Session 37:

TRACING OF PARAMETRIC CURVES

The following rules will help in tracing a Parametric curve$\left(\boldsymbol{x = f(t), y = g(t)}\right)$.

Rule 1: ***<u>Limitations of the curve:</u>***

If possible find the greatest and least values of x & y for a proper value of t.

Rule 2: **<u>Symmetry:</u>**

***(a) Symmetry about X-axis*:** If 'x' is even and 'y' is odd w. r. t 't'
i.e. $f(-t) = f(t)$ & $g(-t) = -g(t)$ then the curve is symmetric about X-axis.

***(b) Symmetry about Y-axis*:**

1. If 'x' is odd and 'y' is even w. r. t 't'
i.e. $f(-t) = -f(t)$ & $g(-t) = g(t)$ then the curve is symmetric about Y-axis.

2. For trigonometric functions if 'x' is odd and 'y' is even w. r. t '$\pi - t$'
i.e. $f(\pi - t) = -f(t)$ & $g(\pi - t) = g(t)$ then the curve is symmetric about Y-axis.

***Symmetry in opposite quadrants*:** . If 'x' and 'y' both are odd w. r. t 't'
i.e. $f(-t) = -f(t)$ & $g(-t) = -g(t)$ then the curve is symmetric about opposite quadrants.

Rule 3: **<u>Points of intersections:</u>**

It will pass through the origin if on putting t = 0 we obtain x = 0 and y = 0 . Also find the points of intersection of the curve and the axes.

Rule 4: **<u>Nature of tangents:</u>**

1) $\dfrac{dy}{dx} = \dfrac{dy}{dt} \Big/ \dfrac{dx}{dt}$
2) Form the table of values of $x, y, \frac{dy}{dx}$ for different values of 't'.

Rule 5: **<u>Asymptotes and region:</u>**

1) Find asymptotes if any.
2) Find region of absence.

Q1. Trace the following curve:

$$x = a\cos^3 t,\ \ y = a\sin^3 t \quad \text{or} \quad x^{2/3} + y^{2/3} = a^{2/3}$$

Solution: We have to trace the curve

$$x = a\cos^3 t,\ \ y = a\sin^3 t \ \text{ or } \ x^{2/3} + y^{2/3} = a^{2/3} \quad \text{---------} \quad (1)$$

We check the following points for tracing of the above curve

1. **Limit:-** $-a \le x \le +a$ and $-a \le y \le +a$
2. **Symmetry:-**
   (i) About X- axis:-
   Since 'x' is even function and y is odd.
   $\therefore$ The curve is symmetry about x-axis.

   (ii) About Y- axis:-
   If we replace t by $\pi - t$, then 'x' is odd function and 'y' is even function w.r.t. $\pi - t$.
   Hence the curve is symmetry about y-axis.
   (iii) About Opposite Quadrant:-
   Since the curve is symmetry about both the axes.
   $\therefore$ It is symmetry about opposite quadrant.
3. **Origin:-**
   For $t = 0,\ x = a$ and $y = 0$.

   Hence the curve does not pass through the origin.

4. **Tangent:-**

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{d}{dt}\left(\mathrm{a}\sin^3 t\right)}{\frac{d}{dt}\left(\mathrm{a}\cos^3 t\right)} = \frac{\cancel{3}\,\cancel{a}\,\sin^{\cancel{2}} t\,\cancel{\cos t}}{\cancel{3}\,\cancel{a}\,\cos^{\cancel{2}} t\left(-\cancel{\sin t}\right)} = -\tan t\,.$$

5. **Asymptotes:-**No asymptotes.
6. **Table values:-**

t	0	$\pi/2$
x	a	0
y	0	a
$dy/dx = \tan\phi$	0 *i.e.* $\phi = 0$	$-\infty$ *i.e.* $\phi = -\pi/2$

It is clear that at $t = 0$, x-axis is tangent. Also when ‘t’ increases from ‘0’ to $\pi/2$ the value of x decreases from ‘a’ to 0 and the value of y increases from ‘0’ to a. Hence we get the curve in first quadrant. Since the curve is symmetry about Y-axis and X-axis. Hence the approximate shape of the curve is as follows: