

$$\dot{x}_a = x_a (\mu_a - M_{aa} x_a - M_{ab} x_b)$$

$$\dot{x}_b = x_b (\mu_b - M_{ba} x_a - M_{bb} x_b)$$

$$\text{let } M_{aa} = M_{bb} = \mu_a = 1$$

$$\begin{cases} \dot{x}_a = x_a (1 - x_a - M_{ab} x_b) \\ \dot{x}_b = x_b (\mu_b - M_{ba} x_a - x_b) \end{cases}$$

$$J = \begin{bmatrix} 1 - 2x_a - M_{ab}x_b & -M_{ab}x_a \\ -M_{ba}x_b & \mu_b - M_{ba}x_a - 2x_b \end{bmatrix}$$

$$\text{SS } (1, 0) \text{ (A)}, (0, \mu_b) \text{ (B)}, \left(\frac{1 - M_{ab}\mu_b}{1 - M_{ab}M_{ba}}, \frac{\mu_b - M_{ba}}{1 - M_{ab}M_{ba}} \right) \text{ (C)}$$

$$\text{eigs}(J|_{(1,0)}) = -1, \mu_b - M_{ba}$$

$$\text{eigs}(J|_{(0,\mu_b)}) = 1 - M_{ab}\mu_b, -\mu_b$$

$$J_C(1,1) = \frac{1 - M_{ab}M_{ba} - 2(1 - M_{ab}\mu_b) - M_{ab}(\mu_b - M_{ba})}{1 - M_{ab}M_{ba}} = \frac{-1 + M_{ab}\mu_b}{1 - M_{ab}M_{ba}}$$

$$J_C(1,2) = \frac{-M_{ab} + M_{ab}^2\mu_b}{1 - M_{ab}M_{ba}}$$

$$J_C(2,1) = \frac{-\mu_b M_{ba} + M_{ba}^2\mu_b}{1 - M_{ab}M_{ba}} \quad J_C(2,2)$$

$$J_C(2,2) = \frac{\mu_b - M_{ab}M_{ba}\mu_b - M_{ba}((1 - M_{ab}\mu_b) - 2(\mu_b - M_{ba}))}{1 - M_{ab}M_{ba}}$$

$$= \frac{-\mu_b + M_{ba}}{1 - M_{ab}M_{ba}}$$

$$\therefore J_C = \frac{1}{1 - M_{ab}M_{ba}} \begin{bmatrix} -1 + M_{ab}\mu_b & -M_{ab} + M_{ab}^2\mu_b \\ -\mu_b M_{ba} + M_{ba}^2\mu_b & -\mu_b + M_{ba} \end{bmatrix}$$

$$\text{let } \mu_b = 1 \quad J_C = \frac{1}{1 - M_{ab}M_{ba}} \begin{bmatrix} -1 + M_{ab} & -M_{ab} + M_{ab}^2 \\ -M_{ba} + M_{ba}^2 & -1 + M_{ba} \end{bmatrix}$$

$$\text{eigenvalue: } (-1 + M_{ab} - \lambda)(-1 + M_{ba} - \lambda) = (M_{ab}^2 - M_{ab})(M_{ba}^2 - M_{ba})$$

$$\lambda^2 - (-1 + M_{ab} - 1 + M_{ba})\lambda + (-1 + M_{ab})(-1 + M_{ba}) = M_{ab}M_{ba}(M_{ab} - 1)$$

$$\lambda^2 - (M_{ab} + M_{ba} - 2)\lambda + (1 - M_{ab}M_{ba})(M_{ab} - 1)(M_{ba} - 1) = 0$$

$$\lambda = \frac{B \pm \sqrt{B^2 - 4C}}{2}$$