252R 기초 데이터 분석 및 실습

5. Mathematics (3)

November 19, 2025

정재준

Welcome To Industrial Engineering

고려대학교
KOREA UNIVERSITY

Contents

252R 기초 데이터 분석 및 실습

Intro

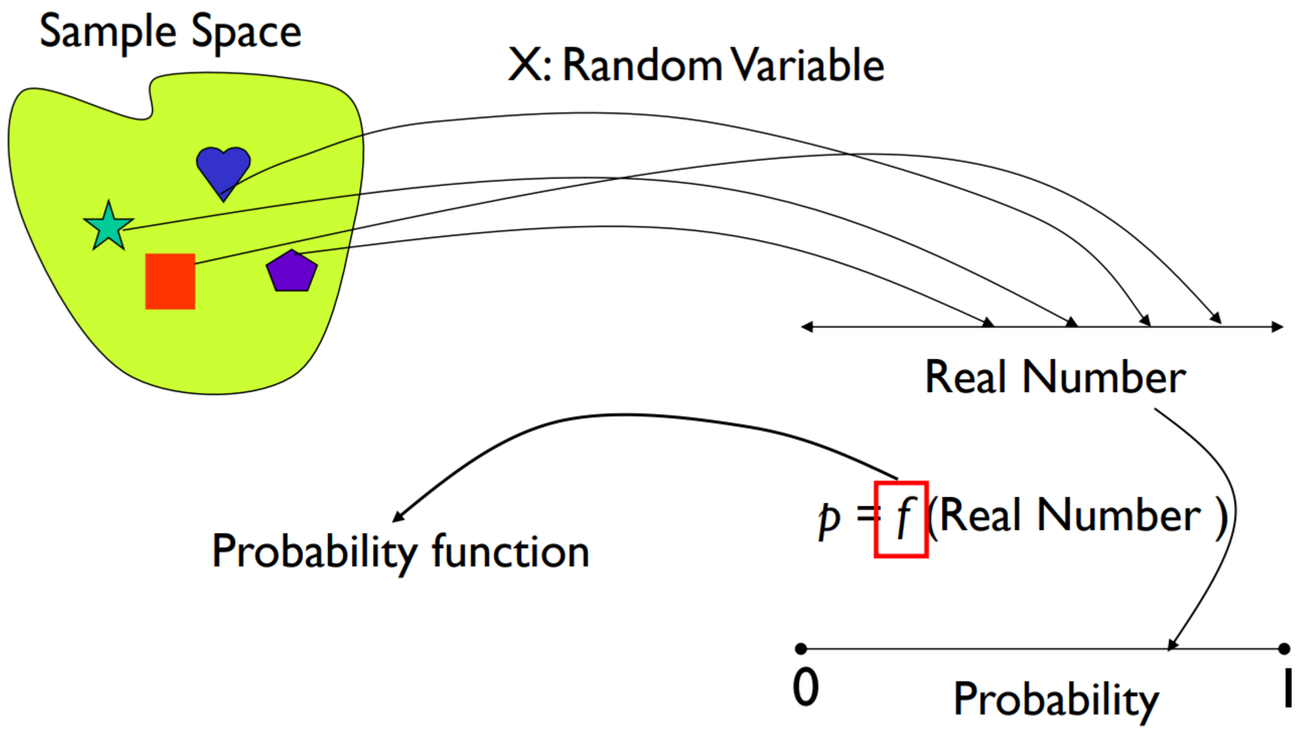
Mathematics (3)

- 확률공간, 사건, 확률변수의 개념을 이해하고 각 관계를 설명할 수 있다.
- 이산/연속 확률변수를 구분하여 각 확률변수의 분포를 PMF/PDF로 나타낼 수 있다.
- 조건부확률을 계산하고 독립성의 정의와 직관을 이해한다.
- 기대값, 분산, 공분산의 정의를 알고, 이를 분포와 연관지어 이해한다.
- 주요 확률 분포의 모수와 특성을 이해하고, 각 분포가 어떤 데이터와 상황을 모델링하는지 설명할 수 있다.

Probability Theory Basics

Mathematics (3)

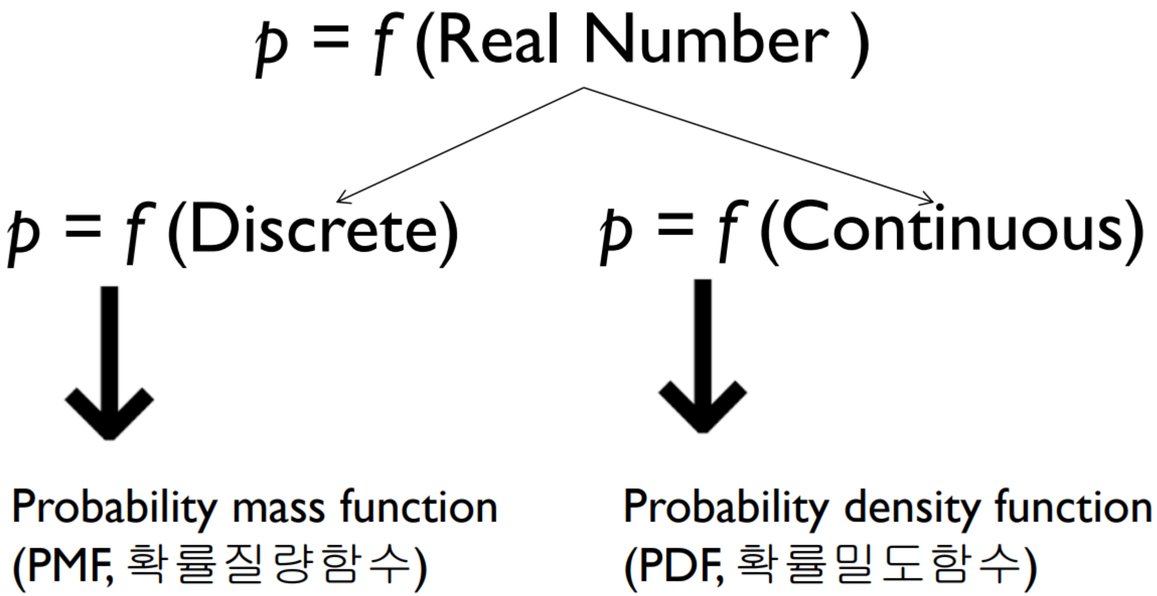
- Sample Space
- Event
- Random Variable
- Probability Function



Probability Theory Basics

Mathematics (3)

- **Discrete or Continuous**



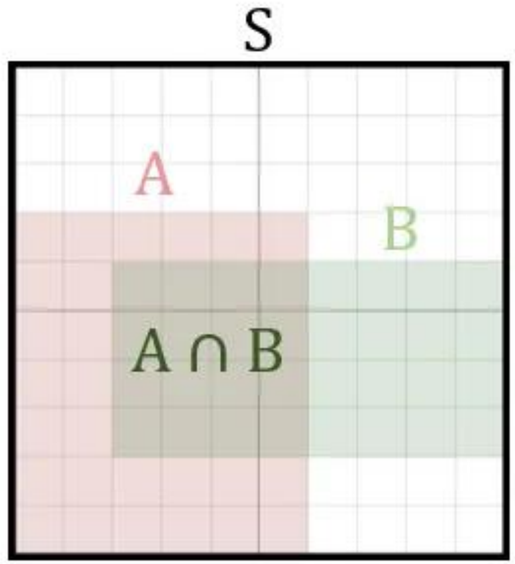
Probability Theory Basics

Mathematics (3)

▪ Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}}$$

$$= \frac{n(A \cap B)}{n(B)}$$



Summary Statistics

Mathematics (3)

▪ Expectation, Variance, and Covariance(Correlation)

- Expectation of a discrete random variable, X

$$\mathrm{E}[X] = \sum_i x_i f_X(x_i)$$

- Expectation of a continuous random variable, X

$$E[X] = \int x\, f_X(x) dx$$

Summary Statistics

Mathematics (3)

▪ Expectation, Variance, and Covariance(Correlation)

$$\begin{aligned} \mathrm{V}(X) &= E[\{X - E(X)\}^2] \\ &= E[X^2 - 2XE(X) + \{E(X)\}^2] \\ &= E(X^2) - 2\{E(X)\}^2 + \{E(X)\}^2 \\ &= E(X^2) - \{E(X)\}^2 \end{aligned}$$

$$\mathrm{SD}[X] = \sqrt{\mathrm{V}[X]}$$

$$Cov(X,Y) = E[\{X - E(X)\}\{Y - E(Y)\}]$$

$$Corr(X,Y) = \frac{Cov(X,Y)}{\sqrt{V(X) \cdot V(Y)}}$$

$$-1 \leq Corr(X,Y) \leq 1$$

Key Probability Rules

Mathematics (3)

- Sum Rule
- Product Rule
- Bayes' Theorem

$$p(\boldsymbol{x}) = \begin{cases} \sum_{\boldsymbol{y} \in \mathcal{Y}} p(\boldsymbol{x}, \boldsymbol{y}) & \text{if } \boldsymbol{y} \text{ is discrete} \\ \int_{\mathcal{Y}} p(\boldsymbol{x}, \boldsymbol{y}) \mathrm{d}\boldsymbol{y} & \text{if } \boldsymbol{y} \text{ is continuous} \end{cases},$$

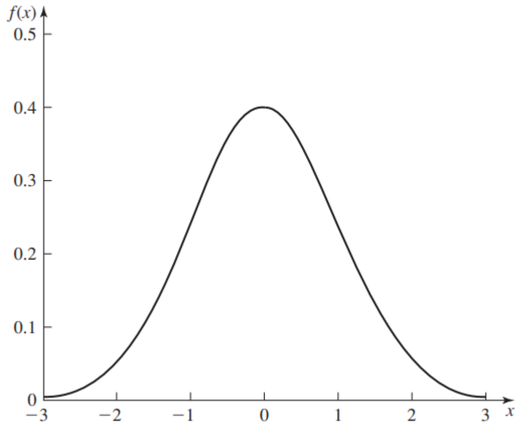
$$p(\boldsymbol{x}, \boldsymbol{y}) = p(\boldsymbol{y} \mid \boldsymbol{x}) p(\boldsymbol{x}) .$$

$$\underbrace{p(\boldsymbol{x} \mid \boldsymbol{y})}_{\text{posterior}} = \frac{\overbrace{p(\boldsymbol{y} \mid \boldsymbol{x})}^{\text{likelihood}} \overbrace{p(\boldsymbol{x})}^{\text{prior}}}{\underbrace{p(\boldsymbol{y})}_{\text{evidence}}}$$

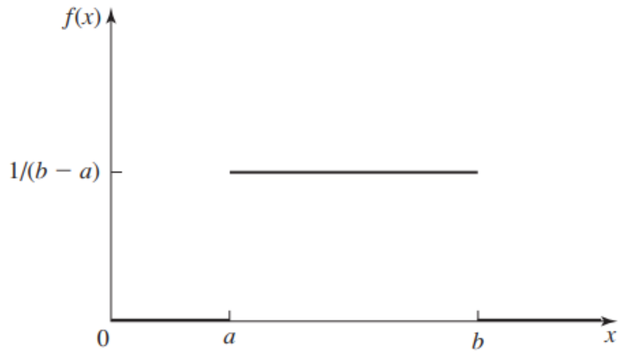
$$p(x_i) = \int p(x_1, \ldots, x_D) \mathrm{d}\boldsymbol{x}_{\setminus i}$$

Examples of Distributions

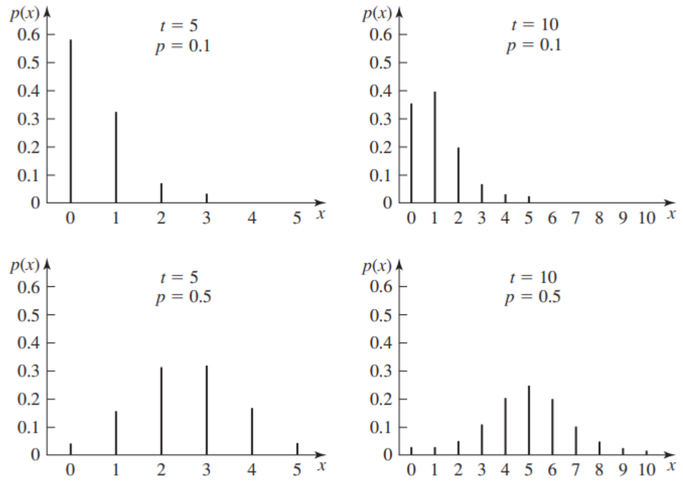
Mathematics (3)



Normal $N(\mu, \sigma^2)$



Uniform $U(a, b)$

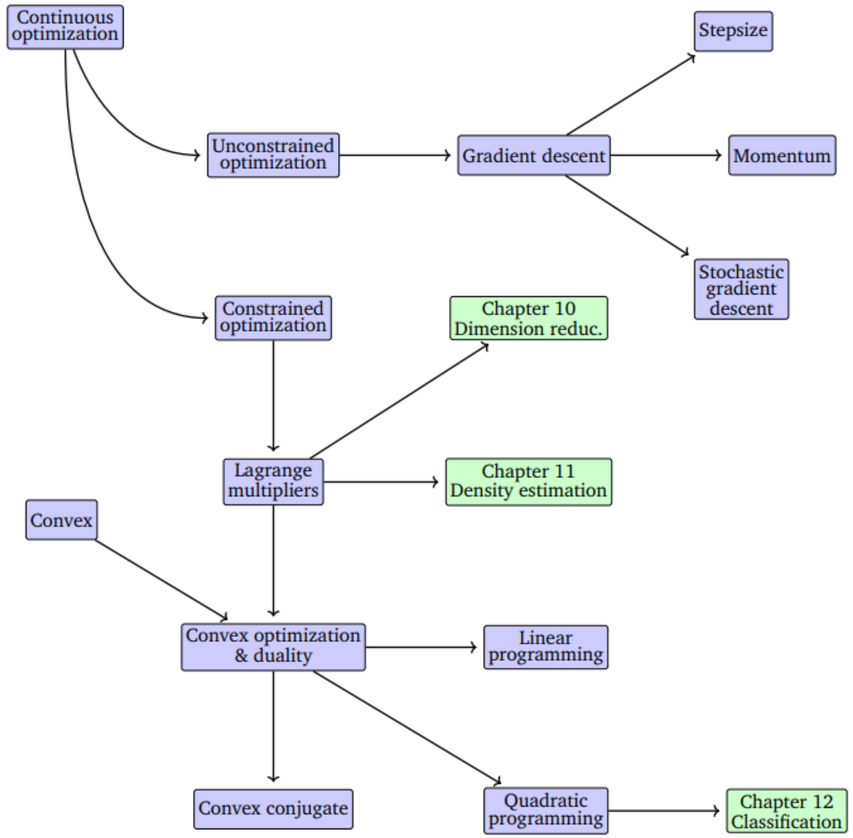


Binomial $bin(t, p)$

Optimization

Mathematics (3)

- Continuous?
- Constrained?
- Convex?



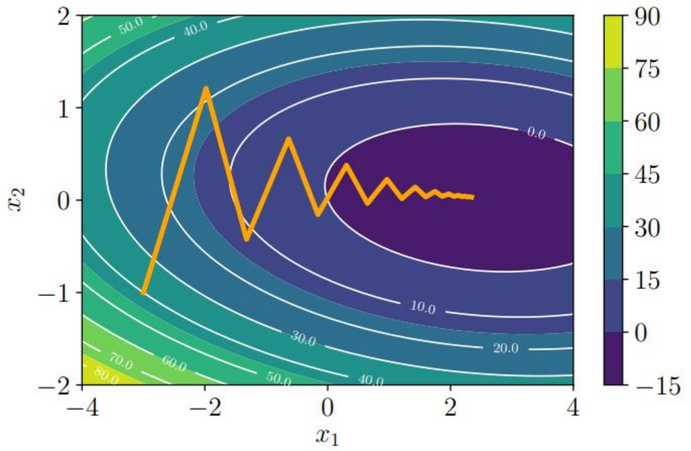
Continuous Optimization

Mathematics (3)

- **Can we differentiate?**
  - No… In general, we can't get analytical solution.

➔Heuristics!

➔There are many algorithms for approximation.

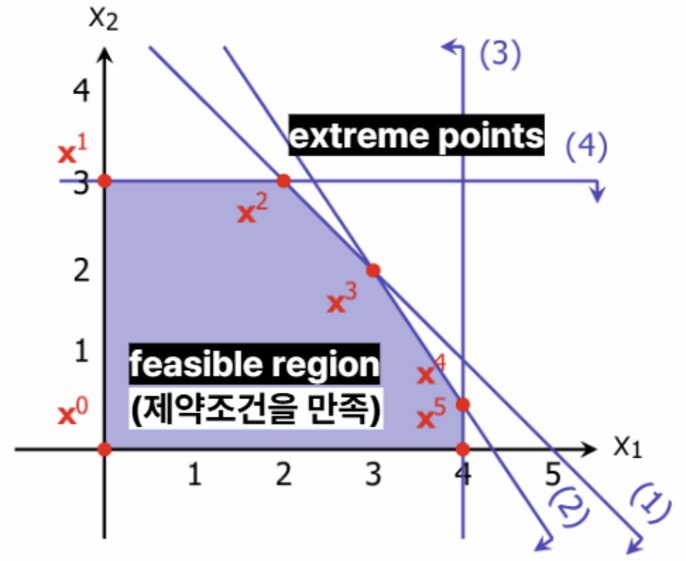


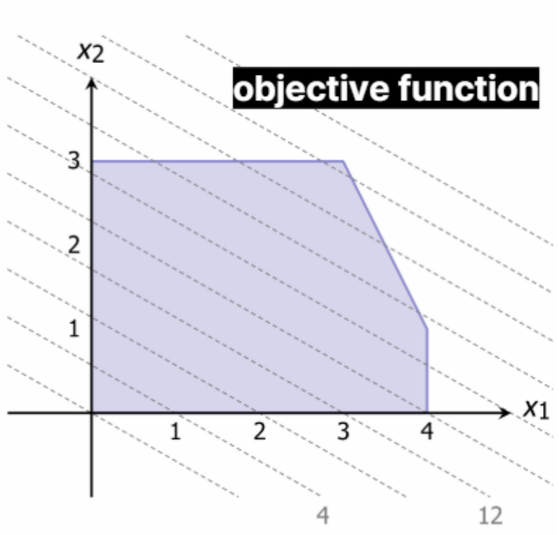
Convex Optimization - Linear Programming

Mathematics (3)

$$
\begin{aligned}
x_1 + x_2 &\le 5 && (1)\\
3x_1 + 2x_2 &\le 13 && (2)\\
x_1 &\le 4 && (3)\\
x_2 &\le 3 && (4)\\
x_1 &\ge 0 && (5)\\
x_2 &\ge 0 && (6)
\end{aligned}
$$

constraints





→ feasible set에서
목적함수를 최대화/최소화하는 해가 최적해(optimal)!

Summary and Bridging

Mathematics (3)

- **선형대수**
  - 데이터와 모델을 설명하는 언어
  - 행렬은 모든 데이터 과학의 근간
- **벡터 미적분**
  - 다변량 함수를 다루는 방법
  - 그래디언트와 헤시안
- **확률과 분포**
  - 생성 모델
  - 베이지안 추론
- **최적화**
  - 학습 알고리즘

Project Description

QnA

References

- Mathematics for Machine Learning Book
- A First Course in Probability
- Introduction to Operation Research

감사합니다