Rectilinear Kinetics(Newton's Second Law of Motion

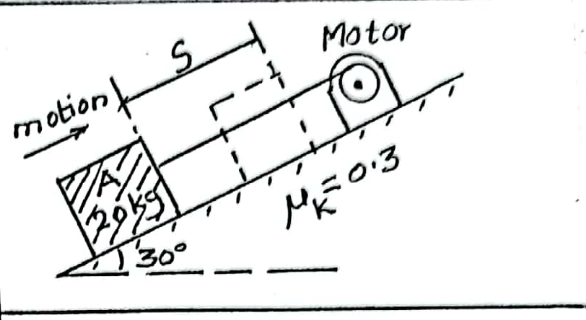
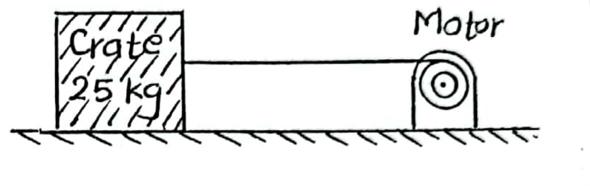
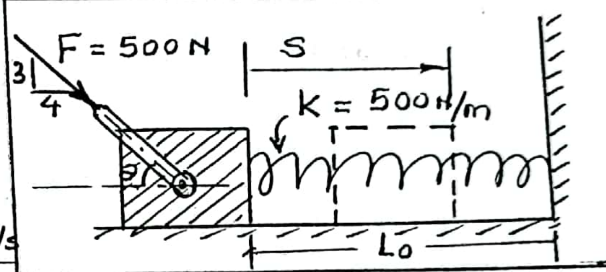
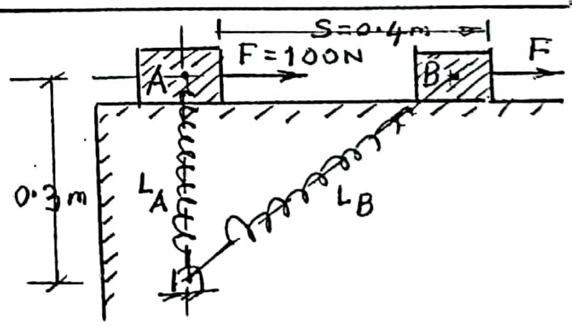
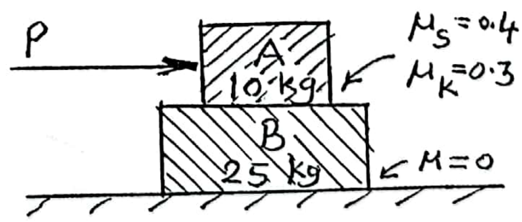
Curvilinear Kinetics (Newton’s Second Law)

work power energy and work energy principle

Conservation of Energy

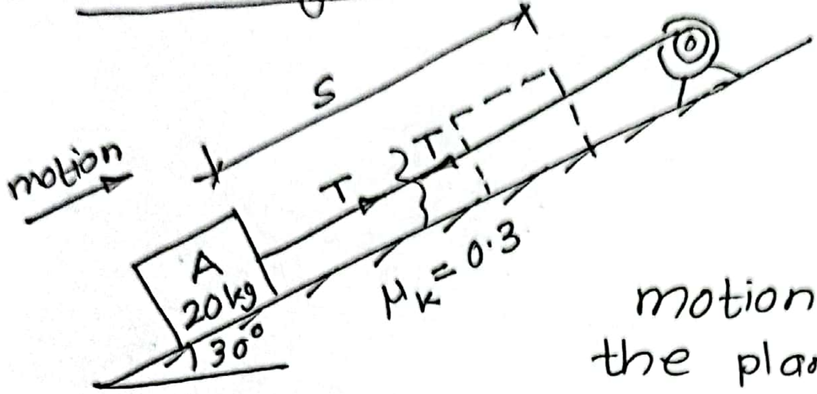
Impulse Momentum Principle

Lecture No: **Rectilinear Kinetics (Newton's Second Law of Motion)**

1	The motor winds in the cable with a constant acceleration, such that the 20-kg. Crate moves a distance s= 6m in 3 sec. starting from rest. Determine the tension developed in the cable. The coefficient of kinetic friction between the crate and the plane is $\mu_k = 0.3$ Ans. : $T = 176$ N	
2	If motor M exerts a force of $F = (10\,t^2 + 100)$ N on the cable, where t is in seconds, determine the velocity of the 25- kg. crate when t = 4 s. The coefficients of static and kinetic friction between the crate and the plane are $\mu_s = 0.3$ and $\mu_k = 0.25$ respectively. The crate is initially at rest. Ans. : $V_4 = 14.7$ m/s (→)	
3	A spring of stiffness k = 500 N/m is mounted against the 10 kg block. If the block is subjected to the force of F = 500 N, determine its velocity at s = 0.5m. When s=0, the block is at rest and the spring is uncompressed. The contact surface is smooth. Ans. : $V = 5.24$ m/s	
4	The 2 Mg. car is being towed by a winch. If the winch exerts a force of T=100 (s + 1) N on the cable where s is the displacement of the car in meters, determine the speed of the car when s= 10 m, starting from rest. Neglect rolling resistance of the car. Ans. : $V = 2.45$ m/s (→)	
5	The spring has a stiffness k = 200 N / m and is unstretched when the 25 kg block is at A. Determine the acceleration of the block when s = 0.4 m. The contact surface between the block and the plane is smooth. Ans. : $a = 2.72$ m/s² (→)	
6	Block B rests upon a smooth surface. If the coefficients of static and kinetic friction between A and B are $\mu_s = 0.4$ and $\mu_k = 0.3$, respectively, determine the acceleration of each block if P = 30N. Ans. : $a_A = a_B = 0.857$ m/s² (→)	

Lecture No: 13 D'Alembert's Principle/Newton's 2nd law (Rectillinear motion)

1 F 13.1/ pg. 742/RCH



$m = 20\,kg$

$u = 0$

a = constant

$s = 6\,m$ at $t = 3\,s$

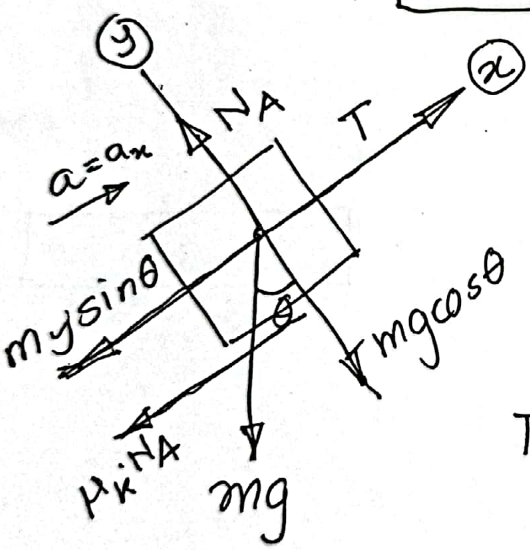
$\theta = 30^\circ$

motion of the crate along the plane is uni. acc. motion

$$s = ut + \frac{1}{2}at^2$$

$$6 = 0 + \frac{1}{2} \times a \times (3)^2$$

$$\boxed{a = 1.333\ m/s^2}$$



Along 'y' axis, $\Sigma F_y = m \cdot a_y$

But, $a_y = 0 \therefore \Sigma F_y = 0$

$\therefore N_A - (20 \times 9.81 \times \cos 30^\circ) = 0$

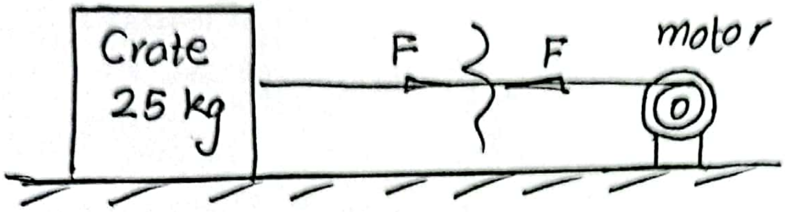
$$\boxed{N_A = 169.914\ N}$$

Along 'x' axis, $\Sigma F_x = m \cdot a_x$

$$T - (20 \times 9.81 \times \sin 30^\circ) - (0.3 \times 169.914) = (20 \times 1.333)$$

$$\therefore \boxed{T = 176\ N}$$

② F13.2/pg.742/RCH



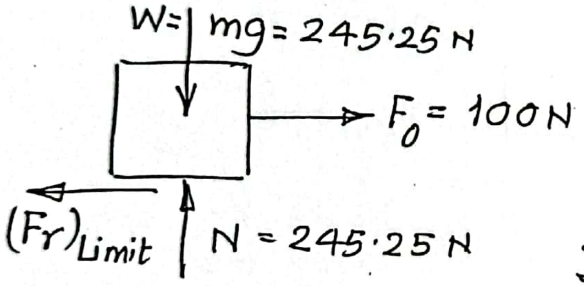
$$\mu_s = 0.30$$
$$\mu_k = 0.25$$

At $t=0$, $v=0$

$$F = (10 \cdot t^2 + 100)\ \text{N}$$

At $t=0$, $F_0 = 100\ \text{N}$

$$(F_r)_{max} = (F_r)_{Limit} = \mu_s \cdot N$$
$$= (0.3 \times 25 \times 9.81) = 73.575\ \text{N}$$



As, $F_0 > (F_r)_{Limit}$ at $t=0$, the crate will start moving immediately after 'F_0' is applied.

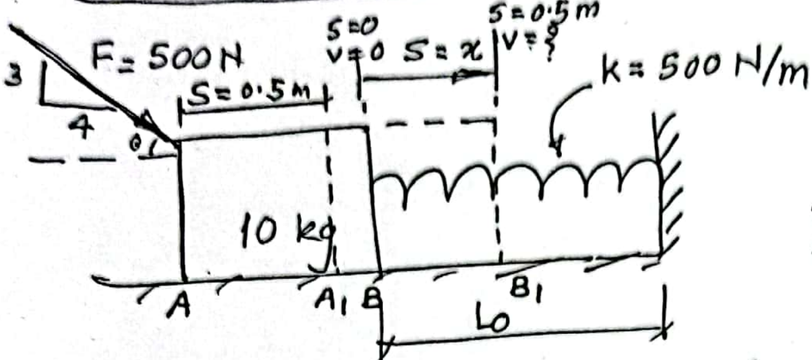
During motion of the crate,

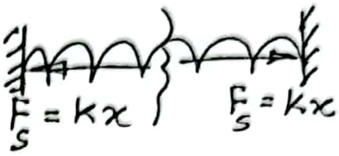
$$\Sigma F_y = m \cdot a_y = 0$$
$$N - 245.25 = 0 \quad \therefore \boxed{N = 245.25\ \text{N}}$$

$$\Sigma F_x = m \cdot a_x$$
$$F - (F_r)_{kinetic} = 25 \cdot a_x$$
$$(10 \cdot t^2 + 100) - (0.25 \times 245.25) = 25 \cdot a_x$$
$$\therefore \quad a_x = a = [(0.4)t^2 + (1.5475)]\ \text{m/s}^2$$
$$dV = a \cdot dt$$
$$\int_0^V dV = \int_0^4 [(0.4)t^2 + (1.5475)]\, dt$$
$$\therefore \boxed{v_4 = 14.72\ \text{m/s}\ (\rightarrow)}$$

Scanned with CamScanner

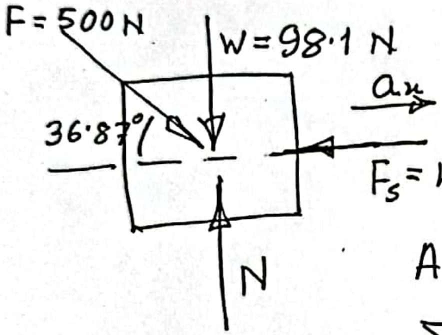
③ F 13·3/ pg· 742/ RCH :





x = deformation of the spring in 'm'

$\tan\theta = \frac{3}{4}$ $\therefore\ \theta = 36{\cdot}87^{\circ}$



$F_s = kx = (500 \times x)$

Along 'x' axis,

$\Sigma F_x = m \cdot a_x$

$(500)(\cos 36{\cdot}87^{\circ}) - (500 \cdot x) = 10 \cdot a_x$

$\therefore\ a_x = (40 - 50 \cdot x)\ m/s^2 = a$ (say)

But, $a \cdot dx = v \cdot dv$

$$\int_0^{0.5\,m} (40 - 50 \cdot x)\,dx = \int_0^{v} v \cdot dv$$

$$\left(40 \cdot x - 25 \cdot x^2\right)_0^{0.5} = \left[\frac{v^2}{2}\right]_0^{v}$$

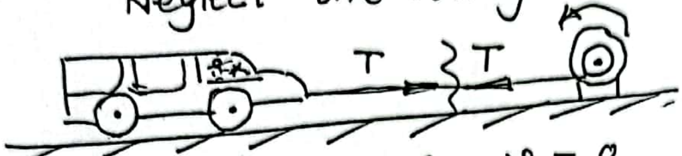
$\therefore\ \boxed{v = 5{\cdot}24\ m/s\ (\rightarrow)}$

④ F 13.4 / pg. 742 / RCH :

$m = 2000$ kg

$T = (100)(s+1)$ N

Neglect the rolling resistance.



when $s = 0$, $v = 0$

$s = 10$ m, $V = ?$

$\Sigma F_x = m \cdot a_x$

$\therefore \quad T = m \cdot a_x$

$\therefore \quad (100)(s+1) = (2000)\, a_x$

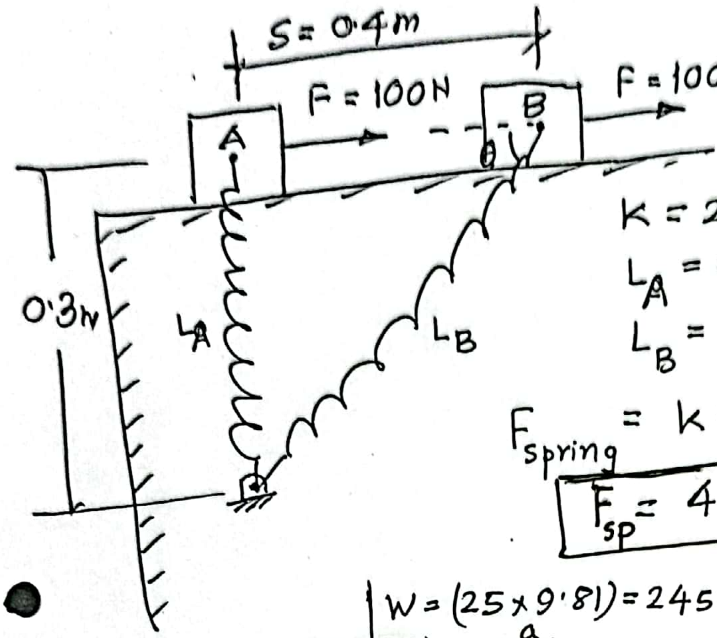
$\therefore \quad a_x = a = [(0.05)x + (0.05)]$ m/s^2

Now, $\quad a \cdot dx = v \cdot dv$

$$\int_0^{10} [(0.05)x + (0.05)]\, dx = \int_0^{v} v \cdot dv$$

$\therefore \quad \boxed{v = 2.45 \text{ m/s} (\rightarrow)}$

5) F 13.5/pg. 742/RCH :



$\tan\theta = \frac{0.3}{0.4}$

$\theta = 36.87°$

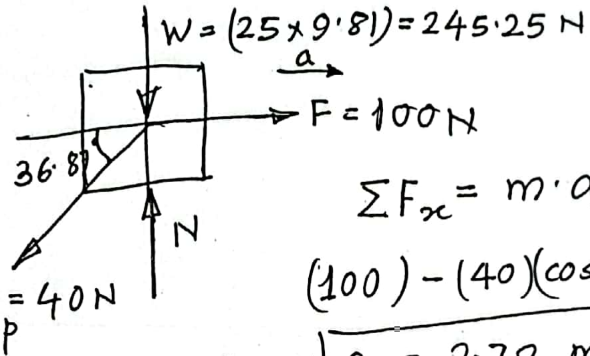
$K = 200 \, N/m$

$L_A = 0.3 \, m = L_0$

$L_B = 0.5 \, m, \quad (L_B - L_0) = x_B$

$F_{spring} = k(L_B - L_0) = (200)(0.5 - 0.3)$

$\boxed{F_{sp} = 40 \, N}$

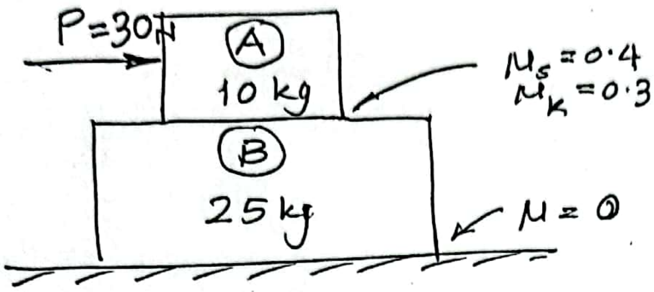


$\Sigma F_x = m \cdot a_x$

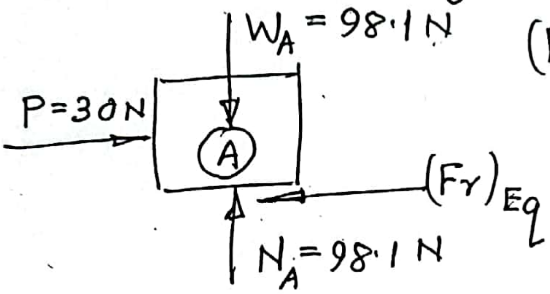
$(100) - (40)(\cos 36.87°) = 25 \cdot a_x$

$\therefore a_x = \boxed{a = 2.72 \, m/s^2 \, (\rightarrow)}$

⑥ F 13·6 / pg. 742 / RCH :



I) Check, if slipping occurs betn A & B :



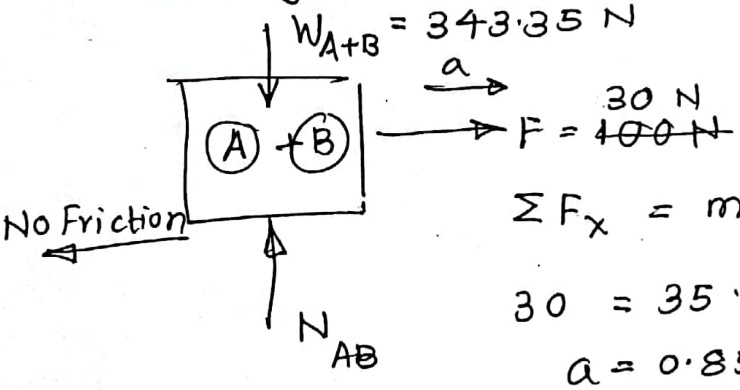
$(Fr)_{Eq} = 30\,N$

$(Fr)_{Limit} = (0.4 \times 98.1) = 39.24\,N$

As, $(Fr)_{Equi.} < (Fr)_{Limit}$

Block A is not sliding at the top of block B.

II) Consider blocks A and B together as one rigid body :



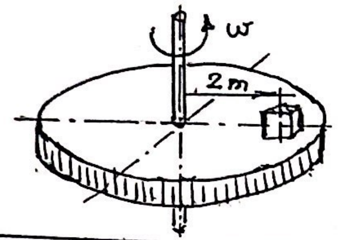
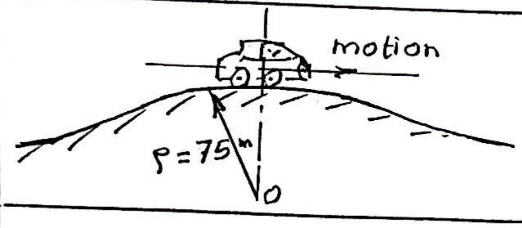
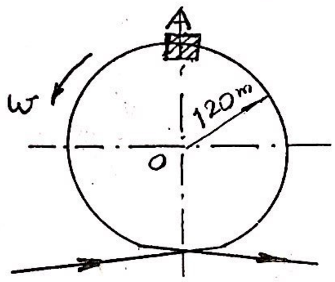
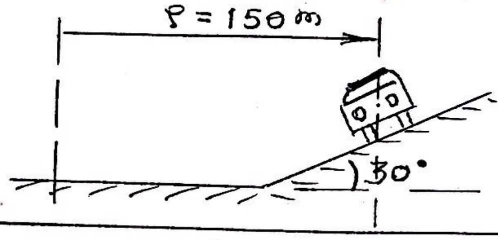
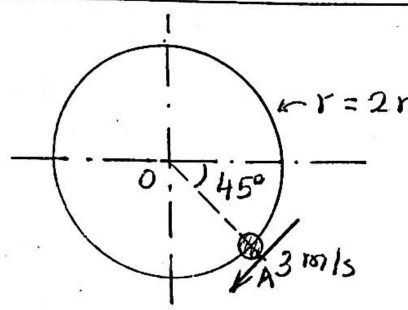
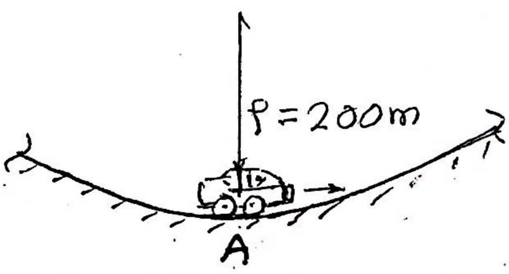
$\Sigma F_x = m \cdot a$

$30 = 35 \cdot a$

$a = 0.857\ m/s^2$

$\therefore \boxed{a_A = a_B = 0.857\ m/s^2 (\rightarrow)}$

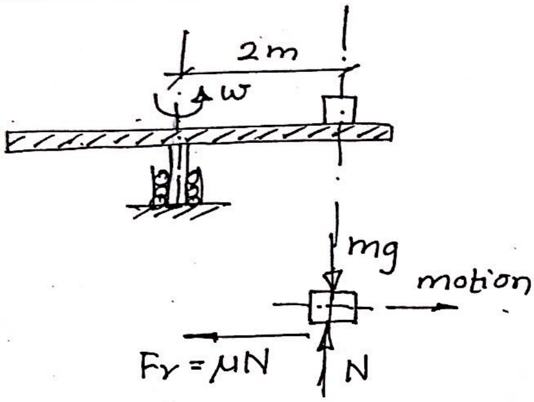
Lecture No: 14 **Curvilinear Kinetics (Newton's Second Law of Motion)**

1	The block rests at a distance of 2m from the center of the platform. If the coefficient of static friction between the block and the platform is $\mu_s = 0.3$, determine the maximum speed which the block can attain before it begins to slip. Assume the angular motion of the disk is slowly increasing Ans.: $V_{max} = 2.426$ m/s	
2	Determine the maximum speed that the jeep can travel over the crest of the hill and not lose contact with the road. Ans, : $V_{max} = 27.125$ m/s	
3	A pilot weighs 70 kg and is travelling at a constant speed of 36 m / s. Determine the normal force he exerts on the seat of the plane when he is upside down at A. The loop has a radius of curvature of 120 m. Ans. ; $N_p = 69.3$ N (↓)	
4	The sports car is travelling along a 30^0 banked road having a radius of curvature of p = 150 m.. If the coefficient of static friction between the tires and the road is $\mu_s = 0.2$, determine the maximum safe speed so no slipping occurs. Neglect the size of the car. Ans.: $V = 35.96$ m/s	
5	If the 10 kg ball has a velocity of 3 m / s when it is at the position A, along the vertical path, determine the tension in the cord and the increase in the speed of the ball at this position. Ans. : T = 114 N, $a_t = 6.94$ m/s²	
6	The motorcycle has a mass of 0.5 Mg. and a negligible size. It passes point A travelling with a speed of 15 m/s, which is increasing at a constant rate of 1.5 m/s². Determine the resultant frictional force exerted by the road on the tires at this instant. Ans.: F = 938 N	

Lecture No. 14 Newton's 2^{nd} Law of Motion

Curvilinear Motion

1) F 13.7 / pg. 759 / RCH (14^{th} Ed.) :

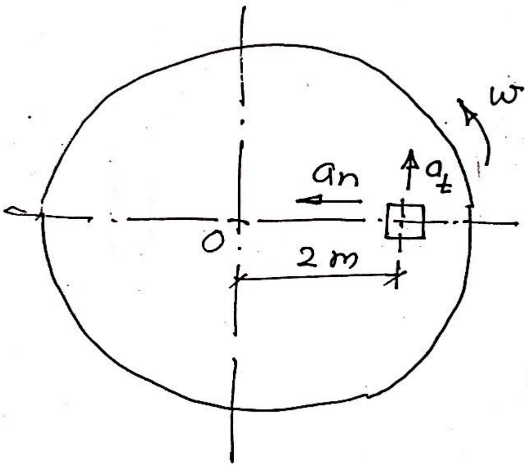


$$\Sigma F_N = m \cdot a_n$$

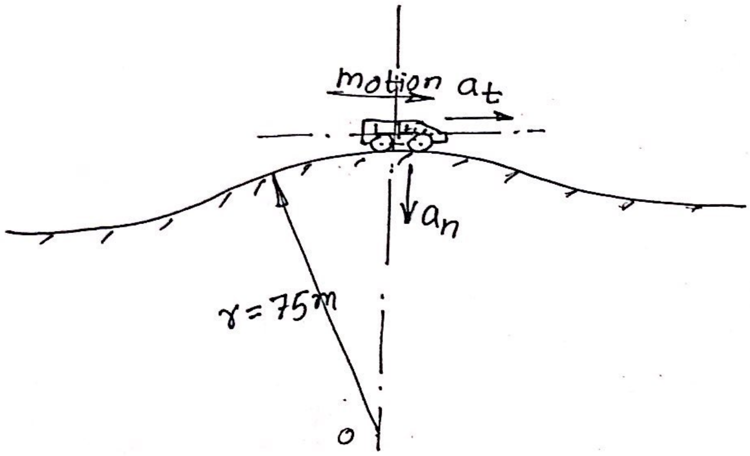
$$(0.3)\, m \times (9.81) = m \cdot \frac{V^2}{r}$$

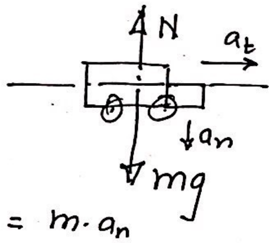
put, $r = 2\ m$

$$\therefore \boxed{V = 2.426\ m/s}$$



② F 13.8 / pg. 759 / RCH (14th Ed.) :





$$\Sigma F_N = m \cdot a_n$$

$$mg - N = \frac{mv^2}{r}$$

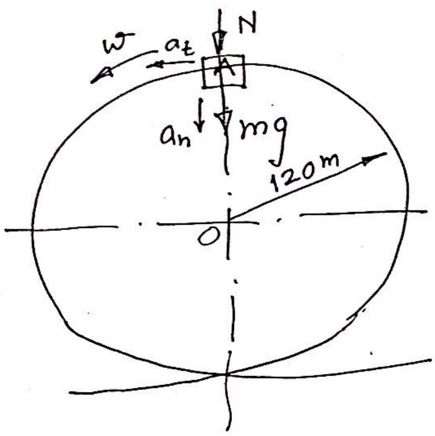
Consider $N = 0$

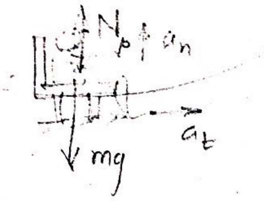
$$\therefore \quad mg = \frac{mv_{max}^2}{r}$$

$$V_{max}^2 = (75 \times 9.81)$$

$$\boxed{V_{max} = 27.125\ m/s}$$

③ F13.9/ pg. 759/ RCH (14th Ed.)





$$\Sigma F_N = m \cdot a_n$$

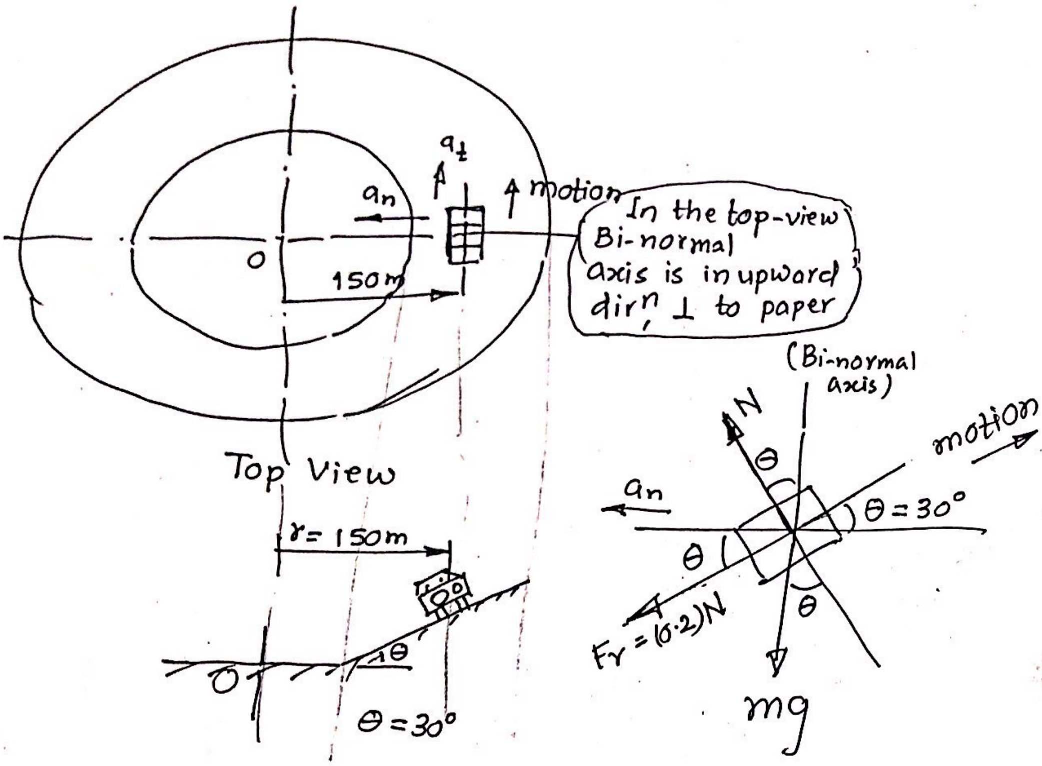
$$mg + N_p = m \cdot \frac{v^2}{r}$$

$$N_p = \frac{mv^2}{r} - mg$$

$$N_p = (70)\left[\frac{36^2}{120} - 9.81\right]$$

$$\boxed{N_p = 69.3\ N}$$

④ F13.10/pg. 759/ RCH(14th Ed.)



$$\Sigma F_N = m \cdot a_n$$

$$N\sin\theta + (0.2)N\cdot\cos\theta = \frac{mv^2}{r} \longrightarrow ①$$

$$\Sigma F_b = 0$$

$$N\cos\theta - (0.2)N\cdot\sin\theta - mg = 0 \longrightarrow ②$$

put, $\theta = 30°$, $(0.766)N - (9.81)m = 0$

$$\boxed{N = (12.807)\,m}$$

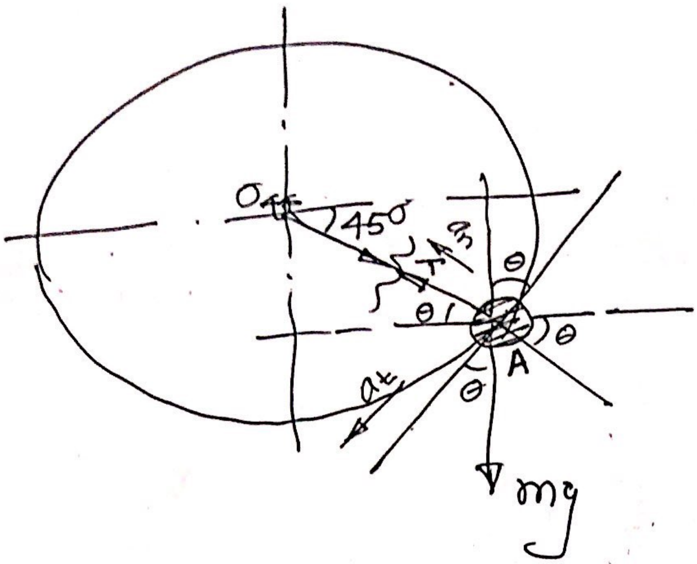
∴ eq^n ① becomes,

$$(12.807)\cancel{m} \times (0.5) + (0.2)(12.807)\cancel{m} \times (0.866) = \frac{\cancel{m}V^2}{150}$$

$$(6.403) + (2.218) = \frac{V^2}{150}$$

$$\boxed{V = 35.96\ m/s}$$

⑤ F13.11/ pg. 759/ RCH (14th Ed) :



$\theta = 45°$

$r = 2\ m$

$V = 3\ m/s$

$m = 10\ kg$

$\Sigma F_t = m \cdot a_t$

$mg\cos\theta = m \cdot a_t$

$\cancel{10} \times 9.81 \times \cos 45° = \cancel{10} \cdot a_t$

$\boxed{a_t = 6.94\ m/s^2}$

$\Sigma F_n = m \cdot a_n$

$T - mg\sin\theta = \frac{mv^2}{r}$

$T = \left(mg\sin\theta + \frac{mv^2}{r}\right)$

$T = (10)\left[(9.81 \times \sin 45°) + \frac{3^2}{2}\right] = 114\ N$

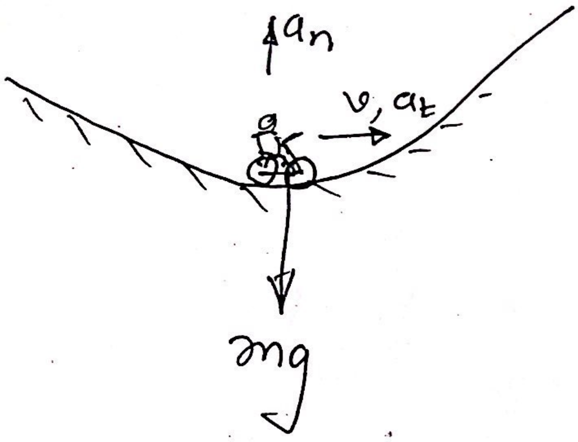
$\boxed{T = 114\ N}$

(6) F13.12/pg.759/RCH(14th Ed),

$m = 500\,kg \quad r = 200\,m$

$V = 15\,m/s$

$a_t = 1.5\,m/s^2$



$$\Sigma F_n = m \cdot a_n$$

$$= \frac{mv^2}{r}$$

$$= \frac{500 \times 15^2}{200}$$

$$= 562.5\ N$$

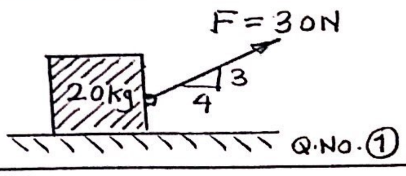
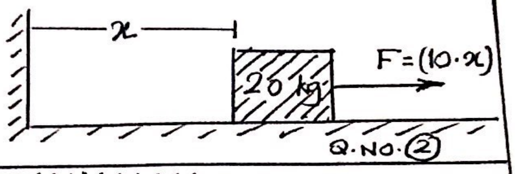
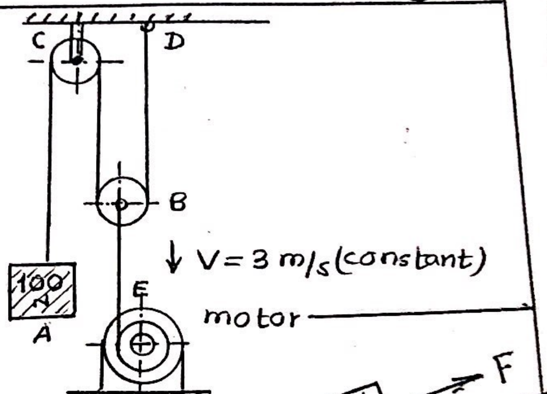
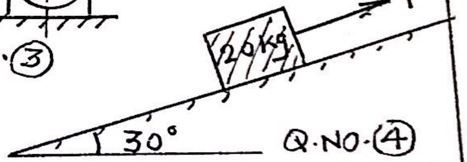
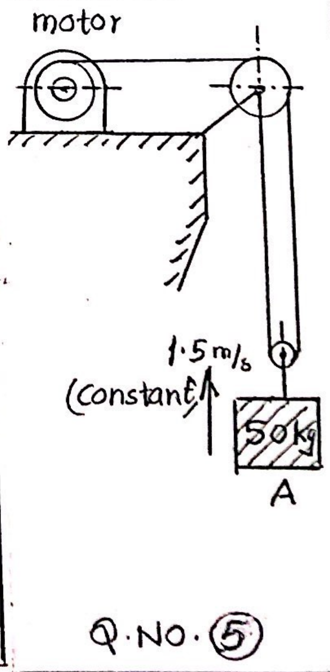
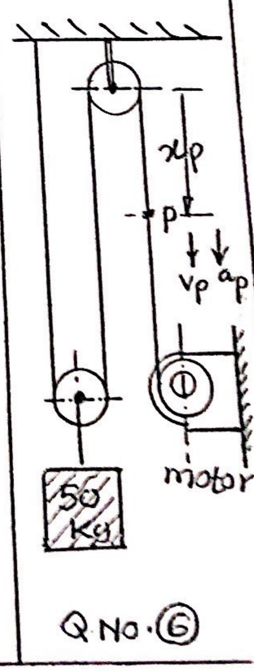
$$\Sigma F_t = m \cdot a_t$$

$$= 500 \times 1.5 = 750\ N$$

$$F = \sqrt{F_t^2 + F_n^2} = \sqrt{750^2 + 562.5^2}$$

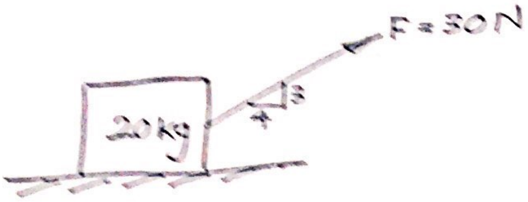
$$\boxed{F = 938\ N}$$

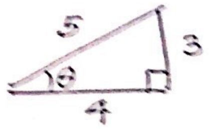
Lecture No: **Work , Power, Energy**

1	If the contact surface between the 20-kg block and the ground is smooth, determine the power of force F when t = 4 s. Initially, the block is at rest.	
2	If F = (10s) N, where s is in meters, and the contact surface between the block and the ground is smooth, determine the power of force F when s = 5 m. When s = 0, the 20 kg block is moving at v = 1 m/s.	
3	If the motor winds in the cable with a constant speed of v = 3 m/s, determine the power supplied to the motor. The load weighs 100 N and the efficiency of the motor is η = 0.8. Neglect the mass of the pulleys.	
4	The coefficient of kinetic friction between the 20-kg block and the inclined plane is μ_k = 0.2. If the block is travelling up the inclined plane with a constant velocity v =5m/s, determine the power of force F.	
5	If the 50 kg-kg load A is hoisted by motor M so that the load has a constant velocity of 1.5 m/s. determine the power input to the motor, which operates at an efficiency η = 0.8	
6	At the instant shown, point P on the cable has a velocity V_p = 12 m/s, which is increasing at a rate of a_p = 6 m/s^2. Determine the power input to motor M at this instant if it operates with an efficiency η = 0.8. The mass of block A is 50 Kg.	

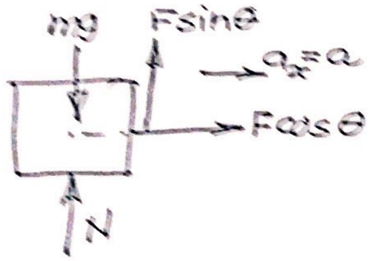
Lecture No (11) Work, Power, Energy

(1) F 14·7 / Pg. 822 / RCH (14th Ed.)





$\theta = 36.87°$



$\Sigma F_x = m \cdot a_x$

$30 \times \cos 36.87° = 20 \times a_x$

$a_x = a = 1.2\ m/s^2$

At $t = 0$, $u = 0$

At $t = 4s$, $v = u + at$

$\therefore v = 0 + (1.2 \times 4) = 4.8\ m/s$

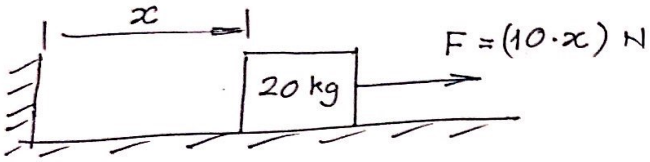
Now, power = force × vel.

$P = F \times V$

$\therefore P = (30 \times \cos 36.87° \times 4.8)$

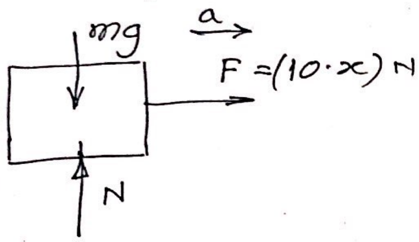
$P = 115$ Watts.

② F 14·8/pg·822/RCH (14th Ed.)



when $x = 0$, $v = 1$ m/s

∴ $x = 5m$, $v = ?$



$$\Sigma F_x = m \cdot a_x$$

$$10 \cdot x = 20 \cdot a_x$$

$$a_x = a = (0.5)x = f(x)$$

Now, $\int v \cdot dv = \int a \cdot dx$

$$\therefore \int_{1\,m/s}^{V} v \cdot dV = \int_{0}^{5m} (0.5)\, x \cdot dx$$

$$V = 3.674 \text{ m/s}$$

Now, Power, $P = F.V$

$$P = (10 \times 5 \times 3.674)$$

$$\therefore P = 184 \text{ Watts}$$

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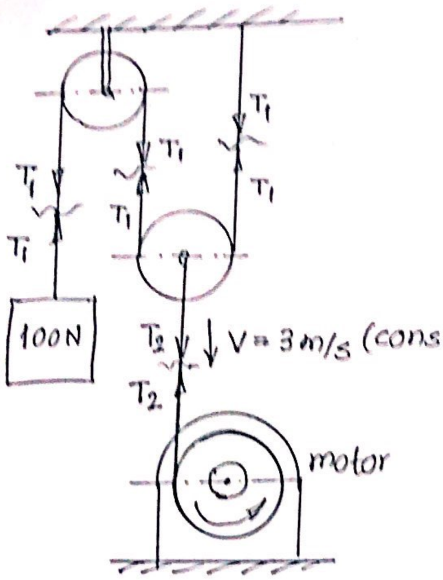
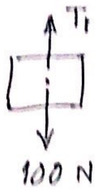
(3) F14·9/pg.822/RCH (14^{th} Ed)



$\eta = 0.8$

For the hanging weight,



$\Sigma F_y = m \cdot a_y = 0$

(as $a_y = 0$)

$T_1 - 100 = 0$

$\therefore T_1 = 100$ N

For the hanging weight,

$T_2 = 2 \cdot T_1 = 200$ N

$\therefore$ Output power $= T_2 \cdot V$

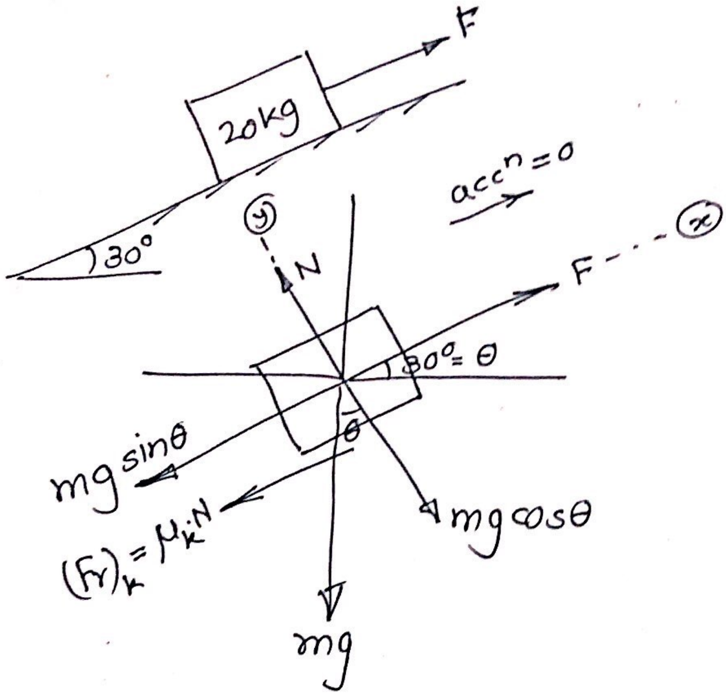
$= (200 \times 3) = 600$ Watt.

Efficiency $= \left(\frac{\text{power output}}{\text{power input}}\right)$

$\therefore$ Input power $= \left(\frac{600}{0.8}\right)$

$= 750$ Watts.

4 F 14·10/ pg. 822/RCH (14th Ed.)



$$\Sigma F_y = m \cdot a_y$$

$$N - (20 \times 9.81 \times \cos 30°) = 0$$

$$N = 169.91 \text{ N}$$

$$\Sigma F_x = m \cdot a_x$$

$$F - (20 \times 9.81 \times \sin 30°) - (0.2 \times 169.91) = 0$$

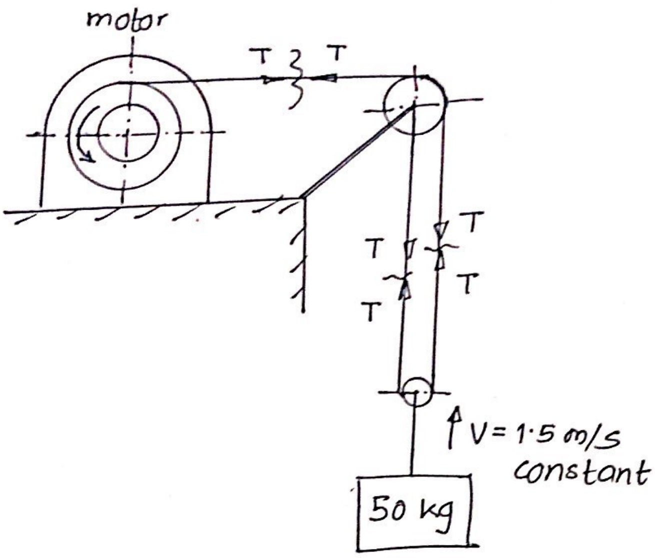
$$\therefore F = 132.08 \text{ N}$$

Power, $P = F \cdot V$

$= (132.08 \times 5)$

$= 660$ Watts.

(5) F 14·11/Pg. 822/RCH (14th Ed.)



For the hanging body, $\Sigma F_y = m \cdot a_y$

$$2T - (50 \times 9.81) = 0$$

$$T = 245.25 \text{ N}$$

$$\text{Output power} = T \cdot V$$
$$= (245.25 \times 1.5)$$
$$= 367.875 \text{ Watts.}$$

$$\text{Input power} = \left(\frac{367.875}{0.8}\right)$$
$$= 459.84 \text{ Watts.}$$

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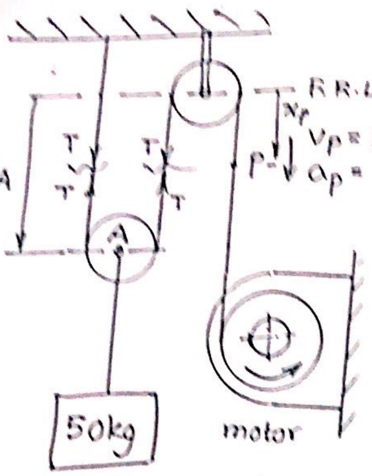
(6) F 14.12/pg. 822/RCH (14^{th} Ed.)



As the length of the rope is constant,

$$2x_A + x_P = L$$

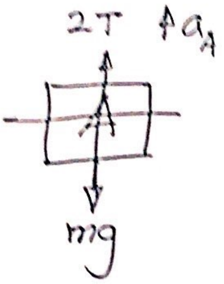
$$\therefore 2V_A + V_P = 0$$

$$\therefore 2a_A + a_P = 0$$

$$\therefore 2 \cdot a_A + 6 = 0$$

$$a_A = -3 \text{ m/s}^2$$

$$= 3 \text{ m/s}^2 (\uparrow)$$



$$\Sigma F_y = m \cdot a_y$$

$$2T - (50 \times 9.81) = (50 \times 3)$$

$$2T - (490.5) = 150$$

$$T = 320.25 \text{ N}$$

Output power $= T \times V$

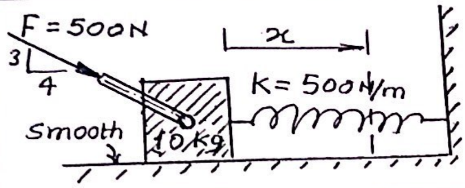
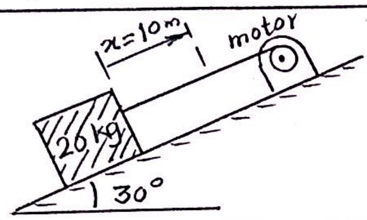
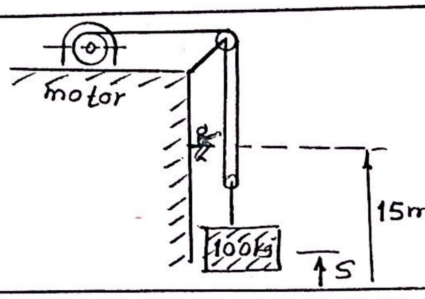
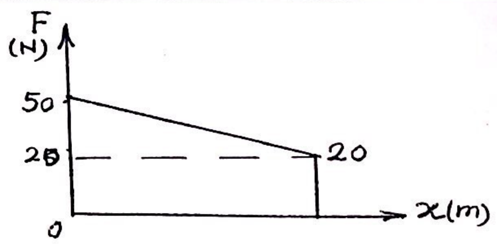
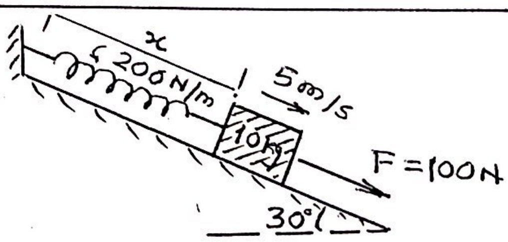
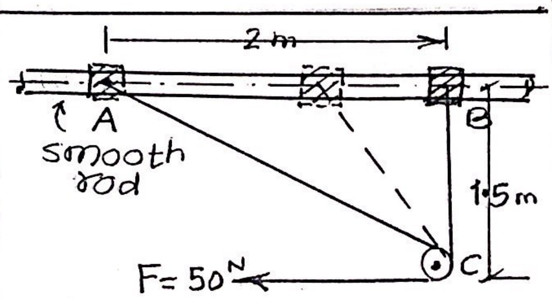
$$= (320.25)(12)$$

$$= 3843 \text{ Watts.}$$

Input power $= \left(\dfrac{3843}{0.8}\right)$

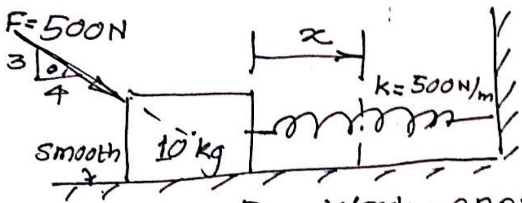
$$= 4803.75 \text{ Watts.}$$

Lecture No: **Work-Energy Principle**

1	The spring is placed between the wall and the 10-kg. block. If the block is subjected to a force of F = 500 N, determine its velocity when s = 0.5 m. When s = 0, the block is at rest and the spring is uncompressed. The contact surface is smooth.	
2	If the motor exerts a constant force of 300 N on the cable, determine the speed of the 20 kg crate when it travels s = 10 m up the plane, starting from rest. The coefficient of kinetic friction between the crate and the plane is μ_k = 0.3	
3	If the motor exerts a force of F = (600 + $2s^2$) N on the cable, determine the speed of the 100-kg crate when it rises to s = 15m. The crate is initially at rest on the ground.	
4	The 1.8 Mg dragster is travelling 125 m/s when, the engine is shut off and the parachute is released. If the drag force of the parachute can be approximated by the graph, determine the speed of the dragster when it has travelled 400m.	
5	When s = 0.6 m, the spring is unstretched and the 10-kg block has a speed of 5 m/s down the smooth plane. Determine the distance s when the block stops.	
6	The 2.5- kg collar is pulled by a cord is that passes around small peg at C. If the cord is subjected to a constant force of F = 50 N, and the collar is at rest when it is at A, determine its speed when it reaches B. Neglect friction.	

Lecture No. 12 Work-Energy Principle

1. F 14.1/pg. 808/RCH (14th Ed)



when $x = 0$, $v = 0$

$x = 0.5\,m$, $v = ?$

$\tan\theta = \frac{3}{4}$

$\theta = 36.87^\circ$

By Work-energy principle,

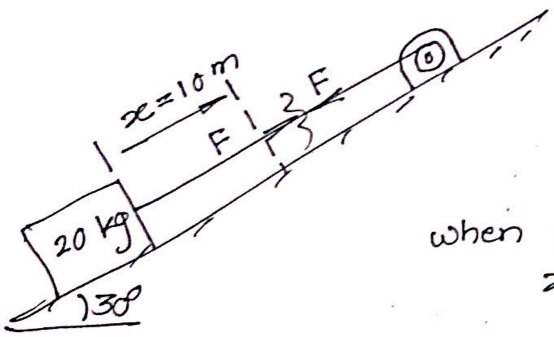
$$U_{1-2} = T_2 - T_1$$

$$(F\cos\theta)(x) - \left(\tfrac{1}{2}kx^2\right) = \tfrac{1}{2}mv^2 - 0$$

$$(500 \cdot \cos 36.87^\circ \times 0.5) - \left(\tfrac{1}{2} \times 500 \times 0.5^2\right) = \tfrac{1}{2} \times 10 \times V^2$$

$$\therefore \quad V = 5.24\ m/s$$

(2) F 14.2 / pg. 808 / RCH (14th Ed.)



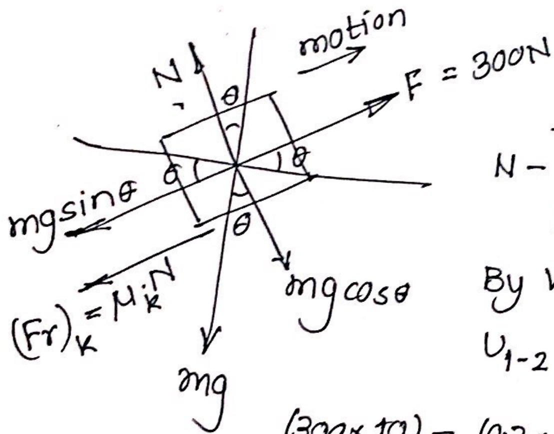
$F = 300\,N$

$m = 20\,kg$

$\mu_k = 0.3$

when $x = 0$, $v = 0$

$x = 10\,m$, $v = ?$



Applying N.S.L.M. along 'y' axis,

$\sum F_y = m \cdot a_y$

$N - (20 \times 9.81 \times \cos 30°) = 0$

$N = 169.91\,N$

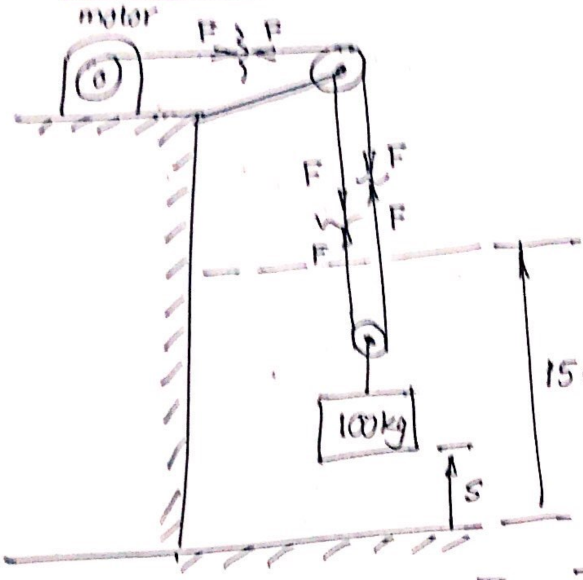
By W-E. Prin.

$U_{1-2} = T_2 - T_1$

$(300 \times 10) - (0.3 \times 169.91)(10) - (20 \times 9.81 \times \sin 30°)(10) = \frac{1}{2} \times 20 \times V^2$

$V = 12.3\,m/s$

(3) F 14.3 / pg. 808 / RCH (14^{th} Ed.)



$F = (600 + 2s^2)$ N

W.D. $= \int F \, ds$

$$U_{1-2} = T_2 - T_1$$

$$2\left[\int_0^{15} (600 + 2s^2) \cdot ds\right] - (100 \times 9.81 \times 15) = \left(\frac{1}{2} \times 100 \times V^2\right)$$

$$V = 12.5 \text{ m/s}$$

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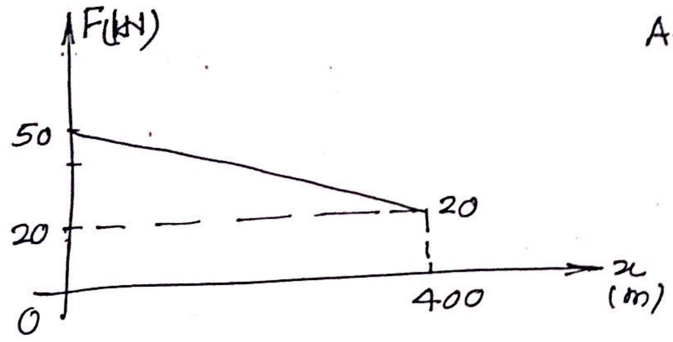
4) F14·4/ pg. 808/ RCH (14th Ed)



At $x = 0$, $V = 125$ m/s

$x = 400$ m, $V = ?$

$m = 1.8$ Mg

$m = 1.8 \times 10^6$ gm

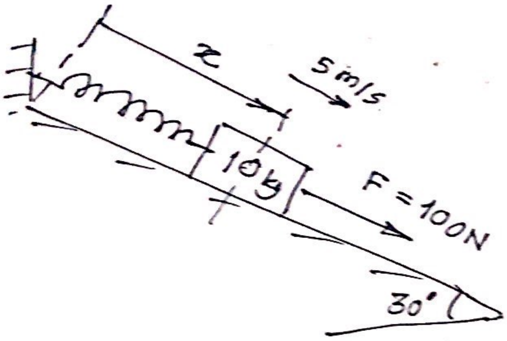
$m = 1800$ kg

$$U_{1-2} = T_2 - T_1$$

$$-\left(\frac{50{,}000 + 20{,}000}{2}\right)(400) = \left(\frac{1}{2} \times 1800 \times V^2\right) - \left(\frac{1}{2} \times 1800 \times 125^2\right)$$

$$V = 8.33 \text{ m/s}$$

(5) F 14.5/Pg. 808/RCH (14th Ed.)



when, $v = 5\,m/s$, $x = 0.6\,m$

$v = 0$, $x = ?$

$$V_{1-2} = T_2 - T_1$$

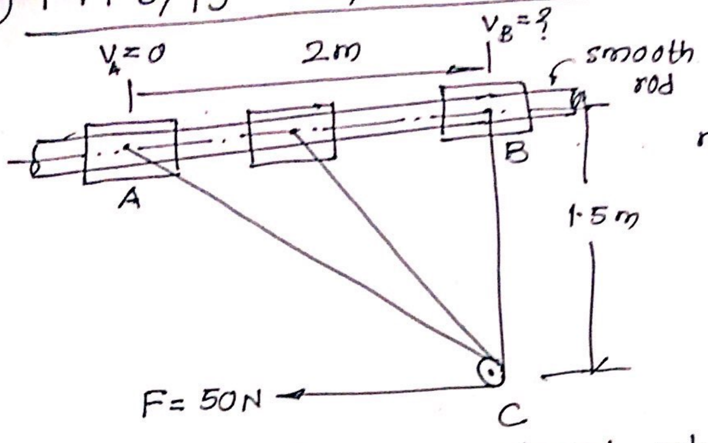
$$(100 \cdot x') + (10 \times 9.81 \times \sin 30^\circ)(x') - \frac{1}{2} \times 200 \times (x')^2 = 0 - \left(\frac{1}{2} \times 10 \times 5^2\right)$$

$x' = 2.09\,m$ (elongation of spring)

$x = 0.6 + 2.09 = 2.69\,m$

(elongated length of the spring)

(6) F 14.6 / pg. 808 / RCH (14^{th} Ed.)



$m = 2.5$ kg

x = dist. travelled by the chord, when the collar moves from A to B = (AC − BC)

$$\therefore \quad x = \left(\sqrt{2^2 + 1.5^2}\right) - (1.5) =$$

$$T_1 = \frac{1}{2} m v_A^2 = 0$$

$$T_2 = \frac{1}{2} \times (2.5) \times V_B^2$$

$$U_{1-2} = F \cdot x = 50 \cdot x$$

$$U_{1-2} = T_2 - T_1$$

$$(50 \cdot x) = \left(\frac{1}{2} \times 2.5 \times V_B^2\right) - 0$$

$$\therefore \quad V_B = 6.32 \text{ m/s}$$

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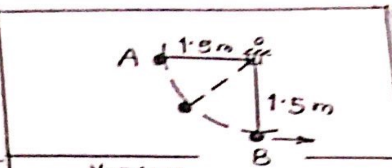
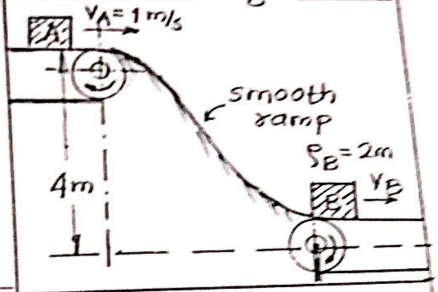
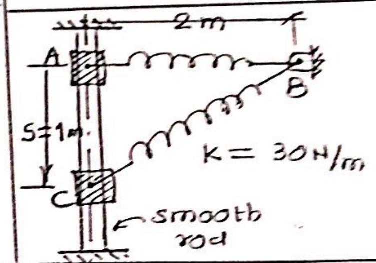
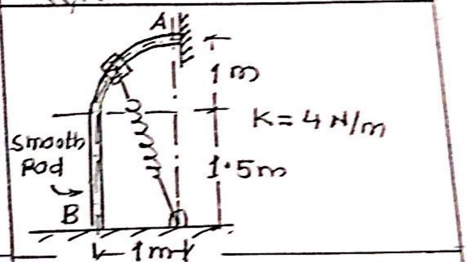
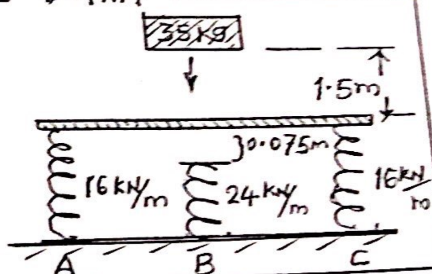
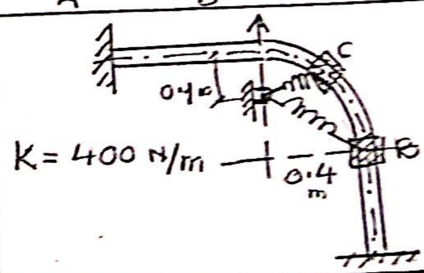
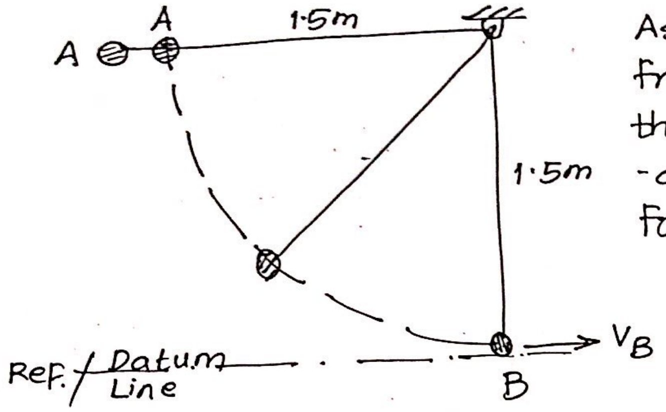
Lecture No: **Conservation of Energy** 17

1	A two kg pendulum bob is released from rest when is at A. Determine the speed of bob and the tension in the chord when the bob passes through its lowest position, B.	
2	A 2 kg package leaves the conveyer belt at A at a speed of v_a=1m/s and slides down the smooth ramp. Determine the required speed of the conveyer belt at so that the package can be delivered without slipping on the belt. Also, find the normal reaction the curved portion the ramp exerts on the package at B if ρ_B=2m.	
3	The 2 kg collar is given a downward velocity of 4 m/s when it is at A. If the spring has an unstretched length 1 m and a stiffness of k=30 N/m, determine the velocity of the collar s=1m	
4	The 5 kg collar is released from rest at A and travels along the frictionless guide. Determine the speed of the collar when it strikes the stop B. The spring has an unstretched length of 0.5m	
5	The 35 kg block is released from rest 1.5m above the plate. Determine the compression of each spring when the block momentarily comes to rest after striking the plate. Neglect the mass of the plate. The springs are initially unstretched.	
6	The 4 kg collar C has a velocity of v_A=2 m/s when it is at A. If the guide rod is smooth, determine the speed of the collar when it is at B. The spring has an unstretched length of l_0=0.2m	

Lecture No. (17) Conservation of Energy

(1) F 14.13/pg. 837/RCH (14th)



As, the force of friction is absent, the system is subjected to conservative force system.

$m = 2\ kg$

By conservation of energy principle,

$$T_A + V_A = T_B + V_B$$

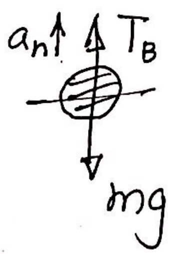
$$(K.E.)_A + (G.P.E.)_A + (E.P.E.)_A = (K.E.)_B + (G.P.E.)_B + (E.P.E.)_B$$

$$\cancel{\tfrac{1}{2}mv_A^2} + mgh_A + \cancel{\tfrac{1}{2}kx_A^2} = \tfrac{1}{2}mv_B^2 + \cancel{mgh_B} + \cancel{\tfrac{1}{2}kx_B^2}$$

$$(2 \times 9.81 \times 1.5) = \tfrac{1}{2} \times 2 \times V_B^2$$

$$\boxed{V_B = 5.42\ m/s}$$

At posi. B : Applying N.S.L.M. along the normal dirn

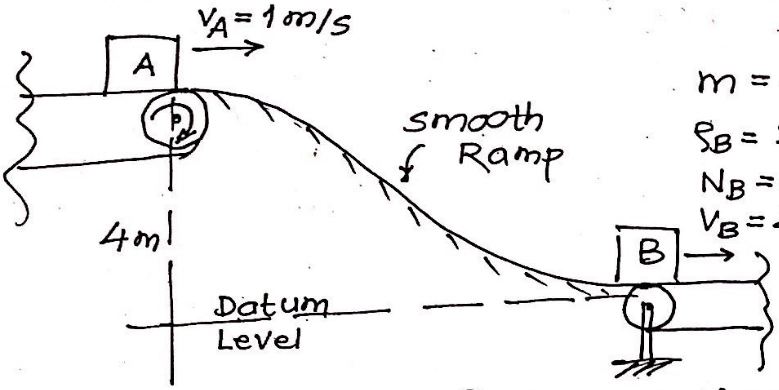


$$\Sigma F_n = m \cdot a_n$$

$$T_B - (2 \times 9.81) = 2 \times \frac{5.42^2}{1.5}$$

$$\boxed{T_B = 58.9\ N}$$

② F 14·14 / pg. 837 / RCH (14th) :



$m = 2\ kg$

$\rho_B = 2\ m$:

$N_B = ?$

$V_B = ?$

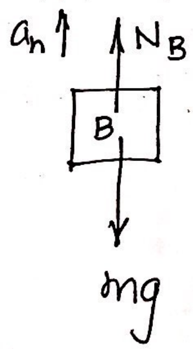
By the principle of conservation of energy,

$$\left(\frac{1}{2} m \cdot v_A^2\right) + (m \cdot g \cdot h_A) = \left(\frac{1}{2} \cdot m \cdot V_B^2\right) + (m g \cdot h_B)$$

$$T_A + V_A = T_B + V_B$$

$$\left(\frac{1}{2} \times 2 \times 1^2\right) + (2 \times 9.81 \times 4) = \left(\frac{1}{2} \times 2 \times V_B^2\right) + 0$$

$$\boxed{V_B = 8.915\ m/s}$$



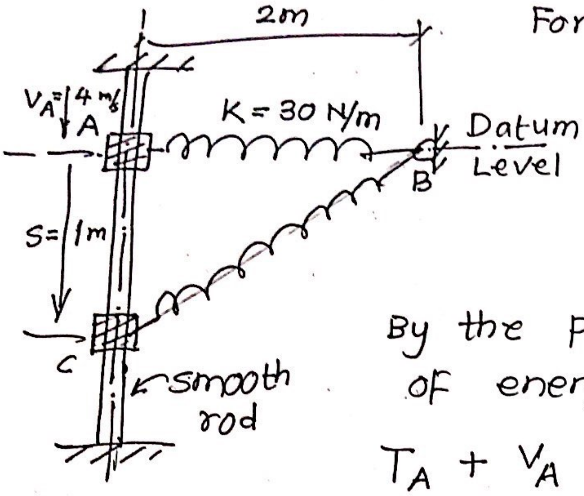
By N.S.L.M. along normal dirⁿ,

$$\Sigma F_n = m \cdot a_n$$

$$N_B - (2 \times 9.81) = 2 \times \frac{8.915^2}{2}$$

$$\boxed{N_B = 99.1\ N}$$

③ F14.15/pg.837/RCH(14^{th})



$m = 2$ kg

For the spring

$L_0 = 1\,m$, $k = 30$ N/m

$L_A = 2\,m$, $x_A = (L_A - L_0) = 1\,m$

$L_C = \sqrt{2^2 + 1^2} = \sqrt{5} = 2.236\,m$

$x_C = (L_C - L_0) = 1.236\,m$

By the principle of conservation of energy,

$$T_A + V_A = T_c + V_c$$

$$\therefore \quad T_A + (V_G)_A + (V_E)_A = T_c + (V_G)_c + (V_E)_A$$

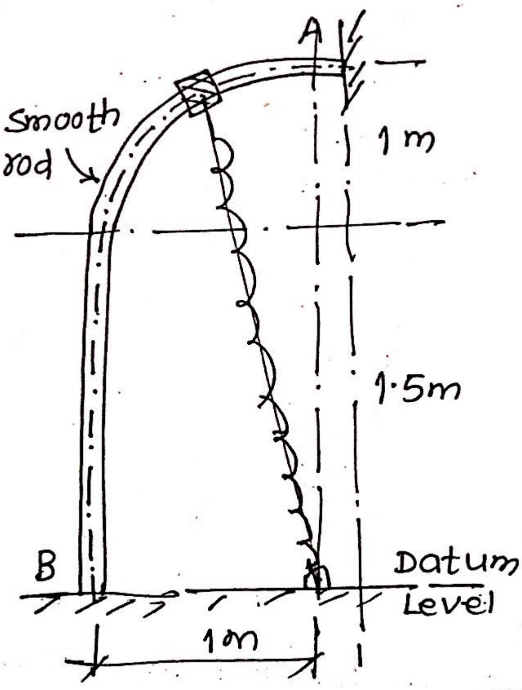
$$(K.E. + G.P.E. + E.P.E.)_A = (K.E. + G.P.E. + E.P.E.)_C$$

$$\frac{1}{2}mv_A^2 + \cancel{mgh_A} + \frac{1}{2}kx_A^2 = \frac{1}{2}mv_c^2 + mgh_c + \frac{1}{2}k \cdot x_C^2$$

$$\left(\frac{1}{2}\times 2\times 4^2\right) + \left(\frac{1}{2}\times 30\times 1^2\right) = \left(\frac{1}{2}\times 2\times V_c^2\right) - (2\times 9.81\times 1) + \left(\frac{1}{2}\times 30\times 1.236^2\right)$$

$$\therefore \boxed{V_c = 5.26 \text{ m/s}}$$

④ F 14·16/pg. 837/RCH (14^{th}):



$m = 5\,kg$

$K = 4\ N/m$

$L_0 = 0{\cdot}5\ m$

$L_A = 2{\cdot}5\ m,\ L_B = 1\ m$

$x_A = (L_A - L_0) = 2{\cdot}0\ m$

$x_B = (L_B - L_0) = 0{\cdot}5\ m$

$V_A = 0$

$V_B = ?$

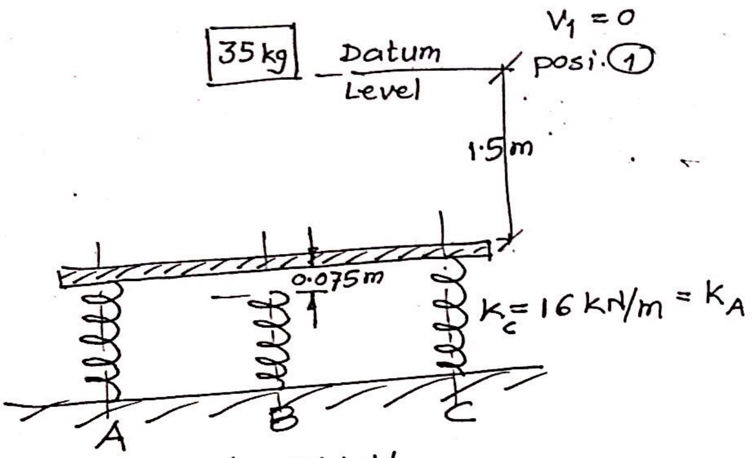
$$T_A + V_A = T_B + V_B$$

$$\cancel{\tfrac{1}{2} m v_A^2} + mgh_A + \tfrac{1}{2} K \cdot x_A^2 = \tfrac{1}{2} m v_B^2 + \cancel{mgh_B} + \tfrac{1}{2} K \cdot x_B^2$$

$$(5 \times 9{\cdot}81 \times 2{\cdot}5) + \left(\tfrac{1}{2} \times 4 \times 2^2\right) = \left(\tfrac{1}{2} \times 5 \times V_B^2\right) + \left(\tfrac{1}{2} \times 4 \times 0{\cdot}5^2\right)$$

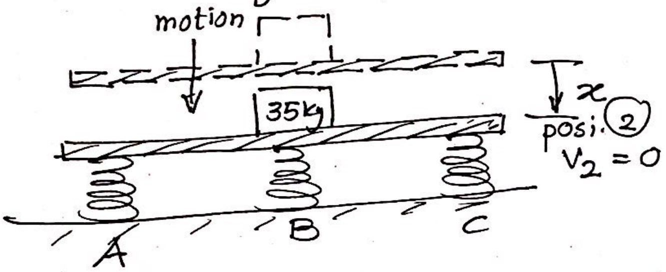
$$\boxed{V_B = 7{\cdot}21\ m/s}$$

⑤ F 14.17/pg. 837/ RCH (14^{th})



$x_A = x_c = x$

$x_B = (x - 0.075)$ m



By the principle of conservation of energy,

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1^2 + mgh_1 + \Sigma\left(\frac{1}{2}kx_1^2\right) = \frac{1}{2}mv_2^2 + mgh_2 + \Sigma\left(\frac{1}{2}kx_2^2\right)$$

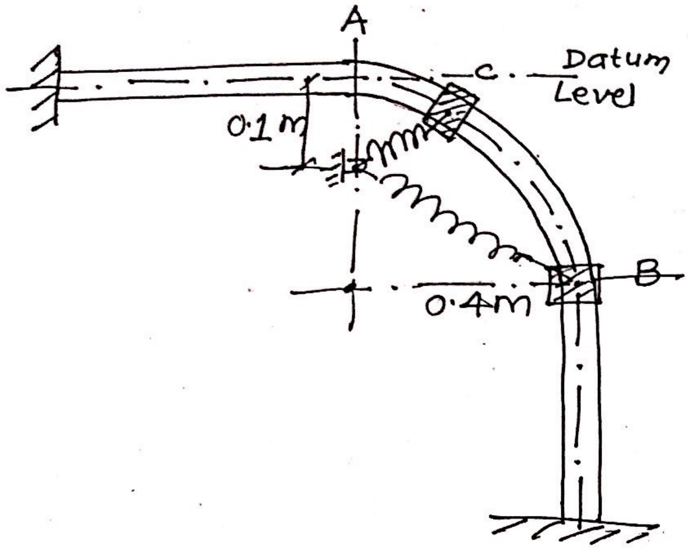
$$0 + 0 + 0 = 0 + (35 \times 9.81)(-1.5 - x) + 2\left(\frac{1}{2} \times 16000 \times x^2\right) + \frac{1}{2} \times 24{,}000 \times (x - 0.0$$

$$\therefore \quad x = x_A = x_c = 0.170 \text{ m}$$

$$x_B = (0.170 - 0.075)$$

$$\boxed{x_B = 0.095 \text{ m}}$$

⑥ F14·18/pg. 837/RCH (14th)



$m = 4\ kg$

At A, $V_A = 2\ m/s$

$L_0 = 0.2\ m$

$x_A = (L_A - L_0)$

$= (0.1 - 0.2)$

$= -0.1\ m$

At B, $V_B = ?$

$L_B = \sqrt{(0.4)^2 + (0.3)^2}$

$= 0.5\ m$

$x_B = (L_B - L_0)$

$= (0.5 - 0.2)$

$= 0.3\ m$

$k = 400\ N/m^2$

$h_A = 0, \quad h_B = -0.4\ m$

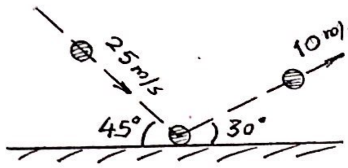
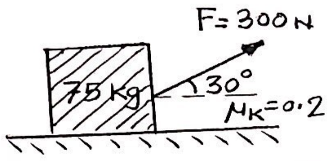
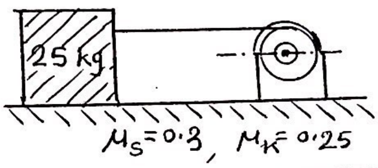
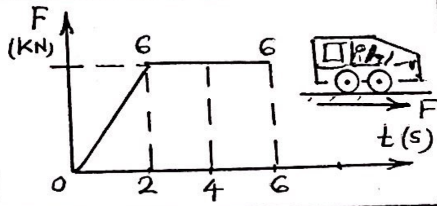
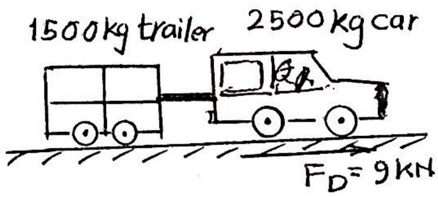
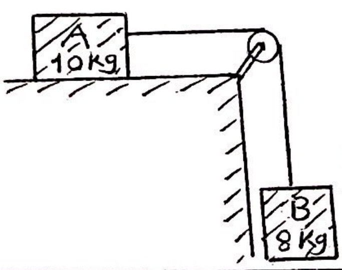
$$T_A + V_A = T_B + V_B$$

$$\left(\tfrac{1}{2} \cdot m \cdot V_A^2\right) + (mgh_A) + \left(\tfrac{1}{2}k \cdot x_A^2\right) = \left(\tfrac{1}{2} \times m \times V_B^2\right) + (mgh_B) + \left(\tfrac{1}{2}k \cdot x_B^2\right)$$

$$\left(\tfrac{1}{2} \times 4 \times 2^2\right) + \cancel{(4 \times 9.81 \times 0)} + \left[\tfrac{1}{2} \times 400 \times (-0.1)^2\right]$$
$$= \left(\tfrac{1}{2} \times 4 \times V_B^2\right) + (4 \times 9.81)(-0.4) + \left(\tfrac{1}{2} \times 400 \times 0.3^2\right)$$

$$\therefore \boxed{V_B = 1.962\ m/s}$$

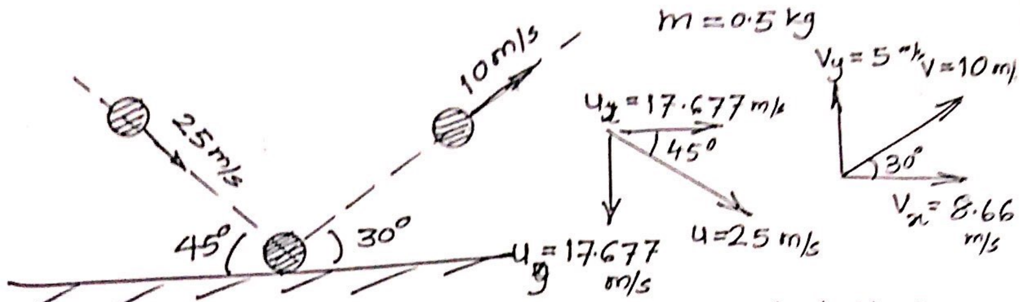
Lecture No: **Impulse Momentum Principle**

1	The 0.5kg ball strikes the rough ground and rebounds with the velocities shown. Determine the magnitude of the impulse the ground exerts on the ball. Assume that the ball does not slip when it strikes the ground, and neglect the size of the ball and the impulse produced by the weight of the ball.	
2	If the coefficient of kinetic friction between the 75 kg crate and the ground is μ=0.2, determine the speed of the crate when t=4s. The crate starts from rest and is towed by the 500N force.	
3	The motor exerts a force $F=20t^2$ N on the cable, where t is in seconds. Determine the speed of the 25 kg crate when t= 4s. The coefficient of static friction and kinetic friction between the crate and the plate are $\mu_s = 0.3$ and $\mu_k = 0.25$ resp.	
4	The wheels of the 1.5mg car generate the traction force F described by the graph. If the car starts from rest, determine its speed when t = 6s.	
5	The 2.5 Mg four wheel drive SUV tows the 1.5Mg trailer. The tractive force developed at the wheels is F_D=9kN. Determine speed of the truck in 20s starting from the rest. Also, determine the tension developed in the coupling A, between the SUV and the trailer. Neglect the mass of wheels.	
6	A 10 kg block A attains velocity of 1m/s in 5 seconds, starting from rest. Determine the tension in cord and the coefficient of kinetic friction between block A and the horizontal plane. Neglect the weight of the pulley. Block B has a mass of 8 kg.	

Lecture No. 18 :

Impulse - Momentum Principle

1 F 15.1 / pg. 860 / RCH (14^{th})



Applying Impulse-Momentum Principle in 'x' dirn, we get,

$$(Imp)_x = (mV_x - m \cdot u_x) = \int_{t_1}^{t_2} F_x \cdot dt$$

$$= (0.5)(8.66 - 17.677)$$

$$= -4.509 \text{ Ns}$$

Similarly, in 'y' dirn,

$$(Imp)_y = (m \cdot V_y - m \cdot u_y) = \int_{t_1}^{t_2} F_y \cdot dt$$

$$= (0.5)(5 + 17.677)$$

$$= 11.339 \text{ Ns}$$

Resultant Impulse on the ball,

$$(Imp.) = \sqrt{(Imp.)_x^2 + (Imp.)_y^2}$$

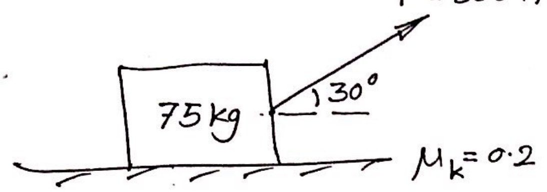
$$= \sqrt{(-4.509)^2 + (11.339)^2}$$

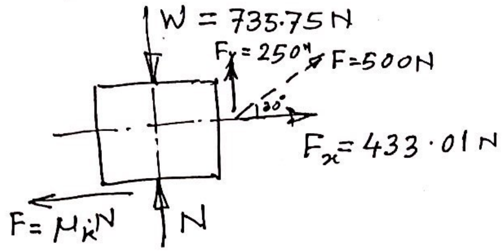
$$= 12.202 \text{ Ns}$$

② F 15·1 / pg. 860 / RCH (14^{th}) :

At $t = 0$, $u = 0$

At $t = 4s$, $V = ?$





$\Sigma F_y = 0$ gives,

$$N + 250 - 735.75 = 0$$

$$N = 485.75 \text{ N}$$

Applying Imp.-mmtm. prin. in 'x' dirn, we get,

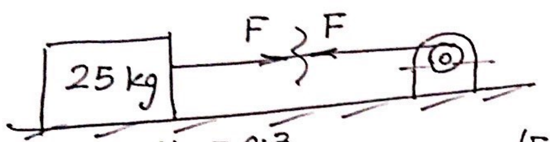
$$(Imp.)_x = \int_{t_1}^{t_2} F_x \, dt = mV_x - m \cdot u_x$$

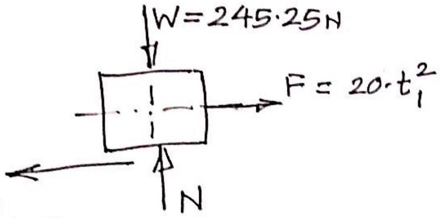
$$(Imp.)_x = (75)(V_x - 0) = (433.01 \times 4) - (0.2 \times 485.75 \times (4))$$

$$\boxed{V_x = 17.912 \text{ m/s}}$$

③ F 15.3/pg.860/RCH (14th) :

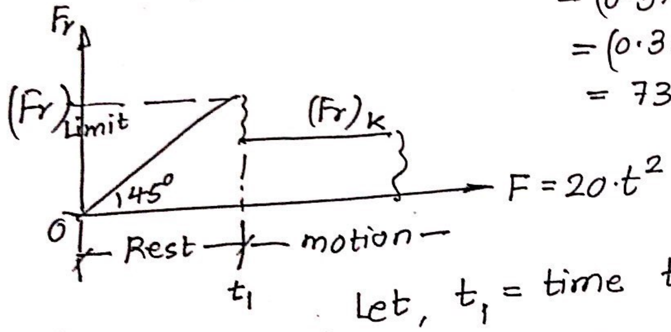
$F = 20 \cdot t^2$ i.e. $f(t)$





$\mu_s = 0.3$
$\mu_K = 0.25$

$(Fr)_{Limit}$
$= (0.3 \times N)$
$= (0.3 \times 245.25)$
$= 73.575$ N



Let, t_1 = time to start the motion

$\therefore \quad (Fr)_{Limit} = 20 \cdot t^2$

$73.575 = 20 \cdot t_1^2 \quad \therefore t_1 = 1.918$ s

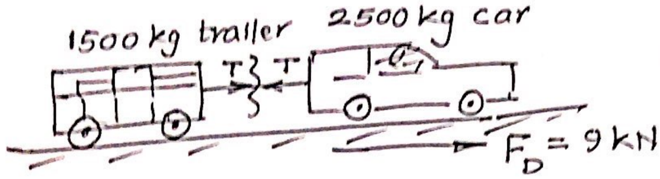
Applying Imp.-mmtm. prin., during motion,

$$(Imp)_x = m \cdot V_x - m \cdot u_x$$

$$\left[\int_{1.918}^{4} (20 \cdot t^2)\, dt - (0.25 \times 245.25)(4 - 1.918)\right] = (25)(V - 0)$$

$$\boxed{V = 10.1 \text{ m/s } (\rightarrow)}$$

⑤ F 15.5/ pg. 860/ RCH (14^{th}) :



At $t=0$, $u=0$
At $t=20\,s$, $V=?$

I) For the car + trailer :

$$(Imp)_x = m(V_x - u_x)$$

$$(9000 \times 20)\ Ns = (2500+1500)(V-0)$$

$$V = 45\ m/s$$

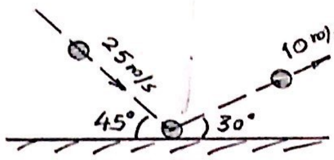
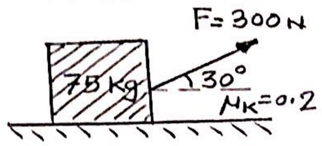
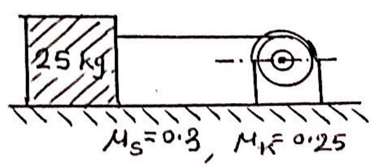
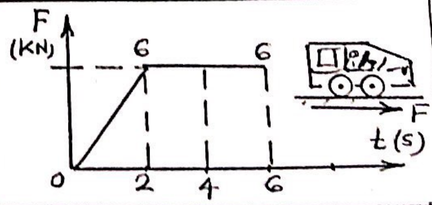
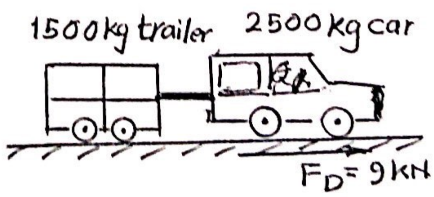
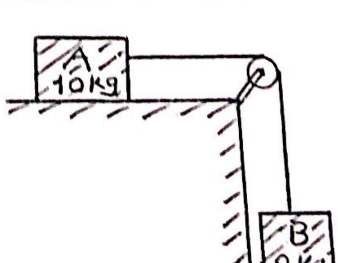
II) For the trailer only :

$$(Imp)_x = m(V_x - u_x)$$

$$(T \times 20)\ Ns = (1500)(45-0)$$

$$T = 3375\ N$$

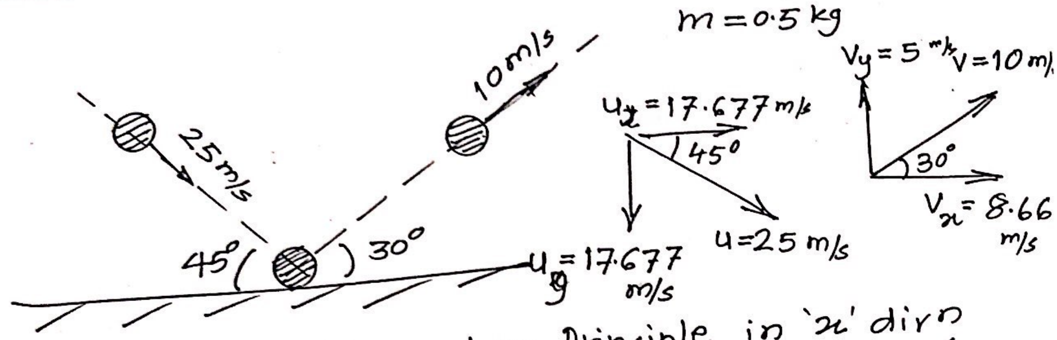
$$\boxed{T = 3.375\ kN}$$

1	The 0.5kg ball strikes the rough ground and rebounds with the velocities shown. Determine the magnitude of the impulse the ground exerts on the ball. Assume that the ball does not slip when it strikes the ground, and neglect the size of the ball and the impulse produced by the weight of the ball.	
2	If the coefficient of kinetic friction between the 75 kg crate and the ground is $\mu=0.2$, determine the speed of the crate when t=4s. The crate starts from rest and is towed by the 500N force.	
3	The motor exerts a force $F=20t^2$ N on the cable, where t is in seconds. Determine the speed of the 25 kg crate when t= 4s. The coefficient of static friction and kinetic friction between the crate and the plate are $\mu_s = 0.3$ and $\mu_k = 0.25$ resp.	
4	The wheels of the 1.5mg car generate the traction force F described by the graph. If the car starts from rest, determine its speed when t = 6s.	
5	The 2.5 Mg four wheel drive SUV tows the 1.5Mg trailer. The tractive force developed at the wheels is F_D=9kN. Determine speed of the truck in 20s starting from the rest. Also, determine the tension developed in the coupling A, between the SUV and the trailer. Neglect the mass of wheels.	
6	A 10 kg block A attains velocity of 1m/s in 5 seconds, starting from rest. Determine the tension in cord and the coefficient of kinetic friction between block A and the horizontal plane. Neglect the weight of the pulley. Block B has a mass of 8 kg.	

Lecture No. 18:

Impulse-Momentum Principle

① F 15.1 / pg. 860 / RCH (14^{th})



Applying Impulse-Momentum Principle in 'x' dirn, we get,

$$(Imp)_x = (mV_x - m \cdot u_x) = \int_{t_1}^{t_2} F_x \, dt$$

$$= (0.5)(8.66 - 17.677)$$

$$= -4.509 \text{ Ns}$$

Similarly, in 'y' dirn,

$$(Imp)_y = (m \cdot V_y - m \cdot u_y) = \int_{t_1}^{t_2} F_y \, dt$$

$$= (0.5)(5 + 17.677)$$

$$= 11.339 \text{ Ns}$$

Resultant Impulse on the ball,

$$(Imp.) = \sqrt{(Imp.)_x^2 + (Imp.)_y^2}$$

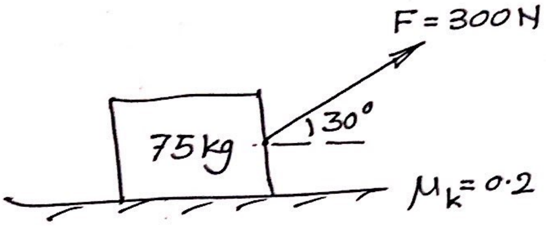
$$= \sqrt{(-4.509)^2 + (11.339)^2}$$

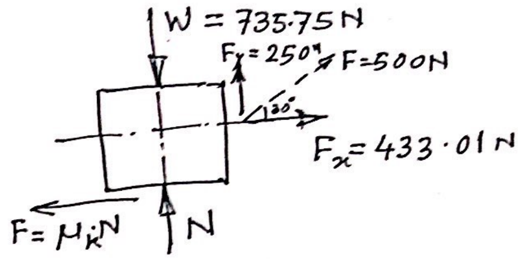
$$= 12.202 \text{ Ns}$$

② F 15.1 / pg. 860 / RCH (14^{th}) :

At $t = 0$, $u = 0$

At $t = 4s$, $V = ?$





$\Sigma F_y = 0$ gives,

$$N + 250 - 735.75 = 0$$

$$N = 485.75 \text{ N}$$

Applying Imp.-mmtm. prin. in 'x' dirn we get,

$$(Imp.)_x = \int_{t_1}^{t_2} F_x \, dt = mV_x - m \cdot u_x$$

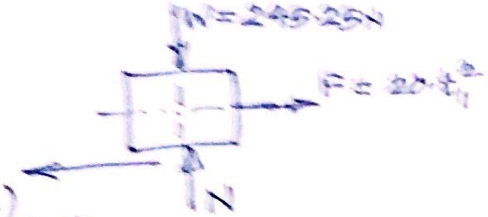
$$(Imp.)_x = (75)(V_x - 0) = (433.01 \times 4) - (0.2 \times 485.75) \times (4)$$

$$\boxed{V_x = 17.912 \text{ m/s}}$$

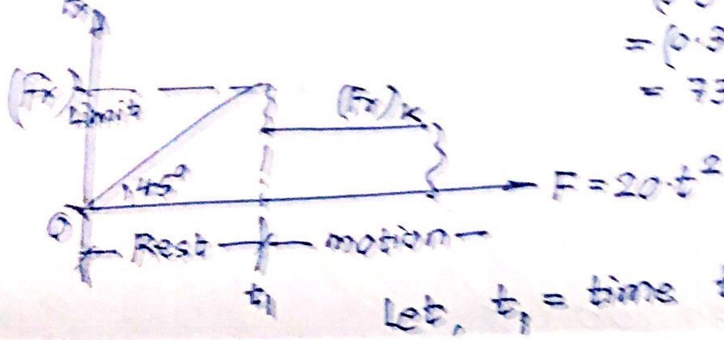
③ F 15.3/pg. 960/RCH (14th) :

$$F = 20 \cdot t^2 \text{ i.e. } f(t)$$

$\mu_s = 0.3$
$\mu_k = 0.25$



$(Fr)_{Limit}$
$= (0.3 \times N)$
$= (0.3 \times 245.25)$
$= 73.575 \text{ N}$



Let, t_1 = time to start the motion

$\therefore \quad (Fr)_{Limit} = 20 \cdot t^2$

$73.575 = 20 \cdot t_1^2 \quad \therefore t_1 = 1.918 \text{ s}$

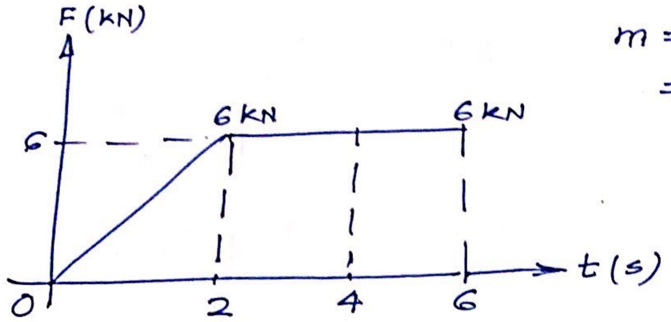
Applying Imp-mmtm prin, during motion,

$$(Imp)_x = m \cdot V_x - m \cdot u_x$$

$$\left[\int_{1.918}^{4} (20 \cdot t^2)\, dt - (0.25 \times 245.25)(4 - 1.918)\right] = (25)(V - 0)$$

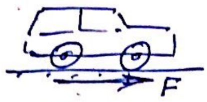
$$V = 10.1 \text{ m/s } (\rightarrow)$$

4) F.15.4/pg.860/RCH(14th):



$m = 1.5\ \text{Mg}$

$= 1500\ \text{kg}$



F = Traction Force

At $t = 0$, $u = 0$

At $t = 6\ s$, $V = ?$

$$(Imp)_{t_1}^{t_2} = \text{Area under } (F-t) \text{ diag. from } t_1 \text{ to } t_2$$

$$= \left(\tfrac{1}{2} \times 2 \times 6000\right) + (6000)(6-2)$$

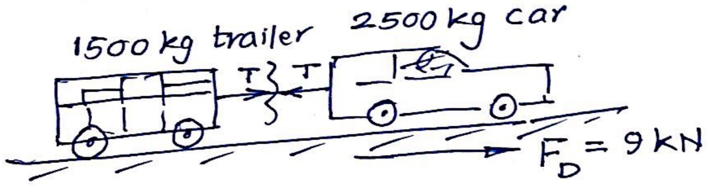
$$= 30{,}000\ \text{Ns}$$

Now, $$(Imp)_x = \int_{t_1}^{t_2} F_x\, dt = m \cdot V_x - m \cdot u_x$$

$$30{,}000 = (1500)(V - 0)$$

At $t = 6\ s$, $V = 20\ m/s$

⑤ F 15.5 / pg. 860 / RCH (14^{th}) :



At $t = 0$, $u = 0$

At $t = 20_s$, $V = ?$

I) For the car + trailer :

$$(Imp)_x = m(V_x - u_x)$$

$$(9000 \times 20) \text{ Ns} = (2500 + 1500)(V - 0)$$

$$V = 45 \text{ m/s}$$

II) For the trailer only :

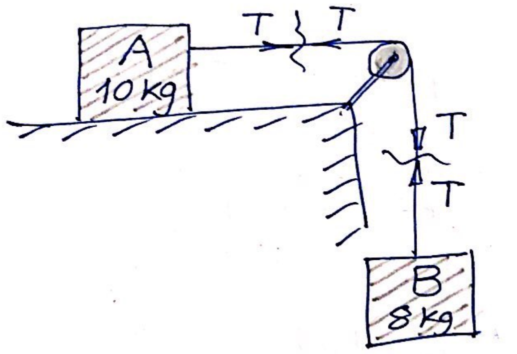
$$(Imp)_x = m(V_x - u_x)$$

$$(T \times 20) \text{ Ns} = (1500)(45 - 0)$$

$$T = 3375 \text{ N}$$

$$= 3.375 \text{ N}$$

⑥ F 15.6/pg. 860/RCH (14^{th}):

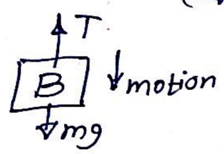


At $t = 0$, $V_1 = 0$

At $t = 5\,s$, $V_2 = 1\ m/s$

For block B :

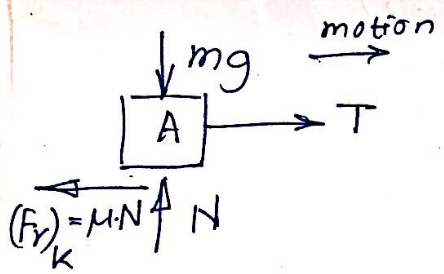
$$(Imp) = \int F \cdot dt = m(V_2 - V_1)$$



$$(8 \times 9.81)(5) - (T \times 5) = 8(1-0)$$

$$T = 76.88\ N$$

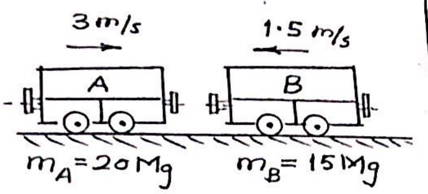
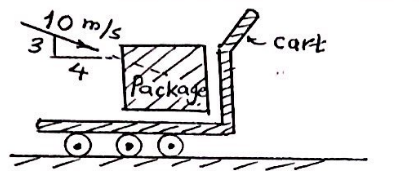
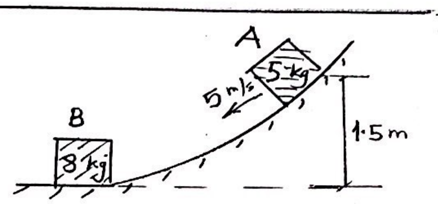
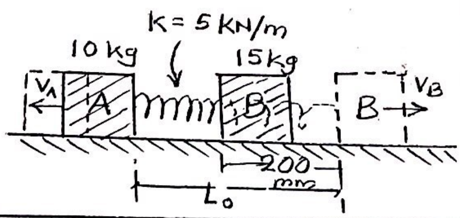
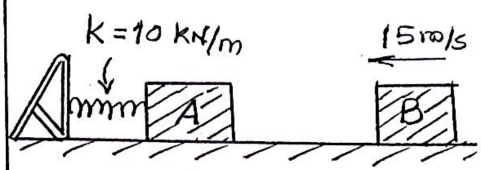
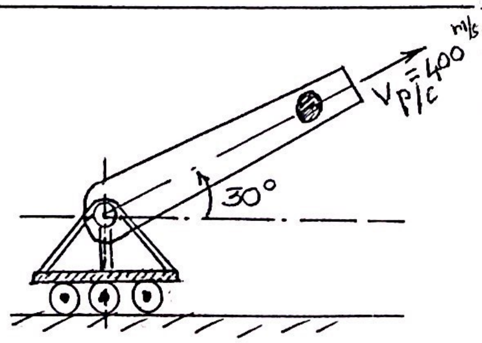
For block A :



$$(Imp) = \int F \cdot dt = m \cdot (V_2 - V_1)$$

$$(76.88 \times 5) - \mu_k (10 \times 9.81)(5) = (10)(1-0)$$

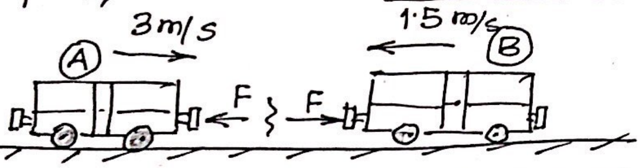
$$\mu_k = 0.763$$

1	The freight cars A and B have a mass of 20 Mg and 15Mg respectively. Determine the velocity of A after collision if the cars collide and rebound, such that B moves to the right with a speed of 2 m/s. If A and B are in contact for 0.5s, find the average impulsive force which acts between them.	
2	The cart and package have a mass of 20 Kg and 5 Kg respectively. If the cart has a smooth surface and it is initially at rest, while the velocity of the package is as shown, determine the final common velocity of the cart and package after impact.	
3	A five Kg block A has an initial speed of 5 m/s as it slides down the smooth ramp, after which it collides with stationary block B of mass 8 kg. If two blocks couple together after collision, determine their common velocity after collision.	
4	A spring is fixed to block A and block B pressed against the spring. If the spring is compressed by s=200mm and then the blocks are released, determine their velocity at the instant block B loses contact with the spring. The masses of block A and B are 10 Kg and 15 kg respectively.	
5	Blocks A and B have a mass of 15 kg and 10 kg respectively. If A is stationary and B has a velocity of 15 m/s just before collision and the blocks coupled together after impact, determine the maximum compression of the spring.	
6	The cannon and support without a projectile have a mass of 250 kg. If a 20kg projectile is fired from the cannon with a velocity of 400 m/s, measured relative to the cannon, determine the speed of projectile as it leaves the barrel of the cannon. Neglect rolling resistance.	

Lecture No. (19) :

Conservation of Momentum

(1) F 15.7 / pg. 875 / RCH (14th) : Here, the two bodies do not move together after impact, hence it is semi-elastic impact.



$m_A = 20{,}000$ kg

$u_A = 3$ m/s

$V_A = ?$

$m_B = 15{,}000$ kg

$u_B = -1.5$ m/s

$V_B = 2$ m/s

Time of contact = 0.5 sec.

By conservation of linear momentum,

$$m_A u_A + m_B u_B = m_A V_A + m_B V_B$$

$$(20{,}000 \times 3) - (15{,}000 \times 1.5) = (20{,}000 \times V_A) + (15000 \times 2)$$

$$\therefore \boxed{V_A = 0.375 \text{ m/s } (\longrightarrow)}$$

Let, F = average impulsive force betn A & B.

Apply Imp.- mmtm. prin. to freight car B.

$$\text{Imp.} = m \cdot V_B - m \cdot u_B$$

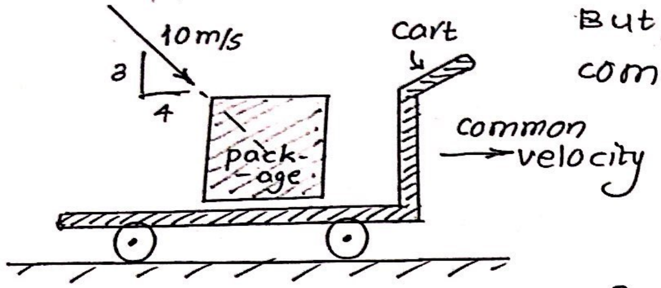
$$(F \times 0.5) \text{ Ns} = (15{,}000)(2 + 1.5)$$

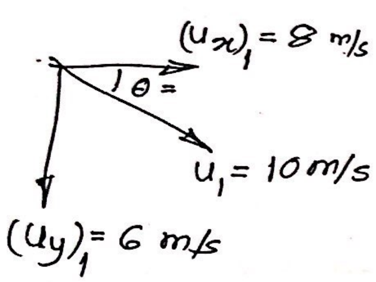
$$\therefore F = 105 \times 10^3 \text{ N}$$

$$\boxed{F = 105 \text{ kN}}$$

② F 15.8 / pg. 875 / RCH (14^{th}) :

In this case, the two bodies move together after impact. They do not separate from each other. But, they move with a common velocity after impact. This is called as Plastic impact.





$\tan\theta = \frac{3}{4}$ $\therefore$ $\theta = 36.87°$

Package : Body ①, $m_1 = 5$ kg

$(u_x)_1 = 8$ m/s ($\rightarrow$)

$(u_y)_1 = 6$ m/s ($\downarrow$)

Cart : Body ②, $m_2 = 20$ kg

$(u_x)_2 = (u_y)_2 = 0$

For both the bodies, $(V_x)_2 = (V_x)_1 = V$ say

$(V_y)_2 = (V_y)_1 = 0$

By conservation of momentum in 'x' dir^n

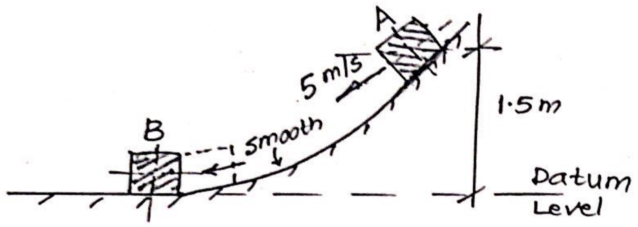
$$m_1 \cdot (u_x)_1 + m_2 (u_x)_2 = (m_1 + m_2) \cdot V$$

$$(5 \times 8) + (20 \times 0) = (20 + 5) \cdot V$$

$$V = \frac{40}{25} = 1.6 \text{ m/s}$$

$$\boxed{V = 1.6 \text{ m/s}}$$

③ F 15.9 / Pg. 875 / RCH(14th) :



$m_A = 5$ kg
$m_B = 8$ kg

As block A is travelling along smooth surface, we can use the law of conservation of energy, for block A, to get its velocity at the lowest level. With that velocity block A will strike block B.

For A : $T_1 + V_1 = T_2 + V_2$

$$\left(\frac{1}{2} m_A \cdot V_A^2\right)_1 + (m_A \cdot g \cdot h_1) = \left(\frac{1}{2} m_A \cdot V_A^2\right)_2 + \cancel{(m_A \cdot g \cdot h_2)}$$

$$\therefore \left(\frac{1}{2} \times 5 \times 5^2\right) + (5 \times 9.81 \times 1.5) = \left(\frac{1}{2} \times 5 \times V_A^2\right)_2$$

$$\boxed{(V_A)_2 = 7.378 \text{ m/s}}$$

As the two blocks move together, after impact, this is a plastic impact. ∴ By conservation of momentum, we get,

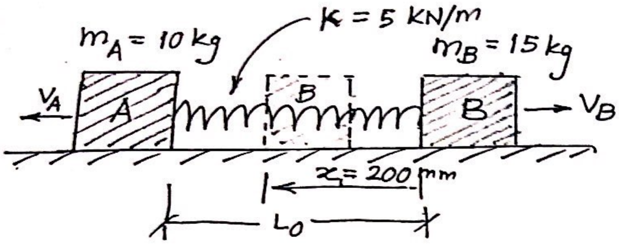
$$m_A \cdot u_A + m_B \cdot u_B = m_A \cdot V_A + m_B \cdot V_B$$

$$(5 \times 7.378) + (8 \times 0) = (5+8) \cdot V$$

$$\therefore V_A = V_B = V = 2.84 \text{ m/s}$$

$$\boxed{V = 2.84 \text{ m/s}}$$

④ F15·10/pg.875/RCH(14th):



$u_A = u_B = 0$

$V_A = ?$

$V_B = ?$

Apply conservation of momentum eq^n, immediatly after the blocks are released.

$$m_A u_A + m_B u_B = m_A \cdot V_A + m_B \cdot V_B$$

$$0 + 0 = 10 \cdot V_A + 15 \cdot V_B$$

$$\therefore \quad 2 \cdot V_A + 3 \cdot V_B = 0 \longrightarrow (1)$$

As, force of friction is absent, we can apply the law of conservation of energy.

$$T_1 + V_1 = T_2 + V_2$$

$$\left(\tfrac{1}{2} m_A u_A^2 + \tfrac{1}{2} m_B u_B^2\right) + \left(\tfrac{1}{2} k x_1^2\right) = \left(\tfrac{1}{2} m_A V_A^2 + \tfrac{1}{2} m_B V_B^2\right) + \cancel{\left(\tfrac{1}{2} k x_2^2\right)}$$

$$0 + 0 + \left(\tfrac{1}{2} \times 5000 \times 0.2^2\right) = \left(\tfrac{1}{2} \times 10 \times V_A^2\right) + \left(\tfrac{1}{2} \times 15 \times V_B^2\right)$$

$$\therefore \quad 5 \cdot V_A^2 + (7.5) V_B^2 = 100 \longrightarrow (2)$$

But, $\quad V_A = -(1.5) V_B$

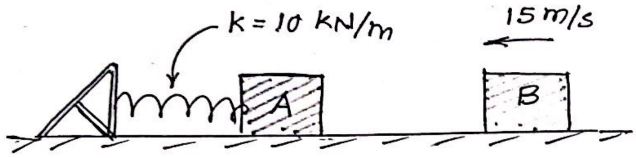
$$\therefore \quad 5(-1.5 \times V_B)^2 + (7.5) V_B^2 = 100$$

$$\therefore \quad (18.75) V_B^2 = 100$$

$$\boxed{V_B = 2.31 \text{ m/s } (\rightarrow)}$$

$$\boxed{\begin{aligned} V_A &= -3.465 \text{ m/s} \\ &= 3.465 \text{ m/s } (\leftarrow) \end{aligned}}$$

(5) F 15.11/pg. 875/RCH (14^{th}):



$m_A = 15$ kg
$m_B = 10$ kg
$u_A = 0$
$u_B = -15$ m/s

After the impact, the blocks are coupled together. Hence, this is a plastic impact. By conservation of momentum,

$$m_A \cdot u_A + m_B \cdot u_B = m_A \cdot V_A' + m_B V_B$$

$$0 + (10)(-15) = (15 + 10)\,V$$

$$\therefore\ V_A' = V_B = V = 6 \text{ m/s}$$

With this velocity the spring is compressed by the two blocks.

Let x_{max} = maximum compression of the spring when the blocks stop.

$\therefore$ By conservation of energy (for the entire system)

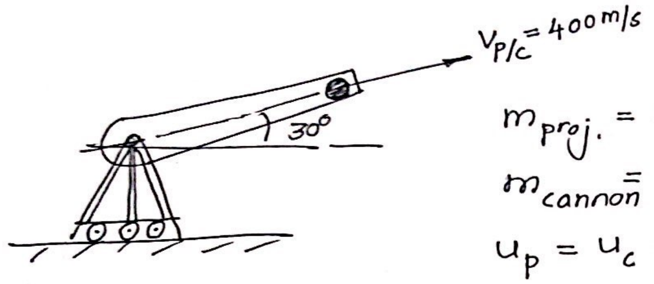
$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(m_A + m_B)V_1^2 + \cancel{\frac{1}{2}k x_1^2} = \cancel{\frac{1}{2}(m_A + m_B)V_2^2} + \frac{1}{2}k x_2^2$$

$$\frac{1}{2}(15 + 10)(6)^2 = \frac{1}{2} \times 10{,}000 \times x_{max}^2$$

$$\therefore\ \boxed{\begin{aligned} x_{max} &= 0.3 \text{ m} \\ &= 300 \text{ mm} \end{aligned}}$$

(6) F 15.12/ pg.875 / RCH (14^{th}) :



$m_{proj.} = 20\ kg$

$m_{cannon} = 250\ kg$

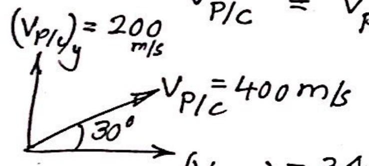
$u_p = u_c = 0$

Applying conservation of momentum in 'x' dirn

$$(\cancel{m_p \cdot u_p + m_c u_c}) = (m_p \cdot V_p + m_c \cdot V_c)$$

$$0 = (20)(V_P)_x - (250 \times V_c) \longrightarrow (1)$$

$$\bar{V}_{P/C} = \bar{V}_p - \bar{V}_c$$



$(V_{P/C})_x = 346.41\ m/s$

$$\therefore (346.41)\hat{i} + (200)\hat{j} = (V_P)_x \hat{i} + (V_P)_y \hat{j} + V_c \cdot \hat{i}$$

$$(V_P)_x + V_c = 346.41\ m/s \longrightarrow (2)$$

$$(V_P)_y = 200\ m/s\ (\uparrow)$$

$$(V_P)_x = (-V_c + 346.41)$$

$\therefore$ Eqn (1) becomes,

$$0 = (20)(-V_c + 346.41) - (250 \times V_c)$$

$$0 = -20 \cdot V_c + 6928.2 - 250 \cdot V_c$$

$$\therefore V_c = 25.66\ m/s\ (\leftarrow)$$

$$\therefore (V_P)_x = 320.75\ m/s\ (\rightarrow),\ (V_P)_y = 200\ m/s\ (\uparrow)$$

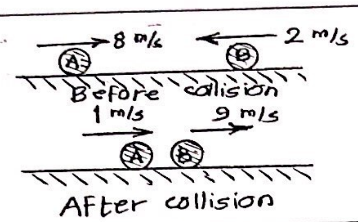
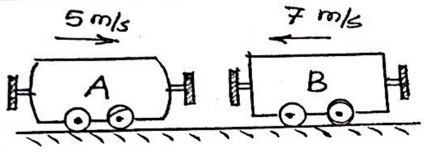
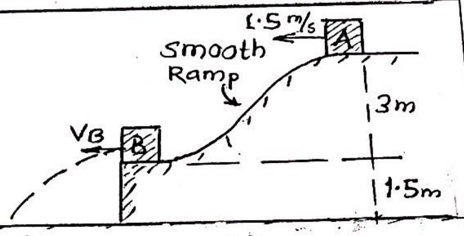
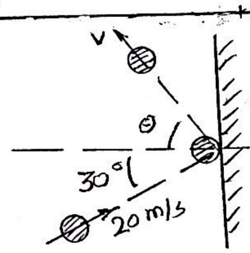
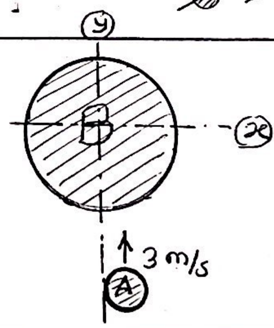
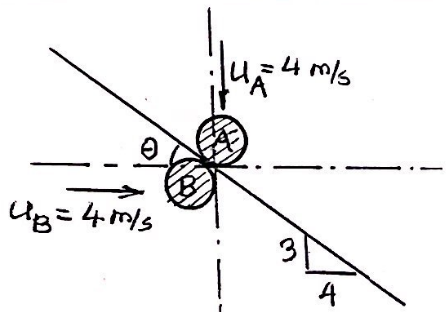
$$V_P = \sqrt{(V_P)_x^2 + (V_P)_y^2} = 378\ m/s$$

$$\boxed{V_P = 378\ m/s}$$

20

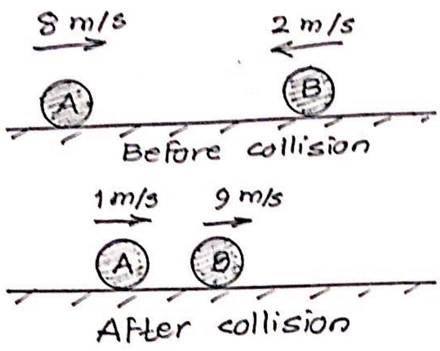
Lecture No: .

Impact

1	Determine the coefficient of restitution e between ball A and ball B. The velocities of A and B before and after collision are shown.	
2	A 15-Mg tank car A and 25-Mg freight car B travel towards each other with the velocities shown. If the coefficient of restitution between the bumpers is e=0.6, determine the velocity of each car just after the collision.	
3	A 15 Kg package A has a speed of 1.5 m/s when it enters the smooth ramp. As it slides down the ramp, it strikes the 40 Kg package B which is initially at rest. If the coefficient of restitution between A and B is e=0.6, determine the velocity of B just after impact.	
4	The ball strikes a smooth wall with a velocity of v_{b1}=20m/s. If the coefficient of restitution between the ball and the wall is e=0.75, determine the velocity of the ball just after impact.	
5	A disk A has a mass of 2 kg and slides on a smooth horizontal plane with a velocity of 3m/s. Disk B has a mass of 11 kg and is initially at rest. If after impact A has velocity of 1 m/s, parallel to positive X axis, determine the speed of disk B after impact.	
6	Two disks A and B each have a mass of 1 kg and initial velocities shown just before they collide. If coefficient of restitution is e=0.5, determine their speed just after impact.	

Lecture No. (20) **Impact**

(1) F15.13/pg. 887/RCH(14th) :



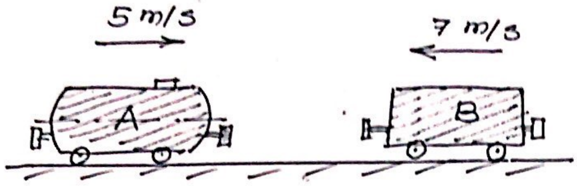
Coefficient of Restitution,

$$e = \left(\frac{V_B - V_A}{u_A - u_B}\right)$$

$$\therefore \quad e = \left[\frac{9-1}{8-(-2)}\right] = 0.8$$

$$\boxed{e = 0.8}$$

② F 15·14/pg.887/RCH (14th) :



$m_A = 1500$ kg $m_B = 2,500$ kg

$u_A = 5$ m/s $u_B = -7$ m/s

$V_A = ?$ $V_B = ?$

$e = 0.6$

(semi-elastic impact)

By conservation of momentum,

$$m_A \cdot u_A + m_B \cdot u_B = m_A \cdot V_A + m_B \cdot V_B$$

$$(1500 \times 5) + (2500)(-7) = 1500 \cdot V_A + 2500 \cdot V_B$$

$$\therefore \quad 3 \cdot V_A + 5 \cdot V_B = -20 \longrightarrow ①$$

By coeff. of Restitution,

$$e = \left(\frac{V_B - V_A}{u_A - u_B} \right)$$

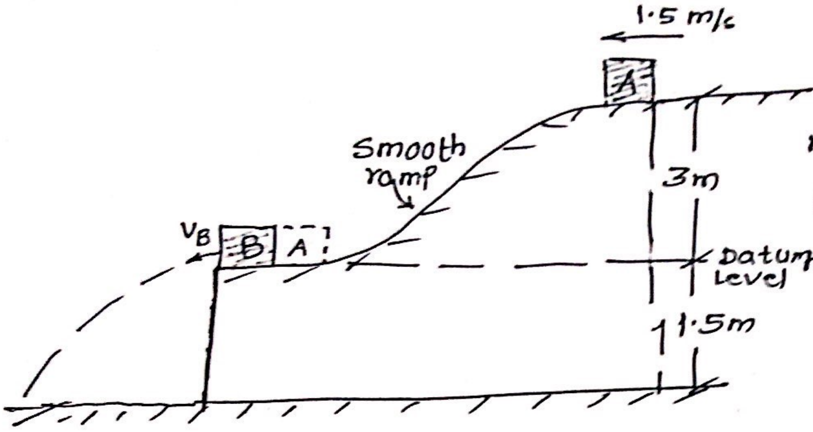
$$0.6 = \left(\frac{V_B - V_A}{5 + 7} \right)$$

$$(V_B - V_A) = 7.2 \longrightarrow ②$$

solving ① & ②, we get,

$V_B = 0.2$ m/s $(\rightarrow)$
$V_A = 7$ m/s $(\leftarrow)$

③ F15.15/pg. 887/RCH (14th):



$m_A = 15$ kg, $h_A = 3$ m
$u_A = 1.5$ m/s
$m_B = 40$ kg
$u_B = 0$, $h_B = 0$
$e_{AB} = 0.6$

By conservation of energy, (only to block A)

$$T_1 + V_1 = T_2 + V_2$$

$$\left(\tfrac{1}{2} \times 15 \times 1.5^2\right) + (15 \times 9.81 \times 3) = \left(\tfrac{1}{2} \times 15 \times V_A^2\right) + 0$$

$$\therefore \quad V_A = 7.817 \text{ m/s } (\leftarrow)$$

This is the striking vel. of A, on B.

By conservation of momentum for A & B,

$$m_A \cdot u_A + m_B \cdot u_B = m_A \cdot V_A + m_B \cdot V_B$$

$$-(15 \times 7.817) + \cancel{(40 \times 0)} = 15 \cdot V_A + 40 \cdot V_B$$

$$\therefore \quad 15 \cdot V_A + 40 \cdot V_B = -117.255 \longrightarrow (1)$$

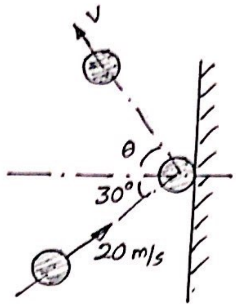
By coeff. of restitution,

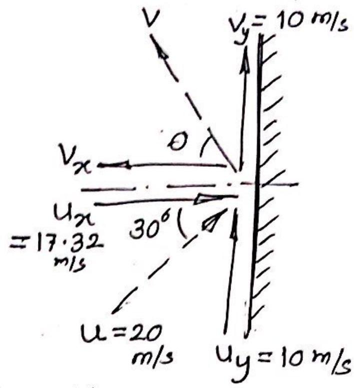
$$e = \left(\frac{V_B - V_A}{u_A - u_B}\right) \quad \therefore 0.6 = \left(\frac{V_B - V_A}{-7.817 - 0}\right)$$

$$\therefore \quad (V_B - V_A) = -4.69 \longrightarrow (2)$$

$$\boxed{\begin{array}{l} V_A = 1.279 \text{ m/s } (\rightarrow) \\ V_B = 3.411 \text{ m/s } (\leftarrow) \end{array}}$$

④ F 15.16 / pg. 887 / RCH (14th) :





For the impact of ball with wall,

coeff. of restitution, $e = \sqrt{\dfrac{h_2}{h_1}} = -\left[\dfrac{V_x}{u_x}\right]$

$\therefore \quad 0.75 = -\left[\dfrac{-V_x}{17.32}\right] \quad \therefore V_x = 13 \text{ m/s } (\leftarrow)$

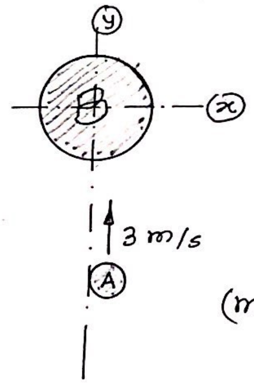
V = Velocity of the ball after impact,

$V = \sqrt{13^2 + 10^2} = 16.4 \text{ m/s}$

$\theta = \tan^{-1}\left(\dfrac{V_y}{V_x}\right) = \tan^{-1}\left(\dfrac{10}{13}\right) = 37.57^\circ$

$\boxed{V = 16.4 \text{ m/s}, \ \theta = 37.57^\circ}$

⑤ F15·17/pg. 887/RCH (14th) :



$m_A = 2 \text{ kg}, \quad u_A = 3 \text{ m/s } (\uparrow)$

$m_B = 11 \text{ kg}, \quad u_B = 0$

$V_A = 1 \text{ m/s } (\rightarrow)$

Applying conservation of momentum, along 'x' axis

$$(m_A \cdot u_A)_x + (m_B \cdot u_B)_x = (m_A \cdot V_A)_x + (m_B \cdot V_B)_x$$

$$0 + 0 = (2 \times 1) + 11 \times (V_B)_x$$

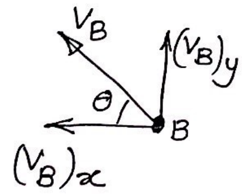
$$\therefore \quad (V_B)_x = -0.1818 \text{ m/s}$$

Applying conservation of momentum, along 'y' axis

$$(m_A \cdot u_A)_y + (m_B \cdot u_B)_y = (m_A \cdot V_A)_y + (m_B \cdot V_B)_y$$

$$(2 \times 3) + 0 = 0 + 11 \times (V_B)_y$$

$$(V_B)_y = 0.545 \text{ m/s}$$



$$\therefore \quad \boxed{\begin{aligned} V_B &= 0.575 \text{ m/s} \\ \theta &= 71.55^\circ \end{aligned}}$$

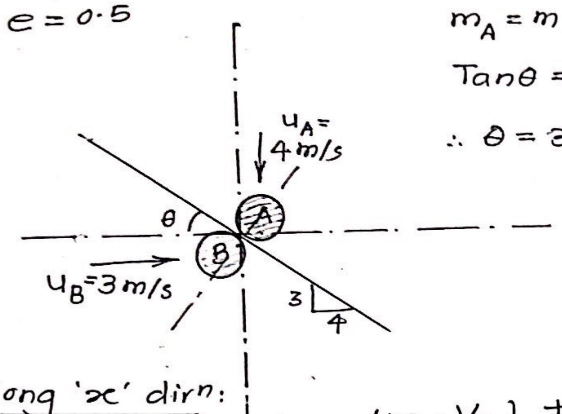
(6) P 15.10 / pg. 887 / RCH ...

$e = 0.5$

$m_A = m_B = 1\ kg$

$\tan\theta = \frac{3}{4}$

$\therefore\ \theta = 36.87^\circ$



I) Along 'x' dir^n:

$$(m_A \cdot u_A)_x + (m_B \cdot u_B)_x = (m_A \cdot V_A)_x + (m_B \cdot V_B)_x$$

$$0 + (1 \times 3) = (V_A)_x + (V_B)_x \longrightarrow (1)$$

$$e = \frac{(V_B)_x - (V_A)_x}{(u_A)_x - (u_B)_x}$$

$$0.5 = \frac{(V_B)_x - (V_A)_x}{0 - 3}$$

$$\therefore\ (V_B)_x - (V_A)_x = -1.5 \longrightarrow (2)$$

$$(V_A)_x = 2.25\ m/s, \quad (V_B)_x = 0.75\ m/s$$

II) Along 'y' dir^n:

$$(m_A \cdot u_A)_y + (m_B \cdot u_B)_y = (m_A \cdot V_A)_y + (m_B \cdot V_B)_y$$

$$0 - (1 \times 4) = (V_A)_y + (V_B)_y \longrightarrow (3)$$

$$e = \frac{(V_B)_y - (V_A)_y}{(u_A)_y - (u_B)_y}$$

$$0.5 = \frac{(V_B)_y - (V_A)_y}{-4 - 0}$$

$$(V_B)_y - (V_A)_y = -2 \longrightarrow (4)$$

$$\therefore\ (V_A)_y = -1\ m/s \qquad (V_B)_y = -3\ m/s$$

23.96° $\qquad$ 75.96°