Worksheet 2

1. 2D gLV: $\dot{x} = x(\mu_x + M_{xx} x + M_{xy} y)$

$\dot{y} = y(\mu_y + M_{yx} x + M_{yy} y)$

Call $x_a = -M_{xx} x$, $x_b = -M_{yy} y$

$\Rightarrow \dot{x}_a = -M_{xx} \dot{x}$, $\dot{x}_b = -M_{yy} \dot{y}$

plug in: $-\frac{\dot{x}_a}{M_{xx}} = -\frac{x_a}{M_{xx}}\left(\mu_x - x_a - \frac{M_{xy}}{M_{yy}} x_b\right)$ define: $M_{ab} = -\frac{M_{xy}}{M_{yy}}$

$-\frac{\dot{x}_b}{M_{yy}} = -\frac{x_b}{M_{yy}}\left(\mu_y - \frac{M_{yx}}{M_{xx}} x_a - x_b\right)$ $M_{ba} = \frac{-M_{yx}}{M_{xx}}$

$$\Rightarrow \quad \dot{x}_a = x_a(\mu_x - x_a + M_{ab} x_b)$$
$$\dot{x}_b = x_b(\mu_y + M_{ba} x_a - x_b)$$

sign conventions can be chosen so that these two terms are negative.

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2. Theorem 1.1.2

condition: 1) $V(\bar{x}) = 0$ and $V(x) > 0$ if $x \neq \bar{x}$.

2) $\dot{V}(x) \leq 0$ in $U - \{\bar{x}\}$

Test condition 2: $V(x_a, x_b) = M_{ba}\frac{1}{2}x_a^2 + M_{ab}\frac{1}{2}x_b^2 - M_{ba} x_a \mu_a - M_{ab} x_b \mu_b + M_{ab} M_{ba} x_a x_b$

$$\dot{V}(x_a, x_b) = M_{ba} x_a \cdot \dot{x}_a + M_{ab} x_b \cdot \dot{x}_b - M_{ba}\mu_a \dot{x}_a - M_{ab}\mu_b \dot{x}_b + M_{ab} M_{ba} x_a \dot{x}_b + M_{ab} M_{ba} \dot{x}_a x_b$$

$$= \dot{x}_a \cdot M_{ba}(x_a - \mu_a + M_{ab} x_b) + \dot{x}_b M_{ab}(x_b - \mu_b + M_{ba} x_a)$$

From the previous problem,

$$\dot{V}(x_a, x_b) = -M_{ba} x_a (x_a - \mu_a + M_{ab} x_b)^2 - M_{ab} x_b (x_b - \mu_b + M_{ba} x_a)^2$$

In domain $[x_a \geq 0, x_b \geq 0$, it is negative.