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LADC Tutorial 3

9.1

(1) If $y = \sin ax + \cos ax$, prove that

$$y_n = a^n \left[1 + (-1)^n \sin^2 ax\right]^{1/2}$$

→

$$y = \sin ax + \cos ax$$

$$y_n = a^n \sin\left[\frac{n\pi}{2} + ax\right] + b^n \cos\left[\frac{n\pi}{2} + ax\right]$$

$$= a^n \left(\sin\left(\frac{n\pi}{2} + ax\right) + \cos\left(n\pi/2 + ax\right)\right)$$

as $\sin ax + \cos ax = \sqrt{1 + \sin 2ax}$

$$y_n = a^n \left\{1 + \sin 2\left(\frac{n\pi}{2} + ax\right)\right\}$$

$$= a^n \left\{1 + \sin\left(\frac{2n\pi}{2} + 2ax\right)\right\}^{1/2}$$

$$= \left\{1 + (-1)^n \sin 2ax\right\}^{1/2}$$

(2) $\sin^2 x \cos^3 x$

$$\left(\frac{1 - \cos 2x}{2}\right)\left(\frac{\cos 3x + 3\cos x}{4}\right)$$

$$= \frac{1}{8}\left[\cos 3x + 3\cos x - \cos 2x \cos 3x - 3\cos x \cos 2x\right]$$

$= \frac{1}{8}(\cos 3x + 3\cos x - \cos 2x \cos 3x =$

$-\frac{1}{2}(\cos x + \cos 5x) - 3/2(\cos x + \cos 3x))$

$= \frac{1}{16}(2\cos 3x + 10\cos x - \cos x - \cos 5x - 3\cos x - 3\cos 3x)$

$= \frac{1}{16}(2\cos x - \cos 3x - \cos 5x)$

$= \frac{1}{8}(\cos x) - \frac{1}{16}\cos 3x - \frac{1}{16}\cos 5x$

$= \frac{1}{8}\cos\left(x + \frac{n\pi}{2}\right) - \frac{3^n}{16}\cos\left(3x + \frac{n\pi}{2}\right)$

$-\frac{5^n}{16}\cos\left(5x + \frac{n\pi}{2}\right)$

(3) $\cos^{-1}\left(\frac{x - x^{-1}}{x + x^{-1}}\right)$

$y = \cos\left(\frac{x - x^{-1}}{x + x^{-1}}\right) = \cos^{-1}\left[\frac{x^2 - 1}{x^2 + 1}\right]$

$= \cos$ let $x = \tan\phi$

then

$y = \cos^{-1}\left(\frac{\tan^2\phi - 1}{\tan^2\phi + 1}\right)$

$= \cos^{-1}(-\cos 2\phi) = \cos^{-1}[\cos(\pi + 2\phi)]$

$\therefore y = \pi + 2\tan^{-1}x$

$\therefore y_n = 2(-1)^{n-1}(n-1)!\sin^n\theta \sin(n\theta)$

$\sin^n\theta$ when $\theta = \tan^{-1}(1/x)$

Q.2. (1) $\sin 4x$

$$y'_n = \frac{1}{8}\cdot\left[4^n\cdot\cos\left(4x+\frac{n\pi}{2}\right) - 4\cdot 2^n\cos\left(2x+\frac{n\pi}{2}\right)\right]$$

(2) $\sin 6x \cos 4x$

$$= \frac{1}{2}\cdot\frac{10^n}{2}\times\left[\sin\left(\frac{n\pi}{2}+10x\right) + \sin\left(\frac{n\pi}{2}+2x\right)\right]$$

(3) $\tan^{-1}\left(\frac{1+x}{1-x}\right)$

$y_n = (-1)^{n-1}\cdot(n-1)!\cdot\sin(n\cot^{-1}x)\sin^n\cot^{-1}x.$

(4) $y = \frac{1}{x^2+a^2}$,

$$y_n = \frac{(-1)^n n!\sin(n+1)\theta\sin^{n+1}\theta}{a^{n+2}}$$ where $\theta = \tan^{-1}\frac{a}{x}$