

Question 5 : For the missing education (ED=3), replace by the mean of E.  
In multiple linear regression, we model the relationship:  $y = 15$

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon$$

where:  $y$ : creditScore

$X_1$ : Age

$X_2$ : Education

$\beta_0$ : Intercept

$\beta_1, \beta_2$ : coefficients for Age and Education

$\varepsilon$ : error term.

Find  $\beta = [\beta_0, \beta_1, \beta_2]$  using the normal solution:

$$\beta = (X^T X)^{-1} X^T y$$

$$y = \begin{bmatrix} 720 \\ 650 \\ 750 \\ 600 \\ 780 \\ 630 \\ 710 \\ 640 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 35 & 16 \\ 1 & 28 & 14 \\ 1 & 45 & 15 \\ 1 & 31 & 12 \\ 1 & 52 & 18 \\ 1 & 29 & 14 \\ 1 & 42 & 16 \\ 1 & 33 & 12 \end{bmatrix}$$

$$X^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 35 & 28 & 45 & 31 & 52 & 29 & 42 & 33 \\ 16 & 14 & 15 & 12 & 18 & 14 & 16 & 12 \end{bmatrix}$$



$$X^T X = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 35 & 28 & 45 & 31 & 52 & 29 & 42 & 33 \\ 16 & 14 & 15 & 12 & 18 & 14 & 16 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 35 & 16 \\ 1 & 28 & 14 \\ 1 & 45 & 15 \\ 1 & 31 & 12 \\ 1 & 52 & 18 \\ 1 & 29 & 14 \\ 1 & 42 & 16 \\ 1 & 33 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 295 & 117 \\ 295 & 11393 & 4409 \\ 117 & 4409 & 1741 \end{bmatrix}$$

$$(X^T X)^{-1} = \frac{1}{\det(X^T X)} \times \text{adj}(X^T X)$$

$$\det(X^T X) = 51424$$

$$\text{co-factor matrix} = \begin{bmatrix} 395932 & 2258 & -32326 \\ 2258 & 239 & -757 \\ -32326 & -757 & 4119 \end{bmatrix}$$

$$\text{adj}(X^T X) = \begin{bmatrix} 395932 & 2258 & -32326 \\ 2258 & 239 & -757 \\ -32326 & -757 & 4119 \end{bmatrix}$$





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No.

$$(X^T X)^{-1} = \frac{1}{51224} \begin{bmatrix} 395932 & 2258 & -32326 \\ 2258 & 239 & -757 \\ -32326 & -757 & 4119 \end{bmatrix}$$

$$= \begin{bmatrix} 7,699 & 0,044 & -0,629 \\ 0,044 & 0,004 & -0,015 \\ -0,629 & -0,015 & 0,080 \end{bmatrix}$$

$$X^T Y = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 35 & 28 & 45 & 31 & 52 & 29 & 42 & 33 \\ 16 & 14 & 15 & 12 & 18 & 14 & 16 & 12 \end{bmatrix} \begin{bmatrix} 720 \\ 650 \\ 750 \\ 600 \\ 780 \\ 630 \\ 710 \\ 640 \end{bmatrix}$$

$$= \begin{bmatrix} 5180 \\ 205520 \\ 80970 \end{bmatrix}$$

$$\beta = (X^T X)^{-1} X^T Y = \begin{bmatrix} 7,699 & 0,044 & -0,629 \\ 0,044 & 0,004 & -0,015 \\ -0,629 & -0,015 & 0,080 \end{bmatrix} \begin{bmatrix} 5180 \\ 205520 \\ 80970 \end{bmatrix}$$

$$\beta = \begin{bmatrix} 303,27 \\ -157,35 \\ -52,12 \end{bmatrix} \begin{bmatrix} 303,27 \\ 2,542 \\ 21,92 \end{bmatrix}$$



Interpret the coefficients

•  $\beta_0 = 337.21$  is the intercept. If Age and Education are 0, then the Credit Score =  $\beta_0$

$\beta_1 = 2.512$  : For each year of Age, If holding Education as constant, the credit score increase by ~~2.09~~  $2.512$  points.  
 $\Rightarrow$  The higher age of individual, the higher Credit Score they have.

$\beta_2 = -0.58$  : For each year of Education, If holding Age as constant, the Credit Score decrease by 0.58 points.

$\beta_2 = 21.32$  : For each year of Education, If holding Age as constant, the Credit Score increase by 21.32 points.  
 $\Rightarrow$  The higher Education year of each individual, the higher Credit Score they have.