Load and Cost Elasticity Defined Fair Resource Allocation(FRA): A Novel Application of Econometric Production Theory

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Abstract—Cloud computing, with its multi-tenancy feature, enables efficient sharing of resources among multiple users. The nature of resource allocation, however could lead to load imbalance and therefore performance degradation. Allocating user demands on servers with least Fixed unit cost would overload them. This leads to poor performance and higher response time which in turn results in an increased average service cost per tenant. The objective of this work is to optimize resource allocation in the cloud environment using a game theoretic approach. The calculation of the average service cost per user in the proposed method, is based both on the Fixed unit cost of the server as well as the servers Load. This approach ensures average service cost per tenant and minimal load imbalance among the cloud servers. Cobb Douglas production function is used to compute the average service cost incurred for each tenant. Series of experiments resulted in a post-allocation load imbalance factor of less than 1 among the servers and an optimum unit cost per tenant, which is less than the maximum unit cost of any server in the cloud environment.

Index Terms—Cloud Resource Allocation, Cobb-Douglas, Fair Resource Allocation, Game Theory, Extended Form Game, Nash Equilibrium, SaaS, IaaS.

1 Introduction

Cloud Computing as reflected in [1], [2] is the new age computing phenomenon, which is widely adopted by the modern day user. A cloud environment provisions resources to multiple clients simultaneously and has adopted the unique pay-as-you-use pricing model. For example, if 'CPU hours' is a resource provided by the service provider(SP), then users are charged per CPU hour used. Cloud Computing offers a cost effective and scalable platform to the computing world.

With the Software-as-a-Service (SaaS) model, many software providers host applications in the cloud for their customers to access [3]. They frequently leverage the use of Infrastructure-as-a-Service (IaaS) from such providers as Amazon Web Services (AWS), Google Compute Engine, Windows Azure, and Rackspace, or private cloud platforms such as CloudStack and OpenStack. Using IaaS resources to host SaaS products provides cost efficiency and decreases time to market. Many enterprises even operate using hybrid infrastructure public and private clouds. These IaaS providers have different cost models for the services they provide [4]. For example, Google offers a lower per-hour cost for shorter duration, but AWS c3 instances offer a lower price per GB of RAM for all purchase options [5].

In such an environment, SaaS providers' profits are generated from the margin between the operational cost of infrastructure and the revenue generated from their customers. One objective of the SaaS providers is to minimize these operational cost. This can be achieved by placing workloads in the lowest cost servers that meet their customers' requirement.

Allocating workloads on servers based only on the cost would lead to overloading of servers. It also means a longer turnaround time for a given task. For instance, the turnaround time for a task on a heavily loaded machine is definitely higher than that on a lightly loaded machine. Jalaparti et al illustrate this statement in [6] with the Illinois Could Computing Testbed. Overloaded servers would result in an increased average service cost per workload. A resource allocation algorithm that considers only the Fixed unit cost of the server will therefore not necessarily give the lowest average service cost per workload. A fair resource allocation algorithm should also consider the current load on the server along with the servers Fixed unit cost.

1.1 Problem Statement

Resource allocation in a cloud environment is a challenge due to the dynamic nature of user requests, heterogeneous environment, various cost models etc. A cloud service provider prefers to choose a resource allocation policy which ensures lower average service cost per user. A resource allocation strategy that considers only the Fixed unit

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cost of the server on which the workload will be allocated is not sufficient because it ignores the load on the server. It is important to note that an overloaded server leads to a higher turnaround time and hence an increased average service cost per user.

Consider a bunch of points (Here, Servers-Abstract sense) in space with co-ordinates (x,y): where, x=Fixed unit cost, y=Load. Objective is to identify the optimal x and y pairs or optimal Fixed cost and Load values that ensure an optimal average service cost per user.

1.2 Related Work

Jalaparti et al in [6] defined a new class of games called Cloud Resource Allocation Games (CRAGs) which models resource allocation problem using game theory. The authors capture the interactions between client-client and client-provider and ensure that the clients are charged optimally for their resource usage. They argue that the existing pricing and scheduling models do not provide the price-to-performance guarantee to clients. Their work demonstrates the variation in response time with respect to resource contention in cloud and compares the cost incurred by clients using linear and exponential cost functions. The performance of the model is compared with that of the Round-Robin mechanism. The authors find equilibrium for the resource allocation problem by using the Stackelberg games. Linlin Wu et al in [7] propose resource allocation algorithms for SaaS providers who want to minimize infrastructure cost and SLA violations. They address the situations where SaaS providers use IaaS to host their services. They have implemented cost driven algorithms which considered various QoS parameters (such as arrival rate, service initiation time and penalty rate) from both the customers and the SaaS providers perspective. Here, the strategy behind cost minimizations is reuse of virtual machines. Adel Nadjaran Toosi et al in [4] have proposed a model for revenue maximization for IaaS. IaaS cloud providers offer diverse purchasing options and pricing plans, namely on-demand, reservation, and spot market plans. They address a novel problem of maximizing revenue through an optimization of capacity allocation to each pricing plan by means of admission control for reservation contracts, in a setting where aforementioned plans are jointly offered to customers. They device an algorithm based on a stochastic dynamic programming formulation. Xu and Yu in [8] also modelled resource allocation in a virtual machine environment using game theory. Their work focuses on fair resource allocation for each user while supporting efficient resource allocation for each physical server. Cloud infrastructure comprises of heterogeneous physical resources like CPU, memory and storage. An intelligent allocation decision should ensure that the resource allocation among various virtual machines is fair. The authors model the resource allocation problem as an extensive form game and identify an optimal resource allocation strategy by finding the games Subgame Perfect Nash Equilibrium (SPNE) using backward induction method. Nahir, Orda and Raz in [9] model workload factoring using game theory. The benefit a user gets from using cloud is related to the usage pattern of other cloud users. Certain heavy cloud users may scare off other users thus resulting in a non-cooperative game. The authors propose a game theoretic approach for fair resource allocation for a wide range of user types. Their results show that there is a unique Nash Equilibrium (NE) strategy which is optimal. Wei et al in [10] discuss the QoS constrained resource allocation problem. They address the parallel computing problem in cloud, where resources across cloud are used for computation. The problem is modelled using game theory and is solved in two stages. In the first stage each participant finds its optimal resource allocation. This optimization problem has been solved using the Binary Integer programming method. In the second stage, initial strategies are multiplexed and an optimal solution is found for these multiplexed strategies. Rao et al in [11] discuss about the initial provisioning of resources and subsequent uninterrupted operation of the infrastructure. The cyber and physical components of a cloud infrastructure are subject to attack. A service provider should ensure that its users continue to get planned aggregate computational capacity even in the event of an attack. Service providers deploy redundant components as a counter measure to mitigate and contain infrastructure degradation or attacks. The authors propose a game theoretic approach for initial infrastructure provisioning and operation under uniform cost models. This entails deciding the number of servers to be deployed at various sites and the reinforcement of selected infrastructure components. Niyato et al in [12] propose a game theory based resource and revenue sharing model with a coalition of cloud service providers. In a coalition model, multiple cloud service providers come together and cooperatively form a resource pool and provide cloud services to users. The authors propose a stochastic linear programming game model to guide the sharing of revenue and resource pool among the service providers.

Literature on Resource allocation in Cloud computing topic is vast and it has various dimensions to it. We have mostly focused on resource allocation strategies that adopt game theoretic approach. The literature on such a topic which addresses SaaS cloud running on IaaS cloud is limited.

2 Proposed Model

This work is aimed at finding the optimal resource allocation strategy for the SaaS provider at a given point in time. Cobb-Douglas production function is used to compute the service cost. Here, the cloud resource allocation problem is modeled using game theory.

Any game theory model consists of 3 elements [13].

- Players who take part in the game. There can be more than 2 players in a game.
- Actions or different options available for each player at each decision point. There can either be one or more options available for a player.
- Preferences, i.e. ordinal preference of the player over the available options.

We map these three components of Game Theory to resource allocation game as below:

Players: In the cloud resource allocation game, objective of proposed model is to achieve optimal average service

cost per user. Thus, users are indirectly playing the game through service demands.
Actions: The distribution strategies among available servers are actions available to the user.
Preferences: Cost incurred by the SP for adopting the demand distribution strategy, will decide the ordinal preference.

Note: In game theory model, equilibrium is computed based on highest utility. However this work aims at finding the resource allocation combinations that result in minimum cost. So utility is computed using $-U_i$.

The model is based on the following assumptions:

- Cloud provides SaaS to users and is built on IaaS. Thus the cloud environment consists of servers whose Fixed unit cost varies based on the IaaS vendor.
- SaaS provider is interested in servicing user demands with optimal average service cost per user.
- User requests are addressed at specific time intervals. All requests pending at the decision moment are considered for resource allocation.

2.1 Essential components of Proposed Model

Game Theory: As discussed by Osborne in [13] and Davis in [14], game theory helps in modelling strategic situations. Game theory assists in determining the best possible outcome for all players resulting in an optimal outcome for each player, thereby achieving Equilibrium. Equilibrium is an action profile for all players where, no player can get a better outcome by unilaterally changing his action from the Equilibrium profile.

This work models cloud resource allocation using game theory where a game is modelled as an Extensive Form Game (EFG). The objective of this work is to distribute client requests among available servers in a cloud environment in such a way that the average service cost incurred per user is optimal and the post-allocation load imbalance among servers is least.

The optimal solution for such an EFG is obtained by finding the Subgame Perfect Nash Equilibrium (SPNE) with backward induction being a natural choice for finding SPNE.

Cobb-Douglas production function: Cobb-Douglas(CD) production function was chosen [15] to compute the cost incurred by SP, on provisioning a resource request in cloud. Cobb-Douglas production function [16] is most extensively used in economics to represent the relationship between the output and the combination of inputs used to obtain it. Following important characteristics of this production function have made it an attractive choice in the economic domain.

- Positive decreasing marginal product
- Constant output elasticity
- Constant returns to scale

The proposed work, models the output $Cost$ to be dependent on input parameters Load and Fixed unit cost and aims to find the optimal cost for the resource allocation. Authors have proved that CD production function is minimized at $\alpha + \beta > 1$ i.e Increasing returns to scale (IRS).

The cost function is defined as

$$C(F, L) = KF^{\alpha}L^{\beta} \tag{1}$$

where,
C = Total cost incurred by the SaaS provider
L = Load on Server
F = Fixed unit cost of the server
K = Is a positive constant which explains technological influence on the model.
α and β are output elasticities of the Fixed unit cost and Load.

α and β are output elasticities of labor and capital, respectively in the classical Econometric model in production [17], modified suitably to serve the context of the problem stated. These values are constants determined by available technology.

Output elasticity measures the responsiveness of output to a change in levels of either labor or capital used in a production. For example, if = 0.15, a 1% increase in labor, would lead to approximately a 0.15% increase in output.

2.2 Proposed Method

Let there be M physical servers $\{1, 2, \ldots, M\}$ in the cloud environment. Each server i has Fixed unit cost F_i, assigned to it. Let L_i be the current load on the server i, which is defined in percentage, highest being 100 and least being 1.

Let there be N $\{1, 2, \ldots, N\}$ users who are simultaneously demanding resources from the cloud. Request from user j is defined as R_j. For all users the resource demand can be represented by the vector $(R_1, R_2, \ldots, R_N)$. Resource demand by each user is distributed among available servers. It can be represented by the vector $(a_j(1), a_j(2), \ldots, a_j(M))$, where $a_j(k)$ represents resource allocated for user j on server k. Distribution of the user's total demand among all the servers should be equal to the total demand by the user.

$$\sum_{k=1}^{M} a_j(k) = R_j \tag{2}$$

Load factor L_f computes the load imbalance incurred among the servers due to the allocation $(a_j(1), a_j(2), \ldots, a_j(M))$. The goal is to achieve low load imbalance after the allocation. A lower load imbalance ensures the most appropriate allocation. For the allocation $(a_j(1), a_j(2), \ldots, a_j(M))$, the load factor L_f is computed using

$$L_f = \frac{\sum_{i=1}^{M-1} \sum_{k=i+1}^{M} (a_j(i)L_i - a_j(k)L_k)}{M} \tag{3}$$

Cobb-Douglas production function is used for computing the cost of using server i.

$$C_i(F, L) = KF_i^{\alpha}L_f^{\beta} \tag{4}$$

For the alloacation $(a_j(1), a_j(2), \ldots, a_j(M))$, the total cost incurred by SP for allocating user j's demand, is computed using

$$TotalCostPerUser_j = \sum_{k=1}^{M} a_j(k)C_k \tag{5}$$

Here, the computation considers both load imbalance created by the given allocation and Fixed unit cost of the server.

The game theory model is interested in the utility gained by the service provider for a given allocation choice. If there are N users, Utility U_j for allocating user j's demand is computed as a ratio of the cost incurred for user j, to the sum of the cost incurred for all users participating in the game. Following formula represents this computation.

$$U_j = -\frac{TotalCostPerUser_j}{\sum_{k=1}^{N} TotalCostPerUser_k} \tag{6}$$

Outlined below are the main steps involved in the proposed method.

1) Read all the input parameters, i.e Load and Fixed unit cost of all the participating servers.
2) Find all possible resource allocation combinations for each user based on demand by user and number of servers available.
3) Find values of α and β for each server using Gradient Descent method.
4) Compute the cost for each allocation combination of all users, using Cobb-Douglas production function (5). Select best combinations based on least cost.
5) Model the game using Extensive Form Game using these best selections. Compute the utility using (6) for each user and for each allocation combination.
6) Optimal solution of the game is SPNE. SPNE is computed using backward induction method.

Refer Appendix A for algorithms to find SPNE.

3 Discussion About Elasticity

Output elasticity of Cobb-Douglas production function is the porcentual change in the output in response to a change in the levels any of the inputs [16] . In (1), α and β are the output elasticities of Fixed unit cost and Load respectively. Accuracy of α and β values is the key to deciding the right resource allocation combination. Different approaches were analysed before arriving at final decision.

3.1 Computing Elasticities via Gradient Descent

Gradient Descent algorithm was used to find the values of α and β.

Gradient Descent is an optimization algorithm used for finding the local minimum of a function. Given a scalar function $F(x)$, gradient descent finds the $\min_x F(x)$ by following slope of the function $\frac{\partial F}{\partial x}$. This algorithm selects initial values for the parameter x and iterates to find the new values of x which minimizes $F(x)$.
Minimum of a function $F(x)$ is computed by iterating through following step,

$$x_{n+1} \leftarrow x_n - \delta \frac{\partial F}{\partial x}$$

TABLE 1
Simulation results: α and β values on Server1 and Server2 with assumption K=1. Results satisfy the condition $\alpha + \beta > 1$

Server Cost	Server Load	α	β
50	10	1.8768	0.0275
50	3	1.664	0.005
5	80	1.963	0.000054
10	40	1.938	0.0007
40	25	1.918	0.0289
30	6	1.8208	0.0056
10	20	1.924	0.00115

TABLE 2
Simulation results: α and β values on Server1 and Server2 without any assumption on K. Results satisfy the condition $\alpha + \beta > 1$

Server Cost	Server Load	α	β	K
50	10	0.611749	0.389197	4.288795
50	3	0.567333	0.434660	4.286146
5	80	0.719920	0.281966	4.288413
10	40	0.685039	0.315826	4.288373
40	25	0.640219	0.360577	4.289915
30	6	0.605103	0.397328	4.286653
10	20	0.670304	0.331262	4.286881

Where,
x_n - initial value for x
x_{n+1} - new value for x
$\frac{\partial F}{\partial x}$ - slope of the function $F(x)$
δ - step size, which is > 0, forces algorithm to make small jumps.

The objective of this paper is to find resource allocation pattern, which incurs optimal cost for the service provider, i.e. to find

$$\min_{\alpha,\beta} C(F, L) \tag{7}$$

s.t.c
$0 < \alpha$
$0 < \beta$
$\alpha + \beta > 1$

Table 1 and Table 2 show values of α and β computed using Gradient Descent method for various combinations of Load and Fixed unit cost.
Refer Appendix A for Gradient Descent algorithm.

3.2 Computing Elasticities via Constrained Optimization

Let the assumed paramteric form of (1) be

$$C = K' + \alpha log(F) + \beta log(L) \tag{8}$$

Consider a set of data points.

$$C_1 = K' + \alpha F_1' + \beta L_1' \tag{9}$$

...

$$C_N = K' + \alpha F'_N + \beta L'_N \tag{10}$$

Where,
$K' = \log(K), F'_i = \log(F_i)$ and $L'_i = \log(L_i)$

Constraints on parameters : This results in a constrained optimization problem. The objective function to be minimized is $(y - Ax)^T(y - Ax)$ and this is a quadratic form in x. If the contraints are linear in x, then the resulting constrained optimization problem is a Quadratic Program (QP). A standard form of a QP is :

$$\min x^T Hx + f^T x \tag{11}$$

s.t.c
$Cx \leq b$ Inequality Constraints
$C_{eq}x = b_{eq}$ Equality Constraints

Suppose the constraints are that α and β are > 0 and $\alpha + \beta > 1$. The quadratic program can be written as (neglecting the constant term $y^T y$).

$$\min x^T(A^T Ax) - 2y^T Ax \tag{12}$$

s.t.c
$\alpha > 0, \beta > 0, \alpha + \beta > 1$

In standard form as given in 11, the objective function can be written as :

$$\min x^T Hx + f^T x \tag{13}$$

where,

$$x = \begin{pmatrix} K' & \alpha & \beta \end{pmatrix}^T$$

$H = A^T A$ and $f = -2A^T y$ The inequality constraints can be specified as

$$C = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & -1 \end{pmatrix}$$

and

$$b = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

By solving above constrained quadratic problem, following values were obtained:
$\log(K) = 0.627368$, $K = 4.2393$
$\alpha = 0.671056$,$\beta = 0.333944$
This result satisfies the condition $\alpha + \beta > 1$

Elasticities α, β and K from both the computations are very close, supporting our choice of α, β and K.

3.3 Optimality of the parameters

The computational exercise to estimate "good fit value of α and β" are further justified by the analytical evidence. This evidence is derived from the functional properties of CD production function, which is modeled and applied as a function of two variables, Fixed unit cost and Load.

Achieving minimum cost is influenced by values of α and β of (4), for the given combination of Fixed unit cost and Load. Here is the proof for convexity of Cobb-Douglas

TABLE 3
Server Configuration

Server	Load	Fixed Unit Cost
1	40	50
2	10	50

w.r.t α and β.
A c^2 function $f : U \subset R^n \to R$ defined on a convex open set U is convex if and only if the Hessian matrix $D^2 f(x)$ is positive semi-definite for all $x \in U$. A matrix H is positive semi-definite if and only if its $2^n - 1$ principal minors are all ≥ 0 [Appendix C].
Cobb-Douglas function for 2 inputs is:

$$f(x, y) = cx^\alpha y^\beta$$

Its Hessian is

$$\begin{bmatrix} cx^\alpha y^\beta (\ln(x))^2 & cx^\alpha y^\beta \ln(x)\ln(y) \\ cx^\alpha y^\beta \ln(x)\ln(y) & cx^\alpha y^\beta (\ln(y))^2 \end{bmatrix}$$

$$\Delta_1 = cx^\alpha y^\beta (\ln(x))^2 \tag{14}$$

$$\Delta_2 = cx^\alpha y^\beta (\ln(y))^2 \tag{15}$$

$$\Delta_3 = c^2 x^{2\alpha} y^{2\beta} (\ln(x))^2 (\ln(y))^2 - c^2 x^{2\alpha} y^{2\beta} (\ln(x))^2 (\ln(y))^2 = 0 \tag{16}$$

Conditions for a function to be convex are,

$$\Delta_1 \geq 0$$
$$\Delta_2 \geq 0$$
$$\Delta_3 \geq 0$$

From the conditions defined for the resource allocation problem we know that,
$x > 0, y > 0, \alpha > 0, \beta > 0, c > 0$

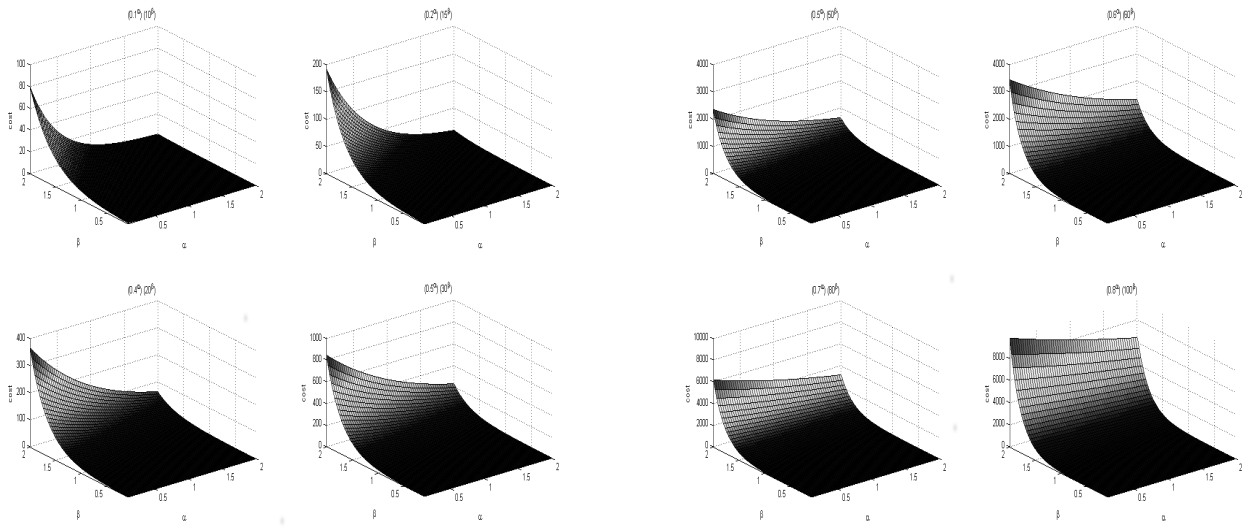
With these conditions on input values,
Δ_1 and Δ_2 as given in (8) and (9) are always ≥ 0
Δ_3 is always zero as given in (10)

Hence it is established that Cobb-Douglas w.r.t α and β is convex.

The 3-D graphs in Fig 1 are plotted for Cobb-Douglas (1) function with α and β against $Cost$. Here, x-axis represents output elasticity α of Fixed unit cost on the server and y-axis represents output elasticity β of Load of the server. The graphs obtained depict the effect of CD function on Cost based on Load and Fixed unit cost of the server. These graphs are plotted by taking different values for L and F. It is evident from the graphs, that the Cobb-Douglas function obtains minimum $Cost$ when $\alpha + \beta$ is greater than 1. Refer Appendix B for the proof for obtaining minimum cost while $\alpha + \beta > 1$

3.4 Illustration of the algorithm

Simulation Environment : The proposed model is simulated using a Python program. Input to this program is
- Number of servers
- Number of users



(a) Case I (b) Case II

Fig. 1. Plot of CD for α and β for minimum cost with different combinations of Load factor and Fixed cost

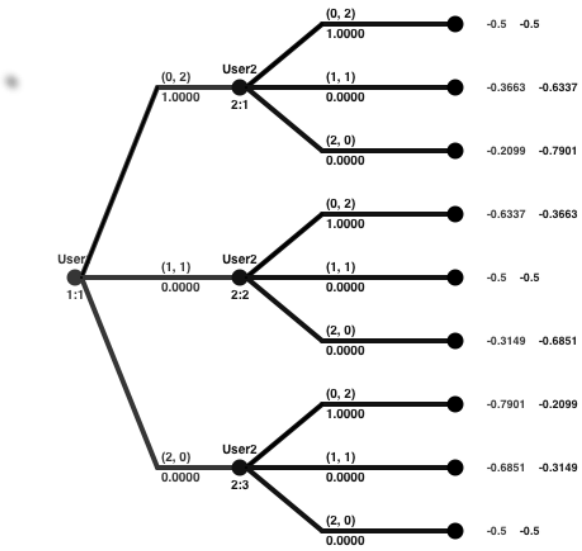


Fig. 2. Resource allocation - EFG

TABLE 4
Simulation results: Allocations, Cost and Utility

User1 Allocation	User2 Allocation	Utility
(2,0)	(2,0)	(-0.5, -0.5)
(2,0)	(1,1)	(-0.6851, -0.3149)
(2,0)	(0,2)	(-0.7901, -0.2099)
(1,1)	(2,0)	(-0.3149, -0.6851)
(1,1)	(1,1)	(-0.5, -0.5)
(1,1)	(0,2)	(-0.6337, -0.3663)
(0,2)	(2,0)	(-0.2099, -0.7901)
(0,2)	(1,1)	(-0.3663, -0.6337)
(0,2)	**(0,2)**	**(-0.5, -0.5)**

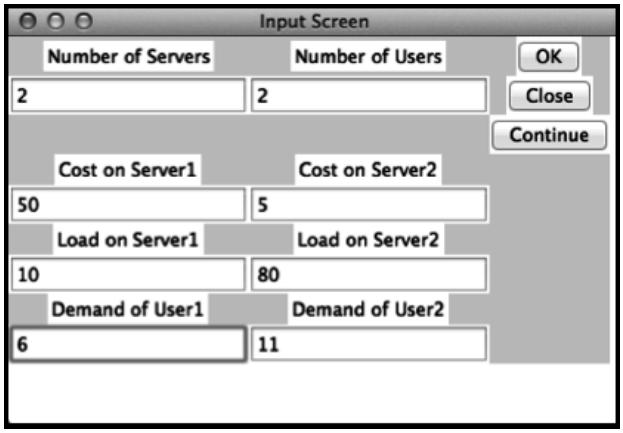


Fig. 3. Simulation Environment : Input Screen

- Fixed unit cost of each server
- Load on each server
- Service demands of all participating users

The program displays results on the screen, which includes
- Average service cost for each user
- Average service cost across the system
- Allocation strategy per user
- Final resource allocation across the system
- Load balancing factor of allocation
- Utility of the allocation strategy

Fig 3 and Fig 4 are the actual screen shots from the

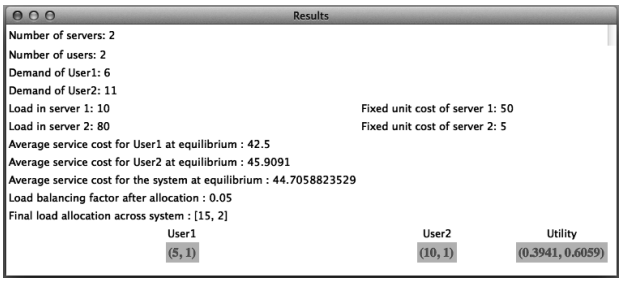


Fig. 4. Simulation Environment : Results

simulation environment.

This program also generates a file in the format that is understood by the tool Gambit [18] and can display the results in the tree format as shown in Fig 2.

Work Flow : Here is an example to show complete working of the algorithm. Let us consider a cloud consisting of two servers, configured as given in Table 3. Two users user1 and user2 are requesting for CPU hours $(2, 2)$ respectively.The utility computed for all possible allocation of user demands is shown in Table 4.

In the table the entry under (1,1) under column User1 Allocation indicates that out of 2 hours of demand from user1, 1 hour is allocated on server1 and another hour is allocated on server2.

Since server2 has lower load, this algorithm allocates more load to server2 and the allocation is (0,2) to user1 and $(0, 2)$ to user2, ie. both users get 2 hours on server2. From the Table 4, it is clear that the allocation $(0, 2)$ for user1 and $(0, 2)$ for user2 charges minimum for these users.

Fig 2 gives the visual representation of the resource allocation for the above example in EFG format. An open source tool Gambit [18] is used for generating this visual representation. In this figure, user1 and user2 are 2 game players. Since both users are demanding 2 hours of CPU, both user1 and user2 have 3 strategies $(0, 2)$, $(1, 1)$ and $(2, 0)$. user1 takes the action first, followed by user2. The value pairs at the leaf node indicate the utilities of user1 and user2 for the actions chosen respectively.

Let us start from the leaf node of the tree. Let us consider the subgame of user2; for any strategy selected by user1, strategy $(0, 2)$ gets maximum utility compared to other strategies. user1 being aware of this, chooses $(0, 2)$ which maximizes its own utility. This is backward induction procedure which finds SPNE for the game. Thus SPNE for the given example is $[(0, 2), (0, 2)]$ which means both users get 2 hours on server2. The edges highlighted in the tree given in Fig. 2 indicate the strategies selected by each user.

4 RESULTS

Goal of this work is to distribute user demands on servers by ensuring optimal service cost per user as well as least load imbalance among the servers. Following components are measured to determine the efficiency of the resource allocation algorithms.

- Average service cost per user
- Average service cost across the system

TABLE 5
Simulation results: Allocations of resources and Load factor, with $\alpha = 1.9$, $\beta = 0.02$ and $K = 1$

Serv1 Cost	Serv2 Cost	Serv1 Load	Serv2 Load	user1	user2	Load Factor
50	50	10	10	(3,3)	(5,6)	0.05
50	50	10	3	(1,5)	(3,8)	0.005
10	50	10	10	(4,2)	(6,5)	0.15
50	5	10	80	(5,1)	(10,1)	0.05
50	10	10	40	(5,1)	(9,2)	0.1
5	30	25	6	(1,5)	(2,9)	0.045

- Load imbalance factor among the servers due to selected allocation strategy.

Average service cost per user and across the system is expected to be lower than or equal to the maximum cost charged per unit time by any server. Load imbalance factor is expected to be lower than 1 among the servers after the allocation.

Actual results meet the expectations. Table 5 shows allocation strategies adopted by the algorithm. Here two users user1 and user2 are requesting for (6,11) hours respectively.

In Table 5, row 1, load and Fixed unit cost on both the servers are same. Algorithm has allocated resources on both the servers equally. In row 2, allocation is influenced by lower load on server2 and hence has allocated more resources on server2. In row 3, allocation is influenced by lower cost on server1 and hence more load is allocated on server1. Though cost is low on server 1, all the load is not allocated on server1 because, such an allocation creates more load imbalance among the servers. Similar intelligent decisions can be noticed in other allocations too.

Table 7 shows the average service cost per user and for the system at equilibrium. In row 3, the load on both the servers is same and average service cost per user is less than average unit cost of the system. Here unit cost is 30. But average service cost for all users and across the system is less than the average 30. Row 4 indicates when the load on servers is different, average service cost per user is greater than the average system unit cost, but is always less than the maximum cost of a single system. Since both load and Fixed unit cost influence the resource allocation, though load is lower on server1, some resources are allocated on server2 which has lower Fixed unit cost. Thus it ensures average service cost per user and across system is less than the maximum unit cost of any single system.

Load balancing factor across the system for the selected allocation $(l(1), l(2), \ldots, l(M))$ is computed using

$$L_f = \frac{\sum_{i=1}^{M-1} \sum_{k=i+1}^{M} (l(i)L_i - l(k)L_k)}{M} \tag{17}$$

where,
L_i is current load on the server i
$l(i)$ is new load allocated on server i

From Table 5, it is clear that after the allocation the load imbalance factor is always lower than 1. Fig 5 is a bar graph which compares Load factor achieved with CD and

TABLE 6
Simulation results: Allocations of resources and Load factor, with $\alpha = 0.6710$, $\beta = 0.3339$ and $K = 4.2393$

Serv1 Cost	Serv2 Cost	Serv1 Load	Serv2 Load	user1	user2	Load Factor
50	50	10	10	(3,3)	(5,6)	0.05
50	50	10	3	(1,5)	(3,8)	0.005
10	50	10	10	(3,3)	(6,5)	0.05
50	5	10	80	(5,1)	(10,1)	0.05
50	10	10	40	(5,1)	(9,2)	0.1
5	30	25	6	(1,5)	(2,9)	0.045

TABLE 7
Simulation results: Average service cost per user and system, with $\alpha = 1.9$, $\beta = 0.02$ and $K = 1$

Serv1 Cost	Serv2 Cost	Serv1 Load	Serv2 Load	Avg.Ser cost usr1	Avg.Ser cost user2	System cost
50	50	10	10	50	50	50
50	50	10	3	50	50	50
10	50	10	10	23.33	28.18	26.47
50	5	10	80	42.5	45.9	44.7
50	10	10	40	43.3	42.72	42.94
5	30	25	6	25.83	25.45	25.58

TABLE 8
Simulation results: Unit cost per user and system, with $\alpha = 0.6710$, $\beta = 0.3339$ and $K = 4.2393$

Serv1 Cost	Serv2 Cost	Serv1 Load	Serv2 Load	User1 cost	User2 cost	System cost
50	50	10	10	50	50	50
50	50	10	3	50	50	50
10	50	10	10	30	28.18	28.82
50	5	10	80	42.5	45.9	44.7
50	10	10	40	43.3	42.72	42.94
5	30	25	6	25.83	25.45	25.58

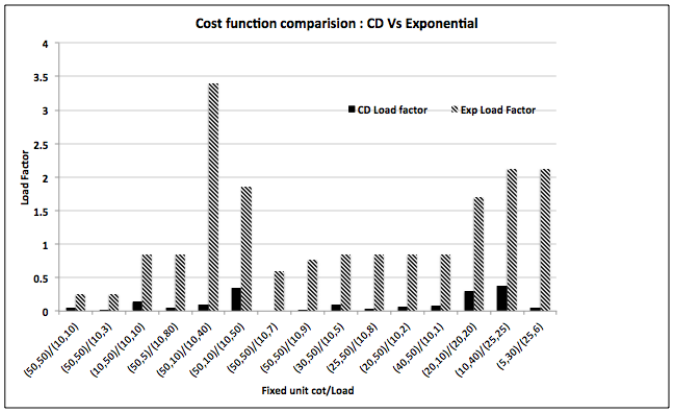


Fig. 5. Bar graph for Load factor behaviour according to difference in cost / load among the servers

exponential cost functions. This shows the impact of fixed cost and load among the two servers on the Load factor after the allocation. X-Axis of the graph represents cost among the

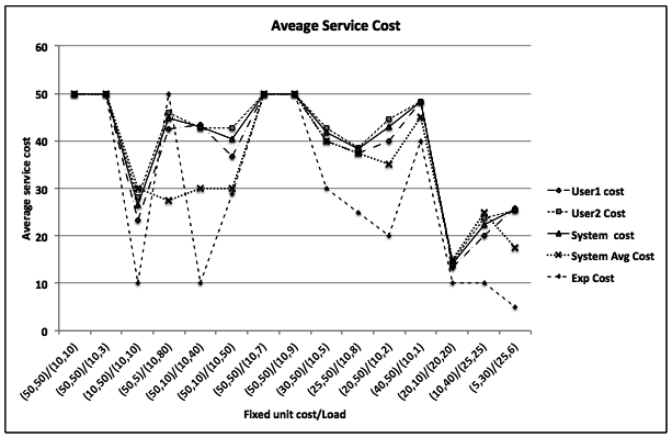


Fig. 6. Line graph of average service cost per user, across the system and system average unit cost

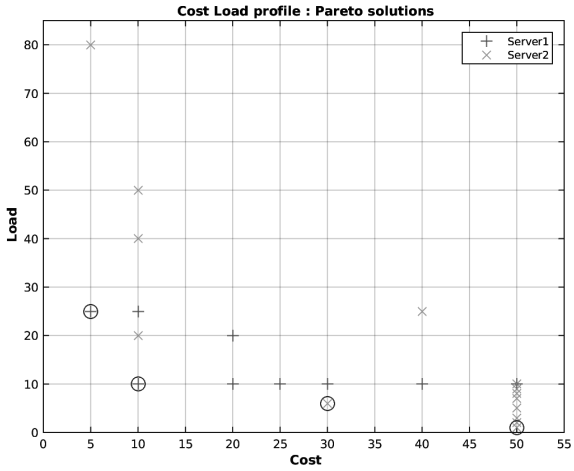


Fig. 7. Pareto optimal (Load, Cost) pairs

servers / load among the servers. (50,5)/(10,80) indicates that cost in server1 is 50 and in server2 is 5; load on server1 is 10 and on server2 is 80. The difference in Load factor with CD and exponential cost functions is evident from the graph.

Similarly, Fig. 6 plots the average service cost per user, unit cost across the system and actual average system cost for different combinations of load and fixed cost of the system. This chart also plots average service cost per user when exponential cost function is used. The exponential cost function allocates user requests on any one of the servers. The allocation is based on the server having least of sum of Fixed unit cost and load.

For the given data i.e. (cost,load) candidates, since there are only two parameters, one can identify the Pareto optimal solutions just by looking at the plot in Fig. 7. Server1 and Server2 candidates considered together. In the plot, the Pareto optimal solutions are circled. These are: (Cost, Load) pairs (10,10), (5,25), (50,1) and (30,6). And in the simulation environment, these servers were considered on priority for resource allocation.

Further, it is established that Cobb-Douglas production function is convex for the possible set of input values of the

allocation problem (Section 2.2). It is also proved that the cost minimization is achieved when

$$\alpha + \beta > 1$$

In Essence, this is an aggregate ranking problem, where ranking on two separate parameters, load and cost need to be merged into a single ranking based on load and cost both, which should turn out to be a reasonably efficient strategy for the service provider. Given the set of (cost, load) candidate solutions, it seems that identifying the Pareto optimal solutions is the best thing to be done. Assume, there exists a fundamental set N of well defined alternatives i.e load-cost pairs, but only a subset of the alternatives are feasible. The standard allocation problem in our case is to be able to estimate if a given allocation is efficient. However, this implies that, it is not dominated/beaten by any other allocation that can actually be achieved with the existing resources. This is a classic Pareto optimality formulation.

PREDICTION AND FORECASTING

Linear regression is an approach for modeling the relationship between a dependent variable y and one or more explanatory variables denoted by x. When one explanatory variable is used, the model is called simple linear regression. When more than one explanatory variables are used to evaluate the dependent variable, the model is called multiple linear regression model. Applying multiple linear equation model to predict a response variable y as a function of predictor variables x_1 and x_2 takes the following form.

$$y = b_0 + b_1 x_1 + b_2 x_2 + e \tag{18}$$

Here, x_1 and x_2 are Load and Fixed unit cost respectively, y is the response variable which determines the allocation, b_0, b_1, b_2 are regression coefficients and e is the error term.

The values for fitting are taken from the simulation to measure the goodness of fit and forecasting efficacy.

Given the sample (x_{11}, x_{21}, $y1$) to (x_{1n}, x_{2n}, yn) of n observations, the model consists of following n equations.

$$y_1 = b_0 + b_1 x_{11} + b_2 x_{21} + e_1 \tag{19}$$

$$y_2 = b_0 + b_1 x_{12} + b_2 x_{22} + e_2 \tag{20}$$

$$\vdots$$

$$y_n = b_0 + b_1 x_{1n} + b_2 x_{2n} + e_n \tag{21}$$

So, we have

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} 1 & x_{11} & x_{21} \\ 1 & x_{12} & x_{22} \\ \vdots & & \\ 1 & x_{1n} & x_{2n} \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} + \begin{pmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{pmatrix}$$

i.e in matrix notation $y = Xb + e$

Parameter estimation:

$$b = (X^T X)^1 (X^T y) \tag{22}$$

Allocation of variation:

TABLE 9
Multiple Linear Regression Results

	K = 1	K=4.3
Number of observations	22	22
Error degrees of freedom	19	19
R-squared	0.935	0.896
Adjusted R-Squared	0.881	0.885
p-value	$3.252e^{-12}$	$4.78e^{-10}$

$$SS0 = n\bar{y}^2 \tag{23}$$

$$SST = SSY - SS0 \tag{24}$$

$$SSE = y^T y - b^T X^T y \tag{25}$$

$$SSE = y^T y - b^T X^T y \tag{26}$$

$$SSR = SST - SSE \tag{27}$$

Where,
SSY=sum of squares of Y
SST=total sum of squares
SS0=sum of squares of y
SSE=sum of squared errrors
SSR= sum of squares given by regression
Coefficient of determination:

$$R^2 = \frac{SSR}{SST} = \frac{SST - SSE}{SST} \tag{28}$$

Coefficient of multiple correlation :

$$R = \sqrt{\frac{SSR}{SST}} \tag{29}$$

Table 9 lists the results obtained by Multiple Linear Regression.

5 CONCLUSION

The work proposes resource allocation model using Extensive Form Game (EFG) and Cobb-Douglas production function. The allocation policy, contrary to the plethora of strategies available in the literature, is not based on a single-point measure and therefore is devoid of single-point failure. Results support that game theoretic approach and Cobb-Douglas production function together provide an optimal solution for resource allocation problem in cloud.

Elasticities α and β are computed based on Fixed unit cost and Load using gradient descent method that ensures optimal average service cost per user. This model ensures minimal load imbalance among the servers after the resource allocation. When there is difference in Fixed unit cost and Load among the servers, results in Table 5 shows that the Load balancing factor after the allocation is always < 1. The cost incurred by SP for unit resource usage is less than

the average unit cost across the system when load among the servers is same as illustrated in Table 7. Cost incurred per unit resource usage is less than the maximum cost of any single server when the load among the servers is different.

Results from Multiple Linear Regression model establishes that the proposed model is a reasonably good fit with R^2 close to the upper bound, 1 for both conditions $K = 1$ and $K = 4.33$. The regression model is a further empirical testimony to the choice of elasticities adopted in the incubation stage of the model. With values of the elasticities matching our heuristics and theoretical guarantees for optimal conditions, the predictive model is able to achieve very good fitting with respect to both types of constant technological progress estimation K=1 and K=4.33. The resource allocation strategy adopted by our model, essentially a two-pronged decision theoretic model, is remarkably similar to the simulation results as reported in Table 5 and Table 6, Table 7 and Table 8. Pareto Optimality conditions further bolstered the strategy by indicating the best pairs in load and cost for allocation. The authors wish to stress on the fact that cost computation is done by formulating cost as a function of Load and Fixed unit cost of the hardware which renders the decision function (cost) as a bi-variate function. This, in turn, endows the decision function with a dynamic handle of two parameters, instead of one which is often done in literature. Cobb Douglas formulation, implementation and theoretical analysis, throughout the paper including the appendices, demonstrates this beyond reasonable doubt.

Future allocation strategies in a scaled up environment could benefit from any of the below:

1) Cost factor can be taken into consideration by the vendor/SP provided he/she has a lot of resources to allocate from. Assume the SP has resources r1,r2, ...rn with the load and cost built in as features of each server/resource, we could do a hierarchical clustering of the resources by using Euclidean distances and allocate jobs to clusters/member of a cluster based on the hierarchies; that way the allocation decision is taken based on both features, load and cost.
2) Recalculate the load on the system at each step of backward induction. In the proposed work, initial load on the system is considered for cost computation. Solution can further be enhanced by calculating the load on the systems after allocation for each user and before selecting best option for the next user.
3) Explore and exploit functionally greater flexible forms for 2 input variables CES production function, another member in the family of Cobb Douglas production function works beautifully for 2 input parameters only, sometimes even better than Cobb Douglas in terms of subtle handling of curvature fluctuations. Marginally better results in cost and load optimization is theoretically possible and begs investigation in the context of the present manuscript.
4) Generalize the model to accommodate more than 2 input variables. Proposed work is based only on the interaction between system load and Fixed unit cost of the server for the cost computation. The Cobb-Douglas production function used in this exercise is scalable and it can be modified to accommodate more than two input parameters. The corresponding formulation with n parameters, $n > 2$ is proved in [17]. An immediate consequence of the theorem cited is the possibility to find optimal resource allocation in cloud based on more than 2 input parameters. Cobb Douglas production is equipped to handle more than two input parameters without experiencing curvature violation and the conditions for minimum(global) could be obtained in an inductive manner as illustrated in [17] .

APPENDIX A
ALGORITHMS

Algorithm to compute α and β using Gradient Descent method

procedure GRADIENTDESCENT()
  $\frac{\partial C}{\partial \alpha} \leftarrow F^{\alpha} L^{\beta} \ln(L)$
  $\frac{\partial C}{\partial \beta} \leftarrow F^{\alpha} L^{\beta} \ln(F)$
  repeat
    $\alpha_{n+1} \leftarrow \alpha_n - \delta \frac{\partial C}{\partial \alpha}$
    $\beta_{n+1} \leftarrow \beta_n - \delta \frac{\partial C}{\partial \beta}$
    $\alpha_n \leftarrow \alpha_{n+1}$
    $\beta_n \leftarrow \beta_{n+1}$
  until $(\alpha_{n+1} > 0)||(\beta_{n+1} > 0)||(\alpha_{n+1} + \beta_{n+1} > 1)$
end procedure

Algorithm to find Optimum Allocation Combinations at equilibrium

procedure FINDALLOCATION()
  $maxSelections \leftarrow 3$
  read $numUsers$, $numServers$
  for $i \leftarrow 1, numServers$ **do**
    read $loadOnServers[i]$
    read $fixedCostOnServers[i]$
  end for
  for $i \leftarrow 1, numServers$ **do**
    compute $loadBalancingFactor[i]$ using (3)
  end for
  for $j \leftarrow 1, numUsers$ **do**
    read $demandByUser[j]$
  end for
  for $j \leftarrow 1, numUsers$ **do**
    $allocPerUser[j] \leftarrow$ FINDALLALLOCATIONCOMBIS(demandByUser[j], numServers)
    COMPUTECOSTPERCOMBINATION(allocPerUser[j], loadBalancingFactor, fixedCostOnServers, numServers)
    sort $allocPerUser$ based on minimum cost
    $finalSelection[j] \leftarrow$ select $maxSelections$ from $allocPerUser$
  end for
  for $j \leftarrow 1, numUsers$ **do**
    $finalCombinations[j] \leftarrow$ GENERATECOMBINATIONS(finalSelection[j])
  end for
  $utilityCombinations \leftarrow$ COMPUTEUTILITY(finalCombinations, numUsers)

24: $equilibriumCombination \leftarrow utilityCombinations[1]$
25: **end procedure**

In the above algorithm, function FINDALLALLOCATIONCOMBIS() finds all possible allocation combinations for each user based on user demand and available servers. The function GENERATECOMBINATIONS(), finds all combinations of allocations possible for all users together on available servers.

Algorithm to compute cost incurred by a given combination

1: **procedure** COMPUTECOSTPERCOMBINATION(allocPerUser[j], loadBalancingFactor, fixedCostOnServers, numServers)
2: **for** $i \leftarrow 1, len(allocPerUser)$ **do**
3: **for** $j \leftarrow 1, numServers$ **do**
4: $(\alpha, \beta) \leftarrow$ GRADIENTDESCENT(fixedCostOnServers,loadBalancingFactor, j)
5: $cost \leftarrow (allocPerUser[i][j]) \times (fixedCostOnServers[j]^{\alpha}) \times (loadBalancingFactor[j]^{\beta})$
6: **end for**
7: **end for**
8: **end procedure**

Algorithm to compute utility of the selected combination

1: **procedure** COMPUTEUTILITY(finalCombinations, numUsers)
2: $totalCost \leftarrow 0$
3: **for** $i \leftarrow 1, numUsers$ **do**
4: $totalCost \leftarrow totalCost + cost[i]$
5: **end for**
6: **for** $i \leftarrow 1, numUsers$ **do**
7: $utility[i] \leftarrow (cost[i] \div totalCost)$
8: **end for**
9: **for** $i \leftarrow 1, numUsers$ **do**
10: $utility[i] \leftarrow (-1) \times utility[i]$
11: **end for**
12: **end procedure**

APPENDIX B
CONDITION FOR COST MINIMIZATION

Cobb-Douglas production function that is used for finding the optimal cost for resource allocation in cloud is,

$$P(F, L) = kF^{\alpha}L^{\beta} \tag{30}$$

Where,
L is Load factor
F is Fixed cost
k is 1
Constraints defined for this problem are

$$L \leq 1$$
$$F \geq 1$$

ie. Load factor is expected to be less than or equal to 1 and Fixed cost is expected to be greater than or equal to 1 unit. (For example 1 unit = 1M USD)
Lagrangian form of the equation (30) is

$$K = F^{\alpha}L^{\beta} - \lambda(L-1) - \mu(1-F)$$
$$\frac{\partial K}{\partial L} = \beta F^{\alpha}L^{\beta-1} - \lambda = 0$$

$$\beta F^{\alpha}L^{\beta-1} = \lambda \tag{31}$$

$$\frac{\partial K}{\partial F} = \alpha F^{\alpha-1}L^{\beta} + \mu = 0$$

$$\alpha F^{\alpha-1}L^{\beta} = -\mu \tag{32}$$

Multiplying (31) by L and (32) by F we get,

$$\beta F^{\alpha}L^{\beta} = \lambda L \tag{33}$$

$$\alpha F^{\alpha}L^{\beta} = -\mu F \tag{34}$$

Dividing (34) by (33) we get,

$$F = -\frac{\alpha\lambda}{\beta\mu}L \tag{35}$$

Substituting (35) in to (31) we get,

$$L = -(\alpha^{\alpha}\beta^{1-\alpha}\lambda^{\alpha-1}\mu^{-\alpha})^{\frac{1}{1-(\alpha+\beta)}} \tag{36}$$

Similarly,

$$F = -(\alpha^{1-\beta}\beta^{\beta}\lambda^{-\beta}\mu^{\beta-1})^{\frac{1}{1-(\alpha+\beta)}} \tag{37}$$

Substituting (36) and (37) in (30) we get,

$$C = (\alpha^{\alpha}\beta^{\beta}\lambda^{-\alpha}\mu^{-\beta})^{\frac{1}{1-(\alpha+\beta)}} \tag{38}$$

For C to be minimum

$$\frac{1}{1-(\alpha+\beta)}$$

must be -ve. Hence,

$$1 - (\alpha + \beta) < 0$$
$$\alpha + \beta > 1 \tag{39}$$

APPENDIX C
ESSENTIAL RESULTS ABOUT CONVEXITY

Definition: (i) Let S be a convex set [19]; $x1$ and $x2$ be any two points in S; then a function $f : S \subset R^n \rightarrow R$ is convex if

$$(1-\lambda)f(x_1) + \lambda f(x_2) \geq f((1-\lambda)x_1 + \lambda x_2)$$

where, $\lambda \in [0, 1]$

(ii) Suppose $f \in C^2$, U is an open set, then $f : U \subset R^n \rightarrow R$ is convex/strictly convex iff the Hessian Matrix $D^2f(x) = H$ is positive semi-definite/positive definite $\forall x \in U$.
Lemma: $f \in C, U \subset R; U$ is a convex open set, $f : R \rightarrow R$, f is convex iff

$$f(x+\theta) \geq f(x) + \nabla f(x)\theta;\ \forall\theta \in R^N;\ x, \theta \in A$$

C: Class of continuous and first order differentiable functions. C^2: Class of continuous and second order differentiable functions [20].
Proof: Using the definition of convex functions [19];

$$f(\alpha(x+\theta)+(1-\alpha)x) \leq \alpha f(x+\theta)+(1-\alpha)f(x)$$

$$\implies f(x+\alpha\theta)-f(x) \leq \alpha(f(x+\theta)-f(x))$$

$$\implies f(x)+\frac{f(x+\alpha\theta)-f(x)}{\alpha} \leq f(x+\theta)$$

$$\implies f(x)+\nabla f(x)\theta \leq f(x+\theta)\ as\ \alpha \to 0$$

Theorem 1: $f \in C^2; x \in R; f : R^2 \to R$ is convex iff the Hessian Matrix $H \equiv D^2 f(x)$ is positive semi-definite $\forall x \in U$.[Necessary and sufficient condition for convexity]
Proof: f is convex; for some $x \in U$ and some $\theta \neq 0$, consider the Taylor expansion;

$$f(x+\alpha\theta) = f(x)+\nabla f(x)(\alpha\theta)+\frac{\alpha^2}{2}\theta D^2 f(x)\theta,$$

for some $\alpha > 0$
By the above Lemma w.k.t

$$\frac{\alpha^2}{2}\theta D^2 f(x)\theta \geq 0$$

Dividing both the sides by α^2

$$\theta^2 D^2 f(x) \geq 0$$

$$\implies D^2 f(x) \geq 0$$

$$\implies H\ is\ positive\ semi-definite$$

Therefore, Convexity implies positive Semi-Definite.

Theorem 2: Global minima result:
Let $f(x_1,x_2) = kx_1^{\alpha}x_2^{\beta} : U \subset R^2 \to R$ be convex function on U; U is an open convex set; the critical point, x^* is a global minimum.
Proof: x^* is a critical point.
$Df(x^*) = 0$ [D: first order partial derivative.]
Using the well known result about convex functions,
$f : R^2 \to R$ is convex iff $f(x_2) - f(x_1) \geq Df(x_1)(x_2-x_1)\ \forall x_1, x_2 \in U$

$$f(x_2)-f(x_1) \geq \frac{\partial f}{\partial x_1}(x_1)(x_2-x_1)+\ldots\frac{\partial f}{(\partial x_2)(x)({x_2}^2-{x_1}^2}$$

Since, $Df(x^*) = 0$, using the inequality

$$f(x_2)-f(x^*) \geq Df(x^*)(x_2-x^*), by\ MVT$$

$$\implies f(x_2) \geq f(x^*)\ \forall x_2 \in U$$

$$f(x2)-f(x*) >= 0; by\ the\ condition\ of\ critical\ point$$

Hence, x^* is a global minima.

Remarks:
Theorem 1: Gives us the optimal α and β values.This defines the condition to be satisfied for convexity of the function.
Theorem 2: Shows, for convex functions, if having a minima, are granted to possess a global minima.

REFERENCES

[1] Amazon elastic compute cloud (ec2). [Online]. Available: http://aws.amazon.com/ec2/
[2] Google app engine. [Online]. Available: http://appengine.google.com/
[3] How to deliver saas using iaas. [Online]. Available: http://www.rightscale.com/blog/enterprise-cloud-strategies/how-deliver-saas-using-iaas
[4] Adel Nadjaran Toosi, Kurt Vanmechelen, Kotagiri Ramamohanarao, and Rajkumar Buyya, "Revenue maximization with optimal capacity control in infrastructure as a service cloud markets," vol. 3, no. 3, SEPTEMBER 2015, pp. 261–274.
[5] Google vs. aws pricing: Google cuts are first of 2015. [Online]. Available: http://www.rightscale.com/blog/cloud-cost-analysis/google-vs-aws-pricing-google-cuts-are-first-2015
[6] V. Jalaparti, G. D. Nguyen, I. Gupta, and M. Caesar, "Cloud resource allocation games," 2010. [Online]. Available: http://hdl.handle.net/2142/17427
[7] Linlin Wu, Saurabh Kumar Garg, and Rajkumar Buyya, "SLA-based resource allocation for software as a service provider (SaaS) in cloud computing environments," 2011, pp. 195–204.
[8] X. Xu and H. Yu, "A game theory approach to fair and efficient resource allocation in cloud computing," vol. 2014, Mathematical Problems in Engineering, 2014, p. 14.
[9] A. Nahir, A. Orda, and D. Raz, "Workload factoring with the cloud: A game-theoretic perspective," in *INFOCOM, 2012 Proceedings IEEE*, IEEE INFOCOM, 2012, p. 25662570.
[10] G. Wei , A. V. Vasilakos, Y. Zheng, and N. Xiong, "A game-theoretic method of fair resource allocation for cloud computing services," vol. 54, pp. 252–269, 2010.
[11] N. S. V. Rao, S. W. Poole, C. Y. T. Ma, F. He, J. Zhuang, and D. K. Y. Yau, "Cloud computing infrastructure robustness: A game theory approach," Proceedings of the International Conference on Computing, Networking and Communications Maui Hawaii, January 2012, pp. 34–38.
[12] D. Niyato, A. V. Vasilakos, and Z. Kun, "Resource and revenue sharing with coalition formation of cloud providers: Game theoretic approach," in *Proceedings of the 2011 11th IEEE/ACM International Symposium on Cluster, Cloud and Grid Computing*, Newport Beach, May 2011, pp. 215–224.
[13] M. J. Osborne, *An Introduction to Game Theory*. New York: Oxford University Press, 2004.
[14] M. D. Davis, *Game Theory, A Nontechnical Introduction*. Dover Publications, 1983.
[15] Cobb, C. W and Douglas, P. H, "A theory of production, american economic review," 1928, pp. 139–165.
[16] Cobb-douglas production function. [Online]. Available: http://economicpoint.com/production-function/cobb-douglas
[17] Snehanshu Saha, Jyotirmoy Sarkar, Nandita Dwivedi, Avantika Dwivedi, Anand MN, and Ranjan Roy, "A novel revenue optimization model to address the operation and maintenance cost of a data center," in *J.Cloud Computing*, vol. 5, no. 1, January 2016, pp. 1–23.
[18] McKelvey, Richard D., McLennan, Andrew M., and Turocy Theodore L., "Gambit: Software tools for game theory," 2014. [Online]. Available: http://www.gambit-project.org
[19] Mokhtar S. Bazaraa, Hanif D. Sherali, and C.M. Shetty, *Nonlinear programming: theory and algorithms*, 3rd ed. Wiley-Interscience, 2006.
[20] Saha, Snehanshu, *Ordinary Differential Equations: A Structured Approach*. Cognella publishing, 2011.
[21] Nandita Dwivedi, Avantika Dwivedi, Snehanshu Saha, Archana Mathur, and Gouri Ginde, "Jimi,journal internationality modelling index- an analytical investigation." 4th national Conference on Scientometrics and Internet Of Things(IoT), pp. 40–49.
[22] Jain, Raj, *The art of computer systems performance analysis*. John Wiley and Sons, 2008.

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