Tutorials on Particle in a Rigid Box

Consider nucleus as a rigid box having width $\sim 10^{-14} m$, calculate the ground state energy of electron if it existed inside the nucleus and hence prove that electron cannot exist inside the nucleus

Solution:

We have

$$E_n = \frac{n^2 h^2}{8mL^2}$$

Considering ground state $(n = 1)$ and taking $L = 10^{-14}\, m$, we have

$$E_1 = \frac{1^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-14})^2}\, J \times \frac{1}{1.6 \times 10^{-19}} \frac{eV}{J} \times \frac{1}{10^6} \frac{MeV}{eV}$$

$$\Rightarrow E_1 = 3774\, MeV \gg 8.8\, MeV$$

As the maximum binding energy of the nucleus is $8.8\,MeV$, the ground state energy of the electron if it existed inside the nucleus is too large to trap it there. Indeed the reality of the absence of the electron inside the nucleus is consistent with almost all principles of quantum mechanics and therefore it can be considered to be a test of validity of quantum ideas

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Calculate the quantum number associated with an electron moving in a rigid box of width $1\,nm$ with a speed $10^6\,\frac{m}{s}$ and a cricket ball of mass 163 g moving on a ground having size $138\,m$ with speed $160\,\frac{km}{h}$. Interpret the results.

Solution:

We have

$$\frac{1}{2}mv^2 = \frac{n^2h^2}{8mL^2}$$

$$\Rightarrow n = \sqrt{\frac{8mL^2 \times mv^2}{2 \times h^2}}$$

$$n = \frac{2mLv}{h}$$

For electron

$$n = \frac{2 \times 9.1 \times 10^{-31} \times 1 \times 10^{-9} \times 10^{6}}{6.63 \times 10^{-34}} = 2.75 \approx 3$$

For cricket ball

$$n = \frac{2 \times 163 \times 10^{-3} \times 160 \times 10^{3} \times \frac{1}{3600} \times 138}{6.63 \times 10^{-34}} \approx 3 \times 10^{36}\ !$$

It can be seen that, the quantum number associated with the electron is low, while that of a cricket ball is extremely high. We recall that when quantum number acquires extremely high value, quantum physics gives the same results as of classical physics. Thus the subatomic particles have lower quantum numbers and they are quantum entities, but the entities in daily life are classical ones.