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LADC Tutorial 5

(1)

Q.1 If $u = e^{xyz}$ prove that $\frac{\partial^3 u}{\partial x \partial y \partial z}$

$$= (1 + 3xyz + x^2y^2z^2)e^{xyz}$$

$$\frac{\partial u}{\partial z} = xy e^{xyz}$$

$$\frac{\partial^2 u}{\partial y \partial z} = e^{xyz}(x) + e^{xyz}(xz)(xy)$$

$$\frac{\partial^2 u}{\partial y \partial z} = e^{xyz}(x + x^2yz)$$

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = \cancel{e^{xyz}(x + x^2yz)}.$$

$$e^{xyz}(1 + 2xyz) + (x + x^2yz)\, yz \cdot e^{xyz}$$

$$= e^{xyz}(1 + 3xyz + x^2y^2z^2)$$

(2) If $u = \sin \frac{x}{y}$ and $x = e^t$, $y = t^2$,

verify $\frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt}$.

→ By actual substitution, we have

$$u = \sin \frac{e^t}{t^2}$$

$$\frac{du}{dt} = \left(\cos \frac{e^t}{t^2}\right) \cdot \frac{t^2 e^t - 2te^t}{t^4}$$

$$= \left(\cos \frac{e^t}{t^2}\right) \cdot \left(\frac{1}{t^2} - \frac{2}{t^3}\right) e^t.$$

Here $u \to xy \to t$, so

u is a composite function of t

$$\frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt}$$

$$= \left(\cos \frac{x}{y}\right) \cdot \frac{1}{y} \cdot e^t + \left(\cos \frac{x}{y}\right) \cdot \frac{-x}{y^2} \cdot 2t$$

$$= \left(\cos \frac{x}{y}\right) \left[\frac{1}{y} e^t - \frac{2x}{y^2} \cdot t\right]$$

$$= \left(\cos \frac{e^t}{t^2}\right) \left[\frac{1}{t^2} \cdot e^t - \frac{2e^t}{t^4} \cdot t\right].$$

$$= \left(\cos \frac{e^t}{t^2}\right) \cdot \left(\frac{1}{t^2} - \frac{2}{t^3}\right) \cdot e^t$$

So LHS = RHS. Therefore True

(3) If $u = (x + y + z)^{-1/2}$, PT.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

Q.2. ① $f(x,y) = \frac{x^3}{y^2}$,

$$f_{xy} = \frac{-6x^2}{y^3}$$

② $f(x,y,z) = x \sin(yz)$

$$f_{yz} = -xyz \sin(yz) + x \cos(yz)$$

③ $f_y = -x^2 \sin(xy)$

④ $f(x,y,z) = x^3 e^{xy} + \cos z$

then

$$f_{zyz} = 0$$