How to stitch them together?

Lecture 2
Local Interest Point Detectors

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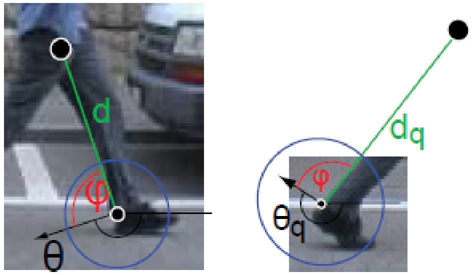
Content

- Local Invariant Features
  - Motivation
  - Requirements
  - Invariance
- Harris Corner Detector
- Scale Invariant Point Detection
  - Automatic scale selection
  - Laplacian-of-Gaussian detector
  - Difference-of-Gaussian detector

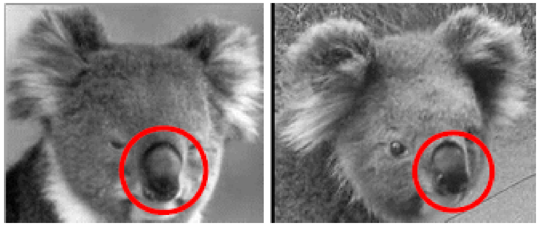
Motivation

- Global representations have major limitations
- Instead, describe and match only local regions
- Increased robustness to
  - Occlusions

  - Articulation



  - Intra-category variations



Motivation

Application: Image Matching

by Diva Sian

by swashford

Motivation

Application: Image Matching

Harder Case

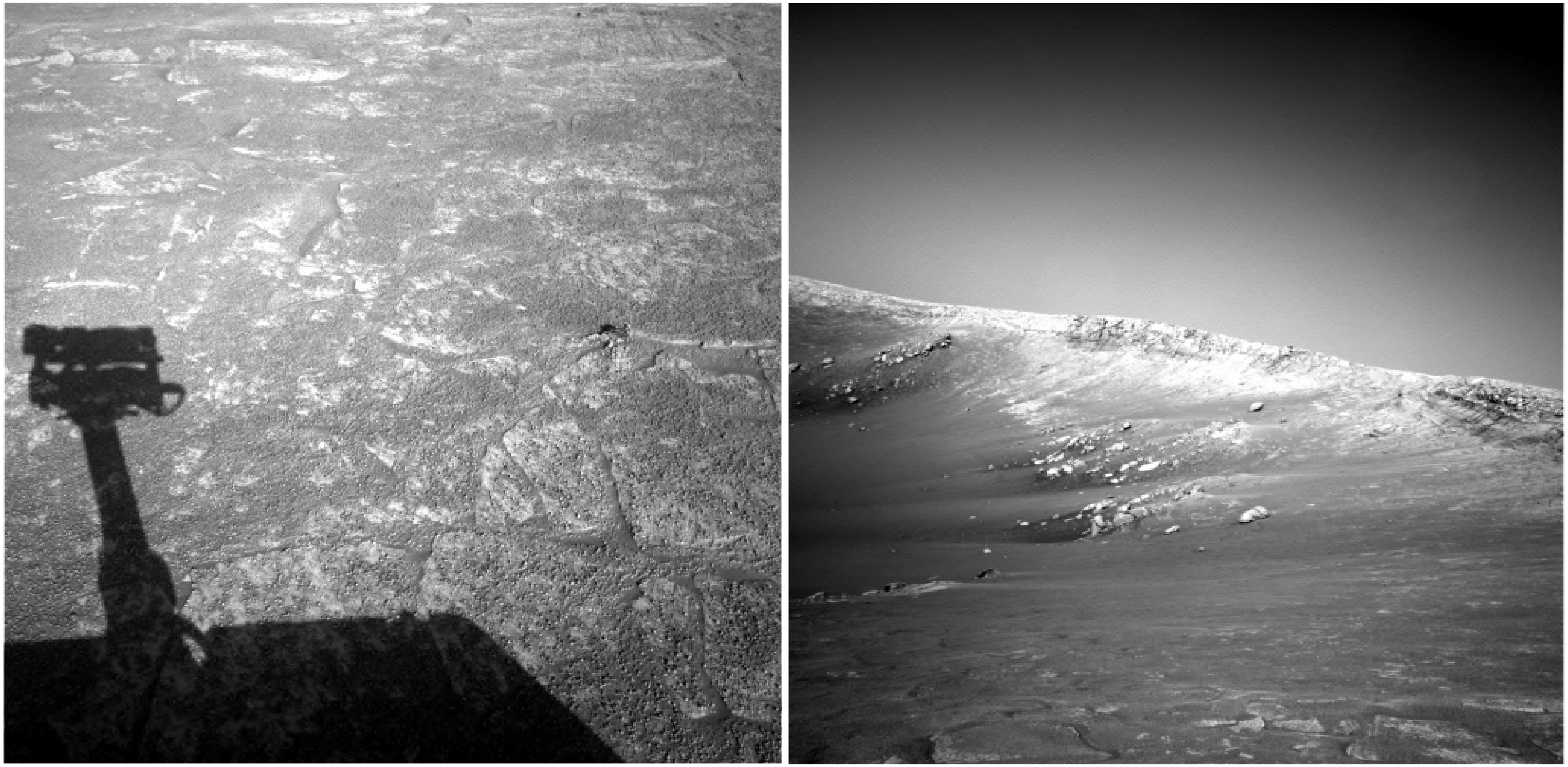
by Diva Sian

by scgbt

Steve Seitz

Motivation

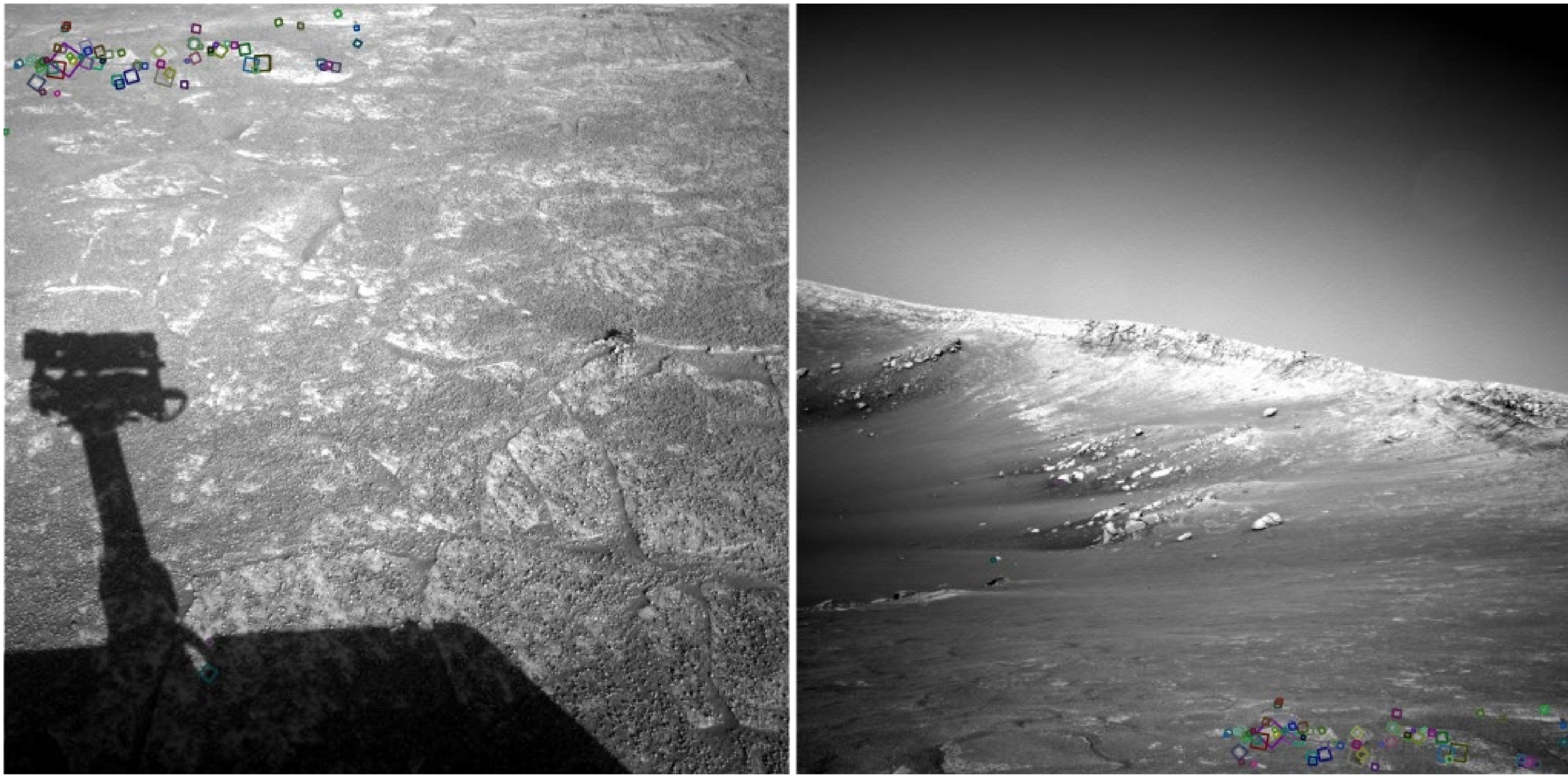
Application: Image Matching



NASA Mars Rover Images

Motivation

Application: Image Matching (Look for tiny colored squares)



NASA Mars Rover images with SIFT matches

Motivation

- Panorama stitching
  - We have two images – how do we combine them?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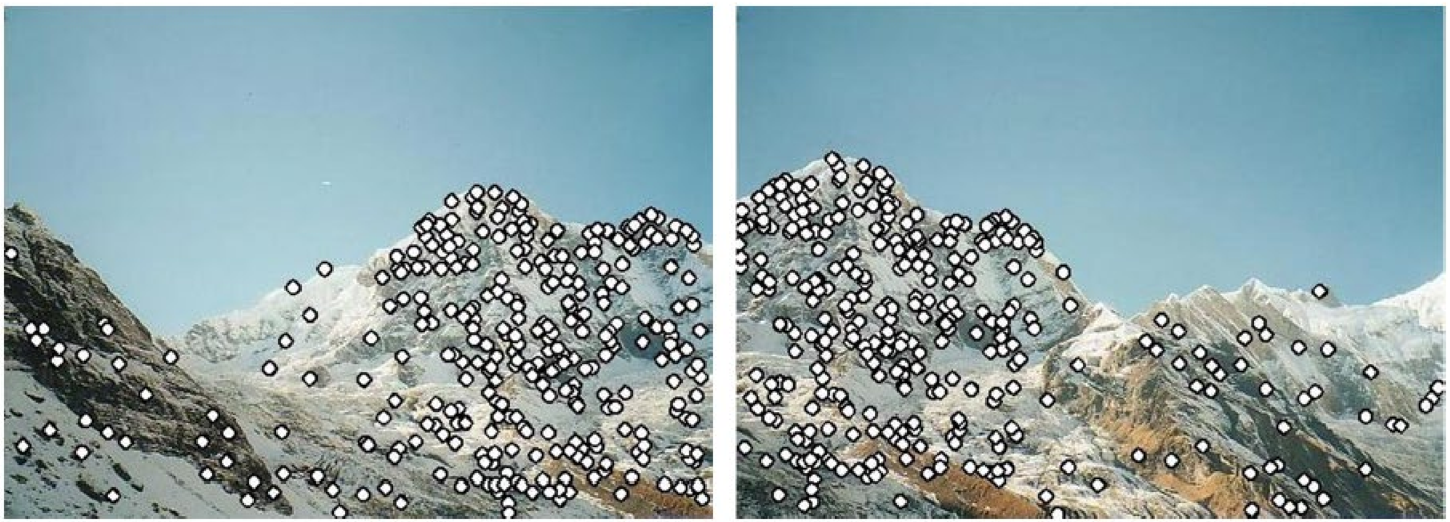
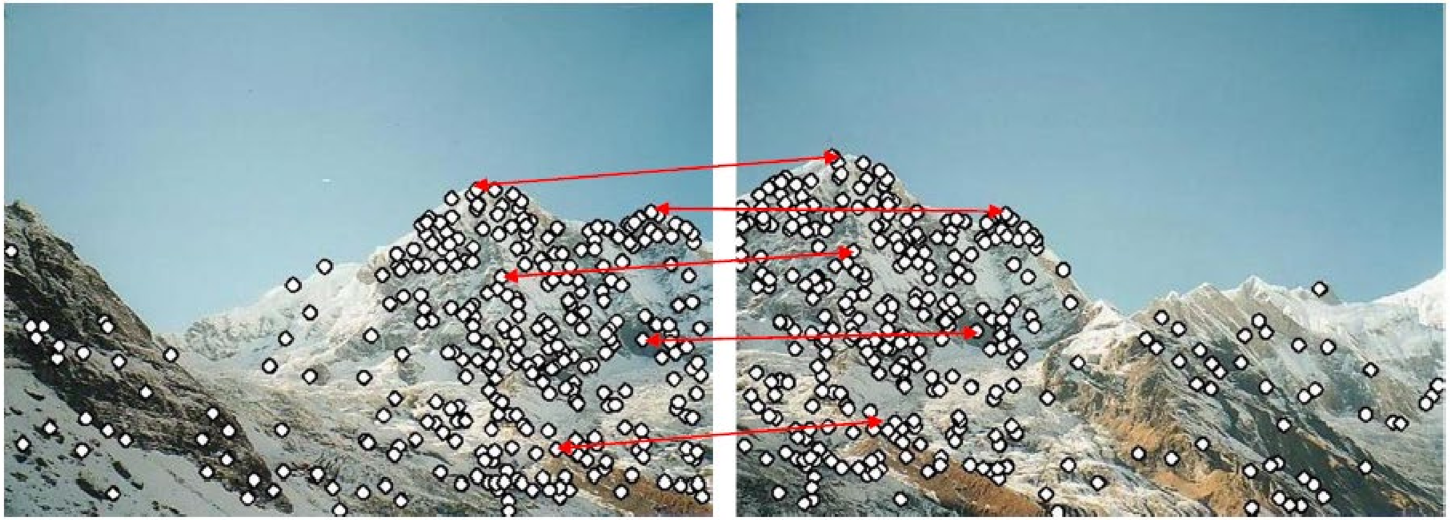


- Procedure:
  – Detect feature points in both images

Motivation

- Panorama stitching
  - We have two images – how do we combine them?



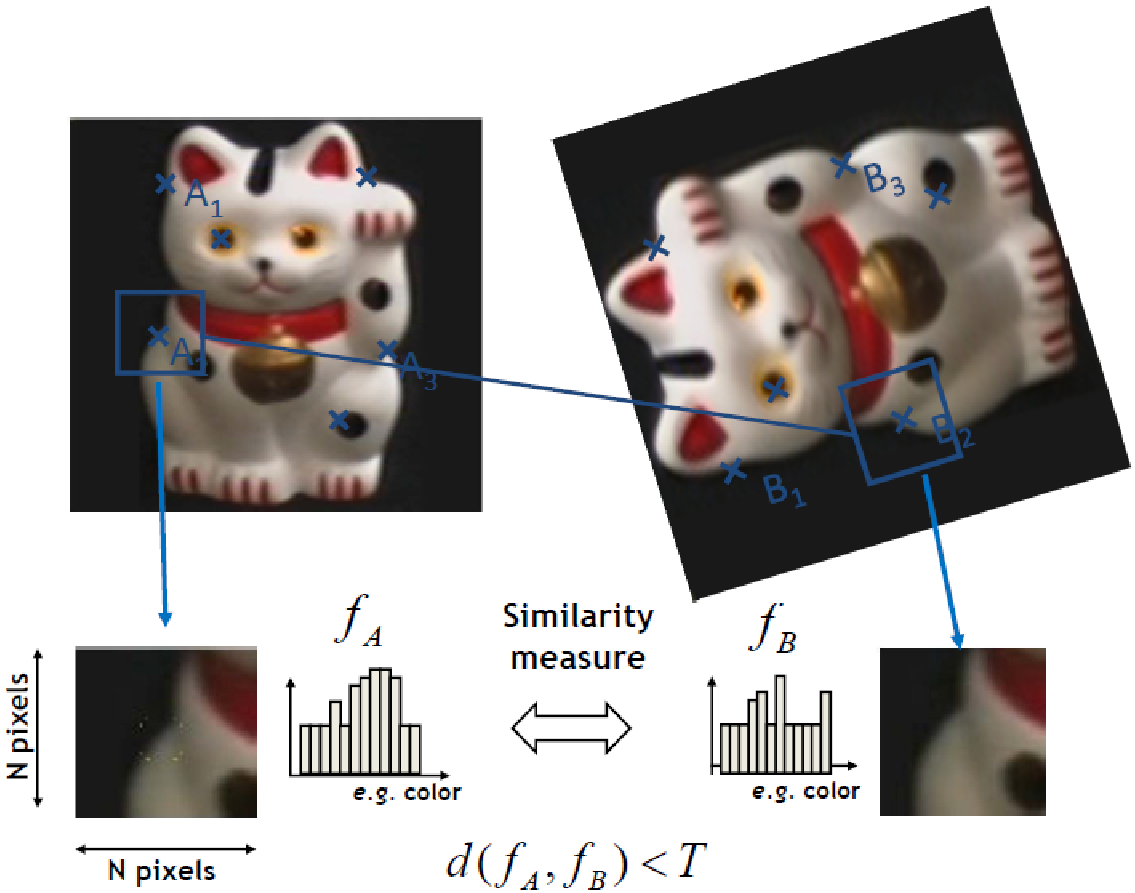
- Procedure:
  - Detect feature points in both images
  - Find corresponding pairs

Motivation

- Panorama stitching
  - We have two images – how do we combine them?

- Procedure:
  - Detect feature points in both images
  - Find corresponding pairs
  - Use these pairs to align the images

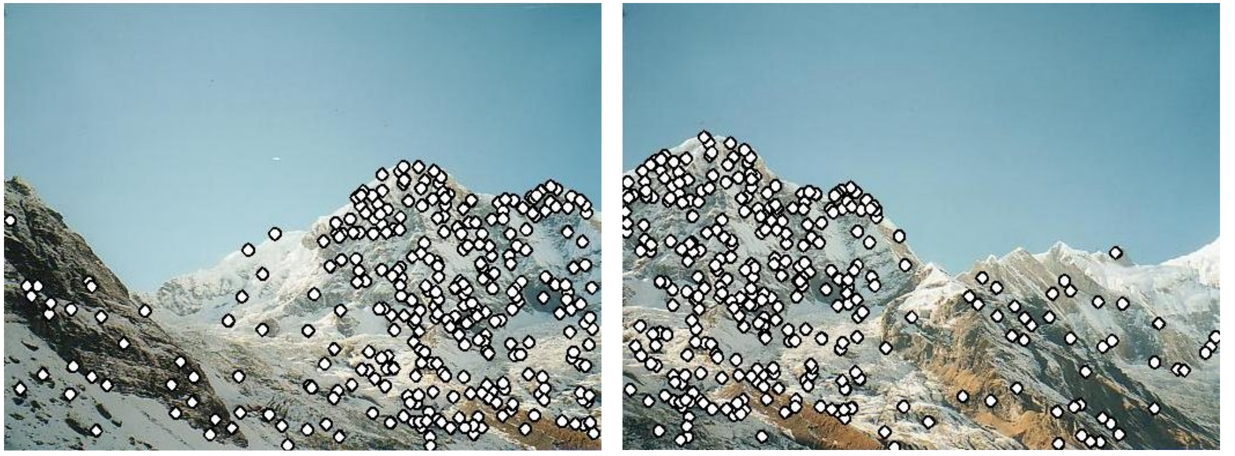
General Approach for Image Matching



1. Find a set of distinctive key-points
2. Define a region around each keypoint
3. Extract and normalize the region content
4. Compute a local descriptor from the normalized region
5. Match local descriptors

Source: B. Leibe

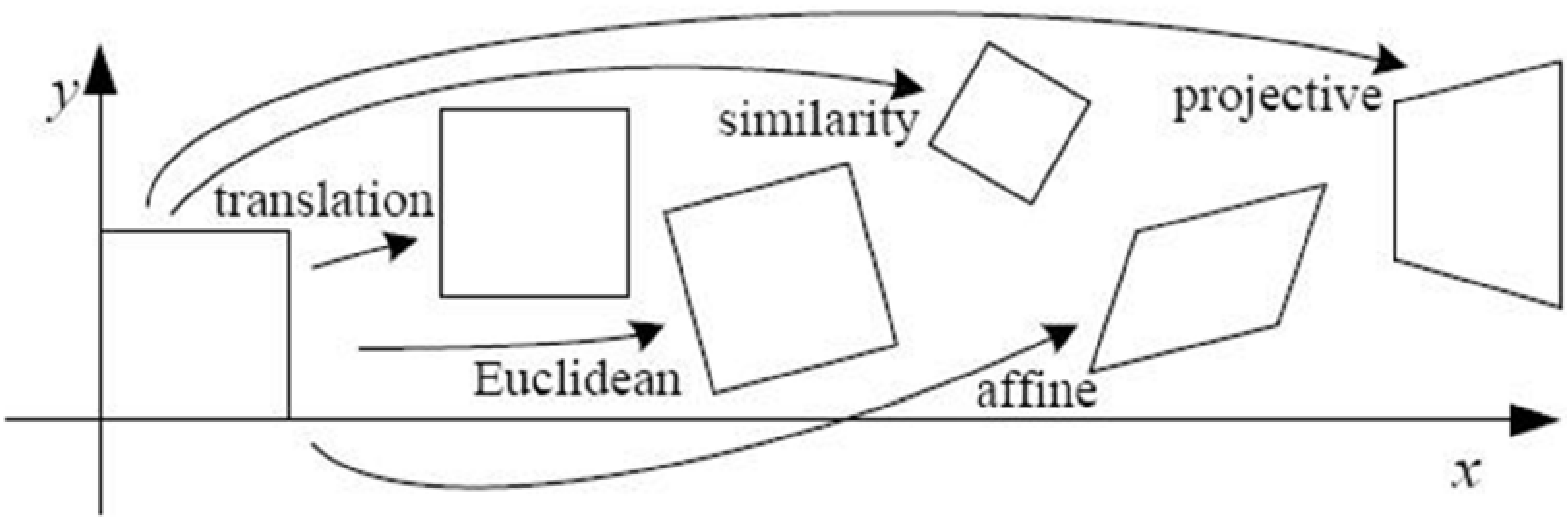
Characteristics of Good Features



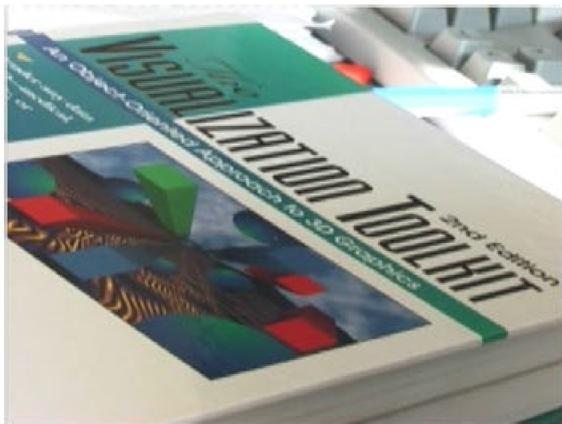
- Repeatability
  - The same feature can be found in several images despite geometric and photometric transformations
- Saliency
  - Each feature has a distinctive description
- Compactness and efficiency
  - Many fewer features than image pixels
- Locality
  - A feature occupies a relatively small area of the image; robust to clutter and occlusion

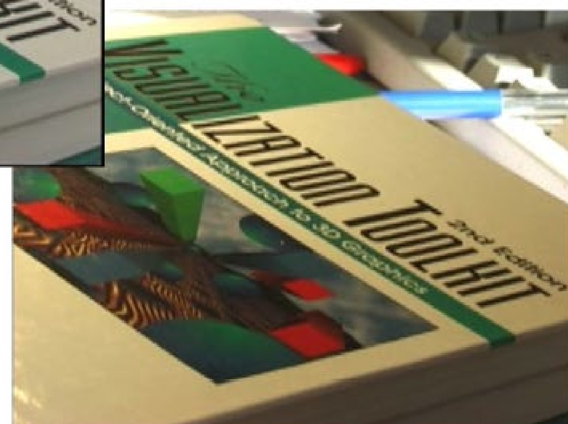
Invariance: Geometric Transformations

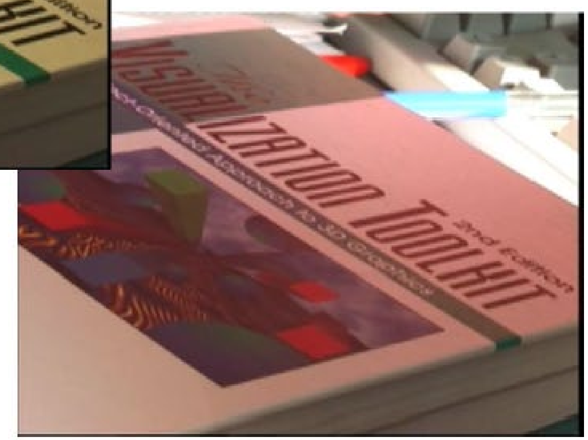
Level of Geometric Invariance



Invariance: Photometric Transformations







- Often modeled as a linear transformation:
  - Scaling + Offset

Applications

Feature points are used for:

- Motion tracking
- Image alignment
- 3D reconstruction
- Object recognition
- Indexing and database retrieval
- Robot navigation

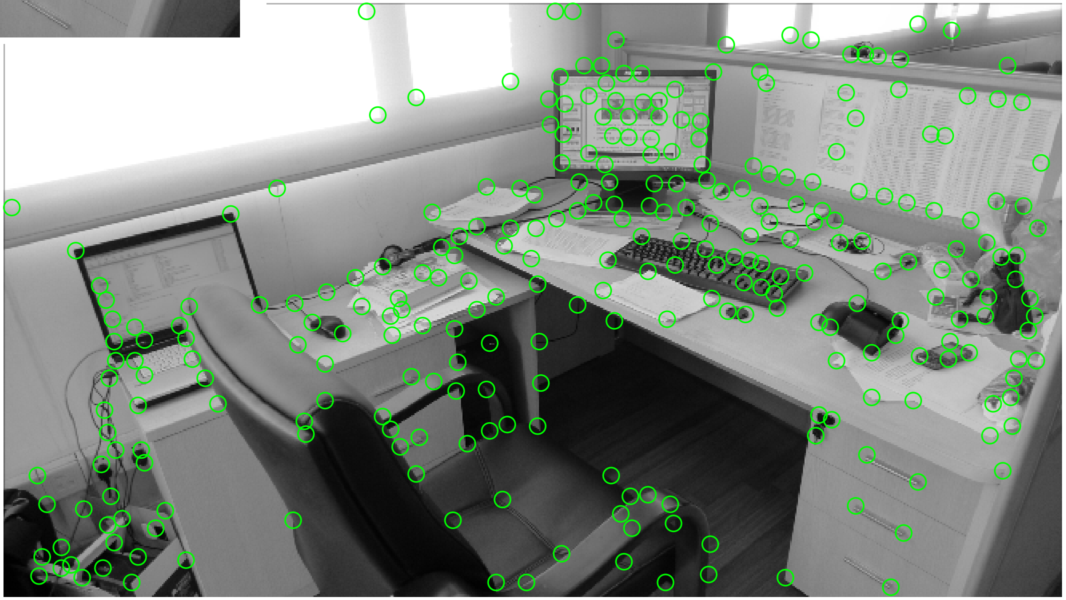
Content

- Local Invariant Features
  - Motivation
  - Requirements
  - Invariance
- Harris Corner Detector
- Scale Invariant Point Detection
  - Automatic scale selection
  - Laplacian-of-Gaussian detector
  - Difference-of-Gaussian detector

Finding Corners

My office,

5:30PM, Sep. 18, 2011



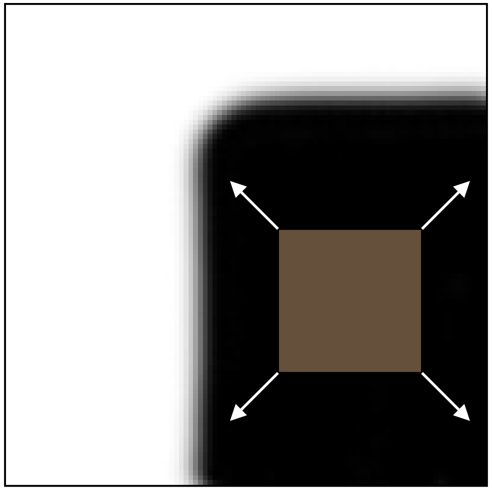
Finding Corners

- Key property: in the region around a corner, image gradient has two or more dominant directions
- Corners are repeatable and distinctive

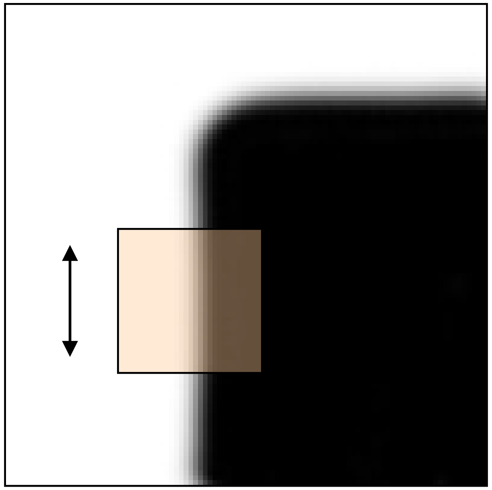
C. Harris and M. Stephens. "A Combined Corner and Edge Detector." *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

Corner Detection: Basic Idea

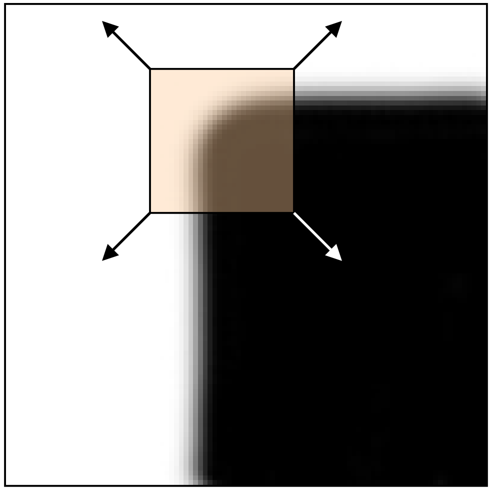
- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity



"flat" region: no change in all directions

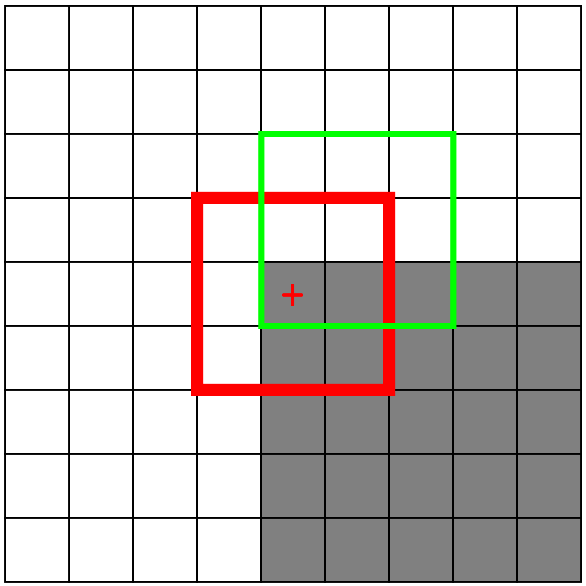


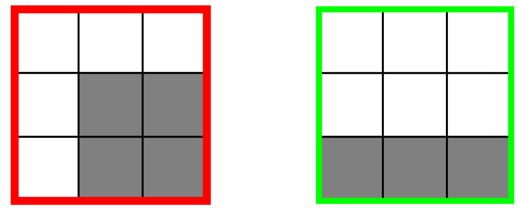
"edge": no change along the edge direction



"corner": significant change in all directions

Harris Detector: Basic Idea



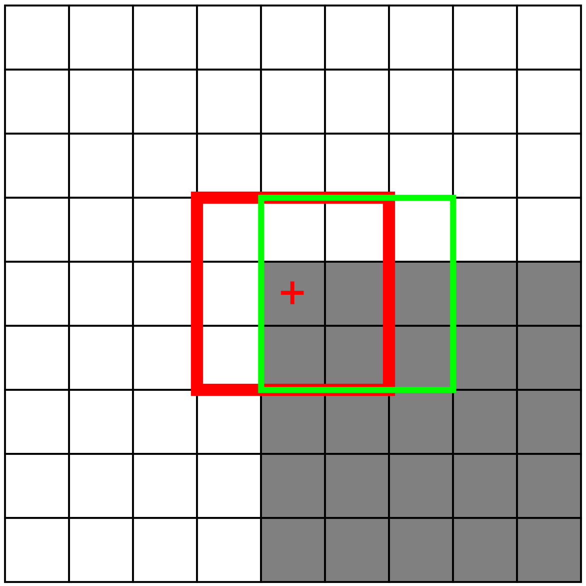


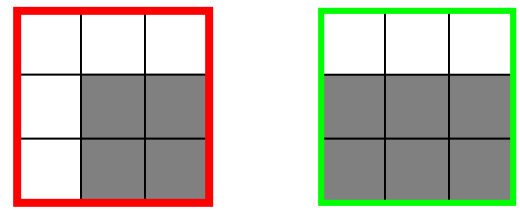
Difference = 3

Demo of a point + with well distinguished neighborhood.

Moving the window in any direction will result in a large intensity change.

Harris Detector: Basic Idea



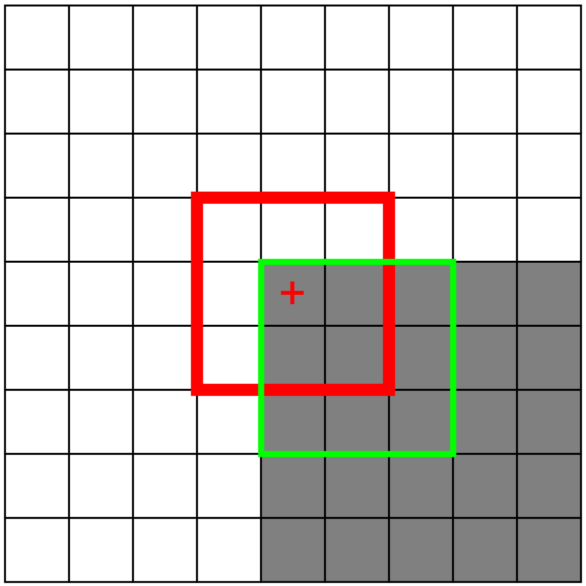


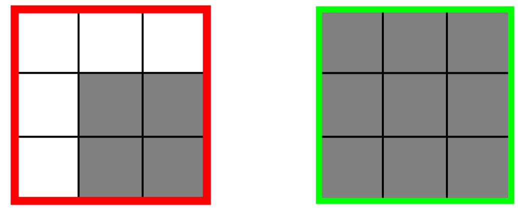
Difference = 2

Demo of a point + with well distinguished neighborhood.

Moving the window in any direction will result in a large intensity change.

Harris Detector: Basic Idea



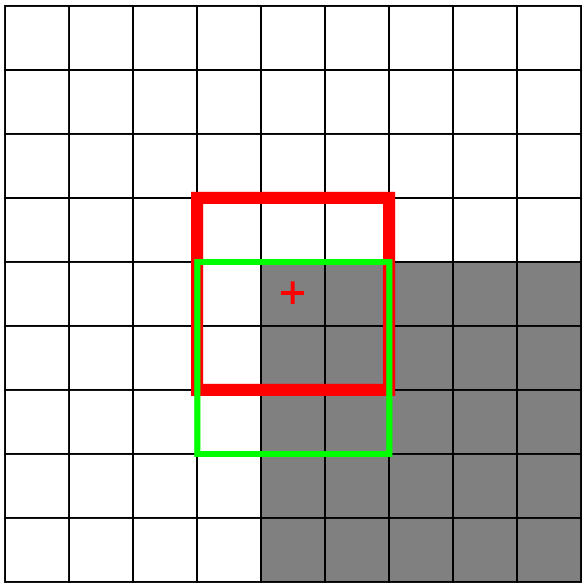


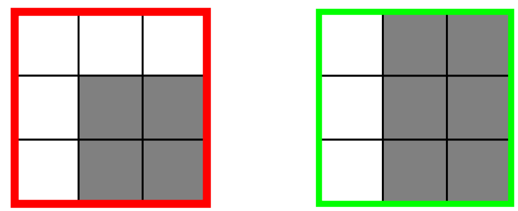
Difference = 5

Demo of a point + with well distinguished neighborhood.

Moving the window in any direction will result in a large intensity change.

Harris Detector: Basic Idea





Difference = 2

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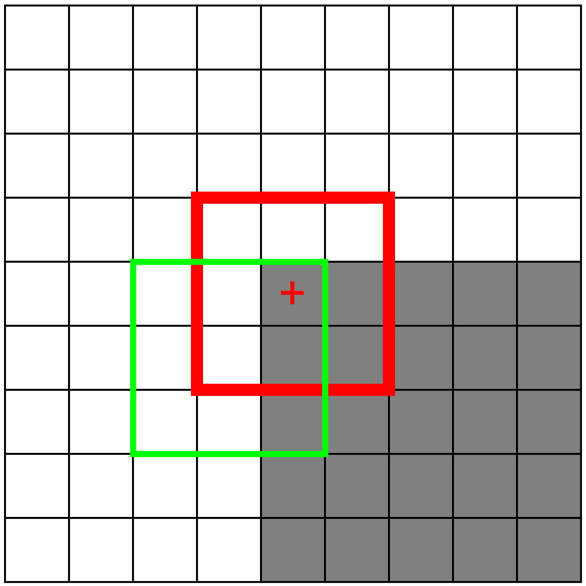
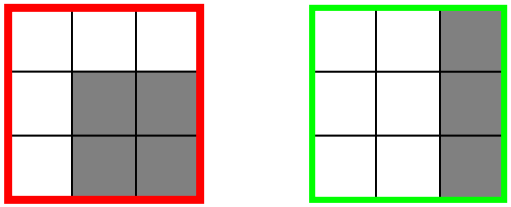
Harris Detector: Basic Idea





Difference = 3

Demo of a point + with well distinguished neighborhood.

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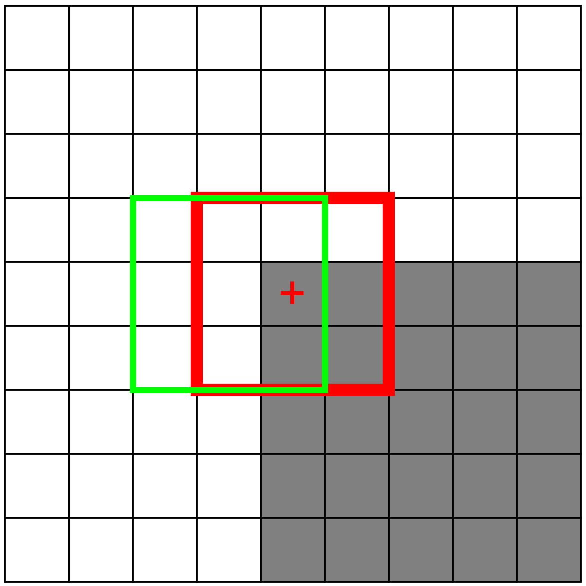
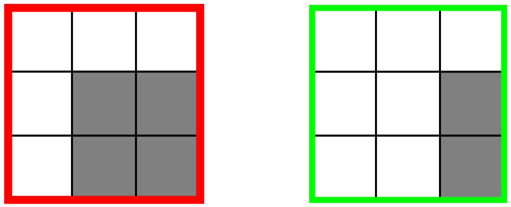
Harris Detector: Basic Idea





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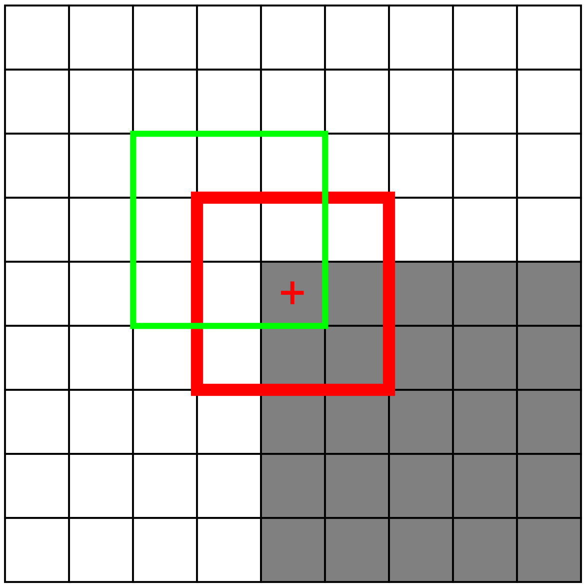
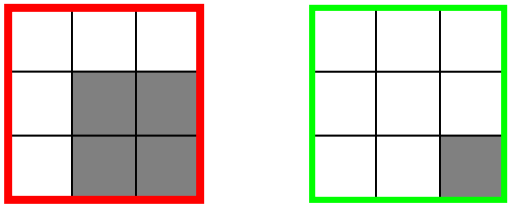
Harris Detector: Basic Idea





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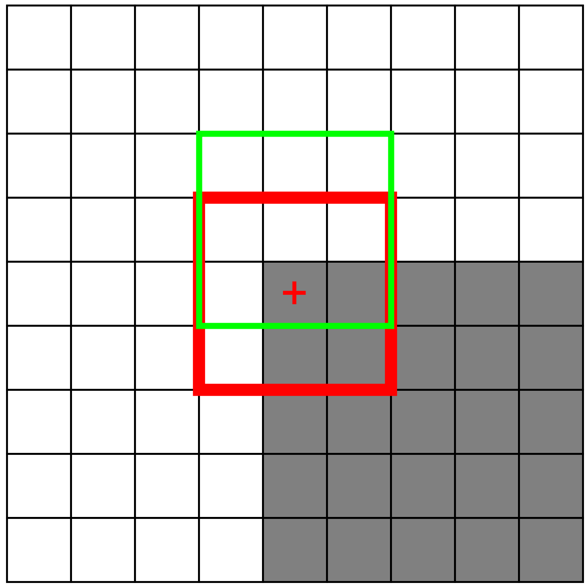
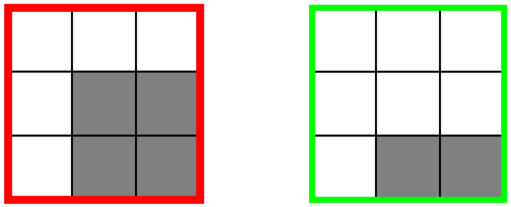
Harris Detector: Basic Idea





Difference = 2

Demo of a point + with well distinguished neighborhood.

Moving the window in any direction will result in a large intensity change.

Harris Corner Detection: Mathematics

Change in appearance of a local patch (defined by a window w) centered at p for the shift $(\Delta x, \Delta y)$:

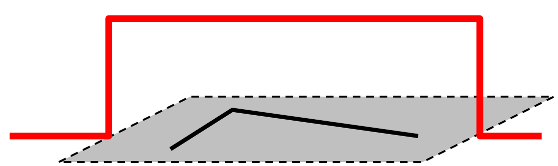
$$S_w(\Delta x, \Delta y) = \sum_{(x_i, y_i) \in w} \left(f(x_i, y_i) - f(x_i + \Delta x, y_i + \Delta y) \right)^2$$

Window function

Intensity

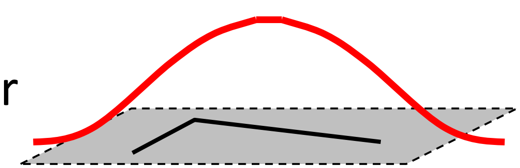
Shifted intensity

Window function W=



1 in window, 0 outside

or



Gaussian

Harris Corner Detection: Mathematics

$$S_w(\Delta x, \Delta y) = \sum_{(x_i, y_i)\in w} \left(f(x_i, y_i) - f(x_i + \Delta x, y_i + \Delta y)\right)^2 \quad (1)$$

$$\approx \sum_{(x_i, y_i)\in w} \left(f(x_i, y_i) - f(x_i, y_i) - \left[\frac{\partial f(x_i, y_i)}{\partial x}, \frac{\partial f(x_i, y_i)}{\partial y} \right] \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \right)^2 \quad (2)$$

$$= \sum_{(x_i, y_i)\in w} \left(\left[\frac{\partial f(x_i, y_i)}{\partial x}, \frac{\partial f(x_i, y_i)}{\partial y} \right] \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \right)^2$$

(Due to $|\mathbf{u}|^2 = \mathbf{u}^T \mathbf{u}$)

$$= [\Delta x, \Delta y] \left(\sum_{(x_i, y_i)\in w} \begin{bmatrix} \frac{\partial f(x_i, y_i)}{\partial x} \\ \frac{\partial f(x_i, y_i)}{\partial y} \end{bmatrix} \left[\frac{\partial f(x_i, y_i)}{\partial x} \quad \frac{\partial f(x_i, y_i)}{\partial y} \right] \right) \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

$$= [\Delta x \; \Delta y] M \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

Harris Corner Detection

$$M =$$

$$\begin{bmatrix} \sum_{(x_i,y_i)\in w} \left(\frac{\partial f(x_i,y_i)}{\partial x} \right)^2 & \sum_{(x_i,y_i)\in w} \left(\frac{\partial f(x_i,y_i)}{\partial x} \cdot \frac{\partial f(x_i,y_i)}{\partial y} \right) \\ \sum_{(x_i,y_i)\in w} \left(\frac{\partial f(x_i,y_i)}{\partial x} \cdot \frac{\partial f(x_i,y_i)}{\partial y} \right) & \sum_{(x_i,y_i)\in w} \left(\frac{\partial f(x_i,y_i)}{\partial y} \right)^2 \end{bmatrix}$$

Harris Corner Detection

$$S(\triangle x, \triangle y) \cong [\triangle x, \triangle y] M \begin{bmatrix} \triangle x \\ \triangle y \end{bmatrix}$$

where, $$M = \begin{bmatrix} \sum_{(x_i, y_i) \in w} (I_x)^2 & \sum_{(x_i, y_i) \in w} (I_x I_y) \\ \sum_{(x_i, y_i) \in w} (I_x I_y) & \sum_{(x_i, y_i) \in w} (I_y)^2 \end{bmatrix}$$

$S(\triangle x, \triangle y) = 1$ actually is the ellipse equation.

The shape of the ellipse is determined by M.

Assignment

M can be proved to be a positive semi-definite matrix

In practice, M is positive definite nearly for sure, then $[\Delta x \; \Delta y] M \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = 1$ represents an ellipse

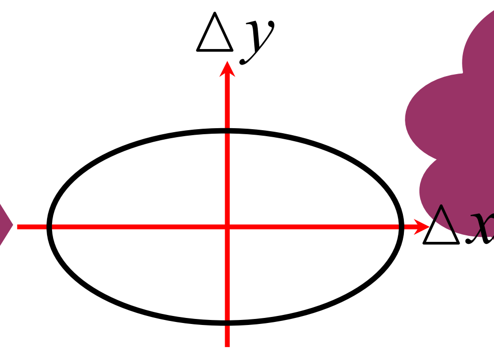
Harris Corner Detection

The "cornerness" of the window w is reflected in M

Suppose there are two local windows w_1 and w_2; consider the cases when the moving of the two windows leads to the intensity change equals to 1. The moving vector $[\Delta x, \Delta y]$ of each window satisfies the ellipse equation. Thus,

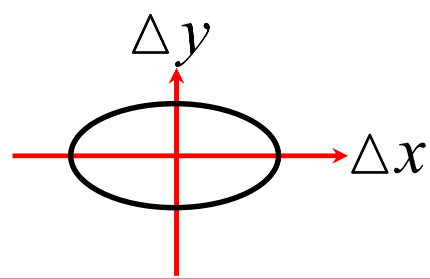
For w_1,

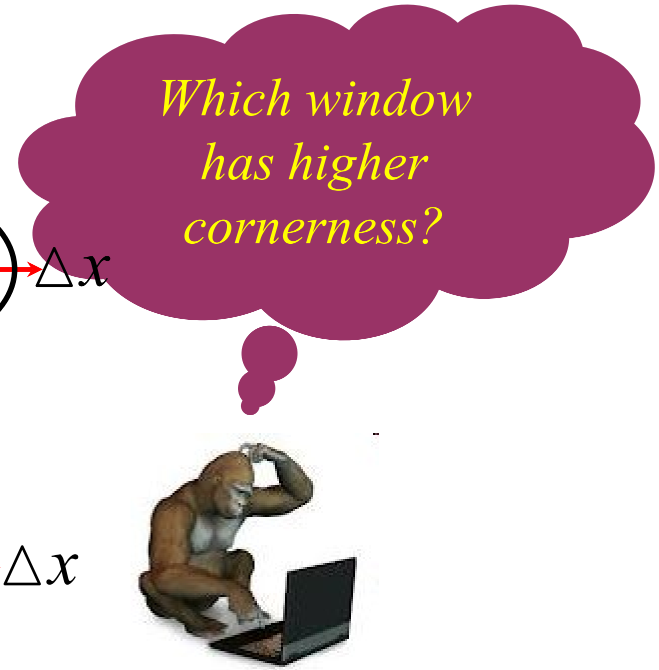
$$[\Delta x, \Delta y] M_1 \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = 1$$



For w_2,

$$[\Delta x, \Delta y] M_2 \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = 1$$



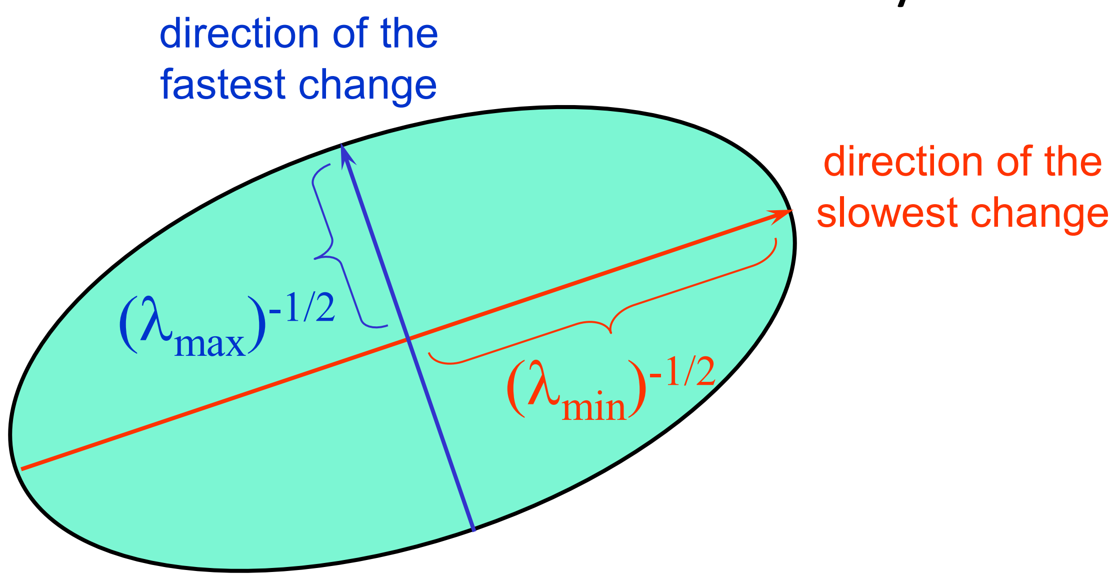


Harris Corner Detection

Diagonalization of M:

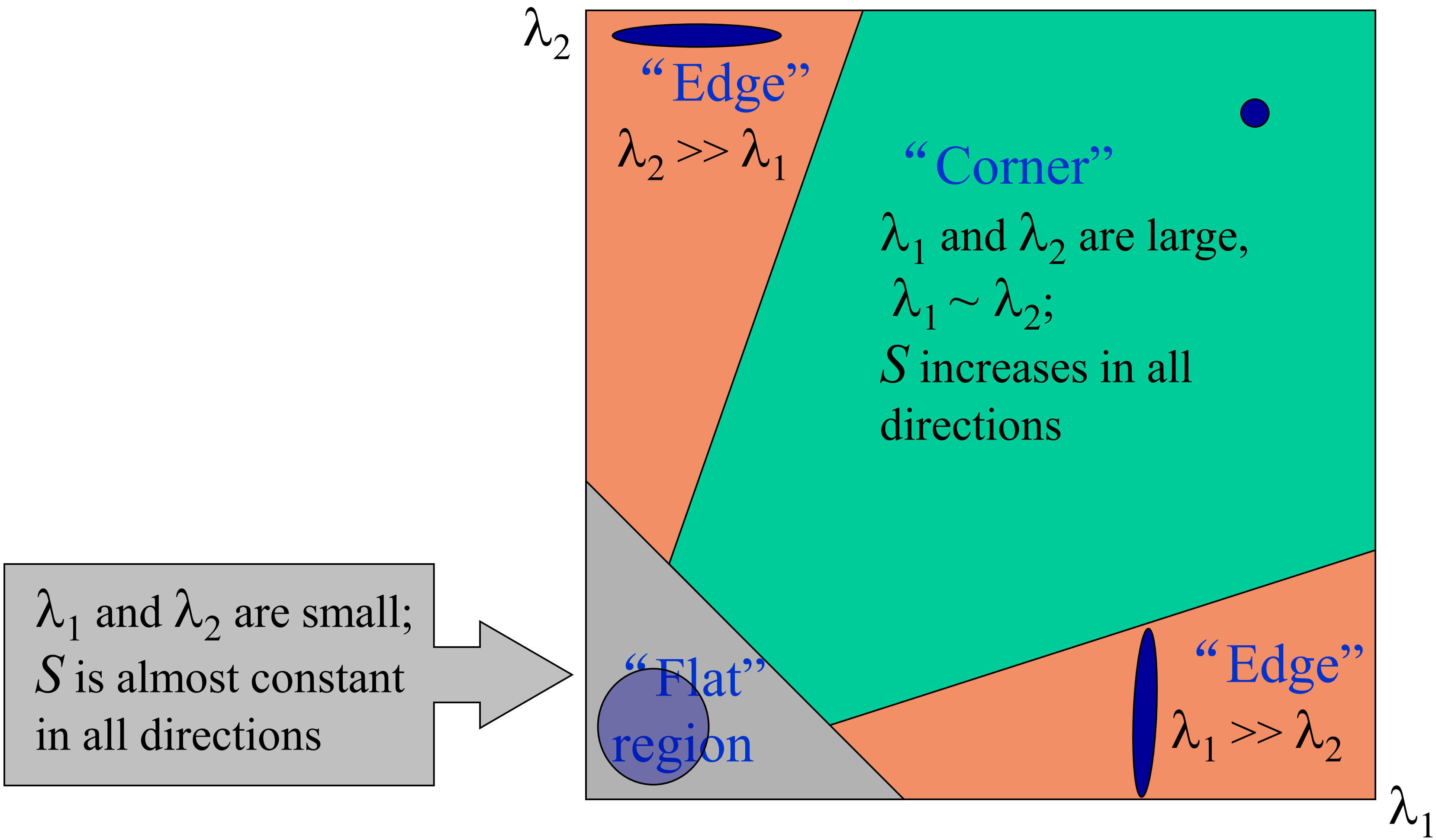
$$M = R^{-1}\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

The axis lengths of the ellipse are determined by the eigenvalues and the orientation is determined by *R*



Interpreting the eigenvalues

Classification of image points using eigenvalues of M:



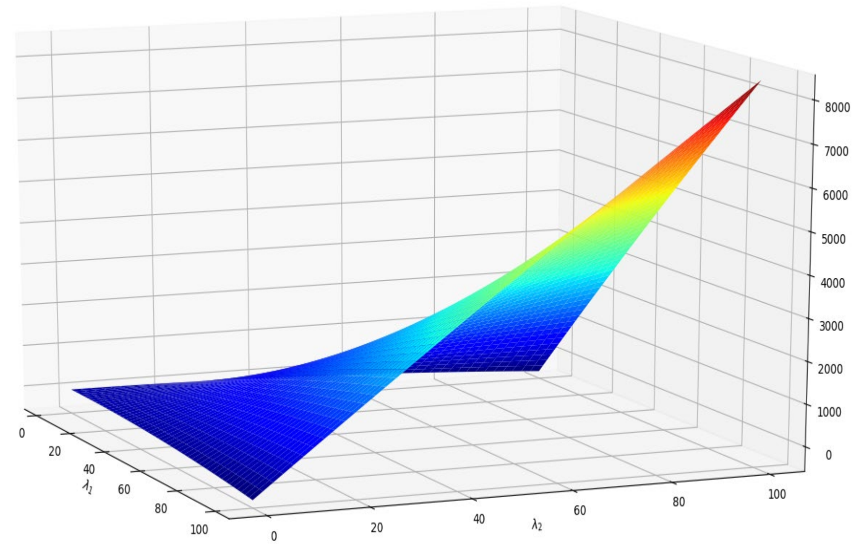
Corner response function

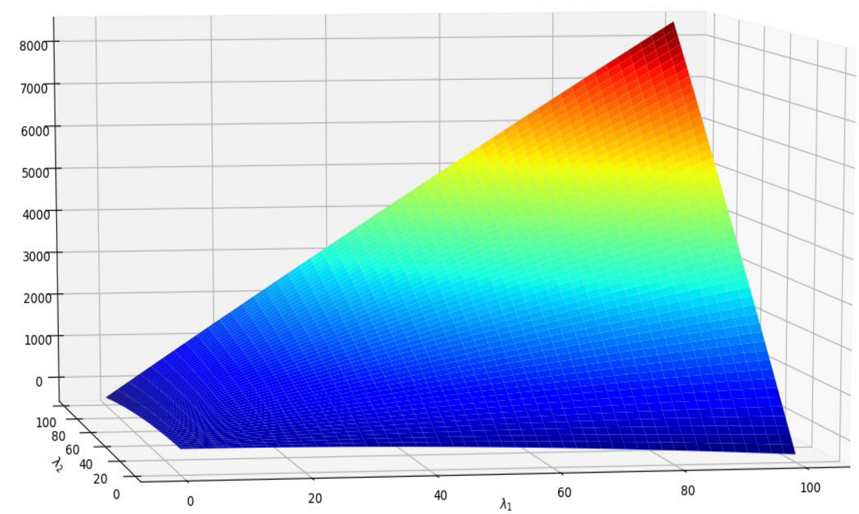
Measure of corner response:

$$R = \det\mathbf{M} - k\left(\mathrm{trace}\mathbf{M}\right)^2$$

$$\det \mathbf{M} = \lambda_1\lambda_2$$
$$\mathrm{trace}\,\mathbf{M} = \lambda_1 + \lambda_2$$

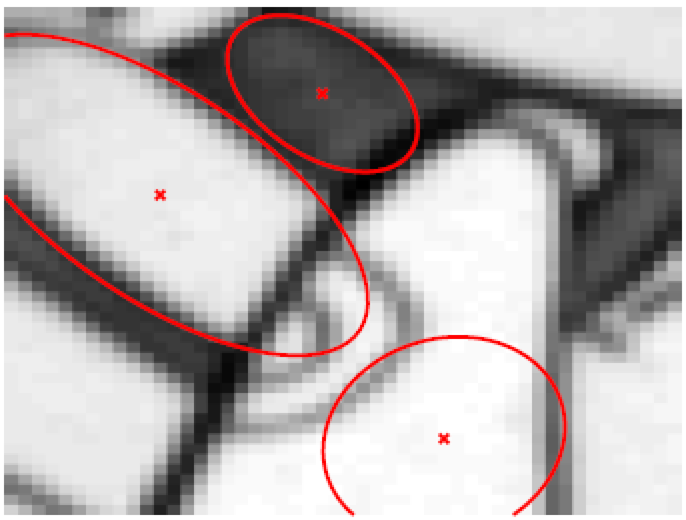
(k – empirical constant, k = 0.04-0.06)



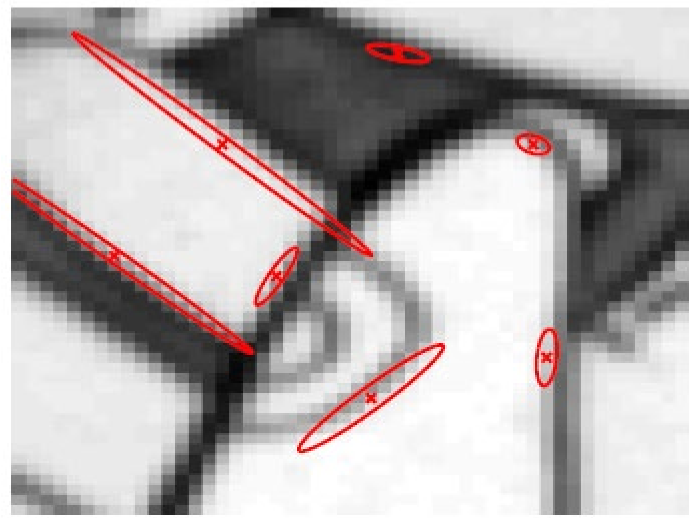


Harris corner detector--illustration

Ellipse with equation : $[\Delta x, \Delta y] M \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = 1$



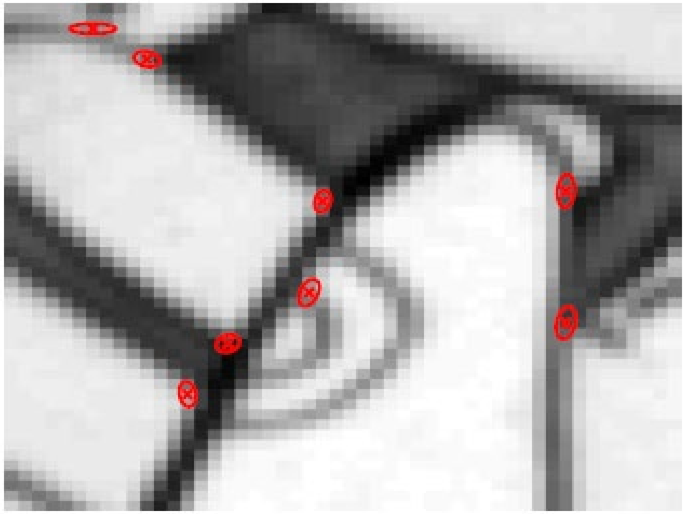
flat region
both eigenvalues small



edge
one small, one large

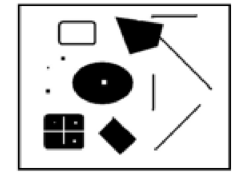
Harris corner detector--illustration

Ellipse with equation : $[\triangle x, \triangle y] M \begin{bmatrix} \triangle x \\ \triangle y \end{bmatrix} = 1$



corner
both eigenvalues large

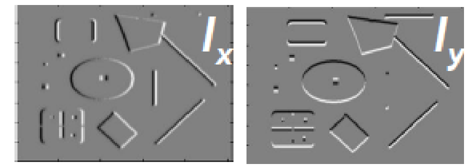
Harris corner detector-Algorithm



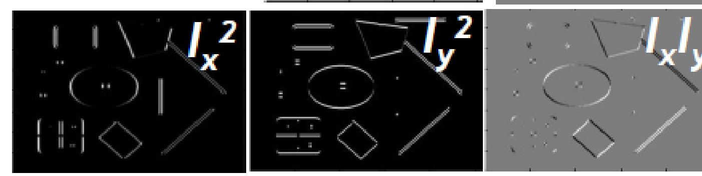
- Compute second moment matrix (autocorrelation matrix)

$$M(\sigma_I,\sigma_D)=g(\sigma_I)*\begin{bmatrix} I_x^2(\sigma_D) & I_xI_y(\sigma_D) \\ I_xI_y(\sigma_D) & I_y^2(\sigma_D)\end{bmatrix}$$

1. Image derivatives



2. Square of derivatives



3. Gaussian filter $g(\sigma_I)$

4. Cornerness function - two strong eigenvalues

$$R=\det[M(\sigma_I,\sigma_D)]-\alpha[\text{trace}(M(\sigma_I,\sigma_D))]^2$$
$$=g(I_x^2)g(I_y^2)-[g(I_xI_y)]^2-\alpha[g(I_x^2)+g(I_y^2)]^2$$

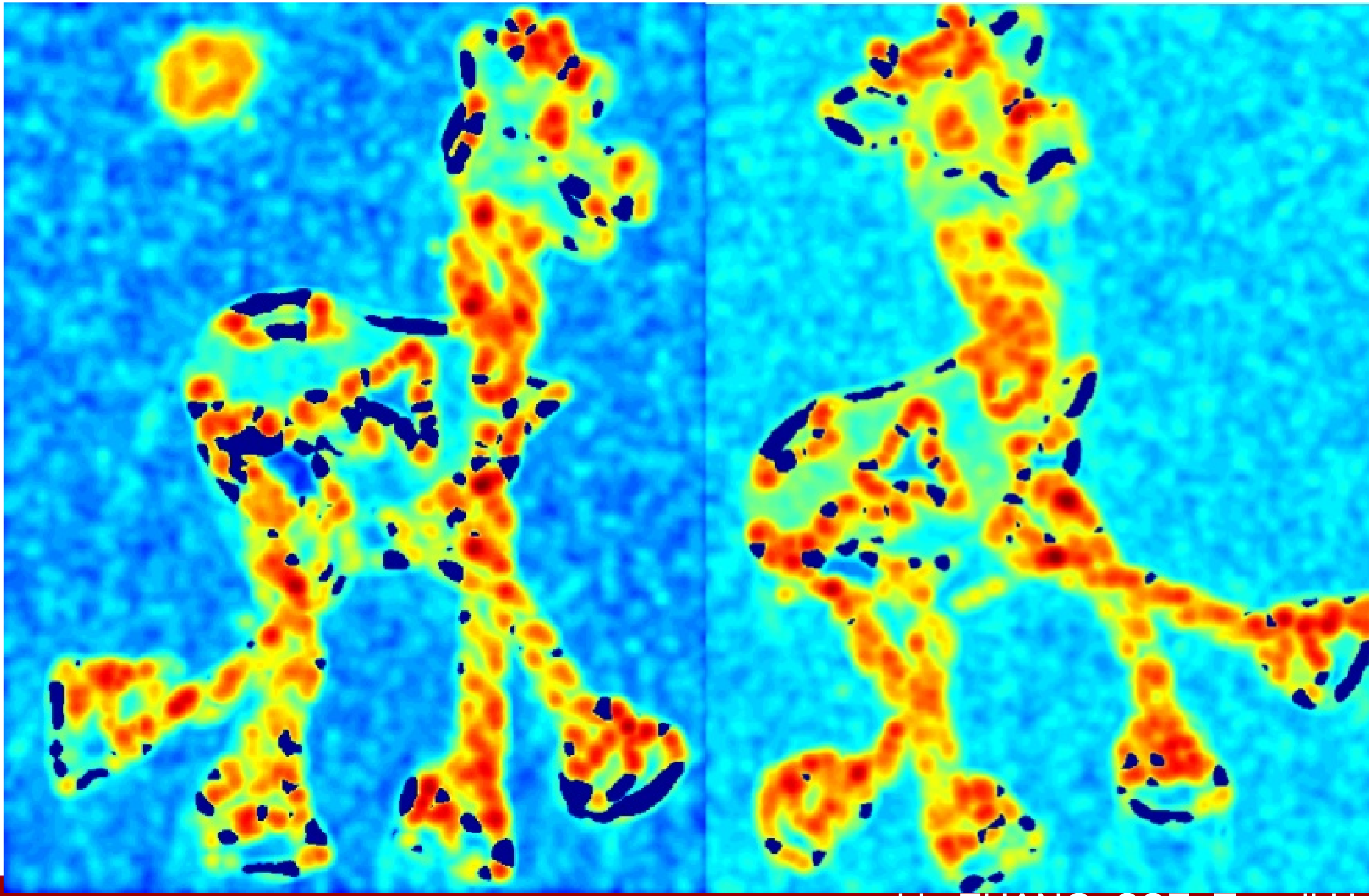
5. Perform non-maximum suppression

Slide credit: Krystian Mikolajczyk

Harris Detector: Steps

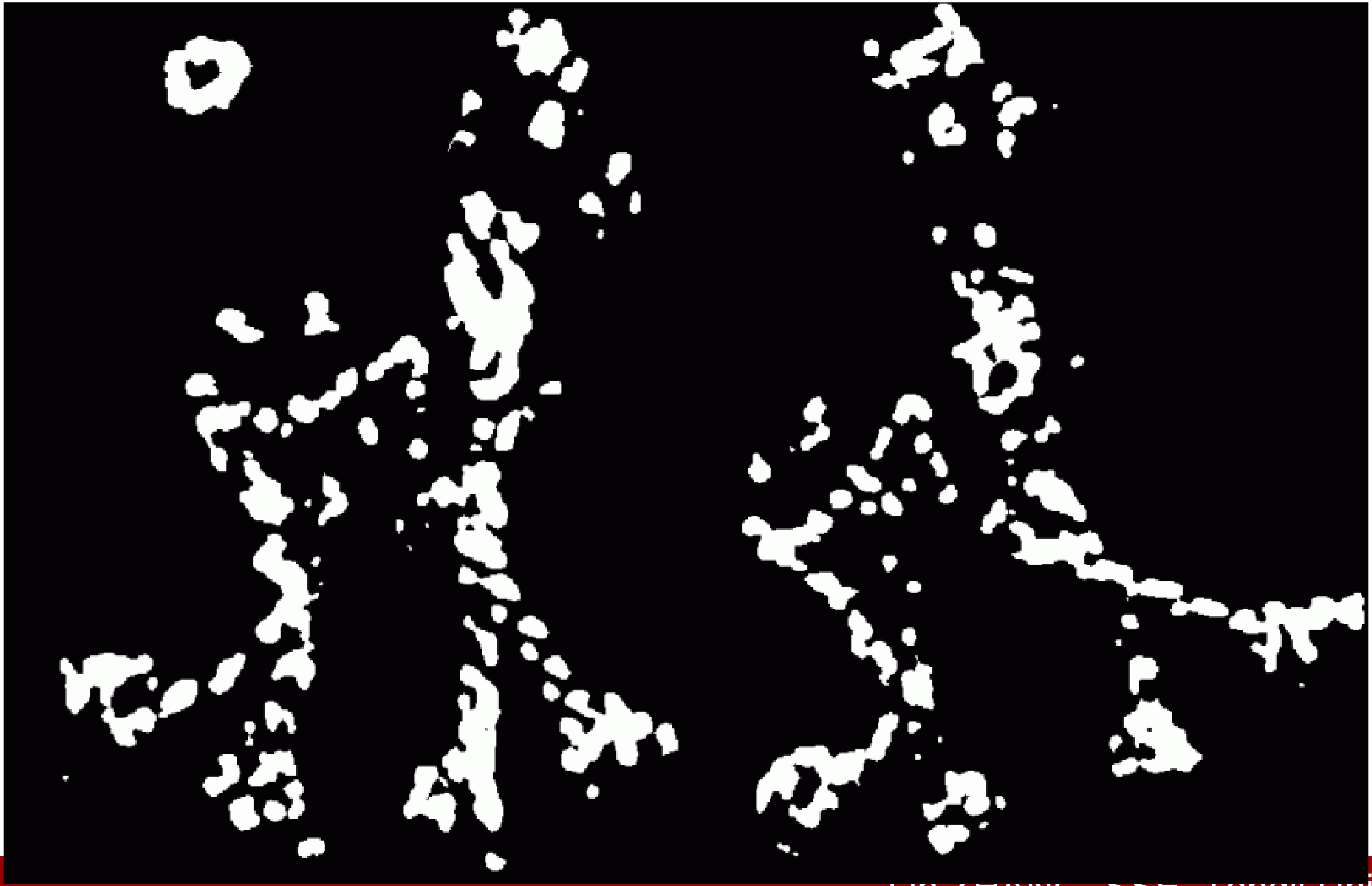
Harris Detector: Steps

Compute corner response R



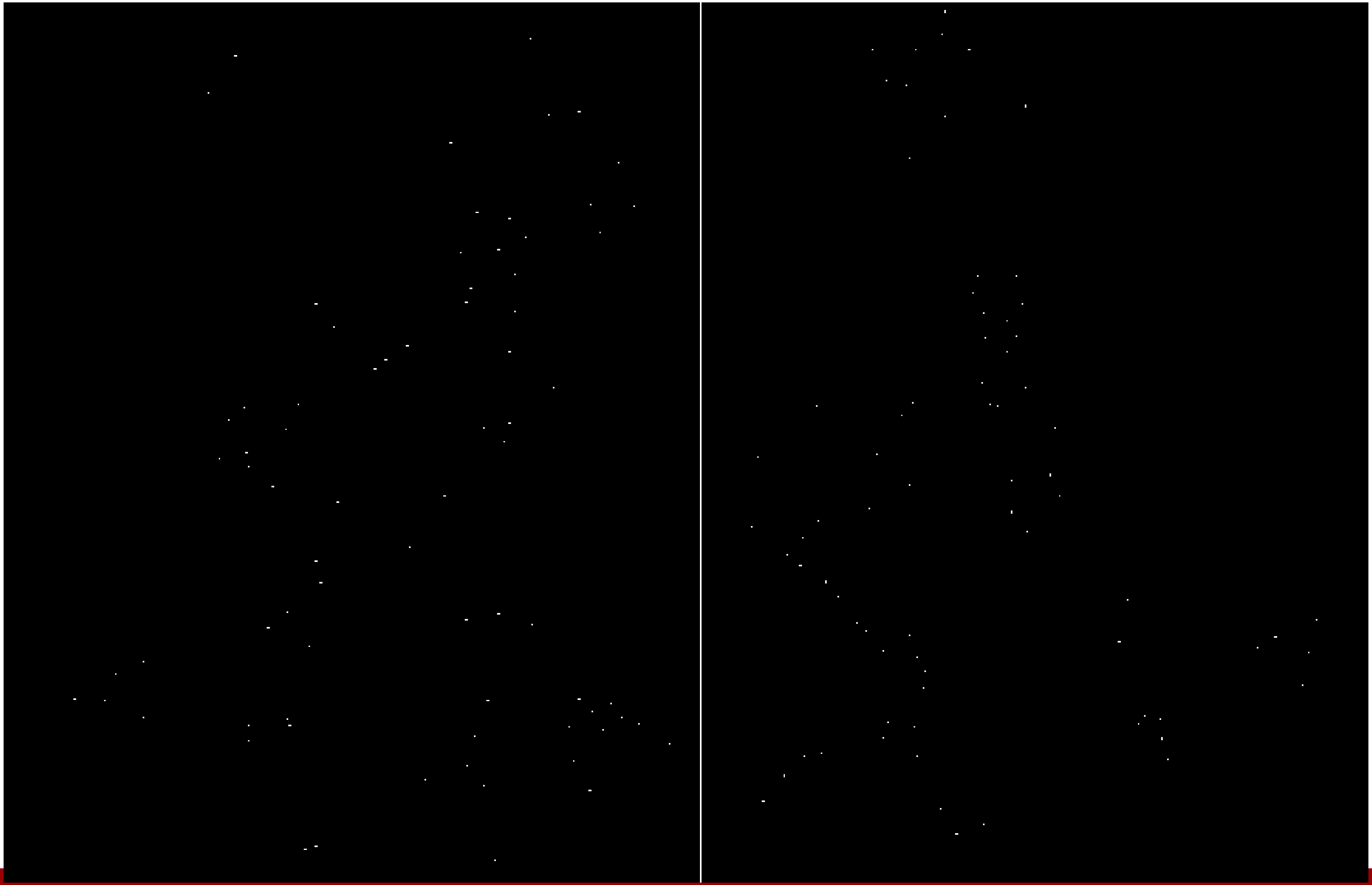
Harris Detector: Steps

Find points with large corner response: *R*>threshold



Harris Detector: Steps

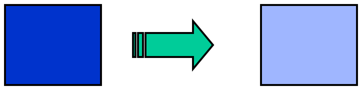
Take only the points of local maxima of *R*



Models of Image Change

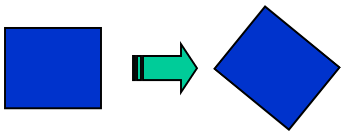
Photometric

- **Affine intensity change** ($I \rightarrow a\, I + b$)

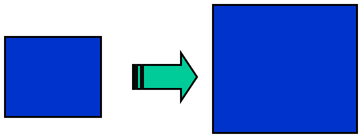


Geometric

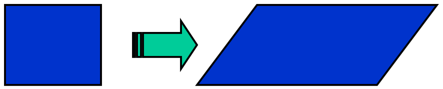
- **Rotation**



- **Scale**

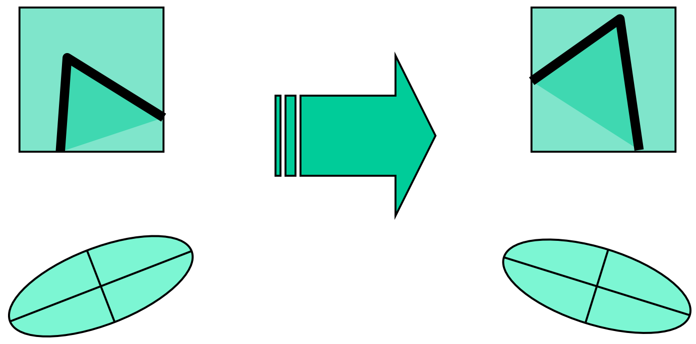


- **Affine**



Harris Detector: Some Properties

Rotation invariance

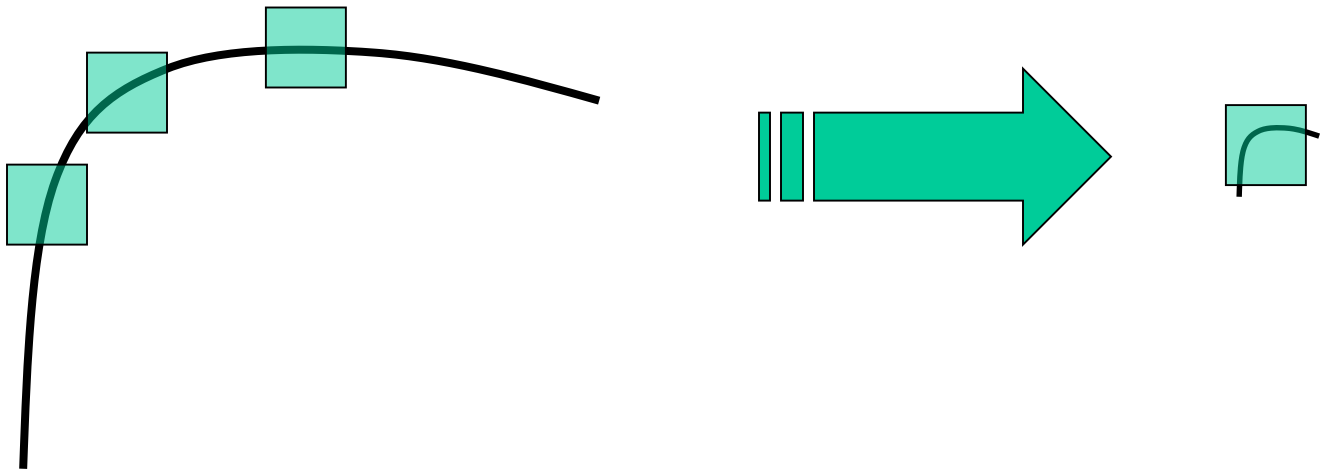


Ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner response R is invariant to image rotation

Harris Detector: Some Properties

Not invariant to *image scale*!



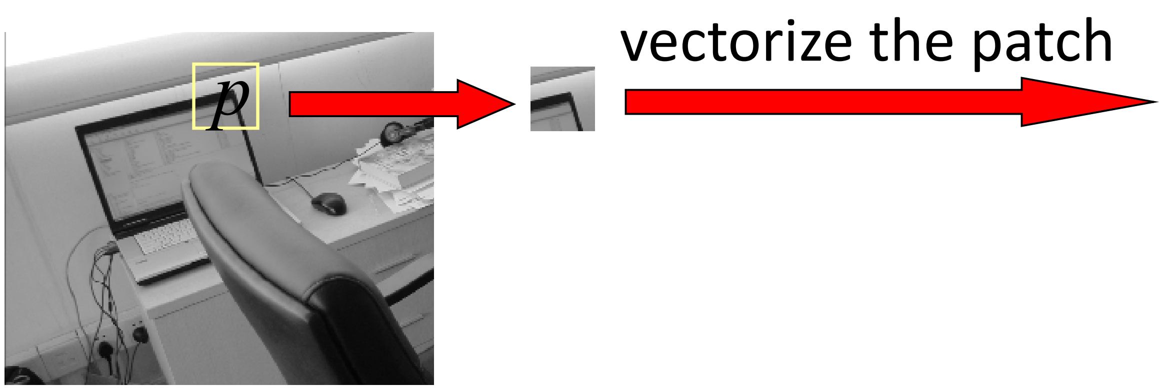
All points will be classified as edges

Corner !

The underlying reason is that Harris corner detection scheme does not provide an automatic and appropriate window size selection method!

Local Descriptors for Harris Corners

- Descriptor for a Harris corner point
  - Take a region with a fixed size around it
  - Stack the region into a vector
  - This vector serves as the descriptor
  - When matching two descriptors in two different images, usually the correlation coefficient is used



descriptor

$$\begin{bmatrix} v1 \\ v2 \\ \vdots \\ v_n \end{bmatrix}$$

Local Descriptors for Harris Corners

- Descriptor for a Harris corner point
  - Take a region with a fixed size around it
  - Stack the region into a vector
  - This vector serves as the descriptor
  - When matching two descriptors in two different images, usually the correlation coefficient is used

Correlation coefficient can be used to measure the similarity of two descriptors

$$\rho = \frac{E[\mathbf{v}_1 - E(\mathbf{v}_1)][\mathbf{v}_2 - E(\mathbf{v}_2)]}{\sqrt{D(\mathbf{v}_1)}\sqrt{D(\mathbf{v}_2)}}$$

Local Descriptors for Harris Corners

- Descriptor for a Harris corner point
  - Take a region with a fixed size around it
  - Stack the region into a vector
  - This vector serves as the descriptor
  - When matching two descriptors in two different images, usually the correlation coefficient is used
- Deficiencies of such simple descriptors
  - Not rotation invariant
  - Not scale invariant

Local Descriptors for Harris Corners

- Descriptor for a Harris corner point
  - Take a region with a fixed size around it
  - Stack the region into a vector
  - This vector serves as the descriptor
  - When matching two descriptors in two different images, usually the correlation coefficient is used
- We want:
  - Rotation and scale invariant feature points
  - Rotation and scale invariant feature descriptors

Content

- Local Invariant Features
  - Motivation
  - Requirements
  - Invariance
- Harris Corner Detector
- Scale Invariant Point Detection
  - Automatic scale selection
  - Laplacian-of-Gaussian detector
  - Difference-of-Gaussian detector

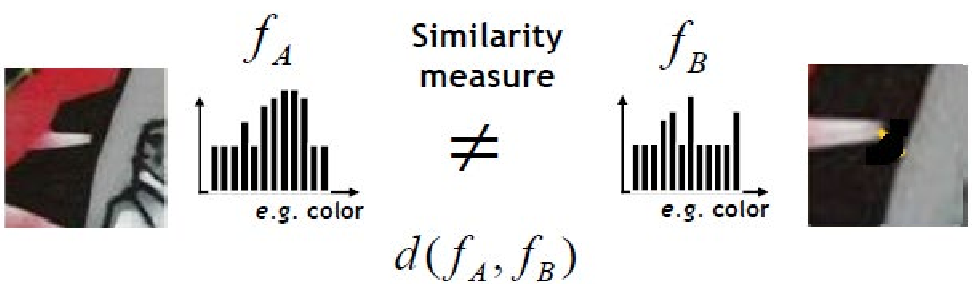
From Points to Regions

- The Harris corner detector defines interest points
  - Precise localization
  - High repeatability

- In order to match those points, we need to compute a descriptor over a region
  - How can we define such a region in a scale invariant manner?
  - That is how can we detect sale invariant regions?

Naïve Approach: Exhaustive Search

- Multi-scale procedure
  - Compare descriptors while varying the patch size



Slide credit: Krystian Mikolajczyk

Naïve Approach: Exhaustive Search

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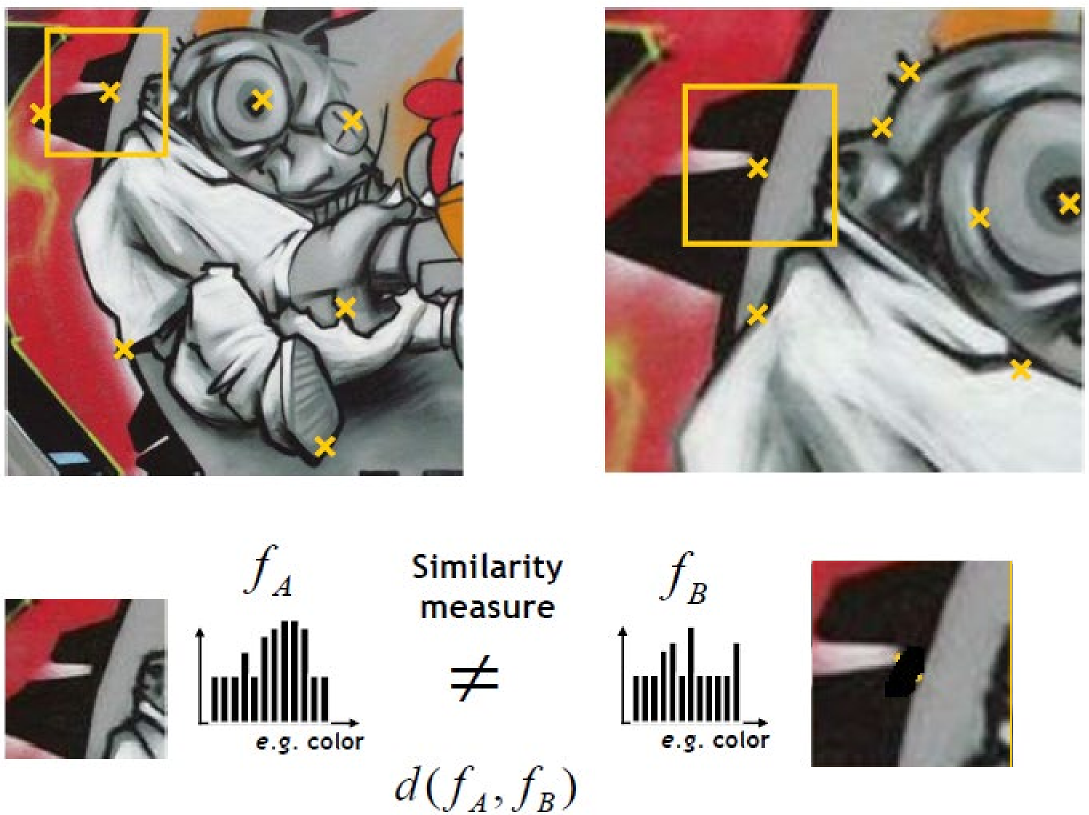

Slide credit: Krystian Mikolajczyk

Naïve Approach: Exhaustive Search

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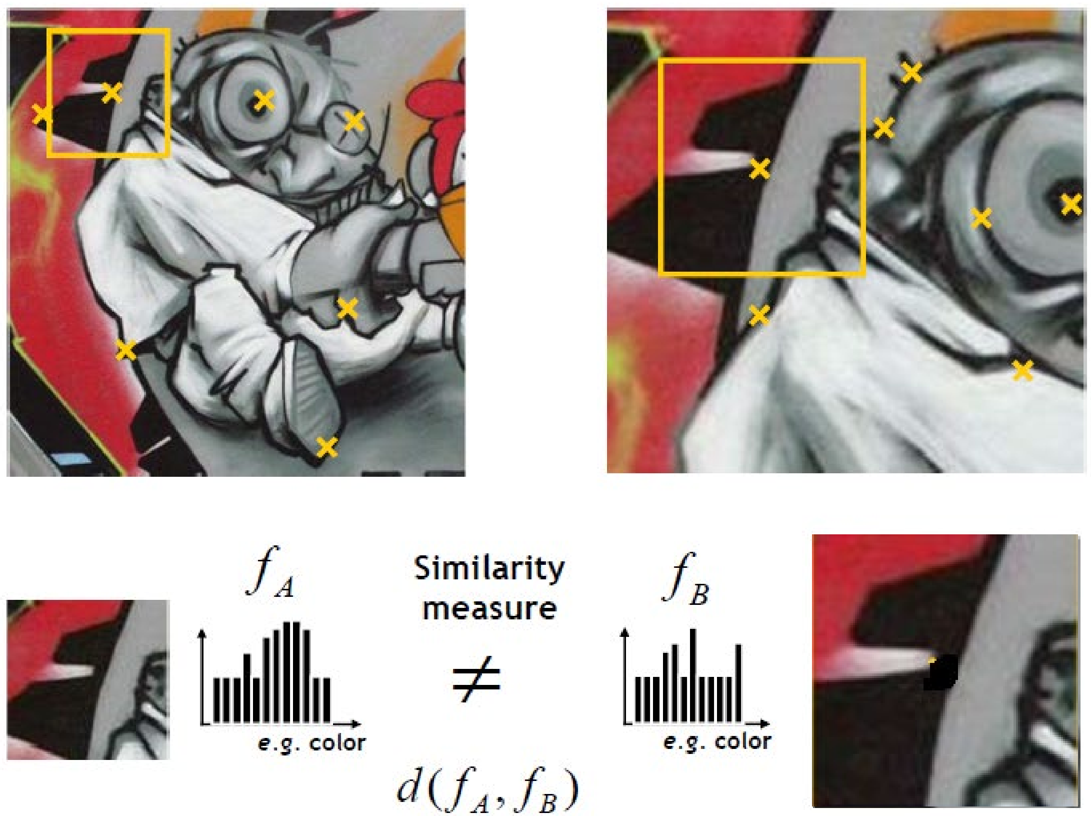
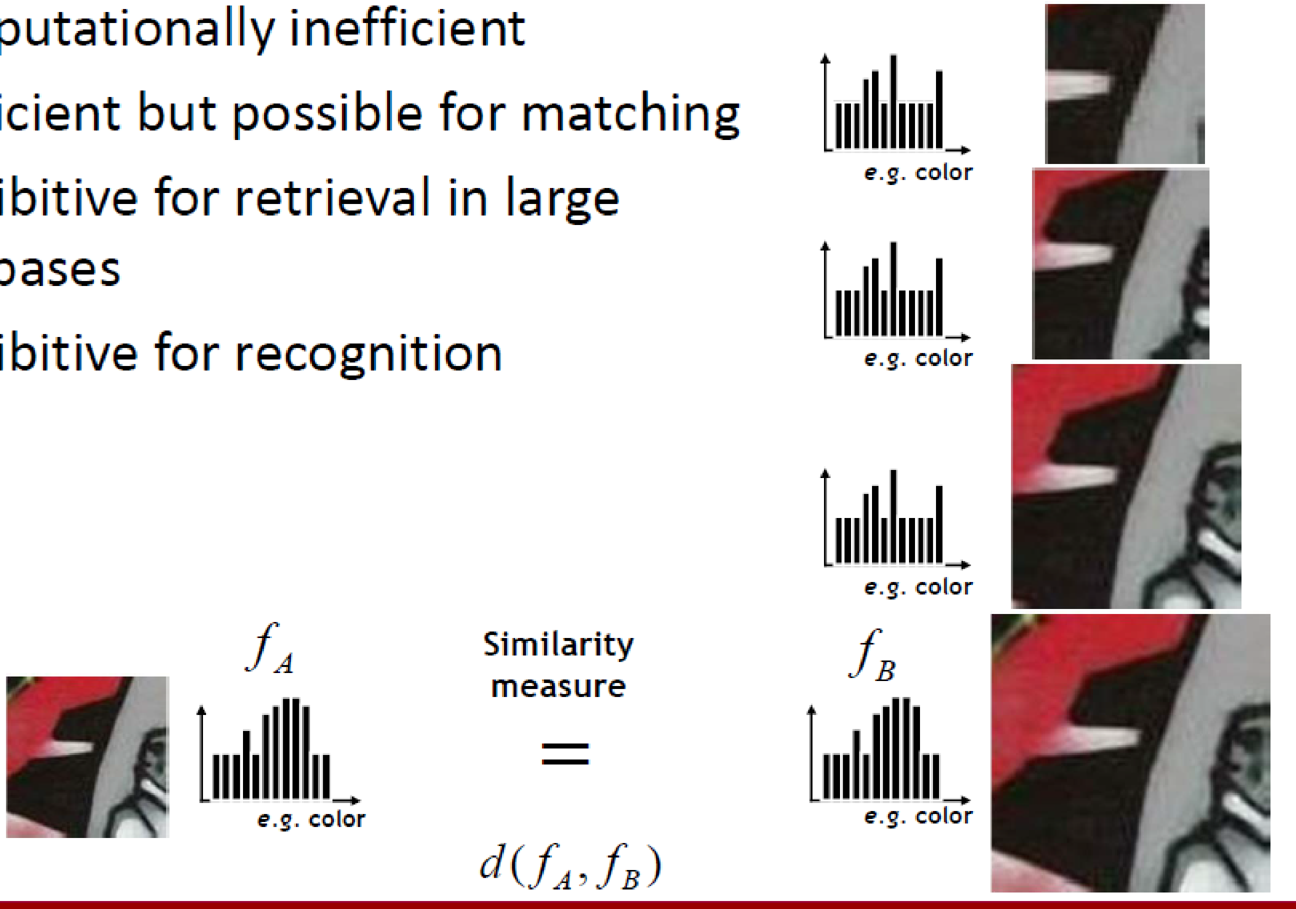


Slide credit: Krystian Mikolajczyk

Naïve Approach: Exhaustive Search

- Comparing descriptors while varying the patch size
  - Computationally inefficient
  - Inefficient but possible for matching
  - Prohibitive for retrieval in large databases
  - Prohibitive for recognition



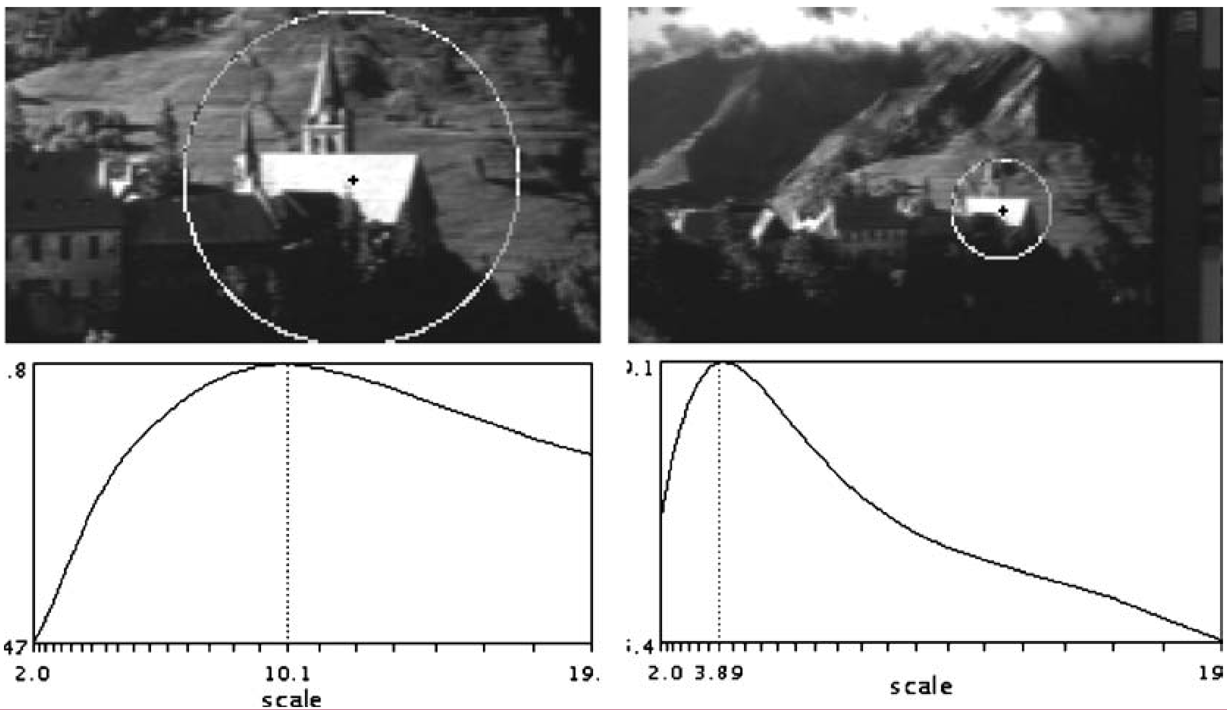
Slide credit: Krystian Mikolajczyk

What do we want to do next?

- Naïve approach for scale invariant local description is not efficient (Detect Harris corners first, and then exhaustively searching for regions with appropriate sizes)
- Now we want to:
  - Find scale invariant points in the image (location)
  - At the same time, we want to know their **characteristic scales** (used to determine the neighborhood for local description)

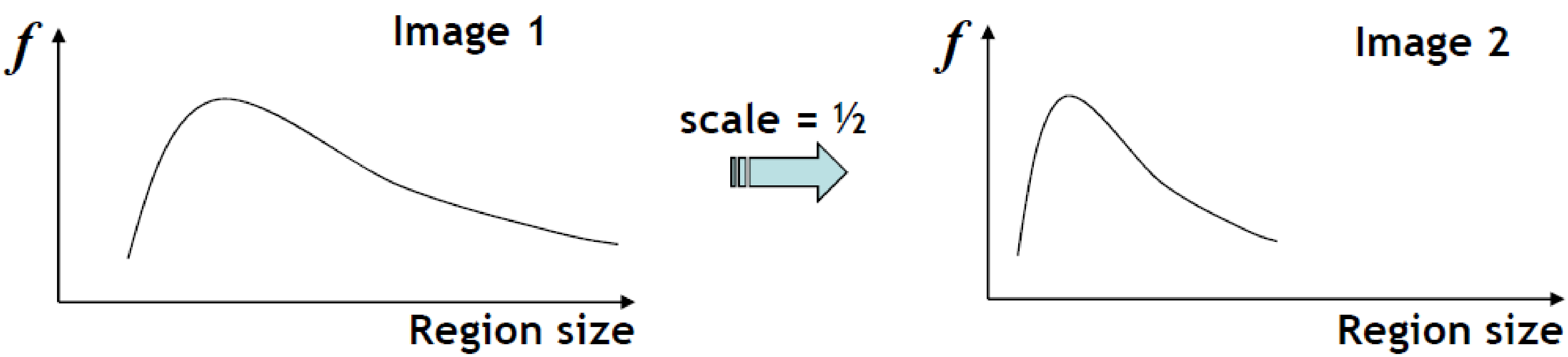
Achieving scale covariance

- Goal: independently detect corresponding regions in scaled versions of the same image
- Need *scale selection* mechanism for finding characteristic region size that is *covariant* with the image transformation



Automatic Scale Selection

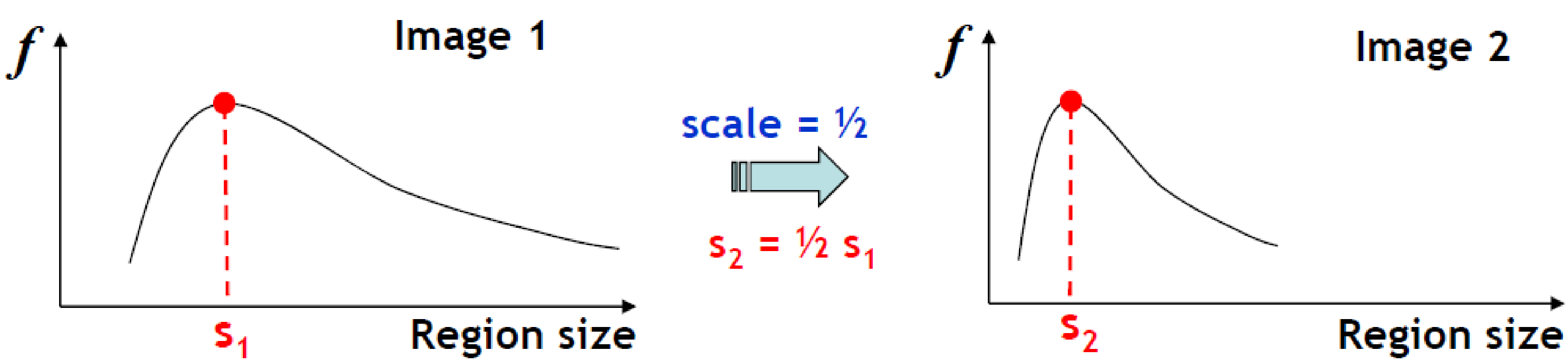
- Solution:
  - Design a function on the region, which is “scale invariant” (*the same for corresponding regions, even if they are at different scales*)
  - For a point in one image, we can consider it as a function of region size (patch width)



Slide credit: Kristen Grauman

Automatic Scale Selection

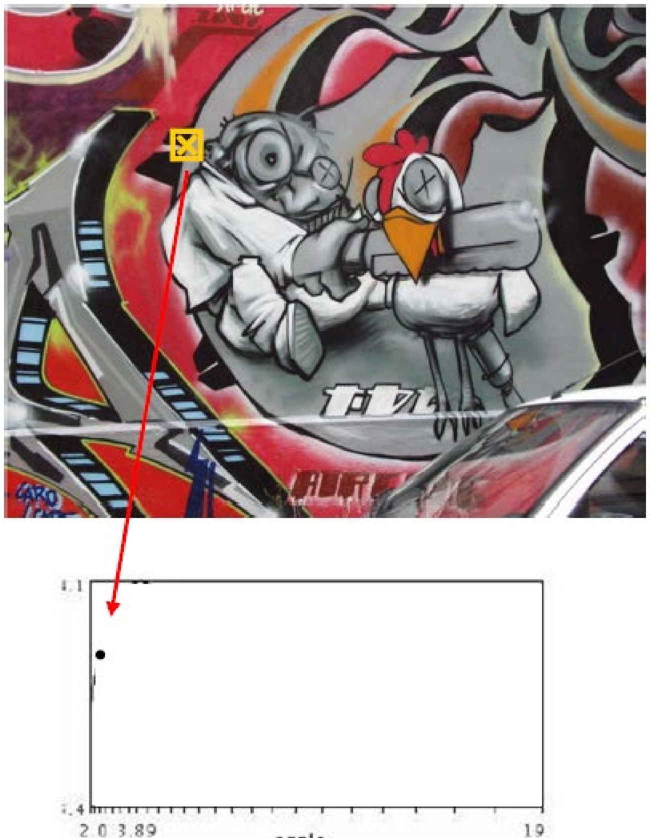
- Common approach
  - Take a local extremum of this function
  - Observation: region size for which the extremum is achieved should be covariant to image scale; this scale covariant region size is found in each image independently



Slide credit: Kristen Grauman

Automatic Scale Selection

- Function responses for increasing scale (scale signature)



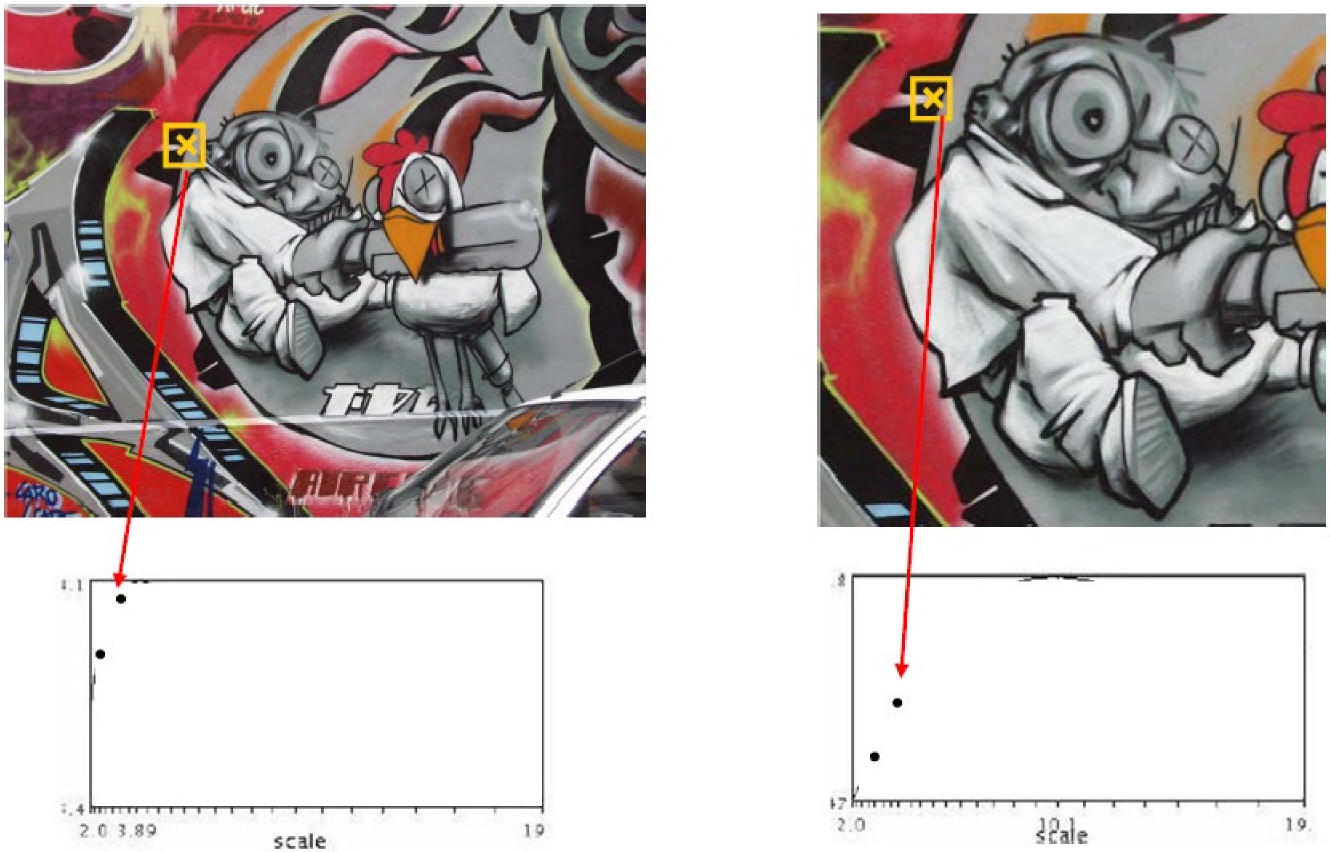
$f(I_{i_1 \ldots i_m}(x, \sigma))$

$f(I_{i_1 \ldots i_m}(x', \sigma))$

Slide credit: Krystian Mikolajczyk

Automatic Scale Selection

- Function responses for increasing scale (scale signature)



$f(I_{i_1 \ldots i_m}(x, \sigma))$

$f(I_{i_1 \ldots i_m}(x', \sigma))$

Slide credit: Krystian Mikolajczyk

Automatic Scale Selection

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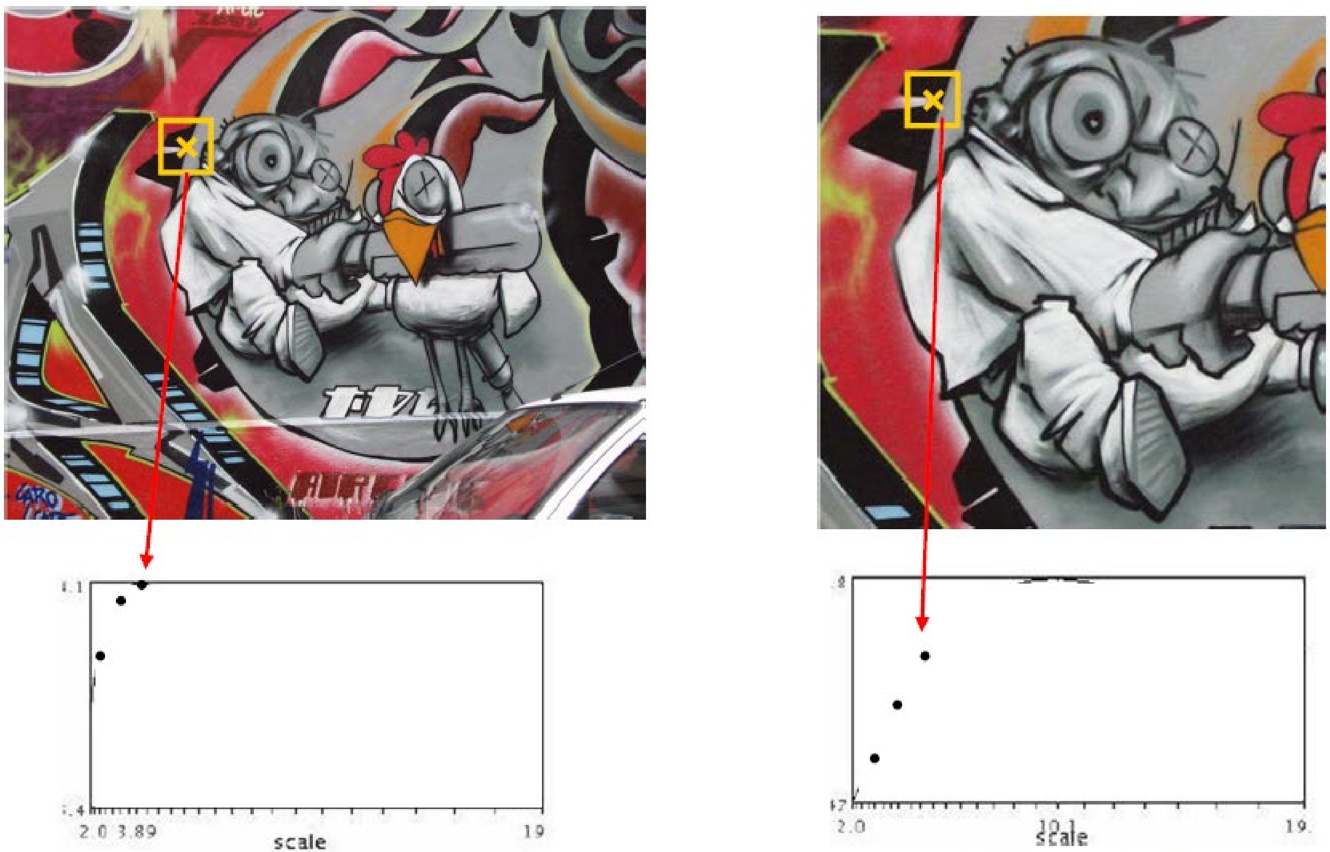
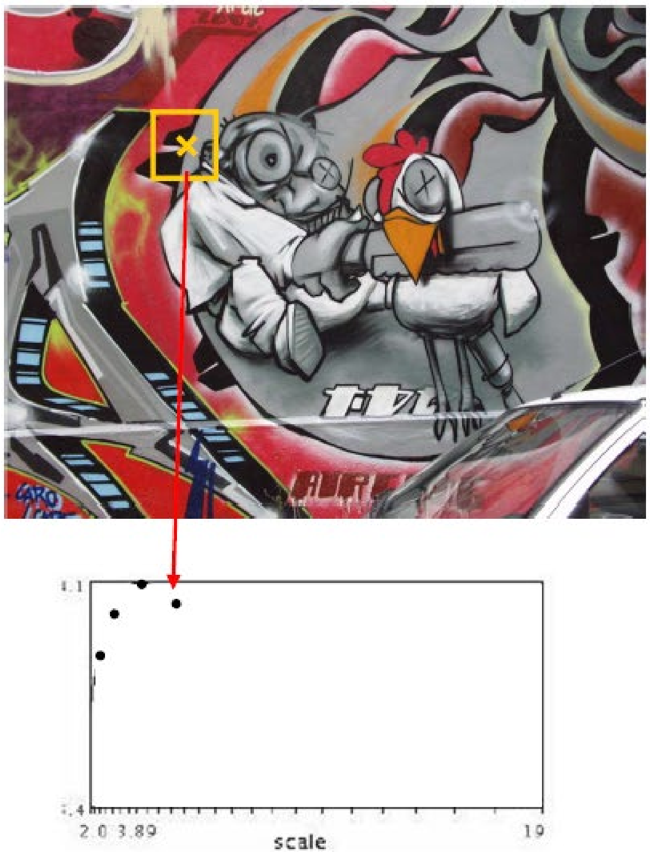


$f(I_{i_1 \ldots i_m}(x, \sigma))$ $\quad$ $f(I_{i_1 \ldots i_m}(x', \sigma))$

Slide credit: Krystian Mikolajczyk

Automatic Scale Selection

- Function responses for increasing scale (scale signature)



$f(I_{i_1 \dots i_m}(x, \sigma))$

$f(I_{i_1 \dots i_m}(x', \sigma))$

Slide credit: Krystian Mikolajczyk

Automatic Scale Selection

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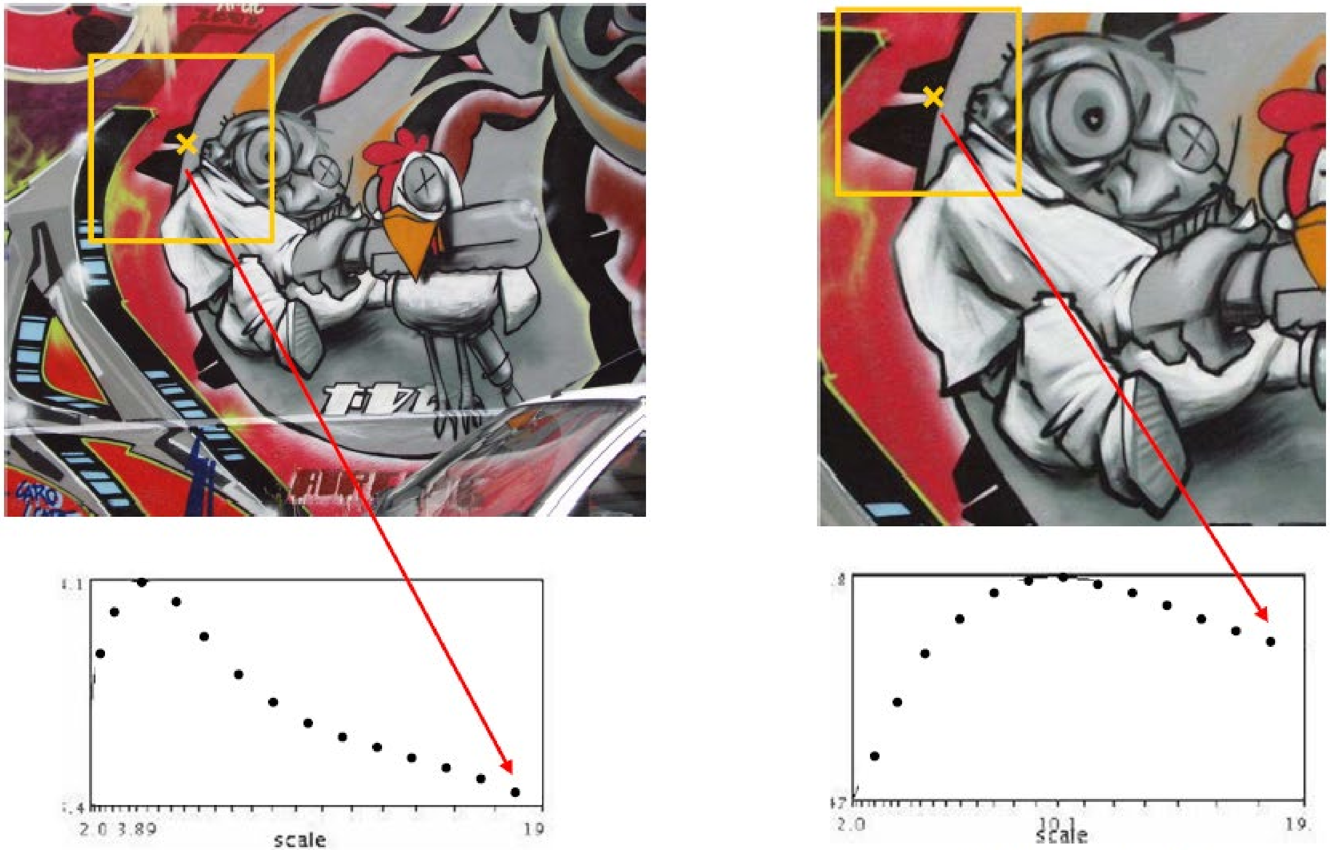
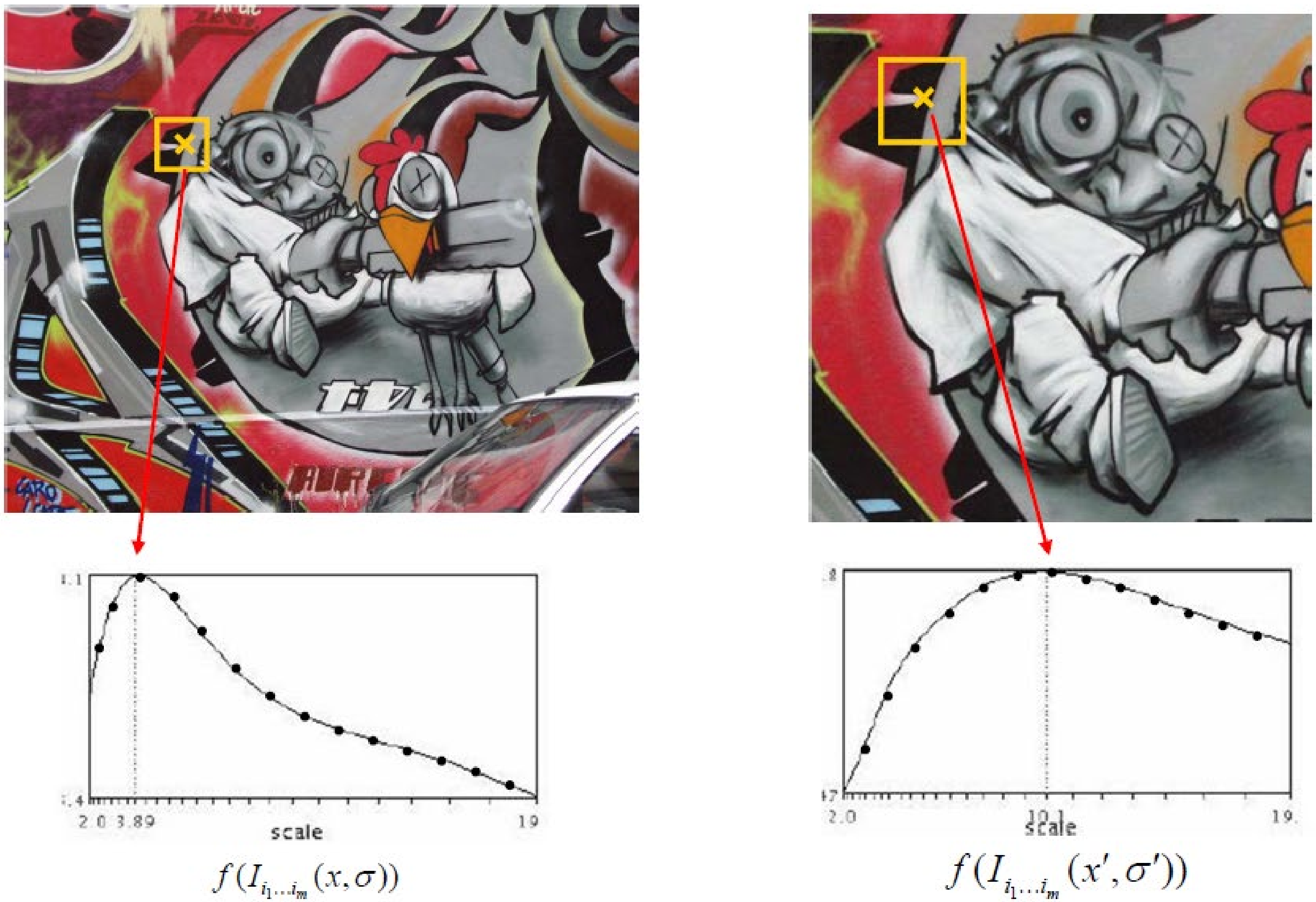


$f(I_{i_1 \dots i_m}(x,\sigma))$ $f(I_{i_1 \dots i_m}(x',\sigma))$

Slide credit: Krystian Mikolajczyk

Automatic Scale Selection

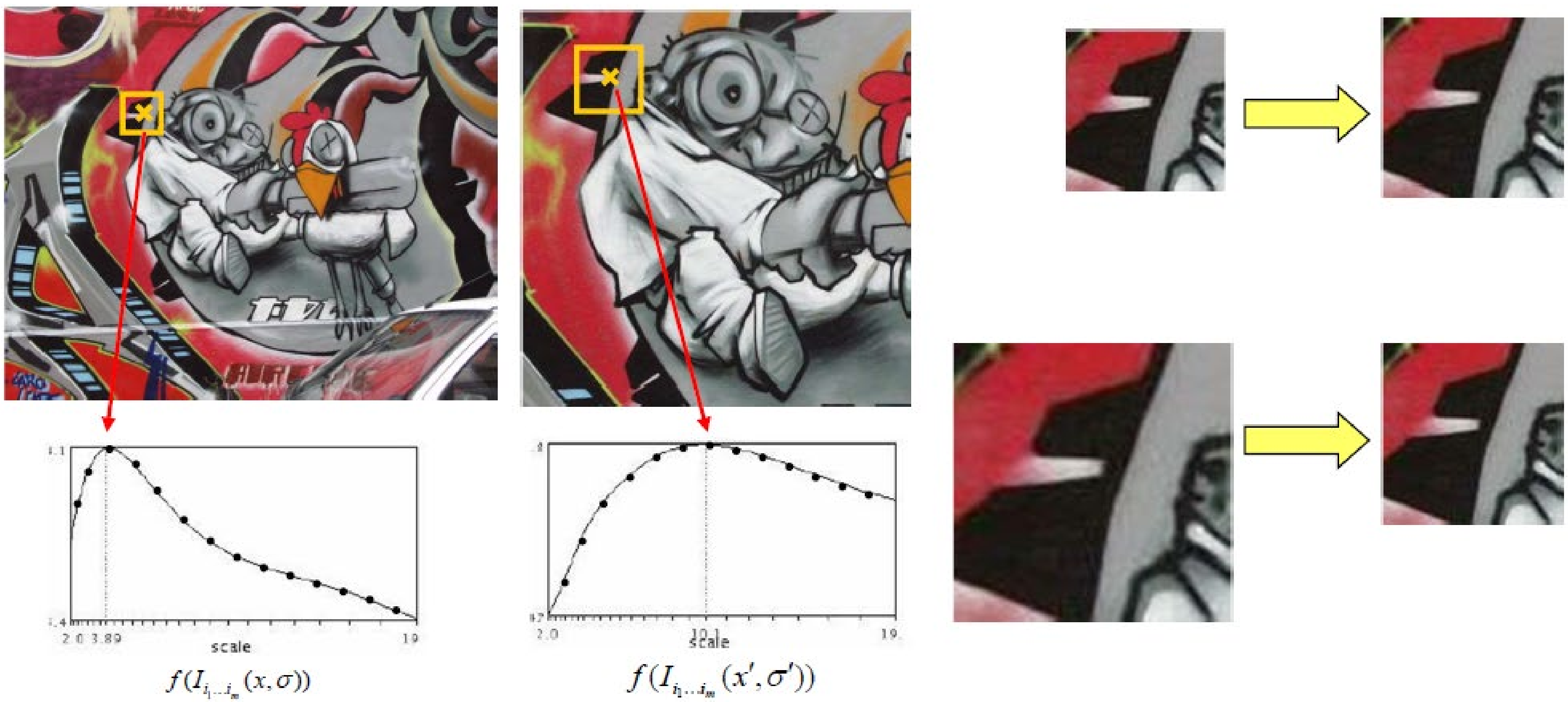
- Function responses for increasing scale (scale signature)



Slide credit: Krystian Mikolajczyk

Automatic Scale Selection

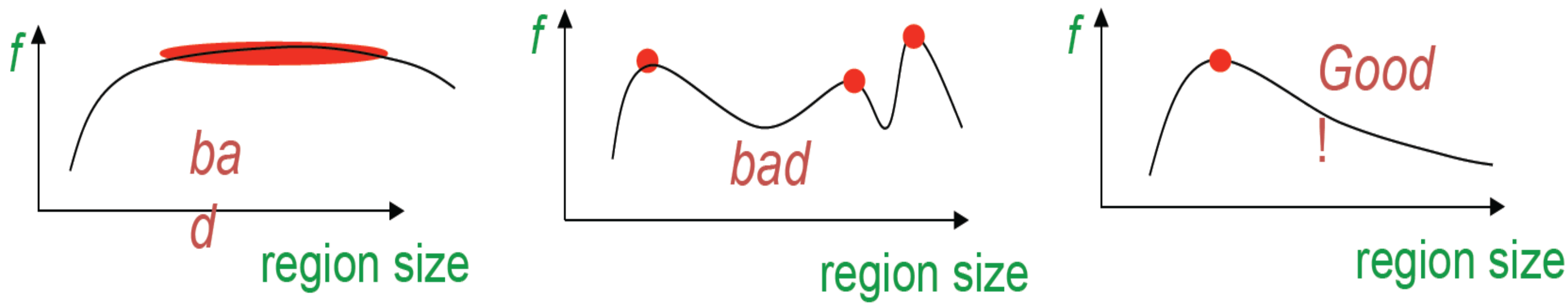
- Normalize: Rescale to fixed size



Slide credit: Tinne Tuytelaars

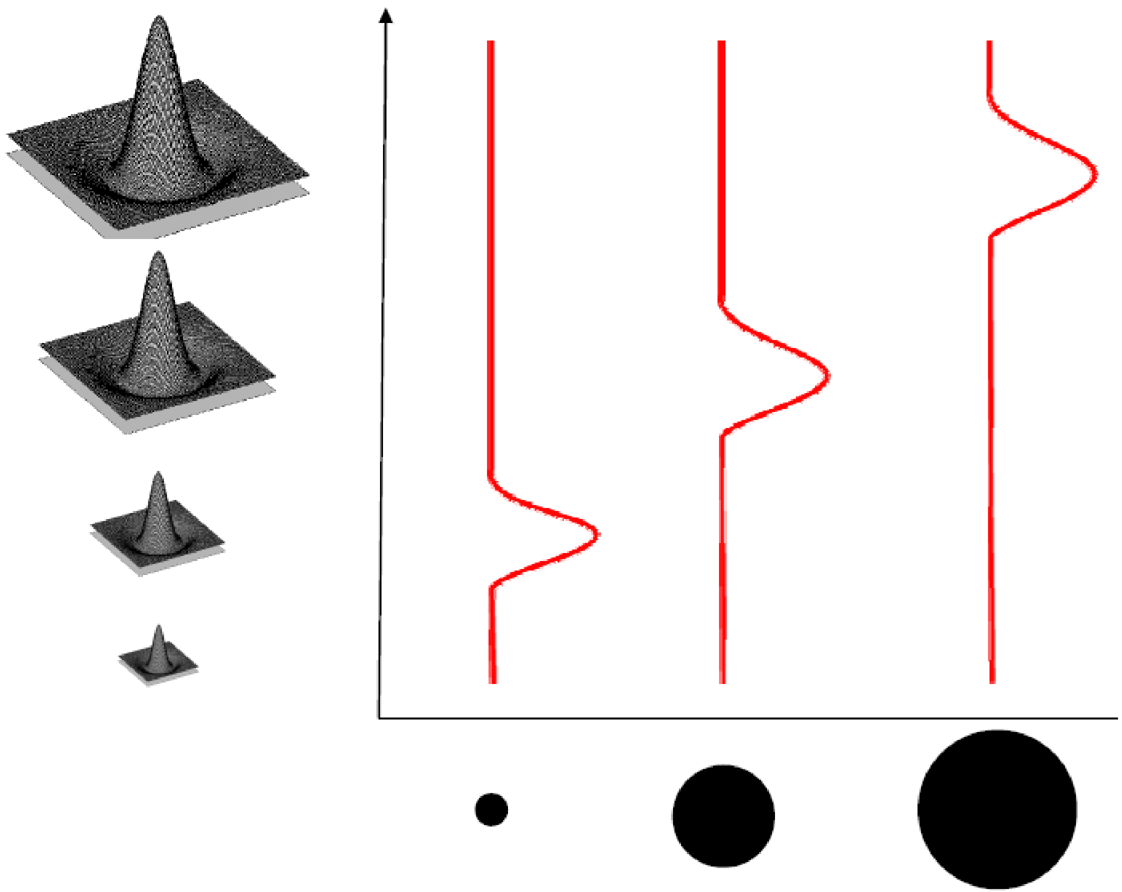
Automatic Scale Selection

- A good function for scale selection
  - It should have one stable sharp peak response to region size



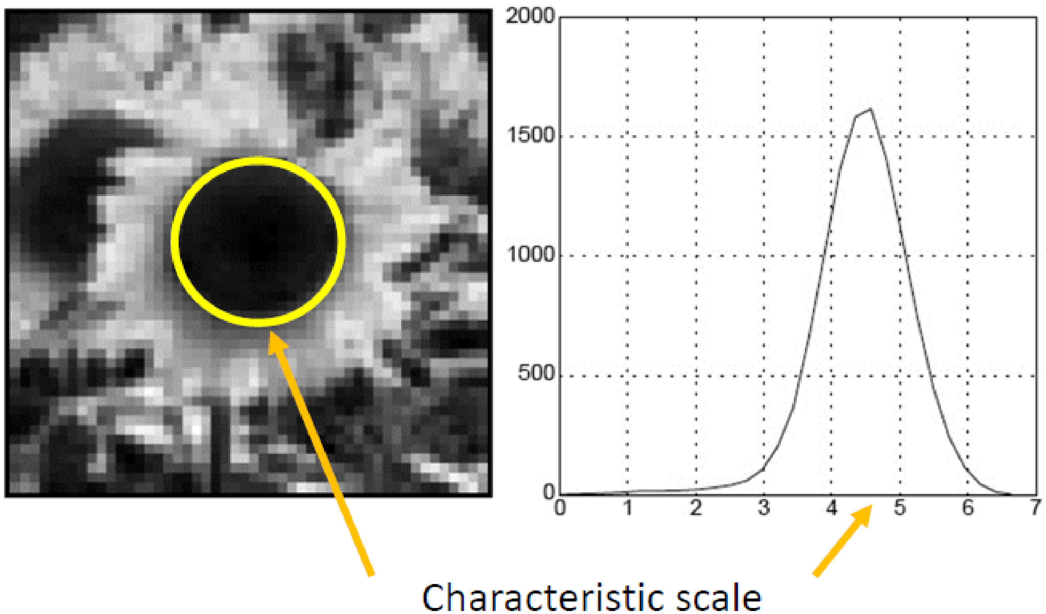
What is a useful signature function for scale?

- **Laplacian-of-Gaussian = "blob" detector**



Characteristic Scale

- We define the characteristic scale as the scale that produces peak of scale-normalized Laplacian-of-Gaussian response



T. Lindeberg (1998). "Feature detection with automatic scale selection." *International Journal of Computer Vision* 30 (2): pp 77--116.

Slide credit: Svetlana Lazebnik

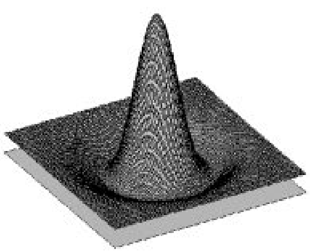
Another Fact

Spatial selection: the magnitude of the scale-normalized Laplacian-of-Gaussian response will achieve an extremum at the center of the blob, provided that its scale is “matched” to the scale of the blob

Scale-Invariant Point Detection

- Interest points:
  - Local extremum in scale space of scale-normalized Laplacian-of-Gaussian

$$\sigma^2\left[G_{xx}(\sigma)+G_{yy}(\sigma)\right]$$



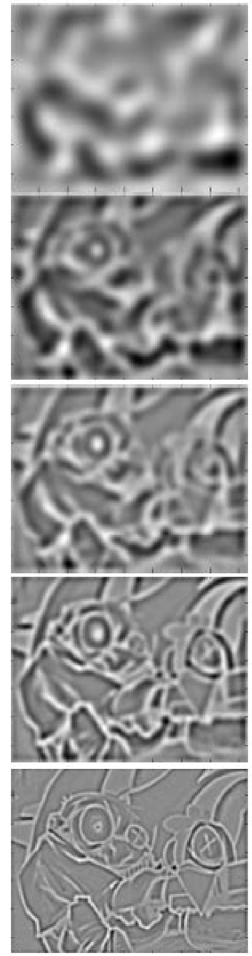
σ^5

σ^4

σ^3

σ^2

σ



Slide adapted from Krystian Mikolajczyk

Scale-Invariant Point Detection

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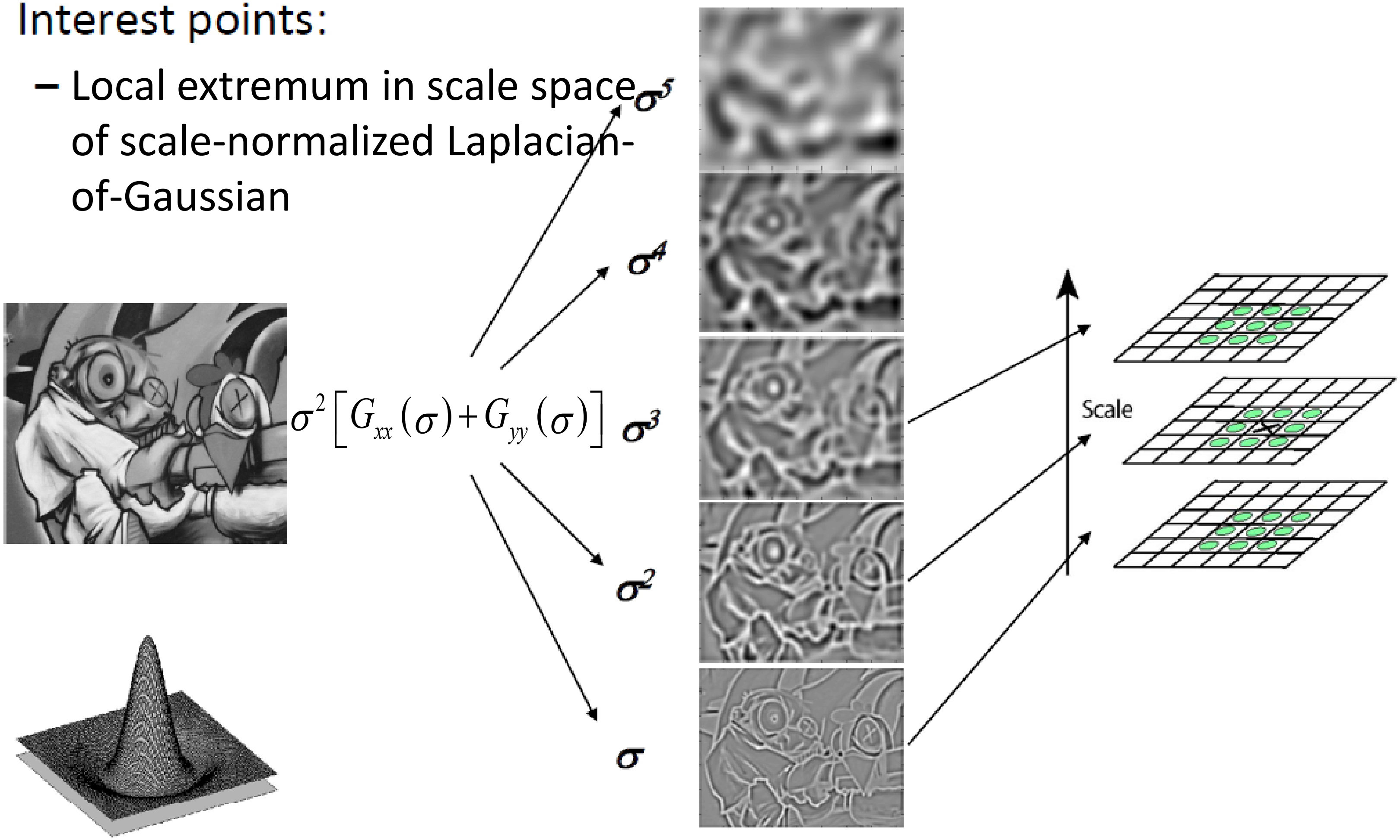
  - Local extremum in scale space of scale-normalized Laplacian-of-Gaussian



Scale-Invariant Point Detection

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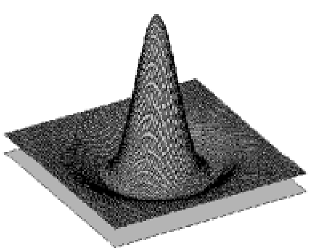
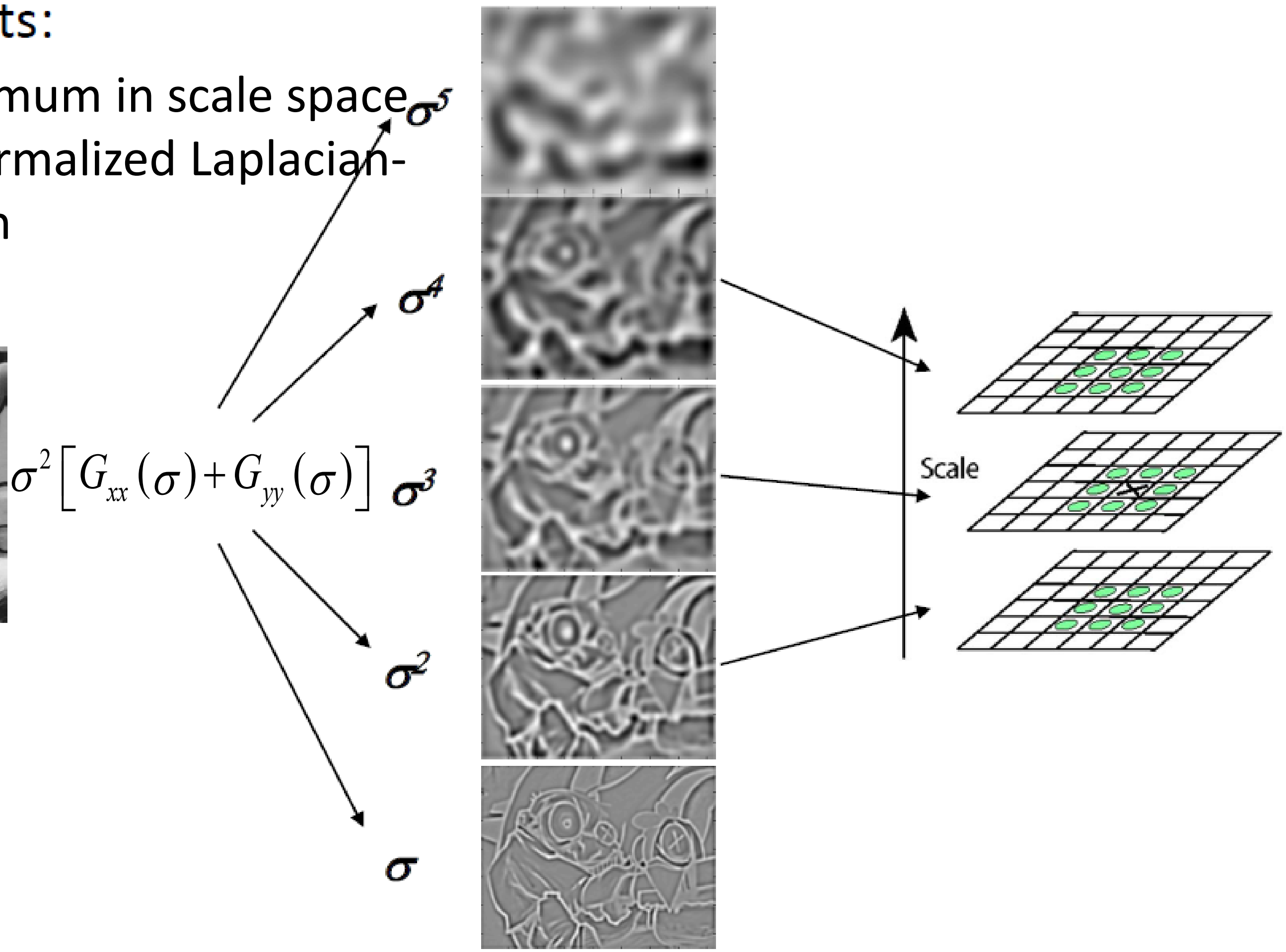




Scale-Invariant Point Detection

- Interest points:
  - Local extremum in scale space of scale-normalized Laplacian-of-Gaussian

$$\sigma^2\left[G_{xx}(\sigma)+G_{yy}(\sigma)\right]$$

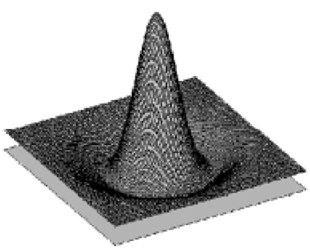
σ^5

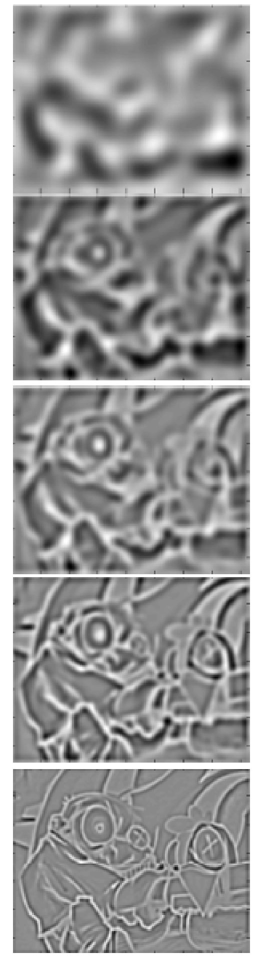
σ^4

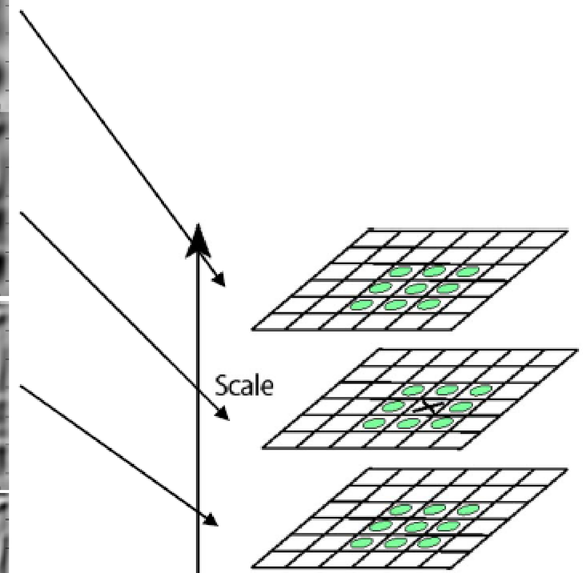
σ^3

σ^2

σ







$\Rightarrow$ **List of (x, y, σ)**

(Positions of extrema in the scale-spatial space)

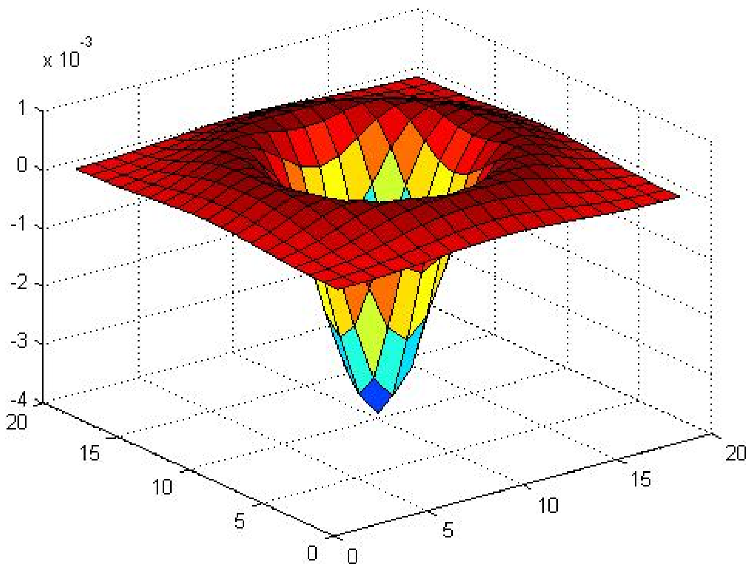
We have got want we want!

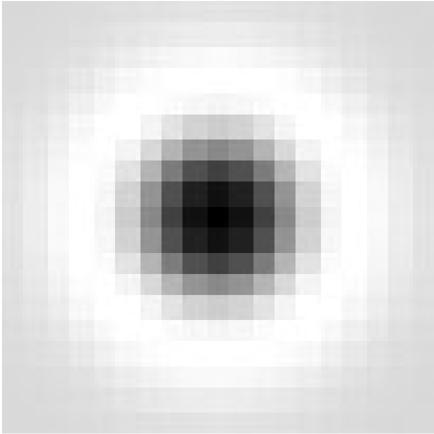
Note: local extrema is obtained by comparing the examined location with all the other 26 points around it in the scale-space

If the local extrema of scale-normalized LoG is achieved at p, two things of p can be determined: its spatial location and characteristic scale!

Scale-normalized LoG

Laplacian of Gaussian: Circularly symmetric operator for blob detection in 2D

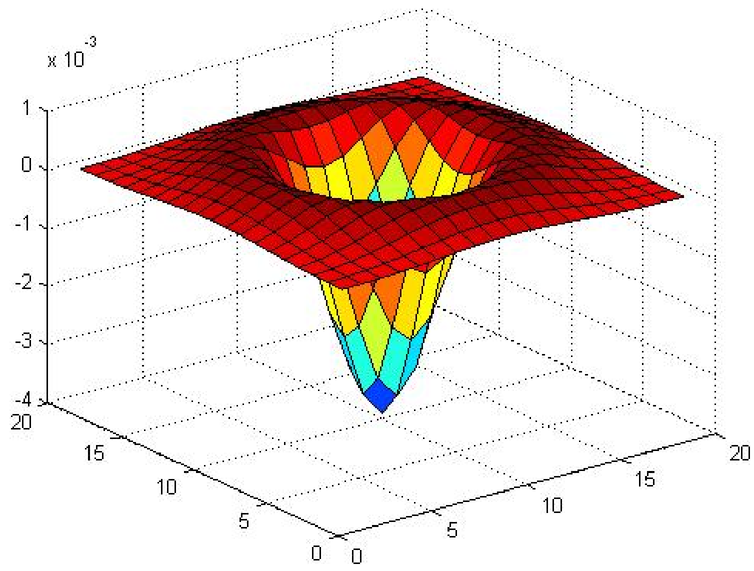


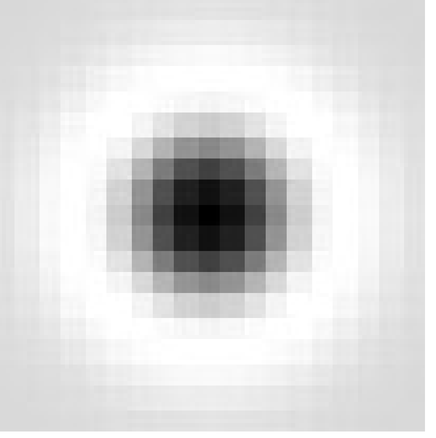


$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$$, g is the Gaussian function

Scale-normalized LoG

Laplacian of Gaussian: Circularly symmetric operator for blob detection in 2D





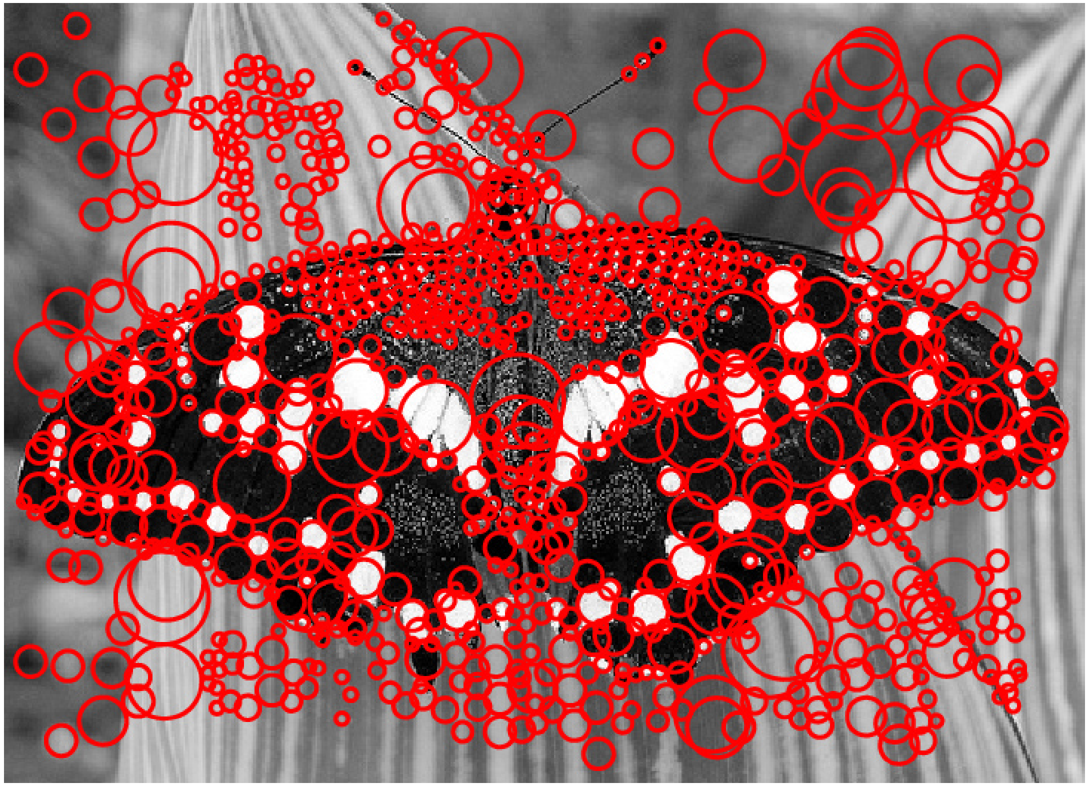
Scale-normalized: $$\nabla^2_{\text{norm}} g = \sigma^2 \left(\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} \right)$$

Scale-Invariant Point Detection: Example

Scale-Invariant Point Detection: Example

sigma = 11.9912

Scale-Invariant Point Detection: Example



Efficient implementation

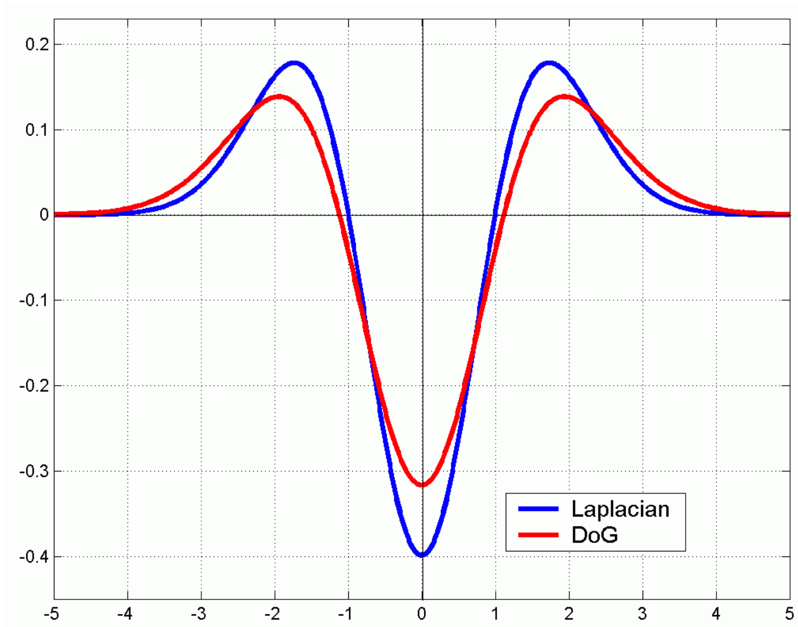
Approximating the scale-normalized LoG with a difference of Gaussians:

$$L = \sigma^2 \left(G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma) \right)$$

(scale-normalized LoG)

$$DoG = G(x, y, k\sigma) - G(x, y, \sigma)$$

(Difference of Gaussians)



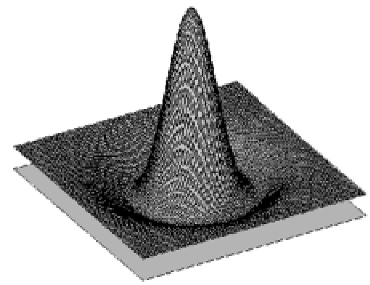
where Gaussian is

$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2} \right)$$

Assignment!

DoG

- Difference of Gaussians as approximation of scale-normalized LoG
  - This is used e.g. in Lowe's SIFT pipeline for feature detection.
- Advantages
  - No need to compute 2nd derivatives
  - Gaussians are computed anyway, e.g. in a Gaussian pyramid.



Slide credit: Bastian Leibe

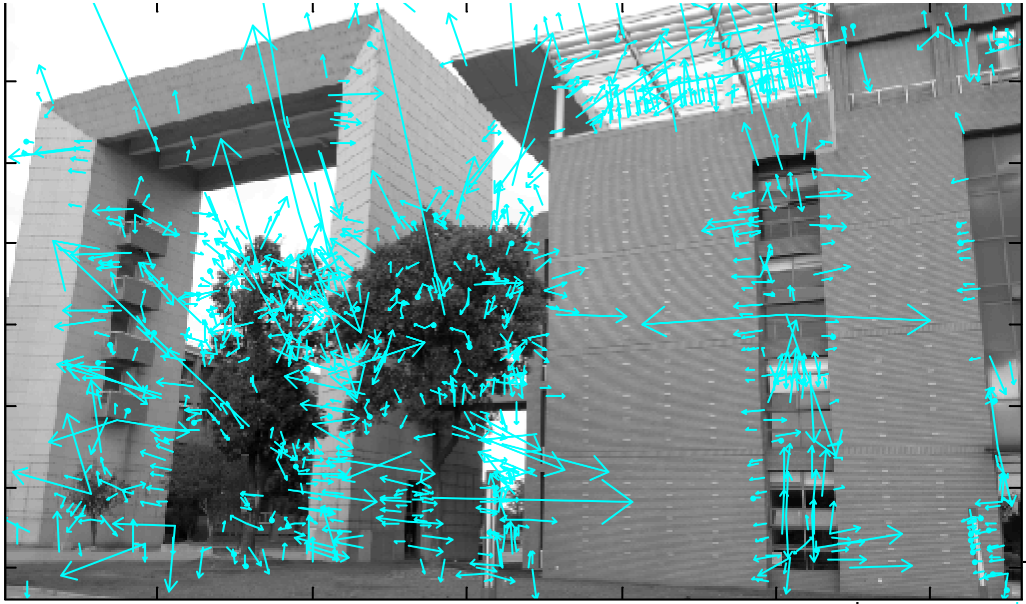
Scale-Invariant Point Detection

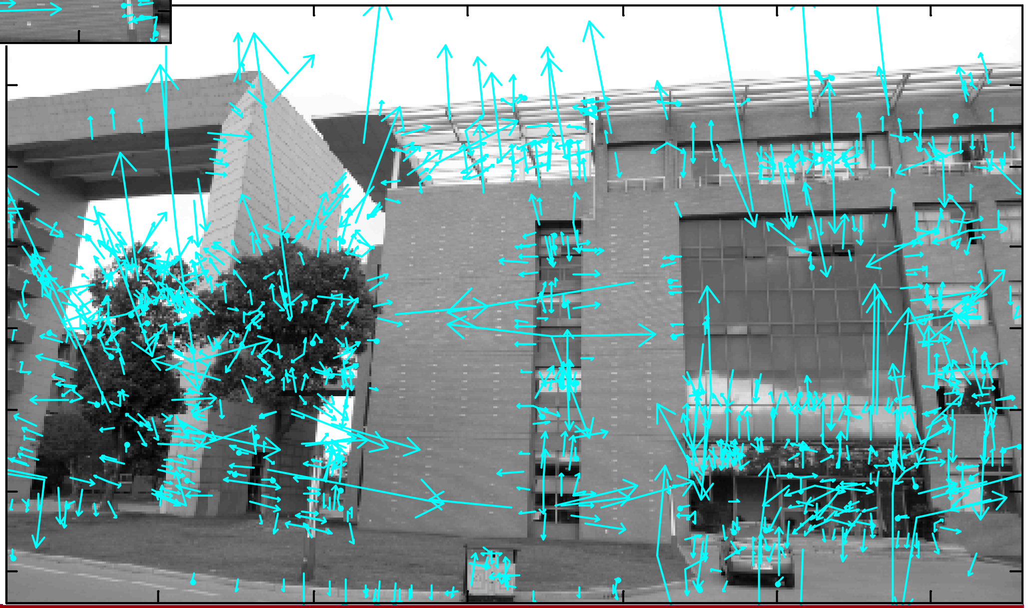
- Given: Two images of the same scene with a large *scale difference* between them.
- Goal: Find *the same* interest points *independently* in each image.
- Solution: Search for *maxima* of suitable functions in *scale* and in *space* (over the image).

- Two strategies
  - scale-normalized LoG
  - Difference-of-Gaussian (DoG) as a fast approximation
  - *These can be used either on their own, or in combinations with single-scale keypoint detectors (Harris, Hessian).*

Examples

Examples





Interest points found by DoG extrema

What does the arrows mean?

Next lecture!!


Q+A