

# Machine Learning

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Neural Network

Dr. Shuang LIANG

# Today's Topics

- Neural Network Introduction
- Neural Network Structure
- How Neural Network Works
- Backpropagation

# Today's Topics

- *Neural Network Introduction*
- Neural Network Structure
- How Neural Network Works
- Backpropagation

# Neural networks are a hot topic



# A bit of history

The **Mark I Perceptron** machine was the first implementation of the perceptron algorithm.

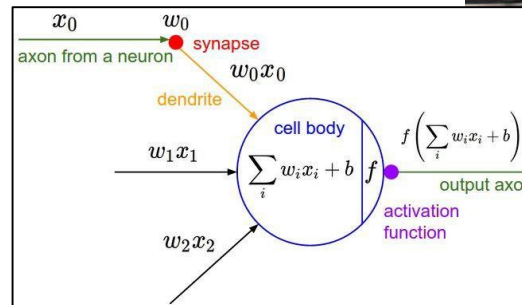
The machine was connected to a camera that used 20×20 cadmium sulfide photocells to produce a 400-pixel image.

recognized  
letters of the alphabet

update rule:

$$w_i(t+1) = w_i(t) + \alpha(d_j - y_j(t))x_{j,i}$$

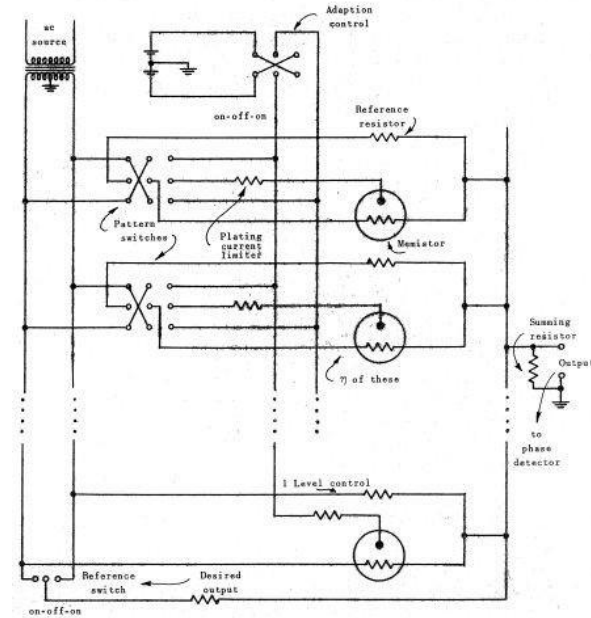
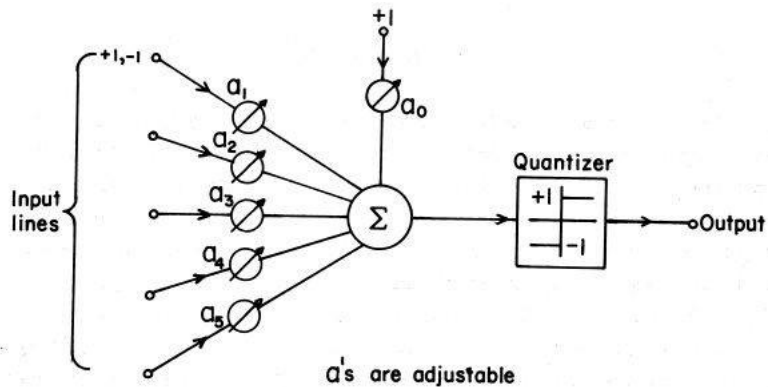
$$f(x) = \begin{cases} 1 & \text{if } w \cdot x + b > 0 \\ 0 & \text{otherwise} \end{cases}$$



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Frank Rosenblatt, ~1957: Perceptron

# A bit of history



Widrow and Hoff, ~1960: Adaline/Madaline

These figures are reproduced from [Widrow 1960, Stanford Electronics Laboratories Technical Report](#) with permission from [Stanford University Special Collections](#).

# A bit of history

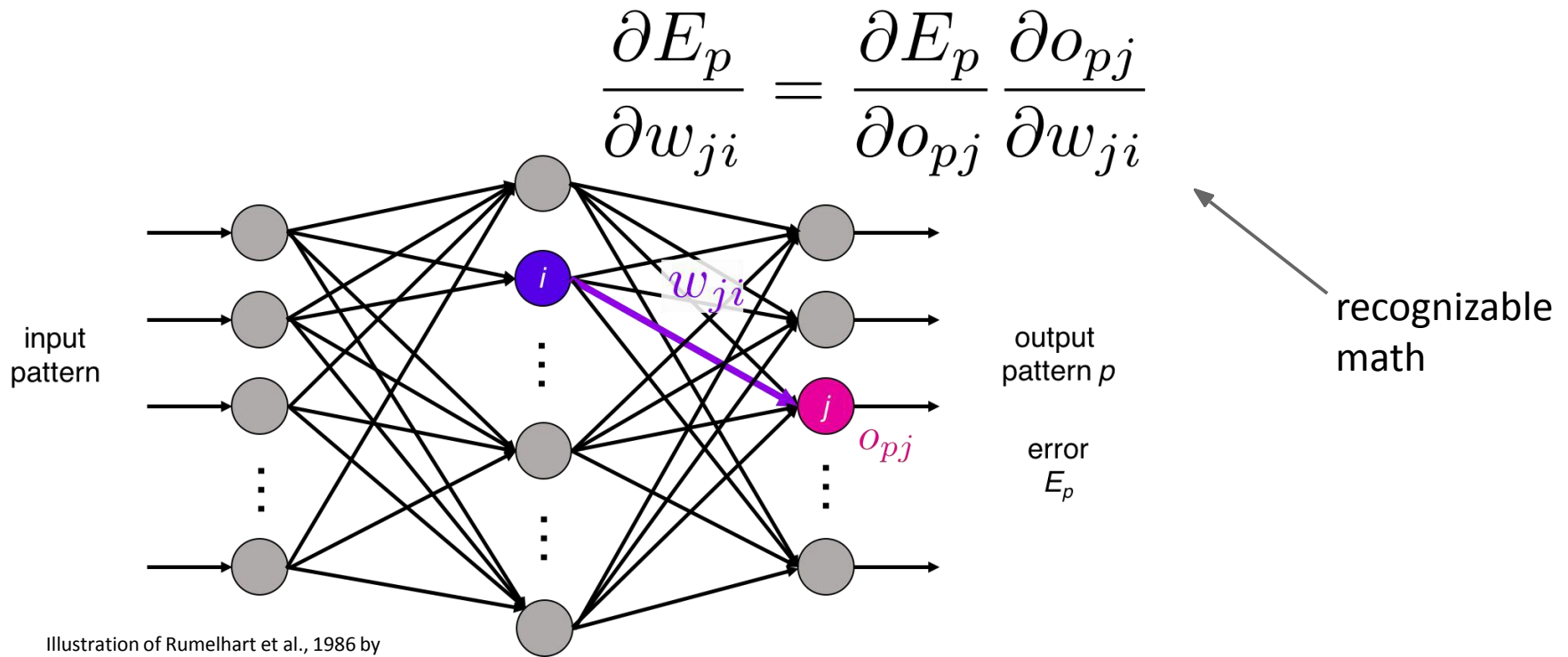


Illustration of Rumelhart et al., 1986 by  
Lane McIntosh, copyright CS231n 2017

Rumelhart et al., 1986: First time back-propagation became popular

# A bit of history

[Hinton and Salakhutdinov 2006]

Reinvigorated research in  
Deep Learning

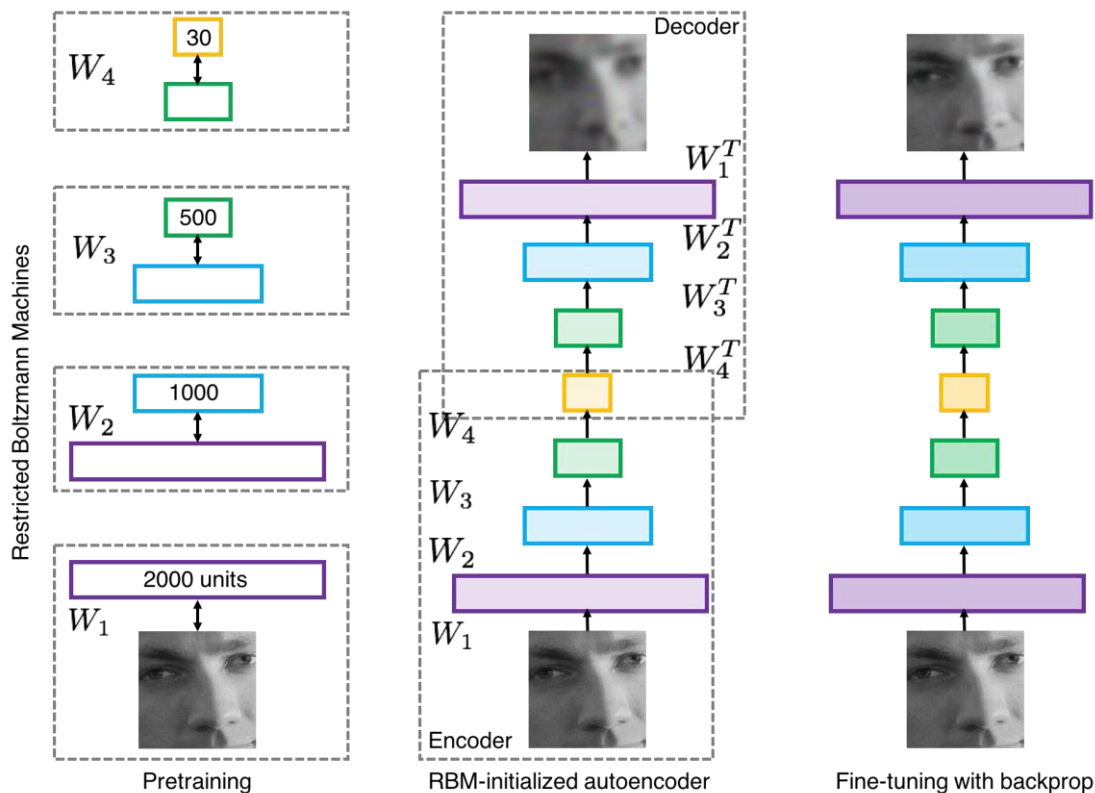


Illustration of Hinton and Salakhutdinov  
2006 by Lane McIntosh, copyright  
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# A bit of history

## First strong results

### ***Acoustic Modeling using Deep Belief Networks***

*Abdel-rahman Mohamed, George Dahl, Geoffrey Hinton, 2010*

### ***Context-Dependent Pre-trained Deep Neural Networks for Large Vocabulary Speech Recognition***

*George Dahl, Dong Yu, Li Deng, Alex Acero, 2012*

### ***Imagenet classification with deep convolutional neural networks***

*Alex Krizhevsky, Ilya Sutskever, Geoffrey E Hinton, 2012*

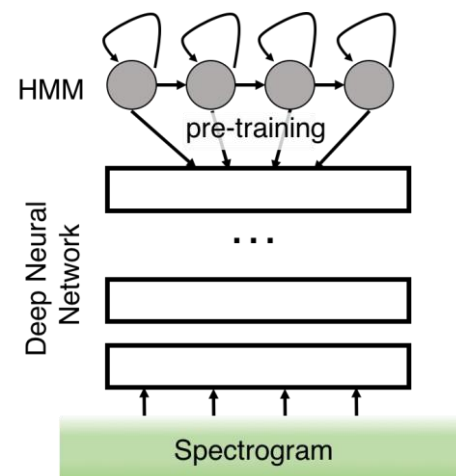
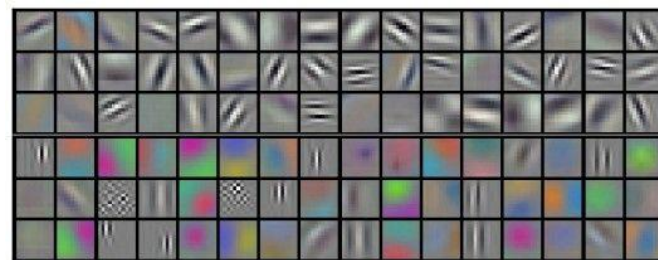
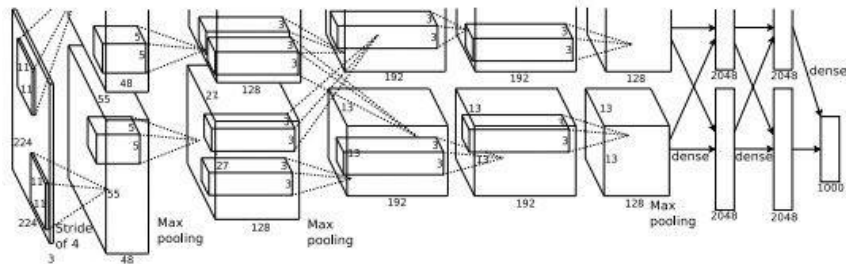


Illustration of Dahl et al. 2012 by Lane McIntosh, copyright CS231n 2017



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# Ups and downs of Neural Networks

- 1958: Perceptron (linear model)
- 1969: Perceptron has limitation
- 1980s: Multi-layer perceptron
  - 1986: Backpropagation
- 1989: 1 hidden layer is “good enough”, why deep?
  - 2006: RBM initialization

# Ups and downs of Neural Networks

- 2009: GPU
- 2011: Start to be popular in speech recognition
  - 2012: win ILSVRC image competition
- 2015.2: Image recognition surpassing human-level performance
  - 2016.3: Alpha GO beats Lee Sedol
- 2016.10: Speech recognition system as good as humans
  - Now: Transformer, BERT, Autopilot...

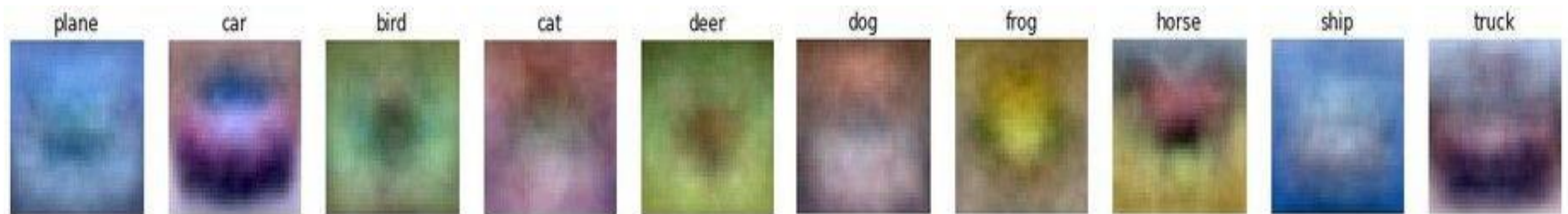
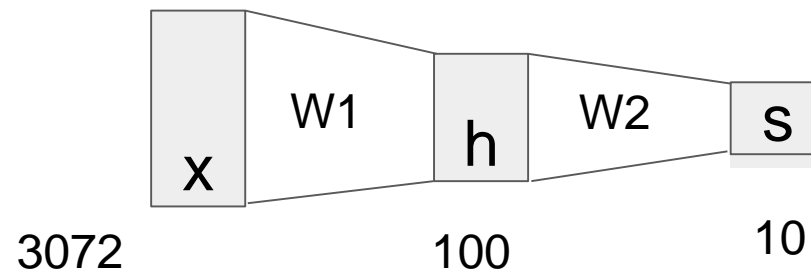
# Neural Networks

(**Before**) Linear score function

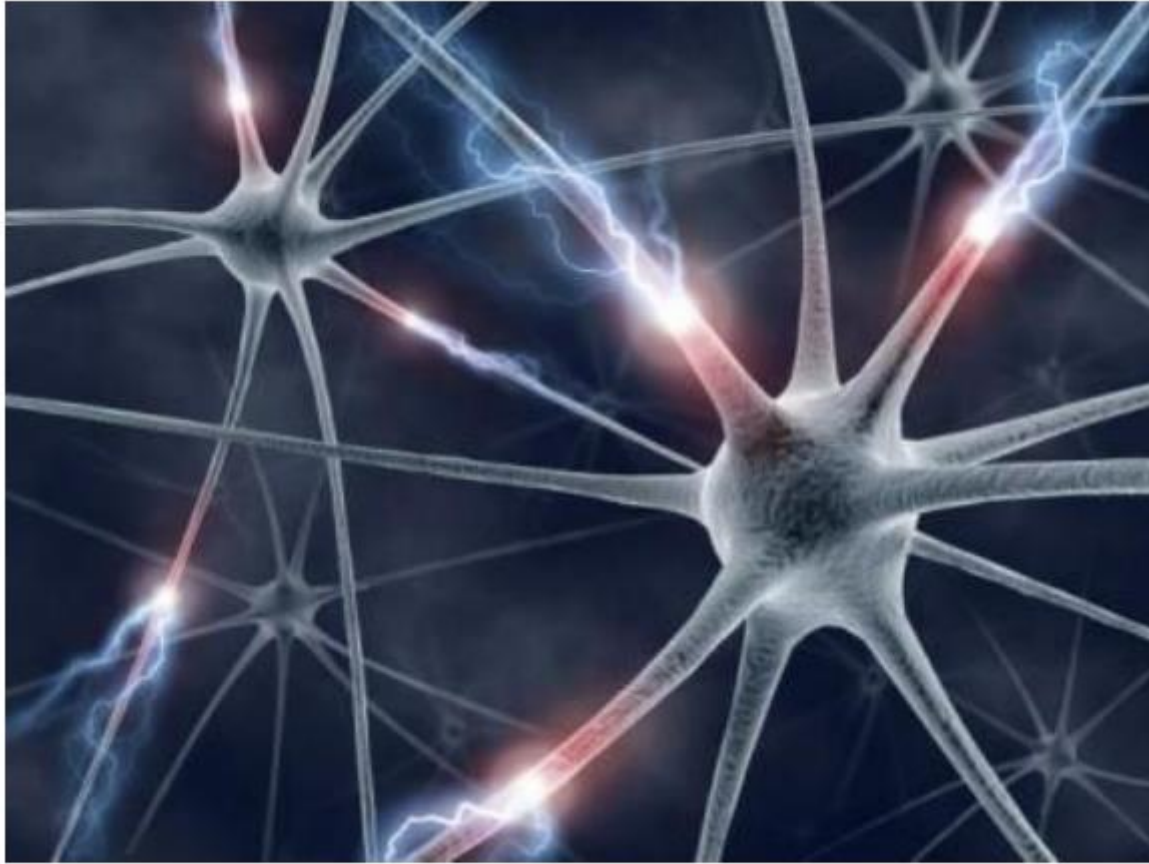
$$f = Wx$$

(**Now**) 2-layer Neural Network

$$f = W_2 \max(0, W_1 x)$$

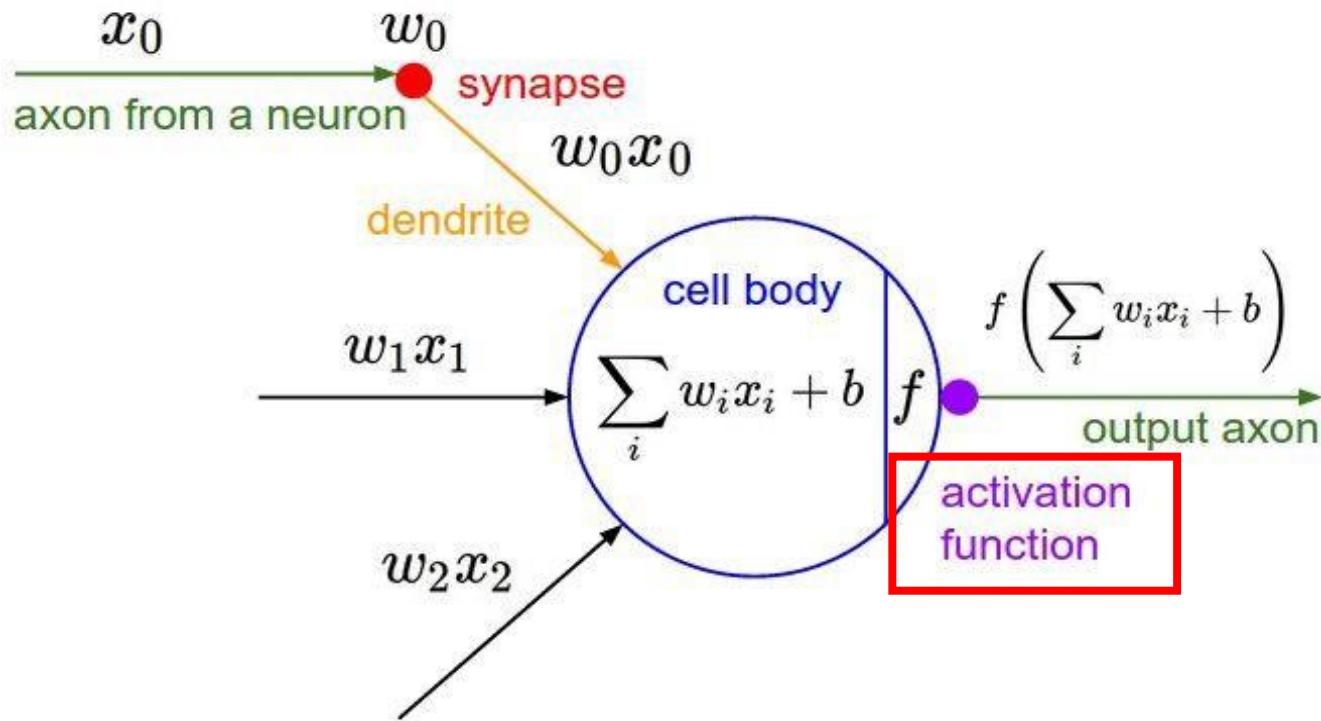


# Biological neuron



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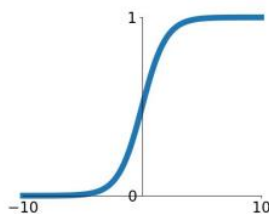
# Artificial neuron



# Activation Function

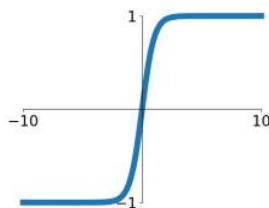
## Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



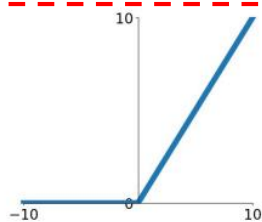
## tanh

$$\tanh(x)$$



## ReLU

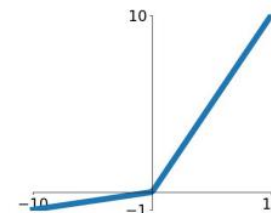
$$\max(0, x)$$



commonly used

## Leaky ReLU

$$\max(0.1x, x)$$

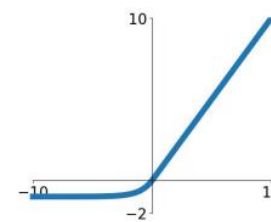


## Maxout

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

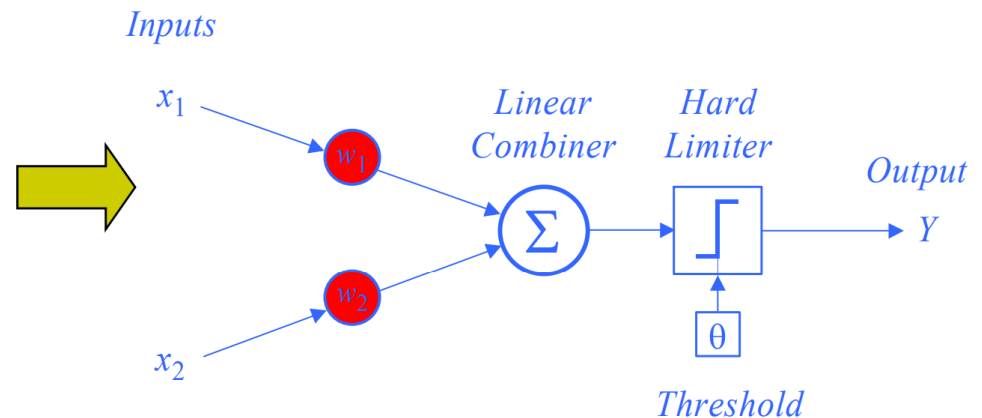
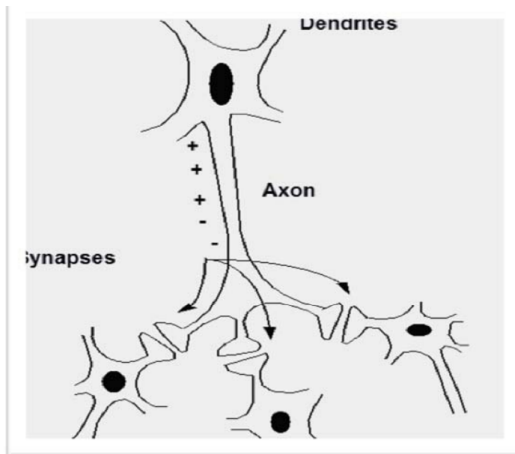
## ELU

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



# Biological v.s. Artificial

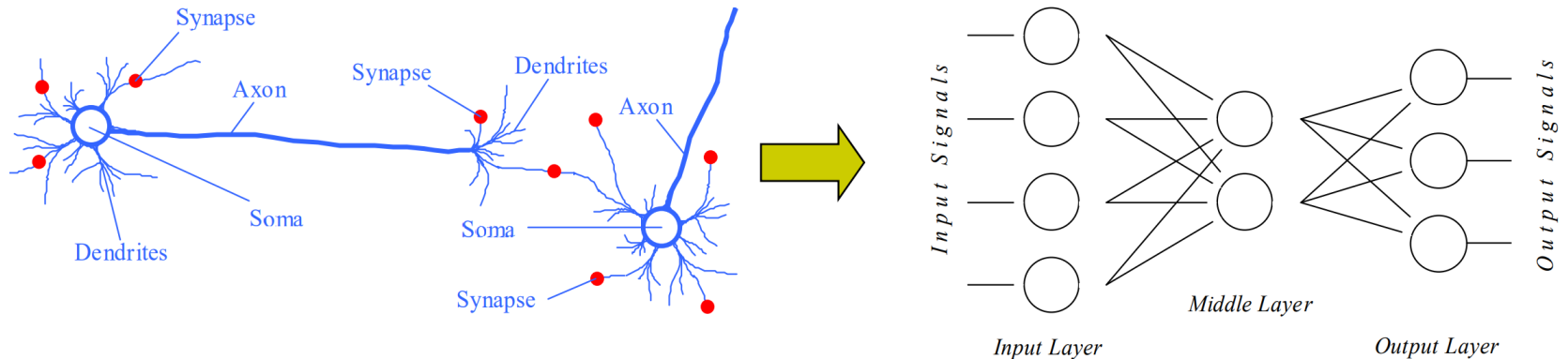
- From biological neuron to artificial neuron (perceptron)





# Biological v.s. Artificial

- From biological neuron network to artificial neuron networks



# Be very careful with your brain analogies!

- **Biological Neurons**

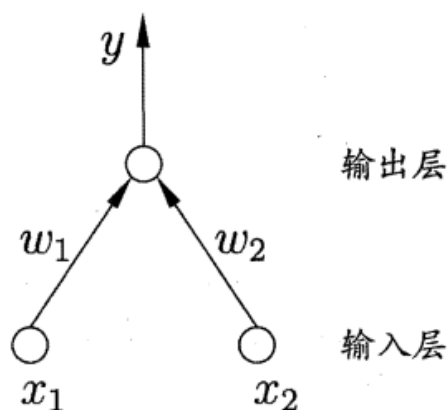
- Many different types
  - Dendrites can perform complex non-linear computations
  - Synapses are not a single weight but a complex non-linear dynamical system
  - Rate code may not be adequate
- We can ignore whether the neural network actually simulates a biological neural network, and just think of a neural network as **a mathematical model with many parameters**

# Today's Topics

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- ***Neural Network Structure***
- How Neural Network Works
- Backpropagation

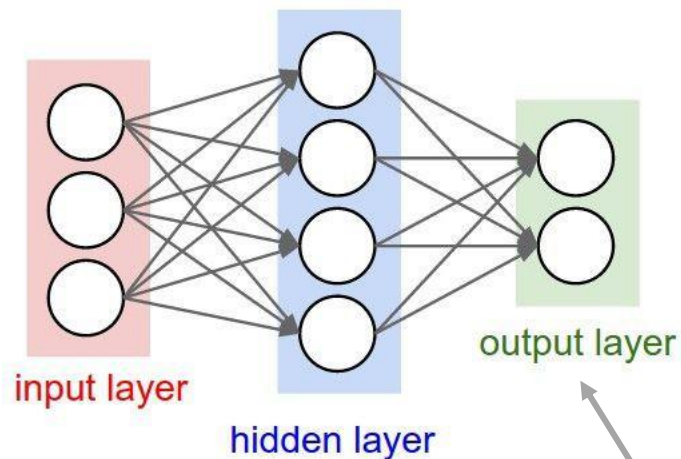
# Perceptron

- It consists of two layers of neurons, the **input layer** and the **output layer**.

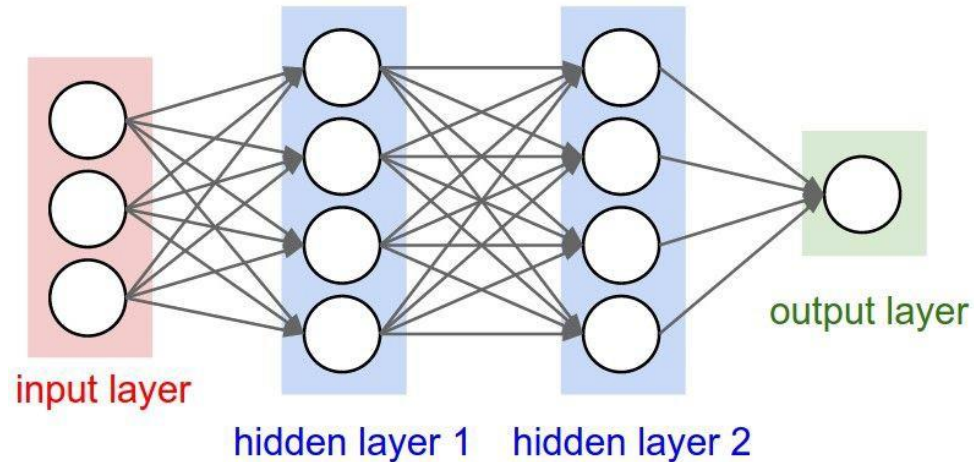


- Can realize logical AND, OR, NOT operation
- Only the neurons in the output layer perform activation function processing, and the learning ability is very limited
- Can't solve problems that are not linear separable, like XOR.

# Multi-layer Network



"2-layer Neural Net", or  
"1-hidden-layer Neural Net"

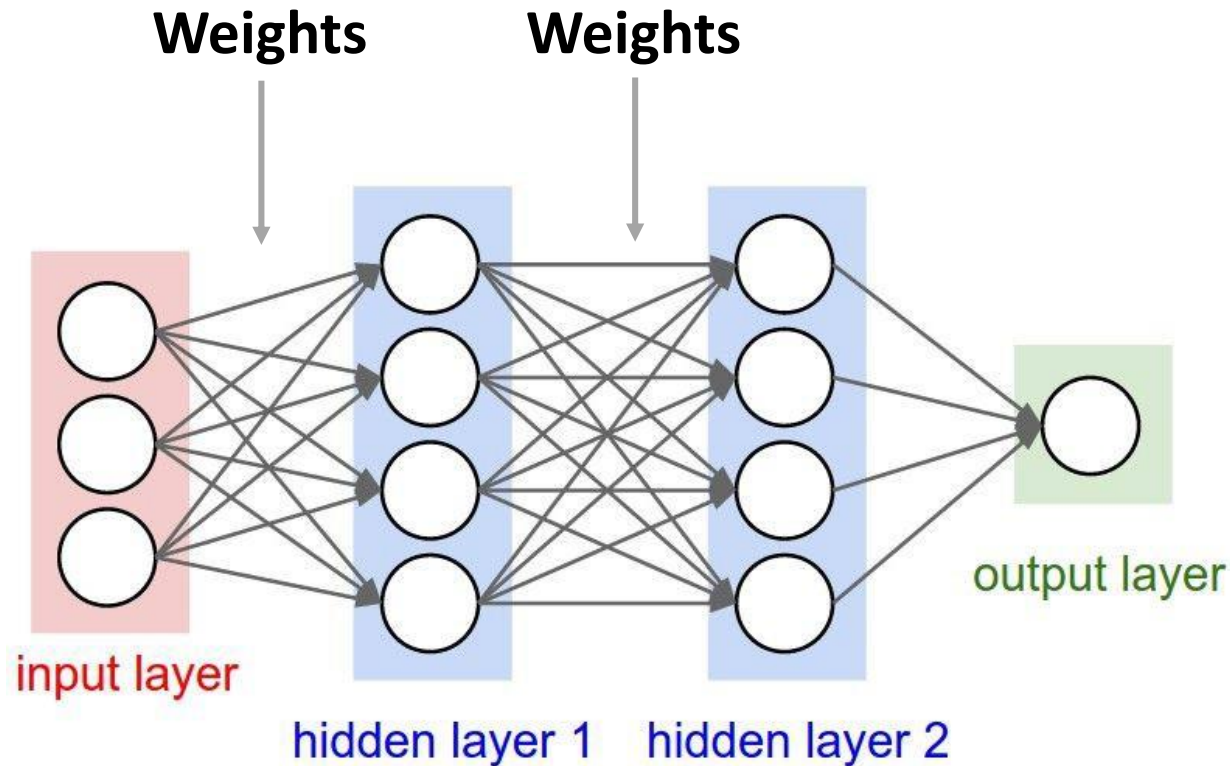


"3-layer Neural Net", or  
"2-hidden-layer Neural Net"

"Fully-connected" layers

Hidden layer and output layer neurons are  
*functional neurons* with activation functions

# Multi-layer Network



The learning process of the neural network is to adjust the **"connection weight"** between neurons and the **threshold** of each functional neuron according to the training data

# Deep Neural Network

Deep = Many hidden layers

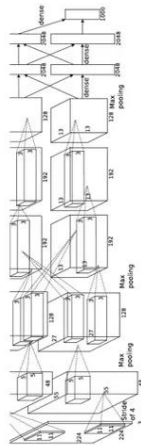
Now the commonly used  
**ResNet** has reached **152** layers

8 layers

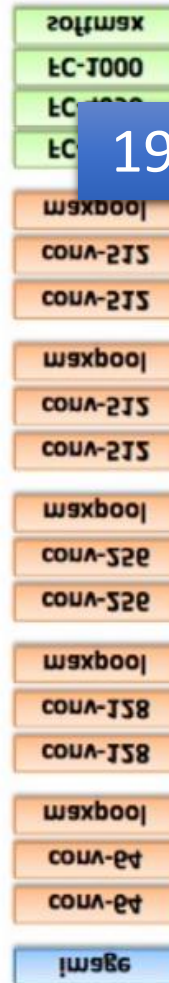
16.4%

7.3%

[http://cs231n.stanford.edu/slides/winter1516\\_lecture8.pdf](http://cs231n.stanford.edu/slides/winter1516_lecture8.pdf)



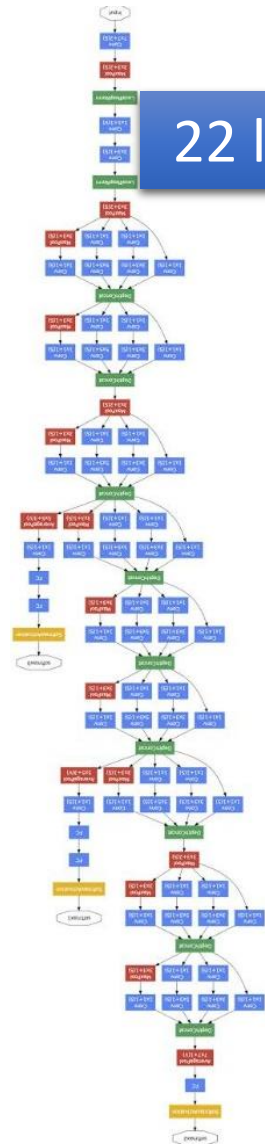
AlexNet (2012)



19 layers

VGG (2014)

6.7%



22 layers

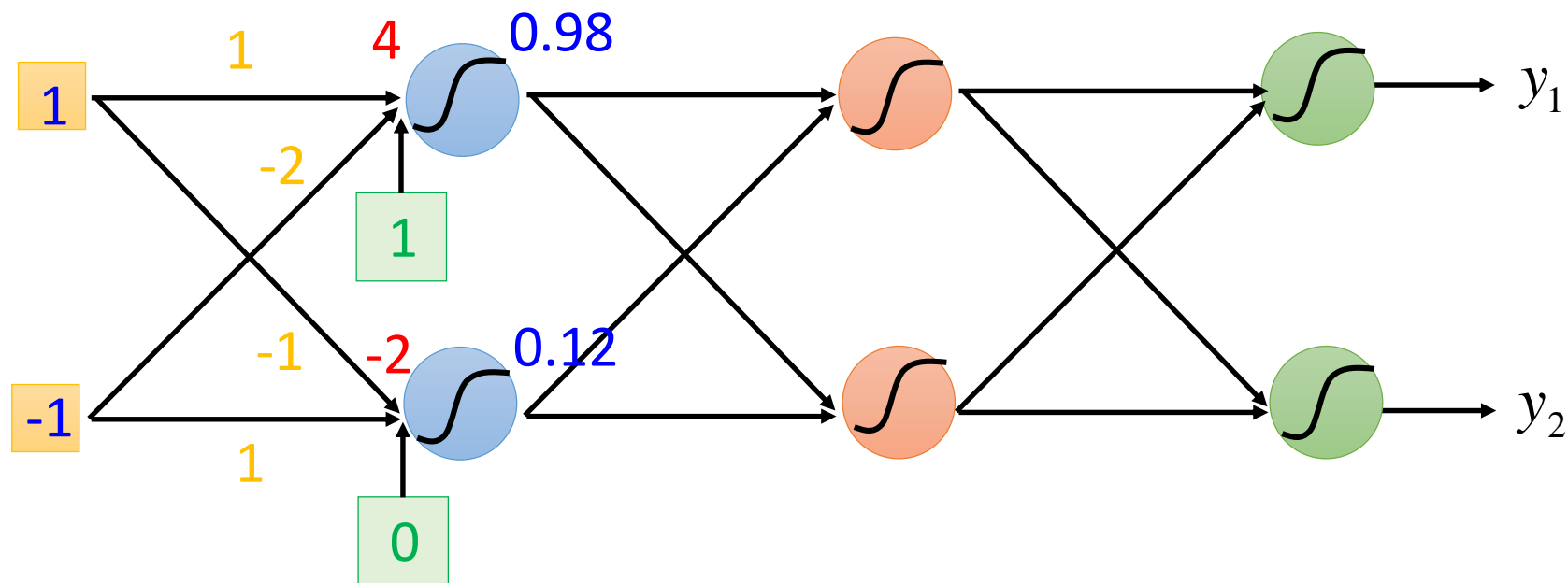
GoogleNet (2014)

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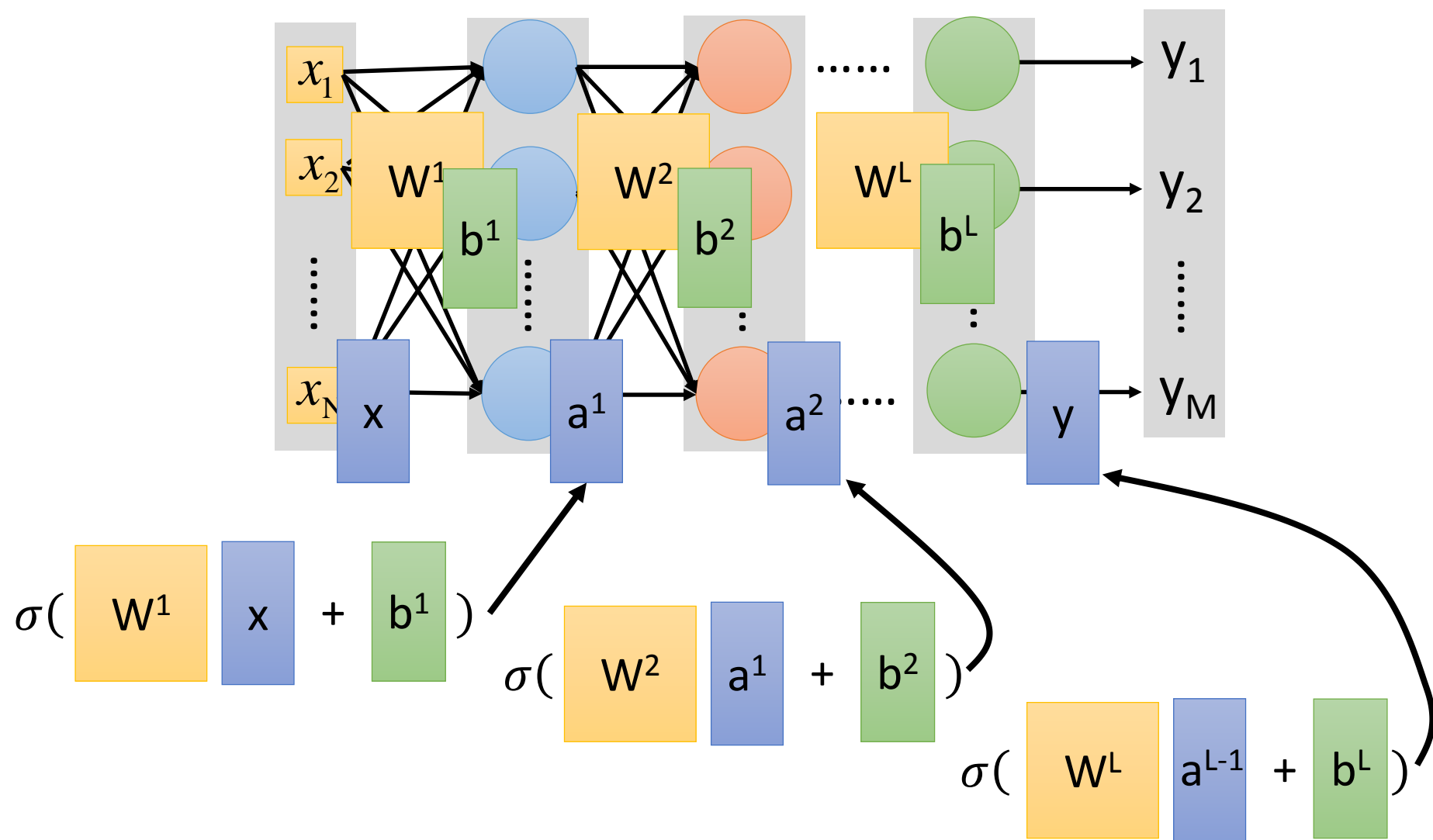


# Matrix Operation

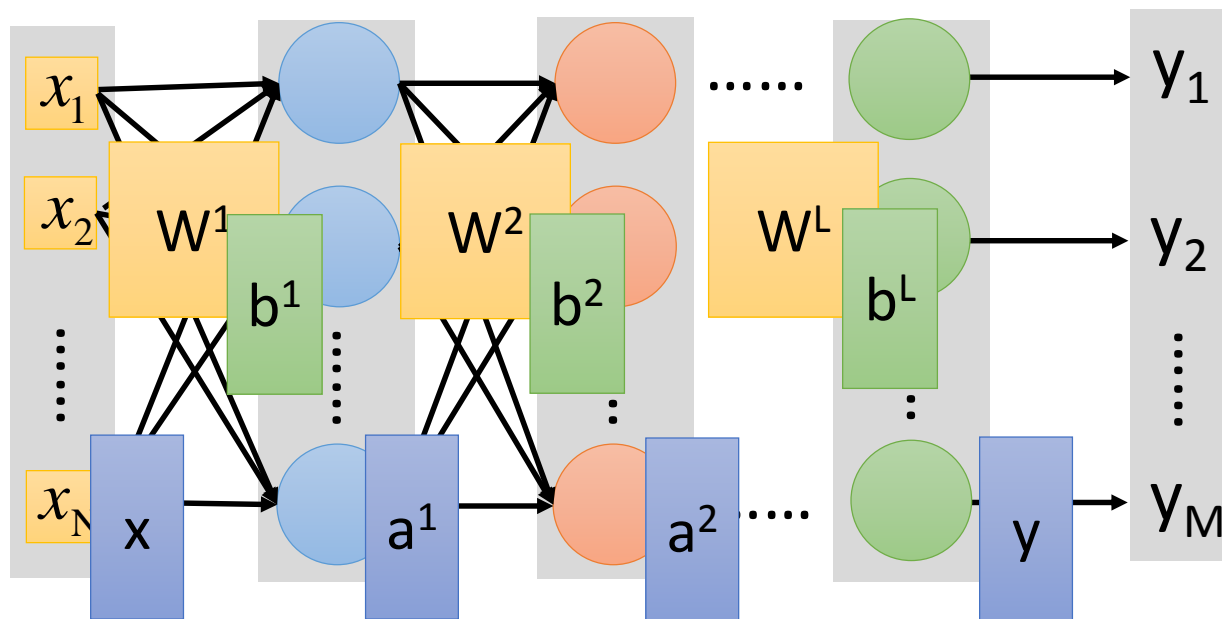


$$\sigma\left( \underbrace{\begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{\begin{bmatrix} 4 \\ -2 \end{bmatrix}} \right) = \begin{bmatrix} 0.98 \\ 0.12 \end{bmatrix}$$

# Function Nesting



# Function Nesting

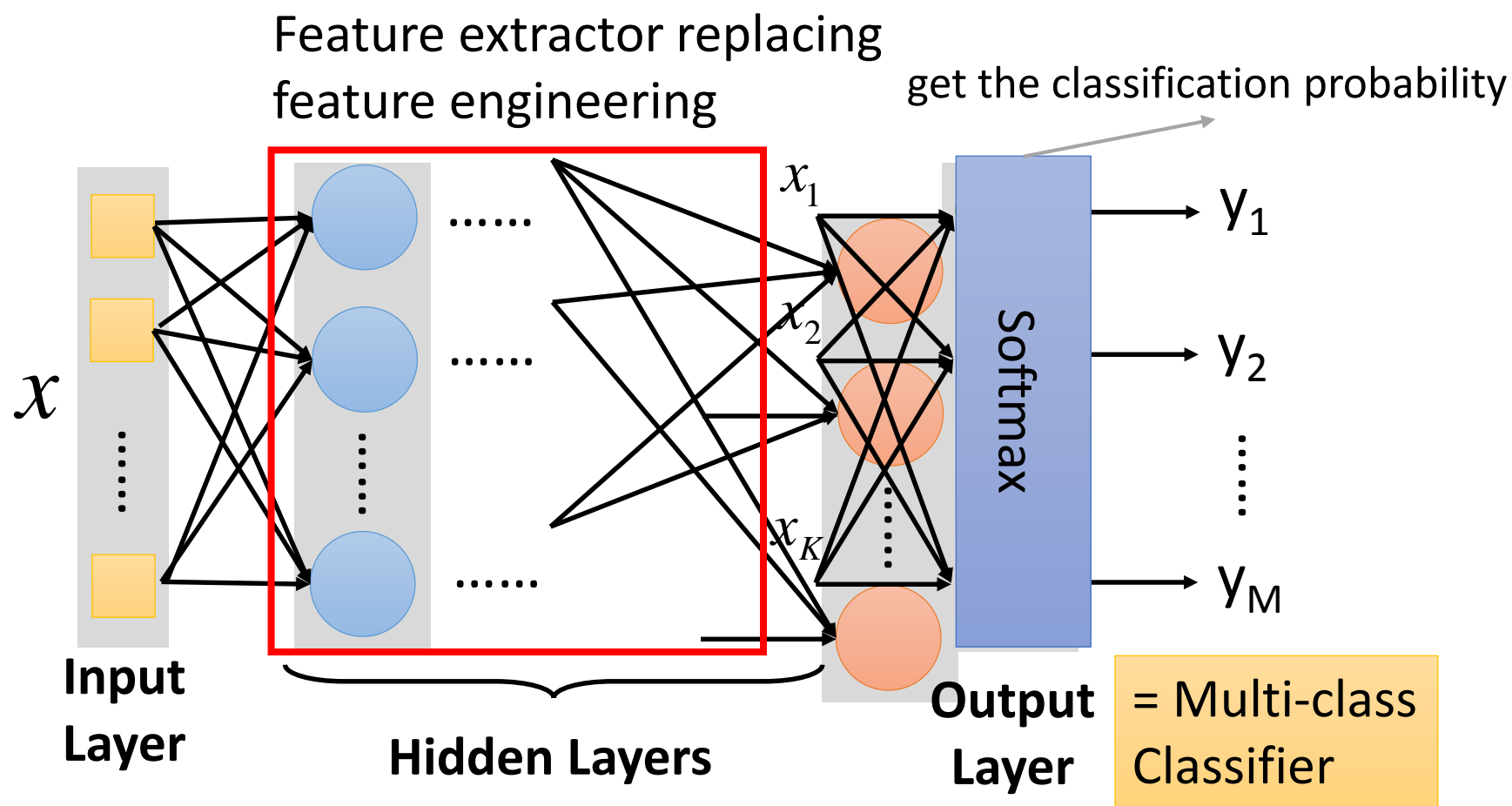


$$y = f(x)$$

Using parallel computing techniques  
to speed up matrix operation

$$= \sigma(W^L \dots \sigma(W^2 \sigma(W^1 x + b^1) + b^2) \dots + b^L)$$

# As a multi-class classifier



# Case study

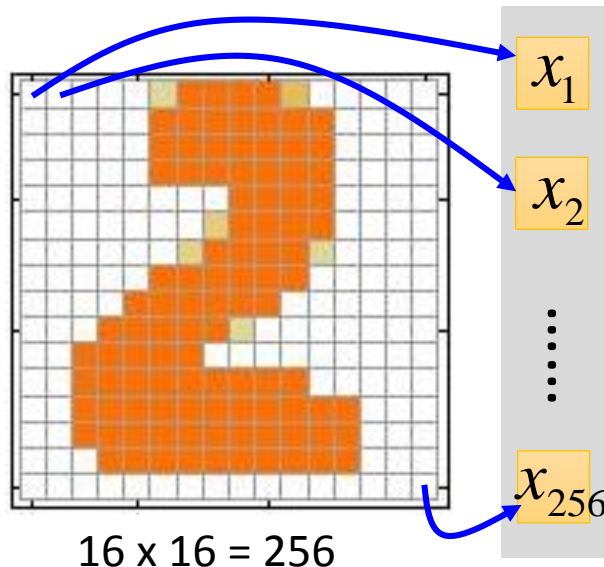
## Handwriting Digit Recognition



# Case study

## Handwriting Digit Recognition

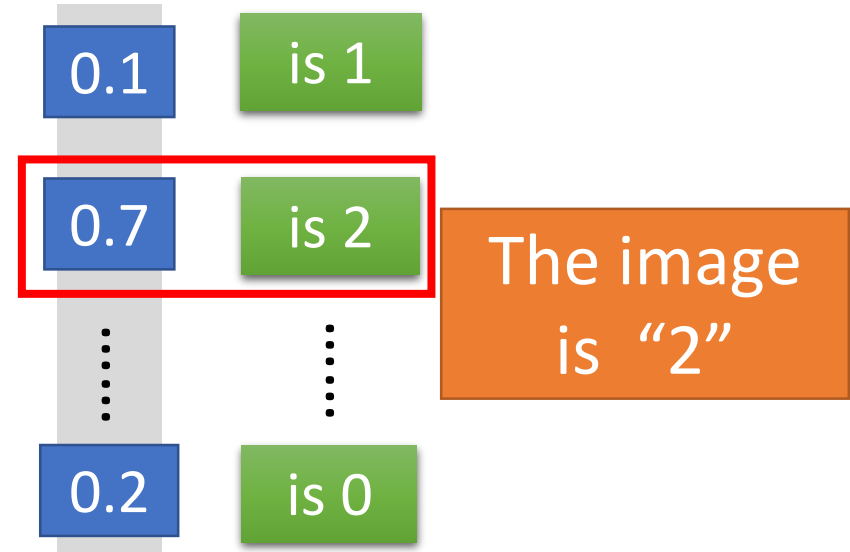
Input



Ink  $\rightarrow$  1

No ink  $\rightarrow$  0

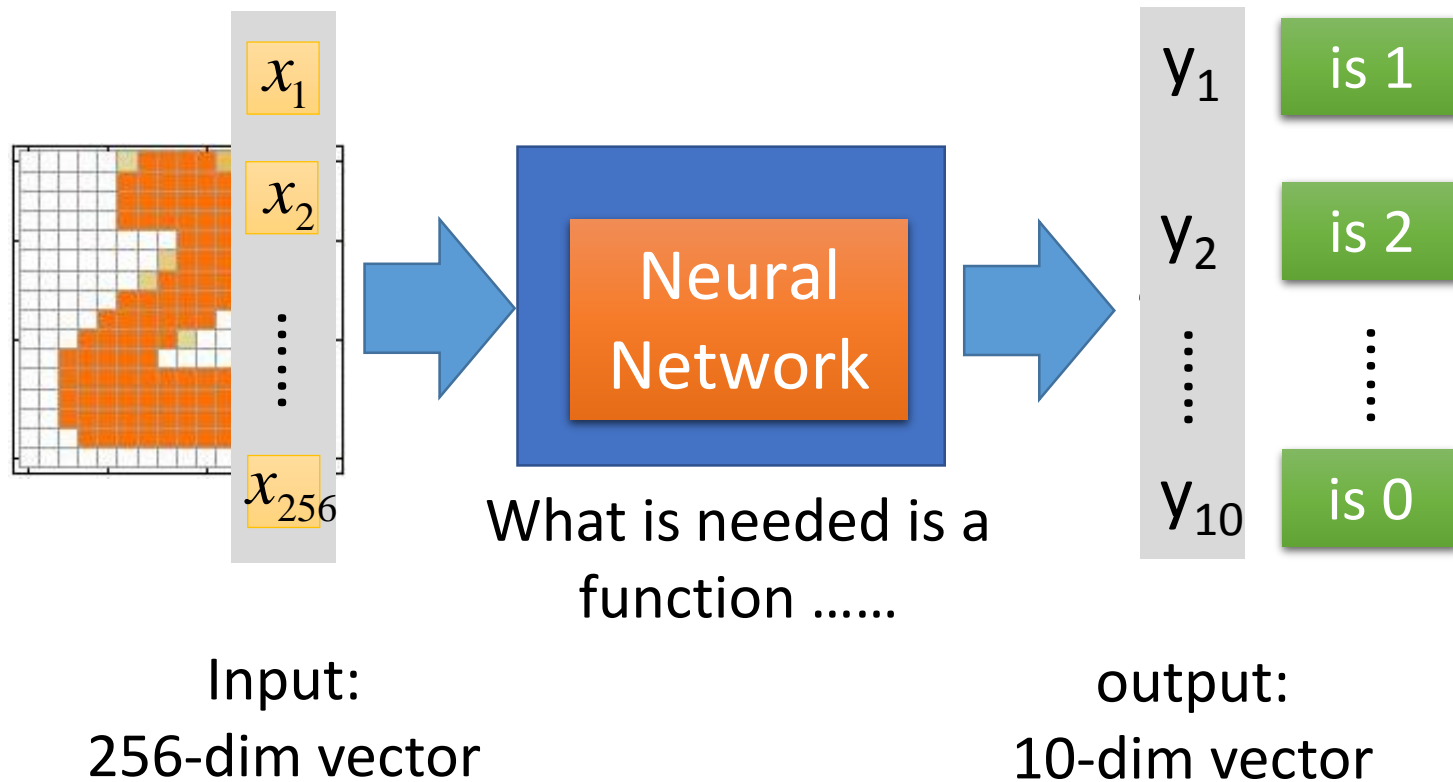
Output



Each dimension represents the confidence of a digit.

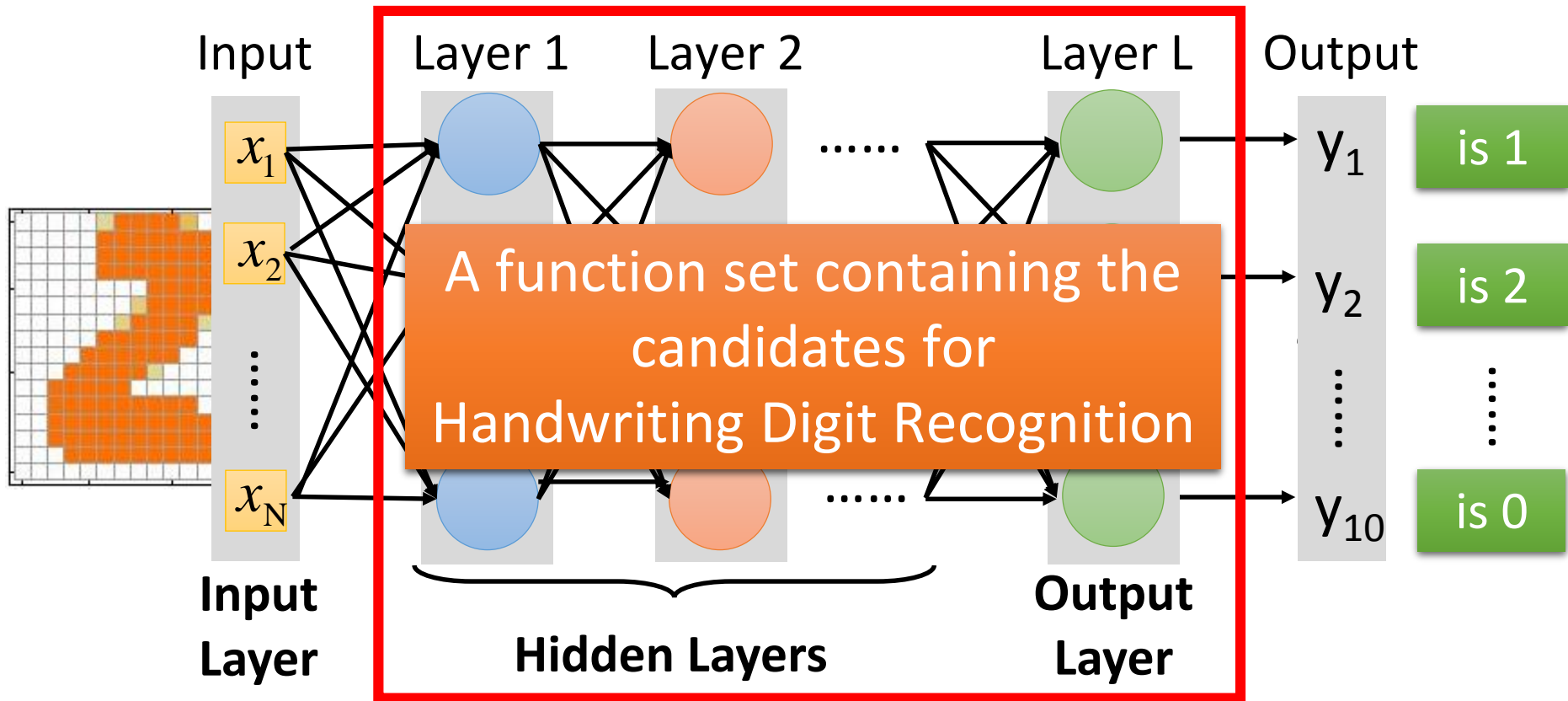
# Case study

## Handwriting Digit Recognition



# Case study

## Handwriting Digit Recognition



**You need to decide the network structure to let a good function in your function set.**



# Case study

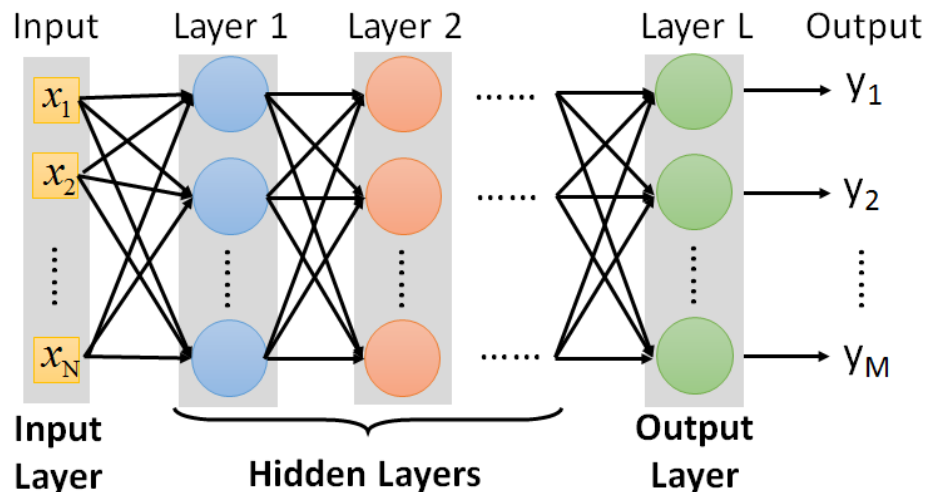
## Handwriting Digit Recognition

- How many layers? How many neurons for each layer?

Trial and Error

+

Intuition



# Today's Topics

- Neural Network Introduction
- Neural Network Structure
- How Neural Network Works
- *Backpropagation*

# Neural Network Optimization

- We have already learned to optimize the learner using the gradient descent method
- Can neural networks also be optimized using gradient descent?

# Case: CNN (AlexNet)

input image

weights

loss

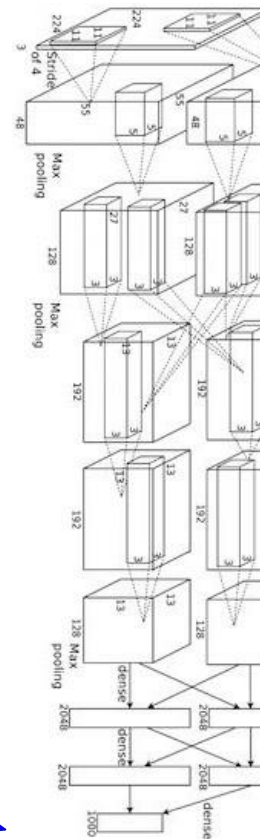


Figure copyright Alex Krizhevsky, Ilya Sutskever, and Geoffrey Hinton, 2012. Reproduced with permission.

# Case: Neural Turing Machine

input image

loss

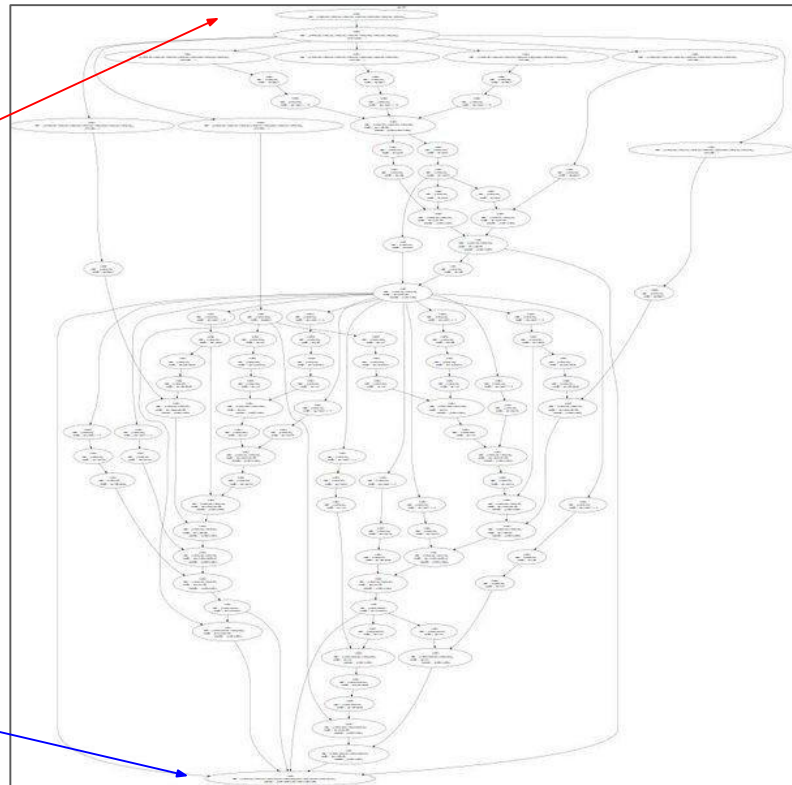


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# Why we need BP

- If we use gradient descent directly

Network parameters  $\theta = \{w_1, w_2, \dots, b_1, b_2, \dots\}$

Starting Parameters  $\theta^0 \longrightarrow \theta^1 \longrightarrow \theta^2 \longrightarrow \dots$

$$\nabla L(\theta) = \begin{bmatrix} \partial L(\theta) / \partial w_1 \\ \partial L(\theta) / \partial w_2 \\ \vdots \\ \partial L(\theta) / \partial b_1 \\ \partial L(\theta) / \partial b_2 \\ \vdots \end{bmatrix}$$

Compute  $\nabla L(\theta^0)$        $\theta^1 = \theta^0 - \eta \nabla L(\theta^0)$

Compute  $\nabla L(\theta^1)$        $\theta^2 = \theta^1 - \eta \nabla L(\theta^1)$

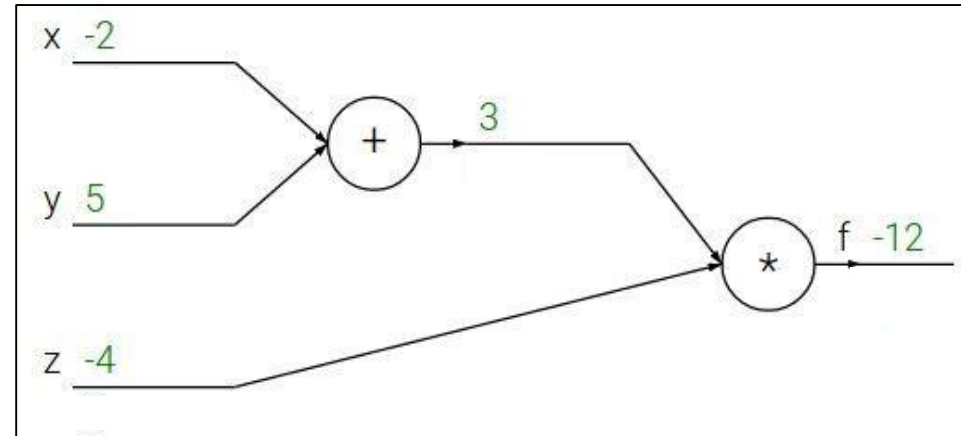
Millions of parameters .....

To compute the gradients efficiently,  
we use **backpropagation**.

# BP: A simple example

$$f(x, y, z) = (x + y)z$$

e.g.  $x = -2, y = 5, z = -4$



# BP: A simple example

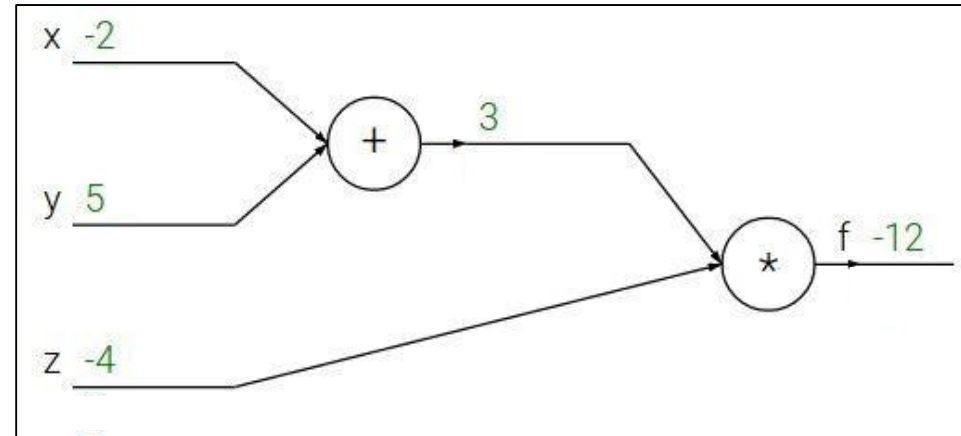
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$$q = x + y \quad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$f = qz \quad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

Want:  $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$





# BP: A simple example

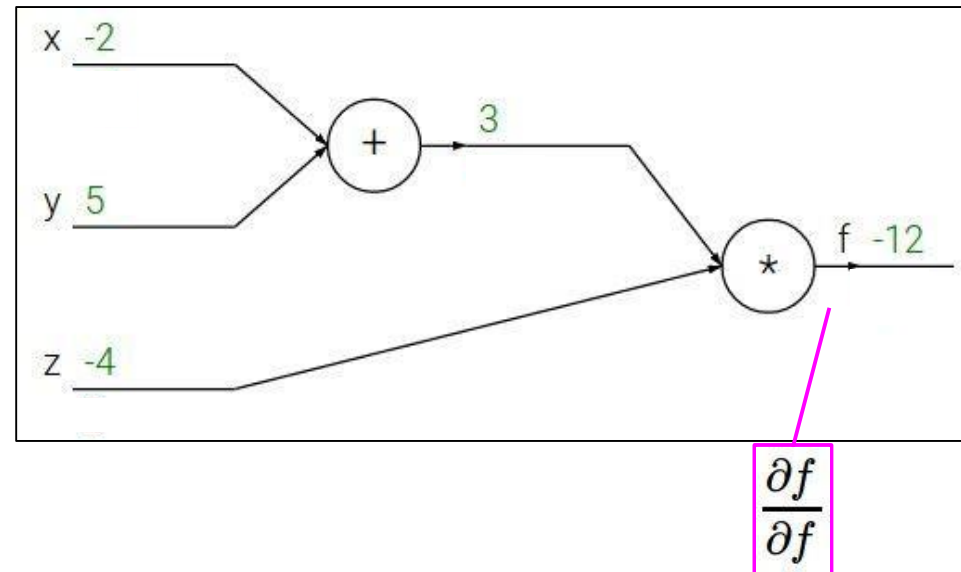
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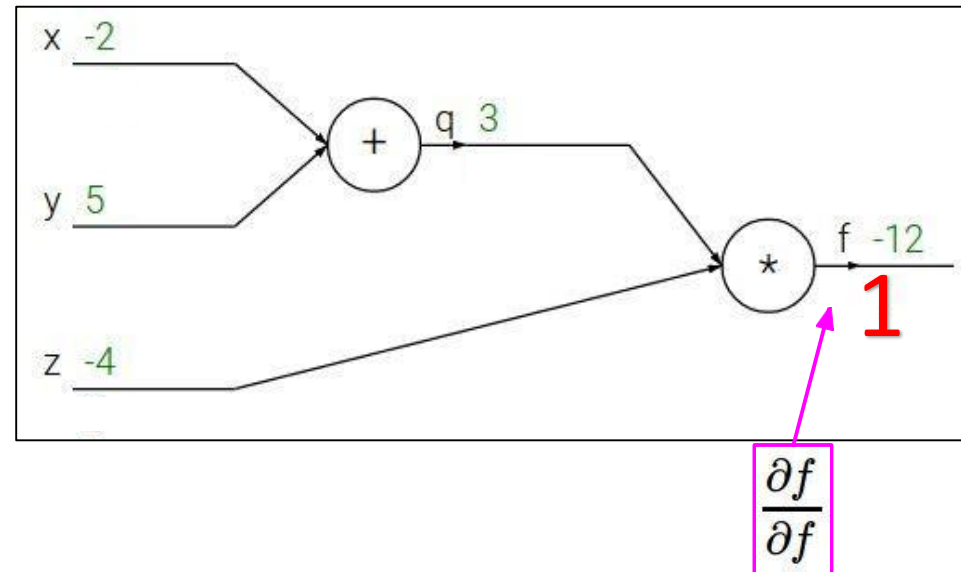
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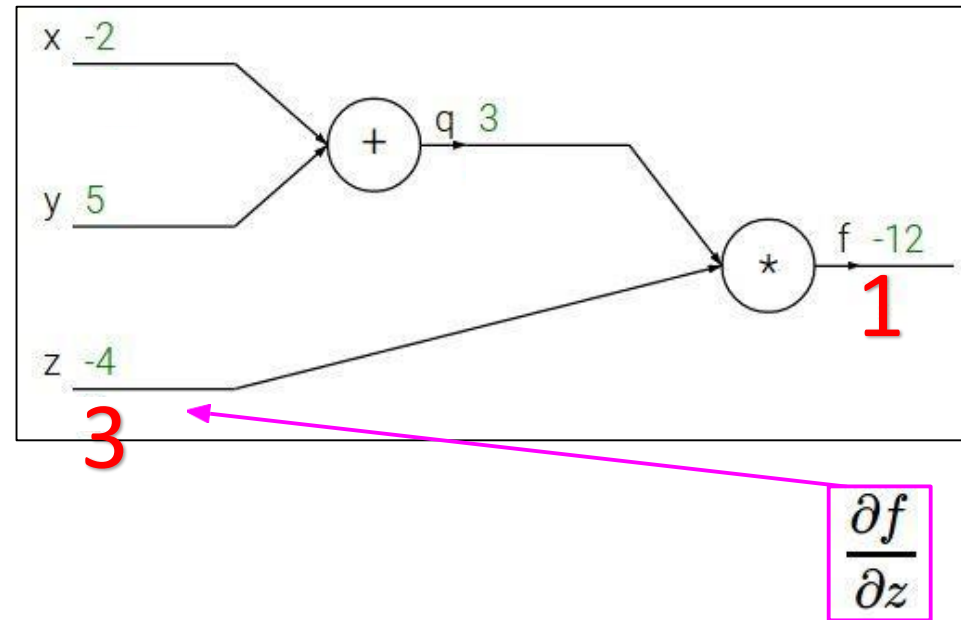
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Want:  $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



$$\frac{\partial f}{\partial z}$$

# BP: A simple example

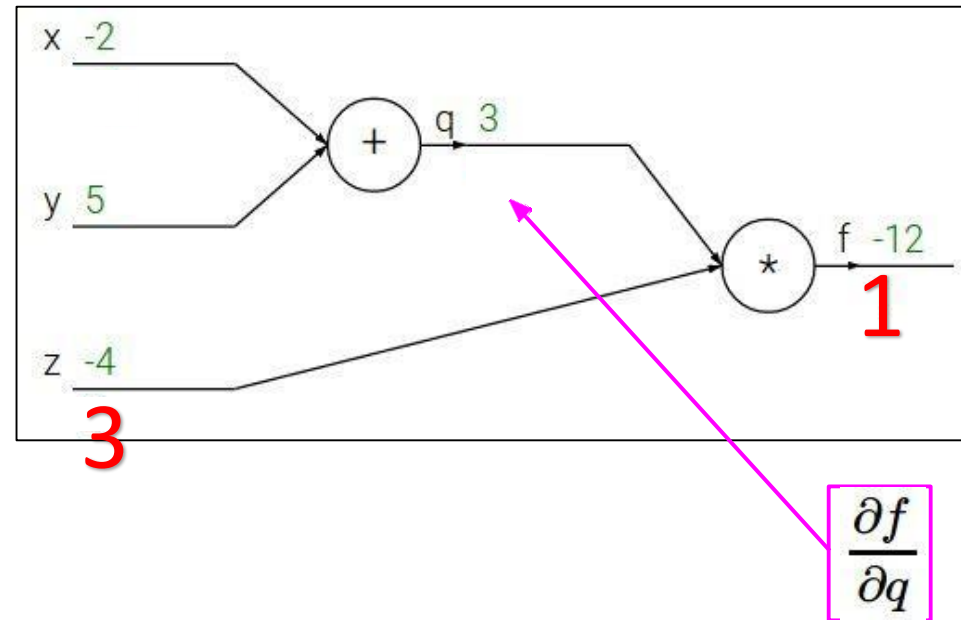
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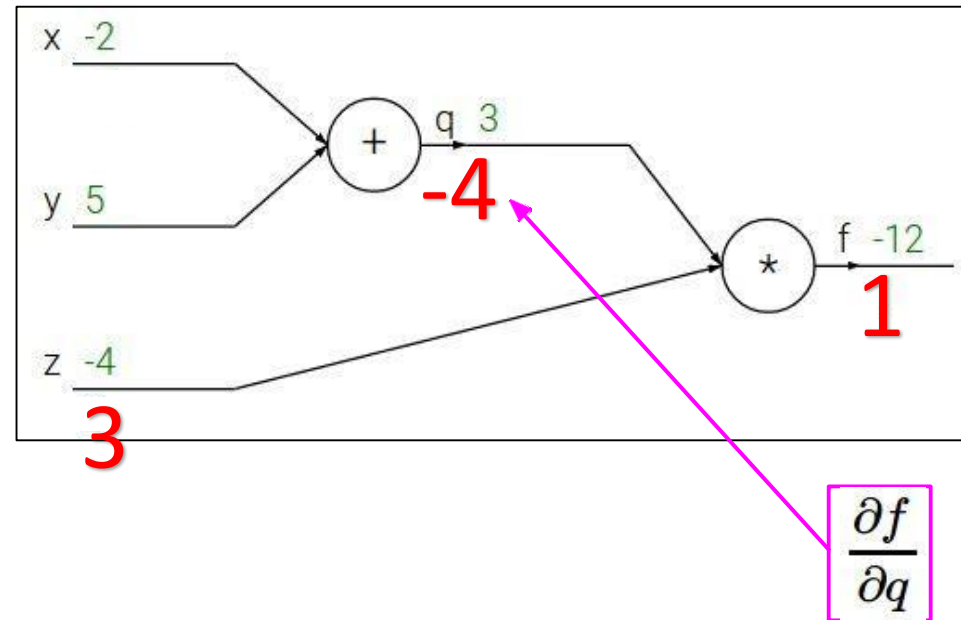
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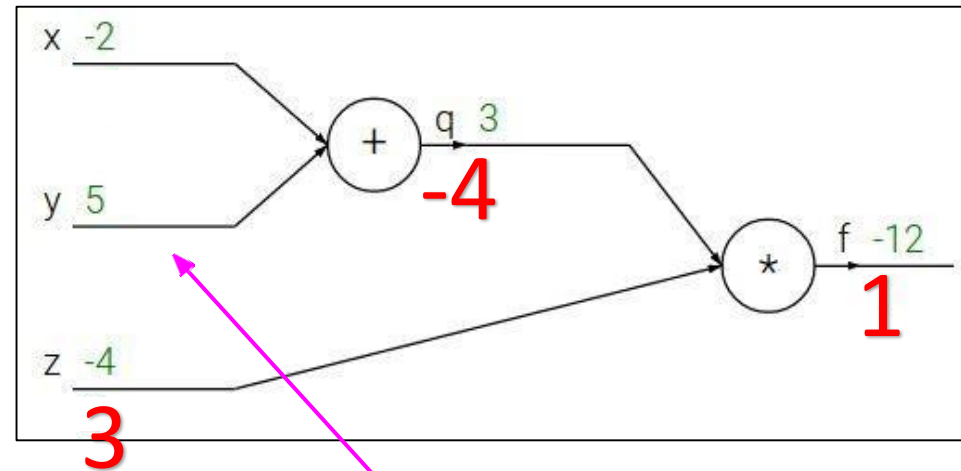
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Want:  $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



$$\frac{\partial f}{\partial y}$$

# BP: A simple example

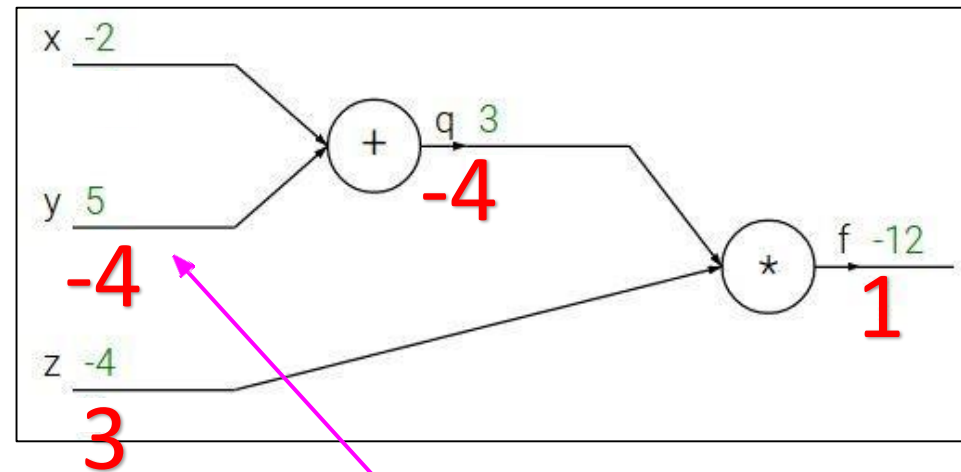
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Want:  $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



Chain rule:

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial y}$$

$$\frac{\partial f}{\partial y}$$

# BP: A simple example

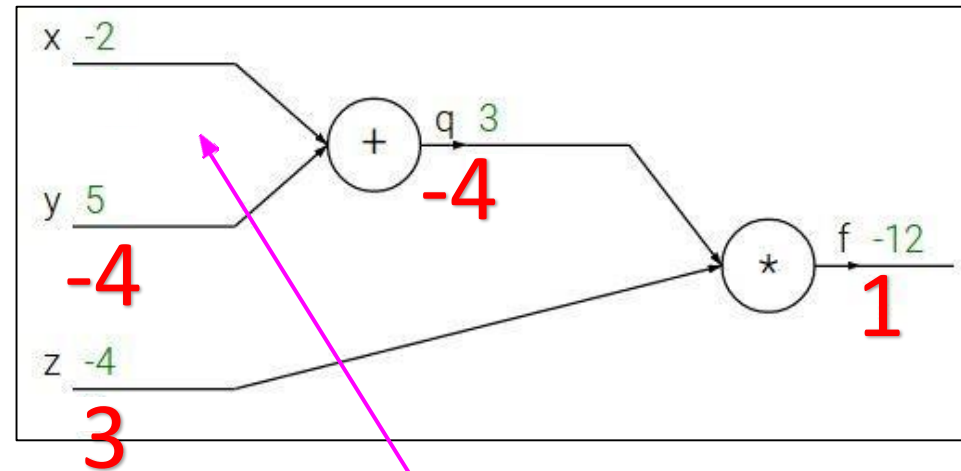
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Want:  $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



$$\frac{\partial f}{\partial x}$$



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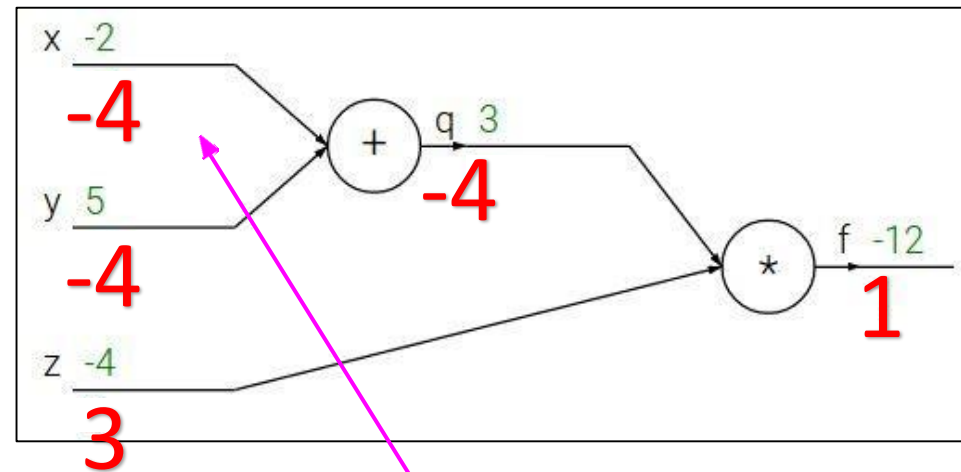
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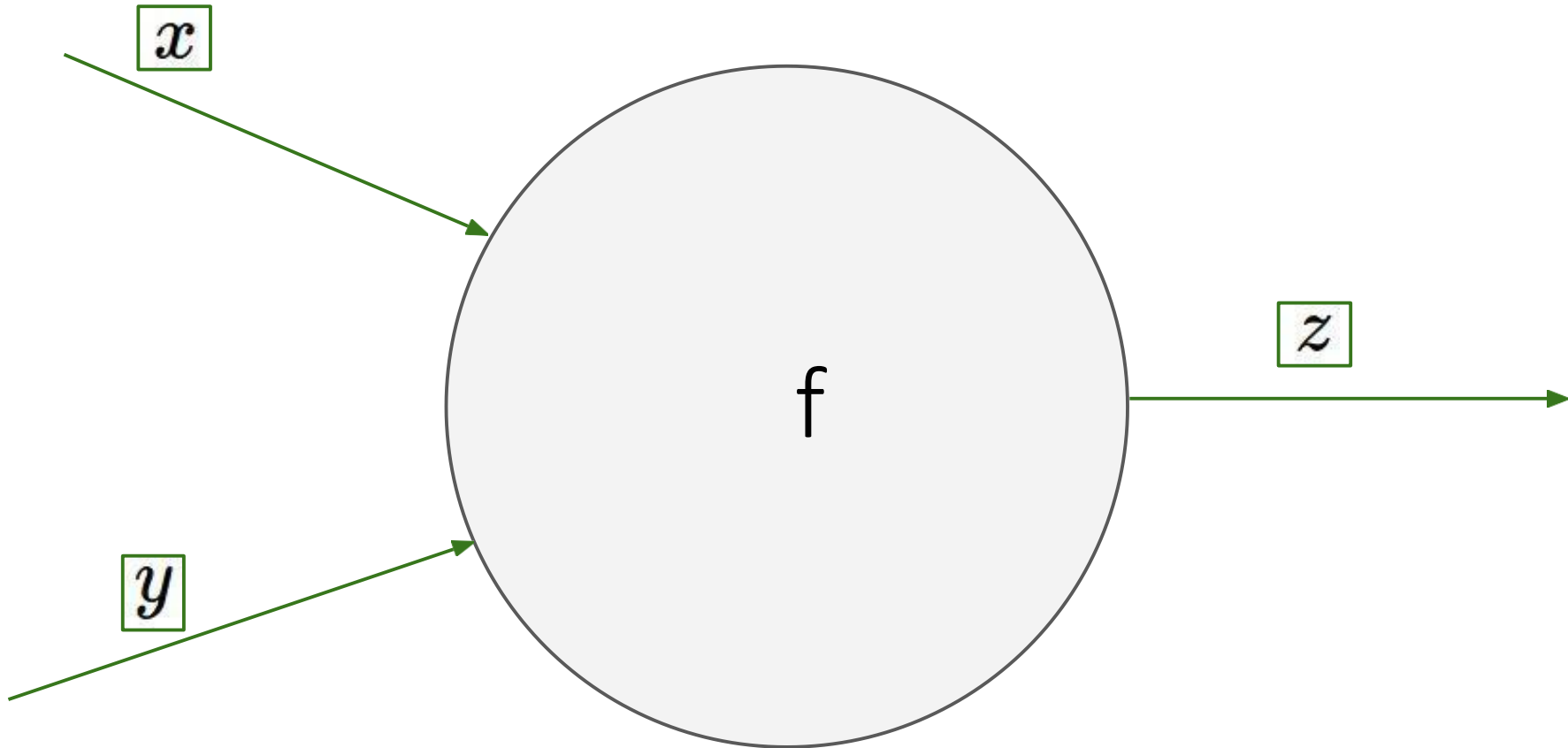
Chain rule:

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial x}$$

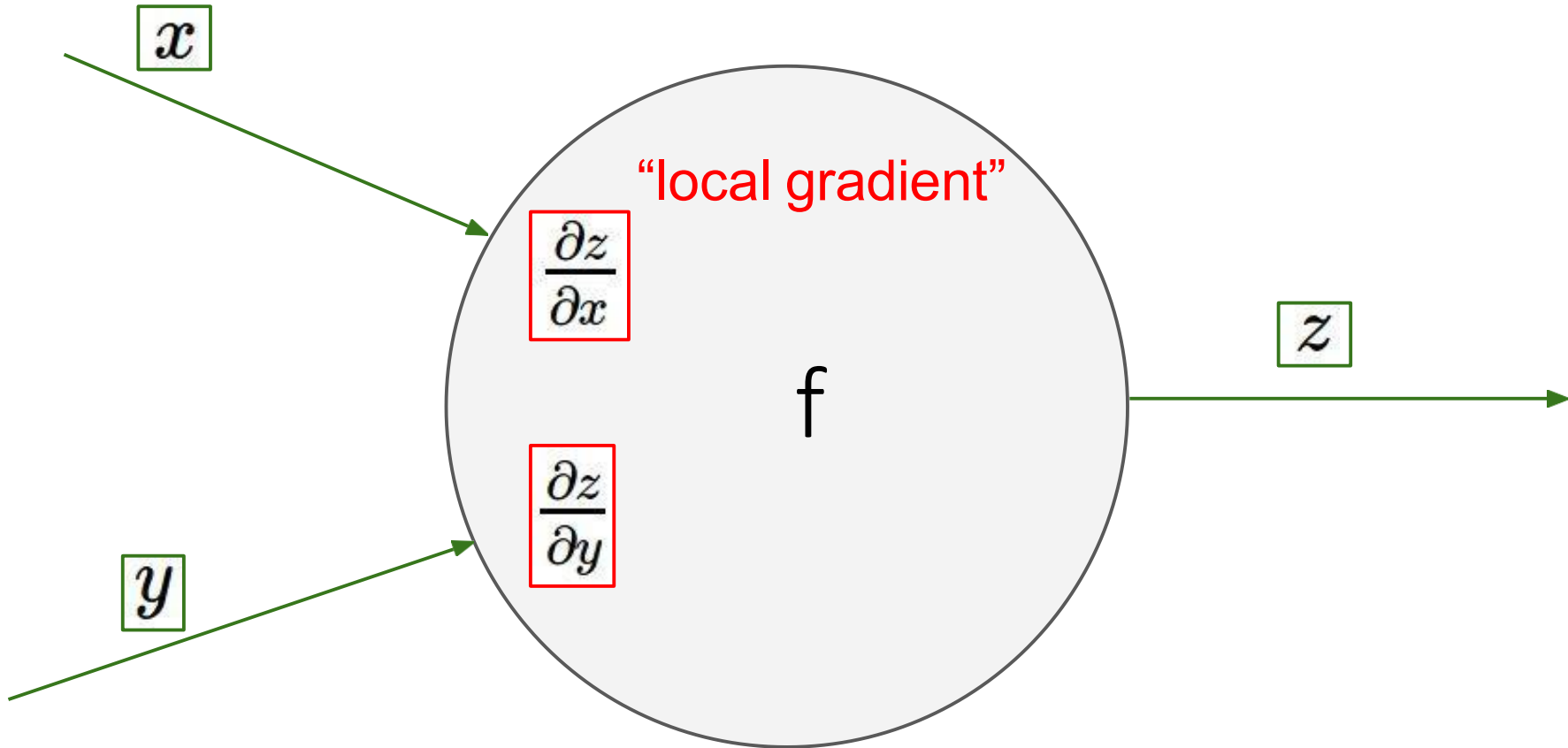


$$\frac{\partial f}{\partial x}$$

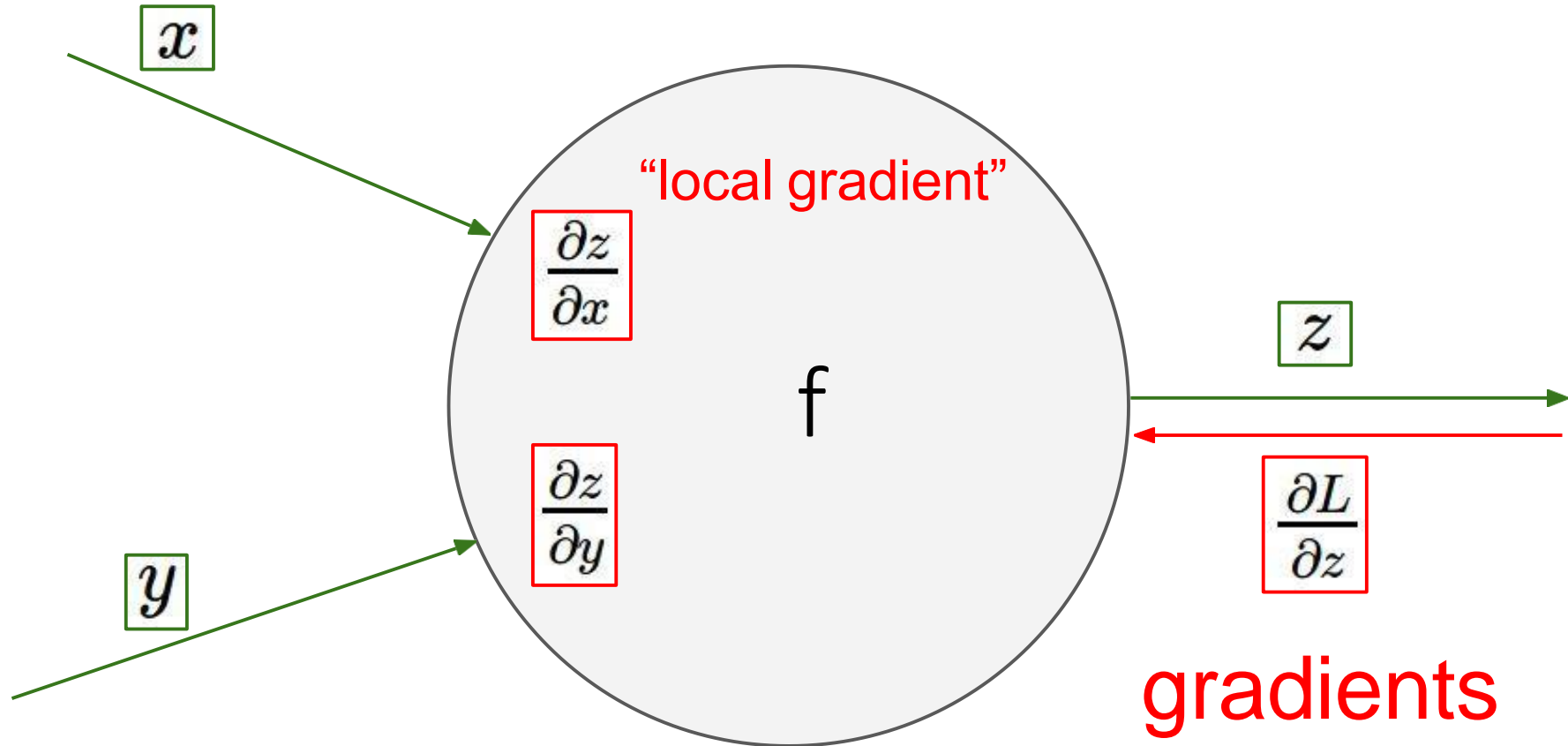
# Backpropagation



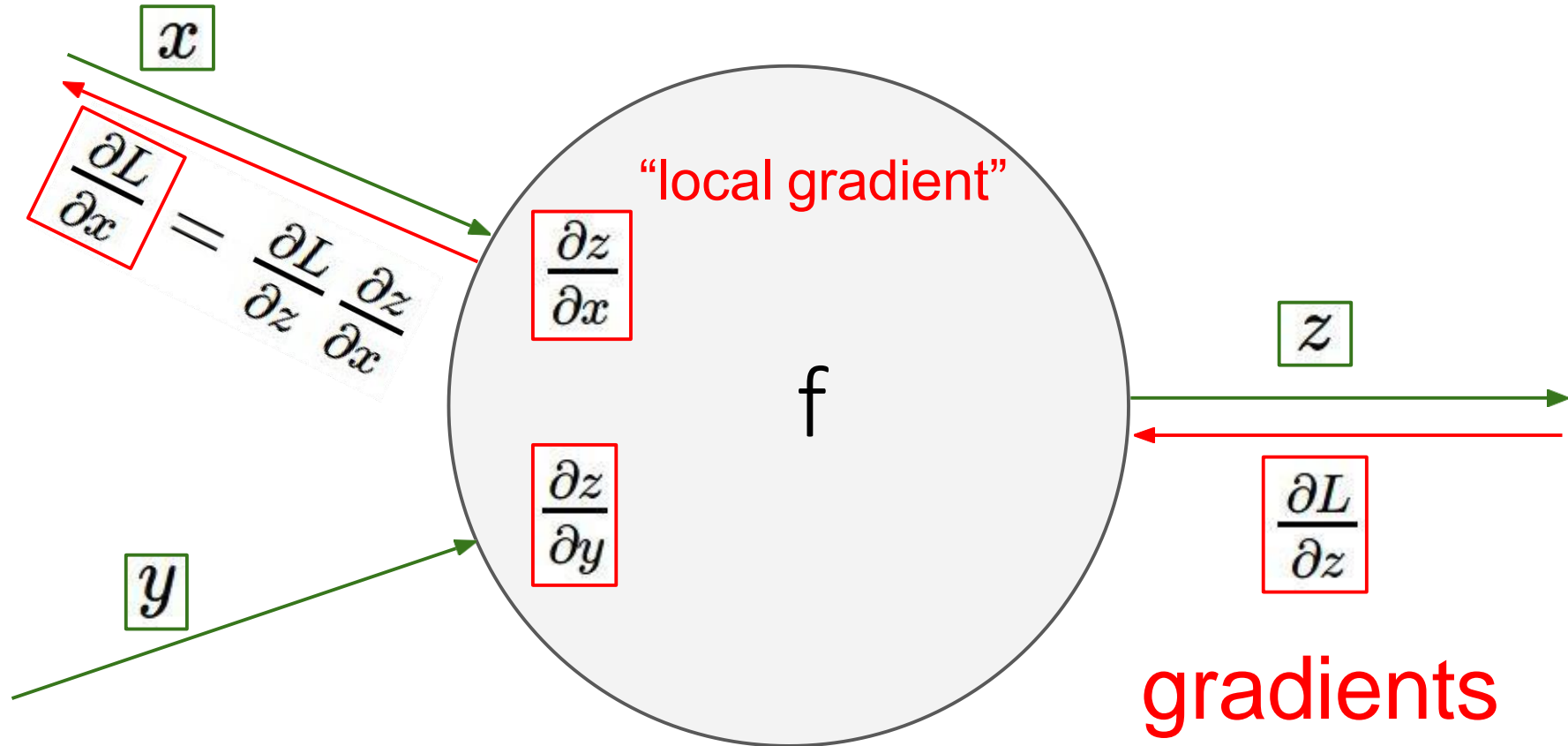
# Backpropagation



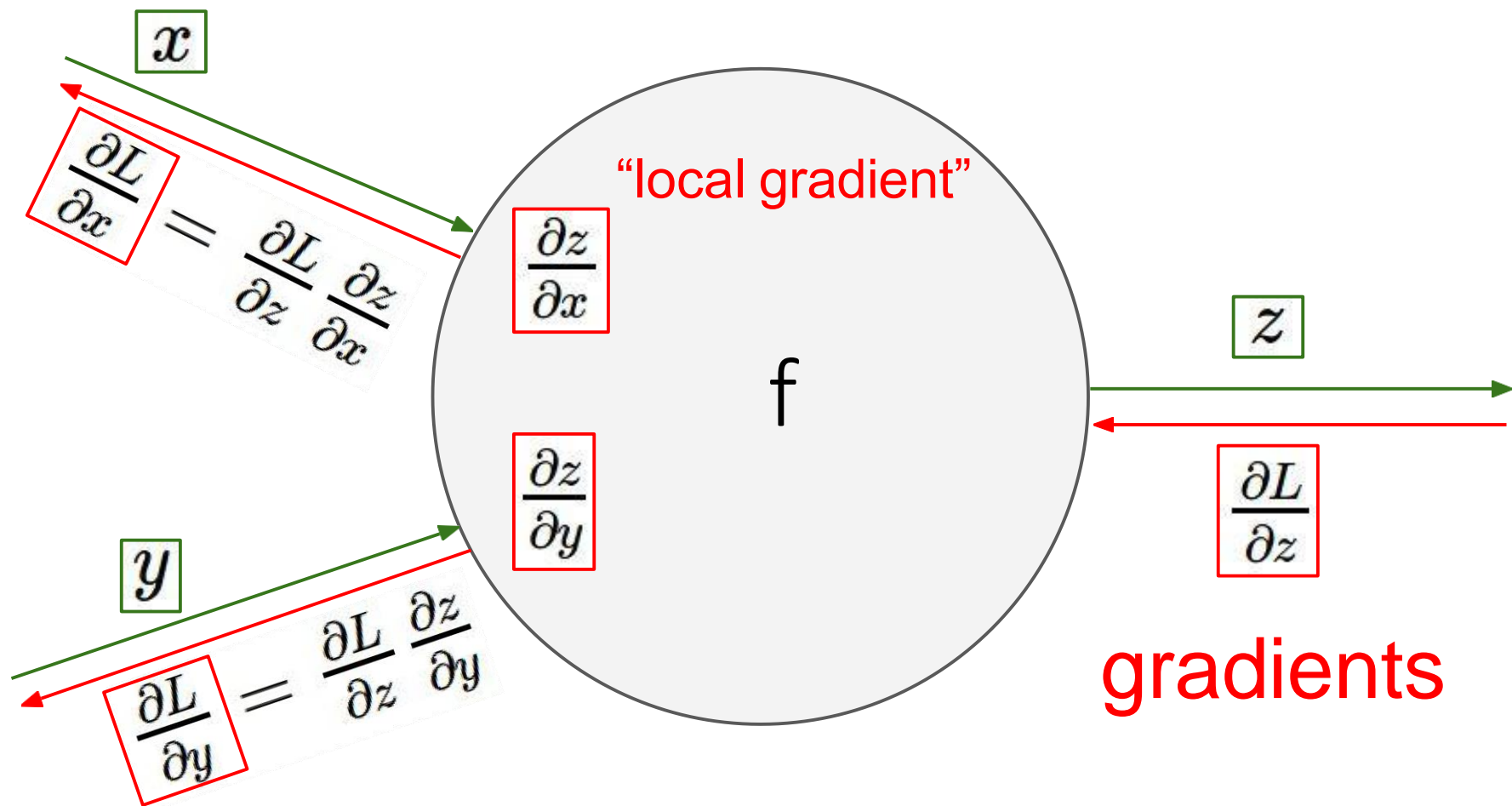
# Backpropagation



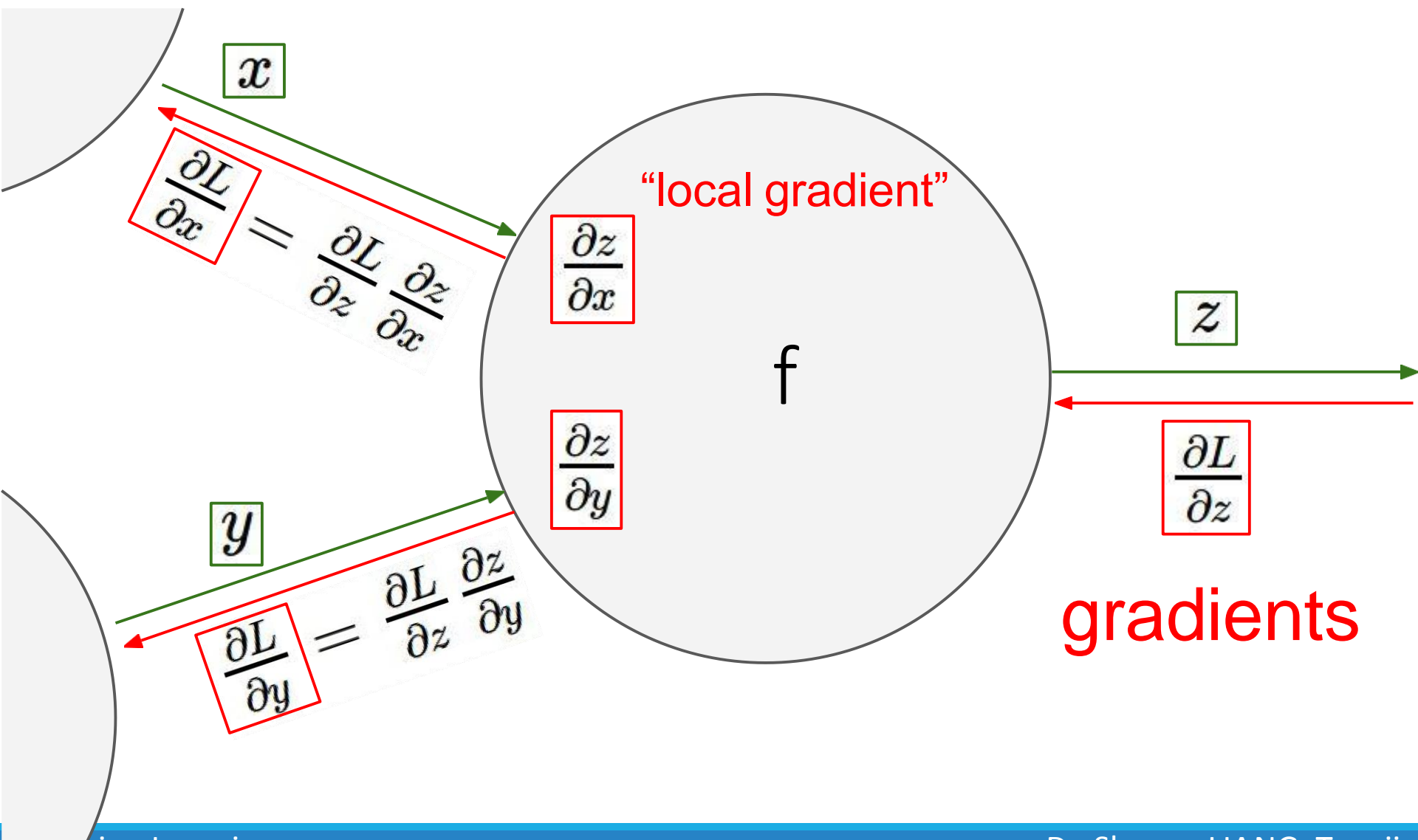
# Backpropagation



# Backpropagation

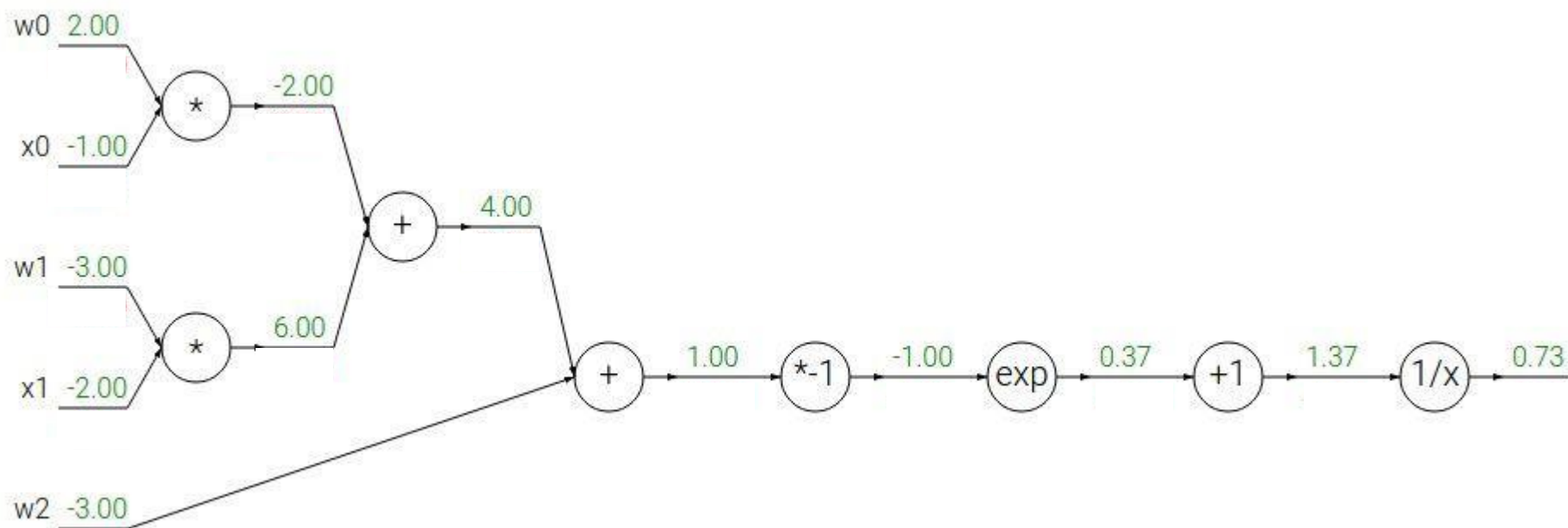


# Backpropagation



# BP: Another example

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2x_2)}}$$



$$f(x) = e^x$$

→

$$\frac{df}{dx} = e^x$$

$$f(x) = \frac{1}{x}$$

→

$$\frac{df}{dx} = -1/x^2$$

$$f_a(x) = ax$$

→

$$\frac{df}{dx} = a$$

$$f_c(x) = c + x$$

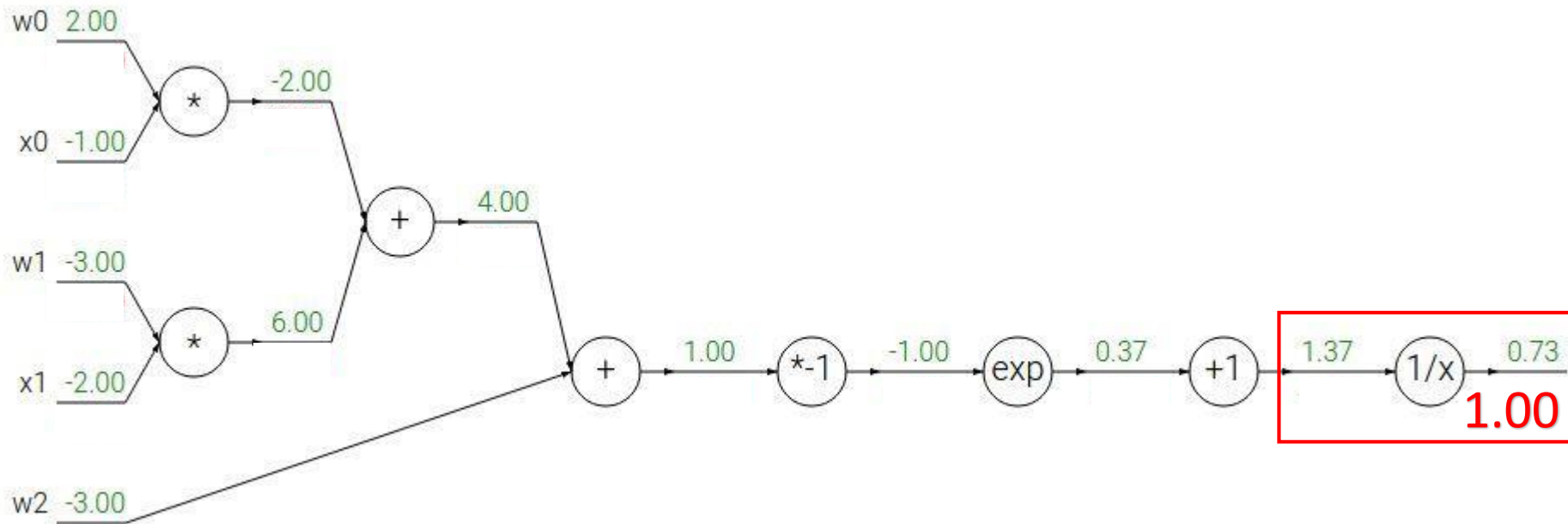
→

$$\frac{df}{dx} = 1$$



# BP: Another example

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2x_2)}}$$



$$f(x) = e^x$$

→

$$\frac{df}{dx} = e^x$$

$$f_a(x) = ax$$

→

$$\frac{df}{dx} = a$$

$$f(x) = \frac{1}{x}$$

→

$$\frac{df}{dx} = -1/x^2$$

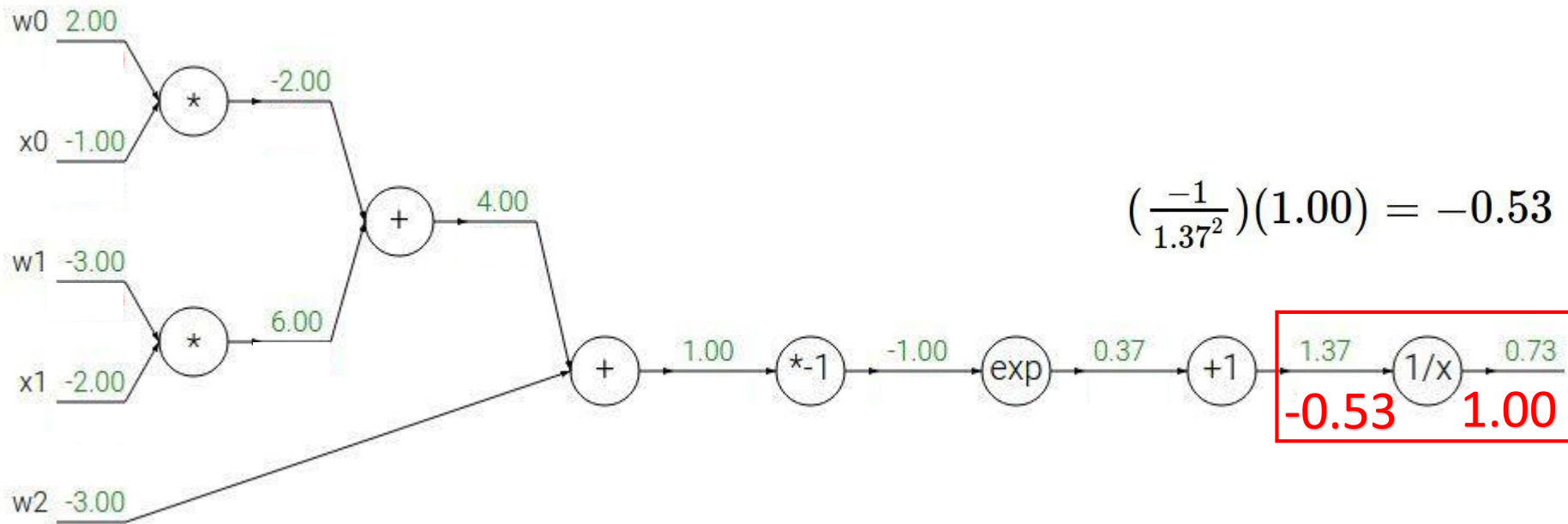
$$f_c(x) = c + x$$

→

$$\frac{df}{dx} = 1$$

# BP: Another example

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2x_2)}}$$



$$f(x) = e^x$$

→

$$\frac{df}{dx} = e^x$$

$$f_a(x) = ax$$

→

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$$f(x) = \frac{1}{x}$$

→

$$\frac{df}{dx} = -1/x^2$$

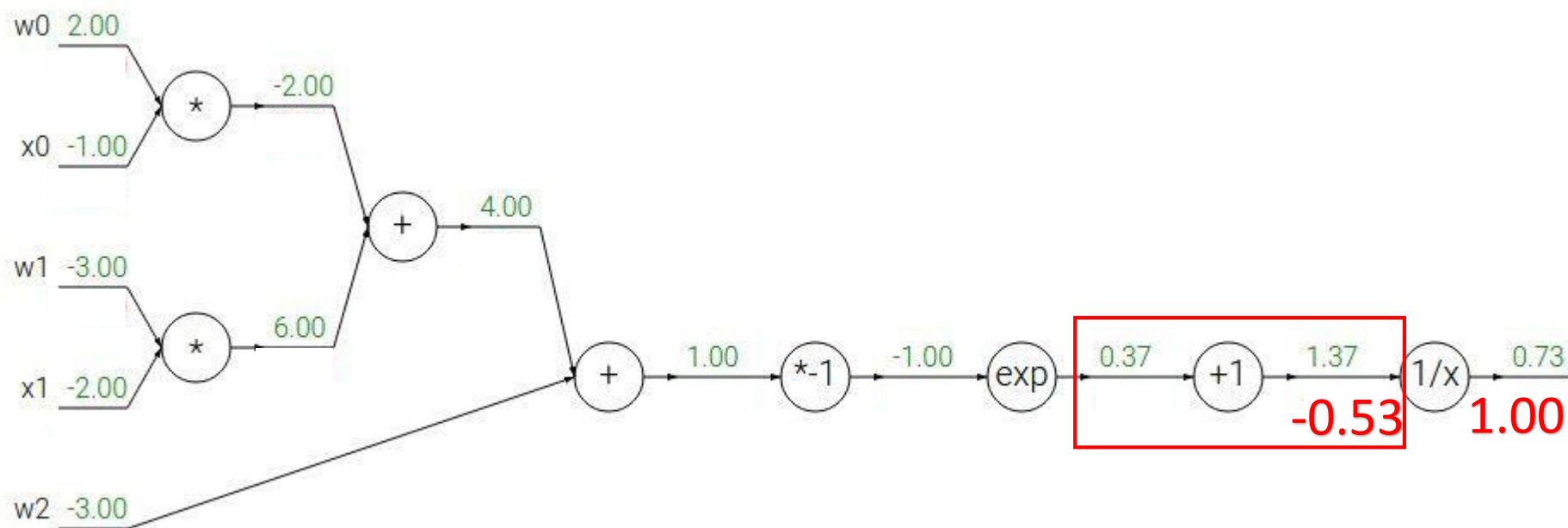
$$f_c(x) = c + x$$

→

$$\frac{df}{dx} = 1$$

# BP: Another example

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2x_2)}}$$



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$$\frac{df}{dx} = e^x$$

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→

$$\frac{df}{dx} = -1/x^2$$

$$f_a(x) = ax$$

→

$$\frac{df}{dx} = a$$

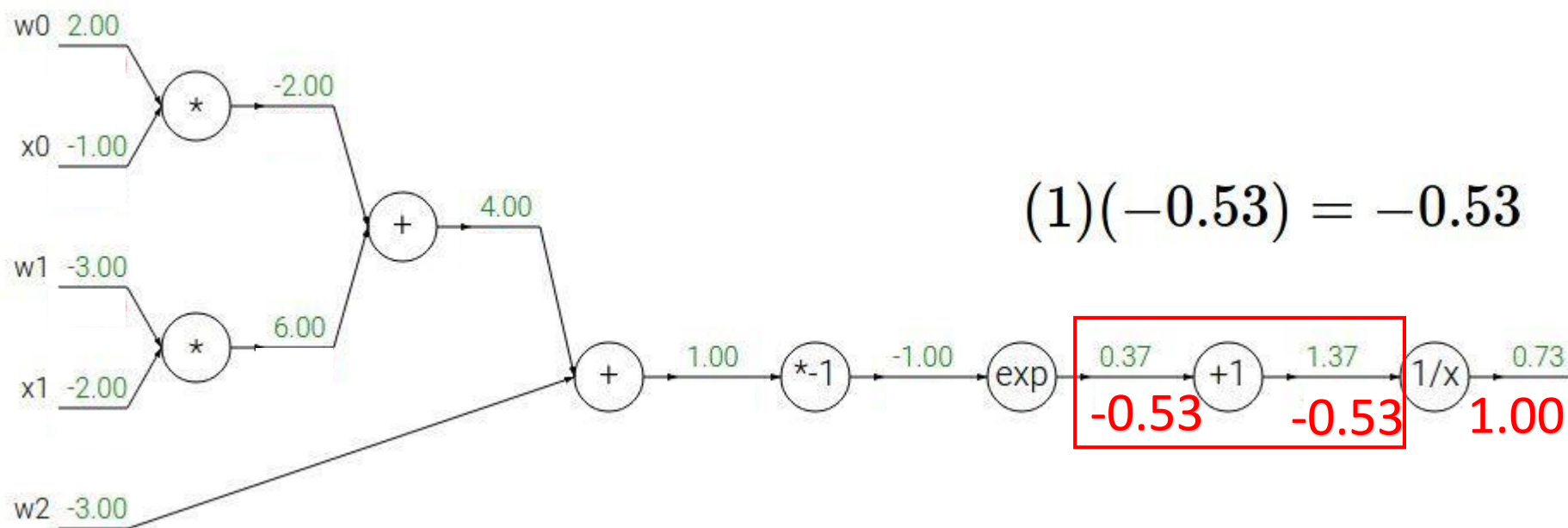
$$f_c(x) = c + x$$

→

$$\frac{df}{dx} = 1$$

# BP: Another example

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2x_2)}}$$



$$(1)(-0.53) = -0.53$$

$$f(x) = e^x$$

→

$$\frac{df}{dx} = e^x$$

$$f(x) = \frac{1}{x}$$

→

$$\frac{df}{dx} = -1/x^2$$

$$f_a(x) = ax$$

→

$$\frac{df}{dx} = a$$

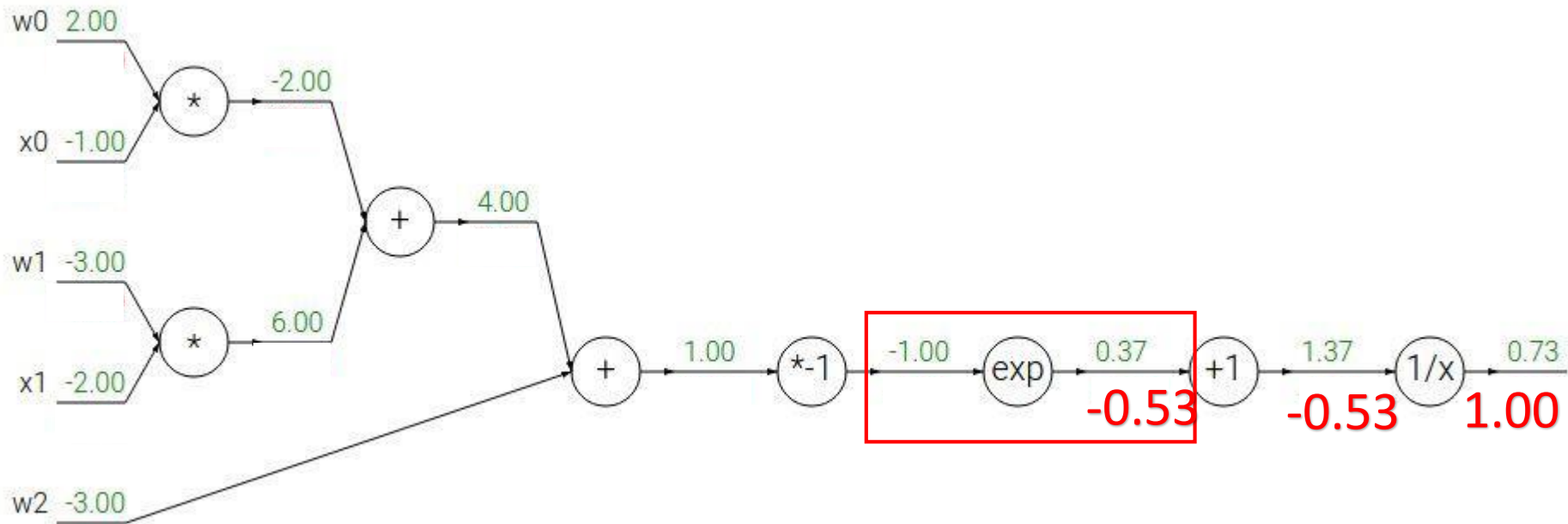
$$f_c(x) = c + x$$

→

$$\frac{df}{dx} = 1$$

# BP: Another example

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2x_2)}}$$



$$f(x) = e^x \rightarrow \frac{df}{dx} = e^x$$

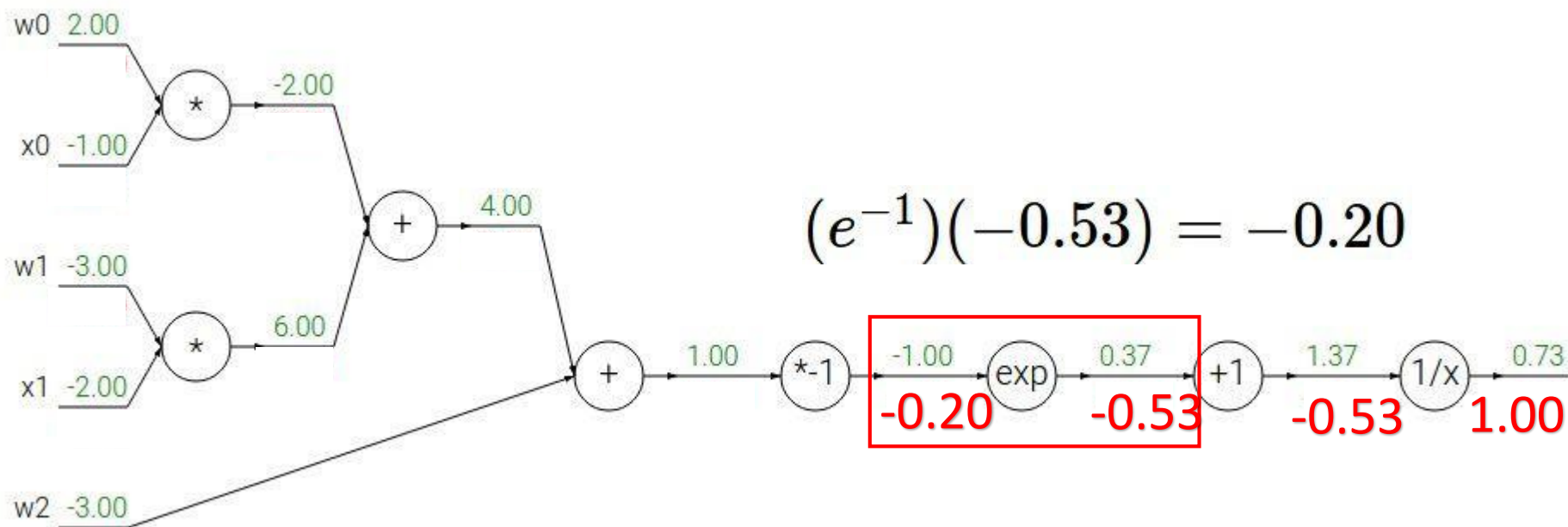
$$f_a(x) = ax \rightarrow \frac{df}{dx} = a$$

$$f(x) = \frac{1}{x} \rightarrow \frac{df}{dx} = -1/x^2$$

$$f_c(x) = c + x \rightarrow \frac{df}{dx} = 1$$

# BP: Another example

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2x_2)}}$$



$$(e^{-1})(-0.53) = -0.20$$

$$f(x) = e^x$$

→

$$\frac{df}{dx} = e^x$$

$$f(x) = \frac{1}{x}$$

→

$$\frac{df}{dx} = -1/x^2$$

$$f_a(x) = ax$$

→

$$\frac{df}{dx} = a$$

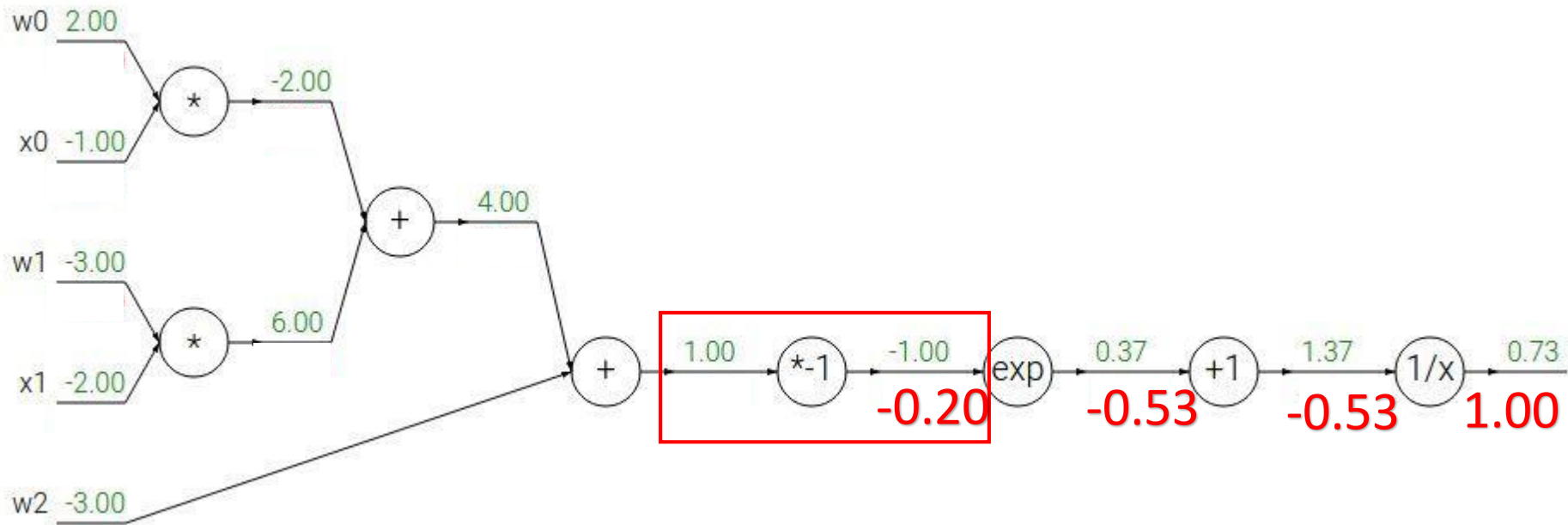
$$f_c(x) = c + x$$

→

$$\frac{df}{dx} = 1$$

# BP: Another example

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2x_2)}}$$



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→

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→

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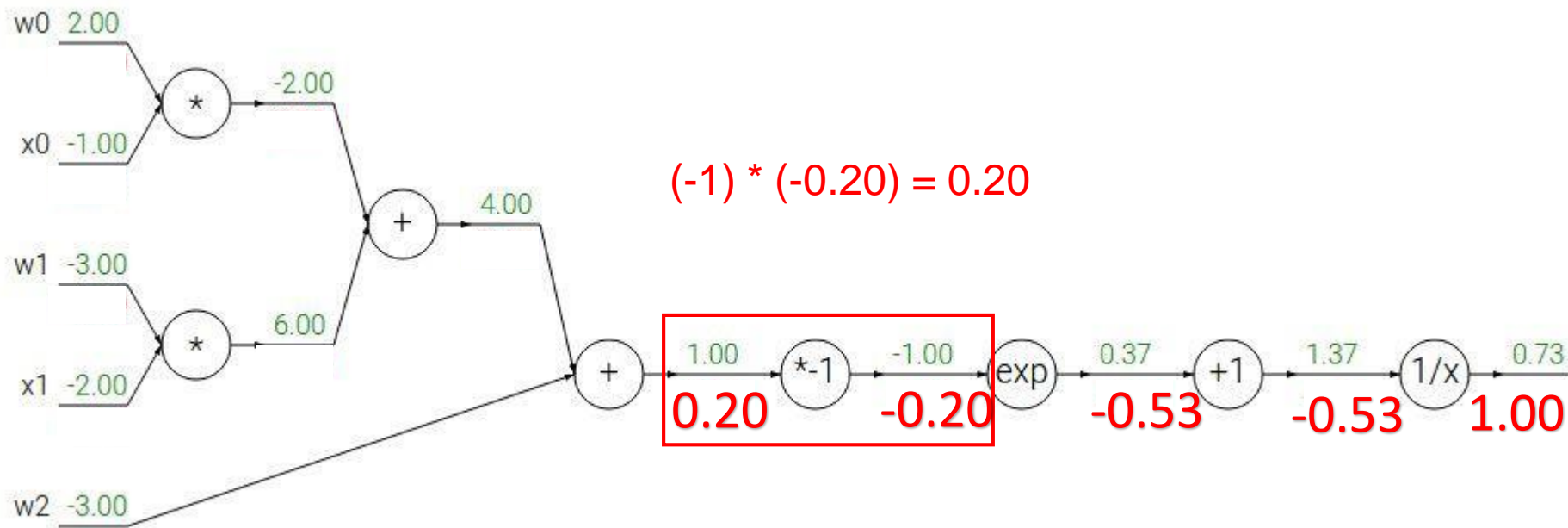
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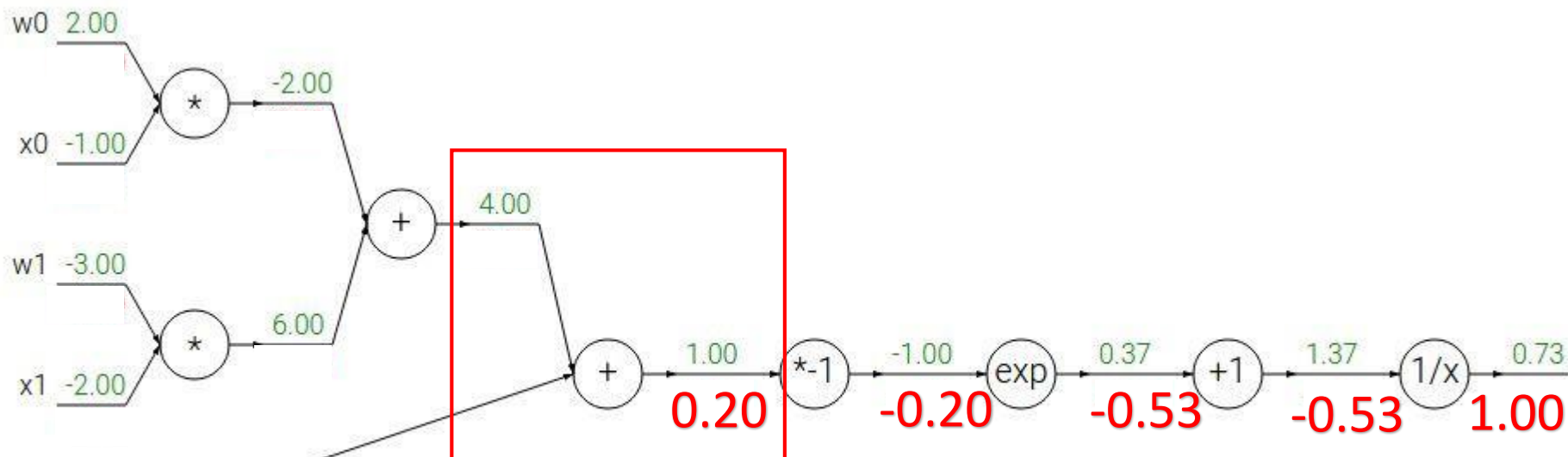
→

$$\frac{df}{dx} = 1$$



# BP: Another example

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2x_2)}}$$



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$$f_c(x) = c + x$$

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$$\frac{df}{dx} = 1$$

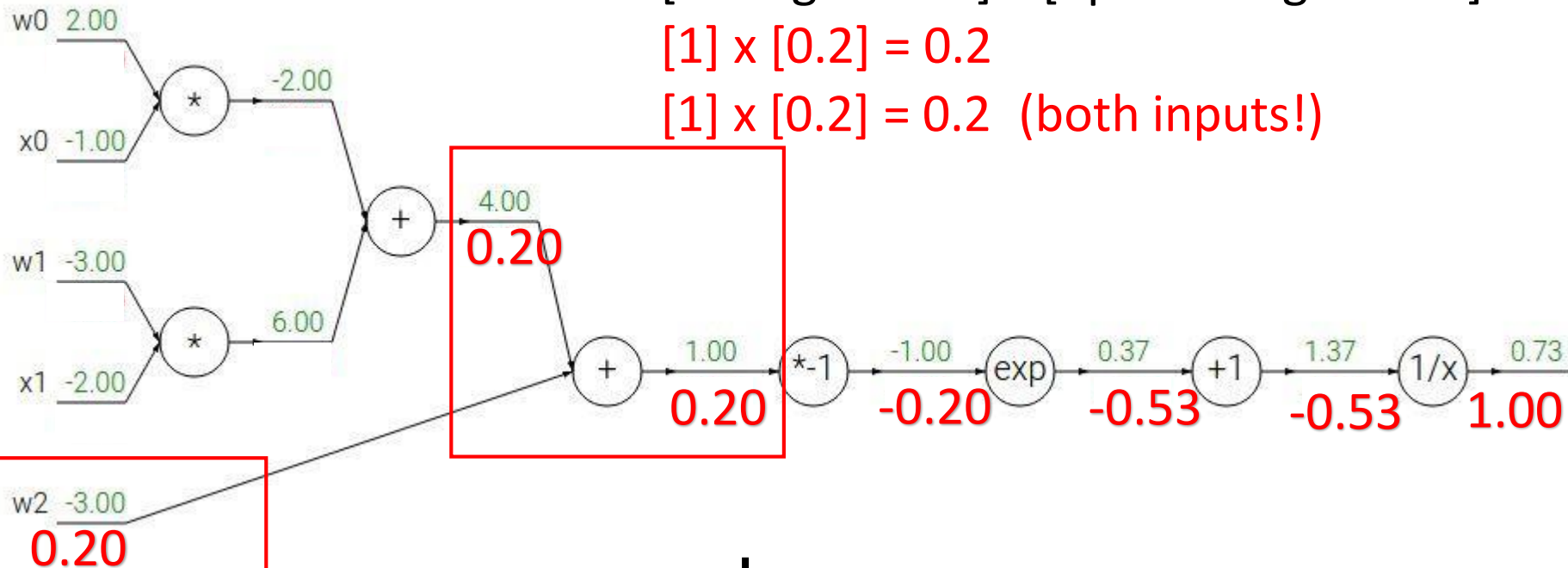
# BP: Another example

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2x_2)}}$$

[local gradient] x [upstream gradient]

$$[1] \times [0.2] = 0.2$$

$$[1] \times [0.2] = 0.2 \text{ (both inputs!)}$$



$$f(x) = e^x$$

→

$$\frac{df}{dx} = e^x$$

$$f(x) = \frac{1}{x}$$

→

$$\frac{df}{dx} = -1/x^2$$

$$f_a(x) = ax$$

→

$$\frac{df}{dx} = a$$

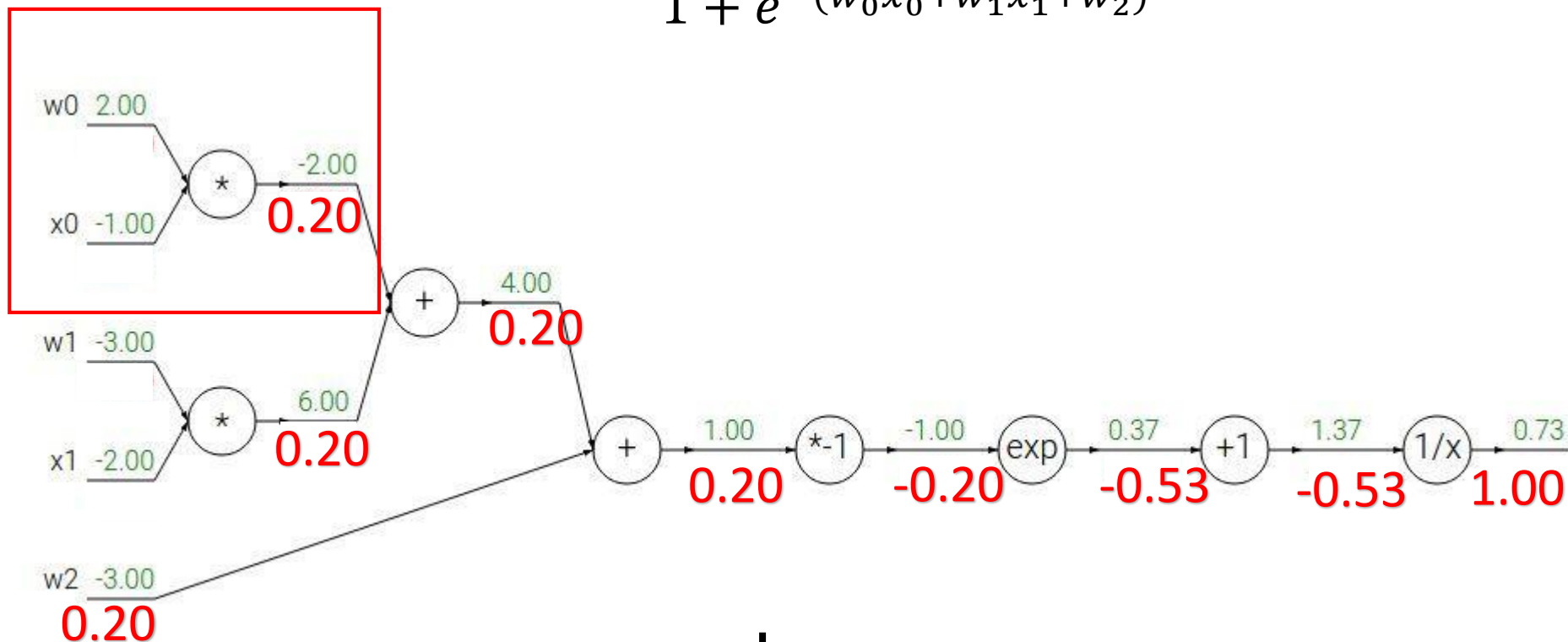
$$f_c(x) = c + x$$

→

$$\frac{df}{dx} = 1$$

# BP: Another example

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2x_2)}}$$



$$f(x) = e^x$$

→

$$\frac{df}{dx} = e^x$$

$$f(x) = \frac{1}{x}$$

→

$$\frac{df}{dx} = -1/x^2$$

$$f_a(x) = ax$$

→

$$\frac{df}{dx} = a$$

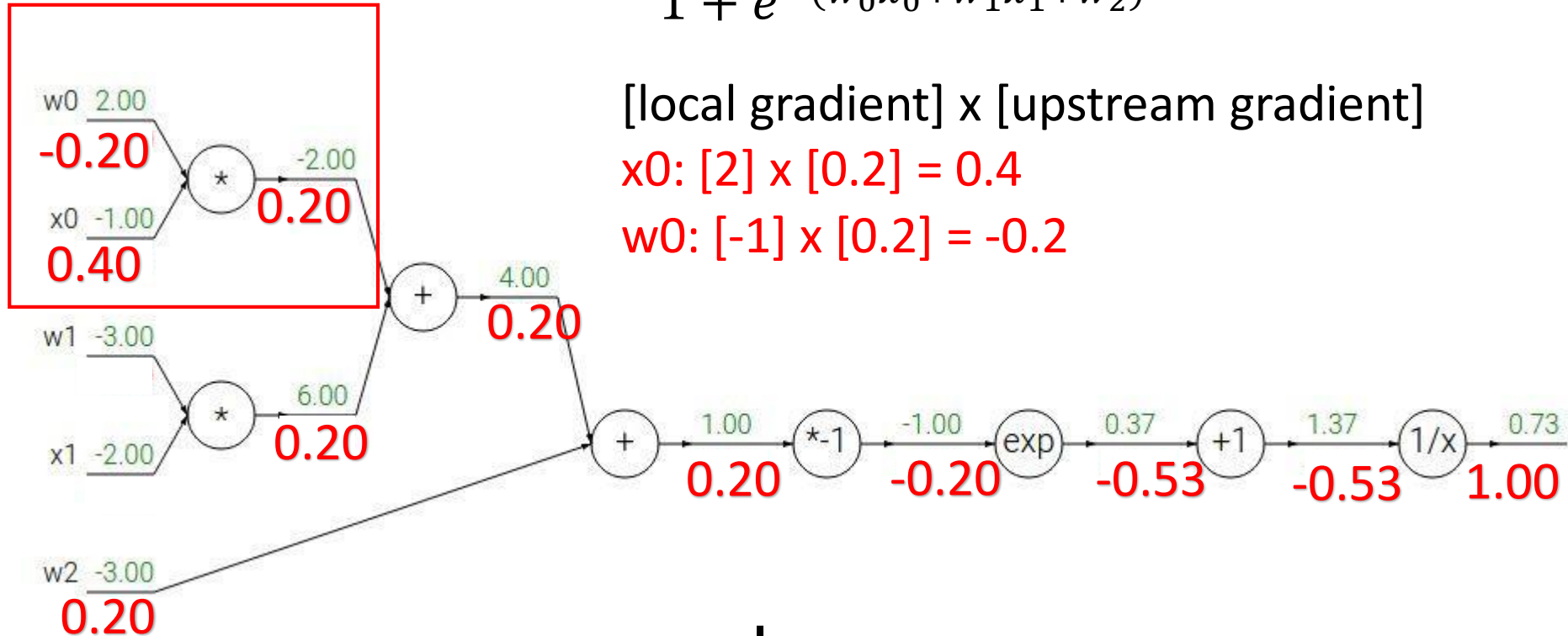
$$f_c(x) = c + x$$

→

$$\frac{df}{dx} = 1$$

# BP: Another example

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2x_2)}}$$



[local gradient] x [upstream gradient]

$$x0: [2] \times [0.2] = 0.4$$

$$w0: [-1] \times [0.2] = -0.2$$

$$f(x) = e^x$$

→

$$\frac{df}{dx} = e^x$$

$$f(x) = \frac{1}{x}$$

→

$$\frac{df}{dx} = -1/x^2$$

$$f_a(x) = ax$$

→

$$\frac{df}{dx} = a$$

$$f_c(x) = c + x$$

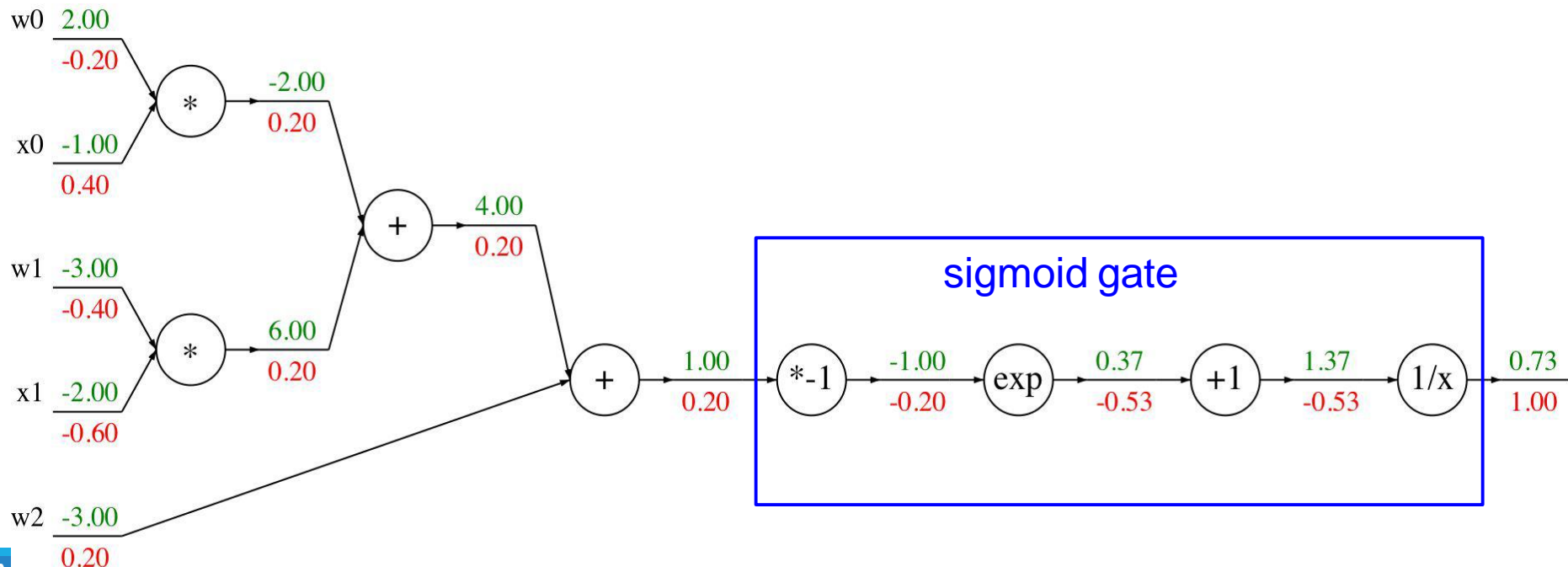
→

$$\frac{df}{dx} = 1$$

# BP: Another example

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}} \quad \sigma(x) = \frac{1}{1 + e^{-x}} \text{ sigmoid function}$$

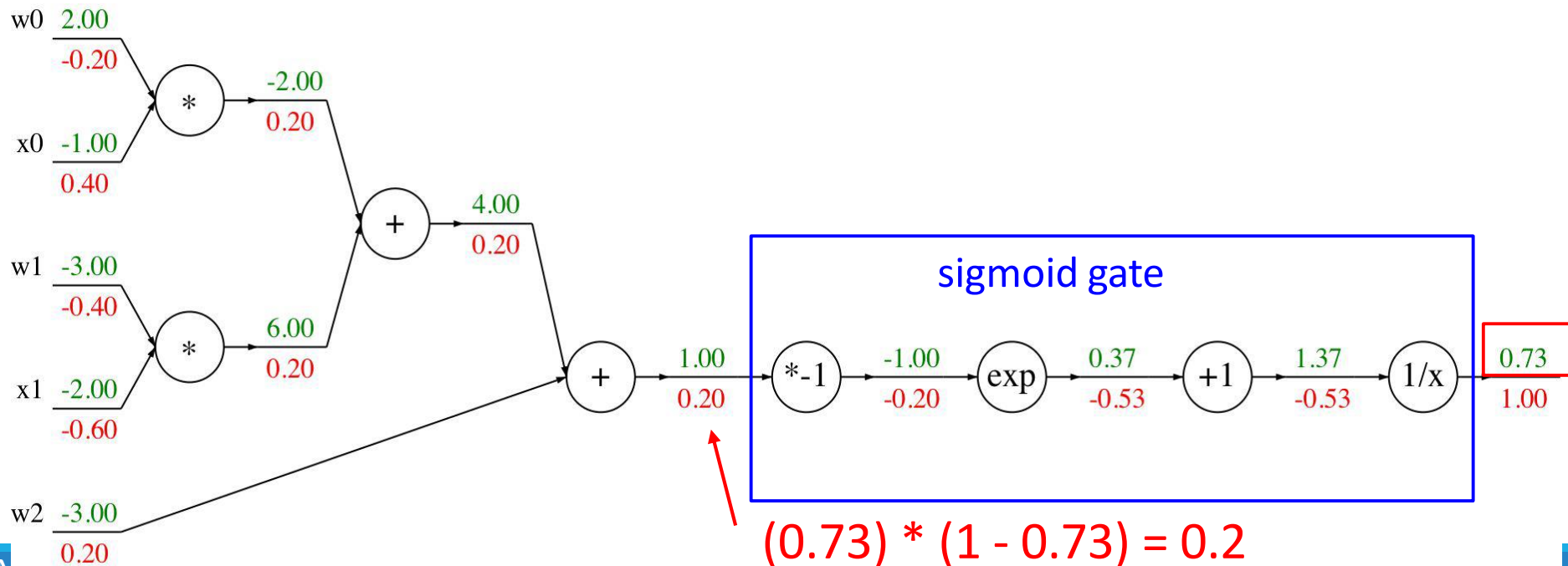
$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1 + e^{-x})^2} = \left( \frac{1 + e^{-x} - 1}{1 + e^{-x}} \right) \left( \frac{1}{1 + e^{-x}} \right) = (1 - \sigma(x)) \sigma(x)$$



# BP: Another example

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}} \quad \sigma(x) = \frac{1}{1 + e^{-x}} \text{ sigmoid function}$$

$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1 + e^{-x})^2} = \left( \frac{1 + e^{-x} - 1}{1 + e^{-x}} \right) \left( \frac{1}{1 + e^{-x}} \right) = (1 - \sigma(x)) \sigma(x)$$



# Summary

- **Neuron**
  - Input and output
- **Neural Networks**
  - Perceptron
  - Multi-layer network
  - Deep neural networks
- **How Neural Network Works**
  - Calculation process
  - Work as a multi-class classifier
- **Backpropagation**

# Thinking

- Are more layers in a neural network better?
- Why shouldn't we use linear function as activation function of neural network?