Q.No.

DMGT

Q.1. Let U be the set of all integers from 1 to 1000

$$U = \{ x \mid 1 \leq x \leq 1000 \}$$
$$x \in \mathbb{Z}$$

→ let set A be the set of all integers divisible by 3

$$A = \{ x \mid x \in U, \cancel{x}\ 3 \text{ divides } x \}$$
$$= \frac{1000}{3} = 333$$

→ Let B be the set of all integers divisible by 5

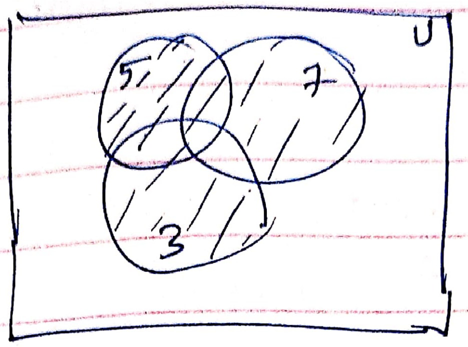
$$B = \{ x \mid x \in U, 5 \text{ divides } x \}$$
$$= \frac{1000}{5} = 200$$

→ let C be the set of all numbers divisible by 7.

$$C = \{ \cancel{xx}\ x \mid x \in U, 7 \text{ divides } x \}$$
$$= \frac{1000}{7} = 142$$

Q.No.

Drawing Venn Diagram



~~$A \cup B \cup C = |A| + |B| + |C| - (A \cup B) - (B \cup C) - |A \cup C| + |A \cap B \cap C|$~~

So set $\left|A \cap B\right| = \dfrac{1000}{3 \times 5} = 66.$

$\left|B \cap C\right| = \dfrac{1000}{5 \times 7} = \dfrac{1000}{42} = 28.$

$\left|A \cap C\right| = \dfrac{1000}{3 \times 7} = 47.$

$\left|A \cap B \cap C\right| = \dfrac{1000}{105} = 9$

$A \cup B \cup C = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

$= 333 + 200 + 142 - 66 - 28 -$

$+ 9 = 543$

So $1000 - 543 = \underline{457}$ = No. not divisible by 3 nor 5 nor 7.

Q.No. Q.2

$f(x) = 3x + 4$

$f : R \rightarrow R$

$g(x) = x^2 - 1$

$f : R \rightarrow R$

(1)

$\rightarrow$ $f \circ g = f(g(x))$

$= 3(g(x)) + 4$

$= 3(x^2 - 1) + 4$

$= 3x^2 - 3 + 4$

$= 3x^2 + 1$

So $\boxed{f \circ g(x) : R \rightarrow R = 3x^2 + 1}$

(2) $g \circ f = g(f(x))$

$g(x) = x^2 - 1$ $\quad g(f(x)) = (3x + 4)^2 - 1$

$= (3x)^2 + 2 \times 3x \times 4 + 4^2 - 1$

$= 9x^2 + 24x + 16 - 1$

$= 9x^2 + 24x + 15$

$= 3(3x^2 + 8x + 5)$

$\boxed{g \circ f(x) = R \rightarrow R = 3(3x^2 + 8x + 5)}$

Q.No. (3) $f \circ f\,(x)$

$= f(f(x)) : R \to R$

$= 3(3x+4) + 4$

$= 9x + 12 + 4$

$= 9x + 16$

$$\boxed{\begin{aligned} f \circ f\,(x) &: R \to R \\ &= 9x + 16 \end{aligned}}$$

(4) $g \circ g\,(x) = g(g(x))$

$= (x^2 - 1)^2 - 1$

$= (x^2)^2 - 2(x^2)\cdot(1) + 1 - 1$

$= x^4 - 2x^2$

$= x^2(x^2 - 2)$

$$\boxed{\begin{aligned} \text{so } g(g(x)) &: R \to R \\ &= x^2(x^2 - 2) \end{aligned}}$$

Q.No. 3

Total men = 10

Total women = 15

Space on Committee = 8

(i) 4 men & 4 women

~~4C~~ $\cancel{{}^{10}C_4}$ ~~$\frac{3}{2}$~~ ${}^{15}C_4$

= ~~210 × 1365 210 + 1365~~

= ~~286,650.~~

$$= {}^{10}C_4 \times {}^{15}C_4$$

$$= 210 \times 1365$$

$$= 286,650 \text{ ways}$$

(ii) at least 2 men :

2 men + 6 w	=	${}^{10}C_2 \times {}^{15}C_6$	=	225,225
3 M + 5 w	=	${}^{10}C_3 \times {}^{15}C_5$	=	360,360
4 M + 4 w	=	${}^{10}C_4 \times {}^{15}C_4$	=	286,650
5 M + 3 W	=	${}^{10}C_5 \times {}^{15}C_3$	=	114,660
6 M + 2 W	=	${}^{10}C_6 \times {}^{15}C_2$	=	22,050
7 M + 1 W	=	${}^{10}C_7 \times {}^{15}C_1$	=	1800
8 M	=	${}^{10}C_8$	=	45

1,010,790 ways

will have atleast 2 men

Q.No. (iii) At least 3 men :

At least 2 men − just 2 men

$= 1,010,790 - {}^{10}C_2 \times {}^{15}C_6$

$= 1,010,790 - 225,225$

$= \underline{\underline{785,565}}$ ways

(b) $x \equiv 3 \mod (4)$

$x \equiv 0 \mod (6)$

given

$m_1 = 4$

$m_2 = 6$

$r_1 = 3$

$r_2 = 0$

$M = 4 \times 6 = 24$

$M_1 = 24/4 = 6$

$M_2 = 24/6 = 4$

So $x = \left(a_1 \cdot M_1 M_1^{-1} + a_2 M_2 M_2^{-1}\right) \mod M$

M_1^{-1} = number such that $r = 3$ with mod 4

for $M_1 = 6$

This is not possible.

Chinese remainder theorem states that for a set of n numbers that are mutually coprime and a corresponding set of R there exists a number x s.t.

$x \,\%\, n_i = R_i \quad \forall\, i \in n, R$

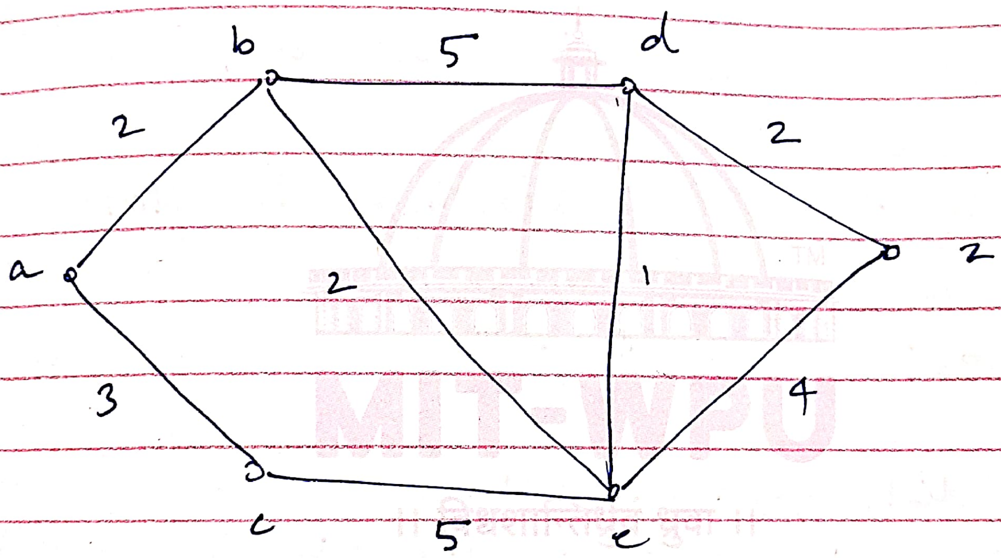
But here set $n = \{4, 6\}$ which are not coprime.

Q.No. They could be made coprime and eq. will change

$$x \equiv 3 \bmod 2$$
$$x \equiv 0 \bmod 2$$

But this set of congruence equations is different from given question.

Q(4) Dijkstra's algorithm



To find shortest path between a and z.

Assign values to ∞

so value of distances now: From (a)

a	b	c	d	e	z
0	∞	∞	∞	∞	∞

vertices adjacent to a = (b, c)

Dist (a, b) = 2 ; dist (a, c) = 3

Q.No.

Now as current value of $b = \infty$

$>$

dist $(a, b) = 2$

we will assign new value to $b = 2$

Similarly as dist $(a, c) < \infty$

new Value of $c = 3$

updated table then :

a	b	c	d	e	z
0	2	3	∞	∞	∞

as $b < c$; visiting b first

From b ; (a, d, e)

dist $(b, d) < \infty$

so $d = 5 +$ dist $(b, a) =$

dist of $(b, e) < \infty$

so $e = 2 +$ dist $(a, e) = 4$

updated values :

a	b	c	d	e	z
0	2	3	5+2	2+2	∞
0	2	3	7	4	∞

Now visiting c (a, e)

Q.No.

current value of e = 4

dist (c, e) + dist (a, c) = 3 + 5 = 8

so no change in value of e.

Now visiting e (b, d, z)

dist (e, d) + dist (a, e) = 4 + 1 = 5

current value of d = 7 > 5

so updating value of d to 5

dist (e, z) + dist (a, e) < current value of z = ∞

so z = 4 + 4 = 8

a	b	c	d	e	z
0	2	3	5	4	8

Now visiting d (b, e, z)

dist (d, z) + dist (a, d) < current value of z

2 + 5 < 8

so new value of z = 7.

Q.No.

so updated value :

a	b	c	d	e
0	2	3	5	4

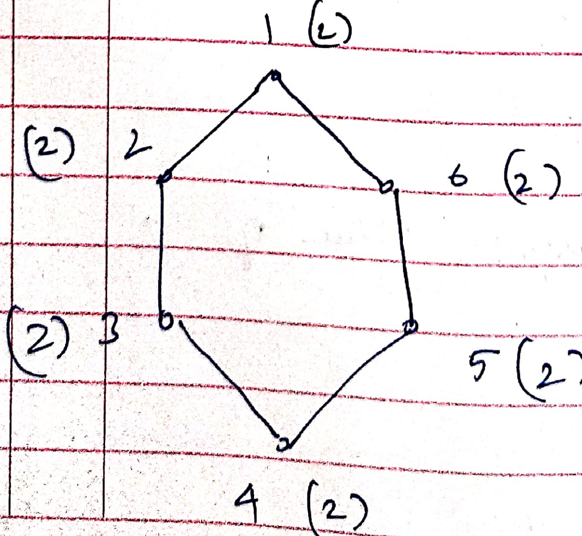
Shortest distance from a to z is therefore 7

path : a → b → e → d →

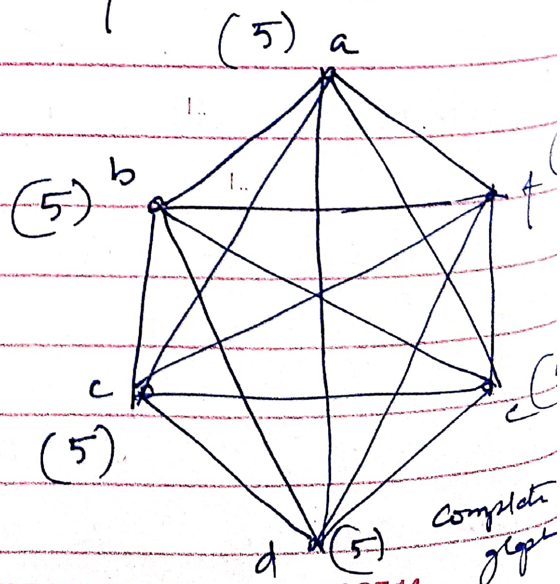
(b) Every Regular graph is not a complete graph ; but every complete graph is a regular graph [(n-1) degree].
eg to disprove given statement

Regular graph : Every vertex has same degree

Complete graph : Every vertex has degree n-1 for n vertices



K_2



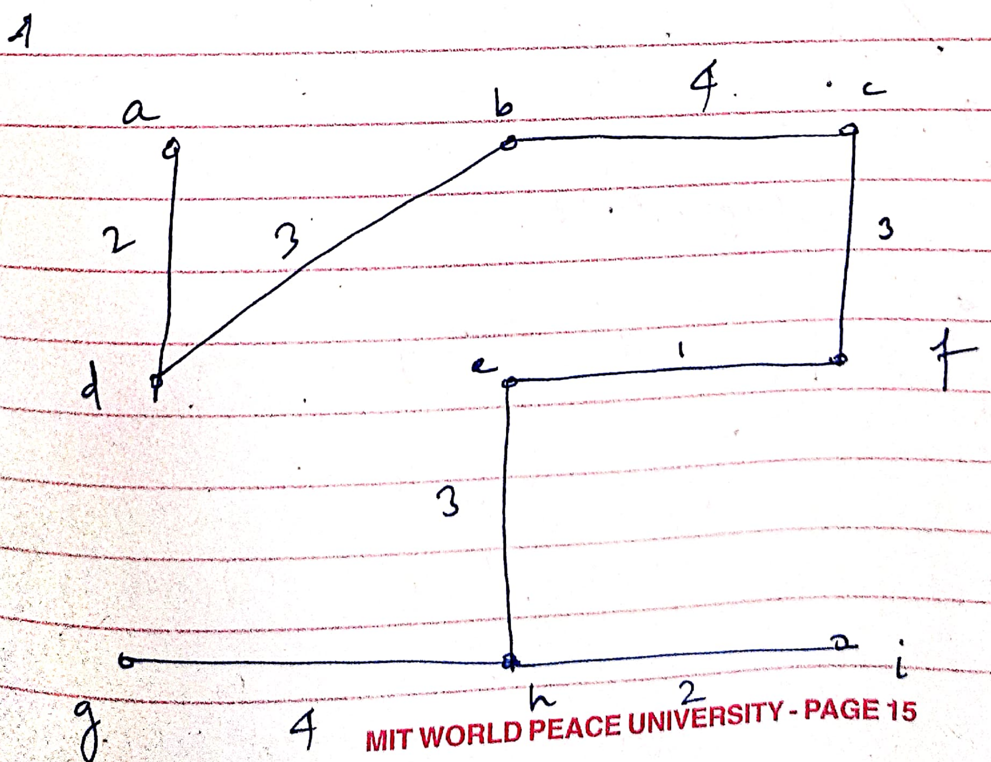
Complete graph K_6

Q.No.

Q(5) b. Kruskal's algorithm to find min cost spanning tree.

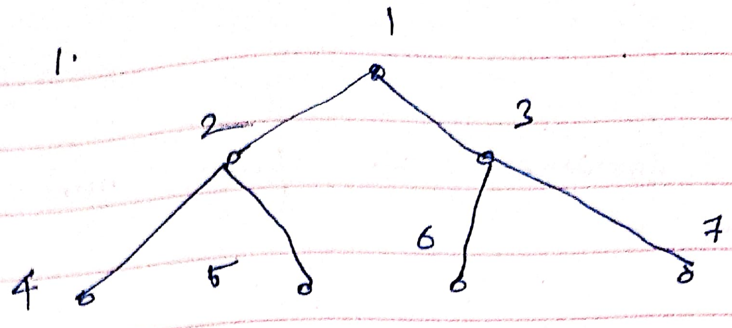
vertices and weights of edges in ascending order.

1. e − f = 1
2. h → i = 2 , a − d = 2
3. c − f = 3
4. d − b = 3 , e − h = 3
5. ~~b~~ e h − f = 4
6. f − i = 4
7. b − c = 4
8. g − h = 4
9. b − e = 5 , a − b = 5
10. d − g = 6
11. b − f = 6
12. b − d = 7
13. d − h = 8



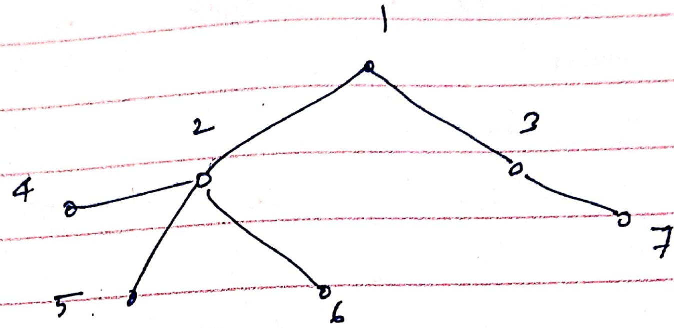
Q.No. (a)

1.

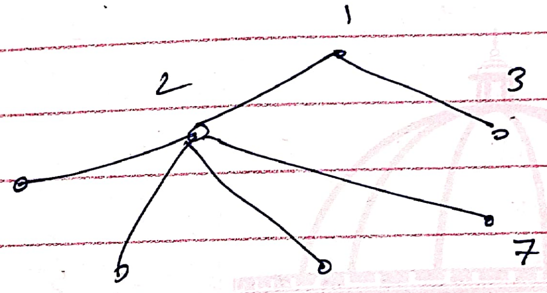


deg = 2
for eac

2.



3.



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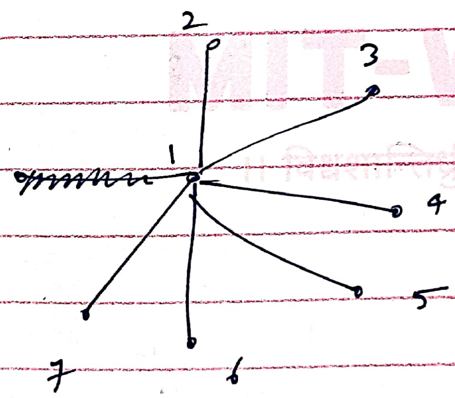
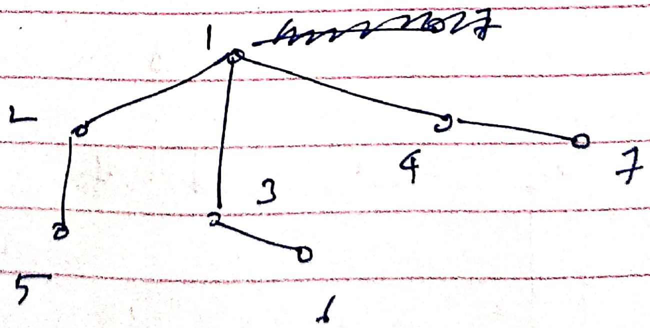


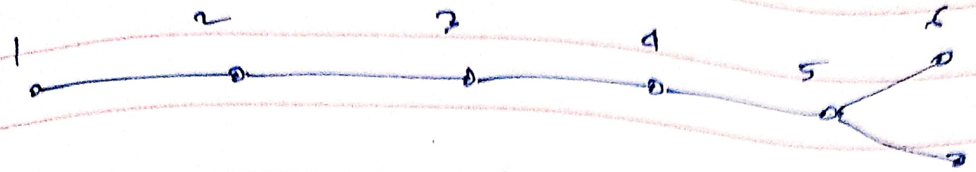
5.



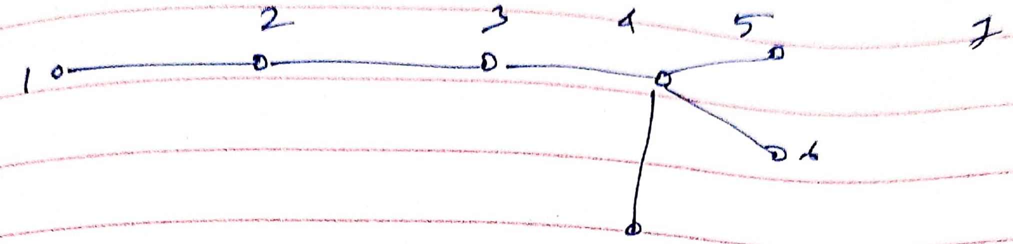
Q.No.

6.

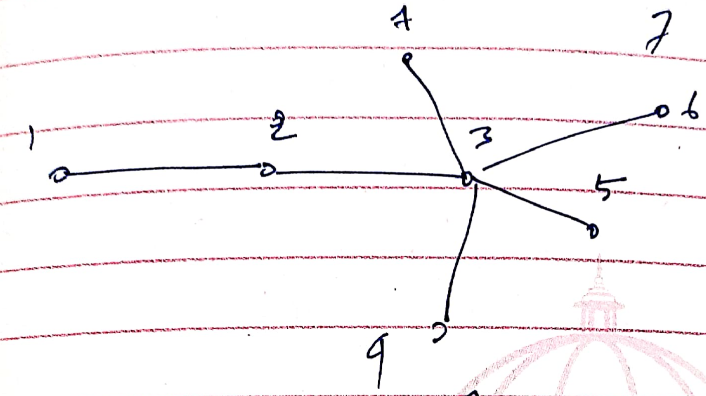
7.



8.



9.



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