beams

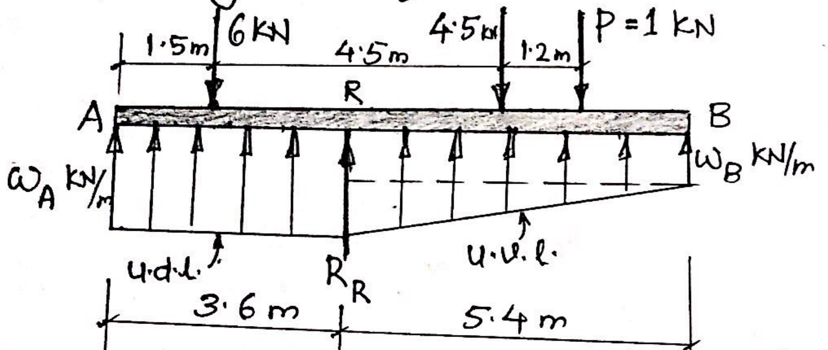
Cables

Frames

(I.)

Beams

Ex. No. (1) A grade beam AB supports three concentrated loads and rests on soil and the top of a large rock. The soil exerts an upward distributed load, and the rock exerts a concentrated load R_R as shown. Knowing that $P = 1$ kN and $\omega_B = (\frac{1}{2} \times \omega_A)$, determine the values of ω_A and R_R corresponding to equilibrium.



Solution: Applying equations of equilibrium to the F.B.D. of the beam,

$\Sigma F_y = 0$ gives,

$$(\omega_A \times 3.6) + R_R + \frac{1}{2}(\omega_A + \omega_B)(5.4) - 6 - 4.5 - 1 = 0$$

But, $\omega_B = (0.5)\omega_A$

$$\therefore \quad (7.65)\,\omega_A + R_R = 11.5 \text{ KN} \longrightarrow (1)$$

$\Sigma M_R = 0$ gives,

$$(\omega_B \times 5.4)(2.7) + \frac{1}{2}(5.4)(\omega_A - \omega_B) \times \left(\frac{5.4}{3}\right) - (\omega_A \times 3.6)(1.8)$$

$$-(1 \times 3.6) - (4.5 \times 2.4) + (6 \times 2.1) = 0$$

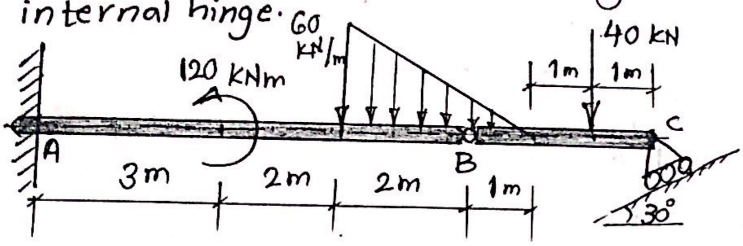
$$(3.24)\,\omega_A - (1.8) = 0 \longrightarrow (2)$$

Ans:

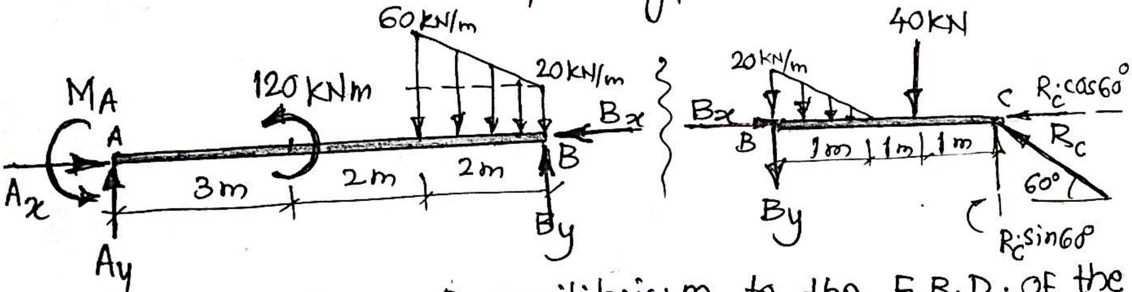
$$\therefore \quad \omega_A = 0.556 \text{ KN/m } (\uparrow)$$

$$\therefore \quad R_R = 7.25 \text{ KN } (\uparrow)$$

Ex. No. 3 Determine the reactions at the supports of the compound beam for the given loading. B is an internal hinge.



Solution: In the compound beam shown in figure, B is the internal hinge, connecting the two beams AB and BC. Consider the F.B.D. of the beams AB and BC separately.



Applying equations of equilibrium to the F.B.D. of the beam BC;

$\Sigma F_x = 0$ gives, $B_x - R_c \cos 60° = 0 \longrightarrow (1)$

$\therefore \quad B_x = (0.5) R_c$

$\Sigma F_y = 0$ gives, $R_c \sin 60° - B_y - (\frac{1}{2} \times 1 \times 20) - 40 = 0$

$\therefore \quad (0.866) R_c - B_y = 50 \longrightarrow (2)$

$\Sigma M_B = 0$ gives, $(R_c \sin 60° \times 3) - (40 \times 2) - (\frac{1}{2} \times 1 \times 20)(\frac{1}{3}) = 0 \longrightarrow (3)$

$\therefore \quad R_c = 32.075$ KN (60°)

$\therefore \quad B_x = 16.0375$ KN, $B_y = -22.223$ KN

Similarly applying equations of equilibrium to the F.B.D. of the beam AB;

$\Sigma F_x = 0$ gives, $A_x - B_x = 0 \longrightarrow (4)$ $\therefore A_x = 16.0375$ KN ($\rightarrow$)

$\Sigma F_y = 0$ gives, $A_y - 22.223 - \frac{1}{2}(60+20)(2) = 0 \longrightarrow (5)$ $\therefore A_y = 102.223$ KN ($\uparrow$)

$\Sigma M_A = 0$ gives, $M_A + 120 - (22.223 \times 7) - (20 \times 2)(6) - (\frac{1}{2} \times 2 \times 40)(5.67) = 0 \longrightarrow (6)$

$\therefore \quad M_A = 502.361$ KNm

P.T.O.

<u>Ans</u> : The reaction at the fixed support consists of;

$$A_x = 16.0375 \text{ kN} (\rightarrow)$$

$$A_y = 102.223 \text{ kN} (\uparrow)$$

and the fixed end moment, $M_A = 502.361 \text{ kNm}$

The reaction at the roller support C,

$$R_c = 32.075 \text{ kN}$$

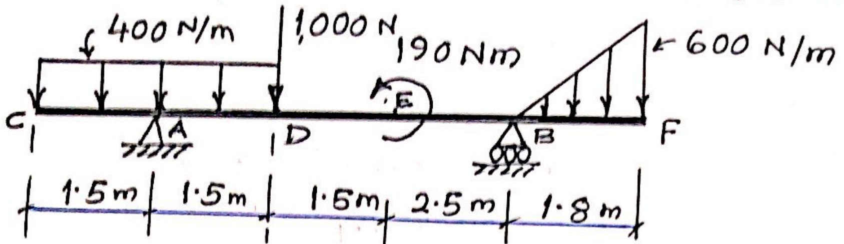
60°

Shear force at the internal hinge B (<u>or</u> force transmitted at B),

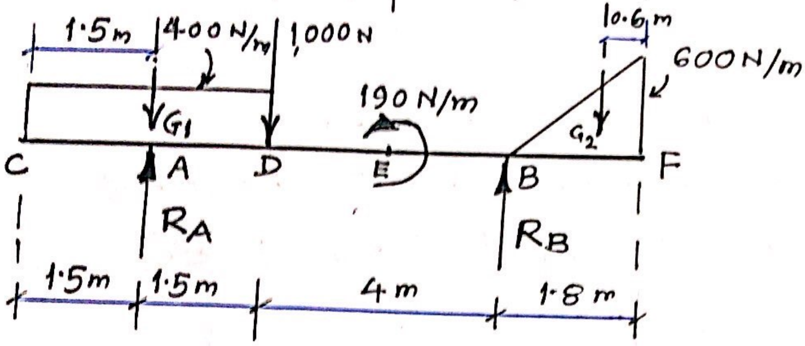
$$F_B = \sqrt{B_x^2 + B_y^2} = \sqrt{(16.0375)^2 + (22.223)^2}$$

$$= 27.4055 \text{ kN}$$

Ex. No. 6 Determine the reactions at supports A and B, for the beam shown in the fig.



Soln.:



G_1 and G_2 are the centroids of load diagrams representing u.d.l. and u.v.l. respectively.

Applying eqns of equilibrium, to the F.B.D. of the beam,

$\Sigma F_y = 0$ gives,

$$R_A + R_B - (400 \times 3) - (1{,}000) - (\tfrac{1}{2} \times 1{\cdot}8 \times 600) = 0$$

$$\therefore \quad R_A + R_B = 2740 \text{ N} \longrightarrow (1)$$

$\Sigma M_c = 0$ gives,

$$(1{\cdot}5)R_A + (7)R_B + (190) - (1{,}000 \times 3) - (1{,}200 \times 1{\cdot}5) - (540 \times 8{\cdot}2) = 0 \longrightarrow (2)$$

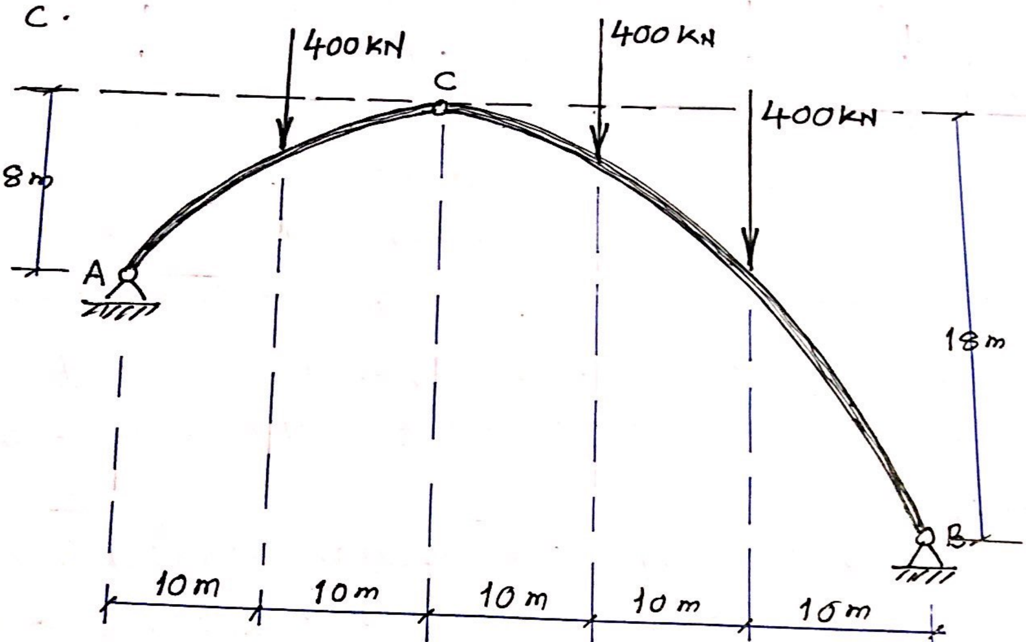
$$\therefore \quad (1{\cdot}5)R_A + 7R_B = 9{,}038$$

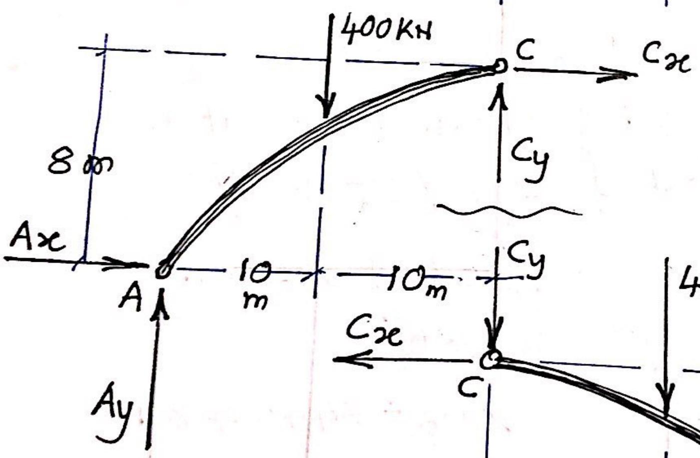
Solving eqns (1) and (2), we get,

Ans: $R_A = 1844 \text{ N} (\uparrow)$

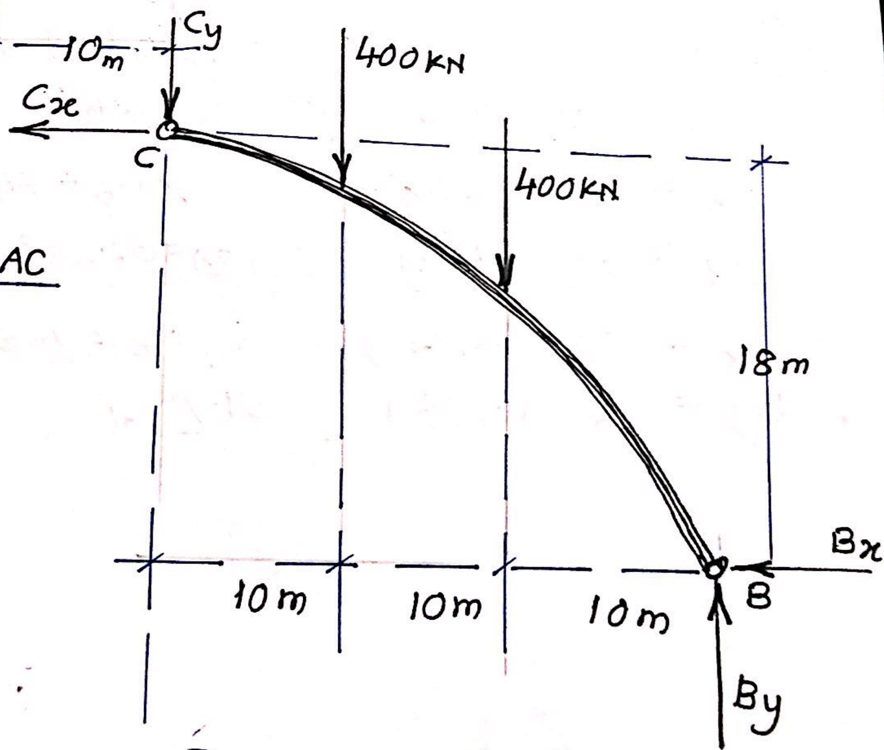
$R_B = 896 \text{ N} (\uparrow)$

Ex. No. 7 For a three-hinged parabolic arch, determine the support reactions and the shear force at C.





F.B.D. OF part AC



F.B.D. OF part CB

Applying eq^{ns} of equilibrium to the F.B.D. of AC,

$\Sigma F_x = 0$ gives, $\quad A_x + C_x = 0 \longrightarrow (1)$

$\Sigma F_y = 0$ gives, $\quad A_y + C_y - (400) = 0 \longrightarrow (2)$

$\Sigma M_A = 0$ gives,

$$-8 \cdot C_x + 20 \cdot C_y - (400 \times 10) = 0$$

$$\therefore \quad -8 \cdot C_x + 20 \cdot C_y = 4{,}000 \longrightarrow (3)$$

Applying eq^{ns} of equilibrium to the F.B.D. of CB,

$\Sigma F_x = 0$ gives, $\quad -B_x - C_x = 0 \longrightarrow (4)$

$\Sigma F_y = 0$ gives, $\quad B_y - C_y - 400 - 400 = 0 \longrightarrow (5)$

$\Sigma M_B = 0$ gives,

$$18 \cdot C_x + 30 \cdot C_y + (400 \times 20) + (400 \times 10) = 0$$

$$\therefore \quad 18 \cdot C_x + 30 \cdot C_y = -(12{,}000) \longrightarrow (6)$$

Solving eq^{ns} (3) and (6) we get,

$$\left.\begin{aligned} C_x &= -600 \text{ kN} \\ C_y &= -40 \text{ kN} \end{aligned}\right\}$$

shear force at C,

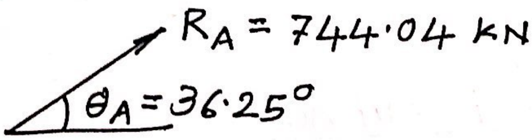
$$F_c = \sqrt{C_x^2 + C_y^2}$$

$$\therefore \quad F_c = 601 \cdot 332 \text{ kN}$$

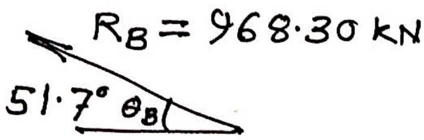
Ans:

From the other eq^{ns} we get,

$$\left.\begin{aligned} A_x &= 600 \text{ kN} (\rightarrow) \\ A_y &= 440 \text{ kN} (\uparrow) \end{aligned}\right\}$$

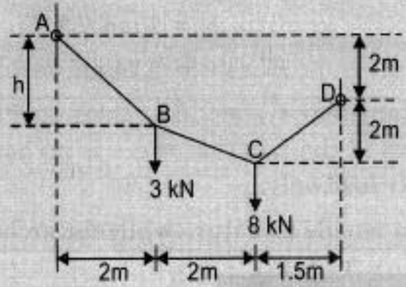


$$\left.\begin{aligned} B_x &= 600 \text{ kN} (\leftarrow) \\ B_y &= 760 \text{ kN} (\uparrow) \end{aligned}\right\}$$



Example 11

Determine the tension in each segment of the cable shown in figure. Also find the dimension of 'h'.?



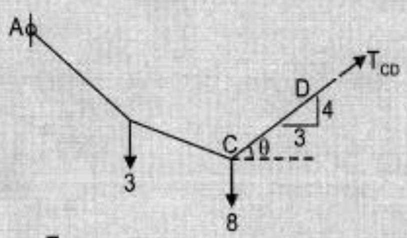
Sol. From the fig. we can see that there are 4-external unknown reaction (R_A, H_A, R_D, H_D), three unknown cable tension and are 'h'.

Equilibrium equations:

$$\Sigma F_x = 0 \; ; \quad \Sigma F_y = 0 \quad \text{at A, B, C and D}$$

$$\Sigma M = 0 \text{ at A and D.}$$

Taking $\Sigma M_A = 0$

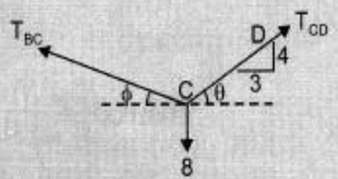


$$T_{CD} \cos\theta \times (2) + T_{CD} \sin\theta \times 5.5 - 3 \times 2 - 8 \times 4 = 0$$

$$T_{CD} \times \left(\frac{3}{5}\right)(2) + T_{CD} \times \left(\frac{4}{5}\right) \times (5.5) - 6 - 32 = 0$$

$$T_{CD} = 6.79 \text{ kN}$$

At joint 'C'



$\Sigma F_x = 0 \Rightarrow$

$$T_{CD} \cos\theta = T_{BC} \cos\phi$$

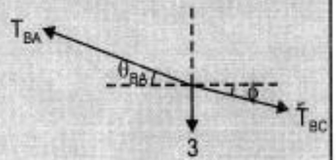
$$T_{CD} \times \frac{3}{5} = T_{BC} \times \cos\phi \qquad \text{... (1)}$$

$\Sigma F_y = 0 \Rightarrow$

$$T_{CD} \sin\theta + T_{BC} \sin\phi = 8$$

$$T_{CD} \times \frac{4}{5} + T_{BC} \sin\phi = 8 \qquad \text{... (2)}$$

After solving (1) and (2)



$$\phi = 32.3°$$

$$T_{BC} = 4.82 \text{ kN}$$

At joint 'B'

$\Sigma Fx = 0 \Rightarrow$

$$T_{BA} \cos\theta_{BA} = T_{BC} \cos\phi$$

$$T_{BA} \cos\theta_{BA} = 4.82 \times \cos(32.3°) \qquad \text{... (3)}$$

$\Sigma F_y = 0 \Rightarrow$

$$T_{BA} \sin\theta_{BA} = 3 + T_{BC} \sin\phi$$

$$T_{BA} \sin \theta_{BA} = 3 + 4.82 \times \sin (32.3) \quad \text{... (4)}$$

After solving (3) and (4)

$$\theta_{BA} = 53.8°$$

$$T_{BA} = 6.90 \text{ kN}$$

Hence,

$$h = 2 \tan (53.8°) \quad \text{From figure}$$

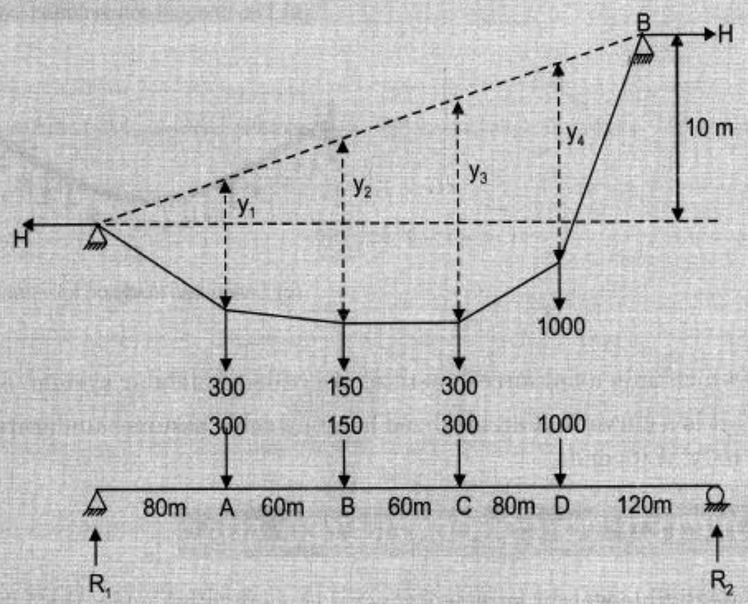
$$= 2.74 \text{ m}$$

* Main difference between funicular system like cables, arches with reference to equivalent simply supported beam lies in the fact that there is no horizontal reaction component in equivalent beam for gravity load only.
* Internal forces in cables is tensile in nature while for arches, it is compressive in nature.

Example 10

A cable of negligible weight is suspended between two points spaced 400 m apart horizontally with the right support being 10 m higher than the left support. Four verticle loads of magnitude 300, 150, 300 and 1000 kN are applied at points A, B, C, D which are 80, 140, 200, 280 m horizontally respectively from left support. The largest sag of the cable will be at which point?

Sol. As per general cable theorem.



$$R_1 = \frac{300 \times 320 + 150 \times 260 + 300 \times 200 + 1000 \times 120}{400}$$

$$= 787.5 \text{ kN}$$

$$R_2 = 962.5 \text{ kN}$$

Using general cable theorem

$$H.y_1 = 787.5 \times 80 = 63000$$

$$H.y_2 = 787.5 \times 140 - 300 \times 60 = 92250$$

$$H.y_3 = 787.5 \times 200 - 300 \times 120 - 150 \times 60$$

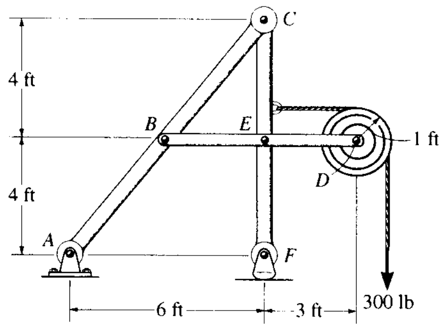
$$= 112500$$

$$H.y_4 = 787.5 \times 280 - 300 \times 200 - 150 \times 140 - 300 \times 80$$

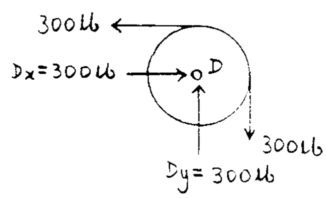
$$= 115500$$

Since 'H' is constant, max. sag will occur at 'D'

6-78. Determine the horizontal and vertical components of force at C which member ABC exerts on member CEF.

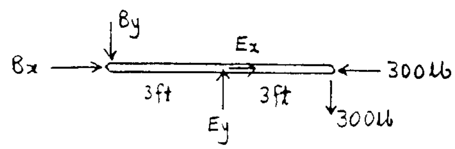


Member BED:



$\circlearrowleft + \Sigma M_B = 0; \quad -300(6) + E_y(3) = 0$

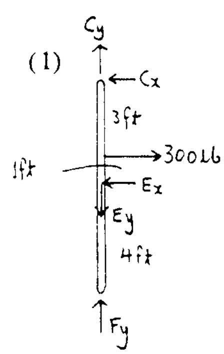
$E_y = 600 \text{ lb}$

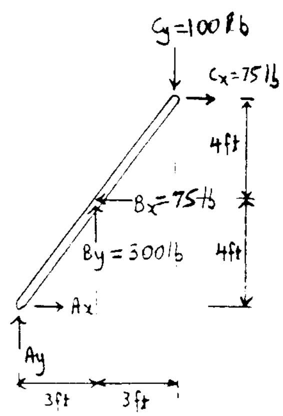


$+\uparrow \Sigma F_y = 0; \quad -B_y + 600 - 300 = 0$

$B_y = 300 \text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad B_x + E_x - 300 = 0 \quad (1)$





Member FEC:

$\circlearrowleft + \Sigma M_C = 0; \quad 300(3) - E_x(4) = 0$

$E_x = 225 \text{ lb}$

From Eq. (1) $\quad B_x = 75 \text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -C_x + 300 - 225 = 0$

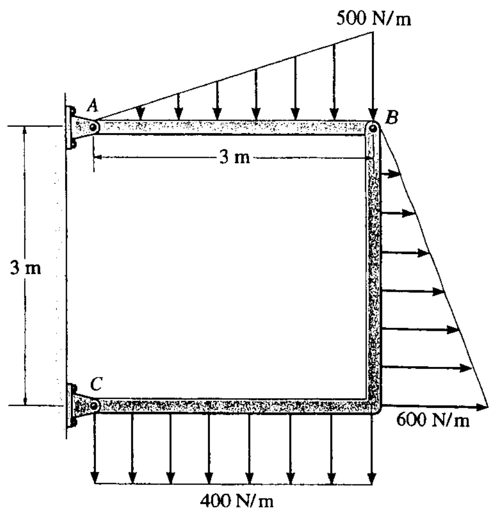
$C_x = 75 \text{ lb}$ **Ans**

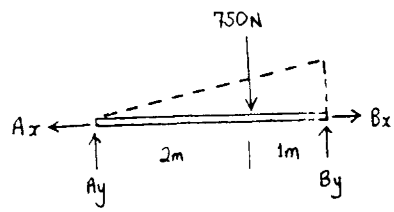
Member ABC:

$\circlearrowleft + \Sigma M_A = 0; \quad -75(8) - C_y(6) + 75(4) + 300(3) = 0$

$C_y = 100 \text{ lb}$ **Ans**

6-130. Determine the horizontal and vertical components of force at pins A and C of the two-member frame.



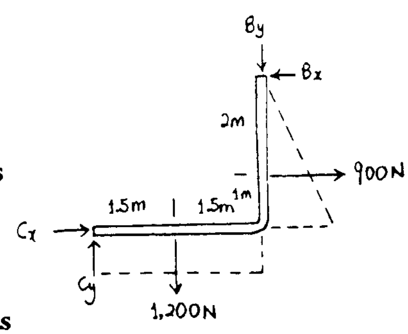


$\circlearrowleft + \Sigma M_A = 0;\qquad -750(2) + B_y(3) = 0$

$B_y = 500\text{ N}$

$\circlearrowleft + \Sigma M_C = 0;\qquad -1200(1.5) - 900(1) + B_x(3) - 500(3) = 0$

$B_x = 1400\text{ N}$



$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad -A_x + 1400 = 0$

$A_x = 1400\text{ N} = 1.40\text{ kN}$ **Ans**

$+\uparrow \Sigma F_y = 0;\qquad A_y - 750 + 500 = 0$

$A_y = 250\text{ N}$ **Ans**

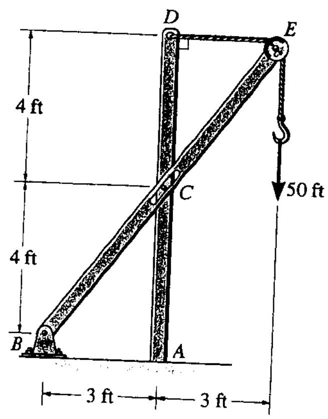
$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad C_x + 900 - 1400 = 0$

$C_x = 500\text{ N}$ **Ans**

$+\uparrow \Sigma F_y = 0;\qquad -500 - 1200 + C_y = 0$

$C_y = 1700\text{ N} = 1.70\text{ kN}$ **Ans**

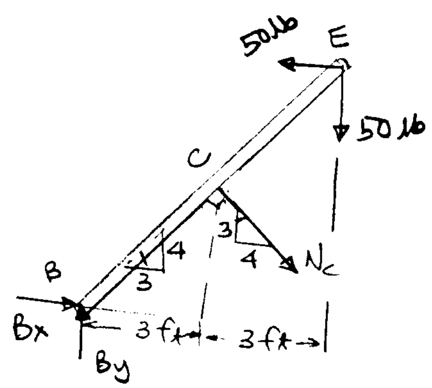
6-106. Determine the horizontal and vertical components of force at pin B and the normal force the pin at C exerts on the smooth slot. Also, determine the moment and horizontal and vertical reactions of force at A. There is a pulley at E.



BCE:

$\circlearrowleft + \Sigma M_B = 0; \quad -50(6) - N_C(5) + 50(8) = 0$

$N_C = 20 \text{ lb}$ **Ans**



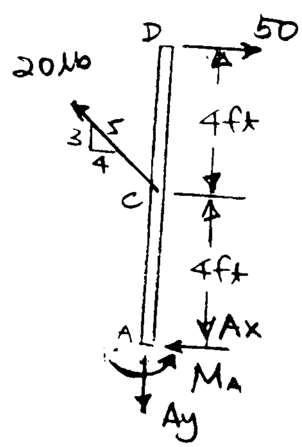
$\xrightarrow{+} \Sigma F_x = 0; \quad B_x + 20(\frac{4}{5}) - 50 = 0$

$B_x = 34 \text{ lb}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad B_y - 20(\frac{3}{5}) - 50 = 0$

$B_y = 62 \text{ lb}$ **Ans**

ACD:



$\xrightarrow{+} \Sigma F_x = 0; \quad -A_x - 20(\frac{4}{5}) + 50 = 0$

$A_x = 34 \text{ lb}$ **Ans**

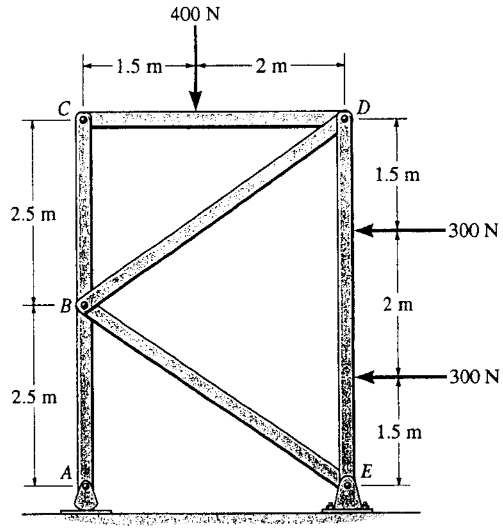
$+\uparrow \Sigma F_y = 0; \quad -A_y + 20(\frac{3}{5}) = 0$

$A_y = 12 \text{ lb}$ **Ans**

$\circlearrowleft + \Sigma M_A = 0; \quad M_A + 20(\frac{4}{5})(4) - 50(8) = 0$

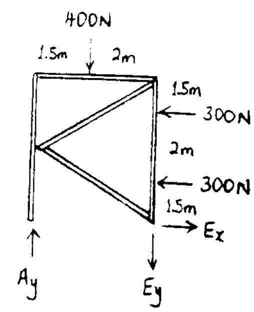
$M_A = 336 \text{ lb}\cdot\text{ft}$ **Ans**

6-91. Determine the horizontal and vertical components of force which the pins at A, B, and C exert on member ABC of the frame.



$\circlearrowleft + \Sigma M_E = 0;$ $\quad -A_y(3.5) + 400(2) + 300(3.5) + 300(1.5) = 0$

$A_y = 657.1 = 657$ N **Ans**



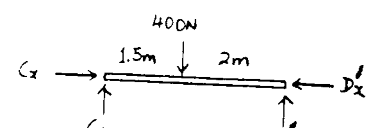
$\circlearrowleft + \Sigma M_D = 0;$ $\quad -C_y(3.5) + 400(2) = 0$

$C_y = 228.6 = 229$ N **Ans**

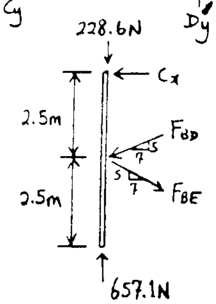
$\circlearrowleft + \Sigma M_B = 0;$ $\quad C_x = 0$ **Ans**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad F_{BD} = F_{BE}$

$+\uparrow \Sigma F_y = 0;$ $\quad 657.1 - 228.6 - 2\left(\frac{5}{\sqrt{74}}\right)F_{BD} = 0$



$F_{BD} = F_{BE} = 368.7$ N



$B_x = 0$ **Ans**

$B_y = \frac{5}{\sqrt{74}}(368.7)(2) = 429$ N **Ans**