Tutorial on Heisenberg’s Uncertainty Principle

Use Heisenberg's uncertainty principle to prove that electron cannot exist inside the nucleus (Size of nucleus $\approx 10^{-14}$ m)

Solution

We know that $\Delta x \Delta p = h$. As the size of nucleus is approximately 10^{-14} m, if electron exist inside the nucleus, then

$$\Delta x \sim 10^{-14}\ \text{m}$$

$$\Delta x \Delta p = h$$
$$\Delta p = \frac{h}{\Delta x}$$
$$\Delta p = \frac{6.63\times10^{-34}}{10^{-14}}$$

$$\Delta p = 6.63\times10^{-20}$$
$$m\Delta v = 6.63\times10^{-20}$$

Thus

$$\Delta v = \frac{6.63\times10^{-20}}{9.1\times10^{-31}}$$
$$\Delta v = 7.29\times10^{10} m/s$$

As the physical quantity always greater than its error

$$v \geq 7.29\times10^{10} m/s$$

Thus if electron is allowed to exist inside the nucleus, its speed will exceed the speed of light. This violates the principle of special theory of relativity. Thus electron cannot exist inside the nucleus.

As an another approach

$$\Delta p = 6.63 \times 10^{-20}\,\mathrm{kg.m/s}$$

The smallest value of a physical quantity can be error in itself. Thus

$$p_{\min} = 6.63 \times 10^{-20}\, kgm/s$$

$$E_{\min} = \frac{p_{\min}^2}{2m}$$

$$E_{\min} = \frac{\left(6.63 \times 10^{-20}\right)^2}{2 \times 9.1 \times 10^{-31}}$$

$$E_{\min} = 2.42 \times 10^{-9}\, J$$

$$E_{\min} = \frac{2.42 \times 10^{-9}\, J}{1.6 \times 10^{-19}\, J/eV}$$

$$E_{\min} = 1.5 \times 10^{10}\, eV$$

$$E_{\min} = 15095\, MeV$$

Thus if electron existed inside the nucleus then its minimum energy would have been 15095 MeV, which is far greater than the maximum binding energy of the nucleus (6.8 MeV).

Thus Heisenberg's uncertainty principle is consistent with the fact that electron does not exist inside the nucleus. (Then what about β rays? We know that β rays are nothing but electrons. Do they exist inside the nucleus? No! β rays are emitted away from the nucleus when a neutron is converted into proton).

Non-existance of the electron inside the nucleus and the discovery of neutron

After the discovery of proton, there was a realization that the number of protons alone could not explain the mass of the nucleus. This is because, it was known that the atomic mass is roughly double the atomic number (atomic number is number of electrons in the atom, and consequently the number of protons in side the nucleus). As almost entire mass of the atom is concentrated inside the nucleus, nucleus would require, as many 'proton-like' but electrically neutral partciles as the number of protrons. It was at first thought that such additional partcile could be proton itself, but surrounded by an electron, so that the net charge of the protron-electron pair would be neutral. But quantum mechnics at that time decisively proved that electron can not exist inside the nucleus. Thus instead of protron-electron pair, an independant particle, as heavy as proton, but electrically neutral would solve the puzzle. Rutherford postulated this partcile as neutron, but the experimental confirmation of the neutron, which was an experimetally difficult task, was done by James Chadwick (as student of Rutherford) who discovred the neutron. The use of neutron in nuclear fission gave birth to the nuclear energy. James Chadwick was awarded a Nobel prize in Physics in 1935 for discovering neutron.

An electron is orbiting around the nucleus with a velocity 2×10^5 m/s. The uncertainty in its velocity is 5%. How much is the corresponding Heisenberg's uncertainty in its position? A cricket ball is moving at the velocity of 20 m/s, with an uncertainty of 5%. How much is the corresponding uncertainty in position? The mass of cricket ball is 0.5 kg.

Solution:

For electron

$$\Delta v = \frac{5 \times 2 \times 10^5}{100}$$

$$\Delta v = 10^3 m/s$$

$$\Delta x \Delta p = h$$

$$\Delta x m \Delta v = h$$

$$\Delta x = \frac{h}{m \Delta v}$$

$$\Delta x = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 1000}$$

$$\Delta x = 7.28 \times 10^{-07} m$$

$$\Delta x = 7286 A^o$$

This error is quite considerable looking at the fact that radii of atomic orbits are in a few A°

For Cricket ball $\Delta v = \frac{5 \times 20}{100}$

$$\Delta v = 1 m/s$$

$$\Delta x \Delta p = h$$

$$\Delta x = \frac{h}{m \Delta v}$$

$$\Delta x = \frac{6.63 \times 10^{-34}}{0.5 \times 1}$$

$$\Delta x = 1.326 \times 10^{-33} m$$

As it can be noticed, for the same 5% uncertainty in the velocity of electron and cricket ball, the uncertainty in the position of electron is considerable and uncertainty in the position of cricket ball is negligible. This indicates that Heisenberg's uncertainty principle (and as we shall learn, every principle in quantum mechanics) works only in world of subatomic particles. The uncertainties indicated by this principle are too small to be considerable for the day to day objects. This is due to the typical (an extremely small) value of Planck's constant. What would happen, if Planck's constant possessed a different value? Let us solve a problem.

A cricket ball is moving at 20 m/s with 5% uncertainty. What would be the corresponding uncertainty in the position, if Planck's constant were 6.63 J-S? Assume mass of cricket ball to be 0.5 kg.

$$\Delta v = \frac{5 \times 20}{100} = 1m/s$$

$$\Delta x \Delta p = h$$

$$\Delta x = \frac{h}{m\Delta v}$$

$$\Delta x = \frac{6.63}{0.5 \times 1} = 1326m$$

This uncertainty is too large to be neglected. It would be extremely difficult to catch the cricket ball, if Planck's had such value. This would happen to every object, if Planck's constant had this value. One can notice that nature has cleverly chosen the value of Planck's constant to be 6.63×10^{-34} J.s, to make our life comfortable!

As a conclusion of problems solved in this chapter, we may state that classical mechanics is the science of motion of day to day objects such as cricket ball, projectiles, airplanes, satellites, planets etc., while quantum mechanics is the science of motion of subatomic particles such as electrons, protons, neutrons, atoms and molecules. And it is the Planck's constant, which differentiates classical mechanics from quantum mechanics. The reason that quantum mechanics and classical mechanics work in their own realms is because the value of Planck's constant is 6.63×10^{-34}. Had it been different, probably our day to day motions would also be determined by quantum laws