Robust Quantum Optimal Control

Thomas Propson*
Department of Physics, University of Chicago, Chicago, Illinois 60637, USA
(Dated: July 13, 2020)

This is a paper about robust quantum optimal control.

Introduction

[Existing work] The leading model of universal quantum computation is gate-based. There are analytic techniques to construct gates [1] [2] [3] [4]. Most methods focus on optimizing a few aspects of the gate, but not all experimentally relevant aspects. There is a growing literature on numerical techniques to construct gates [5] [6]. These methods formulate the quantum optimal control problem as unconstrained. They typically rely on zeroth-order or first-order optimizers. They are not sofisticated enough to handle all of the relevant constraints simultaneously. Although some analytic techniques exist to design pulses robust to decoherence, no numerical techniques have been presented to design pulses robust to decoherence (as far as the author is aware).

[This work] We employ the trajectory optimization literature to formulate the quantum optimal control problem as a constrained optimization problem. We study the quantum optimal control problem on the fluxonium. We outline experimentally realistic constraints and map them to the trajectory optimization framework. For the device we study we achieve a 2x increase in T_1 times. We present two methods for achieving robustness to system parameter deviations, and compare to existing dynamic decoupling methods. We find that our methods beat dynamic decoupling and mitigate dephasing by order X.

[Outline] First we formulate the quantum optimal control problem in the trajectory optimization framework. Then, we introduce the dynamics of the fluxonium device and outline experimental considerations relevant to gate construction. Next we outline a method for making the optimization T_1 aware. Finally, we present some methods for engineering robustness to decoherence and compare them to existing techniques.

* tcpropson@uchicago.edu

QOC + AL-iLQR

[QOC Problem Statement] I have some initial configuration and I want to reach some target configuration, either single- or multi-state transfer, subject to the dynamics Common numerical techniques include approximating the analytic unitary propagator solution, or employing explicit/implicit Runge-Kutta methods of the form The interesting part is that your hamiltonian $H(t)$ has a time-dependent control parameter $u(t)$ that the experimentalist gets to control, e.g. flux threading a superconducting junction. Most often $a_m(u) = u$ but any arbitrary dependence is allowed. The technique of adjusting $|\psi_N\rangle$ based on $\mathbf{u}(t)$ is the goal of the quantum optimal control optimizer. This domain is know as sensitivity analysis. GRAPE does with In general they add more cost functions than target fidelity.

[AL-iLQR Problem Statement] Trajectory optimization gives us gaurantees about our updates via Ricatti recursion and allows us to put constraints on our cost functions.

$$\min_{U:N-1\times m} \mathcal{L}_N(X_N, \lambda_N, \mu_N) + \sum_{k=1}^{N-1} \mathcal{L}_k(X_k, U_k, \lambda_k, \mu_k) \tag{1}$$

$$\mathcal{L}_k(X_k, U_k, \lambda_k, \mu_k) = (\lambda_k + \frac{1}{2} I_{\mu_k} c_k(X_k, U_k))^T c_k(X_K, U_K) \qquad +(X_k \tag{2}$$

$$\tag{3}$$

The important point is that there is an update step (e.g. 17 from ALTRO paper) where we send $\lambda \to \infty$ and get all of the nice convergence properties. The weights are adjusted dynamically between iterations until all of our constraints are satisfied. The Markovian decision structure of the problem allows us to apply differentiable dynamic programming to garauntee that the update for each control U_k is opimal, as apposed to the greedy updates of first-order optimizers like the naive gradient descent.

Our Problem

[Fluxonium + Device Characterization] In the two-level approximation we have

$$H/h = \omega_q \frac{\sigma_z}{2} + A(\Phi_{ext}) \frac{\sigma_x}{2} \tag{4}$$

$$\tag{5}$$

This approximation is good up to the avoided crossing at 0.35 Φ_0. We get A by converting via $\langle g| \hat{\phi} |e\rangle$.

[Constraints] We want constraints on our pulses. We want pulses start and end at zero for concatenation. We want pulses to have zero net flux to mitigate hysteresis in flux bias lines. We want the amplitude to be constrained $\delta\Phi_{\text{ext}} \sim 0.06\Phi_0$ so the two-level approximation stays valid. We want our state to obey normalization conditions, mitigating numerical error in simulaiton.

We test on the basis gates $X/2, Y/2, Z/2$. Universal up to arbitrary Z rotation.

Robustness to T_1-type Noise

Robustness to T_ϕ-type Noise

[1] Z. Huang, P. S. Mundada, A. Gyenis, D. I. Schuster, A. A. Houck, and J. Koch, Engineering dynamical sweet spots to protect qubits from $1/f$ noise, arXiv preprint arXiv:2004.12458 (2020).

[2] J. T. Merrill and K. R. Brown, Progress in compensating pulse sequences for quantum computation, Quantum Information and Computation for Chemistry , 241 (2014).

[3] Z. Han, Y. Dong, B. Liu, X. Yang, S. Song, L. Qiu, D. Li, J. Chu, W. Zheng, J. Xu, *et al.*, Experimental realization of universal time-optimal non-abelian geometric gates, arXiv preprint arXiv:2004.10364 (2020).

[4] Z. Wang, C. Wu, X.-L. Feng, L. C. Kwek, C. Lai, C. H. Oh, and V. Vedral, Nonadiabatic geometric quantum computation, Physical Review A **76**, 044303 (2007).

[5] N. Leung, M. Abdelhafez, J. Koch, and D. Schuster, Speedup for quantum optimal control from automatic differentiation based on graphics processing units, Physical Review A **95**, 042318 (2017).

[6] M. Abdelhafez, D. I. Schuster, and J. Koch, Gradient-based optimal control of open quantum systems using quantum trajectories and automatic differentiation, Physical Review A **99**, 052327 (2019).