MIT World Peace University

Engineering Mechanics
First Year B. Tech, Trimester 1

Graphical Solution of Problems involving Relative Motion between Two Bodies

Experiment 11
Practical Report

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Contents

1 Objective

To find Graphically the resultant of a set of problems involving relative motion between two bodies.

2 Theory

The Following laws and concepts have been used in this experiment.

2.1 Relative Velocity

We encounter occasions where one or more objects move in a frame which is non-stationary with respect to another observer. For example, a boat crosses a river that is flowing at some rate or an aeroplane encountering wind during its motion. In all such instances, in order to describe the complete motion of the object, we need to consider the effect that the medium is causing on the object. While doing so, we calculate the relative velocity of the object considering the velocity of the particle as well as the velocity of the medium. Here, we will learn how to calculate the relative velocity.

Let us consider two objects, A and B moving with velocities V_a and V_b with respect to a common stationary frame of reference, say the ground, a bridge or a fixed platform. The velocity of the object A relative to the object B can be given as,

$$V_{ab} = V_a - V_b$$

Similarly, the velocity of the object B relative to that of object a is given by,

$$V_{ba} = V_b - V_a$$

From the above two expressions, we can see that

$$V_{ab} = -V_{ba}$$

Although the magnitude of both the relative velocities is equal to each other. Mathematically,

$$|V_{ab}| = |V_{ba}|$$

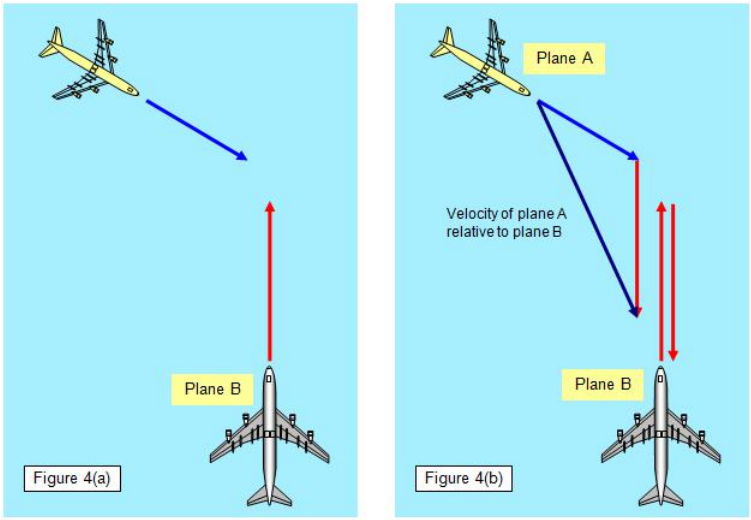


Figure 1: Example of Relative Velocity

3 Graphical Method

Q1. Two chips 'A' and 'B' are at a given instant 4 km away from each other and both are on south east line. Ship 'A' is travelling at 8 km/hr due east and ship 'B' is travelling at 12 km/hr due north. Determine -

1. Velocity of 'B' with Respect to 'A'
2. The Shortest distance between the two ships.
3. Time to get to the shortest distance.

The velocities of 'A' and 'B' are shown in figure:

$\therefore \quad \vec{V}_A = 8\hat{i}$

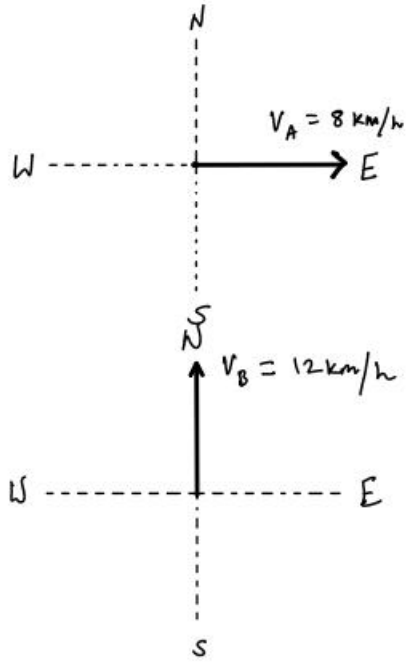
$\vec{V}_B = 12\hat{j}$

$\vec{V}_{B/A} = \vec{V}_B - \vec{V}_A = 12\hat{j} - 8\hat{i}$

$\therefore \quad \vec{V}_{B/A} = -8\hat{i} + 12\hat{j}$

$|V_{B/A}| = \sqrt{8^2 + 12^2} = 14.422 \text{ km/h}$

$\theta = \tan^{-1}\left(\frac{12}{8}\right) = 56.31^\circ$ ∠



The initial positions of A and B and $V_{A/B}$ are shown in figure.

Shortest distance is AC

$AC = 4 \sin 11.31^\circ = 0.784 \text{ Km}$

Time to get shortest distance is :

$t = \frac{BC}{V_{B/A}} = \frac{4 \cos 11.31}{14.422} = 0.272 \text{ hrs} = 16 \text{ min } 19 \text{ sec}$

Q2. Two Planes A and B are flying at the same altitude. If their velocities are $v_A = 600\ kmph$ and $v_b = 500\ kmph$ and the angle between their straight courses is 75°, determine the velocity of plane B with respect to plane A and the shortest distance between them.

$$\vec{V_A} = 600\cos 75\,\hat{i} - 600\sin 75\,\hat{j}$$
$$\vec{V_B} = -500\,\hat{i}$$

$$\vec{V_{B/A}} = \vec{V_B} - \vec{V_A}$$

$$= (-500\,\hat{i}) - (600\cos 75\,\hat{i} - 600\sin 75\,\hat{j})$$

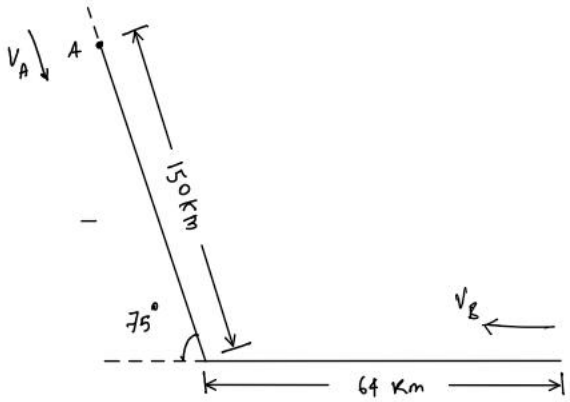
$$\vec{V_{B/A}} = -655.29\,\hat{i} + 579.56\,\hat{j}$$

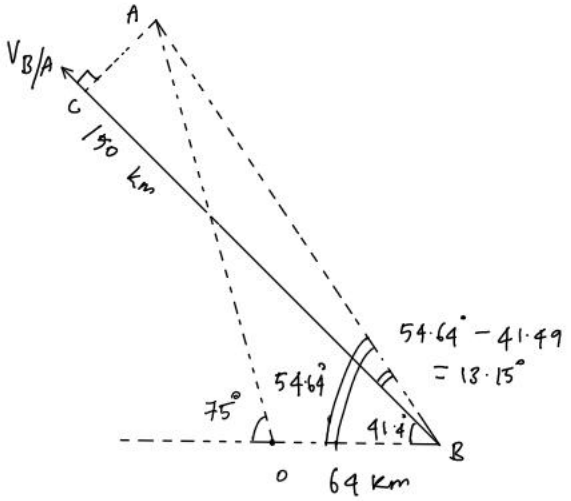
$$|V_{B/A}| = \sqrt{+655.29^2 + 579.56^2}$$

$$\boxed{|V_{B/A}| = 874.81\ km/h}$$

$$\theta = \tan^{-1}\left(\frac{579.56}{655.29}\right)$$

$$\boxed{\theta = 41.49^\circ}$$





∴ The distance AB can be obtained using the cosine rule

$$\cos 105 = \frac{64^2 + 150^2 - AB^2}{2(64)(150)}$$

$$\boxed{AB = 177.67 \text{ km}}$$

By sine rule

$$\frac{150}{\sin \angle ABO} = \frac{177.67}{\sin 105}$$

$$\boxed{\angle ABO = 54.64^\circ}$$

The initial positions of A and B and the direction of $V_{B/A}$ are plotted and shown in the figure.

$$AC = \text{Shortest distance} = AB \sin \angle ABC = 177.67 \sin 13.15^\circ$$

$$\boxed{AC = 40.42 \text{ km}}$$

Q3. At an instant ship A is streaming due East at $20km/h$ and ship B at that instant is $80km$ due South and is streaming at $16km/h$. Determine:

1. $v_{B/A}$
2. Shortest distance between them.
3. time to attain the shortest distance.

Given data:

Velocity of ship A due east at 20 km/h.

Velocity of ship B due along N-W at 16 km/h

At $t=0$, distance between them = 30 km

(1) Relative velocity of ship B with respect to ship A

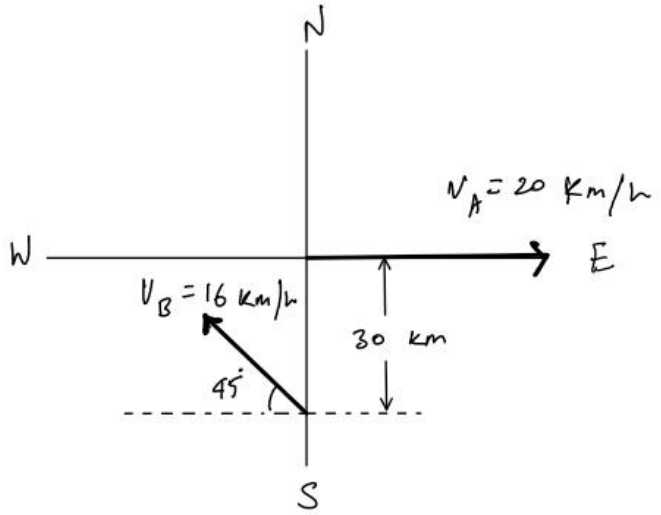
$$V_{B/A} = V_B - V_A$$

$$= (-16\cos 45\,\hat{i} + 16\sin 45\,\hat{j}) - 20\,\hat{i}$$

$$V_{B/A} = (-31.31)\,\hat{i} + 11.31\,\hat{j}$$

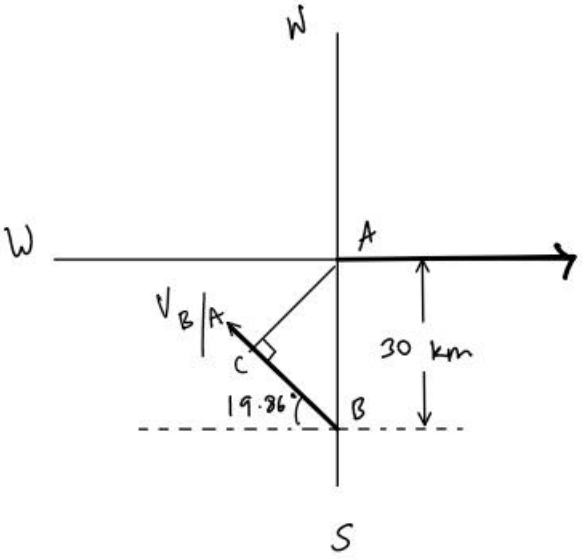
$$|V_{B/A}| = 33.3 \text{ km/h}$$

$$\theta = \tan^{-1}\left(\frac{11.31}{31.31}\right) = 19.86^\circ$$



(2) Shortest distance between them :

AC is the shortest distance between the 2 ships.



$$\cos 19.86^\circ = \frac{AC}{30}$$

$$AC = 30 \times \cos 19.86^\circ$$

$$\boxed{AC = 28.22 \text{ km}}$$

(3) Time required to attain the shortest distance of 28.22 km, ship B travelled from point B to C relative to ship A. Distance BC is given by,

$$\sin 19.86 = \frac{BC}{30}$$

$$BC = 10.19 \text{ km}$$

$$\text{Required Time} = \frac{10.19}{33.3} = \boxed{0.306 \text{ hours}}$$

4 Conclusion

A set of problems involving involving relative motion between two bodies were solved using graphical method.