Krishnaraj P T.
109054 I3

21/01/21

Tutorial - 4

Q.1 $(x^4 e^x - 2mxy^2)dx + (2mx^2y)dy$ — (1)

This is in $M.dx + Ndy = 0$ form.

$$M = x^4e^x - 2mxy^2 \quad \text{and} \quad N = 2mx^2y$$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}[x^4e^x - 2mxy^2]$$

$$= \frac{\partial}{\partial y}(x^4e^x) - \frac{\partial}{\partial y}(2mxy^2)$$

$$= 0 - 2mx(2y)$$

$$\frac{\partial M}{\partial y} = -4mxy \quad \text{— (2)}$$

$$\frac{\partial N}{\partial x} = 2my(2x)$$

$$\frac{\partial N}{\partial x} = 4mxy \quad \text{— (3)}$$

So; $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial y}$ so not exact diff. eq. and not Homogeneous eq.

Also not in $y(f(xy)dx) + x.g(xy)dy = 0$

$$N = 2mx^2y = x(2mxy)$$

$$\frac{1}{N}\left[\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right] = \frac{1}{x(2mxy)}[-4mxy - 4mxy]$$

$$= \frac{-8mxy}{x(2mxy)}$$

$$= \frac{-4}{x} = f(x) \text{ only.}$$

So $IF = e^{\int f(x)dx}$

$= e^{-4\log x} = e^{\log x^{-4}} = x^{-4}$

So $\frac{1}{x^4}\left[\left(x^4 e^x - 2mxy^2\right)dx + 2mx^2y \cdot dy\right] = 0$

$= \left[\frac{x^4 e^x - 2mxy^2}{x^4}\right] dx + \left[\frac{2mx^2y}{x^4}\right] dy = 0$

$\left[e^x - \frac{2my^2}{x^3}\right] dx + \frac{2my}{x^2}(dy) = 0$

$(M_1)\ dx + (N_1)\ dy$

$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$ as eqn is now ~~di~~ exact DE.

$\frac{\partial M_1}{\partial y} = \frac{\partial}{\partial y}(e^x) - \frac{\partial}{\partial y}\left(\frac{2my}{x^3}\right)$

$= \frac{-4my}{x^3}$

$\frac{\partial N_1}{\partial x} = 2my\left[\frac{\partial}{\partial x}\frac{1}{x^2}\right] = 2my(-2x^{-3})$

$= -4my\,x^{-3}$

$= \frac{-4my}{x^3}$

So gen. solution is

$\int^{x} M_1\, dx + \int [\text{Terms of } N \text{ without } x]\, dy = c.$

Rainbow

$$\int \left(e^x - \frac{2my^2}{x^3}\right)dx + \int 0 \, dy = c$$

$$e^x - \left(2my^2 \cdot \frac{x^{-2}}{-2}\right) = c$$

$$= e^x + my^2(x^{-2}) = c$$

$$e^x + \frac{my^2}{x^2} = c$$

Q.2. $(xy - 2y^2)dx - (x^2 - 3xy)dy = 0$

eqn can be written as -

$$(xy - 2y^2)dx = (x^2 - 3xy)dy$$

$$\frac{dy}{dx} = \frac{t^2(\cancel{x^2 - 3x}\ xy - 2y^2)}{t^2(x^2 - 3xy)}$$

$$= \frac{dy}{dx}$$

So it is homogeneous

so

$$IF = \frac{1}{x \cdot M + N \cdot y}$$

$$M = xy - 2y^2$$

$$-N = 3xy + x^2$$

$$IF = \frac{1}{x^2y - 2y^2x - 3xy^2 + x^2y}$$

$$= \frac{1}{xy^2}$$

ainbow

So DE becomes exact

$M = \frac{1}{y} - \frac{2}{x}$

$N = \frac{3}{y} - \frac{x}{y^2}$

$\therefore$ solution

$$\int^{x} \left(\frac{1}{y} - \frac{2}{x}\right) dx + \int \frac{3}{y} = c$$

$$= \cancel{\frac{1}{y} x^2} \quad \boxed{\frac{x}{y} - 2\log x + 3\log y = c}$$

Q.3 $(x^2y^2 + 2)y \cdot dx + (2 - 2x^2y^2)x \cdot dy = 0$

$M = x^2y^3 + 2y$

$N = 2x - 2x^3y^2$

$$IF = \frac{1}{x \cdot M - y \cdot N}$$

$$= \frac{1}{x^3y^3 + 2yx - (2xy - 2x^3y^3)}$$

$$= \frac{1}{x^3y^3 + 2xy - 2xy + 2x^3y^3}$$

$$= \frac{1}{3x^3y^3}$$

$$\boxed{IF = \frac{1}{3x^3y^3}}$$

So $M_1 = M/IF$

$$= \frac{x^{\cancel{2}}\cancel{y^3}}{3x^{\cancel{3}}\cancel{y^3}} + \frac{2y}{3x^3y^{\cancel{3}\,2}}$$

$$= \frac{1}{3x} + \frac{2}{3x^3y^2}$$

$N_1 = N/IF$

$$= \frac{2x}{3x^{\cancel{2}3}y^3} - \frac{2\cancel{x^3}y^{\cancel{2}}}{3\cancel{x^3}y^{\cancel{3}}}$$

$$N_1 = \frac{2}{3x^2y^3} - \frac{2}{3y}$$

$$x \quad \frac{2}{3}$$

So Solution,

$$\int^{x} \frac{1}{3x} + \frac{2}{3x^3y^2} + \int \frac{-(2)}{3y} = c$$

$$= \frac{1}{3}\log x + \frac{\cancel{2}^{-1}}{3y^2}\cdot\frac{x^{-2}}{\cancel{-2}} + \frac{-2}{3}\log y = c$$

$$= \boxed{\frac{\log x}{3} - \frac{1}{3x^2y^2} - \frac{2}{3}\log y = c}$$

inbow

Q.9. $y(2x^2y + e^x)\,dx = (e^x + y^3)\,dy$

$(2x^2y^2 + ye^x)\,dx - (e^x + y^3)\,dy = 0$

which is in $M \cdot dx + N \cdot dy = 0$ form.

when,

$M = 2x^2y^2 + ye^x$

$N = -(e^x + y^3)$

$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}[2x^2y^2 + ye^x]$

$\frac{\partial M}{\partial y} = 4x^2y + e^x$

$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(-e^x - y^3)$

$= -e^x$

$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

$y(2x^2y + e^x)\,dx - (e^x + y^3)\,dy = 0$

$= 2x^2y^2\,dx + ye^x\,dx - e^x\,dy - y^3\,dy = 0$

$= 2x^2y^2\,dx - y^3\,dy + ye^x\,dx - e^x\,dy = 0$

$f(x) = e^x$

$f'(x) = e^x$

Rainbow

$\cancel{\text{If}}$ $\frac{d}{dx} f(x) = e^x \Rightarrow \frac{d(e^x)}{dx} = e^x$

$= \cancel{d} \quad e^x dx$

$2x^2 y^2 dx - y^3 dy + y d(e^x) - e^x dy = 0$

$\left[\because \cancel{y} \cancel{\text{de}} \; \frac{y \cdot d(e^x) - e^x dy}{dy^2} \right.$

$\left. = d\left(\frac{e^x}{\cancel{y}}\right) \right]$

$\Rightarrow \frac{1}{y^2} \left[2x^2 y^2 dx - y^3 dy + y \cdot d(e^x) \right]$

$= 2x^2 dx - y dy + d\left(\frac{e^x}{y}\right) = 0$

$\int 2x^2 dx - \int y dy + \int d\left(\frac{e^x}{y}\right) = 0$

$$\boxed{\frac{2x^3}{3} - \frac{y^2}{2} + \frac{e^x}{y} = c}$$

Rainbow