

Homework/Pop Quiz #1 of the course: Theory of Computer Games.

1. For the game 9x9 Go, estimate its state-space complexity and game-tree complexity.

state-space complexity:  $9 \times 9 = 81 \rightarrow 3^{81} \approx 4.43 \times 10^{38}$

game-tree complexity: The upper bound is  $81! \approx 10^{120}$

(for a closer approximation to real life:  $\therefore$  an average game of  $9 \times 9$  Go uses about 45 steps

$\therefore$  the gametree complexity is  $81!/45! \approx 10^{64}$ )

If we consider Go rule, the situation may be more complicated. All states may not be reached and game-tree complexity may be INF.

2. Prove that for  $N \times (N+1)$  Hex the longer-side wins.

The handwritten text and diagrams provide a proof for the Hex game. It starts by showing two board configurations (A and B) and discusses the complexity of the game. It then proves that the longer side wins by showing that if the longer side is white, white can always win, and if the longer side is black, black can always win. The proof uses the concept of symmetry and the fact that the board is  $N \times (N+1)$ .

i 擇法①和②都會產生  $N(N+1)$  的 board, 兩者是鏡像左右對稱的, 故我以擇法①討論

ii 若以投影片中, 下對應位法來下的話 (White A → Black A → White B → Black B)

iii 若白子想連接兩邊 (N邊到N邊) 其在中間接合處, 必定為如下

iv 這遊戲沒有平手情況, 據iii白子不可能贏, 故黑子必勝

v 不論白子黑子誰先手, 情況皆相同 (短邊想連通必定會如以上討論受阻) 本討論無設定黑子先手

ok:

Assume black first.

If the longer side is white. White player always puts the piece on the symmetric position of black put. When white player ends his/her run, the board must be symmetric. Thus, when black almost win, white have been win (consider the distance of sides, when the length of black is  $N$ , the length of white is also  $N$ ). So, white will win first.

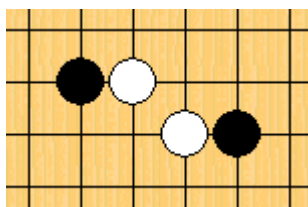
If the longer side is black. The first piece of black can put on random position. Since the more pieces is better in Hex, the condition does not become worse (since the piece only connect our pieces, and cut the pieces of opponent). Next, black always puts the piece on the symmetric

position of white put. When the position has had piece, put the piece on random position. By the same reason which longer side is white case, when the length of white is  $N$ , the length of black is  $N$ , and black must win.

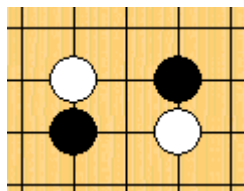
3. Prove that the theoretical value of  $\text{Connect}(m,n,k,p,q+1)$  for Black is not worse than that in  $\text{Connect}(m,n,k,p,q)$ .

Suppose Black has a winning strategy  $S$  in  $\text{Connect}(m, n, k, p, q)$ . The same strategy  $S$  can be applied to  $\text{Connect}(m, n, k, p, q+1)$  because the additional move will not hinder Black's situation in a connect game. Therefore the theoretical value of  $\text{Connect}(m, n, k, p, q+1)$  will always be better than the value of  $\text{Connect}(m, n, k, p, q)$ .

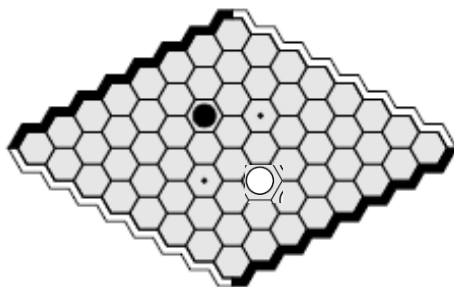
4. The game  $\text{Connect}(6,2,2)$  can be ultra-weakly solved with no win for White. Please solve the following four positions ultra-weakly. If you do not think possible to solve it, answer “no” and explain why.



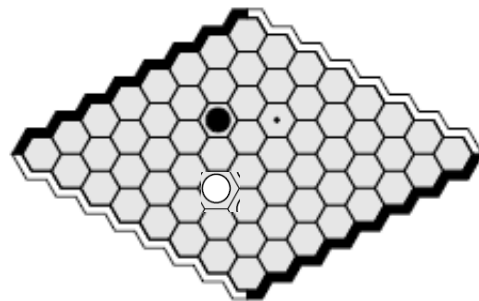
(a)



(b)



(c)



(d)

You need to state the two assumptions for strategy stealing for each case:

- The game is symmetric.
- Extra pieces can never be a disadvantage.

- a) Not symmetric, so it cannot be ultra-weakly solved with strategy stealing.
- b) Symmetric, and extra pieces do not harm a player in  $\text{Connect}(m, n, k, p, q)$ .
- c) Not symmetric.
- d) Symmetric, and extra pieces do not harm a player in Hex.