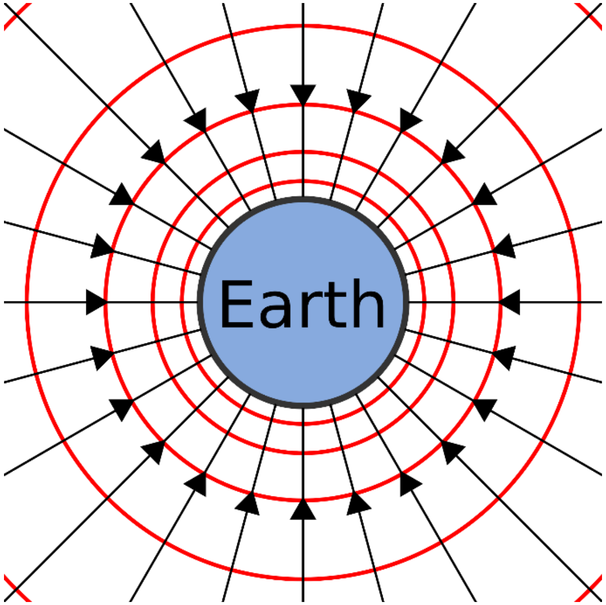
CHAPTER 8
GRAVITATIONAL FIELDS

Outlines

Gravitational fields



An object with mass establishes a **gravitational field** around it.

When another object enters the field, it experiences a gravitational force.

The size of the force depends on **field strength** at that point. Field lines never **cross** each other

A gravitational field is defined as the **force acting per unit mass** when the test mass is placed in a region of gravitational field due to a larger mass.

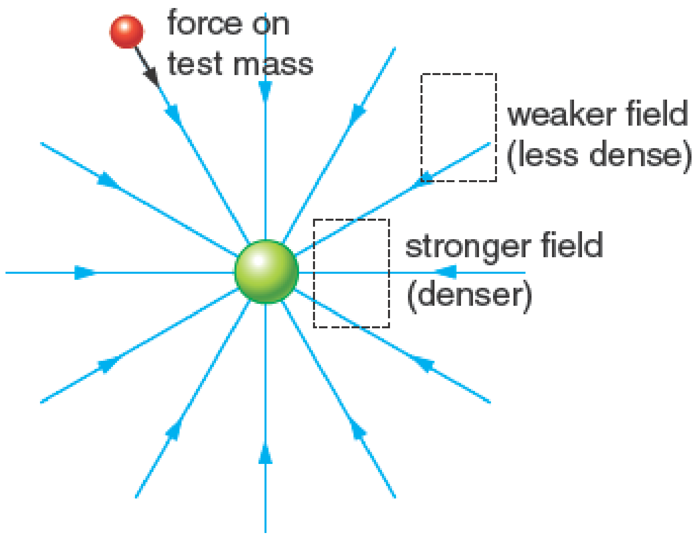
Gravitational field pattern

Field lines help visualize the direction and strength of a field.

- always end on a mass, extend to infinity
- direction of field line = direction of force experienced by a test mass at that position
- closely-spaced $\Rightarrow$ greater field strength
- widely-spaced $\Rightarrow$ weaker field strength

Gravitational field pattern

- For a **uniform spherical** object, its field lines are **radially inwards** towards the centre of the sphere



Gravitational field pattern

Gravitational field pattern of the Earth:

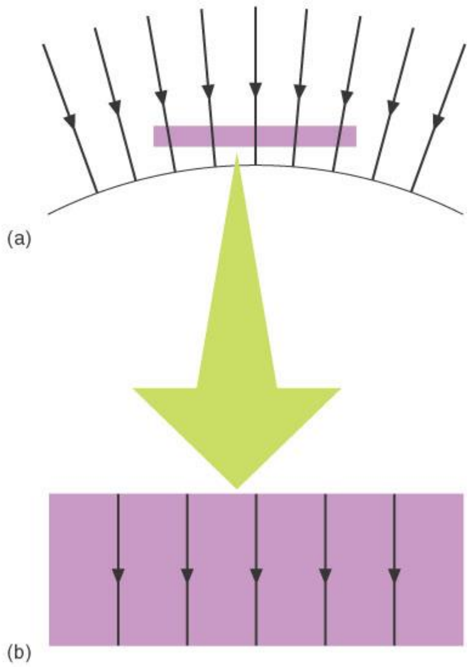
- **densest** at Earth's surface
- more widely spread as distance from the Earth **increases.**

⇒ gravitational force experienced by a mass **decreases** with distance

Gravitational field pattern

At the region very close to the Earth surface, field lines are approximately perpendicular to the ground and parallel to each other.

So, the field strength is assumed to be constant near the surface.

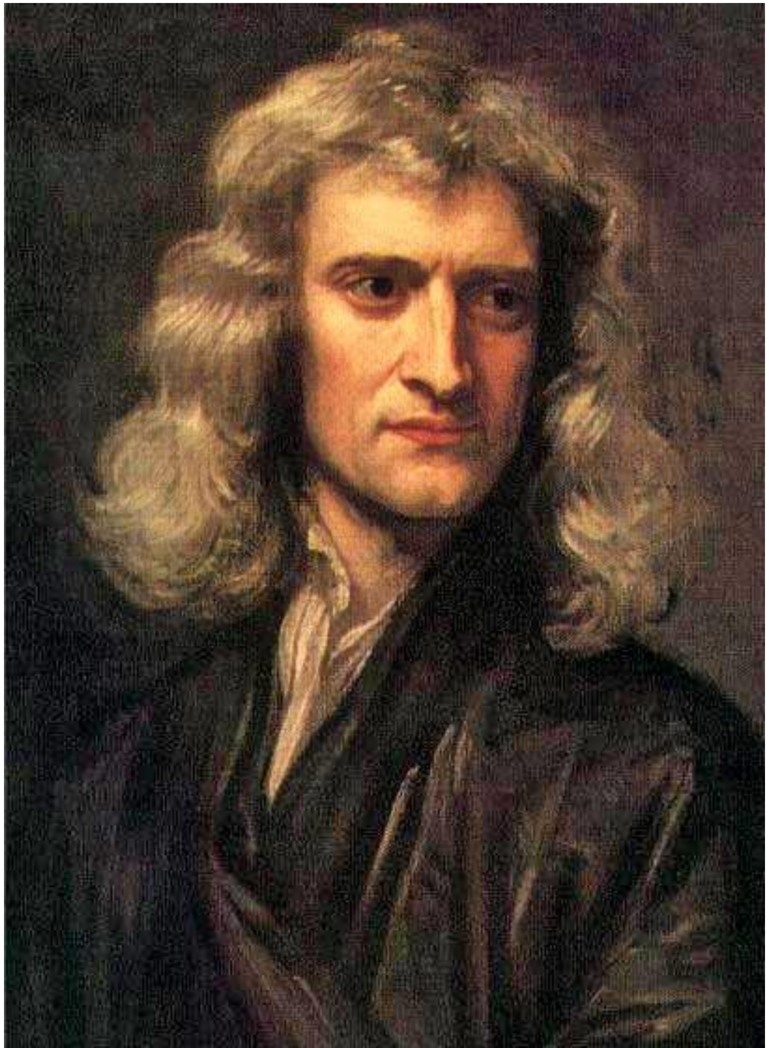


Example Question 1

True or False:

Introduction: Newton’s Law of Universal Gravitation

If the force of gravity is being exerted on objects on Earth, what is the origin of that force?

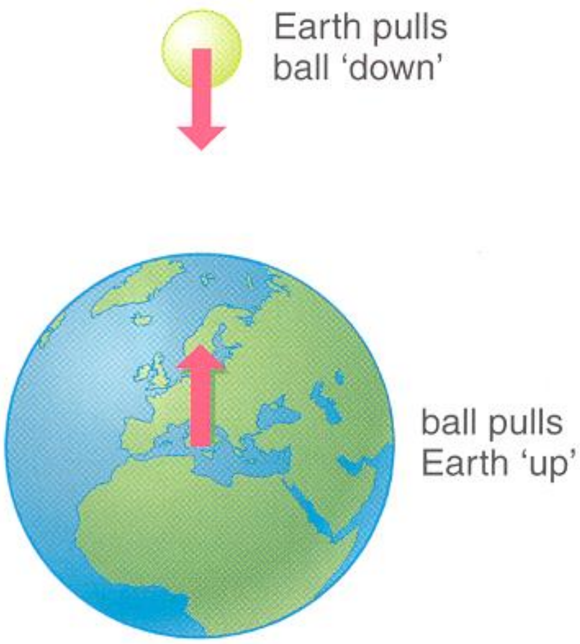


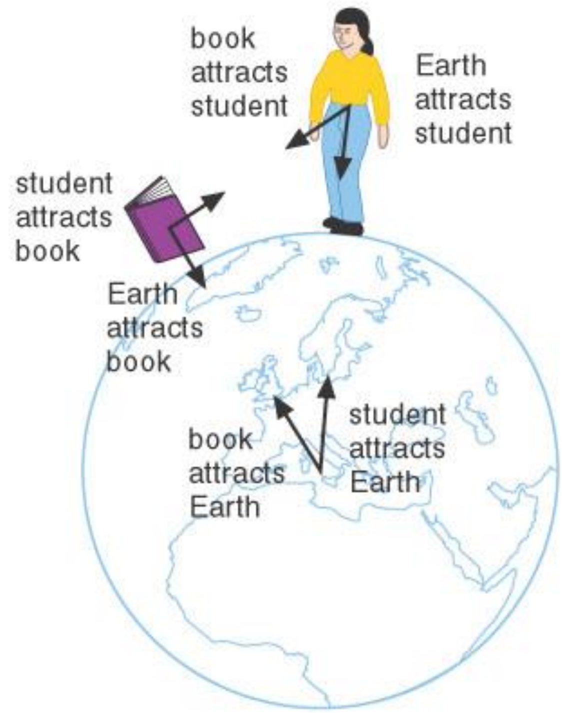
Newton’s realization was that the force must come from the Earth.

He further realized that this force must be what keeps the Moon in its orbit.

Gravitational Force

- A pair of gravitational forces results from the mutual attraction between any two bodies.
- These forces are known as **weight** (not gravity).



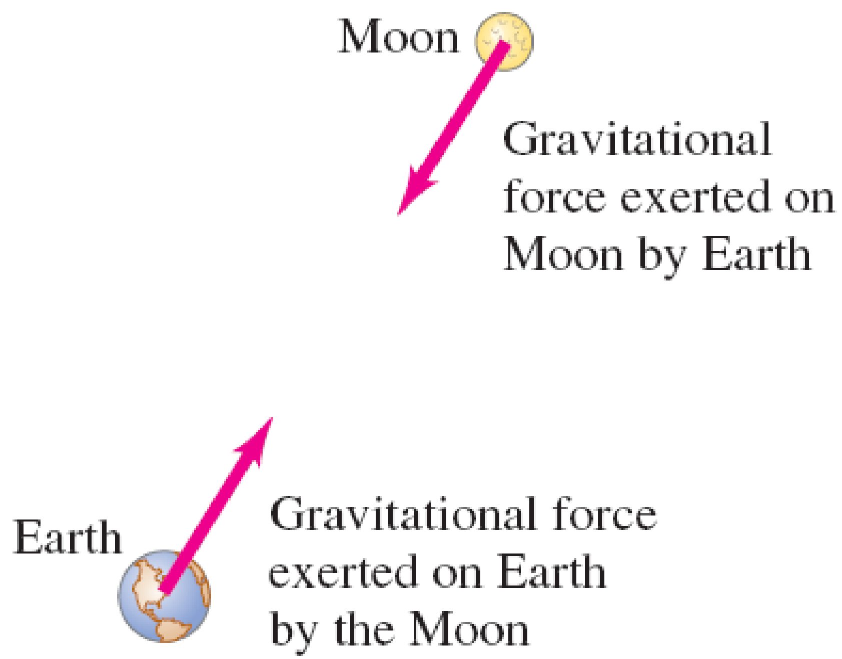


- Why are we not aware of the attractive force we exert on the Earth?

Newton's Law of Universal Gravitation

The gravitational force on you is one-half of a third law pair: the Earth exerts a downward force on you, and you exert an upward force on the Earth.

When there is such a disparity in masses, the reaction force is undetectable, but for bodies more equal in mass it can be significant.



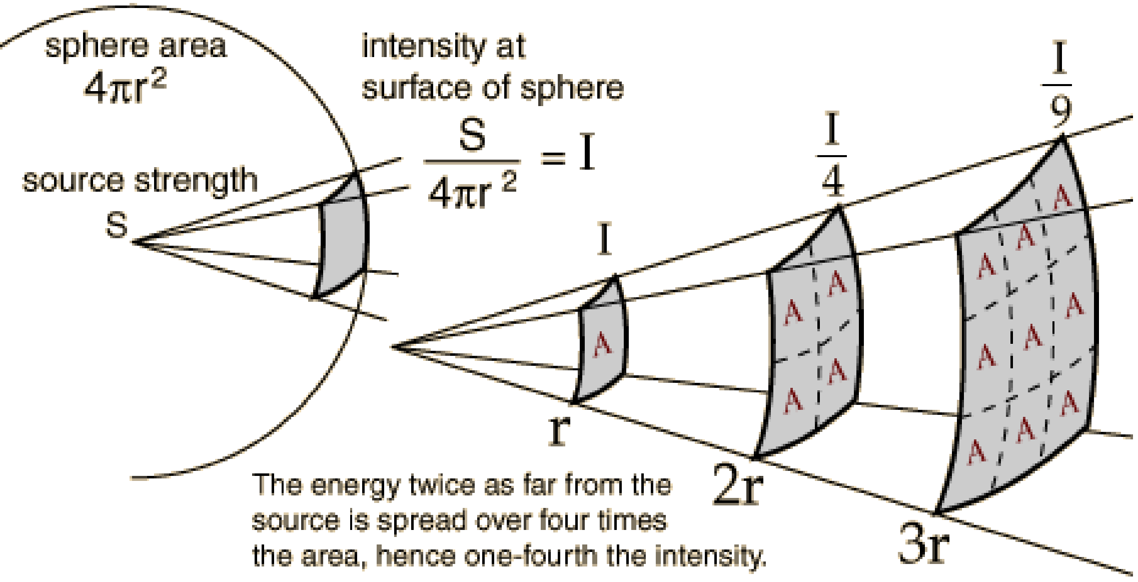
Gravitational force exists wherever there is a gravitational field.

Gravitational force is the **weakest** among all the fundamental forces, as seen in the table below.

Force	Approximate Relative Strength	Range
Strong Nuclear	10^{38}	10^{-15}
Electromagnetic	10^{36}	Infinite
Weak	10^{2}	10^{-18}
Gravitational	1	infinite

Gravitational Forces – Mass and Inverse Square Law

Every object attracts every other object with an attractive force which is **directly proportional** to product of masses and **inversely proportional** to distance squared.



Newton’s Law of Universal Gravitation

Newton’s law of gravitation states that two point masses attract each other with a force that is **proportional** to the product of their masses and **inversely proportional** to the square of their separation.

$$F \propto m_1 m_2 / r^2$$

Or

$$\boxed{F = \frac{G m_1 m_2}{r^2}}$$

Where the constant of proportionality G is called the gravitational constant, the distance from one centre of mass to another is denoted as r.

$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

Example Question 2

Calculate the Earth's gravitational force acting on a person on the Earth's surface using the Newton's law of gravitation, the mass of the person is 60 kg.

Solution:

$R_E = 6.4 \times 10^6$ m

$M_E = 6.0 \times 10^{24}$ kg

$F_g = GMm/r^2$

$$= \frac{(6.67\times10^{-11})(6.0\times10^{24})(60)}{(6.4\times10^6)^2}$$

$= 590\ \mathrm{N}$

Example Question 3

Two small spheres each of mass 20 g hang side by side with their centres 5.00 mm apart. Calculate the gravitational attraction between the two spheres.

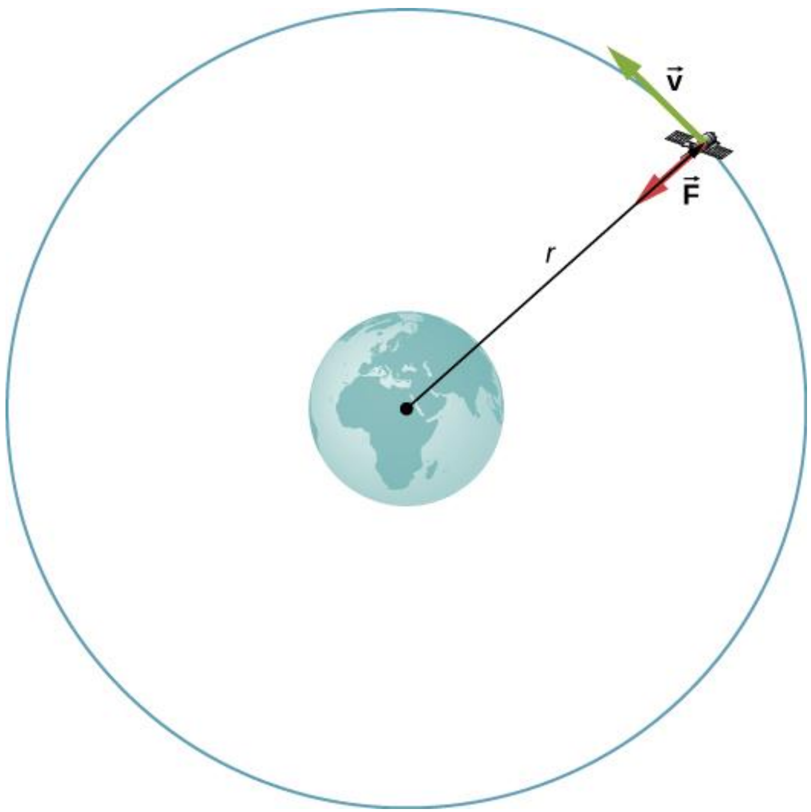
Solution:

$$F_g = Gm_1m_2/r^2$$

$$= \frac{(6.67\times10^{-11})(0.020)(0.020)}{(0.005)^2} = 1.1 \times 10^{-9}\text{N}$$

Gravitational force and Circular Motion

There are always gravitational forces between the Earth and the Sun, why our Earth isn't moving closer to the Sun if there is an accelerating force directing towards the Sun? [hint: centripetal force]



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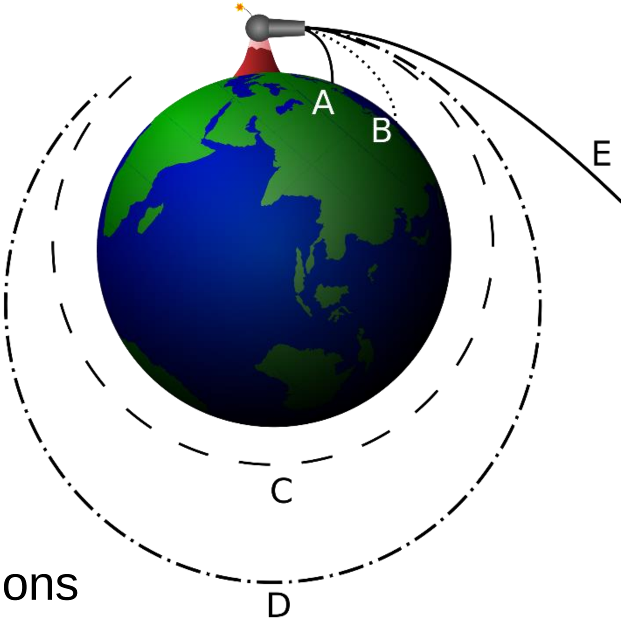
Newton's Thought Experiment

Consider a large cannon on some high point on the Earth's surface, capable of firing objects horizontally.

If the object is fired too slowly, gravity will pull it down towards the ground and it will land at some distance. A faster initial speed results in the object landing further from the cannon.
If we try a bit more faster than this, the object will travel all the way round the Earth.



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Motions of objects in gravitation field

Gravitational force from the Earth

$\Rightarrow$ centripetal force for circular motion

$$\frac{GM_E m}{r^2} = \frac{mu^2}{r}$$

$$u = \sqrt{\frac{GM_E}{r}}$$

Gravitational force and Circular Motion

The fact that our Earth (and other planets) are not getting closer to the Sun despite being subject to the gravitational forces can be explained using these two perspectives:

(i) As the planets orbit the sun, the force of **gravity** acting upon the planets **provides** the **centripetal force** required for circular motion.

(ii) The gravitational force is at the right angles to the velocity of the planet and that causes the circular motion, if the velocity of the planet is high enough, it won’t fall into the sun.

Gravitational force and Circular Motion

Consider a satellite of mass m orbiting the Earth at a distance r from the Earth's centre at a constant speed v. Since it is the gravitational force between the Earth and the satellite which provides this centripetal force, we can write:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

Where M is the mass of the Earth, m is the mass of satellite.

Rearranging gives:

$$v^2 = \frac{GM}{r}$$

Example Question 4

The Moon orbits the Earth at an average distance of 384000 km from the centre of the Earth. Calculate its orbital speed. (The mass of the Earth is 6.0×10^{24} kg)

Solution:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

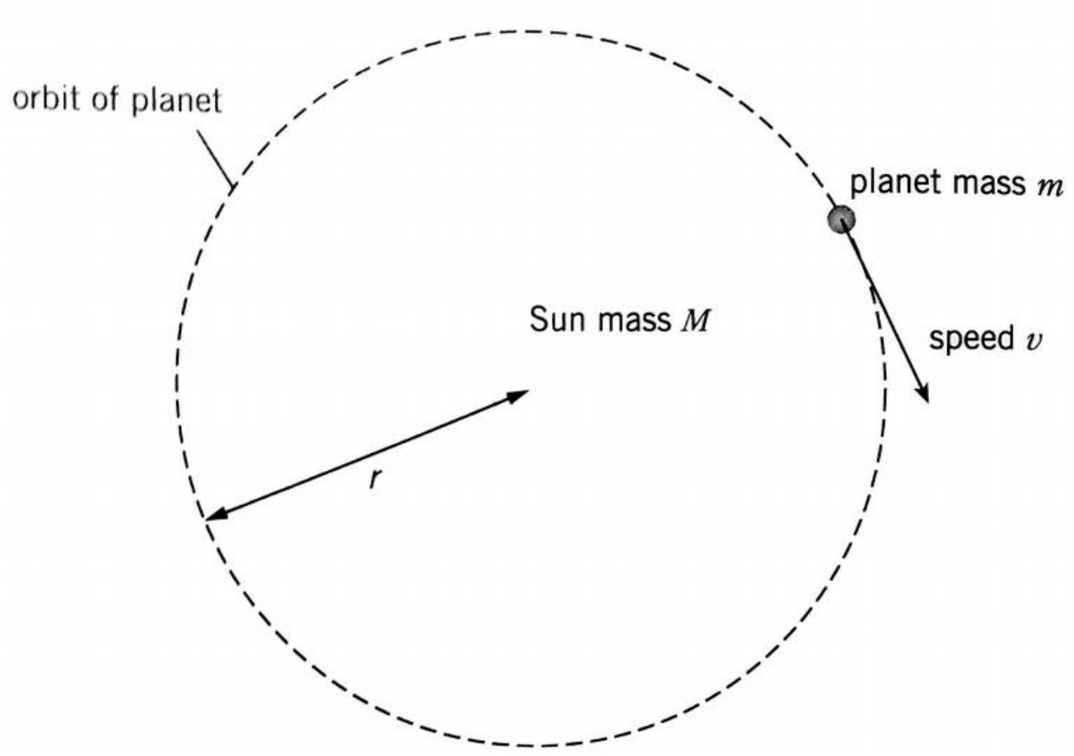
$$\Rightarrow v^2 = \frac{GM}{r}$$

$$v^2 = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{3.84 \times 10^8}$$

$$\Rightarrow v = 1020 \text{ ms}^{-1}$$

Period of Orbit

Consider a planet of mass m in circular orbit about the sun, of mass M, as shown in the figure below.



By Newton's law of gravitation:

$$F_g = GMm/r^2$$

Period of Orbit

The gravitational force provides the centripetal force as the planet moves in its orbit.

The centripetal force is given by

$$F_c = \frac{mv^2}{r} = mr\omega^2$$

Where v is the linear speed of the planet.

Therefore,

$$F_g = F_c$$

$$\frac{GMm}{r^2} = mr\omega^2$$

$$\Rightarrow \omega^2 = \frac{GM}{r^3}$$

Using the identity $\omega = 2\pi/T$:

$$\boxed{\frac{T^2}{r^3} = \frac{4\pi^2}{GM}}$$

Period of Orbit

The relation $\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$ is also known as the Kepler's third law of planetary motion.

Johannes Kepler

Example Question 5

Phobos orbits Mars with an average distance of about 9380 km from the center of the planet and a rotational period of about 7hr 39 min. Use this information to estimate the mass of Mars.

Solution:

r = 9380 km = 9.38×10^6 m
T = 7 hr 39 min = 7.65 hr = 27540 sec

$$(27540)^2 = \frac{4\pi^2}{(6.67 \times 10^{-11})(M)}(9.38 \times 10^6)^3$$

$$M = \frac{(9.38\times10^6)^3}{27540^2} \cdot \frac{4\pi^2}{6.67\times10^{-11}} = 6.44 \times 10^{23} \text{ kg}$$

Geostationary Orbit

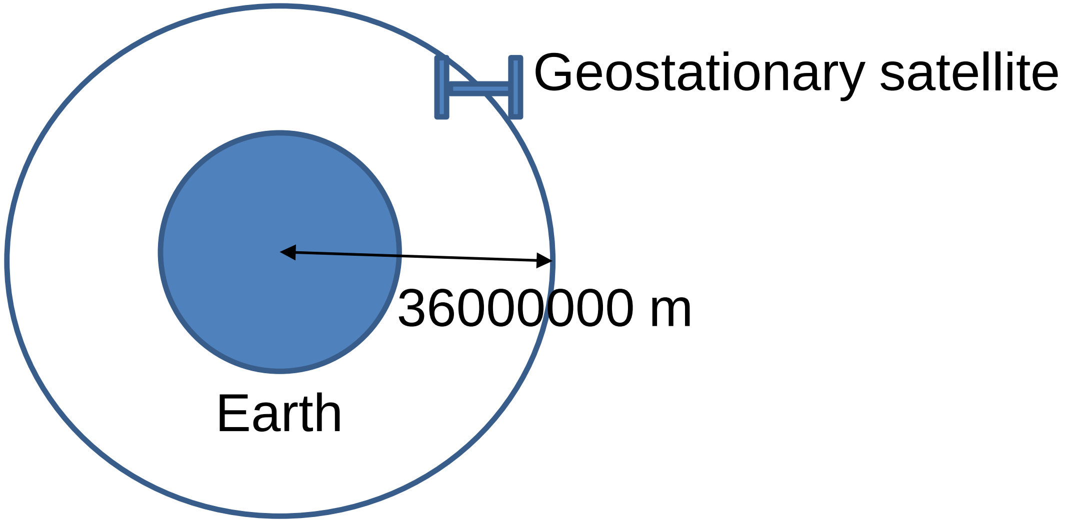
If the **rotational speed** of a satellite in its orbit is the **same** as the rotational speed of the Earth as it turns about its axis, the satellite is said to be in geostationary orbit and will exhibit the following characteristics:

(a) It will revolve in the **same direction** as the Earth.

(b) It will rotate with the **same period** of rotation as the Earth.

(c) It will move **directly above** the Earth's **equator.**

(d) The **centre** of a geostationary orbit is at the centre of the Earth.



Period of Orbit

Example Question 7:

For a geostationary satellite, calculate:

(a) The height above the Earth's surface,

(b) The speed in orbit.

[Radius of Earth = 6.38$\times 10^6$m; mass of Earth = 5.98$\times 10^{24}$kg]

Solution:

(a) The period of the satellite is 24 hours = 8.64 x 10^4s

Using $\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$, we have

$$r^3 = 6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times \frac{(8.64 \times 10^4)^2}{4\pi^2}$$

$\Rightarrow r^3 = 7.54 \times 10^{22}$m^3 $\Rightarrow r = 4.23 \times 10^7$m

The altitude = $4.23 \times 10^7 - 6.38 \times 10^6 = 3.59 \times 10^7$m

(b) Since $v = \frac{2\pi r}{T}$, the speed is given by

$v = 2\pi \times 4.23 \times \frac{10^7}{8.64} \times 10^4 = 3079$ ms^-1

Gravitational Field Strength

Similar to electric field, any mass in a gravitational field experiences a force. The magnitude of this force depends on the gravitational field strength.

The **gravitational field strength** at a point is defined as **the force per unit mass** acting on a small mass placed at that point.

- vector
- unit: N kg^{-1} or m s^{-2}

Gravitational field strength

By Newton's law of universal gravitation,

$$F = \frac{GMm}{r^2}$$

$$g = \frac{F}{m} = \frac{GMm}{r^2} \times \frac{1}{m}$$

$$\Rightarrow \quad \boxed{g = \frac{GM}{r^2}}$$

Gravitational field strength

g above Earth's surface at a height of h :

$$g = \frac{GM_E}{(R_E + h)^2} \qquad (r = R_E + h)$$

M_E : mass of the Earth

R_E : mean radius of the Earth

The value of g near the Earth's surface varies with latitude.

$$R_E = 6.4 \times 10^6 \text{ m}$$
$$M_E = 6.0 \times 10^{24} \text{ kg}$$

Gravitational Field Strength of Various Locations

TABLE 6–1
Acceleration Due to Gravity at Various Locations on Earth

Location	Elevation (m)	g (m/s^2)
New York	0	9.803
San Francisco	0	9.800
Denver	1650	9.796
Pikes Peak	4300	9.789
Sydney, Australia	0	9.798
Equator	0	9.780
North Pole (calculated)	0	9.832

The acceleration due to gravity varies over the Earth's surface due to altitude, local geology, and the shape of the Earth, which is not quite spherical.

We however normally use the average value of 9.81 ms^{-2} when solving problems.

Gravitational field strength

g is used in 2 ways:

1 **Acceleration** of an object at a point when falls freely under gravity.

2 **Gravitational field strength** at a point.

At the Earth's surface, we take the average value of g:

$$g = 9.81 \text{ m s}^{-2} \text{ (or N kg}^{-1}\text{)}$$

Above the Earth's surface, $r\uparrow \Rightarrow g\downarrow$

Gravitational field strength

The equation $g = GM/r^2$ **does not depend** upon the mass of the object that the gravitational force is acting at. This proves that the acceleration due to gravity alone is the same on all objects independent of their mass.

This is true as long as the objects are at the **same** altitude.

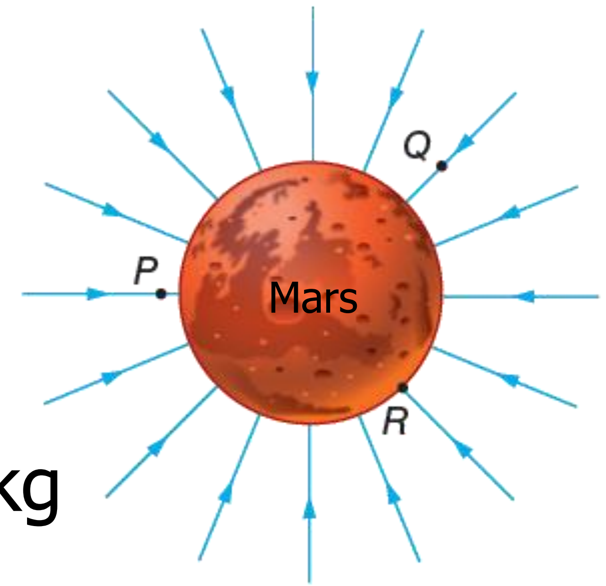
Example 8

X (3 kg), *Y* (9 kg) and *Z* (1 kg) are placed at *P*, *Q* and *R*.

R : at the surface

P : 1610 km above surface

Q : 4610 km above surface



Given: $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

Mass of Mars $M_M = 6.42 \times 10^{23}$ kg

Radius of Mars $R_M = 3390$ km

(a) The gravitational force *F* acting on the particles by Mars = ? Which particle has the largest *F* ?

(b) Find *g* of Mars at *P*, *Q* and *R*. Which position has the largest *g* ?

Solution

(a)

$$F_X = \frac{GM_M m_X}{{r_X}^2} = \frac{6.67\times10^{-11} \times 6.42\times10^{23} \times 3}{(1.61\times10^6 + 3.39\times10^6)^2}$$

$$= 5.14 \text{ N}$$

$$F_Y = \frac{GM_M m_Y}{{r_Y}^2} = \frac{6.67\times10^{-11} \times 6.42\times10^{23} \times 9}{(4.61\times10^6 + 3.39\times10^6)^2}$$

$$= 6.02 \text{ N}$$

$$F_z = \frac{GM_M m_z}{{r_z}^2} = \frac{6.67\times10^{-11} \times 6.42\times10^{23} \times 1}{(3.39\times10^6)^2}$$

$$= 3.73 \text{ N}$$

$\therefore$ Y has the largest gravitational force

Solution

(b) $$g_P = \frac{GM_M}{{r_X}^2} = \frac{6.67\times10^{-11}\times6.42\times10^{23}}{(1.61\times10^6 + 3.39\times10^6)^2}$$

$$= 1.71\ \text{N kg}^{-1}$$

$$g_Q = \frac{GM_M}{{r_Y}^2} = \frac{6.67\times10^{-11}\times6.42\times10^{23}}{(4.61\times10^6 + 3.39\times10^6)^2}$$

$$= 0.669\ \text{N kg}^{-1}$$

$$g_R = \frac{GM_M}{{r_Z}^2} = \frac{6.67\times10^{-11}\times6.42\times10^{23}}{(3.39\times10^6)^2}$$

$$= 3.73\ \text{N kg}^{-1}$$

$\therefore$ R has the largest gravitational field strength.

Example 9

Apollo 11 travelled radially outwards from the Earth. Distance travelled << distance from the Earth during this time interval.

Velocity after rocket was turned off:

Time	v / m s^{-1}
09:58:00	2619
10:08:00	2594

Given: g (at Earth's surface) = 10 m s^{-2}

radius of the Earth = 6370 km

(a) Find g in this time interval.

(b) Find its distance from Earth's surface.

Solution

(a)$\because$ distance travelled << distance from Earth

$\therefore$ assume g to be constant

Take direction towards centre of Earth as +ve.

By $v = u + gt$,

$$-2594 = -2619 + g \times 10 \times 60$$

$$g = 0.0417 \text{ m s}^{-2}$$

Solution

(b) Find its distance from Earth's surface.

By $g = \dfrac{GM_E}{(R_E + h)^2}$,

$$g = \frac{GM_E}{R_E{}^2} \times \frac{R_E{}^2}{(R_E + h)^2}$$

$$0.0417 = 10 \times \frac{(6.37 \times 10^6)^2}{(6.37 \times 10^6 + h)^2}$$

$$h = 9.23 \times 10^7 \text{ m}$$

Weightlessness in Orbit

Try to calculate the gravitational field strength acting on a space station orbiting at an altitude of 400 km: ($R_E = 6.4 \times 10^6$ m
$M_E = 6.0 \times 10^{24}$ kg).

$g = \frac{GM_E}{R_E^2} = (6.67 \times 10^{-11})(6.0 \times 10^{24})/\ (6.4 \times 10^6 + 4.0 \times 10^5)^2$

$= 8.65$ ms^{-2}

It shows that the gravitational field strength at the space station is not so much less than the average gravitational field strength on the Earth's surface.

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ISS: g= 8.73 ms^{-2}

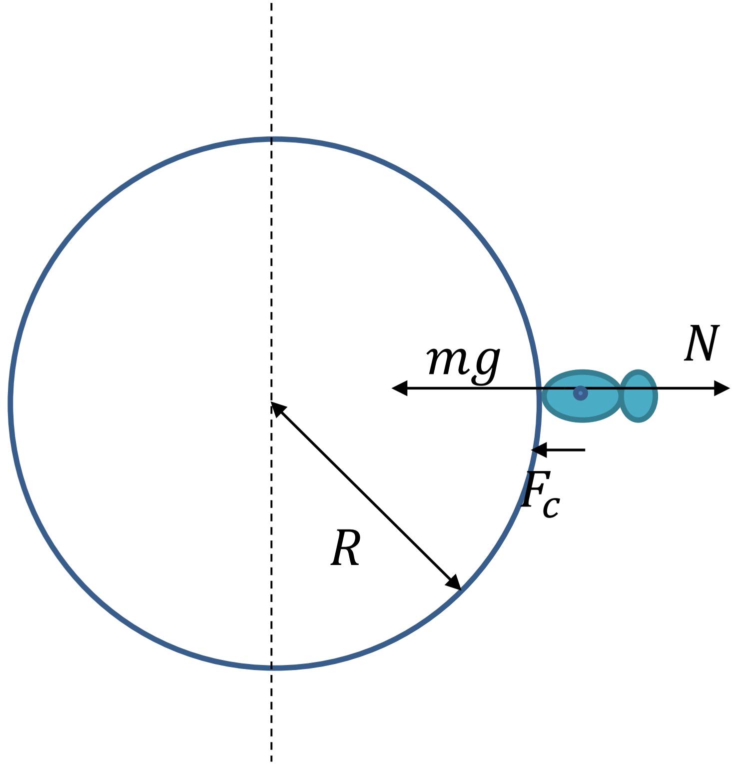
Weightlessness in Orbit

However, the astronauts in the Earth orbit does not experience any gravitational force at all.

- It is due to the fact that the astronauts and the space station are experiencing the **free fall.**
- The reaction force that we normally experience would become **zero** when the centripetal force is equal to the gravitational force acting on the astronauts.

Rotation of the Earth and Apparent Weight

Since the Earth is rotating around its axis, there is a centripetal force directed towards the Earth's centre which is in the same direction as the Earth's gravitational force.
The normal force experience by a body on the Earth is actually slightly smaller than its weight because of the centripetal force.



Rotation of the Earth and Apparent Weight

Example 10:

What is the centripetal acceleration of a 100 kg person standing on our planet at the equator, cause by the rotation of the Earth? (Earth's radius R=6371 km, assuming $g = 9.81$ ms^{-2})

Solution:

$$a_c = \frac{v^2}{R} = \frac{\left(\frac{2\pi r}{T}\right)^2}{R} = \frac{4\pi^2 R}{T^2}$$

$$= \frac{4\pi^2(6371000)}{(3600 \times 24)^2} = 0.0337\mathrm{ms}^{-2}$$

The normal force (apparent weight) experienced by the person, N, can be found by

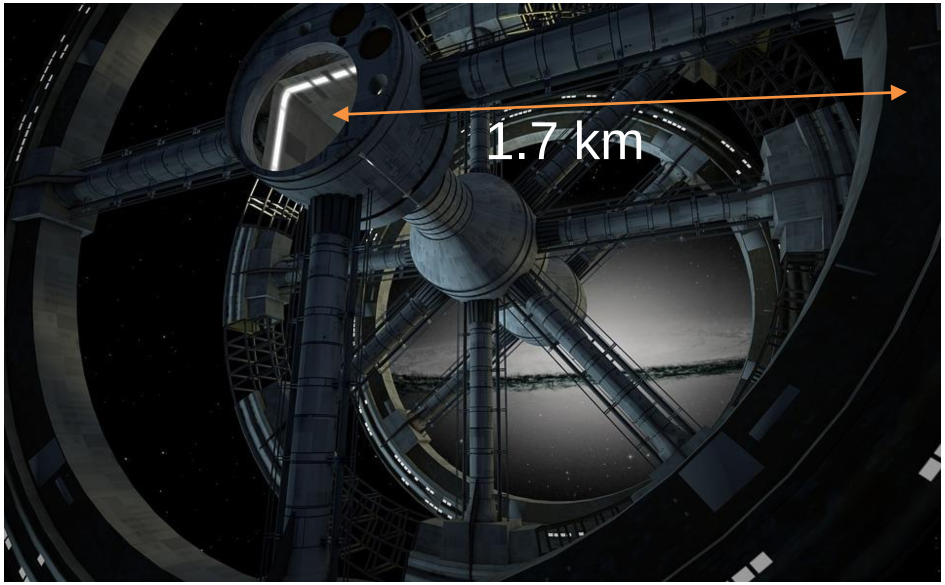
$$\begin{aligned} N &= mg - F_c \\ &= mg - ma_c \\ &= 100(9.81 - 0.0337) \\ &= 978\mathrm{N} \end{aligned}$$

Example 11

Artificial Gravity

At what speed must the surface of the space station move so that the astronaut experiences a push on his feet equal to his weight on earth? The radius is 1700 m.

$$F_c = m\frac{v^2}{r} = mg$$



Source: pixabay

$$v = \sqrt{rg}$$

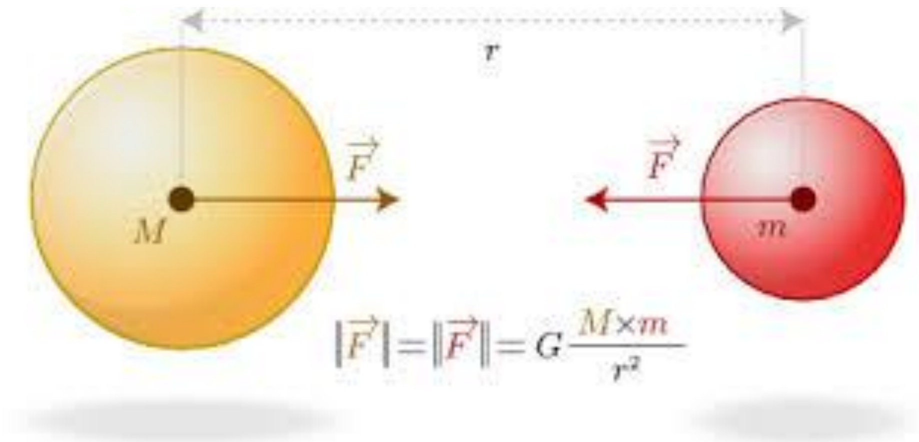
$$= \sqrt{(1700\,\mathrm{m})(9.80\,\mathrm{m/s^2})}$$

$$v = 130\,m/s$$

Gravitational Force due to Point Mass

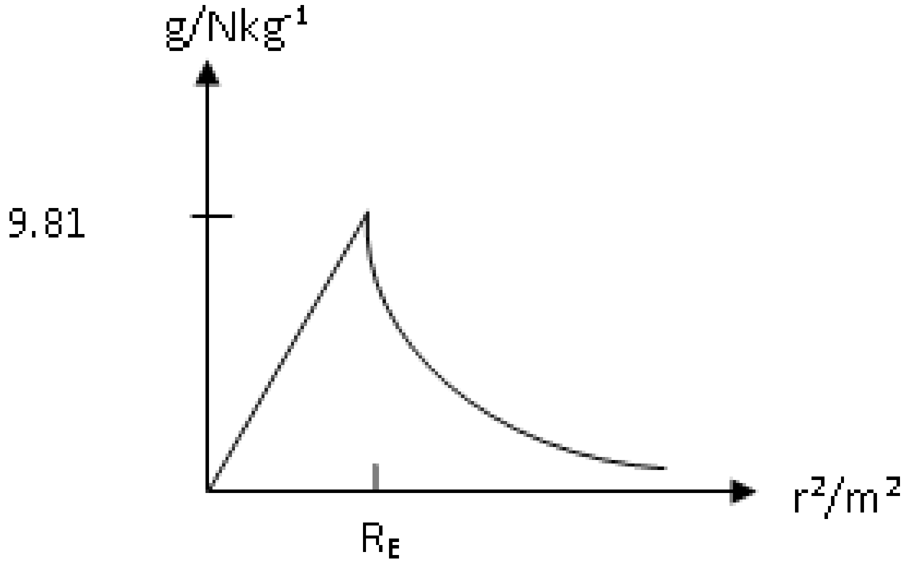
For calculations involving gravitational forces, a mass behaves as if it were a **point mass**, with all the mass of the sphere concentrated at that point.

This is true as long as the point is **outside** of the spherical mass.



Gravitational Field Strength Inside the Sphere

If the point is located **inside** the spherical mass, we can no longer regard the mass as a point mass.



Example 12

The distance from the centre of the sun to the centre of the Earth is $1.5\text{x}10^{11}$m & the masses of the Earth & sun respectively are $6.0\text{x}10^{24}$kg & $2.0\text{x}10^{30}$kg

a) The diameters of the Sun & Earth respectively are $1.4\text{x}10^{9}$m & $1.3\text{x}10^{7}$m why is it reasonable to consider them both to be point masses?

b) Calculate the force of gravitational attraction between the Earth & the Sun

<u>Solution</u>

a) The diameters of both are small compared to the separation. Distance between any part of the sun and Earth is the same within 1%

b) $F = Gm_1m_2 / r^2$

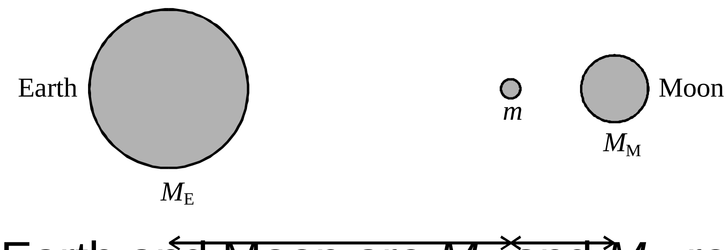
$$F = 6.67\text{x}10^{-11} \text{ x } 6.0\text{x}10^{24} \text{ x } 2.0\text{x}10^{30} /(1.5\text{x}10^{11})^2 = 3.6\text{x}10^{22}\text{N}$$

Example Question 13

What is the force of gravity acting on a 2000-kg spacecraft when it orbits two Earth radii from the Earth's center (that is, a distance r_E = 6380 km above the Earth's surface)? The mass of the Earth is $M_E = 5.98 \times 10^{24}$ kg.

Exam Question 14

2. The diagram shows a body of mass *m* situated at a point which is a distance *R* from the centre of the Earth and *r* from the centre of the Moon.



The masses of the Earth and Moon are M_E and M_M respectively. The gravitational constant is *G.*

Using the symbols given, write down an expression for

(i) the gravitational force of attraction between the body and the Earth.
(ii) the gravitational force of attraction between the body and the Moon.

(2)

The resultant gravitational force exerted upon the body at this point is zero. Calculate the distance R of the body from the centre of the Earth given that r = 3.9 × 107 m and $M_E = 81\ M_M$

(3)

(Total 5 marks)

Forces

(i) $F = GM_E m/R^2$

(ii) $F = GM_M m/r^2$

$$\left.\begin{array}{c} \dfrac{GM_E m}{R^2} = \dfrac{GM_m m}{r^2} \\ \text{OR} \\ \dfrac{M_E}{M_m} = \dfrac{R^2}{r^2} \text{ OR } \left(\dfrac{M_E}{M_m}\right)^{1/2} = \dfrac{R}{r} \end{array}\right\}$$

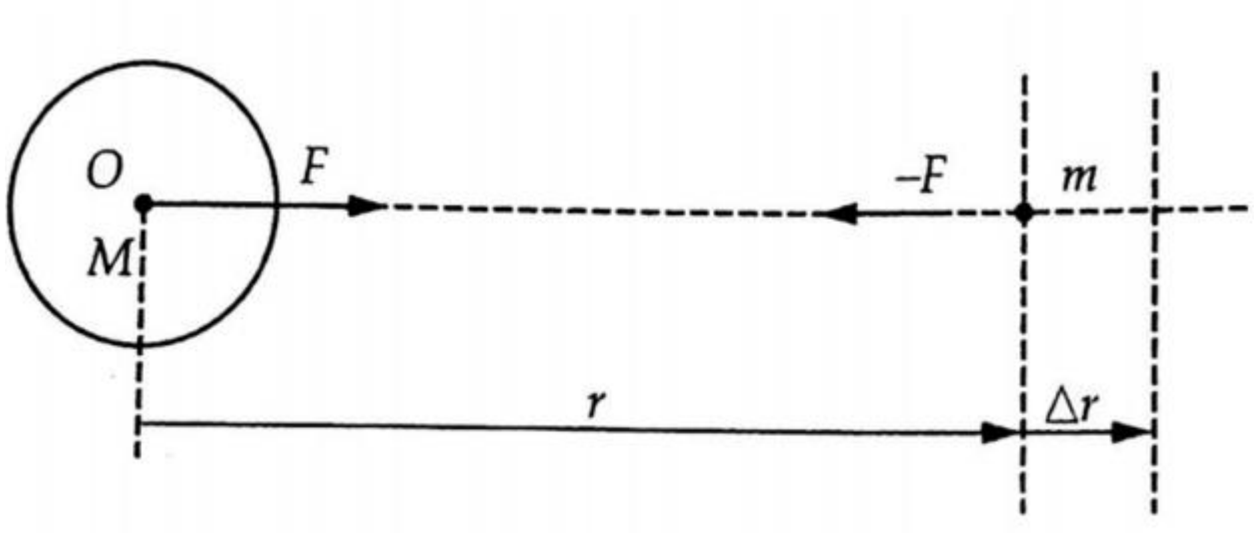
$$\frac{81}{1} = \frac{R^2}{(3.9\times 10^7\ \text{m})^2}$$

Distance *R*

$R = 3.5 \times 10^8$ m

Gravitational Potential Energy

Consider a system of two particles of masses M and m.



Given the definition of potential energy, $U = -\int_{ref}^{r} \vec{F} \cdot \overrightarrow{dr}$

And gravitational force $F = -GMm/r^2$ (negative term denotes the force acting in the opposite direction for m)

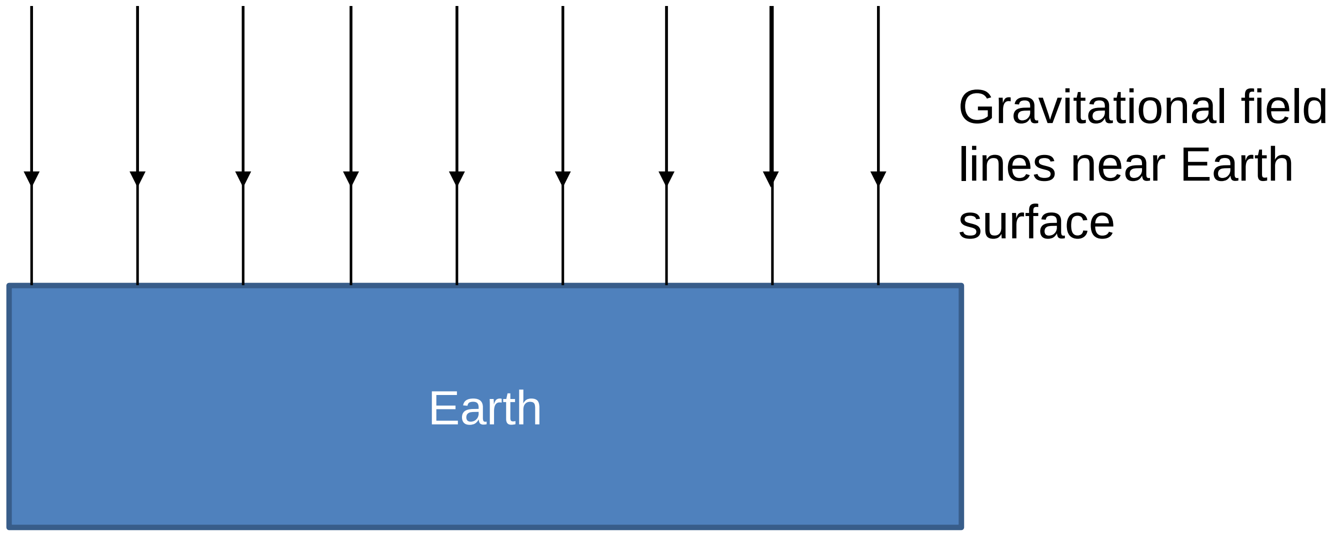
We obtain the gravitational potential energy as:

$$\boxed{U_g = -\frac{GMm}{r}}$$

Gravitational Potential Energy near Earth Surface

Near the Earth surface, g is approximately constant and is called the acceleration of free fall. We can calculate the **change** in gravitational potential energy using this assumption for near Earth surface:

$$\Delta U_g = mg\Delta h$$



Gravitational Potential

The gravitational potential V at a point P is defined as the work done in bringing unit mass from infinity to that point.

Mathematically, V is written as

$$V = \frac{U_g}{m}$$

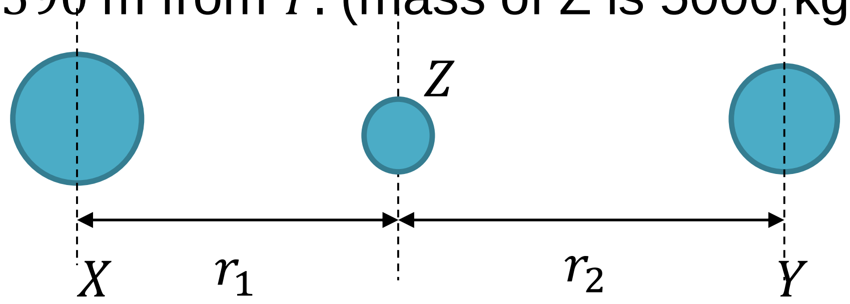
Where V is the gravitational potential at a point and W is the work done in bringing a mass m from infinity to that point.

The S.I. unit for potential is J kg^{-1}.

$$\boxed{V = -\frac{GM}{r}}$$

Example:

A mass X and a mass Y are at a distance from each other, where the mass of X is 90000 kg and the mass of Y is 150000 kg. Calculate the gravitational potential energy of a mass Z, which is placed at distance $r_1 = 500$ m from X, and distance $r_2 = 590$ m from Y. (mass of Z is 5000 kg)



Answer:

Since gravitational potential energy is a scalar,

GPE of mass Z = GPE due to X + GPE due to Y

$$U_Z = U_{XZ} + U_{YZ} = -\frac{Gm_X m_Z}{r_1} + \left(-\frac{Gm_Y m_Z}{r_2}\right)$$

$$= -\frac{(6.67\times10^{-11})(90000)(5000)}{500} - \frac{(6.67\times10^{-11})(150000)(5000)}{590}$$

$$= -1.45 \times 10^{-4}\text{J}$$

Gravitational Potential V

In general

$$g = -\frac{\Delta V}{\Delta r}$$

ΔV = potential difference(J/kg)

For a radial field

$$V = -\frac{GM}{r}$$

V = gravitational potential at a distance r from mass M

A Point in space has Potential
An Object placed there has Potential Energy
Potential Energy = potential x mass

$$E_p = -\frac{GMm}{r}$$

Kinetic Energy of a Satellite

Consider a satellite of mass m orbiting with a speed v round the Earth of mass M in a circular orbit of radius r.
The kinetic energy of the satellite is

$$E_k = \frac{1}{2}mv^2$$

As $\frac{mv^2}{r} = GMm/r^2$(centripetal force = gravitational force)

$$v^2 = \frac{GM}{r}$$

The kinetic energy of the orbiting satellite is given by

$$\boxed{E_k = \frac{1}{2}\frac{GMm}{r}}$$

Total Energy of a Satellite

The total energy E of the satellite is the sum of the kinetic energy and potential energy of the satellite.

$$E = E_k + U$$

$$= \frac{1}{2}\frac{GMm}{r} - \frac{GMm}{r}$$

$$\boxed{E = -\frac{GMm}{2r}}$$

The escape velocity v_{min} from a point in the gravitational field is the **minimum** velocity required to project a mass m to infinity in outer space.

Total Energy of a Satellite

For m to reach infinity,
Kinetic energy of m at infinity ≥ 0
And potential energy of m at infinity $= 0$.

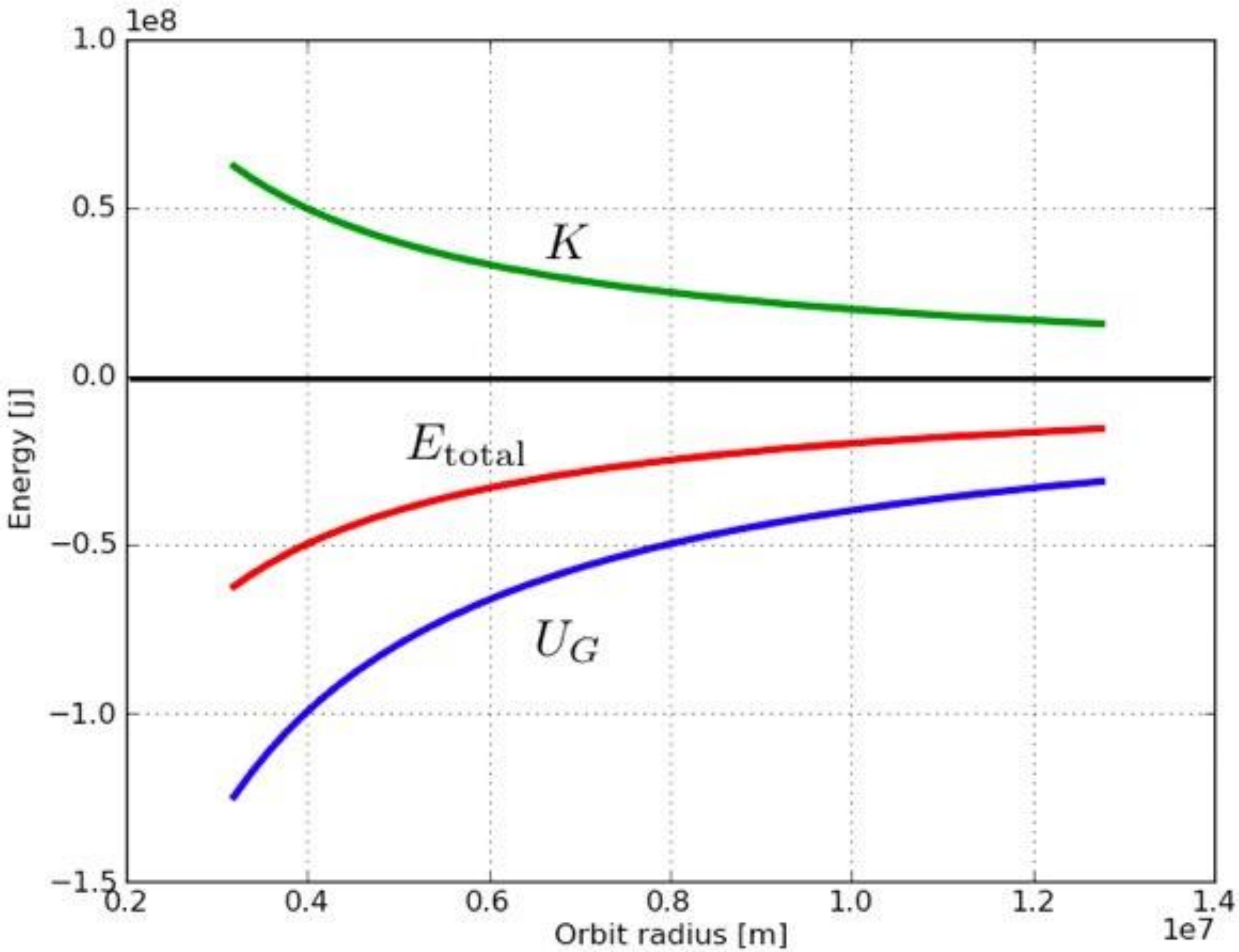
i.e. total energy $E = E_k + U_g \geq 0$
$\therefore \frac{1}{2}mv^2 - \frac{GMm}{r} \geq 0$
Where v is the velocity of projection and
r is the distance from the Earth's centre to the mass m.

$$\boxed{v_{min} = \sqrt{\frac{2GM}{r}}}$$

Or

$$\boxed{v_{min} = \sqrt{2gr}}$$

Relation in Graph



Example 15

1 The Earth may be considered to be a uniform sphere with its mass M concentrated at its centre.

A satellite of mass m orbits the Earth such that the radius of the circular orbit is r.

(a) Show that the linear speed v of the satellite is given by the expression

$$v = \sqrt{\left(\frac{GM}{r}\right)}.$$

(b) For this satellite, write down expressions, in terms of G, M, m and r, for

(i) its kinetic energy,

(ii) its gravitational potential energy,

(iii) its total energy.

(c) The total energy of the satellite gradually decreases.

State and explain the effect of this decrease on

(i) the radius r of the orbit,

(ii) the linear speed v of the satellite.

(a)	centripetal force is provided by gravitational force	B1	
	$mv^2 / r = GMm / r^2$	B1	
	hence $v = \sqrt{(GM / r)}$	A0	**[2]**
(b)	**(i)** E_K (= $\frac{1}{2}mv^2$) = $GMm / 2r$	B1	**[1]**
	(ii) $E_P = - GMm / r$	B1	**[1]**
	(iii) $E_T = - GMm / r + GMm / 2r$	C1	
	$= - GMm / 2r$.	A1	**[2]**
(c)	**(i)** if E_T decreases then $- GMm / 2r$ becomes more negative		
	or $GMm / 2r$ becomes larger	M1	
	so r decreases	A1	**[2]**
	(ii) $E_K = GMm / 2r$ and r decreases	M1	
	so (E_K and) v increases	A1	**[2]**

Review Question 1

(a) Define *gravitational potential* at a point.

....

....

.... [2]

(b) The Moon may be considered to be an isolated sphere of radius 1.74×10^3 km with its mass of 7.35×10^{22} kg concentrated at its centre.

(i) A rock of mass 4.50 kg is situated on the surface of the Moon. Show that the change in gravitational potential energy of the rock in moving it from the Moon's surface to infinity is 1.27×10^7 J.

(ii) The escape speed of the rock is the minimum speed that the rock must be given when it is on the Moon's surface so that it can escape to infinity.
Use the answer in **(i)** to determine the escape speed. Explain your working.

(c) The Moon in **(b)** is assumed to be isolated in space. The Moon does, in fact, orbit the Earth.
State and explain whether the minimum speed for the rock to reach the Earth from the surface of the Moon is different from the escape speed calculated in **(b)**.

Review Question 2

(a) State Newton's law of gravitation.

....

....

.... [2]

(b) A satellite of mass m is in a circular orbit of radius r about a planet of mass M.
For this planet, the product GM is $4.00 \times 10^{14}\,\mathrm{N\,m^2\,kg^{-1}}$, where G is the gravitational constant.
The planet may be assumed to be isolated in space.

(i) By considering the gravitational force on the satellite and the centripetal force, show that the kinetic energy E_K of the satellite is given by the expression

$$E_K = \frac{GMm}{2r}.$$

(ii) The satellite has mass 620 kg and is initially in a circular orbit of radius 7.34×10^6 m, as illustrated in Fig. 1.1.

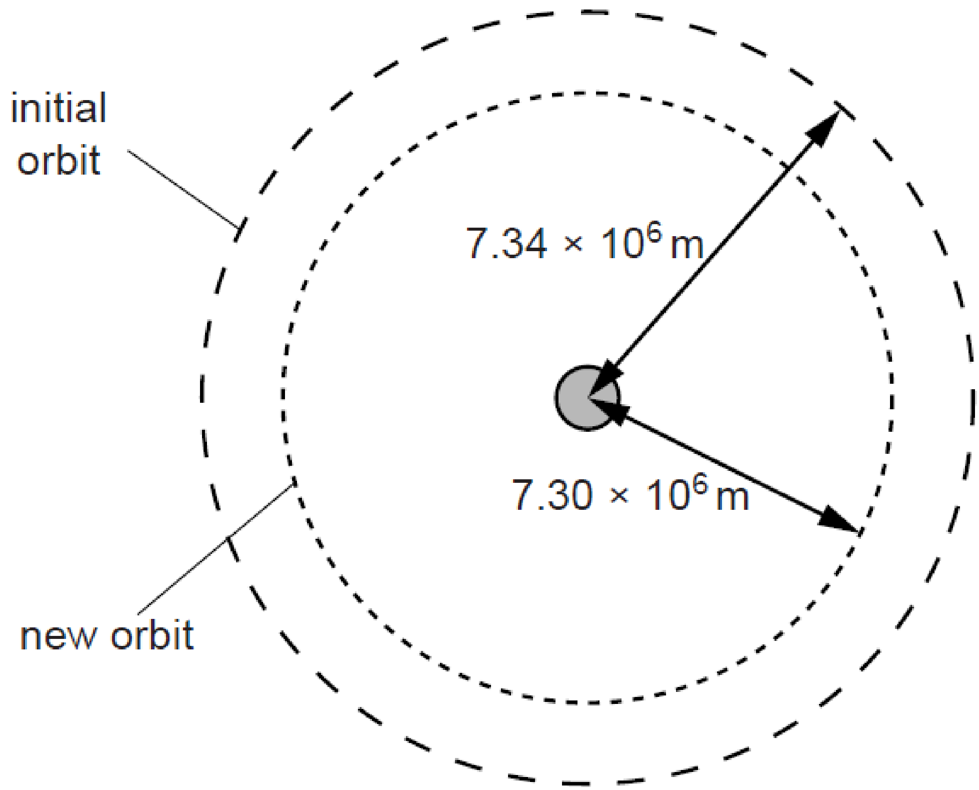


Fig. 1.1 (not to scale)

Resistive forces cause the satellite to move into a new orbit of radius 7.30×10^6 m.

Determine, for the satellite, the change in

1. kinetic energy,

change in kinetic energy = ... J [2]

2. gravitational potential energy.

change in potential energy = ... J [2]

(iii) Use your answers in **(ii)** to explain whether the linear speed of the satellite increases, decreases or remains unchanged when the radius of the orbit decreases.

....

....

.... [2]

Review Question 3

(a) Explain what is meant by a *geostationary orbit*.

....

....

....

.... [3]

(b) A satellite of mass m is in a circular orbit about a planet.
The mass M of the planet may be considered to be concentrated at its centre.
Show that the radius R of the orbit of the satellite is given by the expression

$$R^3 = \left(\frac{GMT^2}{4\pi^2}\right)$$

where T is the period of the orbit of the satellite and G is the gravitational constant.
Explain your working.

(c) The Earth has mass 6.0×10^{24} kg. Use the expression given in **(b)** to determine the radius of the geostationary orbit about the Earth.

radius = ... m [3]

Review Question 4

(b) In the Solar System, the planets may be assumed to be in circular orbits about the Sun. Data for the radii of the orbits of the Earth and Jupiter about the Sun are given in Fig. 1.1.

	radius of orbit / km
Earth Jupiter	1.50×10^8 7.78×10^8

Fig. 1.1

(i) State Newton's law of gravitation.

(ii) Use Newton's law to determine the ratio

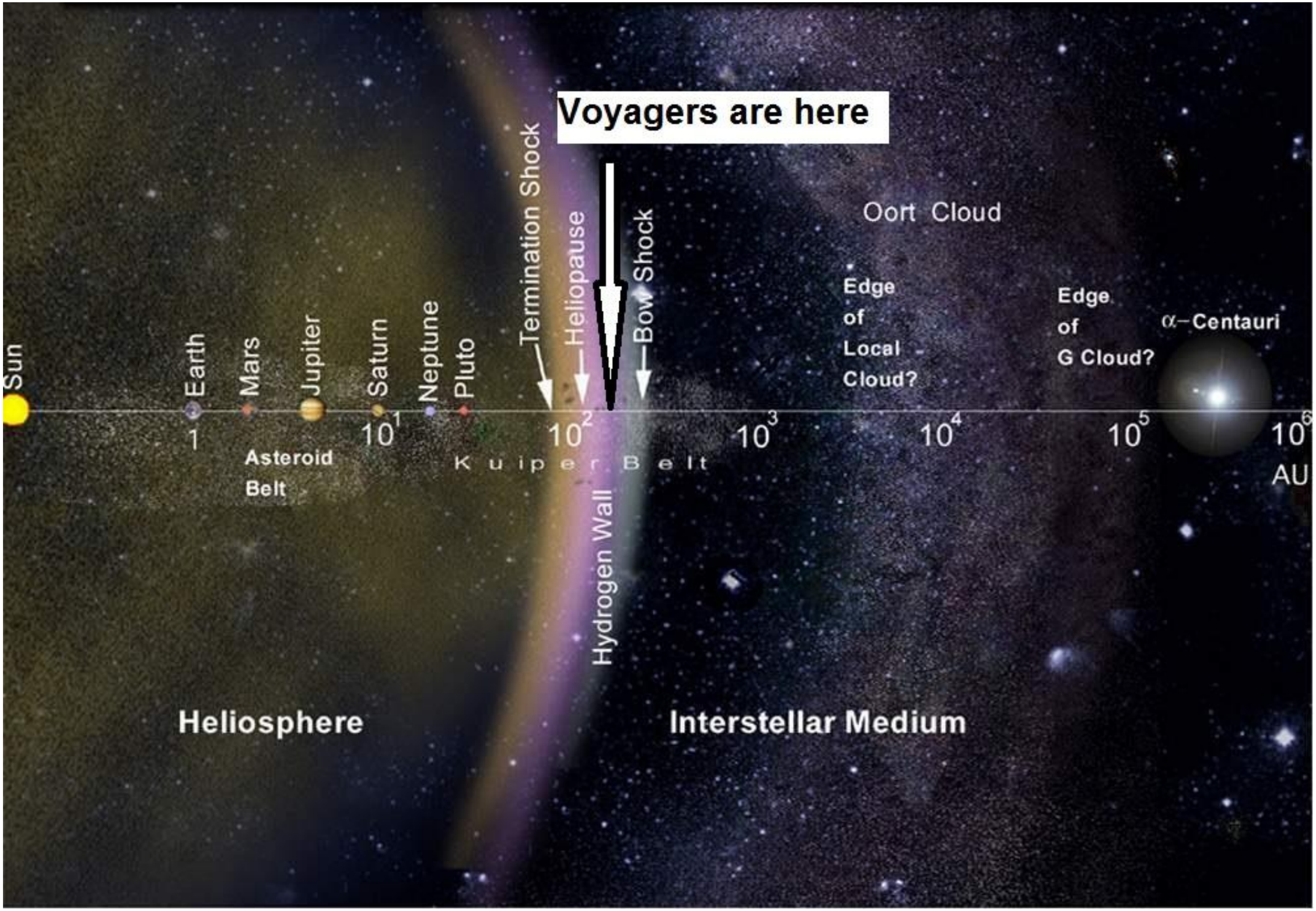
$$\frac{\text{gravitational field strength due to the Sun at orbit of Earth}}{\text{gravitational field strength due to the Sun at orbit of Jupiter}}.$$

ratio = ... [3]

Review Question 5

One object A is placed at the surface of planet X with a mean density ρ and radius r. Another object B is placed on the surface of planet Y which has a radius $5r$ and the same mean density as planet X, calculate the ratio of the escape velocity of the object A to the escape velocity of object B.

How Far Can We Go to Escape the Gravity of Solar System?



Further Examples

Example

4 (a) Artificial satellites are used to monitor weather conditions on Earth, for surveillance and for communications. Such satellites may be placed in a *geo-synchronous* orbit or in a low polar orbit.

Describe the properties of the geo-synchronous orbit and the advantages it offers when a satellite is used for communications.

(b) A satellite of mass m travels at angular speed ω in a circular orbit at a height h above the surface of a planet of mass M and radius R.

(i) Using these symbols, give an equation that relates the gravitational force on the satellite to the centripetal force.

(ii) Use your equation from part (b)(i) to show that the orbital period, T, of the satellite is given by

$$T^2 = \frac{4\pi^2(R+h)^3}{GM}.$$

(iii) Explain why the period of a satellite in orbit around the Earth cannot be less than 85 minutes. Your answer should include a calculation to justify this value.

mass of the Earth $= 6.00 \times 10^{24}$ kg
radius of the Earth $= 6.40 \times 10^{6}$ m

(c) Describe and explain what happens to the speed of a satellite when it moves to an orbit that is closer to the Earth.

Question 4		
(a)	orbits (westwards) over Equator ✓ maintains a fixed position relative to surface of Earth ✓ period is 24 hrs (1 day) or same as for Earth's rotation ✓ offers uninterrupted communication between transmitter and receiver ✓ steerable dish not necessary ✓	**Max 3**
(b) (i)	$G\dfrac{Mm}{(R+h)^2} = m\omega^2(R+h)$ ✓	
(ii)	use of $\omega = \dfrac{2\pi}{T}$ ✓ gives $\dfrac{GM}{(R+h)^3} = \dfrac{4\pi^2}{T^2}$, hence result ✓	**6**
(iii)	limiting case is orbit at zero height i.e. $h = 0$ ✓ $T^2 = \left(\dfrac{4\pi^2 R^3}{GM}\right) = \dfrac{4\pi^2 \times (6.4\times10^6)^3}{6.67\times10^{-11}\times6.0\times10^{24}}$ ✓ $T = 5090$ s ✓ (= 85 min)	
(c)	speed increases ✓ loses potential energy but gains kinetic energy ✓ [or because $v^2 \propto \dfrac{1}{r}$ from $\dfrac{GMm}{r^2} = \dfrac{mv^2}{r}$] [or because satellite must travel faster to stop it falling inwards when gravitational force increases]	**2**
	Total	**11**

Example

1 (a) State the law that governs the magnitude of the force between two point masses.

(b) The table shows how the gravitational potential varies for three points above the centre of the Sun.

distance from centre of Sun/10^8 m	**gravitational potential/10^{10} J kg^{-1}**
7.0 (surface of sun)	–19
16	–8.3
35	–3.8

(i) Show that the data suggest that the potential is inversely proportional to the distance from the centre of the Sun.

(ii) Use the data to determine the gravitational field strength near the surface of the Sun.

(iii) Calculate the change in gravitational potential energy needed for the Earth to escape from the gravitational attraction of the Sun.

mass of the Earth = 6.0×10^{24} kg
distance of Earth from centre of Sun = 1.5×10^{11} m

(iv) Calculate the kinetic energy of the Earth due to its orbital speed around the Sun and hence find the minimum energy that would be needed for the Earth to escape from its orbit. Assume that the Earth moves in a circular orbit.

Question 1 (a)	force is proportional to the product of the two masses force is inversely proportional to the square of their separation (condone radius between masses) **or** equation M0 : masses defined A1 separation defined A1	**B1** **B1**	**2**
(b) (i)	appreciation that potential x distance from centre of sun = constant **or** calculation of Vr for two sets of values (1.33×10^{20}) **or** uses distance ratio to calculate new V or r calculation of all three + conclusion or uses distance ratios twice+ conclusion conclusion must be more than 'numbers are same' (condone 'signs' and no use of powers of 10)	**C1** **A1**	**2**
(ii)	$V = GM/r$ and $g = GM/r^2$ **or** $g = V/r$ (no mark for E or $g = V/d$ or $E = V/r$) substitution of one set of data to obtain GM (1.33×10^{20}) or $19 \times 10^{10}/7 \times 10^{8}$ seen 271 N kg^{-1} (m s^{-2}) (J kg^{-1} m^{-1})	**B1** **B1** **B1**	**3**
(iii)	potential energy of the Earth = $(-)GMm/r$ or potential difference formula + $r_2 = \infty$ or potential at position of Earth = -8.87×10^{8} J kg^{-1} (from $Vr = 1.33 \times 10^{20}$) correct substitution (allow ecf for GM from (ii)) or potential energy = potential x mass of Earth change in PE = 5.32×10^{33} J (cnao) Fd approach is PE so 0 marks	**C1** **C1** **A1**	**3**
(iv)	speed of Earth round Sun = $2\pi r/T$ or $\sqrt{\frac{GM}{r}}$ or 3.0×10^{4} m s^{-1} or KE= $\frac{GMm}{2r}$ KE of Earth = ½ 6×10^{24} x their v^2 (2.68×10^{33}J) energy needed = difference between (iii) and orbital KE (2.64×10^{33} J) or KE in orbit = half total energy needed to escape (−1 for AE)	**B1** **B1** **B1**	**3**

Example

4 (a) State, in words, Newton's law of gravitation.

(b) By considering the centripetal force which acts on a planet in a circular orbit, show that $T^2 \propto R^3$, where T is the time taken for one orbit around the Sun and R is the radius of the orbit.

(c) The Earth's orbit is of mean radius 1.50×10^{11} m and the Earth's year is 365 days long.

(i) The mean radius of the orbit of Mercury is 5.79×10^{10} m. Calculate the length of Mercury's year.

(ii) Neptune orbits the Sun once every 165 Earth years.

Calculate the ratio $\dfrac{\text{distance from Sun to Neptune}}{\text{distance from Sun to Earth}}$.

Question 4			
(a)		attractive **force** between point masses ✓ proportional to (product of) the masses ✓ inversely proportional to square of separation/distance apart ✓	**3**
(b)		$m\omega^2 R = (-)\dfrac{GMm}{R^2}\left(\text{or} = \dfrac{mv^2}{R}\right)$ ✓ (use of $T = \dfrac{2\pi}{\omega}$ gives) $\dfrac{4\pi^2}{T^2} = \dfrac{GM}{R^3}$ ✓ G and M are constants, hence $T^2 \propto R^3$ ✓	**3**
(c)	(i) (ii)	(use of $T^2 \propto R^3$ gives) $\dfrac{365^2}{(1.50\times10^{11})^3} = \dfrac{T_m{}^2}{(5.79\times10^{10})^3}$ ✓ $T_{\mathrm{m}} = 87(.5)$ days ✓ $\dfrac{1^2}{(1.50\times10^{11})^3} = \dfrac{165^2}{R_{\mathrm{N}}{}^3}$ ✓ (gives $R_{\mathrm{N}} = 4.52\times10^{12}$ m) ratio $= \dfrac{4.51\times10^{12}}{1.50\times10^{11}} = 30(.1)$ ✓	**4**