Elements of Photonics, Volume II: For Fiber and Integrated Optics. Keigo Iizuka

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OPTICAL AMPLIFIERS

The word laser is an acronym for lightwave amplification by stimulated emission of radiation. As this acronym includes the word amplification, the optical amplifier and the laser are closely related. They are so closely related that the order of presentation does not really matter. Historically, commercialization of lasers preceded that of optical amplifiers, but for heuristic reasons, the chapter on amplifiers is placed ahead of the chapter on lasers. Optical amplifiers suppress the reflection of light to prevent self-oscillation while lasers enhance the reflection to create oscillation. All other basic principles are common to both devices.

13.1 INTRODUCTION

Optical amplifiers are used as power amplifiers, line amplifiers, and preamplifiers. At the transmitter end, if the light signal is externally modulated or divided into multiple channels, the output from the transmitter is reduced. This reduced light power is amplified by a power amplifier before it is transmitted into the fiber.

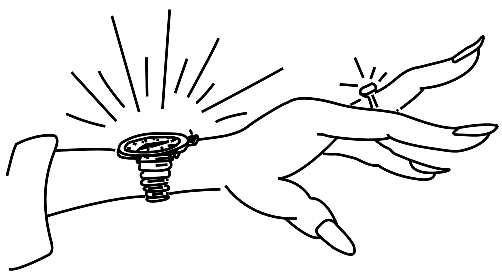
One of the most important applications of the optical amplifier is in repeater stations. Long-haul transmission through a fiber-optic communication line requires repeaters; otherwise, the signal level becomes too low to detect. Repeater design is greatly simplified by using an optical amplifier. Without the optical amplifier, the input light to the repeater is first detected and converted into an electrical signal. The electrical signal is reshaped into a well-defined pulse shape, and the timing of the pulse is readjusted. This new electrical pulse is used to regenerate the light power to send to the next repeater station. In the repeater station, reshaping, retiming, and regeneration, the so-called 3Rs, have to be performed. This rather involved process can be replaced by an optical amplifier. Simplicity and reliability of the repeater compartment are especially important when the lightwave cable is used as an undersea transmission cable.

Another application of optical amplifiers is in the receiver as a preamplifier. When the received light signal is preamplified by an optical amplifier before direct detection, the sensitivity is enhanced.

13.2 BASICS OF OPTICAL AMPLIFIERS

Laser materials are fluorescent, and fluorescence is the key to understanding both lasers and optical amplifiers. As a matter of fact, when searching for a new laser material, the fluorescent properties of the material are the first thing to be checked. A fluorescent substance is one that glows when illuminated, and additionally, the color of the glow is different from the color of the illuminating light. The illuminating light is called the pump light. Even though fluorescence begins as soon as the pump light is turned on, when the pump light is turned off, the fluorescence is sustained. It decays with a characteristic lifetime. Another feature of fluorescence is that the color of the glow is very specific, whereas the color of the pump light does not have to be as specific.

From these observations, scientists have found an explanation using the concept of energy levels and transitions between energy levels. The model shown in Fig. 13.1 will be used as a starting point. This model is known as the three-energy-level model. Pumping takes place between energy levels E_1 and E_3, where $h\nu_{31} = E_3 - E_1$ and ν_{31} is the frequency of the pump light. The fluorescent glow takes place between E_2 and E_1, where $h\nu_{21} = E_2 - E_1$ and ν_{21} is the frequency of the glow light.



Fluorescent crystals are gems for lasers and amplifiers.

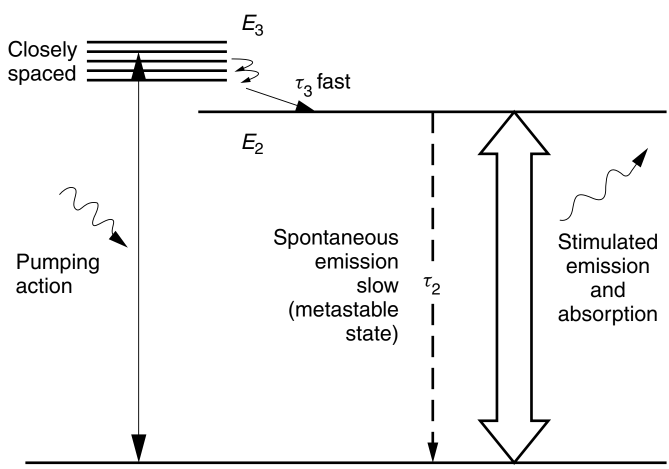


Figure 13.1 Principle of a three-level amplifier.

Energy level E_3 is actually a bundle of many closely spaced energy levels, rather than one discrete level. When the pump light is turned on, upward transitions from E_1 to the E_3 band take place, provided the energies contained in the pump light match the E_1 to E_3 band transition. Almost immediately, downward transitions are initiated between the closely spaced E_3 levels, as well as from E_3 to the nearby level E_2. Because of the narrow spacing, the transitions between the E_3 levels are primarily associated with phonons and nonradiative transitions and occur very quickly over lifetimes of the order of femtoseconds to nanoseconds. The released energy is converted into crystal lattice vibrations or phonons with energy $h\nu_{\text{phonon}} = E_{n+1} - E_n$.

According to Fig. 13.1, there is a large energy gap between E_2 and E_1, which means photons are involved rather than phonons. The downward transition from E_2 to E_1 is responsible for the fluorescent glow. We are particularly interested in materials where E_2 is a metastable state, which means the downward transition from E_2 to E_1 occurs over a lifetime τ_2 of milliseconds to hours. Thus, $\tau_2 > \tau_3$ and the glow lasts a long time after the pump has been turned off.

E_1 and E_2 are discrete levels, whereas E_3 is a band of levels. Hence, the wavelength of the fluorescent glow is very specific (E_2 to E_1 transition), but bands of wavelengths will work for the pump light (E_1 to E_3. band transition).

When the transition from E_2 to E_1 occurs spontaneously, a photon of energy $h\nu_{21}$ is released. This process is called *spontaneous emission*. Spontaneously emitted photons not only travel in different directions but also have different phases. These photons are said to be incoherent.

There is another important mechanism for the emission of light that is called *stimulated emission*. Stimulated emission will be explained by referring once again to the three-level model in Fig. 13.1. The pump light causes the population of atoms in level E_3 to increase. This population increase in E_3, however, is quickly transferred to that of E_2 because of the fast decay from E_3 to E_2, finally resulting in the population buildup in E_2 because of long τ_2. If the material is illuminated by light of frequency ν_{21} while this buildup of population in E_2 exists, a significant increase in the light intensity is observed at frequency ν_{21}. The amount of the increase is proportional to the ν_{21} illumination.

Let's take a closer look at what is going on. Before the buildup in level E_2 has a chance to decay spontaneously, the input photons of energy $h\nu_{21}$ come along. Because these photons happen to have the right frequency, they induce downward transitions from E_2 to E_1. For each induced downward transition, a photon of energy $h\nu_{21}$ is released. Furthermore, both the released photon and the photon that induced the release are not only identical in frequency but also in phase and direction. Hence, the ν_{21} stimulated emission is coherent with the ν_{21} illuminating light.

Stimulated emission is the basis for lasers and optical amplifiers. In a laser, photons capable of causing stimulated emission make multiple passes through a laser cavity, inducing the release of photons of identical energy with each pass. A coherent beam of collimated light with a narrow frequency bandwidth is created. In an optical amplifier, the photons capable of causing stimulated emission (supplied by the signal light) make a single pass through the material. The power of the signal light is amplified by the stimulated emission while traveling through the amplifier.

Emission and the reverse process of absorption can be interpreted as a kind of resonance phenomenon taking place between level E_1 and E_2. A photon is emitted whenever there is a downward transition from E_2 to E_1, and a photon is absorbed whenever there is an upward transition from E_1 to E_2. Moreover, their transition probabilities are identical.

Why then does stimulated emission grow? Would not absorption cancel out the emission? The answer has to do with the populations in levels E_1 and E_2. Under normal circumstances, E_1 is more heavily populated, and absorption dominates. When E_2 is more populated than E_1, this situation is called a population inversion, and in this case, emission dominates. In the three-level model, a population inversion exists between E_1 and E_2 because the pump light keeps E_2 well supplied via the fast decay from E_3 and slow decay to E_1.

If there is no signal light at frequency ν_{21}, then the emission is mainly spontaneous emission. When both a population inversion and input signal light of the right frequency ν_{21} are present, then the output light due to the stimulated emission grows in proportion to the strength of the input signal light, with the population inversion as a proportionality constant.

In the amplifier, there exists not only stimulated emission that is used for amplification, but also spontaneous emission that harms the quality of the amplifier. Spontaneous emission is independent of the input signal light and causes noise problems. Indeed, this noise source is a major concern when using an optical amplifier. Choosing an amplifying medium with a long lifetime is important because the probability of spontaneous emission is $1/\tau_{21}$.

An example of an amplifier that corresponds to the three-level model is the erbium-doped fiber amplifier. This amplifier is used at $\lambda = 1.55$ μm. Not all amplifiers are based on the three-level model. An example of an amplifier based on a four-level model is the neodymium (Nd)-doped fiber amplifier, which is useful at $\lambda = 1.06$ and 1.32 μm.

The four-level model has an extra level E_0 *below* E_1. E_0 is the ground level. The pumping takes place between E_0 and E_3 (see Fig. P13.1b). The population inversion is between E_2 and E_1. As before, the lifetime τ_{32} is short and the lifetime τ_{21} is long. The pumping action populates E_2 via the fast decay from E_3. Compared to the three-level model, the four-level model has a larger population inversion between E_2 and E_1 because E_1 is very quickly emptied out into E_0, which is constantly being evacuated by the pump light. The population inversion between E_2 and E_1 is not only larger but is also less sensitive to the condition of the pump light.

With the three-level material, however, the population in E_1 depends on the strength of the pumping action and its ability to evacuate level E_1. Consequently, the population inversion between E_2 and E_1 is more dependent on the pump power and likewise, the gain of the amplifier is more dependent on the pump power.

13.3 TYPES OF OPTICAL AMPLIFIERS

As shown in Fig. 13.2, there are two major types of optical amplifiers: the fiber amplifier and the semiconductor laser amplifier.

In the fiber amplifier, the doped core of an optical fiber serves as the amplifying medium as shown in Fig. 13.2a. As already mentioned, two examples of fiber amplifiers are the neodymium-doped fiber amplifier [1] that operates at 1.06 and 1.32 μm, and the erbium-doped fiber amplifier (EDFA) that operates in the window of lowest fiber loss at 1.55 μm. A third example of a fiber amplifier is the praseodymium-doped fiber

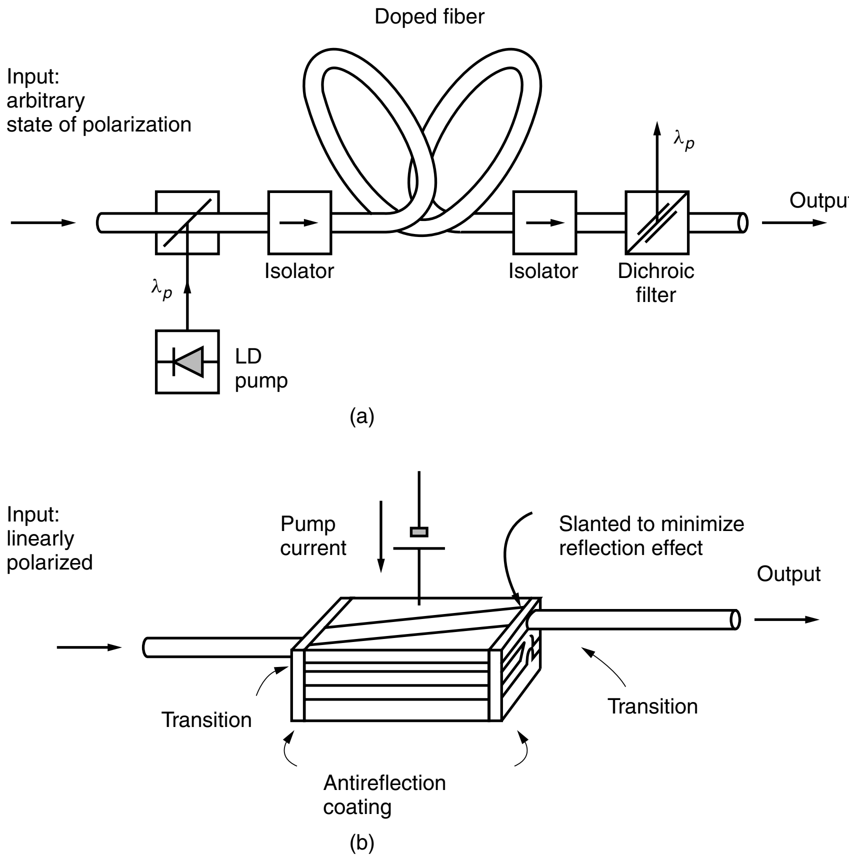


Figure 13.2 Two types of optical amplifiers. (a) Fiber amplifier. (b) Semiconductor laser amplifier.

amplifier (PDFA) that operates in the window of least fiber dispersion at 1.3 μm [2]. Of these three examples, the discussions to follow will feature the EDFA as a typical fiber amplifier.

In order to fabricate an erbium-doped fiber by the two stage process (see Section 11.10.2.1), the core soot of the preform is immersed in alcohol containing erbium chloride ($ErCl_3$). After the alcohol evaporates, the $ErCl_3$ remains in the core soot. The cladding soot is then deposited over the core soot, and the fiber is drawn.

The other type of optical amplifier is the semiconductor laser amplifier (SLA) as shown in Fig. 13.2b. The structure of the SLA is essentially the same as a laser diode, except that end reflections are suppressed in the SLA. The population inversion is achieved by injecting an electric current.

There are advantages and disadvantages to each type of amplifier. The EDFA is ideal for fiber-based systems. Since the EDFA is itself an optical fiber, connections with other optical fibers are easy to make. On the other hand, in systems that employ monolithic integrated optical wafers, the SLA is the amplifier of choice because it can be incorporated directly into the wafer.

The pumping schemes are different for the two types of amplifiers. A light pump is necessary to pump the EDFA. Usually the light pump is the light output from a laser

Table 13.1 Comparison between EDFA and SLA

Characteristic	EDFA	SLA
Structure	Not suitable for monolithic wafer	Suitable for monolithic wafer
Input coupling	To the fiber	To the optical guide
State of polarization	Independent	Dependent
Wavelength	Limited choice in wavelength	Various wavelengths
Method of pumping	Laser diode pump	Electric current
Amplifier gain	Higher	Lower
Saturated output power	Higher	Lower
ASE noise	Present	Present

diode. A coupler is needed to get the pump light into the fiber. An optical filter is required at the end of the fiber to prevent the pump light from emanating out of the amplifier.

By comparison, pumping in the case of the SLA is much simpler. In the SLA, pumping is achieved quite easily with the injection of an electric current.

In the EDFA, the operating wavelength is 1.53–1.55 μm, which matches the wavelength of the lowest fiber transmission loss. For a given fiber, the wavelength is not selectable. A greater range of operating wavelengths is available with the SLA.

The two types of amplifiers behave differently with respect to light polarization. The EDFA is independent of the polarization of the signal light. The same is not true of the SLA. Even when the active region of the SLA has a rectangular shape, the TE mode, whose **E** field is parallel to the surface of the waveguide, is amplified with higher gain than the TM mode. The performance of the SLA is polarization dependent.

Both the EDFA and the SLA generate inherent amplified spontaneous emission (ASE) noise. The gain of the EDFA, typically 30 dB, is higher than the 20-dB gain of the SLA. The level of power saturation is also higher for the EDFA, typically 20 dBm as compared to approximately 15 dBm for the SLA. Distortion of the signal is minimal in the case of the EDFA. Even in the region of gain compression, distortion is practically nonexistent.

In any amplifier, the presence of self-oscillation will cause the operation to become unstable. Self-oscillation is brought on by optical feedback caused by reflections. In continuous fiber systems, reflections inside the fiber are practically nonexistent, and self-oscillation is rarely a problem within the amplifier itself. In the SLA, however, the problem of end reflections can be severe, and elaborate antireflection measures are necessary in the design of the SLA. Both types of amplifiers are vulnerable to reflections that arise from how the various system components are connected together. An isolator is used to prevent oscillations due to the amplified light reflected back from the external components.

Table 13.1 summarizes the comparisons between the EDFA and the SLA.

13.4 GAIN OF OPTICAL FIBER AMPLIFIERS

The expression for the gain of an optical amplifier depends solely on the transitions between the two levels involved in the population inversion. This is true for both three-level and four-level models. In the preliminary discussion of fluorescence, the levels involved in the population inversion were distinctly well-defined levels. However, the description can easily be extended to a population inversion between two *bands* of energy levels, as is often the case with semiconductor materials. Whether the transitions involve molecules, atoms, ions, or electrons in the energy bands depends on the material. For the sake of generality, the term "carrier" will be used to represent an atom, molecule, ion, or electron in a particular level.

Consider the two energy levels E_2 and E_1 shown in Fig. 13.3, disregarding pump level E_3. Energy is released in a downward transition of a carrier, and energy is absorbed in an upward transition. Stimulated transitions, that is, transitions initiated by the presence of an external photon, take place in both directions. In stimulated absorption, the external photon is absorbed as the upward transition takes place, and in stimulated emission, the external photon causes the release of a photon of identical energy as the downward transition takes place. Spontaneous emission is the release of a photon in a downward transition that happens of its own accord, without any external influences. There is no such thing as spontaneous absorption, or a spontaneous upward transition. Since there is only one type of absorption, the qualifier "stimulated' is often dropped, and the process is simply called absorption.

13.4.1 Spectral Lineshape

Every time a carrier makes a transition from E_2 to E_1, a burst of light with frequency $\nu_0 = (E_2 - E_1)/h$ is emitted. Each emission lasts only for a finite time as shown in Fig. 13.4. This finiteness of the duration of emission broadens the power spectrum of the light.

Keep in mind that if the pulse occurrence is completely random, as mentioned in Section 1.4.8, the intensity of the Fourier transform of a train of the pulses is the same as that of a single pulse multiplied by the number of the pulses. Let us find the power spectrum of each emission of light. The light intensity is assumed to decay exponentially with the intensity attenuation constant γ, or the amplitude attenuation constant $\gamma/2$. The amplitude of one of the bursts of light is expressed as

$$E(t) = E_0 e^{-(\gamma/2)t} \cos 2\pi\nu_0 t \tag{13.1}$$

with $t > 0$.

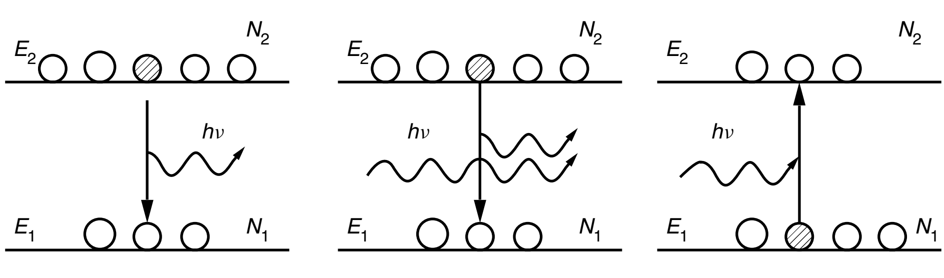


Figure 13.3 Transitions between two energy levels. (a) Spontaneous emission. (b) Stimulated emission. (c) Stimulated absoprtion.

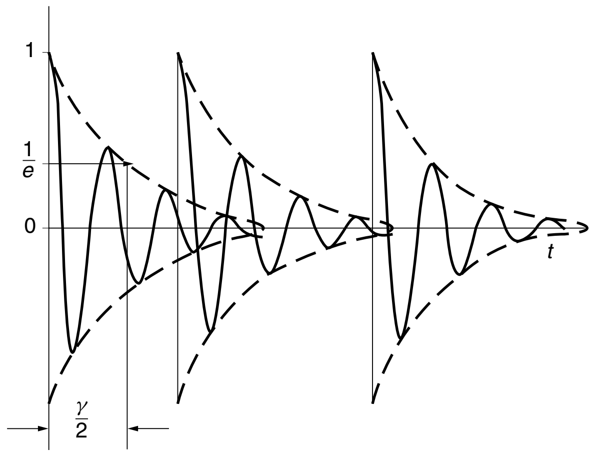


Figure 13.4 Burst of emissions at the times of transitions.

The spectral distribution is obtained from the Fourier transform of Eq. (13.1):

$$E(\nu) = \frac{E_0}{2}\left(\frac{1}{\gamma/2 + j2\pi(\nu - \nu_0)} + \frac{1}{\gamma/2 + j2\pi(\nu + \nu_0)}\right) \tag{13.2}$$

Only the spectrum near ν_0 is considered and the light intensity spectrum is

$$I(\nu) = E(\nu)E^*(\nu) = \frac{I_0}{(\gamma/2)^2 + [2\pi(\nu - \nu_0)]^2} \tag{13.3}$$

The normalized line function $g(\nu)$ will be found by finding I_0 such that

$$\int_{-\infty}^{\infty} \frac{I_0}{(\gamma/2)^2 + [2\pi(\nu - \nu_0)]^2}\, d\nu = 1$$

Using the finite integral formula

$$\int_{-\infty}^{\infty} \frac{a}{x^2 + a^2}\, dx = 2\pi$$

the lineshape function that is normalized as

$$\int_{-\infty}^{\infty} g(\nu)\, d\nu = 1 \tag{13.4}$$

is obtained as

$$g(\nu) = \frac{\gamma/2}{(\gamma/2)^2 + [2\pi(\nu - \nu_0)]^2} \tag{13.5}$$

The lineshape function in Eq. (13.5) is called the *Lorentz lineshape function*. This line broadening is due to the finiteness of the duration of each emission and it takes place even when the medium is homogeneous. There is another kind of broadening that is due to the inhomogeneity of the medium. The former is called homogeneous broadening, and the latter, inhomogeneous broadening. The combination of these two broadening effects determines the lineshape. The overall line broadening will be designated by $g_t(\nu)$ here.

The function $g_t(\nu)$ is sometimes approximated as a constant value g_t and is normalized in a similar manner as Eq. (13.5):

$$g_t(\nu) \doteq g_t$$

with

$$g_t \Delta\nu_t = 1$$

and

$$g_t(\nu) \doteq \frac{1}{\Delta\nu_t} \tag{13.6}$$

The line shape functions are shown in Fig. 13.5.

Having completed a brief description of lineshape functions, the rate of transitions of the carriers will now be calculated. Let N_2 be the number of carriers per unit volume in energy level E_2, and N_1 be the same in energy level E_1. The number of carriers making the downward transition per second per unit volume due to spontaneous emission is

$$-\left.\frac{dN_2}{dt}\right|_{\text{spon}} = AN_2 \tag{13.7}$$

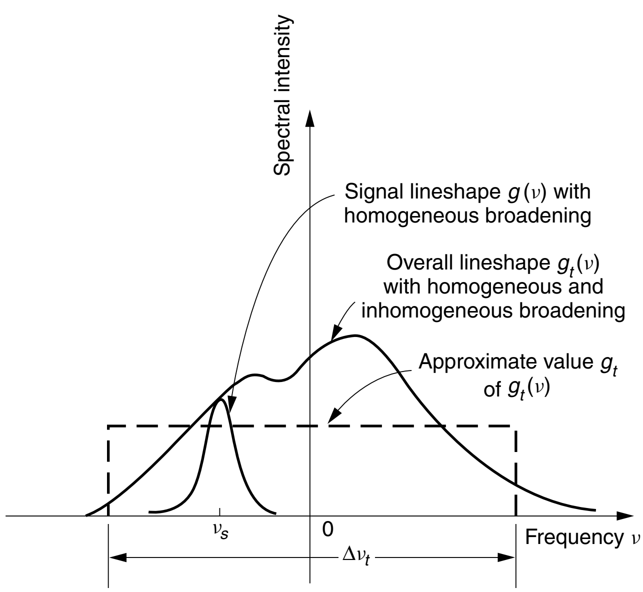


Figure 13.5 Relationships between $g(\nu)$, $g_t(\nu)$, and g_t as a function of frequency.

Let us prove

$$-\frac{dN}{dt} = \frac{N}{\tau}$$

An analogy is made to the calculation of the number of apples falling per day from a tree with a large number N of apples. The lifetime of the apples on the tree is 30 days. The age group of the apples is assumed to be equally distributed. There are $N/30$ apples in each age group and there are $N/30$ apples that are 30 days old and are ready to fall every day.

Thus, the number of apples that fall from the tree each day is

$$-\frac{dN}{dt} = \frac{N}{\tau}$$

where A is Einstein's A coefficient. The spread in frequency of the spontaneous emission is wide and considered constant with frequency. A is a measurable quantity equal to the inverse of the lifetime τ_{21} of the carriers in the E_2 level:

$$A = \frac{1}{\tau_{21}} = \frac{1}{\tau_{\text{spon}}} \tag{13.8}$$

The rate of change in carrier population N_2 per unit time only due to stimulated processes is the difference between the number of carriers that undergo absorption by making the upward transition from E_1 to E_2 and the number of carriers that undergo stimulated emission by taking the downward transition from E_2 to E_1. The induced transition probabilities are identical for both upward and downward transitions and are represented by one probability W_s. The net number of carriers making the downward transition per second per unit volume due to stimulated emission is

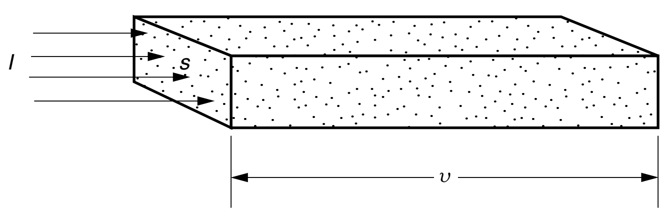
$$-\left.\frac{dN_2}{dt}\right|_{\text{stim}} = W_s(N_2 - N_1) \tag{13.9}$$

The value of W_s is proportional to the light energy density E_d of the stimulating light,

$$W_s = BE_d \tag{13.10}$$

Consider a volume sv defined by light passing through a cross section s m^2 at velocity v in 1 second. The light energy contained in this volume is Is, where I is the light intensity in W/m^2. The light energy density E_d is then

$$E_d = \frac{Is}{sv} = \frac{I}{v} \tag{13.11}$$



where the proportionality constant is known as Einstein's B coefficient. E_d is related to the light intensity I by $E_d = (I/v)$.

By including the spectral lineshape function $g_t(\nu)$, which is a combination of both homogeneous and inhomogeneous line broadenings, W_s can be rewritten as [3,4]

$$W_s(\nu) = B\frac{g_t(\nu)}{v}I \tag{13.12}$$

Next, B will be rewritten in terms of A.

Einstein's A and B coefficients are related to each other. Einstein derived the relationship between A and B from the equilibrium condition of blackbody radiation [3] as

$$\frac{A}{B} = \frac{8\pi n_1^3}{c^3}h\nu^3 \tag{13.13}$$

where n_1 is the index of refraction of a blackbody radiator and c is the speed of light.

Inserting Eqs. (13.8) and (13.13) into Eq. (13.12), W_s becomes

$$W_s(\nu) = \sigma_s\frac{I}{h\nu} \tag{13.14}$$

where

$$\sigma_s = \frac{\lambda^2 g_t(\nu)}{8\pi n_1^2 \tau_{\text{spon}}} \tag{13.15}$$

The quantity σ_s is called the stimulated emission cross section.

Let us further rewrite Eq. (13.13) noting the fact that there are $(8\pi n_1^3\nu^2/c^3)\Delta V \cdot \Delta\nu$ modes in the frequency range between $\nu + \Delta\nu$ and ν in the volume ΔV of a blackbody radiator (see Appendix B of Volume II). With this fact, Eq. (13.13) is described as

$$\frac{A}{B} = m(\nu)h\nu \tag{13.16}$$

where

$$m(\nu) = \frac{8\pi n_1^3}{c^3}\nu^2 = \begin{pmatrix}\text{Number of modes per}\\ \text{unit volume per}\\ \text{unit frequency}\end{pmatrix} \tag{13.17}$$

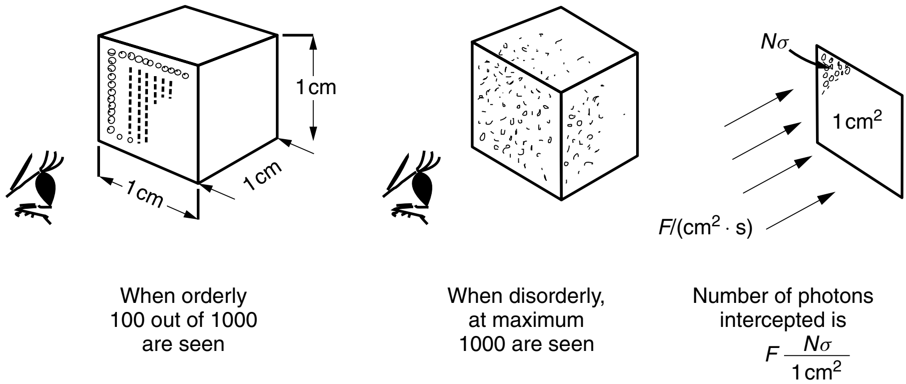
Inserting Eq. (13.17) into Eq. (13.15) or inserting Eq. (13.16) into Eq. (13.12) and comparing with Eq. (13.14) gives

$$\sigma_s = \frac{A g_t(\nu)}{m(\nu)v} \tag{13.18}$$

Equation (13.14) can be interpreted as follows. The number of photons passing through unit area per second is the photon flux density F. Since $I/h\nu$ is the photon flux density F, Eq. (13.9) becomes (see boxed note)

$$\frac{dN_2}{dt} = \sigma_s F(N_2 - N_1)$$

Let us say there are $N = 1000$ carriers in a 1-cm^3 cube. If the carriers are packed in an orderly fashion and viewed straight on, then only the 100 carriers that are in the first face can be seen. But if the carriers are packed in a disorderly manner, then carriers throughout the cube can be seen, and at maximum, all 1000 carriers are seen.



In the maximum disorderly case, the total cross-sectional area that is seen is 1000 σ or $N\sigma$, where σ cm^2 is the cross section of each carrier. Now the number of incident stimulating photons passing through the 1-cm^2 cross-section per second is

$$F = \frac{I}{h\nu}$$

where I is in units of W/cm^2. The ratio of the number of photons that are intercepted by the carriers to that of the incident photons is the ratio of σN to 1 cm^2. The total number of carriers that are stimulated therefore is

$$\frac{dN}{dt} = \left(\frac{\sigma N}{1\ \text{cm}^2}\right) F = \sigma N \frac{I}{h\nu}$$

Including both emission and absorption carriers,

$$\frac{dN_2}{dt} = (N_2 - N_1)\sigma_s \frac{I}{h\nu}$$

The value of σ_s depends on the host material and the wavelength, as shown in Fig. 13.6. The peaks are in the vicinity of 1540 nm, and σ_s tapers off in 1540 ± 40 nm [5].

Now, let us consider the specific case of an erbium-doped fiber amplifier (EDFA). Imagine a fictitious open sided box, such as shown in Fig. 13.7, moving with the photons through a stationary sea of Er^{3+} ions without friction. The increase in the light energy per unit time inside the box is calculated. For each photon having energy $h\nu$, the total contributions from both stimulated and spontaneous emissions are, from Eqs. (13.7) and (13.9),

$$\frac{dE_d}{dt} = \left((N_2 - N_1)\sigma_s \frac{I}{h\nu} + AN_2\right) h\nu \qquad (13.19)$$

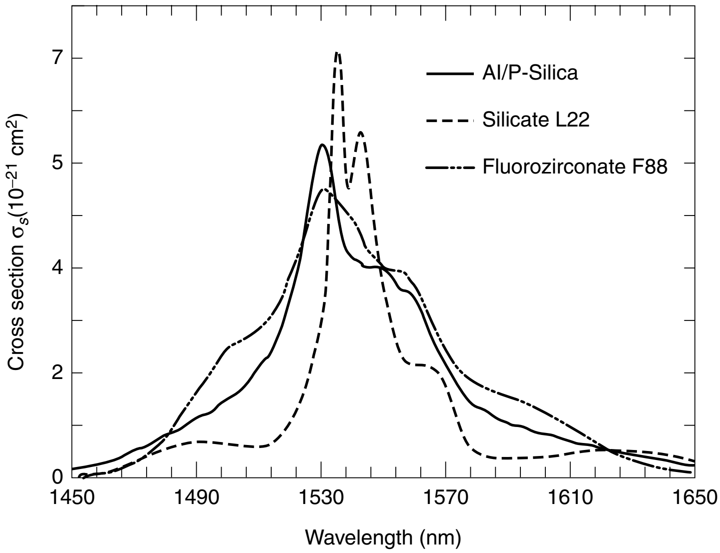


Figure 13.6 Signal cross section σ_s of Er^{3+} hosted in various kinds of glass, as a function of wavelength. (After W. J. Miniscalco [5].)

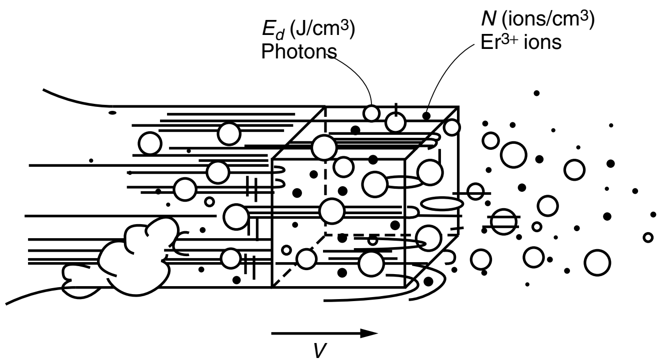


Figure 13.7 Fictitious moving pill box in a fiber amplifier.

From Eqs. (13.11),

$$\frac{dE_d}{dt} = \frac{dI}{v\,dt} = \frac{dI}{dz}$$

and Eq. (13.19) becomes

$$\frac{dI}{dz} = (N_2 - N_1)\sigma_s I + AN_2 h\nu \tag{13.20}$$

The solution of the differential equation is found by letting

$$\begin{aligned} g &= (N_2 - N_1)\sigma_s \\ h &= AN_2 h\nu \end{aligned} \tag{13.21}$$

Equation (13.20) becomes

$$\frac{dI}{dz} = gI + h \tag{13.22}$$

and

$$\int \frac{dI}{gI + h} = \int dz \tag{13.23}$$

Integrating Eq. (13.23) gives

$$\ln(gI + h) = gz + C$$

Solving for I yields

$$I = C' e^{gz} - \frac{h}{g} \tag{13.24}$$

With the boundary condition at $z = 0$,

$$I = I_s \tag{13.25}$$

the solution becomes

$$I = I_s e^{gz} + \frac{h}{g}(e^{gz} - 1) \tag{13.26}$$

Inserting Eq. (13.21) into (13.26) gives the output light intensity I at $z = L$:

$$I = GI_s + (G - 1)\frac{N_2}{N_2 - N_1}\frac{Ah\nu}{\sigma_s} \tag{13.27}$$

where

$$G = e^{gL} \tag{13.28}$$

The first step toward determining the value of σ_s for an erbium-doped fiber is to find the value of $m(\nu)$ specifically for a fiber. Let us consider the entire fiber as one big cavity with length L, a cross section s, and reflectors at each end. The longitudinal modes that are generated by two oppositely traveling waves in the cavity have to satisfy the condition of

$$\frac{\lambda_p}{2} p = L \tag{13.29}$$

In the fiber amplifier there is no reflection and no longitudinal modes, but in order to match the classical approach, we assume there are modes due to the reflected and forward waves. In the end, only one-half of the modes will be used because of the absence of the reflected wave in the fiber amplifier, where λ_p is the wavelength of the pth longitudinal mode. In terms of the frequency f_p, Eq. (13.29) is written as

$$f_p = \frac{v}{2L} p \tag{13.30}$$

where v is the phase velocity in the core. That of the $(p+1)$st mode is

$$f_{p+1} = \frac{v}{2L}(p+1) \tag{13.31}$$

The separation between adjacent modes is

$$f_{p+1} - f_p = \frac{v}{2L} \tag{13.32}$$

The number of modes per unit frequency in this cavity is

$$\frac{1}{f_{p+1} - f_p} = \frac{2L}{v} \tag{13.33}$$

Only the modes that carry the energy forward will be considered. Also, the number m_t of transverse modes has to be accounted for. For a circularly polarized wave, which has two orthogonal modes, m_t is 2. For a linearly polarized mode, m_t is 1. The number $m(\nu)$ per unit volume is obtained by further dividing by sL:

$$m(\nu) = m_t \frac{L}{v}\frac{1}{Ls} = m_t \frac{1}{vs} \tag{13.34}$$

Now, the transition cross section σ_s is found by combining Eqs. (13.18) and (13.34):

$$\sigma_s = \frac{Asg_t(\nu)}{m_t} \tag{13.35}$$

Inserting Eqs. (13.6) and (13.35) into Eq. (13.27) and integrating across the cross section of the fiber gives

$$P = GP_s + (G-1)n_{\text{spon}}\, m_t h\nu\, \Delta\nu_t \tag{13.36}$$

P and P_s are the output and input light powers.

$$n_{\text{spon}} = \frac{N_2}{N_2 - N_1} \tag{13.37}$$

The first term of Eq. (13.36) is the amplified signal power while the second term is the *amplified spontaneous emission* (ASE) noise. The factor n_{spon} is called the *population inversion factor*. The gain factor g of the amplifier can be raised by increasing the population difference $N_2 - N_1$. The ASE noise can be lowered by reducing n_{spon}, but n_{spon} cannot be made zero by making $N_2 = 0$ because the amplification also disappears. A more appropriate minimum value of n_{spon} is unity when $N_1 = 0$ and $N_2 = N$ with $N = N_1 + N_2$. This occurs when all the carriers are emptied out of the energy level E_1. The value of n_{spon} cannot be smaller than unity even for the lossless case. As seen from Eq. (13.36), this noise is amplified by $(G-1)$ with almost the same gain as that for the signal. It is essential for an optical amplifier to be equipped with an optical filter to suppress the ASE noise. The equivalent circuit derived from Eq. (13.36) is shown in Fig. 13.8.

A similar treatment of the gain of the SLA will be given together with lasers in the next chapter.

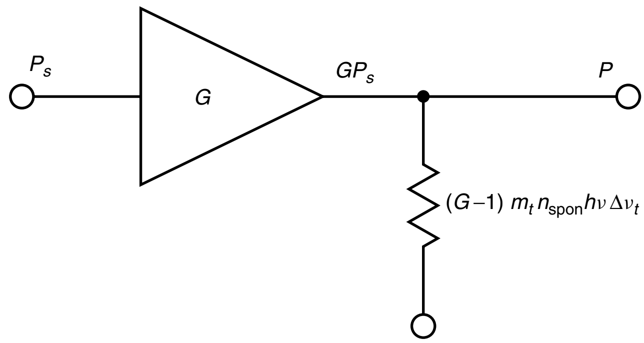


Figure 13.8 Equivalent circuit of an optical amplifier.

13.5 RATE EQUATIONS FOR THE THREE-LEVEL MODEL OF Er^{3+}

The previous section looked at the expression for the amplifier gain, which was derived from the rate equations for the two levels E_2 and E_1 involved in the population inversion. This section explores the solutions of the rate equations for all three levels of a three-level model [6]. The object of this analysis is to reveal the behavior of the threshold and saturation of the gain with respect to the pump light. The example chosen is the three-level Er^{3+} doped fiber amplifier, which is shown in Fig. 13.9. This figure indicates the following quantities: the number of carriers N_i per unit volume in the energy level E_i; the spontaneous transition lifetime τ_{ij}, which is the inverse of the rate of the transition from the ith energy level to the jth energy level (where $i > j$); the stimulated transition probability W_p between E_3 and E_1; and the stimulated transition probability W_s between E_2 and E_1. The quantity W_p is defined similar to Eq. (13.14) as

$$W_p(\nu_p) = \sigma_p \frac{I_p}{h\nu_p} \tag{13.38}$$

The rates of change in the populations in the three levels are

$$\frac{dN_3}{dt} = W_p(N_1 - N_3) - \frac{N_3}{\tau_3} \tag{13.39}$$

$$\frac{dN_2}{dt} = \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}} - W_s(N_2 - N_1) \tag{13.40}$$

$$\frac{dN_1}{dt} = -W_p(N_1 - N_3) + \frac{N_2}{\tau_{21}} + W_s(N_2 - N_1) \tag{13.41}$$

where

$$\frac{1}{\tau_3} = \frac{1}{\tau_{32}} + \frac{1}{\tau_{31}} \tag{13.42}$$

The arguments (ν_s) and (ν_p) are suppressed.

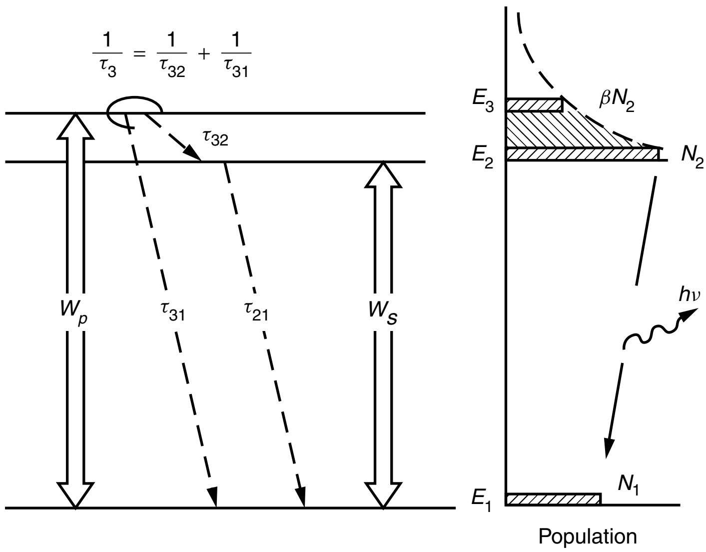


Figure 13.9 Three-level model for Er^{3+} doped fiber amplifier.

The simultaneous rate equations will be solved for the steady-state solution with

$$\frac{d}{dt} = 0 \tag{13.43}$$

The population ratio, the population difference, the gain, and the saturation gain of the amplifier will be found.

13.5.1 Normalized Steady-State Population Difference

The population difference between the E_1 and E_2 levels, which determines the gain of the amplifier, will be found. Since

$$\tau_{31} \gg \tau_{32} \tag{13.44}$$

and with Eqs. (13.42) to (13.44), Eq. (13.39) becomes

$$W_p(N_1 - N_3) = \frac{N_3}{\tau_{32}} \tag{13.45}$$

Putting Eq. (13.45) into (13.40) gives

$$0 = W_p(N_1 - N_3) - \frac{N_2}{\tau_{21}} - W_s(N_2 - N_1) \tag{13.46}$$

The E_2 and E_3 levels of the Er^{3+} doped fiber amplifier pumped by 1.48-μm pump light are very closely spaced, and because of the fast relaxation process, the population ratio between these two levels quickly reaches the Boltzmann population ratio [4]:

$$\beta = \frac{N_3}{N_2} = e^{-(\Delta E/kT)} \tag{13.47}$$

At room temperature, $\beta = 0.38$. Putting Eq. (13.47) into (13.46) gives

$$\frac{N_2}{N_1} = \frac{W_p + W_s}{W_p\beta + 1/\tau_{21} + W_s} \tag{13.48}$$

N is defined as

$$N = N_1 + N_2 \tag{13.49}$$

Let the numerator and denominator of Eq. (13.48) be A and B,* respectively. N becomes

$$N = N_1 + \frac{A}{B}N_1 \tag{13.50}$$

and hence

$$N_1 = \frac{B}{A+B}N \tag{13.51}$$

$$N_2 = \frac{A}{A+B}N \tag{13.52}$$

Equations (13.51) and (13.52) can be rewritten as

$$\frac{N_2 - N_1}{N} = \frac{A-B}{A+B} \tag{13.53}$$

Putting the numerator and denominator of Eq. (13.48) into this equation and letting $\tau_{21} = \tau$ gives

$$\frac{N_2 - N_1}{N} = \frac{(1-\beta)W_p\tau - 1}{(1+\beta)W_p\tau + 2W_s\tau + 1} \tag{13.54}$$

$W_p\tau$ is called the normalized pumping rate.

Recall from Eqs. (13.14) and (13.38) that $W_s\tau$ and $W_p\tau$ are quantities representing the signal and pump light intensities, respectively. The quantity $N_2 - N_1$ represents the gain factor from Eq. (13.21). Thus, Eq. (13.54) is an important equation that relates signal and pump powers to the gain of the amplifier. In Fig. 13.10a, $(N_2 - N_1)/N$ is plotted as a function of the pump power $W_p\tau$ with the signal power $W_s\tau$ and β as parameters. In Fig. 13.10b, the corresponding experimental results [7] are shown.

Example 13.1 Is the following logic true or false?

As shown in the energy level diagram in Fig. 13.3, there exists only the stimulated transition in the upward transition $1 \to 2$, whereas there exist both stimulated and spontaneous transitions in the downward transition $2 \to 1$. As a result, the overall probability of a downward transition $2 \to 1$ is always larger than an upward transition, so that a net downward transition of the carriers always exists.

Solution The conclusion that there is always a net downward transition of the carriers is false. But for Boltzmann's theorem, it could have been true. The population of the

*A and B are *not* Einstein's coefficients.

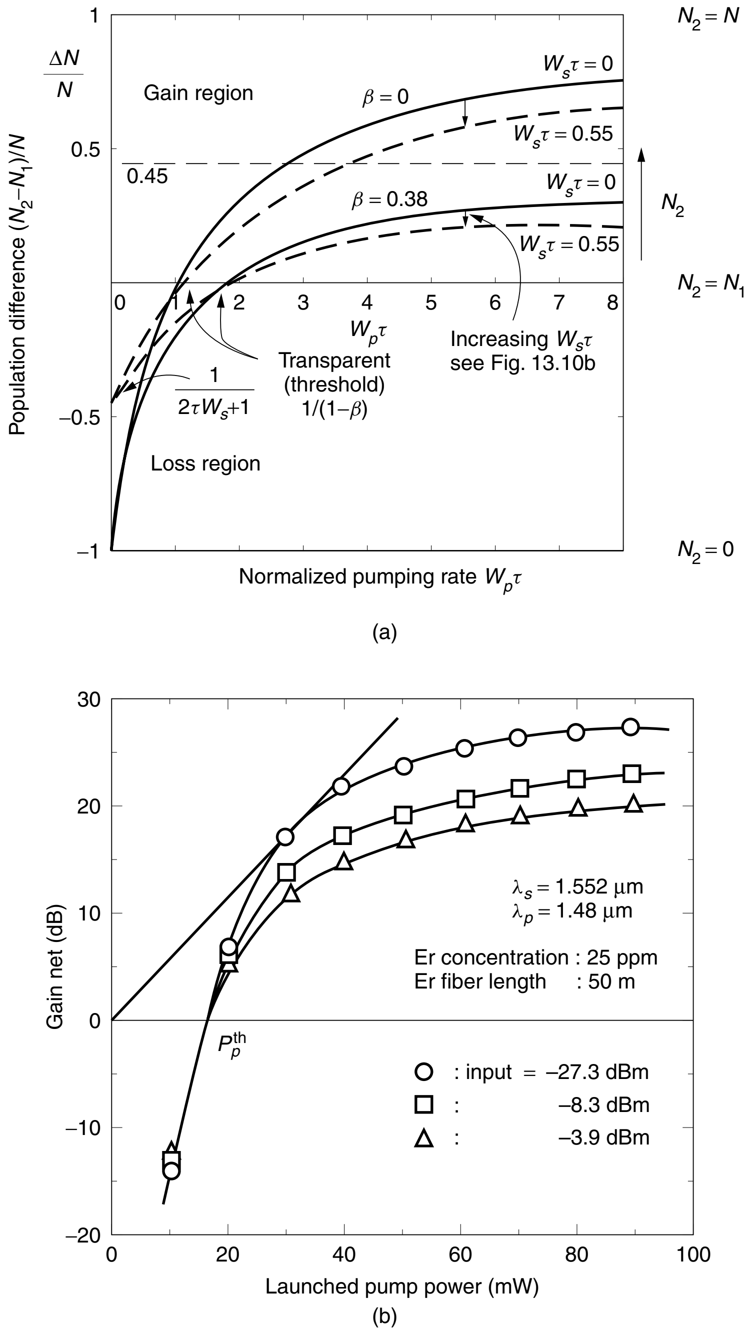


Figure 13.10 Population difference versus pump power with signal power as a parameter. (a) Population difference versus normalized pumping rate with signal power and β as parameters. (b) Signal gain characteristics versus pump power. (After K. Nakagawa et al. [7]).

carriers was overlooked. According to Eq. (13.47), the population N_2 of the upper state is smaller than N_1 of the lower state at equilibrium. □

13.5.2 Gain of the Amplifier

From the expression for the gain factor g of the optical amplifier in Eqs. (13.21) and (13.54), the criteria for whether or not the amplifier experiences gain are

$$\frac{N_2 - N_1}{N} \begin{cases} > 0 & \text{gain} \\ = 0 & \text{transparent} \\ < 0 & \text{loss} \end{cases} \tag{13.55}$$

When a medium starts to experience gain, the medium is said to become active. Equation (13.54) means that the medium does not become active until it reaches the threshold value of

$$W_p^{\text{th}}\tau = \frac{1}{1-\beta} \tag{13.56}$$

This threshold occurs at the same value regardless of the signal power level. The experimental curves in Fig. 13.10b clearly indicate this fact. Equation (13.56) with $\beta = 0$ reduces to

$$W_p^{\text{th}} = \frac{1}{\tau}$$

Thus, the threshold condition is interpreted as the case when the pump power is just enough to sustain the carriers for spontaneous emission.

From Eq. (13.56), we see that the longer the lifetime τ of the metastable state of erbium is, the lower the threshold pump power becomes. This is an additional merit of the long lifetime besides the earlier mentioned reduced spontaneous emission noise.

As seen from Eq. (13.54), the population difference is at its maximum when

$$2W_s\tau = 0 \tag{13.57}$$

Let us define the maximum population difference $\Delta N_{\max}$ from Eqs. (13.54) and (13.57):

$$\frac{\Delta N_{\max}}{N} = \frac{(1-\beta)W_p\tau - 1}{(1+\beta)W_p\tau + 1} \tag{13.58}$$

Inserting Eq. (13.58) into (13.54) gives

$$\frac{N_2 - N_1}{N} = \frac{\Delta N_{\max}}{N} \frac{(1+\beta)W_p\tau + 1}{(1+\beta)W_p\tau + 2W_s\tau + 1}$$

or

$$\frac{N_2 - N_1}{N} = \frac{\Delta N_{\max}}{N} \frac{1}{1 + \dfrac{2W_s\tau}{(1+\beta)W_p\tau + 1}} \tag{13.59}$$

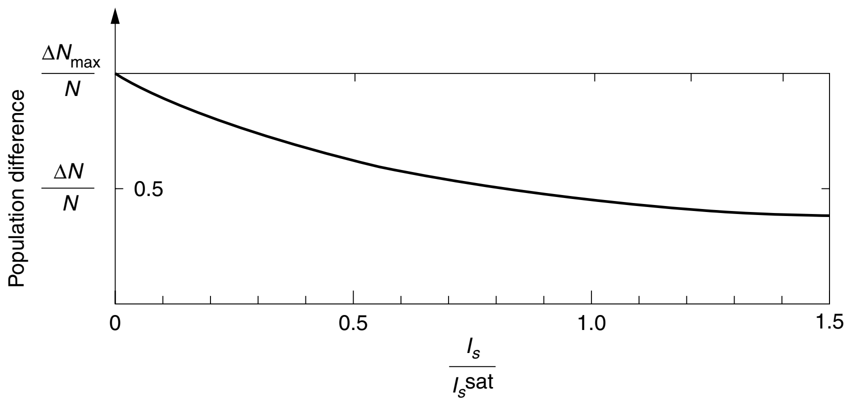


Figure 13.11 Population difference versus normalized signal power density.

The value of $W_s \tau$ (signal power) that reduces $N_2 - N_1$ to one-half of $\Delta N_{\max}$ is designated as W_s^{sat}.

$$\frac{N_2 - N_1}{N} = \frac{\Delta N_{\max}}{N} \frac{1}{1 + W_s / W_s^{\text{sat}}} \tag{13.60}$$

where

$$W_s^{\text{sat}} = \tfrac{1}{2}\left((1+\beta)W_p + \frac{1}{\tau}\right) \tag{13.61}$$

With Eq. (13.56), Eq. (13.61) becomes

$$W_s^{\text{sat}} = \frac{1}{2\tau}\left(1 + \frac{1+\beta}{1-\beta}\frac{W_p}{W_p^{\text{th}}}\right) \tag{13.62}$$

Recalling Eq. (13.14), Eq. (13.60) can be rewritten as

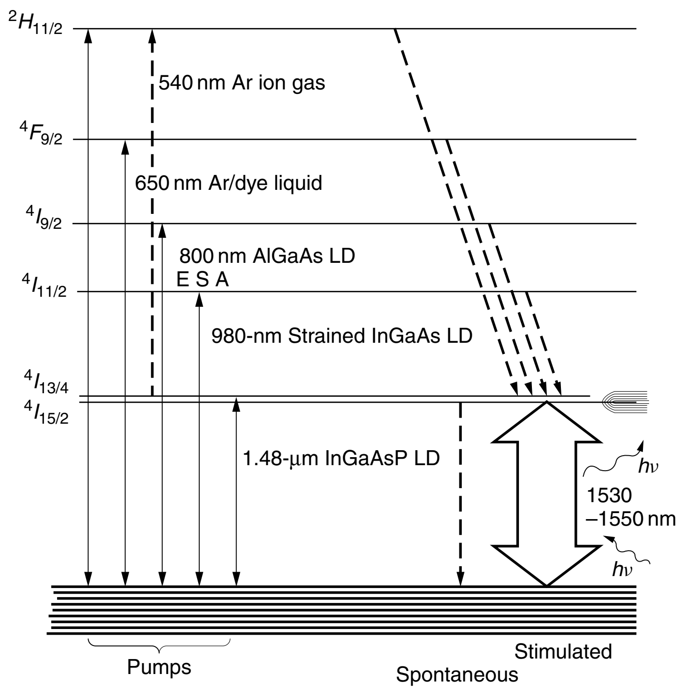
$$\frac{N_2 - N_1}{N} = \frac{\Delta N_{\max}}{N} \frac{1}{1 + I_s / I_s^{\text{sat}}} \tag{13.63}$$

Equation (13.63) is plotted in Fig. 13.11.

The above simple analysis reveals several important conclusions. Recall that the gain factor of the amplifier is proportional to the population difference. An increase in the pump power results in an increase not only in the signal saturation intensity (Eq. (13.62)) but also in the gain of the amplifier (Eq. (13.59)). With an increase in the signal power, the gain of the amplifier gradually decreases and finally reaches zero (Eq. (13.63)).

13.6 PROS AND CONS OF 1.48-μm AND 0.98-μm PUMP LIGHT

Figure 13.12a shows the energy level diagram of Er^{3+}. The types of lasers that may be used as the pump light are indicated in the diagram. Figure 13.12b shows the absorption



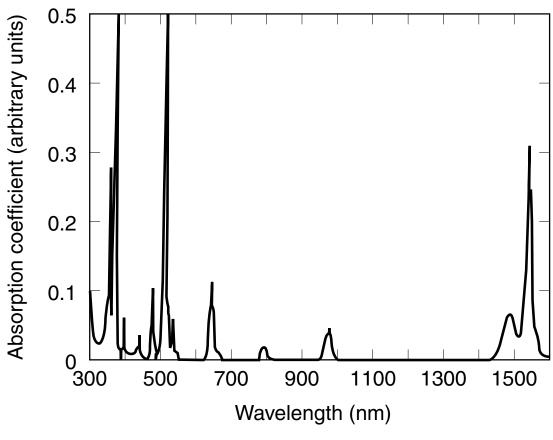


Figure 13.12 Energy level diagram and absorption lines of Er^{3+}. (a) Energy level diagram of Er^{3+}. (b) Absorption spectrum of Er^{3+}-doped silicate glass. The peaks of the strong bands at 380 and 520 nm are 1.5 and 1.0, respectively. (After W. J. Miniscalco [5]).

spectrum of Er^{3+} associated with the transitions shown in Fig. 13.12a. The higher the absorption coefficient is, the larger the absorption cross section σ_p is, and hence the easier to pump. Unfortunately, high absorption lines are in the region of wavelengths shorter than 540 nm where solid state pump lasers are not available. From the viewpoint of reliability, longevity, and compact size, only solid state devices are practical for optical communication, which means the pump light candidates are 800 nm, 980 nm, and 1.48 μm. The 800-nm pump, however, is inefficient due to a process known as excited state absorption (ESA). Figure 13.13 explains the mechanism of ESA loss. If there is still a higher energy level E_4 such that the spacing between E_4 and E_2 is identical with that between E_3 and E_1, there is a possibility that some of the carriers in level E_2 would be brought up further to the E_4 level by the pump light because $E_3 - E_1 = E_4 - E_2$. The attractive candidates now become only 980-nm and 1.48-μm pump light. Comparisons between these two pump wavelengths will be made using the solutions of the rate equations obtained in Section 13.5.

First, the values of β for these two pump wavelengths are considered. The 1.48-μm pump is special in that the ratio N_3/N_2 of the population density is tightly clamped by the Boltzmann distribution because the energy level of the 1.54-μm signal is so close to that of the 1.48-μm pump. At room temperature, β is

$$\beta = \frac{N_3}{N_2} = e^{-\Delta E/kT} = 0.38 \tag{13.64}$$

In contrast, β is practically zero for 0.98-μm pump light. This creates differences between 1.48-μm and 0.98-μm pumping. As seen from Eq. (13.54), the maximum obtainable population difference for a large normalized pumping rate is

$$\left(\frac{N_2 - N_1}{N}\right)_{\max} = \frac{1-\beta}{1+\beta} \tag{13.65}$$

In the case of 1.48-μm pumping, substituting $\beta = 0.38$ in Eq. (13.65) gives

$$\left(\frac{N_2 - N_1}{N}\right)_{\text{max, 1.48-μm pump}} = 0.45 \tag{13.66}$$

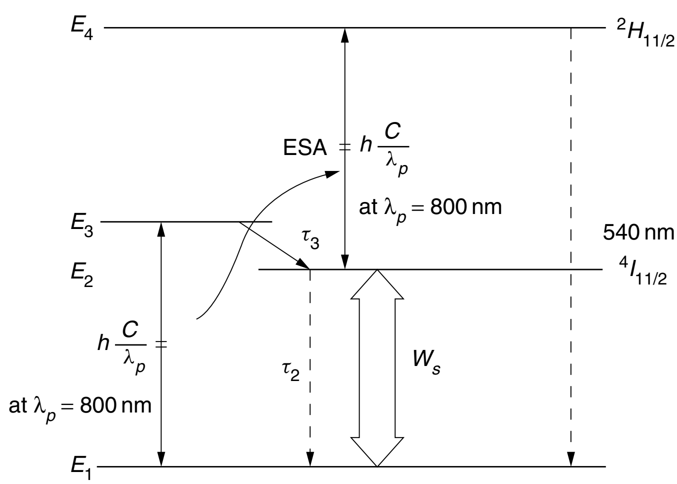


Figure 13.13 Explanation of excited state absorption (ESA) loss: stepwise two-photon absorption.

While for 0.98-μm pumping, Eq. (13.65) evaluates to

$$\left(\frac{N_2 - N_1}{N}\right)_{\text{max, 0.98-μm pump}} = 1 \tag{13.67}$$

Thus, the 0.98-μm pump amplifier has superior gain to its 1.48-μm pump counterpart.

The 0.98-μm pump is also superior to the 1.48-μm pump in terms of the spontaneous noise of the amplifier. As obtained earlier, the amplified spontaneous emission (ASE) noise is proportional to

$$n_{\text{spon}} = \frac{N_2}{N_2 - N_1} \tag{13.68}$$

From Eqs. (13.51) and (13.52), N_{spon} is rewritten as

$$\frac{N_2}{N_2 - N_1} = \frac{A}{A - B} \tag{13.69}$$

and from Eq. (13.48), n_{spon} becomes

$$n_{\text{spon}} = \frac{W_p + W_s}{W_p(1 - \beta) - 1/\tau} \tag{13.70}$$

Increasing the pump power reduces n_{spon} and n_{spon} approaches the limit

$$n_{\text{spon}} = \frac{1}{1 - \beta} \tag{13.71}$$

The 0.98-μm pump can ultimately reduce n_{spon} to 1, but the 1.48-μm pump can only reduce n_{spon} to 1.61.

When comparing the pumping efficiency, which is the gain in dB obtained per milliwatt of pump power, the 0.98-μm pump is also superior to the 1.48-μm pump. The threshold pump power is given by Eq. (13.56). For 1.48 μm, the threshold is defined as $W_p^{\text{th}}\tau = 1.61$, while the same quantity for the 0.98-μm pump is $W_p^{\text{th}}\tau = 1.0$. This difference is related to a measured difference in the energy extraction efficiency from the pump laser. The gain of the amplifier per unit pump power for a 0.98-μm pump laser is 10 dB/mW, while that for a 1.48-μm pump laser is 5 dB/mW.

As for the saturation signal power intensity, Eq. (13.61) contains a factor $(1 + \beta)$. For the same pump power W_p, the saturation signal power intensity W_s^{sat} is higher for the 1.48-μm pump. Thus, the 1.48-μm pump is better than the 0.98-μm pump in this respect.

One of the difficulties associated with the 0.98-μm pump is that the linewidth of the pump transition is narrower than the linewidth of the 1.48-μm transition. The 0.98-μm pump laser diode (LD) needs tight control of its wavelength of emission.

In summary, the 0.98-μm pump is superior with respect to the gain per unit pump power, the amount of ASE noise, and the threshold pump power, while the 1.48-μm pump is superior with respect to saturation signal power and the tolerance of the wavelength of the pump light.

These conclusions are summarized in Table 13.2.

Table 13.2 Comparison of EDFAs pumped by 1.48-μm and 0.98-μm pump light

Characteristic	1.48-μm Pump	0.98-μm Pump
Pump laser material	InGaAs/InP	Strained InGaAs
Typical output power from the pump laser	20–100 mW	10–20 mW
Amplifier differential gain per mW of pump light	5 dB/mW	10 dB/mW
Saturation signal power	20 dBm	5 dBm
Pump transition linewidth	20 nm	2.5 nm
Noise figure (NF)	5 dB	3 dB

13.7 APPROXIMATE SOLUTIONS OF THE TIME-DEPENDENT RATE EQUATIONS

Whether or not a short-duration pulse can be amplified depends on the behavior of the amplifier gain over short time spans. In other words, the time-dependent solutions for N_2 and N_1 have to be found. The time-dependent rate equations will be solved.

The lifetime τ_3 is less than 1 ns and N_3 is large, which means $-N_3/\tau_3$ is a large negative number. This means that N_3 reaches the steady state in less than 1 ns, and it is usually safe to assume after 1 ns that

$$\frac{dN_3}{dt} = 0 \tag{13.72}$$

This assumption means that no significant change in N_3 takes place after 1 ns. This assumption remarkably simplifies the solution, yet gives answers very close to the answers obtained without the assumption [8]. With Eq. (13.72), the time-dependent rate equations — Eqs. (13.40), (13.41), and (13.45) — become

$$\dot{N}_1 = -W_p(N_1 - \beta N_2) + \frac{N_2}{\tau} + W_s(N_2 - N_1) \tag{13.73}$$

$$\dot{N}_2 = W_p(N_1 - \beta N_2) - \frac{N_2}{\tau} - W_s(N_2 - N_1) \tag{13.74}$$

Rewriting Eqs. (13.73) and (13.74) gives

$$\dot{N}_1 = -aN_1 + bN_2 \tag{13.75}$$

$$\dot{N}_2 = aN_1 - bN_2 \tag{13.76}$$

where

$$a = W_p + W_s \quad \text{and} \quad b = \beta W_p + W_s + \frac{1}{\tau} \tag{13.77}$$

The dot means the derivative with respect to time t. The derivative of Eq. (13.76) and the relationship $\dot{N}_1 = -\dot{N}_2$ obtained from Eqs. (13.75) and (13.76) gives

$$\ddot{N}_2 + (a + b)\dot{N}_2 = 0 \tag{13.78}$$

The same form of differential equation is obtained for N_1.

Let's assume the solution of N_2 is

$$N_2 = C_1 e^{\gamma t} + C_2 \tag{13.79}$$

The value of γ is found by putting Eq. (13.79) back into Eq. (13.78) and differentiating:

$$\gamma^2 + (a+b)\gamma = 0 \tag{13.80}$$

The two possible values of γ are obtained as

$$\gamma(\gamma + a + b) = 0 \tag{13.81}$$

$$\gamma = 0 \quad \text{and} \quad \gamma = -(a+b) \tag{13.82}$$

The general solution for $N_2(t)$ is

$$N_2(t) = C_1 e^{-(a+b)t} + C_2 \tag{13.83}$$

With the initial values of $N_2(0)$ and $\dot{N}_2(0)$, C_1 and C_2 are found. The result is

$$N_2(t) = N_2(0) + \frac{\dot{N}_2(0)}{a+b}\left(1 - e^{-t/\tau_{\text{eff}}}\right) \tag{13.84}$$

where

$$\tau_{\text{eff}} = \frac{1}{a+b} \tag{13.85}$$

Invoking the conservation of the number of carriers $N = N_1(t) + N_2(t)$, the time-dependent population difference becomes

$$\frac{N_2(t) - N_1(t)}{N} = 2\frac{N_2(t)}{N} - 1 \tag{13.86}$$

which leads to a time-dependent expression of the gain of the amplifier from Eq. (13.21).

Inserting Eq. (13.77) into (13.85), the effective time constant is obtained as

$$\tau_{\text{eff}} = \frac{\tau}{(1+\beta)W_p\tau + 2W_s\tau + 1} \tag{13.87}$$

Using τ in Eqs. (13.56) and (13.62), Eq. (13.87) can be rewritten as

$$\tau_{\text{eff}} = \frac{\tau}{\dfrac{W_p}{W_p^{\text{th}}}\left(\dfrac{1+\beta}{1-\beta}\right) + \dfrac{W_s}{W_s^{\text{sat}}}\left(1 + \dfrac{1+\beta}{1-\beta}\dfrac{W_p}{W_p^{\text{th}}}\right) + 1} \tag{13.88}$$

The gain factor of the amplifier is proportional to the population difference. Equations (13.83) through (13.88) tell us how quickly the gain of the amplifier reacts to a change of conditions, as, for example, a sudden decrease in $N_2(t)$ due to a sharp high-intensity input pulse.

The lifetime τ of Er^{3+} is quite long and $\tau = 10$ ms. The derived expression of τ_{eff} in Eq. (13.88) shows that τ_{eff} is a slow function of W_s and W_p. Even with the extreme case of $W_s/W_s^{\text{sat}} = W_p/W_p^{\text{th}} = 5$ (= 7 dB), the effective lifetime τ_{eff}

The results without the assumption of $dN_3/dt = 0$ can be found in the literature [8], and the solution is of the form of $N_2(t) = N_0(e^{-t/\tau_{\text{eff}}} + \epsilon e^{-t/\tau_3}) + C$, where $\epsilon = \tau_3/\tau \approx 10^{-7}$ and N_0 and C are constants. The contribution of the term with τ_3 is small and insignificant.

is 14 μs to 0.28 ms, depending on β, and is reasonably slow. This means that as long as the rise time of the input pulse is faster than τ_{eff}, no change in the gain of the amplifier, which is proportional to Eq. (13.86) takes place, and the pulse can be amplified without distortion. Amplification without distortion is an important feature of the erbium-doped fiber amplifier. Provided that the signal rates are higher than $1/(0.28 \times 10^{-3}) = 35.7$ kb/s for RZ coding (see Section 16.2.9), no distortion is generated. This is true even when the amplitude of the signal is beyond saturation because the gain of the amplifier cannot change faster than 0.28 ms. Any signal with a bit rate lower than 35.7 kb/s, however, experiences a change in the gain and, hence, distortion of the signal. Distortionless amplification at saturation power levels is utilized to achieve a stable operating point of the amplifier. Another advantage of setting the operating point at several decibels beyond the point of the saturation power level is that even when the input signal power decreases abruptly for a short time due to unknown causes in the system, the change in the amplifier's gain is minimum. This is also true when the power level increases unexpectedly.

Normally, in the case of a multichannel system, the operating power level is set below gain saturation in order to avoid crosstalk between the channels. Consider two independent channels with slightly different wavelengths that are combined and amplified by the same optical amplifier. Suppose that channel A is the signal channel, and channel B is just providing the operating point. If channel B does not change with time, then channel A is amplified at constant gain and no distortion results. However, if channel B is also changing with time, the operating point of channel A changes with time, and the gain of channel A changes as the operating point fluctuates. The output of channel A is influenced by the channel B signal, and crosstalk between channels A and B occurs. In the case of the EDFA, if the movement of the bias point is fast enough, the gain does not change, and even from the viewpoint of crosstalk between channels, the EDFA has advantages over the SLA.

Example 13.2 From the following measured results, determine the signal stimulated emission cross section σ_s of Er^{3+}.

$\lambda = 1.55$ μm
$\Delta\lambda_t = 50$ nm
$n_1 = 1.46$
$\tau = 10$ ms

The unfiltered ASE noise spectrum was used to measure $\Delta\lambda_t$, which is the full-width half-power point of the ASE spectrum.

Solution From Eq. (13.15)

$$\sigma_s = \frac{\lambda^2 g_t(\nu)}{8\pi n_1^2 \tau_{\text{spon}}}$$

First, $g_t(\nu)$ Will be found.

$$\begin{aligned}
\Delta\nu &= 3 \times 10^8 \left(\frac{1}{1.525} - \frac{1}{1.575} \right) \times 10^6 \\
&= 6.2 \times 10^{12} \text{ Hz} \\
g_t(\nu) \simeq \frac{1}{\Delta\nu} &= 1.6 \times 10^{-13} \text{ Hz}^{-1} \\
\sigma_s &= \frac{(1.55 \times 10^{-6})^2 (1.6 \times 10^{-13})}{8\pi (1.46)^2 (10^{-2})} \\
&= 7.2 \times 10^{-25} \text{ m}^2 = 7.2 \times 10^{-21} \text{ cm}^2
\end{aligned}$$

□

Example 13.3 An EDFA is constructed with a 30-m length of erbium-doped fiber. What is the population difference $N_2 - N_1$ that is required to achieve a 30-dB gain in this amplifier? Use the value of the stimulated emission cross section obtained in Example 13.2.

Solution The gain from Eq. (13.28) is

$$G = e^{gL}$$

where, from Eq. (13.21),

$$g = (N_2 - N_1)\sigma_s$$

The gain (in dB) is

$$\begin{aligned}
G(\text{dB}) &= 10 \; gL \log e \\
g &= \frac{G(\text{dB})}{10L \log e} \\
&= \frac{(30)}{10(30)(0.434)} \\
&= 0.230 \text{ m}^{-1} \\
(N_2 - N_1) &= \frac{g}{\sigma_s} \\
&= \frac{0.230}{7.2 \times 10^{-25}} \\
(N_2 - N_1) &= 3.2 \times 10^{17} \text{ cm}^{-3}
\end{aligned}$$

□

Example 13.4 Calculate the threshold light intensity for a 1.48-μm pumped erbium-doped fiber amplifier with the following parameters:

$\sigma_p = 0.42 \times 10^{-21}$ cm^2
$\tau = 10$ ms

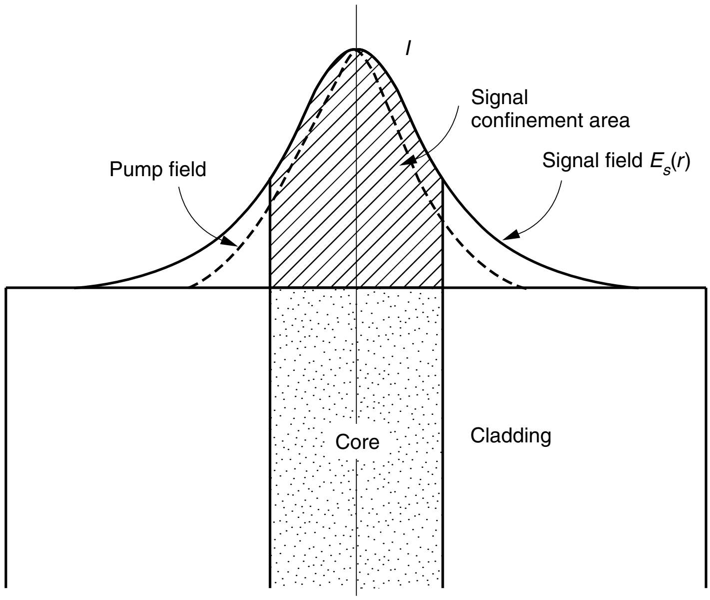


Figure 13.14 Confinement area of light and core.

$h = 6.63 \times 10^{-34}$ J · s
$\beta = 0.38$
$\Gamma = 0.4$
$s = 12.6\ \mu\text{m}^2$

Γ is the confinement factor of the pump light distribution with the shape of the fiber core. It is used to quantify the coupling of the pump light as shown in Fig. 13.14.

Compare the calculated results with the experimental curve in Fig. 13.10b.

Solution The first parameter to be calculated is the frequency of the pump:

$$\nu_p = \frac{c}{\lambda} = 2.03 \times 10^{14} \text{ Hz}$$

As shown in Fig. 13.14, the field $E_s(r)$ is not entirely confined inside the active region (core of the fiber). Only the center portion of the light is inside the active layer and participates in the amplification. The actual gain should be reduced by the *confinement factor*, defined as the ratio of the power inside the core to the entire field:

$$\Gamma_s = \frac{\int_0^a |E_s(r)|^2 r\,dr}{\int_0^\infty |E_s(r)|^2 r\,dr} \tag{13.89}$$

Because of the difference in wavelengths, the confinement factor Γ_s of the signal is not necessarily the same as Γ_p of the pump.

The relationship between the transition probability W_p and the light intensity I_p is, from Eq. (13.38),

$$
\begin{aligned}
W_p &= \sigma_p \frac{I_p}{h\nu_p} \\
&= (0.42 \times 10^{-25}) \frac{I_p}{(6.63 \times 10^{-34})(2.03 \times 10^{14})} \\
&= 0.312 \times 10^{-6} I_p
\end{aligned}
$$

At the threshold,

$$W_p^{\text{th}} = 0.312 \times 10^{-6} I_p^{\text{th}}$$

On the other hand, from Eq. (13.56),

$$
\begin{aligned}
W_p^{\text{th}} &= \frac{1}{(1-\beta)\tau} = \frac{1}{(1-0.38)(10^{-2})} = 161 \text{ s}^{-1} \\
I_p^{\text{th}} &= \frac{161}{0.312 \times 10^{-6}} = 5.16 \times 10^{8} \text{ W/m}^2 \\
&= 5.16 \times 10^{8} \frac{10^3}{10^{12}} \text{ mW/}\mu\text{m}^2 \\
&= 0.516 \text{ mW/}\mu\text{m}^2
\end{aligned}
$$

The confinement factor is $\Gamma = 0.4$, and the cross-sectional area of the fiber is 12.6 μm^2. The threshold pump power is $P_p^{\text{th}} = 0.516 \times (1/0.4) \times 12.6 = 16.3$ mW. This agrees well with experimentally measured values shown in Fig. 13.10b. □

Example 13.5 What is the maximum gain G of a 1.48-μm pumped erbium-doped fiber amplifier when the pump power $P_p = 80$ mW? The maximum gain is the gain near zero signal power. What is the gain when the output light power P is 1 mW? The relevant parameters are:

$$
\begin{aligned}
&\lambda_p = 1.48 \ \mu\text{m} \\
&\lambda_s = 1.55 \ \mu\text{m} \\
&\sigma_p = 0.42 \times 10^{-21} \text{ cm}^2 \\
&\sigma_s = 3.6 \times 10^{-21} \text{ cm}^2 \\
&\tau = 10 \text{ ms} \\
&\Gamma_p = \Gamma_a = 0.4 \\
&s = 12.6 \ \mu\text{m}^2 \\
&L = 35\text{m} \\
&\beta = 0.38 \\
&N = 2.0 \times 10^{18} \text{ cm}^{-3}
\end{aligned}
$$

Ignore the transmission loss.

Solution The gain is

$$G = e^{gL}$$

where

$$g = \sigma_s(N_2 - N_1)$$

The maximum population difference is, from Eq. (13.58),

$$\left(\frac{N_2 - N_1}{N}\right)_{\max} = \frac{(1-\beta)W_p\tau - 1}{(1+\beta)W_p\tau + 1}$$

First, W_p is found from Eq. (13.38):

$$\begin{aligned} W_p &= \sigma_p \frac{1}{h\nu_p}\frac{P_p}{s}\Gamma_p \\ &= 0.42 \times 10^{-25} \frac{8 \times 10^{-2}}{(6.63 \times 10^{-34})(2.03 \times 10^{14})(12.6 \times 10^{-12})}(0.4) \\ &= 793\ \mathrm{s}^{-1} \end{aligned}$$

Next, $(N_2 - N_1)_{\max}$ is calculated.

$$\frac{\Delta N_{\max}}{N} = \frac{(1-0.38)(793)(10^{-2}) - 1}{(1+0.38)(793)(10^{-2}) + 1} = 0.328$$

$$(N_2 - N_1)_{\max} = 2 \times 10^{24} \times 0.328 = 6.56 \times 10^{23}\ \mathrm{m}^{-3}$$

Finally, the maximum gain factor is

$$\begin{aligned} g_{\max} &= \sigma_s(N_2 - N_1) \\ &= (3.6 \times 10^{-25})(6.56 \times 10^{23}) \\ &= 0.24\ \mathrm{m}^{-1} \end{aligned}$$

The maximum gain (in dB) is

$$\begin{aligned} G(\mathrm{dB}) &= 10 \log e^{0.24 \times 35} \\ &= 36.5\ \mathrm{dB} \end{aligned}$$

Next, the gain of the fiber in the region of the nonzero signal light output is calculated. The signal power is not uniform along the fiber but the output signal power $P = 1$ mW is used for the calculation.

$$\frac{N_2 - N_1}{N} = \frac{\Delta N_{\max}}{N}\frac{(1+\beta)W_p\tau + 1}{(1+\beta)W_p\tau + 2W_s\tau + 1}$$

$W_s\tau$ is to be calculated.

$$
\begin{aligned}
W_s &= \sigma_s \frac{1}{h\nu_s} \frac{P}{s} \Gamma_s \\
&= (3.6 \times 10^{-25}) \frac{(10^{-3})}{(6.63 \times 10^{-34})(1.94 \times 10^{14})(12.6 \times 10^{-12})} (0.4) \\
&= 88.9 \ \mathrm{s}^{-1}
\end{aligned}
$$

Note that W_s is one order of magnitude smaller than W_p.

$$
\begin{aligned}
\frac{N_2 - N_1}{N} &= \frac{\Delta N_{\max}}{N} \frac{(1 + 0.38)(793)(10^{-2}) + 1}{(1 + 0.38)(793)(10^{-2}) + 2(88.9)(10^{-2}) + 1} \\
&= \frac{\Delta N_{\max}}{N} 0.87
\end{aligned}
$$

$N_2 - N_1$ drops by 13% from the maximum value, and hence,

$$
\begin{aligned}
g &= 0.24 \times 0.87 = 0.21 \ \mathrm{m}^{-1} \\
G &= e^{gL} = e^{0.21 \times 35} = 1556 \\
G(\mathrm{dB}) &= 10 \log e^{0.21 \times 35} \\
&= 31.9 \ \mathrm{dB}
\end{aligned}
$$

□

13.8 PUMPING CONFIGURATION

The pump light can be injected either from the front or back end of the erbium-doped fiber. Double-clad fibers are used for high-power amplifiers.

13.8.1 Forward Pumping Versus Backward Pumping

The pump light can be injected so that it propagates in the same direction as the signal light, which is called forward pumping, or it can be injected in the opposite direction, which is called backward pumping. There is also bidirectional pumping, which is a combination of forward and backward pumping. All of these configurations are shown in Fig. 13.15. The reason for the different configurations has to do with the distribution of pump light in the fiber. The pump light is attenuated as it propagates.

In the case of the forward pumping, as shown in Fig. 13.15a, $(N_2 - N_1)/N$ is largest at the input end of the fiber. This means the population inversion factor $n_{\text{spon}} = N_2/(N_2 - N_1)$ is smaller at the input, and hence, the ASE noise is smaller at the input end than at the output end. When cascading two amplifiers with different noise figures, the overall noise performance is better when the lower noise amplifier is placed in front of the noisier one. Thus, forward pumping displays better noise performance than backward pumping.

In the case of backward pumping, as shown in Fig. 13.15b, the saturation signal power is higher than forward pumping. Toward the end of the amplifier fiber, the signal intensity has become high and closer to saturation. The saturation signal intensity, however, is increased with pump power by Eq. (13.62). With backward pumping, the pump power is largest toward the end of the fiber, where the high intensity is needed to

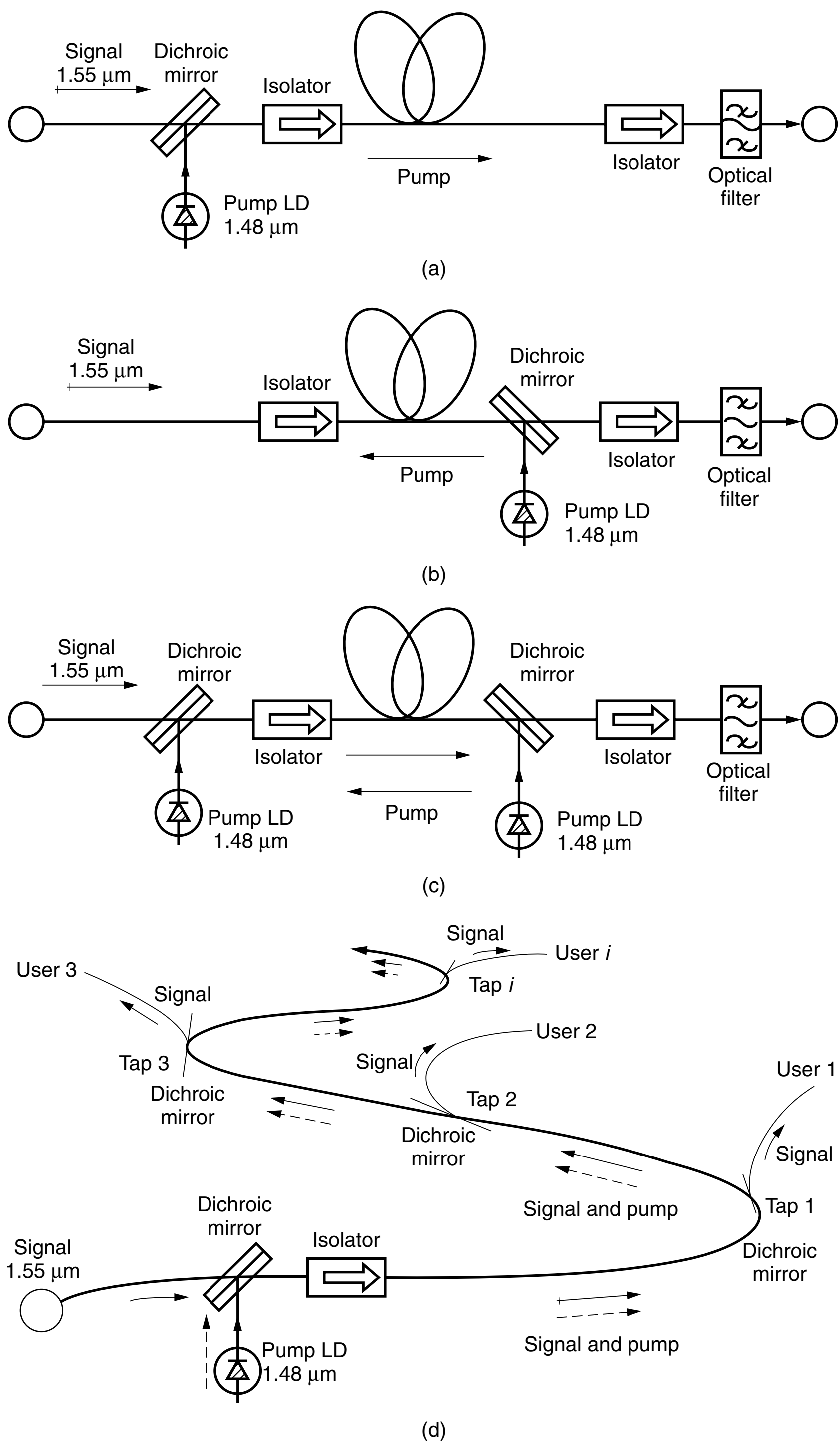


Figure 13.15 EDFA pumping configurations. (a) Forward pumping. (b) Backward pumping. (c) Bidirectional pumping. (d) Remotely pumped concatenated EDFA network.

raise the saturation signal power. Not only is the saturation signal power increased, but the total gain of the amplifier is also increased. If not for the increased pump power, the gain would have been saturated and the overall power would have been reduced. Thus, backward pumping has the advantages of higher saturation power and higher gain.

Finally, bidirectional pumping, as shown in Fig. 13.15c, enjoys the advantages of both at the cost of the complexity.

As shown in Fig. 13.15, the pump light from the 1.48-μm laser diode is coupled to the 1.55-μm signal light path by means of a dichroic mirror. The dichroic mirror is made up of layers of dielectric film whose thicknesses are arranged so as to pass the 1.55-μm light but reflect the 1.48-μm light. Optical isolators, which are not sensitive to the light polarization (see Section 5.5.2) but are sensitive to the direction of propagation, are installed on both ends of the fiber to prevent the reflected wave from going back into the fiber.

An optical filter is installed at the end of the fiber amplifier. This filter is necessary to prevent the pump light from going out of the amplifier and also to remove the effect of the ASE noise in the detector. From the viewpoint of removing the ASE noise, a narrowband filter (<1 nm) is desirable.

As shown in Fig. 13.15d, there is a scheme in which a single pump source supplies pump to concatenated EDFAs. It is a kind of forward pumping. The role of gain in such a network is to overcome propagation losses as well as to compensate for the signal power tapped off by users for detection. Such a scheme is worthy of consideration for managing the densely populated metropolitan area network (MAN) [9].

13.8.2 Double-Clad Fiber Pumping

Confinement of both the signal and pump light in a 5-μm core, such as shown in Fig. 13.14, becomes impractical with a high-power fiber amplifier because of the power density handling capability (nonlinear effects in the core). The double-clad fiber [10,11] subdivides the cladding into inner and outer cladding regions. The signal light propagates as a single mode inside the core, while the pump light propagates as multimodes inside the inner cladding.

Figure 13.16 shows various double-clad fiber geometries.

In Fig. 13.16a, the single-mode core and multimode inner cladding are concentric. Among the higher order modes, there are more skew rays than meridional rays. As shown in Fig. 11.1b, the skew ray goes around the guiding layer and the light energy is concentrated everywhere but the center, where the core is located. The confinement factor is low.

In order to increase the overlap of the skew rays of the pump with the signal in the core, the location of the core is offset in Fig. 13.16b. Figure 13.16c is similar to Fig. 13.16b, but with several offset cores. However, for both the geometries shown in Figs. 13.16b and 13.16c, the offset creates difficulties in light coupling or splicing the fibers.

The geometry in Fig. 13.16d is slightly different. The inner cladding is rectangular. The maximum intensity of the fundamental mode is located at the center as in the case of a rectangular microwave waveguide, and a high confinement factor is achieved.

A D-shaped inner cladding, such as shown in Fig. 13.16e improves the coupling of the skew ray with the signal light in the core.

The geometry of the multiple D fiber is shown in Fig. 13.16f. This geometry not only increases the amount of overlap but also restores the symmetry of the fiber cross section for easier light coupling or splicing.

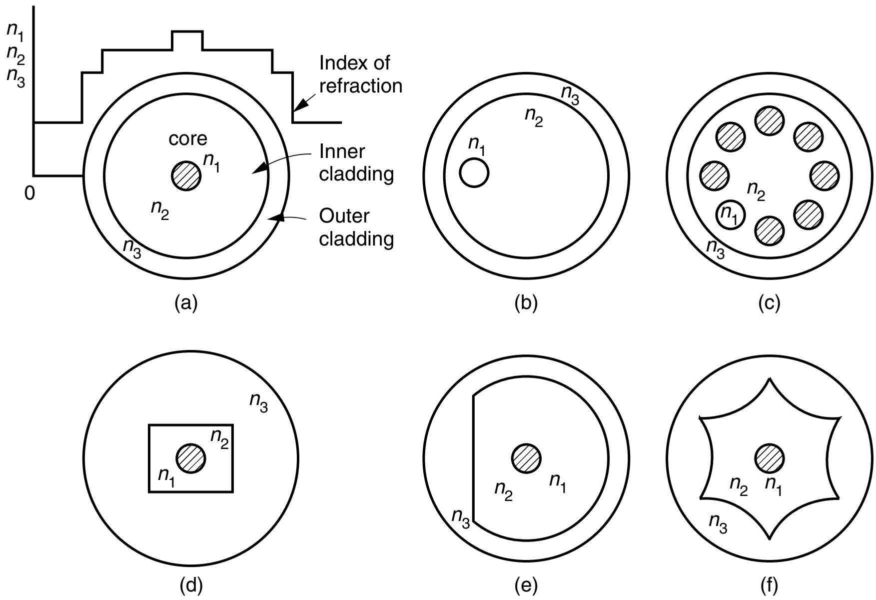


Figure 13.16 Various geometries of a double-clad fiber for high-power fiber amplifiers.

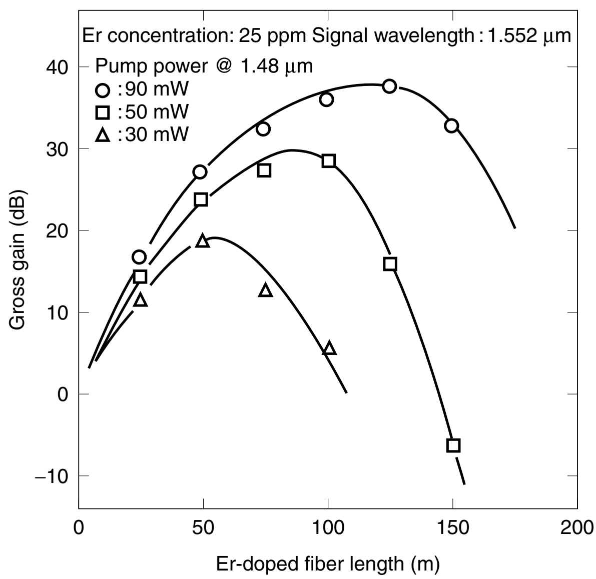


Figure 13.17 Signal gain characteristics versus EDFA length. (After K. Nakagawa et al. [7].)

13.9 OPTIMUM LENGTH OF THE FIBER

Figure 13.17 [7] shows experimental results for the gain of an erbium-doped fiber amplifier as a function of fiber length. The optimum length for maximum gain depends on the pump light power. With an increase in the pump power, the optimum length as well as the maximum gain increases.

If the length of the fiber is too long, there is a region of the fiber where the pump power is too small, the signal has reached saturation intensity, and the gain is decreased.

The ASE noise should also be a consideration when choosing the fiber length. As the fiber length increases, the ASE noise keeps on increasing while the signal is saturated until the ASE noise becomes so large that the ASE noise is the same power level as the signal. From these considerations, the ideal fiber length is in the range of 20–150 meters.

The major applications of the EDFA to fiber-optic communication systems are as a preamplifier to the photodiode detector and as relay amplifiers to stretch the distance of the communication systems. The next two sections will be devoted to noise considerations when the EDFA is used for these applications.

13.10 ELECTRIC NOISE POWER WHEN THE EDFA IS USED AS A PREAMPLIFIER

The electronic noise power when the optical amplifier is used as a preamplifier [4,12,13] to the photodiode detector will be calculated. The layout of the system is shown in Fig. 13.18.

When the incident light to the photodiode detector has already been contaminated by ASE noise, the output current from the detector suffers from excess noise over and above the shot noise and thermal noise of the detector, and the sensitivity of the receiver suffers.

Figure 13.19 shows the measured ASE noise, and Fig. 13.20 compares the size of the frequency bandwidth of the ASE noise $\Delta\nu_t$, the optical band-pass filter passband width $\Delta\nu_f$, and the electronic signal bandwidth $2B$.

In order to reduce the contribution of the ASE noise, a band-pass optical filter is installed at the end of the erbium-doped fiber, as shown in Fig. 13.18. If the ASE noise is uniformly spread over $\Delta\nu_t$, the light output P from the EDFA through the optical filter with passband $\Delta\nu_f$ is, from Eq. (13.36),

$$P = GP_s + GP_{\mathrm{ASE}} \tag{13.90}$$

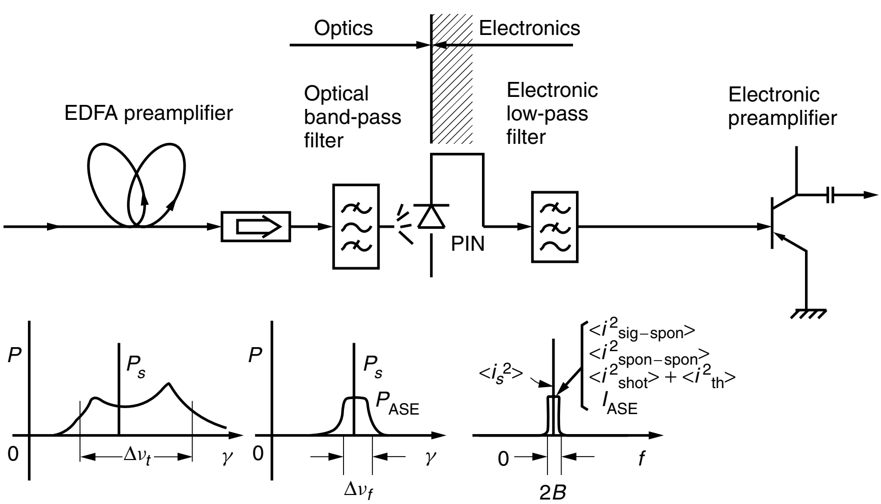


Figure 13.18 Receiver using EDFA as a preamplifier.

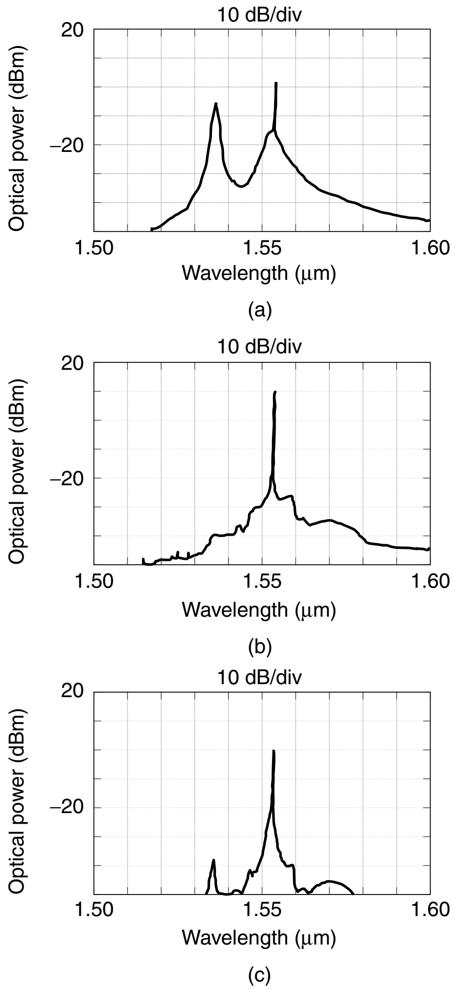


Figure 13.19 Noise spectrum of EDFA output. (a) Unsaturated region (input signal, −27.3 dBm). (b) Saturated region (input signal, −4.1 dBm). (c) Using 1-nm filter (input signal, −27.3 dBm). (After K. Nakagawa et al. [7].)

where

$$P_{\mathrm{ASE}} = m_t n_{\mathrm{spon}}\ h\nu\Delta\nu_f \tag{13.91}$$

$$G - 1 \doteqdot G \tag{13.92}$$

The output from the photodiode when the output P of Eq. (13.90) is fed to the photodiode will be calculated. The amplitude E_s and power GP_s of the light are related by

$$GP_s = \frac{1}{2}\frac{E_s^2}{\eta_0}s \tag{13.93}$$

where η_0 is the intrinsic impedance of air but not that of the photodiode medium (the definition of the quantum efficiency has already accounted for the reflection at the air

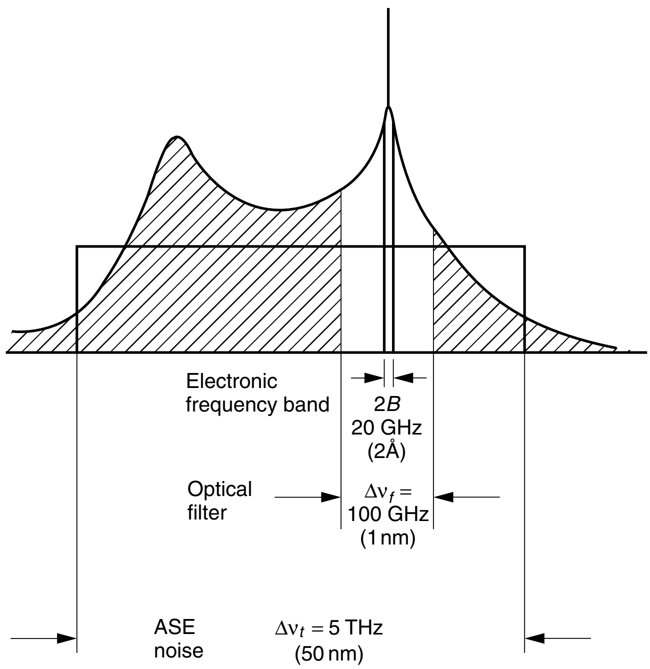


Figure 13.20 Relative frequency ranges compared: ASE noise, optical filter, and electronic frequency band.

and photodiode boundary in Section 12.4.2). The surface area of the photodiode is s. The amplitude of the input light signal into the photodiode is

$$E_s = \sqrt{\frac{2\eta_0}{s} GP_s} \tag{13.94}$$

If the assumption is made that the ASE noise spectrum can be represented as N discrete line spectra with equal powers of GP_{ASE}/N, as shown in Fig. 13.21, then the amplitude of each noise spectrum is

$$E_n = \sqrt{\frac{2\eta_0}{s} GP_{\text{ASE}}/N} \tag{13.95}$$

The photodiode plays the role of the mixer between the signal and the ASE noise. The calculation method is the same as that of heterodyne detection (see Section 12.6) but with the ASE noise as the local oscillator. Using Eqs. (12.17) and (12.18), the sinusoidal wave expression for the instantaneous output electrical current $i(t)$ from a PIN photodiode is

$$i(t) = \frac{\eta e}{h\nu} \frac{s}{\eta_0} \left(E_s \cos(2\pi\nu_s t + \phi_s) + \sum_{n=1}^{N} E_n \cos(2\pi\nu_n t + \phi_n) \right)^2 \tag{13.96}$$

The first term is the signal and the second term is the sum of the N discrete ASE noise spectrum lines. The square operation creates a number of different beat frequencies that belong to the following three categories:

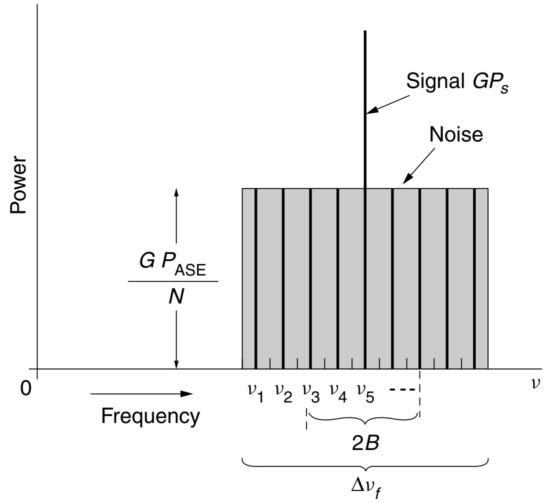


Figure 13.21 Division of continuous ASE noise spectrum into discrete ASE noise line spectra.

1. Signal current.
2. Noise current due to the beats between the signal and the ASE noise whose beat frequencies fall within the electronic frequency bandwidth of the detector. This noise is called signal–spontaneous beat noise.
3. Noise current due to the beats among the line spectra of the ASE noise whose beat frequencies fall within the electronic frequency bandwidth of the detector. This noise is called spontaneous–spontaneous beat noise.

Let us perform some calculations.

(1) The signal current $i(t)$ is

$$i(t) = \frac{\eta e}{h\nu}\frac{s}{\eta_0}E_s^2\cos^2(2\pi\nu_s t + \phi_s) \tag{13.97}$$

$$= \frac{\eta e}{h\nu}\frac{s}{\eta_0}E_s^2\tfrac{1}{2}\left[1 + \cos 2(2\pi\nu_s t + \phi_s)\right] \tag{13.98}$$

The photodiode is not sensitive to the $2\nu_s$ frequency component and the second term of Eq. (13.98) can be ignored. From Eq. (13.94), Eq. (13.98) becomes

$$i(t) = \frac{\eta e}{h\nu}GP_s \tag{13.99}$$

The electrical signal power s thus becomes

$$S = \left(\frac{\eta e}{h\nu}GP_s\right)^2 R_L \tag{13.100}$$

where R_L is the load resistance of the photodiode.

(2) Next, the signal–spontaneous beat noise $i_{\text{sig-spon}}$ is considered. By treating each noise spectrum as a local oscillator, the output current due to the beat between the signal and only the nth noise spectrum is derived from Eq. (13.96).

$$
\begin{aligned}
i_{\text{sig-spon}}(t) &= 2\frac{\eta e}{h\nu}\frac{s}{\eta_0}E_sE_n\sum_{n=1}^{N}\cos(2\pi\nu_s t+\phi_s)\cos(2\pi\nu_n t+\phi_n) \\
&= \frac{\eta_e}{h\nu}\frac{s}{\eta_0}E_sE_n\left(\sum_{n=1}^{N}\cos\left[2\pi(\nu_n-\nu_s)t+\phi_n-\phi_s\right]\right. \\
&\quad \left.+\sum_{n=1}^{N}\cos\left[2\pi(\nu_n+\nu_s)t+\phi_n+\phi_s\right]\right)
\end{aligned}
\tag{13.101}
$$

The second term in the large parentheses is outside the sensitivity range of the photodiode and the output current components from the photodiode are

$$
i_{\text{sig-spon}}(t) = \frac{\eta e}{h\nu}\frac{s}{\eta_0}E_sE_n\sum_{n=1}^{N}\cos\left[2\pi(\nu_n-\nu_s)t+\phi_n-\phi_s\right] \tag{13.102}
$$

$i_{\text{sig-spon}}(t)$ has N discrete spectra that are the beats made by ν_n and ν_s, and they are the components that can pass through the electronic filter. From Eqs. (13.94) and (13.95), the kth frequency current is

$$
\begin{aligned}
i_{\text{sig-spon},k}(t) &= 2\frac{\eta e}{h\nu}\sqrt{GP_s}\sqrt{GP_{\text{ASE}}/N} \\
&\quad \times \cos\left[2\pi(\nu_k-\nu_s)t+\phi_k-\phi_s\right]
\end{aligned}
\tag{13.103}
$$

The number of frequency components of $i_{\text{sig-spon}}(t)$ in Eq. (13.102) that are within $-B$ to B and get through the electronic preamplifier is

$$
N' = N\frac{2B}{\Delta\nu_f} \tag{13.104}
$$

Each spectrum has the same magnitude, and the total of the time averages of the beat current square is N' times and

$$
\langle i^2_{\text{sig-spon}}\rangle = 4\left(\frac{\eta e}{h\nu}\right)^2 GP_sGP_{\text{ASE}}\frac{B}{\Delta\nu_f} \tag{13.105}
$$

The current $\langle i^2_{\text{sig-spon}}\rangle$ is proportional to the product of the output signal power and the ASE noise.

(3) Next, the spontaneous–spontaneous beat noise $i_{\text{spon-spon}}(t)$ is calculated. The noise current due to the jth and kth ASE noise line spectra is obtained from the square of the second term in Eq. (13.96), suppressing ϕ_n:

$$
\begin{aligned}
i_{\text{spon-spon}}(t) &= \frac{\eta e}{h\nu}\frac{s}{\eta_0}\left(E_1\cos 2\pi\nu_1 t+E_2\cos 2\pi\nu_2 t+E_3\cos 2\pi\nu_3 t+\cdots\right) \\
&\quad \times \left(E_1\cos 2\pi\nu_1 t+E_2\cos 2\pi\nu_2 t+E_3\cos 2\pi\nu_3 t+\cdots\right)
\end{aligned}
\tag{13.106}
$$

The result of the multiplication in Eq. (13.106) will be grouped by the frequency components.

$$
\begin{aligned}
i_{\text{spon-spon}}(t) = \frac{\eta e}{h\nu}\frac{s}{\eta_0}
&\left[
\overset{\text{Zeroth group}}{\begin{pmatrix}
E_1^2 \cos^2 2\pi\nu_1 t \\ + \\ E_2^2 \cos^2 2\pi\nu_2 t \\ + \\ E_3^2 \cos^2 2\pi\nu_3 t \\ + \\ \vdots \\ E_N^2 \cos^2 2\pi\nu_N t
\end{pmatrix}}
+ 2
\overset{\text{First group}}{\begin{pmatrix}
E_1E_2 \cos 2\pi\nu_1 t \cos 2\pi\nu_2 t \\ + \\ E_2E_3 \cos 2\pi\nu_2 t \cos 2\pi\nu_3 t \\ + \\ E_3E_4 \cos 2\pi\nu_3 t \cos 2\pi\nu_4 t \\ + \\ \vdots \\ E_{N-1}E_N \cos 2\pi\nu_{N-1} t \cos 2\pi\nu_N t
\end{pmatrix}}
\right. \\
&\left.
+2
\overset{\text{Second group}}{\begin{pmatrix}
E_1E_3 \cos 2\pi\nu_1 t \cos 2\pi\nu_3 t \\ + \\ E_2E_4 \cos 2\pi\nu_2 t \cos 2\pi\nu_4 t \\ + \\ E_3E_5 \cos 2\pi\nu_3 t \cos 2\pi\nu_5 t \\ + \\ \vdots \\ E_{N-2}E_N \cos 2\pi\nu_{N-2} t \cos 2\pi\nu_N t
\end{pmatrix}}
+ \cdots \right]
\end{aligned}
\tag{13.107}
$$

Let the kth term of the zeroth group be

$$
{}_0i_{\text{spon-spon},k}(t) = \frac{\eta e}{h\nu}\frac{s}{\eta_0}E_k^2\tfrac{1}{2}(1 + \cos 4\pi\nu_k t) \tag{13.108}
$$

Terms of this group create dc and $2\nu_k$ frequency components. The $2\nu_k$ frequency components are outside the sensitivity of the photodiode and can be discarded. Thus, there are N terms in this group, all of which have the same magnitude, and using Eqs. (13.95) and (13.96), the total dc current is

$$
{}_0i_{\text{spon-spon}}(t) = \frac{\eta e}{h\nu}\,(GP_{\text{ASE}}/N)N \tag{13.109}
$$

or

$$
{}_0i_{\text{spon-spon}}(t) = GI_{\text{ASE}} \tag{13.110}
$$

where

$$
I_{\text{ASE}} = \frac{\eta e}{h\nu}P_{\text{ASE}} \tag{13.111}
$$

Note that $i_{\text{spon-spon}}(t)$ is independent of time and the contribution of I_{ASE} as noise is indirect in that an increase in the current flow through the photodiode increases the amount of the shot noise of the photodiode (Eq. (16.26)).

Next, the terms in the first group in Eq. (13.107) are considered. Frequencies of these components are all at one quantized frequency, $\Delta\nu_f/N$. For instance, the first term is

$$
\begin{aligned}
{}_1i_{\mathrm{spon-spon},1}(t) &= \frac{\eta e}{h\nu}\frac{s}{\eta_0} 2E_1E_2 \cos 2\pi\nu_1 t \cos 2\pi\nu_2 t \\
&= \frac{\eta e}{h\nu}\frac{s}{\eta_0} 2E_1E_2\ \frac{1}{2}\left(\cos 2\pi\frac{\Delta\nu_f}{N}t + \cos 2\pi(\nu_1+\nu_2)t\right)
\end{aligned}
\tag{13.112}
$$

Again, using the same reason of the insensitivity of the photodiode, the term with $\nu_1 + \nu_2$ is ignored.

$$
{}_1i_{\mathrm{spon-spon},1}(t) = \frac{\eta e}{h\nu}\frac{s}{\eta_0} E_1E_2 \cos 2\pi\frac{\Delta\nu_f}{N}t \tag{13.113}
$$

Using Eq. (13.95) gives

$$
{}_1i_{\mathrm{spon-spon},1}(t) = 2\frac{\eta e}{h\nu}(GP_{\mathrm{ASE}}/N) \cos 2\pi\frac{\Delta\nu_f}{N}t \tag{13.114}
$$

and its current square component is

$$
\langle {}_1i^2_{\mathrm{spon-spon}} \rangle = \left(2\frac{\eta e}{h\nu}GP_{\mathrm{ASE}}/N\right)^2 \left(\frac{1}{2}\right) \tag{13.115}
$$

There are $N - 1$ such terms in the first group and the total current square of the first quantized frequency is

$$
\langle {}_1i^2_{\mathrm{spon-spon}} \rangle = 2\left(\frac{\eta e}{h\nu}GP_{\mathrm{ASE}}/N\right)^2 (N-1) \tag{13.116}
$$

Next, the terms whose frequency is the second quantized frequency, $2\Delta\nu_f/N$, are in the second group in Eq. (13.107). The first term in the second group is

$$
{}_2i_{\mathrm{spon-spon},1}(t) = \frac{\eta e}{h\nu}\frac{s}{\eta_0} 2E_1E_3 \cos 2\pi\nu_1 t \cos 2\pi\nu_3 t \tag{13.117}
$$

If the term with $\nu_1 + \nu_3$ is ignored because of the absence of sensitivity of the photodiode, then

$$
{}_2i_{\mathrm{spon-spon},1}(t) = 2\frac{\eta e}{h\nu}(GP_{\mathrm{ASE}}/N) \cos 4\pi\frac{\Delta\nu_f}{N}t \tag{13.118}
$$

and its current square component is

$$
\langle {}_2i^2_{\mathrm{spon-spon},1} \rangle = 2\left(\frac{\eta e}{h\nu}GP_{\mathrm{ASE}}/N\right)^2 \tag{13.119}
$$

There are $N - 2$ such terms in this group, and the total current square of the second quantized frequency, $2\Delta\nu_f/N$ is

$$
\langle {}_2i^2_{\mathrm{spon-spon}} \rangle = 2\left(\frac{\eta e}{h\nu}GP_{\mathrm{ASE}}/N\right)^2 (N-2) \tag{13.120}
$$

Similarly, the total current square of the third quantized frequency, $3\Delta\nu_f/N$, is

$$\langle {}_3 i^2_{\text{spon-spon}} \rangle = 2\left(\frac{\eta e}{h\nu} G P_{\text{ASE}}/N\right)^2 (N-3) \tag{13.121}$$

Comparing Eqs. (13.116), (13.120), and (13.121), the spectral density distribution of the spontaneous–spontaneous beat noise in the output current from the detector linearly decreases with the quantized frequency as long as the quantized frequency is less than B. It will, however, become zero as soon as the quantized frequency exceeds the electronic filter cutoff frequency.

Since each quantized frequency step is $\Delta\nu_f/N$, the number of frequency steps to reach B is

$$\frac{B}{\Delta\nu_f/N} = N\frac{B}{\Delta\nu_f} \tag{13.122}$$

Thus, the grand total of the spontaneous-to-spontaneous beat current square is

$$\langle i^2_{\text{spon-spon}} \rangle = 2\left(\frac{\eta e}{h\nu} P_{\text{ASE}}/N\right)^2 \times \left(N + (N-1) + (N-2) + (N-3) + \cdots + \left(N - N\frac{B}{\Delta\nu_f}\right)\right) \tag{13.123}$$

Using the expression for the sum of an arithmetic series,

$$\langle i^2_{\text{spon-spon}} \rangle = 2\left(\frac{\eta e}{h\nu} G P_{\text{ASE}}/N\right)^2 \frac{\left(N + N - N\frac{B}{\Delta\nu_f}\right)\left(N\frac{B}{\Delta\nu_f}\right)}{2} \tag{13.124}$$

From Eq. (13.91), the final expression becomes

$$\langle i^2_{\text{spon-spon}} \rangle = \left(\frac{\eta e}{h\nu} G P_{\text{ASE}}\right)^2 \left(2 - \frac{B}{\Delta\nu_f}\right)\frac{B}{\Delta\nu_f} \tag{13.125}$$

Summarizing all beat noises gives

$$\begin{aligned}
\langle i^2_{\text{sig-spon}} \rangle &= 4GI_s \cdot GI_{\text{ASE}} \cdot \frac{B}{\Delta\nu_f} \\
\langle i^2_{\text{spon-spon}} \rangle &= G^2 I^2_{\text{ASE}} \left(2 - \frac{B}{\Delta\nu_f}\right)\frac{B}{\Delta\nu_f} \\
{}_0 i_{\text{spon-spon}}(t) &= GI_{\text{ASE}}
\end{aligned} \tag{13.126}$$

where

$$I_s = \frac{\eta e}{h\nu} P_s \quad \text{and} \quad I_{\text{ASE}} = \frac{\eta e}{h\nu} P_{\text{ASE}}$$

It should be noted that I_s is the photocurrent from the photodiode with an optical amplifier gain of $G = 1$, and similarly I_{ASE} is the ASE current with $G = 1$.

Based on the results of the beat noises summarized in Eq. (13.126), the signal to noise ratio (S/N) of a receiver with an optical amplifier used as a preamplifier, as shown in Fig. 13.18, will be found. Besides the beat noise given by Eq. (13.126), the shot noise and thermal noise from the photodiode have to be included. The shot noise of the PIN diode is caused by the irregularity of the electron flow in the diode, and the shot noise power N_{shot} is given from Eq. (16.26) by

$$N_{\text{shot}} = 2eI_tBR_L \tag{13.127}$$

where e is the electron charge; I_t is the total average current including the signal current GI_s, the ASE noise current GI_{ASE}, and the dark current I_d of the PIN diode; B is the bandwidth of the noise power subjected to the electronic preamplifier; and R_L is the load resistance of the PIN diode (see Chapter 16). The thermal noise is emitted from any medium that has nonzero Kelvin temperature. The thermal noise is given from Eq. (16.27) by

$$N_{\text{th}} = \frac{4kTB}{R_L}R_L \tag{13.128}$$

where k is the Boltzmann constant, and T is the absolute temperature in kelvin.

The electrical S/N of the output from the PIN photodiode shown in Fig. 13.18 is, from Eqs. (13.126), (13.127), and (13.128),

$$\frac{\text{S}}{\text{N}} = \frac{(GI_s)^2}{4G^2 I_s I_{\text{ASE}}\dfrac{B}{\Delta\nu_f} + G^2 I_{\text{ASE}}^2\left(2 - \dfrac{B}{\Delta\nu_f}\right)\dfrac{B}{\Delta\nu_f} + 2e[G(I_s + I_{\text{ASE}}) + I_d]B + \dfrac{4kTB}{R_L}} \tag{13.129}$$

Dividing both denominator and numerator by G^2, the S/N becomes

$$\frac{\text{S}}{\text{N}} = \frac{I_s^2}{4I_s I_{\text{ASE}}\dfrac{B}{\Delta\nu_f} + I_{\text{ASE}}^2\left(2 - \dfrac{B}{\Delta\nu_f}\right)\dfrac{B}{\Delta\nu_f} + 2e\left(\dfrac{1}{G}(I_s + I_{\text{ASE}}) + \dfrac{I_d}{G^2}\right)B + \dfrac{4kTB}{G^2 R_L}} \tag{13.130}$$

As seen from Eq. (13.130), as G is increased, the contribution of the thermal noise is diminished the most, and then that of the shot noise, while the contributions of the signal–spontaneous noise and the spontaneous–spontaneous noise are left unchanged. For large G, the S/N becomes

$$\frac{\text{S}}{\text{N}} = \frac{I_s}{\left[4 + \dfrac{I_{\text{ASE}}}{I_s}\left(2 - \dfrac{B}{\Delta\nu_f}\right)\right] I_{\text{ASE}}\dfrac{B}{\Delta\nu_f}} \tag{13.131}$$

If the input photocurrent I_s is maintained much larger than I_{ASE}, the ultimate S/N value of the receiver when the optical amplifier is used as an optical preamplifier is

$$\frac{\text{S}}{\text{N}} = \frac{I_s}{4I_{\text{ASE}}\dfrac{B}{\Delta\nu_f}} \tag{13.132}$$

where use was made of

$$G \gg 1 \tag{13.133}$$

$$I_s \gg I_{\text{ASE}} \tag{13.134}$$

In terms of the light power using Eqs. (13.91) and (13.126), Eq. (13.132) can be rewritten as

$$\frac{\text{S}}{\text{N}} = \frac{P_s}{4m_t n_{\text{spon}}\ h\nu B} \tag{13.135}$$

and Eq. (13.134) is rewritten as

$$P_s \gg m_t n_{\text{spon}}\ h\nu\ \Delta\nu_f \tag{13.136}$$

Before closing, Eq. (13.131) will be rewritten in terms of light powers, using Eqs. (13.91) and (13.126), as

$$\frac{\text{S}}{\text{N}} = \frac{P_s}{\left[4 + \dfrac{m_t n_{\text{spon}}\ h\nu\ \Delta\nu_f}{P_s}\left(2 - \dfrac{B}{\Delta\nu_f}\right)\right] m_t n_{\text{spon}}\ h\nu B} \tag{13.137}$$

In conclusion, in order to optimize the performance of the optical amplifier as a preamplifier, a high input light power has to be available. In other words, when this detection system is used for fiber-optic communication, the distance between the transmitter and the receiver should be limited such that the input power P_s to the optical amplifier is high enough not only to satisfy Eq. (13.136) but also to achieve the required S/N given by Eq. (13.135).

The value of P_s required for Eq. (13.136) to be satisfied can be reduced by reducing a few quantities. These are the number of modes m_t, $\Delta\nu_f$ of the optical filter, and the population ratio n_{spon}. Since $m_t = 2$ for a circularly polarized wave but $m_t = 1$ for a linearly polarized wave, m_t can be reduced by introducing a polarizer to permit only a linearly polarized wave. The quantity n_{spon} can be reduced by raising the pumping light level.

Example 13.6 Calculate the ASE noise power from the EDFA of Example 13.5 when the wavelength bandwidth of the optical filter is 0.1 nm and $m_t = 2$.

Solution From Eq. (13.36), the ASE noise power output with $m_t = 2$ is

$$GP_{\text{ASE}} = 2n_{\text{spon}}(G - 1)h\nu\ \Delta\nu_f$$

The expression for n_{spon} is found in Eq. (13.70):

$$n_{\text{spon}} = \frac{W_p + W_s}{W_p(1 - \beta) - 1/\tau}$$

From the results of Example 13.5, W_p and W_s are

$$W_p = 793\ \text{s}^{-1} \quad \text{and} \quad W_s = 88.9\ \text{s}^{-1}$$

The frequency interval is

$$\Delta\nu = \frac{3 \times 10^{8}}{10^{-6}}\left(\frac{1}{1.55} - \frac{1}{1.5501}\right) = 1.25 \times 10^{10}\ \text{Hz}$$

$$= 12.5\ \text{GHz}$$

$$n_{\text{spon}} = \frac{793 + 88.9}{(793)(1 - 0.38) - 100} = 2.25$$

$$GP_{\text{ASE}} = 2(1556 - 1)(2.25)(6.63 \times 10^{-34})(1.94 \times 10^{14})(1.25 \times 10^{10})$$

$$= 11.3 \times 10^{-6}\ \text{W} = 11.3\ \mu\text{W}$$

which is a significant amount compared with the signal output of $P_s = 1$ mW. □

Example 13.7 As shown in Fig. 13.22, an EDFA is used as an optical preamplifier followed by a PIN photodiode to demonstrate the improvement of its S/N over the case when a PIN photodiode alone is used as a detector.

(a) Compare the values of the various components of the noise current squares.
(b) Find the S/N with and without the EDFA preamplifier.

The following parameters apply:

Input light power, $P_s = 3.2\ \mu\text{W}$
Wavelength, $\lambda = 1.55\ \mu\text{m}$
Bandwidth of optical filter, $\Delta\nu_f = 12.4$ GHz
Gain of EDFA, $G = 1097$
Number of orthogonal modes, $m_t = 2$
Population inversion factor, $n_{\text{spon}} = 2.25$
Responsivity of the PIN diode, $\eta e/h\nu = 0.5$
Dark current of the PIN diode, $I_d \doteqdot 0$
Load impedance to the PIN diode, $R_L = 50\ \Omega$
Room temperature, $T = 300$ K
Bandwidth of electrical signal, $B = 6.2$ GHz

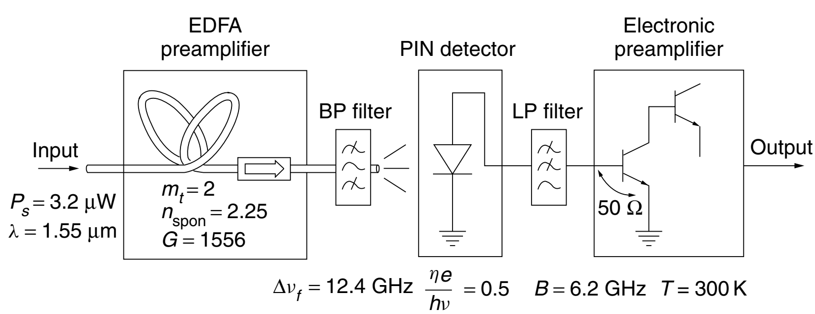


Figure 13.22 Physical layout when an EDFA is used as an optical preamplifier.

Solution The signal to noise ratio (S/N) of the receiver with the EDFA as an optical preamplifier is given by Eq. (13.130). The quantities needed to calculate the S/N are computed below.

$$I_s = \frac{\eta e}{h\nu} P_s = (0.5)(3.2 \times 10^{-6}) = 1.6 \times 10^{-6} A$$

$$\begin{aligned}
P_{\mathrm{ASE}} &= m_t n_{\mathrm{spon}}\, h\nu\, \Delta\nu_f \\
&= (2)(2.25)(6.63 \times 10^{-34})(1.94 \times 10^{14})(1.24 \times 10^{10}) \\
&= 7.18 \times 10^{-9}\ \mathrm{W}
\end{aligned}$$

$$\begin{aligned}
I_{\mathrm{ASE}} = \frac{\eta e}{h\nu} P_{\mathrm{ASE}} &= (0.5)(7.18 \times 10^{-9}) \\
&= 3.59 \times 10^{-9} \mathrm{A}
\end{aligned}$$

$$\frac{B}{\Delta\nu_f} = \frac{6.2 \times 10^9}{12.4 \times 10^9} = \frac{1}{2}$$

$$\begin{aligned}
4\left(I_s I_{\mathrm{ASE}} \frac{B}{\Delta\nu_f}\right) &= 4(1.6 \times 10^{-6})(3.59 \times 10^{-9})\left(\tfrac{1}{2}\right) \\
&= 1.15 \times 10^{-14}\ \mathrm{A}^2
\end{aligned}$$

$$I_{\mathrm{ASE}}^2 \left(2 - \frac{B}{\Delta\nu_f}\right) \frac{B}{\Delta\nu_f} = (3.59 \times 10^{-9})^2 \left(2 - \tfrac{1}{2}\right)\left(\tfrac{1}{2}\right) = 9.67 \times 10^{-18}\ \mathrm{A}^2$$

$$\begin{aligned}
\frac{2e}{G}(I_s + I_{\mathrm{ASE}})B &= \frac{2 \times 1.6 \times 10^{-19}}{1097}(1.6 \times 10^{-6} + 3.59 \times 10^{-9})(6.2 \times 10^9) \\
&= 2.9 \times 10^{-18}\ \mathrm{A}^2
\end{aligned}$$

$$\begin{aligned}
\frac{4kTB}{R_L} &= \frac{4(1.38 \times 10^{-23})(300)(6.2 \times 10^9)}{50} \\
&= 2.06 \times 10^{-12}\ \mathrm{A}^2
\end{aligned}$$

$$\frac{4kTB}{G^2 R_L} = \frac{2.06 \times 10^{-12}}{(1097)^2} = 1.7 \times 10^{-18}\ \mathrm{A}^2$$

Substituting the computed values into Eq. (13.130) gives

$$\begin{aligned}
\frac{\mathrm{S}}{\mathrm{N}} &= \frac{I_s^2}{4 I_s I_{\mathrm{ASE}} \dfrac{B}{\Delta\nu_f} + I_{\mathrm{ASE}}^2 \left(2 - \dfrac{B}{\Delta\nu_f}\right) \dfrac{B}{\Delta\nu_f} + \dfrac{2e}{G}\left(I_s + I_{\mathrm{ASE}} + \dfrac{I_d}{G}\right) B + \dfrac{4kTB}{G^2 R_L}} \\
&= \frac{(1.6 \times 10^{-6})^2}{1.15 \times 10^{-14} + 9.67 \times 10^{-18} + 2.9 \times 10^{-18} + 1.7 \times 10^{-18}} = 223
\end{aligned}$$

Thus, the magnitude of the signal–spontaneous beat noise is significantly larger than the spontaneous–spontaneous beat noise, the shot noise, and the thermal noise.

Next, the S/N without the optical preamplifier is calculated:

$$\frac{\mathrm{S}}{\mathrm{N}} = \frac{I_s^2}{2eI_sB + \dfrac{4kTB}{R_L}}$$

$$= \frac{(1.6 \times 10^{-6})^2}{(2)(1.6 \times 10^{-19})(1.6 \times 10^{-6})(6.2 \times 10^9) + 2.06 \times 10^{-12}}$$

$$= \frac{2.56 \times 10^{-12}}{3.17 \times 10^{-15} + 2.06 \times 10^{-12}} = 1.25$$

The S/N is improved by a factor of nearly 200 by inserting the EDFA preamplifier. □

13.11 NOISE FIGURE OF THE RECEIVER USING THE OPTICAL AMPLIFIER AS A PREAMPLIFIER

The signal to noise ratio S/N and the noise figure F [4,12–14] are among the most important considerations in designing an optical communication system. The optical S/N, which is the ratio between the first and second terms of Eq. (13.90), is seldom used. The output S/N normally means the ratio between the signal current square to the total noise current square if the light is detected by a photodiode detector.

Next, the noise figure of the amplifier will be derived. The noise performance of the amplifier is quantified by the noise figure F defined as the ratio of input and output signal to noise ratios:

$$F = \frac{(\mathrm{S/N})_i}{(\mathrm{S/N})_o} \tag{13.138}$$

The output signal to noise ratio $(\mathrm{S/N})_o$ of the amplifier can be obtained immediately by dividing the input signal to noise ratio $(\mathrm{S/N})_i$ by the noise figure F. The noise figure F is sometimes expressed in dB as

$$10\log(\mathrm{S/N})_o = 10\log(\mathrm{S/N})_i - 10\log F \tag{13.139}$$

10 log F is a measure of the degradation of the output $(\mathrm{S/N})_o$ from that of the input. The smaller F is, the better.

The output signal to noise ratio $(\mathrm{S/N})_o$ of the EDFA is the one that is measured at the output side of the EDFA, as shown in Fig. 13.23a, using an ideal photodiode whose shot noise is the only source of noise. The input signal to noise ratio $(\mathrm{S/N})_i$ is the one that is measured at the input side of the EDFA, as shown in Fig. 13.23b, disconnecting the EDFA preamplifier and using the same ideal photodetector. The output current square from such a detector is

$$\langle i_s^2 \rangle = \left(\frac{\eta e}{h\nu} P_s\right)^2 = I_s^2 \tag{13.140}$$

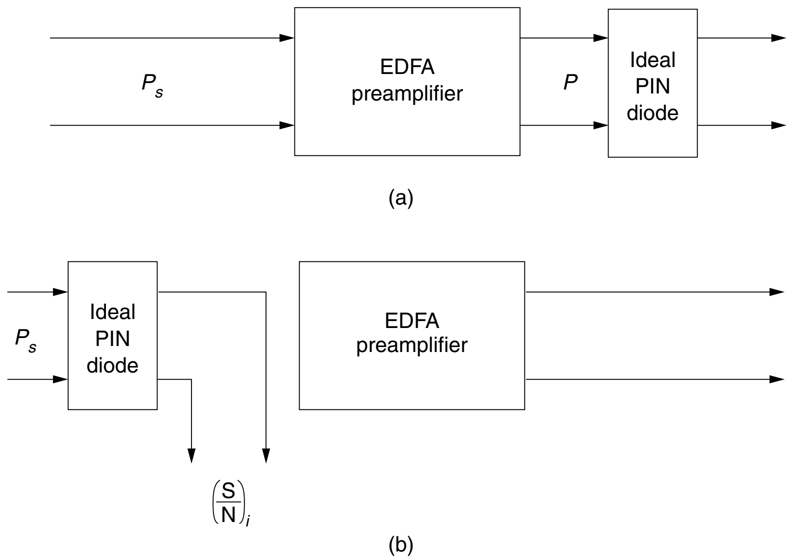


Figure 13.23 Method of measuring the noise figure of the EDFA: $F = (S/N)_i/(S/N)_o$. (a) S/N with the EDFA preamplifier. (b) S/N without the EDFA preamplifier.

The shot noise is

$$\langle i_{\text{shot}}^2 \rangle = 2e\frac{\eta e}{h\nu}P_s B = 2eI_s B \tag{13.141}$$

The input signal to noise ratio is

$$\left(\frac{\text{S}}{\text{N}}\right)_i = \frac{I_s}{2eB} \tag{13.142}$$

From Eqs. (13.131) and (13.142), F is

$$F = \frac{(\text{S/N})_i}{(\text{S/N})_o} = \left[4 + \frac{I_{\text{ASE}}}{I_s}\left(2 - \frac{B}{\Delta\nu_f}\right)\right]\frac{I_{\text{ASE}}}{2e\Delta\nu_f} \tag{13.143}$$

or in terms of the light power

$$F = \frac{\eta}{2}\left[4 + \frac{m_t n_{\text{spon}}\, h\nu\, \Delta\nu_f}{P_s}\left(2 - \frac{B}{\Delta\nu_f}\right)\right] m_t\, n_{\text{spon}} \tag{13.144}$$

The value of the second term in the square brackets can be small compared to the first term. For example, when $m_t = 1$, $n_{\text{spon}} = 3$, $\Delta\nu_f = 100$ GHz, $P_s = -30$ dBm, and $\lambda = 1.55$ μm, then

$$n_{\text{spon}}\, h\nu\, \Delta\nu_f/P_s = (3)(6.63\times 10^{-34}\times 1.94\times 10^{14})(10^{11})/(10^{-6}) = 0.039 \tag{13.145}$$

If this same example is computed with $P_s = -50$ dBm, the second term becomes significant compared to the first term, and hence, small values of P_s should be avoided. As for the value of $n_{\text{spon}} = N_2/(N_2 - N_1)$, it can be made close to unity by pumping hard.

13.12 A CHAIN OF OPTICAL AMPLIFIERS

Optical amplifier chains [4,15–18] are used in optical trunk transmission cables and transoceanic optical submarine cables. Special attention should be paid to the accumulation of ASE noise from the concatenated optical amplifiers.

Figure 13.24 shows a segment of chained optical amplifiers in a trunk line together with an optical signal level diagram. It is assumed that after each amplifier the received signal is amplified to the same output level as that of the previous amplifier. This is accomplished by setting the gain of the amplifier to the same value as the fiber transmission loss between the amplifiers. The distribution of optical power along the line looks like a sawtooth function.

From Eq. (13.36), the output from the first amplifier with an optical filter with bandwidth $\Delta\nu_f$ is

$$P_1 = GP_s + (G-1)m_t n_{\text{spon}}\, h\nu\, \Delta\nu_f \tag{13.146}$$

Only the last amplifier is followed by a detector, so that the beat electrical current noise is accounted for following the last stage. After the attenuation L due to the fiber transmission loss, the input to the second amplifier is

$$P_{s,2} = LGP_s + L(G-1)m_t n_{\text{spon}}\, h\nu\, \Delta\nu_f \tag{13.147}$$

The power is assumed to recover to its original level at each amplifier:

$$GL = 1 \tag{13.148}$$

With Eq. (13.148) and $G \gg 1$, Eq. (13.147) becomes

$$P_{s,2} = P_s + m_t n_{\text{spon}}\, h\nu\, \Delta\nu_f \tag{13.149}$$

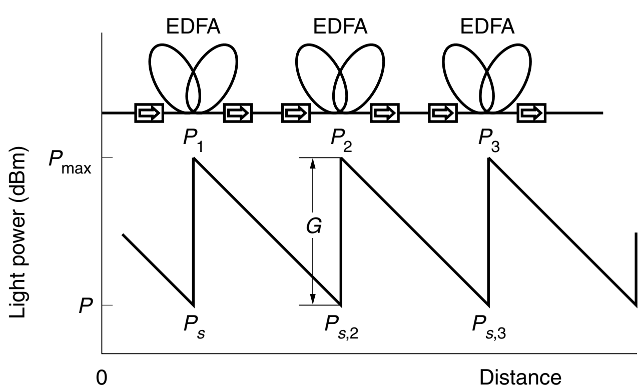


Figure 13.24 Power level diagram of an EDFA chain.

Compared with the input to the first amplifier, the input to the second amplifier contains an ASE noise of $m_t n_{\text{spon}}\ h\nu\ \Delta\nu_f$. The output from the second amplifier is

$$\begin{aligned} P_2 = G(P_s + m_t n_{\text{spon}}\ h\nu\ \Delta\nu_f) \\ + (G-1) m_t n_{\text{spon}}\ h\nu\ \Delta\nu_f \end{aligned} \tag{13.150}$$

With the assumption $G \gg 1$, Eq. (13.150) becomes

$$P_2 = GP_s + 2Gm_t n_{\text{spon}}\ h\nu\ \Delta\nu_f \tag{13.151}$$

After transmission through the kth amplifier, the output power is

$$P_k = GP_s + kGm_t n_{\text{spon}}\ h\nu\ \Delta\nu_f \tag{13.152}$$

If the kth amplifier is followed by another transmission cable with loss L and with a detector at the end, the input power to the detector becomes

$$P_{s,d} = P_s + k\ m_t n_{\text{spon}}\ h\nu\ \Delta\nu_f \tag{13.153}$$

An important conclusion drawn from Eq. (13.153) is that the ASE noise is additive rather than multiplicative. That is, the noise term is $k\ m_t n_{\text{spon}}\ h\nu\Delta\nu_f$, and not $(m_t n_{\text{spon}}\ h\nu\ \Delta\nu_f)^k$. The reason for this is that the ASE noise power, as well as the signal power, is attenuated by L during the transmission in the cable.

Let's evaluate the performance of the chain amplifier combined with a detector at the end. The overall $(\text{S/N})_o$ and noise figure after detection are calculated by replacing n_{spon} by kn_{spon} in Eq. (13.137) and Eq. (13.144), respectively. The noise figure of the k amplifier chain is, therefore,

$$F_k = \frac{\eta}{2}\left[4 + \frac{km_t n_{\text{spon}}\ h\nu\ \Delta\nu_f}{P_s}\left(2 - \frac{B}{\Delta\nu_f}\right)\right] km_t n_{\text{spon}} \tag{13.154}$$

When designing repeater amplifiers in a trunk line, the number k of amplifiers has a significant impact on the overall performance. Suppose that the total distance D of the trunk line and the maximum output power from each amplifier are both fixed. A reduction in k means a reduction of input power P_s to each amplifier, and with a reduction of P_s, the contribution of the spontaneous–spontaneous beat noise of the second term in the square brackets of Eq. (13.154) is increased. There is an optimum number k of amplifiers that minimizes the noise figure.

Example 13.8 Compare the noise figures of the relay amplifiers for the two cases shown in Fig. 13.25. The configuration shown in Fig. 13.25a has higher gain (60 dB) with smaller input power and hence only one amplifier repeater is needed. The configuration shown in Fig. 13.25b has lower gain (30 dB) with larger input power and hence two amplifiers are needed. The maximum signal output in both cases is set at 5 dBm. Fiber transmission loss is 0.6 dB/km. The parameters for each optical amplifier are $\eta = 1$, $m_t = 2$, $n_{\text{spon}} = 3$, $\Delta\nu_f = 100$ GHz, $\lambda = 1.55$ µm, and $B = \Delta\nu_f$.

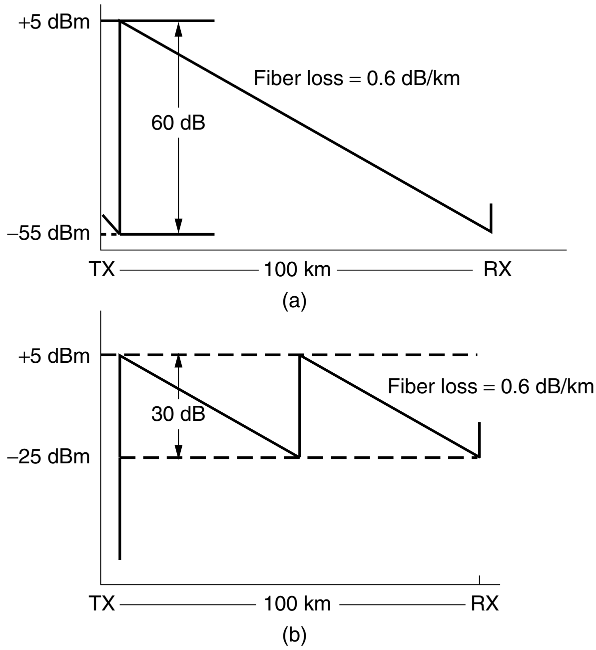


Figure 13.25 Comparison of power level diagrams for two chained amplifier configurations. (a) With 60-dB amplifier, higher gain and longer span, (b) With 30-dB amplifier, lower gain and shorter span.

Solution The input powers for the two cases are

$$P_s = 3.16 \times 10^{-9}\ \mathrm{W} \quad \text{for the 60-dB amplifier}$$
$$P_s = 3.16 \times 10^{-6}\ \mathrm{W} \quad \text{for the 30-dB amplifier}$$

(a) The noise figure F_1 for one amplifier in Fig. 13.25a with the given parameters is

$$\begin{aligned} F_1 &= 2\left[2n_{\text{spon}} + n_{\text{spon}}^2 h\nu\ \Delta\nu_f/P_s\right] \\ &= 2\left[2(3) + (3^2)(6.63 \times 10^{-34})(1.94 \times 10^{14})(10^{11})/3.16 \times 10^{-9}\right] \\ &= 2 \times 6 + 73.3 = 85.3 \end{aligned}$$

The noise figure F_2 for two amplifiers of Fig. 13.25b is

$$\begin{aligned} F_2 &= 2\left[2(2n_{\text{spon}}) + 2^2 n_{\text{spon}}^2 h\nu\ \Delta\nu_f/P_s\right] \\ &= 2\left[2(6) + 2^2(3^2)(6.63 \times 10^{-34})(1.94 \times 10^{14})(10^{11})/3.16 \times 10^{-6}\right] \\ &= 2 \times 12 + 0.29 = 24.29 \end{aligned}$$

Two distributed 30-dB amplifiers are better than one 60-dB amplifier.

In conclusion, a significant increase in the contribution of the spontaneous–spontaneous beat noise is incurred if the input power P_s becomes too low.

□

Example 13.9 An erbium-doped fiber amplifier (EDFA) chain is used in an optical communication trunk line. Each EDFA has a gain of 20 dB and maximum signal output of 5 dBm. Other parameters for each EDFA are $m_t = 2$, $n_{\text{spon}} = 3$, $\Delta\nu_f = 100$ GHz, and $\lambda = 1.55$ μm. The attenuation of the trunk fiber is 0.4 dB/km. It is assumed that after each amplifier the received signal is amplified to the same output level as that of the previous amplifier. Draw a power level diagram of the EDFA chain of *both* the signal and ASE noise power with distance.

Solution The signal power level is

$$P = P_{\max} e^{-2\alpha z}$$

In dB, the above equation is expressed as

$$10 \log P = 10 \log P_{\max} - 20\alpha z \log e$$

where

$$10 \log P_{\max} = 5 \text{ dBm}$$

$$20\alpha \log e = 0.4 \text{ dB/km}$$

Hence,

$$\begin{aligned} 10 \log P &= 5 - 0.4\ z \\ P_{\text{ASE}} &= m_t n_{\text{spon}}\ h\nu\ \Delta\nu_f \\ &= (2)(3)(6.63 \times 10^{-34}) \left(\frac{3 \times 10^{14}}{1.55} \right) (10^{11}) \\ &= 7.7 \times 10^{-8}\ W = -41.1 \text{ dBm} \end{aligned}$$

After the kth amplifier, the ASE noise power becomes kP_{ASE}. The ASE noise (in dB) after the kth amplifier is

$$\begin{aligned} 10 \log kP_{\text{ASE}} &= 10 \log P_{\text{ASE}} + 10 \log k \\ &= -41.1 \text{ dBm} + 10 \log k \end{aligned}$$

The ASE noise decays at the same rate, 0.4 dB/km, as the signal during the transmission. The diagram in Fig. 13.26 shows the power level diagrams of the signal and the ASE noise. □

Example 13.10 An optical communication trunk line is to be built with a separation of D km between the transmitter and the receiver and connected by an optical amplifier chain with parameters $\eta = 1$ and $m_t = 2$.

The maximum signal level is set to $P_{\max}$ Watts. The transmission loss of the optical fiber is l dB/km. The amplifiers are equally spaced.

(a) Find an expression for the noise figure F_k as a function of the number k of repeater amplifiers.

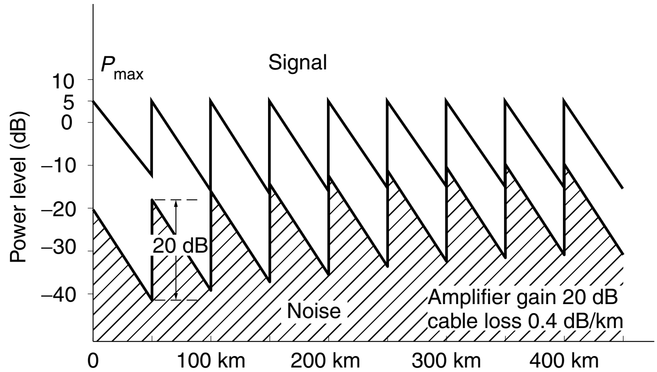


Figure 13.26 Power level diagram of signal and ASE noise of an EDFA.

(b) Derive an expression for numerically optimizing the number of repeater amplifiers.

Solution The power level diagram is shown in Fig. 13.27.

(a) The total power loss between the transmitter and the receiver is lD dB. If this loss is equally split into k relay spans, then each span has a cable loss of lD/k dB and $10 \log(P_s/P_{\max}) = -lD/k$. Hence,

$$P_s = P_{\max} 10^{-lD/10k} \text{ W}$$

(b) The expression for the noise figure with k number of repeaters given by Eq. (13.154) is rewritten as

$$F_k = ax + bx^2 10^{c/x}$$

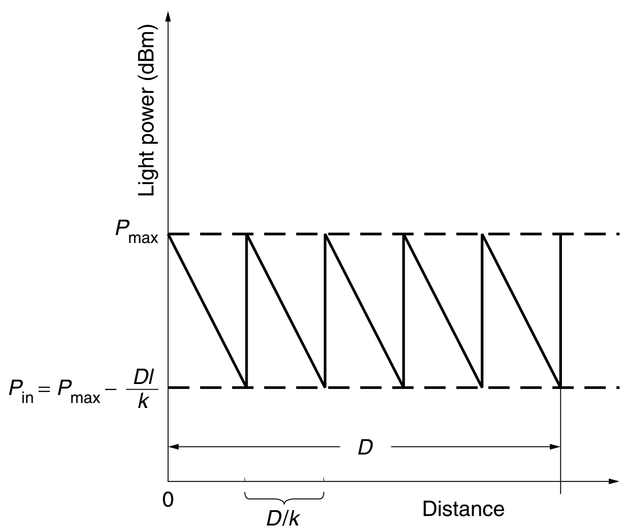


Figure 13.27 The power level diagram for k relay optical amplifiers over a distance D.

where

$$
\begin{aligned}
x &= k \\
a &= 2\eta m_t n_{\text{spon}} \\
b &= \tfrac{1}{2}\eta m_t^2 n_{\text{spon}}^2\ h\nu\ \Delta\nu_f (2 - B/\Delta\nu_f)/P_{\max} \\
c &= \frac{lD}{10}
\end{aligned}
$$

The derivative of F_x with respect to x is set to zero to find the minimum value of F_x:

$$\frac{dF_x}{dx} = a + b(2x - c\ln 10)10^{c/x}$$

where the relationships

$$10^{c/x} = e^{(c/x)\ln 10}$$

and

$$\frac{d}{dx}\left(10^{c/x}\right) = -10^{c/x}\left(\frac{c\ln 10}{x^2}\right)$$

were used. The condition for $dF_x/dx = 0$ is

$$\frac{a}{b}\frac{1}{(c\ln 10 - 2x)} = 10^{c/x}$$

Unfortunately, this is a transcendental equation, but graphical solutions such as shown in Fig. 13.28 can be used. □

Example 13.11 Find the optimum number of EDFAs in the following system:

$$
\begin{aligned}
P_{\max} &= 3\ \text{mW}, \qquad \lambda = 1.55\ \mu\text{m} \\
D &= 500\ \text{km}, \qquad l = 0.3\ \text{dB/km} \\
\eta &= 0.8, \qquad m_t = 2, \qquad n_{\text{spon}} = 1 \\
B &= \Delta\nu_f = 20\ \text{GHz}
\end{aligned}
$$

Solution The noise figure F_k for the system with k amplifiers is, from Eq. (13.154),

$$
\begin{aligned}
F_k &= \frac{\eta}{2}\left[4 + \frac{k m_t n_{\text{spon}}\ h\nu\ \Delta\nu_f}{P_s}\left(2 - \frac{B}{\Delta\nu_f}\right)\right] k m_t n_{\text{spon}} \\
P_s &= P_{\max} 10^{-lD/10k}
\end{aligned}
$$

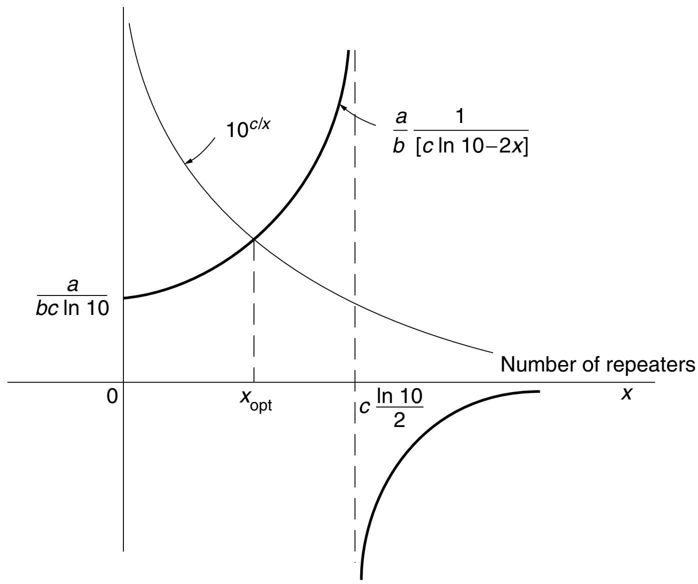


Figure 13.28 Solving a transcendental equation to find the optimum number x of repeater amplifiers.

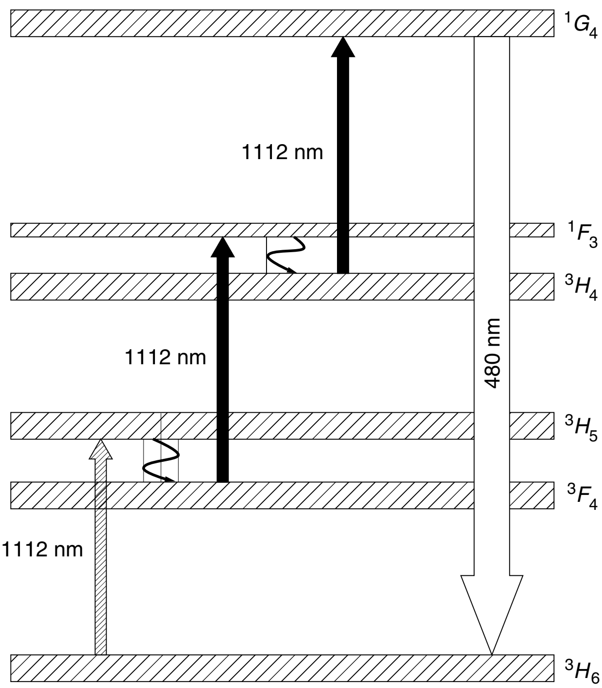


Figure 13.29 Triple-photon excitation of Tm^{3+} in ZBLAN fiber.

where

$$
\begin{aligned}
a &= 2\eta m_t n_{\text{spon}} = 2(0.8)(2)(1) = 3.2 \\
b &= \tfrac{1}{2}\eta m_t^2 n_{\text{spon}}^2 h\nu\ \Delta\nu_f(2 - B/\Delta\nu_f)/P_{\max} \\
&= \tfrac{1}{2}(0.8)(2^2)(1^2)(6.63\times 10^{-34})(1.94\times 10^{14})(2\times 10^{10})/3\times 10^{-3} \\
&= 1.37\times 10^{-6} \\
c &= \frac{Dl}{10} = \frac{(500)(0.3)}{10} = 15
\end{aligned}
$$

With $x = k$, the derivative $dF_x/dx = 0$ leads to the condition

$$
q = \frac{a}{b}\cdot\frac{1}{c\ln 10 - 2x} - 10^{c/x} = 0
$$

$$
\frac{3.2}{1.37\times 10^{-6}}\,\frac{1}{15\ln 10 - 2x} - 10^{15/x}
$$

$$
q\begin{cases} <0 & \text{for } x = 3 \\ >0 & \text{for } x = 4 \end{cases}
$$

and it is approximately $k = 3.3$. The distance between the amplifiers is $500/3 = 167$ km. □

13.13 UPCONVERSION FIBER AMPLIFIER

In Section 13.6, it was pointed out that the 800-nm pump for the EDFA is inefficient because of the excited state absorption (ESA). While this ESA is inefficient for amplification at 1.55 μm, the ESA can be used for amplification of shorter wavelength light at 540 nm. Referring to Fig. 13.13, the down transition from E_4 to E_1 emits 540-nm blue light; thus, a 800-nm infrared pump generates blue light in the Er^{3+}-doped fiber. This is unlike other situations where the pump light wavelength has always been shorter than that of the signal light. This particular excitation is known as upconversion by means of stepwise two-photon absorption.

The case of triple-photon excitation of thulium (Tm^{3+}) doping in a fluorozirconate glass (ZBLAN) fiber, whose energy diagram is shown in Fig. 13.29, can emit 480-nm blue light from 1112-nm infrared pump light [19]. Other types of upconversion mechanisms can be found in Ref. [20].

Upconversion is used for applications such as high-density CD recording by using a shorter wavelength light and to produce coherent light at wavelengths for which no other coherent light sources are available.

PROBLEMS

13.1 Figure P13.1a shows an experimental setup for learning the principles of operation of light amplification by stimulated emission in a neodymium YAG rod. The frequency ν of the signal light source S can be swept over a broad range of optical frequencies. The frequency ν_p of the pump light S_p, however, is fixed rather than swept. The pump light is injected into the rod by means of a half-mirror (HM). There is a choice of detecting the signal light after one pass through the Nd YAG rod or two passes through the rod. The presence or absence of mirror M_1 determines one or two passes. The energy level diagram of the Nd YAG rod is shown in Fig. P13.1b. Even though Nd YAG has many energy levels, its function is normally explained by the four-level model, which consists of the ground level, ${}^4I_{11/2}$, ${}^4F_{3/2}$, and the pump band level.

(a) An experiment was performed with the pump light off and the filter F removed. Draw a qualitative graph of the detected light output from the Nd YAG rod with respect to frequency in terms of the wavenumber $1/\lambda$ (in 10^3 cm^{-1}) of the signal source S.

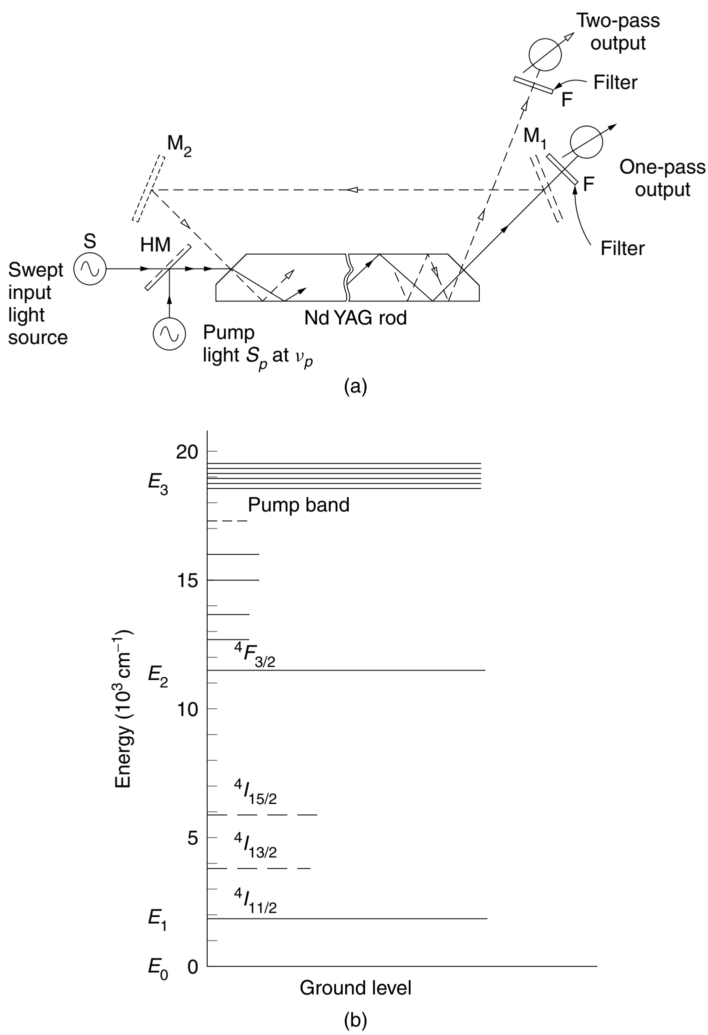


Figure P13.1 Configuration and energy levels of the Nd YAG optical amplifier. (a) Light amplification using stimulated emission. The solid line indicates one pass through the amplifier, and the dashed line indicates two passes through the amplifier. (b) Energy levels of Nd YAG.

(b) The experiment of part (a) is repeated with the pump light turned on. The energy $h\nu$ of the pump light corresponds to the difference in energy levels between the pump band and the ground levels. The filter F is installed to prevent the pump light from reaching the detector. What would be the change in the shape of the absorption lines compared to part (a)?

(c) The pump light is turned on, and signal light with frequency ν_{21} and input power of 2 mW enters the rod. After one pass, the output power was 10 mW. For the same input power, what would the output power be after two passes through the rod? The arrangement for the two passes is shown in Fig. P13.1a. Assume that M_1 and M_2 are perfect reflectors.

13.2 **(a)** What is the length of an erbium-doped fiber amplifier with 35-dB gain?

(b) What is the ASE noise power from this amplifier?

The amplifier parameters are

$N_1 = 1.8 \times 10^{17}\ \text{cm}^{-3}$
$N_2 = 4.8 \times 10^{17}\ \text{cm}^{-3}$
$\sigma_s = 7.0 \times 10^{-25}\ \text{m}^2$
$m_t = 1$
$\Delta\nu_f = 100$ GHz
$\lambda = 1.55\ \mu\text{m}$
$h = 6.63 \times 10^{-34}\ \text{J} \cdot \text{s}$

13.3 With an erbium-doped fiber amplifier having the physical parameters described below, the threshold pump light power was found to be 20 mW. Determine the value of the metastable lifetime of Er^{3+}.

$\sigma_p = 0.42 \times 10^{-21}\ \text{cm}^2$
$\beta = 0.38$
$\Gamma = 0.4$
$r = 2\ \mu\text{m}$ (radius of the core)
$\lambda_p = 1.48\ \mu\text{m}$

13.4 **(a)** Find the optimum number of repeater amplifiers over a transmitter receiver distance of 1000 km from the viewpoint of optimizing the noise.

(b) What is the gain of each amplifier?

(c) What is the noise figure at the optimum condition?

Use the following parameters:

$m_t = 1$
$n_{\text{spon}} = 2$
$\eta = 1$
$\lambda = 1.55\ \mu\text{m}$
$P_{\max} = 10$ mW
$l = 0.2$ dB/km
$B = \Delta\nu_f = 100$ GHz

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