Background: Go over all background sections before going to your actual assigned lab session.

Verification: When you have completed a step and want verification, simply demonstrate the step to the instructor.

Lab Report: It is only necessary to turn in a report on Sections that require you to make graphs and to give short explanations. You are asked to label the axes of your plots and include a title for every plot. If you are unsure about what is expected, ask the instructor.

1 Background

This Lab is about frequency analysis in Matlab.

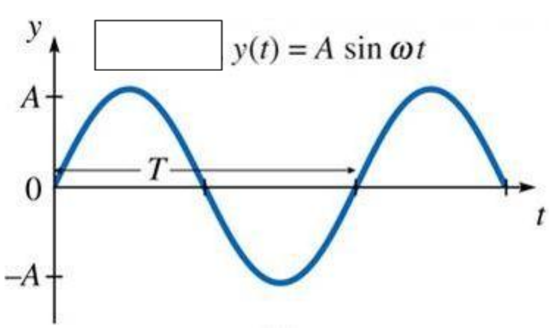
Goal

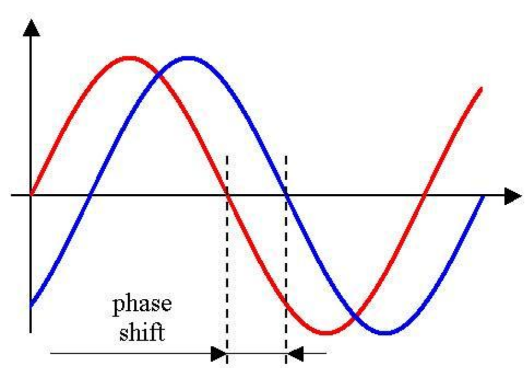
The goal of the laboratory project is to introduce the Fast Fourier Transform (FFT) algorithm for efficient computer calculation of the Fourier transform and to investigate some of the Fourier Transform's properties. The specific goals are:

1. Understanding discrete sampling and the Nyquist frequency.
2. Understanding Discrete Fourier transform.
3. Performing FFT and visualization of the result.

Pure Tones or Sine waves.

A sine wave or pure tone has (I) infinite duration, but only one frequency, (II) is periodic and has a phase and (III) is known as the "harmonic function". A sine wave can be represented by a sine function: $y(t) = A \cdot \sin(2\pi f \cdot t + \phi)$ or $y(t) = A \cdot \sin(\omega \cdot t + \phi)$





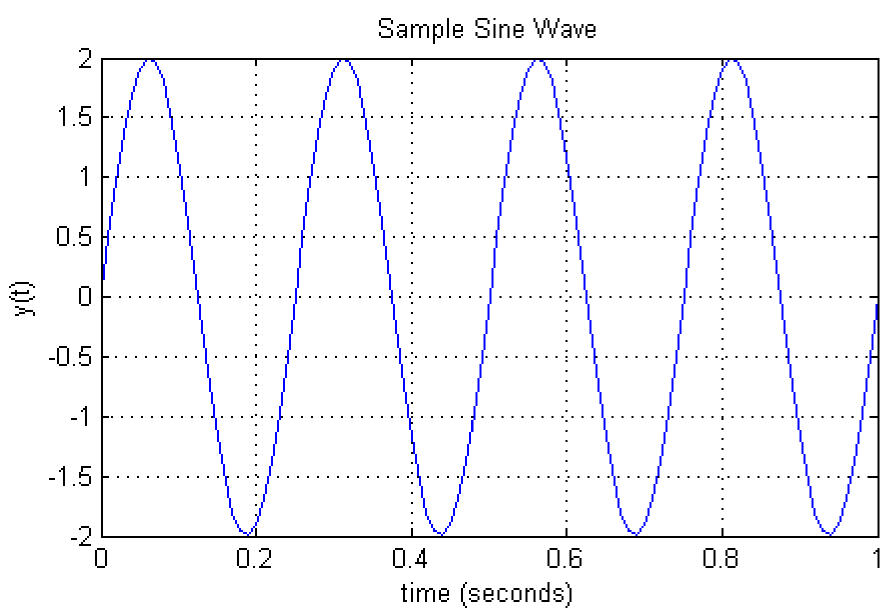
A represents the Amplitude, *f* frequency, ϕ phase and ω is the angular frequency. T is the period of the sine wave, were *f* = 1 / T.

A simple Matlab implementation of example of a pure tone:

```
fo = 4;    %frequency of the sine wave
Fs = 100; %sampling rate
Ts = 1/Fs; %sampling time interval
t = 0:Ts:1-Ts; %sampling period
n = length(t); %number of samples
y = 2*sin(2*pi*fo*t); %the sine curve

%plot the cosine curve in the time domain
sinePlot = figure;
plot(t,y)
xlabel('time (seconds)')
ylabel('y(t)')
title('Sample Sine Wave')
grid
```

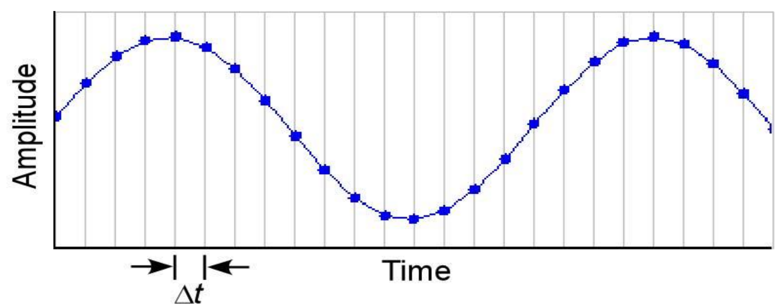
Here's what we get:



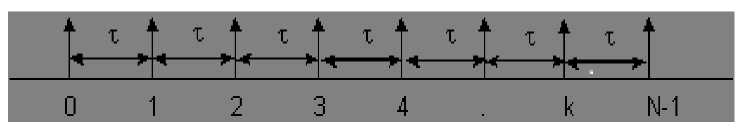
Discrete Sampling and Digital Representation of Sound (Nyquist, Aliasing).

This section provides a brief explanation of how sound is represented digitally. An understanding of the basic principles introduced here will be helpful in creating sound signals in Matlab. Before a continuous, time-varying signal such as sound can be manipulated or analysed with a digital computer, the signal must be *acquired* or *digitized* by a hardware device called an analogue-to-digital (A/D) converter, or *digitizer*. The digitizer repeatedly measures or *samples* the instantaneous voltage amplitude of a continuously varying (analogue) input signal at a particular sampling rate, typically thousands or tens of thousands of times per second (see figure below). In the case of an audio signal, this time-varying voltage is proportional to the sound pressure at a device such as a microphone. The digital representation of a signal created by the digitizer thus consists of a sequence of numeric values representing the amplitude of the original waveform at discrete, evenly spaced points in time.

Sampling to create digital representation of a pure tone signal. The blue sinusoidal curve represents the continuous analog waveform being sampled. Measurements of the instantaneous amplitude of the signal are taken at a sampling rate of $1/\Delta t$. The resulting sequence of amplitude values is the digitized signal.

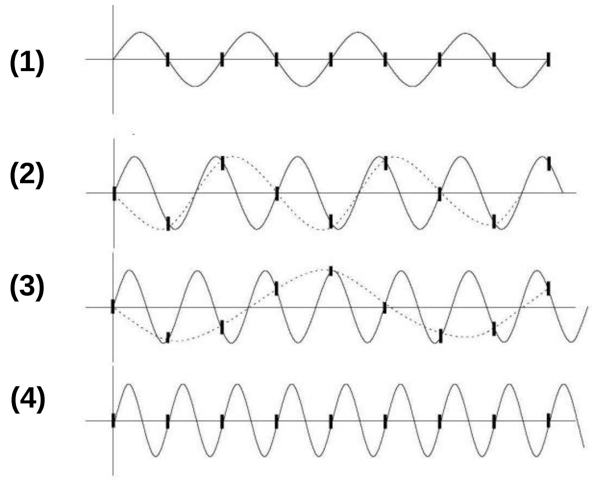


Sampling time Δt or τ. Time between each sample. It is also the inverse of the sampling frequency. If sampling frequency, Fs = 20 samples/sec, then t = 1/Fs = .05 sec Total Number of Samples: N. The total number of samples collected or available = N. Sequence Time Length: T, the time length of the collected sequence and is equal to the product of the sample time and the total number of samples = N Δt Sample index: k. The sequence time is no longer continuous so instead of Δt, we use a discrete time measure called k^{th} sample. This is an index of the samples. Its range extends from 0 to N-1, where N is the last sample. Each k^{th} sample of total N samples, is located at time k times Δt .

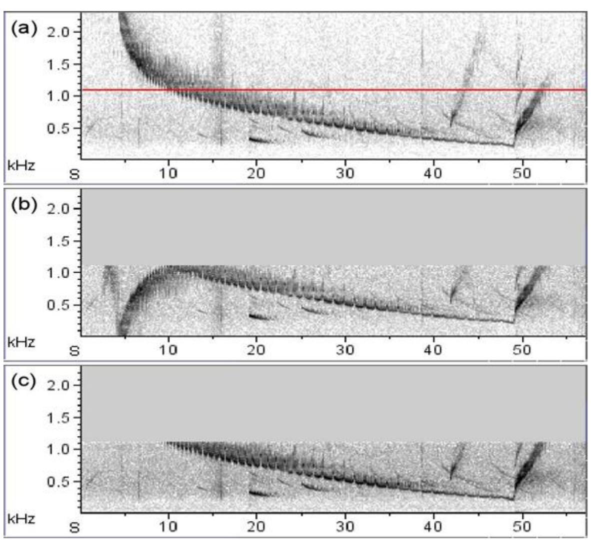


The precision with which the digitized signal represents the continuous signal depends on two parameters of the digitizing process: the rate at which amplitude measurements are made (the *sampling rate* or *sampling frequency, Fs*), and the number of bits used to represent each amplitude measurement (the *sample size* N or *bit depth*). The more frequently a signal is sampled, the more precisely the digitized signal represents temporal changes in the amplitude of the original signal. The sampling rate that is required to make an acceptable representation of a waveform depends on the signal's frequency. More specifically, the sampling rate must be more than twice as high as the highest frequency contained in the signal. Otherwise, the digitized signal will have frequencies represented in it that were not actually present in the original at all. This appearance of phantom frequencies as an artefact of inadequate sampling rate is called *aliasing.*

The highest frequency that can be represented in a digitized signal without aliasing is called the *Nyquist frequency*, and is equal to half the frequency at which the signal was digitized. If the only energy above the Nyquist frequency in the analogue signal is in the form of low-level, broadband noise, the effect of aliasing is to increase the noise in the digitized signal. However, if the spectrum of the analogue signal contains any peaks above the Nyquist frequency, the spectrum of the digitized signal will contain spurious peaks below the Nyquist frequency as a result of aliasing.



Upper trace: A sine wave is sampled near or at the Nyquist frequency. In the example given above, 4 complete sine waves are shown but 8 samples are taken. High-frequency components of the data above the Nyquist frequency will be lost. Traces 2, 3 and 4 show how sampling looks like when occurring below the Nyquist frequency. The dotted lines show the effect of under sampling. Can you explain why we can't see a dotted line in trace 4?

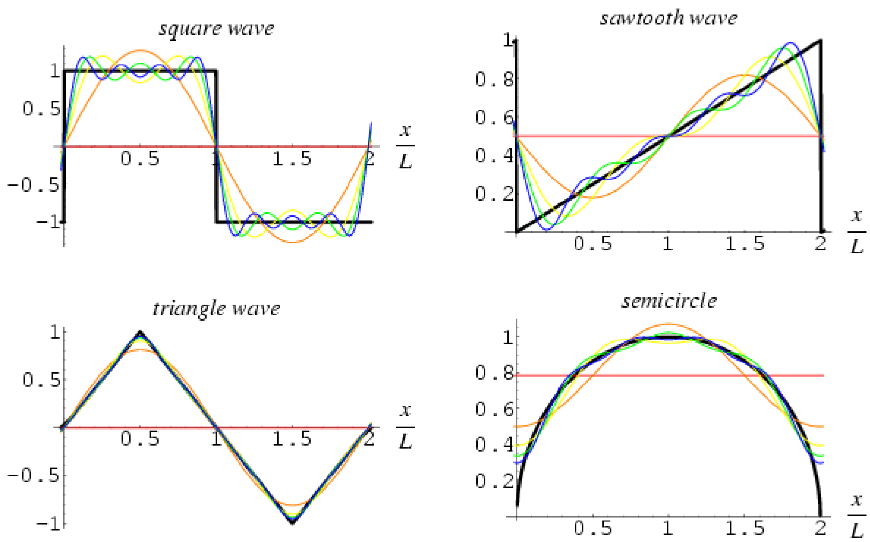


Aliasing in ***spectrograms***. **(a)** Spectrogram of a bearded seal song signal digitized at 11025 Hz. All of the energy in the signal is below the Nyquist frequency (5512.5 Hz); only the lowest 2300 Hz is shown. The red line is at 1103 Hz, one-fifth of the Nyquist frequency. **(b)** The same signal sampled at 2205 Hz (one-fifth of the original rate; Nyquist frequency, 1102.5 Hz) without an anti-aliasing filter. The frequency down sweep in the first ten seconds of the original signal appears in inverted form in this under sampled signal, due to aliasing. **(c)** Same signal as in (b), but now passed through a low-pass (anti-aliasing) filter with a cutoff of 1100 Hz before being digitized. The down sweep in the first 10 sec of the original signal, which exceeds the Nyquist frequency, does not appear because it was blocked by the filter.

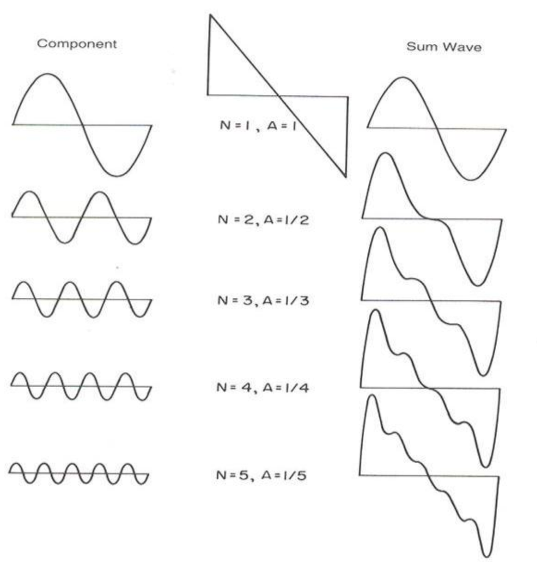
In spectrograms, aliasing is recognizable by the appearance of one or more inverted replicates of the real signal, offset in frequency from the original. Thus, when an analogous signal is sampled at a fixed rate, there is a maximum frequency called the Nyquist frequency (Fn) at which sampling begins to lose information. This critical frequency is two sample points per cycle $Fn = 1 / (2\ Fs)$ of the highest frequency in the original sound signal.

Fourier series and Synthesis (Why use $e^{(j n \omega_o t)}$?).

Up to now we only considered pure tones. Nonetheless, real sounds are broadband and thus contain many pure tones. A Fourier series is an expansion of a harmonic function in terms of an infinite sum of sines and cosines. Fourier series make use of the orthogonal relationships of the sine and cosine functions. The computation and study of Fourier series is known as harmonic analysis and is extremely useful as a way to break up an *arbitrary* periodic function or sound into a set of pure tones. Examples of successive approximations to common functions using Fourier series are illustrated below.



In music, if a note has an fundamental frequency, f, integer multiples of that frequency, $2f, 3f, 4f$ and so on, are known as *harmonics*. As a result, the mathematical study of overlapping waves is called harmonic analysis. An example of a sawtooth wave Fourier series is given below.



The fundamental frequency (or sine wave in this case), N=1, of the sawtooth wave is given by the component (or harmonic) with the lowest frequency. Al other components have higher frequencies and are called higher harmonics of the sawtooth wave, which have higher frequencies. That is, component N=4 has a frequency that is four times higher than that of component N=1. Also notice that a similar relationship occurs for the amplitude of each component. In this case, the fundamental component, N=1, has the largest amplitude, A=I, and e.g., component N=4, has an amplitude that is four times smaller, A=I/4. Finally, a similar relationship occurs for the phase of each component. In this case, however, the phase is zero for all components.