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# **Less Resources, More Outcomes**

# 1. Summary

In this paper, we regard the health care system (HCS) as a system with input and output, representing total expenditure on health and its goal attainment respectively. Our goal is to minimize the total expenditure on health to archive the same or maximize the attainment under given expenditure.

First, five output metrics and six input metrics are specified. Output metrics are overall level of health, distribution of health in the population, etc. Input metrics are physician density per 1000 population, private prepaid plans as % private expenditure on health, etc.

Second, to evaluate the effectiveness of HCS, two evaluation systems are employed in this paper:

- Evaluation of Absolute Effectiveness(EAE)
  - This evaluation system only deals with the output of HCS, and we define Absolute Total Score (ATS) to quantify the effectiveness. During the evaluation process, weighted average sum of the five output metrics is defined as ATS, and the fuzzy theory is also employed to help assess HCS.
- Evaluation of Relative Effectiveness(ERE)
  - This evaluation system deals with the output as well as its input, and also we define Relative Total Score (RTS) to quantify the effectiveness. The measurement to ATS is units of output produced by unit of input.

Applying the two kinds of evaluation system to evaluate HCS of 34 countries (USA included), we can find some countries which rank in a higher position in EAE get a relatively lower rank in ERE, such as Norway and USA, indicating that their HCS should have been able to archive more under their current resources.

Therefore, taking USA into consideration, we try to explore how the input influences the output and archive the goal: less input, more output. Then three models are constructed to our goal:

- Multiple Logistic Regression
  - We model the output as function of input by the logistic equation. In more detains, we model ATS (output) as the function of total expenditure on health system. By curve fitting, we estimate the parameters in logistic equation, and statistical test presents us a satisfactory result.
- Linear Optimization Model on minimizing the total expenditure on health
  - We try to minimize the total expenditure and at the same time archive the same, that is to get a ATS of 0.8116. We employ software to solve the model, and by the analysis of the results. We cut it to 2023.2 billion dollars, compared to the original data 2109.8 billion dollars.
- Linear Optimization Model on maximizing the attainment
- We try to maximize the attainment (absolute total score) under the same total expenditure in 2007. And we optimize the ATS to 0.8823, compared to the original data 0.8116.

Finally, we discuss strengths and weaknesses of our models and make necessary recommendations to the policy-makers  $\circ$ 

### 2. Introduction

Today and every day, the lives of vast numbers of people lie in the hands of health systems. From the safe delivery of a healthy baby to the care with dignity of the frail elderly, health systems have a vital and continuing responsibility to people throughout the lifespan. They are crucial to the healthy development of individuals, families and societies everywhere. Due to the irreplaceable role that the health care systems play in residents' life, better health care system is needed. "Improving performance" is therefore the key words today.

However, nowadays health care systems in many countries do not exhibit enough high effectiveness in guaranteeing residents' good health and a long life expectancy. In some countries, their government invests large amount of money on the health care systems, however, they didn't archive what they should have been to archive. We try to explore an optimized system in this paper.

### 3. Key Terminology

#### Health Care System (HCS)

Health Care System is such a system that has its input and output, representing total expenditure on health and its goal attainment respectively.

#### Evaluation of Absolute Effectiveness of Health Care System (EAE)

It is a kind of evaluation system that only considers the outcomes of the health care system, saying nothing to do with the input (resources), and adapts the outcomes as measurement to effectiveness.

#### Evaluation of Relative Effectiveness of Health Care System (ERE)

It is a kind of evaluation system that considers the outcomes of the health care system as well its inputs, and adapts units of output produced by unit of input as measurement to effectiveness.

#### Absolute Total Score (ATS)

Overall score for the evaluation of absolute effectiveness of health care systems

#### Relative Total Score (RTS)

Overall score for the evaluation of relative effectiveness of health care systems

#### Input Metrics (IM)

Metrics that are specified to assess input of HCS

#### Output Metrics (OM)

Metrics that are specified to assess output of HCS

# 4. Choosing output metrics for measuring health care system

Table 1. Notation for goals and metrics

Goals of Health System	Notation	Metrics for Evaluation	Notation
Health	U <sub>1</sub>	Overall level of health	$u_1$
		Distribution of health in the population	U <sub>2</sub>
Responsiveness	U <sub>2</sub>	Overall level of responsiveness	U <sub>3</sub>
		Distribution of responsiveness	$u_4$
Fairness in Finance	U <sub>3</sub>	Distribution of financial contribution	$u_5$

### 4.1 Goals of Health Care System

According to the World Health Report in 2000, the WHO pointed out the three goal of health care system, each goal with different priority [WHO 2000].

#### Better Health

Better health is unquestionably the primary goal of a health system, with the highest priority.

#### Fairness in financial contribution.

Fairness in financial contribution is the second goal, with a relatively lower priority to health.

#### Responsiveness

Responsiveness to people's expectations in regard to non-health matters reflects the importance of respecting people's dignity, autonomy and the confidentiality of information, and is the third goal ,with the lowest priority.

### 4.2 Characteristics of a good health care system

#### Goodness&&Fairness [WHO 2000]

As the WHO defined what a good health care system was in its World Health Report in 2000, a good health care system is a combination of Goodness and Fairness. A good health system, above all, contributes to good health. But it is not always satisfactory to protect or improve the average health of the population, if at the same time inequality worsens or remains high because the gain accrues disproportionately to those already enjoying better health. The health system also has the responsibility to try to reduce inequalities by preferentially improving the health of the worse-off, wherever these inequalities are caused by conditions amenable to intervention. The objective of good health is really twofold: the best attainable average level – *goodness* – and the smallest feasible differences among individuals and groups – *fairness*. A gain in either one of these, with no change in the other, constitutes an improvement, but the two may be in conflict.

### 4.3 Output metrics for measuring health care system

To assess a health care system, we must measure the following five output metrics:

#### Overall level of health

We use the measure of disability-adjusted life expectancy – DALE to assess the overall level of population health. This measure converts the total life expectancy for a population to the equivalent number of years of 'good health'.

#### Distribution of health in the population

We use the index of equality of child survival to assess distribution of health in the population. It is based on the distribution of child survival across countries, and takes advantage of the widely available and extensive information on complete birth histories in the demographic and health surveys and small area vital registration data on child mortality. WHO defined it as follows[WHO 2000]:

Equality of child survival = 
$$1 - \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} \left| x_i - x_j \right|^3}{2n^2 x^{-0.5}}$$
 (1)

Where x is the survival time of a given child and x is the mean survival time across children

#### Overall level of responsiveness

Responsiveness includes two major components:

- (1) Respect for people (including dignity, confidentiality and autonomy of individuals and families to decide about their own health);
- (2) Client orientation (including prompt attention, access to social support networks during care, quality of basic amenities and choice of provider). The level of responsiveness was based on a survey of key informants in selected countries. And WHO defined the index of Overall level of responsiveness as weighted average of its seven components: [WHO 2000]

Level of Re sponsiveness = 
$$\frac{1}{3}$$
 Dignit +  $\frac{1}{3}$  Confidentiality +  $\frac{1}{3}$  Autonomy  
+  $\frac{1}{5}$  Pr ompt attention +  $\frac{3}{20}$  Quality of amentities  
+  $\frac{1}{10}$  Access to social Support network +  $\frac{1}{20}$  Choice of provider

#### Distribution of responsiveness

We use a simple approach to quantize the distribution of responsiveness. That is respondents in the key informant survey were asked to identify groups who were disadvantaged with regard to responsiveness. The number of times a particular group was identified as disadvantaged was used to calculate a key informant intensity score. Four groups had high key informant intensity scores: poor people, women, old people, and indigenous groups or racially disadvantaged groups (in most instances minorities). The key informant intensity scores for these four groups were multi- plied by the actual percentage of the population within these vulnerable groups in a country to calculate a simple measure of responsiveness inequality ranging from 0 to 1. The total score was calculated taking into account the fact that some individuals belong to more than one disadvantaged group.

#### Distribution of financial contribution

The fair financing measure estimates the degree to which health funding is raised according to the ability to pay for all members of the population. It captures concerns such as progressivity, and protection from catastrophic health costs. Fair financing is only concerned with distribution. In order that complete equality of household contributions is 1 and 0 is below the largest degree of inequality observed across countries, WHO defined the in fairness index. And the index is of the form:[WHO2000]

Where HFC is the financial contribution of a given household and HFC is the average financial contribution across households.

### 5. Determining the weight of the metrics and data processing

### 5.1 Weights from statistical data

The key informant survey, consisting of 1791 interviews in 35 countries, yielded scores (from 0 to 10) on each element of responsiveness, as well as overall scores. A second, Internet-based survey of 1006 participants (half from within WHO) generated opinions about the relative importance of the elements, which were used to combine the element scores into an overall score instead of just taking the mean or using the key informants' overall responses [World Health Report 2000]. See figure 1 and 2:

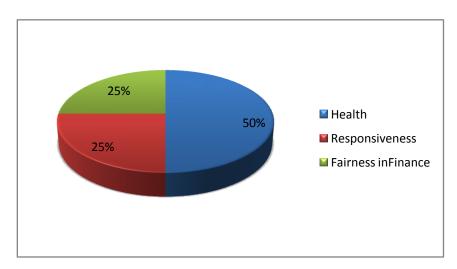


Figure 1 Weights for the three goals of health system

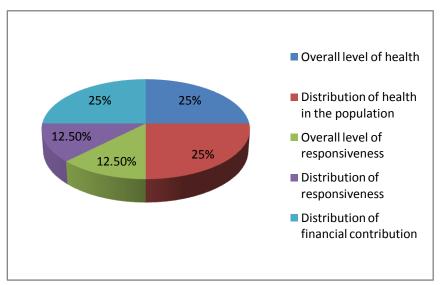


Figure 2 Weights of the five metrics

Figure 1 and figure 2 illustrate the weights of three goals of health system and five metrics respectively.

### 5.2 Data processing

#### Data Source

We get our data from WHO Statistical Information System on the official web site of WHO <a href="http://www.who.int/whosis/en/index.html">(http://www.who.int/whosis/en/index.html)</a>

And data in 'THE WORLD HEALTH STATISTICS REPORT' from 2005 to 2007 and 'World Health Report 'in 2000 is now accessible.

#### Normalization

To ensure comparability of effectiveness of health care system, metrics must be normalized by the following given formulation:

Normalized Data = 
$$\frac{RawData - \min(RawData)}{\max(RawData) - \min(RawData)}$$
(4)

Where max greatest number of Raw Data and min is the least one.

# 6. Input and Output of Health Care System

In this paper, we consider Health Care System a system with both input and output (see fig.3). Five output metrics and six input metrics are specified in this paper.

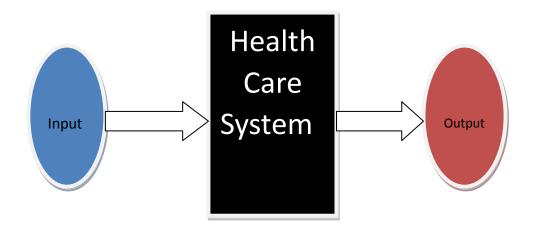


Figure 3: How a health care system works?

### 6.1 Aspects of Input

Table 2 Notation for Input and Output

Input	Notation	Output	Notation
Physician density per 1000 population	m <sub>1</sub>	Overall level of health	u <sub>1</sub>
Nurse density per 1000 population	m <sub>2</sub>	Distribution of health in the population	U <sub>2</sub>
Social Security expenditure on health as			
% of government expenditure on health	$m_3$	Overall level of responsiveness	U <sub>3</sub>
Private prepaid plans as % of	m <sub>4</sub>		
Private expenditure on health		Distribution of responsiveness	U <sub>4</sub>
External resources for health as	m <sub>5</sub>		
% of total expenditure on health		Distribution of financial contribution	<b>U</b> <sub>5</sub>
Out- of- Pocket expenditure as	m <sub>6</sub>		
% of private expenditure on health			

We define Input Vector as a set of the four elements of input, that is  $Input\ Vector = \{m_1, m_2, m_3, m_4, m_5, m_6\}$ 

- Physician density per 1000 population
- Nurse density per 1000 population
- Social Security expenditure on health as % of government expenditure on health
- Private prepaid plans as % of private expenditure on health Physician density per 1000 population
- External resources for health as % of total expenditure on health
- Out- of- Pocket expenditure as % of private expenditure on health

### 6.2 Aspects of Output

Also, we define Output Vector as a set of the five elements of Output, that is  $Output\ Vector = \{u_1\ u_2\ u_3, u_4, u_5\}$ 

- Overall level of health
- Distribution of health in the population
- Overall level of responsiveness
- Distribution of responsiveness
- Distribution of financial contribution

### 7. Evaluation System 1: Absolute Effectiveness of HCS

### 7.1Background

In this part, we deal with the evaluation of health care system by the way of "absolute", a way that only considers the output of the system. Then five typical metrics that can well represent the outcomes of the system are chosen for evaluation. Based on the five metrics, two empirical approaches are employed for evaluation. The former one is weighted average sum as a comprehensive indicator of the effectiveness, and the latter one is based on the theory of fuzzy mathematics.

### 7.2Assumptions

- We consider using output of the health system to evaluate the effectiveness acceptable here.
- The five metrics can represent enough information for evaluation of the health care system, thus we consider it reasonable and enough for us to use the metrics.
- We don't consider the interaction effect of metrics on the results.
- There is simply linear relationship between the metrics and the result of evaluation, thus weighted average sum approach can reasonably reflect how the metrics influence the results.
- As there is no specific definition on how well a health system is or the extent of "effectiveness", thus fuzzy comprehensive based approach employed here is acceptable.
- Most the data collected is reliable, neglecting its accuracy.

# 7.3Two approaches for evaluation

# 1. Approach A: Weighted Average Evaluation Based Model

We define Absolute Total Score (ATS) as an indicator that can be used to describe how a heath system works. Based on the assumptions above, we can formulate the Absolute Total Score as follows:

Absolute Total Score = 
$$\sum_{i=1}^{5} \omega_{i} u_{i}$$
 (5)

Where  $u_i$  represents the *i* th output metric and  $\omega_i$  is the weight corresponding to the metric.

By comparing the Absolute Total Score of a system, we can compare systems among countries. Meanwhile, by calculating the value of five metrics, we can also get the rank of systems with respect to each metric.

### 2. Approach B: Fuzzy Comprehensive Evaluation Based Model [19][20]

As there is no specific definition on how well a health system is or the extent of "effectiveness", we employ the theory of fuzzy mathematics to assess it.

#### Combination of factors

To assess the absolute effectiveness of health care system, we focus on three aspects of health care system that is health, responsiveness and fair financial contribution. Health can be divided into two major parts, the overall level of health; the distribution of health in the population. Responsiveness can be divided into two major part, the overall level of responsiveness; the distribution of responsiveness.

The following figure illustrates the relationships and levels of those indicators.

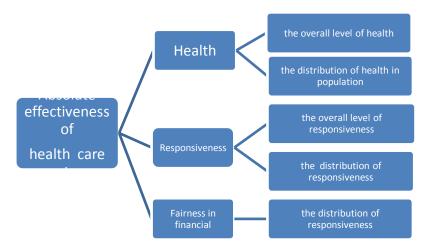


Figure 5: Hierarchy structure of factors

We use fuzzy set

$$U = \{u_1 \ u_2 \ u_3 \dots u_5\},$$

Where  $u_1 u_2 u_3 ... u_5$  is the indication for the five basic metrics respectively,

To include all the five basic metrics, and divided it into three groups, we have

$$U = \{U_1 U_2 U_3\}$$
,

Where fuzzy subset  $\ U_1 \ U_2 \ U_3$  represents health, responsiveness and fair financial contribution respectively.

Then we have

$$U_1 = \{u_1 \ u_2\}$$
 ,  $U_2 = \{u_3 \ u_4\}$  , and  $U_3 = \{u_5\}$ .

The weight set for  $\,U\,$  is

$$W = (\lambda_1 \lambda_2 \lambda_3)$$

Where  $\lambda_1 \lambda_2 \lambda_3$  is the weight of  $U_1$ ,  $U_2$  and  $U_3$  respectively.

And the weight set for  $U_1$  is indicated by  $W_1=(\lambda_{1,1}\ \lambda_{1,2})$ , where  $w_{11}\ w_{12}$  is weight that metrics  $u_1$  and  $u_2$  account for *respectively*. The weight set for  $U_2$  is indicated by  $W_2=(\lambda_{2,1}\ \lambda_{2,2})$ , where  $\lambda_{2,1}\ \lambda_{2,2}$  is weight that metrics  $u_3$  and  $u_4$  account for *respectively*.

#### Determine membership degree for each metric

Assume that there are n countries of to be compared in terms of absolute effectiveness of their health care system. We take normalized form membership functions for each metric so that values of all the metrics of different levels can be constrained between 0 and 1. By the membership degree function

$$\mu_{\tilde{U}}(u_{i,k}) = \frac{u_{i,j} - \min_{1 \le k \le n} (u_{i,k})}{\max_{1 \le k \le n} (u_{i,k}) - \min_{1 \le k \le n} (u_{i,k})} = a_{i,j},$$
(6)

Where  $u_{i,k}$  indicates the i th metric of the k th country.

### Deducing of model

For the fuzzy set  $U_1$ , the single factor judgment matrix

$$R_{\rm l} = \begin{pmatrix} a_{\rm l,1} & a_{\rm l,2} & \dots & a_{\rm l,n} \\ a_{\rm 2,l} & a_{\rm 2,2} & \dots & a_{\rm 2,n} \end{pmatrix}$$

By weighted average method, we can easily have matrix  $B_1$ 

$$B_1 = [b_{1,1} \ b_{1,2} ... b_{1,n}]$$
 , where  $b_{1,j} = \sum_{i=1}^{2} \lambda_{1,i} ... a_{i,j}$   $(j = 1, 2...n)$ 

For the level  $U_1$ , the single factor judgment matrix

$$R_2 = \begin{pmatrix} a_{3,1} & a_{3,2} & \dots & a_{3,n} \\ a_{4,1} & a_{4,2} & \dots & a_{4,n} \end{pmatrix}$$

Similarly, we have matrix  $B_2$ 

$$B_2 = [b_{2,1} \ b_{2,2} ... b_{2,n}]$$
 , where  $b_{2,j} = \sum_{i=3}^4 \lambda_{2,i-2}.a_{i,j}$   $(j=1,2...n)$ 

And

$$B_3 = [b_3, b_3, b_{2n}] = [u_3, u_{n3}]$$

Finally, we perform comprehensive evaluation on the top level. Then the R is

$$R = \begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix} = \begin{pmatrix} b_{1,1} & b_{-1,2} & b_{-n} \\ b_{2,1} & b_{-2,2} & b_{-n} \\ b_{3,1} & b_{-3,2} & b_{-n} \end{pmatrix}_{3,1}^{1,1}$$
(7)

By weighted average method, we have overall synthetic judge matrix

$$B = [b_1 \ b_2 ...b_n]$$
 , where  $b_j = \sum_{i=1}^{3} \lambda_i .b_{i,j}$   $(j = 1, 2...n)$ 

The value of each element in B can be looked on as the absolute effectiveness of health care system for each country. So the larger the value of element in matrix B is, more effective the health care system of the country to which this value is corresponding is.

### 7.4 Applying the Evaluation of Absolute Effectiveness Method

### Applying Approach A

Apply approach A to 34 countries (USA included), and the rank is given in the following table. We focus on the three goals of health system, the five output metrics as well as the overall rank.

Table3 Absolute	Effectiveness	of 34 countries	rank by 5 ou	itnut metrics	estimates for 2007
Tables Hosoluk	LIICCHIVCHCSS	or of countries.	o, rains by 5 ou	itput michics,	Coulinates for 2007

		Health		Resp	onsiveness		Fairness in financial		
Country	Level (DALE)	Distribution	rank	Level	Distribution	rank	Financial Contribution	Score	Rank
Japan	75	0.999	1	7	0.995	3	0.977	0.83988	1
Norway	72	0. 999	2	6.98	0.995	4	0.977	0.82055	2
USA	69	0.966	18	8.1	1	1	0.954	0.81366	3
Iceland	73	0.976	6	6.84	0.995	8	0.976	0.81219	4
Austrialia	73	0.977	5	6.86	0.995	6	0.971	0.81138	5
Canada	72	0.977	9	6.98	0. 995	5	0.974	0.80967	6
Austria	72	0.977	10	6.86	0.995	7	0.976	0.80696	7
France	72	0.978	8	6.82	0.995	9	0.971	0.80442	8
Finland	72	0.975	11	6.76	0.995	11	0.977	0.80357	9
Italy	73	0.978	3	6.65	0. 995	13	0.961	0.80214	10
Denmark	70	0.975	16	7.12	0.995	2	0.979	0.80212	11
UK	71	0.999	7	6.51	0.995	16	0.977	0.80083	12
Belgium	71	0.975	14	6.82	0.995	10	0.979	0.79977	13
New Zealand	71	0.978	13	6.65	0.995	14	0.972	0.79367	14
Israel	71	0.979	12	6.7	0.995	12	0.964	0.79261	15
Spain	73	0.978	4	6.18	0.995	17	0.971	0.79233	16
Ireland	70	0.978	15	6.52	0.995	15	0.978	0.78589	17
Portuga1	70	0.959	17	6	0.981	19	0.951	0.74464	18
Poland	66	0.999	19	5.73	0.97	20	0.896	0.70325	19
Hungary	65	0.941	20	5.47	0.98	21	0.927	0.68082	20
Mexico	66	0.858	21	5.66	0.909	25	0.903	0.60627	21
Turkey	62	0.759	23	5.16	0.969	22	0.958	0.57731	22
Republic of Korea	68	0.632	24	6.12	0.992	18	0.873	0.56781	23
Uzbekistan	66	0.632	25	5.03	0.96	24	0.912	0.52009	24
Mongolia	56	0.624	28	5.79	0.934	23	0.932	0.46831	25
India	63	0.601	26	5.02	0.876	29	0.962	0.45764	26
China	64	0.782	22	5.2	0.911	26	0.638	0.45078	27
Turkmenistan	55	0.684	27	5.2	0.899	27	0.918	0.44672	28
Pakistan	53	0.46	32	4.95	0.897	28	0.949	0.33763	29
Uganda	43	0.796	29	3.74	0.796	34	0.913	0.31907	30
Sudan	49	0.595	31	4.34	0.842	31	0.883	0.29961	31
Nepal	52	0.586	30	3.83	0.792	33	0.714	0.20925	32
Rwanda	38	0.437	34	4.62	0.85	30	0.951	0.1981	33
Zambia	35	0.535	33	4.8	0. 781	32	0.891	0.16673	34

From table 3, we can see:

- With respect to overall health, Japan ranks the first and Rwanda the lowest, while the USA ranks in the lower level.
- With respect to Responsiveness, the USA is leading in the 23 developed countries, while Uganda ranks last.
- With respect to Absolute Effectiveness, Japan leads first, while the USA ranks 3, a relative lower level.

#### Comparison between Approach A and Approach B

By the Evaluation of Absolute Effectiveness(EAE) method, the policy makers and other related department can judge whether the current system approaches its goal, in other words, we can identify whether the system can satisfy residents' requirement of health. And the Evaluation of Relative Effectiveness (ERE) method can evaluate the efficiency of usage of resources, which can give guidance for adjusting and improving health care system.

Table 4 Horizontal and vertical comparison of HCS by EAE, estimates for 2006 and 2007

	Appı	roach A	Appr	oach B		Approach A		Appr	oach B
country	2006	2007	2006	2007	country	2006	2007	2006	2007
Japan	1	1	1	1	Portugal	18	18	18	18
Norway	2	2	2	2	Poland	17	20	17	19
Iceland	3	4	3	3	Hungary	20	21	20	20
Australia	5	5	8	4	Mexico	21	22	21	21
Canada	4	6	4	5	Turkey	23	19	23	22
Austria	6	7	6	6	Republic of Korea	25	26	25	23
USA	8	3	4	7	Uzbekistan	24	23	24	24
Finland	7	9	7	8	India	22	24	22	25
Denmark	9	11	9	9	Mongolia	26	25	26	26
France	12	8	12	10	Turkmenistan	27	27	28	27
UK	11	12	13	11	Pakistan	28	30	27	28
Belgium	10	13	10	12	China	29	29	29	29
Italy	13	10	1	13	Uganda	30	28	30	30
New Zealand	14	14	14	14	Sudan	32	32	32	31
Spain	16	16	16	15	Rwanda	31	33	31	32
Israel	15	15	15	16	Zambia	33	31	33	33
Ireland	19	18	19	17	Nepal	34	34	34	34

#### From table 4, we can see

• Through comparing the ranks of countries using the two approaches respectively in the same year, we find that the results of two different approaches to determine Evaluation of

Absolute Effectiveness (EAE) do not change significantly, with ranks of most countries interested in having not big change. The comparing between the two approaches proves correctness and rationality of each other.

- Through comparing the ranks of countries using the two approaches respectively in the different year, we find the ranks of countries are nearly stable.
- Comparing to Japan which has a quite good health system, the USA's absolute effectiveness of health care system is not as high as J a pan.

# 8. Evaluation system II: Relative Effectiveness of HCS

### 8.1 Only output doesn't work

The overall indicator of attainment, like the five specific metrics which compose it, is an absolute measure. It says how well a country has done in reaching the different goals, but it says nothing about how that outcome compares to what might have been achieved with the resources available in the country. It is *achievement relative to resource* that is the critical measure of a health system's performance.

For example, if Sweden enjoys better health than Uganda – life expectancy is almost exactly twice as long – that is in large part because it spends exactly 35 times as much per capita on its health system. But Pakistan spends almost precisely the same amount per person as Uganda, out of an income per person that is close to Uganda's, and yet it has a life expectancy almost 25 years higher. This is the crucial comparison: why are health outcomes in Pakistan so much better, for the same expenditure? And it is health expenditure that matters, not the country's total income, because one society may choose to spend less of a given income on health than another. Therefore, each health system should be judged according to the resources actually at its disposal, not according to other resources which in principle could have been devoted to health but were used for something else. Therefore, corresponding to the *Evaluation of Absolute Effectiveness*, we introduce another evaluation system, the *Evaluation of Relative Effectiveness (ERE)*.

# 8.2 Assumptions

- We can assess the input of health care system by the total money it needs to operate.
- Total expenditure on health as % of GDP alone can be used to quantify the input of health care system.

# 8.3 Constructing the Model

The concept of Value Engineering was introduced to describe the relationship between costs, function and value [L  $\cdot$ D  $\cdot$ Miles 1943]. It defines value as function of costs and function in the form:

$$Value = \frac{Function}{Costs} \tag{7}$$

Similarly, we define Relative Total Score as:

Relative Total Score = 
$$\frac{\text{Absolute Total Score}}{\text{Input}}$$
 (8)

Where Relative Total Score is defined to assess relative effectiveness of health care system

By comparing the Relative Total Score, we can assess how a *health care system* works according to what it should have been archived. Here, to be simplified, we use Total expenditure on health as % of GDP to quantify the input.

### 8.4 Applying the Evaluation of Relative Effectiveness Method

Table 5 Relative Effectiveness of HCS, ranked by the Relative Evaluation system, estimates for 2007

Country	Total expenditure	Total expenditure R-Rank Country Total expenditure R-R					
Country	on health as % of GDP	IN-INGIIK	Country	on health as % of GDP	R-Rank		
	Off ficaltif as 70 of GDI			Off ficaltif as 70 of GDI			
Pakistan	2.2	1	Netherlands	9.2	18		
Poland	6.2	2	Hungary	7.9	19		
Iceland	7.2	3	Norway	9.7	20		
Ireland	7.1	4	Australia	9.6	21		
Finland	7.4	5	Canada	9.8	22		
Japan	7.8	6	Belgium	9.7	23		
Korea	5.5	7	Austria	10.3	24		
Uzbekistan	5.1	8	Mongolia	6	25		
UK	8.1	9	France	10.5	26		
Spain	8.1	10	Portugal	9.8	27		
China	4.7	11	Turkey	7.7	28		
Mexico	6.5	12	Sudan	4.1	29		
Denmark	8.6	13	USA	15.4	30		
Turkmenistan	4.8	14	Uganda	7.8	31		
Italy	8.7	15	Nepal	5.6	32		
India	5	16	Zambia	6.3	33		
Israel	8.7	17	Rwanda	7.5	34		

From the table (5), we can find that

- Pakistan ranks the first, and Rwanda is last. Especially some developed ones, such as America, ranks in the lower level.
- America has the largest percentage of GDP spent on health care, while Pakistan has only 2.2%.

#### 9. EAE VS ERE: which is better?

Apply the two evaluation system to 34 countries, we focus on the different ranks from the two evaluation

systems.

Table 6 EAE VS ERE, rank comparison

Country	GDP	AST	A-Rank	RST	R-Rank	Country	GDP	AST	A-Rank	RST	R-Rank
Pakistan	2.2	0.33763	28	0.15347	1	Netherlands	9.2	0.79367	14	0.086268	18
Poland	6.2	0.70325	19	0.11343	2	Hungary	7.9	0.68082	20	0.08618	19
Iceland	7.2	0.81219	3	0.1128	3	Norway	9.7	0.82055	2	0.084593	20
Ireland	7.1	0.78589	17	0.11069	4	Austrialia	9.6	0.81138	4	0.084519	21
Finland	7.4	0.80357	8	0.10859	5	Canada	9.8	0.80967	5	0.082619	22
Japan	7.8	0.83988	1	0.10768	6	Belgium	9.7	0.79977	12	0.082451	23
Republic of Korea	5.5	0.56781	23	0.10324	7	Austria	10.3	0.80696	6	0.078346	24
Uzbekistan	5.1	0.52009	24	0.10198	8	Mongolia	6	0.46831	26	0.078052	25
UK	8.1	0.80083	11	0.098868	9	France	10.5	0.80442	10	0.076611	26
Spain	8.1	0.79233	15	0.097819	10	Portugal	9.8	0.74464	18	0.075984	27
China	4.7	0.45078	29	0.095911	11	Turkey	7. 7	0.57731	22	0.074975	28
Mexico	6.5	0.60627	21	0.093272	12	Sudan	4.1	0.29961	31	0.073076	29
Denmark	8.6	0.80212	9	0.09327	13	USA	15.4	0.81366	7	0.052835	30
Turkmenistan	4.8	0.44672	27	0.093067	14	Uganda	7.8	0.31907	30	0.040906	31
Italy	8.7	0.80214	13	0.0922	15	Nepal	5.6	0.20925	34	0.037366	32
India	5	0.45764	25	0.091528	16	Zambia	6.3	0.16673	33	0.026465	33
Israel	8.7	0.79261	16	0.091105	17	Rwanda	7.5	0.1981	32	0.026413	34

From table 6, we can see: Comparing to ranks in terms of Absolutely Evaluation of Effectiveness, the new ranks of these countries change significantly.

 Ranks of countries having large percent of GDP spent on health care such as USA, Norway, Australia, Canada, Austria, France decrease by more then 15, especially for USA of which rank declines from 7 to30. This means that these countries do not make the most of their inputs.

Ranks of countries having small percent of GDP spent on health care such as Pakistan increases from 28 to 1. This means that this country makes the most of its inputs. This may be a good example that those developed countries like the USA should learn from. But for developing countries, especially those having poor health care system, no matter how efficient their health care system is, they still can not supply good enough health service, simply because they have not enough resources to input into health care system.

### 9.1 USA VS Norway

From the aspect of Evaluation of Absolute Effectiveness, we can see that USA ranks 7<sup>th</sup>, while Norway ranks 2, while from the aspect of Evaluation of Relative Effectiveness, the USA ranks 30<sup>th</sup>, and Norway 20<sup>th</sup>.

#### 9.2 USA VS Pakistan

Health care system of the USA is better than Pakistan from the aspect of Evaluation of Absolute Effectiveness obviously. However, Pakistan ranks first from aspect of Evaluation of Relative Effectiveness, while America ranks only 30<sup>th</sup>, a quite low rank.

### 10. Less Resources, more outcomes

### 10.1Multiple Logistic Regression Model

#### 10.1.1 Output as function of Input

We need to determine whether various changes can improve the overall quality of a country's health care system. Thus, we focus on how the output of a system changes due to variation of input. We employ the logistic equation to model the relationship between output and input [Gotelli 1998]. By the equation, we can clearly see how input influences the output.

### 10.1.2Assumptions

- Input can be qualified by weighted average sum of the six input metrics, and the weight reflects how the metric contributes to the input.
- Output can be qualified by weighted average sum of the five output metrics (ATS), and the weight reflects how the metric contributes to the input,
- Relationship between input and output of health system can be quantized as logistic equation, that is the output grows as the inputs growth, and the growth rate is rising at first, but as the output approaches a certain value, its growth rate will gradually decrease to zero.

#### 10.1.3Constructing the model

Here we set the Absolute Total Score as the quantification of output, and the logistical equation is given as:

$$\frac{dATS}{dM} = R\Box ATS(1 - \frac{ATS}{K}) \tag{9}$$

Where R is the growth rate, K is the upper bound of output and M is the quantification of input. For simplicity, we let a=R and b=R/K, so that:

$$\frac{dATS}{dM} = a\Box ATS - b\Box ATS^{2} \tag{10}$$

With the initial condition  $ATS(M_0) = ATS_0$ , the equation has closed-form solutions:

$$ATS(M) = \frac{ae^{aM} \square ATS}{a - b\square ATS + be^{aM} \square ATS}$$
 (1)

According to the assumption that input can be quantified by the linear weighed average sum of input metrics, we can quantify input as:

$$M = \sum_{i=1}^{6} \beta_i m_i + \beta_0 \tag{12}$$

Where  $\beta_i$  is the weight and  $m_i$  is the ith input metric

Then from (11) and (12), we can get

$$ATS(M) = \frac{ae^{a(\sum_{i=1}^{6}\beta_{i}m_{i}+\beta_{0})} \Box ATS}{a-b\Box ATS + be^{a(\sum_{i=1}^{6}\beta_{i}m_{i}+\beta_{0})} \Box ATS}$$

$$(13)$$

The figure below illustrates how output changes as input varies:

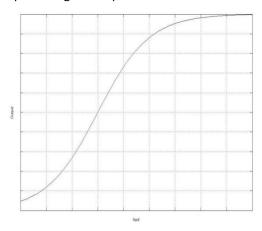


Figure 6 Solution to the logistic equation, with output plotted as a function a input

#### 10.1.4. Estimation of parameters

We estimate the parameters for (13) by curve fit, statistical data collected from the 34 countries mentioned above is employed to help the curve fit, and we get

$$AST(M) = \frac{1.1032}{1.0958 + e^{M}} \tag{14}$$

With 
$$M = 29.982 - 20.498m_1 - 593923m_2 - 57.78m_3 + 8.4232m_4 - 18.556m_5 - 9.9023m_6$$
 (15)

Also, we do statistical tests for our model, and it presents us a satisfactory result: Residual=0.051, and Confidence Degree=1-Residual=0.949, indicating that it passes the statistical test.

#### **10.1.5How the six metrics influence the outcomes?**

Since we have equation (14) and (15), we can consider

$$ATS = f(M) = f(m_1, m_2, ..., m_6)$$
 (16)

Let us consider

$$\frac{\partial A \ T \ S}{\partial m_i} = \frac{\partial A \ T \ S}{\partial M} \frac{M}{\partial m} \stackrel{M}{=} \frac{\partial}{\partial m} \stackrel{M}{\neq}_i$$
(17)

And (11) and (12) can get

$$\frac{\partial ATS}{\partial M} = a\Box ATS - b\Box ATS^2 \tag{18}$$

$$\frac{\partial M}{\partial m_i} = \beta_i \tag{19}$$

Then from (17),(18) and (19), we can get

$$\frac{\partial ATS}{\partial m_i} = \frac{\partial ATS}{\partial M} \frac{\partial M}{\partial m_i} = (a\Box ATS - b\Box ATS^2)\beta_i$$
 (20)

And the value of partial differential  $\frac{\partial ATS}{\partial m_i}$  show how metric  $m_i$  influences the output.

Also, by controlling variable  $m_2$ ,  $m_3$ ,  $m_4$ ,  $m_5$ ,  $m_6$ , and vary variable  $m_1$ , we can see how  $m_1$  influences the output; similarly we can get how  $m_2$ ,  $m_3$ ,  $m_4$ ,  $m_5$ , and  $m_6$  influences the output respectively.

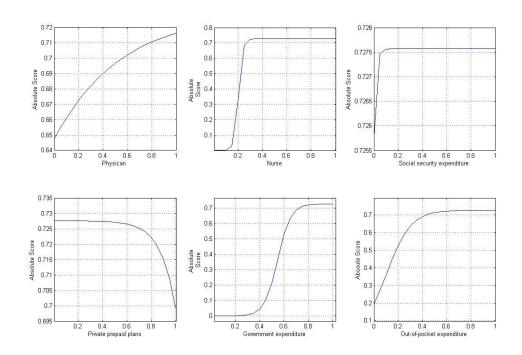


Figure 7 How input metric influences the output

#### As figure 7 illustrates:

With respect to private prepaid plans
 It is negatively correlated to AST. That is as the interest of the property of

It is negatively correlated to AST. That is as the increase of private prepaid plans, AST decreases. The reason for this is mainly due to people of their own country do not trust the health care system, they store a large amount of money to spend by the sick and hospitalized, which reflects the health care system is far from perfect, so lower scores.

• With respect to the other five metrics

We can see that the AST increases as the other five input metrics increase, only that the increasing rate is different.

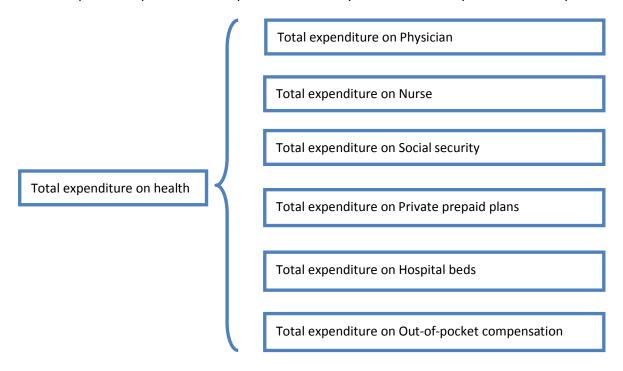
### 10.2 Taking USA into consideration

As we have analyzed above, USA ranks 3rd by the evaluation of absolute effectiveness while ranks 7th by the evaluation of relative effectiveness. The difference between the ranks indicates that health system of USA should have archived more under the current total expenditure on health. In this part, we try to explore an optimized combination of input metrics to minimize the input or maximize the output. Thus we focus the USA in 2007,trying to minimize the total expenditure on health and at the same time archive the same attainment, or to maximize the attainment under the same expenditure.

In 2007, by the evaluation of absolute effectiveness, USA gets an absolute total score 0.8116.We get all concerned data for USA in 2007.See the following table:

#### 10.2.1Assumptions

- The total expenditure on health is employed to as the quantification of the input of health care system.
- Total expenditure on health every year is divided into six parts: expenditure on physician wage, expenditure
  on nurse wage, expenditure on social security, and expenditure on private prepaid plans, expenditure on
  Out-of-pocket expenditure compensation and expenditure on the purchase of hospital beds.



### **10.2.2** Allocation Coefficient $\alpha$

To transit the six input metrics into expenditure, we define the "Allocation Coefficient" as the coefficient that Transits the six input metrics to expenditure:

Table 5 Data collected to determine Allocation Coefficient

Items we concern	Value
Total population	298,213,000
Gross domestic product per capita	46950
Total expenditure on health as % of GDP	15.40%
General government expenditure on health	44.70%
as % of Total expenditure on health	
Private expenditure on health	55.30%
as % of Total expenditure on health	
Social security expenditure on health as %	0.1%
of general government expenditure on health	
Private prepaid plans as %	0.40%
private expenditure on health	
Hospital bed on per 1000 capital	33
Out-of-poke expenditure as % of	84.90%
private expenditure on health	
The average income of Physician per year	181850
The average income of Nurse per year	70000
Physician Density per 1000 population	2.3
Nurse Density per 1000 population	12.12
Average fees for bed	300

- GDP=Total population × GDP per capita
- Total expenditure on health=(GDP×Total expenditure on health as % of GDP)/100
- General government expenditure on health= (General government expenditure on health as % of Total expenditure on health × Total expenditure on health)/100
- Social security expenditure on health= (Social security expenditure on health as % of general government expenditure on health × general government expenditure on health)/100
- Private expenditure on health = (Private expenditure on health as % of Total expenditure on health × Total expenditure on health)/100
- Private prepaid plans expenditure= (Private prepaid plans as % private expenditure on health×Private expenditure on health)/100
- Out-of-pocket expenditure= (Out-of-pocket expenditure as % of private expenditure on health × Private expenditure on health)/100

- Expenditure on payment of Physician= (Total Population × Physician Density per 1000 population × The average income of Physician per year)/1000
- Expenditure on payment of Nurse= (Total Population×Nurse Density per 1000 population×The average income of Nurse per year)/1000
- Expenditure on beds= (Hospital bed on per 1000 capital × Total Population × Average fees for bed)/1000

Input metrics	Allocation Coefficient	Notation
Physician Density per1000population	163.38	$\alpha_{_1}$
Nurse Density per 1000 population	90.544	$lpha_2$
Social security expenditure on health as% of general government expenditure on health	3231.9	$\alpha_3$
Private prepaid plans as % private expenditure on health	3998.5	$lpha_{_4}$
Hospital bed on per 1000 capital	0.4	$lpha_{\scriptscriptstyle 5}$
Out-of-poke expenditure as % of private expenditure on health	39.984	$lpha_{\scriptscriptstyle 6}$

Table 6 Values for allocation coefficient

### 10.3 Scenario 1: Less expenditure to achieve the same goal

In 2007, USA ranks 3<sup>rd</sup> by the evaluation of absolute effectiveness and gets a absolute total score (ATS) 0.8116.We try to minimize the total expenditure on health and at the same time get a ATS of 0.8116.

### 10.3.1 Objective function:

Our goal is to minimize the total expenditure on health by the optimized combination of the six input metrics. Thus we can get the total expenditure M by

Objective function 
$$Min(M = \sum_{i=1}^{6} \alpha_i m_i)$$

Where  $\alpha_{\rm i}$  is the allocation coefficient, see table 6.

#### 10.3.2 Constraints

From the logistic model, we get the relationship between total expenditure and the absolute total score ATS by the logistic equation:

$$AST(M) = \frac{1.1032}{1.0958 + e^{M}}$$

- ATS=0.8116, this guarantees the absolute total score doesn't change.
- Mi>0, this means no metrics is negative.
- $m_{3.4.6}$  < 100 , this is the definition of metrics
- $m_{1,2,5}$  < 1000, this is the definition of metrics

### 10.3.3 Optimization model 1

From the analysis above, we can get an optimization model, shown as follows:

Min (M=
$$\sum_{i=1}^{6} \alpha_i m_i$$
)

st:
$$\begin{cases}
AST(M) = 0.8116 \\
m_i > 0 \\
m_{3,4,6} < 100 \\
m_{1,2,5} < 1000
\end{cases}$$

#### 10.3.4 Solutions of the model

Table 7 Solution to antimization to model 1

	Unit: million\$							
USA	Physicia	Nurse	Social	Private	Hospital	Out-of	AST	Money
	n		security	prepaid	bed	-pocket		Per capita
Current	2.5600	9.370	0	0.4000	3 3	84. 9	0.8116	6263.8
Solution	1.8755	6.510	0	0.1648	50.3	82.9656	0.8116	5992.2

For the condition of America, we recommend the USA spend as less expenditure as possible and at the same time maintains its current AST. Through adjusting each kind of resource according to table (7), we can make each American save 6263.8-5992.2=\$388.18, so the total money saved in America is 107.8 billion. We reduce the cost mainly through decreasing the number of physicians, nurses, and increasing hospital beds.

### 10.4. Scenario2: More outcomes with the same expenditure

In 2007, USA ranks 3<sup>rd</sup> by the evaluation of absolute effectiveness and gets an absolute total score (ATS) 0.8116.We try to maximize absolute total score with the same expenditure on health of 1146 billion dollars.

### 10.4.10bjective function

Our goal is to maximize the absolute total score ATS and it is given as the function of total expenditure on health in the form of logistic equation:

Thus our objective function is Max AST(M)

#### 10.4.2Constraints

- $\sum_{i=1}^{6} \alpha_i m = 2109.8 \times 10^9$ , this guarantees total expenditure on health is 2109.8 billion dollars.
- Mi>0, this means no metrics is negative.
- $m_{3,4,6}$  < 100 , this is the definition of metrics
- $m_{1,2,5} < 1000$  , this is the definition of metrics

### 10.4.3 Optimization model 2

From the analysis above, we can get an optimization model, shown as follows:

Max 
$$AST(M)$$

$$St: \begin{cases} \sum_{i=1}^{6} \alpha_{i} m = 2109.8 \times 10^{9} \\ m_{i} > 0 \\ m_{3,4,6} < 1 \end{cases}$$

#### 10.4.4Solutions to the model

Table8 Solution to optimization to model2

Unite million \$

	Unit: million\$							
USA	Physician	Nurse	Social	Private	Hospita	Out-of	AST	Money
			security	prepaid	1 Bed	-pocket		Per capita
Current	2.5600	9.37	0	0.4000	3 3	84. 9	0.8116	6263. 8
Solution	5.7000	17.2	0.4000	0.1000	6 5	50.0000	0.8823	6263.8

We recommend the USA improve AST as much as possible and at the same time maintain its current expenditure. Through adjusting each kind of resource according to table (8), we can make each AST of America up to 0.8823, increasing by 0.0707. We can improve AST mainly through increasing number of physicians and nurses and hospital beds, at the same time reducing Out-of-pocket expenditure as % of private expenditure on health.

# 15. Strengths and Weaknesses

### **Strengths**

By the Evaluation of Absolute Effectiveness(EAE) method, the policy makes and other related department can judge whether the current system approaches its goal, in other words, we can identify whether the system can satisfy residents' requirement of health. And the Evaluation of Relative Effectiveness (ERE) method can evaluate the efficiency of usage of resources, which can give guidance for adjusting and improving health care system.

#### Weaknesses

- When applying the ERE method, we only choose Total expenditure on health of GDP as metrics, which can evaluate the efficiency of usage of expenditure but can not help to find the concrete reasons for low output of health care system.
- Inefficiency of metrics of some countries limits our choosing of metrics; as a result, sometimes we have to exclude some metrics that may have big influence on the assessment of health care systems.

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