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Determination Of Coefficient of Restitution Between Two Colliding Bodies

Purpose of the experiment:

To study the Impulse Momentum principle, concept of direct central impact and coefficient of restitution (e). To demonstrate direct central impact and to determine the coefficient of restitution between two bodies by using the concept of collision with a body of infinite mass.

Instruments:

Measuring scale, ball, steel plate, Aluminium plate, wooden cabinet of height 1 m attached with measuring scale.

Theory:

Impulse Momentum Principle:

By Newton's 2nd Law of motion, we have

$$F = m\bar{a} = \frac{d\bar{v}}{dt}$$
$$Fdt = md\bar{v}$$
$$\int_{t_1}^{t_2} Fdt = \int_{u}^{v} md\bar{v}$$
$$\bar{F}(t_2 - t_1) = m(\bar{v} - \bar{u})$$

$$now,$$

Let $t_2 - t_1 = t$, *then we have*

$$\boxed{(\bar{F}t) = m \cdot \bar{v} - m \cdot \bar{u}}$$

'F.t' is called as impulse acting on a particle for time 't'.

Thus, the impulse acting on the particle for time 't' is equal to the change in the linear momentum in that time.

This is called as Impulse Momentum Principle.

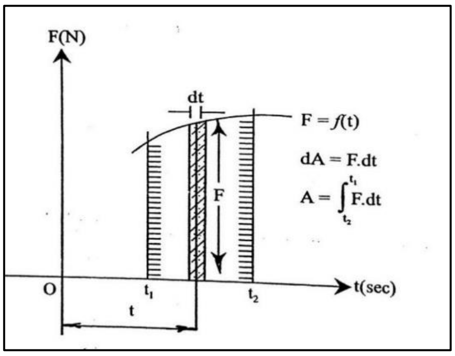

Figure 1. F.t Diagram

Here the force acting on the particle is constant for time ‘t’. If the force acting on the particle is changing and force F = f(t), then the area under the F-t diagram is the impulse acting on the particle for time ‘t’.

Direct Central Impact:

Collision of two bodies in which each body exerts tremendous pressure on the other for a very short interval of time is called as impact. When the mass centres of the colliding bodies are lying on the line of impact and their velocities are collinear to the line of impact then it is called are direct central impact.

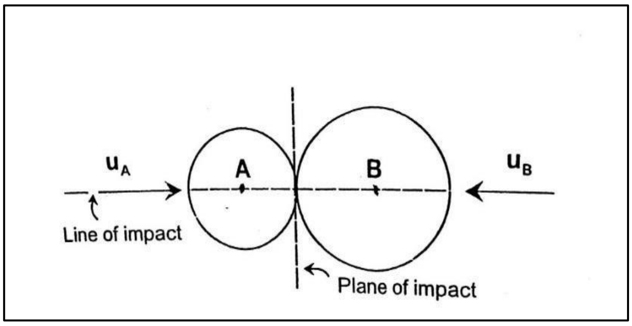

Figure 2. Direct central impact

Impulse of Deformation:

When the two colliding bodies touch each other, initially they have a tendency to push the other body. This stage is called as deformation stage. The impulse acting on the body during deformation stage is called as impulse of deformation.

Impulse of Recovery:

After the deformation stage, the bodies develop the tendency of separating away from each other. This is called as Recovery stage. Recovery may be 100% or 0% or partial. The impulse acting on the body during recovery stage is called as Impulse of recovery.

Time of impact (Δt) = time of deformation (Δtd) + time of recovery (Δtr)

Coefficient of restitution:

The ratio of impulse of recovery to the impulse of deformation is called as coefficient of restitution between the two colliding bodies. It is considered as a constant for given geometries and for a given combination of colliding materials. It also depends on the impact velocity, shape and size of colliding bodies.

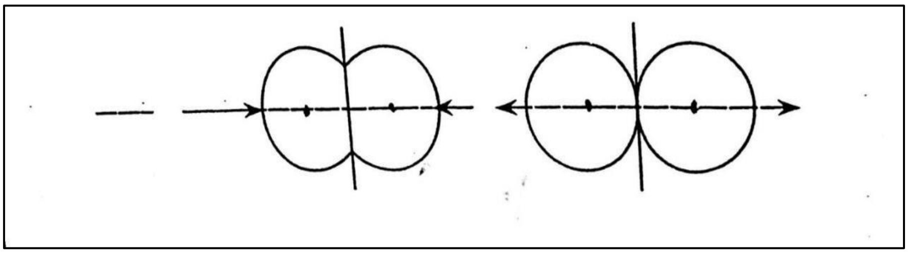

Figure 3. Deformation and recovery stage during impact

Consider two colliding bodies as shown below:

Let

m_1 = *mass of body 1,*

m_2 = *mass of body 2*

u_1 = *Velocity of body 1 before impact,*

u_2 = *Velocity of body 2 before impact*

v_1 = *Velocity of body 1 after impact,*

v_2 = *Velocity of body 2 after impact*

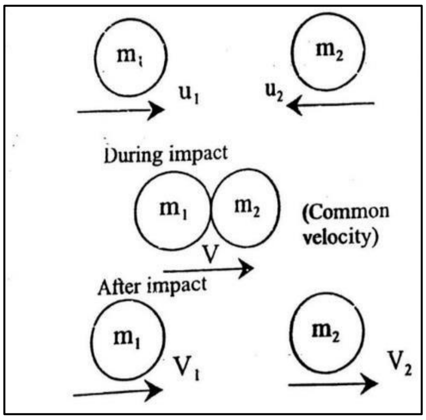

Figure 4. Collision between two bodies

	Impulse of Deformation	Impulse of recovery
Body 1	$\mathrm{m_1 v - m_1 u_1}$	$\mathrm{m_1 v_1 - m_1 v}$
Body 2	$\mathrm{m_2 v - m_2 u_2}$	$\mathrm{m_2\ V_2 - m_2\ V}$

$$e = \frac{m_1 v - m_1 u}{m_1 v - m_1 u_1} = \frac{v_1 - u}{v - u_1} = \frac{m_2 v_2 - m_2 \mathrm{v}}{m_2 v - m_2} = \frac{v_2 - u}{v - v_2}$$

$$e = \frac{v_1 - v_2}{v - v_1} = \frac{v_2 - v}{v - u_2} = \frac{v_1 - v - v_2 + v}{v - u_1 - v + u_2}$$

$$e = \frac{v_1 - v_2}{u_2 - u_1} = \left[\frac{v_1 - v_2}{u_1 - u_2}\right] = \frac{v_{1/2}}{u_{1/2}}$$

$$\boxed{e = \left[\frac{v_1 - v_2}{u_2 - u_1}\right]}$$

$$e = -\left(\frac{\text{relative velocity of 1 wrt 2 after impact}}{\text{relative velocity of 2 wrt 1 before impact}}\right)$$

$$e = -\left(\frac{\text{velocity of separation}}{\text{velocity of approach}}\right)$$

i.e., mass will not have any effect on the coefficient of restitution.

Collision with a body of infinite mass:

Consider the impact between ball and floor.

If a ball (body 1 of finite mass m) be released from height h1.It strikes the floor (body 2 of infinite mass) and rebounds to height h_2 after impact as shown in fig. 5.

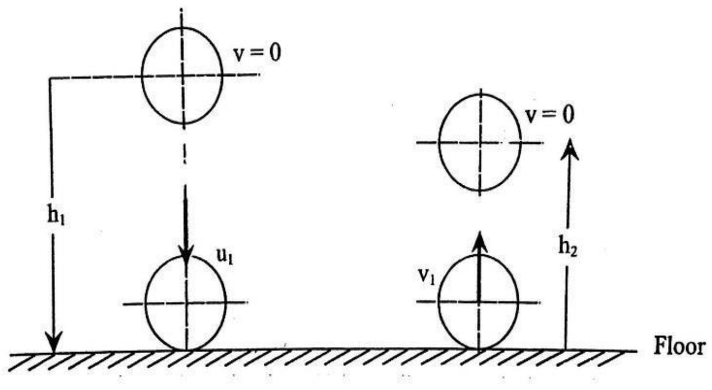

Figure 5: Collision with a body of infinite mass

$u_1 =$ striking velocity $\quad v_1 =$ Rebounding velocity

$u_1 = \sqrt{2g\ h}, (\downarrow) \qquad u_1 = \sqrt{2g\ h_2}(\uparrow)$

For body 2, $u_2 = v_2 = 0$

$$\therefore e = \frac{v_1 - v_2}{u_1 - u_2} = \frac{v_1}{-u_1} = \frac{\sqrt{2gh_2}}{\sqrt{2gh_2}} e = \sqrt{\frac{h_2}{h_1}}$$

Based on coefficient of restitution the phenomenon of impact is classified into three:

1. *Elastic impact*
2. *Semi-elastic impact*
3. *Plastic impact*

The characteristics of these three types of impacts are as under:

Elastic Impact:

1. The two bodies separate after the impact.
2. Coefficient of restitution, e = 1
3. Linear momentum is conserved

$$\boxed{(m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2).}$$

4. Kinetic energy is conserved.

(K.E. of the system before impact) = (K.E. of the system after impact).

$$\boxed{\left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) = \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)}$$

5. Recovery is 100% and the two bodies regain their original shape and size

Semi-elastic Impact:

1. The two bodies separate after the impact.
2. Coefficient of restitution varies between zero and one, $0 < e < 1$.
3. Linear momentum is conserved

$$\boxed{(m_1u_1 + m_2u_2 = m_1u_1 + m_2v_2).}$$

4. Kinetic Energy of system is not conserved.

(K. E. of the system before impact) > (K.E. of the system after impact).

$$\boxed{\left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) > \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)}$$

$$Energy\ lost\ in\ impact = (T_1 - T_2)$$

$$\%\ loss\ in\ energy = \left(\frac{T_1 - T_2}{T_1}\right) \times 100$$

5. The recovery is partial and there is same permanent damage of the bodies.

Plastic Impact:

1. The two bodies do not separate after the impact but they move with a common velocity ' v'.
2. Coefficient of restitution $e = 0$
3. Linear momentum is conserved

$$\boxed{(m_1u_1 + m_2u_2) = (m_1 + m_2)v}$$

4. There is a great loss of Kinetic Energy and it is not conserved.

(K.E. of the system before impact) > (K.E. of the system after impact)

$$\boxed{\left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) > \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)}$$

$$Energy\ lost\ in\ impact = (T_1 - T_2)$$

$$\%\ loss\ in\ energy = \left(\frac{T_1 - T_2}{T_1}\right) \times 100$$

5. The recovery is partial and there is some permanent damage on the colliding bodies.

Procedure: -

1. Place the wooden cabinet of height 1 m on a horizontal plane surface.
2. Release a rubber ball from the central hole at the top of the cabinet i.e., from a height ‘h1’. (h1= 1 m for the lab setup). The ball will fall vertically through 1 m height and strike the base of the cabinet. It will then rebound
3. Measure the height of rebound ‘h2’ from two directions which are perpendicular to each other, with the help of the meter scale attached inside the cabinet on the two perpendicular sides of the cabinet. Take the average of these two readings. This is the final height of rebound ‘h2’.
4. Take three such readings.
5. Calculate the coefficient of restitution (between rubber and wood) using the formula.

$$\boxed{e = \sqrt{\frac{h_2}{h_1}}}$$

6. Place steel plate at the base of the cabinet. Repeat the above procedure and calculate the coefficient of restitution between rubber and steel
7. Now place the Aluminum plate at the base of the cabinet. Repeat the above procedure and calculate the coefficient of restitution between rubber and aluminum.

Observations:

Sr. No.	Materials	Height h1 cm	Height h2 in cm (after impact)				Coefficient of Restitution $e = \sqrt{\frac{h_2}{h_1}}$
			1	2	3	h2 average (cm)	
1.	Rubber and wood	82	42	45	45	44	0.732
2.	Rubber and Steel	82	43	41	43	42.33	0.718
3.	Rubber and Aluminum	82	43	44	44	43.66	0.729

Results: -

Coefficient of restitution	Between Rubber and wood	Between Rubber and steel	Between Rubber and aluminum
e	0.732	0.718	0.729

Calculations:

(4) Coefficient of restitution : (e)

When a body of finite mass coincides with a body of infinite mass, the coefficient of restitution between these 2 bodies is given by :

$$e = \sqrt{\frac{h_2}{h_1}} ,$$

where,
h_1 = hight of drop
h_2 = rebouncing height.

Calculations

(1) For rubber and wood ball :

$$h_2 \text{ avg} = \frac{(42 + 45 + 45)}{3} \text{ cm} = 44 \text{ cm}$$

$$h_1 = 82 \text{ cm} \quad \text{(given)}$$

$$\text{coeff of restitution} = e = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{44}{82}} = 0.732$$

(2) For rubber and steel :

$$h_2 \text{ avg} = \frac{(43 + 41 + 43)}{3} \text{ cm} = 42.33 \text{ cm}$$

$$h_1 = 82 \text{ cm}$$

$$\text{Coeff of restitution} = e = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{42.33}{82}} = 0.718$$

(3) For rubber and aluminium :

$$h_2 \text{ avg} = \frac{(43 + 44 + 44)}{3} \text{ cm} = 43.66 \text{ cm}$$

$$h_1 = 82 \text{ cm} \quad \text{(given)}$$

$$\text{Coeff of restitution} = e = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{43.66}{82}} = 0.729$$

Conclusion:

The Impulse moment principle was studied, and the coefficient of restitution between two bodies by using the concept of collision with a body of infinite mass was calculated to be ***0.732, 0.718, and 0.729*** *respectively for Rubber and wood, Rubber and steel, Rubber and Aluminum respectively.*

Questions: -

1. What is direct central impact?

A. When two bodies apply tremendous forces on one another in the form of collision for an exceptionally short time span, it is referred to as impact.

   Direct impact is seen when the center of gravity/mass of the bodies colliding lies on the line of impact as well as their velocities, hence the impact is experienced along the normal at their point of contact during collision.

2. What is conservation of linear momentum?

A. Conservation of linear momentum states that the momentum of a system remains constant if the net external force acting on the system of bodies remains constant, that is if the system remains isolated, the motion never changes.

3. What is the expression for coefficient of restitution for collision of two bodies of finite masses?

A. Expression is given by:

$$e = \frac{\mathrm{v}_1 - \mathrm{v}_2}{\mathrm{u}_1 - \mathrm{u}_2}$$

   Where e is the coefficient of restitution for two bodies & u1 and u2 are velocities of bodies 1 and 2 before impact while $v1$ and $v2$ are velocities of bodies 1 and 2 after impact.

4. What is the expression for coefficient of restitution for the collision of a body of finite mass with a body of infinite mass?

A. Expression:

$$e = \frac{v_1}{-u_1} = \frac{\sqrt{2gh_2}}{\sqrt{2gh_1}} = \sqrt{\frac{h_2}{h_1}}$$

   This is for coefficient of restitution for the collision of a body of finite mass with a body of infinite mass where he is the coefficient of restitution. Here: h1 is the height from which the body of finite mass is released and $h2$ is the height to which it rebounds after impact to the infinite massed surface/object.

5. What is the difference between elastic, semi elastic and plastic impact?

A. Impacts in colliding objects can be of 3 types:

   Elastic, Semi Elastic, Plastic

   Elastic collisions are usually referred to as ideal collisions too. That's because in elastic impacts, there is no loss of kinetic energy (due to losses caused in the form of heat/sound) that occurs during the collision of bodies. Thus, the deformations caused if any, can be recovered perfectly.

This also implies that the relative velocity of separation after collision is equal to the relative velocity of approach before collision establishing e=1.

On the other hand, in *plastic collisions or impacts*, some portion of the kinetic energy is changed to other forms of energy like heat and sound during the collision. The deformations thus caused are more permanent in nature. Due to this tendency, post collision both objects have same velocity and their relative velocity becomes zero causing e=0.

The intermediate between both an elastic collision and a plastic collision encompasses *semi-elastic collisions*. Here, a little below the perfect 100 percent of the kinetic energy is conserved but it is not as much to force the objects into sticking together post impact. This is the most common one observed in our day to day lives where the relative velocity of separation after collision is less than the relative velocity of approach before collision making e to fall in a range:

$0<e<1$.

[e mentioned above is the ratio of velocity of separation to the velocity of approach is called the coefficient of restitution]