Numeration Systems

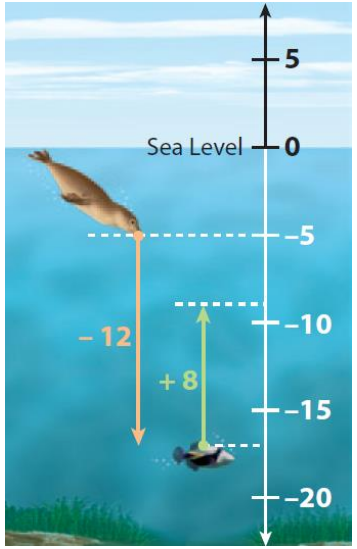
Numeration systems – Overview

- Numbers and symbols
  - Types of real numbers
  - How did these evolve?
- Digital and Analogue
  - Representing measurements
- Representing numbers
  - Basis of Decimal system
  - Binary Numbers
- Converting Decimal to Binary

Numbers and Symbols

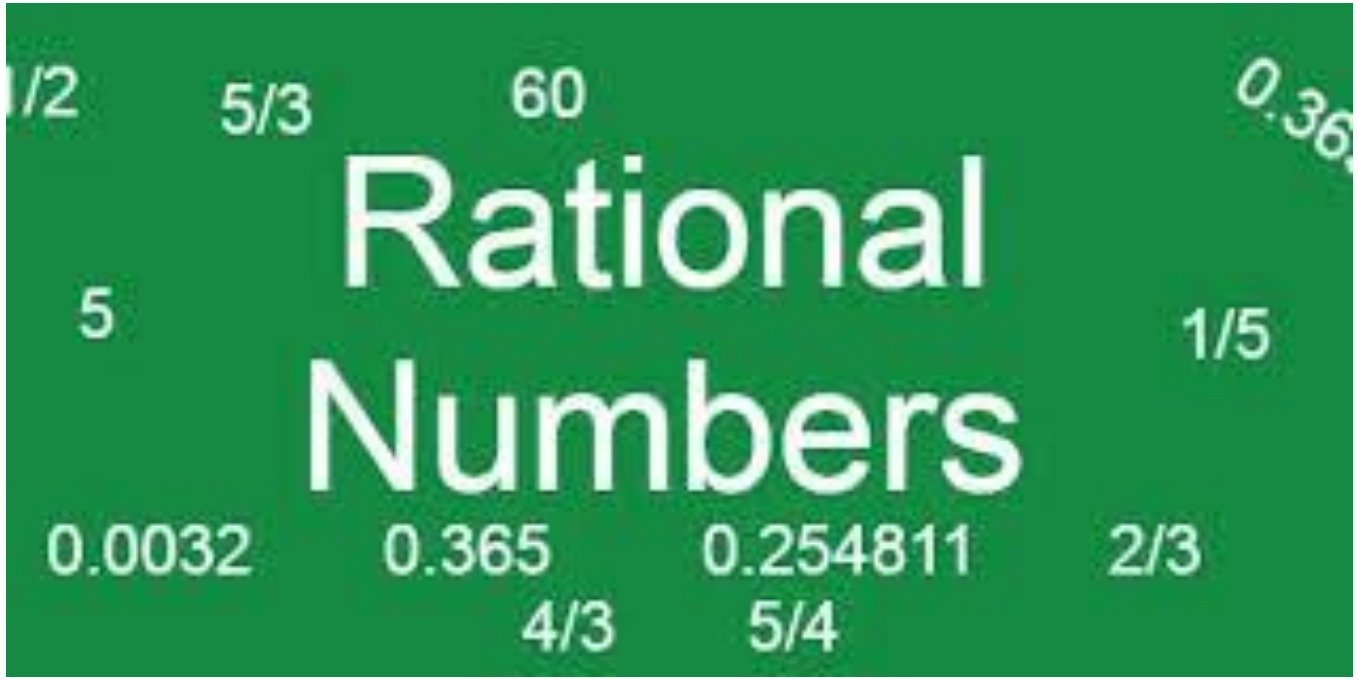
- Different types of number systems
  - serve different purposes in quantifying things in the real world

- E.g Resistor value 
- Whole numbers 1,2, 3, 4, 5 , 6, 7, 8, 9, 10……
- Integers ……-4,-3,-2,-1,0,1,2,3,4,……



Numbers and Symbols

- Rational numbers – can be written as a fraction, i.e. ratio of two integers

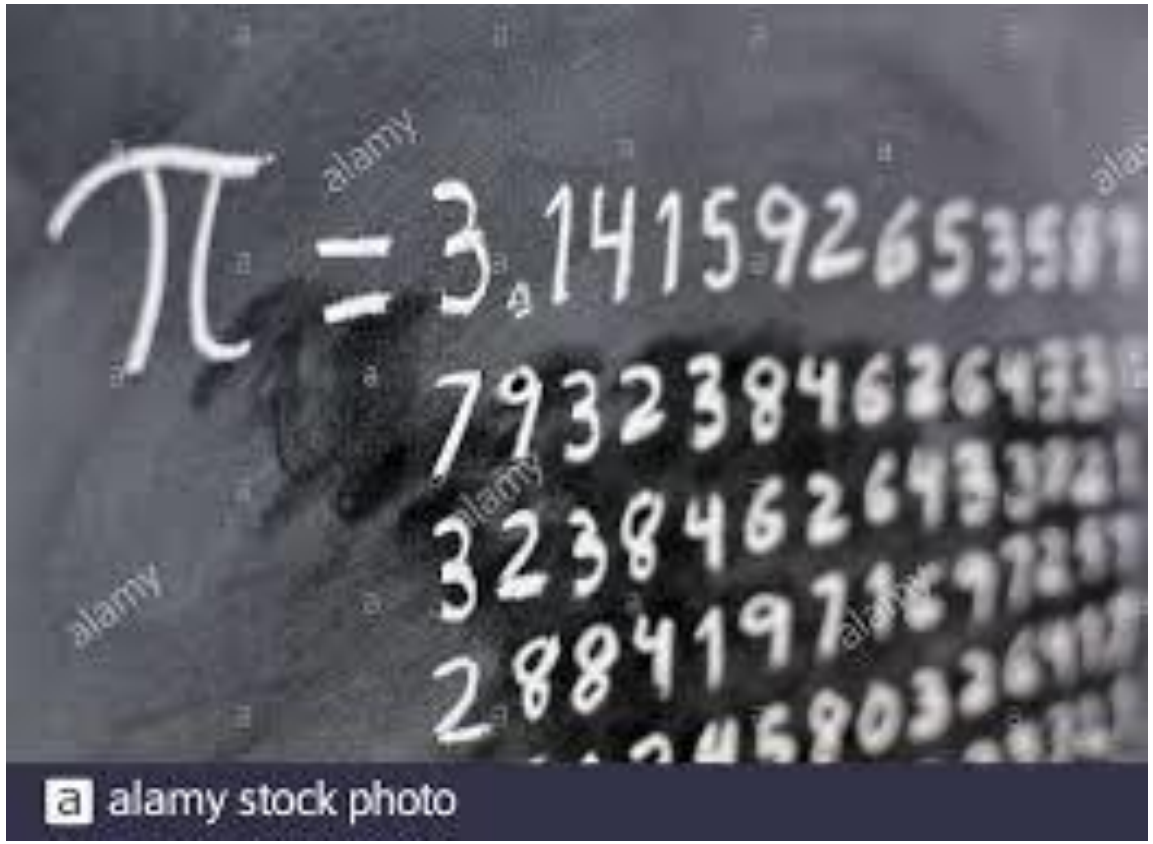


Numbers and Symbols

- Irrational numbers

  π = 3.1415927……

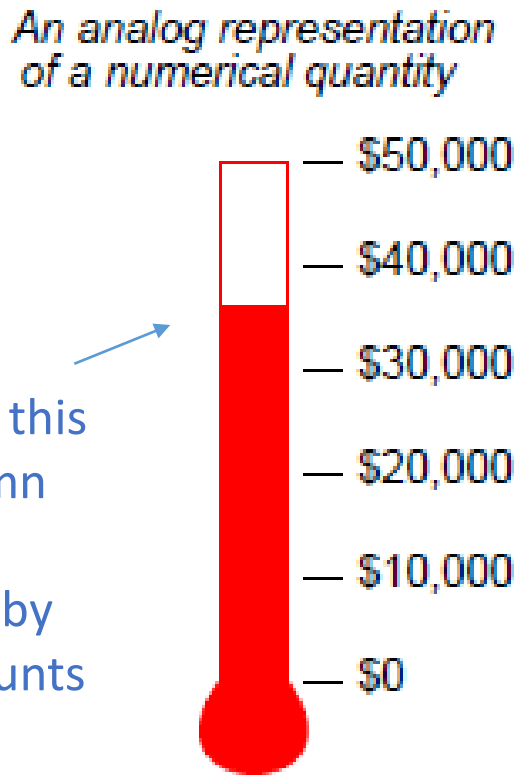
  e = 2.718281828……



Methods of Representing Digital or Analog measurements

Numbers and Measurements

Example from the book “ Lessons in electric circuits “
Two ways to represent the money raised during a fundraiser



height of this red column could be adjusted by tiny amounts

$35,955.38

Digital representation

What if we add 1c ?

Digital vs Analog

Slide Rule

WIRED **Slide Rule Still Rules**

Robert Miles (left), a retired Purdue University civil engineering professor and a Purdue alumnus, and James Alleman, a current Purdue civil engineering professor, hold a 7-foot-long slide rule in front of an exhibit they created that contains about 200 of the pre-digital computational devices. The permanent exhibit is on display in the university's Potter Engineering Center and includes slide rules from astronauts Neil Armstrong and Jerry Ross. **PURDUE NEWS SERVICE/DAVID UMBERGER**

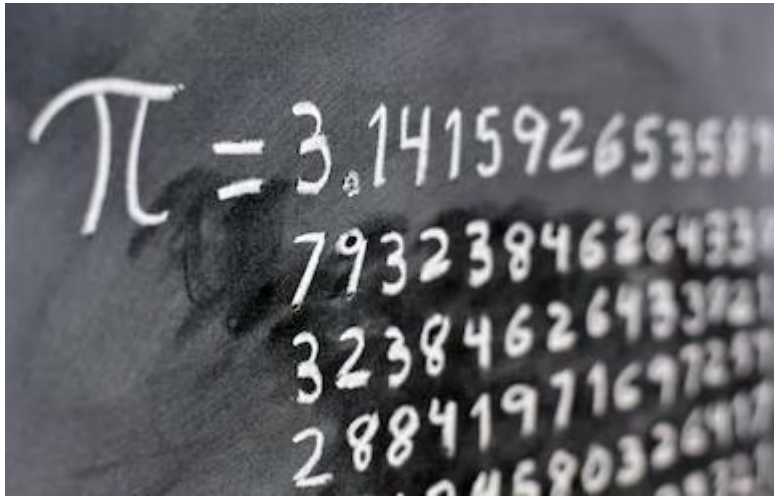
Digital?

Abacus

Each bead symbolizes a numerical quantity

Origins of First Wearable Computer?

Irrational numbers



Analog representations can in theory be used to represent irrational numbers
Slide rule – nudge dial to correct place

Digital representation?
Abacus – need additional bars to increase significant digits (infinite number of bars in theory)

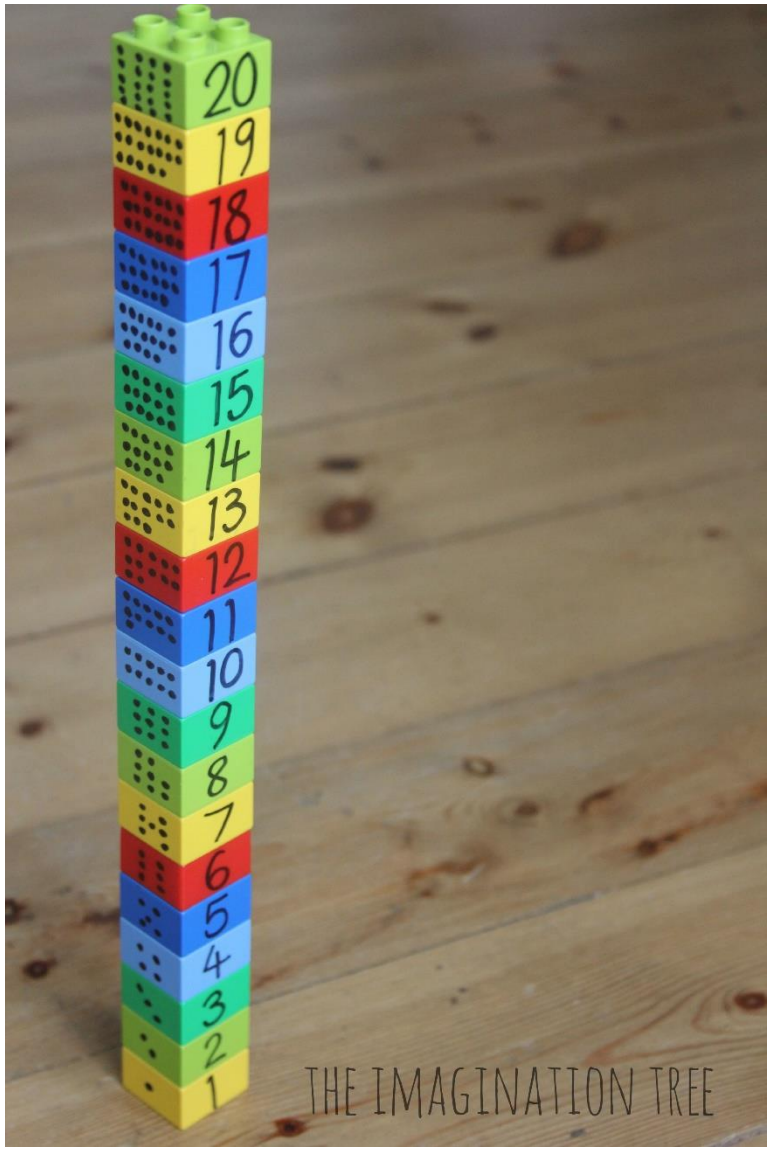
Analog Vs Digital

Analog	Digital
Intuitively understood	Requires training to interpret
Infinitely divisible	Discrete
Prone to errors of precision	Absolute precision

Taking analogue measurements

- Need reference as a standard to compare to
- Use number systems to provide this

Number systems have developed to quantify measurements and represent them symbolically



First Number Symbols

– Hash/tally marks on stones or sticks

1 2 3 4 5 6

12 12

Other tally systems

1 2 3 4 5

Numeration systems
- Roman Numerals

- 7 symbols or “ciphers”
- I, V, X, L, C, D, M

I = 1

V = 5

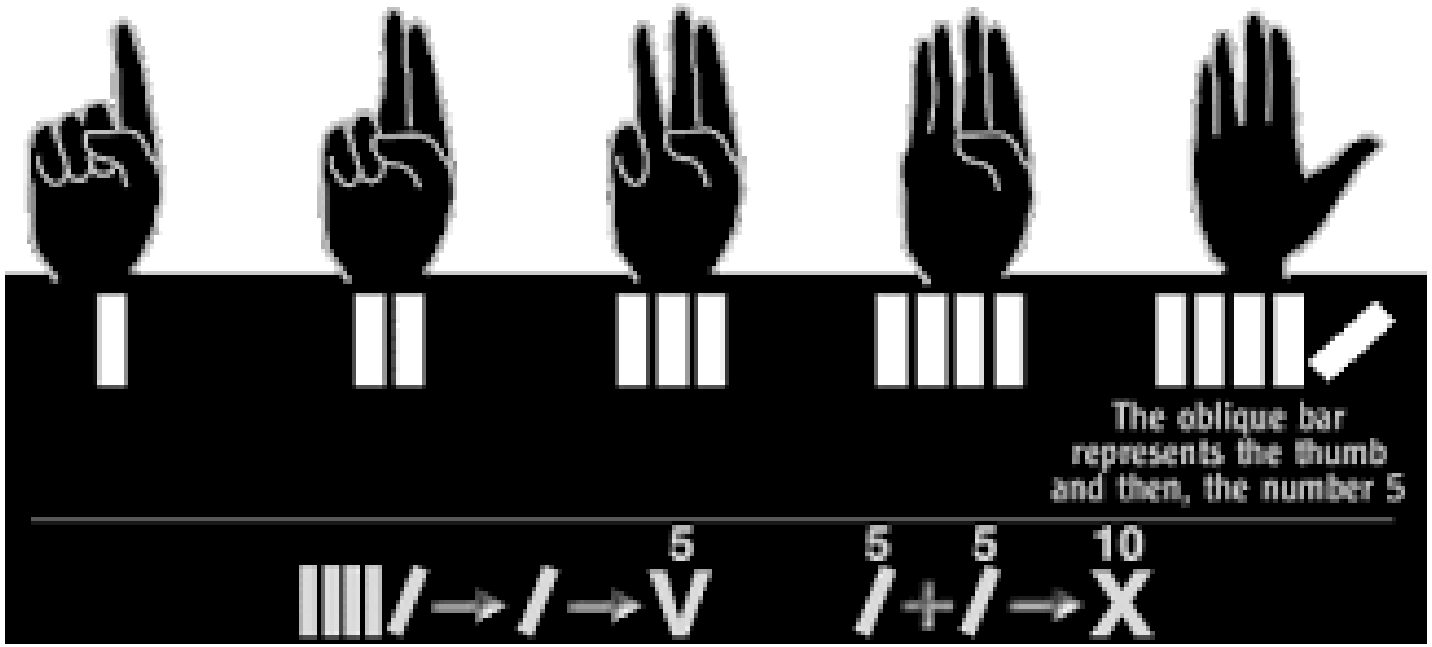
X = 10

L = 50

C = 100

D = 500

M = 1000



Numeration systems – Roman Numerals

XXV = 10 + 10 + 5 = 25

Roman Numerals

If cipher has a number equal or less to the right -> add the values

i.e. XX = 20 , XI = 11

If the cipher to the right is larger then this means “less than”

i.e. XL = 10 less than 50 = 40;

or XC = 90;

Question

Year 2021 would be written MMXXI
(1000 + 1000 + 10 + 10 + 1)

How would you write **1999**?

Solution

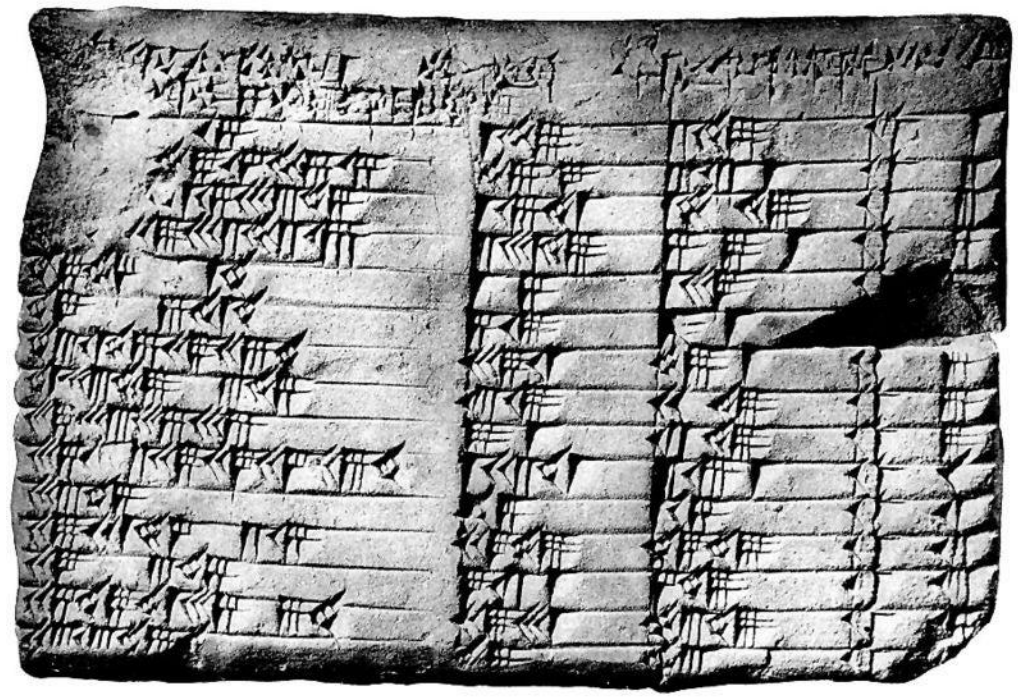
Year 2021 would be MMXXI,
How would you write 1999?

M(1000) + CM (900) + XC(90) + IX (9)
=1999

MCMXCIX

Numeration systems – cipher position

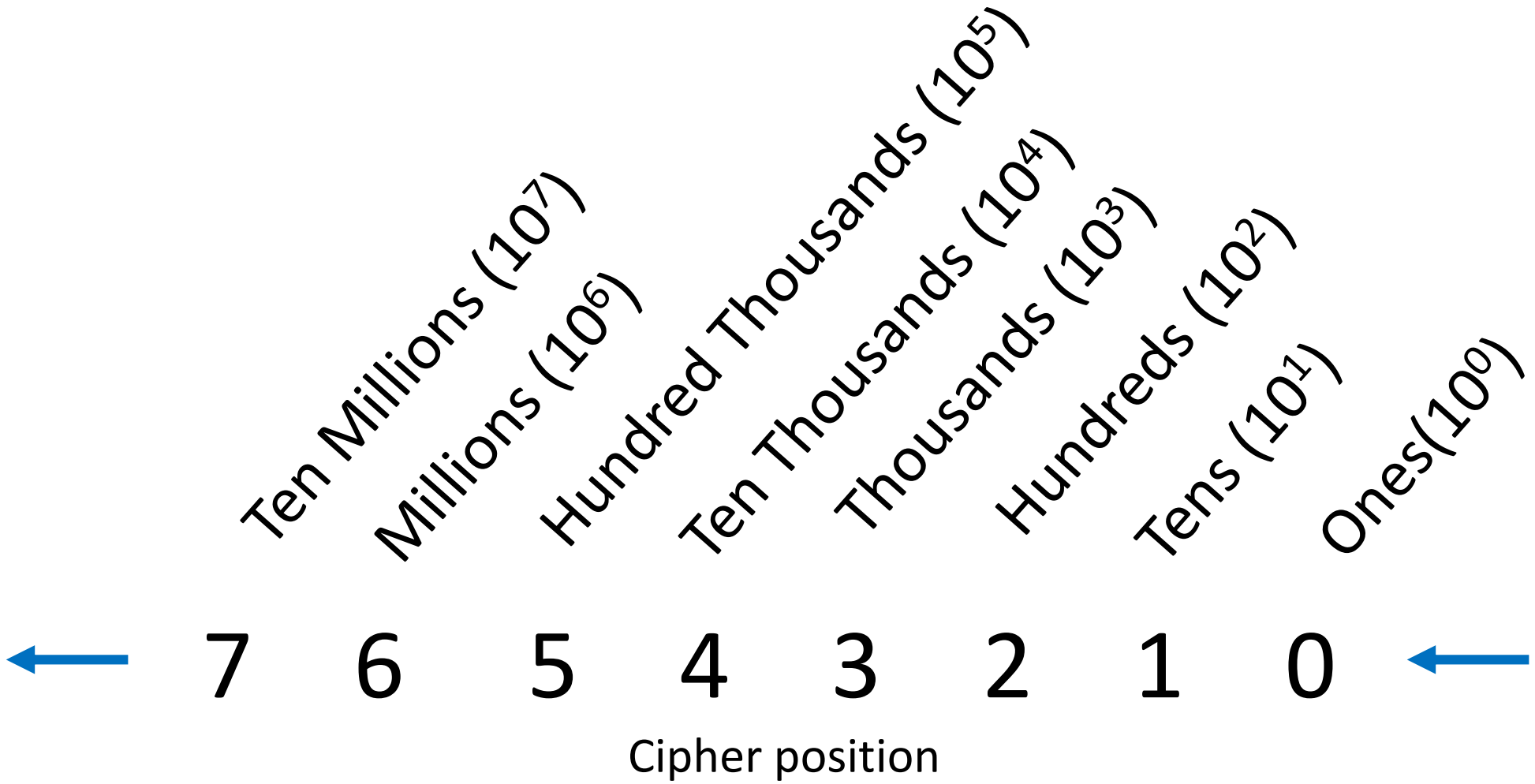
1	11	21	31	41	51
2	12	22	32	42	52
3	13	23	33	43	53
4	14	24	34	44	54
5	15	25	35	45	55
6	16	26	36	46	56
7	17	27	37	47	57
8	18	28	38	48	58
9	19	29	39	49	59
10	20	30	40	50	



1 and 60 have same symbol

Position decides weighting

Decimal systems – cipher position and weighting



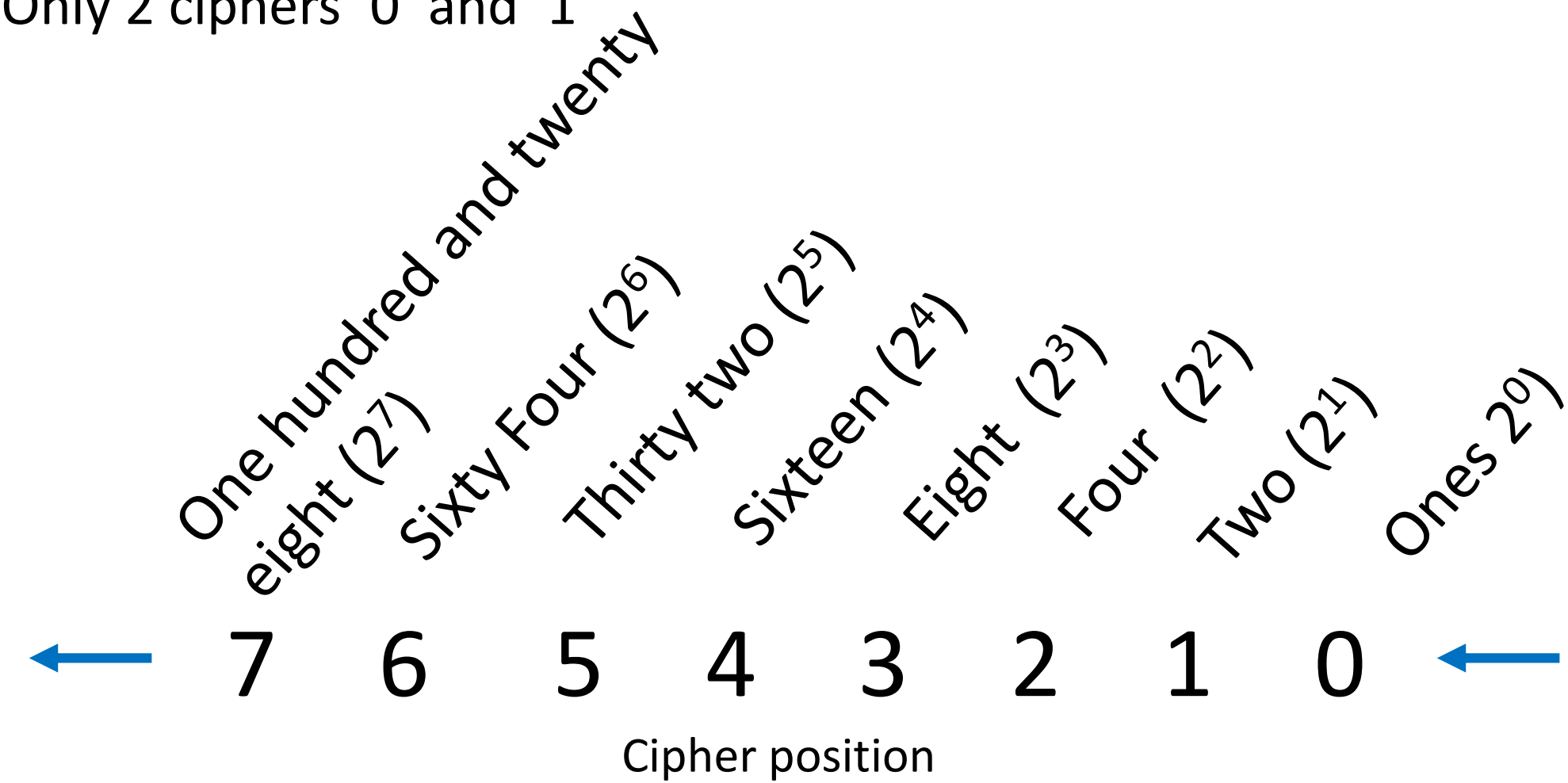
What if we have 2 ciphers?

- 0
- 1

- What would the weighting be of each cipher position?

Binary system

- Only 2 ciphers ‘0’ and ‘1’



Binary system

- Just like the decimal system the ciphers are arranged from right-to-left but in this case each place changes by a factor of 2

0	0
1	1
10	2
11	3
100	4
101	5

Binary system

Lets look at the binary number 111_2

1 1 1

(1 x 4) + (1 x 2) + (1 x 1)

=4+2+1=7

$111_2 = 7_{10}$

Binary system

Lets look at the binary number 11010_2

1 1 0 1 0

(1 x 16) + (1 x 8) + (0 x 4) + (1 x 2) + (0 x 1)

=16+8+2=26

$11010_2 = 26_{10}$

Question?

What is the decimal value of 10101_2 ?

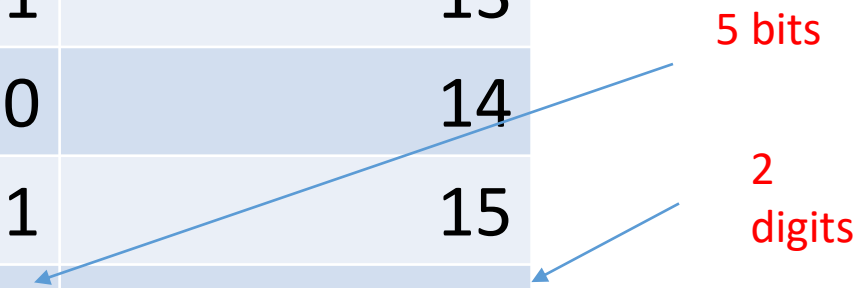
Question?

What is the decimal value of 10101_2 ?

Binary vs Decimal

Binary	Decimal
0	0
1	1
01	2
10	3
11	4
100	5
101	6
111	7
1000	8
1001	9

Binary	Decimal
1010	10
1011	11
1100	12
1101	13
1110	14
1111	15
10000	16
10001	17
10010	18
10011	19



Range of number system?

- If there is a limit on the number of positions for ciphers, then what is the maximum value we can represent?

Cipher positions and range

- 5 digits in decimal (base 10)

Largest number we can represent is 99 999, range is 0 up to 99999,

i.e. 100 000 or 10^5 integer values

- 5 bits in binary (base 2)
- Largest value is 11111 (or 31_{10}) with range of 0 to 31
- 32 or 2^5 integer values

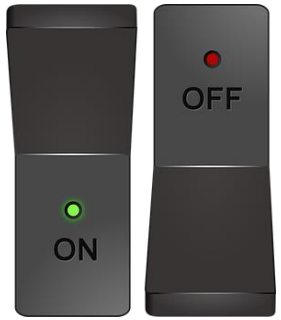
Question

- 8 bits in binary (base 2)

- What is the maximum range?

Why use the binary system?

- Electronically represent ‘0’ or ‘1’ if current is flowing or not
- Bit represented if current is on or off
- Switch from ON/OFF state



- To increase bits we need to add additional circuit

Convert binary to decimal

Convert 11001101_2 to decimal

1	**1**	**0**	**0**	**1**	**1**	**0**	**1**	
128	64	32	16	8	4	2	1	weighting

Most significant bit

Least significant bit

$(1)128_{10} + (1)64_{10} + 0(32_{10}) + 0(16_{10}) + (1)8_{10} + 1(4)_{10} + 0(2)_{10} + 1(1)_{10} = \mathbf{205_{10}}$

Convert binary to decimal

- What about fractions?

- Similar to how we use the decimal point
- Called a “binary point”, follows same principle
- Each point to right is half the value of the one to the left

- So the the first three places past the binary point are ½ , ¼ and 1/8 or 2^{-1} , 2^{-2}, 2^{-3}

Convert binary to decimal

- Example

  Convert 101.011_2 to decimal

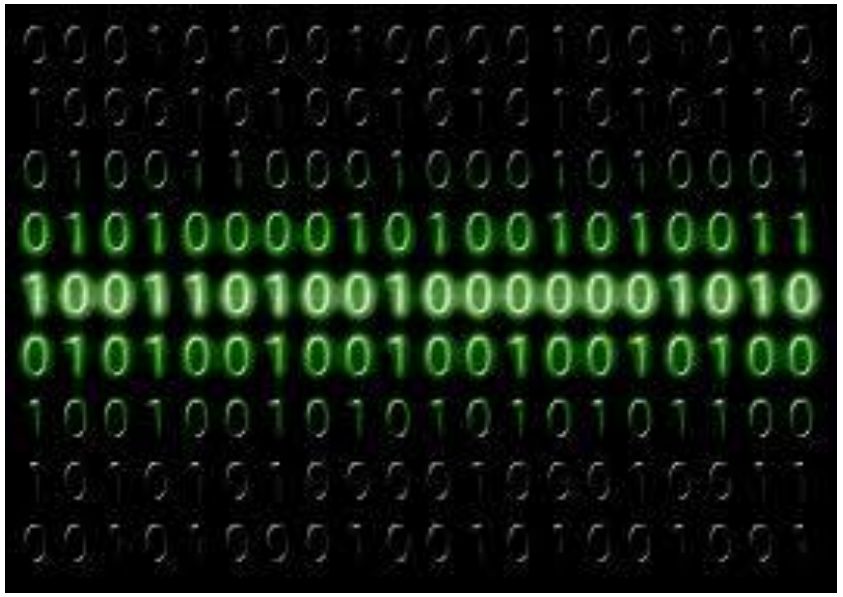
1	**0**	**1**	**.**	**0**	**1**	**1**
4	2	1	.	1/2	1/4	1/8

$$4_{10} + 1_{10} + 0.25_{10} + 0.125_{10} = 5.375_{10}$$

Question

- Convert 11.101 to decimal?

Other number systems



Lots of 0’s and 1’s are difficult for us to work with

Other number systems, multiples of base 2
Easier to use, can be converted to binary

Octal – Base 8
Hexadecimal – Base 16

Summary

- Use of digital and analogue systems for measurements
- Numbers and numeration systems
  - Types of ciphers
  - Position of ciphers
  - Range of values
- Decimal system and binary system
  - Weighting system
  - Conversion from binary to decimal
  - Representing fractions