Module

Curve Tracing:

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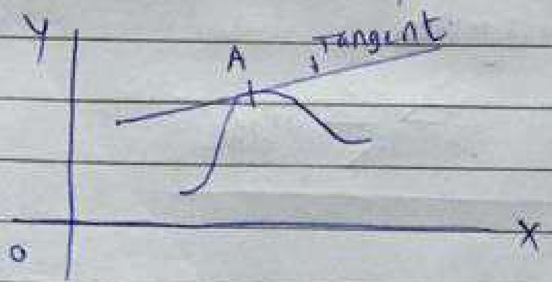
* Basic Def^ns:

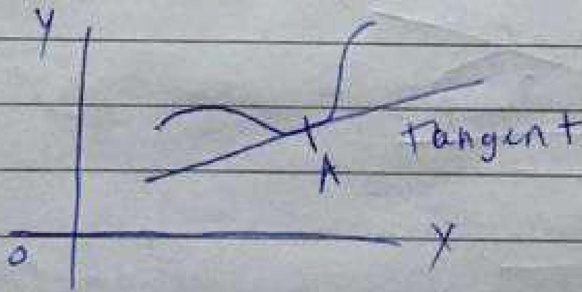
1) Convex upwards:

If the portion of the curve on both sides of 'A' lies below the tangent at A, then the curve is convex upwards.



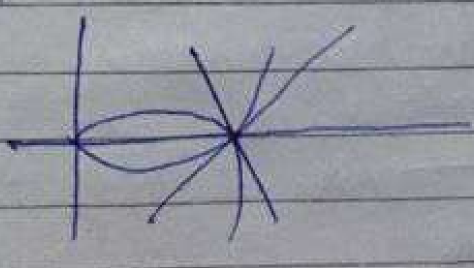
2) Convex Downwards:

If the portion of the curve on both sides of 'A' lies above the tangent at A, then the curve is convex downwards



3) Double point.

A point through which two branches of curve passes.

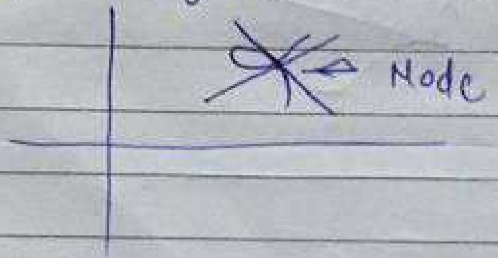


4) Multiple point.

A point through which more than ~~two~~ one branchy pass.

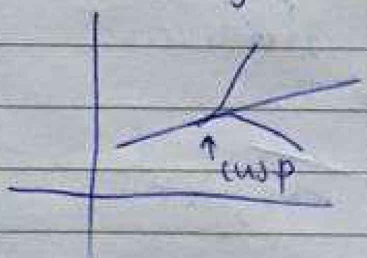
5) Node:

A double point is called node if distinct branches have distinct tangents



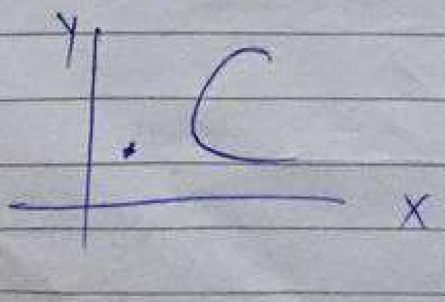
6) Cusp:

A double point is called cusp if two branches have common tangent.



7) Isolated point:

A point P is called isolated or conjugate point if the coordinates of P satisfies the eqn of curve but no branch passes through P.



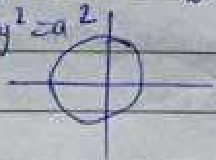
* Rules for tracing the cartesian curves:

Rule 1 Symmetry:

① About X-axis.
If powers of y are even everywhere then curve is symmetric about X-axis.
Ex: $y^2 = 4ax$

② About Y-axis.
If powers of x are even everywhere then curve is symmetric about Y-axis.
Ex: $x^2 = 4ay$

③ Symmetric about both x and y axis:
If powers of both x and y are even then symmetric about both axis
Ex $x^2+y^2=a^2$

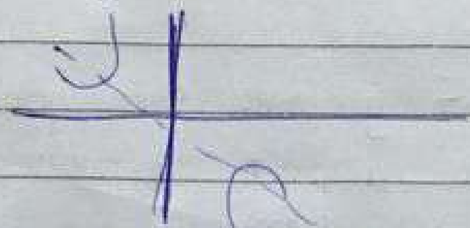


④ Symmetric about origin (opposite quadrant)
If eq^n of curve remains unchanged if x & y are replaced by $-x$ and $-y$.
Ex $xy = c$

⑤ Symmetric about $x=y$.
If on interchanging x & y the eq^n remains unchanged then curve is symmetric about $x=y$.
Ex $xy = c$

⑥ Symmetric about $x=-y$:
If on replacing x by $-y$ and y by $-x$ simultaneously eq^n remains unchanged then curve is symm. about $y=-x$

Ex $xy=-c$



Rule 2 Points of Intersection

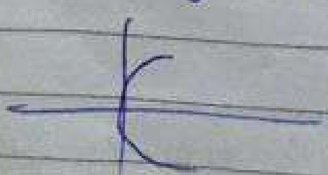
① Origin:
If $f(0,0)=0$ then it will pass through origin.

② Intersection with x-axis.
Put $y=0$ to find $inter^n$ with x-axis

③ Intersection with y-axis:
Put $x=0$ to find $inter^n$ with y-axis

Rule 3 Tangents

① If $(0,0)$ is point on curve then we can find tangent at $(0,0)$
To find tangent at $(0,0)$ equate lowest degree term with zero from the eq^n
Ex $x=y^2$, lowest degree term is x
$\Rightarrow x=0$



i.e. y axis is tgt at $(0,0)$

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② Tangent at any other point p :

find $\frac{dy}{dx}$ at p.

(i) If $\left(\frac{dy}{dx}\right)_p = 0 \Rightarrow$ Tangent at p is parallel to X-axis

(ii) If $\left(\frac{dy}{dx}\right)_p = \infty \Rightarrow$ Tangent at p is ∥ to Y-axis

(iii) If $\left(\frac{dy}{dx}\right) = +ve \Rightarrow$ Tangent at p makes an acute angle with X-axis.

(iv) If $\left(\frac{dy}{dx}\right) = -ve \Rightarrow$ Tangent at P makes an obtuse angle with X-axis

$\boxed{\text{Rule 4 Asymptotes}}$

Tangent to the curve at Infinity

① Asymptote ∥ to Y-axis is obtained by equating coeff of highest degree term in y with zero.

② Asymptote ∥ to X-axis is obtained by equating coeff. of highest degree term in x with zero.

Rule 5 Region of absence:

① For $y = f(x)$, if y becomes imaginary for some values of $x > a$, then curve does not exist beyond $x = a$.

② For $x = f(y)$, if x become imaginary for some values of $y > a$, then curve does not exist beyond $y = a$.

Ex// Trace the curve $y^2(2a-x) = x^3$

$\longrightarrow$ $\Rightarrow$ $y^2 = \dfrac{x^3}{(2a-x)}$

① Eqⁿ of curve contains only even powers of y
$\Rightarrow$ Symmetric about X-axis.

② $f(0,0) = 0$
$\Rightarrow$ It passes through origin.

③ Tangents at the origin are obtained by equating to zero the lowest degree terms in the eqⁿ.

$$y^2(2a - x) = x^3$$
$$\Rightarrow 2ay^2 - xy^2 - x^3 = 0$$
$$\Rightarrow 2ay^2 = 0$$
$$\Rightarrow y = 0$$

$\Rightarrow$ X-axis is tangent at origin.

④ Since two tangents coincides
$\Rightarrow$ The origin is cusp.

(5) For intersection with x-axis put $y=0$

$$\Rightarrow \frac{x^3}{2a-x} = 0$$

$$\Rightarrow x = 0$$

For intersection with y-axis put $x=0$

$\Rightarrow y^2 = 0$

$\Rightarrow y = 0$

Thus the curve meets the co-ordinate axis only at $(0,0)$.

(6) The asymptote ∥ to y-axis can be obtained by eqn to zero the coeff of highest power of y

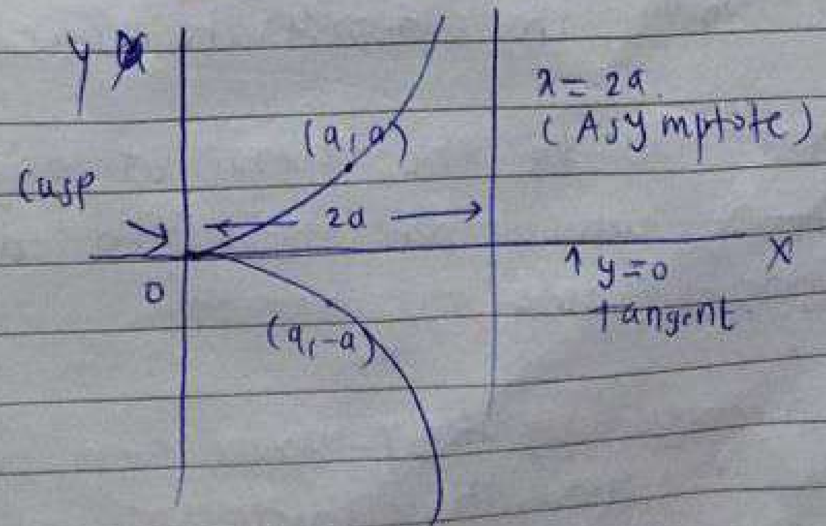
$y^2(2a-x) - x^3 = 0$

$\Rightarrow 2a - x = 0$

$\Rightarrow x = 2a$ is the asymptote ∥ to y axis.

(7) From the eqn of curve and sketch we observe that for $x<0$ and $x>2a$, y^2 becomes -ve

$\Rightarrow$ The curve does not exist for $x<0$ and $x>2a$.



2) Trace the curve $x(x^2+y^2) = a(x^2-y^2)$

→ ① The eqn of curve contains only even powers of y, therefore, it is symmetrical about x-axis

② $f(0,0) = 0$.
⇒ The eq^n passes through origin

③ Tangents at origin are obtained by equating to zero the lowest degree terms in the eq^n

$$ax^2 - ay^2 = 0.$$
$$\Rightarrow y^2 = x^2$$
$$\Rightarrow y = \pm x$$

Hence $y = x$, $y = -x$ are tangents at the origin

④ For intersection with x-axis put $y=0$
$$\Rightarrow x^3 = ax^2$$
$$\Rightarrow x = a$$

For intersection with y-axis put $x=0$
$$\Rightarrow y = 0.$$

Thus curve meets the co-ordinate axes at $(0,0)$, $(a,0)$

⑤ Since tangents to the curve at origin are $y=x$, $y=-x$ which are real and different,
⇒ origin is node.

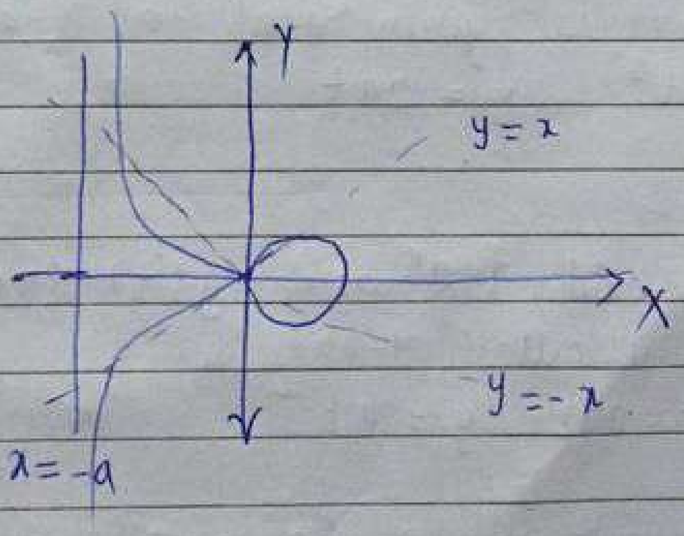
⑥ The asymptotes ∥ to y-axis can be obtained by equating to zero the coefficient of highest powers of y i.e. $a+x=0$

→ $x=-a$ is the asymptote ∥ to y-axis.

(7) For $x < -a$ and $x > a$, y becomes imaginary
∴ curve does not exist for $x < -a$ and $x > a$

(8) Since the curve passes through origin and no branch of curve exists to the right of $x = a$
∴ ∃ exists a loop betn $(0,0)$ and $(a,0)$

$$x^3 + xy^2 = ax^2 - ay^2$$
$$y^2(a+x) = ax^2 - x^3$$
$$y^2 = \frac{ax^2 - x^3}{a+x}$$
$$y^2 = x\sqrt{\frac{(a-x)}{(a+x)}}$$



HW $xy^2 = a^2(a-x)$