fifth harmonic, the values of i and θ^{O} being given as follows 9. Analyse th

se the curi	rent i int	to its con	istituent l	narmonic	s as far a	s the mu	Tilaitio		240	270	300	330	1
θο	0	30	60	90	120	150	180	210	240	275	19.2	12	1
i	0	24	33.5	27.5	18.2	13.0	0	- 24	-33.5	-27.5	-10.2	1 - 13	7

[Ans. $i = (5.7 \cos \theta + 26.85 \sin \theta) - (5.1 \cos 3\theta - 3.2 \sin 3\theta) - (0.6 \cos 5\theta - 0.4 \sin 5\theta)$]

10. Using tabulated values of x and y given in the table, obtain Fourier series upto third harmonic to represent the relation between and y

xo	0	30	60	90	120	150	180	210	240	270	300	330
у	0	1.0	1.732				0	0	0	0	0	0

[Ans. $y = 0.62 + \sin x - 0.46 \cos 2x$] [Note: $y = \frac{2}{\pi} \left[1 + \frac{\pi}{2} \sin x - \frac{2}{3} \cos 2x \right]$

11. Find the harmonics a₀, a₁, a₂, a₃, b₁, b₂, b₃ of the Fourier series of the following data

х	0	π/3	2π/3	π	4π/3	5π/3	2π
у	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(May 2009)

Marks

(1)

(1)

(1)

[Ans. $a_0 = 2.9$, $a_1 = -0.37$, $a_2 = -0.1$, $a_3 = 0.03$, $b_1 = 0.17$, $b_2 = -0.06$, $b_3 = 0$]

12. A function f(x) is given by

$$f(x) \ = \ \left\{ \begin{array}{ll} x, & 0 \leq x \leq 1 \\ \\ 2-x, & 1 \leq x \leq 2 \end{array} \right.$$

Express f(x) as an approximate Fourier series upto fifth harmonic

- (i) taking 12 ordinates
- (ii) taking 24 ordinates.

[Hint: Given function is an even function of x].

 $f(x) = \frac{1}{2} - 0.414 \cos \pi x - 0.056 \cos 3\pi x - 0.03 \cos 5\pi x$ [Ans.

(ii)
$$f(x) = \frac{1}{2} - 0.4045 \cos \pi x - 0.0472 \cos 3\pi x$$

MULTIPLE CHOICE QUESTIONS

Fourier Series and Harmonic Analysis:

1. A function f(x) is said to be periodic of period T if

$$(A)^{c}f(x + T) = f(x)$$
 for all x

(B) f(x + T) = f(T) for all x

(C)
$$f(-x) = f(x)$$
 for all x

(D) f(-x) = -f(x) for all x

If f(x + nT) = f(x) where n is any integer then the fundamental period of f(x) is

(A) 2T

(D) 3T

If f(x) is a periodic function with period T then f(ax), $a \ne 0$ is periodic function with fundamental period

1(x) = 4(x+1) ((x) = 4(x+1)

(C) aT

(D) n

Fundamental period of sin 2x is SINPATTED

(C) 2_π

Fundamental period of cos 2x is cos 201 = 1

(A) $\frac{\pi}{4}$

(B)

ENGINEERING MATHEMATIC 6. Fundamental period of

(A) Z

Fourier series represen

$$\sum_{n=1}^{\infty} (a_n \cos x)$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos x)$$

Fourier series repres

(a)
$$\frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n c_n]$$

(c)
$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos \frac{1}{n} \right]$$

If f(x) is periodic fu

(A)
$$\int_{C}^{C+2L} f(x) dx$$

(C)
$$\frac{1}{L} \int_{C}^{C+2L} f(x) \cos x$$

10. If f(x) is periodic f

(A)
$$\int_{C}^{C+2L} f(x) \cos \left(\frac{1}{2}\right) dx$$

$$\int_{C}^{C+2L} \int_{C}^{C+2L} f(x) co$$

11. If f(x) is periodic

(A)
$$\int_{C}^{C+2L} f(x) \sin x$$

(C)
$$\frac{1}{L} \int_{-L}^{C+2L} f(x)$$

12. A function f(x)

$$(A) f(-x) = f(x)$$

13. A function
$$f(x)$$

(A)
$$f(-x) = f(x)$$

14. Which of the f (A) sin x

15. Which of the



ENGINEERING MATHEMATICS - II

(3.71)

FOURIER SERIES

Fundamental period of tan 3x is

(A)
$$\frac{\pi}{2}$$

$$(B) \frac{\pi}{3}$$

(D) $\frac{\pi}{4}$

Fourier series representation of periodic function f(x) with period 2π which satisfies the Dirichlet's conditions is

(1)

$$(A) \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

(B)
$$\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\pi x + b_n \sin n\pi x)$$

(C)
$$\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx) (b_n \sin nx)$$

(D)
$$\frac{a_0}{2}$$
 + (a_n cos nx + b_n sin nx)

Fourier series representation of periodic function f(x) with period 2L which satisfies the Dirichlet's conditions is

(1)

(a)
$$\frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(n\pi x) + b_n \sin(n\pi x)]$$

(B)
$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_1 \cos \left(\frac{n\pi x}{L} \right) + b_1 \sin \left(\frac{n\pi x}{L} \right) \right]$$

(C)
$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos \left(\frac{n\pi x}{L} \right) \times b_n \sin \left(\frac{n\pi x}{L} \right) \right]$$

$$\sum_{n=1}^{\infty} \left[a_n \cos \left(\frac{n\pi x}{L} \right) + b_n \sin \left(\frac{n\pi x}{L} \right) \right]$$

If f(x) is periodic function with period 2L defined in the interval C to C + 2L then Fourier coefficient ao is

(1)

(A)
$$\int_{C}^{C+2L} f(x) dx$$

(B)
$$\frac{1}{L} \int_{C}^{C+2L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx$$

(C)
$$\frac{1}{L} \int_{C}^{C+2L} f(x) \cos \left(\frac{n\pi x}{L} \right) dx$$

$$\int_{C} \int_{C}^{C+2L} f(x) dx$$

10. If f(x) is periodic function with period 2L defined in the interval C to C + 2L then Fourier coefficient an is

(1)

(A)
$$\int_{C}^{C+2L} f(x) \cos \left(\frac{n\pi x}{L} \right) dx$$

(B)
$$\frac{1}{L} \int_{C}^{C+2L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx$$

$$\int_{C}^{C+2L} \int_{C}^{C+2L} f(x) \cos \left(\frac{n\pi x}{L}\right) dx$$

(D)
$$\frac{1}{L} \int_{C}^{C+2L} f(x) dx$$

11. If f(x) is periodic function with period 2L defined in the interval C to C + 2L then Fourier coefficient b_n is

(1)

(1)

(1)

(1)

(1)

(A)
$$\int_{C}^{C+2L} f(x) \sin \left(\frac{n\pi x}{L} \right) dx$$

$$(B) \int_{L}^{C+2L} \int_{C}^{C+2L} f(x) \sin \left(\frac{n\pi x}{L}\right) dx$$

(C)
$$\frac{1}{L} \int_{C}^{C+2L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx$$

(D)
$$\frac{1}{L} \int_{C}^{C+2L} f(x) dx$$

12. A function f(x) is said to be even if

$$(A) f(-x) = f(x)$$

(B)
$$f(-x) = -f(x)$$

(C)
$$f(x + 2\pi) = f(x)$$

(D)
$$f(-x) = [f(x)]^2$$

13. A function f(x) is said to be odd if

(A)
$$f(-x) = f(x)$$

(B)
$$f(-x) = -f(x)$$

(C)
$$f(x + 2\pi) = f(x)$$

(D)
$$f(-x) = [f(x)]^2$$

14. Which of the following is an odd function?

$$(B)$$
 e^+e^-

(D)
$$\pi^2 - x^2$$

15. Which of the following is an even function

(B)
$$e^{x} - e^{-x}$$

- 16. Which of the following is neither an even function nor an odd function?
 - (A) x sin x

- (B) x2
- LET ex 18 x

- (D) x cos x
- 17. For an even function f(x) defined in the interval $-\pi \le x \le \pi$ and $f(x + 2\pi) = f(x)$ the Fourier series is
 - (A) $\sum_{n=1}^{\infty} b_n \sin nx$

(B) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}$

(e) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$

- (D) $\sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$
- 18. For an odd function f(x) defined in the interval $-\pi \le x \le \pi$ and $f(x + 2\pi) = f(x)$ the Fourier series is
 - (A) $\sum_{n=1}^{\infty} b_n \sin nx$

(B) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}$

(C) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$

- (D) $\sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$
- 19. Fourier coefficients for an even function f(x) defined in the interval $-\pi \le x \le \pi$ and $f(x + 2\pi) = f(x)$ are
 - (A) $a_0 = 0$, $a_n = 0$, $b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx dx$

(B) $a_0 = \frac{2}{\pi} \int_0^{\pi} f(x) dx$, $a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx$, $b_n = 0$

(C) $a_0 = 0$, $a_n = 0$, $b_n = 0$

- (D) $a_0 = 0$, $a_n = 0$, $b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx \, dx$
- 20. Fourier coefficients for an odd function f(x) defined in the interval $-\pi \le x \le \pi$ and $f(x + 2\pi) = f(x)$ are
 - (A) $a_0 = 0$, $a_n = 0$, $b_n = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx$

(B) $a_0 = \frac{2}{\pi} \int_0^{\pi} f(x) dx$, $a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx$, $b_n = 0$

(C) $a_0 = 0$, $a_n = 0$, $b_n = 0$

- (D) $a_0 = 0$, $a_n = 0$, $b_n = \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin nx \, dx$
- 21. Fourier coefficients for an even function f(x) defined in the interval $-L \le x \le L$ and f(x + 2L) = f(x) are
 - (A) $a_0 = 0$, $a_n = 0$, $b_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx$
- (B) $a_0 = \frac{2}{L} \int_0^L f(x) dx$, $a_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi x}{L} dx$, $b_n = 0$

(C) $a_0 = 0$, $a_n = 0$, $b_n = 0$

- (D) $a_0 = 0$, $a_n = 0$, $b_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi x}{L} dx$
- 22. Fourier coefficients for an odd function f(x) defined in the interval $-L \le x \le L$ and f(x + 2L) = f(x) are
 - (A) $a_0 = 0$, $a_n = 0$, $b_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi x}{L} dx$

(B) $a_0 = \frac{2}{L} \int_0^L f(x) dx$, $a_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi x}{L} dx$, $b_n = 0$

(C) $a_0 = 0$, $a_n = 0$, $b_n = 0$

- (D) $a_0 = 0$, $a_n = 0$, $b_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx$
- 23. Half range Fourier cosine series for f(x) defined in the interval $0 \le x \le L$ is
 - (A) $\sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$

(B) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{nx}{L}$

 $(C) \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}$

(D) $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right)$

ENG	SINEERING MATHEMATICS - II				(3.73)				FOU	RIER SERIES
_	Half range Fourier sine serie		defined in	the interval	$0 \le x \le L$ is	Ş				(1)
	(A) $\sum_{n=1}^{\infty} b_n \sin \frac{nx}{L}$					$\sum_{n=1}^{\infty} b_n si$	$n \frac{n\pi x}{L}$			
	$(C) \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}$							+ b _n sin	$\frac{n\pi x}{L}$	
25.	In Harmonic analysis for a f	unction wi	th period	2π, the tern					500	(1)
	(A) second harmonic				/	first harm			*	
	(C) third harmonic				(D)	none of t	hese			
26.	In Harmonic analysis for a f	unction wi	ith period	2π , the am				+ b ₁ sin)	k is	(1)
	(A) $\sqrt{a_1^2 - b_1^2}$			2			~		(D) $\left(a_1^2 + b_1^2\right)^2$	
27.	The value of ao in Fourier se	eries of y v	vith period	6 for the fo	ollowing ta	bulated da	ta			(1)
		х	0	1	2	3	4	5	00= 2 Et	9
		у	9	18	24	28	26	20] , w= ,	
	(A) 17.85	(1	B) 20.83		(C)	35.71			(D) 41.66	
28.	. The value of a ₀ in Fourier so	eries of y v	with period	180° for th	ne following	g tabulated	data is			(1)
	0.	x ⁰	0	30	-60	90	120	150	180	
	, , , , +	у	0	9.2	14.4	17.8	17.3	11.7	N	12
	(A) 23.46	(B) 20.11		(C)	11.73			(D) 10.50 2	7
29	. The values of a ₀ in Fourier	series of y	with perio	d 6 for the	following t	abulated d	ata is		7	16 (1)
		x	0	1	2	3	4	5	20	2 77
		У	4	8	15	7	6	2] MIL-X	
	(A) 3.5	1	B) 14		(C)	6			(D) 10.50 2 (D) 7 3	
30). The value of a ₀ in Fourier s	eries of y	with period	360° for th	ne following	g tabulated	data is		7 7 2	(1)
	CALLEGE CONTRACTOR	х ⁰	0	60	120	180	240	300	eng bias male amin	3 3
		У	1.0	1.4	1.9	1.7	1.5	1.2		-273
	(A) 1.45	(B) 5.8		(0)	2.9			(D) 2.48	72015
31	. Fourier coefficient a ₀ in the	Fourier se	eries expar	nsion of f(x)	$= e^{-x}; 0 \le x$	$\alpha \leq 2\pi$ and	$f(x + 2\pi) = 1$	(x) is	760	(2)
	(a) $\frac{1}{\pi} (1 - e^{-2\pi})$			$-e^{2\pi}$)		2			(D) $\frac{1}{\pi} (1 + e^{2\pi})$	(e - 9)
32	2. Fourier coefficient a ₀ in the	Fourier s	eries expar	nsion of f(x)	$=\left(\frac{\pi-x}{2}\right)^2$	$; 0 \le x \le 2\tau$	τ and f(x + 2	2π) = f(x) i	is	(2)
	(A) $\frac{\pi^2}{3}$	((B) $\frac{\pi^2}{6}$		(C)	0			(D) $\frac{\pi}{6}$	siny]o
33	3. Fourier coefficient a ₀ in the	Fourier s	eries expar	nsion of f(x)	= x sin x; ($0 \le x \le 2\pi a$	$\int dx dx = 2\pi$	= f(x) is	1-2000	(2)
	(A) +2	((B) 0		Jes	-2			(D) -4 _ 27	+0-0.
34	1. $f(x) = \begin{cases} x, & 0 \le x \le \pi \\ 0, & \pi < x \le 2\pi \end{cases}$ and	$f(x + 2\pi) =$	= f(x). Four	rier series is	represente	ed by $\frac{a_0}{2}$	$+ \sum_{n=1}^{\infty} (a_n con)$	os nx + b _r	sin nx), then Fourier	coefficient
	a ₀ is						2			(2)
	(A) 2π	((B) $\frac{\pi}{3}$	17-2)	3) 2 TT	0	(-Ti) - Ti3		$ AD \frac{\pi}{2}$ $ -c^{-\chi} $	7 7-
	1 2 m	(7-8)		-3	(43, 42	3 7	54(e)	(17T)	7 3 x x) - (6.	27 .]
	5° 60) 0 -	in (=	3 47	1/3	4,	+	(**)		
	A STATE OF THE STA		The state of the s	The state of the s					- 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1	The state of the s

(C) $\frac{2}{\pi}$

 $(D) \frac{4}{\pi}$

 $(A) \frac{\pi}{2}$

(A) 2

ao is (A) 2

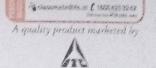
(A) 1

(A) 4

Fourier coefficient ao is

(B) 2





45. For half range sine series of $f(x) = \cos x$, $0 \le x \le \pi$ and period is 2π . Fourier series is represented by $\sum_{n=1}^{\infty} b_n \sin nx$, then Fourier

(A) $\frac{1}{\pi}$

46. For half range cosine series of $f(x) = lx - x^2$, $0 \le x \le l$ and period is 2l. Fourier series is represented by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l}$, then Fourier coefficient ao is

(D) 0 3 (2) -3 (2) 47. For half range sine series of f(x) = x, $0 \le x \le 2$ and period is 4. Fourier series is represented by $\sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{2}$, then Fourier coefficient b₁ is

(A) 4

48. Fourier series representation of periodic function $f(x) = \left(\frac{\pi - x}{2}\right)^2$, $0 \le x \le 2\pi$ is $\left(\frac{\pi - x}{2}\right)^2 = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{1}{n^2} \cos nx$, then value of

(B) $\frac{\pi^2}{12}$

49. Fourier series representation of periodic function $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$ is $f(x) = \frac{8}{\pi^2} \left[\frac{1}{1^2} \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \dots \right]$,

then value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots =$

(C) $\frac{\pi^2}{16}$

50. Fourier series representation of periodic function $f(x) = \pi^2 - x^2$, $-\pi \le x \le \pi$ is $\pi^2 - x^2 = \frac{2\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \cos nx$, then value of Fourier series representation of periodic function (A) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots =$ (A) $\frac{\pi^2}{3}$ (B) $\frac{\pi^2}{4}$ (C) $\frac{\pi^2}{6}$ (C) $\frac{\pi^2}{6}$ (C) $\frac{\pi^2}{6}$ (D) $\frac{\pi^2}{12}$ (2)

(2)

(2)

(2)

(2)

51. Fourier series representation of periodic function $f(x) = \pi^2 - x^2$, $-\pi \le x \le \pi$ is $\pi^2 - x^2 = \frac{2\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \cos nx$, then value of

 $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots =$

(A) $\frac{\pi'}{2}$

(D) 0

52. The value of a₁ in Fourier series of y with period 6 for the following tabulated data is:

0 1 2 3 4 24 9 18 28 26 20 cos TX

2 (000)-1

JAY -8.33

(B) -7.14

(C) -4.16

(D) 0

53. The value of b_1 in Fourier series of v with period π for the following tabulated data is:

er series or y win	n period	π for the ic		90	120	150
x ⁰	0	30	60	90	-	117
y	0	9.2	14.4	17.8	17.3	11.7
sin 2x	0	0.866	0.866	0	-0.866	-0.866

(A) -3.116

(C) -4.16

(D) -1.336

(A) -3.116 (B) -1.558 (C) -4.16

The value of a₁ in Fourier series of y with period 6 for the following tabulated data is:

х	0	1	2	3	4	5
у	4	8	15	7	6	2
$\cos \frac{\pi x}{3}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	1/2

(A) -2.83

(B) -8.32

(D) -10.98

55. The value of b_1 in Fourier series of y with period 2π for the following tabulated data is :

x ⁰	0	60	120	180	240	300
У	1.0	1.4	1.9	1.7	1.5	1.2
sin x	0	0.866	0.866	, 0	-0.866	-0.866

(A) 0.0989

(B) 0.3464

(C) 0.1732

(D) 0.6932

ANSWERS

				ween and the state of the state			
1. (A)	2. (C)	3. (B)	4. (D)	5. (C)	6. (B)	7. (A)	8. (D)
9. (D)	10. (C)	11. (B)	12. (A)	13. (B)	14. (A)		
17. (C)	18. (A)	19. (B)	20. (D)	21. (B)	District the same of the same of	15. (D)	16. (C)
25. (B)	26. (C)	27. (D)	28. (A)	The office of the contract of	22. (D)	23.(C)	24. (B)
33. (C)	34. (D)	35. (A)		29. (B)	30. (C)	31. (A)	32. (B)
			36. (B)	37. (C)	38. (D)	39. (B)	40. (A)
41. (A)	42. (C)	43. (B)	44. (D)	45. (B)	46. (A)	47. (D)	
49. (B)	50. (D)	51. (C)	52. (A)	53. (B)	54. (D)		48. (A)
					J T. (D)	55. (C)	



