

XI

SUPERCONDUCTIVITY

11.1. Summary of properties (cf. Shoenberg, 1952)

In all other parts of this course we have been dealing with phenomena which were, at any rate qualitatively, well understood. This made it possible to present the theoretical treatment in a deductive manner, starting from general principles, although this did not reflect the actual historical development of the subject. In this chapter we are dealing with superconductivity, which cannot yet be claimed to be in a similar state. Within the last few years Fröhlich has proposed a picture of superconductivity which is very attractive, and very probably provides the right explanation, but the mathematical formulation is as yet incomplete, and it is therefore not certain that this model will lead to all the known features of superconductors without bringing in new effects that have at present been neglected.

I shall therefore start by outlining the main observed facts, and thus show the main results that must be derived from a complete theory of superconductivity, and I shall then review the theory of Fröhlich.

Superconducting metals have a transition temperature T_C , usually a few degrees, above which they behave like normal metals. As the temperature is lowered below T_C , the electrical resistance disappears suddenly. For pure metals the transition is extremely sharp, though inhomogeneities in structure or composition may spread it out.

Below the transition point the resistance appears to be zero, not merely very small. This was first demonstrated by inducing a current in a lead ring. The current continued for hours, in fact as long as it was practically possible to maintain the helium bath that kept the temperature low. This suggested that the state in which there was an electric current was a true equilibrium state which would last indefinitely, and this interpretation is borne out by the existence of the Meissner effect.

If an external magnetic field is applied to a superconducting ring, it will induce a current in the ring. The magnitude of this current is obvious from the fact that any change of the magnetic flux through the inside of the ring would result in an electromotive force in the metal. Since no electric field can exist in a substance of infinite conductivity, it follows that the magnetic flux encircled by the ring remains zero, and hence the

flux due to the ring current must be equal and opposite to the applied field times the area of the ring.

Alternatively, if the ring is cooled in an external magnetic field, so that it passes into the superconducting state with a field present, the flux through the ring must remain constant from then on, and if the field is then removed, the flux is 'trapped' and there will be a current (equal and opposite to that of the previous experiment) to maintain this flux.

Meissner was the first to perform such experiments with solid objects, such as a superconducting sphere. Again, if the sphere is first cooled and a field is then applied, the lines of force do not penetrate into the material. The magnetic field inside the sphere therefore remains zero, and this means there must be surface currents flowing which compensate the external field. The sphere behaves like a diamagnetic body of permeability zero, or susceptibility $-1/4\pi$.

This much could have been foreseen from the infinite conductivity. However, if the sphere is cooled in the field, one finds that the final result is the same as before, i.e. surface currents start which reduce the field inside the sphere to zero.

This result, which is known as the Meissner effect, shows that the analogy between the superconducting sphere (or any other singly connected body) and a substance of zero permeability is complete, and that under changes of field and temperature the behaviour is reversible. This again shows that the state in which there are currents is a truly stationary state.

The irreversible behaviour of a ring is due to the fact that, once the ring is superconducting, no further changes of the flux through it are possible.

Superconductivity is destroyed when the field on the surface exceeds a certain critical strength. This critical field, which in general is of the order of a few hundred gauss, is a function of the temperature, and vanishes when the temperature reaches T_C . Its behaviour is approximately described by the relation

$$H_C = A(T_C^2 - T^2), \quad (11.1)$$

where A is a constant.

This also means that a superconducting wire is capable of carrying only a limited current, since stronger currents would produce fields which, on the surface of the wire, would exceed H_C .

The specific heat of a superconductor is discontinuous at the transition point, but, in the absence of a magnetic field, there is no latent heat.

If the transition takes place in a magnetic field, there is a finite latent heat.

These facts can be related to each other by applying thermodynamics. If the free energy per unit volume of the superconducting state without magnetic field is F_s , it will in a magnetic field be

$$F = F_s + \frac{1}{8\pi} H^2. \quad (11.2)$$

This follows from the fact that the induced moment per unit volume, which is

$$M = -\frac{\partial F}{\partial H}, \quad (11.3)$$

must equal $-H/4\pi$, to account for the Meissner effect.

At first sight one might believe that in the magnetic field the energy of a volume in which the field vanishes should be *lower* than that of a normal body because the term $H^2/8\pi$ in the electromagnetic energy density inside the body is missing. However, this reasoning neglects the fact that the flux which would have gone through the body must now take some other path, and therefore will increase the field energy elsewhere; if this is taken into account correctly, the result agrees with (11.2).

The transition to the normal state occurs when the free energy (11.2) equals that of the normal state F_n . Hence

$$F_s - F_n = -\frac{1}{8\pi} H_C^2, \quad (11.4)$$

and by differentiation, using the usual thermodynamic relations,

$$S_s - S_n = \frac{1}{8\pi} \frac{d}{dT} (H_C^2), \quad (11.5)$$

and

$$E_s - E_n = \frac{T^2}{8\pi} \frac{d}{dT} \left(\frac{H_C^2}{T} \right). \quad (11.6)$$

The difference in the specific heats can be obtained by a further differentiation. These relations are well confirmed by experiment, thus justifying the assumption that we are dealing with equilibrium states.

Since H_C decreases with increasing temperature, the entropy and the energy content of the superconducting state are less than those of the normal state. At these low temperatures an important part of the entropy is that due to the conduction electrons, which is proportional to T . From the observed values of H_C , (11.5) contains a part proportional to T , which just cancels the electronic entropy of the normal state, leaving only a part proportional to a higher power of T , probably T^3 . Thus the

entropy of the electrons is reduced, and they are in a more ordered state.

The thermodynamic argument presupposes that as long as the metal is superconducting the magnetic field inside it necessarily vanishes.

In actual fact, the magnetic field must penetrate into a thin surface layer of the metal since the surface currents which shield the inside from the external field cannot be concentrated into a mathematical surface, but will spread over a layer of finite thickness. This penetration depth has been studied in many different experiments, and its magnitude is of the order of 10^{-5} cm.

The only other properties of the metal that show an anomalous behaviour at the transition point are the thermal conductivity, which, for a pure metal, is lower in the superconducting than in the normal state, and the thermoelectric effects, which disappear.

So far the main features to be explained by a theoretical model are:

- (1) The existence of a transition to a state of lower entropy, and of permeability zero.
- (2) The ability of the superconductor to carry circulating currents, if it is not singly connected, or to transmit currents without potential difference, if it is part of a circuit with normal components. It is often stated that (2) is a consequence of (1), but this has never been proved.
- (3) Quantitative relationships, including the magnitude of the transition temperature and the critical field, as well as the penetration depth.

In a study of superconductors of different isotopic composition, it was discovered (Maxwell, 1950; Reynolds and others, 1950) that the transition temperature of a superconducting metal is different for different isotopes, and varies approximately as the inverse root of the nuclear mass. This shows directly that the mechanism of superconductivity must depend in some essential way on the motion of the atoms in the lattice, and this at once disproves all earlier theories which tried to explain the phenomenon in terms of the interaction of free electrons, or electrons in a rigid potential field, which would correspond to infinitely heavy atoms.

On the other hand, a new theory was put forward by Fröhlich just before the discovery of the isotope effect, in which the main mechanism is the interaction between electrons and lattice waves. It is the only type of theory which can account for the isotope effect in a natural manner.

11.2. Outline of the Fröhlich-Bardeen theory

According to Fröhlich (1950), superconductivity is due to the interaction of the electrons with the lattice waves which, indirectly, results in an interaction between the electrons.

Consider first a single electron with wave vector \mathbf{k} in a lattice in which there are no phonons. The interaction with the lattice vibrations which was considered in Chapter VI then gives rise to the possibility of creating a phonon, with a corresponding change in the electron state to \mathbf{k}' . If in this transition energy is conserved, it becomes a real transition of the kind that we found important for the resistance of an ideal metal. Even where such real transitions are not possible, however, transitions in which energy is not conserved must still be allowed for as virtual transitions, i.e. the electron may spend a part of its time in the state \mathbf{k}' , and is then accompanied by a photon.

Such virtual transitions always result in a perturbation of the energy, the change in the energy of the electron state being

$$\Delta E(\mathbf{k}) = -\frac{\hbar}{2\pi} \sum_{\mathbf{k}', s} \frac{|(\mathbf{k}'|A|\mathbf{k}, s)|^2}{E(\mathbf{k}') + \hbar\omega(\mathbf{f}, s) - E(\mathbf{k})}, \quad (11.7)$$

where the matrix element in the numerator is given by (6.48), and \mathbf{f} is the wave vector of a phonon satisfying the conservation law (6.46). I have considered only transitions within the same band and omitted the band suffix l for simplicity.

Equation (11.7) is the standard result for second-order perturbation theory. The terms in the sum for which the denominator is positive correspond to virtual transitions to states in which the sum of phonon and electron energy is higher than $E(\mathbf{k})$, and they tend to make the perturbation energy negative, whereas terms with negative denominators raise the perturbed energy.

A vanishing denominator corresponds to a real transition. If such terms occur, the expression (11.7) is ambiguous, but one knows from general theory that one should interpret the sum by imagining that the denominator has an infinitesimal negative imaginary part. On replacing the summation by an integration, the result then may be expressed as the principal value of the integral, which is real and finite, plus one-half the residue at the pole, which gives an imaginary contribution. An imaginary part of the energy of the state \mathbf{k} represents an exponential decay with time, and therefore expresses the fact that, because of real collisions, the electron only spends a finite time in the state \mathbf{k} .

In discussing the properties of the lowest state of the whole system,

such real collisions must, however, be unimportant, since in the lowest state no energy is available for the emission of a phonon.

As it stands, the energy perturbation (11.7) is of no great interest since it amounts merely to a very small correction to the electronic energy function, which, in any case, is not known to high accuracy.

However, as Fröhlich noticed, the same effect causes an interaction between the electrons. If several electrons are present, we must omit from the sum in (11.7) any states \mathbf{k}' which are already occupied, since by Pauli's principle an electron cannot make a transition, either real or virtual, to an occupied state. Hence the energy correction for all electrons becomes, in this approximation,

$$\frac{\hbar}{2\pi} \sum_{\mathbf{k}, \mathbf{k}', s} \frac{|(\mathbf{k}'|A|\mathbf{k}, s)|^2 n(\mathbf{k}) \{1 - n(\mathbf{k}')\}}{E(\mathbf{k}) - E(\mathbf{k}') - \hbar\omega(\mathbf{f}, s)}, \quad (11.8)$$

where $n(\mathbf{k})$ is the occupation number.

We now divide this expression into two parts. The part due to the 1 in the numerator is just the sum of (11.7) over all occupied states, and is therefore equivalent to a small adjustment in the energy function. This leaves

$$-\frac{\hbar}{2\pi} \sum_{\mathbf{k}, \mathbf{k}', s} \frac{|(\mathbf{k}'|A|\mathbf{k}, s)|^2 n(\mathbf{k}) n(\mathbf{k}')}{E(\mathbf{k}) - E(\mathbf{k}') - \hbar\omega(\mathbf{f}, s)}. \quad (11.9)$$

This expression may be written in a more symmetrical form by combining each term with the one in which \mathbf{k} and \mathbf{k}' are interchanged. These two terms have the same matrix element, and belong to opposite phonon wave vectors, which have the same frequency. This gives

$$-\frac{\hbar}{2\pi} \sum_{\mathbf{k}, \mathbf{k}', s} \frac{\hbar\omega(\mathbf{f}, s) |(\mathbf{k}'|A|\mathbf{k}, s)|^2 n(\mathbf{k}) n(\mathbf{k}')}{\{E(\mathbf{k}') - E(\mathbf{k})\}^2 - \{\hbar\omega(\mathbf{f}, s)\}^2}. \quad (11.10)$$

The value of this expression is now sensitive to the distribution of the electrons. Two electrons of the same energy, but moving in different directions, contribute to (11.10) a positive term. However, if their energy is slightly different, by just the amount which corresponds to a real transition, the denominator changes sign. For all greater energy differences the denominator remains positive, thus giving a reduction of the energy.

This interaction therefore favours electron arrangements in which fewer electrons occupy immediately adjacent points in \mathbf{k} space, and more of them are separated by distances sufficient to make the denominator of (11.10) negative.

Fröhlich has evaluated the integral arising from (11.10) for the case in which the electrons may be regarded as free, and the lattice waves

as those of an isotropic medium. He calculates the interaction energy for a state in which the electrons occupy a sphere about the origin in k space, and in addition a concentric shell, with the inner radius of the shell larger than that of the sphere by a gap a , equal to the thickness of the shell. For $a = 0$, this reduces to the solid sphere, and hence gives the Fermi distribution at $T = 0$. Fröhlich finds that the interaction energy is lower for a finite gap width than for $a = 0$. The most favourable value of a corresponds to energy differences of the same order as found in electron-phonon collisions, and the ratio of the width a of the gap to the radius of the sphere is therefore of the order of the ratio of the velocity of sound to the electron velocity, i.e. of the order of 10^{-3} .

Hence the magnitude of the energy reduction can be estimated in the following way: the denominator in (11.10) is of the order of the square of a phonon energy; in the numerator the first factor is a phonon energy, and the square of the matrix element (including the factor \hbar) is, by the estimate (6.62), of the order of D , where D is an electronic energy. The number of terms making large contributions to (11.10) is given by the number of pairs of states which lie within the shell or within a similar surface layer of the main sphere. This makes the number proportional to $N^2(\hbar\omega)^2/D^3$, and the reduction in interaction energy of the order

$$\frac{N(\hbar\omega)^2}{D}. \quad (11.11)$$

On the other hand, the unperturbed electron energy is lowest for the usual Fermi distribution, i.e. for the solid sphere, and by creating the shell it is increased by an amount which is again of the order of (11.11), since we raised $N\hbar\omega/D$ electrons by energy amounts of the order $\hbar\omega$.

According to whether the change in the interaction energy or in the unperturbed energy is the greater, the sphere with shell or the solid sphere represents the state of lowest energy. It is not known whether it would be possible to construct a state of still lower energy, but the order of magnitude of the reduction would not exceed (11.11).

This argument suggests that certain metals have at $T = 0$ a state in which the electron distribution is rather different from the usual Fermi model. As the temperature is raised, the distribution remains at first similar, but the number of low excited states of the metal will be less than usual, since the energy is now very sensitive to the details of the electron distribution, so that a slight displacement of an electron in k space is of importance. It is therefore reasonable to expect the entropy of this state to be lower than that of the normal state;

as the temperature rises there will come a point where the normal state is stable. This question has not yet been investigated in detail.

The quantity (11.11) is certainly of the same order of magnitude as the observed energy difference (11.6) at $T = 0$. Since the electronic specific heat of the superconducting state is at low temperatures much less than that of the normal state, and since there is no latent heat at the transition point, the electronic energy content of the normal state at T_C should approximately equal the energy difference at $T = 0$. Using (11.11) for the latter, this gives also the right order of magnitude for T_C .

If we compare different isotopes, the quantity (11.11) is proportional to the inverse of the atomic mass, since the vibration frequencies vary inversely as the root of the mass, and since the thermal energy of the electrons in the normal state varies as T^2 , this makes T_C proportional to the inverse square root of the mass. This result is compatible with the observed isotope effect, though the power law $M^{-1/2}$ is not yet established with accuracy.

A theory based on essentially the same mechanism, but using a different method of approximation, has been given by Bardeen (1950) (see also Bardeen, 1951). In this the use of the perturbation formula (11.7), which for small energy denominators is very questionable, is avoided by using instead a variational method. The results are very similar to Fröhlich's. This method, however, proceeds by constructing a separate wave function for each electron in interaction with the lattice vibrations, using wave functions which are not orthogonal to each other; it also cannot be claimed to be rigorous.

11.3. Effect of a magnetic field

A suggestion for describing the Meissner effect is due to London (1948). The interaction of an electron with a magnetic field of axial symmetry may be written (cf. (7.10))

$$\frac{e}{mc} A p_\phi + \frac{e^2 A^2}{2mc^2}, \quad (11.12)$$

where r is the distance from the axis, the vector potential has a component only in the azimuthal direction, and p_ϕ is the angular momentum about the axis. If the potential in which the electron moves has axial symmetry, then p_ϕ has discrete values. If we fix our attention on a state with $p_\phi = 0$, or on a number of electrons for which the average value of p_ϕ vanishes, then their energy in the field is increased by the second term. If their density n in space is approximately constant, the total

energy density would then depend on the field by a term

$$\frac{ne^2A^2}{2mc^2}, \quad (11.13)$$

and the current density would be

$$j = -\frac{\partial E}{\partial A} = -\frac{ne^2A}{mc^2}. \quad (11.14)$$

If this result is inserted in Maxwell's equations, one finds that the vector potential decreases exponentially from the surface of the body inwards, the mean penetration depth being of the order

$$\left(\frac{mc^2}{4\pi ne^2}\right)^{\frac{1}{2}}.$$

This consideration is used in an attempt by Fröhlich (1951) to show that his model leads to a result similar to (11.14). He argues that in a normal metal the term (11.13) is compensated by the fact that electrons are transferred from states of positive p_ϕ to states of negative p_ϕ even though, to satisfy the Pauli principle, this requires an increase in their kinetic energy. The net result is just to cancel the energy change, except for the small diamagnetism of a normal metal. If the electron energy is more sensitive to the electron distribution, it would require more energy to redistribute the electrons, and therefore the term (11.13) would no longer cancel exactly, and a fraction of the current (11.14) would remain, thus giving a Meissner effect with a somewhat greater penetration depth.

This argument overlooks the fact that the interaction of the electrons with the lattice waves no longer leaves the angular momentum of the electrons constant; the first term in (11.12) therefore is no longer just a constant energy shift for each state, but causes a perturbation which gives also second-order contributions proportional to A^2 . It would have to be shown that these do not cancel the remainder of (11.13).

An alternative explanation is proposed by Bardeen (1950) who regards the Meissner effect as the case of a strong electronic diamagnetism. Since, according to Chapter VII, the diamagnetic susceptibility is inversely proportional to the square of the effective mass of the electrons, a decrease of this mass by a factor of the order 10^3 would make the susceptibility of the order of $-1/4\pi$ as required for the Meissner effect. Since on the present model of superconductivity it takes much more energy than usual to change the state of an electron, this is equivalent to a very small effective mass. However, the use of the simple formula for the susceptibility in such circumstances is not justified, since the induced moment is proportional to the actual field at each point. In the language of the phenomenological equations, the induced moment should

in this case be expressed as χB , rather than χH . In the case of ordinary diamagnetism, this difference is unimportant, but it follows that a negative infinite χ would be required to make the permeability zero.

This also follows from the fact that otherwise an even smaller effective mass might make the permeability negative, which would not make sense.

11.4. Objections and difficulties

In spite of the many encouraging features of the model, there remain a number of serious difficulties.

(1) The use of perturbation theory is very doubtful when the perturbation is large enough to reverse the order of different states of the system. This is particularly serious when one is dealing with a continuous spectrum for each electron. The expression (11.9) may be interpreted as a collision in which the virtual phonon emitted by one electron is absorbed by another in such a manner that the electrons exchange their wave vectors. Because of the identity of the electrons, this still leaves the system in the original state. Evidently, however, a collision in which the electrons exchange a phonon and both change their states in a more general way may still conserve energy, and therefore is of equal importance. To take such terms into account one would, however, have to start from a wave function which allows for correlations between the electrons. Even the variation method of Bardeen, which avoids perturbation theory to some extent, does not seem capable of handling this problem.

A more recent paper by Fröhlich (1952) substitutes a canonical transformation for perturbation theory. Since, however, a great number of approximations are made in carrying this out, it is not yet clear whether the result is, in fact, more accurate.

(2) According to Wentzel (1951), the same interaction which leads to (11.7) also leads to a dependence of the perturbed energy on the number of phonons present. If the coupling is strong enough to satisfy Fröhlich's criterion for superconductivity, the perturbation decreases sufficiently strongly with phonon number to outweigh the unperturbed phonon energy. In other words, the energy of a phonon would now be negative, implying an instability of the lattice.

For a one-dimensional model this result was obtained by Wentzel without using perturbation theory. There is a close connexion between this result and the remark in § 5.3 that any deviation from perfect translational symmetry of a linear chain of unsaturated atoms must lead to a decrease of energy. The difference between these points of view is that in § 5.3 we were discussing the effect of a static potential, whereas

Wentzel's argument allows for the motion of the atoms. In the simpler case of atoms at rest, it is clear that, as in Jones's theory of bismuth, the effect will lead merely to a new equilibrium configuration of a less regular structure, and it is likely that the same would happen in Wentzel's model if one were to treat the forces between the atoms more exactly than to the harmonic approximation. However, one would then not be justified in using a treatment in which the atomic displacements from the regular structure were regarded as small.

We saw in § 5.3 that this effect tended to alter the band structure in such a way as to have the actual electron gas just fill one band completely, and at first sight this suggests that the resulting state of lower entropy should have a poor conductivity rather than superconductivity. It is therefore particularly important to examine carefully whether the 'superconducting' state of the Fröhlich model really possesses the electric and magnetic properties of a superconductor.

(3) The model results in an indirect interaction between electrons, with phonons as intermediaries. This is a very weak interaction, and therefore does fit in with the very low transition temperatures and small energy differences observed in practice. However, it seems inconsistent to include this small effect and neglect other causes of interaction, in particular the electrostatic interaction between the electrons, which also produces correlations between them, unless one can see that the indirect interaction has qualitative consequences which are insensitive to the presence of electrostatic forces.

In fact, Heisenberg (1947) had proposed an alternative theory based entirely on the electrostatic interaction, which also seemed capable of producing a change of state at very low temperatures. The method used is perturbation theory of a type similar to that of the preceding section, and therefore open to the same objections. From the isotope effect, it is now clear that the electrostatic effect, which is independent of the motion of the nuclei, cannot by itself be the cause of superconductivity, but its influence on the indirect interaction remains to be discussed.

Wentzel suggests, in fact, that the electrostatic interaction is important to correct the lattice instability to which he has drawn attention, but it is doubtful whether it is necessary for this purpose, and whether it could do this.

These brief remarks on the most fascinating problem of the theory of solids will have to suffice. With the isotope effect as clue, and the Fröhlich-Bardeen model as a starting-point, further progress may be expected in this field.

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For a more complete introduction to large parts of the subject the following books and articles may be used:

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In addition to these, for further information on the subject matter of each chapter, the reader is referred to:

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I to III BORN and HUANG (1954)
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LIST OF SYMBOLS

Symbols are not listed here if they occur only in immediate context with a statement defining them. Conventional mathematical symbols are not listed. References to defining equations are given only when a verbal statement of the meaning is inconvenient, or inadequate.

<i>Symbol</i>	<i>Meaning</i>	<i>Page</i>	<i>Equation</i>	<i>Used only in</i>
a	lattice spacing	1		
\mathbf{a}_n	lattice vector	4		
$A_{n,n'}$	coefficient of harmonic term (linear chain)	11		
$A_{j,n;j',n'}$	coefficient of harmonic term (general)	14		
\mathbf{A}	vector potential			
$(\dots A \dots)$	scattering matrix element	126		
$b(\dots)$	coefficient of cubic term in terms of normal coordinates	36		
$b(\dots)$		38	(2.49)	
$B_{n,n',n''}$	coefficient of cubic term (linear chain)	11		
$B_{j,n;j',n';j'',n''}$	coefficient of cubic term (general)	14		
c	velocity of sound			
c	velocity of light			Chapters III, VII, X
$c(\dots)$	coefficient of quartic term in terms of normal coordinates	39		
$c(\dots)$		39	(2.54)	
O	reduced interaction constant	130		
\mathbf{d}_j	position of atom in cell	4		
D	order of magnitude of band width	130		Chapter VI
D	diffusion coefficient			Chapter X
e	electron charge			
e_i	charge of ion			
E	energy			
$E_1(\mathbf{k})$	energy of electron state	77		
\mathbf{E}	electric vector			§ 9.1
\mathbf{f}	wave vector of lattice wave	15		
$f(E)$	Fermi function	91		
F	free energy			
F	form factor			§ 3.2
F	electric field intensity			
$g(\dots)$	distortion of phonon distribution	49		Chapter II
$g(\dots)$	distortion of electron distribution	128		Chapter VI
$G(\dots)$	distortion of phonon distribution	128		Chapter VI
\mathbf{G}	Fourier transform of force constants	15		
\mathbf{g}	wave vector of spin wave	168		
\hbar	$= \hbar/2\pi$			
h	Planck's constant			
H	magnetic field intensity			
I	electron interaction	132		
j	label of site in cell	4		
\mathbf{J}	total wave vector	42		Chapter II
\mathbf{J}	current density	118		Chapters VI, VII
k	Boltzmann's constant			
\mathbf{k}	light or neutron wave vector	55		Chapter III, § 8.5
\mathbf{k}	electron wave vector	77		
\mathbf{K}	reciprocal lattice vector	16		
L	linear dimension of crystal	15		
l	band label	77		

Symbol	Meaning	Page	Equation	Used only in
m	electron mass			
m^*	effective mass	90		
M	atomic mass	11		
$M(N)$	mass of crystal	19		
M^0	mass of unit cell	19		
n	cell label	1		
$n_i(\mathbf{k})$	number of electrons per state	117		
N	total number of cells			
$N(\dots)$	phonon number	25		
\mathbf{p}	electron momentum	83		§ 4.3
q	normal coordinate	17		
\mathbf{Q}	wave vector difference	59		
Q	normal coordinate of progressive wave	17		
\mathbf{r}	general radius vector			
r	number of atoms per cell	4		
\mathbf{R}	position of atom			
R	Hall constant			§§ 7.4, 10.3
s	type of lattice wave	17		
\mathbf{S}	energy transport	120		
T	kinetic energy			§ 1.7
T	temperature			except in § 1.7
\mathbf{u}	atomic displacement	11		
$\psi_{\mathbf{k},i}$	wave function without phase factor	77	(4.9)	
U	potential energy	7		
$U(\mathbf{r})$	atomic potential	79		
V	volume of crystal	16		
$V(\mathbf{r})$	potential energy of electron	76		
\mathbf{v}	electron velocity	87		
$\mathbf{v}_j(\mathbf{f}, s)$	relative amplitude in normal mode	17		
W	work function	97		
x, y, z	coordinates			
Z	partition function	35		Chapter II
Z	number of electron states	93		except Chapter II
α	interaction integral	166		
β	$= 1/kT$			
β	overlap integral	166		Chapter VIII
γ	exchange integral	166		Chapter VIII
$\delta(x)$	Dirac delta function			
δ_{ik}	Kronecker symbol			
Δ	generalized Kronecker symbol	38	(2.50)	
ϵ	polarization vector			
η	Fermi energy	92		
Θ	Debye temperature	30		Chapters II, VI
Θ	Curie temperature	162		Chapter VIII
κ	thermal conductivity			
λ	mean free path			
μ	Bohr magneton			
μ	component of electron spin	165		§ 8.2
ν	general label for atomic coordinate	19		
ρ	density			
ρ	surface density of electron states	116		§ 6.1
σ	label for progressive lattice waves	17		
σ	electric conductivity			
τ	collision time	116		
ϕ	scalar potential			§ 10.4
ϕ	general label for normal coordinate	19		Chapter I
$\phi(\mathbf{r})$	atomic wave function			§ 4.2, Chapter VIII
ψ	electron wave function	76		
ω	frequency			
$\omega(\mathbf{f}, s)$	frequency of lattice wave	17		
Ω	Larmor frequency	145		Chapter VII

INDEX

(A bold-face page number indicates the main reference or the explanation of the term.)

- absorption: of infra-red radiation, 54; of light, 188; of X-rays, 188.
- edge, 192.
- acceleration, 87, 156.
- acceptor, 197.
- acoustical branch, 30, 56.
- adiabatic approximation, 4, 111.
- alkali halides, 21; *see* NaCl type lattice.
- alkalis, 6, 138, 159.
- anharmonic terms, 31.
- anti-ferromagnetism, 181.
- anti-Stokes line, 69.

- background scattering, 64.
- band, 81, 86, 89, 197.
- basic cell (of reciprocal lattice), 17, 36, 77.
- bismuth, 113, 152.
- Bloch theorem, 75, 126, 130, 131.
- blocking layer, 205.
- body-centred cubic lattice, 2, 16, 82, 104, 171.
- Bohr magneton, 143.
- Boltzmann equation, 45, 117, 127, 156, 160.
- boundary region, 121, 132.
- Bragg's law, 61, 84, 87, 112.
- Brillouin zone, 77, 87.
- bulk modulus, 24, 32.

- centre of symmetry, 7.
- characteristic temperature, 30, 133, 135.
- coherent scattering, 59.
- collective electron model, 173, 178.
- collision time, 115, 116, 120, 133, 139, 156, 184, 201.
- collisions: between electrons, 131, 138; between electrons and phonons, 126, 136, 217; between phonons, 41, 46, 53, 133, 136.
- commutation law, 24, 151.
- compressibility, 24, 101.
- conservation laws, 43, 128.
- contact potential, 97.
- cores, 96, 102.
- correlation, 103, 106, 176.
- critical field, 212.
- cubic terms, 11, 32.
- Curie point, 162, 173.
- cyclic condition, 13, 15, 76, 146.
- Debye model, 29, 33, 136.
- Debye-Scherrer method, 61.
- Debye temperature, *see* characteristic temperature.
- degenerate electron levels, 76, 81.
- Fermi gas, 92, 144, 200.
- density matrix, 47.
- detailed balancing, 116.
- diamagnetic susceptibility, 149, 152, 219.
- diamagnetism, 144.
- diamond, 6, 10.
- dielectric constant, 185.
- diffraction of X-rays, 58, 112.
- diffusion coefficient, 204.
- distorted structure, 108, 112.
- domain, 161, 179.
- donor, 197.
- double layer, 96, 104.
- drift of phonons, 137.
- Dulong-Petit, 30.
- duration of collision, 124.
- dynamical theory, 63.

- effective mass, 91, 106, 152, 199, 219.
- Einstein relation, 204.
- elastic constants, 22, 101, 108.
- electric conductivity, 118, 134, 136.
- — of semi-conductors, 199.
- current, 118.
- field, 88, 120.
- electron diffraction, 86.
- electrostatic forces between ions, 9, 57.
- emission of X-rays, 189.
- energy of vibration, 20, 25.
- surface, 116.
- transfer in collisions, 135.
- transport, 42, 51, 120.
- entropy, 213, 217.
- exchange, 163, 182.
- integral, 167, 174, 181.
- exciton, 194.

- face-centred cubic lattice, 2, 16, 104, 171.
- Fermi-Dirac statistics, 91.
- Fermi energy, 95, 97, 199.
- function, 92.
- surface, 113, 138.
- ferromagnetic susceptibility, 180.
- ferromagnetism, 161.
- form factor, 59, 62.

- Fourier coefficients, 63.
 four-phonon processes, 43, 51.
 fourth-order terms, 36, 39, 43, 51.
 free electrons, 83, 118, 186, 192.
 — energy, 31, 145, 213.
 frequency of lattice vibrations, 13, 17, 28.
 Fröhlich theory of superconductivity, 114, 215.

 graphite, 10.
 Grüneisen's law, 33.

 de Haas-van Alphen effect, 154.
 Hall effect, 155, 202.
 harmonic oscillator, 35.
 Hartree method, 102.
 Heitler-London model, 164, 193.
 hexagonal close-packed lattice, 3, 32.
 hole, 91, 159, 193.
 homopolar bond, 6, 10.
 hysteresis, 180.

 imperfection and impurities, 40, 52, 115, 121, 134.
 impurity semi-conductor, 198.
 infra-red absorption, 54.
 insulator, 89, 197.
 interaction between electrons, 89, 103, 115, 132, 164, 173, 198, 221.
 ionic lattice, 6, 9, 31, 54, 194.
 isotope effect, 214, 218.

 Jones theory of Bi, 113, 155, 221.
 Joule heat, 117.

 Larmor frequency, 145, 157.
 lattice, 1.
 — vector, 1.
 Laue method, 61.
 light waves, 26, 184.
 linear chain, 11, 67, 79, 83, 108, 165, 183.
 longitudinal waves, 21, 43, 44, 51.
 Lorentz-Lorenz factor, 187.

 magnetic field, 88, 143.
 magnetization, 179.
 magneto-resistance, 155.
 mean free path, 40, 52, 116, 202.
 Meissner effect, 211, 218.
 metal, 6, 75, 89, 197.
 metallic bond, 11, 101.
 mobility, 158.
 molecular field, 161.
 — solid, 6, 31.
 mosaic structure, 8.

 NaCl type lattice, 3, 32, 192.
 neutrons, 70, 177.
 non-diagonal terms, 151, 154.

 non-metal, 6, 75.
 normal coordinates, 14, 15, 36, 126.
 — vibrations, 12, 15, 17.

 optical branch, 30, 56.
 orthogonality, 18.
 overlap, 10, 166, 182, 183.

 paramagnetism, 143.
 Pauli principle, 10, 89, 102, 103, 116, 216.
 penetration depth, 214.
 periodic potential, 75.
 perturbation theory, 40, 80, 83, 124, 141, 215, 220.
 phase relations, 47, 123.
 phonon, 26, 41, 54.
 phosphor, 209.
 photoelectric effect, 190.
 photon, 55, 70, 188, 191.
 positron theory, 91.
 potential barrier, 83, 99.

 quantum number, 25.
 quartic terms, *see* fourth-order terms.

 Raman effect, 55, 70.
 reciprocal lattice, 16, 36, 41, 60, 77, 126.
 recombination, 208.
 rectification, 205.
 rotating crystal method, 61.

 scattering: of light, 55, 68; of neutrons, 70, 177; of phonons, 52.
 screening, 131.
 semi-conductor 76, 197.
 shear, 7.
 simple cubic lattice, 1, 16, 171.
 — hexagonal lattice, 2.
 size effect, 53.
 sound velocity, 29, 40.
 — waves, 21, 26, 53.
 specific heat: of lattice, 27, 34; of electrons, 93; of superconductor, 212.
 spin, 89, 143, 161.
 — complex, 169.
 — waves, 164, 178.
 stability of equilibrium, 7, 220.
 standing waves, 85.
 Stokes line, 69.
 Stosszahl-Ansatz, 123.
 strain, 24, 34.
 strongly bound electrons, 79, 164.
 superconductivity, 211.
 surface, 95, 187, 190, 193.
 — waves, 100.

 temperature gradient, 46, 120.
 thermal conductivity: of electrons, 120, 138; of lattice, 40, 45, 128.
 — expansion, 31, 39.

- thermionic emission, 98.
 - transition probability, 40, 55, 115, 122, 126, 189.
 - temperature, 211.
 - translational symmetry, 75.
 - translation group, 1.
 - transverse waves, 21, 43, 44.
 - trapping, 196, 209.
 - two-dimensional crystal, 67, 183.

 - Umklapp process, 41, 50, 53, 137, 139.
 - uncertainty principle, 45.

 - variation principle, 80.
- velocity of electron, 87.
 - volume effect, 192.

 - van der Waals forces, 9.
 - wave number, 13.
 - vector, 15, 43, 126, 188, 193.
 - packet, 45, 87, 120, 156.
 - Weiss model, 161, 168, 181.
 - Wiedemann-Franz law, 121, 134.
 - Wigner-Seitz method, 104, 189.
 - work function, 97.

 - zero-point energy, 26, 27, 135.