Sol. Refer to Fig. 2.21.

Let A, B and C be three pegs.

Let R_A, R_B and R_C be the reactions at A, B and C and T be the tension.

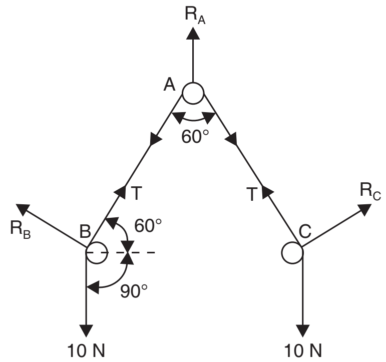


Fig. 2.21

$$T = 10 \text{ N}$$

We know that $R^2 = P^2 + Q^2 + 2PQ \cos \theta$

$$\therefore \quad R_B^2 = (10)^2 + (10)^2 + 2 \times 10 \times 10 \cos (90° + 60°)$$

$$= 100 + 100 - 200 \sin 60°$$

$$= 200 - 200 \times \frac{\sqrt{3}}{2}$$

$$= 200 \left(1 - \frac{\sqrt{3}}{2}\right) = 200\,(1 - 0.866) = 26.8$$

$$\boldsymbol{R_B} = \sqrt{26.8} = \mathbf{5.17\ N.\ (Ans.)}$$

Hence $R_B = R_C = 5.17$ N

Again, $R_A^2 = (10)^2 + (10)^2 + 2 \times 10 \times 10 \times \cos 60°$

$$= 100 + 100 + 200 \times \frac{1}{2} = 300$$

$$\boldsymbol{R_A} = \sqrt{300}$$

$$= \mathbf{17.32\ N.\ (Ans.)}$$

Example 2.8. *Find the components of a force of 150 N into two directions inclined at angle of 45° and 30° with the force.*

Sol. Refer to Fig. 2.22.

$$P = 150 \text{ N}, \angle \alpha = 45°, \angle \beta = 30°$$

$$P_1 = P \,.\, \frac{\sin \beta}{\sin (\alpha + \beta)} = 150 \times \frac{\sin 30°}{\sin (45° + 30°)}$$

$$= 150 \times \frac{0.5}{0.966}$$

$$= \mathbf{77.63\ N.\ (Ans.)}$$

$$P_2 = P \,.\, \frac{\sin \alpha}{\sin (\alpha + \beta)}$$

$$= 150 \, \frac{\sin 45°}{\sin (45° + 30°)} = \frac{150 \times 0.707}{0.966}$$

$$= \mathbf{109.78\ N.\ (Ans.)}$$

Example 2.9. *A particle is acted upon by the following forces :*

(*i*) *A pull of 8 N due North East ;*

(*ii*) *A pull of 10 N due North ;*

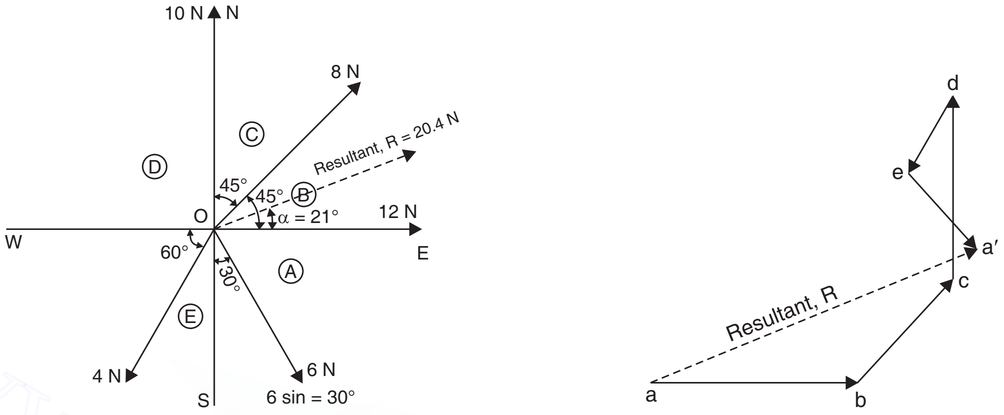
(*iii*) *A pull of 12 N due East ;*

(*iv*) *A pull of 4 N in a direction inclined 60° South of West ;*

(*v*) *A pull of 6 N in a direction inclined 30° East of South.*

Find graphically the magnitude and direction of the resultant force.

Sol. Draw space diagram as in Fig. 2.22 (*a*) showing relative positions of the lines of action of the various forces acting on point *O*.



(a) Space diagram (b) Vector diagram

Fig. 2.22

Draw vectors *ab, bc, cd, de, ea* to represent to some scale, the forces 12 N, 8 N, 10 N, 4 N and 6 N respectively [Fig. 2.22 (*b*)].

Join *aa′* which represents the resultant in magnitude and direction

$$\boldsymbol{R = 20.4\ \text{N}}$$

$$\alpha = \mathbf{21°}.$$

So the resultant is a **20.4 N pull acting at 21° North of East. (Ans.)**

Example 2.10. *Determine analytically the magnitude and direction of the resultant of the following four forces acting at a point :*

(*i*) *10 N pull N 30° E ;*

(*ii*) *12.5 N push S 45° W ;*

(*iii*) *5 N push N 60° W ;*

(*iv*) *15 N push S 60° E.*

Sol. The various forces acting at a point are shown in Fig. 2.23.

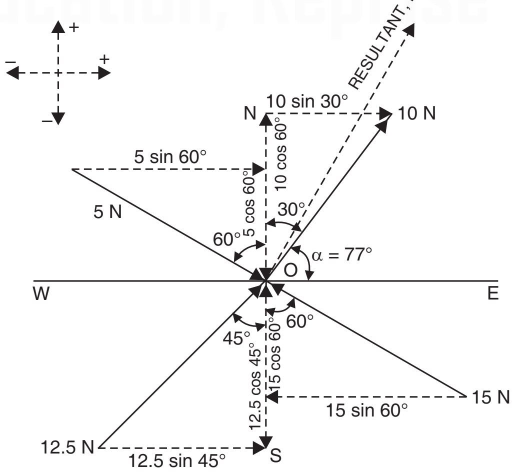


Fig. 2.23

Resolving the forces horizontally, we get

$$\Sigma H = 10 \sin 30° + 5 \sin 60° + 12.5 \sin 45° - 15 \sin 60°$$
$$= 10 \times 0.5 + 5 \times 0.866 + 12.5 \times 0.707 - 15 \times 0.866$$
$$= 5 + 4.33 + 8.84 - 12.99$$
$$= 5.18 \text{ N.}$$

Similarly, resolving forces vertically, we get

$$\Sigma V = 10 \cos 30° - 5 \cos 60° + 12.5 \cos 45° + 15 \cos 60°$$
$$= 10 \times 0.866 - 5 \times 0.5 + 12.5 \times 0.707 + 15 \times 0.5$$
$$= 8.66 - 2.5 + 8.84 + 7.5 = 22.5 \text{ N.}$$

$\therefore$ Resultant,

$$\boldsymbol{R} = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$
$$= \sqrt{(5.18)^2 + (22.5)^2}$$
$$= \sqrt{26.83 + 506.25}$$
$$= \mathbf{23.09 \text{ N. (Ans.)}}$$

$$\tan \alpha = \frac{\Sigma V}{\Sigma H} = \frac{22.5}{5.18} = 4.34$$

$\therefore$ $\alpha = \mathbf{77°.}$ **(Ans.)**

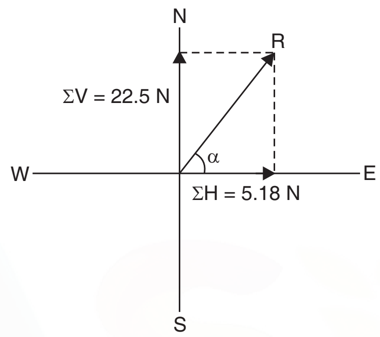


Fig. 2.24

Example 2.11. *The following forces (all pull) act at a point :*

(*i*) *25 N due North ;* (*ii*) *10 N North-East ;*

(*iii*) *15 N due East ;* (*iv*) *20 N 30° East of South* ;

(*v*) *30 N 60° South of West.*

Find the resultant force. What angle does it make with East ?

Sol. The various forces acting at a point are shown in Fig. 2.25.

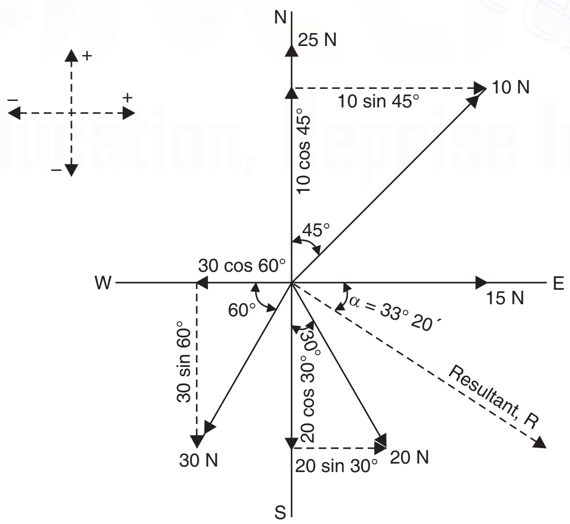


Fig. 2.25

Resolving the forces horizontally, we get

$$\Sigma H = 10 \sin 45° - 30 \cos 60° + 20 \sin 30° + 15$$
$$= 10 \times 0.707 - 30 \times 0.5 + 20 \times 0.5 + 15$$
$$= 7.07 - 15 + 10 + 15$$
$$= 17.07 \text{ N.}$$

Similarly, resolving forces *vertically*, we get

$$\Sigma V = 10 \cos 45° + 25 - 30 \sin 60° - 20 \cos 30°$$
$$= 10 \times 0.707 + 25 - 30 \times 0.866 - 20 \times 0.866$$
$$= 7.07 + 25 - 25.98 - 17.32$$
$$= -\ 11.23 \text{ N.}$$

Resultant, $$\boldsymbol{R} = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$
$$= \sqrt{(17.07)^2 + (-\ 11.23)^2}$$
$$= \sqrt{291.38 + 126.11}$$
$$= \mathbf{20.43\ N\ .\ (Ans.)}$$

$$\tan \alpha = \frac{\Sigma V}{\Sigma H} = \frac{11.23}{17.07} = 0.6578$$

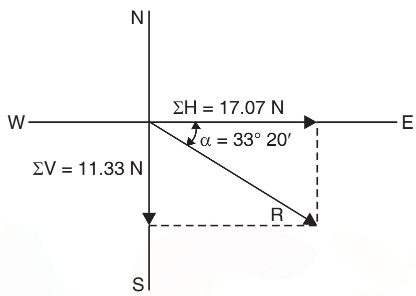


Fig. 2.26

or α = **33° 20′ South of East. (Ans.)**

Example 2.12. *ABCDEF is a regular hexagon. Forces of magnitudes 2, $4\sqrt{3}$, 8, $2\sqrt{3}$ and 4 N act at A in the directions of AB, AC, AD, AE and AF respectively. Determine the resultant completely.*

Sol. Refer to Fig. 2.27.

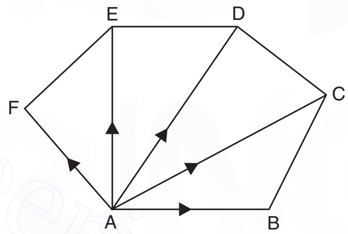


Fig. 2.27

In *ABCDEF* regular hexagon *AE* and *AB* are perpendicular to each other.

Let us resolve the forces along *AB* and *AE*.

Forces along *AB*

$$= 2 + 4\sqrt{3} \cos 30° + 8 \cos 60° + 4 \cos 120°$$
$$= 2 + 6 + 4 - 2 = 10 \text{ N}$$

Forces along $$AE = 2\sqrt{3} + 4\sqrt{3} \sin 30° + 8 \sin 60° + 4 \sin 120°$$
$$= 2\sqrt{3} + 2\sqrt{3} + 4\sqrt{3} + 2\sqrt{3} = 10\sqrt{3} \text{ N.}$$

Now, 10 N act along *AB* and $10\sqrt{3}$ N act along *AE*, and their resultant force

$$= \sqrt{(10)^2 + (10\sqrt{3})^2} = \sqrt{100 + 300}$$
$$= \mathbf{20\ N.\ (Ans.)}$$

Let α be the angle between the resultant force and the horizontal direction *AB*.

Then, $$\tan \alpha = \frac{10\sqrt{3}}{10} = \sqrt{3}\ ;\ \tan \alpha = \tan 60°$$

$\therefore$ α = **60°. (Ans.)**

Similarly, resolving the forces vertically, we get

$$\Sigma V = 700 \sin 72° + 600 \sin 36° - P_1 \sin 36° - P_2 \sin 72°$$

But $\Sigma V = 0$, since the *spokes are in equilibrium* (vertically)

$$\therefore \quad 700 \sin 72° + 600 \sin 36° - P_1 \sin 36° - P_2 \sin 72° = 0$$

or $$700 \times 0.951 + 600 \times 0.588 - P_1 \times 0.588 - P_2 \times 0.951 = 0$$

or $$665.7 + 352.8 - 0.588\, P_1 - 0.951\, P_2 = 0$$

or $$0.588\, P_1 + 0.951\, P_2 = 1018.5$$

or $$P_1 + 1.62\, P_2 = 1732.1 \quad ...(ii)$$

(Dividing both sides by 0.588)

Subtracting (*i*) from (*ii*), we get

$$2.0\, P_2 = 1446.7$$

$$\boldsymbol{P_2 = 723.3 \text{ N. (Ans.)}}$$

Putting the value of P_2 in (*i*), we get

$$P_1 - 0.38 \times 723.3 = 285.4$$

or $$\boldsymbol{P_1 = 560.2 \text{ N. (Ans.)}}$$

Example 2.14. *Three forces keep a particle in equilibrium. One acts towards east, another towards north-west and the third towards south. If the first be 5 N, find the other two.*

Sol. Refer to Fig. 2.31.

Let force P act towards north-west and Q towards south. On applying Lami's theorem, we get

$$\frac{P}{\sin 90°} = \frac{Q}{\sin 135°} = \frac{5}{\sin 135°}$$

$$P = \frac{5 \sin 90°}{\sin 135°}$$

$$= \boldsymbol{5\sqrt{2} \text{ N. (Ans.)}}$$

$$Q = \frac{5 \sin 135°}{\sin 135°}$$

$$= \boldsymbol{5 \text{ N. (Ans.)}}$$

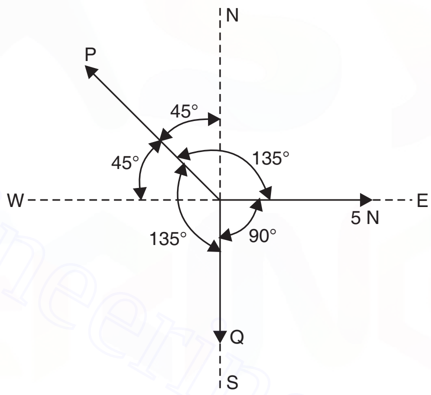


Fig. 2.31

Example 2.15. *A machine weighing 1500 N is supported by two chains attached to some point on the machine. One of these ropes goes to the eye bolts in the wall and is inclined 30° to the horizontal and other goes to the hook in ceiling and is inclined at 45° to the horizontal. Find the tensions in the two chains.*

Sol. The machine is in equilibrium under the following forces :

(*i*) W (weight of the machine) acting vertically down ;

(*ii*) Tension T_1 in the chain OA ;

(*iii*) Tension T_2 in the chain OB.

Now, applying Lami's theorem at O, we get

$$\frac{T_1}{\sin (90° + 45°)} = \frac{T_2}{\sin (90° + 30°)} = \frac{W}{\sin 105°}$$

$$\frac{T_1}{\sin 135°} = \frac{T_2}{\sin 120°} = \frac{1500}{\sin 105°}$$

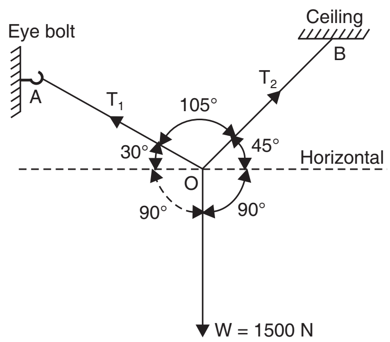


Fig. 2.32

$$\therefore \qquad T_1 = \frac{1500 \sin 135°}{\sin 105°} = \frac{1500 \times 0.707}{0.965}$$

$$= \mathbf{1098.96\ N.\ (Ans.)}$$

$$\text{and} \qquad T_2 = \frac{1500 \times \sin 120°}{\sin 105°} = \frac{1500 \times 0.866}{0.965}$$

$$= \mathbf{1346.11\ N.\ (Ans.)}$$

Example 2.16. *Fig. 2.33 represents a weight of 20 kN supported by two cords, one 3 m long and the other 4 m long with points of support 5 m apart. Find the tensions T_1 and T_2 in kN in the cords.*

Sol. Refer to Fig. 2.33.

In ΔABC, $\angle C$ in a right angle because $AB^2 = AC^2 + BC^2$

i.e., $\left[(5)^2 = (3)^2 + (4)^2 \right.$

or $\left. 25 = 9 + 16 = 25 \right]$

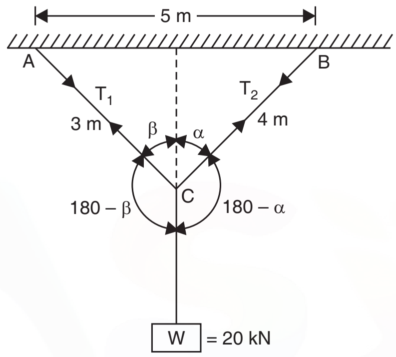


Fig. 2.33

Also by sine rule :

$$\frac{3}{\sin \angle B} = \frac{4}{\sin \angle A} = \frac{5}{\sin \angle C}$$

$$\frac{3}{\sin \angle B} = \frac{4}{\sin \angle A}$$

$$= \frac{5}{\sin 90°}$$

$\therefore$ $\sin \angle B = 3/5 = 0.6$

or $\angle B = 36° \ 52'$

and $\sin \angle A = 4/5 = 0.8$

or $\angle A = 53° \ 8'$

Now, $\angle\alpha = 90 - \angle B = 90 - 36° \ 52'$ or $53° \ 8'$

and $\angle\beta = 90° - \angle A = 90 - 53° \ 8'$ or $36° \ 52'$

Using Lami's theorem at C, we get

$$\frac{T_1}{\sin (180 - \alpha)} = \frac{T_2}{\sin (180 - \beta)} = \frac{W}{\sin (\alpha + \beta)}$$

$$\frac{T_1}{\sin \alpha} = \frac{T_2}{\sin \beta} = \frac{W}{\sin (\alpha + B)}$$

$$\frac{T_1}{\sin 53° 8'} = \frac{T_2}{\sin 36° 52'} = \frac{20}{\sin 90°}$$

$$\therefore \qquad T_1 = \frac{20}{\sin 90°} \times \sin 53° \ 8'$$

$$= \mathbf{16\ kN.\ (Ans.)}$$

$$\text{and} \qquad T_2 = \frac{20}{\sin 90°} \times \sin 36° \ 52'$$

$$= \mathbf{12\ kN.\ (Ans.)}$$

Example 2.17. *A smooth sphere of weight 'W' is supported in contact with a smooth vertical wall by a string fastened to a point on its surface, the end being attached to a point on the wall. If the length of the string is equal to the radius of sphere, find tensions in the string and reaction on the wall.*

Sol. Refer to Fig. 2.34.

The sphere is in equilibrium under the action of following forces :

(*i*) Self weight '*W*' acting vertically downwards through the centre of sphere.

(*ii*) Tension *T* in the string *AC*.

(*iii*) Reaction 'R_B' of the wall at the point of contact *B* of the sphere, acting perpendicular to surface of wall as shown because the wall is smooth.

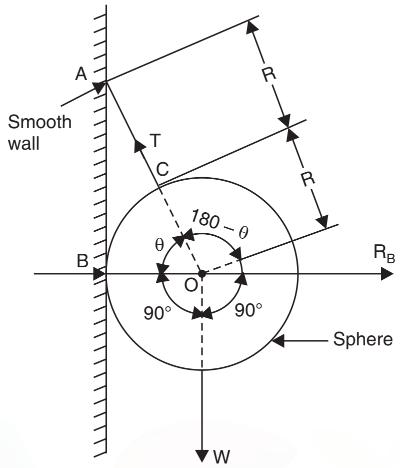


Fig. 2.34

Now, applying Lami's theorem at *O*, we get

$$\frac{T}{\sin 90°} = \frac{R_B}{\sin (90° + \theta)}$$

$$= \frac{W}{\sin (180° - \theta)}$$

$$\left(\because \text{ In } \Delta AOB, \cos\theta = \frac{R}{2R} = \frac{1}{2} \text{ or } \theta = 60° \right)$$

or $$\frac{T}{1} = \frac{R_B}{\sin (90° + 60°)} = \frac{W}{\sin (180° - 60°)}$$

or $$\frac{T}{1} = \frac{R_B}{\sin 150°} = \frac{W}{\sin 120°}$$

$\therefore$ $$T = \frac{W}{\sin 120°} = \frac{W}{\sqrt{3}/2} = \frac{\mathbf{2}}{\sqrt{\mathbf{3}}}\ \mathbf{W.\ (Ans.)}$$

and $$R_B = \frac{W \sin 150°}{\sin 120°} = \frac{W \times 1/2}{\sqrt{3}/2} = \mathbf{W/\sqrt{3}\ .\ (Ans.)}$$

Example 2.18. *A string is tied to two point at the same level and a smooth ring of weight W, which can slide freely along the string, is pulled by horizontal force P. If, in the position of equilibrium the portions of the string are inclined at 60° and 30° to the vertical. Find the value of P and the tension in the string.*

Sol. Refer to Fig. 2.35.

Let *C* be the position of the ring. Considering horizontal equilibrium,

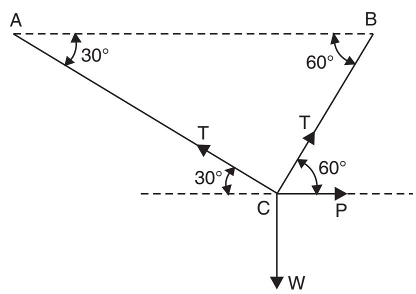


Fig. 2.35

$$T \cos 30° = T \cos 60° + P$$

$$\frac{\sqrt{3}}{2} T = \frac{T}{2} + P$$

$$P = \frac{\sqrt{3} - 1}{2} T \quad \ldots(i)$$

Considering vertical equilibrium

$$T \sin 30° + T \sin 60° = W$$

$$\frac{T}{2} + \frac{\sqrt{3}}{2}\ T = W$$

$$\frac{(1+\sqrt{3})}{2}\ T = W$$

or $$\boldsymbol{T = \frac{2}{\sqrt{3}+1}\ W.} \quad \textbf{(Ans.)} \qquad ...(ii)$$

Substituting the value of T from Eqn. (*ii*) in eqn. (*i*), we get

$$\boldsymbol{P} = \frac{\sqrt{3}-1}{2} \times \frac{2}{\sqrt{3}+1}\ W$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}\ W$$

$$= \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}\ W = \frac{4-2\sqrt{3}}{3-1}\ W$$

$$= \boldsymbol{(2-\sqrt{3})\ W.} \quad \textbf{(Ans.)}$$

Example 2.19. *A body of weight 20 N is suspended by two strings 5 m and 12 m long and other ends being fastened to the extremities of a rod of length 13 m. If the rod be so held that the body hangs immediately below its middle point, find out the tensions in the strings.*

Sol. Refer to Fig. 2.36.

The body is in equilibrium under the action of following forces :

(*i*) W (weight of the rod) acting vertically downwards.

(*ii*) Tension 'T_1' in the string OA.

(*iii*) Tension 'T_2' in the string OB.

Now, in $\Delta\, AOB$, AB = 13 m,

OA = 5 m, OB = 12 m,

$\angle ABO = \theta$

and $\angle AOB = 90°$ $\quad (\because \quad AB^2 = OA^2 + OB^2)$

then $$\sin\theta = \frac{OA}{AB} = \frac{5}{13}$$

and $$\cos\theta = \frac{OB}{AB} = \frac{12}{13}$$

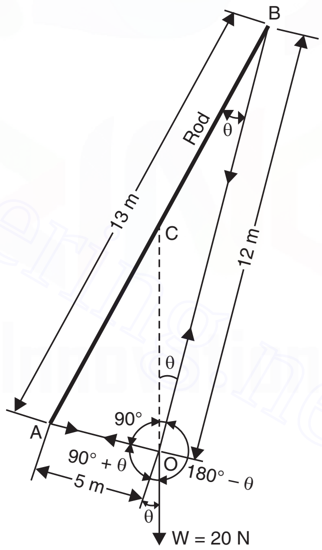


Fig. 2.36

Applying Lami's theorem, at 'O', we have

$$\frac{T_1}{\sin(180° - \theta)} = \frac{T_2}{\sin(90° + \theta)} = \frac{W}{\sin 90°}$$

or $$\frac{T_1}{\sin\theta} = \frac{T_2}{\cos\theta} = \frac{20}{1}$$

or $$\frac{T_1}{5/13} = \frac{T_2}{12/13} = \frac{20}{1}$$

$\therefore$ $$\boldsymbol{T_1} = \frac{20 \times 5}{13} = \textbf{7.69 N.} \quad \textbf{(Ans.)}$$

and $$\boldsymbol{T_2} = \frac{20 \times 12}{13} = \textbf{18.46 N.} \quad \textbf{(Ans.)}$$

Example 2.20. *What axial forces does the vertical load W = 800 N induce in the tie rod and the jib of the jib crane shown in Fig. 2.37 ? Neglect the self-weight of the members.*

Sol. Let P_1 and P_2 be the forces induced in the tie rod and jib respectively. The *tie rod will be under tension* and *jib will be compression* as shown in Fig. 2.37.

Let us now consider equilibrium of the point C and apply Lami's theorem to it.

$$\frac{P_1}{\sin(180° - 30°)} = \frac{P_2}{\sin(30° + 15°)} = \frac{W}{\sin(180° - 15°)}$$

or

$$\frac{P_1}{\sin 30°} = \frac{P_2}{\sin 45°} = \frac{800}{\sin 15°}$$

$\therefore$

$$\boldsymbol{P_1} = \frac{800 \sin 30°}{\sin 15°} = \frac{800 \times 0.5}{0.2588}$$

$$= \mathbf{1545.6\ N.\ (Ans.)}$$

and

$$\boldsymbol{P_2} = \frac{800 \sin 45°}{\sin 15°} = \frac{800 \times 0.707}{0.2588}$$

$$= \mathbf{2185.5\ N.\ (Ans.)}$$

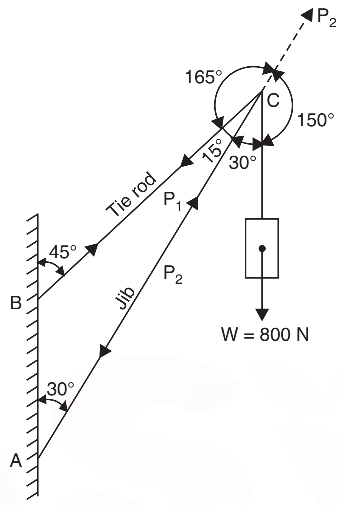


Fig. 2.37

Example 2.21. *A string ABCD hangs from fixed point A and D carrying a weight of 12 N at B and W at C. AB is inclined at 60° to the horizontal, CD is inclined at 30° to the horizontal and BC is horizontal, find W.*

Sol. Refer to Fig. 2.38.

Let T be the tension in BC.

By Lami's theorem,

$$\frac{T}{\sin 150°} = \frac{12}{\sin 120°}$$

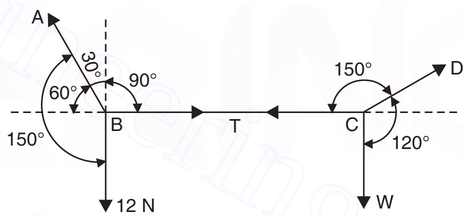


Fig. 2.38

$$T = \frac{12 \sin 150°}{\sin 120°} = \frac{12 \times \frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{12}{\sqrt{3}}$$

Again by Lami's theorem,

$$\frac{W}{\sin 150°} = \frac{T}{\sin 120°}$$

$$\frac{W}{\sin 150°} = \frac{12}{\sqrt{3} \times \frac{\sqrt{3}}{2}}$$

$$\frac{W}{\frac{1}{2}} = \frac{24}{3}$$

$$\boldsymbol{W} = \mathbf{4\ N.\ (Ans.)}$$

Example 2.22. *The extremities A and D of a light inextensible string ABCD are tied to two points in the same horizontal line. Weights W and 3 W are tied to the string at the points B and C respectively. If AB and CD are inclined to the vertical at angles 60° and 30° respectively, show that BC is horizontal and find the tensions in the various parts of the string.*

Sol. Refer to Fig. 2.39.

Let BC makes an angle θ with the vertical and let T_1, T_2 and T_3 are tensions in the three parts AB, BC and CD of the string as shown in Fig. 2.39. At B, the forces in equilibrium are W, T_1 and T_2.

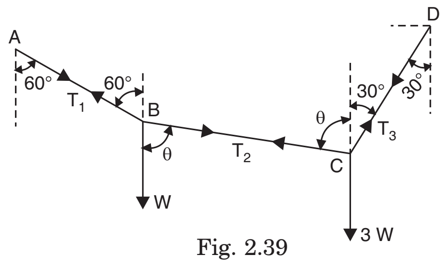


Fig. 2.39

Applying Lami's theorem, we get

$$\frac{T_1}{\sin\theta} = \frac{T_2}{\sin 120^\circ} = \frac{W}{\sin(240^\circ - \theta)} \qquad ...(i)$$

Similarly, applying Lami's theorem to the point C,

$$\frac{T_2}{\sin 150^\circ} = \frac{T_3}{\sin(180^\circ - \theta)} = \frac{3W}{\sin(30^\circ + \theta)} \qquad ...(ii)$$

From eqns. (*i*) and (*ii*),

$$T_2 = \frac{W \sin 120^\circ}{\sin(240^\circ - \theta)} = \frac{3W \sin 150^\circ}{\sin(30^\circ + \theta)}$$

or
$$\frac{\sin 60^\circ}{-\sin(60^\circ - \theta)} = \frac{3 \sin 30^\circ}{\sin(30^\circ + \theta)}$$

or
$$\frac{\sqrt{3}}{2}\sin(30^\circ + \theta) = -3 \times \frac{1}{2}\sin(60^\circ - \theta)$$

or
$$\sin 30^\circ \cos\theta + \cos 30^\circ \sin\theta = -\sqrt{3}\,(\sin 60^\circ \cos\theta - \cos 60^\circ \sin\theta)$$

or
$$\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta = -\frac{3}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta$$

or
$$2\cos\theta = 0$$

or
$$\theta = 90^\circ.$$

So BC makes 90° with the vertical *i.e.*, BC is horizontal.

Putting this value of θ in eqns. (*i*) and (*ii*), we get

$$\boldsymbol{T_1} = \frac{W \sin 90^\circ}{W \sin 150^\circ} = \frac{W}{\frac{1}{2}} = \mathbf{2\ W.\ (Ans.)}$$

$$\boldsymbol{T_2} = \frac{W \sin 120^\circ}{\sin 150^\circ} = \frac{\sqrt{3}/2\, W}{\frac{1}{2}} = \boldsymbol{\sqrt{3}}\ \mathbf{W.\ (Ans.)}$$

$$\boldsymbol{T_3} = \frac{3W \sin 90^\circ}{\sin 120^\circ} = \frac{3W}{\sqrt{3}/2} = \mathbf{2}\boldsymbol{\sqrt{3}}\ \mathbf{W.\ (Ans.)}$$

Example 2.25. *A cast iron sphere 30 cm in diameter rests in 20 cm × 20 cm angle, one leg of which is at an angle of 30° with the horizontal as shown in Fig. 2.42. Assuming all surfaces smooth, compute the reactions on the spheres at A and B. Cast iron weighs 72 kN/m³.*

Sol. Refer to Fig. 2.42.

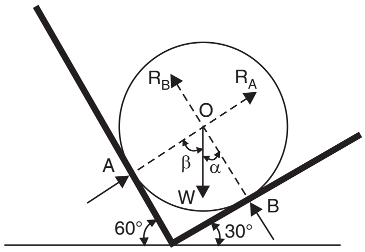


Fig. 2.42

Weight of sphere = volume of sphere × density of cast iron.

$= 4/3\ \pi r^3 \times$ density of cast iron

$= 4/3\ \pi \times (0.15)^3 \times (72 \times 1000)$

$= 1017.9$ N

Since, the surfaces are smooth, so the reactions at A and B will be normal at the points of contact of sphere and the angle. *If the system is in equilibrium the line of action of the weight must pass through O, the point of intersection of the lines of action of the two reactions R_A and R_B.*

Now, $\angle\alpha = 30°$, $\angle\beta = 60°$

Applying Lami's theorem, we get

$$\frac{R_A}{\sin(180° - \alpha)} = \frac{R_B}{\sin(180° - \beta)} = \frac{W}{\sin 90°}$$

$$\frac{R_A}{\sin\alpha} = \frac{R_B}{\sin\beta} = \frac{1017.9}{1}$$

$$\frac{R_A}{\sin 30°} = \frac{R_B}{\sin 60°} = \frac{1017.9}{1}$$

∴ $\boldsymbol{R_A} = 1017.9 \times \sin 30° =$ **508.9 N. (Ans.)**

$\boldsymbol{R_B} = 1017.9 \times \sin 60° =$ **881.5 N. (Ans.)**

Example 2.26. *A uniform wheel 40 cm in diameter rests against a rigid rectangular block 10 cm thick as shown in Fig. 2.43. Find the least pull through the centre of the wheel to just turn it over the corner of the block. All surfaces are smooth. Find also the reaction of the block. The wheel weighs 800 N.*

Sol. Refer to Fig. 2.43.

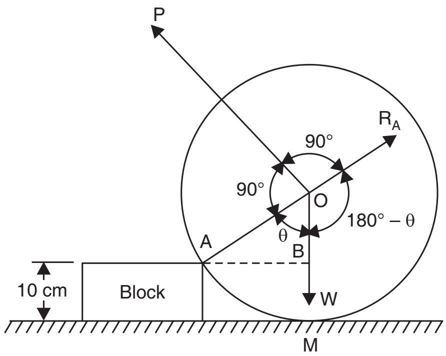


Fig. 2.43

Consider the equilibrium of the wheel when it is just about to turnover the block. In this position, the following forces shall act on the wheel :

(*i*) The pull P ;

(*ii*) Weight of the wheel = 800 N ;

(*iii*) Reaction, R_A.

In this position, there will be no contact between the wheel and the floor, so $R_M = 0$.

If the pull is to be *minimum* it must be applied normal to AO.

Now, applying Lami's theorem, we get

$$\frac{P}{\sin(180° - \theta)} = \frac{R_A}{\sin(90° + \theta)} = \frac{W}{\sin 90°}$$

or

$$\frac{P}{\sin\theta} = \frac{R_A}{\cos\theta} = \frac{800}{1}$$

$$\therefore \quad P = 800 \sin\theta$$

and

$$R_A = 800 \cos\theta$$

But from right-angled $\Delta\, AOE$,

$$\cos\theta = \frac{OB}{OA} = \frac{10}{20} = \frac{1}{2}$$

$$\therefore \quad \theta = 60°$$

$$\therefore \quad P = 800 \times \sin 60°$$

$$= \mathbf{692.8\ N.\ (Ans.)}$$

and

$$\boldsymbol{R_A} = 800 \cos 60°$$

$$= \mathbf{400\ N.\ (Ans.)}$$

Example 2.27. *The cylinders shown in Fig. 2.44 have the same diameter but the cylinder '1' weighs 200 N and cylinder '2' weighs 150 N. Find the reactions at the supports.*

Sol. Refer to Fig. 2.44.

The following forces keep the cylinder '2' *in equilibrium*

(*i*) Weight of cylinder, W_2 (= 150 N) acting vertically downward ;

(*ii*) Reaction (R_s) acting at right angle to OQ ; and

(*iii*) Reaction of cylinder '1' (R_N) in the direction O_1O_2 (action and reaction are equal and opposite).

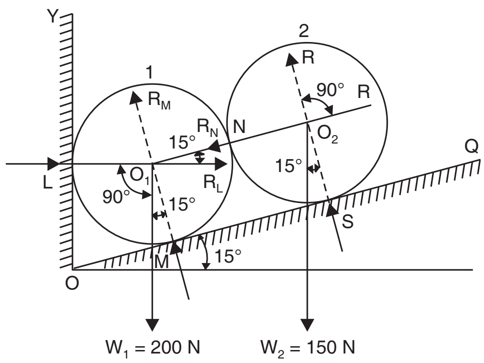


Fig. 2.44

Applying Lami's theorem :

$$\frac{R_N}{\sin(180° - 15°)} = \frac{R_S}{\sin(90° + 15°)} = \frac{W_2}{\sin 90°}$$

or
$$\frac{R_N}{\sin 15°} = \frac{R_S}{\sin 105°} = \frac{150}{1}$$

$\therefore$ $R_N = 150 \sin 15° = 38.82$ N

and $R_S = 150 \sin 105° = 144.89$ N

The forces which keep the cylinder '1' in equilibrium are :

(*i*) Weight of the cylinder W_1 (= 200 N) acting vertically down ;

(*ii*) Reaction R_L acting at right angle to OY ;

(*iii*) Reaction R_M acting at right angle to OQ ;

(*iv*) R_N, the pressure of the cylinder '2' acting in the direction O_2O_1.

Since in this case, number of forces acting are four, so we *cannot apply Lami's theorem here which is applicable in a case where there are only three forces.* The unknown in this case can be determined by resolving the forces along O_1O_2 and in a direction *perpendicular to* O_1O_2.

$\therefore$ $R_L \cos 15° - W_1 \sin 15° - R_N = 0$ ...(*i*)

and $R_M - R_L \sin 15° - W_1 \cos 15° = 0$ ...(*ii*)

From eqn. (*i*),

$R_L \times 0.9659 - 200 \times 0.2588 - 38.82 = 0$

or $R_L \times 0.9659 - 51.76 - 38.82$

or $\boldsymbol{R_L}$ **= 93.78 N. (Ans.)**

Putting the value of R_L in eqn. (*ii*), we get

$R_M - 93.78 \times 0.2588 - 200 \times 0.9659 = 0$

$R_M - 24.27 - 193.18 = 0$

or $\boldsymbol{R_M}$ **= 217.45 N. (Ans.)**

Example 2.28. *Two cylinders '1' and '2' rest in a horizontal channel as shown in Fig. 2.45. The cylinder '1' has a weight of 500 N and radius of 180 mm. The cylinder '2' has a weight of 200 N and a radius of 100 mm. The channel is 360 mm wide at the bottom with one side vertical. The other side is inclined at an angle 60° with the horizontal. Find the reactions.*

Sol. Refer to Figs. 2.45 and 2.46.

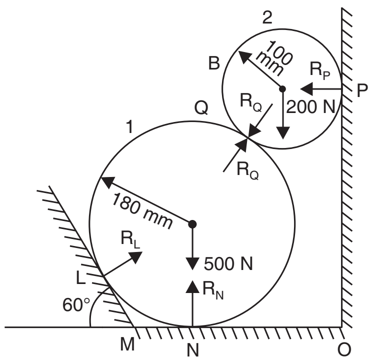


Fig. 2.45

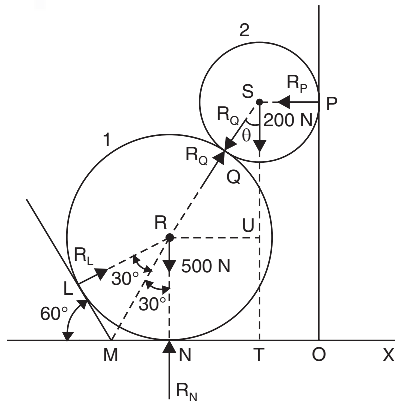


Fig. 2.46

MECHANICS

MODULE - 1 - FORCE SYSTEMS

(*) Practice questions

Q. 28. Find the components of a force of 150 N into 2 directions inclined at an angle of 45° and 30° with the force.

$$\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$$

(using Lami's Theorem)

→ $F = 150\,N$

$\angle\alpha = 45°$, $\angle\beta = 30°$

$$F_1 = \frac{F \cdot \sin\beta}{\sin(\alpha+\beta)} = 150 \cdot \frac{0.5}{0.9} = 77.6\,N$$

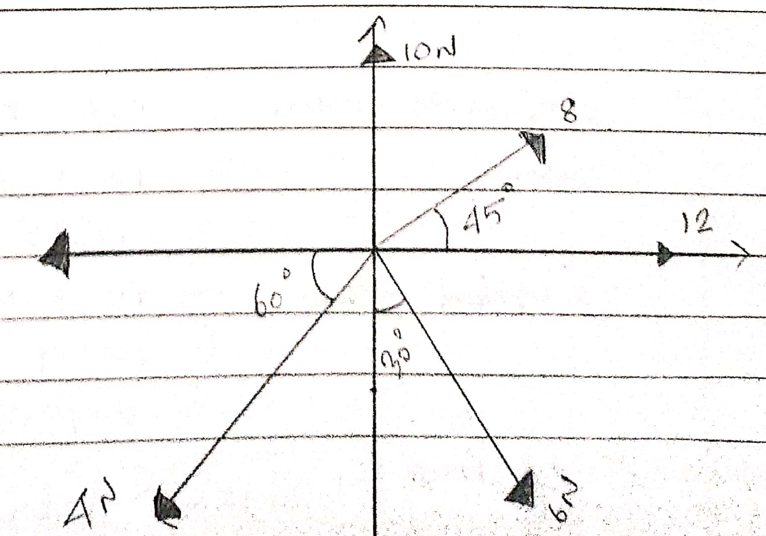
$$F_2 = \frac{F \cdot \sin\alpha}{\sin(\alpha+\beta)} = 150 \cdot \frac{0.7}{0.9} = 109.7\,N$$

Q. 29. A particle is acted upon by the following forces

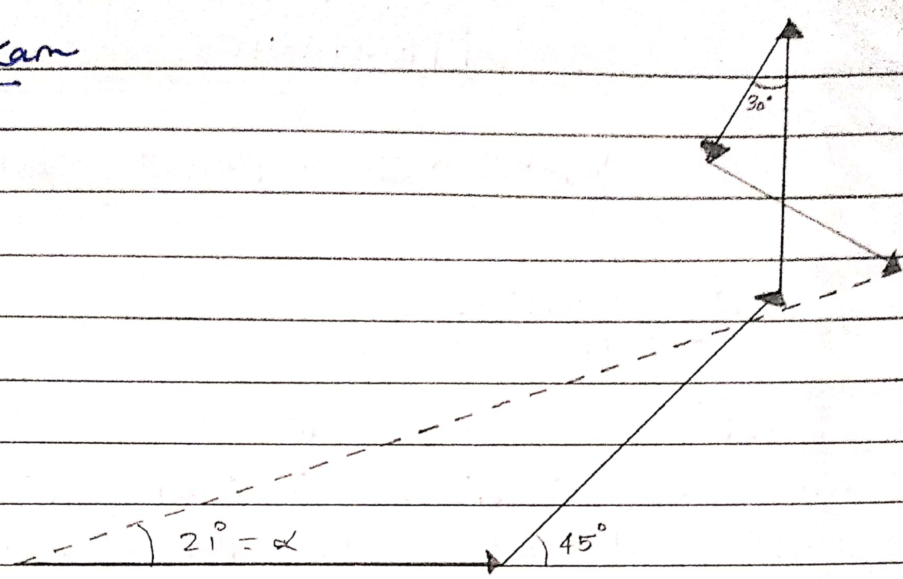
1. Pull 8N due N.E
2. Pull 10N due N.
3. Pull 12N due E
4. Pull 4N 60° SW
5. Pull 6N 30° East of South

Find magnitude and dir of force graphically.

→ Space diagram :

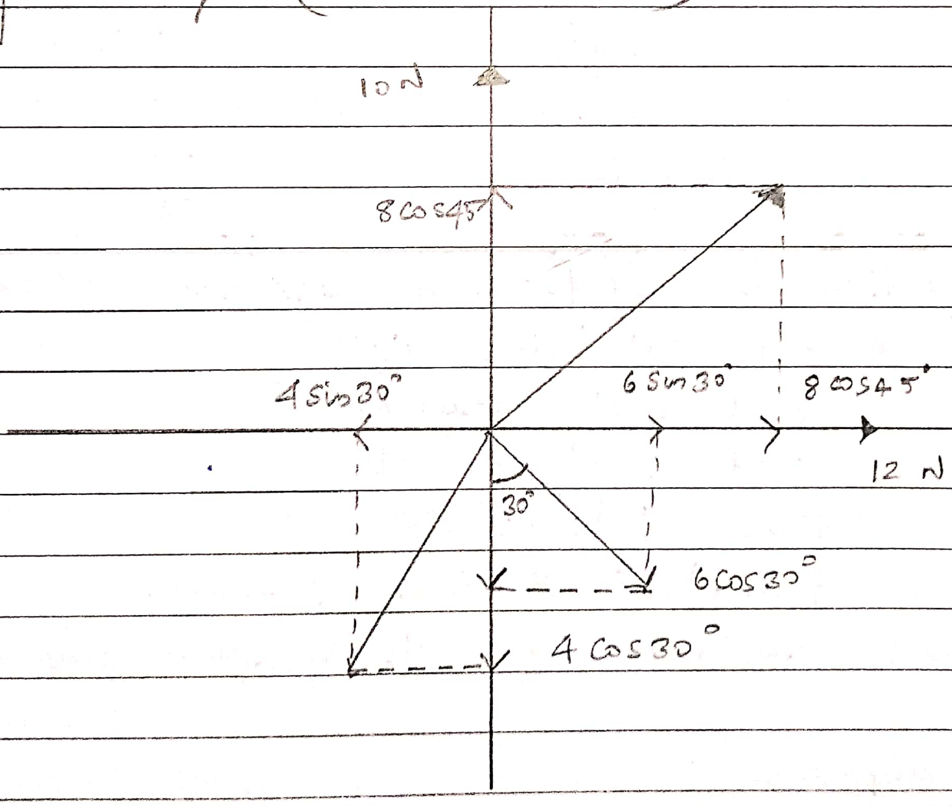


Vector Diagram



∴ Resultant force is 20.04 N at 21° North of East.

Space diagram (Not to scale):



Horizontal Force : $+12 + 8\cos 45 + 6\sin 30° - 4\sin 30$

~~Vertical~~ $= 12 + 5.65 + 3 + -2$

$= 18.65$ N

Vertical Forces $= 10 + 8\cos 45 - 4\sin 60 - 6\cos 30$

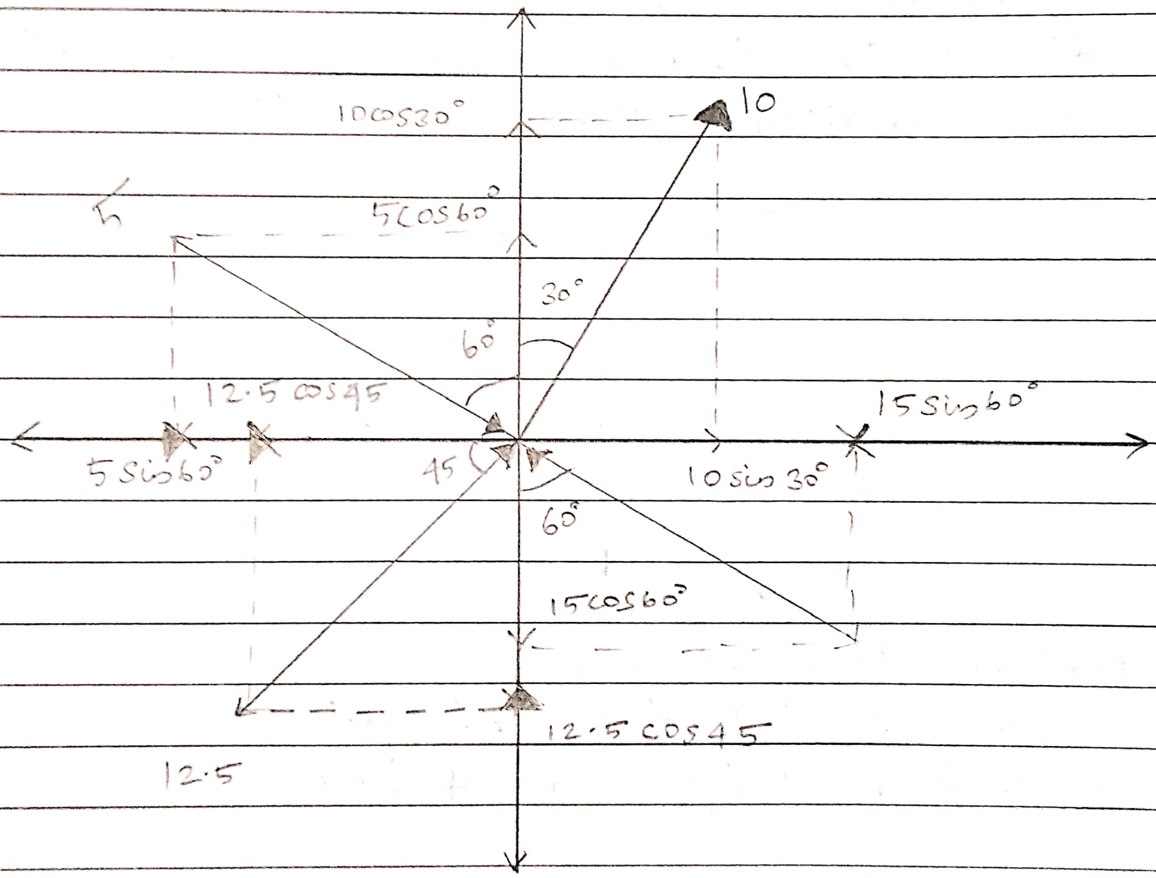
$= 10 + 5.65 - 3.4 - 5.1$

$= 7.1$ N

Resultant $= \sqrt{18.6^2 + 7.1^2} = \underline{20.04 \text{ N}}$

2.10. Find analytically the magnitude and direction of the following 4 forces acting at a point.

1. 10 N pull N 30° E
2. 12.5 N push S 45° W
3. 5 N push N 60° W
4. 15 N push S 60° E



~~Resolv.~~ Resolving Horizontally :

$\Sigma H = 10 \sin \cancel{60}^{30}° \; 15 \sin 60° + 12.5 \cos 45° + 5 \sin 60°$

$= 5.18 N$

~~$\Sigma V =$~~

Resolving vertically :

$\Sigma V = 12.5 \cos 45 + 10 \cos 30 - 5 \cos 60 + 15 \cos 60$

$= 22.5 N$

Resultant $R = \sqrt{\Sigma V^2 + \Sigma H^2}$

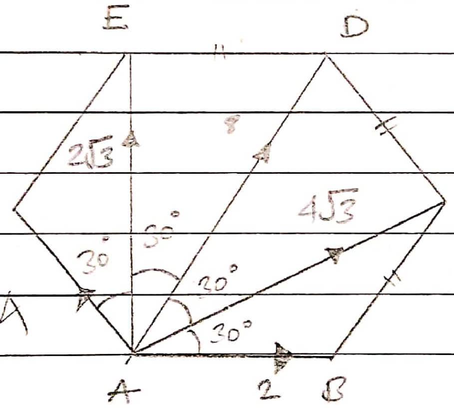
$= \sqrt{(5.18)^2 + (22.5)^2} = \underline{\underline{23.09 N}}$

$$\tan\alpha = \frac{\Sigma V}{\Sigma H} = \frac{22.5}{5.18} = 4.34$$

$$\alpha = \tan^{-1}(4.34)$$

$$\alpha = 77^\circ$$

Q. 2.12 ABCDEF is a regular hexagon. Forces of magnitudes 2, $4\sqrt{3}$, 8, $2\sqrt{3}$, 4 N act at A in direction of AB, AC, AD, AE, AF respectively. Find resultant.



Resolving along AE,

$2\sqrt{3} + 8\cos 30^\circ + 4\sqrt{3}\cos 60^\circ$

$= 2\sqrt{3} + 8\sqrt{3}/2 + \frac{4\sqrt{3}}{2}$

$= 10\sqrt{3}$

Resolving along AB, $2 + 4\sqrt{3}\cos 30^\circ + 8\cos 60^\circ + 4\cos 120^\circ$

$= 2 + 6 + 4 - 2 = 10N$

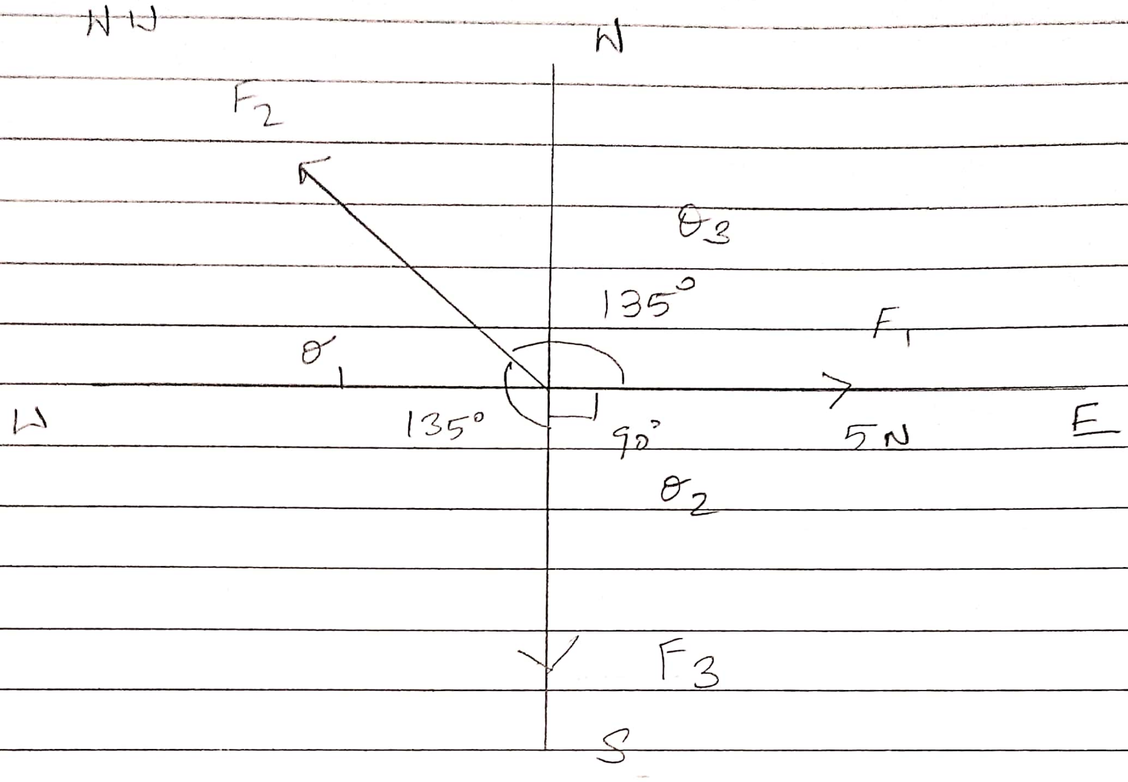
$\therefore$ Resultant $= \sqrt{100 + 300} = 20N$

Let α be angle between resultant Force and AB

$\tan\alpha = \frac{10\sqrt{3}}{10} = \sqrt{3}$

$\therefore \alpha = 60^\circ$

2.14 3 Forces keep a particle in equilibrium as shown. 1st one is 5N. Find other 2



$\theta_1 = 135^\circ$
$\theta_2 = 90^\circ$
$\theta_3 = 135^\circ$

According to Lami's Theorem

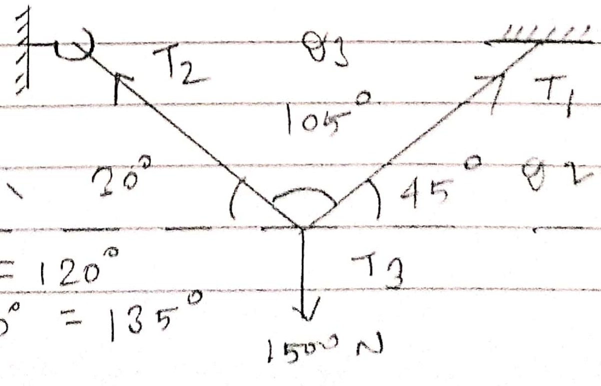
$$\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$$

$$\therefore \frac{5}{\sin 45} = \frac{F_2}{1} = \frac{F_3}{\sin 45}$$

$\therefore F_3 = 5N = F_1$

$F_2 = 5\sqrt{2}\,N$

2.15 A machine is in equilibrium weighing 1500N supported by 2 chains as shown. Find tension.



$\theta_1 = 30^\circ + 90^\circ = 120^\circ$

$\theta_2 = 45^\circ + 90^\circ = 135^\circ$

$\theta_3 = 105^\circ$

inbow

Applying Lami's Theorem;

$$\frac{T_1}{\sin\theta_1} = \frac{T_2}{\sin\theta_2} = \frac{T_3}{\sin\theta_3}$$

$$\frac{T_1}{\sin(120)} = \frac{T_2}{\sin(135)} = \frac{T_3}{\sin(105)}$$

$$\frac{T_1}{\cancel{0.5}\ 0.86} = \frac{T_2}{1/\sqrt{2}} = \frac{1500}{0.96}$$

T_1 = Tension in rope to ceiling

$$= \frac{1500 \times 0.86}{0.96}$$

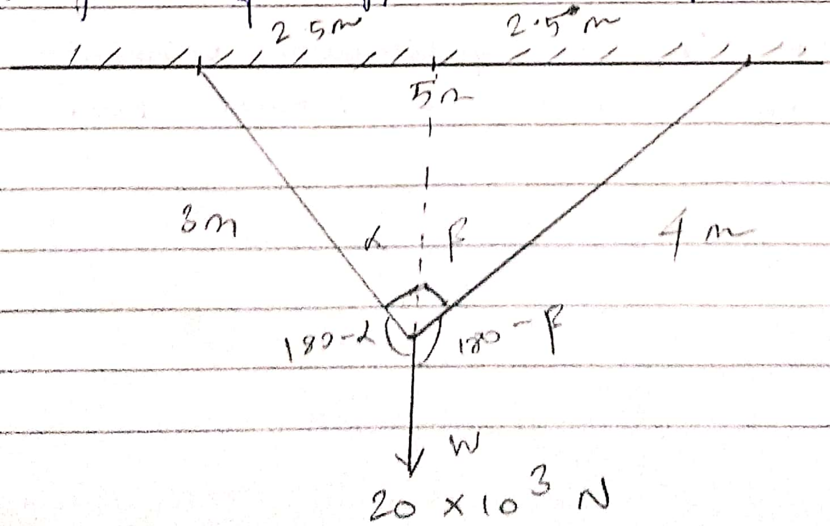
$$= \underline{\underline{1346.11\ N}}$$

T_2 = Tension in rope connected to hook

$$= \frac{1500 \times 0.707}{0.965}$$

$$= \underline{\underline{1098.96\ N}}$$

Q. 2.16 A weight of 20 kN is supported by 2 cords. 1 3m long and other 4 m long. ~~Supported by~~ with points of support 5 m apart. Find T_1, T_2



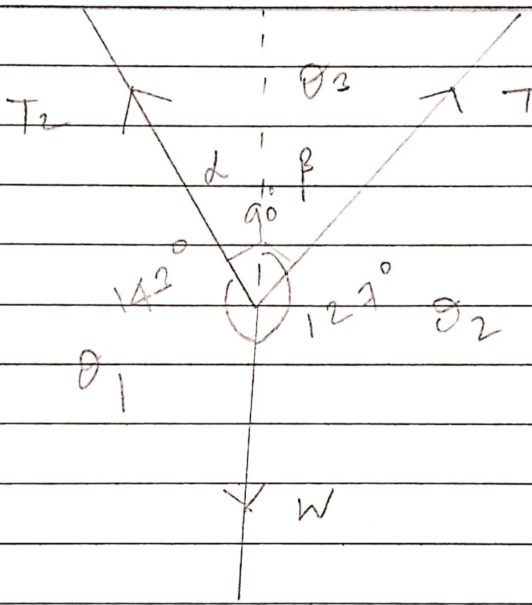
Rainbow

$$\sin\beta = \frac{2.5}{4} = 0.8$$

$$\beta = \sin^{-1}(0.8) = 53^\circ$$

$$\therefore \alpha = 90 - 53 = 37^\circ$$

So now applying Lami's Theorem :



$\theta_1 = 143^\circ$

$\theta_2 = 127^\circ$

$\theta_3 = 90^\circ$

$$\frac{T_1}{\sin(\theta_1)} = \frac{T_2}{\sin\theta_2} = \frac{W}{\sin\theta_3}$$

$$\Rightarrow \frac{T_1}{\sin 143} = \frac{T_2}{\sin 127} = W$$

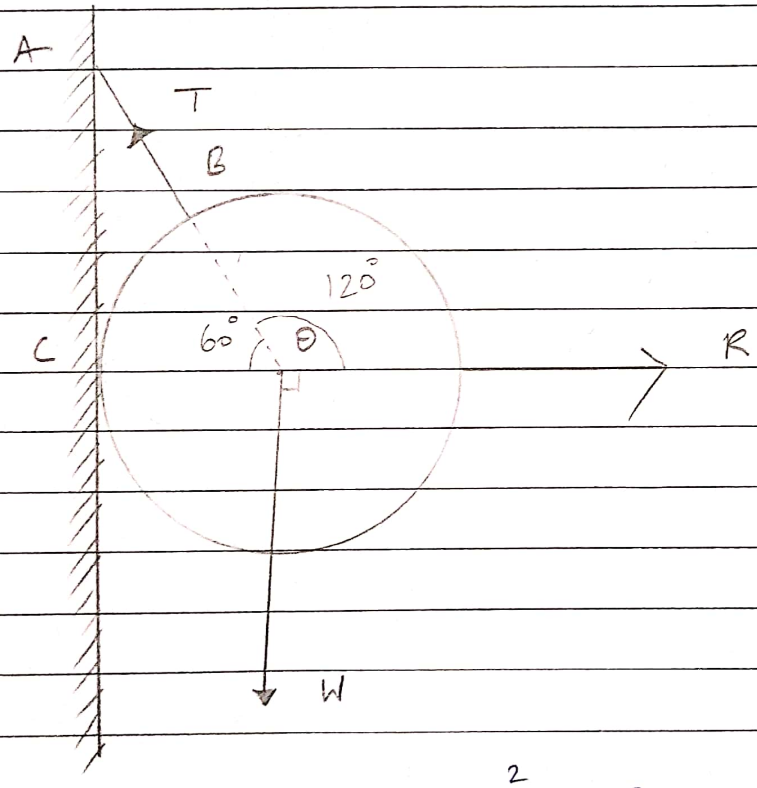
$$\Rightarrow \frac{T_1}{0.6} = \frac{T_2}{0.79} = W$$

$$T_1 = 20 \times 10^3 \times 0.6 \text{ N} = \underline{12 \text{ kN}}$$

$$T_2 = 0.8 \times 20 \times 10^3 \text{ N} = \underline{16 \text{ kN}}$$

Q. 2.17. A smooth sphere of weight W is supported by a ~~a~~ in contact with ~~the~~ a smooth vertical wall. If the length of the string is equal to the radius of the sphere Find tension in string and reaction on the wall.

→



$AO = 2R$, $\quad AO^2 = CO^2 + AC^2$

$CO = R$

$\cos(\angle COA) = \frac{CO}{AO} = \frac{R}{2R} = 0.5$

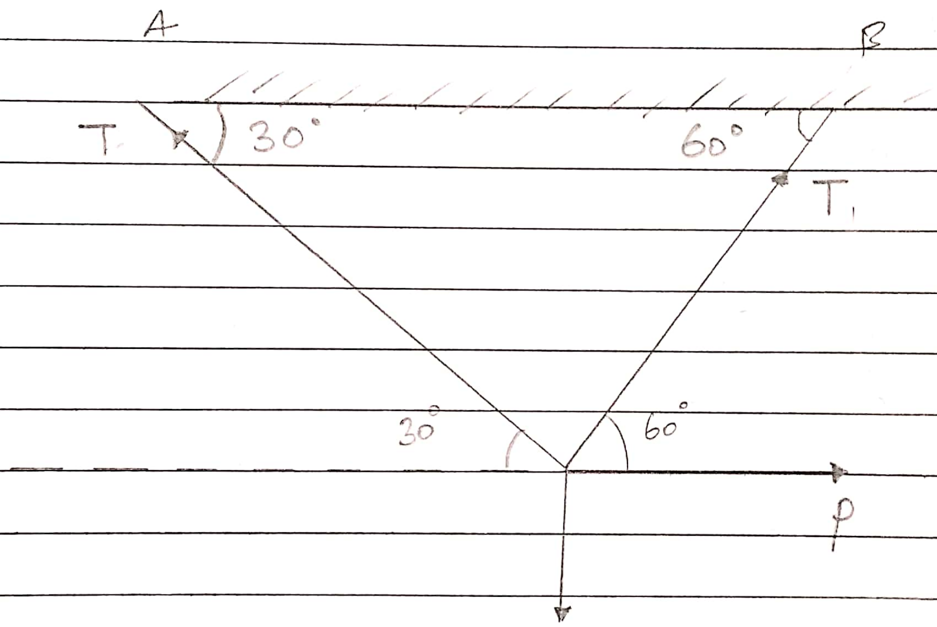
$\angle COA = 60°$

Applying Lami's theorem

$\frac{R}{\sin(150)} = \frac{W}{\sin(120)} = \frac{T}{1}$ $\quad$ $T = \frac{2}{\sqrt{3}} W$

$R = \frac{W \cdot \sin 150}{\sin 120} = \frac{W \cdot 1/2}{\sqrt{3}/2} = \boxed{W/\sqrt{3} = R}$

2.18. A string is tied to two points at the same level and a smooth ring of weight W; which can slide freely along the ring, is pulled by a horizontal force P, if in the position of equilibrium, the portions of the string are inclined at 60° and 30° to the vertical, find P and tension in string.



In horizontal equilibrium,

$$T_2 \cos 30 = T_1 \cos 60 + P$$

But $T_2 = T_1 = T$ (same string)

$\therefore$ $\cancel{P = \frac{\cos 30}{\cos 60} \cdot T = \frac{\sqrt{3}}{2} \div \frac{1}{2} = \sqrt{3}}$

$$T \cos 30 - T \cos 60 = P$$

$$T\left(\frac{\sqrt{3}-1}{2}\right) = P \quad \cdots \quad (1)$$

Vertical equilibrium:

$$T \sin 30 + T \sin 60 = W$$

$$\left(\frac{\sqrt{3}+1}{2}\right) T = W \quad ; \quad \boxed{T = \frac{2}{\sqrt{3}+1} \cdot W}$$

$\cancel{P/T = \left(\frac{\sqrt{3}+1}{2}\right) \cdot W, \left(\frac{\sqrt{3}-1}{2}\right)}$

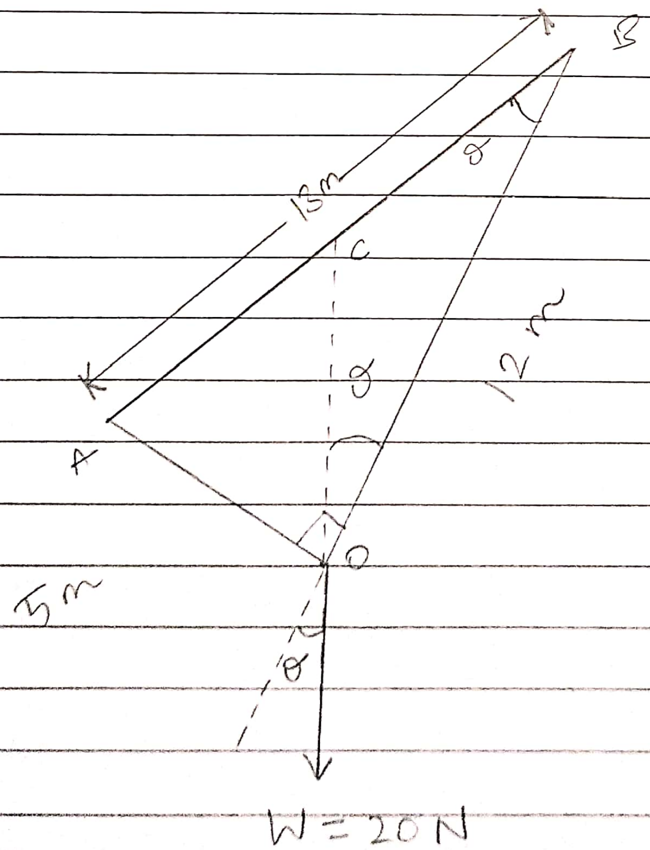
bow

$$P = \frac{\sqrt{3}-1}{\sqrt{3}+1} \cdot W$$

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} \cdot \frac{\sqrt{3}-1}{\sqrt{3}-1} \cdot W$$

$$= \frac{4-2\sqrt{3}}{2} \cdot W = \boxed{(2-\sqrt{3})W = P}$$

Q.2.19. A body of weight 20 N is suspended by 2 strings 5 m and 12 m long. and other ends being fastened to the extremities of a rod of length 13 m. If the body be so hanged such that its immediately below the mid point of the rod, find tension in string.



$$\sin\theta = \frac{OA}{AB} = \frac{5}{13}$$

$$\cos\theta = \frac{OB}{AB} = \frac{12}{13}$$

Applying Lami's Theorem.

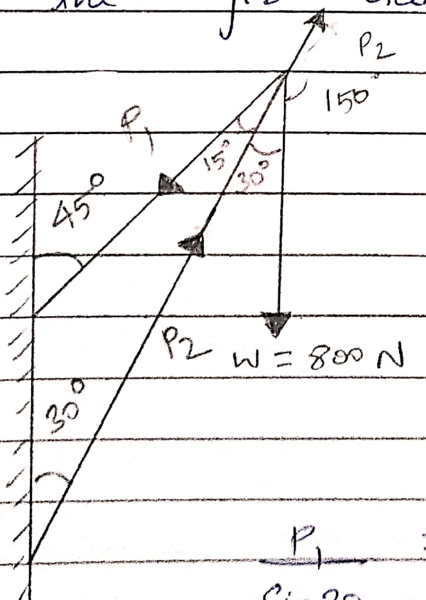
$$\frac{T_1}{\sin(180-\theta)} = \frac{T_2}{\sin(90+\theta)} = \frac{W}{\sin 90}$$

$$\frac{T_1}{\sin\theta} = \frac{T_2}{\cos\theta} = 20$$

$$T_1 = \sin\theta \cdot 20 = \frac{5}{13} \times 20 = \underline{\underline{7.69 N}}$$

$$T_2 = \cos\theta \cdot 20 = \frac{12}{13} \times 20 = \underline{\underline{18.46 N}}$$

Q.20. What axial forces does the vertical load W = 800N induce in the tie rod and the jib of the jib crane shown in figure ?



Apply lamis Theorem

$$\frac{P_1}{\sin(180^\circ - 30^\circ)} = \frac{P_2}{\sin(30^\circ + 15^\circ)} = \frac{W}{\sin(180^\circ - 15^\circ)}$$

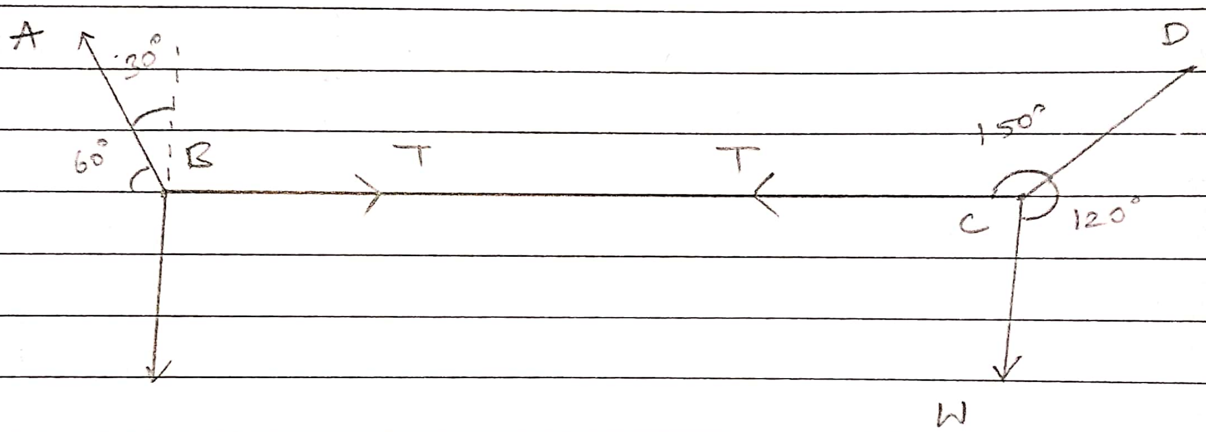
$$\frac{P_1}{\sin 30} = \frac{P_2}{\sin 45} = \frac{800}{\sin 15}$$

$$P_1 = 800 \cdot \frac{\sin 30}{\sin 15} = \underline{\underline{1545.6 N}}$$

$$P_2 = 800 \frac{\sin 45}{\sin 15} = \underline{\underline{2185.5 N}}$$

inbow

Q.21. A string ABCD hangs from a fixed point A and D carrying a weight of 12N at B and W at C. AB is inclined at 60° to horizontal, CD is inclined 30° to horizontal and BC is horizontal, find W.



By Lami's Theorem :

$$\frac{T}{\sin 150} = \frac{12}{\sin 120} = A$$

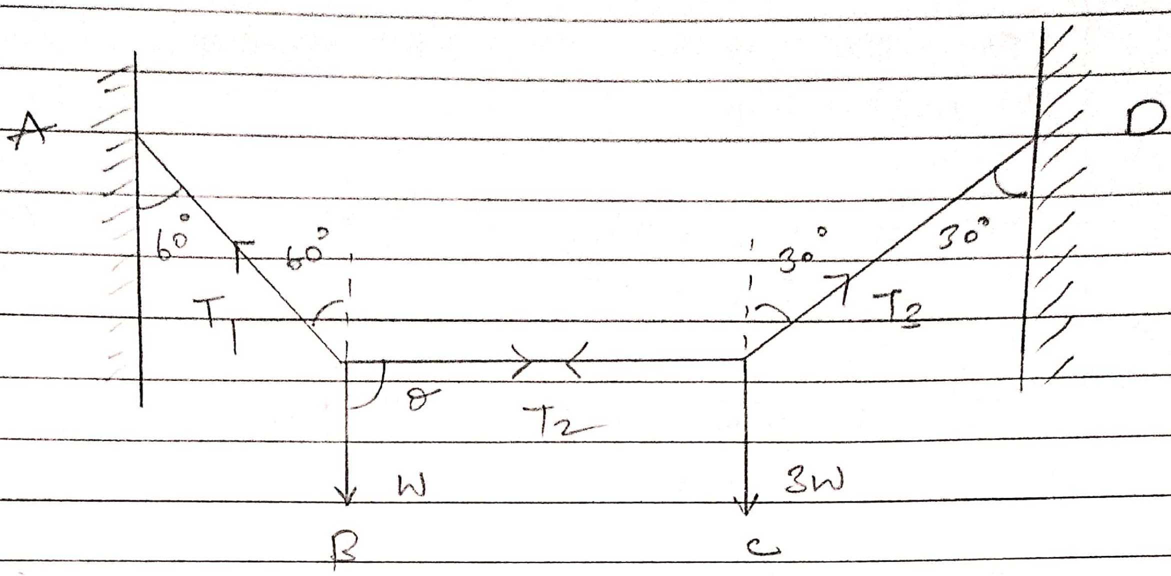
$$T = \frac{12 \cdot \sin 150}{\sin 120} = \frac{12 \times 1/2}{\sqrt{3}/2} = 12/\sqrt{3}$$

$$\frac{W}{\sin 150} = \frac{T}{\sin 120}$$

$$W = \frac{T \cdot \sin 150}{\sin 120}$$

$$= \frac{12 \cdot 1/2}{\sqrt{3} \cdot \frac{\sqrt{3}}{2}} = \underline{\underline{4 \text{ N}}}$$

Q.22. The extremities ~~ABCD~~ A and D of a light extensible string are tied to two points in the same horizontal line. Weights ~~4~~ W and 3W are tied to the string at points B and C. If AB and CD are inclined to the vertical at 60 at 30° respectively, show BC is horizontal and Find T



Applying Lami's Theorem: at B,

$$\frac{T_1}{\sin(\theta)} = \frac{T_2}{\sin 120°} = \frac{W}{\sin(240° - \theta)} \quad \text{—— (1)}$$

Applying Lami's Theorem at C,

$$\frac{T_2}{\sin 150} = \frac{T_3}{\sin(180 - \theta)} = \frac{3W}{\sin(30° + \theta)} \quad \text{—— (2)}$$

From equation (1) and (2)

$$T_2 = \frac{W \sin 120°}{\sin(240 - \theta)} = \frac{3W \sin 150}{\sin(30° + \theta)}$$

$$\frac{\sin 60°}{-\sin(60° - \theta)} = \frac{3 \sin 30°}{\sin(30° + \theta)}$$

$$\frac{\sqrt{3}}{2} \sin(30° + \theta) = -3 \cdot \frac{1}{2} \sin(60° - \theta)$$

$$\sin 30 \cos\theta + \cancel{\cos 30 \sin\theta} = -\sqrt{3}(\sin 60 \cos\theta - \cancel{\cos 60 \sin\theta})$$

$2\cos\theta = 0$, $\theta = 90°$

So BC makes 90° with vertical. ∴ BC is horizontal.

$$T_1 = \frac{W \sin 90}{\sin 150} = \frac{W}{1/2} = \underline{\underline{2W}}$$

$$T_2 = \frac{W \sin 120}{\sin 150} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} \cdot W = \underline{\underline{\sqrt{3}W}}$$

$$T_3 = \frac{3W \sin 90}{\sin 120} = \frac{3W}{\sqrt{3}/2} = \underline{\underline{2\sqrt{3}W}}$$

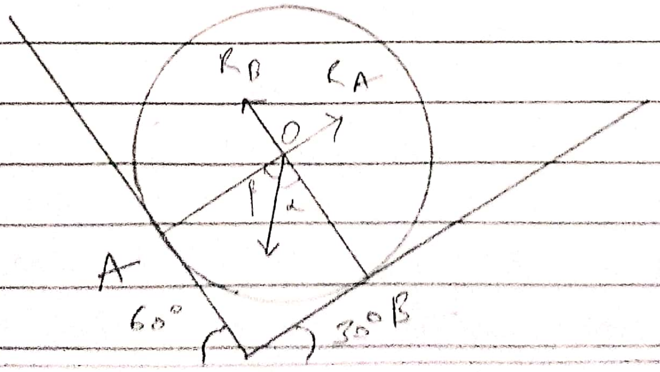
Q2.25. A cast Iron sphere 30 cm in diameter rests in 20 cm to 20 cm angle, one leg of which is at an angle of 30° with the horizontal as shown in 2.42. Find reaction at A and B.

ans. Weight of sphere $= 4/3 \cdot \pi \cdot r^3 \cdot d$

$= 4/3 \cdot 3.14 \cdot (0.15)^3 \cdot \rho$

$= 1.3 \times 3.14 \times 3.3 \times 10^{-3} \times 72 \times 10^3$

$= 1017.9\ N$



$\angle\alpha = 30^\circ$
$\angle\beta = 60^\circ$

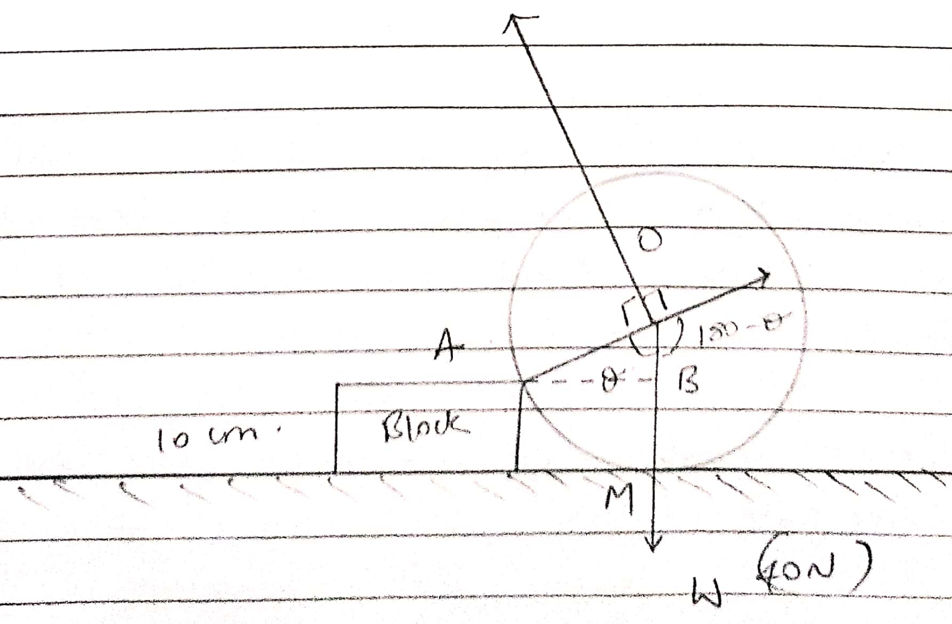
Applying Lami's theorem we get

$$\frac{R_A}{\sin(180^\circ - \alpha)} = \frac{R_B}{\sin(180^\circ - \beta)} = \frac{W}{\sin 90}$$

$$\frac{R_A}{\sin\alpha} = \frac{R_B}{\sin\beta} = 1017.9$$

$R_A = 1017.9 \times \sin 30 = 508.9N$
$R_B = 1017.9 \times \sin 60 = 881.5N.$

Q.2.26 A uniform wheel 40 cm in diameter rests against a rigid rectangular block 10 cm thick as shown. Find the least pull through the centre of the wheel to just turn it over the corner of the block. Also find reaction of block. The wheel weighs 800N.



As wheel is about to turn, there will be no contact from floor. So ~~$R_A = 0$~~. $R_m = 0$

F applied will be $\perp$ to AO

Applying Lami's theorem:

$$\frac{P}{\sin(180-\theta)} = \frac{R_A}{\sin(90°+\theta)} = \frac{W}{\sin 90}$$

$$\frac{P}{\sin\theta} = \frac{R_A}{\cos\theta} = \frac{W}{\sin 90}$$

$$P = 800 \sin\theta$$
$$R_A = 800 \cos\theta$$

But $\cos\theta = \frac{OB}{OA} = \frac{10}{20} = 1/2$

$\therefore \theta = 60°$

$$R_A = \frac{800}{2} = \underline{\underline{400\,N}}$$

$$P = 800 \times \sin 60$$
$$= \underline{\underline{692.8\,N}}.$$

Q.2.27. The cylinders in figure have the same diameter but 1 weighs 200 N and 2 weighs 150 N. Find the reaction at the supports.

→ Cylinder 2 is in equilibrium due to:

1. Reaction force from OQ $\perp$ R_N
2. Reaction force from Cylinder 1 R_S
3. Weight of 2 downwards. W_2.

Applying Lami's theorem :

$$\frac{R_N}{\sin(180^\circ - 15^\circ)} = \frac{R_S}{\sin(90^\circ + 15^\circ)}$$

$$\frac{R_N}{\sin 15} = \frac{R_S}{\sin 105} = 150$$

$R_N = 150 \sin 15 = 38.82\ N.$
$R_S = 150 \sin 105 = 144.89\ N.$

Resolving forces for cylinder 1.

perpendicular to O_1O_2,

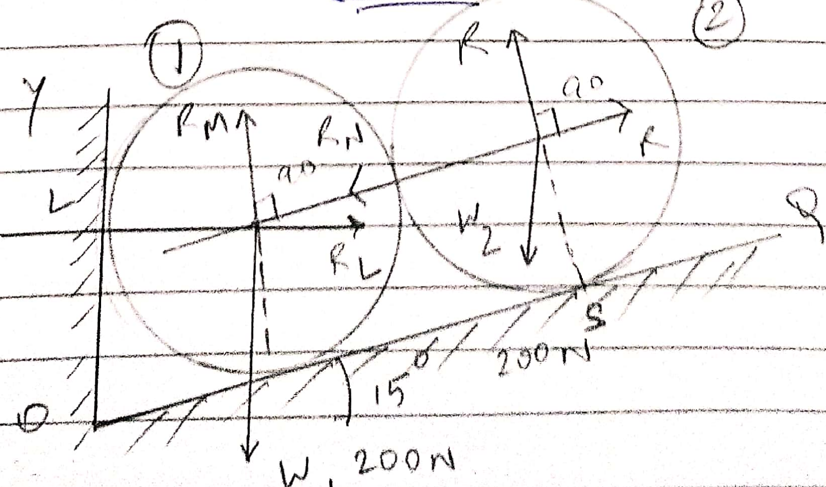
$R_L \cos 15^\circ - W_1 \sin 15^\circ - R_N = 0$ — (1)

and

$R_M - R_L \sin 15^\circ - W_1 \cos 15^\circ = 0$ — (2)

$R_L = 93.78 N$

Putting R_L in 2,
$R_M - 93.78 \times 0.25 - 200 \times 0.96 = 0$
$R_M - 24.27 - 193.1 = 0$
$R_M = 217.45 N.$



Sow