6 Electromagnetic Scattering by Cylindrical Objects on Generic Planar Substrates: Cylindrical-Wave Approach

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Abstract. An analytical-numerical technique for the solution of the two-dimensional scattering of an electromagnetic wave by a set of circular cylinders in the presence of a plane discontinuity for the electromagnetic constants is discussed. Since the interface is only characterized by its reflection coefficient, a wide class of reflecting surfaces can be treated with the same formalism. The solution is obtainable for both the near and the far region, regardless the polarization state of the incident wave. The method exploits the possibility of representing any two-dimensional field in terms of a suitable superposition of cylindrical waves. The expansion coefficients represent the unknowns in a typical scattering problem and can be determined by imposing the boundary conditions. The presence of the interface leads to the necessity of evaluating the reflected cylindrical waves, and this is achieved starting from the Fourier spectrum of the cylindrical functions on a plane.

1 Introduction

Cylindrical waves, expressed as the product of the Hankel function of integer order m [1] times the factor $\exp(\mathrm{i}m\vartheta)$, ϑ being the angular coordinate, are fundamental building blocks in constructing the solution of two-dimensional scattering problems [2]. Due to the fact that these waves are the eigenfunctions of the two-dimensional Laplace operator, they can be assumed as a basis for describing solutions of the two-dimensional electromagnetic problem constituted by the Helmholtz equation and the Sommerfeld radiation condition, although some care has to be used in choosing the convergence domain of the expansions [3,4].

In particular, such solutions are employed to describe the scattered field by cylinders in free space for incident plane waves [2] or Gaussian beams [5–7]. Moreover, circular cylindrical scatterers may be used to simulate the behavior of two-dimensional objects of arbitrary transverse shape, both in the small radius approximation [8] and in the generic radius case [9]. Cylindrical functions play a fundamental role also when numerical procedures are used,

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which allow the case of arbitrarily shaped cylindrical scatterers to be solved [4,10,11].

In all the problems involving cylindrical structures, one has to resort to a suitable representation of the scattered field. A very common approach [12] is to consider this field as a linear combination of the above mentioned cylindrical functions. The coefficients of such expansions are the unknown of the problem and are determined by imposing the boundary conditions on the surfaces of the cylinders.

The problem becomes more complex when a planar interface is present near the cylindrical scatterer. This is the case, for instance, of the radar detection of objects near the ground [13], the design of diffractive optical gratings [14], the study of quasi-optical launchers of lower-hybrid plasma-waves [15–19], and the study of surface contaminations in the semiconductor industry [20].

Several solutions can be found in the literature for problems of this kind. For example, an infinite wire grid parallel to the interface was treated by Wait for cases of both a perfectly conducting plane and a vacuum-dielectric interface [21–23]. Furthermore, for the case of cylinders in front of an interface, the problem has been approached with different techniques, using methods based on integral equations [13], coupled dipoles [24], the extinction theorem [25–28], or the image theory [29]. When the surface coincides with the interface between vacuum and a dielectric homogeneous medium or it is a real conducting plane, numerical methods are available also for non cilindrical objects [30–33]. In addition, partially buried scatterers have recently been considered [34,35].

In general, when a planar interface is present near the cylindrical scatteres, the cylindrical symmetry is broken and an approach based on cylindrical waves seems to be less natural. Nonetheless, cylindrical functions assume an important role if use is made of their Fourier spectrum, inasmuch as the presence of the reflecting surface can be taken into account by considering the reflection of each elementary plane wave. This is the approach used by Wait [21–23] and Clemmow [36], but in their case the diameter of the cylinders is much smaller than the wavelength, so that only the cylindrical wave of zero order is needed to describe the scattered field.

Here, we report the theoretical basis of a method, based on the use of cylindrical waves, for treating the two-dimensional scattering by a set of parallel cylindrical scatterers with arbitrary size, in the presence of a general, planar reflecting interface. In Sect. 3 the study of the scattering of a plane wave by perfectly conducting cylinders is treated, while in Sect. 4 the generalization of the method to dielectric (possibly lossy) diffractive cylinders is given. Section 5 concerns the case in which a general two-dimensional field is considered as the incident field. Finally, in Sect. 6 some examples of application of the method are given for cases of practical interest.

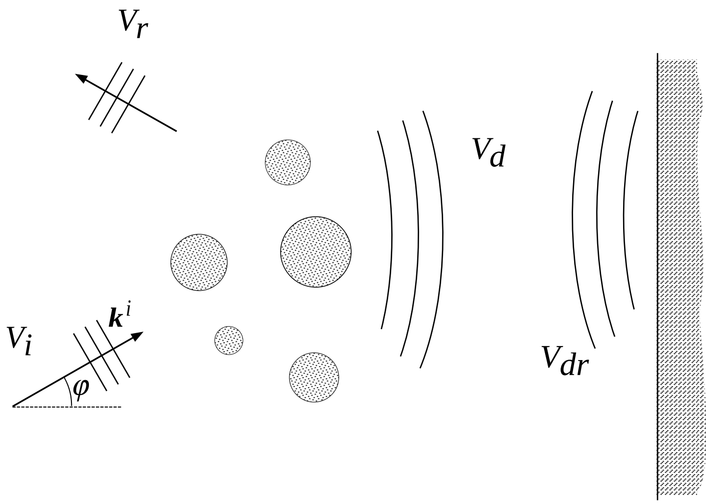

Fig. 1. Fields involved in the scattering process.

2 Preliminaries

From a mathematical viewpoint, the problem consists in solving the two-dimensional Helmholtz equation in the open domain constituted by the half-space to the left side of the planar interface, outside the cylinders (see Fig. 1). Boundary conditions on the various discontinuity surfaces have to be imposed by taking the electromagnetic properties of the interacting bodies into account. As said before, the key for solving this problem is the use of the angular spectrum of the cylindrical waves, which has been recently found for an arbitrary-order cylindrical wave [37]. Indeed, by using such a tool, the reflection of a typical cylindrical wave by a generally reflecting plane surface can be evaluated. The price to be paid is the evaluation of some integrals of oscillating functions. In most cases, these integrals have to be calculated numerically and this represents a computational bottle-neck of the method. However, the presented analysis offers a series of advantages, as will be clear in the following.

Before entering the mathematical details of our procedure, we want to outline in an intuitive way the fundamental ideas constituting the basis of the method. Essentially, the total electromagnetic field resulting from the multiple interaction process is considered as the superposition of several contributions, which are:

- V_i: field due to the incident plane wave;
- V_r: field due to the reflection of the incident plane wave by the planar surface;
- V_d: field diffracted by the cylinders;
- V_{dr}: field due to the reflection of V_d by the planar surface;

- V_c: total field inside the diffracting cylinders.

Each of these contributions have to be expressed by means of suitable modal expansions. Since the diffractive structure consists in one or more circular cylinders with parallel axes, each of the above fields will be expanded as a superposition of cylindrical waves centered on the cylinder axes.

3 Scattering from Perfectly Conducting Cylinders

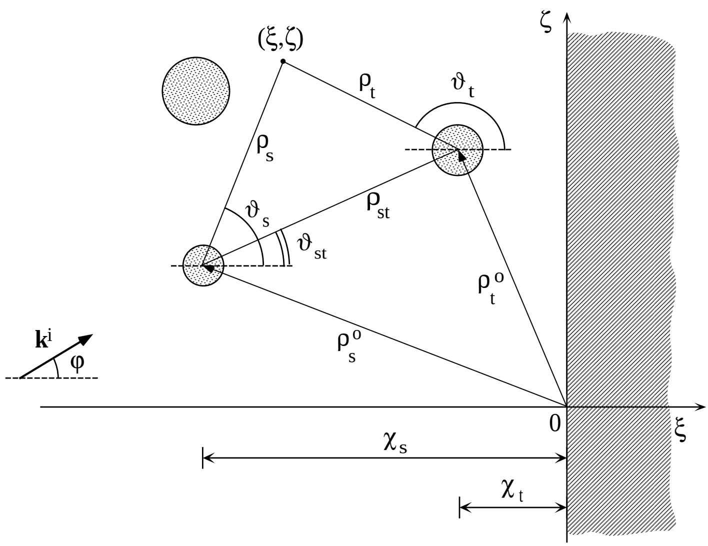

Fig. 2. Geometry of the scattering problem.

In Fig. 2 the geometrical layout of a typical scattering problem is shown: N perfectly conducting circular cylinders with radii a_t $(t = 1, \ldots, N)$ are placed in front of the reflecting surface. The axis of each cylinder is orthogonal to the plane $\xi\zeta$ and the structure is assumed to be infinite along the y direction, so that the problem is reduced to a two-dimensional form. In the following, for brevity, dimensionless coordinates $\xi = kx$, $\zeta = kz$ are used. $V(\xi, \zeta)$ represents the component of the electromagnetic field parallel to the y axis, i.e., $V = E_y$ for TM polarization and $V = H_y$ for TE polarization. In this section we suppose that the incident field is a monochromatic plane wave whose wavevector, say $\mathbf{k}^i$, lies in the $\xi\zeta$ plane and φ is the angle between the wavevector $\mathbf{k}^i$ and the ξ-axis. In Sect. 5 we will generalize the incident field by considering a general, two-dimensional, monochromatic field. The presence of the planar reflecting surface is taken into account by means of its complex plane-wave reflection coefficient $\Gamma\left(n_\parallel\right)$, where $\mathbf{n} = \mathbf{k}/k$, $\mathbf{k}$ being the wavevector of a typical plane wave incident on the surface, and $k = 2\pi/\lambda$ is its wavenumber. Here and in the following, the symbols $\perp$ and $\|$ refer to the orthogonal and parallel components of a vector with respect to the interface, respectively. The time dependence of the field is assumed to be $\exp(-i\omega t)$, where ω is the angular frequency, which will be omitted throughout the paper.

Let $\boldsymbol{\rho}_t^0 \equiv \left(\xi_t^0, \zeta_t^0\right)$ $(t = 1, \dots, N)$ be the position vector of the center of the tth cylinder in the main reference frame (O, ξ, ζ) (MRF from now on), and $\boldsymbol{\rho}_t \equiv (\xi_t, \zeta_t)$ $(t = 1, \dots, N)$ be the position vector in the frame centered on that cylinder (RF_t). It is convenient to choose the ζ axis of MRF lying on the reflecting surface.

As far as the incident field V_i is concerned, the following expansion, in terms of cylindrical waves centered on RF_t, holds [38]:

$$\begin{aligned} V_i(\xi, \zeta) &= V_0 \exp\left(i n_\perp^i \xi_t^0 + i n_\|^i \zeta_t^0\right) \exp\left(i n_\perp^i \xi_t + i n_\|^i \zeta_t\right) \\ &= V_0 \exp\left(i n_\perp^i \xi_t^0 + i n_\|^i \zeta_t^0\right) \sum_{m=-\infty}^{+\infty} i^m \exp(-i m \varphi) \, J_m(\rho_t) \exp(i m \vartheta_t), \end{aligned} \tag{1}$$

where (ρ_t, ϑ_t) are polar coordinates in RF_t, and use has been made of the expansion of a plane wave in terms of Bessel functions J_m [1]. Equation (1) gives the field associated to the point having coordinates (ξ, ζ) in MRF as a function of the coordinates (ξ_t, ζ_t) in RF_t.

A similar expansion can be given for the reflected field V_r, i.e. [38],

$$\begin{aligned} V_r(\xi, \zeta) &= V_0 \,\mathit{\Gamma}\left(n_\|^i\right) \exp\left(-i n_\perp^i \xi_t^0 + i n_\|^i \zeta_t^0\right) \exp\left(-i n_\perp^i \xi_t + i n_\|^i \zeta_t\right) \\ &= V_0 \,\mathit{\Gamma}\left(n_\|^i\right) \exp\left(-i n_\perp^i \xi_t^0 + i n_\|^i \zeta_t^0\right) \\ &\quad \times \sum_{m=-\infty}^{+\infty} i^m \exp(-i m \bar{\varphi}) \, J_m(\rho_t) \exp(i m \vartheta_t), \end{aligned} \tag{2}$$

where $\bar{\varphi} = \pi - \varphi$ is the angle of propagation of the reflected plane wave (see Fig. 2).

The diffracted field V_d can be expressed as a sum of cylindrical functions with unknown coefficients c_{sm} $(s = 1, \dots, N;\ m = 0, \pm 1, \pm 2, \dots)$ in the following way [39]:

$$V_d(\xi, \zeta) = V_0 \sum_{s=1}^{N} \sum_{m=-\infty}^{+\infty} i^m \exp(-i m \varphi) \, c_{sm} \, \mathrm{CW}_m(\xi_s, \zeta_s), \tag{3}$$

where $\mathrm{CW}_m(\xi_s, \zeta_s)$ is the cylindrical function

$$\mathrm{CW}_m(\xi_s, \zeta_s) = H_m^{(1)}(\rho_s) \exp(i m \vartheta_s), \tag{4}$$

$H_m^{(1)}$ is the outgoing Hankel function [1], and (ξ_s, ζ_s) are rectangular coordinates corresponding to (ρ_s, ϑ_s) (see Fig. 2). The last term to be analyzed is V_{dr}. In [39] it was shown that the field due to the reflection of the cylindrical wave $\mathrm{CW}_m(\xi_s, \zeta_s)$ by the interface can be written as $\mathrm{RW}_m(2\chi_s - \xi_s, \zeta_s)$, $\mathrm{RW}_m(\xi, \zeta)$ being the function defined as

$$\mathrm{RW}_m(\xi, \zeta) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \Gamma\left(n_\parallel\right) F_m\left(\xi, n_\parallel\right) \exp\left(i n_\parallel \zeta\right) \mathrm{d}n_\parallel, \tag{5}$$

where $\chi_s = -\xi_s^o$ denotes the distance between the axis of the s-th cylinder and the reflecting surface (see Fig. 2), and $F_m\left(\xi, n_\parallel\right)$ is the angular spectrum of the cylindrical function CW_m, defined as

$$F_m\left(\xi, n_\parallel\right) = \int_{-\infty}^{+\infty} \mathrm{CW}_m(\xi, \zeta) \exp\left(-i n_\parallel \zeta\right) \mathrm{d}\zeta, \tag{6}$$

whose explicit expression is [39,37]

$$F_m(\xi, n_\parallel) = \frac{2 \exp\left(i n_\perp \xi\right)}{n_\perp} \exp\left(-i m \arccos n_\parallel\right). \tag{7}$$

By considering the reflection of each cylindrical wave and summing all the contributions, from (3) we obtain

$$V_{dr}(\xi, \zeta) = V_0 \sum_{s=1}^{N} \sum_{m=-\infty}^{+\infty} i^m \exp(-i m \varphi)\, c_{sm}\, \mathrm{RW}_m(2\chi_s - \xi_s, \zeta_s). \tag{8}$$

Equation (8) shows that the diffracted-reflected field can be thought of as the superposition of fields diffracted by "image" cylinders centered at the points $\left(-\xi_s^0, \zeta_s^0\right)$, for $s = 1, \ldots, N$. These fields are modulated by the presence of the interface through the definition of RW_m functions [see (5)].

Finally, due to the hypothesis of perfectly conducting cylinders, the field must vanish inside them, so that $V_c(\xi, \zeta) = 0$.

In this case, the boundary conditions are expressed by the following equations:

$$V_i + V_r + V_d + V_{dr}\big|_{\rho_t = k a_t} = 0, \tag{9}$$

for TM polarization, and

$$\partial_{\rho_t} V_i + \partial_{\rho_t} V_r + \partial_{\rho_t} V_d + \partial_{\rho_t} V_{\mathrm{dr}}\big|_{\rho_t = k a_t} = 0, \tag{10}$$

for TE polarization. After some algebra, the following linear system for the unknown coefficients $c_{s\ell}$ is obtained [38]:

$$\sum_{s=1}^{N}\sum_{\ell=-\infty}^{+\infty} A_{m\ell}^{st} c_{s\ell} = B_m^t, \qquad \begin{pmatrix} m = 0,\, \pm 1,\, \pm 2, \dots \\ t = 1, \dots, N \end{pmatrix}, \tag{11}$$

where

$$A_{m\ell}^{st} = i^{\ell} \exp\left(-i\ell\varphi\right) \\ \times \left\{ \left[\mathrm{CW}_{\ell-m}\left(\xi_{st}, \zeta_{st}\right)\left(1-\delta_{st}\right) + I_{\ell+m}^{st} \right] T_m\left(ka_t\right) + \delta_{st}\delta_{\ell m} \right\}, \tag{12}$$

$$I_{\ell+m}^{st} = \mathrm{RW}_{\ell+m}\left(\chi_s + \chi_t, \zeta_t^0 - \zeta_s^0\right), \tag{13}$$

$$B_m^t = -i^m \exp\left(i\, n_{\|}^i \zeta_t^0\right) T_m\left(ka_t\right) \\ \times \left[\exp\left(i\, n_{\perp}^i \xi_t^0\right) \exp\left(-im\varphi\right) + \Gamma\left(n_{\|}^i\right) \exp\left(-i\, n_{\perp}^i \xi_t^0\right) \exp\left(-im\bar{\varphi}\right) \right], \tag{14}$$

where the function $T_m(x)$, which brings information about the polarization state of the incident field, is defined as [39]

$$T_m(x) = \begin{cases} \dfrac{J_m(x)}{H_m^{(1)}(x)} & \text{for TM polarization,} \\[2ex] \dfrac{J_m'(x)}{H_m^{(1)\prime}(x)} & \text{for TE polarization,} \end{cases} \tag{15}$$

the prime denoting derivation. The solution of the system (11) leads to the evaluation of the unknown coefficients $c_{s\ell}$ and then to the complete determination of the total field both in the near and in the far region [38].

Some words have now to be spent to illustrate the main computational advantages and limitations of the present method. Although no approximations have been introduced in the theoretical basis of the method, in order to implement it we have unavoidably to truncate the series in (11). Physically, this means that the field diffracted by the sth cylinder is expressed by a finite sum of CW_m functions [see (3)], letting m run from $-M_s$ to M_s, M_s being the truncation order for the sth cylinder. As a matter of fact, M_s depends on the size of the sth cylinder [38] and therefore the algorithm complexity grows with the size and the number of cylinders.

An important problem to be considered is the numerical evaluation of the matrix elements $I^{st}_{\ell+m}$ [see (13)], which actually represents the most limiting factor of the method, at least from a computational point of view. Indeed, these functions are defined by means of integrals of oscillating functions [see (5)], whose amplitude and frequency strongly depend on the order $\ell + m$ and on the mutual distances $\zeta^0_t - \zeta^0_s$ $(s, t = 1, 2, \ldots, N)$, respectively. As can be shown [38], the maximum value of $\ell + m$ is related to the radius of the cylinders, while the transverse (i.e., in the ζ direction) extension of the whole diffractive structure fixes the maximum value of $\zeta^0_t - \zeta^0_s$. High values of $\ell + m$ or $\zeta^0_t - \zeta^0_s$ make the evaluation of the involved integrals rather heavy, and special integration routines, e.g., based on a combined use of Gaussian techniques and extrapolation methods [40], have to be used. We tested the algorithm for values of M_s up to some tens, corresponding to cylinders whose diameters are of the order of some wavelengths. It should be stressed that in such cases neither the Rayleigh nor the Kirchhoff approximation (valid for very small or very large objects, respectively) can be used.

4 Generalization to Dielectric Cylinders

The theory developed in the previous section deals with the scattering problem by a set of perfectly conducting cylinders. However, in many cases such a model does not work properly. This happens, for example, in the case of TE polarization, where the hypothesis of infinite conductivity is often not valid, as well as in dissipative structures.

Essentially, the generalization to the dielectric (possibly dissipative) case consists in considering also the field present inside the tth dielectric cylinder, denoted by V^t_c, which has the form [3]

$$V^t_c\left(\xi, \zeta\right) = V_0 \sum_{m=-\infty}^{+\infty} i^m \exp\left(-im\varphi\right)\, d_{tm}\, J_m\left(n^t_c \rho_t\right) \exp(im\vartheta_t), \tag{16}$$

where d_{tm} $(t = 1, \ldots, N)$ are further unknown coefficients and n^t_c $(t = 1, \ldots, N)$ are the complex refractive indexes of the cylinders. In such a case, boundary conditions are defined in a more complicated way than in the case of perfectly conducting cylinders [see (9)], because the continuity of both the electric and the magnetic field are to be imposed. This leads to a pair of equations for each polarization state, i.e.,

$$\begin{cases} V_i + V_r + V_d + V_{dr}\big|_{\rho_t = ka_t} = V^t_c\big|_{\rho_t = ka_t}\,, \\[2ex] \partial_{\rho_t} V_i + \partial_{\rho_t} V_r + \partial_{\rho_t} V_d + \partial_{\rho_t} V_{dr}\big|_{\rho_t = ka_t} = q_t\; \partial_{\rho_t} V^t_c\big|_{\rho_t = ka_t}\,, \end{cases} \tag{17}$$

where $q_t = 1$ for TM polarization and $q_t = (1/n^t_c)^2$ for TE polarization $(t = 1, \ldots, N)$. By proceeding in the same way as for the perfectly conducting case, the following coupled linear systems are derived [41]:

$$\begin{aligned}
\sum_{s=1}^{N}\sum_{\ell=-\infty}^{+\infty} A_{m\ell}^{st(1)} c_{s\ell} - G_m^{t(1)} d_{tm} &= B_m^{t(1)}, \\
\sum_{s=1}^{N}\sum_{\ell=-\infty}^{+\infty} A_{m\ell}^{st(2)} c_{s\ell} - G_m^{t(2)} d_{tm} &= B_m^{t(2)},
\end{aligned} \tag{18}$$

where superscripts (1) and (2) refer to the boundary conditions on the field and its normal derivative, respectively. In particular

$$G_m^{t(1)} = \exp(-im\varphi)\frac{J_m(n_c^t ka)}{H_m^{(1)}(ka_t)}, \tag{19}$$

$$G_m^{t(2)} = p_t \exp(-im\varphi)\frac{J_m'(n_c^t ka)}{H_m^{(1)\prime}(ka_t)}, \tag{20}$$

with p_t defined as

$$p_t = \begin{cases} n_c^t & \text{for TM polarization,} \\ 1/n_c^t & \text{for TE polarization.} \end{cases} \tag{21}$$

Furthermore, the coefficients of the linear systems (18) are

$$\begin{aligned}
A_{m\ell}^{st(1,2)} &= i^{\ell} \exp\left(-i\ell\varphi\right) \\
&\times \left\{ \left[\mathrm{CW}_{\ell-m}\left(\xi_{st}, \zeta_{st}\right)\left(1-\delta_{st}\right) + I_{\ell+m}^{st} \right] T_m^{(1,2)}\left(ka_t\right) + \delta_{st}\delta_{\ell m} \right\},
\end{aligned} \tag{22}$$

$$I_{\ell+m}^{st} = \mathrm{RW}_{\ell+m}\left(\chi_s + \chi_t, \zeta_t^0 - \zeta_s^0\right), \tag{23}$$

$$\begin{aligned}
B_m^{t(1,2)} &= -i^m \exp\left(i\, n_{\parallel}^i \zeta_t^0\right) T_m^{(1,2)}\left(ka_t\right) \\
&\times \left[\exp\left(i\, n_{\perp}^i \xi_t^0\right) \exp\left(-im\varphi\right) + \Gamma\left(n_{\parallel}^i\right) \exp\left(-i\, n_{\perp}^i \xi_t^0\right) \exp\left(-im\bar{\varphi}\right) \right],
\end{aligned} \tag{24}$$

and the functions $T_m^{(1,2)}(x)$ are defined as

$$T_m^{(1)}(x) = \frac{J_m(x)}{H_m^{(1)}(x)}, \tag{25}$$

$$T_m^{(2)}(x) = \frac{J'_m(x)}{H_m^{(1)\prime}(x)}. \tag{26}$$

A way to solve the systems (18) is to eliminate the coefficients d_{tm}, thus obtaining the following linear system for the sole $c_{s\ell}$ coefficients

$$\sum_{s=1}^{N} \sum_{\ell=-\infty}^{+\infty} D_{m\ell}^{st}\, c_{s\ell} = L_m^t, \tag{27}$$

where

$$D_{m\ell}^{st} = G_m^{t(2)} A_{m\ell}^{st(1)} - G_m^{t(1)} A_{m\ell}^{st(2)}, \tag{28}$$

$$L_m^t = G_m^{t(2)} B_m^{t(1)} - G_m^{t(1)} B_m^{t(2)}. \tag{29}$$

Finally, once the $c_{s\ell}$ coefficients are known, it is possible to evaluate the internal field V_c^t by means of the d_{tm} coefficients in the following way:

$$d_{tm} = -\frac{1}{G_m^{t(1)} - G_m^{t(2)}} \left\{ B_m^{t(1)} - B_m^{t(2)} - \sum_{s=1}^{N} \sum_{\ell=-\infty}^{+\infty} \left[A_{m\ell}^{st(1)} - A_{m\ell}^{st(2)} \right] c_{s\ell} \right\}. \tag{30}$$

5 General Incident Fields

Once the plane-wave scattering by a set of cylinders has been well established, an extension to treat the case of a generic two-dimensional impinging field can be performed. This case is particularly useful when the plane-wave approximation is not valid, for example when the field has been formed by an antenna or an optical system and the scattering elements are in its near-field or intermediate field region.

The general lines closely follow the plane-wave case. Assuming the CW_m functions as the basis for the solution of the Helmholtz equation with Sommerfeld radiation condition, use of (3), (8), and (16) can be made also in this case: therefore, it is sufficient to give suitable expressions for the fields V_i and V_r.

Now, we assume that the y-component of the incident field is represented by means of the Fourier integral

$$V_i(0, \zeta') = \frac{V_0}{2\pi} \int_{-\infty}^{+\infty} \tilde{U}_0(p) \exp(\mathrm{i}\zeta' p) \mathrm{d}p. \tag{31}$$

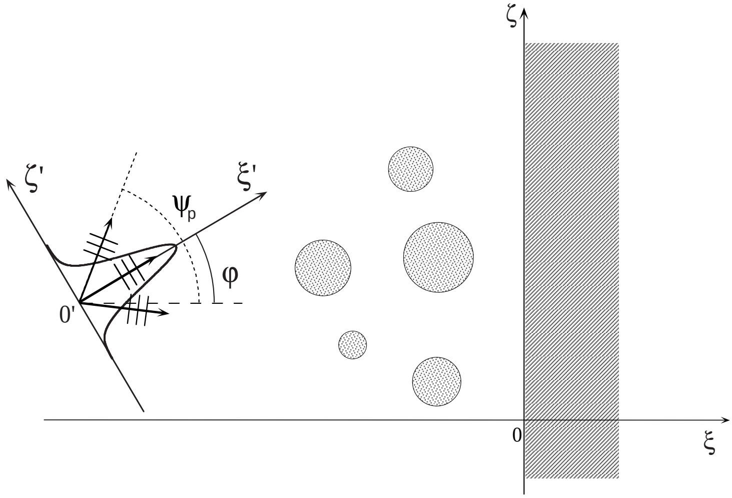

Fig. 3. Plane-wave decomposition of a general incident field.

In order to easily handle different incident fields, we introduce a further reference frame, Incident Field Reference Frame (IFRF, for short). In Fig. 3 a Gaussian profile is sketched as an example. In (31), p is the conjugate variable of ζ' and $\tilde{U}_0(p)$ is the angular spectrum of the incident field. A change of reference frame from IFRF to MRF allows us to obtain the expression of V_i as [18]

$$V_i(\xi,\zeta) = \frac{V_0}{2\pi}\int_{-\infty}^{+\infty} \tilde{U}_0(p)\exp\{\mathrm{i}[(\xi-\xi_0)\cos\psi_p + (\zeta-\zeta_0)\sin\psi_p]\}\mathrm{d}p, \quad (32)$$

where (ξ_0, ζ_0) are the coordinates of the IFRF origin, use has been made of the angle φ formed by the ξ' and ξ axes, and of the angle $\psi_p = \arcsin p + \varphi$ between the propagation direction of the generic plane wave of the spectrum (32) and the ξ axis. Across the plane $\xi = 0$ we have

$$V_r(0,\zeta) = \frac{V_0}{2\pi}\int_{-\infty}^{+\infty} \tilde{U}_0(p)\Gamma(\sin\psi_p)\exp\{\mathrm{i}[-\xi_0\cos\psi_p + (\zeta-\zeta_0)\sin\psi_p]\}\mathrm{d}p. \quad (33)$$

Letting the plane-wave components of $V_r(0,\zeta)$ propagate in the half-plane $\xi < 0$, we obtain the field $V_r(\xi,\zeta)$ as follows:

$$V_r(\xi,\zeta) = \frac{V_0}{2\pi}\int_{-\infty}^{+\infty} \tilde{U}_0(p)\Gamma(\sin\psi_p) \\ \times \exp\{\mathrm{i}[-(\xi+\xi_0)\cos\psi_p + (\zeta-\zeta_0)\sin\psi_p]\}\mathrm{d}p. \tag{34}$$

By performing the same procedures as in the plane-wave case, we obtain a linear system of the same form as in (11) (or (18)), with the same coefficient matrix, but whose matrix B is modified as follows:

$$B_m^t = -i^m \frac{T_m\,(ka_t)}{2\pi}\int_{-\infty}^{+\infty} \tilde{V}_{0,i}(p)\exp[\mathrm{i}(\zeta_t^0-\zeta_0)\sin\psi_p] \\ \times\left\{\exp[\mathrm{i}(\xi_t^0-\xi_0)\cos\psi_p]\exp(-\mathrm{i}m\psi_p)\right. \\ \left.+\Gamma(\sin\psi_p)\exp[-\mathrm{i}(\xi_t^0+\xi_0)\cos\psi_p]\exp(-\mathrm{i}m\bar{\psi}_p)\mathrm{d}p\right\}, \tag{35}$$

with $\bar{\psi}_p = \pi - \psi_p$.

6 Applications

Several practical problems can be studied through the above formalism, and some of them have already been considered. The interaction between two perfectly conducting cylinders placed on a perfect mirror was analyzed in [38], where the effect of the polarization of the incident field on the far-field features was highlighted, together with the influence of the cylinder-cylinder distance. Furthermore, the case of a finite diffraction grating placed near a vacuum-dielectric interface was also treated. The grating consists of N equidistant identical cylinders, whose axes lie on a plane, parallel to the reflecting surface. This structure is of considerable importance in optics and microwaves, as well as in the study of some aspects of the light-matter interaction like, for instance, enhanced backscattering [42] and surface-polaritons interaction [43]. Another important application of the present formalism is relevant to the additional microwave heating of toroidal plasmas [44]. In particular, the study of a finite diffraction grating turns out to be useful in the analysis of the so-called quasi-optical grills. In this case, it is of great importance to launch a quite narrow spectrum of evanescent waves inside the magnetized plasma column. Only these waves, indeed, propagate inside the plasma in the slow-wave lower-hybrid regime and can contribute to the heating of the internal plasma regions. In [15,18] several configurations of quasi-optical launchers have been analyzed, and the corresponding performances in terms of coupling parameters were given, together with the electric field map inside the structure. It is worth stressing that this requires a complete and precise knowledge of the scattered field in the near-zone. As far as the case of dielectric cylinders is concerned, the plane-wave scattering from a single SiO_2 cylinder placed on a flat silicon substrate was reported in [41], where the intensity inside the cylinder was also presented. Researches are currently in progress on the study of the interaction of surface waves with cylindrical objects.

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