

Tutorial - 3

Q.1.  $f(n) = \log(n+y)$

$y^{(1)} = 2$ ;  $h = 0.2$

so  $n_0 = 1$

$y_0 = 2$

$n_{\text{end}} = 1.2 \text{ & } 1.4$

$n_1 = 1.2$ ;  $n_2 = 1.4$

$y_1 = y_0 + h f(n_0, y_0)$

(euler)

$$\begin{aligned} y_1 &= 2 + (0.2) (0.477121) \\ &= 2 + 0.095424 \\ &= 2.095424 \end{aligned}$$

(mod Euler)

$$\begin{aligned} y_1^{(1)} &= y_0 + 0.1 (f(n_0, y_0) + \\ &\quad f(n_1, y_1)) \\ &= 2.0 + 0.1 (0.477121 + 0.5179113) \\ &= 2.0 + 0.9950323 \cdot 0.1 \\ &= 2.09950323 \end{aligned}$$

$$\begin{aligned} y_1^{(11)} &= y_0 + 0.1 (0.477121 + 0.51844) \\ y_1 &= 2 + 0.99856 \cdot (0.99569) 0.1 \\ &= 2.099856 - 2.00 \quad 2.099569 \end{aligned}$$

$$\begin{aligned} y_1^{(111)} &= 2 + 0.1 (0.477121 + 0.51982) \\ y_1 &= 2 + 0. \quad 2.099569 \end{aligned}$$

so for  $n = 1.2$ ;  $y(\text{modif}) = \underline{\underline{2.099569}}$

$$y_2 = y_1 + 0.2 (0.54400) = 2.208304$$

$$= 0.108801 + 2.099569 \quad \underline{\underline{11}}$$

$$y_2^{(1)} = y_1 + 0.1 (f(z_1, y_1^{(2)}) + f(z_2, y_2))$$

$$= 2.099569 + 0.1 (0.51845 + 0.55730)$$

$$= 2.099569 + 0.107575$$

$$= 2.2071447$$

$$y_2^{(1)} = y_1 + 0.1 (f(z_1, 2.099569) + f(z_2, 2.2071447))$$

$$= 2.099569 + 0.1 (0.51845 + 0.557163)$$

$$\approx 2.20713$$

          

$$\text{So } \quad \begin{matrix} z_2 \\ \Rightarrow \end{matrix} \quad z_2 = 1.4 ; \quad y_2 = 2.20713 \quad \underline{\underline{\quad}}$$

$$Q.2. \quad \cancel{h} = h = 0.2$$

$$z_0 = 0$$

$$y_0 = 1$$

$$k_1 = 0.2 (f(0, 1))$$

$$= 0.2 (1) = 0.2$$

$$k_2 = 0.2 (f(0.1, 1.083))$$

$$= 0.1666.$$

$$k_3 = 0.2 (f(0.1, 1.083))$$

$$= 0.2 (0.84509)$$

$$= 0.16901$$

$$k_4 = 0.2 (f(\cancel{0.2}, (0.2, 1.6901))$$

$$= \cancel{0.52901} \quad 0.105814$$

$$K = 0.997039 / 6$$

$$y_1 = \frac{+0.997039}{1} + 1.16283$$

$$\begin{array}{l} y_2 = ? \\ y_1 = \frac{+0.997039}{1.16283} \end{array} \quad | \quad h = 0.2$$
$$K_1 = 0.2 \left( f(0.2, 1.997039) \right)$$

$$\begin{aligned} &= 0.2 (0.5469) \\ &\approx 1.0939 \end{aligned}$$

$$\begin{aligned} K_2 &= 0.2 \cdot f(0.3, 2.17525) \\ &\approx 0.2 (0.40399) \\ &\approx 0.0807 \end{aligned}$$

$$\begin{aligned} K_3 &= 0.2 \cdot f(0.3, 0.16673) \\ &\approx 0.42892 \end{aligned}$$

$$k_4 \approx 1.09 = 0.4070$$

$$K = 0.9196$$

$$y_2 = 2.0479$$

$$h = 0.1 \quad ; \quad y_0 = 1$$

$$3. \quad k_1 = 0.1 ( f(1, 1) ) \\ = 0$$

$$k_2 = 0.1 ( f(1.05, 1) ) \\ = -0.0453$$

$$k_3 = 0.1 ( f(1.05, -0.0226) ) \\ \approx 0.092860$$

$$k_4 = 0.1 ( 1.1, 1.0928 ) \\ = -0.0167$$

$$L = 0.07842 / 6 = 0.01307$$

$$\boxed{y^2 = 1.01307}$$

$$4. \quad h = 0.05$$

$$x_0 = 0$$

$$y_0 = 1$$

$$y_1 = y_0 + h ( f(x_0, y_0) )$$

$$= 1 + 0.05 ( 1 )$$

$$= 1 + 0.05 \cancel{1} + 0.050$$

$$= 1.050$$

$$h_1 = 0.025 = 0.05$$

$$y_1^{(1)} = 1 + 0.025 ( 1 + 1.050 ) \\ = 1.0539$$

$$y_1^{(1)} = 1 + 0.025 (1 + 1.15649)$$

$$= 1.5391$$

~~1.5391~~

$$\cancel{y_2} = n_2 = 0.1 \quad ; \quad n_1 = 0.05$$

$$y_1 = 1.5391$$

$$y_2 = ?$$

$$y_2 = y_1 + 0.025 (\cancel{1.79301})$$

$$= 1.5391 + 0.025 \cancel{1.79301} = 2.372172$$

$$y_2^{(1)} = y_1 + 0.025 (\cancel{1.79301} + 1.666 + 2.7093)$$

$$= \cancel{1.5391} + 2.63292$$

$$y_2^{(1)} = 1.5391 + 0.025 (1.666 + 2.9962)$$

$$= 2.70965$$

$$y_2^{(1)} = 1.5391 + 0.025 (1.666 + 3.075715)$$

$$= 2.724$$

$$y_2^{(1)} = 1.5391 + 0.025 (1.666 + 3.0969)$$

$$= 2.724$$

~~2.724~~

$$f_0 \cdot 3 = 0.1$$