17/04/22

I.C. Tutorial - 2

Q.1: $x^3 dx - y^3 \cdot dy = 3xy (y \cdot dx - x \cdot dy)$

$x^3 dx + 3x^2 y - y^3 dy - 3xy^2 \cdot dx.$

$(x^3 - 3xy^2) dx + (3x^2y - y^3) \cdot dy = 0$

$(x^3 - 3xy^2) dx = (-3x^2y + y^3) dy$

eqn is homogeneous.

$\therefore \quad y = u \cdot x$

$$\frac{dy}{dx} = \frac{x^3 - 3xy^2}{y^3 - 3x^2y}$$

$$u + \frac{x \cdot du}{dx} = \frac{x^3 - 3xy^2}{y^3 - 3x^2y}$$

$$= \frac{x^3 - 3x \cdot u^2 x^2}{u^3 \cdot x^3 - 3 \cdot x^2 \cdot ux}$$

$$= \frac{x^3 (1 - 3u^2)}{x^3 (u^3 - 3u)}$$

$$\frac{x \cdot du}{dx} = \frac{1 - 3u^2}{u^3 - 3u} + (-u)$$

$$= \frac{1 - 3u^2 [-u (u^3 - 3u)]}{u^3 - 3u}$$

$$\frac{x \cdot du}{dx} = \frac{(1 - \cancel{3u^2} - u^4 + \cancel{3u^2})}{u^3 - 3u}$$

Rainbow

$$x \cdot \frac{du}{dx} = \frac{1-u^4}{u^3-3u}$$

$$\frac{x}{dx} = \frac{1-u^4}{(u^3-3u)\,du}$$

$$du \cdot \frac{(u^3-3u)}{1-u^4} = \frac{dx}{x}$$

$$\int \frac{u(u^2-3)}{1-u^4} \cdot dx = \log x + \log c$$

$$= \int \frac{u(u^2-1)}{1^2-(u^2)^2} \cdot du = \log xc$$

$$= \int \frac{u(u^2-1)-2u}{(1-v^2)(1+v^2)} \cdot du$$

$$= \int \frac{u(u^2-1)}{(1-v^2)(1+v^2)} \cdot du - \int \frac{2u}{(1-v^2)(1+v^2)} \cdot dv$$

$$= -\int \frac{u}{1+u^2} \cdot du - \int \frac{2u}{(1-u^2)(1+u^2)} \cdot du$$

let $1+u^2 = t$.

let $1-u^2 = v$

$-2u\,du = dv$

so $2u \cdot du = dt$

$1+u^2 = 1-v+1$

$= 2-v$

$u \cdot du = \frac{dt}{2}$

$$\text{so} \quad -\int \frac{dt}{2t} + \int \frac{dv}{v(2-v)}$$

$$= -\frac{1}{2}\log t + \left.\int \frac{dv}{v(2-v)}\right\} I_2$$

inbow

$$I_2 = \frac{1}{u(\cancel{2}-u} \qquad \frac{1}{v(2-v)} = \frac{A}{u} + \frac{B}{2-v}$$

$$\frac{1}{v(2-v)} = \frac{A\cdot 2 - Au + Bu}{u(2-v)}$$

$$0 = B - A \quad , \quad B = 1/2$$

$$\int \frac{1}{v(2-v)} \cdot dv = \int \frac{1}{2v} \cdot dv + \int \frac{1}{2(2-v)} \cdot du$$

$$= \frac{1}{2}\log v - \frac{1}{2}\log(2-v)$$

$$= \frac{1}{2}(\log v - \log(2-v)$$

$$= \frac{1}{2}(\log v - \log(2-v))$$

$$I_2 = \frac{1}{2}\log\left(\frac{1-u^2}{1+u^2}\right)$$

$$I = -\frac{1}{2}\log t + \frac{1}{2}\log\left(\frac{1-u^2}{1+u^2}\right)$$

$$\log x = -\frac{1}{2}\log(1+u^2) + \frac{1}{2}\log\left(\frac{1-v^2}{1+u^2}\right)$$

$$= \frac{1}{2}\left[\log\left(\frac{1-u^2}{1+v^2}\right) - \log(1+u^2)\right]$$

$$= \frac{1}{2}\log\left(\frac{1-u^2}{1+u^2}\right) = \frac{1}{2}\log\left(\frac{1-u^2}{(1+u^2)^2}\right)$$

Rainbow

So $\log xc = \frac{1}{2} \log \left[\frac{\left\{1 - \frac{y^2}{x^2}\right.}{\left(1+ y^2/x^2\right)^2} \right]$

So $\boxed{\log xc = \frac{1}{2} \log \left[\left(\frac{x^2 (x^2 - y^2)}{(x^2 - y^2)^2} \right] \right.}$

Q.2 $x^3 \cdot \frac{dy}{dx} = y^3 + y^2 \sqrt{y^2 - x^2}$

equation is homogenious.

so put $y = v \cdot x$.

$\therefore \frac{dy}{dx} = \frac{y^3 + y^2 \sqrt{y^2 - x^2}}{x^3}$

$v + x \cdot \frac{dv}{dx} = \frac{x^3 v^3 + x^2 v^2 \sqrt{x^2 v^2 - x^2}}{x^3}$

$= \frac{x^3}{x^3} \left[v^3 + v^2 (v^2 - 1)^{1/2} \right]$

$\therefore v + x \cdot \frac{dv}{dx} = v^3 + v^2 \sqrt{v^2 - 1}$

$x \cdot \frac{dv}{dx} = v^3 + v^2 \sqrt{v^2 - 1} - v$

inbow

$$\therefore \int \frac{dv}{v(v^2 + v\sqrt{v^2-1} - 1)}$$

$$= \int \frac{dx}{x} + c = \log x + c$$

$$I = \int \left(\frac{dv}{v(v^2 - v\sqrt{v^2-1} - 1)} \right)$$

take $v = \sec u$

$dv = \sec u \cdot \tan u \, du$

$$= \int \frac{\sec u \cdot \tan u}{\sec u \left[\sec u \left[\sqrt{\sec^2 u - 1} + \sec^2 u - 1 \right] \right.}$$

$$= \int \frac{\tan u \cdot du}{\sec u \tan u + \sec^2 u - 1}$$

$$= \int \frac{\sin u \cdot du}{\cos u \left[\frac{1}{\cos u} \cdot \frac{\sin u}{\cos u} + \frac{1}{\cos^2 u} - 1 \right]}$$

$$= \int \frac{\sin \cos u \, du}{\sin + 1 - \cos^2 u}$$

$$= \int \frac{\sin u \cos u \, du}{\sin u + \sin^2 u}$$

$$= \int \frac{\cos u \cdot du}{\sin u + 1}$$

let $\sin u + 1 = t$

$\therefore \cos u \cdot du = dt$

Rainbow

$$\int \frac{dt}{t} = \ln . t = \ln . [\sin u + 1]$$

$$= \ln(\sin(\sec^{-1} v) + 1$$

$$= \ln\left[\frac{\sqrt{v^2-1}}{v} + 1\right]$$

(3.) $$\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$$

let $y = v \cdot x$

$$v + \frac{x \cdot dv}{dx} = v + \tan v$$

$$\therefore \cot v \cdot dv = \frac{1}{x} \cdot dx.$$

Integrating, we get

$$\int \cot v \cdot dv = \ln . x + \ln . c.$$

$$\therefore \ln . \sin v = \ln . x . c$$

$$\therefore \sin v = xc.$$

$$\sin y/x = c \cdot x$$

nbow