Module 3 Numericals (1)

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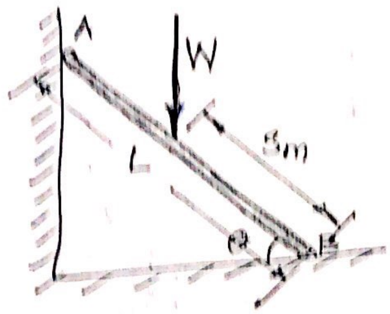
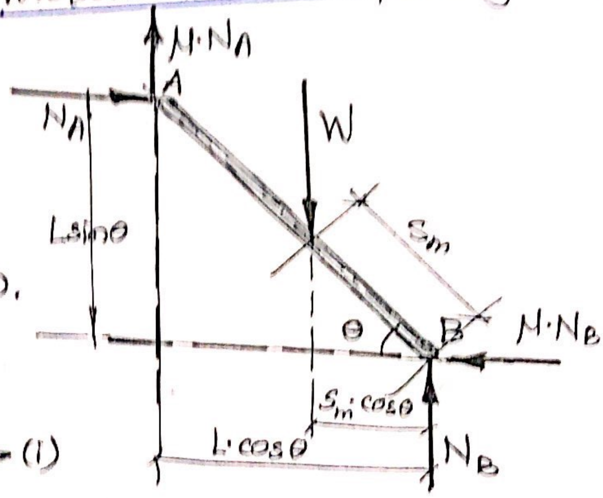
Ex. No. (6) a) A person of weight 'W' ascends the ladder. The weight of the ladder is assumed to be negligible compared to the weight of the person. How far up the ladder, defined by the dimension 's_m' in the figure, may the person climb before sliding motion of the ladder is impending?

b) If $\theta = 50°$, $\mu = 0.30$ and $L = 6$ m, find the value of s_m.



Solution (a) Consider the F.B.D. of the ladder assuming the motion to be impending.



Applying equations of equilibrium to the F.B.D. of the ladder,

$\Sigma F_x = 0$ gives,

$$N_A - \mu \cdot N_B = 0 \longrightarrow (i)$$

$\Sigma F_y = 0$ gives,

$$\mu \cdot N_A + N_B - W = 0 \longrightarrow (ii)$$

$$\therefore \mu^2 \cdot N_B + N_B = W$$

$$\therefore N_B (1 + \mu^2) = W$$

$$\boxed{N_B = \left(\frac{W}{1+\mu^2}\right)}$$

and $$\boxed{N_A = \left(\frac{\mu \cdot W}{1+\mu^2}\right)}$$

Similarly, taking moments about B, $\Sigma M_B = 0$ gives,

$$W \cdot s_m \cos\theta - N_A \cdot L \cdot \sin\theta - \mu N_A \cdot L \cdot \cos\theta = 0 \longrightarrow (iii)$$

$$\therefore s_m = \left[\frac{N_A \cdot L (\sin\theta + \mu \cdot \cos\theta)}{W \cdot \cos\theta}\right]$$

Ans: $$\boxed{s_m = \frac{\mu \cdot L \cdot (\sin\theta + \mu \cos\theta)}{(1+\mu^2) \cdot \cos\theta}}$$

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(b) when, $\theta = 50^\circ$
$\mu = 0.3$
$L = 6m$

$$S_m = \frac{\mu \cdot L \cdot (\sin\theta + \mu \cdot \cos\theta)}{(1+\mu^2)\cos\theta}$$

$$= \frac{0.3 \times 6 \times (\sin 50^\circ + 0.3 \times \cos 50^\circ)}{(1+0.3^2) \cdot \cos 50^\circ}$$

$$= 2.463 \text{ m}$$

Ans: $\boxed{S_m = 2.463 \text{ m}}$

Ex.No.7 In the previous problem, Find the required coefficient of friction at all contact surfaces, if the ladder is to remain stationary for any position of the person on the ladder.

Solution: We have, in the previous probem

$$S_m = \left[\frac{\mu \cdot L(\sin\theta + \mu\cdot\cos\theta)}{(1+\mu^2)\cos\theta}\right]$$

$$\therefore \left(\frac{S_m}{L}\right) = \left(\frac{\mu}{1+\mu^2}\right)\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta}\right)$$

If the ladder is to remain in static equilibrium for any position of the person on it, the distance S_m must satisfy the inequality

$$\left(\frac{S_m}{L}\right) > 1 \quad \text{i.e.} \quad S_m > L$$

Thus, $$\left(\frac{S_m}{L}\right) = \left(\frac{\mu}{1+\mu^2}\right)(\tan\theta + \mu) > 1$$

$$(\mu\cdot\tan\theta + \mu^2) > 1 + \mu^2$$

$$\mu\cdot\tan\theta > 1$$

Ans: $\therefore \boxed{\mu > \frac{1}{\tan\theta}}$ or $\boxed{\mu > \cot\theta}$

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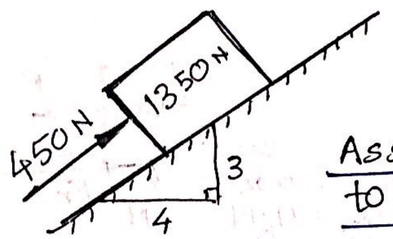
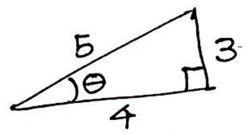
(N). **Ex. No (1)**: A 450 N force acts as shown in figure on a 1350 N block placed on an inclined plane. The coefficients of friction between the block and the plane are $\mu_s = 0.25$ and $\mu_k = 0.20$. Determine whether the block is in equilibrium, and also find the value of the frictional force.

Solution:

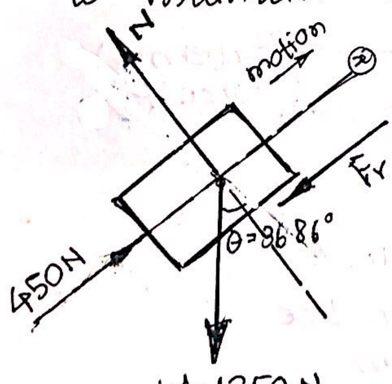


$\tan\theta = \frac{3}{4}$



$\therefore \theta = 36.87^\circ$

Assume the tendency of the block to move up the plane. And determine the frictional force required to maintain equilibrium.



$\Sigma F_x = 0$ gives,

$450 - (1350)\sin 36.87^\circ - F_r = 0$

$\therefore F_r = -360\ N$

$-ve$ sign indicate that the tendency of the block is to move down the plane.

$\Sigma F_y = 0$ gives,

$N - (1350)\cos 36.87^\circ = 0$

$\therefore N = 1080\ N$

The maximum Frictional Force:

The magnitude of the maximum frictional force or limiting frictional force is given by,

$(F_r)_{max} = \mu_s N = (0.25 \times 1080) = 270\ N$

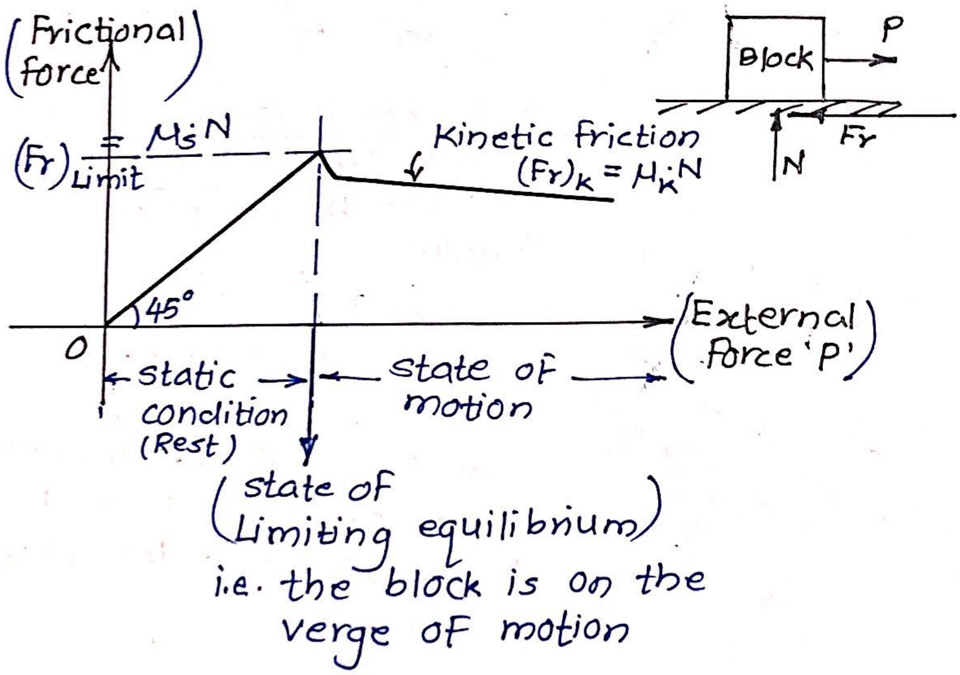
Since the value of the force required to maintain equilibrium (i.e. 360 N) is larger than the limiting frictional force that can be developed (i.e. 270 N), the block is not in equilibrium. And the block is sliding down the plane.

Actual Frictional Force :

The actual Frictional Force experienced by the block is 'kinetic friction' because the block is in motion.

$$(F_r)_{Actual} = (F_r)_{kinetic} = \mu_k N$$

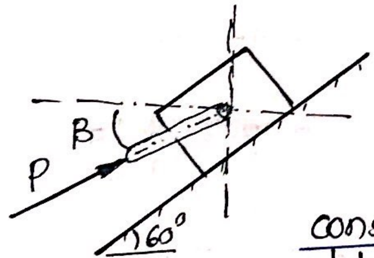
$$= (0.2 \times 1080) = 216\ N \text{ (directed up the plane)}$$



(2) Ex. No. (2) Knowing that the coefficient of friction between the 13.5 kg block and the incline is $\mu_s = 0.25$, determine;

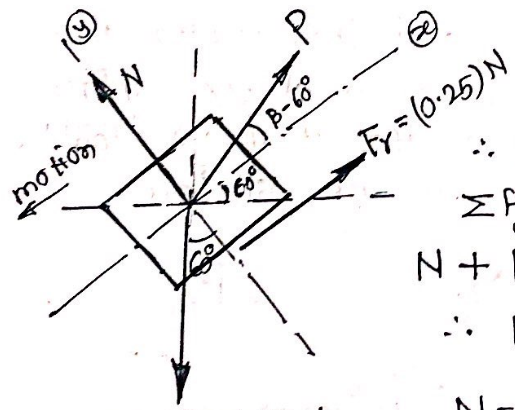
a) the smallest value of P required to maintain the block in equilibrium.

b) the corresponding value of angle β.



Solution: To prevent the possible downward sliding motion of the block, force P_{min} is applied.

Consider the F.B.D. of the block, considering downward motion of the block along the inclined plane.



W = 132.435 N

$\Sigma F_x = 0$ gives,

$(0.25)N + P\cos(\beta - 60°) - (132.435)\sin 60° = 0$

$\therefore (0.25)N + P.\cos(\beta - 60°) = 114.7 \longrightarrow (1)$

$\Sigma F_y = 0$ gives,

$N + P.\sin(\beta - 60°) - (132.435)\cos 60° = 0$

$\therefore N + P.\sin(\beta - 60°) = 66.25 \longrightarrow (2)$

$N = (66.25) - P.\sin(\beta - 60°)$

Substituting this in eq^n (1), we get,

$16.56 - (0.25)P.\sin(\beta - 60°) + P.\cos(\beta - 60°) = 114.7$

$\therefore P\left[\cos(\beta - 60°) - (0.25)\sin(\beta - 60°)\right] = 98.14$

$\therefore P = \left[\dfrac{98.14}{\cos(\beta - 60°) - (0.25)\sin(\beta - 60°)}\right]$

For smallest value of P,

$$\frac{dP}{d\beta} = \frac{-(98.14)\cancel{(0.25)}\left[\sin(\beta - 60°) - (0.25)\cos(\beta - 60°)\right]}{\left[\cos(\beta - 60°) - (0.25)\sin(\beta - 60°)\right]^2} = 0$$

$$\therefore \; -(\sin(\beta-60^\circ) - (0.25)\cos(\beta-60^\circ) = 0$$

$$-\tan(\beta-60^\circ) = 0.25$$

$$\tan(\beta-60^\circ) = -0.25$$

$$\beta - 60^\circ = -14.03$$

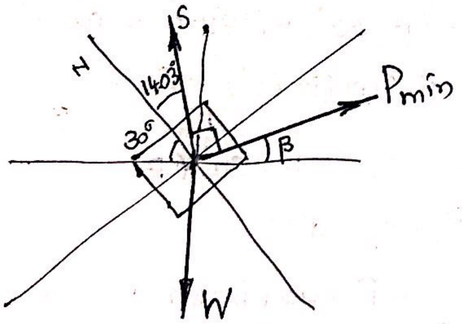
$$\beta = 45.97^\circ \text{ with hz.}$$

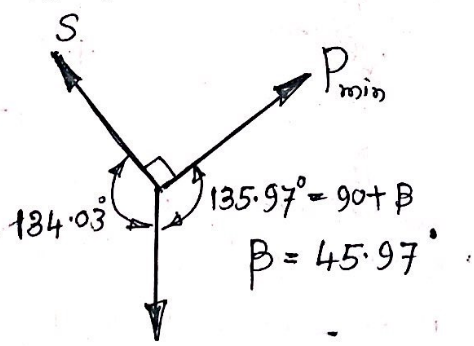
$$\therefore \; P_{min} = \left[\frac{98.14}{0.97 - (0.25)(-0.242)}\right] = 95.235 \text{ N}$$

Ans: $P_{min} = 95.235$ N , $\beta = 45.97^\circ$ with hz.

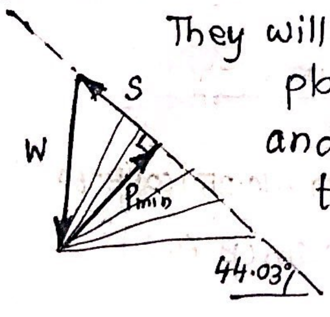
Alternative method :

In the F.B.D. of the block, consider the total reaction 'S' which is the resultant of the normal reaction (N) and the frictional force ($\mu_s N$). This total reaction of the surface will be inclined at an angle of ϕ i.e. ($\tan^{-1}\mu_s = 14.03^\circ$) with the normal to the surface.





The block is acted upon by 3 forces which are in equilibrium.



They will form a closed triangle, when plotted graphically. Directions of 'W' and 'S' are fixed. The shortest side to close the triangle is P_{min}, which is at right angles to 'S'. This gives us $\beta = 45.97^\circ$ and then $P_{min} = 95.235$ N

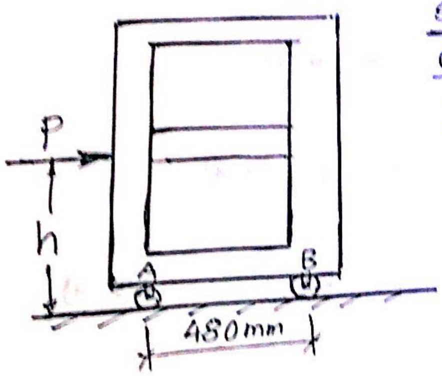
3.)

Ex. No. (3) A 48 kg cabinet is mounted on casters which can be locked to prevent their motion (rotation). The coefficient of friction between the floor and the casters is 0.3. If h = 640 mm, determine the magnitude of force 'P' required for impending motion of the cabinet to the right.

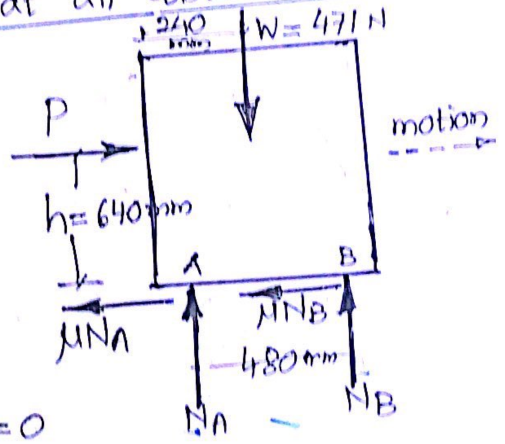
a) If all casters are locked

b) If casters at B are locked and casters at A are free to rotate.

c) If casters at A are locked and casters at B are free to rotate.



Solution : Consider the F.B.D. of the cabinet, considering that all casters are locked.



(a) When all casters are locked :

$\Sigma F_x = 0$ gives,

$P - (0.3)N_A - (0.3)N_B = 0$

$\therefore \quad P = (0.3)(N_A + N_B) \rightarrow (1)$

$\Sigma F_y = 0$ gives,

$N_A + N_B = 471 \text{ N} \rightarrow (2)$

$\therefore \quad P = (0.3)(471) = 141.3 \text{ N} \qquad \therefore \boxed{P = 141.3 \text{ N}}$

(b) Casters at B are locked and at A are free to rotate :

Due to this there will not be any frictional force at A.

$\therefore \Sigma F_x = 0$ gives, $\quad P - (0.3)N_B = 0 \rightarrow (1)$

$\therefore \quad P = (0.3) N_B$

$\Sigma F_y = 0$ gives, $N_A + N_B = 471 \longrightarrow (2)$

$\Sigma M_A = 0$ gives,

$-(P \times 640) + (N_B \times 480) - (471 \times 240) = 0 \longrightarrow (3)$

$\therefore -(P \times 640) + \left(\frac{P}{0.3}\right)(480) - (471 \times 240) = 0$

$\therefore \boxed{P = 118\ N}$

(c) Casters at A are locked and at B are free to rotate:

Due to this there will not be any frictional force at B.

$\therefore \Sigma F_x = 0$ gives,

$P - (0.3) N_A = 0 \longrightarrow (1)$

$\therefore P = (0.3) N_A$

$\Sigma F_y = 0$ gives, $N_A + N_B = 471 \longrightarrow (2)$

$\Sigma M_B = 0$ gives,

$-(P \times 640) - (N_A \times 480) + (471 \times 240) = 0 \longrightarrow (3)$

$\therefore -(P \times 640) - \left(\frac{P}{0.3}\right)(480) + (471 \times 240) = 0$

$\therefore \boxed{P = 50.5\ N}$

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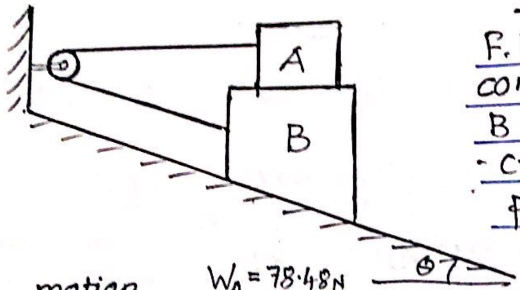
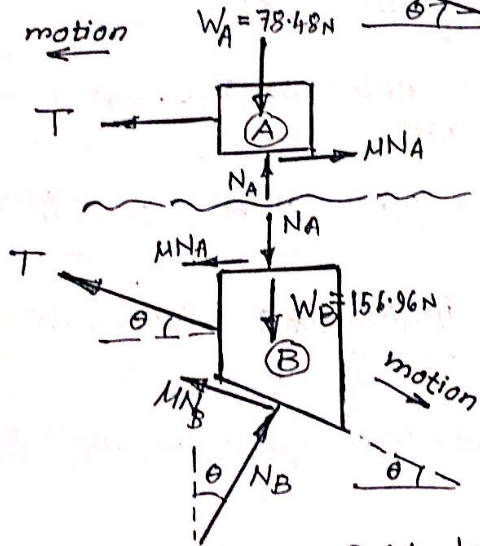
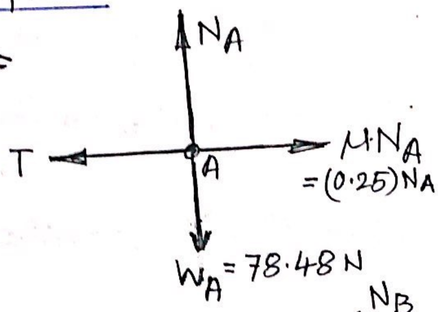
(4)

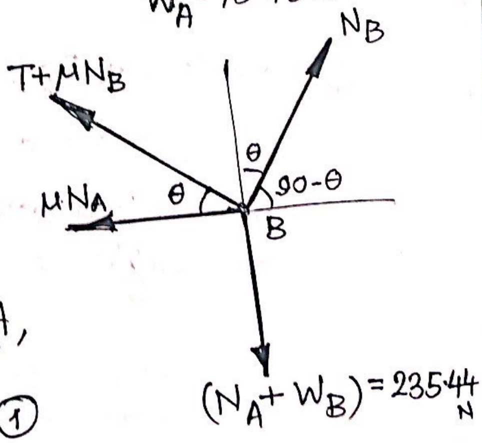
Ex. No (4): The 8 kg block A and 16 kg block B are at rest on an incline as shown. Knowing that $\mu_s = 0.25$ between all surfaces of contact, determine the value of angle 'θ' for which motion is impending.

Solution: Consider the F.B.Ds of blocks A and B, considering the motion of B in the downward direction along the inclined plane.









From the F.B.D. of block A,

$\Sigma F_x = 0$ gives,

$$(0.25) N_A - T = 0 \longrightarrow (1)$$

$\Sigma F_y = 0$ gives,

$$N_A - 78.48 = 0 \longrightarrow (2)$$

$\therefore N_A = 78.48\ N$

$\therefore T = (0.25 \times 78.48) = 19.62\ N$

From the F.B.D. of block B, we get,

$\Sigma f_x = 0$ gives,

$$N_B \cdot \cos(90-\theta) - (0.25)N_A - (T + 0.25 N_B)\cos\theta = 0$$

$$\therefore N_B \cdot \sin\theta - (19.62) - (19.62 + 0.25 N_B)\cos\theta = 0 \rightarrow (3)$$

$\Sigma f_y = 0$ gives,

$$N_B \cdot \sin(90-\theta) + (T + 0.25 N_B)\sin\theta - 235.44 = 0$$

$$\therefore N_B \cdot \cos\theta + (19.62 + 0.25 N_B)\sin\theta - 235.44 = 0 \rightarrow (4)$$

From (3) we get,

$$N_B(\sin\theta - 0.25\cos\theta) = (19.62)(1 + \cos\theta) \longrightarrow (3A)$$

From (4) we get,

$$N_B(\cos\theta + 0.25\sin\theta) = (19.62)(12 - \sin\theta) \longrightarrow (4A)$$

Take the ratio of (3A) and (4A)

$$\left[\frac{\sin\theta - (0.25)\cos\theta}{\cos\theta + (0.25)\sin\theta}\right] = \left[\frac{1 + \cos\theta}{12 - \sin\theta}\right]$$

$$\therefore 12\sin\theta - \sin^2\theta - 3\cos\theta + \cancel{(0.25)\sin\theta \cdot \cos\theta}$$

$$= \cos\theta + \cos^2\theta + \cancel{(0.25)\sin\theta\cos\theta}$$

$$\therefore (11.75)\sin\theta - 4\cos\theta = \sin^2\theta + \cos^2\theta = 1$$

$$\therefore (11.75)\sin\theta - 1 = 4\cos\theta$$

Squaring this we get

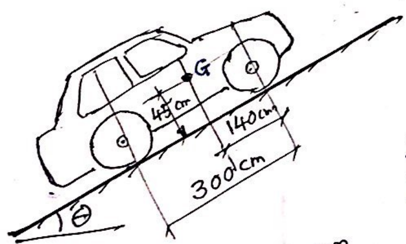
$$(154.0625)\sin^2\theta - (23.5)\sin\theta + 1 = 0$$

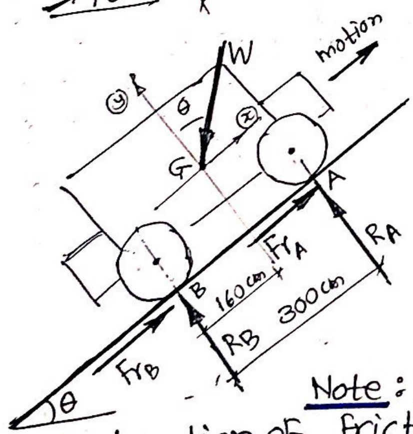
Ans: By trial and error method we get,

$$\boxed{\theta = 23.45^\circ}$$

Ex. No ⑤: The automobile shown in figure is having a mass of 1500 kg. The height of the center of gravity from the ground is 45 cm. The distance between the ~~two~~ rear and front wheel is 300 cm. The C.G. of the car is 140 cm behind the front wheel. The coefficient of friction between the rubber tyres and the dry pavement is 0.8. For the following three cases, find the maximum angle θ for which the vehicle can be driven up.

a) Rear-wheel drive only
b) Front-wheel drive only
c) Four-wheel drive





Solution: Consider the F.B.D. of the car considering four wheel drive.

W = weight of the car

$= (1500 \times 9.81)\ N$

$= 14.715\ kN$

R_A and R_B are the normal reactions at the front and rear wheels.

Fr_A and Fr_B are the frictional forces

Note: For the power driven wheels the direction of frictional force is in the direction of the motion. (not opposite to the motion of the car) Applying eqns of equilibrium to the F.B.D. of the car, we get,

$\Sigma F_x = 0$ gives, $(Fr)_A + (Fr)_B = W \cdot \sin\theta$

$$(0.8)R_A + (0.8)R_B = (14.715)\sin\theta$$

$$\therefore\ R_A + R_B = (18.4)\sin\theta \longrightarrow (i)$$

$\Sigma F_y = 0$ gives, $R_A + R_B = W \cdot \cos\theta = (14.715)\cos\theta \longrightarrow (ii)$

From (i) and (ii) we get,

$$(18.4)\sin\theta = (14.715)\cos\theta$$

$$\tan\theta = \frac{14.715}{18.4} = 0.799$$

$$\boxed{\theta = 38.65^\circ}$$

<u>For the case of rear-wheel drive car,</u>

$$(F_r)_A = 0 \quad \therefore \quad (F_r)_B = W.\sin\theta$$

$$\therefore (0.8) R_B = (14.715)\sin\theta$$

$$R_B = (18.4)\sin\theta \longrightarrow (i)$$

$\Sigma M_A = 0$ gives,

$$(W\cos\theta \times 140) + (W.\sin\theta \times 45) - (300) R_B = 0$$

$$(2060.1)\cos\theta + (662.175)\sin\theta = (300) R_B$$

$$R_B = (6.867)\cos\theta + (2.2)\sin\theta \longrightarrow (ii)$$

$$\therefore (18.4)\sin\theta = (6.867)\cos\theta + (2.2)\sin\theta$$

$$\tan\theta = \frac{6.867}{16.2} = 0.4238$$

$$\therefore \boxed{\theta = 22.97^\circ}$$

<u>For the case of front-wheel drive car,</u>

$$(F_r)_B = 0 \quad \therefore \quad (F_r)_A = W.\sin\theta$$

$$(0.8) R_A = (14.715)\sin\theta$$

$$R_A = (18.4)\sin\theta \longrightarrow (i)$$

$\Sigma M_B = 0$ gives,

$$(300) R_A + (W\sin\theta \times 45) - (W\cos\theta \times 160) = 0$$

$$(300) R_A = (2354.4)\cos\theta - (662.175)\sin\theta$$

$$R_A = (7.848)\cos\theta - (2.2)\sin\theta \longrightarrow (ii)$$

$$(18.4)\sin\theta = (7.848)\cos\theta - (2.2)\sin\theta$$

$$\tan\theta = \frac{7.848}{16.2} = 0.484$$

$$\boxed{\theta = 25.83^\circ}$$

<u>Ans</u>: The above car can be driven up the plane

if a) $\theta = 22.97^\circ$ for rear-wheel drive car

b) $\theta = 25.83^\circ$ for front-wheel drive car

c) $\theta = 38.65^\circ$ for four-wheel drive car.