Krishnaraj P.T.
109054. (I3)

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LADC Tutorial 7

Q.1 $u = \frac{yz}{x} \Rightarrow \frac{\partial u}{\partial x} = \frac{-yz}{x^2}$

$\frac{\partial u}{\partial y} = \frac{z}{x}$; $\frac{\partial u}{\partial z} = \frac{-y}{x}$

$v = \frac{zx}{y} \Rightarrow \frac{\partial v}{\partial x}$; $\frac{\partial v}{\partial y} = \frac{-zx}{yz}$; $\frac{\partial v}{\partial z} = \frac{x}{y}$

$w = \frac{yx}{z} = \frac{\partial w}{\partial x} = \frac{y}{z}$; $\frac{\partial w}{\partial y} = \frac{x}{z}$; $\frac{\partial w}{\partial z} = \frac{-xy}{z^2}$

$$J\frac{(u, v, w)}{x, y, z} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$$

$$= \begin{vmatrix} \frac{-yz}{x^2} & \frac{z}{x} & y/x \\ z/y & \frac{-zx}{y^2} & x/y \\ y/z & x/z & \frac{-yx}{z^2} \end{vmatrix}$$

$$= \frac{-yz}{x^2}\left[\left(\frac{-zx}{yz}\right)\left(\frac{-xy}{z^2}\right) - \frac{x^2}{yz}\right].$$

$$= \frac{-z}{x}\left[\frac{z}{y}\left(\frac{-xy}{z^2}\right) - \frac{xy}{yz}\right]$$

$$= \frac{y^2}{x^2}\left[\frac{x^2}{yz} - \frac{x^2}{y^3}\right] - \frac{z}{x}\left[\frac{-x}{z} - \frac{x}{z}\right]$$

$$+ \, y/x \left[x/y + x/y \right]$$

$$\therefore \; J\left(\frac{u, v, w}{x, y, z}\right) = 2 + 2 = 4.$$

Q.2 $f = x^2 + y^2 + u^2 - v^2 = 0$;

$uv + xy = 0$. PT $\frac{\partial(u, v)}{\partial(x, y)} = \frac{x^2 - y^2}{u^2 + v^2}$

$$\frac{\partial(u,v)}{\partial(x,y)} = \frac{(-1)^2 \left[\frac{\partial(f,g)}{\partial(x,y)}\right]}{\frac{\partial(f,g)}{\partial(u,v)}}$$

$$\therefore \frac{\partial(f,g)}{\partial(x,y)} = \begin{vmatrix} 2x & 2y \\ x & y \end{vmatrix} = 2x^2 - 2y^2$$

$$\therefore \frac{\partial(f,g)}{\partial(u,v)} = \begin{vmatrix} 2u & -2v \\ 2v & u \end{vmatrix} = 2u^2 + 2v^2$$

$$\therefore \partial = \frac{(1)(2)(x^2 - y^2)}{(2)(u^2 + u^2)} = \frac{x^2 - y^2}{u^2 + v^2}$$

Hence Proved.

Q.3. $x + y + z = u$

$x = u - (y+z)$

$u - uv = u(1-v)$

$y = uv - z = uv - uvw$

$z = uvw$,

Now, $$\frac{\partial(x,y,z)}{\partial(u,v,w)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix}$$

$$= \begin{vmatrix} 1-v & -u & 0 \\ v-vw & u-uw & -uv \\ vw & uw & uv \end{vmatrix} \quad R_2 \leftrightarrow R_2 + R_3$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ v & u & 0 \\ vw & wu & uv \end{vmatrix} \quad R_1 \to R_1 + R_2$$

$\Rightarrow 1(u(uv) - 0)$

$= u^2 v$

$$\therefore \frac{(x,y,z)}{\partial(u,v,w)} = \underline{\underline{u^2 v}}$$

Q.4. $u = 2x - y + 3z$

$v = 2x - y - z$

$w = 2x - y + z$

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$$

$$= \begin{vmatrix} 2 & -2 & 1 \\ 2 & -1 & -1 \\ 2 & -1 & 1 \end{vmatrix} = 2(-1-1) + 1(2+2)$$

$$= -4 + 4$$

$$= 0$$

$\therefore$ v, u, w are dependant

Q.5 $\underline{2.1533}$

Q.6.

$$\frac{1}{v} - \frac{1}{u} = \frac{2}{f}$$

$$\therefore \frac{\Delta v}{v^2} + \frac{\Delta u}{u^2} = \frac{2\Delta f}{f^2}$$

$\therefore$ equal errors of measuring u & v

ie $\Delta u = \Delta v = a$

$$\therefore a\left(\frac{1}{u} - \frac{1}{v}\right)\left(\frac{1}{u} + \frac{1}{v}\right) = \frac{-2\Delta f}{f^2}$$

But $\frac{1}{v} - \frac{1}{u} = \frac{2}{f}$

$\therefore \quad a\left(\frac{1}{u} + \frac{1}{v}\right) = \frac{14f}{f}$

(Q.7) To find max value of

$-x^4 + 2x^2 + y^4 - 2y^2$

$\Rightarrow \quad -(x+y^2)(x-y)(x^2+y^2-2)$

$-(x^2-1)^2 + (y^2-1)^2$

$\Rightarrow$ Minimum value of this function can attain is $-\infty$ at $y = 1$, or -1 & $x = -\infty$ or ∞

$\Rightarrow$ Maximum value of this function can attain is ∞ at $x = 1$, or -1.

& $y = -\infty$ or ∞.

(Q.8) The distance of point (x, y) from origin on given curves is $(\sqrt{x^2 - y^2})$

$3x^2 + 4xy + 6y^2 = 140$

$\therefore \quad 3x^2 + 4xy + 6y^2 - 140 = 0.$

$f(x,y) = x^2 + y^2 + \lambda(3x^2 + 4xy + 6y^2 - 140)$

$\frac{\delta f(x,y)}{\delta x} = 2x + \lambda(6x + 4y) = 0$

$= 2x + 6\lambda x + 4\lambda y = 0 \quad -(1)$

$$\frac{\delta f(x,y)}{\delta(y)} = 2y + \lambda(4x + 12y)$$

$$= 2y + 4\lambda x + 12\lambda y = 0 \quad \text{— (2)}$$

Multiplying (1) and (2) with y and x respectively and subtracting (2) from (1),

$$\therefore -6\lambda xy + 4\lambda y^2 - 4\lambda x^2 = 0$$

$$\therefore -2\lambda(3xy - 2y^2 - 2x^2) = 0$$

$$\therefore (3xy - 2y^2 + 2x^2) = 0$$

$$\therefore (x + 2y)(2x - y) = 0$$

$$\therefore \boxed{x = -2y} \text{ — (3)} \quad \& \quad \boxed{x = y/2} \text{ — (4)}$$

Substituting $x = -2y$ in $3x^2 + 4xy + 6y^2 - 140 = 0$.

we get, $y = \pm\sqrt{14}$

$\therefore x = 2\sqrt{14}$ when $y = -\sqrt{14}$

$x = -2\sqrt{14}$ when $y = +\sqrt{14}$.

$\therefore$ Distance of $(2\sqrt{14}, -\sqrt{14})$ from origin, $(2\sqrt{14}, \sqrt{14})$ will be

$$\sqrt{(-2\sqrt{14})^2 + (\sqrt{14})^2} = \sqrt{70} \text{ units.}$$

Now,

Sub $\Rightarrow$ $x = \frac{y}{2}$ in $3x^2 + 4xy + 6y^2 - 140 = 0$.

We get ; $y = \pm 4$.

$\therefore$ $x = 2 \Rightarrow y = 4$

$x = -2 \Rightarrow y = -4$.

The distance of $(2,4)$ from origin, the same with $(-2,-4)$ is $\sqrt{2^2+4^2}$

$= \sqrt{20}$ units.

$\Rightarrow$ Min distance $\Rightarrow$ $\sqrt{20}$ units $(2,4)$ & $(-2,4)$

Max distance $\Rightarrow$ $\sqrt{70}$ units $(2\sqrt{14}, -\sqrt{14})$

& $(-2\sqrt{14}, \sqrt{14})$