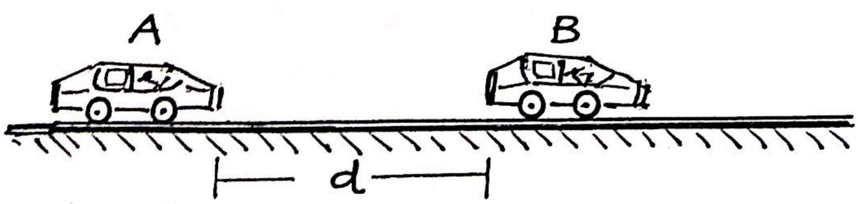
**Lecture no:** **Rectilinear Kinematics**

| | |
|---|---|
| 1. | A Starting from rest, a particle moving in straight line has an acceleration of $a = (2t - 6)\ m/sec^2$, where t is in second. What is the particle's velocity when t = 6 sec, & what is its position when t=11 sec?<br>**Ans: at t=6s (0) & at t=11s (80.67m)** |
| 2. | A Starting from rest, a particle moving in straight line has an acceleration of $a = (60 - 72t^2)\ m/sec^2$, Determine the particle's velocity when it was travelled 110m & the time take by it before it comes to rest again.<br>**Ans: t= 2.444s, v= -203.72m/s, a = -370m/s$^2$** |
| 3. | The car moves in a straight line such that for a short time it's velocity is defined by $v = 0.8(8t^2 + 3t)$ m/sec, where t is in second. Determine it's position & acceleration when t=6secs. Given at t = 0, s = 0<br>**Ans: at t=6s, x=504m, a=79.2 m/s$^2$** |
| 4. | A stone dropped from rest moves a distance equal to one half of the depth of fall in last second of its fall. Find the time of the fall of the stone & the depth of fall.<br>**Ans: t=3.414s , h= 57.166m** |
| 5. | A train travelling with a speed of 110 kmph slows down on account of work in progress, at a retardation of 2.6 kmph per second to 43 kmph. With this it travels 800m thereafter it gains further speed with 1.8 kmph per second till getting original speed. Find the delay caused.<br>**Ans: 60sec** |
| 6. | When to cars A & B are next to one another, they are traveling in the same direction with speed $V_A$ & $V_B$, respectively. IF B maintains it's speed constant, while A begins to deaccelerate at $a_A$, determine the distance d between the cars at the instant A stops.<br>**Ans: $x=[(2V_A.V_B-V_A^2)/2a_A]$**<br> |

# Rectilinear kinematics-I

Ex. No 1 RCH/12.1/ pg. 631

$$\boxed{a = (2t-6)} = f(t) \longrightarrow (1)$$

$$a = \left(\frac{dV}{dt}\right) = (2t-6)\ m/s^2$$

$$\int dv = \int (2t-6) \cdot dt$$

$$\therefore \quad V = t^2 - 6t + C_1$$

At $t = 0$, $V = 0$ $\therefore C_1 = 0$

$$\therefore \quad \boxed{V = (t^2 - 6t)\ m/s} \longrightarrow (2)$$

$$\therefore \quad V = \frac{dx}{dt} = (t^2 - 6t)$$

$$\int dx = \int (t^2 - 6 \cdot t) \cdot dt$$

$$\therefore \quad x = \frac{t^3}{3} - 3t^2 + C_2$$

At $t = 0$, $x = 0$ $\therefore C_2 = 0$

$$\therefore \quad \boxed{x = \frac{1}{3}t^3 - 3 \cdot t^2\ m} \longrightarrow (3)$$

Ans: 1) At $t = 6$ s, $V = (6^2 - 6^2) = 0$

2) At $t = 11$ s, $x = \left(\frac{11^3}{3} - 3 \times 11^2\right) = 80.67$ m

Ex. No. 2 RCH / 12.6 / Pg. 631

$$\boxed{a = (60 - 72 \cdot t^2)\ m/s^2} \longrightarrow (1)$$

$$\therefore a = \frac{dv}{dt} = (60 - 72 \cdot t^2)$$

$$\int dv = \int (60 - 72 \cdot t^2) \cdot dt$$

$$\therefore V = 60 \cdot t - 24 \cdot t^3 + C_1$$

At $t = 0$, $V = 0$ $\therefore C_1 = 0$

$$\therefore \boxed{V = (60 \cdot t - 24 \cdot t^3)\ m/s} \longrightarrow (2)$$

$$\therefore V = \frac{dx}{d} = (60 \cdot t - 24 \cdot t^3)$$

$$\int dx = \int (60 \cdot t - 24 \cdot t^3) dt$$

$$\therefore x = 30 \cdot t^2 - 6 \cdot t^4 + C_2$$

At $t = 0$, $x = 0$ $\therefore C_2 = 0$

$$\therefore \boxed{x = (30 \cdot t^2 - 6 \cdot t^4)\ m} \longrightarrow (3)$$

Now, find the time at which $V = 0$,

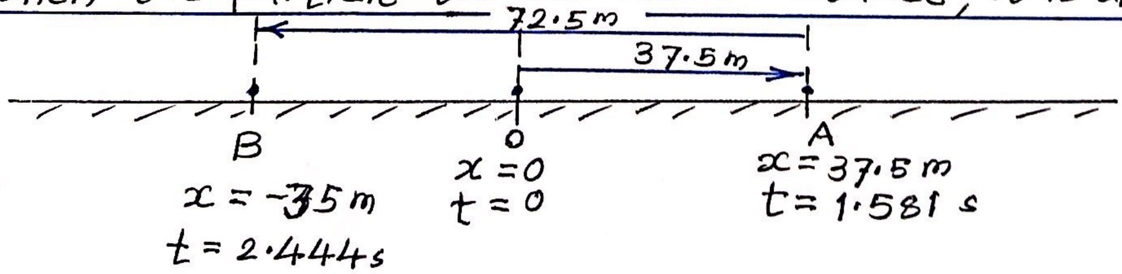
$$\therefore V = (60 \cdot t - 24 t^3) = 0$$

$$\therefore t \cdot (60 - 24 \cdot t^2) = 0$$

$\therefore$ Either $t = 0$ or $t = 1.581$ sec.

At this time, $x = 37.5$ m i.e. at A

Thus, the particle changes it's direction of motion after travelling 37.5 m from the start. Now, when the partizle travels 110 m distance, it is at B.




At B, $x = -35\ m = (30 \cdot t^2 - 6 \cdot t^4)$ $\therefore 6t^4 - 30 \cdot t^2 - 35 = 0$

Solving this, we get, $t = 2.444$ s or $-0.976$ s
Not possible

Ans: $\therefore$ At $t = 2.444$ s, $V = -203.72$ m/s

$a = -370$ m/s$^2$

Ex. No. 3 RCH/12.7/pg. 631

$$\boxed{V = (0.8)\cdot(8t^2 + 3t) \ m/s} \longrightarrow 1$$

$$\therefore \ V = \frac{dx}{dt} = (0.8)(8t^2 + 3t)$$

$$\int dx = \int (0.8)(8t^2 + 3t)\cdot dt$$

$$x = (0.8)\left(\frac{8}{3}t^3 + \frac{3}{2}t^2\right) + c_1$$

At $t = 0, \quad x = 0 \quad \therefore \ c_1 = 0$

$$\therefore \ \boxed{x = (0.8)\left(\frac{8}{3}t^3 + \frac{3}{2}\cdot t^2\right) \ m} \longrightarrow 2$$

$$a = \frac{dV}{dt} \quad \therefore \ \boxed{a = (0.8)(16\cdot t + 3) \ m/s^2} \longrightarrow 3$$

At $t = 6$ sec.

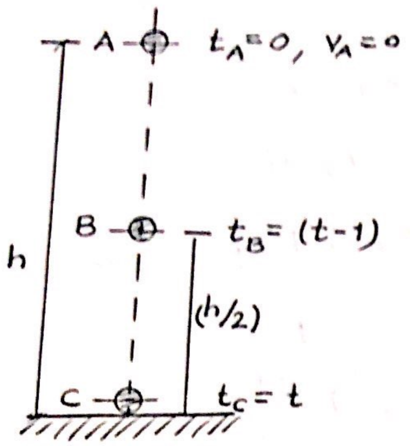
$$x = (0.8)\left(\frac{8}{3}\times 6^3 + \frac{3}{2}\times 6^2\right) = 504 \ m$$

and $a = (0.8)(16\times 6 + 3) = 79.2 \ m/s^2$

Ans.: At $t = 6$ s, $x = 504$ m

$a = 79.2 \ m/s^2$

Ex. No. 4 RCH/12.10/pg. 631




Motion under gravity:

At position B:

$$s = ut + \frac{1}{2}at^2$$

$$-\left(\frac{h}{2}\right) = 0 - \frac{1}{2}g\cdot(t-1)^2$$

$$\therefore \quad h = g(t-1)^2 \longrightarrow (1)$$

At position C:

$$s = ut + \frac{1}{2}gt^2$$

$$-h = 0 - \frac{1}{2}g\,t^2$$

$$\therefore \quad h = \frac{1}{2}g\cdot t^2 \longrightarrow (2)$$

From eqns (1) and (2), we get,

$$g(t-1)^2 = \frac{1}{2}\cdot g\cdot t^2$$

$$\therefore \quad 2t^2 - 4\cdot t + 2 = t^2$$

$$\therefore \quad t^2 - 4t + 2 = 0$$

$\therefore \quad t = 3.414\ s$ or $t = 0.585\ s < 1$ sec.

Hence, not possible.

Ans.: Total time of fall = 3.414 s

Height $h$ = 57.166 m

Ex. No. 5 RCH/12-14/pg. 631

I) Deccelerated motion:

$u = 110$ kmph $= \left(110 \times \frac{5}{18}\right) = 30.55$ m/s

$V = 43$ kmph $= \left(43 \times \frac{5}{18}\right) = 11.95$ m/s

$a_1 = -2.6$ kmph/sec $= -0.72$ m/s$^2$

$V = u + a \cdot t$

$(11.95) = (30.55) - (0.72)t_1 \quad \therefore t_1 = 25.83$ sec.

$S_1 = (30.55 \times 25.83) - (\frac{1}{2} \times 0.72 \times 25.83)^2 = 548.91$ m

II) Uniform motion:

$S = V \cdot t$

$\therefore (800) = (11.95) \cdot t_2 \quad \therefore t_2 = 66.95$ s

$S_2 = 800$ m.

III) Accelerated motion:

$a_2 = 1.8$ kmph/sec $= 0.5$ m/s$^2$

$V = u + at$

$(30.55) = (11.95) + (0.5)t_3 \quad \therefore t_3 = 37.2$ s

$S = ut + \frac{1}{2}at^2$

$\therefore S_3 = (11.95 \times 37.2) + (\frac{1}{2} \times 0.5 \times 37.2^2) = 790.5$ m

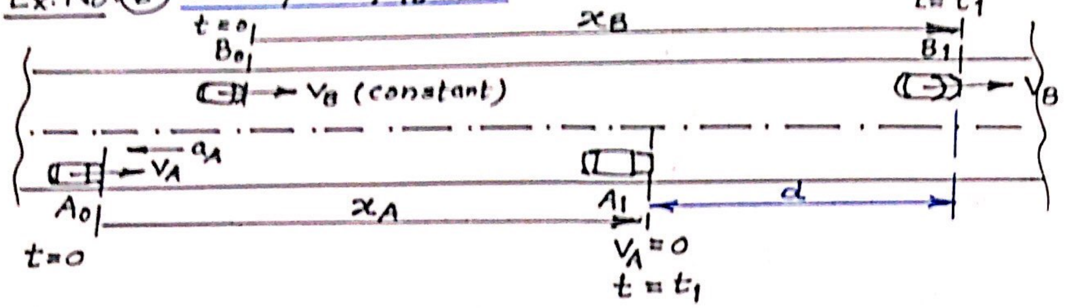
$\therefore$ Total time of travel $= (t_1 + t_2 + t_3) = 130$ s

Total distance travelled $= (S_1 + S_2 + S_3) = 2139.4$ m

If the track repair is not there, the train will take, $t = \left(\frac{2139.4}{30.55}\right) = 70$ sec

Ans. Delay due to track repair $= (130 - 70)$
$= 60$ s.

Ex. No. 6 RCH / 12.9 / pg. 631




For A :

$$v_A - (a_A \cdot t_1) = 0 \qquad \therefore \; t_1 = \left(\frac{v_A}{a_A}\right)$$

$$x_A = \left(v_A \cdot t_1 - \frac{1}{2} \cdot a_A t_1^2\right)$$

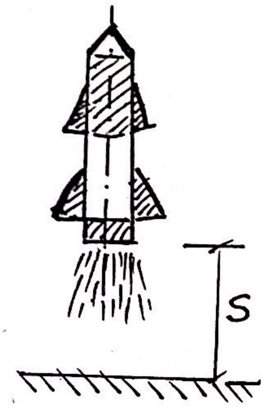
$$= \left(v_A \cdot \frac{v_A}{a_A} - \frac{1}{2} \cdot a_A \cdot \frac{v_A^2}{a_A^2}\right)$$

$$= \left(\frac{v_A^2}{a_A} - \frac{1}{2} \cdot \frac{v_A^2}{a_A}\right) = \frac{1}{2}\left(\frac{v_A^2}{a_A}\right)$$

For B : $x_B = v_B \cdot t_1 = \left(\frac{v_A \cdot v_B}{a_A}\right)$

Ans. : $d$ = distance betn cars A and B when A stops

$$\boxed{d = |x_A - x_B| = \left|\frac{2 \cdot v_A v_B - v_A^2}{2 \cdot a_A}\right|}$$

| | |
|---|---|
| 1 | A particle is moving a with a velocity of $v_0$ when s=0 & t=0. If it is subjected to a deceleration of $a= -kv^3$, where k is the constant, determine its velocity and position as function of time.<br>**Ans: $(2kt+(1/v_0^2)^{-1/2}$** |
| 2 | The acceleration of a rocket traveling upward is given by a=(6 + 0.02s) $m/s^2$ where s is in meters. Determine the rockets velocity when s = 2 km & time needed to reach this altitude. Initially, V=0 & s= 0 when t=0.<br>**Ans:**<br><br> |
| 3 | A particle moving along a straight line such that its acceleration is defined as , a =(-2v) $m/s^2$, where v is in meter/ seconds. If v = 20m/s when s =0 & t = 0, determine the particle's position, velocity and acceleration as function of time.<br>**Ans: $x=10-(0.5)[e^{(2.995-2t)}]$** |
| 4 | If a= (s) $m/s^2$,where 's' is in meter, determine v when s = 5m if v=0 at s= 4m<br>**Ans:V=3m/s** |
| 5 | The acceleration of a particle moving along a straight line is given by the law, a $=3s - 6s^2$. Where 'a' is $m/s^2$ & 's' is in meter. The particles starts from rest.<br>Find (a)Velocity when the displacement is 3m.<br>(b) the displacement when the velocity is again zero &<br>(c) the displacement at maximum velocity.<br>**Ans: x = 0.5m** |
| 6 | The acceleration of a particle is given by a = $-0.02v^{1.75}$ $m/s^2$ performing rectilinear motion knowing at x= 0, v= 20 m/s. Determine ( ) The position where the velocity is 28m/s &(b) acceleration when x = 200m.<br>**Ans: $a=-0.043m/s^2$** |

① RCH/12-17/pg.632:

$$a = -kv^3 \; m/s^2 \longrightarrow ①$$

$$a \cdot dx = v \cdot dv$$

$$\therefore \; -k \cdot v^3 \cdot dx = v \cdot dv$$

$$\therefore \; -k \cdot v^2 \cdot dx = dv$$

$$\therefore \; -k\int_0^x dx = \int_{v_0}^{v} \frac{dv}{v^2}$$

$$-k \cdot x = \left[\frac{v^{-1}}{-1}\right]_{v_0}^{v}$$

$$-kx = -\left[\frac{1}{v}\right]_{v_0}^{v}$$

$$kx = \frac{1}{v} - \frac{1}{v_0}$$

$$\boxed{x = \frac{1}{k}\left[\frac{1}{v} - \frac{1}{v_0}\right]} \Rightarrow \boxed{\frac{1}{k}\left[\left(2kt + \frac{1}{v_0^2}\right)^{\frac{1}{2}} - \frac{1}{v_0}\right]}$$

Ans:

(i) $v = \left(2kt + \frac{1}{v_0^2}\right)^{-1/2}$

(ii) $x = \frac{1}{k}\left[\left(2kt + \frac{1}{v_0^2}\right)^{1/2} - \frac{1}{v_0}\right]$

Now, $a = \frac{dv}{dt} = -kv^3$

$$-k\int_0^t dt = \int_{v_0}^{v} v^{-3} \cdot dv$$

$$-k \cdot t = \left[\frac{v^{-2}}{-2}\right]_{v_0}^{v} = -\left[\frac{1}{2v^2}\right]_{v_0}^{v}$$

$$kt = \frac{1}{2v^2} - \frac{1}{2v_0^2}$$

$$\frac{1}{2v^2} = \frac{1}{2v_0^2} + kt$$

$$\frac{1}{v^2} = \left(\frac{1}{v_0^2} + 2kt\right)$$

$$\therefore V = \frac{1}{\sqrt{2kt + 1/v_0^2}}$$

Ans:

$$\therefore \boxed{V = \left(2kt + \frac{1}{v_0^2}\right)^{-1/2}}$$

② RCH/12.19/pg.632.

$$a = 6 + (0.02)x \quad m/s^2 \longrightarrow ①$$

$$a \cdot dx = v \cdot dv$$

$$\int_0^{2000\,m} [6 + (0.02)x]\,dx = \int_0^{v} v \cdot dv$$

V = 322.49 m/s
t = 19.27 s

$$\left[6 \cdot x + (0.02)\frac{x^2}{2}\right]_0^{2000} = \frac{v^2}{2}$$

$$\therefore v^2 = 2\left[(6 \times 2000) + 40{,}000\right] = (52{,}000 \times 2)$$

Ans: $\boxed{v = 322.49\ m/s}$

NOW, $v^2 = 12 \cdot x + (0.02)x^2$

$$v = \left[12 \cdot x + (0.02)x^2\right]^{1/2} \quad m/s \longrightarrow ②$$

$$\therefore \left(\frac{dx}{dt}\right) = \left[12 \cdot x + (0.02)x^2\right]^{1/2}$$

$$\int_0^{t} dt = \int_0^{2000} \left[12 \cdot x + (0.02)x^2\right]^{-1/2} \cdot dx$$

$$\cancel{t = \frac{1}{2}\left[12 \cdot x + (0.02)x^2\right]^{1/2}(12 + 0.04 \times x)} \longrightarrow ③$$

~~when x = 2000 m~~

$$\cancel{t = \frac{1}{2}(161.245)(92) = 14{,}834.55}\ sec$$

$$\cancel{\frac{v^2}{0.02} = (600)x + x^2}$$

$$\cancel{2v \cdot \frac{dv}{dt} = 12 \cdot v + (0.02)x \cdot v}$$

dV

3 RCH/12.25/pg. 632

$\boxed{a = -2v}$ $m/s^2 \longrightarrow$ 1

At $t=0$, $v = 20\ m/s$ & $x = 0$

$dv = a \cdot dt$

$dv = -2v \cdot dt$

$$\int \frac{dv}{v} = -2 \cdot \int dt$$

$\log_e v = -2t + C_1$

$\ln 20 = 0 + C_1$ $\quad C_1 = 2.995$

$\therefore \log_e v = -2t + 2.995$

$\therefore e^{(2.995-2t)} = v$

$\boxed{v = e^{(2.995-2t)}}$ $m/s \longrightarrow$ 2

$$\int dx = \int e^{(2.995-2t)} dt$$

$$x = e^{(2.995-2t)} \times \frac{1}{(-2)} + C_2$$

$\therefore 0 = -(0.5)(20) + C_2$ $\quad \therefore C_2 = 10$

$\boxed{x = 10 - (0.5)\left[e^{(2.995-2t)}\right]} \longrightarrow$ 3

(4) RCH/ F 12.1(+)/ pg. [illegible] :

$$a = s = x \ m/s^2 \longrightarrow (1)$$

$$a \cdot dx = v \cdot dv$$

$$\therefore \int x \cdot dx = \int v \cdot dv$$

$$\frac{x^2}{2} + C_1 = \frac{v^2}{2}$$

when $x = 4m$, $v = 0$

$$8 + C_1 = 0$$

$$C_1 = -8$$

$$\therefore \frac{x^2}{2} - 8 = \frac{v^2}{2}$$

$$\therefore v^2 = x^2 - 16 \longrightarrow (2)$$

At $x = 5m$,

$$v^2 = 25 - 16 = 09$$

$$\therefore v = 3 \ m/s$$

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(5) RCH/12.13/pg.631 :

$$\boxed{a = 3x - 6x^2}\ m/s^2 \longrightarrow (1)$$

At $t=0$, $v=0$, $s=x=0$

$$a \cdot dx = v \cdot dv$$

$$\int (3x - 6x^2)\,dx = \int v \cdot dv$$

$$\left(\frac{3x^2}{2} - 2x^3\right) = \left(\frac{v^2}{2} + c_1\right)$$

$\therefore c_1 = 0$

$$\therefore v^2 = 3x^2 - 4x^3$$

$$\boxed{v = \sqrt{3x^2 - 4x^3}} \longrightarrow (2)$$

a) At $x = 3m$, $v = \sqrt{27 - 324}$

b) When $v=0$, $3x^2 - 4x^3 = 0$

$$x^2(3 - 4x) = 0$$

$\therefore x = 0$ or $3 - 4x = 0$

$$x = 3/4 = 0.75m$$

$$\boxed{x = 0.75m}$$

c) For $V_{max}$, $\frac{dv}{dx} = 0 = \frac{1}{2}(3x^2 - 4x^3)^{-1/2} \cdot (6x - 12x^2)$

$$\therefore \left[\frac{3x - 6x^2}{\sqrt{3x^2 - 4x^3}}\right] = 0 \qquad \therefore 3x - 6x^2 = 0$$

$\therefore x = 0$ or $x = 0.$

$$\therefore \boxed{x = 0.5\ m}$$

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(6) RCH/12.26/pg.632

$$a = -(0.02)\times v^{1.75}\ m/s^2 \longrightarrow (1)$$

$$a \cdot dx = v \cdot dv$$

$$-(0.02)\cdot v^{1.75}\, dx = v \cdot dv$$

$$-(0.02)\int dx = \int v^{-0.75}\, dv$$

$$-(0.02)x = \left(\frac{v^{0.25}}{0.25}\right) + C_1$$

$$-(0.02)\, x = 4 \cdot v^{1/4} + C_1$$

$$x = -(200)\cdot v^{1/4} + C_1$$

At $x = 0$, $v = 20\ m/s$

$$0 = -(200)(2.115) + C_1$$

$$C_1 = 422.95$$

$$\boxed{x = -(200)\cdot v^{1/4} + 422.95}$$

Ans: (a) when, $v = 28\ m/s$

$$x = -37.115\ m/s$$

(b) when $x = 200\ m$

$$v = 1.544\ m/s$$

$$a = -0.043\ m/s^2$$

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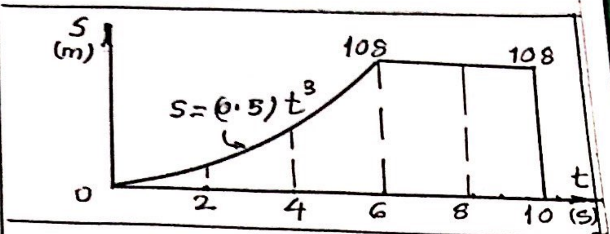
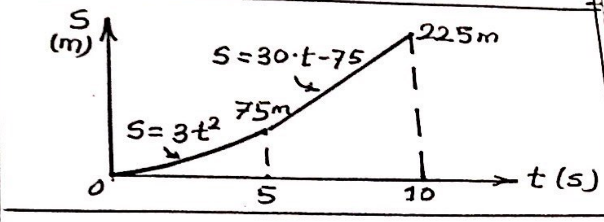
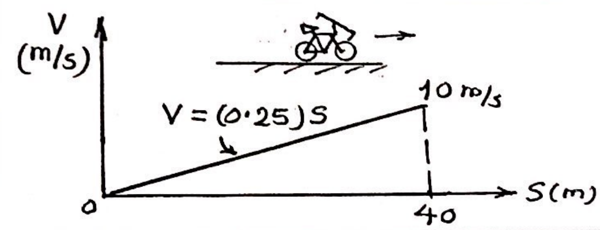
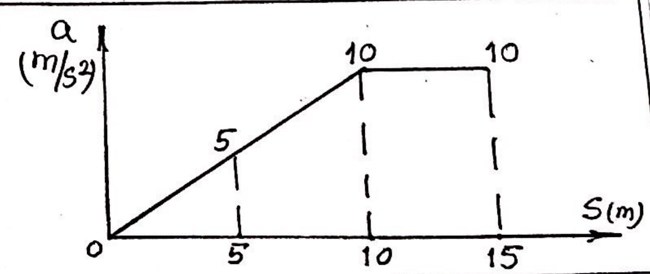
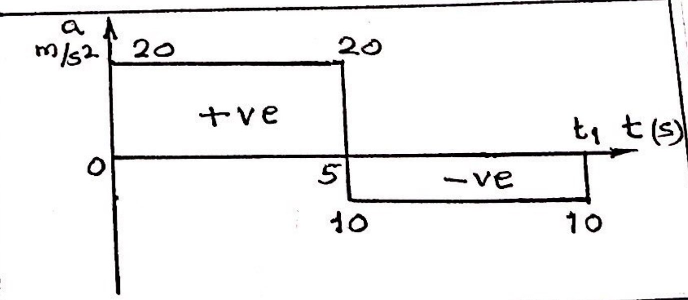
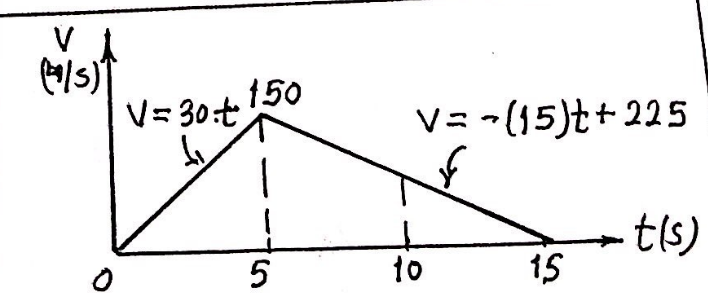
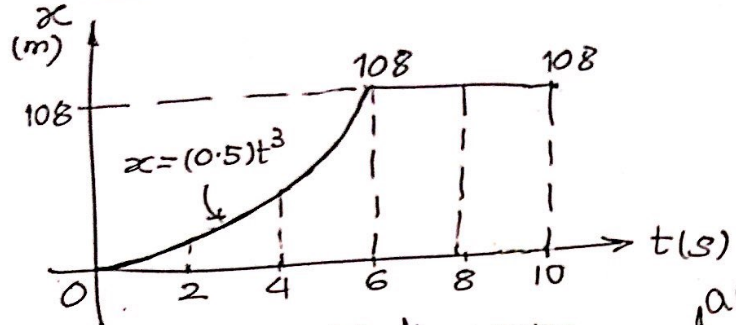
Lecture No: **Motion Curves** 

| | | |
|---|---|---|
| 1 | The particle travels along a straight track such that its position is described by the s-t graph. Construct the v-t graph for the same time interval. |  |
| 2 | The sports car travels along a straight road such that its position is described by the graph. Construct the v-t and a-t graphs for the time interval $0 \le t \le 10s$. |  |
| 3 | A bicycle travels along a straight road where its velocity is described by the v-s graph. Construct the a-s graph for the same interval. |  |
| 4 | The sports car travels along a straight road such that its acceleration is described by the graph. Construct the v-s graph for the same interval and specify the velocity of the car when s=10m and s=15m. |  |
| 5 | The dragster starts from rest and has acceleration described by the graph. Construct the v-t graph for the time interval $0 \le t \le t^1$, where $t^1$ is the time for the car to come to rest. |  |
| 6 | The dragster starts from rest and has a velocity described by the graph. Construct the s-t graph for the time interval $0 \le t \le 15s$. Also, determine the total distance traveled during this time interval. |  |

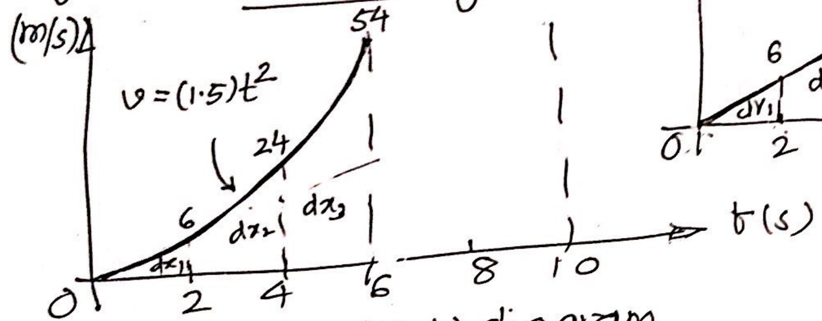
## Lecture No (4)

(1) RCH/F 12·9/pg.641

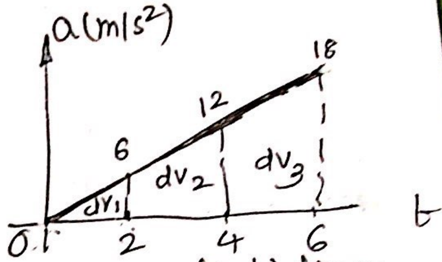
(P1)




(x-t) diagram




(v-t) diagram




(a-t) diagram

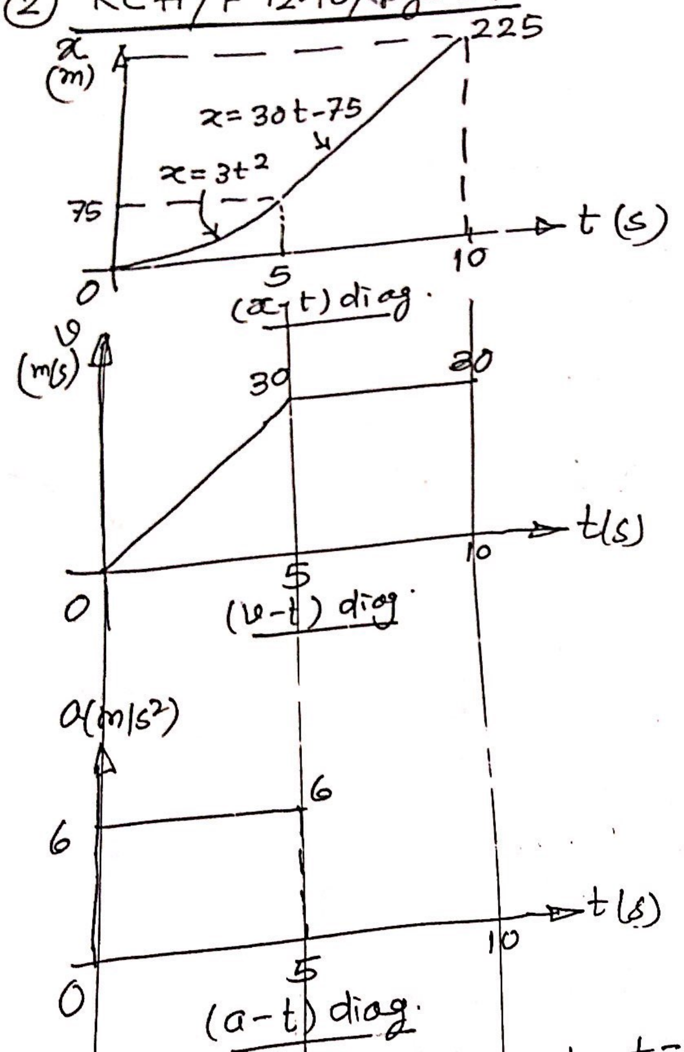
$$x = (0.5)t^3 \longrightarrow (1)$$

$$v = \frac{dx}{dt} = (1.5)t^2 \longrightarrow (2)$$

$$a = \frac{dv}{dt} = 3t \longrightarrow (3)$$

| Time t sec. | a $m/s^2$ | dv m/s | v m/s | dx m | x m |
|---|---|---|---|---|---|
| 0 | 0 | — | 0 | — | 6 |
| 2 | 6 | 6 | 6 | 4 | 4 |
| 4 | 12 | 18 | 24 | 28 | 32 |
| 6 | 18 | 30 | 54 | 76 | 108 |
| 8 | | | 0 | 0 | 108 |
| 10 | | | 0 | 0 | 108 |

② RCH/F 12.10/pg. 641 :




I) Motion from $t = 0$ to $t = 5$ s

$x = 3t^2$ m

$v = 6t$ m/s

$a = 6$ m/s²

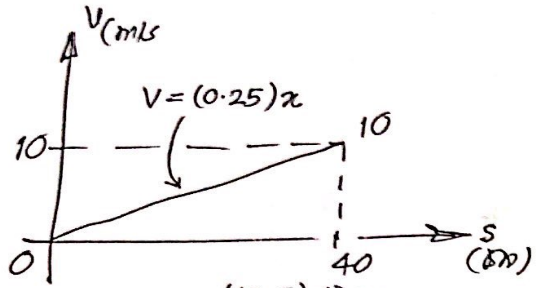
II) Motion from $t = 5$ to $t = 10$ s

$x = (30t - 75)$ m

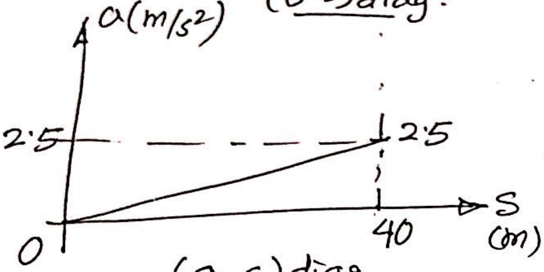
$v = 30$ m/s

$a = 0$ m/s²

③ RCH / F12.11 / pg. 641




(v-s) diag.




(a-s) diag.

$$V = (0.25)x$$

$$a \cdot dx = v \cdot dv$$

$$(0.25)x \cdot dx = v \cdot dv$$

$$a = v \cdot \frac{dv}{dx}$$

$$\therefore a = (0.25)x \cdot (0.25)$$

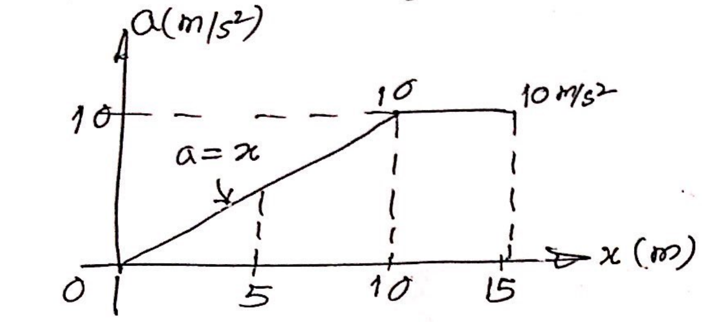
$$\therefore a = (0.0625)x$$

when $x = 40\,m$

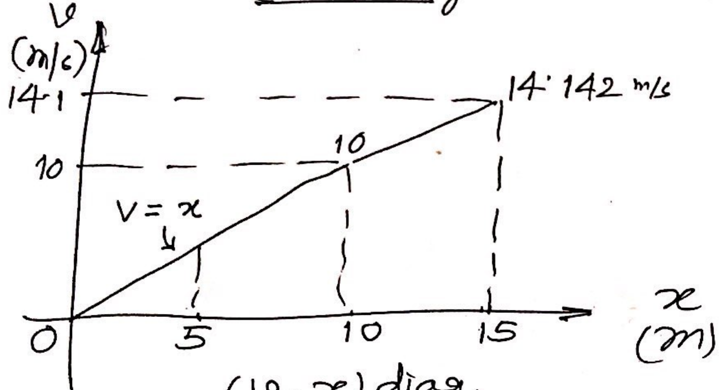
$$a = 0.0625 \times 40$$

$$= 2.5\ m/s^2$$

④ RCH/F12.12/pg.641.




(a-x) diag.




(v-x) diag.

I) Motion from 0 to 10 s:

$$a = x$$

$$a \cdot dx = v \cdot dv$$

$$\int x \cdot dx = \int v \cdot dv$$

$$\frac{V^2}{2} = \frac{x^2}{2} + c_1$$

At $x = 0$, $V = 0$ $\therefore c_1 = 0$

$$\therefore V = x$$

II) Motion from 10 to 15 s:

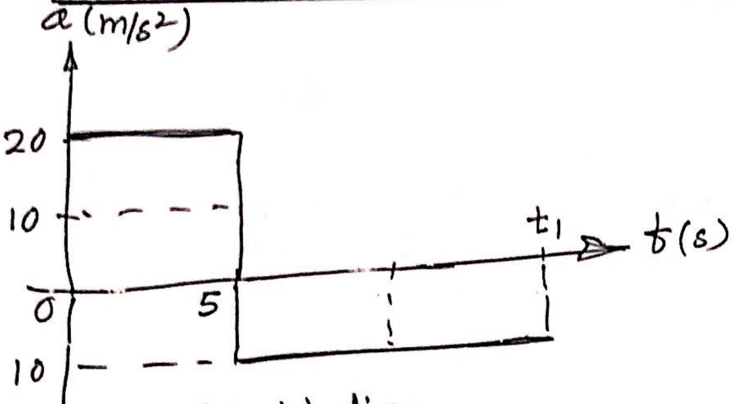
$$V^2 = u^2 + (2a \cdot s)$$

$$= 10^2 + 2(10) \times 5$$

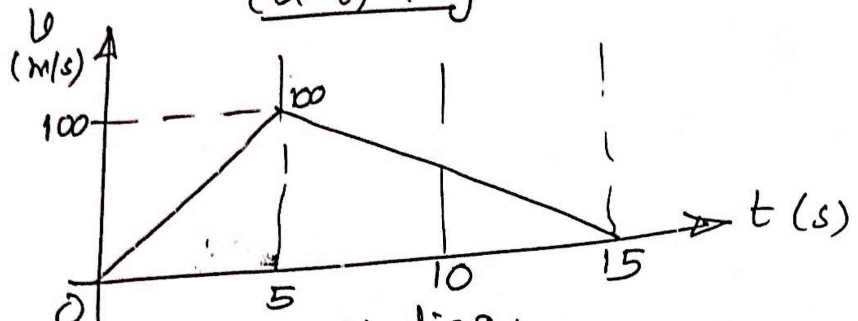
$$= 200$$

$$V = 14.142 \text{ m/s}$$

## (5) RCH/F 12.13/pg.641:




(a–t) diag.




(v–t) diag.

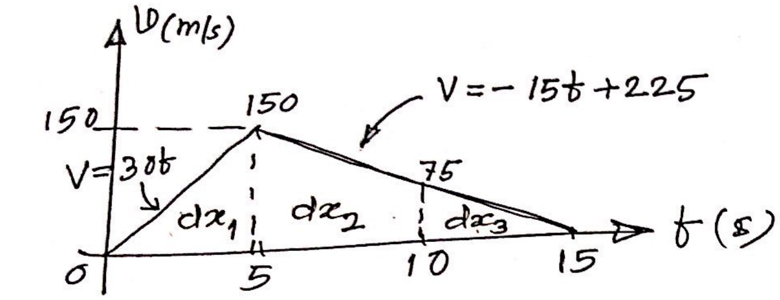
I) Motion from 0 to 5 sec:

$$V = u + at$$
$$= 0 + (20 \times 5) = 100 \text{ m/s}$$

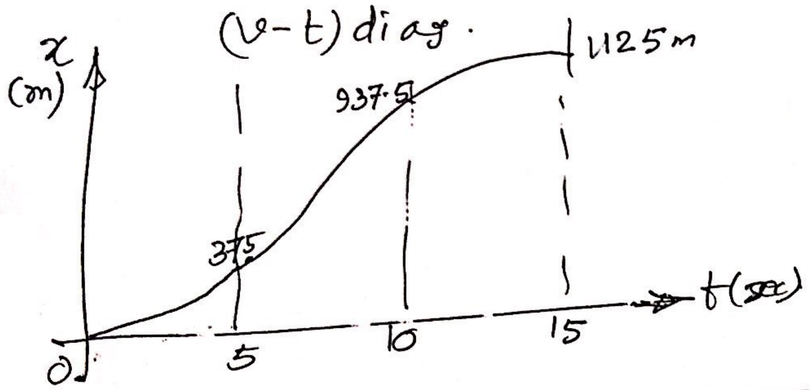
II) Motion from 5 to $t_1$ sec:

$$V = u + at$$
$$0 = 100 - 10 \times t$$
$$\underbrace{t = 10 \text{ sec.}}_{\text{duration}} \quad \therefore t_1 = 15 \text{ sec.}$$

(6) RCH / F 12.14 / pg. 641 :




(v − t) diag.

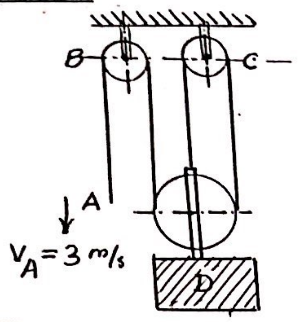



| t (s) | v (m/s) | dx (m) | x (m) |
|---|---|---|---|
| 0 | 0 | – | 0 |
| 5 | 150 | 375 | 375 |
| 10 | 75 | 562.5 | 937.5 |
| 15 | 0 | 187.5 | 1125 |

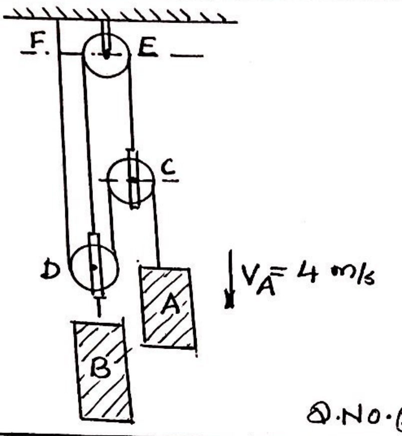
Total distance traveled } $\boxed{1125 \text{ m}}$

**Lecture No:** **Dependant Motion**

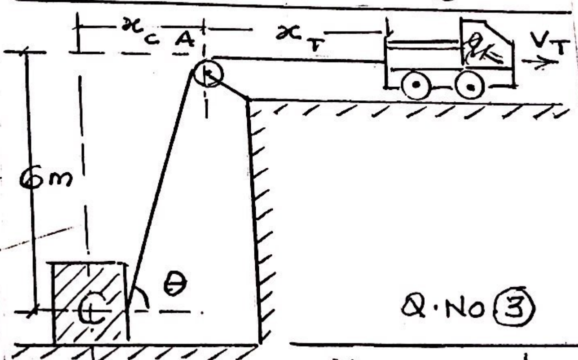
| | |
|---|---|
| 1 | Determine the velocity of block D if end A of the rope is pulled down with a speed of $v_a$ = 3 m/s<br><br>**Ans: $V_D$ = 1 m/s upwards** |
| 2 | Determine the velocity of cylinder B if cylinder A moves downward with a speed of $v_a$ = 4 m/s<br><br>**Ans: $V_B$ = 1 m/s upwards** |
| 3 | If the truck travels at a constant speed of $v_T$ = 1.8 m/s, determine the speed of the crate for any angle θ of the rope. The rope has a length of 30 m and the passes over a pulley of negligible size at A. **Hint:** relate the coordinates $x_T$ & $x_C$ to the length of the rope & take the time derivative. Then substitute the trigonometric relation between θ & $x_C$.<br><br>**Ans: $V_C$ = -1.8 sec θ** |
| 4 | The roller at A is moving with a velocity of $v_A$ = 4 m/s & has as an acceleration of $a_A$ = 2 m/s$^2$ when $X_A$ = 3 m. Determine the velocity & acceleration of block B at this instant.<br><br>**Ans: $V_B$ = 2.4 m/s up; $a_B$ =3.248 m/s$^2$ up** |
| 5 | The motor draws in the cord at B with an acceleration of $a_B$ =2 m/s$^2$. When $s_A$= 1.5 M, $V_B$ = 6 m/s. Determine the velocity & acceleration of the collar at its instant.<br><br>**Ans: $V_A$ = 10 m/s Left ; $a_a$ =46 m/s$^2$ Left** |



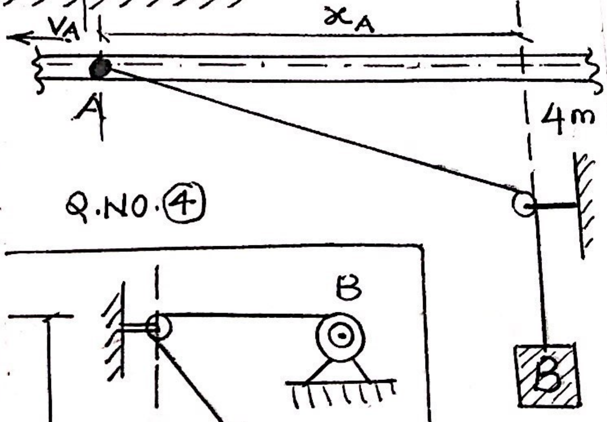

Q.No.1



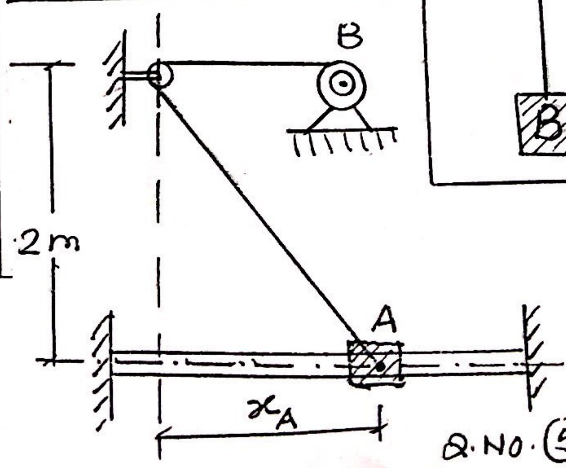

Q.No.2




Q.No 3



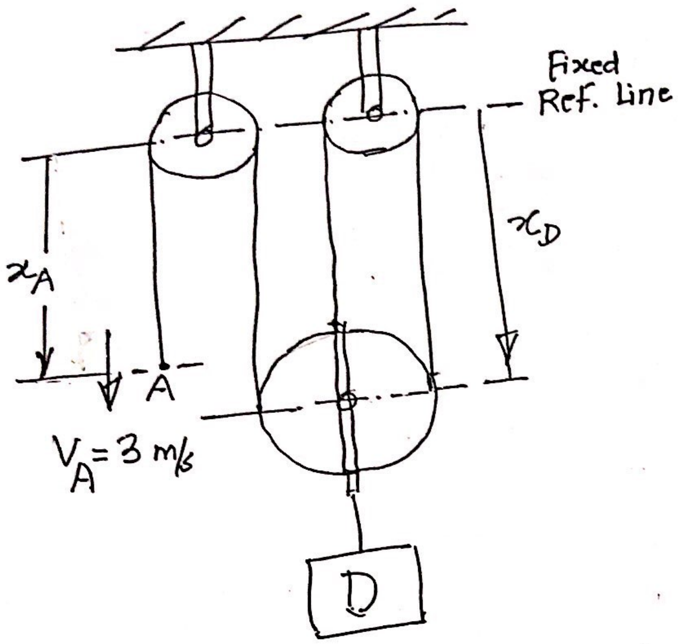

Q.NO.4




Q.NO.5

# Lecture No. (5)

(1) RCH/F 12.39/pg. 710 :




$$x_A + 3 \cdot x_D = \text{constant}$$

$$\therefore V_A + 3 \cdot V_D = 0$$

$$\therefore a_A + 3 \cdot a_D = 0$$

when $V_A = 3$ m/s

$$3 + 3 \cdot V_D = 0$$

$$V_D = -1 \text{ m/s}$$

$$\therefore V_D = 1 \text{ m/s } (\uparrow)$$

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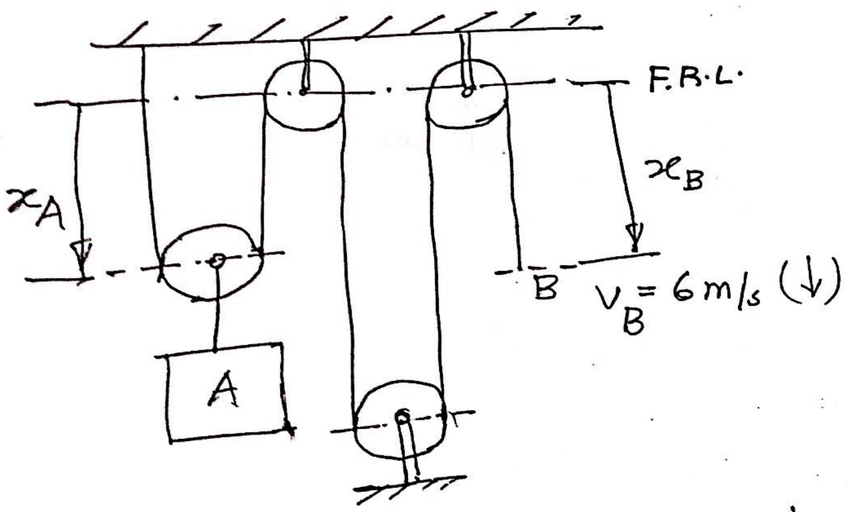
(2) RCH / F 12.40 / pg. 710 :




$$2 \cdot x_A + x_B = \text{constant}$$

$$\therefore 2 \cdot V_A + V_B = 0$$

$$\therefore 2 \cdot a_A + a_B = 0$$

when, $V_B = 6\ m/s$

$$2 \cdot V_A + 6 = 0$$

$$V_A = -3\ m/s$$

$$\therefore V_A = 3\ m/s\ (\uparrow)$$

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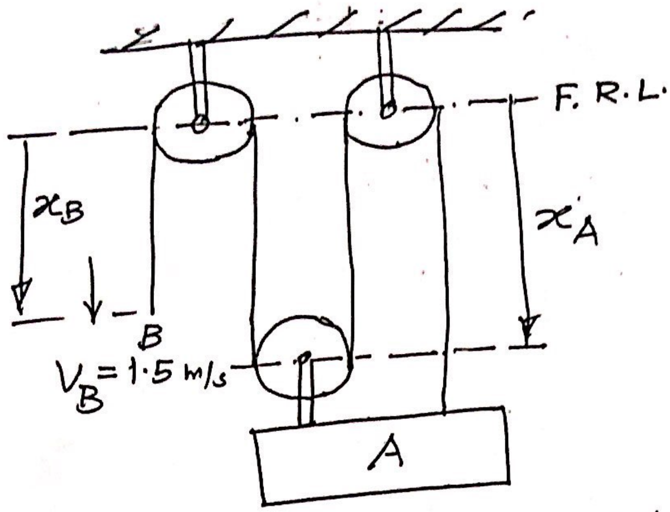
## (3) RCH / F 12.41 / pg. 710




$$x_B + 3 \cdot x_A = \text{constant}$$

$$\therefore \; V_B + 3 \cdot V_A = 0$$

$$\therefore \; a_B + 3 \cdot a_A = 0$$

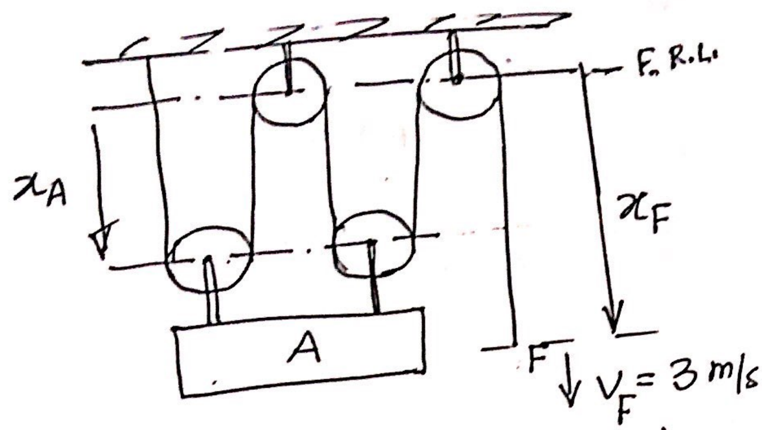
when, $V_B = 1.5$ m/s (↓)

$$1.5 + 3 \cdot V_A = 0$$

$$\therefore \; V_A = -\frac{1.5}{3} = -0.5 \text{ m/s}$$

$$\therefore \; V_A = 0.5 \text{ m/s} \; (\uparrow)$$

④ RCH/F 12.42/pg.710




$$4 \cdot x_A + x_F = \text{constant}$$

$$\therefore\ 4 \cdot V_A + V_F = 0$$

$$\therefore\ 4 \cdot a_A + a_F = 0$$

when $V_F = 3\ m/s$

$$4 \cdot V_A + 3 = 0$$

$$V_A = -\frac{3}{4} = -0.75\ m/s$$

$$\therefore\ V_A = 0.75\ m/s\ (\uparrow)$$

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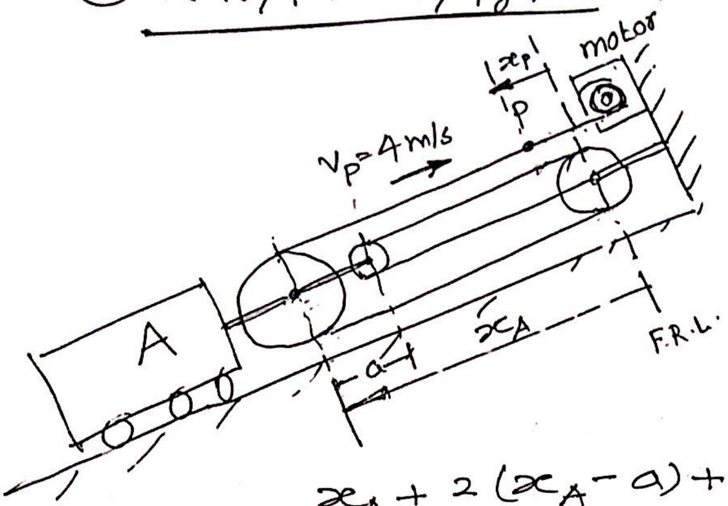
(5) RCH / F 12.43 / pg. 710 :




$$x_A + 2(x_A - a) + (x_A - x_p) = \text{constant}$$

$$x_A + 2 \cdot x_A - 2 \cdot a + x_A - x_p = \text{Const.}$$

$$x_A - x_p = (\text{Const.} + 2a)$$

$$\therefore \quad V_A - V_P = 0$$

$$\therefore \quad a_A - a_P = 0$$

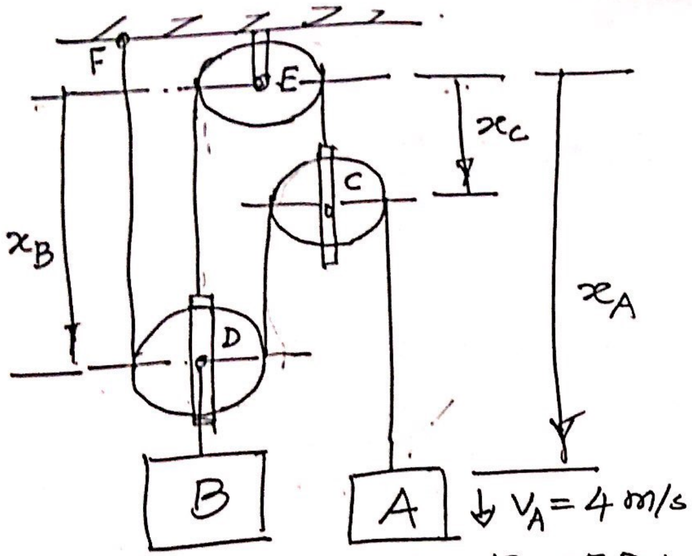
when, $V_P = -4$ m/s

$$V_A + 4 = 0$$

$$V_A = -1 \text{ m/s}$$

$\therefore$ $V_A = 1$ m/s (up the plane)

⑥ RCH/F 12.44/pg. 710




For the rope betn CED:

$$x_B + x_C = \text{constant} \rightarrow \text{(i)}$$

For the rope betn ACDF:

$$(x_A - x_C) + (x_B - x_C) + x_B = \text{const.}$$

$$\therefore x_A + 2x_B - 2x_C = \text{const. (ii)}$$

$$\therefore V_B + V_C = 0 \rightarrow \text{(iii)}$$

$$V_A + 2V_B - 2V_C = 0 \rightarrow \text{(iv)}$$

Eliminating $V_C$,

$$V_A + 4V_B = 0$$

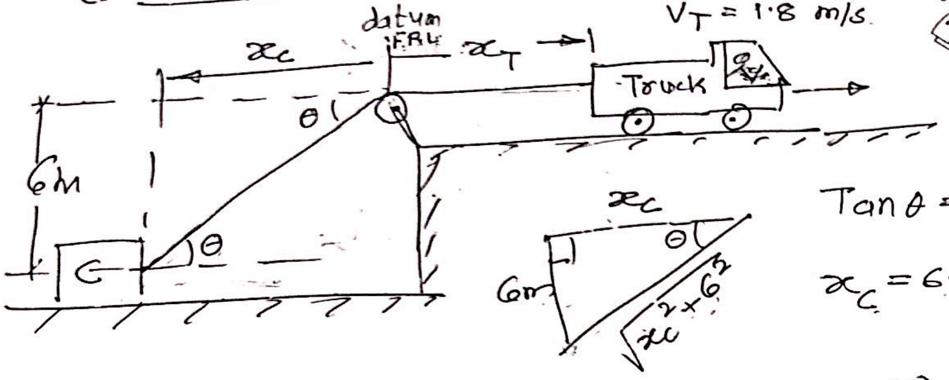
$$4 + 4 \cdot V_B = 0$$

$$V_B = -1 \text{ m/s}$$

$$\therefore V_B = 1 \text{ m/s } (\uparrow)$$

⑦ RCH / 12.210 / pg. 714 :

$a_T = 0$
$V_T = 1.8$ m/s




$\tan\theta = \frac{6}{x_C}$

$x_C = 6 \cdot \cot\theta$

$$\sqrt{x_C^2+36} + x_T = 30 \text{ m} \rightarrow \text{(i)}$$

$$\frac{\cancel{2} \cdot x_C \cdot \dot{x}_C}{\cancel{2}\sqrt{x_C^2+36}} + \dot{x}_T = 0$$

$$\therefore x_C \, \dot{x}_C + (\dot{x}_T)\left(\sqrt{x_C^2+36}\right) = 0 \longrightarrow \text{(ii)}$$

$$V_T = \dot{x}_T = 1.8 \text{ m/s}$$

$$(6 \cdot \cot\theta)(V_C) + \left(\sqrt{36 \cdot \cot^2\theta + 36}\right)(V_T) = 0$$

$$\therefore (\cot\theta)(V_C) + \left(\sqrt{1+\cot^2\theta}\right)(V_T) = 0$$

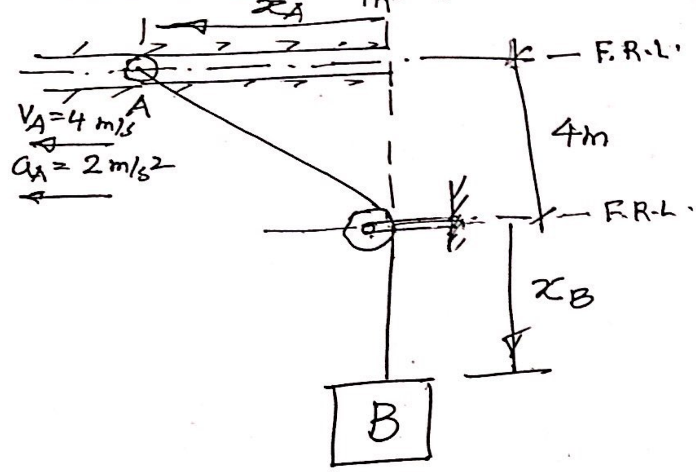
$$\therefore (\cot\theta)(V_C) + (\operatorname{cosec}\theta)(V_T) = 0$$

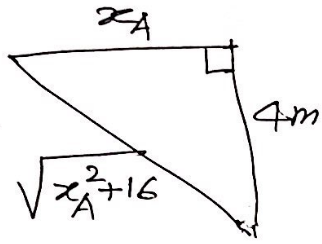
$$(V_C)\left(\frac{\cos\theta}{\cancel{\sin\theta}}\right) = -(V_T)\left(\frac{1}{\cancel{\sin\theta}}\right)$$

$$V_C = -(V_T)(\sec\theta)$$

$$\boxed{V_C = -(1.8)(\sec\theta)}$$

8. RCH/12.212/pg.714 :




$$\sqrt{x_A^2+16} + x_B = \text{constant} \longrightarrow (i)$$

$$\frac{\cancel{2}\cdot x_A \cdot \dot{x}_A}{\cancel{2}\cdot\sqrt{x_A^2+16}} + \dot{x}_B = 0$$

$$x_A \cdot \dot{x}_A + \left(\sqrt{x_A^2+16}\right)(\dot{x}_B) = 0 \longrightarrow (ii)$$

$$\therefore x_A \cdot \ddot{x}_A + \dot{x}_A^2 + \left[\frac{\cancel{2}\cdot x_A \cdot \dot{x}_A \cdot \dot{x}_B}{\cancel{2}\sqrt{x_A^2+16}}\right] + \left(\sqrt{x_A^2+16}\right)(\ddot{x}_B) = 0 \longrightarrow (iii)$$

When $x_A = 3m$

Eq$^n$(ii) gives, $(3\times4) + 5\cdot V_B = 0 \quad \therefore V_B = 2.4$ m/s (↑)

Eq$^n$(iii) gives, $(3\times2) + (16) + \left(\frac{3\times4\times-2.4}{5}\right) + 5\cdot a_B = 0$

$$6 + 16 \cancel{+} -5.76 = -5\cdot a_B$$

$$a_B = \cancel{5.552}\ 3.248 \text{ m/s}^2\ (\uparrow)$$

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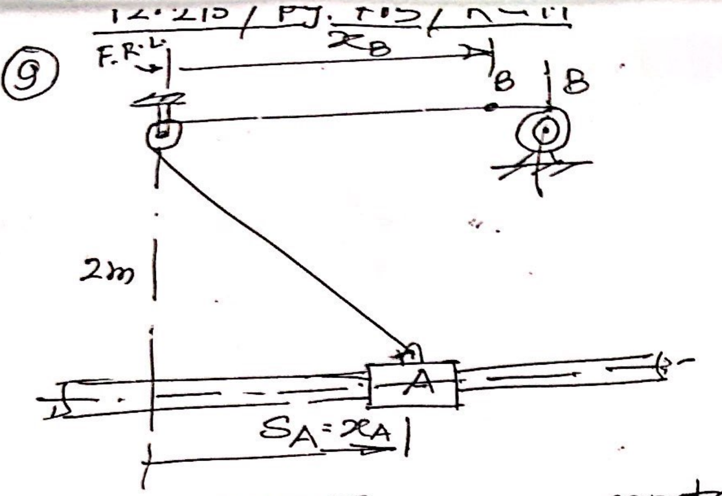
(9)




$$\sqrt{x_A^2+4} + x_B = \text{constant} \longrightarrow (i)$$

$$\frac{\cancel{2}\cdot x_A \cdot \dot{x}_A}{\cancel{2}\cdot\sqrt{x_A^2+4}} + \dot{x}_B = 0 \longrightarrow (ii)$$

$$x_A\cdot\dot{x}_A + (\dot{x}_B)\left(\sqrt{x_A^2+4}\right) = 0 \longrightarrow (ii)$$

$$x_A \cdot \ddot{x}_A + \dot{x}_A^2 + \left[\frac{\cancel{2}\, x_A \cdot \dot{x}_A\, \dot{x}_B}{\cancel{2}\cdot\sqrt{x_A^2+4}}\right] + (\ddot{x}_B)\left(\sqrt{x_A^2+4}\right) = 0 \longrightarrow (iii)$$

Given, $a_B = \ddot{x}_B = 2\ m/s^2$

$x_A = 1.5\ m$, $V_B = 6\ m/s = \dot{x}_B$

$\sqrt{x_A^2+4} = 2.5$

Eq$^n$ (ii) gives, $(1.5)\cdot V_A + (6\times 2.5) = 0$

$\therefore V_A = -10\ m/s$ $\therefore V_A = 10\ m/s\ (\leftarrow)$

Eq$^n$ (iii) gives, $(1.5)(a_A) + (100) + \dfrac{(1.5)(-10)(6)}{(2.5)} + (2\times 2.5) = 0$

$(1.5)\cdot a_A + 69 = 0$ $\quad\therefore a_A = 46\ m/s^2\ (\leftarrow)$