Tutorial - 9

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(1) $\int_0^\infty x^7 \cdot e^{-2x^2} \cdot dx$

Let $2x^2 = t \Rightarrow x = \sqrt{t/2}$

$4x \cdot dx = dt$

$x \cdot dx = \frac{dt}{4}$

$\Rightarrow \int_0^\infty e^{t} \left(\sqrt{t/2}\right)^6 \cdot \frac{dt}{4}$

$= \int_0^\infty \frac{e^{-t}}{4} \cdot \frac{t^{6/2}}{2^{6/2}} \cdot dt$

$= \int_0^\infty \frac{e^{-t} \cdot t^3}{4 \times 2^3} \cdot dt = \frac{1}{32} \int_0^\infty e^{-t} \cdot t^3 \cdot dt$

$= \frac{1}{32} \sqrt{4} = \frac{1}{3} \times 3!$

$= \frac{1}{32} \times 3 \times 2 = \frac{3}{16}$

2. $\int_0^\infty \sqrt{x} \cdot e^{-\sqrt[3]{x}} \cdot dx$

$\sqrt[3]{x} = t$

$x = t^3$

$dx = 3t^2 \cdot dt$

$\int_0^\infty t^{3/2} \cdot e^{-t} \, 3t^2 \cdot dt = 3 \int_0^\infty t^{3/2+2} \cdot e^{-t} \cdot dt$

$$3\int_0^\infty t^{7/2} \cdot e^{-t} \cdot dt$$

$$= 3\int_0^\infty t^{9/2-1} \cdot e^{-t} \cdot dt \quad = 3\,\Gamma(9/2)$$

$$= 3 \times 7/2 \times 5/2 \times 3/2 \times 1/2 \times \sqrt{\pi}$$

$$= \underline{315\sqrt{\pi}}$$
$$16$$

Q.3 $\int_0^\infty \sqrt{y}\; e^{-y^3} dy$

let $y^3 = t \quad \rightarrow y = \sqrt[3]{t}$

$3y^2\, dy = dt$

$$= \int_0^\infty t^{1/6} \cdot e^{-t} \frac{\cdot dt}{3t^{2/3}}$$

$$= \frac{1}{3}\int_0^\infty t^{-1/2}\, e^{-t} \cdot dt$$

$$= \frac{1}{3}\int_0^\infty e^{-t} \cdot t^{1/2-1}\, dt$$

$$= \frac{1}{3}\Gamma(1/2) = \frac{1}{3}\sqrt{\pi} = \frac{\sqrt{\pi}}{3}$$

Q.4 $\int_0^\infty x^{2/3} \cdot e^{-\sqrt[3]{x}} \cdot dx$

$\sqrt[3]{x} = t$

$x = t^3$

$dx = 3t^2 \cdot dt$

$= \int_0^\infty t^2 e^{-t} \left(3t^2 dt\right)$

$= 3\int_0^\infty t^4 e^{-t} \, dt$

$= 3\,\Gamma 3 = 3 \cdot 4! = \underline{72}$