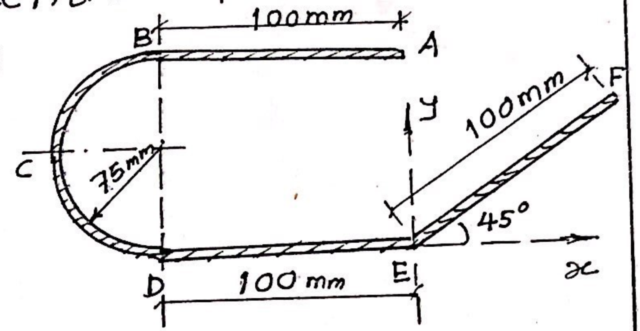
Centroid and CG Numericals

Other Numericals

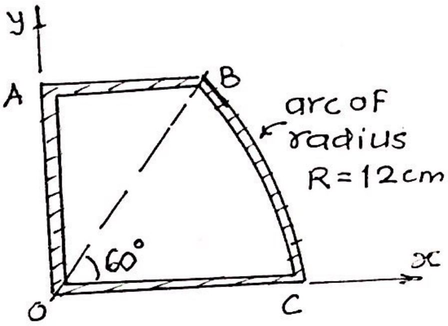
50 Numericals on Module 1

Centroids of Linear Objects (1-D)

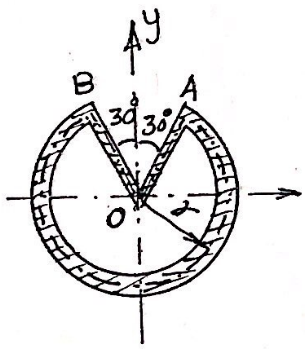
A thin homogeneous uniform wire is bent into the following shapes. Locate the position of the centroid of the wire.

① 

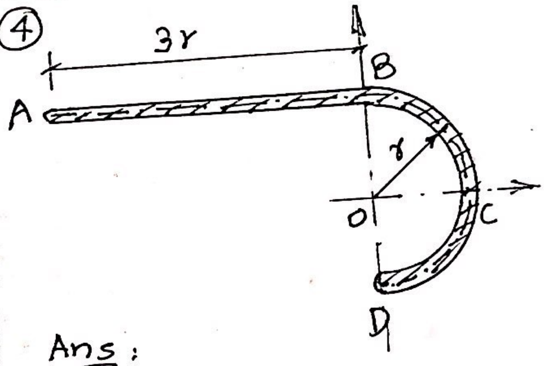
Ans: $G(\bar{x},\bar{y}) \equiv (-77.06, 67.59)$ mm mm

② 

Ans: $G(\bar{x},\bar{y}) \equiv (5.24, 4.59)$ cm

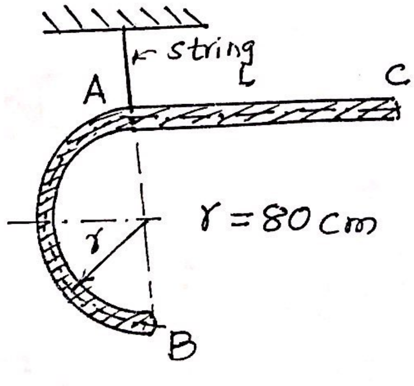
③ 

Ans: $\bar{x} = 0$
$\bar{y} = -(0.018)r$

④ 

Ans: $\bar{x} = -(0.733)r$
$\bar{y} = (0.814)r$

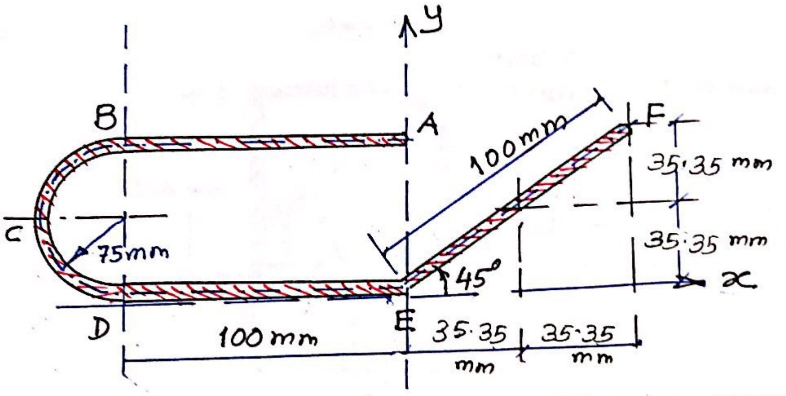
⑤ A rod of uniform thickness is bent in the shape as shown in fig. Find the distance 'L' of member AC, so that it remains in horizontal position when suspended from pt. A with the string.



Ans: L = 1.60 m

Centroids of Linear Objects

(1)

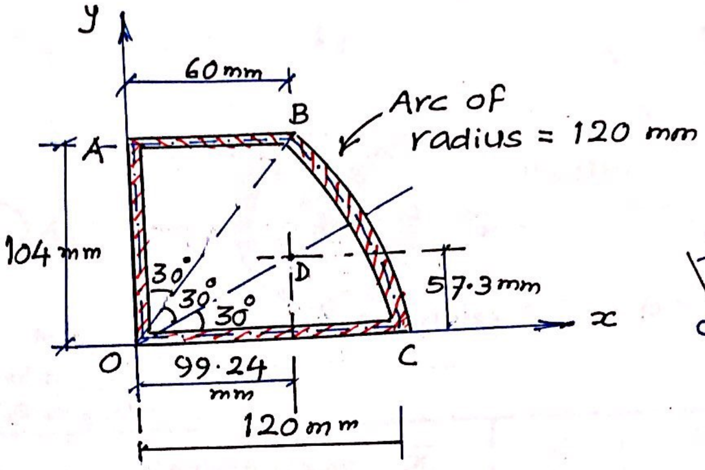


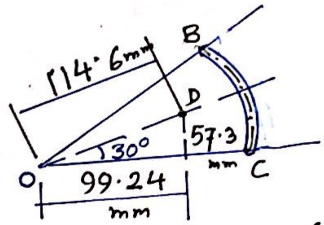
Sr. No.	Name	Length 'L' (mm)	x (mm)	y (mm)	$L \cdot \bar{x}$ (mm^2)	$L \cdot \bar{y}$ $(mm)^2$
1	AB	100	−50	150	−5,000	15,000
2	Semi-circle BCD	$\pi \cdot r$ $=(75)\pi$ $=235.6$	$-(100+\frac{2r}{\pi})$ $=-147.75$	75	-34.8×10^3	17.67×10^3
3	DE	100	−50	0	−5,000	0
4	EF	100	35.35	35.35	3.535×10^3	3.535×10^3
	Total :	$\Sigma L = 535.6$	~	~	$\Sigma L\bar{x} = -41.265 \times 10^3$	$\Sigma L\bar{y} = 36.205 \times 10^3$

$$\bar{x} = \left(\frac{\Sigma L\bar{x}}{\Sigma L}\right) = -\left(\frac{41.265 \times 10^3}{535.6}\right) = -77.04 \text{ mm}$$

$$\bar{y} = \left(\frac{\Sigma L\bar{y}}{\Sigma L}\right) = \left(\frac{36.205 \times 10^3}{535.6}\right) = 67.59 \text{ mm}$$

②





$\alpha = 30° = 0.523^c$

$OD = \left(\frac{r \cdot \sin\alpha}{\alpha}\right)$

$OD = \left(\frac{120 \cdot \sin 30°}{0.523}\right)$

$OD = 114.6$ mm

Length of arc BC

$= \left(\frac{\theta}{360} \times 2\pi r\right) = \left(\frac{60}{360} \times 2\pi \times 120\right)$

$= 125.66$ mm

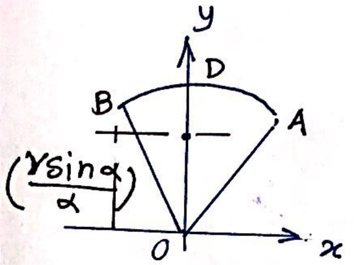
Sr. No.	Name	Length 'L' mm	$\bar{x}$ mm	$\bar{y}$ mm	$L \cdot \bar{x}$ mm^2	$L \cdot \bar{y}$ mm^2
1	AB	60	30	104	1800	6,240
2	OA	104	0	52	0	5,408
3	OC	120	60	0	7,200	0
4	Arc BC	125.66	99.24	57.3	12.47×10^3	7.2×10^3
	Total :	$\Sigma L = 409.66$			$\Sigma L\bar{x} = 21.47 \times 10^3$	$\Sigma L\bar{y} = 18.848 \times 10^3$

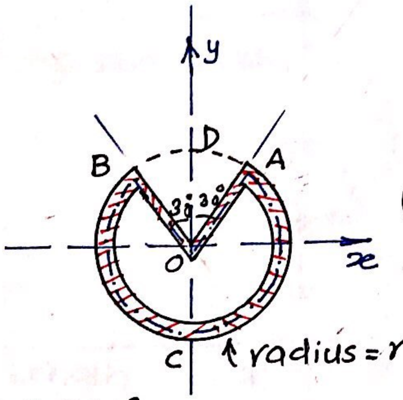
$\bar{X} = \left(\frac{\Sigma L\bar{x}}{\Sigma L}\right) = \left(\frac{21.47 \times 10^3}{409.66}\right) = 52.41$ mm

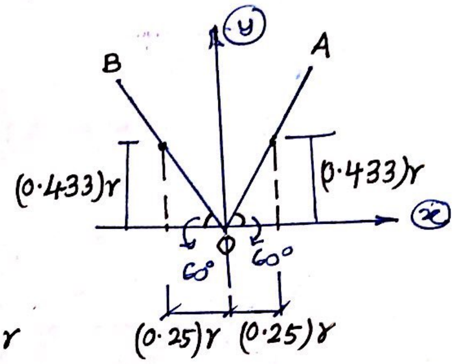
$\bar{Y} = \left(\frac{\Sigma L\bar{y}}{\Sigma L}\right) = \left(\frac{18.848 \times 10^3}{409.66}\right) = 46.0$ mm

Scanned with CamScanner

③







$\alpha = 30^\circ = 0.523^c$

$\theta = 60^\circ$

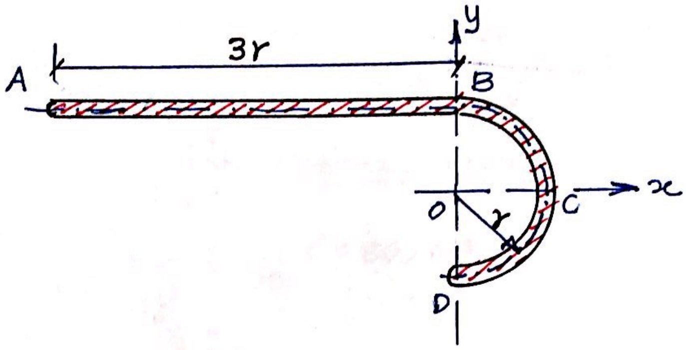
Sr. No.	Name	Length (mm)	$\bar{x}$ mm	$\bar{y}$ mm	$L\cdot\bar{x}$ mm²	$L\cdot\bar{y}$ mm²
1	Circle	$2\pi r = (6.283)r$	0	0	0	0
2	Arc ADB	$-\left(\frac{60}{360}\times 2\pi r\right) = -(1.047)r$	0	$\left(\frac{r\sin 30^\circ}{0.523}\right) = (0.955)r$	0	$-r^2$
3	OA	r	$(0.25)r$	$(0.433)r$	$(0.25)r^2$	$(0.433)r^2$
4	OB	r	$-(0.25)r$	$(0.433)r$	$-(0.25)r^2$	$(0.433)r^2$
	Total :	$\Sigma L = (7.236)r$			$\Sigma L\bar{x} = 0$	$\Sigma L\bar{y} = -(0.134)r^2$

$$\bar{x} = \left(\frac{\Sigma L\bar{x}}{\Sigma L}\right) = 0, \qquad \bar{y} = \left(\frac{\Sigma L\bar{y}}{\Sigma L}\right) = \left[\frac{-(0.134)r^2}{(7.236)r}\right] = -(0.0185)r$$

Note: The given bentup bar is symmetrical @ 'y' axis, hence the centroid will lie on it.

$G \equiv (0, -0.0185\times r)$

4

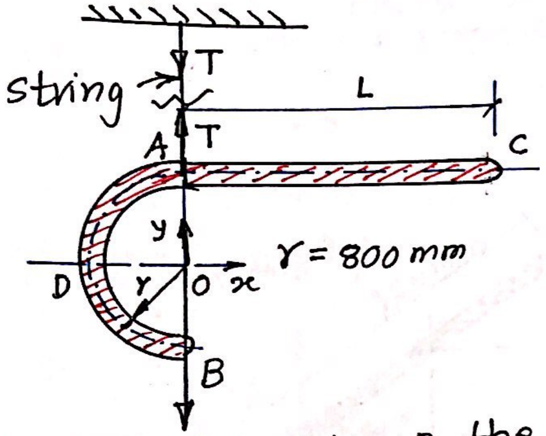


Sr. No.	Name	Length 'L' mm	$\bar{x}$ mm	$\bar{y}$ mm	$L\bar{x}$ mm^2	$L\bar{y}$ mm^2
1	AB	$3r$	$-(1.5)r$	r	$-(4.5)r^2$	$3r^2$
2	semi-circle BCD	$\pi r = (3.14)r$	0	$\frac{2r}{\pi} = (0.64)r$	0	$2r^2$
	Total : $\Sigma L = (6.14)r$				$\Sigma L\bar{x} = -(4.5)r^2$	$\Sigma L\bar{y} = 5r^2$

$$\bar{x} = \left(\frac{\Sigma L\bar{x}}{\Sigma L}\right) = \left[\frac{-(4.5)r^2}{(6.14)r}\right] = -(0.733)r$$

$$\bar{y} = \left(\frac{\Sigma L\bar{y}}{\Sigma L}\right) = \left[\frac{5r^2}{(6.14)r}\right] = (0.814)r$$

5



$\left(W = \text{weight of the bent up bar} \right) = T$

As arm AC is horizontal, the centroid of the bar is lying on 'y' axis. $\therefore \bar{x} = 0$

$$\therefore \quad \bar{x} = \left(\frac{\Sigma L \bar{x}}{\Sigma L} \right) = 0 \qquad \therefore \boxed{\Sigma L \cdot \bar{x} = 0}$$

$$\therefore \quad \left(L \times \frac{L}{2} \right) + (\pi \cdot r)\left(-\frac{2r}{\pi} \right) = 0$$

$$\frac{L^2}{2} - 2r^2 = 0$$

$$L^2 = 4r^2$$

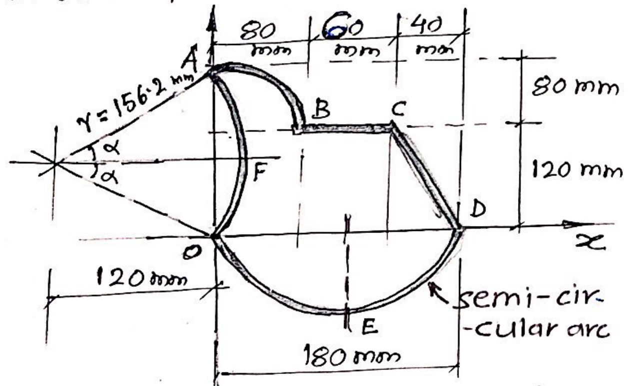
$$L = 2r = (2 \times 800)\ mm$$

$$* \quad L = 1 \cdot 60\ m$$

E

Centroids of linear objects (1-D):

Ex. No (1) A prismatic metalic bar is bent in the shape as shown in figure... Find the co-ordinates of the centroid of it.



$\tan\alpha = \frac{100}{120} = 0.833$

$\alpha = 39.8^\circ$
$= 0.695^c$

Length of arc OFA

$= \frac{39.8 \times 2}{360} \times 2 \times \pi \times 156$

$= 217$ mm

Centroidal distance from the center

$= \frac{r \sin\alpha}{\alpha} = 143.86$ mm

Solution: By tabular method;

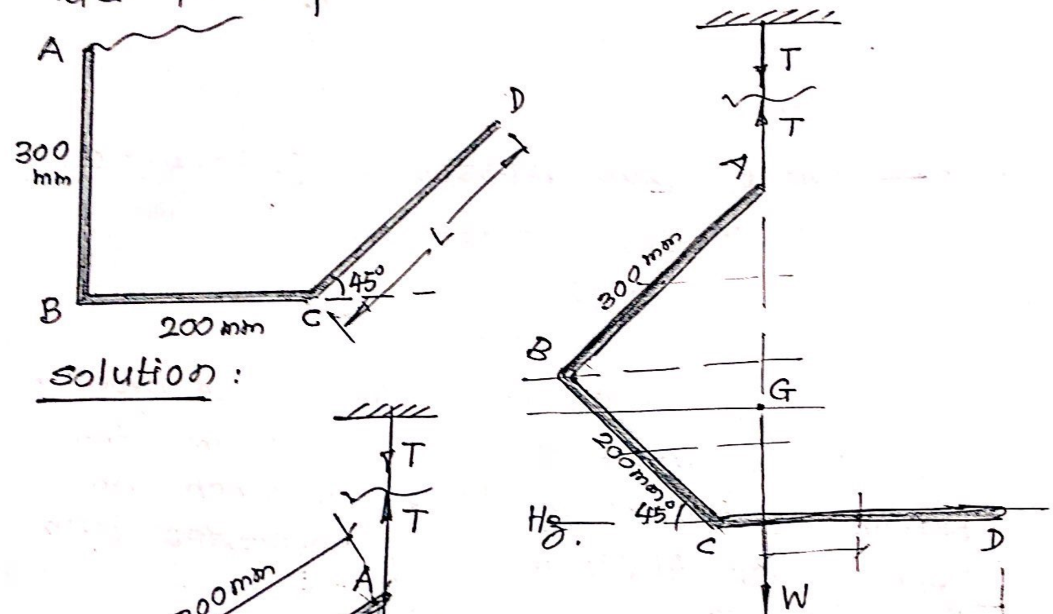
Sr. No.	Name	Length mm	$\bar{x}$ mm	$\bar{y}$ mm	$L\bar{x}$ mm^2	$L\bar{y}$ mm^2
1	Arc of a circle OFA	217	$143.86 - 120 = 23.86$	100	5.17×10^3	21.7×10^3
2	Quarter circle AB	125.66	$\frac{2 \times 80}{\pi} = 50.93$	$120 + 50.93 = 170.93$	6.4×10^3	21.48×10^3
3	Horizontal bar BC	60	$80 + 30 = 110$	120	6.6×10^3	7.2×10^3
4	Inclined bar CD	126.5	$80 + 60 + 20 = 160$	60	20.24×10^3	7.59×10^3
5	Semi-circular bar DEO	282.7	90	$-\frac{2 \times 90}{\pi} = -57.3$	25.44×10^3	-16.2×10^3
	Total	$\Sigma L = 811.86$			$\Sigma L\bar{x} = 63.85 \times 10^3$	$\Sigma L\bar{y} = 41.77 \times 10^3$

$$\therefore \bar{x} = \frac{\Sigma L\bar{x}}{\Sigma L} = \frac{63.85 \times 10^3}{811.86} = 78.65 \text{ mm}$$

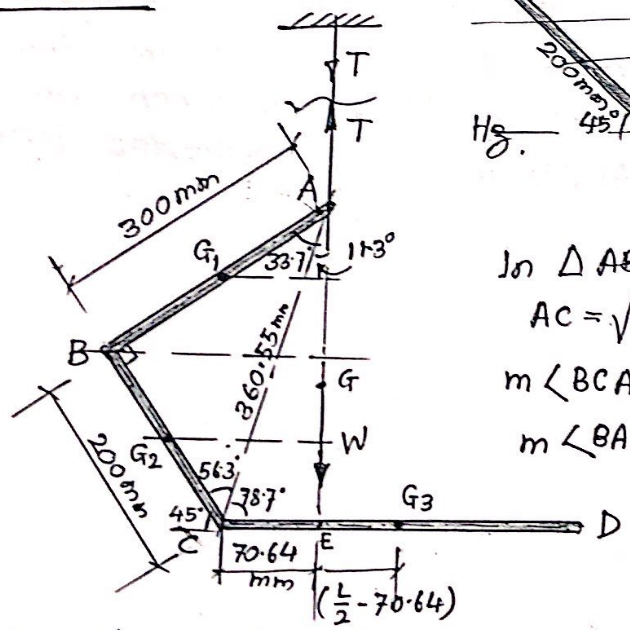
$$\bar{y} = \frac{\Sigma L\bar{y}}{\Sigma L} = \frac{41.77 \times 10^3}{811.86} = 51.45 \text{ mm}$$

Ans: The centroid of the bentup bar is $G \equiv (78.65 \text{ mm}, 51.45 \text{ mm})$

Ex.No. ② A metalic prismatic bar is bent in the shape as shown in Figure .... Find the length of member CD, so that it will rema--in in horizontal position when freely suspen--ded from point A with the help of a string.



solution :



In ΔABC,

$AC = \sqrt{300^2 + 200^2} = 360.55\,mm$

$m\angle BCA = \tan^{-1}\frac{300}{200} = 56.3^\circ$

$m\angle BAC = 90^\circ - 56.3^\circ = 33.7^\circ$

In ΔACE,

$CE = AC \times \cos 78.7^\circ$
$= 70.64\ mm$

Considering line AG as y axis i.e. considering the centroid on y axis we get $\bar{x} = 0$

But, $\bar{x} = \frac{\Sigma L\bar{x}}{\Sigma L} = 0$ $\therefore$ we get $\Sigma L\bar{x} = 0$

i.e. moment @ y axis is zero.

Let G_1, G_2 and G_3 be the midpoints of bars AB, BC and CD respectively.

similarly, let $\bar{x}_1$, $\bar{x}_2$ & $\bar{x}_3$ be the distances of points G_1, G_2 and G_3 from y axis.

$\therefore \quad \bar{x}_1 = (150)[\sin(33.7+11.3)^\circ] = 106.06 \text{ mm}$

$\therefore \quad \bar{x}_2 = 70.64 + 100 \times \cos 45^\circ = 141.35 \text{ mm}$

$\therefore \quad \bar{x}_3 = \left(\frac{L}{2} - 70.64\right) \text{mm}$

Taking moments @ y axis, i.e. $\Sigma L\bar{x}$ =

$= -(300)(106.06) - (200)(141.35) + L \times \left(\frac{L}{2} - 70.64\right) = 0$

$\therefore \quad (0.5)L^2 - (70.64)L - 60088 = 0$

$\therefore \quad L = 424.43 \text{ mm}$

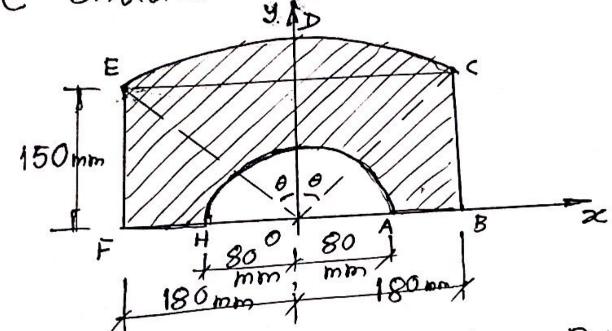
or $\quad L = -283.15 \text{ mm}$

As the length of CD ie. L can not be negative, then

Ans: If, L = Length of CD = 424.43 mm, then portion CD will remain horizontal when the bent up bar ABCD is freely suspended from point A.

Centroids of laminar objects (2-D):

Ex. No ③ Locate the co-ordinates of the centroid of the shaded area shown in figure ...



<u>Solution</u>: $\widehat{EDC}$ is a sector of a circle with center at O and radius $= \sqrt{180^2 + 150^2}$

$= 234.3$ mm

$\theta = m\angle EOD = m\angle COD = \tan^{-1}\left(\frac{180}{150}\right) = 50.19° = 0.876^c$

The shaded area is symmetrical @ y axis, hence the centroid will lie on y axis. Due to that $\bar{x} = 0$. To locate $\bar{y}$, use tabular method;

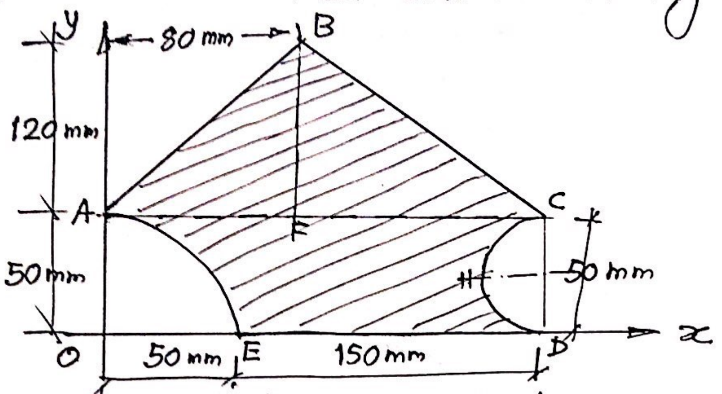
Sr. No.	Name	Area mm^2	$\bar{y}$ mm	$A\bar{y}$ mm^3
1	sector of a circle OEDC	$(234.3)^2 \times 0.876 = 48.09 \times 10^3$	$\frac{2}{3} \times \frac{234.3 \times \sin 50.19°}{0.876} = 136.97$	6.59×10^6
2	Triangle EFO	$\frac{1}{2} \times 150 \times 180 = 13.5 \times 10^3$	$\frac{1}{3} \times 150 = 50$	0.675×10^6
3	Triangle CBO	$\frac{1}{2} \times 150 \times 180 = 13.5 \times 10^3$	$\frac{1}{3} \times 150 = 50$	0.675×10^6
4	Semicircle	$-\left(\frac{\pi \times 80^2}{2}\right) = -10.05 \times 10^3$	$\frac{4 \times 80}{3\pi} = 33.95$	-0.341×10^6
		$\Sigma A = 65.04 \times 10^3$	—	$\Sigma A\bar{y} = 7.599 \times 10^6$

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \left(\frac{7.599 \times 10^6}{65.04 \times 10^3}\right) = 116.84 \text{ mm}$$

<u>Ans</u>: The centroid of the shaded area is

$G \equiv (0, 116.84 \text{ mm})$

Ex. No. (4) Locate the co-ordinates of the centroid of the shaded area shown in figure ...



Solution: By tabular method;

Sr. No.	Name	Area mm^2	$\bar{x}$ mm	$\bar{y}$ mm	$A\bar{x}$ mm^3	$A\bar{y}$ mm^3
1	Rectangle OACD	$200\times50 = 10\times10^3$	100	25	1×10^6	250×10^3
2	Quarter-circle AOE	$-\frac{\pi}{4}\times50^2 = -1.96\times10^3$	$\frac{4\times50}{3\pi} = 21.22$	$\frac{4\times50}{3\pi} = 21.22$	-41.66×10^3	-41.66×10^3
3	semi-circle CHD	$-\frac{\pi}{2}\times25^2 = -981.75$	$200-\frac{4\times25}{3\pi} = 189.39$	25	-185.93×10^3	-24.54×10^3
4	Triangle AFB	$\frac{1}{2}\times80\times120 = 4.8\times10^3$	$\frac{2}{3}\times80 = 53.33$	$50+\frac{1}{3}\times120 = 90$	255.98×10^3	432×10^3
5	Triangle CFB	$\frac{1}{2}\times120\times120 = 7.2\times10^3$	$80+\frac{1}{3}\times120 = 120$	$50+\frac{1}{3}\times120 = 90$	864×10^3	648×10^3
	Total	$\Sigma A = 19.06\times10^3$	—	—	$\Sigma A\bar{x} = 1882.39\times10^3$	$\Sigma A\bar{y} = 1263.8\times10^3$

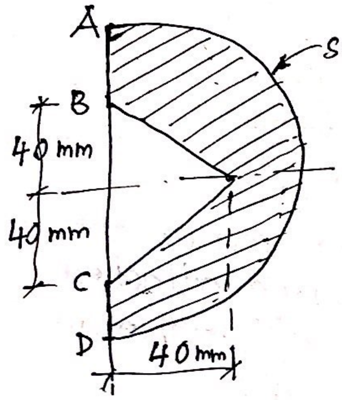
Now, $\bar{x} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{1882.39\times\cancel{10^3}}{19.06\times\cancel{10^3}} = 98.76$ mm

and $\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{1263.8\times\cancel{10^3}}{19.06\times\cancel{10^3}} = 66.31$ mm

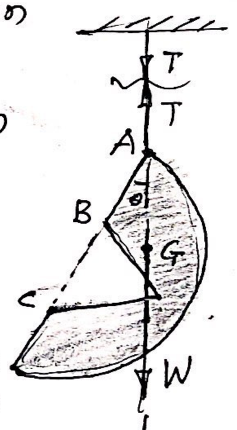
Ans: The centroid of the shaded area is,

$G \equiv (98.76 \text{ mm}, 66.31 \text{ mm})$

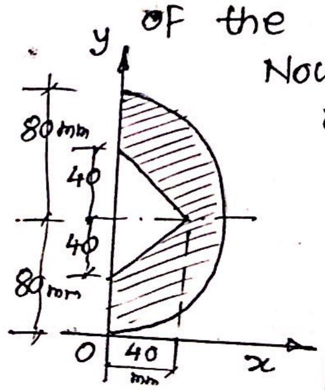
Ex. No. 5 The machine component in the form of a semi-circular plate of uniform thickness is free suspended from point A. Find the angle made by edge AD of the component with vertical in the position of equilibrium.



Solution: When suspended from point A with the help of a string, the machine component will take position as shown in figure ∴ In the position of equilibrium, the centroid will lie on a vertical line colinear to the string attached at A. So that the tension in the string ~~wi~~ (T) will be equal to the weight (W) of the plate. In this position



$\theta = m\angle BAG$ = Angle made by the edge AD of the component with vertical



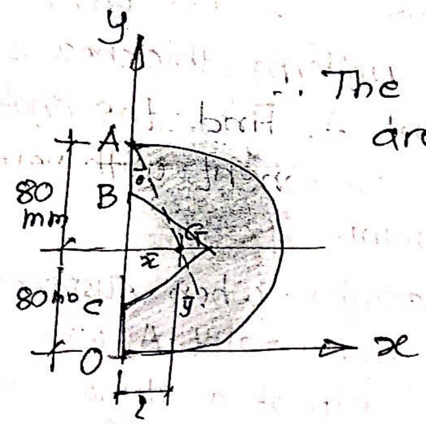
Now, Locate the centroid by tabular method.

Sr. No.	Name	Area mm^2	$\bar{x}$ mm	$\bar{y}$ mm	$A\bar{x}$ mm^3	$A\bar{y}$ mm^3
1	Semi-Circle	$\frac{\pi}{2}\times 80^2 = 10.05\times10^3$	$\frac{4\times80}{3\pi} = 33.95$	80	341.19×10^3	804×10^3
2	Triangle	$-\frac{1}{2}\times80\times40 = -1.6\times10^3$	$\frac{1}{3}\times40 = 13.33$	80	-21.33×10^3	-128×10^3
	Total	$\Sigma A = 8.45\times10^3$	–	–	$\Sigma A\bar{x} = 319.86\times10^3$	$\Sigma A\bar{y} = 676\times10^3$

$$\therefore \bar{x} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{319.86\times\cancel{10^3}}{8.45\times\cancel{10^3}} = 37.85 \text{ mm}$$

$$\therefore \bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{676\times\cancel{10^3}}{8.45\times\cancel{10^3}} = 80 \text{ mm}$$

P.T.O.



$\bar{x} = 37.85$ mm

$\therefore$ The centroid of the shaded area is, $G \equiv (37.85 \text{ mm}, 80 \text{ mm})$

From the figure ... shown here,

$$\tan\theta = \left(\frac{\bar{x}}{160 - \bar{y}}\right)$$

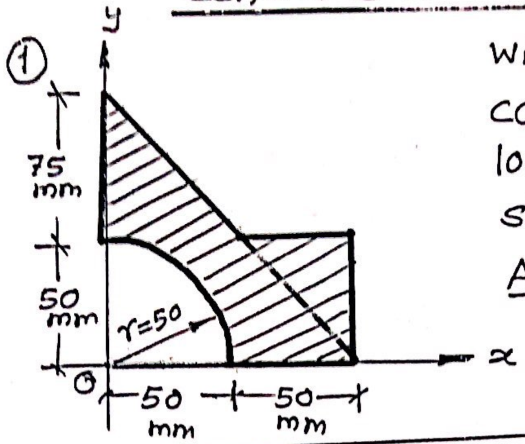
$$\therefore \tan\theta = \left(\frac{37.85}{160 - 80}\right) = 0.473$$

$$\therefore \theta = 25.32^\circ$$

<u>Ans</u>: When the ~~ma~~ given machine component is held in equilibrium by a string attached at point A on it, edge AD will make an angle of 25.32° with the vertical.

Centroids of Laminae (2-D)

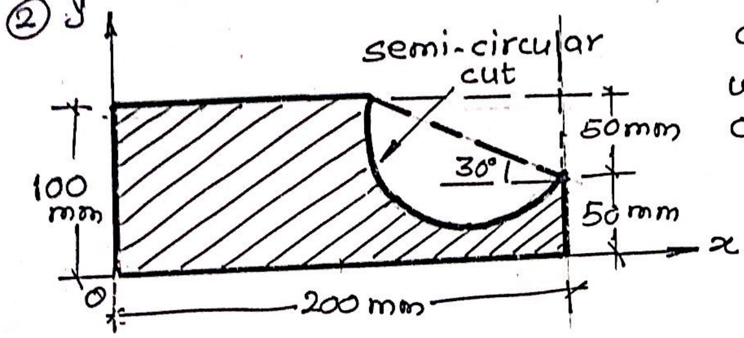
1.



With respect to the co-ordinate axes x and y, locate the centroid of the shaded area shown in fig.

Ans: $\bar{x} = 48$ mm
$\bar{y} = 47.75$ mm

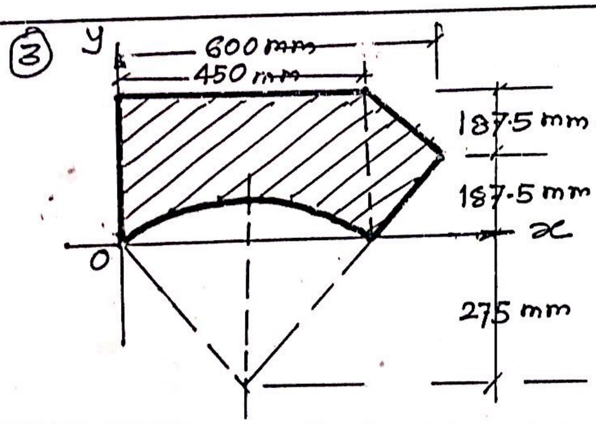
2.



Locate the centroid of shaded lamina w.r.t. the frame of ref. shown in fig.

Ans: $\bar{x} = 75.58$ mm
$\bar{y} = 42.94$ mm

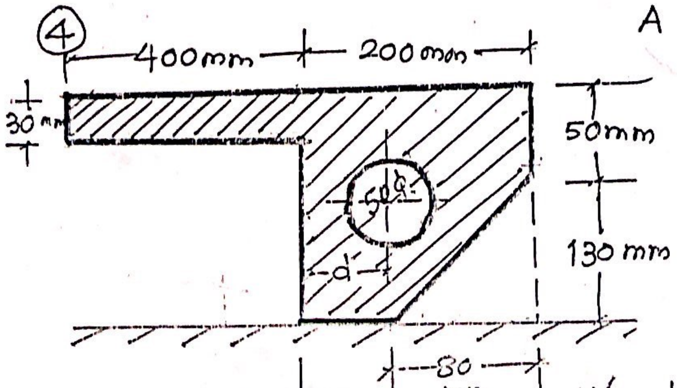
3.



Find the centroidal co-ordinates of the shaded lamina w.r.t. the frame of ref. shown in fig.

Ans: $\bar{x} = 270$ mm
$\bar{y} = 210$ mm

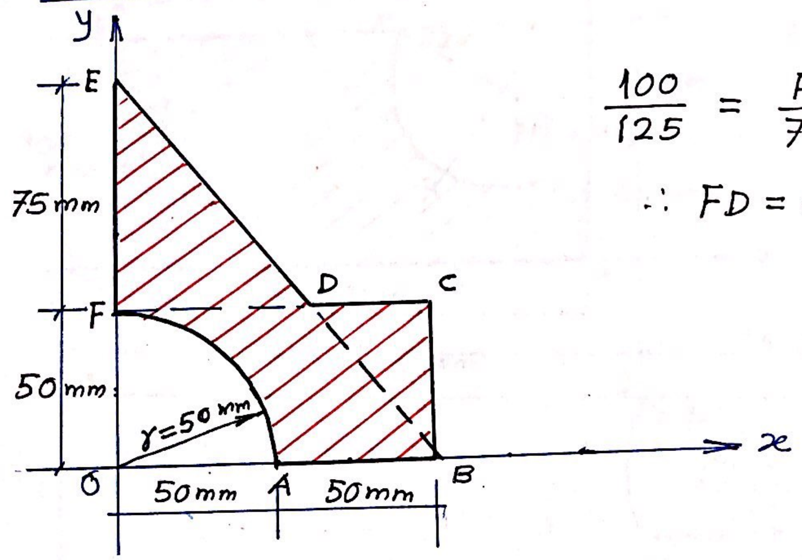
4.



A metallic plate of uniform thickness is to be cut in the dimensions as shown in fig. Find the distance 'd' of the center of the circle of diameter 50 mm from the vertical face of the job so that when the job is placed on the horizontal surface as shown in fig. tipping is prevented. Ans: d = 152.1 mm

Centroids of Laminar Objects

(1)



$$\frac{100}{125} = \frac{FD}{75}$$

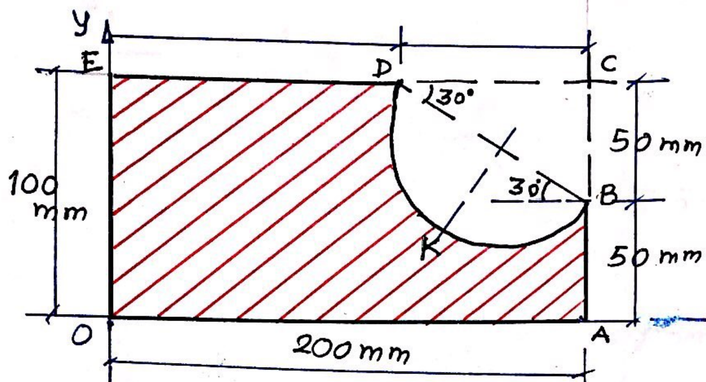
$$\therefore FD = 60\,mm$$

Sr. No.	Name	Area 'A' mm^2	$\bar{x}$ mm	$\bar{y}$ mm	$A\cdot\bar{x}$ mm^3	$A\cdot\bar{y}$ mm^3
1	Rectangle OBCF	$(100\times50) = 5000$	50	25	250×10^3	125×10^3
2	Triangle EFD	$(\frac{1}{2}\times60\times75) = 2,250$	$\frac{2}{3}\times60 = 20$	$50+\frac{75}{3} = 75$	45×10^3	168.75×10^3
3	Quarter circle AOF	$-(\frac{\pi}{4}\times50^2) = -1963.5$	$(\frac{4\times50}{3\pi}) = 21.22$	$(\frac{4\times50}{3\pi}) = 21.22$	-41.66×10^3	-41.66×10^3
	Total	$\Sigma A = 5,286.5$			$\Sigma A\bar{x} = 253.34\times10^3$	$\Sigma A\bar{y} = 252.09\times10^3$

$$\bar{X} = \left(\frac{\Sigma A\bar{x}}{\Sigma A}\right) = \left(\frac{253.34\times10^3}{5,286.5}\right) = 47.922\ mm$$

$$\bar{Y} = \left(\frac{\Sigma A\bar{y}}{\Sigma A}\right) = \left(\frac{252.09\times10^3}{5,286.5}\right) = 47.69\ mm$$

(2)

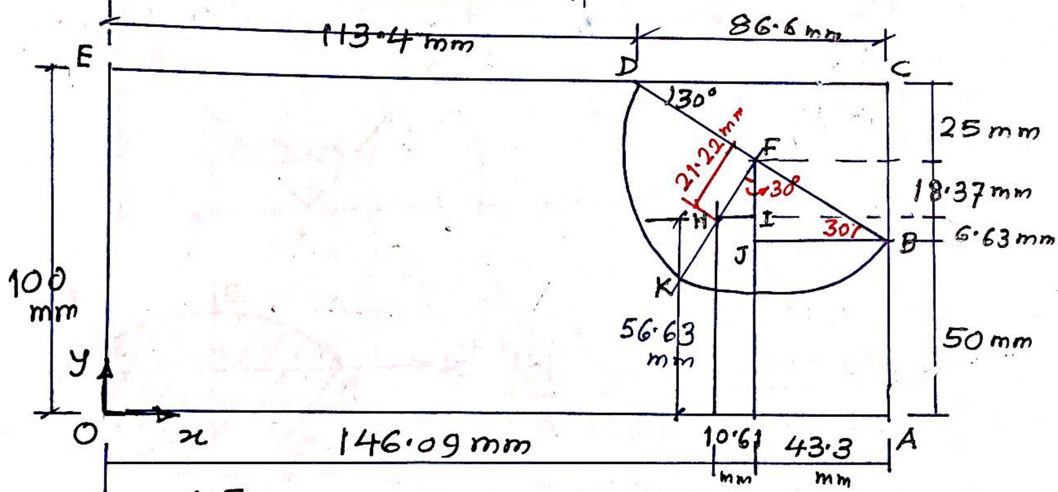


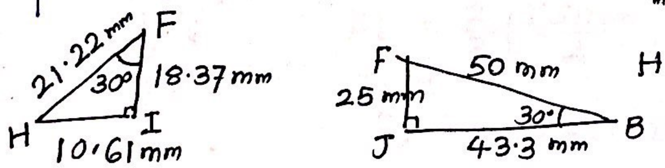
In ΔBCD,

$$\sin 30° = \frac{50}{BD}$$

$BD = 100\,mm$

$\therefore$ Radius of the semi-circle $= 50\,mm$





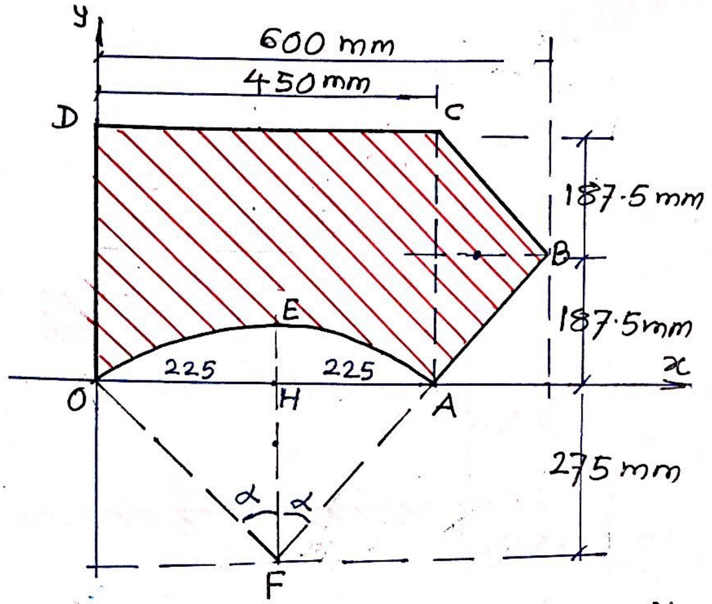
$H \equiv (146.09\,mm,\ 56.63\,mm)$

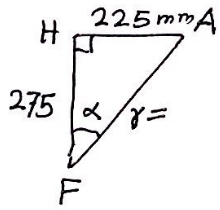
Sr. No.	Name	Area 'A' mm^2	$\bar{x}$ mm	$\bar{y}$ mm	$A\bar{x}$ (mm^3)	$A\bar{y}$ (mm^3)
1	Rectangle OACE	$200\times100 = 20\times10^3$	100	50	2×10^6	1×10^6
2	Triangle BCD	$-(\frac{1}{2}\times86.6\times50) = -2.165\times10^3$	$(200-\frac{86.6}{3}) = 171.13$	$(100-\frac{50}{3}) = 83.31$	-370.5×10^3	-180.36×10^3
3	semi-circle BKD	$-(\pi r^2/2) = -3.93\times10^3$	146.09	56.63	-574.13×10^3	-222.55×10^3
	Total:	$\Sigma A = 13.905\times10^3$			$\Sigma A\bar{x} = 1.055\times10^6$	$\Sigma A\bar{y} = 597.08\times10^3$

$$\bar{x} = \left(\frac{\Sigma A\bar{x}}{\Sigma A}\right) = \left(\frac{1.055\times10^6}{13.905\times10^3}\right) = 75.87\,mm$$

$$\bar{y} = \left(\frac{\Sigma A\bar{y}}{\Sigma A}\right) = \left(\frac{597.08\times10^3}{13.905\times10^3}\right) = 42.94\,mm$$

(3)





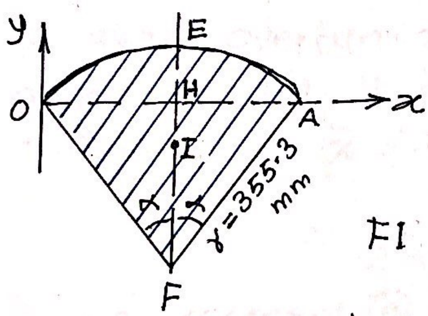
$$\tan\alpha = \left(\frac{225}{275}\right)$$

$$\alpha = 39.29^\circ = 0.686^c$$

$$\theta = 2\alpha = 78.58^\circ = 1.371^c$$

$$\gamma = \sqrt{275^2 + 225^2}$$

$$\gamma = 355.3 \text{ mm}$$



For the sector of a circle:

$\gamma = 355.3$ mm, $\theta = 78.58^\circ$, $\alpha = 39.29^\circ$

$$FI = \frac{2}{3}\left(\frac{\gamma\sin\alpha}{\alpha}\right) = \frac{2}{3}\left(\frac{355.3 \times \sin 39.29^\circ}{0.686}\right)$$

$\therefore$ FI = 218.65 mm $\therefore$ IH = 56.35 mm

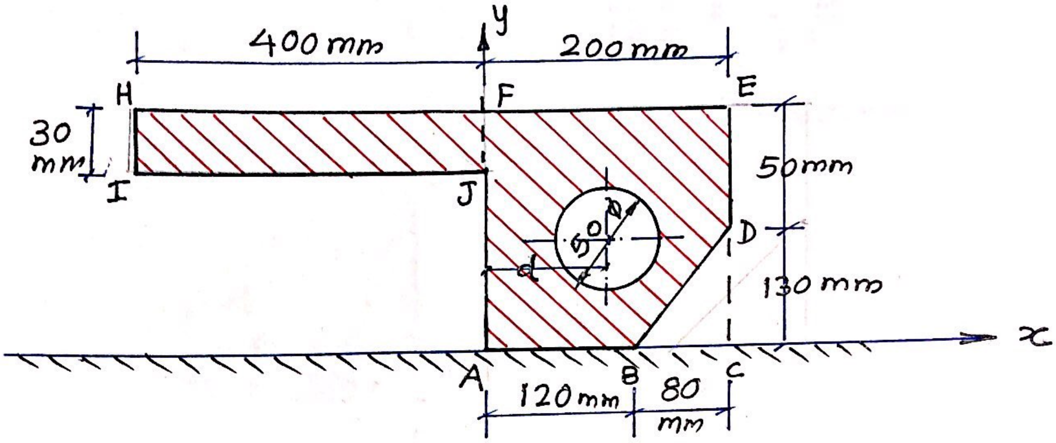
$$\text{Area of sector FOEA} = \left(\frac{78.58}{360} \times \pi \times 355.3^2\right) = 86.56 \times 10^3 \text{ mm}^2$$

Sr. No.	Name	Area 'A' mm^2	x mm	y mm	$A\bar{x}$ mm^3	$A\bar{y}$ mm^3
1	Rectangle OACD	168.75×10^3	225	187.5	37.97×10^6	31.64×10^6
2	Triangle ABC	28.125×10^3	450+50 = 500	187.5	14.06×10^6	5.27×10^6
3	Triangle OFA	61.875×10^3	225	−91.66	13.92×10^6	-5.67×10^6
4	Sector FOEA	-86.56×10^3	225	−56.35	-19.476×10^6	4.88×10^6
	Total:	$\Sigma A = 172.19 \times 10^3$			$\Sigma A\bar{x} = 46.474 \times 10^6$	$\Sigma A\bar{y} = 36.12 \times 10^6$

$$\bar{x} = \left(\frac{\Sigma A\bar{x}}{\Sigma A}\right) = \left(\frac{46.474 \times 10^6}{172.19 \times 10^3}\right) = 269.9 \text{ mm}$$

$$\bar{y} = \left(\frac{\Sigma A\bar{y}}{\Sigma A}\right) = \left(\frac{36.12 \times 10^6}{172.19 \times 10^3}\right) = 209.75 \text{ mm}$$

4



To avoid overturning of the job, the centroid must lie on 'y' axis i.e. on the face AJ of the job. $\therefore\ \bar{x} = \left(\frac{\Sigma A \bar{x}}{\Sigma A}\right) = 0$

$$\therefore\ \Sigma A \cdot \bar{x} = 0$$

i.e. moment @ 'y' axis is zero.

$$\therefore\ (400 \times 30)(-200) + (200 \times 180)(100) - \left(\tfrac{1}{2} \times 80 \times 130\right)\left(120 + \tfrac{2}{3} \times 80\right) - \left(\tfrac{\pi}{4} \times 50^2\right) \cdot d = 0$$

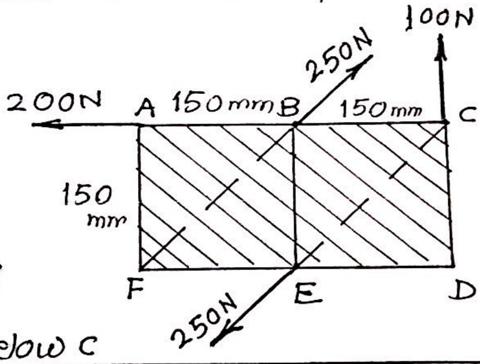
$$0 = -(2.4 \times 10^6) + (3.6 \times 10^6) - (901.33 \times 10^3) - (1.963 \times 10^3) d$$

$$\therefore\ (1.963 \times 10^3) d = 298.67 \times 10^3$$

$$* \ d = 152.1 \text{ mm}$$

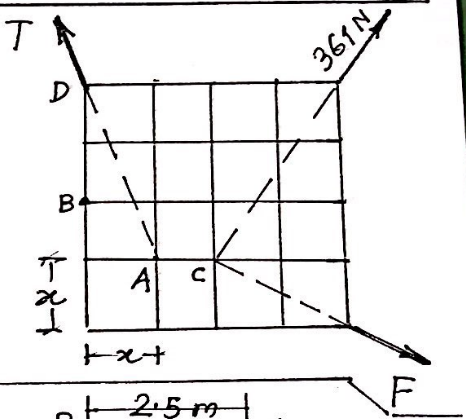
Resultant of General Coplanar Force Systems (6)

(1) Find the resultant of the force system acting on the plate and locate its position w.r.t. corner 'C'.



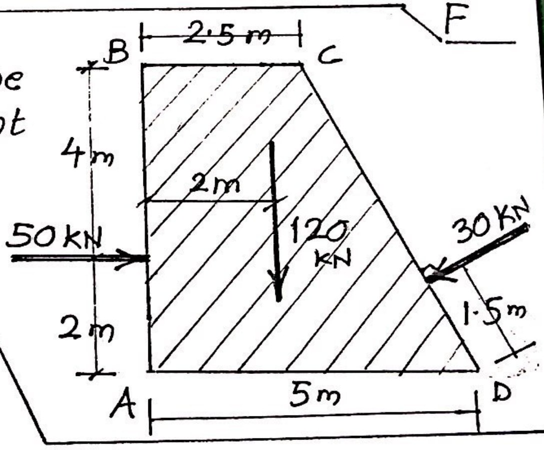
Ans: $R = 223.6\ N, \theta = 26.56^\circ$ in the 2nd quadrent along line PQ. CP = 265.02 mm left on C, CQ = 132.48 mm below C

(2) The three forces acting on the square plate, have vertical resultant force, passing through point B. Find the values of Forces 'T' and 'F'.



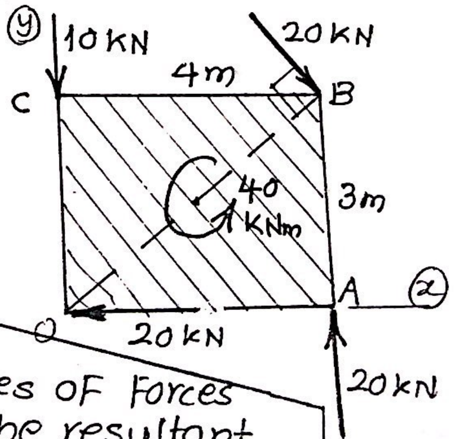
Ans: $T = -1279.53\ N$
$F = -678.89\ N$

(3) A dam is subjected to three forces as shown in fig. Determine the resultant force and its point of intersection with the base AD.

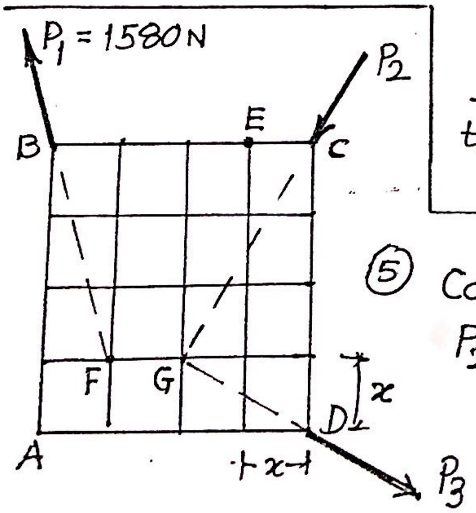


Ans: $R = 133.4\ kN$ at $\theta = 80.4^\circ$
AP = 2.68 m

(4) Find the resultant of the force system acting on the plate and locate its position.



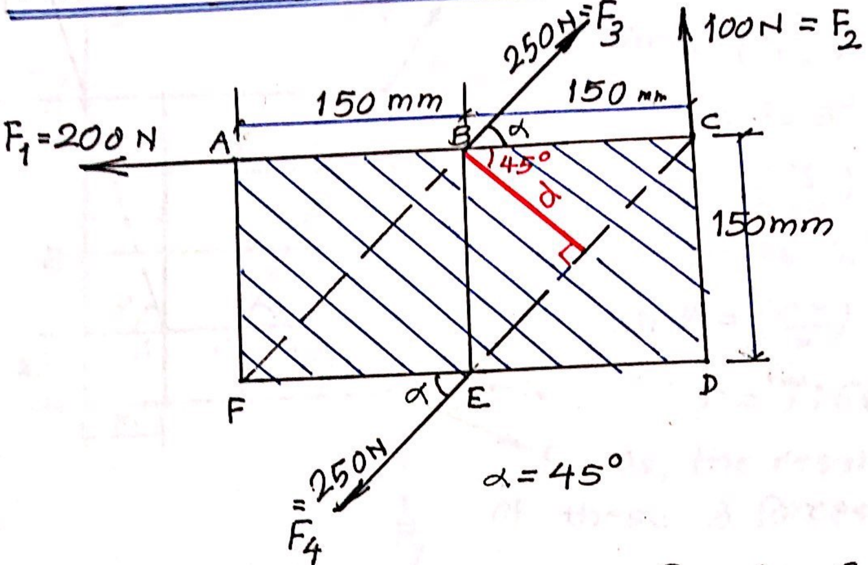
Ans: $R = 10\ kN$ at
$\theta = 36.86^\circ$
3rd quadrent.
$\perp$ dist. $d = 2m$ to the left of O.



(5) Calculate the values of forces P_2 & P_3 so that the resultant of the three forces P_1, P_2 & P_3 will be a horizontal force at E.
Ans: $P_2 = 901\ N$, $P_3 = 1675.7\ N$

Resultant of General Coplanar Force System

(1)



$\alpha = 45^\circ$

Here, forces 'F_3' and 'F_4' are forming a couple.

$\cos 45^\circ = \left(\frac{d}{150}\right)$ $\therefore$ $d = 106.06$ mm

$M = (250 \times 106.06) = 26{,}516.5$ Nmm

Now, $R_x = \Sigma F_x = -200$ N, $R_y = \Sigma F_y = +100$ N } Rectangular component of the resultant force

$\therefore$ $\bar{R} = R_x \hat{i} + R_y \hat{j}$ N

$R = 223.6$ N

$25.56^\circ = \theta$

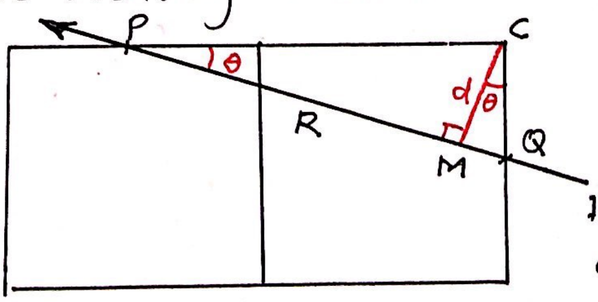
* $\boxed{\bar{R} = -(200)\hat{i} + (100)\hat{j} \text{ N}}$

Resultant force in vector form and polar form

To locate the point of application of the resultant force, use Varignon's thm. of moments. Taking moments @ C, $(223.6)d = -26{,}516.5$ Nmm

$d = -118.58$ mm. As 'd' is $-$ve, the resultant is rotating in clockwise @ C

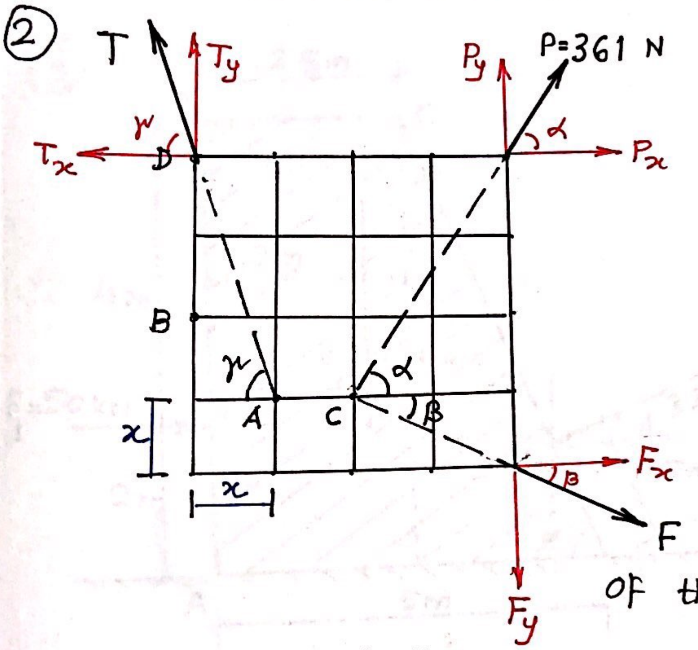
Ans:



In ΔCMP,
$\sin 25.56^\circ = \left(\frac{118.58}{CP}\right)$

$CP = 275$ mm

In ΔCMQ, $CQ = 131.4$ mm
$\cos 25.56^\circ = \left(\frac{118.58}{CQ}\right)$

② 

$\tan\alpha = \left(\frac{3x}{2x}\right)$

$\therefore \alpha = 56.3^\circ$

$\tan\beta = \left(\frac{x}{2x}\right)$

$\therefore \beta = 26.56^\circ$

$\tan\gamma = \left(\frac{3x}{x}\right)$

$\therefore \gamma = 71.56^\circ$

As, the resultant of these 3 forces is vertical in dir^n, $\Sigma f_x = R_x = 0$

$\therefore \Sigma F_x = (361)(\cos 56.3^\circ) + F(\cos 26.56^\circ) - T \cdot \cos(71.56^\circ) = 0$

$\therefore (0.894)F - (0.316)T = -200.298 \rightarrow ①$

As, the resultant is a vertical force, through pt. B, it will also pass through D. Using Varignon's thm. of moments, taking moments @ D,

$0 = (361)(\sin 56.3^\circ)\cancel{(4x)} + (F)(\cos 26.56^\circ)\cancel{(4x)}$

$- (F)(\sin 26.56^\circ)\cancel{(4x)} \longrightarrow ②$

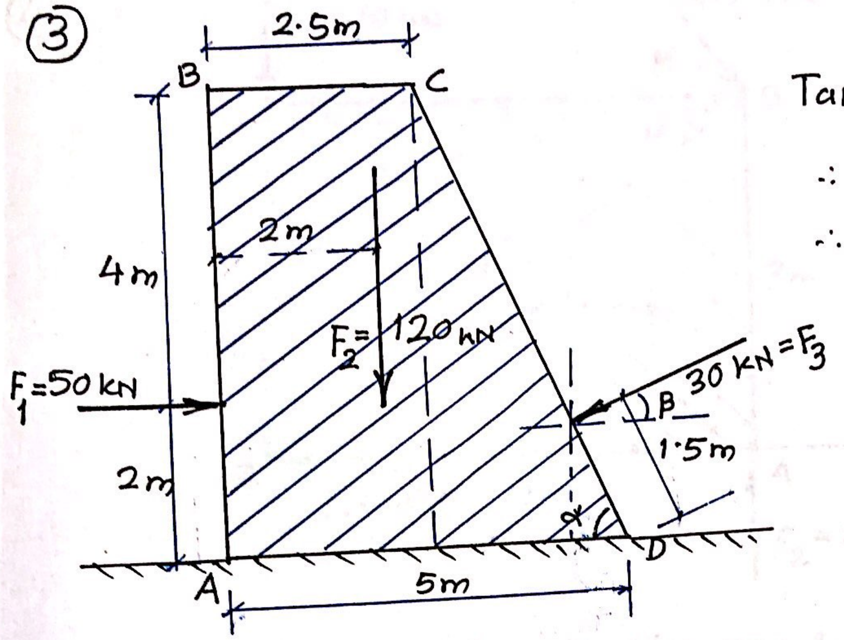
$-(0.447)F + (0.894)F = -300.335$

$(0.447)F = -300.335$

Ans: $\therefore$ * $F = -671.89$ N, and from eq^n ①

we get, $\therefore$ * $T = -1267$ N

(3)



$\tan\alpha = \frac{6}{2.5}$

$\therefore \alpha = 67.38°$

$\therefore \beta = (90-\alpha)$
$= 22.62°$

$R_x = \Sigma F_x = (50) - (30)(\cos 22.62°) = 22.307$ kN $(\rightarrow)$

$R_y = \Sigma F_y = -(120) - (30)(\sin 22.62°) = -131.538$ kN
$= 131.538$ kN $(\downarrow)$

$\therefore$ * $\bar{R} = (22.307)\hat{i} - (131.538)\hat{j}$ kN

$\theta = 80.37°$

* $R = 133.416$ kN

To locate the point of application of the resultant force, use Varignon's thm. of moments.

Take, moment @ A,

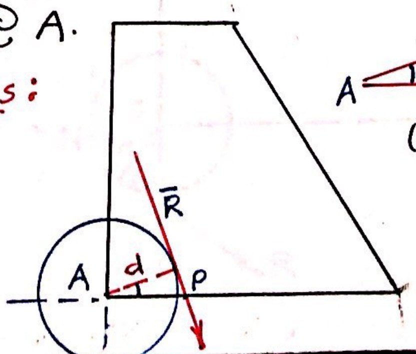
$(133.416)d = -(50\times 2) - (120\times 2) + (30\times\cos 22.62°)(1.5\times\sin 67.38°)$
$- (30\times\sin 22.62°)(5 - 1.5\times\cos 67.38°)$

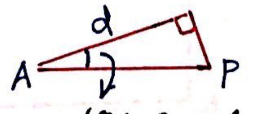
$\therefore (133.416)d = -(100) - (240) + (38.343) - (51.035)$
$= -(352.69)$

$d = -2.643$ m

As 'd' is -ve, the resultant is rotating in clockwise direction @ A.

Ans :

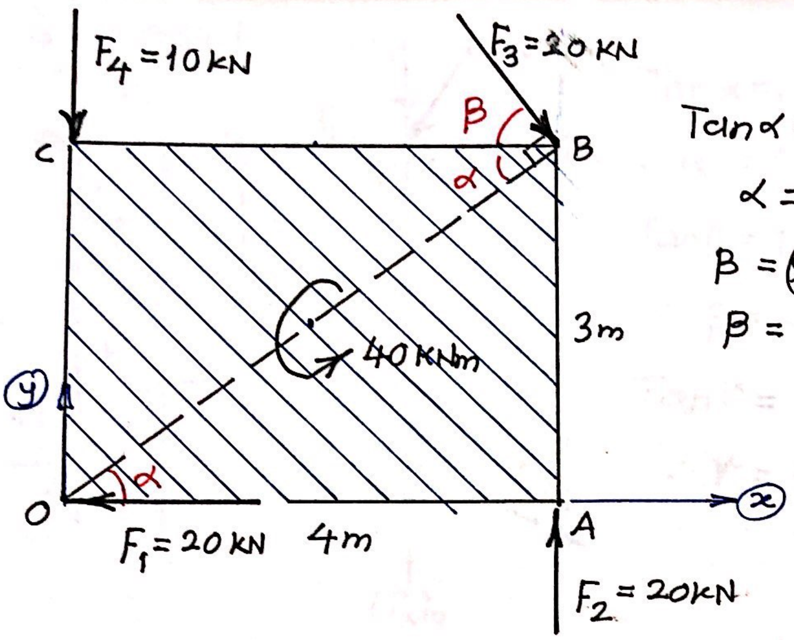




$\cos 9.63° = \frac{2.643}{AP}$

* $AP = 2.68$ m

(4)



$\tan\alpha = \frac{3}{4}$

$\alpha = 36.87°$

$\beta = (90-\alpha)$

$\beta = 53.13°$

$R_x = \Sigma F_x = -20 + (20)(\cos 53.13°) = -8$ kN

$= 8$ kN $(\leftarrow)$

$R_y = \Sigma F_y = 20 - (20)(\sin 53.13°) - 10 = -6$ kN

$= 6$ kN $(\downarrow)$

* $\bar{R} = -8\hat{i} - 6\hat{j}$ kN * $\theta = 36.86°$, $R = 10$ kN

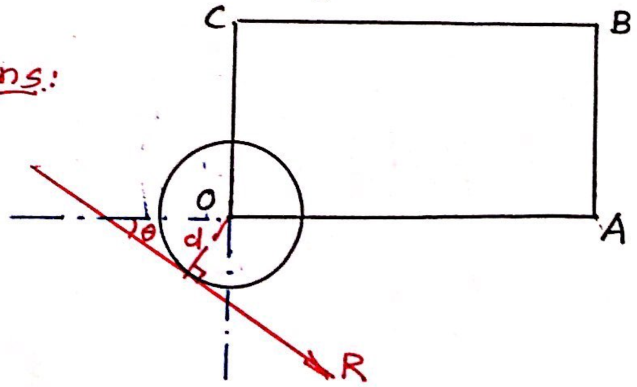
To locate the point of application of the resultant force, use Varignon's thm. of moments. Take moments @ O.

$(10)\cdot d = (20\times4) - (20)(\sqrt{3^2+4^2}) + (40)$

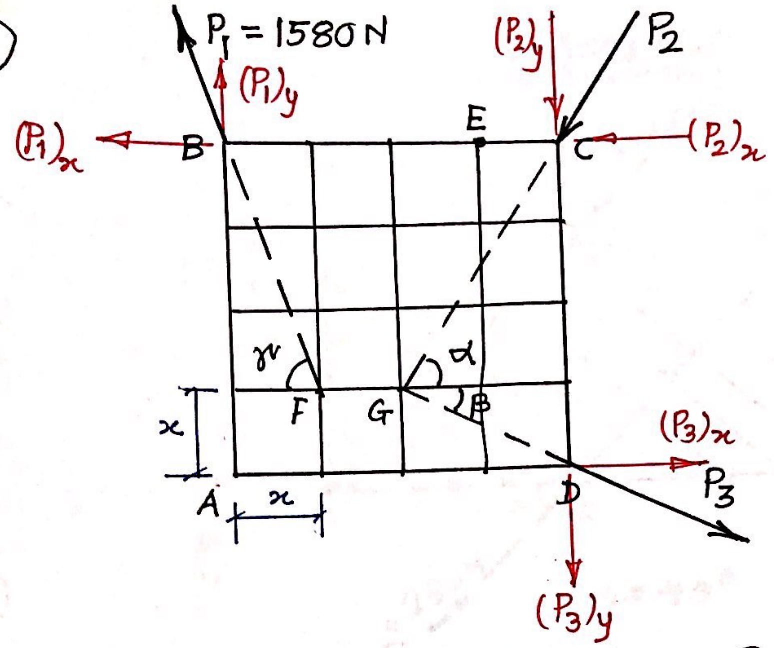
$= (80) - (100) + (40) = 20$ kNm

* $d = 2$ m, As 'd' is +ve, the resultant is rotating in anticlockwise dir^n @ O.

Ans:



5



$\tan\alpha = \left(\frac{3x}{2x}\right)$

$\therefore \alpha = 56.3^\circ$

$\tan\beta = \left(\frac{x}{2x}\right)$

$\therefore \beta = 26.56^\circ$

$\tan\gamma = \left(\frac{3x}{x}\right)$

$\therefore \gamma = 71.56^\circ$

As, the resultant of these 3 forces is horizontal,

$R_y = \Sigma F_y = 0$

$\therefore 0 = (1580)(\sin 71.56^\circ) - (\sin 56.3^\circ) \times P_2 - P_3 \times (\sin 26.56^\circ)$

$-(0.832)P_2 - (0.447)P_3 = -1498.88 \rightarrow (1)$

As, the resultant is a horizontal force through point E, it will also pass through pt. C. Using Varignon's thm. of moments, taking moments @C,

$0 = -(1580)(\sin 71.56^\circ)\cancel{(4x)} + P_3(\cos 26.56^\circ)\cancel{(4x)}$

$0 = -(1498.88) + (0.894)P_3 \rightarrow (2)$

Ans: * $P_3 = 1675.72$ N

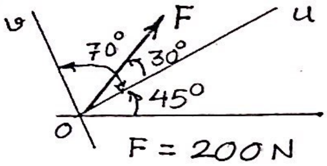
$\therefore$ * $P_2 = 901.24$ N

Scanned with CamScanner

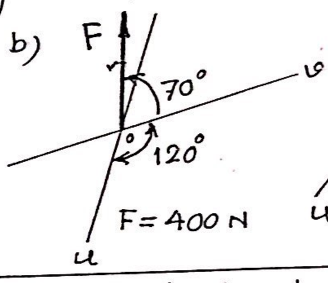
Resolution and Composition of forces

(1) Show how to resolve the force 'F' into components acting along the 'u' and 'v' axes using the Law of Parallelogram of forces.

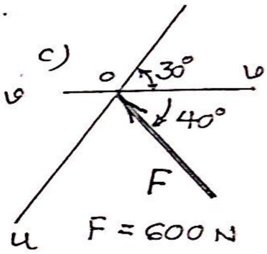
a)



b)

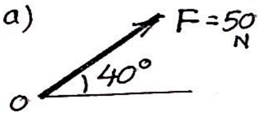


c)

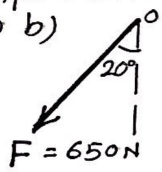


(2) Resolve the given forces into horizontal and vertical components.

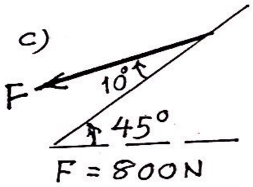
a)



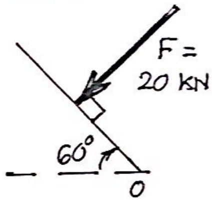
b)



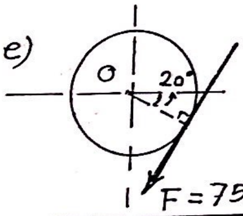
c)



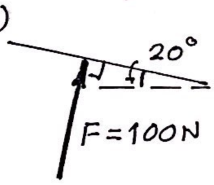
d)



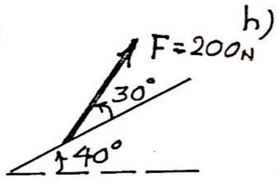
e)



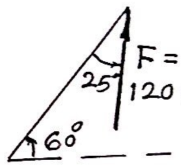
f)



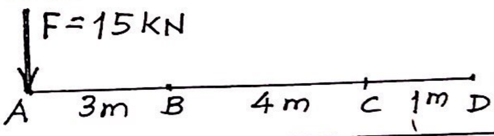
g)



h)



(3) Resolve a force of 15 kN acting at A, into two parallel components at B and C.

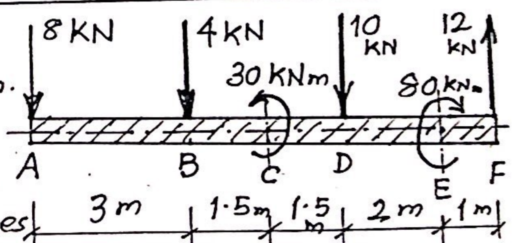


(4) Fig. shows a force system acting on a bar of length 9 m.

a) Replace it by a single force and obtain its location w.r.t. A.

b) Replace it by two parallel forces at B & D

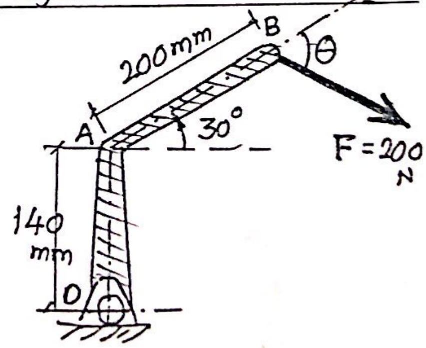
c) Replace it by a force-couple system at D.



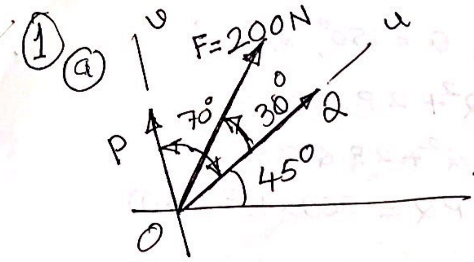
(5) Determine angle θ, which will maximize the moment 'M_O' of the 200 N force about the shaft axis at origin O.

Ans : $\theta = 65.8^\circ$

$M_O = 59.2$ Nm CW



1 (a)



$R = 200\,N$, $\theta = 70^\circ$, $\alpha = 30^\circ$

$$R^2 = P^2 + Q^2 + 2PQ\cdot\cos\theta$$

$$200^2 = P^2 + Q^2 + 2P.Q\cdot\cos 70^\circ$$

$$P^2 + Q^2 + (0.684)PQ = 40\times10^3 \rightarrow (1)$$

$$\tan\alpha = \left(\frac{P\sin\theta}{Q + P\cos\theta}\right) \quad \therefore \tan 30^\circ = \left(\frac{P\sin 70^\circ}{Q + Q\cos 70^\circ}\right)$$

$$\therefore 0.577 = \left[\frac{(0.94)P}{Q + (0.342)P}\right]$$

$$(0.577)Q + (0.197)P = (0.94)P$$

$$(0.577)Q = (0.743)P$$

$$Q = (1.287)P \longrightarrow (2)$$

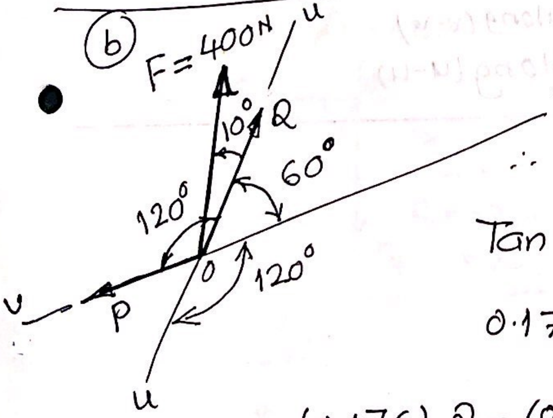
$$\therefore P^2 + (1.656)P^2 + (0.88)P^2 = 40\times10^3$$

$$(3.536)P^2 = 40\times10^3$$

Ans: $P = 106.35\,N$ along line $(v-v)$

$\therefore Q = 136.88\,N$ along line $(u-u)$

(b)



$R = 400\,N$, $\theta = 120^\circ$, $\alpha = 10^\circ$

$$(400)^2 = P^2 + Q^2 + 2\cdot P\cdot Q\cdot\cos 120^\circ$$

$$\therefore P^2 + Q^2 - PQ = 160\times10^3 \longrightarrow (1)$$

$$\tan 10^\circ = \left[\frac{P\sin 120^\circ}{Q + P\cos 120^\circ}\right]$$

$$0.176 = \left[\frac{(0.866)P}{Q - (0.5)P}\right]$$

$$\therefore (0.176)Q - (0.088)P = (0.866)P$$

$$(0.176)Q = (0.954)P$$

$$Q = (5.42)P \longrightarrow (2)$$

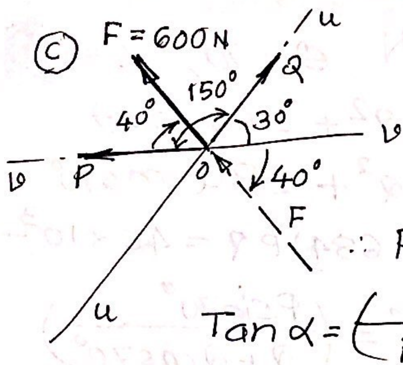
$$\therefore P^2 + (29.38)P^2 - (5.42)P^2 = 160\times10^3$$

$$(24.96)P^2 = 160\times10^3$$

Ans: $P = 80.06\,N$ along $(u-u)$

$Q = 433.94\,N$ along $(v-v)$

(c)



$R = 600N, \ \theta = 150°, \ \alpha = 40°$

$R^2 = P^2 + Q^2 + 2P \cdot Q \cdot \cos\theta$

$(600)^2 = P^2 + Q^2 + 2P \cdot Q \cdot \cos 150°$

$\therefore P^2 + Q^2 - (1.732)PQ = 360 \times 10^3 \longrightarrow (1)$

$\tan\alpha = \left(\frac{Q \cdot \sin\theta}{P + Q\cos\theta}\right)$

$\therefore \tan 40° = \left(\frac{Q \cdot \sin 150°}{P + Q \cdot \cos 150°}\right)$

$0.839 = \left[\frac{(0.5)Q}{P - (0.866)Q}\right]$

$\therefore (0.84)P - (0.723)Q = (0.5)Q$

$(0.84)P = (1.227)Q$

$\therefore Q = (0.685)P \longrightarrow (2)$

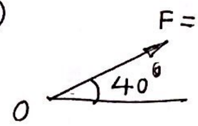
$\therefore P^2 + (0.469)P^2 - (1.186)P^2 = 360 \times 10^3$

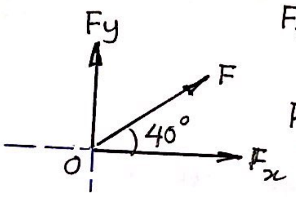
$(0.283)P^2 = 360 \times 10^3$

Ans: $P = 1128.13$ N along (v-v)

$\therefore Q = 772.77$ N along (u-u)

② (a)

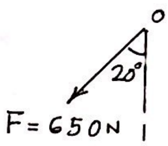


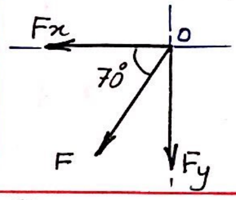


$F_x = F\cos 40°$
$= 38.302\text{ N } (\rightarrow)$

$F_y = F\sin 40°$
$= 32.139\text{ N } (\uparrow)$

(b)

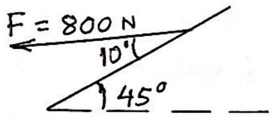


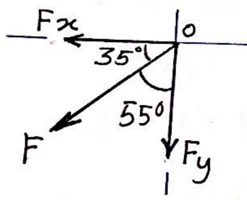


$F_x = -650 \cdot \cos 70°$
$= 222.31\text{ N } (\leftarrow)$

$F_y = -650 \cdot \sin 70°$
$= 618.8\text{ N } (\downarrow)$

(c)

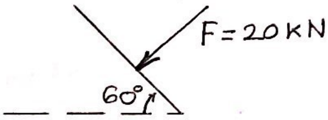


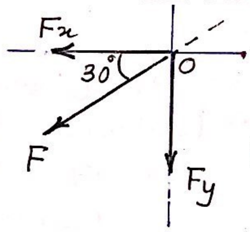


$F_x = -800 \cdot \cos 35°$
$= 655.32\text{ N } (\leftarrow)$

$F_y = -800 \cdot \sin 35°$
$= 458.86\text{ N } (\downarrow)$

(d)

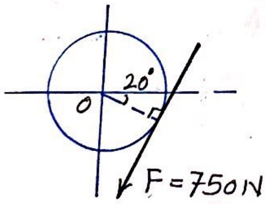


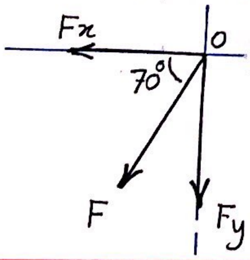


$F_x = -20 \cdot \cos 30°$
$= 17.32\text{ kN } (\leftarrow)$

$F_y = -20 \cdot \sin 30°$
$= 10\text{ kN } (\downarrow)$

(e)

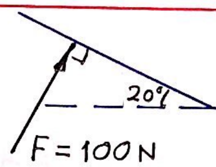


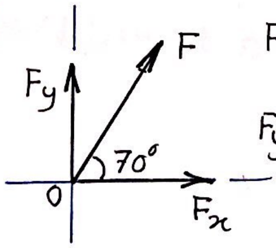


$F_x = -750 \cdot \cos 70°$
$= 256.52\text{ N } (\leftarrow)$

$F_y = -750 \cdot \sin 70°$
$= 704.77\text{ N } (\downarrow)$

(f)

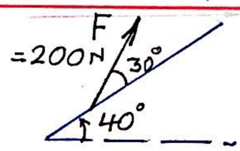


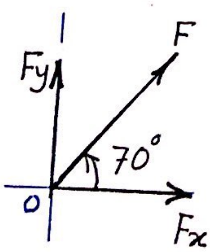


$F_x = 100 \cdot \cos 70°$
$= 34.2\text{ N } (\rightarrow)$

$F_y = 100 \cdot \sin 70°$
$= 93.97\text{ N } (\uparrow)$

(g)

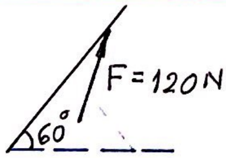


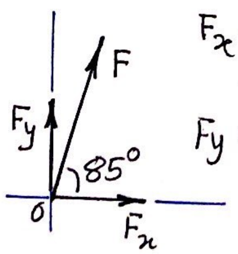


$F_x = 200 \cdot \cos 70°$
$= 68.4\text{ N } (\rightarrow)$

$F_y = 200 \cdot \sin 70°$
$= 187.94\text{ N } (\uparrow)$

(h)

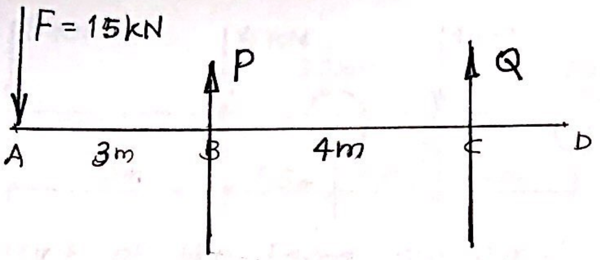




$F_x = 120 \cdot \cos 85°$
$= 10.459\text{ N } (\rightarrow)$

$F_y = 120 \cdot \sin 85°$
$= 119.54\text{ N } (\uparrow)$

(3)



'P' and 'Q' are components of 'F'.
'F' is the resultant of P & Q

$\therefore \quad P + Q = -15 \text{ kN} \longrightarrow (1)$

Using Varignon's theorem of moments,
taking moments @ B,

$$\begin{pmatrix}\text{moment of the} \\ \text{resultant @ B}\end{pmatrix} = \begin{pmatrix}\text{sum of the moments} \\ \text{of individual forces} \\ \text{@ pt. B}\end{pmatrix}$$

$$(15\times3) = (P\times0) + (Q\times4)$$

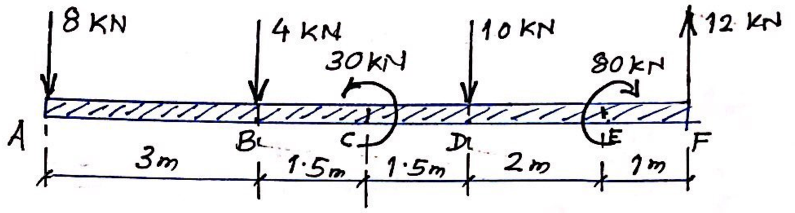
$$45 = 4Q$$

* $Q = 11{\cdot}25 \text{ kN } (\uparrow)$ at pt. C

$P = -15 - 11{\cdot}25 = -26{\cdot}25 \text{ kN}$

* $P = 26{\cdot}25 \text{ kN } (\downarrow)$ at pt. B.

④



a) Resultant of the force system:

$$R = -8-4-10+12 = -10 \text{ kN}$$

i.e. $R = 10$ kN (↓)

To Locate the point of application of the resultant force, Use Varignon's theorem of moments.

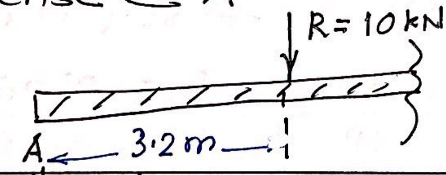
Taking moments @ A,

$$(10 \cdot d) = (8\times 0) - (4\times 3) - (10\times 6) + (10\times 9) + 30 - 80$$

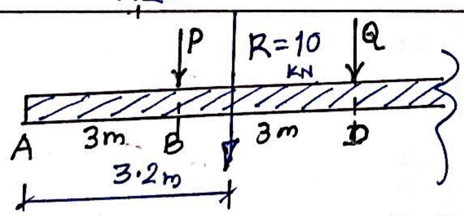
$$\therefore \quad 10 \cdot d = -12 - 60 + 90 + 30 - 80 = -32 \text{ kNm}$$

$$\therefore \quad d = -3.2 \text{ m}$$

As 'd' is −ve, the resultant is rotating in clock-wise sense @ A



b)



$$+P + Q = +10 \text{ kN} \rightarrow \text{(i)}$$

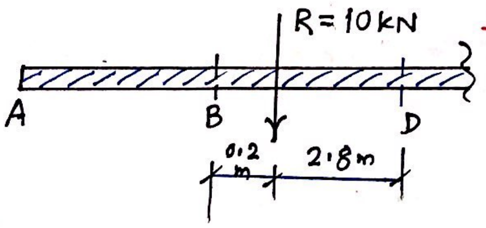
Using Varignon's thm, taking moments @ B

$$-(10\times 0.2) = (P\times 0) - 3Q$$

$$-2 = -3Q$$

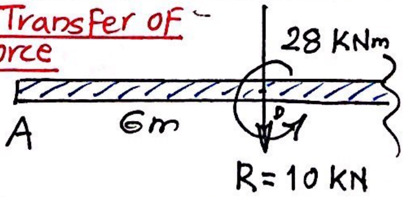
$\therefore$ $Q = 0.67$ kN (↓) at pt. D
$P = 9.33$ kN (↓) at pt B

c)



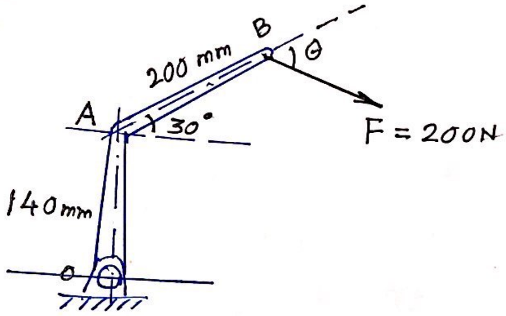
Parallel Transfer of a force

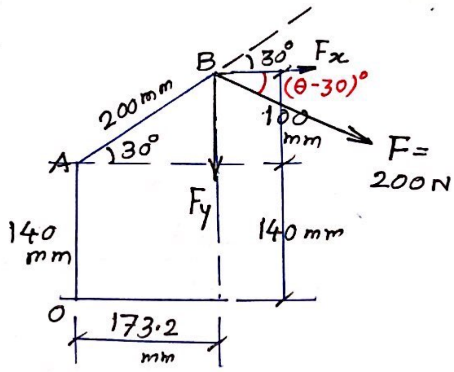
≡



$$M_D = (10\times 2.8) = 28 \text{ kNm}$$

(5)





$F_x = (200) \cdot \cos(\theta - 30)^\circ$, $F_y = (200) \cdot \sin(\theta - 30)^\circ$

Taking moments @ 'origin 'O',

$M_O = -(F_x)(240) - (F_y)(173.2)$

$\therefore M_O = -(48 \times 10^{+3}) \cdot \cos(\theta - 30)^\circ - (34.64 \times 10^{+3}) \cdot \sin(\theta - 30^\circ)$

For maximum M_O, $\frac{dM_O}{d\theta} = 0$

$\therefore (48 \times 10^{+3}) \cdot \sin(\theta - 30)^\circ - (34.64 \times 10^{+3}) \cdot \cos(\theta - 30)^\circ = 0$

$$\therefore \frac{\sin(\theta - 30)^\circ}{\cos(\theta - 30)^\circ} = \left(\frac{34.64 \times 10^{+3}}{48 \times 10^{+3}} \right) = 0.7216$$

$$\tan(\theta - 30)^\circ = 0.7216$$

$$\therefore (\theta - 30) = 35.82^\circ$$

$$\underline{\theta = 65.82^\circ}$$

$\therefore (M_O)_{max} = -(48 \times 10^{+3}) \cdot (\cos 35.82^\circ) - (34.64 \times 10^{+3}) \cdot \sin(35.82^\circ)$

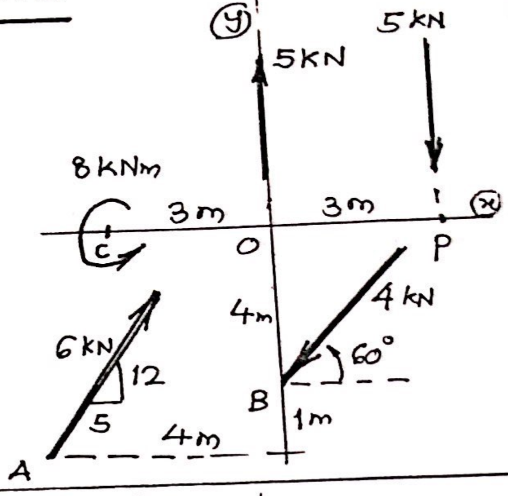
$= -(38.92 \times 10^3) - (20.27 \times 10^3)$

$= -59.192 \times 10^3$ Nmm

$\therefore \underline{(M_O)_{max} = 59.192 \text{ Nm}}$

Equivalent Systems, Resultant of concurrent and Parallel Force systems

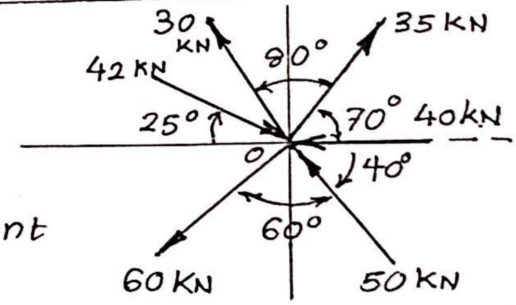
(1) Replace the force-couple system shown in fig. by an equivalent system at point 'P.'



Ans: $R = 2.096$ KN, $\theta = 81.58°$; $M_P = 31.838$ KNm

---

(2) Find the resultant of the force system, acting at O.



Ans: $R = 64.758$ KN
$\theta = 2.824°$ in 2nd quadrent

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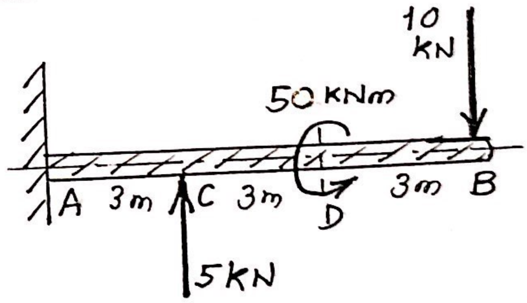
(3) Find the resultant of the force system, acting on the cantilever AB.



Ans: R = 5 KN (↓) at a distance of 5 m to the right of A.

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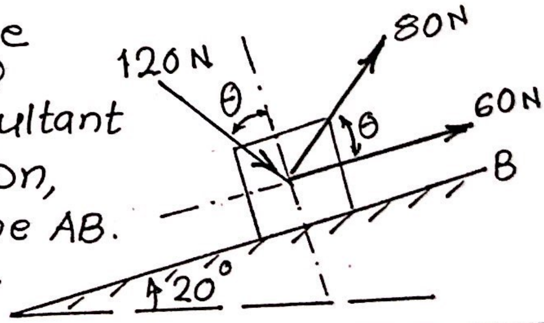
(4) For the block shown in the Fig. determine the required value of angle 'θ', if the resultant of the three forces shown, is to be parallel to the plane AB. Also, find the resultant force.



Ans: $\theta = 56.31°$ $R = 204.22$ N

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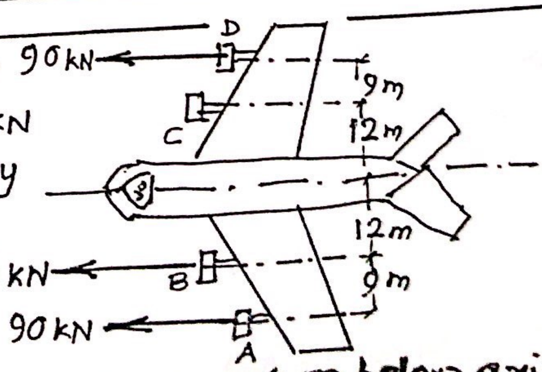
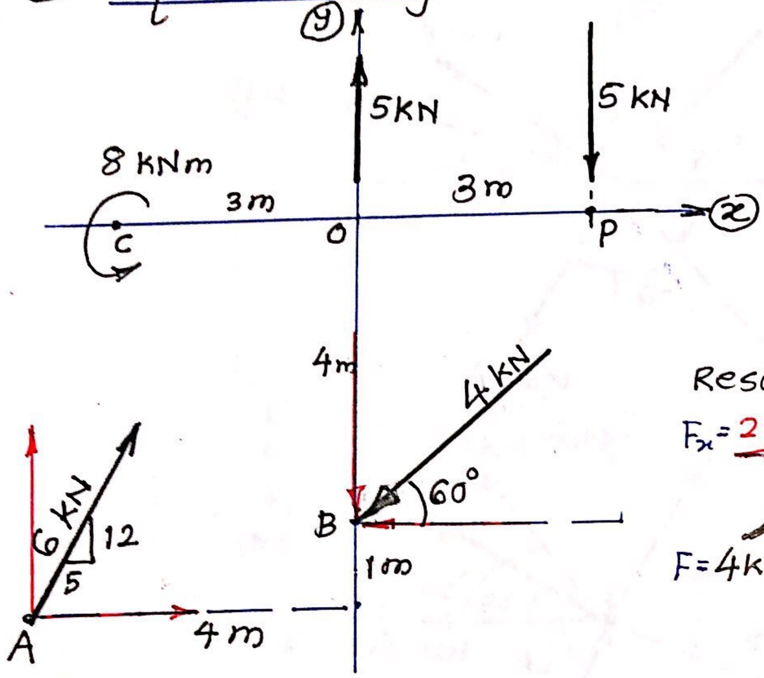
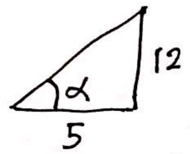
(5) A commercial airliner with four jet enjines, each producing 90 KN of forward thrust is in a steady level cruise, when engine at C suddenly fails. Determine & locate the resultant of the three remaining thrust forces.



Ans: R = 270 KN, 4 m below axis

① Equivalent system at P :

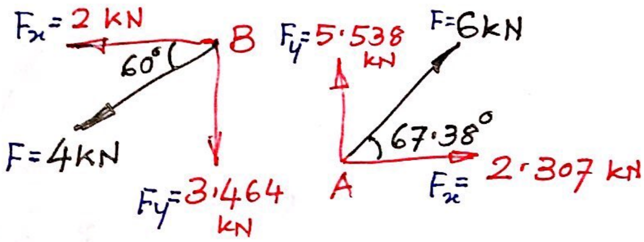




$\tan\alpha = \frac{12}{5}$

$\alpha = 67.38^\circ$

Resolution of forces :



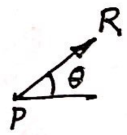
The equivalent system at point 'P' consists of
i) The Resultant Force at pt. P and
ii) The Resultant Moment at pt. P

$\therefore\ R_x = \Sigma F_x = (2.307) - (2.0) = 0.307\ \text{kN}\ (\rightarrow)$

$R_y = \Sigma F_y = (5.538) - (3.464) + 5 - 5 = 2.074\ \text{kN}\ (\uparrow)$

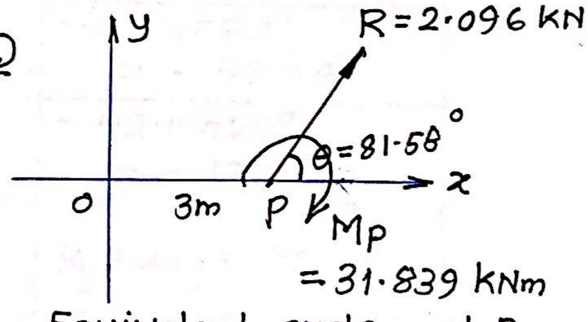
$\therefore$ * $R = \sqrt{R_x^2 + R_y^2} = 2.096\ \text{kN}$

* $\theta = \tan^{-1}(R_y/R_x) = 81.58^\circ$



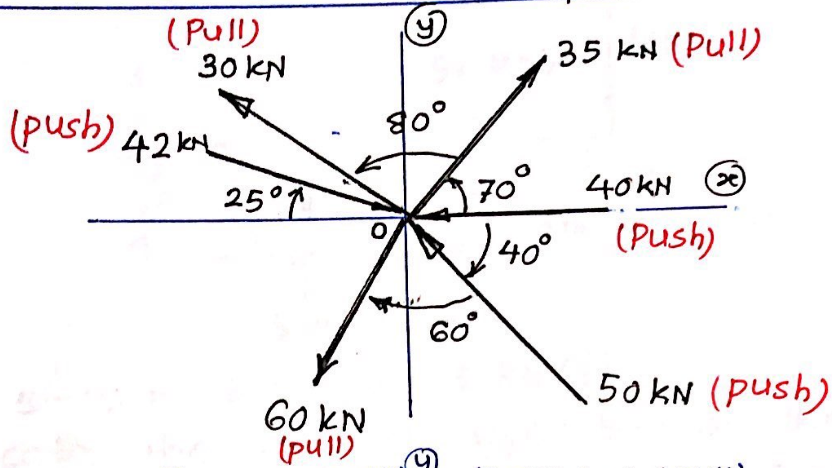
$M_P = +(8.0) - (5\times3) + (2.307\times5) - (5.538\times7)$
$\quad - (2\times4) + (3.464\times3)$

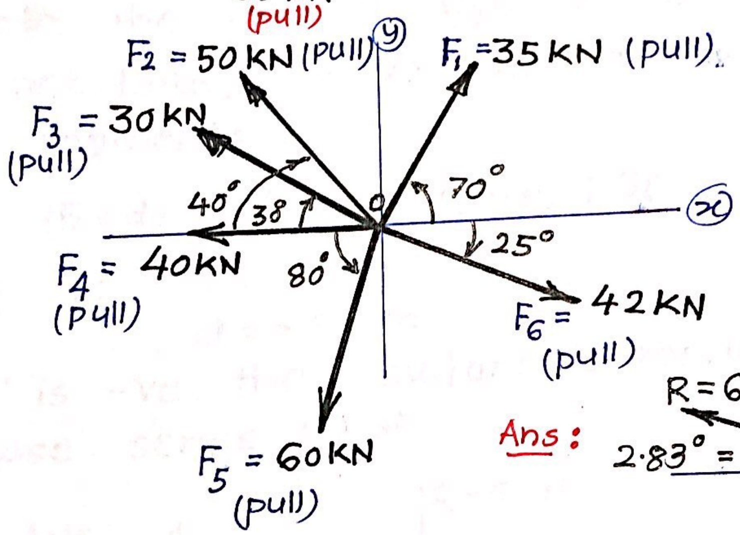
* $M_P = 31.839\ \text{kNm}$ ↺



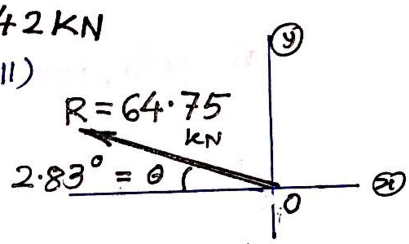
Equivalent system at P

(2) Resutant of concurrent coplonar force system



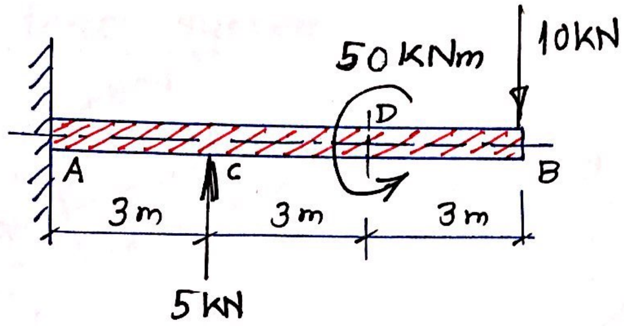


Ans :



Force	x-component	y-component
F_1	$35 \cdot \cos 70° = 11.97$	$35 \cdot \sin 70° = 32.89$
F_2	$-50 \cdot \cos 40° = -38.30$	$50 \cdot \sin 40° = 32.14$
F_3	$-30 \cdot \cos 30° = -25.98$	$30 \cdot \sin 30° = 15.00$
F_4	-40.00	~
F_5	$-60 \cdot \cos 80° = -10.42$	$-60 \sin 80° = -59.08$
F_6	$42 \cdot \cos 25° = 38.06$	$-42 \cdot \sin 25° = -17.75$
Resultant R	$R_x = \Sigma F_x = -64.67$ kN	$R_y = \Sigma F_y = 3.2$ kN

③ Resultant of parallel force system:



$R = 5 - 10 = -5$ KN i.e. 5 KN (↓)

To locate the point of application of the resultant force, use Varignon's thm. of moments.

Take moments @ A,

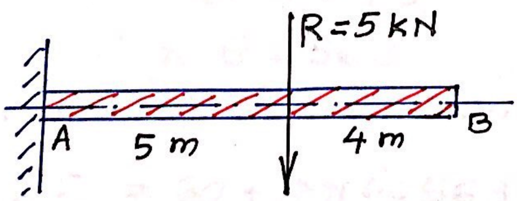
$$(5 \times d) = (5 \times 3) - (10 \times 9) + 50$$

$$\therefore \; 5 d = -25$$

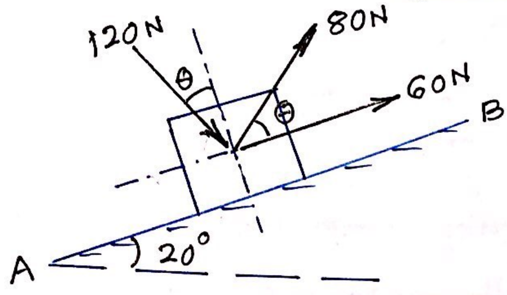
$$d = -5.0 \text{ m}$$

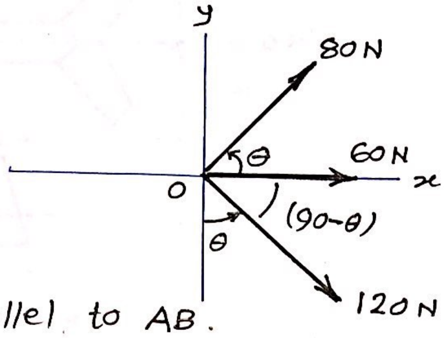
As 'd' is −ve, the resultant is rotating in clockwise sense @ A

Ans:



④ Resultant of concurrent coplanar force system.





Consider 'x' axis, parallel to AB.

As the resultant is parallel to plane AB,

$$\therefore \quad R = \Sigma F_x = R_x \quad \text{and} \quad R_y = \Sigma F_y = 0$$

$$\therefore \quad R_y = \Sigma F_y = 80 \cdot \sin\theta - 120 \cdot \sin(90-\theta) = 0$$

$$\therefore \quad 80 \cdot \sin\theta - 120 \cdot \cos\theta = 0$$

$$\therefore 80 \cdot \sin\theta = 120 \cdot \cos\theta$$

$$\therefore \quad \tan\theta = 1.5$$

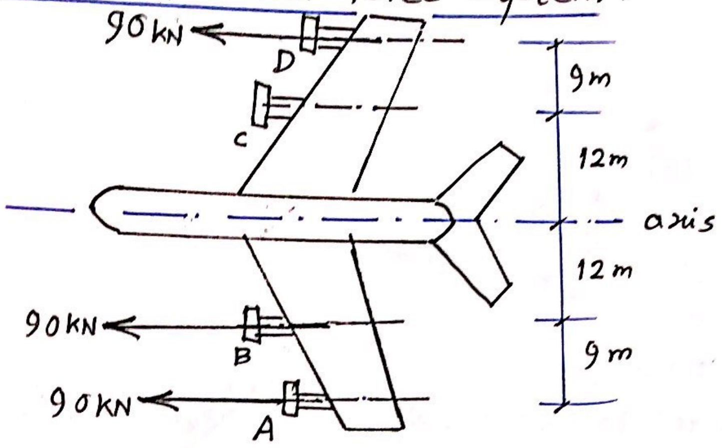
$$\therefore * \ \theta = 56.3^\circ$$

The magnitude of the resultant is given by,

$$R = R_x = \Sigma F_x = 60 + (80)(\cos 56.3^\circ) + (120)(\cos 33.7^\circ)$$

$$* \ R = 204.22 \text{ N along AB}$$

⑤ Resultant of parallel force system :



$R = 90 + 90 + 90 = 270 \text{ kN } (\leftarrow)$

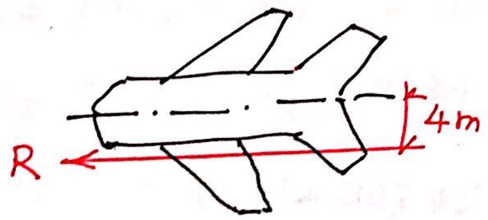
Using Varignon's thm. of moments, taking moment @ the axis of the plane,

$$R \cdot d = 270 \cdot d = \cancel{(90 \times 21)} - (90 \times 12) - \cancel{(90 \times 21)}$$

$$d = -4 \text{ m}$$

∴ As the distance d is -ve, the resultant is rotating in clockwise sense @ the axis of the plane.

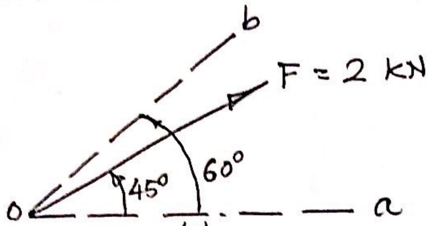
Ans : Resultant thrust = 270 kN acting at a dist. of 4 m below the axis of the plane.



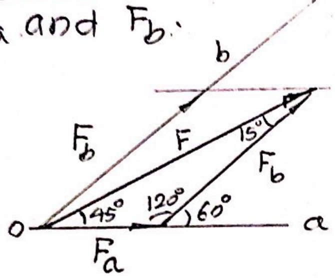
A

Operations with coplanar forces

Ex. No. 1 Determine the components of the 2 kN force along the oblique axes 'a' and 'b'. Also determine the projections of force F on the a and b axes.



Solution: I) Let Components of 'F' along 'a' and 'b' be F_a and F_b.



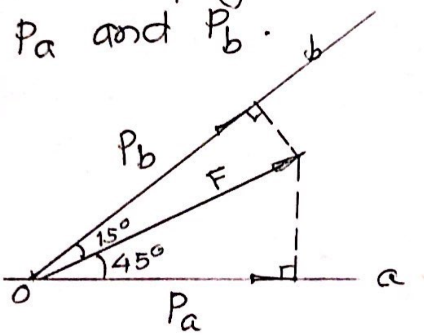
Applying sine rule,

$$\frac{2}{\sin 120^\circ} = \frac{F_a}{\sin 15^\circ} = \frac{F_b}{\sin 45^\circ}$$

$\therefore$ $F_a = 0.598$ kN

$\therefore$ $F_b = 1.633$ kN

II) Let the projections of 'F' along 'a' and 'b' be P_a and P_b.



$P_a = 2 \cdot \cos 45^\circ = 1.414$ kN

$P_b = 2 \cos 15^\circ = 1.932$ kN

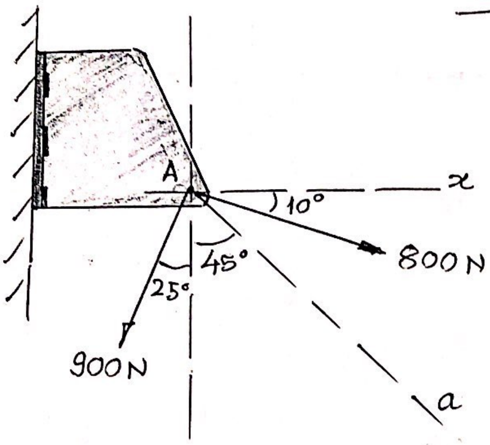
Ans: I) The components of force F along axes a and b are, $F_a = 0.598$ kN

and $F_b = 1.633$ kN

II) The projections of force F on the axes a and b are, $P_a = 1.414$ kN

and $P_b = 1.932$ kN

Ex. No. 2 The gusset plate is subjected to two forces as shown in figure. Replace them by two equivalent forces F_x in x-direction and F_a in a-direction



solution : Let 'R' be the resultant of the given two forces. In ΔACD, by applying cosine rule,

$$R = \sqrt{900^2 + 800^2 - 2 \times 900 \times 800 \times \cos 75^\circ}$$

$\therefore$ $R = 1038$ N

Applying sine rule,

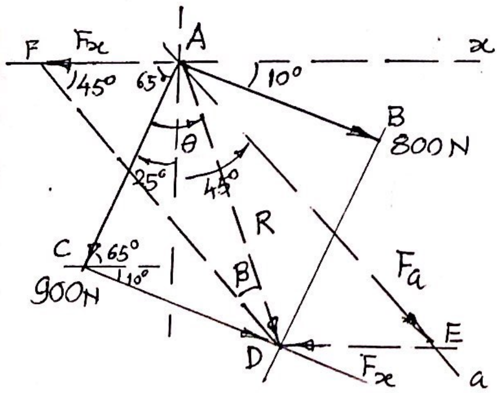
$$\frac{1038}{\sin 75^\circ} = \frac{800}{\sin\theta}$$

$\therefore$ $\theta = 48.1^\circ$

$\therefore$ $\beta = 180^\circ - 45^\circ - (65^\circ + 48.1^\circ)$

$= 21.9^\circ$

Solution :



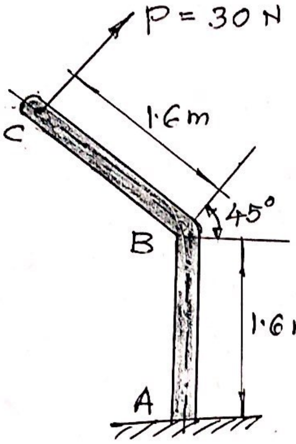
Now, Let 'F_a' and 'F_x' be the components of force 'R' along lines a and x respectively. Then, in ΔAFD, applying sine rule,

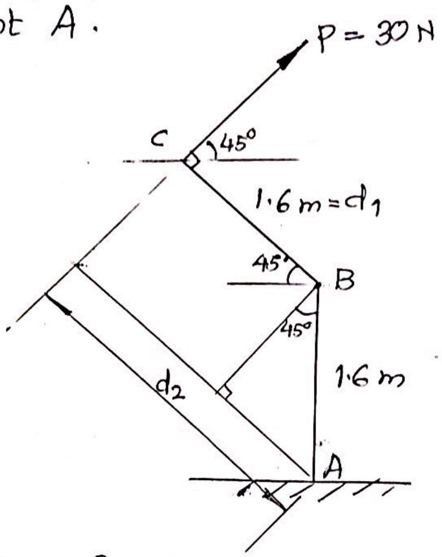
$$\frac{1038}{\sin 45^\circ} = \frac{F_x}{\sin 21.9^\circ} = \frac{F_a}{\sin(65^\circ + 48.1^\circ)}$$

$\therefore$ $F_x = 547$ N , $F_a = 1350$ N

Ans: $F_x = 547$ N and $F_a = 1350$ N

Ex. No. 3 The 30 N force P is applied perpendicular to the portion BC on the bent bar. Determine the moment of force P about point B and also about point A.





I) Moment of the given force about point B :

$$M_B = P \times d_1 = (30 \times 1.6) = 48 \text{ Nm (clockwise)}$$

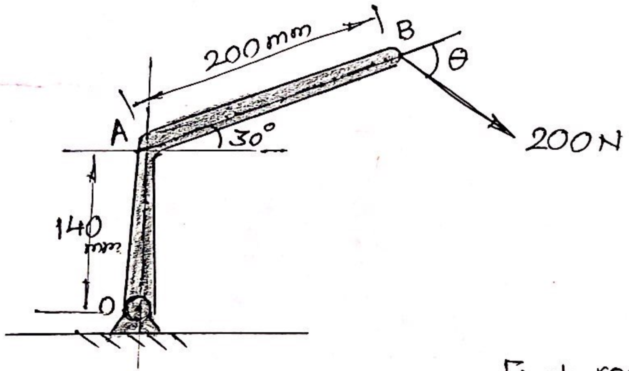
II) Moment of the given force about point A :

$$M_A = P \times d_2 = 30 \times (1.6 + 1.6 \times \sin 45^\circ)$$
$$= 81.94 \text{ Nm (clockwise)}$$

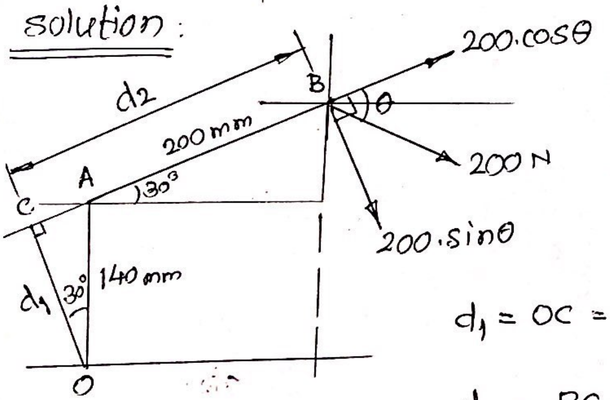
Ans : $M_B = 48$ Nm ↻

$M_A = 81.94$ Nm ↻

Ex. No. 4 Determine angle θ which will maximize the moment M_O of 200N force about the shaft axis at O. Also, comput Mo.



solution:



First resolve the given force of 200 N into two components, one along line AB i.e. $200 \cdot \cos\theta$ and the other perpendicular to line AB i.e. $200 \cdot \sin\theta$

$d_1 = OC = \perp$ distance of O from line AB

$d_2 = BC = \perp$ distance of B from line OC

$$d_1 = (140 \times \cos 30^\circ) = 121.24 \text{ mm}$$

$$d_2 = (200) + (140 \times \sin 30^\circ) = 270 \text{ mm}$$

$$\therefore M_O = -(200 \cdot \cos\theta \times 121.24) - (200 \cdot \sin\theta \times 270) \text{ Nmm}$$

$$= -200 \times (0.1212 \times \cos\theta + 0.27 \times \sin\theta) \text{ Nm}$$

For maximum moment,

$$\frac{dM_O}{d\theta} = -200 \times (-0.1212 \sin\theta + 0.27 \cos\theta) = 0$$

$$\therefore \tan\theta = \frac{0.27}{0.1212} \quad \therefore \theta = 65.83^\circ$$

$$\therefore (M_O)_{max} = -(200)\left[0.1212 \times \cos 65.83^\circ + 0.27 \times \sin 65.83^\circ\right]$$

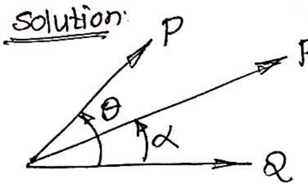
$$= 59.191 \text{ Nm (clockwise)}$$

Ans: For $(M_O)_{max}$, $\theta = 65.83^\circ$ and

$(M_O)_{max} = 59.191$ Nm ↻

Ex. No. 5 Resolve a force of 8 kN into two components of 5 kN each. Is it possible to resolve the same force of 8 kN into two components of 10 kN each, 15 kN each, 20 kN each or even two forces of 100 kN each.

Solution:



By the law of parallelogram of forces,

$$R^2 = P^2 + Q^2 + 2P \cdot Q \cdot \cos\theta$$

If $P = Q$

$$R^2 = 2P^2(1 + \cos\theta) \longrightarrow (1)$$

$$\tan\alpha = \frac{P \cdot \sin\theta}{Q + P \cdot \cos\theta} = \frac{\cancel{P} \cdot \sin\theta}{\cancel{P}(1 + \cos\theta)} \longrightarrow (2)$$

I) when $P = 5$ kN and $R = 8$ kN :

From eqⁿ (1) we get,

$$64 = 2 \times 25 (1 + \cos\theta)$$

$$\cos\theta = 0.28 \quad \therefore \theta = 73.74^\circ$$

From eqⁿ (2) we get, $\tan\alpha = 0.75 \quad \therefore \alpha = 36.87^\circ$

II) when $P = 10$ kN and $R = 8$ kN :

From eqⁿ (1) we get,

$$64 = 2 \times 100 (1 + \cos\theta)$$

$$\therefore \cos\theta = -0.68 \quad \therefore \theta = 132.84^\circ$$

From eqⁿ (2) we get $\tan\alpha = 2.29 \quad \therefore \alpha = 66.42^\circ$

III) when $P = 15$ kN and $R = 8$ kN :

From eqⁿ (1) we get

$$64 = 2 \times 225 (1 + \cos\theta) \quad \therefore \theta = 149.07^\circ$$

$$\therefore \cos\theta = -0.857 \quad \therefore \alpha = 74.54^\circ$$

From eqⁿ (2) we get, $\tan\alpha = 3.614$

IV) when $P = 100$ kN, $R = 8$ kN :

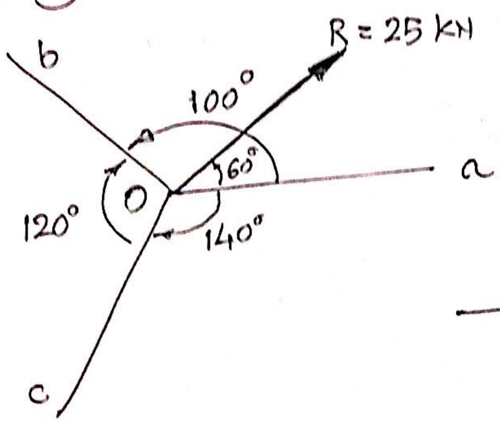
From eqⁿ (1) we get

$$64 = 2 \times 10{,}000 (1 + \cos\theta) \quad \therefore \theta = 175.4^\circ$$

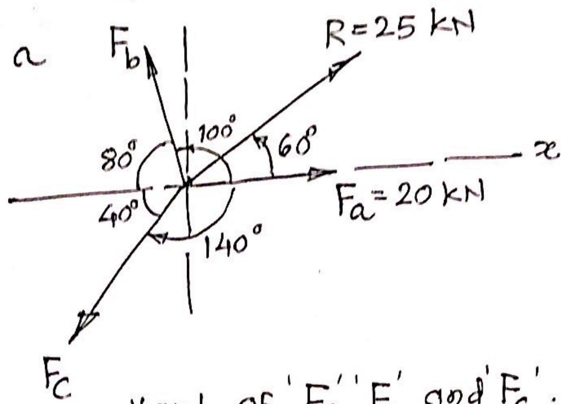
$$\therefore \cos\theta = -0.9968$$

From eqⁿ (2) we get, $\tan\alpha = 24.816 \quad \therefore \alpha = 87.69^\circ$

Ex. No. 6 A force R = 25 kN acting at O has three components A, B and C along the directions a, b and c respectively. The component along line oa is 20 kN and acts away from O. Find the components of R along lines c and b.



Solution: Take the x axis of Cartesian frame of reference along line oa.



Hence, force 'R' is the resultant of 'F_a', 'F_b' and 'F_c'.
Hence, they are called as components of 'R'.

As, $R_x = \Sigma F_x$, we get

$$25 \cdot \cos 60^\circ = 20 - F_b \cdot \cos 80^\circ - F_c \cdot \cos 40^\circ$$

$$\therefore \quad (0.174) F_b + (0.766) F_c = 7.5 \longrightarrow ①$$

As, $R_y = \Sigma F_y$, we get,

$$25 \cdot \sin 60^\circ = F_b \cdot \sin 80^\circ - F_c \cdot \sin 40^\circ$$

$$21.65 = (0.985) F_b - (0.64) F_c \longrightarrow ②$$

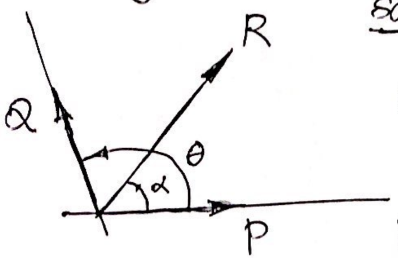
solving eq^{ns} ① and ②, we get,

$$F_b = 24.678 \text{ kN} \quad \text{and} \quad F_c = 4.174 \text{ kN}$$

Ans: $F_b = 24.678$ kN

$F_c = 4.174$ kN

Ex. No. 7 The angle between the lines of action of two forces is 120°. If their resultant makes an angle of 70° with the smaller force whose magnitude is 10 kN. Calculate the magnitude of the larger one and also that of their resultant.



Solution: We have,

$\theta = 120°, \alpha = 70°, P = 10\,kN$

By law of parallelogram of forces,

$$R^2 = P^2 + Q^2 + 2P \cdot Q \cdot \cos\theta$$

$$R^2 = 10^2 + Q^2 + 2 \times 10 \times Q \cos 120°$$

$$\therefore R^2 = Q^2 + 100 + (-10)Q \longrightarrow (1)$$

Now,

$$\tan\alpha = \left[\frac{Q \cdot \sin\theta}{P + Q \cdot \cos\theta}\right]$$

$$\therefore \tan 70° = \frac{Q \cdot \sin 120°}{10 + Q \cdot \cos 120°} \longrightarrow (2)$$

$$(2.747)(10 - 0.5Q) = (0.866)Q$$

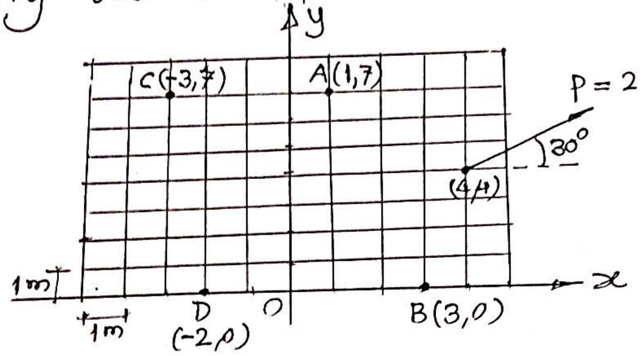
$$27.47 = (2.2395)Q$$

$$Q = 12.266\,kN$$

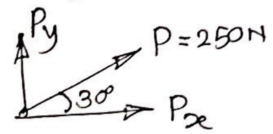
From eqⁿ (1), we get, $R = 11.305\,kN$

Ans: Resultant $R = 11.305\,kN$ and its components are $P = 10\,kN$, $Q = 12.266\,kN$ at 120° with each other.

Ex. No. 8 In the given figure calculate the moment of the force P = 250 N about points A and B. Determine the perpendicular distances of the points C and D from the line of action of P using the concept of the moment of a force.



solution.



$P_x = 250 \cdot \cos 30° = 216.5$ N

$P_y = 250 \cdot \sin 30° = 125$ N

I) Moment of 'P' about A :

$M_A = (P_x \times 3) + (P_y \times 3)$

$= (216.5 \times 3) + (125 \times 3) = 1024.5$ Nm

II) Moment of P about B :

$M_B = -(P_x \times 4) + (P_y \times 1)$

$= -(216.5 \times 4) + (125 \times 3) = -491$ Nm

$= 491$ Nm

III) Moment of P about C :

$M_C = (P_x \times 3) + (P_y \times 7)$

$= (216.5 \times 3) + (125 \times 7) = 1524.5$ Nm

$= 250 \times d_c \quad \therefore d_c = 6.098$ m

d_c = perpendicular distance of point C from the line of action of force P

IV) Moment of P about D :

$M_D = -(P_x \times 4) + (P_y \times 6)$

$= -(216.5 \times 4) + (125 \times 6) = -116$ Nm $= 116$ Nm

W, 116 Nm $= 250 \times d_D \quad \therefore d_D = 0.464$ m

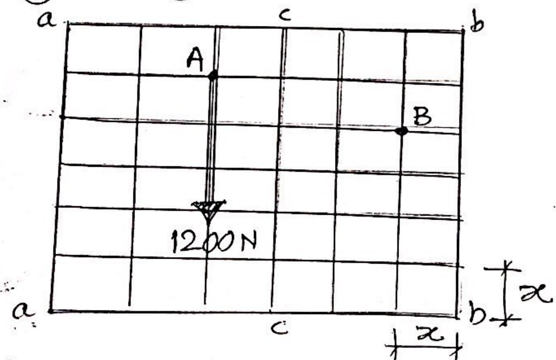
Ans: $M_A = 1024.5$ Nm, $M_B = 491$ Nm

$M_C = 1524.5$ Nm, $M_D = 116$ Nm

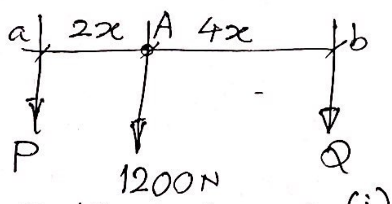
$d_c = 6.098$ Nm, $d_D = 0.464$ m

Ex. No. 9 a) Resolve a force of 1200 N into two parallel components P and Q, acting respectively along lines 'aa' and 'bb'.

b) Resolve the same force into a force P at B and a couple. Represent the couple by forces F acting along lines 'bb' and 'cc'.



Solution: (a)



Let 'P' and 'Q' be the components of force F along lines 'aa' and 'bb' respectively.

$\therefore \quad P + Q = 1200\ N \longrightarrow$ (i)

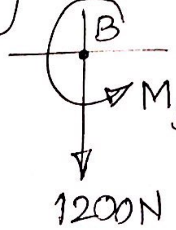
Using Varignon's theorem of moments, taking moments about A,

$$0 = (P \times 2\cancel{x}) - (Q \times 4\cancel{x}) \longrightarrow \text{(ii)}$$

$$\therefore \quad P = 2Q$$

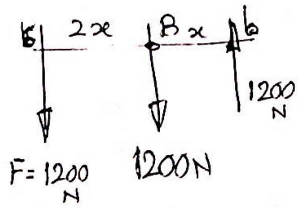
$\therefore$ Q = 400 N along bb, P = 800 N along aa.

(b) Transferring the force of 1200 N at A to point B we get,

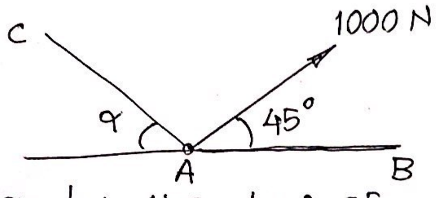


$$M_B = (1200 \times 3x)$$
$$= 3600 \cdot x \text{ Nm}$$

$\equiv$



Ex. No. 10 A 1000 N force is resolved into components along AB and AC. If the component along AB is 700 N, determine angle α and the value of the component along AC.



<u>Solution</u>: Let the components along AB and AC be 'P' and 'Q'.

Then, $P = 700\,N$

$R = 1000\,N$

$\beta = 45^\circ$

$\theta = (180 - \alpha)$

Then by the law of parallelogram of forces,

$$R^2 = P^2 + Q^2 + 2PQ \cdot \cos\theta$$

$$\therefore 1000^2 = 700^2 + Q^2 + 2 \times 700 \times Q \times \cos\theta \longrightarrow (1)$$

$$\tan\beta = \frac{Q \cdot \sin\theta}{P + Q \cdot \cos\theta}$$

$$\therefore \tan 45^\circ = \frac{Q \cdot \sin\theta}{700 + Q \cdot \cos\theta} = 1$$

$$\therefore Q \cdot \sin\theta = 700 + Q \cdot \cos\theta \longrightarrow (2)$$

solving the above equations we get,

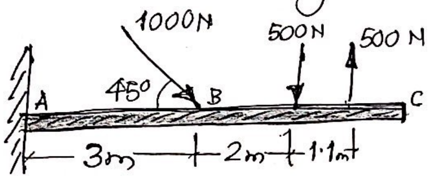
$$Q = 707.19\ N$$

$$\theta = 90^\circ = 180 - \alpha$$

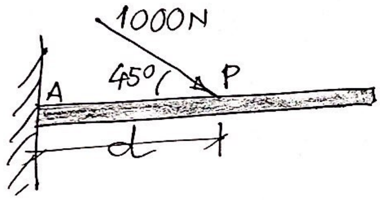
$$\therefore \alpha = 90^\circ$$

<u>Ans</u>: Component of the given force along line AC = 707.19 N and $\alpha = 90^\circ$

Ex. No. 11 A cantilever beam is subjected to a single force and a couple in xy plane. Reduce this system to a single force equivalent to the given system.



<u>solution</u>: As there is one force and one couple acting on the beam. The resultant force is also a similar 1000 N 45° force.



Let 'P' be the point of application of the force on the beam.

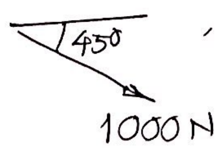
Using Varignon's theorem of moments, taking moments about A, we get,

$$-(1000 \cdot \sin 45^\circ) \cdot d = -(1000 \cdot \sin 45^\circ \times 3) + 550$$

$$-(707.106)\, d = -1571.32 \text{ Nm}$$

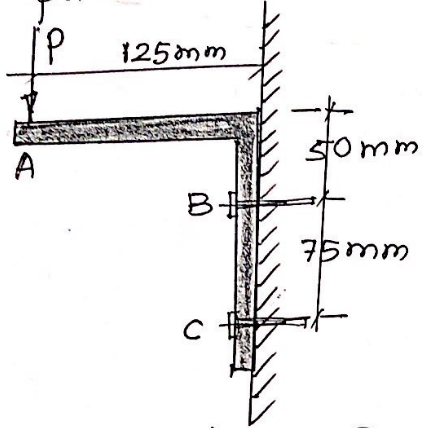
$$\therefore d = 2.222 \text{ m}$$

<u>Ans</u>: Single equivalent force acting on the cantilever beam is



, acting at point P, 2.222m to the right of A.

Ex. No. 12 A 135 N vertical force P is applied at A to the bracket shown in Figure; which is held by screws at B and C. I) Replace force P with an equivalent force-couple system at B. II) Find the two horizontal forces at B and C that are equivalent to the couple obtained in part I.

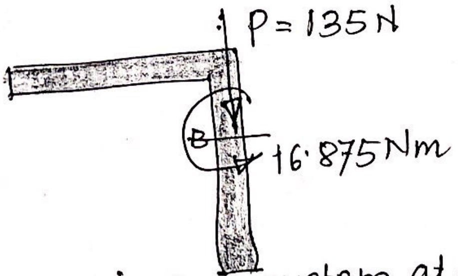


Solution: I) The equivalent system at B consists of,

i) a single force P = 135 N in downward direction at point B, and

ii) a moment of P about B i.e. (135×0.125)

$= 16.875$ Nm



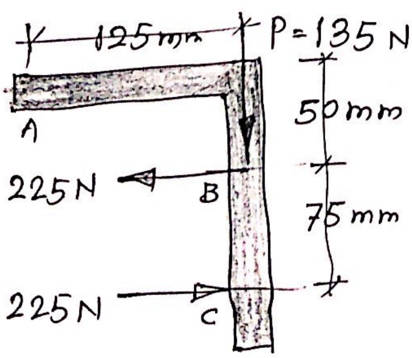
Equivalent system at B.

II) The couple at B

$= 16.875$ Nm

$= (F \times 0.075)$

$\therefore \quad F = 225$ N



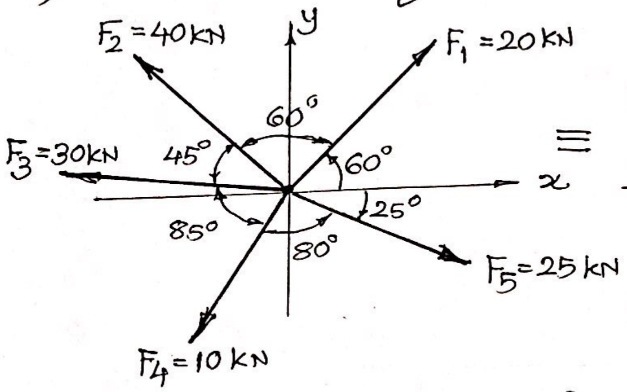
(B)

Resultant of concurrent coplanar forces

Ex. No. 13 I) Find the equilibrant F_6 of the five concurrent forces F_1, F_2, F_3, F_4 and F_5.

II) What is the resultant of F_1, F_2, F_3, F_5 and F_6

III) What is the equilibrant of F_1, F_3, F_4, F_5 and F_6



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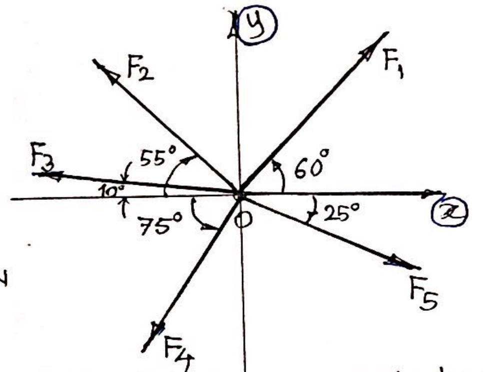


solution : I) First of all, find the acute angle made by each of these forces with horizontal. All the forces must be pulling the particle at O.

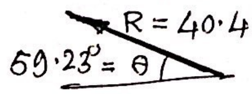
Equilibrant of a given force system is equal, opposite and colinear to the resultant of the given force system. Hence find the resultant of the given five forces. Prepare the table of rectangular components.

Force	x comp.	y comp.
F_1	$20 \cdot \cos 60^\circ = 10.00$	$20 \cdot \sin 60^\circ = 17.32$
F_2	$-40 \cdot \cos 55^\circ = -22.943$	$40 \cdot \sin 55^\circ = 32.766$
F_3	$-30 \cdot \cos 10^\circ = -29.544$	$30 \cdot \sin 10^\circ = 5.209$
F_4	$-10 \cdot \cos 85^\circ = -0.872$	$-10 \cdot \sin 85^\circ = -9.962$
F_5	$25 \cdot \cos 25^\circ = 22.658$	$-25 \cdot \sin 25^\circ = -10.565$
Total	$R_x = \Sigma F_x = -20.701$	$R_y = \Sigma F_y = 34.768$

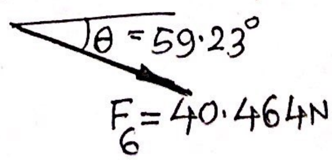
The resultant of the given system is,

$$\bar{R} = (R_x)\hat{i} + (R_y)\hat{j}$$

$$\therefore \bar{R} = -(20.701)\hat{i} + (34.768)\hat{j}$$



Hence the equilibrant is

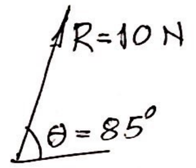


II) For the resultant of F_1, F_2, F_3, F_5 and F_6:

Force	x comp.	y comp.
F_1	10.00	17.32
F_2	−22.943	32.766
F_3	−29.544	5.209
F_5	22.658	−10.565
F_6	20.701	−34.768
Total	$R_x = \Sigma F_x = 0.872$	$R_y = \Sigma F_y = 9.962$

The resultant force is,

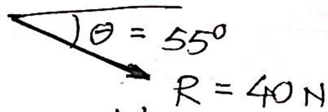
$$\bar{R} = (0.872)\hat{i} + (9.962)\hat{j}$$



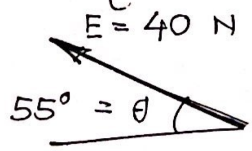
III) For the equilibrant of F_1, F_3, F_4, F_5 and F_6

Force	x-comp.	y-comp.
F_1	10.00	17.32
F_3	−29.544	5.209
F_4	−0.872	−9.962
F_5	22.658	−10.565
F_6	20.701	−34.768
Total	$R_x = \Sigma F_x = 22.943$	$R_y = \Sigma F_y = -32.766$

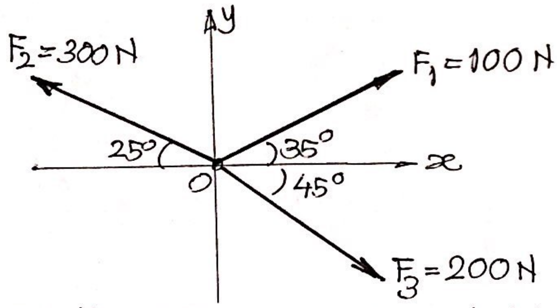
The resultant force is, $\bar{R} = (22.943)\hat{i} - (32.766)\hat{j}$ N



∴ The equilibrant is



Ex. No. 14 Three forces are concurrent at origin as shown in figure. Determine I) the resultant of the three forces, and II) the magnitude and direction of the fourth force, which must be added to make the resultant zero.

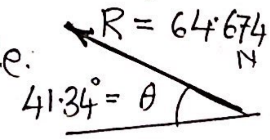


Solution: I) Prepare the table of rectangular components

Force	x-comp.	y-comp.
F_1	$100 \cdot \cos 35^\circ = 81.915$	$100 \cdot \sin 35^\circ = 57.358$
F_2	$-300 \cdot \cos 25^\circ = -271.892$	$300 \cdot \sin 25^\circ = 126.785$
F_3	$200 \cdot \cos 45^\circ = 141.421$	$-200 \cdot \sin 45^\circ = -141.421$
Total	$R_x = \Sigma F_x = -48.555$	$R_y = \Sigma F_y = 42.722$

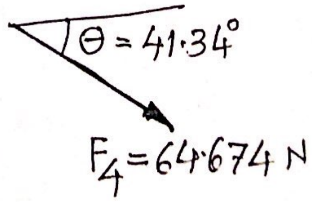
The resultant of the given force system is,

$\bar{R} = -(48.555)\hat{i} + (42.722)\hat{j}$ N i.e.



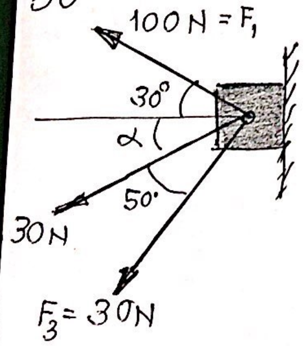
II) The equilibrant of the given force system is the force which is equal, opposite and colinear to the resultant of that system.

$\therefore\ \bar{F_4} = (48.555)\hat{i} - (42.722)\hat{j}$ N i.e.



~~Ex. No. A metallic bar of length 'l' and weight 'W' is resting with one end inside a smooth bowl as shown in figure.~~

$\boxed{\text{Ex. No. 15}}$ Three forces are applied to the bracket as shown in Figure. Determine and show the equilibrant force for $\alpha = 40°$ if the angle between two 30 N forces always remain 50°



Solution : when $\alpha = 40°$, the force system will as shown in figure.

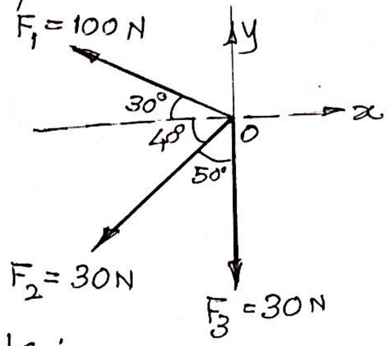


Table of components :

Force	x-comp.	y-comp.
F_1	$-100 \cdot \cos 30° = -86.60$	$100 \cdot \sin 30° = 50.00$
F_2	$-30 \cdot \cos 40° = -22.98$	$-30 \cdot \sin 40° = -19.28$
F_3		-30.00
Total	$R_x = \Sigma F_x = -109.58$ N	$R_y = \Sigma F_y = 0.72$ N

The resultant force is, $\bar{R} = -(109.58)\hat{i} + (0.72)\hat{j}$

$R = 109.58$ N

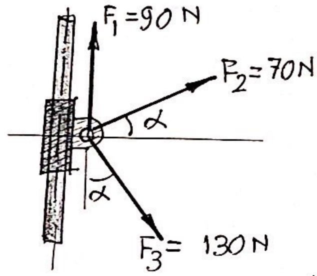
$0.376°$

This is almost horizontal. Take $R_y = 0$

$\therefore R = 109.58$ N $(\leftarrow)$

Thus, the equilibrant is $\bar{E} = 109.58$ N $(\rightarrow)$

Ex. No. 16 A collar can slide on a vertical rod and it is subjected to three forces as shown in figure. Determine I) the value of angle α for which the resultant of the three forces is horizontal.
II) the corresponding magnitude of the resultant.



Solution: As the resultant of the three forces is horizontal,

$R_y = \sum F_y = 0$

$\therefore 90 + 70 \cdot \sin\alpha - 130 \cdot \cos\alpha = 0$

$\therefore 90 + 70 \cdot \sin\alpha = 130 \cdot \cos\alpha$

$\therefore 0.692 + (0.538)\sin\alpha = \cos\alpha$

squarring the above equation,

$0.4788 + (0.745)\sin\alpha + (0.289)\sin^2\alpha = 1 - \sin^2\alpha$

$\therefore (1.289)\sin^2\alpha + (0.745)\sin\alpha - 0.512 = 0$

$\therefore \sin\alpha = 0.409 \quad$ or -0.987

$\therefore \alpha = 24.14^\circ \quad$ or $\quad \alpha = -80.75^\circ$

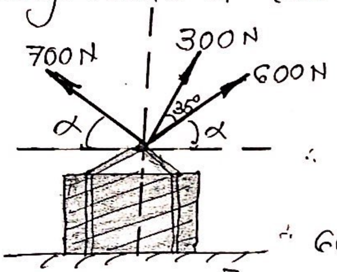
The magnitude of the resultant is given by,

$R = \sum F_x = 70 \cdot \cos\alpha + 130 \cdot \sin\alpha$

Ans: when $\alpha = 24.14^\circ \quad \therefore R = 117.044$ N $(\longrightarrow)$

when $\alpha = -80.75^\circ \quad \therefore R = 117.05$ N $(\longleftarrow)$

Ex. No. 17 For the three forces acting on the block determine I) the required value of angle α if the resultant is to be vertical, II) the corresponding magnitude of the resultant.



Solution: As the resultant is vertical, $R_x = \Sigma F_x = 0$

$\therefore 600 \cdot \cos\alpha + 300 \cdot \cos(35+\alpha) - 700 \cdot \cos\alpha = 0$

$\therefore 600 \cdot \cos\alpha + 300 \cdot [\cos 35 \cdot \cos\alpha - \sin 35^\circ \cdot \sin\alpha] - 700 \cdot \cos\alpha = 0$

$\therefore -100 \cdot \cos\alpha + (245.75)\cos\alpha - (172.07)\sin\alpha = 0$

$(145.75)\cos\alpha = (172.07)\sin\alpha$

$\tan\alpha = 0.847 \quad \therefore \alpha = 40.27^\circ$

The corresponding magnitude of the resultant is,

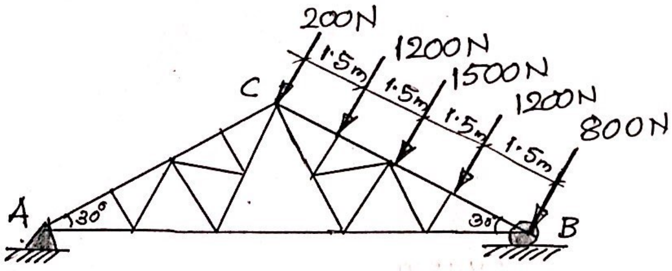
$R = \Sigma F_y = (600)\sin 40.27^\circ + (300)\sin 75.27^\circ + (700)\sin 40.27^\circ$

$\therefore R = 1130.38 \text{ N} (\uparrow)$

Ans: $\alpha = 40.27^\circ$, $R = 1130.38 \text{ N} (\uparrow)$

Resultant of parallel coplanar forces

Ex. No. 18 The forces applied to the roof truss shown in figure represent the effect of a wind pressure against the side of a roof. Find the equivalent single force that can replace the given system without changing its effect on the truss.



Solution: The roof truss is subjected to a parallel force system. Hence, the resultant is given by $R = \Sigma F = 200 + 1200 + 1500 + 1200 + 800$

$\therefore \quad R = 4900\ N \quad 60°$

To locate the point of application of the resultant force on member BC of the truss, use Varignon's theorem of moments, taking moments about B,

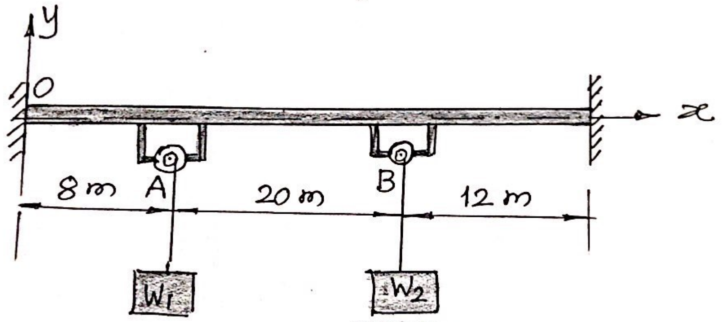
$$(4900 \times d) = (200 \times 6) + (1200 \times 4.5) + (1500 \times 3) + (1200 \times 1.5) + \cancel{(800 \times 0)}$$

$$\therefore \quad d = 2.6326\ m$$

As 'd' is +ve the resultant is rotating in anti-clockwise about B.

Ans: The resultant force is $60°$ $R = 4900N$ acting at point P which is 2.632 m from point B on member BC.

Ex. No. 19 Two hoists are operated on the same overhead track. Hoist A has a 3000 KN load and hoist B has a 4000 kN load. What is the resultant force system at the left end O of the track? Where does the simplest resultant force act?



I) The resultant force system at O consists of a single force $R = \Sigma F$ and a moment $M = \Sigma M_0$

$$R = W_1 + W_2 = 3000 + 4000 = 7000 \text{ kN } (\downarrow)$$

$$M_O = -(3000 \times 8) - (4000 \times 28) = -136{,}000 \text{ KNm}$$

$$\therefore M_O = 136{,}000 \text{ kNm} \circlearrowright$$

II) The simplest resultant of the above system is the force of $R = 7000$ N $(\downarrow)$. To locate the position of the resultant on the track, use Varignon's theorem of moments, taking moments about O,

$$(7000 \times d) = -(3000 \times 8) - (4000 \times 28) = -136000 \text{ kNm}$$

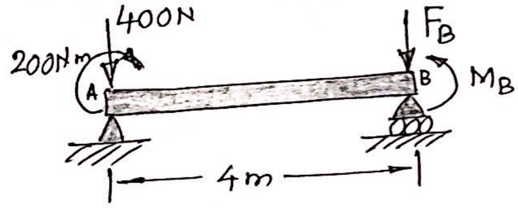
$$\therefore d = -19.428 \text{ m}$$

As 'd' is -ve, the resultant is rotating in clockwise sense about O. The point of application of the resultant is 19.428 m to the right of O, on the track.

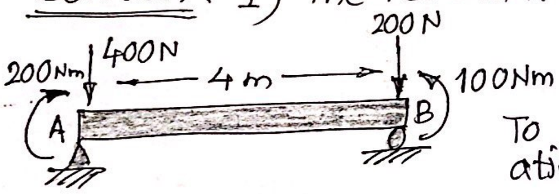
Ans: I) $R = 7000$ kN $(\downarrow)$, $M_O = 136{,}000$ kNm $\circlearrowright$

II) $R = 7000$ kN at 19.428 m to the right of

$\boxed{\text{Ex. No. 20}}$ Determine the single equivalent force and the distance from end A to its line of action for the beam and the loading shown in figure when I) $F_B = 200N (\downarrow)$, $M_B = 100 Nm$ II) $F_B = 100 N (\uparrow)$, $M_B = -600 Nm$



Solution: I) The resultant force on the beam,



$R = \Sigma F = 400 + 200$

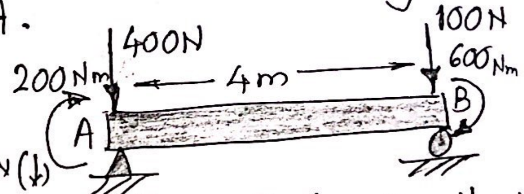
$\therefore R = 600 N (\downarrow)$

To locate the point of application of the resultant force use Varignon's theorem of moments, taking moments about A, $(600)d = \cancel{(400 \times 0)} - (200 \times 4) + 100 - 200$

$d = -1.5 m$ i.e. 1.5 m to the right of A

As 'd' is -ve, the resultant is rotating in clockwise dir^n about A.

II) The resultant force on the beam is,



$R = \Sigma F = 400 + 100 = 500 N (\downarrow)$

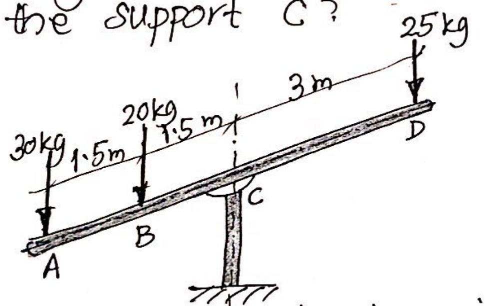
To locate the point of application of the resultant force, use Varignon's theorem of moments, taking moments about A, $(500)d = \cancel{(400 \times 0)} - 200 - (100 \times 4) - 600$

$d = -2.4 m$ i.e. 2.4 m to the right of A

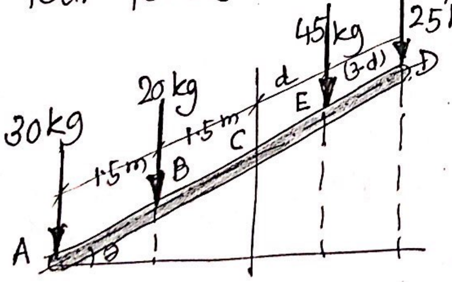
As d is -ve, the resultant is rotating in clock-wise direction about A.

Ans: I) $R = 600 N (\downarrow)$ at 1.5 m to the right of A on the beam, II) $R = 500N (\downarrow)$ at 2.4 m to the right of A on the beam.

Ex. No. 21 Three children of masses 30 kg, 20 kg and 25 kg each are sitting on a seesaw in the garden as shown in Figure. Determine where a fourth child of mass 45 kg should sit on the right side of the support C, so that the resultant of the weights of the four children will pass through the support C?



Solution : As the resultant of the four forces is passing through point C on the seesaw, total moment of the four forces about point C is zero.



$\therefore M_c = (30)(3 \cdot \cos\theta)$
$+ (20)(1.5 \times \cos\theta)$
$- (45)(d \times \cos\theta)$
$- (25)(3 \cdot \cos\theta) = 0$

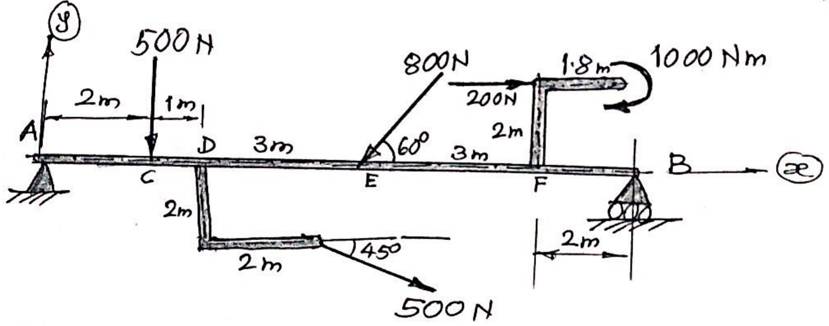
$\therefore 90 + 30 - 45 \cdot d - 75 = 0$

$\therefore d = 1\ m = CE$

Ans : 1 m to the right of C

Resultant of general coplanar forces

Ex. No. 22 What is the simplest resultant of the loads acting on the beam? Where does its line of action crosses x axis?



Solution: $R_x = \Sigma F_x = 500 \cdot \cos 45^\circ - 800 \cdot \cos 60^\circ + 200$

$= 153.55$ N

$R_y = \Sigma F_y = -500 - 500 \cdot \sin 45^\circ - 800 \cdot \sin 60^\circ$

$= -1546.37$ N

The resultant of the given force system or the loadings on the beam is given by,

$\bar{R} = (R_x)\hat{i} + (R_y)\hat{j}$

$\bar{R} = (153.55)\hat{i} - (1546.37)\hat{j}$ N i.e. $\theta = 84.33^\circ$, $R = 1553.97$ N

To locate the point of application of the resultant, use Varignon's theorem of moments. Taking moments about A,

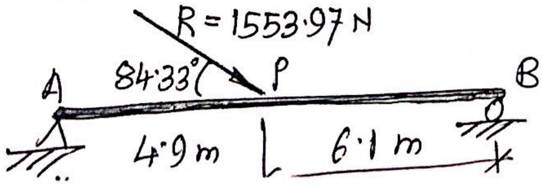
$(1553.97)d = -(500 \times 2) + (500 \cdot \cos 45^\circ)(2) - (500 \cdot \sin 45^\circ)(5) - (800 \cdot \sin 60^\circ)(6) - (200 \times 2) - 1000$

$\therefore (1553.97)d = -7617.58$

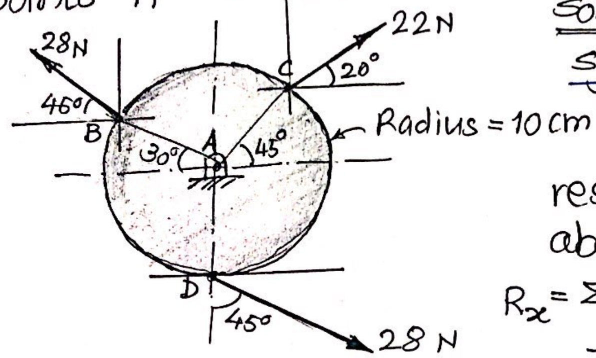
$d = -4.9$ m

As 'd' is $-$ve, the resultant is rotating in clockwise sense about A.

Ans:



Ex. No. 23 Three cables attached to a disc exert on it the forces shown in figure.
I) Replace the three forces by an equivalent force-couple system at A. II) Determine the single force which is equivalent to the force-couple system obtained in part I and specify its point of application on a line drawn through points A and D.



Solution (I) The equivalent system at A consists of a resultant force R at A and a resultant moment M about point A.

$$R_x = \Sigma F_x = 22 \cdot \cos 20^\circ + \cancel{28 \cdot \sin 45^\circ} - \cancel{28 \cdot \cos 45^\circ} = 20.673 \text{ N}$$

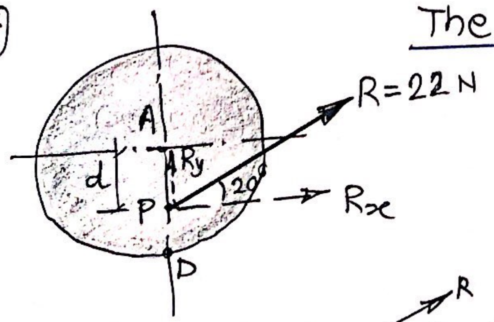
$$R_y = \Sigma F_y = 22 \cdot \sin 20^\circ - \cancel{28 \cdot \cos 45^\circ} + \cancel{28 \cdot \sin 45^\circ} = 7.524 \text{N}$$

$$\bar{R} = R_x \hat{i} + R_y \hat{j} = (20.673)\hat{i} + (7.524)\hat{j} \text{ N}$$

i.e. R = 22N ∠20°

$$M_A = -(22 \cdot \cos 20^\circ)(10 \cdot \sin 45^\circ) + (22 \cdot \sin 20^\circ)(10 \cdot \cos 45^\circ) + (28 \cdot \cos 45^\circ)(10 \cdot \sin 30^\circ) - (28 \cdot \sin 45^\circ)(10 \cdot \cos 30^\circ) + (28 \cdot \sin 45^\circ)(10) = 32.44 \text{ Ncm}$$

(II)



The moment at point A,

$$M_A = (R_x \cdot d) + \cancel{(R_y \times 0)}$$

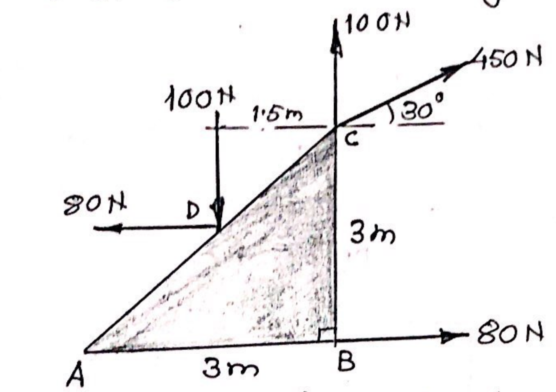
$$32.44 \text{ Ncm} = (20.673) \times d$$

$$d = 1.57 \text{ cm}.$$

Ans: I) R = 22N ∠20° at A and $M_A = 32.44$ Ncm

II) R = 22N ∠20° at P where P is 1.57 cm below point A on line AD.

Ex. No. 24 Replace the force system acting on the triangular plate ABC as shown in figure by a single resultant force. Give the point of intersection of the line of action of this force with the vertical edge BC of the plate.



Solution: The given system consists of one force and two couples. Hence, the resultant of the given force system is the same force i.e.

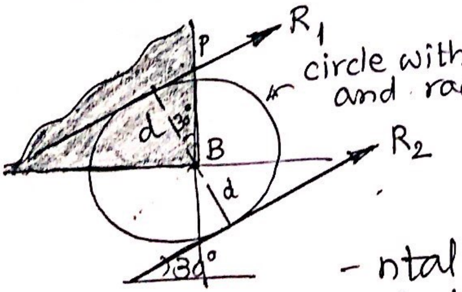
$R = 450\,N$, $30°$

To locate the point of application of the resultant force, use Varignon's theorem of moments, taking moments about B,

$$(450 \times d) = -(450 \cdot \cos 30° \times 3) + (100 \times 1.5) + (80 \times 1.5)$$

$$\therefore d = -1.998\,m \text{ Say } -2\,m$$

As d is -ve the resultant is rotating in clockwise sense about B.



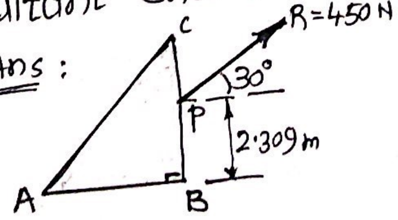
Now, draw a circle with center at B and radius is equal to d i.e. 2 m. Then draw two tangents R_1 and R_2 at an inclination to 30° with the horizontal. Out of these two force 'R_1' is rotating in clockwise sense about B. (Note that 'R_2' is rotating in anticlockwise sense about B)

Hence, R_1 represents the correct resultant. P is the point where the resultant cuts the vertical edge BC of the plate.

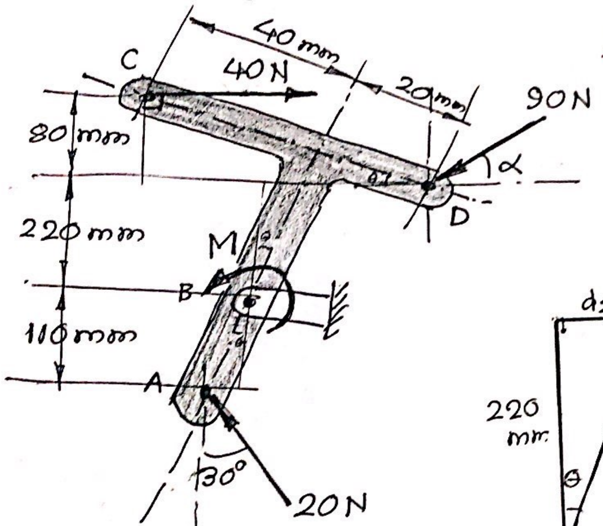
Then $\cos 30° = \dfrac{d}{BP} = \dfrac{2}{BP}$

$BP = 2.309\,m$

Ans :



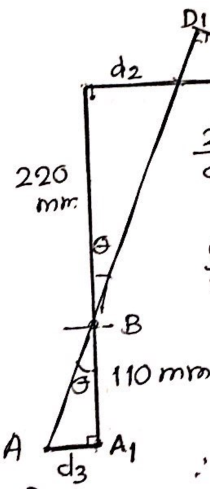
$\boxed{\text{Ex. No. 25}}$ For the force system shown in figure I) Determine the value of M and α so that the resultant of the force system is directed along line BD. II) Also find the magnitude of that resultant force.



Solution :

$\tan\theta = \dfrac{80}{60}$

$\theta = 53.13^\circ$



$\dfrac{20}{d_1} = \cos 53.13^\circ$

$\therefore d_1 = 33.33$ mm

$\dfrac{d_2}{220} = \tan 53.13^\circ$

$\therefore d_2 = 293.33$ mm

$\dfrac{d_3}{110} = \tan 53.13^\circ$

$\therefore d_3 = 146.66$ mm

As the resultant is passing through line BD, $\Sigma M_B = \Sigma M_D = 0$

Taking moments about B,

$(90\cdot\cos\alpha)(220) - (90\cdot\sin\alpha)(326.66) - (40\times 300) - (20\cdot\cos 30^\circ)(146.66) + M = 0$

$-(20\cdot\sin 30^\circ)(110) - (20\cdot\cos 30^\circ)(146.66) + M = 0$

$\therefore (19800)\cos\alpha - (29400)\sin\alpha - 15640.22 + M = 0 \longrightarrow (1)$

Taking moments about D,

$-(40\times 80) - (20\cdot\sin 30^\circ)(330) - (20\cdot\cos 30^\circ)(326.66)$

$+ M = 0 \longrightarrow (2)$

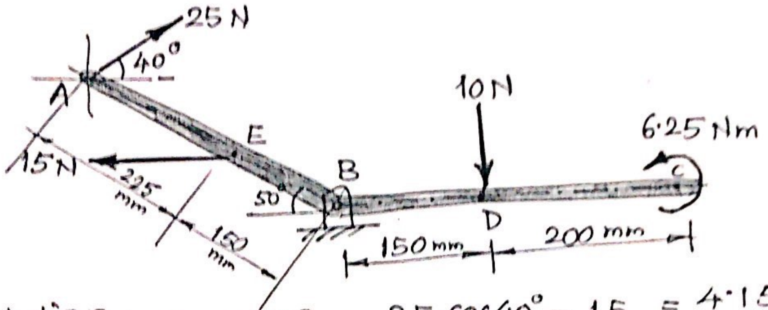
From eq^n (2) we get, $M = 12,158$ Nmm $= 12.158$ Nm

From eq^n (1) we get, $\alpha = 39.67^\circ$ or -28.35°

when $\alpha = 39.67^\circ$ then $\begin{cases} R_x = -39.27\text{ N} \\ R_y = -40.132\text{ N} \end{cases}$ $\cancel{45.6} = \theta$ $R = 56.15$ N

and when $\alpha = -28.35^\circ$ then $\begin{cases} R_x = -49.2\text{ N} \\ R_y = -60.05\text{ N} \end{cases}$ $R = 77.63$ N $\theta = 50.67^\circ$

Ex. No. 26 Three forces and a couple act on crank ABC. Determine the resultant of this force system and locate the point of intersection of the line of action of the resultant force with lines AB and BC.



Solution: $R_x = \Sigma f_x = 25 \cdot \cos 40^\circ - 15 = 4.15$ N

$R_y = \Sigma f_y = 25 \cdot \sin 40^\circ - 10 = 6.07$ N

$\therefore$ The resultant force is,

$\overline{R} = (R_x)\hat{i} + (R_y)\hat{j}$

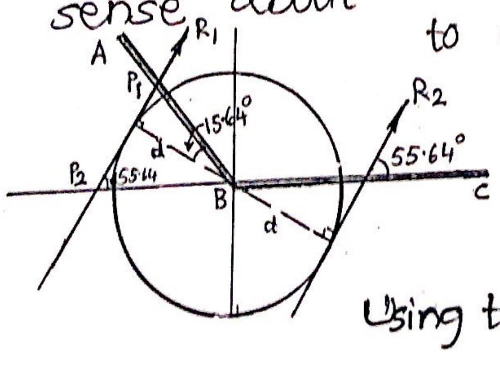
$= (4.15)\hat{i} + (6.07)\hat{j}$ N i.e. $R = 7.353$ N, $\theta = 55.64^\circ$

This gives us the magnitude and direction of the resultant force. To locate the point of application of the resultant, use Varignon's theorem of moments. Taking moments about B,

$$(7.353)d = -(25 \cdot \cos 40^\circ)(0.375 \times \sin 50^\circ) - (25 \cdot \sin 40^\circ)(0.375 \times \cos 50^\circ) - (15 \times 0.15 \times \sin 50^\circ) - (10 \times 0.15) + 6.25$$

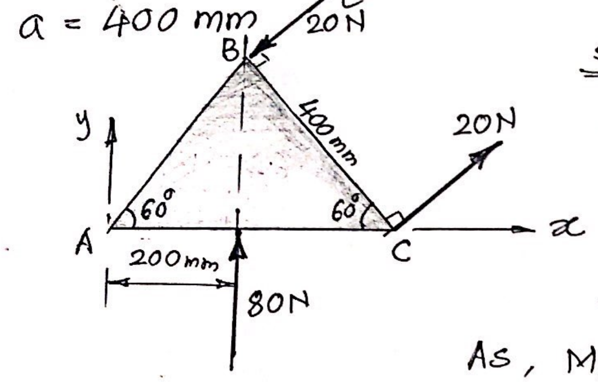
$\therefore d = -0.863$ m

As 'd' is -ve, the resultant is rotating in clockwise sense about B. Now, draw a circle of radius equal to d i.e. 0.863 m and center at B. R_1 and R_2 are the two tangents to that circle inclined at 55.64° to the horizontal. As 'R_1' is rotating in clockwise sense about B, it represents correct resultant.



Using trignometry we get $BP_1 = 896$ mm

Ex. No. 27 Find the resultant of the forces shown in figure. Hence replace this force system by an equipollent force system consisting of three forces acting along the sides of the equilateral triangle ABC of side $a = 400$ mm



Solution: I) For the given force system,

$R_x = \Sigma F_x = 0 \rightarrow$ (i)

$R_y = \Sigma F_y = 80\text{ N} \rightarrow$ (ii)

$\Sigma M_A = (80 \times 0.2) + (20 \times 0.4)$

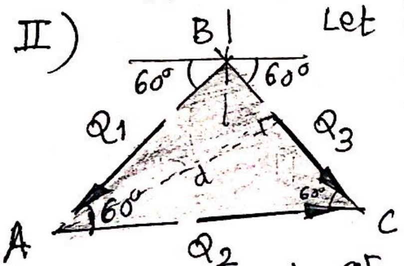
$\therefore M_A = 24 \text{ Nm} \circlearrowright \rightarrow$ (iii)

As, $M_A = 24 \text{ Nm} = 80 \times d$

$d = 0.3$ m from A

$\therefore$ The resultant of the given force system is a force of 80N (↑) acting at point P where AP = 0.3 m on side AC of the triangle.

II)



Let forces Q_1, Q_2 and Q_3 forms an equipollent system consisting of three forces acting along the sides of the equilateral triangle, as shown in figure. Thus the resultant of this system is also the same as the resultant of the original system. Thus, this system will satisfy all three conditions of part I.

$\therefore R_x = \Sigma F_x = -Q_1 \cdot \cos 60° + Q_2 + Q_3 \cdot \cos 60° = 0 \rightarrow$ ①

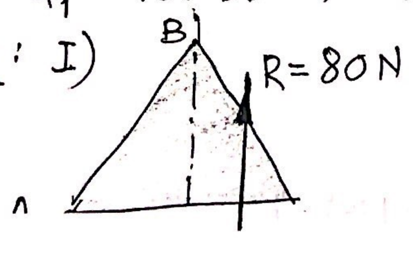
$R_y = \Sigma F_y = -Q_1 \sin 60° - Q_3 \cdot \sin 60° = 80 \text{ N} \rightarrow$ ②

$M_A = -Q_3 \times 0.400 \times \sin 60° = 24 \text{ Nm} \rightarrow$ ③

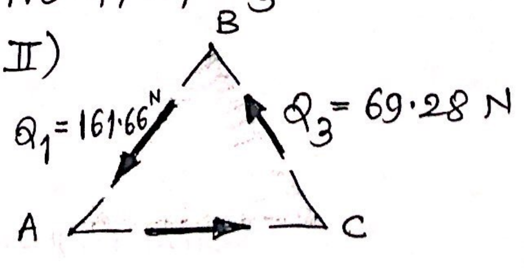
Solving equations ①, ② and ③ we get,

$Q_1 = 161.66$ N, $Q_2 = 115.47$ N, $Q_3 = -69.28$ N

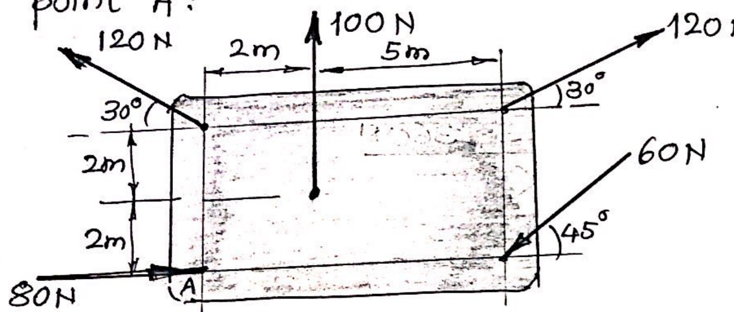
Ans: I)



II)



Ex. No. 28 Determine the resultant of the forces acting in the plane of the plate shown in figure. What is the shortest distance of the line of action of the resultant from point A?



<u>Solution</u>: $R_x = \Sigma F_x = \cancel{(120 \cdot \cos 30^\circ)} + \cancel{(-120 \cdot \cos 30^\circ)} + 80 - (60 \cdot \cos 45^\circ)$

$= 37.574$ N

$R_y = \Sigma F_y = (120 \cdot \sin 30^\circ) + (120 \cdot \sin 30^\circ) + 100 - (60 \cdot \sin 45^\circ)$

$= 177.574$ N

The resultant force is,

$\bar{R} = (37.574)\hat{i} + (177.574)\hat{j}$ N i.e. $R = 181.5$ N, $\theta = 78^\circ$

This gives us the magnitude and direction of the resultant force. To locate the point of application of the resultant force, use Varignon's theorem of moments. Taking moments about A,

$(181.5)d = -(120 \times \cos 30^\circ)(4) + (120 \times \sin 30^\circ)(7) + (100 \times 2) + (120 \times \cos 30^\circ)(4) - (60 \times \sin 45^\circ)(7)$

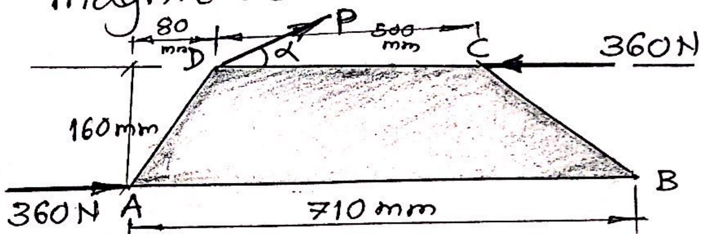
$\therefore d = 1.78$ m

As d is +ve the resultant is rotating in anticlockwise direction about A.

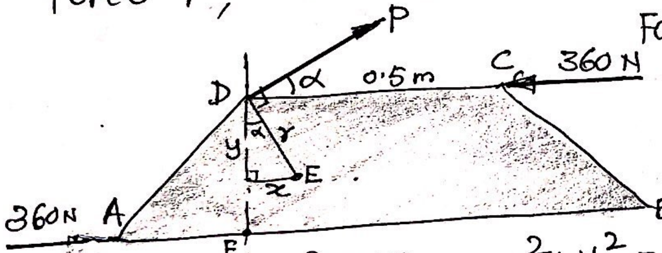
<u>Ans</u> I) The resultant force is $R = 181.5$ N, $\theta = 78^\circ$

II) Perpendicular distance of the resultant force from point A $= 1.78$ m

EX. No. 29 A trapezoidal plate is acted upon by the force P and the couple shown in figure. Determine I) the point of application on the plate of the smallest force F that is equivalent to the given system and II) the magnitude and direction of force F.



__Solution:__ Let E be the point of application of the equivalent single force F. For smallest force F, line DE must be perpendicular to 'P'.



For the resultant force,

$R_x = P\cos\alpha$

$R_y = P\sin\alpha$

$M_D = (360 \times 0.16)$

$= 57.6\ Nm$

From the figure, $x^2 + y^2 = r^2$

$x = (r\sin\alpha)$, $y = (r.\cos\alpha)$

Moment of the given system about E,

$M_E = -(P\cos\alpha).y - (P.\sin\alpha).x + (360 \times 0.16)$

$= -P(r.\cos^2\alpha) - P(r.\sin^2\alpha) + (57.6)$

we have, $r = (x\,\mathrm{cosec}\,\alpha)$ or $r = (y\sec\alpha)$

For r_{min}, $\frac{dr}{d\alpha} = -(x\,\mathrm{cosec}\,\alpha \cdot \cot\alpha) = 0$ or $\frac{dr}{d\alpha} = (y\sec\alpha \cdot \tan\alpha) = 0$

$\alpha = 90^\circ$ or $\alpha = 0$

$\therefore$ This gives us $x = r,\ y = 0$ or $x = 0,\ y = r$

By Varignon's theorem,

$\therefore M_E = 0 = 0 - (P.r) + 57.6 \quad \therefore (P.r) = 57.6\ Nm$

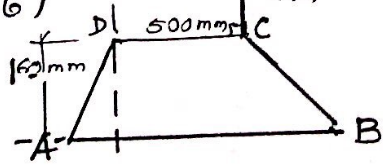
$\therefore P = \left(\frac{57.6}{r}\right) = F$

If $\alpha = 90^\circ$, $F_{min} = \left(\frac{57.6}{r_{max}}\right) = \left(\frac{57.6}{0.5}\right) = 115.2\,N$

If $\alpha = 0^\circ$, $F_{min} = \left(\frac{57.6}{r_{max}}\right) = \left(\frac{57.6}{0.16}\right) = 360\,N$

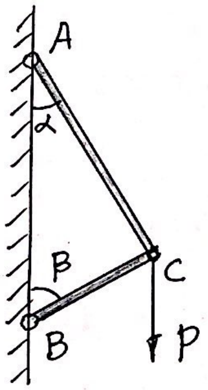
$F_{min} = 115.2\ N$

__Ans:__ Equivalent system



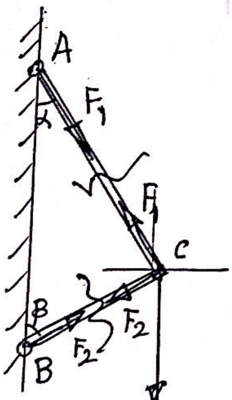
Equilibrium of Concurrent Coplanar Forces

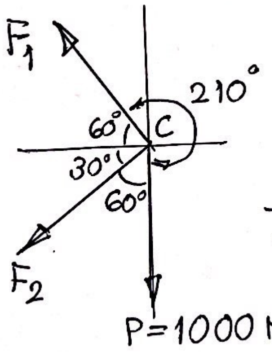
Ex. No. 30 Rods AC and BC are hinged with each other at C and with vertical wall at A and B respectively. If the force P = 1000 N acts on the hinge at C. Define the reactions of these rods on the hinge pin at C. Take $\alpha = 30^\circ$ and $\beta = 60^\circ$



Solution: Due to force 'P' acting at joint C, rods AC and BC will be subjected to axial tensile or compressive forces. Let F_1 and F_2 be the axial tensile forces in rods AC and BC respectively.

Now, consider the F.B.D. of joint C.





Applying Lami's theorem to F.B.D. of joint C, we get,

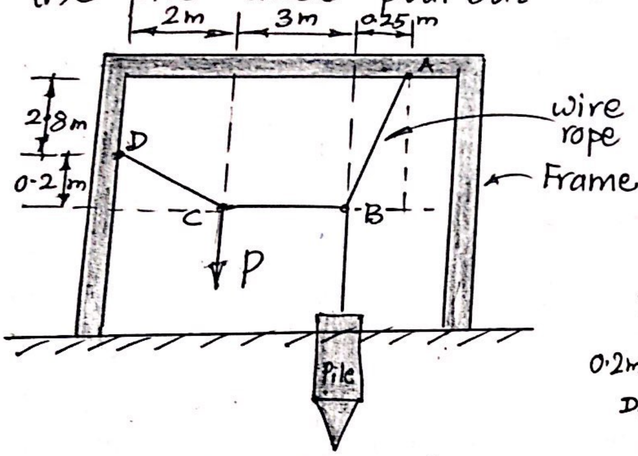
$$\frac{1000}{\sin 90^\circ} = \frac{F_1}{\sin 60^\circ} = \frac{F_2}{\sin 210^\circ}$$

$\therefore F_1 = 866 \text{ N}$

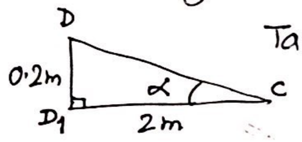
$\therefore F_2 = -500 \text{ N}$

Thus, rod AC is subjected to axial tension of 866 N and rod BC is subjected to axial compression of 500 N.

Ex. No. 31 For extracting a pile driven into the ground, arrangement as shown is used. Calculate the minimum force 'P' to be applied to extract the pile whose pull-out resistance is 200 kN.

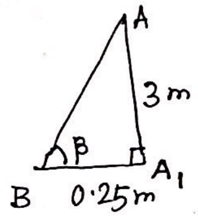


Solution: from the given configuration find the inclination of CD and AB with horizontal.



$\tan\alpha = \frac{0.2}{2}$

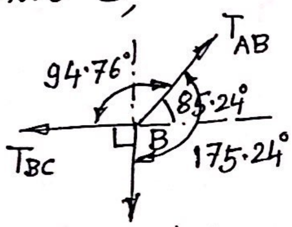
$\therefore \alpha = 5.71°$



$\tan\beta = \frac{3}{0.25}$

$\therefore \beta = 85.24°$

Now, consider F.B.D. of joint B,



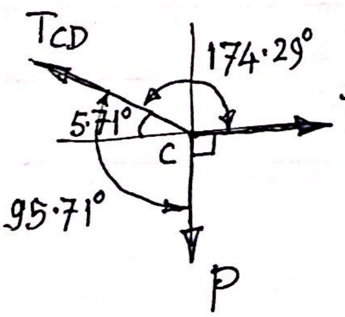
Applying Lami's theorem, we get,

$$\frac{200}{\sin 94.76°} = \frac{T_{AB}}{\sin 90°} = \frac{T_{BC}}{\sin 175.24°}$$

$\therefore T_{AB} = 200.69\,\text{kN}$

$\therefore T_{BC} = 16.65\,\text{kN}$

Now, consider F.B.D. of joint C,



$T_{BC} = 16.65$ N

Applying Lami's theorem, we get,

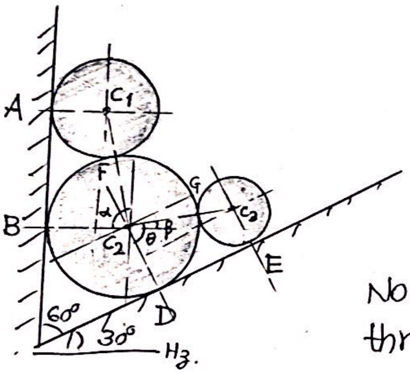
$$\frac{16.65}{\sin 95.71°} = \frac{T_{CD}}{\sin 90°} = \frac{P}{\sin 174.29°}$$

$\therefore T_{CD} = 16.73$ KN

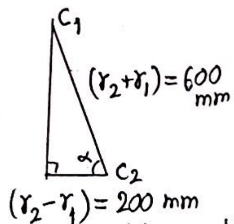
Ans: $P = 1.665$ kN

Ex. No. 32 Three metalic cylinders are stacked as shown in figure. Their radii and their weights are, $r_1 = 200\,mm$, $r_2 = 400\,mm$, $r_3 = 100\,mm$ and $W_1 = 200\,N$, $W_2 = 400\,N$, $W_3 = 100\,N$

Find the reactions at A, B, D and E. Also, Find the contact pressures at F and G.



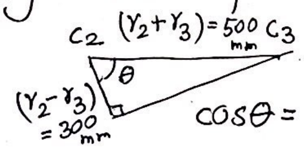
Solution : Find the angles α and β



$$\cos\alpha = \frac{200}{600}$$

$$\therefore \alpha = 70.53^\circ$$

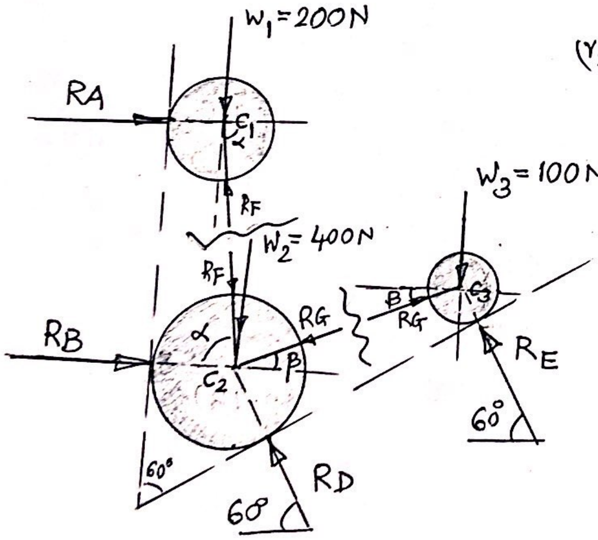
Now, consider the F.B.D. of the three cylinders separately.



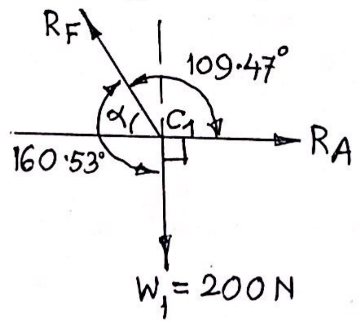
$$\cos\theta = \frac{300}{500}$$

$$\theta = 53.13^\circ$$

$$\beta = \theta - 60^\circ = -6.87^\circ$$



Consider F.B.D. of the first cylinder;



Applying Lami's theorem,

$$\frac{200}{\sin 109.47^\circ} = \frac{R_A}{\sin 160.53^\circ} = \frac{R_F}{\sin 90^\circ}$$

$$\therefore R_A = 70.705\ N\ (\rightarrow)$$

$$\therefore R_F = 212.13\ N$$

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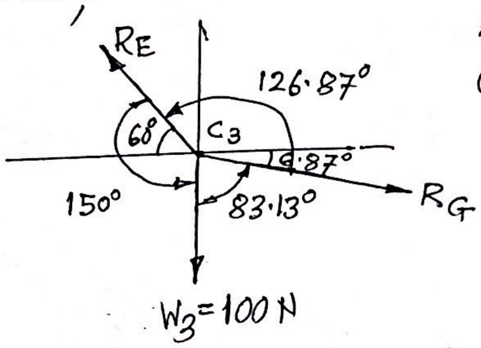
Now, consider the F.B.D. of the third cylinder;



Applying Lami's theorem, we get

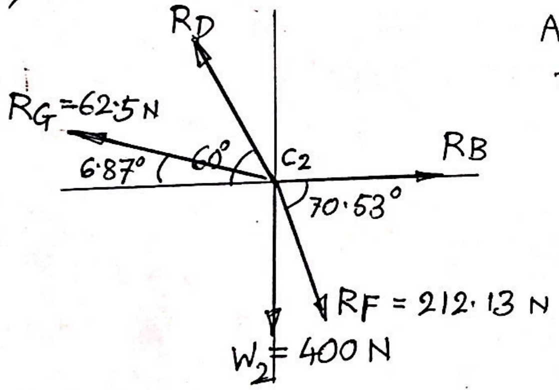
$$\frac{100}{\sin 126.87°} = \frac{R_E}{\sin 83.13°}$$

$$= \frac{R_G}{\sin 150°}$$

$\therefore\ R_G = 62.5\ N$

$\therefore\ R_E = 124.10\ N$ (60°)

Now, consider the F.B.D. of the second cylinder;



As there are more than three concurrent forces in equilibrium, we can not use Lami's theorem. In this case, we can use, the analytical conditions of equilibrium.

i.e. Equations of equilibrium. $(\Sigma F_x = 0,\ \Sigma F_y = 0)$

$\Sigma F_x = 0$ gives,

$$R_B - R_D \cdot \cos 60° - (62.5)\cos 6.87° + (212.13)\cos 70.53° = 0 \quad \longrightarrow (1)$$

$\Sigma F_y = 0$ gives,

$$R_D \cdot \sin 60° + (62.5)\sin 6.87° - 400 - (212.13)\sin 70.53° = 0 \quad \longrightarrow (2)$$

$\therefore\ R_D = 684.2\ N$ (60°)

$\therefore\ R_B = 333.45\ N\ (\rightarrow)$

<u>Ans</u>: $R_A = 70.705\ N\ (\rightarrow)$

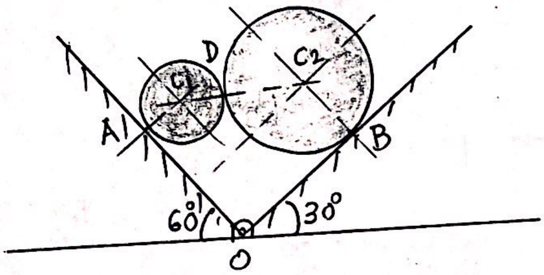
$R_B = 333.45\ N\ (\rightarrow)$

$R_D = 684.2\ N$ (60°)

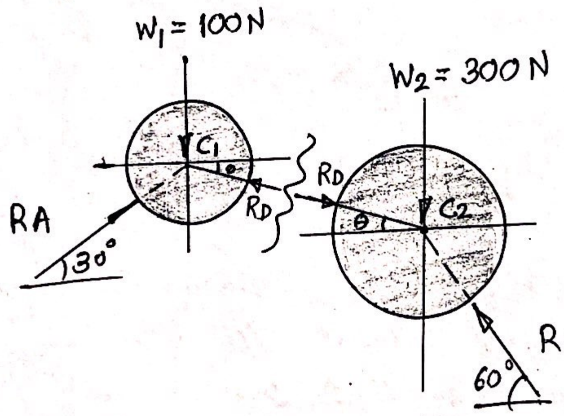
$R_E = 124.10\ N$ (60°)

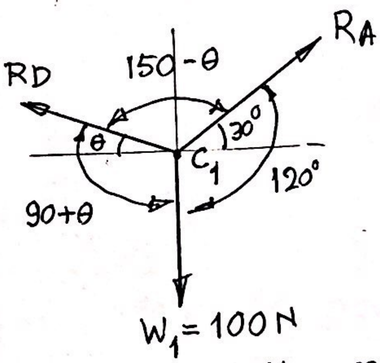
Contact pressures at pt. F and G are 212.13 N and 62.5 N respectively.

$\boxed{\text{Ex. No. 33}}$ Two cylindrical pipes are stacked as shown in figure. Their weights are $W_1 = 100$ N and $W_2 = 300$ N. And their radii are $r_1 = 100$ mm and $r_2 = 200$ mm respec-tively. Calculate the reactions at A, B and D. Also calculate the angle made by C_1C_2 with horizontal.



Consider the F.B.D. of the first pipe.



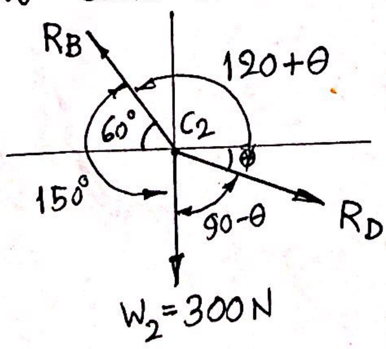


Applying Lami's theorem,

$$\frac{100}{\sin(150-\theta)} = \frac{R_A}{\sin(90+\theta)} = \frac{R_D}{\sin 120^\circ}$$

$$\therefore R_A = \left[\frac{100 \cdot \cos\theta}{\sin(150-\theta)}\right] \qquad \therefore R_D = \left[\frac{86.6}{\sin(150-\theta)}\right] \rightarrow (a)$$

Now consider the F.B.D. of the second pipe,



Applying Lami's theorem, we get,

$$\frac{300}{\sin(120+\theta)} = \frac{R_B}{\sin(90-\theta)} = \frac{R_D}{\sin 150^\circ}$$

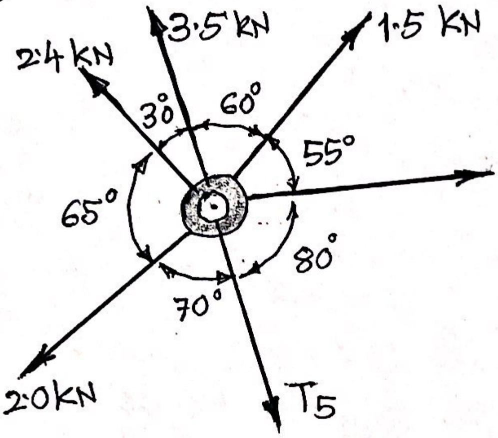
$$\therefore R_D = \left[\frac{150}{\sin(120+\theta)}\right] \rightarrow (b)$$

$$R_B = \left[\frac{300 \cdot \cos\theta}{\sin(120+\theta)}\right]$$

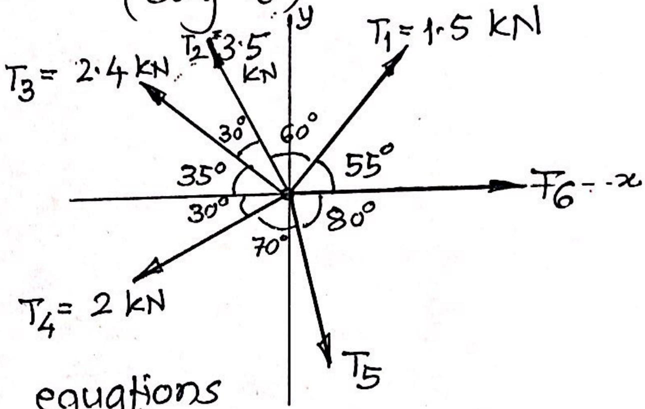
Ans: From (a) and (b) we get $\boxed{\theta = 0}$ i.e. C_1C_2 is horizontal.

$R_A = 200$ N $\angle 30^\circ$, $R_B = 346.4$ N 60°, $R_D = 173.2$ N (hz)

Ex. No. 34 Figure shows the plan view of a floating ring to which six overhead conductor wires (all in the same horizontal plane) of a city electric tram-line are anchored. Values of the tensions in the four of them is shown calculate the tensions T_5 and T_6 in the remaining two.



Solution : Rearrange the forces, considering one of them as horizontal. (say T_6)



As the system is in equilibrium, we can apply equations of equilibrium.

$\Sigma F_x = 0$ gives, $T_6 + (1.5)\cos 55^\circ - (3.5)\cos 65^\circ - (2.4)\cos 35^\circ$
$- (2)\cos 30^\circ + T_5 \cdot \cos 80^\circ = 0$

$\therefore \quad T_6 + (0.173)T_5 = 4.322 \longrightarrow (1)$

$\Sigma F_y = 0$ gives, $(1.5)\sin 55^\circ + (3.5)\sin 65^\circ + (2.4)\sin 35^\circ$
$- (2)\sin 30^\circ - T_5 \sin 80^\circ = 0$

$\therefore \quad (0.985)T_5 = 4.777 \longrightarrow (2)$

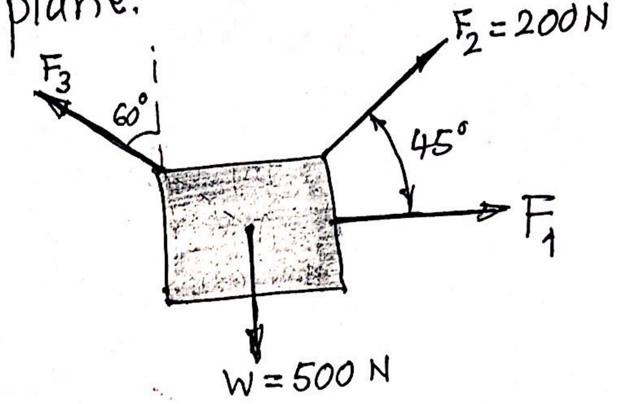
solving equations (1) and (2) we get,

$T_5 = 4.85$ KN

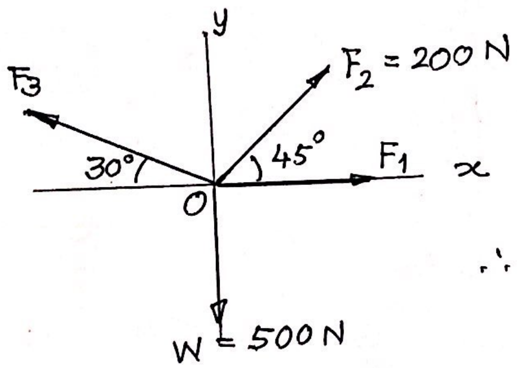
$T_6 = 3.483$ KN

Ans : $T_5 = 4.85$ KN, $T_6 = 3.483$ KN

Ex. No: 35 A 500 N crate is held up by three forces. Clearly the three forces should add up to a force of 500N going upward. What should be forces F_1 and F_3 for this condition? All forces are in same plane.



<u>Solution</u>: Consider the F.B.D. of the crate. Convert the crate into a point and form a concurrent coplanar force system, which is in equilibrium.



Applying equations of equilibrium,

$\Sigma F_x = 0$ gives,

$$F_1 + (200)\cos 45^\circ - F_3 \cdot \cos 30^\circ = 0$$

$$\therefore \quad F_1 + 141.42 = (0.866) F_3 \longrightarrow (1)$$

$\Sigma F_y = 0$ gives,

$$(200)\sin 45^\circ + F_3 \sin 30^\circ - 500 = 0$$

$$\therefore \quad 141.42 + (0.5) F_3 - 500 = 0$$

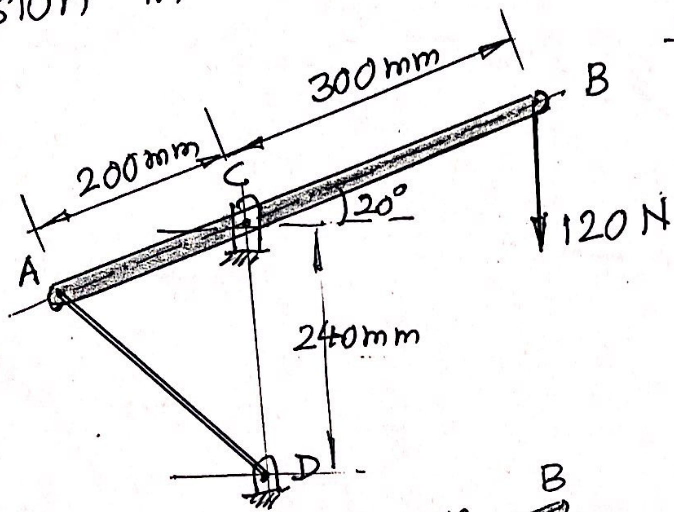
$$(0.5) F_3 = 358.58 \longrightarrow (2)$$

solving equations (1) and (2) we get,

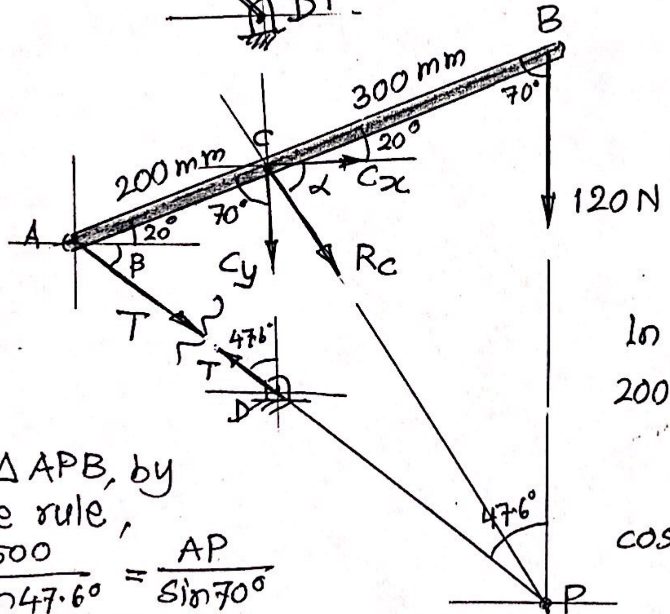
$$F_3 = 717.16 \text{ N} \quad \text{and} \quad F_1 = 479.64 \text{ N}$$

<u>Ans</u>: $F_1 = 479.64$ N $(\longrightarrow)$, $F_3 = 717.16$ N. (30°)

Ex. No. 36 A lever AB is hinged at C and is attached to a control cable at A. If the lever is subjected to a 120 N vertical force at B. Determine (i) the tension in the cable and (ii) the reaction at C



<u>solution</u>: Three non parallel coplanar forces in equilibrium, are always concurrent. Here, bar AB is acted upon by three forces i.e. 120 N vertically downward, tension T along AD and the reaction at C. Under the action of these 3 forces the bar is in equilibrium. These three forces must be concurrent at point 'P' in the same plane.



In ΔACD, by cosine rule,

$200^2 + 240^2 - AD^2 = 2 \times 200 \times 240 \times \cos 70°$

$\therefore AD = 254.5$ mm

$\cos(\beta + 20°) = \left[\dfrac{200^2 + 254.5^2 - 240^2}{2 \times 200 \times 254.5}\right] = 0.463$

$\beta + 20° = 62.4°$

$\beta = 42.4°$

In ΔAPB, by sine rule,

$$\frac{500}{\sin 47.6°} = \frac{AP}{\sin 70°}$$

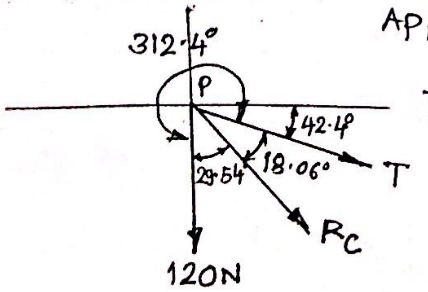
$\therefore AP = 636.25$ mm

In ΔCAP, by cosine rule,

$$\cos 62.4° = \left(\frac{200^2 + 636.25^2 - CP^2}{2 \times 200 \times 636.25}\right) \quad \therefore CP = 571.75 \text{ mm}$$

Now, $\cos\alpha = \left(\dfrac{300 \cdot \cos 20°}{571.75}\right) = 0.493 \quad \therefore \alpha = 60.46°$

Now consider F.B.D. of point P :



Applying Lami's theorem,

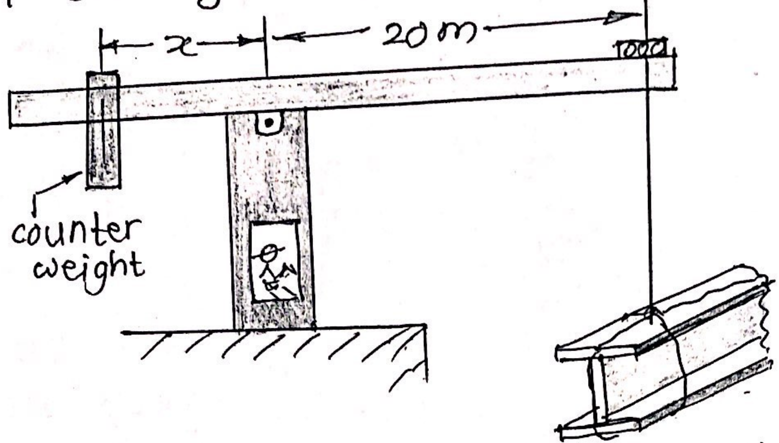
$$\frac{120}{\sin 18.06°} = \frac{T}{\sin 29.54°} = \frac{R_c}{\sin 312.4}$$

$\therefore T = 190.84$ N

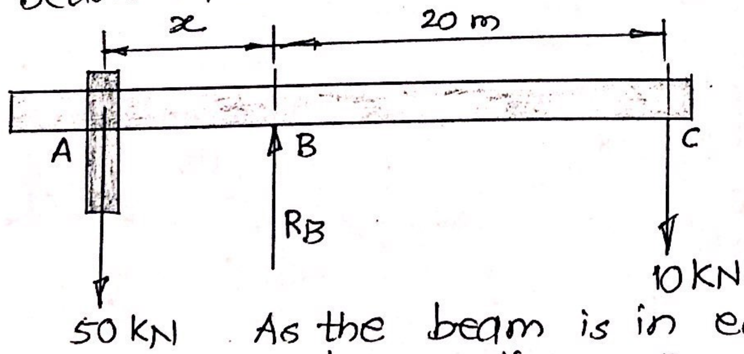
$R_c = -285.84$ N i.e. 285.84 N $60.46°$

Equilibrium of Parallel Coplanar Forces

Ex. No. 37 At what position must the operator of the counterweight crane locate the 50 kN counterweight when he lifts a 10 kN load of steel girder?



Solution: Consider the F.B.D. of the horizontal beam of the crane.



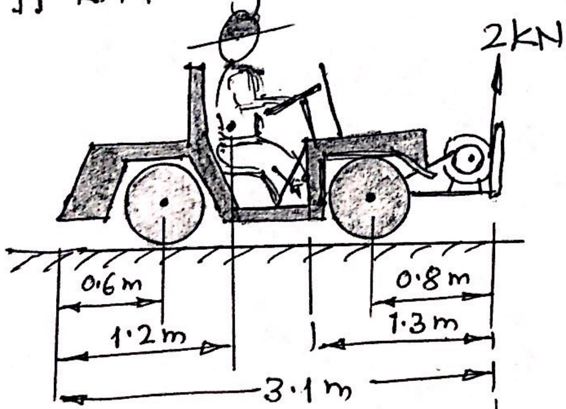
As the beam is in equilibrium, we can apply equations of equilibrium.

$\Sigma F_y = 0$ gives, $R_B - 50 - 10 = 0 \quad \therefore \; R_B = 60 \text{ kN} (\uparrow)$

$\Sigma M_B = 0$ gives, $50 \cdot x - (10 \times 20) = 0 \quad \therefore \; x = 4 \text{ m}$

Ans: $x = 4$ m, $R_B = 60$ kN $(\uparrow)$

Ex. No. 38 A jeep winch is used to raise itself by a force of 2 kN. What are the reactions at the jeep tires with and without the winch load? The driver weighs 800 N and the jeep weighs 11 kN. Weight of the jeep acts at its C.G.



Solution: Consider the F.B.D. of the jeep with driver.

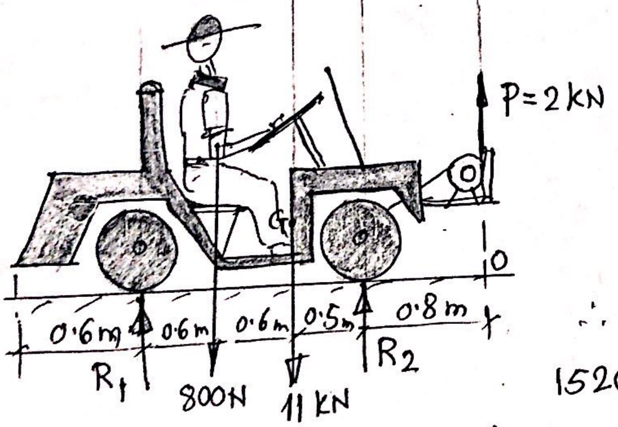
$\Sigma F_y = 0$ gives,

$$R_1 + R_2 + P - 1100 - 800 = 0$$

$$\therefore R_1 + R_2 + P = 1900 \text{ N} \quad \text{(1)}$$

$\Sigma M_O = 0$ gives,

$$-(0.8)R_2 + (1100 \times 1.3) + (800 \times 1.9) - (2.5)R_1 = 0 \quad \text{(2)}$$



$$\therefore (0.8)R_2 + (2.5)R_1 = 2950$$

from eqⁿ (1) we get,

$$R_2 = (1900 - P - R_1)$$

$\therefore$ Eqⁿ (2) becomes,

$$1520 - (0.8)P - (0.8)R_1 + (2.5)R_1 = 2950$$

$$\therefore (1.7)R_1 = 1430 + (0.8)P$$

$$\therefore R_1 = \left(\frac{1430 + (0.8)P}{1.7}\right)$$

and $$R_2 = \left(\frac{1800 - (0.9)P}{1.7}\right)$$

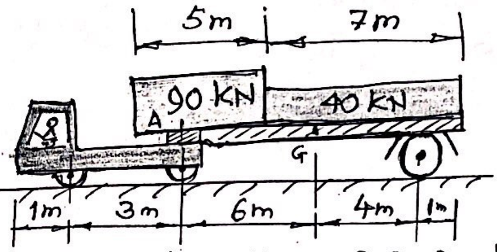
A) With the winch load i.e. $P = 2000$ N

$$R_1 = 1782.35 \text{ N} (\uparrow), \quad R_2 = 0$$

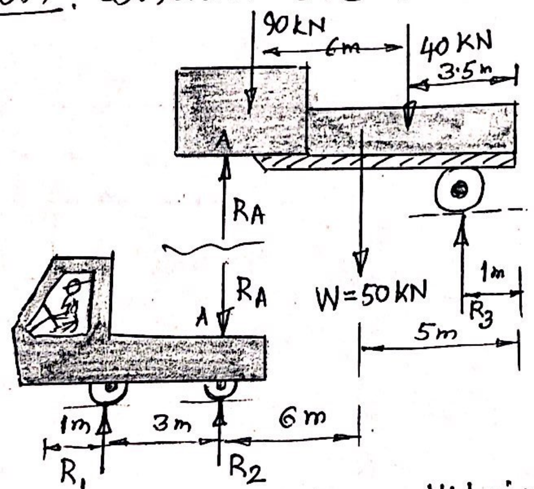
B) Without the winch load i.e. $P = 0$

$$R_1 = 841.18 \text{ N} (\uparrow), \quad R_2 = 1058.82 \text{ N} (\uparrow)$$

Ex. No. 39 The trailer weighs 50 kN and is loaded with crates weighing 90 kN and 40 kN. What are the reactions at the rear wheel and on the tractor at A?



solution: Consider the F.B.D. of the system.



Applying equations of equilibrium,

$\Sigma F_y = 0$ gives, $R_A + R_3 - 90 - 50 - 40 = 0$

$\therefore \quad R_A + R_3 = 180 \text{ kN} \longrightarrow (1)$

$\Sigma M_A = 0$ gives, $10 \cdot R_3 - (40 \times 7.5) - (50 \times 6) - (90 \times 1.5) = 0 \longrightarrow (2)$

$\therefore \quad R_3 = 73.5 \text{ kN} (\uparrow)$

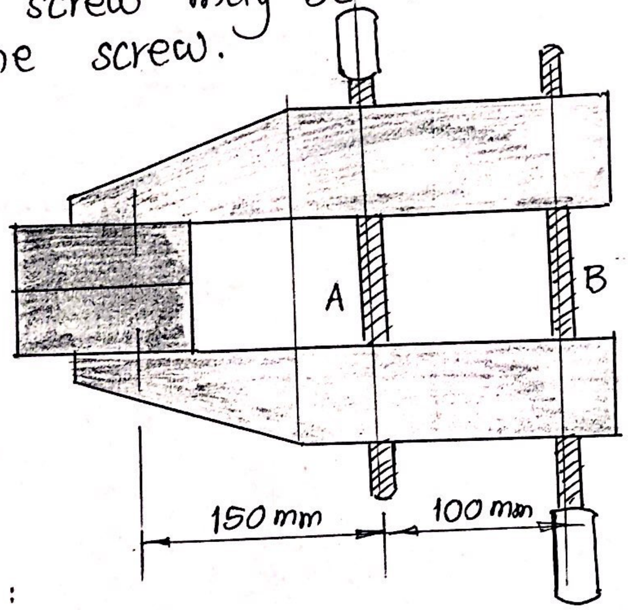
$\therefore \quad R_A = 106.5 \text{ kN}$

Now, since R_2 and R_A are colinear forces,

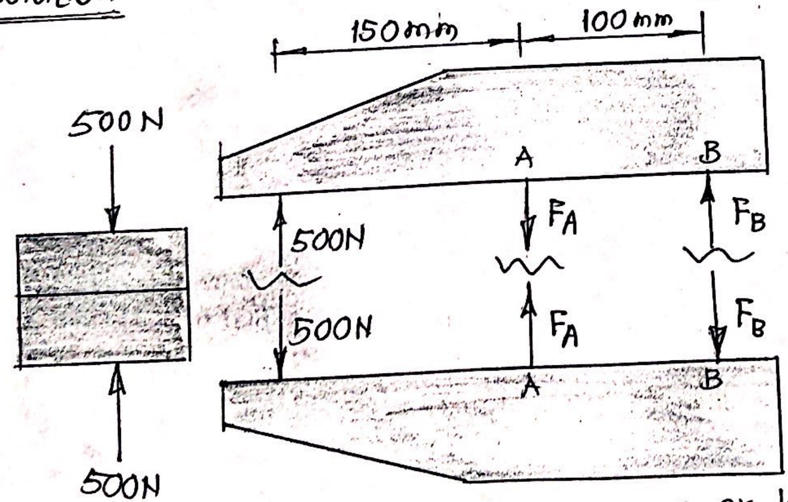
$R_2 = R_A = 106.5 \text{ kN} (\uparrow)$

and $R_1 = 0$

Ex. No. 40 If the screw B of the wood clamp is tightened so that the two blocks are under a compression of 500N. Determine the force in each screw. The force supported by each screw may be taken in the direction of the screw.



Solution :



Consider the F.B.D. of the upper or lower part of the clamp. Applying equations of equilibrium, we get,

$\Sigma F_y = 0$ gives, $500 - F_A + F_B = 0 \longrightarrow (1)$

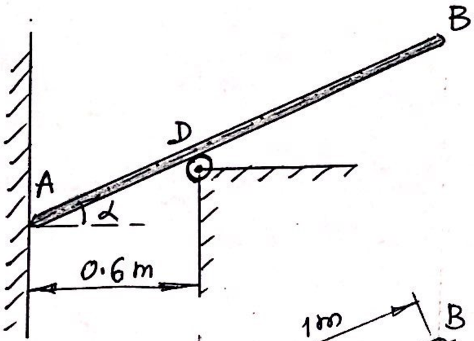
$\Sigma M_B = 0$ gives, $(F_A \times 100) - (500 \times 250) = 0 \longrightarrow (2)$

$\therefore \quad F_A = 1250\ N$ and $F_B = 750\ N$

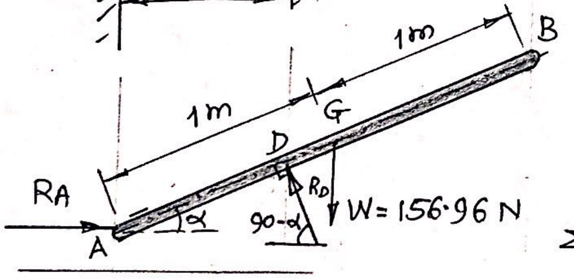
(H)

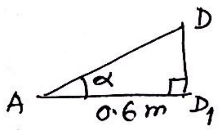
Equilibrium of general coplanar force system

Ex. No. 41 A slender prismatic bar AB of weight 16 kg and length 2 m rests on a very small frictionless roller at D and against a smooth vertical wall at A, as shown in figure find the angle α that the bar must make with the horizontal in the condition of equilibrium



Solution: The bar AB is subjected to three forces i.e. its weight W acting at its midpoint, reaction of the wall at A perpendicular to the wall and the reaction of the roller at D perpendicular to bar AB.





$\cos\alpha = \frac{0.6}{AD}$

$\therefore AD = (0.6)\sec\alpha$

Applying equations of equilibrium

$\Sigma f_x = 0$ gives,

$$R_A - (R_D)\cos(90-\alpha) = 0$$

$$\therefore R_A - R_D \cdot \sin\alpha = 0 \longrightarrow (1)$$

$\Sigma f_y = 0$ gives, $R_D \sin(90-\alpha) - 156.96 = 0$

$$\therefore R_D \cdot \cos\alpha - 156.96 = 0 \longrightarrow (2)$$

$$\therefore R_D = \left(\frac{156.96}{\cos\alpha}\right) \text{ and } R_A = (156.96)\tan\alpha$$

$\Sigma M_A = 0$ gives, $R_D \times 0.6 \times \sec\alpha - (156.96)(1 \times \cos\alpha) = 0$

$$\therefore \left(\frac{156.96 \times 0.6}{\cos^2\alpha}\right) - (156.96)\cos\alpha = 0$$

$$\therefore 0.6 - \cos^3\alpha = 0$$

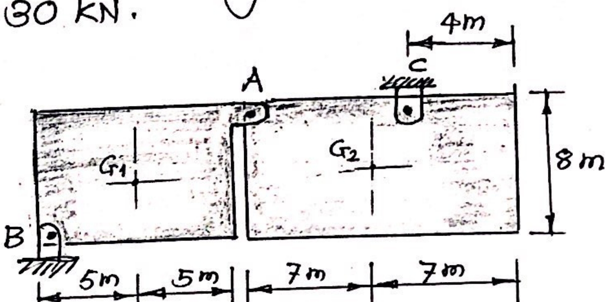
$$\therefore \cos^3\alpha = 0.6$$

$$\cos\alpha = 0.8434$$

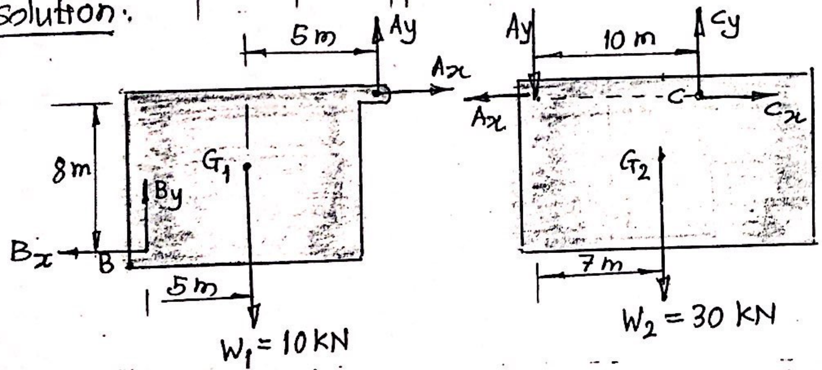
$$\therefore \alpha = 32.5^\circ$$

Ans: $\alpha = 32.5^\circ$

$\boxed{Ex. No. 42}$ Find the components of the forces acting on pin A, B and C connecting and supporting the blocks shown in figure. Block I weighs 10 kN and II weighs 30 kN.



<u>Solution</u>:



consider the F.B.D.s of blocks I and II

Applying equations of equilibrium to f.B.D. of I we get,

$\Sigma F_x = 0$ gives $A_x - B_x = 0 \rightarrow (1)$

$\Sigma F_y = 0$ gives, $A_y + B_y - 10 = 0 \rightarrow (2)$

$\Sigma M_B = 0$ gives, $10 \cdot A_y - 8 \cdot A_x - 50 = 0 \rightarrow (3)$

Similarly applying equations of equilibrium to f.B.D. of II, we get,

$\Sigma F_x = 0$ gives, $C_x - A_x = 0 \rightarrow (4)$

$\Sigma F_y = 0$ gives, $C_y - A_y - 30 = 0 \rightarrow (5)$

$\Sigma M_C = 0$ gives, $(30 \times 3) + 10 \cdot A_y = 0 \rightarrow (6)$

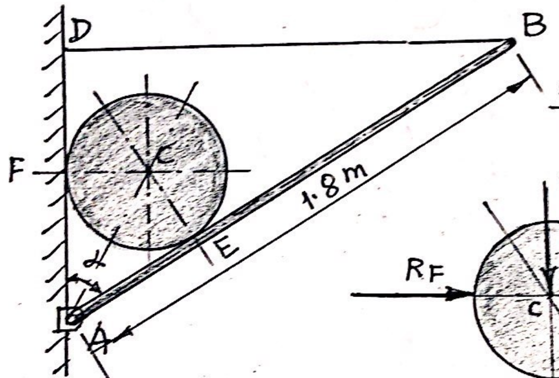
solving the above equation we get

$A_x = -17.5$ kN, $A_y = -9$ kN

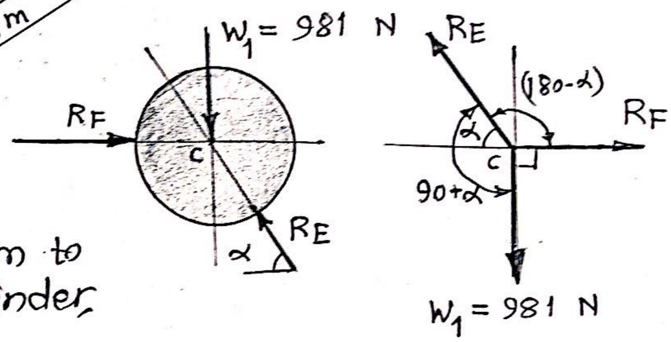
$B_x = 17.5$ kN ($\rightarrow$), $B_y = 19$ kN ($\uparrow$)

$C_x = 17.5$ kN ($\leftarrow$), $C_y = 21$ kN ($\uparrow$)

Ex. No. 43 A prismatic circular cylinder of weight 200 kg and radius 0.3 m is supported in a horizontal position against a vertical wall by two identical brakets like the one shown in figure. Each bar AB is hinged to the wall at A and supported at B by a horizontal cable BD. All the surfaces are smooth. Determine the value of angle α that AB should make with the wall to attain a minimum tension in each cable BD.



Solution : Consider the F.B.D. of the cylinder.

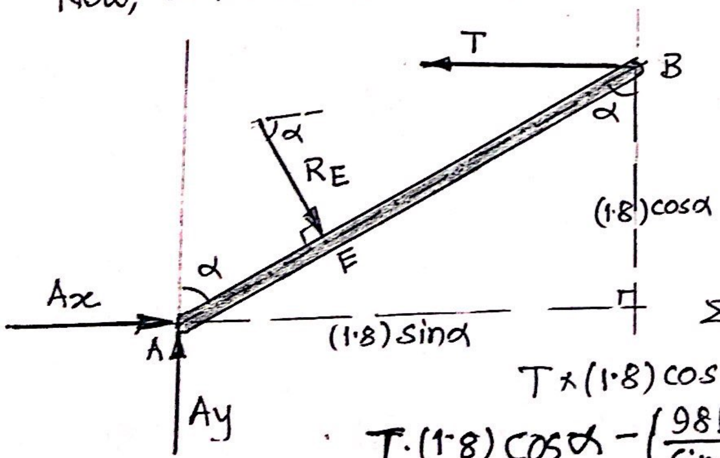


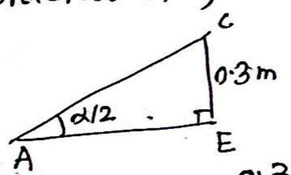
Applying Lami's theorem to the F.B.D. of the cylinder, we get,

$$\frac{981}{\sin(180-\alpha)} = \frac{R_E}{\sin 90^\circ} = \frac{R_F}{\sin(90+\alpha)}$$

$$\therefore \quad R_E = \left(\frac{981}{\sin\alpha}\right), \quad R_F = \left(\frac{981 \cdot \cos\alpha}{\sin\alpha}\right)$$

Now, consider the F.B.D. of the bracket AB;





$$\tan\alpha/2 = \frac{0.3}{AE}$$

$$\therefore AE = \left(\frac{0.3}{\tan\alpha/2}\right)$$

$\Sigma M_A = 0$ gives,

$$T \times (1.8)\cos\alpha - (R_E \times AE) = 0$$

$$T \cdot (1.8)\cos\alpha - \left(\frac{981}{\sin\alpha}\right)\left(\frac{0.3}{\tan\alpha/2}\right) = 0$$

$$\therefore \left(T \times 1.8 \times \cos\alpha\right) = \left[\frac{294.3}{\sin\alpha \cdot \tan\alpha/2}\right]$$

$$\therefore \quad T = \left[\frac{163.5}{\sin\alpha \cdot \cos\alpha \cdot \tan\alpha/2}\right]$$

Here,

$$\sin\alpha = 2 \cdot \sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$$

$$\therefore \sin\alpha \cdot \cos\alpha \cdot \tan\frac{\alpha}{2} = \left(2 \cdot \sin\frac{\alpha}{2} \cdot \cos\frac{\alpha}{2} \cdot \frac{\sin\alpha/2}{\cos\alpha/2} \cdot \cos\alpha\right)$$

$$\therefore \quad T = \left[\frac{81.75}{(\sin^2\alpha/2)(\cos\alpha)}\right]$$

For T_{min}, $\frac{dT}{d\alpha} = 0$

Equating the numerator of the derivative with zero,

we get, $(\sin^2\frac{\alpha}{2})(-\sin\alpha) + (\cos\alpha)(2 \cdot \sin\frac{\alpha}{2} \cdot \cos\frac{\alpha}{2})(\frac{1}{2}) = 0$

$$\therefore \left(\frac{\cancel{\sin\alpha} \cdot \cos\alpha}{2}\right) = \left(\cancel{\sin\alpha} \cdot \sin^2\frac{\alpha}{2}\right)$$

$$\cos\alpha = 2\left(\sin^2\frac{\alpha}{2}\right)$$

$$\therefore \left[1 - 2\sin^2\frac{\alpha}{2}\right] = 2\left(\sin^2\frac{\alpha}{2}\right)$$

$$\therefore \quad 4\sin^2\frac{\alpha}{2} = 1$$

$$\therefore \quad \sin^2\frac{\alpha}{2} = \frac{1}{4}$$

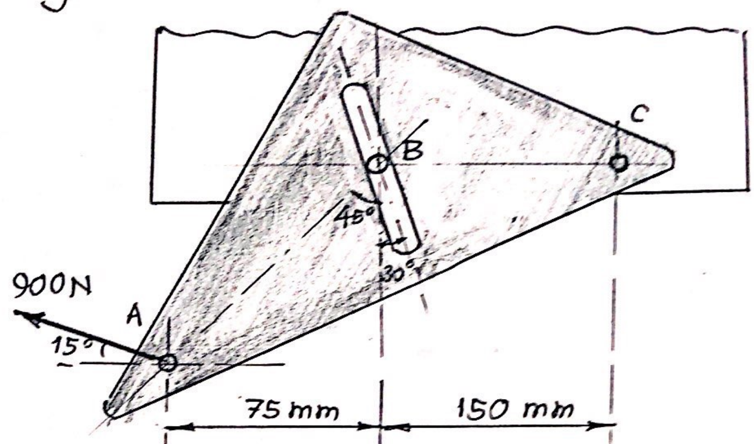
$$\therefore \quad \sin\frac{\alpha}{2} = \frac{1}{2} = 0.5$$

$$\frac{\alpha}{2} = 30^\circ \quad \therefore \boxed{\alpha = 60^\circ}$$

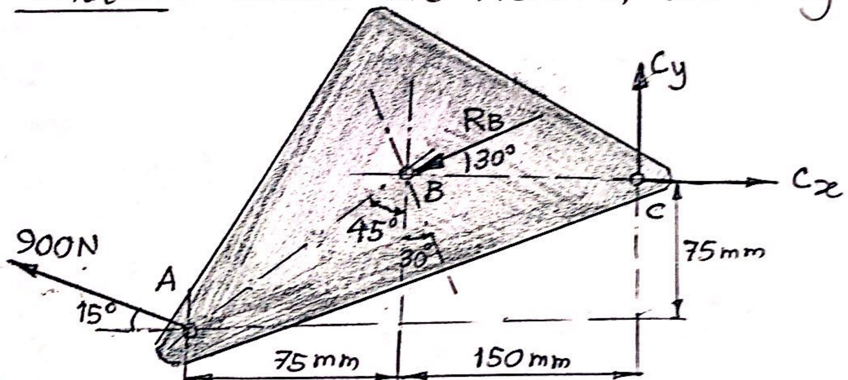
This gives, $T_{min} = \left(\frac{81.75}{\sin^2 30^\circ \times \cos 60^\circ}\right) = 654 \text{ N}$

<u>Ans</u>: For T_{min}, $\alpha = 60^\circ$ and $T_{min} = 654$ N

Ex. No. 44 Calculate the magnitude of the force supported by the pin at C under the action of the 900 N load applied to the bracket. Neglect the friction in the slot.



Solution: Consider the F.B.D. of the triangular plate.



Applying equations of equilibrium, we get,

$\Sigma F_x = 0$ gives, $C_x - R_B \cos 30° - 900 \cdot \cos 15° = 0$

$$C_x - (0.866) R_B - (869.33) = 0 \longrightarrow (1)$$

$\Sigma F_y = 0$ gives, $C_y - R_B \sin 30° + 900 \cdot \sin 15° = 0$

$$C_y - (0.5) R_B + 232.94 = 0 \longrightarrow (2)$$

$\Sigma M_C = 0$ gives,

$$-(900 \cdot \cos 15)(75) - (900 \cdot \sin 15°)(225) + (R_B \sin 30°)(150) = 0$$

$$-65200 - 52410.85 + (75) R_B = 0 \longrightarrow (3)$$

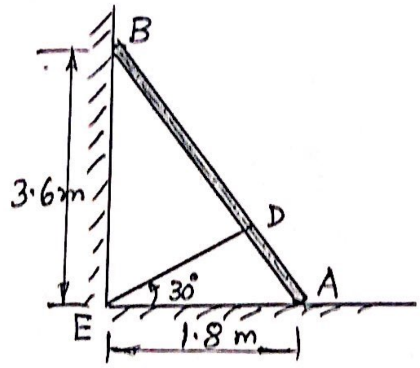
$$R_B = 1568.15 \text{ N } (30°)$$

$$C_x = 2227.40 \text{ N } (\rightarrow)$$

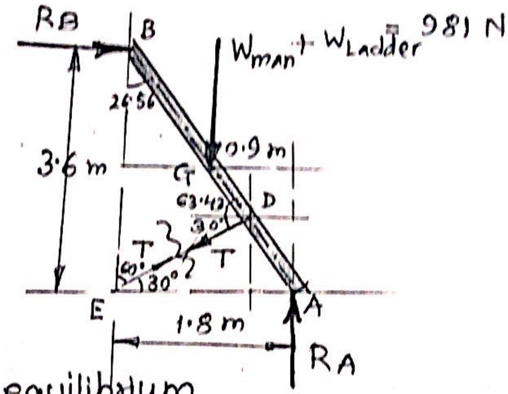
$$C_y = 551.135 \text{ N } (\uparrow)$$

Ans: Reaction at C $R_C = 2294.57$ N $\measuredangle 13.9°$

Ex. No. 45 A 75 kg man stands on the middle rung of a 25 kg ladder as shown in figure. Assuming that the floor and the wall are perfectly smooth and that slipping is prevented by a string DE, Find the tension in the string and the reactions at A and B.



Solution: Consider the F.B.D. of the ladder.



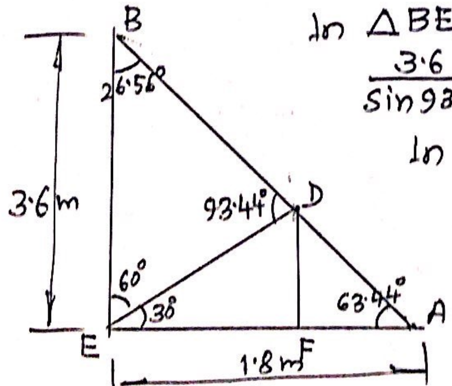
Applying equations of equilibrium,

$\Sigma F_x = 0$ gives, $R_B - T\cdot\cos 30^\circ = 0$

$\therefore \quad R_B - (0.866)T = 0 \longrightarrow (1)$

$\Sigma F_y = 0$ gives, $R_A - T\cdot\sin 30^\circ - 981 = 0$

$\therefore \quad R_A - (0.5)T - 981 = 0 \longrightarrow (2)$



In ΔBED,

$$\frac{3.6}{\sin 93.44^\circ} = \frac{ED}{\sin 26.56^\circ} \quad \therefore ED = 1.612\ m$$

In ΔEDF,

$EF = (1.612)\cos 30^\circ = 1.396\ m$

$DF = (1.612)\sin 30^\circ = 0.806\ m$

$\therefore AF = (1.8 - 1.396) = 0.404\ m$

$\Sigma M_A = 0$ gives,

$$-(R_B \times 3.6) + (981 \times 0.9) + (T\cdot\cos 30^\circ)(0.806) + (T\cdot\sin 30^\circ)(0.404) = 0$$

$\therefore \quad 882.9 - (3.6)R_B + (0.9)T = 0 \longrightarrow (3)$

solving equation (1), (2) and (3), we get,

Ans:

$T = 398.133\ N$

$R_A = 1180.06\ N\ (\uparrow)$

$R_B = 344.78\ N\ (\longrightarrow)$

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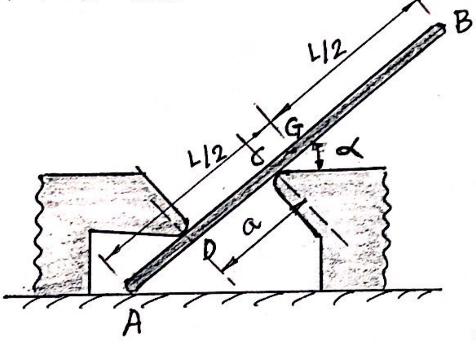
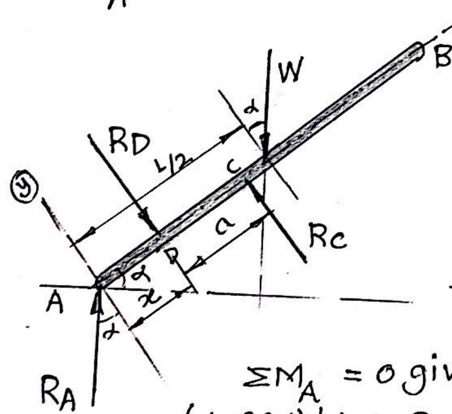
Ex. No. 46 A prismatic bar AB of weight W and length l rests at A against a smooth horizontal floor and under the action of its gravity force W presses against supports at C and D. Neglecting friction determine the reactions at A, C and D.



solution: Consider the x axis of the frame of reference along the bar AB and y axis perpendicular to it. Applying equations of equilibrium to the F.B.D. of the bar,



$\Sigma F_x = 0$ gives,

$$R_A \cdot \sin\alpha - W \cdot \sin\alpha = 0 \quad \text{--- (1)}$$

$$\therefore R_A = W \ (\uparrow)$$

$\Sigma F_y = 0$ gives,

$$\cancel{R_A \cdot \cos\alpha} + R_C - R_D - \cancel{W \cdot \cos\alpha} = 0 \quad \text{--- (2)}$$

$$\therefore R_C = R_D$$

$\Sigma M_A = 0$ gives,

$$-(W \cdot \cos\alpha)\left(\frac{L}{2}\right) + R_C(a + x) - R_D \cdot x = 0$$

$$\therefore -W \cdot \cos\alpha \cdot \frac{L}{2} + R_C \cdot a + \cancel{R_C \cdot x - R_D \cdot x} = 0$$

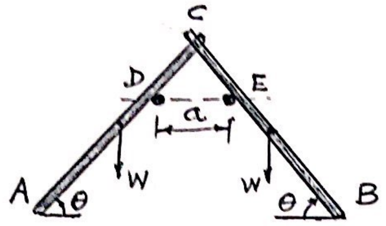
$$R_C \cdot a = W \cdot \cos\alpha \cdot \frac{L}{2}$$

$$\therefore R_C = \left(\frac{W \cdot L}{2 \cdot a}\right)(\cos\alpha) = R_D$$

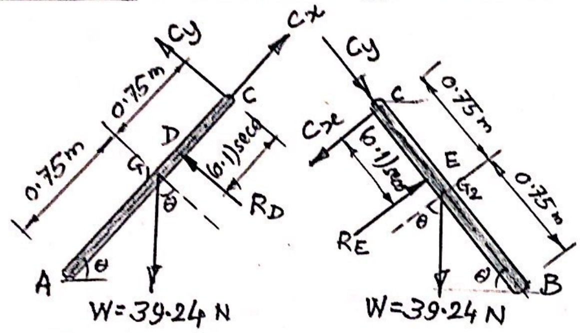
Ans: $R_A = W \ (\uparrow)$

$R_C = R_D = \left(\frac{WL}{2 \cdot a}\right)\cos\alpha$

Ex. No. 47 Two slender prismatic bars AC and BC, each of length 1.5 m and weight 4 kg are hinged together at C and supported in a vertical plane by two pegs at D and E as shown in figure. Neglecting friction, find the angle θ that each bar will make with the horizontal in the condition of equilibrium. Take $a = 0.4$ m



<u>Solution</u> : Consider the F.B.D. of bars AC and CB.



For Bar AC, consider x axis along the bar and y axis perpendicular to it.
For Bar BC, consider y axis along the bar and x axis perpendicular to it.

For bar AC:

$\Sigma F_x = 0$ gives, $C_x - (39.24)\sin\theta = 0 \longrightarrow (1)$

$\Sigma F_y = 0$ gives, $C_y + R_D - (39.24)\cos\theta = 0 \longrightarrow (2)$

$\Sigma M_C = 0$ gives, $(39.24)\cos\theta \times 0.75 - R_D \times 0.1 \times \sec\theta = 0 \longrightarrow (3)$

For bar BC,

$\Sigma F_y = 0$ gives, $C_y + (39.24)\sin\theta = 0 \longrightarrow (4)$

$$\therefore C_y = -(39.24)\sin\theta$$

Eq^n (2) gives, $R_D = (39.24)(\sin\theta + \cos\theta)$

$\therefore$ Eq^n (3) becomes, $\cancel{(39.24)}(0.75)\cos\theta = \dfrac{\cancel{(39.24)}(0.1)}{\cos\theta}(\sin\theta + \cos\theta)$

$$\therefore (7.5)\cos^2\theta - \cos\theta = \sin\theta$$

$$(56.25)\cos^4\theta - (15)\cos^3\theta + \cos^2\theta = 1 - \cos^2\theta$$

$$\therefore (56.25)\cos^4\theta - (15)\cos^3\theta + 2\cos^2\theta - 1 = 0$$

We can get 'θ' by solving the above equation.

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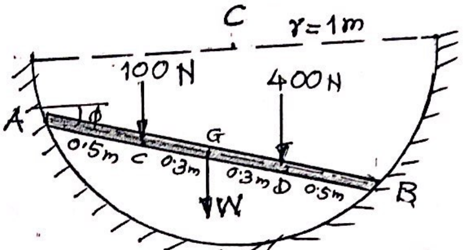
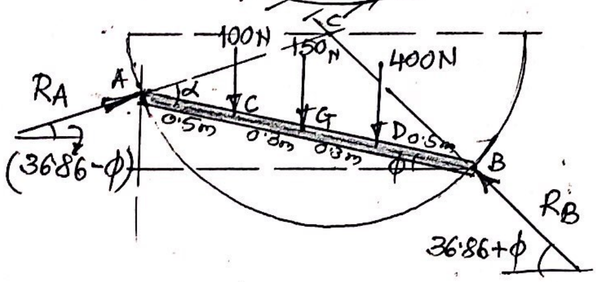
Ex. No. 48 A uniform rod AB of length 1.6 m and weight 150 N rests in equilibrium position on the inner surface of a smooth semicircular channel of radius 1 m. It is carrying two point loads of 100 N and 400 N as shown in figure. Determine its configuration as defined by angle ϕ.





Solution: In ΔACB,

$AC = BC = 1$ m

$\therefore m\angle CAB = m\angle CBA$

$$\cos\alpha = \left(\frac{1^2+1.6^2-1^2}{2\times1\times1.6}\right) = 0.8$$

$\therefore \alpha = 36.86^\circ$

Reactions at A and B are perpendicular to the curved surface. Hence, they are passing through the center of the semicircular channel.

$\Sigma f_x = 0$ gives,

$$R_A\cos(36.86-\phi) - R_B\cos(36.86+\phi) = 0 \longrightarrow (1)$$

$\Sigma f_y = 0$ gives,

$$R_A\sin(36.86-\phi) + R_B\sin(36.86+\phi) - 100 - 150 - 400 = 0 \longrightarrow (2)$$

$\Sigma M_A = 0$ gives,

$$-R_B\cos(36.86+\phi)\times(1.6)\sin\phi + R_B\sin(36.86+\phi)\times1.6\times\cos\phi$$
$$-(100)(0.5)\cos\phi - (150)(0.8)\cos\phi - (400)(1.1)\cos\phi = 0 \longrightarrow (3)$$

$$\therefore (1.6)R_B\left[\sin(36.86+\phi-\phi)\right] - (610)\cos\phi = 0$$

$$(0.96)R_B = (610)\cos\phi$$

$$R_B = (635.42)\cos\phi$$

From eq^n (1) we get,

$$R_A = \frac{R_B\cdot\cos(36.86+\phi)}{\cos(36.86-\phi)}$$

substituting

$$R_B\left[\frac{\sin(36.86-\phi)\cdot\cos(36.86+\phi)}{\cos(36.86-\phi)}\right] + R_B\sin(36.86+\phi) = 650$$

$$\therefore R_B\left[\sin(36.86-\phi)\cdot\cos(36.86+\phi) + \cos(36.86-\phi)\cdot\sin(36.86+\phi)\right] = 650 \times \cos(36.86-\phi)$$

$$\therefore R_B\left[\sin(36.86\cancel{-\phi}+36.86\cancel{+\phi})\right] = 650 \times \cos(36.86-\phi)$$

$$\therefore (0.96)R_B = (650)\cos(36.86-\phi)$$

$$\therefore (0.96)(635.42)\cos\phi = (650)\cos(36.86-\phi)$$

$$\cos\phi = (1.065)\left[(0.8)\cos\phi + (0.6)\sin\phi\right]$$

$$\therefore \cos\phi = (0.852)\cos\phi + (0.639)\sin\phi$$

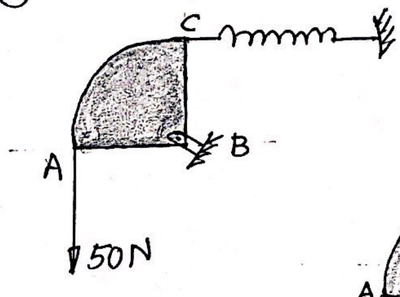
$$\therefore (0.148)\cos\phi = (0.639)\sin\phi$$

$$\therefore \tan\phi = \left(\frac{0.148}{0.639}\right) = 0.2316$$

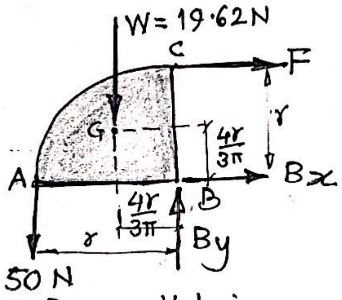
$$\therefore \phi = 13.04^\circ$$

<u>Ans</u> : $\phi = 13.04^\circ$ for equilibrium

Ex. No. 49 A quarter circular plate is supported as shown in figure. The mass of the plate is 2 kg and a force of 50 N is applied vertically downwards at corner A. Determine the force acting on the spring in pulling it. Also, determine the reaction at B.



Solution: Consider the F.B.D. of the quarter circular plate.



Applying equations of equilibrium,

$\Sigma F_x = 0$ gives, $F + B_x = 0$ ⟶ (1)

$\Sigma F_y = 0$ gives, $B_y - 50 - 19.62 = 0$ ⟶ (2)

$\Sigma M_B = 0$ gives, $(50)r + (19.62)\left(\frac{4r}{3\pi}\right) - (F)\cdot r = 0$ ⟶ (3)

$\therefore \quad F = 50 + 8.327 = 58.327$ N

This is the force in the spring.

$B_x = 58.327$ N (←)

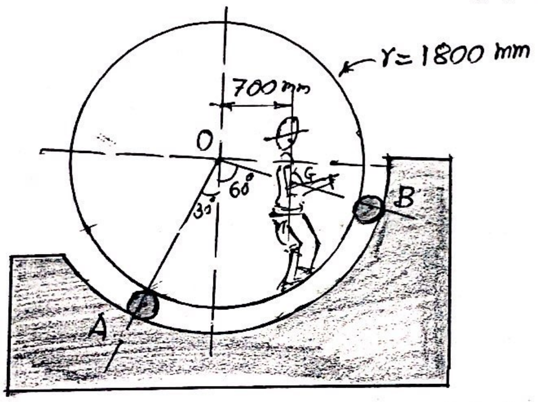
$B_y = 69.62$ N (↑)

Reaction at B, $R_B = 90.824$ N 50°

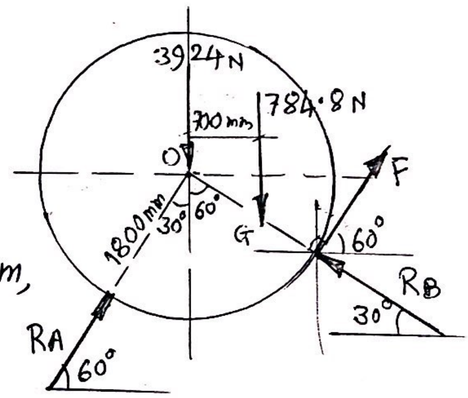
Ans : Force in the spring = 58.327 N

$R_B = 90.824$ N (50° R_B)

Ex. No. 50 The uniform 400 kg drum is mounted on a line of rollers at A and B. An 80 kg man moves slowly a distance of 700 mm from the vertical center before the drum begins to rotate. All rollers are perfectly free to rotate except that at B which must overcome appreciable friction in its bearing. Calculate the frictional force F exerted by the roller at B on the drum for this condition. Also, find the reaction at roller A.



Solution : Consider the F.B.D. of the drum,



Applying equations of equilibrium,

$\Sigma F_x = 0$ gives,

$R_A \cdot \cos 60^\circ + F \cdot \cos 60^\circ - R_B \cdot \cos 30^\circ = 0 \longrightarrow (1)$

$\therefore \quad (0.5) R_A + (0.5) F = (0.866) R_B$

$\Sigma F_y = 0$ gives, $R_A \sin 60^\circ + R_B \sin 30^\circ + F \sin 60^\circ - 3924 - 784.8 = 0$

$\therefore (0.866) R_A + (0.5) R_B + (0.866) F = 4708.8 \longrightarrow (2)$

$\Sigma M_O = 0$ gives, $(F \times 1800) - (784.8)(700) = 0 \longrightarrow (3)$

$\therefore$ (Frictional Force at roller B) $= F = 305.2$ N

solving equations (1) and (2) we get

Ans : $R_A = 3773.4$ N (60°)

$R_B = 2353.46$ N (30°)

Frictional Force at roller B, $F = 305.2$ N