Elements of Photonics, Volume II: For Fiber and Integrated Optics. Keigo Iizuka

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ANSWERS TO PROBLEMS

Chapter 9

9.1 The γ values in the lower half-plane are negative, which means a growing wave and that does not make physical sense. Both $Kd \tan Kd$ and $-Kd \cot Kd$ are even functions of Kd, and only one side is needed.

9.2 From Fig. 9.2 when V is an integral multiple of $\pi/2$,

$$V = Kd \tag{1}$$

Hence,

$$(kn_1)^2 - (kn_2)^2 = K^2 \tag{2}$$

From Fig. 9.5, we have

$$\cos\theta = \frac{K}{n_1 k} \tag{3}$$

Inserting Eq. (3) into (2) gives

$$\sin\theta = \frac{n_2}{n_1} \tag{4}$$

Recall such θ is at the critical angle associated with the core–cladding boundary,

$$\theta = \theta_c$$

9.3 From Fig. 9.2, the value of V that supports five modes is

$$2\pi < V < \tfrac{5}{2}\pi$$

$$\frac{4\pi}{k\sqrt{n_1^2 - n_2^2}} < 2d < \frac{5\pi}{k\sqrt{n_1^2 - n_2^2}}$$

$$9.67\ \mu\text{m} < 2d < 12.09\ \mu\text{m}$$

9.4 The correct word decrease or increase is underlined.

If the thickness of the slab guide is *decreased* with all other physical constants fixed, the normalized thickness *V(increases, <u>decreases</u>)* and K_2d of the TM_2 mode (*increases, <u>decreases</u>*) and the value of K_2 (*<u>increases</u>, decreases*), and this means that the value of β_2 (*increases, <u>decreases</u>*). Thus, in order to obtain the region of a larger effective index of refraction $N = \beta_2/k$, the thickness 2d has to be (*<u>increased</u>, decreased*).

9.5 From Figs. 9.6a and 9.6b,

$$K_{\max} = k\sqrt{n_1^2 - n_2^2}$$

$$n_1^2 = \left(\frac{K_{\max}}{k}\right)^2 + n_2^2$$

$$n_1 = \sqrt{\left(\frac{1.3 \times 1}{2\pi}\right)^2 + 1^2} = 1.02$$

9.6 The angle θ that the incident ray at $x = x_0$ makes with the boundary layer of the guide is (see Fig. A9.6)

$$\sin\theta = \frac{r + x_0}{r + d}$$

The angle θ is smallest for the ray entering at $x_0 = -d$. This has to be larger than the critical angle for the light to remain in the core. At the critical angle we have

$$\sin\theta_c = \frac{n_2}{n_1}$$

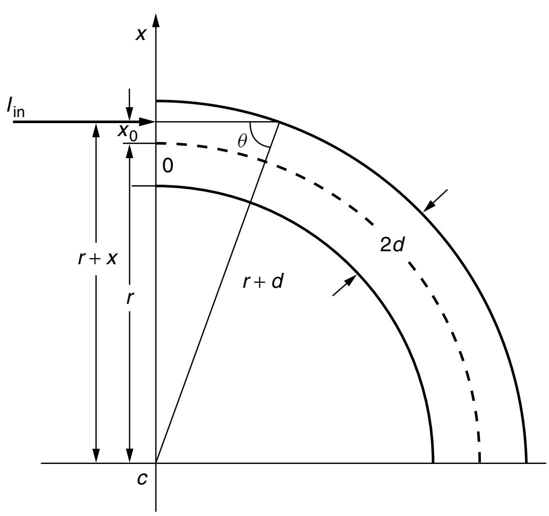

Figure A9.6 Allowed angle of bend.

and hence

$$\frac{n_2}{n_1} = \frac{r-d}{r+d}$$

$$\therefore r = \frac{n_1+n_2}{n_1-n_2} d = \left(\frac{2}{\Delta} - 1\right) d$$

For $d = 50$ μm and $\Delta = 0.055$, $r = 1.77$ mm.

9.7 **(a)** Taking the origin of the x axis in the center of the core layer,

$$\begin{array}{l} H_y(x=d) \\ H_y(x=-d) \\ E_z(x=d) \\ E_z(x=-d) \end{array} \begin{bmatrix} \cos Kd & \sin Kd & -e^{-\gamma_0 d} & 0 \\ \cos Kd & -\sin Kd & 0 & -e^{\gamma_2 d} \\ -K' \sin Kd & K' \cos Kd & \gamma_0' e^{-\gamma_0 d} & 0 \\ K' \sin Kd & K' \cos Kd & 0 & -\gamma_2' e^{-\gamma_2 d} \end{bmatrix} = \Delta$$

with

$$K' = \frac{K}{n_1^2}, \qquad \gamma_0' = \frac{\gamma_0}{n_0^2}, \qquad \gamma_2' = \frac{\gamma_2}{n_2^2}$$

The condition of $\Delta = 0$ is imposed. After some manipulation of the determinant, the characteristic equation becomes

$$\frac{\left(\dfrac{\gamma_0}{n_0'^2} + \dfrac{\gamma_2}{n_2'^2}\right) K}{K^2 + \dfrac{\gamma_0 \gamma_2}{n_0'^2 n_2'^2}} = \tan 2Kd$$

where

$$n_0' = \frac{n_0}{n_1}, \quad n_2' = \frac{n_2}{n_1}$$

(b) Equation (9.100) for the given configuration is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos 2Kd & (1/Z_1) \sin 2Kd \\ -Z_1 \sin 2Kd & \cos 2Kd \end{bmatrix}$$

The characteristic equation (9.108) is

$$Z_2 \cos 2Kd + \frac{Z_0 Z_2}{Z_1} \sin 2Kd - Z_1 \sin 2Kd + Z_0 \cos 2Kd = 0$$

which is rewritten as

$$(Z_0 + Z_2) \cos 2Kd - \left(Z_1 - \frac{Z_0 Z_2}{Z_1}\right) \sin 2Kd = 0$$

Inserting the values of Z_0, Z_2, and Z_1,

$$Z_0 = \frac{j\gamma_0}{\omega\epsilon_0}, \quad Z_2 = \frac{j\gamma_2}{\omega\epsilon_0\epsilon_{r2}}, \quad Z_1 = \frac{jK}{\omega\epsilon_0\epsilon_{r1}}$$

gives

$$\frac{K\left(\dfrac{\gamma_0}{n_0'^2} + \dfrac{\gamma_2}{n_2'^2}\right)}{K^2 - \dfrac{\gamma_0\gamma_2}{n_0'^2 n_2'^2}} = \tan 2Kd$$

Both answers are identical.

9.8 From Eqs. (9.119) and (9.200),

$$\frac{E_{z0}(x)}{H_{y0}(x)} = \frac{Z_1 \sin 2K_1 d - Z_2 \cos 2K_1 d \tanh \gamma_2 s}{\cos 2K_1 d + (Z_2/Z_1) \sin 2K_1 d \tanh \gamma_2 s}$$

Dividing both the denominator and numerator by $\cos 2K_1 d$ gives

$$\frac{E_{z0}(x)}{H_{y0}(x)} = \frac{Z_1 \tan 2K_1 d - Z_2 \tanh \gamma_2 s}{1 + (Z_2/Z_1) \tan 2K_1 d \tanh \gamma_2 s}$$

From T_- in Eqs. (9.170) and the even modes characteristic Eq. (9.172), the right-hand side of the above equation is Z_0.

Chapter 10

10.1 Note that expressions for the ridge guide can be obtained immediately by flipping it upside down and interchanging n_0 and n_2 of the imbedded guide shown in Fig. 10.4 that was treated in Example 10.1 in the text.

The normalized width V_w of the present problem, however, is different from that of Example 10.1.

$$V_w = k\sqrt{n_1^2 - n_0^2}w = 5.47\pi \text{ rad}$$

In every $\pi/2$ radians of V_w, one mode is generated. The total number of modes including the zeroth mode is 11 modes.

For the asymmetric modes, exactly the same results as those of Example 10.1 are applicable. There are two possible modes in the x direction. Hence, the combination of the two modes creates

$$\begin{array}{llll} E^x_{1,1}, & E^x_{1,2}, & E^x_{1,3}, \ldots, E^x_{1,11} \\ E^x_{2,1}, & E^x_{2,2}, & E^x_{2,3}, \ldots, E^x_{2,11} \end{array}$$

There are 22 TM-like modes altogether.

10.2 The light beam will deflect toward the base of the cone, and this device is used as a prism.

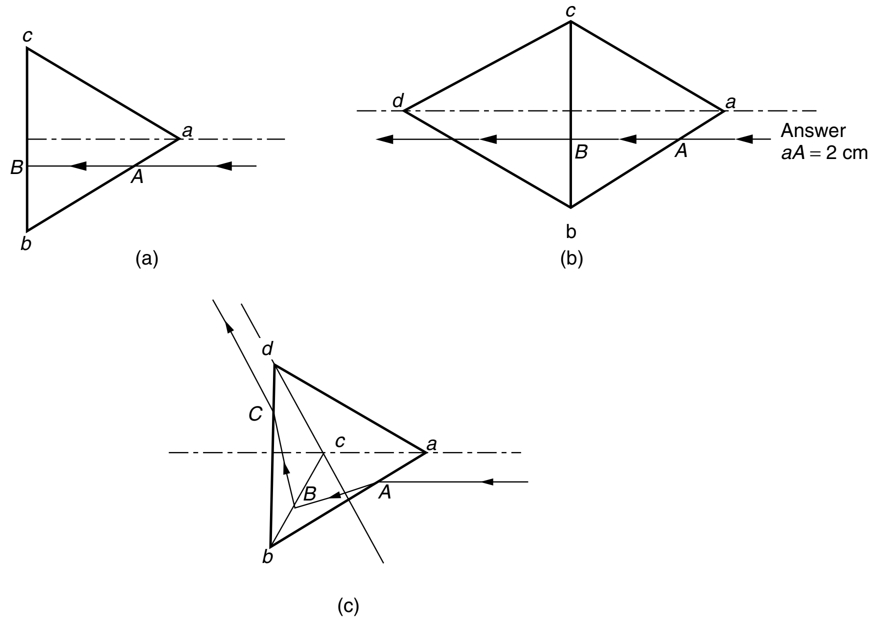

Figure A10.4 Solution of Problem 10.4. (a) Trace *abc*. (b) Traces *abc* and *bcd*. (c) Top view.

10.3 An incident parallel beam is split into two beams, both deflected toward the base of their respective cones. This configuration is used as a beamsplitter.

10.4 The answer is in Fig. A10.4.

10.5 Since the light is Y-directed propagation, the "cross-sectional ellipse" is obtained with $Y = 0$ in Eq. (5.4).

From Eq. (5.10) with $\varepsilon = \varepsilon_z$,

$$a_{11} = \frac{1}{n_o^2} + r_{13}\varepsilon_z$$

$$a_{33} = \frac{1}{n_e^2} + r_{33}\varepsilon_z$$

In a manner similar to Example 5.1, the cross-sectional ellipse is obtained as

$$\frac{x_2}{\left(n_o - \frac{1}{2}r_{13}n_o^3\varepsilon_z\right)^2} + \frac{z^2}{\left(n_e - \frac{1}{2}r_{33}n_e^3\varepsilon_z\right)^2} = 1$$

Since the TE mode is excited, the direction of the polarization is in the Z direction.

Hence, the change $\Delta\phi$ in phase is

$$\Delta\phi = \tfrac{1}{2}k_0 r_{33} n_e^3 \varepsilon_z l$$

with

$$r_{33} = 32.2 \times 10^{-12}$$

$$n_e = 2.2$$

$$\varepsilon_z = 3 \times 10^6 \text{ V/m}$$
$$l = 1 \text{ mm}$$
$$\Delta\phi = 2.48 \text{ rad}$$

Chapter 11

11.1 From the viewpoint of the numerical aperture of the fiber,

$$\sin\phi = \sqrt{n_1^2 - n_2^2} = 0.20$$
$$\phi = 11.5^\circ$$
$$f = \frac{0.4}{\tan 11.50^\circ} = 1.96 \text{ mm}$$

From the viewpoint of the diffraction limit (see Section 1.4.4), the radius r_i of the first null of the Airy diffraction pattern on the end surface of the fiber is obtained.

First zero of Eq. (1.99) appears when

$$a\rho = 0.61$$

or

$$a\frac{r_i}{\lambda z_i} = 0.61$$

with

$$z_i = f$$
$$f = \frac{1}{0.61}\frac{a}{\lambda}r_i$$
$$= \frac{1}{0.61}\frac{400}{1.55}(4.75)$$
$$= 2\,010 \text{ µm}$$

Thus,

$$1.96 \text{ mm} < f < 2.01 \text{ mm}$$

11.2 From Eq. (11.96) with $\nu = 1$,

$$\frac{J_0(Ka)}{KaJ_1(Ka)} = \frac{K_0(\gamma a)}{\gamma a K_1(\gamma a)} = \frac{1}{\gamma a}\left(1 - \frac{1}{2\gamma a}\right)$$

where Eq. (11.101) with $\nu = 1$ was used. With $\gamma a \to \infty$,

$$J_0(Ka) = 0$$

and the asymtotes of the $HE_{1\mu}$ are

$$Ka = 2.4, 5.5, 8.7, \ldots$$

11.3 TM_{02}.

11.4 **(a)** From Table 11.1 the cutoff values ($\gamma a \to 0$) and far from the cutoff values ($\gamma a \to \infty$) are:

	$HE_{3\mu}$	$EH_{1\mu}$
$\gamma a \to 0$	$J_1(Ka) = 0$, except $Ka = 0$	$J_1(Ka) = 0$, except $Ka = 0$
$\gamma a \to \infty$	$J_2(Ka) = 0$	$J_2(Ka) = 0$

The cutoff values are:

Mode	HE_{31}	HE_{32}	HE_{33}	EH_{11}	EH_{12}	EH_{13}
Cutoffs	3.8	7.0	10	3.8	7.0	10

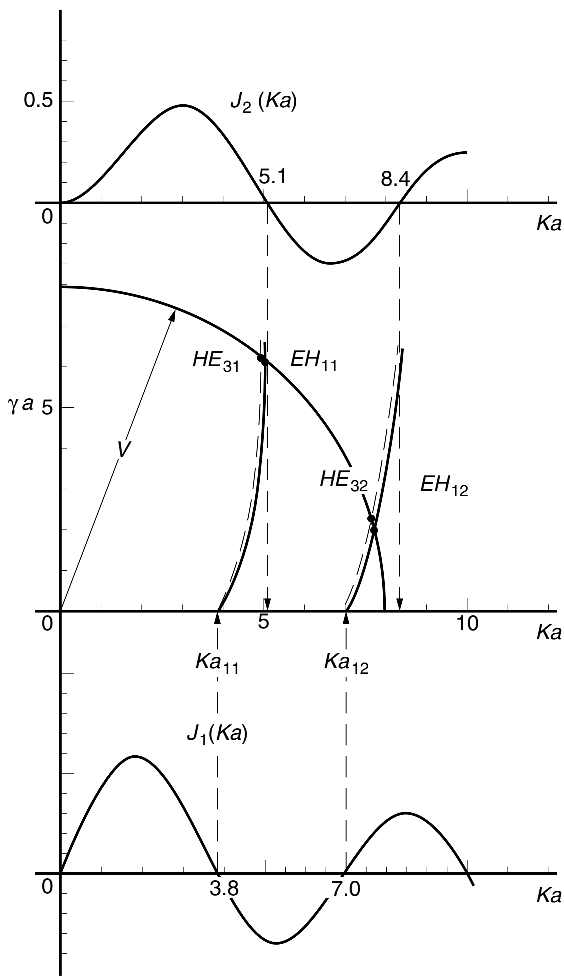

Figure A11.4 Curves of the characteristic equations of $HE_{3\mu}$ and $EH_{1\mu}$ modes.

(b)

	Mode	V_c	Source
(1)	HE_{11}	0	Fig. 11.17
(2)	TE_{01}	2.4	Fig. 11.16
(3)	TM_{01}	2.4	Fig. 11.16
(4)	HE_{21}	2.4	Eq. (11.105)
(5)	HE_{31}	3.8	Fig. A11.4
(6)	EH_{11}	3.8	Fig. A11.4
(7)	HE_{12}	3.8	Fig. 11.17

11.5 Figure A11.5.

11.6 **(a)** LP_{01}, HE_{11}.

(b) LP_{21}, HE_{31}, EH_{11}.

(c) LP_{11}, TE_{01}, TM_{01}, HE_{21}.

(d) LP_{32}, HE_{42}, EH_{22}.

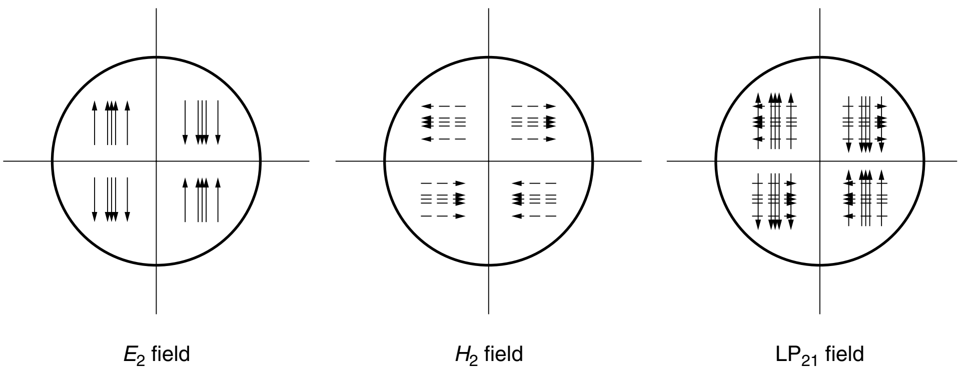

Figure A11.5 Field pattern of the LP_{21} mode.

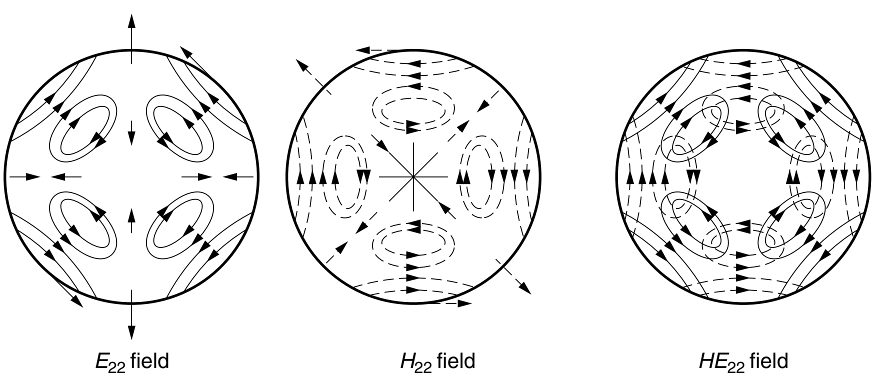

Figure A11.7 Field pattern of the HE_{22} mode.

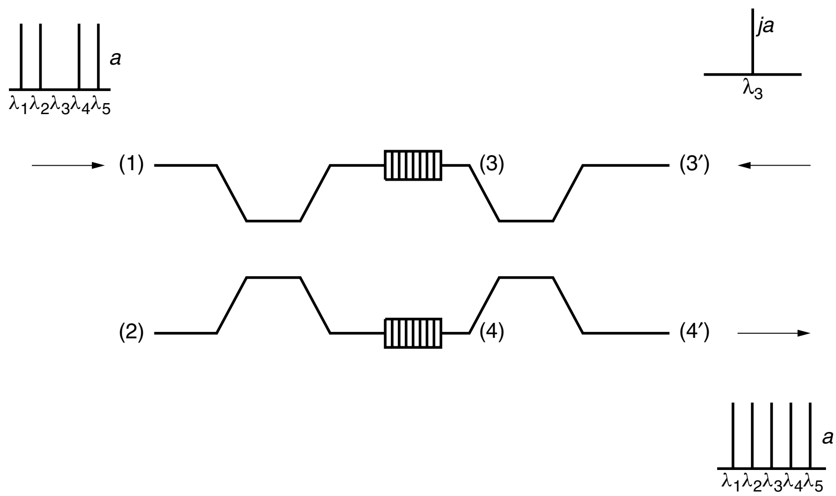

Figure A11.8 Insertion of λ_3 into a WDM transmission stream.

11.7 From Eq. (11.130), the field expressions for HE_{22} are

$$E_r = EJ_1(Kr)\cos 2\phi$$
$$E_\phi = -EJ_1(Kr)\sin 2\phi$$
$$H_r = -\frac{1}{\eta_1}E_\phi$$
$$H_\phi = \frac{1}{\eta_1}E_r$$

The mode pattern is drawn in Fig. A11.7.

11.8 The answer is shown in Fig. A11.8.

Chapter 12

12.1 **(a)**

$$i = \eta\frac{e}{h\nu}P$$
$$\nu = \frac{3 \times 10^{14}}{0.84} = 357 \text{ THz}$$
$$P = 10^{-4.3} = 5 \times 10^{-5} \text{ mW} = 50 \text{ nW}$$

In the theoretical limit ($\eta = 100\%$),

$$i = \frac{(1.6 \times 10^{-19})(5 \times 10^{-8})}{(6.63 \times 10^{-34})(3.57 \times 10^{14})}$$
$$= 3.4 \times 10^{-8} = 33.8 \text{ nA}$$

(b)

$$i = 0.675\eta \text{ A/W}$$

$$\eta = \frac{0.5}{0.675} = 0.74$$

12.2 Let us say that the phase jitter in the transmitting and local oscillators is θ and θ_L, and the phase jitter in the horizontal and vertical waves is α_H and α_V. Then Eq. (12.43) becomes

$$E_H = A(t)\sqrt{\beta}\cos(\omega_c t + \theta + \alpha_H)$$

$$E_V = A(t)\sqrt{1-\beta}\cos(\omega_c t + \theta + \alpha_V)$$

$$E_{LH} = E_{LV} = \frac{1}{\sqrt{2}}E_{\mathrm{LO}}\cos(\omega_L t + \theta_L)$$

The outputs from mixer M_V and M_H are

$$i_H = 2K\left\langle\left[\frac{[A(t)]^2\beta}{2} + \frac{\sqrt{\beta}}{\sqrt{2}}A(t)E_{\mathrm{LO}}\cos(\omega_{\mathrm{IF}}t + \theta - \theta_L + \alpha_H) + \frac{P_L}{2}\right]^2\right\rangle$$

$$i_V = 2K\left\langle\left[\frac{[A(t)]^2(1-\beta)}{2} + \frac{\sqrt{1-\beta}}{\sqrt{2}}A(t)E_{\mathrm{LO}}\cos(\omega_{\mathrm{IF}}t + \theta - \theta_L + \alpha_V) + \frac{P_L}{2}\right]^2\right\rangle$$

The signals are band-pass filtered for the IF frequency components. Applying the square operation, and then low-pass filtering gives

$$i_H^D = [KE_{\mathrm{LO}}A(t)]^2\beta$$

$$i_V^D = [KE_{\mathrm{LO}}A(t)]^2(1-\beta)$$

Thus, the phase jitter has disappeared.

12.3 The outputs from the delay-and-detect circuit for an arbitrary $\Delta\omega\,\tau$ are

$$i_0 = K\cos[\Delta\omega t - \Delta\omega\tau - \Phi(t) - \theta_p] \times \cos[\Delta\omega t - \Phi(t) - \theta_p] \qquad \text{“0” bit}$$

$$i_1 = K\cos[\Delta\omega t - \Delta\omega\tau - \Phi(t) - \pi - \theta_p] \times \cos[\Delta\omega t - \Phi(t) - \theta_p] \qquad \text{“1” bit}$$

The output from the low-pass filter is

$$i_0 = \tfrac{1}{2}K\cos(\Delta\omega\tau) \qquad \text{“0” bit}$$

$$i_1 = \tfrac{1}{2}K\cos(\Delta\omega\tau + \pi) \qquad \text{“1” bit}$$

When $i_0 = i_1$, the function of interrogation is lost. The expression for $i_0 - i_1$ is

$$i_0 - i_1 = K \sin\frac{\pi}{2} \sin\left(\Delta\omega\tau + \frac{\pi}{2}\right)$$

Setting $i_1 - i_0 = 0$ gives the condition

$$\Delta\omega\tau + \frac{\pi}{2} = N\pi$$

which leads to

$$B = \frac{4\Delta f}{2N - 1}$$

with $N = 0, 1, 2, 3, \ldots$.

Chapter 13

13.1 **(a)** Absorption is observed at wavelengths corresponding to each possible transition. The absorption lines are shown in Fig. A13.1b. The depth of each absorption line depends on the atomic structure of Nd YAG as well as the population of the atoms in the levels associated with the transitions.

The task of the spectroscopist is to determine the energy levels conversely from the observed absorption lines such as these.

(b) With the pump light on, the distribution of the population of each level is upset. The atoms in the ground level are pumped to the pump band. Transition rate γ_{32} (the inverse of the lifetime $1/\tau_{32}$) is high and the atoms in level E_3 come down quickly to E_2. The transition rate γ_{10} from E_1 to E_0 is also fast. The transition between E_2 and E_1 is much slower than any of the above. As a result, the population of atoms in E_2 accumulates and that in E_1 diminishes via decay to E_0. These two actions contribute to a population inversion between levels E_2 and E_1. When external light with frequency $\nu_{21} = (E_2 - E_1)/h$ illuminates the rod, stimulated emission is larger than the stimulated absorption and the intensity of the output light becomes larger than that of the input light.

The populations among the levels are controlled by the multilevel rate equations and all absorption lines change to some degree, but the biggest change is a significant increase at $\nu_{21} = (E_2 - E_1)/h$ as indicated by the dotted line in Fig. A13.1b. The deeper absorption at the pump light frequency is due to the filter F.

(c) From

$$P_1 = Ae^{gl}$$

$$10^{-2} = 2 \times 10^{-3} e^{gl}$$

$$\therefore\ e^{gl} = 5$$

$$P_2 = Ae^{2gl} = (2 \times 10^{-3})(5^2) = 50 \text{ mW}$$

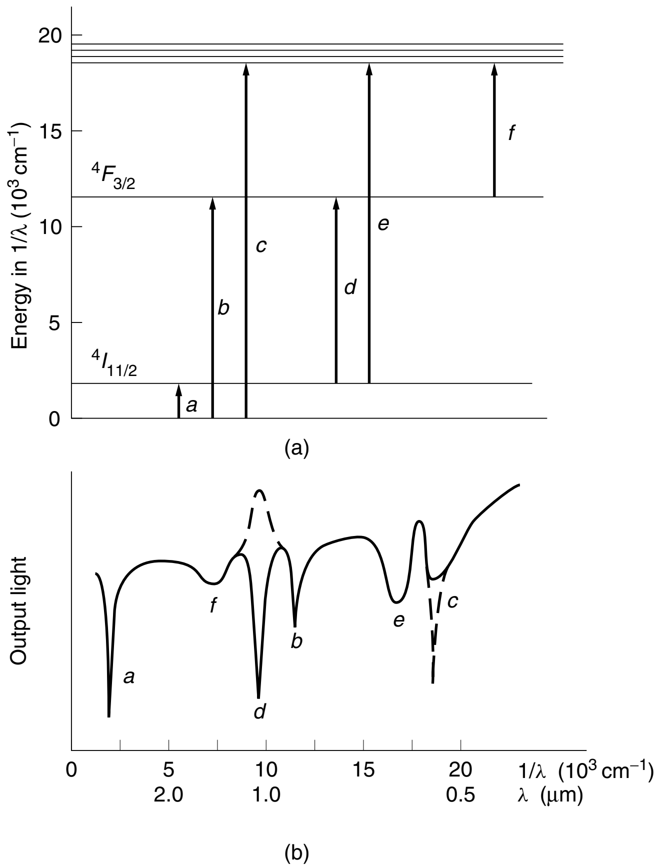

Figure A13.1 Absorption spectrum of Nd YAG.

13.2 (a)

$$G = 10 \log e^{gz}$$

$$z = \frac{G}{10\ g \log e}$$

$$g = \sigma_s(N_2 - N_1)$$

$$= (7.0 \times 10^{-25})[(4.8 - 1.8) \times 10^{23}]$$

$$= 0.21$$

$$z = \frac{35}{10(0.21)(0.434)} = 38.4 \text{ m}$$

(b) The ASE noise $= (G - 1)n_{\text{spon}} m_t h\nu \Delta\nu_f$

$$n_{\text{spon}} = \frac{N_2}{N_2 - N_1} = \frac{4.8 \times 10^{23}}{(4.8 - 1.8) \times 10^{23}} = 1.6$$

G on a linear scale is

$$G = 10^{35/10} = 3162$$

The ASE noise is

$$\begin{aligned}\text{ASE noise} &= (3162 - 1)(1.6)(1)(6.63 \times 10^{-34})(1.94 \times 10^{14})(10^{11})\\ &= 65\ \mu\text{W}\end{aligned}$$

13.3 The threshold pumping rate is $W_p^{\text{th}}\tau$ and τ is expressed as

$$\tau = \frac{1}{(1-\beta)W_p^{\text{th}}}$$

W_p^{th} expressed in terms of the threshold light intensity I_p^{th} is

$$W_p^{\text{th}} = \sigma_p \frac{I_p^{\text{th}}}{h\nu}$$

The value of I_p^{th} has to be determined from the pump power. The pump light actually contributing to pumping is $P_p^{\text{th}}\Gamma$ and the corresponding intensity is

$$I_p^{\text{th}} = \frac{\Gamma P_p^{\text{th}}}{\pi r^2}$$

Thus,

$$\begin{aligned}W_p^{\text{th}} &= \sigma_p \frac{\Gamma P_p^{\text{th}}}{h\nu\pi r^2}\\ &= 0.42 \times 10^{-25} \frac{(0.4)(20 \times 10^{-3})}{(6.63 \times 10^{-34})(2.03 \times 10^{14})\pi(2 \times 10^{-6})^2}\\ &= 198.7\ \text{s}^{-1}\\ \tau &= \frac{1}{(1-\beta)W_p^{\text{th}}}\\ &= \frac{1}{(1-0.38)(198.7)} = 8.1\ \text{ms}\end{aligned}$$

13.4 **(a)** The result of Example 13.10 is used:

$$\begin{aligned}a &= 2\eta m_t n_{\text{spon}} = 2(1)(1)(2) = 4\\ b &= \tfrac{1}{2}\eta m_t^2 n_{\text{spon}}^2 h\nu\Delta\nu_f(2 - B/\Delta\nu_f)/P_{\max}\\ &= \tfrac{1}{2}(1)(1^2)(2^2)(6.63 \times 10^{-34})(1.94 \times 10^{14})(10^{11})\\ &\quad \times (2 - 10^{11}/10^{11})/10^{-2} = 2.57 \times 10^{-6}\end{aligned}$$

$$c = \frac{Dl}{10} = \frac{(1000)(0.2)}{10} = 20$$

$$g_1(k) = \frac{a}{b} \cdot \frac{1}{(c \ln 10 - 2k)} = \frac{7.78 \times 10^5}{(23 - k)}$$

$$g_2(k) = 10^{20/k}$$

k	$g_1(k)$	$g_2(k)$	$g_1(k) - g_2(k)$
1	3.5×10^4	10^{20}	Negative
2	3.7×10^4	10^{10}	Negative
3	3.9×10^4	4.6×10^6	Negative
4	4.1×10^4	10^5	Negative
5	4.3×10^4	10^4	Positive
6	4.6×10^4	2.15×10^3	Positive

The value of k that makes $g_1(k) - g_2(k) = 0$ is between 4 and 5, but much closer to 5 amplifiers.

(b) The gain of each amplifier is

$$\frac{lD}{k} = \frac{(0.2)(1000)}{5} = 40 \text{ dB}$$

(c)

$$\begin{aligned} P_s &= 10^{-2} \times 10^{-4} \\ &= 10^{-6} \text{ W} \\ F_k &= \frac{\eta}{2}\left[4 + \frac{k m_t n_{\text{spon}} h\nu \Delta\nu_f}{P_s}\left(2 - \frac{B}{\Delta\nu_f}\right)\right] k m_t n_{\text{spon}} \end{aligned}$$

With the given parameters of $m_t = 1$ and $\eta = 1$, F_k becomes

$$\begin{aligned} F_k &= 2kn_{\text{spon}} + k^2 n_{\text{spon}}^2 h\nu\Delta\nu_f/2P_s \\ F_5 &= 2(5)(2) + (5^2)(2^2)(6.63 \times 10^{-34})(1.94 \times 10^{14})(10^{11})/(2 \times 10^{-6}) \\ &= 20 + 0.64 = 20.64 \end{aligned}$$

In the case of $k = 4$,

$$\begin{aligned} \frac{lD}{k} &= \frac{200}{4} \text{ dB} \\ P_s &= 10^{-7} \text{ W} \\ F_4 &= 2(4)(2) + (4^2)(2^2)(6.63 \times 10^{-34})(1.94 \times 10^{14})(10^{11})/10^{-7} = 24.23 \end{aligned}$$

In the case of $k = 6$, $F_6 = 24.4$ and $k = 5$ is indeed the optimum.

Chapter 14

14.1 From Eq. (14.81), the relaxation time is

$$\frac{1}{\gamma} = \tau_n \frac{J_{\text{th}}}{J} = 1.18 \text{ ns}$$

From Eq. (14.39),

$$\frac{1}{\tau_s} = \frac{c}{n}\left(\frac{1}{L}\ln\frac{1}{R} + \alpha\right)$$

where $R = [(n-1)/(n+1)]^2$ for a cleaved surface.

$$\frac{1}{\tau_s} = \frac{3\times 10^{10}}{3.5}\left[\frac{1}{0.02}\ln\left(\frac{3.5+1}{3.5-1}\right)^2 + 25\right]$$

$$\tau_s = 1.39 \text{ ps}$$

From Eq. (14.89),

$$\omega_r = \sqrt{\frac{1}{\tau_n\tau_s}\frac{J-J_{\text{th}}}{J_{\text{th}}}} = 15.9\times 10^9 \text{ rad/s}$$

From Eq. (14.88),

$$f_m^r = \frac{1}{2\pi}\omega_r\sqrt{1-\frac{1}{2}\left(\frac{\gamma}{\omega_r}\right)^2} = 2.53 \text{ GHz}$$

14.2 From Eq. (14.68),

$$bS_s = \frac{S_s}{\tau_s}\frac{1}{(N_{\text{th}} - N_\alpha)}$$

Using Eqs. (14.48) and (14.52),

$$bS_s = \frac{1}{\tau_n}\frac{J-J_{\text{th}}}{J_{\text{th}}}\frac{N_{\text{th}}}{N_{\text{th}}-N_\alpha}$$

From Eq. (14.72), the decay constant γ is

$$\gamma = \frac{1}{\tau_n}\left(\frac{m-1}{1-N_\alpha/N_{\text{th}}} + 1\right)$$

where $m = J/J_{\text{th}}$.

14.3 **(a)** Turn-on-delay time. From Eq. (14.61),

$$t_d = \tau_n \ln\left(\frac{J}{J-J_{\text{th}}}\right)$$

$$= 3.7\ln\left(\frac{1.2}{1.2-1}\right) = 6.63 \text{ ns}$$

(b) Cavity length. From Eq. (14.34), $L = \lambda^2/2n\Delta\lambda$ and from Fig. P14.3b,

$$L = \frac{(1.5337)^2}{(2)(3.56)(0.001787)} = 184.9\ \mu\text{m}$$

(c) Threshold electron density. From Eq. (14.30), $R = [(n-1)/(n+1)]^2$. So

$$R = \left(\frac{3.56-1}{3.56+1}\right)^2 = 0.3152$$

From Eq. (14.29),

$$g \geq \frac{1}{L}\ln\frac{1}{R} + \alpha.$$

$$g_{\text{th}} = \frac{1}{0.01849}\ln\frac{1}{0.3152} + 100 = 162.4\ \text{cm}^{-1}$$

From Fig. 14.6, the electron density corresponding to 162.4 cm^{-1} is

$$N_{\text{th}} = 1.8 \times 10^{18}\ \text{cm}^{-3}$$

(d) Threshold current.

$$I_{\text{th}} = J_{\text{th}} L w$$

From Eq. (14.48),

$$I_{\text{th}} = \frac{(0.5 \times 10^{-4})(1.602 \times 10^{-19})(1.8 \times 10^{18})}{3.7 \times 10^{-9}}(0.01849)(4 \times 10^{-4})$$
$$= 29\ \text{mA}$$

(e) Light power versus current. From Eqs. (14.29) and (14.39), $1/\tau_s = v g_{\text{th}}$.

$$\frac{1}{\tau_s} = \frac{3 \times 10^{10}}{3.56}(162.4) = 1.37 \times 10^{12}\ \text{s}^{-1}$$
$$\tau_s = 0.73\ \text{ps}$$

From Eq. (14.55),

$$\tau_c = 1.9\ \text{ps}$$

Using the approximation $E_g \doteqdot h\nu$, we find

$$E_g(J) = \frac{hc}{e\lambda}\ \text{eV}$$
$$E_g = \frac{1.240}{1.5337} = 0.808\ \text{eV}$$

From Eq. (14.58),

$$P_{\text{out}} = (0.808)\left(\frac{0.73}{1.9}\right)(I - I_{\text{th}})$$
$$= 0.31(I - 0.029)\ \text{W}$$

14.4 Figure A14.4 shows the phasor diagram. The diagonal parallelogram *aobc* satisfies

$$2\tilde{E}_s = \tilde{E}_1 + \tilde{E}_3$$

Triangle *aob* satisfies

$$\tilde{E}_a = \tfrac{1}{2}(\tilde{E}_1 - \tilde{E}_3)$$

These are complex forms of Eq. (14.181).

14.5 From Eq. (14.141),

$$\ln I_{\text{th}} = \ln I_{\text{th0}} + \frac{T}{T_0}$$

$\ln I_{\text{th}}$ is plotted against T and the intersect with the vertical axis is found to determine $\ln I_{\text{th0}}$. The slope determines $1/T_0$. They are

$$I_{\text{th0}} = 0.16\ \text{mA}$$
$$T_0 = 50\ \text{K}$$

Similarly,

$$\eta_0 = 1.3\ \text{W/A}$$
$$T_0' = 100\ \text{K}$$

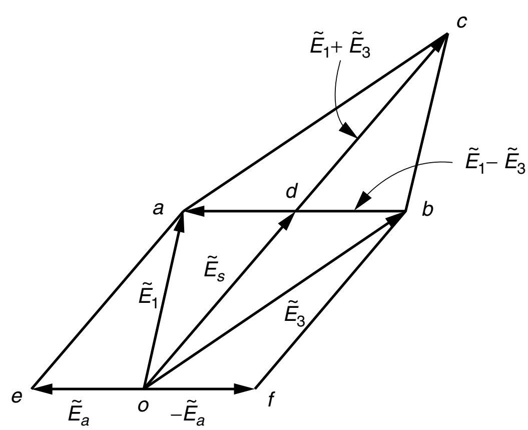

Figure A14.4 Decomposition of the phasor $\tilde{E}_1$ and $\tilde{E}_2$ into $\tilde{E}_s$ and $\tilde{E}_a$.

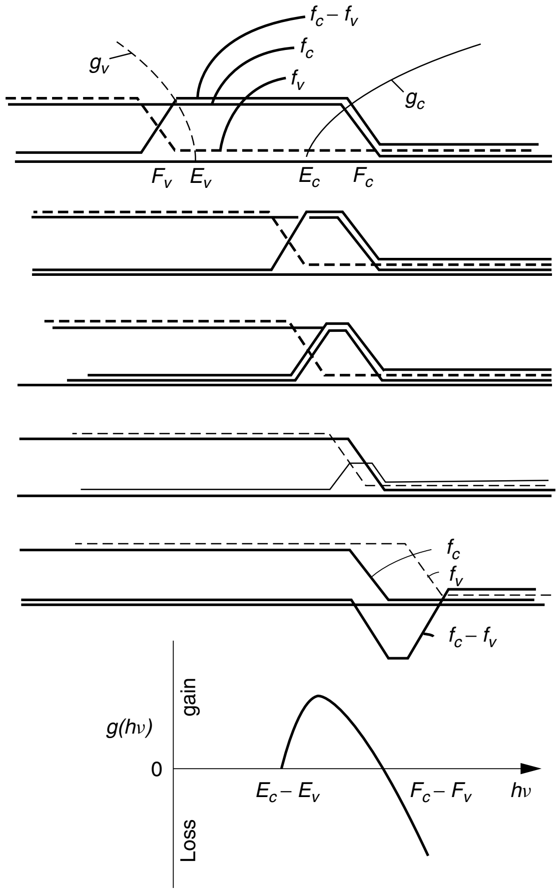

Figure A14.6 Graphical calculation of $g(h\nu)$ (g_v and g_c are suppressed except in the top figure).

Depending on the portions of the experimental curve used, answers may vary. Another way of obtaining η_0 and T_0' is to obtain $\eta = dP/dI$ graphically and then plot it with respect to temperature.

$$\ln\left(\frac{dP}{dI}\right) = \ln \eta_0 - \frac{T}{T_0'}$$

14.6 The Fermi distribution function is represented by a linear ramped curve. The results are shown in Fig. A14.6.

Chapter 15

15.1 Addition of the angles $(K_1 d + \phi)$ and $(-K_1 d + \phi)$ in Eqs. (15.84) and (15.85) gives

$$\tan 2\phi = \frac{K_1(\tilde{\gamma}_0 - \tilde{\gamma}_2)}{K_1^2 + \tilde{\gamma}_0\tilde{\gamma}_2}$$

When $\tilde{\gamma}_0 = \tilde{\gamma}_2$, then

$$\phi = \frac{\pi}{2} q$$
$$q = 0, 1, 2, 3, \ldots$$

In general, $\tilde{\gamma}_0$, $\tilde{\gamma}_2$, and K_1 have to be found first to calculate ϕ.

15.2 Yes, it is possible. Use Fig. 15.12 but with an upside-down pulse shape. You will find that the frequency deviation Δf due to the SPM becomes positive in the leading edge in Fig. 15.12d and the carrier wavelength λ should be chosen at a wavelength in the negative dispersion parameter region ($\lambda < 1.3$ μm).

15.3 From Eq. (15.138),

$$\Delta f_p = -\frac{n_2}{\lambda} L \frac{dI(L, t)}{dt} \tag{1}$$

Inserting

$$I = I_0 e^{-(t/\tau)^2} \tag{2}$$

$$\Delta f_p = \frac{n_2}{\lambda} L I_0 \frac{2t}{\tau^2} e^{-(t/\tau)^2} \tag{3}$$

We need to find the instant at which the value of Δf_p becomes the maximum as well as to rewrite τ in terms of the FWHM τ_s. First, the instant at which Δf_p becomes the maximum is found from

$$\frac{d}{dt}(\Delta f_p) = 0 \tag{4}$$

Inserting Eq. (3) into (4) gives

$$t = \pm \frac{\tau}{\sqrt{2}} \tag{5}$$

Insertion of Eq. (5) with minus sign into (3) gives $\Delta f_{p_{\max}}$ in the left-hand side slope

$$\Delta f_{p_{\max}} = -\frac{n_2}{\lambda} L I_0 \frac{\sqrt{2}}{\tau} e^{-1/2} \tag{6}$$

Next, τ is expressed in terms of the FWHM τ_s.

$$e^{-(\tau_s/2\tau)^2} = \tfrac{1}{2} \tag{7}$$

Solving Eq. (7) for τ gives

$$\tau = \frac{\tau_s}{2\sqrt{\ln 2}} \tag{8}$$

Insertion of Eq. (8) into (6) gives the final result

$$\Delta f_{p_{\max}} = -\frac{n_2}{\lambda} L I_0 \frac{2\sqrt{2\ln 2}}{\tau_s} e^{-1/2} \tag{9}$$

The values of the parameter are inserted

$$\begin{aligned}|\Delta f_{p_{\max}}| &= \frac{3.18 \times 10^{-20}}{1.55 \times 10^{-6}}(200)(1.6 \times 10^{12})\frac{2\sqrt{(2)(0.693)}}{200 \times 10^{-12}}(0.6065)\\ &= 46.9 \text{ GHz}\end{aligned}$$

The total frequency shifts from both sides of the pulse is

$$2\Delta f_{p_{\max}} = 93.8 \text{ GHz}$$

15.4 Rewriting Eq. (15.232) in practical units gives

$$\begin{aligned}P_s &= \frac{0.775(\lambda \times 10^{-6})^3 (D \times 10^{-12}/[(10^3)(10^{-9})])(A_{\text{eff}} \times 10^{-12})}{\pi^2 (3 \times 10^8)(3.18 \times 10^{-20})(\tau_s \times 10^{-12})^2}\\ &= 0.823 \times 10^{-2} \frac{\lambda^3 A D}{\tau_s^2}\end{aligned}$$

Thus

$$\tau_s \sqrt{P_s} = 0.091 \lambda^{3/2} \sqrt{DA}$$

15.5 **(a)** From Eq. (15.152)

$$|\beta^{(2)}| = \frac{\lambda^2}{2\pi c}|D| \tag{1}$$

In MKS units

$$\begin{aligned}|\beta^{(2)}| &= \frac{(1.55 \times 10^{-6})^2 (17 \times 10^{-12}/10^{3-9})}{2\pi(3 \times 10^8)}\\ &= 2.167 \times 10^{-26} \text{ s}^2/\text{m} \qquad (2)\end{aligned}$$

where the dimension of $|\beta^{(2)}|$ is

$$\frac{d^2\beta}{dw^2} = \frac{1/L}{(1/T)^2} = \frac{T^2}{L}$$

Eq. (2) will be converted into practical units

$$\begin{aligned}\beta^{(2)} &= 2.167 \times 10^{-26} \frac{(\text{ps} \times 10^{12})^2}{\text{km} \times 10^{-3}}\\ &= 21.67(\text{ps})^2/\text{km} \qquad (3)\end{aligned}$$

(b) From Eq. (15.231)

$$\Gamma = \frac{2\pi n_2}{\lambda A_{\text{eff}}} \tag{4}$$

In MKS units

$$\Gamma = \frac{2\pi(3.18 \times 10^{-20})}{(1.55 \times 10^{-6})(60 \times 10^{-12})} = 2.15 \times 10^{-3}\ \text{W}^{-1}/\text{m}$$

$$\Gamma = 2.15 \times 10^{-3} \frac{1}{10^{-3}}\ \text{W}^{-1}/\text{km}$$

$$= 2.15\ \text{W}^{-1}/\text{km}$$

where the dimension of Γ is

$$\frac{L^2/P}{LL^2} = P^{-1}/L$$

Let us check the value of peak power using Eq. (15.230) rather than Eq. (15.233).

$$P_s = 3.1 \frac{|\beta^{(2)}|}{\Gamma \tau_s^2}$$

In MKS units, P_s is

$$P_s = 3.1 \frac{2.167 \times 10^{-26}}{2.15 \times 10^{-3} \times (20 \times 10^{-12})^2}$$

$$= 78\ \text{mW} \tag{5}$$

The values of Eq. (5) and (15.234) are within the round-off error.

(c) From Eqs. (15.222) and (15.225)

$$z_c = 0.507 \frac{\tau_s^2}{\beta^{(2)}} \tag{6}$$

From (2) and $\tau_s = 20$ ps

$$z_c = 0.507 \frac{(20 \times 10^{-12})^2}{2.167 \times 10^{-26}} = 9.36\ \text{km}$$

It should be pointed out that Eq. (6) can be rewritten using Eq. (1) as

$$z_c = 0.323 \frac{\pi^2 c \tau_s^2}{\lambda^2 |D|} \tag{7}$$

Chapter 16

16.1 The load resistance R_L is

$$R_L = \frac{1}{2\pi CB} = \frac{1}{2\pi(1.6 \times 10^{-12})(2 \times 10^3)}$$
$$= 4.97 \times 10^7\ \Omega$$

$P_s^a = -23$ dBm $= 5.0 \times 10^{-6}$ W. The receiver system is quantum limited, which can be shown in a variety of ways.

(a) From Eq. (16.55), the receiver is quantum limited if

$$P_s^a > \frac{0.1}{R_L}$$
$$5.0 \times 10^{-6} > \frac{0.1}{4.97 \times 10^7}$$

(b) Being quantum limited can be shown directly from Fig. 16.22.

(c) $N_{\text{shot}} = 2e\dfrac{\eta e}{h\nu}MP_s^aR_LB = 7.95 \times 10^{-14}$ W

$N_{\text{th}} = 4kTB = 3.23 \times 10^{-17}$ W

16.2 The group delay of a step-index fiber is

$$\Delta\tau = \frac{1}{2}\frac{(\text{NA})^2}{cn_1} = \frac{(0.2)^2}{(2)(3 \times 10^5)(1.55)}$$
$$= 4.3 \times 10^{-8}\ \text{s/km}$$
$$t_S = \frac{0.35}{B} = \left(\frac{0.35}{4 \times 10^6}\right) = 8.75 \times 10^{-8}\ \text{s}$$

The rise-time requirement is

$$t_S^2 = t_T^2 + t_R^2 + t_D^2$$
$$(8.75 \times 10^{-8})^2 = (20 \times 10^{-9})^2 + (4.3 \times 10^{-8}x)^2 + (50 \times 10^{-9})^2$$
$$x = 1.6\ \text{km}$$

16.3 Half the bits are "1" and the other half are "0." The "0" is represented by no light, and "1" is represented by the light being on for only half the bit. Thus, for the RZ code, the light is on an average of one-quarter of the time.

$$\frac{S}{N} = \frac{\frac{1}{4}\left(\frac{\eta e}{h\nu}P_s\right)^2}{2e\left(\frac{1}{4}\frac{\eta e}{h\nu}P_s\right)B}$$

For the RZ code, $B = B_t$, and P_s is

$$\begin{aligned} P_s &= 2\frac{h\nu}{\eta e} e \frac{\mathrm{S}}{\mathrm{N}} B_t \\ &= 2(\tfrac{1}{0.5})(1.6 \times 10^{-19})(144)(10^8) \\ &= 9.22 \times 10^{-6}\ \mathrm{mW} \\ &= -50.4\ \mathrm{dBm} \end{aligned}$$

For BER $= 10^{-10}$, 0.5 dBm has to be added. Finally $P_s = -49.9$ dBm.

16.4 The total system loss is

$$2 \times 20 + 2 = 42\ \mathrm{dB}$$

The light power at the APD in dBm is

$$13 - 42 = -29\ \mathrm{dBm}$$

and in watts it is

$$P_s = 10^{-2.9}\ \mathrm{mW} = 1.26\ \mu\mathrm{W}$$

The peak value of the signal current i_s from the APD is

$$\begin{aligned} i_s &= \left(\frac{\eta e}{h\nu}\right) M P_s \\ &= \left(\frac{(0.75)(1.6 \times 10^{-19})}{(6.63 \times 10^{-34})(3.61 \times 10^{14})}\right)(100)P_s \\ &= 50.1 P_s\ \mathrm{A} \end{aligned}$$

This implies that the responsivity matches the value given in the spectral response graph at 830 nm in Fig. 16.21.

(a) For $P_s = 1.26\ \mu$W, the peak value of the signal current is

$$i_s = 50.1 \times 1.26 = 63.1\ \mu\mathrm{A}$$

(b) The load resistance is

$$R_L = \frac{1}{2\pi B C_f} = \frac{1}{2\pi(3 \times 10^8)(3 \times 10^{-12})} = 177\ \Omega$$

The signal power in the load is

$$\begin{aligned} S &= \overline{[i_s A(t)]^2} R_L \\ &= (63.1 \times 10^{-6})^2 \tfrac{1}{2}(177) \\ &= 3.52 \times 10^{-7}\ \mathrm{W} \end{aligned}$$

(c) The shot noise power is

$$N_{\text{shot}} = 2eB(I_s + I_d)M^{2.3}R_L$$

where 0.3 is the typical excess noise index from Fig. 16.21.

$$\begin{aligned}N_{\text{shot}} &= 2(1.6 \times 10^{-19})(3 \times 10^8)(0.631 \times \tfrac{1}{2} + 0.1 \times 10^{-3}) \\ &\quad \times 10^{-6}(100^{2.3})(177) = 2.13 \times 10^{-10}\ \text{W}\end{aligned}$$

Note that if there were no 300-MHz low-pass filter, then 900 MHz from the cutoff frequency of the APD should be used as the value of B.

(d) The thermal noise power is

$$\begin{aligned}N_{\text{th}} &= 4kTB \\ &= 4(1.38 \times 10^{-23})(298)(3 \times 10^8) \\ &= 4.93 \times 10^{-12}\ \text{W}\end{aligned}$$

(e) The signal to noise ratio is

$$\begin{aligned}\frac{S}{N} &= \frac{S}{N_{\text{shot}} + N_{\text{th}}} \\ &= \frac{3.52 \times 10^{-7}}{2.13 \times 10^{-10} + 4.9 \times 10^{-12}} = 1615\end{aligned}$$

The signal to noise ratio in dB is 32.1 dB.

(f) Since $N_{\text{shot}} > N_{\text{th}}$, the system is quantum limited.

(g) If $M = 1$ then $N_{\text{shot}} < N_{\text{th}}$, and the system is thermal noise limited.

(h) The NEP of this APD is

$$\begin{aligned}\text{NEP} &= \frac{h\nu}{\eta e}\left(2eI_dM^x + \frac{4kT}{M^2R_L}\right)^{1/2} \\ &= \frac{1}{0.501}\left[2(1.6 \times 10^{-19})(0.1 \times 10^{-9})(100^{0.3}) \right. \\ &\quad \left. + \frac{4(1.38 \times 10^{-23})(298)}{(100^2)(177)}\right]^{1/2} \\ &= 1.94 \times 10^{-13}\ \text{W}/\sqrt{\text{Hz}}\end{aligned}$$

(i) The minimum detectable power of the APD is

$$\begin{aligned}P_{s\,\min} &= \text{NEP} \times \sqrt{B} \\ &= (1.94 \times 10^{-13})\sqrt{300 \times 10^6}\end{aligned}$$

$$= 3.36 \text{ nW}$$
$$= -54.7 \text{ dBm}$$

(j) The minimum detectable light power $P_{s\,\min}^{\text{ASK}}$ for ASK modulation is, from Eqs. (15.43) and (16.50),

$$P_{s\,\min}^{\text{ASK}} = \sqrt{2} P_{s\,\min}$$
$$P_{s\,\min}^{\text{ASK}} = 1.5 - 54.7 = -53.2 \text{ dBm}$$

The allowable system loss between the source and the detector is

$$13 - (-53.2) = 66.2 \text{ dB}$$
$$66.2 = 2x + 2$$
$$x = 32.1 \text{ km}$$

(k) BER $= 10^{-9}$ requires S/N $= 144$. This system is quantum limited and S/N is proportional to P_s as indicated by Eq. (16.38). In order to increase S/N by 144 times that for S/N $= 1$, P_s has to be increased by 144 times:

$$\begin{aligned} P_s &= 144 P_{s\,\min}^{\text{ASK}} \\ &= (144)(4.75 \times 10^{-9}) \\ &= 6.8 \times 10^{-7} \text{ W} \\ &= -31.7 \text{ dBm} \end{aligned}$$

The allowable system loss between the source and the detector is

$$13 - (-31.7) = 44.7 \text{ dB}$$
$$44.7 = 2x + 2$$
$$x = 21.4 \text{ km}$$

16.5 **(a)**

$$P_s = 1.26\ \mu\text{W}$$
$$i_s = \left(\frac{\eta e}{h\nu}\right) M P_s$$
$$\begin{aligned} i_s &= \frac{(0.75)(1.6 \times 10^{-19})}{(6.63 \times 10^{-34}) \times (3.61 \times 10^{14})}(60) P_s \\ &= 30.1 P_s = 38\ \mu\text{A} \end{aligned}$$

(b)

$$\begin{aligned} S &= \overline{[i_s A(t)]^2} R_L \\ &= (38 \times 10^{-6})^2 \tfrac{1}{2}(50) \\ &= 36 \text{ nW} \end{aligned}$$

(c)

$$\begin{aligned} N_{\text{shot}} &= 2eB(I_s + I_d)M^{2.3}R_L \\ &= 2(1.6 \times 10^{-19})(10^8)[(0.501)(1.26)\tfrac{1}{2} + 1 \times 10^{-3}] \times 10^{-6}(60^{2.3})(50) \\ &= 6.23 \text{ pW} \end{aligned}$$

(d)

$$\begin{aligned} N_{\text{th}} &= 4kTB \\ &= 4(1.38 \times 10^{-23})(298)(10^8) \\ &= 1.65 \text{ pW} \end{aligned}$$

(e)

$$\begin{aligned} \frac{\text{S}}{\text{N}} &= \frac{S}{N_{\text{shot}} + N_{\text{th}}} \\ &= \frac{3.6 \times 10^{-8}}{6.23 \times 10^{-12} + 1.65 \times 10^{-12}} = 4.57 \times 10^3 \end{aligned}$$

The signal to noise ratio in dB is 36.6 dB.

(f) $N_{\text{shot}} > N_{\text{th}}$ and the system is quantum limited.

(g) If $M = 1$, then $N_{\text{shot}} < N_{\text{th}}$, and the system is thermal noise limited.

(h)

$$\begin{aligned} \text{NEP} &= \frac{h\nu}{\eta e}\left(2eI_dM^x + \frac{4kT}{M^2R_L}\right)^{1/2} \\ &= \frac{1}{0.501}\left[2(1.6 \times 10^{-19})(1 \times 10^{-9})(60^{0.3})\right. \\ &\quad \left. + \frac{4(1.38 \times 10^{-23})(298)}{(60^2)(50)}\right]^{1/2} \\ &= 2.00\sqrt{1.09 \times 10^{-27} + 9.14 \times 10^{-26}} \\ &= 6.08 \times 10^{-13}\ \text{W}/\sqrt{\text{Hz}} \end{aligned}$$

(i)

$$\begin{aligned} P_{s\,\text{min}} &= \text{NEP} \times \sqrt{B} \\ &= 6.08 \times 10^{-13}\sqrt{100 \times 10^6} \\ &= 6.08 \text{ nW} \\ &= -52.2 \text{ dBm} \end{aligned}$$

(j)

$$\begin{aligned} P_{s\,\text{min}}^{\text{ASK}} &= \sqrt{2}P_{s\,\text{min}} \\ &= 1.5 - 52.2 = -50.7 \text{ dBm} \end{aligned}$$

The allowable system loss between source and detector is

$$13 - (-50.7) = 63.7 \text{ dB}$$
$$63.7 = 2x + 2$$
$$x = 30.9 \text{ km}$$

(k) Since the system is quantum limited, S/N is proportional to P_s according to Eq. (16.38). The light power P_s needed for S/N = 144 is

$$\begin{aligned} P_s &= 144 P_{s\,\min}^{\text{ASK}} \\ &= 144\sqrt{2} P_{s\,\min} \\ &= 1.24 \times 10^{-6} \text{ W} \\ &= -29.1 \text{ dBm} \end{aligned}$$

The length x of fiber is

$$13 - (-29.1) = 2x + 2$$
$$x = 20 \text{ km}$$

16.6

$$\frac{\text{S}}{\text{N}} = \frac{\frac{1}{2}\left(\frac{\eta e}{h\nu}\right)^2 P_s^2 M^2}{2eB(I_s + I_d)M^{2+x} + \dfrac{4kTB}{R_L}}$$

Let

$$\frac{\text{S}}{\text{N}} = \frac{AM^2}{DM^{2+x} + C}$$

$$\frac{d(\text{S/N})}{dM} = \frac{2AM(DM^{2+x} + C) - AM^2(2+x)DM^{x+1}}{(DM^{2+x} + C)^2}$$

For

$$\frac{d(\text{S/N})}{dM} = 0$$

$$M = \left(\frac{2C}{Dx}\right)^{1/(2+x)}$$

Hence,

$$M = \left(\frac{4kTB/R}{eB(I_s + I_d)x}\right)^{1/(2+x)}$$

16.7 Eq. (16.3) can be rewritten as

$$V_O = -\frac{i}{1/R_f + j\omega C/G}$$

The change due to the presence of C_s is

$$R_f \to \frac{1}{1/R_f + j\omega C_s}$$

$$V_O = \frac{-iR_f}{1 + j\omega(C_s + C/G)R_f}$$

The new cutoff frequency becomes

$$f_{sc} = \frac{1}{2\pi(C_s + C/G)R_f}$$

The ratio of the cutoffs becomes

$$\frac{f_{sc}}{f_c} = \frac{1}{1 + (C_s/C)G}$$

The influence of C_s is amplified by G.

16.8 The probability P_{10} of mistaking "1" as "0" is

$$P_{10} = \frac{1}{\sqrt{2\pi}\sigma_1}\int_{-\infty}^{v_{\text{th}}} e^{-1/2((v-s_1)/\sigma_1)^2} dv$$

Putting

$$-\lambda = \frac{v - s_1}{\sqrt{2}\sigma_1}$$

the integral becomes

$$P_{10} = \frac{1}{\sqrt{\pi}}\int_{\theta_1}^{\infty} e^{-\lambda^2} d\lambda$$

With $\theta_1 = (s_1 - v_{\text{th}})/\sqrt{2}\sigma_1$, P_{10} becomes

$$P_{10} = \tfrac{1}{2}\text{erfc}(\theta_1)$$

Similarly, the probability P_{01} for mistaking "0" as "1" is found as

$$P_{01} = \frac{1}{\sqrt{2\pi}\sigma_0}\int_{v_{\text{th}}}^{\infty} e^{-1/2((v-s_0)/\sigma_0)^2} dv$$

$$= \frac{1}{\sqrt{\pi}}\int_{\theta_0}^{\infty} e^{-\lambda^2} d\lambda$$

$$\theta_0 = \frac{v_{\text{th}} - s_0}{\sqrt{2}\sigma_0}$$

or

$$P_{01} = \tfrac{1}{2}\mathrm{erfc}(\theta_0)$$

For a signal with half "1"s and half "0"s the total probability P of errors is

$$P = \tfrac{1}{2}(P_{10} + P_{01})$$

The value of v_{th} for the minimum P is found from the derivative being zero:

$$\frac{dP}{dv_{\mathrm{th}}} = \frac{dP}{d\theta}\frac{d\theta}{dv_{\mathrm{th}}} = 0$$

Noting

$$\frac{dP}{dv_{\mathrm{th}}} = \frac{1}{2}\left[\frac{dP_{10}}{d\theta_1}\frac{d\theta_1}{dv_{th}} + \frac{dP_{01}}{d\theta_0}\frac{d\theta_0}{dv_{th}}\right]$$

$$\frac{d\theta_1}{dv_{th}} = -\frac{1}{\sqrt{2}\sigma_1}, \qquad \frac{d\theta_0}{dv_{th}} = \frac{1}{\sqrt{2}\sigma_0}$$

$$\frac{dP_{10}}{d\theta_1} = -\frac{e^{-\theta_1^2}}{\sqrt{\pi}}, \qquad \frac{dP_{01}}{d\theta_0} = -\frac{e^{-\theta_0^2}}{\sqrt{\pi}}$$

we have

$$\frac{dP}{dv_{th}} = \frac{1}{2\sqrt{2\pi}}\left(\frac{1}{\sigma_1}e^{-(s_1-v_{\mathrm{th}})^2/2\sigma_1^2} - \frac{1}{\sigma_0}e^{-(v_{\mathrm{th}}-s_0)^2/2\sigma_0^2}\right)$$

With $dP/dv_{\mathrm{th}} = 0$, one obtains

$$2\sigma_0^2\sigma_1^2 \ln\left(\frac{\sigma_0}{\sigma_1}\right) = \sigma_0^2(s_1 - v_{\mathrm{th}})^2 - \sigma_1^2(v_{\mathrm{th}} - s_0)^2$$

which is a quadratic equation of v_{th} and the solution can be found.

For the special case of

$$\sigma_0 = \sigma_1 = \sigma$$

the equation becomes simple and $v_{\mathrm{th}} = \frac{1}{2}(s_0 + s_1)$.

16.9 **(a)** As mentioned in Section 16.6.2, the expression for AM is readily obtained by replacing the "on" state power P_s of the ASK by the average power P_s^a in Eq. (16.53). The required power to each heterodyne detector for a given S/N is calculated from Eq. (16.53) with

$$B_{\mathrm{IF}} = 2B, \qquad M^x = 1, \qquad \frac{\eta e}{h\nu} = 0.5$$

$$P_s^a = 4e\left(\frac{\mathrm{S}}{\mathrm{N}}\right)B$$
$$= 6.4 \times 10^{-19}\left(\frac{\mathrm{S}}{\mathrm{N}}\right)B$$

With

$$\frac{\mathrm{S}}{\mathrm{N}} = 1000$$
$$B = 4 \times 10^6$$

P_s is calculated as

$$P_s = 2(4)(1.6 \times 10^{-19})(10^3)(4 \times 10^6)$$
$$= 5.12 \times 10^{-9}\ \mathrm{W}$$
$$= -52.9\ \mathrm{dBm}$$

The total power loss is

	Unit	Quantity	Total
Fiber loss	1 dB/km	10	10
Connector loss	1 dB	2	2
Splicing loss	0.1 dB	5	0.5
Scrambler loss	0.5 dB	1	0.5
Degradation	3 dB	1	3
Margin of safety	5 dB	1	5
		Total	$P_A = 21$ dB

$$P_T = -52.9 + 21 + 10\log 15$$
$$= -20.1\ \mathrm{dBm}$$

where 10 log 15 is due to splitting the power into 15 households.

(b) The fiber rise time is found. The total frequency bandwidth of the signal in the fiber is

$$9(8 \times 10^9) + 2(4 \times 10^6) \doteqdot 7.2 \times 10^{10}\ \mathrm{Hz}$$

The wavelength spread due to the modulation is

$$\Delta\lambda = 1.15\ \mathrm{nm}$$

The spread of the group delay is, from Fig. 15.14,

$$t_F = 17(10\text{ km})(1.15\text{ nm}) = 2.0 \times 10^{-10}\text{ s}$$

The system rise time is

$$t_S = \frac{0.35}{7.2 \times 10^{10}} = 4.9 \times 10^{-12}\text{ s}$$

Since $t_F > t_S$, the rise-time requirement cannot be met as long as a $\lambda = 1.55$-μm laser is used.

(c) A laser with $\lambda = 1.3$ μm, where the fiber dispersion is almost zero, has to be used. The fiber rise time is $t_F = 0$, and $t_F^2 + t_D^2 < t_S^2$ is satisfied.

(d) The power requirement must be modified. The fiber at 1.3 μm has a higher loss of 3 dB/km and the total fiber loss has to be increased by 20 dB, and the required peak light power is −6.1 dBm.