Solutions Manual to Accompany

SEMICONDUCTOR DEVICES
Physics and Technology
2nd Edition

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Contents

CHAPTER 2

1. (a) From Fig. 11a, the atom at the center of the cube is surround by four equidistant nearest neighbors that lie at the corners of a tetrahedron. Therefore the distance between nearest neighbors in silicon ($a = 5.43$ Å) is

$$1/2\ [(a/2)^2 + (\sqrt{2a}/2)^2]^{1/2} = \sqrt{3a}/4 = 2.35\ \text{Å}.$$

(b) For the (100) plane, there are two atoms (one central atom and 4 corner atoms each contributing 1/4 of an atom for a total of two atoms as shown in Fig. 4a) for an area of a^2, therefore we have

$2/\ a^2 = 2/\ (5.43 \times 10^{-8})^2 = 6.78 \times 10^{14}$ atoms / cm^2

Similarly we have for (110) plane (Fig. 4a and Fig. 6)

$(2 + 2 \times 1/2 + 4 \times 1/4)\ /\ \sqrt{2a}^{\ 2} = 9.6 \times 10^{15}$ atoms / cm^2,

and for (111) plane (Fig. 4a and Fig. 6)

$$(3 \times 1/2 + 3 \times 1/6)\ /\ 1/2(\sqrt{2a})(\sqrt{\left(\dfrac{3}{2}\right)}a\ ) = \dfrac{2}{\left(\dfrac{\sqrt{3}}{2}\right)a^2} = 7.83 \times 10^{14}\ \text{atoms / cm}^2.$$

2. The heights at X, Y, and Z point are $\frac{3}{4},$ $\frac{1}{4},$ and $\frac{3}{4}$.

3. (a) For the simple cubic, a unit cell contains 1/8 of a sphere at each of the eight corners for a total of one sphere.

   — Maximum fraction of cell filled

   = no. of sphere $\times$ volume of each sphere / unit cell volume

   = $1 \times 4\eth(a/2)^3\ /\ a^3 = 52\ \%$

(b) For a face-centered cubic, a unit cell contains 1/8 of a sphere at each of the eight corners for a total of one sphere. The fcc also contains half a sphere at each of the six faces for a total of three spheres. The nearest neighbor distance is $1/2(a\ \sqrt{2}\ )$. Therefore the radius of each sphere is $1/4\ (a\ \sqrt{2}\ )$.

   — Maximum fraction of cell filled

   = $(1 + 3)\ \{4\eth[(a/2)\ /\ 4\ ]^3\ /\ 3\}\ /\ a^3 = 74\ \%.$

(c) For a diamond lattice, a unit cell contains 1/8 of a sphere at each of the eight corners for a total of one sphere, 1/2 of a sphere at each of the six faces for a total of three spheres, and 4 spheres inside the cell. The diagonal distance

between (1/2, 0, 0) and (1/4, 1/4, 1/4) shown in Fig. 9a is

$$D = \frac{1}{2}\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \frac{a}{4}\sqrt{3}$$

The radius of the sphere is $D/2 = \frac{a}{8}\sqrt{3}$

__ Maximum fraction of cell filled

$$= (1 + 3 + 4)\left[\frac{4p}{3}\left(\frac{a}{8}\sqrt{3}\right)\right]^3 / a^3 = \eth\sqrt{3}/16 = 34\%.$$

This is a relatively low percentage compared to other lattice structures.

4. $\left|d_1\right| = \left|d_2\right| = \left|d_3\right| = \left|d_4\right| = d$

$\overline{d_1} + \overline{d_2} + \overline{d_3} + \overline{d_4} = 0$

$\overline{d_1} \cdot (\overline{d_1} + \overline{d_2} + \overline{d_3} + \overline{d_4}) = \overline{d_1} \cdot 0 = 0$

$\left|d_1\right|^2 + \overline{d_1} \cdot \overline{d_2} + \overline{d_1} \cdot \overline{d_3} + \overline{d_1} \cdot \overline{d_4} = 0$

$\_\_d^2 + d^2\cos\ddot{E}_{12} + d^2\cos\ddot{E}_{13} + d^2\cos\ddot{E}_{14} = d^2 + 3\,d^2\cos\ddot{E} = 0$

$$\_\_ \cos\ddot{E} = \frac{-1}{3}$$

$$\ddot{E} = \cos^{-1} \frac{-1}{3} = 109.47^0 .$$

5. Taking the reciprocals of these intercepts we get 1/2, 1/3 and 1/4. The smallest three integers having the same ratio are 6, 4, and 3. The plane is referred to as (643) plane.

6. (a) The lattice constant for GaAs is 5.65 Å, and the atomic weights of Ga and As are 69.72 and 74.92 g/mole, respectively. There are four gallium atoms and four arsenic atoms per unit cell, therefore

$$4/a^3 = 4/(5.65 \times 10^{-8})^3 = 2.22 \times 10^{22} \text{ Ga or As atoms/cm}^2,$$
$$\text{Density} = (\text{no. of atoms/cm}^3 \times \text{atomic weight}) / \text{Avogadro constant}$$
$$= 2.22 \times 10^{22}(69.72 + 74.92) / 6.02 \times 10^{23} = 5.33 \text{ g / cm}^3.$$

   (b) If GaAs is doped with Sn and Sn atoms displace Ga atoms, donors are formed, because Sn has four valence electrons while Ga has only three. The resulting semiconductor is *n*-type.

7. (a) The melting temperature for Si is 1412 °C, and for SiO_2 is 1600 °C. Therefore, SiO_2 has higher melting temperature. It is more difficult to break the Si-O bond than the Si-Si bond.

   (b) The seed crystal is used to initiated the growth of the ingot with the correct crystal orientation.

   (c) The crystal orientation determines the semiconductor's chemical and electrical

properties, such as the etch rate, trap density, breakage plane etc.

(d) The temperating of the crusible and the pull rate.

8. $E_g(T) = 1.17 - \dfrac{4.73 \text{x} 10^{-4} T^2}{(T + 636)}$ for Si

∴ E_g (100 K) = 1.163 eV , and E_g (600 K) = 1.032 eV

$E_g(T) = 1.519 - \dfrac{5.405 \text{x} 10^{-4} T^2}{(T + 204)}$ for GaAs

∴ E_g(100 K) = 1.501 eV, and E_g (600 K) = 1.277 eV .

9. The density of holes in the valence band is given by integrating the product $N(E)[1-F(E)]dE$ from top of the valence band (E_V taken to be $E = 0$) to the bottom of the valence band E_{bottom}:

$$p = \int_0^{E_{bottom}} N(E)[1 - F(E)]dE \qquad (1)$$

where $1 - F(E) = 1 - \left\{ 1 / \left[1 + e^{(E - E_F)/kT} \right] \right\} = \left[1 + e^{(E - E_F)/kT} \right]^{-1}$

If $E_F - E \gg kT$ then

$$1 - F(E) \sim \exp\left[-(E_F - E)/kT \right] \qquad (2)$$

Then from Appendix H and , Eqs. 1 and 2 we obtain

$$p = 4\eth[2m_p / h^2]^{3/2} \int_0^{E_{bottom}} E^{1/2} \exp[-(E_F - E) / kT]dE \qquad (3)$$

Let $x \mapsto E / kT$, and let $E_{bottom} = -\infty$, Eq. 3 becomes

$$p = 4\eth(2m_p / h^2)^{3/2} (kT)^{3/2} \exp[-(E_F / kT)] \int_0^{-\infty} x^{1/2} e^x dx$$

where the integral on the right is of the standard form and equals $\sqrt{p}$ / 2.

$$— \quad p = 2[2\eth m_p kT / h^2]^{3/2} \exp[-(E_F / kT)]$$

By referring to the top of the valence band as E_V instead of $E = 0$ we have,

$$p = 2(2\eth m_p kT / h^2)^{3/2} \exp[-(E_F - E_V) / kT]$$

or $\qquad p = N_V \exp[-(E_F - E_V) / kT]$

where $\quad N_V = 2(2\eth m_p kT / h^2)^3$.

10. From Eq. 18

$$N_V = 2(2\eth m_p kT / h^2)^{3/2}$$

The effective mass of holes in Si is

$m_p = (N_V / 2)^{2/3} (h^2 / 2\eth kT)$

$= \left(\dfrac{2.66 \times 10^{19} \times 10^6 \, \text{m}^{-3}}{2} \right)^{2/3} \dfrac{(6.625 \times 10^{-34})^2}{2p(1.38 \times 10^{-23})(300)}$

$= 9.4 \times 10^{-31}$ kg $= 1.03 \, m_0$.

Similarly, we have for GaAs

$m_p = 3.9 \times 10^{-31}$ kg $= 0.43 \, m_0$.

11. Using Eq. 19

$$E_i = (E_c + E_v)/2 + \left(\frac{kT}{2}\right)\ln\left(N_v/N_c\right)$$

$$= (E_C + E_V)/2 + (3kT/4)\ln\left[(m_p/m_n)(6)^{\frac{2}{3}}\right] \tag{1}$$

At 77 K

$E_i = (1.16/2) + (3 \times 1.38 \times 10^{-23}T) / (4 \times 1.6 \times 10^{-19}) \ln(1.0/0.62)$
$\quad = 0.58 + 3.29 \times 10^{-5}\,T = 0.58 + 2.54 \times 10^{-3} = 0.583$ eV.

At 300 K

$E_i = (1.12/2) + (3.29 \times 10^{-5})(300) = 0.56 + 0.009 = 0.569$ eV.

At 373 K

$E_i = (1.09/2) + (3.29 \times 10^{-5})(373) = 0.545 + 0.012 = 0.557$ eV.

Because the second term on the right-hand side of the Eq.1 is much smaller compared to the first term, over the above temperature range, it is reasonable to assume that E_i is in the center of the forbidden gap.

12. $KE = \dfrac{\displaystyle\int_{E_C}^{E_{top}} (E - E_C)\sqrt{E - E_C}\; e^{-(E - E_F)/kT}\, dE}{\displaystyle\int_{E_C}^{E_{top}} \sqrt{E - E_C}\; e^{-(E - E_F)/kT}\, dE}\; \Bigg|_{x \equiv (E - E_C)}$

$= kT\dfrac{\displaystyle\int_{0}^{\infty} x^{\frac{3}{2}}e^{-x}\,dx}{\displaystyle\int_{0}^{\infty} x^{\frac{1}{2}}e^{-x}\,dx} = kT\,\dfrac{\Gamma\left(\dfrac{5}{2}\right)}{\Gamma\left(\dfrac{3}{2}\right)} = kT\dfrac{1.5 \times 0.5 \times \sqrt{p}}{0.5\sqrt{p}}$

$= \dfrac{3}{2}kT.$

13. (a) $p = mv = 9.109 \times 10^{-31} \times 10^5 = 9.109 \times 10^{-26}$ kg–m/s

$l = \dfrac{h}{p} = \dfrac{6.626 \times 10^{-34}}{9.109 \times 10^{-26}} = 7.27 \times 10^{-9}$ m $= 72.7$ Å

(b) $l_n = \dfrac{m_0}{m_p}\, l = \dfrac{1}{0.063} \times 72.7 = 1154$ Å.

14. From Fig. 22 when $n_i = 10^{15}$ cm^{-3}, the corresponding temperature is $1000 / T = 1.8$. So that $T = 1000/1.8 = 555$ K or 282 .
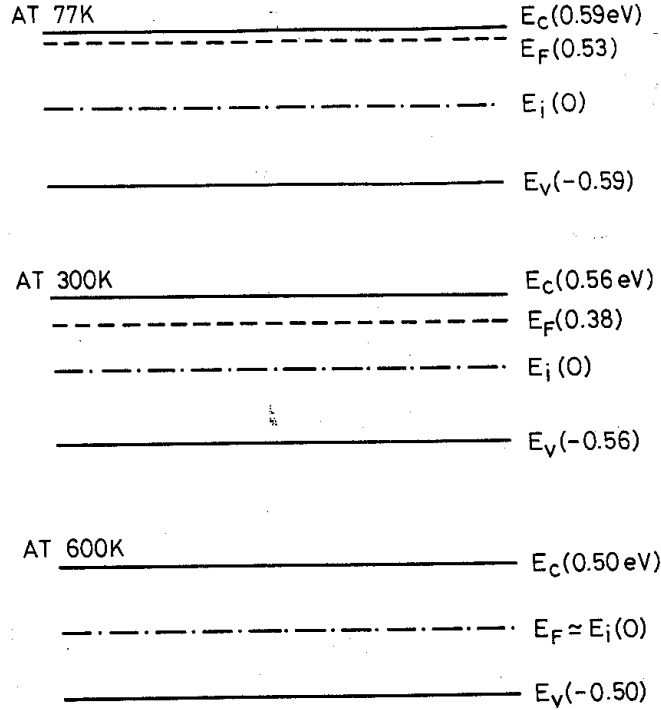
15. From $\qquad E_c - E_F = kT \ln [N_C / (N_D - N_A)]$

which can be rewritten as $\qquad N_D - N_A = N_C \exp [-(E_C - E_F) / kT]$

Then $\qquad N_D - N_A = 2.86 \times 10^{19} \exp(-0.20 / 0.0259) = 1.26 \times 10^{16}$ cm^{-3}

or $\qquad N_D = 1.26 \times 10^{16} + N_A = 2.26 \times 10^{16}$ cm^{-3}

A compensated semiconductor can be fabricated to provide a specific Fermi energy level.

16. From Fig. 28a we can draw the following energy-band diagrams:

AT 77K _____ $E_C(0.59\,eV)$
- $E_F(0.53)$

— · — · — · — · — · — · — · — · — · $E_i(0)$

_____ $E_V(-0.59)$

AT 300K _____ $E_C(0.56\,eV)$

- - - - - - - - - - - - - - - - - - - $E_F(0.38)$

— · — · — · — · — · — · — · — · $E_i(0)$

_____ $E_V(-0.56)$

AT 600K _____ $E_C(0.50\,eV)$

— · — · — · — · — · — · — · — · $E_F \simeq E_i(0)$

_____ $E_V(-0.50)$

17. (a) The ionization energy for boron in Si is 0.045 eV. At 300 K, all boron impurities are ionized. Thus $p_p = N_A = 10^{15}\,cm^{-3}$
$$n_p = n_i^2 / n_A = (9.65 \times 10^9)^2 / 10^{15} = 9.3 \times 10^4 \,cm^{-3}.$$

The Fermi level measured from the top of the valence band is given by:

$$E_F - E_V = kT \ln(N_V/N_D) = 0.0259 \ln (2.66 \times 10^{19} / 10^{15}) = 0.26 \text{ eV}$$

(b) The boron atoms compensate the arsenic atoms; we have

$$p_p = N_A - N_D = 3 \times 10^{16} - 2.9 \times 10^{16} = 10^{15} \,cm^{-3}$$
Since p_p is the same as given in (a), the values for n_p and E_F are the same as in (a). However, the mobilities and resistivities for these two samples are

different.

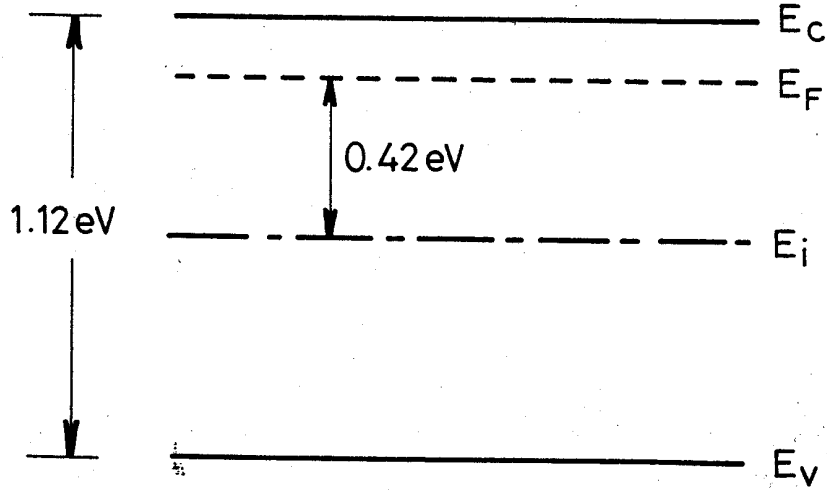18. Since $N_D \gg n_i$, we can approximate $n_0 = N_D$ and
$$p_0 = n_i^2 / n_0 = 9.3 \times 10^{19} / 10^{17} = 9.3 \times 10^2 \,cm^{-3}$$

From $n_0 = n_i \exp\left(\dfrac{E_F - E_i}{kT}\right),$

we have
$$E_F - E_i = kT \ln (n_0 / n_i) = 0.0259 \ln (10^{17} / 9.65 \times 10^9) = 0.42 \text{ eV}$$
The resulting flat band diagram is :

1.12 eV — 0.42 eV — E_C, E_F, E_i, E_V

19. Assuming complete ionization, the Fermi level measured from the intrinsic Fermi level is 0.35 eV for 10^{15} cm^{-3}, 0.45 eV for 10^{17} cm^{-3}, and 0.54 eV for 10^{19} cm^{-3}.

The number of electrons that are ionized is given by

$$n \cong N_D[1 - F(E_D)] = N_D / [1 + \mathrm{e}^{-(E_D - E_F)/kT}]$$

Using the Fermi levels given above, we obtain the number of ionized donors as

$$
\begin{aligned}
n &= 10^{15} \text{ cm}^{-3} &\quad \text{for } N_D = 10^{15} \text{ cm}^{-3}\\
n &= 0.93 \times 10^{17} \text{ cm}^{-3} &\quad \text{for } N_D = 10^{17} \text{ cm}^{-3}\\
n &= 0.27 \times 10^{19} \text{ cm}^{-3} &\quad \text{for } N_D = 10^{19} \text{ cm}^{-3}
\end{aligned}
$$

Therefore, the assumption of complete ionization is valid only for the case of 10^{15} cm^{-3}.

20. $N_D^+ = \dfrac{10^{16}}{1 + e^{-(E_D - E_F)/kT}} = \dfrac{10^{16}}{1 + e^{-0.135}}$

$= \dfrac{10^{16}}{1 + \dfrac{1}{1.145}} = 5.33 \times 10^{15}$ cm^{-3}

The neutral donor $= 10^{16} - 5.33 \times 10^{15}$ cm^{-3} $= 4.67 \times 10^{15}$ cm^{-3}

— The ratio of $\dfrac{N_D^O}{N_D^+} = \dfrac{4.76}{5.33} = 0.876$.

1. (a) For intrinsic Si, $\mu_n = 1450$, $\mu_p = 505$, and $n = p = n_i = 9.65 \times 10^9$

    We have $r = \dfrac{1}{qn\mu_n + qp\mu_p} = \dfrac{1}{qn_i(\mu_n + \mu_p)} = 3.31 \times 10^5$ Ω-cm

   (b) Similarly for GaAs, $\mu_n = 9200$, $\mu_p = 320$, and $n = p = n_i = 2.25 \times 10^6$

    We have $r = \dfrac{1}{qn\mu_n + qp\mu_p} = \dfrac{1}{qn_i(\mu_n + \mu_p)} = 2.92 \times 10^8$ Ω-cm.

2. For lattice scattering, $\mu_n \propto T^{-3/2}$

$$T = 200 \text{ K}, \quad \mu_n = 1300 \times \frac{200^{-3/2}}{300^{-3/2}} = 2388 \ \text{cm}^2/\text{V-s}$$

$$T = 400 \text{ K}, \quad \mu_n = 1300 \times \frac{400^{-3/2}}{300^{-3/2}} = 844 \ \text{cm}^2/\text{V-s}.$$

3. Since $\dfrac{1}{\mu} = \dfrac{1}{\mu_1} + \dfrac{1}{\mu_2}$

    $\therefore \ \dfrac{1}{\mu} = \dfrac{1}{250} + \dfrac{1}{500} \qquad \mu = 167 \ \text{cm}^2/\text{V-s}.$

4. (a) $p = 5 \times 10^{15} \ \text{cm}^{-3}$, $n = n_i^2/p = (9.65 \times 10^9)^2 / 5 \times 10^{15} = 1.86 \times 10^4 \ \text{cm}^{-3}$

    $\mu_p = 410 \ \text{cm}^2/\text{V-s}$, $\mu_n = 1300 \ \text{cm}^2/\text{V-s}$

    $r = \dfrac{1}{q\mu_n n + q\mu_p p} \approx \dfrac{1}{q\mu_p p} = 3 \ \text{Ω-cm}$

   (b) $p = N_A - N_D = 2 \times 10^{16} - 1.5 \times 10^{16} = 5 \times 10^{15} \ \text{cm}^{-3}$, $n = 1.86 \times 10^4 \ \text{cm}^{-3}$

    $\mu_p = \mu_p (N_A + N_D) = \mu_p (3.5 \times 10^{16}) = 290 \ \text{cm}^2/\text{V-s}$,

    $\mu_n = \mu_n (N_A + N_D) = 1000 \ \text{cm}^2/\text{V-s}$

    $r = \dfrac{1}{q\mu_n n + q\mu_p p} \approx \dfrac{1}{q\mu_p p} = 4.3 \ \text{Ω-cm}$

(c) $p = N_A$ (Boron) $- N_D + N_A$ (Gallium) $= 5 \times 10^{15}$ cm^{-3}, $n = 1.86 \times 10^4$ cm^{-3}

$\mu_p = \mu_p (N_A + N_D + N_A) = \mu_p (2.05 \times 10^{17}) = 150$ cm^2/V-s,

$\mu_n = \mu_n (N_A + N_D + N_A) = 520$ cm^2/V-s

$\rho = 8.3$ Ω-cm.

5. Assume $N_D - N_A \gg n_i$, the conductivity is given by

$$\sigma \approx q n \mu_n = q \mu_n (N_D - N_A)$$

We have that

$$16 = (1.6 \times 10^{-19}) \mu_n (N_D - 10^{17})$$

Since mobility is a function of the ionized impurity concentration, we can use

Fig. 3 along with trial and error to determine μ_n and N_D. For example, if we

choose $N_D = 2 \times 10^{17}$, then $N_I = N_D^+ + N_A^- = 3 \times 10^{17}$, so that $\mu_n \approx 510$ cm^2/V-s

which gives $\sigma = 8.16$.

Further trial and error yields

$$N_D \approx 3.5 \times 10^{17} \text{ cm}^{-3}$$

and

$$\mu_n \approx 400 \text{ cm}^2/\text{V-s}$$

which gives

$$\sigma \approx 16 \ (\Omega\text{-cm})^{-1}.$$

6. $\sigma = q(\mu_n n + \mu_p p) = q\mu_p (bn + n_i^2 / n)$

From the condition $d\sigma/dn = 0$, we obtain

$$n = n_i / \sqrt{b}$$

Therefore

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$$\frac{r_m}{r_i} = \frac{\dfrac{1}{q\mu_p(bn_i/\sqrt{b} + \sqrt{b}n_i)}}{\dfrac{1}{q\mu_p n_i(b+1)}} = \frac{b+1}{2\sqrt{b}}.$$

7. At the limit when d $\gg$ s, $CF = \dfrac{p}{\ln 2} = 4.53$. Then from Eq. 16

$$r = \frac{V}{I} \times W \times CF = \frac{10 \times 10^{-3}}{1 \times 10^{-3}} \times 50 \times 10^{-4} \times 4.53 = 0.226 \ \ \Omega\text{-cm}$$

From Fig. 6, $CF = 4.2$ ($d/s = 10$); using the $a/d = 1$ curve we obtain

$$V = r \cdot I /(W \cdot CF) = \frac{0.226 \times 10^{-3}}{50 \times 10^{-4} \times 4.2} = 10.78 \ \ \text{mV}.$$

8. Hall coefficient,

$$R_H = \frac{V_H A}{IB_z W} = \frac{10 \times 10^{-3} \times 1.6 \times 10^{-3}}{2.5 \times 10^{-3} \times (30 \times 10^{-9} \times 10^4) \times 0.05} = 426.7 \ \ \text{cm}^3/\text{C}$$

Since the sign of R_H is positive, the carriers are holes. From Eq. 22

$$p = \frac{1}{qR_H} = \frac{1}{1.6 \times 10^{-19} \times 426.7} = 1.46 \times 10^{16} \ \ \text{cm}^{-3}$$

Assuming $N_A \approx p$, from Fig. 7 we obtain $r = 1.1 \ \Omega$-cm

The mobility μ_p is given by Eq. 15b

$$\mu_p = \frac{1}{qpr} = \frac{1}{1.6 \times 10^{-19} \times 1.46 \times 10^{16} \times 1.1} = 380 \ \ \text{cm}^2/\text{V-s}.$$

9. Since $R \propto r$ and $r = \dfrac{1}{qn\mu_n + qp\mu_p}$, hence $R \propto \dfrac{1}{n\mu_n + p\mu_p}$

From Einstein relation $D \propto \mu$

$$\mu_n / \mu_p = D_n / D_p = 50$$

$$\frac{R_1}{0.5R_1} = \frac{\dfrac{1}{N_D \boldsymbol{m}_n}}{\dfrac{1}{N_D \boldsymbol{m}_n + N_A \boldsymbol{m}_p}}$$

We have $N_A = 50\, N_D$.

10. The electric potential $\boldsymbol{f}$ is related to electron potential energy by the charge $(-q)$

$$\boldsymbol{f} = + \frac{1}{q}(E_F - E_i)$$

The electric field for the one-dimensional situation is defined as

$$\mathrm{E}(x) = -\frac{d\boldsymbol{f}}{dx} = \frac{1}{q}\frac{dE_i}{dx}$$

$$n = n_i \exp\left(\frac{E_F - E_i}{kT}\right) = N_D(x)$$

Hence

$$E_F - E_i = kT \ln\left(\frac{N_D(x)}{n_i}\right)$$

$$\mathrm{E}\ (x) = -\left(\frac{kT}{q}\right)\frac{1}{N_D(x)}\frac{dN_D(x)}{dx}.$$

11. (a) From Eq. 31, $J_n = 0$ and

$$\mathrm{E}\ (x) = -\frac{D_n}{\boldsymbol{m}_n}\frac{dn/dx}{n} = -\frac{kT}{q}\frac{N_0(-a)e^{-ax}}{N_0 e^{-ax}} = +\frac{kT}{q}a$$

  (b) $\mathrm{E}\ (x) = 0.0259\,(10^4) = 259$ V/cm.

12. At thermal and electric equilibria,

$$J_n = q\boldsymbol{m}_n n(x)\mathrm{E} + qD_n\frac{dn(x)}{dx} = 0$$

$$\mathrm{E}\ (x) = -\frac{D_n}{\boldsymbol{m}_n}\frac{1}{n(x)}\frac{dn(x)}{dx} = -\frac{D_n}{\boldsymbol{m}_n}\frac{1}{N_0 + (N_L - N_0)(x/\,L)}\frac{N_L - N_0}{L}$$

$$= -\frac{D_n}{\boldsymbol{m}_n}\frac{N_L - N_0}{LN_0 + (N_L - N_0)x}$$

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$$V = \int_0^L -\frac{D_n}{\mu_n}\frac{N_L - N_0}{LN_0 + (N_L - N_0)x} = -\frac{D_n}{\mu_n}\ln\frac{N_L}{N_0}.$$

13. $\Delta n = \Delta p = \tau_p G_L = 10\times10^{-6}\times10^{16} = 10^{11}$ cm^{-3}

$n = n_{no} + \Delta n = N_D + \Delta n = 10^{15} + 10^{11} \approx 10^{15}$ cm^{-3}

$p = \dfrac{n_i^2}{N_D} + \Delta p = \dfrac{(9.65\times10^9)^2}{10^{15}} + 10^{11} \approx 10^{11}$ cm^{-3}.

14. (a) $\tau_p \approx \dfrac{1}{\sigma_p v_{th} N_t} = \dfrac{1}{5\times10^{-15}\times10^7\times2\times10^{15}} = 10^{-8}$ s

$L_p = \sqrt{D_p \tau_p} = \sqrt{9\times10^{-8}} = 3\times10^{-4}$ cm

$S_{lr} = v_{th}\sigma_s N_{sts} = 10^7\times2\times10^{-16}\times10^{10} = 20$ cm/s

(b) The hole concentration at the surface is given by Eq. 67

$$p_n(0) = p_{no} + \tau_p G_L\left(1 - \frac{\tau_p S_{lr}}{L_p + \tau_p S_{lr}}\right)$$

$$= \frac{(9.65\times10^9)^2}{2\times10^{16}} + 10^{-8}\times10^{17}\left(1 - \frac{10^{-8}\times20}{3\times10^{-4} + 10^{-8}\times20}\right)$$

$$\approx 10^9 \text{ cm}^{-3}.$$

15. $\sigma = qn\mu_n + qp\mu_p$

Before illumination

$$n_n = n_{no}, \ p_n = p_{no}$$

After illumination

$$n_n = n_{no} + \Delta n = n_{no} + \tau_p G,$$

$$p_n = p_{no} + \Delta p = p_{no} + \tau_p G$$

$$\Delta\sigma = [q\mu_n(n_{no} + \Delta n) + q\mu_p(p_{no} + \Delta p)] - (q\mu_n n_{no} + q\mu_p p_{no})$$

$$= q(\mu_n + \mu_p)\tau_p G.$$

16. (a) $J_{p,\text{diff}} = -qD_p \dfrac{dp}{dx}$

$$= -1.6 \times 10^{-19} \times 12 \times \frac{1}{12 \times 10^{-4}} \times 10^{15} \exp(-x/12)$$

$$= 1.6\exp(-x/12) \quad \text{A/cm}^2$$

(b) $J_{n,\text{drift}} = J_{total} - J_{p,\text{diff}}$

$$= 4.8 - 1.6\exp(-x/12) \quad \text{A/cm}^2$$

(c) $\because J_{n,\text{drift}} = qn\mu_n E$

$\diagdown 4.8 - 1.6\exp(-x/12) = 1.6\times10^{-19}\times10^{16}\times1000\times E$

$E \quad = 3 - \exp(-x/12) \quad \text{V/cm.}$

17. For $E = 0$ we have

$$\frac{\partial p}{\partial t} = -\frac{p_n - p_{no}}{t_p} + D_p \frac{\partial^2 p_n}{\partial x^2} = 0$$

at steady state, the boundary conditions are $p_n\,(x = 0) = p_n\,(0)$ and $p_n\,(x = W) = p_{no}$.

Therefore

$$p_n(x) = p_{no} + \left[p_n(0) - p_{no}\right]\left[\frac{\sinh\left(\dfrac{W - x}{L_p}\right)}{\sinh\left(\dfrac{W}{L_p}\right)}\right]$$

$$J_p(x = 0) = -qD_p \left.\frac{\partial p_n}{\partial x}\right|_{x=0} = q\left[p_n(0) - p_{no}\right]\frac{D_p}{L_p}\coth\left(\frac{W}{L_p}\right)$$

$$J_p(x = W) = -qD_p \left.\frac{\partial p_n}{\partial x}\right|_{x=W} = q\left[p_n(0) - p_{no}\right]\frac{D_p}{L_p}\frac{1}{\sinh\left(\dfrac{W}{L_p}\right)}.$$

18. The portion of injection current that reaches the opposite surface by diffusion is

given by

$$a_0 = \frac{J_p(W)}{J_p(0)} = \frac{1}{\cosh(W/L_p)}$$

$$L_p \equiv \sqrt{D_p t_p} = \sqrt{50 \times 50 \times 10^{-6}} = 5 \times 10^{-2} \text{ cm}$$

$$\therefore a_0 = \frac{1}{\cosh(10^{-2}/5 \times 10^{-2})} = 0.98$$

Therefore, 98% of the injected current can reach the opposite surface.

19. In steady state, the recombination rate at the surface and in the bulk is equal

$$\frac{\Delta p_{n,\text{bulk}}}{t_{p,\text{bulk}}} = \frac{\Delta p_{n,\text{surface}}}{t_{p,\text{surface}}}$$

so that the excess minority carrier concentration at the surface

$$\Delta p_{n,\text{ surface}} = 10^{14} \cdot \frac{10^{-7}}{10^{-6}} = 10^{13} \text{ cm}^{-3}$$

The generation rate can be determined from the steady-state conditions in the bulk

$$G = \frac{10^{14}}{10^{-6}} = 10^{20} \text{ cm}^{-3}\text{s}^{-1}$$

From Eq. 62, we can write

$$D_p \frac{\partial^2 \Delta p}{\partial x^2} + G - \frac{\Delta p}{t_p} = 0$$

The boundary conditions are $\Delta p(x = \infty) = 10^{14} \text{ cm}^{-3}$ and $\Delta p(x = 0) = 10^{13} \text{ cm}^{-3}$

Hence $\quad \Delta p(x) = 10^{14}(1 - 0.9 e^{-x/L_p})$

where $\quad L_p = \sqrt{10 \cdot 10^{-6}} = 31.6 \text{ μm}.$

20. The potential barrier height

$$\phi_B = \phi_m - \chi = 4.2 - 4.0 = 0.2 \text{ volts.}$$

21. The number of electrons occupying the energy level between E and $E+dE$ is

$dn = N(E)F(E)dE$

where $N(E)$ is the density-of-state function, and $F(E)$ is Fermi-Dirac distribution function. Since only electrons with an energy greater than $E_F + q\phi_m$ and having a velocity component normal to the surface can escape the solid, the thermionic current density is

$$J = \int q v_x = \int_{E_F + q\phi_m}^{\infty} \frac{4\pi (2m)^{3/2}}{h^3} v_x E^{1/2} e^{-(E-E_F)/kT} dE$$

where v_x is the component of velocity normal to the surface of the metal. Since the energy-momentum relationship

$$E = \frac{P^2}{2m} = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2)$$

13

Differentiation leads to $dE = \dfrac{PdP}{m}$

By changing the momentum component to rectangular coordinates, $4\boldsymbol{p}P^2 dP = dp_x dp_y dp_z$

Hence
$$J = \frac{2q}{mh^3} \int_{P_{x0}}^{\infty} \int_{P_y=-\infty}^{\infty} \int_{P_z=-\infty}^{\infty} p_x e^{-(p_x^2+p_y^2+p_z^2-2mE_f)/2mkT} \, dp_x dp_y dp_z$$

$$= \frac{2q}{mh^3} \int_{P_{x0}}^{\infty} e^{-(p_x^2-2mE_f)/2mkT} p_x dp_x \int_{-\infty}^{\infty} e^{-p_y^2/2mkT} dp_y \int_{-\infty}^{\infty} e^{-p_z^2/2mkT} dp_z$$

where $p_{x0}^2 = 2m(E_F + q\boldsymbol{f}_m)$.

Since $\int_{-\infty}^{\infty} e^{-ax^2} dx = \left(\dfrac{\boldsymbol{p}}{a}\right)^{1/2}$, the last two integrals yield $(2\eth mkT)^{1/2}$.

The first integral is evaluated by setting $\dfrac{p_x^2 - 2mE_F}{2mkT} = u$.

Therefore we have $du = \dfrac{p_x dp_x}{mkT}$

The lower limit of the first integral can be written as
$$\frac{2m(E_F + q\boldsymbol{f}_m) - 2mE_F}{2mkT} = \frac{q\boldsymbol{f}_m}{kT}$$

so that the first integral becomes $mkT \int_{q\boldsymbol{f}_m/kt}^{\infty} e^{-u} du = mkT \, e^{-q\boldsymbol{f}_m/kT}$

Hence $J = \dfrac{4\boldsymbol{p}qmk^2}{h^3} T^2 e^{-q\boldsymbol{f}_m/kT} = A^* T^2 \exp\left(\dfrac{-q\boldsymbol{f}_m}{kT}\right)$.

22. Equation 79 is the tunneling probability
$$\boldsymbol{b} = \sqrt{\frac{2m_n(qV_0 - E)}{\hbar^2}} = \sqrt{\frac{2(9.11\times10^{-31})(20-2)(1.6\times10^{-19})}{(1.054\times10^{-34})^2}} = 2.17\times10^{10}\,\text{m}^{-1}$$

$$T = \left\{1 + \frac{[20\times\sinh(2.17\times10^0\times3\times10^{-10}]^2}{4\times2\times(20-2)}\right\}^{-1} = 3.19\times10^{-6}.$$

23. Equation 79 is the tunneling probability

$$\boldsymbol{b} = \sqrt{\frac{2m_n(qV_0 - E)}{\hbar^2}} = \sqrt{\frac{2(9.11\times10^{-31})(6-2.2)(1.6\times10^{-19})}{(1.054\times10^{-34})^2}} = 9.99\times10^9\,\text{m}^{-1}$$

$$T(10^{-10}) = \left\{1 + \frac{[6\times\sinh(9.99\times10^9\times10^{-10})]^2}{4\times2.2\times(6-2.2)}\right\}^{-1} = 0.403$$

$$T(10^{-9}) = \left\{1 + \frac{[6\times\sinh(9.99\times10^9\times10^{-9})]^2}{4\times2.2\times(6-2.2)}\right\}^{-1} = 7.8\times10^{-9}.$$

24. From Fig. 22

   As $E = 10^3$ V/s

$$v_d \approx 1.3 \times 10^6 \text{ cm/s (Si) and } v_d \approx 8.7 \times 10^6 \text{ cm/s (GaAs)}$$

$$t \approx 77 \text{ ps (Si) and } t \approx 11.5 \text{ ps (GaAs)}$$

   As $E = 5 \times 10^4$ V/s

$$v_d \approx 10^7 \text{ cm/s (Si) and } v_d \approx 8.2 \times 10^6 \text{ cm/s (GaAs)}$$

$$t \approx 10 \text{ ps (Si) and } t \approx 12.2 \text{ ps (GaAs).}$$

25. Thermal velocity $v_{th} = \sqrt{\dfrac{2E_{th}}{m_0}} = \sqrt{\dfrac{2kT}{m_0}}$

$$= \sqrt{\frac{2 \times 1.38 \times 10^{-23} \times 300}{9.1 \times 10^{-31}}}$$

$$= 9.5 \times 10^4 \text{ m/s} = 9.5 \times 10^6 \text{ cm/s}$$

   For electric field of 100 v/cm, drift velocity

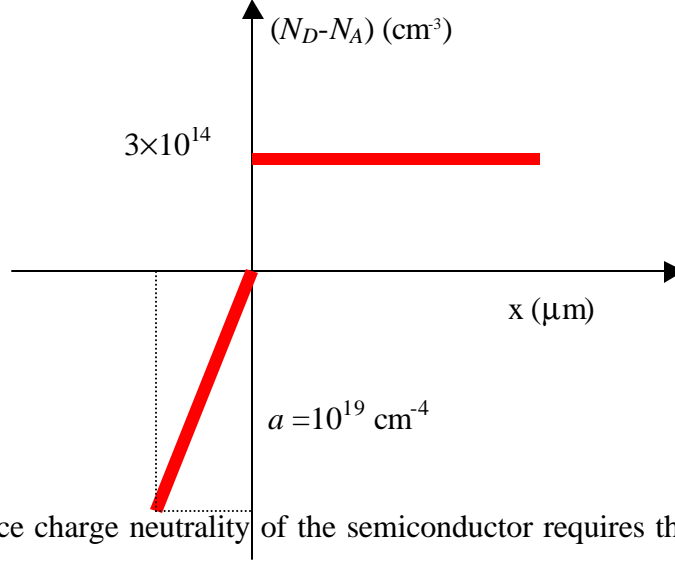$$v_d = \mu_n E = 1350 \times 100 = 1.35 \times 10^5 \text{ cm/s} \ll v_{th}$$

   For electric field of 10^4 V/cm.

$$\mu_n E = 1350 \times 10^4 = 1.35 \times 10^7 \text{ cm/s} \approx v_{th}.$$

   The value is comparable to the thermal velocity, the linear relationship between

   drift velocity and the electric field is not valid.

1. The impurity profile is,

The overall space charge neutrality of the semiconductor requires that the total negative space charge per unit area in the *p*-side must equal the total positive space charge per unit area in the *n*-side, thus we can obtain the depletion layer width in the *n*-side region:

$$\frac{0.8 \times 8 \times 10^{14}}{2} = W_n \times 3 \times 10^{14}$$

Hence, the *n*-side depletion layer width is:

$$W_n = 1.067 \, \mathbf{m}\mathrm{m}$$

The total depletion layer width is 1.867 μm.

We use the Poisson's equation for calculation of the electric field *E(x)*.

In the *n*-side region,

$$\frac{d\mathrm{E}}{dx} = \frac{q}{\boldsymbol{e}_s} N_D \Rightarrow \mathrm{E}\,(\,x_n\,) = \frac{q}{\boldsymbol{e}_s} N_D x + K$$

$$\mathrm{E}\,(\,x_n = 1.067\,\boldsymbol{m}\mathrm{m}\,) = 0 \Rightarrow K = -\frac{q}{\boldsymbol{e}_s} N_D \times 1.067 \times 10^{-4}$$

$$\therefore \mathrm{E}\,(x_n) = \frac{q}{\boldsymbol{e}_s} \times 3 \times 10^{14} (x - 1.067 \times 10^{-4})$$

$$\mathrm{E}_{max} = \mathrm{E}\,(\,x_n = 0\,) = -4.86 \times 10^3 \,\mathrm{V/cm}$$

In the *p*-side region, the electrical field is:

$$\frac{d\mathcal{E}}{dx} = \frac{q}{e_s} N_A \Rightarrow \mathcal{E}(x_p) = \frac{q}{2e_s} \times ax^2 + K'$$

$$\mathcal{E}(x_p = -0.8mm) = 0 \Rightarrow K' = -\frac{q}{2e_s} \times a \times (0.8 \times 10^{-4})^2$$

$$\therefore \mathcal{E}(x_p) = \frac{q}{2e_s} \times a \times \left[x^2 - (0.8 \times 10^{-4})^2\right]$$

$$\mathcal{E}_{max} = \mathcal{E}(x_p = 0) = -4.86 \times 10^3 \text{ V/cm}$$

The built-in potential is:

$$V_{bi} = -\int_{-x_p}^{x_n} \mathcal{E}(x)dx = -\int_{-x_p}^{0} \mathcal{E}(x)dx\Big|_{p-side} - \int_{0}^{x_n} \mathcal{E}(x)dx\Big|_{n-side} = 0.52 \text{ V}.$$

2. From $V_{bi} = -\int \mathcal{E}(x)dx$, the potential distribution can be obtained

   With zero potential in the neutral p-region as a reference, the potential in the p-side

   depletion region is

$$V_p(x) = -\int_0^x \mathcal{E}(x)dx = -\int_0^x \frac{q}{2e_s} \times a \times \left[x^2 - (0.8 \times 10^{-4})^2\right]dx = -\frac{qa}{2e_s}\left[\frac{1}{3}x^3 - (0.8 \times 10^{-4})^2 x - \frac{2}{3}(0.8 \times 10^{-4}\right.$$

$$= -7.596 \times 10^{11} \times \left[\frac{1}{3}x^3 - (0.8 \times 10^{-4})^2 x - \frac{2}{3}(0.8 \times 10^{-4})^3\right]$$
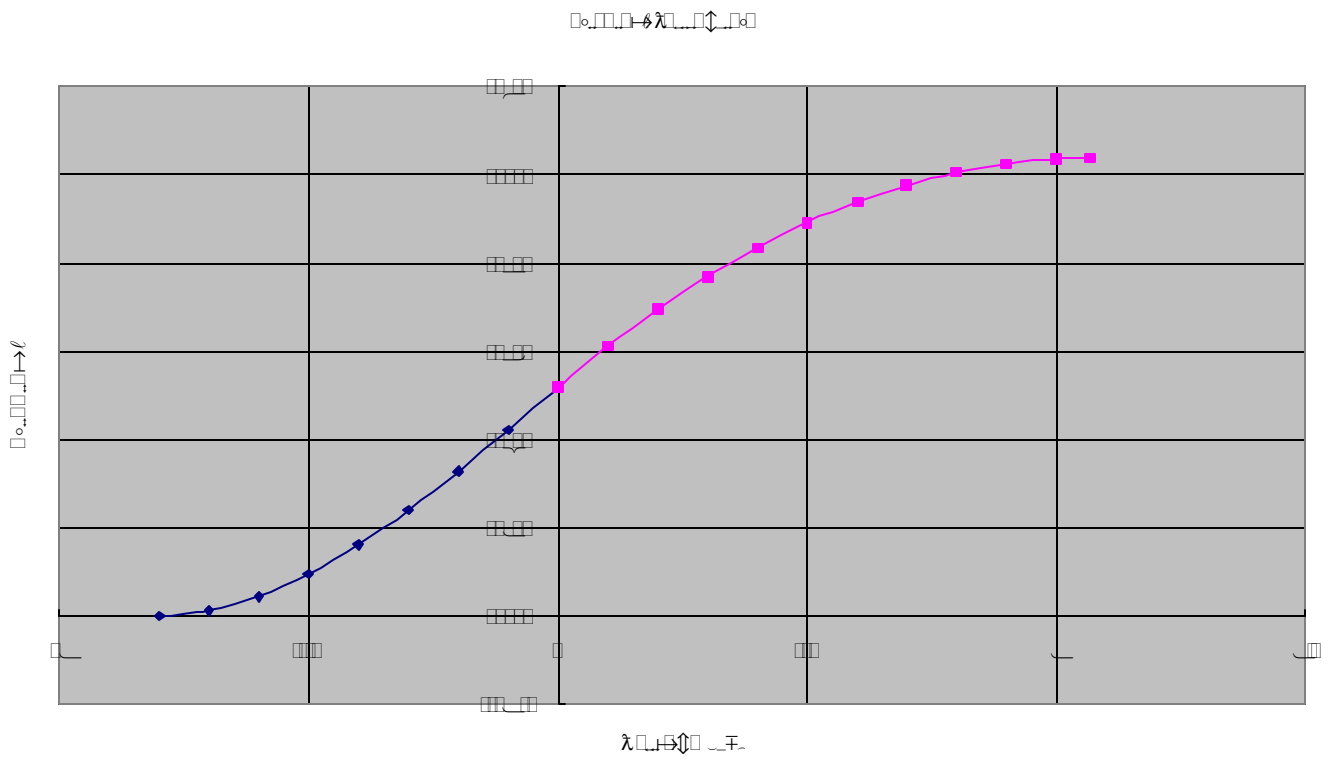
With the condition $V_p(0) = V_n(0)$, the potential in the n-region is

$$V_n(x) = -\frac{q}{e_s} \times 3 \times 10^{14}\left(\frac{1}{2}x^2 - 1.067 \times 10^{-4}x + \frac{0.8^3}{9} \times 10^{-7}\right)$$

$$= -4.56 \times 10^7 \times \left(\frac{1}{2}x^2 - 1.067 \times 10^{-4}x - \frac{0.8^3}{9} \times 10^{-7}\right)$$

The potential distribution is

Distance	p-region	n-region
- 0. __	0. 000	
- 0. __	0. 00__	
- 0. __	0. 0____	
- 0. 5	0. 0____	
- 0. __	0. 0____	
- 0. __	0. ____0	
- 0. __	0. ____	
- 0. __	0. ____	
0	0. __59	0. __59___

17

0.		0. 05 5
0.		0. 0
0.		0. 5 9
0.		0. 55
0. 5		0. 5 9
0.		0. 9 5
0.		0. 9
0.		0. 50 9
0. 9		0. 5 0
		0. 5 9 5
. 0		0. 5 9 5

Potential vs Position

Potential

Line E F

3. The intrinsic carriers density in Si at different temperatures can be obtained by using Fig.22 in

   Chapter 2 :

Temperature (K)	Intrinsic carrier density (n_i)
250	1.50×10^8
300	9.65×10^9
350	2.00×10^{11}
400	8.50×10^{12}
450	9.00×10^{13}
500	2.20×10^{14}

The V_{bi} can be obtained by using Eq. 12, and the results are listed in the following table.

T	ni	Vbi (V)
50	. 500E+0	0.
00	9. 5E+9	0.
50	. 00E+	0. 5
00	. 50E+	0.
50	9. 00E+	0.
500	. 0E+	0. 9

Thus, the built-in potential is decreased as the temperature is increased.

The depletion layer width and the maximum field at 300 K are

$$W = \sqrt{\frac{2e_s V_{bi}}{qN_D}} = \sqrt{\frac{2 \times 11.9 \times 8.85 \times 10^{-14} \times 0.717}{1.6 \times 10^{-19} \times 10^{15}}} = 0.9715 \ \textit{mm}$$

$$E_{max} = \frac{qN_D W}{e_s} = \frac{1.6 \times 10^{-19} \times 10^{15} \times 9.715 \times 10^{-5}}{11.9 \times 8.85 \times 10^{-14}} = 1.476 \times 10^4 \ \text{V/cm.}$$

4. $E_{max} \approx \left[\dfrac{2qV_R}{\varepsilon_s} \left(\dfrac{N_A N_D}{N_A + N_D} \right) \right]^{1/2} \Rightarrow 4 \times 10^5 = \left[\dfrac{2 \times 1.6 \times 10^{-19} \times 30}{11.9 \times 8.85 \times 10^{-14}} \left(\dfrac{10^{18} N_D}{10^{18} + N_D} \right) \right]^{1/2}$

$\Rightarrow 1.755 \times 10^{16} = \dfrac{N_D}{1 + \dfrac{N_D}{10^{18}}}$

We can select n-type doping concentration of $N_D = 1.755 \times 10^{16}$ cm^{-3} for the junction.

5. From Eq. 12 and Eq. 35, we can obtain the $1/C^2$ versus V relationship for doping concentration of 10^{15}, 10^{16}, or 10^{17} cm^{-3}, respectively.

For $N_D = 10^{15}$ cm^{-3},

$$\dfrac{1}{C_j^2} = \dfrac{2(V_{bi} - V)}{q\varepsilon_s N_B} = \dfrac{2 \times (0.837 - V)}{1.6 \times 10^{-19} \times 11.9 \times 8.85 \times 10^{-14} \times 10^{15}} = 1.187 \times 10^{16} (0.837 - V)$$

For $N_D = 10^{16}$ cm^{-3},

$$\dfrac{1}{C_j^2} = \dfrac{2(V_{bi} - V)}{q\varepsilon_s N_B} = \dfrac{2 \times (0.896 - V)}{1.6 \times 10^{-19} \times 11.9 \times 8.85 \times 10^{-14} \times 10^{16}} = 1.187 \times 10^{15} (0.896 - V)$$

For $N_D = 10^{17}$ cm^{-3},

$$\dfrac{1}{C_j^2} = \dfrac{2(V_{bi} - V)}{q\varepsilon_s N_B} = \dfrac{2 \times (0.956 - V)}{1.6 \times 10^{-19} \times 11.9 \times 8.85 \times 10^{-14} \times 10^{17}} = 1.187 \times 10^{14} (0.956 - V)$$

When the reversed bias is applied, we summarize a table of $1/C_j^2$ vs V for various N_D values as following,

V	$N_D=1E15$	$N_D=1E16$	$N_D=1E17$
- __	5. _____E+	5. _____E+ 5	5. _____E+
- __. 5	5. _____E+	5. _____E+ 5	5. ___9E+
- __	__. 555E+	__. ____5E+ 5	__. __9__E+
- __. 5	__. 9___E+	__. 0___E+ 5	__. __0__E+
- __	__. ____E+	__. ____E+ 5	__. 509E+
- __. 5	__. ____E+	__. ____E+ 5	__. 9__5E+
- __	__. ____E+	__. __5__E+ 5	__. ____E+
- 0. 5	__. 5___E+	__. __5__E+ 5	__. ____E+
0	9. 9__5E+ 5	__. 0____E+ 5	__. ____E+

Hence, we obtain a series of curves of $1/C^2$ versus V as following,
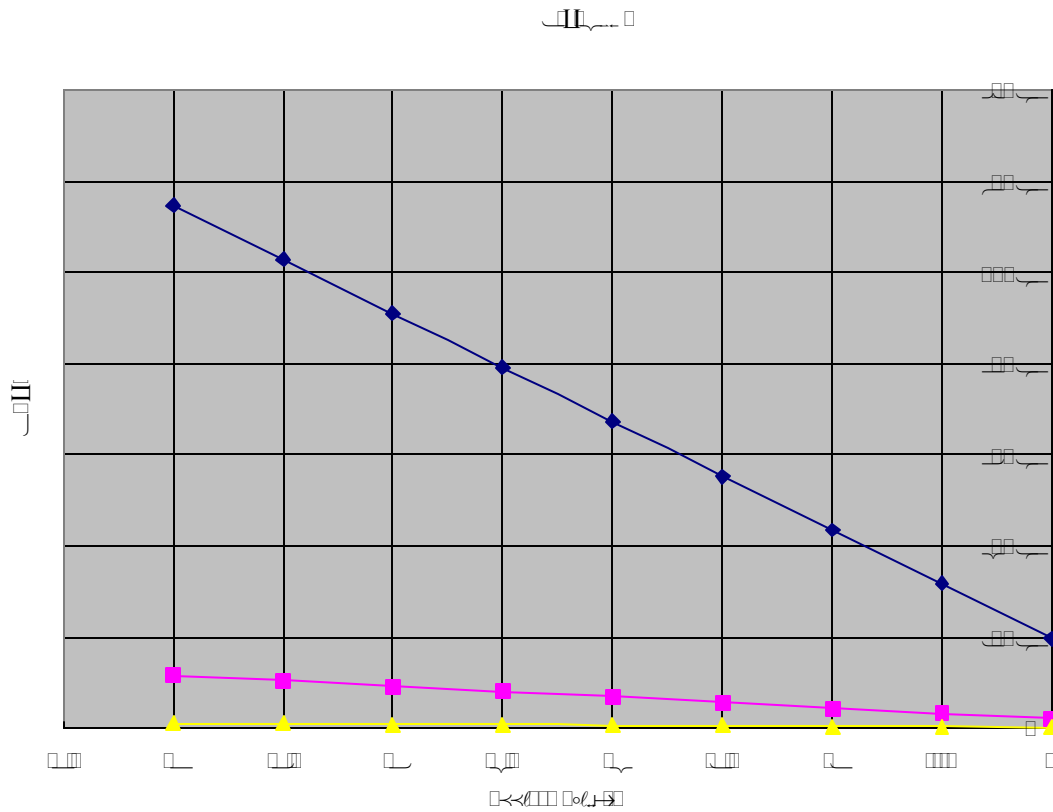
The slopes of the curves is positive proportional to the values of the doping concentration.

The interceptions give the built-in potential of the *p-n* junctions.

6. The built-in potential is

$$V_{bi} = \frac{2}{3}\frac{kT}{q}\ln\left(\frac{a^2 \varepsilon_s kT}{8q^2 n_i^3}\right) = \frac{2}{3}\times 0.0259\times\ln\left(\frac{10^{20}\times 10^{20}\times 11.9\times 8.85\times 10^{-14}\times 0.0259}{8\times 1.6\times 10^{-19}\times\left(9.65\times 10^9\right)^3}\right)$$

$$= 0.5686\ \text{V}$$

From Eq. 38, the junction capacitance can be obtained

$$C_j = \frac{\varepsilon_s}{W} = \left[\frac{qa\varepsilon_s^2}{12(V_{bi}-V_R)}\right]^{1/3} = \left[\frac{1.6\times 10^{-19}\times 10^{20}\times\left(11.9\times 8.85\times 10^{-14}\right)^2}{12(0.5686-V_R)}\right]^{1/3}$$

At reverse bias of 4V, the junction capacitance is 6.866×10^{-9} F/cm^2.

7. From Eq. 35, we can obtain

$$\frac{1}{C_j^2} = \frac{2(V_{bi}-V)}{q\varepsilon_s N_B} \Rightarrow N_D = \frac{2(V_{bi}-V_R)}{q\varepsilon_s}C_j^2$$

$$\because V_R \gg V_{bi} \Rightarrow N_D \cong \frac{2(V_R)}{q\varepsilon_s}C_j^2 = \frac{2\times 4}{1.6\times 10^{-19}\times 11.9\times 8.85\times 10^{-14}}\times(0.85\times 10^{-8})^2$$

$$\Rightarrow N_d = 3.43\times 10^{15}\ \text{cm}^{-3}$$

We can select the n-type doping concentration of 3.43×10^{15} cm^{-3}.

8. From Eq. 56,

$$G = -U = \left[\frac{\sigma_p \sigma_n u_{th} N_t}{\sigma_n \exp\left(\dfrac{E_t-E_i}{kT}\right)+\sigma_p \exp\left(\dfrac{E_i-E_t}{kT}\right)}\right]n_i$$

$$= \left[\frac{10^{-15}\times 10^{-15}\times 10^7\times 10^{15}}{10^{-15}\exp\left(\dfrac{0.02}{0.0259}\right)+10^{-15}\exp\left(\dfrac{-0.02}{0.0259}\right)}\right]\times 9.65\times 10^9 = 3.89\times 10^{16}$$

and

$$W = \sqrt{\frac{2\boldsymbol{e}_s\left(V_{bi}+V\right)}{qN_A}} = \sqrt{\frac{2\times11.9\times8.85\times10^{-14}\times(0.717+0.5)}{1.6\times10^{-19}\times10^{15}}} = 12.66\times10^{-5}\,\text{cm} = 1.266\,\boldsymbol{mm}$$

Thus

$$J_{gen} = qGW = 1.6\times10^{19}\times3.89\times10^{16}\times12.66\times10^{-5} = 7.879\times10^{-7}\,\text{A/cm}^2.$$

9. From Eq. 49, and $p_{no} = \dfrac{n_i^{\,2}}{N_D}$

We can obtain the hole concentration at the edge of the space charge region,

$$p_n = \frac{n_i^{\,2}}{N_D}e^{(\frac{0.8}{0.0259})} = \frac{\left(9.65\times10^9\right)^2}{10^{16}}e^{\left(\frac{0.8}{0.0259}\right)} = 2.42\times10^{17}\ \text{cm}^{-3}.$$

10. $J = J_p\left(x_n\right) + J_n\left(-x_p\right) = J_s\left(e^{qV/kT}-1\right)$

$$\Rightarrow \frac{J}{J_s} = e^{\frac{V}{0.0259}} - 1$$

$$\Rightarrow 0.95 = e^{\frac{V}{0.0259}} - 1$$

$$\Rightarrow V = 0.017\ \text{V}.$$

11. The parameters are

$n_i= 9.65\times10^9\ \text{cm}^{-3}$ $D_n= 21\ \text{cm}^2/\text{sec}$

$D_p=10\ \text{cm}^2/\text{sec}$ $\boldsymbol{t}_{po}=\boldsymbol{t}_{no}=5\times10^{-7}\ \text{sec}$

From Eq. 52 and Eq. 54

$$J_p\left(x_n\right) = \frac{qD_p\,p_{no}}{L_p}\left(e^{qV/kT}-1\right) = q\sqrt{\frac{D_p}{\boldsymbol{t}_{po}}}\times\frac{n_i^{\,2}}{N_D}\times\left[e^{\left(\frac{qV_a}{kT}\right)}-1\right]$$

$\Rightarrow$

$$7 = 1.6\times10^{-19}\times\sqrt{\frac{10}{5\times10^{-7}}}\times\frac{\left(9.65\times10^9\right)^2}{N_D}\times\left[e^{\left(\frac{0.7}{0.0259}\right)}-1\right]$$

$\Rightarrow$

$$N_D = 5.2\times10^{15}\,\text{cm}^{-3}$$

$$J_n(-x_p) = \frac{qD_n n_{po}}{L_n}\left(e^{qV/kT} - 1\right) = q\sqrt{\frac{D_n}{t_{no}}} \times \frac{n_i^2}{N_A} \times \left[e^{\left(\frac{qV_a}{kT}\right)} - 1\right]$$

$\Rightarrow$

$$25 = 1.6 \times 10^{-19} \times \sqrt{\frac{21}{5 \times 10^{-7}}} \times \frac{\left(9.65 \times 10^9\right)^2}{N_A} \times \left[e^{\left(\frac{0.7}{0.0259}\right)} - 1\right]$$

$\Rightarrow$

$$N_A = 5.278 \times 10^{16} \, \text{cm}^{-3}$$

We can select a *p-n* diode with the conditions of $N_A = 5.278 \times 10^{16}\text{cm}^{-3}$ and $N_D = 5.4 \times 10^{15}\text{cm}^{-3}$.

12. Assume $\tau_g = \tau_p = \tau_n = 10^{-6}$ s, $D_n = 21$ cm^2/sec, and $D_p = 10$ cm^2/sec

(a) The saturation current calculation.
From Eq. 55a and $L_p = \sqrt{D_p t_p}$, we can obtain

$$J_s = \frac{qD_p p_{n0}}{L_p} + \frac{qD_n n_{p0}}{L_n} = qn_i^2\left(\frac{1}{N_D}\sqrt{\frac{D_p}{t_{p0}}} + \frac{1}{N_A}\sqrt{\frac{D_n}{t_{n0}}}\right)$$

$$= 1.6 \times 10^{-19} \times \left(9.65 \times 10^9\right)^2 \left(\frac{1}{10^{18}}\sqrt{\frac{10}{10^{-6}}} + \frac{1}{10^{16}}\sqrt{\frac{21}{10^{-6}}}\right)$$

$$= 6.87 \times 10^{-12} \, \text{A/cm}^2$$

And from the cross-sectional area A $= 1.2 \times 10^{-5}$ cm^2, we obtain

$$I_s = A \times J_s = 1.2 \times 10^{-5} \times 6.87 \times 10^{-12} = 8.244 \times 10^{-17}\,\text{A}.$$

(b) The total current density is

$$J = J_s\left(e^{\frac{qV}{kt}} - 1\right)$$

Thus

$$I_{0.7V} = 8.244 \times 10^{-17}\left(e^{\frac{0.7}{0.0259}} - 1\right) = 8.244 \times 10^{-17} \times 5.47 \times 10^{11} = 4.51 \times 10^{-5}\,\text{A}$$

$$I_{-0.7V} = 8.244 \times 10^{-17}\left(e^{\frac{-0.7}{0.0259}} - 1\right) = 8.244 \times 10^{-17}\,\text{A}.$$

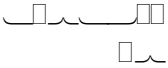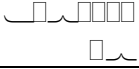13. From $J = J_s \left(e^{\frac{qV}{kt}} - 1 \right)$

we can obtain
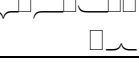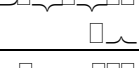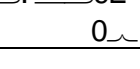
$$\frac{V}{0.0259} = \ln\left[\left(\frac{J}{J_s}\right) + 1 \right] \Rightarrow V = 0.0259 \times \ln\left[\left(\frac{10^{-3}}{8.244 \times 10^{-17}}\right) + 1 \right] = 0.78 \text{ V}.$$

14. From Eq. 59, and assume $D_p = 10 \text{ cm}^2/\text{sec}$, we can obtain

$$J_R \cong q\sqrt{\frac{D_p}{t_p}} \frac{n_i^2}{N_D} + \frac{qn_iW}{t_g}$$

$$= 1.6 \times 10^{-19} \sqrt{\frac{10}{10^{-6}}} \frac{\left(9.65 \times 10^9\right)^2}{10^{15}} + \frac{1.6 \times 10^{-19} \times 9.65 \times 10^9}{10^{-6}} \sqrt{\frac{2 \times 11.9 \times 8.85 \times 10^{-14} \times \left(V_{bi} + V_R\right)}{1.6 \times 10^{-19} \times 10^{15}}}$$

$$V_{bi} = 0.0259\ln\frac{10^{19} \times 10^{15}}{\left(9.65 \times 10^9\right)^2} = 0.834 \text{ V}$$

Thus

$$J_R = 5.26 \times 10^{-11} + 1.872 \times 10^{-7} \sqrt{0.834 + V_R}$$

VR	Js
0	E-0
0.	55E-0
0.	9 E-0
0.	9E-0
0.	E-0
0. 5	50 E-0
0.	9 E-0
0.	E-0
0.	0 5E-0
0. 9	5 E-0
	9E-0
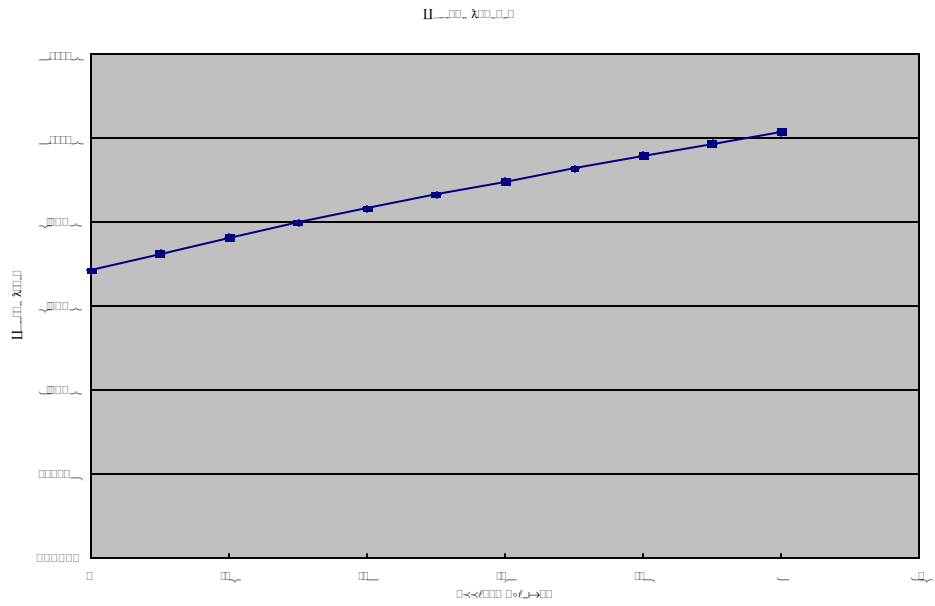
When $N_D = 10^{17}$ cm^{-3}, we obtain

$$V_{bi} = 0.0259 \ln \frac{10^{19} \times 10^{17}}{\left(9.65 \times 10^9\right)^2} = 0.953 \text{ V}$$

$$J_R = 5.26 \times 10^{-13} + 1.872 \times 10^{-8} \sqrt{0.956 + V_R}$$

15. From Eq. 39,
$$Q_p = q \int_{x_n}^{\infty} \left(p_n - p_{no}\right) \, dx$$

$$= q \int_{x_n}^{\infty} p_{no} \left(e^{qV/kT} - 1 \right) e^{-(x-x_n)/L_p} dx$$

The hole diffusion length is larger than the length of neutral region.

$$Q_p = q \int_{x_n}^{x_n'} (p_n - p_{no}) \, dx$$

$$= q \int_{x_n}^{x_n'} p_{no} \left(e^{qV/kT} - 1 \right) e^{-(x-x_n)/L_p} dx$$

$$= q p_{no} (-L_p) \left(e^{\frac{qV}{kT}} - 1 \right) \left(e^{-\frac{x_n' - x_n}{L_p}} - e^{-\frac{x_n - x_n}{L_p}} \right)$$

$$= 1.6 \times 10^{-19} \times \frac{(9.65 \times 10^9)^2}{10^{16}} (-5 \times 10^{-4}) \left(e^{\frac{1}{0.0259}} - 1 \right) \left(e^{-\frac{1}{5}} - e^{-\frac{0}{5}} \right)$$

$$= 8.784 \times 10^{-3} \, \text{C/cm}^2.$$

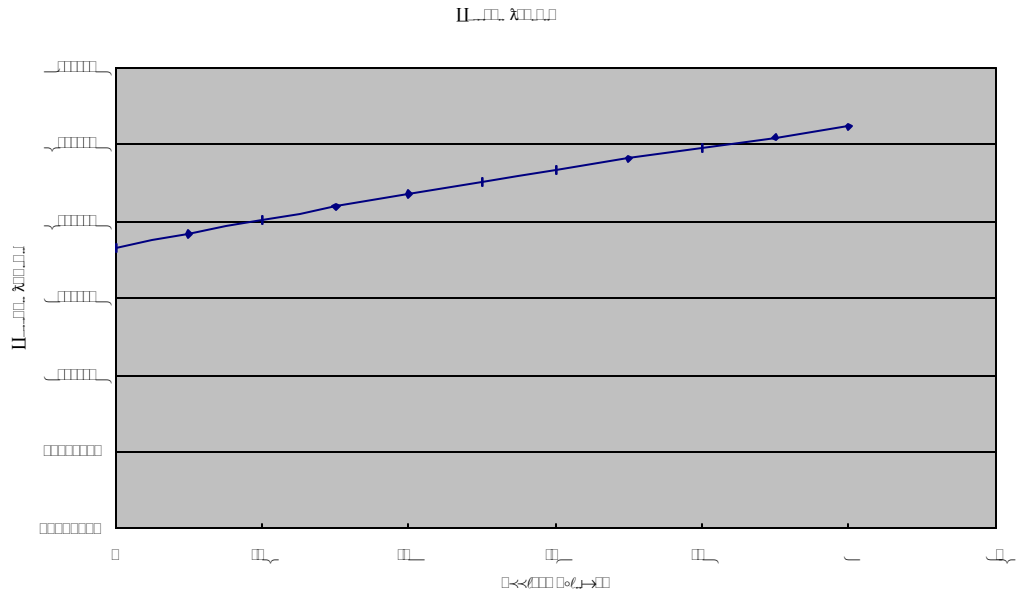16. From Fig. 26, the critical field at breakdown for a Si one-sided abrupt junction is about $2.8 \parallel 10^5$ V/cm. Then from Eq. 85, we obtain

$$V_B (\text{breakdown voltage}) = \frac{E_c W}{2} = \frac{\varepsilon_s E_c^2}{2q} (N_B)^{-1}$$

$$= \frac{11.9 \times 8.85 \times 10^{-14} \times (2.8 \times 10^5)^2}{2 \times 1.6 \times 10^{-19}} (10^{15})^{-1}$$

$$= 258 \ \text{V}$$

$$W = \sqrt{\frac{2\varepsilon_s (V_{bi} - V)}{q N_B}} \cong \sqrt{\frac{2 \times 11.9 \times 8.85 \times 10^{-14} \times 258}{1.6 \times 10^{-19} \times 10^{15}}} = 1.843 \times 10^{-3} \, \text{cm} = 18.43 \pounds \text{gm}$$

When the n-region is reduced to $5\mu\text{m}$, the punch-through will take place first.

From Eq. 87, we can obtain

$$\frac{V_B'}{V_B} = \frac{\text{shaded area in Fig. 29 insert}}{(E_c W_m)/2} = \left(\frac{W}{W_m} \right) \left(2 - \frac{W}{W_m} \right)$$

$$V_B' = V_B \left(\frac{W}{W_m} \right) \left(2 - \frac{W}{W_m} \right) = 258 \times \left(\frac{5}{18.43} \right) \left(2 - \frac{5}{18.43} \right) = 121 \ \text{V}$$

Compared to Fig. 29, the calculated result is the same as the value under the

conditions of $W = 5\ \mu\text{m}$ and $N_B = 10^{15}\ \text{cm}^{-3}$.

17. We can use following equations to determine the parameters of the diode.

$$J_F = q\sqrt{\frac{D_p}{t_p}\frac{n_i^2}{N_D}}\,e^{qV/kT} + \frac{qWn_i}{2t_r}e^{qV/2kT} \cong q\sqrt{\frac{D_p}{t_p}\frac{n_i^2}{N_D}}\,e^{qV/kT}$$

$$V_B = \frac{E_cW}{2} = \frac{e_sE_c^2}{2q}(N_D)^{-1}$$

$$\Rightarrow$$

$$AJ_F = Aq\sqrt{\frac{D_p}{t_p}\frac{n_i^2}{N_D}}\,e^{qV/kT} \Rightarrow A\times1.6\times10^{-19}\times\sqrt{\frac{D_p}{10^{-7}}}\frac{(9.65\times10^9)^2}{N_D}\,e^{\frac{0.7}{0.0259}} = 2.2\times10^{-3}$$

$$V_B = \frac{E_cW}{2} = \frac{e_sE_c^2}{2q}(N_D)^{-1} \Rightarrow 130 = \frac{11.9\times8.85\times10^{-14}E_c^2}{2\times1.6\times10^{-19}}(N_D)^{-1}$$

Let $E_c = 4\times10^5$ V/cm, we can obtain $N_D = 4.05\times10^{15}$ cm^{-3}.

The mobility of minority carrier hole is about 500 at $N_D = 4.05\times10^{15}$

$\therefore D_p = 0.0259\times500 = 12.95$ cm^2/s

Thus, the cross-sectional area A is 8.6×10^{-5} cm^2.

18. As the temperature increases, the total reverse current also increases. That is, the total electron current increases. The impact ionization takes place when the electron gains enough energy from the electrical field to create an electron-hole pair. When the temperature increases, total number of electron increases resulting in easy to lose their energy by collision with other electron before breaking the lattice bonds. This need higher breakdown voltage.

19. (a) The i-layer is easy to deplete, and assume the field in the depletion region is

constant. From Eq. 84, we can obtain.

$$\int_0^W 10^4 \left(\frac{E}{4\times10^5}\right)^6 dx = 1 \Rightarrow 10^4 \left(\frac{E}{4\times105}\right)^6 \times 10^{-3} = 1 \Rightarrow E_{critical} = 4\times10^5 \times (10)^{\frac{1}{6}} = 5.87\times10^5 \text{ V/cm}$$

$$4\,V_B = 5.87\times10^5 \times 10^{-3} = 587 \text{ V}$$

(b) From Fig. 26, the critical field is 5×10^5 V/cm.

$$V_B \text{(breakdown voltage)} = \frac{E_c W}{2} = \frac{e_s E_c^2}{2q}(N_B)^{-1}$$

$$= \frac{12.4\times8.85\times10^{-14}\times(5\times10^5)^2}{2\times1.6\times10^{-19}}(2\times10^{16})^{-1}$$

$$= 42.8 \text{ V}.$$

20. $a = \dfrac{2\times10^{18}}{2\times10^{-4}} = 10^{22}\,\text{cm}^{-4}$

$$V_B = \frac{2E\,W}{3} = \frac{4E_c^{3/2}}{3}\left[\frac{2e_s}{q}\right]^{1/2}(a)^{-1/2}$$

$$= \frac{4E_c^{3/2}}{3}\left[\frac{2\times11.9\times8.85\times10^{-14}}{1.6\times10^{-19}}\right]^{1/2}\times(10^{22})^{-1/2}$$

$$= 4.84\times10^{-8}E_c^{3/2}$$

The breakdown voltage can be determined by a selected E_c.

21. To calculate the results with applied voltage of $V = 0.5\,\text{V}$, we can use a similar calculation

in Example 10 with (1.6-0.5) replacing 1.6 for the voltage. The obtained electrostatic

potentials are $1.1\,\text{V}$ and $3.4\times10^{-4}\,\text{V}$, respectively. The depletion widths are

3.821×10^{-5} cm and 1.274×10^{-8} cm , respectively.

Also, by substituting $V = -5\,\text{V}$ to Eqs. 90 and 91, the electrostatic potentials are $6.6\,\text{V}$

and $20.3\times10^{-4}\,\text{V}$, and the depletion widths are 9.359×10^{-5} cm and 3.12×10^{-8} cm ,

respectively.

The total depletion width will be reduced when the heterojunction is forward-biased from

the thermal equilibrium condition. On the other hand, when the heterojunction is reverse-

biased, the total depletion width will be increased.

22. $E_g(0.3) = 1.424 + 1.247 \times 0.3 = 1.789$ eV

$$V_{bi} = \frac{E_{g2}}{q} - \frac{\Delta E_C}{q} - \left(E_{F2} - E_{V2}\right)/q - \left(E_{C1} - E_{F1}\right)/q$$

$$= 1.789 - 0.21 - \frac{kT}{q}\ln\frac{4.7\times10^{17}}{5\times10^{15}} - \frac{kT}{q}\ln\frac{7\times10^{18}}{5\times10^{15}} = 1.273 \text{ V}$$

$$x_1 = \left[\frac{2N_A\,\mathbf{e}_1\mathbf{e}_2 v_{bi}}{qN_D\left(\mathbf{e}_1 N_D + \mathbf{e}_2 N_A\right)}\right]^{\frac{1}{2}} = \left[\frac{2\times12.4\times11.46\times8.85\times10^{-14}\times1.273}{1.6\times10^{-19}\times5\times10^{15}\left(12.4+11.46\right)}\right]^{\frac{1}{2}}$$

$$= 4.1\times10^{-5}\text{ cm.}$$

Since $N_D x_1 = N_A x_2$ $\therefore x_1 = x_2$

$$\therefore W = 2\,x_1 = 8.2\times10^{-5}\text{ cm} = 0.82\ \boldsymbol{m}\text{m}.$$

CHAPTER 5

1. (*a*) The common-base and common-emitter current gains is given by

$$a_0 = g a_T = 0.997 \times 0.998 = 0.995$$

$$b_0 = \frac{a_0}{1-a_0} = \frac{0.995}{1-0.995}$$

$$= 199 \ .$$

(*b*) Since $I_B = 0$ and $I_{Cp} = 10 \times 10^{-9}$ A , then I_{CBO} is 10×10^{-9} A . The emitter current is

$$I_{CEO} = (1 + b_0) I_{CBO}$$

$$= (1 + 199) \cdot 10 \times 10^{-9}$$

$$= 2 \times 10^{-6} \text{ A} \ .$$

2. For an ideal transistor,

$$a_0 = g = 0.999$$

$$b_0 = \frac{a_0}{1-a_0} = 999 \ .$$

I_{CBO} is known and equals to 10×10^{-6} A . Therefore,

$$I_{CEO} = (1 + b_0) I_{CBO}$$

$$= (1 + 999) \cdot 10 \times 10^{-6}$$

$$= 10 \text{ mA} \ .$$

3. (a) The emitter-base junction is forward biased. From Chapter 3 we obtain

$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.0259 \ln\left[\frac{5 \times 10^{18} \cdot 2 \times 10^{17}}{\left(9.65 \times 10^9\right)^2}\right] = 0.956 \text{ V} \ .$$

The depletion-layer width in the base is

$$W_1 = \left(\frac{N_A}{N_A + N_D} \right) \text{(Total depletion - layer width of the emitter - base junction)}$$

$$= \sqrt{\frac{2e_s}{q} \left(\frac{N_A}{N_D} \right) \left(\frac{1}{N_A + N_D} \right) (V_{bi} - V)}$$

$$= \sqrt{\frac{2 \cdot 1.05 \times 10^{-12}}{1.6 \times 10^{-19}} \left(\frac{5 \times 10^{18}}{2 \times 10^{17}} \right) \left(\frac{1}{5 \times 10^{18} + 2 \times 10^{17}} \right) (0.956 - 0.5)}$$

$$= 5.364 \times 10^{-6} \text{ cm} = 5.364 \times 10^{-2} \text{ } \mu\text{m} .$$

Similarly we obtain for the base-collector function

$$V_{bi} = 0.0259 \ln \left[\frac{2 \times 10^{17} \cdot 10^{16}}{(9.65 \times 10^9)^2} \right] = 0.795 \text{ V} .$$

and

$$W_2 = \sqrt{\frac{2 \cdot 1.05 \times 10^{-12}}{1.6 \times 10^{-19}} \left(\frac{10^{16}}{2 \times 10^{17}} \right) \left(\frac{1}{10^{16} + 2 \times 10^{17}} \right) (0.795 + 5)}$$

$$= 4.254 \times 10^{-6} \text{ cm} = 4.254 \times 10^{-2} \text{ } \mu\text{m} .$$

Therefore the neutral base width is

$$W = W_B - W_1 - W_2 = 1 - 5.364 \times 10^{-2} - 4.254 \times 10^{-2} = 0.904 \text{ } \mu\text{m} .$$

(b) Using Eq. 13a

$$p_n(0) = p_{no} e^{qV_{EB}/kT} = \frac{n_i^2}{N_D} e^{qV_{EB}/kT} = \frac{(9.65 \times 10^9)^2}{2 \times 10^{17}} e^{0.5/0.0259} = 2.543 \times 10^{11} \text{ cm}^{-3} .$$

4. In the emitter region

$$D_E = 52 \text{ cm/s} \quad L_E = \sqrt{52 \cdot 10^{-8}} = 0.721 \times 10^{-3} \text{ cm}$$

$$n_{EO} = \frac{(9.65 \times 10^9)^2}{5 \times 10^{18}} = 18.625 .$$

In the base region

$$D_p = 40 \text{ cm/s} \quad L_p = \sqrt{D_p t_p} = \sqrt{40 \cdot 10^{-7}} = 2 \times 10^{-3} \text{ cm}$$

$$p_{no} = \frac{n_i^2}{N_D} = \frac{(9.65 \times 10^9)^2}{2 \times 10^{17}} = 465.613 \ .$$

In the collector region

$$D_C = 115 \text{ cm/s} \quad L_C = \sqrt{115 \cdot 10^{-6}} = 10.724 \times 10^{-3} \text{ cm}$$

$$n_{CO} = \frac{(9.65 \times 10^9)^2}{10^{16}} = 9.312 \times 10^3 \ .$$

The current components are given by Eqs. 20, 21, 22, and 23:

$$I_{Ep} = \frac{1.6 \times 10^{-19} \cdot 0.2 \times 10^{-2} \cdot 40 \cdot 465.613}{0.904 \times 10^{-4}} e^{0.5/0.0259} = 1.596 \times 10^{-5} \text{ A}$$

$$I_{Cp} \cong I_{Ep} = 1.596 \times 10^{-5} \text{ A}$$

$$I_{En} = \frac{1.6 \times 10^{-19} \cdot 0.2 \times 10^{-2} \cdot 52 \cdot 18.625}{0.721 \times 10^{-3}} \left(e^{0.5/0.0259} - 1 \right) = 1.041 \times 10^{-7} \text{ A}$$

$$I_{Cn} = \frac{1.6 \times 10^{-19} \cdot 0.2 \times 10^{-2} \cdot 115 \cdot 9.312 \times 10^3}{10.724 \times 10^{-3}} = 3.196 \times 10^{-14} \text{ A}$$

$$I_{BB} = I_{Ep} - I_{Cp} = 0 \ .$$

5. (a) The emitter, collector, and base currents are given by

$$I_E = I_{Ep} + I_{En} = 1.606 \times 10^{-5} \text{ A}$$

$$I_C = I_{Cp} + I_{Cn} = 1.596 \times 10^{-5} \text{ A}$$

$$I_B = I_{En} + I_{BB} - I_{Cn} = 1.041 \times 10^{-7} \text{ A} \ .$$

(b) We can obtain the emitter efficiency and the base transport factor:

$$g = \frac{I_{Ep}}{I_E} = \frac{1.596 \times 10^{-5}}{1.606 \times 10^{-5}} = 0.9938$$

$$a_T = \frac{I_{Cp}}{I_{Ep}} = \frac{1.596 \times 10^{-5}}{1.596 \times 10^{-5}} = 1 \ .$$

Hence, the common-base and common-emitter current gains are

$$a_0 = g a_T = 0.9938$$

$$b_0 = \frac{a_0}{1 - a_0} = 160.3 \ .$$

(c) To improve g, the emitter has to be doped much heavier than the base.

To improve a_T, we can make the base width narrower.

6. We can sketch $p_n(x)/p_n(0)$ curves by using a computer program:
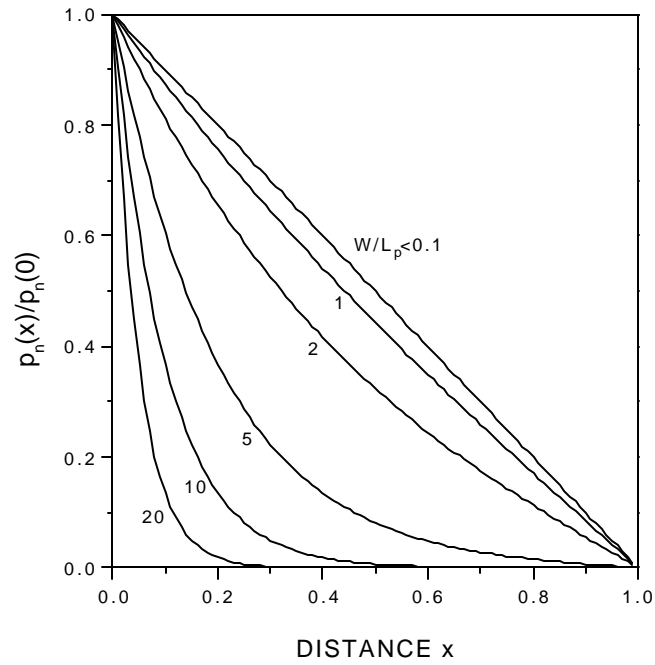
In the figure, we can see when $W/L_p < 0.1$ ($W/L_p = 0.05$ in this case), the minority carrier distribution approaches a straight line and can be simplified to Eq. 15.

7. Using Eq.14, I_{Ep} is given by

$$I_{Ep} = A\left(-qD_p \frac{dp_n}{dx}\bigg|_{x=0}\right)$$

$$= A(-qD_p)\left\{ p_{no}\,(e^{qV_{EB}/kT}-1)\left[\frac{-\dfrac{1}{L_p}\cosh\left(\dfrac{W-x}{L_p}\right)}{\sinh\left(\dfrac{W}{L_p}\right)}\right] + p_{no}\left[\frac{-\dfrac{1}{L_p}\cosh\left(\dfrac{x}{L_p}\right)}{\sinh\left(\dfrac{W}{L_p}\right)}\right]\right\}_{x=0}$$

$$= qA\frac{D_p\,p_{no}}{L_p}\left\{(e^{qV_{EB}/kT}-1)\left[\frac{\cosh\left(\dfrac{W}{L_p}\right)}{\sinh\left(\dfrac{W}{L_p}\right)}\right] + \left[\frac{1}{\sinh\left(\dfrac{W}{L_p}\right)}\right]\right\}$$

$$= qA\frac{D_p\,p_{no}}{L_p}\coth\left(\frac{W}{L_p}\right)\left[(e^{qV_{EB}/kT}-1) + \frac{1}{\cosh\left(\dfrac{W}{L_p}\right)}\right].$$

Similarly, we can obtain I_{Cp} :

$$I_{Cp} = A\left(-qD_p \frac{dp_n}{dx}\bigg|_{x=W}\right)$$

$$= A(-qD_p)\left\{ p_{no}\,(e^{qV_{EB}/kT}-1)\left[\frac{-\dfrac{1}{L_p}\cosh\left(\dfrac{W-x}{L_p}\right)}{\sinh\left(\dfrac{W}{L_p}\right)}\right] + p_{no}\left[\frac{-\dfrac{1}{L_p}\cosh\left(\dfrac{x}{L_p}\right)}{\sinh\left(\dfrac{W}{L_p}\right)}\right]\right\}_{x=W}$$

$$= qA\frac{D_p\,p_{no}}{L_p}\left\{(e^{qV_{EB}/kT}-1)\left[\frac{1}{\sinh\left(\dfrac{W}{L_p}\right)}\right] + \left[\frac{\cosh\left(\dfrac{W}{L_p}\right)}{\sinh\left(\dfrac{W}{L_p}\right)}\right]\right\}$$

$$= qA\frac{D_p\,p_{no}}{L_p}\frac{1}{\sinh\left(\dfrac{W}{L_p}\right)}\left[(e^{qV_{EB}/kT}-1) + \cosh\left(\frac{W}{L_p}\right)\right].$$

8. The total excess minority carrier charge can be expressed by

$$Q_B = qA \int_0^W [p_n(x) - p_{no}] dx$$

$$= qA \int_0^W \left[p_{no} e^{qV_{EB}/kT} (1 - \frac{x}{W}) \right] dx$$

$$= qAp_{no} e^{qV_{EB}/kT} (x - \frac{x^2}{2W}) \Big|_0^W$$

$$= \frac{qAWp_{no} e^{qV_{EB}/kT}}{2}$$

$$= \frac{qAWp_n(0)}{2} .$$

From Fig. 6, the triangular area in the base region is $\frac{Wp_n(0)}{2}$. By multiplying this

value by q and the cross-sectional area A, we can obtain the same expression as Q_B.

In Problem 3,

$$Q_B = \frac{1.6 \times 10^{-19} \cdot 0.2 \times 10^{-2} \cdot 0.904 \times 10^{-4} \cdot 2.543 \times 10^{11}}{2}$$

$$= 3.678 \times 10^{-15} \text{ C} .$$

9.   In Eq. 27,

$$I_C = a_{21} \left(e^{qV_{EB}/kT} - 1 \right) + a_{22}$$

$$\cong \frac{qAD_p p_n(0)}{W}$$

$$= \frac{2D_p}{W^2} \frac{qAQp_n(0)}{2}$$

$$= \frac{2D_p}{W^2} Q_B .$$

Therefore, the collector current is directly proportional to the minority carrier

charge stored in the base.

10.   The base transport factor is

$$\mathbf{a}_T \cong \frac{I_{Cp}}{I_{Ep}} = \frac{\dfrac{1}{\sinh\left(\dfrac{W}{L_p}\right)}\left[\left(e^{qV_{EB}/kT}-1\right)+\cosh\left(\dfrac{W}{L_p}\right)\right]}{\coth\left(\dfrac{W}{L_p}\right)\left[\left(e^{qV_{EB}/kT}-1\right)+\dfrac{1}{\cosh\left(\dfrac{W}{L_p}\right)}\right]} .$$

For $W/L_p \ll 1$, $\cosh(W/L_p) \cong 1$. Thus,

$$\mathbf{a}_T = \frac{1}{\sinh\left(\dfrac{W}{L_p}\right)\cdot\coth\left(\dfrac{W}{L_p}\right)}$$

$$= \operatorname{sech}\left(\dfrac{W}{L_p}\right)$$

$$= 1 - \frac{1}{2}\left(\frac{W}{L_p}\right)^2$$

$$= 1 - \left(W^2/2L_p^2\right).$$

11.   The common-emitter current gain is given by

$$\mathbf{b}_0 \equiv \frac{\mathbf{a}_0}{1-\mathbf{a}_0} = \frac{g\mathbf{a}_T}{1-g\mathbf{a}_T} .$$

Since $g \cong 1$,

$$\mathbf{b}_0 \cong \frac{\mathbf{a}_T}{1-\mathbf{a}_T}$$

$$= \frac{1-\left(W^2/2L_p^2\right)}{1-\left[1-\left(W^2/2L_p^2\right)\right]}$$

$$= \left(2L_p^2/W^2\right) - 1 .$$

If $W/L_p \ll 1$, then $\mathbf{b}_0 \cong 2L_p^2/W^2$.

12. $L_p = \sqrt{D_p t_p} = \sqrt{100 \cdot 3\times10^{-7}} = 5.477\times10^{-3}$ cm $= 54.77\ \mu$m

Therefore, the common-emitter current gain is

$$b_0 \cong 2L_p{}^2 / W^2 = \frac{2(54.77 \times 10^{-4})^2}{(2 \times 10^{-4})^2}$$

$$= 1500 .$$

13. In the emitter region,

$$m_{pE} = 54.3 + \frac{407}{1 + 0.374 \times 10^{-17} \cdot 3 \times 10^{18}} = 87.6$$

$$D_E = 0.0259 \cdot 87.6 = 2.26 \text{ cm/s} .$$

In the base region,

$$m_n = 88 + \frac{1252}{1 + 0.698 \times 10^{-17} \cdot 2 \times 10^{16}} = 1186.63$$

$$D_p = 0.0259 \cdot 1186.63 = 30.73 \text{ cm/s} .$$

In the collector region,

$$m_{pC} = 54.3 + \frac{407}{1 + 0.374 \times 10^{-17} \cdot 5 \times 10^{15}} = 453.82$$

$$D_C = 0.0259 \cdot 453.82 = 11.75 \text{ cm/s} .$$

14. In the emitter region,

$$L_E = \sqrt{D_E t_E} = \sqrt{2.269 \cdot 10^{-6}} = 1.506 \times 10^{-3} \text{ cm}$$

$$p_{EO} = \frac{n_i^2}{N_E} = \frac{(9.65 \times 10^9)^2}{3 \times 10^{18}} = 31.04 \text{ cm}^{-3} .$$

In the base region,

$$L_n = \sqrt{30.734 \cdot 10^{-6}} = 5.544 \times 10^{-3} \text{ cm}$$

$$n_{po} = \frac{(9.65 \times 10^9)^2}{2 \times 10^{16}} = 4656.13 \text{ cm}^{-3} .$$

In the collector region,

$$L_C = \sqrt{11.754 \cdot 10^{-6}} = 3.428 \times 10^{-3} \text{ cm}$$

$$p_{CO} = \frac{\left(9.65 \times 10^9\right)^2}{5 \times 10^{15}} = 18624.5 \text{ cm}^{-3} \ .$$

The emitter current components are given by

$$I_{En} = \frac{1.6 \times 10^{-19} \cdot 10^{-4} \cdot 30.734 \cdot 4656.13}{0.5 \times 10^{-4}} e^{0.6/0.0259} = 526.83 \times 10^{-6} \text{ A}$$

$$I_{Ep} = \frac{1.6 \times 10^{-19} \cdot 10^{-4} \cdot 2.269 \cdot 31.04}{1.506 \times 10^{-3}} \left(e^{0.6/0.0259} - 1\right) = 8.609 \times 10^{-9} \text{ A} \ .$$

Hence, the emitter current is

$$I_E = I_{En} + I_{Ep} = 526.839 \times 10^{-6} \text{ A} \ .$$

And the collector current components are given by

$$I_{Cn} = \frac{1.6 \times 10^{-19} \cdot 10^{-4} \cdot 30.734 \cdot 4656.13}{0.5 \times 10^{-4}} e^{0.6/0.0259} = 526.83 \times 10^{-6} \text{ A}$$

$$I_{Cp} = \frac{1.6 \times 10^{-19} \cdot 10^{-4} \cdot 11.754 \cdot 18624.5}{3.428 \times 10^{-3}} = 1.022 \times 10^{-14} \text{ A} \ .$$

Therefore, the collector current is obtained by

$$I_C = I_{Cn} + I_{Cp} = 5.268 \times 10^{-4} \text{ A} \ .$$

15.   The emitter efficiency can be obtained by

$$g = \frac{I_{En}}{I_E} = \frac{526.83 \times 10^{-6}}{526.839 \times 10^{-6}} = 0.99998 \ .$$

The base transport factor is

$$a_T = \frac{I_{Cn}}{I_{En}} = \frac{526.83 \times 10^{-6}}{526.83 \times 10^{-6}} = 1 \ .$$

Therefore, the common-base current gain is obtained by

$$a_0 = g a_T = 1 \times 0.99998 = 0.99998 \ .$$

The value is very close to unity.

The common-emitter current gain is

$$b_0 = \frac{a_0}{1-a_0} = \frac{0.99998}{1-0.99998} \cong 50000 \; .$$

16. (a) The total number of impurities in the neutral base region is

$$Q_G = \int_W^0 N_{AO}e^{-x/l}dx = N_{AO}l\left(1 - e^{-W/l}\right)$$
$$= 2\times10^{18} \cdot 3\times10^{-5}\left(1 - e^{-8\times10^{-5}/3\times10^{-5}}\right) = 5.583\times10^{13} \; \text{cm}^{-2} \; .$$

(b) Average impurity concentration is

$$\frac{Q_G}{W} = \frac{5.583\times10^{13}}{8\times10^{-5}}$$
$$= 6.979\times10^{17} \; \text{cm}^{-3} \; .$$

17. For $N_A = 6.979\times10^{17} \; \text{cm}^{-3}$, $D_n = 7.77 \; \text{cm}^3/\text{s}$, and

$$L_n = \sqrt{D_n t_n} = \sqrt{7.77 \cdot 10^{-6}} = 2.787\times10^{-3} \; \text{cm}$$

$$a_T \cong 1 - \frac{W^2}{2L_n^2} = 1 - \frac{\left(8\times10^{-5}\right)^2}{2\left(2.787\times10^{-3}\right)^2} = 0.999588$$

$$g = \frac{1}{1 + \frac{D_E}{D_n}\frac{Q_G}{N_E L_E}} = \frac{1}{1 + \frac{1}{7.77}\cdot\frac{5.583\times10^{13}}{10^{19}\cdot10^{-4}}} = 0.99287 \; .$$

Therefore,

$$a_0 = ga_T = 0.99246$$

$$b_0 = \frac{a_0}{1-a_0} = 131.6 \; .$$

18. The mobility of an average impurity concentration of $6.979\times10^{17} \; \text{cm}^{-3}$ is about 300 cm^2/Vs. The average base resistivity $\bar{r}_B$ is given by

$$\bar{r}_B = \frac{1}{qm_n(Q_G/W)} = 0.0299 \; \Omega\text{-cm} \; .$$

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Therefore,

$$R_B = 5 \times 10^{-3} \left(\overline{r}_B / W \right) = 5 \times 10^{-3} \cdot \left(0.0299 / 8 \times 10^{-5} \right) = 1.869 \ \Omega \ .$$

For a voltage drop of kT/q,

$$I_B = \frac{kT}{qR_B} = 0.0139 \ \text{A} \ .$$

Therefore,

$$I_C = b_0 \, I_B = 131.6 \cdot 0.0139 = 1.83 \ \text{A} \ .$$

19.   From Fig. 10b and Eq. 35, we obtain

$I_B \, (\mu A)$	$I_C \, (mA)$	$b_0 = \dfrac{\Delta I_C}{\Delta I_B}$
0	0.20	---
5	0.95	150
10	2.00	210
15	3.10	220
20	4.00	180
25	4.70	140

b_0 is not a constant. At low I_B, because of generation-recombination current, b_0 increases with increasing I_B. At high I_B, V_{EB} increases with I_B, this in turn causes a reduction of V_{BC} since $V_{EB} + V_{BC} = V_{EC} = 5 \, \text{V}$. The reduction of V_{BC} causes a widening of the neutral base region, therefore b_0 decreases.

The following chart shows b_0 as function of I_B. It is obvious that b_0 is not a constant.
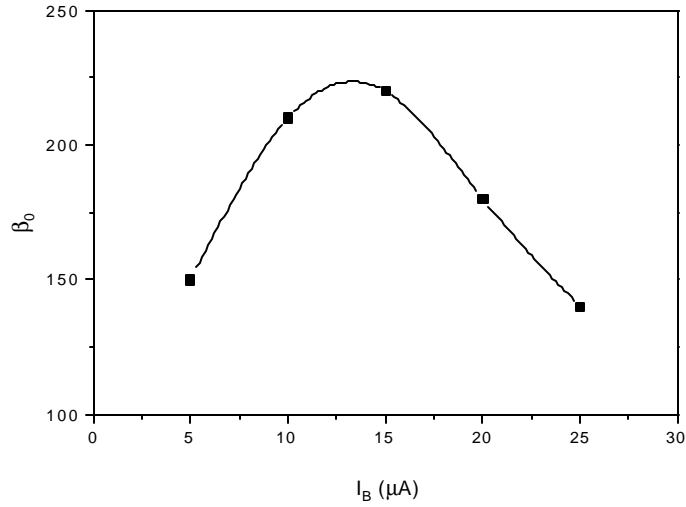
20. Comparing the equations with Eq. 32 gives

$$I_{FO} = a_{11}, \quad \mathbf{a}_R I_{RO} = a_{12}$$

$$\mathbf{a}_F I_{FO} = a_{21}, \text{ and } I_{RO} = a_{22} \quad .$$

Hence,

$$\mathbf{a}_F = \frac{a_{21}}{a_{11}} = \cfrac{1}{1 + \cfrac{W}{L_E} \cdot \cfrac{D_E}{D_p} \cdot \cfrac{n_{EO}}{p_{no}}}$$

$$\mathbf{a}_R = \frac{a_{12}}{a_{22}} = \cfrac{1}{1 + \cfrac{W}{L_C} \cdot \cfrac{D_C}{D_p} \cdot \cfrac{n_{CO}}{p_{no}}} \quad .$$

21. In the collector region,

$$L_C = \sqrt{D_C t_C} = \sqrt{2 \cdot 10^{-6}} = 1.414 \times 10^{-3} \text{ cm}$$

$$n_{CO} = n_i^2 / N_C = \left(9.65 \times 10^9\right)^2 / 5 \times 10^{15} = 1.863 \times 10^4 \text{ cm}^{-3} \quad .$$

From Problem 20, we have

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$$a_F = \cfrac{1}{1+\cfrac{W}{L_E}\cdot\cfrac{D_E}{D_p}\cdot\cfrac{n_{EO}}{p_{no}}} = \cfrac{1}{1+\cfrac{0.5\times10^{-4}}{10^{-3}}\cdot\cfrac{1}{10}\cdot\cfrac{9.31}{9.31\times10^{-2}}}$$

$$= 0.99995$$

$$a_R = \cfrac{1}{1+\cfrac{W}{L_C}\cdot\cfrac{D_C}{D_p}\cdot\cfrac{n_{CO}}{p_{no}}} = \cfrac{1}{1+\cfrac{0.5\times10^{-4}}{1.414\times10^{-3}}\cdot\cfrac{2}{10}\cdot\cfrac{1.863\times10^4}{9.31\times10^{-2}}}$$

$$= 0.876$$

$$I_{FO} = a_{11} = qA\left(\frac{D_p p_{no}}{W}+\frac{D_E n_{EO}}{L_E}\right)$$

$$= 1.6\times10^{-19}\cdot5\times10^{-4}\cdot\left(\frac{10\cdot9.31\times10^2}{0.5\times10^{-4}}+\frac{1\cdot9.31}{10^{-4}}\right)$$

$$= 1.49\times10^{-14}\ \text{A}$$

$$I_{RO} = a_{22} = qA\left(\frac{D_p p_{no}}{W}+\frac{D_C n_{CO}}{L_C}\right)$$

$$= 1.6\times10^{-19}\cdot5\times10^{-4}\cdot\left(\frac{10\cdot9.31\times10^2}{0.5\times10^{-4}}+\frac{2\cdot1.863\times10^4}{1.414\times10^{-3}}\right)$$

$$= 1.7\times10^{-14}\ \text{A}\ .$$

The emitter and collector currents are

$$I_E = I_{FO}\left(e^{qV_{EB}/kT}-1\right)+a_R I_{RO}$$

$$= 1.715\times10^{-4}\ \text{A}$$

$$I_C = a_F I_{FO}\left(e^{qV_{EB}/kT}-1\right)+I_{RO}$$

$$= 1.715\times10^{-4}\ \text{A}\ .$$

Note that these currents are almost the same (no base current) for $W/L_p \ll 1$.

22.   Referring Eq. 11, the field-free steady-state continuity equation in the collector region is

$$D_C\left[\frac{d^2 n_C(x')}{dx'^2}\right]-\frac{n_C(x')-n_{po}}{t_C}=0\ .$$

The solution is given by ($L_C = \sqrt{D_C t_C}$)

$$n_C(x') = C_1 e^{x'/L_C}+C_2 e^{-x'/L_C}\quad.$$

Applying the boundary condition at $x'=\infty$ yields

$$C_1 e^{\infty/L_C} + C_2 e^{-\infty/L_C} = 0 .$$

Hence $C_1 = 0$. In addition, for the boundary condition at $x' = 0$,

$$C_2 e^{-0/L_C} = C_2 = n_C(0)$$

$$n_C(0) = n_{CO}\left(e^{qV_{CB}/kT} - 1\right) .$$

The solution is

$$n_C(x) = n_{CO}\left(e^{qV_{CB}/kT} - 1\right)e^{-x'/L_C} .$$

The collector current can be expressed as

$$
\begin{aligned}
I_C &= A\left(-qD_p\frac{dp_n}{dx}\bigg|_{x=W}\right) + A\left(-qD_C\frac{dn_C}{dx'}\bigg|_{x'=0}\right) \\
&= qA\frac{D_p\, p_{no}}{W}\left(e^{qV_{EB}/kT} - 1\right) - qA\left(\frac{D_p\, p_{no}}{W} + \frac{D_C\, n_{CO}}{L_C}\right)\left(e^{qV_{CB}/kT} - 1\right) \\
&= a_{21}\left(e^{qV_{EB}/kT} - 1\right) - a_{22}\left(e^{qV_{CB}/kT} - 1\right) .
\end{aligned}
$$

23. Using Eq. 44, the base transit time is given by

$$t_B = W^2/2D_p = \frac{\left(0.5\times10^{-4}\right)^2}{2\times10} = 1.25\times10^{-10}\ \text{s} .$$

We can obtain the cutoff frequency :

$$f_T \cong 1/2\pi t_B = 1.27\ \text{GHz} .$$

From Eq. 41, the common-base cutoff frequency is given by:

$$f_a \cong f_T/a_0 = \frac{1.27\times10^9}{0.998} = 1.275\ \text{GHz} .$$

The common-emitter cutoff frequency is

$$f_b = (1-a_0)f_a = (1-0.998)\times1.275\times10^9 = 2.55\ \text{MHz} .$$

Note that f_b can be expressed by

$$f_b = (1-a_0)f_a = (1-a_0)/a_0 \times f_T = \frac{1}{b_0}f_T .$$

24.   Neglect the time delays of emitter and collector, the base transit time is given by

$$t_B = \frac{1}{2\pi f_T} = \frac{1}{2\pi \times 5 \times 10^9} = 31.83 \times 10^{-12}\,\text{s} .$$

From Eq. 44, W can be expressed by

$$W = \sqrt{2D_p t_B} .$$

Therefore,

$$\begin{aligned} W &= \sqrt{2 \times 10 \times 31.83 \times 10^{-12}} \\ &= 2.52 \times 10^{-5}\,\text{cm} \\ &= 0.252\,\mu\text{m} . \end{aligned}$$

The neutral base width should be 0.252 μm.

25. $\Delta E_g = 9.8\% \times 1.12 \cong 110$ meV.

$$b_o \sim \exp\left(\frac{\Delta E_g}{kT} \right)$$

$$\therefore \frac{b_o(100\,°\text{C})}{b_o(0\,°\text{C})} = \exp\left(\frac{110\ \text{meV}}{373\ k} - \frac{110\ \text{meV}}{273\ k} \right) = 0.29.$$

26. $$\frac{b_0(\text{HBT})}{b_0(\text{BJT})} = \exp\left(\frac{E_{gE} - E_{gB}}{kT} \right) = \exp\left[\frac{E_{gE}(x) - 1.424}{0.0259} \right]$$

where

$$\begin{aligned} E_{gE}(x) &= 1.424 + 1.247x, \quad x \le 0.45 \\ &= 1.9 + 0.125x + 0.143x, \quad 0.45 < x \le 1. \end{aligned}$$

The plot of $b_0(\text{HBT})\big/ b_0\ (\text{BJT})$ is shown in the following graph.
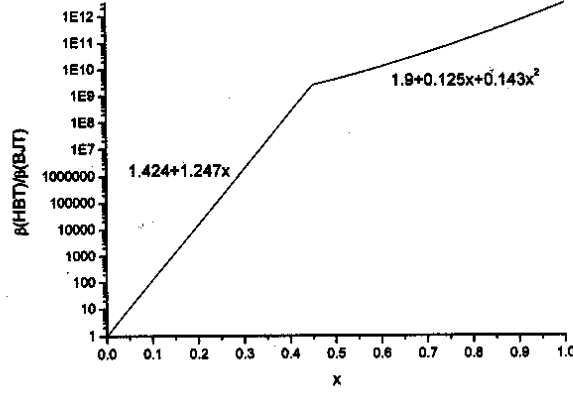
Note that $b_0(\text{HBT})$ increases exponentially when x increases.

27.   The impurity concentration of the $n1$ region is $10^{14}\,\text{cm}^{-3}$. The avalanche breakdown voltage (for $W > W_m$) is larger than 1500 V ($W_m > 100\,\mu\text{m}$). For a reverse block voltage of 120 V, we can choose a width such that punch-through occurs, i.e.,

$$V_{PT} = \frac{qN_D W^2}{2\varepsilon_s}\ .$$

Thus,

$$W = \left(\frac{2\varepsilon_s V_{PT}}{qN_D}\right) = 3.96 \times 10^{-3}\ \text{cm}\ .$$

When switching occurs,

$$a_1 + a_2 \cong 1\ .$$

That is,

$$a_1 = 0.5\sqrt{\frac{L_p}{W}}\ln\left(\frac{J}{J_0}\right)$$
$$= 1 - 0.4 = 0.6$$

$$\ln\left(\frac{J}{J_0}\right) = \frac{0.6}{0.5}\sqrt{\frac{25 \times 10^{-4}}{39.6 \times 10^{-4}}}$$
$$= 1.51\ .$$

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Therefore,

$$J \cong 4.5 J_0 = 2.25 \times 10^{-5} \ \text{A/cm}^2$$

$$\text{Area} = \frac{I_s}{J} = \frac{1 \times 10^{-3}}{2.25 \times 10^{-5}} = 44.4 \ \text{cm}^2 \ .$$

28. In the $n1 - p2 - n2$ transistor, the base drive current required to maintain current conduction is

$$I_1 = (1 - \mathbf{a}_2) I_K \ .$$

In addition, the base drive current available to the $n1 - p2 - n2$ transistor with a reverse gate current is

$$I_2 = \mathbf{a}_1 I_A - I_g \ .$$

Therefore, when use a reverse gate current, the condition to obtain turn-off of the thyristor is given by

$$I_2 < I_1$$

or $\mathbf{a}_1 I_A - I_g < (1 - \mathbf{a}_2) I_K$.

Using Kirchhoff's law, we have

$$I_K = I_A - I_g \ .$$

Thus, the condition for turn-on of the thyristor is

$$I_g > \frac{\mathbf{a}_1 + \mathbf{a}_2 - 1}{\mathbf{a}_2} I_A \ .$$

Note that if we define the ratio of I_A to I_g as turn-off gain, then the maximum turn-off gain $\mathbf{b}_{max}$ is

$$\mathbf{b}_{max} = \frac{\mathbf{a}_2}{\mathbf{a}_1 + \mathbf{a}_2 - 1} \ .$$

1.

$V_G = V_T$

E_C
E_F
E_i
E_V

$\boldsymbol{\psi}_S$
$= 2\boldsymbol{\psi}_B$

METAL   OXIDE   *n*-TYPE SEMICONDUCTOR

2.

E_F
E_C
E_i
E_V

E_C
E_i
E_F
E_V

n^+ POLYSILICON   OXIDE p-TYPE SEMICONDUCTOR

3.

$V_G = V_{FB} = \boldsymbol{f}_{ms}$

E_C
E_i
E_F
E_V

$\boldsymbol{f}_{ms}$

n^+ POLYSILICON   OXIDE p-TYPE SEMICONDUCTOR

4.

(a)

(b)

(c)
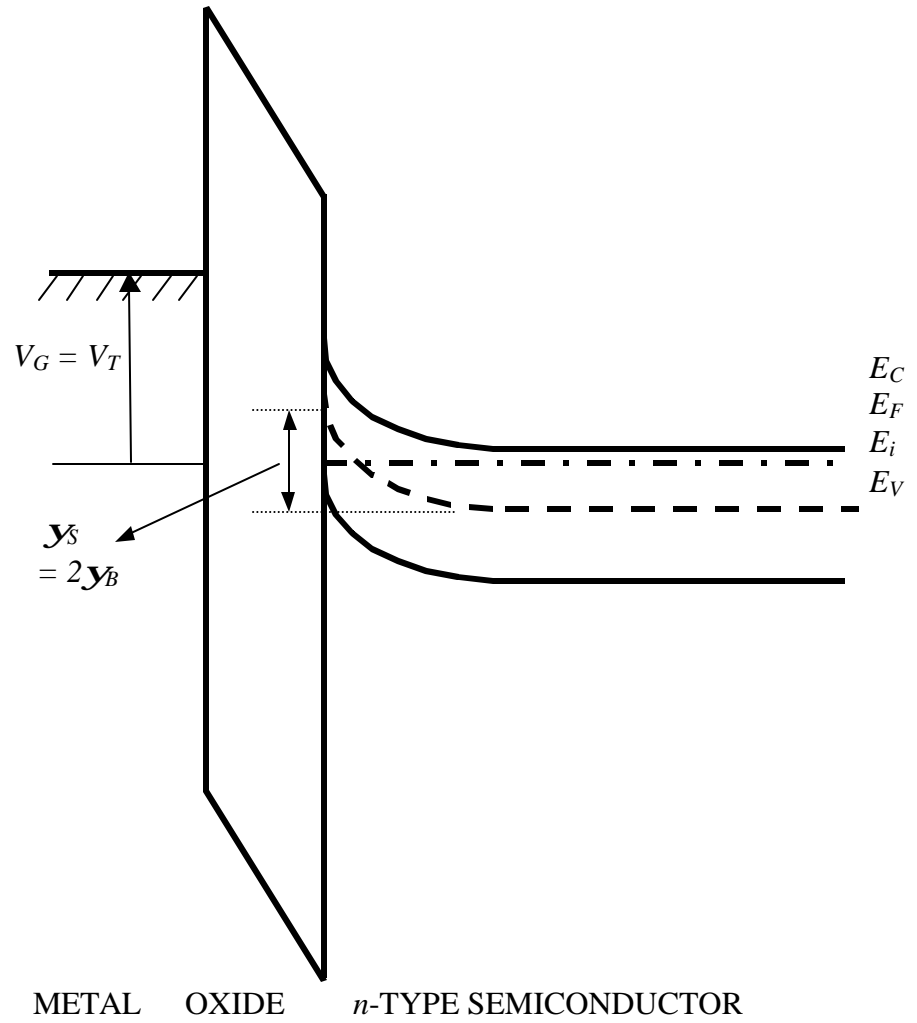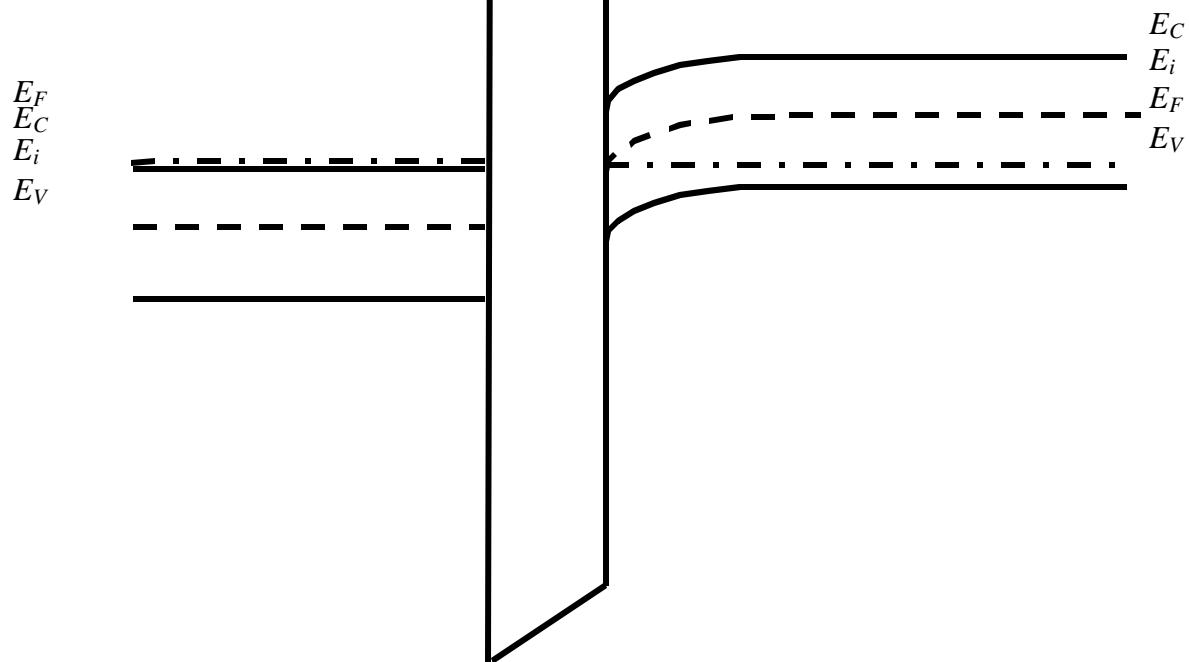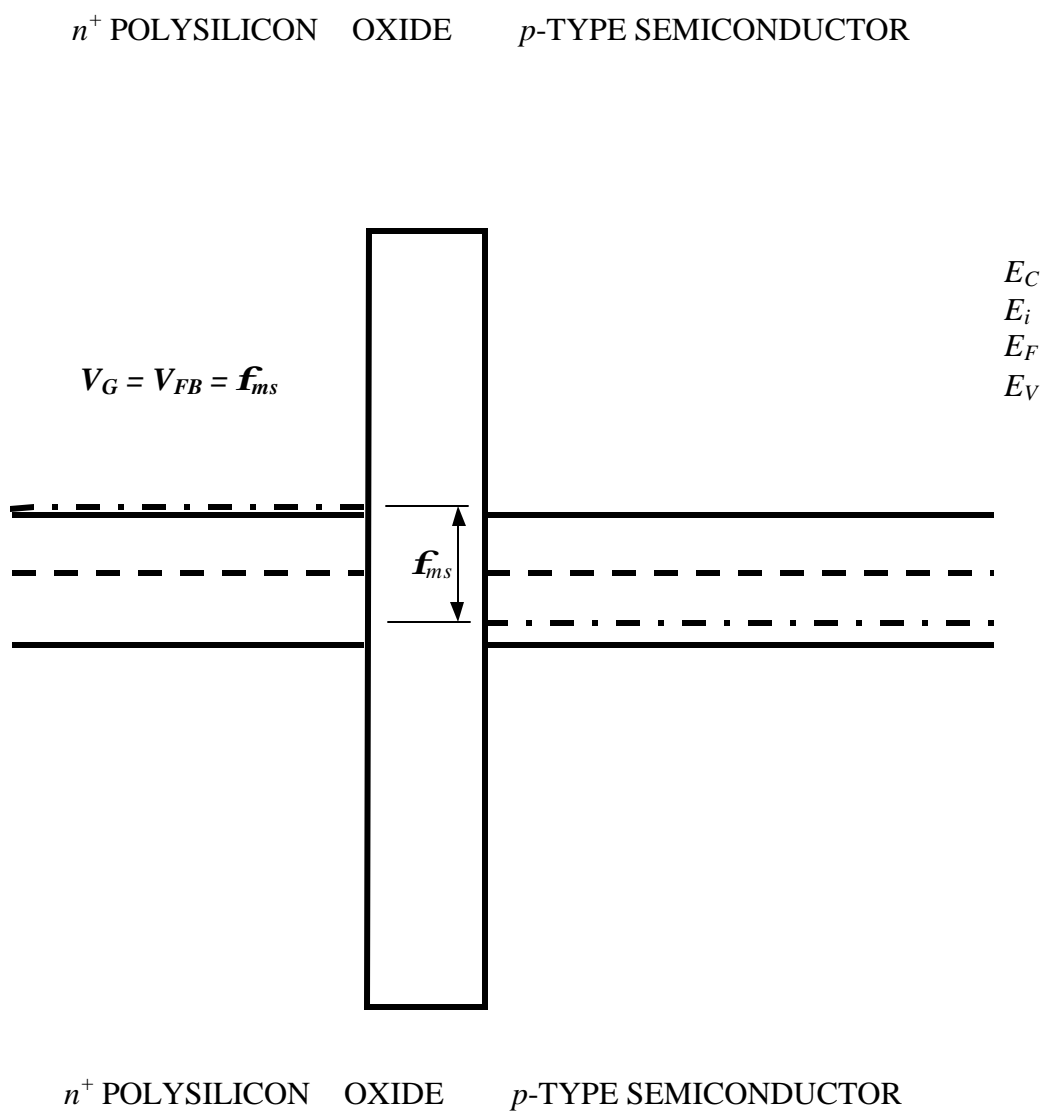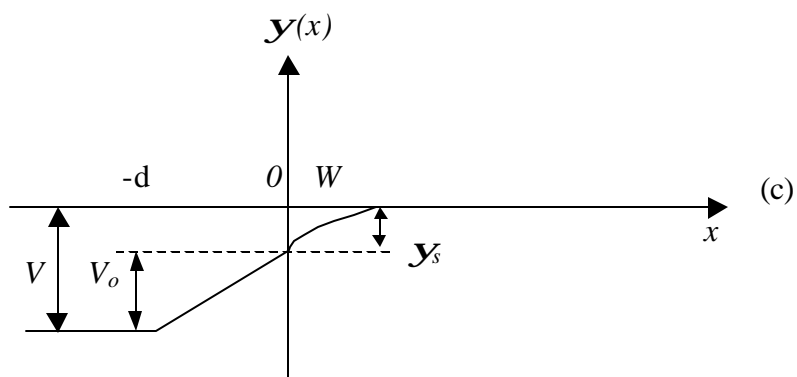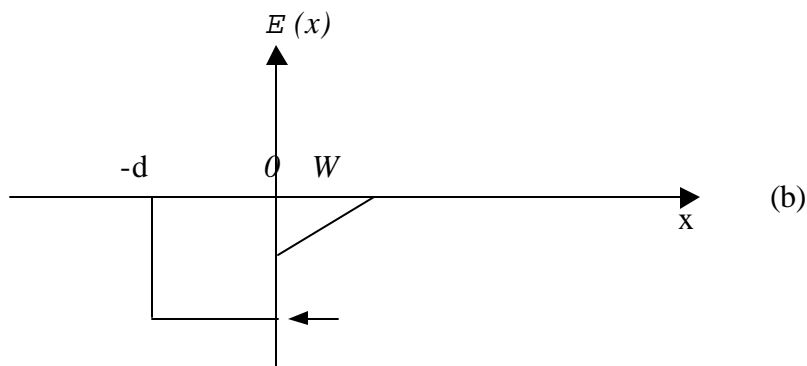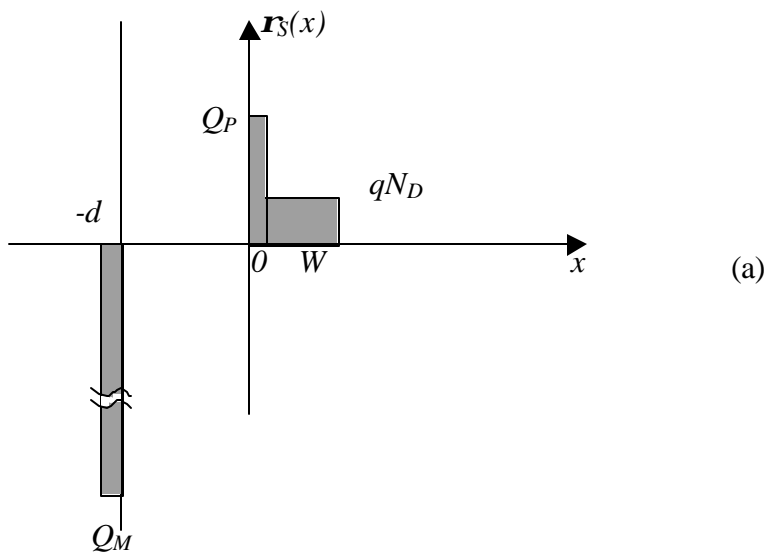
5. $W_m = 2\sqrt{\dfrac{e_s kT \ln\left(\dfrac{N_A}{n_i}\right)}{q^2 N_A}}$

$= 2\sqrt{\dfrac{11.9 \times 8.85 \times 10^{-14} \times 0.026 \ln\left(\dfrac{5 \times 10^{16}}{9.65 \times 10^9}\right)}{1.6 \times 10^{-19} \times 5 \times 10^{16}}}$

$= 1.5 \times 10^{-5}$ cm $= 0.15\ \boldsymbol{m}$m.

6. $C_{\min} = \dfrac{e_{ox}}{d + (e_{ox}/e_s)W_m}$

$W_m = 0.15\,\boldsymbol{m}$m   From Prob. 5

$\therefore C_{min} = \dfrac{3.9 \times 8.85 \times 10^{-14}}{8 \times 10^{-7} + \dfrac{3.9}{11.9} \times 1.5 \times 10^{-5}} = 6.03 \times 10^{-8}$ F/cm^2 .

7. $\boldsymbol{y}_B = \dfrac{kT}{q}\ln\dfrac{N_A}{n_i} = 0.026\ln\dfrac{10^{17}}{9.65 \times 10^9} = 0.42$ V

$E_s = \dfrac{qN_A W}{e_s}$ at intrinsic $\boldsymbol{y}_s = \boldsymbol{y}_B$

$W = \sqrt{\dfrac{2e_s\boldsymbol{y}_s}{qN_A}} = \sqrt{\dfrac{2 \times 11.9 \times 8.85 \times 10^{-14} \times 0.42}{1.6 \times 10^{-19} \times 10^{17}}} = 0.74 \times 10^{-5}$ cm

$E_s = \dfrac{1.6 \times 10^{-19} \times 10^{17} \times 0.74 \times 10^{-5}}{11.9 \times 8.85 \times 10^{-14}} = 1.11 \times 10^5$ V/cm

$E_o = E_s \dfrac{e_s}{e_{ox}} = 1.11 \times 10^5 \times \dfrac{11.9}{3.9} = 3.38 \times 10^5$ V/cm

$V = V_o + \boldsymbol{y}_s = E_o d + \boldsymbol{y}_s = \left(3.38 \times 10^5 \times 5 \times 10^{-7}\right) + 0.42.$
$= 0.59$ V.

8. At the onset of strong inversion, $\boldsymbol{y}_s = 2\boldsymbol{y}_B \Rightarrow V_G = V_T$

thus, $V_G = \dfrac{qN_A W_m}{C_o} + 2\boldsymbol{y}_B$

From Prob. 5, $W_m = 0.15£gm$, $\boldsymbol{y}_B = 0.026\ln\left(\dfrac{5 \times 10^{16}}{9.65 \times 10^9}\right) = 0.4$ V

$C_o = \dfrac{3.9 \times 8.85 \times 10^{-14}}{10^{-6}} = 3.45 \times 10^{-7}$ F/cm^2

$\therefore V_G = \dfrac{1.6 \times 10^{-19} \times 5 \times 10^{16} \times 1.5 \times 10^{-5}}{3.45 \times 10^{-7}} + 0.8 = 0.35 + 0.8 = 1.15$V.

9. $Q_{ot} = \dfrac{1}{d}\int_0^{10^{-6}} yq(10^{17})dy = \dfrac{1.6\times10^{-19}}{10^{-6}}[\dfrac{1}{2}\times10^{17}\times(10^{-6})^2]$

$$= 8\times10^{-9}\ \text{C/cm}^2$$

$$\Delta V_{FB} = \dfrac{Q_{ot}}{C_o} = \dfrac{8\times10^{-9}}{3.45\times10^{-7}} = 2.32\times10^{-2}\ \text{V}.$$

10. $Q_{ot} = \dfrac{1}{d}\int_0^{d} y\boldsymbol{r}_{ot}(y)dy$

$$\boldsymbol{r}_{ot} = q\times5\times10^{11}\boldsymbol{d}(x), \quad \text{where } \boldsymbol{d}(x) = \begin{cases} \infty, & y = 5\times10^{-7} \\ 0, & y \neq 5\times10^{-7} \end{cases}$$

$$\therefore Q_{ot} = \dfrac{1}{10^{-6}}\times5\times10^{11}\times5\times10^{-7}\times1.6\times10^{-19}$$

$$= 4\times10^{-8}\ \text{C/cm}^2$$

$$\therefore \Delta V_{FB} = \dfrac{Q_{ot}}{C_o} = \dfrac{4\times10^{-8}}{3.45\times10^{-7}} = 0.12\ \text{V}$$

$$= \dfrac{1}{10^{-6}}\times1.6\times10^{-19}\times\dfrac{1}{3}\times5\times10^{23}(10^{-6})^3.$$

11. $Q_{ot} = \dfrac{1}{d}\int_0^{10^{-6}} y(q\times5\times10^{23}\times y)dy$

$$= 2.67\times10^{-8}\ \text{C/cm}^2$$

$$\therefore \Delta V_{FB} = \dfrac{Q_{ot}}{C_o} = \dfrac{2.67\times10^{-8}}{3.45\times10^{-7}} = 7.74\times10^{-2}\ \text{V}.$$

12. $\Delta V_{FB} = \dfrac{Q_{ot}}{C_o} = \dfrac{q}{C_o}\times\dfrac{d}{d}\times N_m$, where N_m is the area density of Q_m.

$$\Rightarrow N_m = \dfrac{\Delta V_{FB}\times C_o}{q} = \dfrac{0.3\times3.45\times10^{-7}}{1.6\times10^{-19}} = 6.47\times10^{11}\ \text{cm}^{-2}.$$

13. Since $V_D \ll (V_G - V_T)$, the first term in Eq. 33 can be approximated as

$$\dfrac{Z}{L}\boldsymbol{m}_n C_o(V_G - 2\boldsymbol{y}_B)V_D.$$

Performing Taylor's expansion on the 2$^{\text{nd}}$ term in Eq. 33, we obtain

$$(V_D + 2\boldsymbol{y}_B)^{3/2} - (2\boldsymbol{y}_B)^{3/2} \cong (2\boldsymbol{y}_B)^{3/2} + \dfrac{3}{2}(2\boldsymbol{y}_B)^{1/2}V_D - (2\boldsymbol{y}_B)^{3/2} = \dfrac{3}{2}(2\boldsymbol{y}_B)^{1/2}V_D$$

Equation 33 can now be re-written as

$$I_D \cong \frac{Z}{L} m_n C_o \left[(V_G - 2y_B) V_D - \frac{2}{3} \frac{\sqrt{2e_s q N_A}}{C_o} \times \frac{3}{2} \sqrt{2y_B} V_D \right]$$

$$\cong \frac{Z}{L} m_n C_o \left\{ V_G - \left[2y_B + \frac{\sqrt{2e_s q N_A (2y_B)}}{C_o} \right] \right\} V_D$$

$$\cong (\frac{Z}{L}) m_n C_o (V_G - V_T) V_D$$

where $V_T = 2y_B + \dfrac{\sqrt{2e_s q N_A (2y_B)}}{C_o}$.

14. When the drain and gate are connected together, $V_G = V_D$ and the MOSFET is operated in saturation $(V_D > V_{Dsat})$. I_D can be obtained by substituting $V_D = V_{Dsat}$ in Eq. 33

$$I_D \big|_{V_D = V_G} = \frac{Z}{L} m_n C_o \left\{ \left(\frac{V_{Dsat}}{2} - 2y_B \right) V_{Dsat} - \frac{2}{3} \left(\frac{\sqrt{2e_s q N_A}}{C_o} \right) \left[(V_{Dsat} + 2y_B)^{3/2} - 2(y_B)^{3/2} \right] \right\}$$

where V_{Dsat} is given by Eq. 38. Inserting the condition $Q_n(y = L) = 0$ into Eq. 27 yields

$$V_{Dsat} = \frac{1}{C_o} \sqrt{2e_s q N_A (V_{Dsat} + 2y_B)} + 2y_B - V_G = 0$$

For $V_{Dsat} << 2j_B$, the above equation reduces to

$$V_G = V_D = \frac{\sqrt{2e_s q N_A (2y_B)}}{C_o} + 2y_B = V_T \ .$$

Therefore a linear extrapolation from the low current region to $I_D = 0$ will yield the threshold voltage value.

15. $$y = \frac{kT}{q} \ln(\frac{N_A}{n_i}) = 0.026 \ln \left[\frac{5 \times 10^{16}}{9.65 \times 10^{-9}} \right] = 0.40 \text{ V}$$

$$K \equiv \frac{\sqrt{e_s q N_A}}{C_o} = \frac{\sqrt{11.9 \times 8.85 \times 10^{-14} \times 1.6 \times 10^{-19} \times 5 \times 10^{-16}}}{3.45 \times 10^{-7}}$$

$$= 0.27$$

$$\therefore V_{Dsat} \cong V_G - 2y_B + K^2 \left(1 - \sqrt{1 + \frac{2V_G}{K^2}} \right)$$

$$= 5 - 0.8 + (0.27)^2 [1 - \sqrt{1 + \frac{10}{(0.27)^2}}]$$

$$= 3.42 \text{ V}$$

$$I_{Dsat} = \frac{Z m_n C_o}{2L} (V_G - V_T) = \frac{10 \times 800 \times 3.45 \times 10^{-7}}{2 \times 1} (5 - 0.7)^2$$

$$= 2.55 \times 10^{-2} \text{ A.}$$

16. The device is operated in linear region, since $V_D = 0.1$ V $< (V_G - V_T) = 0.5$ V

Therefore, $g_d = \dfrac{\partial I_D}{\partial V_D}\Big|_{V_G = const.} = \dfrac{Z}{L}\pmb{\mu}_n C_o (V_G - V_T)$

$$= \dfrac{5}{0.25} \times 500 \times 3.45 \times 10^{-7} \times 0.5$$

$$= 1.72 \times 10^{-3} \text{ S.}$$

17. $g_m = \dfrac{\partial I_D}{\partial V_G}\Big|_{V_D = const.} = \dfrac{Z}{L}\pmb{\mu}_n C_o V_D$

$$= \dfrac{5}{0.25} \times 500 \times 3.45 \times 10^{-7} \times 0.1$$

$$= 3.45 \times 10^{-4} \text{ S.}$$

18. $V_T = V_{FB} + 2\pmb{\psi}_B + \dfrac{2\sqrt{\pmb{\varepsilon}_s q N_A \pmb{\psi}_B}}{C_o}$, $\pmb{\psi}_B = 0.026\ln(\dfrac{10^{17}}{9.65 \times 10^9})$

$V_{FB} = \pmb{\phi}_{ms} - \dfrac{Q_f}{C_o} = -\dfrac{E_g}{2} - \pmb{\psi}_B - \dfrac{1.6 \times 10^{-19} \times 5 \times 10^{10}}{3.45 \times 10^{-7}}$

$$= -0.56 - 0.42 - 0.02 = -1 \text{ V}$$

$\therefore V_T = -1 + 0.84 + \dfrac{2\sqrt{11.9 \times 8.85 \times 10^{-14} \times 10^{17} \times 0.42 \times 1.6 \times 10^{-19}}}{3.45 \times 10^{-7}}$

$$= -1 + 0.84 + 0.49$$

$$= 0.33 \text{ V}$$

($\pmb{\phi}_{ms}$ can also be obtained from Fig. 8 to be -0.98 V).

19. $0.7 = 0.33 + \dfrac{q F_B}{3.45 \times 10^{-7}}$

$F_B = \dfrac{0.37 \times 3.45 \times 10^{-7}}{1.6 \times 10^{-19}} = 8 \times 10^{11} \text{ cm}^{-2}$.

20. $\pmb{\phi}_{ms} = -\dfrac{E_g}{2} + \pmb{\psi}_B = -0.56 + 0.42 = -0.14$ V

$V_T = \pmb{\phi}_{ms} - \dfrac{Q_f}{C_o} - 2\pmb{\psi}_B - \dfrac{2\sqrt{\pmb{\varepsilon}_s q N_D \pmb{\psi}_B}}{C_o}$

$$= -0.14 - 0.02 - 0.84 - \dfrac{2\sqrt{11.9 \times 8.85 \times 10^{-14} \times 1.6 \times 10^{-19} \times 10^{17} \times 0.42}}{3.45 \times 10^{-7}}$$

$$= -1.49 \text{ V.}$$

21. $$-0.7 = -1.49 + \frac{qF_B}{3.45 \times 10^{-7}}$$

$$F_B = \frac{0.79 \times 3.45 \times 10^{-7}}{1.6 \times 10^{-19}} = 1.7 \times 10^{12} \text{ cm}^{-2}.$$

22. The bandgap in degenerately doped Si is around 1eV due to bandgap-narrowing effect. Therefore,

$$\boldsymbol{f}_{ms} = -0.14 + 1 = 0.86 \text{ V}$$
$$\therefore V_T = 0.86 - 0.02 - 0.84 - 0.49 = -0.49 \text{ V}.$$

23. $y_B = 0.026 \ln\left(\dfrac{10^{17}}{9.65\times10^9}\right) = 0.42$ V

$$V_T = f_{ms} - \frac{qQ_f}{C_o} + 2y_B + \frac{2\sqrt{e_s qN_A y_B}}{C_o}$$

$$= -0.98 - \frac{1.6\times10^{-19}\times10^{11}}{C_o} + 0.84 + \frac{2\sqrt{11.9\times8.85\times10^{-14}\times10^{17}\times0.42\times1.6\times10^{-19}}}{C_o}$$

$$= -0.14 + \frac{15.2\times10^{-8}}{C_o}$$

$$C_o = \frac{3.9\times8.85\times10^{-14}}{d} = \frac{3.45\times10^{-13}}{d}$$

$$V_T > 20 \Rightarrow \frac{d\times15.2\times10^{-8}}{3.45\times10^{-13}} > 20.14$$

$$\therefore d > 4.57\times10^{-5}\,\text{cm} = 0.457\ \mu\text{m}.$$

24. $V_T = 0.5$ V at $I_d = 0.1 \pounds\text{gA}$

$$\text{Subthreshold swing} = \left(\frac{\log I_D\big|_{V_G=V_T} - \log I_D\big|_{V_G=0}}{V_T - 0}\right)^{-1}$$

$$0.1 = \frac{0.5}{-7 - \log I_D\big|_{V_G=0}}$$

$$\log I_D\big|_{V_G=0} = -12 \quad \therefore I_D\big|_{V_G=0} = 1\times10^{-12}\ \text{A}.$$

25.

$$\Delta V_T = \frac{\sqrt{2qe_s N_A}}{C_o}\left(\sqrt{2y_B + V_{BS}} - \sqrt{2y_B}\right)$$

$$y_B = 0.026 \ln\left(\frac{10^{17}}{9.65\times10^9}\right) = 0.42\ \text{V}$$

$$C_o = \frac{3.9\times8.85\times10^{-14}}{5\times10^{-7}} 6.9\times10^{-7}\,\text{F/cm}^2$$

$\Delta V_T = 0.1$ V if we want to reduce I_D at $V_G = 0$ by one order of magnitude,

since the subthreshold swing is 100 mV/decade.

$$0.1 = \frac{\sqrt{2\times1.6\times10^{-19}\times4.9\times8.85\times10^{-14}\times10^{17}}}{6.9\times10^{-7}}\left(\sqrt{0.84 + V_{BS}} - \sqrt{0.84}\right)$$

$$\therefore V_{BS} = 0.83\ \text{V}.$$

26. Scaling factor $k = 10$

$$\text{Switching energy} = \frac{1}{2}(C\cdot A)V^2$$

$$C' = \frac{\varepsilon_{ox}}{d} = kC$$

$$A' = \frac{A}{k^2}$$

$$V' = \frac{V}{k}$$

∴ scaling factor for switching energy : $k \cdot \dfrac{1}{k^2} \cdot \dfrac{1}{k^2} = \dfrac{1}{k^3} = \dfrac{1}{1000}$

A reduction of one thousand times.

27. From Fig. 24 we have

$$(r_j + \Delta)^2 = (r_j + W_m)^2 - W_m^{\,2}$$

$$\therefore \Delta^2 + 2\Delta r_j - 2W_m r_j = 0$$

$$\Delta = -r_j + \sqrt{r_j + 2W_m r_j}$$

$$L' = L - 2\Delta$$

$$\frac{L + L'}{2L} = \frac{2L - 2\Delta}{2L} = 1 - \frac{\Delta}{L} = 1 - \frac{r_j}{L}\left(\sqrt{1 + \frac{2W_m}{r_j}} - 1\right)$$

From Eq. 17 we have

$$\Delta V_T = \frac{(\text{space charge in the trapezaid al region} - \text{space charge in the rectangula r region})}{C_o}$$

$$= \frac{qN_A W_m}{C_o}\left(\frac{L + L'}{2L}\right) - \frac{qN_A W_m}{C_o} = -\frac{qN_A W_m r_j}{C_o L}\left(\sqrt{1 + \frac{2W}{r_j}} - 1\right)$$

28. Pros:
    1. Higher operation speed.
    2. High device density

   Cons:
    1. More complicated fabrication flow.
    2. High manufacturing cost.

29. The maximum width of the surface depletion region for bulk MOS

$$W_m = 2\sqrt{\frac{\varepsilon_s kT\ln(N_A / n_i)}{q^2 N_A}}$$

$$= 2\sqrt{\frac{11.9 \times 8.85 \times 10^{-14} \times 0.026 \times \ln(5 \times 10^{17} / 9.65 \times 10^9)}{1.6 \times 10^{-19} \times 5 \times 10^{17}}}$$

$$= 4.9 \times 10^{-6} \text{ cm}$$

$$= 49 \text{ nm}$$

For FD-SOI, $d_{si} \le W_m = 49 \text{ nm}$.

30. $V_T = V_{FB} + 2\mathbf{y}_B + \dfrac{qN_A d_{si}}{C_o}$

$V_{FB} = \mathbf{f}_{ms} = -\dfrac{E_g}{2} - \dfrac{kT}{q}\ln(\dfrac{N_A}{n_i}) = -\dfrac{1.12}{2} - 0.026\ln\left(\dfrac{5\times10^{17}}{9.65\times10^9}\right) = -1.02 \text{ V}$

$2\mathbf{y}_B = 2\times\dfrac{kT}{q}\ln(\dfrac{N_A}{n_i}) = 0.92 \text{ V}$

$C_o = \dfrac{3.9\times8.85\times10^{-14}}{4\times10^{-7}} = 8.63\times10^{-7} \text{ F/cm}^2$

$\therefore V_T = -1.02 + 0.92 + \dfrac{1.6\times10^{-19}\times5\times10^{17}\times3\times10^{-6}}{8.63\times10^{-7}}$

$= -0.1 + 0.28$

$= 0.18 \text{ V}.$

31. $\Delta V_T = \dfrac{qN_A \Delta d_{si}}{C_O}$

$= \dfrac{1.6\times10^{-19}\times5\times10^{17}\times5\times10^{-7}}{8.63\times10^{-7}}$

$= 46 \text{ mV}$

Thus, the range of V_T is from (0.18-0.046)= 0.134 V to (0.18+0.046) = 0.226 V.

32. The planar capacitor

$C = A\mathbf{e}_{ox}\big/d = \dfrac{(1\times10^{-4})^2\, 3.9\times8.86\times10^{-14}}{1\times10^{-6}} = 3.45\times10^{-15} \text{ F}$

For the trench capacitor

$A = 4\times7 \ \mu m^2 + 1 \ \mu m^2 = 29 \ \mu m^2$

$C = 29 \times3.45 \times10^{-15} \text{ F} = 100 \times 10^{-15} \text{ F}.$

33. $I = C\dfrac{dV}{d} = 5\times10^{-14} \times\dfrac{2.5}{4\times10^{-3}} = 3.1\times10^{-11} \text{ A}$

34. $g_o = \dfrac{Z}{L}\mathbf{m}_p C_i (V_o - V_T)$

$4\times 10^{-5} = A(-5 - V_T) \quad \therefore V_T = 7V$

$1\times 10^{-5} = A(-5+2)$

$\Delta V = 7 - (-2) = 9 \text{ V}.$

35. $V_T = V_{FB} + 2\mathbf{y}_B + \dfrac{\sqrt{\mathbf{e}_s qN_A \mathbf{y}_B}}{C_0}$

$$C_0 = 3.9 \times \frac{8.854 \times 10^{-14}}{10^{-5}} = 3.45 \times 10^{-8} \ \text{F/cm}^2$$

$$\boldsymbol{\psi}_B = 0.026 \ln\left(\frac{10^{17}}{9.65 \times 10^9}\right) = 0.42 \ \text{V}$$

$$V_{FB} = \boldsymbol{\phi}_{ms} - \frac{Q_f}{2} = -\frac{E_g}{2} - \boldsymbol{\psi}_B - 0$$

$$= -0.56 - 0.42 = -0.98 \ \text{V}$$

$$\therefore V_T = -0.98 + 0.42 + \frac{2\sqrt{11.9 \times 8.85 \times 10^{-14} \times 10^{17} \times 0.42 \times 1.6 \times 10^{-19}}}{3.45 \times 10^{-8}}$$

$$= -0.98 + 0.42 + 4.9$$

$$= 4.34 \ \text{V}$$

$$\Delta V_{FB} = -\frac{Q_f}{C_0} = -\frac{5 \times 10^{11} \times 1.6 \times 10^{-19}}{3.45 \times 10^{-8}}$$

$$= -2.32 \ \text{V}$$

$$V_T = 4.34 - 2.32$$

$$= 2.02 \ \text{V}.$$

CHAPTER 7

1. From Eq.1, the theoretical barrier height is

$$\boldsymbol{f}_{Bn} = \boldsymbol{f}_{m} - \boldsymbol{c} = 4.55 - 4.01 = 0.54 \text{ eV}$$

We can calculate V_n as

$$V_n = \frac{kT}{q} \ln \frac{N_C}{N_D} = 0.0259 \ln(\frac{2.86 \times 10^{19}}{2 \times 10^{16}}) = 0.188 \text{ V}$$

Therefore, the built-in potential is

$$V_{bi} = \boldsymbol{f}_{Bn} - V_n = 0.54 - 0.188 = 0.352 \text{ V}.$$

2. (a) From Eq.11

$$\frac{d(1/C^2)}{dV} = \frac{(6.2 - 4.6) \times 10^{14}}{-2 - 0} = -2.3 \times 10^{14} \ (\text{cm}^2/\text{F})^2/\text{V}$$

$$N_D = \frac{2}{q\boldsymbol{e}_s} \left[\frac{-1}{d(1/C^2)/dV} \right] = 4.7 \times 10^{16} \text{ cm}^{-3}$$

$$V_n = \frac{kT}{q} \ln \frac{N_C}{N_D} = 0.0259 \ln \left(\frac{4.7 \times 10^{17}}{4.7 \times 10^{16}} \right) = 0.06 \text{ V}$$

From Fig.6, the intercept of the GaAs contact is the built-in potential V_{bi},

which is equal to 0.7 V. Then, the barrier height is

$$\boldsymbol{f}_{Bn} = V_{bi} + V_n = 0.76 \text{ V}$$

(b) $J_s = 5 \times 10^{17} \text{ A/cm}^2$

$A^* = 8 \text{ A/K}^2\text{-cm}^2$ for n– type GaAs

$$J_s = A^* T^2 e^{-q\boldsymbol{f}_{Bn}/kT}$$

$$\boldsymbol{f}_{Bn} = \frac{kT}{q} \ln(\frac{A^* T^2}{J_s}) = 0.0259 \ln \left[\frac{8 \times (300)^2}{5 \times 10^{-7}} \right] = 0.72 \text{ eV}$$

The barrier height from capacitance is 0.04 V or 5% larger.

(c) For $V = -1$ V

$$W = \sqrt{\frac{2e_s(V_{bi} + V_R)}{qN_D}} = \sqrt{\frac{2 \times 1.09 \times 10^{-12}(0.7 + 1)}{1.6 \times 10^{-19} \times 4.7 \times 10^{16}}}$$

$$= 2.22 \times 10^{-5} \text{ cm} = 0.222 \text{ } \textbf{m}\text{m}$$

$$E_m = \frac{qN_D W}{e_s} = 1.43 \times 10^5 \text{ V/cm}$$

$$C = \frac{e_s}{W} = 5.22 \times 10^{-8} \text{ F/cm}^2.$$

3. The barrier height is

$$\boldsymbol{f}_{Bn} = \boldsymbol{f}_m - \boldsymbol{c} = 4.65 - 4.01 = 0.64 \text{ V}$$

$$V_n = \frac{kT}{q} \ln \frac{N_C}{N_D} = 0.0259 \times \ln\left(\frac{2.86 \times 10^{19}}{3 \times 10^{16}}\right) = 0.177 \text{ V}$$

The built-in potential is

$$V_{bi} = \boldsymbol{f}_{Bn} - V_n = 0.64 - 0.177 = 0.463 \text{ V}$$

The depletion width is

$$W = \sqrt{\frac{2e_s(V_{bi} - V)}{qN_D}} = \sqrt{\frac{2 \times 11.9 \times 8.85 \times 10^{-14}}{1.6 \times 10^{-19} \times 3 \times 10^{16}} \times 0.463} = 0.142 \text{ ìm}$$

The maximum electric field is

$$\left|E_m\right| = \left|E(x = 0)\right| = \frac{qN_D}{e_s}W = \frac{(1.6 \times 10^{19}) \times (3 \times 10^{16}) \times (1.42 \times 10^{-5})}{11.9 \times (8.85 \times 10^{-14})} = 6.54 \times 10^4 \text{ V/cm.}$$

4. The unit of C needs to be changed from μF to F/cm², so

$$1/C^2 = 1.74 \times 10^{15} - 2.12 \times 10^{15} \text{ } V_a \text{ } (\text{cm}^2/\text{F})^2$$

Therefore, we obtain the built-in potential at $1/C^2 = 0$

$$V_{bi} = \frac{1.57 \times 10^{15}}{2.12 \times 10^{15}} = 0.74 \text{ V}$$

From the given relationship between C and V_a, we obtain

$$\frac{d\left(\dfrac{1}{C^2}\right)}{dV_a} = -2.12 \times 10^{15} \quad (\text{cm}^2/\text{F})^2/\text{V}$$

From Eq.11

$$N_D = \frac{2}{qe_s}\left[\frac{-1}{d(1/C^2)/dV}\right]$$

$$= \frac{2}{1.6 \times 10^{-19} \times 11.9 \times 8.85 \times 10^{-14}}\left(\frac{1}{2.12 \times 10^{15}}\right)$$

$$= 5.6 \times 10^{15} \ \text{cm}^{-3}$$

$$V_n = \frac{kT}{q}\ln\frac{N_C}{N_D} = 0.0259 \ln\left(\frac{2.86 \times 10^{19}}{5.6 \times 10^{16}}\right) = 0.161 \ \text{V}$$

We can obtain the barrier height

$$f_{Bn} = V_{bi} + V_n = 0.74 + 0.161 = 0.901 \ \text{V}.$$

5. The built-in potential is

$$V_{bi} = f_{Bn} - \frac{kT}{q}\ln\frac{N_C}{N_D}$$

$$= 0.8 - 0.0259 \ln\left(\frac{2.86 \times 10^{19}}{1.5 \times 10^{16}}\right)$$

$$= 0.8 - 0.195$$

$$= 0.605 \ \text{V}$$

Then, the work function is

$$f_m = f_{Bn} + c$$

$$= 0.8 + 4.01$$

$$= 4.81 \ \text{V}.$$

6. The saturation current density is

$$J_s = A^* T^2 \exp\left(\frac{-qf_{Bn}}{kT}\right)$$

$$= 110 \times (300)^2 \times \exp\left(\frac{-0.8}{0.0259}\right)$$

$$= 3.81 \times 10^{-7} \text{ A/cm}^2$$

The injected hole current density is

$$J_{po} = \frac{qD_p n_i^2}{L_p N_D} = \frac{1.6 \times 10^{-19} \times 12 \times (9.65 \times 10^9)^2}{1 \times 10^{-3} \times 1.5 \times 10^{16}} = 1.19 \times 10^{-11} \text{ A/cm}^2$$

$$\frac{\text{Hole current}}{\text{Electron current}} = \frac{J_{po}(e^{qV/kT} - 1)}{J_s(e^{qV/kT} - 1)}$$

$$= \frac{J_{po}}{J_s} = \frac{1.18 \times 10^{-11}}{3.81 \times 10^{-3}} = 3 \times 10^{-5}.$$

7. The difference between the conduction band and the Fermi level is given by

$$V_n = 0.0259 \ln\left(\frac{4.7 \times 10^{17}}{1 \times 10^{17}}\right) = 0.04 \text{ V}.$$

The built-in potential barrier is then

$$V_{bi} = 0.9 - 0.04 = 0.86 \text{ V}$$

For a depletion mode operation, V_T is negative. Therefore, From Eq.38a

$$V_T = 0.86 - V_P < 0$$

$$V_P = \frac{qa^2 N_D}{2e_s} = \frac{1.6 \times 10^{-19} a^2 \times 10^{17}}{2 \times 12.4 \times 8.85 \times 10^{-14}} > 0.86$$

$$\frac{1.6 \times 10^{-2}}{2.19 \times 10^{-12}} a^2 > 0.86$$

$$a > 1.08 \times 10^{-5} \text{ cm} = 0.108 \text{ ì m.}$$

8. From Eq.33 we obtain

$$g_m = \frac{I_P}{2V_p^2}\sqrt{\frac{V_P}{V_G + V_{bi}}}\,V_D$$

$$= \frac{Z\mu\varepsilon_s}{aL}\sqrt{\frac{V_P}{V_{bi}}}\,V_D$$

$$V_P = \frac{qN_D a^2}{2\varepsilon_s} = \frac{1.6\times10^{-19}\times7\times10^{16}\times(3\times10^{-5})^2}{2\times12.4\times8.85\times10^{-14}} = 4.62 \text{ V}$$

$$\therefore\, g_m = \frac{5\times10^{-4}\times4500\times12.4\times8.85\times10^{-14}}{0.3\times10^{-4}\times1.5\times10^{-4}}\sqrt{\frac{4.62}{0.84}}\times1$$

$$= 1.28\times10^{-3} \text{ S} = 1.28 \text{ mS.}$$

9. (a) The built-in voltage is

$$V_{bi} = \phi_{Bn} - V_n = 0.9 - 0.025\ln\left(\frac{4.7\times10^{17}}{10^{17}}\right) = 0.86 \text{ V}$$

At zero bias, the width of the depletion layer is

$$W = \sqrt{\frac{2\varepsilon_s V_{bi}}{qN_D}} = \sqrt{\frac{2\times1.09\times10^{-12}\times0.86}{1.6\times10^{-19}\times10^{17}}}$$

$$= 1.07\times10^{-5} \text{ cm}$$

$$= 0.107 \text{ μm}$$

Since W is smaller than 0.2 μm, it is a depletion-mode device.

(b) The pinch-off voltage is

$$V_P = \frac{qN_D a^2}{2\varepsilon_s} = \frac{1.6\times10^{-19}\times10^{17}(2\times10^{-5})}{2\times12.4\times8.85\times10^{-14}} = 2.92 \text{ V}$$

and the threshold voltage is

$$V_T = V_{bi} - V_P = 0.86 - 2.92 = -2.06 \text{ V.}$$

10. From Eq.31b, the pinch-off voltage is

$$V_P = \frac{qN_D a^2}{2e_s} = \frac{1.6 \times 10^{-19} \times 10^{17} (2 \times 10^{-5})}{2 \times 12.4 \times 8.85 \times 10^{-14}} = 0.364 \text{ V}$$

The threshold voltage is

$$V_T = V_{bi} - V_p = 0.8 - 0.364 = 0.436 \text{ V}$$

and the saturation current is given by Eq. 39

$$I_{Dsat} = \frac{Ze_s \mathbf{m}_n}{2aL}(V_G - V_T)^2$$

$$= \frac{50 \times 10^{-4} \times 12.4 \times 8.85 \times 10^{-14} \times 4500}{2 \times (0.5 \times 10^{-4}) \times (1 \times 10^{-4})}(0 - 0.436)^2 = 4.7 \times 10^{-4} \text{ A.}$$

11. $$0.85 - 0.0259 \ln\left(\frac{4.7 \times 10^{17}}{N_D}\right) - \frac{1.6 \times 10^{-19} \times N_D}{2 \times 12.4 \times 8.85 \times 10^{-14}} a^2 = 0$$

For $N_D = 4.7 \times 10^{16}$ cm^{-3}

$$a = \left(0.85 - 0.0259 \ln\frac{4.7 \times 10^{17}}{N_D}\right)^{\frac{1}{2}} \frac{(3.7 \times 10^3)}{\sqrt{N_D}}$$

$$= 1.52 \times 10^{-5} \text{ cm} = 0.152 \text{ μm}$$

For $N_D = 4.7 \times 10^{17}$ cm^{-3}

$$a = 0.496 \times 10^{-5} \text{ cm} = 0.0496 \text{ μm.}$$

12.   From Eq.48 the pinch-off voltage is

$$V_P = \mathbf{f}_{Bn} - \frac{\Delta E_C}{q} - V_T$$

$$= 0.89 - 0.23 - (-0.5)$$

$$= 0.62 \text{ V}$$

and then,

$$V_P = \frac{qN_D d_1^2}{2e_s} = \frac{1.6 \times 10^{-19} \times 3 \times 10^{18} \times d_1^2}{2 \times 12.3 \times 8.85 \times 10^{-14}} = 0.62 \text{ V}$$

$$d_l = 1.68 \times 10^{-6} \text{ cm}$$

$$d_l = 16.8 \text{ nm}$$

Therefore, this thickness of the doped AlGaAs layer is 16.8 nm.

13. The pinch-off voltage is

$$V_P = \frac{qN_D d_1^2}{2e_s} = \frac{1.6 \times 10^{-19} \times 10^{18} \times (50 \times 10^{-7})}{2 \times 12.3 \times 8.85 \times 10^{-4}} = 1.84 \text{ V}$$

The threshold voltage is

$$V_T = f_{Bn} - \frac{\Delta E_C}{q} - V_P$$

$$= 0.89 - 0.23 - 1.84$$

$$= -1.18 \text{ V}$$

When $n_s = 1.25 \times 10^{12} \text{ cm}^{-2}$, we obtain

$$n_s = \frac{12.3 \times 8.85 \times 10^{-14}}{1.6 \times 10^{-19} \times (50 + d_0 + 8) \times 10^{-7}} \times [0 - (-1.18)] = 1.25 \times 10^{12}$$

and then

$$d_0 + 58.5 = 64.3$$

$$d_0 = 5.8 \text{ nm}$$

The thickness of the undoped spacer is 5.8 nm.

14. The pinch-off voltage is

$$V_P = \frac{qN_D d_1^2}{2e_s} = \frac{1.6 \times 10^{-19} \times 5 \times 10^{17} \times (50 \times 10^{-7})^2}{2 \times 12.3 \times 8.85 \times 10^{-14}} = 1.84 \text{ V}$$

The barrier height is

$$f_{Bn} = V_T + \frac{\Delta E_C}{q} + V_P = -1.3 + 0.25 + 1.84 = 0.79 \text{ V}$$

The 2DEG concentration is

$$n_s = \frac{12.3 \times 8.85 \times 10^{-14}}{1.6 \times 10^{-19} \times (50 + 10 + 8) \times 10^{-7}} \times [0 - (-1.3)] = 1.29 \times 10^{12} \, cm^{-2}.$$

15.   The pinch-off voltage is

$$V_P = \frac{qN_D d_1^2}{2\boldsymbol{e}_s} = 1.5 = \frac{1.6 \times 10^{-19} \times 1 \times 10^{18} \times d_1^2}{2 \times 12.3 \times 8.85 \times 10^{-4}}$$

The thickness of the doped AlGaAs is

$$d_1 = \sqrt{\frac{1.5 \times 2 \times 12.3 \times 8.85 \times 10^{-14}}{1.6 \times 10^{-19} \times 10^{18}}} = 4.45 \times 10^{-8} \, cm = 44.5 \, nm$$

$$V_T = \boldsymbol{f}_{Bn} - \frac{\Delta E_C}{q} - V_P = 0.8 - 0.23 - 1.5 = -0.93 \, V.$$

16.   The pinch-off voltage is

$$V_P = \frac{qN_D}{2\boldsymbol{e}_s} d_1^2$$

$$= \frac{1.6 \times 10^{-19} \times 3 \times 10^{18}}{2 \times 12.3 \times 8.85 \times 10^{-14}} \left(35 \times 10^{-7}\right)^2 = 2.7 \, V$$

the threshold voltage is

$$V_T = \boldsymbol{f}_{Bn} - \frac{\Delta E_C}{q} - V_p$$

$$= 0.89 - 0.24 - 2.7$$

$$= -2.05 \, V$$

Therefore, the two-dimensional electron gas is

$$n_s = \frac{12.3 \times 8.85 \times 10^{-14}}{1.6 \times 10^{-19} \times (35 + 8) \times 10^{-7}} \times [0 - (-2.05)] = 3.2 \times 10^{12} \, cm^{-2}.$$

CHAPTER 8

1.
$$Z_0 = \sqrt{\frac{L}{C}}$$

$$L = C \cdot Z_0^2 = 2 \times 10^{-12} \times 75^2 = 11.25 \ \text{nH}.$$

2.
$$f_r = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$$

$$= \frac{c}{2} \sqrt{(10)^2 + 0 + (4)^2}$$

$$= \frac{3 \times 10^8 \ \text{m/s}}{2} \times \sqrt{116} = 1.616 \ \text{GHz}.$$

3.
$$V_{bi} \cong (E_g / q) + V_n + V_p = 1.42 + 0.03 + 0.03 = 1.48 \ \text{V}$$

$$W = \sqrt{\frac{2e_S}{q} \left(\frac{N_A + N_D}{N_A N_D}\right)(V_{bi} + V)}$$

$$= \sqrt{\frac{2 \times 1.16 \times 10^{-12}}{1.6 \times 10^{-19}} \left(\frac{10^{19} + 10^{19}}{10^{19} \times 10^{19}}\right)(1.48 - 0.25)}$$

$$= 1.89 \times 10^{-6} \text{cm} = 18.9 \ \text{nm}$$

$$C = \frac{e_S}{W} = \frac{1.16 \times 10^{-12}}{1.89 \times 10^{-6}} = 6.13 \times 10^{-7} \ \text{F/cm}^2.$$

4. $I = I_P \left(\frac{V}{V_P}\right) \exp\left(1 - \frac{V}{V_P}\right) + I_0 \exp\left(\frac{qV}{kT}\right)$

From Fig. 4, We note that the largest negative differential resistance occurs between $V_p < V < V_V$. The corresponding voltage can be obtained from the condition $d^2 I / d^2 V = 0$. By neglecting the second term in Eq. 5, we obtain

$$\frac{dI}{dV} \cong \left(\frac{I_P}{V_P} - \frac{I_P V}{V_P^2}\right)\exp\left(1 - \frac{V}{V_P}\right)$$

$$\frac{d^2I}{dV^2} = \left(\frac{-2I_P}{V_P^2} + \frac{I_P V}{V_P^3}\right)\exp\left(1 - \frac{V}{V_P}\right) = 0$$

$$\therefore V = 2V_P = 2 \times 0.1 = 0.2\text{V}$$

$$\left.\frac{dI}{dV}\right|_{0.2V} = \left[\frac{10^{-2}}{0.1} - \frac{10^{-2} \times 0.2}{(0.1)^2}\right]\exp\left(1 - \frac{0.2}{0.1}\right) = -0.0367$$

$$R = \left(\left.\frac{dI}{dV}\right|_{0.2V}\right)^{-1} = -27.2 \ \Omega.$$

5. (a) $R_{SC} = \dfrac{1}{I}\displaystyle\int_0^W \Delta E \ dx = \dfrac{1}{I}\displaystyle\int_0^W \dfrac{Ix}{A e_s v_S} dx = \dfrac{W^2}{2A e_s v_S}$

$$= \frac{(12 \times 10^{-4})^2}{2(5 \times 10^{-4})1.05 \times 10^{-12} \times 10^7} = 137 \ \Omega$$

(b) The breakdown voltage for $N_D = 10^{15}$ cm^{-3} and $W = 12$ μm is 250 V (Refer to Chapter 4).

The voltage due to R_{SC} is

$$IR_{SC} = (10^3 \times 5 \times 10^{-4}) \times 137 = 68.5 \text{ V}$$

The total applied voltage is then $250 + 68.5 = 318.5$ V.

6. (a) The dc input power is $100\text{V}(10^{-1}\text{A}) = 10\text{W}$. For 25% efficiency, the power dissipated as heat is $10\text{W}(1-25\%) = 7.5$ W.

$$\Delta T = 7.5\text{W} \times (10^\circ \text{C/W}) = 75^\circ \text{C}$$

(b) $\Delta V_B = (60\text{mV}/^\circ\text{C}) \times 75^\circ \text{C} = 4.5$ V

The breakdown voltage at room temperature is $(100-4.5) = 95.5\text{V}$.

7. (a) For a uniform breakdown in the avalanche region, the maximum electric field is

$E_m = 4.4 \times 10^5$ V/cm. The total voltage at breakdown across the diode is

$$V_B = E_m x_A + \left(E_m - \frac{qQ}{e_s} \right)(W - x_A)$$

$$= 4.4 \times 10^5 \left(0.4 \times 10^{-4} \right) + \left(4.4 \times 10^5 - \frac{1.6 \times 10^{-19} \times 1.5 \times 10^{12}}{1.09 \times 10^{-12}} \right)(3 - 0.4) \times 10^{-4}$$

$$= 17.6 + 57.2 = 74.8 \, \text{V}$$

(b) The average field in the drift region is

$$\frac{57.2}{(3 - 0.4) \times 10^{-4}} = 2.2 \times 10^5 \, \text{V/cm}$$

This field is high enough to maintain velocity saturation in the drift region.

(c) $f = \dfrac{v_S}{2(W - x_A)} = \dfrac{10^7}{2(3 - 0.4)10^{-4}} = 19 \, \text{GHz}$.

8. (a) In the p layer

$$E_1(x) = E_m - \frac{qN_1 x}{e_s} \qquad 0 \le x \le b = 3 \, \mu\text{m}$$

$$E_2(x) = E_m - \frac{qN_1 b}{e_s} \qquad b \le x \le W = 12 \, \mu\text{m}$$

$E_2(x)$ should be larger than 10^5 V/cm for velocity saturation

$$\therefore E_m - \frac{qN_1 b}{e_s} \ge 10^5$$

or $E_m \ge 10^5 + \dfrac{qN_1 b}{e_s} = 10^5 + 4.66 \times 10^{-11} N_1$.

This equation coupled the plot of E_m versus N in Chapter 3 gives

$N_1 = 7 \times 10^{15} \, \text{cm}^{-3}$ for $E_m = 4.2 \times 10^5$ V/cm

$$\therefore V_B = \frac{(E_m - E_2)b}{2} + E_2 W = \frac{(4.2 - 1) \times 10^5 \times 3 \times 10^{-4}}{2} + 10^5 \times 9 \times 10^{-4}$$

$$= 138 \text{ V}$$

(b) Transit time t $= \dfrac{W - b}{v_s} = \dfrac{(12 - 3)\times 10^{-4}}{10^7} = 9\times 10^{-15} = 90 \text{ ps}$.

9. (a) For transit-time mode, we require $n_0 L \geq 10^{12}$ cm^{-2}.

$$n_0 \approx 10^{12} / L = 10^{12} / 1\times 10^{-4} = 10^{16} \text{cm}^{-3}$$

(b) $t = L/v = 10^{-4} / 10^7 = 10^{-11}$ s $= 10$ ps

(c) The threshold field for InP is 10.5 kV/cm; the corresponding applied voltage is

$$V = \left(\frac{10.5\times 10^3}{2}\right)\!\left(1\times 10^{-4}\right) = 0.525 \text{ V}$$

The current is

$$I = JA = \left(q\boldsymbol{m}n_0 E\right)A = \left(1.6\times 10^{-19}\times 4600\times 10^{16}\times 5.25\times 10^3\right)\times 10^{-4} = 3.86 \text{ A}$$

The power dissipated in the device is then

$$P = IV = 2.02 \text{ W} .$$

10. (a) Referring to Chapter 2, we have

$$N_{CU} = 2\left(\frac{2\boldsymbol{p}m_{nU}kT}{\hbar^2}\right)^{3/2} = N_{CL}\left(\frac{m_{nU}}{m_{nL}}\right)^{3/2}$$

$$= 4.7\times 10^{17}\left(\frac{1.2m_0}{0.07m_0}\right)^{3/2} = 4.7\times 10^{17}\times 71 = 3.33\times 10^{19} \text{ cm}^{-3}$$

(b) For $T_e = 300$ K

$$\frac{N_{CU}}{N_{CL}}\exp(-\Delta E / kT_e) = 71\times \exp\!\left(\frac{-0.31\text{eV}}{0.0259}\right) = 71\times \exp\left(-11.97\right)$$
$$= 4.4\times 10^{-4}$$

(c) For $T_e = 1500$ K

$$\frac{N_{CU}}{N_{CL}}\exp(-\Delta E/kT_e) = 71 \times \exp\left(\frac{-0.31\text{eV}}{0.0259\,(1500/300)}\right) = 71 \times \exp(-2.394)$$

$$= 6.5$$

Therefore, at $T_e = 300$ K most electrons are in the lower valley. However, at $T_e = 1500$ K,

87%, i.e., 6.5/(6.5+1), of the electrons are in the upper valley.

11. The energy E_n for infinitely deep quantum well is

$$E_n = \frac{h^2}{8m^*L^2}n^2$$

$$\left|\frac{\Delta E_n}{\Delta L}\right| = \left|\frac{h^2n^2}{8m^*}(-2)(L^{-3})\right| = \frac{2E_1}{L}$$

$$\Delta E_n = \frac{2E_1}{L}\Delta L$$

$$\therefore \Delta E_1 = 3 \text{ meV}, \quad \Delta E_2 = 11 \text{ meV}.$$

12. From Fig. 14 we find that the first excited energy is at 280 meV and the width is 0.8 meV. For

same energy but a width of 8 meV, we use the same well thickness of 6.78 nm for GaAs, but

the barrier thickness must be reduced to 1.25 nm for AlAs.

The resonant-tunneling current is related to the integrated flux of electrons whose energy is in

the range where the transmission coefficient is large. Therefore, the current is proportional to

the width ΔE_n, and sufficiently thin barriers are required to achieve a high current density.

CHAPTER 9

1. $hv\,(0.6\,\textbf{\textit{m}}m) = 1.24/0.6 = 2.07\text{eV (From Eq. 9)}$

   $\textbf{\textit{a}}\,(0.6\ \mu m) = 3 \times 10^4\ \text{cm}^{-1}$

   The net incident power on the sample is the total incident power minus the reflected

   power, or 10 mW.

   $10^{-2}\left(1 - e^{-3 \times 10^4 W}\right) = 5 \times 10^{-3}$

   $W = 0.231\ \mu m.$

   The portion of each photon's energy that is converted to heat is

   $\dfrac{h\textbf{\textit{n}} - E_g}{h\textbf{\textit{n}}} = \dfrac{2.07 - 1.42}{2.07} = 31.4\%$

   The amount of thermal energy dissipated to the lattice per second is

   $31.4\% \times 5 = 1.57$ mW.

2. For $\lambda = 0.898\ \mu m$, the corresponding photon energy is

   $E = \dfrac{1.24}{\textbf{\textit{l}}} = 1.38\quad \text{eV}$

   From Fig. 18, we obtain $\overline{n}_2\,(\text{Al0.3Ga0.7As}) = 3.38$

   $\sin \textbf{\textit{q}}_c = \dfrac{\overline{n}_1}{n_2} = \dfrac{1}{3.38} = 0.2958 \Rightarrow \textbf{\textit{q}}_c = 17^\circ 12'.$

   $Efficiency = \dfrac{4\overline{n}_1\overline{n}_2\left(1 - \cos \textbf{\textit{q}}_c\right)}{\left(\overline{n}_1 + \overline{n}_2\right)^2} = \dfrac{4 \times 1 \times 3.38\left[1 - \cos\left(17^\circ 12'\right)\right]}{\left(1 + 3.38\right)^2}$

   $= 0.0315 = 3.15\%\ .$

3. From Eq. 15

   $$R = \left(\dfrac{3.39 - 1}{3.39 + 1}\right)^2 = 0.296$$

   (a) The mirror loss

$$\frac{1}{L}\ln\left(\frac{1}{R}\right)=\frac{1}{300\times10^{-4}}\ln\left(\frac{1}{0.296}\right)=40.58\ \text{cm}^{-1}.$$

(b) The threshold current reduction is

$$\frac{J_{th}(R=0.296)-J_{th}(R_1=0.296,R_2=0.9)}{J_{th}(R=0.296)}$$

$$\approx\frac{\left[\boldsymbol{a}+\dfrac{1}{L}\ln\left(\dfrac{1}{R}\right)\right]-\left[\boldsymbol{a}+\dfrac{1}{2L}\ln\left(\dfrac{1}{R_1R_2}\right)\right]}{\boldsymbol{a}+\dfrac{1}{L}\ln\left(\dfrac{1}{R}\right)}$$

$$=\frac{40.58-\dfrac{1}{2\cdot300\times10^{-4}}\ln\left(\dfrac{1}{0.296\cdot0.90}\right)}{10+40.58}$$

$$=36.6\%.$$

4. From Eq. 10

$$\sin\boldsymbol{q}_c=\frac{\overline{n}_1}{\overline{n}_2}\Rightarrow\overline{n}_1=\overline{n}_2\cdot\sin\boldsymbol{q}_c$$

From Eq. 14

$$\Gamma\cong1-\exp\left(-C\Delta\overline{n}d\right)=1-\exp\left(-8\times10^5\cdot3.6(1-\sin\boldsymbol{q}_c)\cdot1\times10^{-4}\right)$$

For $\boldsymbol{q}_c=84^{\circ}$ $n_1=3.58$ $\boldsymbol{G}_1=0.794$

$\boldsymbol{q}_c=78^{\circ}$ $n_2=3.52$ $\boldsymbol{G}_2=0.998$.

5. From Eq. 16 we have

$$m\boldsymbol{l}=2\overline{n}L.$$

Differentiating the above equation with respect to $\boldsymbol{l}$, we obtain

$$\boldsymbol{l}\frac{dm}{d\boldsymbol{l}}+m=2L\frac{d\overline{n}}{d\boldsymbol{l}}.$$

Substituting $2\bar{n}L/\boldsymbol{l}$ for m and letting $dm/d\boldsymbol{l} = -\Delta m/\Delta \boldsymbol{l}$, yield

$$\boldsymbol{l}\left(\frac{-\Delta m}{\Delta \boldsymbol{l}}\right) + \frac{2\bar{n}L}{\boldsymbol{l}} = 2L\frac{d\bar{n}}{d\boldsymbol{l}}$$

$$\therefore \Delta \boldsymbol{l} = \frac{\boldsymbol{l}^2 \Delta m}{2\bar{n}L\left[1 - \left(\dfrac{\boldsymbol{l}}{n}\right)\left(\dfrac{d\bar{n}}{d\boldsymbol{l}}\right)\right]}$$

and

$$\Delta \boldsymbol{l} = \frac{(0.89)^2 \times 1}{2(3.58)300\left[1 - \left(\dfrac{0.89}{3.58}\right)(2.5)\right]} = 9.7 \times 10^{-4} \quad \mu m = 0.97 \text{ nm}.$$

6. From Eq. 22

$$g = \frac{1}{\Gamma}\left[\boldsymbol{a} + \frac{1}{L}\ln\left(\frac{1}{R}\right)\right] \text{ and Eq. 15 } R = \left(\frac{n-1}{n+1}\right)^2$$

$R_1 = 0.317$, $R_2 = 0.311$

$$g(84^o) = \frac{1}{0.794}\left[100 + \frac{1}{100\times10^{-4}}\ln\left(\frac{1}{0.317}\right)\right] = 270$$

$$g(78^o) = \frac{1}{0.998}\left[100 + \frac{1}{100\times10^{-4}}\ln\left(\frac{1}{0.311}\right)\right] = 217$$

$$\frac{1}{2L'}\ln\left(\frac{1}{R\cdot0.99}\right) = \frac{1}{100\times10^{-4}}\ln\left(\frac{1}{R}\right)$$

For $R_1 = 0.317$

$$\frac{1}{2L'}\ln\left(\frac{1}{0.317\cdot0.99}\right) = \frac{1}{100\times10^{-4}}\ln\left(\frac{1}{0.317}\right) \Rightarrow L' = 50 \ \boldsymbol{m}\text{m}$$

For $R_2 = 0.311$

$$\frac{1}{2L'_2}\ln\left(\frac{1}{0.311\cdot0.99}\right) = \frac{1}{100\times10^{-4}}\ln\left(\frac{1}{0.311}\right) \Rightarrow L'_2 = 50.43 \ \boldsymbol{m}\text{m}.$$

7. From Eq. 23a

For R = 0.317

$$J_{th} = 10 \times \left[100 + \frac{1}{2 \times 100 \times 10^{-4}} \ln\left(\frac{1}{0.317 \times 0.99} \right) \right] = 1000 \text{ A/cm}^{-2}$$

and so $I_{th} = 1000 \times 100 \times 10^{-4} \times 5 \times 10^{-4} = 5$ mA

For R = 0.311

$$\boldsymbol{b} \propto \boldsymbol{G} \Rightarrow \boldsymbol{b}_2 = 0.1 \times 0.998/0.764 = 0.13$$

$$J_{th} = 7.66 \times \left[100 + \frac{1}{2 \times 100 \times 10^{-4}} \ln\left(\frac{1}{0.311 \times 0.99} \right) \right] = 766 \text{ A/cm}^{-2}$$

and so $I_{th} = 766 \times 100 \times 10^{-4} \times 5 \times 10^{-4} = 3.83$ mA .

8. From the equation, we have for m = 0:

$$\boldsymbol{I}_B^2 \pm 4\overline{n}L\boldsymbol{I}_B \mp 4\overline{n}L\boldsymbol{I}_o = 0 \tag{25a}$$

which can be solved as

$$\boldsymbol{I}_B = \pm \left(\frac{-4\overline{n}L \pm \sqrt{16\overline{n}^2 L^2 + 16\overline{n}L\boldsymbol{I}_o}}{2} \right) \tag{25b}$$

There are several variations of $\pm$ in this solutions. Take the solution which is the only practical one, i.e., $\lambda_B \approx \lambda_o$, gives $\lambda_B = 1.3296$ or 1.3304 µm.

$$d \cong \frac{1.33}{2 \times 3.4} = 0.196 \ \boldsymbol{m}\text{m} \cdot$$

9. The threshold current in Fig. 26b is given by

$I_{th} = I_0 \exp (T/110)$.

Therefore

$$\boldsymbol{x} \equiv \frac{1}{I_{th}} \frac{dI_{th}}{dT} = \frac{1}{110} = 0.0091 \ \left(^{\circ}\text{C} \right)^{-1}.$$

If $T_0 = 50$ °C, the temperature coefficient becomes

$$x = \frac{1}{50} = 0.02 \ (^\circ C)^{-1}.$$

which is larger than that for $T_0 = 110\,^\circ C$. Therefore the laser with $T_0 = 50\,^\circ C$ is worse

for high-temperature operation.

10. (a) $\Delta I = q \ (\mu_n + \mu_p) \ \Delta n E A$

$$\therefore \Delta n = electron-hole \ pairs = \frac{\Delta I}{q(\mu_n + \mu_p)EA}$$

$$= \frac{2.83 \times 10^{-3}}{1.6 \times 10^{-19}(3600 + 1700)(10/0.6)(2 \times 1 \times 10^{-2})} = 10^{13}\,cm^{-3}$$

(b) $t = \frac{2.83 \times 10^{-3}}{23.6} = 120\,ms$

(c) $\Delta n(t) = \Delta n \exp(-t/\tau) = 10^{13} \exp\left(-\frac{10^{-3}}{1.2 \times 10^{-4}}\right) = 2.5 \times 10^9 \ cm^{-3}$.

11. From Eq. 33

$$I_p = q\left(0.85 \cdot \frac{10^{-6}}{3 \times q}\right) \cdot \left(\frac{3000 \cdot 6 \times 10^{-10} \cdot 5000}{10 \times 10^{-4}}\right)$$

$$= 2.55 \times 10^{-6} = 2.55 \ \mu A$$

and from Eq. 35

$$Gain = \frac{\mu_n \tau E}{L} = \frac{3000 \cdot 6 \times 10^{-10} \cdot 5000}{10 \times 10^{-4}} = 9.$$

12. From Eq. 36

$$\eta = \left(\frac{I_p}{q}\right)\left(\frac{P_{opt}}{h\nu}\right)^{-1} = \left(\frac{I_p}{P_{opt}}\right)\cdot\left(\frac{h\nu}{q}\right) = R \cdot \left(\frac{h\nu}{q}\right)$$

The wavelength λ of light is related to its frequency ν by $\nu = c/\lambda$, where c is the velocity of

light in vacuum. Therefore $h\nu/q = hc/\lambda q$ and $h = 6.625 \times 10^{-34}$ J-s, $c = 3.0 \times 10^{10}$ cm/s, q =

1.6×10^{-19} coul, 1 eV = 1.6×10^{-19} J.

Therefore, $h v/q = 1.24 / l$ (μm)

Thus, $h = (R \times 1.24) / l$ and $R = (hl) / 1.24$.

13. The electric field in the p-layer is given by

$$E_1(x) = E_m - \frac{qN_1 x}{e_s} \quad 0 \le x \le b,$$

where E_m is the maximum field. In the p layer, the field is essentially a constant given by

$$E_2(x) = E_m - \frac{qN_1 b}{e_s} \quad b < x \le W$$

The electric field required to maintain velocity saturation of holes is ~ 10^5 V/cm.

Therefore

$$E_m - \frac{qN_1 b}{e_s} \ge 10^5$$

or

$$E_m \ge 10^5 + \frac{qN_1 b}{e_s} = 10^5 + 4.66 \times 10^{-11} N_1.$$

From the plot of the critical field versus doping, the corresponding E_m are obtained :

$N_1 = 7 \times 10^{15}$ cm^{-3}

$E_m = 4.2 \times 10^5$ V/cm

The biasing voltage is given by

$$V_B = \frac{(E_m - E_2)b}{2} + E_2 W = \frac{3.2 \times 10^5 \times (3 \times 10^{-4})}{2} + 10^5 (9 \times 10^{-4})$$

$$= 138 \ \text{V}.$$

The transit time is

$$t \approx \frac{(W-b)}{n_s} = \frac{9 \times 10^{-4}}{10^7} = 9 \times 10^{-11} = 90 \ \text{ps}.$$

14. (a) For a photodiode, only a narrow wavelength range centered at the optical signal wavelength is important; whereas for a solar cell, high spectral response over a broad solar wavelength range are required.

   (b) Photodiode are small to minimize junction capacitance, while solar cells are large-area device.

   (c) An important figure of merit for photodiodes is the quantum efficiency (number of electron-hole pairs generated by incident photon), whereas the main concern for solar cells is the power conversion efficiency (power delivered to the load per incident solar energy).

15. (a) $I_s = AqN_C N_V \left(\dfrac{1}{N_A} \sqrt{\dfrac{D_n}{t_n}} + \dfrac{1}{N_D} \sqrt{\dfrac{D_p}{t_p}} \right) e^{-E_g/kT}$

$$= 2(1.6 \times 10^{-19})(2.86 \times 10^{19})(2.66 \times 10^{19}) \times$$

$$\left(\dfrac{1}{1.7 \times 10^{16}} \sqrt{\dfrac{9.3}{10^{-5}}} + \dfrac{1}{5 \times 10^{19}} \sqrt{\dfrac{2.5}{5 \times 10^{-7}}} \right) \times e^{-1.12eV/kT}$$

$$= 2.43 \times 10^{20} (5.67 \times 10^{-14}) e^{-43.2}$$

$$= 2.28 \times 10^{-12} \ \text{A}$$

$$I = I_s \left(e^{qV/kT} \right) - I_L$$

$$|I| = I_L - I_s \left(e^{qV/kT} - 1 \right)$$

V	0	0.1	0.2	0.3	0.4	0.5	0.6	0.65	0.7	(V)
$I_s(e^{qV/kT} - 1)$	0	1.1×10^{-7}	5×10^{-6}	2.5×10^{-4}	1.1×10^{-2}	0.55	26.8	179	1520	(mA)
I_L	95	95	95	95	95	94.5	68.2	-84	-1425	(mA)

(b) $V_{OC} = \dfrac{kT}{q} \ln \left(\dfrac{I_L}{I_s} \right) = 0.0259 \ln \left(\dfrac{95 \times 10^{-3}}{2.28 \times 10^{-12}} \right) = 0.68 \ \text{V}$
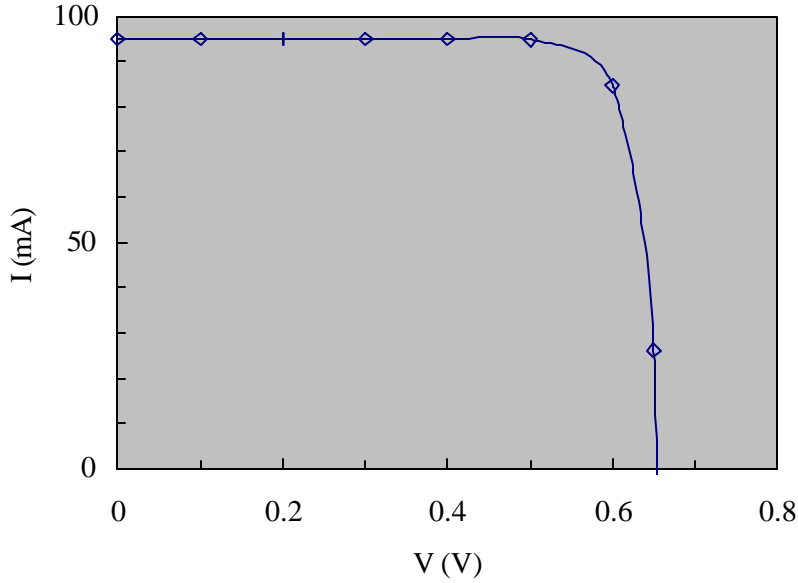
(c) $P = I_s V \left(e^{qV/kT} - 1 \right) - I_L V$

$$\frac{dP}{dV} = 0 = I_s\left(e^{qV/kT} - 1\right) + I_s\frac{qV}{kT}e^{qV/kT} - I_L$$

$$\approx I_s e^{qV/kT}\left(1 + V\right) - I_L$$

$$\therefore e^{qV/kT} = \frac{I_L}{I_s\left(1 + V\right)}$$

$$V_m = 0.64 \quad V$$

$$P_m = I_m V_m = I_L\left[V_{OC} - \frac{kT}{q}\ln\left(1 + \frac{qV_n}{kT}\right) - \frac{kT}{q}\right]$$

$$= 95 \times 10^{-3}\left[0.68 - 0.0259\ln\left(1 + 24.7\right) - 0.0259\right]$$

$$= 52 \quad mW$$

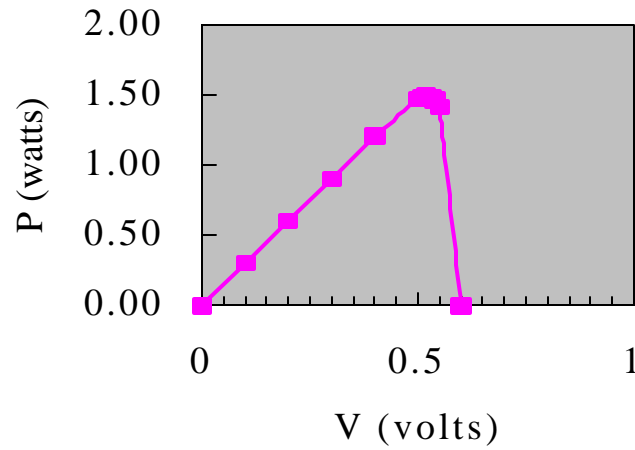16. From Eq. 38 and 39

$$I = I_s\left(e^{qV/kT} - 1\right) - I_L$$
and
$$V_{oc} = \frac{kT}{q}\ln\left(\frac{I_L}{I_s} + 1\right) \cong \frac{kT}{q}\ln\left(\frac{I_L}{I_s}\right)$$

$$I_s = I_L e^{-qV/kT} = 3 \cdot e^{-0.6/0.02585} = 2.493 \times 10^{-10} \quad A$$
$$I = 3 - 2.493 \times 10^{-10} \cdot e^{V/0.02585} \text{ and } P = I\,V$$

V	I	P
0	3.00	0.00
0.1	3.00	0.30
0.2	3.00	0.60
0.3	3.00	0.90
0.4	3.00	1.20
0.5	2.94	1.47
0.51	2.91	1.48
0.52	2.86	1.49
0.53	2.80	1.48
0.54	2.71	1.46
0.55	2.57	1.41
0.6	0.00	0.00

$\therefore$ Maximum power output = 1.49 W

Fill Factor

$$FF \equiv \frac{I_m V_m}{I_L V_{oc}} = \frac{P_m}{I_L V_{oc}} = \frac{1.49}{0.6 \times 3}$$
$$= 0.83.$$

17. From Fig. 40

The output powers for $R_s = 0$ and $R_s = 5\ \Omega$ can be obtained from the area

$P_1\ (R_s = 0) = 95\ \text{mA} \times 0.375\text{V} = 35.6\ \text{mW}$,

$P_2\ (R_s = 5\ \Omega) = 50\ \text{mA} \times 0.18\text{V} = 9.0\ \text{mW}$

$\therefore$ For $R_s = 0 \quad P_1/P_1 = 100\%$

For $R_s = 5\ \Omega \quad P_2/P_1 = 9/35.6 = 25.3\%$.

18. The efficiencies are 14.2% (1 sun), 16.2% (10-sun), 17.8% (100-sun), and 18.5%

(1000-sun).

Solar cells needed under 1-sun condition

$$= \frac{\textbf{h}(\text{concentrat ion}) \times P_{in}(\text{concentrat ion})}{\textbf{h}(1 - sun) \times P_{in}(1 - sun)}$$

$$= \frac{16.2\% \times 10}{14.2\% \times 1} = 11.4 \quad \text{cells} \quad \text{for } 10 \text{-} \text{sun}$$

$$= \frac{17.8\% \times 10}{14.2\% \times 1} = 125 \quad \text{cells} \quad \text{for} \quad 100 \text{-} \text{sun}$$

$$= \frac{18.5\% \times 10}{14.2\% \times 1} = 1300 \quad \text{cells} \quad \text{for} \quad 1000 \text{-} \text{sun.}$$

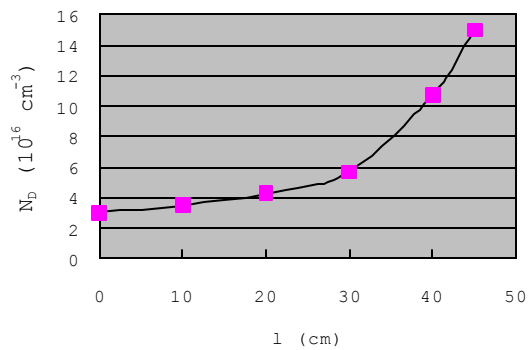# CHAPTER 10

1. $C_0 = 10^{17}$ cm^{-3}
   k_0(As in Si) = 0.3

   $C_S = k_0 C_0 (1 - M/M_0)^{k_0-1}$

   $= 0.3 \times 10^{17} (1 - x)^{-0.7} = 3 \times 10^{16}/(1 - l/50)^{0.7}$

x	0	0.2	0.4	0.6	0.8	0.9
l (cm)	0	10	20	30	40	45
C_S (cm-3)	3×10^{16}	3.5×10^{16}	4.28×10^{16}	5.68×10^{16}	1.07×10^{17}	1.5×10^{17}

2. (a) The radius of a silicon atom can be expressed as

$$r = \frac{\sqrt{3}}{8} a$$

so $r = \frac{\sqrt{3}}{8} \times 5.43 = 1.175 \text{Å}$

(b) The numbers of Si atom in its diamond structure are 8.

So the density of silicon atoms is

$$n = \frac{8}{a^3} = \frac{8}{(5.43\text{Å})^3} = 5.0 \times 10^{22} \text{ atoms/cm}^3$$

(c) The density of Si is

$$r = \frac{M / 6.02 \times 10^{23}}{1 / n} = \frac{28.09 \times 5 \times 10^{22}}{6.02 \times 10^{23}} \, \text{g} / \text{cm}^3 = 2.33 \, \text{g} / \text{cm}^3.$$

3. $k_0 = 0.8$ for boron in silicon

   $M / M_0 = 0.5$

   The density of Si is 2.33 g / cm^3.

   The acceptor concentration for $\rho = 0.01$ Ω–cm is 9×10^{18} cm^{-3}.

   The doping concentration C_S is given by

   $$C_s = k_0 C_0 (1 - \frac{M}{M_0})^{k_0 - 1}$$

   Therefore

   $$C_0 = \frac{C_s}{k_0 (1 - \frac{M}{M_0})^{k_0 - 1}} = \frac{9 \times 10^{18}}{0.8(1 - 0.5)^{-0.2}}$$

   $$= 9.8 \times 10^{18} \text{cm}^{-3}$$

   The amount of boron required for a 10 kg charge is

   $$\frac{10,000}{2.338} \times 9.8 \times 10^{18} = 4.2 \times 10^{22} \text{ boron atoms}$$

   So that

   $$10.8 \text{g/mole} \times \frac{4.2 \times 10^{22} \text{atoms}}{6.02 \times 10^{23} \text{atoms/mole}} = 0.75 \text{g boron}.$$

4. (a) The molecular weight of boron is 10.81.

   The boron concentration can be given as

   $$n_b = \frac{\text{number of boron atoms}}{\text{volume of silicon wa fer}}$$
   $$= \frac{5.41 \times 10^{-3} \text{g} / 10.81 \text{g} \times 6.02 \times 10^{23}}{10.0^2 \times 3.14 \times 0.1}$$
   $$= 9.78 \times 10^{18} \text{atoms/cm}^3$$

   (b) The average occupied volume of everyone boron atoms in the wafer is

$$V = \frac{1}{n_b} = \frac{1}{9.78 \times 10^{18}} \, \text{cm}^3$$

We assume the volume is a sphere, so the radius of the sphere (r) is the average distance between two boron atoms. Then

$$r = \sqrt{\frac{3V}{4p}} = 2.9 \times 10^{-7} \, \text{cm} .$$

5. The cross-sectional area of the seed is

$$p\left(\frac{0.55}{2}\right)^2 = 0.24 \, \text{cm}^2$$

The maximum weight that can be supported by the seed equals the product of the critical yield strength and the seed's cross-sectional area:

$$(2 \times 10^6) \times 0.24 = 4.8 \times 10^5 \, \text{g} = 480 \, \text{kg}$$

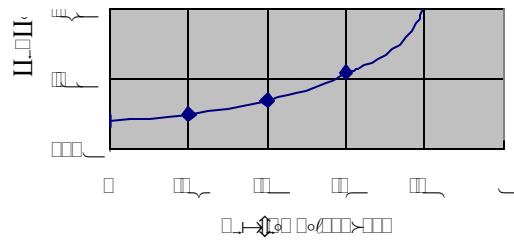The corresponding weight of a 200-mm-diameter ingot with length l is

$$(2.33 \text{g/cm}^3) p\left(\frac{20.0}{2}\right)^2 l = 480000 \, \text{g}$$
$$\therefore \quad l = 656 \, \text{cm} = 6.56 \, \text{m}.$$

6. We have

$$C_s / C_0 = k_0 \left(1 - \frac{M}{M_0}\right)^{k_0 - 1}$$

Fractional solidified	0	0.2	0.4	0.6	0.8	1.0	
C_S / C_0	0.05	0.06	0.08	0.12	0.23	∞	

7. The segregation coefficient of boron in silicon is 0.72. It is smaller than unity, so the solubility of B in Si under solid phase is smaller than that of the melt. Therefore, the excess B atoms will be thrown-off into the melt, then the concentration of B in the melt will be increased. The tail-end of the crystal is the last to solidify. Therefore, the concentration of B in the tail-end of grown crystal will be higher than that of seed-end.

8. The reason is that the solubility in the melt is proportional to the temperature, and the temperature is higher in the center part than at the perimeter. Therefore, the solubility is higher in the center part, causing a higher impurity concentration there.

9. The segregation coefficient of Ga in Si is 8×10^{-3}

   From Eq. 18

   $$C_s / C_0 = 1 - (1-k)e^{-kx/L}$$

   We have

   $$
   \begin{aligned}
   x &= \frac{L}{k} \ln\left(\frac{1-k}{1 - C_s/C_0}\right) \\
   &= \frac{2}{8 \times 10^{-3}} \ln\left(\frac{1 - 8 \times 10^{-3}}{1 - 5 \times 10^{15} / 5 \times 10^{16}}\right) \\
   &= 250 \ln(1.102) \\
   &= 24 \text{ cm.}
   \end{aligned}
   $$

10. We have from Eq.18

    $C_s = C_0[1-(1-k_e)\exp(-k_e x/L)]$

    So the ratio $C_s / C_0 = [1-(1-k_e)\exp(-k_e x/L)]$
    $$
    \begin{aligned}
    &= 1 - (1 - 0.3) \bullet \exp(-0.3 \times 1) = 0.52 \qquad \text{at } x/L = 1 \\
    &= 0.38 \qquad \text{at } x/L = 2.
    \end{aligned}
    $$

11. For the conventionally-doped silicon, the resistivity varies from 120 Ω-cm to 155 Ω-cm. The corresponding doping concentration varies from 2.5×10^{13} to 4×10^{13} cm^{-3}. Therefore the range of breakdown voltages of p^+ - n junctions is given by

$$V_B \cong \frac{\varepsilon_s E_c^2}{2q}(N_B)^{-1}$$

$$= \frac{1.05\times10^{-12}\times(3\times10^5)^2}{2\times1.6\times10^{-19}}(N_B)^{-1} = 2.9\times10^{17}/N_B = 7250 \text{ to } 11600 \text{ V}$$

$$\Delta V_B = 11600 - 7250 = 4350 \text{ V}$$

$$\therefore \left(\frac{\Delta V_B}{2}\right)/7250 = \pm30\%$$

For the neutron irradiated silicon, $\rho = 148 \pm 1.5$ Ω-cm. The doping concentration is 3×10^{13} ($\pm1\%$). The range of breakdown voltage is

$$V_B = 1.3\times10^{17}/N_B = 2.9\times10^{17}/3\times10^{13}(\pm1\%)$$
$$= 9570 \text{ to } 9762 \text{ V}.$$

$$\Delta V_B = 9762 - 9570 = 192 \text{ V}$$

$$\therefore \left(\frac{\Delta V_B}{2}\right)/9570 = \pm1\%.$$

12. We have

$$\frac{M_s}{M_l} = \frac{\text{weight of GaAs at T}_b}{\text{weight of liquid at T}_b} = \frac{C_m - C_l}{C_s - C_m} = \frac{s}{l}$$

Therefore, the fraction of liquid remained f can be obtained as following

$$f = \frac{M_l}{M_s + M_l} = \frac{l}{s + l} \approx \frac{30}{16 + 30} = 0.65.$$

13. From the Fig.11, we find the vapor pressure of As is much higher than that of the Ga. Therefore, the As content will be lost when the temperature is increased. Thus the composition of liquid GaAs always becomes gallium rich.

14. $n_s = N \exp(-E_s / kT) = 5 \times 10^{22} \exp(2.3 \, \text{eV} / kT) = 5 \times 10^{22} \exp\left[\dfrac{-88.8}{(T/300)}\right]$

$$= 1.23 \times 10^{-16} \, \text{cm}^{-3} \approx 0 \quad \text{at} \;\; 27^0 \text{C} \; = 300 \, \text{K}$$

$$= 6.7 \times 10^{12} \, \text{cm}^{-3} \qquad \text{at} \;\; 900^0 \text{C} \; = 1173 \, \text{K}$$

$$= 6.7 \times 10^{14} \, \text{cm}^{-3} \qquad \text{at} \;\; 1200^0 \text{C} = 1473 \, \text{K} \,.$$

15. $n_f = \sqrt{NN^{\grave{}}} \, \exp(-E_f / 2kT)$

$$= \sqrt{5 \times 10^{22} \times 1 \times 10^{27} \, e^{-3.8 eV / kT}} \times e^{-1.1 eV / 2kT} = 7.07 \times 10^{24} \times e^{-94.7/(T/300)}$$

$$= 5.27 \times 10^{-17} \;\; \text{at } 27^{\text{o}}\text{C} = 300 \text{ K}$$

$$= 2.14 \times 10^{14} \;\; \text{at } 900^{\text{o}}\text{C} = 1173 \text{ K}.$$

16. $37 \times 4 = 148$ chips

In terms of litho-stepper considerations, there are 500 μm space tolerance between the mask boundary of two dice. We divide the wafer into four symmetrical parts for convenient dicing, and discard the perimeter parts of the wafer. Usually the quality of the perimeter parts is the worst due to the edge effects.

300 mm
Total Dies: 148
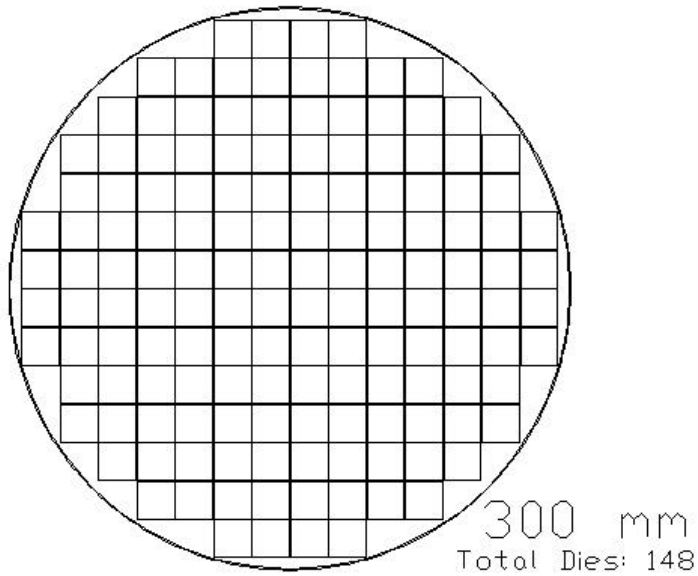
17. $n_{av} = \dfrac{\int_0^\infty v f_v \, dv}{\int_0^\infty f_v \, dv} = \sqrt{\dfrac{8kT}{pM}}$

Where $f_n = \dfrac{4}{\sqrt{p}}\left(\dfrac{M}{2kT}\right)^{3/2} n^2 \exp\left(-\dfrac{Mn^2}{2kT}\right)$

M: Molecular mass

k: Boltzmann constant = 1.38×10^{-23} J/k

T: The absolute temperature

v: Speed of molecular

So that

$n_{av} = \dfrac{2}{\sqrt{p}}\sqrt{\dfrac{2 \times 1.38 \times 10^{-23} \times 300}{29 \times 1.67 \times 10^{-27}}} = 468$ m/sec $= 4.68 \times 10^4$ cm/sec .

18. $l = \dfrac{0.66}{P(\text{in Pa})}$ cm

91

$$\therefore P = \frac{0.66}{I} = \frac{0.66}{150} = 4.4 \times 10^{-3} \text{ Pa}.$$
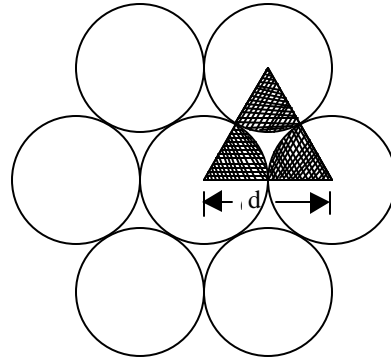
19. For close-packing arrange, there are 3 pie shaped sections in the equilateral triangle.

Each section corresponds to 1/6 of an atom. Therefore

$$N_s = \frac{\text{number of atoms contained in the triangle}}{\text{area of the triangle}} = \frac{3 \times \dfrac{1}{6}}{\dfrac{1}{2} d \times \dfrac{\sqrt{3}}{2} d}$$

$$= \frac{2}{\sqrt{3} d^2} = \frac{2}{\sqrt{3} (4.68 \times 10^{-8})^2}$$

$$= 5.27 \times 10^{14} \text{ atoms/cm}^2.$$

20. (a) The pressure at 970°C (=1243K) is 2.9×10^{-1} Pa for Ga and 13 Pa for As_2. The

arrival rate is given by the product of the impringement rate and $A/\pi L^2$:

$$\text{Arrival rate} = 2.64 \times 10^{20} \left(\frac{P}{\sqrt{MT}} \right) \left(\frac{A}{\pi L^2} \right)$$

$$= 2.64 \times 10^{20} \left(\frac{2.9 \times 10^{-1}}{\sqrt{69.72 \times 1243}} \right) \left(\frac{5}{\pi \times 12^2} \right)$$

$$= 2.9 \times 10^{15} \text{ Ga molecules/cm}^2 \text{--s}$$

The growth rate is determined by the Ga arrival rate and is given by

$(2.9 \times 10^{15}) \times 2.8 / (6 \times 10^{14}) = 13.5$ Å/s $= 810$ Å/min .

(b) The pressure at 700°C for tin is 2.66×10^{-6} Pa. The molecular weight is 118.69.

Therefore the arrival rate is

$$2.64 \times 10^{20} \left(\frac{2.66 \times 10^{-6}}{\sqrt{118.69 \times 973}} \right) \left(\frac{5}{\pi \times 12^2} \right) = 2.28 \times 10^{10} \text{ molecular/ cm}^2 \cdot s$$

If Sn atoms are fully incorporated and active in the Ga sublattice of GaAs, we have an

electron concentration of

$$\left(\frac{2.28\times10^{10}}{2.9\times10^{15}}\right)\left(\frac{4.42\times10^{22}}{2}\right)=1.74\times10^{17}\ cm^{-3}\ .$$

21. The x value is about 0.25, which is obtained from Fig. 26.

22. The lattice constants for InAs, GaAs, Si and Ge are 6.05, 5.65, 5.43, and 5.65 Å, respectively (Appendix F). Therefore, the f value for InAs-GaAs system is

$f = (5.65 - 6.05)/6.05 = -0.066$

And for Ge-Si system is

$f = (5.43 - 5.65)/5.65 = -0.39$.

CHAPTER 11

1. From Eq. 11 (with $\tau = 0$)

$$x^2 + Ax = Bt$$

From Figs. 6 and 7, we obtain $B/A = 1.5$ μm /hr, $B = 0.47$ μm^2/hr, therefore $A = 0.31$ μm. The time required to grow 0.45μm oxide is

$$t = \frac{1}{B}(x^2 + Ax) = \frac{1}{0.47}(0.45^2 + 0.31 \times 0.45) = 0.72 \text{ hr} = 44 \text{ min} .$$

2. After a window is opened in the oxide for a second oxidation, the rate constants are $B = 0.01$ μm^2/hr, $A = 0.116$ μm ($B/A = 6 \times 10^{-2}$ μm /hr).

   If the initial oxide thickness is 20 nm $= 0.02$ μm for dry oxidation, the value of τ can be obtained as followed:

$$(0.02)^2 + 0.166(0.02) = 0.01 (0 + \tau)$$

or

$$\tau = 0.372 \text{ hr.}$$

For an oxidation time of 20 min ($=1/3$ hr), the oxide thickness in the window area is

$$x^2 + 0.166x = 0.01(0.333 + 0.372) = 0.007$$

or

$$x = 0.0350 \text{ μm} = 35 \text{ nm (gate oxide).}$$

For the field oxide with an original thickness 0.45 μm, the effective τ is given by

$$\tau = \frac{1}{B}(x^2 + Ax) = \frac{1}{0.01}(0.45^2 + 0.166 \times 0.45) = 27.72 \text{ hr.}$$

$$x^2 + 0.166x = 0.01(0.333 + 27.72) = 0.28053$$

or $x = 0.4530$ μm (an increase of 0.003μm only for the field oxide).

3.
$$x^2 + Ax = B(t + \tau)$$

$$(x + \frac{A}{2})^2 - \frac{A^2}{4} = B(t + \tau)$$

$$(x + \frac{A}{2})^2 = B\left[\frac{A^2}{4B} + (t + \boldsymbol{t})\right]$$

when $t \gg \boldsymbol{t}$, $t \gg \dfrac{A^2}{4B}$,

then, $x^2 = Bt$

similarly,

when $t \gg \boldsymbol{t}$, $t \gg \dfrac{A^2}{4B}$,

then, $x = \dfrac{B}{A}(t + \boldsymbol{t})$

4. At 980 (=1253K) and 1 atm, $B = 8.5 \times 10^{-3}$ μm^2/hr, $B/A = 4 \times 10^{-2}$ μm /hr (from Figs. 6 and 7). Since $A \mapsto 2D/k$, $B/A = kC_0/C_1$, $C_0 = 5.2 \times 10^{16}$ molecules/cm^3 and $C_1 = 2.2 \times 10^{22}$ cm^{-3}, the diffusion coefficient is given by

$$D = \frac{Ak}{2} = \frac{A}{2}\left(\frac{B}{A} \cdot \frac{C_1}{C_0}\right) = \frac{B}{2}\left(\frac{C_1}{C_0}\right)$$

$$= \frac{8.5 \times 10^{-3}}{2} \frac{2.2 \times 10^{22}}{5.2 \times 10^{16}} \ \boldsymbol{m}m^2 / hr$$

$$= 1.79 \times 10^3 \ \boldsymbol{m}m^2 / hr$$

$$= 4.79 \times 10^{-9} cm^2 / s.$$

5. (a) For SiN_xH_y

$$\frac{Si}{N} = \frac{1}{x} = 1.2$$

$$\underline{\quad} x = 0.83$$

$$\text{atomic \% } H = \frac{100y}{1 + 0.83 + y} = 20$$

$$\underline{\quad} y = 0.46$$

The empirical formula is $SiN_{0.83}H_{0.46}$.

(b) $\dot{O} = 5 \times 10^{28} e^{-33.3 \times 1.2} = 2 \times 10^{11}$ $\ddot{Y}$-cm

As the Si/N ratio increases, the resistivity decreases exponentially.

6. Set Ta_2O_5 thickness $= 3t$, $\varepsilon_1 = 25$

       SiO_2 thickness $= t$, $\varepsilon_2 = 3.9$

       Si_3N_4 thickness $= t$, $\varepsilon_3 = 7.6$, area $= A$

then

$$C_{Ta_2O_5} = \frac{e_1 e_0 A}{3t}$$

$$\frac{1}{C_{ONO}} = \frac{t}{e_2 e_0 A} + \frac{t}{e_3 e_0 A} + \frac{t}{e_2 e_0 A}$$

$$C_{ONO} = \frac{e_2 e_3 e_0 A}{\left(e_2 + 2e_3\right)t}$$

$$\frac{C_{Ta_2O_5}}{C_{ONO}} = \frac{e_1 \left(e_2 + 2e_3\right)}{3e_2 e_3} = \frac{25\left(3.9 + 2 \times 7.6\right)}{3 \times 3.9 \times 7.6} = 5.37 \, .$$

7. Set

       BST thickness $= 3t$, $\varepsilon_1 = 500$, area $= A_1$

       SiO_2 thickness $= t$, $\varepsilon_2 = 3.9$, area $= A_2$

       Si_3N_4 thickness $= t$, $\varepsilon_3 = 7.6$, area $= A_2$

then

$$\frac{e_1 e_0 A_1}{3t} = \frac{e_2 e_3 e_0 A_2}{\left(e_2 + 2e_3\right)t}$$

$$\frac{A_1}{A_2} = 0.0093.$$

8. Let

       Ta_2O_5 thickness $= 3t$, $\varepsilon_1 = 25$

       SiO_2 thickness $= t$, $\varepsilon_2 = 3.9$

       Si_3N_4 thickness $= t$, $\varepsilon_3 = 7.6$

       area $= A$

then

$$\frac{e_1 e_0 A}{3t} = \frac{e_2 e_0 A}{d}$$

$$d = \frac{3e_2 t}{e_1} = 0.468t.$$

9. The deposition rate can be expressed as

$$r = r_0 \exp(-E_a/kT)$$

where $E_a = 0.6$ eV for silane-oxygen reaction. Therefore for $T_1 = 698$ K

$$\frac{r(T_2)}{r(T_1)} = 2 = \exp\left[0.6\left(\frac{1}{kT_1} - \frac{1}{kT_2}\right)\right]$$

$$\ln 2 = \frac{0.6}{0.0259}\left[\left(\frac{300}{698} - \frac{300}{T_2}\right)\right]$$

— $T_2 = 1030$ K= 757 .

10. We can use energy-enhanced CVD methods such as using a focused energy source or UV lamp. Another method is to use boron doped P-glass which will reflow at temperatures less than 900 .

11. Moderately low temperatures are usually used for polysilicon deposition, and silane decomposition occurs at lower temperatures than that for chloride reactions. In addition, silane is used for better coverage over amorphous materials such SiO_2.

12. There are two reasons. One is to minimize the thermal budget of the wafer, reducing dopant diffusion and material degradation. In addition, fewer gas phase reactions occur at lower temperatures, resulting in smoother and better adhering films. Another reason is that the polysilicon will have small grains. The finer grains are easier to mask and etch to give smooth and uniform edges. However, for temperatures less than 575 °C the deposition rate is too low.

13. The flat-band voltage shift is

$$\Delta V_{FB} = 0.5 \text{ V} \sim \frac{Q_{ot}}{C_0}$$

$$C_0 = \frac{e_{ox}}{d} = \frac{3.9 \times 8.85 \times 10^{-14}}{500 \times 10^{-8}} = 6.9 \times 10^{-8} \text{ F/cm}^{-2}.$$

— Number of fixed oxide charge is

$$\frac{0.5C_0}{q} = \frac{0.5 \times 6.9 \times 10^{-8}}{1.6 \times 10^{-19}} = 2.1 \times 10^{11} \, \text{cm}^{-2}$$

To remove these charges, a 450 heat treatment in hydrogen for about 30 minutes is required.

14. $20/0.25 = 80$ sqs.

Therefore, the resistance of the metal line is

$5 \times 50 = 400 \, \Omega$.

15.   For $TiSi_2$

$30 \times 2.37 = 71.1 \text{nm}$

For $CoSi_2$

$30 \times 3.56 = 106.8 \text{nm}.$

16.   For $TiSi_2$:

Advantage: low resistivity

It can reduce native-oxide layers

$TiSi_2$ on the gate electrode is more resistant to high-field-induced hot-electron degradation.

Disadvantage: bridging effect occurs.

Larger Si consumption during formation of $TiSi_2$

Less thermal stability

For $CoSi_2$:

Advantage: low resistivity

High temperature stability

No bridging effect

A selective chemical etch exits

Low shear forces

Disadvantage: not a good candidate for polycides

17. (a) $R = r\dfrac{L}{A} = 2.67 \times 10^{-6} \times \dfrac{1}{0.28 \times 10^{-4} \times 0.3 \times 10^{-4}} = 3.2 \times 10^{3}\,\Omega$

$C = \dfrac{eA}{d} = \dfrac{eTL}{S} = \dfrac{3.9 \times 8.85 \times 10^{-14} \times 0.3 \times 10^{-4} \times 1 \times 10^{4} \times 10^{-6}}{0.36 \times 10^{-4}} = 2.9 \times 10^{-13}\,F$

$RC = 3.2 \times 10^{5} \times 2.9 \times 10^{-15} = 0.93\,\text{ns}$

(b) $R = r\dfrac{L}{A} = 1.7 \times 10^{-6} \times \dfrac{1}{0.28 \times 10^{-4} \times 0.3 \times 10^{-4}} = 2 \times 10^{3}\,\Omega$

$C = \dfrac{eA}{d} = \dfrac{eTL}{S} = \dfrac{2.8 \times 8.85 \times 10^{-14} \times 0.3 \times 10^{-4} \times 1}{0.36 \times 10^{-4}} = 2.1 \times 10^{-13}\,F$

$RC = 2 \times 10^{3} \times 2.1 \times 10^{-13} = 0.42\,\text{ns}$

(c) We can decrease the *RC* delay by 55%. Ratio $= \dfrac{0.42}{093} = 0.45.$

18. (a) $R = r\dfrac{L}{A} = 2.67 \times 10^{-6} \times \dfrac{1}{0.28 \times 10^{4} \times 0.3 \times 10^{4}} = 3.2 \times 10^{3}\,\Omega$

$C = \dfrac{eA}{d} = \dfrac{eTL}{S} = \dfrac{3.9 \times 8.85 \times 10^{-14} \times 0.3 \times 10^{-4} \times 1 \times 3}{0.36 \times 10^{4}} = 8.7 \times 10^{-13}\,F$

$RC = 3.2 \times 10^{3} \times 8.7 \times 10^{-13} = 2.8\text{ ns.}$

.

(b) $R = r\dfrac{L}{A} = 1.7 \times 10^{-6} \times \dfrac{1}{0.28 \times 10^{4} \times 0.3 \times 10^{4}} = 2 \times 10^{3}\,\Omega$

$C = \dfrac{eA}{d} = \dfrac{eTL}{S} = \dfrac{2.8 \times 8.85 \times 10^{-14} \times 0.3 \times 10^{-4} \times 1 \times 3}{0.36 \times 10^{4}} = 6.3 \times 10^{-13}\,F$

$RC = 2 \times 10^{3} \times 8.7 \times 10^{-13} = 2.5\,\text{ns}$

$RC = 3.2 \times 10^{3} \times 8.7 \times 10^{-13} = 2.5\text{ ns.}$

19. (a) The aluminum runner can be considered as two segments connected in series: 20% (or 0.4 mm) of the length is half thickness (0.5 µm) and the remaining 1.6 mm is full thickness (1µm). The total resistance is

$$R = r\left[\dfrac{\ell_1}{A_1} + \dfrac{\ell_2}{A_2}\right] = 3 \times 10^{-6}\left[\dfrac{0.16}{10^{-4} \times 10^{-4}} + \dfrac{0.04}{10^{-4} \times (0.5 \times 10^{-4})}\right]$$
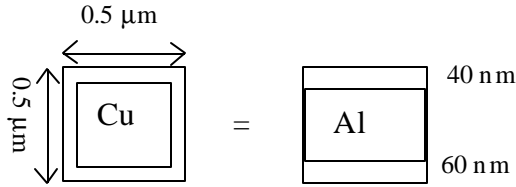
$= 72\ \text{Ù}.$

The limiting current I is given by the maximum allowed current density times cross-sectional area of the thinner conductor sections:

$$I = 5 \times 10^5 \text{ A/cm}^2 \times (10^{-4} \times 0.5 \times 10^{-4}) = 2.5 \times 10^{-3} \text{ A} = 2.5 \text{ mA}.$$

The voltage drop across the whole conductor is then

$$V = RI = 72\Omega \times 2.5 \times 10^{-3} \text{ A} = 0.18\text{V}.$$

20.

h: height , W : width , t : thickness, assume that the resistivities of the cladding layer and TiN are much larger than $r_{A\ell}$ and r_{Cu}

$$R_{Al} = r_{Al} \times \frac{\ell}{h \times W} = 2.7 \frac{\ell}{(0.5 - 0.1) \times 0.5}$$

$$R_{Cu} = r_{Cu} \times \frac{\ell}{h \times W} = 1.7 \frac{\ell}{(0.5 - 2t) \times (0.5 - 2t)}$$

When $R_{Al} = R_{Cu}$

Then $\dfrac{2.7}{0.4 \times 0.5} = \dfrac{1.7}{(0.5 - 2t)^2}$

$\Rightarrow \qquad t = 0.073 \text{ μm} = 73 \text{ nm}$.

CHAPTER 12

1. With reference to Fig. 2 for class 100 clean room we have a total of 3500 particles/m^3 with particle sizes $\geq 0.5\ \mu$m

$$\frac{21}{100} \times 3500 = 735 \text{ particles/m}^2 \text{ with particle sizes} \geq 1.0\ \mu\text{m}$$

$$\frac{4.5}{100} \times 3500 = 157 \text{ particles/m}^2 \text{ with particle sizes} \geq 2.0\ \mu\text{m}$$

Therefore, (a) 3500-735 = 2765 particles/m^3 between 0.5 and 1 μm
   (b) 735-157 = 578 particles/m^3 between 1 and 2 μm
   (c)   157 particles/m^3 above 2 μm.

2. $Y = \prod\limits_{n=1}^{9} e^{-D_1 A}$

   A = 50 mm^2 = 0.5 cm^2

   $Y = e^{-4(0.1 \times 0.5)} \times e^{-4(0.25 \times 0.5)} \times e^{-1(1 \times 0.5)} = e^{-1.2} = 30.1\%$.

3. The available exposure energy in an hour is
   0.3 mW^2/cm^2 × 3600 s = 1080 mJ/cm^2

   For positive resist, the throughput is
   $$\frac{1080}{140} = 7 \text{ wafers/hr}$$
   For negative resist, the throughput is
   $$\frac{1080}{9} = 120 \text{ wafers/hr}.$$

4. (a) The resolution of a projection system is given by
   $$l_m = k_1 \frac{l}{NA} = 0.6 \times \frac{0.193\text{£gm}}{0.65} = 0.178\ \mu\text{m}$$
   $$DOF = k_2 \frac{l}{(NA)^2} = 0.5 \left[\frac{0.193mm}{(0.65)^2} \right] = 0.228\ \mu\text{m}$$

   (b) We can increase NA to improve the resolution. We can adopt resolution enhancement techniques (RET) such as optical proximity correction (OPC) and phase-shifting Masks (PSM). We can also develop new resists that provide lower k$_l$ and higher k$_2$ for better resolution and depth of focus.

   (c) PSM technique changes k$_1$ to improve resolution.

5. (a) Using resists with high g value can result in a more vertical profile but throughput decreases.

   (b) Conventional resists can not be used in deep UV lithography process because these resists have high absorption and require high dose to be exposed in deep UV. This raises the

concern of damage to stepper lens, lower exposure speed and reduced throughput.

6. (a) A shaped beam system enables the size and shape of the beam to be varied, thereby minimizing the number of flashes required for exposing a given area to be patterned. Therefore, a shaped beam can save time and increase throughput compared to a Gaussian beam.

  (b) We can make alignment marks on wafers using e-beam and etch the exposed marks. We can then use them to do alignment with e-beam radiation and obtain the signal from these marks for wafer alignment.

  X-ray lithography is a proximity printing lithography. Its accuracy requirement is very high, therefore alignment is difficult.

  (c) X-ray lithography using synchrotron radiation has a high exposure flux so X-ray has better throughput than e-beam.

7. (a) To avoid the mask damage problem associated with shadow printing, projection printing exposure tools have been developed to project an image from the mask. With a 1:1 projection printing system is much more difficult to produce defect-free masks than it is with a 5:1 reduction step-and-repeat system.

  (b) It is not possible. The main reason is that X-rays cannot be focused by an optical lens. When it is through the reticle. So we can not build a step-and-scan X-ray lithography system.

8. As shown in the figure, the profile for each case is a segment of a circle with origin at the initial mask-film edge. As overetching proceeds the radius of curvature increases so that the profile tends to a vertical line.

9. (a) 20 sec

   $0.6 \times 20/60 = 0.2$ μm…..(100) plane

   $0.6/16 \times 20/60 = 0.0125$ μm……..(110) plane

   $0.6/100 \times 20/60 = 0.002$ μm…….(111) plane

   $W_b = W_0 - \sqrt{2}l = 1.5 - \sqrt{2} \times 0.2 = 1.22$ μm

  (b) 40 sec

   $0.6 \times 40/60 = 0.4$ μm.…(100)plane

$$0.6/16 \times 40/60 = 0.025 \text{ μm}\ldots (110) \text{ plane}$$

$$0.6/100 \times 40/60 = 0.004 \text{ μm}\ldots(111) \text{ plane}$$

$$W_b = W_0 - \sqrt{2}l = 1.5 - \sqrt{2} \times 0.4 = 0.93 \text{ μm}$$

(c)   60 sec

$$0.6 \times 1 = 0.6 \text{ μm}\ldots(100)\text{plane}$$

$$0.6/16 \times 1 = 0.0375 \text{ μm}\ldots (110) \text{ plane}$$

$$0.6/100 \times 1 = 0.006 \text{ μm}\ldots(111) \text{ plane}$$

$$W_b = W_0 - \sqrt{2}l = 1.5 - \sqrt{2} \times 0.6 = 0.65 \text{ μm.}$$

10.   Using the data in Prob. 9, the etched pattern profiles on <100>-Si are shown in below.

(a) 20 sec $l = 0.012$ μm, $W_0 = W_b = 1.5$ μm

(b) 40 sec $l = 0.025$ μm, $W_0 = W_b = 1.5$ μm

(c) 60 sec $l = 0.0375$ μm $W_0 = W_b = 1.5$ μm.

11.   If we protect the IC chip areas (e.g. with Si_3N_4 layer) and etch the wafer from the top, the width of the bottom surface is

$$W = W_1 + \sqrt{2}l = 1000 + \sqrt{2} \times 625 = 1884 \text{ μm}$$

The fraction of surface area that is lost is

$$(W^2 - W_1^2)/W^2 \times 100\% = (1884^2 \text{-}1000^2)/1884^2 \times 100\% = 71.8 \%$$

In terms of the wafer area, we have lost

$$71.8 \% \times p(15/2)^2 = 127 \text{ cm}^2$$

Another method is to define masking areas on the backside and etch from the back. The width of each square mask centered with respect of IC chip is given by

$$W = W_1 - \sqrt{2}l = 1000 - \sqrt{2} \times 625 = 116 \text{ μm}$$

Using this method, the fraction of the top surface area that is lost can be negligibly small.

12. 1 Pa = 7.52 m Torr

PV = nRT

$7.52 / 760 \times 10^{-3}$ = n/V $\times 0.082 \times 273$

n/V = 4.42×10^{-7} mole/liter = $4.42 \times 10^{-7} \times 6.02 \times 10^{23}/1000$ = 2.7×10^{14} cm^{-3}

mean–free–path

$\boldsymbol{l} = 5 \times 10^{-3} / P$ cm = $5 \times 10^{-3} \times 1000 / 7.52$ = 0.6649 cm = 6649 µm

150Pa = 1128 m Torr

PV = nRT

$1128/760 \times 10^{-3} = n/V \times 0.082 \times 273$

$n/V = 6.63 \times 10^{-5}$ mole/liter $= 6.63 \times 10^{-5} \times 6.02 \times 10^{23}/1000 = 4 \times 10^{16}$ cm^{-3}

mean-free-path

$l = 5 \times 10^{-3}/P$ cm $= 5 \times 10^{-3} \times 1000/1128 = 0.0044$ cm $= 44$ μm.

13. Si Etch Rate (nm/min) $= 2.86 \times 10^{-13} \times n_F \times T^{1/2} \times e^{-E_a/RT}$

$$= 2.86 \times 10^{-13} \times 3 \times 10^{15} \times (298)^{1/2} \times e^{\frac{-2.48 \times 10^3}{1.987 \times 298}}$$

$$= 224.7 \text{ nm/min.}$$

14. SiO$_2$ Etch Rate (nm/min) $= 0.614 \times 10^{-13} \times 3 \times 10^{15} \times (298)^{1/2} \times e^{\frac{-3.76 \times 10^3}{1.987 \times 298}} = 5.6$ nm/min

Etch selectivity of SiO$_2$ over Si $= \dfrac{5.6}{224.7} = 0.025$

Or etch rate (SiO$_2$)/etch rate (Si) $= \dfrac{0.614}{2.86} \times e^{(-3.76+2.48)/1.987 \times 298} = 0.025$.

15. A three–step process is required for polysilicon gate etching. Step 1 is a nonselective etch process that is used to remove any native oxide on the polysilicon surface. Step 2 is a high polysilicon etch rate process which etches polysilicon with an anisotropic etch profile. Step 3 is a highly selective polysilicon to oxide process which usually has a low polysilicon etch rate.

16. If the etch rate can be controlled to within 10 %, the polysilicon may be etched 10 % longer or for an equivalent thickness of 40 nm. The selectivity is therefore

40 nm/1 nm $= 40$.

17. Assuming a 30% overetching, and that the selectivity of Al over the photoresist maintains 3. The minimum photoresist thickness required is

$$(1+ 30\%) \times 1 \ \mu m/3 = 0.433 \ \mu m = 433.3 \ nm.$$

18. $$w_e = \frac{qB}{m_e}$$

$$2p \times 2.45 \times 10^9 = \frac{1.6 \times 10^{-19} \times B}{9.1 \times 10^{-31}}$$

$$B = 8.75 \times 10^{-2} (\text{tesla})$$

$$= 875 \ (\text{gauss}).$$

19. Traditional RIE generates low-density plasma (10^9 cm^{-3}) with high ion energy. ECR and ICP generate high-density plasma (10^{11} to 10^{12} cm^{-3}) with low ion energy. Advantages of ECR and ICP are low etch damage, low microloading, low aspect-ratio dependent etching effect, and simple chemistry. However, ECR and ICP systems are more complicated than traditional RIE systems.

20. The corrosion reaction requires the presence of moisture to proceed. Therefore, the first line of defense in controlling corrosion is controlling humidity. Low humidity is essential,. especially if copper containing alloys are being etched. Second is to remove as much chlorine as possible from the wafers before the wafers are exposed to air. Finally, gases such as CF_4 and SF_6 can be used for fluorine/chlorine exchange reactions and polymeric encapsulation. Thus, Al-Cl bonds are replaced by Al-F bonds. Whereas Al-Cl bonds will react with ambient moisture and start the corrosion process , Al-F bonds are very stable and do not react. Furthermore, fluorine will not catalyze any corrosion reactions.

CHAPTER 13

1. E_a(boron) = 3.46 eV, D_0 = 0.76 cm²/sec

   From Eq. 6, $\quad D = D_0 \exp(\dfrac{-E_a}{kT}) = 0.76 \exp\left(\dfrac{-3.46}{8.614 \times 10^{-5} \times 1223}\right) = 4.142 \times 10^{-15}$ cm²/s

   $L = \sqrt{Dt} = \sqrt{4.142 \times 10^{-15} \times 1800} = 2.73 \times 10^{-6}$ cm

   From Eq. 9, $C(x) = C_s \text{erfc}\,(\dfrac{x}{2L}) = 1.8 \times 10^{20} \text{erfc}\left(\dfrac{x}{5.46 \times 10^{-6}}\right)$

   If $x = 0$, $C(0) = 1.8 \times 10^{20}$ atoms /cm³ ; $x = 0.05 \times 10^{-4}$, $C(5 \times 10^{-6}) = 3.6 \times 10^{19}$

   atoms/cm³; $x = 0.075 \times 10^{-4}$, C(7.5×10-6) = 9.4 ×10¹⁸ atoms/cm³; $x = 0.1 \times 10^{-4}$,

   $C(10^{-5}) = 1.8 \times 10^{18}$ atoms/cm³;

   $x = 0.15 \times 10^{-4}$, $C(1.5 \times 10^{-5}) = 1.8 \times 10^{16}$ atoms/cm³.

   The $x_j = 2\sqrt{Dt}\,(\text{erfc}^{-1}\dfrac{C_{sub}}{C_s}) = 0.15\,\pmb{m}$m

   Total amount of dopant introduced $= Q(t)$

   $= \dfrac{2}{\sqrt{\pmb{p}}} C_s L = 5.54 \times 10^{14}$ atoms/cm².

2. $D = D_0 \exp\left(\dfrac{-E_a}{kT}\right) = 0.76 \exp\left(\dfrac{-3.46}{8.614 \times 10^{-5} \times 1323}\right) = 4.96 \times 10^{-14}$ cm²/s

   From Eq. 15, $C_s = C(0,t) = \dfrac{S}{\sqrt{\pmb{p}Dt}} = 2.342 \times 10^{19}$ atoms/cm³

   $C(x) = C_s \text{erfc}\left(\dfrac{x}{2L}\right) = 2.342 \times 10^{19} \text{erfc}\left(\dfrac{x}{2.673 \times 10^{-5}}\right)$

   If $x = 0$, C(0) = 2.342 × 10¹⁹ atoms/cm³; $x = 0.1 \times 10^{-4}$, $C(10^{-5}) = 1.41 \times 10^{19}$ atoms/cm³;

   $x = 0.2 \times 10^{-4}$, $C(2 \times 10^{-5}) = 6.79 \times 10^{18}$ atoms/cm³; $x = 0.3 \times 10^{-4}$, $C(3 \times 10^{-5}) = 2.65 \times 10^{18}$

atoms/cm^3;

$x = 0.4 \times 10^{-4}$, $C(4 \times 10^{-5}) = 9.37 \times 10^{17}$ atoms/cm^3; $x = 0.5 \times 10^{-4}$, $C(5 \times 10^{-5}) = 1.87 \times 10^{17}$ atoms/cm^3;

$x = 0.6 \times 10^{-4}$, $C(6 \times 10^{-5}) = 3.51 \times 10^{16}$ atoms/cm^3; $x = 0.7 \times 10^{-4}$, $C(7 \times 10^{-5}) = 7.03 \times 10^{15}$ atoms/cm^3;

$x = 0.8 \times 10^{-4}$, $C(8 \times 10^{-5}) = 5.62 \times 10^{14}$ atoms/cm^3.

The $x_j = \sqrt{4Dt \ln \dfrac{S}{C_B \sqrt{\pi Dt}}} = 0.72 \ \mu m$.

3. $1 \times 10^{15} = 1 \times 10^{18} \exp\left(\dfrac{10^{-8}}{4 \times 2.3 \times 10^{-13} t}\right)$

   t = 1573 s = 26 min

   For the constant-total-dopant diffusion case, Eq. 15 gives $C_S = \dfrac{S}{\sqrt{\pi Dt}}$

   $S = 1 \times 10^{18} \sqrt{\pi \times 2.3 \times 10^{-13} \times 1573} = 3.4 \times 10^{13}$ atoms/cm^2.

4. The process is called the ramping of a diffusion furnace. For the ramp-down situation, the furnace temperature T is given by

   $$T = T_0 - rt$$

   where T_0 is the initial temperature and r is the linear ramp rate. The effective Dt product during a ramp-down time of t_1 is given by

   $$(Dt)_{eff} = \int_0^{t_1} D(t)dt$$

   In a typical diffusion process, ramping is carried out until the diffusivity is negligibly small. Thus the upper limit t_1 can be taken as infinity:

   $$\frac{1}{T} = \frac{1}{T_0 - rt} \approx \frac{1}{T_0}\left(1 + \frac{rt}{T_0} + ...\right)$$

and

$$D = D_0 \exp\left(-E_a\Big/kT\right) = D_0 \exp\left[\frac{-E_a}{kT_0}(1 + \frac{rt}{T_0} + ...)\right] = D_0 (\exp\frac{-E_a}{kT_0})(\exp\frac{-rE_a t}{kT_0{}^2}...) \approx D(T_0) \exp\frac{-rE_a t}{kT_0{}^2}$$

where $D(T_0)$ is the diffusion coefficient at T_0. Substituting the above equation into the expression for the effective Dt product gives

$$(Dt)_{eff} \approx \int_0^\infty D(T_0) \exp\frac{-rE_a t}{kT_0{}^2} dt = D(T_0)\frac{kT_0{}^2}{rE_a}$$

Thus the ramp-down process results in an effective additional time equal to $kT_0{}^2/rE_a$ at the initial diffusion temperature T_0.

For phosphorus diffusion in silicon at 1000°C, we have from Fig. 4:

$$D(T_0) = D(1273\text{ K}) = 2\times 10^{-14} \text{ cm}^2/\text{s}$$

$$r = \frac{1273 - 773}{20 \times 60} = 0.417 K/s$$

$$E_a = 3.66 \text{ eV}$$

Therefore, the effective diffusion time for the ramp-down process is

$$\frac{kT_0{}^2}{rE_a} = \frac{1.38 \times 10^{-23}(1273)^2}{0.417(3.66 \times 1.6 \times 10^{-19})} = 91s \approx 1.5\text{min} \ .$$

5. For low-concentration drive-in diffusion, the diffusion is given by Gaussian distribution. The surface concentration is then

$$C(0,t) = \frac{S}{\sqrt{\pi Dt}} = \frac{S}{\sqrt{\pi D_0 t}}\exp\left(\frac{E_a}{2kT}\right)$$

$$\frac{dC}{dt} = \frac{S}{\sqrt{\pi D_0}}\exp\left(\frac{E_a}{2kT}\right)\left(\frac{-t^{3/2}}{2}\right) = -0.5 \times \frac{C}{t}$$

or $\dfrac{dC}{C} = -0.5 \times \dfrac{dt}{t}$

which means 1% change in diffusion time will induce 0.5% change in surface

concentration.

$$\dfrac{dC}{dT} = \dfrac{S}{\sqrt{pD_0 t}} \exp\left(\dfrac{E_a}{2kT}\right)\left(\dfrac{-E_a}{2kT^2}\right) = -C\dfrac{E_a}{2kT^2}$$

or $\dfrac{dC}{C} = \dfrac{-E_a}{2kT} \times \dfrac{dT}{T} = \dfrac{-3.6 \times 1.6 \times 10^{-19}}{2 \times 1.38 \times 10^{-23} \times 1273} \times \dfrac{dT}{T} = -16.9 \times \dfrac{dT}{T}$

which means 1% change in diffusion temperature will cause 16.9% change in

surface concentration.

6. At $1100°C$, $n_i = 6\times10^{18}$ cm^{-3}. Therefore, the doping profile for a surface concentration

of 4×10^{18} cm^{-3} is given by the "intrinsic" diffusion process:

$$C(x,t) = C_s \operatorname{erfc}\left(\dfrac{x}{2\sqrt{Dt}}\right)$$

where $C_s = 4\times 10^{18}$ cm^{-3}, $t = 3$ hr $= 10800$ s, and $D = 5\text{x}10^{-14}$ cm^2/s. The

diffusion length is then

$$\sqrt{Dt} = 2.32\times10^{-5} cm = 0.232\,\mu m$$

The distribution of arsenic is $C(x) = 4\times10^{18} \operatorname{erfc}\left(\dfrac{x}{4.64\times10^{-5}}\right)$

The junction depth can be obtained as follows

$$10^{15} = 4\times10^{18} \operatorname{erfc}\left(\dfrac{x_j}{4.64\times10^{-5}}\right)$$

$x_j = 1.2\times 10^{-4}$ cm $= 1.2$ μm.

7. At $900°C$, $n_i = 2\times 10^{18}$ cm^{-3}. For a surface concentration of 4×10^{18} cm^{-3}, given by

the "extrinsic" diffusion process

$$D = D_0 e^{\frac{-E_a}{kT}} \times \frac{n}{n_i} = 45.8 e^{\frac{-4.05 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 1173}} \times \frac{4 \times 10^{18}}{2 \times 10^{18}} = 3.77 \times 10^{-16} \text{cm}^2/\text{s}$$

$$x_j = 1.6\sqrt{Dt} = 1.6\sqrt{3.77 \times 10^{-16} \times 10800} = 3.23 \times 10^{-6} \text{cm} = 32.3 \text{ nm}.$$

8. Intrinsic diffusion is for dopant concentration lower than the intrinsic carrier concentration n_i at the diffusion temperature. Extrinsic diffusion is for dopant concentration higher than n_i.

9. For impurity in the oxidation process of silicon,

$$\text{segregatio n coefficein t} = \frac{\text{equilibriu m concentrat ion of impurity in silicon}}{\text{equilibriu m concentrat ion of impurity in SiO}_2}.$$

10. $$\boldsymbol{k} = \frac{3 \times 10^{11}}{5 \times 10^{13}} = \frac{3}{500} = 0.006.$$

11. ñ0.5 = ñ1.1 + qF_B/C_i

$$C_{ox} = \frac{\boldsymbol{e}_s}{d} = \frac{3.9 \times 8.85 \times 10^{-14}}{10^{-6}} = 3.45 \times 10^{-7}$$

$$F_B = \frac{3.45 \times 10^{-7}}{1.6 \times 10^{-19}} \times 0.6 = 1.3 \times 10^{12} \text{cm}^{-2}$$

$$\frac{10^{-5}}{\boldsymbol{p}(10.16)^2} t = 1.3 \times 10^{12} \times 1.6 \times 10^{-19}$$

the implant time $t = 6.7$ s.

12. The ion dose per unit area is

$$\frac{N}{A} = \frac{\frac{It}{q}}{A} = \frac{\frac{10 \times 10^{-6} \times 5 \times 60}{1.6 \times 10^{-19}}}{\boldsymbol{p} \times (\frac{10}{2})^2} = 2.38 \times 10^{12} \text{ ions/cm}^2$$

From Eq. 25 and Example 3, the peak ion concentration is at $x = R_p$. Figure. 17 indicates the σ_p is 20 nm.

Therefore, the ion concentration is

$$\frac{S}{s_p \sqrt{2p}} = \frac{2.38 \times 10^{12}}{20 \times 10^{-7} \sqrt{2p}} = 4.74 \times 10^{17} \, \text{cm}^{-3}.$$

13. From Fig. 17, the $R_p = 230$ nm, and $\sigma_p = 62$ nm.

The peak concentration is

$$\frac{S}{s_p \sqrt{2p}} = \frac{2 \times 10^{15}}{62 \times 10^{-7} \sqrt{2p}} = 1.29 \times 10^{20} \, \text{cm}^{-3}$$

From Eq. 25,

$$10^{15} = 1.29 \times 10^{20} \exp\left[\frac{-(x_j - R_p)^2}{2s_p^2}\right]$$

$x_j = 0.53$ µm.

14. Dose per unit area $= \dfrac{Q}{q} = \dfrac{C_0 \Delta V_T}{q} = \dfrac{3.9 \times 8.85 \times 10^{-14} \times 1}{250 \times 10^{-8} \times 1.6 \times 10^{-19}} = 8.6 \times 10^{11} \, \text{cm}^{-2}$

From Fig. 17 and Example 3, the peak concentration occurs at 140 nm from the

surface. Also, it is at (140-25) = 115 nm from the Si-SiO$_2$ interface.

15. The total implanted dose is integrated from Eq. 25

$$Q_T = \int_0^\infty \frac{S}{s_p \sqrt{2p}} \exp\left[\frac{-(x - R_p)^2}{2s_p^2}\right] dx = \frac{S}{2}\left\{1 + \left[1 - \text{erfc}\left(\frac{R_p}{s_p \sqrt{2}}\right)\right]\right\} = \frac{S}{2}[2 - \text{erfc}\,(2.3)] = \frac{S}{2} \times 1.9989$$

The total dose in silicon is as follows ($d = 25$ nm):

$$Q_{Si} = \int_d^\infty \frac{S}{s_p \sqrt{2p}} \exp\left[\frac{-(x - R_p)^2}{2s_p^2}\right] dx = \frac{S}{2}\left\{1 + \left[1 - \text{erfc}\left(\frac{R_p - d}{s_p \sqrt{2}}\right)\right]\right\} = \frac{S}{2}[2 - \text{erfc}\,(1.87)] = \frac{S}{2} \times 1.9918$$

the ratio of dose in the silicon $= Q_{Si}/Q_T = 99.6\%$.

16.   The projected range is 150 nm (see Fig. 17).

The average nuclear energy loss over the range is 60 eV/nm (Fig. 16).

$60 \times 0.25 = 15$ eV (energy loss of boron ion per each lattice plane)

the damage volume $= V_D = \pi (2.5 \text{ nm})^2 (150 \text{ nm}) = 3 \times 10^{-18} \text{ cm}^3$

total damage layer $= 150/0.25 = 600$

displaced atom for one layer $= 15/15 = 1$

damage density $= 600/V_D = 2 \times 10^{20} \text{ cm}^{-3}$

$2 \times 10^{20}/5.02 \times 10^{22} = 0.4\%.$

17.   The higher the temperature, the faster defects anneal out. Also, the solubility of

electrically active dopant atoms increases with temperature.

18. $\Delta V_t = 1 \text{ V} = \dfrac{Q_1}{C_{ox}}$

where Q_1 is the additional charge added just below the oxide-semiconductor surface by ion implantation. C_{OX} is a parallel-plate capacitance per unit area given by $C_{ox} = \dfrac{e_s}{d}$

(d is the oxide thickness, e_s is the permittivity of the semiconductor)

$$Q_1 = \Delta V_t C_{ox} = \frac{1V \times 3.9 \times 8.85 \times 10^{-14} \text{F/cm}}{0.4 \times 10^{-6} \text{cm}} = 8.63 \times 10^{-7} \frac{\text{C}}{\text{cm}^2}$$

$$\frac{8.63 \times 10^{-7}}{1.6 \times 10^{-19}} = 5.4 \times 10^{12} \text{ ions/cm}^2$$

$$\text{Total implant dose} = \frac{5.4 \times 10^{12}}{45\%} = 1.2 \times 10^{13} \text{ ions/cm}^2.$$

19.   The discussion should mention much of Section 13.6. Diffusion from a surface film avoids problems of channeling. Tilted beams cannot be used because of shadowing problems. If low energy implantation is used, perhaps with preamorphization by silicon, then to keep the junctions shallow, RTA is also

necessary.

20. From Eq.35

$$\frac{S_d}{S} = \frac{1}{2}\text{erfc}\left(\frac{0.4 - 0.6}{0.2\sqrt{2}}\right) = 0.84$$

The effectiveness of the photoresist mask is only 16%.

$$\frac{S_d}{S} = \frac{1}{2}\text{erfc}\left(\frac{1 - 0.6}{0.2\sqrt{2}}\right) = 0.023$$

The effectiveness of the photoresist mask is 97.7%.

21. $$T = \frac{1}{2\sqrt{p}}\frac{e^{-u^2}}{u} = 10^{-5}$$

$$\therefore u = 3.02$$

$$d = R_p + 4.27\,s_p = 0.53 + 4.27 \times 0.093 = 0.927 \ \mu m.$$

CHAPTER 14

1. Each U-shape section (refer to the figure) has an area of 2500 μm $\times$ 8 μm $= 2 \times$ 10^4 μm^2. Therefore, there are $(2500)^2/2 \times 10^4 = 312.5$ U-shaped section. Each section contains 2 long lines with 1248 squares each, 4 corner squares, 1 bottom square, and 2 half squares at the top. Therefore the resistance for each section is

$$1 \text{ k}\Omega \text{ /}^{\circ} \text{ } (1248 \times 2 + 4 \times 0.65 + 2) = 2500.6 \text{ k}\Omega$$

The maximum resistance is then

$$312.5 \times 2500.6 = 7.81 \times 10^8 \text{ } \Omega = 781 \text{ M}\Omega$$
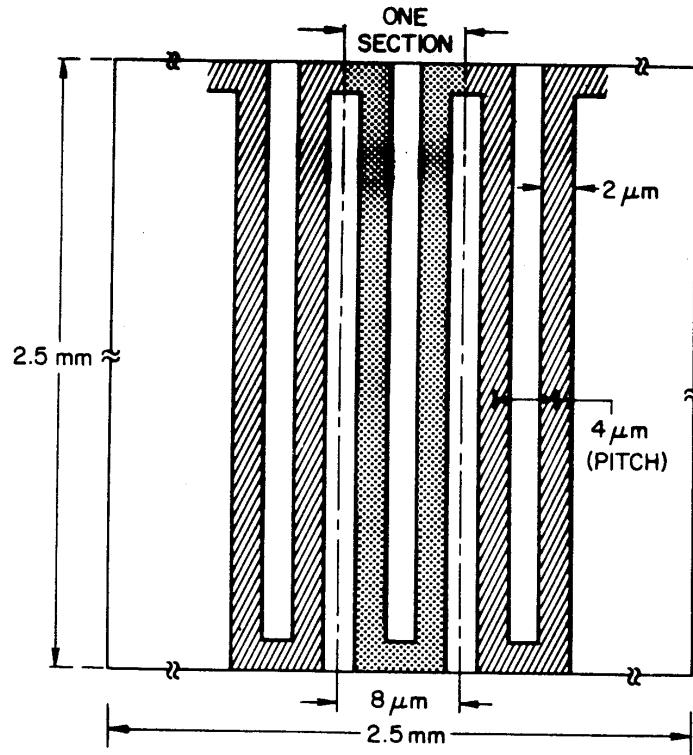
2. The area required on the chip is

$$A = \frac{C_d}{e_{ox}} = \frac{(30 \times 10^{-7})(5 \times 10^{-12})}{3.9 \times 8.85 \times 10^{-14}} = 4.35 \times 10^{-5} \text{ cm}^2$$

$$= 4.35 \times 10^3 \text{ } \mu\text{m}^2 = 66 \times 66 \text{ } \mu\text{m}$$

Refer to Fig.4a and using negative photoresist of all levels

    (a) Ion implantation mask (for p^+ implantation and gate oxide)

    (b) Contact windows (2×10 µm)

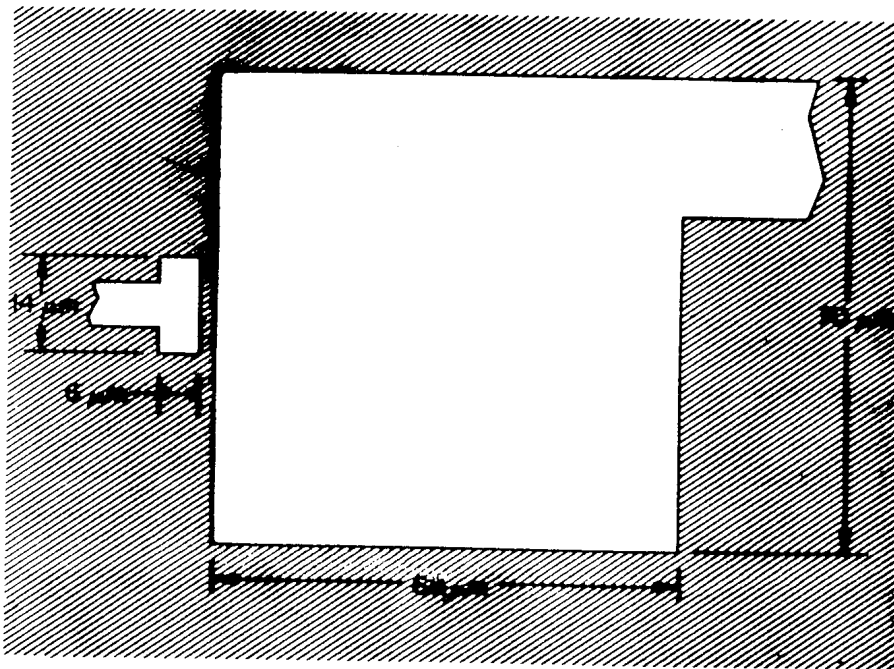   (c) Metallization mask (using Al to form ohmic contact in the contact window and form the MOS capacitor).

Because of the registration errors, an additional 2 µm is incorporated in all critical dimensions.
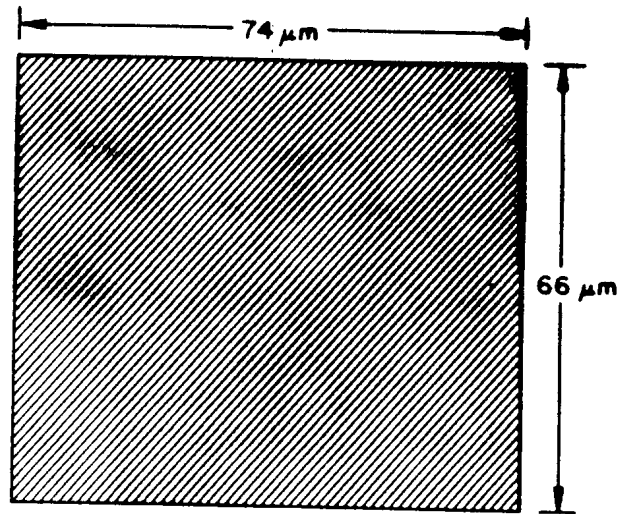
(a)
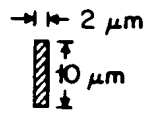
(b)

(c)

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3. If the space between lines is 2 μm, then there is 4 μm for each turn (i.e., 2×n, for

one turn). Assume there are n turns, from Eq.6, $L \approx m_0 n^2 r \approx 1.2 \times 10^{-6} n^2 r$, where r

can be replaced by 2 × n. Then, we can obtain that n is 13.

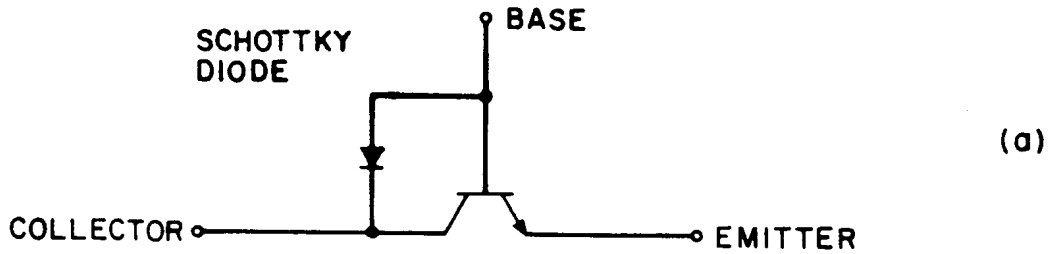4. (a) Metal 1, (b) contact hole, (c) Metal 2.

(a) Metal 1,

(b) contact hole,

(c) Metal 2.
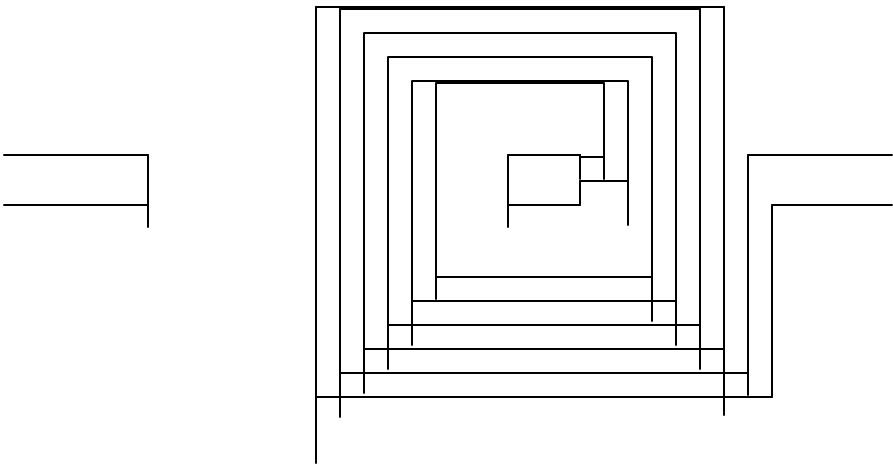
5. The circuit diagram and device cross-section of a clamped transistor are shown in

    (a) and (b), respectively.
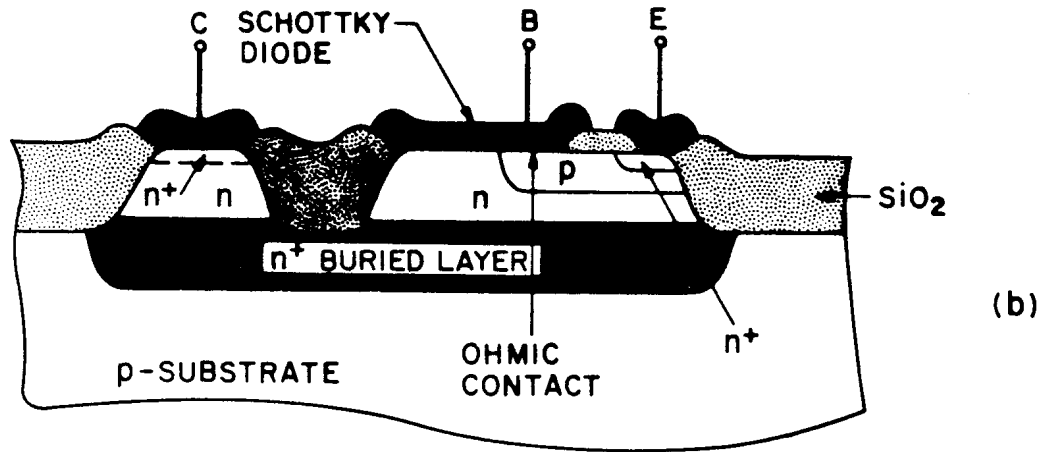
6. (a) The undoped polysilicon is used for isolation.

    (b) The polysilicon 1 is used as a solid-phase diffusion source to form the

        extrinsic base region and the base electrode.

    (c) The polysilicon 2 is used as a solid-phase diffusion source to form the emitter region

        and the emitter electrode.

7. (a) For 30 keV boron, $R_p = 100$ nm and $\Delta R_p = 34$ nm. Assuming that R_p and ΔR_p

    for boron are the same in Si and SiO$_2$ the peak concentration is given by

$$\frac{S}{\sqrt{2p}\Delta R_p} = \frac{8\times10^{11}}{\sqrt{2p}(34\times10^{-7})} = 9.4\times10^{16} \text{ cm}^{-3}$$

The amount of boron ions in the silicon is

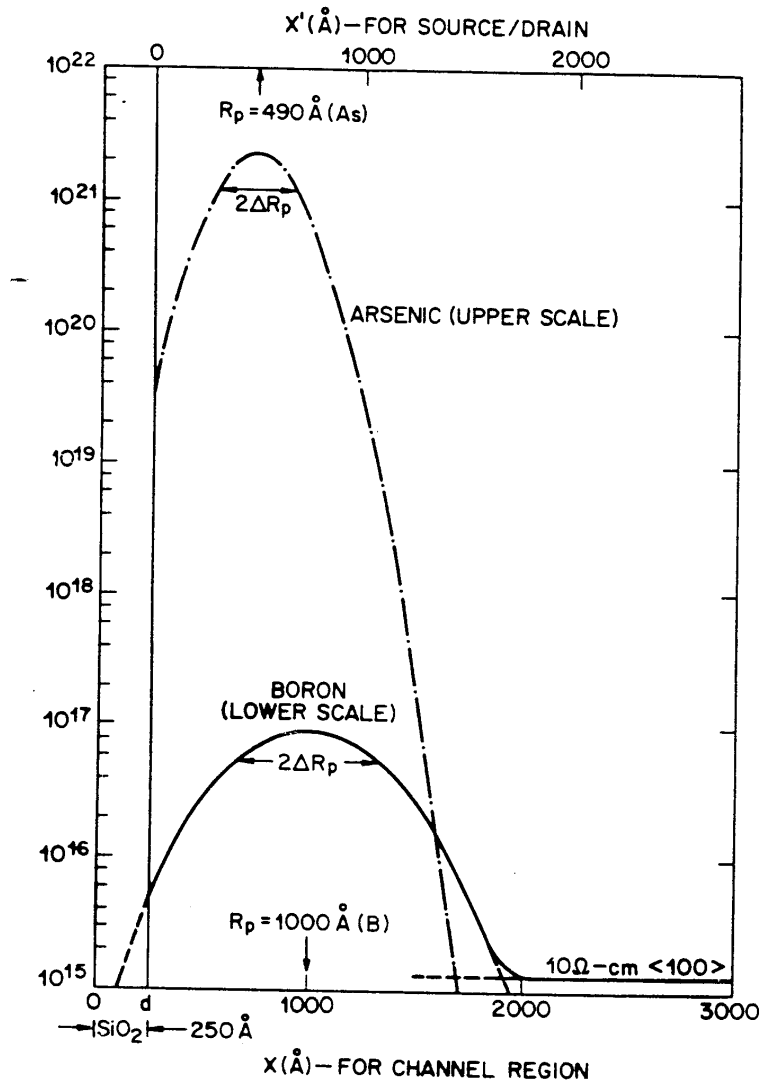$$\frac{Q}{q} = \int_d^\infty \frac{S}{\sqrt{2\pi}\Delta R_p} \exp\left[-\frac{(x-R_p)^2}{2\Delta R_p^2}\right] dx$$

$$= \frac{S}{2}\left[2 - \text{erfc}\left(\frac{R_p - d}{\sqrt{2}\Delta R_p}\right)\right]$$

$$= \frac{8\times10^{11}}{2}\left[2 - \text{erfc}\left(\frac{750}{\sqrt{2}\times340}\right)\right]$$

$$= 7.88\times10^{11}\,\text{cm}^{-2}$$

Assume that the implanted boron ions form a negative sheet charge near the Si-SiO$_2$ interface, then

$$\Delta V_T = q\left(\frac{Q}{q}\right)/C_{ox} = \frac{1.6\times10^{-19}\times(7.88\times10^{11})}{3.9\times8.85\times10^{-14}/(25\times10^{-7})} = 0.91 \text{ V}$$

(b) For 80 keV arsenic implantation, R_p = 49 nm and ΔR_p = 18 nm. The peak arsenic

concentration is $\dfrac{S}{\sqrt{2\pi}\Delta R_p} = \dfrac{10^{16}}{\sqrt{\pi}\times(18\times10^{-7})} = 2.21\times10^{21} \text{ cm}^{-3}$.

X'(Å)—FOR SOURCE/DRAIN

$R_p = 490\,\text{Å}$ (As)

$2\Delta R_p$

ARSENIC (UPPER SCALE)

BORON (LOWER SCALE)

$2\Delta R_p$

$R_p = 1000\,\text{Å}$ (B)

$10\,\Omega$–cm <100>

SiO₂ — 250 Å

X(Å)—FOR CHANNEL REGION

8. (a) Because (100)-oriented silicon has lower (~ one tenth) interface-trapped charge and a lower fixed oxide charge.

(b) If the field oxide is too thin, it may not provide a large enough threshold voltage for adequate isolation between neighboring MOSFETs.

(c) The typical sheet resistance of heavily doped polysilicon gate is 20 to 30 Ω /°, which is adequate for MOSFETs with gate lengths larger than 3 μm. For shorter gates, the sheet resistance of polysilicon is too high and will cause large RC delays. We can use refractory metals (e.g., Mo) or silicides as the gate material to reduce the sheet resistance to about 1 Ω /°.

(d) A self-aligned gate can be obtained by first defining the MOS gate structure, then using the gate electrode as a mask for the source/drain implantation. The self-aligned gate can minimize parasitic capacitance caused by the source/drain regions extending underneath the gate electrode (due to diffusion or misalignment).

(e) P-glass can be used for insulation between conducting layers, for diffusion and ion implantation masks, and for passivation to protect devices from impurities, moisture, and scratches.

9. The lower insulator has a dielectric constant $e_1/e_0 = 4$ and a thickness $d_1 = 10$ nm The upper insulator has a dielectric constant $e_2/e_0 = 10$ and a thickness $d_2 = 100$ nm. Upon application of a positive voltage V_G to the external gate, electric field E_1 and E_2 are established in the d_1 and d_2 respectively. We have, from Gauss' law, that $\varepsilon_1 E_1 = \varepsilon_2 E_2 + Q$ and $V_G = E_1 d_1 + E_2 d_2$

where Q is the stored charge on the floating gate. From these above two equations, we obtain

$$E_1 = \frac{V_G}{d_1 + d_2(e_1/e_2)} + \frac{Q}{e_1 + e_2(d_1/d_2)}$$

$$J = sE_1 = 10^{-7} \left\{ \frac{10 \times 10^7}{10 + 100\left(\frac{4}{10}\right)} + \frac{Q}{\left[4 + 10\left(\frac{10}{100}\right)\right] \times 8.85 \times 10^{-14}} \right\} = 0.2 - 2.26 \times 10^5 |Q|$$

(a) If the stored charge does not reduce E_1 by a significant amount (i.e., $0.2 >> 2.26 \times 10^5 |Q|$, we can write
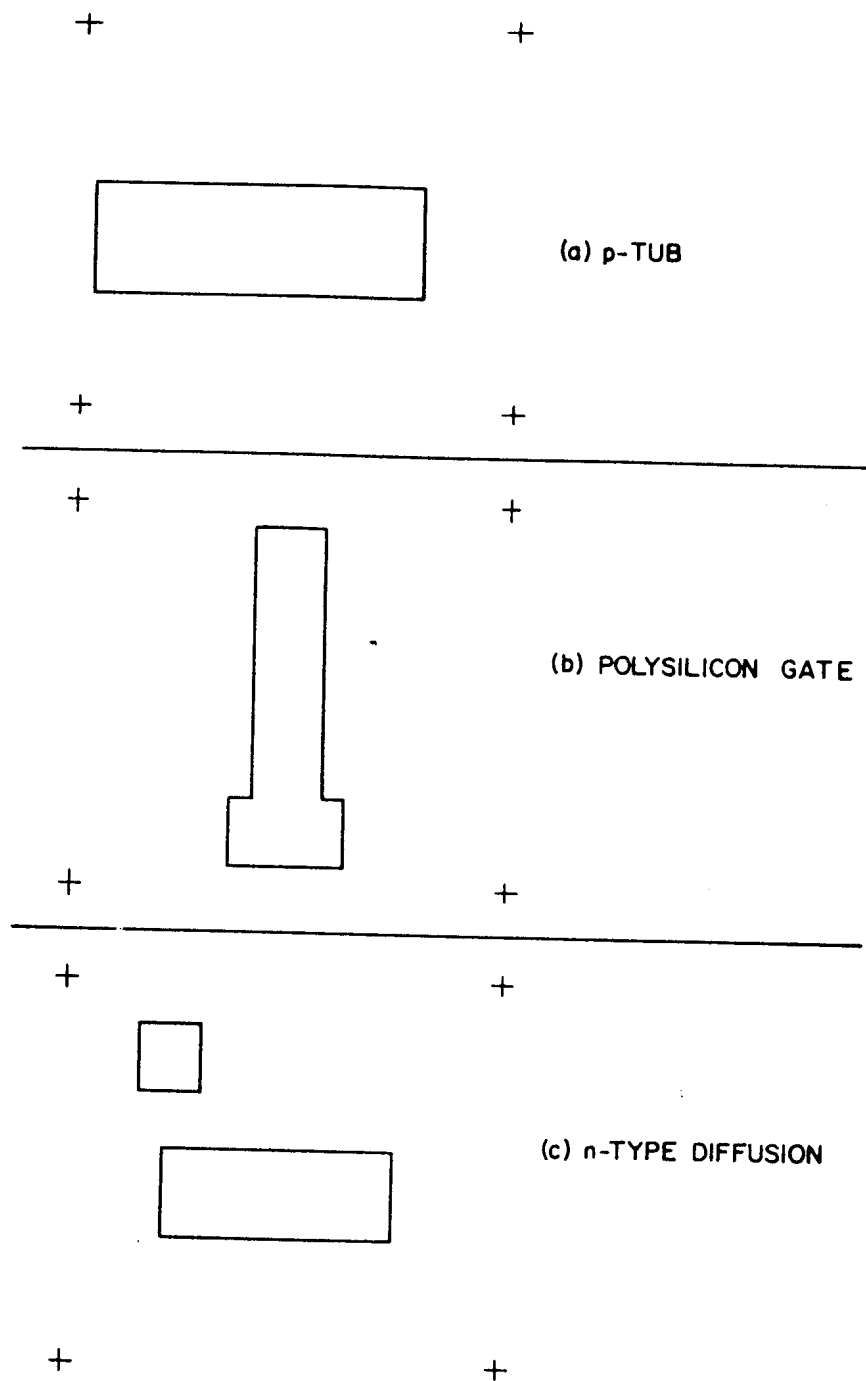
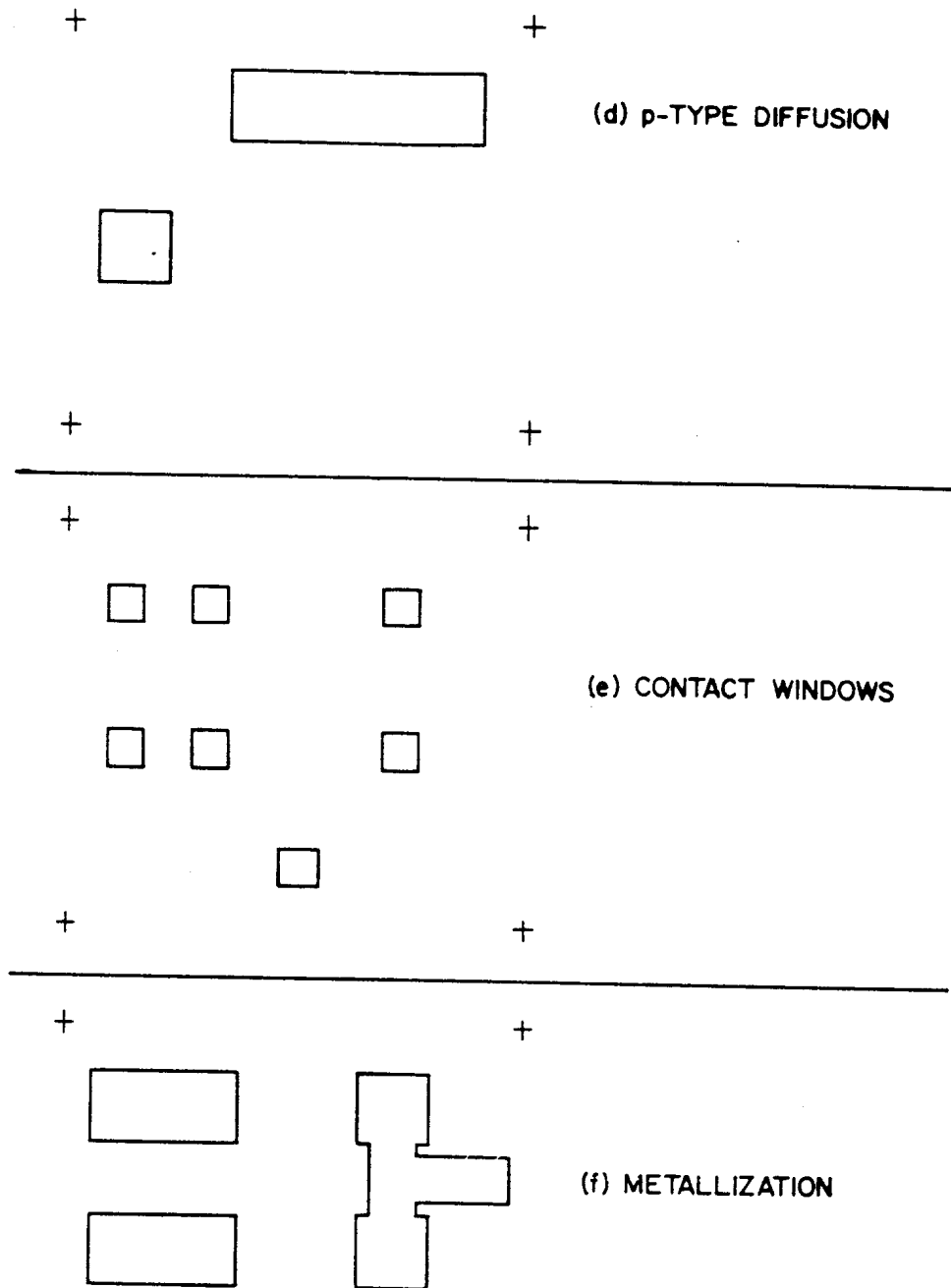$$Q = \int_0^t sE_1 dt' \approx 0.2\Delta t = 0.2 \times \left(0.25 \times 10^{-6}\right) = 5 \times 10^{-8} C$$

$$\Delta V_T = \frac{Q}{C_2} = \frac{5 \times 10^{-8}}{\left(10 \times 8.85 \times 10^{-14}\right)/\left(100 \times 10^{-7}\right)} = 0.565 \text{ V}$$

(b) when $t \to \infty, J \to 0$ we have $|Q| \to 0.2/2.26 \times 10^5 \cong 8.84 \times 10^{-7}$ C.

Then $\Delta V_T = \dfrac{Q}{C_2} = \dfrac{8.84 \times 10^{-7}}{\left(10 \times 8.85 \times 10^{-14}\right)/10^{-5}} = 9.98$ V.

10.

(a) p-TUB

(b) POLYSILICON GATE

(c) n-TYPE DIFFUSION

(d) p-TYPE DIFFUSION

(e) CONTACT WINDOWS

(f) METALLIZATION

11. The oxide capacitance per unit area is given by

$$C_{ox} = \frac{e_{SiO_2}}{d} = 3.5 \times 10^{-7} \, \text{F/cm}^2$$

and the maximum current supplied by the device is

$$I_{DS} \approx \frac{1}{2} \frac{W}{L} \mathbf{m} C_{ox} (V_G - V_T)^2 = \frac{1}{2} \frac{5\,\mathbf{mm}}{0.5\,\mathbf{mm}} 3.5 \times 10^{-7} (V_G - V_T)^2 \approx 5 \, \text{mA}$$

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and the maximum allowable wire resistance is 0.1 V/5 mA, or 20Ω. Then, the length of the wire must be

$$L \leq \frac{R \times Area}{r} = \frac{20\Omega \times 10^{-8} \text{cm}^2}{2.7 \times 10^{-8} \Omega - \text{cm}} = 0.074 \text{ cm}$$

or 740 μm. This is a long distance compared to most device spacing. When driving signals between widely spaced logic blocks however, minimum feature sized lines would not be appropriate.

12.

(a)

(b)

(c)

(d)

(e)

(f)

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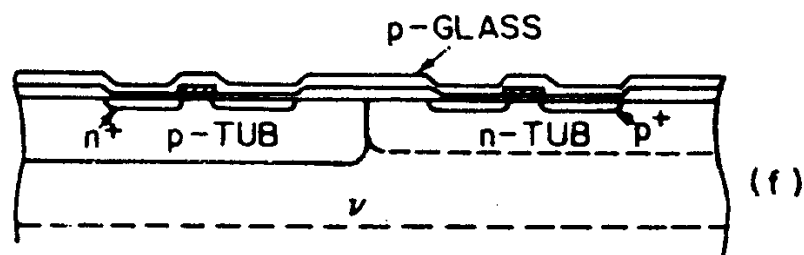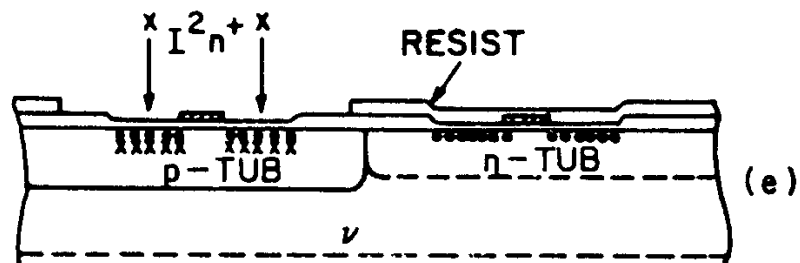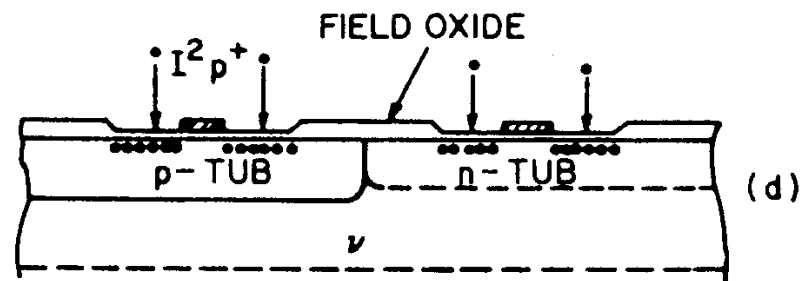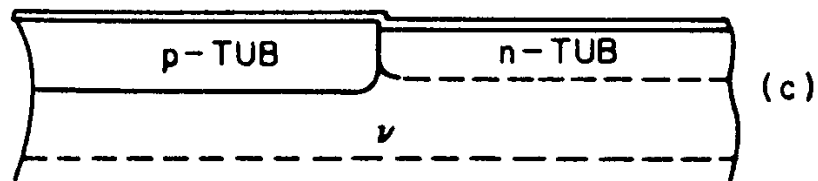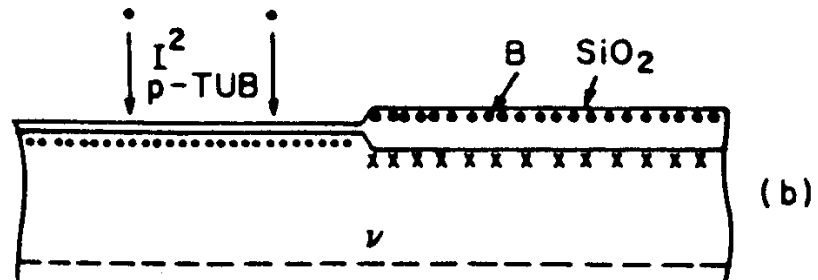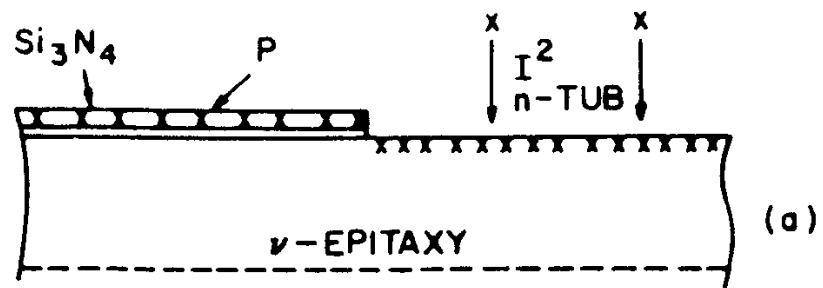
13. To solve the short-channel effect of devices.

14. The device performance will be degraded from the boron penetration. There are methods to reduce this effect: (1) using rapid thermal annealing to reduce the time at high temperatures, consequently reduces the diffusion of boron, (2) using nitrided oxide to suppress the boron penetration, since boron can easily combine with nitrogen and becomes less mobile, (3) making a multi-layer of polysilicon to trap the boron atoms at the interface of each layer.

15. Total capacitance of the stacked gate structure is :

$$C = \frac{e_1}{d_1} \times \frac{e_2}{d_2} \bigg/ \left(\frac{e_1}{d_1} + \frac{e_2}{d_2} \right) = \frac{7}{0.5} \times \frac{25}{10} \bigg/ \left(\frac{7}{0.5} + \frac{25}{10} \right) = 2.12$$

$$\frac{3.9}{d} = 2.12$$

$$\therefore d = \frac{3.9}{2.12} = 1.84 \text{ nm.}$$

16. Disadvantages of LOCOS: (1) high temperature and long oxidation time cause V_T shift, (2) bird's beak, (3) not a planar surface, (4) exhibits oxide thinning effect. Advantages of shallow trench isolation: (1) planar surface, (2) no high temperature processing and long oxidation time, (3) no oxide thinning effect, (4) no bird's beak.

17. For isolation between the metal and the substrate.

18. GaAs lacks of high-quality insulating film.

19. (a)

$$RC = \left(r\frac{L}{A} \right)\left(e_{ox} \frac{A}{d} \right) = \left(10^{-5} \times \frac{1}{1 \times 0.5 \times 10^{-8}} \right)\left[3.9 \times 8.85 \times 10^{-14} \times \frac{1 \times \left(1 \times 10^{-4} \right)}{0.5 \times 10^{-4}} \right]$$
$$= 2000 \times \left(69.03 \times 10^{-14} \right) = 1.38 \times 10^{-9}\,s = 1.38 \text{ ns.}$$

(b) For a polysilicon runner

$$RC = \left(R_{square} \frac{L}{W} \right)\left(e_{ox} \frac{A}{d} \right)$$
$$= 30\left(\frac{1}{10^{-4}} \right)\left(69.03 \times 10^{-14} \right) = 2.07 \times 10^{-7} \text{ s}$$
$$= 207 \text{ ns}$$

Therefore the polysilicon runner's *RC* time constant is 150 times larger than the aluminum runner.

20. When we combine the logic circuits and memory on the chip, we need multiple supply voltages. For reliability issue, different oxide thicknesses are needed for different supply voltages.

21. (a) $\dfrac{1}{C_{total}} = \dfrac{1}{C_{Ta_2O_5}} + \dfrac{1}{C_{nitride}}$

hence $\dfrac{EOT}{3.9} = \dfrac{75}{25} + \dfrac{10}{7} = 17.3$ Å

(b) EOT = 16.7 Å.