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MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

NO. 3

TARGET : JEE (ADVANCED) 2018

Course : VIJETA & VIJAY (ADP & ADR) Date : 17.04.2018

TEST INFORMATION

DATE : 24.04.2018 **PART TEST (PT) - 2**

Syllabus : Indefinite Integration, Definite Integration & Its Application, Differential Equation, Matrices & Determinant, Complex Number

DPP Syllabus: Definite Integration & Its Application, Indefinite Integration, Differential Equation

ANSWER KEY

1.	(B)	**2.**	(C)	**3.**	(A)	**4.**	(B)	**5.**	(C)	**6.**	(C)	**7.**	(B)
8.	(D)	**9.**	(C)	**10.**	(B)	**11.**	(C)	**12.**	(B)	**13.**	(A)	**14.**	(D)
15.	(C)	**16.**	(B)	**17.**	(C)	**18.**	(AB)	**19.**	(BD)	**20.**	(BCD)		
21.	(ACD)	**22.**	(ABD)	**23.**	(BCD)	**24.**	(AC)	**25.**	(BCD)	**26.**	(BD)		
27.	(ABC)	**28.**	3	**29.**	1	**30.**	3	**31.**	08	**32.**	1	**33.**	5
34.	4	**35.**	8	**36.**	9	**37.**	5	**38.**	4	**39.**	7	**40.**	1

Total Marks : 146		**Max. Time : 120 min.**
Comprehension ('–1' negative marking) Q.1 to Q.8	**(3 marks 3 min.)**	**[24, 24]**
Single choice Objective ('-2' negative marking) Q.9 to Q.17	**(3 marks 3 min.)**	**[27, 27]**
Multiple choice objective ('-1' negative marking) Q.18 to Q.27	**(4 marks 3 min.)**	**[40, 30]**
Single Integer Questions ('-1' negative marking) Q.28 to Q. 40	**(3 marks 3 min.)**	**[39, 39]**

Comprehension # 1 (For Q. No. 1 to 2)

Consider the integral $I = \int_0^{10\pi} \frac{\cos 4x \cos 5x \cos 6x \cos 7x}{1+e^{2\sin 2x}} dx$

माना कि समाकलन $I = \int_0^{10\pi} \frac{\cos 4x \cos 5x \cos 6x \cos 7x}{1+e^{2\sin 2x}} dx$

1. If $I = k . \int_0^{\pi/2} \cos 4x \cos 5x \cos 6x \cos 7x \, dx$, then 'k' is equal to

यदि $I = k . \int_0^{\pi/2} \cos 4x \cos 5x \cos 6x \cos 7x \, dx$, तब 'k' का मान है–

(A) 5 (B*) 10 (C) 1 (D) 20

2. If $I = \lambda.\int_{0}^{\pi/4} \cos 2x \cos 4x \cos 6x\, dx$, then 'λ' is equal to

यदि $I = \lambda.\int_{0}^{\pi/4} \cos 2x \cos 4x \cos 6x\, dx$, तब 'λ' का मान है–

(A) 5 (B) 20 (C*) 10 (D) 5/2

Sol.(1 to 2) $I = \int_{0}^{10\pi} \frac{\cos 4x \cos 5x \cos 6x \cos 7x}{1+e^{2\sin 2x}} dx$

$I = \int_{0}^{10\pi} \frac{\cos 4x \cos 5x \cos 6x \cos 7x}{1+e^{-2\sin 2x}} dx$ (from p-5 से)

$2I = \int_{0}^{10\pi} \cos 4x \cos 5x \cos 6x \cos 7x\, dx$

$2I = 10\int_{0}^{\pi} \cos 4x \cos 5x \cos 6x \cos 7x\, dx$ (from p-7 से)

$2I = 20\int_{0}^{\pi/2} \cos 4x \cos 5x \cos 6x \cos 7x\, dx$ (from p-6 से)

$I = 10\int_{0}^{\pi/2} \cos 4x \cos 5x \cos 6x \cos 7x\, dx$ $\therefore$ k = 10

Further, पुनः

$I = 5\int_{0}^{\pi/2} \cos 4x.\cos 6x.(\cos 12x + \cos 2x)$

$I = 5\left(\int_{0}^{\pi/2} \cos 4x \cos 6x \cos 12x\, dx + \int_{0}^{\pi/2} \cos 2x \cos 4x \cos 6x\, dx\right)$

$I = 5\left(0 + 2\int_{0}^{\pi/4} \cos 2x \cos 4x \cos 6x\, dx\right)$ (from p-6 से)

$I = 10\int_{0}^{\pi/4} \cos 2x \cos 4x \cos 6x\, dx$ $\therefore$ $\lambda = 10$

Comprehension # 2 (Q. No. 3 to 5)
अनुच्छेद # 2 (प्र० सं० 3 से 5)

If A is a square matrix and e^A is defined as $e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} \ldots\ldots \infty = \frac{1}{2}\begin{bmatrix} f(x) & g(x) \\ g(x) & f(x) \end{bmatrix}$, where $A = \begin{bmatrix} x & x \\ x & x \end{bmatrix}$, and I being the identity matrix then

यदि A एक वर्ग आव्यूह है तथा e^A इस प्रकार परिभाषित है कि $e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} \ldots\ldots \infty = \frac{1}{2}\begin{bmatrix} f(x) & g(x) \\ g(x) & f(x) \end{bmatrix}$, जहां $A = \begin{bmatrix} x & x \\ x & x \end{bmatrix}$ तथा I द्वितीय क्रम का तत्समक आव्यूह है तब

3. $\int \frac{g(x)}{f(x)} dx =$

(A*) log ($e^x + e^{-x}$) + C
(B) log $|e^x - e^{-x}|$ + C
(C) log $|e^{2x} - 1|$ + C
(D) None of these इनमें से कोई नहीं

4. $\int (g(x)+1) \sin x \, dx =$

(A) $\frac{e^x}{2}$ (sin x – cos x) + C
(B*) $\frac{e^{2x}}{5}$ (2sin x – cos x) + C
(C) $\frac{e^x}{5}$ (sin 2x – cos 2x)
(D) None of these इनमें से कोई नहीं

5. $\int \frac{f(x)}{\sqrt{g(x)}} dx =$

(A) $\frac{1}{2\sqrt{e^x - 1}}$ – $\text{cosec}^{-1}(e^x)$ + C
(B) $\frac{2}{\sqrt{e^x - e^{-x}}}$ – $\sec^{-1}(e^x)$ + C
(C*) $\sqrt{e^{2x} - 1}$ + $\sec^{-1}(e^x)$ + C
(D) None of these इनमें से कोई नहीं

Sol. $e^A = I + A + \frac{A^2}{2!} + \ldots$

$$= \begin{bmatrix} 1 + x + \frac{2x^2}{2\,!} + \frac{2^2x^3}{3\,!} + \ldots & x + \frac{2x^2}{2\,!} + \frac{2^2x^3}{3\,!} + \ldots \\ x + \frac{2x^2}{2\,!} + \frac{2^2x^3}{3\,!} & 1 + x + \frac{2x^2}{2\,!} + \frac{2^2x^3}{3\,!} + \ldots \end{bmatrix} = \frac{1}{2}\begin{bmatrix} e^{2x}+1 & e^{2x}-1 \\ e^{2x}-1 & e^{2x}+1 \end{bmatrix}$$

$\therefore$ f(x) = e^{2x} + 1 & g(x) = e^{2x} – 1

3. $\int \frac{e^{2x} - 1}{e^{2x} + 1} dx = \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$

4. $\int e^{2x} \sin x \, dx = \frac{e^{2x}}{5}$ (2sinx – cos x) + C

5. $\int \frac{e^{2x} + 1}{\sqrt{e^{2x} - 1}} dx = \int \frac{e^{2x}}{\sqrt{e^{2x} - 1}} dx + \int \frac{e^x}{e^x\sqrt{e^{2x} - 1}} dx = \sqrt{e^{2x} - 1} + \sec^{-1}(e^x) + C$

Paragraph : (Q.6 to Q.8)

Consider the curves given by :

$S_1 : \frac{x^2}{25} - \frac{y^2}{9} = 1$

$S_2 : x^2 + y^2 = 8$.

Then answer the following questions :

6. Area of the closed figure formed by common tangents of S_1 and S_2 is equal to :
(A) 16
(B) 64
(C*) 32
(D) 40

7. The area bounded between the common tangents of S_1 and S_2 and the director circle of S_2 is :
(A) $16\pi - 16$
(B*) $16\pi - 32$
(C) $8\pi - 16$
(D) $12\pi - 16$

8. Let S_3 be the conjugate hyperbola of S_1. Let 'A_1' be the area bounded by the upper branch of S_3 and the common tangents of S_1 & S_2 passing through (0, 4) and 'A_2' be the area bounded by the upper branch of S_3 and the common tangent to S_1 and S_2 passing through (0, – 4). Then choose the correct option :
(A) $A_1 > 1$
(B) $B_1 < 49$
(C) $A_1 + B_1 < 49$
(D*) $A_1 + B_1 \geq 49$

अनुच्छेद : (Q.6 to Q.8)

माना कि दिये गये वक्र

$S_1 : \dfrac{x^2}{25} - \dfrac{y^2}{9} = 1$

$S_2 : x^2 + y^2 = 8$.

तब निम्न प्रश्नों के उत्तर दीजिये–

6. S_1 और S_2 की उभयनिष्ठ स्पर्श रेखाओं से बने परिबद्ध क्षेत्र का क्षैत्रफल बराबर है–

(A) 16 (B) 64 (C*) 32 (D) 40

7. S_1 और S_2 की उभयनिष्ठ स्पर्श रेखाओं और S_2 के नियामक वृत्त से परिबद्ध क्षैत्रफल है–

(A) $16\pi - 16$ (B*) $16\pi - 32$ (C) $8\pi - 16$ (D) $12\pi - 16$

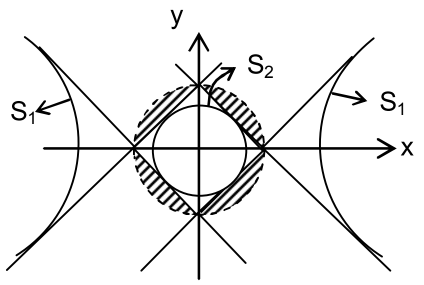
8. माना S_3 ,S_1. का सयुग्मी अतिपरवलय है। माना S_3 के ऊपरी शाखा और (0, 4) से गुजरने वाली S_1 एवं S_2 की उभयनिष्ठ स्पर्श रेखाओं से परिबद्ध क्षैत्रफल 'A_1' है। माना S_3 के ऊपरी शाखा और (0,– 4) से गुजरने वाली S_1 एवं S_2 की उभयनिष्ठ स्पर्श रेखाओं से परिबद्ध क्षैत्रफल 'A_2' है तब सही विकल्प का मिलान कीजिए–

(A) $A_1 > 1$ (B) $B_1 < 49$ (C) $A_1 + B_1 < 49$ (D*) $A_1 + B_1 \geq 49$

Sol. (6) Common tangents उभयनिष्ठ स्पर्श रेखाएं $\Rightarrow y = \pm x \pm 4$

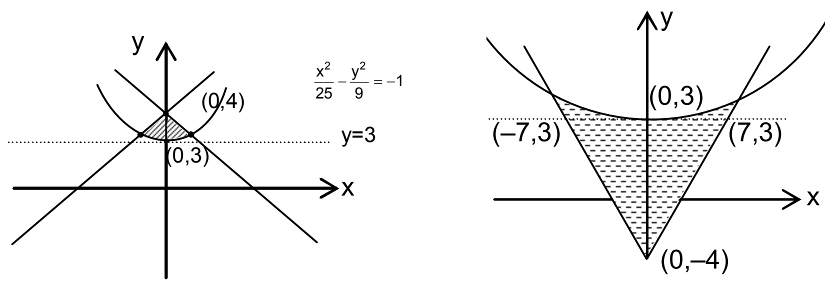
$\Rightarrow$ Area क्षैत्रफल = 1/2 × 8 × 8 = 32

(7)



A (shaded) छायांकित = $16\pi - 32 \cong 18.3$ sq. units

(8)



$A_1 \leq \dfrac{1}{2} \times 2 \times 1$ $\qquad$ $B_1 \geq \dfrac{1}{2} \times 14 \times 7$

$\Rightarrow A_1 \leq 1$ $\qquad$ $\Rightarrow B_1 \geq 49$

A_1 is just close to 1 and B_1 is much larger than 49 $\Rightarrow A_1 + B_1 \geq 49$

A_1, 1 के नजदीक है और B_1 , 49 से बड़ा है। $\Rightarrow A_1 + B_1 \geq 49$

9. The value of $\int_0^{10\pi} \left[\tan^{-1} x\right] dx$, where [.] denotes the greatest integer function, is equal to

$\int_0^{10\pi} \left[\tan^{-1} x\right] dx$ का मान (जहां [.] महत्तम पूर्णांक फलन को व्यक्त करता है) बराबर है–

(A) 10π (B) $\tan 1 - 10\pi$ (C*) $10\pi - \tan 1$ (D) tan 1

Hint. $\left\{\int_0^{\tan 1} 0\,dx + \int_{\tan 1}^{10\pi} 1\,dx\right\}$

10. If $\int_0^\infty \frac{\sin x}{x}dx = a$ then $\int_0^\infty \frac{\sin^3 x}{x}dx$ has a value equal to

यदि $\int_0^\infty \frac{\sin x}{x}dx = a$ तब $\int_0^\infty \frac{\sin^3 x}{x}dx$ का मान होगा–

(A) a (B*) $\frac{a}{2}$ (C) $\frac{a}{3}$ (D) 2a

Sol. $\sin^3 x = \frac{3\sin x}{4} - \frac{1}{4}\sin(3x)$ $\Rightarrow$ $\int_0^\infty \frac{\sin^3 x}{x}dx = \frac{3}{4}\int_0^\infty \frac{\sin x}{x}dx - \underbrace{\frac{1}{4}\int_0^\infty \frac{\sin 3x}{x}dx}_{3x=u} = \frac{3}{4}a - \frac{1}{4}a$

11. If $\int_0^1 e^{x^2}(x-\alpha)dx = 0$, then ($\alpha$ being a real number)

यदि $\int_0^1 e^{x^2}(x-\alpha)dx = 0$, तब ($\alpha$ एक वास्तविक संख्या है)

(A) $\alpha \in (1, 2)$ (B) $\alpha < 0$ (C*) $\alpha \in (0, 1)$ (D) $\alpha = 0$

Hint. $\int_0^1 e^{x^2}.x.dx = \int_0^1 e^{x^2}.\alpha.dx$ $\Rightarrow$ $\alpha\int_0^1 e^{x^2}dx = \frac{e-1}{2}$ As $1 < \int_0^1 e^{x^2}dx < e-1$

also तथा, when जब $x \in [0, 1]$

and $\alpha = \frac{(e-1)}{2\int_0^1 e^{x^2}dx}$ $\Rightarrow$ $\alpha \in \left(\frac{1}{2}, \frac{e-1}{2}\right)$

12. $\int \frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}}dx =$

(A) $e^x . \sqrt{\frac{1-x}{1+x}} + c$ (B*) $e^x\sqrt{\frac{1+x}{1-x}} + c$ (C) $\frac{e^x}{\sqrt{1-x}} + c$ (D) $\frac{e^x}{\sqrt{1+x}} + c$

Sol. $I = \int e^x.\frac{2-x^2}{(1-x)\left(\sqrt{1-x^2}\right)}dx$

$= \int e^x\left(\frac{1}{(1-x)\sqrt{1-x^2}} + \frac{1-x^2}{(1-x)\sqrt{1-x^2}}\right)dx = \int e^x\left(\frac{1}{(1-x)\sqrt{1-x^2}} + \sqrt{\frac{1+x}{1-x}}\right)dx = e^x.\sqrt{\frac{1+x}{1-x}} + c$

13. $\int \frac{8x^{43}+13x^{38}}{\left(x^{13}+x^5+1\right)^4}dx =$

(A*) $\frac{x^{39}}{3\left(x^{13}+x^5+1\right)^3} + c$ (B) $\frac{x^{39}}{\left(x^{13}+x^5+1\right)^3} + c$

(C) $\frac{x^{39}}{5\left(x^{13}+x^5+1\right)^5} + c$ (D) $\frac{x^{52}}{3\left(x^{13}+x^5+1\right)} + c$

Sol. $I = \int \frac{8x^{43}+13x^{38}}{(x^{13}+x^5+1)^4}dx = \int \frac{8x^{-9}+13x^{-14}}{(1+x^{-8}+x^{-13})^4}dx$ Put $1 + x^{-8} + x^{-13} = t$

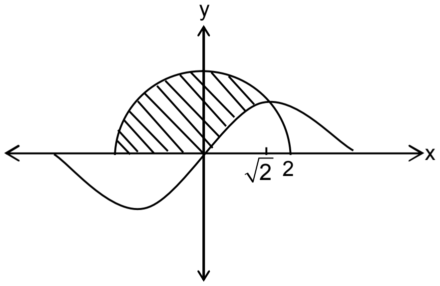
$\therefore \quad I = \int \frac{-dt}{t^4} = \frac{1}{3t^3} + c$

14. The area enclosed by the curve $y \le \sqrt{4-x^2}$, $y \ge \sqrt{2}\ \sin\left(\frac{\pi x}{2\sqrt{2}}\right)$ and the x-axis is divided by y-axis in the ratio

वक्रों $y \le \sqrt{4-x^2}$, $y \ge \sqrt{2}\ \sin\left(\frac{\pi x}{2\sqrt{2}}\right)$ तथा x-अक्ष से परिबद्ध क्षेत्रफल को y-अक्ष किस अनुपात में विभाजित करता है–

(A) $\frac{\pi^2-8}{\pi^2+8}$ (B) $\frac{\pi^2-4}{\pi^2+4}$ (C) $\frac{\pi-3}{\pi+4}$ (D*) $\frac{2\pi^2}{2\pi+\pi^2-8}$

Sol.



Area to the left of y-axis (y-अक्ष के बायीं ओर क्षेत्रफल = π)

Area to the right of y-axis (y-अक्ष के दायीं ओर क्षेत्रफल) = $\int_0^{\sqrt{2}}\left(\sqrt{4-x^2}-\sqrt{2}\sin\left(\frac{\pi x}{2\sqrt{2}}\right)\right)dx$

$= \left(\frac{x\sqrt{4-x^2}}{2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right)_0^{\sqrt{2}} + \left(\frac{4}{\pi}\cos\frac{\pi x}{2\sqrt{2}}\right)_0^{\sqrt{2}} = 1 + \pi/2 - 4/\pi$

15. A wet porous substance in the open air loses its moisture at a rate proportional to the moisture content. If pours substance is hung in the wind loses half its moisture during the first hour, then the time when it would have lost 99.9% of its moisture is : (weather conditions remaining same)

(A) More than 100 hrs (B) More than 10 hrs
(C*) Approximately 10 hrs (D) Approximately 9 hrs

खुली हवा में रखे एक गीले संरध्र पात्र में इसकी आर्द्रता में कमी, उसकी मात्रा के समानुपाती है। हवा में एक चद्दर का लटकाया जाता है तो इसमें आर्द्रता की कमी, एक घंटे के दौरान उसकी मात्रा की आधी होती है, तब वह समय कितना होगा जबकि इसकी आर्द्रता में कमी 99.9% होती है। (मौसम समान रहता है)

(A) 100 घंटे से अधिक (B) 10 घंटे से अधिक
(C*) लगभग 10 घंटे (D) लगभग 9 घंटे

Sol. $\frac{dm}{dt} = -km \quad \Rightarrow \quad m = ce^{-kt}$

when t = 0, $m = m_0 \Rightarrow c = m_0 \quad \Rightarrow \quad m = m_0e^{-kt}$

when t = 1, $m = \frac{m_0}{2} \quad \Rightarrow \quad k = \ell n2$

$\therefore \quad m = m_0e^{-t\ell n2}$

when $m = \frac{m_0}{1000}$, then $t = \log_2 1000$

16. Compute सरल कीजिए: $\int \frac{x^4 - 8x^2 + 11}{(x^2-1)(x^2-4)(x^2-9)}dx$

(A) $\frac{1}{6}\ln(x^2-1) + \frac{1}{3}\ln(x^2-4) + \frac{1}{2}\ln(x^2-9) + C$

(B*) $\frac{1}{12}\ln\left\{\frac{(x-1)(x-2)(x-3)}{(x+1)(x+2)(x+3)}\right\} + C$

(C) $\frac{1}{6}\ln\left(\frac{x-1}{x+1}\right) + \frac{1}{3}\ln\left(\frac{x-2}{x+2}\right) + \frac{1}{2}\ln\left(\frac{x-3}{x+3}\right) + C$

(D) $\frac{1}{12}\ln\{(x^2-1)(x^2-4)(x^2-9)\} + C$

Sol. Using partial fraction : आंशिक भिन्न से

$\frac{1}{6}\int\frac{1}{x^2-1}dx + \frac{1}{3}\int\frac{1}{x^2-4}dx + \frac{1}{2}\int\frac{1}{x^2-9}dx$

$= \frac{1}{12}\ln\left\{\frac{(x-1)(x-2)(x-3)}{(x+1)(x+2)(x+3)}\right\} + C$

17. $f(x) = a_1\sin x + b_1\cos x + a_2\sin 2x + b_2\cos 2x + \ldots\ldots + a_n\sin(nx) + b_n\cos(nx)\ \forall\ x \in R$. All a_i's and b_i's are never simultaneously zero. Then
[2016-17]

(A) $f(x) > 0\ \forall\ x \in R$ if all a_i's and b_i's are positive

(B) $f(x) < 0\ \forall\ x \in R$ if all a_i's and b_i's are negative.

(C*) f(x) cannot have the same sign $\forall\ x \in R$.

(D) none of these

$f(x) = a_1\sin x + b_1\cos x + a_2\sin 2x + b_2\cos 2x + \ldots\ldots + a_n\sin(nx) + b_n\cos(nx)\ \forall\ x \in R$ है। सभी a_i तथा b_i कभी भी एक साथ शून्य नहीं है, तब–

(A) $f(x) > 0\ \forall\ x \in R$, यदि सभी a_i तथा b_i धनात्मक हैं।

(B) $f(x) < 0\ \forall\ x \in R$, यदि सभी a_i तथा b_i ऋणात्मक हैं।

(C*) सभी $x \in R$ के लिए f(x) समान चिन्ह का नहीं हो सकता है।

(D) इनमें से कोई नहीं।

Sol. $\int_0^{2\pi} f(x)\,dx = 0 \Rightarrow$ f(x) can not have same sign $\forall$ x **Hence (C)**

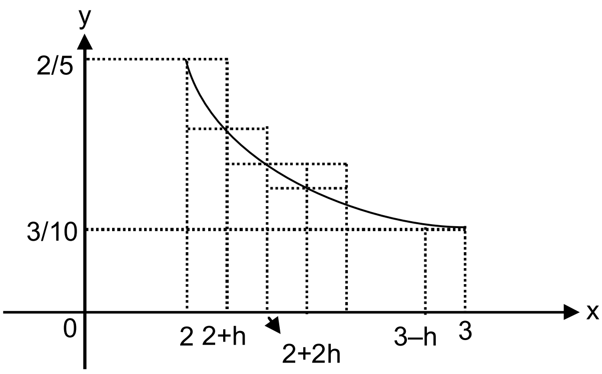
f(x), $\forall$ x के लिए समान चिन्ह नहीं रख सकता है।

18. If $T_n = \sum_{k=2n}^{3n-1}\frac{k}{k^2+n^2}$ and $S_n = \sum_{k=2n+1}^{3n}\frac{k}{k^2+n^2}\ \forall\ n \in \{1, 2, 3, \ldots\}$, then

यदि $T_n = \sum_{k=2n}^{3n-1}\frac{k}{k^2+n^2}$ और $S_n = \sum_{k=2n+1}^{3n}\frac{k}{k^2+n^2}\ \forall\ n \in \{1, 2, 3, \ldots\}$, तब

(A*) $T_n > \ell n\sqrt{2}$ (B*) $S_n < \ell n\sqrt{2}$ (C) $T_n < \ell n\sqrt{2}$ (D) $S_n > \ell n\sqrt{2}$

Sol. Consider $f(x) = \frac{x}{1+x^2}$ लेने पर



Area bounded by f(x) with x-axis $\int_2^3 \frac{x}{x^2+1} = \ell n\sqrt{2}$ f(x) का x-अक्ष के साथ क्षेत्रफल = $\int_2^3 \frac{x}{x^2+1} = \ell n\sqrt{2}$

Clearly, स्पष्टतया $h[f(2)+ f(2 + h) + ... + f(3 – h)] > \ell n\sqrt{2} > h[f(2 + h) + f(2 + 2h) + ... + f(3)]$

19. If $I_1 = \int_0^1 \frac{1+x^8}{1+x^4}dx$ and $I_2 = \int_0^1 \frac{1+x^9}{1+x^3}dx$, then

यदि $I_1 = \int_0^1 \frac{1+x^8}{1+x^4}dx$ और $I_2 = \int_0^1 \frac{1+x^9}{1+x^3}dx$, तब

(A) $I_2 < I_1 < \pi/4$ (B*) $\pi/4 < I_2 < I_1$ (C) $1 < I_1 < I_2$ (D*) $I_2 < I_1 < 1$

Sol. For all सभी $x \in (0, 1)$ के लिए

$\Rightarrow \quad \frac{1}{1+x^2} < \frac{1+x^9}{1+x^3} < \frac{1+x^8}{1+x^4} < 1 \quad \therefore \int_0^1 \frac{1}{1+x^2}dx < I_2 < I_1 < \int_0^1 1\ dx \quad \therefore \pi/4 < I_2 < I_1 < 1$

20. Solution of the differential equation $\frac{dy}{dx} = \frac{y^2-2xy-x^2}{y^2+2xy-x^2}$ is

(A) $x^2 – y^2 + c(x – y) = 0$
(B*) $x^2 + y^2 + c(x + y) = 0$
(C*) a straight line if it passes through (1, –1)
(D*) a circle if it passes through (1, 1)

अवकल समीकरण $\frac{dy}{dx} = \frac{y^2-2xy-x^2}{y^2+2xy-x^2}$ का हल है–

(A) $x^2 – y^2 + c(x – y) = 0$
(B*) $x^2 + y^2 + c(x + y) = 0$
(C*) (1, –1) गामी एक सरल रेखा
(D*) (1, 1) गामी एक वृत्त

Sol. Put y = tx रखनें पर

$t + x\frac{dt}{dx} = \frac{t^2-2t-1}{t^2+2t-1}$

$\Rightarrow \quad \frac{-t^2-2t+1}{(t+1)(t^2+1)}dt = \frac{dx}{x} \quad \Rightarrow \quad \left(\frac{1}{t+1} - \frac{2t}{t^2+1}\right)dt = \frac{dx}{x} \quad \Rightarrow \quad x^2 + y^2 = c(x + y)$

21. Consider a continuous function 'f' such that $x^4 – 4x^2 \le f(x) \le 2x^2 – x^3$ and the area bounded by y = f(x), $g(x) = x^4 – 4x^2$, the y-axis and the line x = t ($0 \le t \le 2$) is twice of the area bounded by y = f(x), $y = 2x^2 – x^3$, y-axis and the line x = t ($0 \le t \le 2$). Then

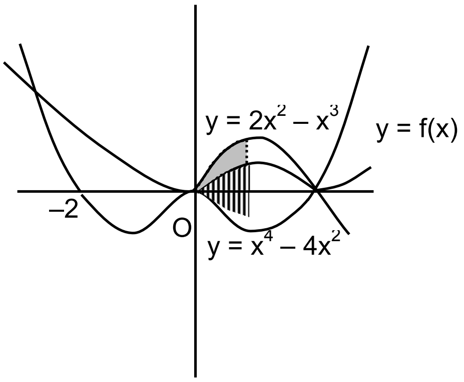
(A*) f(2) = 0 (B) f(1) = 1/3
(C*) f′(1) = –2/3 (D*) f(x) is a many one function

माना 'f' एक सतत् फलन है। यदि $x^4 - 4x^2 \le f(x) \le 2x^2 - x^3$ एवं $y = f(x)$, $g(x) = x^4 - 4x^2$ y-अक्ष और रेखा $x = t\ (0 \le t \le 2)$ के मध्य परिबद्व क्षैत्रफल $y = f(x)$, $y = 2x^2 - x^3$, y-अक्ष और रेखा $x = t\ (0 \le t \le 2)$ के मध्य परिबद्व क्षैत्रफल का दोगुना है। तब

(A*) $f(2) = 0$ (B) $f(1) = 1/3$

(C*) $f'(1) = -2/3$ (D*) f(x)एक बहु एकैकी फलन है

Hint.



$$\int_0^t \left[f(x) - (x^4 - 4x^2)\right] dx = 2\int_0^t \left[\left(2x^2 - x^3\right) - f(x)\right] dx$$

on differentiating with respect to t. t के सापेक्ष अवकलन करने पर

$\Rightarrow \quad f(t) - (t^4 - 4t^2) = 2(2t^2 - t^3 - f(t)) \qquad \Rightarrow \quad f(t) = \dfrac{1}{3}(t^4 - 2t^3)$

22. If $f(x) = \displaystyle\int_0^{\pi/2} \frac{\ell n\left(1 + x\sin^2\theta\right)}{\sin^2\theta} d\theta$, $x \ge 0$, then

(A*) $f(x) = \pi\left(\sqrt{x+1} - 1\right)$ (B*) $f'(3) = \dfrac{\pi}{4}$

(C) f(x) cannot be determined (D*) $f'(0) = \dfrac{\pi}{2}$

यदि $f(x) = \displaystyle\int_0^{\pi/2} \frac{\ell n\left(1 + x\sin^2\theta\right)}{\sin^2\theta} d\theta$, $x \ge 0$, तब

(A*) $f(x) = \pi\left(\sqrt{x+1} - 1\right)$ (B*) $f'(3) = \dfrac{\pi}{4}$

(C) f(x) ज्ञात नहीं कर सकते (D*) $f'(0) = \dfrac{\pi}{2}$

Sol. $f(x) = \displaystyle\int_0^{\pi/2} \frac{\ell n\left(1 + x\sin^2\theta\right)}{\sin^2\theta} d\theta\ ;\ x \ge 0 \qquad \Rightarrow \quad f'(x) = \int_0^{\pi/2} \frac{1}{1 + x\sin^2\theta} d\theta$

$\Rightarrow \quad f'(x) = \displaystyle\int_0^{\pi/2} \frac{\sec^2\theta\ d\theta}{1 + (1+x)\tan^2\theta}$ put $\tan\theta = t$ रखनें पर

$\Rightarrow \quad f'(x) = \displaystyle\int_0^{\infty} \frac{dt}{1 + \left\{\left(\sqrt{1+x}\right)t\right\}^2} \qquad \Rightarrow \quad f'(x) = \frac{1}{\sqrt{1+x}}\left(\tan^{-1}\left(\sqrt{1+x} \times t\right)\right)_0^{\infty}$

$\Rightarrow \quad f'(x) = \dfrac{\pi}{2}\cdot\dfrac{1}{\sqrt{1+x}} \qquad \Rightarrow \quad f(x) = \pi\ .\ \sqrt{1+x} + c$ put x = 0 रखनें पर

$\pi + c = f(0) \qquad \Rightarrow \quad c = -\pi \qquad \therefore \quad f(x) = \pi\left(\sqrt{1+x} - 1\right)$

23^. A real valued function $f(x) : R^+ \to R^+$ satisfies $\int_0^1 f(tx)dt = nf(x)$. If $\lim_{n\to\infty} f(x) = g(x)$, $g(1) = 2$ and area bounded by $y = g(x)$ with x-axis from $x = 3$ to $x = 7$ is S, then

एक वास्तविक मान फलन $f(x) : R^+ \to R^+$, $\int_0^1 f(tx)dt = nf(x)$ को संतुष्ट करता है। यदि $\lim_{n\to\infty} f(x) = g(x)$, $g(1) = 2$ और $y = g(x)$ का x-अक्ष के साथ परिबद्ध क्षेत्रफल $x = 3$ से $x = 7$ के बीच में है, S है,, तब

(A) $S \in \left(2, \frac{8}{3}\right)$ (B*) $S \in \left(\frac{8}{7}, \frac{8}{3}\right)$ (C*) $S < \frac{40}{21}$ (D*) $S > \ell n4$

Sol. $tx = y \Rightarrow \int_0^x f(y)\,dy = xn\,f(x) \Rightarrow f(x) = n[f(x) + xf'(x)] \Rightarrow f(x)(1-n) = nx\,f'(x)$

$\Rightarrow \frac{f'(x)}{f(x)} = \left(\frac{1-n}{n}\right).\frac{1}{x} \Rightarrow \ell n(f(x)) = \left(\frac{1-n}{n}\right)\ell nx + \ell nc$

$\Rightarrow f(x) = c\,x^{\frac{1-n}{n}}$ as $n \to \infty$ $f(x) = cx^{-1} = \frac{c}{x} \Rightarrow g(x) = \frac{2}{x}$

24. If $f(x) = \int\left(\cot\frac{x}{2} - \tan\frac{x}{2}\right)dx$ where $f\left(\frac{\pi}{2}\right) = 0$ then identify which of the following statement(s) is (are) correct?

(A*) $\int_0^\pi f(x)dx = -2\pi\,\ell n2$ (B) $\int_0^\pi f(x)dx = -\pi\,\ell n2$

(C*) $\text{sgn}\left(f\left(\frac{2\pi}{3}\right)\right) = -1$ (D) $\text{sgn}\left(f\left(\frac{2\pi}{3}\right)\right) = 1$

[Note : sgn(y) denotes signum function of y.**]**

यदि $f(x) = \int\left(\cot\frac{x}{2} - \tan\frac{x}{2}\right)dx$ जहाँ $f\left(\frac{\pi}{2}\right) = 0$ तब निम्न में से सत्य कथन को पहचानिए–

(A*) $\int_0^\pi f(x)dx = -2\pi\,\ell n2$ (B) $\int_0^\pi f(x)dx = -\pi\,\ell n2$

(C*) $\text{sgn}\left(f\left(\frac{2\pi}{3}\right)\right) = -1$ (D) $\text{sgn}\left(f\left(\frac{2\pi}{3}\right)\right) = 1$

[Note : sgn(y) ; y के सिग्नम फलन को व्यक्त करता है।**]**

Sol. $f(x) = \int\left(\cot\frac{x}{2} - \tan\frac{x}{2}\right)dx$

$= \int \frac{\cos x}{\sin\frac{x}{2}\cos\frac{x}{2}}dx = 2\int \cot x.dx = 2\ell n\,|\sin x| + C$

at $x = \frac{\pi}{2}$, $f\left(\frac{\pi}{2}\right) = 0 \Rightarrow C = 0$

$f(x) = \ell n\,(\sin^2 x)$

Now, $f\left(\frac{2\pi}{3}\right) = \ell n\left(\frac{3}{4}\right) = -ve$

$\Rightarrow \text{sgn}\left(f\left(\frac{2\pi}{3}\right)\right) = -1$

$$\int_0^{\pi} \ell n\left(\sin^2 x\right)dx = 2.2 \int_0^{\pi/2} \ell n\left(\sin x\right)dx$$

$$= 4\left(\frac{-\pi}{2}\ell n2\right) = 2\pi/\ln 2$$

25. Let $I_n = \int \frac{x^n dx}{\sqrt{x^2+2x+5}}$; then $I_n = \left(\frac{x^{m-1}}{n}\right)\sqrt{x^2+2x+5} - \lambda\, I_{n-1} - \mu\, I_{n-2}$;

where λ and μ are functions of 'n' ; then :

माना $I_n = \int \frac{x^n dx}{\sqrt{x^2+2x+5}}$; तब $I_n = \left(\frac{x^{m-1}}{n}\right)\sqrt{x^2+2x+5} - \lambda\, I_{n-1} - \mu\, I_{n-2}$;

जहां λ और μ ,'n' के फलन है तब–

(A) $m = n - 1$ (B*) $\lambda(2) = \frac{3}{2}$ (C*) $\lambda(1) = 1$ (D*) $\mu(5) = 4$

Sol. $I_n = \int \frac{x^{n-1}(x+1-1)dx}{\sqrt{x^2+2x+5}}$

$I_n = \int \frac{x^{n-1}(x+1)dx}{\sqrt{x^2+2x+5}} - I_{n-1}$

Use integration by parts खण्डश: समाकलन करने पर

$I_n = x^{n-1}\sqrt{x^2+2x+5} - \int \frac{(n-1)x^{n-2}(x^2+2x+5)}{\sqrt{x^2+2x+5}}dx - I_{n-1}$

$I_n = x^{n-1}\sqrt{x^2+2x+5} - [(n-1)(I_n + 2I_{n-1} + 5I_{n-2})] - I_{n-1}$

$I_n = x^{n-1}\sqrt{x^2+2x+5} - (n-1)\, I_n - 2(n-1)\, I_{n-1} - 5(n-1)\, I_{n-2} - I_{n-1}$

Simplifying सरल करने पर ; $m = n$; $\lambda = \frac{2n-1}{n}$; $\mu = \frac{5(n-1)}{n}$

26. The value of the integral,

$$I = \int_{-\sqrt{2}}^{\sqrt{2}} \left(\frac{2x^7+3x^6-10x^5-7x^3-12x^2+x+1}{x^2+2}\right)dx \text{ is}$$

$I = -\frac{a}{b}\sqrt{2} + \frac{c}{2\sqrt{2}}$; where HCF (a, b) = 1. Then

(A) a + b + c = 24 (B*) ab = 80

(C) The equation $C^x = 1 + x\ln\pi$ has 2 solution (D*) a – b = 11

समाकल $I = \int_{-\sqrt{2}}^{\sqrt{2}} \left(\frac{2x^7+3x^6-10x^5-7x^3-12x^2+x+1}{x^2+2}\right)dx$ का मान

$I = -\frac{a}{b}\sqrt{2} + \frac{c}{2\sqrt{2}}$; है– जहां (a, b) का म.स.प. = 1 तब–

(A) a + b + c = 24 (B*) ab = 80

(C) समीकरण $C^x = 1 + x\ln\pi$ के 2 हल है। (D*) a – b = 11

Sol. $I = \int_{-\sqrt{2}}^{\sqrt{2}} \frac{2x^7-10x^5-7x^3+x}{x^2+2}dx + \int_{-\sqrt{2}}^{\sqrt{2}} \frac{3x^2(x^4-4)+1}{x^2+2}$

$\therefore$ I = 0 + 2 $\int_0^{\sqrt{2}}\left(3\left(x^4-2x^2\right)+\frac{1}{x^2+1}\right)dx$

I = $\frac{6}{5}x^5 - 4x^3 + \sqrt{2} + \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\Big|_0^{\sqrt{2}}$

I = $-\frac{16}{5}\sqrt{2} + \frac{\pi}{2\sqrt{2}}$

27. If a differentiable function satisfies $(x - y)f(x + y) - (x + y)f(x - y) = 2(x^2y - y^3)$ $\forall$ x, y $\in$ R, and f(1) = 2, then

(A*) f(x) must be a polynomial function
(B*) area bounded by f(x) with x-axis is $\frac{1}{6}$
(C*) f(3) = 12
(D) f(3) = 13

यदि एक अवकलनीय फलन समीकरण $(x - y)f(x + y) - (x + y)f(x - y) = 2(x^2y - y^3)$ $\forall$ x, y $\in$ R को संतुष्ट करता है एवं f(1) = 2 तो

(A*) f(x) एक बहुपदीय फलन है।
(B*) f(x) एवं x-अक्ष द्वारा परिबद्व क्षैत्रफल $\frac{1}{6}$ है।
(C*) f(3) = 12
(D) f(3) = 13

Sol. Differentiate both sides w.r.t. y, then put y = 0

$2xf'(x) - 2f(x) = 2x^2 \quad \Rightarrow \quad \frac{dy}{dx} - \frac{1}{x}.y = x \quad \Rightarrow \quad y = x^2 + x$

Hindi. y के सापेक्ष दोनो तरफ अवकलन करके y = 0 रखने पर

$2xf'(x) - 2f(x) = 2x^2 \quad \Rightarrow \quad \frac{dy}{dx} - \frac{1}{x}.y = x \quad \Rightarrow \quad y = x^2 + x$

28. Find the value of 4^{th} power of the parameter 'a' (a > 0), for which area bounded by the straight line y = $\frac{a^2-ax}{1+a^4}$ and the parabola y = $\frac{x^2+2ax+3a^2}{1+a^4}$ is maximum.

प्राचल 'a' (a > 0), की चार वी घात का मान होगा जबकि सरल रेखा y = $\frac{a^2-ax}{1+a^4}$ और परवलय y = $\frac{x^2+2ax+3a^2}{1+a^4}$ से परिबद्ध क्षैत्रफल अधिकतम है–

Ans. 3

Sol. Point of intersection of given curves : दिये गये वक्रों का परिच्छेद बिन्दु

$x^2 + 2ax + 3a^2 = a^2 - ax$

$\Rightarrow \quad x^2 + 3ax + 2a^2 = 0$

$x = -a, -2a$

Area क्षैत्रफल = $\int_{-2a}^{-a}\left|\frac{(a^2-ax)-(x^2+2ax+3a^2)}{1+a^4}\right|dx$

$A = \frac{a^3}{6(1+a^4)}$

for maximum area, अधिकतम क्षैत्रफल के लिये $\frac{dA}{da} = 0$

$\Rightarrow \quad a = (3)^{1/4} = (3)^{(1/2)^2}$

29. If यदि m, n $\in$ N evaluate हो तो $\frac{(m+n+1)!}{m!n!}\int_2^3(x-2)^m(3-x)^n dx$ का मान है–

Ans. 1

Sol. $x = 2\cos^2\theta + 3\sin^2\theta$

$$\Rightarrow \quad I = \int_0^{\pi/2} (3-2)^{m+n} \sin^{2m}\theta\cos^{2n}\theta \times 2(3-2)\sin\theta\cos\theta$$

$$= 2(3-2)^{m+n+1}\int_0^{\pi/2} \sin^{2m+1}\theta\cos^{2n+1}\theta d\theta$$

$$= 2\frac{\sqrt{\frac{2m+1+1}{2}}\sqrt{\frac{2n+1+1}{2}}}{2\sqrt{\frac{2m+2n+2+2}{2}}} = \frac{m!n!}{(m+n+1)!}$$

30. Find the sum of order and degree of differential equation of all the conics touching the y-axis at origin and having centre on the x-axis

y-अक्ष को मूल बिन्दु पर स्पर्श करने वाले सभी शांकव, जिनके केन्द्र x-अक्ष पर है से बनी अवकल समीकरण की कोटि और घात का योगफल है–

Ans. 2 + 1 = 3

Sol. equation of conics is शांकव का समीकरण $\frac{(x-a)^2}{a^2} + By^2 = 1$

$$\Rightarrow \quad \frac{x^2}{a^2} - \frac{2x}{a} + By^2 = 1$$

$$\Rightarrow \quad \frac{2x}{a^2} - \frac{2}{a} + By\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{x}{a^2} - \frac{1}{a} + By\frac{dy}{dx} = 0$$

Differential अवकलन करने पर

$$\Rightarrow \quad \frac{1}{a^2} + B\left(\frac{dy}{dx}\right)^2 + By\frac{d^2y}{dx^2} = 0$$

$$\Rightarrow \quad \begin{vmatrix} x^2 & -2x & y^2 \\ x & -1 & ydy/dx \\ 1 & 0 & \frac{dy}{dx} + y\frac{d^2y}{dx^2} \end{vmatrix} = 0$$

31. If $\lim_{n\to\infty} \frac{\left(\sum_{r=1}^{n}\sqrt{r}\right)\left(\sum_{r=1}^{n}\frac{1}{\sqrt{r}}\right)}{\sum_{r=1}^{n} r} = \frac{p}{3}$, then the value of 'p' should be ?

यदि $\lim_{n\to\infty} \frac{\left(\sum_{r=1}^{n}\sqrt{r}\right)\left(\sum_{r=1}^{n}\frac{1}{\sqrt{r}}\right)}{\sum_{r=1}^{n} r} = \frac{p}{3}$, हो तो 'p' का मान होगा–

Ans. 08

Sol. Use limit of sum and evaluate सीमा योग की सहायता से सरल करने पर

32. If यदि m, n $\in$ N evaluate हो तो

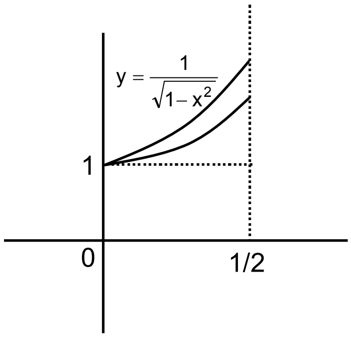
$I = \frac{693}{256}\int_0^{\pi/2} (\sin 2x)^5 (\cos x)dx =$ **Ans.** 1

33. If the range of the integral $I = \int_0^{1/2} \frac{dx}{\sqrt{1-x^{2n}}}$ for $n \ge 1$ is (a, b] then the value of $\frac{\pi}{b}$ $- 2a$ is equal to

यदि समाकल $I = \int_0^{1/2} \frac{dx}{\sqrt{1-x^{2n}}}$ का $n \ge 1$ के लिए परिसर (a, b] है तब $\frac{\pi}{b} - 2a$ का मान बराबर है–

Ans. 5

Hint.



$\sqrt{1-x^2} \le \sqrt{1-x^{2n}} < 1$ when जब $0 < x < 1$ & $n \ge 1$

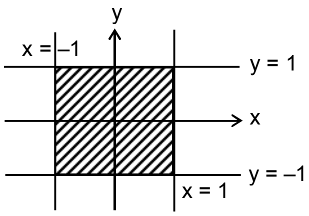
$\therefore \quad \int_0^{1/2} dx < I \le \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}} \quad \Rightarrow \quad \frac{1}{2} < I \le \frac{\pi}{6}$

34. For each positive integer n > 1, let S_n represents the area of the region bounded by $\frac{x^2}{n^2} + y^2 \le 1$ and $x^2 + \frac{y^2}{n^2} \le 1$, then $\lim_{n\to\infty} S_n$ is equal to

प्रत्येक धनात्मक पूर्णांक n > 1 के लिए माना S_n वक्रों $\frac{x^2}{n^2} + y^2 \le 1$ और $x^2 + \frac{y^2}{n^2} \le 1$ द्वारा परिबद्ध क्षेत्रफल को निरूपित करता है तो $\lim_{n\to\infty} S_n$ का मान है–

Ans. 4

Sol. When जब $n \to \infty$ $\quad y^2 \le 1$ & तथा $x^2 \le 1 \Rightarrow -1 \le y \le 1$ & तथा $-1 \le x \le 1$



$\lim_{n\to\infty} S_n = 4$

35. Let f(x) is a continuous function symmetric about the lines x = 1 and x = 2. If $\int_0^2 f(x)dx = 3$ and $\int_0^{50} f(x) \; dx = I$, then $\left[\sqrt{I}\right]$ is equal to (where [.] is G.I.F.)

माना f(x) एक सतत् एवं रेखा x = 1 और x = 2 के सापेक्ष सममित फलन है, यदि $\int_0^2 f(x)dx = 3$ और $\int_0^{50} f(x) \; dx = I$, तब $\left[\sqrt{I}\right]$ का मान है– (जहाँ [.] महत्तम पूर्णांक फलन है–)

Ans. 8

Sol. f(x) = f(2 – x) & f(x) = f(4 – x)

$\Rightarrow$ f(x) = f(x + 2) $\Rightarrow$ f(x) is periodic with period 2 f(x) आवर्तकाल 2 वाला आवर्ती फलन है

Now अब $I = \int_0^{50} f(x)\ dx = 25\int_0^{2} f(x)\ dx = 75$

36. If f : R → R is a continuous function such that f (5 – x) + f (5 + x) = 6 ∀ x ∈ R then the value of $\frac{1}{4}\int_{-1}^{11} f(x)dx$ is

यदि f : R → R सतत फलन इस प्रकार है कि f (5 – x) + f (5 + x) = 6 ∀ x ∈ R तब $\frac{1}{4}\int_{-1}^{11} f(x)dx$ का मान है–

Ans. 9

Sol. f(5 – x) + f(5+x) = 6

putting x = 0 रखने पर, f(5) = 3

∴ f(x) is symmetrical about (5, 3) के सापेक्ष सममित है।

Now अब, $\int_{-1}^{11} f(x)-3)dx \quad \int_{1}^{11} dx = 3(11 - (-1) =$ **36 Ans.**

or या

f(5 – x) + f(5+x) = 6

x → 5 – x

f(x) + f(10 – x) = 6

$$\int_{-1}^{5} f(x)dx + \int_{5}^{11} f(x)dx = \int_{-1}^{5}(6 - f(10-x))dx$$

$$36 - \int_{-1}^{5} f(10-x)dx + \int_{5}^{11} f(x)dx$$

t = 10 – x

$$36 + \int_{11}^{5} f(t)dt + \int_{5}^{11} f(x)dx \quad \Rightarrow \quad \mathbf{36}$$

37. Let $I_n = \int_{-1}^{1} |x|\left(1 + x + \frac{x^2}{2} + \frac{x^3}{3} + ... + \frac{x^{2n}}{2n}\right)dx$ where n ∈ N. If $\lim_{n\to\infty} I_n$ can be expressed as a rational number $\frac{p}{q}$ in the lowest form, then find the value of p + q.

माना $I_n = \int_{-1}^{1} |x|\left(1 + x + \frac{x^2}{2} + \frac{x^3}{3} + ... + \frac{x^{2n}}{2n}\right)dx$ जहाँ n ∈ N तथा $\lim_{n\to\infty} I_n$ को एक परिमेय संख्या $\frac{p}{q}$ के सरलतम रूप में व्यक्त कर सकते है तब p + q का मान ज्ञात कीजिए।

Ans. 5

Sol. We have यहां $I_n = 2\int_0^1 x\left(1 + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + ... + \frac{x^{2n}}{2n}\right)dx \qquad \left(\int_{-1}^{1}(\text{odd})dx = 0\right)$

$$= 2\left[\frac{x^2}{1\cdot 2} + \frac{x^4}{2\cdot 4} + \frac{x^6}{4\cdot 6} + ... + \frac{x^{2n+2}}{2n(2n+2)}\right]_0^1 = 2\left[\frac{1}{1\cdot 2} + \frac{1}{2\cdot 4} + \frac{1}{4\cdot 6} + ... + \frac{1}{2n(2n+2)}\right]$$

$$= 1 + \frac{1}{2}\left[\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + + \left(\frac{1}{3} - \frac{1}{4}\right) + ... + \left(\frac{1}{n} - \frac{1}{n+1}\right)\right]$$

Hence यहाँ $\lim_{n\to\infty}\left[1 + \frac{1}{2}\left(1 - \frac{1}{n+1}\right)\right] = \frac{3}{2}$

∴ p = 3 ; q = 2

38. If the value of the definite integral $\int_0^1 \frac{\sin^{-1}\sqrt{x}}{x^2-x+1}dx$ is $\frac{\pi^2}{\sqrt{n}}$ where $n \in N$, then find the value of '$\frac{n}{27}$'.

यदि निश्चित समाकल $\int_0^1 \frac{\sin^{-1}\sqrt{x}}{x^2-x+1}dx$ का मान $\frac{\pi^2}{\sqrt{n}}$ है जहां $n \in N$, तब '$\frac{n}{27}$' का मान ज्ञात कीजिए।

Ans. 4

Sol.(i) $I = \int_0^1 \frac{\sin^{-1}\sqrt{x}}{x^2-x+1}dx$ ....(1) $I = \int_0^1 \frac{\sin^{-1}\sqrt{1-x}}{x^2-x+1}dx = \int_0^1 \frac{\cos^{-1}\sqrt{x}}{x^2-x+1}dx$ ....(2)

(Applying $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$) ∴ On adding (1) and (2), we get (1) व (2) को जोड़ने पर

$$2I = \int_0^1 \frac{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}{x^2-x+1}dx = \frac{\pi}{2}\int_0^1 \frac{dx}{x^2-x+1}dx = \frac{\pi}{2}\int_0^1 \frac{dx}{\left(x-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dx$$

$$2I = \frac{\pi}{2}\frac{1}{\left(\frac{\sqrt{3}}{2}\right)}\left[\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right]_0^1 = \frac{\pi^2}{3\sqrt{3}}$$

Hence अतः $I = \frac{\pi^2}{6\sqrt{3}} = \frac{\pi^2}{\sqrt{108}} \equiv \frac{\pi^2}{\sqrt{n}}$ $\Rightarrow$ $n = 108$

39. Let C be a curve passing through M (2, 2) such that the slope of the tangent at any point to the curve is reciprocal of the ordinate of that point. If the area bounded by curve C and line $x = 2$ is expressed as a rational number $\frac{p}{q}$ (where p and q are in their lowest form), then find $(p-3q)$.

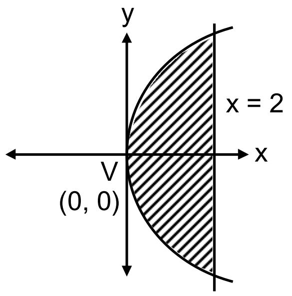
माना कि एक वक्र C बिन्दु M (2, 2) से गुजरता है। यदि वक्र के किसी बिन्दु पर स्पर्श रेखा की प्रवणता उस बिन्दु की कोटि का व्युत्क्रम है तथा वक्र C और रेखा $x = 2$ से परिबद्ध क्षेत्रफल को परिमेय संख्या $\frac{p}{q}$ के रूप में व्यक्त किया जा सकता है (जहां p और q सरलतम रूप में हैं), तब $(p-3q)$ ज्ञात कीजिए।

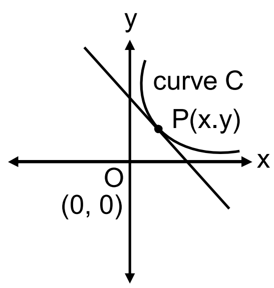
Ans. 7

Sol. Let P (x, y) be any point on the curve C.

Now, $\frac{dy}{dx} = \frac{1}{y}$ $\Rightarrow$ $ydy = dx$ $\Rightarrow$ $\frac{y^2}{2} = x + k$

Since the curve passes through M (2,2), so k = 0 $\Rightarrow$ $y^2 = 2x$



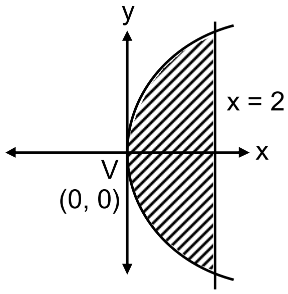


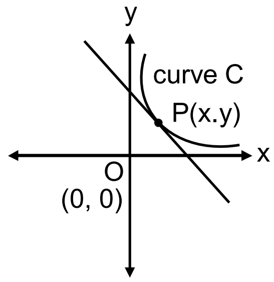
Hence required area = $2\int_0^2 \sqrt{2x}\,dx = 2\sqrt{2}\times\frac{2}{3}\left(x^{3/2}\right)_0^2 = \frac{4}{3}\sqrt{2}\times 2\sqrt{2} = \frac{16}{3}$ (square unit) $\Rightarrow$ p + q = 19

Hindi: माना कि P (x, y) वक्र C पर कोई बिन्दु C है।

Now अब, $\frac{dy}{dx} = \frac{1}{y}$ $\Rightarrow$ ydy = dx $\Rightarrow$ $\frac{y^2}{2} = x + k$

चूंकि वक्र M (2,2) से गुजरता है इसलिए k = 0 $\Rightarrow$ $y^2 = 2x$





अतः अभीष्ट क्षेत्रफल = $2\int_0^2 \sqrt{2x}\,dx = 2\sqrt{2} \times \frac{2}{3}\left(x^{3/2}\right)_0^2 = \frac{4}{3}\sqrt{2} \times 2\sqrt{2} = \frac{16}{3}$ (वर्ग इकाई) $\Rightarrow$ p + q = 19

40. For any $t \in R$ and f being a continuous function.

Let $I_1 = \int_{\sin^2 t}^{1+\cos^2 t} xf\left(x(2-x)\right)dx$

$I_2 = \int_{\sin^2 t}^{1+\cos^2 t} f\left(x(2-x)\right)dx$, then $\frac{I_1}{I_2}$ equal to

$t \in R$ और f एक सतत् फलन है–

माना $I_1 = \int_{\sin^2 t}^{1+\cos^2 t} xf\left(x(2-x)\right)dx$

$I_2 = \int_{\sin^2 t}^{1+\cos^2 t} f\left(x(2-x)\right)dx$, तब $\frac{I_1}{I_2}$ बराबर है

Ans. 1

Sol. $I_1 = \int_{\sin^2 t}^{1+\cos^2 t} xf\left(x(2-x)\right)dx$

$= \int_{\sin^2 t}^{1+\cos^2 t} \left(1+\cos^2 t+\sin^2 t-x\right)f\left\{\left(1+\cos^2 t+\sin^2 t-x\right)\left(2-\left(1+\cos^2 t+\sin^2 t-x\right)\right)\right\}$

$= 2\int_{\sin^2 t}^{1+\cos^2 t} f\{(2-x)x\}dx - \int_{\sin^2 t}^{1+\cos^2 t} xf\{(2-x)x\}dx$

$\Rightarrow$ $I_1 = 2I_2 - I_1$ $\Rightarrow$ $2I_1 = 2I_2$

$\Rightarrow$ $\frac{I_1}{I_2} = 1$