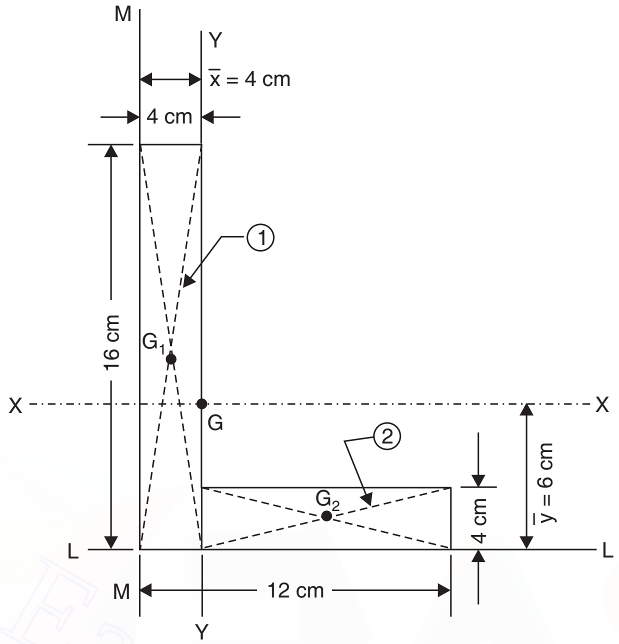

Fig. 4.20

To determine the location of the centroid of the plane figure we have the following table :

Components	*Area a (cm²)*	*Centroidal distance 'x' from MM (cm)*	*Centroidal distance 'y' from LL (cm)*	*ax (cm³)*	*ay (cm³)*
Rectangle (1) Rectangle (2)	16 × 4 = 64 8 × 4 = 32	2 8	8 2	128 256	512 64
	96 (Σa)	—	—	384 (Σ *ax*)	576 (Σ *ay*)

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{384}{96} = \textbf{4 cm. (Ans.)}$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{576}{96} = \textbf{6 cm. (Ans.)}$$

Example 4.2. *Determine the position of the centroid of I-section as shown in Fig. 4.21.*

Sol. Refer to Fig. 4.21.

Divide the composite figure into there simple areas :

(*i*) Rectangle (10 cm × 2 cm) – top flange ...(1)

(*ii*) Rectangle (25 cm × 2 cm) – web ...(2)

(*iii*) Rectangle (15 cm × 3 cm) – bottom flange ...(3)

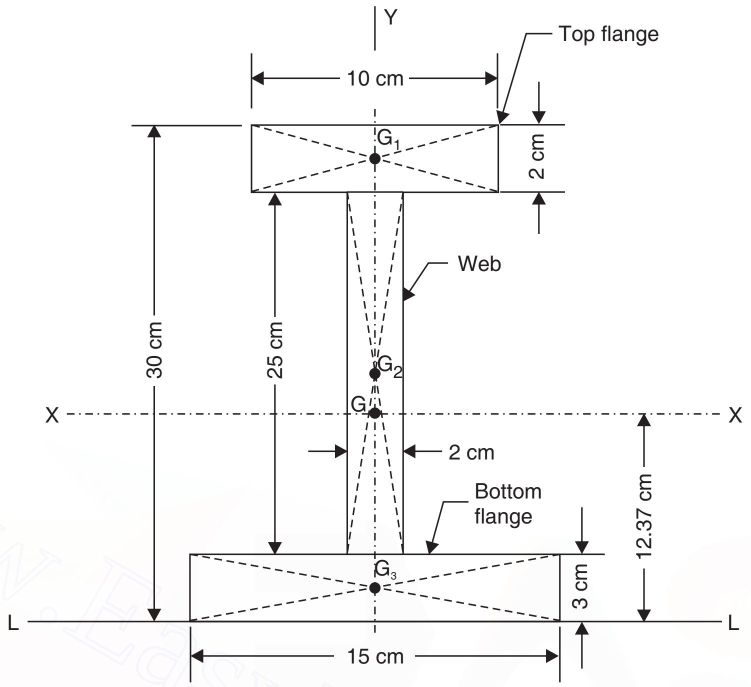

Fig. 4.21

To determine the location of the centroid of the plane figure we have the following table.

Components	*Area 'a'* *(cm²)*	*Centroidal distance 'y' from LL (cm)*	*ay* *(cm³)*
Rectangle (1)	10 × 2 = 20	29	580
Rectangle (2)	25 × 2 = 50	15.5	775
Rectangle (3)	15 × 3 = 45	1.5	67.5
	115 (Σ *a*)	—	1422.5 (Σ *ay*)

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{1422.5}{115} = \mathbf{12.37\ cm.\ (Ans.)}$$

Example 4.3. *Using the analytical method, determine the centre of gravity of the plane uniform lamina shown in Fig. 4.22.*

Sol. Refer to Fig. 4.22.

The lamina may be divided into three parts :

(*i*) A triangle marked (1)

(*ii*) A semi-circle marked (2)

(*iii*) A rectangle marked (3)

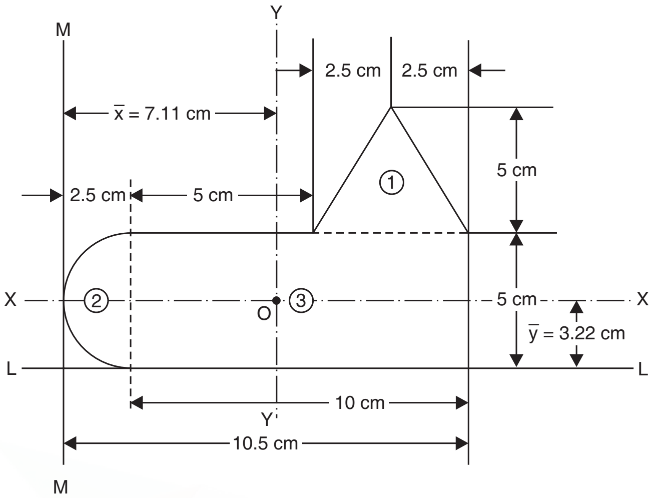

Fig. 4.22

The area of these components, their centroidal distances from the *LL*-axis and *MM*-axis and the moments of the areas of individual components about *LL*-axis and *MM*-axis are tabulated below :

Components	*Area (a) (cm²)*	*Centroidal distance 'x' from MM (cm)*	*Centroidal distance 'y' from LL (cm)*	*ax (cm³)*	*ay (cm³)*
Triangle (1)	$\frac{5 \times 5}{2} = 12.50$	2.5 + 5 + 2.5 = 10	5 + 5/3 = 6.67	125	83.4
Semi-circle (2)	$\frac{\pi \times 2.5^2}{2} = 9.82$	$2.5 - \frac{4 \times 2.5}{3\pi}$ = 1.44	2.5	14.14	24.55
Rectangle (3)	10 × 5 = 50.00	2.5 + 5 = 7.5	2.5	375	125
Total	72.32 (Σa)	—	—	514.14 (Σax)	232.95 (Σay)

Distance of the centroid from *MM*-axis,

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{514.13}{72.32} = \textbf{7.11 cm. (Ans.)}$$

Distance of the centroid from *LL*-axis

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{232.95}{72.32} = \textbf{3.22 cm. (Ans.)}$$

Example 4.4. *Determine the location of the centroid of the plane figure shown in Fig. 4.23.*

Sol. Refer to Fig. 4.23.

Divide the composite figure into three simple areas :

(*i*) a rectangle (7 cm × 5 cm) ...(1)

(*ii*) a quadrant (2 cm radius) ...(2)

(*iii*) a circle (3 cm dia.) ...(3)

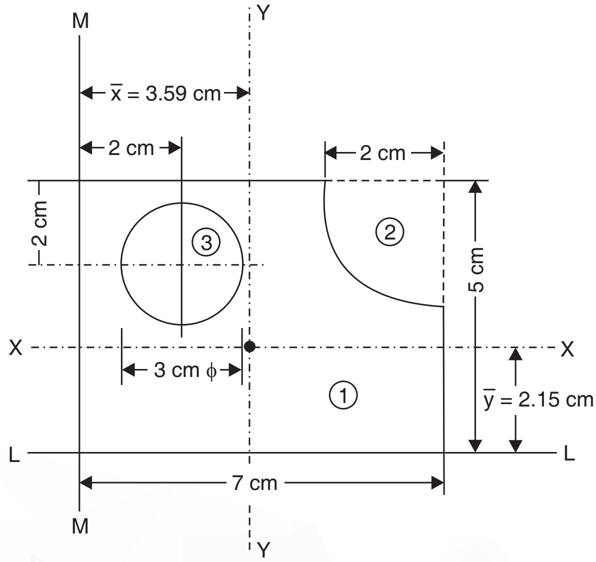

Fig. 4.23

The rectangle is a positive area. The quadrant and hole are treated as negative areas.

To determine the location of the centroid of the plane figure, we have the following table :

Components	*Area 'a' (cm^2)*	*Centroidal distance 'x' from M-M (cm)*	*Centroidal distance 'y' from L-L (cm)*	*ax (cm^3)*	*ay (cm^3)*
Rectangle (1)	$7 \times 5 = 35$ (+)	3.5	2.5	122.5(+)	87.5(+)
Quadrant (2)	$\frac{\pi}{16} d^2 = \frac{\pi}{16} \times 4^2$ = 3.14 (–)	6.15	4.15	19.31(–)	13.03(–)
Circle (3)	$\frac{\pi}{4} d^2 = \frac{\pi}{4} \times 3^2$ = 7.07 (–)	2	3	14.14(–)	21.21(–)
Total	24.79 (Σa)	—	—	89.05 ($\Sigma\, ax$)	53.26 ($\Sigma\, ay$)

Distance of centroid from *MM*-axis

$$\bar{x} = \frac{\Sigma ay}{\Sigma a} = \frac{89.05}{24.79} = \mathbf{3.59\ cm.\ (Ans.)}$$

Distance of centroid from *LL*-axis

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{53.26}{24.79} = \mathbf{2.15\ cm.\ (Ans.)}$$

Example 4.8. *Where must a circular hole of 1 metre radius be punched out of a circular disc of 3 metres radius so that the centre of gravity of the remainder be 2 cm from the centre of the disc ?*

Sol. Refer to Fig. 4.29. Since, the remaining body is symmetrical about *XX*-axis, therefore $\bar{y} = 0$.

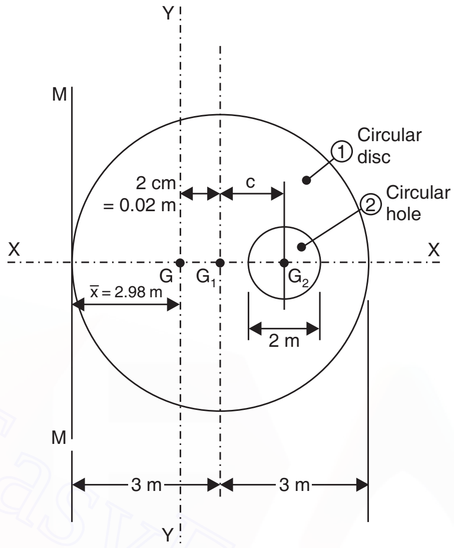

Fig. 4.29

Let '*c*' be the distance of the centre of the circular hole from the centre of the circular disc. For finding out the value of '*c*' we have the following table :

Components	*Area 'a'* (m^2)	*Centroidal distance 'x' from MM (m)*	*ax* (m^3)
Circular disc (1)	$\pi \times 3^2 = 9\pi$	3	27π
Circular hole (2)	$-\pi \times 1^2 = -\pi$	$(3 + c)$	$-\pi(3 + c)$
	$\Sigma a = 8\pi$	—	$\Sigma ax = 27\pi - \pi(3 + c)$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{27\pi - \pi(3 + c)}{8\pi} = \frac{24 - c}{8}$$

But $\bar{x} = 2.98$ m = (given) from *MM*

$$\therefore \quad 2.98 = \frac{24 - c}{8}$$

or $24 - c = 23.84$

or c **= 0.16 m. (Ans.)**

Example 4.9. *A square hole is punched out of a circular lamina, a diagonal of such a square being along any radius of the circle with one vertex at the centre of the circular lamina. It is said that the length of the said diagonal is equal to the radius of circular lamina. Find the centre of gravity of the remainder, if r be the radius of the circle.*

Sol. Refer to Fig. 4.30. Since, the remaining body is symmetrical about X-axis, therefore, $\bar{y} = 0$.

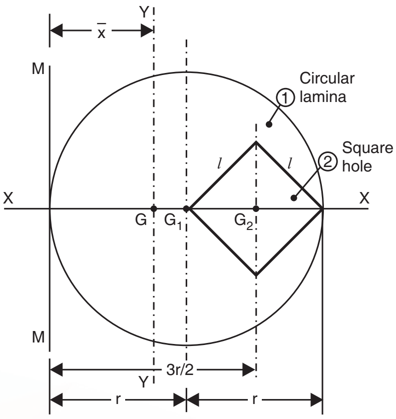

Fig. 4.30

To find out the centre of gravity of the remainder, we have the following table :

Components	*Area 'a'*	*Centroidal distance 'x' from MM*	*ax*
Circular lamina (1)	πr^2	r	πr^3
Square hole (2)	$-\dfrac{r^2}{2}$ $\left[\begin{array}{l} l^2 + l^2 = r^2 \\ \therefore \ l^2 = \dfrac{r^2}{2} \end{array}\right]$	$\dfrac{3r}{2}$	$-\dfrac{3r^3}{4}$
	$\Sigma a = \pi r^2 - \dfrac{r^2}{2}$	—	$\Sigma ax = \pi r^3 - \dfrac{3r^3}{4}$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{\pi r^3 - \dfrac{3r^3}{4}}{\pi r^2 - \dfrac{r^2}{2}} = \frac{r^2\left(\pi - \dfrac{3}{4}\right)}{r^2\left(\pi - \dfrac{1}{2}\right)}$$

$$= \frac{r\,(4\pi - 3)}{2\,(2\pi - 1)} \text{ from } MM$$

$$= \boldsymbol{\frac{r\,(4\pi - 3)}{4\pi - 2}} \textbf{ from MM. (Ans.)}$$

Example 4.10. *From the circular lamina of radius 'a' a smaller circular hole, one quarter the size of the given one is punched out so that its centre bisects a radius of the larger circle. Find the centre of gravity of the remainder.*

Sol. Refer to Fig. 4.31.

$$x^2 = \frac{a^2}{b} y$$

$$\bar{x} = \frac{\int_0^a ydx \,.\, x}{\int_0^a ydx} = \frac{\frac{b}{a^2}\int_0^a x^3\, dx}{\frac{b}{a^2}\int_0^a x^2 dx}$$

$$\frac{a^4/4}{a^3/3} = \mathbf{\frac{3a}{4}}. \textbf{ (Ans.)}$$

$$\bar{y} = \frac{\int_0^a ydx \,.\, \frac{y}{2}}{\int_0^a ydx} = \frac{\frac{1}{2}\int_0^a \frac{b^2}{a^4} x^4\, dx}{\int_0^a \frac{b}{a^2} x^2\, dx}$$

$$\frac{b}{2a^2} \cdot \frac{\frac{a^5}{5}}{\frac{a^3}{3}} = \mathbf{\frac{3b}{10}}. \textbf{ (Ans.)}$$

Example 4.12. *Determine the c.g. of the area of a sector of angle α of a circle of radius r as shown in Fig. 4.33.*

Sol. Refer to Fig. 4.33.

Area $OPQ = \frac{1}{2} r^2\, d\theta$. Its c.g. will be on OP at a distance $\frac{2r}{3}$ from O, so that its distance from OY will be $\frac{2r}{3}\cos\theta$

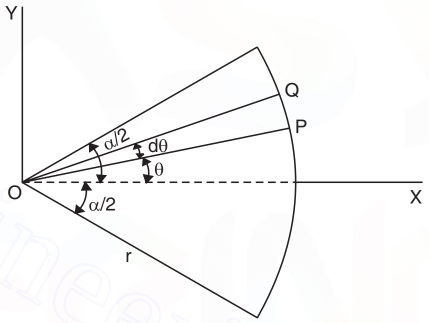

Fig. 4.33

$$\bar{x} = \frac{\int_0^{\alpha/2} \frac{2r}{3}\cos\theta\, \frac{r^2}{2}\, d\theta}{\int_0^{\alpha/2} \frac{r^2}{2}\, d\theta} = \frac{\frac{2r}{3}\int_0^{\alpha/2} \cos\theta\, d\theta}{\int_0^{\alpha/2} d\theta}$$

$$= \frac{2r}{3} \cdot \frac{\sin\alpha/2}{\alpha/2} = \mathbf{\frac{4r}{3\alpha}\,.\,\sin\frac{\alpha}{2}}. \textbf{ (Ans.)}$$

$\bar{\mathbf{y}}$ **= 0. (Ans.)**

Example 4.13. *Find the position of centre of gravity of the plane lamina in the form of a quarter of an ellipse, shown in Fig. 4.34.*

Sol. Refer to Fig. 4.34.

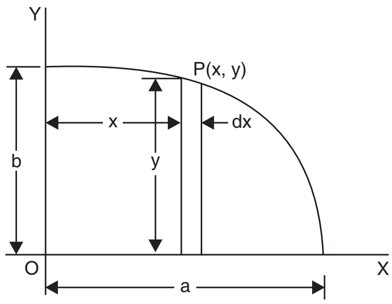

Fig. 4.34

$$\bar{x} = \frac{\int_0^a xy\, dx}{\int_0^a y\, dx}$$

$$\bar{y} = \frac{\int_0^a \frac{y}{2}\, y dx}{\int_0^a y\, dx}.$$

The equation of the ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

From which $\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} = \frac{a^2 - x^2}{a^2}$

$$y = b/a \sqrt{a^2 - x^2}$$

$\therefore$ $\int_0^a xy\, dx = \int_0^a x \,.\, \frac{b}{a} \sqrt{a^2 - x^2}\; dx$

$$= -\frac{b}{2a} \int_0^a (a^2 - x^2)^{1/2} (-2x)\, dx$$

$$= -\frac{b}{2a} \cdot \frac{2}{3} \left| (a^2 - x^2)^{3/2} \right|_0^a = \frac{b}{3a} \,.\, a^3 = \frac{a^2 b}{3}$$

$$\int_0^a y\, dx = \int_0^a \frac{b}{a} \sqrt{a^2 - x^2}\; dx$$

$$= \frac{b}{a} \left| \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right|_0^a = \frac{b}{a} \cdot \frac{a^2}{2} \quad \frac{\pi}{2} = ab\, \frac{\pi}{4}$$

$\therefore$ $\bar{x} = \frac{a^2 b}{3} \times \frac{4}{ab\pi}$

or $\bar{x} = \mathbf{\frac{4a}{3\pi}}$. **(Ans.)**

Similarly, $\bar{y} = \mathbf{\frac{4b}{3\pi}}$. **(Ans.)**

Example 4.14. *A hemisphere of diameter 60 mm is placed on the top of a cylinder, whose diameter is also 60 mm. The height of the cylinder is 75 mm. Find the common C.G. of the composite body.*

Sol. Refer to Fig. 4.35.

Since, the composite solid is symmetrical about Y-axis, so the c.g. will lie on Y-axis and we shall, therefore, find out $\bar{y}$ only.

Volume of hemisphere,

$$V_1 = \frac{2}{3} \pi r^3 = \frac{2}{3} \times \pi \times 30^3 = 18000\, \pi \text{ mm}^3$$

(*ii*) **Free-body diagram of roller P.** Free-body diagram of roller P is shown in Fig. 4.16 (*c*). The roller P has points of contact at A and D. The forces acting on the roller P are :

(*a*) Weight W

(*b*) Reaction R_A at point A

(*c*) Reaction R_D at point D.

The reactions R_A and R_D will pass through point F, *i.e.*, centre of roller P. These two reactions are unknown. If W is given, then these reactions can be calculated.

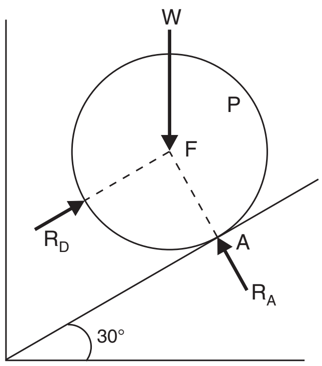

Fig. 4.16 (*c*)

(*iii*) **Free-body diagram of rollers P and Q taken together.** When the rollers P and Q are taken together, then points of contacts are A, B and C. The free-body diagram of this case is shown in Fig. 4.16 (*d*). The forces acting are :

(*a*) Weight W on each roller

(*b*) Reaction R_A at point A

(*c*) Reaction R_B at point B

(*d*) Reaction R_C at point C.

In this case there will be no reaction at point D.

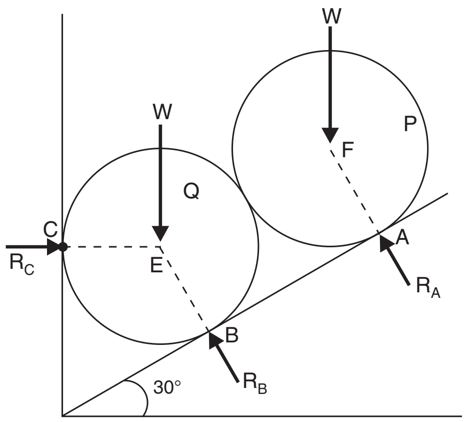

Fig. 4.16 (*d*)

Problem 4.14. *Two identical rollers, each of weight W = 1000 N, are supported by an inclined plane and a vertical wall as shown in Fig. 4.17 (a). Find the reactions at the points of supports A, B and C. Assume all the surfaces to be smooth.*

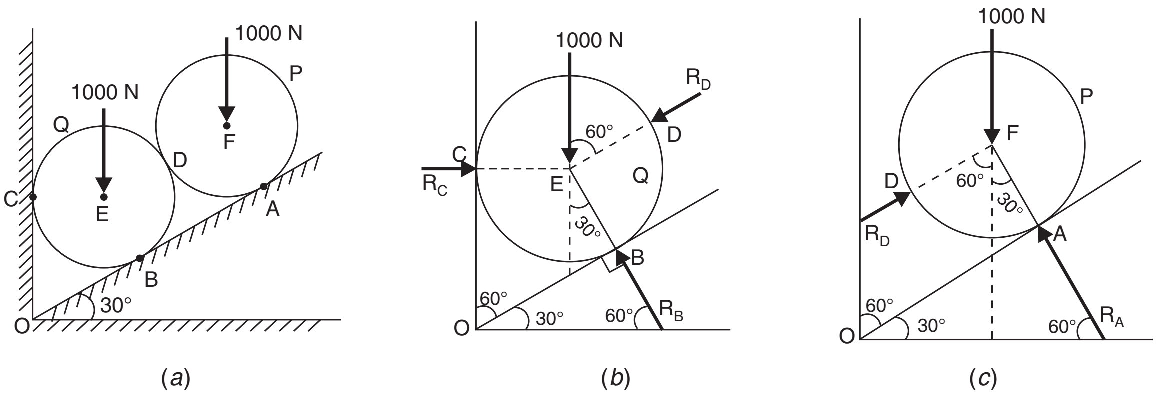

Fig. 4.17

Sol. Given :

Weight of each roller = 1000 N

Radius of each roller is same. Hence line EF will be parallel to AB.

Equilibrium of Roller P

First draw the free-body diagram of roller P as shown in Fig. 4.17 (c). The roller P has points of contact at A and D. Hence the forces acting on the roller P are :

(i) Weight 1000 N acting vertically downward.

(ii) Reaction R_A at point A. This is normal to OA.

(iii) Reaction R_D at point D. This is parallel to line OA.

The resultant force in x and y directions on roller P should be zero.

For $\Sigma F_x = 0$, we have

$$R_D \sin 60° - R_A \sin 30° = 0 \quad \text{or} \quad R_D \sin 60° = R_A \sin 30°$$

$$\therefore \qquad R_D = R_A \frac{\sin 30°}{\sin 60°} = 0.577\, R_A \qquad ...(i)$$

For $\Sigma F_y = 0$, we have

$$R_D \cos 60° + R_A \cos 30° - 1000 = 0$$

$$(0.577\, R_A) \cos 60° + R_A \cos 30° = 1000 \qquad (\because \; R_D = 0.577\, R_A)$$

or $$0.577 \times 0.5\, R_A + R_A \times 0.866 = 1000$$

$$1.1545\, R_A = 1000 \quad \text{or} \quad R_A = \frac{1000}{1.1545} = \mathbf{866.17\ N.} \quad \textbf{Ans.}$$

Substituting this value in equation (i), we get

$$R_D = 0.577 \times 866.17 = 499.78$$

Equilibrium of Roller Q

The free-body diagram of roller Q is shown in Fig. 4.17 (b). The roller Q has points of contact at B, C and D.

The forces acting on the roller Q are :

(i) Weight W = 1000 N ;

(ii) Reaction R_B at point B and normal to BO ;

(iii) Reaction R_C at point C and normal to CO ; and

(iv) Reaction R_D at point D and parallel to BO.

For $\Sigma F_x = 0$, we have

$$R_B \sin 30° + R_D \sin 60° - R_C = 0$$

or $$R_B \times 0.5 + 449.78 \times 0.866 - R_C = 0$$

or $$R_C = 0.5\, R_B + 432.8 \qquad ...(ii)$$

For $\Sigma F_y = 0$, we have

$$R_B \times \cos 30° - 1000 - R_D \times \cos 60° = 0$$

or $$R_B \times 0.866 - 1000 - 499.78 \times 0.5 = 0 \qquad (\because \; R_D = 499.78)$$

or $$0.866\, R_B - 1249.89 = 0 \quad \text{or} \quad R_B = \frac{1249.89}{0.866} = \mathbf{1443.3\ N.} \quad \textbf{Ans.}$$

Substituting this value in equation (ii), we get

$$R_C = 0.5 \times 1443.3 + 432.8 = \mathbf{1154.45\ N.} \quad \textbf{Ans.}$$

The forces acting on the given body ABC are :

(i) Force F acting at point C.

(ii) Reaction R_A at a point A.

Only two forces are acting on the body and the body is in equilibrium. But the two forces can be in equilibrium only if they are equal in magnitude, opposite in direction and have the same line of action. As the force F is acting at point C, hence reaction R_A should also pass through point C. Therefore, the line of action of reaction R_A is along line AC. The line of action of F should also be along AC, *i.e.*, the force F will be making an angle α with the horizontal.

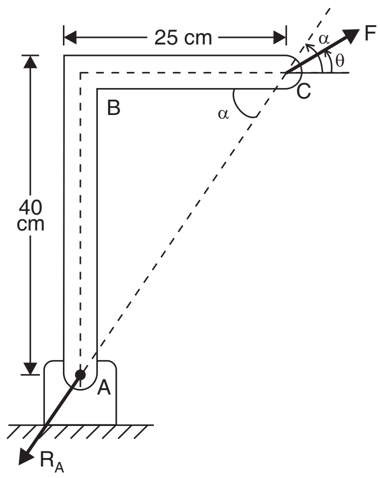

Fig. 4.23

Now from ΔABC, $\tan \alpha = \dfrac{AB}{BC} = \dfrac{40}{25} = 1.6$

$\therefore \quad \alpha = \tan^{-1} 1.6 = 57.99°$

$\therefore \quad \theta = \alpha = \mathbf{57.99°}$. **Ans.**

Problem 4.20. *A horizontal force 200 N is applied to the sloping bar BCD whose bottom rests on a horizontal plane, as shown in Fig. 4.24. Its upper end is pinned at B to the horizontal bar AB which has a pinned support at A. What couple M must be applied to AB to hold the system in equilibrium ? What is the magnitude of the pin reaction at B ? Assume the bars to be weightless and pins at A and B to be smooth.*

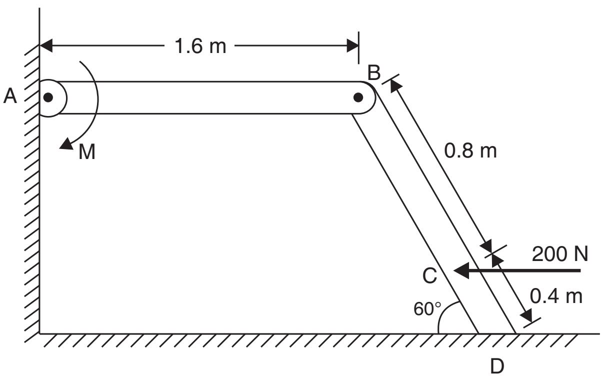

Fig. 4.24

Sol. Given :

Length $AB = 1.6$ m, Length $BD = 1.2$ m, $BC = 0.8$ m and $CD = 0.4$ m

Horizontal force at $C = 200$ N

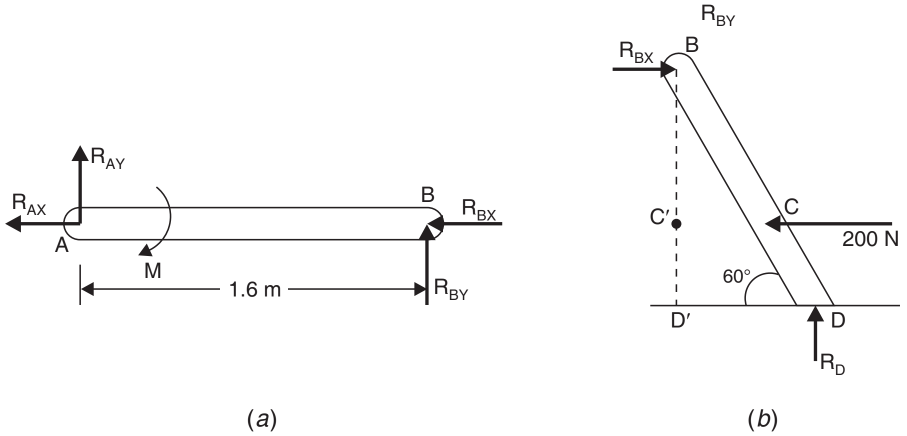

Fig. 4.25

Let M = Couple applied A to bar AB to keep the system in equilibrium.

R_B = Reaction at B

R_{BX} and R_{BY} are the horizontal and vertical components of reaction R_B.

The free-body diagram of the sloping bar BCD is shown in Fig. 4.25 (*b*). The reaction R_D at point D is vertical. Also show R_{BX} and R_{BY} at B.

Since the bar is in equilibrium, first apply $\Sigma F_x = 0$, which gives

$$R_{BX} = 200 \text{ N}$$

Now apply $\Sigma F_y = 0$, then $R_{BY} = R_D$ ...(*i*)

The magnitude of R_{BY} and R_D is unknown. To find their values apply ΣM at the pin B.

Taking moments of all forces at point B, we get

$$R_D \times DD' = 200 \times BC'$$

or $$R_D \times BD \cos 60° = 200 \times BC \sin 60° \quad (\because DD' = BD \cos 60° \text{ and } BC' = BC \sin 60°)$$

or $$R_D \times 1.2 \cos 60° = 200 \times 0.8 \sin 60°$$

or $$R_D = \frac{200 \times 0.8 \times 0.866}{1.2 \times \cos 60°} = 230.93 \text{ N}$$

$\therefore$ From equation (*i*), $R_{BY} = R_D = 230.93$ N

Reaction at B, $R_B = \sqrt{R_{BX}^2 + R_{BY}^2} = \sqrt{200^2 + 230.93^2}$ = **305.4 N. Ans.**

Now draw the free-body diagram of the horizontal bar AB as shown in Fig. 4.25 (*a*). Show the reactions R_{BX} and R_{BY} at B. Also show the reactions R_{AY} and R_{AX} at A. The applied couple M is also shown in the figure.

Apply $\Sigma M_A = 0$

$$M = R_{BY} \times 1.6 = 230.93 \times 1.6 = \mathbf{369.44 \text{ Nm.}} \quad \textbf{Ans.}$$

4.4. EQUILIBRIUM OF A BODY UNDER THREE FORCES

The three forces acting on a body which is in equilibrium may be either concurrent or parallel. Let us first consider that the body is in equilibrium when three forces, acting on the body, are concurrent. This is shown in Fig. 4.26.

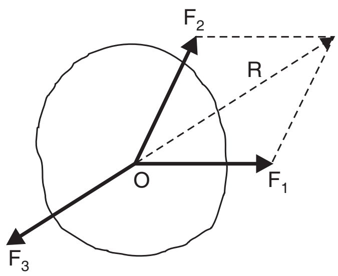

Fig. 4.26

(*a*) *When three forces are concurrent.* The three concurrent forces F_1, F_2 and F_3 are acting on a body at point O and the body is in equilibrium. The resultant of F_1 and F_2 is given by R. If the force F_3 is collinear, equal and opposite to the resultant R, then the body will be in equilibrium. The force F_3 which is equal and opposite to the resultant R is known as *equilibrant.* Hence for three concurrent forces acting on a body when the body is in equilibrium, the resultant of the two forces should be equal and opposite to the third force.

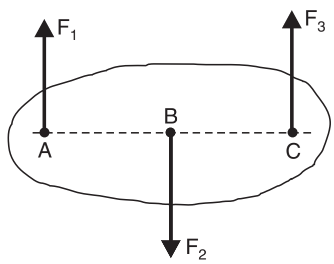

Fig. 4.26(*a*)

(*b*) *When three forces are parallel.* Fig. 4.26 (*a*) shows a body on which three parallel forces F_1, F_2 and F_3 are acting and the body is in equilibrium. If three forces F_1, F_2 and F_3 are acting in the same direction, then there will be a resultant $R = F_1 + F_2 + F_3$ and body will not be in equilibrium. The three forces are acting in opposite direction and their magnitude is so adjusted that there is no resultant force and body is in equilibrium. Let us suppose that F_2 is acting in opposite direction as shown in Fig. 4.26 (*a*).

Problem 4.21. *A body weighing 2000 N is suspended with a chain AB 2 m long. It is pulled by a horizontal force of 320 N as shown in Fig. 4.28. Find the force in the chain and the lateral displacement (i.e., x) of the body.*

Sol. Given :

Weight suspended at B = 2000 N

Length AB = 2 m

Horizontal force at B = 320 N

Find : Force in AB and value of x

Let F = Force in chain AB

θ = Angle made by AB with horizontal.

The free-body diagram of the point B is shown in Fig. 4.28 (*b*).

The point B is in equilibrium under the action of three forces. Hence using Lami's theorem, we get

$$\frac{F}{\sin 90°} = \frac{2000}{\sin (180° - \theta)} = \frac{320}{\sin (90° + \theta)}$$

or $$\frac{F}{1} = \frac{2000}{\sin \theta} = \frac{320}{\cos \theta} \qquad [\because \sin (180° - \theta) = \sin \theta, \sin (90° + \theta) = \cos \theta]$$

or $$F \sin \theta = 2000 \qquad ...(i)$$

and $$F \cos \theta = 320 \qquad ...(ii)$$

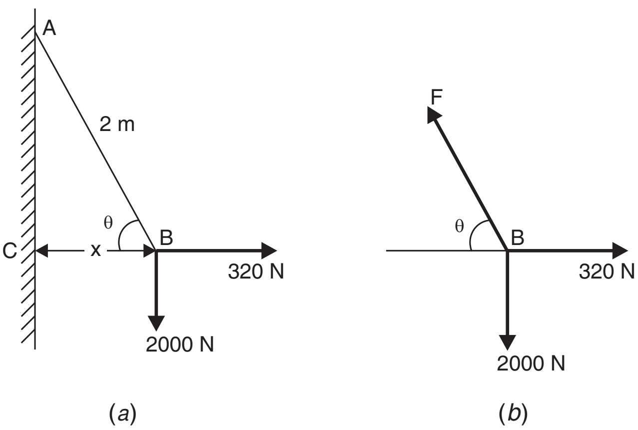

Fig. 4.28

Dividing equation (*i*) by equation (*ii*), we get

$$\tan \theta = \frac{2000}{320} = 6.25$$

$\therefore$ $$\theta = \tan^{-1} 6.25 = 80.9°$$

Substituting this value of θ in equation (*i*), we get

$$F \sin 80.9° = 2000 \quad \text{or} \quad F = \frac{2000}{\sin 80.9°} = \mathbf{2025.5\ N.} \quad \textbf{Ans.}$$

Now from Fig. 4.23 (*a*), $\cos \theta = \frac{x}{2}$ or $x = 2 \times \cos \theta = 2 \times \cos 80.9°$

$$= \mathbf{0.3163\ m.} \quad \textbf{Ans.}$$

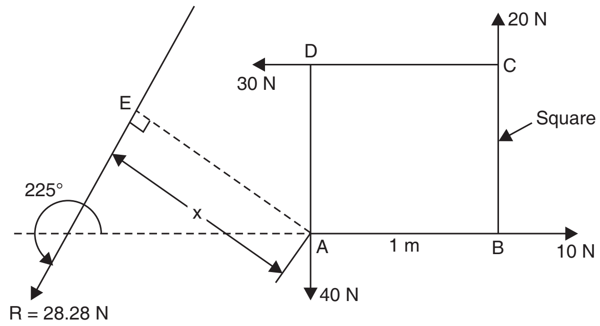

Fig. 3.35

Example 3.9. *A body is under the action of four coplanar forces as shown in Fig. 3.36. Find the magnitude, direction and position of resultant of the given force system.*

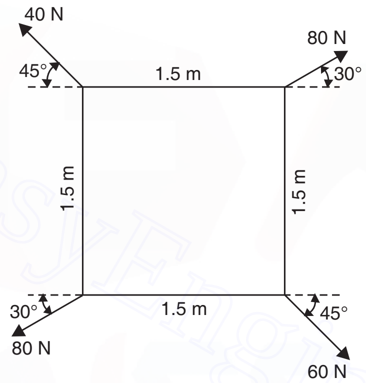

Fig. 3.36

Sol. Refer to Fig. 3.37.

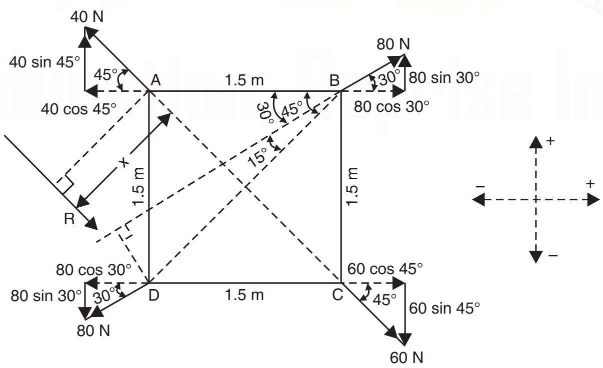

Fig. 3.37

Magnitude of resultant, *R* = ?

Resolving the forces *horizontally*,

$$\Sigma H = 80 \cos 30° + 60 \cos 45° - 80 \cos 30° - 40 \cos 45°$$
$$= 80 \times 0.866 + 60 \times 0.707 - 80 \times 0.866 - 40 \times 0.707 = 14.14 \text{ N}.$$

Resolving the forces *vertically* :

$$\Sigma V = 80 \sin 30° - 60 \sin 45° - 80 \sin 30° + 40 \sin 45° = -14.14 \text{ N}$$

Resultant, $$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(14.14)^2 + (-14.14)^2}$$

$$= 20 \text{ N}$$

i.e., $\boldsymbol{R = 20}$ **N. (Ans.)**

Direction of resultant, α = ?

The resultant will act at an angle with the horizontal so that,

$$\tan \alpha = \frac{\Sigma V}{\Sigma H} = \frac{14.14}{14.14} = 1$$

or $\alpha = 45°$. (Fig. 3.38).

Position of the resultant = ?

The position of the resultant 'R' can be determined by using the relation :

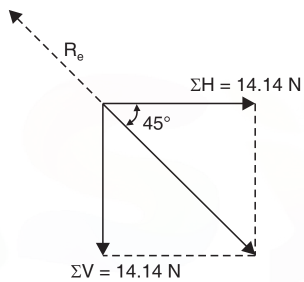

Fig. 3.38

Moment of resultant about A = algebraic sum of moments of the rectangular components of all forces about A.

$$- R \times x = 40 \sin 45° \times 0$$
$$+ 40 \cos 45° \times 0$$
$$+ 80 \sin 30° \times 1.5$$
$$+ 80 \cos 30° \times 0$$
$$+ 60 \cos 45° \times 1.5 - 60 \sin 45° \times 1.5 - 80 \cos 30° \times 1.5$$
$$+ 80 \sin 30° \times 0$$

$$- 20 \times x = 0 + 0 + 60 + 0 + 63.63 - 63.63 - 103.92 + 0$$

or $-20\,x = -43.92$

or $\boldsymbol{x = 2.196}$ **m from A (Fig. 3.37). (Ans.)**

Example 3.10. *The lever LMN of a component of a machine is hinged at M, and is subjected to a system of coplanar forces as shown in Fig. 3.39. Neglecting friction determine :*

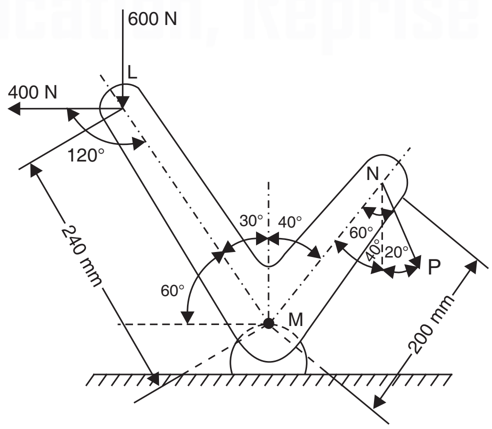

Fig. 3.39

(*i*) *The magnitude of the force P to keep the lever in equilibrium.*

(*ii*) *The magnitude and direction of the reaction at M.*

Sol. Magnitude of force *P* = ?

Taking moments about the hinge M, we get

$$\Sigma M : P \times 200 \sin 60° = 600 \times 240 \cos 60° + 400 \times 240 \sin 60°$$

or
$$P \times 200 \times 0.866 = 600 \times 240 \times 0.5 + 400 \times 240 \times 0.866$$
$$173.2\,P = 155136$$

$\therefore$ $\mathbf{P = 895\ N.}$ **(Ans.)**

Magnitude of reaction at M, R_M = ?

Resolving the forces horizontally,

$$\Sigma H = +\,400 - P \sin 20°$$
$$= +\,400 - 895 \times 0.342 = 93.9 \text{ N}$$

Resolving the forces vertically,

$$\Sigma V = 600 + P \cos 20°$$
$$= 600 + 895 \times 0.939 = 1440.4 \text{ N}$$

$\therefore$
$$R_M = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(93.9)^2 + (1440.4)^2}$$
$$= \mathbf{1443.46\ N.}\ \textbf{(Ans.)}$$

Direction of the reaction at *M* = ?

Let θ = angle, which the reaction at M makes with the horizontal.

Then
$$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{93.9}{1440.4} = 0.065$$

or θ = **3.73° or 3°44′. (Ans.)**

Example 3.11. *A square LMNS has forces acting along its sides as shown in Fig. 3.40. Find the values of F_1 and F_2, if the system reduces to a couple. Also find magnitude of the couple, if the side of the square is 2 m.*

Sol. Refer to Fig. 3.40.

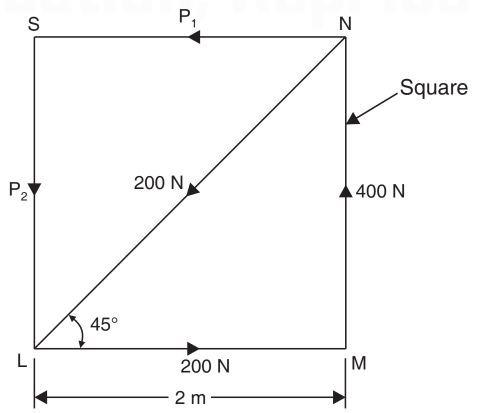

Fig. 3.40

Values of P_1 and P_2 = ?

We know that if *the system reduces to a couple, the resultant force in horizontal and vertical directions is zero.* Therefore, resolving the forces horizontally, we get

$$200 - 200 \cos 45° - P_1 = 0$$

$\therefore$ $P_1 = 200 - 200 \times 0.707$ **= 58.6 N. (Ans.)**

Now resolving the forces vertically

$$400 - 200 \sin 45° - P_2 = 0$$

$\therefore$ $P_2 = 400 - 200 \times 0.707$ **= 258.6 N. (Ans.)**

Magnitude of the couple = ?

We know that moment of the couple is equal to the algebraic sum of the moments about any corner. Therefore, moment of the couple (taking moments about L)

$$= -400 \times 2 - P_1 \times 2 = -800 - 58.6 \times 2$$

= – 917.2 Nm. (Ans.)

(Minus sign taken due to anti-clockwise moments).

Example 3.12. *ABCDEF is a regular hexagon ; forces P, 2P, 3P, 2P, 5P, 6P act along AB, BC, DC, ED, EF, AF respectively. Show that the six forces are equivalent to a couple, and find the moment of this couple.*

Sol. Refer to Fig. 3.41.

Let A be the origin and let X and Y-axis be along AB and AE respectively.

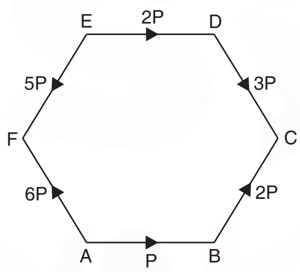

Fig. 3.41

Then,
$$\Sigma H = P + 2P \cos 60° + 3P \cos 60° + 2P - 5P \cos 60° - 6P \cos 60°$$

$$= P + P + \frac{3P}{2} + 2P - \frac{5P}{2} - 3P = 0$$

$$\Sigma V = 0 + 2P \sin 60° - 3P \sin 60° + 0 - 5P \sin 60° + 6P \sin 60°$$

$$= (2P - 3P - 5P + 6P)\,\frac{\sqrt{3}}{2} = 0$$

Hence, there is no resultant force.

If there be a resultant couple its moment

= the sum of moments of the forces about A. (Each side = 1)

$$= 2P\left(\frac{\sqrt{3}}{2}\right) - 3P\,(\sqrt{3}) - 2P\,(\sqrt{3}) + 5P\left(\frac{\sqrt{3}}{2}\right)$$

$$= \sqrt{3}P - 3\sqrt{3}P - 2\sqrt{3}P + \frac{5}{2}\sqrt{3}P$$

$$= \sqrt{3}P\ (1 - 3 - 2 + 5/2)$$

$$= \frac{-3\sqrt{3}\,P}{2}$$

Hence, **moment of couple = $\mathbf{\dfrac{-3\sqrt{3}P}{2}}$. (Ans.)**

Sol. Refer to Fig. 3.46.

Let h = height of telegraph pole AB, having two portions BC and BD of telegraphic wire attached to it at B such that $\angle CBD = 60°$

T = tension in each portion of the telegraphic wire in N

T' = tension in the supporting wire LM.

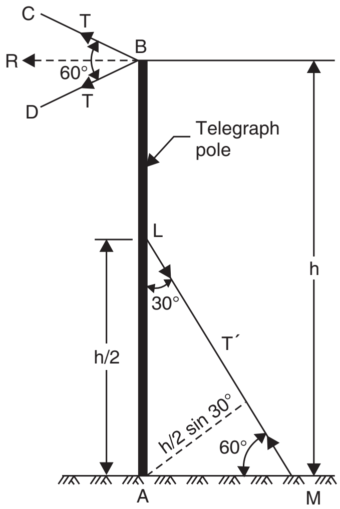

Fig. 3.46

Let us first find out the resultant of tensions T each in portions BC and BD of telegraphic wire by using the relation,

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

$$= \sqrt{T^2 + T^2 + 2 \times T \times T \cos 60°}$$

$$= \sqrt{2T^2 + 2T^2 \times 0.5}$$

$$= T\sqrt{3} \text{ N.}$$

The pole is in equilibrium under the action of the following forces :

(*i*) Resultant of tensions T each in portions BC and BD, R,

(*ii*) Tension in the supporting wire, T'.

Now taking moments about bottom A of the pole, we get

$$T' \times \frac{h}{2} \sin 30° = R \times h$$

$$T' \times \frac{h}{2} \times \frac{1}{2} = T\sqrt{3} \times h$$

$\therefore$ $\boldsymbol{T' = 4\sqrt{3}\, T}$ **......Proved.**

Hence, tension in the supporting wire is $4\sqrt{3}$ times the tension in the telegraph wire.

Example 3.18. *A ladder rests at an angle of 30° to the horizontal with its ends resting on a smooth floor and against a smooth vertical wall. The lower end being attached to the junction of the wall and floor by a string, find the tension in the string.*

Find also tension in the string when a man whose weight is one-half of the weight of ladder, has ascended the two-third of its length.

Sol. Case I. Refer to Fig. 3.47.

The ladder is in equilibrium under the action of following forces :

(*i*) Weight of the ladder, W ;

(*ii*) Reaction at the floor, R_A ;

(*iii*) Reaction at the wall, R_B ; and

(*iv*) Tension in the string,

Considering horizontal equilibrium of the ladder ($\Sigma H = 0$)

$$R_B = T \quad ...(i)$$

Considering vertical equilibrium of the ladder ($\Sigma V = 0$)

$$R_A = W \quad ...(ii)$$

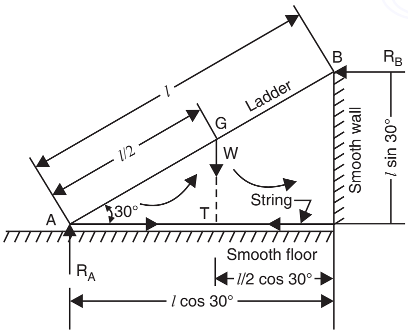

Fig. 3.47

Also $\Sigma M = 0$

Taking moments about B, we get

$$R_A \times l \cos 30° = W \times \frac{l}{2} \cos 30° + T \times l \sin 30°$$

or $\quad R_A \times l \times 0.866 = W \times l/2 \times 0.866 + T \times l \times 0.5$

or $\quad R_A \times 0.866 = W \times 0.433 + 7 \times 0.5 \qquad [\because\ R_A = W$ from eqn. $(ii)]$

or $\quad W \times 0.866 = W \times 0.433 + 0.5\, T$

or $\quad$ **T = 0.866 W. (Ans.)**

Case II. Refer to Fig. 3.48. M shows the position of man having weight $\frac{W}{2}$ (one-half the weight of ladder W). The ladder is in equilibrium under the action of the following forces :

(i) Weight of the ladder, W ;

(ii) Weight of the man, $\frac{W}{2}$;

(iii) Reaction at the floor, R_A ;

(iv) Reaction of the wall R_B ; and

(v) Tension in the string T.

Considering horizontal equilibrium of the ladder ($\Sigma H = 0$)

$$R_B = T \qquad ...(i)$$

Considering vertical equilibrium of the ladder ($\Sigma V = 0$),

$$R_A = W + \frac{W}{2} = \frac{3W}{2} \qquad ...(ii)$$

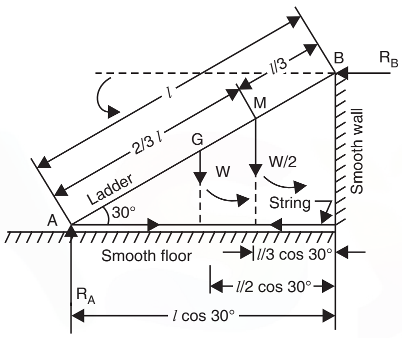

Fig. 3.48

Also, $\Sigma M = 0$

Taking moments about B, we get

$$R_A \times l \cos 30° = W \times \frac{l}{2} \cos 30° + \frac{W}{2} \times \frac{l}{3} \cos 30° + T \times l \sin 30°$$

$$\frac{3W}{2} \times l \times 0.866 = W \times \frac{l}{2} \times 0.866 + \frac{W}{2} \times \frac{l}{3} \times 0.866 + T \times l \times 0.5$$

$$1.3\ W = 0.433\ W + 0.144\ W + 0.5T$$

$\therefore$ **T = 1.446 W. (Ans.)**

Example 3.19. *A beam simply supported at both the ends carries load system as shown in Fig. 3.49. Find the reactions at the two ends.*

Sol. Refer to Fig. 3.49. Since, all the loads acting on the beam are vertically downwards therefore, the reactions at the ends shall be vertically upwards. Let R_A and R_B be the reactions at the ends A and B respectively.

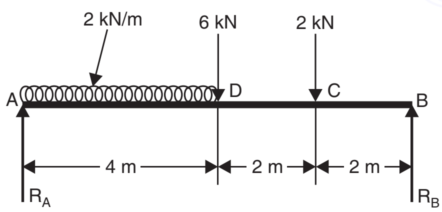

Fig. 3.49

Since, the beam AB is in equilibrium,

$\therefore \quad \Sigma H = 0, \Sigma V = 0$ and $\Sigma M = 0$.

There is no horizontal force.

$\therefore$ Considering vertical equilibrium of the beam ($\Sigma V = 0$)

$$R_A + R_B = 2 \times 4 + 6 + 2$$

or $\quad R_A + R_B = 16 \qquad ...(i)$

(*ii*) *Distance of F from A*

Take the moments of all forces about point A.

Moment of force 50 N about $A = 0$ ($\because$ Force 50 N is passing through)

Moment of force F about $A = F \times x$ (anti-clockwise)

Moment of force 100 N about $A = 100 \times AD = 100 \times 7 = 700$ Nm (anti-clockwise)

$\therefore$ Algebraic sum of moments of all forces about A

$= 0 + F \times x + 700$ Nm

$= F \times x + 700$ Nm (anti-clockwise)

Moment of resultant R about $A = R \times 4 = 250 \times 4 = 1000$ Nm (anti-clockwise)

But algebraic sum of moments of all forces about A must be equal to the moment of resultant R about A.

$\therefore$ $F \times x + 700 = 1000$ or $F \times x = 1000 - 700 = 300$

or $x = \dfrac{300}{F} = \dfrac{300}{100}$ ($\because$ $F = 100$ N)

$=$ **3 m. Ans.**

Problem 3.5. *Four parallel forces of magnitudes 100 N, 150 N, 25 N and 200 N are shown in Fig. 3.14. Determine the magnitude of the resultant and also the distance of the resultant from point A.*

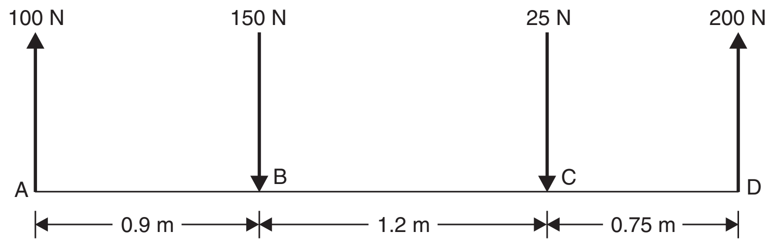

Fig. 3.14

Sol. Given :

Forces are 100 N, 150 N, 25 N and 200 N.

Distances $AB = 0.9$ m, $BC = 1.2$ m, $CD = 0.75$ m.

As all the forces are acting vertically, hence their resultant R is given by

$R = 100 - 150 - 25 + 200$

(Taking upward force +ve and downward as –ve)

$= 300 - 175 = 125$ N

+ve sign shows that R is acting vertically upwards. To find the distance of R from point A, take the moments of all forces about point A.

Let x = Distance of R from A in metre.

As the force 100 N is passing through A, its moment about A will be zero.

Moment of 150 N force about $A = 150 \times AB$

$= 150 \times 0.9$ (clockwise) (–) $= -135$ Nm

Moment of 25 N force about $A = 25 \times AC = 25 \times (0.9 + 1.2)$

$= 25 \times 2.1$ (clockwise) (–) $= -52.5$ Nm.

Moment of 200 N force about $A = 200 \times AD$
$= 200 \times (0.9 + 1.2 + 0.75)$
$= 200 \times 2.85$ (anti-clockwise) (+) = 570 Nm

Algebraic sum of moments of all forces about A
$= -135 - 52.5 + 570 = 382.5$ Nm ...(i)

+ve sign shows that this moment is anti-clockwise. Hence the moment of resultant R about A must be 382.5 Nm, *i.e.*, moment of R should be anti-clockwise about A. The moment of R about A will be anti-clockwise if R is acting upwards and towards the right of A.

Now moment of R about $A = R \times x$. But $R = 125$
$= 125 \times x$ (anti-clockwise) (+)
$= +125 \times x$ ...(ii)

Equating (i) and (ii),
$382.5 = 125 \times x$

or $x = \dfrac{382.5}{125}$ = **3.06 m. Ans.**

∴ Resultant (R = 125 N) will be 125 N upwards and is acting at a distance of 3.06 m to the right of point A as shown in Fig. 3.14 (a).

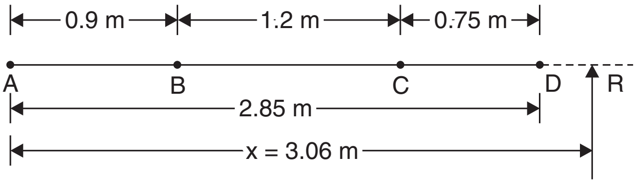

Fig. 3.14 (*a*)

3.6. RESOLUTION OF A FORCE INTO A FORCE AND A COUPLE

A given force F applied to a body at any point A can always be replaced by an equal and parallel force applied at another point B together with a couple which will be equivalent to the original force. This is proved as given below :

Let the given force F is acting at point A as shown in Fig. 3.15 (a).

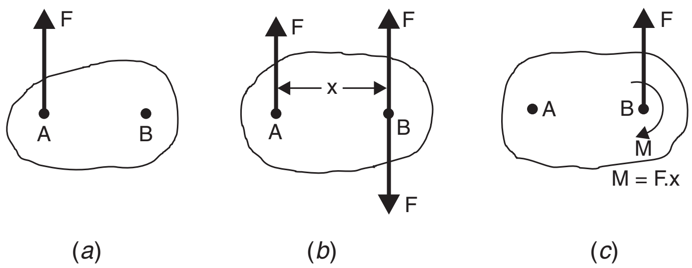

Fig. 3.15

This force is to be replaced at the point B. Introduce two equal and opposite forces at B, each of magnitude F and acting parallel to the force at A as shown in Fig. 3.15 (b). The force system of Fig. 3.15 (b) is equivalent to the single force acting at A of Fig. 3.15 (a). In Fig. 3.15 (b) three equal forces are acting. The two forces *i.e.*, force F at A and the oppositely directed force F at B (*i.e.*, vertically downward force at B) from a couple. The moment of this couple is $F \times x$ clockwise where x is the perpendicular distance between the lines of action of forces at A and B. The third force is acting at B in the same direction in which the force at A is acting. In Fig. 3.15 (c), the couple is shown by curved arrow with symbol M. The force system of Fig. 3.15 (c) is equivalent to Fig. 3.15 (b). Or in other words the Fig. 3.15 (c) is equivalent to Fig. 3.15 (a). Hence the given force F acting at A has been replaced by an equal and parallel force applied at point B in the same direction together with a couple of moment $F \times x$.

Thus a force acting at a point in a rigid body can be transferred to an equal and parallel force at any other point in the body, and a couple.

Problem 3.6. *A system of parallel forces are acting on a rigid bar as shown in Fig. 3.16. Reduce this system to :*

(*i*) *a single force*

(*ii*) *a single force and a couple at A*

(*iii*) *a single force and a couple at B.*

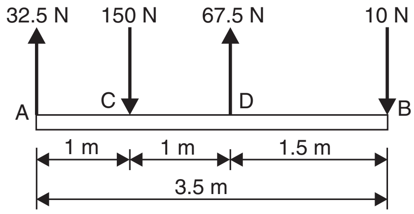

Fig. 3.16

Sol. Given :

Forces at A, C, D and B are 32.5 N, 150 N, 67.5 N and 10 N respectively.

Distances $AC = 1$ m, $CD = 1$ m and $BD = 1.5$ m.

(*i*) *Single force system.* The single force system will consist only resultant force in magnitude and location. All the forces are acting in the vertical direction and hence their resultant (R) in magnitude is given by

$$R = 32.5 - 150 + 67.5 - 10 = -\mathbf{60\ N.}\quad \textbf{Ans.}$$

Negative sign shows that resultant is acting vertically downwards.

Let x = Distance of resultant from A towards right. To find the location of the resultant take the moments of all forces about A, we get moment of resultant about A.

= Algebraic sum of moments of all forces about A

or $$R \times x = -150 \times AC + 67.5 \times AD - 10 \times AB$$

(Taking clockwise moment –ve and anti-clockwise moment +ve)

or $$(-60)x = -150 \times 1 + 67.5 \times 2 - 10 \times 3.5$$

(The moment due to R at A is clockwise and hence it is –ve)

or $$-60x = -150 + 135 - 35 = -50$$

$\therefore$ $$x = \frac{-50}{-60} = \mathbf{0.833\ m.}\quad \textbf{Ans.}$$

Hence the given system of parallel forces is equivalent to a single force 60 N acting vertically downwards at point E at a distance of 0.833 m from A shown in Fig. 3.16 (*a*).

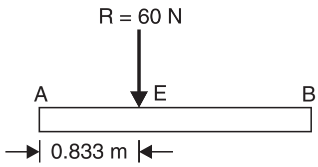

Fig. 3.16 (*a*)

(*ii*) *A single force and a couple at A.* The resultant force R acting at point E as shown in Fig. 3.16 (*a*) can be replaced by an equal force applied at point A in the same direction together with a couple. This is shown in Fig. 3.16 (*c*).

The moment of the couple = 60 × 0.833 Nm (clockwise)

$$= -\mathbf{49.98\ Nm.}\quad \textbf{Ans.}$$ (–ve sign is due to clockwise)

(*iii*) *A single force and a couple at B.* First find distance BE. But from Fig. 3.16 (*b*), the distance

$$BE = AB - AE = 3.5 - 0.833 = 2.667 \text{ m.}$$

Now if the force $R = 60$ N is moved to the point B, it will be accompanied by a couple of moment 60 × BE or 60 × 2.667 Nm. This is shown in Fig. 3.16 (*e*).

The moment of the couple = 60 × 2.667 Nm (anti-clockwise)

$$= \mathbf{160\ Nm.}\quad \textbf{Ans.}$$

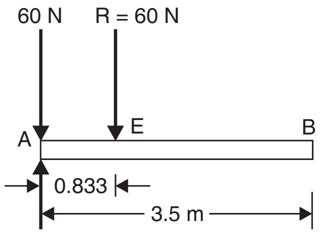

Fig. 3.16 (*b*)

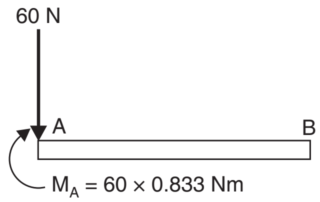

Fig. 3.16 (*c*)

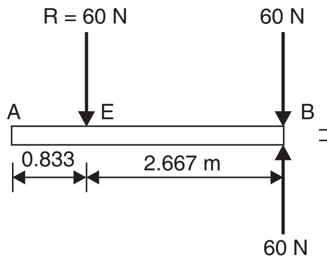

Fig. 3.16 (*d*)

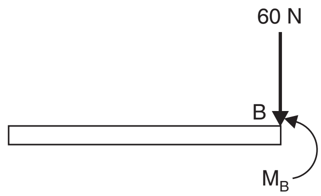

Fig. 3.16 (*e*)

Problem 3.6 (A). *A rigid bar is subjected to a system of parallel forces as shown in Fig. 3.16 (f). Reduce this system to :*

(*i*) *a single force system*,

(*ii*) *a single force moment system at B*.

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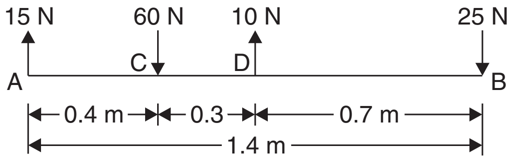


Fig. 3.16 (*f*)

Sol. (*i*) **A Single Force System**

A single force will consist only a resultant force in magnitude and location. All the forces are acting in the vertical direction and hence their resultant (R) in magnitude is given by

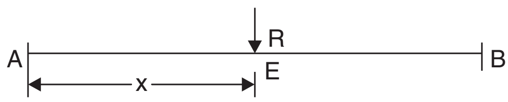


Fig. 3.16 (*g*)

$$R = -\,15 + 60 - 10 + 25 = \mathbf{60\ N.}\quad \textbf{Ans.}$$

(Here upward direction is considered –ve whereas downward direction +ve).

As R is +ve, hence it acts downward as shown in Fig. 3.16 (*g*).

To find the distance of the resultant from end A, take the moments of all forces about A, then

Moment of R about A = Algebraic sum of moments of all forces about A

$$-\,R \times x = -\,60 \times 0.4 + 10(0.4 + .3) - 25(0.4 + .3 + .7)$$

$$-\,R \times x = -\,24 + 7 - 35$$

or
$$-\,60x = -\,52$$

or
$$x = \frac{-\,52}{-\,60} = \mathbf{0.867\ m.}\quad \textbf{Ans.}$$

Hence the given system of parallel forces is equivalent to a single force 60 N acting vertically downward at point E at a distance of 0.867 m from A as shown in Fig. 3.16 (*g*).

Problem 5.3. *A simply supported beam of length 10 m, carries the uniformly distributed load and two point loads as shown in Fig. 5.15. Calculate the reactions R_A and R_B.*

Sol. Given :

Length of beam = 10 m

Length of U.D.L. = 4 m

Rate of U.D.L. = 10 kN/m

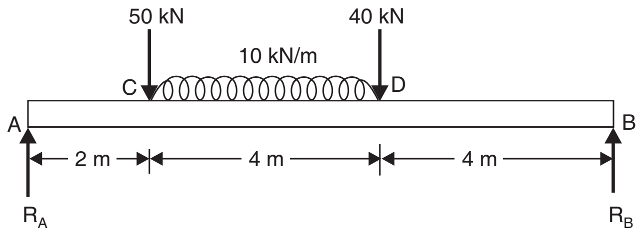


Fig. 5.15

∴ Total load due to U.D.L. = 4 × 10 = 40 kN

This load of 40 kN due to U.D.L. will be acting at the middle point of *CD, i.e.,* at a distance of $\frac{4}{2}$ = 2 m from *C* (or at a distance of 2 + 2 = 4 m from point *A*).

Let R_A = Reaction at *A*

and R_B = Reaction at *B*

Taking the moments of all forces about point *A* and equating the resultant moment to zero, we get

$$R_B \times 10 - 50 \times 2 - 40 \times (2 + 4) - (10 \times 4)\left(2 + \frac{4}{2}\right) = 0$$

or $$10R_B - 100 - 240 - 160 = 0$$

or $$10R_B = 100 + 240 + 160 = 500$$

∴ $$R_B = \frac{500}{10} = \mathbf{50\ kN.}\quad \textbf{Ans.}$$

Also for equilibrium of the beam, $\Sigma F_y = 0$

∴ $R_A + R_B$ = Total load on the beam = 50 + 10 × 4 + 40 = 130

∴ $R_A = 130 - R_B = 130 - 50 =$ **80 kN. Ans.**

Problem 5.3 (A). *Find the support reactions in the beam shown in Fig. 5.15 (a).*

(U.P. Tech. University, 2000–2001)

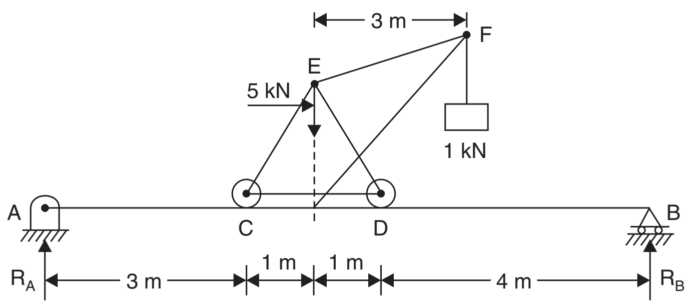


Fig. 5.15 (*a*)

Sol. Let R_A = Reaction at A and

R_B = Reaction at B

Vertical load at E = 5 kN and at F = 1 kN

As the load at E and F are vertical, hence reactions R_A and R_B will be vertical.

For equilibrium, the moment of all forces about any point should be zero.

Taking moments about A and equating to zero, we get ($M_A = 0$)

$$5 \times 4 + 1 \times 7 - R_B \times 9 = 0$$

or $$20 + 7 - 9R_B = 0$$

or $$9R_B = 20 + 7 = 27$$

$\therefore$ $$R_B = \frac{27}{9} = \mathbf{3\ kN.} \quad \textbf{Ans.}$$

Also $\Sigma F_y = 0$ $\quad\therefore\quad$ $R_A + R_B = 5 + 1 = 6$ kN

$\therefore$ $$R_A = 6 - R_B = 6 - 3 = \mathbf{3\ kN.} \quad \textbf{Ans.}$$

Problem 5.4. *A simply supported beam of span 9 m carries a uniformly varying load from zero at end A to 900 N/m at end B. Calculate the reactions at the two ends of the support.*

Sol. Given :

Span of beam = 9 m

Load at end $A = 0$

Load at end B = 900 N/m

Total load on the beam

= Area of right-angled triangle ABC

$$= \frac{AB \times BC}{2} = \frac{9 \times 900}{2} = 4050 \text{ N}$$

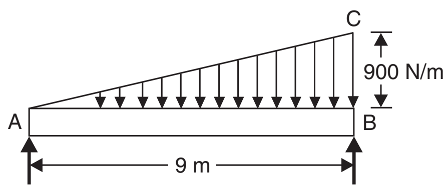


Fig. 5.16

This load will be acting at the C.G. of the ΔABC, *i.e.*, at a distance of $\frac{2}{3} \times AB = \frac{2}{3} \times 9 = 6$ m from end A.

Let R_A = Reaction at A

and R_B = Reaction at B.

Taking the moments of all forces about point A and equating the resultant moment to zero, we get

$$R_B \times 9 = (\text{Total load on beam}) \times \text{Distance of total load from } A$$
$$= 4050 \times 6$$

$\therefore$ $$R_B = \frac{4050 \times 6}{9} = \mathbf{2700\ N.} \quad \textbf{Ans.}$$

Also for equilibrium of the beam, $\Sigma F_y = 0$

or $$R_A + R_B = \text{Total load on beam} = 4050$$

$\therefore$ $$R_A = 4050 - R_B = 4050 - 2700 = \mathbf{1350\ N.} \quad \textbf{Ans.}$$

Problem 5.5. *A simply supported beam of length 5 m carries a uniformly increasing load of 800 N/m at one end to 1600 N/m at the other end. Calculate the reactions at both ends.*

Sol. Given :

Length of beam = 5 m

Load at A = 800 N/m

Load at B = 1600 N/m

Total load on the beam

= Area of load diagram $ABDC$

= Area of rectangle $ABEC$

+ Area of ΔCED

$= AB \times AC + \dfrac{CE \times ED}{2}$

$= 5 \times 800 + \frac{1}{2} \times 5 \times 800$

($\because$ $CE = AB = 5$ m, $ED = 1600 - 800 = 800$)

$= 4000 + 2000 = 6000$ N

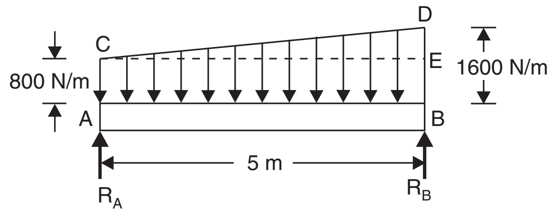


Fig. 5.17

The C.G. of the rectangle $ABEC$ will be at a distance of $\dfrac{5}{2} = 2.5$ m from A, whereas the C.G. of the triangle CED will be at a distance of $\dfrac{2}{3} \times 5 = 3.33$ m from A.

Let R_A = Reaction at A

and R_B = Reaction at B.

Taking the moments of all forces about point A and equating the resultant moment to zero, we get

$R_B \times 5$ – (Load due to rectangle) × Distance of C.G. of rectangle from A

– (Load due to triangle) × Distance of C.G. of triangle from A = 0

or $$5R_B - (5 \times 800) \times 2.5 - \left(\frac{1}{2} \times 5 \times 800\right) \times \left(\frac{2}{3} \times 5\right) = 0$$

or $$5R_B - 10000 - 6666.66 = 0$$

or $$5R_B = 10000 + 6666.66 = 16666.66$$

or $$R_B = \frac{16666.66}{5} = \mathbf{3333.33\ N.}\quad \textbf{Ans.}$$

Also for the equilibrium of the beam, $\Sigma F_y = 0$

$\therefore$ $R_A + R_B$ = Total load on the beam

= 6000 ($\because$ Total load on beam = 6000 N)

$\therefore$ $R_A = 6000 - R_B = 6000 - 3333.33 =$ **2666.67 N. Ans.**

Problem 5.5 (A). *Determine the reactions at A, B and D of the system shown in Fig. 5.17 (a).*

(U.P. Tech. University, 2001–2002)

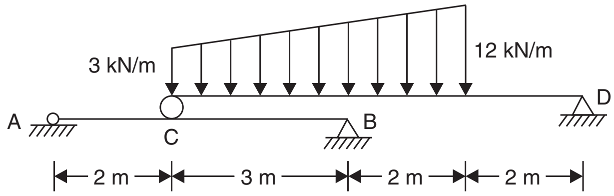


Fig. 5.17 (*a*)

Sol. Since the load on the beam CD is vertical, the reaction at C and D will be vertical. Also the reaction at A and B will be vertical.

Rate of U.D.L. $= 2$ kN/m

Total load due to U.D.L. $= 2 \times 6 = 12$ kN

The load of 12 kN (*i.e.*, due to U.D.L.) will act at the middle point of AC, *i.e.,* at a distance of 3 m from A.

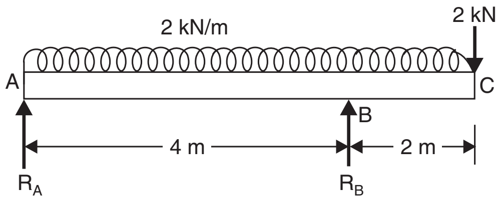


Fig. 5.20

Let R_A = Reaction at A

and R_B = Reaction at B.

Taking the moments of all forces about point A and equating the resultant moment to zero, we get

$$R_B \times 4 - (2 \times 6) \times 3 - 2 \times (4 + 2) = 0$$

or $$4R_B - 36 - 12 = 0$$

or $$4R_B = 36 + 12 = 48$$

$\therefore$ $$R_B = \frac{48}{4} = \mathbf{12\ kN.} \quad \textbf{Ans.}$$

Also for equilibrium, $\Sigma F_y = 0$ or $R_A + R_B = 12 + 2 = 14$

$\therefore$ $$R_A = 14 - R_B = 14 - 12 = \mathbf{2\ kN.} \quad \textbf{Ans.}$$

5.4.5. Problems on Roller and Hinged Supported Beams. In case of roller supported beams, the reaction on the roller end is always *normal* to the support. All the steel trusses of the bridges is generally having one of their ends supported on rollers. The main advantage of such a support is that beam, due to change in temperature, can move easily towards left or right, on account of expansion or contraction.

In case of a hinged supported beam, the reaction on the hinged end may be either vertical or inclined, depending upon the type of loading. If the load is acting vertically downwards, the reaction at the hinge will be vertical. But if acting load is inclined to the beam then reaction will also be inclined. This reaction will have horizontal and vertical component. The main advantage of a hinged end is that the beam remains stable. Hence all the steel trusses of the bridges, have one of their end on rollers and the other end as hinged.

Problem 5.8. *A beam AB 1.7 m long is loaded as shown in Fig. 5.21. Determine the reactions at A and B.*

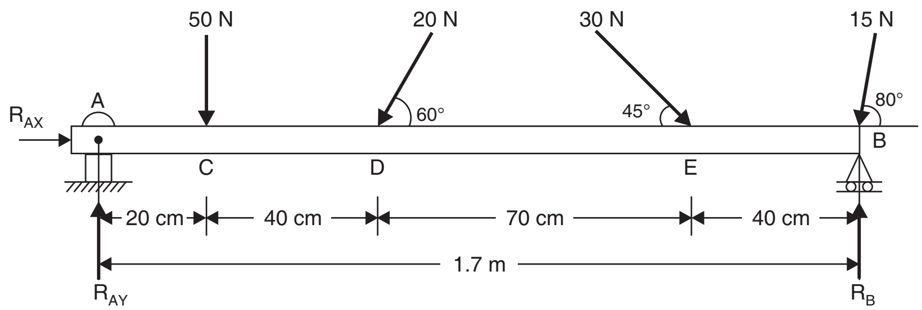


Fig. 5.21

Sol. Given :

Length of beam = 1.7 m

Let R_A = Reaction at A

and R_B = Reaction at B.

Since the beam is supported on rollers at B, therefore the reaction R_B will be vertical.

The beam is hinged at A, and is carrying inclined load, therefore the reaction R_A will be inclined. This means reaction R_A will have two components, *i.e.*, vertical component and horizontal component.

Let R_{AX} = Horizontal component of reaction R_A

R_{AY} = Vertical component of reaction R_A.

First resolve all the inclined loads into their vertical and horizontal components.

(*i*) Vertical component of load at D

$= 20 \sin 60° = 20 \times 0.866 = 17.32$ N

and its horizontal component $= 20 \cos 60° = 10 \text{ N} \leftarrow$

(*ii*) Vertical component of load at E

$= 30 \sin 45° = 21.21$ N

and its horizontal component $= 30 \cos 45° = 21.21 \text{ N} \rightarrow$

(*iii*) Vertical component of load at B

$= 15 \sin 80° = 14.77$ N

and its horizontal component $= 15 \cos 80° = 2.6 \text{ N} \leftarrow$

From condition of equilibrium, $\Sigma F_x = 0$

or $R_{AX} - 10 + 21.21 - 2.6 = 0$

or $R_{AX} = 10 - 21.21 + 2.6 = -8.61$ N

–ve sign shows that the assumed direction of R_{AX} (*i.e.*, horizontal component of R_A) is wrong. Correct direction will be opposite to the assumed direction. Assumed direction of R_{AX} is towards right. Hence correct direction of R_{AX} will be towards left at A.

$\therefore$ $R_{AX} = 8.61 \text{ N} \leftarrow$

To find R_B, take moments* of all forces about A.

For equilibrium, $\Sigma M_A = 0$

$\therefore$ $50 \times 20 + (20 \sin 60°) \times (20 + 40) + (30 \times \sin 45°)$
$\times (20 + 40 + 70) + (15 \sin 80°) \times (170) - 170 R_B = 0$

or $1000 + 1039.2 + 2757.7 + 2511 - 170 R_B = 0$

or $7307.9 - 170 R_B = 0$

$\therefore$ $R_B = \dfrac{7307.9}{170} = $ **42.98 N. Ans.**

To find R_{AY}, apply condition of equilibrium, $\Sigma F_y = 0$

or $R_{AY} + R_B = 50 + 20 \sin 60° + 30 \sin 45° + 15 \sin 80°$

or $R_{AY} + 42.98 = 50 + 17.32 + 21.21 + 14.77 = 103.3$

$\therefore$ $R_{AY} = 103.3 - 42.98 = 60.32 \text{ N} \uparrow$

$\therefore$ Reaction at A, $R_A = \sqrt{R_{AX}^2 + R_{AY}^2}$

$= \sqrt{8.61^2 + 60.32^2} = 60.92$ N

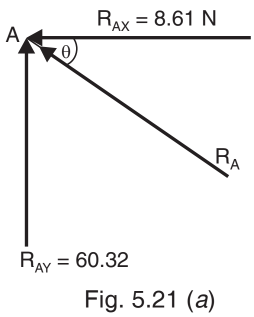


Fig. 5.21 (*a*)

The angle made by R_A with x-direction is given by

$$\tan \theta = \frac{R_{AY}}{R_{AX}} = \frac{60.32}{8.61} = 7.006$$

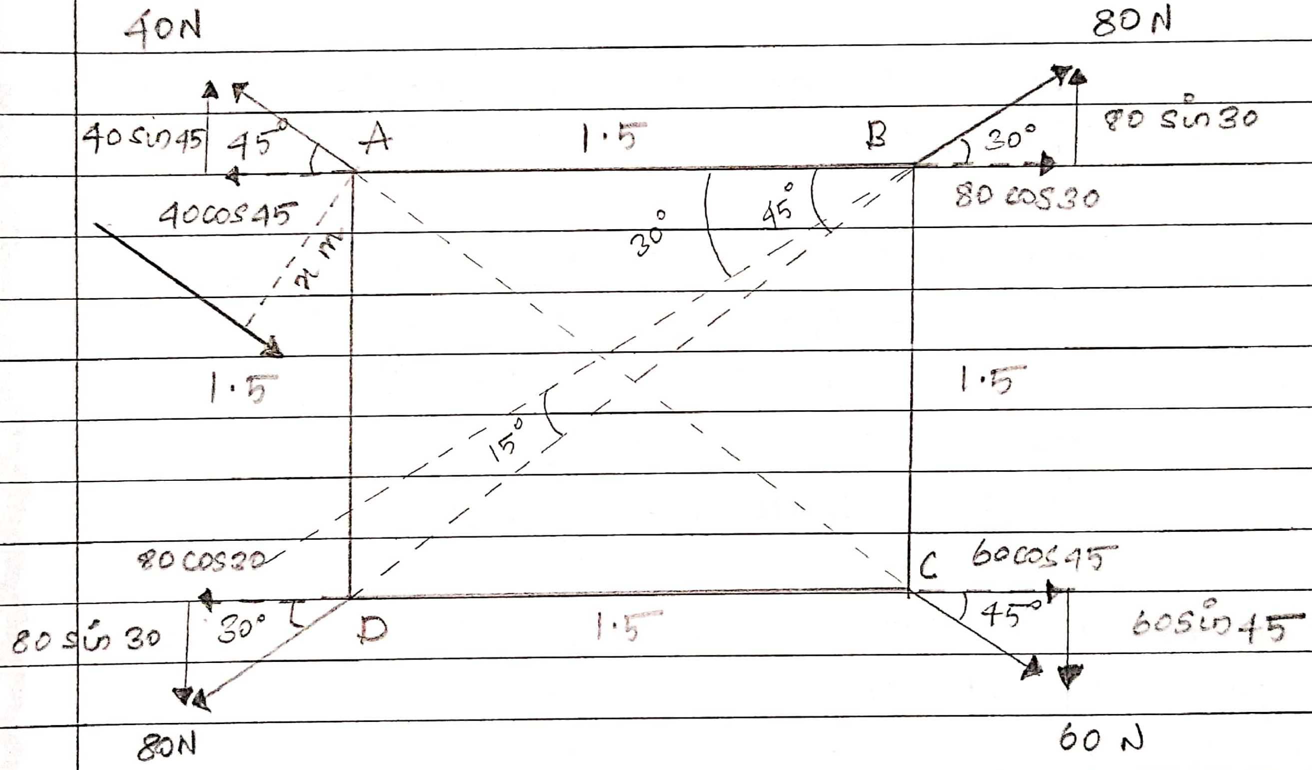
$\therefore$ $\theta = \tan^{-1} 7.006 = $ **81.87°. Ans.**

*The moment of all horizontal components about point A, will be zero.

PRACTICE SET-2

I Couples and Moments

Q.3.9 A body is under the action of four coplanar forces as shown in figure 3.36. Find the magnitude, direction and position of the resultant of the given force system.



Resolving forces horizontally,

$$\Sigma H = 80\cos 30^\circ + 60\cos 45^\circ - 80\cos 30^\circ - 40\cos 45^\circ$$

$$= 80 \times 0.86 + 60 \times 0.707 - 80 \times 0.866 - 40 \times 0.707$$

$$= 14.14\ N$$

Resolving forces vertically,

$$\Sigma V = 80\sin 30^\circ - 60\sin 45^\circ - 80\sin 30^\circ + 40\sin 45^\circ =$$

$$= -14.14\ N$$

$$\text{Resultant} = \sqrt{(14.14)^2 + (-14.14)^2}$$

$$\boxed{R = 20N}$$

Direction of resultant $\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$

$= \tan^{-1}(1)$

$\boxed{\alpha = 45^\circ}$

To find $\perp$ distance b/w resultant and point A,

we can take moments about point A.

Forces on A, C get cancelled due to them being in line of action.

$\therefore \quad -R \cdot x = (-1.5)\, 80 \cos 30 + (1.5)\, 80 \sin 30$

$= (1.5)(80)(\sin 30 - \cos 30)$

$= (-0.36)(1.5)(80)$

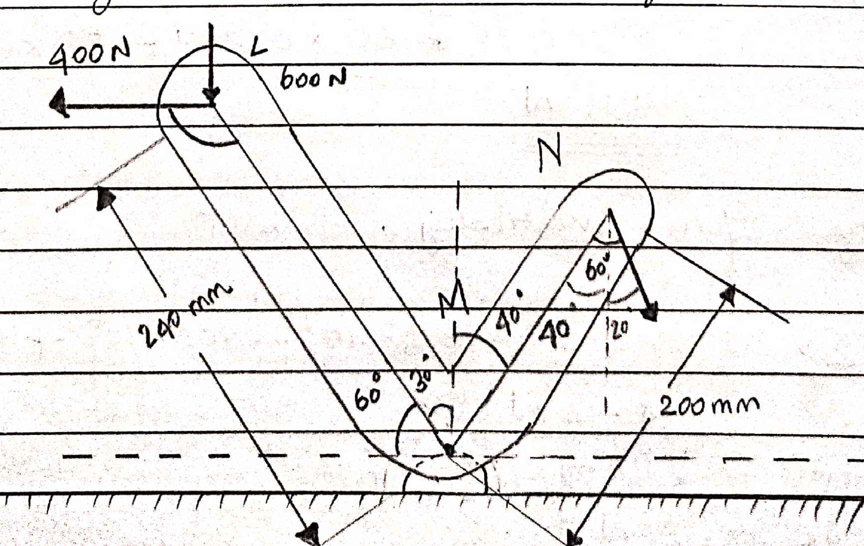
$-20 \cdot x = -43.92 \text{ N}$

$x = 2.196$ m from A

Q 3.10 The lever LMN of a component of a machine is hinged at M, and is subjected to a system of coplanar forces as shown in Figure. Find,

(1) The magnitude of the force P to keep the lever in equilibrium.

(2) The magnitude and direction of reaction at M.



→ Taking moments about the hinge M, we get

$\Sigma M = P \times 200 \sin 60^\circ = 600 \times 240 \cos 60^\circ$
$+ 400 \times \sin 60^\circ$

$P \times 200 \times 0.86 = 600 \times 240 \times 0.5 + 400 \times 240 \times 0.86$

$173.2\, P = 155136$

$P = 895\, N$

Resolving the forces horizontally,

$\Sigma H = 400 - P \sin 20^\circ$

$= 400 - 895 \times 0.34$

$= 93.9 N$

Resolving the forces vertically,

$\Sigma V = 600 + P \cos 20$

$= 600 + 895 \times 0.939 = 1440.4 N$

$R_M = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= \sqrt{93.9^2 + (1440.4)^2}$

$= 1443.46\, N$

Direction at M,

$\tan \theta = \frac{\Sigma V}{\Sigma H} = \frac{93.9}{1440.4} = 0.065$

$\theta = 3.73^\circ$

Q.3.11. A square LMNS has forces acting along its sides as shown in Figure. Find the values of F_1 and F_2, if the system reduces to a couple. Also find magnitudes of the couple, if the side of the square is 2m.

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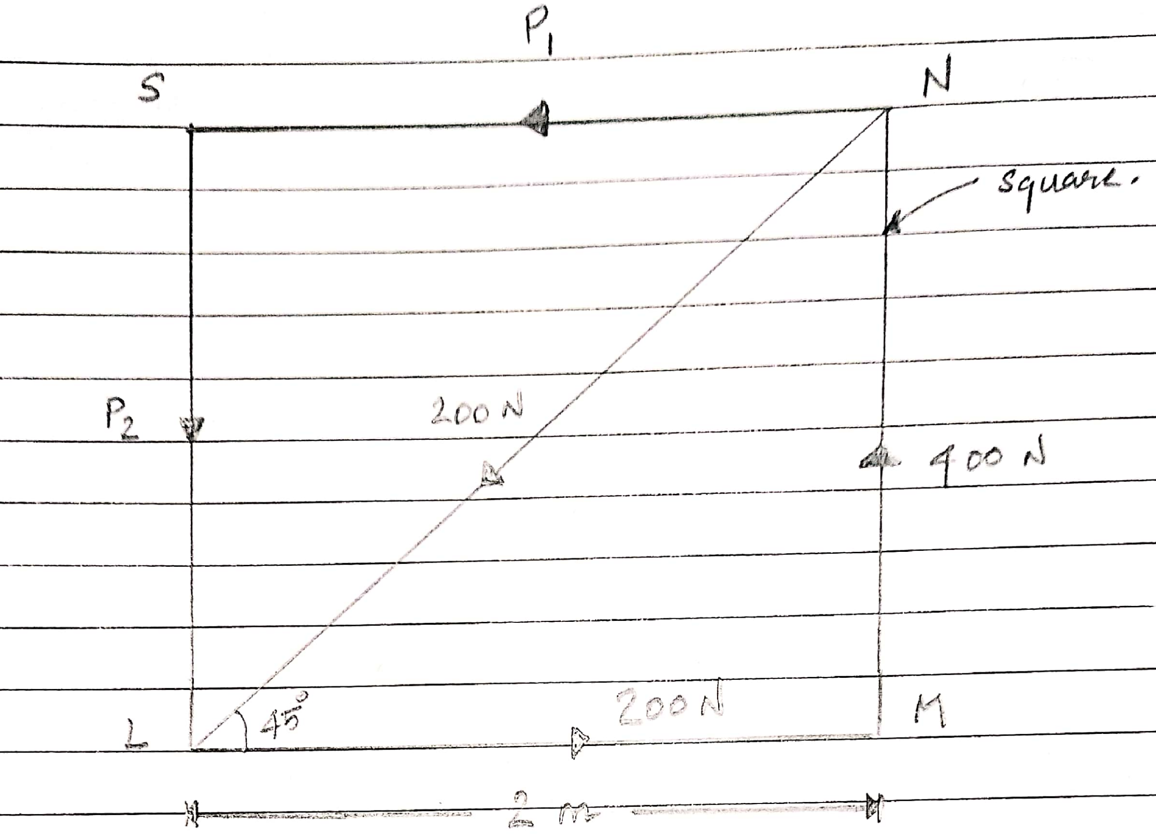


Values of P_1 and P_2.

As system reduces to a couple, $\Sigma H = 0$, $\Sigma V = 0$,

∴ Resolving forces horizontally,

$$200 - 200\cos 45^\circ - P_1 = 0$$

$$P_1 = 200 - 200 \times 0.707$$

$$\underline{P_1 = 58.6\,N}$$

Resolving forces vertically,

$$400 - 200\sin 45 - P_2 = 0$$

$$P_2 = 400 - 200 \times 0.707 = \underline{\underline{258.2\,N}}$$

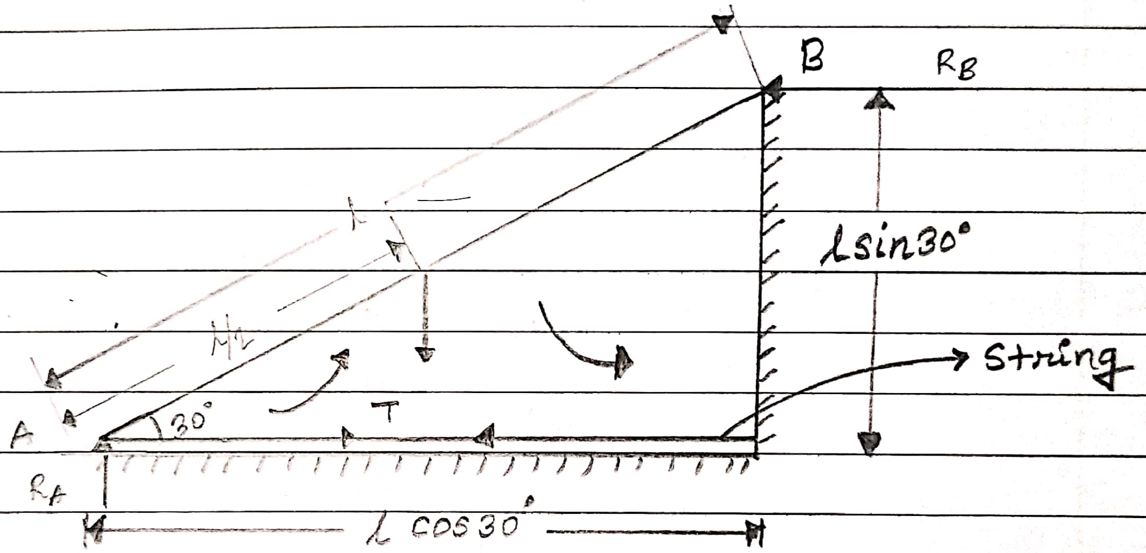
Taking moments about L,

$$-400 \times 2 - P_1 \times 2 = -800 - 58.6 \times 2$$

$$= \underline{\underline{-917.2\ Nm}}$$

~~Q 312~~

Q 3.15 A ladder rests at an angle of 30° to the horizontal with its ends resting on a smooth floor and against a smooth vertical wall. The lower end being attached to the junction of the wall and floor by a string, find the tension in the string. Find also tension in the string when a man whose weight is one half of the weight of ladder has ascended 2/3 of its length.



→ Case - 1 : Ladder ~~is~~ is in equilibrium under the following forces.

1) Weight of ladder - W
2) Reaction at the floor - R_A
3) Reaction at the wall, R_B
4) Tension in the string

→ Consider horizontal equilibrium in the ladder.

$$\Sigma H = 0$$

$$R_B = T$$

Consider vertical equilibrium of the ladder ($\Sigma V = 0$)

$$R_A = W.$$

Considering horizontal equilibrium of ladder

$(\Sigma H = 0)$

$R_B = T.$

Considering vertical equilibrium of ladder

$R_A = W + W/2 = 3W/2$

also $\Sigma M = 0,$

Taking Moments about point B,

$R_A \times l\cos 30^\circ = W \times l/2 \cos 30^\circ + W/2 \cdot l/3 \cos 30^\circ$
$+ T \cdot l \cdot \sin 30^\circ$

$\frac{3W}{2} \times l \times 0.86 = W \cdot l/2 \times 0.86 + W/2 \times l/3 \times 0.86$
$+ T \times l \times 0.5$

$1.3W = 0.43W + 0.144W + 0.5T.$

$\underline{\underline{T = 1.446W.}}$

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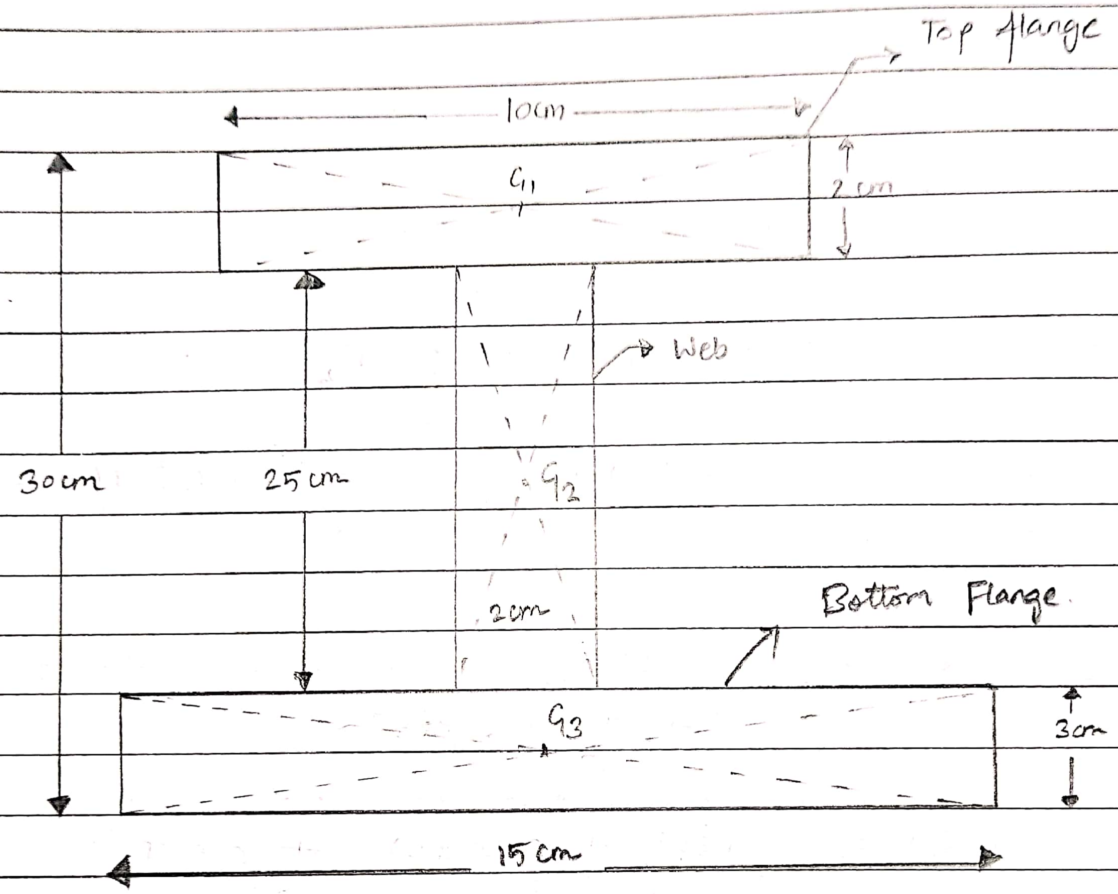
Practice - Set - 2. — CG and Centroid Problems.

Q.4.2. Determine the centroid of I section as shown in the figure.



→ Let us divide the figure into 3 simple areas:

① Rectangle (10cm x 2cm) – Top flange

② Rectangle (25 cm x 2 cm) – web

③ Rectangle (15 x 3 cm) – Bottom Flange.

To determine location of centroid we need to follow this table.

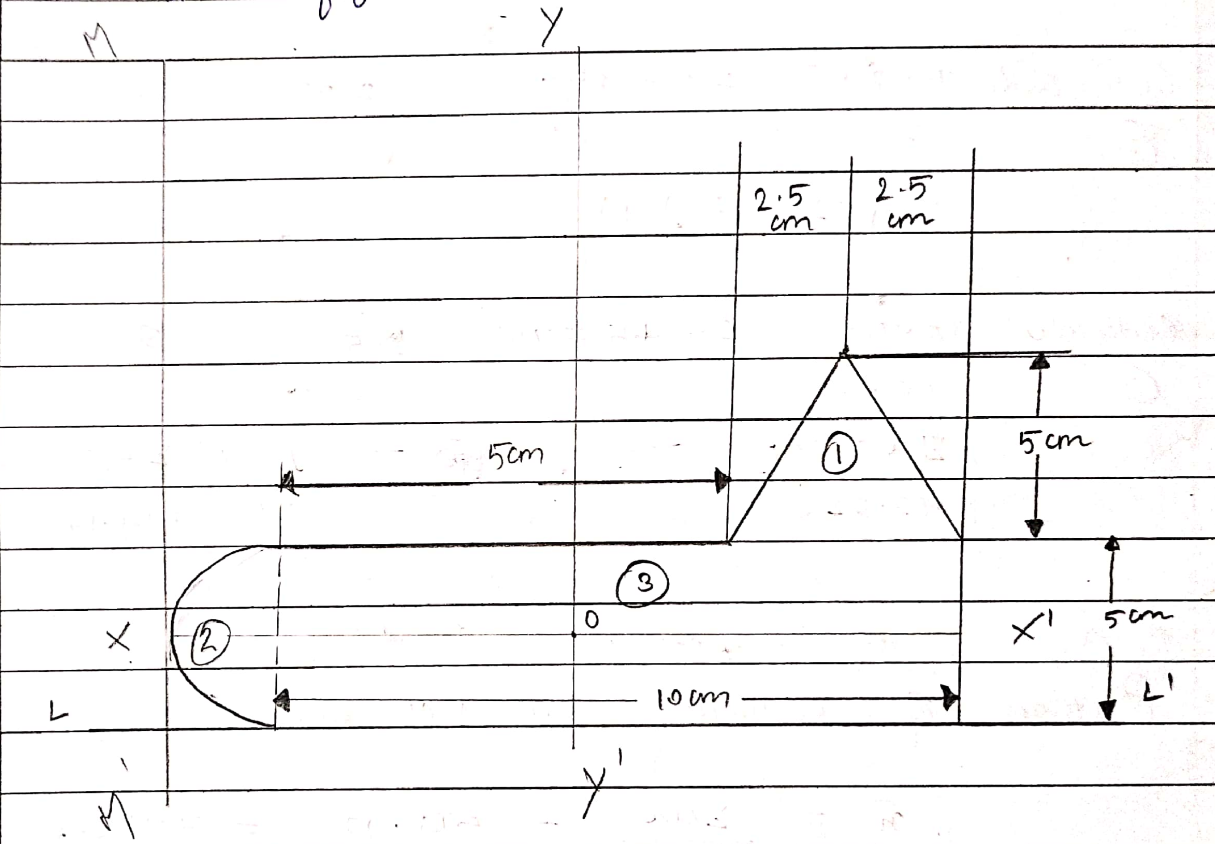
Components	Area 'a'	Centroidal distance y from LL	ay
Rectangle ①	$10 \times 2 = 20$	29 cm	
Rectangle ②	$25 \times 2 = 50$	15.5 cm	
Rectangle ③	$15 \times 3 = 45$	1.5 cm.	
	$\Sigma a = 115$ cm^2	–	$\Sigma ay = 1422.5$

$$y = \frac{\Sigma ay}{\Sigma ya} = \frac{1422.5}{115}$$

$$\boxed{y = 12.37\ cm}$$

Q.4.3. Using the analytical method, determine the center of gravity of the plane uniform lamina shown in the figure.

→



The laminar may be divided into three parts:

① A Triangle marked ①
② A semi-circle marked ②
③ A Rectangle marked ③

The area of these components, their centroidal distance from the LL axis and MM axis and the moments of the areas of individual component about LL axis and MM axis are tabulated below:

	Components	Area (a) (cm²)	Centroidal distance 'x' from MM — (cm)	Centroidal distance 'y' from LL — (cm)	ax (cm³)	ay (cm³)
①	Triangle ①	$\frac{5\times5}{2}$ = 12.50	2.5 + 5 + 2.5 = 10	5 + 5/3 = 6.67	125	83.4
②	Semi-circle ②	$\frac{\pi \cdot 2.5^2}{2}$ = 9.82	$2.5 - \frac{4\times2.5}{3\pi}$ = 1.44	2.5	14.14	24.55
③	Rectangle ③	10×5 = 50.00	2.5 + 5 = 7.5	2.5	375	125
		Σa = 72.32	—	—	(Σax) 514.14	Σay 232.95

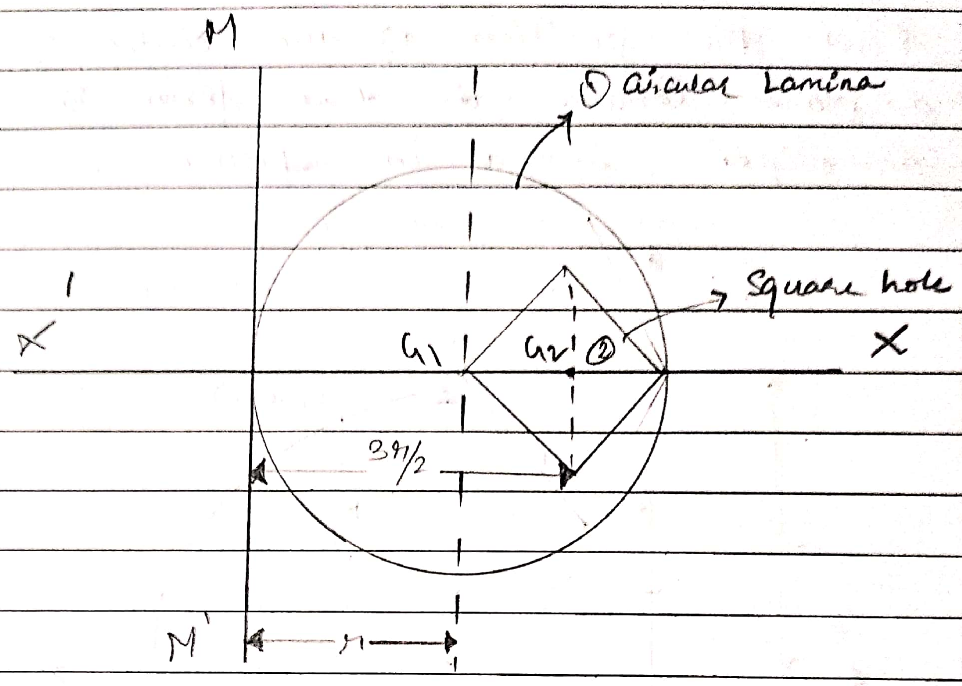
Distance of centroid from MM axis

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{514.13}{72.32} = 7.11 \text{ cm}$$

Distance of centroid from LL axis

$$y = \frac{\Sigma ay}{\Sigma a} = \frac{232.95}{72.32} = 3.22 \text{ cm}$$

Q.4.9. A square hole is punched out of a circular lamina. A diagonal of such a square being along any radius of the circle with one vertex at the centre of the circular lamina. It is said that the length of the said diagonal is equal to the radius of circular lamina. Find 'cog' if r is the radius of circle.



To find out the centre of gravity of the remaining part, we have the following table.

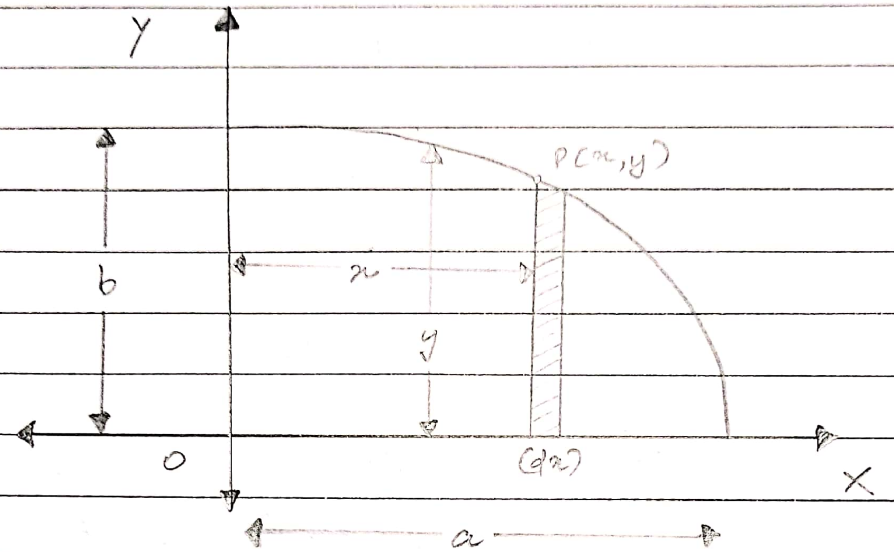
Components	Area 'a'	Centroidal distance 'x' from MM	a·x
① Circular Lamina ①	πr^2	r	$a r^3$
② Square hole ②	$-\frac{r^2}{2}$ $\left[\begin{array}{l} l^2 + l^2 = r^2 \\ \therefore (l^2 = \frac{r^2}{2}) \end{array}\right]$	$\frac{3r}{2}$	$-\frac{3r^3}{4}$
	$\Sigma a = \pi r^2 - \frac{r^2}{2}$	—	$\Sigma ax = \pi r^3 - 3r^3/4$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{\pi r^3 - \frac{3r^3}{4}}{\pi r^2 - \frac{r^2}{2}} = \frac{r^3(\pi - 3/4)}{2r^2(\pi - 1/2)}$$

$$= \frac{r(4\pi - 3)}{2(2\pi - 1)} \text{ from MM}$$

$$= \boxed{\frac{r(4\pi - 3)}{4\pi - 2}} \text{ from MM}$$

Q. 4.13 Find the position of the centre of gravity of a plane lamina in the form of quarter of an ellipse, shown in figure.



$$\bar{x} = \frac{\int_0^a xy \cdot dx}{\int_0^a y \cdot dx}, \quad \bar{y} = \frac{\int_0^b y/2 \cdot y \cdot dx}{\int_0^a y \cdot dx}$$

Equation of the ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

From which, $\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} = \frac{a^2 - x^2}{a^2}$

$$y = b/a \sqrt{a^2 - x^2}$$

$$\therefore \int_0^a xy \cdot dx = \int_0^a x \cdot b/a \sqrt{a^2 - x^2} \cdot dx$$

$$= \frac{-b}{2a} \int_0^a (a^2 - x^2)^{1/2} (-2x) \cdot dx$$

$$= \frac{-b}{2a} \cdot \frac{2}{3} \left| (a^2 - x^2)^{3/2} \right|_0^a = \frac{b}{3a} \cdot a^3 = \frac{a^2 b}{3}$$

Rainbow

$$\int_0^a y\cdot dx = \int_0^a b/a\,\sqrt{a^2-x^2}\cdot dx$$

$$= \frac{b}{a}\left|\frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\sin^{-1}(x/a)\right|_0^a$$

$$= \frac{b}{a}\cdot\frac{a^2}{2}\left(\frac{\pi}{2}\right) = \frac{ab\pi}{4}.$$

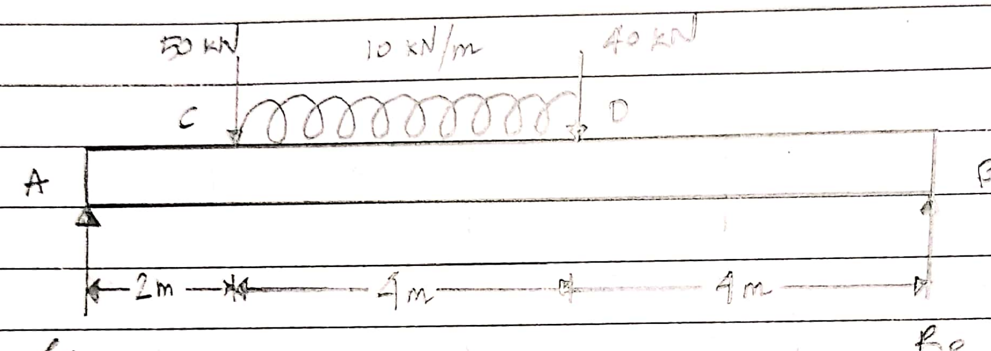
$$\bar{x} = \frac{a^2b}{3} \times \frac{4}{ab\pi} = \boxed{\frac{4a}{\cancel{ab}\,3\pi}}$$

$$\boxed{\bar{y} = \frac{4b}{3\pi}}$$

Practice set - 2 - Support Reaction Problems.

Q.5.3. A simply supported beam of length 10 m, carries the uniformly distributed load and 2 point loads as shown in figure. Calculate Reactions R_A and R_B.

Ans.



Total load due to U.D.L = $4 \times 10 = 40$ kN

will at on mid point of CD, at 2 m from C. or 4 m from A.

let R_A = Reaction at A

R_B = Reaction at B.

Taking the moments of all forces about point A and equating the resultant moment to zero,

$\Rightarrow$ $R_B \times 10 - 50 \times 2 - 40 \times (6) - 10 \times 4\ (2+2) = 0$

$\Rightarrow$ $10 R_B - 50 \times 2 - 40 \times (2+4) - 10 \times 4\ (4) = 0$

$10\ R_B = 100 + 240 + 160 = 500$

$R_B = \frac{500}{10} = 50$ KN

Also Beam is in equilibrium, so

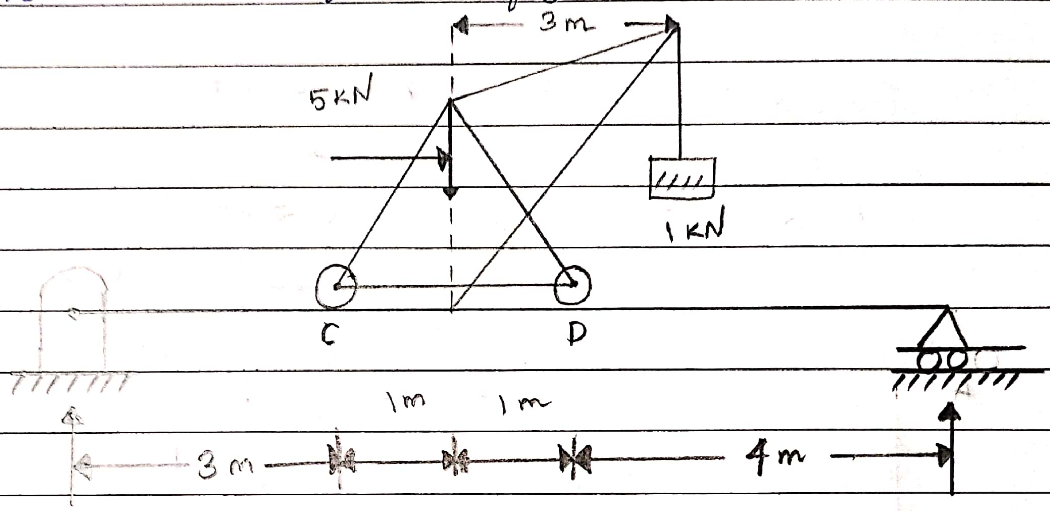
$\Sigma F_y = 0.$

Rainbow

$\therefore \quad R_A + R_B = \text{Total Load} = 130$

$R_A = 130 - 50 = \underline{80\ kN}.$

3.5.3 · (A) Find support reactions in the beam shown in the figure.



→ Let R_A → Reaction at A
R_B = Reaction at B.

→ Vertical load at $E = 5\ kN$ and $F = 1\ kN$

As the load at E and F are vertical, hence reactions R_A and R_B will be vertical. For equilibrium, moment of all forces about point A will be zero. Therefore,

$$5 \times 4 + 7 \times 1 - R_B \times 9 = 0$$

$$20 + 7 - 9R_B = 0$$

$$9R_B = 20 + 7 = 27$$

$$R_B = \frac{27}{9} = \underline{3\ kN}$$

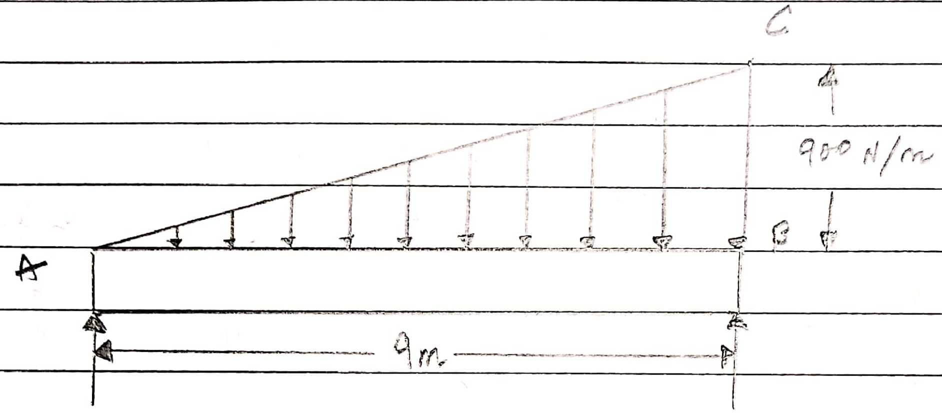
also $\Sigma F_y = 0$, $\therefore R_A + R_B = 5 + 1 = 6\ kN$

$R_A = 6 - R_B = 6 - 3 = \underline{3\ kN}$

nbow

Q.5.4. A simply supported beam of span 9 m carries a uniformly varying load from zero at end A to 900 N/m at end B. Calculate reactions at 2 ends of the support.

→



Span of Beam = 9 m
Load at end A = 0
Load at end B = 900 N/m

Total load on Beam :

$= \text{Area of } \Delta ABC$

$= \dfrac{AB \times BC}{2} = \dfrac{9 \times 900}{2} = \underline{\underline{4050 \text{ N}}}$

This load will act at 'cog' of Δ ABC. that is at a distance of $2/3 \cdot AB = 2/3 \times 9 = 6$ m from end A

let R_A = Reaction A
R_B = Reaction at B

Taking moments of all forces about a point A and equating the resultant moment to zero,
~~be~~ ∵ eq ($\Sigma M = 0$)

$R_B \times 9 = (\text{Total load})\,(\text{Distance of total load from end } A)$

$= 4050 \times 6$

$R_B = \dfrac{4050 \times 6}{9} = \underline{\underline{2700\ N}}$

For equilibrium of Beam, $\Sigma F_y = 0$,

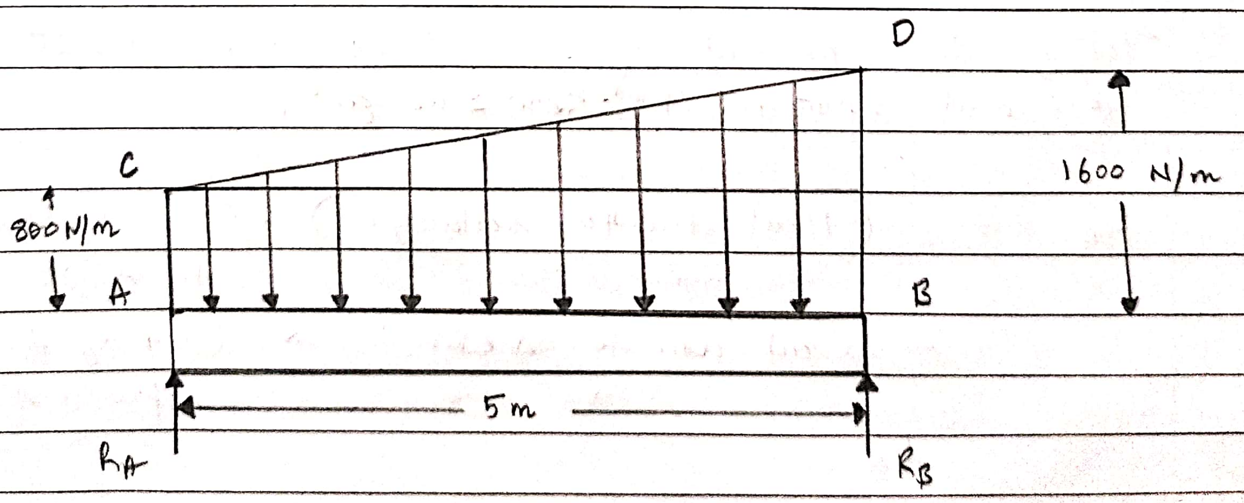
So $R_A + R_B$ = Total load on Beam = 4050

$R_A = 4050 - R_B$
$= 4050 - 2700$

$\boxed{R_A = 1350\ N}$

5.5 A simply supported beam of length 5m carries a uniformly increasing load of 800 N/m at one end to 1600 N/m at other end. Find Reactions at both ends.

→ ~~Soln~~



→ length of Beam = 5m
Load at A = 800 N/m
Load at B = 1600 N/m

Total load on Beam,

= Area of Rectangle ABDC

= Area of rectangle ABEC

+ Area of triangle CED

$$= AB \times EC + \frac{CE \times DE}{2}$$

$$= 5 \times 800 + \frac{5 \times 800}{2}$$

$(\because CE = AB = 5\,m$

$ED = 1600 - 800 = 800)$

$$= 4000 + 2000 = \underline{\underline{6000\,N}}$$

The ~~CG~~ COG of Rectangle ABEC will be at a distance of $5/2 = 2.5$ m from A, whereas COG of Δ CED will be at a distance of $2/3 \times 5 = 3.33$ from A

let R_A = Reaction at A

R_B = Reaction at B

Taking the moments of all forces about point A and equating to zero we get,

$R_B \times 5$ − (load due to rectangle) × (CG of Rectangle to A)

− (load due to Δ CED) × (CG of Δ CED from A)

= 0

$$5R_B - 5 \times 800 \times (2.5) - \left(\frac{1}{2} \times 5 \times 800\right) \times \left(2/3 \times 5\right) = 0.$$

Rainbow

$$5R_B - 10000 - 6666.66 = 0$$

$$5R_B = 16666.66$$

$$R_B = \frac{16666.66}{5} = \underline{\underline{3333.33\ N}}$$

for equilibrium of beam, $\Sigma F_y = 0$

$R_A + R_B$ = Total load on the beam.

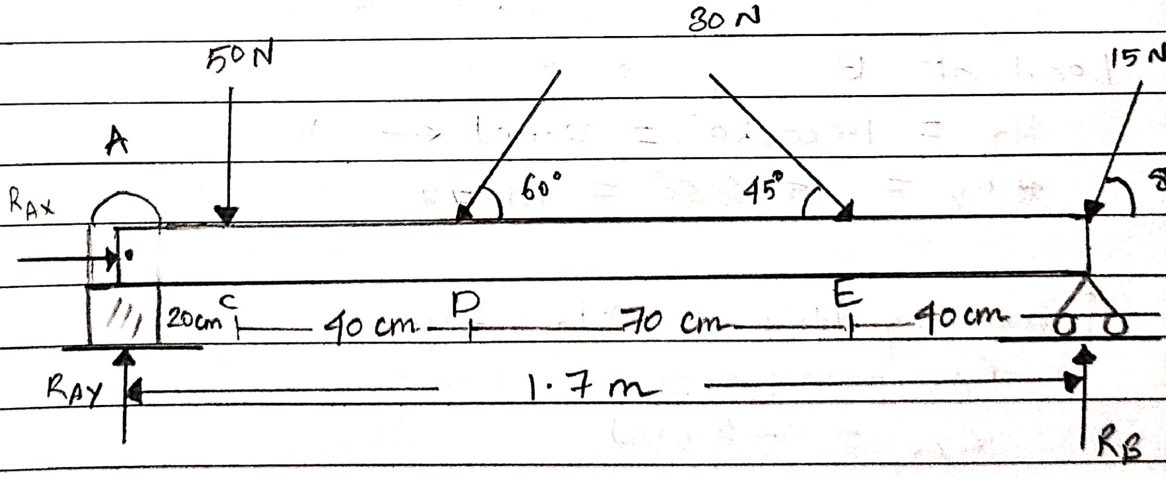
$R_A + R_B = 6000$

$R_A = 6000 - 3333.3\ N$

$R_A = \underline{\underline{2666.67\ N}}$

5.5 ~~(A) Find reactions at A, B, D of the system shown below.~~

5.8 A beam AB 1.7 m long is loaded as shown. Determine Reactions at A and B.



Length of Beam = 1.7 m

let R_A = Reaction at A

R_B = Reaction at B.

as Beam is on rollers at B, reaction at B will be vertical.

Now

The beam is hinged at A, and is carrying inclined load, therefore the reaction RA will be inclined. This means RA will have 2 components.

let

R_{AX} – horizontal component of reaction R_A

R_{AY} – Vertical component of reaction R_A

Resolving the forces,

(1) Load at D.

$D_y = 20 \sin 60° = 20 \times 0.86 = 17.32\ N$

$D_x = 20 \cos 60 = 10\ N \leftarrow$

(2) Load at E

$E_x = 30 \cos 45° = 21.21 N$

$E_y = 30 \sin 45° = 21.21 N \rightarrow$

(3) Load at B

$B_x = 15 \cos 80° = 2.6 N \leftarrow$

$B_y = 15 \sin 80° = 14.77$

From equilibrium $\Sigma F_x = \Sigma F_y = 0$

$R_{AX} - 10 + 21.21 - 2.6 = 0$

$R_{AX} = -8.61 N$

So 8.61 N towards left

also, $\Sigma M_A = 0$

$\therefore (50 \times 20) + (20 \sin 60°) \times (20 + 40) + (30 \times \sin 45°) \times (20 + 40 + 70) + (15 \sin 80°) \times 170 - 170 R_B = 0$

Rainbow

$\Rightarrow$ $1000 + 1039.2 + 2757.7 + 2511 - 170 R_B = 0$

$7309 - 170 R_B = 0$

$R_B = \frac{7309}{170} = \underline{\underline{42.98\ N}} \uparrow$

For ~~Ray~~ R_{AY},

$\Sigma F_y = 0$,

$\therefore$

$R_{AY} + R_B = 50 + 20\sin 60° + 30\sin 45° + 15\sin 80°$

$R_{AY} + 42.98 = 50 + 17.32 + 21.21 + 14.77$
$= 103.3$

$\underline{\underline{R_{AY}}} = 60.32\ N \uparrow$

Reaction at A, $R_A = \sqrt{R_{AY}^2 + R_{AX}^2}$

$= \sqrt{8.61^2 + 60.32^2}$

$= \underline{\underline{60.92\ N}}$

Angle of RA with horizontal

$\tan\theta = \frac{R_{AY}}{R_{AX}} = \frac{60.32}{8.61} = 7.006$

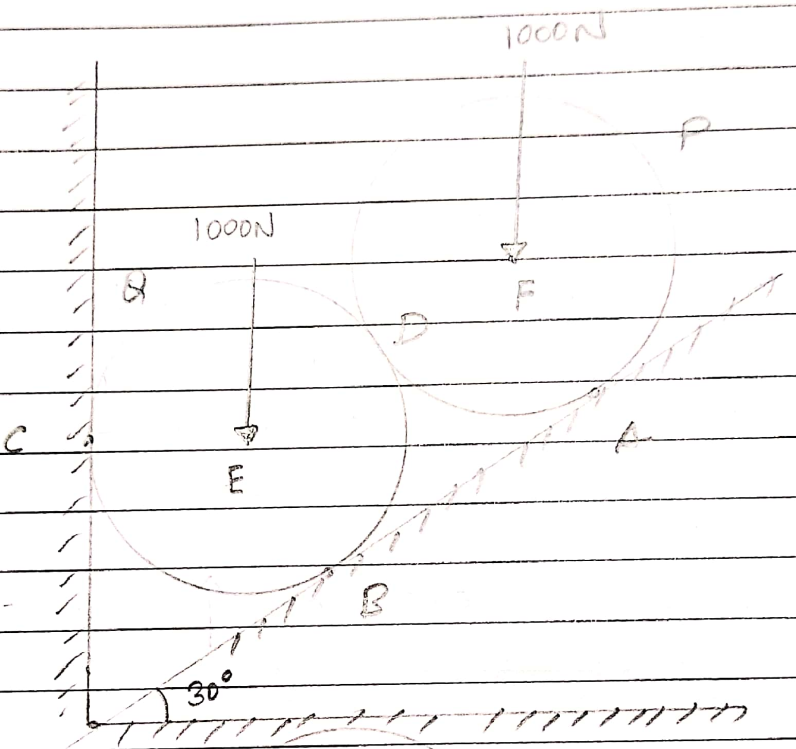
$\theta = \tan^{-1}(7.006) = \underline{\underline{81.87°}}$

ibow

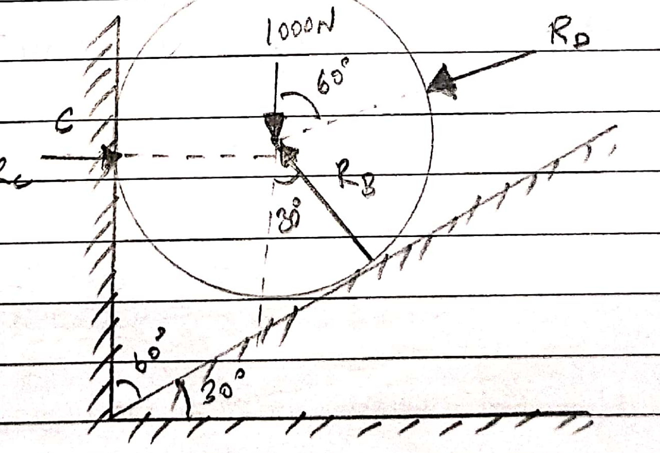
Practice set -2- FBD and equilibrium

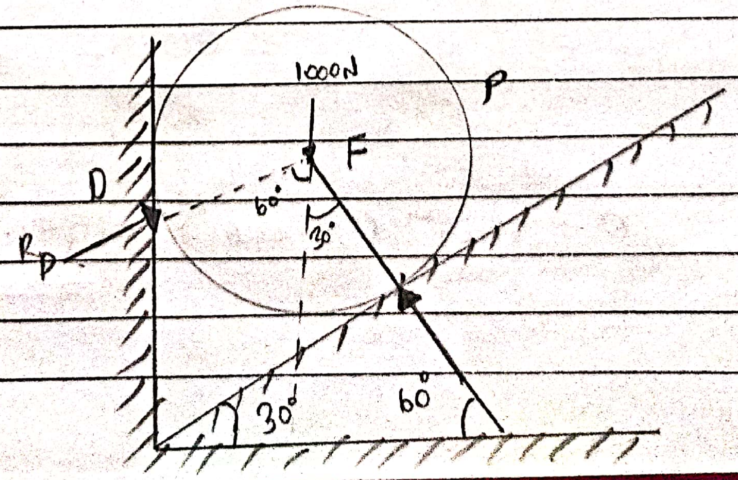
Q.4.14 Two Identical Rollers, each of weight 1000 N, are supported by an inclined plane and a vertical wall as shown. Find reactions at points of support A, B, C.

(a)



(b)





Rainbow

→ Weight of each roller = 1000 N

→ Equilibrium of Roller P

→ Forces acting on roller P are :

(1) Weight 1000N acting vertically downward
(2) Reaction R_A at A, ⊥ to OA
(3) Reaction R_D at D, ∥ to OA.

The resultant force in x and y directions must be zero.

⇒ $\Sigma F_x = 0$, we have

$R_D \sin 60° - R_A \sin 30° = 0$, or
$R_D \sin 60 = R_A \sin 30°$
$R_D = \frac{\sin 30}{\sin 60} \cdot R_A = 0.577 \, R_A$ .... (1)

⇒ $\Sigma F_y = 0$, we have

$R_D \cos 60° + R_A \cos 30° - 1000 = 0$
$(0.577 \, R_A) \cos 60° + R_A \cos 30° = 1000$. (From (1))

$R_A = \frac{1000}{1.1545} = 866.17$ N.

Putting this in (1), we get,

$R_D = 0.577 \times 866.17 = 499.78$ N.

ibow

→ Equilibrium of Roller Q.

Forces acting on roller Q, are

1. Weight $W = 1000N$
2. Reaction R_B at B $\perp$ BO
3. Reaction R_C at C $\perp$ to CO
4. Reaction R_D at D $\parallel$ to BO.

$\Sigma F_x = 0$,

$\therefore R_B \sin 30° + R_D \sin 60° - R_C = 0$

$R_B \times 0.5 + 499.78 \times 0.866 - R_C = 0$

$R_C = 0.5 R_B + 432.8.$

For $\Sigma F_y = 0$, we have,

$R_B \times \cos 30° - 1000 - R_D \cos 60° = 0$

$R_B \times 0.866 - 1000 - 499.78 \times 0.5 = 0$

$0.866 R_B - 1249.89 = 0$

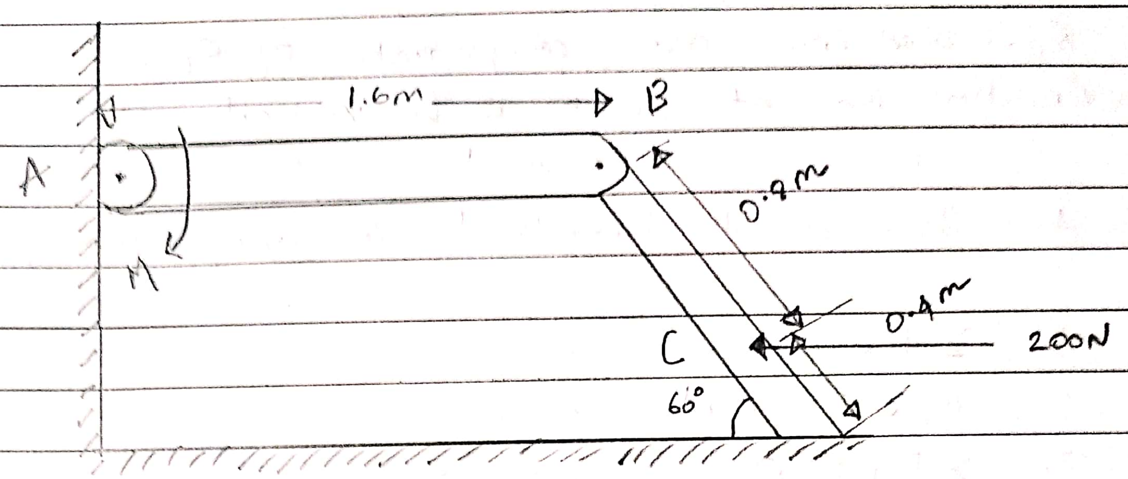
$R_B = \frac{1249.89}{0.866} = 1443.3 N.$

$R_C = 0.5 \times R_B + 432.8$

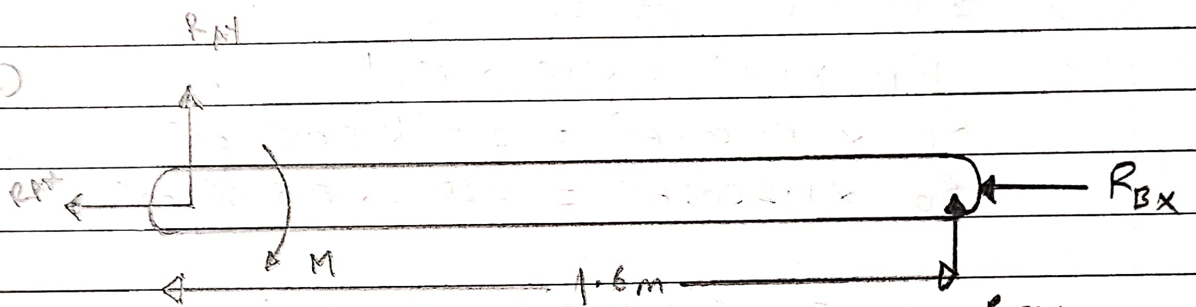
$= 0.5 \times 1443.3 + 432.8$

$R_C = 1154.45 N$

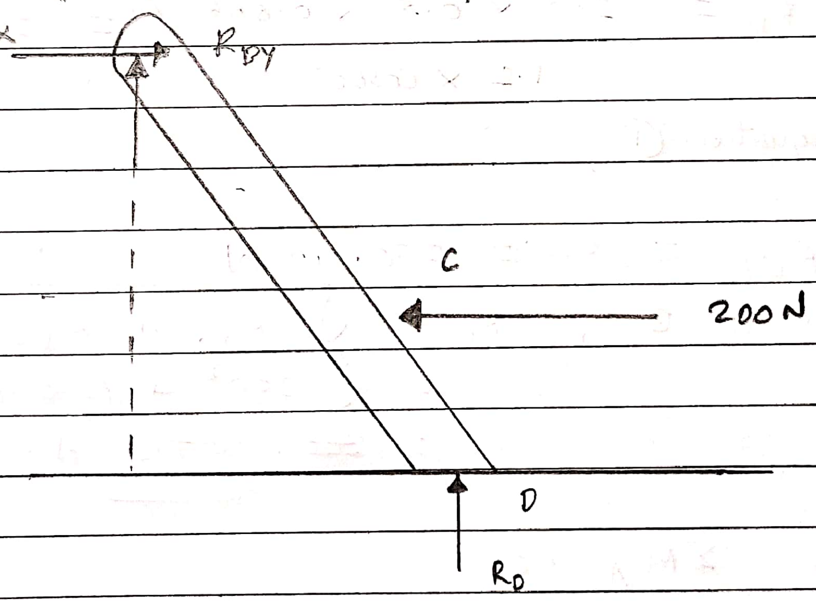
Q.4.20. A horizontal force 200N is applied to the sloping bar BCD, whose bottom rests on a horizontal plane, as shown. Its upper end is pinned at B to the horizontal bar AB which has a pinned support at A. What couple M must be applied to AB, to hold the system in equilibrium? Bars weightless, ~~surface smooth~~, pins smooth.



(a)



(b)



Length AB = 1.6 m, BD = 1.2 m; BC = 0.8 m; CD = 0.4 m
horizontal force C = 200N.

let M → couple applied about A to bar AB to keep the system in equilibrium.
let R_B → Reaction at B.

R_{BX} and R_{BY} are components of R_B.
Reaction R_D at point D is vertical.

As bar is to be held in equilibrium,

$\Rightarrow \Sigma F_X = 0$

$\therefore R_{BX} = 200N$

$\Rightarrow \Sigma F_y = 0$

$\therefore R_{BY} = R_D. \quad \ldots \quad (1)$

$\Rightarrow \Sigma M = 0$ at B.

$\therefore R_D \times DD' = 200 \times BC'$

$R_D \times BD \cos 60^\circ = 200 \times BC \sin 60^\circ$

$R_D \times 1.2 \cos 60^\circ = 200 \times 0.8 \sin 60^\circ$

$$R_D = \frac{200 \times 0.8 \times 0.866}{1.2 \times \cos 60^\circ} = 230.93\ N.$$

From equation (1),

$R_{BY} = R_D = 230.93\ N.$

Reaction at B, $R_B = \sqrt{R_{BX}^2 + R_{BY}^2}$

$= \sqrt{200^2 + (230.93)^2}$

$= \cancel{300}.\ 305.4\ N.$

also, $\Sigma M_A = 0$

$M = R_{BY} \times 1.6 = 230.93 \times 1.6$

$= 369.44\ Nm$

Q. 4.21. A body weighing 2000 N is suspended with a chain AB 2m long. It is pulled by a horizontal force of 320 N as shown. Find Force in chain and lateral displacement in body.

→ Weight suspended at B = 2000N

length AB = 2 m.

horizontal F at B = ~~200N~~ 320 N.

Let :

F = Force in chain AB

θ = Angle made by AB with horizontal

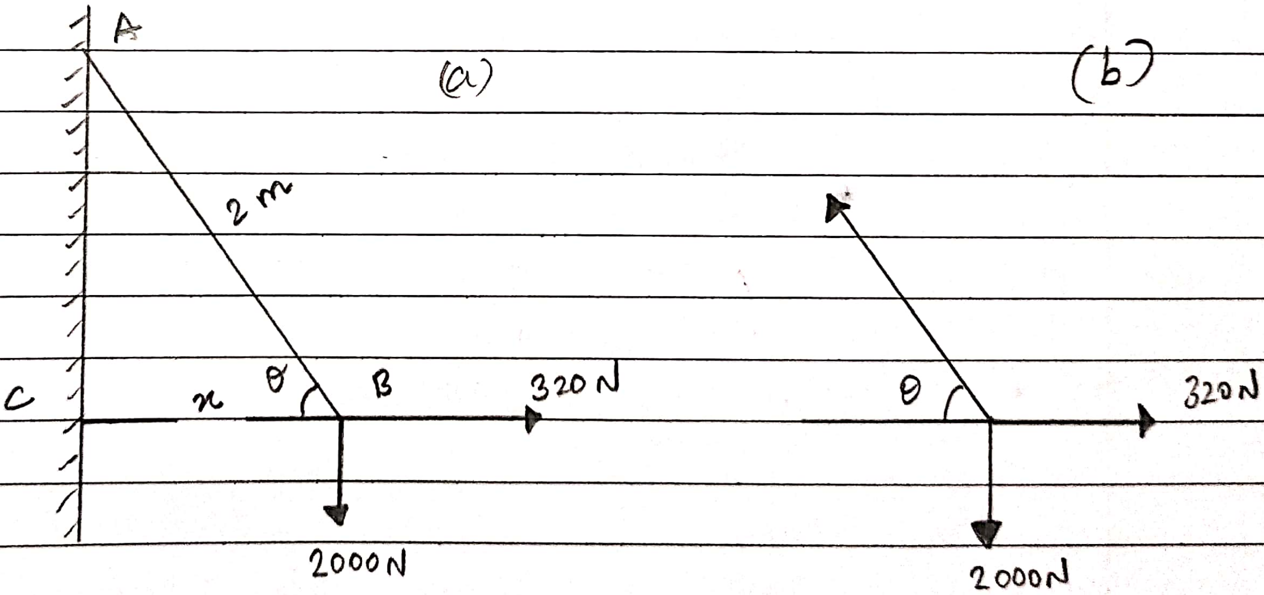
Using Lami's Theorem. (Body in equilibrium.

$$\frac{F}{\sin 90} = \frac{2000}{\sin(180^\circ - \theta)} = \frac{320}{\sin(90^\circ + \theta)}$$

$$\frac{F}{1} = \frac{2000}{\sin\theta} = \frac{320}{\cos\theta}$$

$F\sin\theta = 2000$ —— (1)

$F\cos\theta = 320$ —— (2)



Dividing (1) by (2)

$$\tan\theta = \frac{2000}{320} = 6.25$$

$$\theta = \tan^{-1}(6.25) = 80.9^\circ$$

$\therefore \quad F \sin(80.9^\circ) = 2000$ os

$F = 2025.5\,N$

Now, $\cos\theta = \frac{x}{2}$

$x = 2 \times \cos\theta$

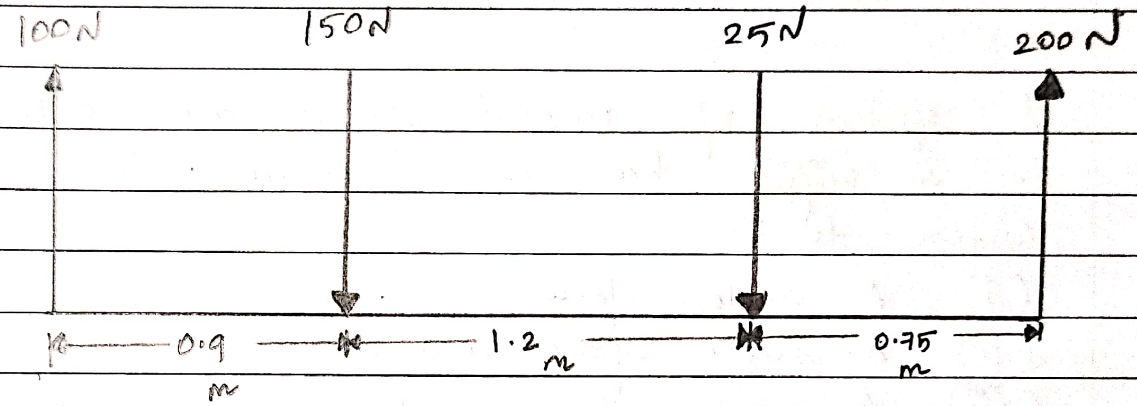
$= 2 \times \cos 80.9$

$= 0.3163\ m$

Rainbow

Practice Set 2 – Parallel Forces

3.5. Four parallel forces of magnitudes 100 N, 150 N, 25 N, 200 N are shown in figure. Determine the magnitude of the resultant and also the distance of the resultant from point A.



→ Forces are 100N, 150N, 25N and 200N.

Distances AB = 0.9 m, BC = 1.2 m. CD = 0.75 m.

As all forces are acting vertically,

$R = 100 - 150 - 25 + 200$

$= 300 - 175 = 125$ N.

Taking moments about point A,
(let x – distance from R to A)

$= 150 \times AB + 25 \times AC + 200 \times AD$
$\quad (-) \qquad\qquad (-) \qquad\qquad (+)$

$= -150 \times 0.9 - 25 \times 2.1 + 200 \times 2.85$

$= -135 - 52.5 + 570 = 382.5$ Nm.

∴ Moment of R must be 382.5 Nm, anti clockwise.

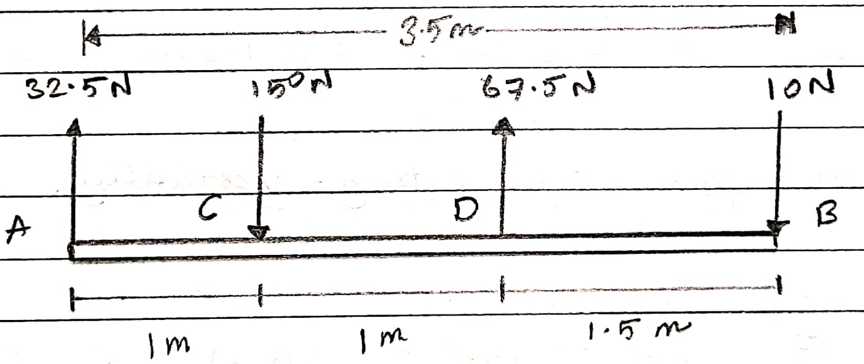
Moment of R about A,

$382.5 = R \times x = 125 \times x$

$x = \frac{382.5}{125} = \underline{\underline{3.06\ m}}$

$\therefore$ Resultant $R = 125N$ will act upwards at a distance 3.06 m from A.

Q.3.6 A system of parallel forces are acting on a rigid bar as shown. Reduce this system to:
(i) A single force
(ii) A single force and a couple
(iii) A single force and a couple B.



→ Forces at A, C, D, B are 32.5N, 150N, 67.5N.
AC = 1m, CD = 1m, BD = 1.5m.

(i) Single force system: The single force system will only have resultant force and direction.

$R = 32.5 - 150 + 67.5 - 10 = \underline{\underline{-60\ N}}$.

Negative sign shows F is clockwise. (down)

Rainbow

let x be the distance between A and R.

Taking moments about A,

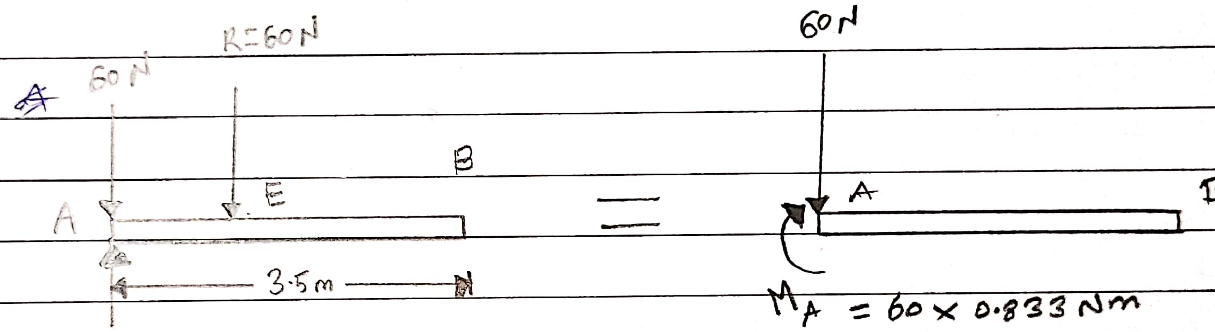
$R \times x = -150 \times AC + 67.5 \times AD - 10\, AB$

$-60\, x = -150 + 67.5 \times 2 - 10 \times 3.5$

$= -50$

$x = \frac{50}{60} = \underline{\underline{0.833}}\ m$

i)



ii) A single force and couple at A

We can replace the resultant force R with another Force at A with magnitude 60N and a couple

$= 60 \times 0.833$ Nm

$= \underline{\underline{-50}}$ Nm. (Clockwise)

iii) A single force and a couple at B.

$BE = AB - AE = 3.5 - 0.83 = 2.66$ m.

If R is moved to point B it will be accompanied with a couple of $60 \times BE$

nbow

$= 60 \times 2.66\ Nm$

$= \underline{160\ Nm}$ (anti clockwise)