MIT-WPU

T.Y. B.Tech

Theory of Computation

(CET2008B)

Course Objective & Course Outcomes

- **Course Objectives:**

By participating in and understanding all facets of this Course a student will be able:

1. To learn Automata theory, Regular Expression from the perspective of formal languages
2. To learn Context Free Grammar, Pushdown Automata, Turing Machine and Complexity Theory
3. To Aquent with various applications of Automata Theory
4. To realize Industry relevance of Automata
5. To apply automata theory concepts in design and c implementation of Compilers.
6. To apply automata theory concepts in various domains.

- **Course Outcomes:**

After successful completion of this course students will be able:

1. To construct Finite Automata to solve problems in computing
2. To build regular expressions and understand regular language
3. To construct context-free grammar and Push Down Automata
4. To design computational models and classify the problems of decidability

Text Books & Reference Books

- **Text Books**

1. Vivek Kulkarni, Theory of Computation, Oxford University Press, ISBN-13: 978-0-19-808458-7
2. K.L.P Mishra, N. Chandrasekaran, Theory of Computer Science (Automata, Languagesand Computation), Prentice Hall India, 2nd Edition.

- **Reference Books**

1. Introduction to the Theory of Computation, Michael Sipser.
2. Introduction to Languages and the Theory of Computation, John Martin.
3. Computers and Intractability: A Guide to the Theory of NP Completeness, M. R. Garey and D. S. Johnson

Important Instructions

- Be ready Be Attentive
- Keep a separate notebook for TOC
- Add your profile picture and use only MITWPU email-id for TOC communications and submissions.
- ***Basic Instructions:***

➢Submission of Theory Assignments and relevant components on Ruled pages

➢Before uploading label your pages with your Rollno., Name, Component Name.

➢Upload pdf with name :ComponentName_FirstNameLastname

➢You maybe asked to keep your video ON anytime during the Sessions.

Unit I

- Introduction to Automata: Computability and Complexity theory:
- Concepts of Automata Theory: Alphabet, languages and grammars, productions and derivation,
- Introduction to Finite Automata,
- Simplified notation: State transition graph, Transition table, Acceptance of a string, Acceptance of a Language,
- Deterministic finite Automata: (DFA)-Formal Definition,
- Non Deterministic Finite Automata(NFA)-Formal Definition,
- Non-Deterministic Finite Automata (NFA) with epsilon transition, Equivalence of NFA and DFA, Conversion from NFA to DFA, Conversion from NFA with epsilon transition to DFA, Minimization of finite automata.
- Finite Automata with output : Moore and Mealy Machine, Moore to Mealy conversion, Mealy to Moore conversion

Planner - Lectures

Lecture No	Topics Covered
1	Introduction to TOC, Introduction to Automata: Computability and Complexity theory, Basic concepts: Alphabet, languages and grammars, productions and derivation
2	Introduction to FA, State transition graph, Transition table, Acceptance of a string, Acceptance of a Language,
3	DFA -Formal Definition, NFA-Formal Definition, NFA with epsilon transition, Equivalence of NFA and DFA
4	Conversion from NFA to DFA
5	Conversion from NFA to DFA (cont), Conversion from NFA with epsilon transition to DFA
6	Conversion from NFA with epsilon transition to DFA (cont)
7	Minimization of FA
8	Applications and Limitations of FA.
9	Finite Automata with output : Moore and Mealy Machine, Moore to Mealy conversion
10	Mealy to Moore conversion

Topics

About TOC

- Course is about models of computation, their power, and relationships.
- Hierarchy of models of increasing power:

  -DFA < PDA < TM.

For each model, we study:

-What can be computed.

-What cannot be computed.

- Automata theory is the study of abstract computing devices or "machines"

History of TOC

- 1930-45: Alan Turing studied Turing machine

  Describe boundary between what a computing machine could do and what it could not do

- 1940-50: Researchers studies Finite Automata
- Late 1950s: N. Chomsky studied formal "Grammars"
- 1969-70: S. Cook extended Turing's study

  Separated problems that can be solved efficiently by computers and the problems that can be solved theoretically, but takes so much time by computers to solve

Complexity theory and Computability theory

- Computability theory is concerned with what can be computed versus what cannot.
- Complexity theory is concerned with the resources required to compute the things that are computable.
- Computability is about what can be computed.
- Complexity is about how efficiently can it be computed.

Automata theory

- The study of **abstract 'mathematical' machines** or systems and the computational problems that can be solved using these machines.
- These abstract machines are called automata.
- Automata theory is also closely related to **formal language** theory, as the automata are often classified by the class of formal languages they are able to recognize.
- An automaton can be a finite representation of a formal language that may be an infinite set.
- Automata are used as theoretical models for computing machines, and are used for proofs about computability.

Basic Definitions

- **Difference Between Formal Language and Natural Language**

Formal Language	**Natural Language**
1. All the rules are stated explicitly	1. Not necessarily rules to be stated explicitly. It is a medium of communication.
2. Considered as Symbols on papers	2. Expressions of ideas in Natural way.
3. In formal Languages we are interested in the form of strings of symbols and not meaning	3. We are interested in Meaning than only strings.

Basic Definitions

- **Alphabet** - a finite set of symbols.
  - ✓ An alphabet is a finite set of symbols. These symbols are the basic elements from which strings are formed.
  - ✓ In the context of automata theory, an alphabet is used to define the input and output symbols for a machine or a formal language.
  - ✓ Notation: Σ .
  - ✓ E.g.: Binary alphabet {0,1},
    English alphabet {a,...,z,!,?}

- **String over an alphabet Σ** - a finite sequence of symbols from Σ.

E.g: 0100 is a string over the binary alphabet.
a!? is a string over the English alphabet.

Basic Definitions (cont…)

- **Languages:**
  - ✓ In automata theory, a language is a set of strings over a given alphabet.
  - ✓ Languages can be classified into various types, such as regular languages, context-free languages, context-sensitive languages, and recursively enumerable languages, based on the complexity of the rules defining them.
- **Language over alphabet Σ - a set of strings over Σ.**

  Notation: L.

  E.g:{0, 00, 000, ...} is an "infinite" language over the binary alphabet.

  {a, b, c} is a "finite" language over the English alphabet.
- **Empty Language:** Empty set of strings.

  Notation: Φ

Basic Definitions (cont…)

- **Regular Language-** a finite set of symbols.

  L is called regular Language if there is some automaton that accepts L.

  E.g.Σ^* , Even length of strings, strings containing abba are regular Languages.

- **Empty string:** e or ε denotes the empty sequence of symbols.

- **Grammars:**
  - ✓ A grammar is a set of production rules that describe how strings from a language can be generated.
  - ✓ Formal languages are often defined by grammars. Different types of grammars are used to generate different classes of languages.

Basic Definitions (cont…)

- **Productions:**
  - ✓ Productions are the rules in a grammar that specify how symbols can be replaced by other symbols or strings.
  - ✓ In the context of context-free grammars, a production typically takes the form of a non-terminal symbol being replaced by a sequence of terminal and/or non-terminal symbols.
- **Derivation:**
  - ✓ Derivation is the process of applying production rules to generate strings in a language.
  - ✓ Starting with an initial symbol, successive applications of production rules lead to the generation of strings in the language defined by the grammar.

Basic Machine and Finite State Machine

- **Basic Machine:**
  - A machine that recognizes input set I and produces output set O,
    where I & O are finite.
  - Machine Function(MAF): I-> O
  - Deals with **what** output is generated, not **how** output is generated

E.g.: Switch

Basic machine

It is the most primitive machine with a set of inputs I and set of output O.

E.g. AND,OR,NOR… gates , Vending machine, Decimal to binary converter, Electric fan…

Weighing machine : I/p- Coin

O/p- Printed weight ticket

It just maps input set to output set.

Mapping function is called Machine function MAF I -> O

E.g.for AND gate MAF:

I	O
(0,0)	0
(0,1)	0
(1,0)	0
(1,1)	1

Limitations:

Cannot remember the intermediate states.

Finite State machine(FSM)

In FSM the internal state of machine changes when it receives I/P.

It has a pair of functions:

Machine Fn MAF : I x S -> 0

State Fn STF : I x S -> S

Where S: finite or internal state of machine
I: set of i/p symbols
O: set of o/p symbols

For a given i/p there will be always only one final state.

e.g. Binary Adder, Divisibility by 3 tester..

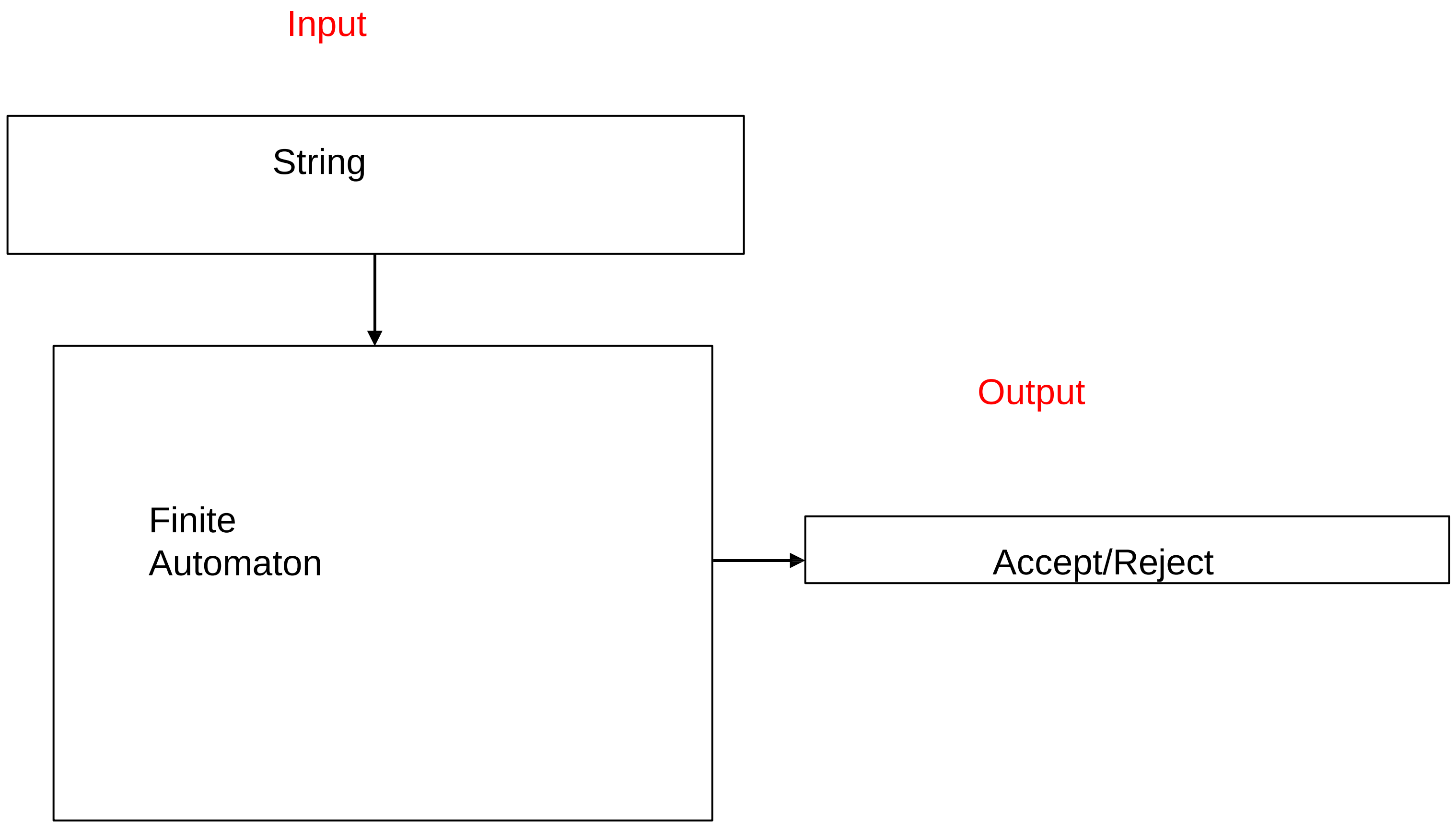
Finite Automata

- **5-tuple:**
- M= (Q,Σ, δ, q_o, F)

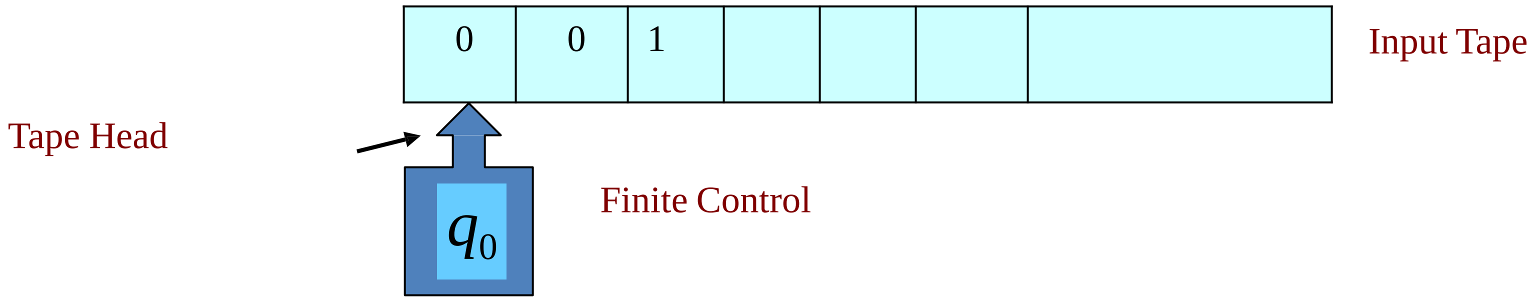
where,

- Q: Finite set of states
- Σ: Finite input alphabet
- δ: STF that maps Q X Σ to Q, i.e. Q X Σ $\rightarrow$ Q
- $q_{o:}$ Initial state of FA, $q_o \in Q$
- F: Set of final states, $F \subseteq Q$

Finite Automata (cont…)



Block diagram of FA



- Tape is divided into units ,each containing one symbol of the i/p string from Σ.
- Read the current letter of input under the tape head.
- Transit to a new state depending on the current input and the current state, as dictated by the transition function $\delta(q_i,a)=q_j$.
- Halt after consuming the entire input.

Acceptance of a String

- A string ‘x’ is said to be accepted by an FA (given by):

$M = (Q, \Sigma, \delta, q_o, F)$

if $\delta(q_o, x) = p$,

for some $p \in F$ (i.e. p is a member of F)

Acceptance of a Language

- If there is a language L such that:

$$L = \{x \mid \delta(q_o, x) = p, \text{ for some } p \in F\},$$

Then it is said to be accepted by the FA, M

Finite Automata (cont…)

- Finite number of states
- No memory to store intermediate results
- Limited power due to lack of memory
- Input string is considered to be accepted, if the FA resides in any of the final states
- Input string is considered to be rejected, if the FA resides in any of the non-final states

Preferred Notation for FA

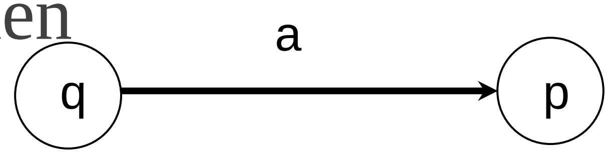
- ❑Transition Diagram
- ❑Transition Table

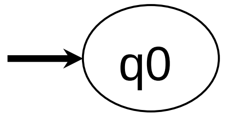
Transition Diagram

Transition diagram for FA($Q,\Sigma,q_{0,}F,\delta$) is a graph defined as:

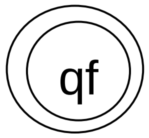
- For each state there is a node.
- For each state q in Q and for input symbol in Σ .

let δ(q,a)=p then


$$\begin{tikzcd} q \arrow[r, "a"] & p \end{tikzcd}$$


$$\begin{tikzcd} {} \arrow[r] & q0 \end{tikzcd}$$

- There is arrow into start state.



- Final states marked by double circle.

Transition Table

- Transition table is a conventional, tabular representation of a function like δ(delta) that takes two arguments and return a state.
- The rows of the table correspond to the states, and the columns correspond to the input.
- The entry for one row corresponding to the states q and the column corresponding to the input a is the state δ (q,a)

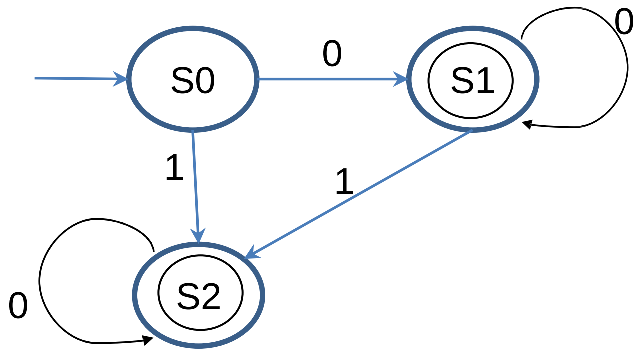
for example:

	0	1
q0	q2	q0
q1	q1	q1
q2	q2	q1

Deterministic Finite Automata

- An FA is said to be deterministic, if for every state there is a unique input symbol, which takes the state to the required next unique state

- Given a state Sj, the same input symbol does not cause the FA to move into more than one state-there is always a unique next state for an input symbol

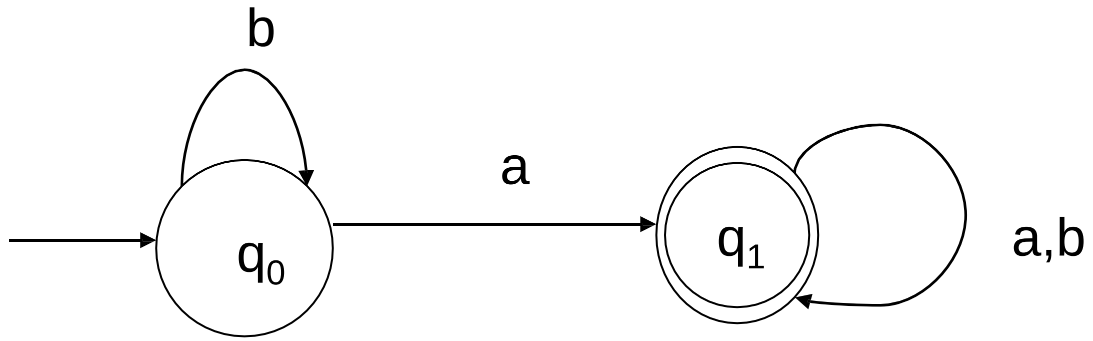


Designing FA

- Accept strings consisting of a & b, that has at least one ‘a’

i)Min string :a

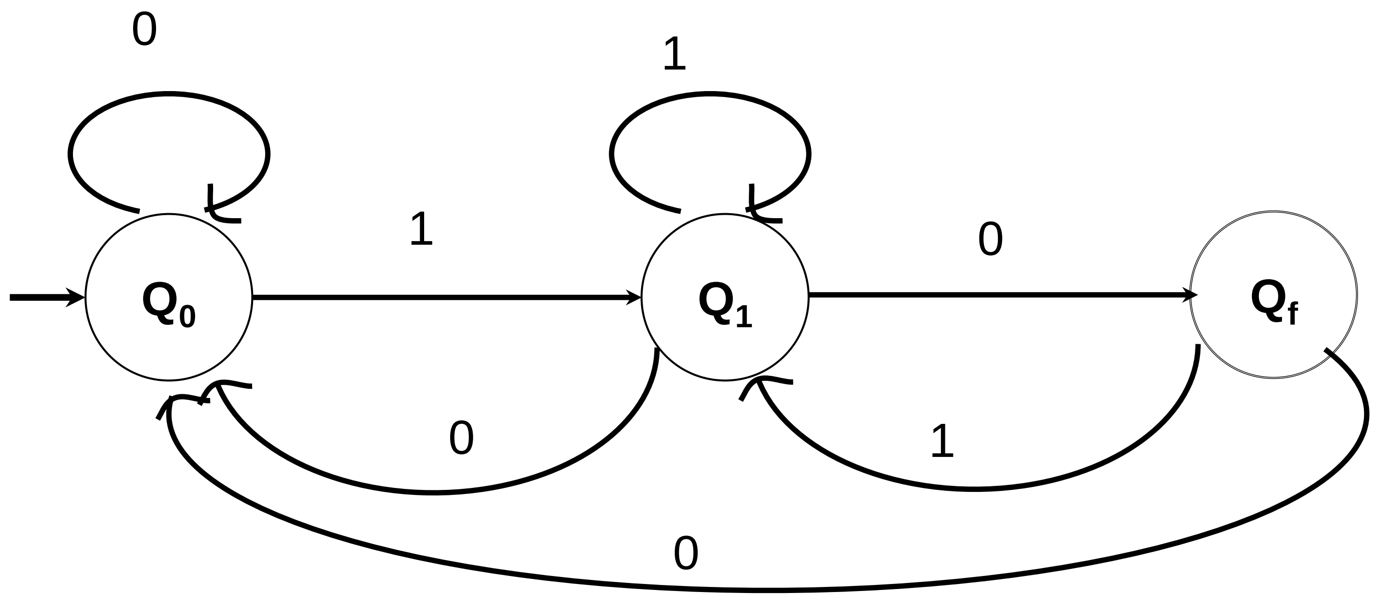
ii)∑ ={a,b}



$$\begin{tikzcd} \arrow[r] & q_0 \arrow[loop above, "b"] \arrow[r, "a"] & q_1 \arrow[loop right, "{a,b}"] \end{tikzcd}$$

Finite Automata Example

FA for language over {1,0} in which every string ends with 10.



Deterministic finite automata (D.F.A)- Formal definition

- A deterministic finite automaton M is a 5-tuple, **(Q, Σ, δ, q0, F)** consisting of-
  - a finite set of states (Q)
  - a finite set of input symbols called the alphabet (Σ)
  - a transition function ($\delta : Q \times \Sigma \rightarrow Q$)
  - a start state ($q0 \in Q$)
  - a set of accept states ($F \in Q$)
- Let w = a1a2 ... an be a string over the alphabet if a sequence of states, r
- r0 = q0
- ri+1 = δ(ri, ai+1), for i = 0, ..., n-1
- rn∈ F.

FA(DFA & NFA) Examples

1. Construct a FA to recognize language containing strings starting with ‘10’
2. Construct a FA to recognize language containing strings ending with ‘101’
3. Construct a FA to recognize language of strings with exactly two 0s anywhere
4. Construct a FA to recognize language containing strings starting with ‘a’
5. Construct a FA to identify string containing even number of ‘a’s over $\sum = \{a\}$

Non-Deterministic Finite Automata

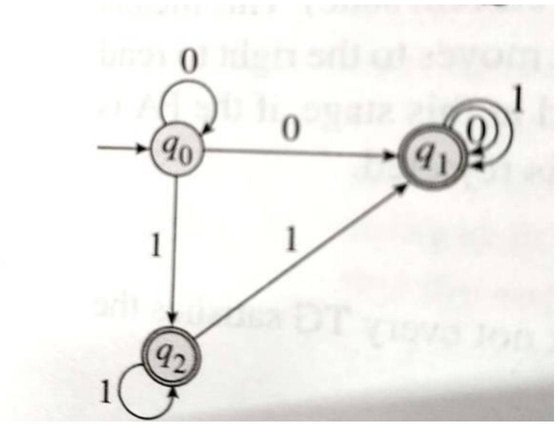
- NFA has the power to be in several states at once
- Each NFA accepts a language that is also accepted by some DFA
- NFAs are often easier than DFAs
- We can always convert an NFA to a DFA, but the latter may have exponentially more states than the NFA (a rare case)
- The difference between the DFA and the NFA is the type of transition function δ
- For a NFA δ is a function that takes a state and input symbol as arguments (like the DFA transition function), but returns a set of zero or more states (rather than returning exactly one state, as the DFA must)

NFA Definition

- **5-tuple:**
- M= (Q,Σ, δ, q_o, F)

  where,

  - Q: Finite set of states
  - Σ: Finite input alphabet
  - **δ: STF that maps Q X Σ to Q, i.e. Q X Σ $\longrightarrow$ 2^Q**
    - (Here the power set of Q (2^Q) has been taken because in case of NDFA, from a state, transition can occur to any combination of Q states)
  - $q_{o:}$ Initial state of FA, $q_o \in Q$
  - F: Set of final states, $F \subseteq Q$

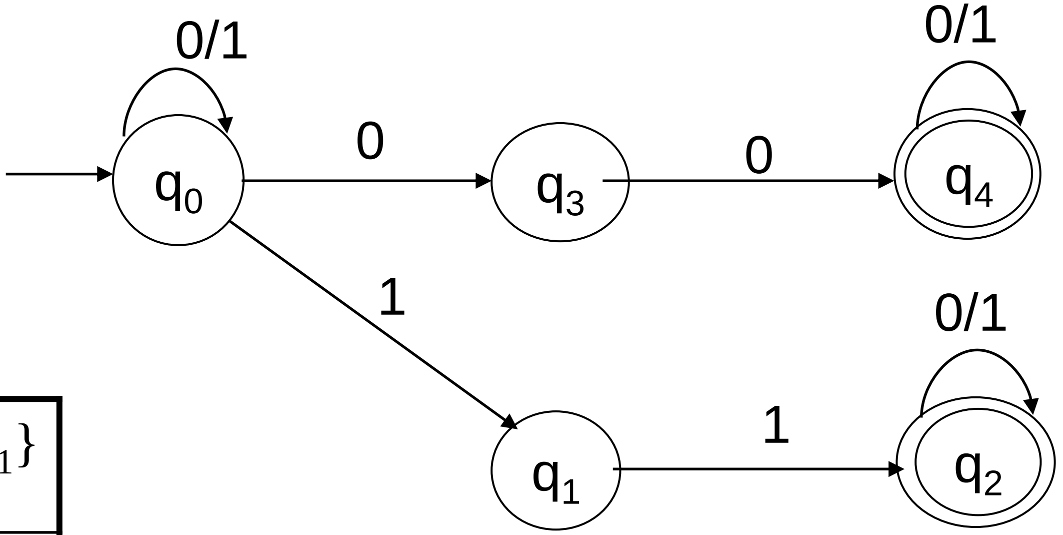


Example : pair of 0’s or pair of 1’s

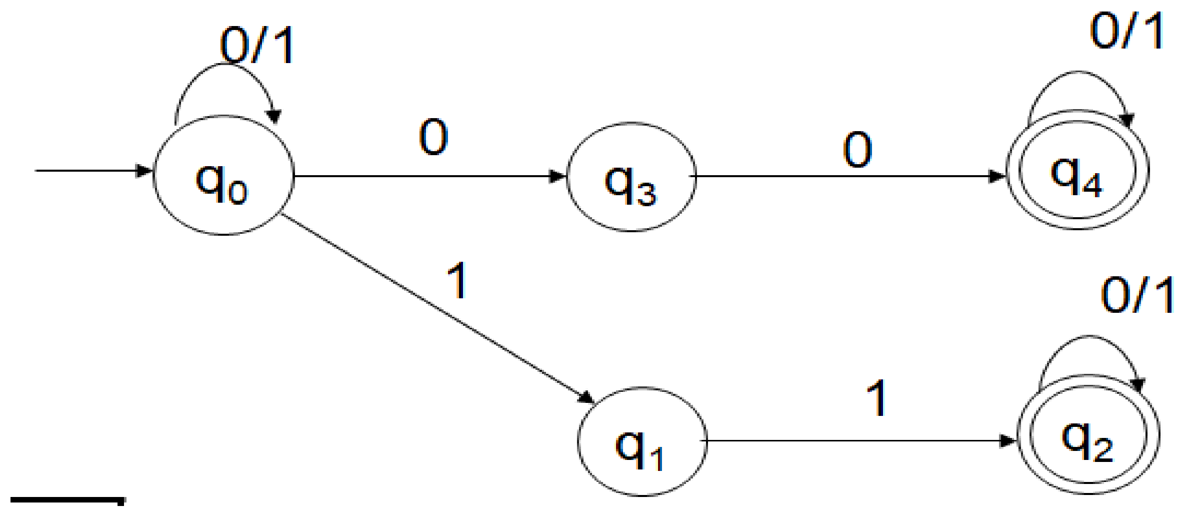
$Q = \{q_0, q_1, q_2, q_3, q_4\}$
$\Sigma = \{0, 1\}$
Start state is q_0
$F = \{q_2, q_4\}$

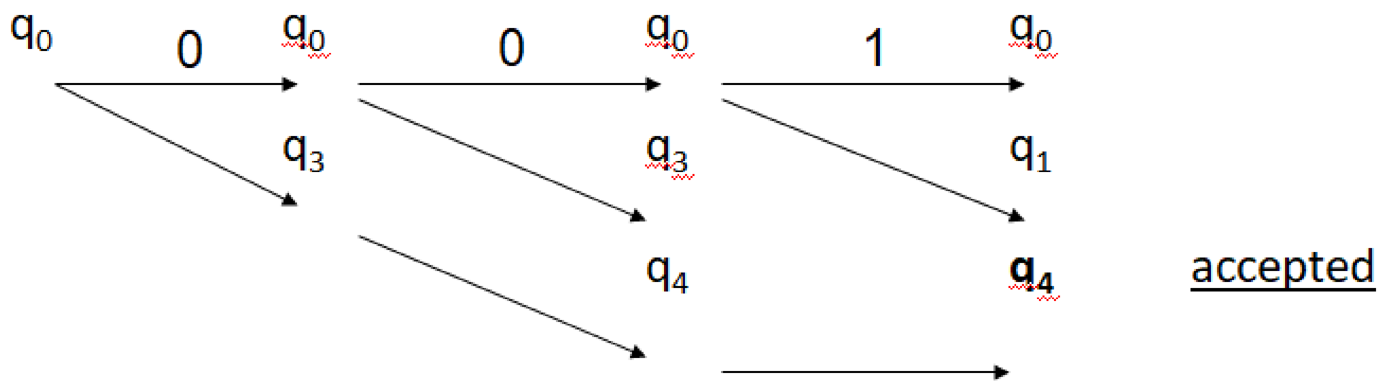
δ:

	0	1
q_0	$\{q_0, q_3\}$	$\{q_0, q_1\}$
q_1	$\{\}$	$\{q_2\}$
q_2	$\{q_2\}$	$\{q_2\}$
q_3	$\{q_4\}$	$\{\}$
q_4	$\{q_4\}$	$\{q_4\}$



NFA



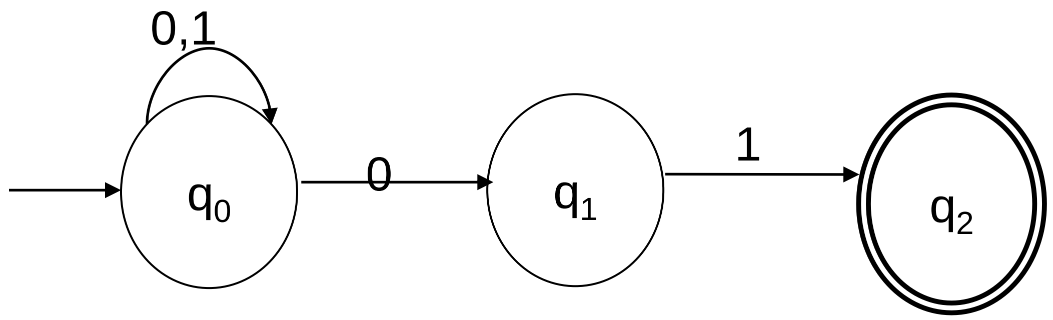


NFA Examples

1. Construct a FA to recognize language containing strings starting with ‘10’
2. Construct a FA to recognize language containing strings ending with ‘101’
3. Construct a FA to recognize language of strings with exactly two 0s anywhere
4. Construct a FA that recognizes the language of strings that end in 01

NFA

Example: NFA that recognizes the language of strings that end in 01


$$\begin{tikzcd}
\arrow[r] & q_0 \arrow[loop above, "0,1"] \arrow[r, "0"] & q_1 \arrow[r, "1"] & q_2
\end{tikzcd}$$

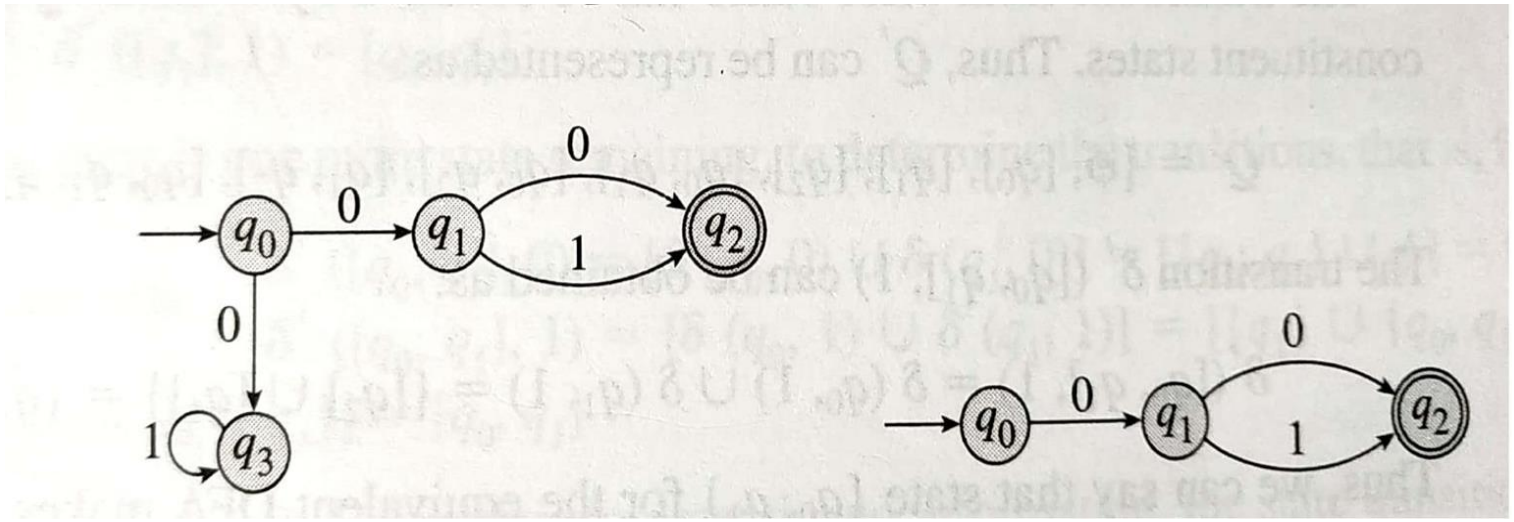
Equivalence of NFA and DFA

For every NFA, there exists an equivalent DFA.

NFA and DFA have equal powers

- **Example:**

NFA vs DFA



Example NFA

Equivalent DFA

Identify the language

Conversion of NFA to DFA
(Method I)

- For every NFA, there is an equivalent DFA
- **Let us consider Q'= 2^Q, set of states for resulting DFA**

  Q' X $\boldsymbol{\Sigma} \rightarrow$ **Q'**
- **Find all transitions from every state in Q' on reading each input symbols**
- Every combination of states can be considered as a new state in the resulting DFA and can be given a new label

Conversion of NFA to DFA (Method I) (Cont..)

- In the resulting DFA, there should be a unique state resulting from an input to the previous state
- The initial state remains unchanged for the resultant DFA
- **All possible subsets of Q (i.e. 2^Q) are the possible states for the resultant DFA**
- Final state in resulting DFA is all the states containing final state of given NFA

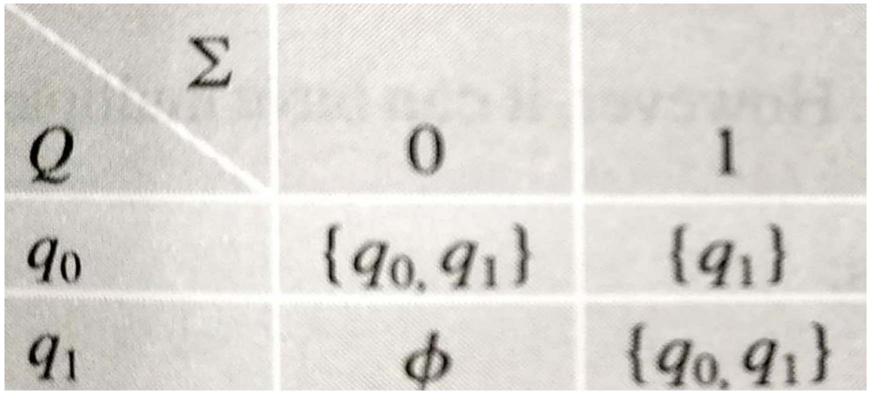
Conversion of NFA to DFA

(Method I) (Cont..)

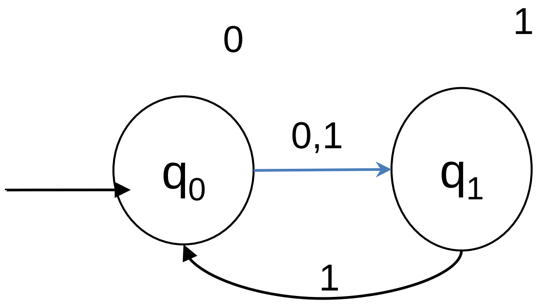
Example:

Convert the NFA : M = ({q_0, q_1}, {0, 1}, δ, q_0, {q_1}) to its equivalent DFA

State Transition Table δ for example NFA



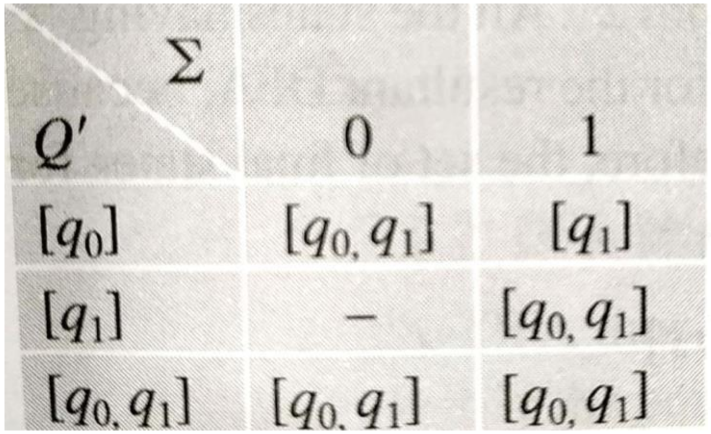
Q \ Σ	0	1
q_0	$\{q_0, q_1\}$	$\{q_1\}$
q_1	ϕ	$\{q_0, q_1\}$



Conversion of NFA to DFA

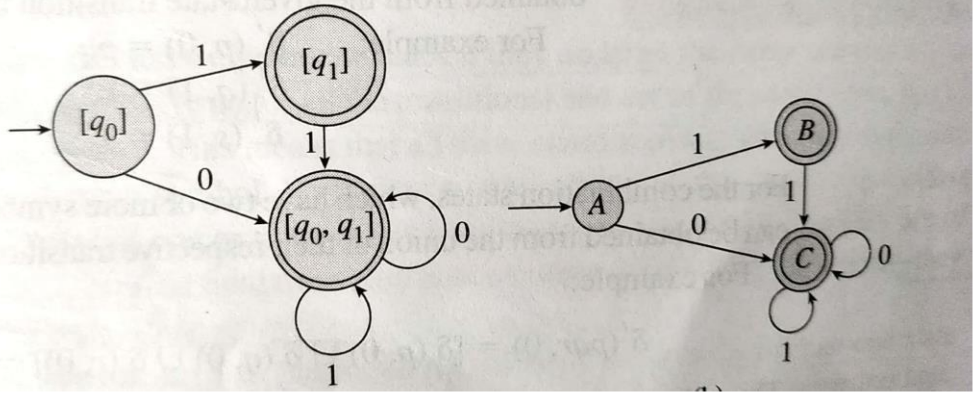
(Method I) (Cont..)

State Transition table for resultant NFA



Q' \ Σ	0	1
$[q_0]$	$[q_0, q_1]$	$[q_1]$
$[q_1]$	–	$[q_0, q_1]$
$[q_0, q_1]$	$[q_0, q_1]$	$[q_0, q_1]$

State Transition Graph for resultant NFA



Conversion of NFA to DFA

(Method II)

- Easier and direct
- Takes lesser number of steps and time to build an equivalent DFA
- Instead of considering the set Q’=2^Q and then removing non-required states, consider only those states that are required

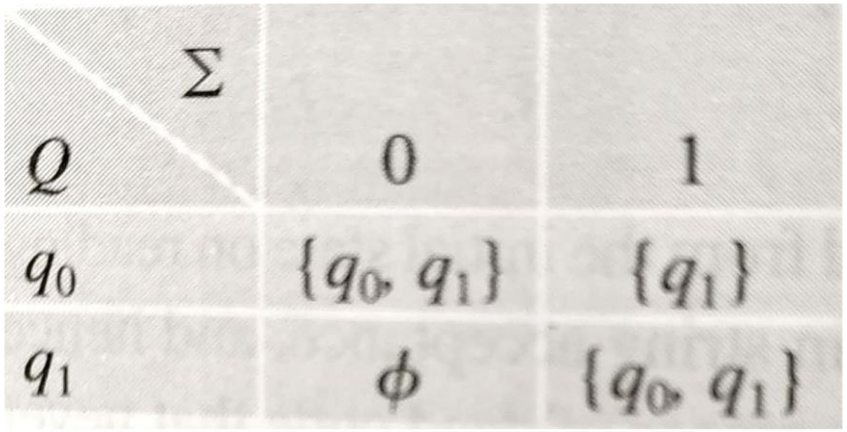
Conversion of NFA to DFA
(Method II) (Cont..)

- Start building an equivalent DFA with the transition diagram of given NFA
- Find the transition, one state at a time
- If the next state of a given transition is a new combination, add that to the set of states for the resultant DFA

Conversion of NFA to DFA
(Method II) (Cont..)

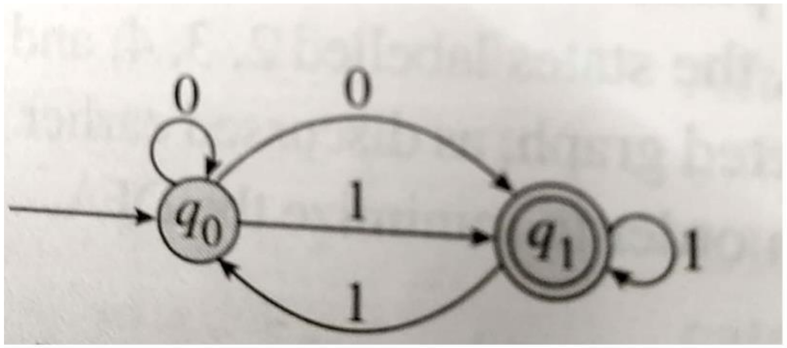
- **Example:**
  - Convert the NFA: M = ($\{q_0, q_1\}, \{0, 1\}, \delta, q_0, \{q_1\}$) to its equivalent DFA

State Transition Table δ for example NFA



Σ / Q	0	1
q_0	$\{q_0, q_1\}$	$\{q_1\}$
q_1	ϕ	$\{q_0, q_1\}$

Transition Graph for example NFA

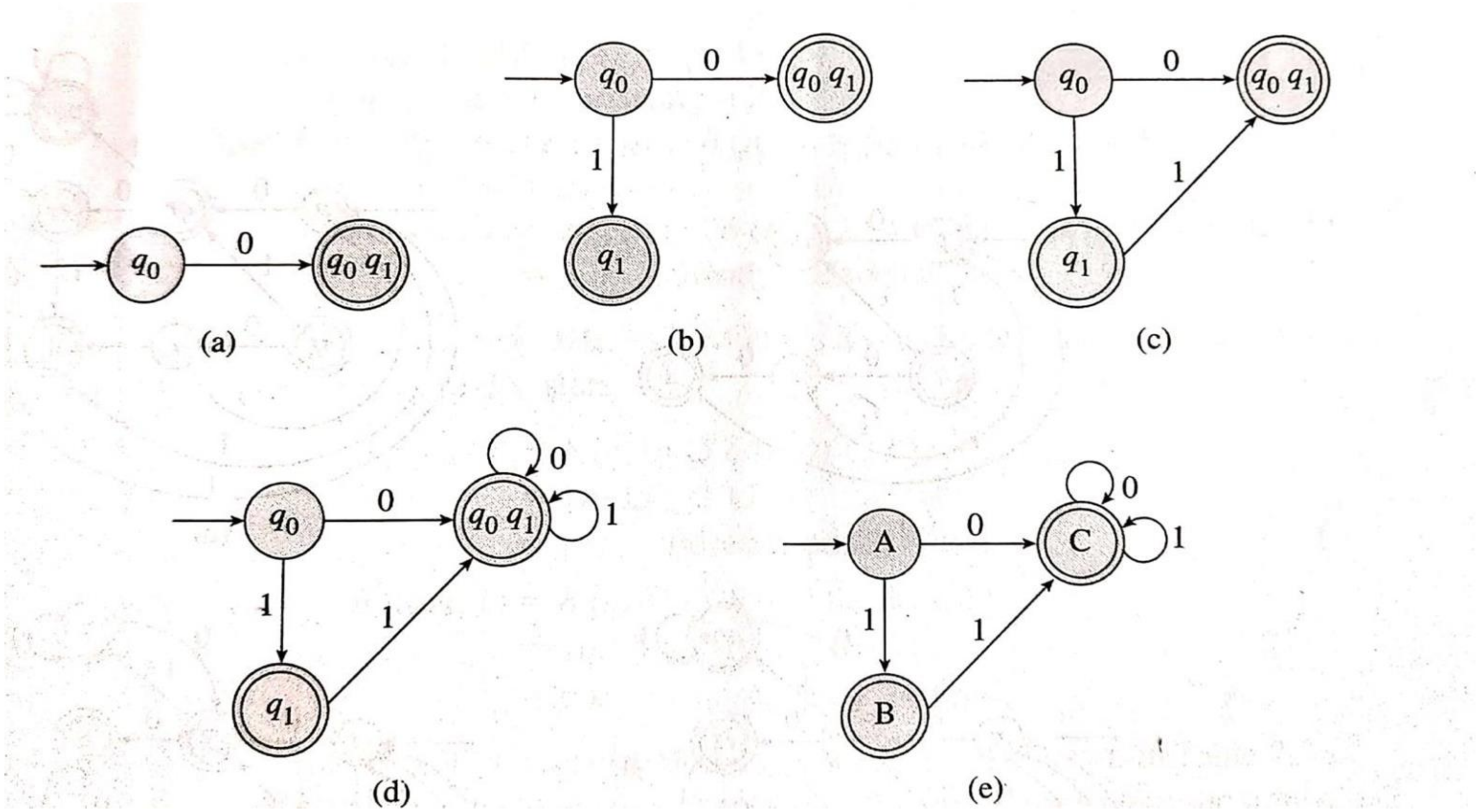


$$\begin{tikzcd} \arrow[r] & q_0 \arrow[loop above, "0"] \arrow[r, bend left, "0"] \arrow[r, "1"] & q_1 \arrow[loop right, "1"] \arrow[l, bend left, "1"] \end{tikzcd}$$

Conversion of NFA to DFA
(Method II) (Cont..)

DFA construction from the given NFA:

(a) Step 1 (b) Step 2 (c) Step 3 (d) Step 4 (e)Final DFA



DFA Minimization

- Identify equivalent states and keep any one of them.
- Replace remaining states by that one.
- Identify unreachable states and remove them
- Identify dead (or trap) states and remove them

DFA Minimization (cont..)

- **Equivalent states:**
  - go to same next state on reading the same input symbol
  - same type (final or non-final)
- **Unreachable states**:
  - Can not be reached from initial state on reading any input symbol
- **Dead (or Trap) states**:
  - Have no outgoing transitions, except to themselves

DFA Minimization Rules

- One non-final state can be replaced by its equivalent non-final state only

- One final state can be replaced by its equivalent final state only

- Initial state can not be replaced by any other state

- One final state can not be replaced by a non- final state or one non-final state can not be replaced by a final state

DFA Minimization Rules (cont…)

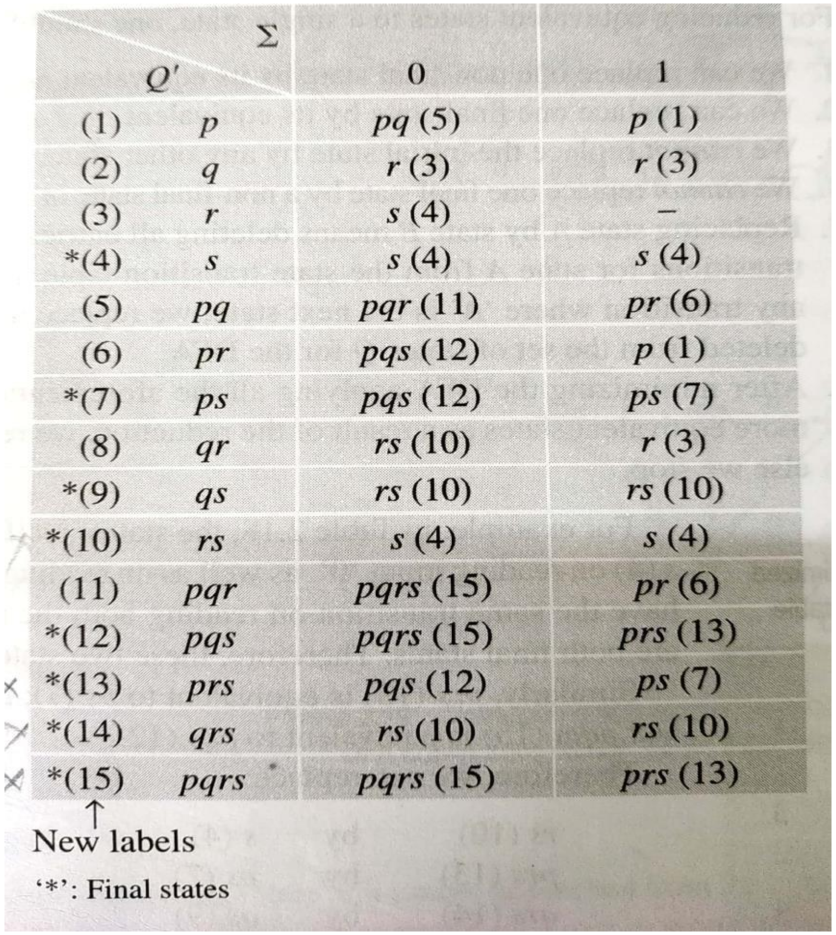
- Replacing state A by state B means deleting all entries related to state A i.e. deleting all the transitions for state A from the state transition table.
- If more equivalent states are found after applying all the rules, repeat the same five steps to further reduce the DFA.

Example

Q \ Σ	0	1
p	p,q	p
q	r	r
r	s	-
s	s	s

DFA Minimization (cont…)

State Transition Table of the DFA to be minimized



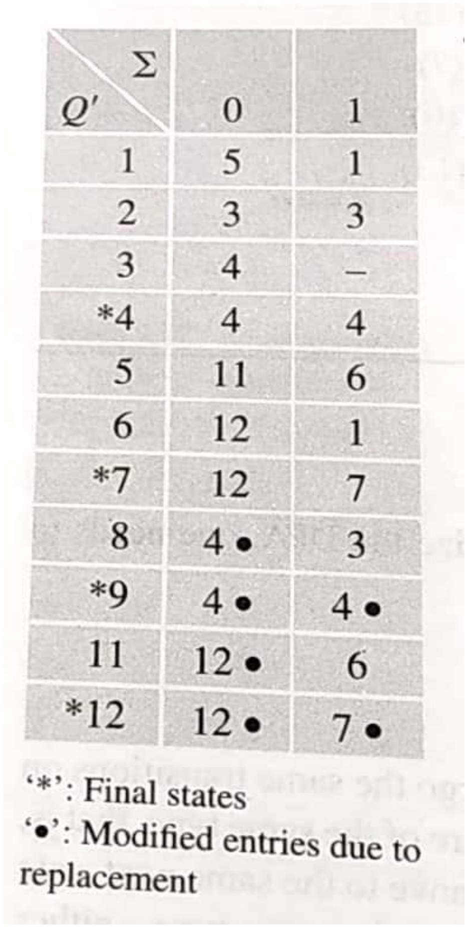
	Q' \ Σ	0	1
(1)	*p*	*pq* (5)	*p* (1)
(2)	*q*	*r* (3)	*r* (3)
(3)	*r*	*s* (4)	–
*(4)	*s*	*s* (4)	*s* (4)
(5)	*pq*	*pqr* (11)	*pr* (6)
(6)	*pr*	*pqs* (12)	*p* (1)
*(7)	*ps*	*pqs* (12)	*ps* (7)
(8)	*qr*	*rs* (10)	*r* (3)
*(9)	*qs*	*rs* (10)	*rs* (10)
*(10)	*rs*	*s* (4)	*s* (4)
(11)	*pqr*	*pqrs* (15)	*pr* (6)
*(12)	*pqs*	*pqrs* (15)	*prs* (13)
*(13)	*prs*	*pqs* (12)	*ps* (7)
*(14)	*qrs*	*rs* (10)	*rs* (10)
*(15)	*pqrs*	*pqrs* (15)	*prs* (13)

↑
New labels

'*': Final states

DFA Minimization (cont…)

Minimized DFA

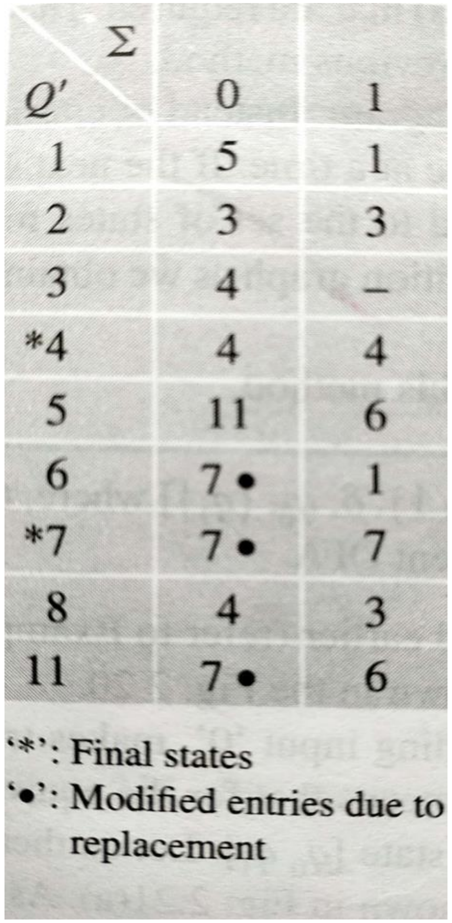


Σ / Q′	0	1
1	5	1
2	3	3
3	4	–
*4	4	4
5	11	6
6	12	1
*7	12	7
8	4 •	3
*9	4 •	4 •
11	12 •	6
*12	12 •	7 •

‘*’: Final states
‘•’: Modified entries due to replacement

DFA Minimization (cont…)

Further minimized DFA



Q′ \ Σ	0	1
1	5	1
2	3	3
3	4	–
*4	4	4
5	11	6
6	7 •	1
*7	7 •	7
8	4	3
11	7 •	6

‘*’: Final states
‘•’: Modified entries due to replacement

NFA with ε – Transitions

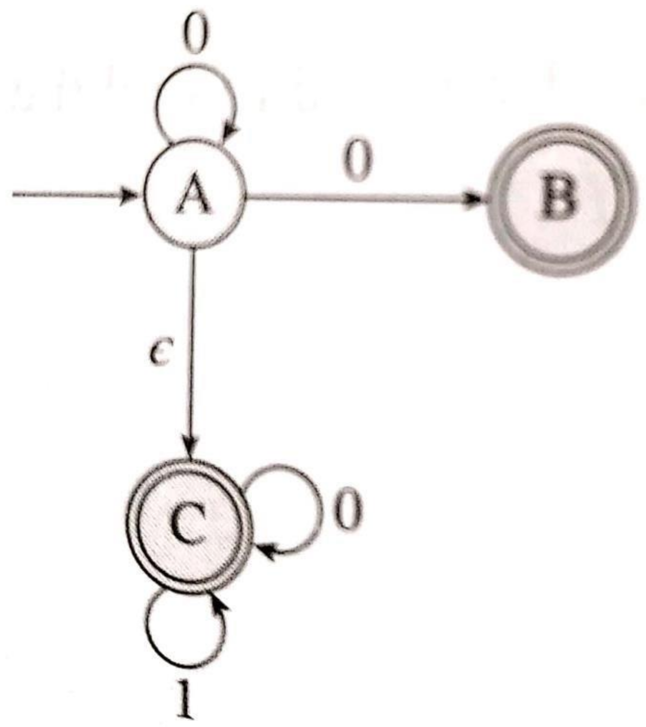
- **5-tuple:**
- M= $(Q,\Sigma, \delta, q_o, F)$ where,
  - Q: Finite set of states
  - Σ: Finite input alphabet
  - **$\boldsymbol{\delta}$: STF that maps Q x $(\boldsymbol{\Sigma} \cup \{ \boldsymbol{\varepsilon} \})$ to 2^Q**

    i.e. Q X $(\boldsymbol{\Sigma} \cup \{\boldsymbol{\varepsilon}\})$ 2^Q
- $q_{o:}$ Initial state of FA, $q_o \in Q$
- F: Set of final states, $F \subseteq Q$

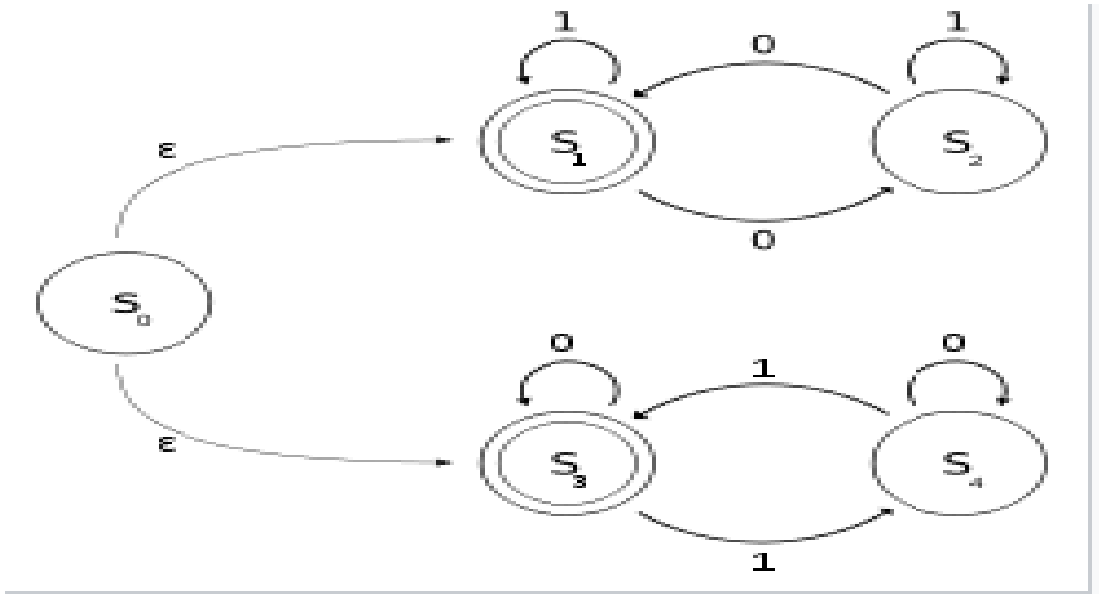
NFA with ε – Transitions (cont..)

- **Example:**

Transition Graph for example NFA with ε – transitions



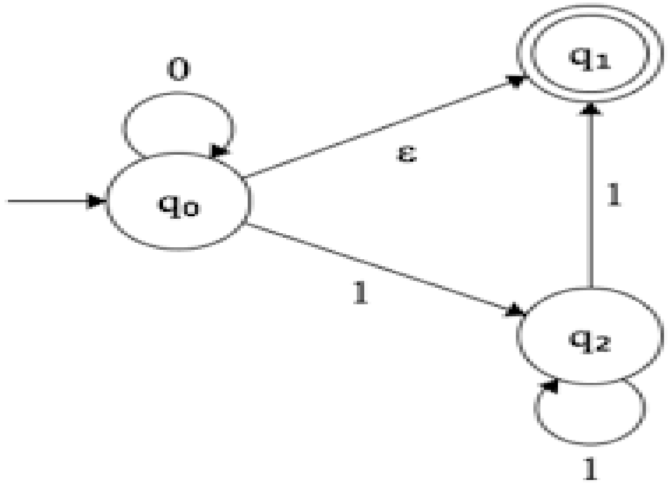
NFA with Ɛ moves Example



NFA with ε – Transitions (Examples)

- **Example:**

Transition Graph for example NFA with ε – transitions

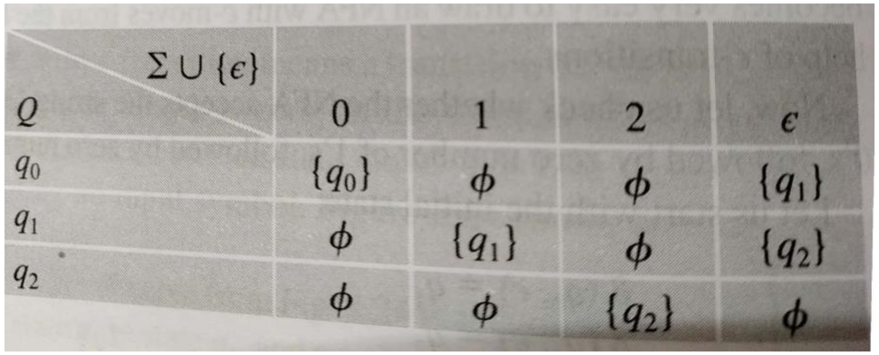


NFA with $\varepsilon -$ Transitions (Examples)

- **Example:**

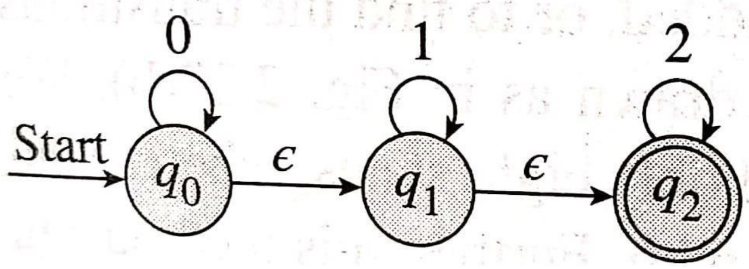
  Convert the NFA with $\varepsilon -$transitions given in the figure to its equivalent DFA

State Transition table for example NFA with ε −transitions



Q \ $\Sigma \cup \{\epsilon\}$	0	1	2	ϵ
q_0	$\{q_0\}$	ϕ	ϕ	$\{q_1\}$
q_1	ϕ	$\{q_1\}$	ϕ	$\{q_2\}$
q_2	ϕ	ϕ	$\{q_2\}$	ϕ

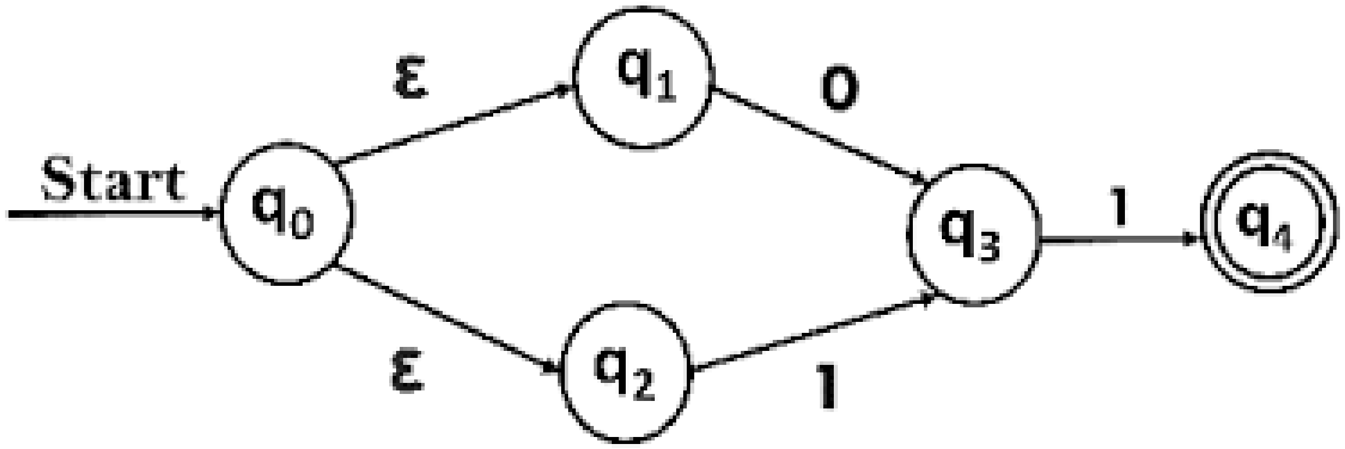
State Transition Graph for example NFA with ε −transitions



NFA with ε – Transition (Examples)

- **Example:**

Transition Graph for example NFA with ε – transitions

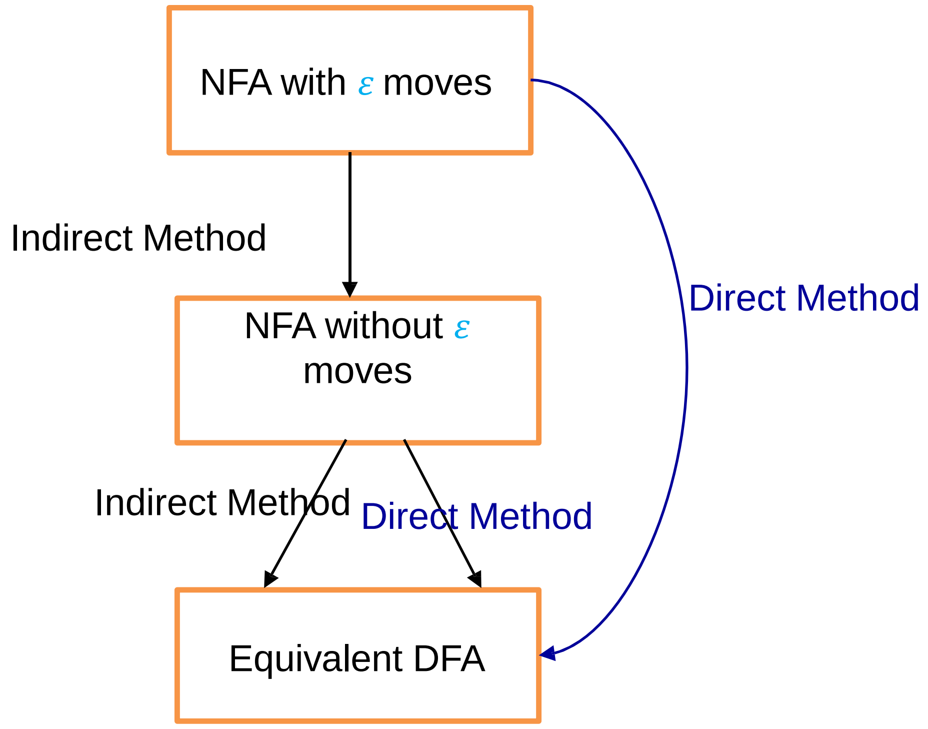


Significance of NFA with ε – Transitions

- Helps to split complex language acceptance problems into smaller once.
- Solution to these problems can be integrated with the help of ϵ -transition.
- Concatenation ,parallel path can be connected with the ϵ -transition.

Conversion of NFA with ε – Transitions to DFA

- Indirect Conversion Method
- Direct Conversion Method



Indirect Conversion Method

1. Construct the NFA without $\varepsilon - transitions$ from the given NFA with $\varepsilon -$ move.
2. Construct an equivalent DFA with this newly constructed NFA without $\varepsilon - transitions$ (using existing methods)

Indirect Conversion Method (cont…)

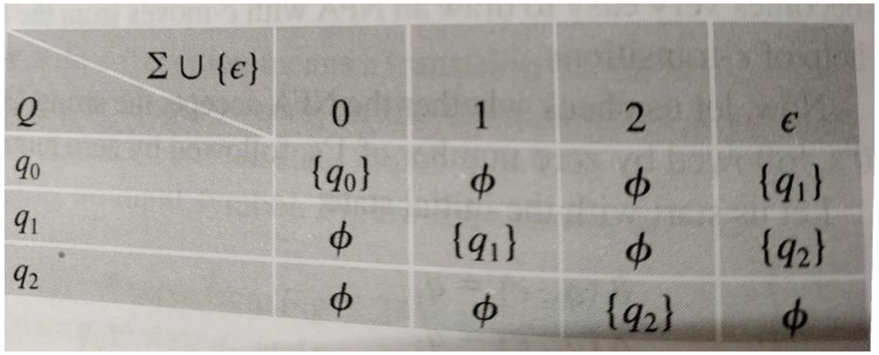
- ε-closure of a state:
    - Set of all states p, such that there is a path from state q to state p labeled 'ε', is known as ε-closure (q)
    - Set of all the states having distance zero from state q
    - Every state is at distance zero from itself
    - Denoted by δ^

Indirect Conversion Method (cont…)

- **Example:**

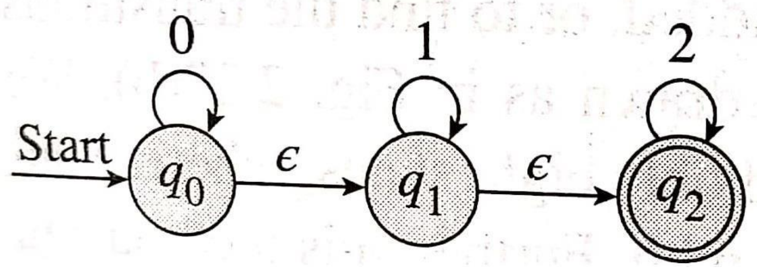
  Convert the NFA with ε –transitions given in the figure to its equivalent DFA

State Transition table for example NFA with ε –transitions



$Q \backslash \Sigma \cup \{\epsilon\}$	0	1	2	ϵ
q_0	$\{q_0\}$	ϕ	ϕ	$\{q_1\}$
q_1	ϕ	$\{q_1\}$	ϕ	$\{q_2\}$
q_2	ϕ	ϕ	$\{q_2\}$	ϕ

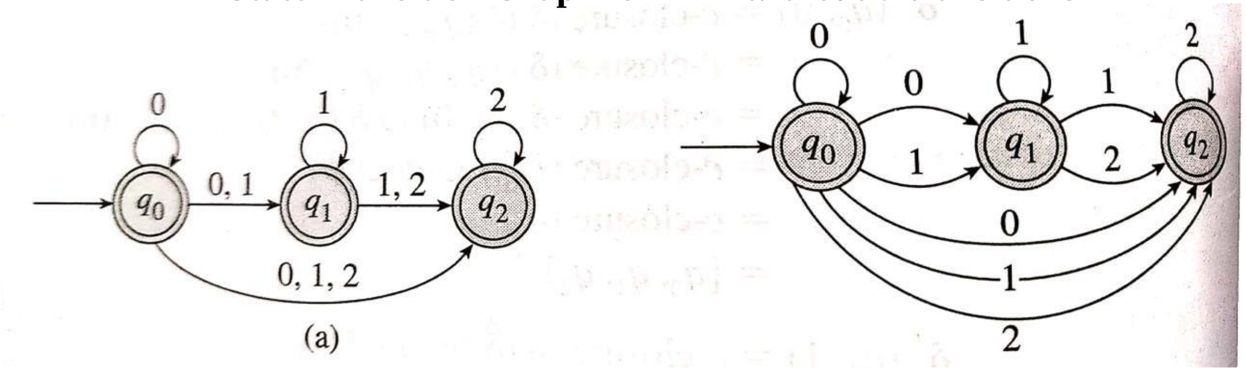
State Transition Graph for example NFA with ε –transitions



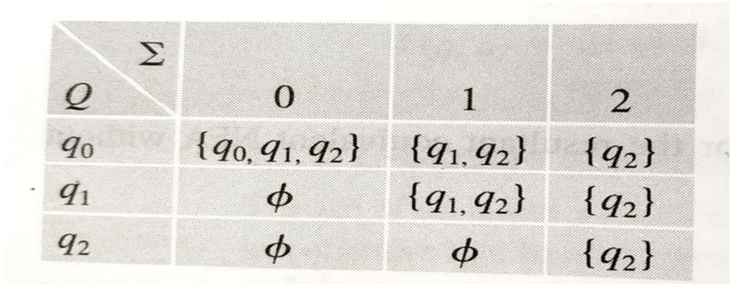
Indirect Conversion Method (cont…)

1. **Conversion of NFA with ε –transitions to NFA without ε –transitions:**

State Transition Graph for NFA without ε-transitions



State Transition Table for NFA without ε-transitions

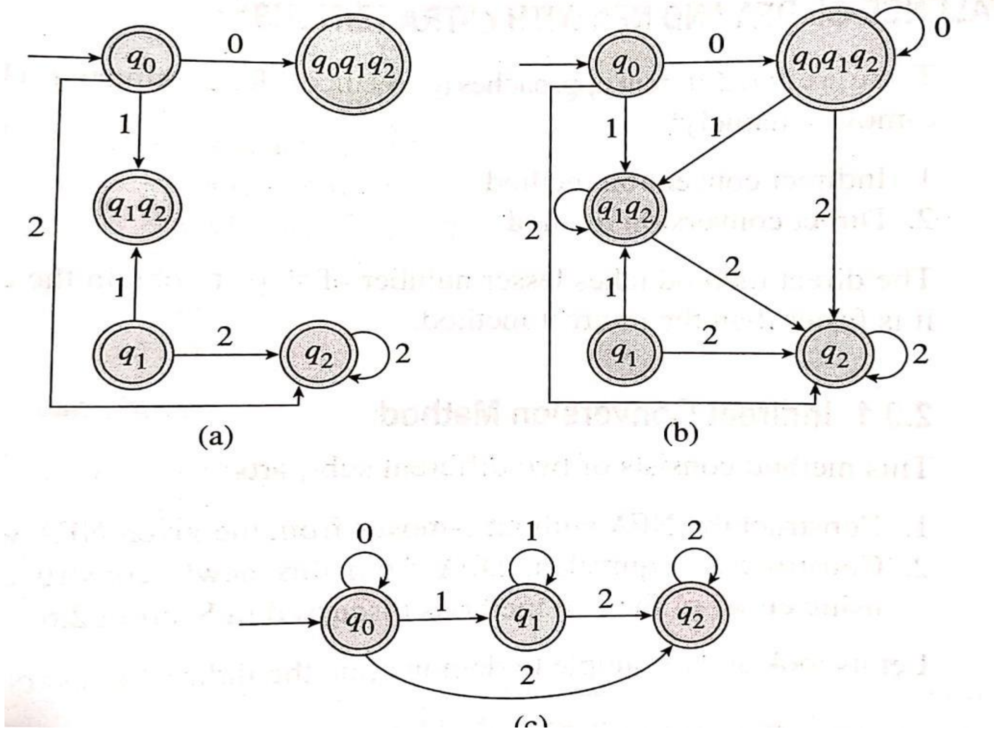


Q \ Σ	0	1	2
q_0	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
q_1	ϕ	$\{q_1, q_2\}$	$\{q_2\}$
q_2	ϕ	ϕ	$\{q_2\}$

Indirect Conversion Method (cont…)

2. Conversion of NFA without ε –transitions to an equivalent DFA:

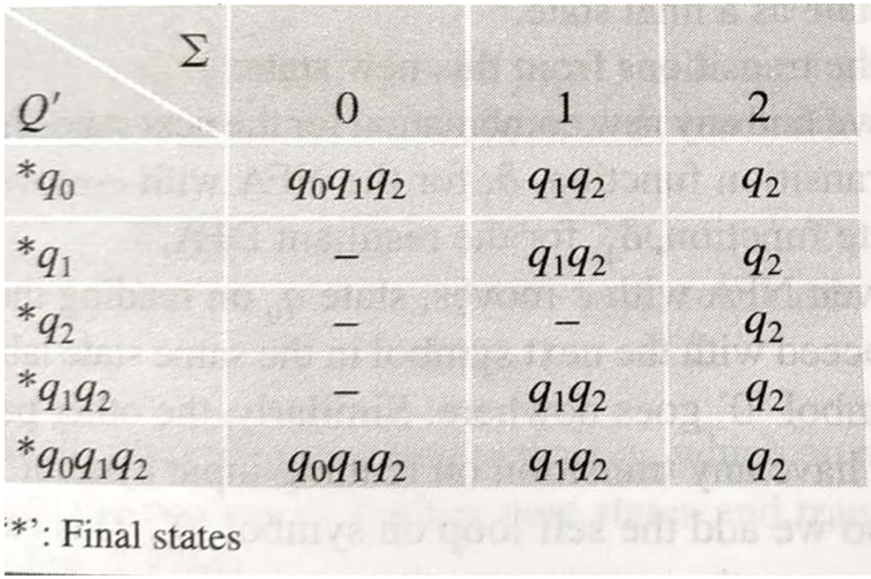
Step 1 (b) Step 2 (c) Final DFA



Indirect Conversion Method (cont…)

2. **Conversion of NFA without ε −transitions to an equivalent DFA:**

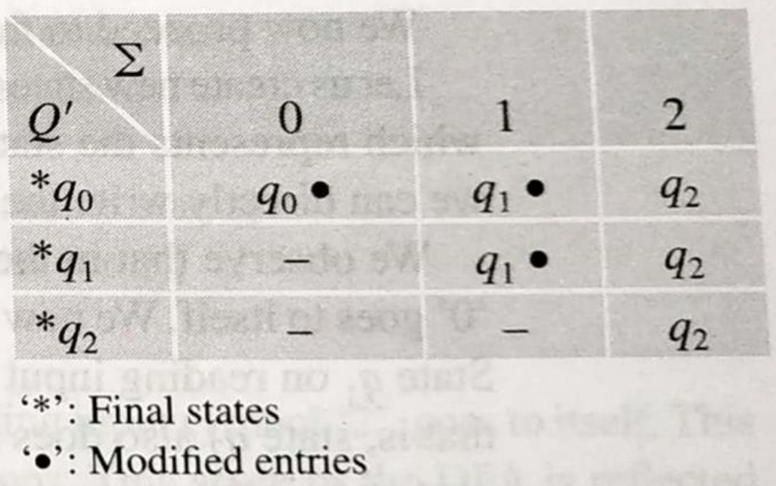
State Transition table for the equivalent DFA



Q' \ Σ	0	1	2
$*q_0$	$q_0q_1q_2$	q_1q_2	q_2
$*q_1$	–	q_1q_2	q_2
$*q_2$	–	–	q_2
$*q_1q_2$	–	q_1q_2	q_2
$*q_0q_1q_2$	$q_0q_1q_2$	q_1q_2	q_2

'*': Final states

Reduced State Transition table for the equivalent DFA



Q' \ Σ	0	1	2
$*q_0$	q_0 ●	q_1 ●	q_2
$*q_1$	–	q_1 ●	q_2
$*q_2$	–	–	q_2

'*': Final states
'●': Modified entries

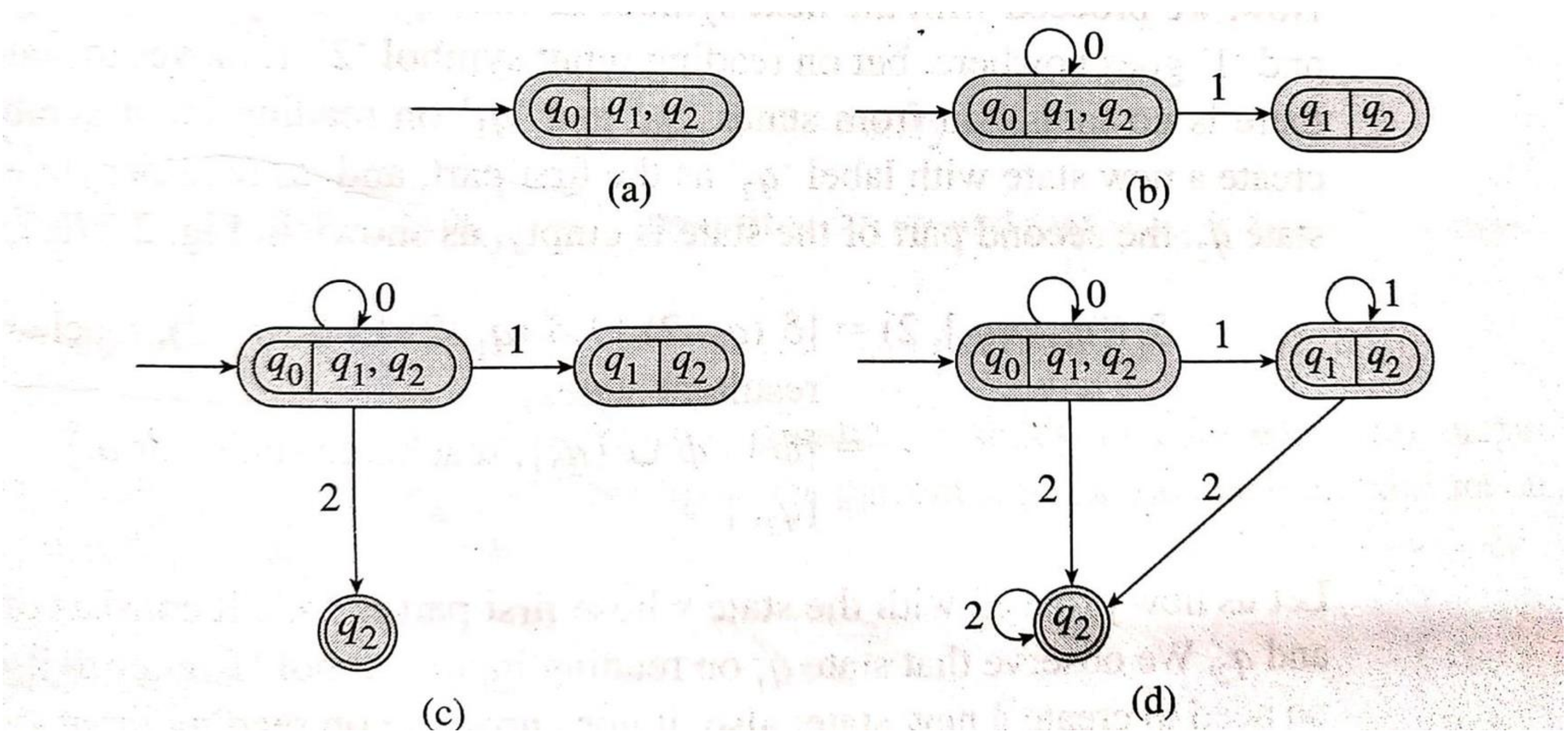
Direct Conversion Method

- Begin with the initial state
- Go on adding the states as & when required to the diagram
- Follow the same process until there exists no state without having the transitions specified
- Each state label consists of:
    - Name of state label
    - Combination of all the state symbols, which are reachable from the given state
- Start building an equivalent DFA with the transition diagram of given NFA
- Find the transition, one state at a time
- If the next state of a given transition is a new combination, add that to the set of states for the resultant DFA

Direct Conversion Method (cont…)

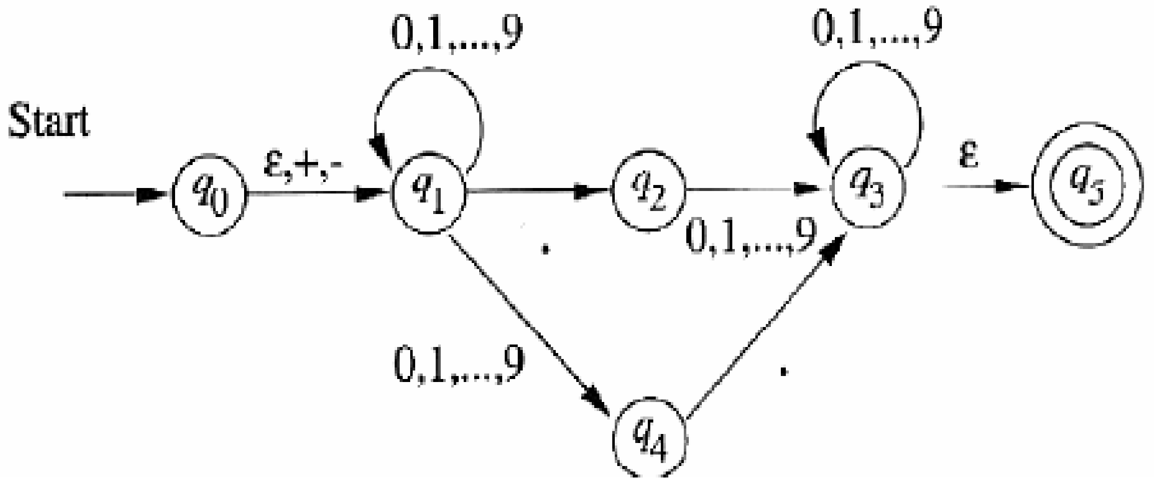
- **Example (Refer slide no. 53 for example NFA with ε −transitions**

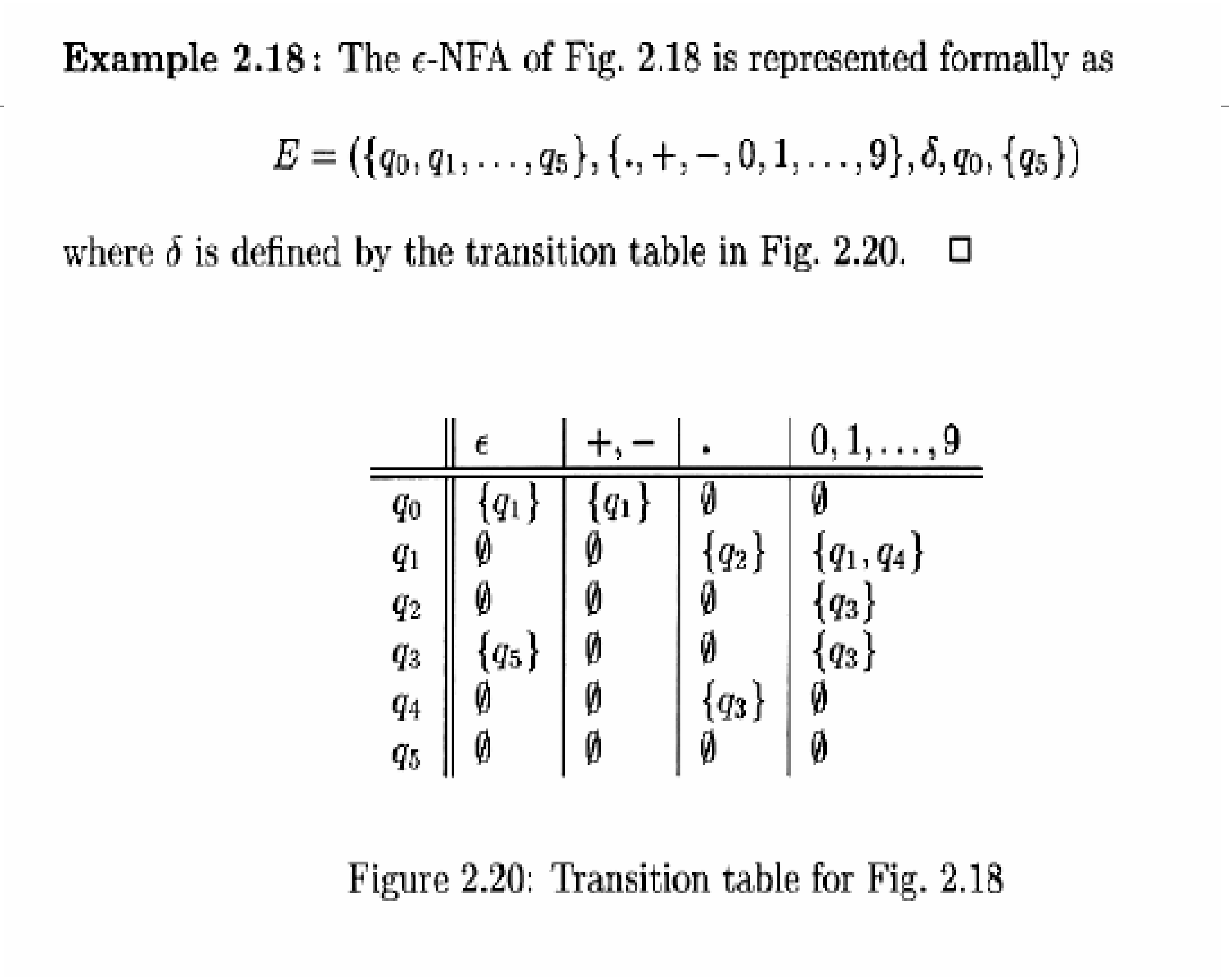
  Resultant DFA (a) Step 1 (b) Step 2 (c) Step 3 (d) Final DFA



Example 2.16: In Fig. 2.18 is an ϵ-NFA that accepts decimal numbers consisting of:

1. An optional + or − sign,
2. A string of digits,
3. A decimal point, and
4. Another string of digits. Either this string of digits, or the string (2) can be empty, but at least one of the two strings of digits must be nonempty.





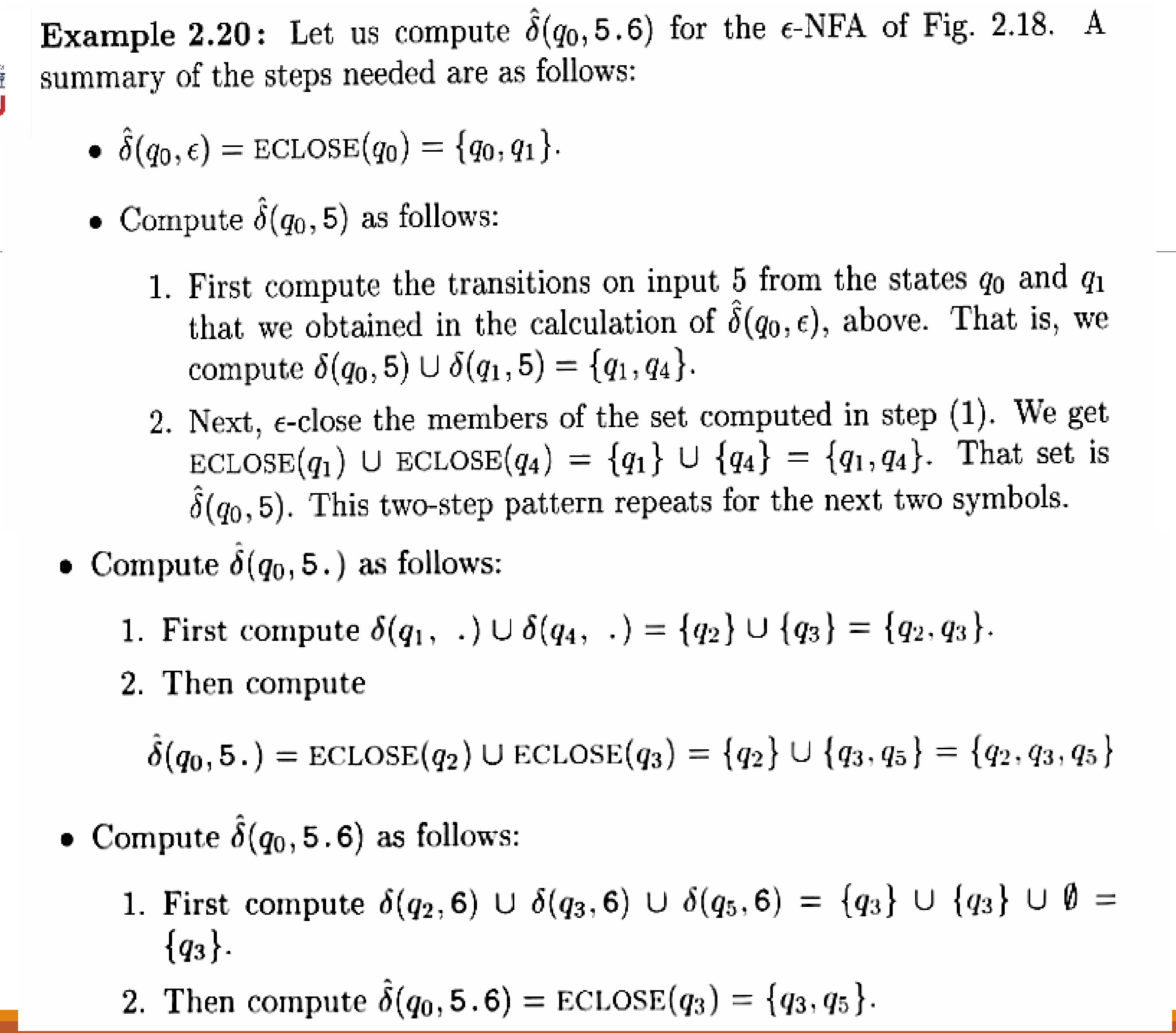
Example 2.18: The ϵ-NFA of Fig. 2.18 is represented formally as

$$E = (\{q_0, q_1, \ldots, q_5\}, \{., +, -, 0, 1, \ldots, 9\}, \delta, q_0, \{q_5\})$$

where δ is defined by the transition table in Fig. 2.20. □

	ϵ	$+, -$	$.$	$0, 1, \ldots, 9$
q_0	$\{q_1\}$	$\{q_1\}$	$\emptyset$	$\emptyset$
q_1	$\emptyset$	$\emptyset$	$\{q_2\}$	$\{q_1, q_4\}$
q_2	$\emptyset$	$\emptyset$	$\emptyset$	$\{q_3\}$
q_3	$\{q_5\}$	$\emptyset$	$\emptyset$	$\{q_3\}$
q_4	$\emptyset$	$\emptyset$	$\{q_3\}$	$\emptyset$
q_5	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

Figure 2.20: Transition table for Fig. 2.18

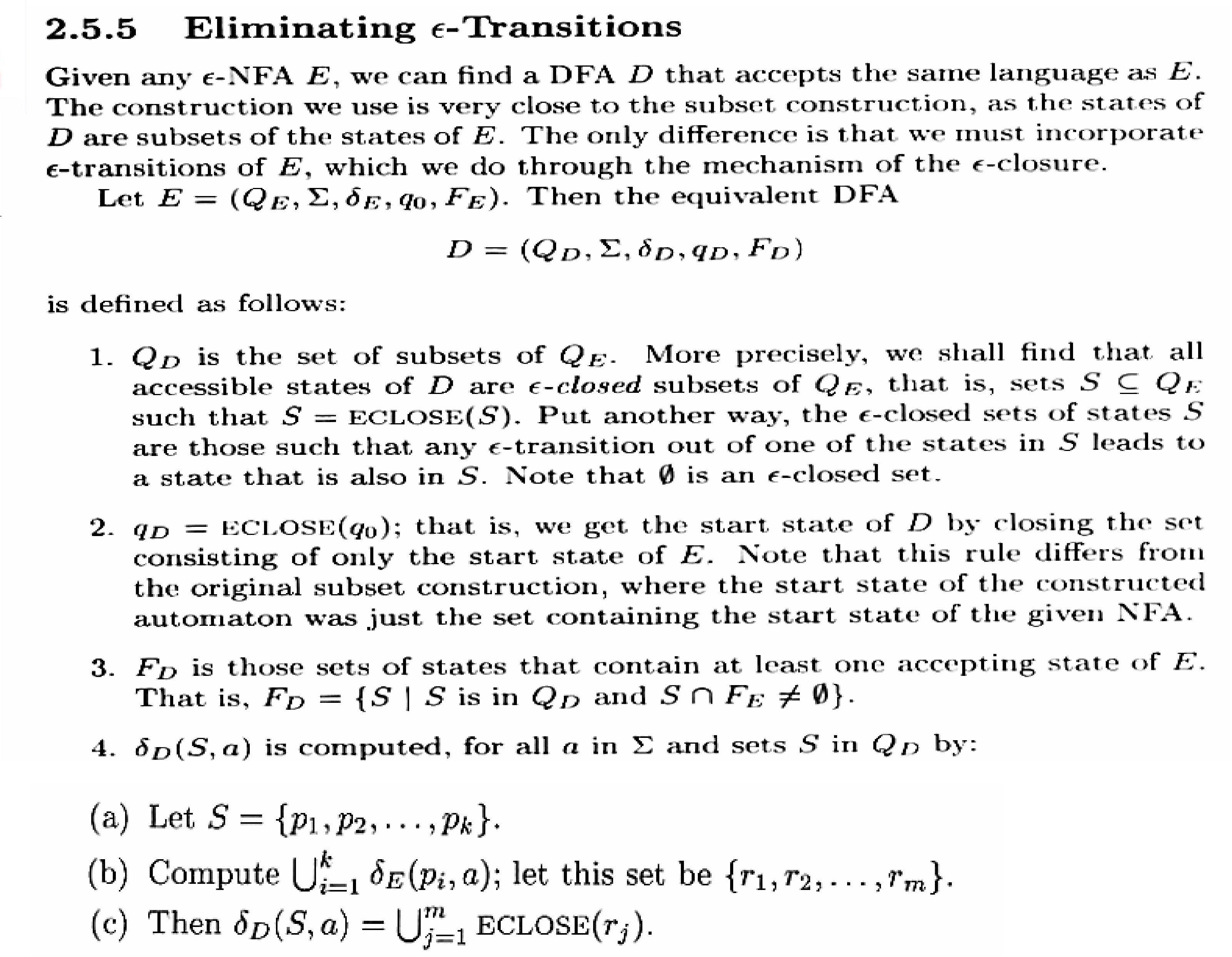


Example 2.20: Let us compute $\hat{\delta}(q_0, 5.6)$ for the ϵ-NFA of Fig. 2.18. A summary of the steps needed are as follows:

- $\hat{\delta}(q_0, \epsilon) = \text{ECLOSE}(q_0) = \{q_0, q_1\}$.
- Compute $\hat{\delta}(q_0, 5)$ as follows:
  1. First compute the transitions on input 5 from the states q_0 and q_1 that we obtained in the calculation of $\hat{\delta}(q_0, \epsilon)$, above. That is, we compute $\delta(q_0, 5) \cup \delta(q_1, 5) = \{q_1, q_4\}$.
  2. Next, ϵ-close the members of the set computed in step (1). We get $\text{ECLOSE}(q_1) \cup \text{ECLOSE}(q_4) = \{q_1\} \cup \{q_4\} = \{q_1, q_4\}$. That set is $\hat{\delta}(q_0, 5)$. This two-step pattern repeats for the next two symbols.
- Compute $\hat{\delta}(q_0, 5.)$ as follows:
  1. First compute $\delta(q_1, .) \cup \delta(q_4, .) = \{q_2\} \cup \{q_3\} = \{q_2, q_3\}$.
  2. Then compute

$$\hat{\delta}(q_0, 5.) = \text{ECLOSE}(q_2) \cup \text{ECLOSE}(q_3) = \{q_2\} \cup \{q_3, q_5\} = \{q_2, q_3, q_5\}$$

- Compute $\hat{\delta}(q_0, 5.6)$ as follows:
  1. First compute $\delta(q_2, 6) \cup \delta(q_3, 6) \cup \delta(q_5, 6) = \{q_3\} \cup \{q_3\} \cup \emptyset = \{q_3\}$.
  2. Then compute $\hat{\delta}(q_0, 5.6) = \text{ECLOSE}(q_3) = \{q_3, q_5\}$.



2.5.5 Eliminating ϵ-Transitions

Given any ϵ-NFA E, we can find a DFA D that accepts the same language as E. The construction we use is very close to the subset construction, as the states of D are subsets of the states of E. The only difference is that we must incorporate ϵ-transitions of E, which we do through the mechanism of the ϵ-closure.

Let $E = (Q_E, \Sigma, \delta_E, q_0, F_E)$. Then the equivalent DFA

$$D = (Q_D, \Sigma, \delta_D, q_D, F_D)$$

is defined as follows:

1. Q_D is the set of subsets of Q_E. More precisely, we shall find that all accessible states of D are *ϵ-closed* subsets of Q_E, that is, sets $S \subseteq Q_E$ such that $S = \text{ECLOSE}(S)$. Put another way, the ϵ-closed sets of states S are those such that any ϵ-transition out of one of the states in S leads to a state that is also in S. Note that $\emptyset$ is an ϵ-closed set.

2. $q_D = \text{ECLOSE}(q_0)$; that is, we get the start state of D by closing the set consisting of only the start state of E. Note that this rule differs from the original subset construction, where the start state of the constructed automaton was just the set containing the start state of the given NFA.

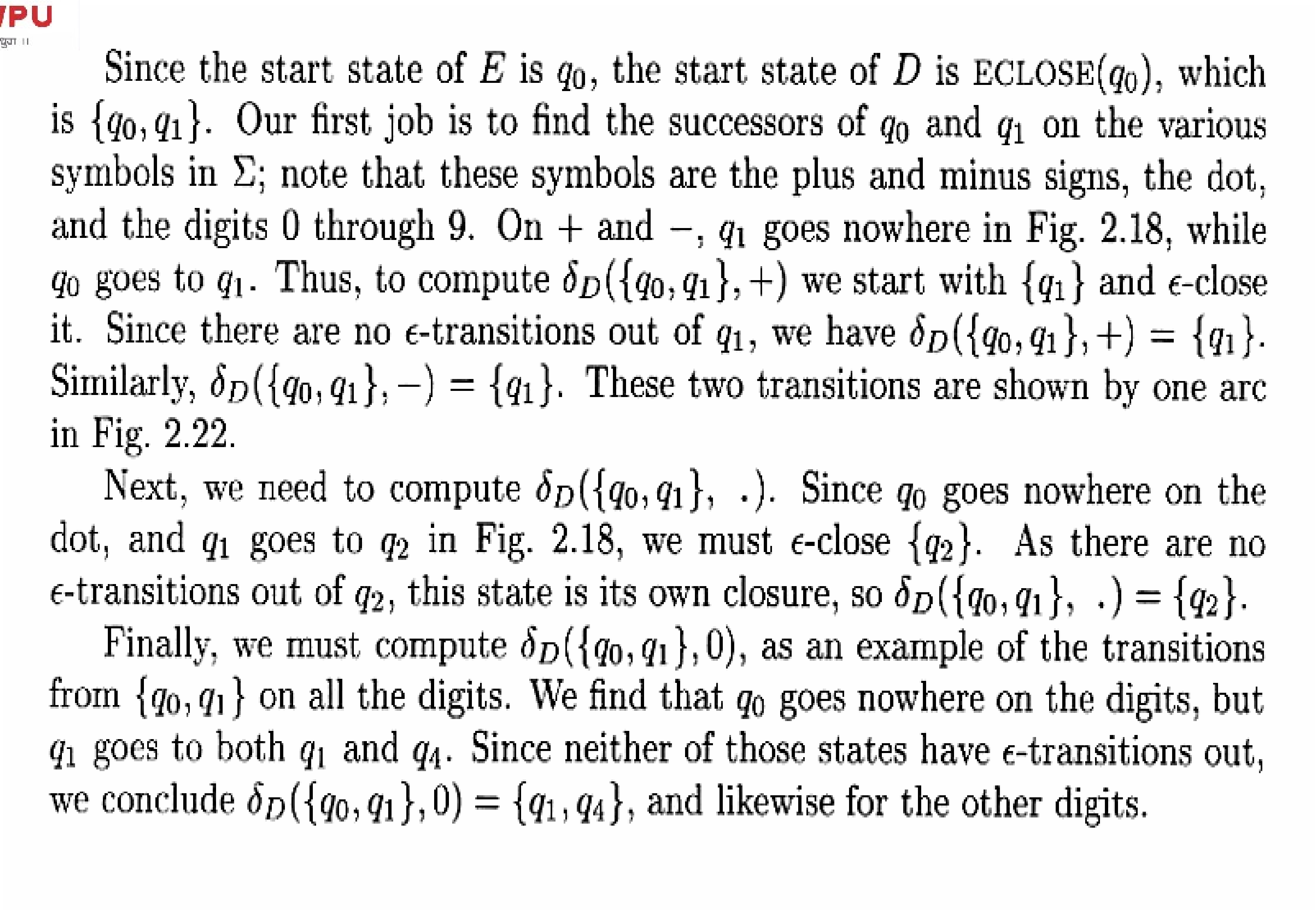
3. F_D is those sets of states that contain at least one accepting state of E. That is, $F_D = \{S \mid S \text{ is in } Q_D \text{ and } S \cap F_E \neq \emptyset\}$.

4. $\delta_D(S, a)$ is computed, for all a in Σ and sets S in Q_D by:

   (a) Let $S = \{p_1, p_2, \ldots, p_k\}$.

   (b) Compute $\bigcup_{i=1}^{k} \delta_E(p_i, a)$; let this set be $\{r_1, r_2, \ldots, r_m\}$.

   (c) Then $\delta_D(S, a) = \bigcup_{j=1}^{m} \text{ECLOSE}(r_j)$.



Since the start state of E is q_0, the start state of D is ECLOSE(q_0), which is $\{q_0, q_1\}$. Our first job is to find the successors of q_0 and q_1 on the various symbols in Σ; note that these symbols are the plus and minus signs, the dot, and the digits 0 through 9. On $+$ and $-$, q_1 goes nowhere in Fig. 2.18, while q_0 goes to q_1. Thus, to compute $\delta_D(\{q_0, q_1\}, +)$ we start with $\{q_1\}$ and ϵ-close it. Since there are no ϵ-transitions out of q_1, we have $\delta_D(\{q_0, q_1\}, +) = \{q_1\}$. Similarly, $\delta_D(\{q_0, q_1\}, -) = \{q_1\}$. These two transitions are shown by one arc in Fig. 2.22.

Next, we need to compute $\delta_D(\{q_0, q_1\}, .)$. Since q_0 goes nowhere on the dot, and q_1 goes to q_2 in Fig. 2.18, we must ϵ-close $\{q_2\}$. As there are no ϵ-transitions out of q_2, this state is its own closure, so $\delta_D(\{q_0, q_1\}, .) = \{q_2\}$.

Finally, we must compute $\delta_D(\{q_0, q_1\}, 0)$, as an example of the transitions from $\{q_0, q_1\}$ on all the digits. We find that q_0 goes nowhere on the digits, but q_1 goes to both q_1 and q_4. Since neither of those states have ϵ-transitions out, we conclude $\delta_D(\{q_0, q_1\}, 0) = \{q_1, q_4\}$, and likewise for the other digits.

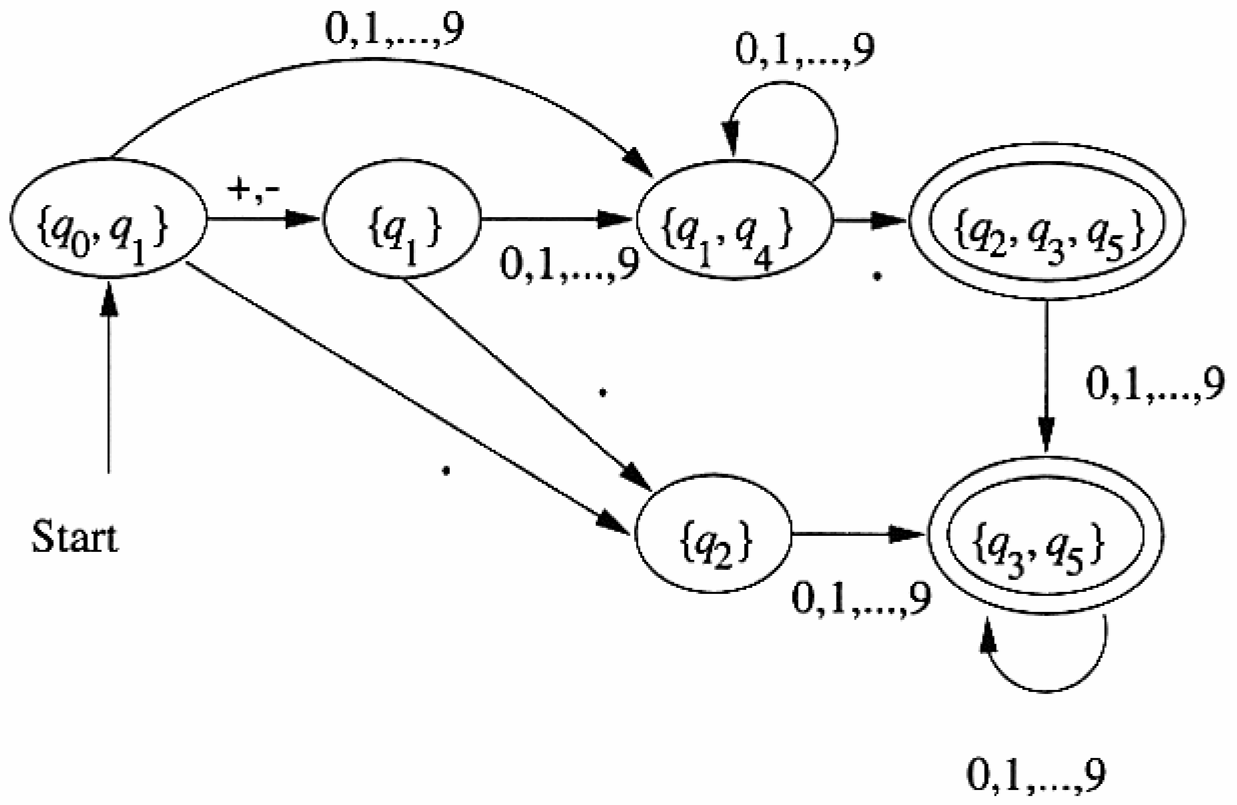


Figure 2.22: The DFA D that eliminates ϵ-transitions from Fig. 2.18

Convert Є-NFA to DFA example

	a	b	c	Є
p	{P}	{q}	{r}	∅
q	{q}	{r}	∅	{p}
***r**	{r}	∅	{p}	{q}

Solution
NFA

	a	b	c
p	{P}	{p,q}	{p,q,r}
q	{p,q}	{p,q,r}	{p,q,r}
***r**	{p,q,r}	{p,q,r}	{p,q,r}

Є-closure(p) = { p}
Є-closure(p) = { p,q}
Є-closure(p) ={ p,q,r}

DFA using direct method

	a	b	c
[p]	[p]	[pq]	[pqr]
[pq]	[pq]	[pqr]	[pqr]
*[pqr]	[pqr]	[pqr]	[pqr]

Convert Є-NFA to DFA example

	0	**1**	**Є**
A	{A}	∅	{B}
B	{C}	∅	{D}
C	∅	{B}	{D}
***D**	{D}	{D}	∅

Solution

NFA

	0	**1**
A	{A,B,C,D}	{D}
B	{C,D}	{D}
C	---	{B,D}
***D**	{D}	{D}

DFA using direct method

	0	1
[A]	[ABCD]	[D]
[ABCD]	[ABCD]	[BD]
*[D]	-[D]	[D]
[BD}	[CD]	[D]
[CD}	[D]	[BD]

Applications and Limitations of FA

- Finite Automata Applications:
  - Software for designing and checking behavior of digital circuits
  - The “Lexical Analyzer” of a compiler
  - Software for scanning large bodies of text

    E.g. to find occurrences of words in collections of web pages
  - Software for verifying systems of all types that have a finite number of distinct states

    E.g. communication Protocols
  - Used in Text Editor and Spell Checkers.

Applications and Limitations of FA

- Limitations:
- Cannot recognize Palindrome of string.
- Set of strings over "(" and ")" & have balanced parenthesis.
- No Memory, Only memory it has is State and State.

Applications of TOC

- Design of the "Lexical Analyzer" of a compiler.
- Use of the Mealy and Moore Machines for designing the combination and sequential circuits.
- Used in diverse fields, including the management of tap water, Pharmaceutical field, Environmental Field etc..
- CFG n PDA can be used in defining n implementing programming languages.

- So on many more…

Application of TOC (cont…)

- Turing Machine:
  - Prototype/Abstract of modern computer
  - To find out what a computer can do and what it can not do
  - Help us understand what we can expect from our software
  - To decide, whether we can solve the problem, which requires more time to solve, in less amount of time.

Thank You…!!!