I.C. Tutorial - 3

Q.1. $\left(\frac{y^2}{(y-x)^2} - \frac{1}{x}\right) dx + \left(\frac{1}{y} - \frac{x^2}{(x-y)^2}\right) dy = 0.$

$M \cdot dx + N dy = 0.$

$M = \frac{y^2}{(y-x)^2} - \frac{1}{x}$

$N = \frac{1}{y} - \frac{x^2}{(x-y)^2}$

$\frac{\partial M}{\partial y} = y^2 \cdot (y-x)^{-2}$

$= y^2 \cdot (-2)(y-x)^{-3} + (y-x)^{-2} \cdot 2y$

$= 2y^2 (x-y)^{-3} + 2y (x-y)^{-2}$

$\frac{\partial N}{\partial x} = \frac{1}{y} - \frac{x^2}{(x-y)^2} = x^2 \cdot (x-y)^{-2}$

$= -\left(x^2 \cdot (-2)(x-y)^{-3} + (x-y)^{-2} \cdot 2x\right)$

$= 2x^2 (x-y)^{-3} - 2x (x-y)^{-2}$

So $\frac{\partial M}{\partial y} = \frac{2y^2}{(x-y)^3} + \frac{2y}{(x-y)^2}$

$= \frac{2y^2 + (2y)(x-y)}{(x-y)^3}.$

$$\frac{\partial M}{\partial y} = \frac{2y^2 + 2xy - 2y}{(x-y)^3} = \frac{2xy}{(x-y)^3}$$

$$\frac{\partial N}{\partial x} = \frac{2x^2}{(x-y)^3} - \frac{2x}{(x-y)^2}$$

$$= \frac{2x^2 - 2x(x-y)}{(x-y)^3}$$

$$= \frac{2xy}{(x-y)^3}$$

So it is an exact dE.

So soln:

$$\int_{y\ \text{const}} M\,dx + \int N\,dy \quad (\text{only term free of } x) = c$$

$$\int \left(\frac{y^2}{(y-x)^2} - \frac{1}{x}\right) dx + \int \frac{1}{y}\,dy = c$$

$$\frac{y^2}{(y-x)} \cdot (-1) - \log x + \log y = \log c$$

$$= \log xyc = \log c$$

$$\frac{-y^2}{(y-x)} + \log(y/x) = c$$

$$= \frac{-y^2}{(y-x)} - \log(y/x) = c$$

nbow

Q.2. $[y^2 e^{xy^2} - 4x^3]dx + [2xy \cdot e^{xy^2} - 3y^2]dy = 0$

Here $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = y^2(2xy) \cdot e^{xy^2} + 2ye^{xy^2}$

$\therefore \int M \cdot dx + \int N\, dy = c$

$\int [y^2 e^{xy^2} + 4x^3]dx + \int -3y^2 \cdot dy = c$

$\frac{2e^{xy^2}}{x^2} + x4 - y^3 = c$

Q.3 $\frac{dy}{dx} = \frac{y+1}{(y+2)e^y - x}$

$(y+2)e^y - x = y+1 \cdot \frac{dx}{dy}$

$(y+1)\frac{dx}{dy} + x = (y+2)e^y$

$\frac{dx}{dy} + \frac{x}{y+1} = \frac{(y+2)e^y}{(y+1)}$

This is of the form $\frac{dx}{dy} + Px = Q$.

$P = \frac{1}{y+1}$, $Q = \frac{(y+2)}{y+1} e^y$

Thus $IF = e^{\int P dy} = e^{\int \frac{1}{y+1} dy}$

$= e^{\log y+1}$

$IF = y+1$

Rainbow

$$x(y+1) = \int \frac{y+2}{(y+1)} e^y (y+1)\, dy$$

$$x(y+1) = \int y \cdot e^y + 2e^y \, dy$$

$$x(y+1) = y e^y - e^y + 2e^y + c$$

$$= y e^y + e^y + c$$

$$= (y-1) e^y + c$$

$$x(y+1) = e^y (y+1) + c$$

$$\boxed{x = \frac{e^y + c}{y+1}}$$