A Robust Asymptotic Control Model to Analyze Climate Policy with CDR Options

Frédéric Babonneau[1], Alain Haurie[2], Marc Vielle[3]

[a] *Kedge Business School, Bordeaux, France*
[b] *University of Geneva, Geneva, Switzerland*
[c] *EPFL-LEURE, Lausanne, Switzerland*

Abstract

A three-region optimal economic growth model is proposed to represent the global energy transition to net-zero emissions when carbon dioxide removal (CDR) technologies are available. The main features of the model are (i) the representation of the economy and energy use with nested CES production functions; (ii) the representation of climate policy through the use of a safety cumulative emissions budget concept; and (iii) the introduction of an international emissions trading scheme for the implementation of climate policy. Using an infinite horizon optimal control paradigm, several contrasting scenarios are analyzed both in an asymptotic steady state or "turnpike" point, and in an optimal transition to sustainability. This very compact model produces dynamic path simulations that are consistent with the main recommendations from IPCC for long term climate policies. The potential use of this simple model in future developments in climate and economic modeling is discussed.

Keywords: Optimal Net-zero emissions, Economic Growth, Asymptotic Steady State, Sustainability, CDR Options, Carbon Market, Robustness.

1. Introduction

In this paper, asymptotic control theory (Carlson et al., 1991) is employed to develop an optimal economic growth model with an infinite time horizon.

[1]frederic.babonneau@kedgebs.com
[2]ahaurie@gmail.com
[3]marc.vielle@epfl.ch

Preprint submitted to Elsevier *November 18, 2024*

This model can reproduce, in a simple and compact three-region framework, the IPCC's main recommendations for long-term climate policy and international development. The model also provides an overview of the potential contribution of carbon dioxide removal (CDR) technologies to achieving the objectives of the Paris Climate Agreement (Masson-Delmotte et al., 2021).

Climate and development policies are closely linked. The challenge facing the economy is to find a path for economic growth that is compatible both with the development needs of the least favored nations and with the fight against climate change. Recent integrated modelling exercises have shown that achieving the objectives of the Paris Agreement may require reaching a regime of net zero emissions before the end of the century, by 2050 or 2080 at the latest (Rogelj et al., 2018; Bouckaert and et al., 2021). CDR technologies could play an important role in generating negative emissions to offset emissions from the remaining use of fossil fuels once the ZNE regime has been reached. This was recognized at COP28, which ended with an agreement marking the "beginning of the end" of the fossil fuel era, but also leaving the door open for CDR technologies, championed in particular by oil-producing countries, to limit global warming.

Recently endogenous growth and directed technical change theories have developed to analyze climate policy, see e.g. Acemoglu et al. (2012); Papageorgiou et al. (2017); Bretschger (2024). These theories allowed for the consideration of non convex abatement costs, while the endogenous technical progress and population growth would yield more proactive policies for a transition to low carbon economy. Our approach which is computational and based on dynamic optimization over long (infinite) horizon requires strong convexity of the dynamic system. Therefore, we rather stay with a neoclassical optimal economic growth model based on the Ramsey paradigm Ramsey (1928) of optimal capital accumulation, with the economy, energy and negative emissions production described by CES functions (Uzawa, 1962; McFadden, 1963), in a manner very similar to computable general equilibrium models (CGE), such as EPPA (Paltsev et al., 2005) or GEMINI-E3 (Bernard and Vielle, 2003). We thought that the introduction of a global emissions budget to be shared between different coalitions of nations, coupled with a constraint of carbon neutrality at a certain date, would force the development of renewable energies and the asymptotic use of CDR technologies. The model we present here confirms this intuition and allows us to envisage a more equitable long-term future exempt of ecological catastrophe, without invoking endogenous growth and directed technical change.

To take into account the efficiency of different forms of energy, as in Casey (2024) the model represents conversion of useful and secondary energy, as well as the production of negative emissions by three types of CDR technologies, through a mix of capital (plants), labor and the primary fossil energy carriers, coal, oil and gas, which are the sources of CO_2 emissions. To deal with the very long term, the model exploits the global asymptotic stability of economic growth paths optimized over an infinite time horizon, described in the work of Cass and Shell (1976) and Rockafellar (1976), and summarized in the book of Carlson et al. (1991). The asymptotic levels of state variables, i.e. capital stocks, can be calculated, defining the so-called "turnpike" attractor. These turnpike values can then be used as terminal conditions for a dynamic optimization over a long (e.g. 175-year) horizon to give an approximation of the transition to sustainability.

Direct carbon air capture and storage (DACCS) technologies have already been assessed in integrated assessment models that include an optimal economic growth paradigm à la Ramsey, see Chen and Tavoni (2013) and Marcucci et al. (2017). Our modeling approach takes long-term analysis of the impact of CDR/DACCS technologies on economic growth a step further. Ramsey's model of optimal capital accumulation (Ramsey, 1928) is one of the most widely used paradigms for representing and studying long-term economic growth (Cass, 1965). In the context of climate policy, several authors of integrated assessment models have adopted this paradigm to represent economic growth and the resulting GHG emissions. This is particularly the case for the DICE/RICE (Nordhaus and Boyer, 2000), MERGE (Manne et al., 1995), REMIND-R (Baumstark et al., 2021; Bauer et al., 2016), and WITCH (Bosetti et al., 2008) models, which have provided insights into the economics of climate policies. The model proposed in this paper belongs to the same strand of research, its originality lies in its compactness, the systematic use of nested CES production functions to represent the economic structure, and the exploration of the long term consequence on growth and development, using asymptotic "turnpike" properties of optimal economic growth models.

This model is a complement to the general economic equilibrium approach presented in Babonneau et al. (2021a), concerning the possible role of CDR and DACCS development in the climate policy of oil and gas producing countries (Babonneau et al., 2023), and (Babonneau et al., 2021b), where an oligopoly game of CDR technology development in a steady-state net-zero emissions climate regime is proposed. The compact optimal economic growth model presented herein can also be related to the stochastic control (Bahn et al., 2008) and differential game (Bahn and Haurie, 2016, 2008) models proposed to analyse global climate policy. As in these previous works, climate policy is represented by the sharing of a safety cumulative emissions budget (SEB) remaining (Ohndorf et al., 2015; Allen, 2015). A remaining carbon budget, typically of 1070 Gt of CO_2, is used, which is consistent with the estimation made by Rogelj et al. (2019). A representation of an international emissions trading scheme is also proposed, where the global supply of emission rights (permits) will determine the permit price, and emissions will be such that, in each region the marginal cost of reduction equals this price.

We consider three different CDR technologies, namely direct air capture with sequestration DACCS, bio-energy with carbon capture and storage (BECCS) and enhanced rock weathering (ERW). ERW, like biochar and afforestation/reforestation may have a potential similar to that of BECCS and DACCS as stated in IPCC report (Shukla and Pathak, 2022). The roles of advanced technologies in energy-economy models for climate change assessment has been studied using general equilibrium models in Morris et al. (2019). The potential offered by DACCS has been assessed in (Keith et al., 2018; Socolow et al., 2011; Desport et al., 2024), in a recent IEA report (McGlade, 2023) and in a most recent modeling effort (Edwards et al., 2024). BECCS potential has been recently assessed with the TIAM and TIMES models (Selosse and Ricci, 2014) and (Emenike et al., 2021). For a most recent techno-economic assessment of BECCS we refer to (Pratama et al., 2023). ERW has been assessed in (Kantzas et al., 2022).

The rest of the paper is organized as follows: the model is formulated and the optimization approach is discussed in Section 2; different scenario simulations obtained with this model are presented in Section 3. Section 4 presents the insight into the long term effects of CDR development gained from the simulation results and concludes.

2. The optimal economic growth paradigm with infinite horizon optimization

The formulation of optimal economic growth as infinite horizon dynamic optimization problems has been clearly explained by K. Arrow an M. Kurz in (Arrow and Kurz, 1970a,b). The use of infinite horizon optimal control in climate integrated assessment models was proposed in (Haurie, 2003) as a way to deal with the very long term and the sustainability issue. In an infinite horizon optimization formulation one can exploit the global asymptotic stability of economic growth paths, as described by Cass & Shell Cass and Shell (1976), Rockafellar Rockafellar (1976), and summarized in the book of Carlson et al. (1991). The asymptotic levels of state variables, typically capital stocks, can be calculated, defining the so-called "turnpike" attractor. These turnpike values can then be used as terminal conditions for an optimal control over a long (e.g. 175-year) horizon to give an approximation of the transition to sustainability. Hence, our approach in this paper has the following two steps. First, one computes the "Turnpike" using an associated fixed point problem based on an implicit programming problem (Feinstein and Luenberger, 1981) to define the long-term economic steady state under a steady net-zero emissions regime. Then, the resulting asymptotic values will define the terminal conditions for capital stocks in an optimal economic growth model for the design of an optimal state trajectory.

We regroup the world countries in three "coalitions", with label j, called BRIC (Brazil, Russia, India and China), OECD (Organisation for Economic Co-operation and Development) and ROW (Rest Of the World), respectively. They represent groups of nations often seen as natural coalitions climate negotiations. For each group, an optimal economic growth model is built around a representation of the economy, energy and negative emissions through nested CES production functions (Solow, 1956; McFadden, 1963; Uzawa, 1962, 1963), as schematised in Figure 1. The sectors corresponding to different types of physical capital are indexed as follows

- $i = 0$: General economic good productive capital;
- $i = 1$: Fossil energy productive capital;
- $i = 2$: Renewable energy productive capital;
- $i = 3$: DACCS productive capital;
- $i = 4$: BECCS productive capital;
- $i = 5$: ERW productive capital.

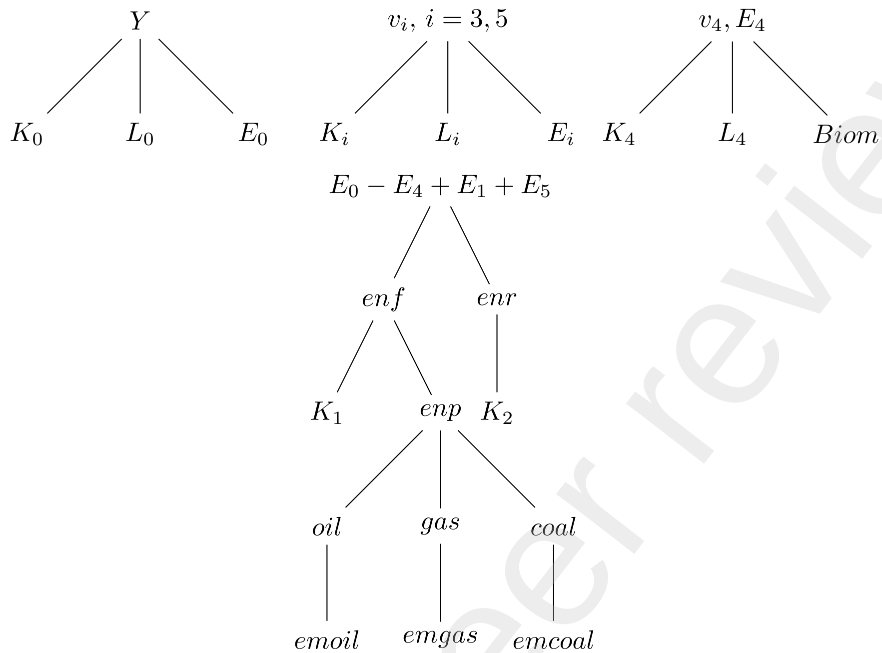


Figure 1: Nested production structure for general economy, energy, emissions and negative emissions

Useful energy (E_0) minus the energy produced by BECCS (E_4) and the energy (E_3, E_5) used to produce negative emissions (v_i, $i = 3, 5$) in three CDR technologies labelled $i = 3, 4, 5$, are obtained from a mix of fossil (enf) and renewable (enr) secondary energy forms. Fossil energy is produced using capital (e.g. refineries K_1) and extracted primary energy (enp), which is itself composed of oil, gas and coal, with their respective CO_2 emission factors (em). Renewable energy is produced using capital (K_2) only. Labor is used to produce the general economic good (L_0) and the negative emissions in CDR technologies (L_i, $i = 3, 4, 5$). Capital is a production factor for the general economic good (K_0) and negative emissions (K_i, $i = 3, 4, 5$).

Trading in goods between the three coalitions is not represented in this model, except for the emission rights that will be exchanged on an international market.

2.1. The optimal economic growth problem

We formulate an optimal economic growth model for each coalition, adapted to the study of climate policy issues. We consider a time set $t \in \{0, 1, \ldots, T\}$, where each period t contains a number of years Ny. In this application, we take five-year periods ($Ny = 5$).

2.1.1. Objective function

The performance criterion is $\Phi = \sum_j \phi(j)$, where for each coalition j, $\phi(j)$ represents the discounted sum of utility derived from consumption for the population of coalition j.

$$\phi(j) = \sum_{t=0}^{T-1} \beta(t) PV \cdot L(t,j) \log(C(t,j)/L(t,j)), \quad j = \mathrm{BRIC}, \mathrm{OECD}, \mathrm{ROW}. \quad (1)$$

In this expression

$PV = \sum_{s=1}^{Ny} (1+r)^{(1-s)}$	is the present value factor for one period,
$\beta(t) = 1/(1+r)^{Ny \cdot t}$	is the periodic discount factor, at time t,
$r = 3\%$	is the time preference rate,
$L(t,j)$	is the population of coalition j at time t,
$\log(C(t,j)/L(t,j))$	is the utility of per-capita consumption.

The consumption $C(t,j)$ by coalition j at period t is given by

$$C(t,j) = Y(t,j)\,(1 - DAMFRAC(t)) - \sum_{i=0}^{5} I_i(t,j) - Pcoal(t,j)coal(t,j) - Poil(t,j)oil(t,j) - Pgas(t,j)gas(t,j), \quad (2)$$

where $Y(t,j)$ is the output of the general economy, $I_i(t,j)$ is the yearly investment in sector i at period t, and

$$Pcoal(t,j)coal(t,j) + Poil(t,j)oil(t,j) + Pgas(t,j)gas(t,j)$$

is the expenditure in primary energy resources at period t.

2.1.2. Capital stocks and emissions budget dynamics

In an optimal control format, the model has six state variables for the capital stocks $K_i(t,j), i = 0,\dots,5$, the remaining emission budget $b(t,j)$, for coalition j at period t, five control variables, which are the investment levels $I_i(t,j); i = 0,1,2,3$, and the supply $\omega(t,j)$ of emission permits by coalition j at period t.

The state equations for capital stocks $K_i(t,j)$ are given in (3)-(4), where μ_j ($\equiv 4.5\% \, \forall j$) is annual depreciation rate for capital in coalition j. In (5) the parameter $IB_i(t,j)$ is an upper bound for yearly investment in CDR technology $i = 3,4,5$.

$$
\begin{aligned}
K_i(t,j) &= K_i(t-1,j)(1-\mu_j)^{Ny} + Ny \cdot I_i(t-1,j), \quad \forall t > 0 && (3)\\
K_i(j,0) &= K_i^0(j), \quad i = 0,\dots,5, \quad \forall j, && (4)\\
I_i(t,j) &\le IB_i(t,j), \quad i = 3,4,5. && (5)
\end{aligned}
$$

The cumulative emissions budget, since the beginning of the industrial revolution, compatible with a 60% probability of limiting the temperature increase below 2°C has been evaluated at 1 trillion tons of carbon Allen et al. (2009). Based on this figure we define a remaining SEB, B of $1'070$ Gt of CO_2. This budget is shared among the coalitions; $\theta_j \in [0,1]$ is the share of the SEB given to the coalition j, ($\sum_j \theta_j = 1$.) We assume the parameters θ_j have been fixed in the climate negotiations, with the creation of an international CO_2 emissions trading system.

Table 1: Shares of the emissions budget (in %)

Share	OECD	BRIC	ROW
θ	0.10	0.40	0.50

A justification for this sharing of the budget, based on fair burden sharing arguments, can be found in (Babonneau et al., 2021a). The remaining SEB, $b(t,j)$ for each coalition j at time t, will decrease by the amount of emissions permits $Ny \cdot \omega_i(t-1,j)$ supplied by the coalition on the CO_2 market, but the SEB $b(t,j)$ will be replenished by the amount of negative emissions $Ny \cdot v(t-1,j)$ extracted by coalition j. In summary the SEB dynamics is, for all coalitions j

$$\begin{aligned}
b(t,j) &= b(t-1,j) - Ny \cdot \omega_i(t-1,j) + Ny \cdot v(t,j) \quad t = 1 \dots T, &(6)\\
b(0,j) &= \theta_j B, &(7)\\
\sum_j b(t,j) &\geq 0, \quad t = 1 \dots T, &(8)
\end{aligned}$$

where the total negative emissions for coalition j at period t is given by the sum of negative emissions created by the three technologies, DAC, BECCS and ERW.

$$v(t,j) = \sum_{i=3}^{5} v_i(t,j). \tag{9}$$

By keeping the global remaining SEB nonnegative (Eq. 8), we exclude overshooting in the climate policy. We would prohibit overshooting for each coalition by imposing $b(t,j) \geq 0$, $\forall t$.

2.1.3. Production functions

CES production functions are introduced in the following constraints (for coalition j and period t).

General economic good production.

$$Y(t,j) - A_0(j) tg(t,j) \left[\alpha_{0K} K_0(t,j)^{\frac{s_0(j)-1}{s_0(j)}} + \alpha_{0L} L_0(t,j)^{\frac{s_0(j)-1}{s_0(j)}} + \alpha_{0E} E_0(t,j)^{\frac{s_0(j)-1}{s_0(j)}} \right]^{\frac{s_0(j)}{s_0(j)-1}} \leq 0, \tag{10}$$

where $Y(t,j)$ is the annual output of the general economic good that can be consumed or invested, α_{0K}, α_{0L} and α_{0E} are the input share parameters, $A_0(j)$ the factor productivity parameter, $s_0(j)$ the elasticity of substitution between inputs and $tg(t,j)$ a disembodied technical progress.

Negative emissions production. Negative emissions $v_i(t,j)$ are produced by three CDR technology i (3=DAC, 4= BECCS, 5=ERW). For DACCS and ERW production is defined by a CES function with three factors (capital, labor and energy), an elasticity of substitution $s_i(j)$ and a disembodied technical progress represented by $tgv_i(t,j)$:

$$v_i(t,j) - A_i(j)tgv_i(t,j)\left[\alpha_{i,K}(j)K_i(t,j)^{\frac{s_i(j)-1}{s_i(j)}} + \alpha_{iL}(j)L_3(t,j)^{\frac{s_i(j)-1}{s_i(j)}} + \alpha_{iE}(j)E_i(t,j)^{\frac{s_i(j)-1}{s_i(j)}}\right]^{\frac{s_i(j)}{s_i(j)-1}} \leq 0, \quad i = 3,5. \tag{11}$$

For BECCS ($i = 4$) the production of negative emissions is represented by a CES function with three inputs, capital (K_4), labor (L_4), and biomass ($Biom$):

$$v_4(t,j) - A_4(j)tgv_4(t,j)\left[\alpha_{4,K}(j)K_i(t,j)^{\frac{s_4(j)-1}{s_4(j)}} + \alpha_{4,L}(j)L_4(t,j)^{\frac{s_4(j)-1}{s_4(j)}} + \alpha_{4,B}(j)Biom(t,j)^{\frac{s_4(j)-1}{s_4(j)}}\right]^{\frac{s_4(j)}{s_4(j)-1}} \leq 0. \tag{12}$$

The useful energy (electricity) produced by BECCS is a function of the efficiency of BECCS plant and caloric content of biomass:

$$E_4(t,j) = eff_{BECCS} \cdot Biom(t,j). \tag{13}$$

Labour use. Labor supply $L(t,j)$ is always greater or equal to the labor demand:

$$L(t,j) \geq L_0(t,j) + \sum_{i=3}^{5} L_i(t,j). \tag{14}$$

Bounds on sequestration. The negative emissions $v_3(t,j)$ by DAC, $v_4(t,j)$ by BECCS, and $v_5(t,j)$ by ERW are limited by the potential for capture and sequestration of each type of technology, in the different regions of the world:

$$v_3(t,j) \leq BCCS_3(j), \tag{15}$$
$$v_4(t,j) \leq BCCS_4(j), \tag{16}$$
$$v_5(t,j) \leq BCCS_5(j). \tag{17}$$

We also introduce a robustification constraint

$$v(t,j) \leq ROBUSTv(j), \tag{18}$$

where $v(t,j) = \sum_{i=3,4,5} v_i(t,j)$ is the total negative emissions of coalition j at time t, and $ROBUSTv(j)$ is the robust total negative emissions computed in the turnpike. This concept of robustification will be discussed later in the paper.

Limiting the speed of penetration of CDR technologies. We project technology adoption using the logistic substitution model as suggested in Edwards et al. (2024). Let $Cap_i(t,j)$ be a cap on negative emissions of type $i = 3,4,5$ that could be deployed at time t.

$$v_i(t,j) \leq Cap_i(t,j). \tag{19}$$

This upper bound is assumed to grow according to the following recurrence

$$Cap_i(t+1,j) = Cap_i(t,j)\left(1 + k_i(j)\left(1 - \frac{Cap_i(t,j)}{\bar{Cap}_i(j)}\right)\right), \tag{20}$$

where $k_(j)$ is the growth rate (taken from a historical analog) and $\bar{Cap}_i(j)$ is the saturation level or market pull which corresponds to the value of this parameter or variable in the turnpike point.

In the simulation presented in forthcoming section, we take an initial value $Cap_i(0,j) = 0.5$ Gt for all i and j and growth rates of 0.2 for DACCS and BECCS, 0.3 for ERW, per 5 year periods.

Useful energy production. The useful energy is a production factor of the general economic good and negative emissions by DACCS and ERW. It is obtained from BECCS activity (E_4) and from a mix of renewable energy and secondary fossil energy. A CES function describes the possibility of substitution between renewable and fossil energy,

$$E_0(t,j) + \sum_{i=3,5} E_i(t,j) - E_4(t,j) - A_e(j)\Big[\alpha_{Ef}(j)enf(t,j)^{\frac{s_e(j)-1}{s_e(j)}} + \alpha_{Er}(j)enr(t,j)^{\frac{s_e(j)-1}{s_e(j)}}\Big]^{\frac{s_e(j)}{s_e(j)-1}} \leq 0, \tag{21}$$

where $enf(t,j)$ is the fossil fuel energy input to deliver useful energy, and $enr(t,j)$ is the renewable energy input to deliver useful energy, for coalition j at period t. In this representation the elasticity $s_e(j)$ plays a central role as it measures the ease with which one can switch between the two forms of secondary energy. As advocated in Acemoglu et al. (2012) an elasticity larger than 1 may be chosen. For example, if $s_e(j) = 2$, the production function takes the form

$$A_e(j)\left[\alpha_{Ef}(j)enf(t,j)^{\frac{1}{2}}+\alpha_{Er}(j)enr(t,j)^{\frac{1}{2}}\right]^2$$
$$= A_e(j)\left[\alpha_{Ef}(j)^2 enf(t,j)+\alpha_{Er}(j)^2 enr(t,j)\right.$$
$$\left.+2alpha_{Ef}(j)\alpha_{Er}(j)\sqrt{enf(t,j)enr(t,j)}\right]. \quad (22)$$

For comparison purpose, we will also run simulations with a low elasticity, $s_e(j) = 0.9$.

Fossil secondary energy conversion Fossil secondary energy conversion is represented by a CES function that combines capital and primary fossil energy

$$enf(t,j) - A_1(j)\left[\alpha_{1K}(j)(tgenf(t,j)K_1(t,j))^{\frac{s_1(j)-1}{s_1(j)}}\right.$$
$$\left.+\alpha_{1em}(j)enp(t,j)^{\frac{s_1(j)-1}{s_1(j)}}\right]^{\frac{s_1(j)}{s_1(j)-1}} \leq 0. \quad (23)$$

where $K_1(t,j)$ is the stock of capital, and $enp(t,j)$ is the fossil energy source used to produce fossil secondary energy, for coalition j at period t.

The parameter $(tgenf(t,j)$ is a productivity loss coefficient for fossil energy capita, which may traduce the depletion of cheap oil and gas deposits.

Renewable secondary energy conversion. Finally, renewable secondary energy is supposed to be obtained from capital only,

$$enr(t,j) - A_2(j)(tgenr(t,j)K_2(t,j))^{s_2(j)} \leq 0. \quad (24)$$

The elasticities $(s.)$ and share parameters $(\alpha.)$, obtained from calibration are shown in Table A.7. The parameters $tg(t,j)$, $tgv(t,j)$, $tgenf(t,j)$, $tgenr(t,j)$ are exogenously defined productivity growth factors.

Emissions from primary fossil energy (for coalition j at period t)

$$em(t,j) = CO2coal \cdot coal(t,j) + CO2oil \cdot oil(t,j) + CO2gas \cdot gas(t,j), \quad (25)$$

where $CO2coal, CO2oil, CO2gas$ are the respective emission factors for the three fossil primary energy sources.

2.1.4. Carbon market equilibrium

One assumes that at each period the three coalitions compete on an international carbon market. The strategic variable, for coalition j at time t is the supply $\omega(t,j)$ of emissions rights on the international market. At market equilibrium, the firms, in each coalition, will set their emission at a level where carbon price equals the marginal productivity of emissions (or marginal abatement cost). The market equilibrium is determined by the following two sets of conditions:

Total supply of permits is greater or equal to total emissions (at period t)

$$\sum_j \omega(t,j) - \sum_j em(t,j) \geq 0. \tag{26}$$

Efficiency (at period t)

$$\begin{aligned} p(t) &= \frac{\partial Y(t,j)}{\partial em(t,j)} && (27)\\ &= \frac{\partial Y(t,j)}{\partial E_0(t,j)} \frac{\partial E_0(t,j)}{\partial enp_1(t,j)} \frac{\partial enp_1(t,j)}{\partial em(t,j)}. && (28) \end{aligned}$$

The expression of the derivatives are given below.

$$\begin{aligned} \frac{\partial Y(t,j)}{\partial E_0(t,j)} = A_0(t,j)tg(t.j) \Bigg[& \alpha_{0K} K_0(t,j)^{\frac{s_0(j)-1}{s_0(j)}} + \alpha_{0L} L_0(t,j)^{\frac{s_0(j)-1}{s_0(j)}} \\ & + \alpha_{0E} E_0(t,j)^{\frac{s_0(j)-1}{s_0(j)}} \Bigg]^{\frac{s_0(j)}{s_0(j)-1}-1} \alpha_{0E} E_0(t,j)^{\frac{s_0(j)-1}{s_0(j)}-1}, \end{aligned} \tag{29}$$

$$
\begin{aligned}
\frac{\partial E_0(t,j)}{\partial enp_1(t,j)} = \frac{\partial E_0(t,j)}{\partial enf(t,j)} \times \frac{\partial enf(t,j)}{\partial enp_1(t,j)} = \\
\frac{A_e(j)}{Coeff(j)} \left[\alpha_{Ef}(j) enf(t,j)^{\frac{s_e(j)-1}{s_e(j)}} + \alpha_{Er}(j) enr(t,j)^{\frac{s_e(j)-1}{s_e(j)}}\right]^{\frac{s_e(j)}{s_e(j)-1}-1} \\
\alpha_{Ef}(j) enf(t,j)^{\frac{s_e(j)-1}{s_e(j)}-1} \\
\times \\
A_2 \left[\alpha_{1K}(j)(enf(t,j)K_1(t,j))^{\frac{s_1(j)-1}{s_1(j)}} + \alpha_{1em}(j)(j) enp(t,j)^{\frac{s_1(j)-1}{s_1(j)}}\right]^{\frac{s_1(j)}{s_1(j)-1}-1} \\
\alpha_{1em}(j)(j) enp(t,j)^{\frac{s_1(j)-1}{s_1(j)}-1}
\end{aligned}
\tag{30}
$$

$$
\begin{aligned}
\frac{\partial enp_(t,j)}{\partial em(t,j)} = Aef(j)(alphacoal(j) \cdot coal(t,j)^{((sef(j)-1)/sef(j))} + \\
alphaoil(j) \cdot oil(t,j)^{((sef(j)-1)/sef(j))} + \\
alphagas(j) \cdot gas(t,j)^{((sef(j)-1)/sef(j))})^{(sef(j)/(sef(j)-1)-1)} \\
(alphacoal(j) \cdot coal(t,j)^{((sef(j)-1)/sef(j)-1)}/CoEcoal(j) \\
+ alphaoil(j) \cdot oil(t,j)^{((sef(j)-1)/sef(j)-1)}/CoEoil(j) \\
+ alphagas(j) \cdot gas(t,j)^{((sef(j)-1)/sef(j)-1)}/CoEgas(j))
\end{aligned}
\tag{31}
$$

2.1.5. Climate module.

To evaluate the impact of an emission schedule on surface average temperature (SAT) change we have borrowed the climate module of DICE-2013R Nordhaus (2014); Nordhaus and Sztorc (2013). The damage function of DICE-2013R is also used to run the model in a pure cost-benefit analysis format. The use of a cumulative safety emissions budget enables us to run the model in a cost-effective way that better reflects the objectives of the Paris Agreement, in addition to the introduction of an economic damage function.

2.2. Robust asymptotic steady-state (Turnpike)

We consider now the asymptotic steady-state or "turnpike", reached in the long term, when all parameters have attained their asymptotic values. The environmental constraint, in the asymptotic scenario is defined by the condition that net emissions must be kept equal to 0. The turnpike is an attractor for the optimal state trajectory. It can be obtained by solving an "implicit optimization problem", which involves a fixed point calculation. The optimization problem is formulated using steady-state variables $\bar{Y}$, $\bar{C}$, etc. For the sake of clarity, the mathematical formulation is given in Appendix Appendix C. The optimization is called implicit, because in this problem, the constraints (C.6) involve the parameters $TNPK_i(j)$, $i = 0, \ldots, 5.$, which should be equal to the solution values in the optimization. Therefore, to solve the problem one has to find a fixed point in a procedure that will update the $TNPK_i(j)$ parameters after each optimization. We refer to Carlson et al. (1991); Feinstein and Luenberger (1981) for a detailed description of this approach.

Robustification of sequestration potential. Now let us introduce a robustication of the capacity constraints on individual sequestration potentials (C.22). The actual sequestration potentials $B\tilde{C}CS_i(j)$ are indeed highly uncertain. Taking inspiration from (Babonneau et al., 2012), we model randomness in a broad sense, meaning that we do not consider the risk on each potential separately but rather for the entire potential of sequestration. For each coalition a worst case is considered globally for the three capture technologies together. For that purpose we introduce the new constraints (32) that concern the total sequestration potential in each coalition

$$\sum_{i=3,4,5} \bar{v}_i(j) \leq \sum_{i=3,4,5} B\tilde{C}CS_i(j), \quad j = OECD, BRIC, ROW. \tag{32}$$

Let assume that each individual sequestration potential $B\tilde{C}CS_i(j)$ has a nominal value $BCCS_i(j)$ and a variability $B\hat{C}CS_i(j)$ such that

$$B\tilde{C}CS_i(j) = BCCS_i(j) + \xi_i^j B\hat{C}CS_i(j), \quad i = 3, 4, 5 \tag{33}$$

in which ξ_i^j is a set of independent random variables with support $[-1, 1]$. We may now rewrite the constraints (32) as follows:

$$\sum_{i=3,4,5} \bar{v}_i(j) \leq \sum_{i=3,4,5} BC\tilde{C}S_i(j) + \sum_{i=3,4,5} \xi_i^j B\hat{C}CS_i(j). \tag{34}$$

The first summation of the right hand side of constraint (34) is a linear deterministic expression and the second term is a random variable.

We introduce robustness in constraint (34) applying robust optimization as proposed in (Ben-Tal et al., 2009) and explained in (Babonneau et al., 2010). For each coalition j an uncertainty set is defined as follows

$$\Xi_j = \{\xi^j_{\cdot} \mid \sum_{i=3,4,5} \mid \xi^j_i \mid^2 \leq k^2\} \tag{35}$$

where k is a user-defined parameter. Using robust optimization techniques, the worst case of the CDR potential constraint is given by

$$\sum_{i=3,4,5} \bar{v}_i(j) \leq \sum_{i=3,4,5} BCCS_i(j) - k\sqrt{\sum_{i=3,4,5} (B\hat{C}CS_i(j))^2}. \tag{36}$$

The second component in the RHS above corresponds to a safety factor ensuring that (34) is satisfied for all realizations of ξ within the uncertainty set Ξ. According to Ben-Tal et al. (2009), one can derive a probability of satisfying the constraint for any realizations of ξ that depends on the radius k of the uncertainty set. The probabilistic interpretation depends on the support of the random variables, their expectation and their assumed independence. No other assumption concerning the form of the probability distributions is made. In this paper, we set $k = 2$ that can be viewed as a coefficient defining a confidence interval evaluated in number of standard deviation units.

2.3. Model calibration

The CES functions are calibrated from GTAP 10 database for the reference year 2014 (Aguiar et al., 2019). All economic variables are expressed in US$_{2020}$ using market exchange rates. Energy consumptions from fossil or renewable sources are expressed in physical terms (Exa-joule, EJ); for calibration purpose they are obtained from the energy balances published online by the International Energy Agency (IEA)[4]. The CO_2 emissions from fuel combustion are also obtained from the IEA (International Energy Agency, 2020).

Population levels from 2014 to 2100, expressed in million of people in Table 2, are based on the World Population Prospects 2019 done by the United Nations (United Nations, 2019). We use the medium variant scenario. For

[4]https://www.iea.org/reports/world-energy-balances-overview

the whole world, it varies from 7.295 billion in 2014 up to 10.875 billion people in 2100. After 2100 we assume a steady state for population in different regions. The pure time preference rate is set at 3%.

Table 2: Population levels in million of inhabitants

	2014	2020	2030	2040	2050	2060	2070	2080	2090	2100
BRIC	3042	3251	3375	3417	3387	3303	3182	3043	2904	2822
OECD	1273	1331	1365	1385	1388	1382	1372	1363	1354	1350
ROW	2980	3527	4081	4626	5141	5602	6001	6331	6587	6703
World	7295	8109	8821	9428	9916	10287	10555	10737	10845	10875

The cost estimates for a DACCS is based on a system using the sodium/calcium hydroxide option (see Table 2.5 in (Socolow et al., 2011)). We retain a total cost of $430 per ton of CO_2 captured. BECCS is calibrated using (Selosse and Ricci, 2014; Fajardy et al., 2018; Emenike et al., 2021; Bakkaloglu et al., 2023; Fajardy et al., 2021). We retain a total cost of $250 per ton of CO_2 captured. ERW is calibrated using (Kantzas et al., 2022). We retain a total cost of $180 per ton of CO_2 captured. Table 3 shows the decomposition of CDR costs into factors costs.

Table 3: Decomposition of CDR costs (in $/t$CO_2$)

	Total Cost	Capital Cost	Energy Cost	Other cost (i.e. L)
DACCS	430	327	81	22
BECCS	250	150	50	90
ERW	180	50	60	70

The values of the CES function parameters are given in Appendix Appendix A.

3. Numerical simulations

We present now numerical simulations performed with this model. We show first the computation of Turnpikes (Section 3.1), that are the asymptotic steady states for the three regions. Then we show the optimal transition paths toward the Turnpikes (Section 3.2).

We have defined four contrasted scenarios to assess the potential impact of the safety emissions budget, the elasticity of substitution between fossil and renewable energies and the carbon sequestration potential on the international climate policy and economic performance for the different groups of countries. Their settings are given in Table 4. The first two scenarios, called

Bud_{1070}-CCS_9-$Elas_2$ and Bud_{1070}-CCS_9-$Elas_{0.9}$, respectively, impose an emissions budget compatible with a 2°C warming limit (i.e., 1'070 Gt of CO_2) and a sequestration potential limited to 9 Gt CO_2 per year (i.e., corresponding to 1 Gt per CDR type and per coalition). In the first scenario, we assume a high elasticity of substitution of 2 while in the second one it is limited to 0.9. The second case mainly reflects the assumption used by CGE models applied to climate change, in which the elasticity between fossil fuels and electricity is less than 1 (Antimiani et al., 2015; Chen et al., 2016), while the first case follows recent econometric work which tends to show that this elasticity greatly exceeds unity (Papageorgiou et al., 2017). In the third scenario, named Bud_{2000}-CCS_9-$Elas_2$, we relax the safety emissions budget to 2'000 Gt of CO_2 to verify the effect of high elasticity of substitution without strong environmental constraints. Finally, the last simulation (Bud_{600}-CCS_{18}-$Elas_2$) corresponds to the most ambitious scenario in terms of temperature rise (1.7°C, corresponding to a budget of 600 Gt of CO_2), with an elasticity of substitution $s_e = 2$, and sequestration potential of 18 Gt of CO_2 per year (i.e., corresponding to 2 Gt per CDR type and per coalition). In Section 3.3, we analyze in more details the 600Gt emissions scenario under different settings of carbon capture and elasticity of substitution.

To represent the uncertainty in CO_2 storage potential, we use a confidence level of 2 σ, where the standard deviation σ_j is defined by a coefficients of variation (ratio standard deviation over mean) of 10% of each CDR type and coalition j.

3.1. Asymptotic Steady States

We look first at the turnpike values, which define an attractor for the economic growth trajectory. The turnpike provides an abstract representation of sustainable development. The global steady state results are reported in Table 4.

First, we observe that the 1'070 Gt budget maintains the temperature rise below 2°C as expected and the associated damages to less than 3.8% of GDP. The highest and lowest budgets lead to a temperature increase of 2.5°and 1.7°C, respectively, with associated damages of 5% and 3% of GDP. The first conclusion of our numerical experiments is that, under our model assumptions, the Paris objective of 1.5°C is unreachable even with a high elasticity of substitution s_e from fossil fuels to renewable energy and under a favorable CDR scenario. Considering uncertainty on carbon storage capacity for CDR, the robust solutions use only around 8.47 Gt and 15.92 Gt out of the 9 Gt and 18 Gt defining the global potentials.

The most important factor to achieve a climate target of 2°C is the elasticity of substitution s_e between fossil and renewable secondary energy forms when it takes a value higher than 1; this would reflect the facility to move from an economy based on fossil fuels to one based on renewable energies. In scenario 2, a low elasticity of substitution, $s_e = 0.9$ translates into a very high CO_2 price of \$1'847 (in comparison with the \$447 CO_2 price of scenarios 1 and 3). Our analyses also show that access to carbon capture and sequestration can help to smooth the transition and mitigate its economic effects. In particular, in Scenario 4 the CO_2 price is reduced to \$274 even with a more stringent objective.

Table 4: Scenario settings and global Steady State results

Scenario	1	2	3	4
	Scenario settings			
CO_2 safety budget (Gt)	1'070	1'070	2'000	600
Elasticity of substitution (se)	2	0.9	2	2
Sequestration potential (Gt/Year)	9	9	9	18
	Global Steady State results			
Temperature increase (°C)	2	2	2.5	1.7
CO_2 price (\$)	447	1'823	447	274
Damage loss factor (%)	3.8	3.8	5	3
CO_2 sequestrated (Gt/Year)	8.47	8.47	8.48	15.92

Scenario 1- Bud_{1070}-CCS_9-$Elas_2$, Scenario 2- Bud_{1070}-CCS_9-$Elas_{0.9}$

Scenario 3- Bud_{2000}-CCS_9-$Elas_2$, Scenario 4- Bud_{600}-CCS_{18}-$Elas_2$

The detailed results on emissions, capital stocks, total consumption and per capita consumption, shown in the Table 5, confirm these observations. The low elasticity of substitution between fossil fuels and renewable energies in scenario 2 has a significant impact on consumption, particularly for the BRIC and ROW countries, with a drop of around 40% in per capita consumption compared with the other scenarios. This shows how important it is for countries to implement policies that foster the development of renewable energies. There is also a slight increase in consumption in scenario 4, where the climate objective is the strongest, which, is due in particular to the reduction in damage caused by climate change. With regard to emissions, we observe that although negative emissions are uniformly distributed between the three coalitions, emissions remain at significant levels for the BRIC and the ROW countries, which seems logical in terms of financial compensation.

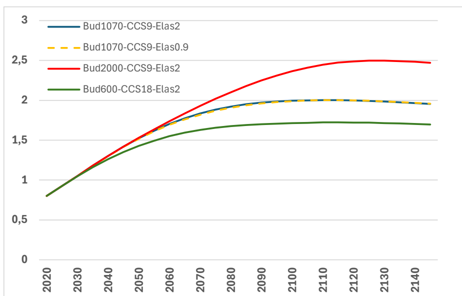
Table 5: Detailed robust Steady State results

	OECD	BRIC	ROW	OECD	BRIC	ROW
	1 - $\mathbf{Bud}_{1070}$-$\mathbf{CCS}_{9}$-$\mathbf{Elas}_{2}$			2 - $\mathbf{Bud}_{1070}$-$\mathbf{CCS}_{9}$-$\mathbf{Elas}_{0.9}$		
Emissions (Gt)	0.44	1.56	6.47	0.72	2.08	5.67
Negative emi. (Gt)	2.82	2.82	2.82	2.82	2.82	2.82
K0 (10^9\$)	386'287	655'549	745'671	272'946	353'706	406'076
K1 (10^9\$)	991	2'476	15'015	1'846	4'669	16'441
K2 (10^9\$)	101'691	224'019	423'908	23'670	40'647	68'618
K3 (10^9\$)	1'737	1'737	1'348	1'643	1'643	1'226
K4 (10^9\$)	700	766	607	698	758	600
K5 (10^9\$)	256	297	236	328	375	284
Consumption (10^9\$)	67'250	84'419	138'230	50'503	48'873	80'846
Per cap. Cons. (10^3\$)	49.8	29.9	20.6	37.4	17.3	12.0
	3 - $\mathbf{Bud}_{2000}$-$\mathbf{CCS}_{9}$-$\mathbf{Elas}_{2}$			4 - $\mathbf{Bud}_{600}$-$\mathbf{CCS}_{18}$-$\mathbf{Elas}_{2}$		
Emissions (Gt)	0.45	1.57	6.46	0.98	3.38	11.56
Negative emi. (Gt)	2.83	2.83	2.83	5.31	5.31	5.31
K0 (10^9\$)	378'913	639'748	729'783	400'086	688'487	789'124
K1 (10^9\$)	992	2'484	14'951	2'078	4'961	24'915
K2 (10^9\$)	99'617	218'208	413'834	99'685	223'149	416'493
K3 (10^9\$)	1'734	1'734	1'346	3'478	3'478	2'698
K4 (10^9\$)	700	766	607	1'401	1'535	1'216
K5 (10^9\$)	255	295	235	514	475	378
Consumption (10^9\$)	65'857	82'234	135'036	69'791	88'879	146'028
Per cap. Cons. (10^3\$)	48.7	29.1	20.1	51.7	31.5	21.8

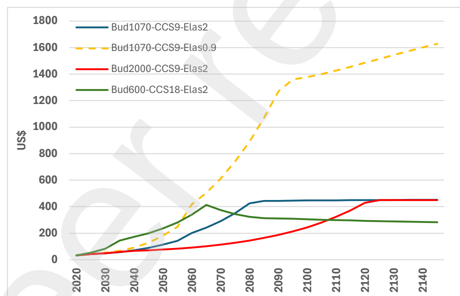
3.2. Optimal transition pathways

In this section, we consider the transition path that has been computed by forcing the economic system to reach the turnpike values by year 2195. In the figures below we shall limit the display to the horizon 2020-2145, which is already quite long. Figure 2 displays the evolution of temperature increase, CO_2 prices and net emissions for the four scenarios. Figures 3, 4 and 5 shows the pathways for negative emissions, emissions and Per capita consumption, respectively. The detailed GDP results are displayed in Appendix Appendix B (See Figure B.7).

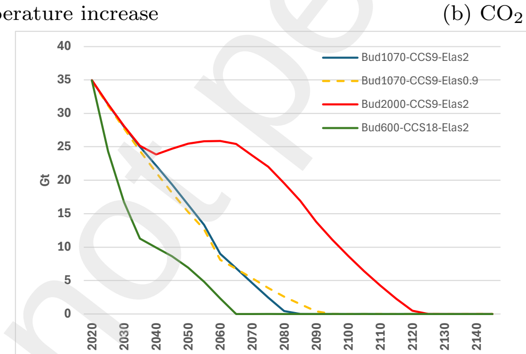
We observe that net-zero emissions regimes are achieved in all cases, but at different times depending on the budget available, i.e., 2065 for scenario 4, 2080 for scenario 1, 2090 for scenario 2 and, 2120 for scenario 3. In scenario 3, the least constrained one, we see a rebound in emissions in 2040, with the highest availability of negative emissions as it can also be seen on Figures 3 and 4. As expected, in all cases the achievement of net-zero emissions is associated with a stabilization of the CO_2 price to its turnpike value. In Scenario 4, CO_2 price tops \$400 to then decreases to its \$274 steady state value. This is due the continuous development of CDR capacities as seen in Figure 3.



(a) Temperature increase



(b) CO_2 price



(c) Net emissions

Figure 2: Temperature increase, CO_2 price and net emissions

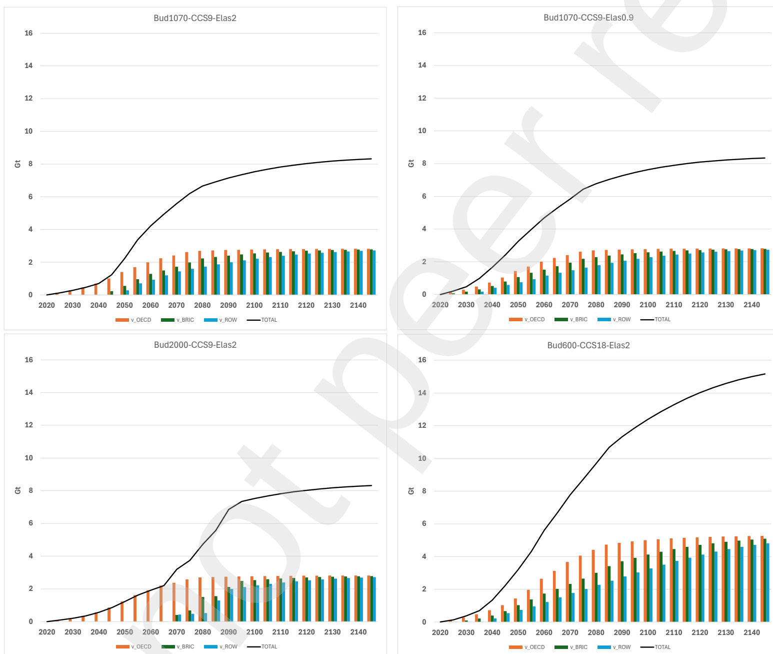


Figure 3: Negative emissions

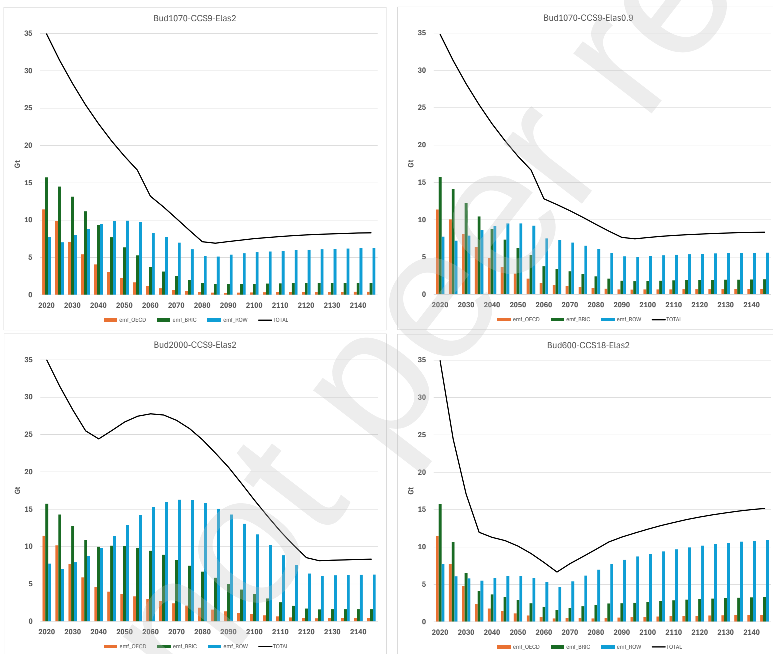


Figure 4: CO_2 emissions

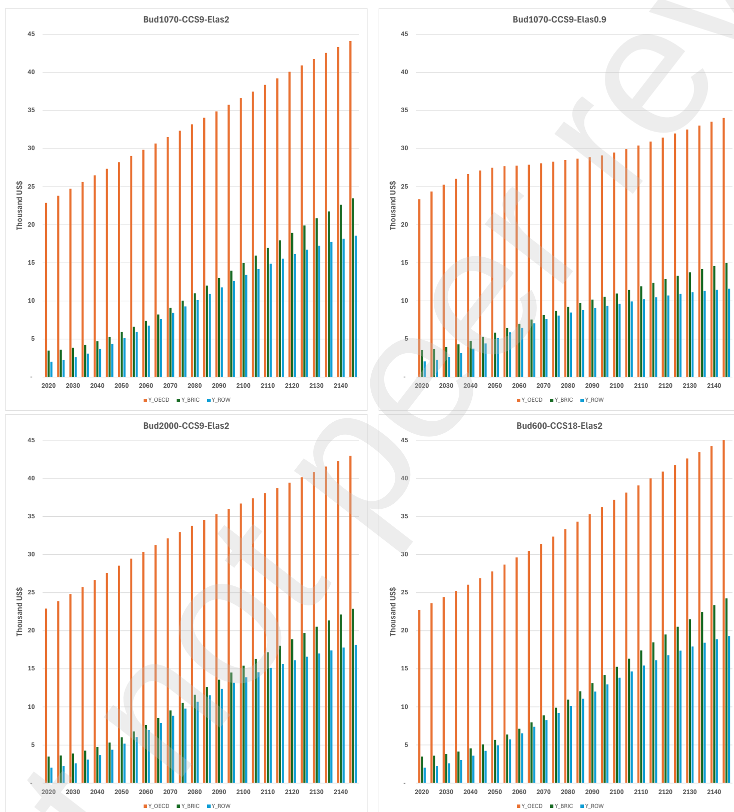


Figure 5: Per capita consumption

3.3. Focus on the 600Gt emissions budget

Our results above have shown that in the scenario with a limited budget of 600 Gt of CO_2 and a storage capacity of 18 Gt of CO_2 per year, the long-term situation is the most favorable. The atmospheric surface temperature remains below 1.7°C. The price of carbon settles at $274 per ton of CO_2. However, before reaching this "bliss point", the energy transition is costly, particularly for developing regions. Table 6 gives the steady state results for this specific 600Gt emissions budget scenario under different settings for carbon capture potentials, i.e., 1 and 18 Gt, and elasticity of substitution, i.e., s_e equal to 0.9 and 2.

From Table 6, we observe, on one hand, that if the budget is small (600 Gt of CO_2) and the storage capacity is also small, i.e., 1 Gt of CO_2 per year, the price of carbon rises to over $2'000 per ton of CO_2. However, the high elasticity of substitution (s_e=2) ensures that economic welfare conditions remain acceptable. On the other hand, if the storage capacity is low (e.g. 1 Gt of CO_2 per year), with a low elasticity of substitution (s_e=0.9), the economic situation will be much worse. In the long term (turnpike point), per capita consumption is very close to current levels, reflecting a lack of potential growth. The price of carbon is reaching astronomical levels ($12'000 per ton of CO_2). The transition to carbon neutrality is marked by widespread economic stagnation, as shown in Figure 6.

Table 6: Detailed robust Steady State results for the 600 Gt budget scenarios

	OECD	BRIC	ROW	OECD	BRIC	ROW
	4 - **Bud$_{600}$-CCS$_{18}$-Elas$_2$**			5 - **Bud$_{600}$-CCS$_{18}$-Elas$_{0.9}$**		
Emissions (Gt)	0.98	3.38	11.56	1.36	4.04	10.52
Negative emi. (Gt)	5.31	5.31	5.31	5.31	5.31	5.31
Per cap. Cons. ($10^3$$)	51.7	31.5	21.8	40.9	20.21	14.24
Price of carbon ($/t$CO_2$)	274			992		
	6 - **Bud$_{600}$-CCS$_1$-Elas$_2$**			7 - **Bud$_{600}$-CCS$_1$-Elas$_{0.9}$**		
Emissions (Gt)	0.03	0.11	0.66	0.07	0.19	0.54
Negative emi. (Gt)	0.27	0.27	0.27	0.27	0.27	0.27
Per cap. Cons. ($10^3$$)	49.8	29.6	19.69	26.51	9.45	5.98
Price of carbon ($/t$CO_2$)	2'028			12'000		

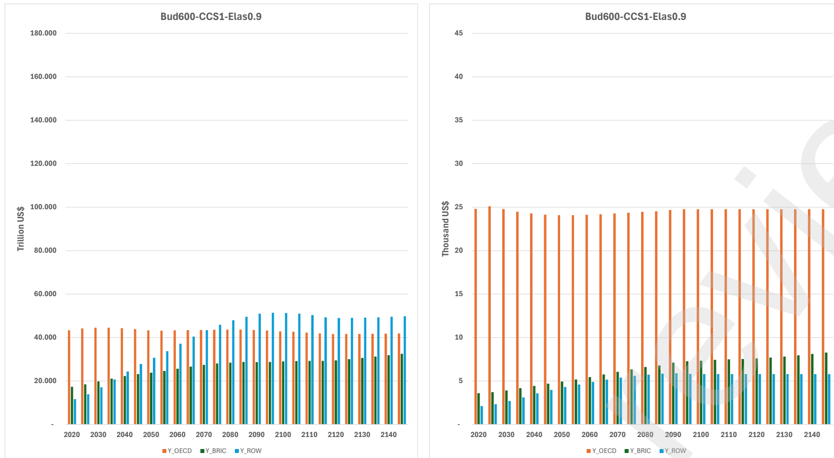


Figure 6: GDP and Per Capita Consumption for Scenario $\mathbf{Bud}_{600}$-$\mathbf{CCS}_1$-$\mathbf{Elas}_{0.9}$

4. Conclusion

The model we have presented is an archetype of optimal control problem used to analyze economic growth. We have exploited the global asymptotic stability of these dynamic systems to characterise long-term sustainable regimes using turnpike attractors. To take account of the uncertainty that exists regarding the possibility of using the full potential of CO_2 capture, we have introduced constraints that allow the calculation of "robust" asymptotic stationary states. While retaining a structure inspired by CGE models, based on the use of CES production functions, this model distinguishes between primary, secondary and useful forms of energy, as in technology-rich bottom-up partial equilibrium models.

To represent climate policies, we used the concept of a cumulative emissions budget and exploited the DICE-2013 climate module. We found that a budget of 1'070 Gt of CO_2 was compatible with a temperature rise of 2°C. A much lower budget of 600 Gt of CO_2 would limit the temperature rise to 1.7°C.

This model explores the support that CDR technologies can provide to achieve the objectives of the Paris Agreement. It shows that these direct capture and sequestration technologies are essential when the elasticity of substitution s_e between fossil and renewable energies in the production of useful energy is low, i.e., with a value less than 1. When the elasticity of substitution is much higher than 1, e.g., $s_e = 2$, CDR technologies still provide support, but this does not translate into a big difference in terms of economic welfare.

For the implementation of climate policies, we have assumed the existence of an international market in emission rights. The long-term price of carbon ($\$_{2020}$ per ton of CO_2) varies between \$274, when $s_e = 2$ and the storage potential is very high (18 Gt per year), and \$12'000 when $s_e = 0.9$ and the storage potential is very low (1 Gt per year). Indeed, the simulations show that a combination of high elasticity of substitution ($se = 2$) and low cumulative CO_2 budget (600 Gt), with high CDR potential (18 Gt/Year) would tend to a much lower asymptotic price of CO_2 (\$274/ton) and the highest levels of per-capita consumption for the three coalitions.

In summary, the simulations carried out using this model suggest that, in order to reach the objectives of Paris-agreement (temperature increase less than 1.7°C) a proactive policy to develop the use of renewable energies would be supported by the massive development of CDR technologies, which would make it possible to maintain a reasonable carbon price and strong growth in purchasing power, particularly for developing countries.

One might question the point of developing a new Ramsey-type model, given the DICE, RICE, MERGE, WITCH and REMIND-R models already available. We believe that by reducing the size of the model as much as possible and building it according to the principles established by the originators of the theory of optimal economic growth, Solow, Uzawa, Arrow, Cass, Shell, etc., we can concentrate the analysis of climate policies on a few fundamental economic parameters, such as the flexibility of fossil fuel use, the speed of penetration of CDR technologies, and the cumulative emissions budget. As the model is small, we can envisage developing a robust analysis, taking into account the uncertainty that may affect these different parameters. This model can also serve as a basis for developing a model of conflicts and cooperation in the establishment of global climate policies. In short, what we are proposing in this model is an abstract tool of computational economics for discussing long-term climate policy issues in the language of optimal economic growth theory.

Appendix A. Parameters in CES functions

Table A.7: Parameters in CES functions

Y	OECD	BRIC	ROW
$A_0(\cdot)$	4.084	1.00	1.394
$\alpha_{0K}(\cdot)$	0.469	0.537	0.397
$\alpha_{0L}(\cdot)$	0.448	0.337	0.413
$\alpha_{0E}(\cdot)$	0.083	0.127	0.190
$s_0(\cdot)$	0.9	0.9	0.9

v_3	OECD	BRIC	ROW
$A_3(\cdot)$	3.6E-4	3.3E-4	0.002
$\alpha_{3K}(\cdot)$	0.997	0.998	0.997
$\alpha_{3L}(\cdot)$	3.3E-6	3.9E-5	6.91E-5
$\alpha_{3E}(\cdot)$	0.003	0.002	0.003
$s_3(\cdot)$	0.45	0.45	0.45

$E_0 + E_3$	OECD	BRIC	ROW
$A_e(\cdot)$	1.956	1.9959	1.905
$\alpha_{Ef}(\cdot)$	0.575	0.573	0.612
$\alpha_{Er}(\cdot)$	0.425	0.427	0.388
$s_e(\cdot)$	2.0	2.0	2.0

v_4	OECD	BRIC	ROW
$A_4(\cdot)$	0.001	0.001	0.001
$\alpha_{4K}(\cdot)$	0.995	0.994	0.991
$\alpha_{4L}(\cdot)$	1.17E-4	0.001	0.002
$\alpha_{4E}(\cdot)$	0.005	0.004	0.006
$s_3(\cdot)$	0.45	0.45	0.45

$enf0$	OECD	BRIC	ROW
$A_1(\cdot)$	1.010	0.967	1.086
$\alpha_{1K}(\cdot)$	0.483	0.566	0.307
$\alpha_{1enp}(\cdot)$	0.517	0.434	0.693
$s_1(\cdot)$	0.2	0.2	0.2

v_5	OECD	BRIC	ROW
$A_5(\cdot)$	0.005	0.005	0.006
$\alpha_{5K}(\cdot)$	0.920	0.901	0.859
$\alpha_{5L}(\cdot)$	0.003	0.03	0.051
$\alpha_{5E}(\cdot)$	0.078	0.069	0.091
$s_3(\cdot)$	0.45	0.45	0.45

enp	OECD	BRIC	ROW
$A_{ef}(\cdot)$	1.913	2.418	1.884
$\alpha_{coal}(\cdot)$	0.036	0.263	0.032
$\alpha_{oil}(\cdot)$	0.786	0.640	0.791
$\alpha_{gas}(\cdot)$	0.178	0.097	0.176
$s_{ef}(\cdot)$	0.9	0.9	0.9

enr	OECD	BRIC	ROW
$A_2(\cdot)$	0.095	0.087	0.114
$s_2(\cdot)$	1	1	1

Table A.8: Asymptotic values of time-varying parameters

Parameters	OECD	BRIC	ROW	Unit
L_{SS}	1'350	2'822	6'703	million
tg	1.442	1.731	2.077	-
$tg3$	4.292	4.292	4.292	-
$tg4$	4.292	4.292	4.292	-
$tg5$	4.292	4.292	4.292	-
$tgenr$	1.854	1.854	1.854	-
$tgenf$	0.685	0.685	0.685	-

Appendix B. Evolution of coalition GDP

Figure Appendix B displays the detailed evolution of coalition GDP for the different scenario.

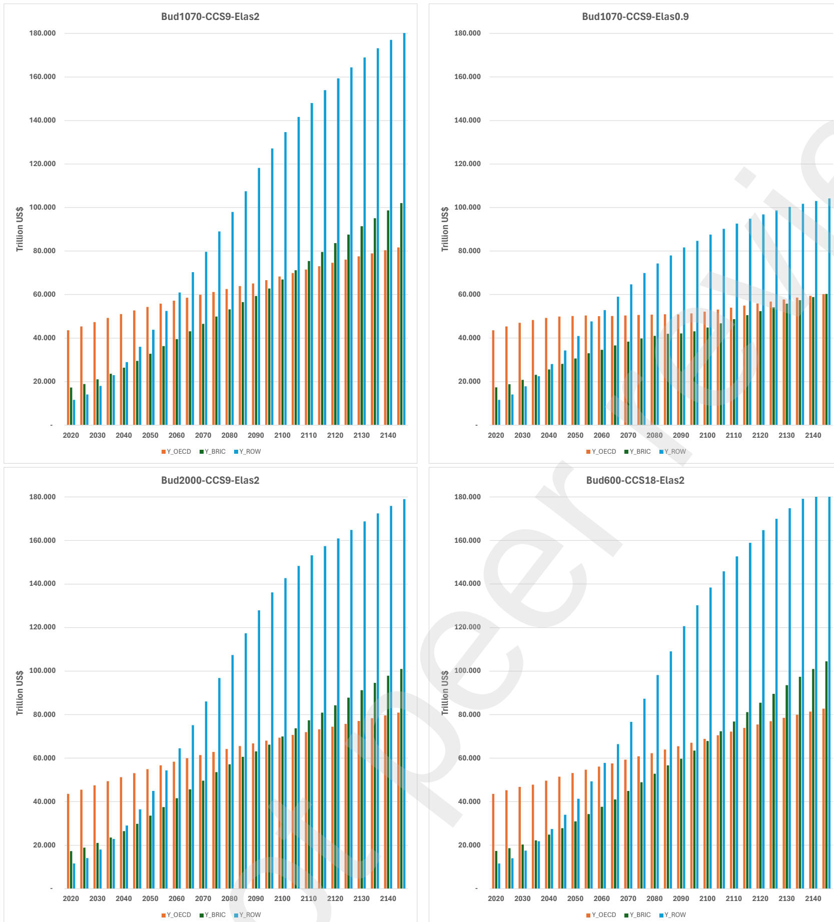


Figure B.7: Coalition GDP

Appendix C. The equations to compute a turnpike steady state

Steady-State Criterion.

$$\bar{Y}(j) = A_0(j)\bar{tg}(j)\left(\alpha_{0K}(j)\bar{K}_0(j)^{\frac{s0(j)-1)}{s0(j)}} + \alpha_{0E}(j)\bar{E}_0(j)^{\frac{s0(j)-1)}{s0(j)}}\right.$$
$$\left. +\alpha_{0L}(j)\bar{L}_(j)^{\frac{s0(j)-1)}{s0(j)}}\right)^{\frac{s0(j)}{s0(j)-1}} \quad \text{(C.1)}$$

$$\bar{C}(j) = \bar{Y}(j)(1 - d\bar{a}m) - \big(\bar{I}_0(j) + \bar{I}_1(j) + \bar{I}_2(j) + \bar{I}_3(j) + \bar{I}_4(j) + \bar{I}_5(j) \\ + Pcoal(j)co\bar{a}l(j) + Poil(j)o\bar{i}l(j) + Pgas(j)g\bar{a}s(j)\big) \quad (C.2)$$

$$C\bar{P}C(j) = \bar{C}(j)/\bar{L}(j) \quad (C.3)$$

$$\bar{U}(j) = \bar{L}(j)\log(\bar{C}(j)/\bar{L}(j)) \quad (C.4)$$

$$\max T\bar{O}TU = \sum_j \bar{U}(j) \quad (C.5)$$

Steady-State capital equations.

$$0 = \bar{I}_i(j) - cmu\bar{K}_i(j) - rSS(\bar{K}_i(j) - TNPK_i(j)), \quad i = 0, \ldots, 5. \quad (C.6)$$

Steady-State emissions.

$$e\bar{m}f(j) = (CO2_{coal}\,Cexa_{coal}(j)co\bar{a}l(j) + CO2oil\,Cexa_{oil}(j)o\bar{i}l(j) \\ + CO2_{gas}\,Cexa_{gas}(j)g\bar{a}s(j))/1000 \quad (C.7)$$

Net emissions.

$$\sum_j (e\bar{m}f(j) - \bar{v}(j)) + et\bar{r}ee = N\bar{E} = 0. \quad (C.8)$$

Energy use.

$$\bar{E}_0(j) + \bar{E}_3(j) + \bar{E}_4(j) + \bar{E}_5(j) \\ = Ae(j)\,(\alpha_{Ef}(j)\,e\bar{n}f_0(j)^{\frac{se(j)-1}{se(j)}} + \alpha_{Er}(j)\,e\bar{n}r_0(j)^{\frac{se(j)-1}{se(j)}})^{\frac{se(j)}{se(j)-1)}} \quad (C.9)$$

Production of fossil energy.

$$e\bar{n}f_0(j) = A1(j)\,(\alpha_{1K}(j) * (tg\bar{e}nf(j)\,\bar{K}_1(j))^{\frac{s1(j)-1}{s1(j)}} \\ + \alpha_{1em}(j)\,e\bar{n}f_1(j)^{\frac{s1(j)-1}{s1(j)}})^{\frac{s1(j)}{s1(j)-1}} \quad (C.10)$$

Extraction of primary fossil energy.

$$e\bar{n}f_1(j) = Aef(j)\,(\alpha_{coal}(j)\,co\bar{a}l(j)^{\frac{sef(j)-1}{sef(j)}} + \alpha_{oil}(j)\,o\bar{i}l(j)^{\frac{sef(j)-1}{sef(j)}} \\ + \alpha_{gas}(j)\,g\bar{a}s(j)^{\frac{sef(j)-1}{sef(j)}})^{\frac{sef(j)}{sef(j)-1}} \quad (C.11)$$

Production of renewable energy.

$$e\bar{n}r_0(j) = A_2(j)\, tg\bar{e}nr(j)\, \bar{K}_2(j)^{s_2}(j) \tag{C.12}$$

CO_2 market equilibrium (efficiency price = marginal cost of emissions).

$$\bar{p}(j) = D\bar{e}r_0(j)\, D\bar{e}r_1(j)\, D\bar{e}r_2(j)\, D\bar{e}r_3(j) \tag{C.13}$$

$$\begin{aligned} D\bar{e}r_0(j) = e = A_0(j)\, t\bar{g}(j)\, (\alpha_{0K}(j)\bar{K}_0(j)^{\frac{s0(j)-1)}{s0(j)}} + \alpha_{0E}(j)\, \bar{E}_0(j)^{\frac{s0(j)-1)}{s0(j)}} \\ + \alpha_{0L}(j)\, (t\bar{g}(j)\, \bar{L}0(j))^{\frac{s0(j)-1)}{s0(j)}})^{\frac{s0(j))}{s0(j)-1}-1}\, \alpha_{0E}(j)\, \bar{E}_0(j)^{\frac{s0(j)-1)}{s0(j)}-1} \end{aligned} \tag{C.14}$$

$$\begin{aligned} D\bar{e}r_1(j) = Ae(j)\, (\alpha_{Ef}(j)\, e\bar{n}f_0(j)^{\frac{se(j)-1)}{se(j)}} \\ + \alpha_{Er}(j)\, e\bar{n}r_0(j)^{\frac{se(j)-1)}{se(j)}})^{\frac{se(j))}{se(j)-1}-1}\, \alpha_{Ef}(j)\, e\bar{n}f_0(j)^{\frac{se(j)-1)}{se(j)}-1} \end{aligned} \tag{C.15}$$

$$\begin{aligned} D\bar{e}r_2(j) = A1(j)\, (\alpha_{1K}(j)\, (tg\bar{e}nf(j)\, \bar{K}_1(j))^{\frac{s1(j)-1}{s1(j)}} \\ + \alpha_{1em}(j)\, e\bar{n}f_1(j)^{\frac{s1(j)-1}{s1(j)}})^{\frac{s1(j)}{s1(j)-1}-1} + \alpha_{1em}(j)\, e\bar{n}f_1(j)^{\frac{s1(j)-1}{s1(j)}-1} \end{aligned} \tag{C.16}$$

$$\begin{aligned} D\bar{e}r_3(j) = Aef(j)\, (\alpha_{coal}(j)\, co\bar{a}l(j)^{\frac{sef(j)-1}{sef(j)}} + \alpha_{oil}(j)\, o\bar{i}l(j)^{\frac{sef(j)-1}{sef(j)}} \\ + \alpha_{gas}(j)\, g\bar{a}s(j)^{\frac{sef(j)-1}{sef(j)}})^{\frac{sef(j)}{sef(j)-1}-1}(\alpha_{coal}(j)\, co\bar{a}l(j)^{\frac{sef(j)-1}{sef(j)}-1}\, \frac{Cexa_{coal}(j)}{CoE_{coal}(j)} \\ + \alpha_{oil}(j)\, o\bar{i}l(j)^{\frac{sef(j)-1}{sef(j)}-1}\, \frac{Cexa_{oil}(j)}{CoE_{oil}(j)} \\ + \alpha_{gas}(j)\, g\bar{a}s(j)^{\frac{sef(j)-1}{sef(j)}-1}\, \frac{Cexa_{gas}(j)}{CoE_{gas}(j)} \end{aligned} \tag{C.17}$$

CO_2 market price. Equalizes marginal productivity of emissions

$$p\bar{m} = \bar{p}(j). \tag{C.18}$$

Trading of emission permits.

$$\bar{emtrade}(j) = (\bar{v}(j) - \bar{emf}(j))\,\bar{p}(j). \tag{C.19}$$

Negative emissions.

$$\bar{v}(j) = \bar{v}_3(j) + \bar{v}_4(j) + \bar{v}_5(j) \tag{C.20}$$

$$\begin{aligned}\bar{v}_i(j) \leq A_i(j)\,\bar{tgi}(j)\,(\alpha_{iK}(j)\,\bar{K}_i(j)^{\frac{si(j)-1}{si(j)}} + \alpha_{iL}(j)\,\bar{L}_i(j)^{\frac{si(j)-1}{si(j)}} \\ + \alpha_{iE}(j)\,\bar{E}_i(j)^{\frac{si(j)-1}{si(j)}})^{\frac{si(j)}{si(j)-1}}, \quad i = 3,4,5.\end{aligned} \tag{C.21}$$

$$\bar{v}_i(j) \leq BCCS_i(j), \quad i = 3,4,5. \tag{C.22}$$

References

Acemoglu, D., Aghion, P., Bursztyn, L., Hémous, D., 2012. The environment and directed technical change. American Economic Review 102, 131–166.

Aguiar, A., Chepeliev, M., Corong, E., McDougall, R., van der Mensbrugghe, D., 2019. The GTAP data base: Version 10. Journal of Global Economic Analysis 4, 1–27.

Allen, M., 2015. Climate 2020. Witan Media, London. chapter The scientific case for a cumulative carbon budget. pp. 118–120.

Allen, M., Frame, D., Huntingford, C., Jones, C., Lowe, J., Meinshausen, M., N., M., 2009. Warming caused by cumulative carbon emissions towards the trillionth tonne. Nature 458, 1163–1166.

Antimiani, A., Costantini, V., Paglialunga, E., 2015. The sensitivity of climate-economy CGE models to energy-related elasticity parameters: Implications for climate policy design. Economic Modelling 51, 38–52.

Arrow, K., Kurz, M., 1970a. Public Investment, the Rate of Return and Optimal Fiscal Policy. Resources for the Future. Johns Hopkins University.

Arrow, K.J., Kurz, M., 1970b. Optimal growth with irreversible investment in a Ramsey model. Econometrica 38, 331–344. URL: `http://www.jstor.org/stable/1913014`.

Babonneau, F., Badran, A., Benlahrech, M., Haurie, A., Schenckery, M., Vielle, M., 2021a. Economic assessment of the development of CO_2 direct reduction technologies in long-term climate strategies of the Gulf Countries. Climatic Change 165, 1–18.

Babonneau, F., Badran, A., Haurie, A., Schenckery, M., Vielle, M., 2023. GCC countries strategic options in a global transition to zero-net emissions. Environmental Modelling and Assessment 28, 709–733.

Babonneau, F., Bahn, O., Haurie, A., Vielle, M., 2021b. An oligopoly game of CDR strategy deployment in a steady-state net-zero emission climate regime. Environmental Modelling and Assessment 26, 969–984.

Babonneau, F., Kanudia, A., Labriet, M., Loulou, R., Vial, J.P., 2012. Energy security: A robust optimization approach to design a robust european energy supply via TIAM-WORLD. Environmental Modeling & Assessment 17, 19–37. URL: https://doi.org/10.1007/s10666-011-9273-3, doi:10.1007/s10666-011-9273-3.

Babonneau, F., Vial, J.P., Apparigliato, R., 2010. Robust optimization for environmental and energy planning, in: Filar, J.A., Haurie, A. (Eds.), Uncertainty and Environmental Decision Making: A Handbook of Research and Best Practice. Springer US, Boston, MA. chapter 3, pp. 79–126. URL: https://doi.org/10.1007/978-1-4419-1129-2_3, doi:10.1007/978-1-4419-1129-2_3.

Bahn, O., Haurie, A., 2008. A class of games with coupled constraints to model international ghg emission agreements. International Game Theory Review Vol. 10, 337–362.

Bahn, O., Haurie, A., 2016. A cost-effectiveness differential game model for climate agreements. Dynamic Games and Applications 6, 1–19.

Bahn, O., Haurie, A., Malhamé, R., 2008. A stochastic control model for optimal timing of climate policies. Automatica 44, 1545–1558.

Bakkaloglu, S., Mersch, M., Sunny, N., Markides, C., Shah, N., Hawkes, A., 2023. How far should the UK go with negative emission technologies?, in: Proceedings of ECOS 2023 - the 36th international conference on efficiency, cost, optimization, simulation and environmental impact of energy systems 25-30 june, 2023, las Palmas de Gran Canaria, Spain.

Bauer, N., Mouratiadou, I., Luderer, G., Baumstrak, L., Brecha, R., Edenhofer, O., Kriegler, E., 2016. Global fossil energy markets and climate change mitigation-an analysis with REMIND. Climatic Change 136, 59–82.

Baumstark, L., Bauer, N., Benke, F., Bertram, C., Bi, S., Gong, C.C., Dietrich, J.P., Dirnaichner, A., Giannousakis, A., Hilaire, J., Klein, D., Koch, J., Leimbach, M., Levesque, A., Madeddu, S., Malik, A., Merfort, A., Merfort, L., Odenweller, A., Pehl, M., Pietzcker, R.C., Piontek, F., Rauner, S., Rodrigues, R., Rottoli, M., Schreyer, F., Schultes, A., Soergel, B., Soergel, D., Strefler, J., Ueckerdt, F., Kriegler, E., Luderer, G., 2021. Remind2.1: transformation and innovation dynamics of the energy-economic system within climate and sustainability limits. Geoscientific Model Development 14, 6571–6603. URL: https://gmd.copernicus.org/articles/14/6571/2021/, doi:10.5194/gmd-14-6571-2021.

Ben-Tal, A., El Ghaoui, L., Nemirovski, A., 2009. Robust Optimization. Princeton University Press, Princeton USA.

Bernard, A., Vielle, M., 2003. Measuring the welfare cost of climate change policies: a comparative assessment based on the computable general equilibrium model GEMINI-E3. Environmental Modeling and Assessment 8, 199–217.

Bosetti, V., Carraro, C., Massetti, E., Tavoni, M., 2008. International energy R&D spillovers and the economics of greenhouse gas atmospheric stabilization. Energy Economics 30, 2912–2929. URL: https://www.sciencedirect.com/science/article/pii/S0140988308000625, doi:https://doi.org/10.1016/j.eneco.2008.04.008. technological Change and the Environment.

Bouckaert, S., et al., 2021. Net Zero by 2050: a Roadmap for the Global Energy Sector. International Energy Agency.

Bretschger, L., 2024. Energy transition and climate change abatement: A macroeconomic analysis. Resource and Energy Economics 76, 101423.

Carlson, D.A., Haurie, A., Leizarowitz, A., 1991. Infinite horizon optimal control : deterministic and stochastic systems. Springer.

Casey, G., 2024. Energy efficiency and directed technical change: Implications for climate change mitigation. Review of Economic Studies 91, 192–228.

Cass, D., 1965. Optimum Growth in an Aggregative Model of Capital Accumulation1. The Review of Economic Studies 32, 233–240. URL: https://doi.org/10.2307/2295827, doi:10.2307/2295827.

Cass, D., Shell, K., 1976. The structure and stability of competitive dynamical systems. Journal Economic Theory 12, 31–70.

Chen, C., Tavoni, M., 2013. Direct air capture of CO_2 and climate stabilization: a model based assessment. Climatic Change 118, 59–72.

Chen, Y.H.H., Paltsev, S., Reilly, J.M., Morris, J.F., Babiker, M.H., 2016. Long-term economic modeling for climate change assessment. Economic Modelling 52, 867–883.

Desport, L., Gurgel, A., Morris, J., Herzog, H., Chen, Y.H.H., Selosse, S., Paltsev, S., 2024. Deploying direct air capture at scale: How close to reality? Energy Economics 129, 107244. URL: https://www.sciencedirect.com/science/article/pii/S0140988323007429, doi:https://doi.org/10.1016/j.eneco.2023.107244.

Edwards, M.R., Thomas, Z.H., Nemet, G.F., Rathod, S., Greene, J., Surana, K., Kennedy, K.M., Fuhrman, J., McJeon, H.C., 2024. Modeling direct air carbon capture and storage in a 1.5° C climate future using historical analogs. Proceedings of the National Academy of Sciences 121, e2215679121.

Emenike, O., Michailos, S., Hughes, K.J., Ingham, D., Pourkashanian, M., 2021. Techno-economic and environmental assessment of BECCS in fuel generation for FT-fuel, bioSNG and OMEx. Sustainable Energy Fuels 5, 3382–3402. URL: http://dx.doi.org/10.1039/D1SE00123J, doi:10.1039/D1SE00123J.

Fajardy, M., Chiquier, S., Mac Dowell, N., 2018. Investigating the BECCS resource nexus: delivering sustainable negative emissions. Energy Environ. Sci. 11, 3408–3430. URL: http://dx.doi.org/10.1039/C8EE01676C, doi:10.1039/C8EE01676C.

Fajardy, M., Morris, J., Gurgel, A., Herzog, H., Mac Dowell, N., Paltsev, S., 2021. The economics of bioenergy with carbon capture and storage (BECCS) deployment in a 1.5° C or 2° C world. Global Environmental Change 68, 102262. URL: https://www.sciencedirect.com/science/article/pii/S0959378021000418, doi:https://doi.org/10.1016/j.gloenvcha.2021.102262.

Feinstein, C., Luenberger, D., 1981. Analysis of the asymptotic behavior of optimal control trajectories: The implicit programming problem. SIAM Journal on Control and Optimization 19. doi:10.1137/0319035.

Haurie, A., 2003. Integrated assessment modeling for global climate change: An infinite horizon optimization viewpoint. Environmental Modeling & Assessment 8, 117–132. URL: https://doi.org/10.1023/A:1025534905304, doi:10.1023/A:1025534905304.

International Energy Agency, 2020. CO_2 Emissions from Fuel Combustion.

Kantzas, E.P., Val Martin, M., Lomas, M.R., Eufrasio, R.M., Renforth, P., Lewis, A.L., Taylor, L.L., Mercure, J.F., Pollitt, H., Vercoulen, P.V., Vakilifard, N., Holden, P.B., Edwards, N.R., Koh, L., Pidgeon, N.F., Banwart, S.A., Beerling, D.J., 2022. Substantial carbon drawdown potential from enhanced rock weathering in the United Kingdom. Nature Geoscience 15, 382–389. URL: https://doi.org/10.1038/s41561-022-00925-2, doi:10.1038/s41561-022-00925-2.

Keith, D.W., Holmes, G., Angelo, D.S., Heidel, K., 2018. A process for capturing CO_2 from the atmosphere. Joule 2, 1573–1594.

Manne, A., Mendelsohn, R., Richels, R., 1995. MERGE: A model for evaluating regional and global effects of GHG reduction policies. Energy Policy 23, 17–34. URL: https://www.sciencedirect.com/science/article/pii/030142159590763W, doi:https://doi.org/10.1016/0301-4215(95)90763-W.

Marcucci, A., Panos, V., Kypreos, S., 2017. The road to achieving the long-term Paris targets: energy transition and the role of direct air capture. Climatic Change 144, 181–193.

Masson-Delmotte, V., Zhai, P., Pirani, A., Connors, S., Péan, C., Berger, S., Caud, N., Chen, Y., Goldfarb, L., Gomis, M., M. Huang, K.L., Lonnoy, E., Matthews, J., Maycock, T., T. Waterfield, O. Yelekçi, R.Y., Zhou, B. (Eds.), 2021. Climate Change 2021: The Physical Science Basis. Contribution of Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change. Cambridge University Press, Cambridge UK. chapter Summary for Policymakers.

McFadden, D., 1963. Constant elasticity of substitution production functions. The Review of Economic Studies 30, 73–83.

McGlade, C., 2023. The Oil and Gas Industry in Net Zero Transition. IEA, Paris, France.

Morris, J.F., Reilly, J.M., Chen, Y.H.H., 2019. Advanced technologies in energy-economy models for climate change assessment. Energy Economics 80, 476–490.

Nordhaus, W., 2014. Estimates of the social cost of carbon: Concepts and results from the DICE-2013R model and alternative approaches. Journal of the Association of Environmental and Resource Economists , Vol. 1, No. 1/2, 273–312.

Nordhaus, W., Boyer, J., 2000. Warming the world : economic models of global warming. MIT Press, Cambridge, USA. URL: https://doi.org/10.7551/mitpress/7158.001.0001, doi:10.7551/mitpress/7158.001.0001.

Nordhaus, W., Sztorc, P., 2013. DICE 2013R: Introduction and User's Manual. http://acdc2007.free.fr/dicemanual2013.pdf ed.

Ohndorf, M., Blasch, J., Schubert, R., 2015. Emission budget approaches for burden sharing: some thoughts from an environmental economics point of view. Climatic Change 133, 385–395.

Paltsev, S., Reilly, J.M., Jacoby, H.D., Eckaus, R.S., McFarland, J., Sarofim, M., Asadoorian, M., Babiker, M., 2005. The MIT Emissions Prediction and Policy Analysis (EPPA) Model: Version 4. Technical Report 125. MIT Joint Program Report on the Science and Policy of Global Change.

Papageorgiou, C., Saam, M., Schulte, P., 2017. Substitution between clean and dirty energy inputs: A macroeconomic perspective. The Review of Economics and Statistics 99, 281–290.

Pratama, M.R., Muthia, R., Purwanto, W.W., 2023. Techno-economic and life cycle assessment of the integration of bioenergy with carbon capture and storage in the polygeneration system (BECCS-PS) for producing green electricity and methanol. Carbon Neutrality 2, 26. URL: https://doi.org/10.1007/s43979-023-00069-1, doi:10.1007/s43979-023-00069-1.

Ramsey, F.P., 1928. A mathematical theory of saving. Economic Journal 38, 543–559.

Rockafellar, R.T., 1976. Saddle points of hamiltonian systems in convex Lagrange problems having a nonzero discount rate. Journal of Economic Theory 12, 71–113. URL: https://www.sciencedirect.com/science/article/pii/0022053176900284, doi:https://doi.org/10.1016/0022-0531(76)90028-4.

Rogelj, J., Forster, P.M., Kriegler, E., Smith, C.J., Séférian, R., 2019. Estimating and tracking the remaining carbon budget for stringent climate targets. Nature 571, 335–342. URL: https://doi.org/10.1038/s41586-019-1368-z, doi:10.1038/s41586-019-1368-z.

Rogelj, J., Popp, A., Calvin, K.V., Luderer, G., Emmerling, J., Gernaat, D., Fujimori, S., Strefler, J., Hasegawa, T., Marangoni, G., Krey, V., Kriegler, E., Riahi, K., van Vuuren, D.P., Doelman, J., Drouet, L., Edmonds, J., Fricko, O., Harmsen, M., Havlík, P., Humpenöder, F., Stehfest, E., Tavoni, M., 2018. Scenarios towards limiting global mean temperature increase below 1.5 °c. Nature Climate Change 8, 325–332. URL: https://doi.org/10.1038/s41558-018-0091-3, doi:10.1038/s41558-018-0091-3.

Selosse, S., Ricci, O., 2014. Achieving negative emissions with BECCS (bioenergy with carbon capture and storage) in the power sector: New insights from the TIAM-FR (TIMES Integrated Assessment Model France) model. Energy 76, 967–975.

Shukla, P., Pathak, M. (Eds.), 2022. Climate Change 2022. Mitigation of Climate Change. Contribution of Working Group III to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change. Cambridge University Press, Cambridge, UK and New York, USA. chapter Technical Summary.

Socolow, R., Desmond, M., Aines, R., Blackstock, J., Bolland, O., Kaarsberg, T., Lewis, N., Mazzotti, M., Pfeffer, A., Sawyer, K., Siirola, J., Smit, B., Wilcox, J., 2011. Direct Air Capture of CO_2 with Chemicals: A Technology Assessment for the APS Panel on Public Affairs.

Solow, R.M., 1956. A contribution to the theory of economic growth. The Quarterly Journal of Economics 70, 65–94. URL: http://www.jstor.org/stable/1884513.

United Nations, 2019. World population prospects: The 2019 revision. Population Division, Department of Economic and Social Affairs.

Uzawa, 1962. Production functions with constant elasticities of substitution. Review of Economic Studies 29, 291–299.

Uzawa, H., 1963. On a two-sector model of economic growth II. The Review of Economic Studies 30, 105–118. URL: http://www.jstor.org/stable/2295808.