Fundamentals of Photonics
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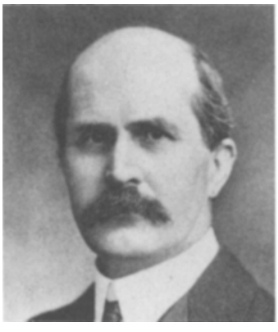
CHAPTER

20

ACOUSTO-OPTICS



Sir William Henry Bragg (1862–1942, left) and **Sir William Lawrence Bragg (1890–1971**, right), a father-and-son team, were awarded the Nobel Prize in 1915 for their studies of the diffraction of light from periodic structures, such as those created by sound.

The refractive index of an optical medium is altered by the presence of sound. Sound therefore modifies the effect of the medium on light; i.e., *sound can control light* (Fig. 20.0-1). Many useful devices make use of this **acousto-optic effect**; these include optical modulators, switches, deflectors, filters, isolators, frequency shifters, and spectrum analyzers.

Sound is a dynamic strain involving molecular vibrations that take the form of waves which travel at a velocity characteristic of the medium (the velocity of sound). As an example, a harmonic plane wave of compressions and rarefactions in a gas is pictured in Fig. 20.0-2. In those regions where the medium is compressed, the density is higher and the refractive index is larger; where the medium is rarefied, its density and refractive index are smaller. In solids, sound involves vibrations of the molecules about their equilibrium positions, which alter the optical polarizability and consequently the refractive index.

An acoustic wave creates a perturbation of the refractive index in the form of a wave. The medium becomes a *dynamic* graded-index medium—an inhomogeneous medium with a time-varying stratified refractive index. The theory of acousto-optics deals with the perturbation of the refractive index caused by sound, and with the propagation of light through this perturbed time-varying inhomogeneous medium.

The propagation of light in static (as opposed to time-varying) inhomogeneous (graded-index) media was discussed at several points in Chaps. 1 and 2 (Secs. 1.3 and 2.4C). Since optical frequencies are much greater than acoustic frequencies, the variations of the refractive index in a medium perturbed by sound are usually very slow in comparison with an optical period. There are therefore two significantly different time scales for light and sound. As a consequence, it is possible to use an adiabatic approach in which the optical propagation problem is solved separately at every instant of time during the relatively slow course of the acoustic cycle, always treating the material as if it were a static (frozen) inhomogeneous medium. In this quasi-stationary approximation, acousto-optics becomes the optics of an inhomogeneous medium (usually periodic) that is controlled by sound.

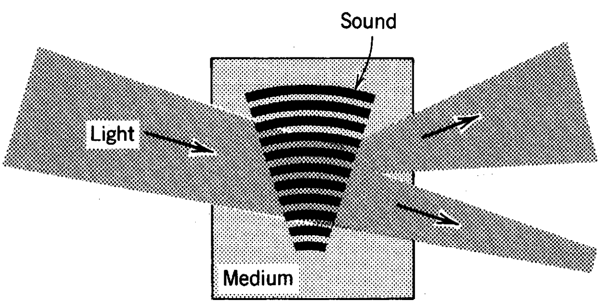


Figure 20.0-1 Sound modifies the effect of an optical medium on light.

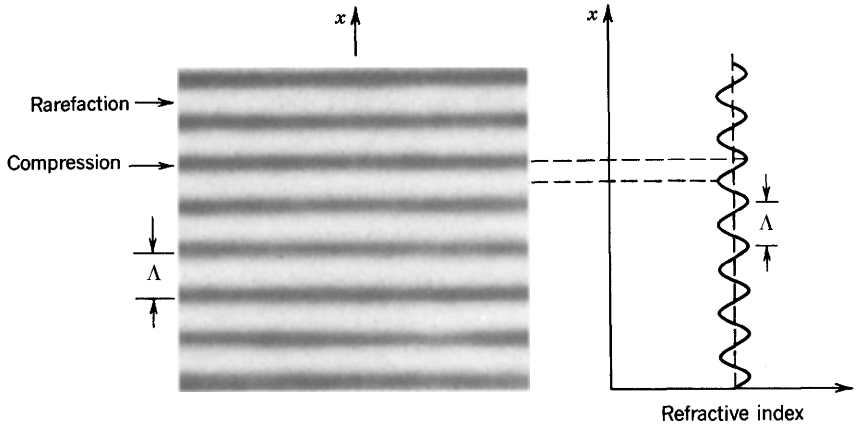


Figure 20.0-2 Variation of the refractive index accompanying a harmonic sound wave. The pattern has a period Λ, the wavelength of sound, and travels with the velocity of sound.

The simplest form of interaction of light and sound is the partial reflection of an optical plane wave from the stratified parallel planes representing the refractive-index variations created by an acoustic plane wave (Fig. 20.0-3). A set of parallel reflectors separated by the wavelength of sound Λ will reflect light if the angle of incidence θ satisfies the Bragg condition for constructive interference,

$$\boxed{\sin\theta = \frac{\lambda}{2\Lambda},} \qquad (20.0\text{-}1)$$

Bragg Condition

where λ is the wavelength of light in the medium (see Exercise 2.5-3). This form of light-sound interaction is known as **Bragg diffraction**, Bragg reflection, or Bragg scattering. The device that effects it is known as a Bragg reflector, a Bragg deflector, or a **Bragg cell**.

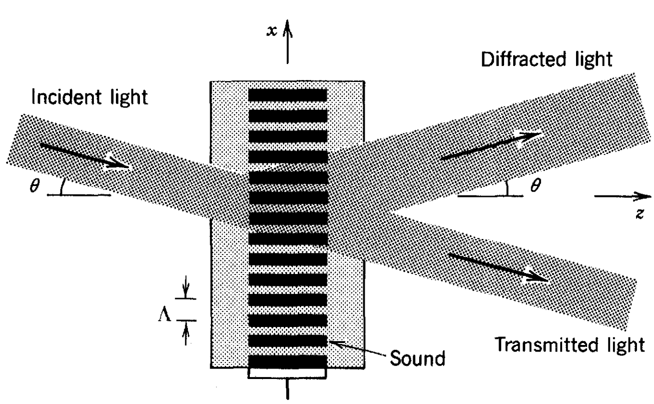


Figure 20.0-3 Bragg diffraction: an acoustic plane wave acts as a partial reflector of light (a beamsplitter) when the angle of incidence θ satisfies the Bragg condition.

Bragg cells have found numerous applications in photonics. This chapter is devoted to their properties. In Sec. 20.1, a simple theory of the optics of Bragg reflectors is presented for linear, nondispersive media. Anisotropic properties of the medium and the polarized nature of light and sound are ignored. Although the theory is based on wave optics, a simple quantum interpretation of the results is provided. In Sec. 20.2, the use of Bragg cells for light modulation and scanning is discussed. Section 20.3 provides a brief introduction to anisotropic and polarization effects in acousto-optics.

20.1 INTERACTION OF LIGHT AND SOUND

The effect of a scalar acoustic wave on a scalar optical wave is described in this section. We first consider optical and acoustic plane waves, and subsequently examine the interaction of optical and acoustic beams.

A. Bragg Diffraction

Consider an acoustic plane wave traveling in the x direction in a medium with velocity v_s, frequency f, and wavelength $\Lambda = v_s/f$. The strain (relative displacement) at position x and time t is

$$s(x,t) = S_0 \cos(\Omega t - qx), \tag{20.1-1}$$

where S_0 is the amplitude, $\Omega = 2\pi f$ is the angular frequency, and $q = 2\pi/\Lambda$ is the wavenumber. The acoustic intensity (W/m^2) is

$$I_s = \tfrac{1}{2}\varrho v_s^3 S_0^2, \tag{20.1-2}$$

where ϱ is the mass density of the medium.

The medium is assumed to be optically transparent and the refractive index in the absence of sound is n. The strain $s(x,t)$ creates a proportional perturbation of the refractive index, analogous to the Pockels effect in (18.1-4),

$$\Delta n(x,t) = -\tfrac{1}{2}\mathfrak{p} n^3 s(x,t), \tag{20.1-3}$$

where $\mathfrak{p}$ is a phenomenological coefficient known as the **photoelastic constant** (or strain-optic coefficient). The minus sign indicates that positive strain (dilation) leads to a reduction of the refractive index. As a consequence, the medium has a time-varying inhomogeneous refractive index in the form of a wave

$$\boxed{n(x,t) = n - \Delta n_0 \cos(\Omega t - qx),} \tag{20.1-4}$$

with amplitude

$$\Delta n_0 = \tfrac{1}{2}\mathfrak{p} n^3 S_0. \tag{20.1-5}$$

Substituting from (20.1-2) into (20.1-5), we find that the change of the refractive index is proportional to the square root of the acoustic intensity,

$$\boxed{\Delta n_0 = \left(\tfrac{1}{2}\mathscr{M} I_s\right)^{1/2},} \tag{20.1-6}$$

where

$$\mathscr{M} = \frac{\mathfrak{p}^2 n^6}{\varrho v_s^3} \tag{20.1-7}$$

is a material parameter representing the effectiveness of sound in altering the refractive index. $\mathscr{M}$ is a figure of merit for the strength of the acousto-optic effect in the material.

EXAMPLE 20.1-1. ***Figure of Merit.*** In extra-dense flint glass $\varrho = 6.3 \times 10^3$ kg/m^3, $v_s = 3.1$ km/s, $n = 1.92$, $\mathfrak{p} = 0.25$, so that $\mathscr{M} = 1.67 \times 10^{-14}$ m^2/W. An acoustic wave of intensity 10 W/cm^2 creates a refractive-index wave of amplitude $\Delta n_0 = 2.89 \times 10^{-5}$.

Consider now an optical plane wave traveling in this medium with frequency ν, angular frequency $\omega = 2\pi\nu$, free-space wavelength $\lambda_o = c_o/\nu$, wavelength in the unperturbed medium $\lambda = \lambda_o/n$ corresponding to a wavenumber $k = n\omega/c_o$, and wavevector $\mathbf{k}$ lying in the x–z plane and making an angle θ with the z axis, as illustrated in Fig. 20.0-3.

Because the acoustic frequency f is typically much smaller than the optical frequency ν (by at least five orders of magnitude), an adiabatic approach for studying light–sound interaction may be adopted: We regard the refractive index as a static "frozen" sinusoidal function

$$n(x) = n - \Delta n_0 \cos(qx - \varphi), \tag{20.1-8}$$

where φ is a fixed phase; we determine the reflected light from this inhomogeneous (graded-index) medium and track its slow variation with time by taking $\varphi = \Omega t$.

To determine the amplitude of the reflected wave we divide the medium into incremental planar layers orthogonal to the x axis. The incident optical plane wave is partially reflected at each layer because of the refractive-index change. We assume that the reflectance is sufficiently small so that the transmitted light from one layer approximately maintains its original magnitude (i.e., is not depleted) as it penetrates through the following layers of the medium.

If $\Delta r = (dr/dx)\,\Delta x$ is the incremental complex amplitude reflectance of the layer at position x, the total complex amplitude reflectance for an overall length L (see Fig. 20.1-1) is the sum of all incremental reflectances,

$$r = \int_{-L/2}^{L/2} e^{j2kx\sin\theta}\,\frac{dr}{dx}\,dx. \tag{20.1-9}$$

The phase factor $e^{j2kx\sin\theta}$ is included since the reflected wave at a position x is

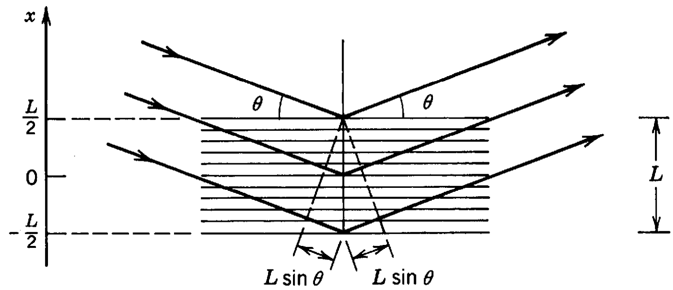


Figure 20.1-1 Reflections from layers of an inhomogeneous medium.

advanced by a distance $2x \sin\theta$ (corresponding to a phase shift $2kx \sin\theta$) relative to the reflected wave at $x = 0$ (see Fig. 20.1-1). The wavenumbers for the incident and reflected waves are taken to be the same, for reasons that will be explained later.

An expression for the incremental complex amplitude reflectance Δr in terms of the incremental refractive-index change Δn between two adjacent layers at a given position x may be determined by use of the Fresnel equations (see Sec. 6.2). For TE (orthogonal) polarization, (6.2-4) is used with $n_1 = n + \Delta n$, $n_2 = n$, $\theta_1 = 90° - \theta$, and Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2$ is used to determine θ_2. When terms of second order in Δn are neglected, the result is

$$\Delta r = \frac{-1}{2n\sin^2\theta}\,\Delta n. \tag{20.1-10}$$

Equation (6.2-6) is similarly used for the TM (parallel) polarization, yielding

$$\Delta r = \frac{-\cos 2\theta}{2n\sin^2\theta}\,\Delta n.$$

In most acousto-optic devices θ is very small, so that $\cos 2\theta \approx 1$, making (20.1-10) approximately applicable to both polarizations.

Using (20.1-8) and (20.1-10), we obtain

$$\frac{dr}{dx} = \frac{dr}{dn}\frac{dn}{dx} = \frac{-1}{2n\sin^2\theta}\left[q\,\Delta n_0 \sin(qx - \varphi)\right] = r'\sin(qx - \varphi), \tag{20.1-11}$$

where

$$r' = \frac{-q}{2n\sin^2\theta}\,\Delta n_0. \tag{20.1-12}$$

Finally, we substitute (20.1-11) into (20.1-9), and use complex notation to write $\sin(qx - \varphi) = [e^{j(qx-\varphi)} - e^{-j(qx-\varphi)}]/2j$, thereby obtaining

$$r = \tfrac{1}{2}jr'e^{j\varphi}\int_{-\frac{1}{2}L}^{\frac{1}{2}L} e^{j(2k\sin\theta - q)x}\,dx - \tfrac{1}{2}jr'e^{-j\varphi}\int_{-\frac{1}{2}L}^{\frac{1}{2}L} e^{j(2k\sin\theta + q)x}\,dx. \tag{20.1-13}$$

The first term in (20.1-13) has its maximum value when $2k\sin\theta = q$, whereas the second is maximum when $2k\sin\theta = -q$. If L is sufficiently large, these maxima are sharp, so that any slight deviation from the angles $\theta = \pm\sin^{-1}(q/2k)$ makes the corresponding term negligible. Thus only one of these two terms may be significant at a time, depending on the angle θ. For reasons to become clear shortly, the conditions $2k \sin\theta \approx q$ and $2k \sin\theta \approx -q$ are called the upshifted and downshifted reflections, respectively. We first consider the upshifted condition, $2k \sin\theta \approx q$, for which the second term is negligible, and comment on the downshifted case subsequently. Performing the integral in the first term of (20.1-13) and substituting $\varphi = \Omega t$, we obtain

$$\boxed{r = \tfrac{1}{2} j r' L \operatorname{sinc}\left[(q - 2k \sin\theta)\frac{L}{2\pi}\right] e^{j\Omega t},} \tag{20.1-14}$$

Amplitude Reflectance (Upshifted Case)

where $\operatorname{sinc}(x) = \sin(\pi x)/(\pi x)$. We proceed to discuss several important conclusions based on (20.1-14).

Bragg Condition

The sinc function in (20.1-14) has its maximum value of 1.0 when its argument is zero, i.e., when $q = 2k \sin\theta$. This occurs when $\theta = \theta_B$, where $\theta_B = \sin^{-1}(q/2k)$ is the **Bragg angle**. Since $q = 2\pi/\Lambda$ and $k = 2\pi/\lambda$,

$$\boxed{\sin\theta_B = \frac{\lambda}{2\Lambda}.} \tag{20.0-1}$$

Bragg Angle

The Bragg angle is the angle for which the incremental reflections from planes separated by an acoustic wavelength Λ have a phase shift of 2π so that they interfere constructively (see Exercise 2.5-3).

EXAMPLE 20.1-2. ***Bragg Angle.*** An acousto-optic cell is made of flint glass in which the sound velocity is $v_s = 3$ km/s and the refractive index is $n = 1.95$. The Bragg angle for reflection of an optical wave of free-space wavelength $\lambda_o = 633$ nm ($\lambda = \lambda_o/n = 325$ nm) from a sound wave of frequency $f = 100$ MHz ($\Lambda = v_s/f = 30$ μm) is $\theta_B = 5.4$ mrad $\approx 0.31°$. This angle is internal (i.e., inside the medium). If the cell is placed in air, θ_B corresponds to an external angle $\theta'_B \approx n\theta_B = 0.61°$. A sound wave of 10 times greater frequency ($f = 1$ GHz) corresponds to a Bragg angle $\theta_B \approx 3.1°$.

The Bragg condition can also be stated as a simple relation between the wavevectors of the sound wave and the two optical waves. If $\mathbf{q} = (q, 0, 0)$, $\mathbf{k} = (-k\sin\theta, 0, k\cos\theta)$, and $\mathbf{k}_r = (k\sin\theta, 0, k\cos\theta)$ are the components of the wavevectors of the sound wave, the incident light wave, and the reflected light wave, respectively, the condition $q = 2k\sin\theta_B$ is equivalent to the vector relation

$$\boxed{\mathbf{k}_r = \mathbf{k} + \mathbf{q},} \tag{20.1-15}$$

illustrated by the vector diagram in Fig. 20.1-2.

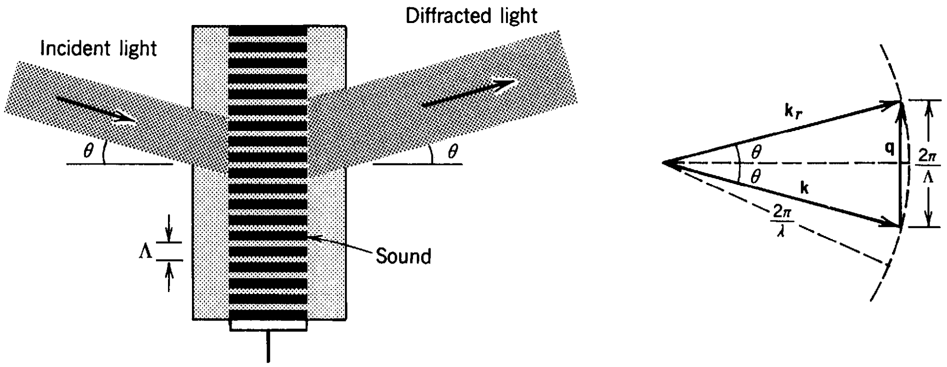


Figure 20.1-2 The Bragg condition $\sin\theta_B = q/2k$ is equivalent to the vector relation $\mathbf{k}_r = \mathbf{k} + \mathbf{q}$.

Tolerance in the Bragg Condition

The dependence of the complex amplitude reflectance on the angle θ is governed by the symmetric function $\text{sinc}[(q - 2k\sin\theta)L/2\pi] = \text{sinc}[(\sin\theta - \sin\theta_B)2L/\lambda]$ in (20.1-14). This function reaches its peak value when $\theta = \theta_B$ and drops sharply when θ differs slightly from θ_B. When $\sin\theta - \sin\theta_B = \lambda/2L$ the sinc function reaches its first zero and the reflectance vanishes (Fig. 20.1-3). Because θ_B is usually very small, $\sin\theta \approx \theta$, and the reflectance vanishes at an angular deviation from the Bragg angle of approximately $\theta - \theta_B \approx \lambda/2L$. Since L is typically much greater than λ, this is an extremely small angular width. This sharp reduction of the reflectance for slight deviations from the Bragg angle occurs as a result of the destructive interference between the incremental reflections from the sound wave.

Doppler Shift

In accordance with (20.1-14), the complex amplitude reflectance r is proportional to $\exp(j\Omega t)$. Since the angular frequency of the incident light is ω [i.e., $E \propto \exp(j\omega t)$], the reflected wave $E_r = rE \propto \exp[j(\omega + \Omega)t]$ has angular frequency

$$\boxed{\omega_r = \omega + \Omega.} \tag{20.1-16}$$

Doppler Shift

The process of reflection is therefore accompanied by a frequency shift equal to the

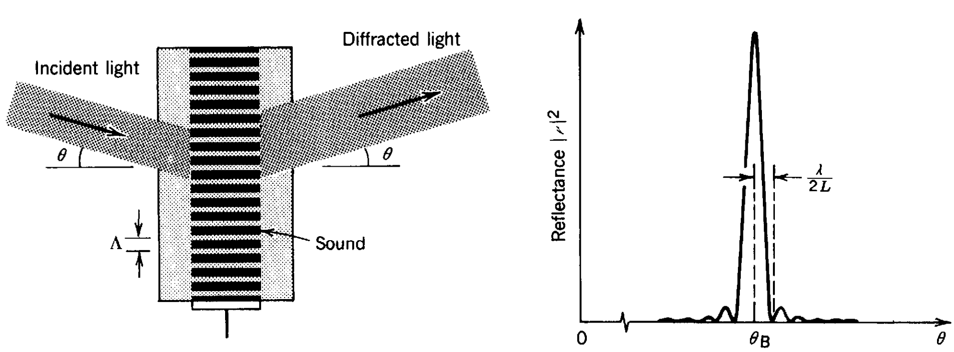


Figure 20.1-3 Dependence of the reflectance $|r|^2$ on the angle θ. Maximum reflection occurs at the Bragg angle $\theta_B = \sin^{-1}(\lambda/2\Lambda)$.

frequency of the sound. This can also be thought of as a Doppler shift (see Exercise 2.6-1 and Sec. 12.2D). The incident light is reflected from surfaces that move with a velocity v_s. Its Doppler-shifted angular frequency is therefore $\omega_r = \omega(1 + 2v_s \sin\theta/c)$, where $v_s \sin\theta$ is the component of velocity of these surfaces in the direction of the incident and the reflected waves. Using the relations $\sin\theta = \lambda/2\Lambda$, $v_s = \Lambda\Omega/2\pi$, and $c = \lambda\omega/2\pi$, (20.1-16) is reproduced. The Doppler shift equals the sound frequency.

Because $\Omega \ll \omega$, the frequencies of the incident and reflected waves are approximately equal (with an error typically smaller than 1 part in 10^5). The wavelengths of the two waves are therefore also approximately equal. In writing (20.1-9) we have implicitly used this assumption by using the same wavenumber k for the two waves. Also, in drawing the vector diagram in Fig. 20.1-2 it was assumed that the vectors $\mathbf{k}_r$ and $\mathbf{k}$ have approximately the same length $n\omega/c_o$.

Reflectance

The reflectance $\mathscr{R} = |r|^2$ is the ratio of the intensity of the reflected optical wave to that of the incident optical wave. At the Bragg angle $\theta = \theta_B$, (20.1-14) gives $\mathscr{R} = |r'|^2 L^2/4$. Substituting for r' from (20.1-12),

$$\mathscr{R} = \frac{\pi^2}{\lambda_o^2}\left(\frac{L}{\sin\theta}\right)^2 \Delta n_0^2, \tag{20.1-17}$$

and using (20.1-6), we obtain

$$\boxed{\mathscr{R} = \frac{\pi^2}{2\lambda_o^2}\left(\frac{L}{\sin\theta}\right)^2 \mathscr{M} I_s.} \tag{20.1-18}$$

Reflectance

The reflectance $\mathscr{R}$ is therefore proportional to the intensity of the acoustic wave I_s, to the material parameter $\mathscr{M}$ defined in (20.1-7) and to the square of the oblique distance $L/\sin\theta$ of penetration of light through the acoustic wave.

Substituting $\sin\theta = \lambda/2\Lambda$ into (20.1-18), we obtain

$$\mathscr{R} = 2\pi^2 n^2 \frac{L^2\Lambda^2}{\lambda_o^4} \mathscr{M} I_s.$$

Thus the reflectance is inversely proportional to λ_o^4 (or directly proportional to ω^4). The dependence of the efficiency of scattering on the fourth power of the optical frequency is typical of light-scattering phenomena.

The proportionality between the reflectance and the sound intensity poses a problem. As the sound intensity increases, $\mathscr{R}$ would eventually exceed unity, and the reflected light would be more intense than the incident light! This unacceptable result is a consequence of violating the assumptions of this approximate theory. It was assumed that the incremental reflection from each layer is too small to deplete the transmitted wave which reflects from subsequent layers. Clearly, this assumption does not hold when the sound wave is intense. In reality, a saturation process occurs, ensuring that $\mathscr{R}$ does not exceed unity. A more careful analysis (see Sec. 20.1C), in which depletion of the incident optical wave is included, leads to the following

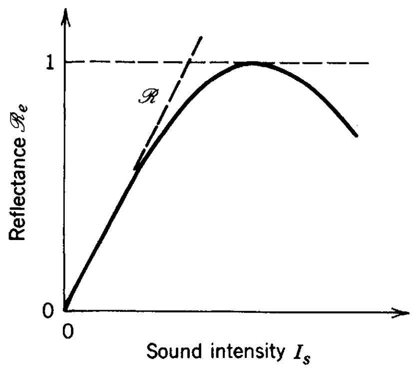


Figure 20.1-4 Dependence of the reflectance $\mathscr{R}_e$ of the Bragg reflector on the intensity of sound I_s. When I_s is small $\mathscr{R}_e \approx \mathscr{R}$, which is a linear function of I_s.

expression for the reflectance:

$$\boxed{\mathscr{R}_e = \sin^2 \sqrt{\mathscr{R}},} \tag{20.1-19}$$

where $\mathscr{R}$ is the approximate expression (20.1-18) and $\mathscr{R}_e$ is the exact expression. This relation is illustrated in Fig. 20.1-4. Evidently, when $\mathscr{R} \ll 1$, $\sin \sqrt{\mathscr{R}} \approx \sqrt{\mathscr{R}}$, so that $\mathscr{R}_e \approx \mathscr{R}$.

EXAMPLE 20.1-3. *Reflectance.* A Bragg cell is made of extra-dense flint glass with material parameter $\mathscr{M} = 1.67 \times 10^{-14}$ m^2/W (see Example 20.1-1). If $\lambda_o = 633$ nm (wavelength of the He–Ne laser), the sound intensity $I_s = 10$ W/cm^2, and the length of penetration of the light through the sound is $L/\sin\theta = 1$ mm, then $\mathscr{R} = 0.0206$ and $\mathscr{R}_e = 0.0205$, so that approximately 2% of the light is reflected. If the sound intensity is increased to 100 W/cm^2, then $\mathscr{R} = 0.206$, $\mathscr{R}_e = 0.192$ (i.e., the reflectance increases to $\approx$ 19%).

Downshifted Bragg Diffraction

Another possible geometry for Bragg diffraction is that for which $2k \sin\theta = -q$. This is satisfied when the angle θ is negative; i.e., the incident optical wave makes an acute angle with the sound wave as illustrated in Fig. 20.1-5. In this case, the second term of (20.1-13) has its maximum value, whereas the first term is negligible. The complex amplitude reflectance is then given by

$$r = -\tfrac{1}{2} j r' L e^{-j\Omega t}. \tag{20.1-20}$$

In this geometry, the frequency of the reflected wave is downshifted, so that

$$\omega_s = \omega - \Omega \tag{20.1-21}$$

and the wavevectors of the light and sound waves satisfy the relation

$$\mathbf{k}_s = \mathbf{k} - \mathbf{q}, \tag{20.1-22}$$

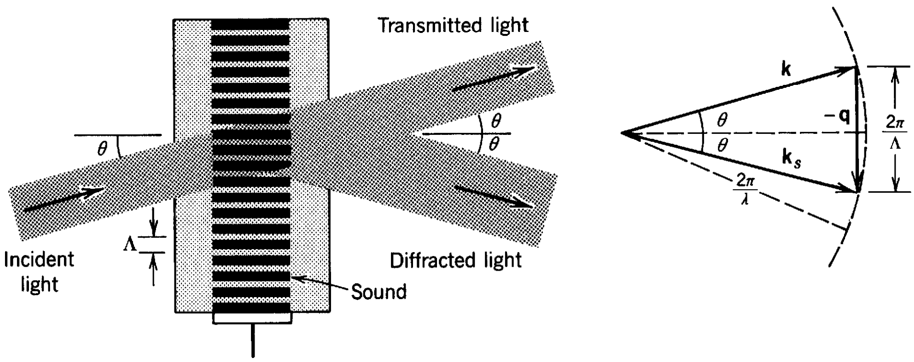


Figure 20.1-5 Geometry of downshifted reflection of light from sound. The frequency of the reflected wave is downshifted.

illustrated in Fig. 20.1-5. Equation (20.1-22) is a phase-matching condition, ensuring that the reflections of light add in phase. The frequency downshift in (20.1-21) is consistent with the Doppler shift since the light and sound waves travel in the same direction.

B. Quantum Interpretation

In accordance with the quantum theory of light (see Chap. 11), an optical wave of angular frequency ω and wavevector $\mathbf{k}$ is viewed as a stream of photons, each of energy $\hbar\omega$ and momentum $\hbar\mathbf{k}$. An acoustic wave of angular frequency Ω and wavevector $\mathbf{q}$ is similarly regarded as a stream of acoustic quanta, called **phonons**, each of energy $\hbar\Omega$ and momentum $\hbar\mathbf{q}$.

Interaction of light and sound occurs when a photon combines with a phonon to generate a new photon of the sum energy and momentum. An incident photon of frequency ω and wavevector $\mathbf{k}$ interacts with a phonon of frequency Ω and wavevector $\mathbf{q}$ to generate a new photon of frequency ω_r and wavevector $\mathbf{k}_r$, as illustrated in Fig. 20.1-6. Conservation of energy and momentum require that $\hbar\omega_r = \hbar\omega + \hbar\Omega$ and $\hbar\mathbf{k}_r = \hbar\mathbf{k} + \hbar\mathbf{q}$, from which the Doppler shift formula $\omega_r = \omega + \Omega$ and the Bragg condition, $\mathbf{k}_r = \mathbf{k} + \mathbf{q}$, are recovered.

*C. Coupled-Wave Theory

Bragg Diffraction as a Scattering Process

Light propagation through an inhomogeneous medium with dynamic refractive index perturbation $\Delta n(x, t)$ my also be regarded as a light-scattering process and the Born approximation (see Sec. 19.1) may be used to describe it. A perturbation $\Delta\mathscr{P}$ of the

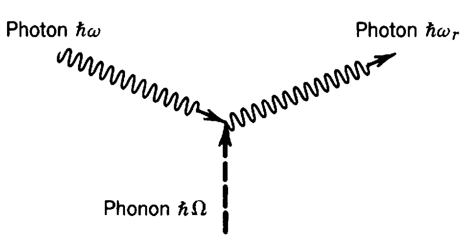


Figure 20.1-6 Bragg diffraction: a photon combines with a phonon to generate a new photon of different frequency and momentum.

electric polarization density acts as a source of light

$$\mathscr{S} = -\mu_o \frac{\partial^2 \Delta \mathscr{P}}{\partial t^2} \qquad (20.1\text{-}23)$$

[see (5.2-19) and the discussion following (19.1-7)]. Since $\mathscr{P} = \epsilon_o \chi \mathscr{E} = \epsilon_o(\epsilon/\epsilon_o - 1)\mathscr{E} = \epsilon_o(n^2 - 1)\mathscr{E}$, where $\mathscr{E}$ is the electric field, the perturbation Δn corresponds to $\Delta \mathscr{P} = \epsilon_o \, \Delta(n^2 - 1)\mathscr{E} = 2\epsilon_o n \, \Delta n \mathscr{E}$, so that

$$\boxed{\mathscr{S} = -2\mu_o \epsilon_o n \frac{\partial^2}{\partial t^2}(\Delta n \mathscr{E}).} \qquad (20.1\text{-}24)$$

Thus the source $\mathscr{S}$ is proportional to the second derivative of the product $\Delta n \mathscr{E}$. To determine the scattered field we solve the wave equation (19.1-6), $\nabla^2 \mathscr{E} - (1/c^2)\partial^2 \mathscr{E}/\partial^2 t = -\mathscr{S}$, together with (20.1-24) and $\Delta n = -\Delta n_0 \cos(\Omega t - \mathbf{q} \cdot \mathbf{r})$.

The idea of the first Born approximation is to assume that the source $\mathscr{S}$ is created by the incident field only and to solve the wave equation for the scattered field. Substituting $\mathscr{E} = \mathrm{Re}\{A \exp[j(\omega t - \mathbf{k} \cdot \mathbf{r})]\}$ into (20.1-24), where A is a slowly varying envelope, we obtain

$$\mathscr{S} = -\left(\frac{\Delta n_0}{n}\right)\left(k_r^2 \, \mathrm{Re}\{A \exp[j(\omega_r t - \mathbf{k}_r \cdot \mathbf{r})]\} + k_s^2 \, \mathrm{Re}\{A \exp[j(\omega_s t - \mathbf{k}_s \cdot \mathbf{r})]\}\right), \qquad (20.1\text{-}25)$$

where $\omega_r = \omega + \Omega$, $\mathbf{k}_r = \mathbf{k} + \mathbf{q}$, $k_r = \omega_r/c$; and $\omega_s = \omega - \Omega$, $\mathbf{k}_s = \mathbf{k} - \mathbf{q}$, $k_s = \omega_s/c$. We thus have two sources of light of frequencies $\omega \pm \Omega$, and wavevectors $\mathbf{k} \pm \mathbf{q}$, that may emit an upshifted or downshifted Bragg-reflected plane wave. Upshifted reflection occurs if the geometry is such that the magnitude of the vector $\mathbf{k} + \mathbf{q}$ equals $\omega_r/c \approx \omega/c$, as can easily be seen from the vector diagram in Fig. 20.1-2. Downshifted reflection occurs if the vector $\mathbf{k} - \mathbf{q}$ has magnitude $\omega_s/c \approx \omega/c$, as illustrated in Fig. 20.1-5. Obviously, these two conditions may not be met simultaneously.

We have thus independently proved the Bragg condition and Doppler-shift formula using a scattering approach. Equation (20.1-25) indicates that the intensity of the emitted light is proportional to $\omega_r^4 \approx \omega^4$, so that the efficiency of scattering is inversely proportional to the fourth power of the wavelength. This analysis can be pursued further to derive an expression for the reflectance by determining the intensity of the wave emitted by the scattering source (see Problem 20.1-2).

Coupled-Wave Equations

To go beyond the first Born approximation, we must include the contribution made by the scattered field to the source $\mathscr{S}$. Assuming that the geometry is that of upshifted Bragg diffraction, the field $\mathscr{E}$ is composed of the incident and Bragg-reflected waves: $\mathscr{E} = \mathrm{Re}\{E \exp(j\omega t)\} + \mathrm{Re}\{E_r \exp(j\omega_r t)\}$. With the help of the relation $\Delta n = -\Delta n_0 \cos(\Omega t - \mathbf{q} \cdot \mathbf{r})$, (20.1-24) gives

$$\mathscr{S} = \mathrm{Re}\{S \exp(j\omega t) + S_r \exp(j\omega_r t)\} + \text{terms of other frequencies},$$

where

$$S = -k^2 \frac{\Delta n_0}{n} E_r, \qquad S_r = -k_r^2 \frac{\Delta n_0}{n} E. \qquad (20.1\text{-}26)$$

Comparing terms of equal frequencies on both sides of the wave equation, $\nabla^2\mathscr{E} - (1/c^2)\partial^2\mathscr{E}/\partial^2 t = -\mathscr{S}$, we obtain two coupled Helmholtz equations for the incident wave and the Bragg-reflected wave,

$$(\nabla^2 + k^2)E = -S, \qquad (\nabla^2 + k_r^2)E_r = -S_r. \tag{20.1-27}$$

These equations, together with (20.1-26), may be solved to determine E and E_r.

Consider, for example, the case of small-angle reflection ($\theta \ll 1$), so that the two waves travel approximately in the z direction. Assuming that $k \approx k_r$, the fields E and E_r are described by $E = A\exp(-jkz)$ and $E_r = A_r\exp(-jkz)$, where A and A_r are slowly varying functions of z. Using the slowly varying envelope approximation (see Sec. 2.2C), $(\nabla^2 + k^2)A\exp(-jkz) \approx -j2k(dA/dz)\exp(-jkz)$, (20.1-26) and (20.1-27) yield

$$\boxed{\frac{dA}{dz} = j\tfrac{1}{2}\gamma A_r} \tag{20.1-28a}$$

$$\boxed{\frac{dA_r}{dz} = j\tfrac{1}{2}\gamma A,} \tag{20.1-28b}$$

where

$$\gamma = k\frac{\Delta n_0}{n}. \tag{20.1-29}$$

If the cell extends between $z = 0$ and $z = d$, we use the boundary condition $A_r(0) = 0$, and find that equations (20.1-28) have the harmonic solution

$$A(z) = A(0)\cos\frac{\gamma z}{2} \tag{20.1-30a}$$

$$A_r(z) = jA(0)\sin\frac{\gamma z}{2}. \tag{20.1-30b}$$

These equations describe the rise of the reflected wave and the fall of the incident wave, as illustrated in Fig. 20.1-7. The reflectance $\mathscr{R}_e = |A_r(d)|^2/|A(0)|^2$ is therefore given by $\mathscr{R}_e = \sin^2(\gamma d/2)$, so that $\mathscr{R}_e = \sin^2\sqrt{\mathscr{R}}$, where $\mathscr{R} = (\gamma d/2)^2$. Using

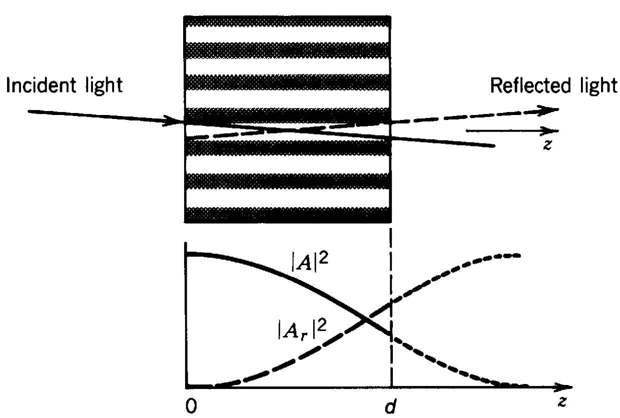


Figure 20.1-7 Variation of the intensity of the incident optical wave (solid curve) and the intensity of the Bragg-reflected wave (dashed curve) as functions of the distance traveled through the acoustic wave.

(20.1-29), we obtain $\mathscr{R} = (\pi^2/\lambda_o^2)\,\Delta n_0^2 d^2$. This is exactly the expression for the weak-sound reflectance in (20.1-17) with $d = L/\sin\theta$.

D. Bragg Diffraction of Beams

It has been shown so far that an optical *plane wave* of wavevector $\mathbf{k}$ interacts with an acoustic *plane wave* of wavevector $\mathbf{q}$ to produce an optical plane wave of wavevector $\mathbf{k}_r = \mathbf{k} + \mathbf{q}$, provided that the Bragg condition is satisfied (i.e., the angle between $\mathbf{k}$ and $\mathbf{q}$ is such that the magnitude $k_r = |\mathbf{k} + \mathbf{q}| \approx k = 2\pi/\lambda$). Interaction between a *beam* of light and a *beam* of sound can be understood if the beam is regarded as a superposition of plane waves traveling in different directions, each with its own wavevector (see the introduction to Chap. 4).

Diffraction of an Optical Beam from an Acoustic Plane Wave

Consider an optical *beam* of width D interacting with an acoustic *plane wave*. In accordance with Fourier optics (see Sec. 4.3A), the optical beam can be decomposed into plane waves with directions occupying a cone of half-angle

$$\delta\theta = \frac{\lambda}{D}. \tag{20.1-31}$$

There is some arbitrariness in the definition of the diameter D and the angle $\delta\theta$, and a multiplicative factor in (20.1-31) is taken to be 1.0. If the beam profile is rectangular of width D, the angular width from the peak to the first zero of the Fraunhofer diffraction pattern is $\delta\theta = \lambda/D$; for a circular beam of diameter D, $\delta\theta = 1.22\lambda/D$; for a Gaussian beam of waist diameter $D = 2W_0$, $\delta\theta = \lambda/\pi W_0 = (2/\pi)\lambda/D \approx 0.64\lambda/D$ [see (3.1-19)]. For simplicity, we shall use (20.1-31).

Although there is only one wavevector $\mathbf{q}$, there are many wavevectors $\mathbf{k}$ (all of the same length $2\pi/\lambda$) within a cone of angle $\delta\theta$. As Fig. 20.1-8 illustrates, there is only one direction of $\mathbf{k}$ for which the Bragg condition is satisfied. The reflected wave is then a plane wave with only one wavevector $\mathbf{k}_r$.

Diffraction of an Optical Beam from an Acoustic Beam

Suppose now that the acoustic wave itself is a beam of width D_s. If the sound frequency is sufficiently high so that the wavelength is much smaller than the width of

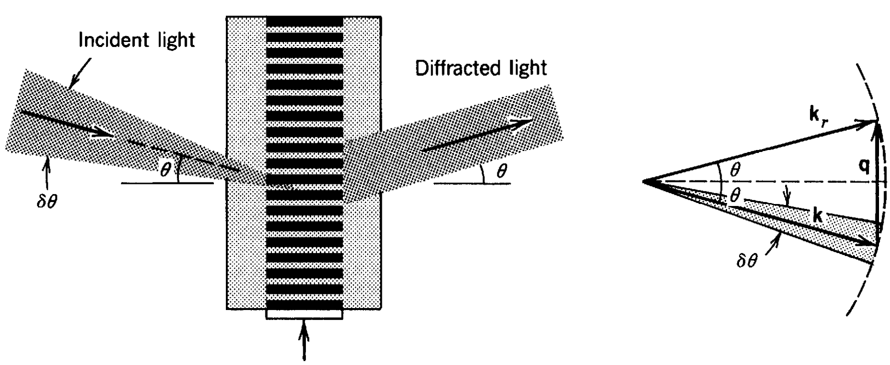


Figure 20.1-8 Diffraction of an optical *beam* from an acoustic *plane wave*. There is only one plane-wave component of the incident light beam that satisfies the Bragg condition. The diffracted light is a plane wave.

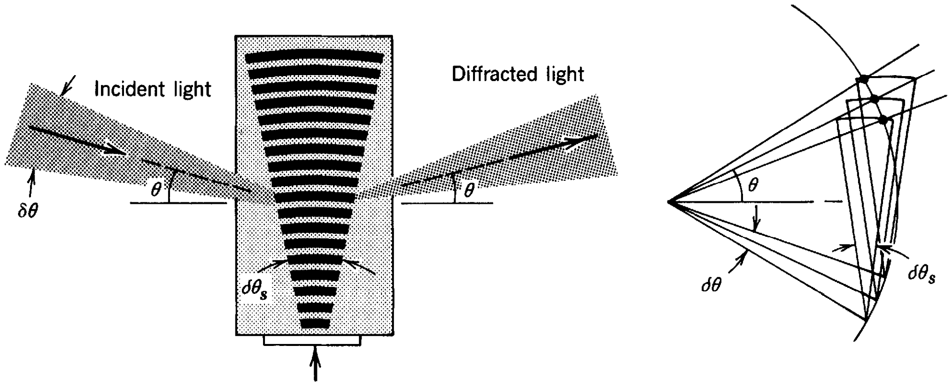


Figure 20.1-9 Diffraction of an optical *beam* from a sound *beam*.

the medium, sound propagates as an unguided (free-space) wave and has properties analogous to those of optical beams, with angular divergence

$$\delta\theta_s = \frac{\Lambda}{D_s}. \tag{20.1-32}$$

This is equivalent to many plane waves with directions lying within the divergence angle.

The reflection of an optical beam from this acoustic beam can be determined by finding matching pairs of optical and acoustic plane waves satisfying the Bragg condition. The sum of the reflected waves constitutes the reflected optical beam. There are many vectors $\mathbf{k}$ (all of the same length $2\pi/\lambda$) and many vectors $\mathbf{q}$ (all of the same length $2\pi/\Lambda$); only the pairs of vectors that form an isoceles triangle contribute, as illustrated in Fig. 20.1-9.

If the acoustic-beam divergence is greater than the optical-beam divergence ($\delta\theta_s \gg \delta\theta$) and if the central directions of the two beams satisfy the Bragg condition, every incident optical plane wave finds an acoustic match and the reflected light beam has the same angular divergence as the incident optical beam $\delta\theta$. The distribution of acoustic energy in the sound beam can thus be monitored as a function of direction, by using a probe light beam of much narrower divergence and measuring the reflected light as the angle of incidence is varied.

Diffraction of an Optical Plane Wave from a Thin Acoustic Beam; Raman–Nath Diffraction

Since a thin acoustic beam comprises plane waves traveling in many directions, it can diffract light at angles that are significantly different from the Bragg angle corresponding to the beam's principal direction. Consider, for example, the geometry in Fig. 20.1-10 in which the incident optical plane wave is perpendicular to the main direction of a thin acoustic beam. The Bragg condition is satisfied if the reflected wavevector $\mathbf{k}_r$ makes angles $\pm\theta$, where

$$\sin\frac{\theta}{2} = \frac{\lambda}{2\Lambda}. \tag{20.1-33}$$

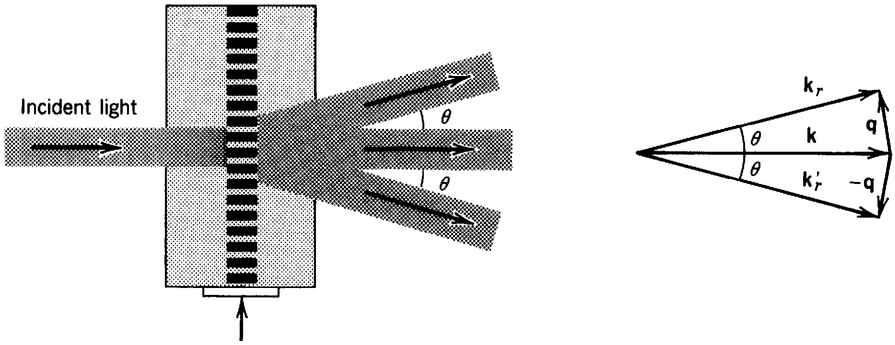


Figure 20.1-10 An optical plane wave incident normally on a thin-beam acoustic standing wave is partially deflected into two directions making angles $\approx \pm\lambda/\Lambda$.

If θ is small, $\sin(\theta/2) \approx \theta/2$ and

$$\boxed{\theta \approx \frac{\lambda}{\Lambda}.} \tag{20.1-34}$$

The incident beam is therefore deflected into either of the two directions making angles $\pm\theta$, depending on whether the acoustic beam is traveling upward or downward. For an acoustic standing-wave beam the optical wave is deflected in both directions.

The angle $\theta \approx \lambda/\Lambda$ is the angle by which a diffraction grating of period Λ deflects an incident plane wave (see Exercise 2.4-5). The thin acoustic beam in fact modulates the refractive index, creating a periodic pattern of period Λ confined to a thin planar layer. The medium therefore acts as a thin diffraction grating. This phase grating diffracts light also into higher diffraction orders, as illustrated in Fig. 20.1-11(*a*).

The higher-order diffracted waves generated by the phase grating at angles $\pm 2\theta$, $\pm 3\theta, \ldots$ may also be interpreted using a quantum picture of light–sound interaction. One incident photon combines with two phonons (acoustic quantum particles) to form a photon of the second-order reflected wave. Conservation of momentum requires that $\mathbf{k}_r = \mathbf{k} \pm 2\mathbf{q}$. This condition is satisfied for the geometry in Fig. 20.1-11(*b*). The second-order reflected light is frequency shifted to $\omega_r = \omega \pm 2\Omega$. Similar interpretations apply to higher orders of diffraction.

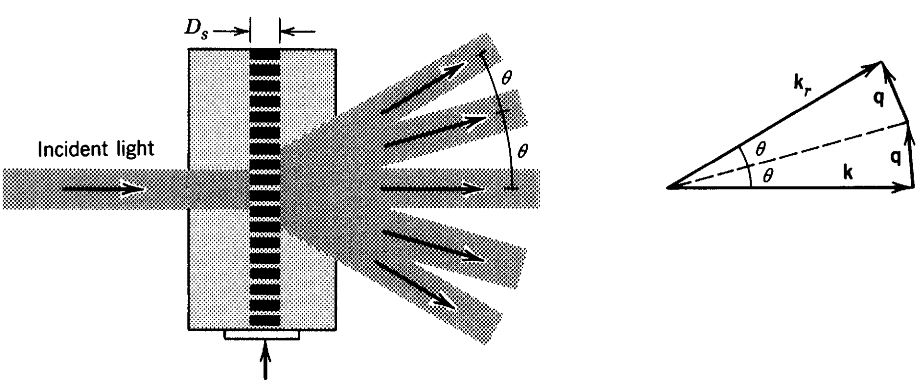


Figure 20.1-11 (*a*) A thin acoustic beam acts as a diffraction grating. (*b*) Conservation-of-momentum diagram for second-order acousto-optic diffraction.

The acousto-optic interaction of light with a *perpendicular thin* sound beam is known as **Raman–Nath** or **Debye–Sears scattering** of light by sound.†

20.2 ACOUSTO-OPTIC DEVICES

A. Modulators

The intensity of the reflected light in a Bragg cell is proportional to the intensity of sound, if the sound intensity is sufficiently weak. Using an electrically controlled acoustic transducer [Fig. 20.2-1(*a*)], the intensity of the reflected light can be varied proportionally. The device can be used as a linear analog modulator of light.

As the acoustic power increases, however, saturation occurs and almost total reflection can be achieved (see Fig. 20.1-4). The modulator then serves as an optical switch, which, by switching the sound on and off, turns the reflected light on and off, and the transmitted light off and on, as illustrated in Fig. 20.2-1(*b*).

Modulation Bandwidth

The bandwidth of the modulator is the maximum frequency at which it can efficiently modulate. When the amplitude of an acoustic wave of frequency f_0 is varied as a function of time by amplitude modulation with a signal of bandwidth B, the acoustic wave is no longer a single-frequency harmonic function; it has frequency components within a band $f_0 \pm B$ centered about the frequency f_0 (Fig. 20.2-2). How does monochromatic light interact with this multifrequency acoustic wave and what is the maximum value of B that can be handled by the acousto-optic modulator?

When both the incident optical wave and the acoustic wave are plane waves, the component of sound of frequency f corresponds to a Bragg angle,

$$\theta = \sin^{-1}\frac{\lambda}{2\Lambda} = \sin^{-1}\frac{f\lambda}{2v_s} \approx \frac{\lambda}{2v_s}f \tag{20.2-1}$$

(assumed to be small). For a fixed angle of incidence θ, an incident monochromatic optical plane wave of wavelength λ interacts with one and only one harmonic component of the acoustic wave, the component with frequency f satisfying (20.2-1), as illustrated in Fig. 20.2-3. The reflected wave is then *monochromatic* with frequency

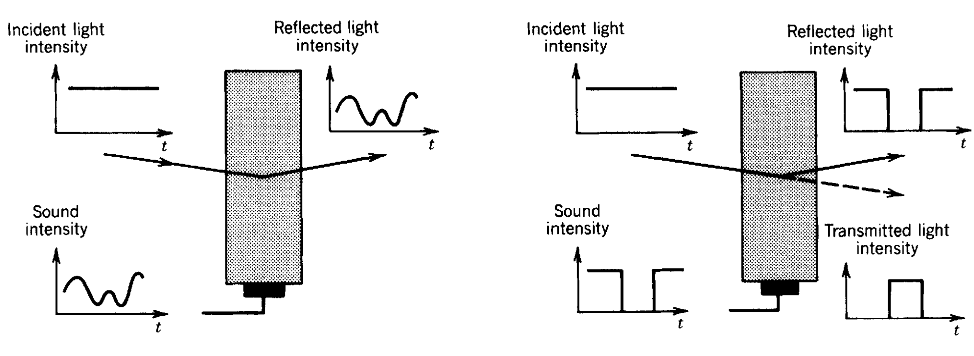


Figure 20.2-1 (*a*) An acousto-optic modulator. The intensity of the reflected light is proportional to the intensity of sound. (*b*) An acousto-optic switch.

†For further details, see, e.g., M. Born and E. Wolf, *Principles of Optics*, Pergamon Press, New York, 6th ed. 1980, Chap. 12.

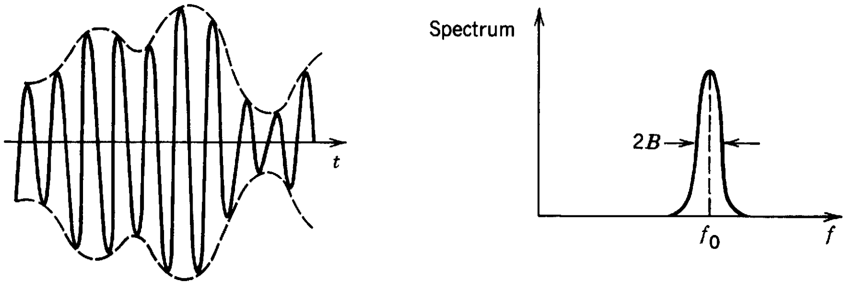


Figure 20.2-2 The waveform of an amplitude-modulated acoustic signal and its spectrum.

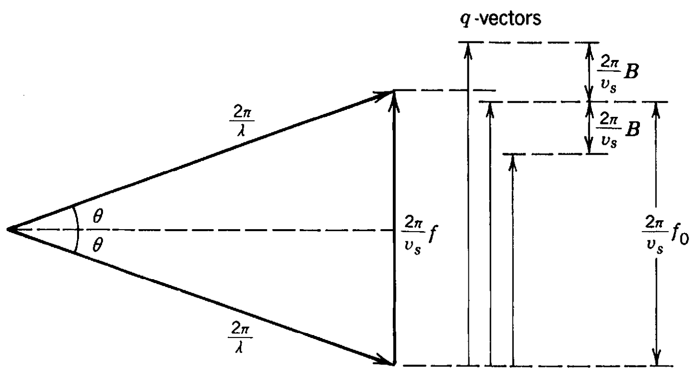


Figure 20.2-3 Interaction of an optical plane wave with a modulated (multiple frequency) acoustic plane wave. Only one frequency component of sound reflects the light wave. The reflected wave is monochromatic and not modulated.

$\nu + f$. Although the acoustic wave is modulated, the reflected optical wave is not. Evidently, under this idealized condition the bandwidth of the modulator is zero!

To achieve modulation with a bandwidth B, each of the acoustic frequency components within the band $f_0 \pm B$ must interact with the incident light wave. A more tolerant situation is therefore necessary. Suppose that the incident light is a beam of width D and angular divergence $\delta\theta = \lambda/D$ and assume that the modulated sound wave is planar. Each frequency component of sound interacts with the optical plane wave that has the matching Bragg angle (Fig. 20.2-4). The frequency band $f_0 \pm B$ is matched by an optical beam of angular divergence

$$\delta\theta \approx \frac{(2\pi/v_s)B}{2\pi/\lambda} = \frac{\lambda}{v_s}B.$$

The bandwidth of the modulator is therefore

$$B = v_s\frac{\delta\theta}{\lambda} = \frac{v_s}{D}, \tag{20.2-2}$$

or

$$\boxed{B = \frac{1}{T}, \qquad T = \frac{D}{v_s},} \tag{20.2-3}$$

Bandwidth

where T is the transit time of sound across the waist of the light beam. This is an

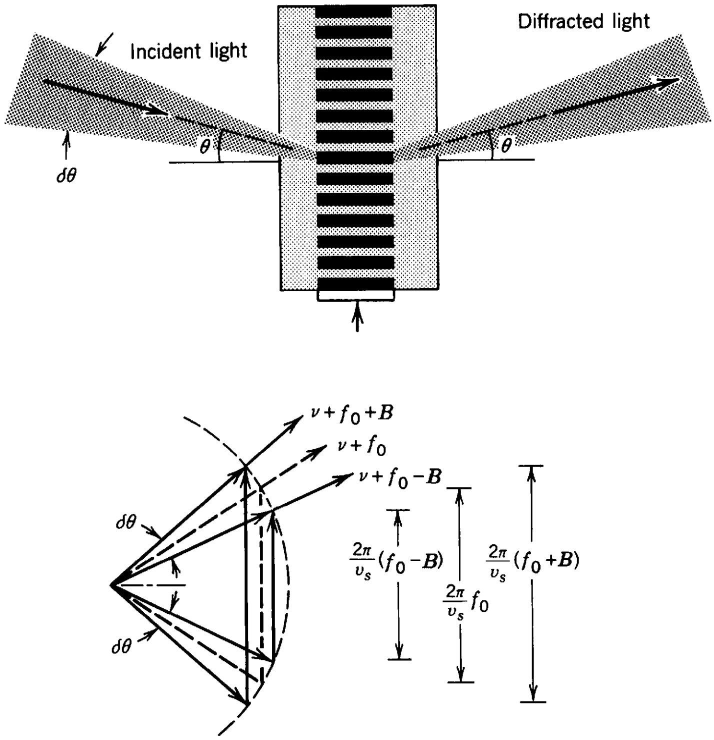


Figure 20.2-4 Interaction of an optical beam of angular divergence $\delta\theta$ with an acoustic plane wave of frequency in the band $f_0 \pm B$. There are many parallel $\mathbf{q}$ vectors of different lengths each matching a direction of the incident light.

expected result since it takes time T to change the amplitude of the sound wave at all points in the light–sound interaction region, so that the maximum rate of modulation is $1/T$ Hz. To increase the bandwidth of the modulator, the light beam should be focused to a small diameter.

EXERCISE 20.2-1

Parameters of Acousto-Optic Modulators. Determine the Bragg angle and the maximum bandwidth of the following acousto-optic modulators:

Modulator 1

Material:	Fused quartz ($n = 1.46$, $v_s = 6$ km/s)
Sound:	Frequency $f = 50$ MHz
Light:	He–Ne laser, wavelength $\lambda_o = 633$ nm, angular divergence $\delta\theta = 1$ mrad

Modulator 2

Material:	Tellurium ($n = 4.8$, $v_s = 2.2$ km/s)
Sound:	Frequency $f = 100$ MHz
Light:	CO_2 laser, wavelength $\lambda_o = 10.6$ μm, and beam width $D = 1$ mm

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B. Scanners

The acousto-optic cell can be used as a scanner of light. The basic idea lies in the linear relation between the angle of deflection 2θ and the sound frequency f,

$$2\theta \approx \frac{\lambda}{v_s} f, \tag{20.2-4}$$

where θ is assumed sufficiently small so that $\sin\theta \approx \theta$. By changing the sound frequency f, the deflection angle 2θ can be varied.

One difficulty is that θ represents both the angle of reflection and the angle of incidence. To change the angle of reflection, both the angle of incidence and the sound frequency must be changed simultaneously. This may be accomplished by tilting the sound beam. Figure 20.2-5 illustrates this principle. Changing the sound frequency requires a frequency modulator (FM). Tilting the sound beam requires a sophisticated system that uses, for example, a phased array of acoustic transducers (several acoustic transducers driven at relative phases that are selected to impart a tilt to the overall generated sound wave). The angle of tilt must be synchronized with the FM driver.

The requirement to tilt the sound beam may be alleviated if we use a sound beam with an angular divergence equal to or greater than the entire range of directions to be scanned. As the sound frequency is changed, the Bragg angle is altered and the incoming light wave selects only the acoustic plane-wave component with the matching direction. The efficiency of the system is, of course, expected to be low. We proceed to examine some of the properties of this device.

Scan Angle

When the sound frequency is f, the incident light wave interacts with the sound component at an angle $\theta = (\lambda/2v_s)f$ and is deflected by an angle $2\theta = (\lambda/v_s)f$, as Fig. 20.2-6 illustrates. By varying the sound frequency from f_0 to $f_0 + B$, the deflection angle 2θ is swept over a scan angle

$$\boxed{\Delta\theta = \frac{\lambda}{v_s} B.} \tag{20.2-5}$$

Scan Angle

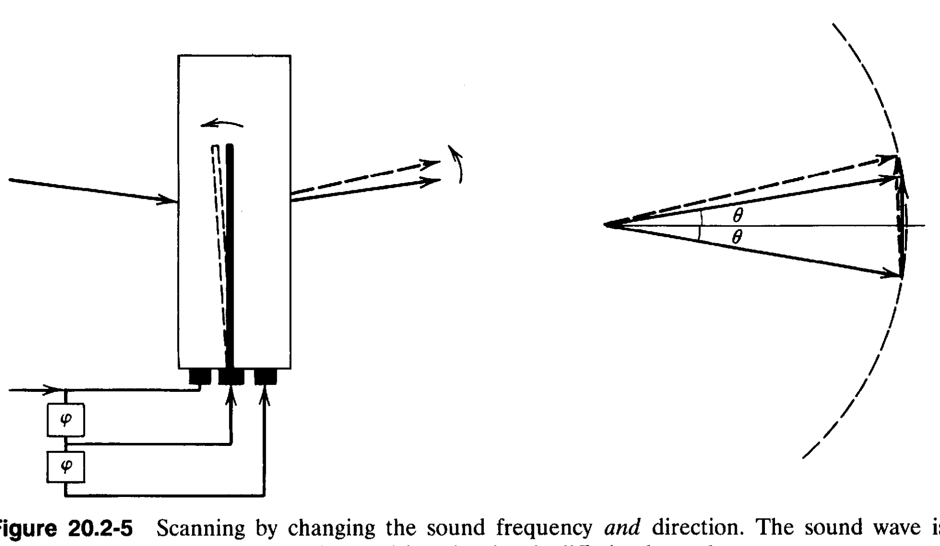

Figure 20.2-5 Scanning by changing the sound frequency *and* direction. The sound wave is tilted by use of an array of transducers driven by signals differing by a phase φ.

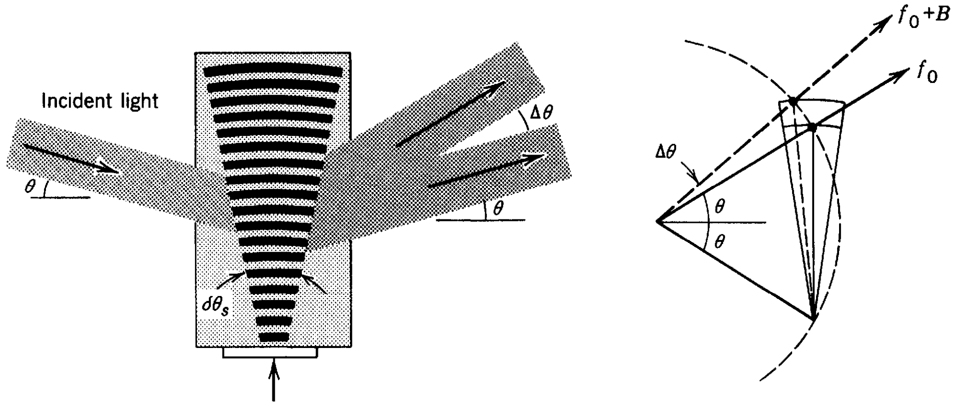

Figure 20.2-6 Scanning an optical wave by varying the frequency of a sound beam of angular divergence $\delta\theta_s$ over the frequency range $f_0 \leq f \leq f_0 + B$.

This, of course, assumes that the sound beam has an equal or greater angular width $\delta\theta_s = \Lambda/D_s \geq \Delta\theta$. Since the scan angle is inversely proportional to the speed of sound, larger scan angles are obtained by use of materials for which the sound velocity v_s is small.

Number of Resolvable Spots

If the optical wave itself has an angular width $\delta\theta = \lambda/D$, and assuming that $\delta\theta \ll \delta\theta_s$, the deflected beam also has a width $\delta\theta$. The number of resolvable spots of the scanner (the number of nonoverlapping angular widths within the scanning range) is therefore

$$N = \frac{\Delta\theta}{\delta\theta} = \frac{(\lambda/v_s)B}{\lambda/D} = \frac{D}{v_s}B,$$

or

$$\boxed{N = TB,} \tag{20.2-6}$$

Number of Resolvable Spots

where B is the bandwidth of the FM modulator used to generate the sound and $T = D/v_s$ is the transit time of sound through the light beam (Fig. 20.2-7).

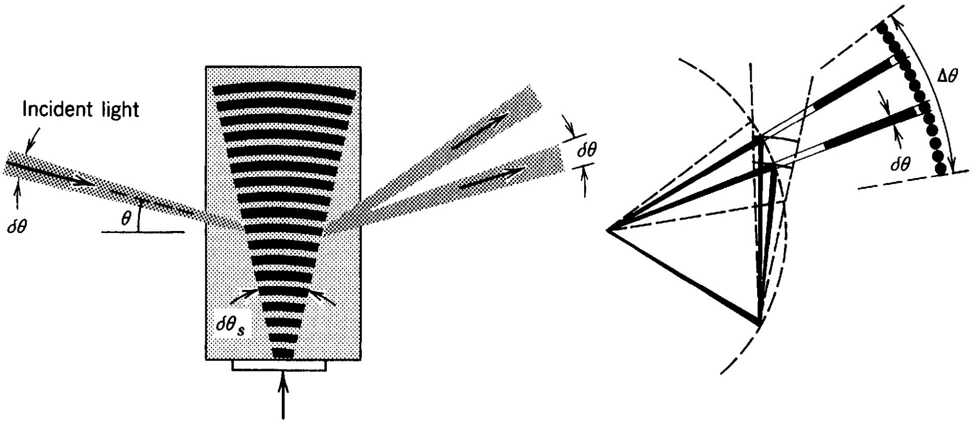

Figure 20.2-7 Resolvable spots of an acousto-optic scanner.

The number of resolvable spots is therefore equal to the time–bandwidth product. This number represents the degrees of freedom of the device and is a significant indicator of the capability of the scanner. To increase N, a large transit time T should be used. This is the opposite of the design requirement in an acousto-optic modulator, for which the modulation bandwidth $B = 1/T$ is made large by selecting a small T.

EXERCISE 20.2-2

Parameters of an Acousto-Optic Scanner. A fused-quartz acousto-optic scanner ($v_s = 6$ km/s, $n = 1.46$) is used to scan a He–Ne laser beam ($\lambda_o = 633$ nm). The sound frequency is scanned over the range 40 to 60 MHz. To what width should the laser beam be focused so that the number of resolvable points is $N = 100$? What is the scan angle $\Delta\theta$? What is the effect of using a material in which sound is slower, flint glass ($v_s = 3.1$ km/s), for example?

The Acousto-Optic Scanner as a Spectrum Analyzer

The proportionality between the angle of deflection and the sound frequency can be utilized to make an acoustic spectrum analyzer. A sound wave containing a spectrum of different frequencies disperses the light in different directions with the intensity of deflected light in a given direction proportional to the power of the sound component at the corresponding frequency (Fig. 20.2-8).

C. Interconnections

An acousto-optic cell can be used as an interconnection optical switch that routes information carried by one or more optical beams to one or more selected directions. Several interconnection schemes are possible:

- An acousto-optic cell in which the frequency of the acoustic wave is one of N possible values, $f_1, f_2, \ldots,$ or f_N, deflects an incident optical beam to one of N

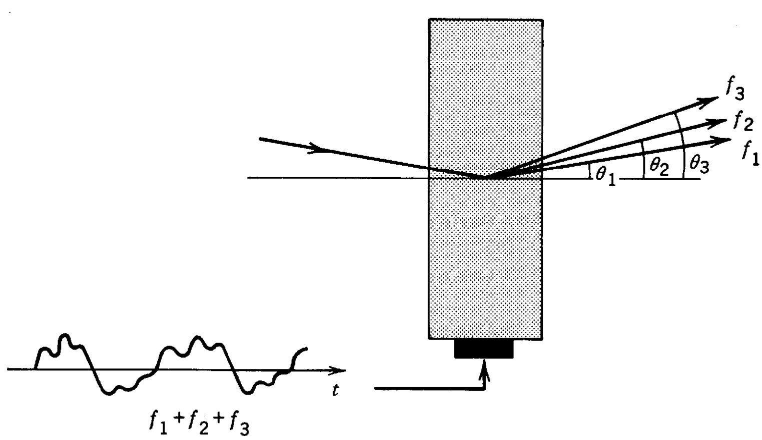

Figure 20.2-8 Each frequency component of the sound wave deflects light in a different direction. The acousto-optic cell serves as an acoustic spectrum analyzer.

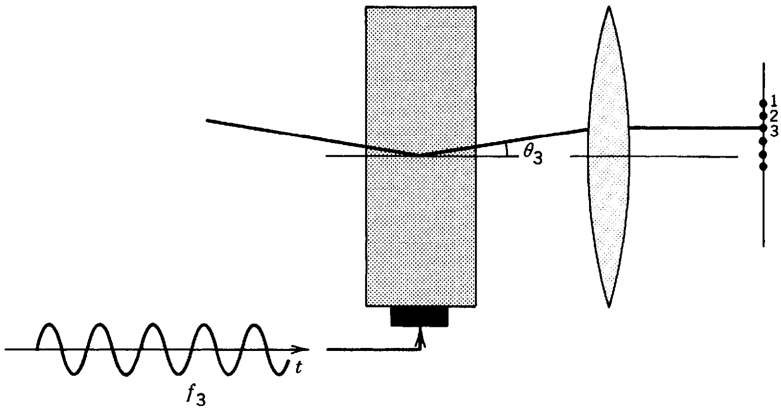

Figure 20.2-9 Routing an optical beam to one of N directions. By applying an acoustic wave of frequency f_3, for example, the optical beam is deflected by an angle θ_3 and routed to point 3.

corresponding directions, $\theta_1, \theta_2, \ldots,$ or θ_N, as illustrated in Fig. 20.2-9. The device routes one beam to any of N directions.

- By using an acoustic wave comprising two frequencies, f_1 and f_2, simultaneously, the incident optical beam is reflected in the two corresponding directions, θ_1 and θ_2, simultaneously. Thus one beam is connected to any pair of many possible directions as illustrated in Fig. 20.2-10. Similarly, by using an acoustic wave with M frequencies the incoming beam can be routed simultaneously to M directions. An example is the acoustic spectrum analyzer for which an incoming light beam is reflected from a sound wave carrying a spectrum of M frequencies. The light beam is routed to M points, with the intensity at each point proportional to the power of the corresponding sound-frequency component.
- The length of the acousto-optic cell may be divided into two segments. At a certain time, an acoustic wave of frequency f_1 is present in one segment and an acoustic wave of frequency f_2 is present in the other. This can be accomplished by generating the acoustic wave from a frequency-shift-keyed electric signal in the form of two pulses: a pulse of frequency f_1 followed by another of frequency f_2, each lasting a duration $T/2$, where $T = W/v_s$ is the transit time of sound through the cell length W (see Fig. 20.2-11). When the leading edge of the acoustic wave

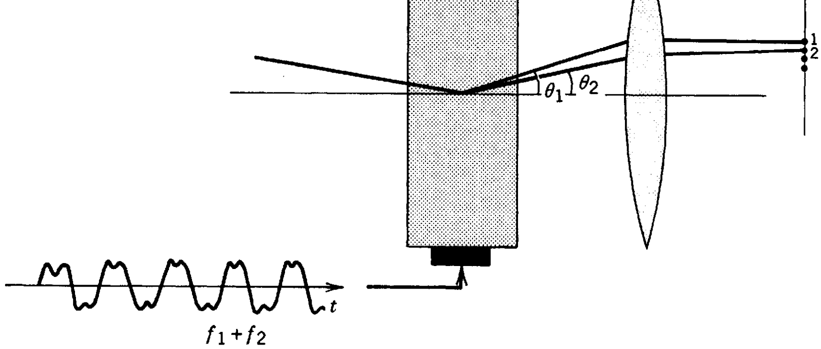

Figure 20.2-10 Routing a light beam simultaneously to a number of directions.

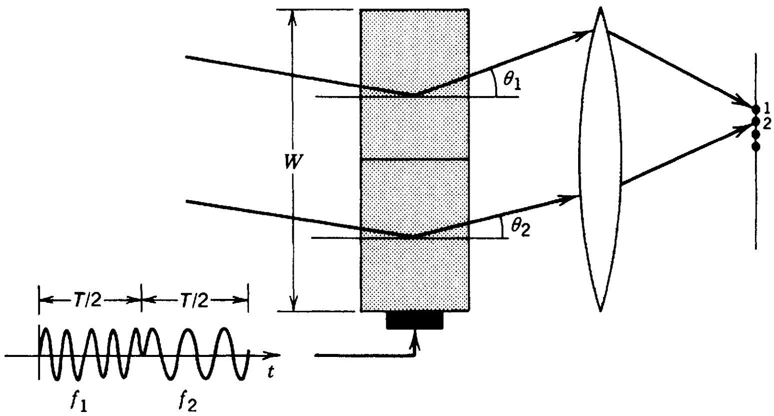

Figure 20.2-11 Routing each of two light beams to a set of specified directions. The acoustic wave is generated by a frequency-shift-keyed electric signal.

reaches the end of the cell, the cell processes two incoming optical beams by deflecting the top beam to the direction θ_1 corresponding to f_1, and the bottom beam to the direction θ_2 corresponding to f_2. This is a switch that connects each of two beams to any of many possible directions. By placing more than one frequency component in each segment, each of the two beams can itself be routed simultaneously to several directions.

- The cell may also be divided into N segments, each carrying a harmonic acoustic wave of the same frequency f but with a different amplitude. The result is a **spatial light modulator** that modulates the intensities of N input beams (Fig. 20.2-12). Spatial light modulators are useful in optical signal processing (see Sec. 21.5).

- The most general interconnection architecture is one for which the cell is divided into L segments, each of which carries an acoustic wave with M frequencies. The device acts as a random access switch that routes each of L incoming beams to M directions simultaneously (Fig. 20.2-13).

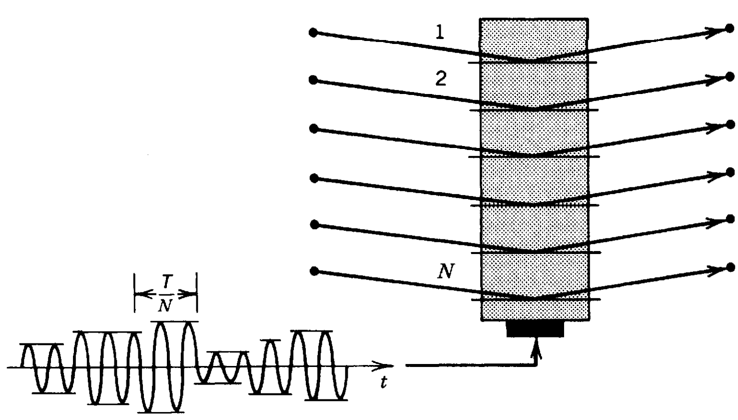

Figure 20.2-12 The spatial light modulator modulates N optical beams. The acoustic wave is driven by an amplitude-modulated electric signal.

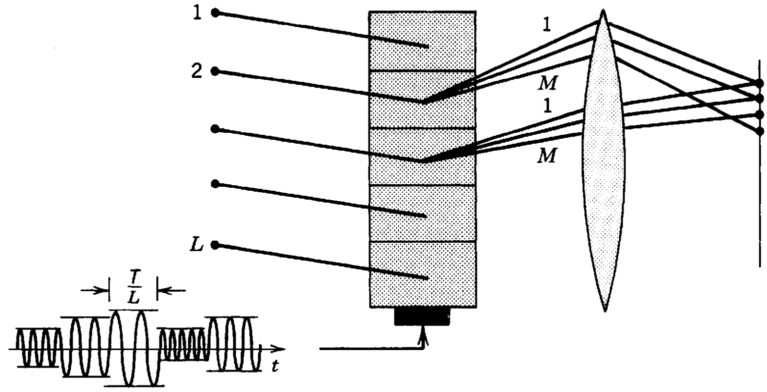

Figure 20.2-13 An arbitrary-interconnection switch routes each of L incoming light beams for the random access of M points.

Interconnection Capacity

There is an upper limit to the number of interconnections that may be established by an acousto-optic device, as will be shown subsequently. If an acousto-optic cell is used to route each of L incoming optical beams to a maximum of M directions simultaneously, then product ML cannot exceed the time–bandwidth product $N = TB$, where T is the transit time through the cell and B is the bandwidth of the acoustic wave,

$$\boxed{ML \leq N.} \tag{20.2-7}$$

Interconnection Capacity

This upper bound on the number of interconnections is called the interconnection capacity of the device.

An acousto-optic cell with L segments uses an acoustic wave composed of L segments each of time duration T/L. For each segment to address M independent points the acoustic wave must carry M independent frequency components per segment. For a signal of duration T/L there is an inherent frequency uncertainty of L/T hertz. The M frequency components must therefore be separated by at least that uncertainty. For the M components to be placed within the available bandwidth B, we must have $M(L/T) \leq B$, from which $ML \leq TB$, and hence (20.2-7) follows.

A single optical beam ($L = 1$), for example, can be connected to any of $N = TB$ points, but each of two beams can be connected to at most $N/2$ points, and so on. It is a question of dividing an available time–bandwidth product $N = TB$ in the form of L time segments each containing M independent frequencies. Examples of the possible choices are illustrated in the time–frequency diagram in Fig. 20.2-14.

D. Filters, Frequency Shifters, and Isolators

The acousto-optic cell is useful in a number of other applications, including filters, frequency shifters, and optical isolators.

Tunable Acousto-Optic Filters

The Bragg condition $\sin\theta = \lambda/2\Lambda$ relates the angle θ, the acoustic wavelength Λ, and the optical wavelength λ. If θ and Λ are specified, reflection can occur only for a single optical wavelength $\lambda = 2\Lambda\sin\theta$. This wavelength-selection property can be used to

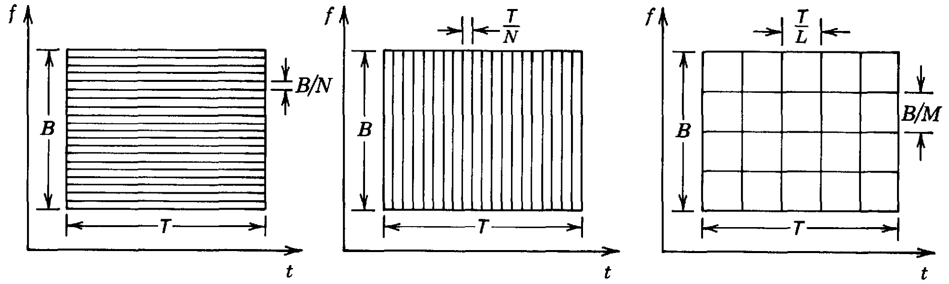

Figure 20.2-14 Several examples of dividing the time–bandwidth region TB in the time–frequency diagram into $N = TB$ subdivisions (in this diagram $N = 20$). (*a*) A scanner: a single time segment containing N frequency segments. (*b*) A spatial light modulator: N time segments each containing one frequency component. (*c*) An interconnection switch: L time segments each containing $M = N/L$ frequency segments (in this diagram, $N = 20$, $M = 4$, and $L = 5$).

filter an optical wave composed of a broad spectrum of wavelengths. The filter is tuned by changing the angle θ or the sound frequency f.

EXERCISE 20.2-3

Resolving Power of an Acousto-Optic Filter. Show that the spectral resolving power $\lambda/\Delta\lambda$ of an acousto-optic filter equals fT, where f is the sound frequency, T the transit time, and $\Delta\lambda$ the minimum resolvable wavelength difference.

Frequency Shifters

Optical frequency shifters are useful in many applications of photonics, including optical heterodyning, optical FM modulators, and laser Doppler velocimeters. The acousto-optic cell may be used as a tunable frequency shifter since the Bragg reflected light is frequency shifted (up or down) by the frequency of sound. In a heterodyne optical receiver, a received amplitude- or phase-modulated optical signal is mixed with a coherent optical wave from a local light source, acting as a local oscillator with a different frequency. The two optical waves beat (see Sec. 2.6B) and the detected signal varies at the frequency difference. Information about the amplitude and phase of the received signal can be extracted from the detected signal (see Sec. 22.5A). The acousto-optic cell offers a practical means for imparting the frequency shift required for the heterodyning process.

Optical Isolators

An optical isolator is a one-way optical valve often used to prevent reflected light from retracing its path back into the original light source (see Sec. 6.6C). Optical isolators are sometimes used with semiconductor lasers since the reflected light can interact with the laser process and create deleterious effects (noise). The acousto-optic cell can serve as an isolator. If part of the frequency-upshifted Bragg-diffracted light is reflected onto itself by a mirror and traces its path back into the cell, as illustrated in Fig. 20.2-15, it undergoes a second Bragg diffraction accompanied by a second frequency upshift.

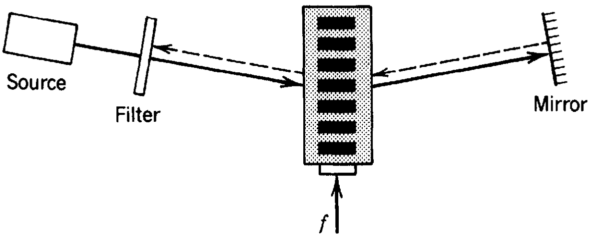

Figure 20.2-15 An acousto-optic isolator.

Since the frequency of the returning light differs from that of the original light by twice the sound frequency, a filter may be used to block it. Even without a filter, the laser process may be insensitive to the frequency-shifted light.

*20.3 ACOUSTO-OPTICS OF ANISOTROPIC MEDIA

The scalar theory of interaction of light and sound is generalized in this section to include the anisotropic properties of the medium and the effects of polarization of light and sound.

Acoustic Waves in Anisotropic Materials

An acoustic wave is a wave of material strain. Strain is defined in terms of the displacements of the molecules relative to their equilibrium positions. If $\mathbf{u} = (u_1, u_2, u_3)$ is the vector of displacement of the molecules located at position $\mathbf{x} = (x_1, x_2, x_3)$, the strain is a symmetrical tensor with components $s_{ij} = \frac{1}{2}(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$, where the indices $i, j = 1, 2, 3$ denote the coordinates (x, y, z). The element $s_{33} = \partial u_3/\partial x_3$, for example, represents tensile strain (stretching) in the z direction [Fig. 20.3-1(a)], whereas s_{13} represents shear strain since $\partial u_1/\partial x_3$ is the relative movement in the x direction of two incrementally separated parallel planes normal to the z direction, as illustrated in Fig. 20.3-1(b).

An acoustic wave can be longitudinal or transverse, as illustrated in the following examples.

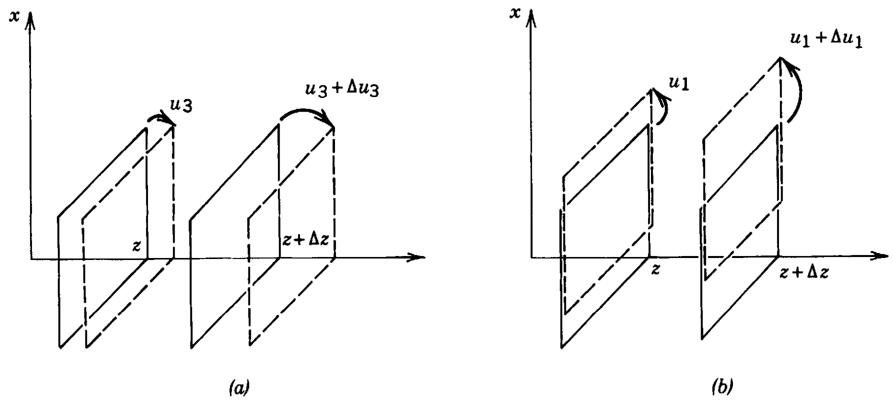

Figure 20.3-1 (a) Tensile strain. (b) Shear.

EXAMPLE 20.3-1. ***Longitudinal Wave.*** A wave with the displacement $u_1 = 0$, $u_2 = 0$, $u_3 = A_0 \sin(\Omega t - qz)$, where A_0 is a constant, corresponds to a strain tensor with all components vanishing except

$$s_{33} = S_0 \cos(\Omega t - qz), \tag{20.3-1}$$

where $S_0 = -qA_0$. This is a wave of stretching in the z direction traveling in the z direction. Since the vibrations are in the direction of wave propagation, the wave is longitudinal.

EXAMPLE 20.3-2. ***Transverse Wave.*** The displacement wave, $u_1 = A_0 \sin(\Omega t - qz)$, $u_2 = 0$, $u_3 = 0$, corresponds to a strain tensor with all components vanishing except

$$s_{13} = s_{31} = S_0 \cos(\Omega t - qz), \tag{20.3-2}$$

where $S_0 = -\frac{1}{2}qA_0$. This wave travels in the z direction but vibrates in the x direction. It is a transverse (shear) wave.

The velocities of the longitudinal and transverse acoustic waves are characteristics of the medium and generally depend on the direction of propagation.

The Photoelastic Effect

The optical properties of an anisotropic medium are characterized completely by the electric impermeability tensor $\boldsymbol{\eta} = \epsilon_0 \boldsymbol{\epsilon}^{-1}$ (see Sec. 6.3). Given $\boldsymbol{\eta}$, we can determine the index ellipsoid and hence the refractive indices for an optical wave traveling in an arbitrary direction with arbitrary polarization.

In the presence of strain, the electric impermeability tensor is modified so that η_{ij} becomes a function of the elements of the strain tensor, $\eta_{ij} = \eta_{ij}(s_{kl})$. This dependence is called the **photoelastic effect**. Each of the nine functions $\eta_{ij}(s_{kl})$ may be expanded in terms of the nine variables s_{kl} in a Taylor's series. Maintaining only the linear terms,

$$\eta_{ij}(s_{kl}) \approx \eta_{ij}(0) + \sum_{kl} \mathfrak{p}_{ijkl} s_{kl}, \qquad i, j, l, k = 1, 2, 3, \tag{20.3-3}$$

where $\mathfrak{p}_{ijkl} = \partial \eta_{ij} / \partial s_{kl}$ are constants forming a tensor of fourth rank known as the **strain-optic tensor.**

Since both $\{\eta_{ij}\}$ and $\{s_{kl}\}$ are symmetrical tensors, the coefficients $\{\mathfrak{p}_{ijkl}\}$ are invariant to permutations of i and j, and to permutations of k and l. There are therefore only six instead of nine independent values for the set (i, j) and six independent values for (k, l). The pair of indices (i, j) is usually contracted to a single index $I = 1, 2, \ldots, 6$ (see Table 18.2-1 on page 714). The indices (k, l) are similarly contracted and denoted by the index $K = 1, 2, \ldots, 6$. The fourth-rank tensor $\mathfrak{p}_{ijkl}$ is thus described by a 6×6 matrix $\mathfrak{p}_{IK}$.

Symmetry of the crystal requires that some of the coefficients $\mathfrak{p}_{IK}$ vanish and that certain coefficients are related. The matrix $\mathfrak{p}_{IK}$ of a cubic crystal, for example, has the structure

$$\mathfrak{p}_{IK} = \begin{bmatrix} \mathfrak{p}_{11} & \mathfrak{p}_{12} & \mathfrak{p}_{12} & 0 & 0 & 0 \\ \mathfrak{p}_{12} & \mathfrak{p}_{11} & \mathfrak{p}_{12} & 0 & 0 & 0 \\ \mathfrak{p}_{11} & \mathfrak{p}_{12} & \mathfrak{p}_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathfrak{p}_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathfrak{p}_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathfrak{p}_{44} \end{bmatrix}. \tag{20.3-4}$$

Strain-Optic Matrix (Cubic Crystal)

This matrix is also applicable for isotropic media, with the additional constraint $\mathfrak{p}_{44} = \frac{1}{2}(\mathfrak{p}_{11} + \mathfrak{p}_{12})$, so that there are only two independent coefficients.

EXAMPLE 20.3-3. ***Longitudinal Acoustic Wave in a Cubic Crystal.*** The longitudinal acoustic wave described in Example 20.3-1 travels in a cubic crystal of refractive index n. By substitution of (20.3-1) and (20.3-4) into (20.3-3) we find that the associated strain results in an impermeability tensor with elements,

$$\eta_{11} = \eta_{22} = \frac{1}{n^2} + \mathfrak{p}_{12} S_0 \cos(\Omega t - qz)$$

$$\eta_{33} = \frac{1}{n^2} + \mathfrak{p}_{11} S_0 \cos(\Omega t - qz)$$

$$\eta_{ij} = 0, \qquad i \neq j.$$

Thus the initially optically isotropic cubic crystal becomes a uniaxial crystal with the optic axis in the direction of the acoustic wave (z direction) and with ordinary and extraordinary refractive indices, n_o and n_e, given by

$$\frac{1}{n_o^2} = \frac{1}{n^2} + \mathfrak{p}_{12} S_0 \cos(\Omega t - qz) \tag{20.3-5}$$

$$\frac{1}{n_e^2} = \frac{1}{n^2} + \mathfrak{p}_{11} S_0 \cos(\Omega t - qz). \tag{20.3-6}$$

The shape of the index ellipsoid is altered periodically in time and space in the form of a wave, but the principal axes remain unchanged (see Fig. 20.3-2). Since the change of the refractive indices is usually small, the second terms in (20.3-5) and (20.3-6) are small, so that the approximation $(1 + \Delta)^{-1/2} \approx 1 - \Delta/2$, when $|\Delta| \ll 1$, may be applied to approximate (20.3-5) and (20.3-6) by

$$n_o \approx n - \tfrac{1}{2} n^3 \mathfrak{p}_{12} S_0 \cos(\Omega t - qz) \tag{20.3-7}$$

$$n_e \approx n - \tfrac{1}{2} n^3 \mathfrak{p}_{11} S_0 \cos(\Omega t - qz). \tag{20.3-8}$$

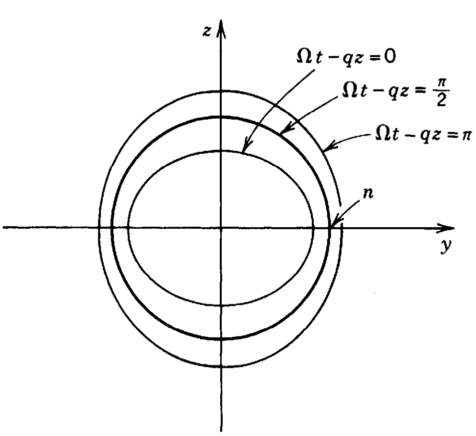

Figure 20.3-2 A longitudinal acoustic wave traveling in the z direction in a cubic crystal alters the shape of the index ellipsoid from a sphere into an ellipsoid of revolution with dimensions varying sinusoidally with time and an axis in the z direction.

EXERCISE 20.3-1

Transverse Acoustic Wave in a Cubic Crystal. The transverse acoustic wave described in Example 20.3-2 travels in a cubic crystal. Show that the crystal becomes biaxial with principal refractive indices

$$n_1 \approx n - \tfrac{1}{2} n^3 \mathfrak{p}_{44} S_0 \cos(\Omega t - qz) \tag{20.3-9}$$

$$n_2 \approx n \tag{20.3-10}$$

$$n_3 \approx n + \tfrac{1}{2} n^3 \mathfrak{p}_{44} S_0 \cos(\Omega t - qz). \tag{20.3-11}$$

In Example 20.3-3 and Exercise 20.3-1, the acoustic wave alters the index ellipsoid's principal values but not its principal directions, so that the ellipsoid maintains its orientation. Obviously, this is not always the case. Acoustic waves in other directions and polarizations relative to the crystal principal axes result in alteration of the principal refractive indices as well as the principal axes of the crystal.

Bragg Diffraction

The interaction of a linearly polarized optical wave with a longitudinal or transverse acoustic wave in an anisotropic medium can be described by the same principles discussed in Sec. 20.1. The incident optical wave is reflected from the acoustic wave if the Bragg condition of constructive interference is satisfied. The analysis is more complicated, in comparison with the scalar theory, since the incident and reflected waves travel with different velocities and, consequently, the angles of reflection and incidence need not be equal.

The condition for Bragg diffraction is the conservation-of-momentum (phase-matching) condition,

$$\mathbf{k}_r = \mathbf{k} + \mathbf{q}. \tag{20.3-12}$$

The magnitudes of these wavevectors are $k = (2\pi/\lambda_o)n$, $k_r = (2\pi/\lambda_o)n_r$, and $q = (2\pi/\Lambda)$, where λ_o and Λ are the optical and acoustic wavelengths and n and n_r are the refractive indices of the incident and reflected optical waves, respectively.

As illustrated in Fig. 20.3-3, if θ and θ_r are the angles of incidence and reflection, the vector equation (20.3-12) may be replaced with two scalar equations relating the z and x components of the wavevectors in the plane of incidence:

$$\frac{2\pi}{\lambda_o} n_r \cos\theta_r = \frac{2\pi}{\lambda_o} n \cos\theta$$

$$\frac{2\pi}{\lambda_o} n_r \sin\theta_r + \frac{2\pi}{\lambda_o} n \sin\theta = \frac{2\pi}{\Lambda},$$

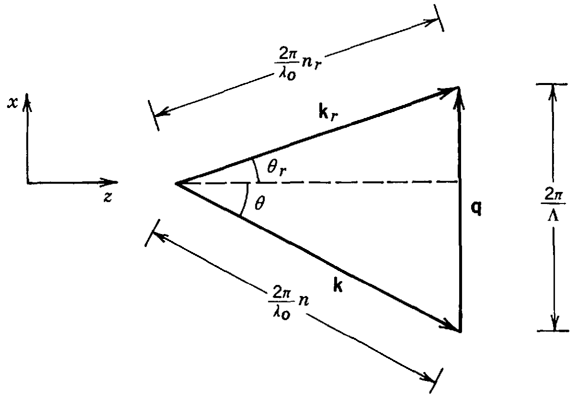

Figure 20.3-3 Conservation of momentum (phase-matching condition, or Bragg condition) in an anisotropic medium.

from which

$$n_r \cos \theta_r = n \cos \theta \tag{20.3-13a}$$

$$n_r \sin \theta_r + n \sin \theta = \frac{\lambda_o}{\Lambda}. \tag{20.3-13b}$$

Given the wavelengths λ_o and Λ, the angles θ and θ_r may be determined by solving equations (20.3-13). Note that n and n_r are generally functions of θ and θ_r that may be determined from the index ellipsoid of the unperturbed crystal.

Equations (20.3-13) can be easily solved when the acoustic and optical waves are collinear, so that $\theta = \pm\pi/2$ and $\theta_r = \pi/2$. The + and − signs correspond to back and front reflections, as illustrated in Fig. 20.3-4. The conditions (20.3-13) then reduce to one condition,

$$n_r \pm n = \frac{\lambda_o}{\Lambda}. \tag{20.3-14}$$

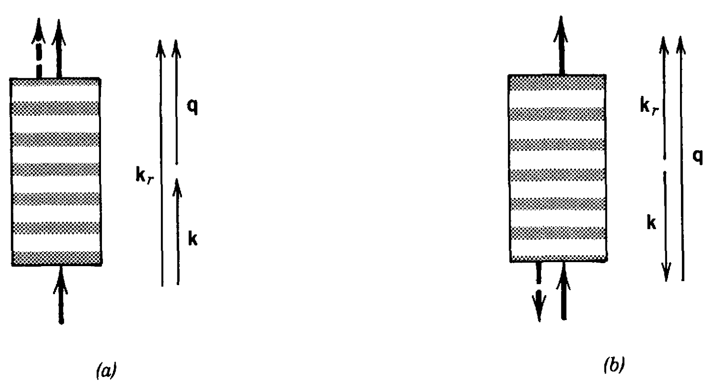

Figure 20.3-4 Wavevector diagram for front and back reflection of an optical wave from an acoustic wave.

For back reflection ($+$ sign), Λ must be smaller than λ_o, which is unlikely except for very high frequency acoustic waves. For front reflection ($-$ sign), the incident and reflected waves must have different polarizations so that $n_r \neq n$.

READING LIST

Books

C. S. Tsai, *Guided-Wave Acoustooptics*, Springer-Verlag, Berlin, 1990.

L. N. Magdich and V. Ya. Molchanov, *Acoustooptic Devices and Their Applications*, Gordon and Breach, New York, 1989.

A. Korpel, *Acousto-Optics*, Marcel Dekker, New York, 1988.

A. Yariv, *Optical Electronics*, Holt, Rinehart and Winston, New York, 1971, 3rd ed. 1985.

A. Yariv and P. Yeh, *Optical Waves in Crystals*, Wiley, New York, 1984.

J. F. Nye, *Physical Properties of Crystals: Their Representation by Tensors and Matrices*, Clarendon Press, Oxford, 1957; Oxford University Press, New York, 1984.

M. Gottlieb, C. L. M. Ireland, and J. M. Ley, *Electro-Optic and Acousto-Optic Scanning and Deflection*, Marcel Dekker, New York, 1983.

N. J. Berg and J. N. Lee, eds., *Acousto-Optic Signal Processing*, Marcel Dekker, New York, 1983.

T. S. Narasimhamurty, *Photoelastic and Electro-Optic Properties of Crystals*, Plenum Press, New York, 1981.

M. Born and E. Wolf, *Principles of Optics*, Pergamon, New York, 1959, 6th ed. 1980, Chap. 12.

D. F. Nelson, *Electric, Optic, and Acoustic Interactions in Dielectrics*, Wiley, New York, 1979.

J. Sapriel, *Acousto-Optics*, Wiley, New York, 1979.

M. J. P. Musgrave, *Crystal Acoustics*, Holden-Day, San Francisco, 1970.

M. V. Berry, *The Diffraction of Light by Ultrasound*, Academic Press, New York, 1966.

Articles

A. C. Tam, Applications of Photoacoustic Sensing Techniques, *Reviews of Modern Physics*, vol. 58, pp. 381–431, 1986.

Special issue on acoustooptic signal processing, *Proceedings of the IEEE*, vol. 69, no. 1, 1981.

A. Korpel, Acousto-Optics, in *Applied Optics and Optical Engineering*, vol. 6, R. Kingslake and B. J. Thompson, eds., Academic Press, New York, 1980.

E. G. Lean, Interaction of Light and Acoustic Surface Waves, in *Progress in Optics*, vol. 11, E. Wolf, ed., North-Holland, Amsterdam, 1973.

E. K. Sittig, Elastooptic Light Modulation and Deflection, in *Progress in Optics*, vol. 10, E. Wolf, ed., North-Holland, Amsterdam, 1972.

R. W. Damon, W. T. Maloney, and D. H. McMahon, Interaction of Light with Ultrasound: Phenomena and Applications, in *Physical Acoustics: Principles and Methods*, vol. 7, W. P. Mason and R. N. Thurston, eds., Academic Press, New York, 1970.

R. Adler, Interaction between Light and Sound, *IEEE Spectrum*, vol. 4, no. 5, pp. 42–54, 1967.

E. I. Gordon, A Review of Acoustooptical Deflection and Modulation Devices, *Proceedings of the IEEE*, vol. 54, pp. 1391–1401, 1966.

PROBLEMS

20.1-1 **Diffraction of Light from Various Periodic Structures.** Discuss the diffraction of an optical plane wave of wavelength λ from the following periodic structures, indicating in each case the geometrical configuration and the frequency shift(s):

(a) An acoustic traveling wave of wavelength Λ.

(b) An acoustic standing wave of wavelength Λ.

(c) A graded-index transparent medium with refractive index varying sinusoidally with position (period Λ).
(d) A stratified medium made of parallel layers of two materials of different refractive indices, alternating to form a periodic structure of period Λ.

*20.1-2 **Bragg Diffraction as a Scattering Process.** An incident optical wave of angular frequency ω, wavevector $\mathbf{k}$, and complex envelope A interacts with a medium perturbed by an acoustic wave of angular frequency Ω and wavevector $\mathbf{q}$, and creates a light source $\mathscr{S}$ given by (20.1-25). The angle θ corresponds to upshifted Bragg diffraction, so that the scattering light source is $\mathscr{S} = \mathrm{Re}\{S_r(\mathbf{r})\exp(j\omega_r t)\}$, where $S_r(\mathbf{r}) = -(\Delta n_0/n)k_r^2 A\exp(-j\mathbf{k}_r\cdot\mathbf{r})$, $\omega_r = \omega + \Omega$, and $\mathbf{k}_r = \mathbf{k} + \mathbf{q}$. This source emits a scattered field E. Assuming that the incident wave is undepleted by the acousto-optic interaction (first Born approximation, i.e., A remains approximately constant), the scattered light may be obtained by solving the Helmholtz equation $\nabla^2 E + k^2 E = -S$. This equation has the far-field solution (see Problem 19.2-3)

$$E(\mathbf{r}) \approx \frac{\exp(-jkr)}{4\pi r}\int_V S_r(\mathbf{r}')\exp(jk\hat{\mathbf{r}}\cdot\mathbf{r}')\,d\mathbf{r}',$$

where $\hat{\mathbf{r}}$ is a unit vector in the direction of $\mathbf{r}$, $k = 2\pi/\lambda$, and V is the volume of the source. Use this equation to determine an expression for the reflectance of the acousto-optic cell when the Bragg condition is satisfied. Compare the result with (20.1-18).

20.1-3 **Condition for Raman–Nath Diffraction.** Derive an expression for the maximum width D_s of an acoustic beam of wavelength Λ that permits Raman–Nath diffraction of light of wavelength λ (see Fig. 20.1-10).

20.1-4 **Combined Acousto-Optic and Electro-Optic Modulation.** One end of a lithium niobate ($LiNbO_3$) crystal is placed inside a microwave cavity with an electromagnetic field at 3 GHz. As a result of the piezoelectric effect (the electric field creating a strain in the material), an acoustic wave is launched. Light from a He–Ne laser ($\lambda_o = 633$ nm) is reflected from the acoustic wave. The refractive index is $n = 2.3$ and the velocity of sound is $v_s = 7.4$ km/s. Determine the Bragg angle. Since lithium niobate is also an electro-optic material, the applied electric field modulates the refractive index, which in turn modulates the phase of the incident light. Sketch the spectrum of the reflected light. If the microwave electric field is a pulse of short duration, sketch the spectrum of the reflected light at different times indicating the contributions of the electro-optic and acousto-optic effects.

20.2-1 **Acousto-Optic Modulation.** Devise a system for converting a monochromatic optical wave with complex wavefunction $U(t) = A\exp(j\omega t)$ into a modulated wave of complex wavefunction $A\cos(\Omega t)\exp(j\omega t)$ by use of an acousto-optic cell with an acoustic wave $s(x,t) = S_0\cos(\Omega t - qx)$. *Hint*: Consider the use of upshifted and downshifted Bragg reflections.

20.2-2 **Frequency-Shift-Free Bragg Reflector.** Design an acousto-optic system that deflects light without frequency shifting it. *Hint*: Use two Bragg cells. (Reference: F. W. Freyre, *Applied Optics*, vol. 22, pp. 3896–3900, 1981.)

*20.3-1 **Front Bragg Diffraction.** A transverse acoustic wave of wavelength Λ travels in the x direction in a uniaxial crystal with refractive indices n_o and n_e and optic axis in the z direction. Derive an expression for the wavelength λ_o of an incident optical wave, traveling in the x direction and polarized in the z direction, that satisfies the condition of Bragg diffraction. What is the polarization of the front reflected wave? Determine Λ if $\lambda_o = 633$ nm, $n_e = 2.200$, and $n_o = 2.286$.