$\boxed{1.}$ (b.) $\dot{x} = \mu x - M x^2 = x(\mu - Mx)$

Steady state: $\dot{x} = 0 \Rightarrow x(\mu - Mx) = 0$

$\Rightarrow \boxed{x = 0}$ or $\boxed{x = \frac{\mu}{M}}$

$f(x) = \mu x - Mx^2$, $f'(x) = \mu - 2Mx$

$x = 0$: $f'(x)|_{x=0} = \mu$ If $\mu > 0$, $x=0$ is unstable.
If $\mu < 0$, $x=0$ is stable.

$x = \frac{\mu}{M}$: $f'(x)|_{x=\frac{\mu}{M}} = -\mu$ If $\mu > 0$, $x = \frac{\mu}{M}$ is stable
If $\mu < 0$, $x = \frac{\mu}{M}$ is unstable

$\boxed{2}$

$$\begin{cases} \dot{x} = x(\mu_x + M_{xx} x + M_{xy} y) = f(x,y) \\ \dot{y} = y(\mu_y + M_{yx} x + M_{yy} y) = g(x,y) \end{cases}$$

(a) $(\bar{x}, 0)$: $\bar{x}(\mu_x + M_{xx}\bar{x}) = 0$, $(0, \bar{y})$: $\bar{y}(\mu_y + M_{yy}\bar{y}) = 0$

$$\boxed{\bar{x} = -\frac{\mu_x}{M_{xx}}, \quad \bar{y} = -\frac{\mu_y}{M_{yy}}}$$

For $(\bar{x}, \bar{y})$; since $\bar{x} \neq 0$, $\bar{y} \neq 0$,

$$\boxed{\begin{bmatrix} \mu_x \\ \mu_y \end{bmatrix} + \begin{bmatrix} M_{xx} & M_{xy} \\ M_{yx} & M_{yy} \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = 0}$$

(b) Jacobian $= \begin{bmatrix} f_x, & f_y \\ g_x, & g_y \end{bmatrix}$

$f_x(x,y) = \mu_x + 2M_{xx}x$.
$f_y(x,y) = M_{xy}x$
$g_x(x,y) = M_{yx}y$
$g_y(x,y) = \mu_y + 2M_{yy}y$.

at $(0,0)$: Jacobian $= \begin{bmatrix} \mu_x, & 0 \\ 0, & \mu_y \end{bmatrix}$

$\boxed{\text{stable if } \mu_x < 0 \text{ and } \mu_y < 0, \text{ otherwise unstable.}}$

at $(\bar{x}, 0)$:

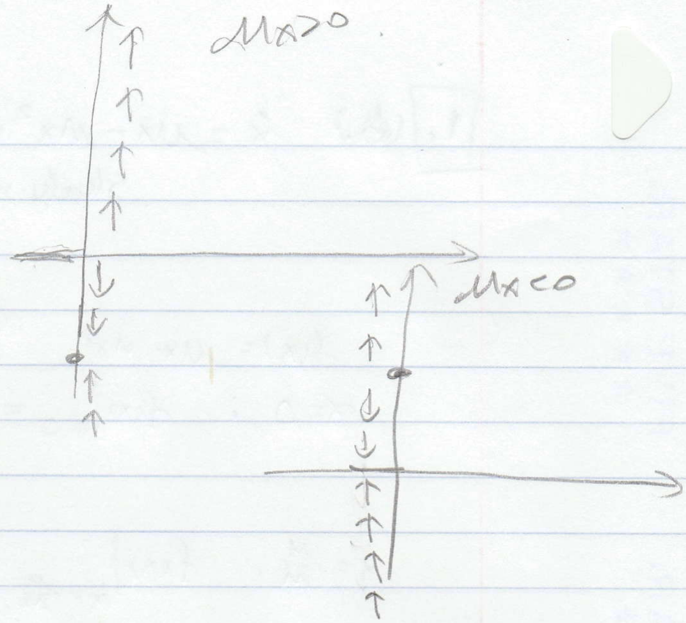
Jacobian $= \begin{bmatrix} -\mu_x, & -\mu_x \frac{M_{xy}}{M_{xx}} \\ 0, & \mu_y \end{bmatrix}$

eigenvalues are $-\mu_x$, μ_y.

$\boxed{\text{stable if } \mu_x > 0 \text{ and } \mu_y < 0, \text{ otherwise unstable.}}$

at $(0, \bar{y})$: Jacobian $= \begin{bmatrix} \mu_x, & 0 \\ -\frac{M_{yx}}{M_{yy}}\mu_y, & -\mu_y \end{bmatrix}$

$\boxed{\text{Stable if } \mu_x < 0 \text{ and } \mu_y > 0, \text{ otherwise unstable.}}$



[If we neglect the effect of y]

(C) $M_{xx} > 0$; $\bar{x} = -\frac{\mu_x}{M_{xx}}$ [If $\mu_x > 0$,

x will diverge if $x > 0$,

converge if $x < 0$.

If $\mu_x < 0$; x will diverge if

$x > -\frac{\mu_x}{M_{xx}}$,

converge if $x < -\frac{\mu_x}{M_{xx}}$

$\boxed{3}$ (a) when $N=2$:

$$\dot{x}_1 = x_1(\mu_1 + M_{11}x_1 + M_{12}x_2)$$
$$\dot{x}_2 = x_2(\mu_2 + M_{21}x_1 + M_{22}x_2)$$

same as (2).

(b) 4 steady states for Eq (2). $(0,0)$, $(\bar{x},0)$, $(0,\bar{y})$, and $(\bar{x},\bar{y})$.

$$\dot{x}_i = x_i(\mu_i + \sum_{j=1}^{N} M_{ij}x_j)$$

There are N functions, each one having two possibilities ; $x_i = 0$ or $\mu_i + \sum_{j=1}^{N} M_{ij}x_j = 0$.

$\therefore$ There are 2^N possibilities for the total group of functions. All 2^N possible groups are linear equations, having 1 solution.

$\Rightarrow$ There are 2^N steady states,

$\boxed{4}$ simulation!

$$\dot{x} = x(1-0.5x)$$