Intro to Mechanics

Centroid and CG Theory

Coplanar Forces Theory

STATICS

Lecture Notes

By

S.R.BENDALE

Chapter 1

Introduction to Engineering Mechanics

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1) Introduction to Engineering Mechanics:

Engineering Mechanics is the study of the effects that forces produce on bodies. Mechanics is that branch of physical science which deals with the state of rest or motion of bodies under the action of forces. Modern research and development in the fields of vibrations, stability and strength of structures and machines, robotics, rocket and spacecraft design, automatic control, engine performance, fluid flow, electrical machines and molecular, atomic and subatomic behaviour are highly dependant upon the basic principle of mechanics.

Mechanics has two major subdivisions:

Statics which deals with the conditions of equilibrium of bodies acted upon by forces.
Dynamics which deals with bodies that are in motion when acted upon by forces.

Statics is one of the beginning courses in the fields of aeronautical, civil and mechanical engineering. A through understanding of its fundamental principle is a prerequisite for further study in dynamics, strength of materials, structural engineering, stress analysis and mechanical design and analysis.

The principles of dynamics has direct useful application in itself and is a prerequisite for further study in vibrations, dynamics of machinery and mechanical design and analysis. The basic principles of mechanics are relatively few in number but they have infinitely wide applications and the methods employed in mechanics carryover into many fields of engineering endeavor.

2) Basic Concepts:

Space: The region occupied by the bodies is called as space. The position of the body in the space can be defined by linear and angular measurements w.r.t. a co-ordinate system. For 3-dimensional problems the space requires 3 independents co-ordinates and for 2 dimensional problems we require 2 independent co-ordinates.

Time: Time is the measure of succession of events. It is the basic quantity in dynamics but it is not involved in statics.

Mass: Mass is a measure of inertia of the body. It is the property of every body by virtue of which each body is attracted by the other. Inertia is that property of every body by virtue of which every body resists the change in its state.

Force: Action of one body on the other which changes or tends to changes the state of the body is called as a force. It is that action exerted by one body over another which is at rest which tries to change the state of the other body. This is called as external force. The effect of the external force on the body at rest is to produce internal reactions and deformations. (sometimes the deformations being too small to cause any change in the geometrical dimensions of the body.)

There are many kinds of forces such as
1) gravity forces exerted by our earth on the bodies in the world
2) simple push or pull that we can exert upon a body with our hands.
3) Gravitational attraction between the sun and the planets.
4) Tractive force of a locomotive or an automobile
5) The force of magnetic attraction
6) Steam or gas pressure in a cylinder.
7) Wind pressure
8) Atmospheric pressure
9) Frictional resistance between the surfaces in contact

Force is a vector quantity. There are four characteristics of a force:

1) Magnitude of a force: It represents the numerical value of fix force.

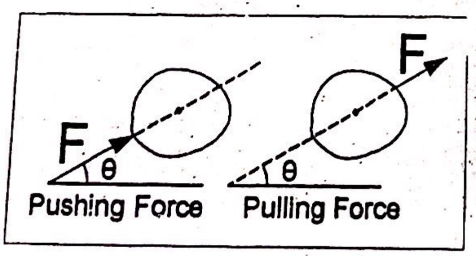
2) Unit of a force: In SI system of units, the unit of a force is 'N' (i.e. Newton)

1N(force) = 1 kg (mass) x 1 m/s^2 (acceleration)

Thus one Newton is the force required to produce an acceleration of 1 m/s^2 on a mass of 1 kg.

3)The direction and sense of a force:

The direction of a force is represented by the angle made by the line of action of the force with horizontal. The arrowhead represents the sense of the force.



4) Point of application:

The point of application of a force acting upon a body is that point in the body at which the force can be assumed to be concentrated. Physically it will be impossible to concentrate a force at a single point. There are some forces which can be explained completely without the point of application. These forces are called as **free vectors**. But there are some forces which can not be explained completely without their point of application. Hence point of application is must for them. These forces are called as **localized vectors.**

Graphical Representation of force:

Force is a vector quantity. It can be represented graphically by drawing an arrow. The length of the arrow is in proportion with the magnitude of the force. The direction of the force is represented by the inclination of the arrow line with the horizontal. The sense of the force is represented by the arrow head. For graphical representation of any force system we require to select a suitable scale for the forces as well as for the dimensions of the body under consideration.

3) Idealization of bodies in Engineering Mechanics:

1) **Particle Body:**
There are some situations in engineering Mechanics in which we do not consider the dimensions of the body under consideration. We consider the mass of the body and the forces acting on that body can be considered as a particle body. Particle is a dimensionless mass. The forces acting on a particle are always central forces. Due to this a particle body is subjected to only motion of translation. A particle can not rotate.

2) **Rigid Body:**
There are some situations in Engineering Mechanics in which we do consider the dimensions of the body in addition to its mass and the forces acting on it. In these situations the body is said to be a rigid body. Thus, the rigid body is the one which is having mass as well as dimensions and it is considered as a non deformable body. The bodies with which we deal with in engineering design of structures and machine parts are never absolutely rigid but deform slightly under the action of loads which thy have to carry. The forces acting on a rigid body can be central or eccentric also. Eccentricity of a force gives rise to rotation. Due to the this the rigid bodies are subjected to motion of translation as well as rotation.

4) Basic Units:

Mechanics deals with four fundamental quantities – length, mass, force and time. The units used to measure these quantities cannot all be chosen independently because they must be consistent with Newton's second law of motion. The international System of Units (SI) is termed as an absolute system. Since the measurement of the base quantity mass is independent of its environment (i.e. position w.r.t. sea level, longitude and latitude etc.)

Quantity	Dimensional symbol	Unit in SI system	Symbol
Mass	M	Kilogram	Kg
Length	L	Meter	M
Time	T	Second	T
Force	F	Newton	N

Scalars and Vectors:

In engineering mechanics we come across two types of quantities – scalars and vectors. Quantities which can be explained completely by only magnitude and unit are called as scalars. For example time, volume, density, speed, energy and mass etc. Quantities which can be explained completely by magnitude unit and direction and sense are called as vector quantities. For example force, moment, momentum, displacement, velocity and acceleration etc. Vectors can be classified into three categories.

Free Vectors:

Those vectors whose action is not confined to a unique line in space is called as free vectors.

Sliding vectors:

Those vectors whose action is confined to a unique line in space along which the quantity acts are called as sliding vectors.

Fixed vectors:

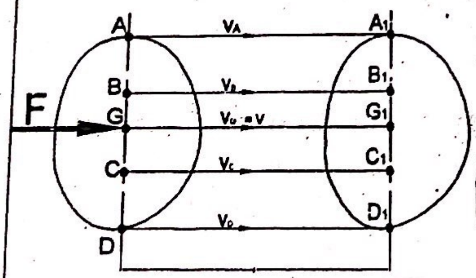
Those vectors whose action is confined to a unique point of application in space re called as fixed vectors or localized vectors.

Effect of forces on bodies:

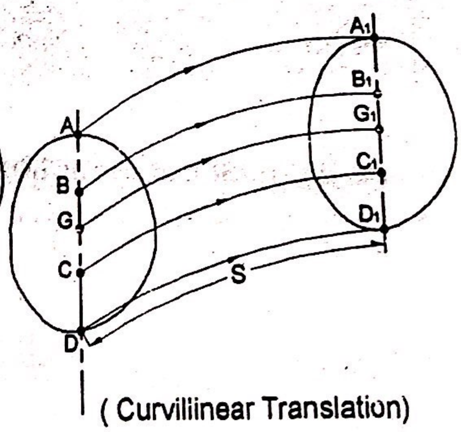
A) External Effects :

1) Motion of Translation:

Under the action of external forces the body gets translated from one position to the other. The position co-ordinates of the body changes in this case. The paths of various particles of the body are parallel to each other. The magnitudes of linear velocities of all the particles are equal. Thus there is a common linear velocity to the body. If the body is translating along a rectilinear path, it is called as rectilinear translation. If the body is translating along a curvilinear path, it is called as curvilinear translation (plane curve or space curve).



VA =VB=VC=VD=VG =V (say)
i.e. Linear Velocity of the body
(Rectilinear Translation)

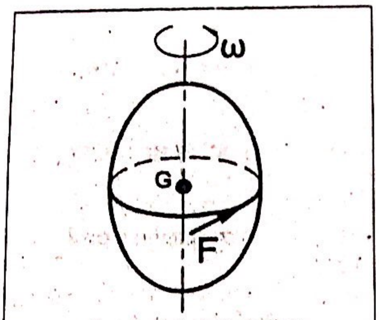


(Curvilinear Translation)

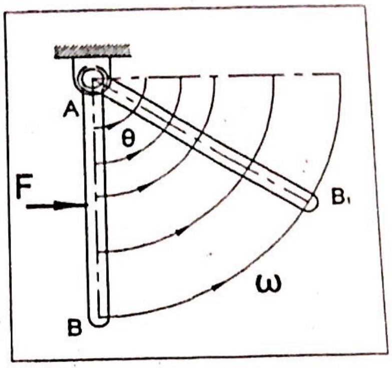
Scanned with CamScanner

2) Motion of Rotation:

Sometimes due to external forces, instead of changing the position the body starts rotating at its position about an axis passing through the body. The paths of the particles of the body are arcs of concentric circles. There is a common angular velocity to the body. If the axis of rotation is passing through the body it is called as centroidal rotation otherwise it is called as non-centroidal rotation. Since, the axis of rotation is fixed in both the cases it is called as 'rotation about a fixed axis'.



Centroidal Rotation

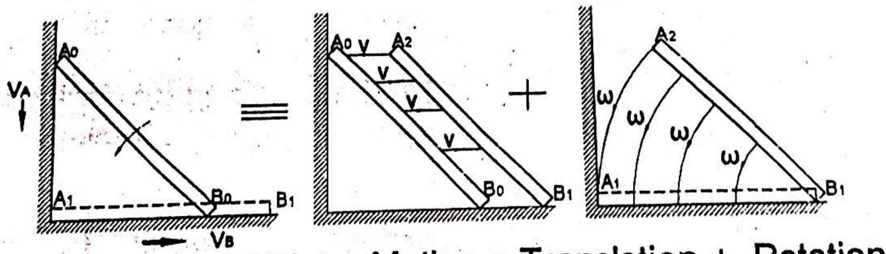


3) General Plane motion:

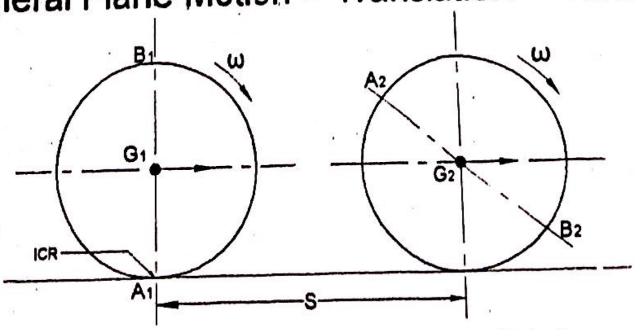
Under the action of external forces, sometimes the body is subjected to a motion which can be analyzed as combination of translation and rotation. This is called as general plane motion. In this motion the particles of the body move in one plane during the entire motion.

For example, i) a ladder sliding against a vertical wall

ii) Rolling of a wheel without sleeping



A) General Plane Motion = Translation + Rotation



B) Rolling of a wheel without slipping

(V= Linear velocity of the centre & ω = Angular Velocity of the wheel)

4) Equilibrium:

Under the action of external forces, sometimes, the body is neither subjected to translation nor subjected to rotation, but it continues to be in the state of rest or the state of uniform rectilinear motion. This is achieved by offering equal and opposite balancing reactions by the body. Thus to develop the balancing relations under the action of external forces is also the property of the bodies in some situations. This is called as equilibrium.

Study of equilibrium is called as statics.

Study of bodies in motion or the study of bodies in motion or the study of the motion of the body is called as Dynamics.

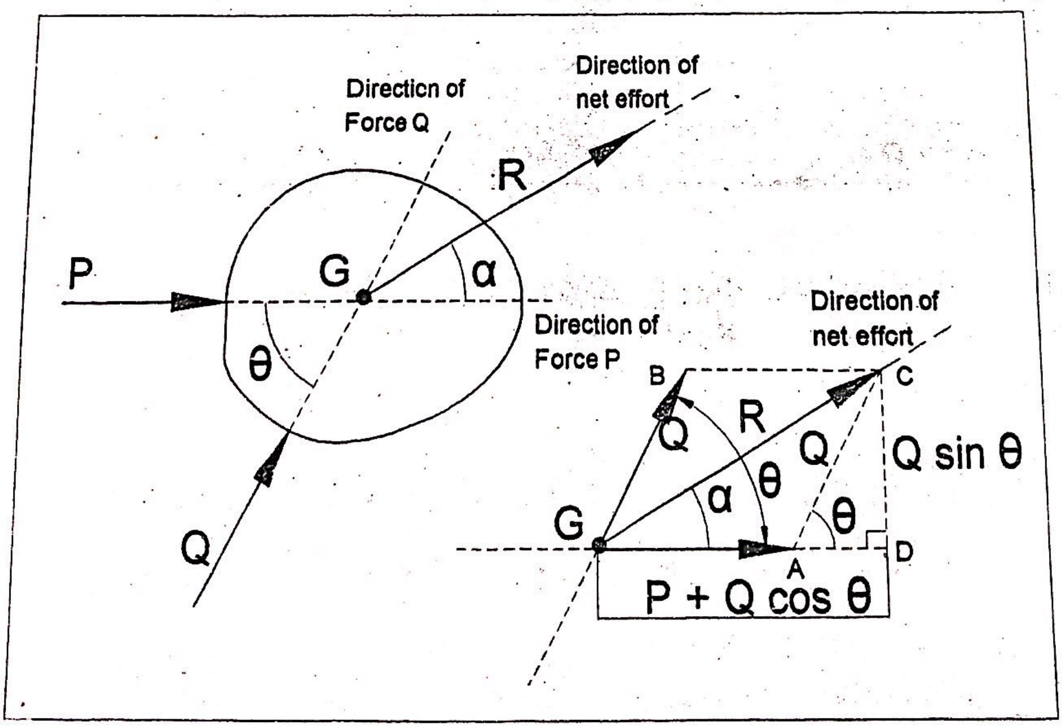
B) Internal Effects:

Sometimes, under the action of external forces, even though the body is in equilibrium, it changes its shape and size. This is called as deformation. If the load is within the elastic limits, this deformation gets cancelled on removal of the loads. The deformation due to external forces is called as strain. Resistance offered by the body to the deformation is called as stress. The study of relationship between stresses and strains is called as mechanics of deformable bodies or strength of materials.

Axioms in Engineering Mechanics:

1) The law of parallelogram of forces:

If a body is simultaneously acted upon by two forces 'P' and 'Q' then the net effect of these forces is given by a force represented by the diagonal of a parallelogram passing through the body whose adjacent sides are the given two forces.



The net effect of the two forces 'P' and 'Q' acting simultaneously on a body is also a force, called as 'Resultant force' (R). The two forces 'P' and 'Q' are called as the components of force R,

Thus,

Assuming the statement of the parallelogram law to be true, the geometrical construction is drawn.

$$In \Delta OCD,$$

$$OC^2 = OD^2 + DC^2$$

$$= (OA^2 + AD^2) + DC^2$$

$$= OA^2 + 2OA.OD + AD^2 + DC^2$$

$$= OA^2 + AC^2 + 2.P.Q.COS\theta$$

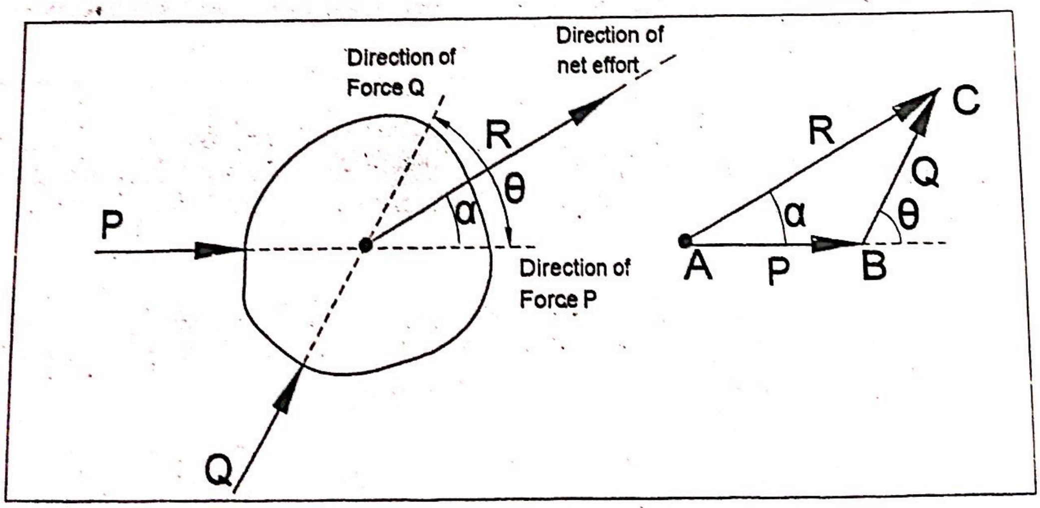
$$\therefore R = \sqrt{P^2 + Q^2 + 2.P.Q.COS\theta} \text{ --------- } (1)$$

$$\tan\alpha = \frac{CD}{OD} = \frac{CD}{OA + AD}$$

$$\tan\alpha = \frac{Q.\sin\theta}{P + Q.\cos\theta} \text{ ---------------- } (2)$$

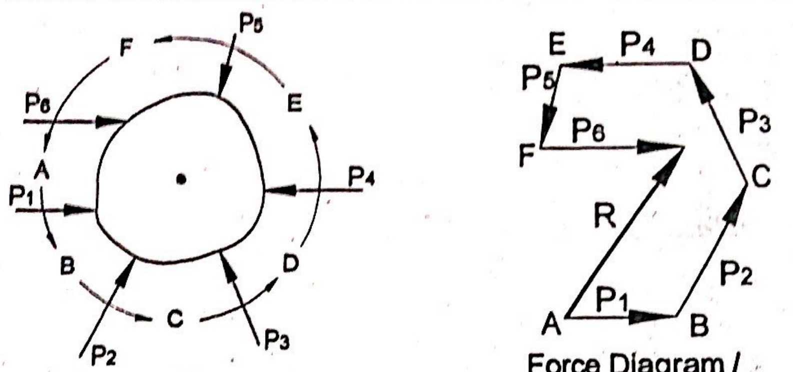
Equation (1) gives us the magnitude of the resultant force and equation (2) gives us the direction of the resultant force.

2) Triangle law of addition of forces:



This is a graphical representation of the law of parallelogram of forces. This is also called as a 'tip-tail' addition. In this case any one force is plotted graphically to some scale on paper and the second force is drawn parallel to itself at the tip of the first force. Thus at the tip of first force the tail of the second force joined graphically. Then the vector joining the tail of graphically. Then the vector joining the tail of the first force to the tip of the second force is representing the resultant of the two forces.

3) Polygon law of addition of forces:



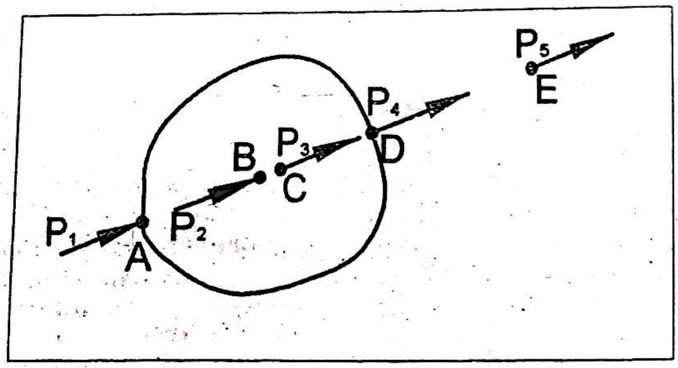
Space Diagram

Force Diagram / Bow's Notation

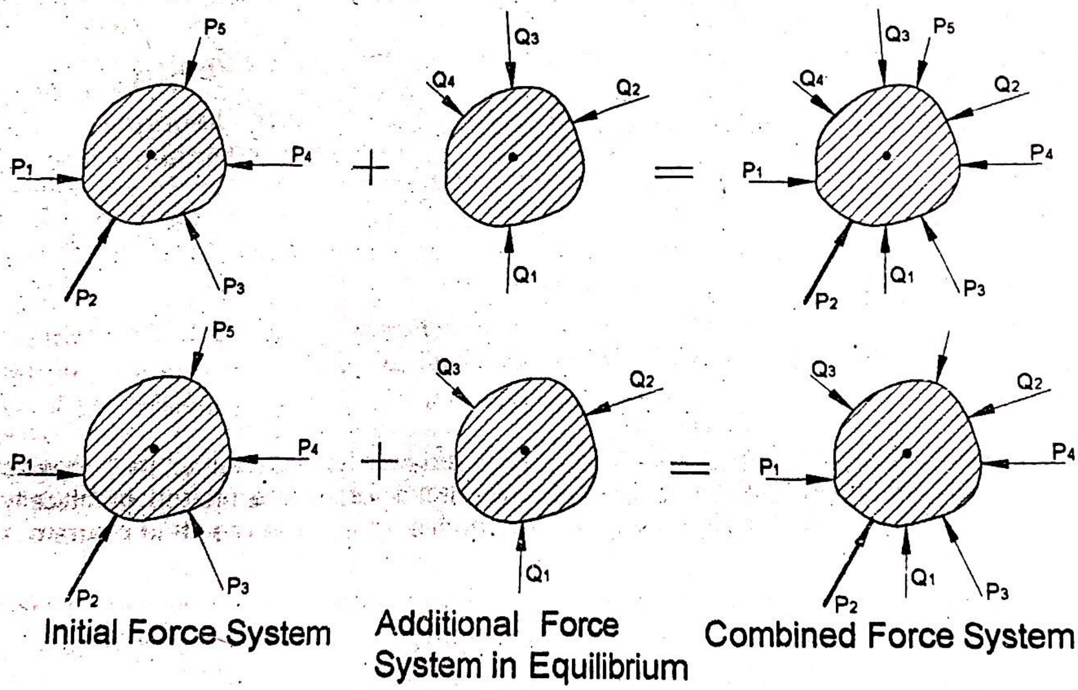
Extension of the triangle law to many coplanar forces is called as polygon law of addition of forces. In this case first we draw a neat sketch of the isolated body and the external forces acting on it at their respective points of application. The spaces between the forces are then named by using Bow's notation. For this purpose take a round around the body either in clockwise sense or in anticlockwise sense. Then all the forces are plotted graphically parallel to themselves as shown in the above figure. The resulting diagram is called as the force diagram. The force vector joining the tail of the first force to the tip of the last force is representing the resultant of the given force system acting on the body.

4) The principle of transmissibility of a force:

The effect of the force acting on the body remains unchanged even if it is translated along the same line of action on the body. In the above figure the forces P1, P2, P3 and P4 are having same magnitude, same line of action and same sense. They all are acting on the body at points A, B, C and D respectively. Because of this when they are acting independently, they produce the same effect on the body. But force P5 having same magnitude, same sense, same line of action but point of application not in contact with the body, then it will not have any effect on the body. Force P1, P2, P3 and P4 are called as 'sliding vectors'.

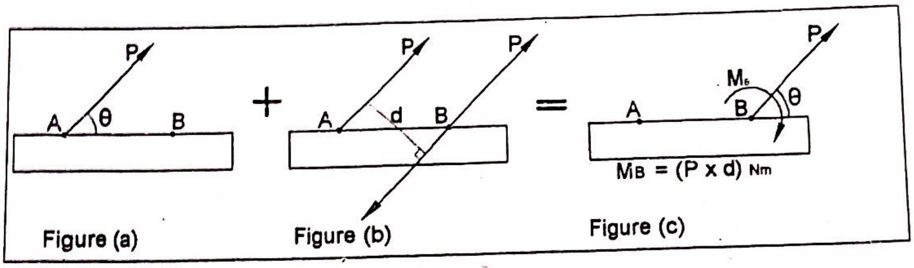


5) The principle of superposition of forces:



When a body acted upon by one force system is superimposed by another system of forces then their combined effect is the addition of their individual effects.

The effect of the force system acting on the body remains unchanged even if we superimpose another force system onto it which itself is in equilibrium. The above principle is used to develop the 'principle of parallel transfer of a force.'



In figure (a): A and B are two different points of a body. The given system consists of single force 'P' acting at point A.

In figure (b): On the force system in figure(a), we are adding another force system consisting of two equal and opposite collinear forces parallel to the force 'P' at A but acting at point B. Thus, we are adding a zero force system to the original force system.

In figure (c): We have got a force-couple system acting at B. Here, in this system the force at B is identical to the original force at A and it is accompanied by a couple of moment 'M_B' obtained by taking moment of force 'P' about point B. $M_B = (P \times d)$ Nm

Thus the three force system in the above three figures are having same effect on the body on which they are acting such force systems are called as 'equivalent systems'.

In the above example we have transferred the force 'P' acting at point A to point B on the same body without changing its original effect on the body. Thus, to transfer a force acting at one point on the body to the other point on the same body without changing its effect, it is transferred by a force-couple system.

6) Newton's Laws of Motion:

A) Newton's First Law:

A body continues to be in the state of rest or in the state of uniform rectilinear motion unless and until it is acted upon by an external unbalanced force.

B) Newton's Second Law:

The force acting on the body is directly proportional to the rate of change of linear momentum.

$$\overline{F} \,\alpha\, \frac{d}{dt}(m\,\overline{v})$$

In SI units,

$$\overline{F} = \frac{d}{dt}(m\,\overline{v})$$

$$\overline{F} = m \cdot \frac{d\overline{v}}{dt}$$

$$\overline{F} = m \cdot \overline{a}$$

Where,

V = velocity of the body

a = acceleration of the body

m = mass of the body

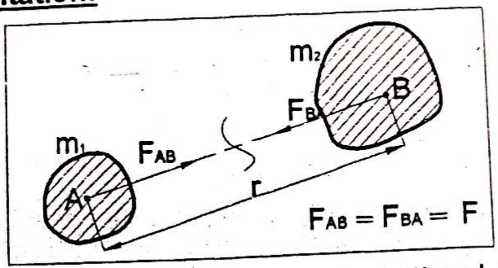
C) Newton's Third Law:

To every action, there is equal and opposite reaction.

7) Newton's Law Of Gravitation:

$$F \,\alpha \left(\frac{m_1.m_2}{r^2}\right)$$

$$F = \left(\frac{G.m_1.m_2}{r^2}\right)$$

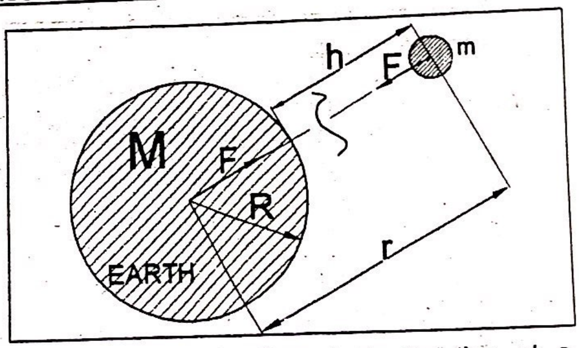


The force of attraction between two bodies of masses 'm1' and 'm2' separated by distance 'r' directly proportional to the product of their masses and inversely proportional to the square of the distance between them. 'G' is called as Universal Gravitational Constant'.

Concept of Gravitational Acceleration:

Consider a body of mass 'm' at a distance of 'h' from the surface of the earth.

Let M=mass of the earth,
R=radius of the earth



Then, $F=\dfrac{GMm}{(R+h)^2}=g'm$

Where $g'=\dfrac{GM}{(R+h)^2}$ is called as earth's gravitational acceleration at a distance of 'h' from its surface. When the body is on the surface of the earth, i.e. when h=0.

$$\text{gravitational acceleration} = g = \left(\frac{GM}{R^2}\right) = 9.81 m/s^2$$

Chapter 6

Centroid and Centre of Gravity

6.1 Introduction

We know every body is attracted to the centre of the earth by a force of attraction, known as the weight of the body.

The weight being a force acts through a point known as the centre of gravity of the body. In Chapter 2 we have emphasised in article 2.2 that the point of application of the force is one of the necessary data to define a force. Hence the location of centre of gravity becomes important while dealing with the weight force.

In this chapter we will learn to find the centre of gravity of bodies, plane areas and lines. We will also study the approach using *integration method* to find the centre of gravity of figures bounded by curve. Finally we will study the application of the location of centre of gravity to certain engineering problems.

6.2 Centroids and Centre of Gravity defined

6.2.1 *Centre of Gravity*

It is defined as a point through which the whole weight of the body is assumed to act. It is a term used for all actual physical bodies of any size, shape or dimensions e.g. book, cupboard, human beings, dam, car, etc.

6.2.2 *Centroid*

The significance of centroid is same as centre of gravity. It is a term used for centre of gravity of all plane geometrical figures. For example, two dimensional figures (Areas) like a triangle, rectangle, circle, and trapezium or for one dimensional figures (Lines) like circular arc, straight lines, bent up wires, etc.

6.3 Relation for Centre of Gravity

Consider a body of weight W whose centre of gravity is located at G $(\overline{X}, \overline{Y})$ as shown. If the body is split in n parts, each part will have its elemental weight W_i acting through its centre of gravity located at G_i (x_i, y_i). Refer Fig. 6.1

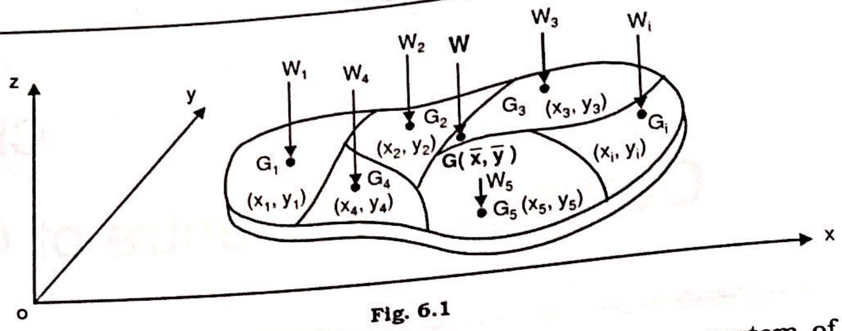


Fig. 6.1

The individual weights W_1, W_2, W_3, ... W_i .... W_n form a system of parallel forces. The resultant weight of the body would then be

$$W = W_1 + W_2 + W_3 \ldots\ldots + W_i \ldots\ldots W_n$$

$$= \Sigma W_i$$

To locate the point of application of the resultant weight force W using Varignon's theorem (discussed earlier in Chapter 2).

Taking moments about y axis

Moments of individual weights about y axis = Moment of the total weight about y axis

$$W_1 \times x_1 + W_2 \times x_2 + \ldots + W_i \times x_i \ldots\ldots + W_n \times x_n = W \times \overline{X}$$

$$\Sigma W_i x_i = W \times \overline{X}$$

$$\overline{\mathbf{X}} = \frac{\Sigma \mathbf{W_i x_i}}{\mathbf{W}} = \frac{\Sigma \mathbf{W_i x_i}}{\Sigma \mathbf{W_i}} \qquad \ldots\ldots\ \mathbf{6.1\ (a)}$$

Similarly if the moments are taken about x axis

Moments of individual weights about x axis = Moment of the total weight about x axis

$$W_1 \times y_1 + W_2 \times y_2 + \ldots + W_i \times y_i \ldots\ldots + W_n \times y_n = W \times \overline{Y}$$

$$\Sigma W_i y_i = W \times \overline{Y}$$

$$\overline{\mathbf{Y}} = \frac{\Sigma \mathbf{W_i y_i}}{\mathbf{W}} = \frac{\Sigma \mathbf{W_i y_i}}{\Sigma \mathbf{W_i}} \qquad \ldots\ldots\ \mathbf{6.1\ (b)}$$

Using the relations 6.1 (a) and 6.1 (b), the centre of gravity G of a body having co-ordinates $(\overline{X}, \overline{Y})$ can be located.

6.3.1 Relation for Centroid

We recall that weight = mass × g

= (Density × Volume) × g

= (Density × Area × Thickness) × g

$W = \rho \times A \times t \times g$

$= (\rho \times t \times g)\, A$

For uniform bodies i.e. of same density and thickness throughout the body, we get,

$$\overline{X} = \frac{\Sigma(\rho \times t \times g) A_i\, x_i}{\Sigma(\rho \times t \times g) A_i} = \frac{\Sigma A_i\, x_i}{\Sigma A_i} \qquad \text{......... 6.2 (a)}$$

Similarly,

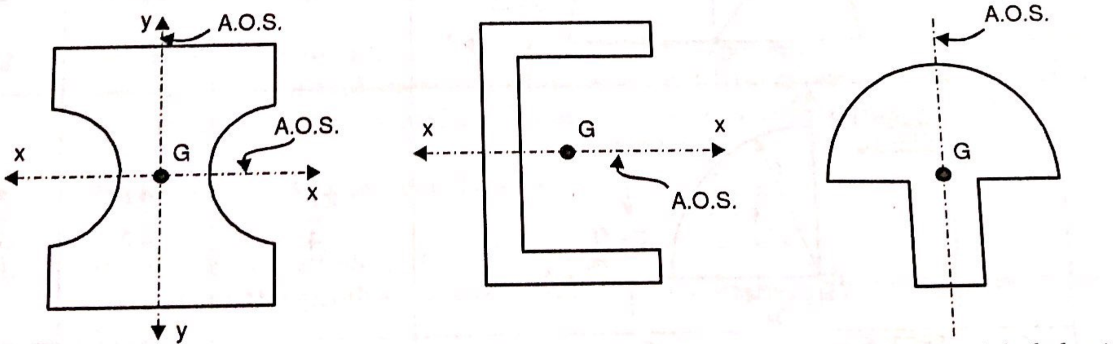
$$\overline{Y} = \frac{\Sigma(\rho \times t \times g) A_i\, y_i}{\Sigma(\rho \times t \times g) A_i} = \frac{\Sigma A_i\, y_i}{\Sigma A_i} \qquad \text{......... 6.2 (b)}$$

Using the relation 6.2 (a) and 6.2 (b), the centroid G having co-ordinates $(\overline{X}, \overline{Y})$ of a plane area can be located.

6.4 Axis Of Symmetry (A. O. S.)

Axis Of Symmetry is defined as the line which divides the figure into two equal parts such that each part is a mirror image of the other.

If the geometrical figure whose centroid has to be located is a symmetrical figure, then the centroid will lie on the axis of symmetry (A.O.S.). If the figure has more than one axis of symmetry, the centroid will lie on the intersection of the axis of symmetry. Fig 6.2 below shows the importance of identifying the axis of symmetry.



(a) Fig. having more than one A.O.S., hence centroid lies on their intersection

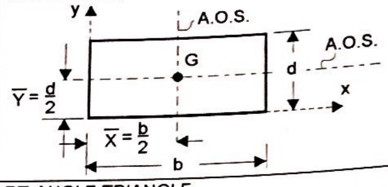
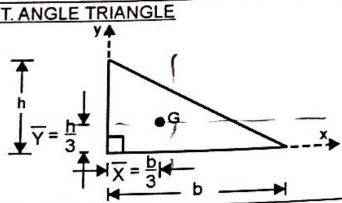
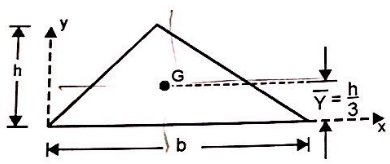
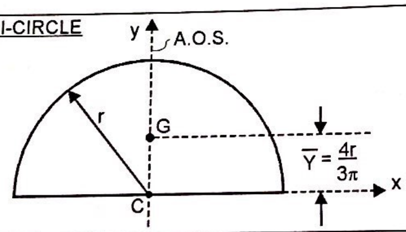
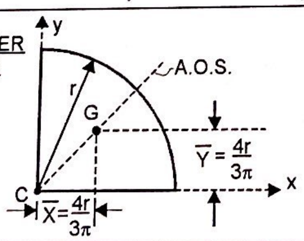
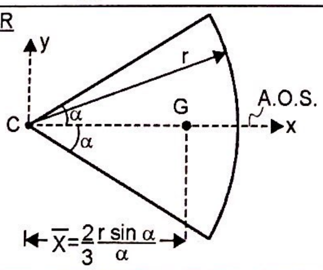
(b) Fig. is symmetrical about a horizontal axis, hence centroid lies on the horizontal axis.

(c) Fig. is symmetrical about a vertical axis, hence centroid lies on vertical axis.

Fig. 6.2

6.5 Centroids of Regular Plane Areas

Table 6.1 shows the centroids of regular plane areas. The co-ordinates $(\overline{X}, \overline{Y})$ of the centroid 'G' are with respect to the axis shown in the figure.

SR. NO.	FIGURE	AREA	$\overline{X}$	$\overline{Y}$
1.	RECTANGLE 	$b \times d$	$\frac{b}{2}$	$\frac{d}{2}$
2.	RT. ANGLE TRIANGLE 	$\frac{1}{2} \times b \times h$	$\frac{b}{3}$	$\frac{h}{3}$
3.	ANY TRIANGLE 	$\frac{1}{2} \times b \times h$	–	$\frac{h}{3}$
4.	SEMI-CIRCLE 	$\frac{\pi r^2}{2}$	0	$\frac{4r}{3\pi}$
5.	QUARTER CIRCLE 	$\frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
6.	SECTOR 	$r^2 \alpha$ *	$\frac{2}{3} \frac{r \sin \alpha}{\alpha^{\#}}$	0

* α is in radians

Table 6.1

α in the denominator is in radians

6.6 Centroid of Composite Area

An area made up of number of regular plane areas is known as a Composite Area.

To locate the centroid of a composite area, adopt the following procedure.

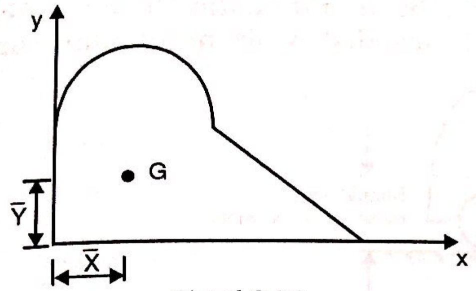


Fig. 6.3 (a)

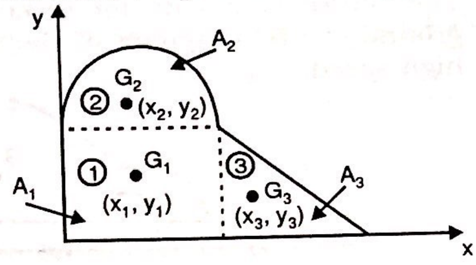


Fig. 6.3 (b)

PART	AREA A_i	CO-ORDINATES X_i	Y_i	$A_i.X_i$	$A_i.Y_i$
1.RECTANGLE	A_1	X_1	Y_1	$A_1.X_1$	$A_1.Y_1$
2. SEMI-CIRCLE	A_2	X_2	Y_2	$A_2.X_2$	$A_2.Y_2$
3.RT. ANGLE TRIANGLE	A_3	X_3	Y_3	$A_3.X_3$	$A_3.Y_3$
	ΣA_i			$\Sigma A_i.X_i$	$\Sigma A_i.Y_i$

Table 6.2

1. Divide the composite area into regular areas as in Fig. 6.3 (b)

2. Mark the centroids G_1, G_2, G_3, ... on the composite figure as shown in Fig. 6.3 (b) and find their co-ordinates w. r. t. the given axis. Let the area of a regular part be A_i and the co-ordinates be X_i and Y_i.

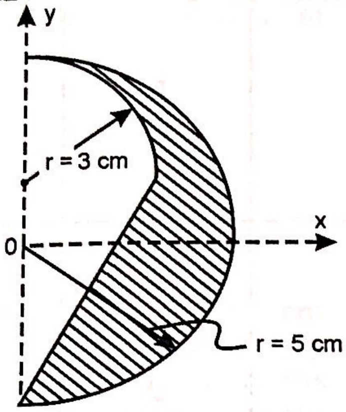
3. Prepare a table as shown (Table 6.2)

4. a) Add up the areas of the different parts to get ΣA_i
   b) Add up the product of area and x co-ordinate of different parts to get $\Sigma A_i.X_i$
   c) Add up the product of area and y co-ordinate of different parts to get $\Sigma A_i.Y_i$

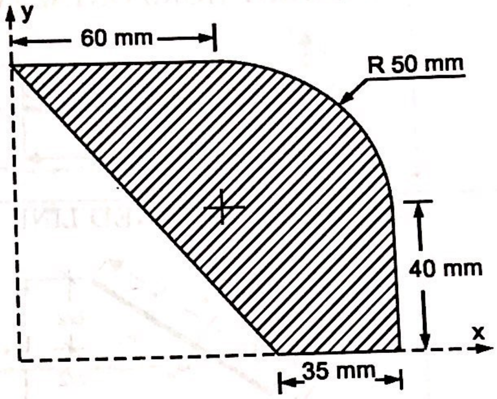
5. The co-ordinates of the centroid of the composite figure are obtained by using relations 6.2 (a) and 6.2 (b) viz.

$$\overline{X} = \frac{\Sigma A_i x_i}{\Sigma A_i} \quad \text{and} \quad \overline{Y} = \frac{\Sigma A_i Y_i}{\Sigma A_i}$$

P16. Determine the centroid of the shaded area shown.



P17. Determine the centre of gravity of the shaded area ***(M.U. May 14)***



6.8 Centroid of Lines

So far we have studied to locate the centroid of plane areas. Now let us learn to find out centroid of lines which are also geometrical figures.

Lines are one dimensional figures whose length (L) is prominent than its thickness (b). The thickness also is uniform throughout the length. We therefore modify relation 6.2 (a) and (b) to obtain the relation of centroid of lines.

We recall $$\overline{X} = \frac{\Sigma A_i x_i}{\Sigma A_i} \quad \text{......... 6.2 (a)}$$

$$\therefore \quad \overline{\mathbf{X}} = \frac{\Sigma (\mathbf{L} \times \mathbf{b})_i \, \mathbf{x}_i}{\Sigma (\mathbf{L} \times \mathbf{b})_i} = \frac{\Sigma \mathbf{L}_i \, \mathbf{x}_i}{\Sigma \mathbf{L}_i} \quad \text{......... } \mathbf{6.3\ (a)}$$

also $$\overline{Y} = \frac{\Sigma A_i y_i}{\Sigma A_i} \quad \text{......... 6.2 (b)}$$

$$\therefore \quad \overline{\mathbf{Y}} = \frac{\Sigma (\mathbf{L} \times \mathbf{b})_i \, \mathbf{y}_i}{\Sigma (\mathbf{L} \times \mathbf{b})_i} = \frac{\Sigma \mathbf{L}_i \, \mathbf{y}_i}{\Sigma \mathbf{L}_i} \quad \text{......... } \mathbf{6.3\ (b)}$$

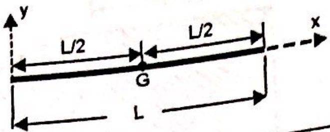
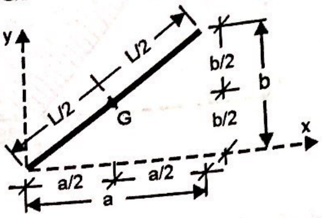
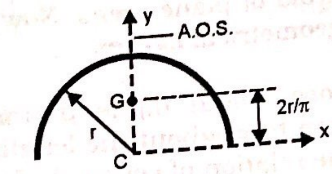
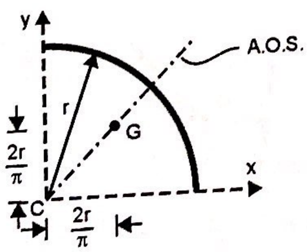
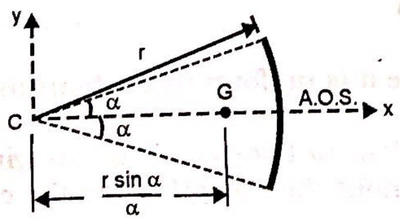
Note that thickness b cancels out since it is uniform throughout the length

The physical bodies which are equivalent to lines are bent up wires, pipe lines etc. If these bodies are uniform throughout their length, then the centroid would coincide with the centre of gravity.

6.8.1 Centroids of Regular Lines

Table 6.3 shows the centroid of regular lines. The co-ordinates $(\overline{X}, \overline{Y})$ of the centroid 'G' are with respect to the axis shown in the figure.

Scanned with CamScanner

SR. NO	FIGURE	LENGTH	Co-ordinates	
			X	Y
1.	STRAIGHT HORIZONTAL LINE 	L	$\frac{L}{2}$	0
2.	STRAIGHT INCLINED LINE 	L	$\frac{a}{2}$	$\frac{b}{2}$
3.	SEMI-CIRCULAR ARC 	πr	0	$\frac{2r}{\pi}$
4.	QUARTER-CIRCULAR ARC 	$\frac{\pi r}{2}$	$\frac{2r}{\pi}$	$\frac{2r}{\pi}$
5.	CIRCULAR ARC 	$2\,r\,\alpha$*	$\frac{r\sin\alpha}{\alpha^{\#}}$	0

* α is in radians

\# α in the denominator is in radians

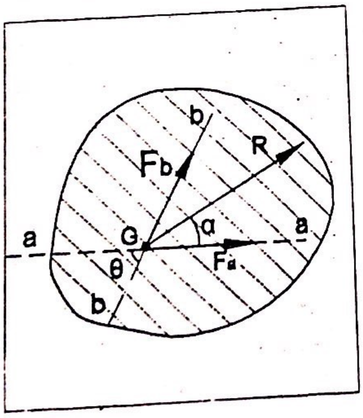
Table 6.3

Chapter 2 - COPLANAR FORCES

1. Resolution of a force
2. Rectangular components of a force
3. Moment of a force
4. Varignon's theorem of moments
5. couples
6. Force systems
7. composition of forces i.e. Resultant of a force system
8. Concept of Equilibrium
9. Free-Body Diagram
10. Types of Supports
11. Equilibrium of two forces
12. Equilibrium of three forces
13. Sample Free-body diagrams

Coplanar Forces

(I) Resolution of a force:



It is always necessary to replace a given fore by its vector components which act in specified directions. This is called as resolution of a force. A given force can be resolved into two or more vector components.

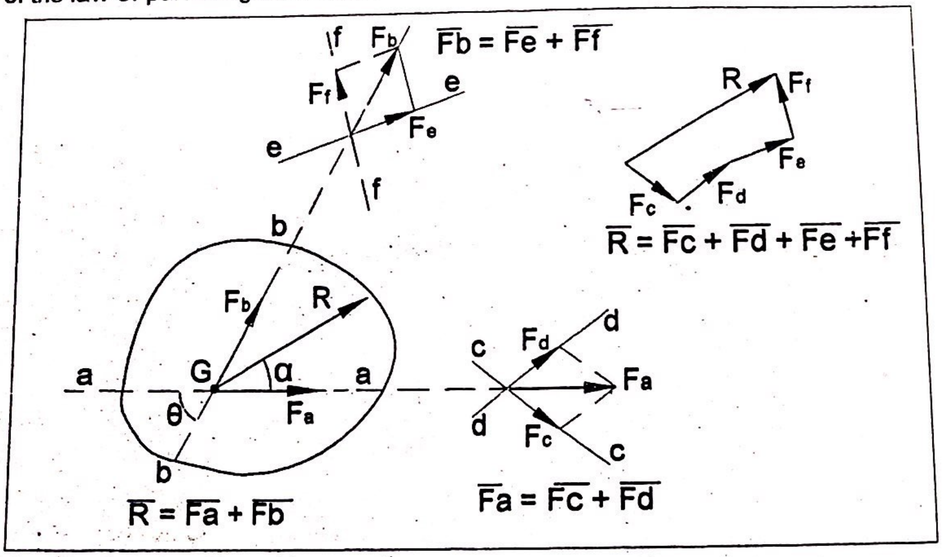
To resolve the given force 'R', into two components 'Fa' and 'Fb' along the lines (a-a) and (b-b), we use the law of parallelogram of forces. Thus in the above figure,

$$R=\sqrt{F_a^2+F_b^2+2.F_a.F_b.\cos\theta} \quad \text{——(1)}$$

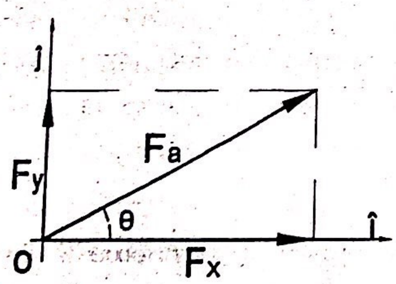
$$\tan\alpha=\left(\frac{F_b.\sin\theta}{F_a+F_b.\sin\theta}\right) \quad \text{——(2)}$$

solving the above two equations, the components Fa and Fb along the two given lines can be obtained.

A single force can be resolved into many vector components by repeated use of the law of parallelogram of forces. This is explained in the following figure.



Rectangular components:



Polar Form	Rectangular Form
θ	F = Fx î +Fy ĵ
θ	F = - Fx î +Fy ĵ
θ	F = - Fx î -Fy ĵ
θ	F = Fx î -Fy ĵ

This is the most common resolution of a force vector into two mutually perpendicular component vectors. This can also be considered as a special case of the law of parallelogram of forces where the angle between the two component vectors is 90°. In this case $\overline{Fx}$ and $\overline{Fy}$ are the vector components of a force vector $\overline{F}$.

Fx and Fy are called as x and y scalar components force F. Depending upon the quadrant into which force 'F' points, the components Fx and Fy can be taken as positive or negative.

Thus, a force can be expressed in two different forms (i) Rectangular form using Cartesian frame of reference (ii) Polar form using polar frame of reference

Conversion from polar to rectangular (p to R)

$$Fx = F.\cos\theta$$

$$Fy = F.\sin\theta$$

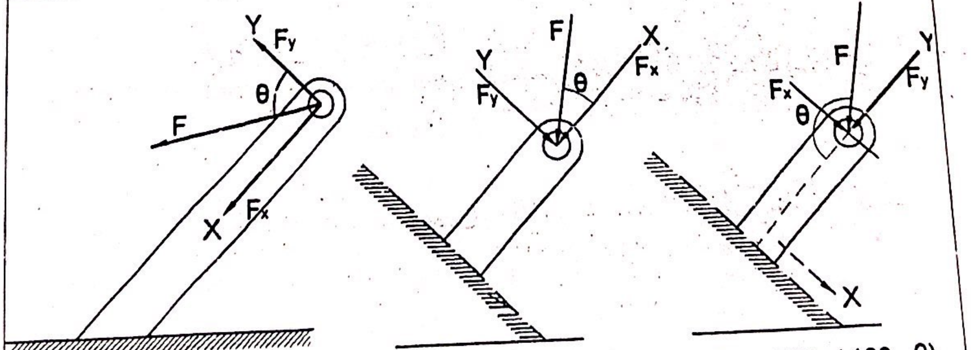
conversion from Rectangular to Polar (R to P)

$$F = \sqrt{F_x^2 + F_y^2}$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

Note:

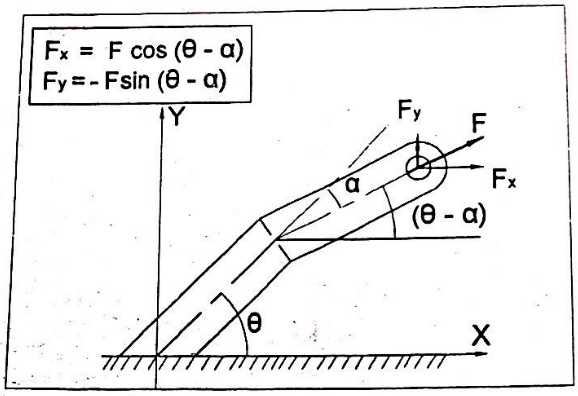
It should be noted that θ is the acute angle made by the line of action of the force with the horizontal. Hence, do not use negative quantities while calculating θ. Use only $|F_x|$ and $|F_y|$ i.e. only numerical values. Choice of reference axes: Actual problem do not come with reference axes; so their assignment is a matter of arbitrary convenience; and the choice is up to the individual.



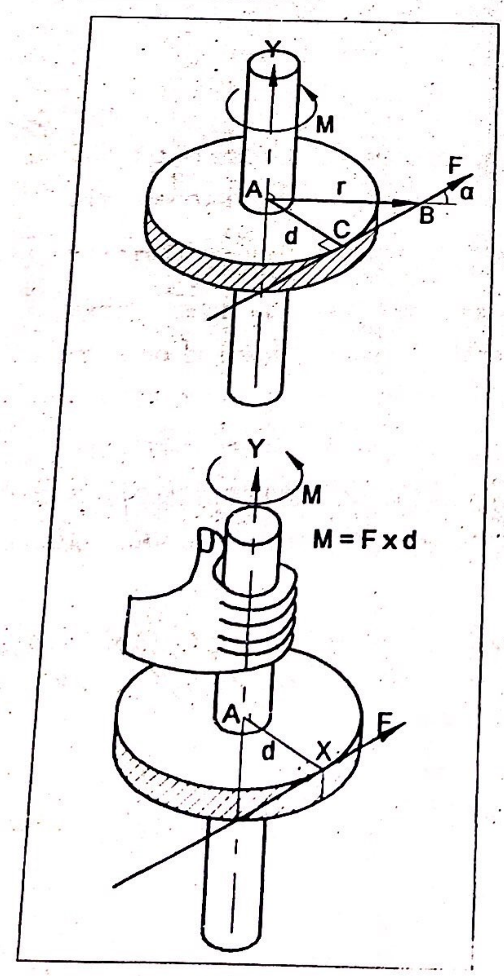
F_x = F sin θ
F_y = Fcos θ

F_x = - F cos θ
F_y = - Fsinθ

F_x = F Sin (180 - θ)
F_y = - Fcos (180 - θ)



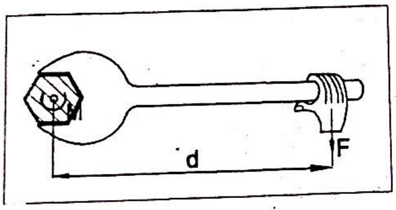
Moment of a Force:



A force acting on a rigid body can produce mainly two effects. These are (i) Tendency of translation of a body in the direction of its application (ii) Tendency of rotation of a body about an axis. The axis may be any line which neither intersects nor is parallel to the line of action of a force.

The rotation producing capacity of a force is called as moment or torque.

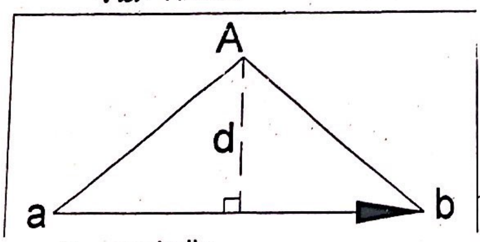
The moment of a force about a point is the product of the magnitude of the force and the perpendicular distance of that point from the line of action of the force. The point about which the rotational effect is produced is called as 'moment center'. The perpendicular distance of the axis of rotation from the line of action of the force is called as 'moment arm'. Moment of a plane force about a point is a vector quantity the direction of the moment vector is always perpendicular to the plane containing the force vector and the moment center. The sense of the moment vector is identified by the right-hand rule. The moment vector is a sliding vector with a line of action coinciding with the moment axis. Moment with anticlockwise sense of rotation are considered as positive and moments with clockwise sense of rotation are considered as negative. Unit of the moment in S.I. units is N-m. we can also use vector approach for calculation of moments. The moment of force $\overline{F}$ about point A may be represented by the cross-product expression, $\overline{M} = \overline{r} \times \overline{F}$



Where, $\overline{r}$ = position vector of ay point on the line of action of a force from the moment center. The magnitude of this cross-product is

$$M=(F\times r\times \sin\alpha)=F\times d$$

$$\therefore d=r.\sin\alpha$$



Geometrically,

$$Area\ \Delta aAb = \frac{1}{2}\times F\times d$$

$$\therefore (F\times d) = 2.A(\Delta aAb)$$

But F x d = M_A = moment of force ‘F’ about point A.

Thus, the magnitude of the moment of a force about a point is twice the area of the triangle formed by the force vector and the moment center.

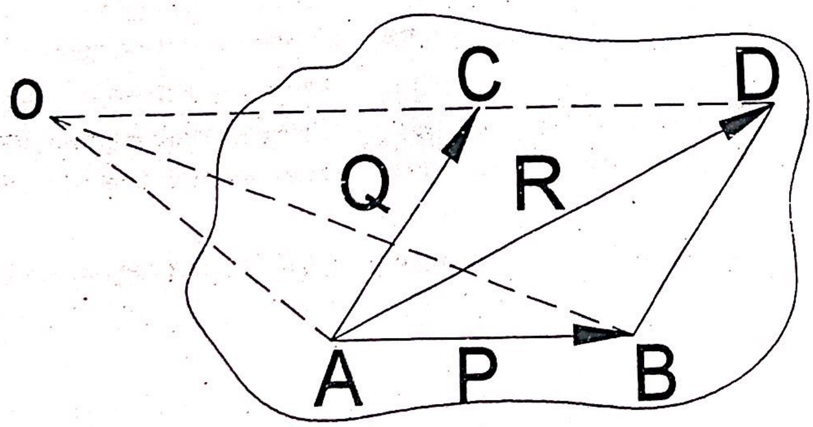
Varignon's Theorem:

This is also called as the principle of moments. It states that the moment of a force about any point is equal to the sum of the moments of the components of the force about the same point.

In this figure, $\overline{R} = \overline{P} + \overline{Q}$

Then force $\overline{R}$ is called as the resultant of forces $\overline{P}$ and $\overline{Q}$. And Forces $\overline{P}$ and $\overline{Q}$ are called as the components of the force $\overline{R}$. Then by the Varignon's theorem of moments, the moment of the resultant force about any point in the plane of the force is equal to the sum of the moments of the individual forces about the same point.

To prove this theorem, let us make use of the geometrical interpretation of the moment of a force about a point.



Moment of force $\overline{R}$ about point O = 2.Area(ΔAOD)

$= 2.Area(\Delta AOD + \Delta ACD)$

But A(ΔACD) = A(ΔABD) = A(ΔAOB)

$\therefore$ Moment of force $\overline{R}$ about point O

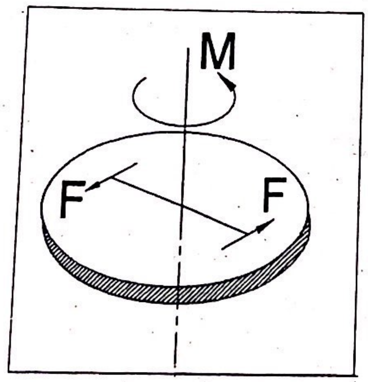
= 2. Area(ΔAOC + ΔAOB)

=2. Area(ΔAOC)+2.Area(ΔAOB)

=moment of forces $\overline{Q}$ about point O + moment of force $\overline{P}$ about point O

Varignon's theorem of moments is useful in locating the position of the point of application of the resultant force, in case of non-concurrent coplanar force systems.

Couples:



Two forces having same magnitude, parallel lines of action but acting in opposite directions are said to form a couple. The perpendicular distance between the lines of actions of the two forces forming the couple is called as the 'moment arm' of the couple.

The two equal and opposite forces with moment arm will not tend to introduce any translator motion on the body. The only effect of a couple acting on the body is the tendency to rotate the body about an axis perpendicular to the plane of the couple. The resultant force of the couple is always zero.

The magnitude of the couple moment is the product of the magnitude of the force and the moment arm. The sense of the couple can be clockwise or anticlockwise. Anti-clockwise couple moments are considered as positive and clockwise couple moments are considered as negative.

The moment of a couple is a constant quantity irrespective of the axis about which it is taken. Thus, moment of a couple is a free vector. There is no specific point of application for the couple. Due to this we can change the position of the couple on the body to any other point on the same body without changing its effect on the body.

Two couples are said to be equivalent, if they have same magnitude & same sense irrespective of the forces forming the couples.

Moments and couples both give turning, twisting or rotational effects. But there is a significant difference between a moment of a force and a couple. The couple is a pure tuning effect which may be moved anywhere in its own plane, without change of its effect on the body. The definition of a moment requires a statement of the reference axis about which this effect occurs whereas couple does not require a reference axis for its definition. Thus couple is a free vector. The magnitude to the moment of a force depends upon the position of the axis about which moment is taken whereas the magnitude of the couple moment is always constant regardless of the axis about which moment is taken.

Force Systems:

In the previous articles we have studied the properties of force, moment and couples. In most of the common engineering problems the bodies are always subjected to group of forces and couples. This is called as a force system. Depending upon their features the force systems are classified as under,

Resultant of a force system:

Most problems in mechanics deal with a system of forces and it is generally necessary to reduce the given force system to its simplest form in describing its action. This is called as composition of forces.

The resultant of the given force system is the simplest force system that can replace the given force system without changing its external effect on the body. Thus the resultant of the given force system implies the net external action on the body. The force acting on a body can have the following actions on the body.

i) translator action ii) rotary action

iii) translator as well as rotary action

iv) continuation of the state of rest or the state of uniform rectilinear motion.

Accordingly the resultant of the given force system can be,

i) a single force or
ii) a single couple or
iii) a force-couple system or
iv) neither a force nor a couple

if the resultant of the given force system is neither a force nor a moment and the value of the resultant force as well as the resultant couple is zero, then we may say that the force system acting on the body is balanced and the body on which it acts Is in balanced and the body on which it acts is in equilibrium. If the resultant of the given force system is not zero then the body will be subjected to linear or angular acceleration.Thus, the determination of the resultant is basic to both statics and dynamics.

For any system of coplanar forces the process of obtaining the resultant can be summarized in equation form as under,

$$\overline{R} = \overline{F_1} + \overline{F_2} + \overline{F_3} + \ldots\ldots\ldots + \overline{F_n} = \sum_1^n \overline{F}$$

$$\overline{R} = R_x\hat{i} + R_y\hat{j}$$

$$R_x = \sum Fx, \quad Ry = \sum Fy, \quad R=\sqrt{(\sum Fx)^2 + (\sum Fy)^2}$$

$$\theta = \tan^{-1}\left(\frac{Ry}{Rx}\right) = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$$

These equations give us the magnitude and direction of the resultant force. To locate the point of application of the resultant force on the body under consideration, we use principle of moment (i.e. Varignon's theorem of moments). For this purpose a convenient reference point O is selected. Now,

Mo= $\sum M = \sum (F.d)$ = sum of the moments of the individual forces about point O.

Let, R.d = the moment of the resultant force about point O.

Then, R.d=Mo ----------(Where, d= perpendicular distance of the line of action of the resultant force from the reference point O.)

If the resultant force R for a given force system is zero, the check for the resultant moment Mo. If the resultant moment Mo is not zero then the given force system is having the resultant as a couple. If the resultant moment Mo is also zeros then the given force system is balanced and the body on which it acts is in equilibrium.

Equilibrium:

Whenever a body is subjected to number of external forces and couples simultaneously and due to the supports or neighboring bodies or constraints the body is subjected to external reactions which are canceling the effects of external forces and couples, then the body is said to be in equilibrium. The total force system acting on the body consisting of external applied forces and external developed reactions is then called as balanced force system. For a balanced force system the magnitude of the resultant force as well as the resultant moment is zero. Due to this whenever a body is in equilibrium it is neither subjected to translation nor subjected to rotation. These are called as physical conditions of equilibrium.

Mathematically, $\overline{R} = \sum \overline{F}$ and $\overline{Mo} = \sum \overline{M} = 0$. These are called as **equations of equilibrium.** These requirements are both necessary and sufficient conditions for equilibrium. The six scalar equations obtained thereby are the analytical conditions of equilibrium or the equations of equilibrium.

For **coplanar force systems,** the above set of equations reduces to three independent equations. These are $\sum Fx = 0$, $\sum Fy = 0$, $\sum Mz = Mo = 0$

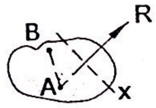
For a complete equilibrium in two dimensions all the above three equations must be satisfied independently i.e. one may hole good without the other.

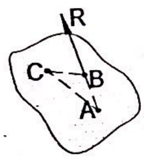
Alternative equilibrium Equations:

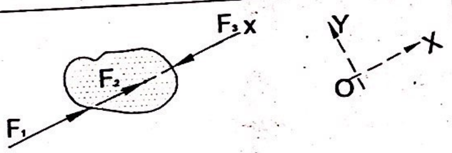
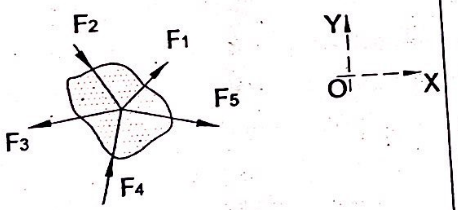
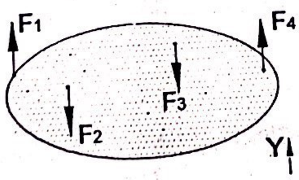
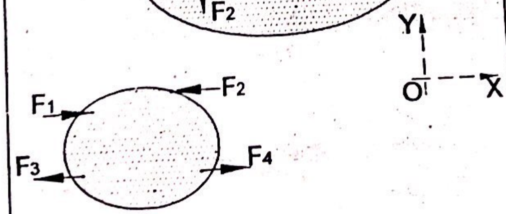
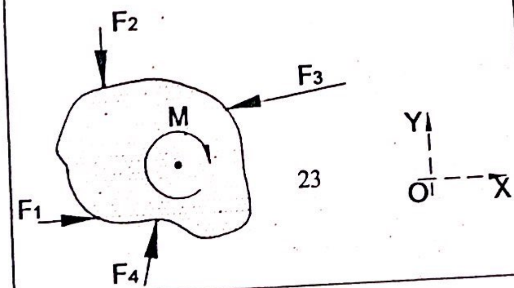
There are two additional ways in which we may express the general conditions for the equilibrium of coplanar forces.

1) $\sum Fx = 0$, $\sum M_A = 0$, $\sum M_B = 0$ ---------(where two points A and B do not lie on a line perpendicular to X-direction.)

2) $\sum M_A = 0, \quad \sum M_B = 0, \quad \sum M_C = 0$ ----------(where three points A, B and C are non –collinear points.)

① 

② 

Force System	Free Body Diagram	Independant equations of equilibrium
Colliner		$\sum F_x = 0$
Concurrent at a point		$\sum F_x = 0$ $\sum F_y = 0$
Parallel		$\sum F_y = 0$ $\sum M_z = 0$
		$\sum F_x = 0$ $\sum M_z = 0$
Non Concurrent Non Parallel i.e. General force System		$\sum F_x = 0$ $\sum F_y = 0$ $\sum M_z = 0$

Free-Body Diagram:

In the analysis of the equilibrium of the bodies, it is necessary to know all the forces acting on the body in all respects. This can be achieved by drawing the 'free-body diagram' of the body under consideration. The free-body diagram is the most important step in solution of problems in mechanics. While drawing the free-body diagram following steps are used-

i) A neat diagram of an isolated body is to be drawn to the scale showing all the important dimensions.

ii) All the external forces acting on the body are to be shown at their respective points of application with their magnitudes as well as directions and sense.

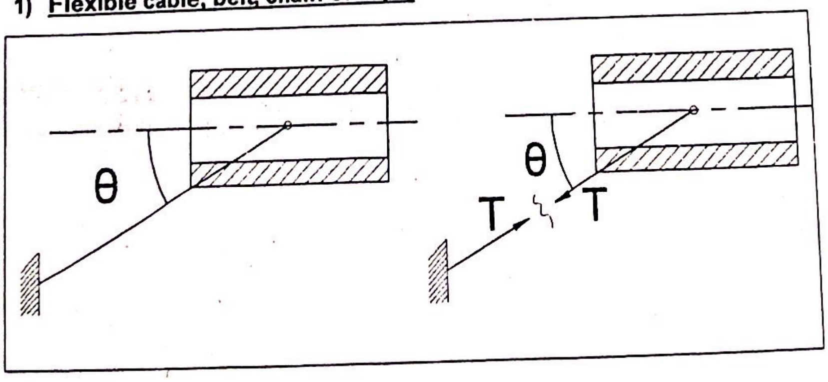
iii) The supports of body or the neighboring bodies or the constraints are to be removed and to be replaced by their appropriate reactions.

iv) The force system formed thereby consisting of all the applied forces and the external reactions is then classified and analyzed using the equations of equilibrium. Thus we develop the analytical model of an isolated mechanical system to which equations of equilibrium are applied.

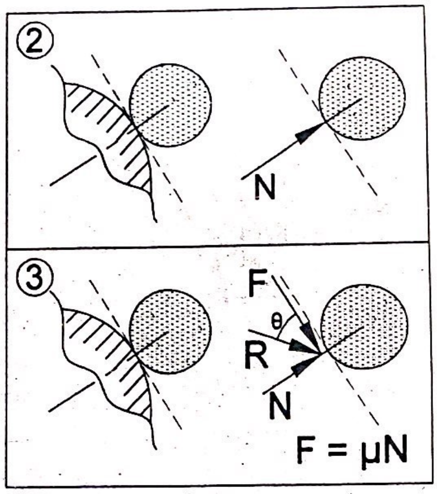
For drawing the free body diagram one must study the various types of supports and the force application on mechanical systems and their corresponding reactions.

Types of supports:

1) **Flexible cable, belt, chain or rope:**



Force exerted by the cable is always the tension away from the body in the direction of the cable. Cables, belts, chains or ropes can not be subjected to compression.



2) Smooth surfaces:

Contact force is compressive and normal to the contact surface.

3) Rough surfaces:

Due to the roughness this surface offers frictional resistance (F) as well as normal reaction N. thus the total reaction R is the resultant of F and N.

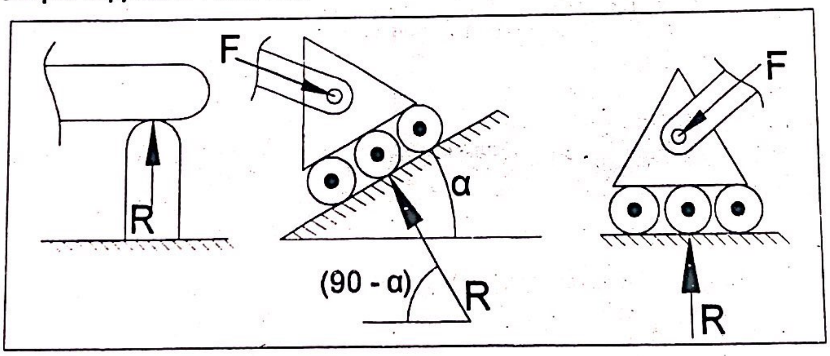
$$\overline{R} = \sqrt{F^2 + N^2}$$

$$\theta = \tan^{-1} \frac{N}{F}$$

$F = \mu N$

where μ = coefficient of friction between the two surfaces

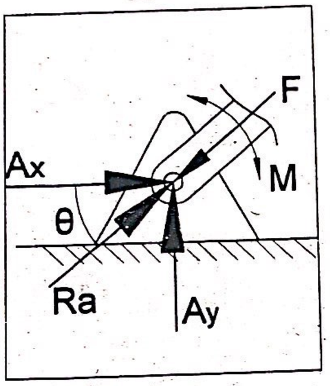
4) Simple support or roller support:



Reactions are provided to prevent motion. In case of simple support or a roller support motion is possible in all the directions except against the surface on which the roller is resting. Because of this the reaction R is compressive and perpendicular to the surface on which the roller is resting.

Number of unknown is one i.e. magnitude of the reaction.

5) Hinged support or simple pin connection:



In this case translation is completely prevented but rotation about the axis of the pin and in the plane of the forces is possible. To prevent translation the connection offers the reaction R_A in any direction. (but only in the plane of the forces) Thus magnitude as well as the direction of the reaction is unknown. Thus there are two unknowns. (R_A , θ_A) while solving problems, only for the sake of convenience we resolve the reaction into its rectangular components (A_x and A_y).

Here, A_x = Horizontal components of the reaction at A.

A_y = Vertical component of the reactions at A.

$$R_A = \sqrt{A_x^{\ 2} + A_y^{\ 2}}$$

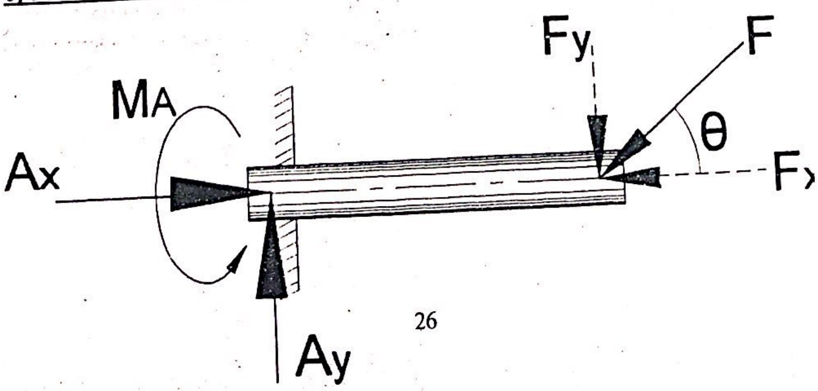
$$\theta_A = \tan^{-1}\left(\frac{Ay}{Ax}\right)$$

conversion from rectangular to polar form

$$Ax = R_A.\cos\theta_A$$
$$Ay = R_A.\sin\theta_A$$

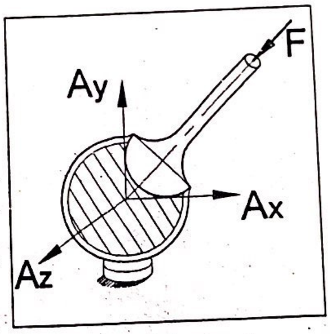
conversion from polar to rectangular form.

<u>6) Fixed support or Built-in support or Encastre:</u>



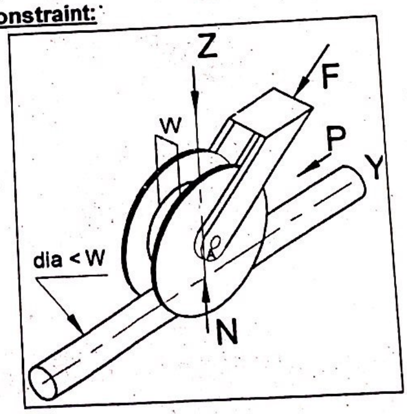
In case of a fixed support, we achieve total fixity by completely. The reaction components 'Ax' and 'Ay' are developed to prevent translation in horizontal and vertical direction. The fixed end moment 'M_A ' is developed on. The fixed end moment 'M_A ' is developed to prevent rotational motion. Thus, there are three unknowns.

7) Ball and Socket Joint:



A ball and socket joint is a hinge in three dimensions. In this connection translator motion in all the directions is prevented by the development of the three reaction components Ax, Ay and Az. But rotational motion about any axis passing through the center of the ball is possible. Thus there are three unknowns.

8) Roller or a wheel supports with lateral constraint:



This is a guided rolled or a wheel. A lateral force 'P' is exerted by the guide on the wheel. And the surface offers the normal reaction 'N'.

Let, P = Ay

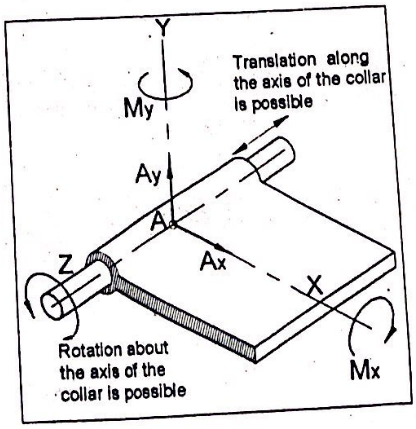
And N = Az

If the motion is possible in X – direction, then reaction 'R_A' can be expressed as,

$$\overline{R_A} = Ay\hat{j} + Az\hat{k}$$

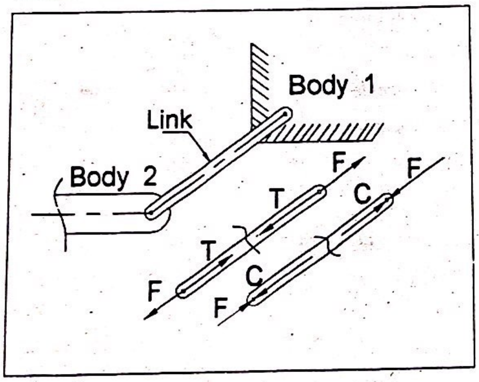
9) Collar joint or Collar Bearing :

$$\overline{R_A} = Ax\hat{j} + Ay\hat{j}$$
$$\overline{M_A} = Mx\hat{j} + My\hat{j}$$



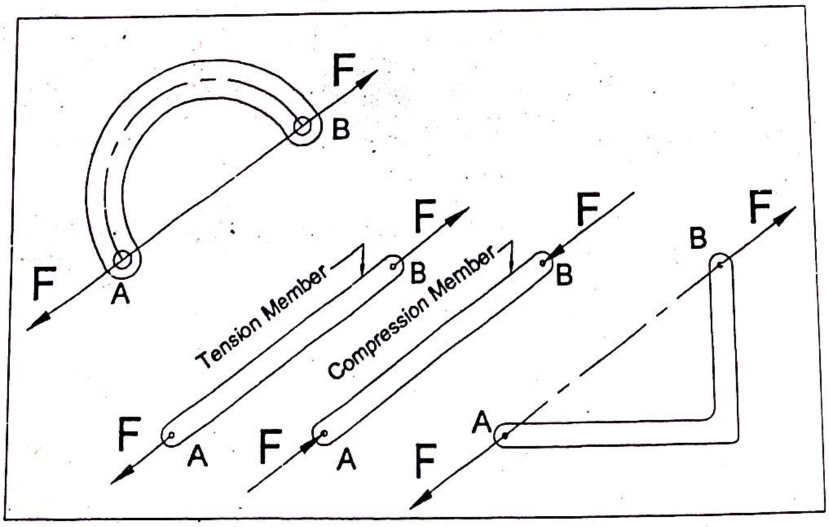
in this type of a joint translation along the axis of the collar and rotation about the axis of the collar is possible. To prevent the translation in other two directions two reaction components are developed
(say Ax and Ay) and to prevent the rotation about the other two directions two moment components are developed (say Mx and My).
Hence total number of unknowns are four.

10) Metalic link:



when two bodies are connected by a metallic link. It can be subjected to axial tension or compression. T = Axial tension in the link. C = axial compression in the link.

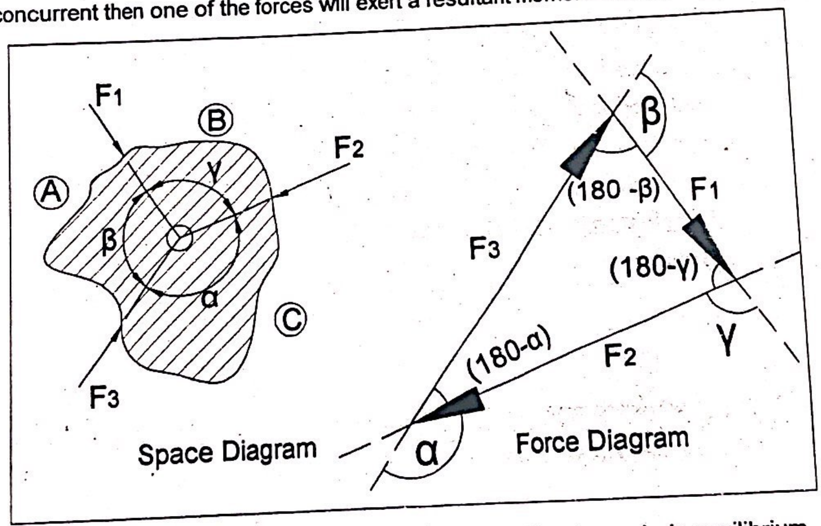
Equilibrium of two forces:



When a body is acted upon by two equal, opposite and collinear forces then it is in equilibrium. The shape of the body does not have any effect on the requirement of the equilibrium. If the body is acted upon by such a force system then it is called as a two force member. If such a body is linear in shape then it is axially loaded body subjected to axial tension or axial compression. Here weights of the member are neglected.

Equilibrium of three forces:

When a body is subjected to three coplanar forces and it is equilibrium then the lines of action of these three forces must be concurrent. If they are not concurrent then one of the forces will exert a resultant moment about the point of



concurrency of the other two and the body will not remain in equilibrium. The only exception occurs when the three forces are parallel. Because of the condition of concurrency when plotted graphically the three non parallel coplanar forces in equilibrium always form a closed triangle as a force polygon.

Applying sine rule to triangle abc, we get

$$\frac{F_1}{\sin(180-\alpha)} = \frac{F_2}{\sin(180-\beta)} = \frac{F_3}{\sin(180-\gamma)}$$

$$\therefore \frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

this is also called as **Lami's theorem**.

From the above discussion we can conclude that, three non-parallel coplanar forces in equilibrium are always concurrent.

As the given forces are equilibrium,

We can write, $\overline{F}_1 + \overline{F}_2 + \overline{F}_3 = 0$

From this we get three conditions, i.e.

$\overline{F}_1 + \overline{F}_2 = -\overline{F}_3$ ---------------------- (i)

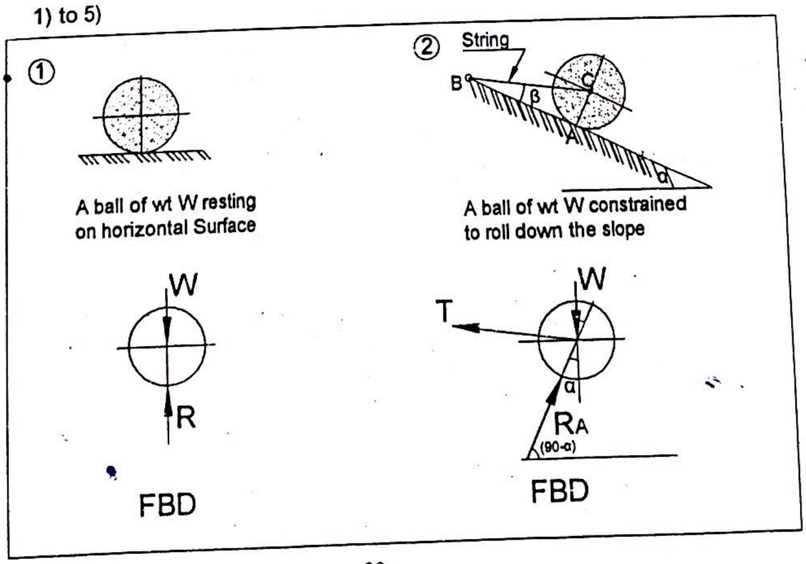
$\overline{F}_2 + \overline{F}_3 = -\overline{F}_1$ ---------------------- (ii)

$\overline{F}_3 + \overline{F}_1 = -\overline{F}_2$ ---------------------- (iii)

the above conditions are such that the resultant of any two forces must be equal, opposite and collinear to the third force. All the above three conditions are simultaneously valid only when all the three forces are concurrent any one point in the plane of the forces.

Sample free Body diagrams :

1) to 5)



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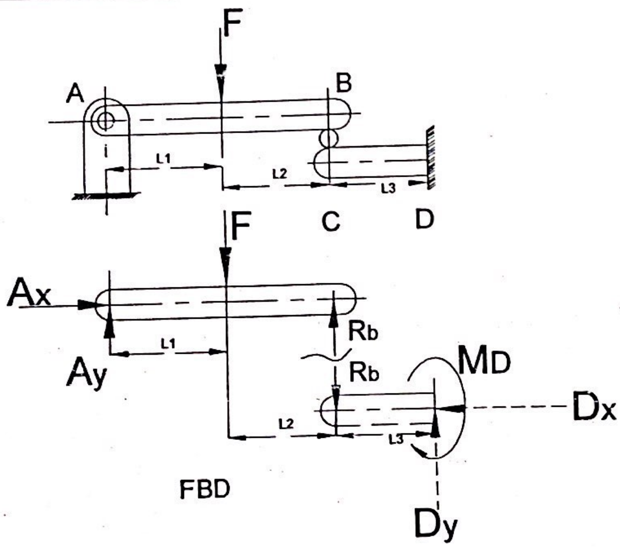

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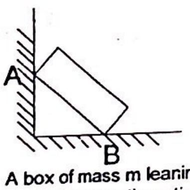
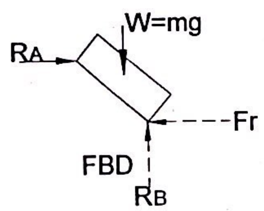


A box of mass m leaning against a smooth vertical wall & resting on a rough horizontal surface



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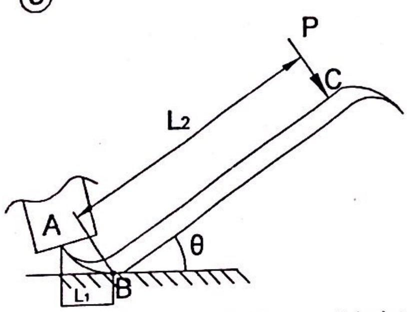

A pry bar lifting a body having smooth horizontal base & is resting on a rough horizontal surface