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Tutorial - 4 - LAOC

Q.1. (1) $e^x \sin x$ upto x^4.

$y(0) = 0$; using Maclaurin's theorem

~~$y_1(1) = e^x$~~

$y_1 = e^x \sin x + \cos x \cdot e^x$

$y_1(0) = 1$

$y_2 = 2\cos x \cdot e^x \Rightarrow y_2(0) = 2.$

$y_3 = 2e^x(\cos x - \sin x) = y_3(0) = 2$

$y_4 = -4e^x \sin x = y_4(0) = 0$

$$e^x \sin x = 0 + x.1 + \frac{2 \times x^2}{2!} + \frac{2 \times x^3}{3!}$$

$$+ \frac{0 \cdot x^4}{4!}$$

$$= x + x^2 + \frac{x^3}{3} + \cancel{\frac{x^4}{4}} \cdot 0$$

Q.2. $x^4 - 3x^3 + 2x^2 - x + 1$ in powers of $(x-3)$

$\rightarrow$ $f = x^4 - 3x^3 + 2x^2 - x + 1$

$f' = 4x^3 - 9x^2 + 4x - 1$

$f'' = 12x^2 - 18x + 4$

$f''' = 24x - 18$

$f'''' = 24$

$$f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)(x-c)^2}{2!} + \frac{f'''(c)(x-c)^3}{3!} + \frac{f''''(c)(x-c)^4}{4!}$$

let $c = 3$

then

$$f(x) = f(3) + f'(3)(x-3) + \frac{f''(3)(x-3)^2}{2!} + \frac{f'''(3)(x-3)^3}{3!} + \frac{f^{iv}(3)(x-3)^4}{4!}$$

$f(3) = 81 - 81 + 18 - 3 + 1 = 16$

$f'(3) = 108 - 81 + 12 - 1 = 38$

$f''(3) = 108 - 54 + 4 = 58$

$f'''(3) = 72 - 18 = 54$

$f^{(iv)}(3) = 24.$

$$= 16 + 38(x-3) + \frac{58}{2}(x-3)^2$$

$$+ \frac{54}{6}(x-3)^3 + \frac{24(x-3)^4}{4!}$$

$$= 16 + 38(x-3) + 29(x-3)^2$$
$$+ 27(x-3)^2 + (x-3)^4$$

(3) If $y = a \cos \log(x) + b \sin \log(x)$

ST. $x^2 y_{n+2} + (2n+1) x y_{n+1} + n(n+1) y_n = 0$

$\rightarrow$ $y = a \cos(\log x) + b \sin(\log x)$

$$y_1 = \frac{a(-\sin \log x)}{x} + \frac{b \cos \log x}{x}$$

$$x y_1 = -a \sin(\log x) + b \cos(\log x)$$

differentiating both sides,

$\Rightarrow \quad xy_2 + y_1 = \dfrac{-a\cos(\log x)}{x} - \dfrac{b\sin(\log x)}{x}$

$x^2 y_2 + xy_1 = -\{a\cos\log x + b\sin\log x\}$

$= -y$

So

$x^2 y_2 + xy_1 + y = 0. \quad \text{--- (1)}$

Using leibnitz theorem,

$\rightarrow \quad (uv)_n = uv_n + {}^nC_1 u_1 v_{n-1} + {}^nC_2 u_2 v_{n-2} \cdots$

$u_n v.$

let $u = x^2$, $v = y_2$

$(x^2 y_2)_n = x^2 (y_2)_n + {}^nC_1 (x^2)' (y_2)_{n-1}$

$+ {}^nC_2 (x^2)'' (y_2)_{n-2}$

$= x^2 y_{n+2} + n\cdot(2x)\cdot y_{n+1}$

$+ \dfrac{n(n-1)}{2}\cdot 2\cdot y_n + 0 + 0 \ldots$

$= \cancel{x^2 y_n + \ldots +}$

$\cancel{x^2 y_{n+2} + n\cdot(2x) y_{n+1}} + \cancel{\dfrac{n(n-1)}{2}\cdot 2\, y_n}$

$$(x^2 y_2)_n = x^2 y_{n+2} + 2nx y_{n+1} + (n^2-n) y_n \quad -(1)$$

Similally, $(xy_1)_n = xy_{n+1} + ny_n \cdots (2)$

usin (1) and (2) in (0)

$$\{x^2 y_{n+2} + 2nx y_{n+1} + (n^2-n) y_n\}$$

$$+ \{x y_{n+1} + n y_n\} + \{y_n\} = 0$$

$$\Rightarrow x^2 y_{n+2} + x(2n+1) y_{n+1}$$

$$+ (n^2+1) y_n = 0$$

Q.2. (1.) $y_n = (-1)^{n-2} \dfrac{(n-2)!}{x^{n-1}} \cdot \ln 10$

(2) $y_n = e(2x+3)^2 + 4n(2x+3) \cdot e^x$
$+ 4(n-1)n \cdot e^x$

(3.) $y_n = \cancel{x/2} + \pi/2 + 2x + 2x^2 - \dfrac{2x^3}{3}$

(4). 3.019933