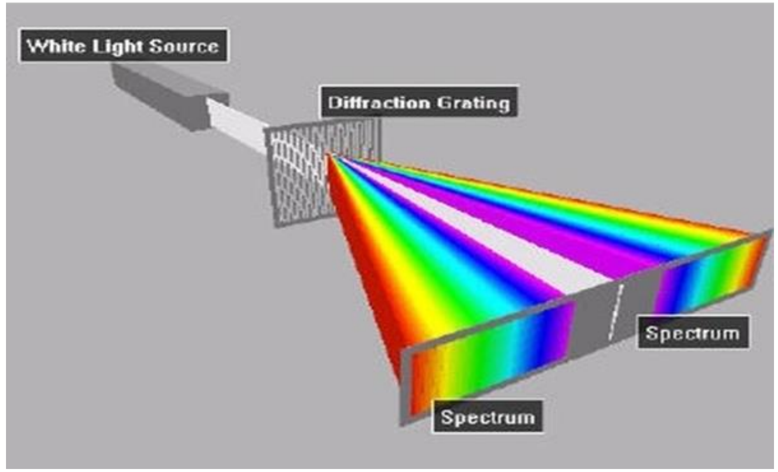
Diffraction Grating



Diffraction: Lecture #3
(Tutorial)

Single Slit Diffraction

minima — $a\sin\theta = m\lambda$

Secondary maxima — $a\sin\theta = \left(m+\frac{1}{2}\right)\lambda$

a = Slit width

θ = diffraction angle

λ = wavelength of light

m = Order of Spectral line

Diffraction Grating

Principle maxima — $d\sin\theta = m\lambda$

or, $(a+b)\sin\theta = m\lambda$

$d = (a+b)$

a = slit width

separated by

b = Opaque Space

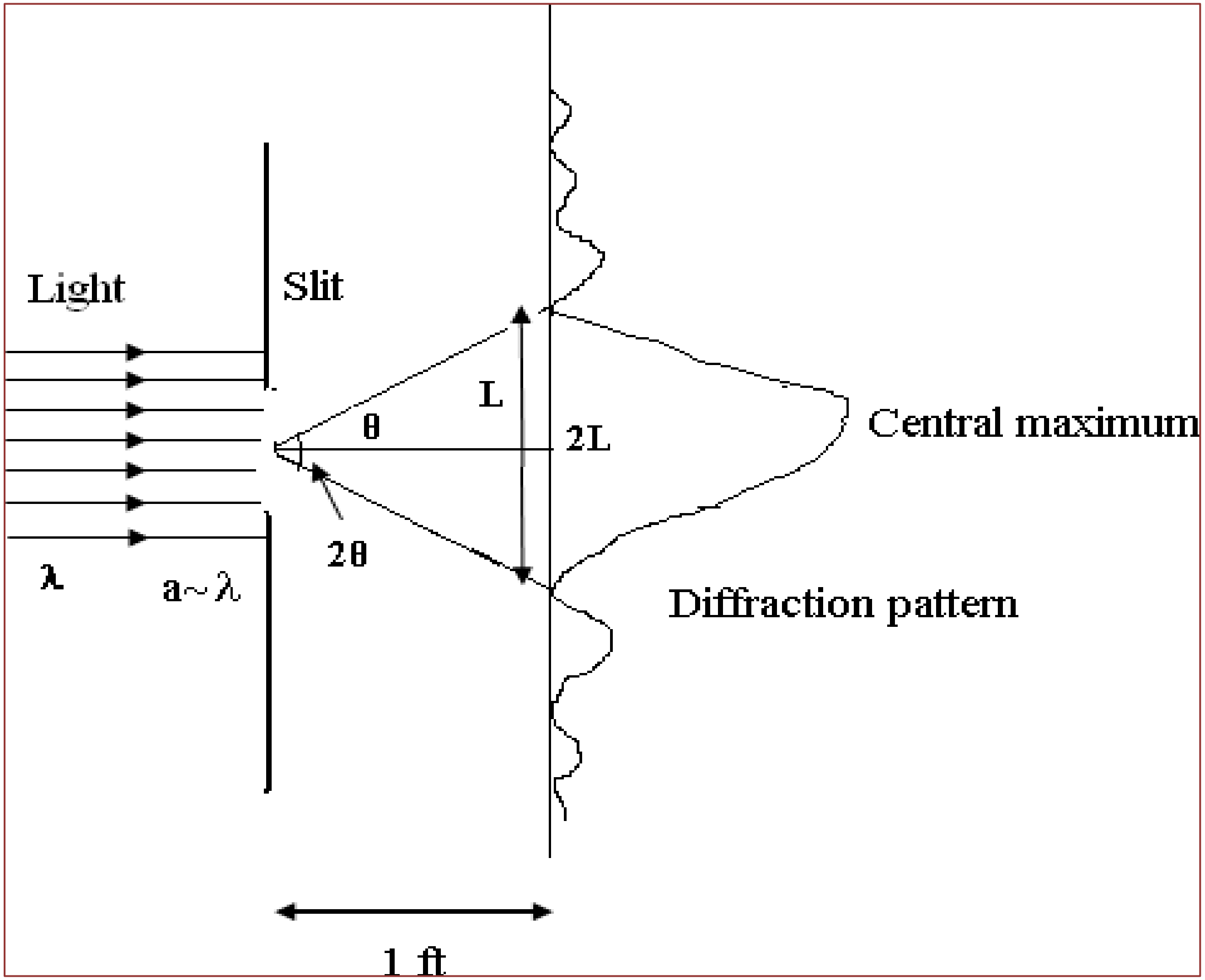
N = number of slits

$(a+b) = \frac{1}{N}$ N lines per inch

$= \frac{2.54}{N}$ N lines per cm

$= \frac{2.54\times10^{8}}{N}$ Å

Example 1: A slit having width 1.6933 μm is illuminated by sodium light having average wavelength 5893 A°. Calculate the angular width of the central maximum. Also calculate the linear width of the central maximum if the screen is placed at a distance of 1ft from the slit



Characteristics (I) of Diffraction: Diffraction results in to widening of images (or even shadows!)

Solution: The figure (3.5) shows that the width of central maximum is *2θ*, where *θ* is the position of the first minimum. We have

$$a sin\theta \ = m\lambda$$

$$\theta = sin^{-1}\left(\frac{m\lambda}{a}\right)$$

$$\theta = sin^{-1}\left(\frac{1 \times 5893 \times 10^{-10}}{1.6933 \times 10^{-6}}\right)$$

$$\theta = sin^{-1}\left(\frac{1 \times 5893 \times 10^{-10}}{1.6933 \times 10^{-6}}\right)$$

$$\theta \ = 20.37^{o}$$

Angular width of the central maximum = $2\theta = 40.74^{o}$

From the Fig (3.5) it can be seen that *2L* is the linear width of the central maximum and

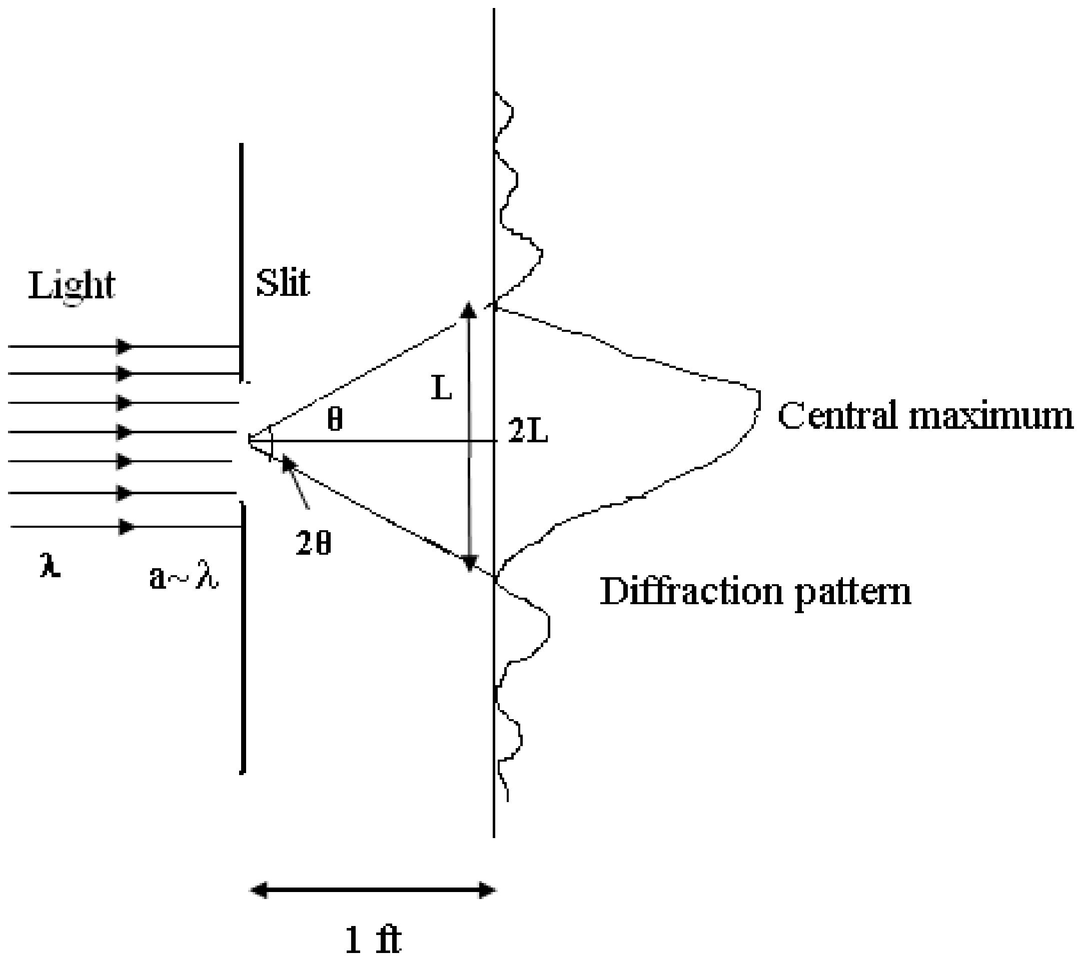
$L = 1ft \times tan\theta$

$L = 1 \times 12 \times 2.54 \times tan 20.37$

$L = 11.32\, cm$

Linear width of the central maximum is = $2L = 22.63\ cm$

Example 2: Calculate the angular width of the central maximum, when slits having widths $a = \lambda$, $a = 2\lambda$ and $a = 20\lambda$ are used



Minima is given by

$$a\sin\theta = m\lambda$$

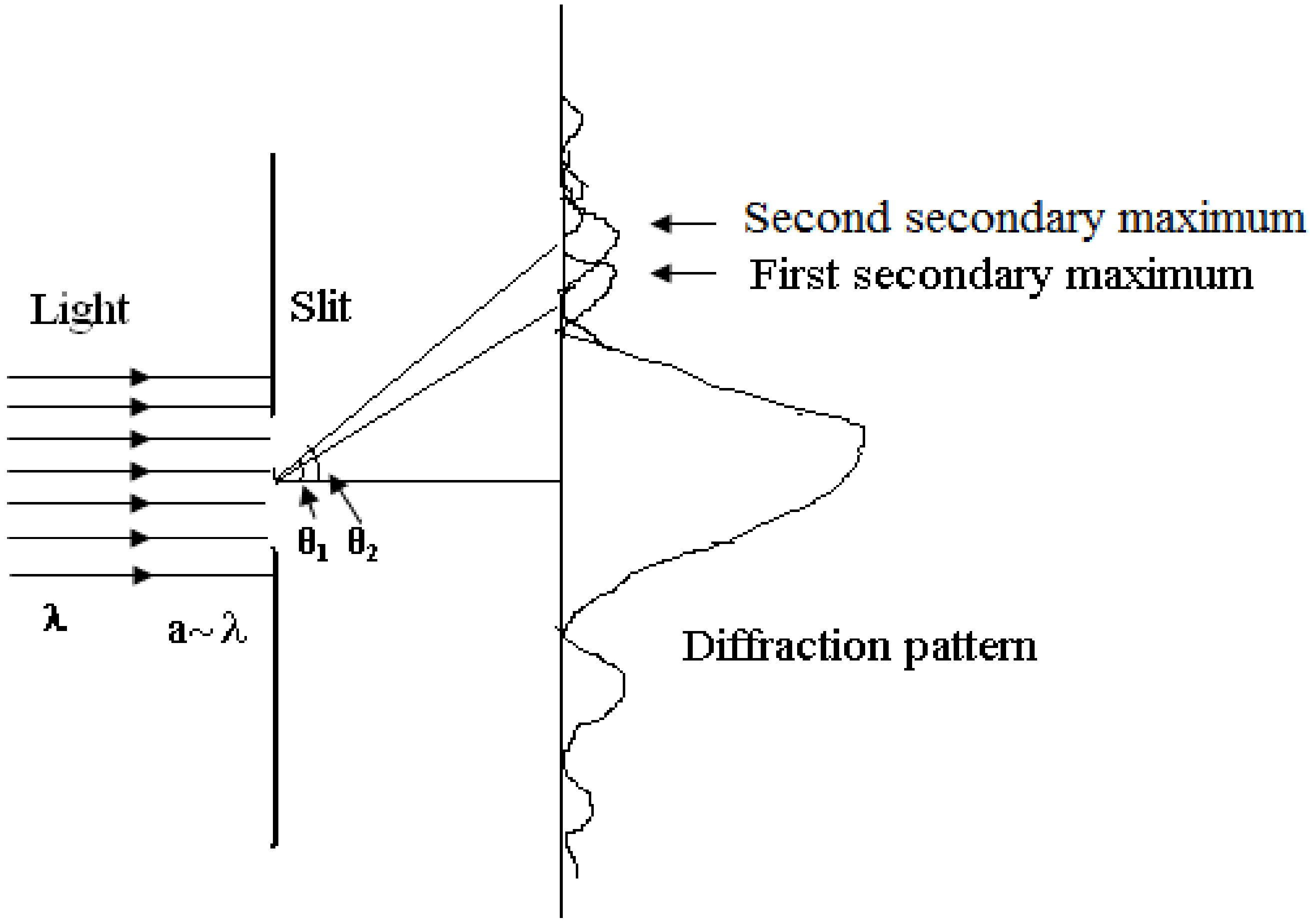
As the central maximum is bounded by first minimum on upper as well as lower side, we take *m =1*.

Then for $a = \lambda$, $a = 2\lambda$ and $a = 20\lambda$ we get $\sin\theta = 1, 0.5\ and\ 0.05$. Correspondingly θ are *90°*, *30°*, *3.86°*.

Characteristics (II) of Diffraction

Diffraction effects become weak as the obstacle become bigger that the wavelength of wave being diffracted. On the contrary, diffraction is strengthened when the width of obstacle approaches the wavelength of wave being diffracted

Example 3: Calculate the angular positions of the first secondary maximum, when a slit of 1.6933 μm is illuminated by sodium source emitting two wavelengths 5890 A^o and 5896 A^o.



Characteristics (III) of Diffraction

The angle of diffraction changes with the wavelength. Thus diffraction separates the colors of light.

The position of the secondary maximum is given by

$$asin\theta = \left(m + \frac{1}{2}\right)\lambda$$

For first secondary maximum, m=1

For 5890 A^o

$$1.6933 \times 10^{-6} sin\theta_1 = \left(1 + \frac{1}{2}\right) \times 5890 \times 10^{-10}$$

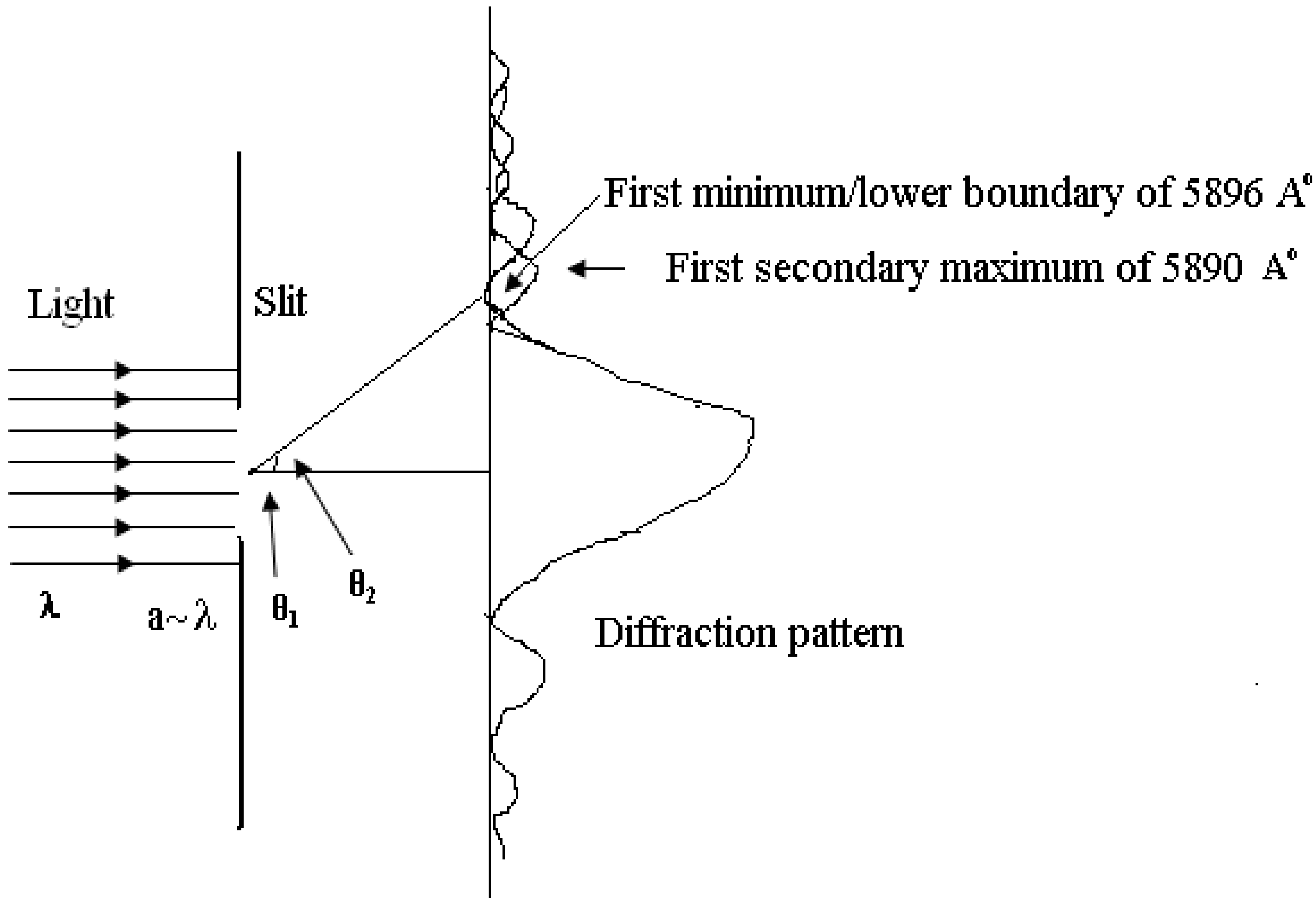
$$\theta_1 = 31.45^o$$

For 5896 A^o

$$1.6933 \times 10^{-6} asin\theta_2 = \left(1 + \frac{1}{2}\right) \times 5896 \times 10^{-10}$$

$$\theta_2 = 31.49^o$$

Example 4: Calculate the angular width of the first secondary maximum, when a slit of 1.6933 μm is illuminated by sodium source emitting two wavelengths 5890 A^{o}and 5896 A^o.



The first secondary maximum is bounded by first minimum and the second minimum.

The minimum is given by

$$a sin\theta = m\lambda$$

$$\theta = sin^{-1}\left(\frac{m}{a}\right)$$

Thus we have for 5890 A^o

$$\theta_1 = \frac{1\times5890\times10^{-10}}{1.6933\times10^{-6}} = 20.26^o$$

$$\theta_2 = \frac{2\times5890\times10^{-10}}{1.6933\times10^{-6}} = 44.08^o$$

Thus the angular width of the first secondary maximum for 5890 A^o is 44.08^o – 20.26^o = 23.82^o.

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Characteristics (IV) of Diffraction

A single slit can separate the colors of light but the spectral lines are too broad to be resolvable

Example 5: Calculate the dispersive power of a slit having width 1.6933 μm

Solution: From previous problem, we know that this slit diffracts 5890 A^o at 31.45^o and 5896 A^o at 31.49^o. Thus the dispersive power is

$$Dispersive power = D.P. = \frac{\theta_2 - \theta_1}{\lambda_2 - \lambda_1} = \frac{31.49^o - 31.45^o}{5896A^o - 5890A^o} = 0.0067 \deg / A^o$$

We note that dispersive power is extremely small

Characteristics (V) of Diffraction

Though principally a single slit can be considered as a dispersive device, it's dispersive power is too small for it to be practically useful

Example 6: Calculate the relative intensities of first and second secondary maxima in the single slit diffraction

The intensity at any point in the single slit diffraction is given by

$$I_\theta = I_m \left(\frac{sin\alpha}{\alpha}\right)^2 \Rightarrow Relative\ intensity = \frac{I_\theta}{I_m} = \left(\frac{sin\alpha}{\alpha}\right)^2$$

Eqn (3.5) indicates that α for the secondary maximum is given by

$$\alpha = \left(m + \frac{1}{2}\right)\pi$$

For first secondary maximum, $m = 1$, thus $\alpha = (1.5)\pi$

Thus

$$\frac{I_{\theta SM1}}{I_m} = \left(\frac{sin 1.5}{1.5\pi}\right)^2 = \left(\frac{-1}{1.5\times 3.14}\right)^2 = 0.045 = 4.5\ \%$$

For first secondary maximum, $m = 2$, thus $\alpha = (2.5)\pi$

$$\frac{I_{\theta SM2}}{I_m} = \left(\frac{sin 2.5}{2.5\pi}\right)^2 = \left(\frac{-1}{2.5\times 3.14}\right)^2 = 0.0162 = 1.62\ \%$$

Characteristics (VI) of Diffraction

Though, a single slit can be used to form the spectrum of light, the intensity of these spectra are too weak for a single slit to be used as a dispersive device

Example 7: Calculate the angular width of the first order maximum, if a grating having 15000 slits per inch is exposed to a laser having wavelength 6328 ^{o}. Repeat the calculations for a grating having 20000 slits per inch and record your conclusions.

$d sin\theta = m\lambda$ (maxima) and $d sin\theta = \frac{m'}{N}\lambda$ (minima)

15000th minima is forbidden as there 1st maxima is laying

Thus for the above mentioned minima

$d sin\theta = \frac{14999}{15000} 6328 = 21.94^{o}$.

And

$d sin\theta = \frac{15001}{15000} 6328 = 21.95^{o}$.

Thus the angular width for the 1st principle maximum is 0.01^{o}.

Example 8: A diffraction grating having 15000 slits per inch is exposed to a Krypton source emitting following wavelengths. Calculate the angle of diffraction of these lines in the first order.

Color	Wavelength
Violet	4000 A^o
Blue	4500 A^o
Green	5000 A^o
Yellow	5790 A^o
Orange	6000 A^o
Red	6500 A^o
Brown	7000 A^o

The condition for the maxima of the grating is

$d sin\theta = m\lambda$

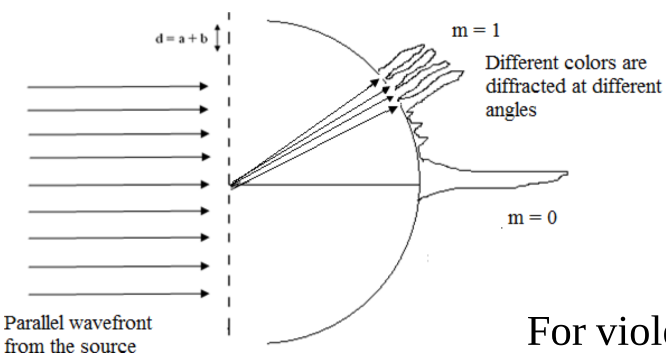
The grating has 15000 slits in 1 inch

Thus it has 15000 grating elements (***d***) is 1 inch

Thus 1 grating element $d = \frac{1''}{15000} = \frac{2.54}{15000}$ cm = $\frac{2.54\times10^8}{15000}$A^o = 16933.33 A^o

Considering first order i.e. $m = 1$

$$\theta = sin^{-1}\frac{\lambda}{d}$$



For violet

$\theta = sin^{-1}\ \frac{4000}{16933.33}$ = 13.66^o

For blue

$\theta = sin^{-1}\ \frac{4500}{16933.33}$ = 15.41^o