10/7/23

AA – Theory Assignment – 1

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Q.1. Towers of Hanoi

* It is a classic problem where you try to move all disks from one peg to another using only 3 pegs

The rules are as follows :

1. Only 1 disk can be moved at a time

2. Each disk consists of taking the upper disk from one of the stacks and placing it ~~of~~ on top of another stack

3. No disk may be placed on top of a smaller ~~disks.~~ peg disc.

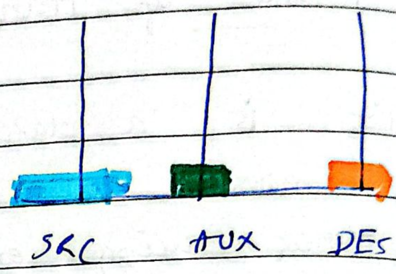
eg. Let us consider 3 disks stacked on one peg.

The steps to solve it would be:

$2^{no-of-start pegs} = 2^3 = \cancel{8}\ 7$

(1)

(2)



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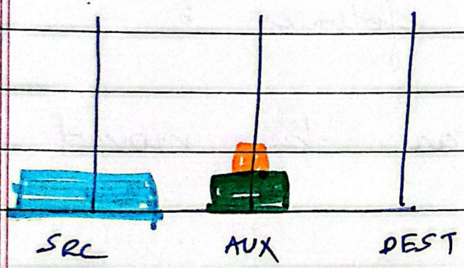

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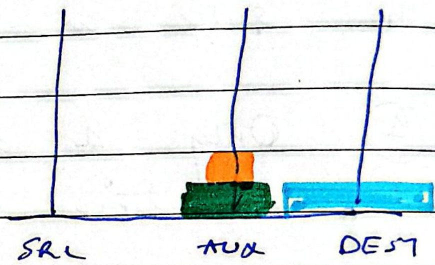

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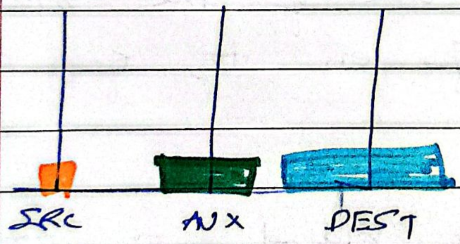

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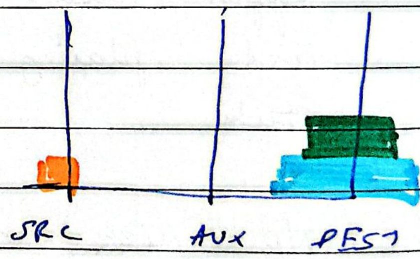

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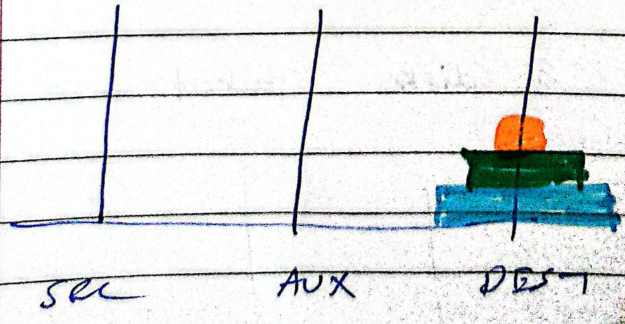

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→ The method is straight forward

1. we move $(n-1)$ disks from 1st (A) peg to ~~second~~ last peg (C)

This takes $T(n-1)$ steps

2. Shift last disk from A to C
takes $T(1)$

3. Shift $(n-1)$ disks again from B to C using A

also takes $T(n-1)$ steps.

so total time $= T(n-1) + 1 + T(n-1)$

so

$$T(n) = 2T(n-1) + 1 \quad \text{— (1)}$$

using substitution method,

put $n = (n-1)$ in eq. (1)

$$T(n-1) = 2T(n-2) + 1 \quad \text{— (2)}$$

putting (2) in (1)

$$T(n) = 2^2 T(n-2) + 2 + 1 \quad \text{— (3)}$$

Solving again,

let $n = n-2$ in eq (1)

so $T(n-2) = 2T(n-3) + 1$ — (4)

put (4) in (3),

$T(n) = 2^3 T(n-3) + 2^2 + 2 + 1$ — (5)

This is a gp.

we know $T(1) = 1$

so,

as $\boxed{T(n) = 2^i T(n-i) + 2^{i-1} + 2^{i-2} \ldots 2^0}$

So $n-1 = i$ for stopping condition

$T(n) = 2^i (1) + 2^{i-1} + 2^{i-2} \ldots 2^0$

$s_n = \frac{a(1-r^n)}{1-r}$

$T(n) = \frac{1(1-2^{i+1})}{(1-2)} = \frac{2^{i+1} - 1}{1}$

$T(n) = 2^{i+1} - 1$

put $i = n-1$

so $2^{n-1+1} - 1 = T(n)$

so $2^n - 1 = T(n)$

which is the required complexity.

eg problem; $n = 3$

so steps $= 2^3 - 1$
$= 8 - 1 = 7$ steps.

Pseudo code

function MoveTower (disk, source, dest, spare) :

```
{
    If disk == 0 :
        move disk from src to dest

    Else :
//1     MoveTower (disk -1, source, spare, dest)
//2     Move disk from source to destination
//3     MoveTower( disk -1, spare, dest, source)
}
```

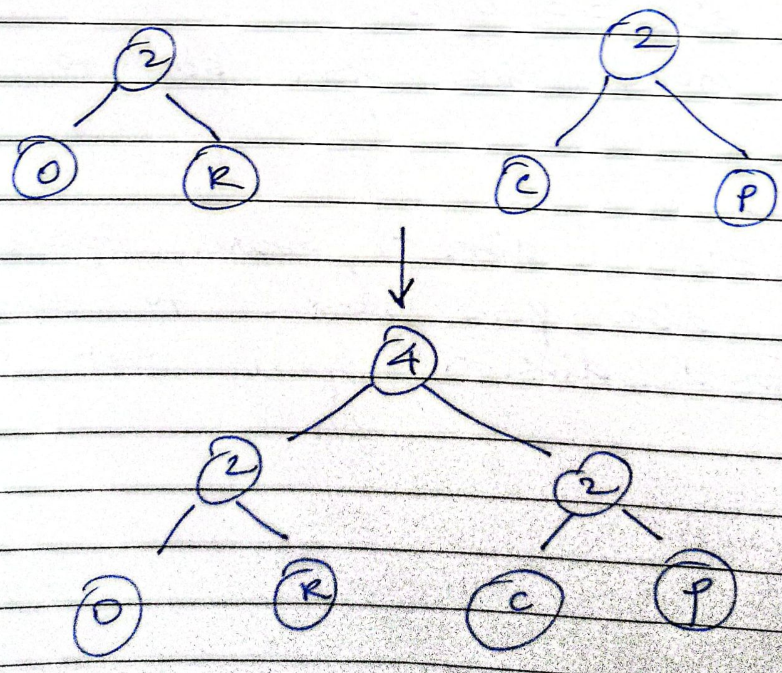
Q.2. Huffman codes

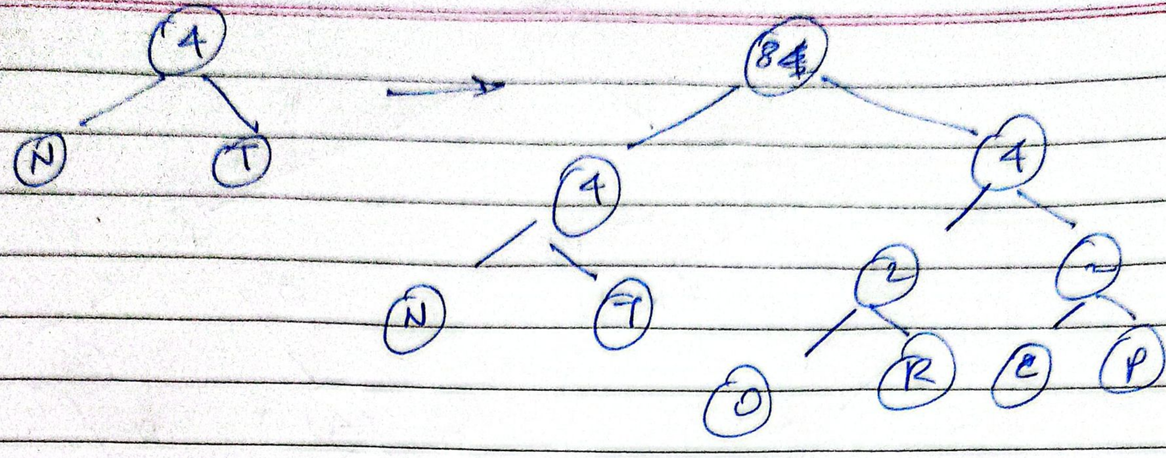
→ WE WANT PEACE NOTWAR

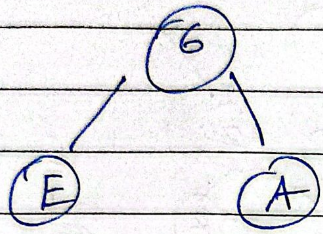
Let us count the frequency of each letter

W — 3
E — 3
A — 3
N — 2
T — 2
P — 1
C — 1
O — 1
R — 1

Let us now make trees







Now 3, 3 → 6 is smaller

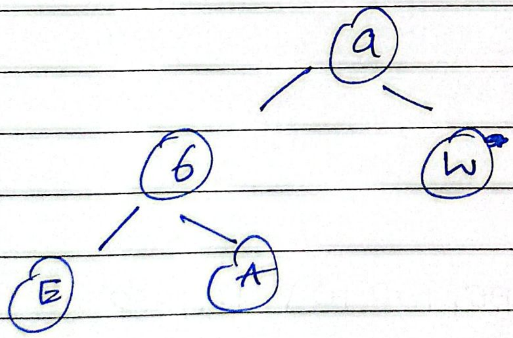
+ 6

= 12

than

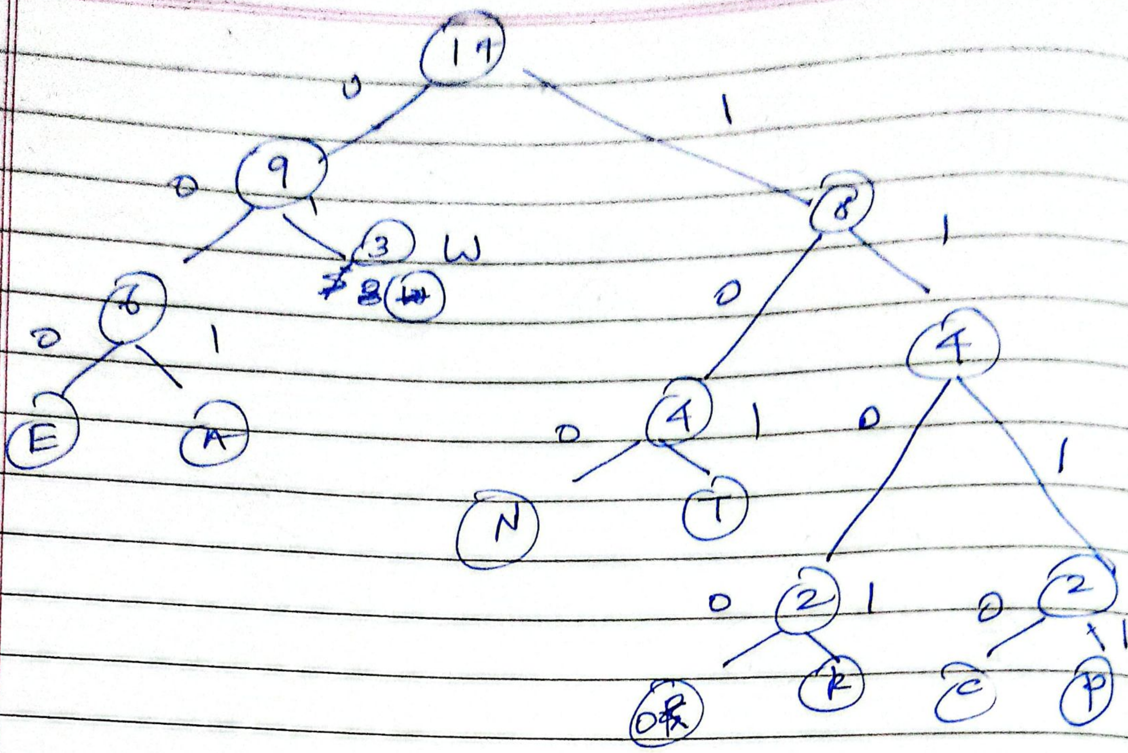
6 + 8 = 14

So



Finally combining everything,

we get,



So

W — 01
E — 000
A — 001
N — 100
T — 101
P — 1111
C — 1110
O — 1100
R — 1101

Code : 01 000 01 001 100 101 1111 000 001 1110 000
100 1100 01 001 1101