Fundamentals of Photonics
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APPENDIX

C

MODES OF LINEAR SYSTEMS

Every linear system is characterized by special inputs that are invariant to the system, i.e., inputs that are not altered (except for a multiplicative constant) upon passage through the system. These inputs are called the modes, or the eigenfunctions, of the system. The multiplicative constants are the eigenvalues; they are the attenuation or amplification factors of the modes.

A linear system is completely characterized by its eigenfunctions and eigenvalues. An arbitrary input function may be expanded as a combination of the eigenfunctions, each of which is multiplied by the corresponding eigenvalue upon transmission through the system, and the output is the sum of the resultant components. The modes are transmitted through the system without mixing among themselves.

The linear system that operates on the two-dimensional function $f(x, y)$ in accordance with (B.2-1), for example, is characterized by a number of modes satisfying the integral equation

$$\iint_{-\infty}^{\infty} h(x, y; x', y') f_q(x', y')\, dx'\, dy' = \lambda_q f_q(x, y), \qquad q = 1, 2, \ldots . \tag{C.1-1}$$

The functions $f_q(x, y)$ and the constants λ_q are the eigenfunctions and eigenvalues of the system, respectively. When $f_q(x, y)$ is the input to the system, the output is $\lambda_q f_q(x,y)$, which is identical to the input, except for the multiplicative factor λ_q.

An example (discussed in Sec. 9.2E) is light traveling a single round trip between two mirrors in a laser resonator. The distributions of light in the transverse plane at the beginning and at the end of the trip are the input and output to the system. The modes of the resonator are those light distributions that maintain their shape after one round trip, except for a multiplicative factor representing propagation and reflection losses. The modes are therefore the stationary distributions that remain unchanged after many round trips.

Another example (discussed in Chap. 7) is light traveling in an optical waveguide. The modes of the waveguide are those distributions in the transverse plane (the x–y plane) that are not altered as the light travels along the axis of the waveguide (the z direction). The eigenvalues are the phase factors $\exp(-j\beta_q z)$, where β_q is the propagation constant of mode q.

The concept of modes applies also to one-dimensional linear systems operating on functions $f(t)$. The modes of a linear shift-invariant system are the harmonic functions $\exp(j2\pi\nu t)$, since these functions maintain their harmonic nature (including the frequency) when they are transmitted through the system. The eigenvalue associated with the harmonic function of frequency ν is the system's transfer function $\mathcal{H}(\nu)$. In this case there is a continuum of modes indexed by the frequency ν.

Discrete linear systems are also important in optics. The linear system operating on vectors of size N (sets of numbers $X_1, X_2, \ldots, X_N$ arranged in a column matrix $\mathbf{X}$) is characterized by a square matrix $\mathbf{M}$ of size $N \times N$, which operates on an input vector $\mathbf{X}$ to generate an output vector $\mathbf{Y} = \mathbf{MX}$. The modes of such discrete systems are those input vectors that remain parallel to themselves upon transmission through the system, i.e., that obey $\mathbf{MX}_q = \lambda_q \mathbf{X}_q$, $q = 1, 2, \ldots, N$, where λ_q is a scalar. Thus the modes of the system are the eigenvectors $\mathbf{X}_q$ of the matrix $\mathbf{M}$, and the scalars λ_q are the corresponding eigenvalues.

The special case of discrete systems operating on vectors of size $N = 2$ is particularly important in optics. This system performs the matrix operation

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix},$$

with the input and output represented by vectors of size 2. There are two independent modes of the system, the eigenvectors of the *ABCD* matrix. Such systems describe the transformation of the polarization of light transmitted through an optical system (see Sec. 6.1B). The vector (X_1, X_2) represents the components of the input electric field in two orthogonal directions (the Jones vector), and (Y_1, Y_2) similarly describes the output electric field. The modes of the optical system are the vectors (X_1, X_2) (polarization states) that change only by a multiplicative factor on passing through the optical system. They represent the polarization states that are maintained as light travels through the system.

READING LIST

See the reading list in Appendix A.