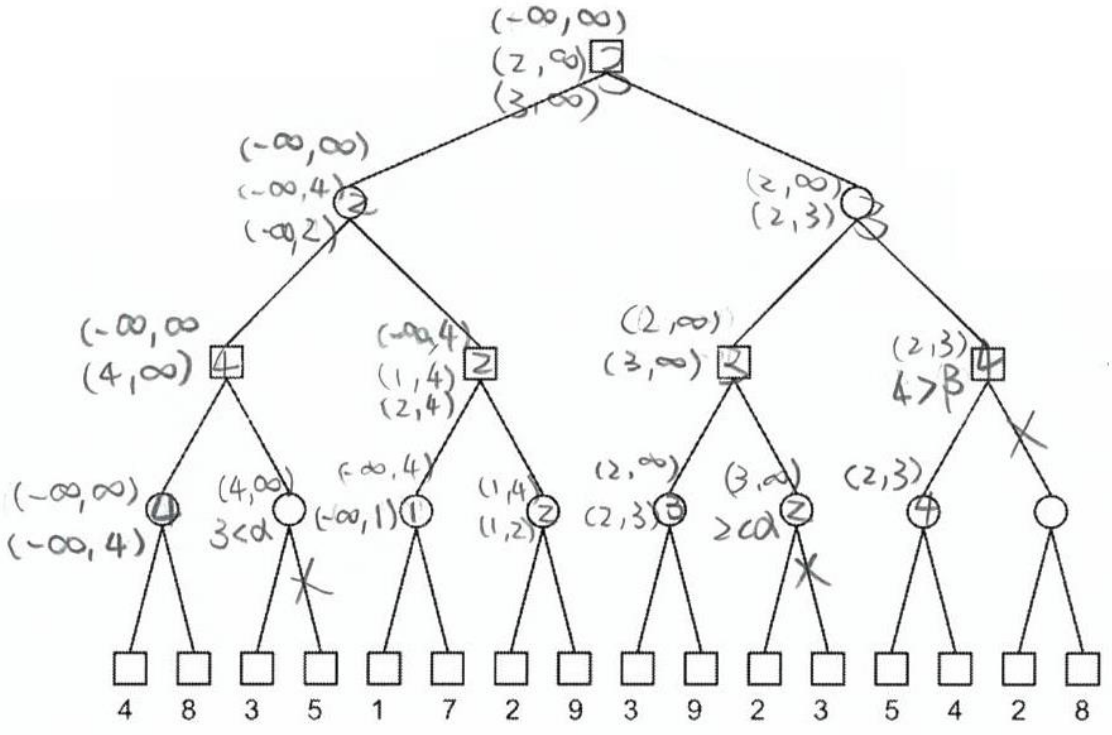
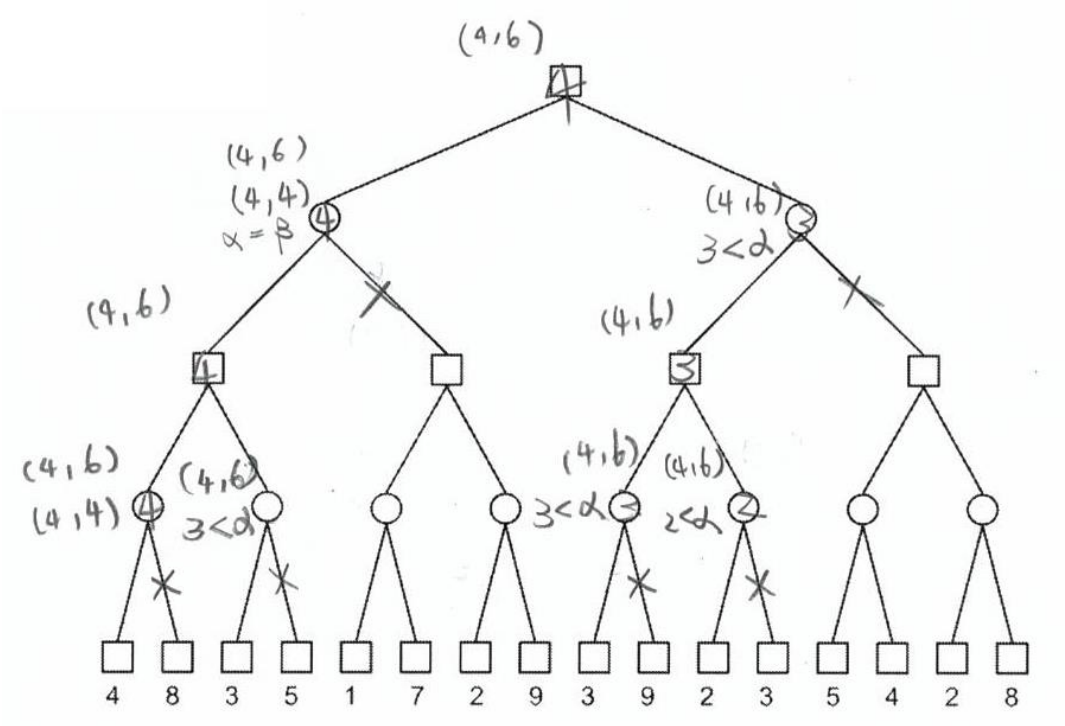
Homework and Pop Quiz #2 of the course: Theory of Computer Games.

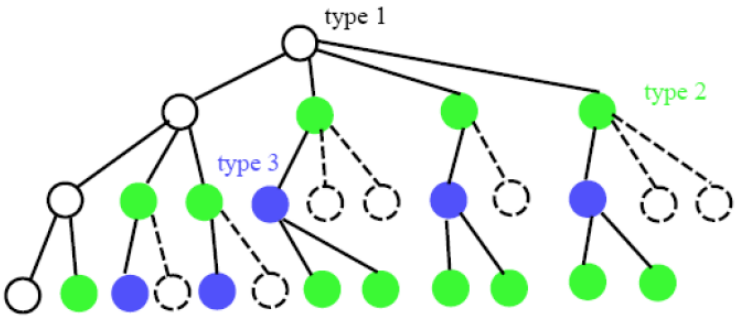
1. Do the alpha-beta search to find the minimax value of the following search tree. You must mark windows at each node and indicate the pruned nodes.



2. For the above problem, assume that the alpha-beta window at the root is (4, 6). Do the alpha-beta search again.



3. Please prove why Corollary 1 is correct. Note that Corollary 1 says: "In the best case, the alpha-beta procedure examines exactly $d^{\lceil l/2 \rceil} + d^{\lfloor l/2 \rfloor} - 1$ positions on level l, where l is even."



a-b search tree has three types of nodes in the best case as above:

- Type1 – White (W): This node will search all the children.
- Type2 – Green (G): This node will prune all the children expect the first one.
- Type3 – Blue (B): This node will search all the children.

(The difference between W and B is : W will have one child in W, but B will not.)

For each level, assume

- d means the branch factor.
- w means the number of W on **last level**.
- g means the number of G on **last level**.
- b means the number of B on **last level**.

Then,

- The number of W on this level is: 1
- The number of G on this level is: w*(d-1) + b*d
- The number of B on this level is: g

Thus,we have a table which shows the number of each node:

Level	W	G	B	Total
0	1	0	0	1
1	1	d-1	0	d
2	1	d-1	d-1	2d-1
3	1	d^2-1	d-1	d^2+d-1
4	1	d^2-1	d^2-1	2d^2-1
5	1	d^3-1	d^2-1	d^3 + d^2-1
6	1	d^3-1	d^3-1	2d^3-1
7	1	d^4-1	d^3-1	d^4 + d^3-1
n	1	$d^{\lceil n/2 \rceil}$-1	$d^{\lfloor n/2 \rfloor}$-1	$d^{\lceil n/2 \rceil} + d^{\lfloor n/2 \rfloor} - 1$

Thus, the number of nodes on each level l is $d^{\lceil l/2 \rceil} + d^{\lfloor l/2 \rfloor} - 1$.

Or you can use mathematical induction.