Session36:

TRACING OF ROSE CURVES ($r = a\sin n\theta \ or \ r = a\cos n\theta$).

Rule 1: No. of loops :

1. If n is odd then number of loops in the curve = n.
2. If n is even then number of loops in the curve = 2n.

Rule 2: Symmetry

(a) *Symmetry about initial line*: If the equation of the curve remains unchanged by replacing $\theta \ by - \theta$, then the curve is symmetric about the initial line $\theta = 0$.

(b) Symmetry about the line $\theta = \frac{\pi}{2}$ **:**

1. If the equation of the curve remains unchanged by replacing $\theta \ by - \theta$ and $r \ by - r \ respectively$, then curve is symmetric about the line $\theta = \frac{\pi}{2}$.

2. If the equation of the curve remains unchanged by replacing $\theta \ by \ \ \pi - \theta$ then curve is symmetric about the line $\theta = \frac{\pi}{2}$.

Rule 3: Pole: Find in particular values of θ, which give r = 0.

Rule 4: Tangents: To find tangents at the pole, put r = 0 in the equation, the values of θ gives the tangent at the pole.

Rule 5: Angle between radius vector and tangent [ϕ] :

Use the formula $\tan\phi = r\frac{d\theta}{dr}$ and find ϕ and also the points where $\phi = 0 \ or \ \infty$.

Rule 6: Form the table showing values of r for some values of θ

Q1. Trace the following curve:

$$r = a\sin 2\theta$$

Solution: We check the following points for tracing of the above curve

1. **Limit:-** $|r| \le a$ i.e. total curve will lie inside the circle of radius 'a'.
2. **No. of loops:-** The curve contains 4 loops because $n = 2$ is even.
3. **Symmetry:-**

(i) **About the line perpendicular to initial line** $\theta = 0$ i.e. **the line** $\theta = \pi/2$ **:-**

If we replace θ by $-\theta$ and r by $-r$ then the equation of the curve is remains unchanged.

$\therefore$ The curve is symmetry about the line $\theta = \pi/2$.

4. **Pole:-**

(i) For $\theta = 0 \Rightarrow r = 0$.

Hence the curve passes through the pole.

(ii) Tangent at pole:- If we put $r = 0$, then we get the tangent at pole.

Putting $\mathrm{r} = 0$ in (1), we have

$$a \sin 2\theta = 0$$

$$\Rightarrow \quad \sin 2\theta = 0$$

$$\Rightarrow \quad 2\theta = \sin^{-1} 0$$

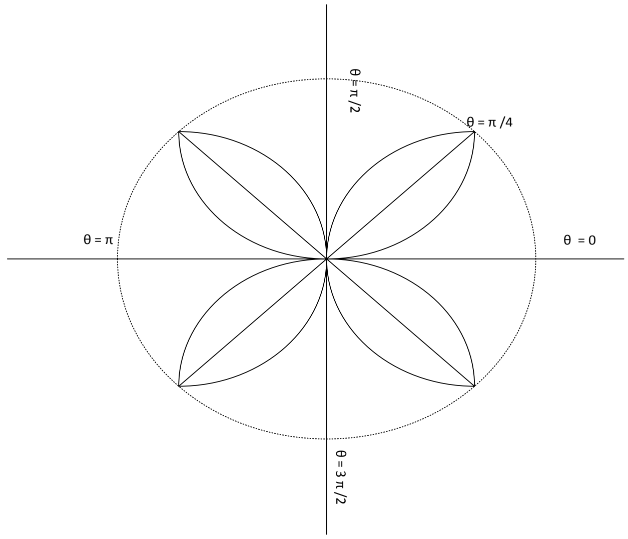
$$\Rightarrow \quad 2\theta = 0,\ \pi,\ 2\pi,\ 3\pi$$

$$\Rightarrow \quad \theta = 0,\ \frac{\pi}{2},\ \pi,\ \frac{3\pi}{2}$$

5. **Asymptotes:-**No asymptotes.
6. **Table values:-**

θ	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
r	0	a	0	a	0	a	0	a	0

It is clear that for $\theta = 0,\ \frac{\pi}{2},\ \pi,\ \frac{3\pi}{2}$, the value of r is zero therefore these are tangents at pole and for $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ the value of r is maximum i.e. 'a'. Hence we get four loops at those points. Hence the approximate shape of the curve is as follows.



Q. 2 Trace the following curve:

$$r = a\cos 3\theta$$

Solution: We check the following points for tracing of the above curve

1. **Limit:-** $|r| \le a$ i.e. total curve will lie inside the circle of radius 'a'.
2. **No. of loops:** The curve contains 3 loops because $n = 3$ is odd.
3. **Symmetry:-**

   (i) About initial line $\theta = 0$:-

   If we replace θ by $-\theta$, then the equation of the curve is remains unchanged.

   $\therefore$ The curve is symmetry about the initial line $\theta = 0$.
4. **Pole:-**

   (i) For $\theta = \frac{\pi}{6} \Rightarrow r = 0$

   Hence the curve passes through the pole.

   (ii) Tangent at pole:- If we put $r = 0$, then we get the tangent at pole.

   Putting $\mathrm{r} = 0$ in (1), we have

   $$a\cos 3\theta = 0$$

   $$\Rightarrow \quad \cos 3\theta = 0$$

$$\Rightarrow \qquad 3\theta = \cos^{-1} 0$$

$$\Rightarrow \qquad 3\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}$$

$$\Rightarrow \qquad \theta = \frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{9\pi}{6}, \frac{11\pi}{6}$$

$$\Rightarrow \qquad \theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$$

5. **Asymptotes:-**No asymptotes.
6. **Table values:-**

θ	0	$\pi/6$	$\pi/2$	$2\pi/3$	π	$4\pi/3$	$3\pi/2$	$11\pi/6$	2π
r	a	0	0	a	0	a	0	0	a

It is clear that for $\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$, the value of r is zero therefore these are tangents at pole and for $\theta = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$ the value of r is maximum i.e. 'a'. Hence we get three loops at those points. Hence the approximate shape of the curve is as follows.