1. De Broglie's Wavelength

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$h = 6.63 \times 10^{-34}$ J.s

mass of electron $= 9.1 \times 10^{-31}$ kg

h = Planck's Constant

m = Mass of Particle

v = Velocity of Particle

P = Momentum of Particle

2. Energy of Photon

$E = h\nu = h\frac{c}{\lambda}$

$\lambda = h\frac{c}{E}$

Energy of Electron

$K = \frac{h^2}{2m\lambda^2} = \frac{p^2}{2m}$ $\left(p = \frac{h}{\lambda}\right)$

$\lambda = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2meV}}$

$c = 3 \times 10^8$ m/s

$e = 1.6 \times 10^{-19}$ C

ν = frequency of Light/Photon

c = Speed of photon

λ = Wavelength of photon

e = charge of electon

V = Applied potential

Calculate the De Broglie wavelength of the (a) electron moving at 2×10^6 m/s and a cricket ball of mass 200gm moving at 20 m/s. Which of this entity particle behaves more like a wave and which of the entity behaves more like a particle?

Solution:

For electron

$$\lambda = \frac{h}{p}$$

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 2 \times 10^{6}}$$

$$\lambda = 3.64 \times 10^{-10} m$$

$$\lambda = 3.64 A^{o}$$

For cricket ball

$$\lambda = \frac{h}{p}$$

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{200 \times 10^{-3} \times 20}$$

$$\lambda = 1.6575 \times 10^{-10} m$$

$$\lambda = 1.6575 \times 10^{-34} m$$

Let us note two observations. The De Broglie wavelength of electron, though small w.r.t. our day to day standards, is fairly comparable with its own size (10^{-16} m) and quite comparable with the size of the region (i.e. atom or a molecule) in which it exists. Further, like X rays, the wavelength of this order can be easily measured using electron diffractometers. Thus electron certainly behaves like a wave

This is not true for cricket ball; its wavelength is extremely small as compared to its own size as well as the size of the region in which it exits (i.e. cricket ground). Further, an experimental set up to measure such a small wavelength is yet to be invented. Thus cricket ball, though principally a wave, appears like an object.

Thus results of this problem lead to an extremely important principle of quantum mechanics, which asserts that

The wave-like properties are more conspicuous in case of only subatomic entities.

Calculate the wavelengths of photons of energies 1 eV, 1 keV and 1 MeV. Comment of the results

For photons

$$E = h\nu = h\frac{c}{\lambda}$$

$$\lambda = h\frac{c}{E}$$

$$\lambda = 6.63\times10^{-34}\frac{3\times10^{8}}{1.6\times10^{-19}\times E}\times10^{10}\,A^{o}$$

$$\lambda = \frac{12431}{E(eV)}A^{o}$$

For 1 eV photon $\lambda = 12431$ A°

Foe 1 KeV photon: $\lambda = 12.431$ A°

Foe 1 MeV photon $\lambda = 0.12431$ A°

Thus amongst the electromagnetic radiations, low energy radiations like radio waves (1 eV) have higher wavelengths and the high energy radiations like gamma rays (1 MeV) have considerably smaller wavelength. Indeed one may be tempted to conclude that as we move from radio waves to gamma rays, the *'waveness'* of the radiation decreases and its *'particleness'* increases.

Calculate the De Broglie wavelengths of 1 keV photon and 1 keV electron. Compare them and interpret the results

Now for 1 keV electron

$$\lambda = \frac{h}{\sqrt{2mK}}$$

$$\lambda = \frac{6.63\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1000\times1.6\times10^{-19}}}$$

$$\lambda = 3.88\times10^{-11}m$$

$$\lambda = 0.388A^{o}$$

Thus the De Broglie wavelength of a 1 keV electron is much smaller than the wavelength of photon of the same energy. Thus one may be tempted to conclude that, amongst the photon and electron of same energy, photon has more *'waveness'* and less *'particleness'* and electron has less *'waveness'* and more *'particleness'*

Calculate the energy of electron and photon both having wavelength 1 A°

Solution:

We have for electrons

$$\lambda = \frac{h}{\sqrt{2mK}}$$

$$\lambda^2 = \frac{h^2}{2mK}$$

$$K = \frac{h^2}{2m\lambda^2}$$

$$K = \frac{\left(6.63\times10^{-34}\right)^2}{2\times9.1\times10^{-31}\times\left(10^{-10}\right)^2}$$

$$K = \frac{4.40\times10^{-67}}{1.82\times10^{-50}}$$

$$K = 2.42\times10^{-17}\,J$$

$$K = \frac{2.42\times10^{-17}\,J}{1.6\times10^{-19}\,J/eV}$$

$$K = 151eV$$

For Photon

$$E = h\nu = h\frac{c}{\lambda}$$

$$E = 6.63\times10^{-34}\,\frac{3\times10^{8}}{10^{-10}}$$

$$E = 1.989\times10^{-15}\,J$$

$$E = 12431\,eV$$

Thus for possessing same wavelength the photon has to carry much more energy than an electron. This also indicates that it is more difficult for a photon to acquire *'particleness'* than an electron. Wave properties of photons are stronger than that of electrons. Particle properties of electrons are stronger than that of photons

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Use De Broglie's hypothesis to prove that electron cannot exist inside the nucleus

Solution

If electron had existed inside the nucleus then its maximum De Broglie wavelength would not exceed the size of nucleus i.e. 10^{-14} m

Thus for $\lambda_{max} = 10^{-14}\, m$

$$P_{\min} = \frac{h}{\lambda_{\max}}$$

$$P_{\min} = \frac{6.63\times10^{-34}}{10^{-14}}$$

$$P_{\min} = 6.63\times10^{-20}\, kgm/\sec$$

$$mv_{\min} = 6.63\times10^{-20}$$

$$v_{\min} = \frac{6.63\times10^{-20}}{m}$$

$$v_{\min} = \frac{6.63\times10^{-20}}{9.1\times10^{-31}}$$

$$v_{\min} = 7.28\times10^{10} > c, \text{Speed of light}$$

Thus if electron had existed inside the nucleus, its minimum velocity would exceed the speed of light. This would violate the special theory of relativity. Thus electron can’t exist inside the nucleus

OR

$$P_{\min} = 6.63 \times 10^{-20} kgm/\sec$$

$$E_{\min} = \frac{P_{\min}^2}{2m}$$

$$E_{\min} = \frac{\left(6.63 \times 10^{-20}\right)^2}{2m}$$

$$E_{\min} = \frac{4.39 \times 10^{-39}}{9.1 \times 10^{-31} \text{ x } 2}$$

$$E_{\min} = 2.4 \times 10^{-09} J$$

$$E_{\min} = \frac{2.4 \times 10^{-09} J}{1.6 \times 10^{-19} J/eV}$$

$$E_{\min} = 1.5 \times 10^{10} eV$$

$$E_{\min} = 15000\, MeV >> 8.8 \text{ MeV (the maximum B.E. of the nucleus)}$$

Thus if electron had existed inside the nucleus, its energy would be far greater than maximum binding energy of the nucleus. Thus nucleus can never trap an electron.

De Broglie’s hypothesis suggests that material objects have to be restless. How, then the objects in our day to day life can be at the rest?

Solution

Let us solve this problem by assuming a suitable data. Consider a cricket ball of mass 0.5 kg existing in a room of length 10 m. The maximum De Broglie wavelength of such cricket ball can be $\lambda = 10$ m. The corresponding minimum momentum is then

$$p = \frac{h}{\lambda}$$

$$p = \frac{6.63 \times 10^{-34}}{10}$$

$$p = 6.63 \times 10^{-35} kgm/s$$

This momentum is too small to be considered. As can be noticed, this is due to an extremely small value of the Planck's constant. What would happen, if Planck's constant possessed a different value?

Example (6.8):

What would be the minimum momentum of the cricket ball in the above problem, if Planck's constant were 6.63 J.s?

Solution:

The calculation shows that the minimum momentum would be $66.3\ kg\frac{m}{s}$. This suggests that all the objects in our daily life would be restless if Planck's constant were really 6.63 J.s. But nature has cleverly chosen an appropriate and an extremely small value of Planck's constant to avoid the complications in our life!

Extremely small value of Planck's constant allows us to take rest!