Can We Travel More Freely?

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Abstract

To reduce the interruption time caused by tollbooths and minimize the dissatisfactory of customers, we design the optimal number of tollbooths for different toll plazas. We set up an optimization model based on Queuing Theory and Hydrodynamics Theory to make research on the optimal setting number of tollbooths in a toll plaza and the opening number of it in different hours. In the model, the number of tollbooths is the decisive parameter and the objective function is the delay time of a vehicle passing through the whole toll plaza. The delay time contains the waiting time for queuing and the congestion time. When the traffic flow gets too large, we should set more tollbooths to reduce the delay time and upgrade the service level. On the other hand, if we set too many tollbooths, the congestion would become serious after vehicles leave the tollbooths and before driving on the travel lane, and then the delay time would be increasing apparently. So we consider the two aspects in the optimization model. We also define the Service Level to assess the performance of tollbooths.

The result of the model shows that the setting tollbooths can deal with the flow demand of the peaking hour in northbound direction or southbound direction, and the traffic administration can make the decision to open the corresponding number of tollbooths with the optimal number of tollbooths.

We analyze the sensitivity and stability of the result to such factors as vehicle arrival intensity, average service rate, free vehicle velocity, jam density**.** The critical flow and average service rate are considered to analyze the more or less effective condition, and the result of the model and current practice are compared in the end.

Introduction

To design the optimal tollbooth number for a toll plaza, we set up a two- phase model based on queuing theory and traffic flow theory. In the first phase, we use the M/M/c queuing theory to describe the process of the vehicles going into the plaza, paying service, and determine the delay time of the process. In the second phase, we utilize traffic flow theory to discuss the time spent on congestion when vehicles leave tollbooths before going on to the normal speed way. And we set up the bottleneck model to work it out. Put the processes of two phases in consideration, we could decide the average delay time when vehicles pass the toll plaza, which is the objective function of our model. In our objective function, the number of lanes and tollbooths is decisive parameters. Simultaneously, introducing the lever of service (LOS) into our model to assess the results is an important approach. Then we could apply the model to calculate the optimal number of 5 different toll plazas on Garden State Parkway, and the results are exciting that the average Service Level stays above B， which is pleasing to traffic administration. Also we can offer the optimal tollbooths number for different times of a day.

Background

There were many articles discussing about the traffic flow problem. In 1955, England scholar M.J.Lighthill and GB.Whitham assimilate the traffic flow to a fluid. They use hydrodynamics theory to discuss the problem, and form the Hydrodynamics Simulation Theory, which is used to settle the traffic flow problem with high vehicle density. Their theory set up the continuity equation for vehicle similar to hydrodynamics. With the theory they could analyze the relationship between velocity and density of the traffic flow and then describe the congestion-scattering process.

Many articles have discussed about the Service Level of tollbooths and give its definition. Using Neural Network to classify and assess the Service Level [Liu etc. 2003], Woll and Heol defined Service Level on the basis of V/C (V is the traffic density and C is the traffic flux)[Zheng etc. 2000].Another model, the TPS (Toll Plaza Simulation) model, comprehensively considered the waiting length and waiting time [Zheng etc. 2000]. And the lever of service (LOS) criteria is usually defined to have 6 grades listed below (**Table 1**) [Jack and Haitham 2002]

Table 1: Toll Plaza Lever of Service Criteria

Lever of Service(LOS)	85th-percentile delay(seconds/vehicle)
A	≤ 14
B	$>14-28$
C	$>28-49$
D	$>49-77$
E	$>79-112$
F	>112

Terms and Definitions

Traffic Flow：The number of cars passing a given point per unit time per lane (vehicles per hour per lane, unless otherwise specified).
Speed：The rate of movement of a single car (miles per hour, unless otherwise specified).
Traffic density：The number of cars per unit length of roadway (vehicles per mile per lane, unless otherwise specified).
Delay time：Time from the moment when vehicles reach the toll plaza to the moment they depart from the exit, which is a decisive parameter in describing the level of service.
Arrival intensity：the number of vehicles reaching the toll plaza unite time (vehicles per hour).
Average service rate：the number of vehicles served by a tollbooth in unit time (vehicles per hour).
Recovery zone：the length from the point where cars leave the tollbooth to the point where cars enter the travel lane.
Jam density：the limited traffic flow density.
Free velocity：the velocity that cars run freely.

Assumptions and Hypotheses

About the Traffic Flow

- For there are differences between vehicles in kinds, sizes and velocity, which will have important effects on traffic capability, safety and controlling, we could not compare one vehicle with another at different places and different time. So in our model, we take car for the criteria, and transform the data of other vehicles into car to calculate the traffic flow according to the area and velocity of other vehicles.
- The variety of traffic flow with time reflects the growing of economy and growing demands for more roads. As society develops, the traffic flow increases. But in our model, the traffic flow will stay stable in a period of time. (a month in a year, etc.)
- The provided statistic data of the traffic flow in different period of the day at Garden State Parkway is accurate and available.
- In common occasion, the traffic flow in center of the roadway is greater than that near the sideward. In our model, we do not take the differences between lanes into consideration and we regard all lanes as identical.

About the Toll Plaza

- In our model, we do not take into account the shape of toll plaza which actually have influence on the number of tollbooths.
- There is exactly one tollbooth per incoming travel lane.

About the Vehicles

- The velocity of cars varies from different grades of roads. See Freely moving velocity in terms and definitions for more detail.
- Velocity has relations with climate, geography, etc. But in our model we just assume that all cars lie in the same situation.
- All cars are approximately 10-16 *ft* long.
- In a traffic jam, there is an average of 1 ft of space between cars.
- The two above assumptions lead to a maximum traffic density of 5280(ft/mile)/11(17)(ft/car)=480(310) cars/mile/lane. We apply our model with an average value of jam density (an average maximum density), 402 cars/mile/lane.

We list the symbols used in our model in **Table 2**.

Table 2.

Nomenclature

Symbol	Property	Units	Abbreviation
c	Number of tollbooths	none	
λ	Average arrival intensity of traffic	vehicles/hour	vph
μ	Average service rate	vehicles/hour	vph
ρ	Parameter divide λ by $c\mu$	none	
P_i	Probability of a random state in the queuing system	none	

L_q	Expected value of queuing length in queue	vehicles	veh
L_s	Expected value of queuing length in system	vehicles	veh
T_s	Mean delay time of vehicle at tollbooths	seconds	s
V_s	Traffic velocity	miles/hour	mi/h
V_f	Free speed of vehicles on the road	miles/hour	mi/h
r	Number of lanes	none	
V_w	Velocity on the congestion section	miles/hour	mi/h
K	The traffic density	cars/mile/lane	
K_j	Jam density	cars/mile/lane	
T_r	Delay time of each vehicle on bottleneck	seconds	s
T_{total}	Average delay time for a vehicle passed toll plaza	seconds	s
l_{rec}	Length of the recovery zone	miles	mi

Motive of the Model

For the delay time reflects the Service Level, we convert our aim function into considering the average delay time of vehicles passing toll plazas. o predigest this problem, we set the process of vehicles passing tollbooths in two phases：Queuing phase and leaving phase. We first compute each phase's delay time, then consider all factors through the whole process with two phases.

Model Design

Phase 1: $M/M/n/\infty/\infty$ Queuing Model

From **Figure 1**, we assume that there are c tollbooths in the system and they are independent to each other. Let λ be the mean intensity of arriving vehicle and μ be the mean service rate for each vehicle passing the tollbooth, which conform to the exponential distribution, so $c\mu$ is the mean service for the whole system. Then we define $\rho = \lambda / c\mu$, and we could demonstrate that when $\rho < 1$, the system is stable and we can get the state probability of the system (**see Appendix B for detail**)

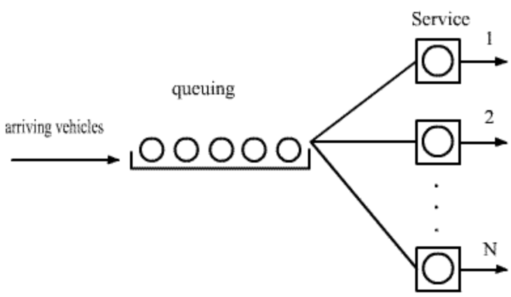

Figure 1. The M/M/c queuing system

$$P_0 = \left[\sum_{k=0}^{c-1} \frac{1}{k!} \left(\frac{\lambda^k}{\mu} \right) + \frac{1}{c!} \frac{1}{1-\rho} \left(\frac{\lambda}{\mu} \right)^c \right]^{-1} \quad (1)$$

$$P_n = \begin{cases} \dfrac{1}{n!}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & (n \le c) \\ \dfrac{1}{c!c^{n-c}}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & (n > c) \end{cases} \tag{2}$$

Where P_0 is the probability when there is no vehicle in the system while P_n is the probability when there are n vehicles in the system.

$$L_q = \sum_{n=c+1}^{\infty}(n-c)P_n = \frac{(c\rho)^c \rho}{c!(1-\rho)^2} P_0 \tag{3}$$

$$L_s = L_q + \lambda/\mu \tag{4}$$

Where L_q is the expected value of queuing length.

L_s is the expected value of queue length, which could be computed.

To apply the model the mean delay time of vehicle could be expressed as below:

$$T_s = L_s/\lambda \tag{5}$$

Phase 2: Bottleneck Model

Theory of Traffic Flow

We can measure the traffic density of a section of a road in vehicles per mile per lane. The traffic velocity V_s at a point on the road can be calculated from the density according the formula

$$V_s = V_f(1-\frac{K}{K_j}) \tag{6}$$

Where K is the traffic density, V_f is the free speed of any car on the road, and K_j is the maximum traffic density (with no space between cars). We define the flow of traffic on the road as the number of cars passing the given point in a unit of time in a single lane. The flow Q can be easily calculated as

$$Q = \lambda = rV_sK \tag{7}$$

Where r is the number of lanes and λ is the mean arrival intensity.

Bottleneck Model

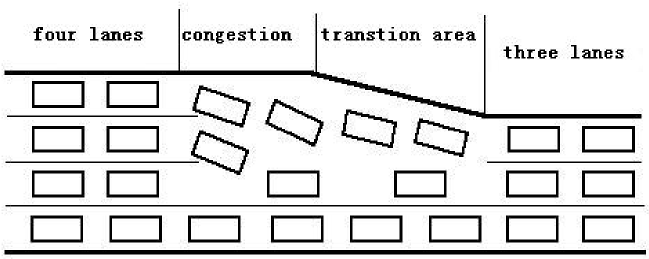

Figure 2. Traffic state at bottleneck area

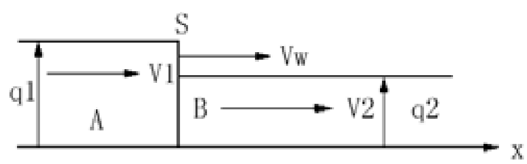

Figure 3. State of movement

From **Figures 2 and 3**, we could see that when traffic meeting bottleneck, here comes the traffic jam. Assume that the velocity on the section S is V_w, and the velocity beside the bottleneck section are V_1, V_2 separately, K_1, q_1, r_1 stands for density, flow and number of lanes of part A and so is K_2, q_2, r_2 for part B. Then we can get the formula (See **Appendix C** for proof)

$$V_w = \frac{V_1 K_1 r_1 - V_2 K_2 r_2}{K_1 r_1 - K_2 r_2} \qquad (8)$$

As to the system which has r lanes and c tollbooths, we could see that there would be bottleneck when vehicle leave the toll plaza and go on to the parkway. The arrival intensity λ' at the tollbooth can be computed as

$$\lambda' = \frac{c}{r}\lambda \qquad (9)$$

Where λ is the rate of vehicle leaving the tollbooth, which equals to the arriving rate.

Integrating equation (6) and (7), we can calculate the traffic velocity and the traffic density at the tollbooth for a given arrival intensity λ, then we can go further to get the value of velocity on the section V_w.If the designed length of the recovery zone is l_{rec}, the delay time of each vehicle (T_r)spent on bottleneck is

$$T_r = l_{rec} / V_w \qquad (10)$$

Determining the Optimal Objective Function

From the analysis and discussion on phase 1 and 2, the total delay time of vehicles could be calculated as equation (11)：

$$T_{total} = T_s + T_r \qquad (11)$$

where T_r is the delay time resulted from bottleneck when a vehicle departs from the toll plaza， which is the function of travel lanes and tollbooths number. And T_s is the delay time as a result of queuing when vehicles come to the tollbooths and wait for service，which is the function of vehicles flux and average service time. Then come to the optimal model

Minimize $T_{total} = T_s + T_r$

Subject to $T_s = L_s / \lambda$

$T_r = l_{rec} / V_w$

now the main task is to determine the optimal tollbooth number to get the shortest delay time.

Applying the Model

Data of the Model

We apply our model constructed above to discuss the number of tollbooths at the Bergen Toll Plaza, one of the several toll plazas on the Garden Stated Parkway. The **Figure 4** shows the traffic flow at different period of time of a day from 6 AM to 10 AM and from 3 PM to 7 PM in northbound and southbound direction. We can see that in each period of time the traffic flow in northbound and southbound direction are approximately equivalent, while the summation has some differences. Accordingly, we apply our model to analyze the required optimal value of tollbooths at different periods of time so as to maximize the level of service.

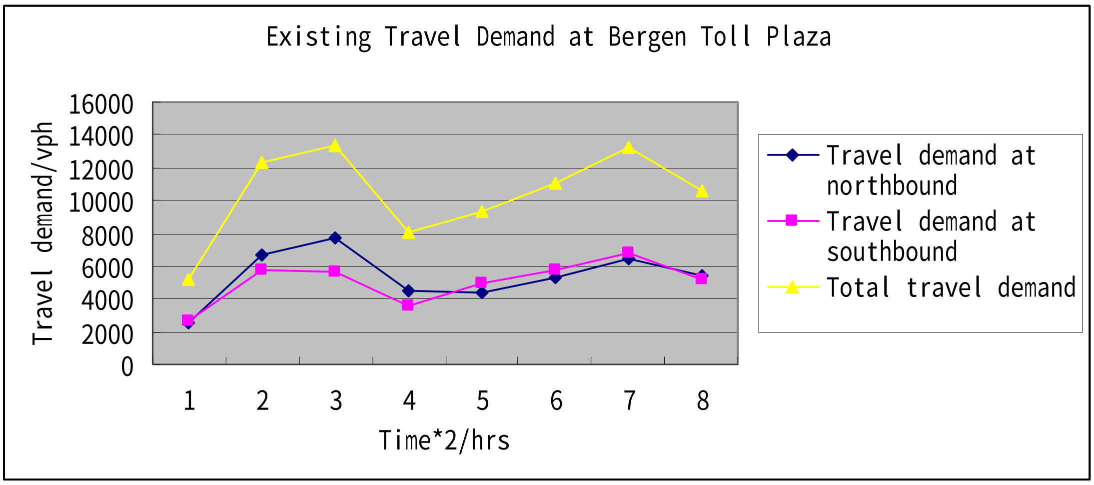

Figure 4. Existing travel demand at Bergen Toll Plaza

Determining the Parameters

- The average arrival intensity λ : utilizing the statistic data, we can calculate the value of λ.
- The average service rate μ : the average service rate varies from different toll instruments: for ETC (Electronic Toll Collection), 1200 vph per lane (vphpl); Manual, 350 vphpl; ACM (Automation Coin Machine), 550 vphpl [David and Robert 2001]. For most of modern toll plazas adopt ETC, we set the average service rate at the value of 1100 vphpl.
- The free velocity V_f :the free velocity can be divided into four ranks according to the design of American freeway, which is 129 km/h to 139km/h in plain areas; 96 km/h in hill areas; 80 km/h in mountainous areas; 80 km/h to 113 km/h in city freeway.
- Jam density K_j : for different freeway, the jam density is not the same. According to our assumption, K_j =402 cars/mile/lane.
- The number of lanes r : for American freeway, there are 6,8,10 lanes freeway around New York, Washington and Los Angles freeways, but a majority of them is with 4 lanes. There are 11 toll plazas along the Garden State Parkway and we list the incoming traveling lanes of each plaza in **Table 3**.

Table 3. Travel lanes in different direction of different toll plazas

Toll Plazas	Northbound (NB)	Southbound (SB)
Hillsdale	6	6
Bergen	6	7
Essex	6	6
Union	10	8
Raritan	6	6
Asbury Park	8	5
Toms River	5	6
Barnegat	4	4
New Gretna	4	4

Great Egg Harbor	4	4
Cape May	4	4

Experimental Result

Mean Delay Time vs. Number of Tollbooths

The graph of mean delay time versus the number of tollbooths gives more information **(Figure 5)**.We can get some essential conclusions from the figure：

- The delay time first decreases then increases as the number of tollbooths grow.

- If the number of tollbooths doesn't exceed the optimal point, the mean delay time increase quickly; while the actual tollbooth number exceeds the optimal point, the mean delay time increases much slower, which accords with the actual situation probably. So when traffic flow gets too large, we would usually propose that set up more tollbooths to help reduce the delay time, so as to upgrade the service level.

- On the other hand, if we set up too many tollbooths, the congestion would become serious when vehicles leave the tollbooths driving on the buffer way, and complaints from customers would be increasing apparently. So we have to do something with it, such as reducing the service time for each vehicle, changing the geometry shape to upgrade the service level etc.

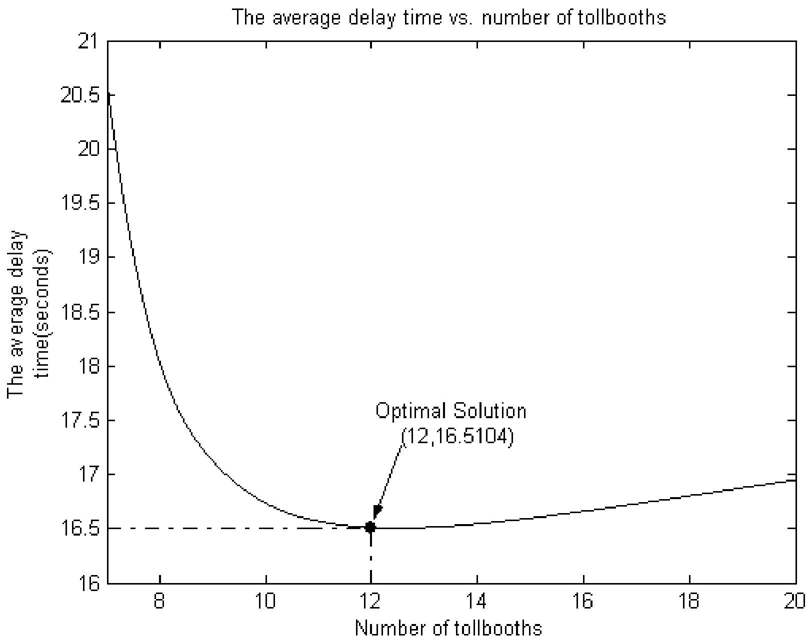

Figure 5. The relationship of mean delay time and number of tollbooths

Applying the Model to Bergen Toll Plaza

We utilize the analysis data to assess the Bergen Toll Plaza. **Table 4** shows the required optimal tollbooths at different period of time in north-south direction. And the lever of service criterion is clearly demonstrated in this table.

Table 4. The optimal number of opening tollbooths, delay time and LOS for different traffic flow with 6 lanes

Time	Direction							
	Northbound(NB)				Southbound(SB)			
	Travel Demand(vph)	Optimal tollbooth number	Delay time(sec)	LOS	Travel Demand(vph)	Optimal tollbooth number	Delay Time(sec)	LOS
6-7AM	2539	5	12.6372	A	2678	5	12.6591	A
7-8AM	6621	13	18.5202	B	5735	11	16.3213	B
8-9AM	7680	14	20.5145	B	5687	11	16.1223	B
9-10AM	4485	9	14.541	B	3542	7	14.163	B
3-4PM	4421	9	14.1457	B	4926	10	15.5392	B
4-5PM	5325	10	15.5262	B	5722	11	16.5215	B
5-6PM	6476	12	16.5104	B	6743	13	19.5164	B
6-7PM	5430	10	15.7288	B	5176	10	16.0268	B

From the table, we could get the following crucial conclusions：

- According to the results of the tollbooths model with six lanes, the average delay time is no more than 20 seconds, and the service level is above B grade. So, the optimized results are quite efficient and practical to enhance traffic service.

- For different traffic flows at different moments, the optimal tollbooth number for Bergen Toll Plaza varies in a series of time, which could be got in **Table 4**.We recommend setting 14 tollbooths which can deal with the flow demand of the peaking hour in northbound direction, then utilize optimal number of tollbooths according to the table to open the corresponding number of tollbooths. Therefore, the conclusion from our model is important for traffic administration to make decision.

- The traffic flow is so low that the number of operating tollbooth windows is smaller than the number of lanes led to tollbooths around 6 AM in north-south direction.

Applying the Model to Other Toll Plazas at Garden State Parkway

Utilizing the data, we get the optimal results under our model, we could get the suitable tollbooth number and average delay time. We could even assess toll plazas through the Service Level (LOS).**Table 5** provides us with the simulative results of different toll plazas.We could come to some important conclusions:

- The optimal number of tollbooths in five stations of Garden State Parkway varies, which is shown in **Appendix A**. It supplies traffic administration with clear number of tollbooths, makes it easy for them to ease congestion. Given the average traffic flux, the optimal number of tollbooths could be decided with the highest Service Level.

- To Raritan Toll Plaza and Union Toll Plaza, they should set large amounts of tollbooths for their great traffic flux at the peak moment. So we suggest removing the two stations and to adopt more advanced equipments like ETC in order to reduce the service time. And then to get a higher Service Level with less construction.

Table 5. The setting number of tollbooths, mean delay time and Service Level for different toll plazas

Location	Direction					
	Northbound(NB)			Southbound(SB)		
	Tollbooth number	Average delay time(second)	LOS	Tollbooth number	Average delay time(second)	LOS
Raritan Toll Plaza	16	19.4162	B	16	19.7924	B
Union Toll Plaza	17	23.5157	B	16	20.5434	B
Essex Toll Plaza	14	18.5196	B	14	18.5196	B
Bergin Toll Plaza	14	18.5414	B	13	17.5462	B
Hillsdale Toll Plaza	9	15.1606	B	9	14.9000	B

Stability and Sensibility Analysis

- ***The effects of arrived vehicle intensity*** λ ***on the results***：as we all know the number of vehicles increases with the development of society, **Figure 6** gives us information that the average increase rate of traffic flux stays at about 0.5% after 2004, so we could compute the effects of λ on delay time. From the chat we know as the traffic flux increases, the average delay time becomes longer. So the administration should take care and take action to ease the congestion.

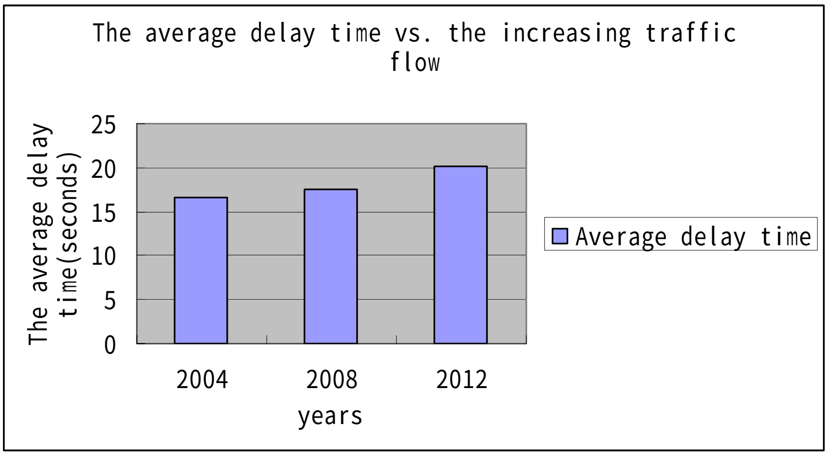

Figure 6. Changes of average delay time with increasing number of vehicle

- ***The effects of average service rate*** μ ***on our model***：for different charge measures there are different μ, and we take $\mu \in [250,1200]$.Then we use the selected μ to check the model. From **Figure 7** and **Table 6**， we could see as μ increases, the average delay time decreases. And we propose that the toll plazas use as many ETC as possible to upgrade the Service

Level.

Table 6. Different service rate with its corresponding delay time

Service rate(vph)	Delay Time(second)
250	85.63
350	56.2675
450	40.4464
550	33.9248
650	29.3247
750	26.8173
850	24.0299
950	21.5804
1100	18.3826
1200	16.4625

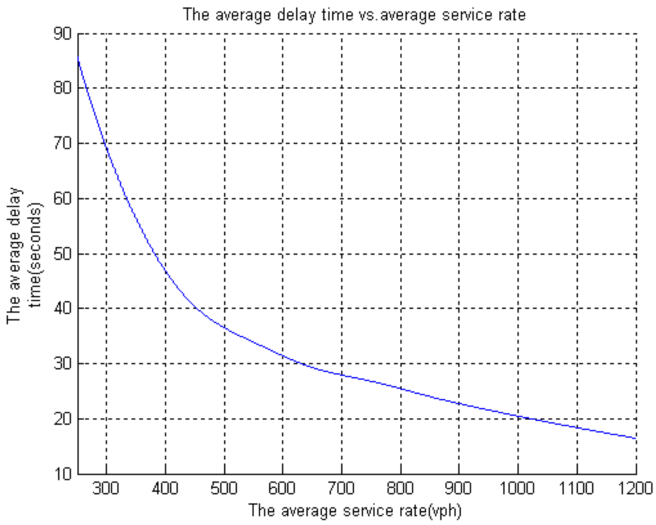

Figure 7. The average delay time vs. the average service rate

- ***The effects of free vehicle velocity*** V_f ***on our model***：the graph of the average delay time versus vehicle velocity is as below，we could recognize the average delay time changes slowly with variable velocity. So our model is stable with the change of free velocity.

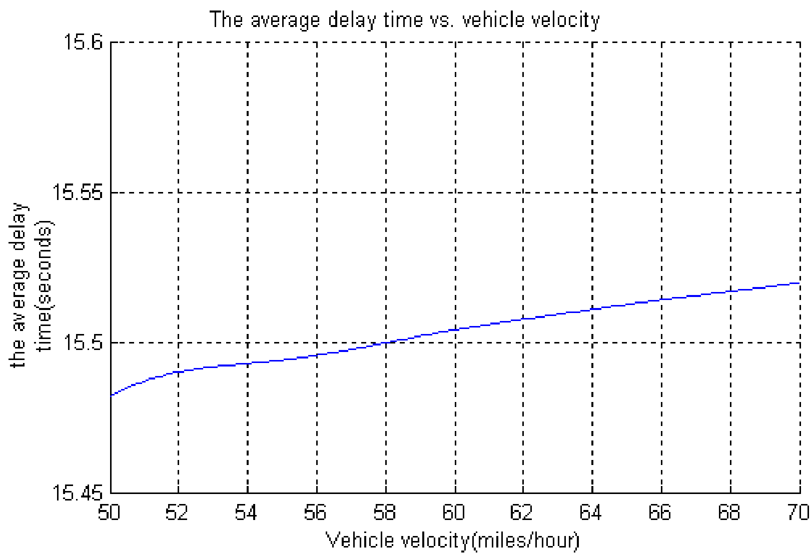

Figure 8. The average delay time vs. vehicle velocity

- ***The effects of jam density on our model***：jam density describes the capability of a speed way. The graph, **Figure 9,** shows us that as congestion density increases, the average delay time decreases, which is accordant with the practice. However, the average changes slightly with jam density, so we could say it's stable of our model with the change of jam density.

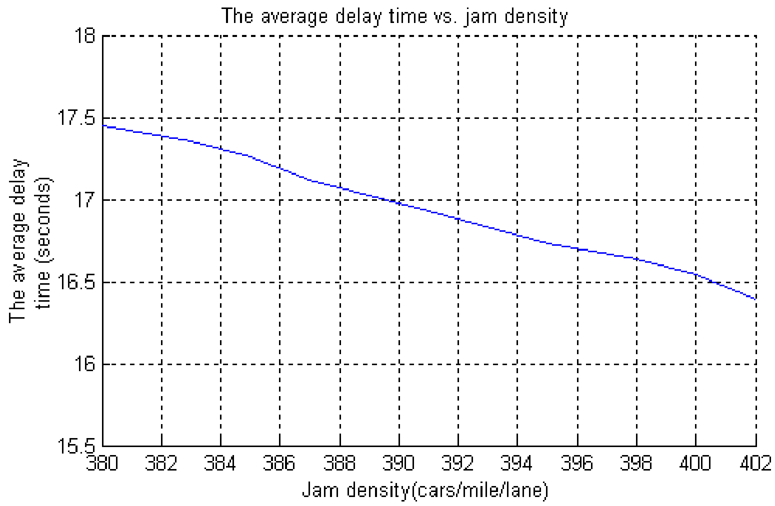

Figure 9. The average delay time vs. jam density

Under What Condition is More or Less Effective?

Through sensitivity and stability analysis, we could see the main factors act on our model involves average traffic flow and service time of the tollbooth. Accordingly, we separately take average traffic flow and service time in consideration. Take the configuration of north-facing tollbooths' number for example, now there are 10 tollbooths of Raritan Toll Plaza facing north with 6 north-facing lanes while we would design 16 north-facing tollbooths under our optimal model.

Critical Traffic Flow

Read the figure below then we get the critical traffic flow $Q_c = 6465.4$ (veh/h).When traffic flow goes less than the critical flow, the optimal average delay time would be longer than the practical data, i.e. the efficiency in optimal model is below the actual system efficiency. And it is true with the reverse situation, so we could get a higher efficiency from the optimal model when the average traffic flow goes beyond the critical traffic flow. All these situations mentioned above exactly match the fact. So as to get the highest utilizing rate we have to reduce the number of tollbooths with the fall of traffic flow.

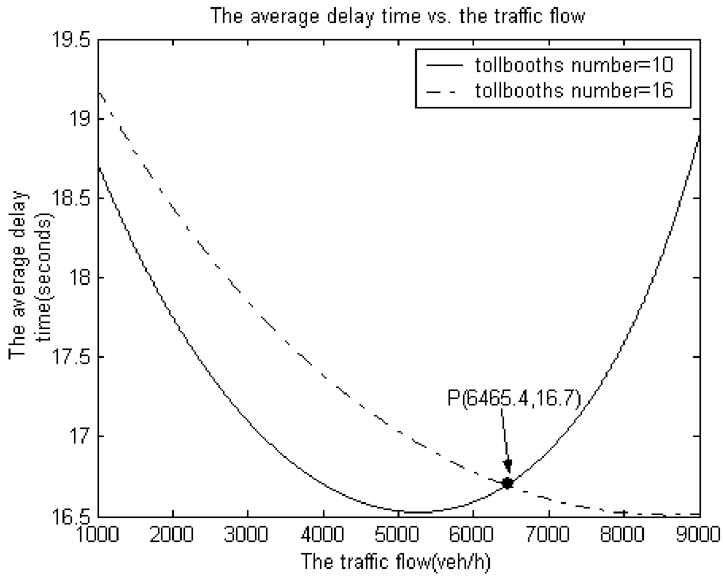

Figure 10. The average delay time vs. the traffic flow with tollbooth number values 10 and 16 separately

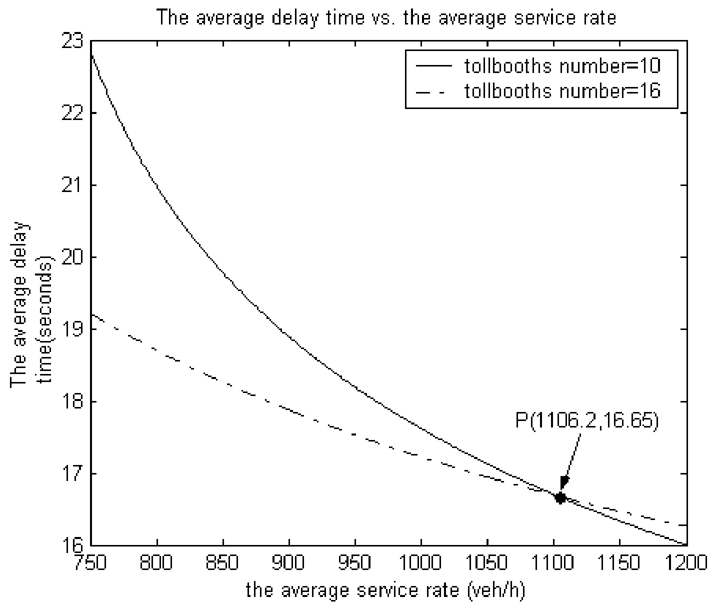

Figure 11. The average delay time vs. the average service rate with tollbooths number values 10 and 16 separately

Critical Tollbooth Service Rate

As informed by the **Figure 11**, we get the critical service rate $\mu_c = 1106.2$ (veh/h).When the service rate goes under the critical service, we get the optimal number of tollbooths 16 with a shorter average delay time than the practice, which leads to higher tollbooth efficiency under our model. And if the tollbooth service rate goes above the critical service rate, efficiency from our model would be lower than the actual efficiency, which rightly meets the practice. if the service rate falls, the administration would set up new tollbooths to ease congestion.

Strengths and Weakness

Strengths

- The model can be easily implemented and flexible. For example, we can change the value of parameters to get the optimal number of tollbooths for a specific traffic flow at random period of time. So it is convenient for the department of transportation to make precise decision.
- Application in different toll plazas at the Garden State Parkway proves the practicability and validity of the model. We adopt a great amount of data about Garden State Parkway to run the program and get some significant conclusions and results.
- The model is stable with regards to all parameters tested. The stability and sensitivity analysis indicate that our model can commendably deal with all kind of different situation.
- The model can handle both situations when congestion only appears on the departure from the toll plaza and when congestion builds on the entrance and exit of the toll plaza.

Weaknesses

- In our model, we do not take into account the shape of toll plazas which actually has influence on the optimal number of tollbooths.
- In the same plaza, we just consider that all tollbooths use the same measure, that's to say we just thought the Service Level to be the same in a plaza. However, a actual toll plaza could adopt mixed service measures, it may choose the suitable service measure, for example using ETC together with ACM and Manual may get a higher integrated efficiency.

Conclusion and Recommendation

- When vehicle number changes, the tollbooths needed is different, but after the construction, the number of tollbooths is unchangeable. We recommend traffic management to build the maximum tollbooths needed in peak hours, then only use some of them when traffic is small.
- Most time, we just consider to set tollbooths at each direction on travel lanes, thus vehicles will be delayed both at departing and coming back periods. We recommend to eliminate the tollbooths in one direction and to install free-flow E-ZPass in the opposite，which will relief congestion greatly.
- It can be seen from the model that the tollbooths number is the main factor of the delay time. But when the traffic is very heavy, it also costs much time upon departure from the toll plaza. Widening one or more lanes will be better relatively.

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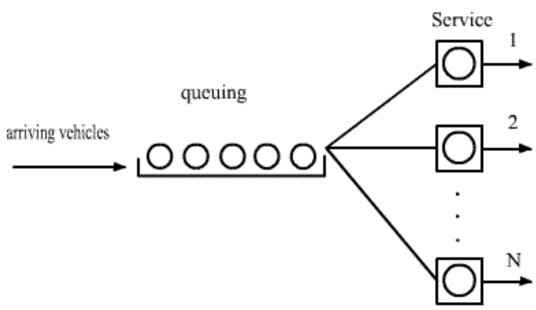
Appendices

Appendix A The Optimal Opening Number of Tollbooths Table

Time		6-7AM	7-8AM	8-9AM	9-10AM	3-4PM	4-5PM	5-6PM	6-7PM
Bergen	#	5	10	11	9	9	9	10	9
	*	16.646	16.514	16.511	16.534	16.537	16.531	16.518	16.526
	#	5	11	11	7	10	11	13	10
	*	16.681	16.521	16.521	16.563	16.53	16.521	16.514	16.526
Hillsdale	#	4	7	9	5	5	7	8	7
	*	16.731	16.563	16.535	16.647	16.672	16.601	16.549	16.579
	#	4	7	8	5	5	7	8	7
	*	16.757	16.573	16.548	16.647	16.657	16.563	16.549	16.603
Essex	#	10	14	13	10	12	12	12	11
	*	16.526	16.516	16.519	16.5294	16.520	16.519	16.516	16.5205
	#	9	13	11	9	11	11	11	11
	*	16.535	16.516	16.522	16.5342	16.521	16.525	16.522	16.5223
Union	#	13	17	15	13	12	14	14	11
	*	16.517	16.509	16.512	16.5194	16.519	16.514	16.512	16.5207
	#	10	15	16	12	15	16	15	15
	*	16.526	16.512	16.512	16.5215	16.510	16.509	16.511	16.511
Raritan	#	20	20	20	17	10	11	11	9
	*	16.786	17.004	16.724	16.5103	16.526	16.527	16.520	16.5336
	#	5	8	10	9	20	20	20	20
	*	16.689	16.551	16.527	16.5352	16.508	16.810	16.949	16.5589

Notice: these rows began with # show us the optimal tollbooths number needed in corresponding lane, the rows began with * show us the average delay time of vehicles. Besides, the data under shadow are calculated when traffic move northward, others are values of southbound lanes.

Appendix B M/M/n Queuing Model Formula Deduction



Fiugre 1.A M/M/1 queuing system

Figure 1 is a M/M/n queuing system.When analyzing this queue system, we should start from the transform between several states, as shown in Figure 2.From state 1 to state 0, the possibility for each one to be serviced is μP_1, then the situation turns from state 2 to 1, and the customer who has received service would depart from the station.So the state possibility will be cumulated to $2\mu P_2$.The condition is same for the turning from state n to state $n-1$.If n is no more than

c, the state transform possibility is $n\mu P_n$, others the possibility is $c\mu P_n$.

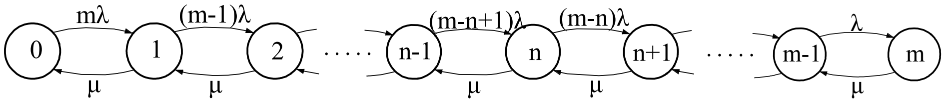


Figure 2. The state transformation graph

From the graph we can get these equations:

$$\begin{cases} \mu P_1 = \lambda P_0 \\ (n+1)\mu P_{n+1} + \lambda P_{n-1} = (\lambda + n\mu)P_n \quad (1 \le n \le c) \\ c\mu P_{n+1} + \lambda P_{n-1} = (\lambda + c\mu)P_n \qquad (n > c) \end{cases}$$

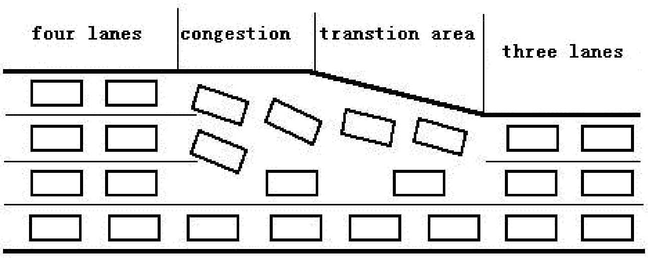
here $$\sum_{i=0}^{\infty} P_i = 1, \quad and\, \rho \le 1.$$

Deducing from these equations step by step, we could get the state possibility shown blow

$$P_0 = \left[\sum_{k=0}^{c-1} \frac{1}{k!} \left(\frac{\lambda^k}{\mu} \right) + \frac{1}{c!} \frac{1}{1-\rho} \left(\frac{\lambda}{\mu} \right)^c \right]^{-1}$$

$$P_n = \begin{cases} \dfrac{1}{n!} \left(\dfrac{\lambda}{\mu} \right)^n P_0 & (n \le c) \\ \dfrac{1}{c! c^{n-c}} \left(\dfrac{\lambda}{\mu} \right)^n P_0 & (n > c) \end{cases}$$

Appendix C Bottleneck Model Duduction



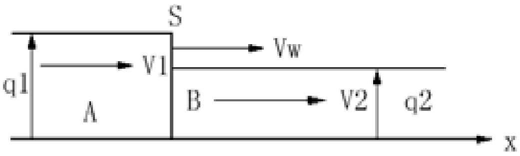


Figure 3.Traffic state at bottleneck area **Figure 4**.State of movement

As can be seen from **Figure 3** and **Figure 4**, the road is divided by segment S into two parts : A and B. Assumed that the speed at S is V_W, and V_1, V_2 are used to represent each of the average speed in A and B segment. K_1, q_1, r_1 stands for density, flux and number of lanes of part A and so is K_2, q_2, r_2 for part B. Therefore we can get the formula as following:

$$N = (V_1 - V_W)Kr_1 t = (V_2 - V_W)K_2 r_2 t$$

thus $$(V_1 - V_W)Kr_1 = (V_2 - V_W)K_2 r_2$$

then we can obtain $$V_w = \frac{V_1 K_1 r_1 - V_2 K_2 r_2}{K_1 r_1 - K_2 r_2}$$

Appendix D Program Code

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Wait.m
function t1=wait(lamda,mu,tollnum)
ru=lamda/(tollnum*mu);
prob1=0;
for k=0:tollnum-1
    prob1=prob1+(lamda/mu)^k/(factorial(k));
end
prob2=prob1+(lamda/mu)^tollnum/((1-ru)*factorial(tollnum));
prob=prob2^(-1);
lenque=prob*(tollnum*ru)^tollnum*ru/(factorial(tollnum)*(1-ru)^2);
lensca=lenque+lamda/mu;
t1=lensca/lamda;
if prob2<=0
    t1=0.3;
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Choke.m
function t2=choke(lamda,mu,jamfactor,tollnum,lanes,freevel)
lamda2=tollnum*lamda/lanes;
recovery= 0.011;
k1=lamda/(lanes*freevel);
vs=zeros(2,1);
vs(1)=0.5*freevel+sqrt(jamfactor^2*tollnum^2*freevel^2-4*lamda*freevel*jamfactor*tollnum)/(
2*jamfactor*tollnum);
vs(2)=0.5*freevel-sqrt(jamfactor^2*tollnum^2*freevel^2-4*lamda*freevel*jamfactor*tollnum)/(2
*jamfactor*tollnum);
%if min(vs)<0
%       selectvs=max(vs);
%else
%       selectvs=min(vs);
%end
selectvs=min(vs)+0.15;
k2=lamda2/(selectvs*tollnum);
v2=selectvs;
velw=abs((k2*v2*tollnum-k1*freevel*lanes)/(k1*lanes-k2*tollnum))+2.2;
t2=recovery/velw;
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Total.m
function [totaltime,bestnum]=total(lamda,mu,jamfactor,lanes,freevel)
recovery=0.011;
tollnum=4:20;
tollnum(find(tollnum==lanes))=[];
%t1=zeros(size(tollnum));t2=zeros(size(tollnum));t=zeros(size(tollnum));
for j=1:length(lamda)
   for i=1:length(tollnum)
        t1(j,i)=wait(lamda(j),mu,tollnum(i));
        t2(j,i)=choke(lamda(j),mu,jamfactor,tollnum(i),lanes,freevel);
        t(j,i)=t1(j,i)+t2(j,i);
   end
end
%plot(tollnum,t);
for i=1:size(t,1)
   totaltime1(i)=min(t(i,:));
   bestnum(i)=tollnum(find(t(i,:)==totaltime1(i)));
   totaltime(i)=totaltime1(i)*3600;
end
%plot(tollnum,t(5,:)*3600,'k-')
%hold on
%plot(bestnum(5),totaltime(5),'k.','Markersize',18)
%hold on
%y=50:0.1:totaltime(5);
%x=bestnum(4)*ones(size(y));
%plot(x,y,'k-.')
%hold on
%xx=5:0.1:bestnum(5);
%yy=totaltime(5)*ones(size(xx));
%plot(xx,yy,'k-.')
%xlabel('Number of tollbooths')
%ylabel('Average delay time(second)')
%title('Average delay time Vs.Number of tollbooth')
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Condition.m
function [waittime]=condition(lamda,tollnum,lanes,jamfactor,mu,freevel)
for i=1:length(lamda)
      t1(i)=wait(lamda(i),mu,tollnum);
      t2(i)=choke(lamda(i),mu,jamfactor,tollnum,lanes,freevel);
      waittime(i)=(t1(i)+t2(i))*3600;
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```