1 Introduction to Light Scattering from Microstructures

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When electromagnetic radiation interacts with microstructures (either isolated or located on a surface) whose size is of the order of the incident wavelength, scattered radiation can be detected in all directions. Furthermore, the electromagnetic characteristics of the scattered wave in a given direction may be changed due to the interaction of the incident radiation with the microstructure. This constitutes an important electromagnetic problem since long time ago and, in order to solve it some information has to be obtained about the physical scattering structure (i.e. size, shape, optical properties, etc). The interest in solving this problem is not only from a basic point of view but for the applications in many fields. For instance, microstructures in volume are common in areas like meteorology, astronomy, biology, radar, atmospheric contamination, optical oceanography, etc.; whereas microstructures on surfaces are common for instance in the semiconductor industry, high reflectance surface contamination, SERS, optical particle sizing, etc. Because of these applications, many researchers have developed theories and numerical methods, and have carried out experiments in order to solve those electromagnetic scattering problems. Recent scientific meetings in close relation with the scattering of electromagnetic waves from microstructures confirm the current interest of the scientific community [1]. A number of issues have been devoted to the subject in some journals [2–4], and also several books of collected works (theoretical and applied) have been edited [5,6].

Obviously, a review of all the theories developed for solving the problem of scattering from microstructures, or even a list of the works related with the topic, is an ambitious undertaking. The aim of this chapter is to show only some basic aspects of its solution and to describe some of the methods most frequently used, depending on the size, shape and optical constants of the microstructures. We think this introduction will help the reader to understand the different contributions of the book, each one focused in a particular aspect. We distinguish between the isolated particle and the particle or microstructure on a substrate. For the first case it is worthwhile mentioning here some excellent reviews [7,8] in which some of the most relevant works are described.

The chapter is structured as follows: Section 1 is devoted to describe the basic electromagnetic tools. In Sects. 2 and 3 we survey the most commonly

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used methods in scattering from, respectively, isolated regular and irregular microstructures. In particular, in Sect. 3 we shall describe three methods that, to our opinion, are the most powerful from the calculus point of view: T-matrix method, Discrete Dipole Approximation method and Finite-Difference Time-Domain method. Finally, Sect. 4 is entirely devoted to describe the most important models in the analysis of the scattering from microstructures on surfaces.

1 Electromagnetic Theory

The basic tools for the solution of the scattering of electromagnetic waves from microstructures are the Maxwell equations. Integral theorems applied to these equations and appropriate boundary conditions of the problem constitute a general operational scheme. As it is well known, an electromagnetic wave propagating in a medium is characterised by its electric field vector **E** and its induction magnetic vector **B**. In the medium, the response to these fields is given by the electric displacement vector **D** and the magnetic field vector **H**. These macroscopic magnitudes satisfy Maxwell's equations:

$$\nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t} \tag{1}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{2}$$

$$\nabla \cdot \mathbf{D} = \rho \tag{3}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{4}$$

where **j** and ρ are the electric current density and the charge density, respectively. **D** and **H** are related to **E** and **B** by means of the relations

$$\mathbf{D} = \epsilon_0 \epsilon_r \mathbf{E} \tag{5}$$

$$\mathbf{H} = \frac{1}{\mu_0 \mu_r} \mathbf{B} \tag{6}$$

where $\epsilon = \epsilon_0 \epsilon_r$ is the dielectric constant of the medium, ϵ_0 is the dielectric constant of vacuum and ϵ_r is the relative dielectric constant. $\mu = \mu_0 \mu_r$ is the magnetic permeability of the medium, μ_0 is the permeability of vacuum and μ_r is the relative permeability. Furthermore, $\mathbf{j}$ and $\mathbf{E}$ are related by Ohm's Law:

$$\mathbf{j} = \sigma_c \mathbf{E} \tag{7}$$

σ_c being the electric conductivity of the medium.

The solution of a problem in a given region of space requires the solution of the Maxwell equations in the medium, which is somehow limited by other media. Around the surface S separating regions 1 and 2 constituted by different media, some boundary conditions must be satisfied by $\mathbf{E}$, $\mathbf{B}$, $\mathbf{D}$, and $\mathbf{H}$. Let $\mathbf{n}_{12}$ be the unitary surface vector, from medium 1 to medium 2 in a given point of surface S. The boundary conditions are:

$$\mathbf{n}_{12} \times (\mathbf{E}_2 - \mathbf{E}_1) = 0 \tag{8}$$

$$\mathbf{n}_{12} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{j}_S \tag{9}$$

$$\mathbf{n}_{12} \cdot (\mathbf{D}_2 - \mathbf{D}_1) = \rho_S \tag{10}$$

$$\mathbf{n}_{12} \cdot (\mathbf{B}_2 - \mathbf{B}_1) = 0 \tag{11}$$

where $\mathbf{j}_S$ and ρ_S are the electric current density and the charge density at the surface, respectively.

For the case of a homogeneous and isotropic medium, and if the electric current and charge densities are null at the surface, $\mathbf{j}_S = 0$, $\rho_S = 0$, the Helmholtz equation is derived from eqs. 1-4:

$$(\nabla^2 + k^2)\psi = 0 \tag{12}$$

where $k = \omega\sqrt{\epsilon\mu}$, ψ is the spatial part of the electric field, ω is the angular frequency, $\mathbf{E}(\mathbf{r})$, or magnetic field, $\mathbf{H}(\mathbf{r})$, and the time dependence is assumed $\exp(-i\omega t)$. The solution of the Helmholtz equation, with the boundary conditions of (8)-(11) constitute a description of the electromagnetic field in any region of space.

2 Isolated Regular Particles

We refer to a regular, or simple, particle as one whose interfaces can be defined by having one of the components of a coordinate system set equal to a constant; e.g., a sphere can be defined by $r = C$, a cylinder by $\rho = C$, a plane by $z = C$, etc. The simplest microstructure is a sphere. For such a system it is most convenient to express the wave equation in spherical coordinates and carry out a separation of variables. The incident, internal and scattered electromagnetic fields can be expressed in terms of vector spherical harmonics, and the coefficients of the harmonic expansions, which represent the scattering solution, depend on the composition and morphology of the microparticle. The first studies, carried out by Mie, Lorentz and Debye, dealt with the scattering of an incident plane wave by a non-magnetic, isotropic, homogeneous, spherical particle placed in a non-absorbent medium. This is often referred to as Mie theory. A detailed study of this solution can be found in the classic literature of the field (van de Hulst [10], Kerker [11], and Bohren and Huffman [12]). For nonspherical particles, rigorous analytic scattering solutions exist only for a few special cases: very small particles compared with the wavelength or infinitely long cylinders. More recently [13–16], exact solutions were found for spheroidal particles by means of first expressing the vectorial wave equation in spheroidal coordinates for the separation of variables, and second, expanding the incident, internal and scattered field vectors in spheroidal wave functions. As before, the coefficients in the expansion are the unknowns to be determined by the boundary conditions.

These analytic solutions may be applied to particles of all sizes, but numerical results are not always trivially calculated. Convergence and truncation criteria contribute to the difficulties. Algorithms and numerical methods have been developed to assist in the calculations [17–20]. In [20] the programs are based on the T-matrix method that is described below.

Though analytic solutions are possible only for simple geometries, numerous works focus on scattering by inhomogeneous, anisotropic particles having irregular geometries, and also consider different profiles of the incident beam (Gaussian, for instance). A solution for a coated sphere (homogeneous core covered by a uniform layer of a different material) was first obtained by Aden and Kerker [21]. A reformulation of that theory was employed by Toon and Ackerman [22] to perform a numerical method for the calculation of scattering from concentrically coated spheres. The radiation of a spherical core-shell system was also calculated by Fuller [23], using the vector spherical harmonics. A complete analytical study of the multilayered spherical particle as an extension of the Mie solution was carried out by Bhandari [24], although the computational scheme of his results require the implementation of complex algorithms. More recently Wu and Wang [25], Kai and Massoli [26], and Wu *et al.* [27] proposed more stable and accurate recursive procedures for numerically computing of the scattering coefficients.

Another particle system of interest is the case of host particles having eccentric inclusions. The use of multiple-multipole expansions have allowed researchers to satisfy boundary conditions for these cases [28–31]. In this method, vector harmonics are expanded about multiple coordinate locations corresponding to the center of each inclusion. These expansions can be translated to the host coordinate system and the boundary conditions can be satisfied on all locations simultaneously. The same procedure has been used to calculate the scattering from aggregates [32–35].

3 Isolated Irregular Particles

In many situations where the knowledge of light scattered by microstructures has some relevance, the structures have a non-spherical shape and/or are nonhomogeneous. Sometimes the information provided by the solution to less complicated cases (like those discussed above) may approximate the system sufficiently that the results are meaningful, but sometimes the system is too irregular, and we need to resort to a specific solution. In these cases, the required computational methods are directly based on the numerical solution of Maxwell's equations, either in differential or integral form. Here we refer to the most powerful methods from the calculus point of view, though the computational requirements may be very high in some cases. In particular we shall describe briefly the T-matrix method and the Discrete Dipole Approximation (DDA) based on the integral form of the Maxwell equations, and the Finite-Difference Time-Domain (FDTD) technique, based on the differential form of the Maxwell equations.

3.1 Integral Methods

One approach to the scattering problem utilizes an integral formulation of the Maxwell equations. The starting point is the differential form of Maxwell's equations. By applying the appropriate boundary conditions one obtains more compact equations by means of the application of well known integral (Green) theorems. An example is the so called extinction theorem (ET) of the physical optics, which is derived from the Green theorem [9]. In this method two surface integral equations are derived. The first is used to calculate the sources along a surface S induced by the incident field

$$\mathbf{E}(\mathbf{r_s}) = \mathbf{E_o}(\mathbf{r_s}) + \frac{1}{4\pi\epsilon k_o^2}\nabla\times\nabla\times\int_S \left[\mathbf{E}(\mathbf{r}')\frac{\partial G(\mathbf{r}_s,\mathbf{r}')}{\partial\widehat{\mathbf{n}}} - G(\mathbf{r}_s,\mathbf{r}')\frac{\partial\mathbf{E}(\mathbf{r}')}{\partial\widehat{\mathbf{n}}}\right] dS \tag{13}$$

where $\mathbf{E_o}(\mathbf{r_s})$ is the incident field at the point $\mathbf{r}_s$ belonging to the surface S, $\mathbf{E}(\mathbf{r}')$ and $\mathbf{E}(\mathbf{r_s})$ are the total fields (incident plus scattered) at points $\mathbf{r}'$

and $\mathbf{r_s}$ respectively, $G(\mathbf{r}_s, \mathbf{r}')$ is the Green's function, $\widehat{\mathbf{n}}$ is the unit outward-pointing normal of the surface, k_o is the wavenumber in vacuum. In order to calculate the total field at any point in space due to the the sources, the following equation is evaluated:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E_o}(\mathbf{r}) + \frac{1}{4\pi\epsilon k_o^2}\nabla\times\nabla\times\int_S \left[\mathbf{E}(\mathbf{r}')\frac{\partial G(\mathbf{r},\mathbf{r}')}{\partial\widehat{\mathbf{n}}} - G(\mathbf{r},\mathbf{r}')\frac{\partial\mathbf{E}(\mathbf{r}')}{\partial\widehat{\mathbf{n}}}\right] dS \tag{14}$$

3.2 T-Matrix

The T-matrix method, also known as the extended-boundary-condition method (EBCM) provides a general formulation of the scattering from obstacles of arbitrary size and shape, with size ratios from the Rayleigh region to the geometric optics limit. An excellent review of the current status of the T-matrix method has been published by Mishchenko *et al.* [36]. This method is a numerical technique based on the integral formulation of the Maxwell equations. The electromagnetic fields (incident, internal and scattered) are expanded in vector spherical harmonics. After applying the boundary conditions, a matrix is obtained, relating the known expansion coefficients of the incident field (a, b) to the unknown expansion coefficients of the scattered field (f, g):

$$\begin{bmatrix} f \\ g \end{bmatrix} = -\,[T]\begin{bmatrix} a \\ b \end{bmatrix}. \tag{15}$$

This matrix is called the transition or T-matrix and depends only on the physical properties of the scattering object (shape, size parameter, and optical constants) as well as on its orientation with respect to the coordinate system. One enormous advantage of obtaining a T-matrix is that it is necessary to compute it only once and the results can be used in computations at any incident or scattering direction. Orientation averaging can also be performed.

The T-matrix method was developed in a series of papers by Waterman [37,38], and has been widely used in scattering by microstructures. Barber and Hill [20] have published a book including the T-matrix software for spheres, cylinders and spheroidal particles. The T-matrix method has been used in many scattering investigations of many types of particles: non-spherical homogeneous [16,39–42] and inhomogeneous [43], polydisperse samples of non-spherical homogeneous [44,45] and inhomogeneous [46] particles, bispheres [47] and arbitrarily shaped scatterers [48–52].

3.3 Discrete Dipole Approximation (DDA) Method

The discrete-dipole approximation (DDA) for computing scattering and absorption by particles was originally developed by Purcell and Pennypacker

[53]. It is a very general method for calculating the scattering and absorption of electromagnetic waves by particles of arbitrary shape and size comparable to the wavelength. In the DDA method, the scattering system is discretized and represented by an array of N point dipoles, with the spacing between dipoles small compared to the wavelength. Each dipole j is characterized by a polarizability tensor α_j such that $\mathbf{p}_j = \alpha_j \mathbf{E}_j$, where $\mathbf{p}_j$ is the instantaneous dipole moment and $\mathbf{E}_j$ is the total instantaneous electric field at position j due to the incident field on the dipole $\mathbf{E}^{\text{inc}}$ plus the effect of the other $N-1$ dipoles

$$\mathbf{E}_j = \mathbf{E}^{\text{inc}}\left(\mathbf{r}_j\right) + \sum_{j \neq k} \mathbf{A}_{jk} \alpha_k \mathbf{E}_k \tag{16}$$

where the matrix $\mathbf{A}_{jk}$ includes the geometric factors relating the electric field at $\mathbf{r}_j$ due to the radiating dipole at $\mathbf{r}_k$. This equation may be solved using a variety of techniques including matrix inversion and from this knowledge the scattering properties of the particle may be evaluated. The total scattered field at the observation point $\mathbf{r}_{\text{obs}}$ is then

$$\mathbf{E}^{\text{sca}}\left(\mathbf{r}_{\text{obs}}\right) = \sum_j \mathbf{A}_j\left(\mathbf{r}_{\text{obs}}\right) \alpha_j \mathbf{E}_j \tag{17}$$

where the matrix $\mathbf{A}_j\left(\mathbf{r}_{\text{obs}}\right)$ includes the geometric factors relating the electric field at $\mathbf{r}_{\text{obs}}$ due to the radiating dipole at $\mathbf{r}_j$.

The DDA Method appears in the literature as the Coupled Dipole Method (CDM), used for instance by Singham and Bohren [54,55] and Dungey and Bohren [56] to calculate the scattering by a homogeneous spherical or non-spherical particle and by Bourrely *et al.* [57] to introduce a new formulation to discretize a homogeneous or inhomogeneous particle of arbitrary shape in terms of multiple partitions.

An excellent review concerning the DDA method has been published by Draine and Flatau [58], that includes a comparison with other methods. An important conclusion reached in that work is that for targets with $|m| \leq 2$ (m being the complex refractive index), scattering and absorption cross-sections can be evaluated with accuracies of a few percent, provided that the number of dipoles N satisfies the relation

$$N > \frac{4\pi}{3} |m|^3 \left(k a_{eff}\right)^3 \tag{18}$$

a_{eff} being the effective radius of the target of volume V:

$$a_{eff} = \left(\frac{3V}{4\pi}\right)^{1/3} \tag{19}$$

The DDA method also has been employed to analyze the scattering by interstellar graphite grains [59], by two spheres in contact [60], rectangular solids

[61], porous dust particles [62], bianisotropic scatterers [63], by human white blood cells [64], etc.

3.4 Finite-Difference Time-Domain (FDTD) Method

This method is broadly used in computational electromagnetism [65]. It is similar to the DDA in that the scattering system is descritized, but in other respects it is unique. The FDTD technique is a direct implementation of the Maxwell time-dependent curl equations to solve the temporal variations of electromagnetic waves within a finite space that contains an object of arbitrary shape and properties. In practice, the space including the scatter object is descritized by a grid. The basic element of this descritization is the Yee cell [66]. In general, any function of space and time u can be evaluated as

$$u(i\Delta x, j\Delta y, k\Delta z, n\Delta t) = u^n_{i,j,k} \tag{20}$$

Here Δx, Δy, Δz are the lattice space increments in the coordinate directions, and i, j, k are integers. Δt is the time increment, assumed uniform over the observation interval, and n is an integer. The fields for the case of an absorptive scatterer are computed by a time-marching scheme:

$$\begin{aligned}\mathbf{E}^{n+1}(x,y,z) = {} & \exp(-\tau\Delta t)\,\mathbf{E}^{n}(x,y,z) \\ & + \exp(-\tau\Delta t/2)\,\frac{\Delta t}{\varepsilon_r}\nabla\times\mathbf{H}^{n+\frac{1}{2}}(x,y,z)\end{aligned} \tag{21}$$

$$\mathbf{H}^{n+\frac{1}{2}}(x,y,z) = \mathbf{H}^{n-\frac{1}{2}}(x,y,z) - \frac{\Delta t}{\mu}\nabla\times\mathbf{E}^{n}(x,y,z) \tag{22}$$

where $\varepsilon = \varepsilon_r + \mathrm{i}\varepsilon_i$ is the complex dielectric constant, $\tau = \omega\varepsilon_i/\varepsilon_r$, and μ the permeability of the medium. The unknown value of each component of the electric field $\mathbf{E}^{n+1}$ depends on the previous value of the electric field $\mathbf{E}^n$ and on the previous values of the other components of the magnetic field $\mathbf{H}^{n+\frac{1}{2}}$ at the sides of the point (i, j, k). By alternating these computations and stepping through time, the fields can be propagated through the entire volume, and the near or far-field scattering solutions can be obtained.

In the FDTD method the grid used must be smaller than the wavelength and the time steps must be chosen such that some stability criteria is ensured [65]. Traditionally, FDTD results have suffered due to spurious, nonphysical reflections occuring at the boundary of the matrix. These problems have largely been solved with the introduction of perfectly matched boundary conditions [67–69]. The method has been used extensively to study the scattering, absorption and polarization features of large hexagonal ice crystals [70,71]. Recently, Videen *et al.* [72] compared the results with those given by multipole techniques when examining the scattering from aggregated spheres and used the method to study the changes that occur when the internal structure of the system is modified.

There are still other techniques developed to solve the problem of scattering from microstructures, for instance, the method of moments (MOM) [73], the multiple multipole (MMP) method [74], the finite element (FE) method [75], etc. The method of choice depends on the desired accuracy, the computing resources, and on the intrinsic size and complexity of the scattering system. With the development of so many techniques, the method of choice is often the one that is most readily available. Recent studies have been undertaken to compare different methods. Recent work by Wriedt and Comberg [76] compares the DDA, FDTD and EBCM methods for the same scattering system, a cube. Cooper *et al.* [77] compared the MOM, MMP and the Finite Integration Technique (FIT) for the sphere canonical problem.

4 Microstructures on Surfaces

4.1 Introduction

The scattered electromagnetic fields change significantly when an object is brought in close proximity to a planar or near-planar surface or substrate. The combination of a microstructure and a smooth surface constitute by itself a rough surface, and its study is of practical interest in many areas: surface defect detection in the semiconductor industry [78], surface degradation by particle contamination [79], biosensors [80], optical particle sizing [81], etc. Many theoretical models have been developed and many experimental measurements have been carried out in recent years. In this section a short review of recent works in the field is presented.

4.2 Dipole Methods

The simplest system occurs when the microstructure size is much smaller than the wavelength, so the microstructure can be modelled by a dipole. This approximation is valid when the field within the particle is nearly constant. This approximation holds when the scatterer is not too close to the interface. On decreasing the particle-substrate separation, the assumption begins to fail because higher order multipoles become significant due to particle-substrate interaction.

Lindell *et al.* [82,83] used exact image theory (EIT) to derive the scattering of electromagnetic waves from a small object above an interface separating two isotropic and homogeneous media. EIT is used for deriving a generalized Green function to take into account the presence of a nearby interface. Other simpler models have been shown to provide quite accurate results [84].

Most simple models assume the dipole is illuminated by the a superposition of the direct and reflected plane waves. The total scattered field from the dipole is the superposition of the direct and image scattered fields. No multiple interactions occur between the dipole and the substrate. Jakeman showed that analytical expressions for the field scattered from small particles (spheres, discs, and needles) distributed on a tilted interface agreed to first order with those of Rayleigh-Rice theory [85]. Videen *et al.* used similar expressions to study the polarization state of small spheres near substrates [86]. Moreno *et al.* [87] studied multiple scattering effects between two small spherical metallic particles on a flat conducting surface. Germer [88] used a similar theory to study the angular dependent polarization of out-of-plane optical scattering from dipole-like particles above and below a substrate. An exact calculation of the substrate effect in the scattering is considered by Ortiz *et al.* [89,90] and Valle *et al.* [91]. These works use the exact solution for the emission of a dipole close to a plane interface and include the surface waves components of the scattered field that are identified as the surface plasmon polaritons in the case of metallic substrates.

4.3 Mie Methods

The double-interaction model of Nahm and Wolfe [92] has been used to calculate the scattering by a sphere over a perfectly conducting mirror. In this model, the sphere is illuminated by the beam both directly and after a specular reflection from the surface. This secondary reflected beam is partially obscured by the particle. The interaction between the sphere and mirror is assumed to be zero, so the sphere scatters light as if it were isolated. Each of these two beams generates two contributions to the total scattered field: one directly and another after being reflected by the mirror surface. Four components make up the total scattered field. Similar models include Fresnel reflection coefficients to handle nonperfectly conducting mirrors [93,94].

In a series of papers, Spyak and Wolfe [95–97] compare calculated light scattered by spheres on a mirror with experimental data. They assume that the particles scatter radiation according to Mie theory, and do so independently. The lack of agreement between theory and experiment for the backscatter may be due to the radiation reflected off the mirror before illuminating the sphere not considered in the model.

Bobbert and Vlieger [98,99] reduce the problem of light scattering by a sphere on a substrate to the problem of scattering by a sphere in a homogeneous medium and that of the reflection of spherical waves by the substrate. The first problem is solved using the Mie theory, for the second one they use an extension of Weyl's method for the calculation of the reflection of dipole radiation by a flat surface. They introduce a matrix characterizing the reflection of spherical waves by the substrate. The matrix elements are integrals over a complex angle that can only be analytically evaluated in the case of a perfectly conducting substrate and in the case of very small particles.

Videen expanded the interaction field about the image location to solve the scatter from a sphere in front of [100] and behind [101] a smooth arbitrary substrate and later provided an exact theory for an arbitrary particle system in front of [102] and behind [103] a perfectly conducting substrate. Similar derivations for spheres resting on substrates have been provided by Johnson [104–106] and Fucile *et al.* [107]. In this methodology, the boundary conditions at the particle and at the surface are satisfied simultaneously by projecting the fields in the half-space region not including the particle onto the half-space region including the particle. For nonperfectly conducting substrates some numerical method or simplifying assumption must be used. Theoretical results for a cylinder above a substrate [108] calculated using this method were compared with experimental results [109]. Videen was able to show that the polarization state of highly irregular dust particles on a substrate was primarily dependent on the size of the particle and could be approximated by a single dipole placed at the center of the particle above the substrate [110].

Moreno *et al.* [111] and Peña *et al.* [81] developed a modified double interaction model for the light scattered from spheres and cylinders on a substrate. The Fresnel coefficients for the reflection at the surface are considered as well as a geometrical shadowing factor to take into account the shadowing effect in the incoming and outgoing beams.

Borghi *et al.* [112] presented a method for treating the two-dimensional scattering of a plane wave by an arbitrary configuration of perfectly conducting circular cylinders in front of a plane surface with general reflection properties. This is based on the plane-wave spectrum of cylindrical functions involved in the decomposition of the field scattered by a cylinder. The substrate can be dielectric, metallic, anisotropic or lossy media, as well as multilayered.

4.4 Integral Methods

A number of theories have been formulated in terms of exact integral equations for the electromagnetic fields that are solved by standard numerical methods. It may be stressed that the formulation is exact in the sense that no additional physical assumptions or approximations are needed; i.e., all limitations come from the numerical procedure. Integral equations are derived from integral theorems that combine differential Maxwell equations and appropriate boundary conditions. One of the methods most widely used is the Extinction Theorem (ET) of Physical Optics [9] which produces a surface integral equation that relates the incident field with the sources on the surface , and these with the scattered fields. Though the ET method was initially used to calculate the scattering by random rough surfaces, Valle *et al.* extended the method to calculate the far-field [113] and near-field [114] scattering by small metallic particles on flat conducting substrates. Due to the exact character of the formulation, multiple interaction between particle and substrate is taken into account. Saiz *et al.* [115] and Valle *et al.* [116] have studied the effect of particle size, separation and the effect of the optical constants on the light scattering. The method has also been used to study surface plasmon polariton generation with small particles on real metallic substrates [117]. Madrazo and Nieto-Vesperinas have established the ET method for multiply connected domains. This improvement allows the numerical simulation of the scattering from systems composed of surfaces belonging to separated bodies of arbitrary shape [118–120]. A different integral method is that proposed by Greffet *et al.* [121] In this case they obtain a volume integral extended to the particle volume, including the substrate, by means of adequate Green functions. They have calculated the far-field and near-field scattering produced by 2-D particles deposited on a dielectric planar waveguide, with special attention to particle interaction [122], and the near field corresponding to a dielectric rod below a metallic surface under surface plasmon generation conditions.

4.5 Other Methods

Many other approaches to the problem of scattering from microstructures on surfaces may be found in the literature. Wriedt and Doicu [123] solved the scattering from an axisymmetric particle on or near a surface with a formalism based on the EBCM and the integral representation of spherical vector wave function over plane waves. Special attention was put in the interaction between a particle and its image. Taubenblatt [124] used a modified version of the coupled dipole method to calculate the far field scattering intensity from a dielectric cylinder on a surface, when illuminated with a plane wave with field vector along the cylinder axis. Schmehl *et al.* [125] used the same method to analyze the scattering by features as contaminants on surfaces. These works show important efforts in the improvement of numerical techniques in order to accelerate the computation or obtain good convergence rates. Wojcik *et al.* [126] showed numerical solutions of Maxwell's equations for problems involving scattering from submicron objects on silicon wafers by considering time-domain finite elements. Kolbehdary *et al.* [127] analyzes the problem of scattering of electromagnetic waves from a dielectric cylinder partially embedded in a perfectly conducting ground plane. A field equivalence theorem is employed to derive an integral equation for the equivalent magnetic current on the dielectric interface, which is later solved using the Galerkin's method. The equivalent magnetic currents are then used to determine the scattered field everywhere outside and inside the dielectric cylinder.

Finally, a very simple approach is the ray-tracing solution or, in other words, the application of the geometrical optics approximation. A plane wave incident on a metallic object is, from a geometrical point of view, a beam of parallel rays of uniform density that is reflected by the sphere-substrate system. The scattered field is obtained as the coherent sum of the group of rays emerging from the surface with a common angle. This method has been showed to produce surprisingly good fits to the experiment when the observation is far from the specular direction (where the diffraction effects are too strong), even for particles with size of the order of the wavelength. For instance Saiz *et al.* [128] analyzed by this means the backscattering from particles on a substrate or the effect of the different particle density on the scattering patterns [129].

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