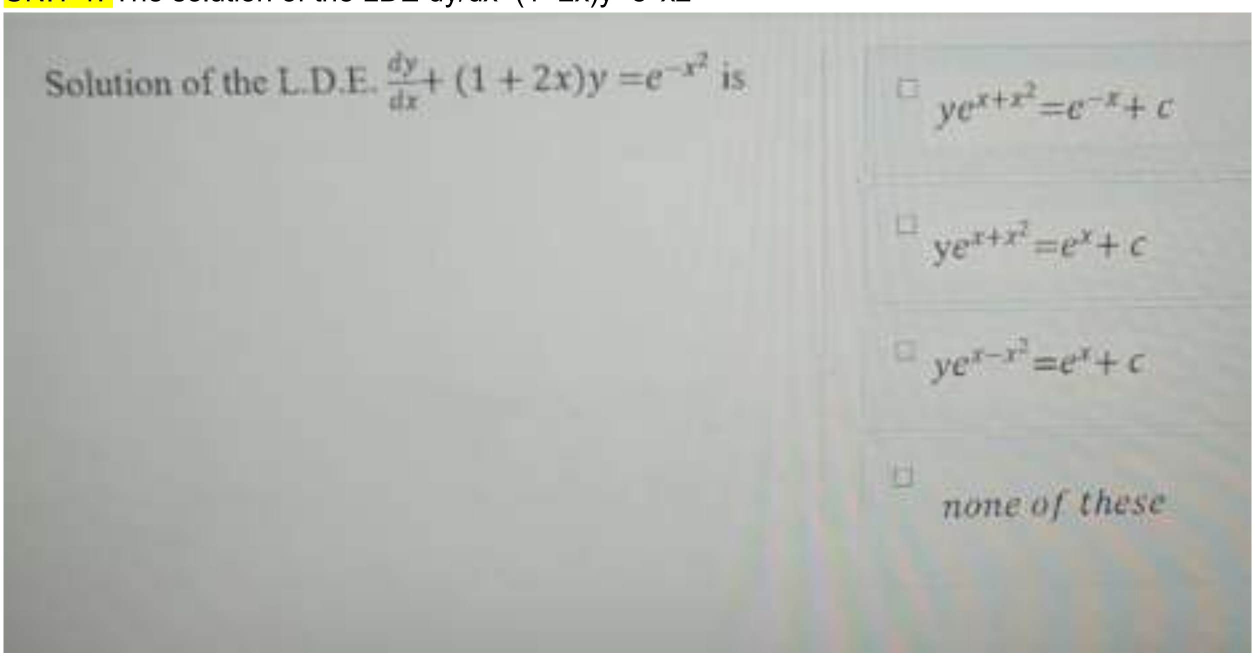


Variable separable form of the differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$

- $\frac{dy}{dx}+\frac{1}{\sqrt{x}}y=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}$
- None of these
- $\frac{dy}{dx}+\frac{1}{\sqrt{x}}y=\frac{e^{2\sqrt{x}}}{\sqrt{x}}$
- $\frac{dy}{dx}-\frac{1}{\sqrt{x}}y=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

sAns: A (none of these if wanna go for variable separable form of the eq)

UNIT 1: The solution of the LDE dy/dx+(1+2x)y=e^x2

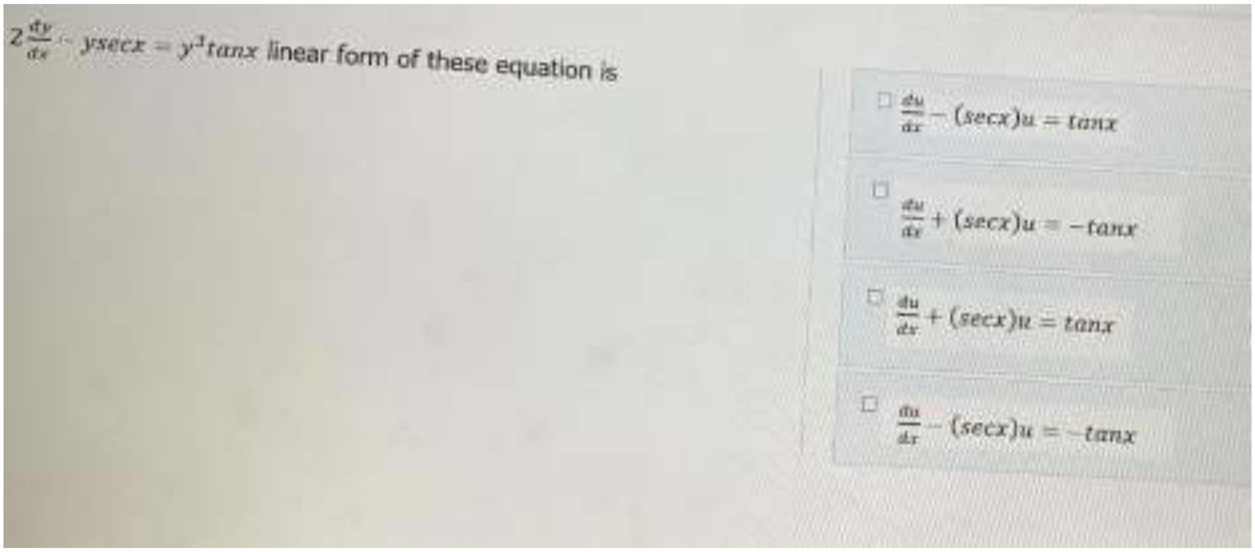


Solution of the L.D.E. $\frac{dy}{dx}+(1+2x)y=e^{-x^2}$ is

- $ye^{x+x^2}=e^{-x}+c$
- $ye^{x+x^2}=e^{x}+c$
- $ye^{x-x^2}=e^{x}+c$
- none of these

Ans: B

Unit:1 2dy/dx - ysecx=y^3 tanx Linear form of these equation is

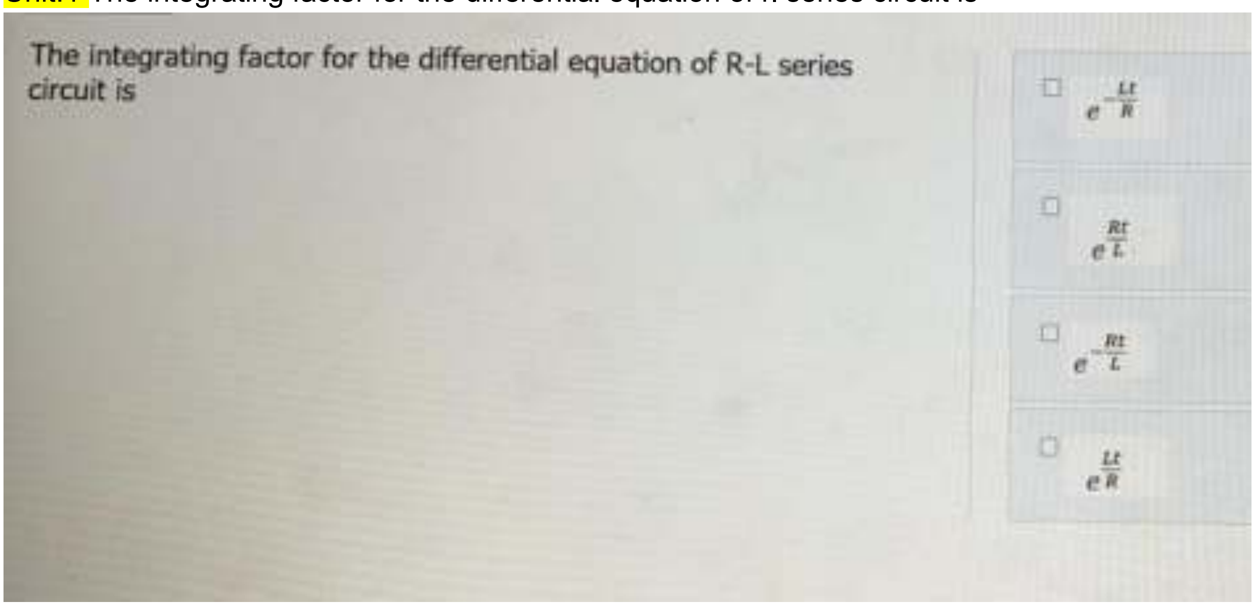


$2\frac{dy}{dx} - y\sec x = y^3\tan x$ linear form of these equation is

- $\frac{du}{dx} - (\sec x)u = \tan x$
- $\frac{du}{dx} + (\sec x)u = -\tan x$
- $\frac{du}{dx} + (\sec x)u = \tan x$
- $\frac{du}{dx} - (\sec x)u = -\tan x$

Ans: B

Unit:1 The integrating factor for the differential equation of rl series circuit is

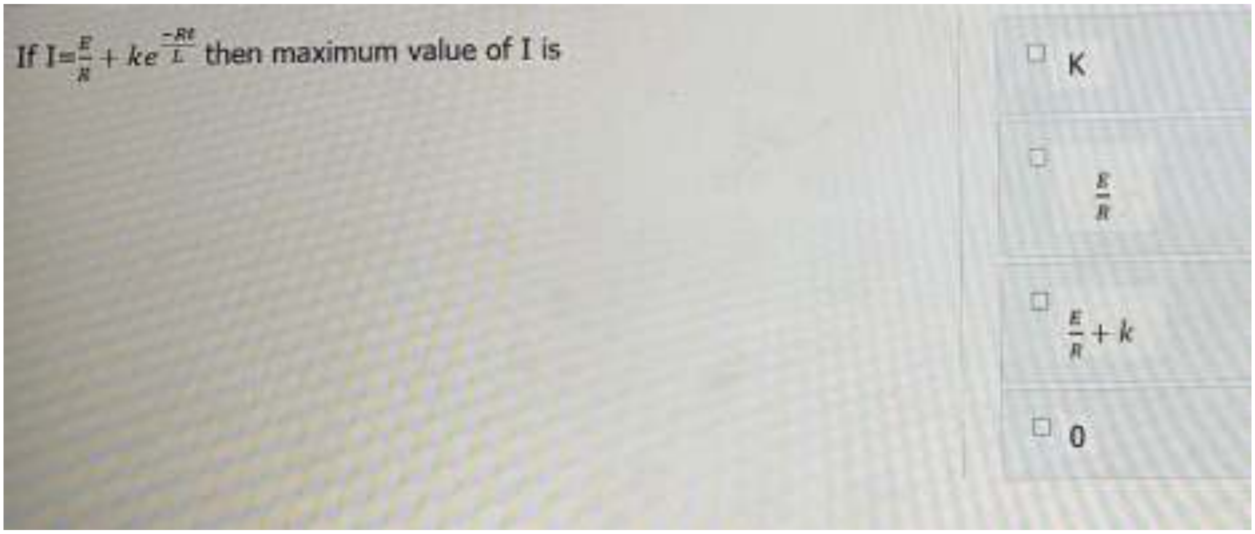


The integrating factor for the differential equation of R-L series circuit is

- $e^{-\frac{Lt}{R}}$
- $e^{\frac{Rt}{L}}$
- $e^{-\frac{Rt}{L}}$
- $e^{\frac{Lt}{R}}$

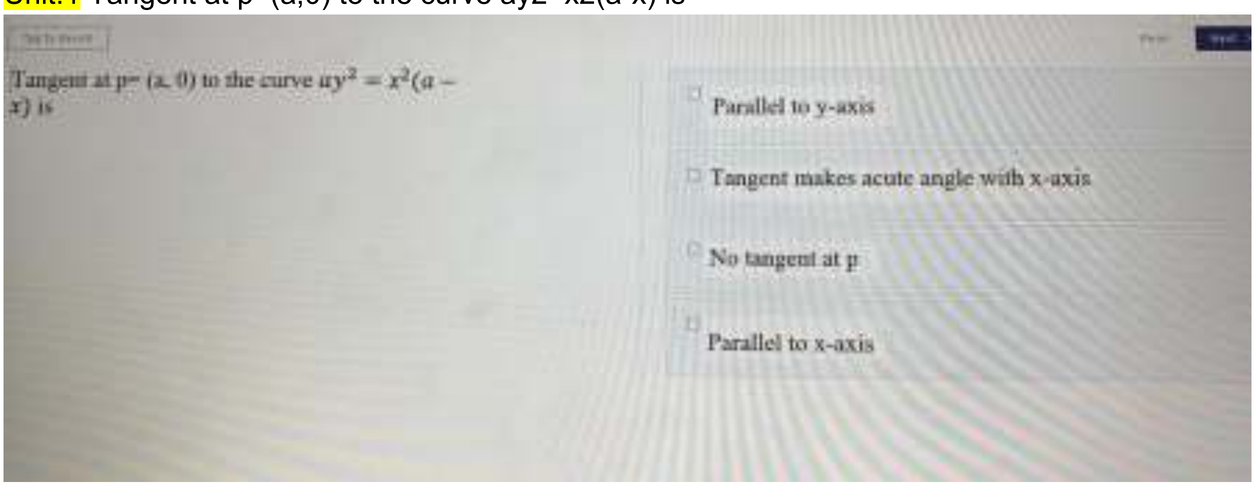
Ans:B e^RT/L

Unit:1: If I=e/r+ke^-rt/l then maximum value of I is



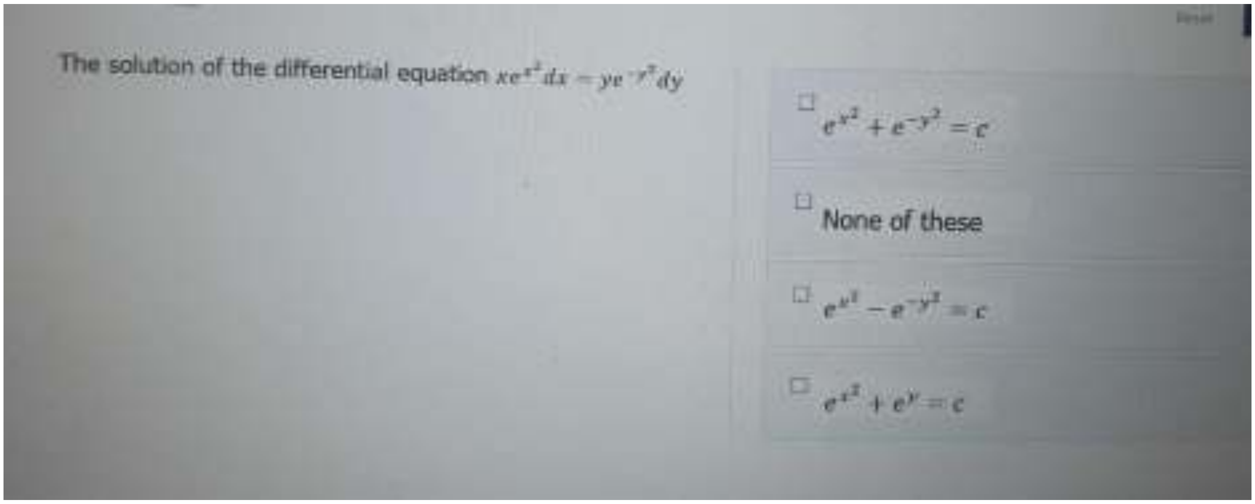
Ans:B(E/R)

Unit:1 Tangent at p=(a,0) to the curve ay2=x2(a-x) is



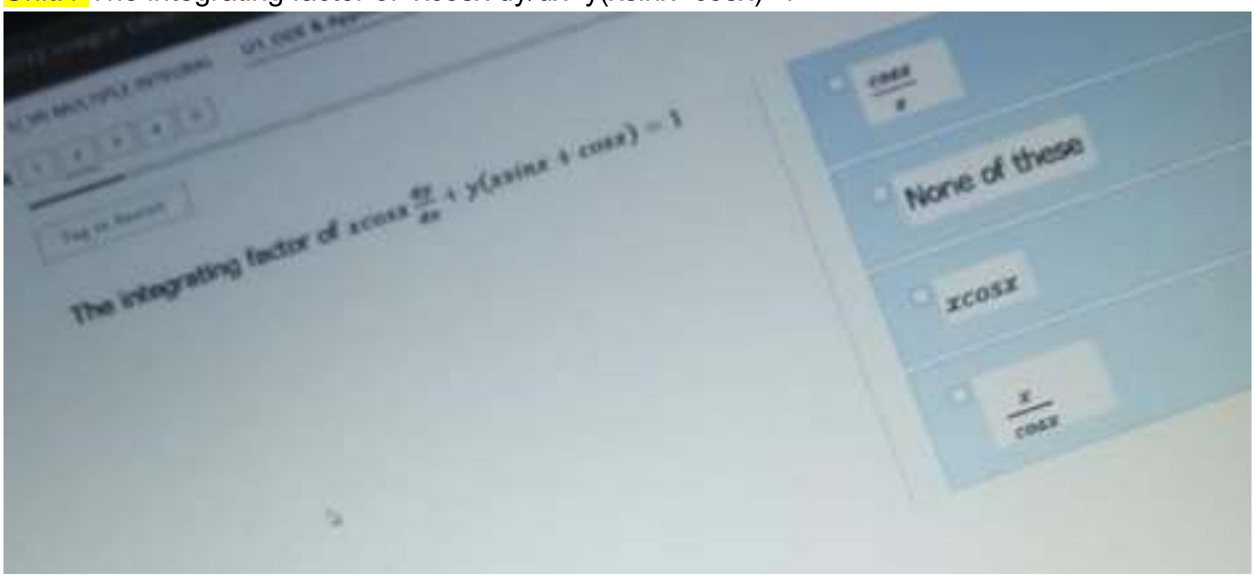
Ans:A (parallel to y axis)

Unit:1 The solution of the differential equation xe^x2.dx=y.e^y2.dy



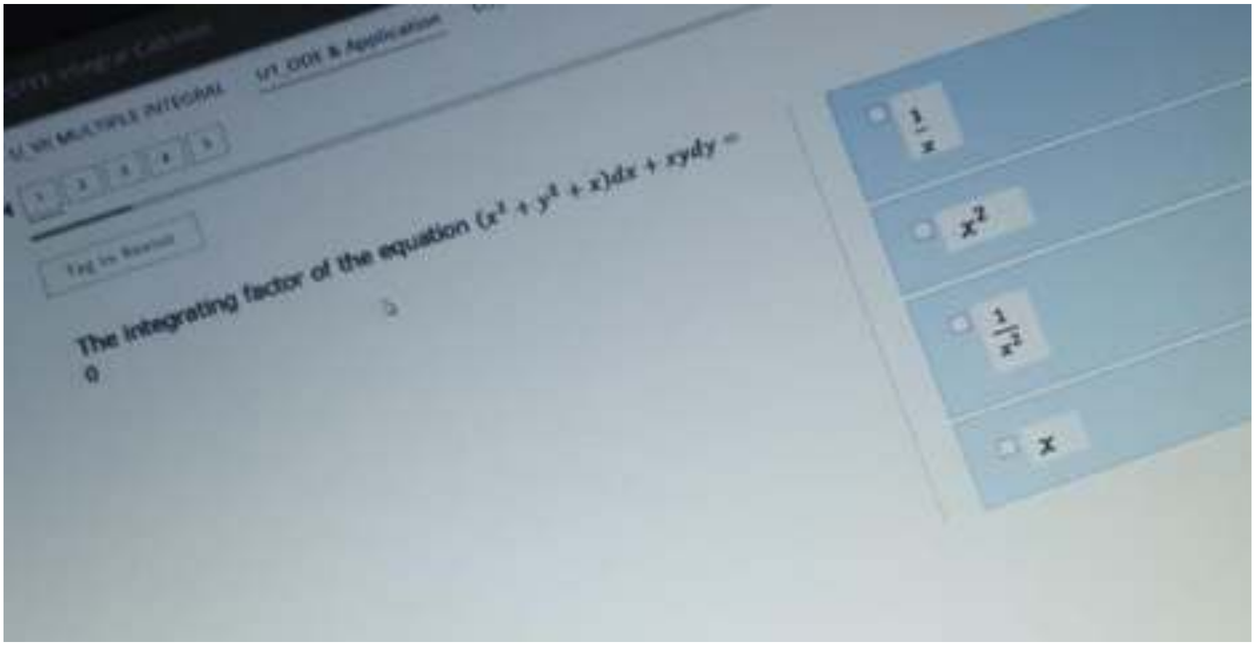
Ans:A

Unit:1 The integrating factor of xcosx dy/dx+y(xsinx+cosx)=1



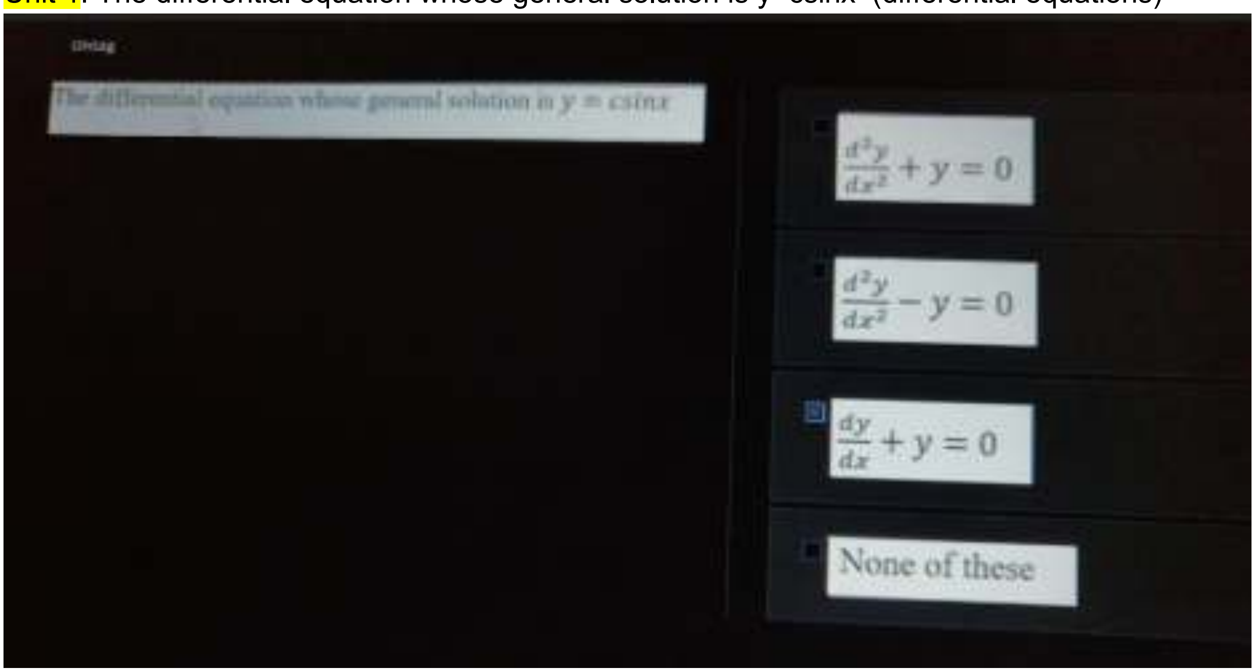
Ans:d

Unit:1 The integrating factor of equation (x^2+y^2+x)dx+xydy=0



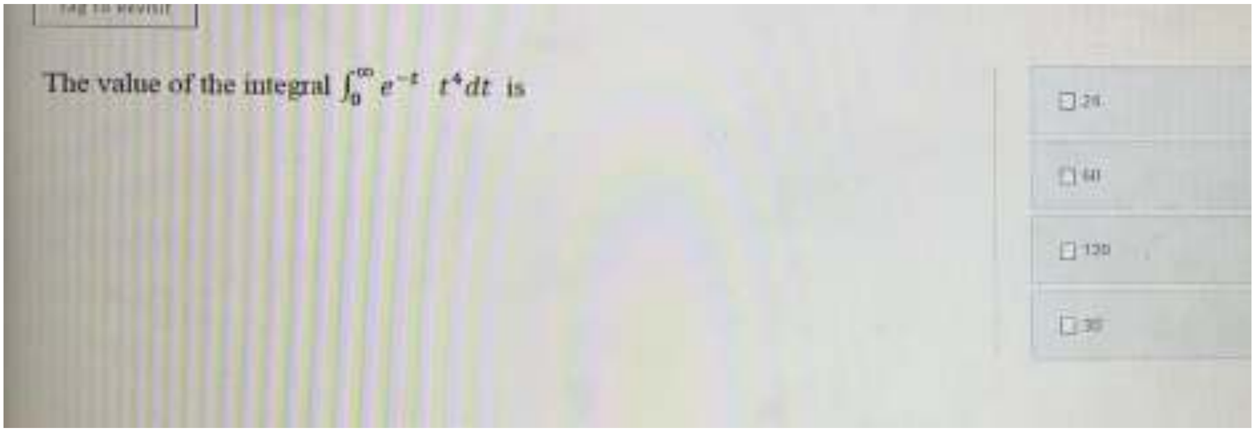
Ans: x (check just in case)

Unit 1: The differential equation whose general solution is y=csinx (differential equations)



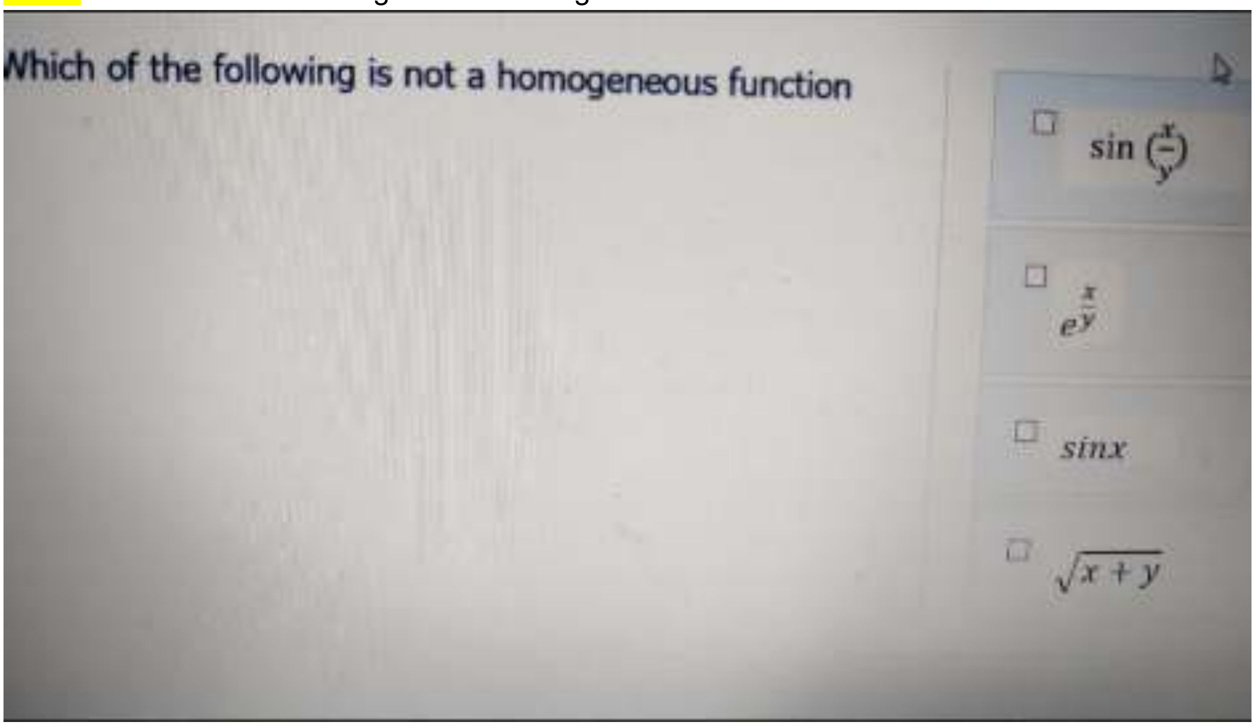
Ans: a

UNIT 1 The value of the integral e^-t t^4dt is



Ans: 24

Unit 1 : Which of the following is not a homogeneous function



Ans : sin x

Unit 1 :The differential equation whose general solution is y=Acos(nx+B) is

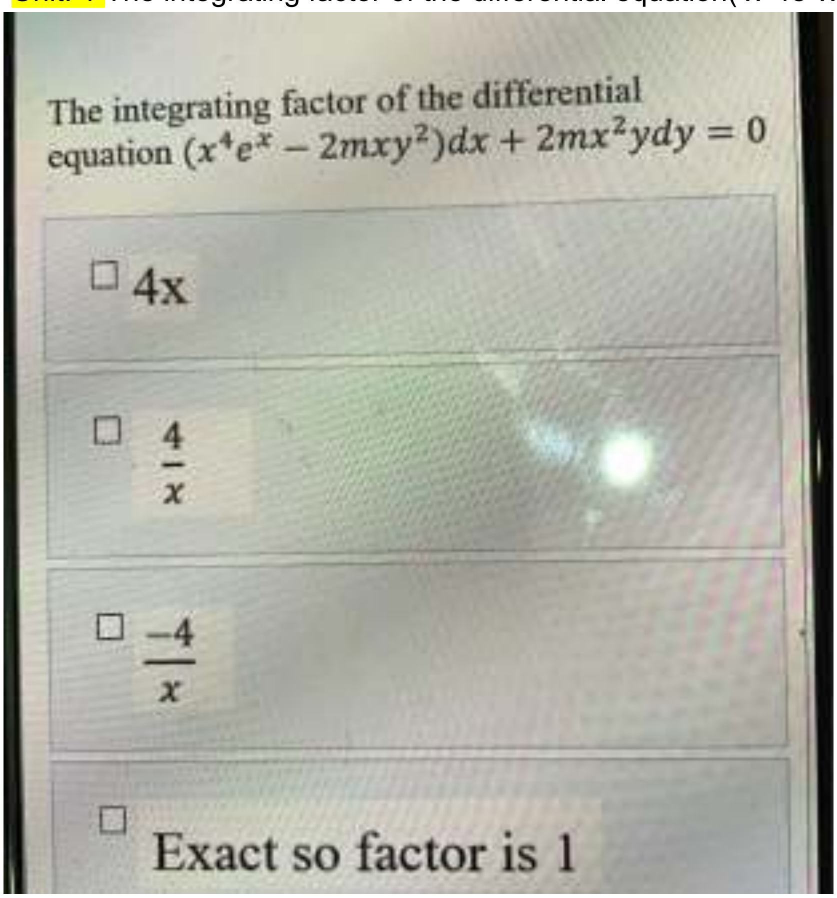
$\frac{d^2y}{dx^2} + ny = 0$

$\frac{d^2y}{dx^2} - n^2y = 0$

$\frac{d^2y}{dx^2} + n^2y = 0$

Ans:C

Unit: 1 The integrating factor of the differential equation(x^4e^x-2mxy)dx



The integrating factor of the differential equation $(x^4e^x - 2mxy^2)dx + 2mx^2ydy = 0$

☐ 4x

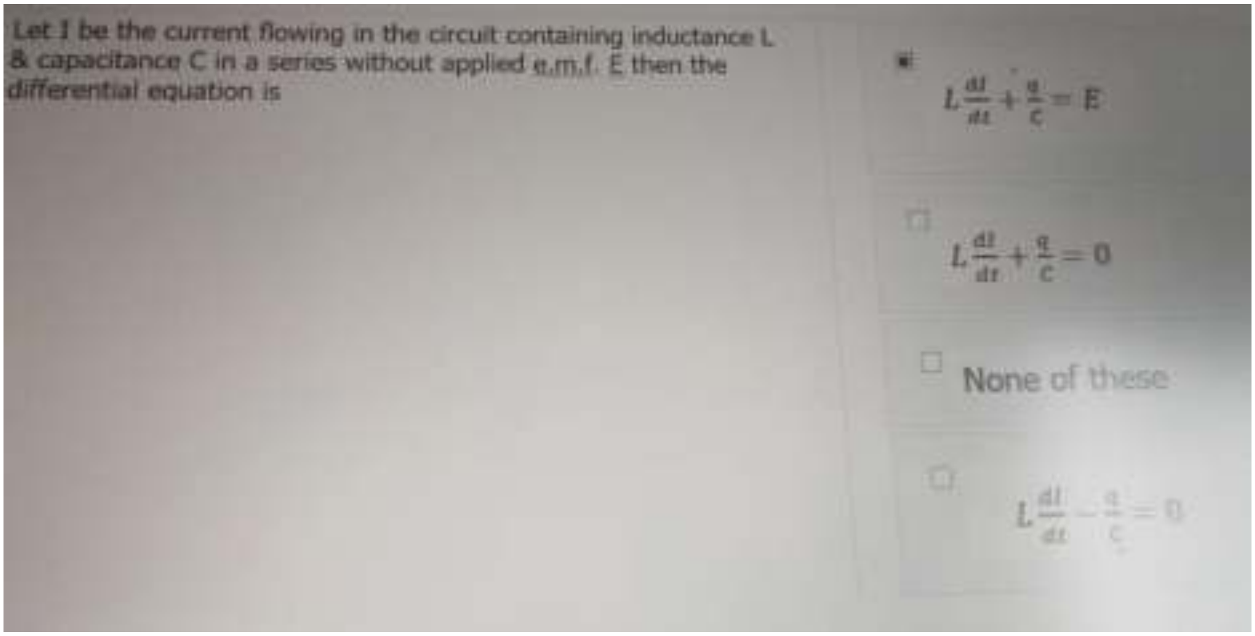
☐ $\frac{4}{x}$

☐ $\frac{-4}{x}$

☐ Exact so factor is 1

Ans:

Unit 1 Let I be the current flowing in the circuit containing inductance


Let I be the current flowing in the circuit containing inductance L & capacitance C in a series without applied e.m.f. E then the differential equation is

$L\frac{dI}{dt}+\frac{q}{C}=E$

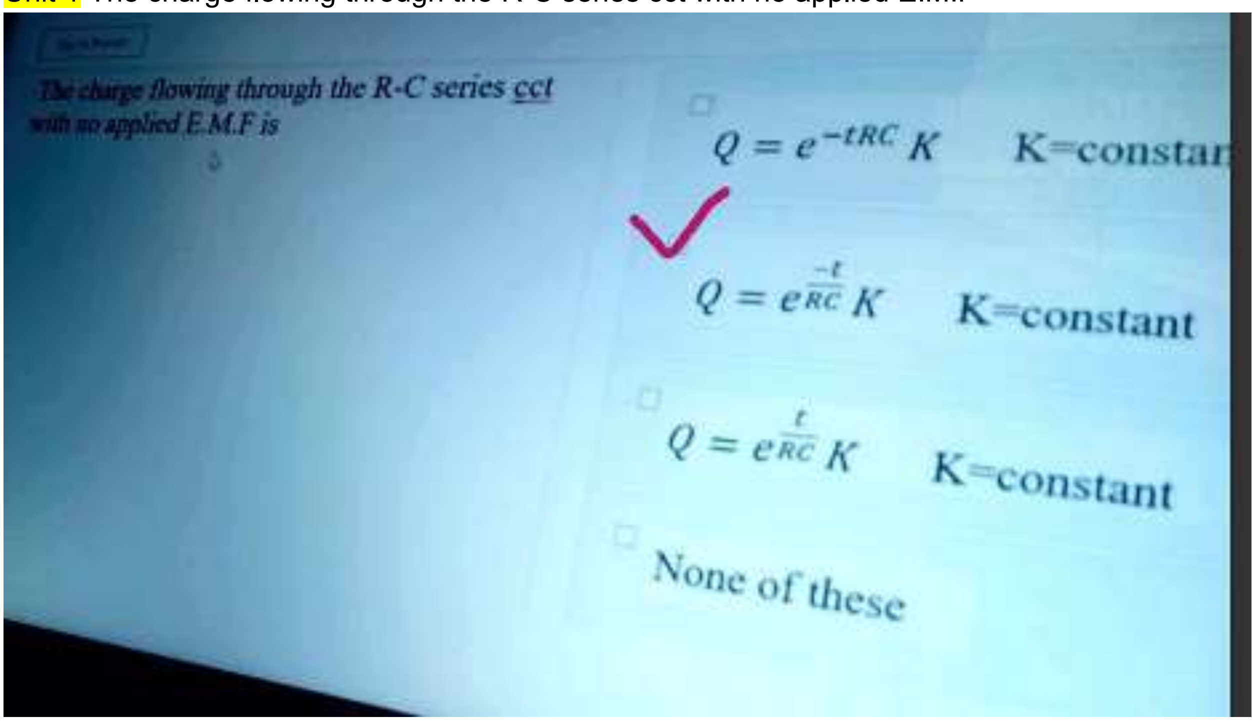
$L\frac{dI}{dt}+\frac{q}{C}=0$

None of these

$L\frac{dI}{dt}-\frac{q}{C}=0$

Ans: B

Unit 1 The charge flowing through the R-C series cct with no applied E.M.F


The charge flowing through the R-C series cct with no applied E.M.F is

$Q=e^{-tRC}K$ K=constan

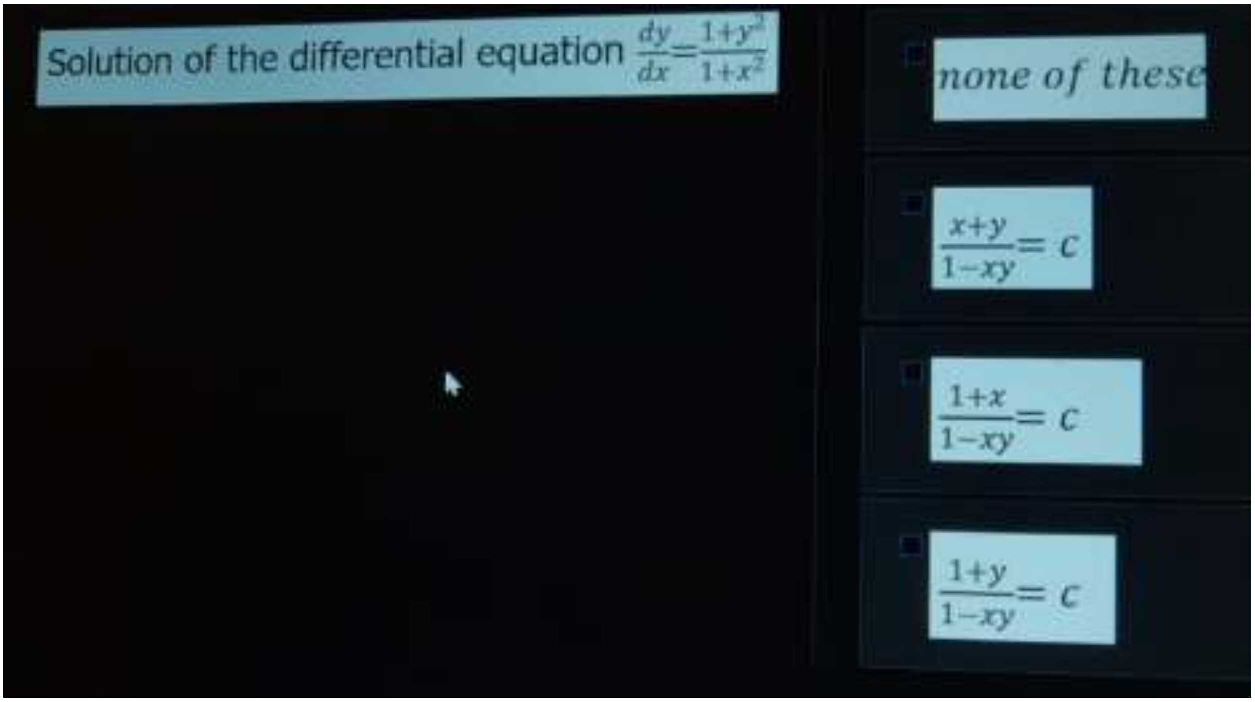
$Q=e^{\frac{-t}{RC}}K$ K=constant

$Q=e^{\frac{t}{RC}}K$ K=constant

None of these

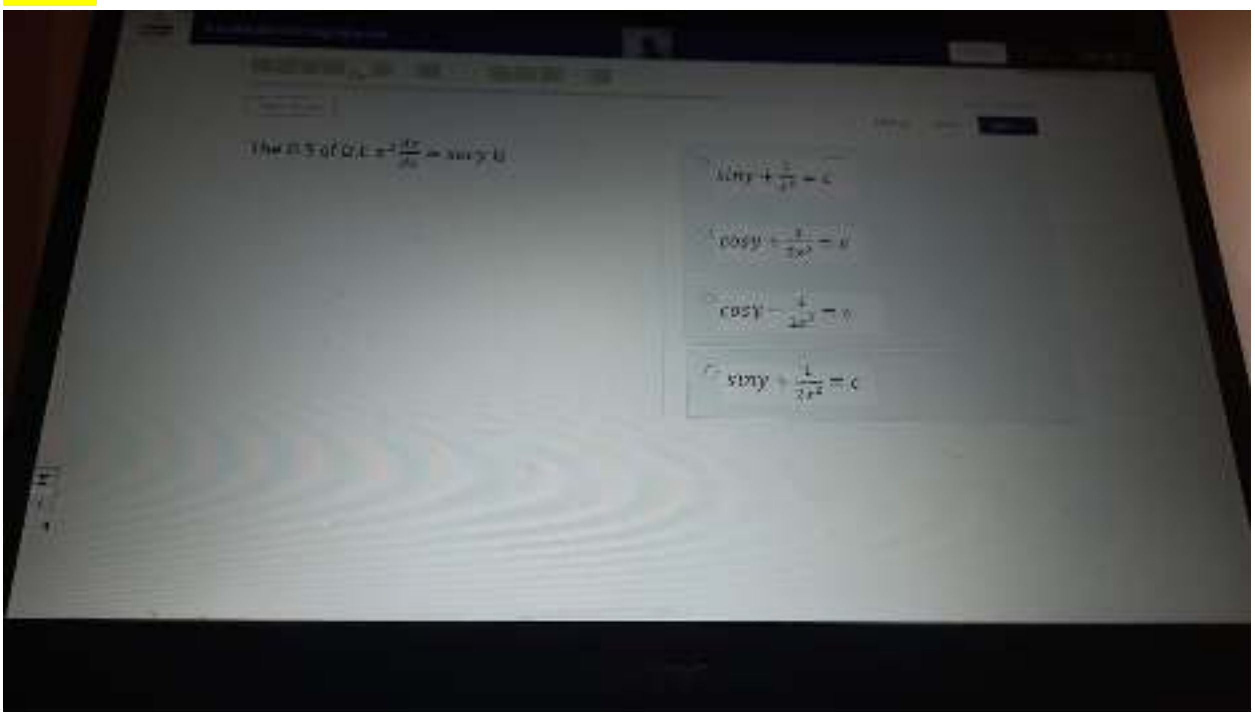
Ans: B

Unit 1 : Solution of the differential equation dy/dx=1+y^2/1+x^2



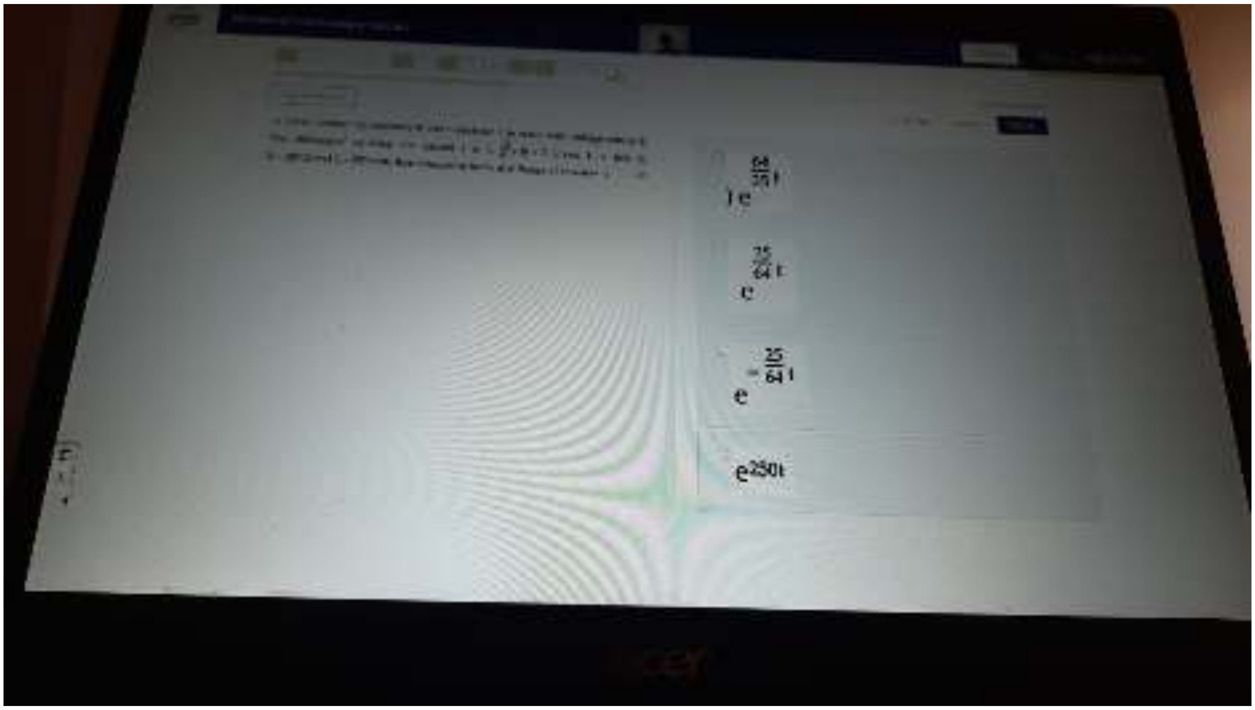
Ans: none of these (ans- $\left[\frac{y-x}{1+yx}\right]$

Unit 1 : The G.S of D.E



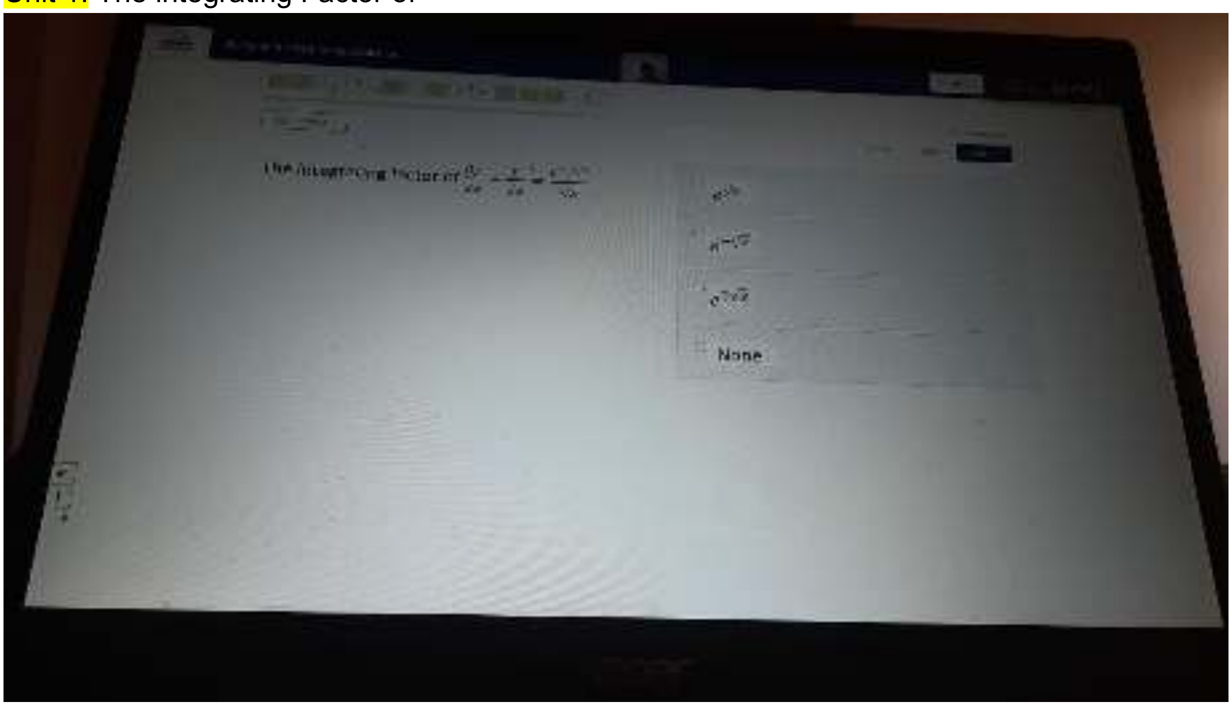
Ans. D

Unit 1 : A circuit containing R and inductance R and Inductance L in series with Voltage source E.



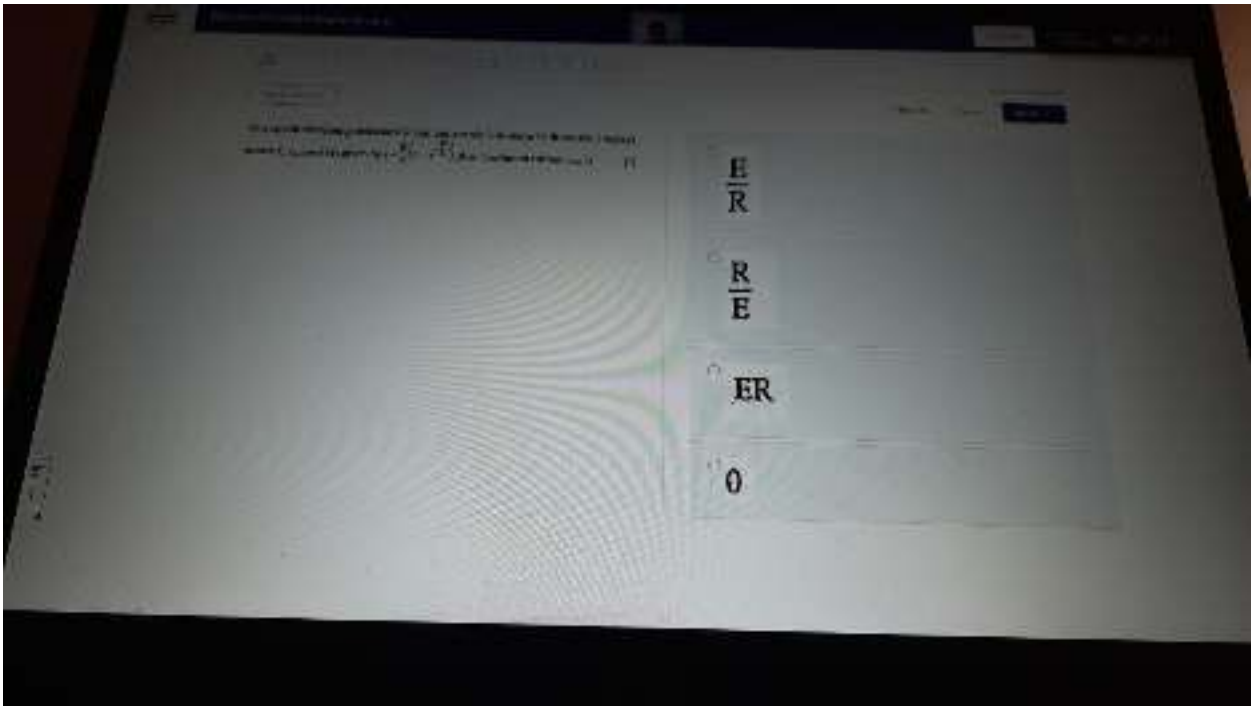
Ans. B

Unit 1: The integrating Factor of



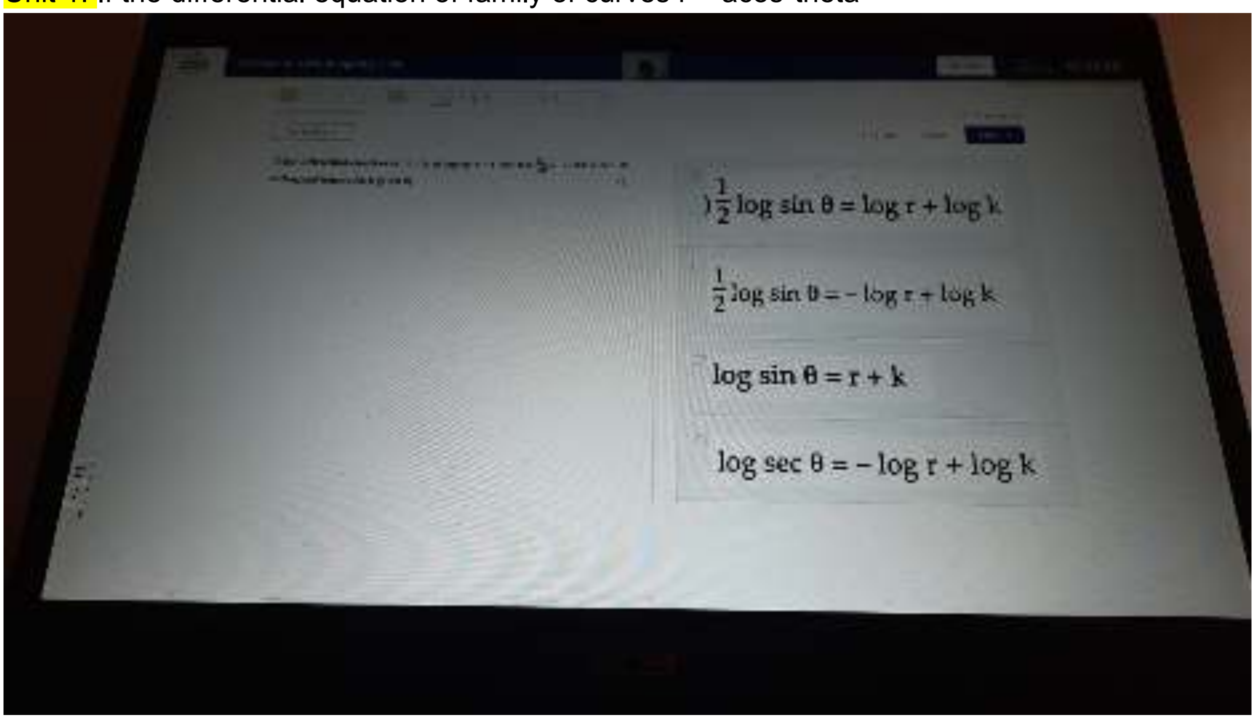
Ans. C

Unit 1 : In a circuit containing resistance R and inductance L in series with constant voltage Source E



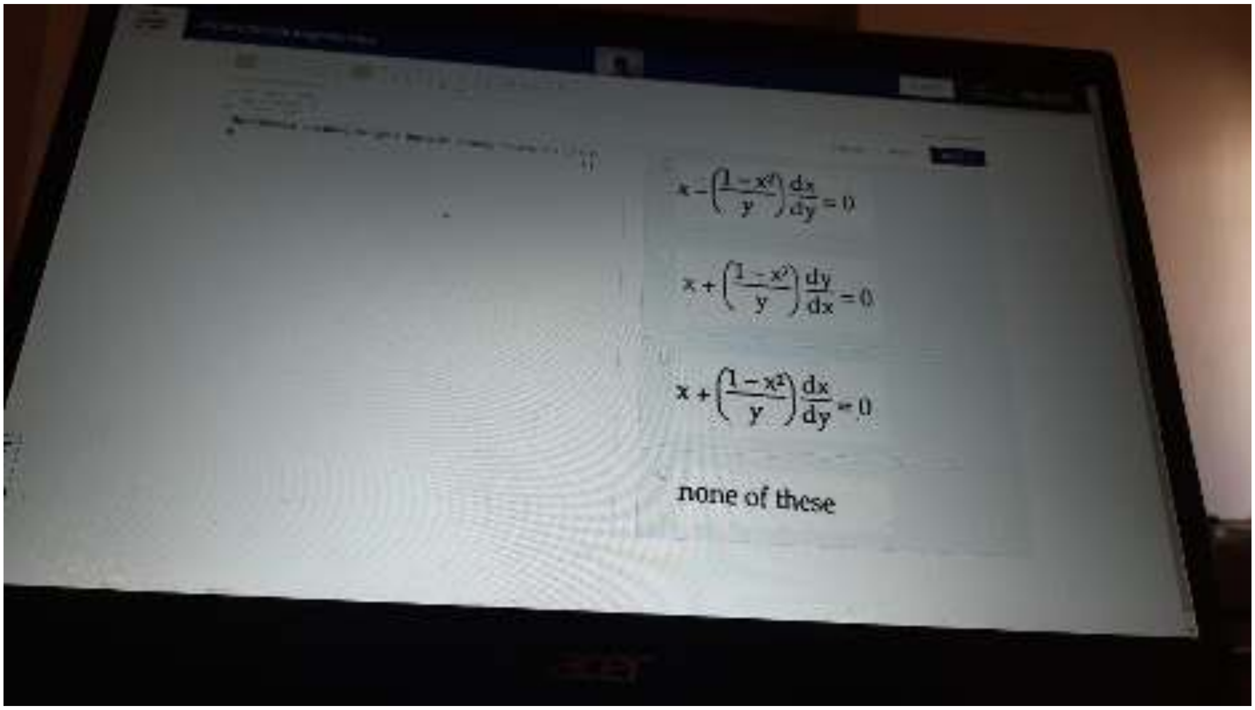
Ans. option A : E/R

Unit 1: If the differential equation of family of curves r = $acos^2$theta



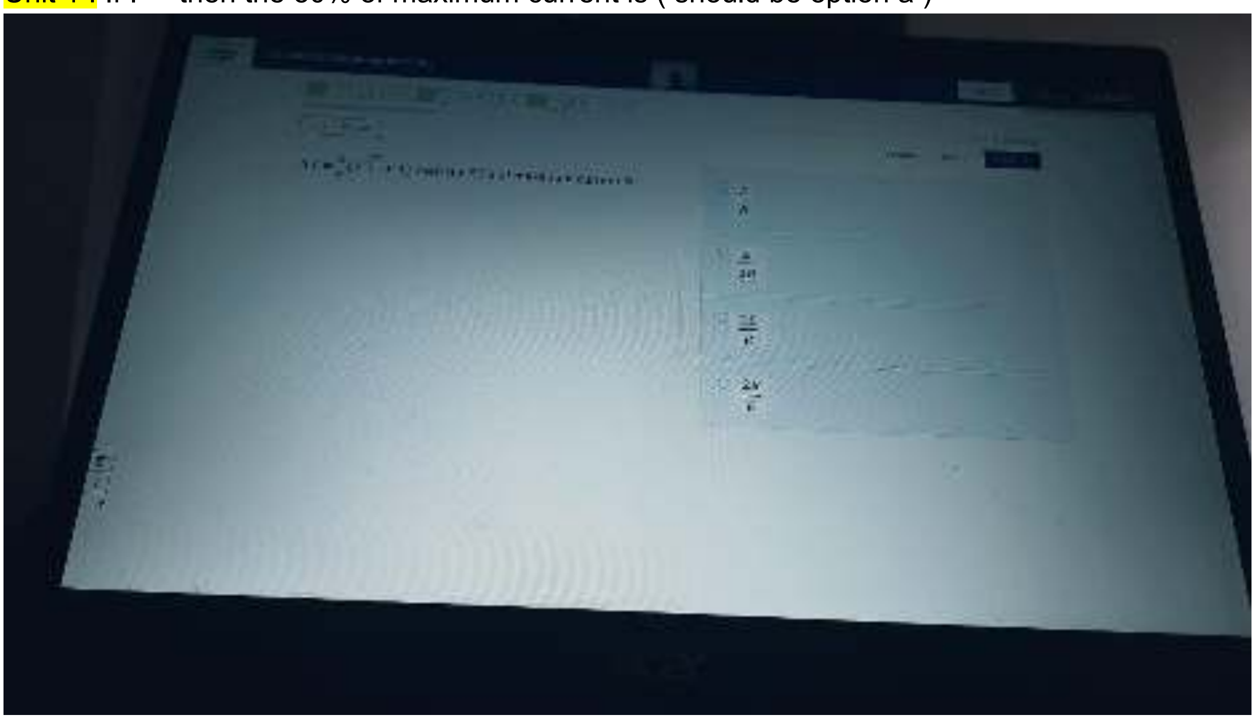
Ans. option B : ½ log sin theta = -log r + log k

Unit 1 : The differential equation of orthogonal trajectories of family of curves



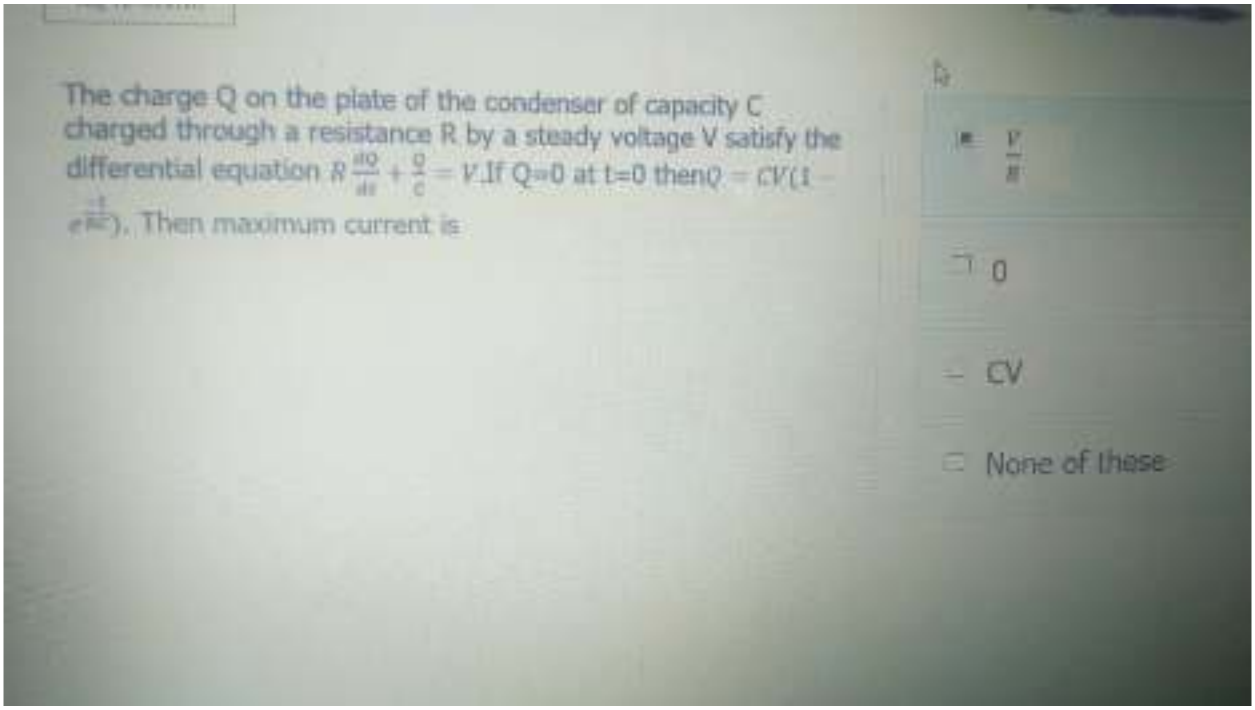
Ans. A

Unit 1 : If I = then the 50% of maximum current is (should be option a)



Ans. A

Unit 1: The charge Q on the plate of the condenser of capacity C charged through resstance R by a steady voltage R by a steady voltage V satisfy the differential equation



Ans. V/R

Unit 1:A steam pipe 20cm in diameter in protected with covering 6cm

A steam pipe 20 cm in diameter is protected with covering 6 cm thick for which thermal conductivity k = 0.0003 in steady state. The inner surface of the pipe is at 200°C and outer surface of the covering is at 30°C and differential equation of conduction of heat is $dT = -\frac{Q}{2\pi k}\frac{dx}{x}$. The amount of heat loss Q is [2]

$$\frac{170\,(2\pi k)}{\log(1.6)}$$

$$-\frac{170\,(2\pi k)}{\log(1.6)}$$

$$\frac{\log(1.6)}{170\,(2\pi k)}$$

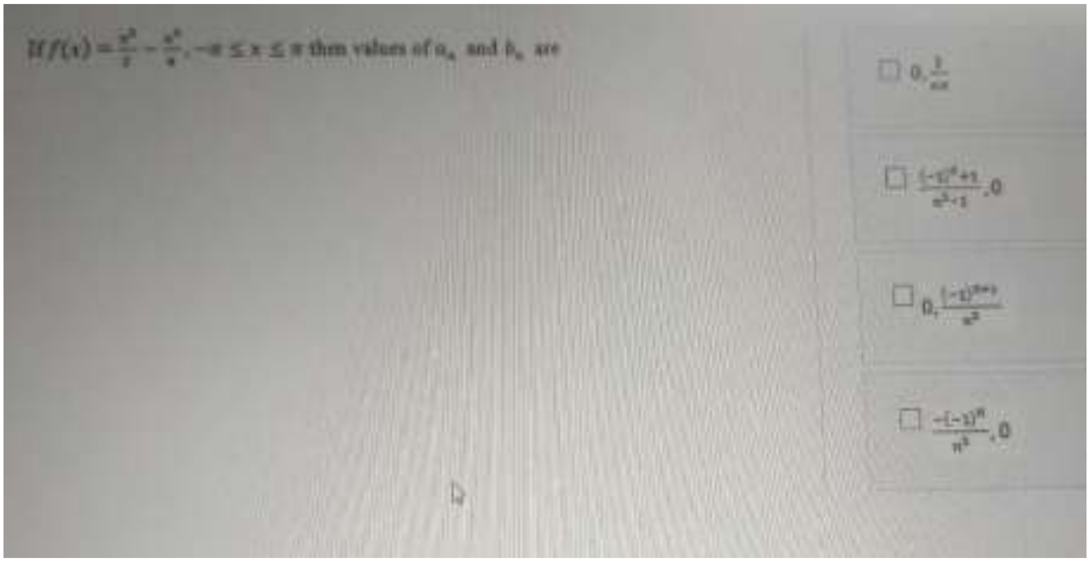
$$\frac{170}{\log(1.6)}$$

Ans: A

Unit: 2

28 questions

Unit:2 If F(x)= , then the values of an and bn are

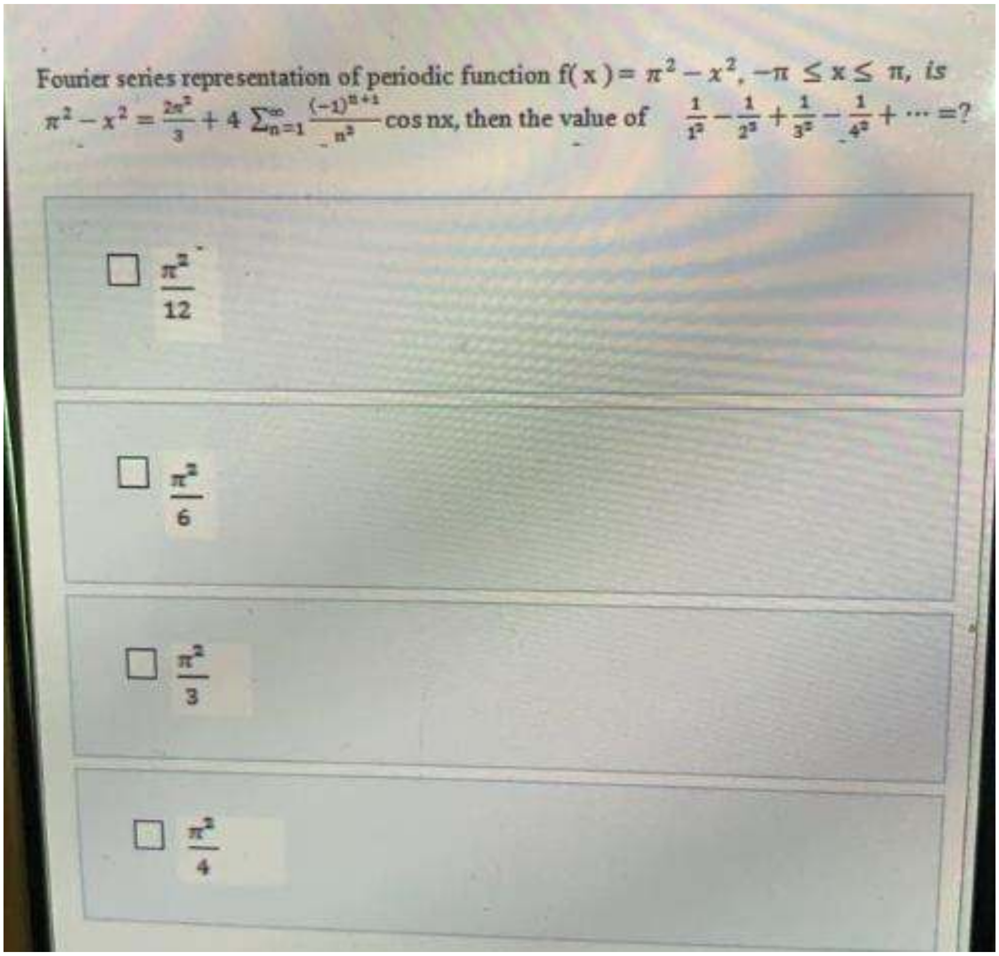


Ans: D

Unit 2: If f(x)=e^x,-1… then constant term of f(x) in fourier expansion is

Ans A

Unit:2 Fourier series representation of periodic function f(x)=pi^2-x^2 Then the value of:1/(1^2)-1/(2^)2+1/(3^2)+...=?

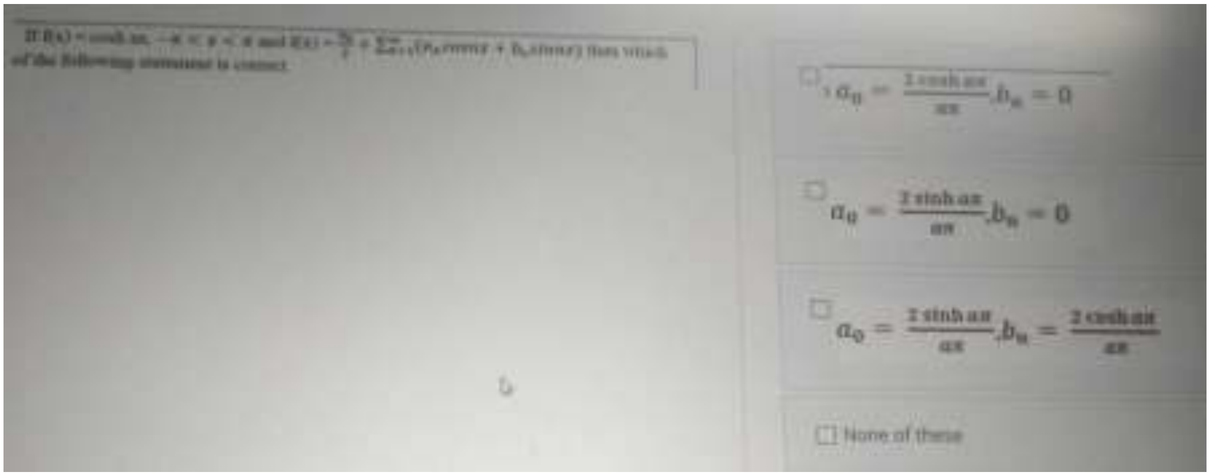


Fourier series representation of periodic function $f(x) = \pi^2 - x^2, -\pi \leq x \leq \pi$, is $\pi^2 - x^2 = \frac{2\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \cos nx$, then the value of $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = ?$

- ☐ $\frac{\pi^2}{12}$
- ☐ $\frac{\pi^2}{6}$
- ☐ $\frac{\pi^2}{3}$
- ☐ $\frac{\pi^2}{4}$

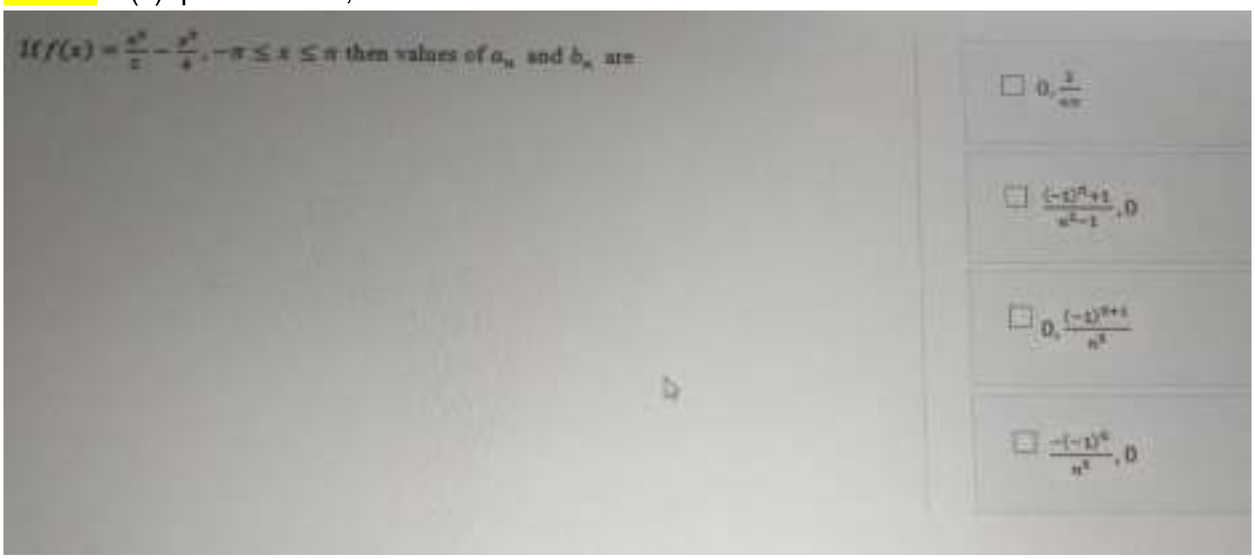
Ans: C

Unit 2: If f(x)=cosh ax,   then which of the following statements is correct.



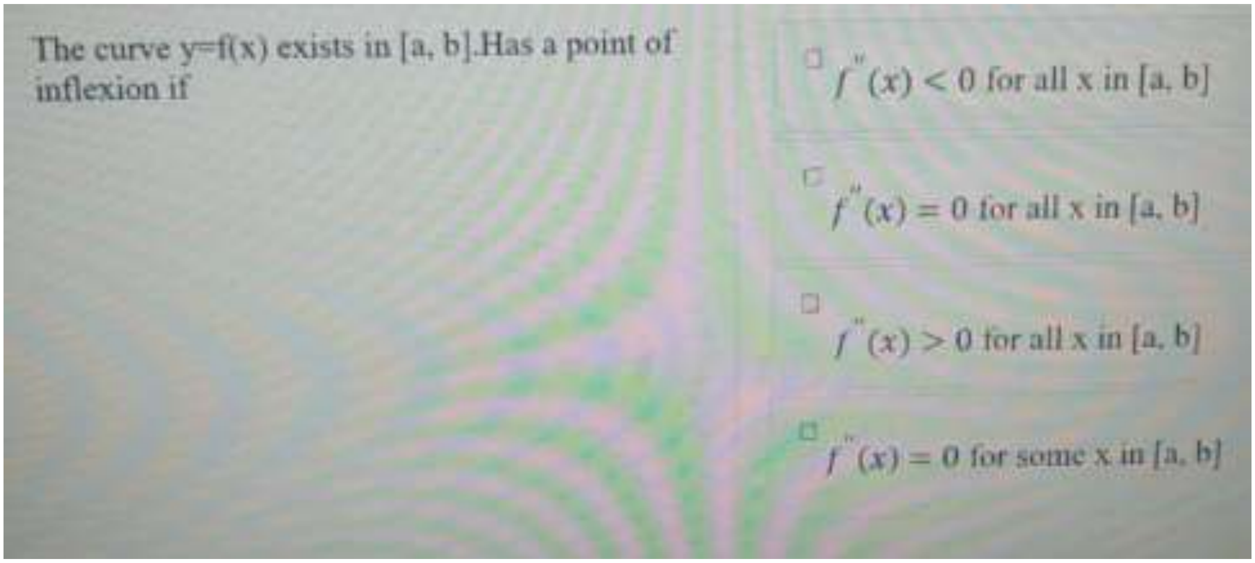
Answer→ option B

Unit 2: If f(x)=pi^2/2 -x^2/4, then values of an and bn are



Ans: option D

UNIT 2 :The curve y+f(x) exists in [a,b] .Has a point of inflexion if

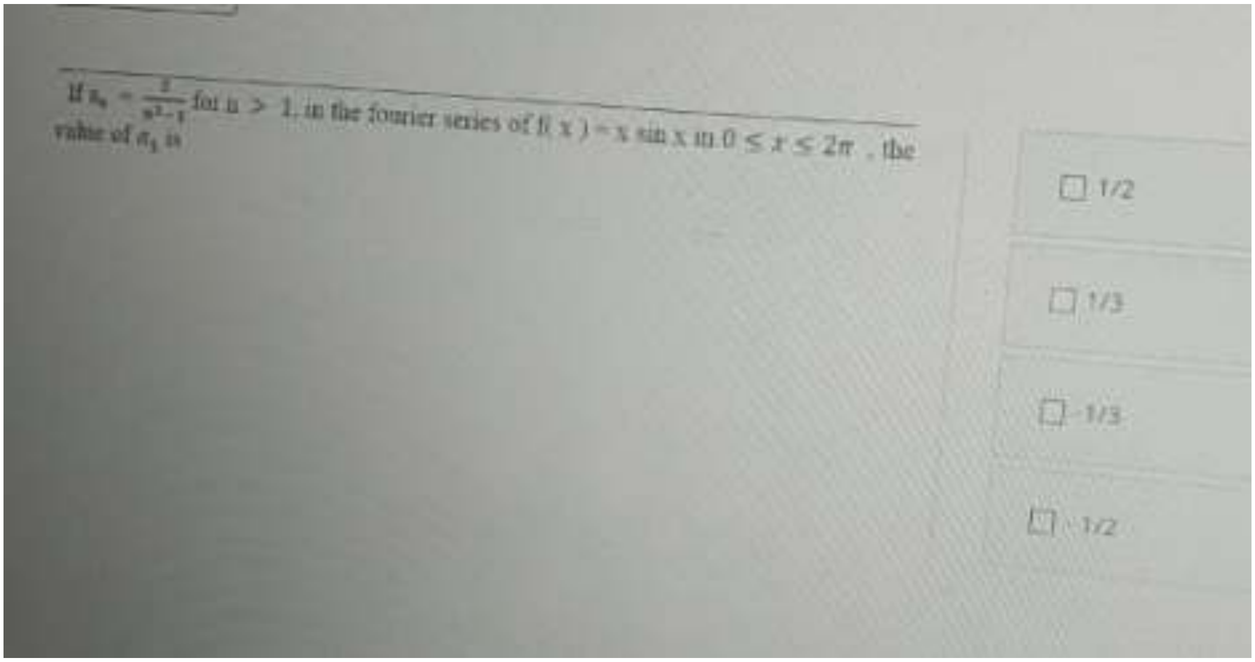


Ans:D f”x = 0 for some x in [a,b]

Unit:2 If f(x) the constant term of f(x)= in fourier expansion

Ans:A

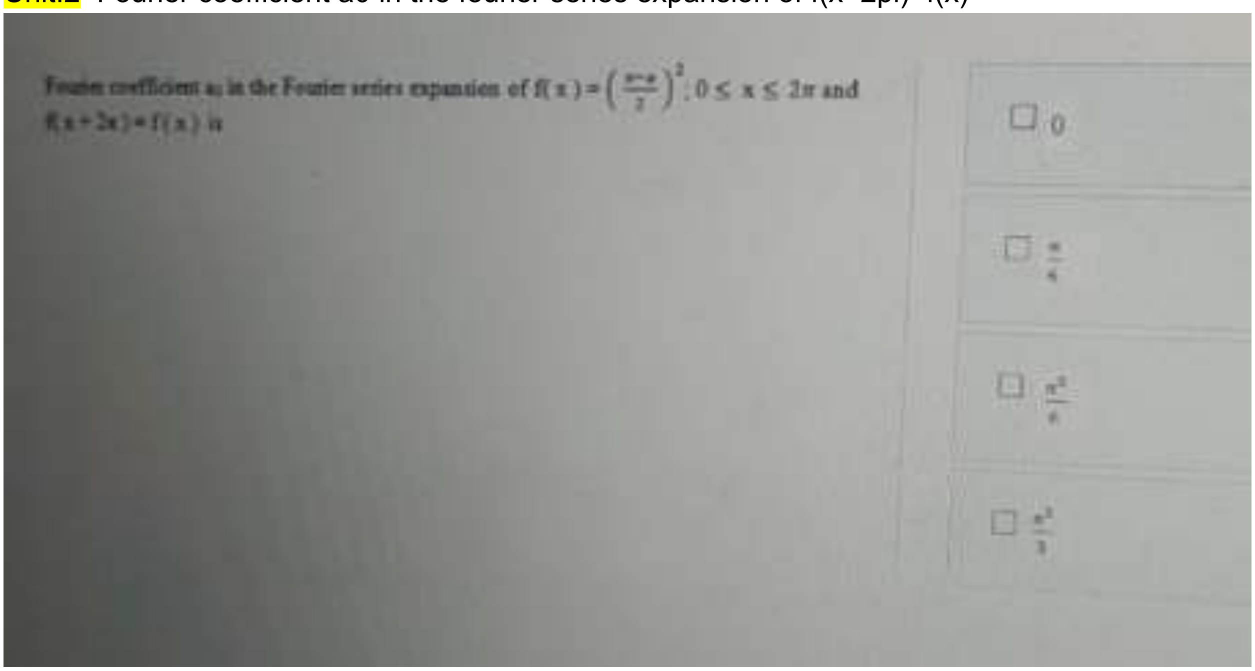
Unit:2 IF $a_n=2/n^2-1$ for u > 1, the the fourier series of f(X)=x sin x in 0 less than= x less than equal to 2pi, the value of a_1 is


If $a_n = \frac{2}{n^2-1}$ for u > 1, in the fourier series of f(x) = x sin x in $0 \le x \le 2\pi$, the value of a_1 is

- 1/2
- 1/3
- -1/3
- -1/2

Ans: -½

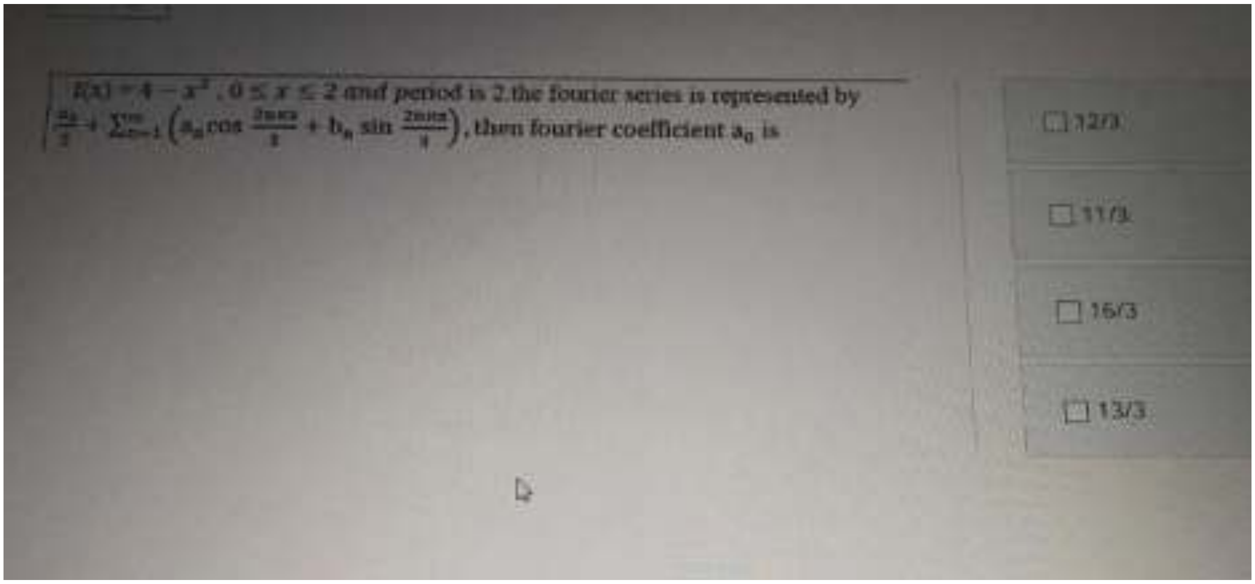
Unit:2 Fourier coefficient a0 in the fourier series expansion of f(x+2pi)=f(x)


Fourier coefficient a_0 in the Fourier series expansion of $f(x) = \left(\frac{\pi - x}{2}\right)^2, 0 \le x \le 2\pi$ and $f(x+2\pi) = f(x)$ is

- 0
- $\frac{\pi}{6}$
- $\frac{\pi^2}{6}$
- $\frac{\pi^2}{3}$

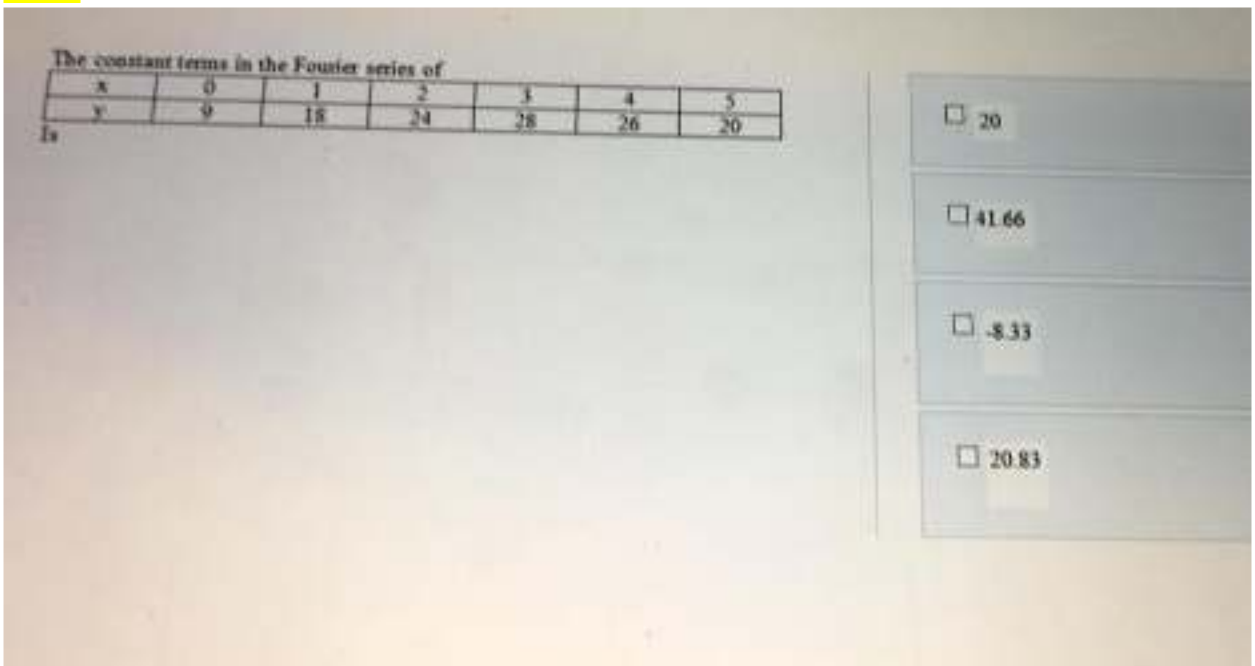
Ans: (pi^2)/12 answer aa rha he

Unit:2 period is 2, the fourier series is represented by, then the fourier coefficient a0 is



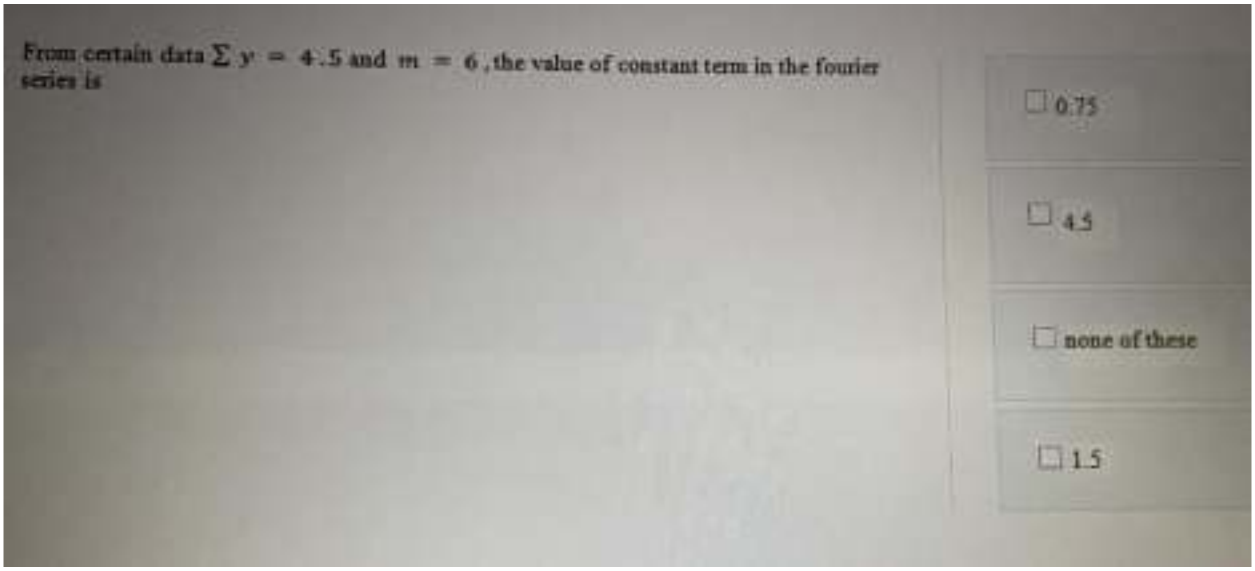
Ans: c(16/3)

Unit:2 The Constant terms in the fourier series of



Ans:D(20.83)

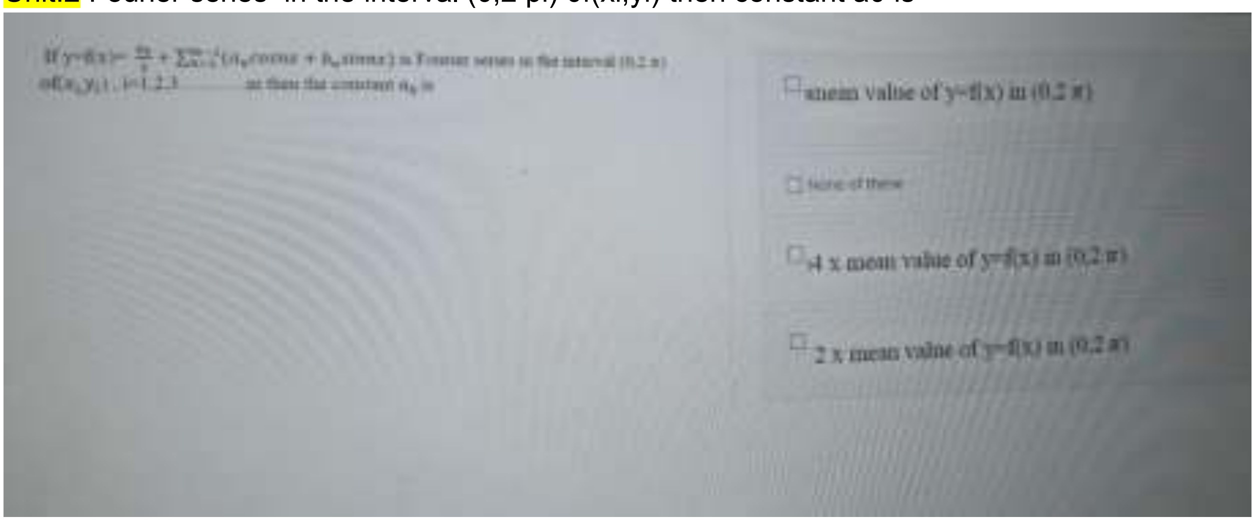
Unit:2 From certain Data sigma y=4.5 and m=6, the value of constant term in the fourier series is


From certain data $\sum y = 4.5$ and $m = 6$, the value of constant term in the fourier series is

- 0.75
- 4.5
- none of these
- 1.5

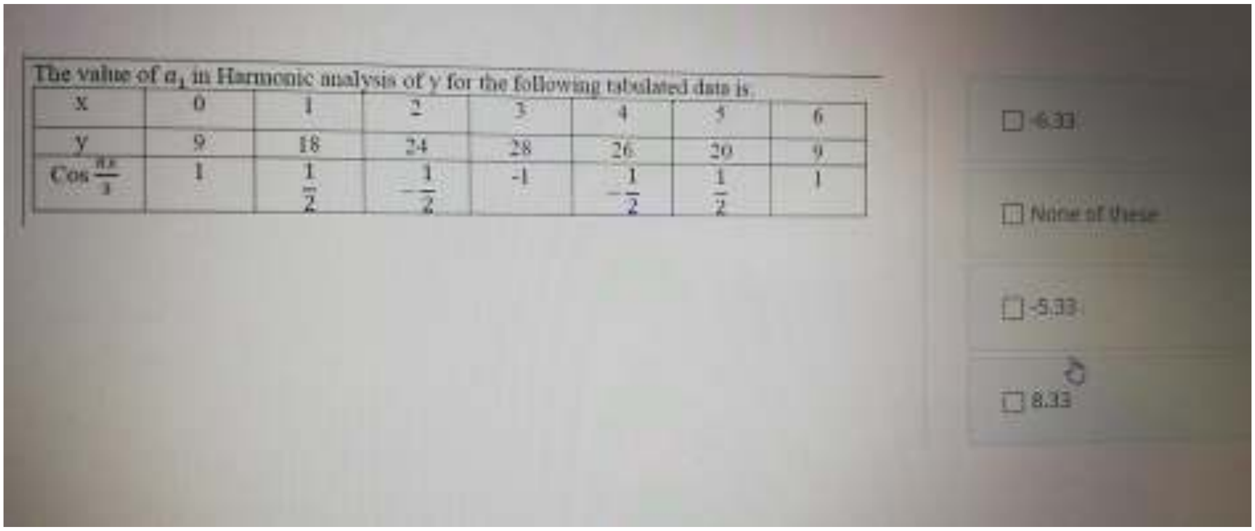
Ans:A(0.75)

Unit:2 Fourier series in the interval (0,2 pi) 0f(xi,yi) then constant a0 is


- mean value of y=f(x) in (0,2 π)
- none of these
- 4 x mean value of y=f(x) in (0,2 π)
- 2 x mean value of y=f(x) in (0,2 π)

Ans: D

Unit:2 The value of a_1 in harmonic analysis of y for the following tabulated data is



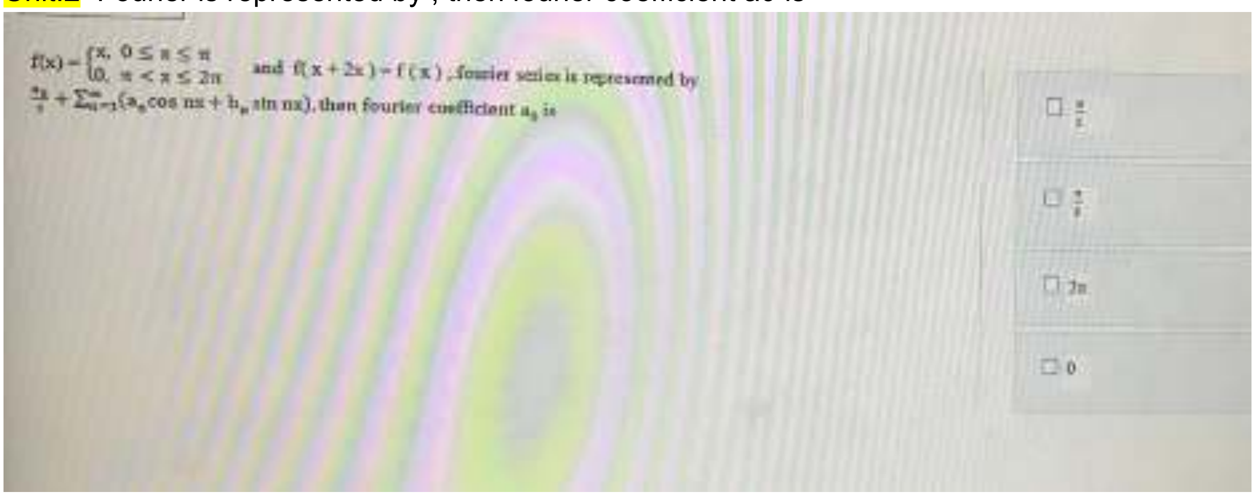
The value of a_1 in Harmonic analysis of y for the following tabulated data is.

x	0	1	2	3	4	5	6
y	9	18	24	28	26	20	9
$\cos\frac{\pi x}{3}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1

- -6.33
- None of these
- -5.33
- 8.33

Ans: B (-1)

Unit:2 Fourier is represented by , then fourier coefficient a0 is

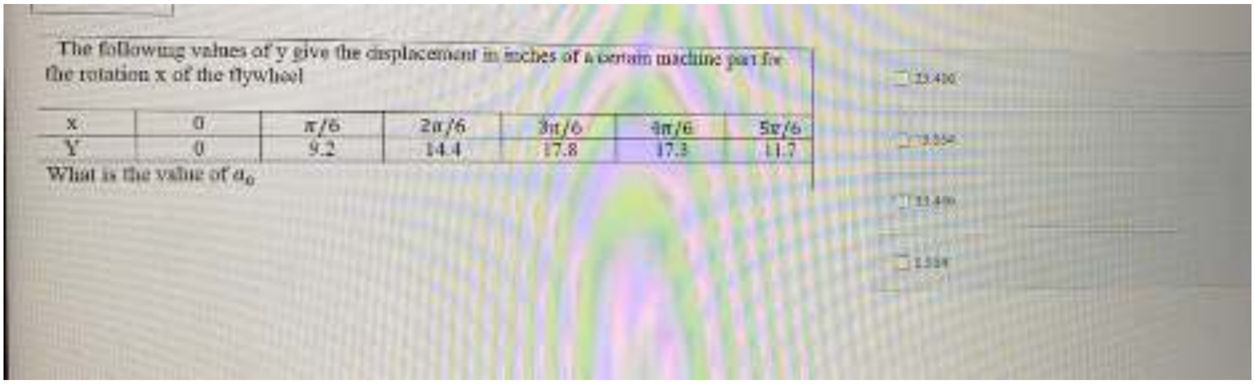


$f(x) = \begin{cases} x, & 0 \le x \le \pi \\ 0, & \pi < x \le 2\pi \end{cases}$ and $f(x + 2\pi) = f(x)$, fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$, then fourier coefficient a_0 is

- [illegible]
- [illegible]
- 2π
- 0

Ans: Pi/2

UNIT 2 The following values of y give the displacement in inches of a certain machine part for the rotation



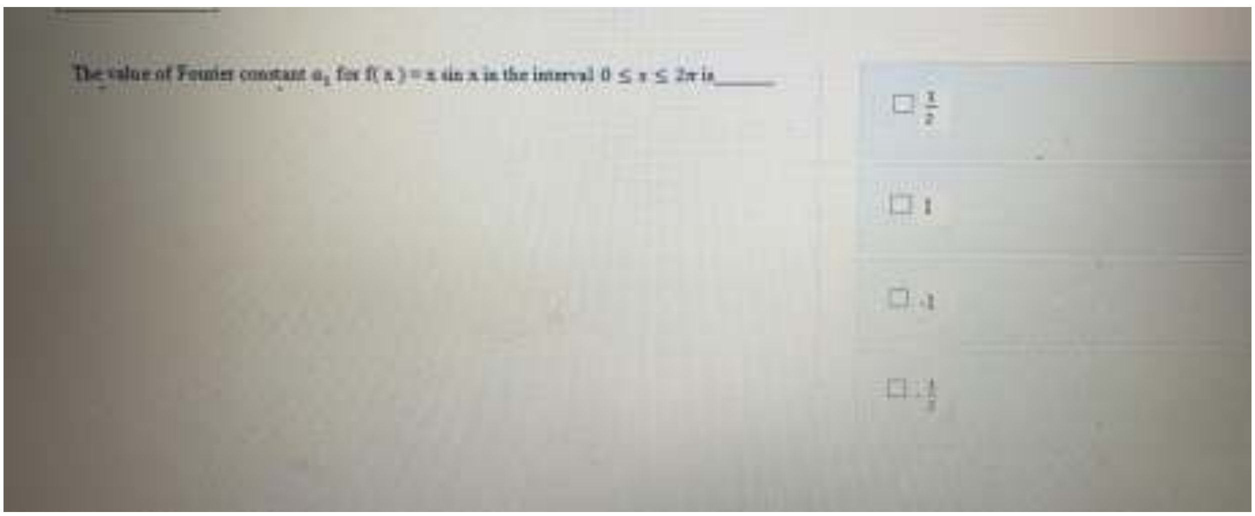
The following values of y give the displacement in inches of a certain machine part for the rotation x of the flywheel

x	0	$\pi/6$	$2\pi/6$	$3\pi/6$	$4\pi/6$	$5\pi/6$
Y	0	9.2	14.4	17.8	17.3	11.7

What is the value of a_0

Ans: a

Unit 2 : The value of fourier constant a1 for f(x)=xsinx in the interval

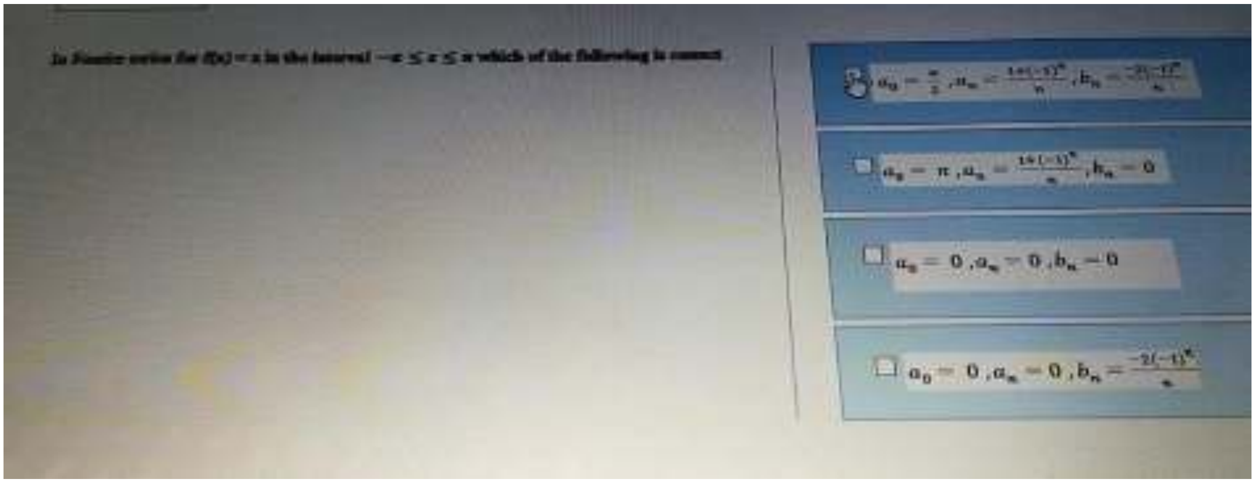


The value of Fourier constant a_1 for $f(x) = x \sin x$ in the interval $0 \le x \le 2\pi$ is______

- $\frac{1}{2}$
- 1
- -1

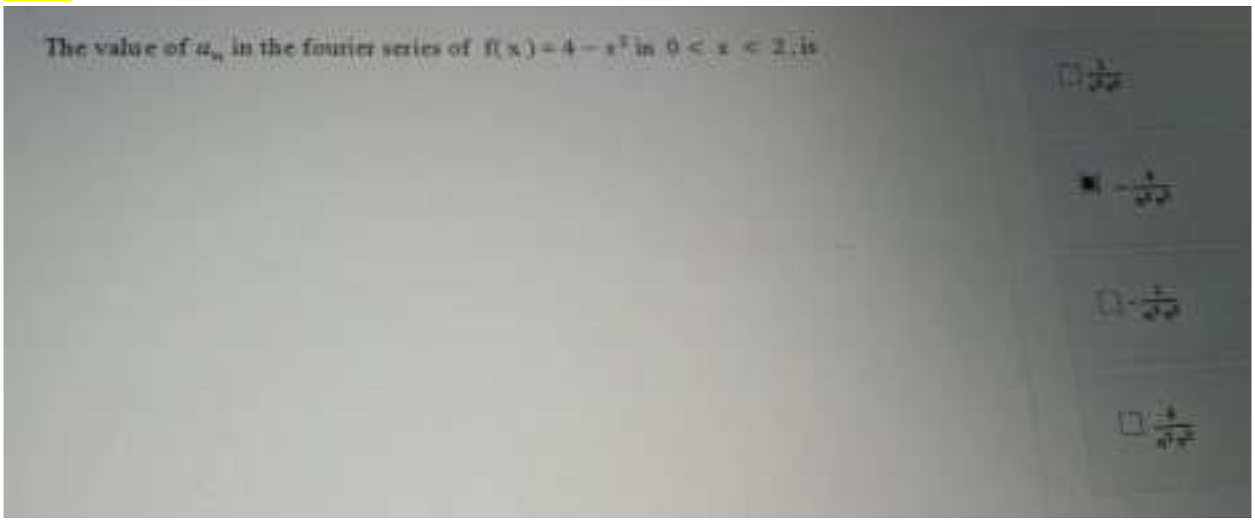
Ans: A = ½

Unit 2 IN fourier series



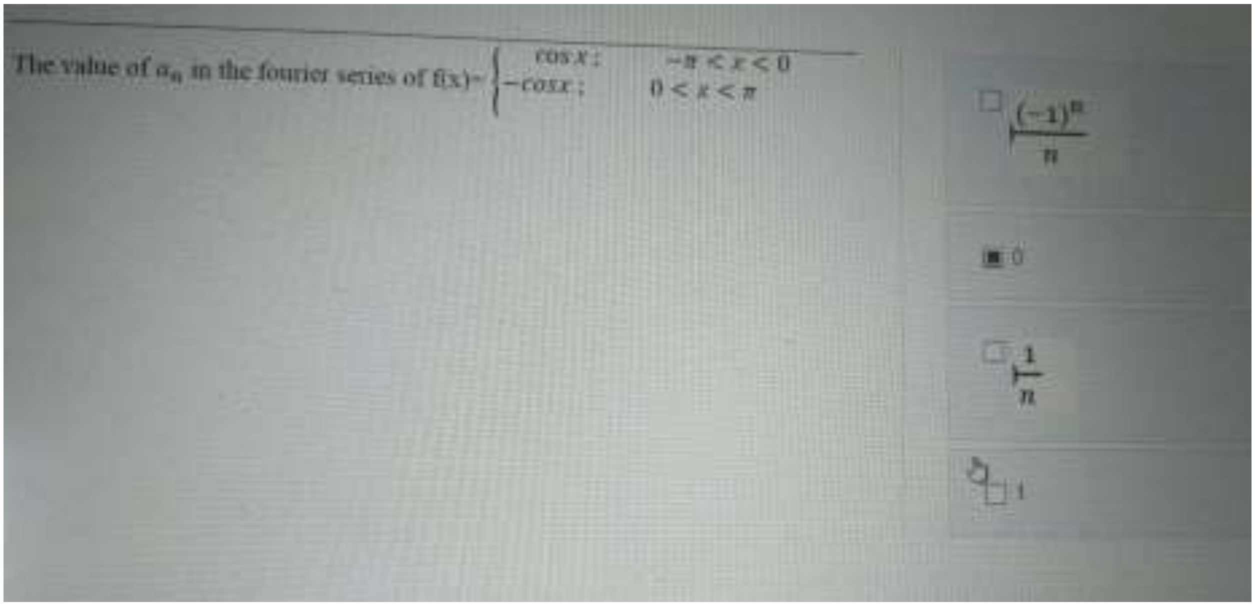
Ans:D

Unit2 The value of a_n in the fourier series of



Ans: $- 4/n^2n^2$

Unit: 2 The value of a_n in the fourier series

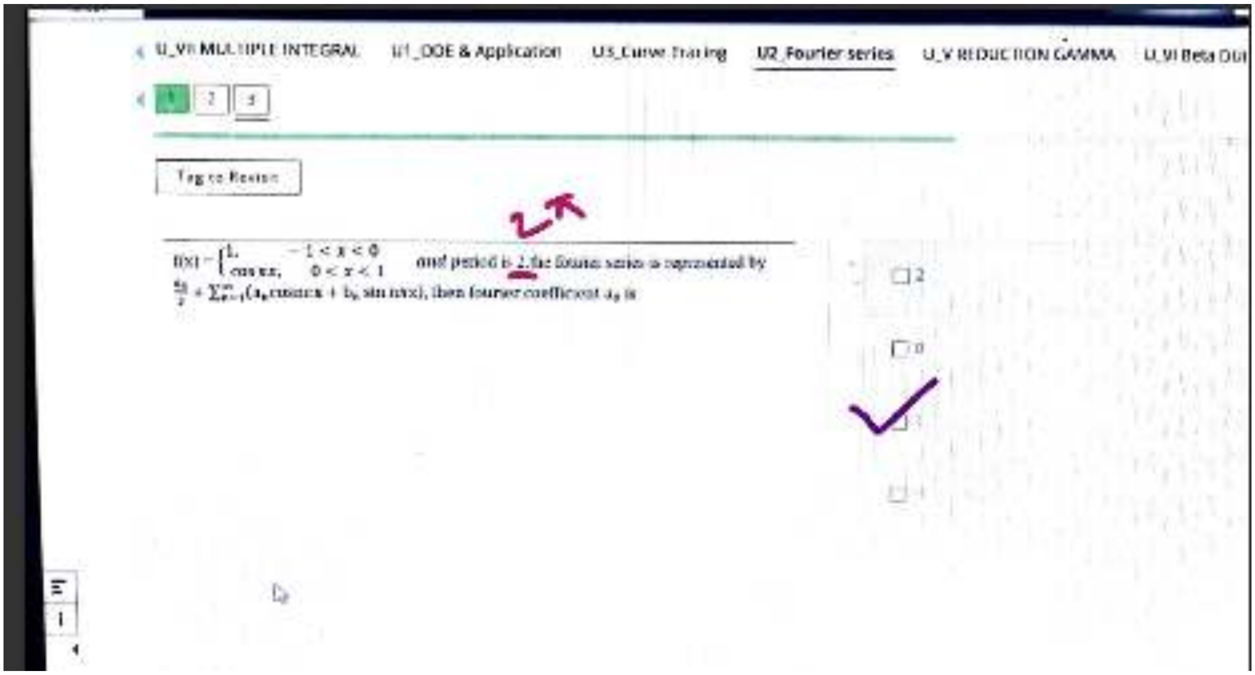


The value of a_n in the fourier series of $f(x)=\begin{cases} \cos x ; & -\pi < x < 0 \\ -\cos x ; & 0 < x < \pi \end{cases}$

- $\frac{(-1)^n}{n}$
- 0
- $\frac{1}{n}$
- 1

Ans: 0

Unit 2 And period is 2. The fourier series is represented by


U_VII MULTIPLE INTEGRAL | U1_ODE & Application | U3_Curve Tracing | U2_Fourier series | U_V REDUCTION GAMMA | U_VI Beta Dui

1 2 3

Tag to Review

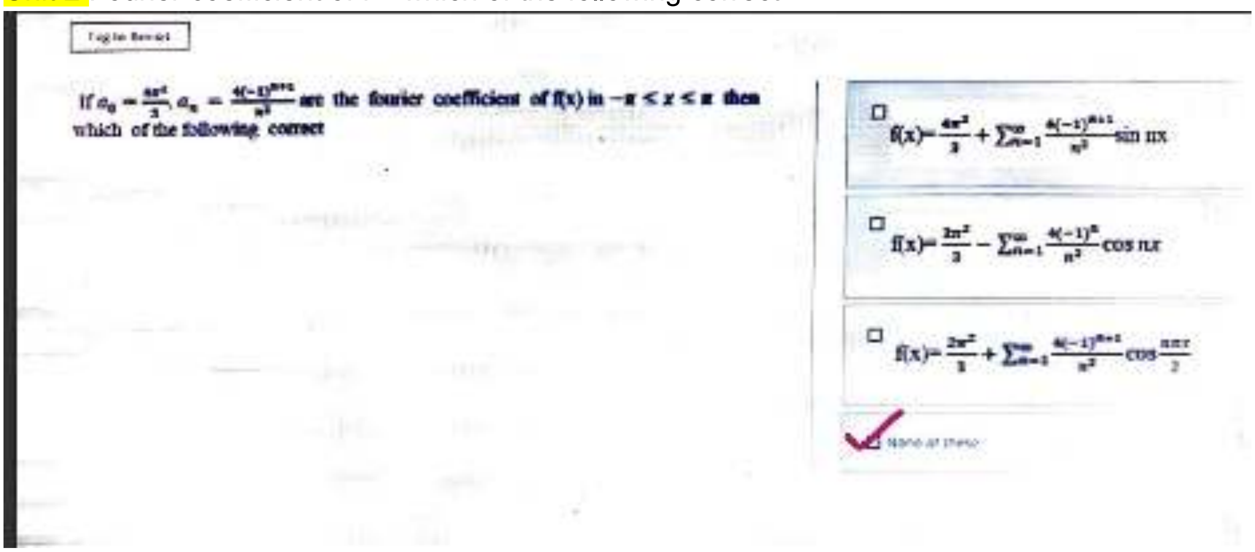
2π

$f(x)=\begin{cases}1, & -1<x<0\\ \cos\pi x, & 0<x<1\end{cases}$ and period is 2 the fourier series is represented by $\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos n\pi x+b_n\sin n\pi x)$, then fourier coefficient a_0 is

- 2
- 0

Answer: 0

Unit 2 Fourier coefficient of f which of the following correct.

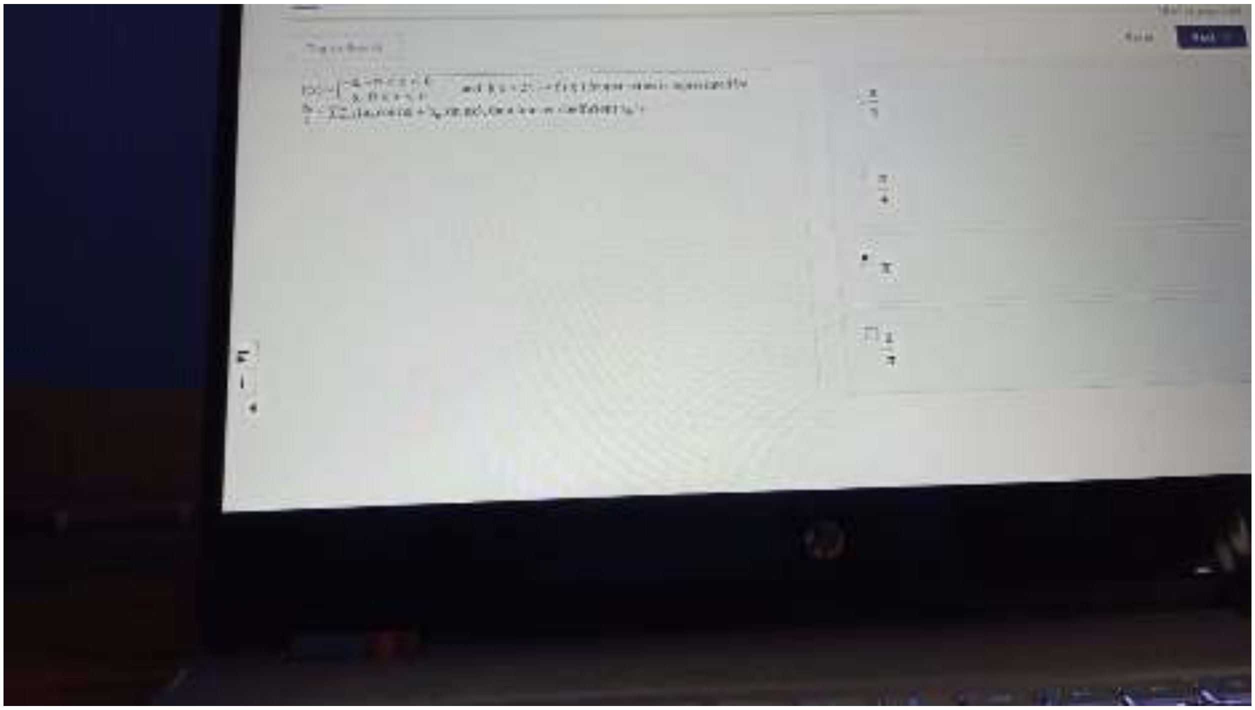

Tag to Review

If $a_0=\frac{4\pi^2}{3}$, $a_n=\frac{4(-1)^{n+1}}{n^2}$ are the fourier coefficient of f(x) in $-\pi\le x\le\pi$ then which of the following correct

- $f(x)=\frac{4\pi^2}{3}+\sum_{n=1}^{\infty}\frac{4(-1)^{n+1}}{n^2}\sin nx$
- $f(x)=\frac{2\pi^2}{3}-\sum_{n=1}^{\infty}\frac{4(-1)^{n}}{n^2}\cos nx$
- $f(x)=\frac{2\pi^2}{3}+\sum_{n=1}^{\infty}\frac{4(-1)^{n+1}}{n^2}\cos\frac{n\pi x}{2}$
- None of these

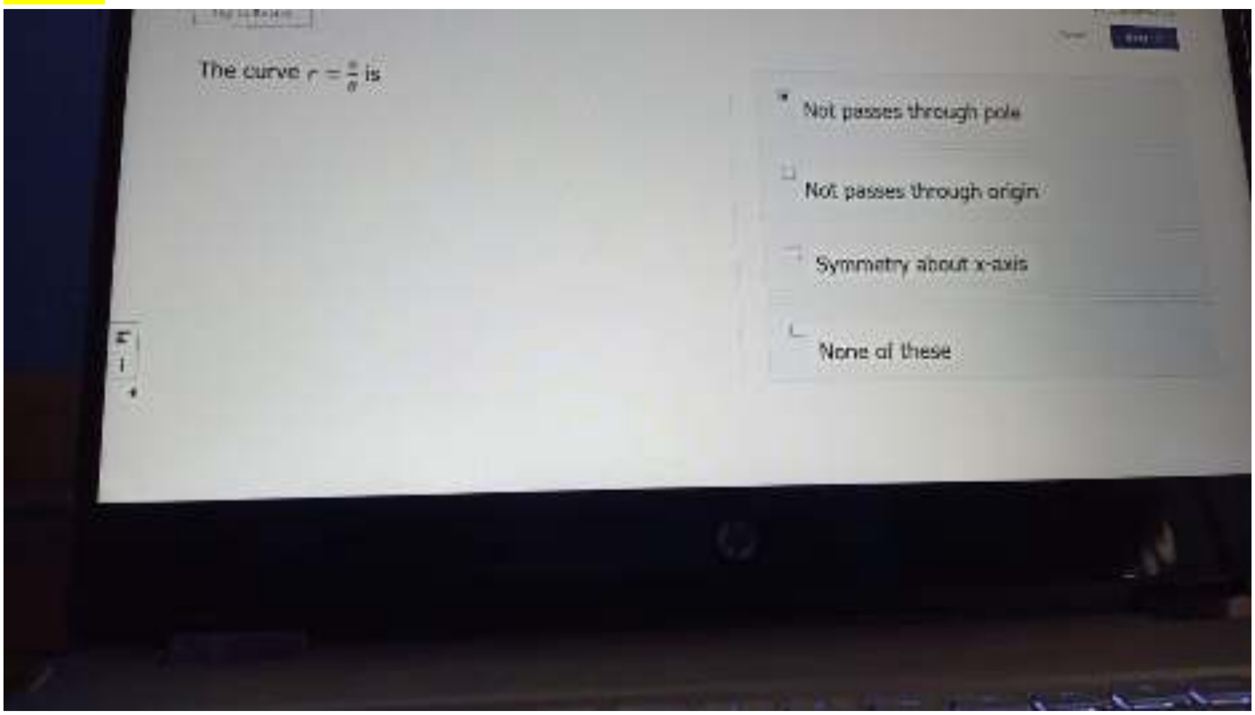
Ans:D

Unit 2 : Fourier series is represented by



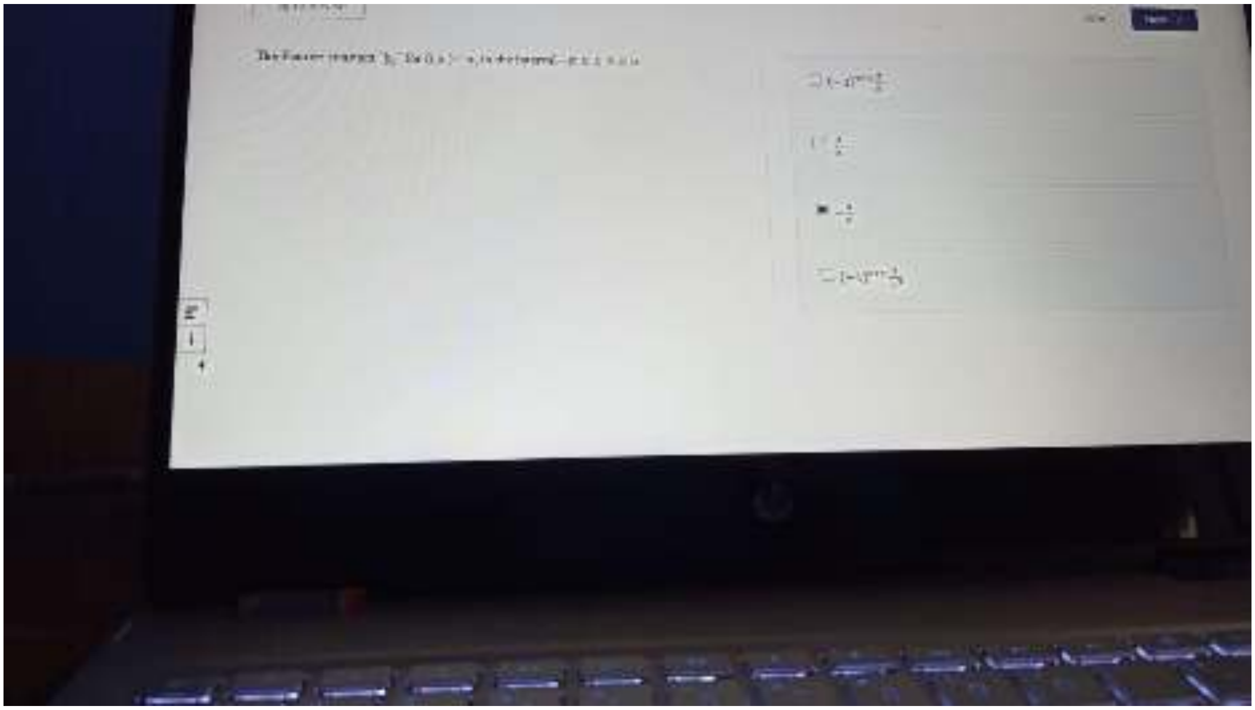
Ans.

Unit 2 : The curve r = a/theta is



Ans. option A

Unit 2 : The Fourier constant ‘b_a’ for f(x) = x,



Ans. Option A

Unit 2: IF a_0 are the fourier coefficient of f(x) in then which of the following correct

Ans: D

Unit 2: The second harmonic fourier series of

The second harmonic Fourier series of

x	0	1	2	3	4	5
y	9	18	24	28	26	20

is ____

- $a_2 \cos \frac{2\pi x}{3} + b_2 \sin \frac{2\pi x}{3}$
- $a_2 \cos \pi x + b_2 \sin \pi x$
- None of the above
- $a_2 \cos 2\pi x + b_2 \sin 2\pi x$

Ans: D

Unit 2: If f(x)=

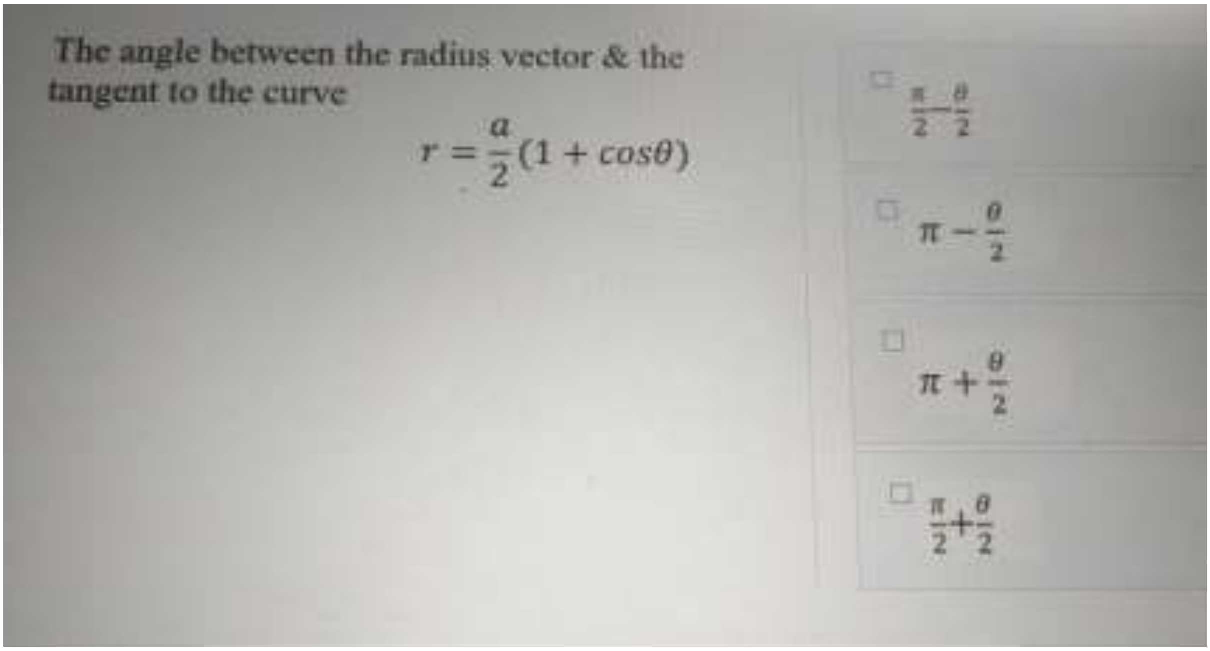
If $f(x) = \begin{cases} -x, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$, then f(x) is

- Even function
- none of these
- Neither even nor odd
- odd function

Ans: Even function

Unit: 3 (22 Questions)

Unit:3 The Angle between the radius vector and the tangent to the curve r=a/2 (1+cos theta)



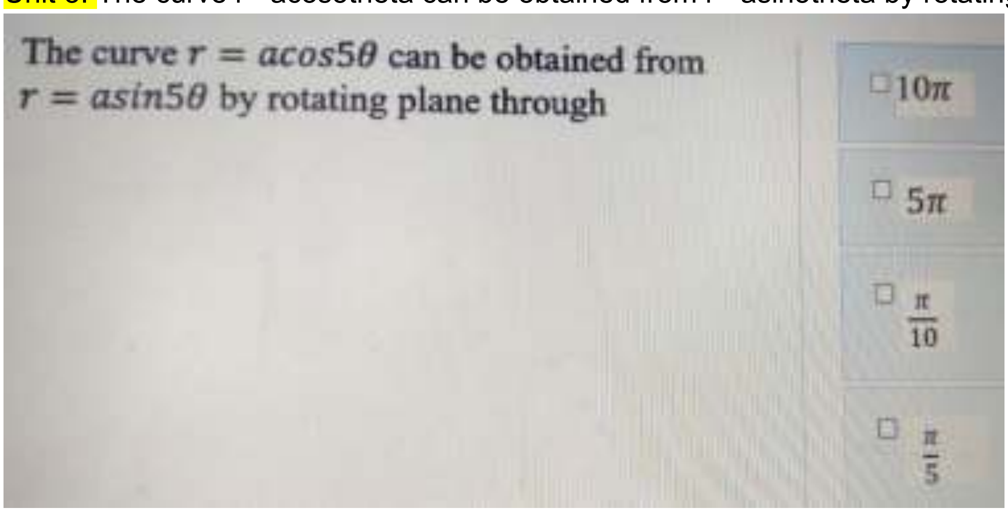
The angle between the radius vector & the tangent to the curve

$$r = \frac{a}{2}(1 + cos\theta)$$

- $\frac{\pi}{2} - \frac{\theta}{2}$
- $\pi - \frac{\theta}{2}$
- $\pi + \frac{\theta}{2}$
- $\frac{\pi}{2} + \frac{\theta}{2}$

Ans: D

Unit 3: The curve r =acos5theta can be obtained from r =asin5theta by rotating plane through



The curve $r = acos5\theta$ can be obtained from $r = asin5\theta$ by rotating plane through

- 10π
- 5π
- $\frac{\pi}{10}$
- $\frac{\pi}{5}$

Ans: option C : pi/10

UNIT 3 The curve y=f(x) exists in [a, b]. Has a point of inflexion if

The curve y=f(x) exists in [a, b].Has a point of inflexion if

- $f''(x) < 0$ for all x in [a, b]
- $f''(x) = 0$ for all x in [a, b]
- $f''(x) > 0$ for all x in [a, b]
- $f''(x) = 0$ for some x in [a, b]

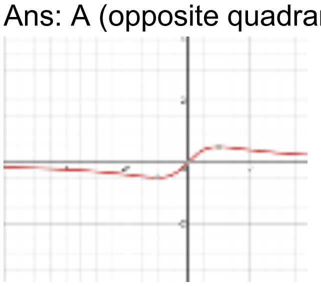
Ans: D

Unit:3 The curve y(x2+1)= x is symmetric about

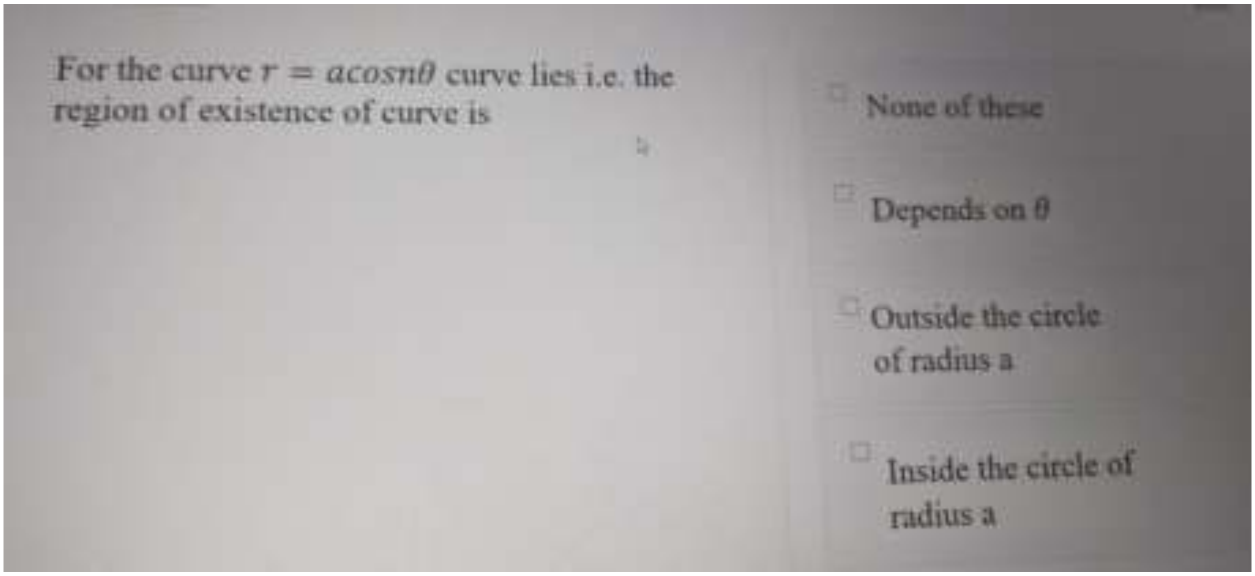
The curve $y(x^2 + 1) = x$ is symmetric about

- opposite quadrant
- y-axis
- y=x
- x-axis

Ans: A (opposite quadrant)



Unit:3 For the curve r=a cos ntheta curve lines i.e the region of existence of curve is

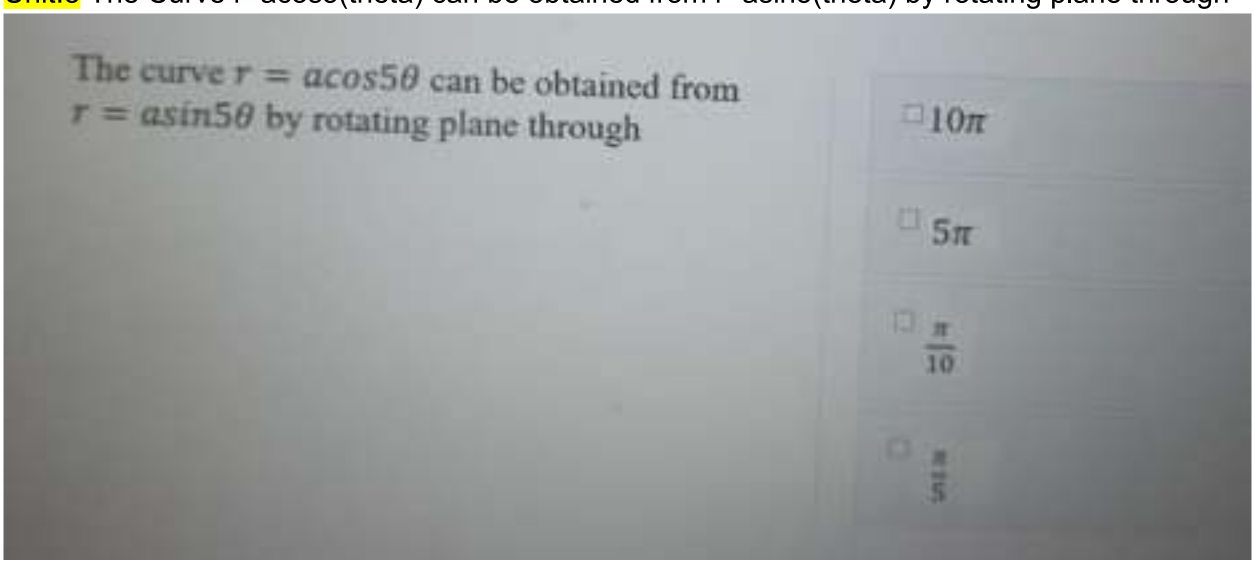


For the curve $r = acosn\theta$ curve lies i.e. the region of existence of curve is

- None of these
- Depends on θ
- Outside the circle of radius a
- Inside the circle of radius a

Ans: Inside the circle of radius a (D)

Unit:3 The Curve r=acos5(theta) can be obtained from r=asin5(theta) by rotating plane through

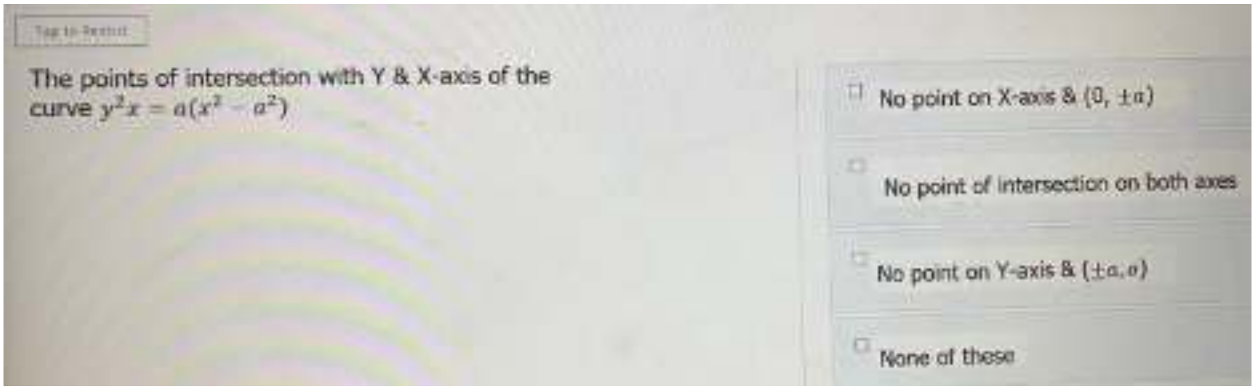


The curve $r = acos5\theta$ can be obtained from $r = asin5\theta$ by rotating plane through

- 10π
- 5π
- $\frac{\pi}{10}$
- $\frac{\pi}{5}$

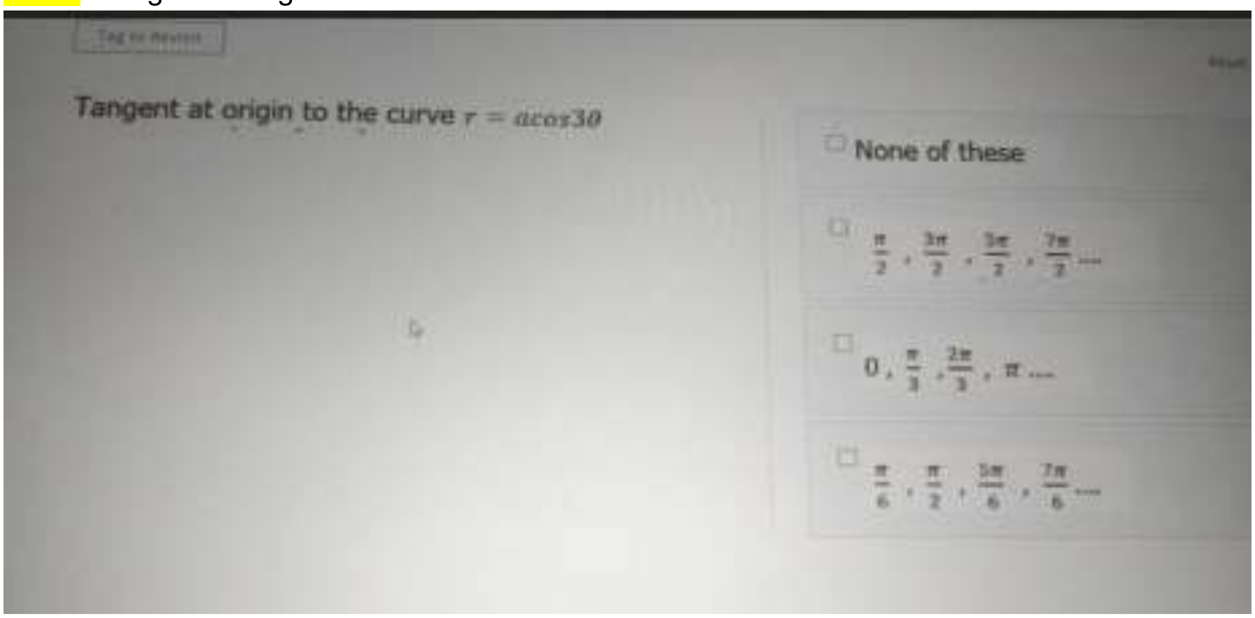
Ans: C
Pi/10

UNIT 3 The points of intersection with y and x axis of the curve y^2x=a(x^2-a^2)



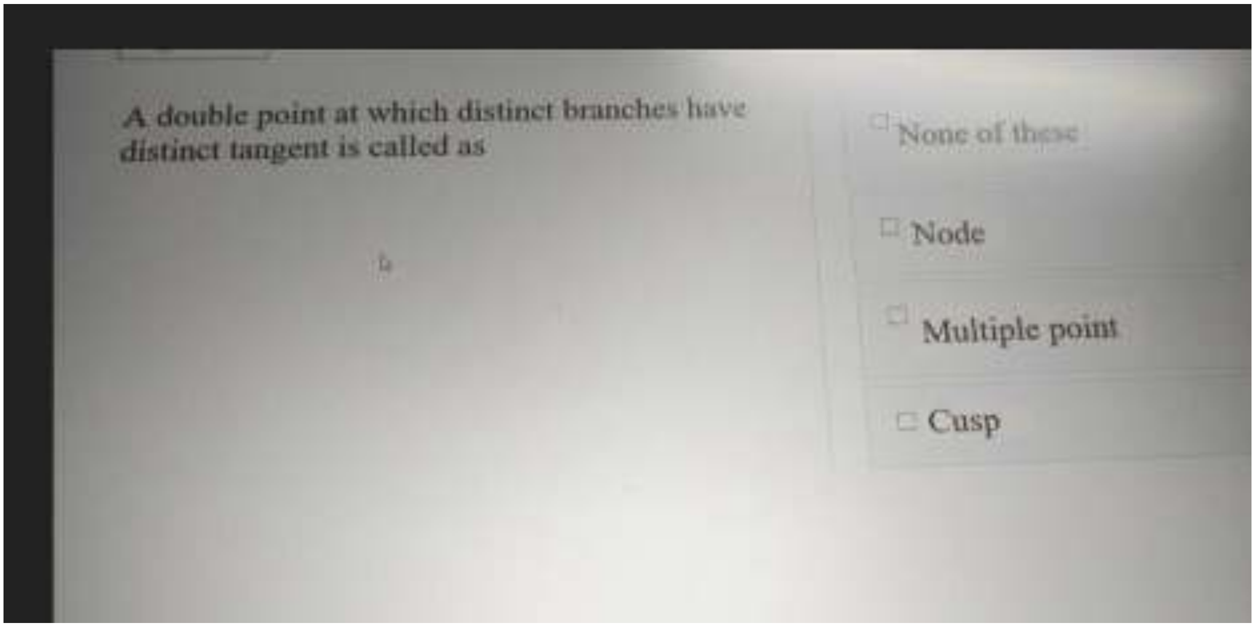
Ans: None of these

Unit 3 :Tangent at origin to the curve r=acos3theta



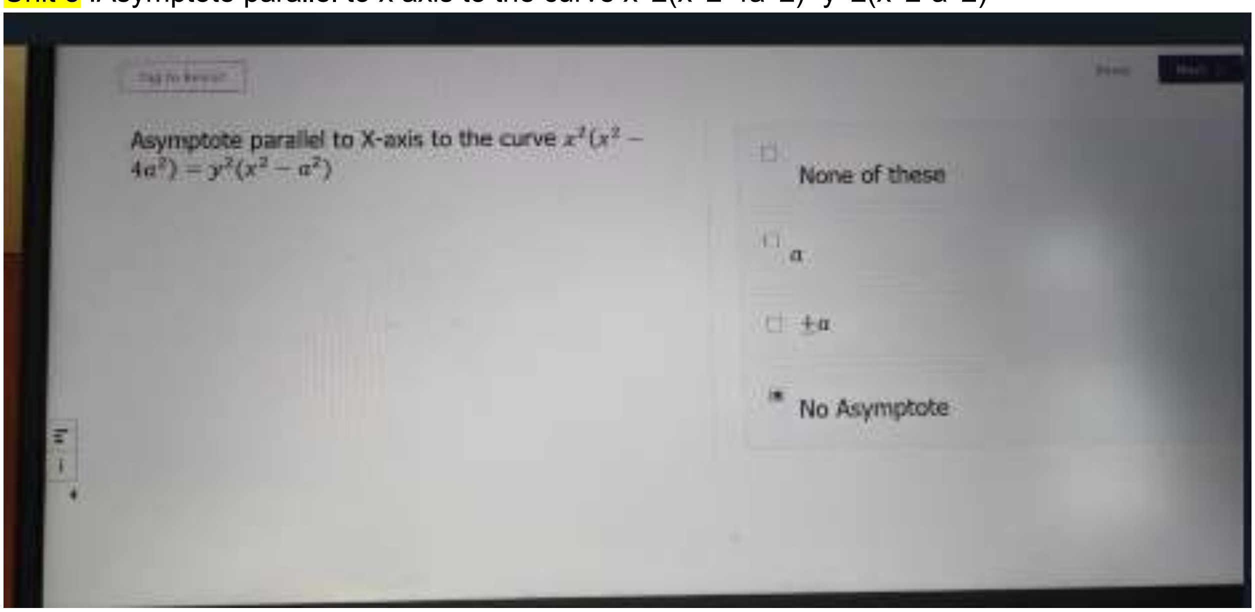
Ans: d

Unit 3 : A double point at which distinct branches have distinct tangent is called as



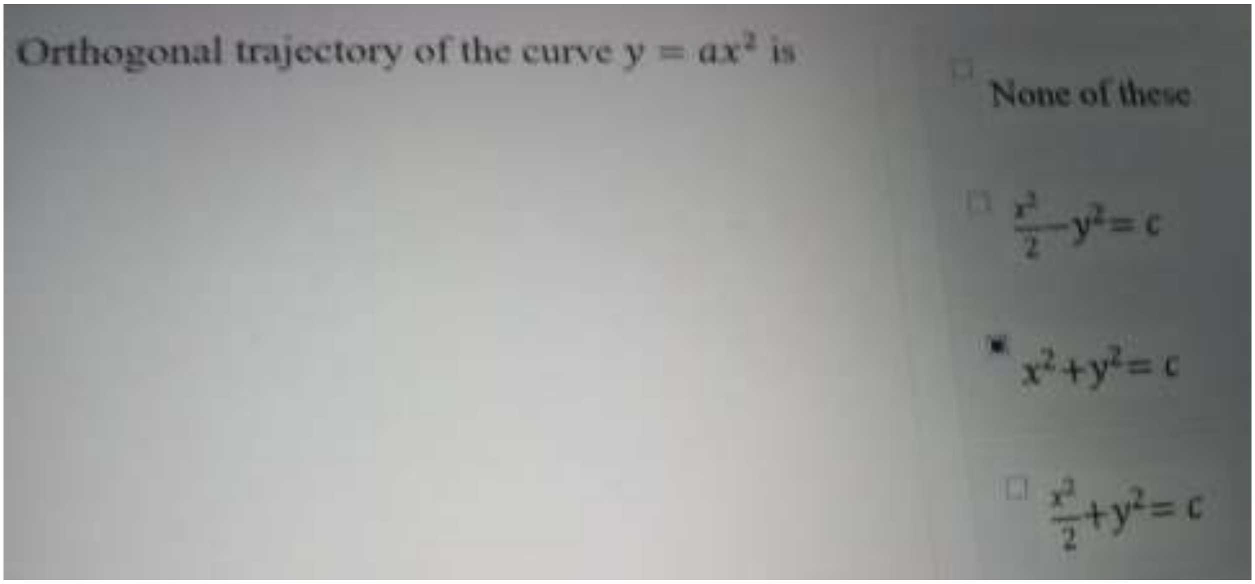
Ans: Node

Unit 3 :Asymptote parallel to x axis to the curve x^2(x^2-4a^2)=y^2(x^2-a^2)



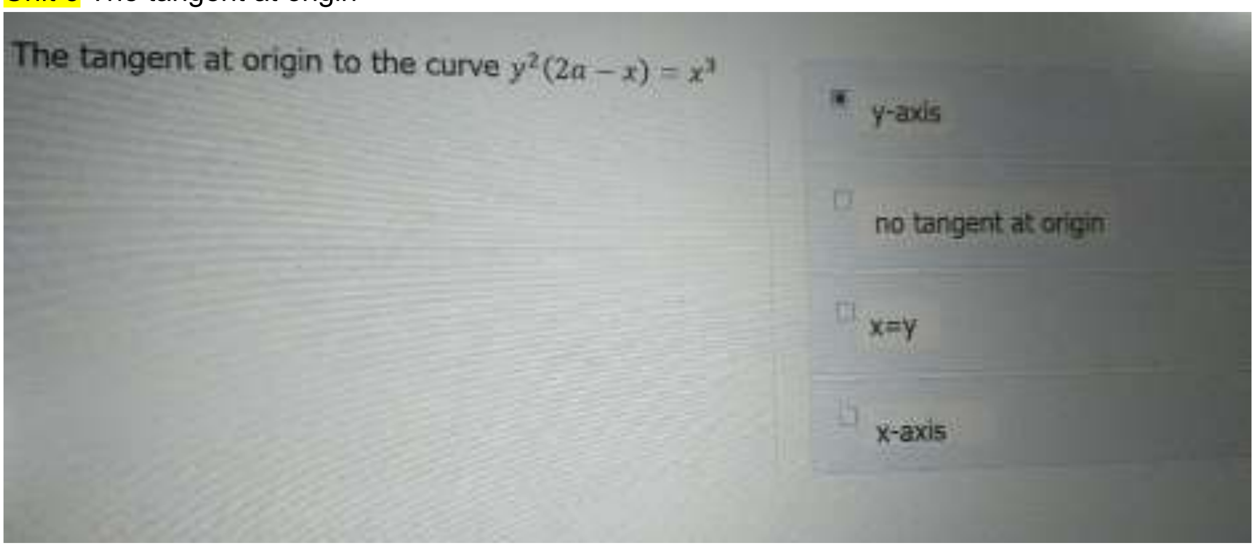
Ans (+- a)

Unit 3 Orthogonal trajectory of the curve y= ax^2



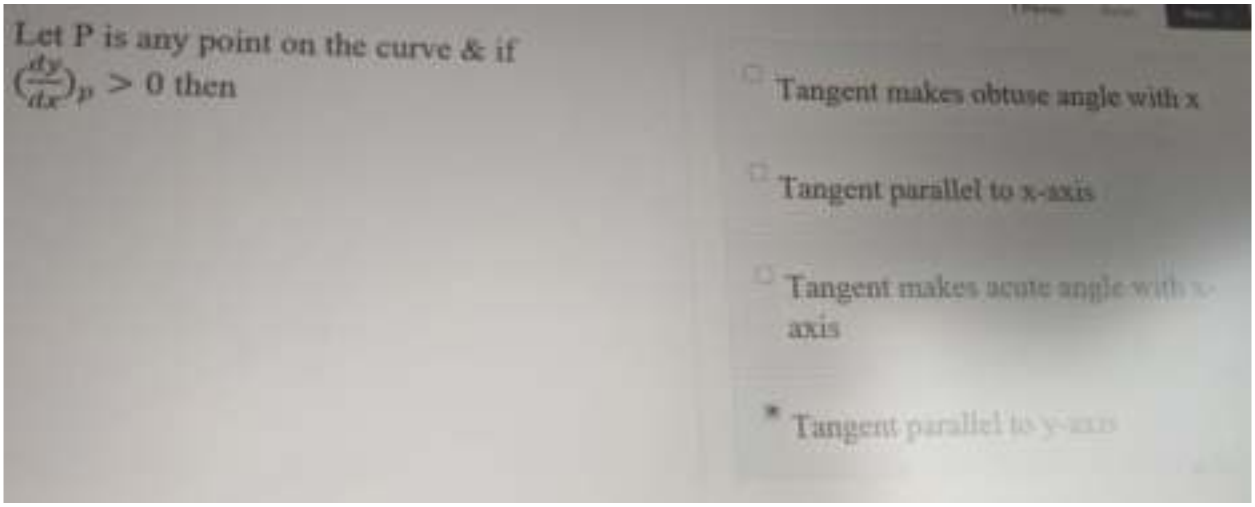
Ans: X^2+Y^2=3

Unit 3 The tangent at origin



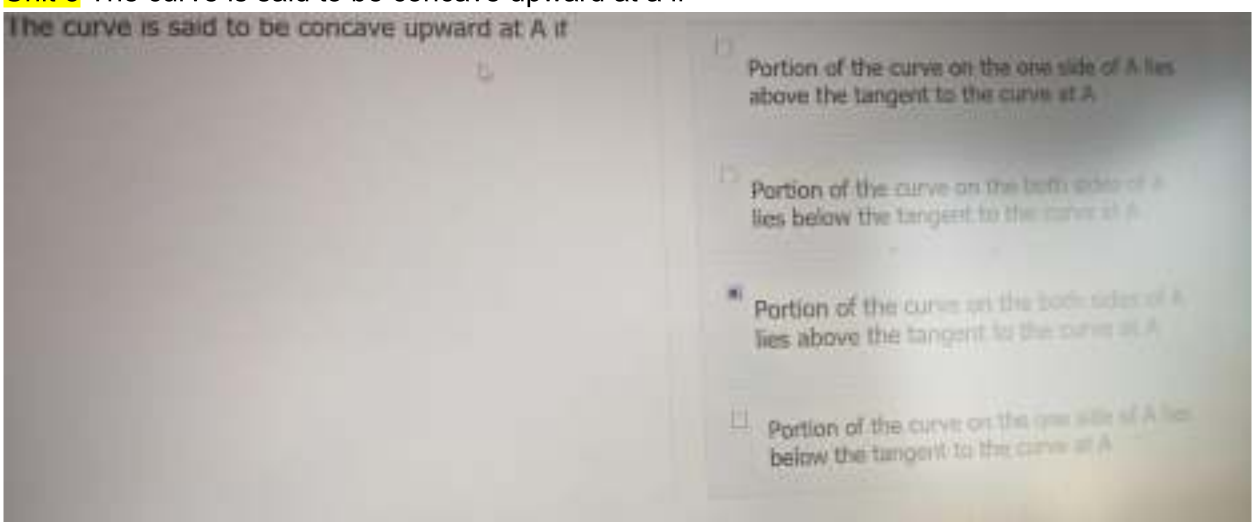
Ans: y-axis

Unit 3 Let P is any point on the curve and if



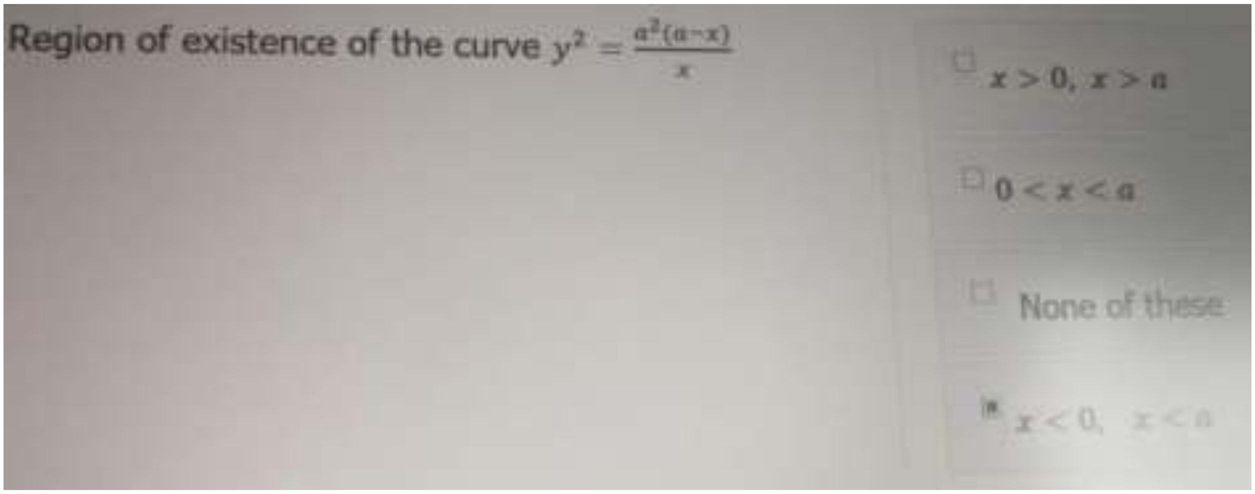
Ans: tangent parallel to y axis

Unit 3 The curve is said to be concave upward at a if



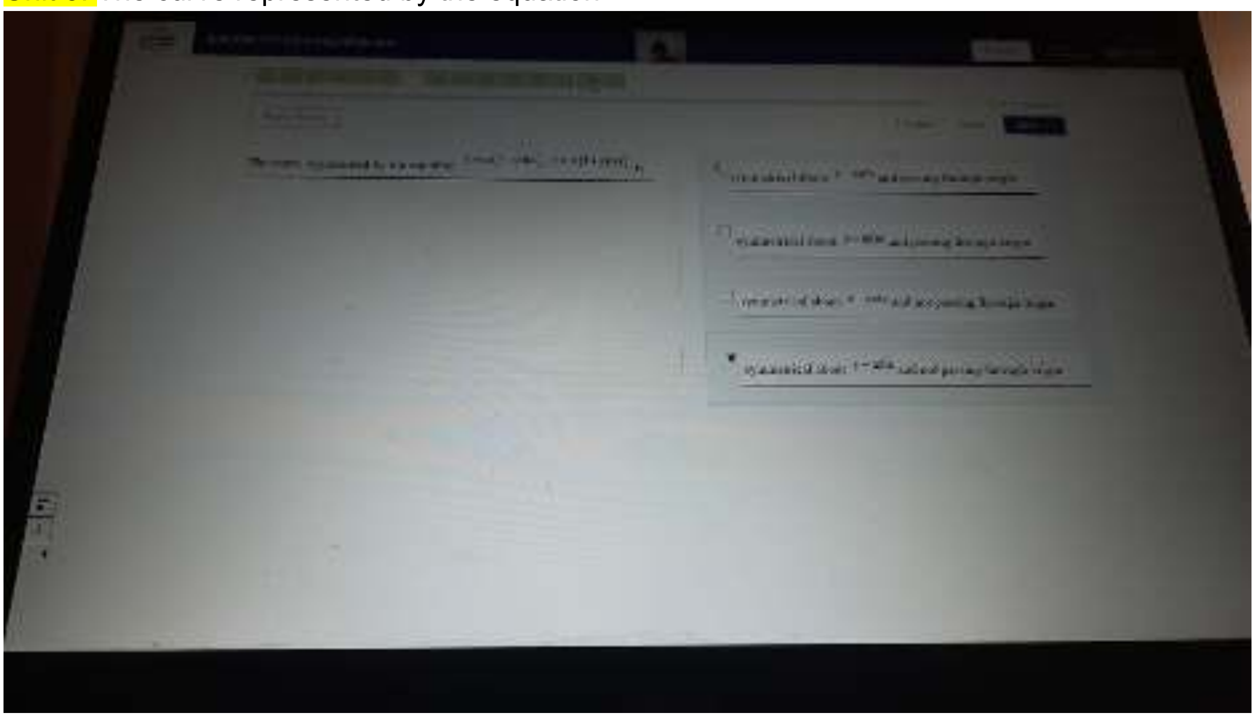
Ans: c

Unit 3 Region of existence



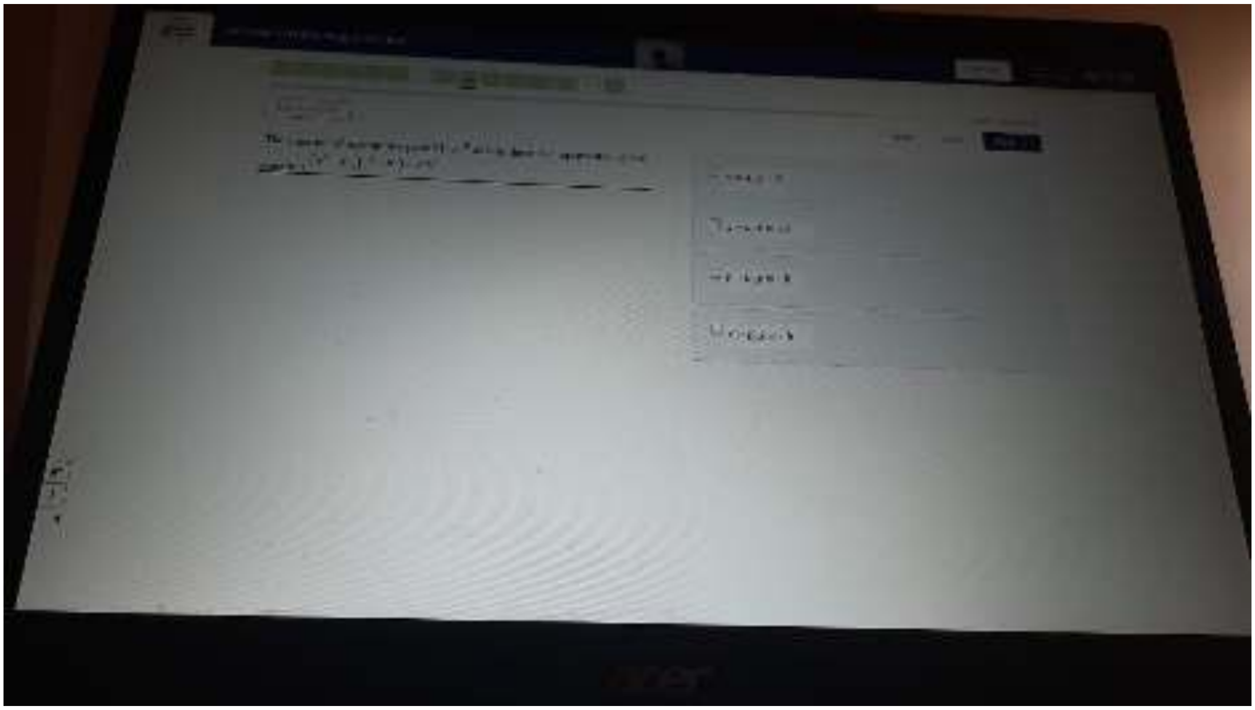
Ans: C kyuki y square 0 bhi ho skta he at x=a

Unit 3: The curve represented by the equation



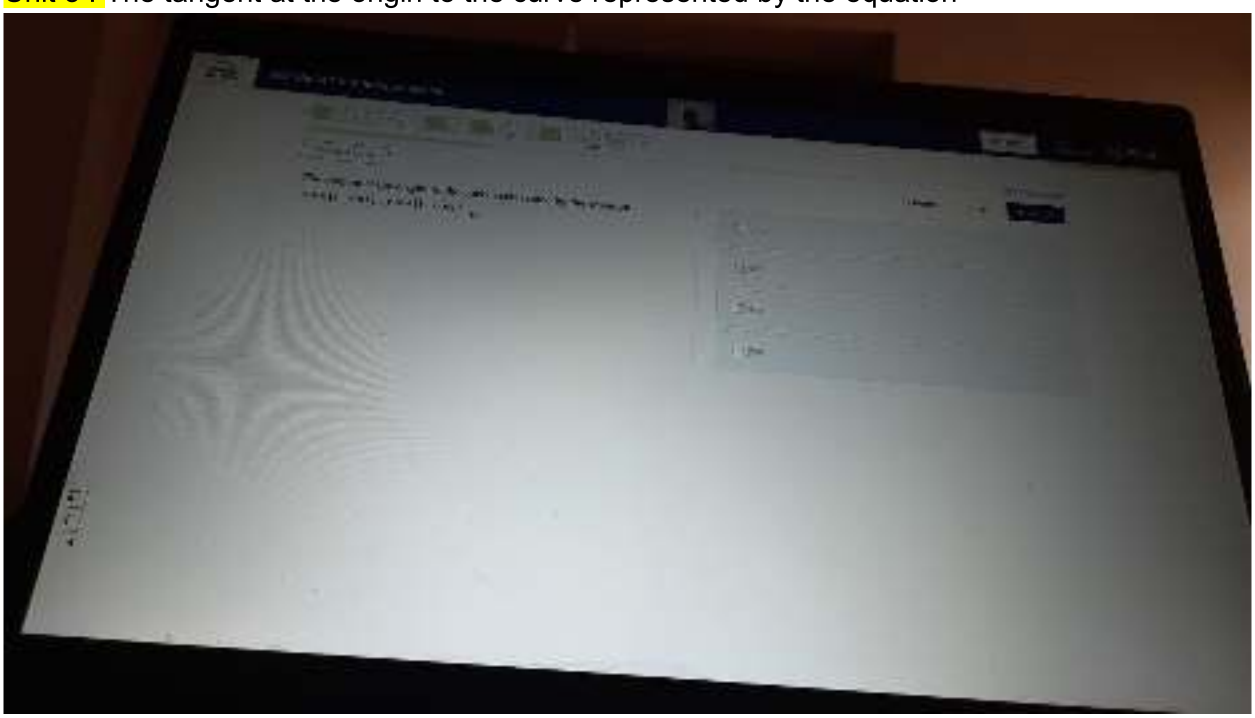
Ans. D

Unit 3 : The equation of asymptotes parallel to y axis to the curve represented by the equation



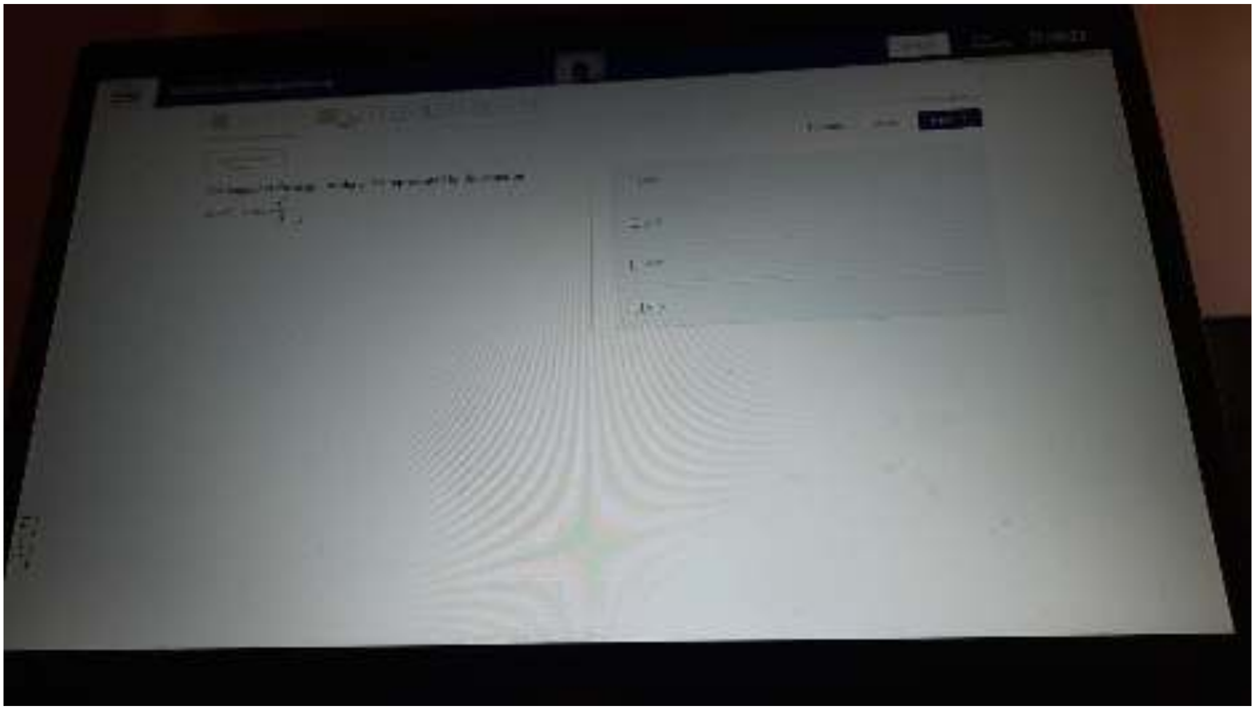
Ans. b

Unit 3 : The tangent at the origin to the curve represented by the equation



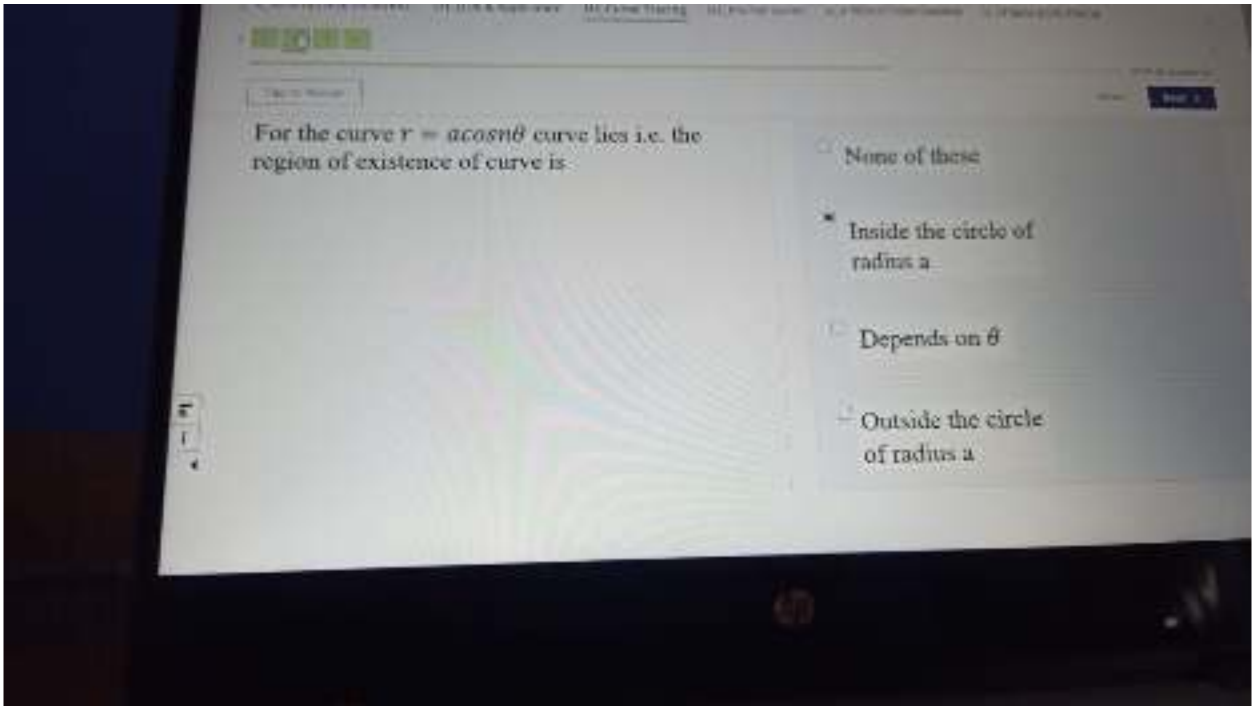
Ans. (option B)

Unit 3 : The tangent at the origin to the curve represented by the equation



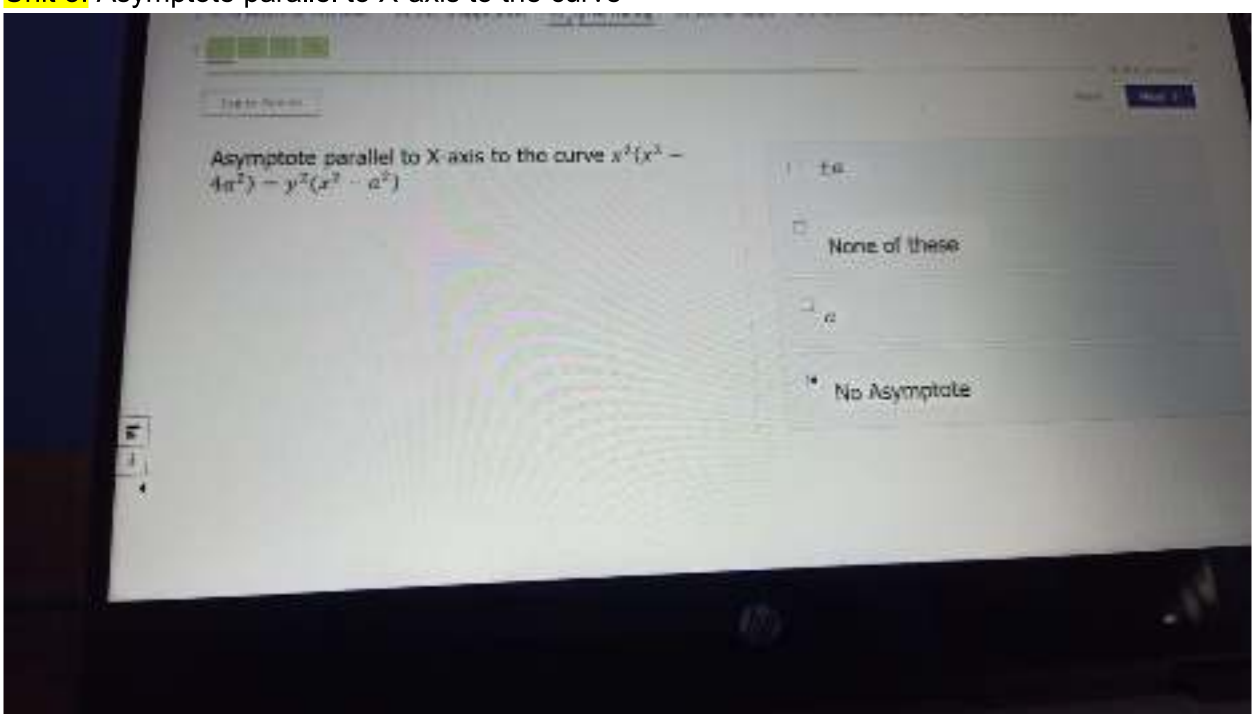
Ans. correct ans not in options : x=0

Unit 3: For curve r= acos(theta) curve lies i.e. the region of existence of curve is



Ans. B

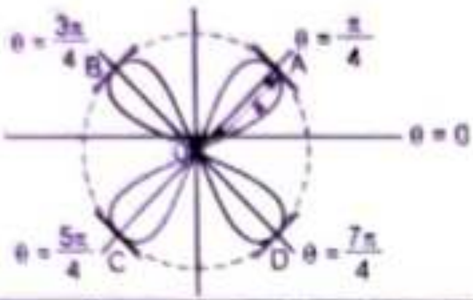
Unit 3: Asymptote parallel to X-axis to the curve



Ans
D

Unit 3: The following figure represents the curve whose equation is

The following figure represents the curve whose equation is



☐ $r = a\sin 2\theta$

☐ $r = a\sin 3\theta$

☐

RESET

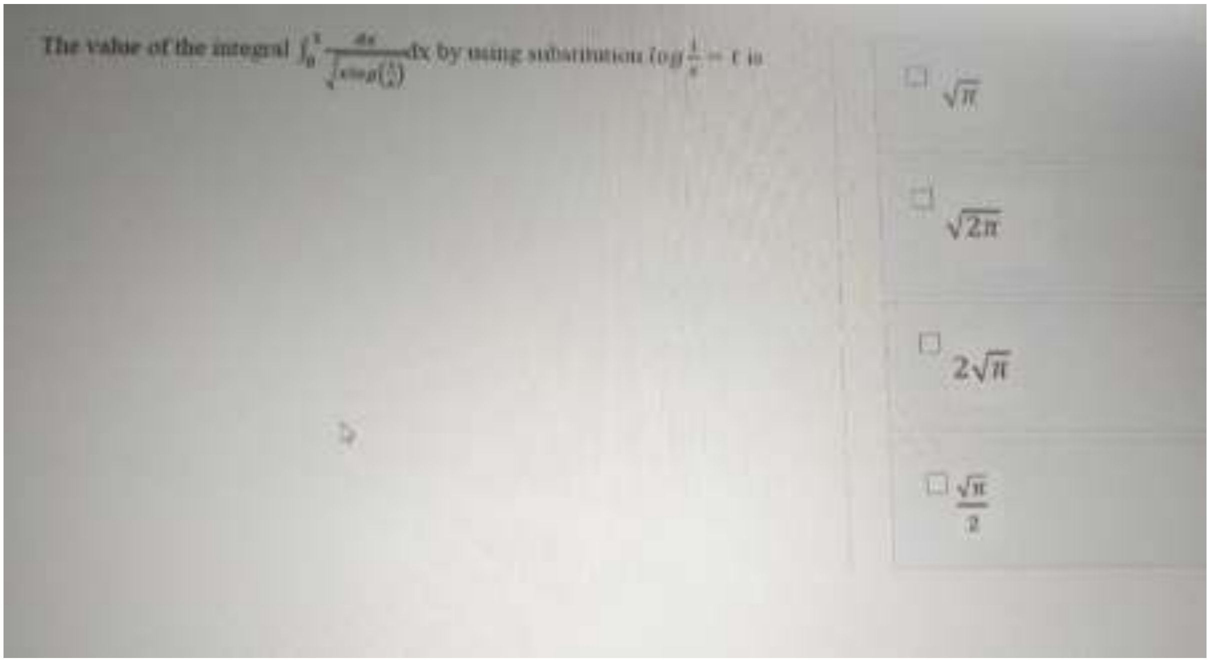
NEXT

Ans: A

Unit: 4

Unit: 5(13 questions)

Unit:5 The value of the integral by using substitution log(1/x)= t

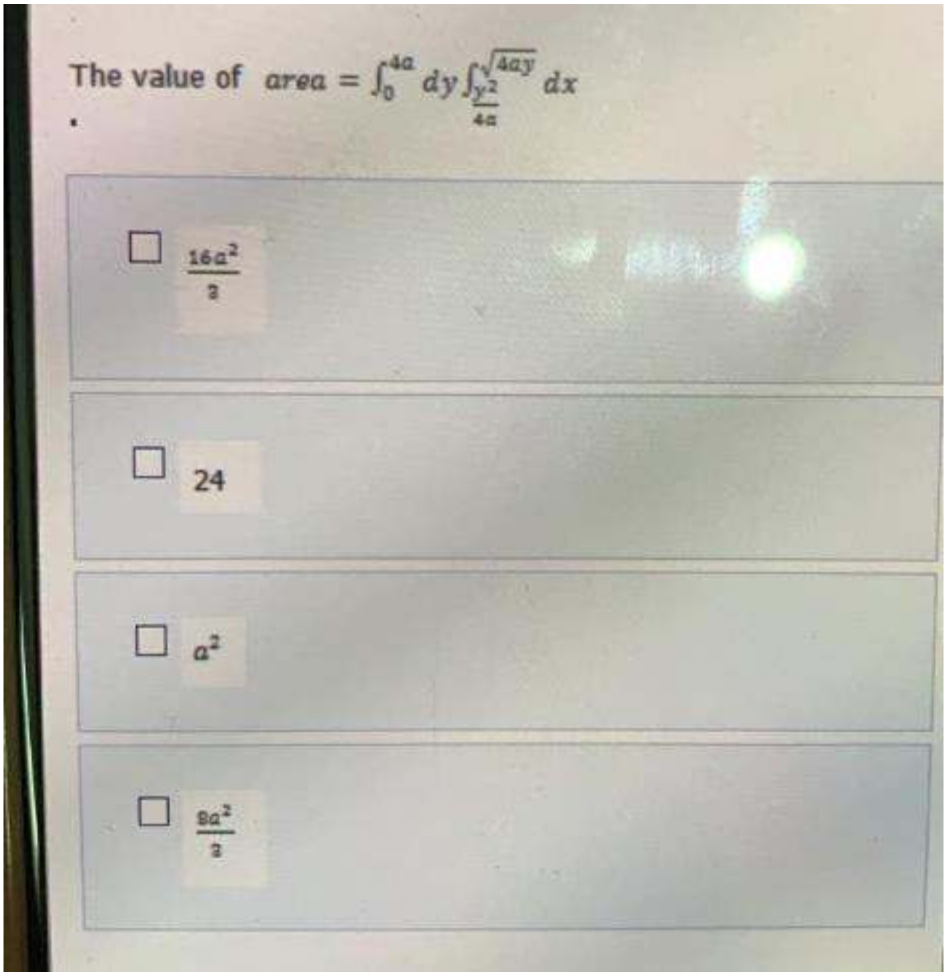


Ans: sqrt(2pi) (option b)

Unit 5: The value of gamma fxn (1/4)*gamma fxn(3/4) is

Ans: option a : pi root(2)

Unit:5 The value of area=



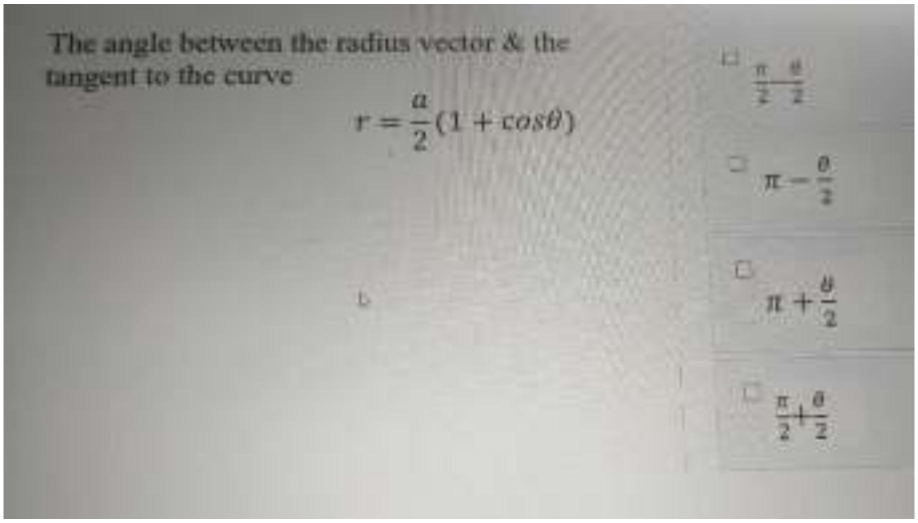
The value of $area = \int_0^{4a} dy \int_{\frac{y^2}{4a}}^{\sqrt{4ay}} dx$

.

- $\frac{16a^2}{3}$
- 24
- a^2
- $\frac{8a^2}{3}$

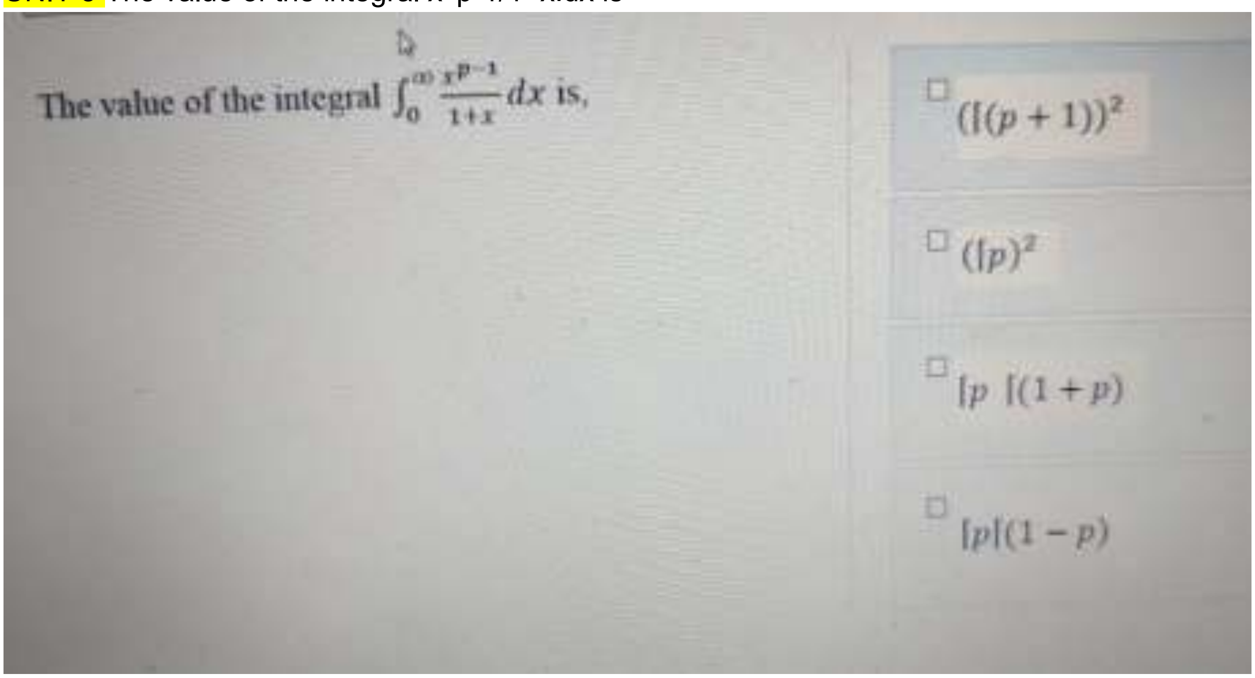
Ans:A

Unit:5 The angle between the radius vector and the tangent to the curve r= a/2(1+cos theta)



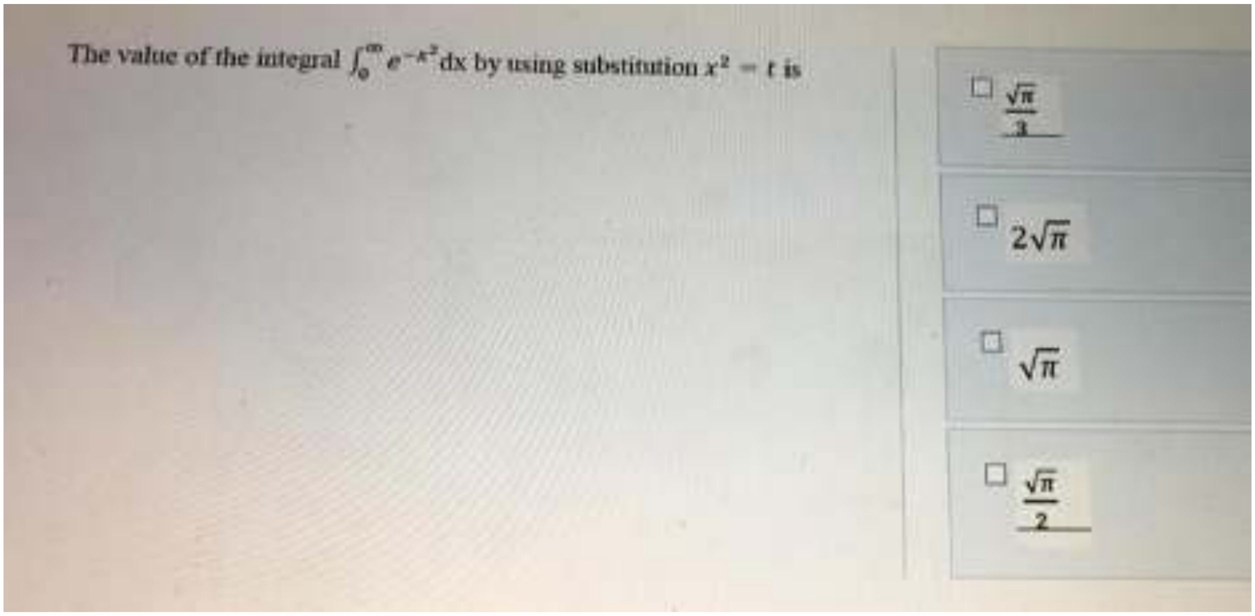
Ans: option A : pi/2-theta/2

UNIT 5 The value of the integral x^p-1/1+x.dx is



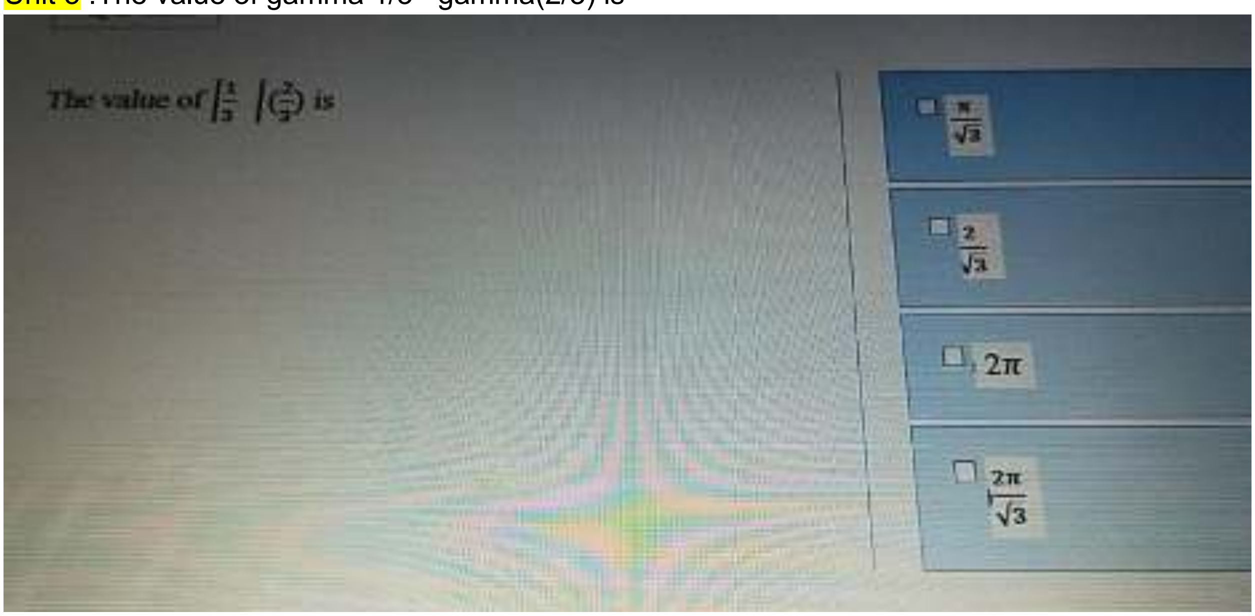
Ans:D [Ꞁ p * Ꞁ 1-p

Unit:5 The value of the integral e^-x2dx by using substitution x^2=t is



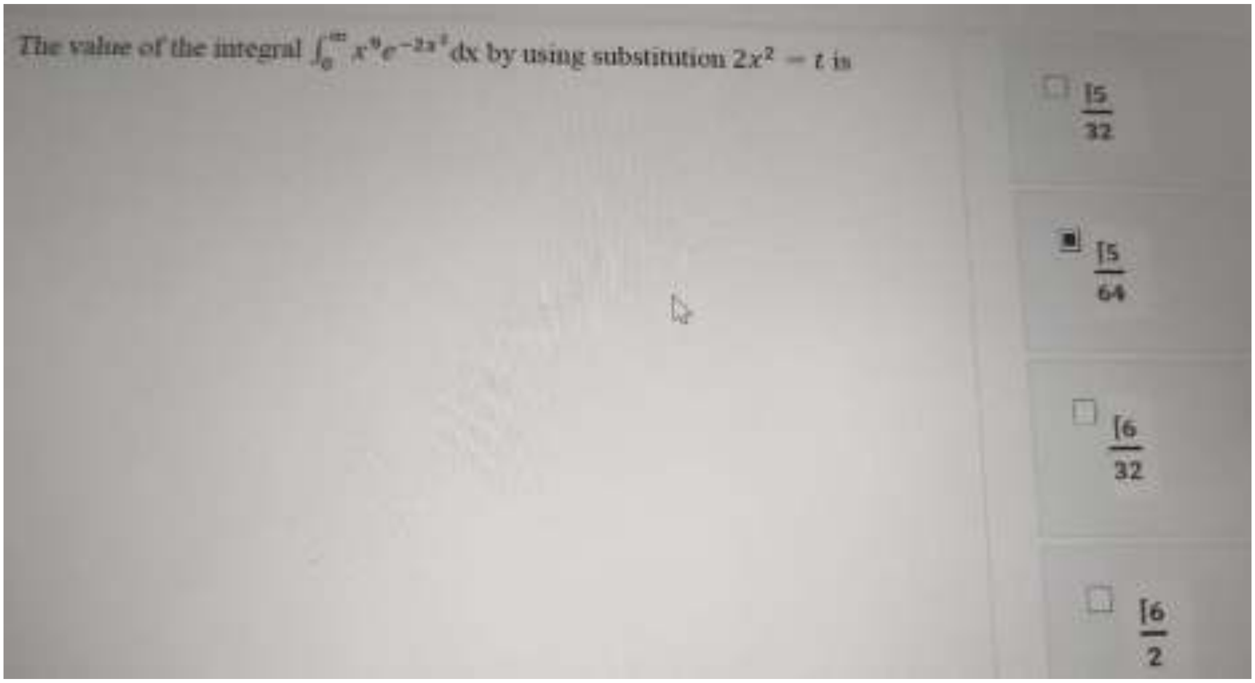
Ans:D

Unit 5 :The value of gamma 1/3 gamma(2/3) is



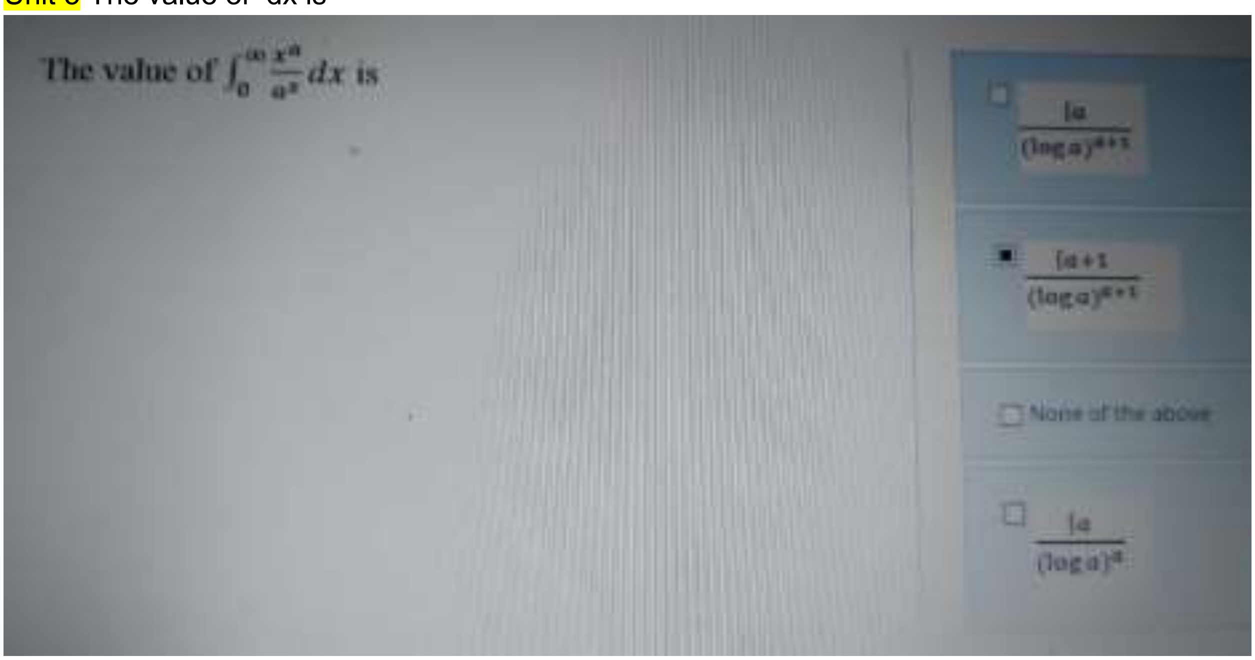
Ans: d 2pi/root 3

Unit 5 The value of the integral



Ans: 5/64

Unit 5 The value of dx is



Ans: B

Unit 5 The value of the integral by using substitution

The value of the integral $\int_0^1 x^{m-1}(1-x^2)^{n-1}\,dx$ by using substitution $x=\sqrt{t}$ is

- $B\left(\frac{m}{2}, n\right)$
- $B\left(\frac{m-1}{2}, n-1\right)$
- $\frac{1}{2}B\left(\frac{m}{2}, n\right)$
- $\frac{1}{2}B\left(\frac{m-1}{2}, n-1\right)$

Ans: C

Unit:5 The value of integral by using substitution method

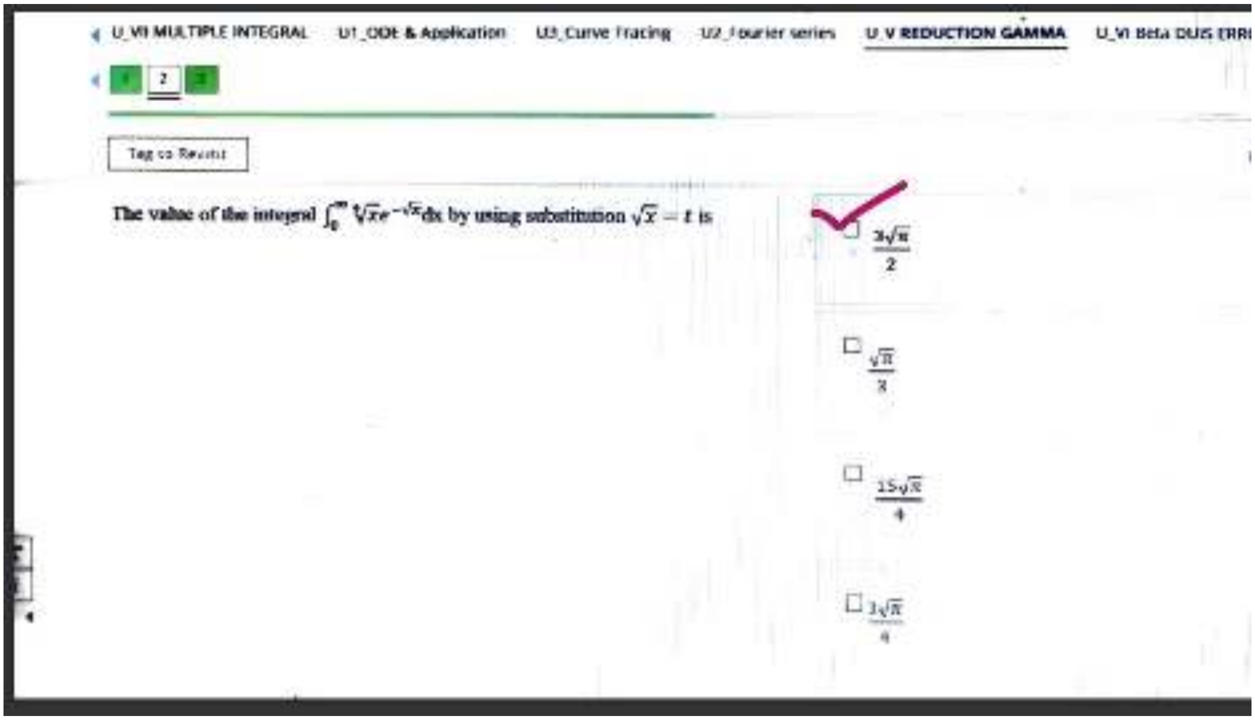
The value of the integral $\int_0^1 (1-x^{1/n})^m\,dx$ by using substitution $x^{1/n}=t$ is

$\int_0^1 (1-x^{1/n})^m$

- $B(n, m+1)$
- $mB(m, n+1)$
- ✓ $nB(n, m+1)$
- $B(m, n+1)$

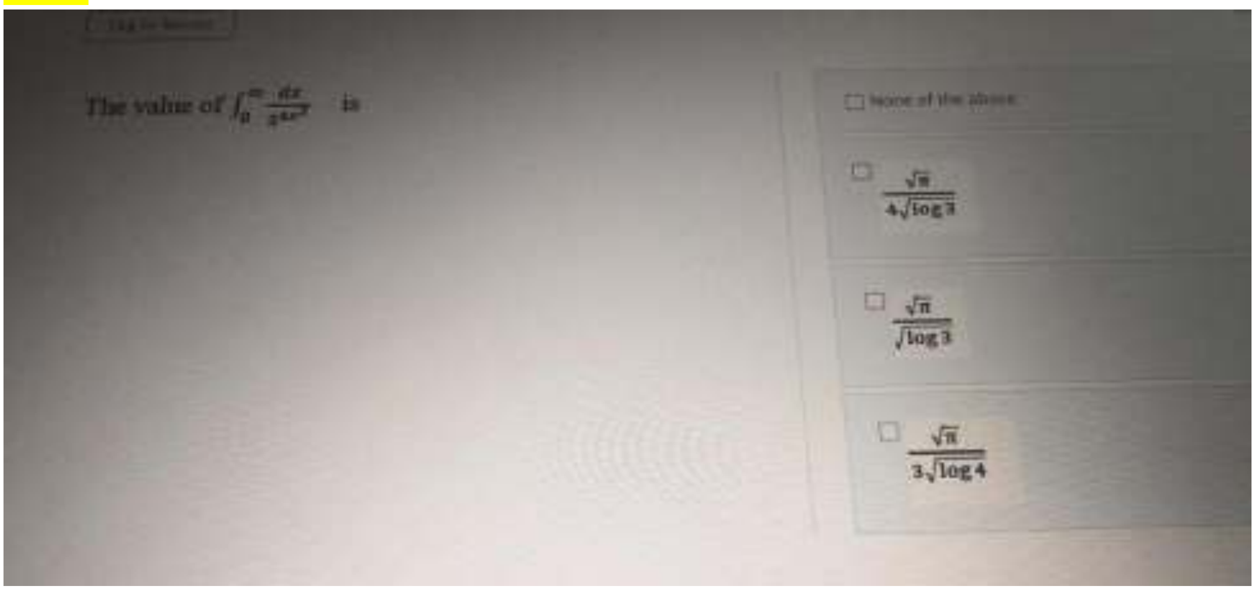
Ans:C

Unit 5 The value of the integral by using substitution



Ans: A

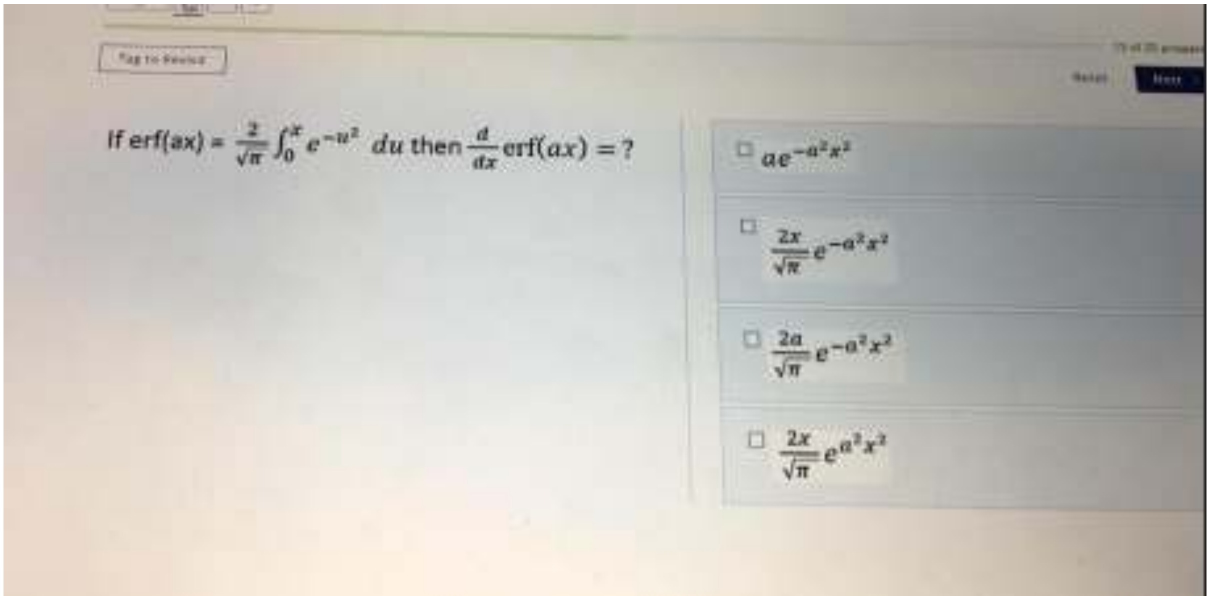
Unit 5: The value of dx/3^4x2



Ans: B

Unit: 6 (34 Questions)

Unit:6 If erf(ax)=2/root pi … then d/dx erf(ax) =?

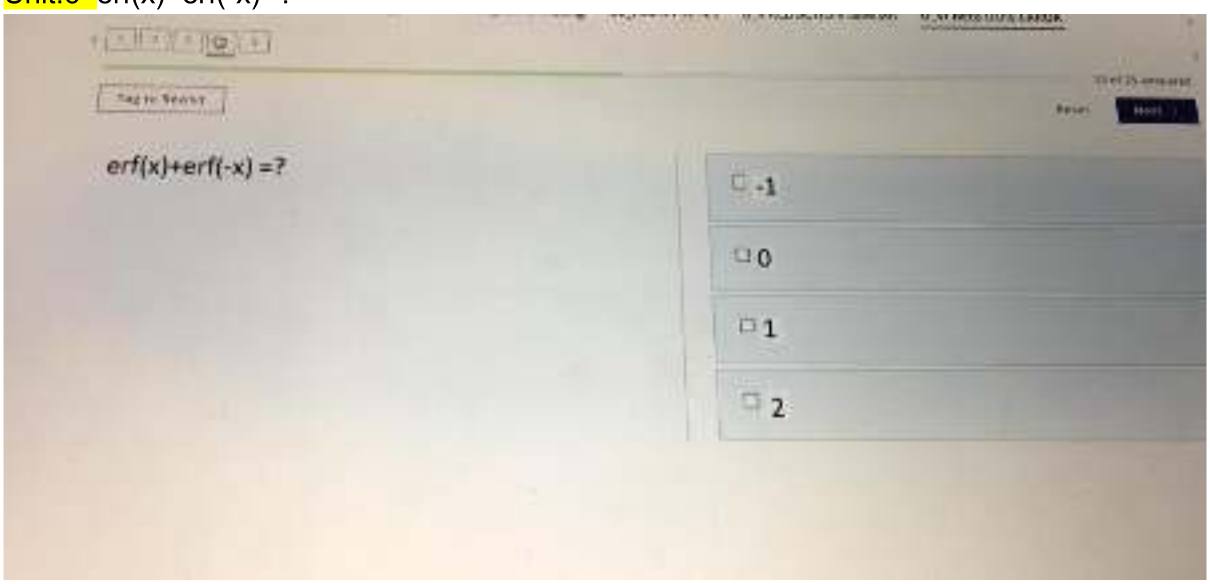


Example 54. *Prove that*

$$\frac{d}{dx}[erf_c(a\,x)] = \frac{-2a}{\sqrt{\pi}}e^{-a^2x^2}$$

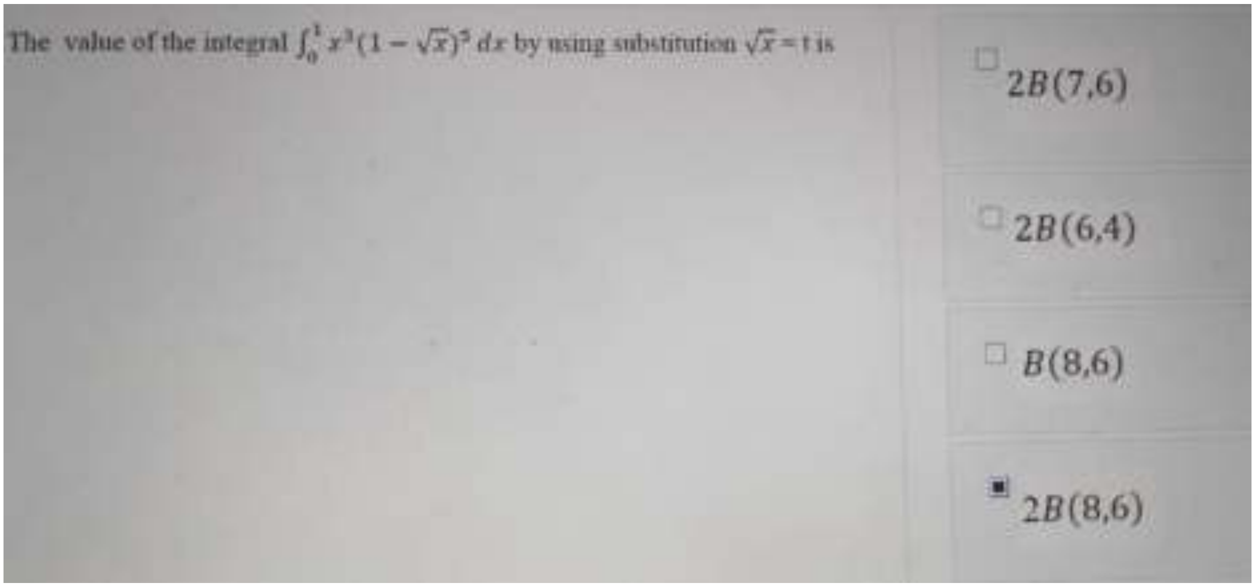
Ans:

Unit:6 erf(x)+erf(-x)=?



Ans: option B (= 0) (Erf is odd function)

Unit 6: The value of the integral ... by using substitution root(x)= t

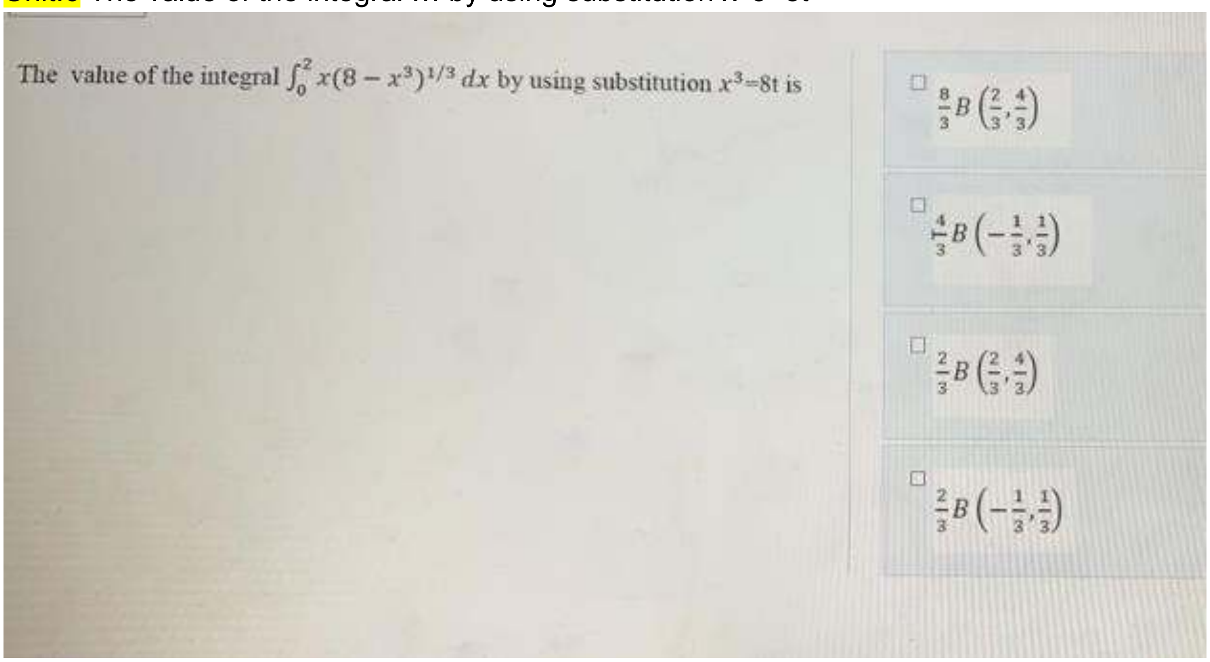


The value of the integral $\int_0^1 x^3(1-\sqrt{x})^5\,dx$ by using substitution $\sqrt{x}=t$ is

- [ ] $2B(7,6)$
- [ ] $2B(6,4)$
- [ ] $B(8,6)$
- [x] $2B(8,6)$

Ans: 2B(8,6)

Unit:6 The value of the integral … by using substitution x^3=8t

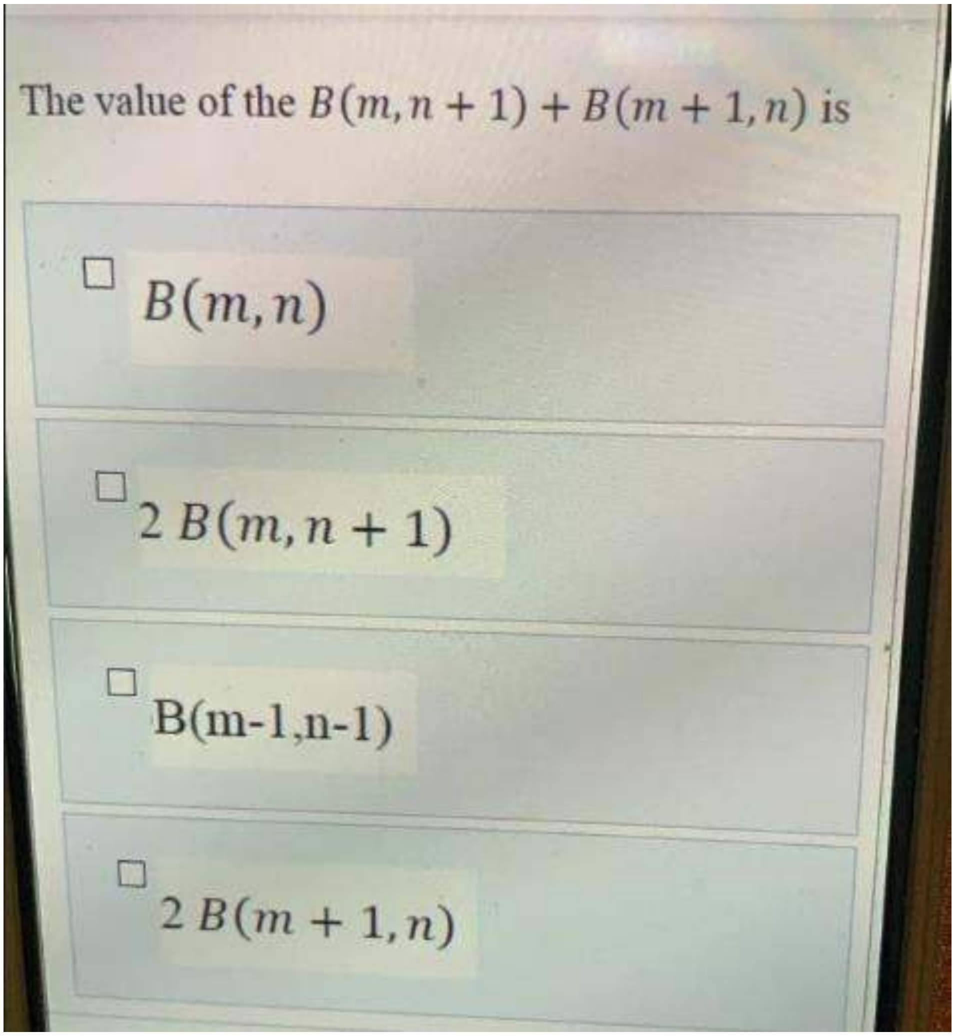


The value of the integral $\int_0^2 x(8-x^3)^{1/3}\,dx$ by using substitution x^3=8t is

- [ ] $\frac{8}{3}B\left(\frac{2}{3},\frac{4}{3}\right)$
- [ ] $\frac{4}{3}B\left(-\frac{1}{3},\frac{1}{3}\right)$
- [ ] $\frac{2}{3}B\left(\frac{2}{3},\frac{4}{3}\right)$
- [ ] $\frac{2}{3}B\left(-\frac{1}{3},\frac{1}{3}\right)$

Ans: option A (8/3)(B(2/3,4/3))

Unit 6: The value of the B(m,n+1) + B(m+1, n)

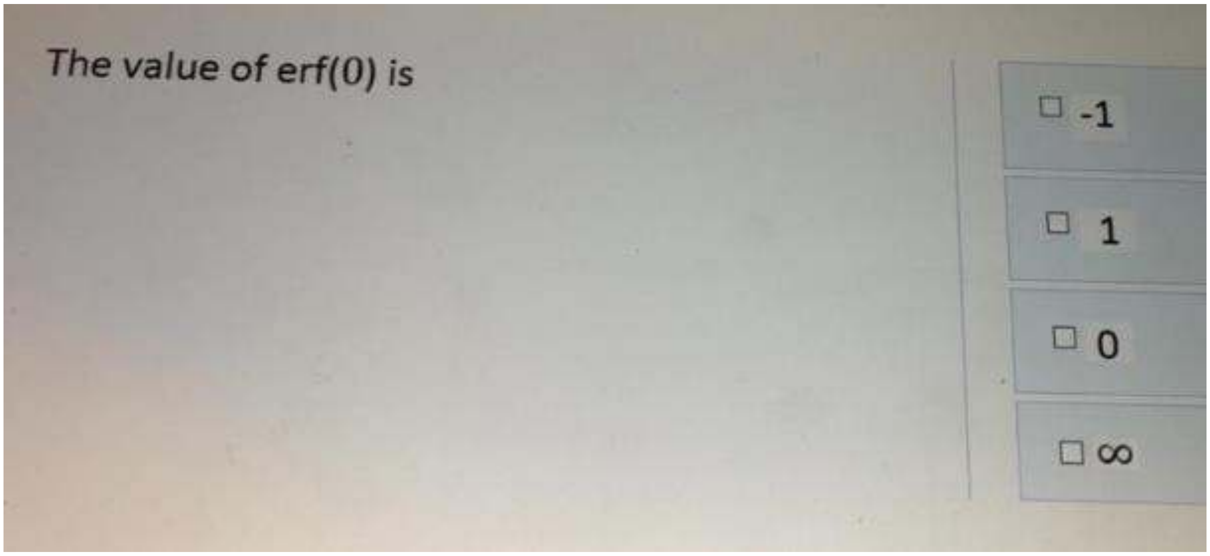


The value of the $B(m, n+1) + B(m+1, n)$ is

- $B(m, n)$
- $2\,B(m, n+1)$
- B(m-1,n-1)
- $2\,B(m+1, n)$

Ans: A (B(m,n))

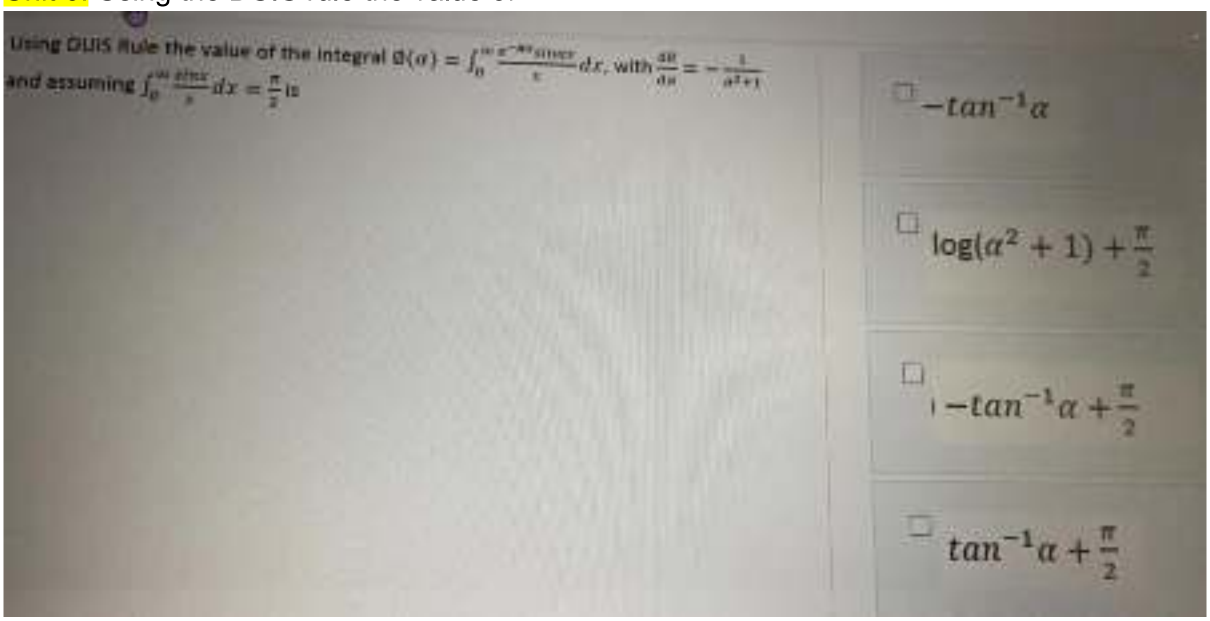
Unit:6 The Value of erf(0) is


The value of erf(0) is

- [ ] -1
- [ ] 1
- [ ] 0
- [ ] ∞

Ans: C (0)

Unit 6: Using the DUIS rule the value of


Using DUIS Rule the value of the integral $\phi(\alpha) = \int_0^\infty \frac{e^{-\alpha x}\sin x}{x}dx$, with $\frac{d\phi}{d\alpha} = -\frac{1}{\alpha^2+1}$ and assuming $\int_0^\infty \frac{\sin x}{x}dx = \frac{\pi}{2}$ is

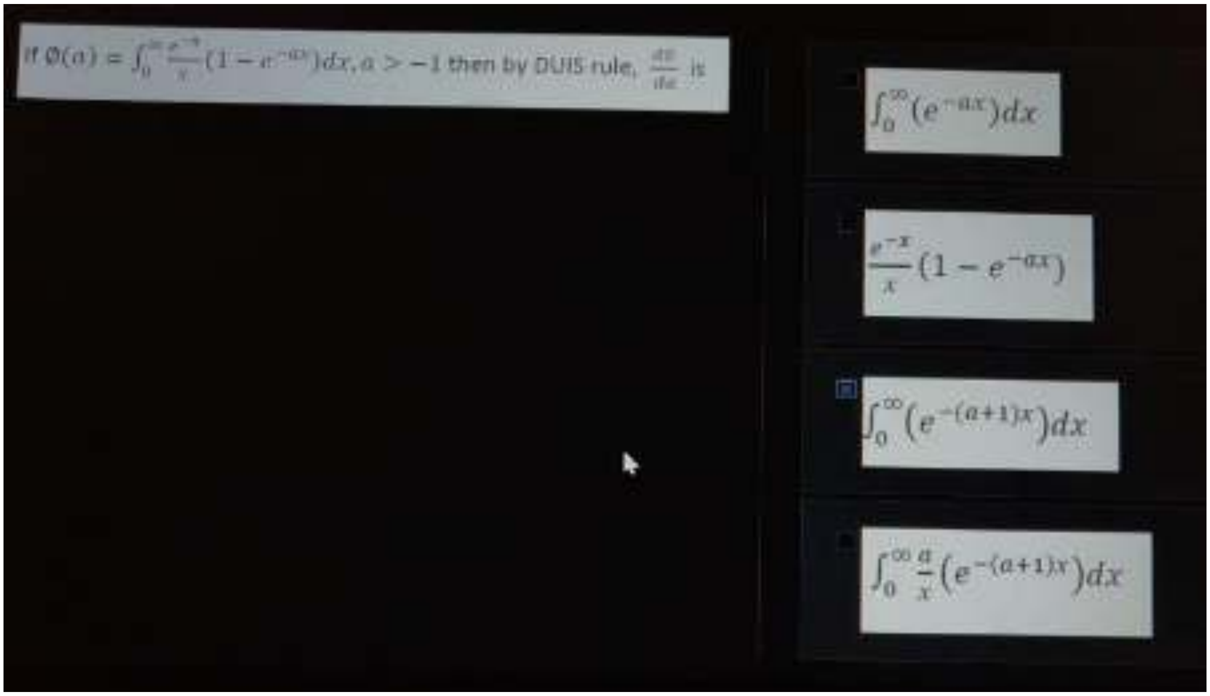
- [ ] $-\tan^{-1}\alpha$
- [ ] $\log(\alpha^2+1)+\frac{\pi}{2}$
- [ ] $-\tan^{-1}\alpha+\frac{\pi}{2}$
- [ ] $\tan^{-1}\alpha+\frac{\pi}{2}$

Ans: option a

Unit 6: B(¼ ,¾) is equal to

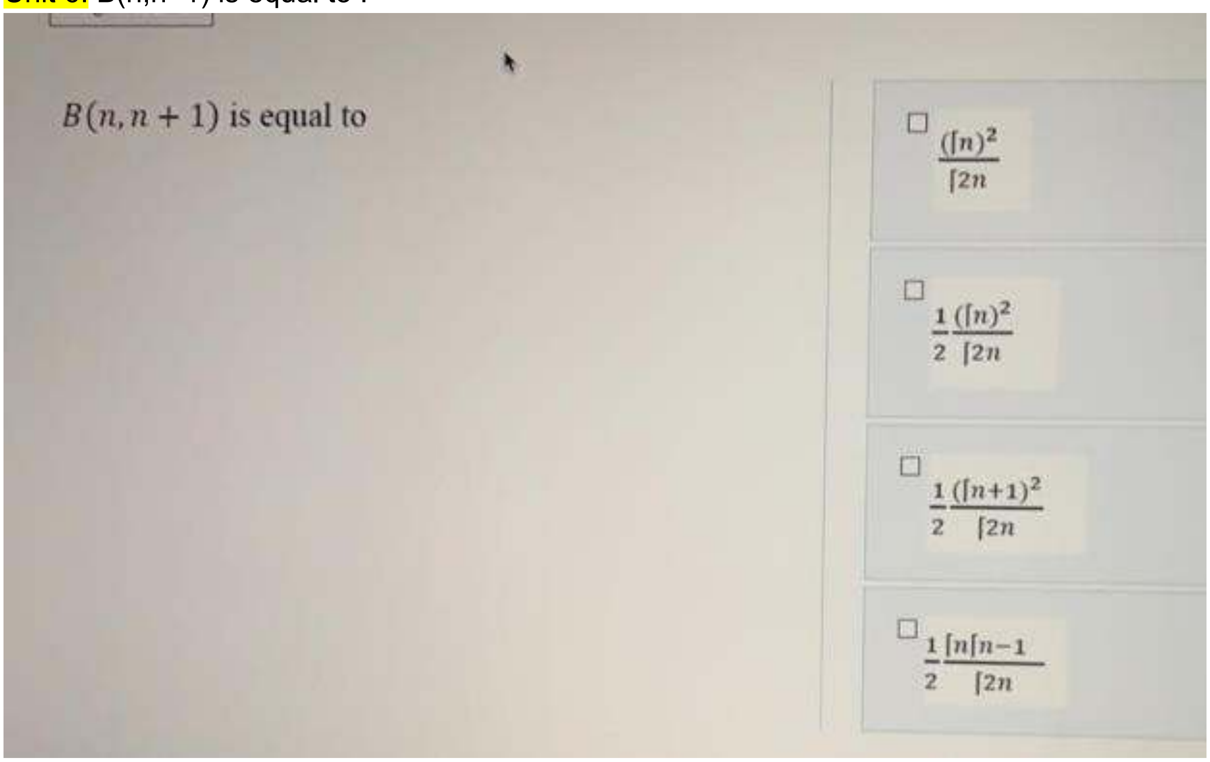
Ans: option D root 2 pi

Unit 6: IF (1-e^-ax)dx, a>-1 then by DUIS rule, dtheta/da



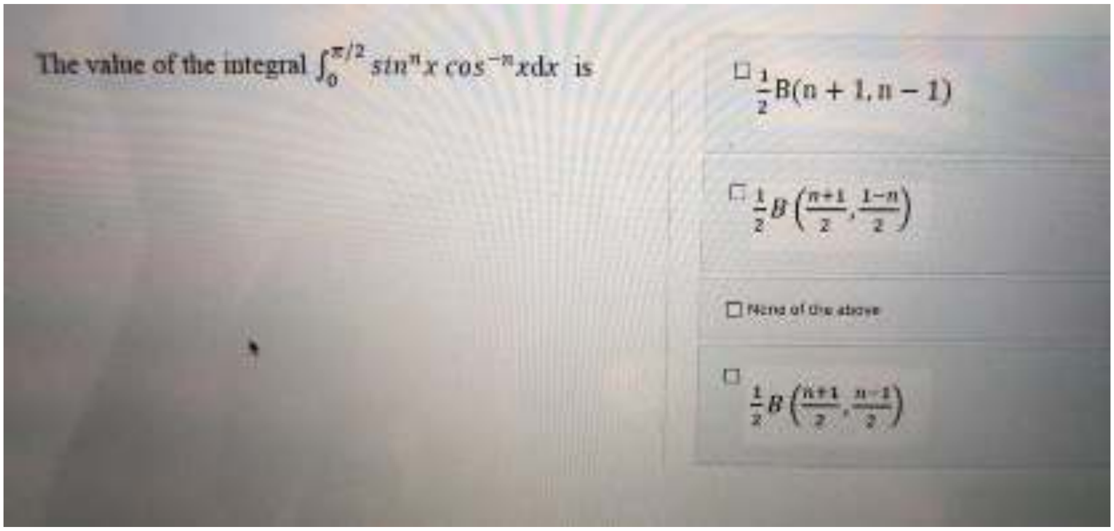
Ans: D

Unit 6: B(n,n+1) is equal to :



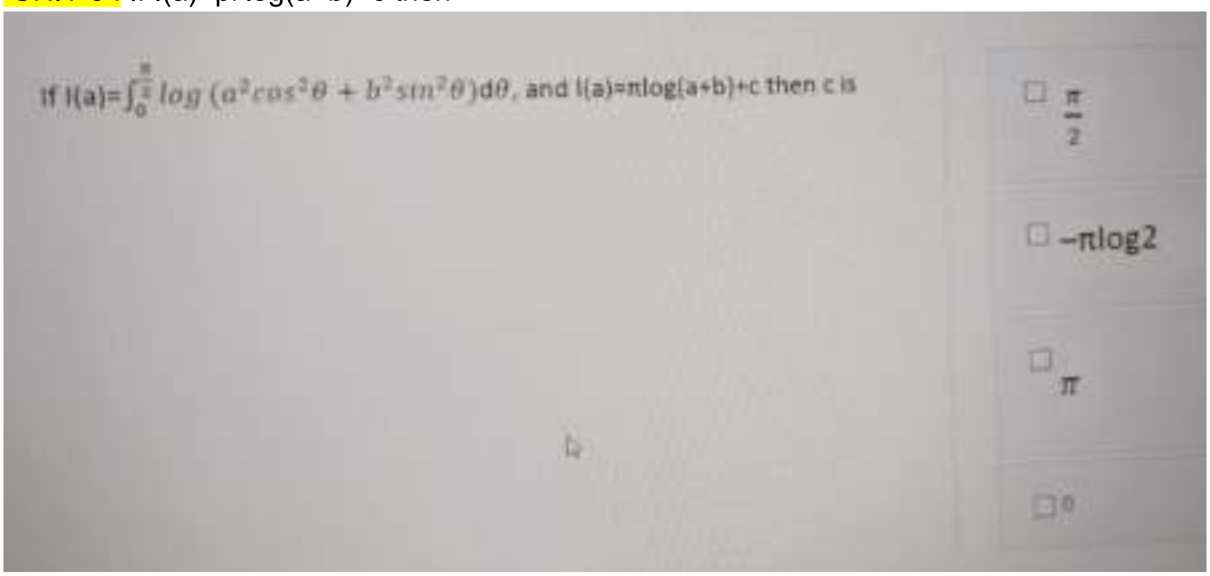
Ans: **B**

Unit 6: The value of integral sin^n x cos^-n x dx is



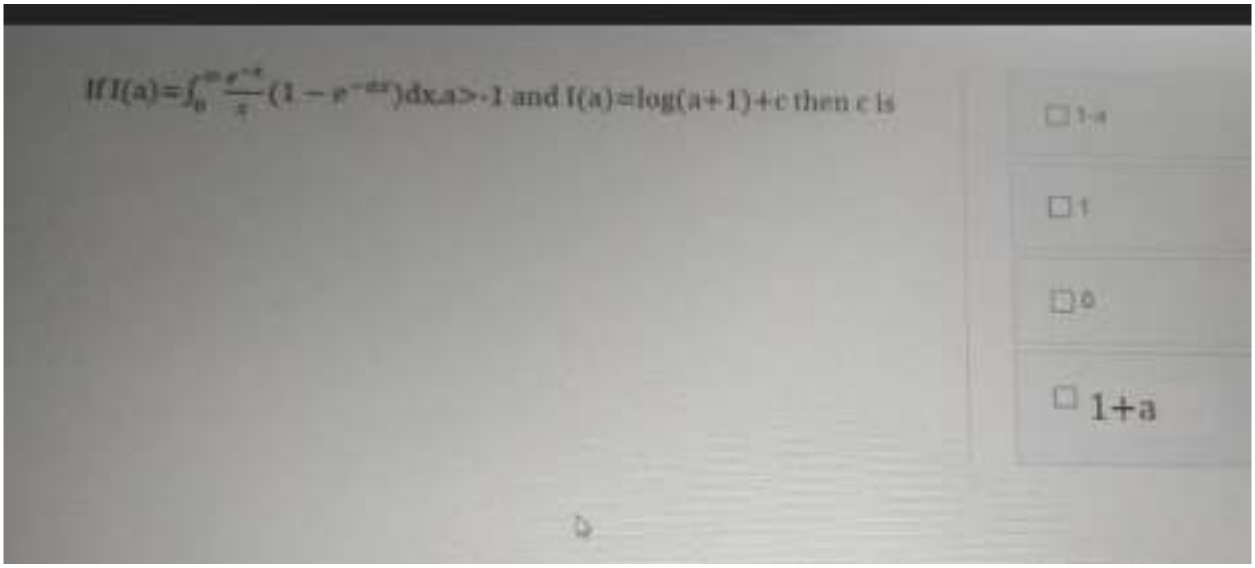
Ans: option B : ½ B ((n+1)/2 , (1-n)/2)

UNIT 6 : If I(a)=pi log(a+b)+c then



Ans: B (-pi log2)

UNIT 6 If I(A) = log(a+1)+c is



If $I(a)=\int_0^{\infty}\frac{e^{-x}}{x}(1-e^{-ax})dx, a>-1$ and $I(a)=\log(a+1)+c$ then c is

☐ 1-a

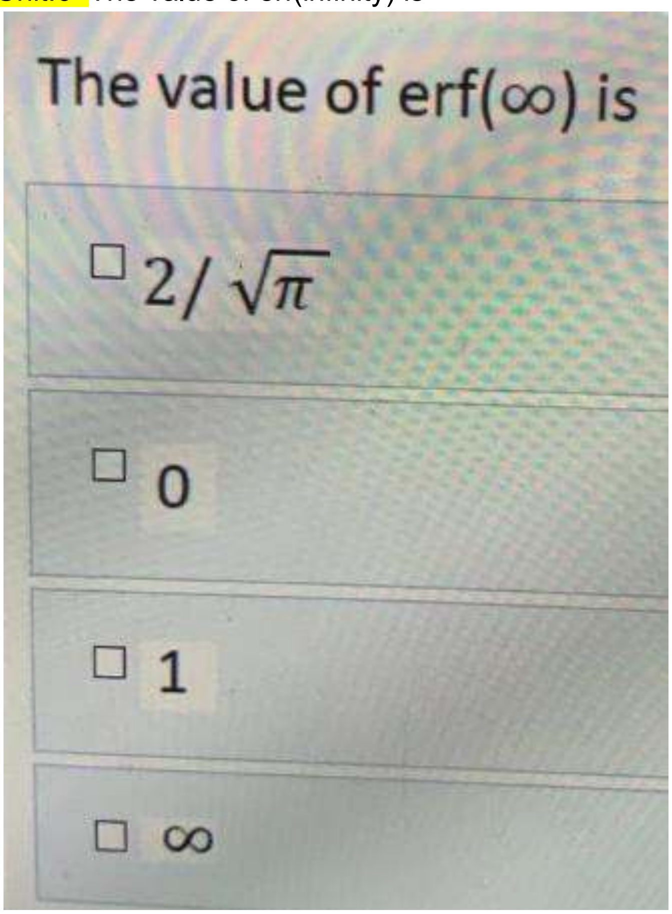
☐ 1

☐ 0

☐ 1+a

Ans: 0

Unit:6 The value of erf(infinity) is



The value of $\mathrm{erf}(\infty)$ is

☐ $2/\sqrt{\pi}$

☐ 0

☐ 1

☐ ∞

Ans : C

Unit:6 The value of integral by using substitution x^1/n=t is

The value of the integral $\int_0^1 (1 - x^{1/n})^m \, dx$ by using substitution $x^{1/n}=t$ is

- $mB(m, n+1)$
- $nB(n, m+1)$
- $B(n, m+1)$
- $B(m, n+1)$

Ans: B

Unit:6 Erf(-x)

$Erf(-x)$ is equals to

- $-\text{erfc}(x)$
- $\text{erf}(x)$
- $\text{erfc}(x)$
- $-\text{erf}(x)$

Ans: D

Unit:6 The value of the integral by using substitution x=root(t)

The value of the integral $\int_0^1 x^{m-1}(1-x^2)^{n-1}\,dx$ by using substitution $x=\sqrt{t}$ is

☐ $B\left(\frac{m}{2},n\right)$

☐ $\frac{1}{2}B\left(\frac{m-1}{2},n-1\right)$

☐ $\frac{1}{2}B\left(\frac{m}{2},n\right)$

☐ $B\left(\frac{m-1}{2},n-1\right)$

Ans:C(1/2B(m/2,n))

Unit:6 B(m,n)* B(m+n,p) is equal to (beta function)

$B(m,n)\times B(m+n,p)$ is equal to

☐ $\frac{\Gamma m\,\Gamma n}{\Gamma m+n+p}$

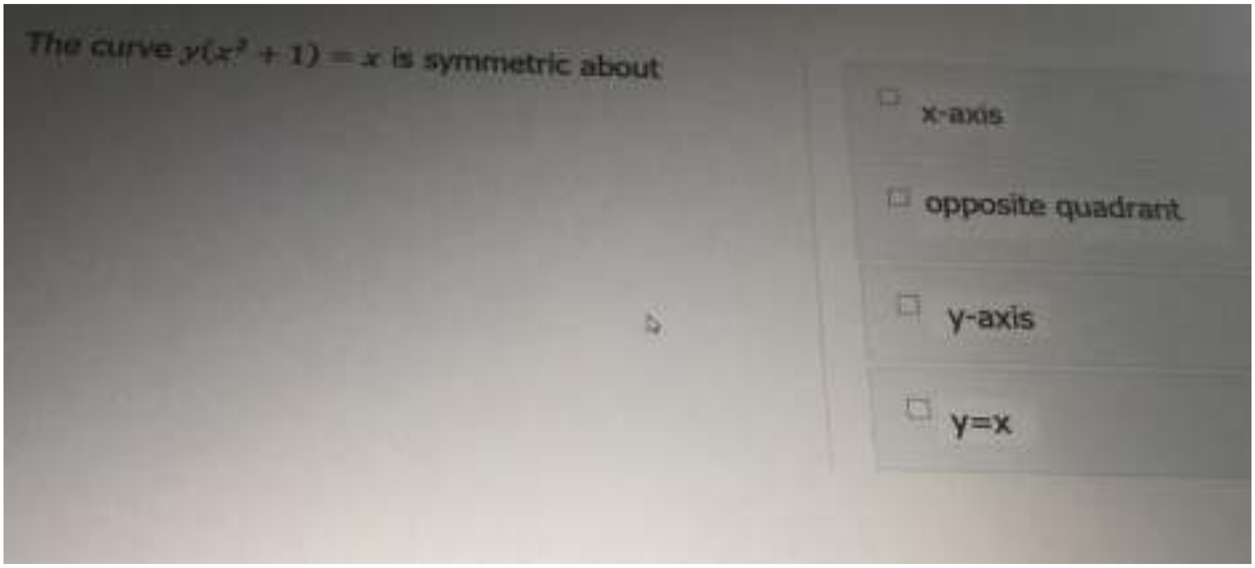
☐ $\frac{\Gamma m\,\Gamma p}{\Gamma m+n+p}$

☐ $\frac{\Gamma m\,\Gamma n\,\Gamma p}{\Gamma m-n-p}$

☐ $\frac{\Gamma m\,\Gamma n\,\Gamma p}{\Gamma m+n+p}$

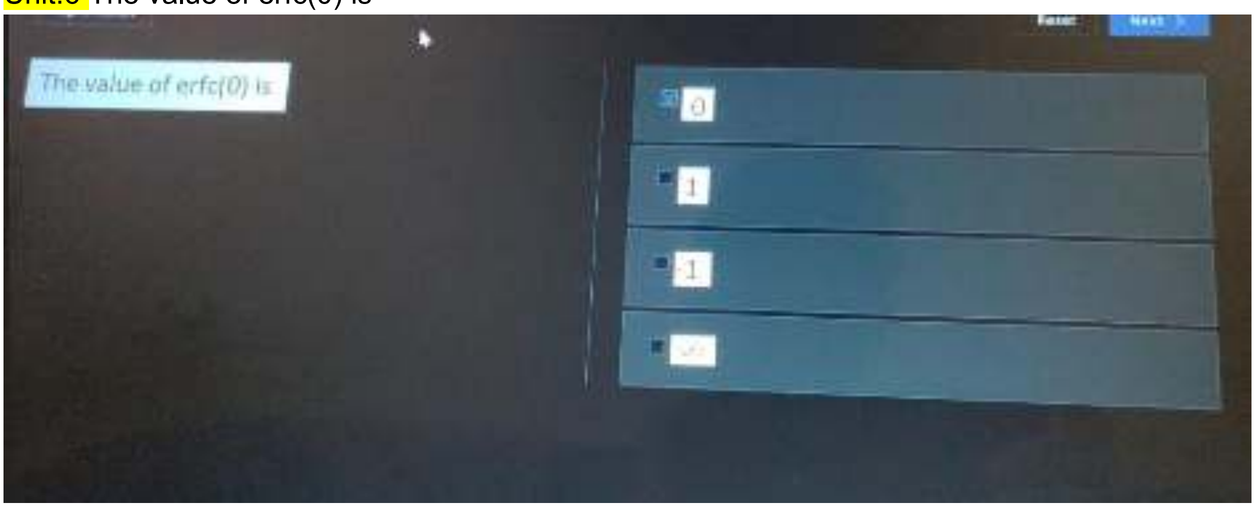
Ans: D

Unit:6 The curve y(x^2+1)=x is symmetric about



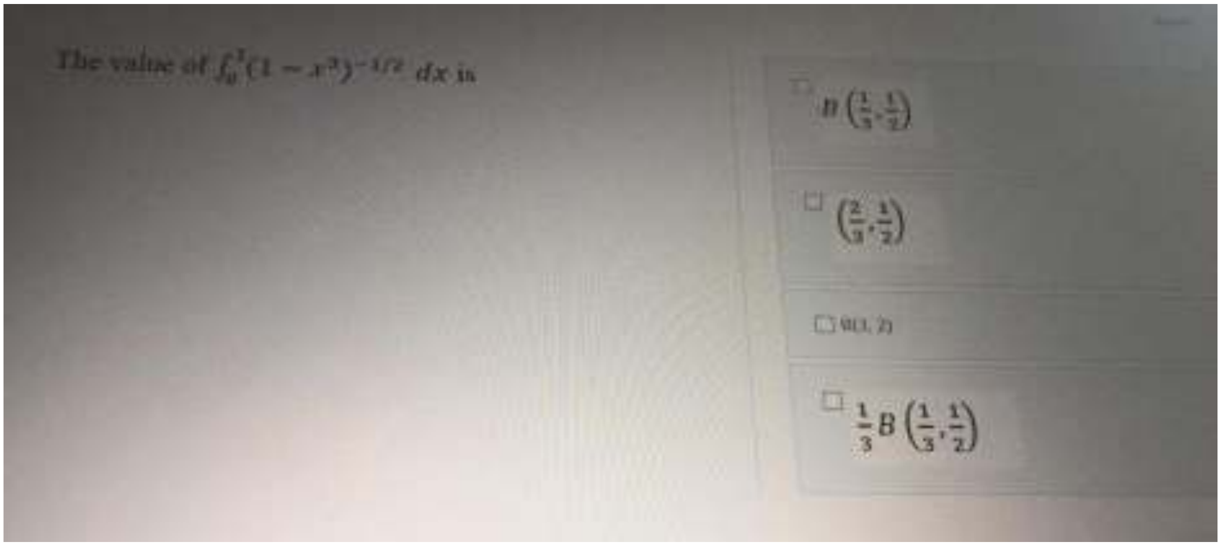
Ans: B opposite quadrant

Unit:6 The value of erfc(0) is



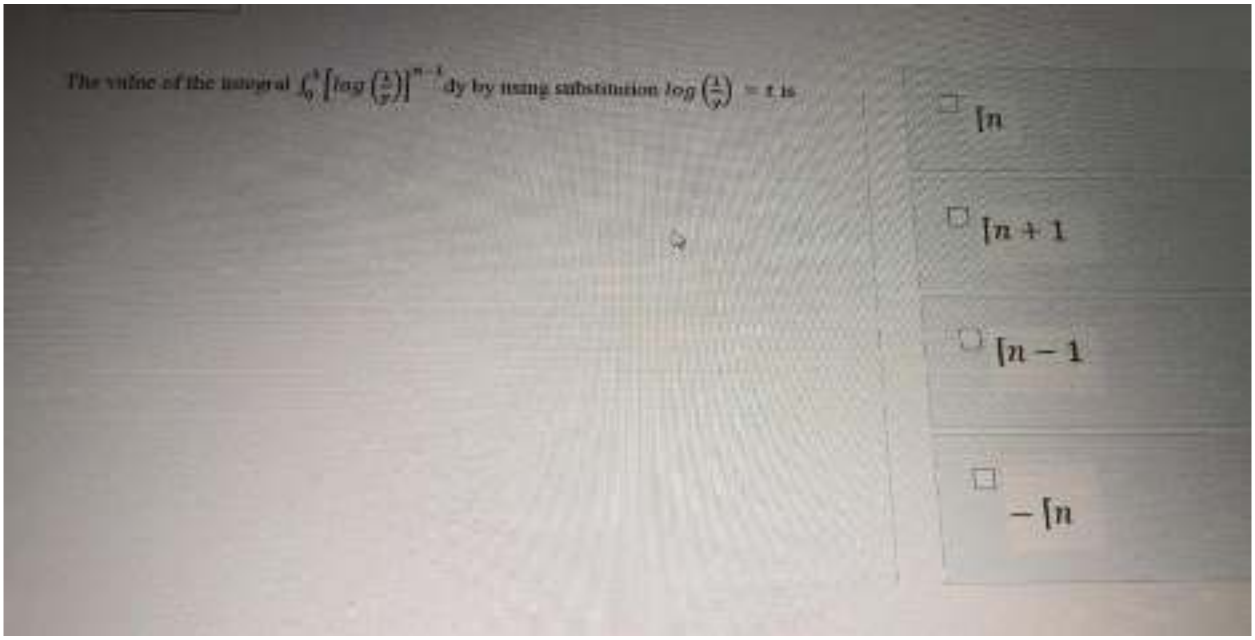
Ans: Should be 1 (B)

Unit 6: The value of (1-x^3)^-½ dx



Ans: d

Unit 6 :The value of the integral log 1/y^(n-1) dy by using substitution log(1/y)=t


The value of the integral $\int_0^1 \left[\log\left(\frac{1}{y}\right)\right]^{n-1} dy$ by using substitution $\log\left(\frac{1}{y}\right) = t$ is

[n

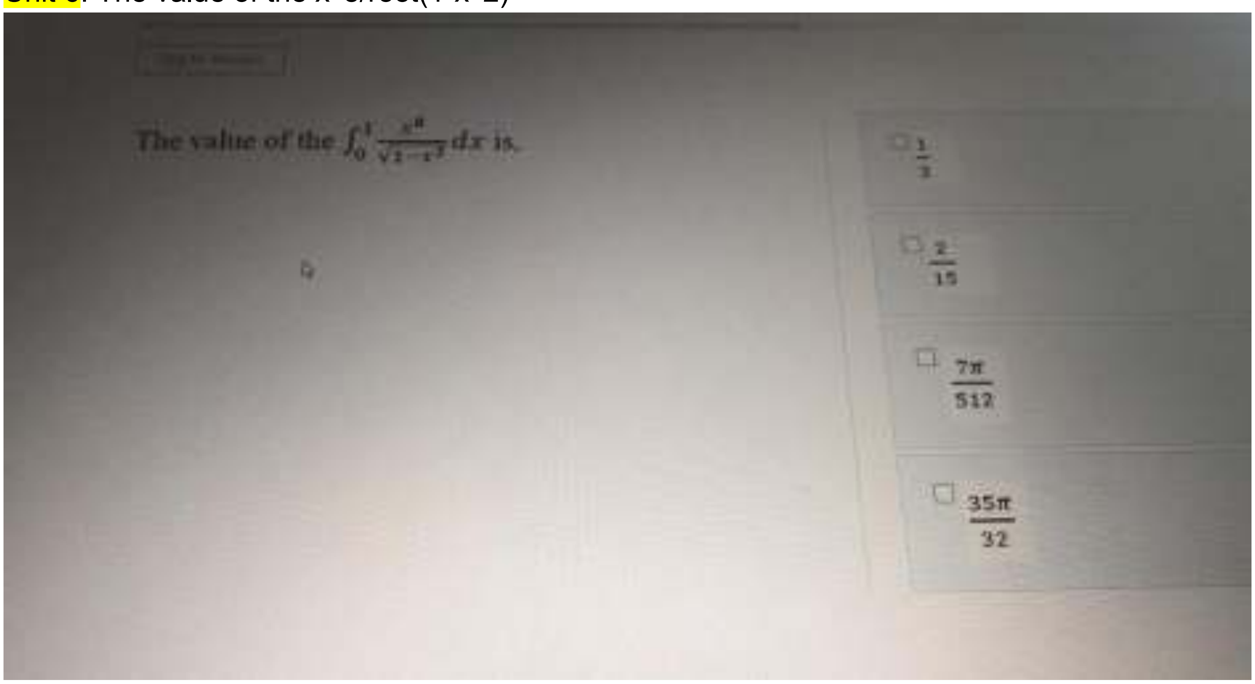
[n + 1

[n − 1

− [n

Ans: [n-1

Unit 6: The value of the x^8/root(1-x^2)


The value of the $\int_0^1 \frac{x^8}{\sqrt{1-x^2}} dx$ is.

$\frac{1}{3}$

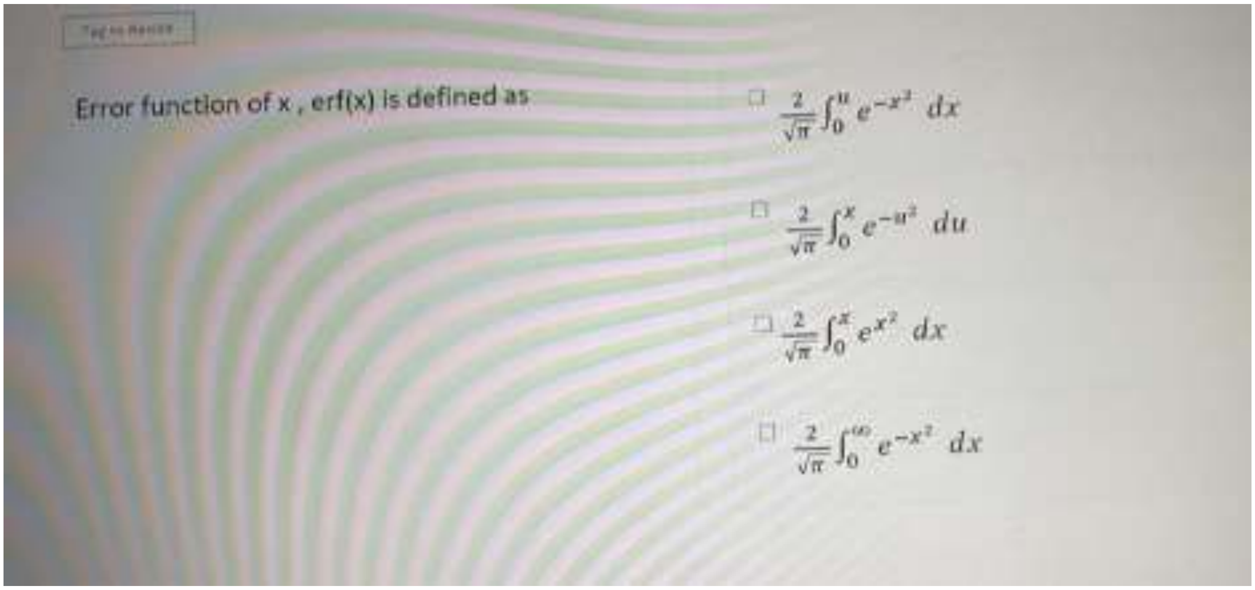
$\frac{2}{15}$

$\frac{7\pi}{512}$

$\frac{35\pi}{32}$

Ans: c :

Unit 6 : Error function of X, erf(x) is defined as

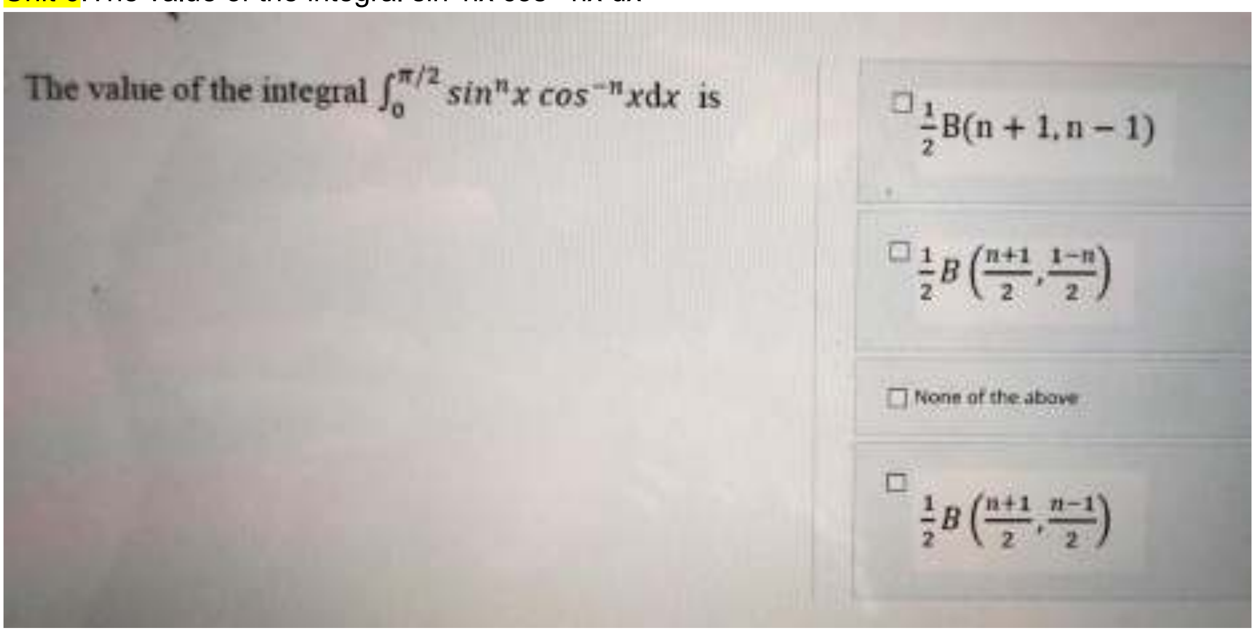


Error function of x , erf(x) is defined as

- $\frac{2}{\sqrt{\pi}}\int_0^{u} e^{-x^2}\, dx$
- $\frac{2}{\sqrt{\pi}}\int_0^{x} e^{-u^2}\, du$
- $\frac{2}{\sqrt{\pi}}\int_0^{x} e^{x^2}\, dx$
- $\frac{2}{\sqrt{\pi}}\int_0^{\infty} e^{-x^2}\, dx$

Ans: b

Unit 6:The value of the integral sin^nx cos^-nx dx

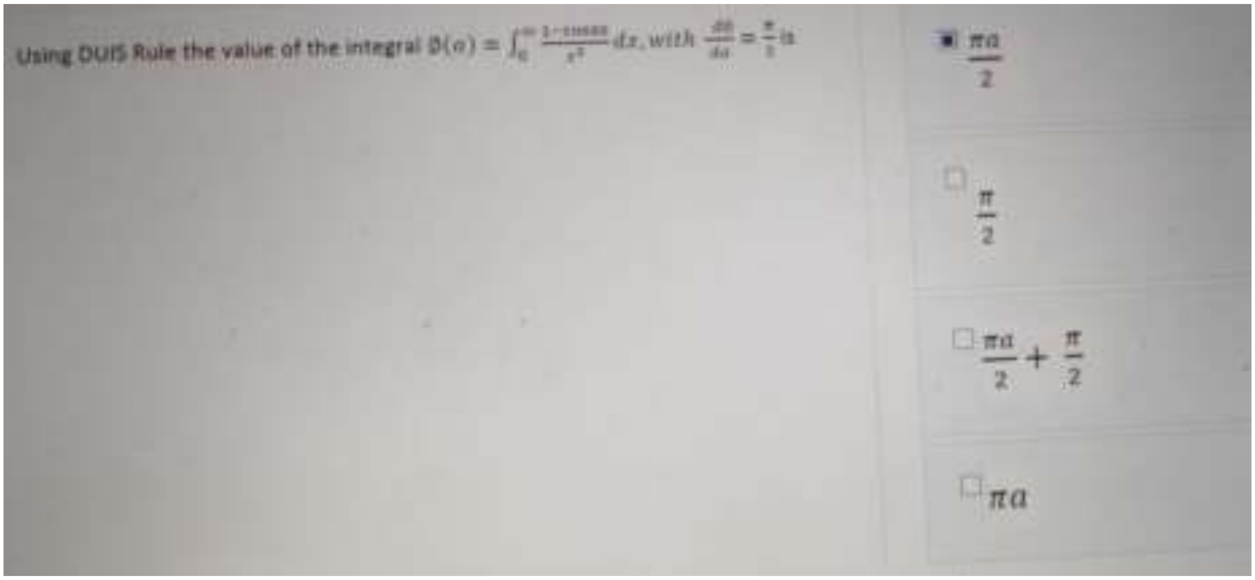


The value of the integral $\int_0^{\pi/2} sin^n x\, cos^{-n} x dx$ is

- $\frac{1}{2}\mathrm{B}(\mathrm{n}+1,\mathrm{n}-1)$
- $\frac{1}{2}B\left(\frac{n+1}{2},\frac{1-n}{2}\right)$
- None of the above
- $\frac{1}{2}B\left(\frac{n+1}{2},\frac{n-1}{2}\right)$

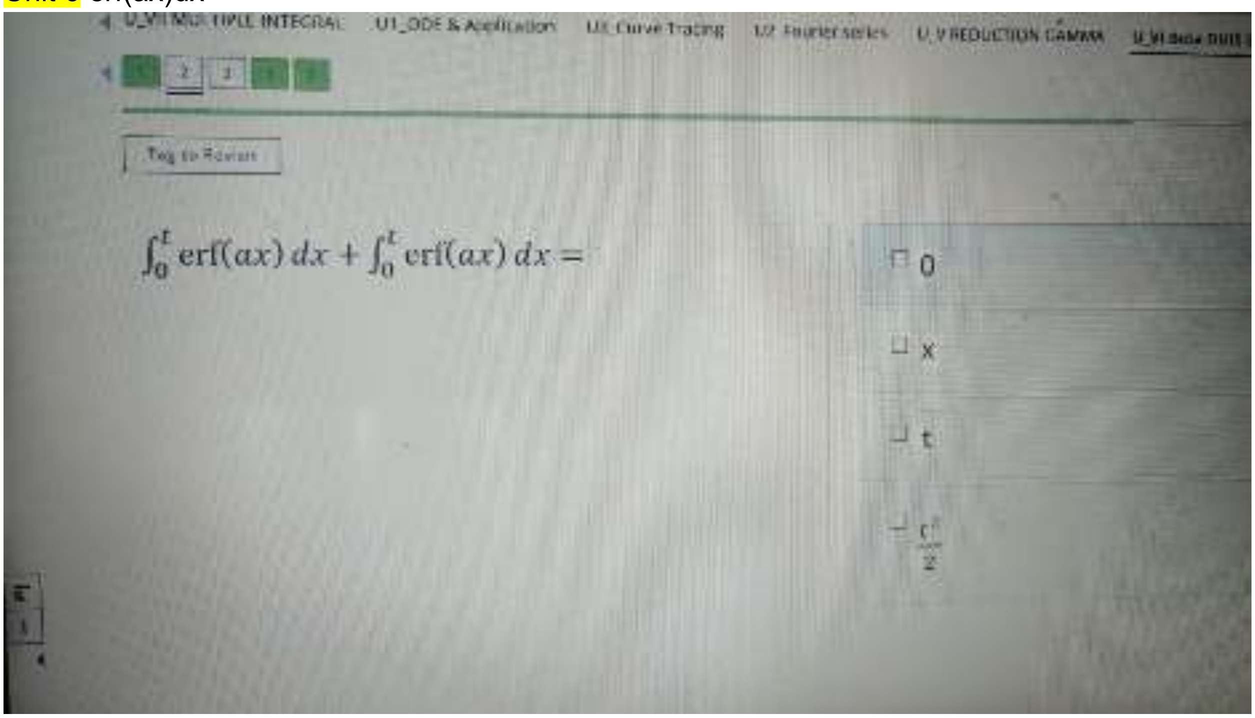
Ans: b

Unit 6 Using DUIS rule of the value of integral



Ans: Pi *a*/ 2

Unit 6 erf(ax)dx



Ans- option c : t

UNIT 6: Using DUIS rule the value of the integral e^-ax sin ax/x.dx from infinity to zero

Using DUIS Rule the value of the integral $\emptyset(\alpha) = \int_0^\infty \frac{e^{-\alpha x}\sin x}{x}dx$, with $\frac{d\emptyset}{d\alpha} = -\frac{1}{\alpha^2+1}$ and assuming $\int_0^\infty \frac{\sin x}{x}dx = \frac{\pi}{2}$ is

- ☐ $\log(\alpha^2+1)+\frac{\pi}{2}$
- ☐ $-\tan^{-1}\alpha+\frac{\pi}{2}$
- ☐ $-\tan^{-1}\alpha$
- ☐ $\tan^{-1}\alpha+\frac{\pi}{2}$

Ans: D

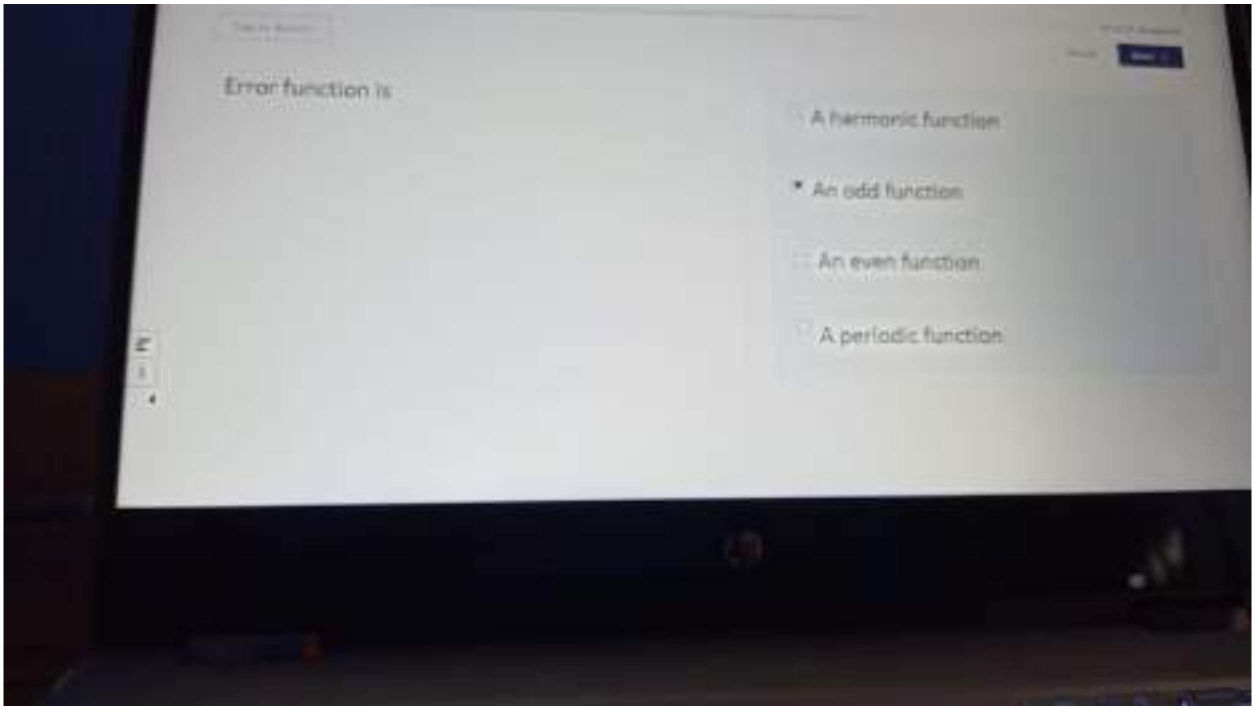
Unit:6 The value of dx/3^4x2 is

The value of $\int_0^\infty \frac{dx}{3^{4x^2}}$ is

- ☐ $\frac{\sqrt{\pi}}{3\sqrt{\log 4}}$
- ☐ $\frac{\sqrt{\pi}}{4\sqrt{\log 3}}$
- ☐ $\frac{\sqrt{\pi}}{\sqrt{\log 3}}$
- ☐ None of the above

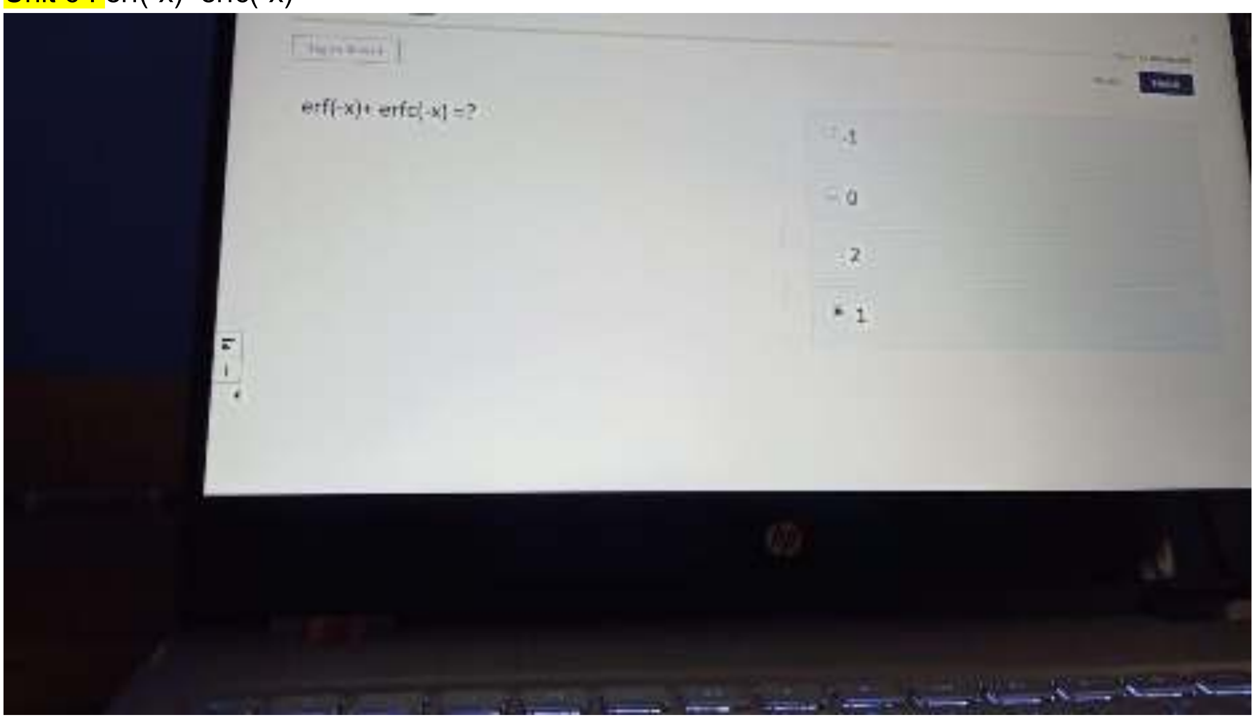
Ans: B

Unit 6 : Error Function is



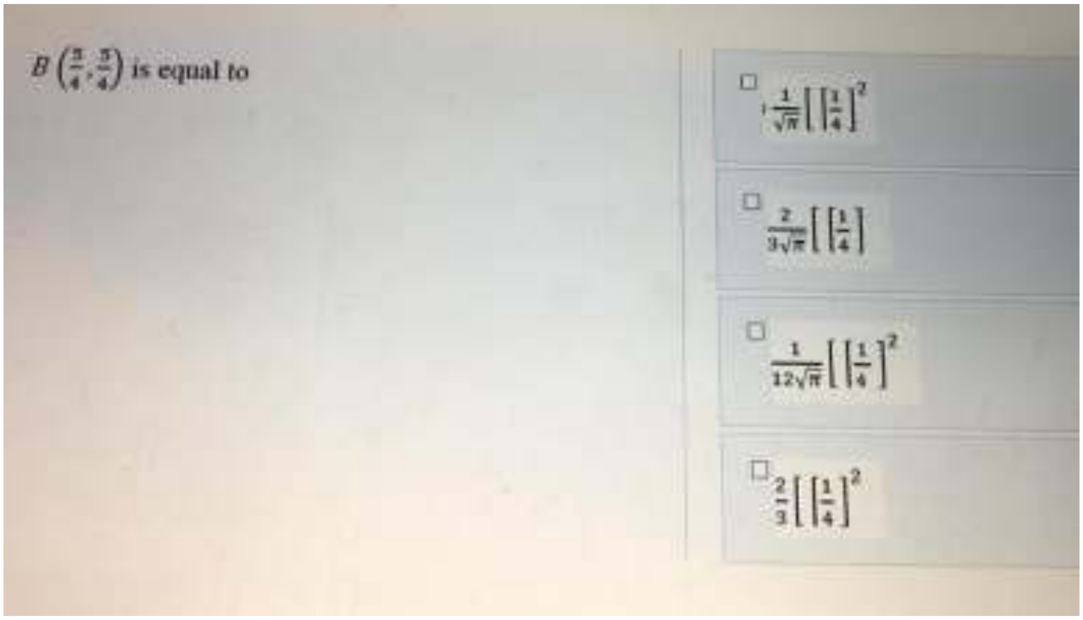
Ans. Option B : An odd function

Unit 6 : erf(-x)+erfc(-x)=



.Ans. option D : 1

Unit 6: B (5/4,5/4) is equal to

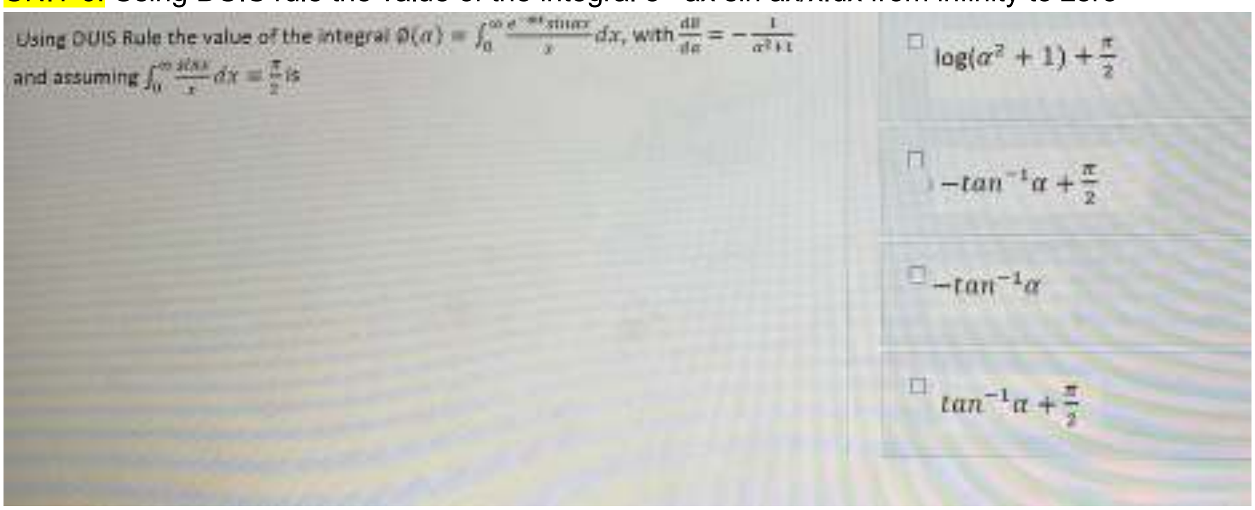


$B\left(\frac{3}{4},\frac{5}{4}\right)$ is equal to

- $\frac{1}{\sqrt{\pi}}\left[\Gamma\left(\frac{1}{4}\right)\right]^2$
- $\frac{2}{3\sqrt{\pi}}\left[\Gamma\left(\frac{1}{4}\right)\right]$
- $\frac{1}{12\sqrt{\pi}}\left[\Gamma\left(\frac{1}{4}\right)\right]^2$
- $\frac{2}{3}\left[\Gamma\left(\frac{1}{4}\right)\right]^2$

Ans: C

UNIT 6: Using DUIS rule the value of the integral e^-ax sin ax/x.dx from infinity to zero

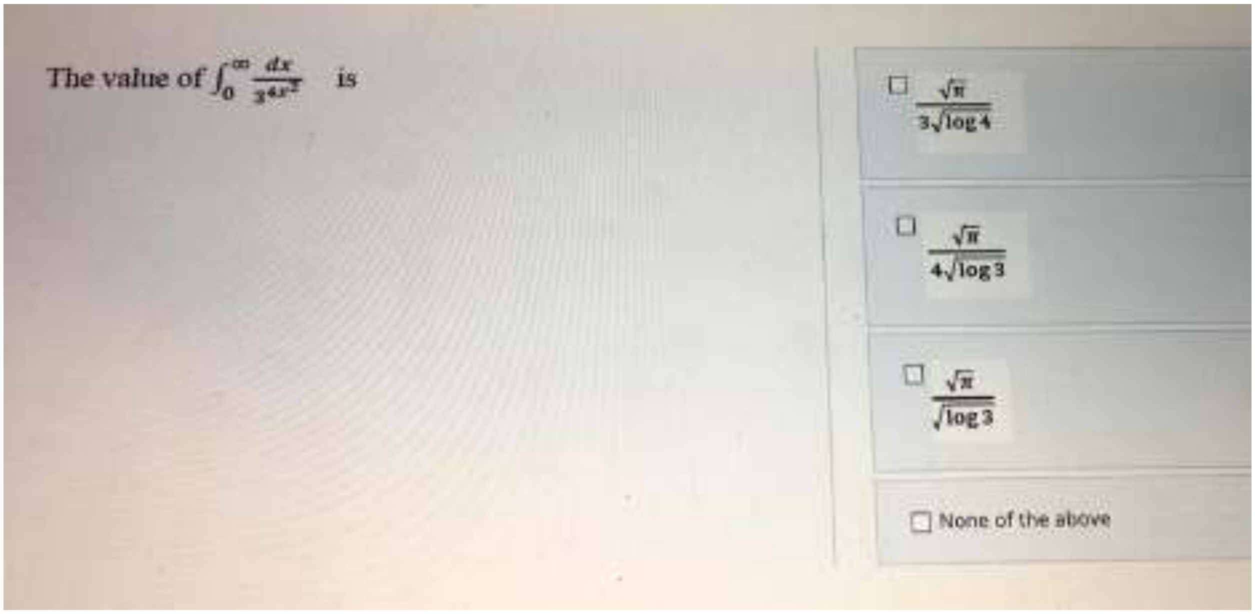


Using DUIS Rule the value of the integral $\emptyset(\alpha) = \int_0^{\infty} \frac{e^{-\alpha x}\sin\alpha x}{x}dx$, with $\frac{d\emptyset}{d\alpha} = -\frac{1}{\alpha^2+1}$ and assuming $\int_0^{\infty}\frac{\sin x}{x}dx = \frac{\pi}{2}$ is

- $\log(\alpha^2+1)+\frac{\pi}{2}$
- $-tan^{-1}\alpha+\frac{\pi}{2}$
- $-tan^{-1}\alpha$
- $tan^{-1}\alpha+\frac{\pi}{2}$

Ans: D

Unit:6 The value of dx/3^4x2 is



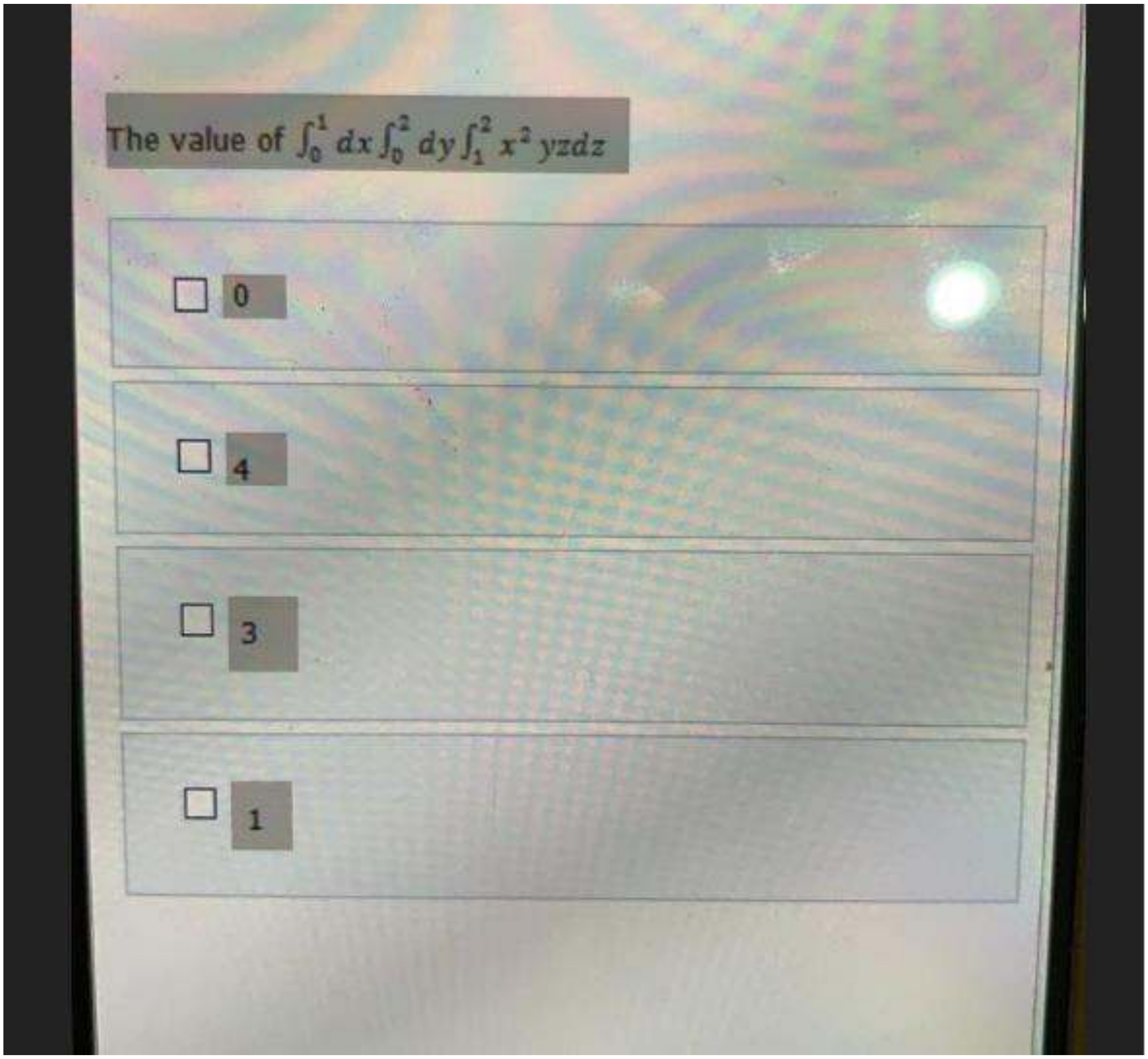
The value of $\int_0^{\infty} \frac{dx}{3^{4x^2}}$ is

- [ ] $\frac{\sqrt{\pi}}{3\sqrt{\log 4}}$
- [ ] $\frac{\sqrt{\pi}}{4\sqrt{\log 3}}$
- [ ] $\frac{\sqrt{\pi}}{\sqrt{\log 3}}$
- [ ] None of the above

Ans: B

Unit: 7 (21 questions)

Unit:7 The Value of integral x^2 yz dz



The value of $\int_0^1 dx \int_0^2 dy \int_1^2 x^2 yzdz$

- 0
- 4
- 3
- 1

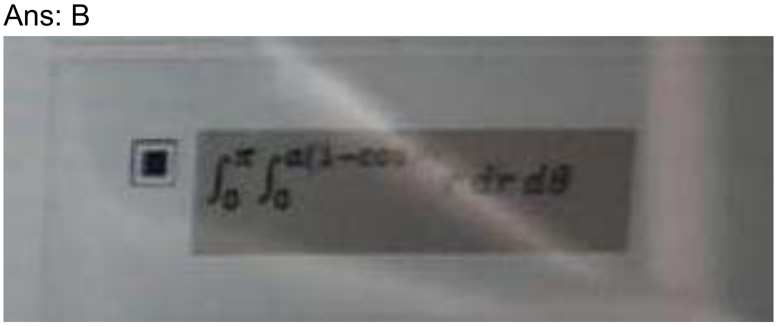
Ans: d (1)

Unit 7: The value of integral r d theta dr

Ans: 3pi/2

Unit 7: The area of upper half of cardiode r=a(1-cos theta) is given by following double integral

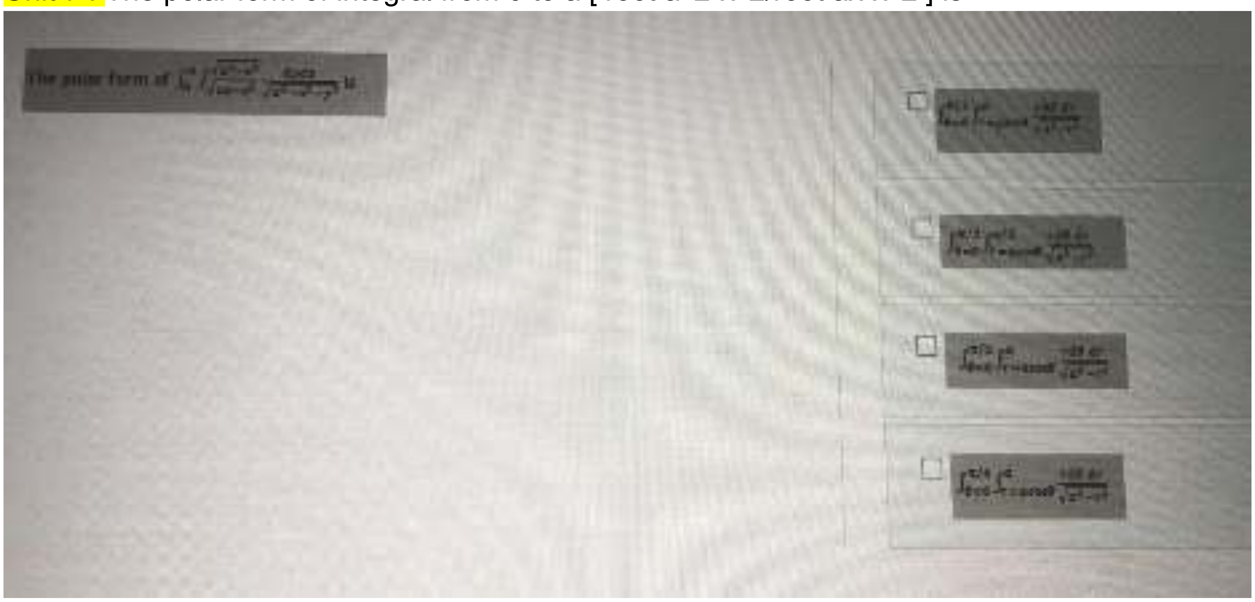
Ans: B



Unit 7: The value of integral xya^x+y dx dy

Ans:B

Unit 7 : The polar form of integral from 0 to a [root a^2-x^2/root ax-x^2] is

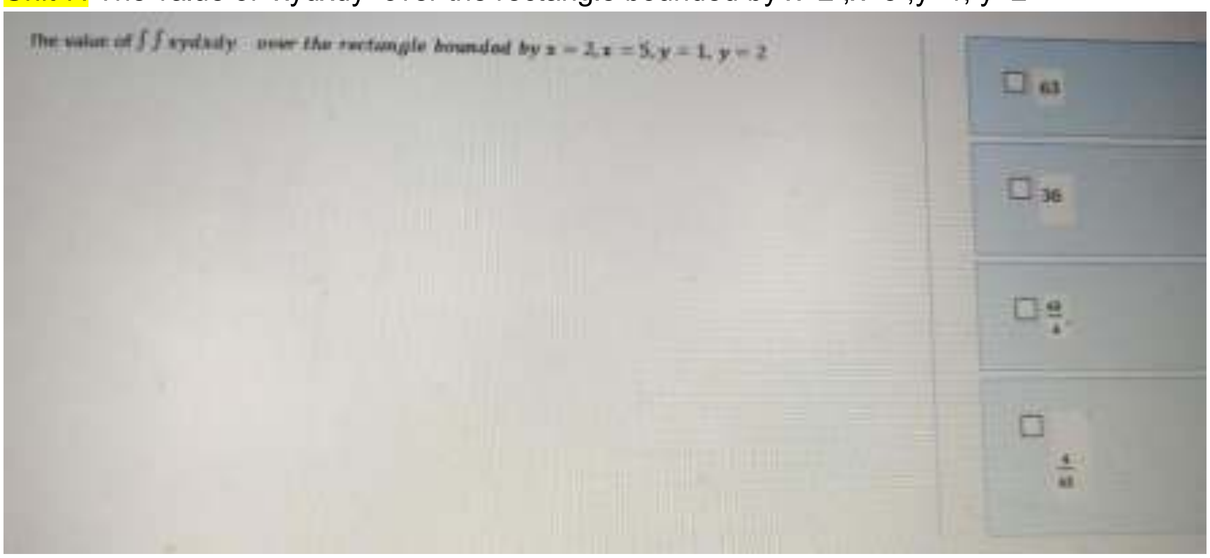


Ans: D

Unit 7: The Value of the integral : dx dy

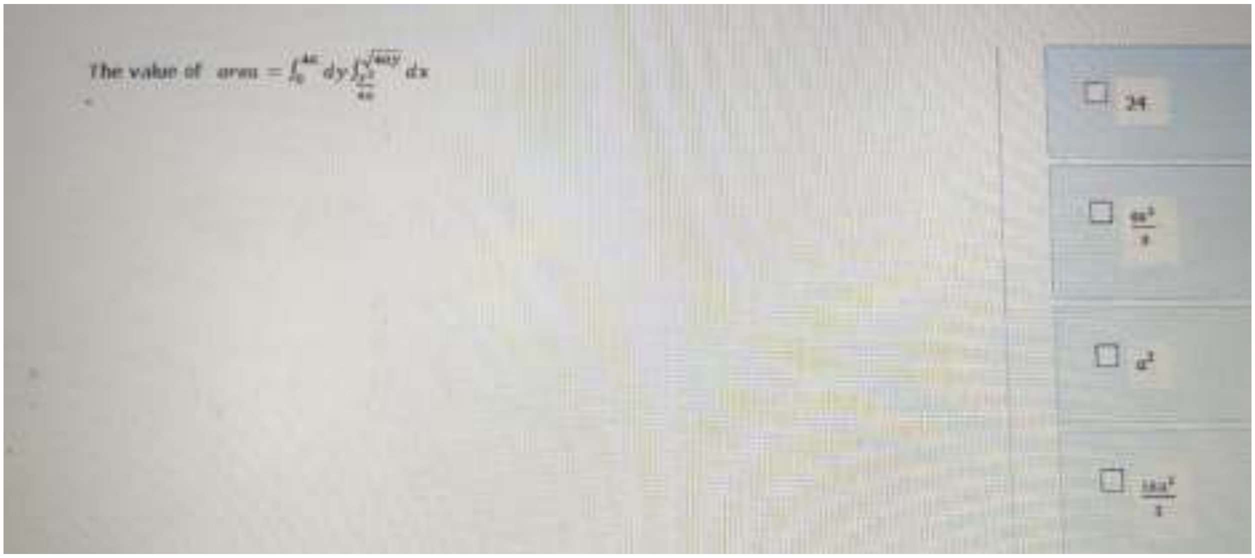
Ans:e^2-1

Unit 7: The value of xydxdy over the rectangle bounded by x=2 ,x=5 ,y=1, y=2



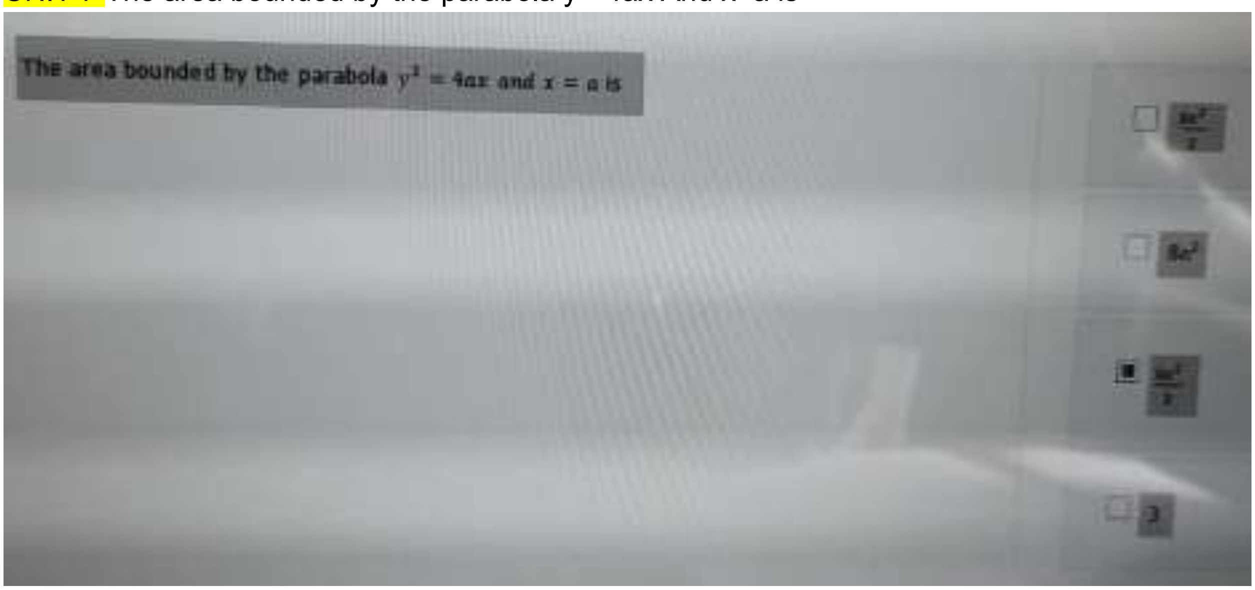
Ans: 63/4

Unit:7 The value of area 4a to 0



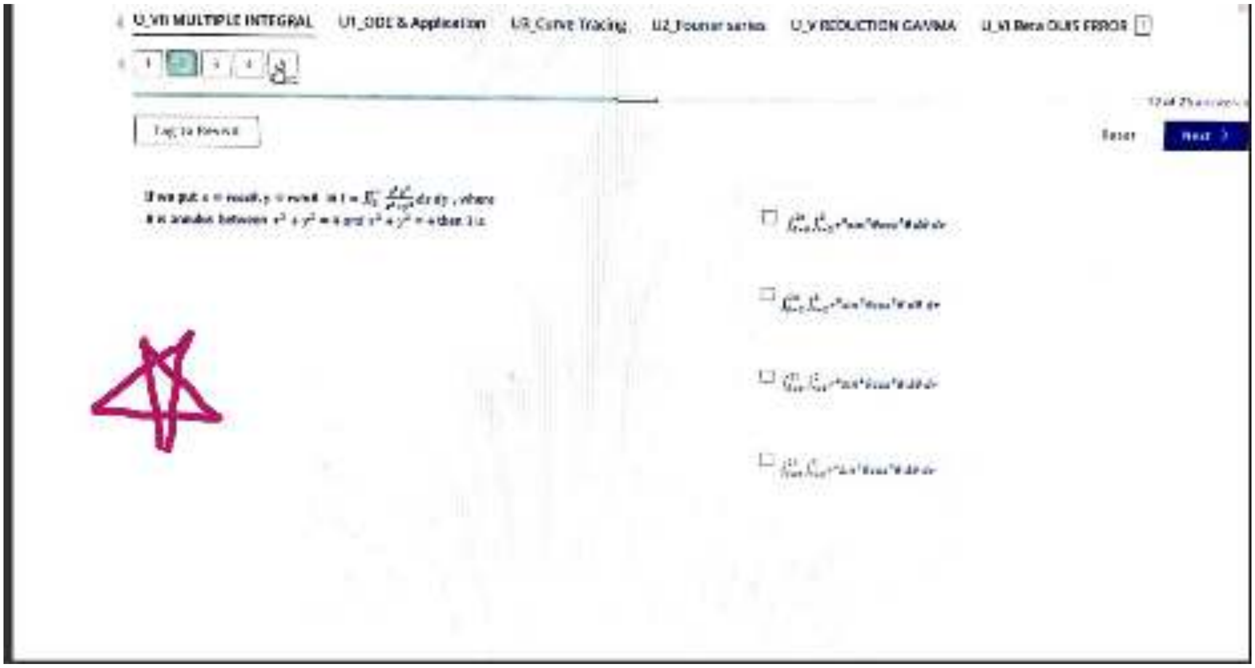
Ans: D

UNIT 7 The area bounded by the parabola y^2=4ax And x=a is



Ans:C(8a^2 / 3)

Unit:7 If we put x= r cos theta, where r is annulus between



Ans: question galat he but answer A hi hoga

UNIT 7: The value of (x^2+y^2+x^2) dxdydz

Ans: 12

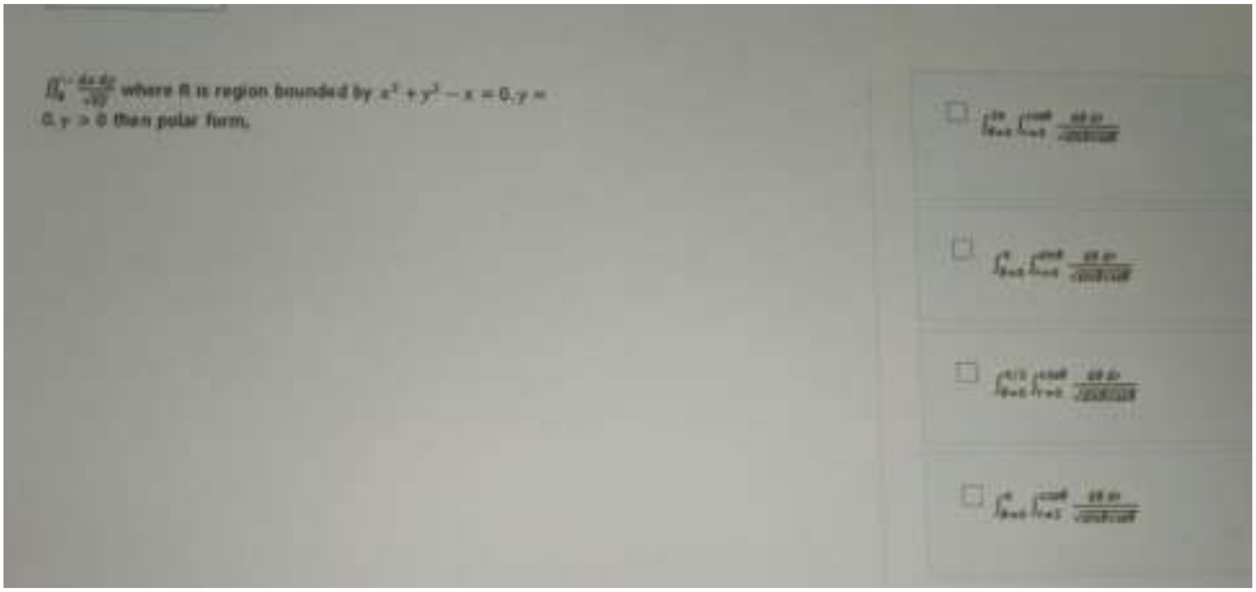
UNIT 7:IF we change the order of integration then the new limits of x and y are

If $I = \int_0^a \int_y^a \frac{x}{x^2+y^2}\,dxdy$, if we change the order of integration then new limits of x and y are

- ☐ $0 \le x \le a,\ 0 \le y \le a$
- ☐ $0 \le x \le \frac{a}{2},\ 0 \le y \le x$
- ☐ $0 \le x \le \frac{a}{2},\ 0 \le y \le \frac{a}{2}$
- ☐ $0 \le x \le a,\ 0 \le y \le x$

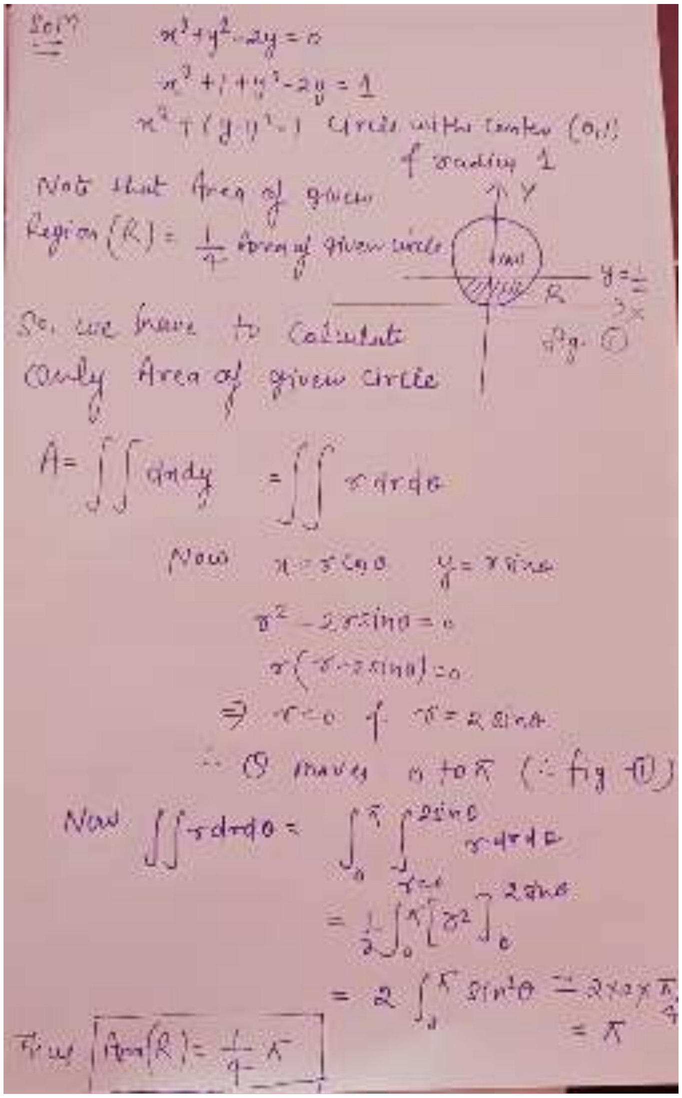
Ans: D

UNIT 7 Where r is region bounded by X^2+Y^2-X=0, y=0,y>0 then polar form,



Ans: c (best guess)[still not very sure]

Sauce



Sol^n ⇒ $x^2+y^2-2y=0$

$x^2+1+y^2-2y=1$

$x^2+(y-1)^2=1$ Circle with center (0,1) & radius 1

Note that Area of given Region (R) = $\frac{1}{4}$ Area of given circle

So, we have to calculate only Area of given circle

$A=\iint dx\,dy = \iint r\,dr\,d\theta$

Now $x=r\cos\theta$ $y=r\sin\theta$

$r^2-2r\sin\theta=0$

$r(r-2\sin\theta)=0$

$\Rightarrow r=0$ & $r=2\sin\theta$

∴ θ varies 0 to π (∵ fig (1))

Now $\iint r\,dr\,d\theta = \int_0^{\pi}\int_{r=0}^{2\sin\theta} r\,dr\,d\theta$

$= \frac{1}{2}\int_0^{\pi}\left[r^2\right]_0^{2\sin\theta}$

$= 2\int_0^{\pi}\sin^2\theta = 2\times2\times\frac{\pi}{4}$

$= \pi$

Thus $\boxed{Area(R)=\frac{1}{4}\pi}$

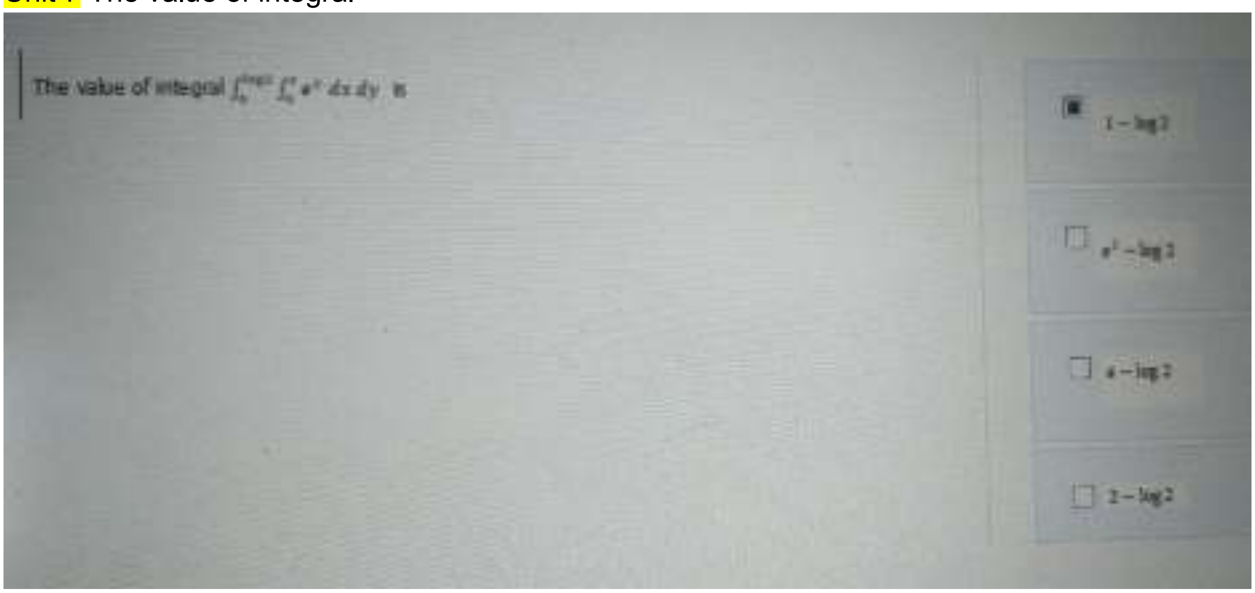
Unit 7 The value of xsinydx dy

Ans: pi^2/2 +2

Unit7 The value of the integral dzdydx is

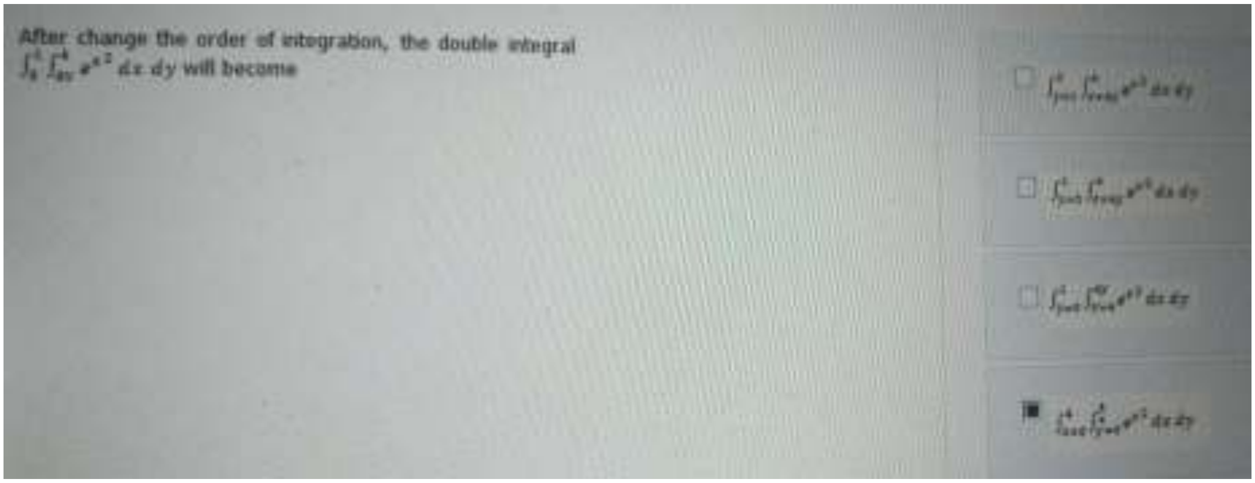
Ans: ⅛

Unit 7 The value of integral

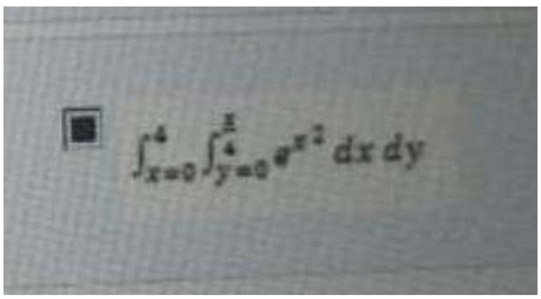


Ans: 1-log2

Unit 7 After change the order of integration, the double integral dx dy will become



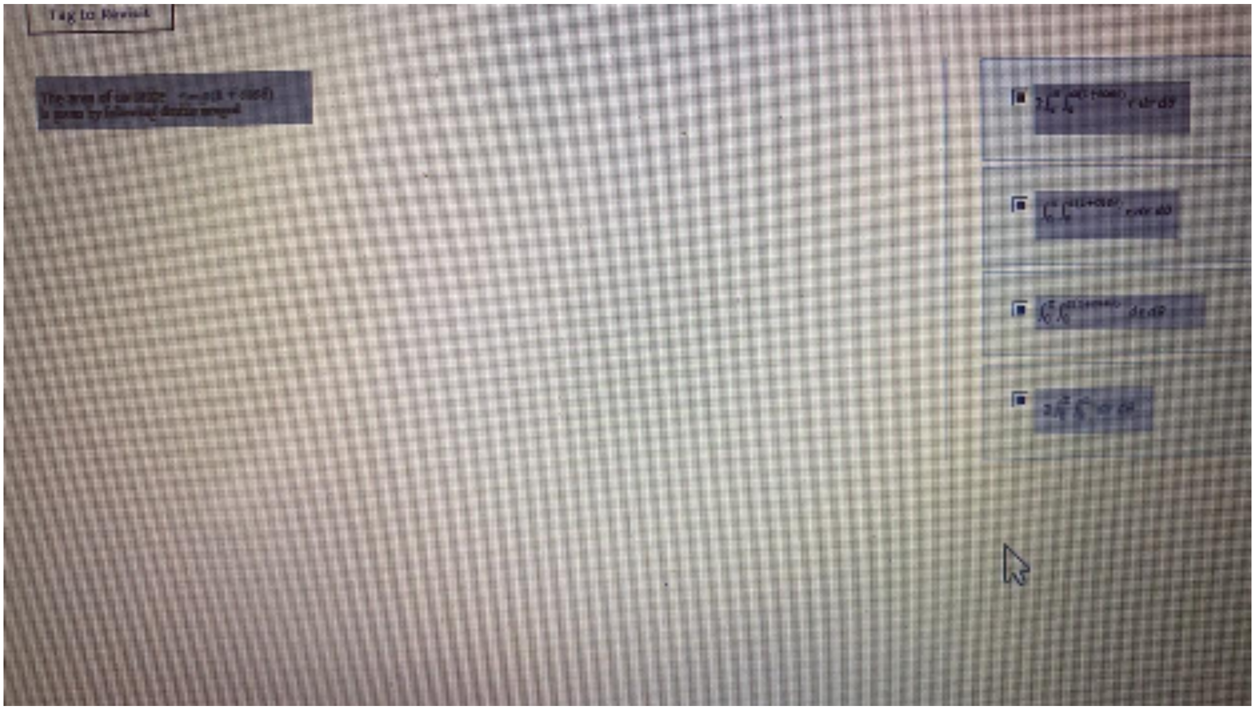
Ans: d



Unit 7 IF I we change the order of integration then new limits of x and y are

Ans: c

Unit 7 The area of cardiode



Ans A

Unit 7: Area of the region bounded by y=x

Ans: C

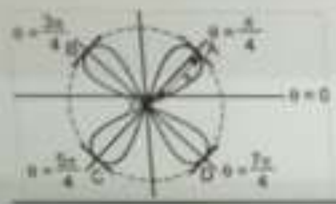
Unit 7: The value of sintheta dtheta dr

The value of $\int_{r=0}^{a}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} r^2 \sin\theta \, d\theta \, d\phi \, dr$

- ☐ $8a^3 \frac{\pi}{3}$
- ☐ $2a^3 \frac{\pi}{3}$
- ☐ $4a^3 \frac{\pi}{3}$
- ☐ $a^3 \frac{\pi}{4}$

Ans B

The following figure represents the curve whose equation is



- $r = a\cos 3\theta$
- $r = a(1+\cos\theta)$
- $r = a\sin 2\theta$
- $r = a\sin 3\theta$

The solution of the differential equation $\frac{dy}{dx} = e^{x+y} + x^2 e^y$

- $e^y + e^x + \frac{x^3}{3} + c = 0$
- None of these
- $e^{-y} + e^x + \frac{x^3}{3} + c = 0$
- $e^{-y} + e^x - \frac{x^3}{3} + c = 0$

The points of intersection with Y & X-axis of the curve $y^2x = a(x^2 - a^2)$

- No point of intersection on both axes
- No point on Y-axis & $(\pm a, o)$
- No point on X-axis & $(0, \pm a)$
- ✓ None of these

If the differential equation of family of curves [illegible] then its orthogonal trajectories is given by

- $r^2 = \log \sec 2\theta + k$
- $r^2 = k \sin 2\theta$
- $r^2 = k \cos 2\theta$
- $\log r = -\frac{1}{2}\sec^2 2\theta + k$

Tag to Review

The region of absence for the curve represented by the equation $a^2x^2 = y^3(2a - y)$

- ☐ $y < 0$ and $y < 2a$
- ☐ $y > 0$ and $y < 2a$
- ☐ $y < 0$ and $y > 2a$
- ☐ $x < 0$ and $y < 2a$

Tag to Review

1 Points Reset Next

The curve represented by the equation $x = a(t - \sin t)$, $y = a(1 - \cos t)$ is

- ☐ symmetric about x axis and not passing through origin.
- ☐ symmetric about y axis and not passing through origin.
- ☐ symmetric about y axis and passing through origin.
- ☐ symmetric about x axis and passing through origin.

Tag to Revisit

The tangent at the origin to the curve represented by the equation
$x = a(t - \sin t)$, $y = a(1 - \cos t)$ is

- x=a
- y=a
- x=0
- y=0

Tag to Revisit

The D.E. $(x + y - 5)dx + (x - y + 4)dy$ is

- Homogeneous
- Linear
- Exact
- Non Exact

Tag to Revisit

1 Points

The equation of tangent to the curve at origin represented by the equation $y^2(4-x)=x(x-2)^2$ is

- x=2
- x=0
- y=0
- x=4

Tag to Revisit

1 Points

The differential equation for the current i in an electric circuit containing resistance 100 ohm and an inductance of 0.5 henry connected in series with battery of 20 volts is (1)

- $0.5\frac{di}{dt}+100i=0$
- $0.5\frac{di}{dt}+100i=20$
- $100\frac{di}{dt}+0.5i=20$
- $100\frac{di}{dt}+0.5R=0$

Tag to Review

If $I = \frac{E}{R}(1 - e^{\frac{-Rt}{L}})$ & E=500volts R=250Ω L=640 H. Then maximum value of I is

- 0.5
- 2
- 0
- None of these

Tag to Review

The region of presence for the curve represented by the equation

$x = t^2,\ y = t - \frac{t^3}{3}$

- $x<0$
- $x>0$
- $y>0$
- $y<0$

Tag for Review

The angle between the radius vector & the tangent to the curve

$$r = \frac{a}{2}(1 + cos\theta)$$

- [ ] $\frac{\pi}{2} - \frac{\theta}{2}$
- [ ] $\pi - \frac{\theta}{2}$
- [ ] $\frac{\pi}{2} + \frac{\theta}{2}$
- [ ] $\pi + \frac{\theta}{2}$

Tag for Review

The charge Q on the plate of the condenser of capacity C charged through a resistance R by a steady voltage V satisfy the differential equation $R\frac{dQ}{dt} + \frac{Q}{C} = V$ If Q=0 at t=0 then $Q = CV(1 - e^{\frac{-t}{RC}})$. Then maximum current is

- [ ] CV
- [ ] None of these
- [x] $\frac{V}{R}$
- [ ] 0

The G.S. of $\log \frac{dy}{dx} = ax + by$

- $ae^{-by} + be^{-ax} + c = 0$
- $ae^{-by} + be^{ax} + c = 0$
- $ae^{by} + be^{ax} + c = 0$
- NONE

Let P is any point on the curve & if $\left(\frac{dy}{dx}\right)_P > 0$ then

- Tangent makes obtuse angle with x
- Tangent parallel to y-axis
- Tangent makes acute angle with x-axis
- Tangent parallel to x-axis

Tag to Revisit

1 Points Reset Next

The integrating factor of the D.E. $\frac{dy}{dx}=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}$

- $2\sqrt{x}$
- ✓ $e^{2\sqrt{x}}$
- $e^{-2\sqrt{x}}$
- $e^{\frac{1}{\sqrt{x}}}$

Tag to Revisit

The curve $a^2y^2=a^2x^2-x^4$ has

- ☐ two asymptotes
- ☐ one asymptote
- ☐ origin is node
- ☐ origin is cusp

Tag to Revisit

1 Points Reset

The tangents at pole to the polar curve $r = a\cos 2\theta$ are

- [ ] $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots\ldots$
- [ ] $\theta = 0, \pi, 2\pi, 3\pi, \ldots\ldots$
- [ ] $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots\ldots$
- [ ] $\theta = \frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \ldots\ldots$

Tag to Revisit

1 Points

The angle ϕ between radius vector and tangent line using $\tan\phi = r\dfrac{d\theta}{dr}$ for the polar equation $r^2 = a^2\cos 2\theta$ is equal to

- [ ] $\dfrac{\pi}{4} + 2\theta$
- [ ] $\pi + 2\theta$
- [ ] $\dfrac{\pi}{2} + \theta$
- [ ] $\dfrac{\pi}{2} + 2\theta$

A steam pipe 20 cm in diameter is protected with covering 6 cm thick for which thermal conductivity k = 0.0003 in steady state. The inner surface of the pipe is at 200°C and outer surface of the covering is at 30°C and differential equation of conduction of heat is $dT = -\frac{Q}{2\pi k}\frac{dx}{x}$. The amount of heat loss Q is (2)

- $\dfrac{170\,(2\pi k)}{\log(1.6)}$
- $-\dfrac{170\,(2\pi k)}{\log(1.6)}$
- $\dfrac{\log(1.6)}{170\,(2\pi k)}$
- 170

Tag to Revisit

The D.E. of orthogonal trajectory of $r = a\cos\theta$ is $\frac{dr}{r} = \cot\theta d\theta$ then orthogonal trajectory is

- $r = c\sin\theta$
- $r = c\cos\theta$
- $r = \cos\theta$
- None

Untag

1 Points Rese

The differential equation $(x^3 + 3y^2x)dx(y^3 + 3x^2y) = 0$

- Only Exact
- Exact and Homogeneous
- Only Homogeneous
- None

Tag to Revisit

1 Points Reset

The integrating factor of the differential equation $(x^4e^x - 2mxy^2)dx + 2mx^2ydy = 0$

$\int \frac{-4}{x} dx$

e

- [ ] $\frac{4}{x}$
- [ ] 4x
- [ ] $\frac{-4}{x}$
- [ ] Exact so factor is 1

1 Points

The equation of tangent to the curve at origin represented by the equation $y = x(x^2-1)$ is

- [ ] x=0
- [ ] y=x
- [ ] y=o
- [ ] y=-x

1 Points

The charge flowing through the R-C series cct with no applied E.M.F is

- [ ] $Q = e^{\frac{t}{RC}} K$ K=constant
- [ ] None of these
- [ ] $Q = e^{\frac{-t}{RC}} K$ K=constant
- [ ] $Q = e^{-tRC} K$ K=constan

1 Points

The linear form of D.E. $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$ by putting $e^y = v$

- $\frac{dv}{dx} + e^x = e^{2x}$
- $\frac{dv}{dx} + ve^x = e^{2x}$
- $\frac{dv}{dx} + ve^x = e^x$
- None

Tag to Revisit

1 Points

The tangent at the origin to the curve represented by the equation $x=t^2,\ y=t-\frac{t^3}{3}$ is

- ☐ y=x
- ☐ y=-x
- ☐ y=x
- ☐ y=0

RESET

NEXT

Tag to Revisit

The equation of tangents to the curve at origin represented by the equation $x^2(x^2-4a^2)=y^2(x^2-a^2)$, where $a>0$ is

- $y=2x, y=-2x$
- $x=a, x=-a$
- $x=2a, x=-2a$
- $y=x, y=-x$

Asymptote parallel to X-axis to the curve $x^2(x^2-4a^2)=y^2(x^2-a^2)$

- $\pm a$
- No Asymptote
- a
- None of these

Tag to Revisit

1 Points

The equation of asymptotes parallel to $y-$axis to the curve represented by the equation $y(1+x^2)=x$ is

- [ ] x=0
- [ ] $x=1,\ x=-1$
- [ ] y=1
- [ ] none of the above

Tag to Revisit

1 Points Reset

Let I be the current flowing in the circuit containing inductance L & capacitance C in a series without applied e.m.f. E then the differential equation is

- None of these
- $L\frac{dI}{dt}+\frac{q}{C}=E$
- $L\frac{dI}{dt}+\frac{q}{C}=0$
- $L\frac{dI}{dt}-\frac{q}{C}=0$

Tag to Revisit

1 Points Reset

The differential equation of orthogonal trajectories of family of curves $2x^2+y^2=cx$ is

- $4x+2y\frac{dy}{dx}=\frac{2x^2+y^2}{x}$
- $4x-2y\frac{dx}{dy}=\frac{2x^2+y^2}{x}$
- $4x+2y\frac{dx}{dy}=\frac{2x^2+y^2}{x}$
- none of these

Tag to Revisit

1 Points

The solution of D.E. $(x^2+e^x)dx+ydy=0$

- $\frac{x^3}{3}+e^x+\frac{y^2}{2}=c$
- $x^3+e^x+y^2=c$
- $x^3+e^x+\frac{y^2}{2}=c$
- None of these

Tag to Revisit

The differential equation of orthogonal trajectories of family of straight lines $y = mx$ is (1)

- $\frac{dy}{dx} = \frac{y}{x}$
- $\frac{dx}{dy} = -\frac{y}{x}$
- $\frac{dx}{dy} = -\frac{x}{y}$
- $\frac{dy}{dx} = m$

Tag to Revisit

The integrating factor for the differential equation of R-L series circuit is

- $e^{\frac{Lt}{R}}$
- $e^{\frac{Rt}{L}}$
- $e^{\frac{Lt}{R}}$
- $e^{\frac{Rt}{L}}$

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Tag to Revisit

1 Points

The curve $r = a\cos 5\theta$ can be obtained from $r = a\sin 5\theta$ by rotating plane through

- 5π
- 10π
- $\frac{\pi}{5}$
- $\frac{\pi}{10}$

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Tag to Revisit

1 Points

Region of existence of the curve $y^2 = \frac{a^2(a-x)}{x}$

- None of these
- $x < 0, \ x < a$
- $0 < x < a$
- $x > 0, x > a$

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Tag to Revisit

1 Points

$2\frac{dy}{dx} - y\sec x = y^3 \tan x$ linear form of these equation is

- [ ] $\frac{du}{dx} + (\sec x)u = \tan x$
- [ ] $\frac{du}{dx} - (\sec x)u = \tan x$
- [ ] $\frac{du}{dx} + (\sec x)u = -\tan x$
- [ ] $\frac{du}{dx} - (\sec x)u = -\tan x$

Tag to Revisit

Tangent at origin to the curve $r = acos3\theta$

- $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}....$
- None of these
- $0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi ....$
- $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}....$

Tag to Revisit

1 Points Reset Finish

The G.S of D.E $x^3 \frac{dy}{dx} = secy$ is

- $siny + \frac{1}{x^2} = c$
- $cosy + \frac{1}{2x^2} = c$
- $cosy - \frac{1}{2x^2} = c$
- $siny + \frac{1}{2x^2} = c$

Ordinary Differential Equations

1. Form a differential equation whose general solution is

$i)\ y = ae^{-2x} + be^{-3x}$ **(Ans : $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$)**

$ii)\ y = e^{x}(Acosx + Bsinx)$ **(Ans : $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$)**

Solve the differential equations-

1. $\frac{dy}{dx} = \frac{x\,sinx}{2e^{y}sinhy}$ **(Ans: $\frac{e^{2y}}{2} - y + xcosx - sinx = C$)**
2. $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$ **(Ans : $\log[1 + \tan(\frac{x+y}{2})] - x = C$)**
3. $\frac{dy}{dx} = \frac{x^3-y^3}{yx^2}$ **(Ans: $cy = e^{\frac{-x^3}{3y^3}}$)**
4. (x + y cot $\frac{x}{y}$)dy – y dx =0 (Ans : y cos $\frac{x}{y}$=C)
5. $\frac{dy}{dx} = \frac{\tan y - 2xy - y}{x^2 - x\tan^2 y + \sec^2 y}$ (Ans : x.tany – xy – x^2y – tany = C)
6. $ylogy\, dx + (x - \log y)dy = 0$ **(Ans :2xlogy – $(logy)^2$= C)**
7. $(2y + y^4)dx + (2y^4 + xy^3 - 4x)dy = 0$ **(Ans : $xy + \frac{x}{y^2} + y^2 = C$)**
8. $xcos\,x\frac{dy}{dx} + (\cos x - xsin\,x)y = 1$ **(Ans : $xycosx - x = C$)**
9. $\frac{dy}{dx} - xy = -y^3e^{x^2}$ **$(Ans : \frac{e^{x^2}}{y^2} = 2x + C)$**
10. $(y - 2x^3)dx - x(1 - xy)dx = 0$ **(Ans: $\frac{y}{x} + x^2 - \frac{y^2}{2} = C$)**
11. $(x^2y - 2xy^2)dx = (x^3 - 3x^2y)dy$ **Ans : $\frac{x}{y} - 2logx + 3logy = C$**
12. $ye^{y}dx = (y^3 + 2xe^{y})dy$ **(Ans : $\frac{x}{y^2} + e^{-y} = C$)**
13. $sin\,y\frac{dy}{dx} - cosx(2cosy - \sin^2 x)\,y = 0$
(Ans : $4cosy = 2\sin^2 x + 2sinx = 1 = Ce^{-2sinx}$)
14. $\cos y - xsin\,y\frac{dy}{dx} = \sec^2 x$ **(Ans: $xcosy = tanx + C$)**
15. $\left(xy^2 - e^{\frac{1}{x^3}}\right)dx + x^2\,y\,dy = 0$ **(Ans : $\frac{3y^2}{x^2} - 2e^{-x^{-3}} = C$)**

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APPLICATIONS OF DIFFERENTIAL EQUATIONS

Orthogonal Trajectories

13. Find the orthogonal trajectories of the family of $y^2 = 4ax$. **(Ans : $2x^2 + y^2 = C$)**

14. Find the orthogonal trajectory of the family of ellipses $\frac{1}{2}x^2 + y^2 = C$, where C > 0
(Ans : $x^2 = ky$)

[**Ref: Kreyszig, page-36**]

15. Find the orthogonal trajectory of the family of r = a cos θ. (Ans : $r = Csin\theta$)

16. Find the orthogonal trajectory of the family of r = a (1- cos θ). (Ans: r = C(1+cosθ))

17. Find the orthogonal trajectory of the family of $e^x + e^{-y} = c$. (Ans: $e^y - e^{-x} = C$)

Electric Circuits

20. An electric circuit contains an inductance of 0.5 henry and a resistance of 10 ohms in series with electro motive force of 20 volts. Find the current at any time t, it is zero at t=0. (Ans : $\frac{1}{5}(1 - e^{-200t})$)

21. A circuit consist of resistance R ohms and condenser C farads connected to constant electromotive force E, if $\frac{q}{C}$ is the voltage of condenser at time t after closing the circuit. Show that the voltage at time t, is $E\left(1 - e^{-\frac{t}{RC}}\right)$. Also find current flowing into the circuit.

22. The charge Q on the plate of a condenser of capacity $'C'$ charged through a resistance $'R'$ by steady voltage $'V'$ satisfies the differential equation $R\frac{dQ}{dt} + \frac{Q}{C} = V$. If $Q = 0$ at $t = 0$ then show that $Q = CV\left[1 - e^{-t/RC}\right]$. Find the current flowing into the plate. (Ans: $i = \frac{V}{R}\, e^{-\frac{t}{RC}}$)

23.Find the current i in the circuit having resistance R and condenser of capacity C in series with emf E sin ωt.

24.The equation of L-R circuit is given by L$\frac{dI}{dt}$. + RI = 10 sin t .If I=0, at t = 0 , express I as a function of t . (Ans: $I = \frac{10}{\sqrt{R^2+L^2}}[\sin(t - \emptyset) + sin\emptyset\, e^{\frac{-Rt}{L}}$)

Heat Conduction

23. A steam pipe 20 cm in diameter is protected with a covering 6 cm thick for which the coefficient of thermal conductivity is k = 0.0003 cal/cm in steady state. Find the heat lost per hour through a meter length of the pipe, if the inner surface of the pipe is at 200 ℃ and the outer surface of the covering is at 30 ℃. (Ans : q=245443.3861)

24. A long hollow pipe has an inner diameter of 10 cm and outer diameter of 20 cm the inner surface is kept at 200 ℃ and outer surface at 50 ℃. The thermal conductivity is 0.12. How much heat is lost per minute from a portion of the pipe 20 m long and also find temperature at distance x=7.5 cm from the centre of pipe. (Ans: T=113 C)

Tracing of Curve

Trace the following curves

1) $y^2(2a-x)=x^3$
2) $(x^2+y^2)x=(x^2-y^2)$
3) $xy^2=a^2(a-x)$
4) $x^2y^2=a^2(y^2-x^2)$
5) $(x^2+a^2)y^2=a^2x^2$
6) $(x^2+4a^2)y=8a^3$
7) $x=a(t+sint)\,,y=a(1-cost)$
8) $x=a(t-sint)\,,y=a(1-cost)$
9) $x=a(t+sint)\,,y=a(1+cost)$
10) $r^2=a^2cos2\theta$
11) $r=a\,cos2\theta$
12) $r=a\,cos5\theta$
13) $r=a\,(1-cos\theta)$
14) $r=a\,sin2\theta$
15) $r=2\,sin5\theta$

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Reduction Formulae, Beta and Gamma

1. Evaluate $\int_0^{\pi} x \sin^5 x \cos^8 x\, dx$ **Ans.** $\frac{8\pi}{1287}$
2. Evaluate $\int_0^{2a} x^3 (2ax - x^2)^{\frac{3}{2}} dx$ **Ans.** $\frac{9\pi a^7}{16}$
3. Find the reduction formula for $\int_0^{\frac{\pi}{3}} cos^n x\, dx$ and using it evaluate $\int_0^{\frac{\pi}{3}} \cos^6 x\, dx$.

   Ans. $\boldsymbol{I_n = \frac{\sqrt{3}}{n2^n} + \frac{n-1}{n} I_{n-2}}$, $\boldsymbol{\frac{3\sqrt{3}}{32} + \frac{5\pi}{48}}$
4. If $I_n = \int_0^{\frac{\pi}{4}} \frac{\sin(2n-1)x}{\sin x} dx$ then prove that $n(I_{n+1} - I_n) = \sin \frac{n\pi}{2}$ and hence find I_3.

   Ans. $1 + \frac{\pi}{4}$
5. If $I_n = \int_0^{\infty} e^{-x} \sin^n x\, dx$, Obtain the relation between I_n and I_{n-2} .

   Ans. $I_n = \frac{n(n-1)}{n^2+1} I_{n-2}$
6. Evaluate $\int_0^{\infty} x^7 e^{-2x^2} dx$ **Ans.** $3/16$
7. Evaluate $\int_0^{\infty} 3^{-4x^2} dx$ **Ans.** $\frac{\sqrt{\pi}}{4\sqrt{\log 3}}$
8. Evaluate $\int_0^{\infty} \frac{x^4}{4^x} dx$ **Ans.** $\frac{24}{(\log 4)^5}$
9. Evaluate $\int_0^1 \frac{dx}{\sqrt{-\log x}}$ **Ans.** $\sqrt{\pi}$
10. Evaluate $\int_0^1 x^3 (\log x)^4 dx$ **Ans.** $\frac{3}{128}$
11. Show that $\int_0^1 \frac{dx}{\sqrt[3]{1-x^3}} = \frac{2\pi}{3\sqrt{3}}$
12. Show that $\int_0^{\infty} \frac{x^6 - x^3}{(1+x^3)^5} x^2 dx = 0$
13. Evaluate $\int_3^7 (x-3)^{\frac{1}{4}} (7-x)^{\frac{1}{4}}\, dx$ **Ans.** $\frac{2}{3\sqrt{3}} \left(\gamma\left(\frac{1}{4}\right)\right)^2$
14. Prove that $\int_0^{\infty} \frac{dx}{1+x^4} = \frac{\pi}{2\sqrt{2}}$
15. Show that $\int_0^{\frac{\pi}{2}} \sqrt{\tan\theta}\, d\theta \int_0^{\frac{\pi}{2}} \sqrt{\cot\theta}\, d\theta = \frac{\pi^2}{2}$.

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Differentiation Under Integral Sign (DUIS)

1. Show that $\int_0^1 \frac{x^a-1}{logx} = \log(a+1)\,, \;\; a \geq 0$
2. Show that $\int_0^\infty \frac{e^{-x}-e^{-ax}}{xsecx} dx = \frac{1}{2}\log(\frac{a^2+1}{2})$
3. Find $\int_0^\infty \frac{e^{-ax}sinmx}{x} dx$ and hence evaluate $\int_0^\infty \frac{sinx}{x} dx$
4. Prove that $\int_0^\infty \frac{1-cosax}{x^2} dx = \frac{\pi a}{2}$
5. If $y = \int_0^x f(t)sina(x-t)dt$ then show that $\frac{d^2y}{dx^2} + a^2y = af(x)$
6. If $\emptyset(a) = \int_a^{a^2} \frac{sinax}{x} dx$ then find $\frac{d\emptyset}{da}$
7. Verify the DUIS rule for the $\int_a^{a^2} logaxdx$

Error Function

1. Prove that $erfc(-x) + erfc(x) = 2$
2. Show that $\int_a^b e^{-x^2}\, dx = \frac{\sqrt{\pi}}{2}[\text{erf}(b) - \text{erf}(a)]$
3. Find $\frac{d}{dx} erfc(ax^n)$
4. Prove that $\text{erf}(x)$ is an odd function. Deduce that $erfc(-x) - \text{erf}(x) = 1$
5. Show that $\int_0^\infty e^{-st}\,\text{erf}(\sqrt{t})\, dt = \frac{1}{s\sqrt{s+1}}$
6. Show that $\int_0^\infty e^{-(x+a)^2}\, dx = \frac{\sqrt{\pi}}{2}[1 - \text{erf}(a)]$
7. Show that $\frac{d}{dt}\text{erf}(\sqrt{t}) = \frac{e^{-t}}{\sqrt{\pi t}}$ and hence evaluate $\int_0^\infty e^{-t}\,\text{erf}(\sqrt{t})\, dt$.
8. Show that $\int_0^t \text{erf}(ax)\, dx + \int_0^t erfc(ax)dx = t$.

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Practice Problems

Double Integral and Applications

1. $\int_1^2 \int_{-\sqrt{2-y}}^{\sqrt{2-y}} 2x^2\, y^2 dx\, dy$ (Ans: $\frac{856}{945}$)
2. $\iint \sqrt{4x^2 - y^2}\;\; dx\, dy$ over the area of triangle $y = 0\,, y = x\; \&\; x = 1$

   Ans: $\frac{1}{3}\left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\right)$
3. $\iint_R xy\,\sqrt{1 - x - y}\, dxdy$ over the region $x \geq 0\,, y \geq 0\;\&\;\; x + y \leq 1$ (Ans: $\frac{16}{945}$)
4. Evaluate $\iint_R x^2 + y^2\; dxdy$ over area of triangle whose vertices are $(0{,}1)$ $(1\,{,}1)$ & $(1,\,2)$. (Ans: $\frac{7}{6}$)
5. Show that $\int_0^a \int_0^{a-\sqrt{a^2-y^2}} \frac{xyiog(x+a)}{(x-a)^2} dxdy = \frac{a^2}{8}(2loga + 1)$
6. Evaluate by changing the order

   I) $\int_0^1 \int_{x^2}^{2-x} xy\, dxdy$ (Ans: $\frac{3}{8}$)

   II) $\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{e^y}{(e^y+1)\sqrt{1-x^2-y^2}} dxdy$ (Ans : $\frac{\pi}{2}\log\left(\frac{e+1}{2}\right)$
7. Express the following integral as a single integral

   $\int_0^1 \int_0^y f(x,y)dxdy + \int_1^\infty \int_0^{\frac{1}{y}} f(x,y)dxdy$ (Ans: $\int_0^1 \int_x^{\frac{1}{x}} f(x,y)dxdy$)
8. Evaluate

   I) $\int_0^{\frac{a}{\sqrt{2}}} \int_0^{\sqrt{a^2-y^2}} \ln(\,x^2 + y^2)\; dxdy$ (Ans: $\frac{\pi}{2}\left[\frac{a^2}{2}loga - \frac{a^2}{4}\right]$)

   II) $\int_0^{\frac{a}{\sqrt{2}}} \int_x^{\sqrt{a^2-x^2}} \frac{x}{\sqrt{x^2+y^2}} dxdy$ (Ans: $\frac{a^2}{2}\left(1 - \frac{1}{\sqrt{2}}\right)$)
9. Evaluate over one loop of $r^2 =\; a^2cos2\theta \quad \iint_R \frac{rdrd\theta}{\sqrt{a^2+r^2}}$ (Ans : 2a$(1 - \frac{\pi}{4}$))
10. Find area bounded by curve $y^2\;(2a - x) = x^3$ & its Asymptote (Ans :$3\pi a^2$)
11. Find area of cardioid $r = a(1 + cos\theta)$ (Ans: $\frac{3\pi a^2}{2}$)
12. Find area bounded by curve $y^2x = 16(4 - x)$ & its Asymptote. (Ans:16π)
13. Find area bounded by curves $y^2 = 4x\;\;\&\;\; 2x - y - 4 = 0$ (Ans: 9)
14. Find area bounded by curves $y^2 = x\;\;\&\;\;\; x^2 = -8y$ (Ans: $\frac{8}{3}$)
15. Evaluate $\int_0^{\frac{\pi}{2}} \int_0^y cos2y\,\sqrt{1 - a^2sin^2x}\;\, dxdy$ (Ans: $\frac{1}{3a^2}[(1 - a^2)^{\frac{3}{2}} - 1$)

Triple Integral and Applications

1. Evaluate $\iiint xyz\ dx\,dy\,dz$ Over positive octant of a sphere $x^2 + y^2 + z^2 = a^2$.

   Ans: $\frac{a^6}{48}$
2. Evaluate $\int_0^2 \int_0^y \int_{x-y}^{x+y} (x + y + z)dzdx\,dy$ Ans: 16
3. Evaluate $\iiint x^2yz\ dxdydz$ throughout the volume bounded by planes $x = 0, y = 0, z = 0$ and $\frac{x}{2} - y + z = 1$. Ans: $\frac{8}{2520}$
4. Evaluate $\iiint \frac{z^2dxdydz}{x^2+y^2+z^2}$ over the volume of sphere $x^2 + y^2 + z^2 = 2$ Ans: $\frac{8\pi\sqrt{2}}{9}$
5. Evaluate $\iiint z^2dxdydz$ over the volume bounded by the cylinder $x^2 + y^2 = a^2$ and the paraboloid $x^2 + y^2 = z$ and the plane $z = 0$. Ans: $\frac{\pi a^8}{12}$
6. Evaluate $\int_0^{\pi/2} \int_0^{a\sin\theta} \int_0^{(a^2-r^2)/a} rdzdrd\theta$ Ans: $\frac{5a^3}{64}$
7. Evaluate $\iiint \sqrt{1 - \frac{x^2}{4} - \frac{y^2}{9} - \frac{z^2}{64}}\ dxdydz$ throughout the volume of Ellipsoid $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{64} = 1$. Ans: $12\pi^2$
8. Evaluate $\iiint (x^2 + y^2 + z^2)^{3/2}\ dx\,dy\,dz$ Over the hemisphere defined by $x^2 + y^2 + z^2 = 9 \quad z \geq 0$. Ans: 243π
9. Calculate the volume of the solid bounded by the following surfaces $z = 0,\ x^2 + y^2 = 1,\ x + y + z = 3$. Ans: 3π
10. Find by triple integration the volume of region bounded by paraboloid $az = x^2 + y^2$ and the cylinder $x^2 + y^2 = r^2$. Ans: $\frac{\pi r^4}{2a}$
11. A cylindrical hole of radius 4 is bored through a sphere of radius 6. Find the volume of the remaining solid. Ans: $\frac{4\pi}{3}(20)^{3/2}$
12. Find the volume bounded by coordinate planes and the plane $lx + my + nz = 1$. Ans: $\frac{1}{6mln}$
13. Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$. Ans: 16π
14. Find the volume bounded by $y^2 = x, x^2 = y$ and the planes $z = 0, x + y + z = 1$. Ans: $\frac{1}{30}$
15. Find the volume of the solid bounded by the cylinder $x^2 + y^2 = 4y$, the paraboloid $x^2 + y^2 = 2z$ and the plane $z = 0$ Ans: 12π

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Practice Problems

Fourier series

Q.1) Find the Fourier series expansion for $f(x) = a(2 - x)$ in the interval $0 \leq x \leq 2$

Q.2) Find Fourier series for $f(x) = x + x^2$ in the interval $-\pi < x < \pi$

and $f(x) = f(x + 2\pi)$ hence show that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} \ldots$

Q.3) Find Fourier series for $f(x) = \frac{\pi^2}{12} - \frac{x^2}{4} in\ (-\pi, \pi)$.

Q.4) Obtain Fourier series expansion for $f(x) = 2 - \frac{x^2}{2}, 0 \leq x \leq 2$.

Q.5) Find the Fourier series expansion for $f(x) = \frac{1}{2}(\pi - x)$ in the interval $0 \leq x \leq 2\pi$.

Q.8) Find the Fourier series of the function $f(x) = \begin{cases} -1; & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases}$

where $(x) = f(x + 2\pi)$.

Q.9) Determine the Fourier series for the following function

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ \sin x & 0 < x < \pi \end{cases}$$

Q.10) Expand the function $f(x) = x \sin x$, as a Fourier series

in the interval $-\pi < x < \pi$. Hence deduce that $\frac{\pi - 2}{4} = \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \cdots$

Q.11) Expand the function $f(x) = x \cos x$, as a Fourier series

in the interval $-\pi < x < \pi$.

Harmonic Analysis

Q.12) Determine the first two harmonics of the Fourier series for the following values:

x^0	0	30	60	90	120	150	180	210	240	270	300	330
y	2.34	3.01	3.68	4.15	3.69	2.20	0.83	0.51	0.88	1.09	1.19	1.64

Express T as a Fourier series up to first two harmonics.

Q.15) Obtain the constant term and the coefficient of the first harmonic in the

Fourier series of $f(x)$ as given in the following table

x	0	1	2	3	4	5
f(x)	9	18	24	28	26	20

<table>
<tr><th>Q.N.</th><th>Question</th><th>ANS</th></tr>
<tr><td>1</td><td>Fourier coefficient 'a_0' in the Fourier series expansion of $f(x) = e^{-x}$; $0 \leq x \leq 2\pi$ and $f(x + 2\pi) = f(x)$ is
a) $\frac{1}{\pi}(1 - e^{-2\pi})$ b) $\frac{1}{2\pi}(1 - e^{2\pi})$ c) $\frac{2}{\pi}(e^{-2\pi} - 1)$ d) $\frac{1}{\pi}(1 + e^{2\pi})$</td><td>A</td></tr>
<tr><td>2</td><td>Fourier coefficient a_0 in the Fourier series expansion of $f(x) = \left(\frac{\pi - x}{2}\right)^2$; $0 \leq x \leq 2\pi$ and $f(x + 2\pi) = f(x)$ is
a) $\frac{\pi^2}{3}$ b) $\frac{\pi^2}{6}$ c) 0 d) $\frac{\pi}{6}$</td><td>B</td></tr>
<tr><td>3</td><td>$f(x) = x$, $-\pi \leq x \leq \pi$ and period is 2π the fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$, then fourier coefficient b_1 is
a) 2 b) -1 c) 0 d) $\frac{2}{\pi}$</td><td>A</td></tr>
<tr><td>4</td><td>$f(x) = \begin{cases} 1, & -1 < x < 0 \\ \cos \pi x, & 0 < x < 1 \end{cases}$ and period is 2 the fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\pi x + b_n \sin n\pi x)$, then fourier coefficient a_0 is
a) 2 b) -1 c) 1 d) $\frac{2}{\pi}$</td><td>C</td></tr>
<tr><td>5</td><td>$f(x) = x - x^3$, $-2 < x < 2$ and period is 4 the fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\pi x + b_n \sin n\pi x)$, then fourier coefficient a_0 is
a)1 b)0 c)-2 d)-1</td><td>B</td></tr>
<tr><td>6</td><td>For the half range cosine series of $f(x) = \sin x$, $0 \leq x < \pi$ and period is 2π the fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx)$ fourier coefficient a_0 is
a) 4 b) 2 c) $\frac{2}{\pi}$ d) $\frac{4}{\pi}$</td><td>D</td></tr>
<tr><td>7</td><td>The value of b_1 in Harmonic analysis of y for the following tabulated data is:
<table>
<tr><td>x</td><td>0</td><td>60</td><td>120</td><td>180</td><td>240</td><td>300</td><td>360</td></tr>
<tr><td>y</td><td>1.0</td><td>1.4</td><td>1.9</td><td>1.7</td><td>1.5</td><td>1.2</td><td>1.0</td></tr>
<tr><td>Sin x</td><td>0</td><td>0.866</td><td>0.866</td><td>0</td><td>-0.866</td><td>-0.866</td><td>0</td></tr>
</table>
a) 0.0989 b) 0.3464 c) 0.1732 d) 0.6932</td><td>C</td></tr>
<tr><td>8</td><td>The value of the constant term in the fourier series of $f(x) = e^{-x}$ in $0 \leq x \leq 2\pi$, $f(x + 2\pi) = f(x)$ is
a) $\frac{1}{\pi}(1 - e^{-2\pi})$ b) $\frac{1}{2\pi}(1 - e^{-2\pi})$ c) $1 - e^{-2\pi}$ d) $2(1 - e^{-2\pi})$</td><td>B</td></tr>
<tr><td>9</td><td>The value of the constant term in the fourier series of $f(x) = x \sin x$ in $0 \leq x \leq 2\pi$, is
a) -2 b) 2 c) $-\frac{1}{2}$ d) -1</td><td>D</td></tr>
</table>

10	If $a_n = \frac{2}{n^2-1}$ for $n > 1$, in the fourier series of f(x) = x sin x in $0 \le x \le 2\pi$, then the value of a_1 is a) -2 b) $\frac{2}{n^2-1}$ c) $-\frac{1}{2}$ d) $\frac{1}{2}$	C
11	The value of the constant term in the fourier series of $f(x)=\begin{cases} -\pi & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$, is a) $-\frac{\pi}{4}$ b) $-\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{2}$	A
12	The value of a_n in the fourier series of $f(x)=\begin{cases} \cos x; & -\pi < x < 0 \\ -cosx; & 0 < x < \pi \end{cases}$ a)$\frac{(-1)^n}{n}$ b)$\frac{1}{n}$ c)$\frac{(-1)^n}{n^2-1}$ d)0	D
13	The value of b_n in the fourier series of f(x) = x in $-\pi < x < \pi$, is a) 0 b) $\frac{cosn\pi}{n}$ c) $-\frac{2cosn\pi}{n}$ d) $-\frac{cos\, n\pi}{n}$	C
14	The Fourier constant 'a_n' for f (x) = $4 - x^2$ in the interval $0 < x < 2$ is_______ a) $-\frac{4}{\pi^2 n^2}$ b)$\frac{4}{\pi n^2}$ c)$\frac{4}{\pi^2 n^2}$ d)$\frac{2}{\pi^2 n^2}$	A
15	If f(x)=sin ax defined in the interval (-l, l) then value of 'a_n'is_________ a)$\frac{2}{\pi n^2}$ b)$\frac{1}{n^2}$ c)0 d) $-\frac{1}{n^2}$	C
16	The function $f(x) = \begin{cases} 0 & -2<x<-1 \\ 1+x & -1<x<0 \\ 1-x & 0<x<1 \\ 0 & 1<x<2 \end{cases}$ Is a)an odd function b) an even function c) neither even nor odd function d)cannot be decided	B
17	The Fourier constant 'a_n'for f(x)=x^2 in the interval$-1 \le x \le 1$ is a) $\frac{4(-1)^n}{\pi^2 n^2}$ b)$\frac{4(-1)^{n+1}}{\pi^2 n^2}$ c) $\frac{4}{\pi^2 n^2}$ d)$-\frac{4}{\pi^2 n^2}$	A
18	If $f(x) = \begin{cases} -x, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$, then f(x) is a) Even function b) odd function c)Neither even nor odd d) none of these	A
19	In fourier series for f(x) = x in the interval $-\pi \le x \le \pi$ which of the following is correct a) $a_0 = \pi, a_n = \frac{1+(-1)^n}{n}, b_n = 0$ b) $a_0 = 0, a_n = 0, b_n = \frac{-2(-1)^n}{n}$ c) $a_0 = \frac{\pi}{2}, a_n = \frac{1+(-1)^n}{n}, b_n = \frac{-2(-1)^n}{n}$ d) $a_0 = 0, a_n = 0, b_n = 0$	B

20	The Fourier constant $'b_n'$ for f(x)=$2-\frac{x^2}{2}$ in the interval $0 \le x \le 2$ is a) $\frac{-2}{n\pi}$ b) $\frac{2}{n\pi}$ c) $\frac{2}{\pi^2 n^2}$ d) none of these.	B
21	If $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ is discontinuous at x = 0.Then value of f(0) is a) $\pi/2$ b) 0 c) $-\frac{\pi}{2}$ d) π	C
22	The function $f(x) = \begin{cases} 0 & -2<x<-1 \\ 1+x & -1<x<0 \\ 1-x & 0<x<1 \\ 0 & 1<x<2 \end{cases}$ Find a_0 a)2 b) 1/4 c) 1/2 d)0	D
23	Which of the following is the half range sine series of f(x)in the interval, $0 \le x \le \pi$? a)f(x)=$\sum_{n=1}^{\infty} b_n sin\frac{n\pi x}{L}$ b) f(x)=$\frac{a_0}{2}\sum_{n=1}^{\infty} b_n sinnx$ c) f(x)=$\sum_{n=1}^{\infty} b_n sinnx$ d) none of these.	C
24	For the function $f(x) = \begin{cases} -\pi, & 0 < x < \pi \\ x-\pi, & \pi < x < 2\pi \end{cases}$ what is the value of a_0 a) $\frac{1}{2}$ b) $-\frac{\pi}{2}$ c) $\frac{\pi}{2}$ d) π	B
25	If f(x) = $e^x, -1 \le x \le 1$ then constant term of f(x) in fourier expansion is a) $\frac{e}{2}$ b) $\frac{e-e^{-1}}{2}$ c) $\frac{e+e^{-1}}{2}$ d) $\frac{1+e}{2}$	B
26	If $f(x) = \begin{cases} -\pi, & 0 < x < \pi \\ x, & \pi < x < 2\pi \end{cases}$ is discontinuous at x= 0 then value of f(0) is a) $-\pi$ b) 0 c) $-\frac{\pi}{2}$ d) π	C
27	If $\sum y = 42$,n=6.$\sum ycos\theta = -8.5, \sum ycos2\theta$= -1.5,what arethe values of a_0, a_1, a_2 a)7,-2.8,-2.8 b)14,-2.8,1.5 c) 7,-1.5,-2.8 d)none of these	D
28	If f(x) = x^4 in (-1,1) then the fourier coefficient b_n is a) $\frac{24(-1)^n}{n^3\pi^3}$ b) $6\left[\frac{(-1)^n+1}{n^4\pi^4}\right]$ c)0 d)None of these.	C
29	For the function $f(x) = 2x - x^2, 0 \le x \le 3$ the value of b_n is, a) 0 b) $\frac{-9}{n^2\pi^2}$ c) $\frac{3}{n\pi}$ d) none of these	C
30	In the Fourier expansion of the function $f(x) = \frac{1}{2}(\pi - x)$ in the interval $0 \le x \le 2\pi$ the value of a_n is, a) $\frac{1}{n^2\pi}$ b) $\frac{\pi}{n^2}$ c) 0 d) $\frac{(-1)^n}{n^2\pi}$	C

<table>
<tr><td>31</td><td>If $f(x) = \frac{\pi^2}{2} - \frac{x^2}{4}, -\pi \le x \le \pi$ then values of a_n and b_n are
a) $0, \frac{3}{n\pi}$ b) $0, \frac{(-1)^{n+1}}{n^2}$ c) $\frac{(-1)^n+1}{n^2-1}, 0$ d) $\frac{-(-1)^n}{n^2}, 0$</td><td>D</td></tr>
<tr><td>32</td><td>The following values of y give the displacement in inches of a certain machine part for the rotation x of the flywheel
<table><tr><td>x</td><td>0</td><td>$\pi/6$</td><td>$2\pi/6$</td><td>$3\pi/6$</td><td>$4\pi/6$</td><td>$5\pi/6$</td></tr><tr><td>Y</td><td>0</td><td>9.2</td><td>14.4</td><td>17.8</td><td>17.3</td><td>11.7</td></tr></table>What is the value of a_0
a)11.733 b)14.4 c)23.466 d) none of these</td><td>C</td></tr>
<tr><td>33</td><td>If $\sin x = \frac{2}{\pi} - \frac{2}{\pi}\sum_{n=2}^{\infty}\frac{1+(-1)^n}{(n-1)(n+1)}\cos nx$ for $0 \le x \le \pi$ then which of the following correct
a) $\frac{1}{2} = \frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \cdots$ b) $\frac{\pi^2}{2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$
c) $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$ d) none of these</td><td>A</td></tr>
<tr><td>34</td><td>If $a_0 = \frac{4\pi^2}{3}, a_n = \frac{4(-1)^{n+1}}{n^2}$ are the fourier coefficient of f(x) in $-\pi \le x \le \pi$ then which of the following correct
a) f(x)= $\frac{2\pi^2}{3} + \sum_{n=1}^{\infty}\frac{4(-1)^{n+1}}{n^2}\cos\frac{n\pi x}{2}$ b) f(x)= $\frac{4\pi^2}{3} + \sum_{n=1}^{\infty}\frac{4(-1)^{n+1}}{n^2}\sin nx$
c) f(x)= $\frac{2\pi^2}{3} - \sum_{n=1}^{\infty}\frac{4(-1)^n}{n^2}\cos nx$ d) none of these</td><td>C</td></tr>
<tr><td>35</td><td>If f(x) = x^2, $0 < x < 2$ then in half range cosine series $\frac{a_0}{2}$ is
a) 4 b) 12 c) $\frac{8}{3}$ d) 8</td><td>C</td></tr>
<tr><td>36</td><td>For the half range cosine series f(x) = $\left(\frac{\pi-x}{2}\right)^2, 0 \le x \le \pi$, if $a_0 = \frac{\pi^2}{6}, a_n = \frac{1}{n^2}, b_n = 0$, then which of the following statement is correct
a) $\frac{\pi^2}{6} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2}$...... b) $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2}$ − − −
c) $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}$...... d) $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2}$ − − −</td><td>C</td></tr>
</table>

Q.N.	Question	Ans
1	If $\emptyset(a) = \int_0^{\infty}\frac{e^{-x}}{x}(1 - e^{-ax})dx, a > -1$ then by DUIS rule, $\frac{d\emptyset}{da}$ is a) $\frac{e^{-x}}{x}(1 - e^{-ax})$ b) $\int_0^{\infty}\frac{a}{x}\left(e^{-(a+1)x}\right)dx$ c) $\int_0^{\infty}(e^{-ax})dx$ d) $\int_0^{\infty}\left(e^{-(a+1)x}\right)dx$	D
2	If $\emptyset(a) = \int_0^1\frac{x^a-1}{logx}dx, a \ge 0$ then by DUIS rule, $\frac{d\emptyset}{da}$ is a) $\int_0^1\frac{x^a loga}{logx}dx$ b) $\int_0^1\frac{ax^{a-1}}{logx}dx$ c) $\int_0^1 x^a dx$ d) $\frac{x^a-1}{logx}$	C

3	If $\emptyset(a) = \int_0^{\infty} \frac{e^{-x}sinax}{x} dx$ then by DUIS rule, $\frac{d\emptyset}{da}$ is a) $\int_0^{\infty} e^{-x}sinax\, dx$ b) $\int_0^{\infty} e^{-x}cosax\, dx$ c) $\int_0^{\infty} \frac{ae^{-x}sinax}{x} dx$ d) $\frac{e^{-x}sinax}{x}$	B
4	If $\emptyset(a) = \int_0^{\frac{\pi}{2}} \frac{\log(1+asin^2x)}{sin^2x} dx, a > 0$, then by DUIS rule, $\frac{d\emptyset}{da}$ is a) $\int_0^{\frac{\pi}{2}} \frac{2sinxcosx}{(1+asin^2x)} dx$ b) $\int_0^{\frac{\pi}{2}} \frac{1}{(1+asin^2x)sin^2x} dx$ c) $\int_0^{\frac{\pi}{2}} \frac{1}{1+asin^2x} dx$ d) $\int_0^{\frac{\pi}{2}} \frac{sin^2x}{(1+asin^2x)} dx$	C
5	If $\emptyset(a) = \int_a^{a^2} \log(ax)dx$, then by DUIS rule II, $\frac{d\emptyset}{da}$ is a) $\int_a^{a^2} \frac{\partial}{\partial a}\log(ax)\, dx + 2aloga^3$ b) $\int_a^{a^2} \frac{\partial}{\partial a}\log(ax)\, dx$ c) $\int_a^{a^2} \frac{\partial}{\partial a}\log(ax)\, dx + 2aloga^3 - 2loga$ d) $\int_a^{a^2} \frac{\partial}{\partial a}\log(ax)\, dx - 2aloga^3 + 2loga$	A
6	If $\emptyset(a) = \int_0^{a^2} tan^{-1}\left(\frac{x}{a}\right) dx$, then by DUIS rule II, $\frac{d\emptyset}{da}$ is a) $\int_0^{a^2} \frac{\partial}{\partial a} tan^{-1}\left(\frac{x}{a}\right) dx + 2atan^{-1}a$ b) $\int_0^{a^2} \frac{\partial}{\partial a} tan^{-1}\left(\frac{x}{a}\right) dx$ c) $\int_0^{a^2} \frac{\partial}{\partial a} tan^{-1}\left(\frac{x}{a}\right) dx + a^2tan^{-1}a$ d) $\int_0^{a^2} \frac{\partial}{\partial a} tan^{-1}\left(\frac{x}{a}\right) dx + a^2tan^{-1}a - tan^{-1}\left(\frac{x}{a}\right)$	A
7	IF $I(a)=\int_a^{a^2} \frac{1}{x+a} dx$, then by DUIS rule II, $\frac{dI}{da}$ is a) $\int_a^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx - \frac{1}{a^2+a}(2a) + \frac{1}{2a}$ b) $\int_a^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx + \frac{1}{a^2+a}(2a) - \frac{1}{2a}$ c) $\int_a^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx - \frac{1}{a^2+a}$ d) $\int_a^{a^2} \frac{\partial}{\partial a}\left(\frac{1}{x+a}\right) dx$	B
8	Using DUIS Rule the value of the integral $\emptyset(a) = \int_0^{\infty} \frac{e^{-x}}{x}(1 - e^{-ax})dx, a > -1$, given $\frac{d\emptyset}{da} = \frac{1}{a+1}$ is a) log(a+1) b) $-\frac{1}{(a+1)^2}$ c) log(a+1) + π d) $-\frac{1}{(a+1)^2} + 1$	A
9	Using DUIS Rule the value of the integral $\emptyset(a) = \int_0^{\frac{\pi}{2}} \frac{\log(1+asin^2x)}{sin^2x} dx$ with $\frac{d\emptyset}{da} = \frac{\pi}{2}\frac{1}{\sqrt{a+1}}$ is a) $\pi\sqrt{a+1}$ b) $\pi\sqrt{a+1} + \pi$ c) $\pi\sqrt{a+1} - \pi$ d) $3\pi(a+1)^{\frac{3}{2}} - \pi$	C
10	Using DUIS Rule the value of the integral $\emptyset(a) = \int_0^{\infty} \frac{1-cosax}{x^2} dx$, with $\frac{d\emptyset}{da} = \frac{\pi}{2}$ is a) $\frac{\pi}{2}$ b) $\frac{\pi a}{2}$ c) πa d) $\frac{\pi a}{2} + \frac{\pi}{2}$	B
11	If I(a)=$\int_0^1 \frac{x^a - x^b}{logx}$dx then the value of I(a) is a) $\log\left(\frac{a}{b}\right)$ b) $\log\left(\frac{a+1}{b+1}\right)$ c) $\log\left(\frac{b+1}{a+1}\right)$ d) $\log\left(\frac{b}{a}\right)$	B

12	If erf($\sqrt{t}$) =$\frac{2}{\sqrt{\pi}}\int_0^{\sqrt{t}} e^{-u^2}\,du$ then $\frac{d}{dt}$ erf($\sqrt{t}$) is a) $\frac{e^{-t}}{2\sqrt{t}}$ b) $\frac{e^{-t^2}}{\sqrt{\pi t}}$ c) $\frac{e^{-t}}{\sqrt{\pi}}$ d) $\frac{e^{-t}}{\sqrt{\pi t}}$	D
13	$\int_0^t \mathrm{erf}(ax)\,dx + \int_0^t \mathrm{erfc}(ax)\,dx =?$ a) t b) x c) 0 d) $\frac{t^2}{2}$	A
14	If $\frac{d}{dx}$ erf(ax)=$\frac{2ae^{-a^2x^2}}{\sqrt{\pi}}$, the value of $\int_0^t \mathrm{erf}(ax)\,dx$ is a) t erf(at)+$\frac{1}{a\sqrt{\pi}}e^{-a^2t^2}-\frac{1}{a\sqrt{\pi}}$ b) t erf(at)$-\frac{1}{a\sqrt{\pi}}e^{-a^2t^2}-\frac{1}{a\sqrt{\pi}}$ c) erf(at)+$\frac{1}{a\sqrt{\pi}}e^{-a^2t^2}+\frac{1}{a\sqrt{\pi}}$ d) t erf(at)$-\frac{1}{a\sqrt{\pi}}e^{-a^2t^2}+\frac{1}{a\sqrt{\pi}}$	A
15	The integral for "erf(b)-erf(a)" is, a) $\frac{2}{\sqrt{\pi}}\int_a^b e^{-t^2}\,dt$ b) $\sqrt{\frac{2}{\pi}}\int_a^b e^{-t^2}\,dt$ c) $\int_a^b e^{-t^2}\,dt$ d) none of these	A

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(1) The differential equation of all circles touching Y-axis at the origin & centres on X-axis, is

(A) $x^2 - y^2 = 2x\left[x + y\frac{dy}{dx}\right]$

(B) $x^2 + y^2 = 2x\left[x + y\frac{dy}{dx}\right]$

(C) $x^2 + y^2 = 2x\left[x - y\frac{dy}{dx}\right]$

(D) None of these

2] Integrating factor of $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$

(A) $\frac{1}{x^2y^2}$

(B) $\frac{1}{xy}$

(C) $\frac{1}{x^2y}$

(D) $\frac{1}{xy^2}$

3] If $I = \frac{E}{R}\left[1 - e^{-Rt/L}\right]$ then the value of q is

A) $\frac{E}{R}\left[t + \frac{L}{R}e^{-Rt/L}\right]$

B) $\frac{E}{R}\left[t + \frac{R}{L}e^{-Rt/L}\right]$

C) $\frac{E}{R}\left[t - \frac{R}{L}e^{-Rt/L}\right]$

(D) None of these

(4) The curve $r = a e^{m\theta}$

(A) Not passes through the pole

(B) passes through the pole

(C) symmetry about y-axis

(D) None of these

(5) The curve $a^2 y^2 = x^2 (2a - x)(x - a)$ is

(A) symmetry about y-axis

(B) symmetry about $y = x$

(C) symmetry about x-axis

(D) symmetry about $y = -x$

(6) Tangent at origin to the curve $x^3 + y^3 = 3ax$ is

(A) $x = 0$

(B) $y = 0$

(C) $x = 0, y = 0$

(D) None of these

Q.1) $\int_0^1\int_0^1 \frac{dx\,dy}{\sqrt{1-x^2}\sqrt{1-y^2}} = \cdots$

a) $\frac{\pi^2}{3}$ b) ✓ $\frac{\pi^2}{4}$ c) π d) 1

Q.2) $\int_0^{2\pi}\int_{a\sin\theta}^{a} r\,d\theta\,dr = \cdots$

✓ a) $\frac{\pi a^2}{2}$ b) $\frac{\pi}{2}$ c) $\frac{\pi^2 a}{2}$ d) 5

Q.3) $\int_0^1\int_0^x e^{x+y}\,dx\,dy = \cdots$

✓ a) $\frac{(e-1)^2}{2}$ b) $\frac{e-1}{2}$ c) $\frac{e}{2}$ d) e

Q.4) After changing the order of integration $I = \int_0^1\int_{4y}^4 e^{x^2}\,dx\,dy$, the new limits of x & y are

✓ a) $0\le x\le 4,\ 0\le y\le \frac{x}{4}$ b) $4\le x\le 0,\ \frac{x}{4}\le y\le 0$

c) $0\le x\le \frac{y}{4},\ 0\le y\le 4$ d) $0\le x\le 1,\ 0\le y\le 4$

Q.5) After changing the order of integration of $I = \int_0^\infty\int_x^\infty \frac{e^{-y}}{y}\,dx\,dy$ the new limits of x & y are

✓ a) $0\le x\le y \quad 0\le y<\infty$ b) $0\le y\le x,\ 0\le x<\infty$

c) $0\le x\le 1 \quad x\le y\le 1$ d) $0\le x\le 1,\ 0\le y\le 2$

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Q.1) $\int_0^1 \int_0^1 \frac{dx\,dy}{\sqrt{1-x^2}\sqrt{1-y^2}} = \ldots$

a) $\frac{\pi^2}{3}$ b) $\frac{\pi^2}{4}$ ✓ c) π d) 1

Q.2) $\int_0^{2\pi} \int_{a\sin\theta}^{a} r\,d\theta\,dr = \ldots$

a) $\frac{\pi a^2}{2}$ ✓ b) $\frac{\pi}{2}$ c) $\frac{\pi^2 a}{2}$ d) 5

Q.3) $\int_0^1 \int_0^x e^{x+y}\,dx\,dy = \ldots$

a) $\frac{(e-1)^2}{2}$ ✓ b) $\frac{e-1}{2}$ c) $\frac{e}{2}$ d) e

Q.4) After changing the order of integration $I = \int_0^1 \int_{4y}^4 e^{x^2}\,dx\,dy$, the new limits of x & y are

✓ a) $0 \le x \le 4,\ 0 \le y \le \frac{x}{4}$ b) $4 \le x \le 0,\ \frac{x}{4} \le y \le 0$

c) $0 \le x \le \frac{y}{4},\ 0 \le y \le 4$ d) $0 \le x \le 1,\ 0 \le y \le 4$

Q.5) After changing the order of integration of $I = \int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dx\,dy$ the new limits of x & y are

✓ a) $0 \le x \le y\ \ 0 \le y < \infty$ b) $0 \le y \le x,\ 0 \le x < \infty$

c) $0 \le x \le 1\ \ x \le y \le 1$ d) $0 \le x \le 1,\ 0 \le y \le 2$

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1. For non-singular matrix A of order $n \times n$, rank r of A is
   (A) $r > n$ (B) $r = n$
   (C) $r < n$ (D) None of these.

2. Normal form of matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ is
   (A) $[I_2\ 0]$ (B) $[I_2]$
   (C) $[I_3]$ (D) $\begin{bmatrix} I_1 \\ 0 \end{bmatrix}$

3. Nonhomogeneous system of linear equations $AX = B$ is inconsistent if
   (A) rank of A = rank of (A|B)
   (B) rank of A ≠ rank of (A|B)
   (C) rank of A > number of unknowns
   (D) None of these

4. Given system of linear equations $x - 4y + 5z = 0$, $2x - y + 3z = 0$, $3x + 2y + z = 0$ has
   (A) No solution (B) only trivial solution
   (C) Infinite solutions (D) None of these

5. If $Y = AX$ is orthogonal transformation then its inverse transformation is
   (A) $X = A^{-1}Y$ (B) $Y = A^{-1}X$
   (C) $X = YA^{-1}$ (D) Does not exist.

6. The matrix of linear transformation,
   $y_1 = 2x_1 + x_2 + x_3$ $y_2 = x_1 + x_2 + 2x_3$,
   $y_3 = x_1 - 2x_3$

(A) $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 0 & -2 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & -2 \end{bmatrix}$

(D) $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 0 & 0 \end{bmatrix}$

7) For what value of b, the matrix $A = \frac{1}{3}\begin{bmatrix} b & -5 \\ 5 & b \end{bmatrix}$ is an orthogonal ?

(A) ± 5

(B) ± 13

(C) ± 12

(D) ± 16

8) If $\lambda_1, \lambda_2, \lambda_3$ are non-zero eigen value of A then trace of A^{-1} is

(A) $\frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3}$

(B) $\lambda_1 + \lambda_2 + \lambda_3$

(C) $\lambda_1^2 + \lambda_2^2 + \lambda_3^2$

(D) $\lambda_1 \times \lambda_2 \times \lambda_3$

9) For the matrix $A = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 3 & 2 \\ 0 & 0 & 5 \end{bmatrix}$ the eigen value of A are

(A) $1, -2, -1$

(B) $1, -3, -5$

(C) $-1, -3, -5$

(D) $1, 3, 5$.

10) The sum of the eigen value of $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ is

(A) 7

(B) 5

(C) 6

(D) 8

(11) Cayley Hamilton theorem states that

(A) sum of eigen values is equal to trace of matrix

(B) the product of the eigen values of a matrix A is equal to determinant of the matrix

(C) every square matrix satisfies its own characteristic equation

(D) eigen values of a matrix & its transpose is same

(13) The linear transformation $Y = \begin{bmatrix} 4 & -5 & 1 \\ 3 & 1 & -2 \\ 1 & 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ is

(A) non-singular (B) composite
(C) singular (D) None of these

(14) For the matrix $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 4 & 2 \end{bmatrix}$ then eigen values of A^2 are

(A) 1, 4 (B) -1, -1, 4
(C) -1, -1, 2 (D) None of these

(15) If the characteristic equation of matrix A of order 2×2 is $\lambda^2 - 9\lambda - 1 = 0$ then A^{-1} is

(A) $A - 9I$ (B) $A + 9I$
(C) $-A - 9I$ (D) $A^2 - 9A - I$

Date

1) If $y = \frac{1}{(x+1)^2}$, then $y_n = ?$

a) $\frac{(-1)^n (n+1)!}{(x+1)^{n+2}}$ b) $\frac{(-1)^n (n)!}{2(x+1)^n}$

c) $\frac{(-1)^{n+1} n!}{(x+1)^{n+2}}$ d) None of the above

2) If $y = 2^x$ then $y_n = ?$

a) $2^x (\log 2)^n$ b) $x^2 (\log x)^n$

c) $x^2 (\log x)^n$ d) $2^x (\log x)^n$

3) If $y = \sin(bx+c)$ then $y_n = ?$

a) $\sin(bx+c+n)$ b) $\sin(bx+c+n\pi/2)$

c) $b^n \sin(bx+c+n\pi/2)$ d) $\cos(bx+c+n\pi/2)$.

4) If $(1+x^2) y_1^2 = 4y$, then which of the following is true ?

a) $(1-x^2) y_{n+2} + (2n+1) y_n = 0$

b) $(1+x^2) y_{n+2} + (2n+1) x y_{n+1} + n^2 y_n = 0$

c) $(2n+1) y_{n+1} = n^2 y_n$

d) None of the above

5) If $y = e^{ax}$ then $y_n = ?$

a) $a e^{ax}$ b) $a^n e^{ax+1}$

c) $a^n e^{ax}$ d) $a^x e^n$

6) If $u = y^x$ then $\frac{\partial^2 u}{\partial x \partial y} = ?$

a) $y^{x-1}[1 + x\log y]$ b) $x^{y-1}[1 + y\log x]$

c) $y^x \log y$ d) None of the above

7) If $u = \log(x^3 + y^3 - x^2y - xy^2)$ then $u_y = ?$

a) $\frac{2}{x-y}$ b) $\frac{1}{x+y}$ c) $\frac{-2}{x-y} + \frac{1}{x+y}$

8) If $x = r\cos\theta$, $y = r\sin\theta$. then $\left(\frac{\partial x}{\partial r}\right)_\theta = ?$

a) $x\sin\theta$ b) r c) 1 d) $x\cos\theta$

9) $f(x,y) = \frac{x^2 + y^2}{\sqrt{x} + \sqrt{y}}$ is

a) Non-homogenous
b) Homogenous with degree 1
c) Homogenous with dregree 2
d) Homogenous with dregree 3/2

10) If z is a homogenous function of two variables x, y of degree 'n' then,

a) $x^2 z_{xx} + 2xy z_{xy} + y^2 z_{yy} = n(n-1)z$
b) $z_{xx} + z_{yy} = n(n-1)z$
c) $x^2 z_{yy} + 2xy z_{xy} + y^2 z_{xx} = nz$
d) None of the above

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11) If $u = ax+by$, $v = bx-ay$ then $\left(\frac{\partial v}{\partial y}\right)_u = ?$

a) $-\frac{a^2+b^2}{a}$ b) $\frac{a^2+b^2}{a}$ c) $\frac{a^2+b^2}{b}$ d) 0

12) If $z^3 + xz - y = 4$ then $\frac{\partial z}{\partial x} = ?$

a) $\frac{z}{3z^2-x}$ b) $\frac{z}{3z^2+x}$ c) xy d) 0

13) If $u \to x, y, z \to t$ then total derivative is given by

a) $\frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial u}{\partial z} \cdot \frac{dz}{dt}$

b) $\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} + 0$

c) $\frac{du}{dt} = \frac{du}{dx} \frac{dx}{dt} + \frac{du}{dy} \cdot \frac{dy}{dt} + \frac{du}{dz} \frac{dz}{dt}$

14) If $f(x,y) = 0$ represents implicit relation then $\frac{dy}{dx} = ?$

a) $\frac{-\partial f/\partial x}{\partial f/\partial y}$ b) $\frac{\partial f/\partial x}{\partial f/\partial y}$ c) $\frac{-\partial f/\partial y}{\partial f/\partial x}$

d) $\frac{\partial f/\partial y}{\partial f/\partial x}$

15) If $y = (2x+3)^3$ then $y_n = ?$

a) 1 b) 0 c) $\frac{(-1)^n (n+1)!}{(2x+3)^{n+3}}$ d) None

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Practice MCQs.

① Expansion of $\frac{1}{1+x}$ in ascending powers of x is

a) $-1-x-x^2-x^3-\cdots$ b) $1-x+x^2-x^3+\cdots$

c) $1+\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots$ d) $1+x+x^2+x^3+\cdots$

② First two terms in expansion of $e^x \sec x$ by Maclaurin's theorem is

a) $x+x^2+\cdots$ b) $x-x^2+\cdots$

c) $1+x+\cdots$ d) $1-x+\cdots$

③ First two terms in expansion of $(x+2)^5+3\cdot(x+2)^4$ by Taylor's theorem in ascending powers of x is

a) $48+98x$ b) $80+176x+\cdots$

c) $80+98x$ d) $48+176x+\cdots$

④ First two terms in expansion of $\tan^{-1}x$ by Taylor's theorem in ascending powers of $(x-1)$ is

a) $\frac{\pi}{4}-\frac{1}{2}(x-1)+\cdots$ b) $\frac{\pi}{4}+\frac{1}{2}(x-1)+\cdots$

c) $1+\frac{1}{2}(x-1)+\cdots$ d) $1-\frac{1}{2}(x-1)$

5) If $u=x^2-y$. $v=xy$ then $\frac{\partial(u,v)}{\partial(x,y)}$ is

a) $2x^2+y$ b) $2x^2-y$ c) x^2+y d) $x+y$

6) If $x=1-v$, $y=uv$ then $\frac{\partial(x,y)}{\partial(u,v)}$ is

a) u b) $-u$ c) $-v$ d) v

7) If $u=\sin^{-1}x+\sin^{-1}y$, $v=x\sqrt{1-y^2}+y\sqrt{1-x^2}$ are functionally dependent then relation betⁿ u & v is

a) $v=\sin u$ b) $u=\sin v$. c) $\sqrt{v}=\sin u$ d) $u+v=\sin u$

8) If $u = \frac{x-y}{x+y}$, $v = \frac{x+y}{x}$ are functionally dependent, then relation betn u & v is

a) $uv = 2+v$ b) $u = 2-v$ c) $uv = 2-v^2$ d) $uv = 2-v$

9) Find the percentage error in the area of an ellipse when an error of 4% is made in measuring its major & minor axes. Given area of ellipse = πab

a) 4% b) 2% c) 4π% d) 8%

10) In calculating volume of right circular cylinder errors of 3% & 4% are found in measuring height & base radius respectively. Find the percentage error in calculating volume of cylinder. Given volume of right circular cylinder $V = \pi r^2 h$.

a) 7% b) 1% c) 11% d) 4%

11) critical (stationary) point & nature of the function $f(x,y) = x^2 - 2x + 2y^2 + 4y - 2$ at critical point is

a) (1,1) & maxima b) (1,-1) & maxima

c) (1,1) & minima d) (1,-1) & minima.

12) Using Lagrange's method of undetermined multiplier Find maximum value of function $f(x,y,z) = xy^2z^3$ on the plane $x+y+z=3$, given that $x = \frac{1}{\lambda}$, $y = \frac{2}{\lambda}$, $z = \frac{3}{\lambda}$

a) $\frac{27}{16}$ b) $\frac{27}{8}$ c) $\frac{1}{2}$ d) $-\frac{27}{8}$