Glassy Behavior of Random Field Magnets

A. A. Pastor and V. Dobrosavljević

Department of Physics and National High Magnetic Field Laboratory
Florida State University
Tallahassee, FL 32306

The key unresolved feature of the random field Ising model is the complex, history dependent behavior found both in experiments and numerical simulations. It appears that for strong enough disorder, and low temperatures, the system falls out of equilibrium, displaying anomalously slow relaxation and other features characteristic of glassy materials. Recently, it has been proposed that this behavior reflects the emergence of a low-temperature glassy phase, which can be mathematically described as a replica-symmetry breaking instability. In this study, we use a $1/z$ expansion approach (z is the coordination number), and examine the glassy behavior of the random-field Ising model.

The random field Ising model (RFIM) is one of the simplest models of disordered interacting systems. It is believed to describe the behavior of diluted antiferromagnets in strong magnetic fields [1], but it is also likely to be crucial for understanding some aspects of the metal-insulator transition in disordered electronic systems [2]. In recent years, there have been many theoretical [3] and experimental [4] indications that the essential difficulties in unraveling the behavior of the RFIM may be related to the existence of many metastable states and the associated glassy phase, similar as that found in spin glass materials [5].

Standard approaches, such as the Bragg-Williams mean field theory (MFT), when used to examine the RFIM predicted only the existence of the ferromagnetic (FM) and the paramagnetic (PM) phases, but failed to identify a spin-glass (SG) phase. Very recently, by including fluctuations beyond MFT using a $1/N$ approach, Mezard and Young [3] were successful in predicting the SG phase, based on an appropriate replica symmetry breaking scheme. However, this formulation is fairly complicated, making it difficult to obtain simple analytical results or to extend the theory to more complex situations.

In this work, we propose an approach based on a $1/z$ expansion [6] (z is the coordination number), that is technically simpler and more convenient for generalizations, but reduces to standard MFT when $z \to \infty$. Our approach is able to identify a SG phase and allows us to study its properties for high disorder. For simplicity, we concentrate on a Bethe lattice, but to leading nontrivial order in $1/z$, the results will be valid [6] for *any lattice*.

We consider an Ising Hamiltonian

$$H_{int} = -\sum_{\langle ij \rangle} J_{ij} S_i S_j - \sum_i h_i S_i. \tag{1}$$

Here, $S_i = \pm 1$, and $J_{ij} = \frac{J}{m}$ are uniform exchange interactions between nearest neighbor sites, rescaled with $m = z - 1$ in order to obtain a finite result as $z \to \infty$. The random magnetic fields h_i are assumed to be Gaussian distributed, with zero mean and a variance $\langle h_i^2 \rangle = H_{RF}^2$.

Using standard replica methods[5], we can formally average over disorder, and the resulting partition function takes the form ($\alpha = 1, ..., n;\ \ n \to 0$)

$$Z^n = \mathrm{Tr} \exp \left[\frac{\beta J}{m} \sum_{\alpha=1}^{n} \sum_{\langle ij \rangle} S_i^\alpha S_j^\alpha + \frac{1}{2} (\beta H_{RF})^2 \sum_i (\sum_{\alpha=1}^{n} S_i^\alpha)^2 \right]. \tag{2}$$

We proceed by taking advantage of the tree-like structure of the Bethe lattice, by formally summing over all the degrees of freedom in one branch. The resulting expression $\Phi(S_0^\alpha)$ is a function only of the variable S_0^α at the branch origin, and can be easily seen to obey the following self-consistency equation

$$\Phi(S_0^\alpha) = \mathrm{Tr}_{S_1^\alpha} \left(\exp \left[\frac{\beta J}{m} \sum_{\alpha=1}^{n} S_0^\alpha S_1^\alpha + \frac{1}{2} (\beta H_{RF})^2 (\sum_{\alpha=1}^{n} S_1^\alpha)^2 \right] \Phi^m(S_1^\alpha) \right). \tag{3}$$

In order to generate an $1/m$ expansion, it is convenient to define a single-site effective action by

$$L[S^\alpha] \equiv -\frac{1}{2} (\beta H_{RF})^2 (\sum_{\alpha=1}^{n} S_1^\alpha)^2 - \ln(\Phi^m(S^\alpha)), \tag{4}$$

which to first order in $1/m$ (dropping constant terms) takes the form

$$L[S^\alpha] = -\beta J \sum_{\alpha=1}^{n} S^\alpha h^\alpha - (\beta H_{RF})^2 \sum_{\alpha<\beta} S^\alpha S^\beta - \frac{1}{m} (\beta J)^2 \sum_{\alpha<\beta} S^\alpha S^\beta \left[q^{\alpha\beta} - (h^\alpha)^2 \right].$$

This procedure automatically defines the order parameters h^α and $q^{\alpha\beta}$, which from Eq. (3) satisfy the following set of self-consistency conditions

$$h^\alpha = \langle S^\alpha \rangle_{L[S]}; \quad q^{\alpha\beta} = \langle S^\alpha S^\beta \rangle_{L[S]}. \tag{5}$$

It is not difficult to see that these order parameters in fact coincide (to the considered order in $1/m$) with the magnetization and the Edwards-Anderson order parameter [5], respectively.

We continue by concentrating on the replica symmetric solution of these equations, which take the form

$$h = \int Dz \tanh(\sqrt{(\beta H_{RF})^2 + \frac{(\beta J)^2}{m}(q-h^2)}z + \beta J h)$$
$$q = \int Dz \tanh^2(\sqrt{(\beta H_{RF})^2 + \frac{(\beta J)^2}{m}(q-h^2)}z + \beta J h), \qquad (6)$$

where $Dz = dz \exp(-z^2/2)/\sqrt{2\pi}$. The FM phase boundary is obtained by locating at a given value of H_{RF} the temperature where the magnetization h vanishes.

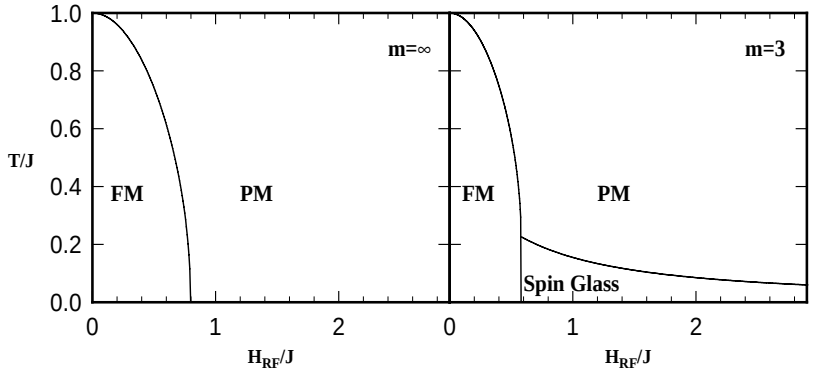

Figure 1: Phase diagram for coordination numbers $z = 4, \infty$ was obtained by solving set of equations(6,7) numerically. Notice that the SG phase does not exist for $z = \infty$.

Identifying the SG phase is more difficult. Similarly as for spin glass models in a uniform external field [5], in our case the Edwards-Anderson order parameter is nonzero for any temperature, and thus cannot be used to determine the phase boundary. Instead, we follow Mezard and Young [3], and look for an instability to replica symmetry breaking (RSB) within the PM phase. To do this, we can set $h = 0$, and note that the remaining equations for $q^{\alpha\beta}$ are very similar to those describing infinite-range spin glass models [5]. In particular, it is not difficult to see that they are in fact *identical* to those describing an infinite range (IR) model that is also described with the Hamiltonian of Eq. (1), but this time with J_{ij}'s being Gaussian random variables with zero mean and

variance $\langle J_{ij}^2 \rangle \equiv \frac{\tilde{J}^2}{N}$. To get precisely Eq. (4) we need to specify $\tilde{J} = J/\sqrt{m}$, where J is the interaction of the Bethe lattice model.

The main advantage of mapping our Bethe lattice model onto the described IR model follows from the fact that for the latter it proves easier to obtain an expression for the free energy as a function of the order parameters, and perform the required RSB stability analysis. The rest of the calculation can be carried out using standard methods [5,7], and we finally obtain the desired RSB stability boundary, that takes the form

$$1 = \frac{(\beta J)^2}{m} \int Dz \cosh^{-4}(\sqrt{(\beta H_{RF})^2 + \frac{(\beta J)^2 q}{m}} z). \qquad (7)$$

As expected, in the large coordination ($m \to \infty$) limit, the glass transition temperature vanishes, and our results reduce to standard MFT predictions. For finite m, the SG phase emerges, in agreement with the results of Mezard and Young [3] and experiments [4]. The above equations can easily be solved numerically, and the results are shown in Fig. 1. It is interesting that our glass transition temperature decreases only *slowly* with the random field strength, as $T_G \approx \frac{4J^2}{3mH_{RF}} \sim 1/H_{RF}$, as $H_{RF} \to \infty$, in contrast to the form of the de Almeida-Thouless line [7,5](IR spin glasses in a uniform field) where $T_G \sim \exp(-H^2/2J^2)$. This fact could be particularly significant for strongly disordered electronic systems [2,8], where it would suggest the possibility to observe the glassy behavior of electrons at finite temperatures.

In summary, we have argued that a $1/z$ expansion to first non-trivial order establishes the existence of a SG phase for the RFIM. This method would be easier to extend to quantum mechanical models than the $1/N$ approaches[3] and thus may be useful for the study of metal-insulator transitions in disordered electronic systems [8].

References

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