

Conservation of Energy

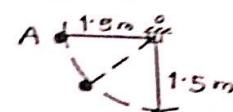
Impulse Momentum Principle

Lecture No:

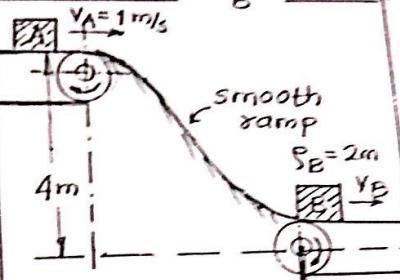
Conservation of Energy

17

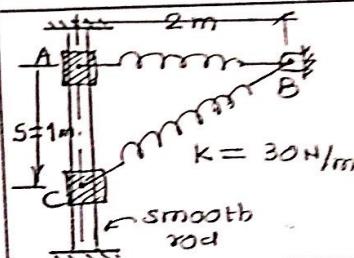
- 1 A two kg pendulum bob is released from rest when it is at A. Determine the speed of bob and the tension in the chord when the bob passes through its lowest position, B.



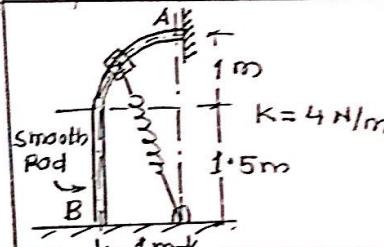
- 2 A 2 kg package leaves the conveyor belt at A at a speed of $v_A = 1 \text{ m/s}$ and slides down the smooth ramp. Determine the required speed of the conveyor belt at so that the package can be delivered without slipping on the belt. Also, find the normal reaction the curved portion of the ramp exerts on the package at B if $\rho = 2 \text{ m}$.



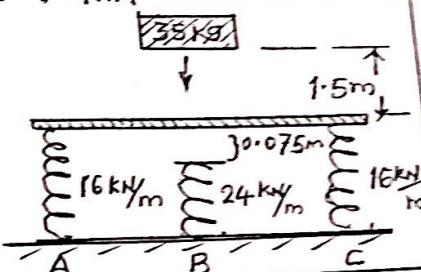
- 3 The 2 kg collar is given a downward velocity of 4 m/s when it is at A. If the spring has an unstretched length 1 m and a stiffness of $k = 30 \text{ N/m}$, determine the velocity of the collar at B.



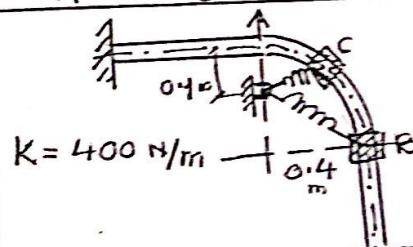
- 4 The 5 kg collar is released from rest at A and travels along the frictionless guide. Determine the speed of the collar when it strikes the stop B. The spring has an unstretched length of 0.5m



- 5 The 35 kg block is released from rest 1.5m above the plate. Determine the compression of each spring when the block momentarily comes to rest after striking the plate. Neglect the mass of the plate. The springs are initially unstretched.

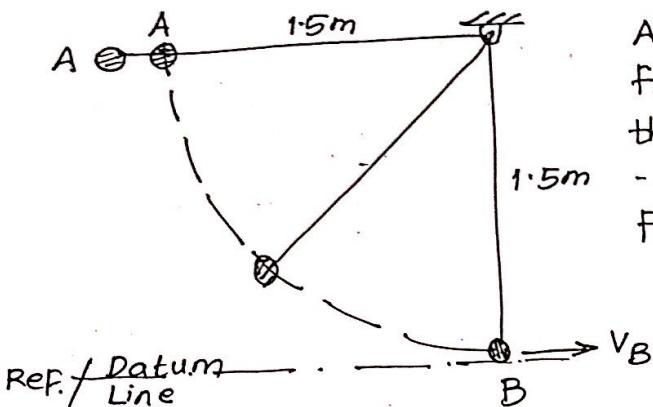


- 6 The 4 kg collar C has a velocity of $v_A = 2 \text{ m/s}$ when it is at A. If the guide rod is smooth, determine the speed of the collar when it is at B. The spring has an unstretched length of $l_0 = 0.2 \text{ m}$



Lecture No. (17) Conservation of Energy

① F 14-13/pg. 837/RCH (14th)



As, the force of friction is absent, the system is subjected to conservative force system.
 $m = 2 \text{ kg}$

By conservation of energy principle,

$$T_A + V_A = T_B + V_B$$

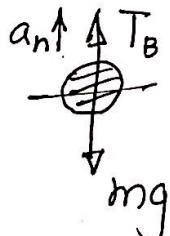
$$(K.E.)_A + (G.P.E.)_A + (E.P.E.)_A = (K.E.)_B + (G.P.E.)_B + (E.P.E.)_B$$

$$\cancel{\frac{1}{2}mv_A^2} + mgh_A + \cancel{\frac{1}{2}Kx_A^2} = \cancel{\frac{1}{2}mv_B^2} + mgh_B + \cancel{\frac{1}{2}Kx_B^2}$$

$$(2 \times 9.81 \times 1.5) = \frac{1}{2} \times 2 \times v_B^2$$

$$v_B = 5.42 \text{ m/s}$$

At pos. B : Applying N.S.L.M. along the normal dirn

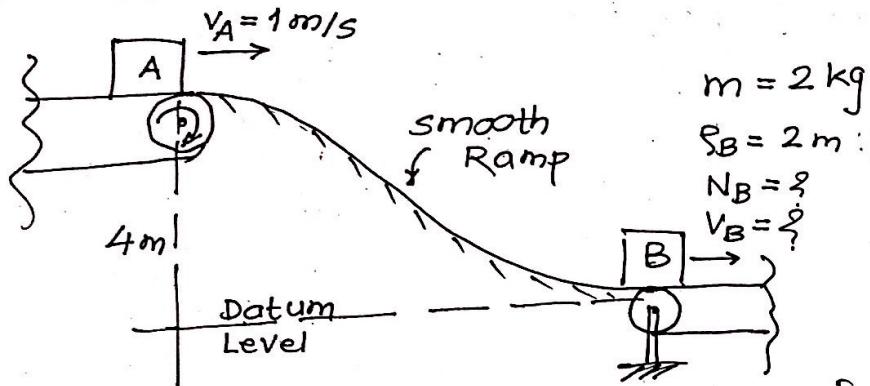


$$\sum F_n = m \cdot a_n$$

$$T_B - (2 \times 9.81) = 2 \times \frac{5.42^2}{1.5}$$

$$T_B = 58.9 \text{ N}$$

(2) F 14.14 / pg. 837 / RCH (14th):



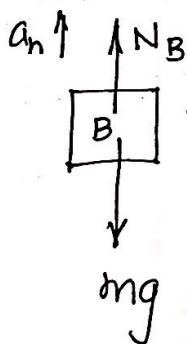
By the principle of conservation of energy,

$$\left(\frac{1}{2}m \cdot v_A^2\right) + (m \cdot g \cdot h_A) = \left(\frac{1}{2}m \cdot v_B^2\right) + (m \cdot g \cdot h_B)$$

$$T_A + V_A = T_B + V_B$$

$$\left(\frac{1}{2} \times 2 \times 1^2\right) + (2 \times 9.81 \times 4) = \left(\frac{1}{2} \times 2 \times v_B^2\right) + 0$$

$$v_B = 8.915 \text{ m/s}$$



By N.S.L.M. along normal dir?

$$\sum F_n = m \cdot a_n$$

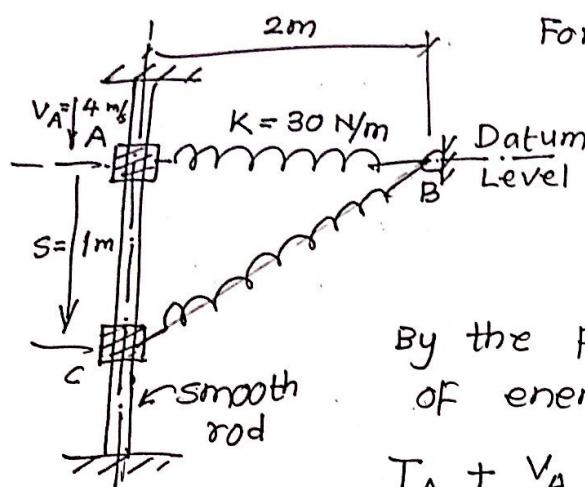
$$N_B - (2 \times 9.81) = 2 \times \frac{8.915^2}{2}$$

$$N_B = 99.1 \text{ N}$$

(3)

F 14.15 / pg. 837 / RCH (14th)

$$m = 2 \text{ kg}$$



For the spring

$$L_0 = 1 \text{ m}, K = 30 \text{ N/m}$$

$$L_A = 2 \text{ m}, x_A = (L_A - L_0) = 1 \text{ m}$$

$$L_C = \sqrt{2^2 + 1^2} = \sqrt{5} = 2.236 \text{ m}$$

$$x_C = (L_C - L_0) = 1.236 \text{ m}$$

By the principle of conservation
of energy,

$$T_A + V_A = T_C + V_C$$

$$\therefore T_A + (V_G)_A + (V_E)_A = T_C + (V_G)_C + (V_E)_A$$

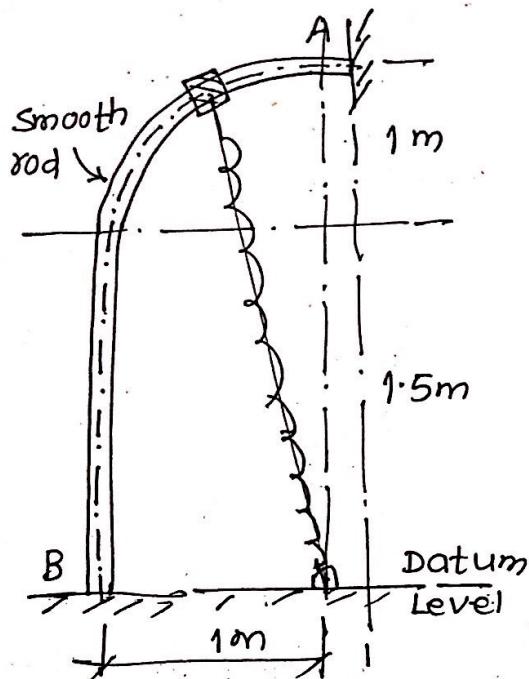
$$(K.E. + G.P.E. + E.P.E.)_A = (K.E. + G.P.E. + E.P.E.)_C$$

$$\frac{1}{2}mv_A^2 + mgh_A + \frac{1}{2}kx_A^2 = \frac{1}{2}mv_C^2 + mgh_C + \frac{1}{2}kx_C^2$$

$$\left(\frac{1}{2} \times 2 \times 4^2 \right) + \left(\frac{1}{2} \times 30 \times 1^2 \right) = \left(\frac{1}{2} \times 2 \times V_C^2 \right) - (2 \times 9.81 \times 1) + \left(\frac{1}{2} \times 80 \times 1.236^2 \right)$$

$$\therefore V_C = 5.26 \text{ m/s}$$

④ F 14.16 / Pg. 837 / RCH (14th) :



$$m = 5 \text{ kg}$$

$$K = 4 \text{ N/m}$$

$$L_0 = 0.5 \text{ m}$$

$$L_A = 2.5 \text{ m}, L_B = 1 \text{ m}$$

$$x_A = (L_A - L_0) = 2.0 \text{ m}$$

$$x_B = (L_B - L_0) = 0.5 \text{ m}$$

$$V_A = 0$$

$$V_B = ?$$

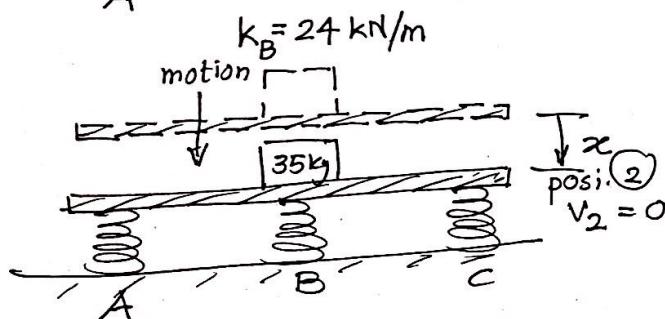
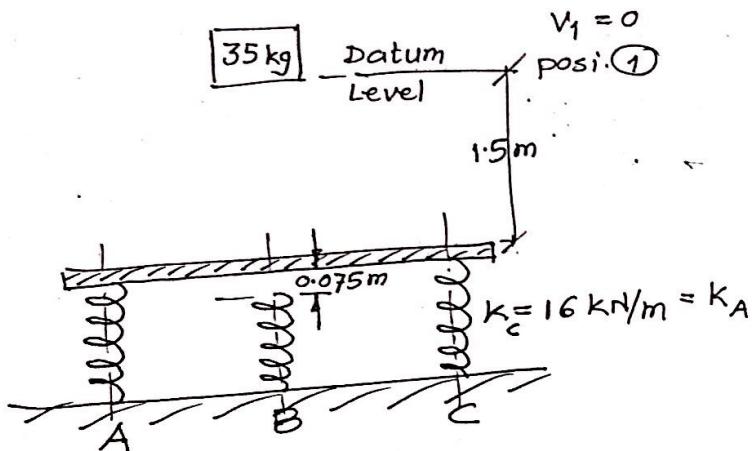
$$T_A + V_A = T_B + V_B$$

$$\cancel{\frac{1}{2}mv_A^2 + mgh_A + \frac{1}{2}k \cdot x_A^2} = \frac{1}{2}mv_B^2 + \cancel{mgh_B + \frac{1}{2}k \cdot x_B^2}$$

$$(5 \times 9.81 \times 2.5) + (\frac{1}{2} \times 4 \times 2^2) = (\frac{1}{2} \times 5 \times V_B^2) + (\frac{1}{2} \times 4 \times 0.5^2)$$

$$V_B = 7.21 \text{ m/s}$$

⑤ F 14.17 / pg. 837 / RCH (14th)



$$x_{c_A} = x_{c_C} = x \\ x_B = (x - 0.075) \text{ m}$$

By the principle of conservation of energy,

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mV_1^2 + mgh_1 + \sum\left(\frac{1}{2}kx_1^2\right) = \frac{1}{2}mV_2^2 + mgh_2 + \sum\left(\frac{1}{2}kx_2^2\right)$$

$$0 + 0 + 0 = 0 + (35 \times 9.81)(-1.5 - x)$$

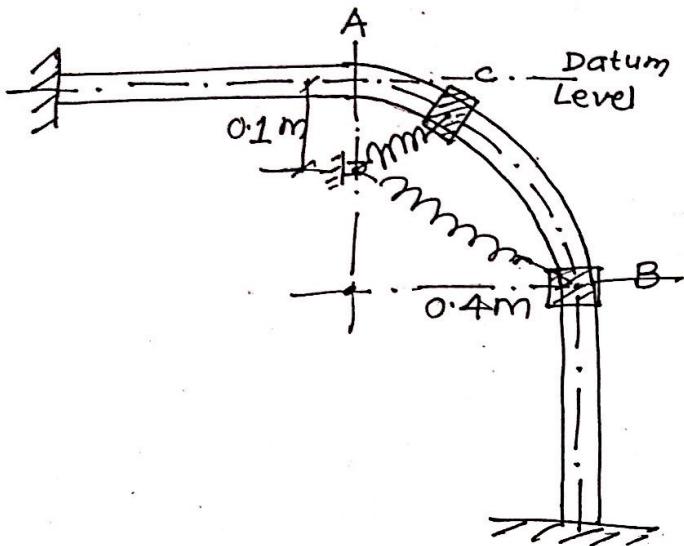
$$+ 2\left(\frac{1}{2} \times 16,000 \times x^2\right) + \frac{1}{2} \times 24,000 \times (x - 0.075)^2$$

$$\therefore x = x_{c_A} = x_{c_C} = 0.170 \text{ m}$$

$$x_B = (0.170 - 0.075)$$

$$\boxed{x_B = 0.095 \text{ m}}$$

⑥ F14.18/pg. 837/RCH (14th)



$$m = 4 \text{ kg}$$

$$\text{At } A, V_A = 2 \text{ m/s}$$

$$L_0 = 0.2 \text{ m}$$

$$x_A = (L_A - L_0)$$

$$= (0.1 - 0.2)$$

$$= -0.1 \text{ m}$$

$$\text{At } B, V_B = ?$$

$$L_B = \sqrt{(0.4)^2 + (0.2)^2}$$

$$= 0.5 \text{ m}$$

$$x_B = (L_B - L_0)$$

$$= (0.5 - 0.2)$$

$$= 0.3 \text{ m}$$

$$k = 400 \text{ N/m}^2$$

$$h_A = 0, h_B = -0.4 \text{ m}$$

$$T_A + V_A = T_B + V_B$$

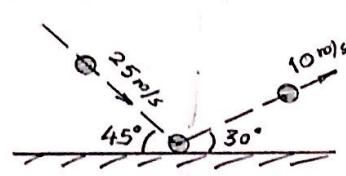
$$\left(\frac{1}{2} \cdot m \cdot V_A^2 \right) + (mgh_A) + \left(\frac{1}{2} k \cdot x_A^2 \right) = \left(\frac{1}{2} \cdot m \cdot V_B^2 \right) + (mgh_B) + \left(\frac{1}{2} k \cdot x_B^2 \right)$$

$$\left(\frac{1}{2} \times 4 \times 2^2 \right) + \cancel{(4 \times 9.81 \times 0)} + \left[\frac{1}{2} \times 400 \times (-0.1)^2 \right]$$

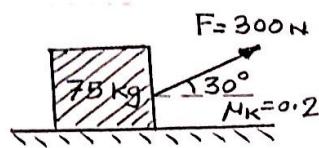
$$= \left(\frac{1}{2} \times 4 \times V_B^2 \right) + (4 \times 9.81)(-0.4) + \left(\frac{1}{2} \times 400 \times 0.3^2 \right)$$

$$\therefore \boxed{V_B = 1.962 \text{ m/s}}$$

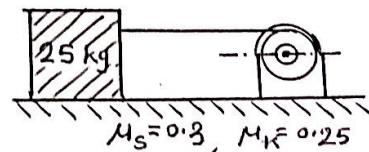
- 1 The 0.5kg ball strikes the rough ground and rebounds with the velocities shown. Determine the magnitude of the impulse the ground exerts on the ball. Assume that the ball does not slip when it strikes the ground, and neglect the size of the ball and the impulse produced by the weight of the ball.



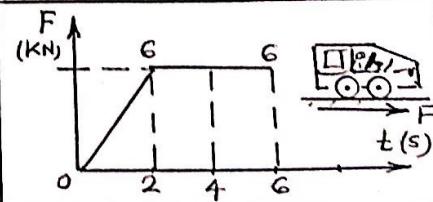
- 2 If the coefficient of kinetic friction between the 75 kg crate and the ground is $\mu_k = 0.2$, determine the speed of the crate when $t = 4s$. The crate starts from rest and is towed by the 500N force.



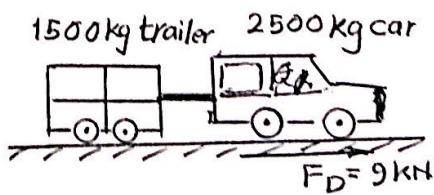
- 3 The motor exerts a force $F = 20t^2$ N on the cable, where t is in seconds. Determine the speed of the 25 kg crate when $t = 4s$. The coefficient of static friction and kinetic friction between the crate and the plate are $\mu_s = 0.3$ and $\mu_k = 0.25$ resp.



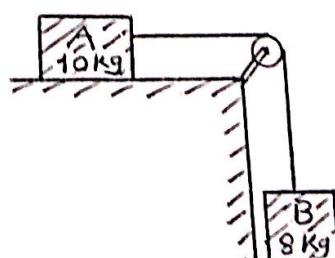
- 4 The wheels of the 1.5mg car generate the traction force F described by the graph. If the car starts from rest, determine its speed when $t = 6s$.



- 5 The 2.5 Mg four wheel drive SUV tows the 1.5Mg trailer. The tractive force developed at the wheels is $F_D = 9\text{kN}$. Determine speed of the truck in 20s starting from the rest. Also, determine the tension developed in the coupling A, between the SUV and the trailer. Neglect the mass of wheels.



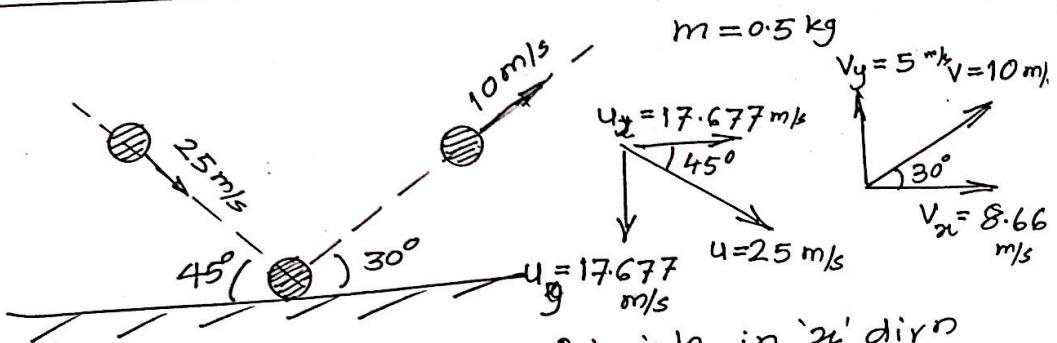
- 6 A 10 kg block A attains velocity of 1m/s in 5 seconds, starting from rest. Determine the tension in cord and the coefficient of kinetic friction between block A and the horizontal plane. Neglect the weight of the pulley. Block B has a mass of 8 kg.



Lecture No. (18):

Impulse-Momentum Principle

① F 15.1 / pg. 860 / RCH (14th)



Applying Impulse-Momentum principle in 'x' dir,
we get,

$$(Imp)_x = (m v_x - m \cdot u_x) = \int_{t_1}^{t_2} F_x dt \\ = (0.5)(8.66 - 17.677) \\ = -4.509 \text{ Ns}$$

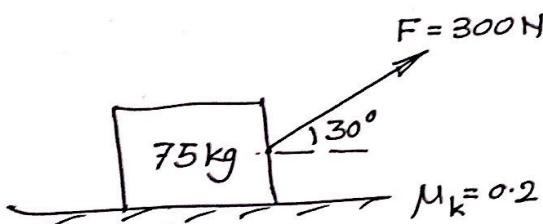
similarly, in 'y' dir,

$$(Imp)_y = (m \cdot v_y - m \cdot u_y) = \int_{t_1}^{t_2} F_y dt \\ = (0.5)(5 + 17.677) \\ = 11.339 \text{ Ns}$$

Resultant Impulse on the ball,

$$(Imp.) = \sqrt{(Imp.)_x^2 + (Imp.)_y^2} \\ = \sqrt{(-4.509)^2 + (11.339)^2} \\ = 12.202 \text{ Ns}$$

② F 15.1 / pg. 860 / RCH (14th) :



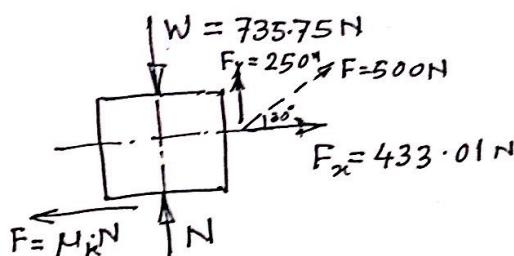
$$\sum F_y = 0 \text{ gives,}$$

$$N + 250 - 735.75 = 0$$

$$N = 485.75 \text{ N}$$

$$\text{At } t=0, u=0$$

$$\text{At } t=4s, v=?$$



Applying Imp.-mmtm. prin. in 'x' dirn, we get,

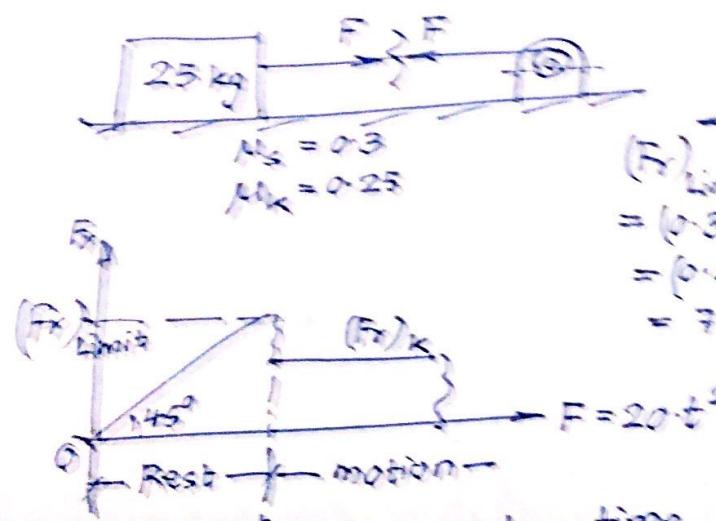
$$(\text{Imp.})_x = \int_{t_1}^{t_2} F_x dt = mV_x - m \cdot u_x$$

$$(\text{Imp.})_x = (75)(V_x - 0) = (433.01 \times 4) - (0.2 \times 485.75 \times 4)$$

$$\boxed{V_x = 17.912 \text{ m/s}}$$

Q) F15.3/pg. 860/RCH (14*) :

$$F = 20t^2 \text{ N} \quad f(t)$$



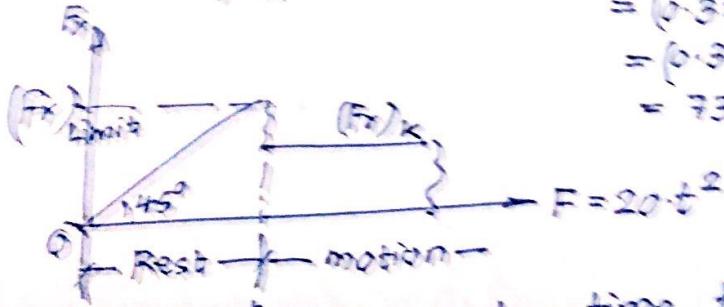
$$N = 245.25 \text{ N}$$

$$F = 20t_1^2$$

$$(F_x)_{\text{limit}} = (0.3 \times N)$$

$$= (0.3 \times 245.25)$$

$$= 73.575 \text{ N}$$



Let, t_1 = time to start the motion

$$(F_x)_{\text{limit}} = 20t_1^2$$

$$73.575 = 20t_1^2 \therefore t_1 = 1.918 \text{ s}$$

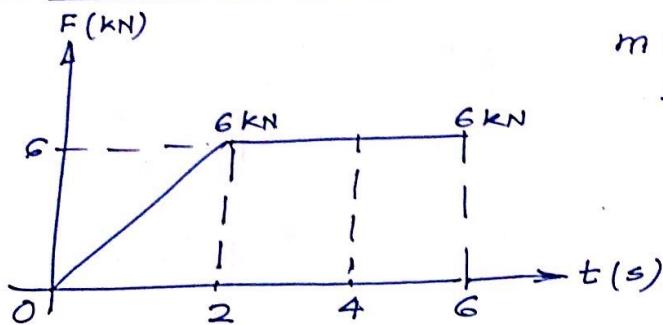
Applying Imp-momt.m. prin; during motion,

$$(Imp)_x = m \cdot V_x - m \cdot U_x$$

$$\left[\int_{1.918}^4 (20t^3) dt - (0.25 \times 245.25)(4 - 1.918) \right] = (25)(V - 0)$$

$$V \approx 10.1 \text{ m/s} (\rightarrow)$$

(4) F.15.4/pg.860/RCH(14th):



$$m = 1.5 \text{ Mg} \\ = 1500 \text{ kg}$$



F = Traction Force

At $t = 0$, $u = 0$

At $t = 6 \text{ s}$, $v = ?$

$(\text{Imp})_{t_1}^{t_2} = \text{Area under } (F-t) \text{ diag.}$
from t_1 to t_2

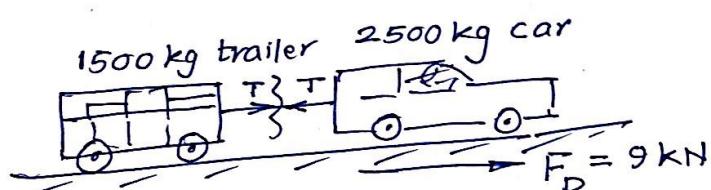
$$= \left(\frac{1}{2} \times 2 \times 6000 \right) + (6000)(6-2) \\ = 30,000 \text{ Ns}$$

Now, $(\text{Imp})_x = \int_{t_1}^{t_2} F_x dt = m \cdot V_x - m \cdot U_x$

$$30,000 = (1500)(V - 0)$$

$$\text{At } t = 6 \text{ s}, V = 20 \text{ m/s}$$

(5) F 15.5 / pg. 860 / RCH (14th) :



At $t=0, u=0$
At $t=20s, v=?$

I) For the car + trailer :

$$(Imp)_x = m(v_x - u_x)$$

$$(9000 \times 20) \text{ Ns} = (2500 + 1500)(v - 0)$$

$$v = 45 \text{ m/s}$$

II) For the trailer only :

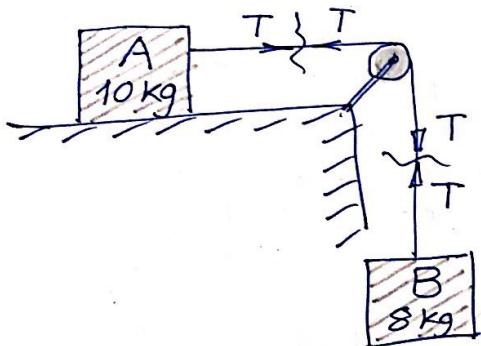
$$(Imp)_x = m(v_x - u_x)$$

$$(T \times 20) \text{ Ns} = (1500)(45 - 0)$$

$$T = 3375 \text{ N}$$

$$= 3.375 \text{ N}$$

⑥ F 15.6 / pg. 860 / RGH (14th) :



$$\text{At } t=0, v_1 = 0$$

$$\text{At } t=5\text{s}, v_2 = 1 \text{ m/s}$$

For block B :

$$(\text{Imp}) = \int F \cdot dt = m(v_2 - v_1)$$

$$\begin{array}{c} \text{Block B} \\ \uparrow T \\ \downarrow mg \\ \downarrow \text{motion} \end{array} \quad (8 \times 9.81)(5) - (T \times 5) = 8(1-0)$$

$$T = 76.88 \text{ N}$$

For block A :

$$\begin{array}{c} \text{Block A} \\ \downarrow mg \\ \rightarrow \text{motion} \\ \leftarrow (F_r)_k = \mu_k N \end{array} \quad (\text{Imp}) = \int F \cdot dt = m(v_2 - v_1)$$

$$(76.88 \times 5) - \mu_k (10 \times 9.81)(5) = (10)(1-0)$$

$$\mu_k = 0.763$$

Lecture No:

Conservation of Momentum

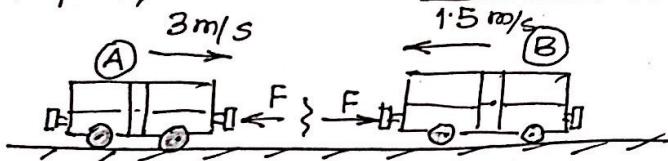
(19)

1 The freight cars A and B have a mass of 20 Mg and 15Mg respectively. Determine the velocity of A after collision if the cars collide and rebound, such that B moves to the right with a speed of 2 m/s. If A and B are in contact for 0.5s, find the average impulsive force which acts between them.	<p>$m_A = 20 \text{ Mg}$ $m_B = 15 \text{ Mg}$</p>
2 The cart and package have a mass of 20 Kg and 5 Kg respectively. If the cart has a smooth surface and it is initially at rest, while the velocity of the package is as shown, determine the final common velocity of the cart and package after impact.	
3 A five Kg block A has an initial speed of 5 m/s as it slides down the smooth ramp, after which it collides with stationary block B of mass 8 kg. If two blocks couple together after collision, determine their common velocity after collision.	
4 A spring is fixed to block A and block B pressed against the spring. If the spring is compressed by $s=200\text{mm}$ and then the blocks are released, determine their velocity at the instant block B loses contact with the spring. The masses of block A and B are 10 Kg and 15 kg respectively.	
5 Blocks A and B have a mass of 15 kg and 10 kg respectively. If A is stationary and B has a velocity of 15 m/s just before collision and the blocks coupled together after impact, determine the maximum compression of the spring.	
6 The cannon and support without a projectile have a mass of 250 kg. If a 20kg projectile is fired from the cannon with a velocity of 400 m/s, measured relative to the cannon, determine the speed of projectile as it leaves the barrel of the cannon. Neglect rolling resistance.	

Lecture No. (19) :

Conservation of Momentum

- ① F 15.7 / pg. 875 / RCH (14th): Here, the two bodies do not move together after impact, hence it is semi-elastic impact.



$$m_A = 20,000 \text{ kg}$$

$$u_A = 3 \text{ m/s}$$

$$v_A = ?$$

$$m_B = 15,000 \text{ kg}$$

$$u_B = -1.5 \text{ m/s}$$

$$v_B = 2 \text{ m/s}$$

Time of contact = 0.5 sec.

By conservation of linear momentum,

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$(20,000 \times 3) - (15,000 \times 1.5) = (20,000 \times v_A) + (15,000 \times 2)$$

$$\therefore v_A = 0.875 \text{ m/s} (\rightarrow)$$

Let, F = average impulsive force bet'n A & B.

Apply Imp.- mmtm. prin. to freight cor B.

$$\text{Imp.} = m_B v_A - m_B u_B$$

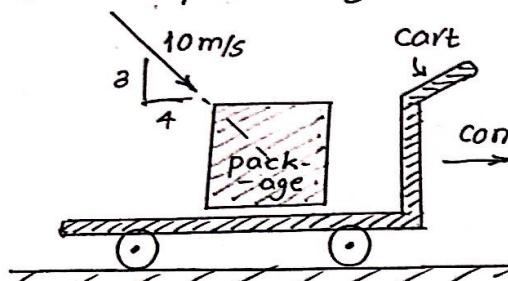
$$(F \times 0.5) \text{ Ns} = (15,000)(2 + 1.5)$$

$$\therefore F = 105 \times 10^3 \text{ N}$$

$$\boxed{F = 105 \text{ kN}}$$

(2) F 15.8 / pg. 875 / RCH (14th) :

In this case, the two bodies move together after impact. They do not separate from each other.



But, they move with a common velocity, after impact. This is called as Plastic impact.

$$(u_x)_1 = 8 \text{ m/s} \quad \tan \theta = \frac{3}{4} \therefore \theta = 36.87^\circ$$

$$\begin{array}{l} \text{At impact, } u_i = 10 \text{ m/s} \\ (u_x)_1 = 8 \text{ m/s} \\ (u_y)_1 = 6 \text{ m/s} \end{array}$$

Package: Body ①, $m_1 = 5 \text{ kg}$
 $(u_x)_1 = 8 \text{ m/s} (\rightarrow)$
 $(u_y)_1 = 6 \text{ m/s} (\downarrow)$

Cart: Body ②, $m_2 = 20 \text{ kg}$
 $(u_x)_2 = (u_y)_2 = 0$

For both the bodies, $(v_x)_2 = (v_x)_1 = V$ say
 $(v_y)_2 = (v_y)_1 = 0$

By conservation of momentum in 'x' dirn

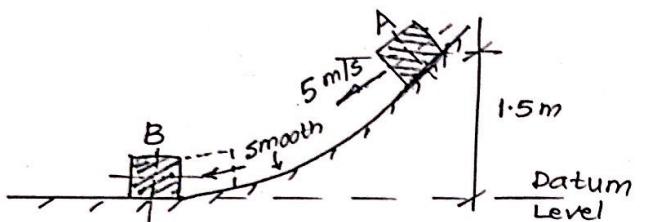
$$m_1(u_x)_1 + m_2(u_x)_2 = (m_1 + m_2) \cdot V$$

$$(5 \times 8) + (20 \times 0) = (20 + 5) \cdot V$$

$$V = \frac{40}{25} = 1.6 \text{ m/s}$$

$V = 1.6 \text{ m/s}$

③ F 15.9 / pg. 875 / RCH⁽¹⁴⁺¹⁵⁾:



$$m_A = 5 \text{ kg}$$

$$m_B = 8 \text{ kg}$$

As block A is travelling along smooth surface, we can use the law of conservation of energy, for block A, to get its velocity at the lowest level. With that velocity block A will strike block B.

For A: $T_1 + V_1 = T_2 + V_2$

$$\left(\frac{1}{2}m_A V_A^2\right)_1 + (m_A g \cdot h_1) = \left(\frac{1}{2}m_A V_A^2\right)_2 + (m_A g \cdot h_2)$$

$$\therefore \left(\frac{1}{2} \times 5 \times 5^2\right) + (5 \times 9.8 \times 1.5) = \left(\frac{1}{2} \times 5 \times V_A^2\right)_2$$

$$\boxed{(V_A)_2 = 7.378 \text{ m/s}}$$

As the two blocks move together, after impact, this is a plastic impact. \therefore By conservation of momentum, we get,

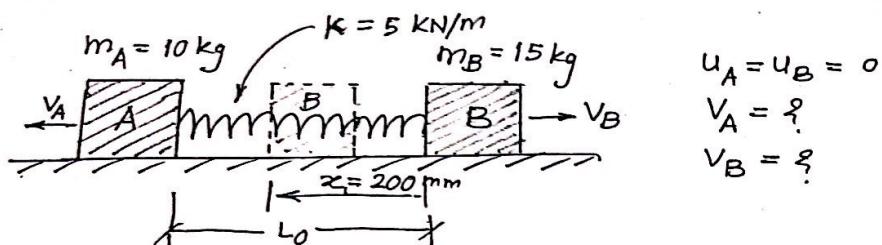
$$m_A u_A + m_B u_B = m_A V_A + m_B V_B$$

$$(5 \times 7.378) + (8 \times 0) = (5+8) \cdot V$$

$$\therefore V_A = V_B = V = 2.84 \text{ m/s}$$

$$\boxed{V = 2.84 \text{ m/s}}$$

④ F15.10 / pg. 875 / RCH (14th):



Apply conservation of momentum eqⁿ, immediately after the blocks are released.

$$m_A u_A + m_B u_B = m_A V_A + m_B V_B$$

$$0 + 0 = 10 \cdot V_A + 15 \cdot V_B$$

$$\therefore 2 \cdot V_A + 3 \cdot V_B = 0 \rightarrow (1)$$

As, force of friction is absent, we can apply the law of conservation of energy.

$$T_1 + V_1 = T_2 + V_2$$

$$\left(\frac{1}{2} m_A u_A^2 + \frac{1}{2} m_B u_B^2 \right) + \left(\frac{1}{2} k x_0^2 \right) = \left(\frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2 \right) + \left(\frac{1}{2} k x_2^2 \right)$$

$$0 + 0 + \left(\frac{1}{2} \times 5000 \times 0.2^2 \right) = \left(\frac{1}{2} \times 10 \times V_A^2 \right) + \left(\frac{1}{2} \times 15 \times V_B^2 \right)$$

$$\therefore 5 \cdot V_A^2 + (7.5) V_B^2 = 100 \rightarrow (2)$$

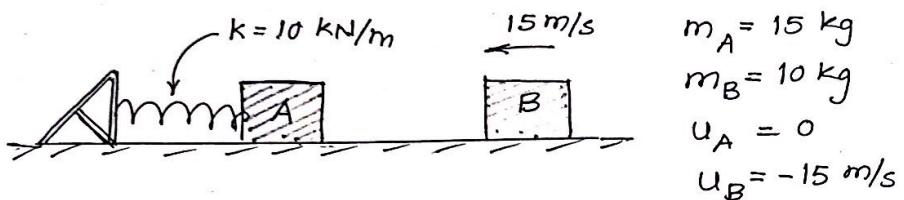
$$\text{But, } V_A = - (1.5) V_B$$

$$\therefore 5 (-1.5 \times V_B)^2 + (7.5) V_B^2 = 100$$

$$\therefore (18.75) V_B^2 = 100$$

$V_B = 2.31 \text{ m/s} (\rightarrow)$
$V_A = - 3.465 \text{ m/s}$ $= 3.465 \text{ m/s} (\leftarrow)$

(5) F 15.11 / Pg. 875 / RCH (14th):



After the impact, the blocks are coupled together. Hence, this is a plastic impact. By conservation of momentum,

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$0 + (10)(-15) = (15+10)V$$

$$\therefore V_A = V_B = V = 6 \text{ m/s}$$

With this velocity the spring is compressed by the two blocks.

Let x_{\max} = maximum compression of the spring when the blocks stop.

∴ By conservation of energy (for the entire system)

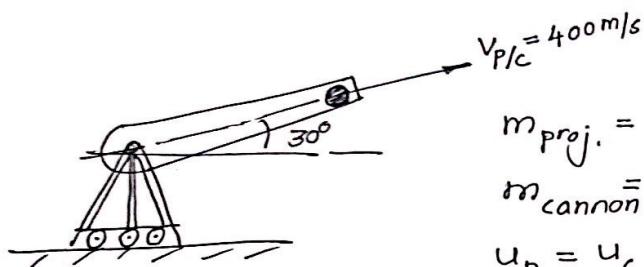
$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(m_A + m_B)V_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}(m_A + m_B)V_2^2 + \frac{1}{2}kx_2^2$$

$$\frac{1}{2}(15+10)(6)^2 = \frac{1}{2} \times 10,000 \times x_{\max}^2$$

$$\begin{aligned} \therefore x_{\max} &= 0.3 \text{ m} \\ &= 300 \text{ mm} \end{aligned}$$

(6) F 15.12 / Pg. 875 / RCH (14th) :



$$m_{\text{proj.}} = 20 \text{ kg}$$

$$m_{\text{cannon}} = 250 \text{ kg}$$

$$u_p = u_c = 0$$

Applying conservation of momentum in 'x' dir?

$$(m_p u_p + m_c u_c) = (m_p V_p + m_c V_c)$$

$$0 = (20)(V_p)_x - (250 \times V_c) \rightarrow ①$$

$$(V_{p/c})_y = 200 \text{ m/s} \quad \overline{V}_{p/c} = \overline{V}_p - \overline{V}_c$$

$$V_{p/c} = 400 \text{ m/s}$$

$$(V_{p/c})_x = 346.41 \text{ m/s}$$

$$\therefore (346.41)\hat{i} + (200)\hat{j} = (V_p)_x \hat{i} + (V_p)_y \hat{j} + V_c \cdot \hat{i}$$

$$(V_p)_x + V_c = 346.41 \text{ m/s} \rightarrow ②$$

$$(V_p)_y = 200 \text{ m/s} (\uparrow)$$

$$(V_p)_x = (-V_c + 346.41)$$

\therefore Eqⁿ ① becomes,

$$0 = (20)(-V_c + 346.41) - (250 \times V_c)$$

$$0 = -20 \cdot V_c + 6928.2 - 250 \cdot V_c$$

$$\therefore V_c = 25.66 \text{ m/s}$$

$$\therefore (V_p)_x = \underset{(\leftarrow)}{320.75} \text{ m/s} (\rightarrow), (V_p)_y = 200 \text{ m/s} (\uparrow)$$

$$V_p = \sqrt{(V_p)_x^2 + (V_p)_y^2} = 378 \text{ m/s}$$

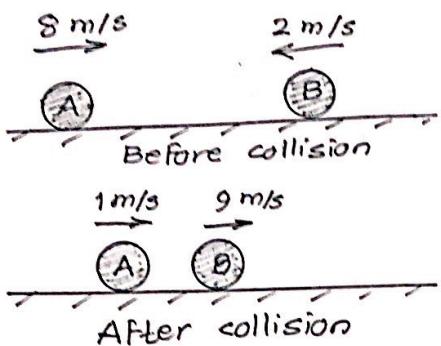
$V_p = 378 \text{ m/s}$

Lecture No.:

Impact

(2c)

1 Determine the coefficient of restitution e between ball A and ball B. The velocities of A and B before and after collision are shown.	<p>Before collision: Ball A: 8 m/s to the right Ball B: 2 m/s to the left After collision: Ball A: 1 m/s to the right Ball B: 9 m/s to the right</p>
2 A 15-Mg tank car A and 25-Mg freight car B travel towards each other with the velocities shown. If the coefficient of restitution between the bumpers is $e=0.6$, determine the velocity of each car just after the collision.	<p>Car A: 5 m/s to the right Car B: 7 m/s to the left</p>
3 A 15 Kg package A has a speed of 1.5 m/s when it enters the smooth ramp. As it slides down the ramp, it strikes the 40 Kg package B which is initially at rest. If the coefficient of restitution between A and B is $e=0.6$, determine the velocity of B just after impact.	<p>Smooth Ramp Height: 3 m Length: 1.5 m Package A: 1.5 m/s</p>
4 The ball strikes a smooth wall with a velocity of $v_{b1}=20 \text{ m/s}$. If the coefficient of restitution between the ball and the wall is $e=0.75$, determine the velocity of the ball just after impact.	<p>Wall Approach Velocity: 20 m/s Angle: 30° Velocity after impact: v Angle: θ</p>
5 A disk A has a mass of 2 kg and slides on a smooth horizontal plane with a velocity of 3 m/s. Disk B has a mass of 11 kg and is initially at rest. If after impact A has velocity of 1 m/s, parallel to positive X axis; determine the speed of disk B after impact.	<p>Disk A: 3 m/s to the right Disk B: 0 m/s Velocity after impact: v_B</p>
6 Two disks A and B each have a mass of 1 kg and initial velocities shown just before they collide. If coefficient of restitution is $e=0.5$, determine their speed just after impact.	<p>Initial Velocity: $U_A = 4 \text{ m/s}$ to the right Initial Velocity: $U_B = 4 \text{ m/s}$ to the left Angle: 30°</p>

Lecture No. (20)Impact① F1513/pg. 887/RCH (14th) :

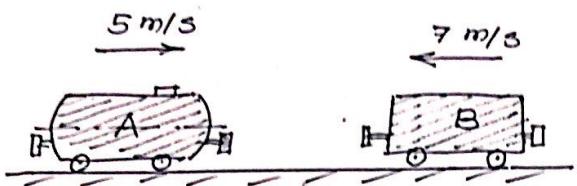
Coefficient of Restitution,

$$e = \left(\frac{V_B - V_A}{U_A - U_B} \right)$$

$$\therefore e = \left[\frac{9 - 1}{8 - (-2)} \right] = 0.8$$

$$\boxed{e = 0.8}$$

(2) F 15.14 / pg. 887 / RCH (14th) :



$$m_A = 1500 \text{ kg} \quad m_B = 2500 \text{ kg}$$

$$u_A = 5 \text{ m/s} \quad u_B = -7 \text{ m/s}$$

$$V_A = ? \quad V_B = ?$$

$$e = 0.6$$

(semi-elastic impact)

By Conservation of momentum,

$$m_A u_A + m_B u_B = m_A V_A + m_B V_B$$

$$(1500 \times 5) + (2500)(-7) = 1500 \cdot V_A + 2500 \cdot V_B$$

$$\therefore 3 \cdot V_A + 5 \cdot V_B = -20 \longrightarrow ①$$

By coeff. of Restitution,

$$e = \left(\frac{V_B - V_A}{u_A - u_B} \right) =$$

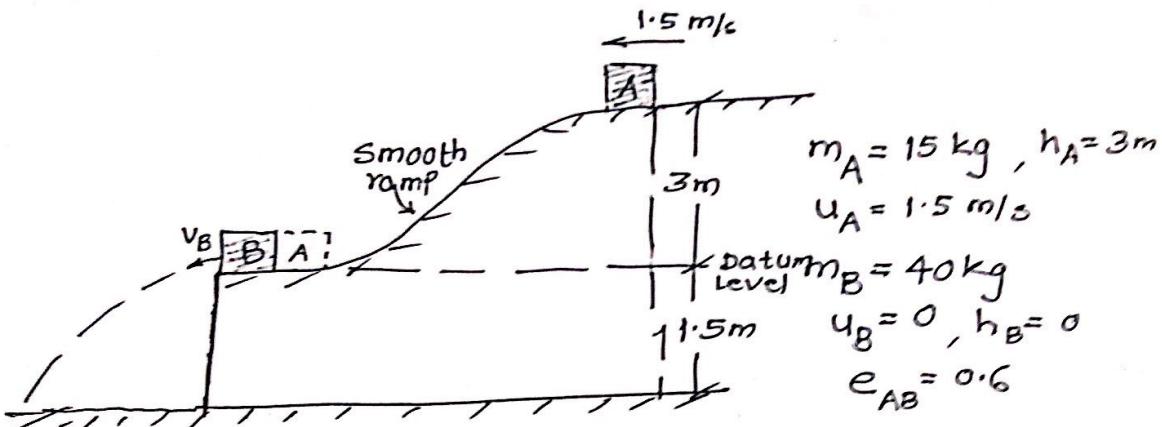
$$0.6 = \left(\frac{V_B - V_A}{5 + 7} \right)$$

$$(V_B - V_A) = 7.2 \longrightarrow ②$$

Solving ① & ②, we get,

$V_B = 0.2 \text{ m/s } (\rightarrow)$
$V_A = 7 \text{ m/s } (\leftarrow)$

③ F15.15/pg. 887/RCH (14th) :



By conservation of energy, (only to block A)

$$T_1 + V_1 = T_2 + V_2$$

$$\left(\frac{1}{2} \times 15 \times 1.5^2\right) + (15 \times 9.81 \times 3) = \left(\frac{1}{2} \times 15 \times V_A^2\right) + 0$$

$$\therefore V_A = 7.817 \text{ m/s } (\leftarrow)$$

This is the striking vel. of A, on B.

By conservation of momentum for A & B,

$$m_A u_A + m_B u_B = m_A V_A + m_B V_B$$

$$-(15 \times 7.817) + (40 \times 0) = 15 \cdot V_A + 40 \cdot V_B$$

$$\therefore 15 \cdot V_A + 40 \cdot V_B = -117.255 \rightarrow ①$$

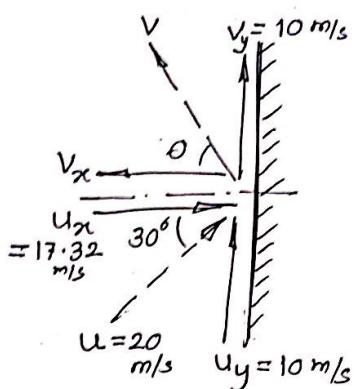
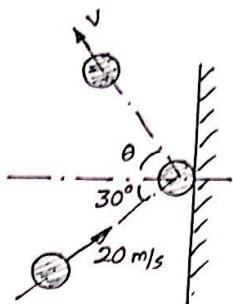
By coeff. of restitution,

$$e = \left(\frac{V_B - V_A}{u_A - u_B} \right) \therefore 0.6 = \left(\frac{V_B - V_A}{-7.817 - 0} \right)$$

$$\therefore (V_B - V_A) = -4.69 \rightarrow ②$$

$V_A = 1.279 \text{ m/s } (\rightarrow)$
$V_B = 3.411 \text{ m/s } (\leftarrow)$

Q) F 15-16 / pg. 887 / RCH (14th):



For the impact of ball with wall,
coeff. of restitution,

$$e = \sqrt{\frac{h_2}{h_1}} = -\left[\frac{V_x}{U_x} \right]$$

$$\therefore 0.75 = -\left[\frac{-V_x}{17.32} \right] \therefore V_x = 13 \text{ m/s} (\leftarrow)$$

V = Velocity of the ball after impact,

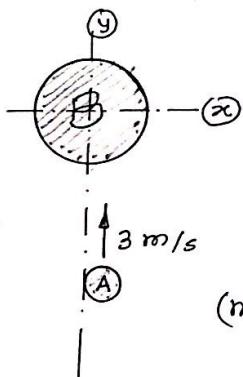
$$V = \sqrt{13^2 + 10^2} = 16.4 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right) = \tan^{-1}\left(\frac{10}{13}\right) = 37.57^\circ$$

$V = 16.4 \text{ m/s}, \theta = 37.57^\circ$

NAME	DEPARTMENT	SUBJECT	ACADEMIC YEAR	CLASS	ROLL NO.
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⑤ F15.17 / Pg. 887 / RCH (14th) :



$$m_A = 2 \text{ kg}, \quad u_A = 3 \text{ m/s} (\uparrow)$$

$$m_B = 11 \text{ kg}, \quad u_B = 0$$

$$v_A = 1 \text{ m/s} (\rightarrow)$$

Applying conservation of momentum, along 'x' axis

$$(m_A \cdot u_A)_x + (m_B \cdot u_B)_x = (m_A \cdot v_A)_x + (m_B \cdot v_B)_x$$

$$0 + 0 = (2 \times 1) + 11 \times (v_B)_x$$

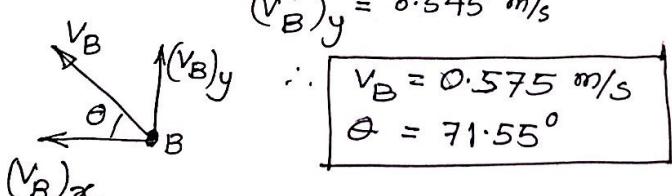
$$\therefore (v_B)_x = -0.1818 \text{ m/s}$$

Applying conservation of momentum, along 'y' axis

$$(m_A \cdot u_A)_y + (m_B \cdot u_B)_y = (m_A \cdot v_A)_y + (m_B \cdot v_B)_y$$

$$(2 \times 3) + 0 = 0 + 11 \times (v_B)_y$$

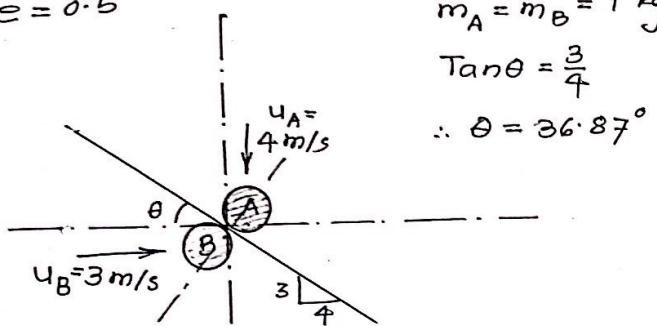
$$(v_B)_y = 0.545 \text{ m/s}$$



$$\boxed{\begin{aligned} v_B &= 0.575 \text{ m/s} \\ \theta &= 71.55^\circ \end{aligned}}$$

(6) 10/10/1988 / KC UNIT

$$e = 0.5$$



$$m_A = m_B = 1 \text{ kg}$$

$$\tan \theta = \frac{3}{4}$$

$$\therefore \theta = 36.87^\circ$$

I) Along 'x' dirn:

$$(m_A \cdot u_A)_x + (m_B \cdot u_B)_x = (m_A \cdot v_A)_x + (m_B \cdot v_B)_x$$

$$0 + (1 \times 3) = (v_A)_x + (v_B)_x \rightarrow ①$$

$$e = \frac{(v_B)_x - (v_A)_x}{(u_A)_x - (u_B)_x}$$

$$0.5 = \frac{(v_B)_x - (v_A)_x}{0 - 3}$$

$$\therefore (v_B)_x - (v_A)_x = -1.5 \rightarrow ②$$

$$(v_A)_x = 2.25 \text{ m/s}, (v_B)_x = 0.75 \text{ m/s}$$

II) Along 'y' dirn:

$$(m_A \cdot u_A)_y + (m_B \cdot u_B)_y = (m_A \cdot v_A)_y + (m_B \cdot v_B)_y$$

$$0 - (1 \times 4) = (v_A)_y + (v_B)_y \rightarrow ③$$

$$e = \frac{(v_B)_y - (v_A)_y}{(u_A)_y - (u_B)_y}$$

$$0.5 = \frac{(v_B)_y - (v_A)_y}{-4 - 0}$$

$$(v_B)_y - (v_A)_y = -2 \rightarrow ④$$

$$\therefore (v_A)_y = -1 \text{ m/s} \quad (v_B)_y = -3 \text{ m/s}$$

$\sqrt{23.96^\circ}$

$\sqrt{75.96^\circ}$