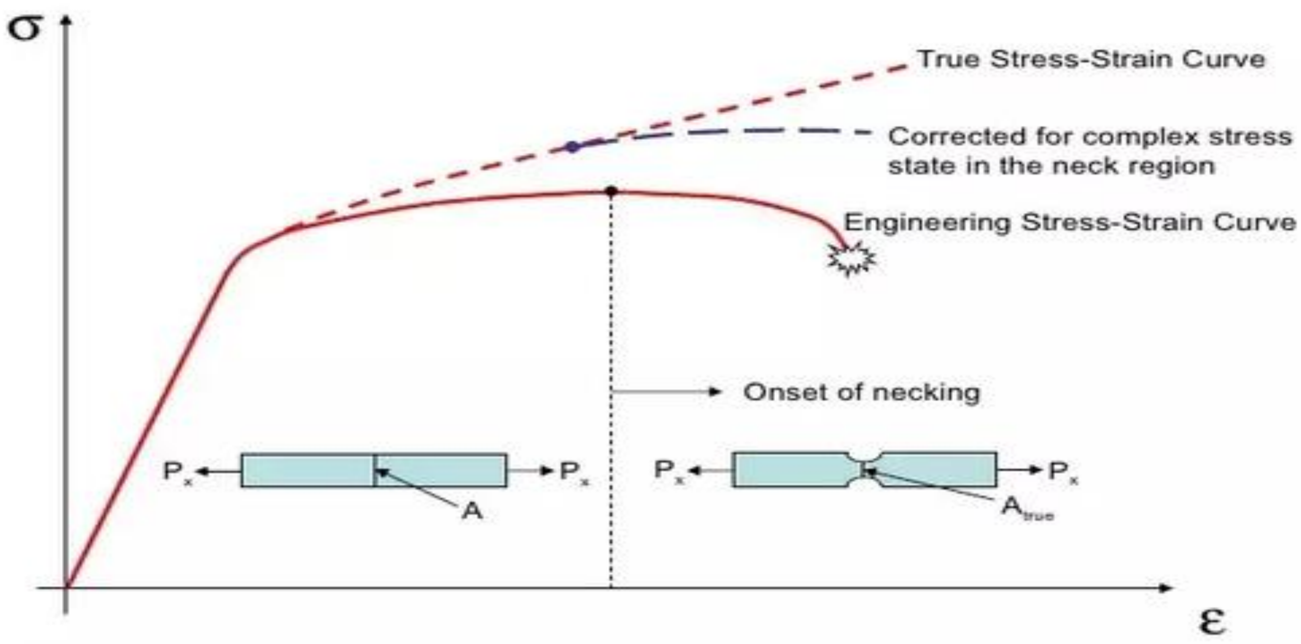
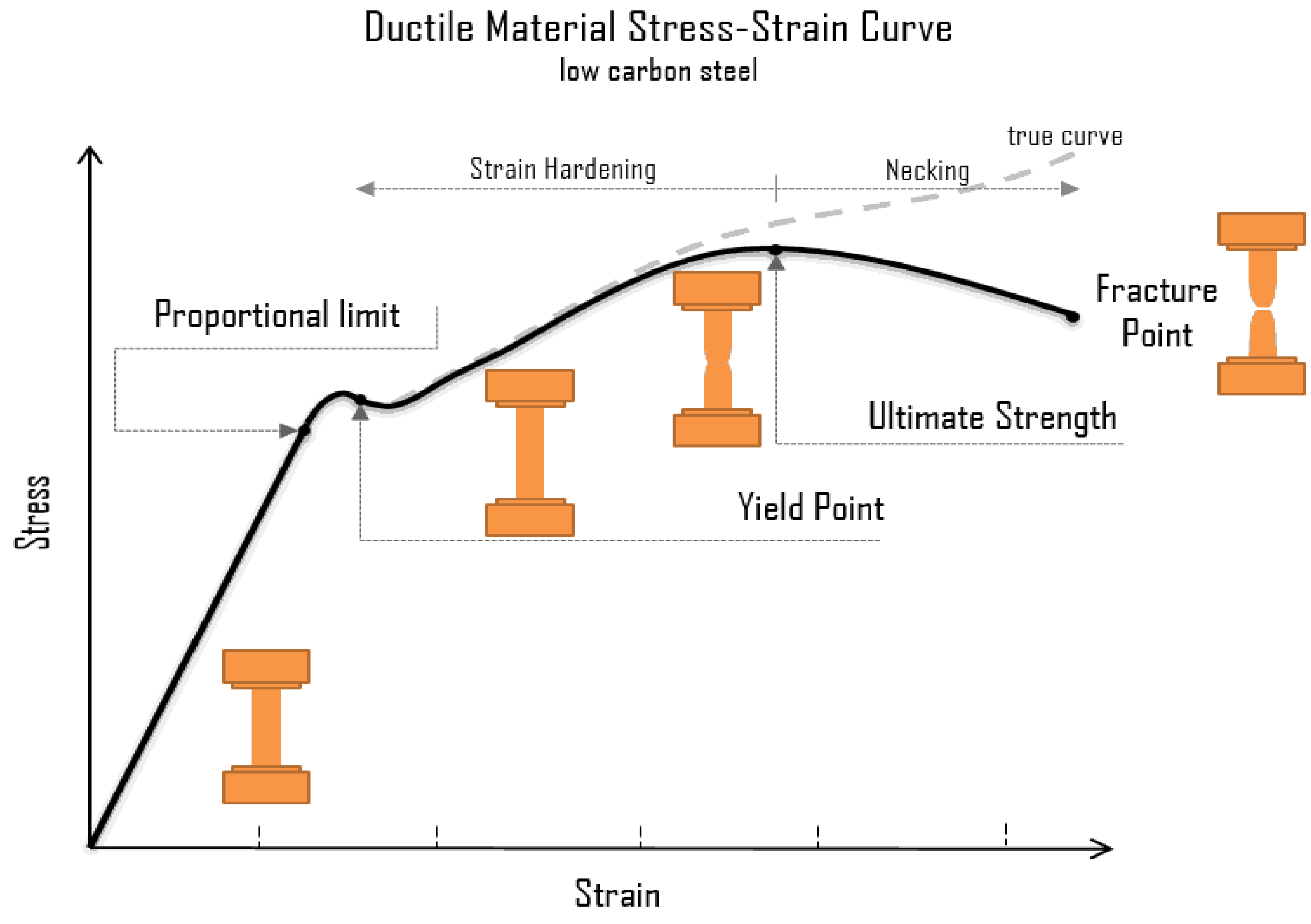
MATERIAL SCIENCE (MEE102B)

True Stress-Strain Curve

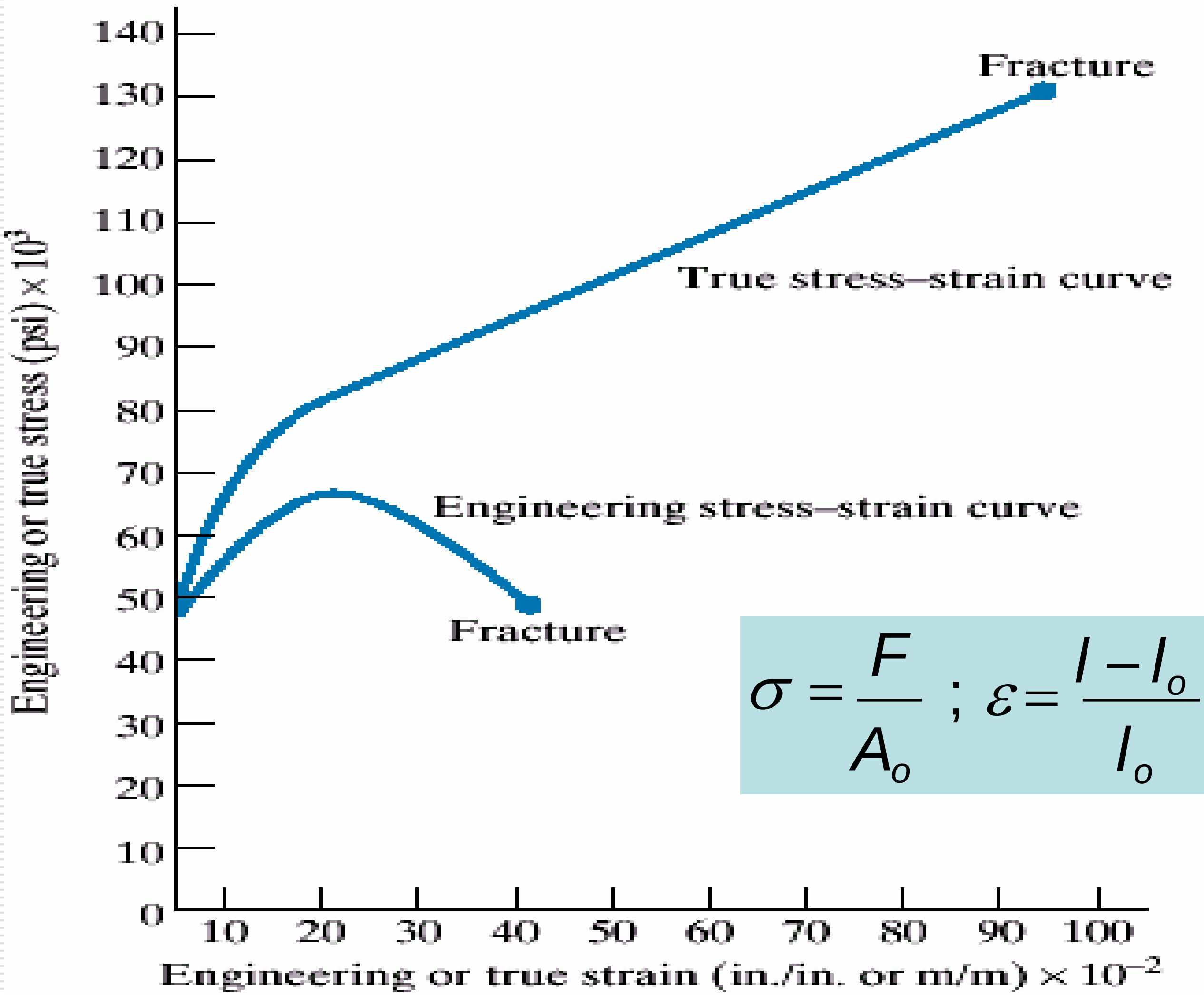


True Stress (σ_T)

True stress is the stress determined by the instantaneous load acting on the instantaneous cross-sectional area


Ductile Material Stress-Strain Curve
low carbon steel
true curve
Strain Hardening
Necking
Proportional limit
Fracture Point
Ultimate Strength
Yield Point
Stress
Strain

True Stress-Strain Curve



$$\sigma = \frac{F}{A_o} \; ; \; \varepsilon = \frac{l - l_o}{l_o}$$

$$\sigma_T = \frac{P}{A_o}(1+\varepsilon) = \sigma(1+\varepsilon)$$

$$\varepsilon_T = \int \frac{d\ell}{\ell} = \ln\left(\frac{\ell}{\ell_o}\right)$$

Engineering stress-strain & True stress-strain

Let

σ = Engineering stress

$\sigma = \frac{P}{A_o}$, P is applied load
A_o = original cross section Area

ε = Engineering strain

$= \frac{\delta l}{L_o}$ L_o = original length

$= \left(\frac{L_f - L_o}{L_o}\right)$ L_f = final length
δl = change in length

σ_T = True stress

$= \frac{P}{A_I}$ P is applied load
, A_I = instantaneous cross section Area

ε_T = True strain

True stress is the stress determined by the instantaneous load acting on the instantaneous cross-section area.

Relation between True & Engineering stress-strain

Engineering stress $\sigma = \dfrac{P}{A_0}$ — (1)

Assuming material volume remain constant

$\therefore \quad A_0 L_0 = A_I L_I$, A_I = instantaneous Area

$\dfrac{A_0}{A_I} = \dfrac{L_I}{L_0}$ — (2) , L_I = — " — Length

$$\sigma_T = \frac{P}{A_I}$$

$$= \frac{P}{A_I} \times \frac{A_0}{A_0}$$

$$= \frac{P}{A_0} \times \frac{A_0}{A_I}$$

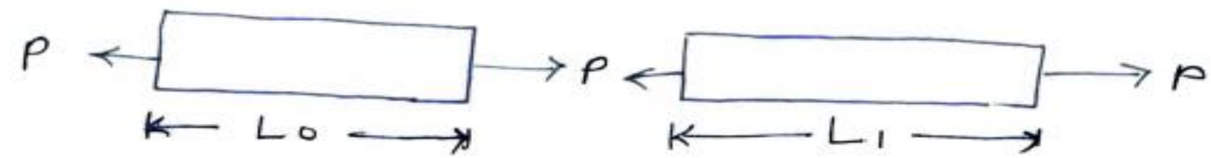
$= \sigma \times \left(\dfrac{L_I}{L_0}\right)$ — from eqn (1) & (2)

$= \sigma \left[\dfrac{L_0 + \delta l}{L_0}\right]$, $L_I = (L_0 + \delta l)$

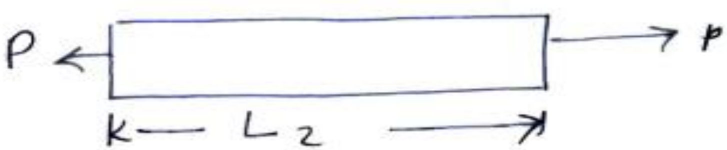
$$= \sigma \left[1 + \frac{\delta l}{L_0}\right]$$

$$\boxed{\sigma_T = \sigma\,[1 + \varepsilon]}$$

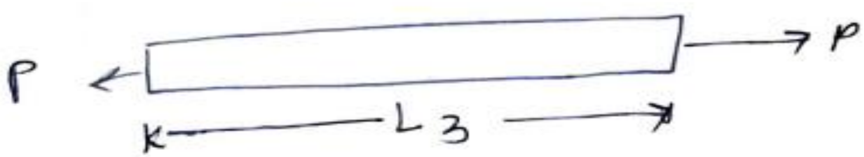
True strain



True strain $\varepsilon_{T_1} = \dfrac{L_1 - L_0}{L_0}$ — (1)



Now $\varepsilon_{T_2} = \dfrac{L_2 - L_1}{L_1}$ — (2)



Now $\varepsilon_{T_3} = \dfrac{L_3 - L_2}{L_2}$ — (3)

Now if you want to calculate total strain in material from original length L_0 to final length L_f

$$\varepsilon_T = \int_{L_0}^{L_f} \frac{dl}{L}$$

$$= \ln[L]_{L_0}^{L_f}$$

$$= \ln\left[\frac{L_f}{L_0}\right], \quad L_f = L_0 + \delta l$$

$$= \ln\left[\frac{L_0 + \delta l}{L_0}\right]$$

$$\boxed{\varepsilon_T = \ln[1+\varepsilon]}$$

Numericals based on Tensile Test

Example 1 : Tensile test was conducted on a steel specimen of diameter 12.5 mm and gauge length 50 mm, the loads at lower and upper yield points were 4500 kg and 4600 kg respectively. The maximum and fracture loads were 7500 kg and 5000 kg respectively. The gauge length after fracture was 62.5 mm. the diameter of fracture place was found to be 8.0 mm.

Determine the following
(i) Lower yield stress (ii) Upper yield stress (iii) Ultimate tensile stress
(iv) Fracture Stress (v) True fracture stress (vi) Percent elongation
(vii) Percent reduction in cross sectional area.

Solution :

(1) Lower yield stress = Lower yield Load/original cross sectional area

(2) Upper yield stress = Upper yield Load/original cross sectional area

(3) U.T.S. = Ultimate tensile load/ Load/original cross sectional area

(4) Fracture stress = Fracture Load/ original cross sectional area

(5) True fracture stress = Fracture Load/ area of cross section at fracture

(6) % elongation = (Final length – original length)/ original length x 100

(7) % reduction in cross sectional are = (original area- Final area) / original areax100

Ans:-

Given

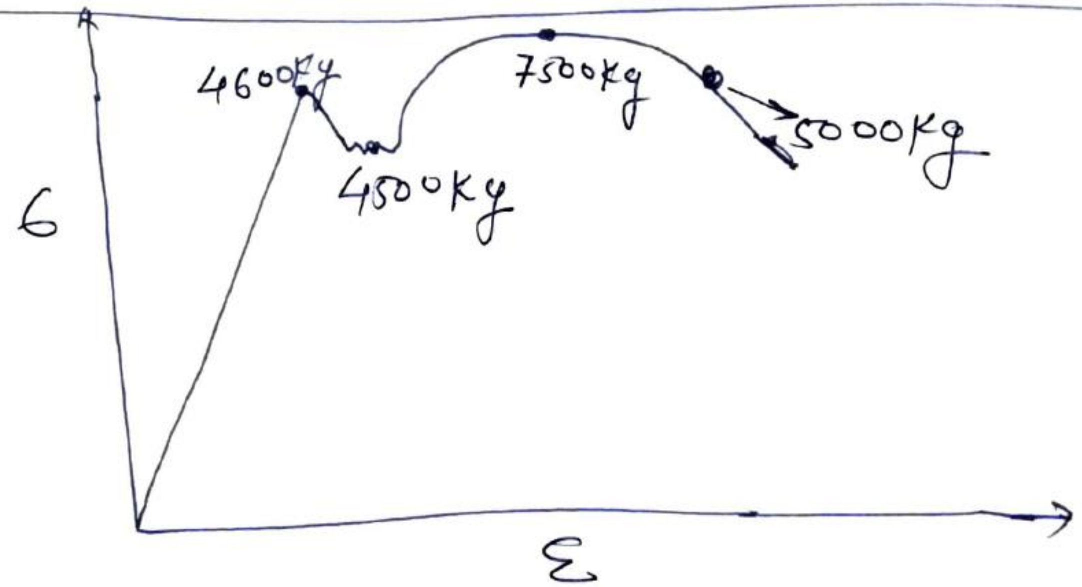
$P_L = 4500 kg$

$P_U = 4600 kg$

$P_{max} = 7500 kg$

$P_f = 5000 kg$

$d_o = 12.5 mm$



Given

$A_o = \frac{\pi}{4} d_o^2$

$A_f = \frac{\pi}{4} d_f^2$

(i) Lower yield stress $(\sigma_{yL}) = \frac{P_L}{A_o} = \frac{4500}{\frac{\pi}{4} d_o^2} = \frac{4500}{\frac{\pi}{4}(12.5)^2} =$

$= 36.669 \ kg/mm^2$

$= 366.69 \ N/mm^2 \ (\text{if } g = 10 \ m/s^2)$

$\boxed{\sigma_{yL} = 366.69 \ MPa}$

(ii) upper yield stress $(\sigma_{yu}) = \frac{P_u}{A_0}$

$$= \frac{4600}{\frac{\pi}{4}(12.5)^2}$$

$$= 37.48 \ kg/mm^2$$

$$= 374.8 \ N/mm^2 \ (g = 10 m/s^2)$$

$$= 374.8 \ N/10^{-6} m^2$$

$$= 374.8 \times 10^6 \ N/m^2$$

$$\boxed{\sigma_{yu} = 374.8 \ MPa}$$

(iii) Ultimate tensile stress (σ_{max} or σ_{UTS}) $\sigma(UTS) = \frac{P_{max}}{A_0}$

$$\sigma(UTS) = \frac{P_{max}}{\frac{\pi}{4} d_0^2}$$

$$= \frac{7500}{\frac{\pi}{4}(12.5)^2} = 61.115 \ kg/mm^2$$

$$= 611.15 \ N/mm^2$$

$$\boxed{\sigma_{UTS} = 611.15 \ MPa}$$

(iv) Fracture stress $(\sigma_f) = \dfrac{P_f}{\frac{\pi}{4} d_0^2}$

$$= \frac{5000}{\frac{\pi}{4}(12.5)^2} = 40.74 \text{ kg/mm}^2$$

$$= 407.4 \text{ N/mm}^2$$

$$\boxed{\sigma_f = 407.4 \text{ MPa}}$$

(v) True fracture stress $(\sigma_{Tf}) = \dfrac{P_f}{A_f} = \dfrac{5000}{\frac{\pi}{4} d_f^2}$

$$= \frac{5000}{\frac{\pi}{4}(8)^2} = 99.47 \text{ kg/mm}^2$$

$$= 994.7 \text{ N/mm}^2$$

$$\boxed{\sigma_{Tf} = 994.7 \text{ MPa}}$$

(vi) Percent elongation $= \left(\frac{L_f - L_o}{L_o}\right) \times 100$

$$= \left(\frac{62.5 - 50}{50}\right) \times 100$$

$$\boxed{\%\ \text{Elongation} = 25\%}$$

(vii) Percent reduction in cross section area

$$= \left(\frac{A_o - A_f}{A_o}\right) \times 100$$

$$= \left[\frac{\frac{\pi}{4}(d_o)^2 - \frac{\pi}{4}(d_f)^2}{\frac{\pi}{4}(d_o)^2}\right] \times 100$$

$$= \left[\frac{d_o^2 - d_f^2}{d_o^2}\right] \times 100$$

$$= \left[\frac{(12.5)^2 - (8)^2}{(12.5)^2}\right] \times 100$$

$$\boxed{\%\ \text{reduction in C/S Area} = 59.04\%}$$

Example 2

A test rod of 15 mm diameter was failed at 50 kN during tensile test but reached to maximum load of 63kN. The specimen had 75 mm gauge length and yielding at 45 kN and was elongated to 80 mm.

Calculate a. Yield stress b. ultimate tensile stress c. % Elongation.

Ans

Given $d_0 = 15\,mm$, $A_0 = \frac{\pi}{4} d_0^2$

$P_f = 50\,KN$

$P_{max} = 63\,KN$

$L_0 = 75\,mm$

$P_y = 45\,KN = 45 \times 10^3\,N$

$L_f = 80\,mm$

(a) $\text{yield stress } (\sigma_y) = \frac{P_y}{A_0} = \frac{45 \times 10^3}{\frac{\pi}{4}(15)^2} = 254.647\,N/mm^2$

$= 254.647\,N/10^{-6}\,m^2$

$= 254.647 \times 10^6\,N/m^2$

$\boxed{\sigma_y = 254.647\,MPa}$

(b) $$\sigma_{UTS} = \frac{P_{max}}{A_o} = \frac{63 \times 10^3}{\frac{\pi}{4}(15)^2} = 356.507 \; N/mm^2$$

$$\boxed{\sigma_{UTS} = 356.507 \; MPa}$$

(c) $$\% \text{ Elongation} = \left(\frac{L_f - L_o}{L_o}\right) \times 100$$

$$= \left[\frac{80 - 75}{75}\right] \times 100$$

$$\boxed{\% \text{ Elongation} = 6.67\%}$$

Example 3

A 20 cm long rod with a diameter of 0.30 cm is loaded with 4000N weight. If the diameter decreases to 0.27 cm determine,

(i) Engineering stress

(ii) True stress

- Solution:

Length of rod (l) = 20 cm = 200 mm

Original diameter (d_0) = 0.30 cm = 3 mm

Weight (P) = 4000 N

Change diameter (d_i) = 0.27 cm = 2.7 mm

(1) Engineering stress (σ_E) = Applied Load / original cross sectional area

(2) True stress = Applied Load / Actual cross sectional area

Example 4

The tensile test specimen of mild steel of 8 mm diameter and 40 mm gauge length was tested with the following result

(i) Maximum load = 3212 kg (ii) yield load = 1750 kg

(iii) increase in gauge length after fracture = 50mm (iv) diameter at fracture = 5.4 mm .

Calculate : i) UTS in kg/mm^2 ii) Y.S. in kg/mm^2

iii) % elongation iv) % reduction in area

Example 5

The following deta was obtained during, tensile test conducted on a mild steel specimen 42 mm in diameter and 210 mm long.

Elongation with 45 kN load δl = 0.0404 mm, Yield load = 163 kN,

Maximum load = 245 kN, length of specimen of fracture = 250 mm.

Find out (i) Youngs modulus of elasticity (ii) Yield point (iii) Ultimate stress (iv) Percentage elongation.

Example 6

- A steel bar of 13.7 mm diameter breaks with a load of 15 kN. It's final diameter is 7.98 mm. Find out (i) True breaking strength (ii) Nominal breaking strength

- **Solution :**

Original or nominal diameter =d_0 = 13.7 mm.

Instantaneous or final diameter =d = 7.98 mm

Load applied = 15 kN

Example 7

- A steel bar 100 mm long & square cross section 20 mm on an edge is pulled in tension with load of 89000N & experiences an elongation of 0.1 mm. Assuming that the deformation is entirely elastic, calculate the elastic modulus of the steel.

Example 8

A steel rod of 2 cm^2 are & 1m in length is subjected to an axial pull of 40000N. If young Modulus is 2x10^5 N/mm^2, find the elongation of the rod in mm.

THANK YOU