Inter IIT 25' Practice Test-1 (100 points)
SciMathSoc
November 1, 2025 Roll Number: ______________________________

Important Instructions :

1. There are a total of **7 questions**:
   - All questions are worth 20 marks
   - Only 5 out of 7 questions have to be done.
2. The paper is of **3 hours long**.
3. Before submission, you must clearly indicate the 5 questions that you want to have checked. We will consider only those five questions mentioned.
4. **NOTE:** Solution of each problem must be in a separate page. Don't start the solution from the mid of the page.
5. **Multiple solutions** for the same problem are **allowed and encouraged**. Note that for multiple submissions we will consider the solution with highest mark.
6. Write neat proofs for each question attempted, even if you're not entirely sure. Partial marks will be given generously to some logical conclusions.

Important : Leave this table blank

Question	**Q1**	**Q2**	**Q3**	**Q4**	**Q5**	**Q6**	**Q7**	**Total**
Marks								

1. Given a real number a, we define a sequence by
$$x_0 = 1, \quad x_1 = x_2 = a, \quad x_{n+1} = 2x_n x_{n-1} - x_{n-2} \quad \text{for } n \geq 2.$$
Prove that if $x_n = 0$ for some n, then the sequence is periodic.

2. The following game is played on an infinite chessboard. Initially, each cell of an $n \times n$ square is occupied by a chip. A move consists of a jump of a chip over another chip in a horizontal or vertical direction onto a free cell directly behind it. The chip that is jumped over is then removed. Find all values of n for which the game ends with one chip left over.

3. Among any 15 co-prime positive integers > 1 and ≤ 1992, there is at least one prime.
OR
There are n students in each of three schools. Any student has altogether $n+1$ acquaintances from the other two schools. Prove that one can select one student from each school so that the three selected students know each other.

4. Let $1 \leq r \leq n$ and consider all subsets of r elements of the set $\{1, 2, \ldots, n\}$. Each of these subsets has a smallest member. Let $F(n, r)$ denote the arithmetic mean of these smallest numbers. Prove that
$$F(n, r) = \frac{n+1}{r+1}.$$

5. Consider a circular row of n seats. A child sits on each. Each child may move by at most one seat. Find the number a_n of ways they can rearrange.

6. Let $n \geq 2$. Determine the number of permutations π of $\{1, 2, \ldots, n\}$ such that for every $1 \leq i < n$,
$$|\pi(i+1) - \pi(i)| \neq 1.$$

7. Find the sum of all fractions $\dfrac{1}{xy}$ such that $\gcd(x, y) = 1$, $x \leq n$, $y \leq n$, and $x + y > n$.