Beta Function

Def^n :- A Beta function of m, n is denoted by $\beta(m,n)$ & defined as

$$\beta(m,n) = \int_0^1 x^{m-1} (1-x)^{n-1} \, dx \quad ; \quad m>0, \; n>0$$

The Beta function is also called as Euler's integral of the first kind.

Note That :- Beta function of negative number is not defined.

* Properties of Beta function :-

① $\beta(m,n) = \beta(n,m)$

Proof :- We know that $\beta(m,n) = \int_0^1 x^{m-1} (1-x)^{n-1} \, dx$ —— ①

put $1-x = t \Rightarrow x = 1-t$

$\Rightarrow dx = -dt \Rightarrow$

when $x =$	0	1
then $t =$	1	0

$\therefore$ eq^n ① becomes.

$$\beta(m,n) = \int_1^0 (1-t)^{m-1} \cdot t^{n-1} \, -dt$$

$$= \int_0^1 t^{n-1} (1-t)^{m-1} \, dt \quad \because \int_a^b = -\int_b^a$$

$$\beta(m,n) = \beta(n,m)$$

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2] $\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cdot \cos^{2n-1}\theta \, d\theta$

proof :-

We know that, $\beta(m,n) = \int_0^1 x^{m-1} (1-x)^{n-1} \, dx$ — (1)

put $x = \sin^2\theta$

$\Rightarrow dx = 2\sin\theta \cdot \cos\theta \, d\theta$

When $x =$	0	1
then $\theta =$	0	$\pi/2$

By eqn (1),

$$\beta(m,n) = \int_0^{\pi/2} (\sin^2\theta)^{m-1} \cdot (1-\sin^2\theta)^{n-1} \, 2\sin\theta \cdot \cos\theta \, d\theta$$

$$= 2\int_0^{\pi/2} \sin^{2m-2}\theta \cdot \cos^{2n-2}\theta \cdot \sin\theta \cdot \cos\theta \, d\theta$$

$$\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \, \cos^{2n-1}\theta \, d\theta$$

if we put $m = \frac{p+1}{2}$, $n = \frac{q+1}{2}$ or $\begin{bmatrix} 2m-1 = p \\ 2n-1 = q \end{bmatrix}$

$$\beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right) = 2\int_0^{\pi/2} \sin^p\theta \cdot \cos^q\theta \, d\theta$$

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3] $\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\, dx$

Proof :

We know that $\beta(m,n) = \int_0^1 x^{m-1} (1-x)^{n-1}\, dx$ — (1)

put $x = \frac{t}{1+t}$,

$$dx = \frac{(1+t)\,dt - t \cdot dt}{(1+t)^2}$$

$$dx = \frac{dt + t\,dt - t\,dt}{(1+t)^2}$$

$$dx = \frac{dt}{(1+t)^2}$$

When $x =$	0	1
then $t =$	0	∞

so eq^n (1) becomes,

$$\beta(m,n) = \int_0^\infty \left(\frac{t}{1+t}\right)^{m-1} \left(1 - \frac{t}{1+t}\right)^{n-1} \frac{dt}{(1+t)^2}$$

$$= \int_0^\infty \frac{t^{m-1}}{(1+t)^{m-1}} \cdot \frac{(1+t-t)^{n-1}}{(1+t)^{n-1}} \frac{dt}{(1+t)^2}$$

$$= \int_0^\infty \frac{t^{m-1}}{(1+t)^{m+n}}\, dt$$

$$\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\, dx$$ ... As the variable is immaterial in definite integral.

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4]

Relation between Beta and Gamma function :-

$$\beta(m,n) = \frac{\Gamma m \cdot \Gamma n}{\Gamma m+n}$$

5] prove that using Beta function $\Gamma 1/2 = \sqrt{\pi}$

proof :-

We know that

$$\int_0^{\pi/2} \sin^p\theta \cdot \cos^q\theta \, d\theta = \frac{1}{2}\beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$$

put $p = 0 = q$

$$\int_0^{\pi/2} d\theta = \frac{1}{2}\beta\left(\frac{1}{2}, \frac{1}{2}\right)$$

$$2\left(\frac{\pi}{2}\right) = \frac{\Gamma 1/2 \, \Gamma 1/2}{\Gamma 1/2 + 1/2}$$

$$\pi = \left(\Gamma 1/2\right)^2 \Rightarrow \boxed{\Gamma \frac{1}{2} = \sqrt{\pi}}$$

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Examples on Beta Function

Type - 1

Evaluate $\int_0^{1/2} x^3 (1-4x^2)^{1/2} \, dx$ — (1)

put $4x^2 = t \Rightarrow x = \frac{\sqrt{t}}{2}$

$\Rightarrow dx = \frac{+1}{4\sqrt{t}} dt$

when $x =$	0	1/2
then $t =$	0	1

$\therefore$ eqn (1) becomes

$$\int_0^{1/2} x^3 (1-4x^2)^{1/2} \, dx = \int_0^1 \left(\frac{\sqrt{t}}{2}\right)^3 (1-t)^{1/2} \frac{+1}{4\sqrt{t}} dt$$

$$= \int_0^1 \frac{(t)^{3/2}}{(t)^{1/2}} \cdot \frac{1}{(2)^3 \cdot 4} (1-t)^{1/2} \, dt$$

$$= \frac{1}{32} \int_0^1 t \cdot (1-t)^{1/2} \, dt$$

$$= \frac{1}{32} \int_0^1 t^{2-1} (1-t)^{\frac{3}{2}-1} \, dt$$

$$= \frac{1}{32} \beta\left(2, \frac{3}{2}\right)$$

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But we know that

$$\beta(m,n) = \frac{\Gamma m \, \Gamma n}{\Gamma m+n}, \quad \text{Here, } m=2, \; n=\frac{3}{2}$$

$$\int_0^{1/2} x^3 (1-4x^2)^{1/2} \, dx = \frac{1}{32}\left(\frac{\Gamma 2 \; \Gamma 3/2}{\Gamma 2+3/2}\right)$$

$$= \frac{1}{32}\left(\frac{\Gamma 2 \cdot 1/2 \cdot \Gamma 1/2}{\Gamma 7/2}\right)$$

$$= \frac{1}{32}\left(\frac{1 \cdot 1/2 \cdot \cancel{\sqrt{\pi}}}{5/2 \cdot 3/2 \cdot 1/2 \cdot \cancel{\sqrt{\pi}}}\right) \quad \because \Gamma n+1 = n\Gamma n$$

$$= \frac{1}{\cancel{32}\,4}\left(\frac{\cancel{8}\,1}{30}\right)$$

$$\int_0^{1/2} x^3 (1-4x^2)^{1/2} \, dx = \frac{1}{120}$$

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2] Evaluate $\int_0^2 x \cdot \sqrt[3]{8-x^3}\, dx$ — (1)

put $x^3 = 8t \Rightarrow x = 2(t)^{1/3}$

$\Rightarrow dx = \frac{2}{3}(t)^{-2/3}\, dt$

Now

when $x =$	0	2
then $t =$	0	1

so, eqn (1) becomes,

$$\int_0^2 x \cdot \sqrt[3]{8-x^3}\, dx = \int_0^1 2(t)^{1/3} \cdot (8-8t)^{1/3} \cdot \frac{2}{3} t^{-2/3}\, dt$$

$$= \frac{4}{3}\int_0^1 (t)^{1/3} \cdot (8)^{1/3} (1-t)^{1/3} \cdot t^{-2/3}\, dt$$

$$= \frac{8}{3}\int_0^1 t^{-1/3} \cdot (1-t)^{1/3}\, dt$$

$$= \frac{8}{3}\int_0^1 t^{2/3-1} \cdot (1-t)^{4/3-1}\, dt$$

$$= \frac{8}{3}\beta\left(\frac{2}{3}, \frac{4}{3}\right)$$

$$= \frac{8}{3}\left(\frac{\Gamma 2/3 \; \Gamma 4/3}{\Gamma 2/3 + 4/3}\right)$$

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$$\int_0^2 x \cdot \sqrt[3]{8-x^3}\, dx = \frac{8}{3}\left(\frac{\Gamma 2/3 \;\; \Gamma 4/3}{\Gamma 6/3}\right)$$

$$= \frac{8}{3}\left(\frac{\Gamma 2/3 \cdot \Gamma 4/3}{\Gamma 2}\right)$$

$$= \frac{8}{3}\left(\Gamma\frac{2}{3} \cdot \Gamma\frac{4}{3}\right)$$

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3] Evaluate $\int_0^1 \frac{dx}{\sqrt[3]{1-x^3}}$ —— (1)

put $x^3 = t \Rightarrow x = t^{1/3}$

$\Rightarrow dx = \frac{1}{3} t^{-2/3} dt$

Now,

when $x =$	0	1
then $t =$	0	1

so eq^n (1) becomes

$$\int_0^1 \frac{dx}{(1-x^3)^{1/3}} = \int_0^1 \frac{1}{(1-t)^{1/3}} \cdot \frac{1}{3} t^{-2/3} dt$$

$$= \frac{1}{3} \int_0^1 (1-t)^{-1/3} \cdot t^{-2/3} dt$$

$$= \frac{1}{3} \int_0^1 t^{1/3-1} (1-t)^{2/3-1} dt$$

$$= \frac{1}{3} \beta\left(\frac{1}{3}, \frac{2}{3}\right)$$

$$= \frac{1}{3} \left(\frac{\Gamma 1/3 \cdot \Gamma 2/3}{\Gamma 1/3 + 2/3} \right)$$

$$= \frac{1}{3} \left(\frac{\Gamma 1/3 \cdot \Gamma 2/3}{\Gamma 1} \right)$$

$$= \frac{1}{3} \left(\Gamma \frac{1}{3} \cdot \Gamma \frac{2}{3} \right)$$

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But we know that

$$\Gamma p \cdot \Gamma 1-p = \frac{\pi}{\sin p\pi} \quad \text{where } 0 < p < 1$$

Here $p = \frac{1}{3}$

$$\int_0^1 \frac{dx}{(1-x^3)^{1/3}} = \frac{1}{3}\left(\Gamma\frac{1}{3} \cdot \Gamma 1-\frac{1}{3}\right) \quad \ldots \quad 0 < \frac{1}{3} < 1$$

$$= \frac{1}{3}\left(\frac{\pi}{\sin(\pi/3)}\right) \quad \ldots \quad \text{Using above Formula}$$

$$= \frac{1}{3}\frac{\pi}{(\sqrt{3}/2)}$$

$$= \frac{2\pi}{3\sqrt{3}}$$

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Examples For Home work

① Evaluate

$$\int_0^1 x^3 (1-\sqrt{x})^5 \, dx$$

Answer $= \dfrac{1}{5148}$

② prove that

$$\int_0^1 (1-x^{1/n})^m \, dx = \frac{m!\,n!}{(m+n)!}$$

Hint :- $x^{1/n} = t$

③ Evaluate :-

$$\int_0^3 x^3 (3-x)^2 \, dx$$

Hint put $x = 3t$

Answer $= 3^6 \cdot \beta(4,3)$

$$= \frac{3^6 \cdot 3!\,2!}{6!}$$

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Type : 2

Evaluate $\int_0^\infty \frac{dx}{1+x^4}$ ——— (i)

put $x^2 = \tan\theta \Rightarrow x = \sqrt{\tan\theta} = (\tan\theta)^{1/2}$

$\Rightarrow dx = \frac{1}{2}(\tan\theta)^{-1/2} \cdot \sec^2\theta \, d\theta$

Now

when $x =$	0	∞
then $\theta =$	0	$\pi/2$

So, eq^n (i) becomes

$$\int_0^\infty \frac{dx}{1+x^4} = \int_0^{\pi/2} \frac{1}{1+\tan^2\theta} \cdot \frac{1}{2}(\tan\theta)^{-1/2} \cdot \sec^2\theta \, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} \frac{\sec^2\theta}{\sec^2\theta} \cdot \left(\frac{\sin\theta}{\cos\theta}\right)^{-1/2} d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} \sin^{-1/2}\theta \cdot \cos^{1/2}\theta \, d\theta$$

Now we use, $\int_0^{\pi/2} \sin^p\theta \cdot \cos^q\theta \, d\theta = \frac{1}{2}\beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$

Here, $p = -1/2$, $q = 1/2$

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$$\therefore \int_0^{\pi/2} \frac{dx}{1+x^4} = \frac{1}{2}\left[\beta\left(\frac{-1/2+1}{2}, \frac{1/2+1}{2}\right)\right]$$

$$= \frac{1}{2}\cdot\frac{1}{2}\beta\left(\frac{1}{4}, \frac{3}{4}\right)$$

$$= \frac{1}{4}\left[\frac{\Gamma 1/4 \; \Gamma 3/4}{\Gamma 1/4 + 3/4}\right] \quad \ldots \quad \beta(m,n) = \frac{\Gamma m \; \Gamma n}{\Gamma m+n}$$

$$= \frac{1}{4}\left[\frac{\Gamma 1/4 \; \Gamma 1-1/4}{\Gamma 1}\right]$$

$$= \frac{1}{4}\left[\Gamma\frac{1}{4} \; \Gamma 1-\frac{1}{4}\right] \quad \because 0 < \frac{1}{4} < 1$$

$$= \frac{1}{4}\left[\frac{\pi}{\sin(\pi/4)}\right] \quad \ldots \because \Gamma p \cdot \Gamma 1-p = \frac{\pi}{\sin p\pi}, \; 0<p<1$$

$$= \frac{1}{4}\left(\frac{\pi}{1/\sqrt{2}}\right)$$

$$= \frac{\sqrt{2}\cdot\pi}{4} = \frac{\sqrt{2}\;\pi}{2\cdot\sqrt{2}\cdot\sqrt{2}}$$

$$= \frac{\pi}{2\sqrt{2}}$$

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2]

Prove that, $\int_0^{\pi/2} \tan^n x \, dx = \frac{\pi}{2} \sec\left(\frac{n\pi}{2}\right)$, $-1 < n < 1$

$$L.H.S = \int_0^{\pi/2} \tan^n x \, dx = \int_0^{\pi/2} \left(\frac{\sin x}{\cos x}\right)^n dx$$

$$= \int_0^{\pi/2} \sin^n x \cdot \cos^{-n} x \, dx$$

$$= \frac{1}{2} \beta\left(\frac{1+n}{2}, \frac{1-n}{2}\right)$$

$$= \frac{1}{2}\left(\frac{\Gamma\frac{1+n}{2} \, \Gamma\frac{1-n}{2}}{\Gamma\left(\frac{1+n}{2} + \frac{1-n}{2}\right)}\right)$$

$$= \frac{1}{2}\left(\Gamma\frac{1+n}{2} \cdot \Gamma\frac{1-n}{2}\right)$$

$$= \frac{1}{2}\left(\Gamma\frac{1+n}{2} \, \Gamma\left(1 - \left(\frac{1+n}{2}\right)\right)\right) \qquad \because p = \frac{1+n}{2}$$

$$= \frac{1}{2}\left(\frac{\pi}{\sin\left(\frac{1}{2} + \frac{n}{2}\right)\pi}\right)$$

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$$= \frac{1}{2} \frac{\pi}{\sin\left(\frac{\pi}{2} + \frac{n\pi}{2}\right)}$$

$$= \frac{\pi}{2\cos\left(\frac{n\pi}{2}\right)}$$

$$= \frac{\pi}{2} \sec\left(\frac{n\pi}{2}\right)$$

$$= \text{R.H.S.}$$

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3] Evaluate $\int_0^\infty \left(\frac{x}{1+x^2}\right)^6 dx$

put $x = \tan\theta$

$dx = \sec^2\theta \, d\theta$

When $x =$	0	∞
then $\theta =$	0	$\pi/2$

Now eqn (1) becomes,

$$\int_0^\infty \left(\frac{x}{1+x^2}\right)^6 dx = \int_0^{\pi/2} \left(\frac{\tan\theta}{1+\tan^2\theta}\right)^6 \cdot \sec^2\theta \, d\theta$$

$$= \int_0^{\pi/2} \frac{\tan^6\theta}{\sec^{12}\theta} \cdot \sec^2\theta \, d\theta$$

$$= \int_0^{\pi/2} \frac{\sin^6\theta}{\cos^6\theta} \cdot \cos^{10}\theta \, d\theta$$

$$= \int_0^{\pi/2} \sin^6\theta \cos^4\theta \, d\theta$$

$$= \frac{1}{2} \beta\left(\frac{6+1}{2}, \frac{4+1}{2}\right)$$

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$$\int_0^\infty \left(\frac{x}{1+x^2}\right)^6 dx = \frac{1}{2}\beta\left(\frac{7}{2}, \frac{5}{2}\right)$$

$$= \frac{1}{2}\left[\frac{\Gamma 7/2 \cdot \Gamma 5/2}{\Gamma 7/2 + 5/2}\right]$$

$$= \frac{1}{2}\left[\frac{5/2 \cdot 3/2 \cdot \frac{1}{2}\sqrt{\pi} \cdot 3/2 \cdot 1/2 \sqrt{\pi}}{\Gamma 6}\right]$$

$$= \frac{1}{2 \cdot 5!}\left[\frac{45}{2^5}\pi\right]$$

$$= \frac{45\pi}{120 \cdot 2^6}$$

$$= \frac{3\pi}{8 \cdot 64}$$

$$\int_0^\infty \left(\frac{x}{1+x^2}\right)^6 dx = \frac{3\pi}{512}$$

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Home work

① Evaluate following

① $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta$ Answer $= \frac{\pi}{\sqrt{2}}$

② $\int_0^1 \sqrt{1-x^4}\, dx$ Answer $= \frac{1}{12}\sqrt{\frac{2}{\pi}} \cdot \left(\Gamma\frac{1}{4}\right)^2$

Hint put $x^4 = \sin^2\theta$

③ prove that

$$\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\, dx = \beta(m,n)$$

Hint put $x = \tan^2\theta$

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Type - 3

Problems having general Form,

$$\int_a^b (x-a)^l (b-x)^m \, dx$$

In this type, use standard substitution

$$\boxed{(x-a) = (b-a)t}$$

Examples. 1

Evaluate $\int_3^7 (x-3)^{1/4} (7-x)^{1/4} \, dx$ ——— (1)

use standard substitution, $(x-3) = (7-3)t$

$x-3 = 4t \Rightarrow x = 4t+3$

$\Rightarrow dx = 4\,dt$

=

when $x =$	3	7
then $t =$	0	1

so, eqn (1) becomes,

$$\int_3^7 (x-3)^{1/4} (7-x)^{1/4} \, dx = \int_0^1 (4t)^{1/4} \cancel{(7-(x-3))} (7-(4t+3))^{1/4} \, 4\,dt$$

$$= (4)^{1/4} \cdot 4 \int_0^1 t^{1/4} (4-4t)^{1/4} \, dt$$

$$= (4)^{1/4} \cdot 4 \cdot (4)^{1/4} \int_0^1 t^{1/4} (1-t)^{1/4} \, dt$$

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$$= 4^{3/2} \int_0^1 t^{5/4-1} \cdot (1-t)^{5/4-1} \, dt$$

$$= (4)^{3/2} \, \beta\left(\frac{5}{4}, \frac{5}{4}\right)$$

$$= (4)^{3/2} \left(\frac{\Gamma 5/4 \; \Gamma 5/4}{\Gamma 5/4 + 5/4} \right)$$

$$= (4)^{3/2} \frac{\left(\Gamma 5/4\right)^2}{\Gamma 5/2}$$

$$= (4)^{3/2} \frac{\left(\Gamma 5/4\right)^2}{3/2 \cdot 1/2 \cdot \sqrt{\pi}}$$

$$= \frac{(4)^{3/2} \cdot 4 \left(\Gamma 5/4\right)^2}{3\sqrt{\pi}}$$

$$= \frac{(2)^3 \cdot 4 \left(\Gamma 5/4\right)^2}{3\sqrt{\pi}}$$

$$= \frac{32 \left(\Gamma 5/4\right)^2}{3\sqrt{\pi}}$$

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* Examples for Home work

① $\int_3^7 \sqrt{(x-3)(7-x)}\, dx$ Answer is 2π

② $\int_5^9 \sqrt[4]{(9-x)(x-5)}\, dx$ Answer is $\frac{8\left(\Gamma 5/4\right)^2}{3\sqrt{\pi}}$

③ Evaluate $\int_2^5 (x-2)^{1/4} (5-x)^{1/4}\, dx$

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Type - 4

Revision :- $\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx$

Examples

Evaluate $\int_0^\infty \frac{x^8(1-x^6)}{(1+x)^{24}}\,dx$

$$I = \int_0^\infty \frac{x^8 - x^{14}}{(1+x)^{24}}\,dx$$

$$= \int_0^\infty \frac{x^8}{(1+x)^{24}} - \int_0^\infty \frac{x^{14}}{(1+x)^{24}}$$

$$= \int_0^\infty \frac{x^{9-1}}{(1+x)^{9+15}} - \int_0^\infty \frac{x^{15-1}}{(1+x)^{15+9}}$$

We know that, $\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} = \beta(m,n)$

$$= \beta(9,15) - \beta(15,9)$$

$$= \beta(9,15) - \beta(9,15) \quad \because \beta(m,n) = \beta(n,m)$$

$$= 0.$$

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2] Evaluate $\int_0^\infty \frac{x^6 - x^3}{(1+x^3)^5} \cdot x^2 \, dx$ ——— (1)

Sol^n put $x^3 = t$

$\Rightarrow 3x^2 \, dx = dt \Rightarrow x^2 \, dx = \frac{dt}{3}$

Now

when $x =$	0	∞
then $t =$	0	∞

So, eq^n (1)

$$\int_0^\infty \frac{x^6 - x^3}{(1+x^3)^5} x^2 \, dx = \int_0^\infty \frac{t^2 - t}{(1+t)^5} \cdot \frac{dt}{3}$$

$$= \frac{1}{3}\left[\int_0^\infty \frac{t^2}{(1+t)^5} dt - \int_0^\infty \frac{t}{(1+t)^5} dt\right]$$

$$= \frac{1}{3}\left[\int_0^\infty \frac{t^{3-1}}{(1+t)^{3+2}} dt - \int_0^\infty \frac{t^{2-1}}{(1+t)^{2+3}} dt\right]$$

$$= \frac{1}{3}\left[\beta(3,2) - \beta(2,3)\right]$$

$$= 1/3 \left[\beta(2,3) - \beta(2,3)\right]$$

$$= 0.$$

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Evaluate

3]

$$\int_0^\infty \frac{x - x^3}{(1+x)^5}\, dx$$

$$I = \int_0^\infty \frac{x}{(1+x)^5}\, dx - \int_0^\infty \frac{x^3}{(1+x)^5}\, dx$$

$$= \int_0^\infty \frac{x^{2-1}}{(1+x)^{2+3}}\, dx - \int_0^\infty \frac{x^{4-1}}{(1+x)^{4+1}}\, dx$$

$$= \beta(2,3) - \beta(4,1)$$

$$= \frac{\Gamma 2 \cdot \Gamma 3}{\Gamma 2+3} - \frac{\Gamma 4 \, \Gamma 1}{\Gamma 4+1}$$

$$= \frac{1 \cdot 2}{4!} - \frac{3! \cdot 1}{4!}$$

$$= \frac{2}{24} - \frac{6}{24}$$

$$= \frac{-1}{6}$$

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* Examples For Home work.

(1) Evaluate $\int_0^\infty \frac{x^8 - x^5}{(1+x^3)^5}\, dx$ — Answer = 0.

(2) Evaluate $\int_0^\infty \frac{x + x^{1/2}}{(1+x)^4}\, dx$ Answer $= \frac{1}{6} + \frac{\pi}{16}$

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