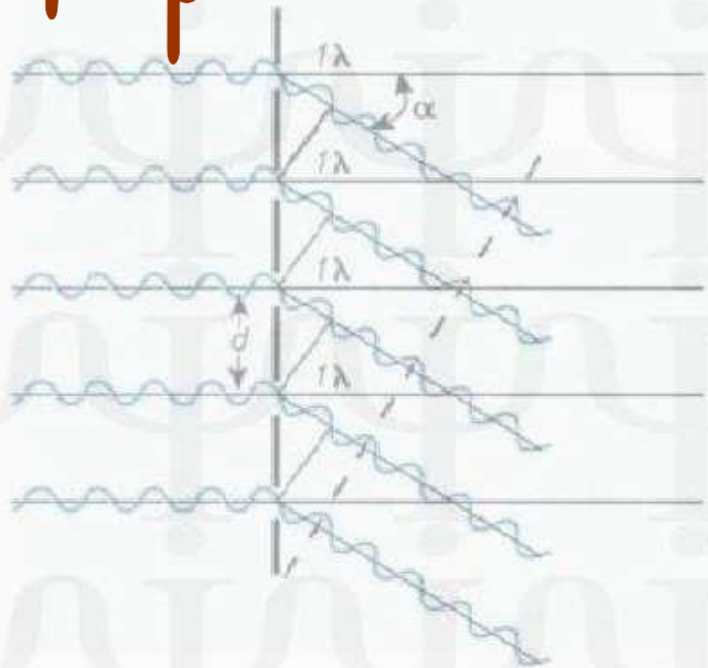
# Wave nature of particles




$$E\Psi = -\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + V\Psi$$

# Nature of a wave

- A wave is described by frequency $\nu$, wavelength $\lambda$, phase velocity $u$ and intensity $I$

- A wave is spread out and occupies a relatively large region of space

# Nature of a particle

- A particle is specified by mass $m$, velocity $v$, momentum $p$, and energy $E$

- A particle occupies a definite position in space.

  In order for that it must be small

# Light

- Interference and Diffraction experiments showed the wave nature of light

- Blackbody radiation and Photoelectric effect can be explained only by considering light as a stream of particles

# Basics of Quantum Mechanics
# - Particle-Wave Duality -

- The behavior of a "microscopic" particle is very different from that of a classical particle:
  - ➔ in some experiments it resembles the behavior of a classical wave (not localized in space)
  - ➔ in other experiments it behaves as a classical particle (localized in space)
- Corpuscular theories of light treat light as though it were composed of particles, but can not explain DIFRACTION and INTERFERENCE.
- **Maxwell's theory of electromagnetic radiation can explain these two phenomena**, which was the reason why the corpuscular theory of light was abandoned.

# Basics of Quantum Mechanics
# - Particle-Wave Duality -

- **Waves as particles:**

  - **Max Plank work on black-body radiation, in which he assumed that the molecules of the cavity walls, described using a simple oscillator model, can only exchange energy in quantized units.**

  - **1905 Einstein proposed that the energy in an electromagnetic field is not spread out over a spherical wavefront, but instead is localized in individual clumbs - quanta. Each quantum of frequency n travels through space with speed of light, carrying a discrete amount of energy and momentum =photon => used to explain the photoelectric effect, later to be confirmed by the *x*-ray experiments of Compton.**

- **Particles as waves**

  - **Double-slit experiment, in which instead of using a light source, one uses the electron gun. The electrons are diffracted by the slit and then interfere in the region between the diaphragm and the detector.**

# Waves as Particles

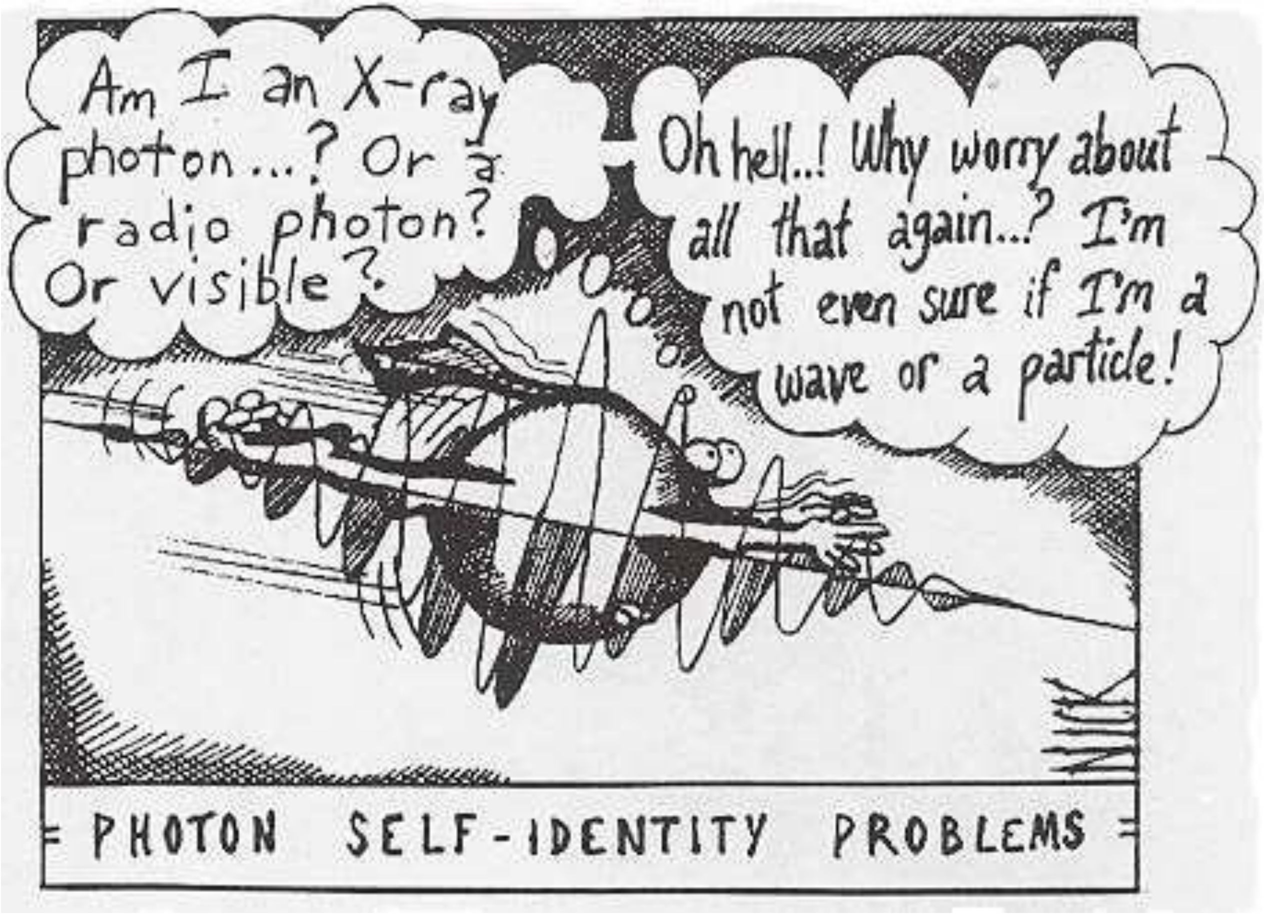
# Blackbody?

- A material is constantly exchanging heat with its surrounding (to remain at a constant temperature):
  - It absorbs and emits radiations
  - Problem: it can reflect incoming radiations, which makes a theoretical description more difficult (depends on the environment)
- A blackbody is a perfect absorber:
  - Incoming radiations is totally absorbed and none is reflected

# The Photoelectric Effect

- Electrons are ejected from a metal surface by absorption of a photon
- Electron ejection depends on frequency <u>not</u> on intensity
- The threshold frequency corresponds to $h\nu_o = \phi$
  - $\phi$ is the work function (essentially equal to the ionization potential of the metal)
- The kinetic energy of the ejected particle is given by:
- $\frac{1}{2}mv^2 = h\nu - \phi$
- The photoelectric effect shows that the incident radiation is composed of photons that have energy proportional to the frequency of the radiation

# So is light a wave or a particle ?



# How are they related?

$$\boxed{E = h\nu}$$

- $E$ – energy of the photon
- $\nu$ – frequency of the wave
- $h$ – plank's constant

$$\boxed{p = h/\lambda}$$

- $p$ – momentum of the particle
- $\lambda$ - wavelength of the photon

# DE BROGLIE HYPOTHESIS

## In the Year 1924 Louis de Broglie made the bold suggestion


Prince Louis de Broglie (1892-1987)

*" If radiation which is basically a wave can exhibit particle nature under certain circumstances, and since nature likes symmetry, then entities which exhibit particle nature ordinarily, should also exhibit wave nature under suitable circumstances"*

**The reasoning used might be paraphrased as follows**

1. **Nature loves symmetry**
2. **Therefore the two great entities, matter and energy, must be mutually symmetrical**
3. **If energy (radiant) is undulatory and/or corpuscular, matter must be corpuscular and/or undulatory**

# The de Broglie Hypothesis

- If light can act like a wave sometimes and like a particle at other times, then *all matter*, usually thought of as particles, should exhibit wave-like behaviour

- The relation between the momentum and the wavelength of a photon can be applied to material particles also

Prince Louis de Broglie
(1892-1987)

# de Broglie Wavelength

wavelength

$$\lambda = \frac{h}{p}$$

momentum

Relates a particle-like property ($p$)
to a wave-like property ($\lambda$)

# DE BROGLIE WAVELENGTH

**The Wave associated with the matter particle is called Matter Wave.**

**The Wavelength associated is called de Broglie Wavelength.**

$$\text{de Broglie wavelength } \lambda = \frac{h}{p} = \frac{h}{mv}$$

h is Planck's Constant

m is the mass of the particle

v is the velocity of the particle

*for an electron with Kinetic Energy 'E'*

*accelerated by a Potential difference 'V'*

$$\text{Then} \quad \lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2meV}}$$

*substituting for h, m, and e we get*

$$\lambda = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 1.602 \times 10^{-19} \times V}} = \frac{1.226}{\sqrt{V}} \; nm$$

*thus for* $V = 100$ *Volts*

$$\lambda = \frac{1.226}{\sqrt{100}} = 0.1226 \; nm$$

# DE BROGLIE WAVELENGTH

**de-Brogile wavelength.** The expression of the wavelength associated with a material particle can be derived on the analogy of radiation as follows:

Considering the Planck's theory of radiation, the energy of a photon (quantum) is given by

$$\boxed{E = h\nu = \frac{hc}{\lambda}} \quad \text{...(1)}$$

where $c$ is the velocity of light in vacuum and $\lambda$ is its wavelength.

According to Einstein energy mass relation

$$\boxed{E = mc^2} \quad \text{...(2)}$$

From equations (1) and (2), we get

$$mc^2 = \frac{hc}{\lambda} \quad \text{or} \quad \lambda = \frac{hc}{mc^2} \quad \text{or} \quad \lambda = \frac{h}{mc} \quad \text{...(3)}$$

where $mc = p$ (momentum associated with photon).

# DE BROGLIE WAVELENGTH

## Kinetic Energy

If we consider the case of a material particle of mass $m$ and moving with a velocity $v$, *i.e.*, momentum $mv$, then the wavelength associated with this particle is given by

$$\boxed{\lambda = \frac{h}{m\,v} = \frac{h}{p}} \qquad \ldots(4)$$

If $E$ is the kinetic energy of the material particle, then

$$E = \frac{1}{2} m\,v^2 = \frac{1}{2}\frac{m^2 v^2}{m} = \frac{p^2}{2\,m}$$

or $p = \sqrt{(2\,m\,E)}$

$\therefore$ de-Broglie wavelength

$$\boxed{\lambda = \frac{h}{\sqrt{2\,m\,E}}} \qquad \ldots(5)$$

# DE BROGLIE WAVELENGTH

## For Electron

**de-Broglie's wavelength associated with electrons.** Let us consider the case of an electron of rest mass $m$, and charge $e$ which is accelerated by a potential $V$ volts from rest to velocity $v$, then

$$\frac{1}{2} m_0 v^2 = eV \quad \text{or} \quad v = \sqrt{\frac{2eV}{m_0}}$$

$$\boxed{\lambda = \frac{h}{m_0 v} = \frac{h\sqrt{m_0}}{m_0\sqrt{2eV}} = \frac{h}{\sqrt{2eVm_0}}}$$

$$\text{or} \quad \lambda = \frac{6.625 \times 10^{-34}}{\sqrt{(2 \times 1.632 \times 10^{-19} V \times 9.1 \times 10^{-31})}}$$

$$\boxed{\lambda = \frac{12.26}{\sqrt{V}} \text{ Å}}$$

If $V = 100$ volts, then

$$\lambda = 1.226 \text{ Å} \qquad \ldots(6)$$

Equation (6) shows that the wavelength associated with an electron accelerated to 100 volts is 1.226 Å

# PROPERTIES/CHARACTERISTICS OF MATTER WAVES

1. Matter waves are produced by the motion of the particles and are independent of the charge. Therefore, they are neither electromagnetic nor acoustic waves but are new kind of waves.
2. They can travel through vacuum and do not require any material medium for their propagation.
3. The smaller the velocity of the particle, the longer is the wavelength of the matter wave associated with it.
4. The lighter the particle, the longer is the wavelength of the matter wave associated with it.
5. The velocity of matter waves depends on the velocity of the material particle and is not a constant quantity.
6. The velocity of matter waves is greater than the velocity of light.
7. They exhibit diffraction phenomenon as any other waves.

# WAVE VELOCITY OF MATTER WAVE

If we consider a harmonic wave, the wave has a single wavelength and a single frequency.

Then the Velocity of Propagation of the Wave is given by:

$u = \nu \lambda$, where $\nu$ is the frequency of matter waves. According to Planck's theory, $E = h\nu$.

and from Einstein's relation, $E = mc^2$

$$\therefore \qquad h\nu = mc^2 \text{ or } \nu = \frac{mc^2}{h} \qquad \ldots(1)$$

$$\text{For matter waves, } \boxed{\lambda = \frac{h}{mv}} \qquad \ldots(2)$$

Substituting these values, we get

$$\boxed{u = \frac{mc^2}{h} \times \frac{h}{mv} = \frac{c^2}{v}} \qquad \ldots(3)$$

where $v$ is the speed of material particle which is less than the speed of light $c$.

# WAVE VELOCITY OF MATTER WAVE

## PHASE VELOCITY

*Phase velocity:* The velocity with which a wave travels is called Phase velocity or wave velocity. It is denoted by $v_{p.}$ It is given by

$$v_p = \frac{c^2}{v}$$

Where c = velocity of light and v = is velocity of the particle.

The above equation gives the relationship between the phase velocity and particle velocity.

It is clear from the above equation that, Phase velocity is not only greater than the velocity of the particle but also greater than the velocity of light.

According to theory of Relativity it is not possible that the Phase velocity of the particle wave be greater than or equal to velocity of light. So a harmonic wave of wavelength λ can't represent a moving atomic particle.

De Broglie Waves can't be harmonic waves.

Thus a concept of group velocity was Introduced.

# WAVE VELOCITY OF MATTER WAVE

## GROUP VELOCITY

However a wave spreads over a large region of space and can't represent a highly localized particle.

Schrodinger postulated that a wave packet rather than a single harmonic wave represents a particle.

It consists of a group of harmonic waves.

Each wave has slightly different wavelength.

*" Matter wave is regarded as the resultant of the superposition of large number of component waves all traveling with different velocities. The resultant is in the form of a packet called wave packet or wave group.*

*The velocity with which this wave group travels is called Group velocity." The group velocity is represented by $v_g$.*

*Group Velocity* ( $v_g$ ) = Particle Velocity ( v)