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LAOC Tutorial - 6

Q.1. $x = u\tan v$, $y = u\sec v$; prove,

$$\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial v}{\partial x}\right)_y = \left(\frac{\partial u}{\partial y}\right)_x \left(\frac{\partial v}{\partial y}\right)_x$$

LHS :

$$\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial v}{\partial x}\right)_y \longrightarrow \left(\frac{\partial (x\cot v)}{\partial x}\right)_y \cdot \left[\frac{\partial(\tan^{-1} x/u)}{\partial x}\right]_y$$

$$= \cot v \times \frac{u}{x^2+u^2}$$

put $x = u\tan v$,

$$= \cot v \times \frac{u}{u^2(\tan^2 v + 1)} = \cancel{\cot v}$$

$$= \cot v \times \frac{1}{u\sec^2 v} \qquad (\tan^2 x + 1 = \sec^2 x)$$

$$= \frac{\cos^2 v\cot v}{u}$$

RHS

$$\left(\frac{\partial u}{\partial y}\right)_x \left(\frac{\partial v}{\partial y}\right)_x = \left[\frac{\partial (y\cos v)}{\partial y}\right]_x \left[\frac{\partial (\sec^{-1}(y/u))}{\partial y}\right]_x$$

$$= \cos v \times \frac{u}{y\sqrt{y^2-u^2}}$$

put $y = u\sec v$

$$= \cos v \times \frac{u}{u\sec v\sqrt{u^2(\sec^2 v - 1)}} = \frac{\cos^2 v \cot v}{u}$$

Since, LHS = RHS,

$$\frac{\cos^2 v \cot v}{u} = \frac{\cos^2 v \cot v}{u}$$

(2) If $u = x\log xy$, where $x^3 + y^3 + 3xy = 1$, then find $\frac{du}{dx}$

Ans. $u = x\log xy$

$x^3 + y^3 + 3xy = 1$

By using total differentiation concept,

$$du = \left(\frac{\partial u}{\partial x}\right)dx + \left(\frac{\partial u}{\partial y}\right)\cdot dy$$

$$\frac{\partial u}{\partial x} = \left(\frac{\partial u}{\partial x}\right) + \left(\frac{\partial u}{\partial y}\right)\cdot\left(\frac{dy}{dx}\right)$$

$u = x\log(xy)$

Partial differentiation with respect to x,

$$\cancel{\frac{\partial x}{\partial y}} \frac{\partial u}{\partial x} = \log xy + \frac{x}{xy}(y) = 1 + \log xy$$

Partial differentiation with respect to y,

$$\frac{\partial u}{\partial y} = x\left(\frac{1}{xy}\right)(x) = \frac{x}{y}.$$

$x^3 + y^3 + 3xy = 1.$

Differentiating with respect to x we get.

$$3x^2 + 3y^2\left(\frac{dy}{dx}\right) + 3y + 3x\left(\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx}\left[3x + 3y^2\right] = -\left[3x^2 + 3y\right]$$

$$\frac{dy}{dx} = -\left[\frac{x^2+y}{x+y^2}\right]$$

Now, $$\frac{du}{dx} = \left(\frac{\partial u}{\partial x}\right) + \left(\frac{\partial u}{\partial y}\right)\left(\frac{du}{dx}\right)$$

$$= 1 + \log xy + \left(\frac{x}{y}\right)\left(\frac{-(x^2+y)}{(x+y^2)}\right)$$

$$\therefore \boxed{\frac{dy}{dx} = 1 + \log xy - \frac{x(x^2+y)}{y(x+y^2)}}$$

3. If $u = \sin^{-1}\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)$, show that,

$$\frac{\partial u}{\partial x} = \frac{-y}{x} \cdot \frac{\partial u}{\partial y}.$$

$$u = x^0 \sin^{-1}\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)$$

$$= x^0 \sin^{-1}\left[\frac{1-\sqrt{y/x}}{1+\sqrt{y/x}}\right]$$

$$= x^0 f(y/x)$$

so u is a homogeneous function of degree 0. Hence by Euler's theorem,

$$x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = 0(u) = 0$$

$$\therefore \; x\left(\frac{\partial u}{\partial x}\right) = -y\left(\frac{\partial u}{\partial y}\right)$$

Q·2 Fill in the Blanks

(1) $u = x^3 + y^3$, $x = a\cos t$, $y = b\sin t$,

$$\frac{du}{dt} = -3a^3\cos^2 t \sin t + 3b^3 \sin^2 t \cos t$$

(2) $f(x, y, z) = (x^2 + y^2 - 2z^2)(y^2 + z^2)$

at $x = 2, y = 1, z = 3$,

$\Rightarrow$ 40

(3) $x = r\cos\theta$, $y = r\sin\theta$. $\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = ?$

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = 0$$

(4) If $(\cos x)^y = (\sin y)^x$ then $f\left(\frac{dy}{dx}\right) = ?$

$$\boxed{f\left(\frac{dy}{dx}\right) = \frac{\log(\sin y) + y\tan x}{\log(\cos x) - x\cot y}}$$