CHAPTER 5

Analysis of Structures

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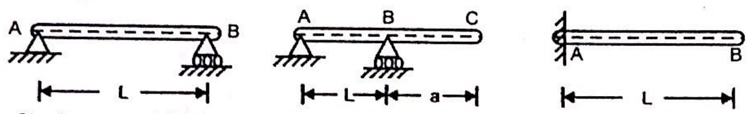
Beams

A linear structural member designed to support, loads applied at various points along the member is called as a beam. Normally, the loads are perpendicular to the axis of the beam and they cause only shearing and bending effect in the beam. When the loads are not at right angles to the beam, they will also produce axial forces in the beam. Beams are long straight prismatic bars. The distance between the supports is called as span of the beam. Beams are classified according to the way in which they are supported.

Statically determinate beams

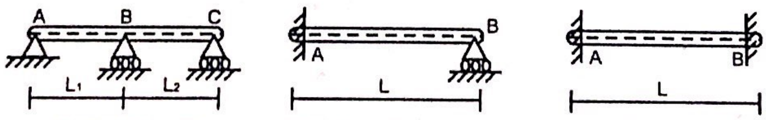
The beams whose reactions can be obtained by using only the equations of equilibrium from statics, are called as statically determinate beams. For these beams the no. of reaction unknowns are equal to three.



Simply supported Beam without overhang | Simply supported Beam with overhang | Contilever Beam

Statically indeterminate beams

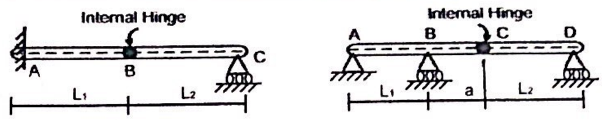
The beams whose reactions can not be obtained by using only the equations of equilibrium from statics are called as statically indeterminate beams. For these beams no. of reaction unknowns are more than three. To calculate the reactions of these beams, in addition to the load equations (equations of equilibrium) the deflection equations are also required to use. e.g.



Continuous beam | Propped cantilever beam | Fixed beam

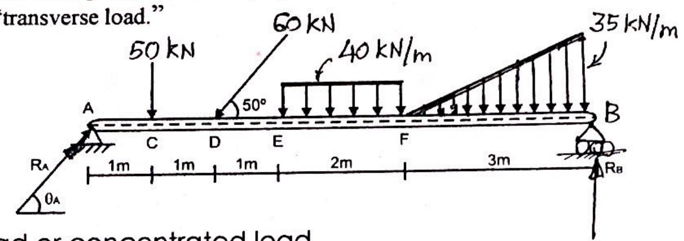
Compound beams

Sometimes two or more beams are connected to form a single continuous structure. These are called as compound beams. For finding the reactions at the supports free body diagrams of the individual beams (separated from each other) are to be considered.



Types of loads on beams

When the load acting on the beam is perpendicular to the longitudinal axis of the beam, it is called as "transverse load."



i) Point load or concentrated load

When the entire load is supposed to be acting at one point on the beam, it is called as a 'point load' or a 'concentrated load.' In the above beam, the loads 50 KN, and the reactions R_A and R_B are the point loads. Load 60 kN is also a point load.

ii) Uniformly distributed load (u.d.l.)

When the load acting on the beam is spread over some length of the beam along the span and the intensity of the load is constant then it is called as uniformly accelerated load. In this case the load diagram is rectangular and the total load is supposed to be acting at the midpoint of the span of the uniformly distributed load. For e.g. in the above beam 40 KN/m is the u.d.l. on the span EF. The total load (40x2) = 80 KN is acting at the midpoint of EF on the beam.

iii) Uniformity varying load (u.v.l.)

When the load acting on the beam is spread over some length of the beam along the span and the intensity of the load varies uniformly then it is called as uniformly varying load. In this case the load diagram is either triangular or trapezoidal. For e.g. in the above beam the load on portion FB of the beam is u.v.l.

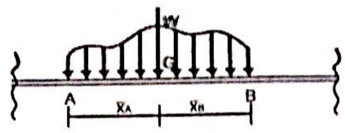
Total load on FB = Area of the load diagram

= (1/2x 3 x 35) = 52.5 KN

This load of 52.5 KN is supposed to be acting at the centroid of the triangle i.e. at a distance of 2 m from point B.

iv) Randomly distributed load

In this case,



Total load on AB = Area of the load diagram = W

And it is supposed to be acting at the centroid of the load diagram (G).

The analysis of beams consists of

Finding the support reactions and determining the shear force and bending moments produced by the given loads and to plot their variations along with the length of the beam (S.F.D. and B.M.D.)

The topic of shear force and bending moment diagrams is normally included in the subject of strength of materials and it is to be studied in the second year of Engineering. At first year of Engineering, we only study to find the reactions at the supports of determinate beams.

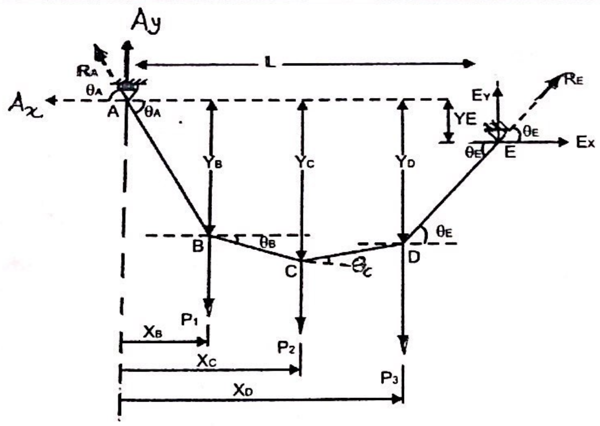
Cables

A cable is defined as a structural member by means of which a tensile force (pull) can be **resisted and can be transmitted from one end of the cable to the other end.** Cables are used **in many engineering applications such as suspension bridges, transmission** lines, aerial **tramways, guy wires for high towers etc. Cables may be divided into two types** according to their loading; i) cables supporting concentrated loads, ii) cables supporting distributed loads. Here, we are going to study only the cables supporting concentrated loads.

Assumptions

1. A cable is assumed to be perfectly flexible (very easily bent) but inextensible member. The resistance due to bending is negligible.
2. The self weight of the cable is assumed to be negligibly small (and hence ignored as compared to the loads it carries)

Due to this any portion of the cable between successive loads can therefore be considered as a "two-force member," and the internal forces at any point in the cable reduce to a tensile force directed along the cable. Thus, a cable can not resist a compressive force (the cable would crimple up if attempt to apply any compressive force is made). The ends of the cable and the loads acting on it are considered in one vertical plane unless otherwise specified. The loads acting on the cable are normally vertical. The cables are supported at their ends by pin or hinged connections and loads are applied on it which the cable resists. The supports of the cable may be at the same level or at different levels.



The analysis of cables consists of

1. Determination of reactions at the supports where the ends of the cable are unchored.
2. Determination of tensions in all the segments of the cable formed due to point loads on it.
3. Determination of missing co-ordinates at the load points. i.e. to complete the configuration of the cable.

In the analysis of cables following points are to be noted;

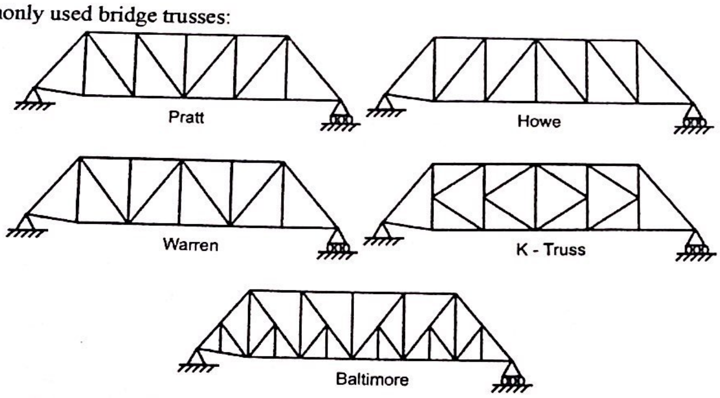
1. **The support reactions of the cable are always collinear with the segments** adjacent to **the supports. The magnitude of the reaction is equal to the magnitude of** the tension in **those segments.**
2. The segment having maximum inclination with horizontal is the segment having maximum tension.
3. If the external loads are purely vertical then the horizontal component of the tension force is the same at any point of the cable and is equal to the horizontal component of the support reactions.

Trusses

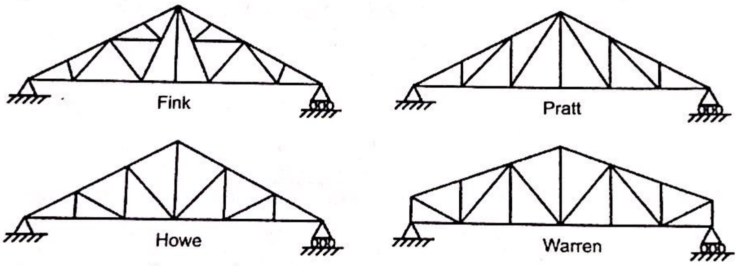
A truss is a structure or a framework made up of straight, rigid bars connected with each other at their ends by smooth frictionless pins.

Trusses are extensively used as structures for supporting loads. They are used to support the roofing loads, to support the bridge decks, for transmission line towers, for industrial assemblies, for chassis of vehicles etc. Depending upon their geometrical shapes trusses are named as fink, pratt, howe, warren, K-truss etc. When the members of the truss lie essentially in a single plane, the truss is known as a "plane truss" otherwise it is called as a "space truss."

Commonly used bridge trusses:



Commonly used roof truss:



In bridges trusses act as beams, in transmission line towers they act principally as columns and cantilevers supported at the bottom and subjected to axial and lateral forces at the top. In masts, they may act as columns or posts which are subjected to axial forces. The material used for trusses are timber or steel. Now-a-days timber trusses are obsolete. In steel trusses rolled steel sections like angles, channels, joists or compound sections fabricated out of them. The joints between members are generally made by welding, riveting or bolting them to a single steel plate called as "gusset plate", one at each joint.

The essential requirement of a truss is that no system of forces acting in the same plane as that of the truss can be capable of distorting its shape and geometry.

Fig. (a) Fig. (b) Fig. (c)

[illegible]

Now, consider a triangular frame ABC shown in figure (c). [illegible]

Thus, the simplest form of a truss that satisfies the geometric requirement is the triangular frame.

When external loads are applied on the truss, its members are subjected to axial forces (i.e. axial tension or axial compression).

Assumptions in the theory of trusses:

1. The given truss is a perfect truss.
2. All the members are lying in one plane, hence the force systems formed thereby are coplanar force systems.
3. The weights of the members are very small compared to the applied loads and hence they are neglected.
4. All the members are connected to each other only at their ends by frictionless pins. The pin at the joint can, therefore, exert only forces but no moments on the members connected by it.
5. All the external forces are applied to the truss only at the pins of the joints in the plane of the truss and never on a member at a point in between its two ends.

The effect of the above assumptions on the statics of the truss is as under:

1. Every member of a truss becomes a 2-force body in equilibrium. Hence they are called as "two force" members. They are subjected to axial tension or compression. Members of a truss are never subjected to bending.
2. Every joint becomes the center of a concurrent force system in equilibrium. Hence we can apply two equations of equilibrium i.e. $\Sigma F_x = 0$ and $\Sigma F_y = 0$ to each joint.

Perfect, redundant and deficient truss

Let, m = number of members in the truss

j = number of joints in the truss

As every joint of a truss is representing a concurrent coplanar force system in equilibrium, we can apply two equations of equilibrium to each joint.

$\therefore$ 2j = total number of equations of equilibrium.

Let, R = number of reaction components at the supports of the truss (R = 3 if one support is hinged and other roller and R = 4 if both the supports are hinged)

Then, m + R = total number of unknowns.

For determinate trusses,

(total number of unknowns) = (total number of equations of equilibrium)

$\therefore m + R = 2j$

OR $\boxed{M = 2j - R}$

For simply supported trusses, one support is a hinge and the other roller, then we get,

$\boxed{M = 2j - 3}$

For cantilever trusses both the supports are hinges, then we get,

$\boxed{M = 2j - 4}$

The trusses satisfying the above condition are called as "perfect trusses". They ~~are~~ statically determinate trusses. They are geometrically stable trusses.

If m > (2j – R), then the truss is statically indeterminate and the number of extra members are said to be "redundant", such a truss is called as "redundant truss."

If m < (2j – R), the truss does not form a perfect frame and the resulting truss is then called as "deficient truss." Deficient trusses are not geometrically stable.

The analysis of a truss consists of

1. Finding the support reactions
2. Determination of the internal force in all the members of a truss with magnitude as well as nature.

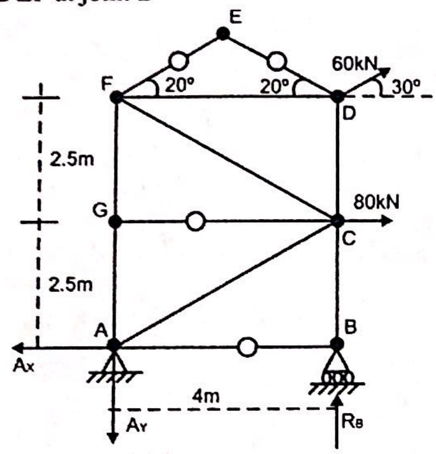
The results obtained are to be presented in a tabular format.

Zero force members of a truss

Under the action of the given set of loads, there are some members of the truss which are neither subjected to tension nor to compression. These members are called as zero force members. These members can be located by careful inspection of the given truss and given loading on it applying the following rules;

1. If at a particular joint, there are only two members connected together and there is no external load acting on that joint, then both the members are zero force members.

For eg. member ED and EF at joint E



2. If at a particular joint, there are two members connected together and the load at that joint is collinear to one of the member, then the second member is a zero force member. For e.g. at joint B, roller reaction, R_B is collinear to BC. Then AB is a zero force member.
3. If at a particular joint, there are three members connected together two out of which are collinear and there is no external load acting on that joint then the third member is a zero force member. For e.g. At joint G, FG and AG are collinear members there is no load on the joint then member CG is a zero force member.

But these members are not useless. They do not carry any load under the given loading conditions. But the same members would probably carry loads under new loading conditions. These members are required to support the weight of the truss and to maintain the truss in the desired shape.

There are three methods for the analysis of truss

1. method of joints
2. method of sections
3. graphical method

Method of joints

A truss can be analysed by method of joints by following the steps as under:

1. Check for the perfect truss using the equation,

   $\boxed{M = 2j - R}$

   Where, R = 3 for one support hinged and other roller

   And R = 4 for both the supports hinged.

2. Considering the free body diagram of the entire truss find the reactions at the supports of the truss.

   If the truss is having both the supports as hinged then R = 4 and the truss becomes "externally indeterminate." In this case we can not get all 4 unknown reaction components by using 3 equations of equilibrium. But in such type of a truss, we always get at least one joint at which there are maximum two unknowns. Hence, we can start the solution from that joint. Thus, in this case, we need not calculate the support reactions.

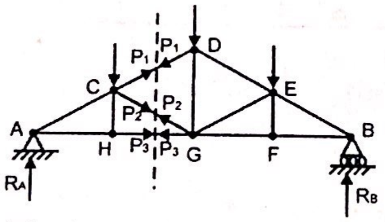
3. Locate the zero force members, if any. For this use the three rules mentioned above.
4. Assume the forces in all the members to be tensile and name them as P_1, P_2, P_3, ...
5. Select a joint at which maximum number of unknowns is two. Solve that joint using two equations of equilibrium i.e. $\Sigma F_x = 0$ and $\Sigma F_y = 0$. Prepare a table representing name of the member and the internal force in it with its nature. Enter the result in the table.
6. Select a new joint at which maximum number of unknown is two and solve it. Repeat the procedure, till forces in all the members are obtained.
7. Finally, when forces in all the members are obtained, wherever the internal forces are positive, it indicates tensile force and wherever the internal forces are negative it indicates 'compressive force.' While mentioning compression in the column for nature of the force, see that negative sign of the magnitude of the force is cancelled or converted to positive.

Method of sections

This method is adopted in the following situations:

1. At every joint of a truss, more than two members are connected together due to which we can not get a joint at which maximum number of unknowns is two, to start the solution.
2. When the truss is having too many members and we require to calculate the internal forces in only few of them, then solution by method of joint becomes lengthy.

In the method of sections, we take a section line cutting the truss in two parts. As the entire truss is in equilibrium, the part of it is also in equilibrium. Hence, the equilibrium of any one part of the truss is considered. It forms a non-concurrent coplanar force system in equilibrium. Hence, applying the three equations of equilibrium i.e. $\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M = 0$, we can get the required unknowns.



While selecting the section line, following points are to be noted;

1. The section line should not pass through any joint.
2. The section line should not cut one member twice.
3. The section line should not cut all the members which are concurrent at a point in the plane of the truss.
4. As far as possible, the section line should cut minimum three non-concurrent members forming letter "Z".

Graphical method

This method is explained in details in the chapter of "Graphic Statics."

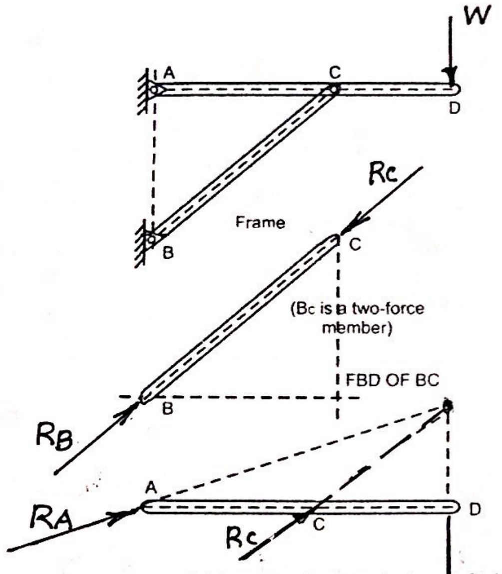
Frames

Frame is a structure consisting of a number of structural members, but not necessarily loaded at their ends to transmit loads. (Remember, in the case of trusses, the loading is always at the ends of the members which constitute the joint). The loads acting on the frame may or may not be acting at the joints. The loads acting on a frame may act at an intermediate point along the length (longitudinal axis) of a member of a frame. Due to this the members of a frame are not necessarily axial force members but are also subjected to transverse (at right angles to the longitudinal axis) forces which cause bending. (Note that, in case of trusses, the members are "two-force members" subjected to axial tension or compression)

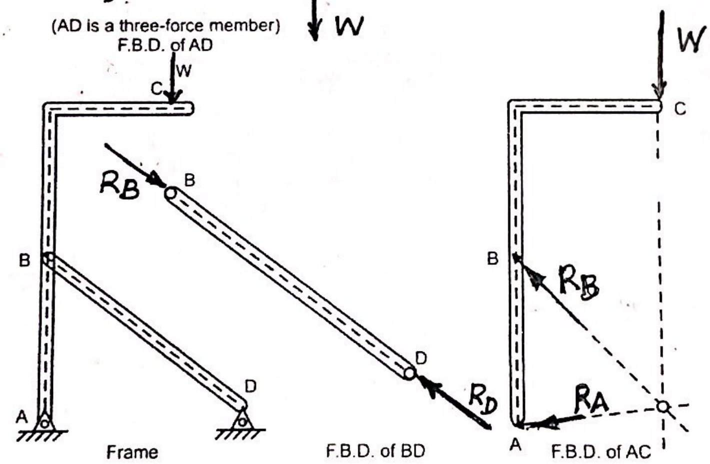
A member of a frame on which forces act at three (or more points) is called as "three-force member" (or "multi-force member"). In a three-force member some forces necessarily act transversely (at right angles to the axis of the member).

A frame must have at least one member which is a three-force member. The internal force-resistance in a three-force member is complex, being usually combination of axial, bending and shear stresses. While in a two-force member it is purely axial. Because of this the three-force member should never be cut for the purpose of analysis as was done for method of joints or method of section used for the analysis of truss.

In the figure, member BC is clearly a two-force member since forces exerted by the pins on the member at their ends act at its two end points B and C. And there is no other force acting on member BC, in between the two pins.



(AD is a three-force member)
F.B.D. of AD



Frame F.B.D. of BD F.B.D. of AC

In the above frame, member AD is a three-force member. Force 'W' act transversely to member AD, due to which reactive forces R_A and R_C must have transverse components.

It is not necessary that a three-force member is necessarily straight member as shown in the above frame. It may be also a bent one.

The reactive force at any support of a three-force member never lies parallel to the axis of that member. In case of a two-force member, the reactive forces at its ends must lie parallel to the axis of that member.

The analysis of a frame consists of:

1. Finding the support reactions of the frame
2. Finding the reactions at the pins at the connecting points (internal hinges). These reactive forces are called as shear forces at the internal hinges

Method of members

We use "method of members" for the analysis of the frames. In this method, various members of the frame are isolated and they are considered as independent members in equilibrium, Free-body diagram of each member gives a non-concurrent coplanar force-system in equilibrium, Hence, for each member, we can use, three equations of equilibrium i.e. $\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M_z = 0$ and determine the required unknowns

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