Tollbooths

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Abstract

Toll plazas are major bottlenecks of heavily-traveled toll roads, causing delay and congestion. Our task is to minimize motorist annoyance by determining the optimal number of tollbooths to deploy in a barrier-toll plaza.

Three models are presented in this paper to solve the problem. Firstly we develop a basic model called Traffic Flow Graphic Model which examines the traffic flowing through the plaza. In this model, motorist annoyance is viewed as the combination of two factors: how many motorists are affected within the plaza and how long they take to pass it. So the product of vehicle number and the time span is taken as the measure of motorist annoyance. We express this measure in terms of 4 variables: traffic flow rate at plaza entrance, number of tollbooths, tollbooth service time span, and the exit flow capacity. The optimal number of tollbooths is just what minimizes the motorist annoyance measure function. The result is that the optimal number of tollbooths is proportional to tollbooth service time span, and it is also proportional to exit flow capacity during rush hours, or proportional to arrival flow rate during normal hours.

We explicitly apply the model to the scenario where there is exactly one tollbooth per incoming travel lane. The results are: this is more effective than the current practice (more tollbooths less lanes) during normal hours, but less effective during rush hours.

We develop further a delicate traffic flow model which allows the flow rate to vary, and take the probability of the event that passing tollbooths rate exceeds departure flow rate, as the motorist satisfaction measure. This measure is based on the fact that if the flow rate at tollbooths is larger than that of the exit, the annoyance will be thought by motorists to be the short of lanes rather than the tollbooths.

Combining the two models above results in a better estimation of the optimal number of tollbooths. The lower bound of the tollbooths number can be obtained with the basic flow model, which is then improved

with the delicate flow model.
In addition, we propose the third model called the Combined Queuing Model to simulate the behavior of individual vehicles. This model is essentially a sequential combination of two queuing systems, representing the entrance and the exit of the plaza, respectively. The motorist annoyance measure in this model is the average waiting time of the vehicles within the plaza.
Test data are based on some historical and empirical data about the road capacity, the traffic flow rates of rush hour and normal hour, etc. Applying the same test data to each of the models described above, the results are quite consistent.

1 Introduction

Toll plazas are major bottlenecks of heavily-traveled toll roads, causing delay and congestion. Collecting tolls are usually unpopular, it is desirable to minimize motorist annoyance by limiting the amount of traffic disruption caused by the toll plazas. Thus, our task is to determine the optimal number of tollbooths to deploy in a barrier-toll plaza.

A toll plaza and the vehicles it serves form a typical queuing system. Certainly, we can apply a queuing model to simulate the behavior of individual vehicles, and use the average delay time to determine the optimal number of tollbooths. In fact, this is what we do later. However, we first examine the behavior of the traffic flow from a more global viewpoint. Our first model, the analytic Traffic Flow Graphic Model displays how a plaza influences the traffic flow, how the motorist annoyance can be measured by both congestion and delay, and how an estimated optimal number of tollbooths is determined by minimizing the annoyance. Then in our second model, the Refined Traffic Flow Model, we optimize the estimated optimal number of tollbooths through a delicate point of view, and obtain the ultimate results. Finally, we test the ultimate results in our third model, which is based on a combination of sequential queuing models, to see whether they coincide with the cases in real world.

2 Assumptions

1. There is no accidents in our analysis.

2. Any two toll plazas along the same road are much far away apart, and do not affect each other. Thus we involve only one plaza in our analysis.

Assumptions $3--8$ is required in the Traffic Flow Graphic Model:

3. The traffic flow rate is defined as the number of vehicles passing through an ideal horizontal line on the road per unit time.

4. The congestion on departure is not so serious as to influence the service at the tollbooths. This enable us to analyze the congestion at the entry and congestion at the exit separately, with tollbooths as their separating boundary.

5. During a reasonably short time interval T, the arrival traffic flow rate is constant. Typically, if T represents a rush hour of the day, we assume the arrival flow rate to be constant during the rush hour.

6. The exit of a plaza has a maximum permissible departure flow rate Q_2. Its value is determined by the maximum speed the vehicles can reach after acceleration, the road capacity of the exit lanes, etc.Q_2 is variable because the maximum speed of every vehicle is different. Q_2 varies as the maximum speed of the vehicles that travel through the exit changes.

7. The entry lanes to the plaza also has a maximum permissible arrival flow rate Q_0. We calculate Q_0 using maximum speed permitted and the necessary interspace between vehicles in real world, and assume that the incoming flow rate will never exceeds this limit.

8. The flow passing through tollbooths does not exceed a maximum permissible rate Q_1. The maximum rate $Q_1$is proportional to the number of tollbooths m, and inversely proportional to the average toll service time tau(the time a vehicle is served at a tollbooth), that is, $Q_1 = m/\tau$.

Assumptions $9--10$ is required in the Combined Queuing Model:

9. The toll service time follow some distribution over some interval.

10. The interarrival time(the time between successive arriving vehicles at the entry) follows some distribution during some time interval.

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2.1 List of Symbols

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Figure 1:

symbol	discription
L	the number of lanes of the road
q_0	the arrival flow rate
q_1	the passing flow rate
q_2	the departure flow rate
Q_0	the limit of the arrival flow rate
Q_1	the limit of the passing flow rate
Q_2	the limit of the departure flow rate
n_1	the number of vehicles congested at the entrance of the plaza
n_2	the number of vehicles congested at the exit of the plaza
A_1	the total motorist annoyance before the tollbooths of the plaza
A_2	the total motorist annoyance at the exit of the plaza
m	the number of tools in the plaza
m^*	the number of tools in the plaza
τ	the average vehicle passing time at the tollbooths
t_c	the average departure time of the vehicles

3 Traffic Flow Graphic Model

3.1 Analysis

We devise the following experiment to measure how much a plaza influences a flow of vehicles passing through it: we let a flow enter the plaza at a constant flow rate of q_0, and the flow lasts a time period T. Assume the plaza is empty before the flow, and no other vehicles enter the plaza during or after the period T. In the worst case, $Q_2 < Q_1 < q_0$, vehicles begin to accumulate both at the entry and the exit to the plaza, congestion is build up; in some other cases, such as $Q_2 < q_0 \leq Q_1$, congestion is formed only at the exit. Let $n_1(t)$ and $n_2(t)$ denote the number of blocked vehicles at the entry and at the exit, respectively. For simplicity, assumption 4 allows us to analyze $n_1(t)$ and $n_2(t)$ separately. The key point here is that the motorist annoyance can be measured by the area A_i under the curve $n_i(t)$(explained later), and this area can be expressed as a function of q_0, Q_1 and Q_2. To minimize the total annoyance, We only need to minimize the total area $A = A_1 + A_2$, and find out where the minimum can be reached. Given a certain toll road, Q_2 is fixed, Q_1 is directly related to the number of tollbooths m(see assumption 8), therefore we can find the optimal m^* that minimize annoyance, under various values of q_0(such as the rush hour q_0 and the normal hour q_0).

Figure 2:

Let's explain why the annoyance can be measured by a area. 1 unit of annoyance can be measured as 1 unit number of blocked vehicles multiplied by 1 unit of delay time. Consider an amount q of gasoline is stored at time $t = 0$, and each day an equally small amount is used, and after T time it is used up.(See figure 2) We know the storage cost per unit quantity per day is s, then what's the total cost? We take the average daily storage $q/2$, multiply by the time period T, and multiply that by the storage cost per quantity per day s, yielding total cost $s * q/2 * t$. We can also compute the area under the curve:

$$s \int_0^T q(t)dt = \frac{s}{2}qt$$

Similarly, if $n(t)$ is the number of blocked vehicles at the entry or exit of a plaza, and if it stays over a positive time period T, then the total annoyance A is the area under the curve. It take both congestion(quantity) and delay(time) into account.

3.2 Model Formulation

We will derive the formula for $n_1(t)$(blocked vehicles at the entry) and $n_2(t)$(blocked vehicles at the exit, and find the total annoyance $A = A_1 + A_2$, under various conditions, where A_i is the area under the function curve $n_i(t)$.

See figure 1. $n_1(t)$ depends on q_0 and q_1, where q_1 is the actual passing flow rate at the tollbooths. If $q_0 > Q_1$, then $q_1 = Q_1$, q_1 reaches the maximum permissible flow rate Q_1; otherwise, $q_1 = q_0$. So $q_1 = min(q_0, Q_1)$. Similarly, $n_2(t)$ depends on q_1 and q_2, where $q_2 = min(q_1, Q_2)$. So the total annoyance is of the form

$$A = f(T, q_0, Q_1, Q_2)$$

Then we will find the optimal Q_1^* that minimize A, under various conditions. Since $Q_1^* = m^*/\tau$(because $1/\tau$ number of vehicles pass through a tollbooth, and there are m tollbooths), we will finally reach the optimal number of tollbooths m^*. We carefully develop these formula as follows.

There are $3! = 6$ permutations of q_0, Q_1, and Q_2, so there are 6 cases to consider.

case 1: $Q_2 < q_0 \leq Q_1$ In this case, $n_1(t)$ remains 0, and $n_2(t)$ is increasing with the slope $q_0 - Q_2$. See figure 3. The area under the curve $n_2(t)$ is

$$A_2 = \frac{T^2 q_0 (q_0 - Q_2)}{2Q_2}$$

.

Figure 3:

The total annoyance is

$$A = 0 + A_2 = \frac{T^2 q_0 (q_0 - Q_2)}{2Q_2}$$

case 2: $Q_2 < Q_1 < q_0$ Both $n_1(t)$ and $n_2(t)$ are increasing, congestion occur at both sides of the plaza. See figure 4.

The area under the curve $n_1(t)$ is

$$A1 = \frac{T^2 q_0 (q_0 - Q_1)}{2Q_1}$$

The area under the curve $n_2(t)$ is

$$A2 = \frac{q_0^2 T^2 (Q_1 - Q_2)}{2Q_1 Q_2}$$

The total area is

$$A = A_1 + A_2 = \frac{(q_0 - Q_2) T^2 q_0}{2Q_2}$$

.

Figure 4:

case 3: $Q_1 \leq Q_2 < q_0$ Congestion only occur at the entry. See figure c.

The total area is the area under the curve $n_1(t)$:

$$A = \frac{T^2 q_0 (q_0 - Q_1)}{2Q_1}$$

.

Figure 5:

case 4: $Q_1 < q_0 <= Q_2$ Congestion only occur at the entry, the area is the same as case 3.

case 5: $q_0 \leq Q_1 \leq Q_2$ No congestion occurs, the annoyance is 0.

case 6: $q_0 \leq Q_2 < Q_1$ No congestion occurs, the annoyance is 0.

Now let's conduct the traffic flow experiment on a given toll road to determine its optimal number of tollbooths m^*, or its corresponding optimal Q_2^*. We know the total annoyance

$$A = f(T, q_0, Q_1, Q_2)$$

Once the road is given, Q_2 is a constant; let q_0 equal the flow rate of rush hours($q_0 > Q_2$), which is also a constant; also fix T for the moment. Then the annoyance totally depends on Q_1:

$$A = f(Q_1) = \begin{cases} \frac{T^2 q_0(q_0 - Q1)}{2Q_1} & Q_1 \leq Q_2 \leq q_0 \\ \\ \frac{T^2 q_0(q_0 - Q2)}{2Q_2} & Q_2 < Q_1 \leq q_0 \\ \\ \frac{T^2 q_0(q_0 - Q2)}{2Q_2} & Q_2 < q_0 \leq Q_1 \end{cases} \tag{1}$$

When Q_1 ranges from 0 to $+\infty$,(equivalent to the number of tollbooths ranges from 0 to $+\infty$), we expect a minimum to occur at some value of Q_1.

For convenience, we draw the graph of the function in figure 6.

Figure 6:

Obviously, A reaches its minimum when $Q_1^* = Q_2$.

Similarly, let q_0 be the flow rate of normal hour($q_0 \leq Q_2$), other variables also remain constant, we draw the graph of the function

$$A = f(Q_1) = \begin{cases} \frac{T^2 q_0 (q_0 - Q_1)}{2Q_1} & Q_1 \leq q_0 \leq Q_2 \\ 0 & q_0 < Q_1 \leq Q_2 \\ 0 & q_0 \leq Q_2 \leq Q_1 \end{cases} \tag{2}$$

in figure 7.

Figure 7:

Apparently, A reaches its minimum when $Q_1^* = q_0$.

3.3 Model Solution

To get the optimal m^*, we proceed as follows.

Q_1 is the sum of m flow rates:

$$Q_1 = m/\tau, \text{ or } m = Q_1\tau$$

Thus, the optimal number of tollbooths is

$$m^* = Q_1^*\tau$$

On the same road, the optimal numbers of tollbooths during rush hour and during normal hour are usually different, we choose the larger one to satisfy both cases. Thus the optimal number of a toll road is

$$m^* = Q_2\tau$$

Values of these parameters can be easily obtained from real world, so the model is easily applicable. For example, on a typical toll road with 5 lanes, during rush hour, the incoming flow rate q_0 is 1.4 vehicles/second, and average toll service time τ is 8 seconds, and Q_2 is 1.0 vehicles/second, since $q_0 > Q_2$, then the optimal number of tollbooths is

$$m^* = Q_2\tau = 8$$

if during normal hour q_0 is 0.65 vehicles/second, and other parameters remain unchanged, then, since $q_0 \leq Q_2$, the optimal number of tollbooths is

$$m^* = q_0\tau = 5$$

As noted above, we deploy 8 tollbooths in this plaza.

3.4 Interpretation of the Model

Equation $m^* = Q_2\tau$ suggests the optimal number of tollbooths m^* is proportional to toll service time τ and departure flow rate during rush hours, and proportional to arrival flow rate and toll service time during normal hours. Intuitively, we would expect m^* to increases as τ increases and to decrease as τ decreases, thus our model makes common sense.

We explicitly apply the model to the scenario where there is exactly one tollbooth per incoming travel lane. In this case, Q_1 is usually smaller than Q_2, and $Q_1\tau$ is the number of tollbooths, which equals the number of lanes. The optimal number of tollbooths for this road for rush hour is $Q_2\tau$, bigger than the number of lanes, thus the practice of more tollbooths than lanes is more effective; on the other hand, the optimal number for this road for normal hour is $q_0\tau$, which is relatively small, thus the scenario of equal number of tollbooths and lanes is more effective.

3.5 Strengths and Weaknesses

1. Simple. Observing the traffic flow as a whole and assuming constant flow rate greatly simplifies calculation.

2. Applicable. The required optimal number of tollbooths turns out to be a product of toll service time and maximal permissible departure flow rate. Since these parameters can be easily obtained, our model is easily applied to real world.

3. Reasonable. The annoyance is measured by the integral of the number of blocked vehicles with respect to delay time, quite reasonable.

4. Not so realistic. Constant flow rate and separately consider the entry and exit of the plaza are not so realistic. And our model also ignores many other factors such as the density of the number of vehicles blocked around the plaza.

4 Refined Traffic Flow Model

The Traffic Flow Graphic Model is a ideal model in which we ignore many factors that have important influence on the traffic flow. These factors are:

1. τ is variable at any time, because the service time for every car is different. τ follows some distribution which is difficult to be regarded as some common distribution.

2. Q_2 is also a variable, every car's speed at the exit may be different because of many factors.

But we can consider the Traffic Flow Graphic Model as the solution in a very short time interval. In a very short period of time, τ and Q_2 can be considered to be a certain value and remain constant. Now we only consider the case in rush hours since the optimal number of tollbooths is determined by its required number during rush hours. So $Q_2 \leq q_0$ holds. To get the optimal solution for some short time interval, we can simply let $Q_1 = Q_2$, and calculate m by $m = \tau \cdot Q_2$.

If in some time interval, $Q_1 \geq Q_2$, we can say that the Q_1 at this time is suitable for the Q_2 at this time. Interpret this theory to real case is that: if the flow rate at tollbooths is larger than that of the exit, the annoyance will be thought by motorists to be the short of lanes rather than the tollbooths.

But from equation $Q_1 = \frac{m}{\tau}$, as τ is varying from time to time, we know that when given a certain m, Q_1 is also a variable.

The equation

$$Q_2 = \frac{L}{t_c} \tag{3}$$

reflects the relationship between Q_2 and other parameters. L is the number of the lanes and t_c is the average time for a car to pass the exit at that time. It is obvious that t_c is varying from time to time. t_c follows some distribution. If given the distribution of t_c, we can get the distribution of Q_2.

τ and t_c are different on different road based on the its position, capability(number of lanes), road surface condition etc.

So our goal is : when given distributions of τ and t_c, decide the set of m, which maximize the probability of event $Q_1 \geq Q_2$ and choose the one with least value. This m^* is the optimal value for the road.

4.1 Results

We assign τ and t_c with many distribution such as Gauss Distribution and Uniform Distribution and etc to test our model. When $Q_1 \geq Q_2$, it is obvious that enlarging m can make the probability of $Q_1 \geq Q_2$ increase. When m reaches a certain value, the probability will remains 1. But in some cases, the value is so big, that it is unrealistic to apply it to the number of tollbooths. However, when m reaches some point m', the probability of $Q_1 \geq Q_2$ increases very slowly. Enlarge the m which is larger than m' will have little effect on the probability. Then we only need to find the optimal m', which is the m^* we want. In the figure 8 we can see the slope of the curve representing probability is large when m is small, and become smaller when m increases. It means that the increasing rate of the probability is slowing down. If this is a continues function, we can simply calculate the second derivative of the function to find where the slope decreases most quickly. But for the discrete point, it is hard for us to get the function and estimate the approximate second derivative. If we calculate the slope between every 2 points, and then calculate the $slope'$ between two slopes as the second derivative of the curve, the $slope'$ may fluctuate between positive and negative values when m increase. So we can not get the estimated approximate second derivative directly. As a result, we draw many figures of m and its related probability, and find that, when the probability is around 0.7, when enlarging m, the probability of event $Q_1 \geq Q_2$ increases very slowly. So we conclude that m^* is the optimal value of m when m^* is the least value that makes the probability of event $\frac{m^*}{\tau} \geq \frac{L}{t_c}$ be no less than 0.7.

We use this principle to get the results of the simulation when lane number is 4,6,8, and τ and t_c follow the Gauss distribution and draw the figures below:

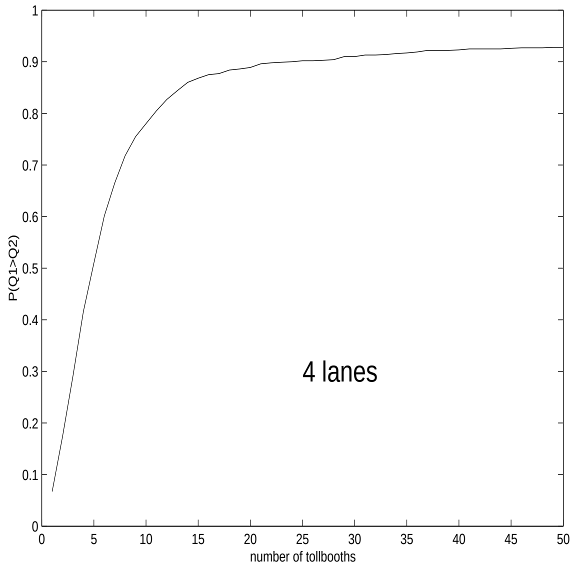

Figure 8: $\tau \sim$ Gauss(9,6), $t_c \sim$ Gauss(8,4), L = 4, $m^* = 8$

asd

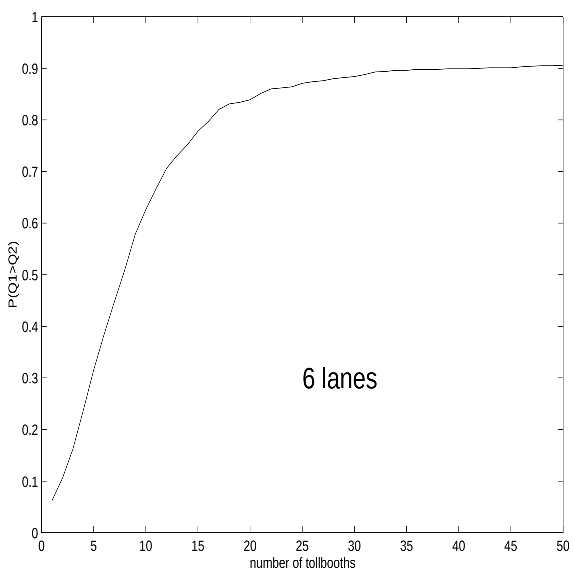

Figure 9: $\tau \sim$ Gauss(9,6), $t_c \sim$ Gauss(8,4), L = 6 , $m^* = 11$

asdf

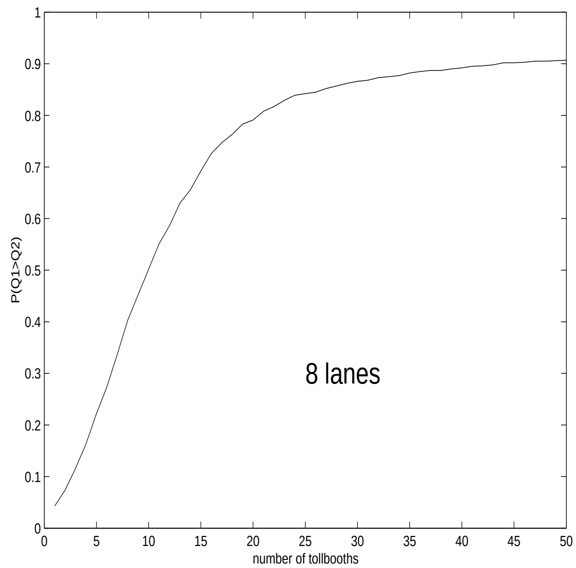

Figure 10: $\tau \sim$ Gauss(9,6), $t_c \sim$ Gauss(8,4), L = 8, $m^* = 16$

Figures above are the results based on Gauss distribution, but in real world, the distribution of τ and t_c may not follow Gauss distribution. We search for the real data of τ and t_c, and find in Jack[1] the data set of service time for each car during rush hours (06:59:36 am $\sim$ 08:00:53 am) at some plaza . The data set has a mean value 9 and standard deviation 6. The data set is listed in the appendix. Then we apply the data set to our model, with t_c still following Gauss distribution and get the following figures.

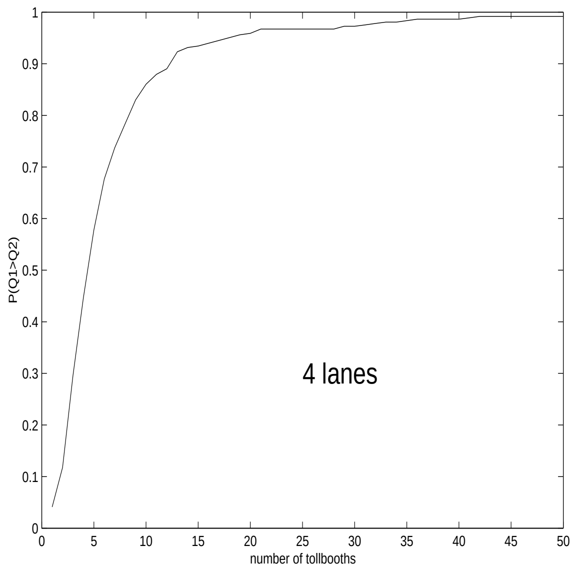

Figure 11: $\tau =$ the real data set , $t_c \sim$ Gauss(8,4), L = 4, $m^* = 7$

adf

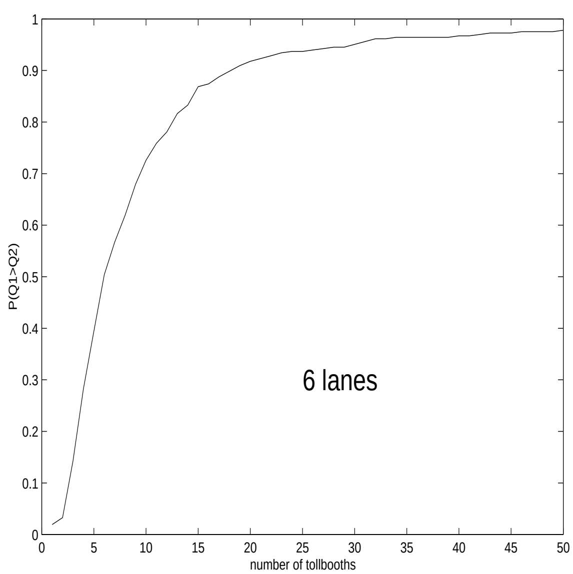

Figure 12: $\tau =$ the real data set , $t_c \sim$ Gauss(8,4), L = 6, $m^* = 10$

asdf

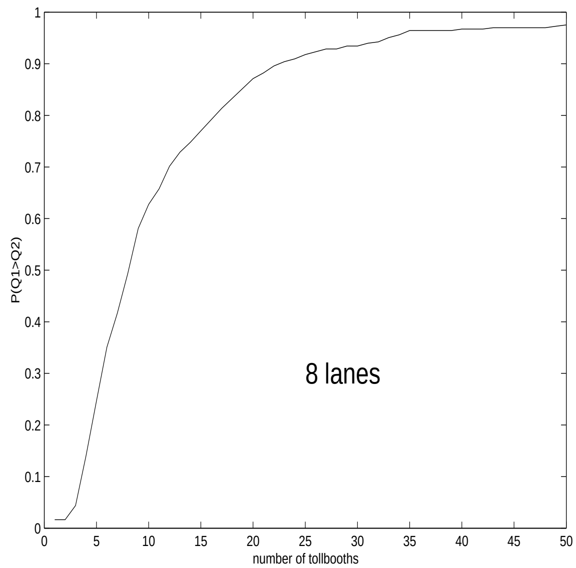

Figure 13: τ = the real data set , $t_c \sim$ Gauss(8,4), L = 8, $m^* = 13$

The figures above show that, when using the real data, the optimal value of m is much smaller than the case with Gauss distribution. This may indicate that the real data does not follow Gauss distribution. But the data is only collected from one particular plaza on 1 day, these data can only reflect the condition in that plaza at that particular time.

But There are errors between the results and the real cases. We attribute it to 2 reasons.

1. First, In Traffic Flow Rate Model, we simplify the arrival rate , passing rate and departure rate to be invariable during the time interval to get the total annoyance. Then we assume the annoyance to be the instantaneous annoyance. This way of thinking is just like the deduction of differential equation. But the flow rate is discrete and undifferentiable. So the result of this model will not coincide with the result derived from Figure 6 and Figure7.

2. The distribution of τ and t_c are not common distributions, Gauss distribution is an approximate simulation of distribution of τ and t_c. Further more, the mean and standard deviation is assigned using empirical data. This may not fit the real world situation.

But the error between our model and the simulation results is slim, we can also get optimal results from the simulation. We test many data sets in our simulation, and draw the results in figures.

4.2 Testing of the Model

Different distribution will lead to different results using the Refined Traffic Flow Model. Then we design some data sets to test the model to see how the distribution will affect the results of the model.

	(8,0)	(8,4)	(8,5)	(8,7)
(15,0)	5	6	6	7
(15,5)	6	7	7	8
(15,10)	7	8	9	10
(15,14)	9	10	11	11

The horizontal heading represents the Gauss distribution of t_c and the vertical heading represents the Gauss distribution of τ. The table is the optimal m value under these distributions. It reveals that when the service time and the departure time fluctuate in a larger range, the more tollbooths are needed to minimize the annoyance.

In the case $\tau \sim$ Gauss(9,0) and $t_c \sim$ Gauss(8,0), when m ≥ 5, the probability of event $Q_1 \geq Q_2$ will always be 1, and when m < 5, always be 0. So, 5 is the optimal value of m.

On the other hand, When $\tau \sim$ Gauss(15,0) and $t_c \sim$ Gauss(8,0), it means that τ is constant 15, and t_c is constant 8. This case is the same as the assumption in Traffic Flow Graphic Model, in which τ and t_c are constants. Using Traffic Flow Graphic Model we have m $= \tau \times \frac{L}{t_c} = 9 \times \frac{4}{8} = 4.5$ the least integer larger than it is 5. This exactly complies with the result of the Refined Traffic Flow Model.

4.3 Strength and Weakness of the Refined Traffic Flow Model

This model is derived from the Traffic Flow Graphic Model and apply its conclusion to the real world. It cut the time into small pieces, and assume that Q_1 and Q_2 are constant during the time small pieces of time, and deduce that the probability of $Q_1 \geq Q_2$ is the core to the solution. It changes the problem to be a probability model. This model simplify the problem and get the results under certain conditions. The results are to some extent reasonable in real life. But But the flow rate is discrete and undifferentiable. This is the reason of the error between the model and the real life.

5 Combined Queuing Model

After investigating the traffic flow from a global perspective, we adopt a delicate perspective, observing the individual behavior of the vehicles using a queuing model. Through computer simulations, we can obtain some statistics reflecting the level of service, such as the average waiting time and maximum waiting time of the vehicles under varying conditions.

5.1 Basic structure of the model

Our model is essentially a sequential combination of two multiple-server queuing models, each representing the congestion at the entrance and exit of the plaza.

Customers: the coming vehicles are the customers of the queuing system, and the interarrival time(the time between successive arrivals) are assumed to follow some distribution.

Queue1: represents the congestion at the entrance to the toll plaza. A vehicle is assumed to wait in queue when no tollbooth is currently available.

Queue discipline: the first-come-first-served principle is assumed.

Service facility 1: parallel servers are the tollbooths, and the service time are assumed to follow some distribution.

Queue2: represents the congestion at the exit of the toll plaza. A vehicle is assumed to wait in the queue when no travel lane is currently available, since the vehicles compete for the limited number of lanes.

Service facility 2: parallel servers are the travel lanes, and service time are assumed to follow some distribution. We assume a vehicle on departure takes no time to reach a lane, and service time at the entrance of the lane, and immediately leave the queueing system.

5.2 Simulation algorithm

We can easily apply the standard queueing algorithm twice sequentially. We ignore the list of the algorithm here, since it can be found in many mathematical modeling textbooks, such as Frank[5].

5.3 Simulation results

We use the real world data and get the following result:

When the incoming flow rate reaches the maximum: 70mph
the average service time follows Gauss(15,10) .

the average departure time follows Gauss(8,4)

number of lanes	optimal number of tollbooths
4	12
6	19
8	26

The results are close to the results of Refined Traffic Flow Model under the same conditions.

5.4 Strengths and Weaknesses

1. This model is more realistic, because it simulates the individual behavior of vehicles and collect their average waiting time.

2. In the simulation, we assign some distributions to the random variables, but these distribution may not reflect the fact. And different roads have parameters with different distributions.

6 Conclusions and Recommendations

According to the three models above, we propose a method to determine the optimal number of tollbooths on a road by the following steps:

1. Get the rush hour data for arrival flow rate , the estimated service time, and the estimated departure flow rate.

2. Analyze the data to get a distribution for τ and t_c. And calculate the mean of τ and t_c as τ^* and t_c^*.

3. Use the Traffic Flow Graphic Model to get the estimated optimal of m by m $= \tau \times Q_2 = \tau^* \times \frac{L}{t_c^*}$

4. Use the refined traffic flow model to search the optimal value of m setting the estimated m as the lower bound of m. Search the number larger than m to get the ultimate optimal value of m.

5. Test the result in the simulation using queuing model.

If the designers of the toll plaza feel that Q_2 is so small that it is difficult to optimize the total Annoyance, they can extend the plaza to let the vehicles to have a sufficient space to accelerate when it reach the exit, which can enlarge the departure rate.

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[5] Frank R. Giordano Maurice D.Weir. William P. Fox. 2003. *A First Courser In Mathematical Modeling*. Third Edition. Machine Press.

Appendix A

real data set of service time in seconds for each car during rush hours (06:59:36 am $\sim$ 08:00:53 am) from Jack[1].

5	5	6	8	7	4	8	8	15	15	13	7	8	4	16	18	6	6	4	4
8	8	16	8	4	1	15	5	10	6	8	6	7	6	9	5	6	5	9	6
4	6	4	9	4	14	7	19	8	31	7	8	12	5	7	4	29	6	4	15
7	4	6	5	9	5	8	6	5	9	5	8	6	5	21	25	13	7	11	12
24	7	7	6	17	9	7	8	28	7	5	6	8	11	7	6	6	7	10	10
9	6	18	17	7	6	6	6	7	9	7	6	19	14	9	6	7	7	8	10
7	5	10	7	6	4	9	5	7	12	8	10	6	7	10	9	6	5	4	16
6	10	10	8	10	4	11	8	10	15	18	6	5	5	15	11	8	5	5	4
5	7	5	11	9	11	8	8	5	27	5	16	17	11	13	5	5	11	6	13
33	6	5	4	20	8	6	5	4	5	4	7	7	8	48	12	12	5	6	7
19	8	7	5	6	7	19	9	18	9	6	7	5	7	6	22	7	6	8	12
9	19	13	7	11	12	8	8	11	21	8	6	6	10	5	14	6	12	9	9
8	6	8	7	4	12	9	5	14	7	23	6	4	4	22	5	5	6	7	6
5	7	9	9	7	5	13	5	6	9	15	5	7	7	5	6	19	9	8	11
7	11	8	5	6	7	9	20	8	20	6	4	9	7	8	8	7	6	6	9
7	6	6	7	7	11	8	7	6	7	6	13	6	4	4	7	16	10	10	16
4	5	17	10	8	6	9	6	8	18	16	5	72	9	30	7	10	9	6	6
9	13	11	19	05	11	7	6	5	8	11	6	7	10	5	26	7	18	19	13
5	11	5	22	11															

average = 9,   standard deviation = 6,   max number = 72