ENGINEERING MECHANICS

MODULE - 1 - ASSIGNMENT - 1

Conventional Questions

Q.1. 2 Spheres weighing 125 N and 100N respectively and with corresponding radii are placed in a container as shown below. Find support reactions. ($r_1 = 100$ mm, $r_2 = 75$mm)

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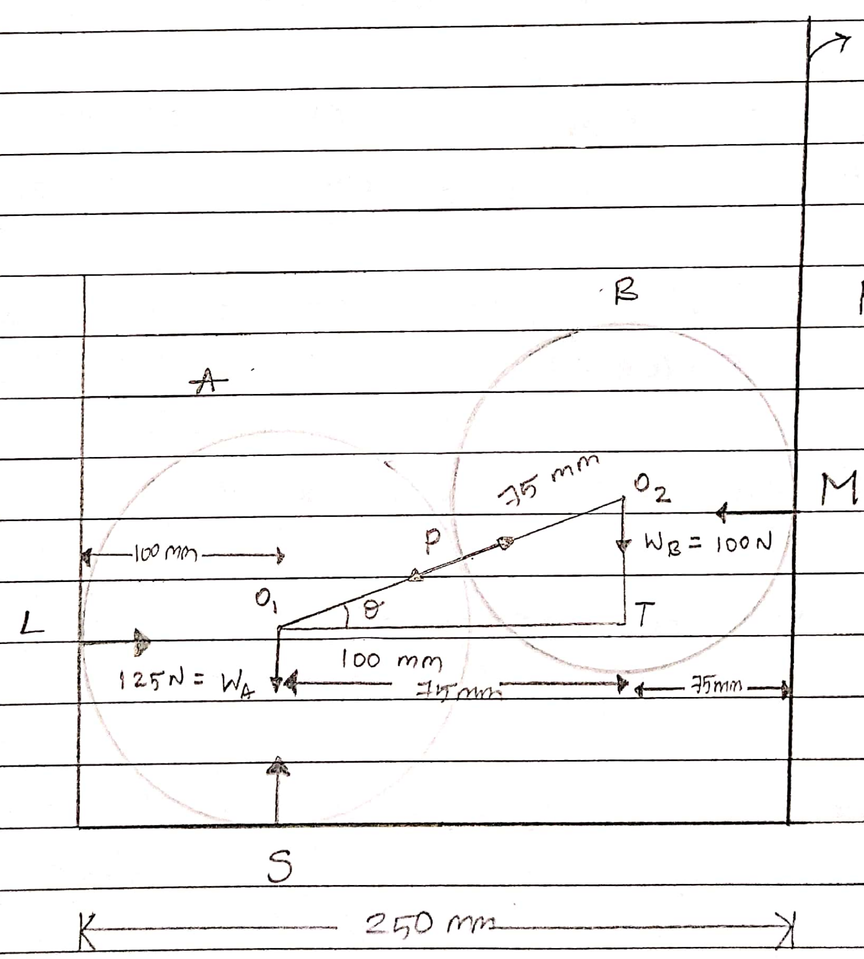

From Figure,

$O_1T = 75$ mm

$O_1O_2 = 175$ mm

$\cos\theta = \frac{O_1T}{O_1O_2}$

$\cos\theta = \frac{75}{175}$

$\cos\theta = 0.42$

$\theta = \cos^{-1}(0.42)$

$\theta = 64.6°$

Let us consider them in equilibrium, then,

Let us draw a free Body diagram for Sphere B.

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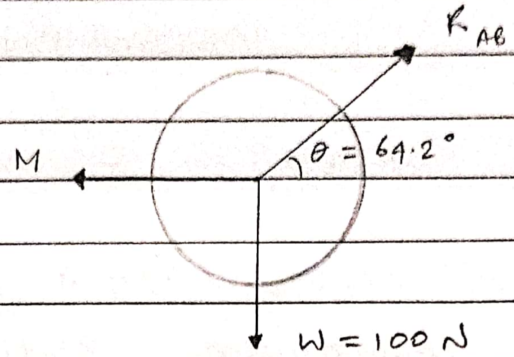
Sphere - B



Applying Lami's theorem

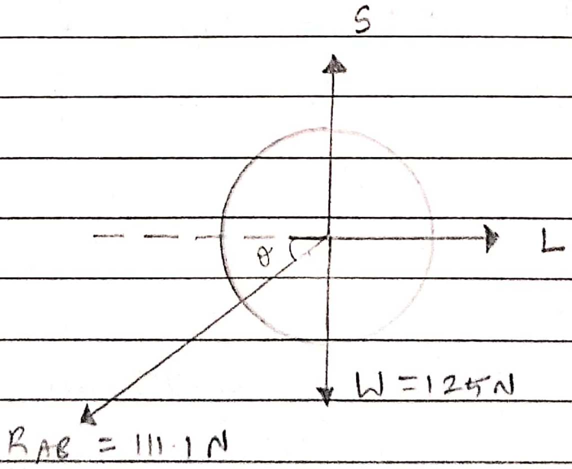
$$\frac{W}{\sin\theta} = \frac{R_{AB}}{\sin 90} = \frac{M}{\sin(90+\theta)}$$

$$\Rightarrow \frac{W}{\sin\theta} = R_{AB} = \frac{M}{\cos\theta} = \frac{M}{0.42}$$

$$R_{AB} = \frac{W}{\sin(64.2)} = \frac{100}{0.9} = \underline{\underline{111.1\ N}}$$

$$M = R_{AB} \cdot \cos\theta = 0.42 \times 111.1 = \underline{\underline{46.6\ N}}$$

Drawing FBD for Sphere A,



Applying Lami's theorem to R_{AB}, S and L,

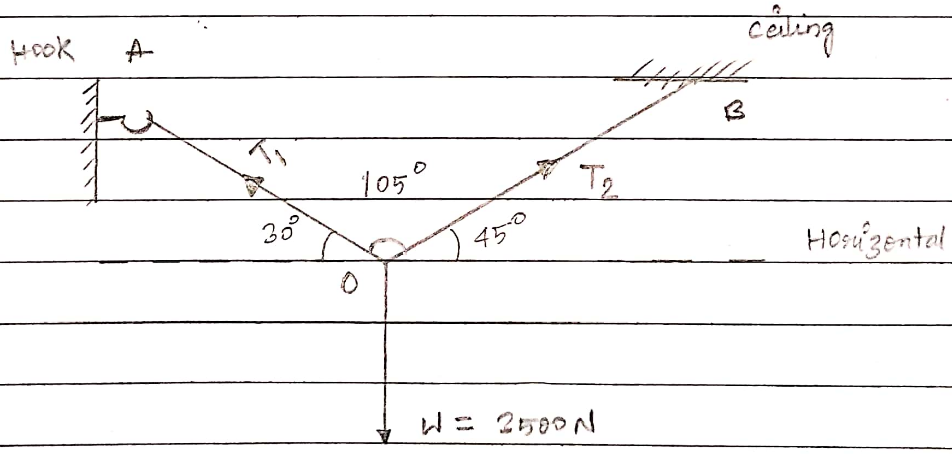
$$\frac{(S-W)}{\sin\theta} = \frac{L}{\cos\theta} = \frac{R_{AB}}{\sin 90}.$$

$$L = R_{AB} \cos\theta$$
$$S = R_{AB} \sin\theta$$

$$L = 111.1 \times 0.42 = 46.6 \text{ N}$$
$$S - W = 111.1 \times 0.90 = 99.9 \text{ N}$$

$$S = 99.9 + 125 \text{ N} = \underline{\underline{224.9 \text{ N}}}$$

Q.2. A machine weighing 3500N is supported by 2 chains attached to some point on the machine. One of these chains goes to the eye bolts on the wall and is inclined at 30° to the horizontal. The other goes to the hook in the ceiling and is inclined at 45° to horizontal. Find Tensions in the 2 chains.



Consider the machine is in equilibrium, by forces:

1. T_1 pulling at 30° to horizontal
2. T_2 pulling at 45° to horizontal
3. W Pulling at 0° to vertical, downwards.

Then by applying Lami's Theorem,

$$\frac{W}{\sin(105)} = \frac{T_1}{\sin(45+90)} = \frac{T_2}{\sin(90+30^\circ)}$$

$$\frac{W}{\sin(105)} = \frac{T_1}{\cos 45} = \frac{T_2}{\cos 30^\circ} = \frac{3500}{0.96} = 3645 \text{ N}.$$

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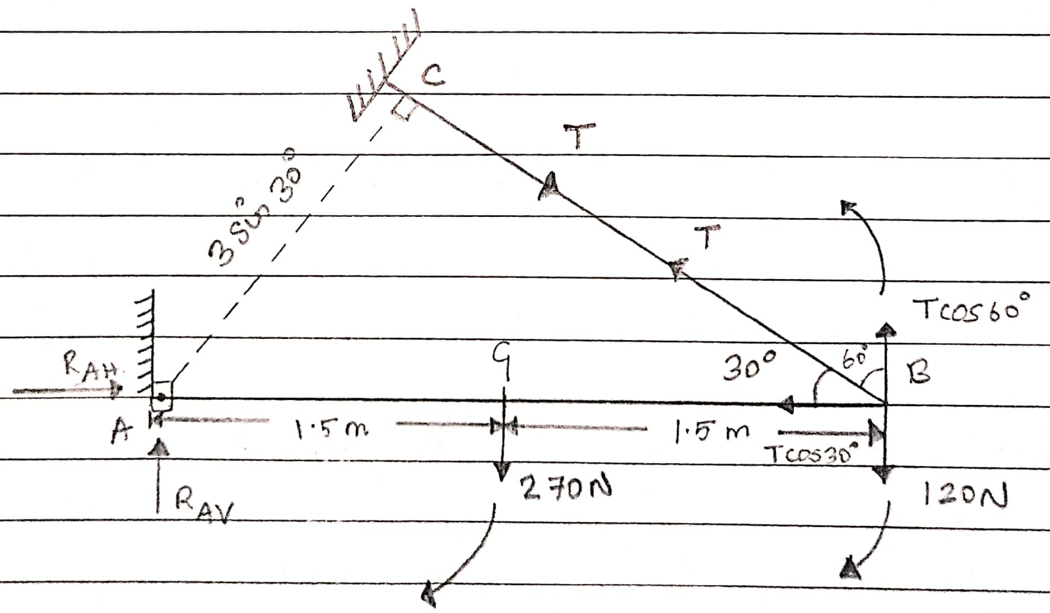
$$T_1 = \cos 45 \cdot (3645\,N)$$
$$= 0.709 \cdot (3645)$$
$$= 2577.4\,N$$

$$T_2 = \cos 30 \cdot (3645\,N)$$
$$= 0.86 \times 3645\,N$$
$$= 3156.6\,N$$

Q.3. A uniform rod AB of length 3 m and weight 270 N, is hinged at point A. At B, a weight of 120 N is hung. The rod is kept in a horizontal position by a string BC which makes an angle of 30° with the rod. Find the tension in the string at reaction at A.



Considering the rod is at equilibrium, let us take moments about point A.

$$T\cos 60^\circ \cdot 3 = 270 \cdot (1.5) + 120 \cdot (3)$$

$$T\cos 60^\circ = \frac{405 + 360}{3} = \frac{765}{3}$$

$$T = \frac{765}{3} \times 2 = 510\,N$$

As A has a hinge, at equilibrium,

$\Sigma F = 0$, $\Rightarrow$ $\Sigma F_x = 0$

$\therefore$ taking horizontal equilibrium,

$$R_{AH} - T\cos 30 = 0$$
$$R_{AH} = T\cos 30 = T \cdot (0.86)$$
$$= (510) \cdot (0.86)$$
$$= \underline{\underline{438.6N}}.$$

taking vertical equilibrium,

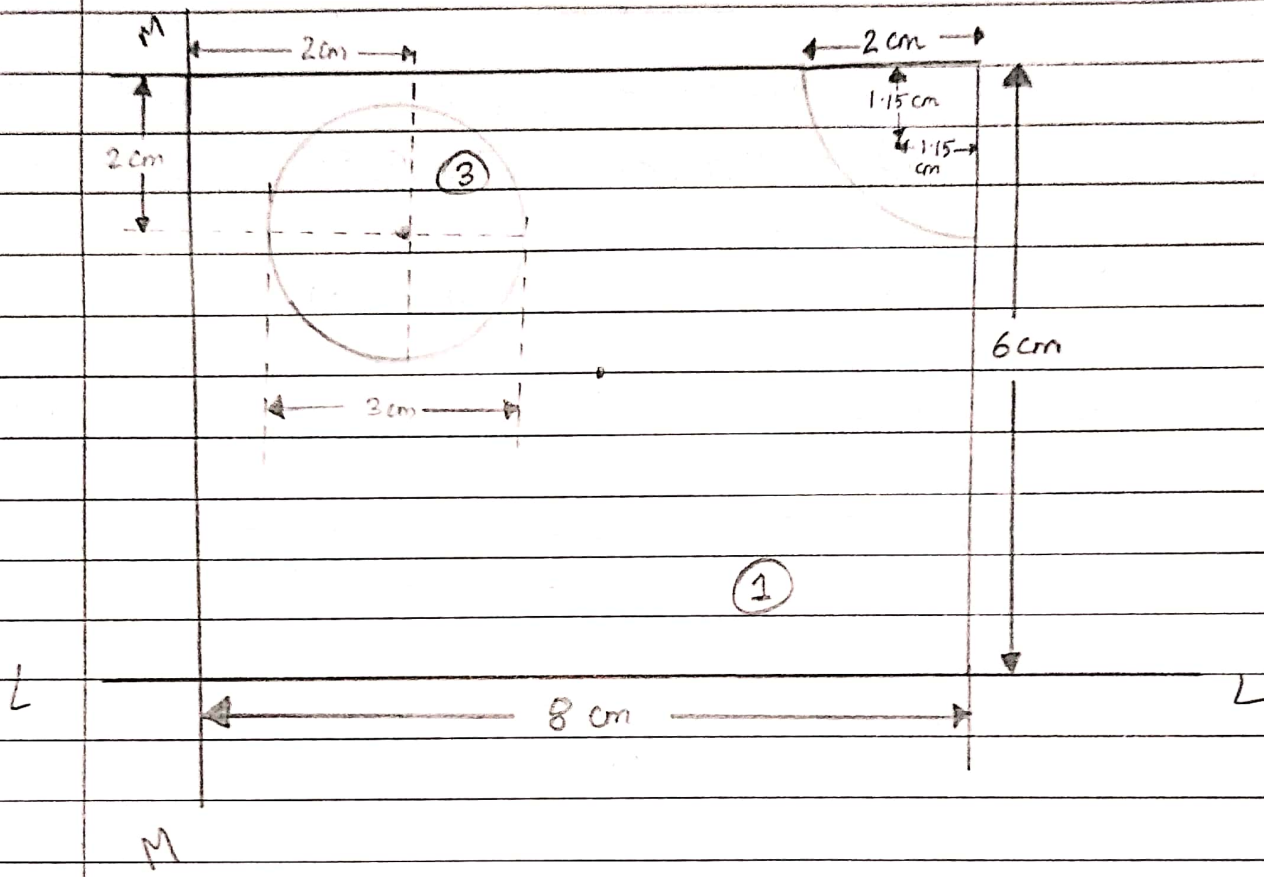
$$R_{AV} - 270 - 120 + \cancel{510}\ T\sin 30 = 0$$
$$R_{AV} - 390 + \frac{510}{2} = 0$$

$$R_{AV} = 390 - 255$$
$$= \underline{\underline{135\ N}}$$

$\therefore$ Reaction at A $= \sqrt{R_{AV}{}^2 + R_{AH}{}^2}$
$$= \sqrt{441^2 + 135^2}$$
$$= \underline{461.8N}$$

bow

Q.4 Determine the location of the centroid of the plan figure shown below.



The circle and quarter are negative space, while the rectangle is a + space.
Finding the individual centroids of each shape,

(1) Circle (radius = 3cm)

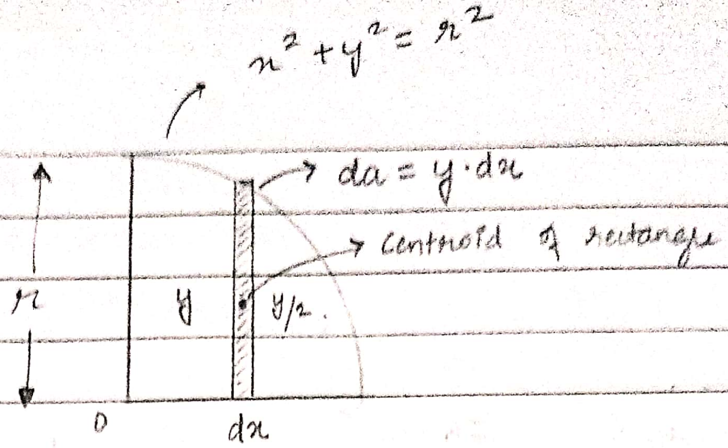
Position of x of centroid = 2 cm from MM
position of Y of centroid = 4 cm. from LL
Area $= \pi \cdot r^2 = \pi \cdot \left(\frac{3}{2}\right)^2 = \frac{9\pi}{4} = 7.06\ cm^2$

(2) Rectangle ($l \times b$ = 8cm x 6cm)

Position of X of centroid from MM = 4 cm
Position of Y of centroid from LL = 3 cm.
Area $= lb = 8 \times 6 = 48\ cm^2$

Rainbow

(3) Quarter:



We know, $\bar{Y} = \dfrac{\int y \cdot da}{\int da}$, $\bar{X} = \dfrac{\int x \cdot da}{\int da}$

$$\therefore \bar{Y} = \frac{\int_0^r \frac{y}{2} \cdot y \cdot dx}{\frac{\pi r^2}{4}} = \frac{\int_0^r y^2 \cdot dx}{\frac{\pi r^2}{2}}$$

$$\Rightarrow \frac{2}{\pi r^2} \int_0^r (r^2 - x^2) \cdot dx$$

$$= \frac{2}{\pi r^2} \left[r^2 x - \frac{x^3}{3} \right]_0^r = \frac{2}{\pi r^2} \cdot r^3 - \frac{r^3}{3}$$

$$= \frac{2}{\pi r^2} \cdot \frac{2r^3}{3} = \frac{4r}{3\pi} = \bar{Y}$$

Similarly, $\bar{X} = \dfrac{4r}{3\pi}$. given $r = 2$ cm

$\therefore \bar{Y} = \dfrac{8}{3 \cdot \pi} = \bar{X} = 1.15$ cm from centre.

$\therefore$ Position of $\bar{X}$ of centroid from MM = 7.15 cm
Position of $\bar{Y}$ of centroid from LL = 5.15 cm
Area = $\dfrac{\pi r^2}{4}$ = 3.14 cm^2

To then determine the location of the centroid of the given figure we can draw the following ~~T~~ Table with the calculated values from above.

	Components	Area (cm²)	Centroid distance X from MM (cm)	Centroid distance Y from LL (cm)	ax (cm³)	ay (cm³)
1	Rectangle	48 (+)	4	3	192 (+)	144 (+)
2	Circle	7.06 (−)	2	4	14.12 (−)	28.24 (−)
3	Quarter	3.14 (−)	7.15	5.15	22.4 (−)	16.17 (−)
	Total	37.8 Σa	-	-	155.48 Σax	99.59 Σay

∴ Distance of centroid co-ordinate X of the entire given figure from MM axis is

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{155.48}{37.8} = \underline{4.11 \text{ cm}}$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{99.59}{37.8} = \underline{2.64 \text{ cm.}}$$

Rainbow