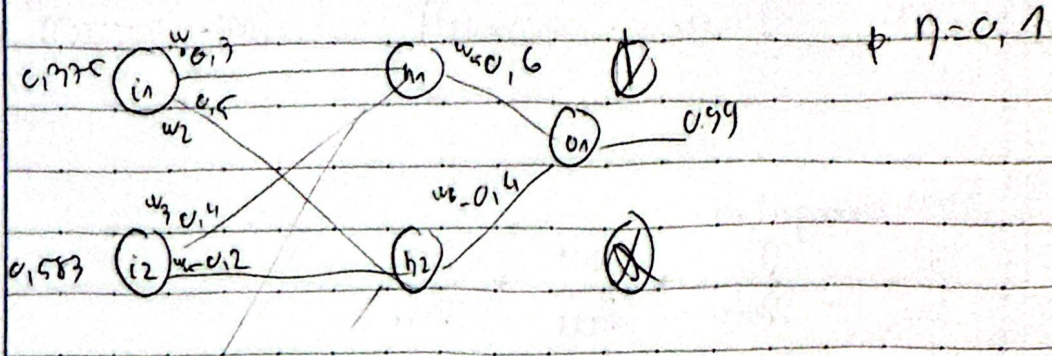




Question 10



$$b_1 = [0,1, -0,1]$$

$$b_2 = 0,2$$

Forward pass

$$\begin{aligned} \text{net}_{h_1} &= w_{11} \cdot i_1 + w_{12} \cdot i_2 + b_{h_1} = 0,375 \cdot 0,3 + 0,4 \cdot 0,583 + 0,1 = 0,4457 \\ &= 0,3 \cdot 0,375 + 0,4 \cdot 0,583 + 0,1 = 0,4457 \end{aligned}$$

$$\text{out}_{h_1} = \frac{1}{1 + e^{-\text{net}_{h_1}}} = \frac{1}{1 + e^{-0,4457}} = 0,3904$$

$$\text{net}_{h_2} = w_{21} \cdot i_1 + w_{22} \cdot i_2 + b_{h_2} =$$

$$= 0,5 \cdot 0,375 + 0,2 \cdot 0,583 + 0,1 = -0,0291$$

$$\text{out}_{h_2} = \frac{1}{1 + e^{-\text{net}_{h_2}}} = \frac{1}{1 + e^{-0,0291}} = 0,5073$$

Output o_1

$$\text{net}_{o_1} = w_{31} \cdot \text{out}_{h_1} + w_{32} \cdot \text{out}_{h_2} + b_{o_1}$$

$$\begin{aligned} \text{net}_{o_1} &= w_{31} \cdot \text{out}_{h_1} + w_{32} \cdot \text{out}_{h_2} + b_{o_1} \\ &= 0,6 \cdot 0,3904 + 0,4 \cdot 0,5073 + 0,2 = 0,2313 \end{aligned}$$

$$\text{out}_{o_1} = \frac{1}{1 + e^{-\text{net}_{o_1}}} = \frac{1}{1 + e^{-0,2313}} = 0,5576$$

Calculating the Error

$$E = \frac{1}{2} (\text{target} - \text{output})^2 = \frac{1}{2} (0.99 - 0.5571)^2$$

Backpropagation

$$\frac{\partial E}{\partial w_5} = \frac{\partial E}{\partial \text{out}} * \frac{\partial \text{out}}{\partial \text{net}} * \frac{\partial \text{net}}{\partial w_5}$$

$$\frac{\partial E}{\partial \text{out}} = -(\text{target} - \text{out}) = -(0.99 - 0.5571) = -0.4329$$

$$\frac{\partial \text{out}}{\partial \text{net}} = \text{out} (1 - \text{out}) = 0.5576 (1 - 0.5576) = 0.2467$$

$$\frac{\partial \text{net}}{\partial w_5} = \text{out to out}_{n_1} = 0.3904$$

$$\frac{\partial E}{\partial w_5} = -0.4329 * 0.2467 * 0.3904 = -0.0416$$

$$w_5' = w_5 - \eta \frac{\partial E}{\partial w_5} = 0.6 - 0.01 * (-0.0416) = 0.6004$$

$$\frac{\partial E}{\partial w_6} = \frac{\partial E}{\partial \text{out}} * \frac{\partial \text{out}}{\partial \text{net}} * \frac{\partial \text{net}}{\partial w_6}$$

$$= -(\text{target} - \text{out}) * \text{out}(1 - \text{out}) * \text{out}_{n_2}$$

$$= -0.4329 * 0.2467 * 0.5073 = -0.0541$$

$$w_6' = w_6 - \eta \frac{\partial E}{\partial w_6} = -0.4 - 0.01 * (-0.0541) = -0.3994$$

$$\frac{\partial E}{\partial w_1} = \frac{\partial E}{\partial out_h} \cdot \frac{\partial out_h}{\partial net_h} \cdot \frac{\partial net_h}{\partial w_1}$$

$$\frac{\partial E}{\partial out_h} = -(target - out_h) =$$

Hidden layer

$$\frac{\partial E}{\partial w_1} = \frac{\partial E}{\partial out_h} \cdot \frac{\partial out_h}{\partial net_h} \cdot \frac{\partial net_h}{\partial w_1}$$

w_1 before update

$$\frac{\partial E}{\partial out_h} = \frac{\partial E}{\partial net_o} \cdot \frac{\partial net_o}{\partial out_h} = \frac{\partial E}{\partial out} \cdot \frac{\partial out}{\partial net_o} \cdot \frac{\partial net_o}{\partial out_h}$$

$$= -0,1321 \times 0,2467 \times 0,6 = -0,0640$$

$$\frac{\partial out_h}{\partial net_h} = out_h (1 - out_h) = 0,3904 (1 - 0,3904) = 0,2379$$

$$\frac{\partial net_h}{\partial w_1} = 1 = 0,375$$

$$\Rightarrow \frac{\partial E}{\partial w_1} = -0,0640 \times 0,2379 \times 0,375 = -0,0057$$

$$w_1' = w_1 - \eta \frac{\partial E}{\partial w_1} = 0,3 - 0,01 (-0,0057) = 0,300057$$

$$\frac{\partial E}{\partial w_2} = \frac{\partial E}{\partial out_{h2}} \cdot \frac{\partial out_{h2}}{\partial net_{h2}} \cdot \frac{\partial net_{h2}}{\partial w_2}$$

$$\frac{\partial E}{\partial out_{h2}} = \frac{\partial E}{\partial net_o} \cdot \frac{\partial net_o}{\partial out_{h2}} = \frac{\partial E}{\partial out_o} \cdot \frac{\partial out_o}{\partial net_o} \cdot \frac{\partial net_o}{\partial out_{h2}}$$

$$= -0,4324 \times 0,2467 \times (-0,4) = 0,0428$$

$$\frac{\partial out_{h2}}{\partial net_{h2}} = out_{h2}(1 - out_{h2}) = 0,5073(1 - 0,5073) = 0,2499$$

$$\frac{\partial net_{h2}}{\partial w_2} = i_1 = 0,375$$

$$\Rightarrow \frac{\partial E}{\partial w_2} = 0,0428 \times 0,2499 \times 0,375 = 0,004$$

$$w_2' = w_2 - \eta \frac{\partial E}{\partial w_2} = 0,5 - 0,01 \times 0,004 = 0,49996$$

$$w_3' = \frac{\partial E}{\partial w_3} = \frac{\partial E}{\partial out_o} \cdot \frac{\partial out_o}{\partial net_o} \cdot \frac{\partial net_o}{\partial out_{h1}} \cdot \frac{\partial out_{h1}}{\partial net_{h1}} \cdot \frac{\partial net_{h1}}{\partial w_3} \rightarrow i_2$$

$$= -0,4324 \times 0,2467 \times 0,6 \times 0,2379 \times 0,583$$

$$= -0,0089$$

$$w_3' = w_3 - \eta \frac{\partial E}{\partial w_3} = 0,4 - 0,01 \times (-0,0089) = 0,400089$$

$$\frac{\partial E}{\partial w_4} = \frac{\partial E}{\partial out_o} \cdot \frac{\partial out_o}{\partial net_o} \cdot \frac{\partial out_o}{\partial net_{h2}} \cdot \frac{\partial net_{h2}}{\partial out_{h2}} \cdot \frac{\partial out_{h2}}{\partial net_{h2}} \cdot \frac{\partial net_{h2}}{\partial w_4} \rightarrow i_2$$

$$= -0,4324 \times 0,2467 \times (-0,4) \times 0,2499 \times 0,583$$

$$= 0,0062$$

$$w_4' = w_4 - \eta \frac{\partial E}{\partial w_4} = 0,2 - 0,01 \times (0,0062) = 0,199938$$