More Profit, Less Complaint

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Summary

The present paper aims at providing the optimal overbooking strategy for the Chief-Executive-Officer (CEO) of an aviation corporation. In more detail, its aim is to determine the number of tickets which are overbooked to achieve the maximum possible economic profit while taking various (economic and social) factors into consideration. First of all, the mutual relationship among the elements such as profit, ticket-price, number of passengers, possibility of booking-and-arriving, and the maximum number of tickets which are permissible to reserve, is revealed. Based on this, three typical mathematical models are proposed, which are exposed one after another below.

The first model, which is the simplest one, aims simply at maximizing the single-airliner profit rate for the single-price situation. Considering the number of passengers as a random variable, this model makes use of the average profit (i.e., mathematical expectation) as the measure of the profit rate. Based on this model, we discuss three typical strategies of how to pay the passengers who have booked a specific airliner but have to give up the right for taking that airliner due to the limited transportation capacity of the airplane, respectively.

Although the first model provides a good starting-point for further study, thorough analysis indicates that it deviates from real situations to a certain extent. Due to this and referring to the multiple-price strategy which is commonly used by most aviation corporations, a model is presented based on double-price strategy which is simplification of multiple-price strategy. In order to reduce the economic loss incurred by the compensation for the booking-but-not-arriving customers, this model divides all tickets into two classes: full-price-ticket with which the customer can acquire preferential treatments of some sort such as choosing to take later airliner when he or she misses the reserved flight, and discount-price-ticket, with which there is no any compensation if the customer misses his airliner. Further research of this model reveals the reason why taking the double-price strategy can effectively reduce the potential economic risk borne by aeronautic companies.

Next, a multiple-element optimization model is devised, which simultaneously takes various elements into consideration such as profit rate, honor of corporation, safety, and extent of customers' satisfaction. Based on this model and applying the Analytic Hierarchy Process, a better overbooking strategy is presented. Furthermore, the effect of some main parameters on the optimal overbooking strategy is discussed.

The applications of all the three models to the optimal overbooking strategies are illustrated. Based on the resulting data and taking some practical economic and social elements into consideration, several helpful suggestions and practical strategies are proposed.

Finally, all our conclusions are briefly summarized and are submitted to CEO of an aviation corporation as a reference when making practical overbooking strategy.

BASIC ASSUMPTIONS

I. This model assumes the up-to-date most advanced airliner: Boeing 777, whose maximum transportation amounts to 375 people.
II. Fix the cost for one flight, that is, it does not change with the number of passengers.
III. The passengers arrive for taking the reserved airliner in a statistically independent way.
IV. Any given overbooking level is theoretically achievable.
V. A person, who has made reservation for a specific flight but dose not arrive, is called "active-discarder". A person, who has made reservation for a specific flight but passively gives up the right due to the limited transportation capacity of the airplane, is called "passive-discarder".
VI. The passive-discarders who have the right for later flights have the highest priority.

PROBLEM ANALYSIS

The problem aims at proposing an effective overbooking strategy for maximizing the expected profit of an aviation corporation.

Intuitively, due to the fixed flight cost (assumption 2), an airplane should be fully loaded in order to achieve the maximum possible profit. One may take it as granted that an airplane is fully loaded if its maximum possible reservation amount is achieved. But this is not the case because of the potential possibility that part of reservation maker do not arrive at the airport, leading to reduction of the profit. As a result, the majority of aviation corporations practically adopt some kinds of overbooking mechanism in order to achieve the maximum possible average profit.

One thing has to be considered: it is likely that a small part of reservation-maker can not take the reserved airplane due to the limited transportation capacity and the overbooking mechanism, which leads to both explicit economic loss and implicit honor loss for the corporation, and, even more seriously, endangers the security of the airport. Therefore, the maximum booking level is practically limited.

The present problem is to determine the highest permissible overbooking level such that the corporation can gain the maximum economic profit while at the same time considering the social effect, which is a typical constrained optimization problem.

THE SINGLE-PRICE MODEL

First, a simplified model is proposed which assumes that all airplane tickets have exactly the same price. The design problem of overbooking strategy is expressed as the following optimization problem:

$$\text{Maximize } (C(m)\,/\,f) \tag{1}$$

where m denotes the maximum permissible reservation amount for a specific flight; $C(m)$ denotes the average profit corresponding to m; f denotes the cost for a single flight. A reasonable formula for $C(m)$ is of the form

$$C(m) = \sum_{k=0}^{m-N-1} P_k[(Ng - f) - (m-k-N)b] + \sum_{k=m-N}^{m} P_k[(m-k)g - f] \tag{2}$$

where b denotes the compensation fee for each passive-discarder; k denotes the amount of active-discarders; P_k denotes the probability of occurrence of the event that there are totally k active-discarders; g denotes each passenger's traveling fee; and N denoted the capacity of the airplane. This formula takes into consideration both the cases when the number of practical passengers, $m - k$, exceeds or is less than the airport capacity N. In practice, the amount of active-discarders k is approximately subject to a binomial distribution with parameters m and q, that is,

$$P_k = C_m^k q^k (1-q)^{m-k} \tag{3}$$

where q denotes the probability of occurrence of the event that any reservation-maker actively gives up the right for taking the reserved flight. $\sum_{k=0}^{m} kP_k$ denotes the expectation of k, so $\sum_{k=0}^{m} kP_k$ = mq. It is well-known that a binomial distribution can be numerically well approximated by a Poisson distribution when (a) q is very small, (b) m is very large, and (c) mq is medium-sized. By replacing $C(m)$ in (1) by (2), the objective function in (1) is converted into the form

$$\frac{C(m)}{f} = \frac{g}{f}\left[(1-q)m - (\frac{b}{g}+1)\sum_{k=0}^{m-N-1} P_k (m-N-k)\right] - 1 \tag{4}$$

which depends on the parameters g, b, f, m, q, and N. The three factors g, b, and f, are usually specified by IATA, N is a performance parameter of the airplane, and q is a natural parameter, indicating that they can not be determined by the aviation corporation. In fact, only the parameter m can be determined by the corporation. The value of m when achieving the maximum profit can be found through an exhaustive search over all possible values of m.

For instance, a Boeing 777 is typically assumed to meet the conditions: $N = 375$, $f / g = 0.6N$, $b / g = 0.2$. The optimization problem (1) is solved. Four typical values of q, and the results are listed in Table I.

Table I: The results obtained by solving the optimization problem for different values of q

q	**Expected profit**	**Maximum Reservation Amount** m
0.050	**0.6605**	**399**
0.075	**0.6591**	**411**
0.100	**0.6577**	**425**
0.125	**0.6566**	**437**
0.150	**0.6555**	**450**

THINKING OVER THE PARAMETER *b*

Now let us make a somewhat detailed discussion on the parameter b in the previously presented model. Below are three typical compensation strategies corresponding to different values of b.

The first compensation strategy simply assumes $b = 0$, implying that the passive-discarders can not acquire any form of compensation. This strategy has the following obvious disadvantages: (1) It is harmful to the honour of the corporation, leading to potential economic loss; (2) It may incur chaos and even threaten the safety of the airport.

The second strategy allows the active-discarders to choose to take a later flight. This means that part of the expected profit of the corporation is decreased by half, i.e., $b = 0.5g$, which is a sort of high compensation strategy.

The third strategy pays each passive-discarder a sum of money. In reference to international conventions, $b = 0.2g$.

The similarity of the last two compensation strategies lies in that the active-discarders will be compensated in some way. To gain an insight into their distinction, let us consider the case when $N = 375$, $f / g = 0.6N$, and $q = 0.05$. The related data for the two compensation strategies are listed in Table II

Table II The effect of two typical compensation strategies

Compensation Strategy	Average Profit	Average Passive Discarders
b=0.5g	**0.6557**	**2.4494**
b=0.2g	**0.6605**	**4.5271**

It can be seen from Table II that the average profits are near for the two compensation strategies, but the second strategy is superior to the third one due to its lower average passive discarders. In fact, the second compensation strategy is widely applied in aviation corporations.

THE DOUBLE-PRICE MODEL

The first model assumes that all tickets have exactly the same price. In practice, nearly all aviation corporations adopt multiple-price strategy. Now let us present a more practical double-price strategy below:

There are two kinds of tickets available: full-price ticket and discount ticket. A full-price-ticket reservation-maker could still choose to take another flight even if he actively misses the right flight. In contrast, a discount-ticket reservation-maker could not get any compensation if he actively gives up the right for taking the reserved flight. In our model, a discount-ticket reservation-maker is assumed to always arrive at the airport in time. Based on this, our double-price model is expressed in the form

$$\text{Maximize } (C(m) / f) \tag{6}$$

with

$$C(m) = \sum_{k=0}^{m-N-1} P_k[Ng - j(1-r)g - f - (m-k-N)b] + \sum_{k=m-N}^{m} P_k[(m-k)g - j(1-r)g - f] \tag{7}$$

where j denotes the amount of discount-ticket reservation-maker, r is the discount rate for the discount ticket, and all the remaining parameters are of the same meaning as those in the single-price model. Due to the assumption that all the k active-discarders fall into the set of the $m - j$ full-price-ticket reservation makers, the probability of occurrence of the event that there are k active-discarders is of the form:

$$P_k = C_{m-j}^{k} q^{k} (1-q)^{m-j-k} . \tag{8}$$

Therefore, the average profit rate is

$$\frac{E(m)}{f} = \frac{g}{f}[pm-(1-r)j-(1+\frac{b}{g})\sum_{k=0}^{m-N-1} P_k (m-N-k)]-1 . \tag{9}$$

The analysis and simulation of the double-price strategy are similar to those of the single-price strategy, and hence are omitted.

MULTIPLE ELEMENTS OPTIMIZATION MODEL

Both the previously mentioned two models are high simplification of the real cases. The practical model should reflect more elements such as social effect and the total number of flights. Since there are a large number of elements to be considered, the traditional analytic hierarchy process (AHP) is helpful for the design of the model. The principle of AHP is to construct a hierarchical diagram, which represents the top-down successive refinement process for the analysis of the effect of various elements on the objective function where the effect of a specific element is denoted by a weighing factor. For our case, the AHP diagram is shown in Fig.1:

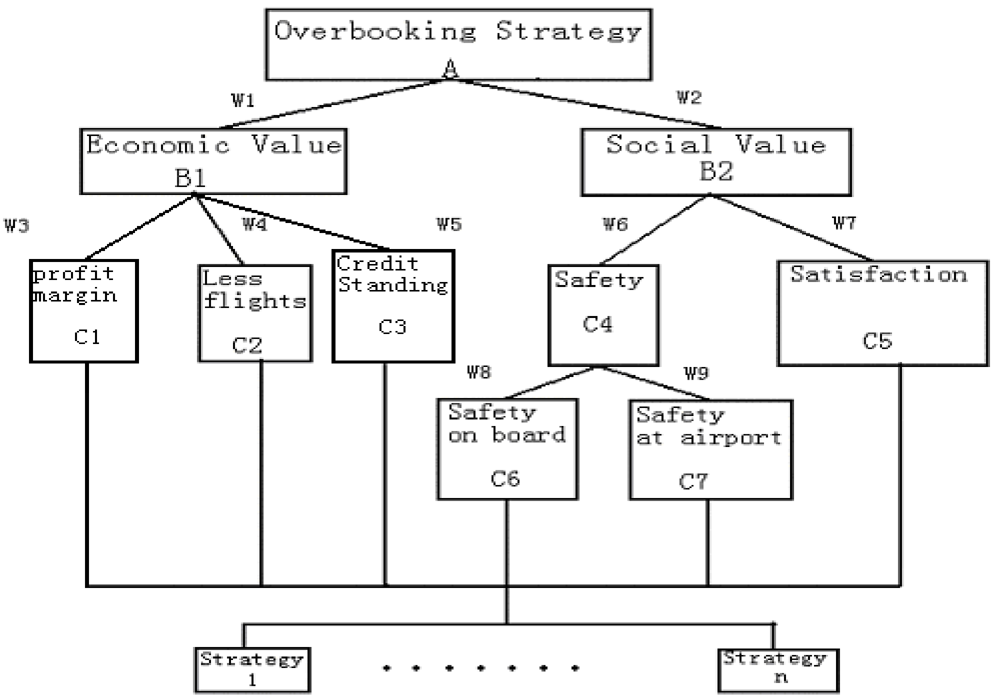


Fig.1. The AHP diagram for our model

where

A------------------------------- strategy goodness

B_1 --------------------------------economic element

B_2 --------------------------------social element

C_1--------------------------------single flight element

C_2--------------------------------number of flights element

C_3--------------------------------honour of corporation element
C_4--------------------------------safety element
C_5--------------------------------satisfaction of customer
C_6--------------------------------airplane safety
C_7--------------------------------airport safety

Obviously, such a more complicated model has remarkable advantages such as the huge flexibility of choosing more practical elements to consider. By the way, all the elements should be normalized before numerical calculations. In more detail, the AHP model is formulated below:

$$\text{Maximize } A(m) = W_3\times W_1\times C_1+W_1\times W_4\times C_2+W_1\times W_5\times C_3+ W_2\times W_6\times W_8\times C_6+W_2\times W_6\times W_9\times C_7+W_2\times W_7\times C_5 \quad (10)$$

$$s.t. \quad A = W_1\times B_1+W_2\times B_2 \quad (11)$$

$$B_1 = W_3\times C_1+W_4\times C_2+W_5\times C_3 \quad (12)$$

$$B_2 = W_6\times C_4+W_7\times C_5 \quad (13)$$

$$C_4 = W_8\times C_6+W_9\times C_7 \quad (14)$$

$$W_1+W_2=1,\ W_3+W_4+W_5=1,\ W_6+W_7=1,\ W_8+W_9=1 \quad (15)$$

$$0\le W_i\le 1 \quad i=1,2\ldots 9 \quad (16)$$

$$m > N, \quad (17)$$

where the meanings of all the parameters are stated below:

(a) W_i are the weighing factors of the parameters, which reflect their effect on the objective function and are assigned empirically or according to practical applications. For convenience, let us introduce several related notions:

element a and b are identically important: $W_a/W_b=1$;
element a is slightly more important than element b: $1\le W_a/W_b<1.3$;
element a is significantly more important than element b: $1.3\le W_a/W_b<4$;
element b can be neglected compared to element a: $W_a/W_b \ge 4$;

where W_a and a W_b are the weighing factor for the two elements a and b, respectively. According to (15), all the W_i can be evaluated when some ratios of them are given. An example is like this: given that $W_1/W_2=2.5$, $W_3/W_4=3$, $W_4/W_5=1.2$, $W_6/W_7=2$, and $W_8/W_9=1$, the values of all the factors can be derived, which are listed in Table III.

Table III A set of factors

W_i	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	W_9
Value	**0.714**	**0.28**	**0.552**	**0.185**	**0.263**	**0.67**	**0.33**	**0.5**	**0.5**

(b) Our main task is to determine the values of C_2, C_3, C_5, C_6, and C_7 (C_1 is determined with the help of the aforementioned double-price model).

(c) Number of flights element C_2: Let $\overline{F}_{num}$ denote the average number of the flights. Let *Need* denote the required transportation amount. Let $\overline{A}_{mount}$ denote the average transportation amount. Then $\overline{F}_{num}$ is of the form

$$\overline{F}_{num} = \frac{Need}{\overline{A}_{mount}}. \tag{18}$$

The real transportation amount is $m - k$ or N depending on whether $m - k < N$ or not. Thus, $\overline{A}_{mount}$ is expressed as

$$\overline{A}_{mount} = \sum_{k=0}^{m-N-1} P_k \times N + \sum_{k=m-N}^{m} P_k \times (m-k) \tag{19}$$

$$P_k = C_{m-j}^{k} q^k (1-q)^{m-j-k} \tag{20}$$

where q denotes the probability of occurrence of the event that any reservation-maker actively gives up his right. Given the total required transportation amount *Need*, C_2 obviously increases with $\overline{A}_{mount}$. In this paper, it is assumed that C_2 is a linear function in $\overline{A}_{mount}$, which is shown below:

$$C_2 = t \times \overline{A}_{mount} + s \tag{21}$$

where t and s are usually empirically determined, and are assumed to take the values 1 and 0, respectively.

(d) Honour of corporation element C_3: C_3 is dominated by the amount of passive-discarders. Let $\overline{R}_l$ denote the probability of occurrence of the event when a reservation-maker is forced to give up. In real situations, C_3 is not simply a linear function in $\overline{R}_l$ due to the fact that, if $\overline{R}_l$ is large enough, the honour of the corporation would be seriously damaged. In our model, C_3 is assumed as a quadratic function in $\overline{R}_l$, which is of the form

$$C_3 = a \times (\overline{R}_l)^2 + b \times \overline{R}_l + c \tag{22}$$

$$\overline{R}_l = \frac{\sum_{k=0}^{m-N-1} P_k (m-N-k)}{m-N} \tag{23}$$

where a, b, and c are assumed to take the values 1, 0, and 0, respectively.

(e) Satisfaction of customer element C_5: Intuitively, C_5 depends on the compensation rate b / g and the average amount of passive-discarders $\overline{U}$. The higher b / g and the lower $\overline{U}$, the higher the satisfaction of customers would be. Let E denote the compensation rate. In our model, C_5 is assumed to linearly depend on E and $\overline{U}$, which is of the form:

$$C_5 = x \times E + y \times \overline{U} \tag{24}$$

where x and y are determined depending on the applications, and take the values 1 and -1, respectively. (Remark: E and $\overline{U}$ should be normalized according to the method provided in Appendix I).

(f) Airplane safety elements C_6: C_6 depends on the average transportation amount $\overline{A}_{mount}$. For instance, the reliability of safety check would decline and the rescue work would become more difficult with the increase of the amount of passengers. Practical experiences tell us that C_6 approximately a quadratic function in $\overline{A}_{mount}$, which is of the form

$$C_6 = s \times (\overline{A}_{mount})^2 + t \times \overline{A}_{mount} + h \tag{25}$$

where, in our model, the parameters s, t, and h take their values -1, 0, and 0, respectively.

(g) Airport safety element C_7: Analogously to the analysis of C_6, C_7 depends on the average amount of customers arriving at the airport $\overline{V} = \sum_{k=0}^{m} P_k \times (m-k)$, that is, $C_7 = F(\overline{V})$.

Having exposed all the parameters in the third model, we may find the optimal maximum reservation amount by solving the corresponding optimization problem through exhaustive search (see Appendix II). It is expected that the resulting data is of great reference value when an aviation corporation makes its own overbooking strategy. (Remark: C_1, C_2, C_3, C_5, C_6, C_7 are also normalized using the method in Appendix I). Below is such an instance:

Given that $q = 0.05$, $E = 0.2$, $j = 100$, $r = 0.75$, and N=375, the strategy goodness A is determined for different values of m (A is multiplied by 10^4). The results are listed in Table IV(a)-(d).

Table IV The strategy goodness A for different values of m

(a)

m	10^4A
392	80.324
393	82.659
394	84.938
395	87.041
396	89.012
397	90.868
398	92.744
399	94.426

(b)

m	10^4A
400	96.065
401	97.747
402	99.329
403	100.826
404	102.344
405	103.726
406	105.089
407	106.500

(c)

m	10^4A
408	107.924
409	109.336
410	110.604
411	111.916
412	113.254
413	114.499
414	115.822
415	117.055

(d)

m	10^4A
416	118.169
417	119.204
418	119.939
419	119.803
420	117.106
421	106.588
422	74.559
423	-24.336

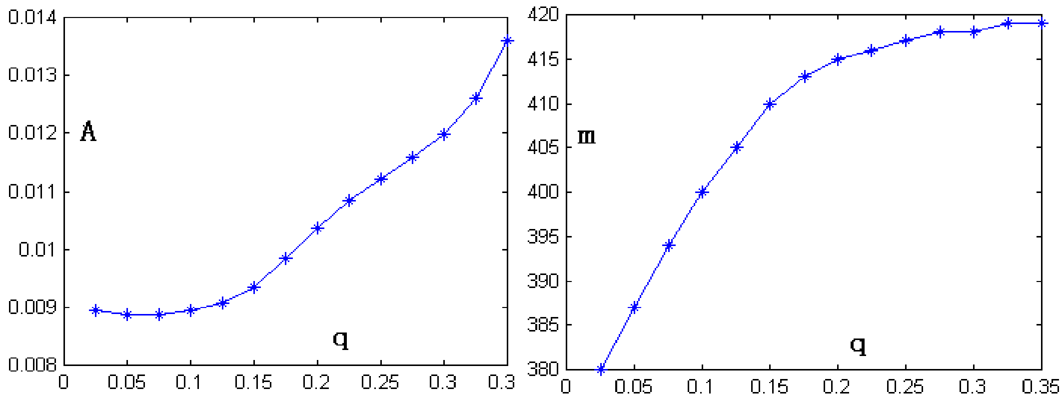
As it can be seen from Table IV, when m=418, A achieves its maximum value (≈0.012). This indicates that it is the best choice to fix the maximum reservation amount as 418.

The weight ratio of various elements change with time, place, customer, and corporation. As an instance, the 9.11. terrorism attack has render most governments to greatly enhance the effect of the safety element on the goodness. The third model can adapts itself to various senarios through the adjustment of various weighing factors.

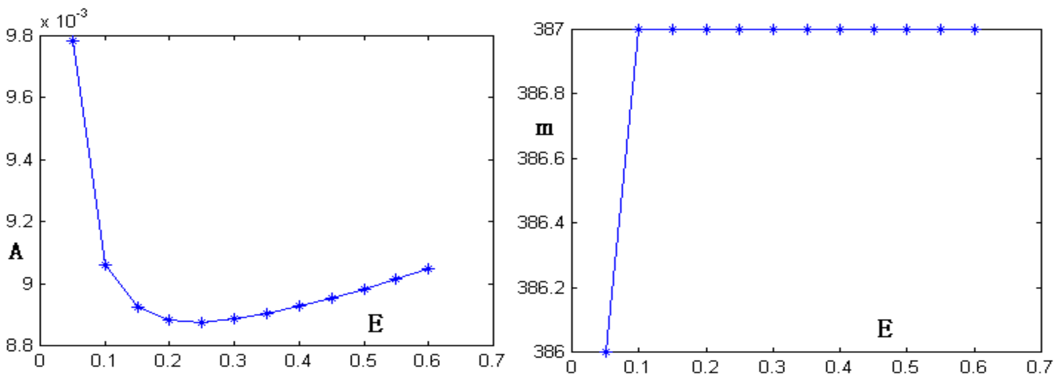
Sensitivity Analysis

In order to determine which elements have significant effects on the goodness, we need to analyze the sensitivity of goodness with respect to each element, that is, how heavily the goodness depends on the element. Below is concentrated on the sensitivity of the maximum value of A (and the corresponding m) with respect to the typical four parameters q, E, r, and j.

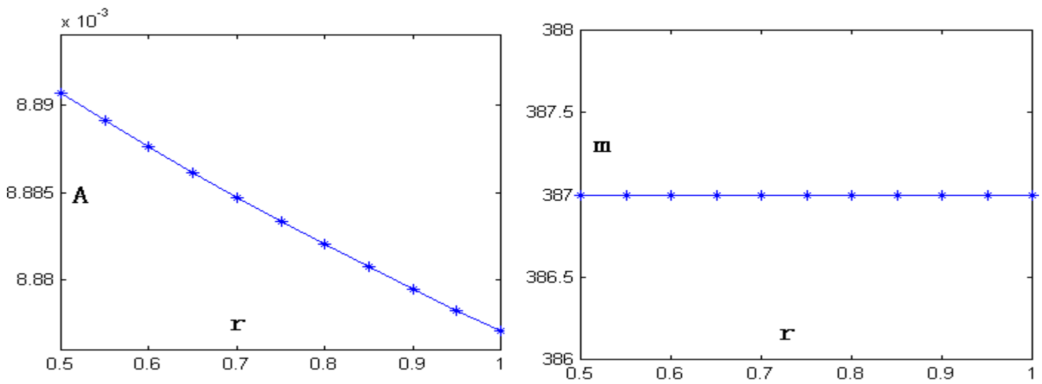
For simplicity, let us fix three of the four elements and consider only the remaining one for sensitivity. Through exhaustive search (see Appendix III), four collections of data are obtained (one for each parameter) and are plotted in Fig.2.(a)-(d).



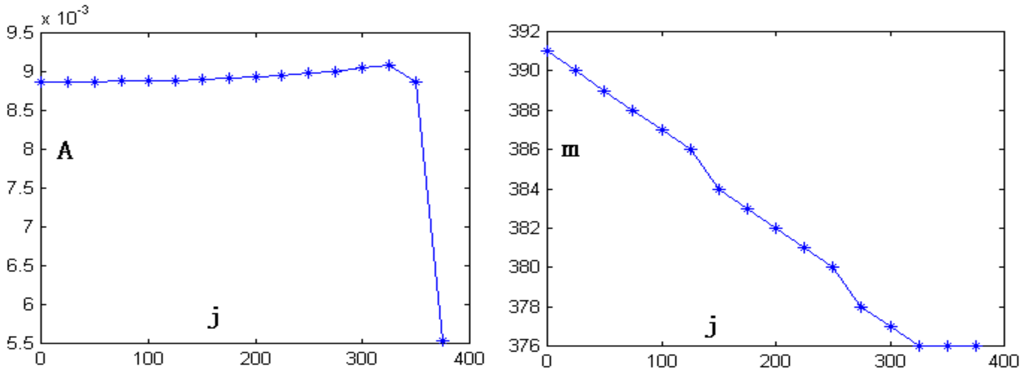
(a) q is tested in the interval [0, 0.3], E=0.2, r=0.75, j=100



(b) E is tested in the interval [0, 0.7], q=0.05, r=0.75, j=100



(c) r is tested in the interval [0, 1], q=0.05, E=0.2, j=100



(d) j is tested in the interval [0, 375], q = 0.05, E=0.2, r=0.75

Fig.2. Sensitivity experiments for four typical elements

It is found from Fig.2 that the maximum value of the goodness A i.e. max(A) heavily increases with q and drastically decreases when j > 350, but only slightly change with r or E. Thus, we can conclude that max(A) (and the optimal m) is sensitive to the change of q or j (>350), but is insensitive to the change of r or E. Furthermore, the amount of discount tickets should be limited in order to achieve the optimal goodness. In addition, the reason why the optimal m is not sensitive to the change of r resides in our assumption that all the discount-ticket reservation-maker will certainly arrive at the airport in time.

In real life, it is a challenging problem to make an optimal overbooking strategy due to the large amount of relevant elements. In most cases, we may find a sub-optimal strategy instead of the optimal one.

FURTHER DISCUSSIONS

Having been seriously affected by the global economic regression and the 9.11. hijacking event, the aviation industry is undergoing a severe winter. Although airplanes have many irreplaceable advantages such as very high speed, people tend to prefer other forms of transportation vehicles than airplane due to the cheaper expense as well as the higher safety.

Although Aviation Safety has been improved dramatically (**see Fig.3(a)**), accidents still happen every year (**see Fig.3(b) and Fig.3(c)**).

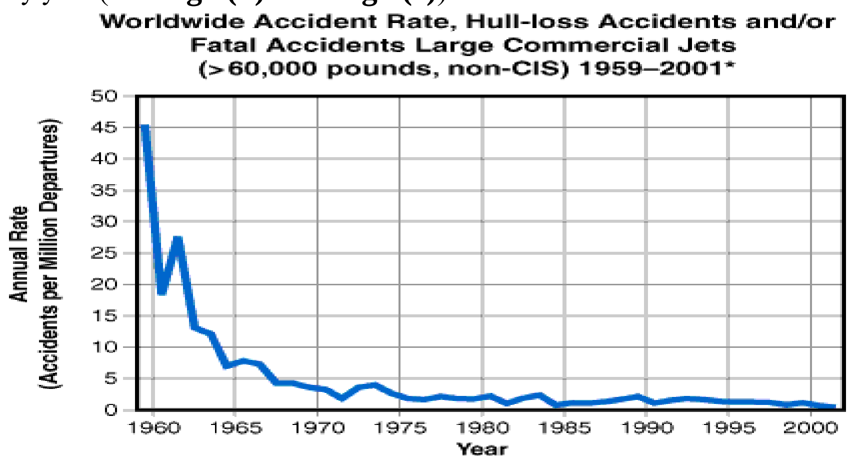


Fig.3(a)

Ranking of safest and worst fatal civil airliner accident; total casualties (excl. ground casualties) and total # of fatal accidents since 1970

RANK	safest	fatalities	worst	fatalities	safest	#	worst	#
1	1984	624	1972	2556	2001	34	1972	73
2	1999	674	1985	2362	1984	34	1979	69
3	1990	781	1973	2135	1983	35	1970	69
4	1981	920	1974	2082	1982	35	1973	66
5	1986	926	1996	1945	2000	36	1988	63
6	2001	1118	1979	1855	1990	39	1989	61
7	2000	1134	1989	1855	1998	40	1978	61
8	1991	1161	1976	1807	1997	40	1974	57
9	1982	1164	1977	1736	1981	40	1976	57
10	1995	1167	1988	1734	1985	40	1992	57

Fig.3(b)

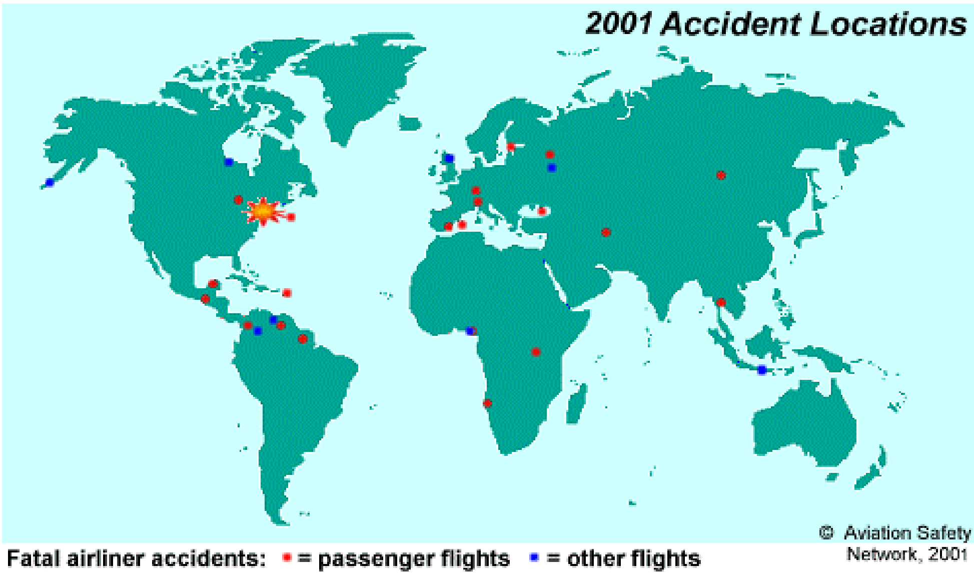


Fig.3(c)

Specialists have figure out significant safety issues that lead to accidents as follow:

•Aircraft component/system failure

•Collision between aircraft

•Collision with birds

•Engine malfunctions

•Fire

•Mistakes of flightcrew

•Loss of control

•Sabotage/hijack

•Bad weather

•Taking off/landing failure

Consequently, what the aviation corporations need to do is to take effective measures to avoid those safety issues and to struggle to prevent the happening of catastrophe.

Based on the research of the three models, we propose the following helpful suggestions for aviation corporations:

- Widely disseminate the advantages of taking airplanes.
- Strengthen aviation safety education
- Strengthen security and protection measures such as installing advanced safety check equipments and increasing the amount of relevant clerk.
- Effectively divide the ticket price levels.
- Prevent terroristic attack such as hijacking and sabotage
- Specify the maximum amount of tickets which are permissible to overbook.

In addition, the ticket price should be enhanced within an appropriate extent in order to compensate for the higher cost paid for strengthening the safety of the customers. Let Δg denote the incremental amount of the ticket price, then the new price g' is

$$g' = g + \Delta g$$

where g is the old price. In this case, the profit rate is modified as follows:

$$\frac{\bar{s}}{f} = \frac{1}{0.6(N-(1-r)j)}[pm-(1-r)j-(1+\frac{b}{g+\Delta g})\sum_{k=0}^{m-N-1} p_k(m-N-k)]-1$$

with all the other formulas left intact.

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MEMORANDOM

To whom it may concern:

Dear Sir/Madam:

Although reading this letter may take you a few minutes, it is worthwhile reading.

We have already noticed such a phenomenon: when some customers who have made reservation for a specific arrival at the airport in time, they hear such a bad news: “The airplane you have reserved is full now. Please wait for the next flight. We apologize for bringing you great inconvenience”.

Indeed, fully occupied airplanes will bring you the maximum profit, but this profit is at the expense of loss of honour of your corporation and potential economic loss. As one of the decision makers of your corporation, you must have found the very importance of making a reasonable overbooking strategy. This paper will provide you with a novel angle of viewing the problem as well as a few valuable overbooking mechanisms in order for you to make better tradeoff between profit and customer’s satisfaction. Exactly as the title of the paper says: “More Profit, Less Complaint”.

We propose an optimization model which takes into consideration many elements such as economic profit, social profit, and safety. The combined profit of your corporation is taken as the objective function, followed by a successively refined list of relevant elements. The refinement process is repeated until a satisfactory top-down hierarchical structure is constructed. The effect of each element on the profit is flexibly adjusted by a weighing factor according to practical applications. The original optimization problem is solved through a bottom-up calculation process with the help of an exhaustive search.

Once you understand our model, you may use it for your purpose immediately. If you have any questions when reading this paper, please feel free to contact us.