```
let rec c fold (e: expr): expr =

match e with

(* If there are two numbers, produce a constant! *)

Perinz(Plus, ENum(n1), ENum(n2)) -> ENum(n1 + n2)

(* Otherwise, just recur *)

| EPrinz(Plus, e1, e2) -> EPrinz(Plus, c_fold e1, c_fold e2)

| EIf(EBool(true), thn, els)) -> C_fold e1, c_fold e2)

| EIf(Ebool(false), thn, els)) -> C_fold e1, c_fold e2)

| EIf(Ebool(false), thn, els)) -> C_fold e1, c_fold e2, c_fold e1, c_fold
```

```
let rec c_prop (e : expr) : expr =

match e with

| ELet(x, e, body) ->
begin match e with

| ENum(n) -> replace (c_prop body) & e

| EBool(b) ->

| EId(x) ->

| --> Elet(x, c_prop e, c_prop body)

end

... other cases just recur ...
```

```
let rec replace (e : expr) (x : string) (with : expr) : expr =
    | EId(y) -> if x=y then with alse e
    | EPrim2(0, e1, e2) -> EPrin2(0, replace el x with, replace el x with)
    | ELet(y, e, body) -> if x=y then Elet(y, replace e x with, body)
                   else replace in both e, body
 ... and other recursive cases ...
                                          Don't replace! (let (x 7)
  let rec improve_expr_<del>first_try</del> (e : expr) : expr =
                                                        (let (x (+12))
    let folded = c_fold expr in
    let propped = c_prop folded in
                                     (let (x 3)
                                                         (+ 1 x))
   -propped-
   if propped = e then e
   else inflore-extr brobbed
                                                         Il improve - expr-ft
                                         (+ 1 ×))
                                           Tr-biob
Why does this ternitate?
                                                       A: (let (x 3)
(+ 1 x))
                                       (+1 3)
Lenna 1:
    c-fold and c-prop make e smaller
or leave it unchanged
                                                       B: 4
Therefore, by induction on size (e),
                                                       (:(+1 3)
       it reads a fixed point at some size
                                                       D: Somety alse
   Case n= 2 -> expr is a constant and does not change, so terminates
   (ase n > 1 - Assum ternimers (e') where size(e') = n', n' < n (IH)
         Cuse 1.7 -> size (propped) = size (e) -> by Lemna 1, e= propped
         Case 1.2 -> size (propped) < size (e) -> by Inductive Hypothesis
         Case 7.3 -> size (propped > size(e) -> Impossible by Lemna 1
     1 mon -- > Ollott
```

```
(def (contains bst key)
  (let ((t1 (tup-len bst))
        (t2 (= t1 0)))
    (if t2 false
        (let ((k (tup-get bst 0))
              (t3 (= key k))
(if t3
                  true
                  (let ((t4 (< key k)))
                    (if t4
                       (let ((t5 (tup-get bst 1)))
   t1
              t2
                         (contains t5 key))
                       (let ((t6 (tup-get bst 2)))
                         (contains t6 key))))))))))
   t3
              t4
              t6
```

```
(def (insert bst key)
  (let ((t1 (tup-len bst 0))
        (t2 (= t1 0)))
    (if t2
        (let ((t3 (tup)) (t4 (tup))) (tup key t3 t4))
        (let ((k (tup-get bst 0))
              (t5 (= key k))
              (if t5
                  bst
                  (let ((t6 (< key k)))
                    (if t6
                        (let ((t7 (tup-get bst 1))
                              (t8 (insert t7 key))
                              (t9 (tup-get bst 2)))
                          (tup k t8 t9)))
                        (let ((t10 (tup-get bst 1))
                              (t11 (insert t10 key))
                              (t12 (tup-get bst 2)))
                          (tup k t11 t12))))))))
                         t3
                                                t5
    t1
               t2
                                    t4
                                                          t6
                               k
    t7
               t8
                         t9
                                     t10
                                                t11
                                                           t12
```