

Graphical MS Windows user interface for ConsoleApp\_DistributionFunctions (Schrausser, 2024b).

The following functions were realized, German notation:

### 1. Wahrscheinlichkeits-Verteilung (probability distribution)

- Binomial-Funktion  $f(X = k|n)$ .
- Poisson-Funktion  $f(X = k|n, p)$ .
- Geometrische-Funktion  $f(X = r|p)$ .
- Hypergeometrische-Funktion  $f(X = k|n, K, N)$ .
- Exakt binomialer 2-Felder Test  $f(X = b|b, c)$ .
- Exakt hypergeometrischer 4-Felder Test  $f(X = a|a, b, c, d)$ , Fisher Exact (Fisher, 1922, 1954; s. Agresti, 1992).

The fundamental *binomial distribution* was derived by Bernoulli (1713), s. Schneider (2005a) and above all de Moivre (1711, 1718) with the discovery of the first instance of central limit theorem, to approximate the *binomial distribution* with the *normal distribution*, further developed by Gauss (1809, 1823), see Hahn (1970), Hald (1990) or Schneider (2005b).

### 2. Theta-Verteilung $\theta$ (characteristic value or $\theta$ distribution)

- z-Dichte Funktion  $f(x = z)$ .
- z-Funktion  $F(x = z)$ .
- t-Funktion  $F(x = t)$ .
- $\chi^2$ -Funktion  $F(x = \chi^2)$ .
- F-Funktion  $F(x = F)$ .
- Effekt-Stärke  $\epsilon$ , Cohen (1977).

The *t-distribution* was first derived by Lüroth (1876), later in a more general form defined as *Pearson Type IV* (Pearson, 1895), commonly known as *Student's t-distribution*, from William Sealy Gosset (1908).

Helmert (1876) first described the  $\chi^2$ -distribution, independently rediscovered by Pearson (1900), c.f. also Elderton (1902), Pearson (1914) or Plackett (1983), for the *F-distribution* by Fisher (1924) see Snedecor (1934) and Scheffé (1959).

Statistical power  $1 - \beta$  and effect size  $\epsilon$  (Cohen, 1977, 1992) layed foundations for statistical meta-analysis and methods of estimation statistics, see e.g. Borenstein et al. (2001) for related software applications.

### 3. Transformationen (transformation functions)

- Fisher Z Funktion  $F(x = r)$ , Fisher (1915).
- Gamma  $F(x) = \Gamma$ .

*Gamma  $\Gamma$* , to solve the problem of extending the factorial to non-integer arguments, was first considered in a letter from Bernoulli to Goldbach (Bernoulli, 1729), introduced later by Euler (1738) - of fundamental definitional importance for the formulation of approximate probability distribution functions such as  $\chi^2$ , *t* or *F* (c.f. Meyberg & Vachenauer, 2001; Cuyt et al., 2008; Beals & Wong, 2020; Little et al., 2022).

See further e.g. Bortz (1984), Bortz and Weber (2005), Bortz and Schuster (2010), Döring (2023), Pascucci (2024a, b) and Schrausser (2024a).

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