MIT World Peace University

Engineering Mechanics
First Year B. Tech, Trimester 1

Graphical Solution of Resultant of Concurrent Co-planar Force System.

Practical Report

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Contents

1 Objective

To find Graphically and Analytically the resultant of a set of problems involving Concurrent Co-Planar Force System, and to compare the results thereby finding the Percentage Error.

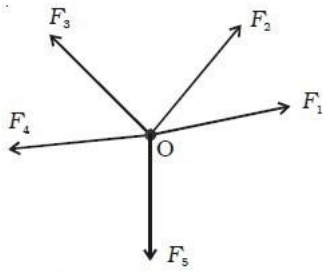
2 Theory

The Following laws and concepts have been used in this experiment.

2.1 Concurrent Co-Planar Force System

Co-planar force systems have all the forces acting in in one plane. They may be concurrent, parallel, non-concurrent or non-parallel. All of these systems can be resolved by using graphic statics or algebra.

A concurrent coplanar force system is a system of two or more forces whose lines of action ALL intersect at a common point. However, all of the individual vectors might not actually be in contact with the common point. These are the most simple force systems to resolve with any one of many graphical or algebraic options.



Concurrent forces

2.2 Parallelogram Law of Vector Addition

The Parallelogram Law of Vector Addition is law that helps us to resolve a given vector into its respective components, or find the resultant of 2 vectors inclined to each other at an angle.

It States that:

"If two vectors acting simultaneously at a point can be represented both in magnitude and direction by the adjacent sides of a parallelogram drawn from a point, then the resultant vector is represented both in magnitude and direction by the diagonal of the parallelogram passing through that point."

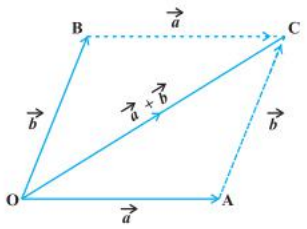


Fig. 5

Figure 1: Parallelogram Law of Vector Addition

2.3 Polygon Law of Vector Addition

Polygon law of vector addition states that:

If a number of vectors can be represented in magnitude and direction by the sides of a polygon taken in the same order, then their resultant is represented in magnitude and direction by the closing side of the polygon taken in the opposite order.

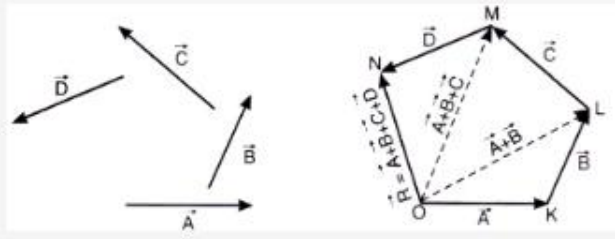


Figure 2: Polygon Law of Vector Addition

3 Procedure

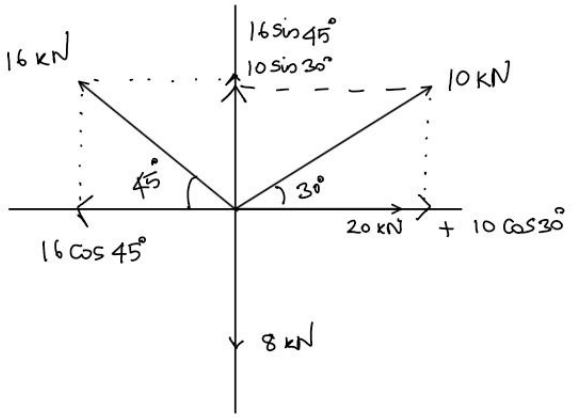
1. Find the Analytical Solution of the Given 3 Problems.
2. Find the Graphical Solution of the Given 3 Problems by plotting to scale the figures and applying Polygon and Parallelogram Laws of Vector Addition.
3. Tabulate the Results
4. Find the Error Percentage of the graphical Solution in comparison to the Analytical Solution using the formula:

$$\text{Percentage Error } (\eta) = \frac{|\text{Graphical Value} - \text{Analytical Value}|}{\text{Analytical Value}} * 100$$

4 Analytical Method

Q1. Find the Resultant of the Force system shown below.

A.1.



Equating Forces along X direction :

$$R_X = 20 + 10\cos 30^\circ - 16\cos 45^\circ$$
$$= 20 + 8.660 - 11.313$$
$$= 17.347 \text{ kN}$$

Equating Forces along Y direction :

$$R_Y = -8 + 10\sin 30^\circ + 16\sin 45^\circ$$
$$= -8 + 5 + 11.313$$
$$= 8.313 \text{ kN}$$

Therefore, Resultant $R = \sqrt{R_X^2 + R_Y^2}$

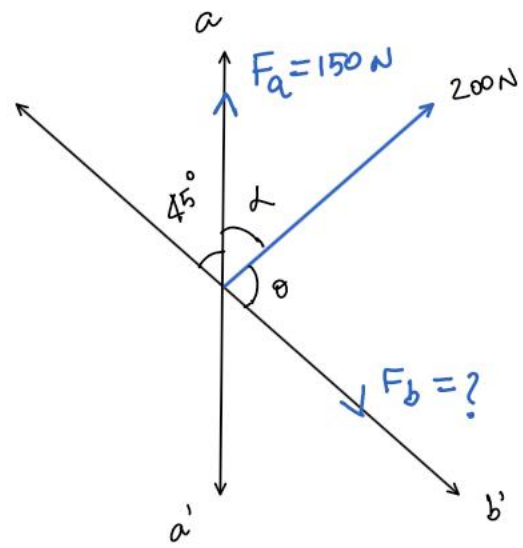
$$R = \sqrt{17.347^2 + 8.313^2}$$
$$= 19.23 \text{ kN}$$

$$\therefore \tan\theta = \frac{17.343}{8.313} = 0.479 ;$$

$$\theta = \tan^{-1}(0.479) = 25.60^\circ$$

Q2. A 200 N force is to be resolved into components along lines a-a' and $b-b'$ as shown in the figure. Determine the angle α knowing that the component along $a-a'$ is to be 150N. What is the corresponding value of the component along $b-b'$?

A2.



we know,

$45 + \alpha + \theta = 180^\circ$

$\alpha + \theta = 135^\circ$ ——— (1)

Resultant = 200N ; so using Resultant Formula,

$$R = \sqrt{F_A^2 + F_B^2 + 2 F_A F_B \cos\theta}$$

[here θ is $(\theta + \alpha) = (135^\circ)$]

$200^2 = 150^2 + F_B^2 + 300 \cos 135^\circ \cdot F_B$

$F_B^2 = 200^2 - 150^2 - 300 \cos 135 \cdot F_B$

$= 17500 - 212.132 F_B$

$\therefore \quad F_B^2 + 212.82 F_B - 17500 = 0$

Solving the quadratic we get,

$F_B = 275.62\ N$

Now that we know F_B and F_a, we can use the formula

$$\tan\theta = \frac{F_A \sin(\alpha+\theta)}{F_A \cos(\alpha+\theta) + F_B} = \frac{F_A \sin 135}{F_A \cos 135 + F_B} = \frac{F_A(0.707)}{F_A(-0.707) + F_B}$$

$$\Rightarrow \tan\theta = \frac{150\ (0.707)}{150\ (-0.707) + 275.62} = \frac{106.5}{169.57} = 0.628 \ ;$$

$$\therefore \ \theta = \tan^{-1}(0.628)$$

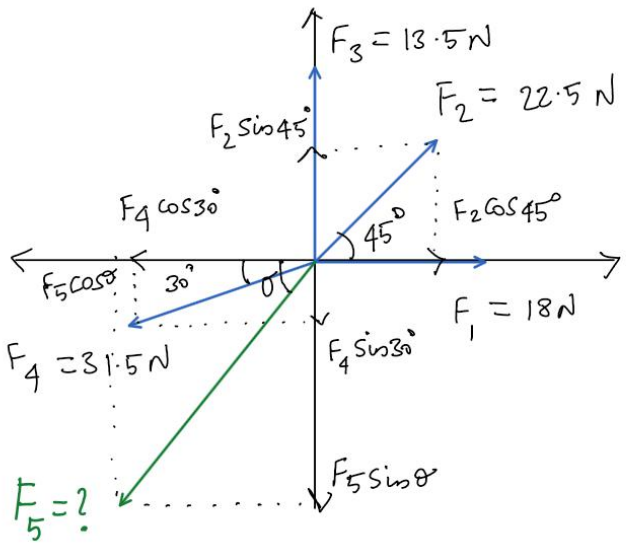
$$\theta = 32.13^\circ$$

As $\theta + \alpha = 135^\circ$, $\alpha = 135^\circ - \theta = 135 - 32.13$

$$\therefore \ \boxed{\alpha = 102.86^\circ}$$

Q3. Under the action of five forces, the following system is in equilibrium. Determine the magnitude and direction of the fifth force.

A.3.



Equating Forces along x axis $(\sum F_x = 0)$

$$-F_{5_x} + F_2 \cos 45^\circ + 18 + -F_4 \cos 30^\circ = 0$$

$$-F_{5_x} = 31.5 \cos 30^\circ - 18 - 22.5 \cos 45^\circ$$

$$= -6.630\,N = F_{5_x}$$

Equating Forces along y axis $(\sum F_y = 0)$

$$-F_{5y} + 13.5 - 31.5 \sin 30^\circ + 22.5 \cos 45^\circ$$

$$F_{5_y} = -13.60$$

$$F_5 = \sqrt{F_{5_y}^2 + F_{5_x}^2} = \sqrt{(-13.6)^2 + (-6.630)^2}$$

$$F_5 = 15.13\,N$$

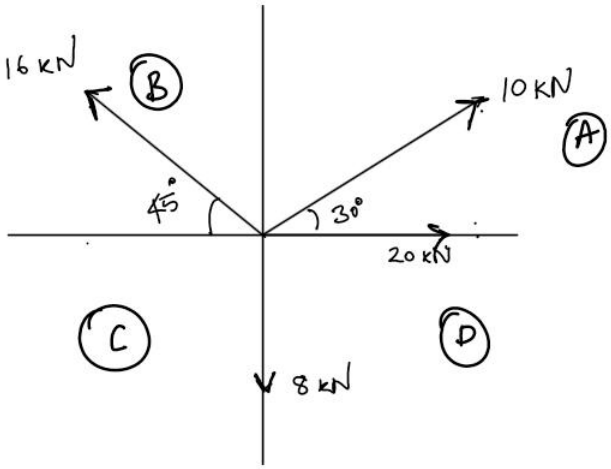
$$\tan\theta = \frac{-13.6}{-6.60} = 2.06 \;;\; \theta = \tan^{-1}(2.06)$$

$$\theta = 64.11^\circ$$

5 Graphical Method

Q1. Find the Resultant of the Force system shown below.

A1. Space Diagram:



As $aa' = 4.8\text{ cm}$,

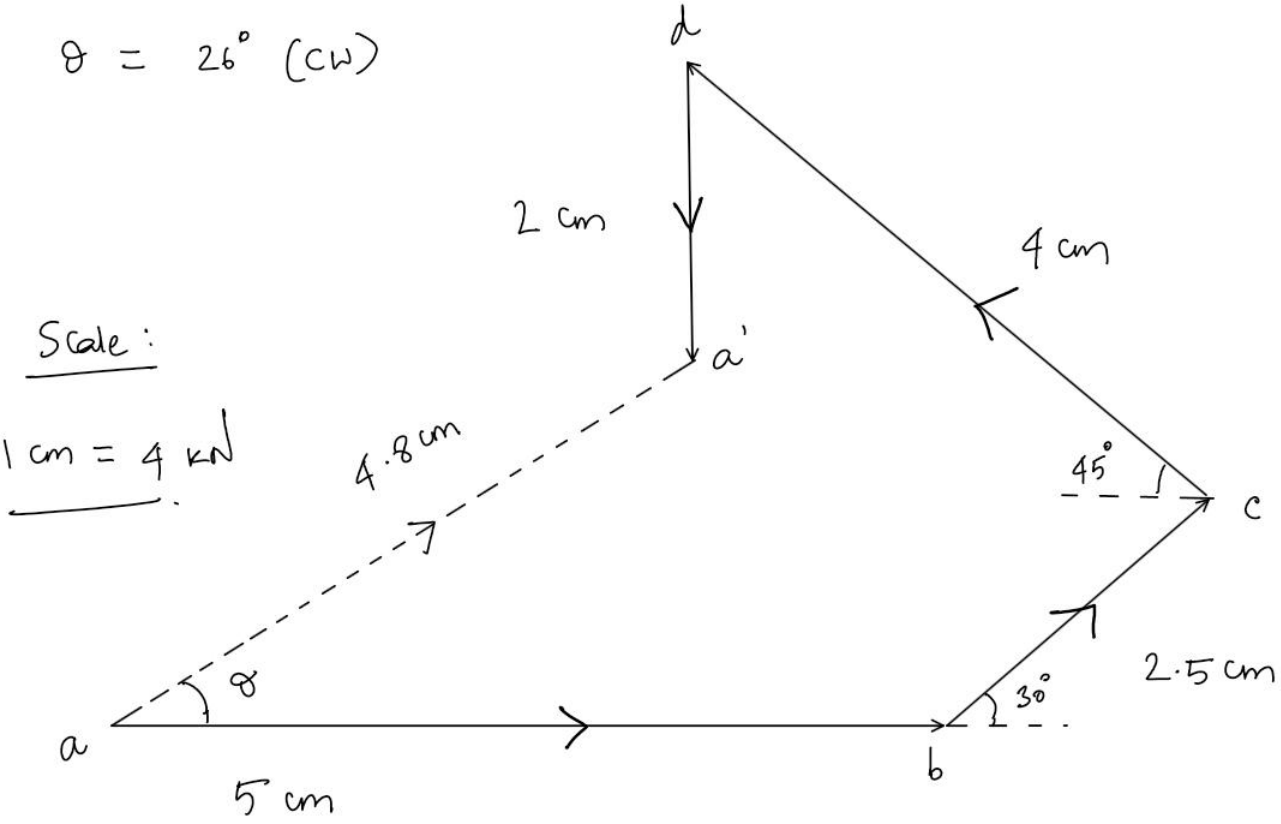
$R = 4.8 \times 4 = 19.2\text{ kN}$

$R = 19.20\text{ kN}$

$\theta = 26^\circ\text{ (CW)}$

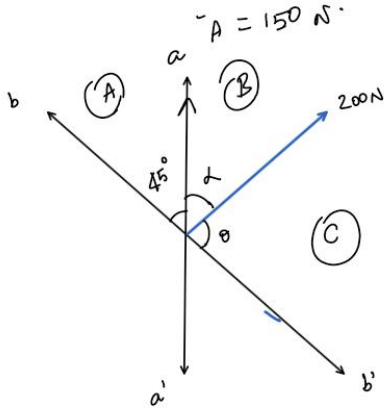
Scale:

1 cm = 4 kN



Q2. A 200 N force is to be resolved into components along lines a-a' and $b-b'$ as shown in the figure. Determine the angle α knowing that the component along $a-a'$ is to be 150N. What is the corresponding value of the component along $b-b'$?

A2. (1) Space Diagram



(2) Draw ab => F_a = 7.5 cm.

(3) From b, draw a line at 45°

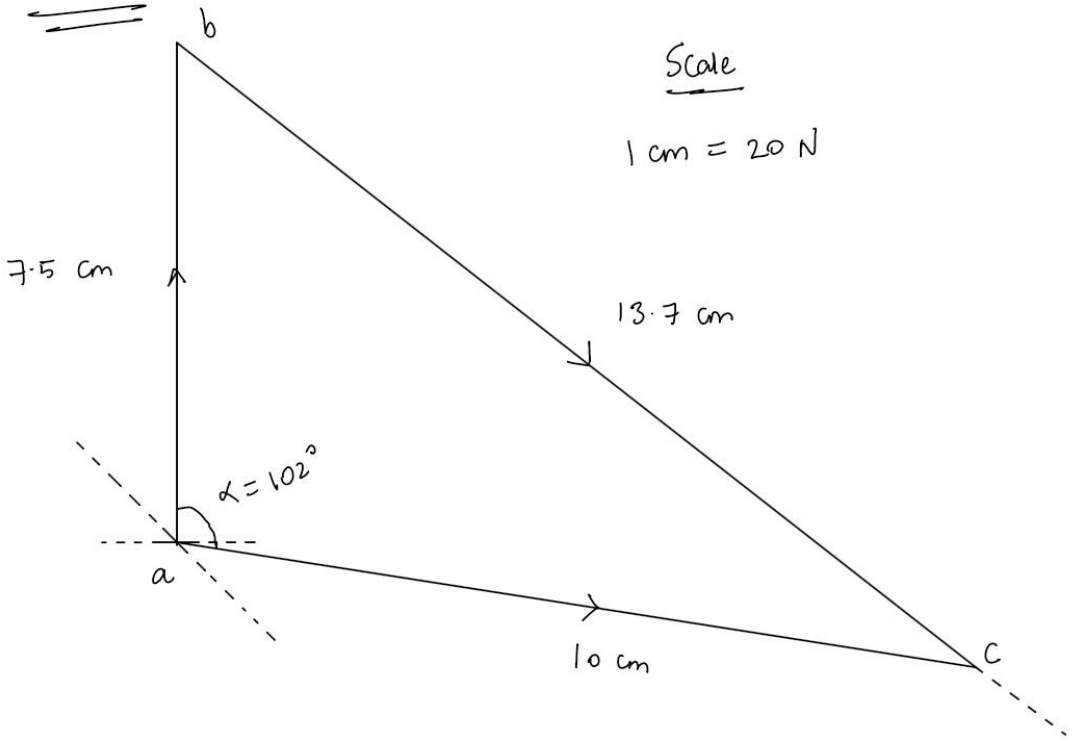
(4) on this line, cut distance of 10 cm from a, which represents 200 N Force. This point is c.

(5) bc = 13.7 cm

$F_b = 13.7 \times 20$

$F_b = 274$ N

$\alpha = 102°$



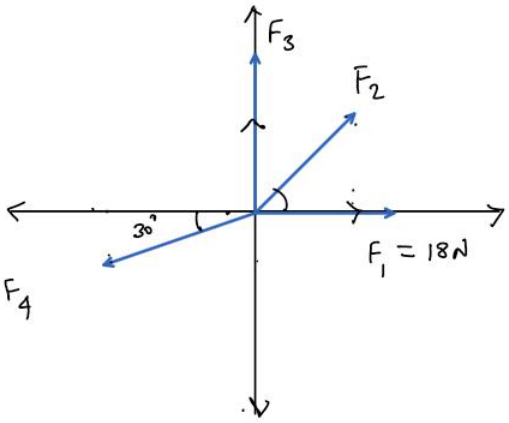
Q3. Under the action of five forces, the following system is in equilibrium. Determine the magnitude and direction of the fifth force.

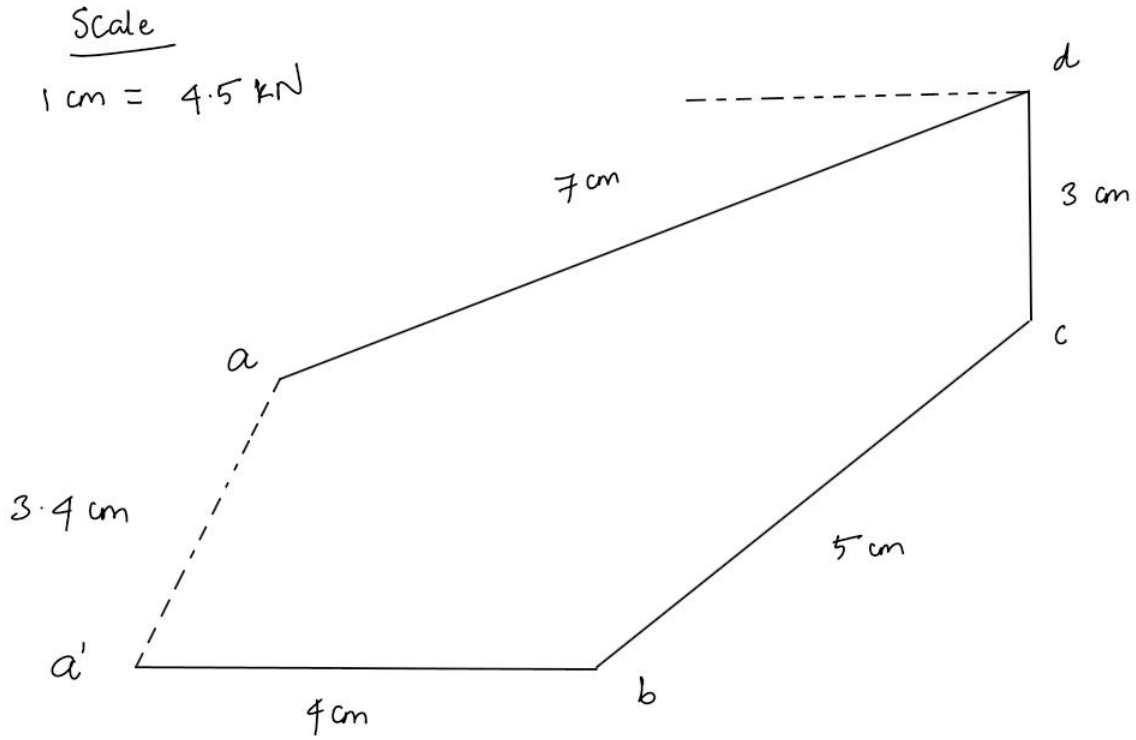
A3. Space diagram :

$F_5 = 3.4 \times 4.5$

$F_5 = 15.3\ N$

$\theta = 64^\circ$





6 Results

Question Number	Analytical Solution	Graphical Solution	Percent Error
1	R = 19.23 kN θ = 25.60°	R = 19.20 kN θ = 26.00 °	$\eta_R = 0.31\%$ $\eta_\theta = 1.56\%$
2	$F_B = 275.62N$ α = 102.86°	$F_B = 274.00N$ α = 102.00°	$\eta_{F_B} = 0.58\%$ $\eta_\alpha = 0.83\%$
3	$F_5 = 15.13N$ $\theta = 64.11°$	$F_5 = 15.3N$ $\theta = 64.00°$	$\eta_{F_5} = 0.85\%$ $\eta_\theta = 0.11\%$

7 Conclusion

A set of problems involving concurrent Co-planar Force system were solved using graphical as well as analytical methods. The Percentage error between the 2 answers was found.