IC Tutorial-6

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Q.1: $E = Ri + \int \frac{i}{c} \, dt$

Putting $E = E_0 \sin \omega t$ ~~in~~ in the equation

$$E_0 \sin \omega t = Ri + \int \frac{i}{c} \, dt$$

Differentiating wrt t,

$$R \cdot \frac{di}{dt} + \frac{i}{RC} = \frac{E_0}{R} \omega \cos \omega t$$ (linear eqn of type $\frac{dy}{dx} + Py = Q$)

$$i e^{\int (1/RC) \, dt} = \int e^{(1/RC)} \cdot dt \cdot \frac{E_0 \cdot \omega}{R} \cos \omega t \, dt + k$$

$$\therefore \quad i \cdot e^{t/RC} = \frac{E_0 \omega}{R} \int e^{t/RC} \cos \omega t \cdot dt + k$$

$$\therefore \quad i \cdot e^{t/RC} = \frac{E_0 \omega}{R} \cdot \frac{e^{t/RC}}{\sqrt{(1/R^2c^2) + \omega^2}} \cdot \cos(\omega t + \phi) + k$$

where $\tan \phi = \frac{\omega}{1/RC} = \underline{\underline{RC\omega}}$

$$\boxed{i = \frac{E_0 \omega c}{\sqrt{1 + R^2 c^2 \omega^2}} \cdot \cos(\omega t + \phi) + k \cdot e^{-t/RC}}$$

Q.2

$A = 2\pi x \; cm^2$

$$\phi = -KA\frac{dT}{dx} = -K \cdot 2\pi x \cdot \frac{dT}{dx}$$

$$dT = \frac{-Q}{2\pi K} \cdot \frac{dx}{x}$$

Integrating, we have,

$$T = \frac{-Q}{2\pi K} \log_e x + c$$

(1) $T = 200^\circ c$, $x = 5$ cm.

$$200 = \frac{-Q}{2\pi K} \ln 5 + c \quad — (1)$$

(2) $T = 50^\circ c$, $x = 10$ cm

$$50 = \frac{-Q}{2\pi K} \ln 10 + c \quad — (2)$$

Subtracting (2) From (1)

$$150 = \frac{Q}{2\pi K} (\ln 10 - \ln 5)$$

$$\therefore 150 = \frac{Q}{2\pi K} \ln 2$$

$$Q = \frac{2\pi K \times 150}{\ln 2} = \frac{300\pi \cdot (0.12)}{\ln 2} = 163 \text{ cal/sec}$$

(ii) Heat lost per min $= 60 \times 2000 \times Q$

$= 60 \times 2000 \times 163$

$\Rightarrow 19560$ K.cal.

Now, let , $T = t$ when $x = 7.5$

$$t = \frac{-9}{2\pi K} \ln 7.5 + c \quad -(3)$$

Subtracting (1) from (3)

$$t - 200 = \frac{-9}{2\pi K} (\ln 7.5 - \ln 5)$$

$$t - 200 = \frac{-9}{2\pi K} (\ln 1.5)$$

Dividing,

$$\frac{t - 200}{150} = \frac{-\ln 1.5}{\ln 2}$$

$$t = 200 - 150 \times 0.58 = 113$$

(i) $\therefore$ when $x = 7.5$ cm, $\underline{T = 113^{\circ}C}$

Q.3. $y^2(x^2 + a^2) = a^2 x^2$

i. Symmetry — Symmetric about x axis
$\because$ (even powers of y)

(ii) Put $x=0, y=0$ — $f(0,0) = 0$.
$\therefore$ curve passes through origin.

(iii) Intersection x axis $(y=0)$ $a^2x^2 = 0$. $\therefore x = 0$.
Y axis $(x=0)$ $a^2y^2 = 0$. $\therefore y = 0$.

$\therefore$ curve will intersect at $(0,0)$ origin.

(iii) Tangents – (equating lowest degree ~~#~~ to 0)

$y^2(x^2+a^2) = a^2x^2 = 0$

$\Longrightarrow x = 0$

$\therefore$ Y axis is tangent at origin.

asymptote is $\parallel$ to x axis.

(iv) <u>asymptote</u>

(equating coeff of highest power of x to 0)

$y^2(x^2+a^2) = a^2x^2 \Rightarrow x^2(y^2-a^2)+a^2y^2 = 0.$

$y^2-a^2 = 0 \Rightarrow y^2 = a^2$

$y = \pm a$ is an asymptote $\parallel$ to x axis.

$\Rightarrow$ Region of absence is $y < -a$; $y > a$.