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Determination of Forces in A Space Force System

Purpose of the Experiment:

To introduce the concept of a force as a vector in space, concept of equilibrium of concurrent space force system. To determine the non-coplanar concurrent forces experimentally and verify them analytically.

Instruments:

Space force apparatus, ropes, spring balances, weights, hangers.

Theory:

This experiment is based upon the equilibrium of non-coplanar concurrent forces, it, the equilibrium of concurrent space forces. Like coplanar forces, this system of forces also can be resolved. The resolution of forces will be along three mutually perpendicular directions called as X, Y, Z axes.

Thus, force can be expressed in the form of a vector such as

$$\boxed{F = \vec{F}_x\hat{\imath} + \vec{F}_y\hat{\jmath} + \vec{F}_z\hat{k}}$$

Where $\vec{F}_x, \vec{F}_y, \vec{F}_z$ are the components of the force F in X, Y, and Z directions respectively and $\hat{\imath}, \hat{\jmath}, \hat{k}$ are the unit vectors along X, Y, Z directions respectively. If the force $\vec{F}$ is defined by the coordinates of the two points M (x_1, y_1, z_1) and N (x_2, y_2, z_2) located on its line of action, then force F is defined as

$$\boxed{\vec{F} = F.\hat{\lambda}}$$

where F, is the magnitude and $\hat{\lambda}$ is the unit vector in the direction $\vec{F}$

$$\lambda = \left[\frac{(x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}\right]$$

$$l = \left[\frac{(x_2 - x_1)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}\right]$$

$$m = \left[\frac{(y_2 - y_1)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}\right]$$

$$n = \left[\frac{(z_2 - z_1)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}\right]$$

Where l, m, n are the direction cosines of the line of action of the force

Also

$$\boxed{l^2 + m^2 + n^2 = 1}$$

$F_x = F_l$ = X component of the force, $F_y = F_m$ = Y component of the force, $F_z = F_n$ = Z component of the force.

If the system of forces in in equilibrium, the algebraic sum of the components in three mutually perpendicular directions must be zero.

$$\boxed{\sum F_x = \sum F_y = \sum F_z = 0}$$

These are the analytical conditions of equilibrium of concurrent force system in space.

Procedure:

1. Attach three strings AD, BC and CD with spring balances to the different points on the space force apparatus as shown in the figure. Ensure that the strings are just tight and that the spring balance reading is zero.
2. Suspend Weight W at the point of concurrency (Point D) of the three ropes.
3. With respect to a fixed origin, and observing the right-hand screw rule, decide the orientation of X, Y and Z axes. Find the coordination of A, B, C and D with respect to this frame of reference.
4. Read the Spring Balances and get the experimental values of the tensions $T_{DA,}$ T_{DB}, and T_{DC}.
5. Using the conditions of equilibrium at D, find the analytical values of $T_{DA,}$ T_{DB}, and T_{DC}.
6. Repeat the procedure with different combinations of Weight W and locations of A, B, C and D.

Observations:

Sr. No.	Weight (W) in N	Coordinates in (m)					Tensions in N						%Error		
							Experimental			Analytical					
		Co-od(m)	A	B	C	D	T_{DA}	T_{DB}	T_{DC}	T_{DA}	T_{DB}	T_{DC}	T_{DA}	T_{DB}	T_{DC}
1	24.525	X	0	1	0.6	0.54	17.5	13.5	12.5	15.69	16.13	10.35	11.54	16.31	20.77
		Y	0.65	0.75	0	0.53									
		Z	1	1	1	0.62									
2	29.43	X	0	1	0.9	0.54	20.6	10.59	9.96	38.7	44.25	4.94	46.77	76.07	101.62
		Y	0.9	0.1	0	0.48									
		Z	1	1	1	0.74									
3	34.335	X	0	0.5	1	0.52	25.1	27.5	25	21.45	26.52	23.48	17.02	3.7	6.47
		Y	0.1	1	0.1	0.45									
		Z	1	1	1	0.68									

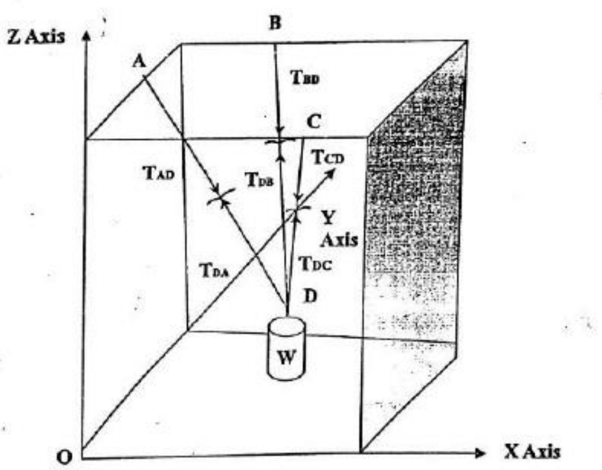

Fig. 1: Concurrent space force system

Calculations:

In the above arrangement AD, BD and CD are the three strings supporting the weight W at the joint D. Thus, load W is freely suspended with the help pf strings AD, BD, and CD. Thus, at point D, there are 4 forces meeting together. Point D is in equilibrium, Hence,

$$\boxed{\overrightarrow{T_{DA}} + \overrightarrow{T_{DB}} + \overrightarrow{T_{DC}} + \vec{w} = 0}$$

Experimental values of the three tensions are directly recorded from the spring balances attached along the strings DA, DB and DC.

Analytical Values of these tensions are calculated by using the equations of equilibrium.

$$\boxed{\sum F_x = 0, \qquad \sum F_y = 0, \qquad \sum F_z = 0}$$

For First Reading :

Vector	$(x_2 - x_1)\hat{\imath}$	$(y_2 - y_1)\hat{\jmath}$	$(z_2 - z_1)\hat{k}$	Length (m)
DA	-0.54	0.12	0.03	0.67
DB	0.46	0.22	0.38	0.63
DC	0.06	0.53	0.38	0.65

Unit Vector	i	j	k
$\hat{e}_{DA} = \frac{\overrightarrow{DA}}{L_{DA}}$	-0.804	0.178	0.566
$\hat{e}_{DB} = \frac{\overrightarrow{DB}}{L_{DB}}$	0.723	0.346	0.597
$\hat{e}_{DC} = \frac{\overrightarrow{DC}}{L_{DC}}$	0.092	-0.809	0.58

Force Vector	i	j	k
$\overrightarrow{T_1} = T_1 \cdot \hat{e}_{DA}$	(-0.804)T1	(0.178)T1	(0.566)T1
$\overrightarrow{T_2} = T_2 \cdot \hat{e}_{DB}$	(0.723)T2	(0.346)T2	(0.597)T2
$\overrightarrow{T_3} = T_3 \cdot \hat{e}_{DC}$	(0.092)T3	(-0.809)T3	(0.58)T3
$\vec{w}$	0	0	-24.525
Total	$\sum F_x = 0$	$\sum F_y = 0$	$\sum F_z = 0$

Writing the Equations of Equilibrium for Sr. 1,

$$-(0 \cdot 804)T_1 + (0 \cdot 723)T_2 + (0.092)T_3 = 0$$

$$(0 \cdot 178)T_1 + (0 \cdot 346)T_2 - (0 \cdot 804)T_3 = 0$$

$$(0 \cdot 566)T_1 + (0 \cdot 597)T_2 + (0 \cdot 58)T_3 = 24 \cdot 525$$

Solving them, we get

$$\boxed{\begin{aligned} T_1 &= T_{DA} = 15.693 \\ T_2 &= T_{DB} = 16.137 \\ T_3 &= T_{DC} = 10.355 \end{aligned}}$$

For Second Reading:

Vector	$(x_2 - x_1)\hat{\imath}$	$(y_2 - y_1)\hat{\jmath}$	$(z_2 - z_1)\hat{k}$	Length (m)
DA	-0.54	0.42	0.26	0.73
DB	0.46	-0.38	0.26	0.65
DC	0.36	-0.48	0.26	0.65

Unit Vector	i	j	k
$\hat{e}_{DA} = \frac{\overrightarrow{DA}}{L_{DA}}$	-0.737	0.574	0.355
$\hat{e}_{DB} = \frac{\overrightarrow{DB}}{L_{DB}}$	0.706	-0.584	0.399
$\hat{e}_{DC} = \frac{\overrightarrow{DC}}{L_{DC}}$	0.55	-0.734	0.398

Force Vector	i	j	k
$\overrightarrow{T_1} = T_1 \cdot \hat{e}_{DA}$	(-0.737)T1	(0.574)T1	(0.355)T1
$\overrightarrow{T_2} = T_2 \cdot \hat{e}_{DB}$	(0.706)T2	(-0.584)T2	(0.399)T2
$\overrightarrow{T_3} = T_3 \cdot \hat{e}_{DC}$	(0.55)T3	(-0.734)T3	(0.398)T3
$\vec{w}$	0	0	-29.43
Total	$\sum F_x = 0$	$\sum F_y = 0$	$\sum F_z = 0$

Writing the Equations of equilibrium for the second reading, and then solving them, we get the values of T1, T2 and T3 as given below.

$$T_1 = T_{DA} = 38.704$$
$$T_2 = T_{DB} = 44.25$$
$$T_3 = T_{DC} = 4.94$$

For Third Reading:

Vector	$(x_2 - x_1)\hat{\imath}$	$(y_2 - y_1)\hat{\jmath}$	$(z_2 - z_1)\hat{k}$	Length (m)
DA	-0.52	-0.35	0.32	0.703
DB	-0.02	0.55	0.32	0.636
DC	0.48	0.35	0.32	0.675

Unit Vector	i	j	k
$\hat{e}_{DA} = \frac{\overrightarrow{DA}}{L_{DA}}$	-0.74	-0.5	0.46
$\hat{e}_{DB} = \frac{\overrightarrow{DB}}{L_{DB}}$	-0.03	0.86	0.503
$\hat{e}_{DC} = \frac{\overrightarrow{DC}}{L_{DC}}$	0.71	-0.52	0.474

Force Vector	i	j	k
$\overrightarrow{T_1} = T_1 \cdot \hat{e}_{DA}$	(-0.74)T1	(-0.5)T1	(0.46)T1
$\overrightarrow{T_2} = T_2 \cdot \hat{e}_{DB}$	(-0.03)T2	(0.86)T2	(0.503)T2
$\overrightarrow{T_3} = T_3 \cdot \hat{e}_{DC}$	(0.71)T3	(-0.52)T3	(0.474)T3
$\vec{w}$	0	0	-34.335
Total	$\sum F_x = 0$	$\sum F_y = 0$	$\sum F_z = 0$

Writing the Equations of equilibrium for the second reading, and then solving them, we get the values of T1, T2 and T3 as given below.

$$T_1 = T_{DA} = 21.45$$

$$T_2 = T_{DB} = 26.52$$

$$T_3 = T_{DC} = 23.48$$

Conclusion:

The Values of Tension for First Reading are:

$$\boxed{\begin{aligned} T_1 &= T_{DA} = 15.693 \\ T_2 &= T_{DB} = 16.137 \\ T_3 &= T_{DC} = 10.355 \end{aligned}}$$

For Second Reading are:

$$\boxed{\begin{aligned} T_1 &= T_{DA} = 38.704 \\ T_2 &= T_{DB} = 44.25 \\ T_3 &= T_{DC} = 4.94 \end{aligned}}$$

And For third Reading are:

$$\boxed{\begin{aligned} T_1 &= T_{DA} = 21.45 \\ T_2 &= T_{DB} = 26.52 \\ T_3 &= T_{DC} = 23.48 \end{aligned}}$$

The Percentage Errors are:

Sr. No.	Weight (W) in N	% Error in Tension		
		TDA	TDB	TDC
1	24.525	11.54	16.31	20.77
2	29.43	46.77	76.07	101.62
3	34.335	17.02	3.7	6.47

The general reasons for errors are:

1. Parallax error in taking reading from the spring Balance
2. Strings are not tied properly which may reduce the tension
3. Keeping the weighing balance and strings tied for a long time may affect the tension in the string
4. Error in taking reading of the Load on the weighing balance.
5. Instrumental error of the springs balance due to fault in the internal springs.

Questions:

1. How do you decide the orientation of coordinate axes in a space force system?
   A. With respect to a fixed origin, and observing the right-hand screw rule, decide the orientation of X, Y and Z axes. Find the coordination of A, B, C and D with respect to this frame of reference.

2. What are the conditions of the equilibrium of non-coplanar concurrent force system?
   A. For a non-coplanar concurrent force system, all the forces are acting on the same point, and therefore there are no moments generated, Therefore for Equilibrium, the conditions are:

$$\boxed{\sum f_x = 0, \qquad \sum f_y = 0, \qquad \sum F_z = 0}$$

3. What are the conditions of the equilibrium of non-coplanar parallel force system?
   A. For a non-coplanar parallel force system, the forces don't all act on the same point, but act parallel to each other, and therefore create various moments about a single axis. For the system to be in equilibrium, the forces along that axis must be zero, and the moments about that axis must be zero. Assuming the Y axis to be the axis perpendicular to which the forces are acting,

$$\boxed{\sum M_x = 0, \qquad \sum M_z = 0, \qquad \sum F_y = 0}$$

4. How to express a force as a vector Quantity?

A. The resolution of forces will be along three mutually perpendicular directions called as X, Y, Z axes. Thus, force can be expressed in the form of a vector such as

$$\boxed{F = \vec{F}_x\hat{\imath} + \vec{F}_y\hat{\jmath} + \vec{F}_z\hat{k}}$$

Where $\vec{F}_x$, $\vec{F}_y$, $\vec{F}_z$ are the components of the force F in X, Y, and Z directions respectively and $\hat{\imath}, \hat{\jmath}, \hat{k}$ are the unit vectors along X, Y, Z directions respectively

5. What do you mean by the direction cosines of a force vector?
   A. The direction cosines of a force vector are the cosines of the angles between the vector and the three positive coordinate axes. Equivalently, they are the contributions of each component of the basis to a unit vector in that direction. They are represented by l, m and n.

$$\boxed{\begin{aligned}
l &= \left[\frac{(x_2 - x_1)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}\right] \\
m &= \left[\frac{(y_2 - y_1)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}\right] \\
n &= \left[\frac{(z_2 - z_1)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}\right]
\end{aligned}}$$