Metal-insulator transition in Si:X (X=P,B): Anomalous response to a magnetic field

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The zero-temperature magnetoconductivity of just-metallic Si:P scales with magnetic field H and dopant concentration n lying on a single universal curve: $\sigma(n,H)/\sigma(n,0)=G[H^{-\delta}\Delta n]$ with a magnetic-field crossover exponent $\delta\approx 2$. We note that Si:P, Si:B, and Si:As all have unusually large crossover exponents near 2, and suggest that this anomalously weak response to a magnetic field, $\Delta n_c \propto H^{\delta}$, is a common feature of uncompensated doped semiconductors. [S0163-1829(98)07036-2]

The transition from metallic to insulating behavior that occurs in doped semiconductors and amorphous metal-semiconductor mixtures as a function of dopant concentration, stress, or magnetic field is a continuous one.[1–4] Since thermal excitations blur the distinction between conducting and insulating behavior at any finite temperature, a continuous metal-insulator transition (MI) is a quantum phase transition that occurs only in the limit of zero temperature. The two basic mechanisms that cause the localization of charge carriers are the correlations[5] among the electrons and disorder.[6] Either mechanism has been shown to produce a metal-insulator transition in the absence of the other and their interplay has been the subject of intensive study.[3]

A major step was taken with the introduction of the scaling ideas[7] that have proved so successful for understanding phase transitions driven by temperature. Here the behavior is controlled by a diverging correlation length that is much larger than any of the microscopic lengths in the problem. Within this formulation, the electrical conductivity is expected to scale with temperature T and dopant concentration n, obeying the form $\sigma(n,T)=T^{\mu/\nu z}F(\Delta n/T^{1/\nu z})$. Here Δn is the difference between n and its critical value n_c, μ is the exponent that describes the approach to the transition of the zero temperature conductivity, $\sigma(0)\propto(\Delta n)^{\mu}$, ν is the critical exponent that characterizes the divergence of the length scale, and z is the ''dynamical'' exponent associated with the divergent time scale at the transition.[3] In the limit of zero temperature, the conductivity is expected to scale with magnetic field[8,9] as $\sigma(n,H)=\sigma(n,0)G[H^{-\delta}\Delta n]$, where δ is the magnetic-field crossover exponent. The approach to the transition and the values of the critical exponents are expected to be governed by the symmetry of the system which determines its universality class (associated, for example, with spin-orbit effects, magnetic field, or scattering by magnetic impurities).[3,4] This theoretical framework has provided an adequate description for many experimental studies. However, it is not yet clear what determines the universality classes, or how many such classes there are. The description of the metal insulator transition in the presence of both disorder and interactions remains a central unsolved problem in condensed-matter physics.[10]

Despite the success of the scaling formulation in some regards, simple scaling does not appear to hold in doped semiconductors in a wide region of temperature and concentration. Scaling with temperature and concentration has been shown to hold for Si:P in the presence of a magnetic field of 75 kOe.[3] However, the conductivity does not appear to scale with temperature in the absence of a magnetic field: scaling is obtained only if one chooses a critical conductivity exponent, $\mu\approx 0.29$,[11] considerably smaller than the value found experimentally.[12–14] On the other hand, scaling of the zero-temperature conductivity has been demonstrated with magnetic field for p-type Si:B,[15] albeit with an unexpectedly large magnetic field crossover exponent near 2, in disagreement with the theoretically expected value of $\frac{1}{2}$. This raises the issue whether the anomalously weak response to a magnetic field is due to the spin-orbit scattering present in boron-doped silicon, or whether it is a general feature of uncompensated doped semiconductors near the metal-insulator transition. Si:P is considered the archetypical strongly correlated disordered system, and is used as a standard against which newer materials are compared.[16] It is therefore of great fundamental interest to determine the functional form of the magnetoresistance close to its MIT.

In this paper, we report measurements of the magnetoconductivity of Si:P. Detailed analysis of data taken over a broad range of temperature and magnetic field allowed the identification of separate field- and temperature-dependent components, yielding reliable determinations of the zero-temperature conductivity. Our results demonstrate that the zero-temperature conductivity scales with magnetic field and dopant concentrations, $\sigma(n,H)/\sigma(n,0)=G(H^{-\delta}\Delta n)=F(H/H^{*})$, with a crossover exponent, $\delta\approx 2$, comparable to the anomalously large crossover exponent of Si:B.[15] Earlier data of Shafarman *et al.*[17] indicate that the magnetoconductance of Si:As strongly resembles that of Si:P, scaling with a similar crossover function and exponent. We conclude that all the silicon-based doped semiconductors measured to date have an unusually large crossover exponent near 2, signaling an anomalously weak response to a magnetic field. We point out that this is a feature of the universality class of

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silicon-based doped semiconductors that is currently not understood and warrants theoretical attention.

Four Czochralski-grown Si:P samples were used in our studies with dopant concentrations 3.60, 3.66, 3.95 and 4.21×10^{18} cm^{-3}. Based on a critical concentration $n_c=3.46\times10^{18}$ cm^{-3},[14] this corresponds to $1.04n_c$, $1.06n_c$, $1.14n_c$, and $1.22n_c$. Measurements were taken at temperatures between 0.037 and 0.5 K in magnetic fields up to 90 kOe. Sample characterization and measurement techniques are described elsewhere.[14,18]

At temperatures sufficiently low that corrections due to localization are small, finite-temperature corrections due to interactions are expected to yield a conductivity in the absence of a magnetic field[19,20,4] given by $\sigma(n,T)=\sigma(n,0)+A(n)T^{1/2}$. For doped semiconductors the slope $A(n)$ is net positive near the transition and changes sign as one moves away toward higher dopant concentration n. For the four specimens used in our study, the slope is positive only for the sample closest to the transition ($n=1.04n_c$). The slope $A\approx0$ for the sample with $n=1.06n_c$, and the two samples furthest from the transition have net negative slopes A. The samples used in our experiments thus span concentrations that include temperature coefficients for the conductivity that are positive, zero, and negative in the absence of a field. We note that it has been suggested that the critical region is restricted to dopant concentrations for which the slope is positive.[13]

In order to demonstrate scaling we must obtain dependable extrapolations of the conductivity to $T=0$. We describe in detail below a careful empirical analysis that yields reliable zero-temperature values of the conductivity as a function of magnetic field. The data and analysis for the three other samples were similar.

Data are shown for the sample with negative slope $A(n)$ in Figs. 1 and 2(a). Figure 1 shows the conductivity as a function of temperature at various fixed fields. The inset shows the conductivity in large magnetic field on an expanded scale. The curves are clearly parallel to each other in fields above 10 or 20 kOe, indicating that the temperature dependence in high magnetic fields is independent of the field. This is also demonstrated in Fig. 2(a), where the conductivity of the same sample is shown as a function of magnetic field at four different temperatures. Again, it is clear that the curves at different temperatures are parallel to each other in fields above 10 or 20 kOe. Thus, in sufficiently high magnetic fields the conductivity can be expressed as the sum of two terms, one of which depends only on the temperature and one of which depends only on the field: $\sigma(H,T)=\sigma_T(T)+\sigma_H(H)$. As shown in Fig. 2(b), all of the curves can be brought into coincidence in high magnetic fields by subtracting $\sigma_T(T)$ from the total conductivity. Deviations are apparent in small fields, becoming increasingly pronounced as the temperature increases: the conductivity flattens below a magnetic field H given approximately by $g\mu_B H\approx k_B T$.

The curves shown in Fig. 2(b) for 37, 50, and 70 mK differ from each other by small amounts compared to the overall change of the conductivity with magnetic field; the lowest-temperature data are thus close to the $T=0$ curve on this scale. Setting $\sigma(H,0)\approx\sigma(H,37\text{ mK})$, we show $\sigma(H,0)/\sigma(0,0)$ plotted as a function of H for all four samples in Fig. 3(a). These four very similar curves can be

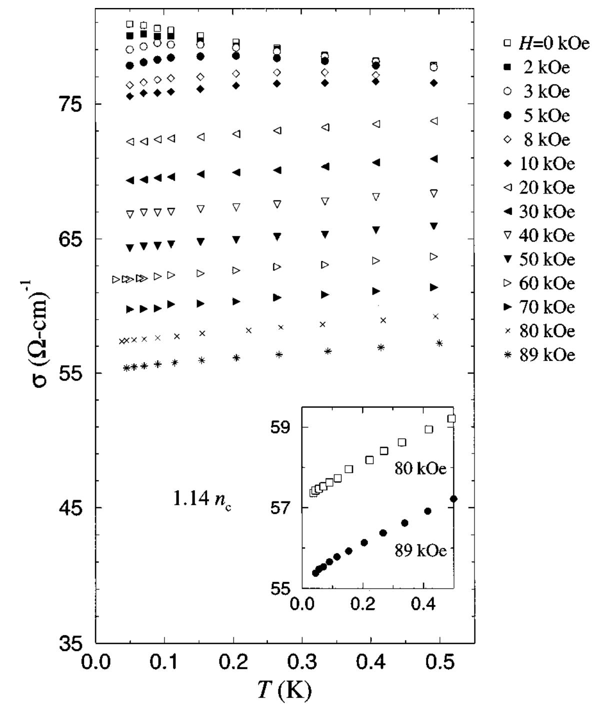

FIG. 1. Conductivity σ vs temperature in various magnetic fields for dopant concentration $n=1.14n_c$. The inset shows data in high magnetic fields on an expanded scale.

collapsed onto a single curve by appropriate choices of a scaling parameter H^*, as shown in Fig. 3(b). We have thereby demonstrated that the zero-temperature conductivity scales with magnetic field, taking on the form[8,9]

$$\frac{\sigma(n,H)}{\sigma(n,0)}=G(H^{-\delta}\Delta n)=F\left(\frac{H}{H^*}\right) \tag{1}$$

with a crossover function $G(H^{-\delta}\Delta n)$, a magnetic-field crossover exponent δ, and a scaling parameter H^* that should obey a power law in the critical region, $H^*\propto\Delta n^{1/\delta}$.

We note that scaling appears to hold quite well for the samples that have negative as well as positive zero-field slopes A. The crossover function of Fig. 3(b) exhibits complex behavior at low fields, and becomes linear with magnetic field at high fields. The inset to Fig. 3(b) shows H^* versus $\Delta n\equiv(n-n_c)$ on a log-log scale, using $n_c=3.46\times10^{18}$ cm^{-3}.[14] (Data are included in the inset for samples measured earlier and not otherwise presented in this paper.) There appear to be deviations from a straight line (power-law behavior) at the highest dopant concentration, perhaps indicating that the last sample is not in the critical region. Additional careful studies are needed to determine the breadth of the critical regime. The (inverse) slope yields a magnetic-field crossover exponent $\delta\approx2$, which is unusually large. A similar analysis yields the curves shown for Si:B (Ref. 15) in Fig. 4(a), and scaling is obtained for appropriate choices of the scaling parameter H^*, as shown in Fig. 4(b). The inset is a plot of H^* versus Δn on a log-log scale. Again, deviations from power-law behavior are apparent at the higher dopant concentrations, implying that these are outside the critical range; the crossover exponent $\delta\approx1.9$.

Si:P and Si:B both have large δ's near 2. However, their crossover functions are quite different, as illustrated in Fig.

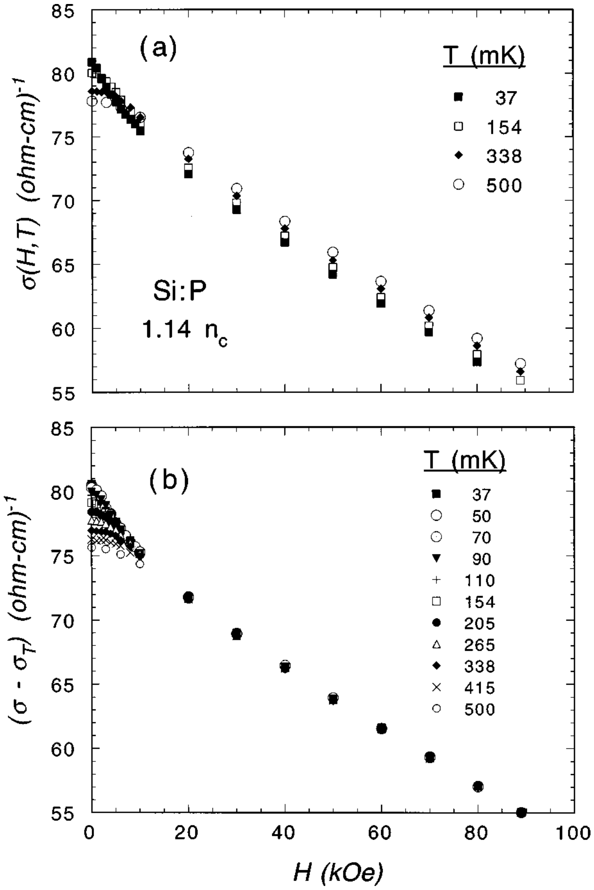

FIG. 2. (a) For dopant concentration $n=1.14n_c$, the conductivity σ vs magnetic field H at four different temperatures, as labeled. (b) The conductivity minus its high-field temperature-dependent term, $[\sigma(H,T)-\sigma_T(T)]$ (see text). Note that all curves now coincide at above 10 or 20 kOe.

4(b) where both are shown for comparison. The effect of a magnetic field is considerably stronger in the case of Si:B: a 90-kOe field easily drives a just-metallic sample into the insulating phase. The magnetoconductance of Si:B exhibits

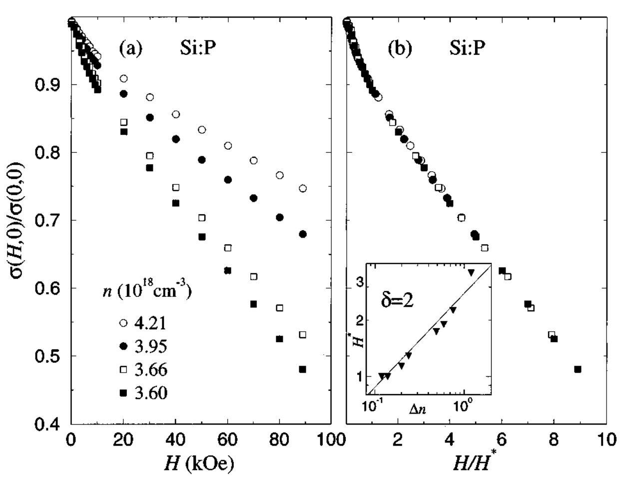

FIG. 3. (a) The ratio $\sigma(H,0)/\sigma(0,0)$ versus magnetic field H for four just-metallic Si:P samples with dopant concentrations as labeled; $n_c=3.46\times10^{18}$ cm^{-3}. (b) Scaled curves of $\sigma(H,0)/\sigma(0,0)$ vs H/H^* (using $n=3.60\times10^{18}$ cm$^{-3}=1.04n_c$ as the reference sample for which $H^*=1$). The inset shows H^* vs $\Delta n=(n-n_c)$ on a log-log scale, and includes additional samples measured earlier.

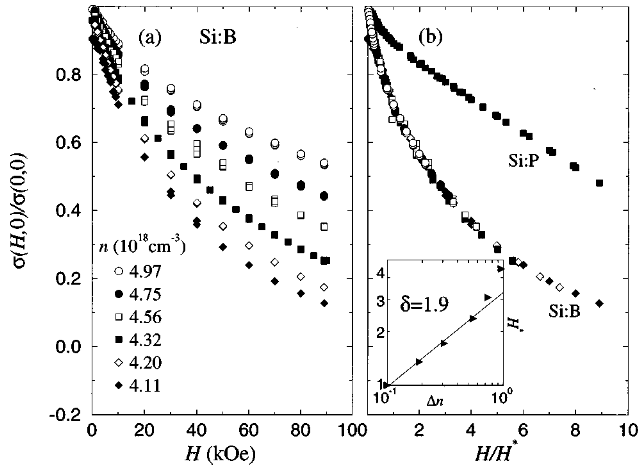

FIG. 4. (a) The ratio $\sigma(H,0)/\sigma(0,0)$ versus magnetic field H for just-metallic samples of Si:B with dopant concentrations as labeled; $n_c\approx4.01\times10^{18}$ cm^{-3}. (b) Scaled curves for Si:B of $\sigma(H,0)/\sigma(0,0)$ vs H/H^*. For comparison, the upper curve shows data for Si:P. The inset shows H^* vs $\Delta n=(n-n_c)$ on a log-log scale (using $n=4.11\times10^{18}$ cm$^{-3}=1.025n_c$ as the reference sample for which $H^*=1$).

the theoretically expected $H^{1/2}$ behavior[4,21] over a broad range of fields. On the other hand, the magnetoconductance of Si:P displays $H^{1/2}$ dependence only in moderate magnetic fields and becomes strictly linear in H at higher fields; the conductivity of Si:As exhibits very similar behavior[17] to Si:P. We note that the theory of Refs. 4 and 21 is valid only outside the critical region, and the behavior near the transition is not known. The observed deviation from $H^{1/2}$ may simply reflect the fact that sufficiently high magnetic fields drive our samples toward the transition and into the critical range. The scaling found in this paper suggests that a magnetic field drives samples into the critical regime for $H>H^*\propto\Delta n^{1/\delta}$.[22]

To summarize, the zero-temperature conductivities of Si:P and Si:B both scale as a function of magnetic field and dopant concentration, $\sigma(n,H)/\sigma(n,0)=F(H/H^*)=G(H^{-\delta}\Delta n)$, with an anomalously large crossover exponent near 2. An analysis of published data for Si:As (Ref. 17) indicates that scaling is also obeyed in this system, again with a crossover exponent substantially larger than 1. In contrast, theory predicts crossover exponents considerably smaller than 1: orbital effects are expected to give a magnetic-field crossover exponent $\delta=1/2$,[8] and calculations done to date indicate that coupling of the magnetic field to the electrons' spin yields an even smaller value.[23,24] Since $(\Delta n_c)\propto H^{\delta}$, this implies that very near the transition, a small magnetic field should induce a very large change in critical concentration. The large crossover exponent found in our experiments signals instead that for both n-type and p-type uncompensated doped silicon, a small magnetic field induces a small change in the critical concentration. This unexpectedly weak response to a magnetic field exhibited by all silicon-based material studied to date may be characteristic of the universality class of all doped semiconductors and may provide an important clue in the continuing efforts to understand the metal-insulator transition.

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