Indian Institute of Technology
Madras

ED5330:
Control of Automotive Systems

Project 2 – Heading Angle Control

ED17B005 Anuj Khandalikar
ED17B008 Ayush Chauhan

Contents

1 Introduction

The objective of the project is to design a a heading angle controller for an autonomous ground vehicle (AGV) taking into account the vehicle dynamics. We used Control System Toolbox from MATLAB and Simulink for Proportional controller and Proportional-Integral controller design.

The following sections describe the tasks involved and their results.

2 [TASK 1] System modeling and open loop response analysis

Equations involved in are:

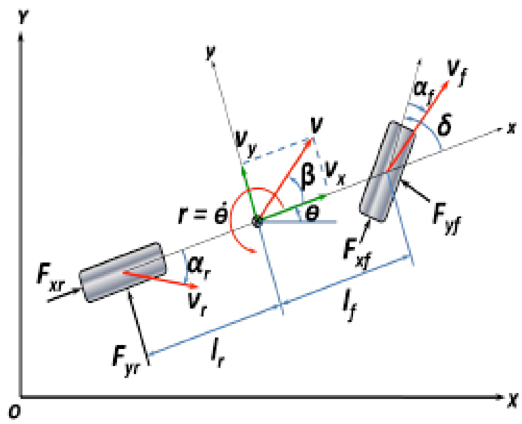
$$\dot{V}_y = \frac{C_y \cdot \delta}{m} - \frac{(C_f + C_r) \cdot V_y}{m \cdot V_x} - \frac{1}{m} \cdot (m \cdot V_x + \frac{C_f \cdot l_f - C_r \cdot l_r}{V_x}) \cdot r$$

and

$$\dot{r} = \frac{C_f \cdot l_f \cdot \delta}{I_r} + \frac{(C_r \cdot l_r - C_f \cdot l_f)}{I_2 \cdot V_x} \cdot V_y - \frac{(C_f \cdot l_f^2 + C_r \cdot l_r^2)}{I_2 \cdot V_x} \cdot r$$

The derivations for the above equations are as follows:
The two degrees of freedom are represented by the vehicle lateral position, y, and the vehicle yaw angle, . The vehicle lateral position is measured along the lateral axis of the vehicle and the vehicle yaw angle is measured with respect to the global X-axis. The lateral force at the tire-road interface depends on the slip angle. Figure 2 illustrates the bicycle model for a vehicle with no roll motion. It is assumed that only the front wheel is steerable. The lateral motion of the vehicle is described by

$$m\dot{v}_y = F_{xy} sin\delta + F_{yf} cos\delta + F_{yr} \tag{1}$$



The equation governing the yaw motion is

$$I_z\dot{r} = l_f F_{xf} sin\delta + l_f F_{yf} cos\delta - l_r F_{yr} \tag{2}$$

Considering to be small, (1) and (2) can be written as

$$ma_y = F_{yf} + F_{yr} \tag{3}$$

$$I_z\dot{r} = l_f F_{yf} - l_r F_{yr} \tag{4}$$

The velocity vector v can be written as:

$$v = v_x i + v_y j \tag{5}$$

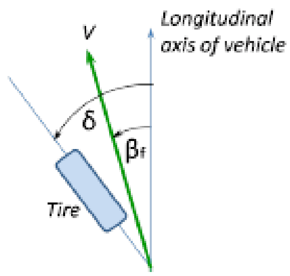
where i and j are the unit vectors in x and y directions respectively. Here, vx and vy are the velocity components in the x and y directions respectively. The x–y coordinate system is fixed to the vehicle. The acceleration could be written as

$$a = (\dot{v}_x - v_y r)i + (\dot{v}_y + v_x r)j \tag{6}$$

Substituting y-component of the acceleration from (6) into (3), the lateral motion of the vehicle is described by

$$m(\dot{v}_y + v_x r) = F_{yf} + F_{yr} \tag{7}$$

Experimental results show that the lateral tire force of a tire is proportional to the slip-angle for small slip-angles [11]. The slip angle of a tire is defined as the angle between the orientation of the tire and the orientation of the velocity vector of the wheel as shown in Fig



The slip angles of the front and rear wheels are:

$$\alpha_f = \delta - \beta_f \tag{8}$$

$$\alpha_r = -\beta_r \tag{9}$$

where f and r are the angles that the velocity vectors of the front and rear wheels make with the longitudinal axis of the vehicle respectively. When a side-slip angle is negative, Fyf and Fyr act in the positive y-direction. For small slip angles, the lateral forces acting on the front and rear wheels can be written as

$$F_{xf} = C_f \alpha_f \tag{10}$$

$$F_{yr} = C_r \alpha_r \tag{11}$$

The side-slip angles are given by

$$tan\beta_f = \frac{v_y + l_f r}{v_x}$$

OR

$$\beta_f \approx \frac{v_y + l_f r}{v_x} \tag{12}$$

Substituting f from (12) in (8) results in

$$\alpha_f = \delta - \frac{v_y + l_f r}{v_x} \tag{13}$$

Similarly, the slip angle for rear wheel can be written as:

$$\alpha_r = \frac{v_y - l_r r}{v_x} \tag{14}$$

Using (10), (11), (13) and (14), the governing equations (4) and (7) become:

$$\dot{V}_y = \frac{C_y \cdot \delta}{m} - \frac{(C_f + C_r) \cdot V_y}{m \cdot V_x} - \frac{1}{m} \cdot (m \cdot V_x + \frac{C_f \cdot l_f - C_r \cdot l_r}{V_x}) \cdot r \tag{15}$$

and

$$\dot{r} = \frac{C_f \cdot l_f \cdot \delta}{I_r} + \frac{(C_r \cdot l_r - C_f \cdot l_f)}{I_2 \cdot V_x} \cdot V_y - \frac{(C_f \cdot l_f^2 + C_r \cdot l_r^2)}{I_2 \cdot V_x} \cdot r \tag{16}$$

Putting the equations in Steady State Representation:

$$\begin{bmatrix} \dot{V}_y \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -\frac{(C_f + C_r)}{m \cdot V_x} & -V_x + \frac{C_r \cdot l_r - C_f \cdot l_f}{m \cdot V_x} \\ \frac{C_r \cdot l_r - C_f \cdot l_f}{I_2 \cdot V_x} & -\frac{(C_f \cdot l_f^2 + C_r \cdot l_r^2)}{I_2 \cdot V_x} \end{bmatrix} \begin{bmatrix} V_y \\ r \end{bmatrix} + \begin{bmatrix} \frac{C_f}{m} \\ \frac{C_f \cdot l_f}{I_2} \end{bmatrix} [\delta]$$

Parameter	*Value*
Wheelbase (*l*)	2 m
Distance of front tire from vehicle CG (l_f)	1.2 m
Vehicle mass (*m*)	1000 kg
Yaw moment of inertia of the vehicle (I_z)	792 kgm^2
Cornering stiffness of the front wheel (C_f)	35973 N/rad
Cornering stiffness of the rear wheel (C_r)	53959 N/rad

Putting the known variables in the above equation:

$$\begin{bmatrix} \dot{V}_y \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{-89.932}{V_x} & -V_x + \frac{-0.4}{1000 \cdot V_x} \\ \frac{0.4}{792 \cdot V_x} & -\frac{109}{V_x} \end{bmatrix} \begin{bmatrix} V_y \\ r \end{bmatrix} + \begin{bmatrix} 35.973 \\ 54.5 \end{bmatrix} [\delta]$$

which is of the form:

$$\dot{X} = AX + B[\delta]$$

Consider dθ/dt *as r as output* :

$$\begin{bmatrix} d\theta \\ dt \end{bmatrix} = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} V_y \\ r \end{bmatrix}$$

This is form y = Cx + D

We don't directly go to the desired delta to theta transfer function but delta to d(theta)/dt transfer because it's readily available:

$$TF = C(SI - A)^{-1}B$$

$$A = \begin{bmatrix} -35.973 & -2.5 \\ -0.0002 & -43.6 \end{bmatrix}$$

$$B = \begin{bmatrix} 35.975 \\ 54.5 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

substituting all known values in, we get

$$TF = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 3 + 35.97 & 2.50016 \\ 0.0002 & 5 + 43.6 \end{bmatrix}^{-1} \begin{bmatrix} 35.975 \\ 54.5 \end{bmatrix}$$

Solving on, we arrive at

$$\frac{d\theta(s)}{dt} = \frac{54.5 \cdot s + 1960.357}{\Delta} \cdot \delta(s)$$

Also, we know

$$\frac{d\theta(s)}{dt} = s \cdot \theta(s)$$

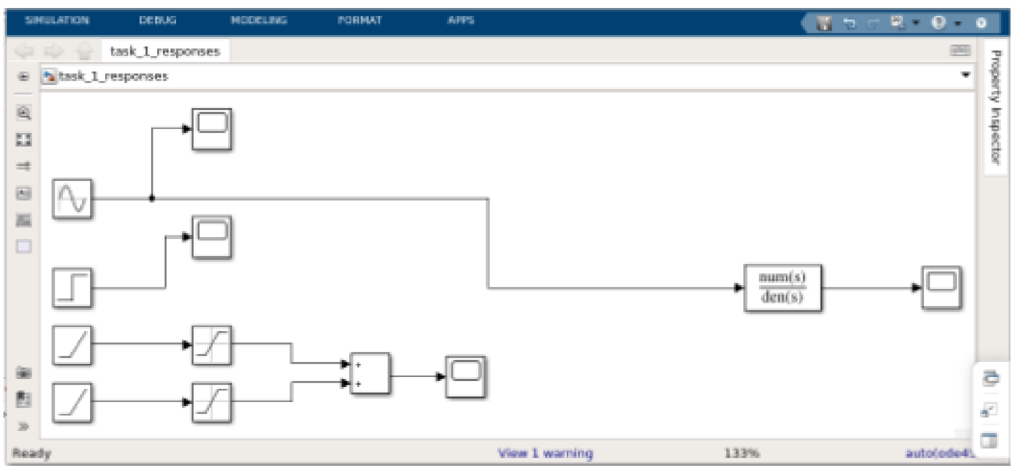
Hence,

$$s \cdot \theta(s) = \frac{54.5 \cdot s + 1960.357}{\Delta} \cdot \delta(s)$$

Therefore,

$$TF_{req} = \frac{\theta(s)}{\delta(s)} = \frac{54.5 \cdot s + 1960.357}{s \cdot \Delta}.$$

We have obtained the transfer function for the open-loop system in previous steps. Now we are going to use Simulink for obtaining the heading angle response.



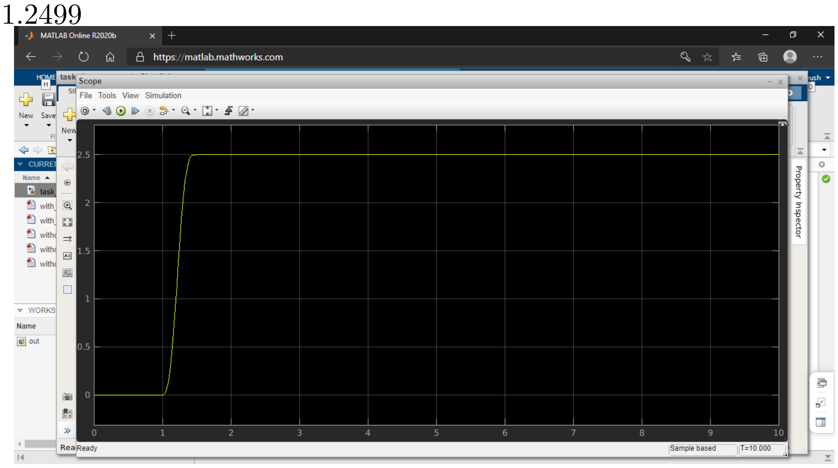
Plots for Heading Angle Response:

1. Pulse steering angle input of 10°

By hand

$$y(t) = L^{-}1((54.5 * s + 1960.357)/(s^3 + 79.57 * s^2 + 1568.3 * s))$$

$= 1.2499$



2. A Step steering angle input of 10°

Heading response for a step steering angle input of 10°

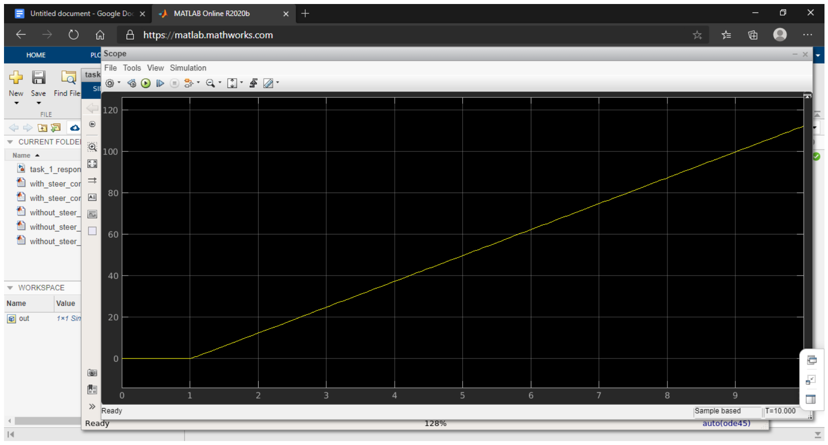
By hand

$$Q(s)/(s) = TF$$

Here (s) = 10/s Heading angle

$Q(t) = L^{-}1((545*s+19603.57)/(s^4+79.57*s^3+1568.3*s^2)) = 12.499t-0.286$

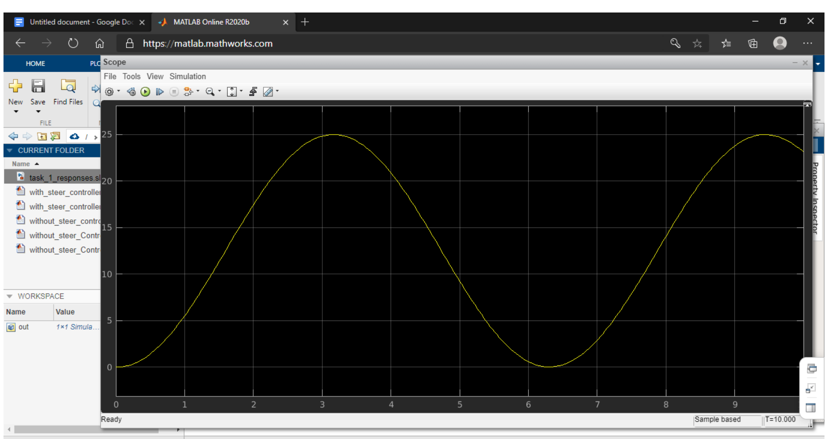
(eliminating super small terms)



3. A Sinusoidal steering angle input with a magnitude of 10° at
a) frequency = 1 rad/s

$$Byhand$$
$$Q(s)/(s) = TF$$
$$Here(s) = 10/(s^2+1)$$
$$HeadingangleQ(s) = TF * (10/s^2+1)$$
$$Q(t) = L^-1((545*s+19603.57)/(s^5+79.57*s^4+1569.3*s^3+79.57*s^2+1568.3*s))$$

$$(-6.24666 + 0.14327\,i)\,e^{-i\,t}\left((0.998948 + 0.0458467\,i) + (1 + 0\,i)\,e^{(2\,i)\,t}\right) - 0.00657367\,e^{-43.599\,t} + 1.83366\times 10^{-6}\,e^{-35.971\,t} + 12.4999$$



b) frequency = 2 rad/s
By hands

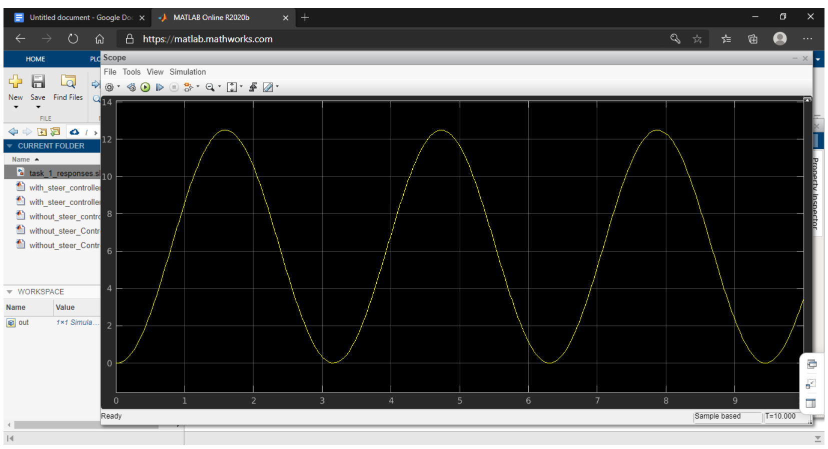
$$Q(s)/(s) = TF$$

$$Here(s) = 20/(s^2 + 4)$$

$$HeadingangleQ(s) = tf * (20/s^2 + 4)$$

$$Q(t) = L^-1((1090{*}s{+}39207.14)/(s^5{+}79.57{*}s^4{+}1572.3{*}s^3{+}318.28{*}s^2{+}6273.2{*}s))$$

$$(-3.11841 + 0.143044\, i)\, e^{(-2\, i)\, t} \left((0.995801 + 0.091549\, i) + (1 + 0\, i)\, e^{(4\, i)\, t}\right) - 0.0131266\, e^{-43.599\, t} + 3.65885 \times 10^{-6}\, e^{-35.971\, t} + 6.24994$$



c) frequency = 5 rad/s

Observations:
By hands

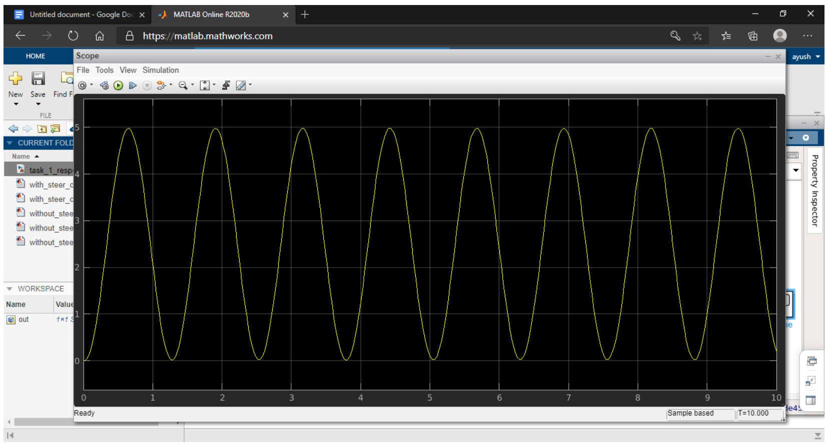
$$Q(s)/(s) = TF$$

$$Here(s) = 50/(s^2 + 25)$$

$$HeadingangleQ(s) = TF * (50/s^2 + 25)$$

$$Q(t) = L^-1((2725{*}s{+}98017.85)/(s^5{+}79.57{*}s^4{+}1593.3{*}s^3{+}1989.25{*}s^2{+}39207.5{*}s))$$

$$(-1.23376+0.141484\,i)\,e^{(-5\,i)\,t}\left((0.97404+0.226377\,i)+(1+0\,i)\,e^{(10\,i)\,t}\right)-0.0324588\,e^{-43.599\,t}+9.00147\times10^{-6}\,e^{-35.971\,t}+2.49998$$



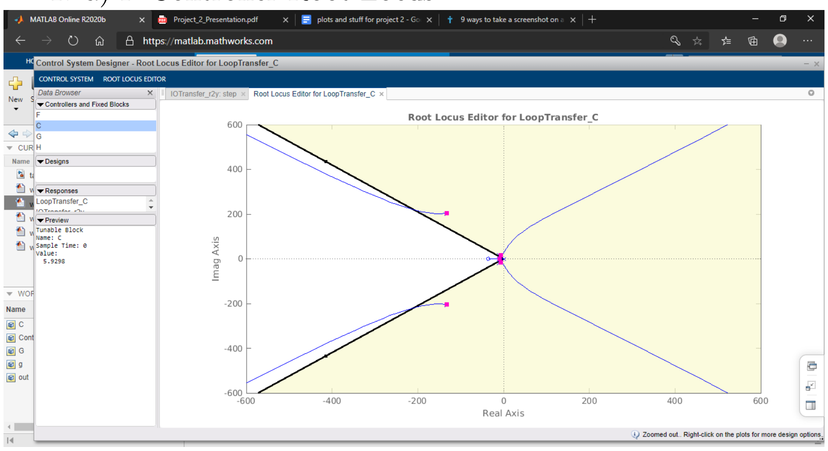
Observations:
We observe that in case of an impulse input at steering, we get a heading angle in form of a step-like output signal. In the case of a step input, we see that the output signal is a ramp function.
For the sinusoidal inputs, we see that we get shifted sinusoidal output. The frequency of the output signal increases with input signal frequency.
All these inputs were taken in the frequency domain and response function in the time domain was calculated by finding the inverse Laplace transform of the product of input and transfer function. In all the cases we got response functions that are same as what we get in Simulink and are also logically interpretable.

3 [TASK 2] Controller design

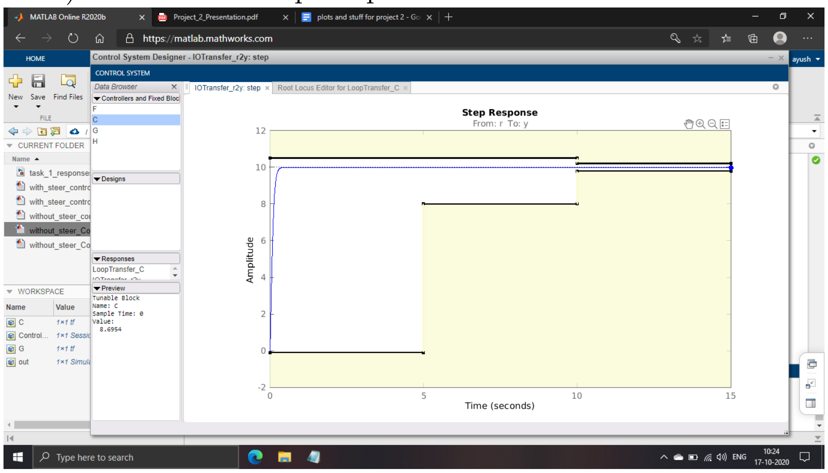
Process Description:
We observe that in case of an impulse input at steering, we get a heading angle in form of a step-like output signal. In the case of a step input, we see that the output signal is a ramp function.
For the sinusoidal inputs, we see that we get shifted sinusoidal output. The frequency of the output signal increases with input signal frequency.

All these inputs were taken in the frequency domain and response function in the time domain was calculated by finding the inverse Laplace transform of the product of input and transfer function. In all the cases we got response functions that are same as what we get in Simulink and are also logically interpretable.

1. a) P Controller Root Locus

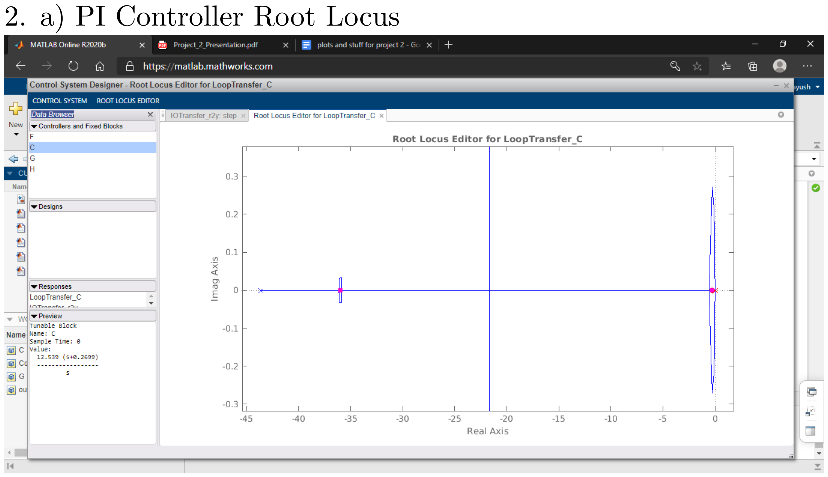


b) P Controller Step Response

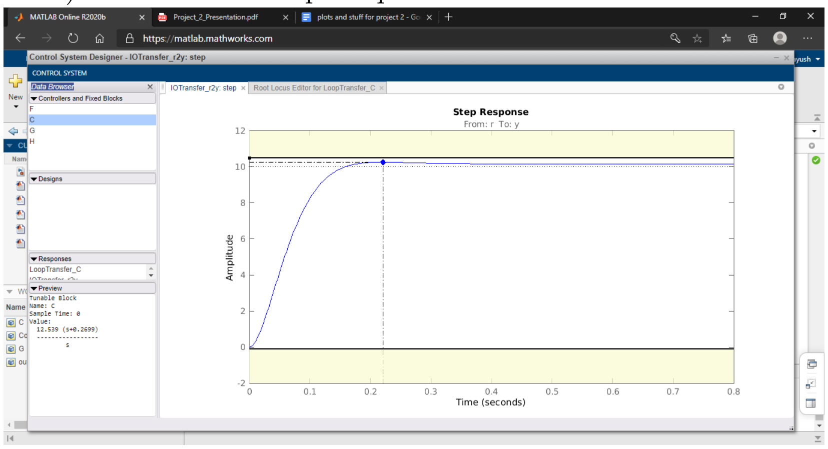


No possible pole value in root locus corresponds to instability (as it's all in left half s-plane). The only active performance requirement is maximum peak overshoot and no pole location gives any overshoot. So here we focused on making system more responsive by minimizing the settling time. Which doesn't go below 0.35 sec. The corresponding P controller gain is 8.6954

2. a) PI Controller Root Locus



b) PI Controller Step Response



The Controller transfer function that we got by arranging poles in order to satisfy the stability and performance requirement of max peak overshoot of 5% is

$$C = \frac{1.2539(s + 0.2699)}{s}$$

and the peak overshoot that we observe for this controller is 2.55%

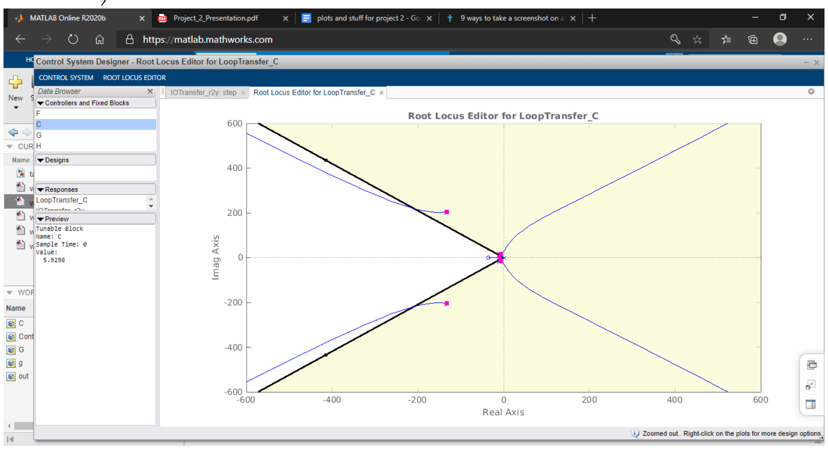
4 [TASK 3] Effect of steering actuator dynamics

The transfer function of the steering actuator is

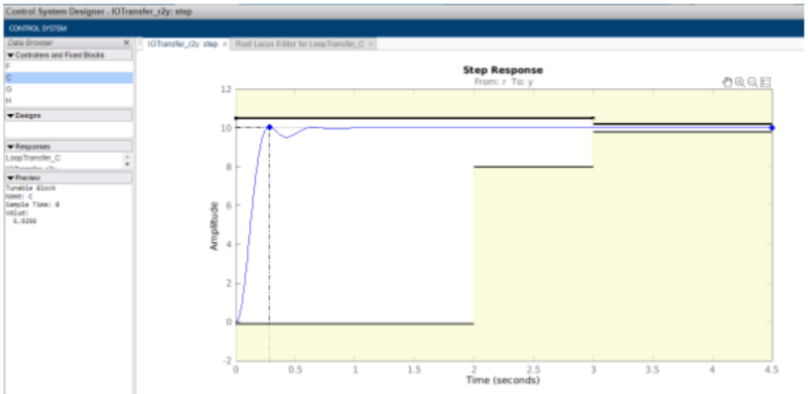
$$G_m(s) = \frac{604}{0.044 \cdot s^2 + 9.1634 \cdot s + 604}$$

In this task, we incorporated the steering actuator in our a closed-loop system for which we already had a steering transfer function. To use this for controller design in the control system designer app we made a new transfer function that was the multiplication of previous transfer function with this steering actuator transfer function. Now we did the same thing as we did for task 2 to for finding P and PI controller gains.

1. a) P Controller Root Locus

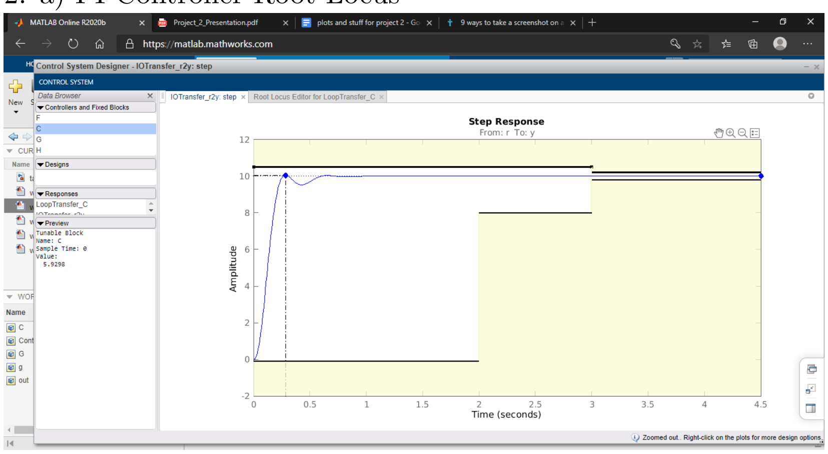


b) P Controller Step Response

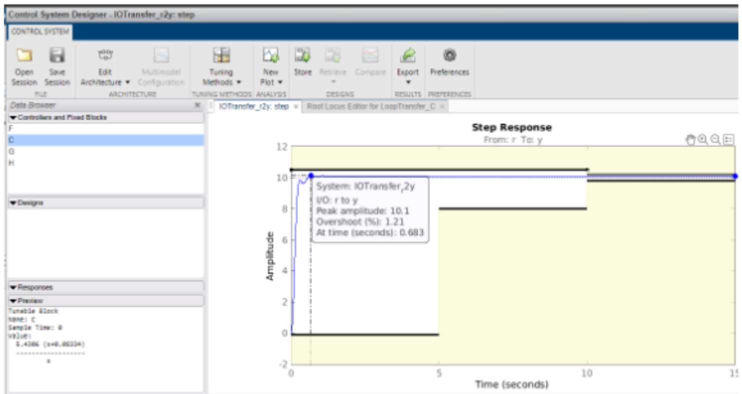


P controller gain is 5.9298 for the performance requirements with a peak overshoot of 0.638%

2. a) PI Controller Root Locus



b) PI Controller Step Response



The Controller transfer function that we got by arranging poles in order to satisfy the stability and performance requirement of max peak overshoot of 5% is

$$C = \frac{5.43(s + 0.033)}{s}$$

And the peak overshoot that we observe for this controller is 1.21%

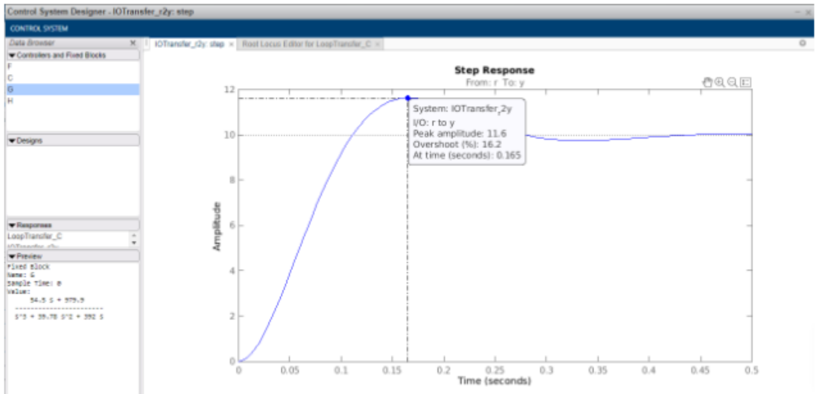
5 [TASK 4] Effect of variation in vehicle longitudinal speed

In this part, we try to look at the variations in close loop stability and performance with a change in vehicle longitudinal speed Vx.

For Velocity=5 m/s

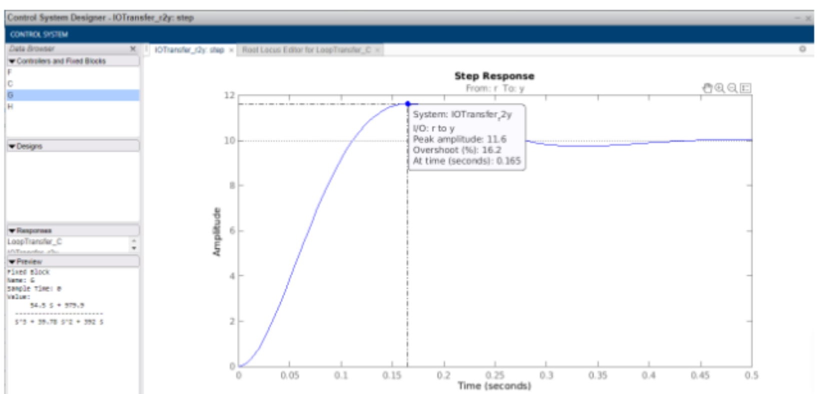
$$TF = \frac{54.5 \cdot s + 979.9}{s^3 + 39.78 \cdot s^2 + 392 \cdot s}$$

P controller with same gain (8.695) for this new transfer function



PI controller with the same Controller transfer function

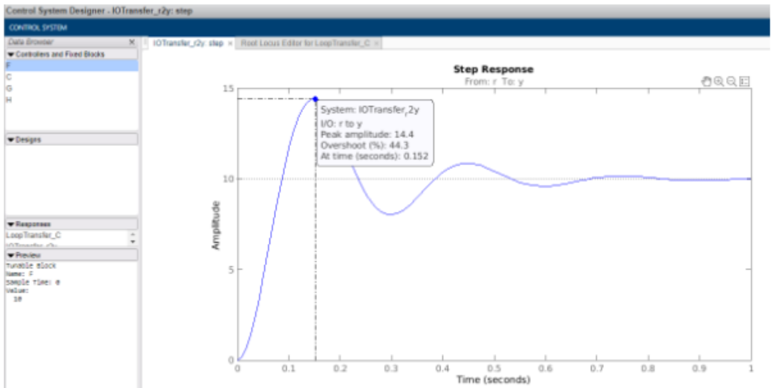
$$\frac{1.254 \cdot s + 0.3384}{s}$$



For Velocity=10 m/s:

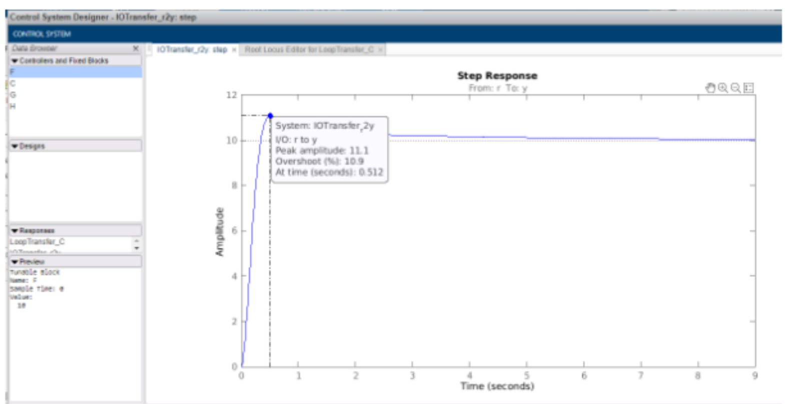
$$TF = \frac{54.5 \cdot s + 489.9}{s^3 + 19.89 \cdot s^2 + 97.99 \cdot s}$$

P controller with same gain (8.695) for this new transfer function



PI controller with the same Controller transfer function

$$\frac{1.254 \cdot s + 0.3384}{s}$$



Observations:
We see that with an increment in longitudinal velocity, there is an increase in max peak overshoot for the same controller gains. While we were able to restrict the peak overshoot under 5 percent for the same controller gains at a slower longitudinal speed of 2.5 m/s, now we see an increased max peak overshoot for a longitudinal velocity of 5 m/s. As the longitudinal speed is doubled, we see that overshoot gets even higher.
Another observation is that the peak overshoot for the PI controller is less

sensitive to longitudinal speed change than that for the P controller.
To conclude

- For both the controllers, the peak overshoot increases with an increase in longitudinal velocity.
- The peak overshoot for the P controller is more sensitive to change in longitudinal speed than that for PI controller.