Tutorial 10

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Q.1 $\int_0^\infty \frac{dx}{3^{4x^2}} = \int_0^\infty 3^{4x^2} \cdot dx$

take $3^{-4x^2} = e^{-t}$

$\therefore 4x^2 \log 3 = t$

$\therefore x^2 = \frac{t}{4\log 3} \Rightarrow x = \frac{\sqrt{t}}{2\sqrt{\log 3}}$

$dx = \frac{1}{2\sqrt{t}} \times \frac{1}{2\sqrt{\log 3}}$

$= \int_0^\infty e^{-t} \cdot \frac{t^{-1/2} \cdot dx}{4\sqrt{\log 3}} = \frac{1}{4\sqrt{\log 3}} \int_0^\infty e^{t} \cdot t^{1/2 - 1} \cdot dx$

$= \frac{\sqrt{1/2}}{4\sqrt{\log 3}} = \frac{\sqrt{\pi}}{4\sqrt{\log 3}}$ //

Q.2 $\int_0^\infty \frac{x^2}{2^x} \cdot dx$

Put $2^x = e^t$

$x \log 2 = t$

$dx = \frac{dt}{\log 2}$

$$\int_0^\infty \frac{t^2}{\log 2} \cdot e^{-t} \frac{dt}{\log 2} = \frac{1}{(\log 2)^3} \int_0^\infty t^2 e^{-t} \cdot dt$$

$$= \frac{1}{(\log 2)^3} \int_0^\infty e^{-t} \cdot t^{3-1} \cdot dt$$

$$= \frac{\Gamma 3}{(\log 2)^3} = \frac{2}{(\log 2)^3}$$

Q.3 $\displaystyle\int_0^\infty \frac{x^4}{4^x} \cdot dx.$

Put $4^x = e^t$

$x \log 4 = t \quad \Rightarrow x = \dfrac{t}{\log 4}$

$\Rightarrow dx = \dfrac{dt}{\log 4}$

$$= \int_0^4 \frac{t^4}{(\log 4)^4} e^{-t} \cdot \frac{dt}{\log 4}$$

$$= \frac{1}{(\log 4)^5} \int_0^\infty e^{-t} \cdot t^{5-1} dt$$

$$= \frac{\Gamma 5}{(\log 4)^3} = \frac{4 6}{(\log 4)^5} = \frac{24}{(\log 4)^5}$$

(4) $\int_0^\infty 7^{-4x^2} dx$

Put $7^{-4x^2} = e^{-t}$

$$4x^2 \log 7 = t$$

$$x = \sqrt{\frac{t}{4\log 7}}$$

$$dx = \frac{1}{2\sqrt{t}} \times \frac{1}{2\sqrt{\log 7}}$$

$$= \int_0^\infty e^{-t} \cdot t^{1/2} \frac{dt}{4\sqrt{\log 7}}$$

$$= \frac{1}{4\sqrt{\log 7}} \int_0^\infty e^{-t} \cdot t^{1/2-1} dt$$

$$= \frac{\Gamma 1/2}{4\sqrt{\log 7}} = \boxed{\frac{\sqrt{\pi}}{4\sqrt{\log 7}}}$$