PURPOSE-LED
PUBLISHING™

PAPER • OPEN ACCESS

Gradient measure and Jacobi sets for estimation of interrelationship between geophysical multifields

View the article online for updates and enhancements.

You may also like

Gradient measure and Jacobi sets for estimation of interrelationship between geophysical multifields

I V Artamonova[1,2,4]**, V V Alekseev**[3]** and N G Makarenko**[1]

[1] Central Astronomical Observatory of the Russian Academy of Sciences at Pulkovo, St.-Petersburg, Russia
[2] Voeikov Main Geophysical Observatory, St.-Petersburg, Russia
[3] Yaroslavl State P. G. Demidov University, 150000 Sovietskaya st., 14, Yaroslavl, Russia

E-mail: artirin@yandex.ru, vladislav.alekseev.yar@gmail.com, ng-makar@mail.ru

Abstract. We consider a problem of estimation of relationship between scalar 2D fields using gradient measure and Jacobi sets. The gradient measure method is based on estimation of alignment of gradients in every point of fields. The Jacobi set is the set of critical points of the restrictions of one function to the intersection of level sets of the other functions. We present the results of a numerical experiment for the case of multifield containing two fields: geopotential height on isobaric level 300 hPa and total ozone column, under influence of disturbing factor — intensive solar proton events in January 2005. Estimation of interrelationship by gradient measure indicates strengthening of interaction between fields for period $16^{th} - 22^{th}$ January 2005. The estimation made by Jacobi sets computation also shown strengthening of relation between analysed fields during solar proton events.

1. Introduction
The investigation of atmospheric feedback on the solar activity is an actual task of Solar-Earth physics. In this work we consider intensive solar proton events (SPE) with energies of particles up to ~100 MeV as factor of solar activity. Solar protons generated by chromospheric flares penetrate into the Earth's atmosphere and cause atmospheric ionisation with maximum on heights $20 - 80$ km. This process leads to growth of NOx and HOx aerosol components, which destroy ozone in catalytic chemical reactions. It is known that on high and middle latitudes the total ozone field is closely related with geopotential height of the tropopause level ($8 - 12$ km). Therefore, the aim of this work is to analyse the interrelation of total ozone and geopotential height fields during influence of disturbing factor – intensive SPE in January 2005. For estimation of intensity of interrelationship between these geophysical fields we used gradient measures [1, 2] and Jacobi sets [3, 4].

2. Quick start to methods
Let $F = \{f_1, f_2, \ldots, f_n\}$ be a set of functions defined on a smooth manifold $\mathbb{M}$. In experiments it might be a set of scalar fields measured on a common equidistant grid in a compact area $A \subseteq \mathbb{M}$. The set F is known as a multifield. Let's consider two ways of topological evaluation of the relationship between the components of the multifield.

---

[4] To whom any correspondence should be addressed.

2.1. Gradient comparison measure

Let $\mathbf{d}F = \left[\frac{\partial f_i}{\partial x_j}\right], i = 1, 2, \ldots n; j = 1, 2, \ldots m \, (m \geq n)$ be the matrix of partial derivatives. Then multifield comparison measure $\mu_F(p)$ at point p is defined as norm

$$||\mathbf{d}F|| = \max_{|x|=1} ||\mathbf{d}Fx||, x \in \mathbb{R}^m. \tag{1}$$

In the case $n = 2$ and $F = \{f, g\}$, the multifield comparison measure takes higher value when the gradients $\nabla f(p)$ and $\nabla g(p)$ are parallel or anti-parallel at the point p, and takes zero values if gradients are orthogonal [1].

Also it may be convenient to use different gradient measure:

$$\mu_{f,g}(p) = ||\nabla f(p) \times \nabla g(p)||, \tag{2}$$

which takes zero values if gradients are collinear, and takes higher values if they are orthogonal [2]. The growth of matrix norm corresponds to the amplification of interrelation between f and g fields.

2.2. Jacobi sets

Heuristically, let the field $f(x)$, $x \in \mathbb{R}^2$ describe a 2D topography of the land. Let g be a snow density field defined on the same domain. Consider a skier going down from a local maximum of f. On his way, we introduce the level lines $f^{-1}(h) = \{x \mid f(x) = h\}$ and consider restriction of snow density field to the level lines $g|_{f^{-1}(h)}$. Now let a skier go down by the path that follows critical points of $g|_{f^{-1}(h)}$ on each level h (i. e., a skier prefers paths with the most dense or loose snow). The set of points of all such skier's paths is Jacobi set.

Formally, Jacobi set $\mathbf{J}(f, g)$ of two Morse functions f, g defined on a common manifold $\mathbb{M}$ is a set of critical points of the restrictions of one function to the level sets of the other function [3]. On the other hand, criticality of a point $x \in \mathbb{M}_h$, $\mathbb{M}_h = g^{-1}(h)$ determined by the condition $\nabla f(x) = 0$. Therefore Jacobi set $\mathbf{J}(f, g)$ is the closure of a set of points x where for some real λ [3]

$$\lambda \nabla f(x) + \nabla g(x) = 0 \text{ or } \nabla f(x) + \lambda \nabla g(x) = 0. \tag{3}$$

According to definition 3, the simplification of Jacobi set topology corresponds to increasing of interrelationship between f and g fields.

The result of calculations by both methods in MatLab are presented in the next section.

3. Experimental data analysis

To combine the geophysical multifield $F = (O_3, HGT_{300})$ we used the total ozone column data on the grid $1° \times 1.25°$ (database provided by Bodecker Scientific Group [5]) and geopotential height on isobaric level 300 hPa on the grid $2.5° \times 2.5°$ by NCEP/NCAR reanalysis [6]. Previously we normalized both analysed fields and reduced them to the same grid step.

The period under consideration is $10-24$ January 2005. According to data from GOES-11 satellite [7] about density of solar protons, period $10-15$ January corresponded to the undisturbed conditions. Two intensive solar proton events with maxima on 17 and 20 January were observed for the period $16-21$ January and during the $22-24$ January the proton flux density again corresponded to the undisturbed conditions.

The result of calculations for 12, 16, 18 and 20 January 2005 is presented on figure 1. Multifield comparison measure (i.e. the maximum of matrix norm) for geophysical multifield F is on the left panel, Jacobi set for the multifield F is on the right panel. Black boxes mark areas of noticeable feedback of both geophysical fields to the influence of solar proton events revealed by both methods. These areas include a part of the North America, North Atlantic and Europe.
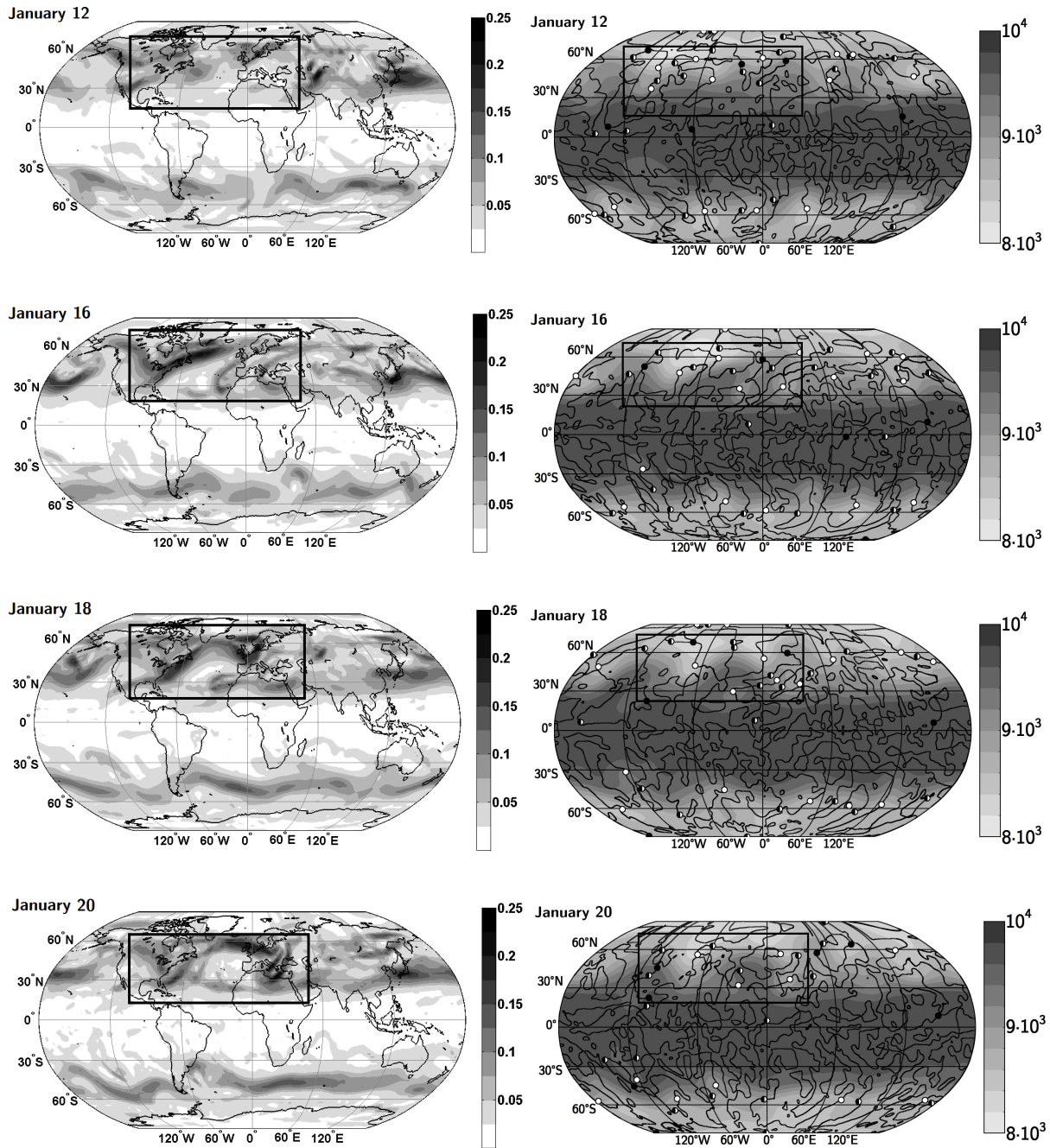
Figure 1: Maximum of matrix norm for 2D scalar fields: the geopotential height on the level 300 hPa and total ozone column in the course of intensive solar proton events in January 2005 (left panel). Jacobi set of two 2D scalar fields for the same period: the geopotential height on the level 300 hPa is on background, critical points located on the Jacobi set for total ozone field are marked by ● — maxima, ◖ — saddles, ○ — minima (right panel).

The first map plotted for 12 January shows the intensity of interrelationship between analysed geophysical fields under undisturbed conditions, the maximum of matrix norm on 12 January has amplitude ∼0.15. On the 16 January, at the beginning of the first solar proton event, the noticeable growth of matrix norm up to ∼0.2 − 0.25 was observed over the eastern coast

of North America and the North Atlantic. On the next map for 18 January the area of high values of matrix norm $\sim 0.2-0.25$ spreads in the east direction and covers Western Europe and Scandinavia. The last map corresponds to the second solar proton event on 20 January, the amplitude of matrix norm decreased over North America to values $\sim 0.1-0.15$ and the noticeable growth of matrix norm up to ~ 0.25 was observed over the North Atlantic and Europe. During $22-24$ January, the value of matrix norm corresponded to the undisturbed level.

The Jacobi sets for the multifield $F = (O_3, HGT_{300})$ for the same days 12, 16, 18 and 20 January are presented on the right panel of figure 1. The geopotential height on the isobaric level 300 hPa is on the background; critical points of total ozone field located on the Jacobi set curves. Black boxes mark areas on right panel of figure 1, which correspond to the high values of matrix norm on the left panel. Now there is no suitable numerical measure to estimate the complication or simplicity of Jacobi set topology. One of possible variants of numerical measure may be an integral curvature of Jacobi set lines. In the areas marked by black boxes a simplification of Jacobi sets topology and decreasing of integral curvature of lines during solar proton events $16-20$ January 2005 is observed.

The growth of matrix norm and simplification of Jacobi sets topology observed in the course of intensive solar proton events in January 2005 related with the intensification of interrelationship between total ozone field and geopotential height field on the isobaric level 300 hPa in the North Atlantic region.

4. Discussion and conclusions

Topological methods of estimation of interrelation between analysed geophysical fields are revealed an existence of connection between total ozone and geopotential height fields under undisturbed conditions and a noticeable strengthening of this connection under influence of the external disturbing factor — solar proton event. Numerical estimation made by multifield comparison measure showed the growth of matrix norm in ~ 2 times in the course of intensive solar proton events of January 2005 in comparison with undisturbed conditions. Numerical measure for Jacobi sets now is under discussion. Suitable quantitative measure may be an integrated curvature of the line between two critical points.

The physical mechanism of studied phenomenon is very complicated and not considered in this paper. To understand the mechanism it is necessary to use 3D models of atmospheric circulation, which consider the ozone photochemistry and variations of atmospheric chemical composition under ionization produced by cosmic rays.

References

[1] Nagaraj S *et al.* 2011 A gradient-based comparison measure for visual analysis of multifield data *Comput. Graph. Forum* **30** 1101-10
[2] Edelsbrunner H *et al.* 2004 Local and global comparison of continuous functions *In Proc. IEEE Conf. Visualization* 275-80.
[3] Edelsbrunner H and Harer J 2002 Jacobi set of multiple Morse functions. *Cambridge Univ. Press* 37-57.
[4] Huettenberger L and Garth C 2015 *Topological and Statistical Methods for Complex Data, Mathematics and Visualization* 125-41
[5] Bodecker Scientific Group database, *www.bodekerscientific.com/data/total-column-ozone*
[6] NCEP/NCAR reanalysis database, *http://www.esrl.noaa.gov/psd/data/gridded/data.ncep.reanalysis.html*
[7] Space Weather Prediction Center NOAA, *www.swpc.noaa.gov/products/goes-proton-flux*