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APPENDIX C

PERTURBATION THEORY

We will deal only with the case when F is expressed in a form such as

$$F(x, y, z, t) = F(x, y)e^{-j\omega t + j\beta z}$$

Perturbation theory allows us to calculate the change in the propagation constant β due to a small change in the index of refraction of the medium [1].

A scalar wave equation before it is perturbed is designated by the subscript 1 and that after the perturbation due to Δk resulting in the change $\Delta\beta$ in β is designated by the subscript 2.

$$\nabla_t^2 F_1 + (k_1^2 - \beta_1^2)F_1 = 0 \tag{C.1}$$

$$\nabla_t^2 F_2^* + (k_2^2 - \beta_2^2)F_2^* = 0 \tag{C.2}$$

were the operator ∇_t is two dimensional. The subscript t will be suppressed.

Introduction of the divergence identity

$$\boldsymbol{\nabla}\cdot(F_2^*\boldsymbol{\nabla}F_1 - F_1\boldsymbol{\nabla}F_2^*) = F_2^*\boldsymbol{\nabla}^2 F_1 - F_1\boldsymbol{\nabla}^2 F_2^* \tag{C.3}$$

suggests that Eq. (C.1) be multiplied by F_2^* and Eq. (C.2) by F_1, and the results be subtracted from each other.

$$\boldsymbol{\nabla}\cdot(F_2^*\boldsymbol{\nabla}F_1 - F_1\boldsymbol{\nabla}F_2^*) = F_1F_2^*(k_1^2 - k_2^2 + \beta_2^2 - \beta_1^2) \tag{C.4}$$

The surface integral is performed on both sides of Eq. (C.4). The left-hand side is converted into a line integral by the two-dimensional Green's theorem [2].

$$\oint_c (F_2^*\nabla F_1 - F_1\nabla F_2^*)\cdot d\mathbf{l} = \int_S F_1F_2^*[k_1^2 - k_2^2 + \beta_2^2 - \beta_1^2)\,dS \tag{C.5}$$

S for the surface integral is taken in a circle in the cross-sectional plane (perpendicular to the fiber axis) centered at the fiber axis with radius much larger than the core radius of the fiber, and $\mathbf{l}$ for the line integral is taken in the circumference of this circle as shown in Fig. C.1. If the field on the boundary of the fiber core decays and can be approximated as zero, the left-hand side of Eq. (C.5) becomes zero, and

$$\int (\beta_2^2 - \beta_1^2) F_1 F_2^* \, dS = \int (k_2^2 - k_1^2) F_1 F_2^* \, dS \tag{C.6}$$

The change in β on the left-hand side is related to that in k on the right-hand side.

Now, the propagation constants before and after the nonlinear effect are

$$k_1^2 = \omega^2 \mu \epsilon_0 \epsilon = k^2 \epsilon \tag{C.7}$$

$$k_2^2 = k^2 (\epsilon + \delta\epsilon) \tag{C.8}$$

where ϵ is the relative dielectric constant of the medium. Hence

$$k_2^2 - k_1^2 = k^2 \delta\epsilon \tag{C.9}$$

$$\beta_2^2 - \beta_1^2 \doteqdot 2\beta\Delta\beta \tag{C.10}$$

Inserting Eqs. (C.9) and (C.10) into (C.6) gives

$$\Delta\beta = \frac{k^2 \int \delta\epsilon F_1 F_2^* \, dS}{2\beta \int F_1 F_2^* \, dS} \tag{C.11}$$

Equation (C.11) can be expressed in terms of the refractive index by noting

$$\epsilon + \delta\epsilon = (n_1 + \delta_n)^2 = n_1^2 + 2n_1\delta_n \tag{C.12}$$

and

$$\delta\epsilon = 2n_1 \delta n \tag{C.13}$$

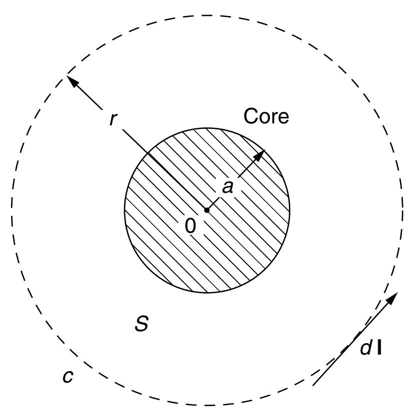

Figure C.1 Geometry of surface and line integrals.

Insertion of Eq. (C.13) in (C.11) finally leads to

$$\Delta\beta = \frac{k^2 \iint n_1 \delta n |F(x, y)|^2 \, dx \, dy}{\beta \iint |F(x, y)|^2 \, dx \, dy} \tag{C.14}$$

where

$$F_1 = F_2 = F(x, y) \tag{C.15}$$

we assumed.

Equation (C.14) is useful for calculating the amount of change $\Delta\beta$ in the propagation constant due to the change δn in the refractive index of the fiber core. The change δn may be attributed to strain (fiber strain sensor), bending (fiber displacement sensor) temperature (fiber temperature sensor) or nonlinearity due to high intensity of light (self phase modulation of the soliton wave).

1. H. A. Haus, *Waves and Fields in Optoelectronics*, Prentice-Hall Inc., Englewood Cliffs, NJ, 1984.
2. W. Hauser, *Introduction to the Principles of Electromagnetism*, Addison-Wesley, Reading, Massachusetts, 1971.