Homework #4 of the course: Theory of Computer Games.

1. Based on Z-hashing, we can distinguish a position A from another B. Now, we want to use a variant of the Zobrist function to encode into a transposition table key a path, a sequence of moves (placing a piece like Go or Go-Moku), say (m_1, m_2, m_3, …, m_k). For example, the key encoded from the path (m_1, m_2, m_3) must be different from that for (m_3, m_2, m_1).
   Hint: you can assume the length of a path is no more than 10.

A. 使用多個 Z-Hashing table，每一步都有各自的黑或白的 table。
B. 位移 bits，將第 k 步的原始 z-table 值循環位移 (k-1) bit。(I think it is non-uniform and should be careful about the use of the z-value bits)
C. 線性函式轉換使得第 k 步進行 XOR 的值不一與原始的不一樣(Non-uniform)
D. 第 k 步採用z^{k}來進行 XOR，但要小心$\mathrm{z}^{\mathrm{k}} \neq z^{k'}$。
E. 族繁不及備載

實際應用上還是希望採用計算速度快、Hash 分布均勻減少碰撞機率等

2. Assume that each position is represented by a 64-bit hash key. Assume to maintain at most 10^7 positions in a transposition table. Derive the approximate probability of collision. Hint: refer to the birthday problem at https://en.wikipedia.org/wiki/Birthday_problem.

A. $\bar{\mathrm{p}} = \prod_{m=0}^{10^7-1}\left(1-\frac{m}{2^{64}}\right) \cong \prod_{m=0}^{10^7-1} e^{-\frac{m}{2^{64}}} = e^{-\frac{10^7(10^7-1)}{2\times 2^{64}}}$

$$1-\bar{\mathrm{p}} = 1 - e^{-\frac{10^7(10^7-1)}{2\times 2^{64}}} = 1-\mathrm{e}^{-(2.71050516016321796364252350031 17e-6)}$$

$$= 0.0000027105$$