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A Trajectory Planning and Fuzzy Control for Autonomous Intelligent Parking System

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Abstract

This paper proposed a two-section trajectory planning algorithm. In this trajectory planning, sigmoid function is adopted to fit two tangent arcs to meet limited parking spaces by reducing the radius of turning. Then the transverse preview model is established and the path tracking errors including distance error and angle error are estimated. The weight coefficient is considered to distribute the impact factor of traverse distance error or traverse angle error in the total error. The fuzzy controller is designed to track the two-section trajectory in autonomous intelligent parking system. The fuzzy controller is developed due to its real-time and robustness in the parking process. Traverse errors and its first-order derivative are selected as input variables and the outer wheel steering angle is selected as the output variable in fuzzy controller. They are also divided into seven fuzzy sets. Finally, forty rules are decided to achieve effective trajectory tracking. The detailed description of the proposed trajectory planning is demonstrated. The design aspects of Fuzzy Logic Controller and kinematic/dynamic theories in intelligent parking system are investigated in details. A combination of PreScan and Matlab Simulink is used to develop a numerical simulation model in order to verify the effectiveness of the proposed trajectory planning algorithm and Fuzzy Logic Controller in parking system. A superior performance is demonstrated and concluded for the proposed autonomous intelligent parking system.

Keywords

two-section trajectory planning, Sigmoid function, function fitting, preview fuzzy control, PreScan, MATLAB

Introduction

Intelligent Transportation System (ITS) has aroused great research interest in the past few decades. Multiple research tracks have been identified in this area, among which autonomous parking has attracted most attention. The main reason is that parking is a very difficult task even for a skilled driver because of limited space, incoming vehicles,

and fixed and moving obstacles such as pedestrians. Therefore, the development of parking assistant systems and autonomous parking systems have become important issues. Furthermore, an autonomous vehicle must perform numerous tasks to park the vehicle in a narrow space safely, including precise environment detection and parking maneuvers. Generally, autonomous parking systems consist of three steps: detection of the free parking space, the trajectory planning and the trajectory tracking control on the steering angle and the velocity in accelerating or braking. The detection of the free parking space involves sensing and calculating the surrounding environment before the path generation to park the vehicle. Then an ideal trajectory is planned and the control algorithm is designed to perform path tracking and automatic parking [1].

Two significant issues in automatic parking are trajectory generation and path tracking. The increasingly control precise demands an ideal path as accuracy as possible. Paromtchik [2] generated regression algorithm to fit sine curve to calculate the ideal path, however, it requires a large parking space. Patrik Z. [14] proposed a multi working condition method to generate the ideal path for parallel, bay and angle parking under a tiny space on the basis of tree extension guides and local optimization. Combining with the priori-knowledge, the algorithm demonstrated the robustness and wide range of application. Z. Lv [15] composed the straight line and arc curve to generate the parking trajectory and determined the range of initial position, in which vehicle could parking in appropriate location favoringly. Li [3] proposed the method of multi-constraint optimization to fit the inverse tangent curve to design the ideal path. Research in [16, 17, 18, 19] emerged the fitting optimization methods in parking path programming, in general, beta-spline, Bezier curve and fifth-order polynomial curve were usually adopted to fit the arc path, which would face off the path as well as make steering smooth. On the other hand, literature review on controller designing was demonstrated. Zhao and Chang [4-5] applied the parking experience to design a fuzzy controller to control the parking process. Wu [6] developed the particle swarm method combining with the RBF neural network to realize parking path planning automatically. K. Demirli [20] presented a neuro-fuzzy algorithm, in details, the vehicle

obtained the information of local obstacles, then the parameters of neural network had been trained by fitting fifth-order polynomial, which aimed to optimize the membership function of fuzzy controller. However, it should be trained repeatedly and the controller was open-loop methods. B. Li [21] established the kinematic differential equation of parking vehicle. Considering the collision constraints, they decomposed the time and space to control the parking process. Interior-point method was implemented to optimize the controller, which realized the multiple operation for parking in, especially for narrow parking lot. Aforementioned researches require a large parking space, meanwhile, the computation cost is high in parking tracking. In this paper, surrounding spatial information about parking space has already been acquired. The research is limited to trajectory planning and parking tracking.

The purpose of this paper is to provide a method for an autonomous parking system that includes plan panning and plan tracking when the parking space is limited. It can consider a minimum distance required to avoid collision with obstacles and space limit as well as other kinematic constraints in the path planning. On the other hand, lateral motion is controlled to implement plan tracking accurately to desired position in the road.

Path Planning Algorithm

An autonomous intelligent parking system consists of four steps [7], that are environmental perception, path planning, path tracking, and human-machine-interface (HMI) respectively. This parking system is shown as Figure 1. Sensors of the parking system will perceive the ambient environment after the switch of parking system is turn on. Once a proper space [8] has been found, HMI will prompt the driver and conduct the path planning automatically. If an ideal path exists, the system will remind the driver who can undertake parking action. Otherwise the driver should give up the current location and detect new parking location. Then the steering input and the velocity of the vehicle is controlled, the parking action is implemented autonomously.

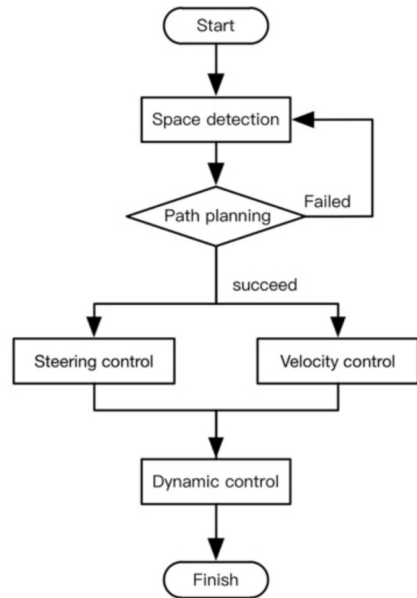


Figure 1. The process of automatic parking

Kinematic Model

This part will reveal the principle of the parking process. Firstly the kinematic model of the vehicle based on Ackerman linkage is put up as Figure 2 [9].

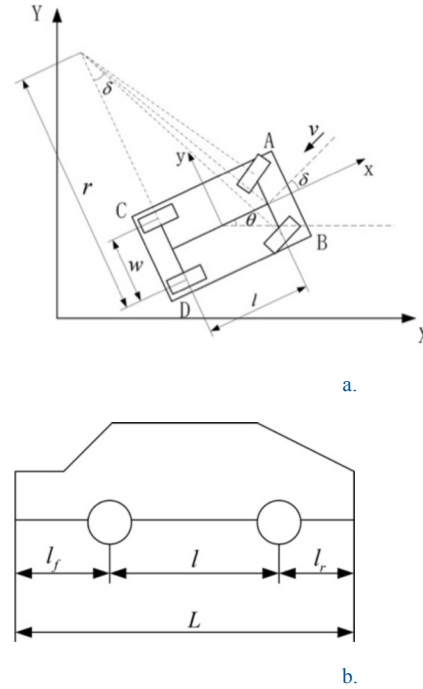


Figure 2. Kinematic model

Generally, the parking velocity is very small (about 5km/h), consequently the slip of the rear wheel nearly zero which means the velocity in y direction is zero. Therefore, the coordinates of the central point on rear axle can be expressed as:

$$y_r \cos \theta - x_r \sin \theta = 0 \quad (1)$$

The relationship between the central point on front axle and rear axle is written as:

$$\begin{aligned} x_r &= x_f - l \cos \theta \\ y_r &= y_f - l \sin \theta \end{aligned} \quad (2)$$

Derivation of Eq. (2) is yielded:

$$\begin{aligned} x_r &= x_f + \theta l \sin \theta \\ y_r &= y_f - \theta l \cos \theta \end{aligned} \quad (3)$$

Substituting Eq. (3) into Eq. (1):

$$x_f \sin \theta - y_f \cos \theta + \theta l = 0 \quad (4)$$

Setting δ as the equivalent steering angle of the central point on front axle in Ackerman linkage, the velocity of the front axle center can be expressed as follows:

$$\begin{aligned}x_f &= v \cos(\theta + \delta) \\y_f &= v \sin(\theta + \delta)\end{aligned}\quad (5)$$

Substituted Eq. (5) into Eq. (4), θ can be derived

$$\theta = \frac{v \sin \delta}{l} \quad (6)$$

Furthermore, substituting Eq. (5) and Eq. (6) into Eq. (3), the velocity of rear wheel can be obtained:

$$\begin{aligned}x_r &= v \cos \theta \cos \delta \\y_r &= v \sin \theta \cos \delta\end{aligned}\quad (7)$$

Integrating Eq. (7), the trajectory of rear wheel is derived:

$$\begin{aligned}x_r(t) &= l \cot \delta \sin\left(\frac{v \sin \delta}{l} t\right) \\y_r(t) &= -l \cot \delta \cos\left(\frac{v \sin \delta}{l} t\right) + l \cot \delta \\x_r^2 + (y_r - l \cot \delta)^2 &= (l \cot \delta)^2\end{aligned}\quad (8)$$

The trajectory of left rear wheel is further generated according geometric relationship in vehicle:

$$\begin{aligned}x_{rl}(t) &= (l \cot \delta - \frac{w}{2}) \sin\left(\frac{v \sin \delta}{l} t\right) \\y_{rl}(t) &= -(l \cot \delta - \frac{w}{2}) \cos\left(\frac{v \sin \delta}{l} t\right) + l \cot \delta \\x_{rl}^2 + (y_{rl} - l \cot \delta)^2 &= (l \cot \delta - \frac{w}{2})^2\end{aligned}\quad (9)$$

The trajectory of right rear wheel is written as:

$$\begin{aligned}x_{rr}(t) &= (l \cot \delta + \frac{w}{2}) \sin\left(\frac{v \sin \delta}{l} t\right) \\y_{rr}(t) &= -(l \cot \delta + \frac{w}{2}) \cos\left(\frac{v \sin \delta}{l} t\right) + l \cot \delta \\x_{rr}^2 + (y_{rr} - l \cot \delta)^2 &= (l \cot \delta + \frac{w}{2})^2\end{aligned}\quad (10)$$

Here, θ is the heading. l is the axle distance. w is the wheel distance. Eq. (8), Eq. (9), Eq. (10) provide the theoretical basis of two-section parking path.

Fitting of Two-Section Path

Two tangent arcs can be determined by the spaces detected. Two-section parking path could keep a low parking space by reducing the radius of turning. Figure 3 shows the principle of two-section parking path.

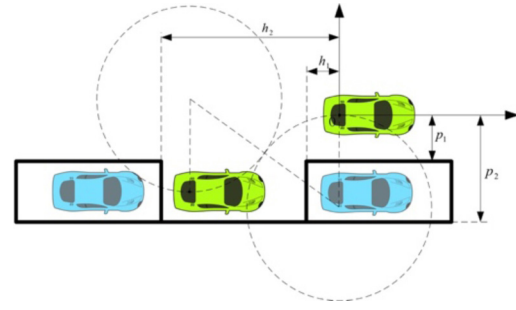


Figure 3. Two-section parking path

The relationship between radius of turning and steering angle in the light of principles of Ackerman linkage is shown as following equations.

$$r = \frac{l}{\tan \alpha_{out}}, \quad \tan \delta = \frac{l}{r - w/2} \quad (11)$$

α_{out} is the steering angle of front outer wheel. If the car parks with the minimum steering angle, heading will reach to the maximum at the tangent point of two-section arc path, that is

$$\theta_{max} = \arccos\left(1 - \frac{p_2 - \frac{B}{2} - \Delta}{2r_{min} - w}\right) \quad (12)$$

Δ is safety threshold. p_2 is the distance in Figure 3. The minimum radius of turning is expressed as r_{min} . B is the total width of vehicle. Especially, to realize the minimum turning, the vehicle cannot swerve immediately because of the risk of collision on the right side of the car to the corner of the garage. Hence, the vehicle should retreat a certain distance to start turning. The determination of this distance employs the following iteration logic.

```

S=0;
While  $x_B \geq -h_1 - \Delta$  &  $y_B \leq -p_1 + \Delta$ 
.....
S=S+0.1;
End

```

Here, S is the retreat distance, x_B and y_B are coordinates of point B referring to Figure 2. (a). Above iteration logic could ensure that the end location in the garage is as front as possible, which means the vehicle has more space to adjustment, simultaneously, the vehicle would not crash the garage.

The original two-section path is an ideal path that the car could not achieve for its mutation of the curvature along the trajectory. Curve fitting is an effective tool to work it. The trajectory in two-section parking path is similar to "S" which is usually fitted by Tangent function $y = a_1 \arctan(a_2 + a_3 x) + a_4$, fifth-order polynomial curve $y = a_1 x^5 + a_2 x^4 + a_3 x^3 + a_4 x^2 + a_5 x^1 + a_6$ and Sigmoid function $y = \frac{a_1}{1 + e^{a_2(x - a_3)}} + a_4$, and so on. Considering the real-time of calculating and the demand of path curvature, namely, initial curvature is zero and the curvature should be continuous, therefore, Sigmoid function is selected to fit the two-section parking path.

$$y = \frac{a_1}{1 + e^{a_2(x-a_3)}} + a_4 \quad (13)$$

In Eq. (13), there are four fitting parameters a_1 , a_2 , a_3 , a_4 , and the second-order derivative of the Sigmoid function is written as

$$y = \frac{2a_1a_2^2e^{-2a_2(a_3-x)}}{(1+e^{-a_2(a_3-x)})^3} - \frac{a_1a_2^2e^{-a_2(a_3-x)}}{(1+e^{-a_2(a_3-x)})^2} \quad (14)$$

In Eq. (14), the curvature is continuous and the value of $-a_2a_3$ determines that it tends to zero at $x = 0$ in terms of mathematic.

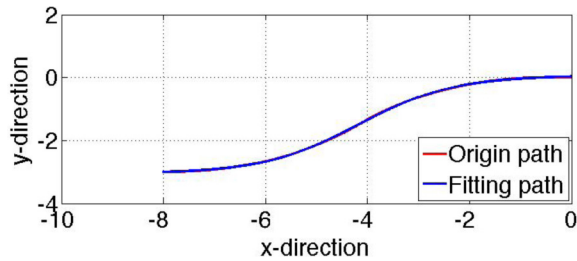
Three simulating cases are designed to verify the fitting performance as shown in Table 1, and the fitting parameters are shown in Table 2.

Table 1. Simulation conditions settings

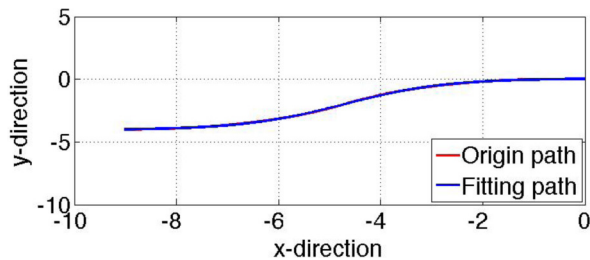
Distance (m)	Case 1	Case 2	Case 3
h_1	2	3	4
h_2	9	10	11
p_1	2	3	2.5
p_2	4.2	5.2	5

Table 2. Fitting parameters

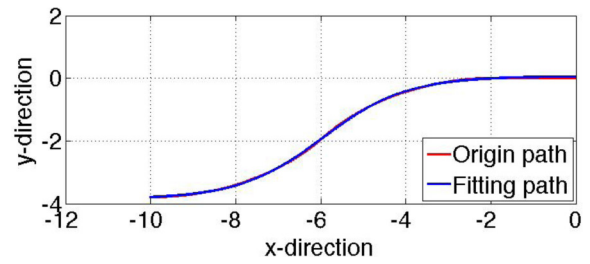
Fitting parameters	Case 1	Case 2	Case 3
a_1	-3.1220	-4.1199	-3.9174
a_2	1.0710	1.0047	1.0163
a_3	-4.1715	-4.7218	-5.9428
a_4	0.0655	0.0643	0.0615



a. Case 1



b. Case 2



c. Case 3

Figure 4. Fitting results

In order to inspect the performance of fitting, the coefficient of determination is defined and calculated.

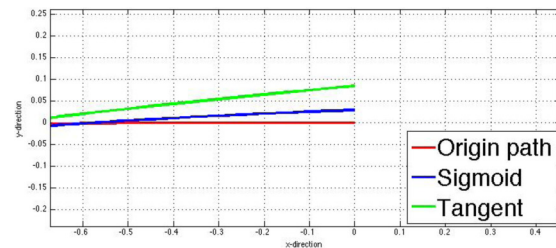
$$r^2 = \left(\frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}} \right)^2 \quad (15)$$

The coefficients of determination remain between 0 and 1, and the bigger the value is, the more similarities between two curves. The Table 3. illustrates the comparison among Tangent function, fifth-order polynomial and Sigmoid curves.

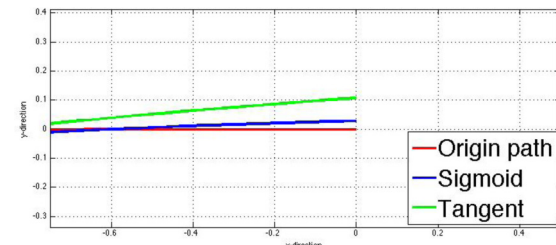
Table 3. Fitting performance

Curves	Case 1	Case 2	Case 3
Tangent	0.9998	0.9998	0.9996
Fifth-order polynomial	0.9995	0.9992	0.9987
Sigmoid	0.9999	0.9998	0.9998

It is evident that Sigmoid curve has a higher value of coefficient of determination. It worth noting that the slope of Tangent curve at initial position is very large as shown Figure 5., which is incongruent for initial condition obviously. To conclude, Sigmoid curve shows better performance in parking path smoothing and for its fewer fitting parameters, the cost of calculation can be ignored.

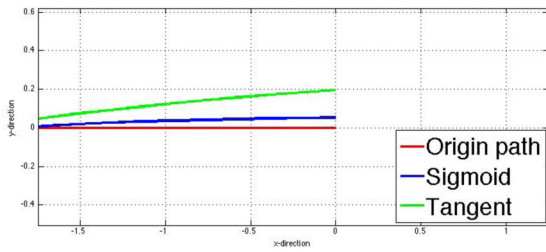


a. Case 1



b. Case 2

Figure 5. Comparison of Fitting curve



c. Case 3

Figure 5 (cont). Comparison of Fitting curve

Path Tracking Algorithm

Path tracking control is a core technology in unmanned intelligent vehicle. The common methods include PID control, fuzzy reasoning, fuzzy-PID, neural network or integrated control with various methods. No matter what methods being selected, path tracking model should be established at first.

Traverse Preview Model

Preview in vehicle control means setting up a driver model to achieve the closed-loop control. Distance-angle Bias Preview was proposed in Stanford Artificial Intelligence Laboratory [10]. CLA developed the Annular Preview methods [11]. Follow the Carrot mainly aimed to control the front wheel steering to eliminate the heading bias [12]. Pure Pursuit model [13] designed an arc where the vehicles could drive along with to destination, then the front wheel angle bias could be calculated. All these models are applicable to intelligent vehicle which runs ahead. However, automatic parking is a little bit different from such conditions, because of its low speed, small space, and negative velocity. All above preview models could not reach a quick and accurate calculation for errors under parking condition. This paper proposed the traverse preview model based on Distance-angle Bias Preview as shown in Figure 6.

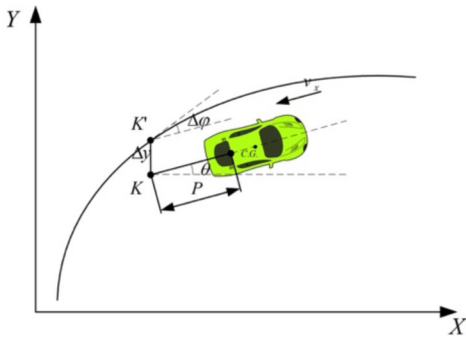


Figure 6. Traverse preview model

The ground Cartesian coordinate is designed based on the current position and heading after spaces detected seeing Figure 3. Once the coordinate in Figure 3 has been determined, the automatic parking process will implement under this coordinate. The bias Δy from K (a constant distance front of back direction) to K' (corresponding point to K on ideal path) called traverse distance error, and the bias $\Delta\phi$ of θ (heading of preview point or current position) and heading of K' called traverse angle error. Traverse distance error and traverse angle error collectively called traverse errors. A weight was set to distribute the impact factor of traverse distance error and traverse angle error. They are:

$$\begin{aligned} X_K &= X_r - P \cos \theta \\ \Delta y &= Y(X_K) - (Y_r - P \sin \theta) \\ \Delta \phi &= \arctan(Y(X_K)) - \theta \\ e &= (1-i)\Delta y + i\Delta \phi \end{aligned} \quad (16)$$

X_r and Y_r are the x-coordinate and y-coordinate of rear axle center respectively. X_K is the x- coordinate of preview point. The preview distance is defined as P . $Y(X)$ indicates the expression of fitting function. i is the weight coefficient, which is adopted as $i=0.5$ for the same important impact of traverse distance error and traverse angle error.

Fuzzy Control

Fuzzy control suits for nonlinear and coupling vehicle dynamic system for its premium character of real-time and robustness, furthermore, it takes advantages of the experience and knowledge to obtain a highly precise. Traverse errors and its first-order derivative are considered as inputs, and the outer wheel steering angle is considered as the output in this paper. The fuzzy control frame is shown as Figure 7.

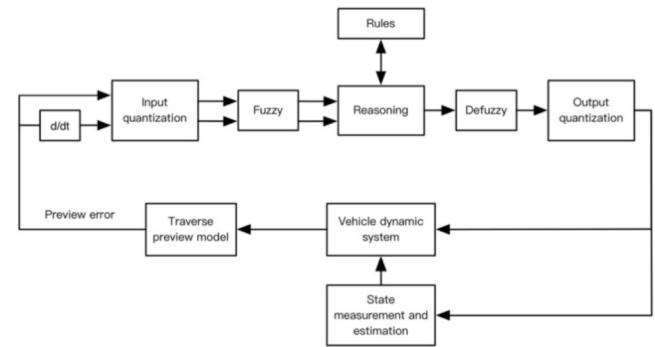
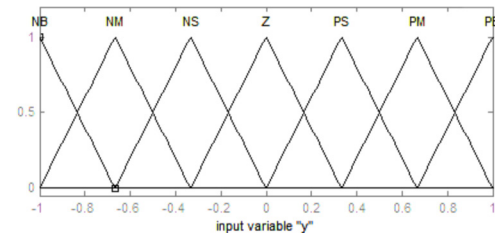
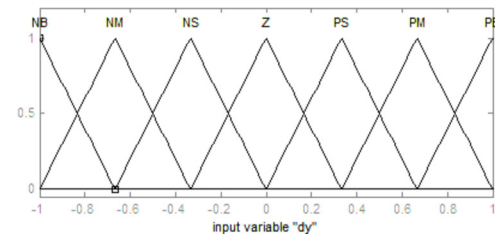


Figure 7. Preview closed-loop control

Error input, rate of error input and steering angle output is divided into seven fuzzy sets, respectively. The fuzzy domains are all set $[-1,1]$, and the membership functions are shown as Figure 8.

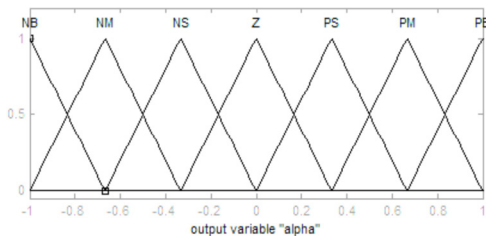


a. Traverse preview error



b. Rate of traverse preview error

Figure 8. Membership function



c. Outer wheel steering angle

Figure 8 (cont). Membership function

NB, NM, NS, Z, PS, PM, PB respectively means the value is “negative big”, “negative middle”, “negative small”, “zero”, “positive small”, “positive middle”, “positive big”. “if E and EC then U” rules are developed to generate 49 rules as Table 4.

Table 4. Fuzzy rules

	NB	NM	NS	Z	PS	PM	PB
NB	NB	NB	NM	NM	NM	NS	Z
NM	NB	NB	NM	NS	NS	Z	PS
NS	NM	NM	NS	NS	Z	PS	PS
Z	NM	NS	NS	Z	PS	PS	PM
PS	NS	NS	Z	PS	PS	PM	PM
PM	NS	Z	PS	PS	PM	PB	PB
PB	Z	PS	PM	PM	PM	PB	PB

Figure 9. gives the fuzzy control surface.

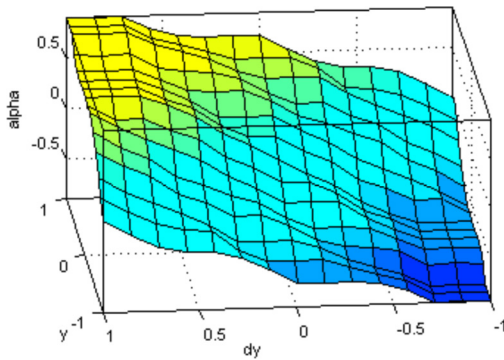


Figure 9. Fuzzy control surface

Simulations

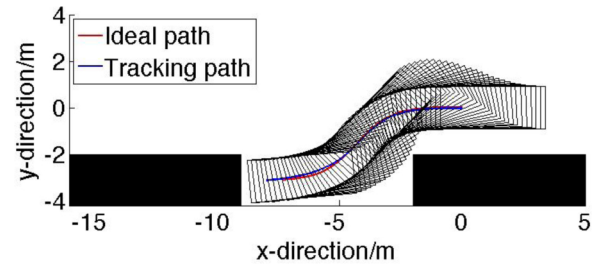
The shape parameters of a certain passenger car are shown in Table 5.

Table 5. The shape parameters of a certain passenger car

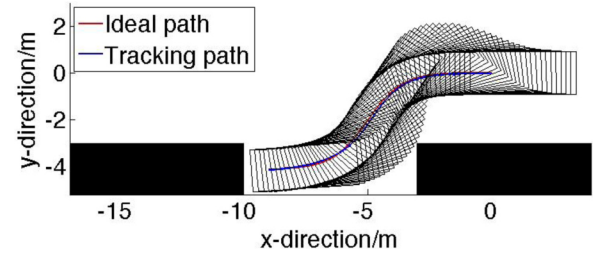
Name	Symbol	Value (m)
Header length	l_f	0.82
Tail length	l_r	0.88
Axle distance	l	2.58
Total length	L	4.28
Total width	B	1.82
Wheel distance	w	1.57

Matlab Simulation

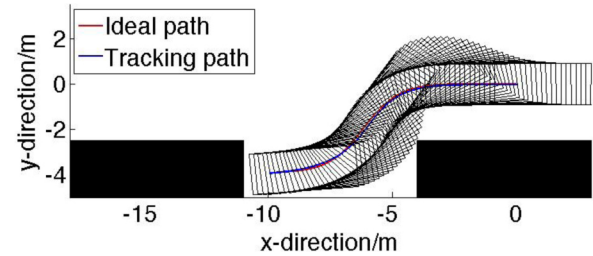
According to two-section parking path planning and preview fuzzy control, Matlab simulation model is established to verify the proposed algorithm. The results of previous three cases are shown as Figure 10.



a. Case 1



b. Case 2



c. Case 3

Figure 10. Tracking results

The results indicate that the preview fuzzy control could track the ideal path accurately.

Moreover, the change of steering angle meets the requirement of continuity. All these results illustrate that the preview fuzzy control is an engineering practical method for its accuracy and robustness.

PreScan simulation

PreScan is an active safety designing and environment with sensors modeling software. This paper establishes the real-world simulation taking use of ultrasonic sensors coming with the PreScan, then verifies the preview fuzzy control algorithm under real-time process. Figure 11. is the simulation scenario and Figure 12. is the frame of co-simulation of PreScan and Matlab.



Figure 11. PreScan scenarios

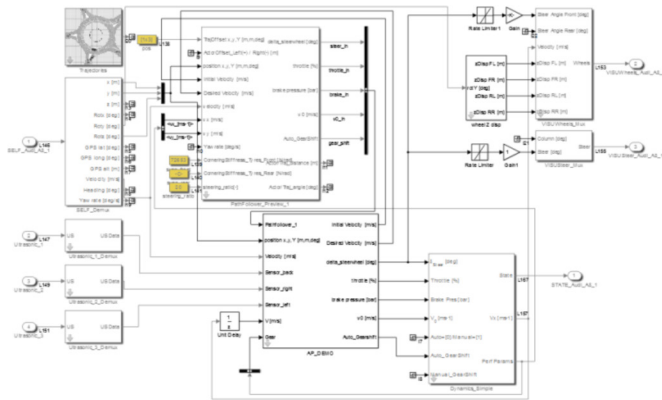


Figure 12. Co-simulation model of PreScan and Matlab

Figure 13. shows the real-time simulation process of case 2. It can be divided into 5 steps to park the vehicle.

- Vehicle runs ahead with automatic parking switch on, and simultaneously, the ultrasonic sensors is working to detect the parking spaces.
- Vehicle slows down after garage has been found, then ground Cartesian coordinate is set up and path planning is implementing at the same time. After all calculation finished, vehicle starts hanging in reverse gear and parking.
- Steering angle is calculated by preview fuzzy control at real-time, and the vehicle is parking automatically.
- Vehicle successfully parks into the garage; however, it is not at the center of the garage, then it needs to run ahead to locate a proper position.
- Parking finishes.
- GUI simulation illustrates the parking process and space information update from ultrasonic sensors. The black line is the trajectory of vehicle, and blue line is the contour line of the obstacles.

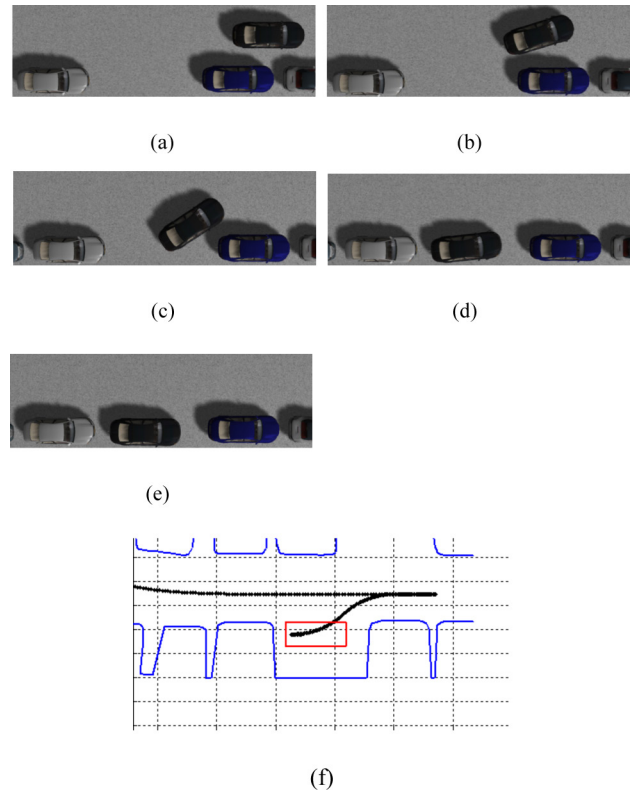


Figure 13. Real-time simulation process

Conclusions

This paper proposes a path planning fitting method which is based on two-section path and aims to solve the discontinuous steering of conventional path and small parking space. later on, the traverse preview model is established and fuzzy logic controller is design to track the ideal two-section path. Finally, the simulation test is implemented based on a combination of PreScan and Matlab softwares. The main conclusions include following three points.

- The two-section path planning is proposed. Sigmoid function can fit the two-section path perfectly. The curve meets the demand of continuity.
- Preview fuzzy controller is designed to track the ideal path. the robustness and adaptability are demonstrated.
- The co-simulation of PreScan and Matlab verifies the proposed two-section path planning and fuzzy logic control strategy in tracking control process.

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