FUNDAMENTALS OF DIELECTRIC WAKE FIELD ACCELERATION

S.V. Shchelkunov and T.C. Marshall, Columbia University, 2004

1. WAKE FIELD PHYSICS BACKGROUND[a]

If a charged particle passes through a transparent medium at a speed greater than the speed of light in the medium, this particle emits light called ***Cerenkov radiation*** (for P. A. Cerenkov [1]). The effect was discovered by Cerenkov in 1934 while he was studying the effects of gamma rays on liquids, and explained by I. E. Tamm and I. M. Frank in 1937. Cerenkov radiation is analogous to the creation of a sonic boom when an object exceeds the speed of sound in a medium. It is emitted only in directions inclined at a certain angle to the direction of the particle motion. This angle depends upon the particle momentum. By measuring the angle between the radiation and the path of a particle, the particle speed may be determined. Commonly, this effect is used in the *Cerenkov counter*, a device for detecting fast particles and determining their speeds or distinguishing between particles of different speeds [1].

To confine the radiation, a medium such as a solid dielectric [2] or plasma [3] can be surrounded by metal walls. If a vacuum channel is manufactured inside a solid medium, a charged particle will propagate along this channel without experiencing direct interaction with the medium, and consequently it will lose its energy only to Cerenkov radiation which will occupy the entire region behind the charge [2].

Such a structure (see Fig.1) is called the ***dielectric-lined waveguide*** (or, sometimes, ***dielectric-filled*** or ***dielectric-loaded waveguide***) because, as in any waveguide, both structure ends are open, and only a discreet set of eigenmodes can propagate along the structure.

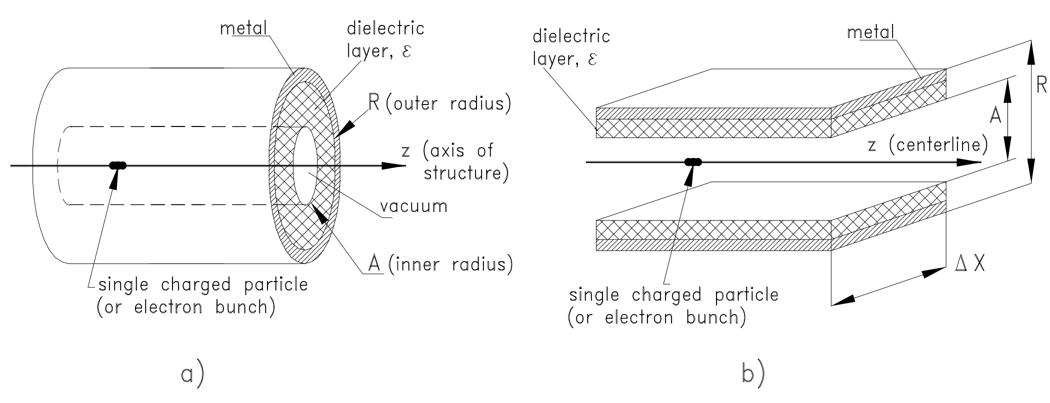

FIG.1. a) Cylindrically symmetric dielectric- lined waveguide (usually $R >> A$), and b) rectangular dielectric- lined waveguide (usually $\Delta X >> R >> A$).

---

[a] A reader familiar with this subject may proceed to the next section "DIELECTRIC WAKE FIELD ACCELERATION" on page 3.

The examples in Fig.1 are structures with a dielectric layer of only one type. However, the dielectric layer can be a ***composite dielectric***, a material containing several dielectric layers, each with a different dielectric constant. A dielectric material may also be dispersive, i.e. with the dielectric constant that depends on frequency.

In a cylindrical structure with one nondispersive dielectric layer (ε = *const*), the frequencies of the TM axisymmetric eigenmodes must satisfy the dispersion relationship [4]

$$\frac{I_1(t_1\omega)}{I_0(t_1\omega)} - \varepsilon\frac{\gamma_k}{\gamma}\frac{J_0(t_R\omega)N_1(t_A\omega) - J_1(t_A\omega)N_0(t_R\omega)}{J_0(t_R\omega)N_0(t_A\omega) - J_0(t_A\omega)N_0(t_R\omega)} = 0 \qquad (1)$$

where[b] $t_1 = \frac{A}{v_\phi}\sqrt{1-\left(\frac{v_\phi}{c}\right)^2}$, $t_A = \frac{A}{v_\phi}\sqrt{\varepsilon\left(\frac{v_\phi}{c}\right)^2 - 1}$, and $t_R = \frac{R}{v_\phi}\sqrt{\varepsilon\left(\frac{v_\phi}{c}\right)^2 - 1}$ with ν_ϕ being the phase velocity (the propagation velocity of an eigenmode in the z- direction) and c being the speed of light in vacuum ($2.99792458 \cdot 10^8$ m/sec). J_0 is the zero- order Bessel function of the first kind, J_1 is the first- order Bessel function of the first kind, I_0 is the zero- order modified Bessel function of the first kind, I_1 is the first- order modified Bessel function of the first kind, N_0 is the zero- order Bessel function of the second kind, and N_1 is the first- order Bessel function of the second kind.

In the limit of $A/R \ll 1$, one finds that the phase velocity can be less than the speed of light in vacuum, i.e. $\nu_\phi < c$, only if

$$\omega > \frac{2.40}{\sqrt{\varepsilon - 1}}\frac{c}{R}\left[1 + 1.85\left(1 - \frac{1}{\varepsilon}\right)\frac{A^2}{R^2}\right] \qquad (2)$$

Thus, the dielectric-lined waveguide is a *slow-wave structure* (supports waves with phase velocities less than the speed of light in vacuum). Every wave with $\nu_\phi < c$ can effectively interact with a bunch of charged particles (electrons) if $\nu_\phi = \nu$ (where ν is the bunch velocity). This effect can be used for either generation/ amplification of electromagnetic waves (RF) or acceleration of charged particles[c].

Setting the phase velocity equal to the bunch (particle) velocity, $\nu_\phi = \nu$, one finds from Eq.(1) the frequencies of all eigenmodes which interact via E_z with this bunch. The

---

[b] Later, we will be interested in the axisymmetric eigenmodes whose phase velocity, ν_ϕ, equals the particle (bunch) velocity, ν (we assume here that the bunch is not displaced from the z-axis and excites/interacts with only the axisymmetric eigenmodes). Then, $\nu_\phi = \nu = c\beta$ with β being the usual relativistic beta-factor; and $\sqrt{1-\left(\frac{v_\phi}{c}\right)^2} = \gamma^{-1}$ with γ being the usual relativistic gamma- factor. The coefficients t_1, t_A, and t_R can be rewritten as $t_1 = A/c\beta\gamma$, $t_A = A/c\beta\gamma_k$, and $t_R = R/c\beta\gamma_k$, where $\gamma_k = (\varepsilon\cdot\beta^2 - 1)^{-1/2}$

[c] One of the first descriptions of such use of the dielectric- lined waveguide was given in the book "A Set of Electrodynamics Problems" by two Russian famous physics teachers, V. Batygin and I. Toptygin (the first edition was published in 1961, and the second edition was published in 1969).

solution of Eq.(1) is a discrete set of eigen- frequencies ω_m, with $1 \leq m \leq \infty$. The second row of Table 1 gives an example for the first 6 TM_{0m} eigen- frequencies when $v/c = 0.9999476$ (the gamma- factor $\gamma = 97.66$), and the third row gives an example when $v/c = 0.99999417$ ($\gamma = 292.97$).

Table 1. The first 6 TM_{0m} eigen-frequencies, when $\varepsilon = 9.65$, A = 1.5mm, R = 19.31mm; $v/c = 0.9999476$ (γ=97.66) for the 2nd row; $v/c = 0.9999942$ (γ=292.97) for the 3rd row

m	1	2	3	4	5	6
$\omega_m/2\pi$, [GHz]	2.04166	4.73880	7.49345	10.2785	13.0821	15.8978
$\omega_m/2\pi$, [GHz]	2.04165	4.73877	7.49341	10.2784	13.0820	15.8977

One discovers that the eigen- frequencies negligibly depend on γ for large gamma factors

$$\gamma \geq 50 \tag{3}$$

or, equivalently, when the electron bunch energy $\geq$ 25 MeV. Thus, an ultra-relativistic bunch can effectively interact with an eigenmode electromagnetic field even if this bunch gains or loses its energy.

2. DIELECTRIC WAKE FIELD ACCELERATION

The full electromagnetic field excited by a bunch (particle) when it passes through a dielectric- lined waveguide is called the ***wake field***. It is a superposition of eigenmodes whose eigen- frequencies satisfy the dispersion relationship (*DRS*). The DRS is given by Eq.(1) if a bunch propagates along the z-axis of a cylindrically symmetric waveguide; the functional form of the electromagnetic fields excited by a bunch in a cylindrically symmetric dielectric-lined waveguide is given in appendix **A**.

Figure 2 gives an example of the electric flux lines [5] in a cylindrically symmetric dielectric-lined waveguide, which demonstrates that the wake field has a near-periodic pattern

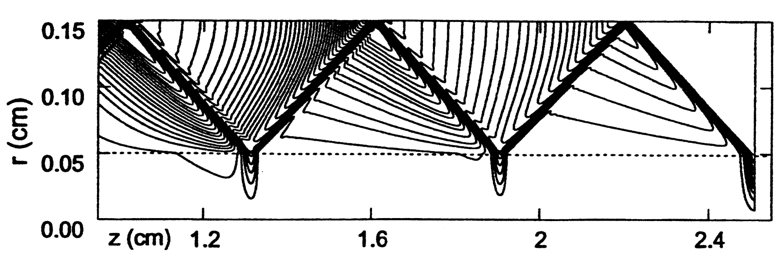

FIG.2. The wake field has a near-periodic pattern (see Fig.6 on p.1277 in [5]). This picture displays the electric flux lines of the wake field excited by a bunch which moves from left to right.

The period with which the wake field pattern nearly repeats itself is called the ***wake-field period*** *L*.

Figure 3 gives an example of the E_z – profile along the z-axis of a cylindrically symmetric waveguide. One observes that the axial E_z - field is nearly periodic.

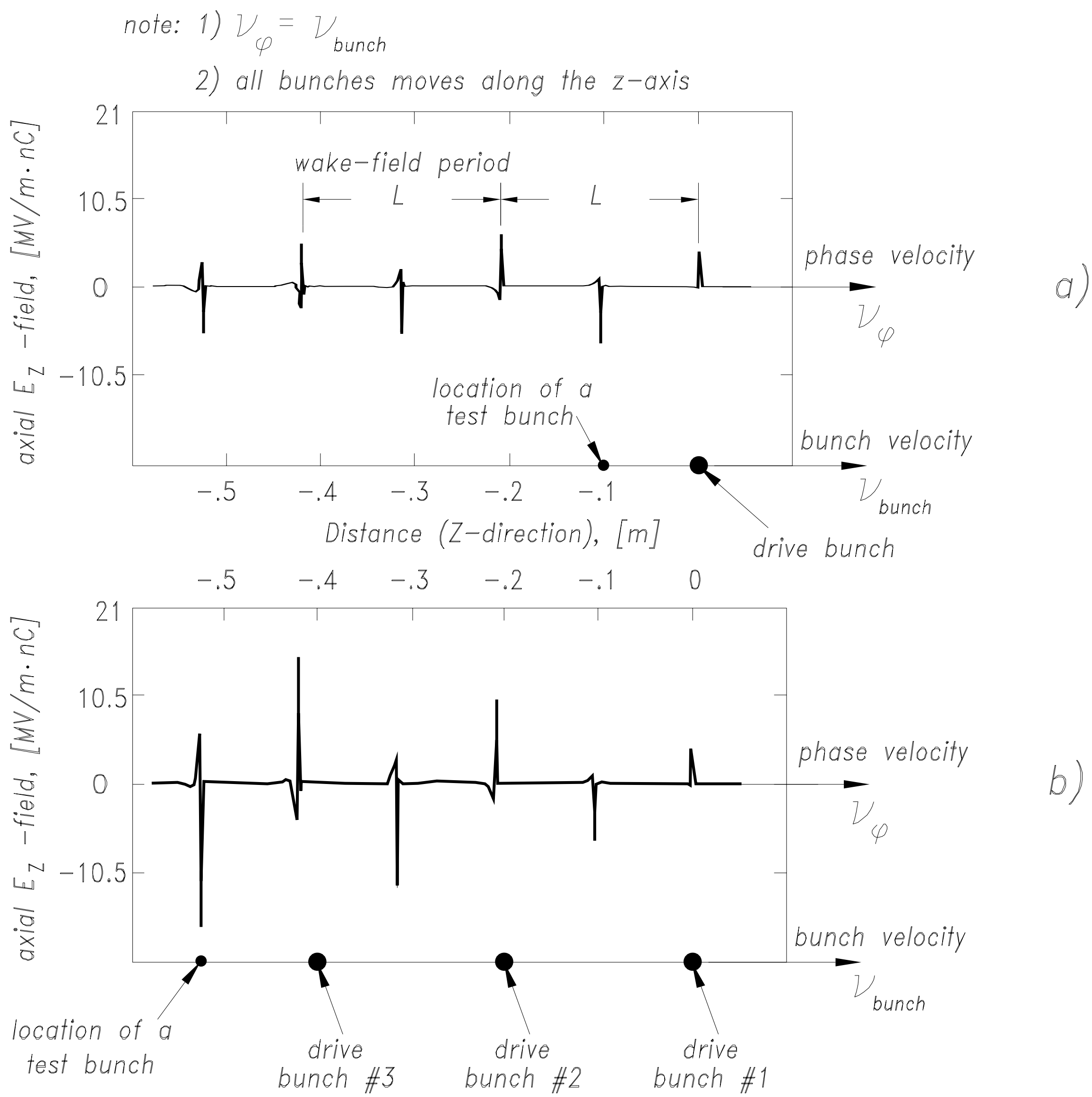

FIG.3. a) The axial E_z- field after one bunch (called a *drive bunch*), and b) after 3 identical drive bunches. In Fig. 3.a, one observes that the axial E_z- field is nearly periodic (over many *L*) with the *wake-field period L*. In Fig.3.b, every consecutive drive bunch is placed at the distance *L* behind the previous one, and amplifies the wake field. If one places a relativistic bunch (called a *test bunch*) at the distance *L*/2 behind the last drive bunch, it will be accelerated.

The bunch[d] that creates the wake field is called the ***drive*** bunch. If one places several more bunches, each at the distance L behind the previous one, such bunches will amplify the wake field. These bunches are also called drive bunches. Together, all drive bunches form the ***drive train***[e]. If one places a relativistic bunch (called a ***test bunch***) at the distance $L/2$ behind the last drive bunch (i.e. at the distance $L/2$ behind the driving train), it will be accelerated (see Fig.3). If several drive bunches are used, the E_z – field at the test bunch location will have a much higher amplitude than that after only one bunch; consequently, the test bunch introduced after a train of several drive bunches will gain significantly more energy. In this acceleration scheme, all bunches must be relativistic: the drive bunches can be lower energy but relativistic, and the test bunch when introduced can be lower energy but relativistic as well (but it will become higher energy due to acceleration).

Depending on the dielectric-lined waveguide parameters, and bunch charges and dimensions, an energy gain in the scale from a few hundreds MeV/m to 1 GeV/m might become available.

Using a dielectric-lined waveguide in which several drive bunches create and amplify the wake field, followed by a test bunch which gains energy from the wake field, we have the typical colinear ***dielectric wake-field accelerator*** (*DWA*).

In a cylindrically symmetric waveguide excited by drive bunches moving along the z-axis, the axial E_z – field profile at the location of a test bunch resembles a hump. This will be broad but with low amplitude if only a few eigenmodes of the structure are excited by the bunch [6][7]. However, if many (hundred or more) eigenmodes are excited, the axial E_z- field profile at the test bunch location will be narrow but with high amplitude [5][8][9]. One can excite many eigenmodes and achieve a high amplitude wake field if relatively short drive bunches are used [4] [5] [10].

The experimental studies of wake fields which are superposition of many eigenmodes (and thus, have a narrow but high amplitude E_z – field profile on the structure axis) were one of the prime goals of our work. These studies have been undertaken to demonstrate a novel acceleration technique proposed by T.C. Marshall and collaborators [4] which utilizes ***constructive superposition*** [f] of wake-fields produced by short drive bunches so that a broadband frequency spectrum is excited which will synthesize a high-amplitude localized accelerating field. A similar study has also been made by a group at Argonne National Laboratory [11]. The difference between these two experiments, as well as the difference in the obtained data, will be illustrated.

[d] All bunches here are relativistic.

[e] Sometimes, they are referred to as *driving* bunches and a *driving* train.

[f] I.e. when the first drive bunch creates the wake-field, and the consecutive drive bunches amplify it

APPENDIX A

The functional form of the electromagnetic fields excited by a bunch or several bunches in a cylindrically symmetric dielectric-lined waveguide is given in this appendix. The bunches are assumed to propagate along the z-axis. The charge distribution is represented as:

$$\rho(r', z'_0) = Q_0 \frac{\delta(r')}{2\pi \cdot r'} \sum_i q_i F_i(z'_0)$$

with $\int F_i(\xi) d\xi = 1$, and $F_i(\xi) \to 0$, when $|\xi| \to \infty$

where the function F_i describes the longitudinal shape of the i^{th} bunch, whose full charge is:

$$Q_i = q_i \cdot Q_0 \tag{A.1}$$

with Q_0 being some reference charge; $\delta(r')$ is the usual Dirac delta-function; $z'_0 = z' - vt$, r', z' are the radial and longitudinal coordinates, t is time, and $v = c\beta$ is the bunch velocity. Note that $q_i \geq 0$ always.

Searching the fields in the form of TM_{0m} eigenmode decomposition [4][5]:

$$E_z = \sum_{m=1}^{\infty} E_0^m \cdot e_z^m, \qquad E_r = \sum_{m=1}^{\infty} E_0^m \cdot e_r^m, \qquad H_\theta = \sum_{m=1}^{\infty} \frac{E_0^m}{\sqrt{\mu_0 / \varepsilon_0}} \cdot h_\theta^m \tag{A.2}$$

where $E_0^m = \dfrac{Q_0}{2\pi\varepsilon_0 A^2} f_m(0)$, and m is mode number, one obtains that:

$$e_z^m = -\sum_i q_i \frac{f_m(r)}{\alpha_m} \int_{-\infty}^{\infty} dz'_0 \cdot F_i(z'_0) \theta_{hv}(z'_0 - z_0) \cos\left(\frac{\omega_m}{v} \cdot (z'_0 - z_0) \right)$$

$$e_r^m = \sum_i q_i \frac{c\beta}{\omega_m \left(1 - \varepsilon(r)\beta^2\right)} \cdot \frac{\partial f_m(r) / \partial r}{\alpha_m} \int_{-\infty}^{\infty} dz'_0 \cdot F_i(z'_0) \theta_{hv}(z'_0 - z_0) \sin\left(\frac{\omega_m}{v} \cdot (z'_0 - z_0) \right)$$

$$h_\theta^m = \sum_i q_i \frac{\varepsilon(r) c\beta^2}{\omega_m \left(1 - \varepsilon(r)\beta^2\right)} \cdot \frac{\partial f_m(r) / \partial r}{\alpha_m} \int_{-\infty}^{\infty} dz'_0 \cdot F_i(z'_0) \cdot \theta_{hv}(z'_0 - z_0) \cdot \sin\left(\frac{\omega_m}{v} \cdot (z'_0 - z_0) \right)$$

where $z'_0 = z' - vt$, $z_0 = z - vt$, α_m is the normalization constant, θ_{hv} is the Heaviside function, ω_m is the m^{th} eigen- frequency, and $f_m(r)$ is the m^{th} eigen- function [4]. In the vacuum channel, one obtains eigen- functions:

$$f_m(r) = I_0(k_{1m} r)$$

Then, the normalization constant is:

$$\alpha_m = \frac{1}{2}\cdot\left\{\frac{1}{\varepsilon}\cdot\left(\frac{\gamma}{\gamma_k}\right)^2 I_1^2(k_{1m}A)\cdot\left[\frac{R^2}{A^2}\left(\frac{P_1(k_{2m,}R,R)}{P_1(k_{2m},R,A)}\right)^2 - 1\right] - (\varepsilon-1)I_0^2(k_{1m}A) - I_1^2(k_{1m}A)\right\}$$

where $k_{1m} = \frac{\omega_m}{c\cdot\beta\cdot\gamma} = k_{2m}\frac{\gamma_k}{\gamma}$, with γ being the relativistic gamma- factor, $\gamma_k = 1/\sqrt{\varepsilon\cdot\beta^2 - 1}$, and $P_1(k,R,r) = J_0(kR)N_1(kr) - J_1(kr)N_0(kR)$.

J_m, N_m are the Bessel, and Neumann functions (Bessel equation functions of the 1st and 2nd kinds) respectively. I_m is the modified Bessel function.

The DRS (dispersion relationship) and analysis of wake fields for a semi-infinite cylindrically symmetric waveguide excited by a bunch that moves along the z-axis can be found in [12]. The DRS and analysis of wake fields for a cylindrically symmetric waveguide excited by a point charge (or a short bunch with the rectangular or Gaussian distribution) that moves parallel to the z-axis but can be displaced from the z-axis can be found in [5][13][14]. The analysis of the wake field of a point charge, moving in a circular cross-section vacuum tunnel in a dielectric material which is uniform in the direction parallel to the motion of the particle (the particle can be off axis), but has an arbitrary but axisymmetric geometry in the transverse direction (such as a *photonic crystal* or *Bragg structure*), can be found in [15]. The DRS and analysis of wake fields excited in a rectangular waveguide can be found in [16][17][18].

REFERENCES

[1] See, e.g., Columbia Encyclopedia, 6th edition

[2] B.M. Bolotovskii, Usp. Fiz. Nauk. 75, 295, (1961); English version in: Sov. Phys. Usp. 4, 781 (1962).

[3] "A Plasma Wake Field Accelerator", by R.D. Ruth, A.W. Chao, P.L. Morton, and P.B. Wilson, Particle Accelerators, (1985), vol. 17, pp 171-189

[4] "The Stimulated Dielectric Wake-Field Accelerator: A Structure with Novel Properties", by T.C. Marshall, T-B. Zang, and J.L. Hirshfield, AIP Conf. Proc., 472, 27, (1999), edited by W. Lawson, C. Bellamy, and D. Brosius, and references therein

[5] "Theory of Wakefields in a Dielectric- Lined Waveguide", by S.Y. Park and J.L. Hirshfield, Phys. Rev. E 62, 1266- 1283, (2000)

[6] "Transformer Ratio Enhancement Using a Ramped Bunch Train In a Collinear Wakefield Accelerator", by J. G. Power, W. Gai, and A. Kanareykin, AIP Conf. Proc. 569, 605, (2001), and references therein

[7] "Experimental Demonstration Of Wake-Field Effects In Dielectric Structures", by W. Gai, P. Schoessow, B. Cole, R. Konecny, J. Norem, J. Rosenzweig, and J. Simpson, Phys. Rev. Letters 61, p.2756, (1988)

[8] "An Experimental Test Of the Theory Of the Stimulated Dielectric Wake-Field Accelerator", by J-M. Fang, T.C. Marshall, J.L. Hirshfield, M.A. LaPointe, T-B. Zhang, and X.J. Wang, Proc. of the 1999 Particle Accelerator Conf, p. 3627, (1999)

[9] "Measurement Of the Longitudinal Wakefields In a Multimode, Delectric Wakefield Accelerator Driven By a Train Of Electron Bunches', by J.G. Power, M.E. Conde, W.Gai, R. Konecny, and P. Schoessow, Phys. Rev. ST Accel. Beams 3, 101302, (2000)

[10] Experimental And Numerical Studies Of Dielectric Wake Field Acceleration Devices", by S.V. Shchelkunov, J-M. Fang, T.C. Marshall, and J.L. Hirshfield, Proc. of the 2003 Particle Accelerator Conf, vol.3, p.1924, (2003)

[11] “Measurement Of the Longitudinal Wakefields In a Multimode, Delectric Wakefield Accelerator Driven By a Train Of Electron Bunches’, by J.G. Power, M.E. Conde, W.Gai, R. Konecny, and P. Schoessow, Phys. Rev. ST Accel. Beams 3, 101302, (2000)
[12] “Structure of Electromagnetic Field Excited By an Electron Bunch In a Semi- Infinite Dielectric-Filled Waveguide”, by I.N. Onishchenko, D. Yu. Sidorenko, and G.V. Sotnikov, Phys. Rev. E 65, 066501, (2002)
[13] "Wake Fields In a Dielectric Lined Waveguide", by K-Y. Ng, Phys. Rev. D 42, pp. 1819-1828, (1990)
[14] "Longitudinal- and Transverse- Wake-Fields Effects in Dielectric Structures", by M. Rosing and W. Gai, Phys. Rev. 42, pp.1829-1834, (1990)
[15] "Wake Field In Dielectric Acceleration Structures", by L. Schächter, R.L. Byer, and R.H. Siemann, Phys. Rev. E 68, 036502, (2003)
[16] “Strong Wake Fields Generated By a Train of Femtosecond Bunches In a Planar Dielectric Microstructure”, by C. Wang, J. L. Hirshfield, J.-M. Fang, and T. C. Marshall, Phys. Rev. ST Accel. Beams 7, 051301, (2004), and references therein
[17] "Field Analysis Of a Dielectric- Loaded Rectangular Waveguide Accelerating Structure”, by L. Xiao, W. Gai, and X. Sun, Phys. Rev. E 65, 016505, (2001)
[18] "Electromagnetic Wake Fields and Beam Stability In Slab-Symmetric Dielectric Structures", by A Tremaine and J. Rosenzweig, Phys. Rev. 56, pp. 7204-7216, (1997)