Swap Regression with US GDP and Public Debt

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Contents

1 Motivation

The idea for SWAP Regression came from a paper published by Mosuk Chow et al, to use alternating predictors to find the relation between human tolerance to temperature and pressure.[1] In such situations, it is not possible to get certain measurements such as pressure, with a set temperature, and vice verse. In such cases, we have to remove the assumption that one variable is regressing the other, as in such cases, for some data points, the role of regressors and predictors switches.

2 Theory for Data Collection

Our idea is that SWAP Regression holds for any Bi Directional causal data points. In this discussion, we limit ourselves to Economic indicators. Glauco De Vita el at concluded that Public Debt and GDP Growth are Bi Directional Causal for the American market.[2] However, in our research, we took US Public Debt and Real GDP.

3 Data Collection

The data was found easily on Kaggle, US Public Debt Quarterly Data(in Billion $) and US GDP Quarterly Data(in Billion $), in quarters. Public Debt data was available from 1966-01-01 to 2023-01-01, and Real GDP data from 1947-01-01 to 2023-04-01.

4 Testing the Data

4.1 Testing the Bi Directional Causality

To test Bi Directional Causality, we made use of the Python function, **grangercausalitytests** from the **statsmodels.tsa.stattools** library.
While testing the Null Hypothesis that GDP does not causes Public Debt, we get a p-value of 0.00094 at lag 4, which means that we can reject the Null Hypothesis, and accept the Alternate Hypothesis that GDP does cause Public Debt.
While testing the Null Hypothesis that Public Debt does not causes GDP, we get a p-value of 2.2984 x 10^{-8} at lag 4, which means that we can reject the Null Hypothesis, and accept the Alternate Hypothesis that Public Debt does cause GDP.
This shows that the data we have is Bi directional causal.

4.2 Testing Stationarity of Data

Implementing Augmented Dickey Fuller Test of the data does not give any evidence of stationarity.

5 Breaking Point

Here, visually, the breaking point seems to be 15300, the point at which roles are swapped. Mathematically, Howard Lee proposed a method using the Chow Test to detect the breaking point. [3]

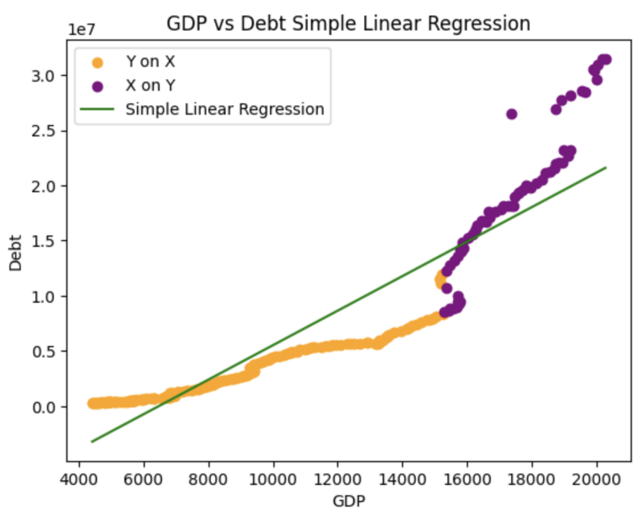
We have $\frac{\frac{e^Te - e_1^Te_1 - e_2^Te_2}{p}}{\frac{e_1^Te_1 + e_2^Te_2}{n+m-2p}} \sim F_{m,n-p}$, where n is number of observations in first dataset, and m in second dataset, and p is number of parameters. e is error of regression on total dataset, e_1 is error of regression on first dataset, and e_2 is error of regression on second dataset. Using this, we have implemented a Quadratic model, and find the value of x such that the p-value is lowest. In this case, we get x = 9404.494 and p-value as 1.11 x 10^{-16}.

6 Visual Breaking Point

6.1 Simple Linear Regression

First, let us treat them as linearly related. So, we can draw a simple linear regression for the model, all together.

With that, we get the line as y = 1561.373428004712x -10088309.55260129.

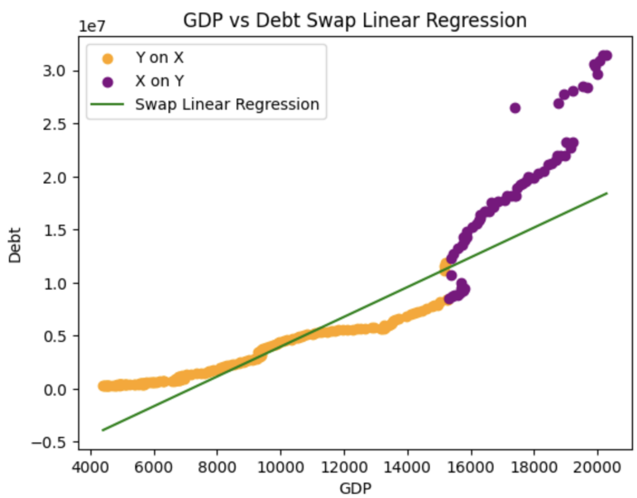


6.2 SWAP Linear Regression

We can implement SWAP Regression on the linear model with degree 1.

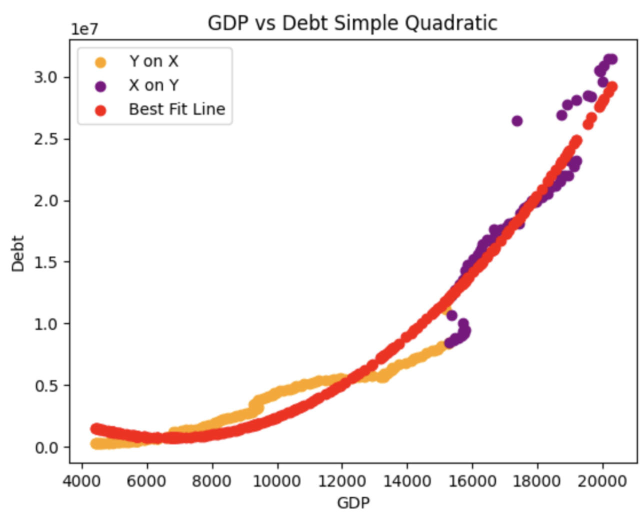
We use $L(a, b, c) = \Sigma_i(Y_{1i} - mX_{1i} - c)^2 + \Sigma_i(X_{0i} - \frac{Y_{0i} - c}{m})^2$ as the loss function, and minimize it with respect to m and c, using Gradient Descent.

With that, we get the line as y = 1404.4404395648457x - 10088309.496300373.



6.3 Simple Quadratic Regression

If we try to get this in a Quadratic Model, we will get the quadratic as y = $0.1542209902091852x^2$ - $2066.5291468890855x$ + 7658628.78216592.

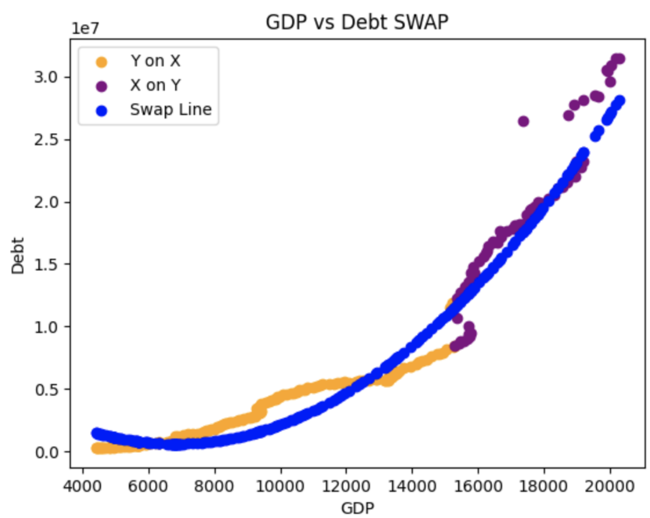


6.4 SWAP Regression

$L(a,b,c) = \Sigma_i(Y_{1i} - aX_{1i}^2 - bX_{1i} - c)^2 + \Sigma_i(X_{0i} + \frac{b}{2a} - \frac{\sqrt{b^2+4ac-4aY_{0i}}}{2a})^2$

The inverse of the Quadratic over here is taking as such because we know the

nature of the function. We can reverse the sign if the data set is shifted in another direction.
We can optimize this using Gradient Descent. However, the problem arises due to the square root. Over here, we can use the trick of only considering the points where the square root is defined. For considering the final gradient, we can average out the sum with the total number of points which were defined.
We get the final quadratic as
y = 0.15165658015046404x^2 - 2066.5290692327685x + 7658628.78216592

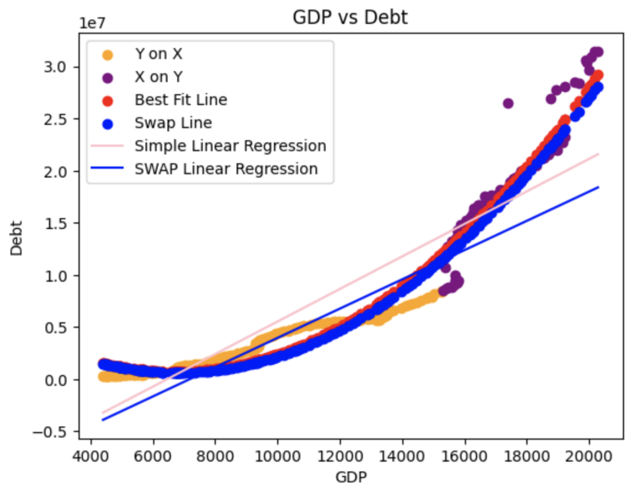


6.5 Error Analysis

The Gradient of the error, i.e. $(\frac{\partial L}{\partial a})^2 + (\frac{\partial L}{\partial b})^2 + (\frac{\partial L}{\partial c})^2$ is decreasing, and appears to tend to 0, as iterations increases.

6.6 Comparison and Analysis

Like in the paper, we have defined the error as
$L(a, b, c) = \Sigma_i(Y_{1i} - aX_{1i}^2 - bX_{1i} - c)^2 + \Sigma_i(X_{0i} + \frac{b}{2a} - \frac{\sqrt{b^2+4ac-4aY_{0i}}}{2a})^2$



Model	Error
Simple Linear Regression	5009776144566.557
SWAP Linear Regression	3410938979942.837
Simple Quadratic Regression	1258791879684.8152
SWAP Regression	1202759267483.3918

7 Chows Test Breaking Point

7.1 Simple Linear Regression

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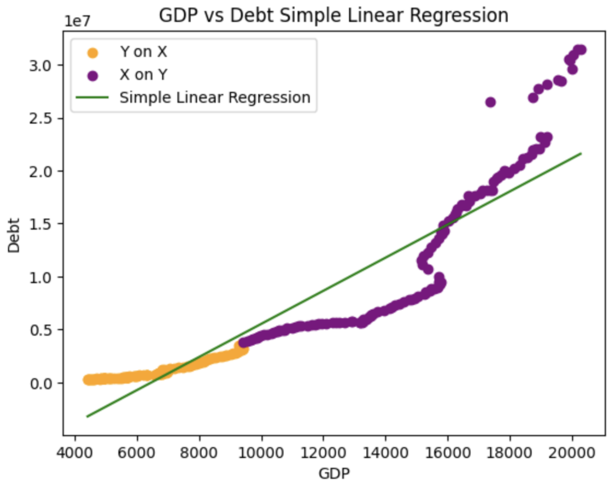
With that, we get the line as y = 1561.373428004712x -10088309.55260129.



7.2 SWAP Linear Regression

We can implement SWAP Regression on the linear model with degree 1.
We use $L(a,b,c) = \Sigma_i(Y_{1i} - mX_{1i} - c)^2 + \Sigma_i(X_{0i} - \frac{Y_{0i}-c}{m})^2$ as the loss function, and minimize it with respect to m and c, using Gradient Descent.
With that, we get the line as y = 1636.2686725322055x - 10088309.519003594.

7.3 Simple Quadratic Regression

If we try to get this in a Quadratic Model, we will get the quadratic as
y = 0.1542209902091852x^2 - 2066.5291468890855x + 7658628.78216592.

7.4 SWAP Regression

$$L(a,b,c) = \Sigma_i(Y_{1i} - aX_{1i}^2 - bX_{1i} - c)^2 + \Sigma_i(X_{0i} + \frac{b}{2a} - \frac{\sqrt{b^2+4ac-4aY_{0i}}}{2a})^2$$

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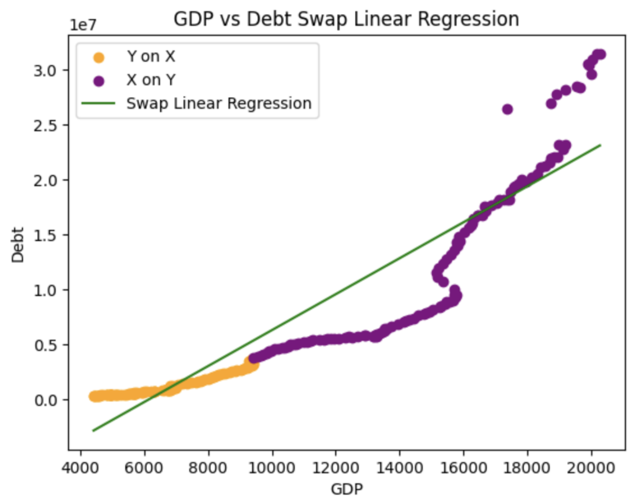
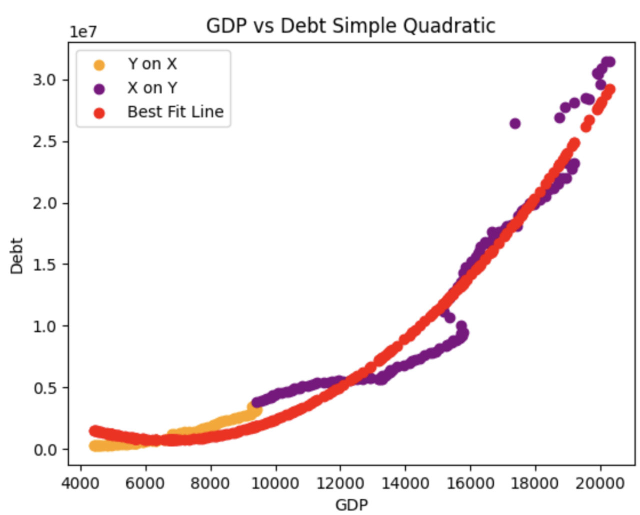
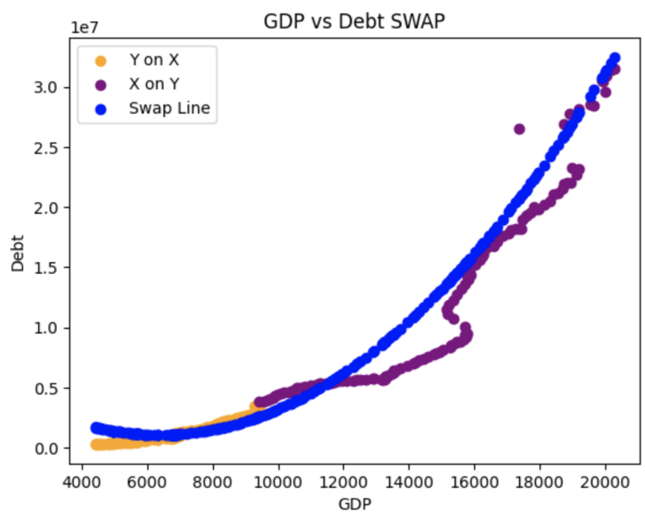

GDP vs Debt Swap Linear Regression
1e7
Y on X
X on Y
Swap Linear Regression
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Debt
4000
6000
8000
10000
12000
14000
16000
18000
20000
GDP


GDP vs Debt Simple Quadratic
1e7
Y on X
X on Y
Best Fit Line
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Debt
4000
6000
8000
10000
12000
14000
16000
18000
20000
GDP

to the square root. Over here, we can use the trick of only considering the points where the square root is defined. For considering the final gradient, we can average out the sum with the total number of points which were defined. We get the final quadratic as
y = 0.16217313566174235x^2 - 2066.529178646319x + 7658628.78216592



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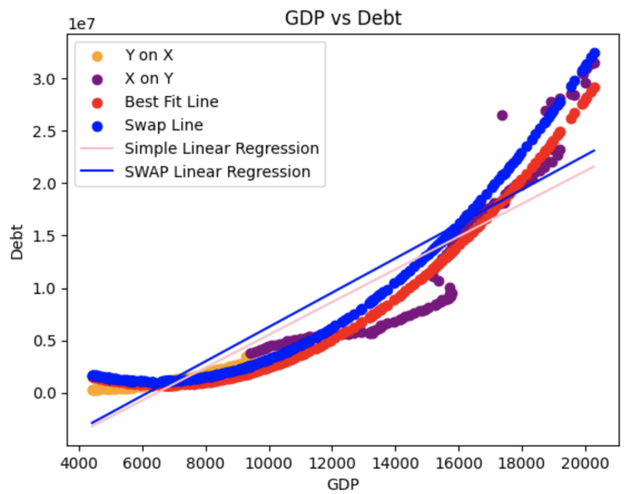
The Gradient of the error, i.e. $(\frac{\partial L}{\partial a})^2 + (\frac{\partial L}{\partial b})^2 + (\frac{\partial L}{\partial c})^2$ is decreasing, and appears to tend to 0, as iterations increases.

7.6 Comparison and Analysis

Like in the paper, we have defined the error as
$L(a, b, c) = \Sigma_i (Y_{1i} - aX_{1i}^2 - bX_{1i} - c)^2 + \Sigma_i (X_{0i} + \frac{b}{2a} - \frac{\sqrt{b^2+4ac-4aY_{0i}}}{2a})^2$ for Quadratic Case
and
$L(a, b, c) = \Sigma_i (Y_{1i} - mX_{1i} - c)^2 + \Sigma_i (X_{0i} - \frac{Y_{0i}-c}{m})^2$ for Linear Case.



Model	Error
Simple Linear Regression	1377304220480.1267
SWAP Linear Regression	1245587741204.4058
Simple Quadratic Regression	299920259363.1452
SWAP Regression	224786657072.94373

8 Final Analysis

Here, we can see, in this type of data, we can see that SWAP is working better than the other Linear Models for this Bi Directional Causal Dataset.

References

[1] M. Chow, B. Li, and J. Q. Xue, "On regression for samples with alternating predictors and its application to psychrometric charts," *Statistica Sinica*, vol. 25, no. 3, pp. 1045–1064, 2015.

[2] G. De Vita, E. Trachanas, and Y. Luo, "Revisiting the bi-directional causality between debt and growth: Evidence from linear and nonlinear tests," *Journal of International Money and Finance*, vol. 83, pp. 55–74, 2018.

[3] H. Lee, "Using the chow test to analyze regression discontinuities," *Tutorials in Quantitative Methods for Psychology*, vol. 4, 09 2008.