* Harmonic analysis :- In most of practical egs. what we get is not the funct^n $f(x)$ but we have the numerical data i.e. the value of the funct^n & independent variables. The process of finding the fourier series for the available numerical data is called harmonic analysis.

Let $y = f(x)$ be periodic funct^n of period $2L$ defined in the range $[0, 2L]$. Let (x_i, y_i) $i = 0, 1, 2, \cdots, m-1$ be the given set of values where x_i are equispaced.

$\therefore$ the range $[0, 2L]$ is divided into 'm' equal parts given by pt's $x_0, x_1, x_2, \cdots, x_m$.

interval width $= \frac{2L}{m}$

$$\therefore \quad x_i = x_0 + \frac{2Li}{m}$$

The coefficients a_0, a_n, b_n in the Fourier series are obtained by applying trapezoidal rule of approximate integrat^n we get,

[Trapezoidal rule : $\int_a^b f(x)\,dx = \frac{\Delta x}{2}\left[f(x_0) + 2[f(x_1) + \cdots + f(x_{m-1})] + f(x_m)\right]$]

$$a_0 = \frac{1}{L}\int_0^{2L} y\,dx$$

$$= \frac{1}{L}\,\frac{2L}{m}\,[y_0 + y_1 + y_2 + \cdots + y_{m-1}]$$

$$a_0 = \frac{2}{m}\sum_{i=0}^{m-1} y_i$$

$= 2$ [mean value of $y = f(x)$ in $[0, 2L)$]

$$a_n = \frac{1}{L}\int_0^{2L} y\cos\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{1}{L} \cdot \frac{2L}{m} \sum_{i=0}^{m-1} y_i \cos\left(\frac{n\pi x_i}{L}\right)$$

$$= \frac{2}{m} \sum_{i=0}^{m-1} y_i \cos\left(\frac{n\pi x_i}{L}\right)$$

$a_n = 2$ [mean value of $y\cos\left(\frac{n\pi x}{L}\right)$ in $[0, 2L]$]

& $$b_n = \frac{1}{L}\int_0^{2L} y\sin\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{1}{L} \cdot \frac{2L}{m} \sum_{i=0}^{m-1} y_i \sin\left(\frac{n\pi x_i}{L}\right).$$

$b_n = 2$ [Mean value of $y\sin\left(\frac{n\pi x}{L}\right)$ in $[0, 2L]$]

The Fourier series is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{m-1}\left[a_n\cos\left(\frac{n\pi x}{L}\right) + b_n\sin\left(\frac{n\pi x}{L}\right)\right]$$

* Note → 1] The Fourier series of (x_i, y_i) $i = 0, 1, 2, \dots, m$ in the interval $[0, 2\pi]$ with period 2π is given by,

$$y = f(x) = \frac{a_0}{2} + \sum_{n=1}^{m-1}\left[a_n\cos nx + b_n\sin nx\right]$$

where fourier coefficients are

$a_0 = 2$ [Mean value of $y = f(x)$ in $[0, 2\pi]$]

$a_n = 2$ [Mean value of $y\cos(nx)$ in $[0, 2\pi]$]

$b_n = 2$ [Mean value of $y\sin(nx)$ in $[0, 2\pi]$]

2] The term $a_1\cos\left(\frac{\pi x}{L}\right) + b_1\sin\left(\frac{\pi x}{L}\right)$ is called fundamental / 1^{st} harmonic.

The term $a_2\cos\left(\frac{2\pi x}{L}\right) + b_2\sin\left(\frac{2\pi x}{L}\right)$ is

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called 2^{nd} harmonic & so on ..

3] The amplitude of n^{th} harmonic is $+\sqrt{a_n^2+b_n^2}$

4] % n^{th} harmonic $= \dfrac{\text{Amplitude of } n^{th} \text{ harmonic}}{\text{Amplitude of } 1^{st} \text{ harmonic}} \times 100$

eg① Obtain the constant term & the coefficients of 1^{st} cosine & sine terms in the expansion of y from the table

x	0	1	2	3	4	5	
y	9	18	24	28	26	20	

→ Here period $= 6$ we have,

$(0, 2L) = (0, 6) \Rightarrow L = 3$

$\therefore \dfrac{n\pi x}{L} = \dfrac{n\pi x}{3}$

$\therefore$ the Fourier series to represent y is $(0,5)$ is $y = \frac{1}{2} a_0 + a_1 \cos \frac{\pi x}{3} + b_1 \sin \frac{\pi x}{3}$

$+ a_2 \cos \frac{2\pi x}{3} + b_2 \sin\left(\frac{2\pi x}{3}\right) + \cdots$

$a_0 = 2$ [Mean value of y in $(0,5)$]

$a_1 = 2$ [Mean value of $y \cdot \cos \frac{\pi x}{3}$ in $(0,5)$]

$b_1 = 2$ [Mean value of $y \cdot \sin \frac{\pi x}{3}$ in $(0,5)$]

i.e. $a_0 = 2 \dfrac{\Sigma y}{n}$

$a_1 = \dfrac{2 \Sigma y \cos \pi x/3}{n}$

$b_1 = \dfrac{2 \Sigma y \sin(\pi x/3)}{n}$

x	$\pi x/3$	y	$y \sin(\pi x/3)$	$y \cdot \cos(\pi x/3)$
0	0	9	0	9
1	$\pi/3$	18	15.589	9
2	$2\pi/3$	24	20.785	-12
3	$3\pi/3 = \pi$	28	0	-28
4	$4\pi/3$	26	-22.517	-13
5	$5\pi/3$	20	-17.321	
		$\Sigma = 125$	$\Sigma = -3.404$	$\Sigma = -25$

$n = 6$

$$a_0 = \frac{2\Sigma y}{n} = 2 \times \frac{125}{6} = 41.66$$

$$a_1 = \frac{2 \Sigma y \cdot \cos(\pi x/3)}{n} = \cancel{\frac{2/\times 125}{6}} \Rightarrow \cancel{41.66}$$

$$= 2 \times \frac{-25}{6} = -8.33$$

$$b_1 = \frac{2 \ \Sigma y \sin(\pi x/3)}{6} = \frac{-3.404}{3} = -1.15$$

$$\therefore y = 20.83 - 8.33 \cos\frac{\pi x}{3} - 1.15 \sin\frac{\pi x}{3}$$

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2] The following table gives variatⁿ of periodic current over a period.

t sec.	0	T/6	T/3	T/2	2T/3	5T/6
A (amp.)	1.98	1.30	1.05	1.30	-0.88	-0.25
	T					
	1.98					

Show that there is a direct current part of 0.75 amp in variable current & obtain the amplitude of 1st harmonic.

→ Here, $n = 6$ period $= T$

$\therefore (0, 2L) = (0, T) \Rightarrow \frac{n\pi x}{L} = \frac{n\pi t}{T/2}$

$a_0 =$ 2 × Mean value of A

$a_1 =$ 2 × Mean value of $A \times \cos\left(\frac{\pi t}{T/2}\right)$

$b_1 =$ 2× Mean value of $A \times \sin\left(\frac{\pi t}{T/2}\right)$

t	$2\pi t/T$	A	$A\cos(\pi t/T/2)$	$A\sin(\pi t/T/2)$
0	0	1.98	1.98	0
T/6	$\pi/3$	1.30	0.65	1.1258
T/3	$2\pi/3$	1.05	-0.525	0.909
T/2	π	1.30	-1.30	0
2T/3	$4\pi/3$	-0.88	0.44	0.762
5T/6	$5\pi/3$	-0.25	-0.125	0.2165
Total		4.5	1.12	3.0133

$\therefore a_0 = 2 \times \frac{\Sigma A}{6} = \frac{2 \times 4.5}{6} = 1.5$

$a_1 = \frac{2 \times \Sigma A\cos(2\pi t/T)}{6} = \frac{2 \times 1.12}{6} = 0.373$

$b_1 = \frac{2 \times \Sigma A\sin(2\pi t/T)}{6} = \frac{2 \times 3.0133}{6} = 1.004$

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$\therefore$ Required series representat^n is

$$A = \frac{a_0}{2} + a_1 \cos\frac{2\pi t}{T} + b_1 \sin\frac{2\pi t}{T} + \dots$$

$$A = 0.75 + 0.373 \cos\frac{2\pi t}{T} + 1.004 \sin\frac{2\pi t}{T} + \dots$$

where direct current = 0.75 amp

amplitude of 1st harmonic is $= \sqrt{a_1^2 + b_1^2}$

$$= \sqrt{(0.373)^2 + (1.004)^2}$$

$$= 1.07.$$

3] Find 1st two harmonics of the Fourier series for y from the data.

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°
y	2.34	3.01	3.69	4.15	3.69	2.20	0.83	0.51	0.88	1.09
	300°	330°								
	1.19	1.64								

→ the Fourier series expans^n upto 2nd harmonic is given by,

$$f(x) = \frac{a_0}{2} + (a_1 \cos x + b_1 \sin x) + (a_2 \cos 2x + b_2 \sin 2x)$$

We will form the table

$n = 12 \qquad (0, 2L) = (0, 360°) \Rightarrow L = \pi$

$a_0 =$ 2 × Mean value of $f(x)$ in $(0, (0, \pi)$

$a_1 =$ 2 × Mean value of $y \cos x$

$b_1 =$ 2 × Mean value of $y \sin x$

$a_2 =$ 2 × Mean value of $y \cos 2x$

$b_2 =$ 2 × Mean value of $y \sin 2x$

x	$y=f(x)$	$y\cos x$	$y\cos 2x$	$y\sin x$	$y\cdot\sin 2x$
0°	2.34	2.34	0	0	0
30°	3.01	2.6067	1.505	1.505	2.6067
60°	3.69	1.845	−1.845	3.1955	3.1955
90°	4.15	0	−4.15	4.15	0
120°	3.69	−1.845	−1.845	3.1955	−3.1955
150°	2.20	−1.9052	1.1	1.1	−1.9052
180°	0.83	−0.083	0.083	0	0
210°	0.51	−0.4417	0.225	−0.255	0.4417
240°	0.88	−0.44	0.44	−0.7621	0.7621
270°	1.09	0	−1.09	−1.09	0
300°	1.19	0.595	−0.595	−1.0305	−1.0305
330°	1.64	1.4202	0.82	−0.82	−1.4202
	$\Sigma=24.473$	$\Sigma=4.092$	$\Sigma=-3.862$	$\Sigma=9.1884$	$\Sigma=-0.5454$

$n=12$, $a_0 = 2\times\frac{\Sigma f(x)}{n} = \frac{2\times 24.473}{12} = 4.078$

$a_1 = \frac{2}{12}\Sigma y\cos x = \frac{-4.092}{6} = 0.682$

$a_2 = \frac{2\times \Sigma y\cos 2x}{12} = -0.6437$

$b_1 = \frac{2\times \Sigma y\sin x}{12} = 1.5314$

$b_2 = \frac{2\times \Sigma y\sin 2x}{12} = -0.0909$.

eg 1] In a machine the displacement $f(x)$ of a given pt. is given for a certain angle (x°)

x°	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°
$f(x)$	7.9	8.0	7.2	5.6	3.6	1.7	0.5	0.2	0.9	2.5	4.7	6.8

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Find the coefficient of $\sin 2x$ in the Fourier series representat^n

2] the displacement $f(x)$ of a part of a machine tabulated with corresponding angular moment 'x' of the crank. Express $f(x)$ as a Fourier series upto 2^{nd} harmonic. Find amplitude of 1^{st} harmonic & o/o 2^{nd} harmonic.

x^0	0^0	30	60	90	120	150	180	210	240	270	300	330
$f(x)$	1.80	1.10	0.30	0.16	0.50	1.30	2.16	1.25	1.30	1.52	1.76	2.00