

init solve

$$Ax = b \Rightarrow LUx = b$$

$A$  LU new-matrix LU fact repeat backsub w/  $b_i$

Heritable factorized

$$\text{cache} = \text{init}(\text{prob}, \text{alg}) \quad A^{(p)}x = b$$

$\Rightarrow \text{solve}!(\text{cache}) \rightarrow$  if not LU, then LU

$\cdot \text{new\_matrix } A!(\text{cache}, A)$  then backsub  
 $\cdot \text{new\_matrix } b!(\text{cache}, b)$  prec =  $(P, I, LU)$   
 $P = (u, p, t)$

new-matrix  $\text{set-p}!(\text{cache}, p)$   
 $\text{update-prec}!(\text{cache}, \text{left/right})$



$$A = QSQ^T \quad \text{or } LHL^T$$

$$rI + A = Q(S + rI)Q^T$$

Root Problem Linear Problem  $(A, b, \gamma = \text{false})$

Shifted Linear Problem  $(A, b, \delta)$  mass-matrix = I



$$u' = f(u)$$

$$J = J_1 J_2$$

$$(I - \delta J_1)(I - \delta J_2) = I - \delta J_1 - \delta J_2 - \delta^2 J_1 J_2$$

ADI = Alternating Direction Implicit

$$\Delta = D_x D_y u$$

Linear Problem( A, b, p, <sup>u0</sup> ~~pp~~ = nothing )

Solve( ... ; abstol, reltol

GMRES( ... ; prec-right = ... )



What did we make?

Linear Problem( A, b, p, u0 = nothing ) <sup>kwargs---</sup>  $\nwarrow$  \*  
Shifted Linear Problem( A,  $\gamma$ , b, p, u0 = nothing ) = Linear Problem( wcp )

solve( prob, alg, reltol, abstol, weights )  
<sub>prec?</sub>

Factorize <sup>in cache</sup>: Struct fields:  
A factorization PR, PL <sup>nothing</sup>

cache = init( prob, alg ; kwargs )  
<sub>isfresh</sub>

Prec: it's upto  
the alg to  
use it or not

set\_A!( cache, A )  
set\_prec!( cache, PR, PL )  
set\_b!( cache, b )  
set\_p!( cache, p )

update\_prec!( cache ) = cache.prec( cache, ... )  
<sub>set\_prec!( cache,</sub>

Iterative Solvers Alg: struct fields  
end <sup>iterable</sup>

Preconditioner function: (optimal)  
@ set cache.prec = f( Val { :int }, cache )

$$f^{proc}(\text{cache}) \rightarrow (p_l, p_r)$$