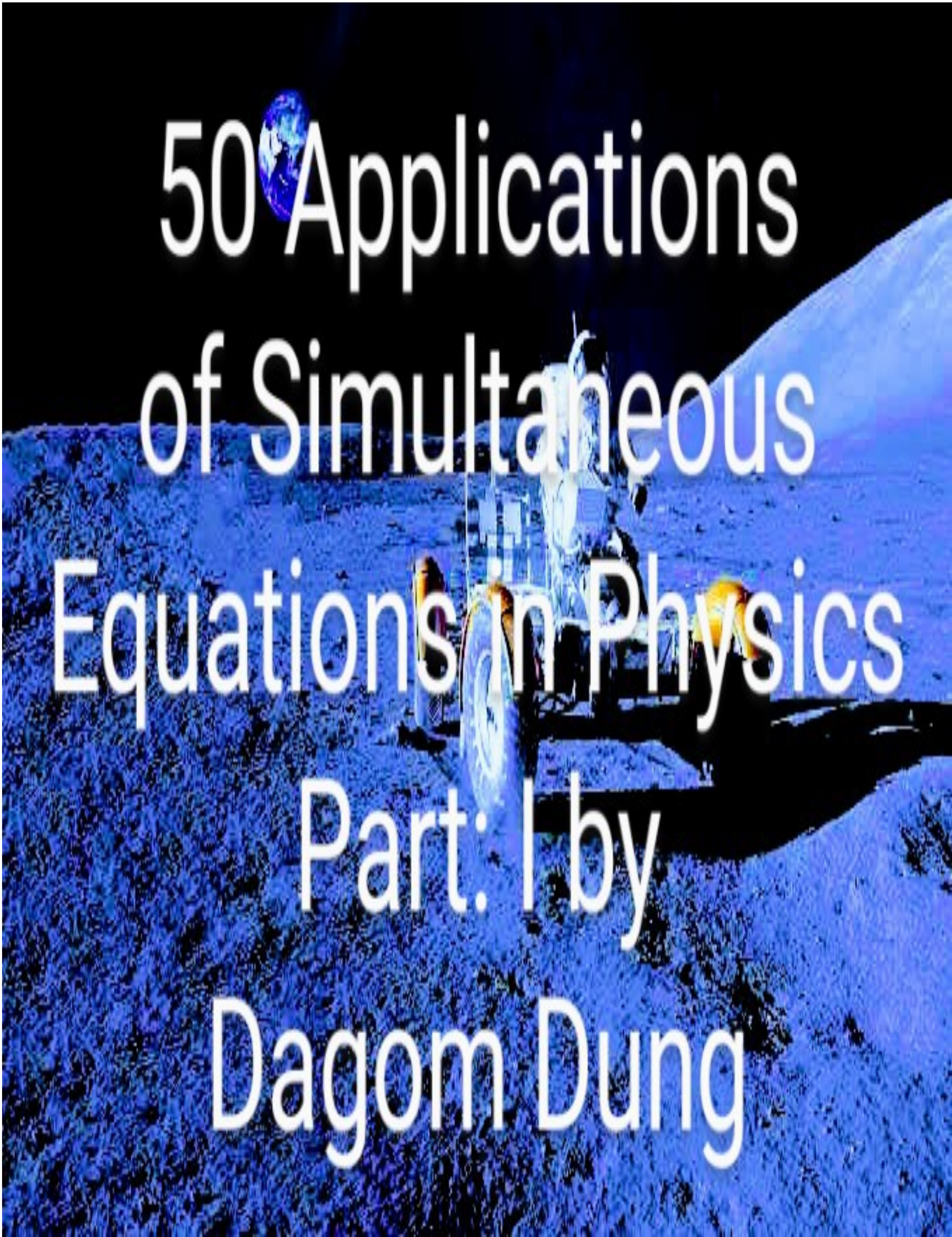

50 Applications
of Simultaneous
Equations in Physics
Part: I by
Dagom Dung

Pre-face:

This book was written to demonstrate how simultaneous equations can be applied to physics problems, in order to derive solutions from them.

This book was written to encourge critical thinking and problem solving skills, it was written to inspire physicists, mathematicians and engineers.

By reason of this book three mathematical methods for solving simultaneous equations were motivated to solve various physics problems were possible.

The first and second chapters of this book were motivated to solve linear systems of equations, whereas the third and final chapter was motivated to solve non linear systems of equtions in physics.

In the first chapter Gauss-Jordan elimination was motivated to solve linear systems of equations in physics which where derived from physics problems, by elementary row operations, and the final solutions: in reduced row echelon form.

The final solutions in reduced row echelon form were interpeted in physical terms, hence the solutions of the physics problems were obtained.

In the second chapter the method of determinants was motivated to solve resulting linear systems of equations by evaluating their determinants to find their consistency, and that of their unknowns.

Finally Cramer's rule was used to determine the numerical values of the unknowns, ergo the values were interpreted in physical terms.

In the third and final chapter the substitution method was motivated to solve non linear systems of equations derived from non linear equations in physics, and finally variable substitutions were used to find the solutions to the systems of equations, the obtained solutions were interpreted in physical terms were inference was possible.

This book was written to promote *excellentia scientiarum* and to contribute to and share scientific knowledge, and to incite methodical and philosophical reasoning: "*scientiam mentis operandi*".

Pre-requisites:

- Be able to solve simultaneous equations by the method of substitution, Gauss-Jordan elimination, and the method of determinants.
- Be able to solve quadratic equations by completing the square.
- Be able to solve simultaneous linear and quadratic equations.
- Be able to solve determined non linear systems of equations.
- Knowledge of measurement, mechanics, and electronics.
- Knowledge and understanding of Hooke's law and Ohm's law.
- Knowledge and understanding of fractions and percentages.

- Knowledge of the law of indices and logarithms.
- Have a general understanding of mathematics and physics.

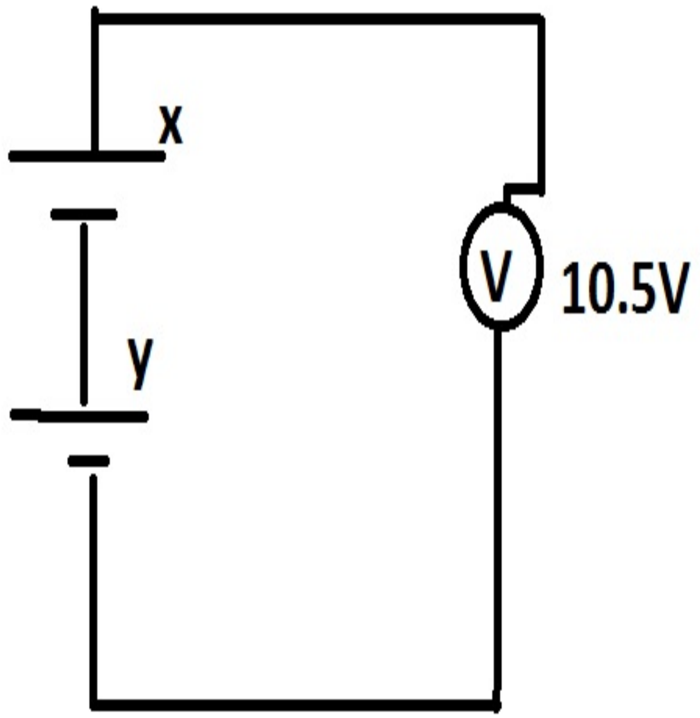
Table of Contents:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

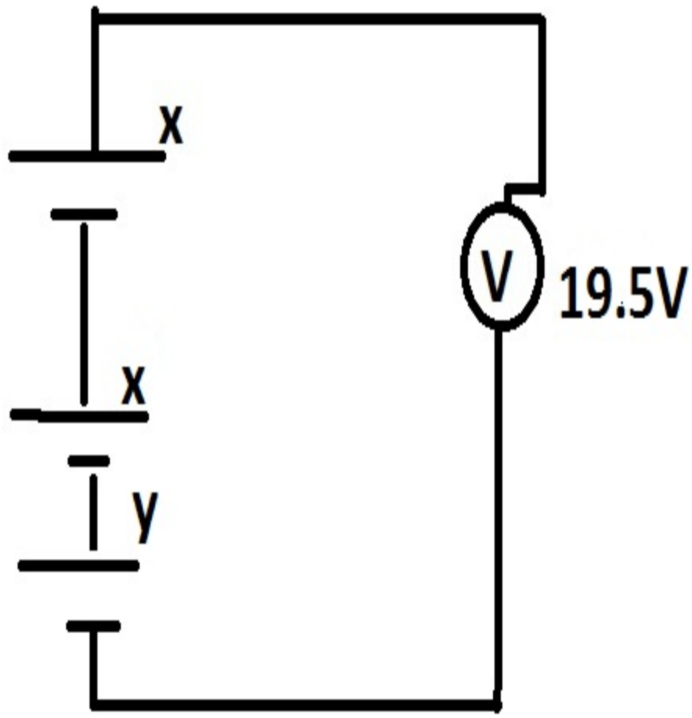
Chapter 1: Motivation for Gauss-Jordan Elimination in Physics

Problem I:

1st Schematic diagram:



2nd Schematic diagram:



From the above schematic diagrams determine the e.m.fs of cells x, and y.

Solution I:

From the 1st schematic diagram:

$x + y = 10.5 \, (1)$

From the 2nd schematic diagram:

$x + x + y = 19.5$

$2x + y = 19.5 \, (2)$

Writing an augmented matrix that describes the system of equations:

$$\left[\begin{array}{cc|c} 1 & 1 & 10.5 \\ 2 & 1 & 19.5 \end{array}\right]$$

By Gauss-Jordan elimination:

$2R_1 - R_2 \rightarrow R_2$:

$$\left[\begin{array}{cc|c} 1 & 1 & 10.5 \\ 0 & 1 & 1.5 \end{array}\right]$$

$R_1 - R_2 \rightarrow R_1$:

$$\left[\begin{array}{cc|c} 1 & 0 & 9 \\ 0 & 1 & 1.5 \end{array}\right]$$

From the matrix above which is augmented, in reduced row echelon form. The battery labeled x has an e.m.f of 9 V and the battery labeled y has an e.m.f of 1.5 V.

Problem II:

When a 1 N load is applied to two springs in series the springs extend $\frac{6}{5}$ meters, but when the second spring is doubled the springs extend $\frac{7}{5}$ meters find the elastic constants of the first, and second springs.

Solution II:

$$\frac{1}{k_1} + \frac{1}{k_2} = \frac{1}{\left[\frac{1}{(6/5)}\right]}$$

$$\frac{1}{k_1} + \frac{1}{k_2} = \frac{1}{\left(\frac{5}{6}\right)}$$

$$\frac{1}{k_1} + \frac{1}{k_2} = \frac{6}{5} \; (1)$$

$$\frac{1}{k_1} + \frac{2}{k_2} = \frac{1}{\left[\frac{1}{(7/5)}\right]}$$

$$\frac{1}{k_1} + \frac{2}{k_2} = \frac{1}{\left(\frac{5}{7}\right)}$$

$$\frac{1}{k_1} + \frac{2}{k_2} = \frac{7}{5} \; (2)$$

If $\frac{1}{k_1} = x$ and $\frac{1}{k_2} = y$ then:

$x + y = \frac{6}{5}$ (i)

$x + 2y = \frac{7}{5}$ (ii)

Writing an augmented matrix for the system of equations:

$$\left[\begin{array}{cc|c} 1 & 1 & 6/5 \\ 1 & 2 & 7/5 \end{array}\right]$$

By Gauss-Jordan elimination:

$R_1 - R_2 \rightarrow R_1$:

$$\left[\begin{array}{cc|c} 0 & -1 & -1/5 \\ 1 & 2 & 7/5 \end{array}\right]$$

$R_1/-1 \rightarrow R_1$:

$$\left[\begin{array}{cc|c} 0 & 1 & 1/5 \\ 1 & 2 & 7/5 \end{array}\right]$$

$2R_1 - R_2 \rightarrow R_2$:

$$\left[\begin{array}{cc|c} 0 & 1 & 1/5 \\ -1 & 0 & -1 \end{array}\right]$$

$R_2/-1 \rightarrow R_2$:

$$\left[\begin{array}{cc|c} 0 & 1 & 1/5 \\ 1 & 0 & 1 \end{array}\right]$$

In reduced row echelon form:

$$\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1/5 \end{array}\right]$$

Since $x = 1$ and $y = \frac{1}{5}$ then: $k_1 = \frac{1}{1} = 1$ and $k_2 = \frac{1}{(\frac{1}{5})}$,

$k_2 = 5$

Therefore: the elastic constant of the first spring (k_1) is 1 N/ m and the elastic constant of the second spring (k_2) is 5 N/ m.

Problem III:

A truck has two compartments for rice and flour.The rice compartment is one meter above the ground, and the flour compartment is $1\frac{1}{2}$ meters above the ground, if the work done in lifting a bag of rice and flour to their respective compartments must be 95 Joules, and the total weight of the bags of rice and flour must be 70 N for transportational reasons. What will be the respective weights of the bags of rice and flour that will meet these criteria?

Solution III:

Work done by lifting the rice to the rice compartment:

$$W_1 = w_1 \cdot h_1$$

Since $h_1 = 1$:

$$W_1 = w_1$$

Work done by lifting the flour to the flour compartment:

$$W_2 = w_2 \cdot h_2$$

Since $h_2 = \frac{3}{2}$:

$$W_2 = \frac{3}{2}w_2$$

The total work done in lifting the rice and flour to their respective compartments which is 95 Joules:

$$W_1 + W_2 = 95$$

$$w_1 + \frac{3}{2}w_2 = 95$$

The total weight must be 70 N for transportational reasons:

$$w_1 + w_2 = 70$$

From the above process we obtain these simultaneous equations where w_1 is the weight of the bag of rice and w_2 that of flour:

$$w_1 + \frac{3}{2}w_2 = 95\,(1)$$

$$w_1 + w_2 = 70\,(2)$$

Writing an augmented matrix that expresses the system:

$$\left[\begin{array}{cc|c} 1 & 3/2 & 95 \\ 1 & 1 & 70 \end{array}\right]$$

By Guass-Jordan elimination:

$R_1 - R_2 \rightarrow R_1$:

$$\left[\begin{array}{cc|c} 0 & 1/2 & 25 \\ 1 & 1 & 70 \end{array}\right]$$

$R_1 - R_2/2 \rightarrow R_2$:

$$\left[\begin{array}{cc|c} 0 & 1/2 & 25 \\ -1/2 & 0 & -10 \end{array}\right]$$

$R_1/(1/2) \rightarrow R_1$:

$$\left[\begin{array}{cc|c} 0 & 1 & 50 \\ -1/2 & 0 & -10 \end{array}\right]$$

$R_2/(-1/2) \rightarrow R_2$:

$$\left[\begin{array}{cc|c} 0 & 1 & 50 \\ 1 & 0 & 20 \end{array}\right]$$

In reduced row echelon form:

$$\left[\begin{array}{cc|c} 1 & 0 & 20 \\ 0 & 1 & 50 \end{array}\right]$$

Therefore: the weight of the bag of rice (w_1) must be 20 N and the weight of the bag of flour (w_2) must be 50 N that will meet these criteria.

Problem IV:

If the difference of two complementary angles α, and β is 60° find the values of α, and β.

Solution IV:

Since the angles are complementary their sum equals 90°:

$\alpha + \beta = 90\,(1)$

Since the difference of α and β is 60°:

$\alpha - \beta = 60\,(2)$

Writing an augmented matrix for the system:

$$\left[\begin{array}{cc|c} 1 & 1 & 90 \\ 1 & -1 & 60 \end{array}\right]$$

By Gauss-Jordan elimination:

$R_1 + R_2 \to R_2$:

$$\left[\begin{array}{cc|c} 1 & 1 & 90 \\ 2 & 0 & 150 \end{array}\right]$$

$R_2/2 \rightarrow R_2$:

$$\left[\begin{array}{cc|c} 1 & 1 & 90 \\ 1 & 0 & 75 \end{array}\right]$$

$R_1 - R_2 \rightarrow R_1$:

$$\left[\begin{array}{cc|c} 0 & 1 & 15 \\ 1 & 0 & 75 \end{array}\right]$$

In reduced row echelon form:

$$\left[\begin{array}{cc|c} 1 & 0 & 75 \\ 0 & 1 & 15 \end{array}\right]$$

Therefore: the angle β is 15° and the angle α is 75°.

Problem V:

$$G^x \cdot G^y = G^3$$

$$\frac{G^x}{G^y} = \frac{1}{G}$$

Find the values of x and y if G is the constant of universal gravitation ($6.67 \cdot 10^{-11} Nm^2Kg^{-2}$).

Solution V:

Since x and y are expressed in the same base (G) in both sides of each equation:

$$x + y = 3 \ (1)$$

$$x - y = -1 \ (2)$$

Writing an augmented matrix for the system of equations:

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 1 & -1 & -1 \end{array}\right]$$

By Gauss-Jordan elimination:

$R_1 + R_2 \rightarrow R_2$:

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 2 & 0 & 2 \end{array}\right]$$

$R_2/2 \rightarrow R_2$:

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 1 & 0 & 1 \end{array}\right]$$

$R_1 - R_2 \rightarrow R_1$:

$$\left[\begin{array}{cc|c} 0 & 1 & 2 \\ 1 & 0 & 1 \end{array}\right]$$

In reduced row echelon form:

$$\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array}\right]$$

Therefore: x=1 and y=2

Problem VI:

When a 1.1 kN weight is applied to two springs in parallel they extend 1 m, but when the first spring is doubled, and a load of 10 N is applied to the springs in parallel they extend $(\frac{10}{2.1 \cdot 10^3})$ of a meter, find the extension of the first spring when a 100 N load is applied to it, and the extension of the second spring when a 50 N load is applied to it.

Solution VI:

$$k_1 + k_2 = \frac{1100}{1}$$

$$k_1 + k_2 = 1100 \, (1)$$

$$2k_1 + k_2 = \frac{10}{(\frac{10}{2.1 \cdot 10^3})}$$

$$2k_1 + k_2 = 2100 \, (2)$$

Writing an augmented matrix of the system of equations:

$$\left[\begin{array}{cc|c} 1 & 1 & 1100 \\ 2 & 1 & 2100 \end{array}\right]$$

By Gauss-Jordan elimination:

$$\left[\begin{array}{cc|c} 2 & 1 & 2100 \\ 1 & 1 & 1100 \end{array}\right]$$

$R_1 - R_2 \rightarrow R_1$:

$$\left[\begin{array}{cc|c} 1 & 0 & 1000 \\ 1 & 1 & 1100 \end{array}\right]$$

$R_1 - R_2 \rightarrow R_2$:

$$\left[\begin{array}{cc|c} 1 & 0 & 1000 \\ 0 & -1 & -100 \end{array}\right]$$

$R_2 / -1 \rightarrow R_2$:

$$\left[\begin{array}{cc|c} 1 & 0 & 1000 \\ 0 & 1 & 100 \end{array}\right]$$

Therefore: the elastic constant of the first spring (k_1) is $1\,\mathrm{kN/m}$, and the elastic constant of the second spring (k_2) is $100\,\mathrm{N/m}$.

According to Hooke's law:

$F \propto e$

$F = ke$

$e = \frac{F}{k}$

Therefore:

$e_1 = \frac{F_1}{k_1}$

Substituting the value of F_1 for 100, and k_1 for 1000 to find the extension of the first spring when a 100 N load is applied to it:

$e_1 = \frac{100}{1000}$

$e_1 = 0.1$

And:

$e_2 = \frac{F_2}{k_2}$

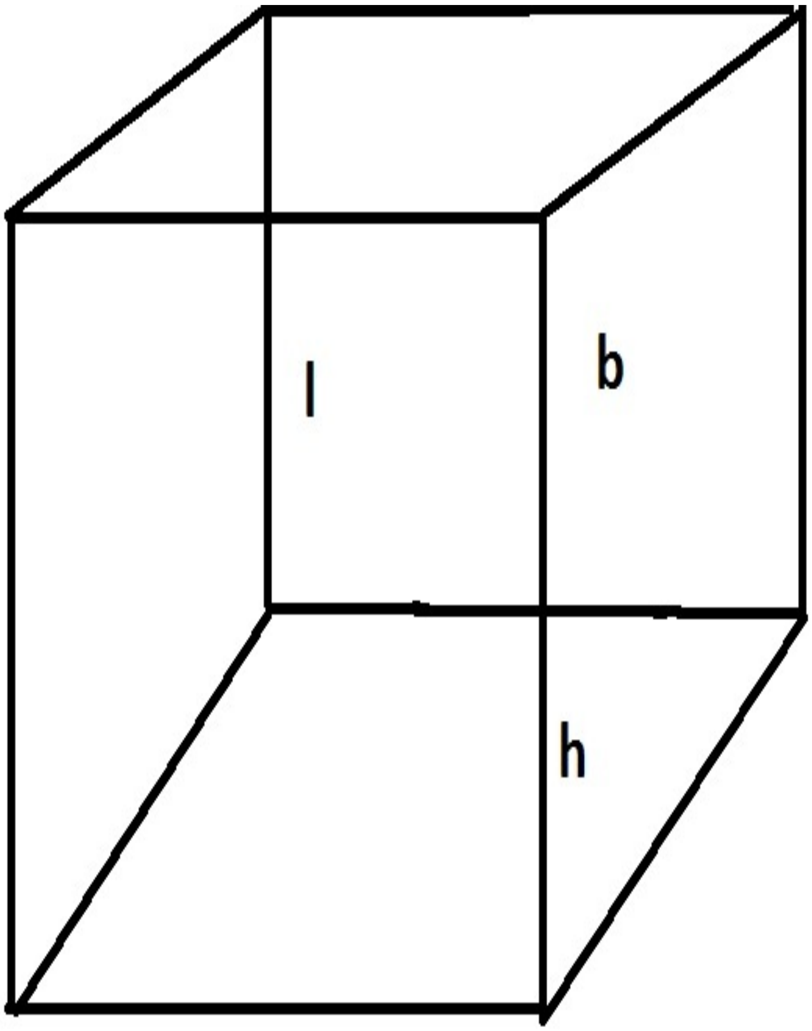
Substituting the value of F_2 for 50, and k_2 for 100 to find the extension of the second spring when a 50 N load is applied to it:

$$e_2 = \frac{50}{100}$$

$$e_2 = 0.5$$

Therefore: the extension of the first spring (e_1) when a 100 N load is applied to it is 0.1 m, and the extension of the second spring (e_2) when a 50 N load is applied to it is 0.5 m.

Problem VII:



Not drawn to scale

If the coefficients of linear, and area expansivity of the cube α, and β are given by these simultaneous equations:

$\alpha + \beta = 2.55 \cdot 10^{-5} k^{-1}$

$\alpha - 2\beta = -2.55 \cdot 10^{-5} k^{-1}$

find the coefficients of linear, area, and volumetric expansivity, and by using linear expansivity coefficient tables determine the possible materials the cube could be made of.

Solution VII:

Writing an augmented matrix of the system:

$$\left[\begin{array}{cc|c} 1 & 1 & 0.0000255 \\ 1 & -2 & -0.0000255 \end{array}\right]$$

By Gauss-Jordan elimination:

$R_1 - R_2 \rightarrow R_2$:

$$\left[\begin{array}{cc|c} 1 & 1 & 0.0000255 \\ 0 & 3 & 0.000051 \end{array}\right]$$

$R_2/3 \rightarrow R_2$:

$$\left[\begin{array}{cc|c} 1 & 1 & 0.0000255 \\ 0 & 1 & 0.000017 \end{array}\right]$$

$R_1 - R_2 \rightarrow R_1$:

$$\left[\begin{array}{cc|c} 1 & 0 & 0.0000085 \\ 0 & 1 & 0.000017 \end{array}\right]$$

Therefore: the coefficient of linear expansivity (α) is $0.85 \cdot 10^{-5}K^{-1}$, and the coefficient of area expansivity (β) of the cube is $1.7 \cdot 10^{-5}K^{-1}$.

Using the relation $\gamma = 3\alpha$ to find the coefficient of volumetric expansion of the cube:

$$\gamma = 3 \cdot 0.85 \cdot 10^{-5}K^{-1}$$

$$\gamma = 2.55 \cdot 10^{-5}K^{-1}$$

From the linear expansivity coefficient tables:

Substance	Linear expansivity(/K)
Platinum	0.000009
Iron	0.000012
Copper	0.000017
Brass	0.000018
Aluminium	0.000023
Lead	0.000029
Zinc	0.000030
Invar(alloy)	0.000001
Glass	0.0000085
Silica	0.0000004

Since the linear expansivity (α) of the cube is $8.5 \cdot 10^{-6} K^{-1}$ using the above table the cube should be made of glass.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} -$$

$$a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}.$$

Chapter 2: Motivation for the Method of Determinants in Physics

Problem VIII:

$$K^{2x} \cdot K^{y} = K^{4} \cdot K^{3}$$

$$\frac{K^{x}}{K^{2y}} = K^{-4}$$

If k is the rate of a spring which is $3N/m$ find the values of x and y.

Solution VIII:

Substituting the value of k for 3:

$$3^{2x} \cdot 3^{y} = 3^{4} \cdot 3^{3}$$

$$\frac{3^{x}}{3^{2y}} = 3^{-4}$$

Since their bases are the same, which is 3, on both sides of each equation:

$$2x + y = 4 + 3$$

$$x - 2y = -4$$

Therefore:

$$2x + y = 7 \, (1)$$

$$x - 2y = -4 \, (2)$$

Solving for x and y by the method of determinants:

$$\Delta = \begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix}$$

$$\Delta = -5$$

$$\Delta_x = \begin{vmatrix} 7 & 1 \\ -4 & -2 \end{vmatrix}$$

$$\Delta_x = -10$$

$$\Delta_y = \begin{vmatrix} 2 & 7 \\ 1 & -4 \end{vmatrix}$$

$$\Delta_y = -15$$

$$x = \frac{\Delta_x}{\Delta}$$

$$y = \frac{\Delta_y}{\Delta}$$

$$x = \frac{-10}{-5}$$

$$x = 2$$

$$y = \frac{-15}{-5}$$

$$y = 3$$

Problem IX:

Two containers of machine oil, container x, and container y, the total volume of machine oil of x and y when put into one container z is $80cm^3$, but when 20% of x is taken from z the total volume is $74cm^3$, find the volumes of x and y.

Solution IX:

$z = x + y$

$z = 80$

$x + y = 80 \, (1)$

$z - \frac{1}{5}x = 74$

$x + y - \frac{1}{5}x = 74$

$\frac{4}{5}x + y = 74 \, (2)$

Solving the simultaneous linear equations by the method of determinants:

$$\Delta = \begin{vmatrix} 1 & 1 \\ 4/5 & 1 \end{vmatrix}$$

$$\Delta = \frac{1}{5}$$

$$\Delta_x = \begin{vmatrix} 80 & 1 \\ 74 & 1 \end{vmatrix}$$

$$\Delta_x = 6$$

$$\Delta_y = \begin{vmatrix} 1 & 80 \\ 4/5 & 74 \end{vmatrix}$$

$$\Delta_y = 10$$

$$x = \frac{\Delta_x}{\Delta}$$

$$x = \frac{6}{(1/5)}$$

$$x = 6 \cdot 5$$

$$x = 30$$

$$y = \frac{\Delta_y}{\Delta}$$

$$y = \frac{10}{(1/5)}$$

$$y = 10 \cdot 5$$

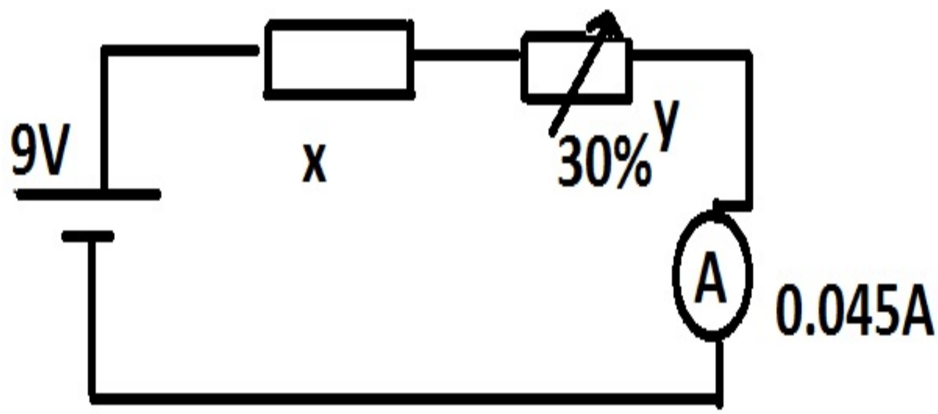
$$y = 50$$

Therefore: the volume of container x is $30cm^3$ and container y, $50cm^3$.

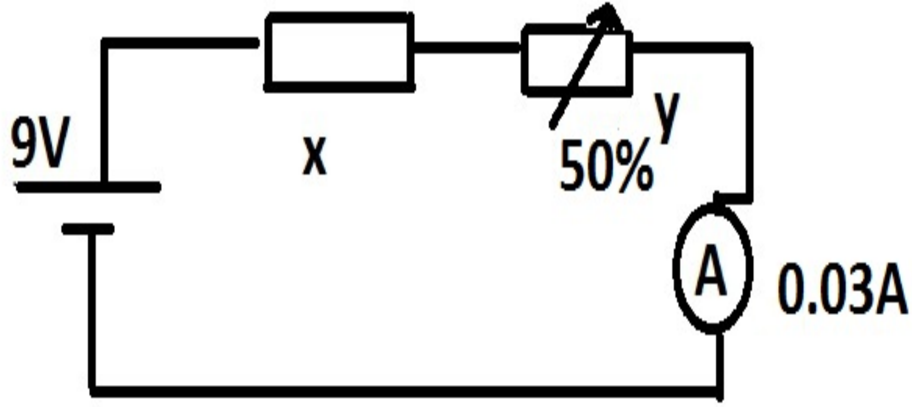
Problem X:

From the schematic diagrams below determine the resistance of the fixed resistor and the initial resistance of the rheostat in ohms.

1st Schematic diagram:



2nd Schematic diagram:



Solution X:

$$\frac{9}{x+0.3y} = 0.045$$

$$9 = 0.045(x + 0.3y)$$

$$9 = 0.045x + 0.0135y\,(1)$$

$$\frac{9}{x+0.5y} = 0.03$$

$9 = 0.03(x + 0.5y)$

$9 = 0.03x + 0.015y \, (2)$

Solving the simultaneous equations by the method of determinants:

$$\Delta = \begin{vmatrix} 0.045 & 0.0135 \\ 0.03 & 0.015 \end{vmatrix}$$

$$\Delta = 0.00027$$

$$\Delta_x = \begin{vmatrix} 9 & 0.0135 \\ 9 & 0.015 \end{vmatrix}$$

$$\Delta_x = 0.0135$$

$$\Delta_y = \begin{vmatrix} 0.045 & 9 \\ 0.03 & 9 \end{vmatrix}$$

$$\Delta_y = 0.135$$

$$y = \frac{\Delta_y}{\Delta}$$

$$x = \frac{\Delta_x}{\Delta}$$

$$y = \frac{0.135}{0.00027}$$

$$y = 500$$

$$x = \frac{0.0135}{0.00027}$$

$$x = 50$$

Therefore: the resistance of the fixed resistor denoted by x is 50 Ω, and the maximum resistance of the rheostat is 500 Ω, and the initial resistance of the rheostat is its resistance, in the first diagram. If the initial resistance of the resistor is denoted by R_1 then: $R_1 = y \cdot \frac{3}{10}$. Since $y = 500$ substituting it in the equation: $R_1 = 500 \cdot \frac{3}{10}$, $R_1 = \frac{1500}{10}$, $R_1 = 150$, therefore the resistance of the rheostat in the first diagram is 150 Ω.

Problem XI:

If the rate of a spring (k) is given by: $\frac{k^x}{k^y} = 5^{-5}$ and the values of x and y are given by these simultaneous equations:

$2x - y = -3\,(1)$

$7y - 8x = 33\,(2)$.

Find the elastic constant of this spring.

Solution XI:

Since k is the rate of the spring:

$$\frac{k^x}{k^y} = 5^{-5}$$

$$k^{(x-y)} = 5^{-5}$$

$$k = \sqrt[(x-y)]{5^{-5}}\,(3)$$

Solving the simultaneous equations by the method of determinants to find x, and y:

$$\Delta = \begin{vmatrix} 2 & -1 \\ -8 & 7 \end{vmatrix}$$

$$\Delta = 6$$

$$\Delta_x = \begin{vmatrix} -3 & -1 \\ 33 & 7 \end{vmatrix}$$

$$\Delta_x = 12$$

$$\Delta_y = \begin{vmatrix} 2 & -3 \\ -8 & 33 \end{vmatrix}$$

$$\Delta_y = 42$$

$$x = \frac{\Delta_x}{\Delta}$$

$$y = \frac{\Delta_y}{\Delta}$$

$$x = \frac{12}{6}$$

$$x = 2$$

$$y = \frac{42}{6}$$

$$y = 7$$

Substituting the value of x for 2 and y for 7 in (3):

$$k = \sqrt[-5]{5^{-5}}$$

$$k = 5$$

Therefore: the elastic constant of the spring is 5 N/ m.

Problem XII:

Two spring balances are supporting two separate masses if the sum of the readings of the spring balances is 5 Newtons, and the difference of their readings is − 1 Newton. Find the weights of the two masses.

Solution XII:

Since the weights of the two masses are the same as the reading of each spring balance, the reading of the first spring balance will be x, and the reading of the second spring balance will be y.

$x + y = 5 \,(1)$

$x - y = -1 \,(2)$

Solving the simultaneous equations by the method of determinants:

$$\Delta = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$$

$\Delta = 2$

$$\Delta_x = \begin{vmatrix} 5 & 1 \\ -1 & -1 \end{vmatrix}$$

$$\Delta_x = 4$$

$$\Delta_y = \begin{vmatrix} 1 & 5 \\ 1 & -1 \end{vmatrix}$$

$$\Delta_y = 6$$

$$x = \frac{\Delta_x}{\Delta}$$

$$y = \frac{\Delta_y}{\Delta}$$

$$x = \frac{4}{2}$$

$$x = 2$$

$$y = \frac{6}{2}$$

$$y = 3$$

Therefore: the reading of the first spring balance is 2 Newtons, which is equal to the weight attached to it, and the reading of the second spring balance is 3 Newtons, which is equal to the weight attached to it.

Problem XIII:

If the change in temperature of a body is 20° C, and the sum of the initial and final temperatures is 80° C, find the final and initial temperatures in Kelvins.

Solution XIII:

Let the change in temperature be denoted by $\Delta\,\theta$.

Let the final temperature be denoted by θ_2.

Let the initial temperature be denoted by θ_1.

$\theta_2 - \theta_1 = \Delta\,\theta\,(1)$

$\theta_2 + \theta_1 = 80\,(2)$

Solving the resulting simultaneous equations by the method of determinants:

Finding the determinant of the system:

$$\Delta = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$$

$\Delta = 2$

Finding the final temperature determinant:

$$\Delta_{\theta_2} = \begin{vmatrix} \Delta\theta & -1 \\ 80 & 1 \end{vmatrix}$$

$$\Delta_{\theta_2} = \Delta\theta + 80$$

Finding the initial temperature determinant:

$$\Delta_{\theta_1} = \begin{vmatrix} 1 & \Delta\theta \\ 1 & 80 \end{vmatrix}$$

$$\Delta_{\theta_1} = 80 - \Delta\theta$$

Evaluating the initial and final temperatures of the body:

$$\theta_2 = \frac{\Delta_{\theta_2}}{\Delta}$$

$$\theta_1 = \frac{\Delta_{\theta_1}}{\Delta}$$

$$\theta_2 = \frac{\Delta\theta + 80}{2}$$

$$\theta_2 = \frac{1}{2}\Delta\theta + 40$$

$$\theta_1 = \frac{80 - \Delta\theta}{2}$$

$$\theta_1 = 40 - \frac{1}{2}\Delta\theta$$

Since the change in temperature of the body ($\Delta\theta$) is 20° C, then:

$$\theta_2 = \frac{20}{2} + 40$$

$$\theta_2 = 10 + 40$$

$$\theta_2 = 50$$

$$\theta_1 = 40 - \frac{20}{2}$$

$$\theta_1 = 40 - 10$$

$$\theta_1 = 30$$

Converting the initial and final temperatures of the body in Celsius to Kelvins:

$$K_1 = \theta_1 + 273$$

$$K_2 = \theta_2 + 273$$

$K_1 = 30 + 273$

$K_1 = 303$

$K_2 = 50 + 273$

$K_2 = 323$

Therefore: the initial and final temperatures of the body are 303 K and 323 K respectively.

Problem XIV:

The sum, and difference of the teeth of a driving, and driven spur gear are 45, and 5 respectively. Find the number of teeth of the driving and driven spur gears.

Solution XIV:

Let the number of teeth of the driving and driven spur gears be given by x and y respectively:

$$x + y = 45 \quad (1)$$

$$x - y = 5 \quad (2)$$

Solving the resulting simultaneous equations by the method of determinants:

$$\Delta = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$$

$$\Delta = -2$$

$$\Delta_x = \begin{vmatrix} 45 & 1 \\ 5 & -1 \end{vmatrix}$$

$$\Delta_x = -50$$

$$\Delta_y = \begin{vmatrix} 1 & 45 \\ 1 & 5 \end{vmatrix}$$

$$\Delta_y = -40$$

$$x = \frac{\Delta_x}{\Delta}$$

$$y = \frac{\Delta_y}{\Delta}$$

$$x = \frac{-50}{-2}$$

$$x = 25$$

$$y = \frac{-40}{-2}$$

$$y = 20$$

Therefore: the driving and driven spur gears have 25 and 20 teeth respectively.

Problem XV:

The sum, and difference of the number of turns of the primary, and secondary of a transformer are 150, and 50 respectively.

(i) Find the number of turns in the primary, and secondary of the transformer.

(ii) Is the transformer a step up or step down?

(iii) Find the output of the transformer when 50 V AC is applied to the primary coil of the transformer.

Solution XV:

(i)

$N_p + N_s = 150 \, (1)$

$N_p - N_s = 50 \, (2)$

Solving the system of equations by the method of determinants:

$$\Delta = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$$

$$\Delta = -2$$

$$\Delta_{N_p} = \begin{vmatrix} 150 & 1 \\ 50 & -1 \end{vmatrix}$$

$$\Delta_{N_p} = -200$$

$$\Delta_{N_s} = \begin{vmatrix} 1 & 150 \\ 1 & 50 \end{vmatrix}$$

$$\Delta_{N_s} = -100$$

$$N_p = \frac{\Delta_{N_p}}{\Delta}$$

$$N_s = \frac{\Delta_{N_s}}{\Delta}$$

$$N_p = \frac{-200}{-2}$$

$$N_p = 100$$

$$N_s = \frac{-100}{-2}$$

$N_s = 50$

Therefore: the number of turns in the primary and secondary of the transformer are 100, and 50 turns respectively.

(ii)

If:

$N_s > N_p$

then it's a step up transformer.

But if:

$N_p > N_s$

then it's a step down transformer.

Since: $N_p = 100$ and $N_s = 50$, then:

$100 > 50$

$N_p > N_s$

Therefore: it's a step down transformer.

(iii)

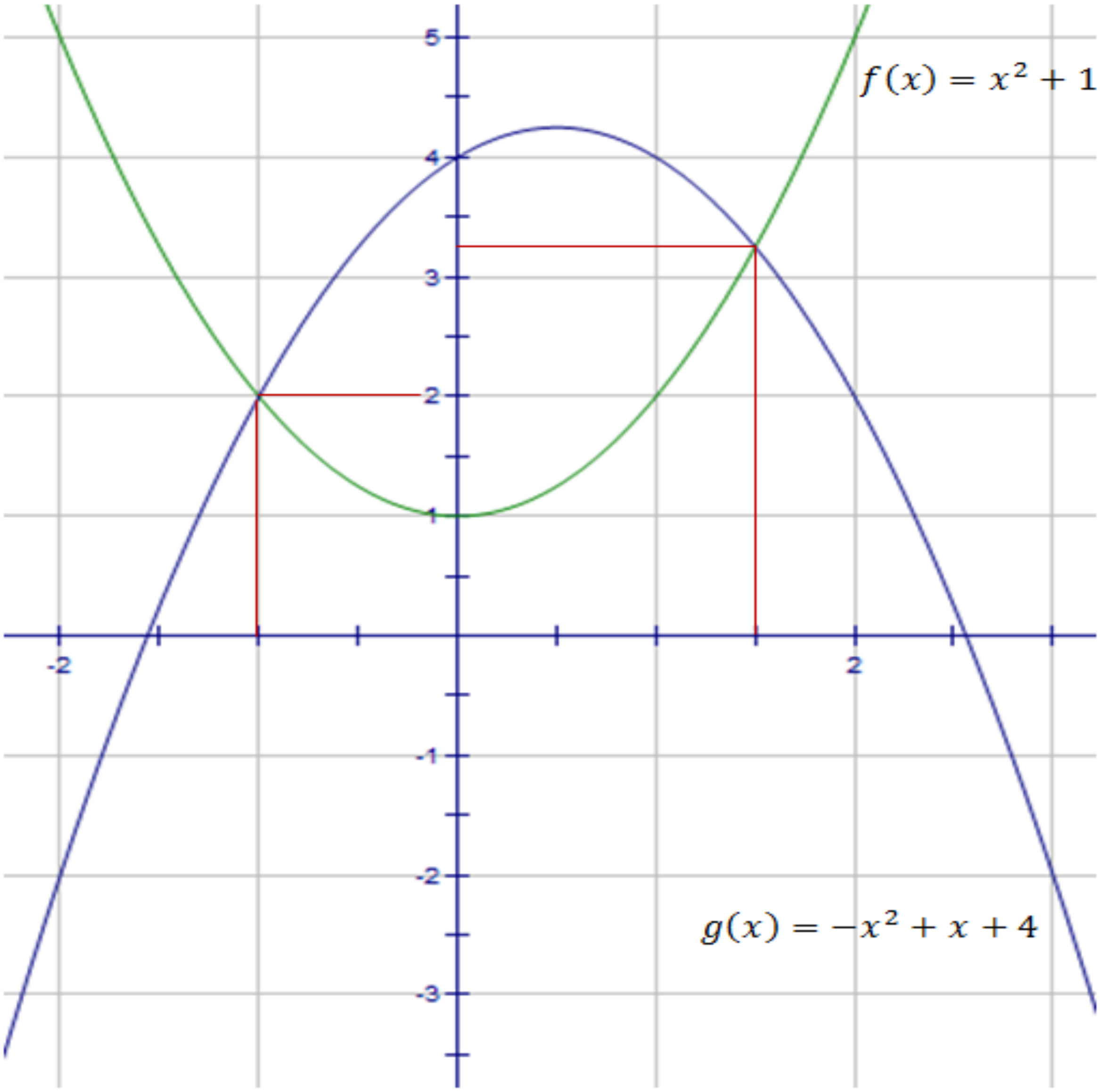
If the output voltage is denoted by V_o then:

$$V_o = \frac{N_s}{N_p} \cdot 50$$

$$V_o = \frac{50}{100} \cdot 50$$

$$V_o = \frac{1}{2} \cdot 50$$

$$V_o = 25$$



Chapter 3: Motivation for the Substitution Method in Physics

Problem XVI:

Two resistors x, and y when in series is 20 Ω, and when in parallel is 5 Ω. Find the resistances of x, and y in ohms.

Solution XVI:

$x + y = 20\,(1)$

$\frac{1}{x} + \frac{1}{y} = \frac{1}{5}\,(2)$

Isolating x from (1):

$x = 20 - y\,(3)$

Substituting the value of x for $20 - y$ in (2):

$$\frac{1}{(20-y)} + \frac{1}{y} = \frac{1}{5}$$

$$\frac{1y + 1(20-y)}{(20-y)y} = \frac{1}{5}$$

$$\frac{y + 20 - y}{20y - y^2} = \frac{1}{5}$$

$$\frac{20}{20y - y^2} = \frac{1}{5}$$

$$\frac{20y - y^2}{20} = 5$$

$$y - \frac{1}{20}y^2 = 5$$

$$\frac{-1}{20}y^2 + y = 5$$

$$\frac{1}{20}y^2 - y = -5$$

$$\frac{1}{20}y^2 - y + 5 = 0$$

$$1y^2 - 20y + 100 = 0$$

$$y^2 - 20y = -100$$

$$(y - 10)^2 - (10)^2 = -100$$

$$(y - 10)^2 - 100 = -100$$

$$(y - 10)^2 = -100 + 100$$

$$(y - 10)^2 = 0$$

$$y - 10 = \pm\sqrt{0}$$

$$y - 10 = \pm 0$$

$y = 10 \pm 0$

$y = 10$

Substituting the value of y for 10 in (3):

$x = 20 - 10$

$x = 10$

Therefore: resistor x is 10 Ω, and resistor y is 10 Ω.

Problem XVII:

x, and y are capacitance values of two capacitors in μF

$x + y = 10$, $\frac{1}{x} + \frac{1}{y} = \frac{10}{21}$, find x, and y.

Solution XVII:

$$x + y = 10 \ (1)$$

$$\frac{1}{x} + \frac{1}{y} = \frac{10}{21} \ (2)$$

Isolating x from (1):

$$x = 10 - y \ (3)$$

Substituting (3) in (2):

$$\frac{1}{(10 - y)} + \frac{1}{y} = \frac{10}{21}$$

$$\frac{1(10 - y) + 1y}{(10 - y)y} = \frac{10}{21}$$

$$\frac{10 - y + y}{10y - y^2} = \frac{10}{21}$$

$$\frac{10}{10y - y^2} = \frac{10}{21}$$

$$\frac{10y - y^2}{10} = \frac{21}{10}$$

$$y - \frac{1}{10}y^2 = \frac{21}{10}$$

$$10y - y^2 = 21$$

$$-y^2 + 10y = 21$$

$$y^2 - 10y = -21$$

$$y^2 - 10y + 21 = 0$$

$$(y - 5)^2 - (-5)^2 = -21$$

$$(y - 5)^2 - 25 = -21$$

$$(y - 5)^2 = -21 + 25$$

$$(y - 5)^2 = 4$$

$$y - 5 = \pm\sqrt{4}$$

$$y - 5 = \pm 2$$

$y = 5 \pm 2$

$y = 7$

or

$y = 3$

Substituting the value of y for 7 and 3 in (3):

$x = 10 - 7$

$x = 3$

or

$x = 10 - 3$

$x = 7$

Therefore: we have two sets of capacitance values. Capacitor x and y, may be $3\mu F$ and $7\mu F$ respectively, or $7\mu F$, and $3\mu F$ respectively.

Problem XVIII:

The length of the hypothenuse of an inclined plane is

$\sqrt{40}$ cm, and the velocity ratio of the inclined plane is $\frac{\sqrt{40}}{6}$. Find the lengths of the other two sides of the inclined plane.

Solution XVIII:

$$VR = \frac{1}{\sin\theta}$$

$$\sin\theta = \frac{y}{\sqrt{40}}$$

$$VR = \frac{1}{(y/\sqrt{40})}$$

$$VR = \frac{\sqrt{40}}{y}$$

$$VR = \frac{\sqrt{40}}{6}$$

$$\frac{\sqrt{40}}{y} = \frac{\sqrt{40}}{6}$$

$$y = 6$$

From the Pythagoras' theorem:

$$x^2 + y^2 = (\sqrt{40})^2$$

$x^2 + y^2 = 40$

Substituting the value of y for 6:

$x^2 + 6^2 = 40$

$x^2 + 36 = 40$

$x^2 = 40 - 36$

$x^2 = 4$

$x = \sqrt{4}$

$x = 2$

Or it could have been solved this way:

$\sqrt{x^2 + y^2} = \sqrt{40}$ (i)

$\frac{\sqrt{x^2 + y^2}}{y} = \frac{\sqrt{40}}{6}$ (ii)

Isolating x from (i):

$x^2 + y^2 = 40$

$$x^2 = 40 - y^2$$

$$x = \sqrt{40 - y^2} \text{ (iii)}$$

Substituting (iii) in (ii):

$$\frac{\sqrt{[(\sqrt{40 - y^2})^2 + y^2]}}{y} = \frac{\sqrt{40}}{6}$$

$$\frac{\sqrt{(40 - y^2 + y^2)}}{y} = \frac{\sqrt{40}}{6}$$

$$\frac{\sqrt{40}}{y} = \frac{\sqrt{40}}{6}$$

$$y = \frac{\sqrt{40}}{(\sqrt{40}/6)}$$

$$y = \sqrt{40} \cdot \frac{6}{\sqrt{40}}$$

$$y = 6$$

Substituting the value of y for 6 in (iii):

$$x = \sqrt{(40 - 6^2)}$$

$$x = \sqrt{(40 - 36)}$$

$x = \sqrt{4}$

$x = 2$

Since: x is the adjacent of the hypothenuse, and y is the opposite of the hypothenuse, therefore the adjacent of the hypothenuse, and the opposite of the hypothenuse are 2 cm, and 6 cm respectively.

Problem XIX:

A person is standing before the window of a building, and the person uses an angle sextant to find the angle of elevation from the window to the top of the building, and the angle sextant reads 20°, but when the person moves closer to the building by one meter the angle sextant reads 30°, find the distance from the window to the top of the building.

Solution XIX:

Let the distance from the window of the building to the top of the building be h.

Let the total distance from the person to the building be y.

Let the remaining distance to the building be x.

$$\tan 20 = \frac{h}{(1+x)} \ (1)$$

$$\tan 30 = \frac{h}{x} \ (2)$$

Isolating h from (2):

$$h = x \tan 30 \ (3)$$

Substituting (3) in (1):

$$\tan 20 = \frac{x\tan 30}{(1+x)}$$

$$(1 + x)\tan 20 = x\tan 30$$

$$\tan 20 + x\tan 20 = x\tan 30$$

$$\tan 20 = x\tan 30 - x\tan 20$$

$$\tan 20 = x(\tan 30 - \tan 20)$$

$$x = \frac{\tan 20}{(\tan 30 - \tan 20)}$$

$x \approx 1.70$ to 2 d.p

Substituting the value of x for $\frac{\tan 20}{(\tan 30 - \tan 20)}$ in (3):

$$h = [\frac{\tan 20}{(\tan 30 - \tan 20)}]\tan 30$$

$$h = \frac{\tan 30\tan 20}{(\tan 30 - \tan 20)}$$

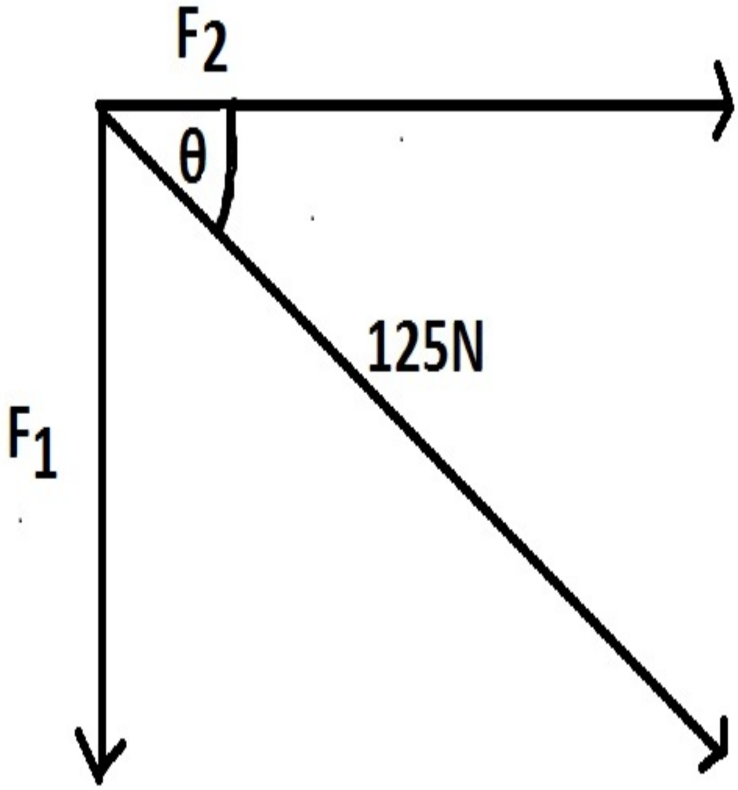
$h \approx 0.98$ to 2 d.p

Therefore: the distance from the window of the building to the top is $\frac{\tan 30 \tan 20}{(\tan 30 - \tan 20)}$ of a meter or approximately 0.98 of a meter.

Problem XX:

Two forces F_1, and F_2 are acting at right angles, and the resultant force of F_1, and F_2 is 125 N if F_1 is acting in the opposite direction, and F_2 is acting in the adjacent direction and the tangent ratio is 0.5 what are the values of F_1, and F_2, and find the angle θ formed between the resultant and F_2 if F_1 and F_2 are positive.

Solution XX:



Not drawn to scale

$\tan\theta = 0.5$

$\theta = \tan^{-1}(0.5)$

$\theta \approx 26.6°$ to 1 d.p

From Pythagoras' theorem:

$F_1{}^2 + F_2{}^2 = 125^2$ (1)

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan\theta = 0.5$$

$$0.5 = \frac{\text{opposite}}{\text{adjacent}}$$

Since: F_1 is acting in the opposite direction and F_2 is acting in the adjacent direction, therefore:

$$0.5 = \frac{F_1}{F_2} \quad (2)$$

Solving for F_1 in (2):

$$F_1 = 0.5F_2 \quad (3)$$

Substituting F_1 for $0.5F_2$ in (1):

$$(0.5F_2)^2 + F_2{}^2 = 125^2$$

$$0.25F_2{}^2 + F_2{}^2 = 125^2$$

$$1.25F_2{}^2 = 125^2$$

$$F_2{}^2 = \frac{125^2}{1.25}$$

$F_2^{\,2} = 12500$

$F_2 = \sqrt{12500}$

$F_2 \approx 111.80$ to 2 d.p

Substituting the value of F_2 for $\sqrt{12500}$ in (3):

$F_1 = 0.5\sqrt{12500}$

$F_1 \approx 55.90$ to 2 d.p

Therefore: the force acting in the opposite direction (F_1) is $\sqrt{12500}$ N, or approximately to 2 d.p 111.80 N, and the force acting in the adjacent direction (F_2) is $0.5\sqrt{12500}$ N or, approximately to 2 d.p 55.90 N, and the angle (θ) formed between F_2 which is $\sqrt{12500}$ N and the resultant which is 125 N is $[\tan^{-1}(0.5)]°$ or approximately to 1 d.p 26.6°.

Problem XXI:

The velocity ratio of a wheel, and axle is $\frac{5}{2}$, and the sum of the radii of the wheel and axle is 7 cm.

(i) Find the radii of the wheel, and axle.

(ii) Find the circumferences of the wheel, and the axle in terms of π in cm.

Solution XXI:

(i)

Let the radius of the wheel be R, and the radius of the axle be r.

Ergo:

$$VR = \frac{R}{r}$$

Since: $VR = \frac{5}{2}$

Then:

$\frac{R}{r} = \frac{5}{2}$ (1)

And $R + r = 7$:

$R + r = 7$ (2)

Isolating R from (2):

$R = 7 - r$ (3)

Substituting (3) in (1):

$$\frac{5}{2} = \frac{7 - r}{r}$$

$$\frac{5}{2}r = 7 - r$$

$$\frac{5}{2}r + r = 7$$

$$\frac{7}{2}r = 7$$

$$7r = 14$$

$$r = \frac{14}{7}$$

$$r = 2$$

Substituting the value of r for 2 in (3):

$R = 7 - 2$

$R = 5$

Hence: the radii of the wheel, and axle are 5, and 2 centimeters respectively.

(ii)

arc of wheel $= 2\pi R$ (i)

arc of axle $= 2\pi r$ (ii)

Since: $R = 5$ and $r = 2$, by substituting in (i) and (ii):

arc of wheel$= 2\pi \cdot 5$

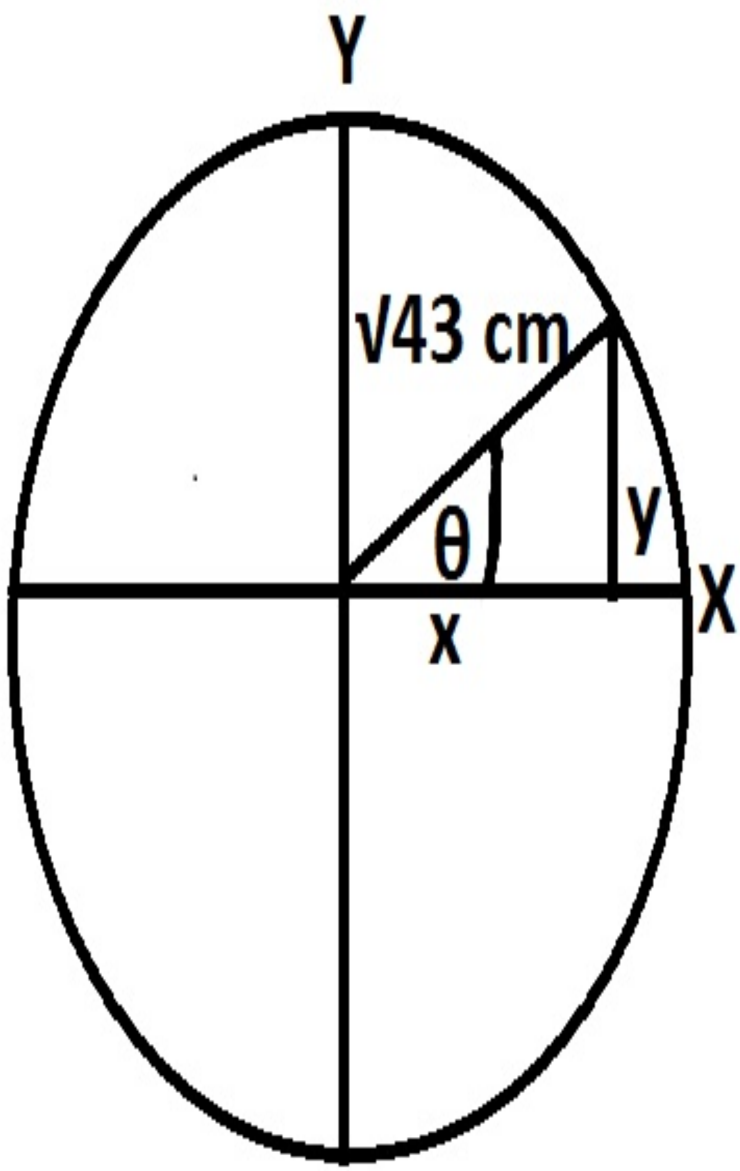
arc of wheel $= 10\pi$

arc of axle $= 2\pi \cdot 2$

arc of axle $= 4\pi$

Therefore: the circumferences of the wheel, and axle in terms of π are 10π cm, and 4π cm respectively.

Problem XXII:



Not drawn to scale

If the difference of x and y is 2 cm find the values of x and y in cm.

Solution XXII:

From Pythagoras' theorem:

$$\sqrt{34} = \sqrt{x^2 + y^2}$$

By squaring both sides:

$$34 = x^2 + y^2$$

$$x^2 + y^2 = 34 \, (1)$$

Because the difference of x and y is 2:

$$x - y = 2 \, (2)$$

Isolating x from (2):

$$x = 2 + y \, (3)$$

Substituting (3) in (1):

$$(2 + y)^2 + y^2 = 34$$

$$4 + 4y + y^2 + y^2 = 34$$

$$4 + 4y + 2y^2 = 34$$

$4y + 2y^2 = 34 - 4$

$4y + 2y^2 = 30$

Making the coefficient of y^2 1:

$y^2 + 2y = 15$

Converting to the standard form of quadratic equations:

$y^2 + 2y - 15 = 0$

Solving the quadratic equation by completing the square:

$(y + 1)^2 - 1^2 = 15$

$(y + 1)^2 = 16$

$y + 1 = \pm\sqrt{16}$

$y + 1 = \pm 4$

$y = -1 \pm 4$

$y = 3$ or $y = -5$

Substituting the values of y in (3):

$x = 2 + 3$

$x = 5$

or

$x = 2 - 5$

$x = -3$

Hence we obtain two sets of solutions:

$(5, 3)$ and $(-3, -5)$

Since the triangle lies on the first quadrant the true set of solutions is:

$(5, 3)$

Problem XXIII:

If the force(F) and Young's modulus (Y) of a system are given by these equations:

$F = ke$

$Y = \frac{F/A}{e/1}$

find the value of Y in terms of k, l, and A, and the value of e in terms of F, L, A and Y.

Solution XXIII:

$F = ke$ (1)

$Y = \frac{F/A}{e/1}$ (2)

Isolating e from (1):

$e = F/k$ (3)

Substituting (3) in (2):

$$Y = \frac{F/A}{(F/K)/1}$$

$$Y = \frac{F / A}{(F / kl)}$$

$$Y = \frac{(F / A)kl}{F}$$

$$Y = \frac{F\,kl / A}{F}$$

$$Y = \frac{F\,kl}{F\,A}$$

$$Y = \frac{kl}{A}$$

Therefore: $Y = \frac{kl}{A}$ is the Young's modulus equation in terms of k, l, and A.

Making k the subject of the formula in the Young's modulus equation:

$$Y = \frac{kl}{A}$$

$$kl = AY$$

$$k = \frac{AY}{l}$$

Substituting the value of k for $\frac{AY}{l}$ in (3):

$$e = \frac{F}{(AY/l)}$$

$$e = \frac{Fl}{AY}$$

Therefore: the value of e in terms of F , l, A and Y is $\frac{Fl}{AY}$.

Problem XXIV:

If a system containing several derivatives of unknown functions is given by:

$$\frac{dF}{de} + \frac{d^2W}{de^2} = 2k$$
$$\frac{dF}{de} - \frac{d^2W}{de^2} = 0$$

find: $\frac{dF}{de}$, $\frac{d^2W}{de^2}$, $\frac{dW}{de}$, F and W.

And then infer meaning from the system.

Solution XXIV:

$$\frac{dF}{de} + \frac{d^2W}{de^2} = 2k\,(1)$$

$$\frac{dF}{de} - \frac{d^2W}{de^2} = 0\,(2)$$

Isolating $\frac{dF}{de}$ in (1):

$$\frac{dF}{de} = 2k - \frac{d^2W}{de^2}\,(3)$$

Substituting (3) in (2):

$$2k - \frac{d^2W}{de^2} - \frac{d^2W}{de^2} = 0$$

$$2k - 2\left(\frac{d^2W}{de^2}\right) = 0$$

$$-2\left(\frac{d^2W}{de^2}\right) = -2k$$

$$\frac{d^2W}{de^2} = k$$

Substituting the value of $\frac{d^2W}{de^2}$ for k in (3):

$$\frac{dF}{de} = 2k - k$$

$$\frac{dF}{de} = k$$

Solving the first and second order differential equations, to find F and W respectively:

$$\frac{dF}{de} = k$$

$$F = \int_0^e k\,de$$

$$F = ke$$

$$\frac{d^2W}{de^2} = k$$

$$\frac{dW}{de} = \int_0^e k\,de$$

$$\frac{dW}{de} = ke$$

$$W = \int_0^e ke\, de$$

$$W = \frac{1}{2}ke^2$$

Inference:

Since the derivatives of W higher than the second order results to zero this therefore draws the conclusion that:

$$\frac{dF}{de} - \frac{d^2W}{de^2} = 0 = \frac{d^3W}{de^3} = \frac{d^2F}{de^2} = \frac{d^4W}{de^4} = \frac{d^5W}{de^5} = \frac{d^3F}{de^3} \ldots$$

ad infinitum.

Since $\frac{d^2W}{de^2}$ and $\frac{dF}{de}$ are both equall to k this therefore draws the parallel that:

$$\frac{dF}{de} = \frac{d^2W}{de^2}.$$

And $\frac{dF}{de} + \frac{d^2W}{de^2} = 2k$ represents the stiffness(k) of two springs that have the same stiffness constant in parallel.

Since $\frac{dW}{de} = ke$ and F=ke, then: $\frac{dW}{de} = F$, this means that the first order derivative of work is force.

Problem XXV:

The reactance of two capacitors in parallel connected to a 220 V 50 Hz AC supply is: $\frac{1}{5000\pi}$ MΩ, but when in series is: $\frac{1}{1200\pi}$ MΩ, find the values of the first and second capacitors in nano Farads.

Solution XXV:

$$X_{c_p} = \frac{1}{2\pi f C_p}$$

$$X_{c_p} = \frac{1}{2\pi f (C_1 + C_2)}$$

$$\frac{1}{5000\pi} = \frac{1}{100\pi(C_1 + C_2)}$$

$$5000\pi = 100\pi(C_1 + C_2)$$

$$C_1 + C_2 = 50 \, (1)$$

$$X_{c_s} = \frac{1}{2\pi f C_s}$$

$$X_{c_s} = \frac{1}{2\pi f \left(\frac{C_1 C_2}{C_1 + C_2}\right)}$$

$$\frac{1}{1200\pi} = \frac{1}{100\pi\left(\frac{C_1C_2}{C_1+C_2}\right)}$$

$$\frac{C_1C_2}{C_1+C_2} = 12\ (2)$$

Isolating C_1 from (1):

$$C_1 = 50 - C_2\ (3)$$

Substituting (3) in (2):

$$\frac{(50-C_2)C_2}{50-C_2+C_2} = 12$$

$$\frac{50C_2-C_2^2}{50} = 12$$

$$50C_2 - C_2^2 = 600$$

$$-C_2^2 + 50C_2 = 600$$

$$C_2^2 - 50C_2 = -600$$

$$C_2^2 - 50C_2 + 600 = 0$$

Solving the quadratic equation by completing the square:

$$C_2^2 - 50C_2 = -600$$

$$(C_2 - 25)^2 - (-25)^2 = -600$$

$$(C_2 - 25)^2 - 625 = -600$$

$$(C_2 - 25)^2 = -600 + 625$$

$$(C_2 - 25)^2 = 25$$

$$C_2 - 25 = \pm\sqrt{25}$$

$$C_2 - 25 = \pm 5$$

$$C_2 = 25 \pm 5$$

$C_2 = 30$ or $C_2 = 20$

Substituting the value of y for 30 and 20 in (3):

$$C_1 = 50 - 30$$

$$C_1 = 20$$

or

$$C_1 = 50 - 20$$

$$C_1 = 30$$

Ergo: the values of the first and second capacitors can either be 30 and 20 nano Farads respectively, or 20 and 30 nano Farads respectively.

Contact:

ScientiamMentisOperandi@gmail.com