

MIT WORLD PEACE UNIVERSITY

Engineering Mechanics
First Year B. Tech, Trimester 1

GRAPHICAL SOLUTION TO FIND REACTIONS OF
BEAMS UNDER THE INFLUENCE OF A CO-PLANAR
NON-CONCURRENT FORCE SYSTEM

EXPERIMENT 9
PRACTICAL REPORT

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1 Objective

To find Graphically and Analytically the resultant of a set of problems involving Non-Concurrent Co-Planar Force System, and to compare the results thereby finding the Percentage Error.

2 Theory

The Following laws and concepts have been used in this experiment.

2.1 Force law of equilibrium

When all the forces that act upon an object are balanced, then the object is said to be in a state of equilibrium. The forces are considered to be balanced if the rightward forces are balanced by the leftward forces and the upward forces are balanced by the downward forces. This however does not necessarily mean that all the forces are equal to each other.

2.2 Moment of a Force

The Moment of a force is a measure of its tendency to cause a body to rotate about a specific point or axis. This is different from the tendency for a body to move, or translate, in the direction of the force.

In order for a moment to develop, the force must act upon the body in such a manner that the body would begin to twist. This occurs every time a force is applied so that it does not pass through the centroid of the body. Moment τ can be defined as

$$\tau = F \times r$$

2.3 Equilibrium and Rigid Body in Equilibrium

Knowledge of the forces required to maintain an object in equilibrium is essential in understanding the nature of bodies at rest and in motion. To determine if a body is in equilibrium, the overall effect of all the forces acting on it must be assessed. All the forces that act on an object result in essentially one force that influences the object's motion. The force which results from all the forces acting on a body is defined as the net force.

Since an object in equilibrium is considered to be in a state of balance, it can be summarised that the net force on the object is equal to zero. That is, if the vector sum of all the forces acting on an object is equal to zero, then the object is in equilibrium. The first condition of equilibrium, a consequence of Newton's first law, may be written in vector form, "A body will be in translational equilibrium if and only if the vector sum of forces exerted on a body by the environment equals zero." For example, if three forces act on a body it is necessary for the following to be true for the body to be in equilibrium.

$$F_1 + F_2 + F_3 = 0 \text{ or } \Sigma F = 0$$

This condition applies to objects in motion with constant velocity and to bodies at rest or in static equilibrium.

A rigid body is in equilibrium when it is not undergoing a change in rotational or translational motion. This equilibrium requires that two conditions must be met. The forces acting on it must add up to zero, and the moments must also add up to zero. It can be represented as:

$$\Sigma F = 0 \text{ and } \Sigma M = 0$$

2.4 Beams

Beam is a horizontal member used to carry vertical load, shear load and sometime horizontal load. It is the major component of building structures. It mainly used in construction of bridges, trusses, and other structures which carry vertical load.

2.5 Space and Polar Diagrams

2.5.1 Space Diagram

A drawing of a structure that indicates its form as well as means of its support and loading conditions.

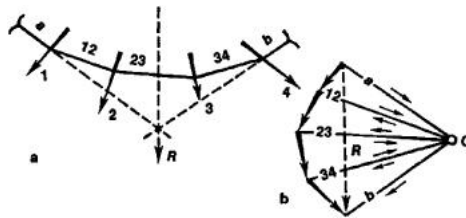
2.5.2 Polar Diagram

In case of non-concurrent or parallel force system the point of application of resultant can be found out by constructing polar diagram. Polar diagram is obtained from the vector diagram. To construct a polar diagram, any point "O" known as pole is chosen near the vector diagram and the points on the vector diagram are joined to it. The lines joined in this way are known as rays.

2.6 Funicular Polygon

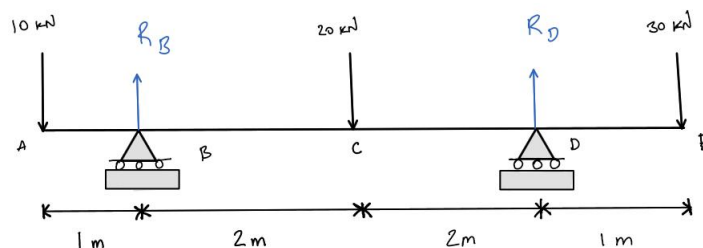
a graphical method of finding the reactive forces and resultants of a system of forces, constructing bending-moment diagrams, determining the rational shapes of arched and suspended systems, and solving other problems of the statics of two-dimensional systems.

The construction of funicular polygons is based on the conception of a polygon formed by an axis that is fixed at the ends of a weightless thread (string), which is tensioned by the forces acting on it. The construction of a funicular polygon together with a force polygon (see Figure) is also used to determine the geometric characteristics of planar cross sections and to solve certain problems in engineering hydraulics, economics, and other fields.



2.7 Coplanar and non-concurrent force system

These forces do not meet at a common point; however, they lie in a single plane, for example, forces acting on a beam as shown in Figure.



3 Procedure

In non-concurrent force system, the following procedure is adopted to find resultant:

1. Draw space diagram to suitable scale as discussed for concurrent force system.
2. Draw vector diagram as discussed for concurrent force system.
3. Select a convenient point in the vector diagram called the pole 'o'. From this pole, draw rays oa, ob, oc, \dots etc. to points a, b, c, \dots etc. of the vector diagram. This diagram is called the polar diagram.
4. Draw lines parallel to the rays in polar diagram by maintaining their order in the respective spaces of space diagram. The first line oa is to be drawn in space A. The next line parallel to ob is drawn from the point of intersection of the first line with the first force. Each subsequent line is drawn from the point of intersection of previous line with its corresponding force. This diagram is known as funicular polygon or link polygon.
5. Resultant is obtained by finding magnitude and direction from vector diagram. Its point of application is obtained by extending the first and last lines of funicular polygon to locate their point of intersection. From this point, a line is drawn parallel to the resultant obtained in vector diagram. The point where this line intersects the given object is the point of application of the resultant.
6. If the object is in equilibrium, then the vector diagram and the funicular polygon must be closed figures. This concept is used to find reactions in beams. The procedure discussed above is illustrated in subsequent problem.

3.1 Problem 1

1. Draw the given beam ABCD to scale as shown in Fig. Represent the spaces according to Bow's notation as (A), (B), (C) (D) and (E).
2. Draw vector diagram. As all the forces are vertical all the vectors and their resultant will be in one line. In Figure, a scale of $1 \text{ cm} \equiv 5 \text{ kN}$ is chosen. Choose any point a in the plane. So, $ab = 2 \text{ cm} \equiv 10 \text{ kN}$ represents the force at A. From b , a downward line bc of 3 cm is drawn representing 15 kN force. From C , upward line $cd = 2 \text{ cm}$ representing 10 kN force at C and from d , upward line of $de = 4 \text{ cm}$ representing 20 kN force at D are drawn. ae represents resultant force in magnitude and direction. By measurement,

$$ae = 5 \text{ cm}$$

Converting to scale,

$$R = 5 \times 5$$

$$R = 25 \text{ kN} \uparrow$$

3. Choose an arbitrary point o the pole and draw rays oa, ob, \dots etc. to form the polar diagram.
4. Draw line oa in space diagram parallel to ray oa in polar diagram. From the point of intersection of this line with line of action of 10 kN force at A, draw line ob parallel to ob . Similarly draw oc, od , and oe .

5. Extend oa and oe and from the point of their intersection draw a line parallel to ae in polar diagram which represents the resultant R . The point of intersection of R with the beam AD gives point of application. By measurement,

$$x = 9.1 \text{ cm}$$

Converting to scale,

$$x = 9.1 \times 0.5 = 4.55 \text{ m}$$

$$x = 4.55 \text{ m from A}$$

By analytical calculation, $R = 25\text{kN} \uparrow$ and $x = 4.6 \text{ m from A}$.

3.2 Problem 2

1. Draw the space diagram by drawing the beam to scale and naming the spaces between only the loads as A, B, (C), and D as shown in Fig.
2. Draw vector diagram for the loads, choose a pole and draw rays to complete the polar diagram.
3. Draw a line parallel to the first ray on in space diagram starting from any point on line of action of R_B and intersecting line of action of 10kN force acting at A. From point of intersection of oa with line of action of 10kN, draw line parallel to ob to intersect line of action of 20kN at C. From this point of intersection, draw line parallel to oc to intersect line of action of 30kN force acting at E. From this point of intersection, draw line parallel to od to intersect line of action of R_D . Draw line from this point of intersection to the point of intersection of on K. R_B . This is the closing line for funicular polygon.
4. Draw ray from pole in polar diagram parallel to the closing line for funicular polygon to intersect the line ad at point p .

3.3 Problem 3

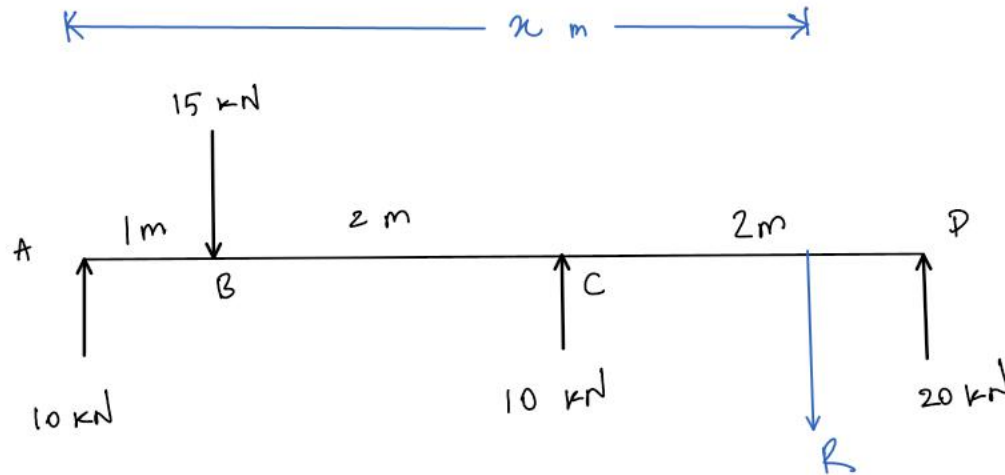
1. Draw the space diagram by drawing the beam to scale and naming the spaces between the loads as A, B, and (C) as shown in Fig.
2. Draw vector diagram for the loads, choose a pole and draw rays to complete the polar diagram.
3. When there is hinge, always start from the hinge. Draw line parallel to ray oc from D in space diagram to intersect line of action of 15kN force acting at C. From this point of intersection, draw line parallel to ray ob to intersect line of action of 10kN force acting at B. From this point of intersection, draw line parallel to on to intersect line of action of R_A . Draw closing line of funicular polygon from D to this point of intersection.
4. Draw ray parallel to closing line in polar diagram. Draw a line parallel to R_A which is at $60^\circ \simeq$ from point a . Let P be the point of intersection of these lines then pa represents R_A and cp represents R_D . By measurement,

$$pa = 3 \text{ cm and } cp = 2.9 \text{ cm}$$

4 Analytical Method

Q1. Determine the magnitude, direction and point of application of resultant from A for the force system shown in Fig. which acts on beam AD.

A.1



As Beam is in equilibrium, $\sum F_y = 0$, $\sum M = 0$

\therefore

$$\sum F_y = 10 - 15 + 10 - R + 20 = 0$$

$$0 = -R + 20 + 20 - 15$$

$$0 = -R + 25$$

$$\boxed{R = 25 \text{ kN} \downarrow}$$

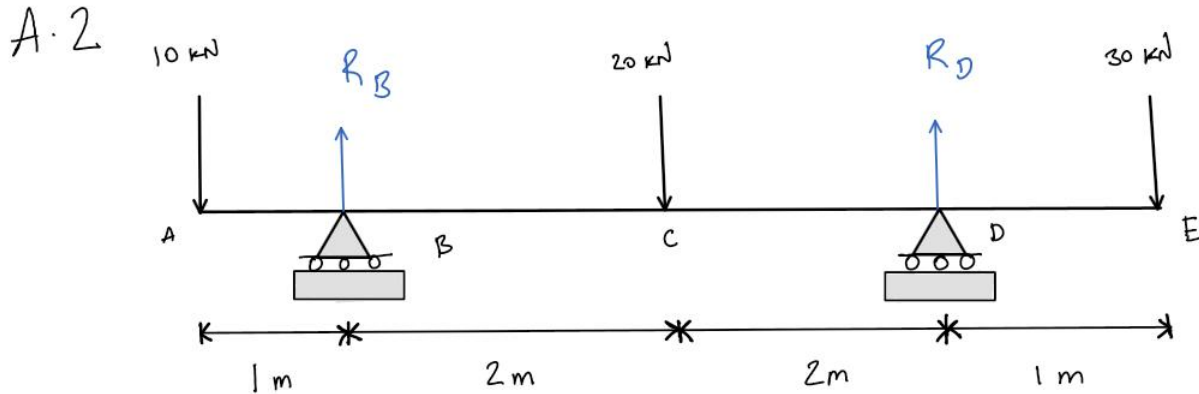
Taking Moments about point A,

$$M_A = -Rx - 15 + 30 + 100 = 0$$

$$Rx = 115 \text{ kNm}$$

$$x = \frac{115 \text{ kNm}}{25 \text{ kN}} = \boxed{4.6 \text{ m From A}}$$

Q2. Determine the beam reactions for the beam shown in the figure.



As Beam is in equilibrium, $\sum F_y = 0$, $\sum M = 0$

$$\therefore R_D + R_B - 10 - 20 - 30 = 0$$

$$R_D + R_B = 60 \quad \text{--- (1)}$$

Taking Moment about Point B,

$$\sum M_B = 0, \Rightarrow$$

$$10 - 40 + 4R_D - 150 = 0$$

$$R_D = \frac{180}{4} = 45 \text{ kN} \uparrow$$

Taking Moment about Point D,

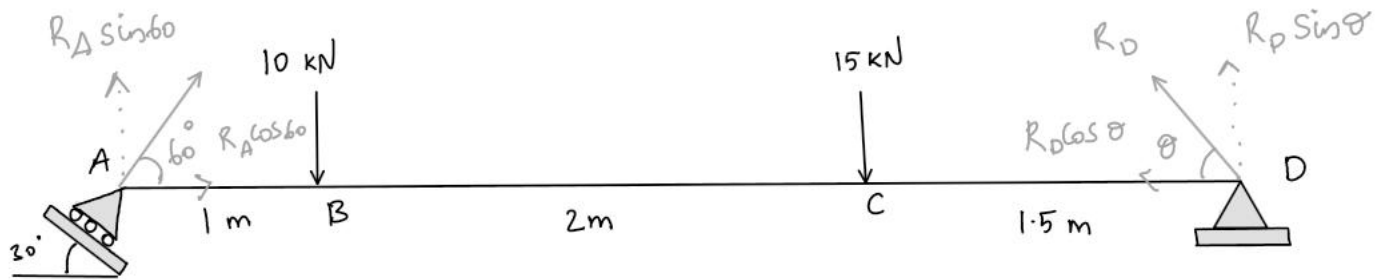
$$\sum M_D = 0$$

$$30 - 40 - 50 + 4R_B = 0$$

$$R_B = \frac{60}{4} = 15 \text{ kN} \uparrow$$

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Q3. Determine support reactions for the beam loaded and supported as shown in the figure.



As Beam is in equilibrium, $\sum F_y = 0$, $\sum M = 0$

Taking Moments about Point D,

$$4.5 (R_A \sin 60) + 22.5 - 35 = 0$$

$$4.5 (R_A \sin 60) = 57.5$$

$$R_A = \frac{57.5}{4.5 \sin 60} = 14.75 \text{ kN } \angle 60^\circ$$

Taking Moments about Point A,

$$4.5 R_D \sin \theta - 45 - 10 = 0$$

$$R_D = \frac{55}{4.5 \sin \theta}$$

$$R_D \sin \theta = 12.22 \text{ kN} \quad \text{--- (1)}$$

But $\sum F_x = 0$ gives

$$R_D \cos \theta + R_A \cos 60 = 0$$

$$R_D \cos \theta = 14.75 \cdot \frac{1}{2}$$

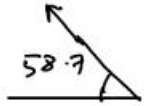
$$R_D \cos \theta = 7.32 \text{ kN} \quad \text{--- (2)}$$

From equations ① and ② ,

$$\frac{\textcircled{1}}{\textcircled{2}} \quad \frac{R_D \sin \theta = 12.22 \text{ kN}}{R_D \cos \theta = 7.32 \text{ kN}} = \tan \theta = 1.65$$

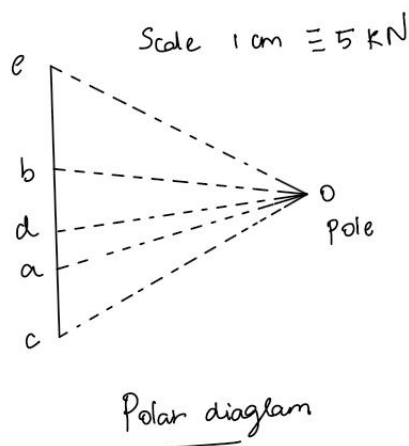
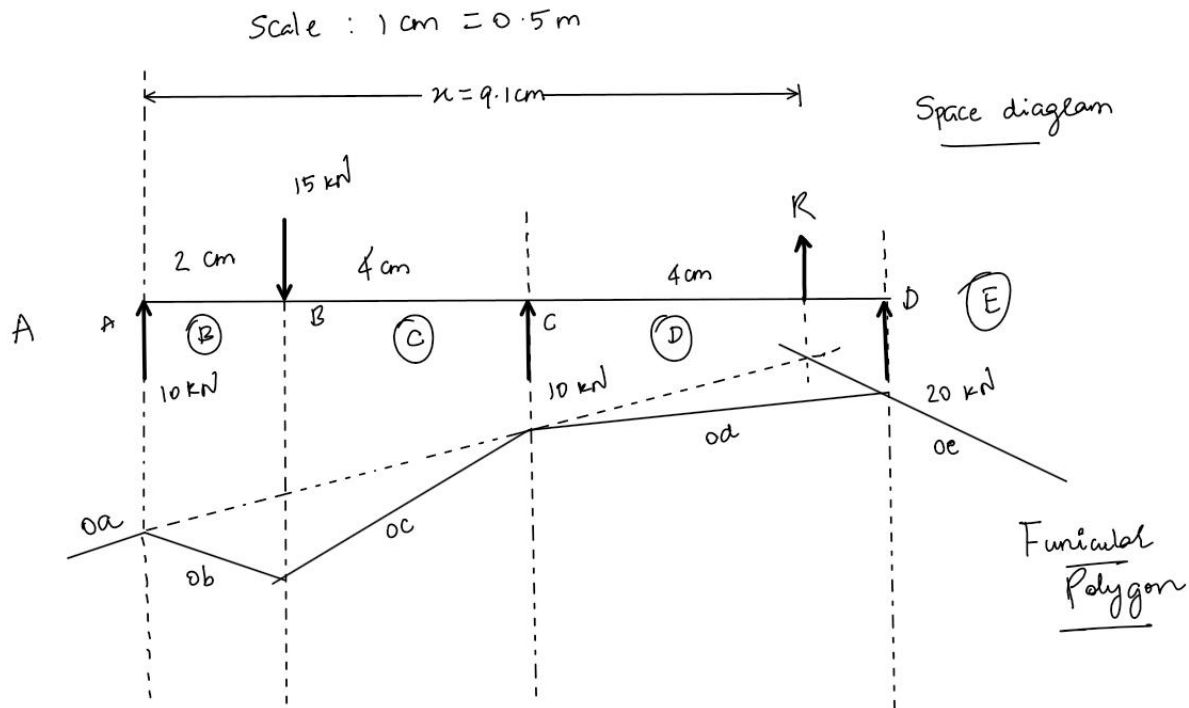
$$\Rightarrow \quad \boxed{\theta = \tan^{-1}(1.65) = 58.7^\circ} \quad - \textcircled{3}$$

Putting ③ in ① and ②

$$\boxed{R_D = \frac{12.22}{\sin(58.7)} = 14.30 \text{ kN}}$$


5 Graphical Method

Q1. Determine the magnitude, direction and point of application of resultant from A for the force system shown in Fig. which acts on beam AD.



ae = 5 cm
Converting to scale

$$R = 5 \times 5 \text{ cm}$$

$$R = 25 \text{ kN} \uparrow$$

$$x = 9.1 \text{ cm}$$

converting to scale

$$x = 9.1 \times 0.5 = 4.55 \text{ m}$$

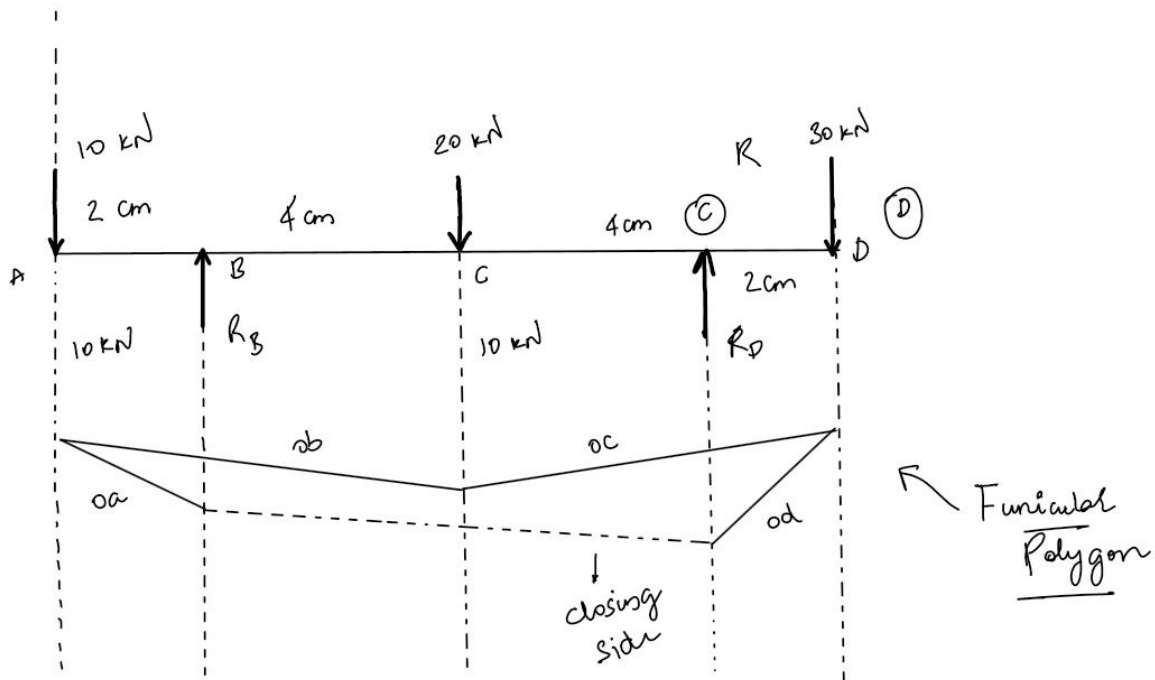
$$x = 4.55 \text{ m from A}$$

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Q2. Determine the beam reactions for the beam shown in the figure.

Space diagram

Scale : 1 cm = 0.5 m



$$ap = 1.5 \text{ cm} \Rightarrow R_B = 1.5 \times 10$$

$$\therefore \boxed{R_B = 15 \text{ kN} \uparrow}$$

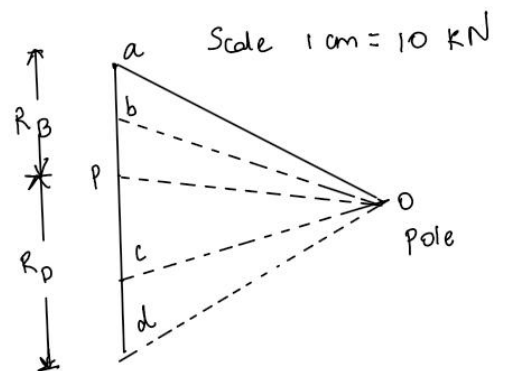
$$pd = 4.5 \text{ cm} \Rightarrow R_D = 4.5 \times 10$$

$$\boxed{R_D = 45 \text{ kN} \uparrow}$$

By analytical calculations,

$$R_B = 15 \text{ kN} \uparrow$$

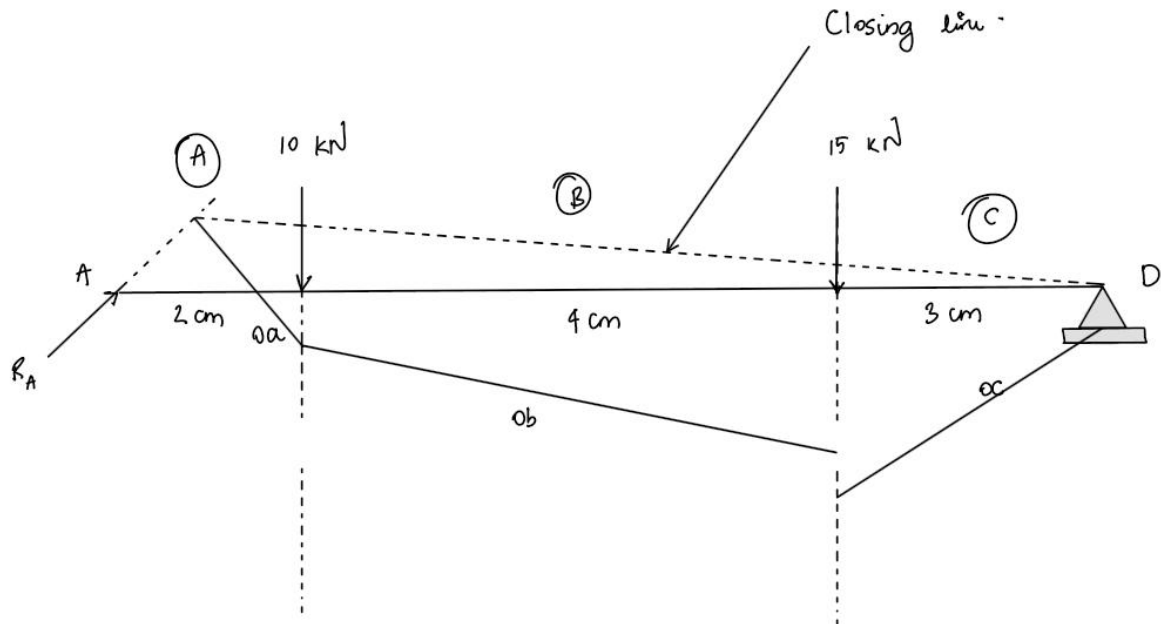
$$R_D = 45 \text{ kN} \uparrow$$



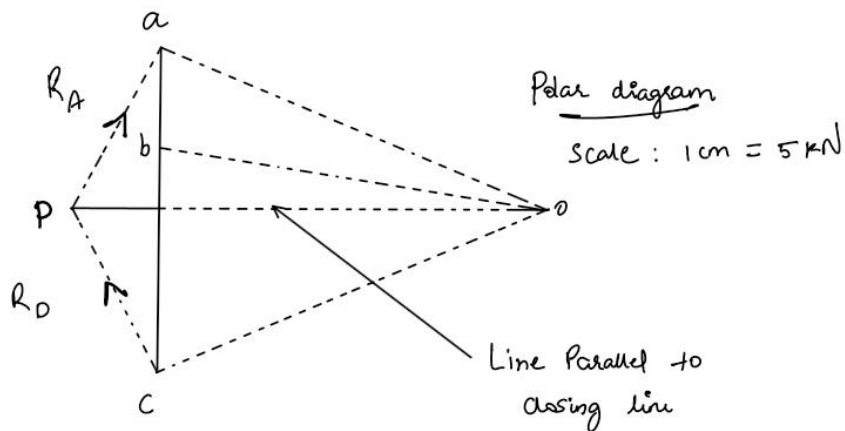
Polar diagram

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Q3. Determine support reactions for the beam loaded and supported as shown in the figure.



Space diagram
Scale 1 cm = 0.5



Polar diagram

Scale: 1 cm = 5 kN

$$R_A = 3 \times 5$$

$$\therefore R_A = 15 \text{ kN}, 60^\circ \swarrow$$

$$R_D = 2.9 \times 5$$

$$\therefore R_D = 14.5 \text{ kN}, 57.7^\circ \searrow$$

6 Results

Question Number	Analytical Solution	Graphical Solution	Percent Error
1	$R = 25 \text{ kN}$ $x = 4.6 \text{ m}$	$R = 25 \text{ kN}$ $x = 4.55 \text{ m}^\circ$	$\eta_R = 0\%$ $\eta_x = 0.016\%$
2	$R_A = 45 \text{ kN}$ $R_B = 15 \text{ kN}$	$R_A = 45 \text{ kN}$ $R_B = 15 \text{ kN}$	$\eta_{R_A} = 0\%$ $\eta_{R_B} = 0\%$
3	$R_A = 14.75 \text{ kN}$ $\theta = 58.7^\circ$	$R_A = 15 \text{ kN}$ $\theta = 57.7^\circ$	$\eta_{R_A} = 0.01\%$ $\eta_\theta = 0.01\%$

7 Conclusion

A set of problems involving Non-concurrent Co-planar Force system were solved using graphical as well as analytical methods. The Percentage error between the two answers was found.