

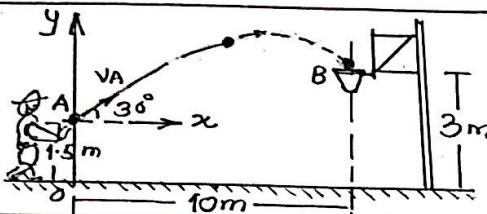
Curvilinear system and Rectangular Coordinate System

Path variables and Polar coordinates

Relative Motion

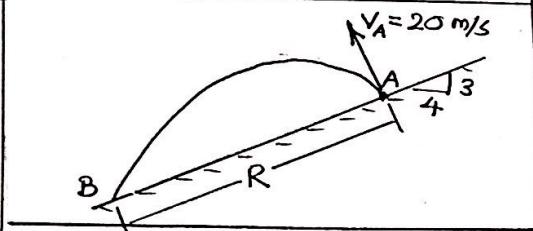
- 1 Determine the speed at which the basketball at A must be thrown at the angle of 30° so that it makes it to the basket at B.

$$\text{Ans: } u = 12.373 \text{ m}$$



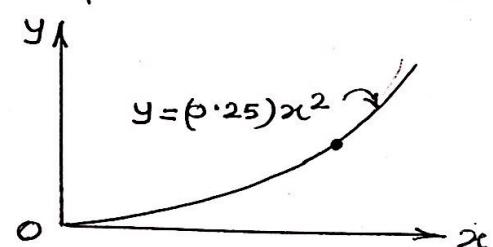
- 2 Water is sprayed at an angle of 90° from the slope at 20 m/s . Determine the range R.

$$\text{Ans: } R = 33.864 \text{ m}$$



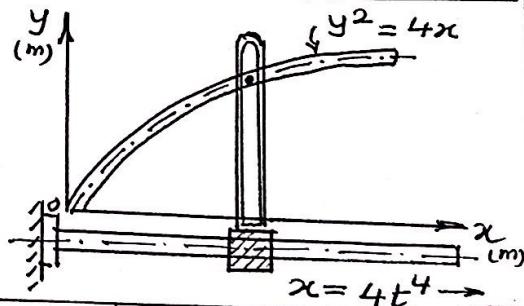
- 3 A particle is traveling along the parabolic path $y = 0.25x^2$. If $x = 8 \text{ m}$, $V_x = 8 \text{ m/s}$, $a_x = 4 \text{ m/s}^2$ when $t = 2 \text{ s}$, determine the magnitude of particle's velocity and acceleration at this instant.

$$\text{Ans: } v = 32.985 \text{ m/s; } a = 48.2 \text{ m/s}^2$$



- 4 A particle constraint to travel along the path. If $x = (4t^4) \text{ m}$, where t is in seconds, determine the magnitude of particle's velocity and acceleration when $t = 0.5 \text{ seconds}$.

$$\text{Ans: } v = 4.472 \text{ m/s; } a = 14.422 \text{ m/s}^2$$

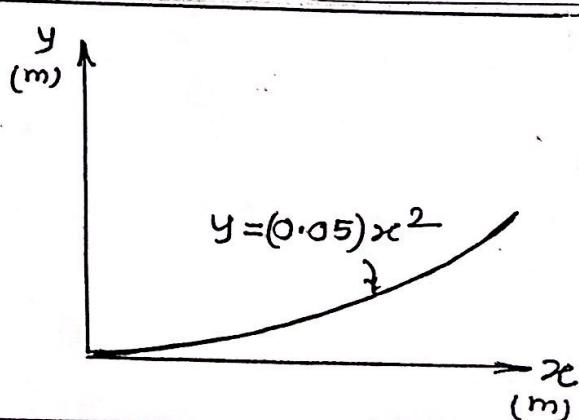


- 5 If x & y components of a particle's velocity are $V_x = (32t) \text{ m/s}$, $V_y = 8 \text{ m/s}$, determine the equation of the path $y = f(x)$, if $x = 0$ & $y = 0$ when $t = 0$.

$$\text{Ans: } y^2 = 4x$$

- 6 The box slides down the slope described by the equation $y = (0.05x^2) \text{ m}$, where x is in meters. If the box has x component of velocity and acceleration of $V_x = -3 \text{ m/s}$ and $a_x = -1.5 \text{ m/s}^2$ at $x = 5 \text{ m}$, determine the y components of the velocity and the acceleration of the box at this instant.

$$\text{Ans: } V_y = -1.5 \text{ m/s; } a_y = 0.15 \text{ m/s}^2$$



Lecture No. (7)

5 ① RCH/F 12.15/pg. 661:

$$v_x = (32)t = \frac{dx}{dt}$$

$$\int dx = \int (32)t \cdot dt$$

$$x = 16t^2 \rightarrow ①$$

$$\therefore x = 16 \times \frac{y^2}{64} = \frac{y^2}{4}$$

$$\underline{\text{Ans: } y^2 = 4x}$$

$$v_y = 8 \text{ m/s} = \frac{dy}{dt}$$

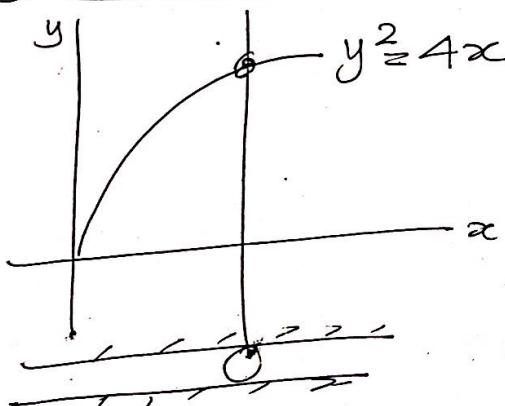
$$\int dy = \int 8 \cdot dt$$

$$y = 8t \rightarrow ②$$

$$\therefore t = y/8$$

$$\underline{\text{or } y = 2\sqrt{x}}$$

② RCH/F 12.17/pg. 661:



At $t = 0.5 \text{ sec.}$

$$v_x = 2 \text{ m/s}, v_y = 4 \text{ m/s}$$

$$\vec{v} = 2\hat{i} + 4\hat{j} \text{ m/s}$$

$$V = 4.472 \text{ m/s}$$

$$63.43^\circ$$

$$\alpha_x = 12 \text{ m/s}^2, \alpha_y = 8 \text{ m/s}^2$$

$$\vec{\alpha} = 12\hat{i} + 8\hat{j} \text{ m/s}^2$$

$$\alpha = 14.422 \text{ m/s}^2$$

$$33.69^\circ$$

3 ③ RCH/F 12.19/pg.661:

$$y = (0.25)x^2$$

$$\text{At } t = 2 \text{ s, } x = 8 \text{ m}$$

$$v_x = \dot{x} = 8 \text{ m/s}$$

$$a_x = \ddot{x} = 4 \text{ m/s}^2$$

$$\text{As, } y = (0.25)x^2$$

Differentiating w.r.t. x , we get,

$$\dot{y} = (0.5)x\dot{x} = v_y$$

$$\ddot{y} = (0.5)\dot{x}^2 + (0.5)x\ddot{x}$$

At $t = 2 \text{ sec.}$ $v_x = 8 \text{ m/s}$

$$v_y = 0.5 \times 8 \times 8 = 32 \text{ m/s}$$

$$v = \sqrt{8^2 + 32^2} = 32.985 \text{ m/s}$$

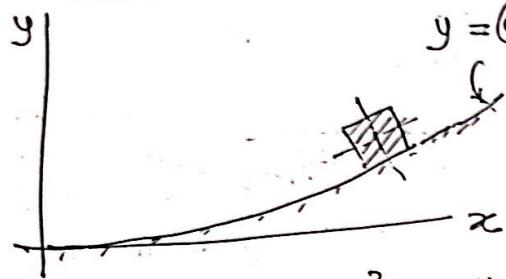
$$a_x = 4 \text{ m/s}^2$$

$$a_y = (0.5 \times 64) + (0.5 \times 8 \times 4)$$

$$= 32 + 16 = 48 \text{ m/s}^2$$

$$a = \sqrt{4^2 + 48^2} = 48.2 \text{ m/s}^2$$

6. ④ RCH/F 12.20/pg. 661



$$y = (0.05)x^2$$

At $x = 5 \text{ m}$

$$v_x = \dot{x} = -3 \text{ m/s}$$

$$v_y = ?$$

$$a_x = -1.5 \text{ m/s}^2$$

$$a_y = ?$$

$$y = (0.05)x^2 \rightarrow (i)$$

$$\therefore \dot{y} = (0.10)x \cdot \dot{x} \rightarrow (ii)$$

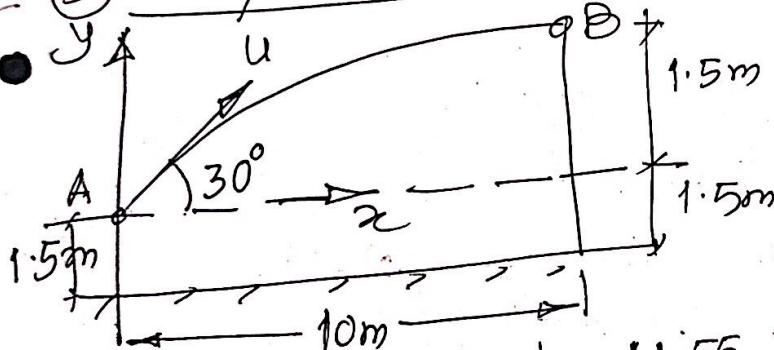
$$\ddot{y} = (0.10)\dot{x}^2 + (0.1)x\ddot{x} \rightarrow (iii)$$

$$\therefore \text{At } x = 5 \text{ m}, v_y = \dot{y} = 0.1 \times 5 \times (-3) = -1.5 \text{ m/s}$$

i.e. 1.5 m/s (↑)

$$a_y = (0.10)(-3)^2 + (0.1)(5)(-1.5) = 0.15 \text{ m/s}^2 (↑)$$

1. ⑤ RCH/F 12.23/pg. 662 :



$$B \equiv (10, 1.5 \text{ m})$$

$$x = (4 \cos 30^\circ)t$$

$$10 = (4 \cos 30^\circ)t \rightarrow (1)$$

$$y = (4 \sin 30^\circ)t - \frac{1}{2}gt^2$$

$$1.5 = (4 \sin 30^\circ)t - (4.905)t^2 \rightarrow (2)$$

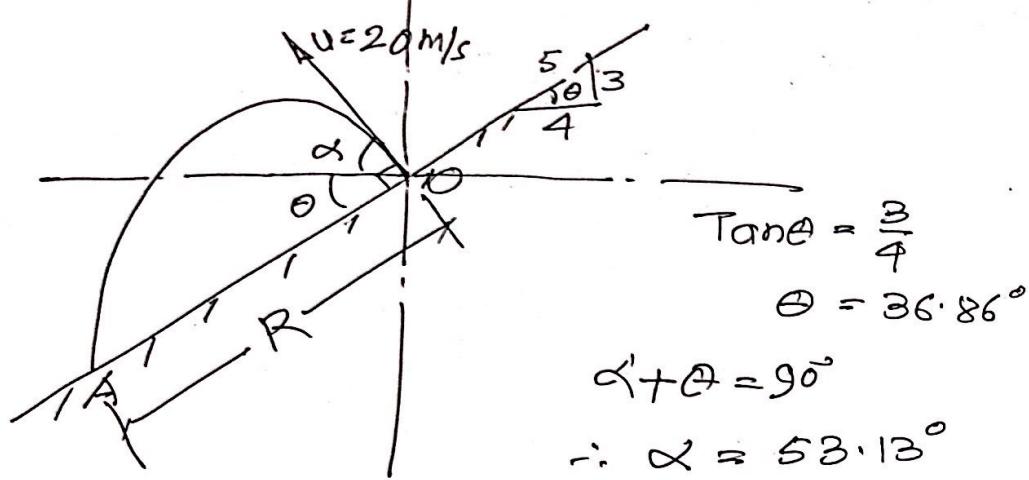
$$ut = 11.55 \quad \therefore u = \left(\frac{11.55}{t} \right)$$

$$y = \left(\frac{11.55}{t} \times 0.5 \times t \right) - (4.905)t^2$$

$$y = (5.775) - (4.905)t^2 = 1.5$$

$$t = 0.9834 \text{ s}, u = 12.373 \text{ m/s}$$

2 (6) RCH/F 12.24/pg.662



$$\tan \theta = \frac{3}{4}$$

$$\theta = 36.86^\circ$$

$$\alpha + \theta = 90^\circ$$

$$\therefore \alpha = 53.13^\circ$$

$$\text{At pt. A, } x = -R \cos \theta = -(0.8)R$$

Eqs of projectile $y = -R \sin \theta = -(0.6)R$

are, $x = -(20)(\cos 53.13^\circ)t = -(12.0 \cancel{\text{m/s}})t \text{ m/s} \rightarrow ①$

$$y = (20)(\sin 53.13^\circ)t - \frac{1}{2}(9.81)t^2$$

$$y = (16)t - (4.905)t^2 \rightarrow ②$$

At A,

$$x = -(0.8)R = -(12)t$$

$$\therefore R = 15. \boxed{t}$$

$$y = 16t - (4.905)t^2 = -(0.6)R$$

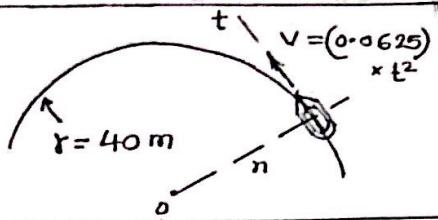
$$16t - (4.905)t^2 = -9t$$

$$(4.905)t^2 - (25)t = 0$$

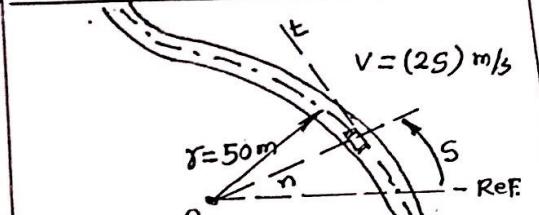
$$t = 2.257 \text{ sec.} \quad 5.0968 \text{ sec.}$$

$$\therefore R = 33.864 \text{ m} \quad 76.452 \text{ m}$$

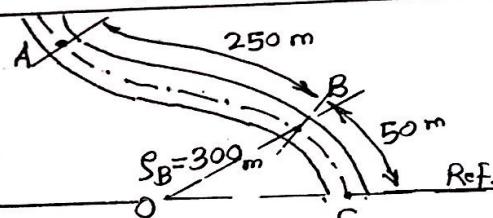
- 1 The boat is traveling along the circular path with a speed of $v = (0.0625t^2) \text{ m/s}$, where t is in seconds. Determine the magnitude of its acceleration when $t=10\text{s}$.
Ans: $a = 1.586 \text{ m/s}^2$



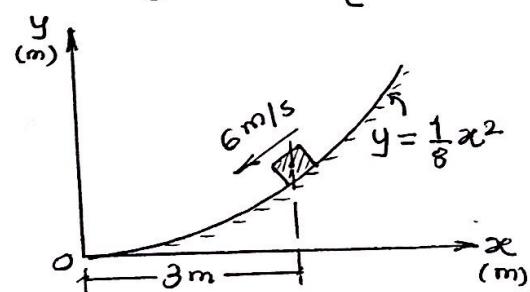
- 2 The car is traveling along the road with a speed of $v = (2s) \text{ m/s}$, where s is in meters. Determine the magnitude of its acceleration when $s=10\text{m}$.
Ans: $a = 40.792 \text{ m/s}^2$



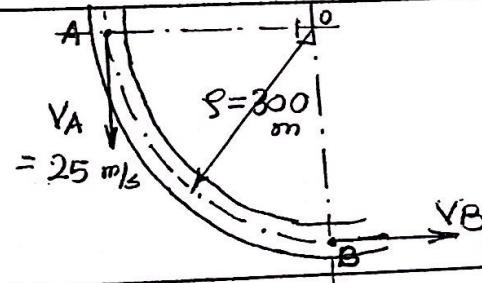
- 3 If the car decelerates uniformly along the curved road from 25m/s at A to 15m/s at C, determine the acceleration of the car at B.
Ans: $a_B = 1.178 \text{ m/s}^2$



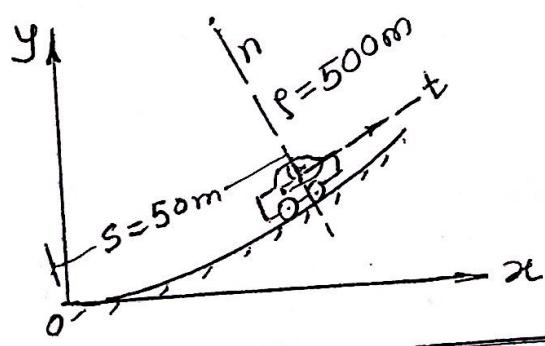
- 4 When $x=3\text{m}$, the crate has a speed of 6m/s which is increasing at 2m/s^2 . Determine the direction of the crate's velocity and the magnitude of the crate's acceleration at this instant.
Ans: $a = 5.02 \text{ m/s}^2$



- 5 If the motorcycle has a deceleration of $a_t = - (0.001s) \text{ m/s}^2$ and its speed at position A is 25m/s , determine the magnitude of its acceleration when it passes point B.
Ans: $a_B = 1.423 \text{ m/s}^2$



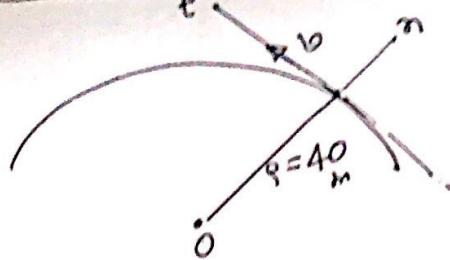
- 6 The car travels up the hill with a speed of $v = (0.2s) \text{ m/s}$, where s is in meters, measured from O. Determine the magnitude of its acceleration when it is at point s=50m, where p=500m.
Ans: $a = 2.01 \text{ m/s}^2$



Lecture No 8



① RCH/F 12.27/ Pg. 678:



$$v = (0.0625)t^2 \text{ m/s}$$

$$\therefore a_t = \frac{dv}{dt} = (0.125)t \text{ m/s}^2$$

$$g = 40 \text{ m}$$

$$a_n = \frac{v^2}{s}$$

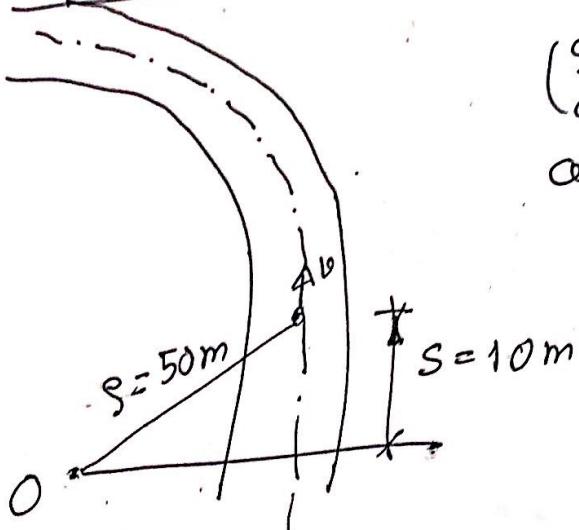
At $t = 10 \text{ s}$:

$$v = 6.25 \text{ m/s}, a_t = 1.25 \text{ m/s}^2$$

$$a_n = \frac{6.25^2}{40} = 0.9765 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = 1.586 \text{ m/s}^2$$

② RCH/F 12.28/ Pg. 678:



$$v = 2 \text{ s} \text{ m/s}$$

$$\left(\frac{dv}{ds}\right) = 2$$

$$a_t = v \cdot \frac{dv}{ds} = 4 \text{ s} \text{ m/s}^2$$

$$a_n = \frac{v^2}{s} = \frac{4 \cdot 2^2}{50} \text{ m/s}^2$$

$$\text{when } s = 10 \text{ m}$$

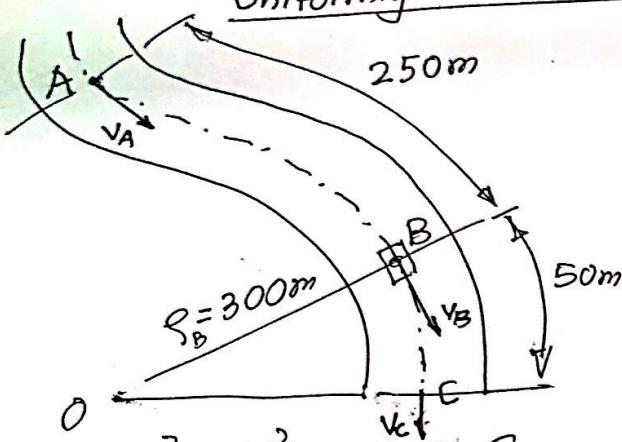
$$a_t = 40 \text{ m/s}^2$$

$$a_n = 8 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = 40.792 \text{ m/s}^2$$

③ RCH/F 12.29/pg. 678:

Uniformly accelerated curvilinear motion



$$V_A = 25 \text{ m/s}$$

$$V_C = 15 \text{ m/s}$$

$$S_c = 300 \text{ m}$$

$$V_C^2 = V_A^2 + 2 \cdot a_t \cdot S_c$$

$$15^2 = 25^2 + 2 \cdot a_t \cdot 300$$

$$\boxed{a_t = -0.666 \text{ m/s}^2}$$

$$V_B^2 = V_A^2 + 2 \cdot a_t \cdot S_B$$

$$= 25^2 - (2 \times 0.666 \times 250)$$

$$\therefore V_B = 17.078 \text{ m/s}$$

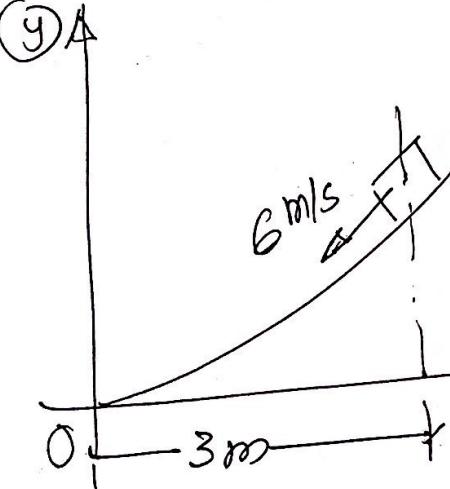
$$(a_n)_B = \frac{V_B^2}{S_B} = \left(\frac{17.078^2}{300} \right) = 0.972 \text{ m/s}^2$$

$$\therefore a_B = \sqrt{a_t^2 + a_n^2} = \sqrt{(-0.666)^2 + (0.972)^2}$$

$$\therefore a_B = 1.178 \text{ m/s}^2$$

④

RCH/F 12.30/pg. 678:



$$y = \frac{1}{8} \cdot x^2$$

$$\left(\frac{dy}{dx} \right) = y' = \frac{1}{4} x, \quad \left(\frac{d^2y}{dx^2} \right) = y'' = \frac{1}{4}$$

$$\text{At } x = 3 \text{ m}, \quad \left(\frac{dy}{dx} \right) = \frac{3}{4} = \tan \theta$$

$$\therefore \theta = 36.87^\circ$$

$$\textcircled{2} \quad f = \left| \frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}}{\frac{d^2y}{dx^2}} \right| = \frac{\left(1 + 0.75^2 \right)^{3/2}}{(1/4)} \quad 3/2$$

$$\therefore f = 7.8125 \text{ m}$$

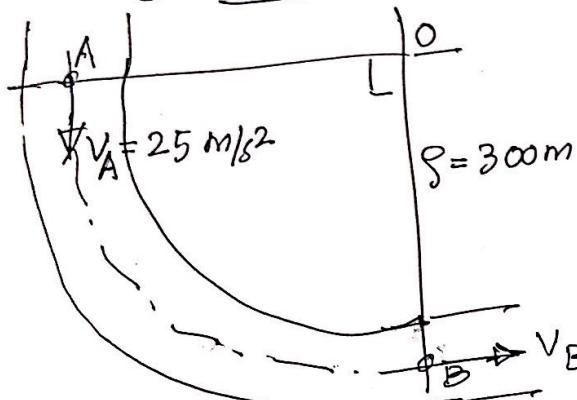
$$a_n = \frac{V^2}{S} = \left(\frac{6^2}{7.8125} \right) = 4.608 \text{ m/s}^2$$



$$a_t = 2 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{2^2 + 4.608^2} = 5.02 \text{ m/s}^2$$

(5) RCH/F 12.31 / pg. 678 :



$$a_t = -(0.001)s \text{ m/s}^2$$

$$\text{At } B, s = \theta = 300 \times \frac{\pi}{2}$$

$$s = 471.238 \text{ m}$$

$$(a_t)_B = -(0.001 \times 471.238) \\ = -0.4712 \text{ m/s}^2$$

$$a_t \cdot ds = v \cdot dv$$

$$-\int_0^{471.238} (0.001) s \cdot ds = \int_{25}^{v_B} v \cdot dv$$

$$-(0.001) \left(\frac{471.238^2}{2} \right) = \frac{1}{2} (v_B^2 - 625)$$

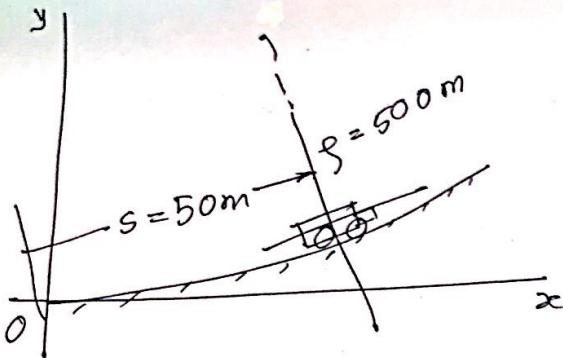
$$v_B = 20.07 \text{ m/s}$$

$$(a_n)_B = \frac{v_B^2}{S} = \left(\frac{20.07^2}{300} \right) = 1.343 \text{ m/s}^2$$

$$\therefore a_B = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{(-0.4712)^2 + (1.343)^2}$$

$$\therefore a_B = 1.423 \text{ m/s}^2$$

⑥ RCH/F 12.32 / Pg. 678 :



$$V = (0.2)S$$

$$a \cdot da = v \cdot dv$$

$$\text{Hence, } a_t ds = v \cdot dv$$

$$a_t = v \cdot \frac{dv}{ds} = (0.2 \times S)(0.2)$$

$$a_t = (0.04)S \text{ m/s}^2$$

$$\text{when } S = 50 \text{ m}, \quad a_t = (0.04 \times 50) = 2 \text{ m/s}^2$$

$$v = (0.2 \times 50) = 10 \text{ m/s}$$

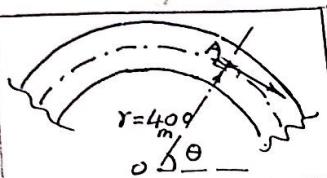
$$a_n = \frac{v^2}{r} = \frac{10^2}{500} = 0.2 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{2^2 + 0.2^2}$$

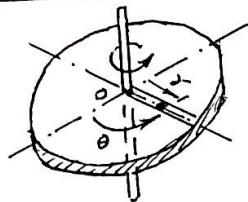
$$\approx 2.01 \text{ m/s}^2$$

(9)

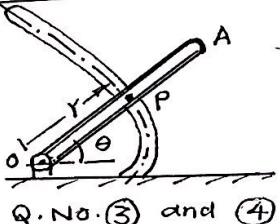
- 1) The car has a speed of 15m/s. Determine the angular velocity $\dot{\theta}$ of the radial line OA at this instant.
 Ans: $\dot{\theta} = 0.0375 \text{ rad/s}$



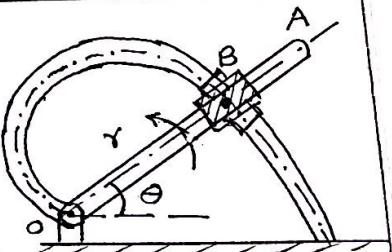
- 2) The platform is rotating about the vertical axis such that at any instant its angular position is $\Theta = (4t^{3/2}) \text{ rad}$, where t is in seconds. A ball rolls outward along the radial groove so that its position is $r = (0.1t^3) \text{ m}$, where t is in seconds. Determine the magnitudes of the velocity and acceleration of the ball when $t=1.5\text{s}$.
 Ans: $v = 2.57 \text{ m/s}; a = 20.4 \text{ m/s}^2$



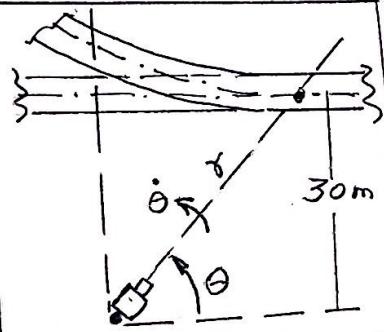
- 3) Peg P is driven by the fork line OA along the curved path described by $r = (2\Theta) \text{ m}$. At the instant $\Theta = \pi/4 \text{ rad}$, the angular velocity and angular acceleration of the link are $\dot{\theta} = 3 \text{ rad/s}$ and $\ddot{\theta} = 1 \text{ rad/s}^2$. Determine the magnitude of the pegs acceleration at this instant.
 Ans: $a = 39.5 \text{ m/s}^2$



- 4) Peg P is driven by the forked link OA along the path described by $r = e^\theta$, where r is in meters. When $\Theta = \pi/4 \text{ rad}$, the angular velocity and angular acceleration of the link are $\dot{\theta} = 2 \text{ rad/s}$ and $\ddot{\theta} = 4 \text{ rad/s}^2$. Determine the radial and transverse components of pegs acceleration at this instant.
 Ans: $a_r = 8.774 \text{ m/s}^2; a_\theta = 26.316 \text{ m/s}^2$



- 5) Collars are pin connected at B and are free to move along rod OA and the curved guide OC having the shape of a cardioid, $r = [0.2(1+\cos\Theta)] \text{ m}$. At $\Theta = 30^\circ$, the angular velocity of OA is $\dot{\theta} = 3 \text{ rad/s}$. Determine the magnitude of the velocity of the collars at this point.
 Ans: $v = 1.16 \text{ m/s}$



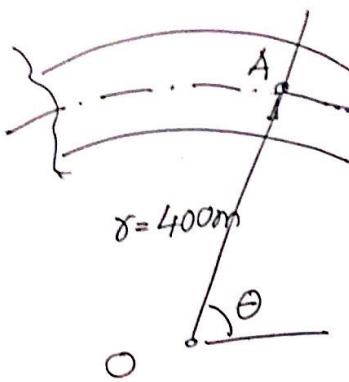
- 6) 2) At the instant $\Theta = 45^\circ$, the athlete is running with a constant speed of 2 m/s. Determine the angular velocity at which the camera must turn in order to follow the motion.
 Ans: $\dot{\theta} = 0.0333 \text{ rad/s}$

NAME	DEPARTMENT	SUBJECT	ACADEMIC YEAR	CLASS	ROLL NO.
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Lecture No. 0

① F 12.33 / RCH / Pg. 693



$$\bar{V} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$V_r = \dot{r} = 0$$

$$V_\theta = r\dot{\theta} = 400 \cdot \dot{\theta}$$

$$V = \sqrt{V_r^2 + V_\theta^2}$$

$$15 = \sqrt{0 + (400 \cdot \dot{\theta})^2}$$

$$\therefore \dot{\theta} = 0.0375 \text{ rad/s}$$

② RCH / F 12.34 / Pg. 693

$$r = (0.1)t^3 \quad \text{At } t = 1.5 \text{ s}$$

$$\dot{r} = (0.3)t^2 = 0.675 \text{ m/s}$$

$$\ddot{r} = (0.6)t = 0.900 \text{ m/s}^2$$

$$\theta = 4t^{3/2} = 7.348 \text{ rad}$$

$$\dot{\theta} = 6t^{1/2} = 7.348 \text{ rad/s}$$

$$\ddot{\theta} = 3t^{-1/2} = 2.449 \text{ rad/s}^2$$

$$\alpha_r = (\ddot{r} - r\dot{\theta}^2) = (0.9 - 0.3375 \times 7.348^2) \quad \boxed{\therefore V = 2.57 \text{ m/s}}$$

$$= -17.325 \text{ m/s}^2$$

$$\alpha_\theta = (r\ddot{\theta} + 2\dot{r}\dot{\theta}) = (0.3375 \times 2.449) + (2 \times 0.675 \times 7.348)$$

$$= 10.747 \text{ m/s}^2$$

$$\alpha = \sqrt{\alpha_r^2 + \alpha_\theta^2} = \sqrt{(-17.325)^2 + (10.747)^2}$$

$$\therefore \boxed{\alpha = 20.4 \text{ m/s}^2}$$

③ RCH/F 12.35/pg. 693 :

$$\begin{aligned} r &= 2\theta & \dot{\theta} &= 3 \text{ rad/s} \\ \dot{r} &= 2\dot{\theta} & \ddot{\theta} &= 1 \text{ rad/s}^2 \\ \ddot{r} &= 2\ddot{\theta} & \theta &= (\pi/4) \text{ Rad} \end{aligned}$$

$$\therefore r = \left(2 \times \frac{\pi}{4}\right) = 1.57 \text{ m}$$

$$\dot{r} = (2 \times 3) = 6 \text{ m/s}$$

$$\ddot{r} = (2 \times 1) = 2 \text{ m/s}^2$$

$$a_r = (\ddot{r} - r\dot{\theta}^2) = 2 - (1.57 \times 3^2) = -12.14 \text{ m/s}^2$$

$$a_\theta = (r\ddot{\theta} + 2\dot{r}\dot{\theta}) = (1.57 \times 1) + (2 \times 6 \times 3) \\ = 37.57 \text{ m/s}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-12.14)^2 + (37.57)^2}$$

$$a = 39.5 \text{ m/s}^2$$

④ RCH/F 12.36/pg. 693

$$r = e^\theta$$

$$\theta = \pi/4 \text{ rad}$$

$$\dot{r} = (e^\theta)(\dot{\theta})$$

$$\dot{\theta} = 2 \text{ rad/s}$$

$$\begin{aligned} \ddot{r} &= (e^\theta)(\dot{\theta}') + (e^\theta)(\dot{\theta}^2) \\ &= e^\theta(\dot{\theta}^2 + \ddot{\theta}) \end{aligned}$$

$$\ddot{\theta} = 4 \text{ rad/s}^2$$

$$\therefore \dot{r} = 2e^{\pi/4} = 2 \times 2.193 \\ = 4.386 \text{ m/s}$$

$$\begin{aligned} \therefore a_r &= (\ddot{r} - r\dot{\theta}^2) = (17.546 - 2.193 \times 4) \\ &= 8.774 \text{ m/s}^2 \end{aligned}$$

$$\dot{\theta} = \frac{\pi}{4}(4+4) \\ = 17.546 \text{ m/s}$$

$$\begin{aligned} a_\theta &= (r\ddot{\theta} + 2\dot{r}\dot{\theta}) \\ &= (2.193 \times 4 + 2 \times 4.386 \times 2) \\ &= 26.316 \text{ m/s}^2 \end{aligned}$$

$$r = e^{\pi/4} = 2.193 \text{ m}$$

(5) RCH/F 12.37 / Pg. 693

$$r = (0.2)(1 + \cos\theta) \text{ m}$$

$$\dot{r} = -(0.2)(\sin\theta)\dot{\theta} \text{ m/s}$$

$$\ddot{r} = -(0.2)(\sin\theta)\ddot{\theta} + (\cos\theta)(\dot{\theta}^2) \text{ m/s}^2$$

$$\text{when } \theta = 30^\circ \quad r = 0.2732 \text{ m}$$

$$\dot{r} = -0.3 \text{ m/s}$$

$$V_r = \dot{r} = -0.3 \text{ m/s}$$

$$V_\theta = r\dot{\theta} = (0.2732 \times 3) = 1.120 \text{ m/s}$$

$$\therefore V = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$V = -(0.3)\hat{e}_r + (1.120)\hat{e}_\theta \text{ m/s}$$

$$V = \sqrt{(-0.3)^2 + (1.120)^2} = 1.16 \text{ m/s}$$

(6) RCH/F 12.38 / Pg. 693

$$r = 30 \cdot \operatorname{cosec}\theta = \frac{30}{\sin\theta}$$

$$\theta = 45^\circ$$

$$\dot{\theta} = ?$$

$$\ddot{r} = -30 \cdot \operatorname{cosec}\theta \cdot \cot\theta \cdot \dot{\theta}$$

$$\text{when } \theta = 45^\circ = 0.7854 \text{ rad}$$

$$r = 42.426 \text{ m}, \dot{r} = \frac{-80}{\sin 45^\circ} \times \frac{1}{\tan 45^\circ} \cdot \dot{\theta}$$

$$V_r = \dot{r} = -(42.426)\dot{\theta} = -(42.426)\dot{\theta}$$

$$V_\theta = r\dot{\theta} = (42.426)\dot{\theta}$$

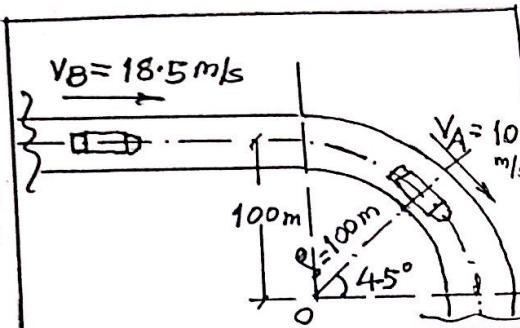
$$V = 2 \text{ m/s} = \sqrt{V_r^2 + V_\theta^2}$$

$$2 = \sqrt{(-42.426\dot{\theta})^2 + (42.426\dot{\theta})^2}$$

$$\dot{\theta} = 0.0333 \text{ rad/s}$$

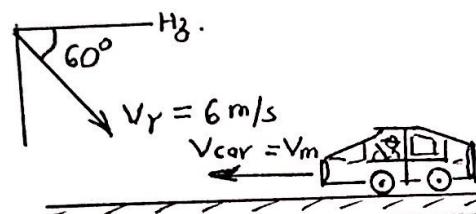
- 1 At the instant shown, the car at A is traveling at 10 m/s around the curve while increasing its speed at 5 m/s^2 . The car at B is travelling at 18.5 m/s along the straightway & increasing its speed at 2 m/s^2 . Determine the relative velocity & relative acceleration of A with respect to B at this instant.

$$\text{Ans: } v_{A/B} = 13.44 \text{ m/s}, a_{A/B} = 4.322 \text{ m/s}^2$$



- 2 The car is travelling at a constant speed of 100km/h. if the rain is falling at 6 m/s in the direction shown, determine the velocity of the rain as seen by the driver.

$$\text{Ans: } v_{r/m} = 31.212 \text{ m/s}$$

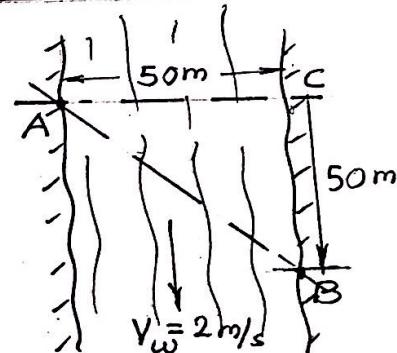


- 3 The car is travelling north along a straight at 50 kmph. An instrument in the car indicates that the wind is coming from the East. If the car's speed is 80kmph, the instrument indicates that the wind coming from north east. Determine the speed and direction of the wind.

$$\text{Ans: } V_w = - (30) i + 50 j; V_w = 58.309 \text{ kmph}$$

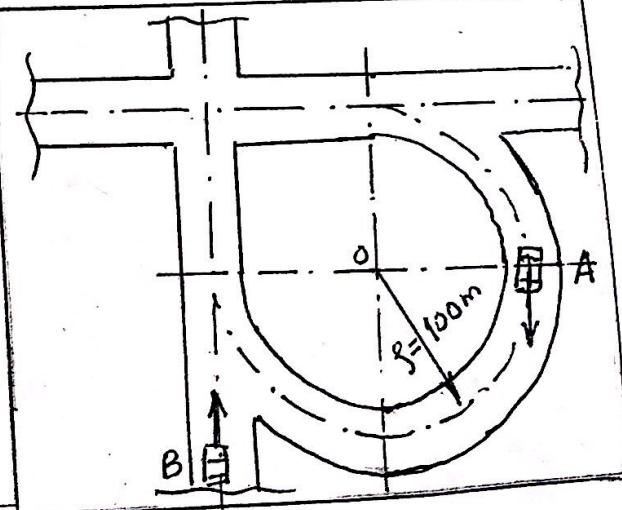
- 4 A man can row a boat at 5m/s in still water. He wishes to cross a 50-m-wide river to point B, 50 m downstream. If the river flows with a velocity of 2m/s, determine the speed of the boat and the time needed to make the crossing.

$$\text{Ans: } V_{RB} = 6.21 \text{ m/s}, t = 11.385 \text{ s}$$

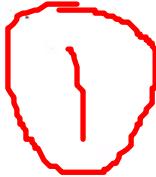
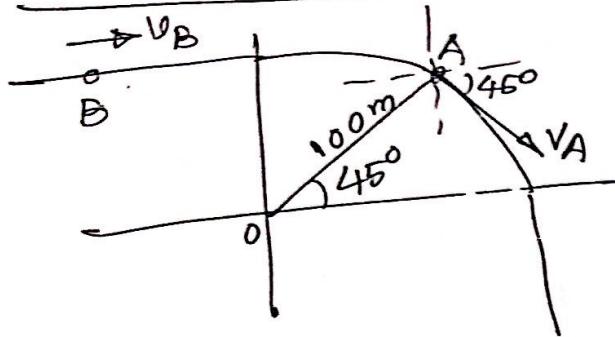


- 5 At the instant shown, car A has a speed of 20kmph, which is been increased at the rate of 300 km/h^2 as the car enters the expressway. At the same instant, car B is decelerating at 250 km/h^2 while travelling forward AT 100kmph. Determine the velocity & acceleration of A with respect to B.

$$\text{Ans: } V_{A/B} = -33.332 \text{ m/s} = 0 \text{ kmph}; a_{A/B} = 0.3085 \text{ m/s}^2$$



RCH / 12.217 / Pg. 715



$$v_B = 18.5 \text{ m/s} \quad \therefore \bar{v}_B = (18.5) \hat{i}$$

$$a_B = 2 \text{ m/s}^2 \quad \therefore \bar{a}_B = (2.0) \hat{i}$$

$$v_A = 10 \text{ m/s} \angle 45^\circ \quad \therefore \bar{v}_A = (7.07) \hat{i} - (7.07) \hat{j} \text{ m/s}$$

$$a_A \Rightarrow (a_t)_A = 5 \text{ m/s}^2 \angle 45^\circ$$

$$(a_n)_A = \frac{V^2}{r} = \frac{100^2}{100} = 1 \text{ m/s}^2 \angle 45^\circ$$

$$(\bar{a}_t)_A = (3.535) \hat{i} - (3.535) \hat{j}$$

$$(\bar{a}_n)_A = -(0.707) \hat{i} - (0.707) \hat{j}$$

$$\bar{a}_A = (2.828) \hat{i} - (4.242) \hat{j} \text{ m/s}^2$$

$$\bar{v}_{A/B} = (\bar{v}_A - \bar{v}_B) = -(11.43) \hat{i} - (17.07) \hat{j}$$

$$31.74^\circ \quad v_{A/B} = 13.44 \text{ m/s}$$

$$\bar{a}_{A/B} = (\bar{a}_A - \bar{a}_B) = + (0.828) \hat{i} - (4.242) \hat{j} \text{ m/s}^2$$

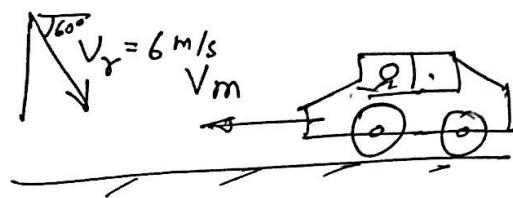
$$78.95^\circ$$

$$a_{A/B} = 4.322 \text{ m/s}^2$$

RCH / 12. 219 / Pg. 715 :



1



$$V_{man} = 100 \text{ kmph} = 27.777 \text{ m/s}$$

$$\vec{V}_m = -(27.777) \hat{i}$$

$$\vec{V}_{rain/man} = \vec{V}_{r/m} = \vec{V}_r - \vec{V}_m$$

$$\vec{V}_{r/m} = (3) \hat{i} - (5.196) \hat{j} + (27.777) \hat{k}$$
$$= (3) \hat{i} - (32.973) \hat{j} \text{ m/s}$$

$$\sqrt{34.8^2} = 33.109 \text{ m/s}$$

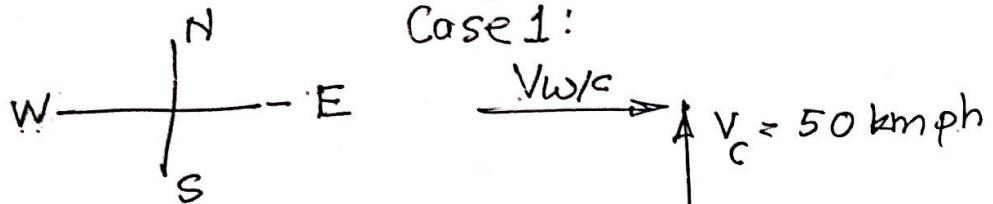
$$= (30.777) \hat{i} - (5.196) \hat{j} \text{ m/s}$$

$$53.48^\circ$$

$$V_{r/m} = 31.212 \text{ m/s}$$

$$9.58^\circ$$

RCH/12.221/pg.716 :



Case 2:

$V_c = 80 \text{ km/h}$

For case ①

The diagram shows a vertical axis representing velocity and a horizontal dashed line representing wind velocity. A vector V_w/c is shown at an angle of 45° above the horizontal dashed line. A vector V_c is shown vertically upwards along the vertical axis.

$$\bar{V}_{\omega/c} = \bar{V}_\omega - \bar{V}_c$$

$$\bar{V}_\omega = \bar{V}_{\omega/c} + \bar{V}_c$$

$$\bar{V}_\omega = (V_{\omega/c})^i + (B) j$$

$$\text{For case ② } \bar{V}_{\infty} = - (0.707 \times V_{w/c_2})^{\hat{i}} + (0.707 \times V_{w/c_2})^{\hat{j}} + 80 \hat{j}$$

Equating the coeff. of \hat{x}^j

$$(\nu_{w/c})_1 = - (0.707) (\nu_{w/c})_2 \rightarrow ①$$

$$50 = -(707)(V_w/c_2) + 80$$

$$\therefore +30 = (0.707)(V_{WTC_2})$$

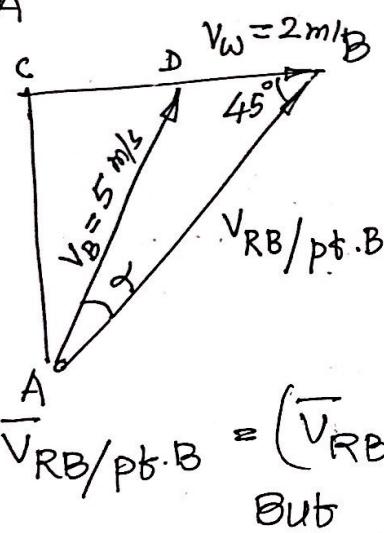
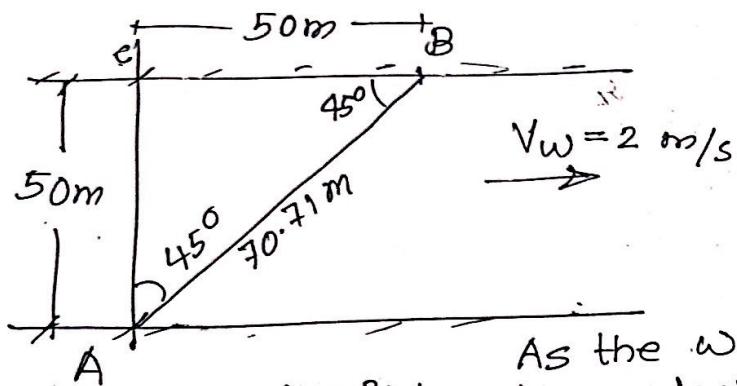
$$\therefore (\omega/c)_2 = 42.432 \text{ rad/s}$$

$$(w/c)_1 = -30 \text{ m/s}$$

$$V_w = 58.309 \text{ kmph}$$

$$\underline{Ans}: \bar{V}_W = -(30)\hat{i} + (50)\hat{j}$$

14



As the water current has velocity of 2 m/s, boat can never cross the river in \perp direction. But, it will make some angle α , with the resultant velocity.

$$\overline{V}_{RB/pt.B} = \left(\overline{V}_{RB} - \cancel{\overline{V}_{pt.B}} \right) = \overline{V}_B + \overline{V}_w$$

$$\text{But } V_{pt.B} = 0$$

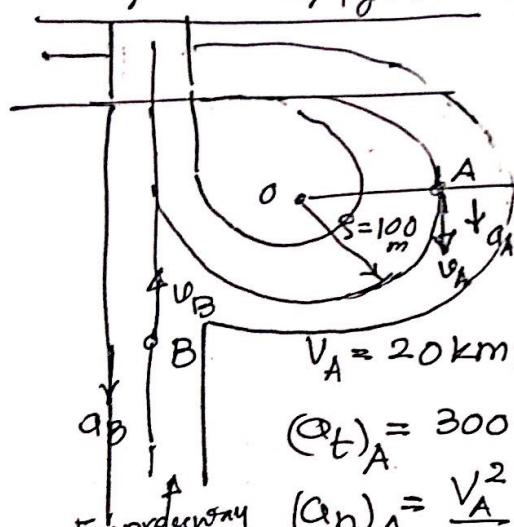
$$\text{In } \triangle ADB, \frac{5}{\sin 45^\circ} = \frac{2}{\sin \alpha} \therefore \alpha = 16.43^\circ$$

$$\text{and } \frac{5}{\sin 45^\circ} = \frac{V_{RB}}{\sin(180^\circ - 45^\circ - 16.43^\circ)}$$

$$\therefore V_{RB} = 6.21 \text{ m/s}$$

Time reqd. to cross the river & to reach pt. B, $t = \frac{70.71 \text{ m}}{6.21 \text{ m/s}} = 11.385 \text{ sec.}$

RCH/12.225/pg. 716 :



5

$$V_A = 20 \text{ kmph} = 5.555 \text{ m/s}$$

$$(\alpha_t)_A = 300 \text{ kmph}^2 = 0.023 \text{ m/s}^2$$

$$\text{Expression for } (\alpha_n)_A = \frac{V_A^2}{r} = \left(\frac{5.555^2}{100} \right) = 0.3085 \text{ m/s}^2$$

$$V_B = 100 \text{ kmph} = 27.777 \text{ m/s}$$

$$a_B = 250 \text{ kmph}^2 = 0.019 \text{ m/s}^2$$

$$\therefore \bar{V}_A = 0\hat{i} + (5.555)\hat{j} \text{ m/s}$$

$$\bar{V}_B = 0\hat{i} + (27.777)\hat{j} \text{ m/s}$$

$$\therefore \bar{V}_{A/B} = (\bar{V}_A - \bar{V}_B) = -(33.332)\hat{j} \text{ m/s}$$

i.e. 0 kmph (\downarrow)

$$\bar{\alpha}_A = -(0.3085)\hat{i} + (0.023)\hat{j} \text{ m/s}^2$$

$$\bar{\alpha}_B = 0\hat{i} - (0.019)\hat{j} \text{ m/s}^2$$

$$\therefore \bar{\alpha}_{A/B} = (\bar{\alpha}_A - \bar{\alpha}_B)$$

$$= -(0.3085)\hat{i} - (0.004)\hat{j} \text{ m/s}^2$$

~~$$\alpha_{A/B} = 0.31134 \text{ m/s}^2$$~~

0.74°

$$\alpha_{A/B} = 0.3085 \text{ m/s}^2$$