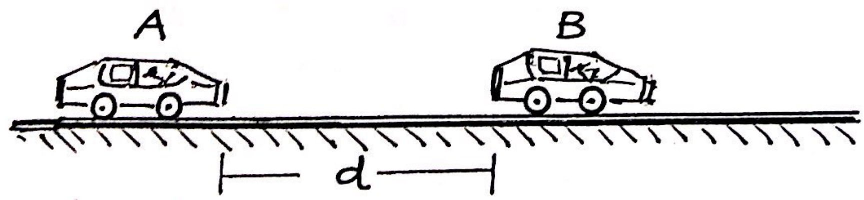
Lecture no: **Rectilinear Kinematics**

1.	A Starting from rest, a particle moving in straight line has an acceleration of a= $(2t - 6)$ m/sec^2, where t is in second. What is the particle's velocity when t = 6 sec, & what is its position when t=11 sec? **Ans: at t=6s (0) & at t=11s (80.67m)**
2.	A Starting from rest, a particle moving in straight line has an acceleration of a= $(60 - 72t^2)$ m/sec^2, Determine the particle's velocity when it was travelled 110m & the time take by it before it comes to rest again. **Ans: t= 2.444s, v= -203.72m/s, a = -370m/s^2**
3.	The car moves in a straight line such that for a short time, it's velocity is defined by $v = 0.8(8t^2 + 3t)$ m/sec, where t is in second. Determine it's position & acceleration when t=6secs. Given at t = 0, s = 0 **Ans: at t=6s, x=504m, a=79.2 m/s^2**
4.	A stone dropped from rest moves a distance equal to one half of the depth of fall in last second of its fall. Find the time of the fall of the stone & the depth of fall. **Ans: t=3.414s , h= 57.166m**
5.	A train travelling with a speed of 110 kmph slows down on account of work in progress, at a retardation of 2.6 kmph per second to 43 kmph. With this it travels 800m thereafter it gains further speed with 1.8 kmph per second till getting original speed. Find the delay caused. **Ans: 60sec**
6.	When to cars A & B are next to one another, they are traveling in the same direction with speed V_A & V_B, respectively. IF B maintains it's speed constant, while A begins to deaccelerate at a_A, determine the distance d between the cars at the instant A stops. **Ans: x=[(2V_A.V_B-V_A^2)/2a_A]** 

Rectilinear kinematics-I

Ex. No (1) RCH/12.1/ pg. 631

$$\boxed{a=(2t-6)} = f(t) \longrightarrow (1)$$

$$a=\left(\frac{dV}{dt}\right) = (2t-6) \ m/s^2$$

$$\int dv = \int (2t-6)\cdot dt$$

$$\therefore \quad V = t^2 - 6t + C_1$$

At $t=0$, $V=0$ $\therefore$ $C_1 = 0$

$$\therefore \quad \boxed{V = (t^2 - 6t) \ m/s} \longrightarrow (2)$$

$$\therefore \quad V = \frac{dx}{dt} = (t^2 - 6t)$$

$$\int dx = \int (t^2 - 6\cdot t)\cdot dt$$

$$\therefore \quad x = \frac{t^3}{3} - 3t^2 + C_2$$

At $t = 0$, $x = 0$ $\therefore$ $C_2 = 0$

$$\therefore \quad \boxed{x = \frac{1}{3}t^3 - 3\cdot t^2 \ m} \longrightarrow (3)$$

Ans: 1) At $t = 6$ s, $V = (6^2 - 6^2) = 0$

2) At $t = 11$ s, $x = \left(\frac{11^3}{3} - 3\times 11^2\right) = 80.67$ m

Ex. No. 2 RCH / 12.6 / Pg. 631

$$\boxed{a = (60 - 72 \cdot t^2)\ m/s^2} \longrightarrow (1)$$

$$\therefore\ a = \frac{dv}{dt} = (60 - 72 \cdot t^2)$$

$$\int dv = \int (60 - 72 \cdot t^2) \cdot dt$$

$$\therefore\ V = 60 \cdot t - 24 \cdot t^3 + C_1$$

At $t = 0$, $V = 0$ $\therefore\ C_1 = 0$

$$\therefore\ \boxed{V = (60 \cdot t - 24 \cdot t^3)\ m/s} \longrightarrow (2)$$

$$\therefore\ V = \frac{dx}{d} = (60 \cdot t - 24 \cdot t^3)$$

$$\int dx = \int (60 \cdot t - 24 \cdot t^3) dt$$

$$\therefore\ x = 30 \cdot t^2 - 6 \cdot t^4 + C_2$$

At $t = 0$, $x = 0$ $\therefore\ C_2 = 0$

$$\therefore\ \boxed{x = (30 \cdot t^2 - 6 \cdot t^4)\ m} \longrightarrow (3)$$

Now, find the time at which $V = 0$,

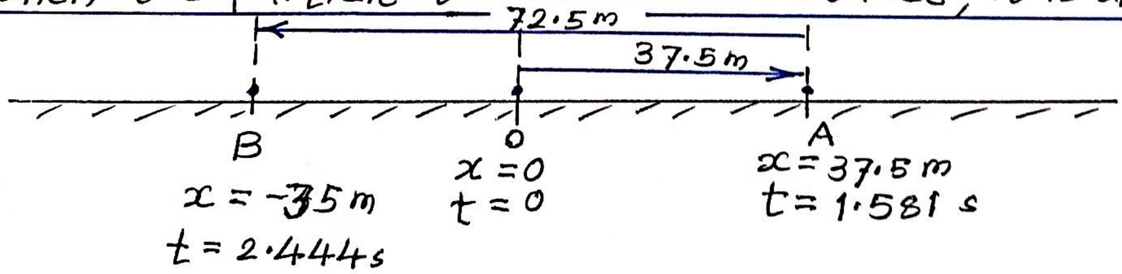
$$\therefore\ V = (60 \cdot t - 24 t^3) = 0$$

$$\therefore\ t \cdot (60 - 24 \cdot t^2) = 0$$

$\therefore$ Either $t = 0$ or $t = 1.581$ sec.

At this time, $x = 37.5\ m$ i.e. at A

Thus, the particle changes it's direction of motion after travelling 37.5 m from the start. Now, when the partizle travels 110 m distance, it is at B.



At B, $x = -35\ m = (30 \cdot t^2 - 6 \cdot t^4)$ $\therefore\ 6t^4 - 30 \cdot t^2 - 35 = 0$

solving this, we get, $t = 2.444\ s$ or $-0.976\ s$ Not possible

Ans: $\therefore$ At $t = 2.444\ s$, $V = -203.72\ m/s$

$a = -370\ m/s^2$

Ex. No. 3 RCH/12.7/pg. 631

$$\boxed{V = (0.8)\cdot(8t^2 + 3t)\ m/s} \longrightarrow (1)$$

$$\therefore \quad V = \frac{dx}{dt} = (0.8)(8t^2 + 3t)$$

$$\int dx = \int (0.8)(8t^2 + 3t)\cdot dt$$

$$x = (0.8)\left(\frac{8}{3}t^3 + \frac{3}{2}t^2\right) + c_1$$

At $t = 0$, $x = 0$ $\therefore$ $c_1 = 0$

$$\therefore \boxed{x = (0.8)\left(\frac{8}{3}t^3 + \frac{3}{2}\cdot t^2\right)\ m} \longrightarrow (2)$$

$$a = \frac{dV}{dt} \quad \therefore \boxed{a = (0.8)(16\cdot t + 3)\ m/s^2} \longrightarrow (3)$$

At $t = 6$ sec.

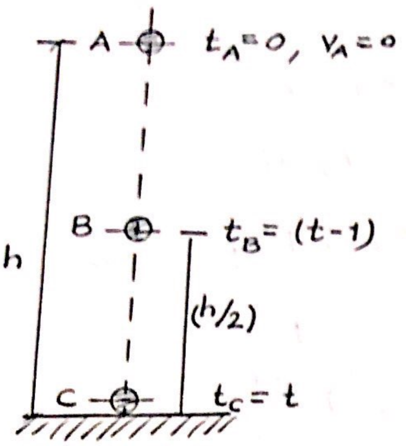
$$x = (0.8)\left(\frac{8}{3}\times 6^3 + \frac{3}{2}\times 6^2\right) = 504\ m$$

and $a = (0.8)(16\times 6 + 3) = 79.2\ m/s^2$

Ans.: At $t = 6$ s, $x = 504$ m

$a = 79.2\ m/s^2$

Ex. No. 4 RCH/12.10/pg. 631



Motion under gravity:

At position B:

$$s = ut + \frac{1}{2}at^2$$

$$-\left(\frac{h}{2}\right) = 0 - \frac{1}{2}g \cdot (t-1)^2$$

$$\therefore \quad h = g(t-1)^2 \longrightarrow (1)$$

At position C:

$$s = ut + \frac{1}{2}gt^2$$

$$-h = 0 - \frac{1}{2}g\,t^2$$

$$\therefore \quad h = \frac{1}{2}g \cdot t^2 \longrightarrow (2)$$

From eqns (1) and (2), we get,

$$g(t-1)^2 = \frac{1}{2} \cdot g \cdot t^2$$

$$\therefore \quad 2t^2 - 4 \cdot t + 2 = t^2$$

$$\therefore \quad t^2 - 4t + 2 = 0$$

$\therefore \quad t = 3.414\ s$ or $t = 0.585\ s < 1$ sec.

Hence, not possible.

Ans.: Total time of fall = 3.414 s

Height h = 57.166 m

Ex. No. 5 RCH/12-14/pg. 631

I) Decelerated motion:

$u = 110$ kmph $= \left(110 \times \frac{5}{18}\right) = 30.55$ m/s

$V = 43$ kmph $= \left(43 \times \frac{5}{18}\right) = 11.95$ m/s

$a_1 = -2.6$ kmph/sec $= -0.72$ m/s^2

$$V = u + a \cdot t$$

$(11.95) = (30.55) - (0.72)t_1$ $\quad \therefore t_1 = 25.83$ sec.

$S_1 = (30.55 \times 25.83) - (\frac{1}{2} \times 0.72 \times 25.83)^2 = 548.91$ m

II) Uniform motion:

$$S = V \cdot t$$

$\therefore (800) = (11.95) \cdot t_2$ $\quad \therefore t_2 = 66.95$ s

$S_2 = 800$ m.

III) Accelerated motion:

$a_2 = 1.8$ kmph/sec $= 0.5$ m/s^2

$$V = u + at$$

$(30.55) = (11.95) + (0.5)t_3$ $\quad \therefore t_3 = 37.2$ s

$$S = ut + \frac{1}{2}at^2$$

$\therefore S_3 = (11.95 \times 37.2) + (\frac{1}{2} \times 0.5 \times 37.2^2) = 790.5$ m

$\therefore$ Total time of travel $= (t_1 + t_2 + t_3) = 130$ s

Total distance travelled $= (S_1 + S_2 + S_3) = 2139.4$ m

If the track repair is not there, the train will take, $\quad t = \left(\frac{2139.4}{30.55}\right) = 70$ sec

Ans. Delay due to track repair $= (130 - 70)$

$= 60$ s.

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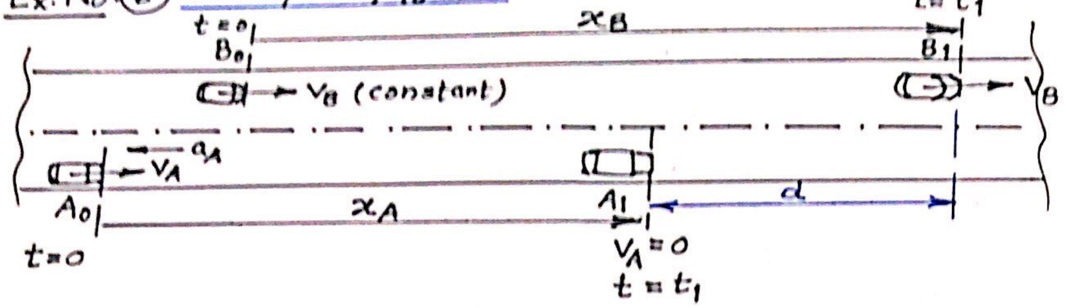
Ex. No. 6 RCH / 12.9 / pg. 631



For A :

$$V_A - (a_A \cdot t_1) = 0 \qquad \therefore \; t_1 = \left(\frac{V_A}{a_A}\right)$$

$$x_A = \left(V_A \cdot t_1 - \frac{1}{2} \cdot a_A t_1^2\right)$$

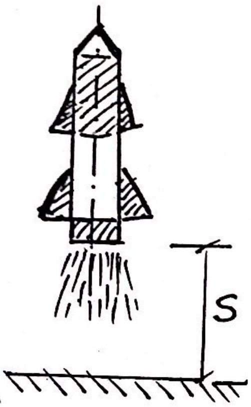
$$= \left(V_A \cdot \frac{V_A}{a_A} - \frac{1}{2} \cdot a_A \cdot \frac{V_A^2}{a_A^2}\right)$$

$$= \left(\frac{V_A^2}{a_A} - \frac{1}{2} \cdot \frac{V_A^2}{a_A}\right) = \frac{1}{2}\left(\frac{V_A^2}{a_A}\right)$$

For B : $x_B = V_B \cdot t_1 = \left(\frac{V_A \cdot V_B}{a_A}\right)$

Ans. : d = distance betn cars A and B when A stops

$$\boxed{d = |x_A - x_B| = \left|\frac{2 \cdot V_A V_B - V_A^2}{2 \cdot a_A}\right|}$$

1	A particle is moving a with a velocity of v_0 when s=0 & t=0. If it is subjected to a deceleration of $a= -kv^3$, where k is the constant, determine its velocity and position as function of time. **Ans: $(2kt+(1/v_0^2)^{-1/2}$**
2	The acceleration of a rocket traveling upward is given by $a=(6 + 0.02s)\ m/s^2$ where s is in meters. Determine the rockets velocity when s = 2 km & time needed to reach this altitude. Initially, V=0 & s= 0 when t=0. **Ans:** 
3	A particle moving along a straight line such that its acceleration is defined as , a $=(-2v)\ m/s^2$, where v is in meter/ seconds. If v = 20m/s when s =0 & t = 0, determine the particle's position, velocity and acceleration as function of time. **Ans: $x=10-(0.5)[e^{(2.995-2t)}]$**
4	If $a= (s)\ m/s^2$,where 's' is in meter, determine v when s = 5m if v=0 at s= 4m **Ans:V=3m/s**
5	The acceleration of a particle moving along a straight line is given by the law, a $=3s - 6s^2$. Where 'a' is m/s^2 & 's' is in meter. The particles starts from rest. Find (a)Velocity when the displacement is 3m. (b) the displacement when the velocity is again zero & (c) the displacement at maximum velocity. **Ans: x = 0.5m**
6	The acceleration of a particle is given by $a = -0.02v^{1.75}\ m/s^2$ performing rectilinear motion knowing at x= 0, v= 20 m/s. Determine ( ) The position where the velocity is 28m/s &(b) acceleration when x = 200m. **Ans: $a=-0.043m/s^2$**

① RCH/12-17/pg. 632:

$a = -kv^3 \ m/s^2 \longrightarrow ①$

$a \cdot dx = v \cdot dv$

$\therefore \ -k \cdot v^3 \cdot dx = v \cdot dv$

$\therefore \ -k \cdot v^2 \cdot dx = dv$

$\therefore \ -k\int_0^x dx = \int_{v_0}^{v} \frac{dv}{v^2}$

$-k \cdot x = \left[\frac{v^{-1}}{-1}\right]_{v_0}^{v}$

$-kx = -\left[\frac{1}{v}\right]_{v_0}^{v}$

$kx = \frac{1}{v} - \frac{1}{v_0}$

$\boxed{x = \frac{1}{k}\left[\frac{1}{v} - \frac{1}{v_0}\right]} \Rightarrow \boxed{\frac{1}{k}\left[\left(2kt + \frac{1}{v_0^2}\right)^{\frac{1}{2}} - \frac{1}{v_0}\right]}$

Ans:

(i) $v = \left(2kt + \frac{1}{v_0^2}\right)^{-1/2}$

(ii) $x = \frac{1}{k}\left[\left(2kt + \frac{1}{v_0^2}\right)^{1/2} - \frac{1}{v_0}\right]$

Now, $a = \frac{dv}{dt} = -kv^3$

$-k\int_0^t dt = \int_{v_0}^{v} v^{-3} \cdot dv$

$-k \cdot t = \left[\frac{v^{-2}}{-2}\right]_{v_0}^{v} = -\left[\frac{1}{2v^2}\right]_{v_0}^{v}$

$kt = \frac{1}{2v^2} - \frac{1}{2v_0^2}$

$\frac{1}{2v^2} = \frac{1}{2v_0^2} + kt$

$\frac{1}{v^2} = \left(\frac{1}{v_0^2} + 2kt\right)$

$\therefore V = \left[\frac{1}{\sqrt{2kt + 1/v_0^2}}\right]$

Ans:

$\therefore \boxed{V = \left(2kt + \frac{1}{v_0^2}\right)^{-1/2}}$

② RCH/12.19/pg.632:

$$a = 6 + (0.02)x \quad m/s^2 \longrightarrow ①$$

$$a \cdot dx = v \cdot dv$$

$$\int_0^{2000\,m} [6 + (0.02)x]\, dx = \int_0^{v} v \cdot dv$$

$V = 322.49$ m/s
$t = 19.27$ s

$$\left[6 \cdot x + (0.02)\frac{x^2}{2}\right]_0^{2000} = \frac{v^2}{2}$$

$$\therefore v^2 = 2\left[(6 \times 2000) + 40{,}000\right] = (52{,}000 \times 2)$$

Ans: $\boxed{v = 322.49\ m/s}$

Now,
$$v^2 = 12 \cdot x + (0.02)x^2$$

$$v = \left[12 \cdot x + (0.02)x^2\right]^{1/2} \ m/s \longrightarrow ②$$

$$\therefore \left(\frac{dx}{dt}\right) = \left[12 \cdot x + (0.02)x^2\right]^{1/2}$$

$$\int_0^{t} dt = \int_0^{2000} \left[12 \cdot x + (0.02)x^2\right]^{-1/2} \cdot dx$$

(3) RCH/12.25/pg. 632

$\boxed{a = -2v}$ m/s^2 $\longrightarrow$ (1)

At $t = 0$, $v = 20\ m/s$ & $x = 0$

$dv = a \cdot dt$

$dv = -2v \cdot dt$

$$\int \frac{dv}{v} = -2 \cdot \int dt$$

$\log_e v = -2t + C_1$

$\ln 20 = 0 + C_1$ $\quad C_1 = 2.995$

$\therefore \log_e v = -2t + 2.995$

$\therefore e^{(2.995 - 2t)} = v$

$\boxed{v = e^{(2.995 - 2t)}}$ m/s $\longrightarrow$ (2)

$$\int dx = \int e^{(2.995 - 2t)} dt$$

$$x = e^{(2.995 - 2t)} \times \frac{1}{(-2)} + C_2$$

$\therefore 0 = -(0.5)(20) + C_2$ $\quad \therefore C_2 = 10$

$\boxed{x = 10 - (0.5)\left[e^{(2.995 - 2t)}\right]}$ $\longrightarrow$ (3)

4 RCH/ P 12.1 (+)/ pg. [illegible] :

$$a = s = x \ m/s^2 \longrightarrow (1)$$

$$a \cdot dx = v \cdot dv$$

$$\therefore \int x \cdot dx = \int v \cdot dv$$

$$\frac{x^2}{2} + C_1 = \frac{v^2}{2}$$

when $x = 4m, \ v = 0$

$$8 + C_1 = 0$$

$$C_1 = -8$$

$$\therefore \frac{x^2}{2} - 8 = \frac{v^2}{2}$$

$$\therefore v^2 = x^2 - 16 \longrightarrow (2)$$

At $x = 5\,m$,

$$v^2 = 25 - 16 = 09$$

$$\therefore v = 3 \ m/s$$

(5) RCH/12.13/pg.631 :

$$\boxed{a = 3x - 6x^2} \ m/s^2 \longrightarrow (1)$$

At $t=0$, $v=0$, $s=x=0$

$$a \cdot dx = v \cdot dv$$

$$\int (3x - 6x^2)\,dx = \int v \cdot dv$$

$$\left(\frac{3x^2}{2} - 2x^3\right) = \left(\frac{v^2}{2} + c_1\right)$$

$\therefore c_1 = 0$

$$\therefore v^2 = 3x^2 - 4x^3$$

$$\boxed{v = \sqrt{3x^2 - 4x^3}} \longrightarrow (2)$$

a) At $x = 3m$, $v = \sqrt{27 - 324}$

b) When $v=0$, $3x^2 - 4x^3 = 0$

$$x^2 (3 - 4x) = 0$$

$\therefore x = 0$ or $3 - 4x = 0$

$$x = 3/4 = 0.75\,m$$

$$\boxed{x = 0.75\,m}$$

c) For v_{max}, $\frac{dv}{dx} = 0 = \frac{1}{2}(3x^2 - 4x^3)^{-1/2} \cdot (6x - 12x^2)$

$$\therefore \left[\frac{3x - 6x^2}{\sqrt{3x^2 - 4x^3}}\right] = 0 \qquad \therefore 3x - 6x^2 = 0$$

$\therefore x = 0$ or $x = 0.5$

$$\therefore \boxed{x = 0.5\,m}$$

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(6) RCH/12.26/pg.632

$$a = -(0.02)\times v^{1.75} \ m/s^2 \longrightarrow (1)$$

$$a\cdot dx = v\cdot dv$$

$$-(0.02)\cdot v^{1.75}\, dx = v\cdot dv$$

$$-(0.02)\int dx = \int v^{-0.75} dv$$

$$-(0.02)x = \left(\frac{v^{0.25}}{0.25}\right) + C_1$$

$$-(0.02)\,x = 4\cdot v^{1/4} + C_1$$

$$x = -(200)\cdot v^{1/4} + C_1$$

At $x = 0$, $v = 20 \ m/s$

$$0 = -(200)(2.115) + C_1$$

$$C_1 = 422.95$$

$$\boxed{x = -(200)\cdot v^{1/4} + 422.95}$$

Ans: (a) when, $v = 28 \ m/s$

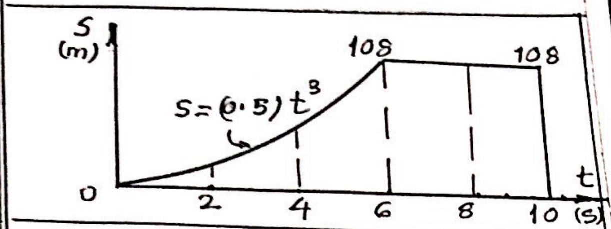
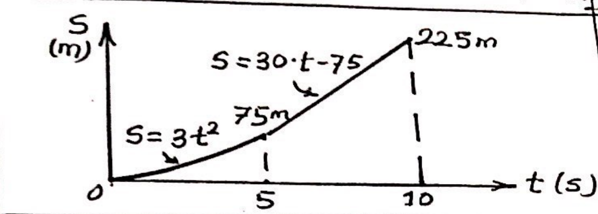
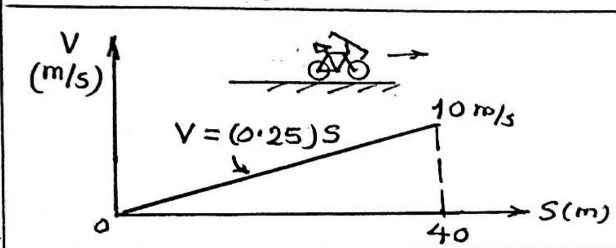
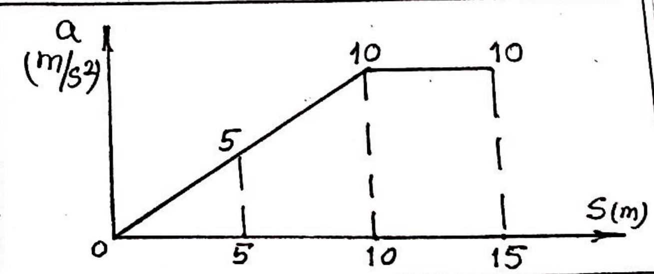
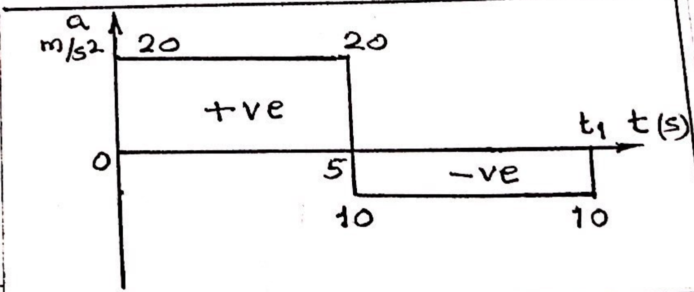
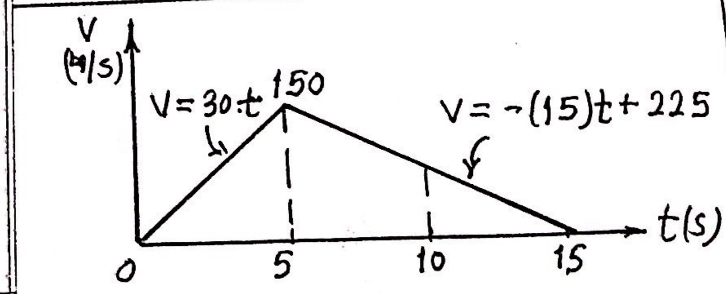
$$x = -37.115 \ m/s$$

(b) when $x = 200 \ m$

$$v = 1.544 \ m/s$$

$$a = -0.043 \ m/s^2$$

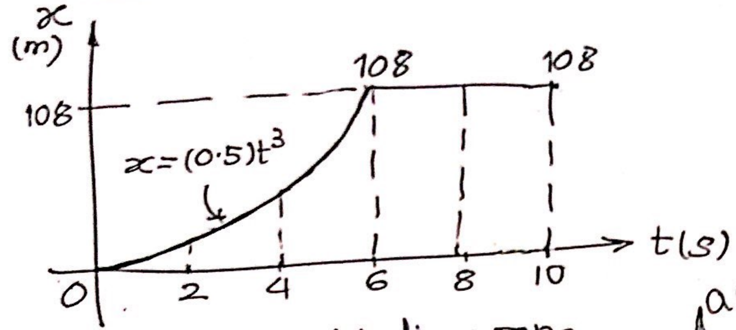
Lecture No: **Motion Curves**

1	The particle travels along a straight track such that its position is described by the s-t graph. Construct the v-t graph for the same time interval.	
2	The sports car travels along a straight road such that its position is described by the graph. Construct the v-t and a-t graphs for the time interval $0 \le t \le 10s$.	
3	A bicycle travels along a straight road where its velocity is described by the v-s graph. Construct the a-s graph for the same interval.	
4	The sports car travels along a straight road such that its acceleration is described by the graph. Construct the v-s graph for the same interval and specify the velocity of the car when s=10m and s=15m.	
5	The dragster starts from rest and has acceleration described by the graph. Construct the v-t graph for the time interval $0 \le t \le t^1$, where t^1 is the time for the car to come to rest.	
6	The dragster starts from rest and has a velocity described by the graph. Construct the s-t graph for the time interval $0 \le t \le 15s$. Also, determine the total distance traveled during this time interval.	

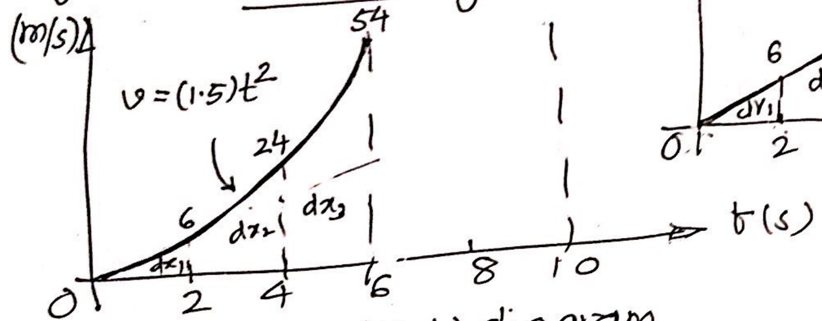
Lecture No (4)

P1

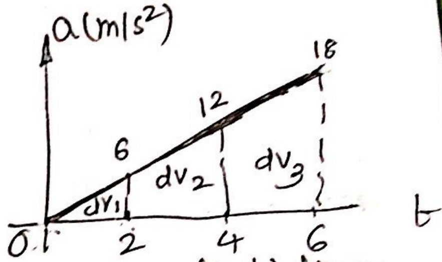
(1) RCH/F 12.9/pg.641



(x-t) diagram



(v-t) diagram



(a-t) diagram

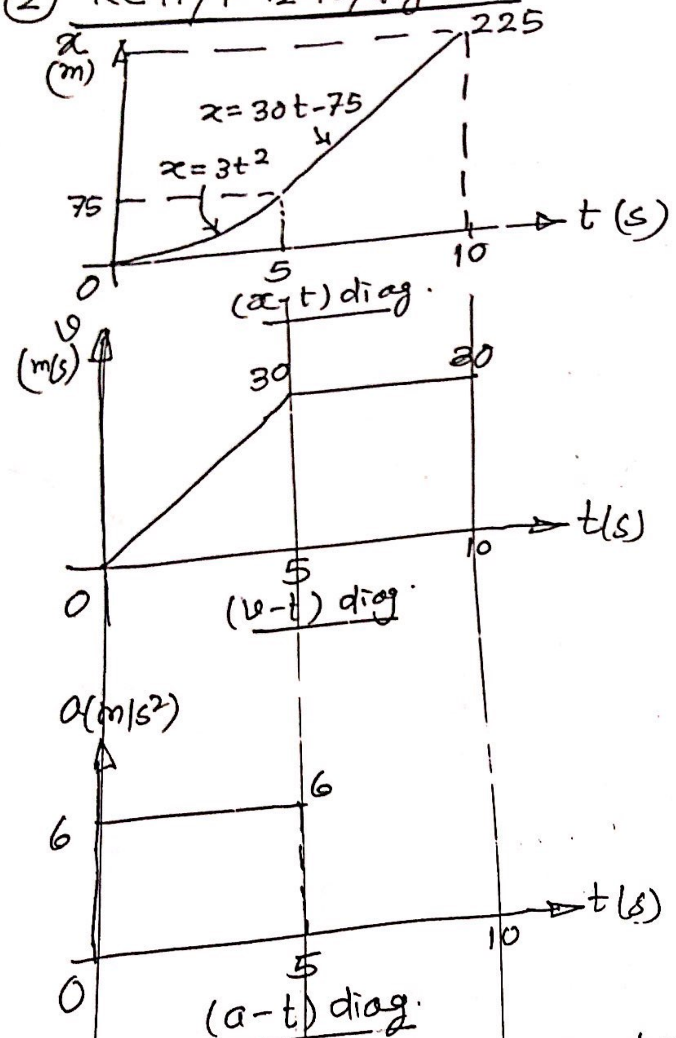
$$x = (0.5)t^3 \longrightarrow (1)$$

$$v = \frac{dx}{dt} = (1.5)t^2 \longrightarrow (2)$$

$$a = \frac{dv}{dt} = 3t \longrightarrow (3)$$

Time t sec.	a m/s^2	dv m/s	v m/s	dx m	x m
0	0	—	0	—	0
2	6	6	6	4	4
4	12	18	24	28	32
6	18	30	54	76	108
8			0	0	108
10			0	0	108

② RCH/F 12.10/pg. 641 :



I) Motion from t = 0 to t = 5 s

$x = 3t^2$ m

$v = 6t$ m/s

$a = 6$ m/s²

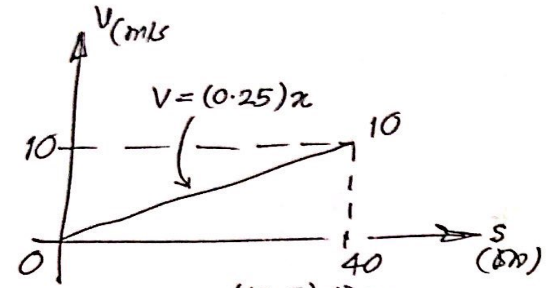
II) Motion from t = 5 to t = 10 s

$x = (30t - 75)$ m

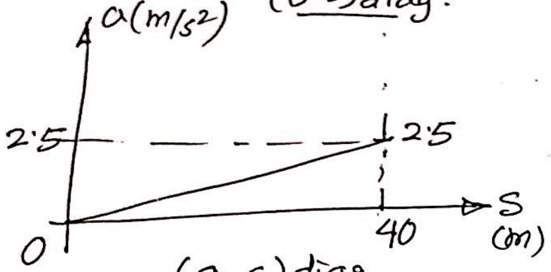
$v = 30$ m/s

$a = 0$ m/s²

③ RCH/F12.11/pg. 641.



(v-s) diag.



(a-s) diag.

$$V = (0.25)x$$

$$a \cdot dx = v \cdot dv$$

$$(0.25)x \cdot d\cancel{x} = v \cdot dv$$

$$a = v \cdot \frac{dv}{dx}$$

$$\therefore a = (0.25)x \cdot (0.25)$$

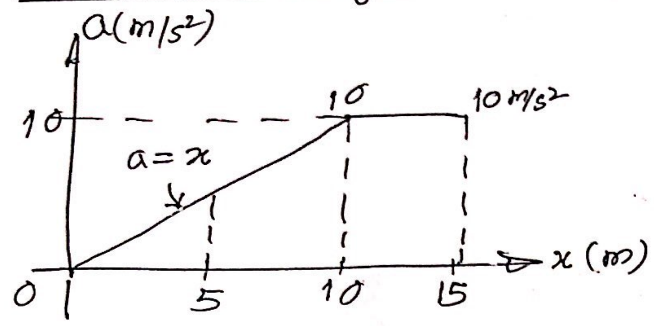
$$\therefore a = (0.0625)x$$

when $x = 40 m$

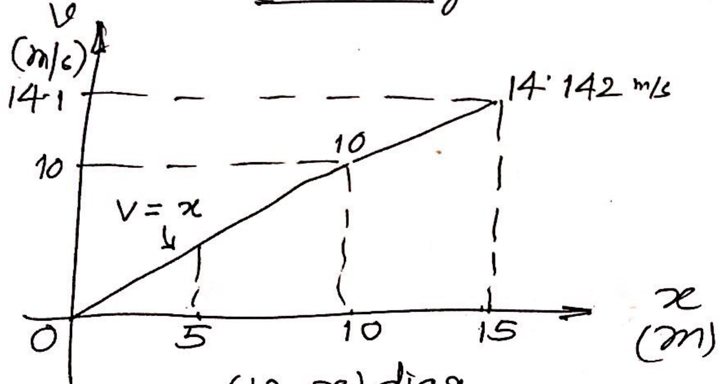
$$a = 0.0625 \times 40$$

$$= 2.5 \ m/s^2$$

④ RCH/F12.12/pg.641.



(a-x) diag.



(v-x) diag.

I) Motion from 0 to 10 s:

$$a = x$$

$$a \cdot dx = v \cdot dv$$

$$\int x \cdot dx = \int v \cdot dv$$

$$\frac{V^2}{2} = \frac{x^2}{2} + c_1$$

At $x = 0$, $V = 0 \therefore c_1 = 0$

$$\therefore V = x$$

II) Motion From 10 to 15 s:

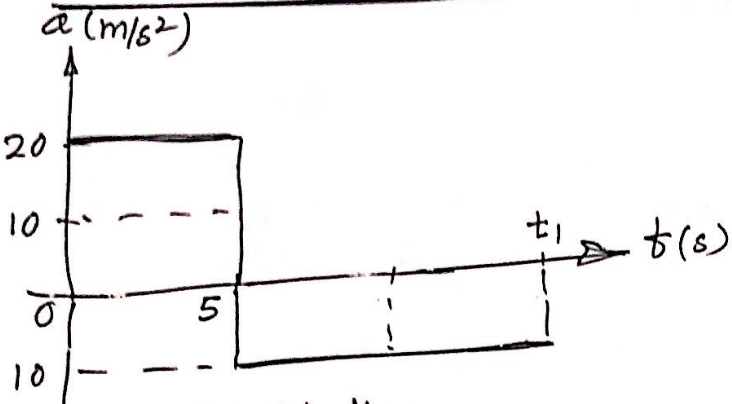
$$V^2 = U^2 + (2a \cdot s)$$

$$= 10^2 + 2(10) \times 5$$

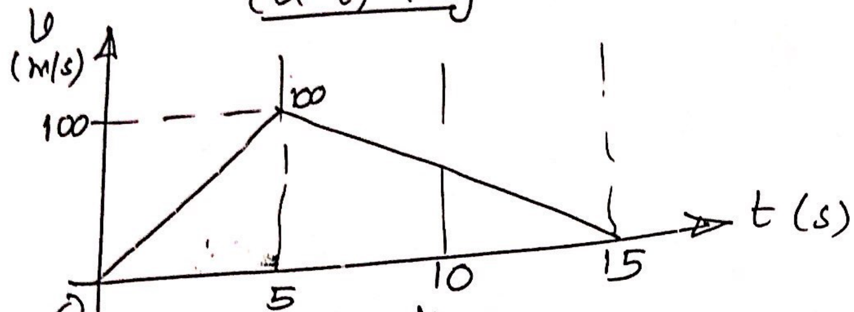
$$= 200$$

$$V = 14.142 \text{ m/s}$$

⑤ RCH/F 12.13/pg. 641 :



(a-t) diag.



(v-t) diag.

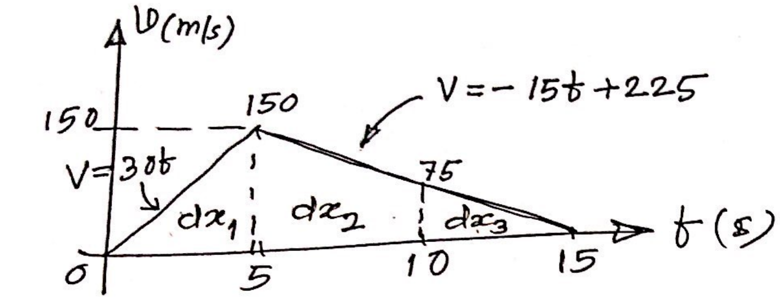
I) Motion from 0 to 5 sec:

$$V = u + at$$
$$= 0 + (20 \times 5) = 100 \text{ m/s}$$

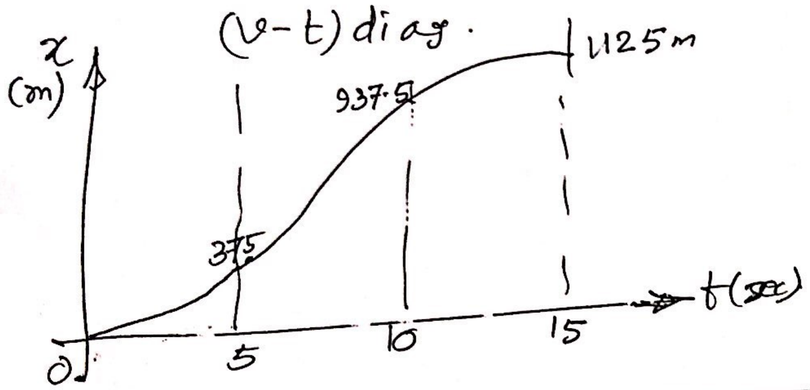
II) Motion from 5 to t_1 sec. :

$$V = u + at$$
$$0 = 100 - 10 \times t$$
$$\underbrace{t = 10 \text{ sec.}}_{\text{duration}} \quad \therefore t_1 = 15 \text{ sec.}$$

(6) RCH / F 12.14 / pg. 641 :



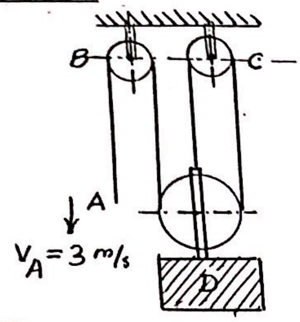
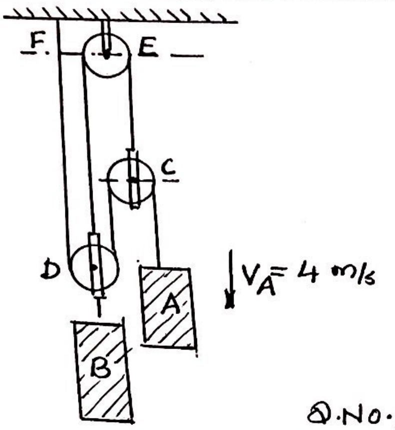
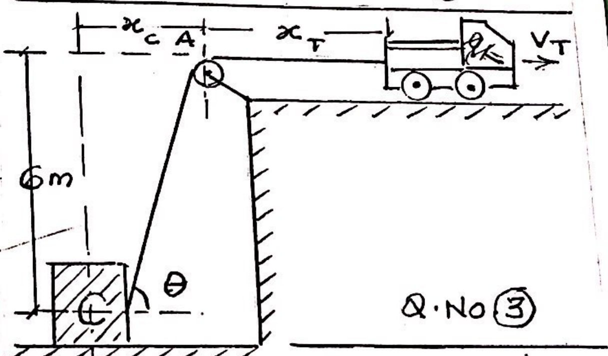
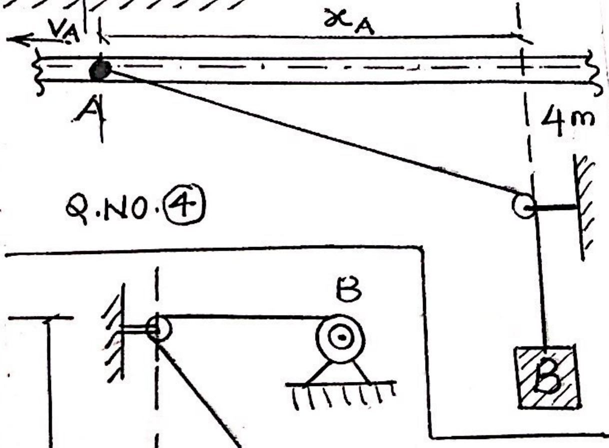
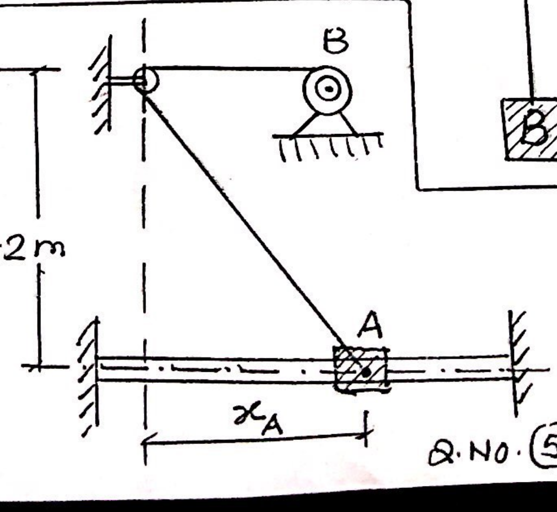
(v-t) diag.



t (s)	v (m/s)	dx (m)	x (m)
0	0	–	0
5	150	375	375
10	75	562.5	937.5
15	0	187.5	1125

Total distance traveled } $\boxed{1125\,m}$

Lecture No: **Dependant Motion**

1	Determine the velocity of block D if end A of the rope is pulled down with a speed of v_a = 3 m/s **Ans: V_D = 1 m/s upwards**	Q.No. 1
2	Determine the velocity of cylinder B if cylinder A moves downward with a speed of v_a = 4 m/s **Ans: V_B = 1 m/s upwards**	Q.No. 2
3	If the truck travels at a constant speed of v_T = 1.8 m/s, determine the speed of the crate for any angle θ of the rope. The rope has a length of 30 m and the passes over a pulley of negligible size at A. **Hint:** relate the coordinates x_T & x_C to the length of the rope & take the time derivative. Then substitute the trigonometric relation between θ & x_C. **Ans: V_C = -1.8 sec θ**	Q.No 3
4	The roller at A is moving with a velocity of v_A = 4 m/s & has as an acceleration of a_A = 2 m/s^2 when X_A = 3 m. Determine the velocity & acceleration of block B at this instant. **Ans: V_B = 2.4 m/s up; a_B =3.248 m/s^2 up**	Q.NO. 4
5	The motor draws in the cord at B with an acceleration of a_B =2 m/s^2. When s_A= 1.5 M, v_B = 6 m/s. Determine the velocity & acceleration of the collar at its instant. **Ans: V_A = 10 m/s Left ; a_a =46 m/s^2 Left**	Q.NO. 5

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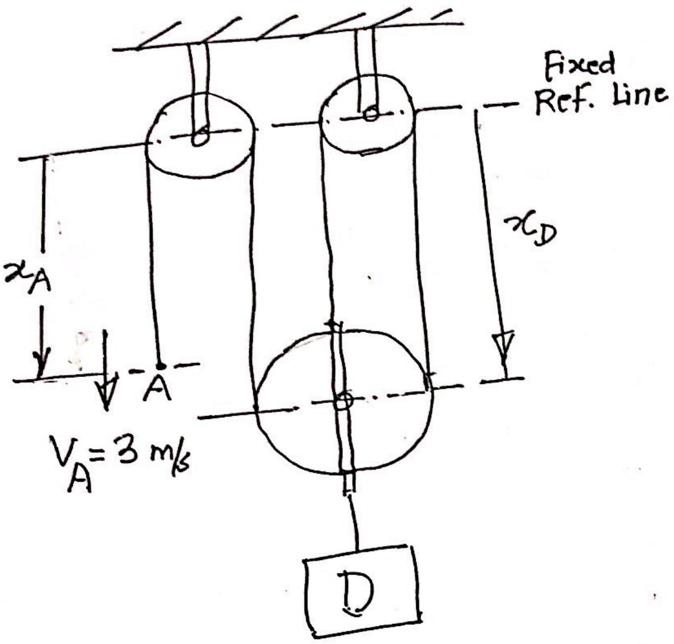
Lecture No. (5)

(1) RCH/F 12.39/pg. 710 :



$$x_A + 3 \cdot x_D = \text{constant}$$

$$\therefore \; V_A + 3 \cdot V_D = 0$$

$$\therefore \; a_A + 3 \cdot a_D = 0$$

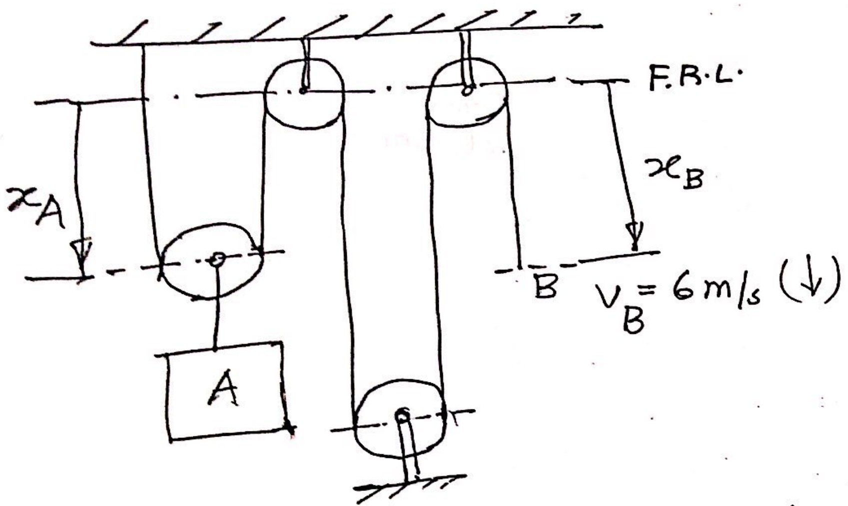
when $V_A = 3$ m/s

$$3 + 3 \cdot V_D = 0$$

$$V_D = -1 \text{ m/s}$$

$$\therefore \; V_D = 1 \text{ m/s } (\uparrow)$$

(2) RCH / F 12.40 / pg. 710 :



$$2 \cdot x_A + x_B = \text{constant}$$

$$\therefore 2 \cdot V_A + V_B = 0$$

$$\therefore 2 \cdot a_A + a_B = 0$$

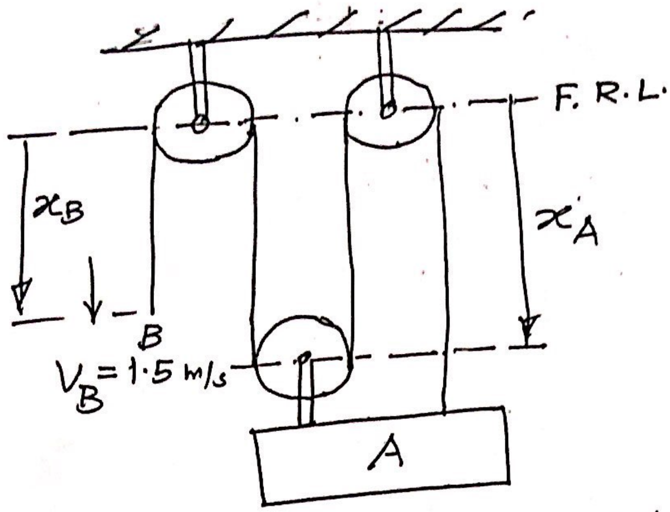
when, $V_B = 6\ m/s$

$$2 \cdot V_A + 6 = 0$$

$$V_A = -3\ m/s$$

$$\therefore V_A = 3\ m/s\ (\uparrow)$$

(3) RCH / F12.41 / pg. 710



$$x_B + 3 \cdot x_A = \text{constant}$$

$$\therefore V_B + 3 \cdot V_A = 0$$

$$\therefore a_B + 3 \cdot a_A = 0$$

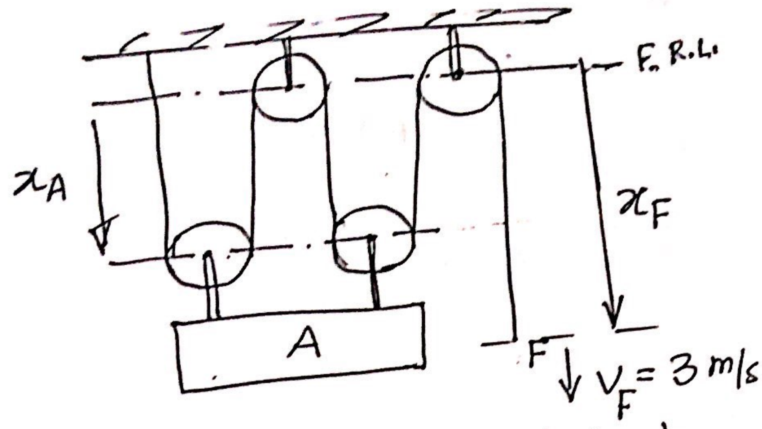
when, $V_B = 1.5$ m/s ($\downarrow$)

$$1.5 + 3 \cdot V_A = 0$$

$$\therefore V_A = -\frac{1.5}{3} = -0.5 \text{ m/s}$$

$$\therefore V_A = 0.5 \text{ m/s} (\uparrow)$$

④ RCH/F 12.42/pg.710



$$4 \cdot x_A + x_F = \text{constant}$$

$$\therefore \; 4 \cdot V_A + V_F = 0$$

$$\therefore \; 4 \cdot a_A + a_F = 0$$

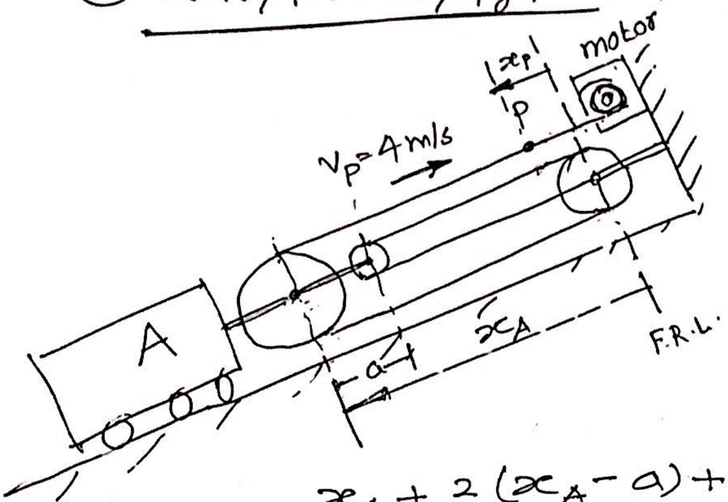
when $V_F = 3\,m/s$

$$4 \cdot V_A + 3 = 0$$

$$V_A = -\frac{3}{4} = -0.75\,m/s$$

$$\therefore \; V_A = 0.75\,m/s \; (\uparrow)$$

⑤ RCH / F 12·43 / pg. 710 :



$$x_A + 2(x_A - a) + (x_A - x_p) = \text{constant}$$

$$x_A + 2 \cdot x_A - 2 \cdot a + x_A - x_p = \text{Const.}$$

$$x_A - x_p = (\text{Const.} + 2a)$$

$$\therefore \quad V_A - V_p = 0$$

$$\therefore \quad a_A - a_p = 0$$

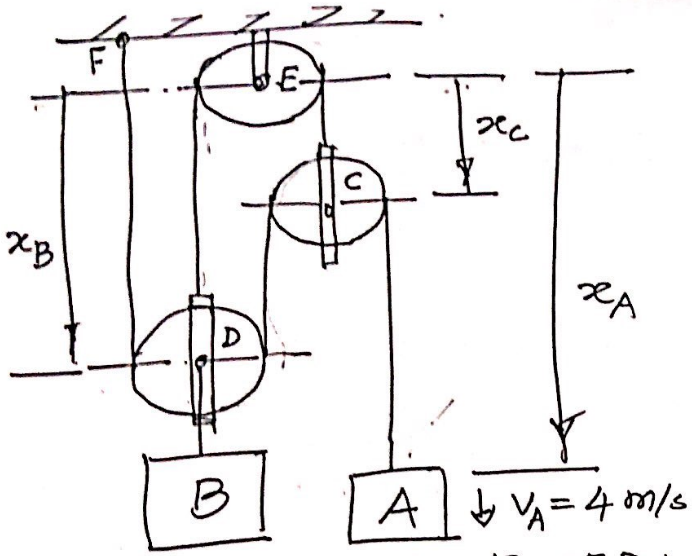
when, $V_p = -4$ m/s

$$V_A + 4 = 0$$

$$V_A = -1 \text{ m/s}$$

$$\therefore \quad V_A = 1 \text{ m/s (up the plane)}$$

⑥ RCH/F 12.44/pg. 710



For the rope betn CED:

$$x_B + x_C = \text{constant} \rightarrow (i)$$

For the rope betn ACDF:

$$(x_A - x_C) + (x_B - x_C) + x_B = \text{const.}$$

$$\therefore x_A + 2x_B - 2x_C = \text{const.} \quad (ii)$$

$$\therefore V_B + V_C = 0 \rightarrow (iii)$$

$$V_A + 2V_B - 2V_C = 0 \rightarrow (iv)$$

Eliminating V_C,

$$V_A + 4V_B = 0$$

$$4 + 4 \cdot V_B = 0$$

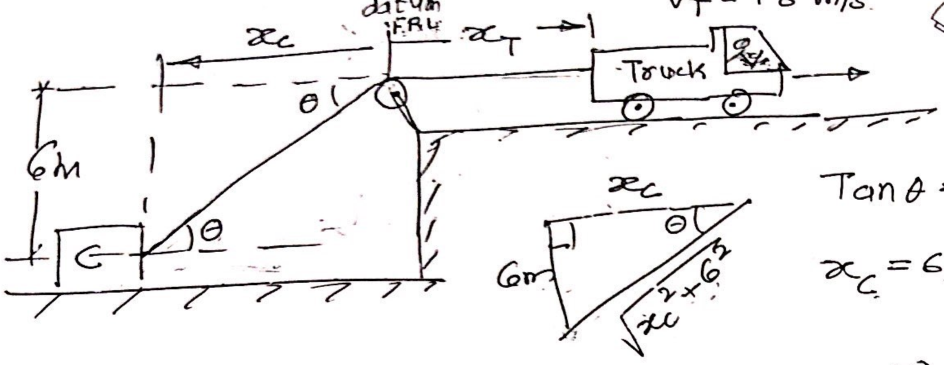
$$V_B = -1 \text{ m/s}$$

$$\therefore V_B = 1 \text{ m/s } (\uparrow)$$

(7) RCH / 12.210 / pg. 714:

$a_T = 0$

$V_T = 1.8 \text{ m/s}$



$$\tan\theta = \frac{6}{x_c}$$

$$x_c = 6\cdot\cot\theta$$

$$\sqrt{x_c^2+36} + x_T = 30 \text{ m} \longrightarrow \text{(i)}$$

$$\frac{\cancel{2}\cdot x_c\cdot \dot{x}_c}{\cancel{2}\sqrt{x_c^2+36}} + \dot{x}_T = 0$$

$$\therefore\ x_c\,\dot{x}_c + (\dot{x}_T)\left(\sqrt{x_c^2+36}\right) = 0 \longrightarrow \text{(ii)}$$

$$V_T = \dot{x}_T = 1.8 \text{ m/s}$$

$$(6\cdot\cot\theta)(V_c) + \left(\sqrt{36\cdot\cot^2\theta+36}\right)(V_T) = 0$$

$$\therefore\ (\cot\theta)(V_c) + \left(\sqrt{1+\cot^2\theta}\right)(V_T) = 0$$

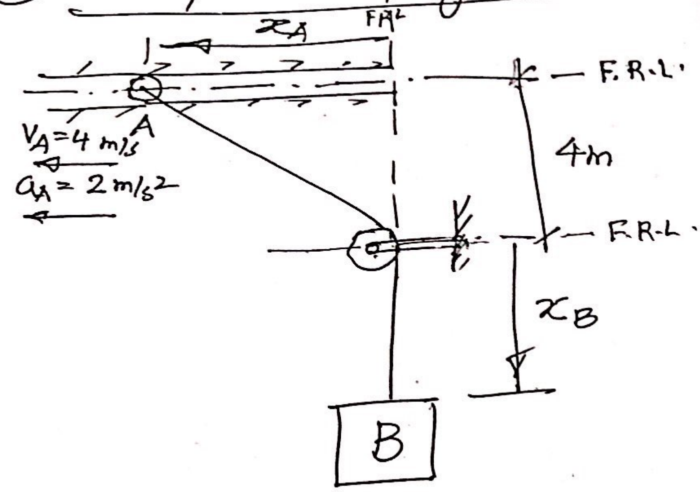
$$\therefore\ (\cot\theta)(V_c) + (\operatorname{cosec}\theta)(V_T) = 0$$

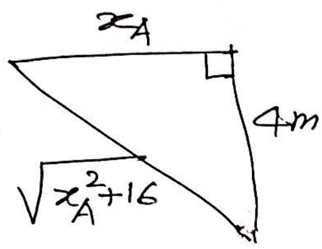
$$(V_c)\left(\frac{\cos\theta}{\cancel{\sin\theta}}\right) = -(V_T)\left(\frac{1}{\cancel{\sin\theta}}\right)$$

$$V_c = -(V_T)(\sec\theta)$$

$$\boxed{V_c = -(1.8)(\sec\theta)}$$

8 RCH/12.212/pg.714 :





$$\sqrt{x_A^2+16} + x_B = \text{constant} \longrightarrow (i)$$

$$\frac{\cancel{2}\cdot x_A \cdot \dot{x}_A}{\cancel{2}\cdot\sqrt{x_A^2+16}} + \dot{x}_B = 0$$

$$x_A \cdot \dot{x}_A + \left(\sqrt{x_A^2+16}\right)(\dot{x}_B) = 0 \longrightarrow (ii)$$

$$\therefore x_A \cdot \ddot{x}_A + \dot{x}_A^2 + \left[\frac{\cancel{2}\cdot x_A \cdot \dot{x}_A \cdot \dot{x}_B}{\cancel{2}\sqrt{x_A^2+16}}\right] + \left(\sqrt{x_A^2+16}\right)(\ddot{x}_B) = 0 \longrightarrow (iii)$$

When $x_A = 3m$

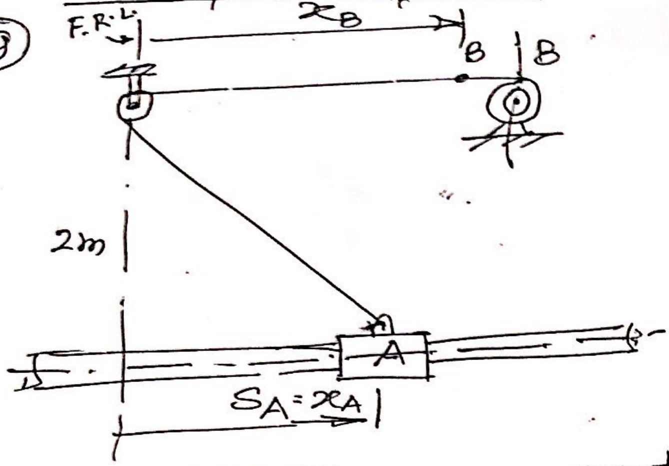
Eqⁿ(ii) gives, $(3 \times 4) + 5 \cdot V_B = 0 \quad \therefore V_B = 2.4$ m/s (↑)

Eqⁿ(iii) gives, $(3\times2) + (16) + \left(\frac{3\times4\times-2.4}{5}\right) + 5\cdot a_B = 0$

$$6 + 16 \cancel{+} -5.76 = -5 \cdot a_B$$

$a_B = \cancel{5.552}$ 3.248 m/s² (↑)

⑨



$$\sqrt{x_A^2+4} + x_B = \text{constant} \longrightarrow (i)$$

$$\frac{\cancel{2}\cdot x_A \cdot \dot{x}_A}{\cancel{2}\cdot\sqrt{x_A^2+4}} + \dot{x}_B = 0 \longrightarrow (ii)$$

$$x_A \cdot \dot{x}_A + (\dot{x}_B)(\sqrt{x_A^2+4}) = 0 \longrightarrow (ii)$$

$$x_A \ddot{x}_A + \dot{x}_A^2 + \left[\frac{\cancel{2} x_A \dot{x}_A \dot{x}_B}{\cancel{2}\sqrt{x_A^2+4}}\right] + (\ddot{x}_B)(\sqrt{x_A^2+4}) = 0 \longrightarrow (iii)$$

Given, $a_B = \ddot{x}_B = 2\ m/s^2$

$x_A = 1.5\ m$, $V_B = 6\ m/s = \dot{x}_B$

$\sqrt{x_A^2+4} = 2.5$

Eq^n (ii) gives, $(1.5)\cdot V_A + (6\times 2.5) = 0$

$\therefore V_A = -10\ m/s$ $\therefore V_A = 10\ m/s\ (\leftarrow)$

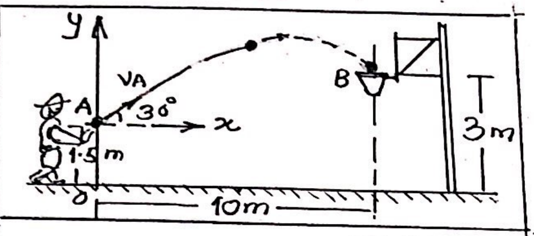
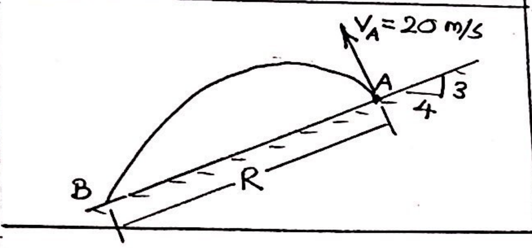
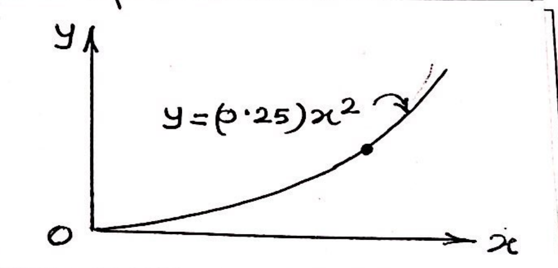
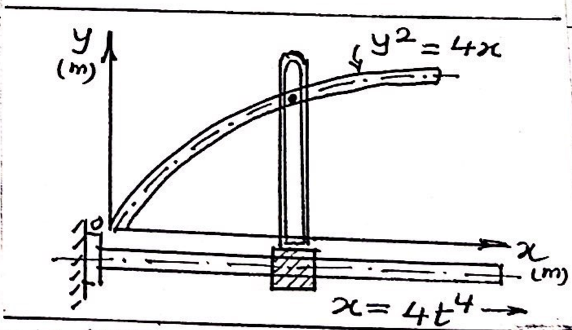
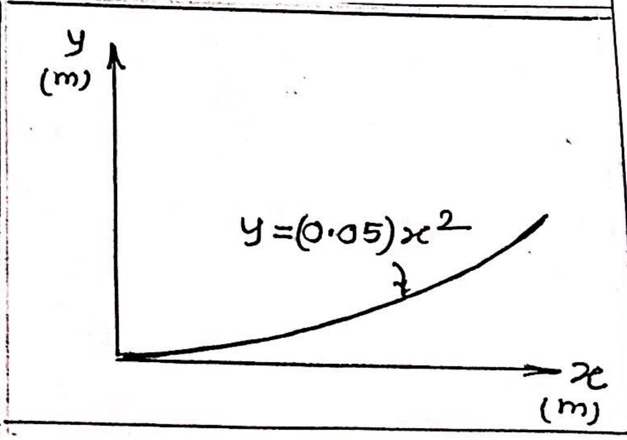
Eq^n (iii) gives, $(1.5)(a_A) + (100) + \frac{(1.5)(-10)(6)}{(2.5)}$

$+ (2\times 2.5) = 0$

$(1.5)\cdot a_A + 69 = 0$ $\quad \therefore a_A = 46\ m/s^2\ (\leftarrow)$

Lecture No: ..

Curvilinear Motion , Rectangular Co-ordinate System

1	Determine the speed at which the basketball at A must be thrown at the angle of 30^0 so that it makes it to the basket at B. **Ans: u= 12.373 m**	
2	Water is sprayed at an angle of 90^0 from the slope at 20m/s. Determine the range R. **Ans: R=33.864 m**	
3	A particle is traveling along the parabolic path $y=0.25x^2$.If x=8m, V_x=8m/s, a_x=4 m/s^2 when t=2s, determine the magnitude of particle's velocity and acceleration at this instant. **Ans: v =32.985 m/s; a =48.2 m/s^2**	
4	A particle constraint to travel along the path. If x= $(4t^4)$m, where t is in seconds, determine the magnitude of particle's velocity and acceleration when t=0.5seconds. **Ans: v =4.472 m/s; a =14.422 m/s^2**	
5	If x & y components of a particle's velocity are V_x=(32t) m/s, V_y=8m/s, determine the equation of the path y=f(x), if x=0 & y=0 when t=0. **Ans: y^2=4x**	
6	The box slides down the slope described by the equation $y=(0.05x^2)$m, where x is in meters. If the box has x component of velocity and acceleration of V_x= -3m/s and a_x= -1.5m/s^2 at x=5m, determine the y components of the velocity and the acceleration of the box at this instant. **Ans: V_y= -1.5m/s; a_y= 0.15m/s^2**	

Lecture No. 7

5 (1) RCH/F 12.15/pg. 661:

$$V_x = (32)t = \frac{dx}{dt}$$

$$\int_0^x dx = \int_0^t (32)t \cdot dt$$

$$x = 16 \cdot t^2 \longrightarrow (1)$$

$$V_y = 8 \text{ m/s} = \frac{dy}{dt}$$

$$\int_0^y dy = \int_0^t 8 \cdot dt$$

$$y = 8t \longrightarrow (2)$$

$$\therefore t = y/8$$

$$\therefore \quad x = 16 \times \frac{y^2}{64} = \frac{y^2}{4}$$

Ans: $y^2 = 4 \cdot x$ or $\boxed{y = 2\sqrt{x}}$

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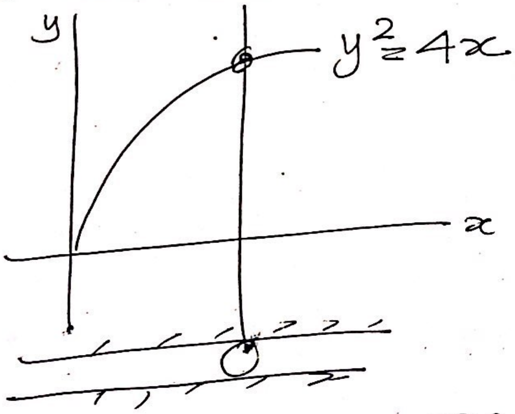
4 (2) RCH/F 12.17/pg. 661:



$$x = 4 \cdot t^4 \text{ m}$$

$$\dot{x} = V_x = 16t^3 \text{ m/s}$$

$$\ddot{x} = a_x = 48t^2 \text{ m/s}^2$$

$$y = 2\sqrt{x} = 4t^2$$

$$\dot{y} = 8t = V_y \text{ m/s}$$

$$\ddot{y} = a_y = 8 \text{ m/s}^2$$

At $t = 0.5$ sec.

$V_x = 2$ m/s, $V_y = 4$ m/s

$\bar{V} = 2\hat{i} + 4\hat{j}$ m/s

$V = 4.472$ m/s, 63.43°

$a_x = 12$ m/s², $a_y = 8$ m/s²

$\bar{a} = 12\hat{i} + 8\hat{j}$ m/s²

$a = 14.422$ m/s, 33.69°

3 (3) RCH/F12.19/pg.661:

$y = (0.25)x^2$

At $t = 2$ s, $x = 8$ m

$V_x = \dot{x} = 8$ m/s

$a_x = \ddot{x} = 4$ m/s^2

As, $y = (0.25)x^2$

Differenting w.r.t. x, we get,

$\dot{y} = (0.5)x\dot{x} = V_y$

$\ddot{y} = (0.5)\dot{x}^2 + (0.5)x\ddot{x}$

At $t = 2$ sec.: $V_x = 8$ m/s

$V_y = 0.5 \times 8 \times 8 = 32$ m/s

$V = \sqrt{8^2 + 32^2} = 32.985$ m/s

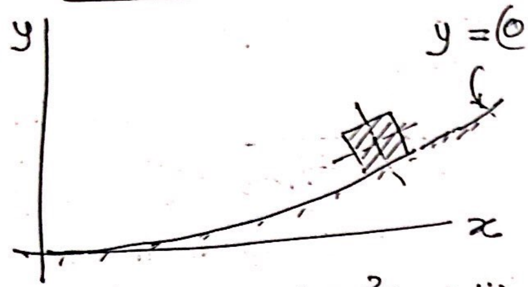
$a_x = 4$ m/s^2

$a_y = (0.5 \times 64) + (0.5 \times 8 \times 4)$

$= 32 + 16 = 48$ m/s^2

$a = \sqrt{4^2 + 48^2} = 48.2$ m/s^2

6. (4) RCH / F12.20 / pg. 661



$y = (0.05)x^2$

At $x = 5\,m$

$V_x = \dot{x} = -3\ m/s$

$V_y = ?$

$a_x = -1.5\ m/s^2$

$a_y = ?$

$y = (0.05)x^2 \longrightarrow (i)$

$\therefore \dot{y} = (0.10)x \cdot \dot{x} \longrightarrow (ii)$

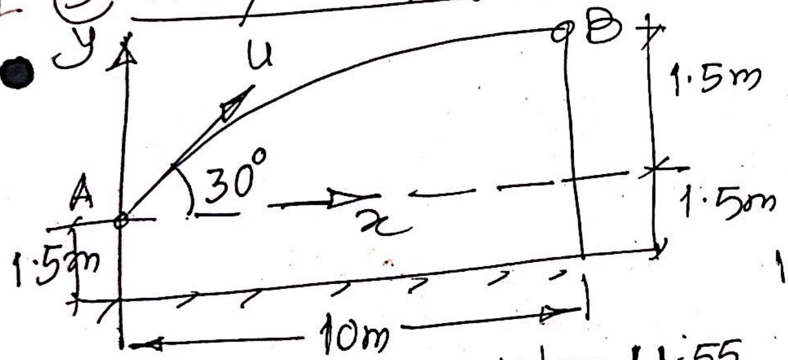
$\ddot{y} = (0.10)\dot{x}^2 + (0.1)x\ddot{x} \longrightarrow (iii)$

$\therefore$ At $x = 5\,m$, $V_y = \dot{y} = 0.1 \times 5 \times (-3) = -1.5\ m/s$

i.e. $1.5\ m/s\ (\downarrow)$

$a_y = (0.10)(-3)^2 + (0.1)(5)(-1.5) = 0.15\ m/s^2\ (\uparrow)$

1. (5) RCH / F12.23 / pg. 662 :



$B \equiv (10\,m, 1.5\,m)$

$x = (u\cos\alpha)t$

$10 = (u\cos 30^\circ)t \rightarrow (1)$

$y = (u\sin\alpha)t - \frac{1}{2}gt^2$

$1.5 = (u\sin 30^\circ)t - (4.905) \rightarrow (2)$

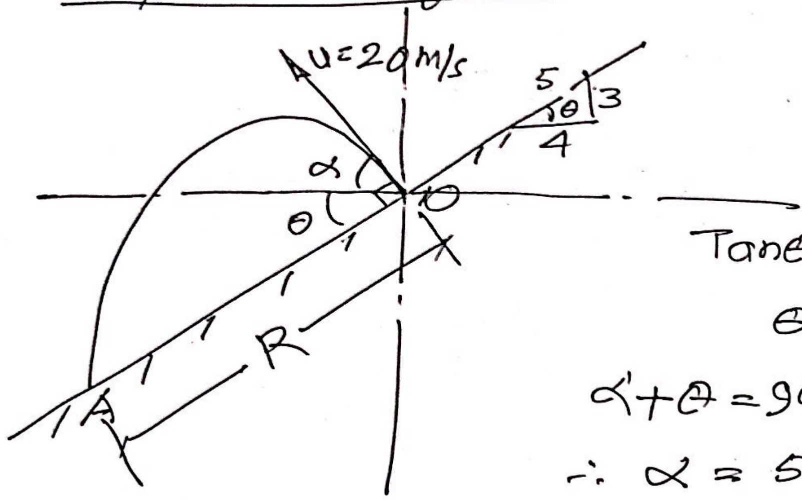
$ut = 11.55 \quad \therefore u = \left(\frac{11.55}{t}\right)$

$y = \left(\frac{11.55}{t} \times 0.5 \times t\right) - (4.905)t^2$

$y = (5.773) - (4.905)t^2 = 1.5$

$t = 0.9334\,s$, $u = 12.373\ m/s$

2 6 RCH/ F 12.24/ pg.662 :



$\tan\theta = \frac{3}{4}$

$\theta = 36.86°$

$\alpha + \theta = 90°$

$\therefore \alpha = 53.13°$

At pt. A, $x = -R\cos\theta = -(0.8)R$

Eqns of projectile $y = -R\sin\theta = -(0.6)R$

are,

$x = -(20)(\cos 53.13°)t = -(12.0)\cdot t$ m/s $\rightarrow$ (1)

$y = (20)(\sin 53.13°)t - \frac{1}{2}(9.81)t^2$

$y = (16)t - (4.905)t^2 \longrightarrow$ (2)

At A,

$x = -(0.8)R = -(12)t$

$\therefore \boxed{R = 15.t}$

$y = 16\cdot t - (4.905)t^2 = -(0.6)R$

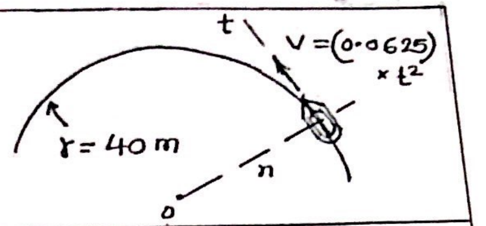
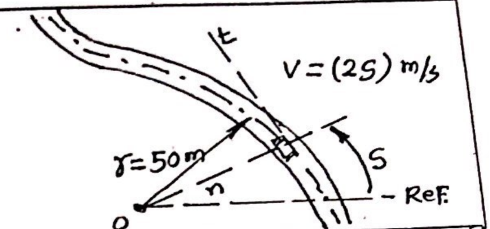
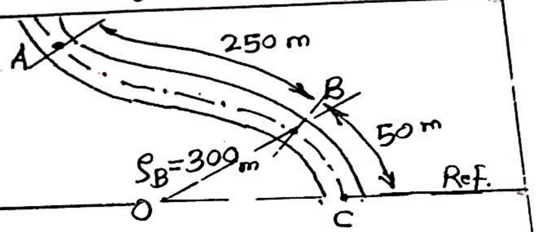
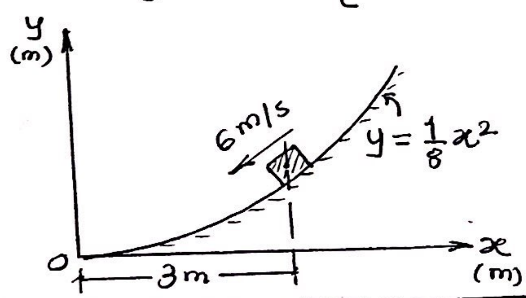
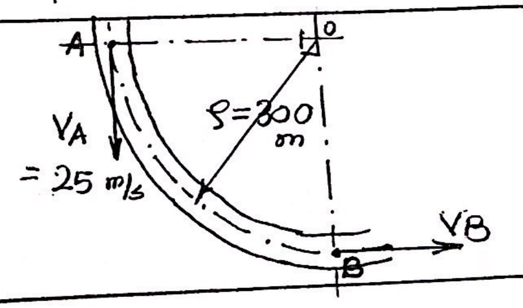
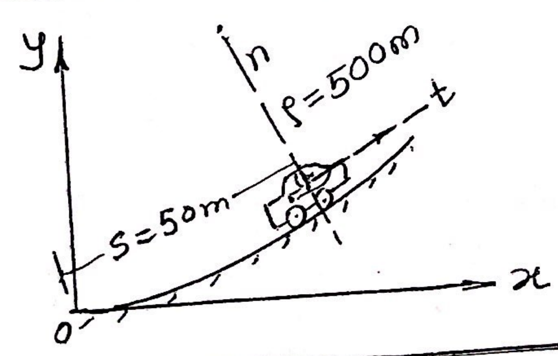
$16\cdot t - (4.905)t^2 = -9\cdot t$

$(4.905)t^2 - (25)t = 0$

$t = 2.257$ sec. 5.0968 sec.

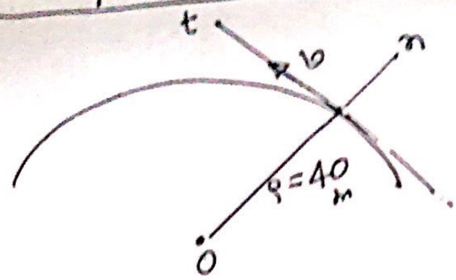
$\therefore R = 33.864$ m 76.452 m

Lecture No: **Path Variables**

1	The boat is traveling along the circular path with a speed of $v=(0.0625t^2)$m/s, where t is in seconds. Determine the magnitude of its acceleration when t=10s. **Ans: a=1.586 m/s^2**	
2	The car is traveling along the road with a speed of v=(2s)m/s, where s is in meters. Determine the magnitude of its acceleration when s=10m. **Ans: a= 40.792 m/s^2**	
3	If the car decelerates uniformly along the curved road from 25m/s at A to 15m/s at C, determine the acceleration of the car at B. **Ans: a$_B$= 1.178 m/s^2**	
4	When x=3m, the crate has a speed of 6m/s which is increasing at 2m/s^2. Determine the direction of the crate's velocity and the magnitude of the crate's acceleration at this instant. **Ans: a= 5.02 m/s^2**	
5	If the motorcycle has a deceleration of $a_t = -(0.001s)$m/s^2 and its speed at position A is 25m/s, determine the magnitude of its acceleration when it passes point B. **Ans: a$_B$ = 1.423 m/s^2**	
6	The car travels up the hill with a speed of v=(0.2s)m/s, where s is in meters, measured from O Determine the magnitude of its acceleration when it is at point s=50m, where ρ=500m. **Ans: a= 2.01 m/s^2**	

Lecture No (8)

(1) RCH/ F 12.27/ pg. 678 :



$V = (0.0625)t^2$ m/s

$\therefore a_t = \frac{dv}{dt} = (0.125)t$ m/s²

$\rho = 40$ m

$\therefore a_n = \frac{V^2}{\rho}$

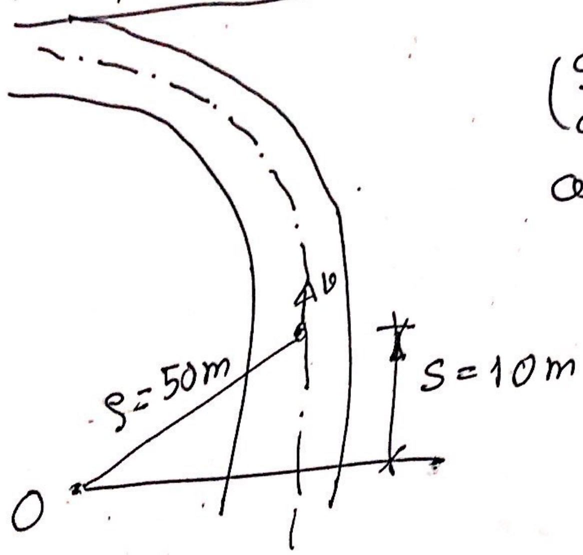
At t = 10 s:

$V = 6.25$ m/s, $a_t = 1.25$ m/s²

$a_n = \frac{6.25^2}{40} = 0.9765$ m/s²

$a = \sqrt{a_t^2 + a_n^2} = 1.586$ m/s²

(2) RCH/ F 12.28/ Pg 678 :



$v = 2s$ m/s

$\left(\frac{dv}{ds}\right) = 2$

$a_t = v.\frac{dv}{ds} = 4s$ m/s²

$a_n = \frac{V^2}{\rho} = \frac{4s^2}{50}$ m/s²

when s = 10m

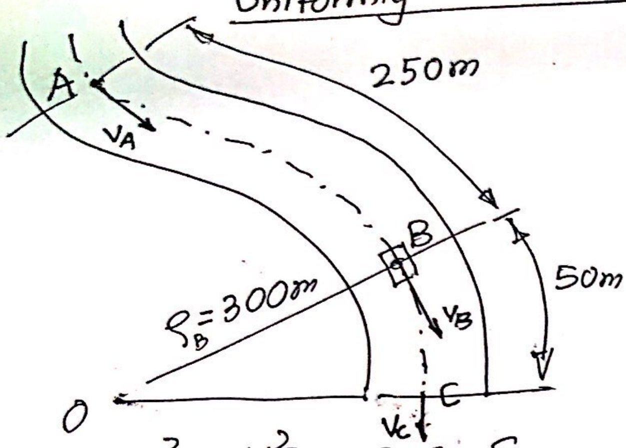
$a_t = 40$ m/s²

$a_n = 8$ m/s²

$a = \sqrt{a_t^2 + a_n^2} = 40.792$ m/s²

③ RCH/F.12.29/pg. 678 :

Uniformly accelerated curvilinear motion



$V_A = 25\ m/s$

$V_C = 15\ m/s$

$S_C = 300\ m$

$V_C^2 = V_A^2 + 2\,a_t\,S_C$

$15^2 = 25^2 + 2 \cdot a_t \cdot 300$

$\therefore \boxed{a_t = -0.666\ m/s^2}$

$V_B^2 = V_A^2 + 2 \cdot a_t \cdot S_B$

$= 25^2 - (2 \times 0.666 \times 250)$

$\therefore V_B = 17.078\ m/s$

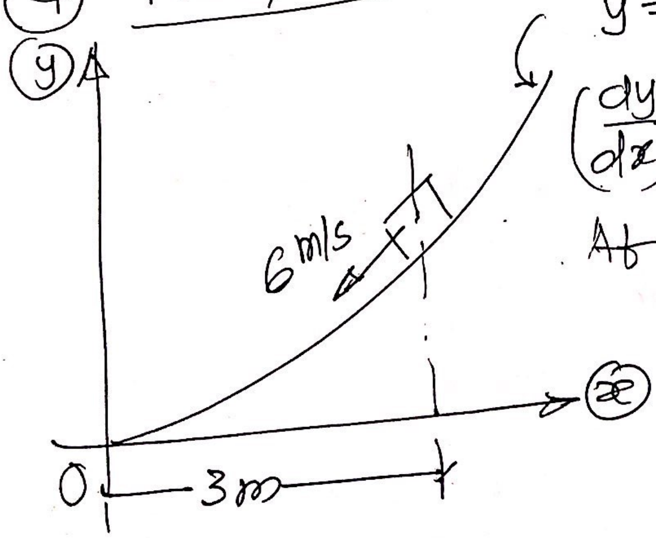
$(a_n)_B = \dfrac{V_B^2}{\rho_B} = \left(\dfrac{17.078^2}{300}\right) = 0.972\ m/s^2$

$\therefore a_B = \sqrt{a_t^2 + a_n^2} = \sqrt{(-0.666)^2 + (0.972)^2}$

$\therefore a_B = 1.178\ m/s^2$

RCH/F 12.30/pg.678 :

④



$y = \frac{1}{8} \cdot x^2$

$\left(\dfrac{dy}{dx}\right) = y' = \frac{1}{4}x, \quad \left(\dfrac{d^2y}{dx^2}\right) = y'' = \frac{1}{4}$

At $x = 3\ m$, $\left(\dfrac{dy}{dx}\right) = \dfrac{3}{4} = \operatorname{Tan}\theta$

$\therefore \theta = 36.87^\circ$

$\rho = \left| \dfrac{\left[1 + (dy/dx)^2\right]^{3/2}}{d^2y/dx^2} \right| = \dfrac{(1 + 0.75^2)^{3/2}}{(1/4)}$

$\therefore \rho = 7.8125\ m$

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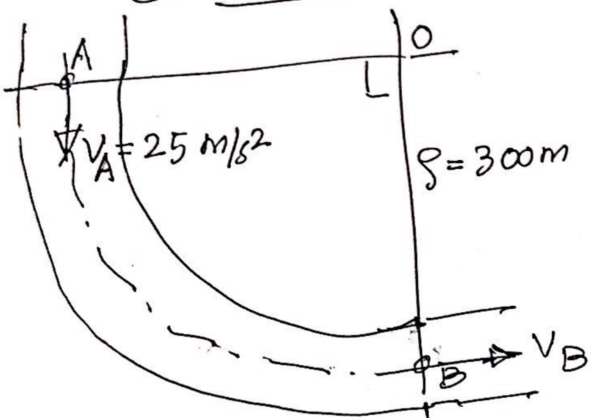
$$a_n = \frac{V^2}{\rho} = \left(\frac{6^2}{7.8125}\right) = 4.608 \ m/s^2$$

$$a_t = 2 \ m/s^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{2^2 + 4.608^2} = 5.02 \ m/s^2$$

⑤ RCH/F 12.31/pg.678 :



$a_t = -(0.001)s \ m/s^2$

At B, $s = \rho\theta = 300 \times \frac{\pi}{2}$

$s = 471.238 \ m$

$(a_t)_B = -(0.001 \times 471.238)$

$= -0.4712 \ m/s^2$

$$a_t \cdot ds = v \cdot dv$$

$$-\int_0^{471.238} (0.001) s \cdot ds = \int_{25}^{V_B} v \cdot dv$$

$$-(0.001)\left(\frac{471.238^2}{2}\right) = \frac{1}{2}(V_B^2 - 625)$$

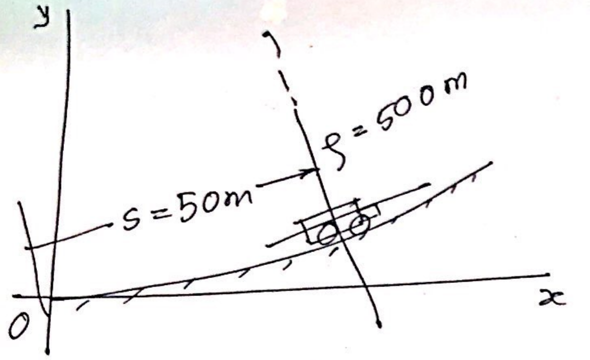
$$V_B = 20.07 \ m/s$$

$$(a_n)_B = \frac{V_B^2}{\rho} = \left(\frac{20.07^2}{300}\right) = 1.343 \ m/s^2$$

$$\therefore a_B = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{(-0.4712)^2 + (1.343)^2}$$

$$\therefore a_B = 1.423 \ m/s^2$$

⑥ RCH/F12.32/~~pg~~.678 :



$$V = (0.2)s$$

$$a \cdot dx = v \cdot dv$$

Here, $a_t ds = v \cdot dv$

$$a_t = v \cdot \frac{dv}{ds} = (0.2 \times s)(0.2)$$

$$a_t = (0.04)s \ \ m/s^2$$

when $s = 50$ m, $a_t = (0.04 \times 50) = 2 \ m/s^2$

$$v = (0.2 \times 50) = 10 \ m/s$$

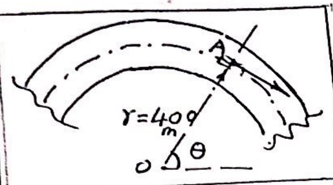
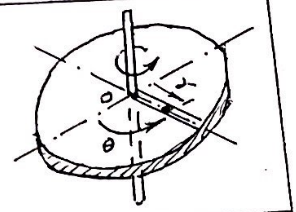
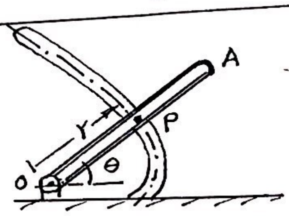
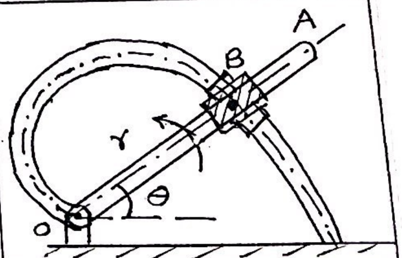
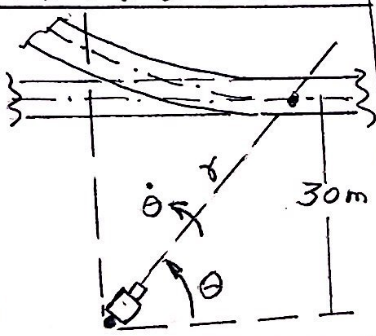
$$a_n = \frac{V^2}{\rho} = \frac{10^2}{500} = 0.2 \ \ m/s^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{2^2 + 0.2^2}$$

$$= 2.01 \ m/s^2$$

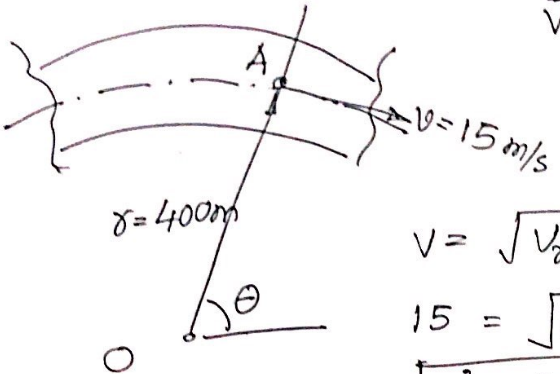
Lecture No: .

Polar Co-ordinate System

1	1) The car has a speed of 15m/s. Determine the angular velocity $\dot{\theta}$ of the radial line OA at this instant. **Ans: $\dot{\theta}$ = 0.0375 rad/s**	
2	The platform is rotating about the vertical axis such that at any instant its angular position is $\theta = (4t^{3/2})$rad, where t is in seconds. A ball rolls outward along the radial groove so that its position is $r = (0.1t^3)$m, where t is in seconds. Determine the magnitudes of the velocity and acceleration of the ball when t=1.5s. **Ans: v = 2.57m/s; a= 20.4 m/s^2**	
3	Peg P is driven by the fork line OA along the curved path described by $r = (2\theta)$m. At the instant $\theta = \pi/4$ rad, the angular velocity and angular acceleration of the link are $\dot{\theta}$ = 3 rad/s and $\ddot{\theta}$= 1 rad/s^2. Determine the magnitude of the pegs acceleration at this instant. **Ans: a= 39.5 m/s^2**	 Q.No. 3 and 4
4	Peg P is driven by the forked link OA along the path described by $r = e^{\theta}$, where r is in meters. When $\theta = \pi/4$ rad, the angular velocity and angular acceleration of the link are $\dot{\theta}$ = 2 rad/s and $\ddot{\theta}$= 4 rad/s^2. Determine the radial and transverse components of pegs acceleration at this instant. **Ans: a_r = 8.774 m/s^2; a_θ = 26.316 m/s^2**	
5	Collars are pin connected at B and are free to move along rod OA and the curved guide OC having the shape of a cardioid, $r = [0.2\,(1+\cos\theta)]$m. At $\theta = 30^0$, the angular velocity of OA is $\dot{\theta}$ = 3 rad/s. Determine the magnitude of the velocity of the collars at this point. **Ans:v = 1.16 m/s**	
6	2) At the instant $\theta = 45^0$, the athlete is running with a constant speed of 2 m/s. Determine the angular velocity at which the camera must turn in order to follow the motion. **Ans: $\dot{\theta}$ = 0.0333 rad/s**	

Lecture No. 1

1) F 12.33/RCH/pg. 693



$\bar{V} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$

$V_r = \dot{r} = 0$

$V_\theta = r\dot{\theta} = 400 \cdot \dot{\theta}$

$V = \sqrt{V_r^2 + V_\theta^2}$

$15 = \sqrt{0 + (400 \cdot \dot{\theta})^2}$

$\therefore \boxed{\dot{\theta} = 0.0375 \text{ rad/s}}$

2) RCH/F 12.34/pg. 693

At $t = 1.5$ s

$r = (0.1)t^3 = 0.3375$ m

$\dot{r} = (0.3)t^2 = 0.675$ m/s

$\ddot{r} = (0.6)t = 0.900$ m/s²

$\theta = 4 \cdot t^{3/2} = 7.348$ rad

$\dot{\theta} = 6 \cdot t^{1/2} = 7.348$ rad/s

$\ddot{\theta} = 3 \cdot t^{-1/2} = 2.449$ rad/s²

$V_r = \dot{r} = 0.675$ m/s

$V_\theta = r\dot{\theta} = (0.3375 \times 7.348)$

$= 2.480$ m/s

$V = \sqrt{V_r^2 + V_\theta^2}$

$= \sqrt{0.675^2 + 2.48^2}$

$\therefore \boxed{V = 2.57 \text{ m/s}}$

$a_r = (\ddot{r} - r\dot{\theta}^2) = (0.9 - 0.3375 \times 7.348^2)$

$= -17.325$ m/s²

$a_\theta = (r\ddot{\theta} + 2\dot{r}\dot{\theta}) = (0.3375 \times 2.449) + (2 \times 0.675 \times 7.348)$

$= 10.747$ m/s²

$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-17.325)^2 + (10.747)^2}$

$\therefore \boxed{a = 20.4 \text{ m/s}^2}$

③ RCH/F 12.35/pg.693 :

$r = 2\theta$ $\qquad \dot{\theta} = 3$ rad/s

$\dot{r} = 2\dot{\theta}$ $\qquad \ddot{\theta} = 1$ rad/s²

$\ddot{r} = 2\ddot{\theta}$ $\qquad \theta = (\pi/4)$ Rad

$\therefore r = \left(2 \times \frac{\pi}{4}\right) = 1.57$ m

$\dot{r} = (2\times3) = 6$ m/s

$\ddot{r} = (2\times1) = 2$ m/s²

$a_r = (\ddot{r} - r\dot{\theta}^2) = 2 - (1.57\times3^2) = -12.14$ m/s²

$a_\theta = (r\ddot{\theta} + 2\dot{r}\dot{\theta}) = (1.57\times1) + (2\times6\times3)$

$= 37.57$ m/s²

$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-12.14)^2 + (37.57)^2}$

$\boxed{a = 39.5 \text{ m/s}^2}$

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④ RCH/F 12.36/pg.693

$r = e^\theta$ $\qquad \theta = \pi/4$ rad

$\dot{r} = (e^\theta)(\dot{\theta})$ $\qquad \dot{\theta} = 2$ rad/s

$\ddot{r} = (e^\theta)(\ddot{\theta}) + (e^\theta)(\dot{\theta}^2)$ $\qquad \ddot{\theta} = 4$ rad/s²

$= e^\theta(\dot{\theta}^2 + \ddot{\theta})$ $\qquad \therefore \dot{r} = 2 e^{\pi/4} = 2\times2.193$

$= 4.386$ m/s

$\therefore a_r = (\ddot{r} - r\dot{\theta}^2) = (17.546 - 2.193\times4)$ $\qquad \ddot{r} = e^{\pi/4}(4+4)$

$= 8.774$ m/s² $\qquad = 17.546$ m/s

& $a_\theta = (r\ddot{\theta} + 2\dot{r}\dot{\theta})$ $\qquad r = e^{\pi/4} = 2.193$ m

$= (2.193\times4 + 2\times4.386\times2)$

$= 26.316$ m/s²

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(5) RCH/F 12.37/ pg. 693

$r = (0.2)(1+\cos\theta)$ m $\qquad \theta = 30° = 0.5235^c$

$\dot{r} = -(0.2)(\sin\theta)\dot{\theta}$ m/s $\qquad \dot{\theta} = 3$ rad/s

$\ddot{r} = -(0.2)(\sin\theta)\ddot{\theta} + (\cos\theta)(\dot{\theta}^2)$ m/s²

when $\theta = 30°$ $\quad r = 0.3732$ m

$\dot{r} = -0.3$ m/s

$V_r = \dot{r} = -0.3$ m/s

$V_\theta = r\dot{\theta} = (0.3732 \times 3) = 1.120$ m/s

$\therefore \quad V = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$

$V = -(0.3)\hat{e}_r + (1.120)\hat{e}_\theta$ m/s

$V = \sqrt{(-0.3)^2 + (1.120)^2} = 1.16$ m/s

---

(6) RCH/F 12.38/ pg. 693

$r = 30 \cdot \mathrm{cosec}\,\theta = \dfrac{30}{\sin\theta}$ $\qquad \theta = 45°$, $\dot{\theta} = ?$

$\dot{r} = -30 \cdot \mathrm{cosec}\,\theta \cdot \cot\theta \cdot \dot{\theta}$

$\therefore$ when $\theta = 45° = 0.7854$ rad

$r = 42.426$ m, $\quad \dot{r} = \dfrac{-30}{\sin 45°} \times \dfrac{1}{\tan 45°} \cdot \dot{\theta}$

$= -(42.426)\dot{\theta}$

$V_r = \dot{r} = -(42.426)\dot{\theta}$

$V_\theta = r\dot{\theta} = (42.426)\dot{\theta}$

$V = 2$ m/s $= \sqrt{V_r^2 + V_\theta^2}$

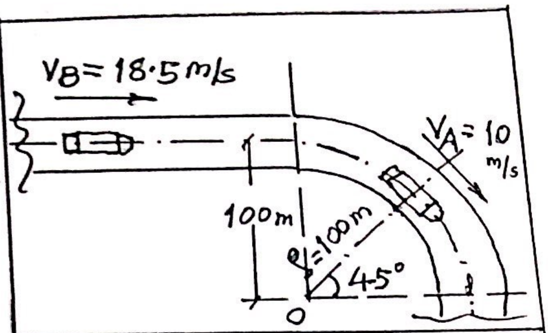
$2 = \sqrt{(-42.426\dot{\theta})^2 + (42.426\dot{\theta})^2}$

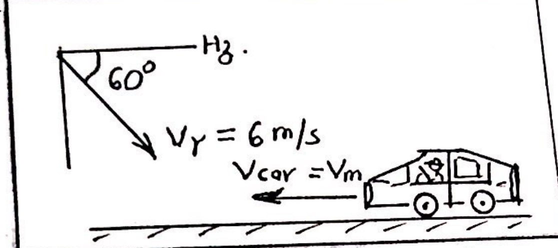
$\dot{\theta} = 0.0333$ rad/s

Lecture No:.

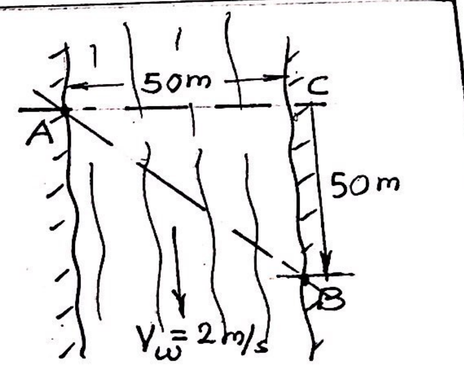
Relative Motion

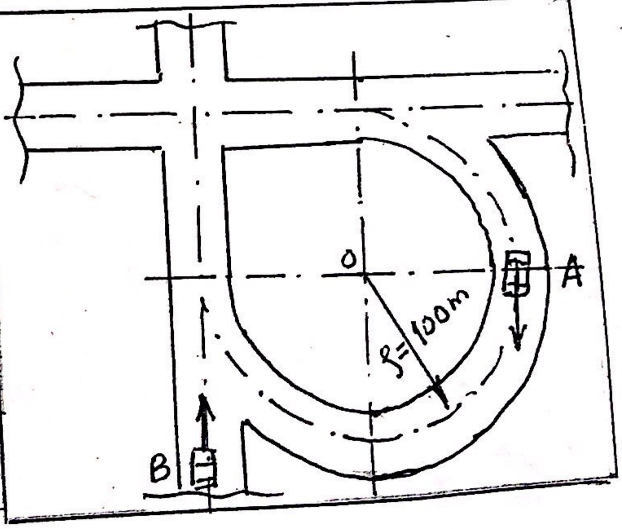
<table>
<tr>
<td>1</td>
<td>At the instant shown, the car at A is traveling at 10 m/s around the curve while increasing its speed at $5 m/s^2$. The car at B is travelling at 18.5 m/s along the straightway & increasing its speed at $2 m/s^2$. Determine the relative velocity & relative acceleration of A with respect to B at this instant.

Ans: $v_{A/B} = 13.44$ m/s, $A_{A/B} = 4.322 m/s^2$</td>
<td>
</td>
</tr>
<tr>
<td>2</td>
<td>The car is travelling at a constant speed of 100km/h. if the rain is falling at 6 m/s in the direction shown, determine the velocity of the rain as seen by the driver.

Ans: $v_{r/m} = 31.212$ m/s</td>
<td>
</td>
</tr>
<tr>
<td>3</td>
<td colspan="2">The car is travelling north along a straight at 50 kmph. An instrument in the car indicates that the wind is coming from the East. If the car's speed is 80kmph, the instrument indicates that the wind coming from north east. Determine the speed and direction of the wind.

Ans: V_w= - (30) i+ 50 j ; V_w= 58.309kmph</td>
</tr>
<tr>
<td>4</td>
<td>A man can row a boat at 5m/s in still water. He wishes to cross a 50-m-wide river to point B, 50 m downstream. If the river flows with a velocity of 2m/s, determine the speed the speed of the boat and the time needed to make the crossing.

Ans: V_{RB}= 6.21 m/s, t= 11.385 m/s</td>
<td>
</td>
</tr>
<tr>
<td>5</td>
<td>At the instant shown, car A has a speed of 20kmph, which is been increased at the rate of $300 km/h^2$ as the car enters the expressway. At the same instant, car B is decelerating at $250 km/h^2$ while travelling forward AT 100kmph. Determine the velocity & acceleration of A with respect to B.

Ans: $V_{A/B}$= -33.332 m/s = 0 kmph; $a_{A/B}$= $0.3085 m/s^2$</td>
<td>
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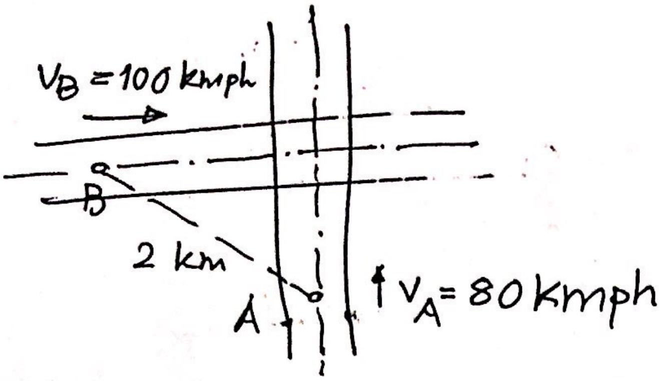
Lecture No 5

RCH/F 12.45/pg. 711 :

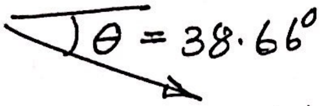


$\bar{V}_A = 80\hat{j}$ kmph

$\bar{V}_B = 100 \cdot \hat{i}$ kmph

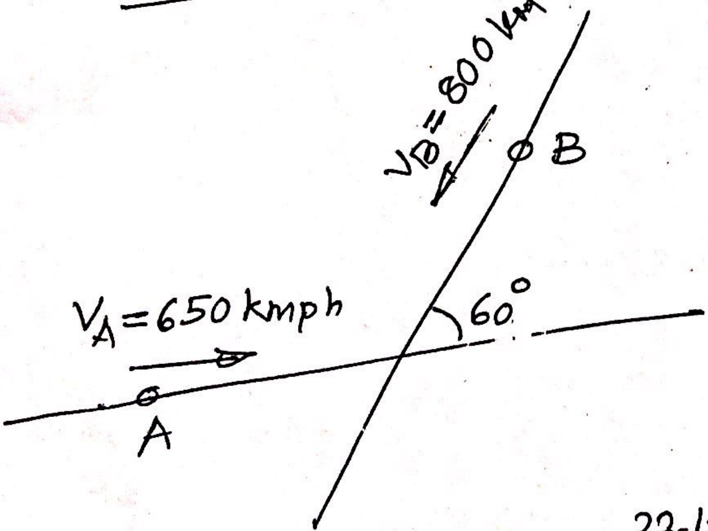
$\bar{V}_{B/A} = \bar{V}_B - \bar{V}_A$

$= (100)\hat{i} - (80)\hat{j}$



$V_{B/A} = 128.06$ kmph

RCH/F 12.46/pg. 711

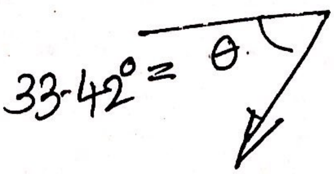


$\bar{V}_A = (650)\hat{i}$ kmph

$\bar{V}_B = -(400)\hat{i} - (692.82)\hat{j}$ kmph

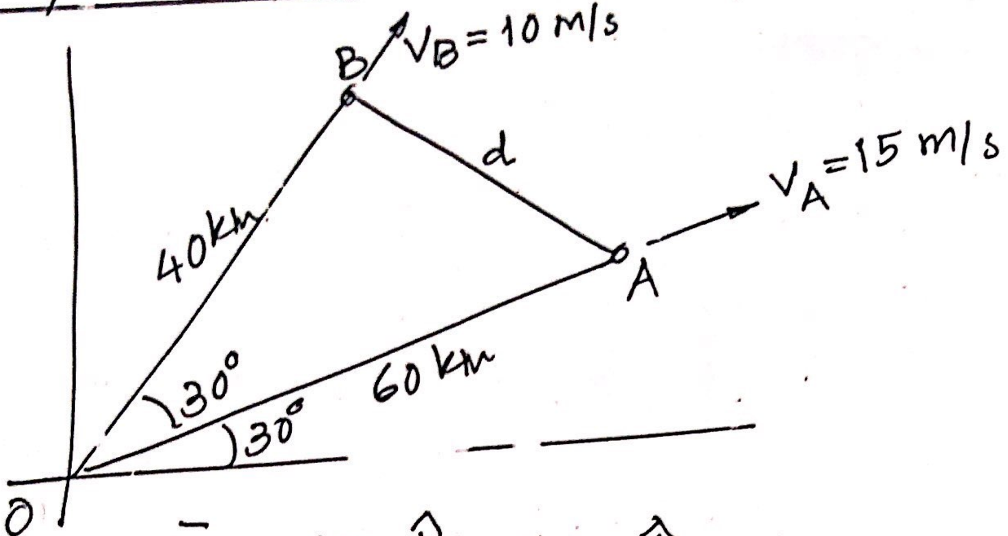
$\bar{V}_{B/A} = \bar{V}_B - \bar{V}_A$

$\bar{V}_{B/A} = -(1050)\hat{i} - (692.82)\hat{j}$



$V_{B/A} = 1257.97$ kmph

RCH/F 12.47/pg.710



$$\bar{V}_A = 13\hat{i} + 7.5\hat{j} \text{ m/s}$$

$$\bar{V}_B = 5\hat{i} + 8.66\hat{j} \text{ m/s}$$

$$\bar{V}_{B/A} = \bar{V}_B - \bar{V}_A = -8\hat{i} + 1.16\hat{j}$$

$V_{B/A} = 8.084$ kmph

$8.25^\circ = \theta$

At $t = 4$ s,

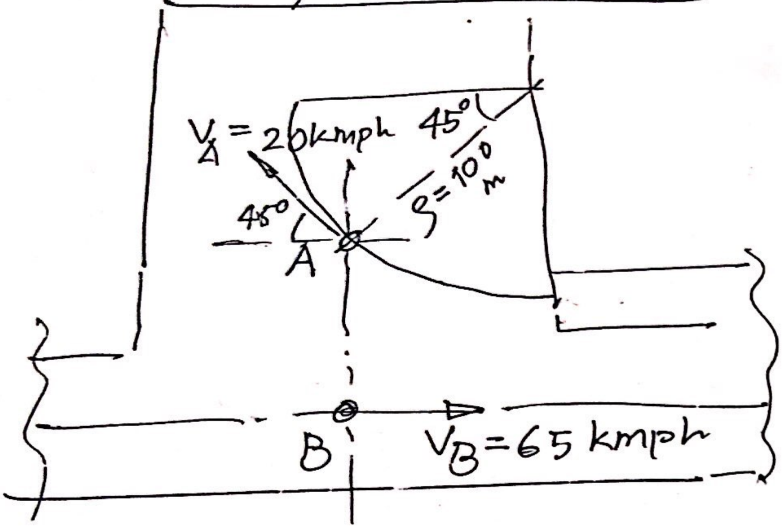
$$d(AB) = 8.084 \times 4 = 32.336 \text{ m}$$

OR By Cosine-rule

$$\cos 30^\circ = \frac{60^2 + 40^2 - d^2}{2 \times 60 \times 40}$$

$$\therefore d = 32.296 \text{ km}.$$

RCH/F 12.48/pg. 711



$V_A = 20$ kmph
$= \left(20 \times \frac{5}{18}\right)$
$= 5.555$ m/s

V_A 45°

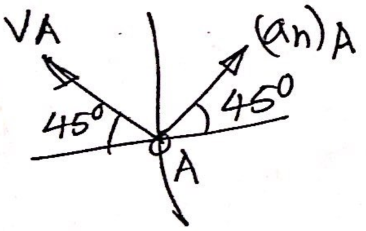
$\bar{V}_A = -(3.928)\hat{i} + (3.928)\hat{j}$ m/s

$\bar{V}_B = \left(65 \times \frac{5}{18}\right)\hat{i}$
$= (18.055)\ \hat{i}$ m/s

$$\bar{V}_{A/B} = \bar{V}_A - \bar{V}_B$$
$$= (21.983)\hat{i} + (3.928)\hat{j}$$

$V_{A/B} = 22.331$ m/s
$= 80.392$ kmph
$\theta = 10.13^\circ$

Now, $(a_t)_A = 0$ as its speed is constant
but $(a_n)_A = \frac{V_A^2}{\rho} = \frac{5.555^2}{100} = 0.3085$ m/s²



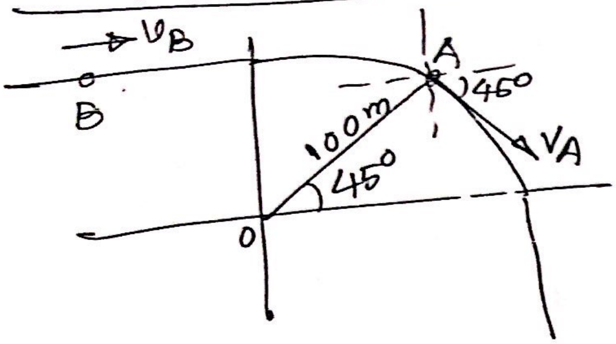
$\bar{a}_A = (0.2182)\hat{i} + (0.2182)\hat{j}$ m/s²

$\bar{a}_B = 1200$ km/h² $\hat{i}$
$= \left(\frac{1200 \times 1000}{3600 \times 3600}\right)$ m/s² $\hat{i}$
$= (0.0925)\hat{i}$ m/s²

$$\bar{a}_{A/B} = \bar{a}_A - \bar{a}_B$$
$$= (0.1256)\hat{i} + (0.2182)\hat{j}$$

$a_{A/B} = 0.2517$ m/s² $= 3262$ km/h²
$\theta = 60^\circ$

RCH/12.217/~~pg~~.715



$v_B = 18.5$ m/s $\quad \therefore \bar{v}_B = (18.5)\hat{i}$

$a_B = 2$ m/s^2 $\quad \therefore \bar{a}_B = (2.0)\hat{i}$

$V_A = 10$ m/s 45° $\quad \therefore \bar{V}_A = (7.07)\hat{i} - (7.07)\hat{j}$ m/s

$a_A \Rightarrow (a_t)_A = 5$ m/s^2 45°

$$(a_n)_A = \frac{V^2}{\rho} = \frac{100^2}{100} = 1 \text{ m/s}^2 \; 45°$$

$(\bar{a}_t)_A = (3.535)\hat{i} - (3.535)\hat{j}$

$(\bar{a}_n)_A = -(0.707)\hat{i} - (0.707)\hat{j}$

$\bar{a}_A = (2.828)\hat{i} - (4.242)\hat{j}$ m/s^2

$\bar{V}_{A/B} = (\bar{V}_A - \bar{V}_B) = -(11.43)\hat{i} - (7.07)\hat{j}$

31.74° $\quad V_{A/B} = 13.44$ m/s

$\bar{a}_{A/B} = (\bar{a}_A - \bar{a}_B) = +(0.828)\hat{i} - (4.242)\hat{j}$ m/s

78.95° $\quad a_{A/B} = 4.322$ m/s^2

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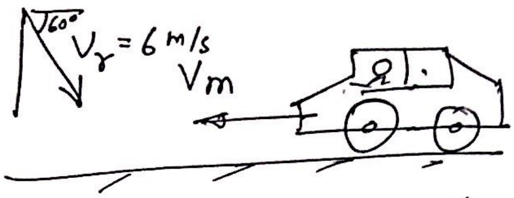
RCH/12.219/pg.715 :



$$V_{man} = 100 \text{ kmph} = 27.777 \text{ m/s}$$

$$\bar{V}_m = -(27.777)\hat{i}$$

$$\bar{V}_{rain/man} = \bar{V}_{r/m} = \bar{V}_r - \bar{V}_m$$

$$V_{r/m} = (3)\hat{i} - (5.196)\hat{j} + (27.777)\hat{i}$$

$$\cancel{= (3)\hat{i} - (32.973)\hat{j} \text{ m/s}}$$

$$\cancel{84.8^\circ}$$

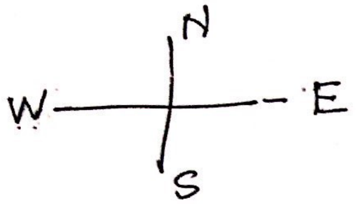
$$\cancel{V_{r/m} = 33.109 \text{ m/s}}$$

$$= (30.777)\hat{i} - (5.196)\hat{j} \text{ m/s}$$

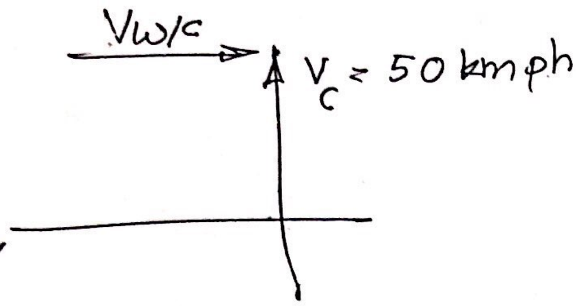
$$9.58^\circ$$

$$V_{r/m} = 31.212 \text{ m/s}$$

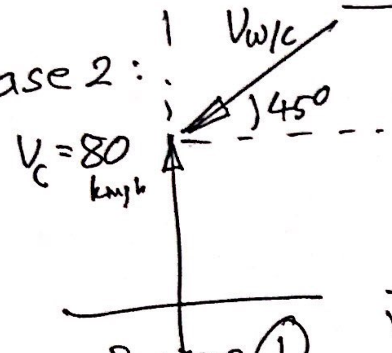
RCH/12.221/pg.716 :



Case 1:



Case 2:



$$\bar{V}_{w/c} = \bar{V}_w - \bar{V}_c$$

$$\bar{V}_w = \bar{V}_{w/c} + \bar{V}_c$$

For case ① $\vec{V}_w = (V_{w/c_1})\hat{i} + (50)\hat{j}$

For case ② $\bar{V}_w = -(0.707 \times V_{w/c_2})\hat{i}$
$- (0.707 \times V_{w/c_2})\hat{j} + 80\hat{j}$

Equating the coeff. of $\hat{i}$ & $\hat{j}$

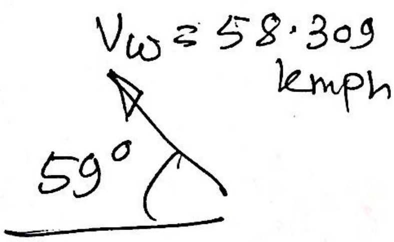
$$(V_{w/c})_1 = -(0.707)(V_{w/c})_2 \longrightarrow ①$$

$$50 = -(707)(V_{w/c_2}) + 80$$

$$\therefore +30 = (0.707)(V_{w/c_2})$$

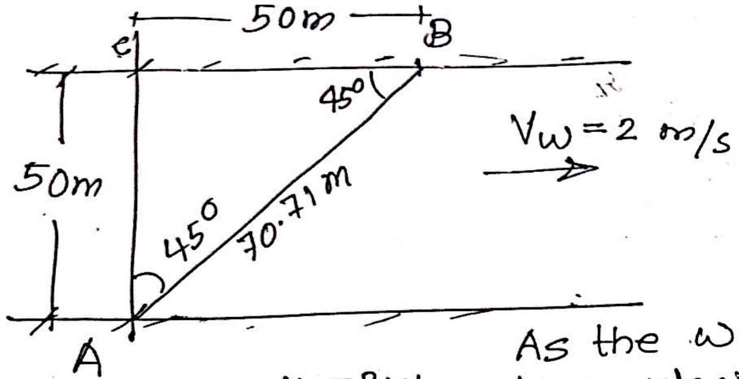
$$\therefore (V_{w/c})_2 = 42.432 \text{ m/s}$$

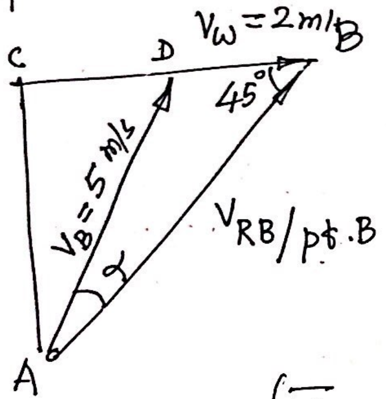
$$(V_{w/c})_1 = -30 \text{ m/s}$$



Ans: $\vec{V}_w = -(30)\hat{i} + (50)\hat{j}$

RCH/12.223/pg. 716





As the water current has velocity of 2 m/s, boat can never cross the river in ⊥ direction. But, it will make some angle α, with the resultant velocity.

$$\bar{V}_{RB/pt.B} = \left(\bar{V}_{RB} - \cancel{\bar{V}_{pt.B}}\right) = \bar{V}_B + \bar{V}_w$$

But $V_{pt.B} = 0$

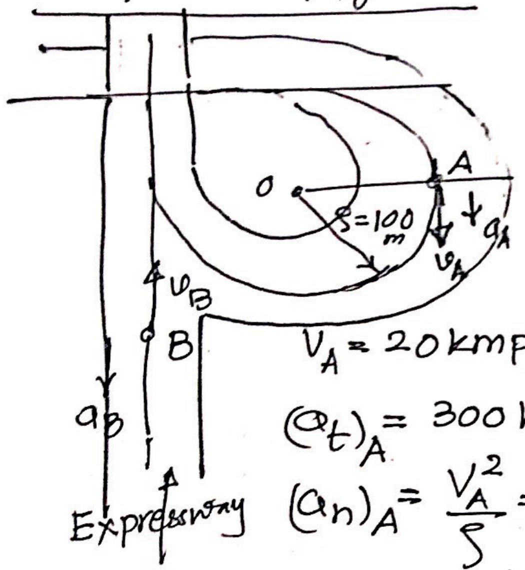
In ΔADB, $\dfrac{5}{\sin 45^\circ} = \dfrac{2}{\sin\alpha}$ $\therefore \alpha = 16.43^\circ$

and $\dfrac{5}{\sin 45^\circ} = \dfrac{V_{RB}}{\sin(180^\circ - 45^\circ - 16.43^\circ)}$

$\therefore \boxed{V_{RB} = 6.21 \text{ m/s}}$

Time reqd. to cross the river & to reach pt. B, $t = \dfrac{70.71 \text{ m}}{6.21 \text{ m/s}} = 11.385$ sec.

RCH/12.225/pg.716 :



$$V_A = 20 \text{ kmph} = 5.555 \text{ m/s}$$

$$(a_t)_A = 300 \text{ kmph}^2 = 0.023 \text{ m/s}^2$$

$$(a_n)_A = \frac{V_A^2}{\rho} = \left(\frac{5.555^2}{100}\right) = 0.3085 \text{ m/s}^2$$

$$V_B = 100 \text{ kmph} = 27.777 \text{ m/s}$$

$$a_B = 250 \text{ kmph}^2 = 0.019 \text{ m/s}^2$$

$$\therefore \bar{V}_A = 0 \cdot \hat{i} - (5.555)\hat{j} \text{ m/s}$$

$$\bar{V}_B = 0 \cdot \hat{i} + (27.777)\hat{j} \text{ m/s}$$

$$\therefore \bar{V}_{A/B} = (\bar{V}_A - \bar{V}_B) = -(33.332)\hat{j} \text{ m/s}$$

i.e. 0 kmph (↓)

$$\bar{a}_A = -(0.3085)\hat{i} - (0.023)\hat{j} \text{ m/s}^2$$

$$\bar{a}_B = 0\,\hat{i} - (0.019)\hat{j} \text{ m/s}^2$$

$$\therefore \bar{a}_{A/B} = (\bar{a}_A - \bar{a}_B)$$

$$= -(0.3085)\hat{i} - (0.004)\hat{j} \text{ m/s}^2$$

$\cancel{a_{A/B} = 0.31134 \text{ m/s}^2}$

$\cancel{7.75^\circ}$

$0.74°$

$a_{A/B} = 0.3085 \text{ m/s}^2$

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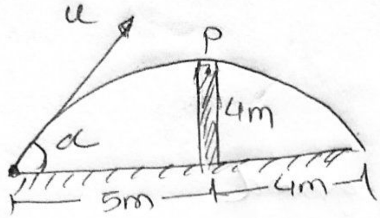
① Find the least initial velocity with a projectlie is to be projected so that it cleers a wall 4m high located at a distance of 5m and strikes the ground at a distance 4m beyond the wall as shown in figure. The point of projections is at the same level as the foot of the wall.



Range = 9m, $x = 5m$, $y = 4m$

$R = \frac{u^2 \sin 2\alpha}{g}$, $\boxed{u^2 = \frac{9g}{\sin 2\alpha}}$ — ①

Eq.n of Trajectory, $\boxed{y = x\tan\alpha - \frac{1}{2}\frac{gx^2}{u^2\cos^2\alpha}}$

$\therefore \quad 4 = 5\tan\alpha - \frac{1}{2}\frac{gx^2}{u^2\cos^2\alpha}$ — Put value of u^2 we get,

$4 = 5\tan\alpha - \frac{25}{18\frac{\cos^2\alpha}{2\sin\alpha \cdot \cos\alpha}}$

$4 = 5\tan\alpha - \frac{50}{18}\tan\alpha$

$= 2.222\tan\alpha$

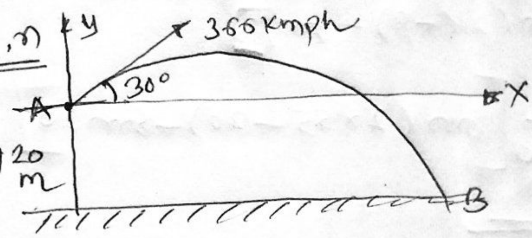
$\therefore \tan\alpha = 1.8$

$\alpha = \underline{60.95^\circ}$

from eq.① $u^2 = \frac{9g}{\sin(2\times 60.95^\circ)} = \underline{10.20 \text{ m/sec}}$

② A Bullet is fired from a height of 120m at a velocity of 360 kmph at an angle of 30° upwords. Neglecting air resistance.
find ⓐ Total time of flight ⓑ Horrizontal ~~angle~~ Range of the bullet
ⓒ maximum height reached by the bullet
ⓓ find velocity of the bullet just before reaching the ground

Sol.n



$u = 360 \text{ kmph} = \frac{360\times 1000}{3600}$
$= 100 \text{ m/sec}$

① Total time of flight

$y_v = \underline{-120m}$

Consider vertical motions

$y = u\sin\alpha \times t - \frac{1}{2}gt^2$

$-120 = 100\sin 30^\circ \times t - \frac{1}{2}\times 9.81\times t^2$

$\therefore t^2 - 10.194t - 24.465 = 0$

$\therefore \boxed{t = 12.20 \text{ sec}}$

(Negative value of t is Neglected)

(b) Maximum height reached by the bullet

$$h = \frac{u^2 \sin^2\alpha}{2g} = \frac{100^2 \sin^2 30°}{2 \times 9.81} = 127.42 \text{ m above point A.}$$

from ground, $127.42 + 120 = 247.42$ m above the ground

(c) Horizontal Range $= u\cos\alpha \times t$

$= 100 \cos 30 \times 12.2$

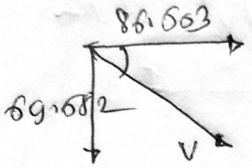
$= 1058.55$ m

(d) Velocity of the bullet just before striking the ground

$V_y = u\sin\alpha - gt = 100 \sin 30° - 9.81 \times 12.2$

$= -69.682$ m/s

$= 69.682$ downwards



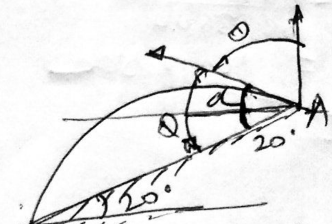
$V_x = 100 \cos\alpha = 86.603$ m/sec

$V = \sqrt{69.682^2 + 86.603^2}$

$V = 111.16$ m/sec

$\theta = \tan^{-1}\left(\frac{69.682}{86.603}\right) = 38.82°$

(3) A person can throw a ball at a maximum velocity of 30 m/sec. If he wants to get maximum range on the plane inclined at 20° to horizontal at what angle should the ball be projected and what would be maximum range (a) up the plane (b) down the plane?



Up the plane

$\theta = \frac{90 - 20}{2} = 35°$ (∵ Range is maximum, when angle of projection bisects the angle bet.n vertical and inclined plane)

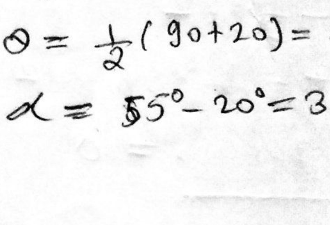
$\alpha = \theta + 20°$

$= 55°$

$$R_{max} = \frac{u^2}{g\cos^2\beta}\left[\sin(2\alpha - \beta) - \sin\beta\right]$$

$$= \frac{30 \times 30}{9.81 \cos^2 20}\left[\sin(2 \times 55 - 20) - \sin 20\right]$$

$= 68.362$ m



Down the plane

$\theta = \frac{1}{2}(90 + 20) = 55°$

$\alpha = 55° - 20° = 35°$

$$R_{max} = \frac{30 \times 30}{9.81 \cos^2(-20)}\left[\sin(2 \times 35 + 20) - \sin(-20)\right]$$

$= 139.432$ m