Tutorial - 11

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Div-9

Q.1 $\int_0^1 [\log(1/x)]^{n-1} \, dx$

$\log(1/x) = t$

$\frac{1}{x} = e^t$

$x = e^{-t}$

$x = 0$	1
$t = \infty$	0

$dx = -e^{t} \cdot dt$

$\Rightarrow \int_\infty^0 t^{n-1} (-e^{-t}) \cdot dt$

$= \int_0^\infty e^{-t} \cdot t^{n-1} \cdot dt$

$= \Gamma n$ (By definition of the Gamma function)

Q.2 $\int_0^1 \frac{x}{\sqrt{\log(1/x)}} \cdot dx$

Put $\log 1/x = t$

$1/x = e^t$

$x = e^{-t}$

$x = 0$	1
$t = \infty$	0

$$= \int_0^{\infty} e^{-t} \cdot t^{-1/2} (-e^{-t} dt)$$

$$= \int_0^{\infty} e^{-2t} \, t^{-1/2} . dt$$

As $\int_0^{\infty} e^{-kx} t^{n-1} dt = \frac{\sqrt{n}}{k^n}$

$$= \frac{\sqrt{1/2}}{2^{1/2}} = \frac{\sqrt{\pi}}{\sqrt{2}} = \underline{\underline{\sqrt{\pi/2}}}$$

Q.3 $\int_0^{\pi} x^9 (\log x)^3 . dx$

$\log x = -t$

$x = e^{-t}$

$dx = -e^{-t} dt$

$x = 0$	1
$t = \infty$	0

$$= \int_{\infty}^{0} e^{-3t} \, -t^3 \, -e^{-t} . dt$$

$$= -\int_0^{\infty} e^{-4t} . t^3 . dt$$

$$= \frac{-\sqrt{4}}{4^4} = \frac{3!}{-(4)^4}$$

Q.4 $\int_0^1 x^3 (\log 1/x)^4 \cdot dx$

Put $\log 1/x = t$

$x = e^{-t}$

$dx = -e^{-t} \cdot dt$

$x = 0$	1
$x = \infty$	0

$$= \int_\infty^0 -e^{-3t} \, t^4 \cdot dt \cdot e^{-t}$$

$$= \int_0^\infty e^{-4t} \cdot t^{5-1} \cdot dt = \frac{\Gamma 5}{(4)^5} = \frac{4!}{(4)^5} = \frac{3}{128}$$

Q.5 $\int_0^1 (\log x)^n \cdot dx$

put $\log x = -t \Rightarrow x = e^{-t}$

$dx = -e^{-t} \cdot dt$

$x = 0.$	1
$t = \infty$	0

$$\int_\infty^0 (-t)^n - e^{-t} \cdot dt = \int_0^\infty (-1)^n t^n e^{-t} . dt$$

$$= (-1)^n \cdot \Gamma n+1$$