Multi 13.7 Notes

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January 10, 2025

Contents

List of Figures

0.1 Finding the equation of a plane

To find the equation of a plane, you need a normal vector and a point on the plane

$$a(x - x0) + b(y - y0) + c(z - z0) = 0 \tag{1}$$

The normal vector is ∇ F(x0, y0, z0), which is the gradient The normal vector is ;greek('nabla'); F(x0, y0, z0), which is the gradient To find the equation of a line, you need a direction vector and point on the line. The normal vector is the gradient as well, and you can write a parametric equation

$$x = x0 + at \tag{2}$$

$$y = y0 + bt \tag{3}$$

$$z = z0 + ct \tag{4}$$

0.2 Tangent plane and normal line to a surface

If given $z =$ f(x, y), then rewrite as:

$$F(x, y, z) = f(x, y) - z \tag{5}$$

0.2.1 Writing a surface equation

For F(x, y, z), if ∇ F(x, y, z) · r(t) where r(t) is the tangent vector For F(x, y, z), if ;greek('nabla'); F(x, y, z) ;greek('dot'); r(t) where r(t) is the tangent vector This dotproduct is equal to zero, so the gradient and tangent vector are orthogonal refer to the example surface below This dotproduct is equal to zero, so the gradient and tangent vector are orthogonal; refer to the example surface below

The gradient is also the direction vector of the normal line, so you can write the equation of the normal line as well given a point on that line

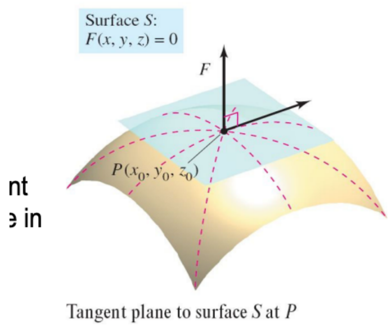
0.2.2 Finding the equation of a tangent plane

Given $z^2 - 2x^2 - 2y^2 = 12$, find a tangent plane at point (1, -1, 4)

$$F(x, y, z) = z^2 - 2x^2 - 2y^2 - 12 = 0 \tag{6}$$

$$\frac{\partial F}{\partial x} = -4x \tag{7}$$

Figure 1: There are infinitely many tangent vectors orthogonal to the gradient, which is the normal vector, perpendicular to the plane



$$\frac{\partial F}{\partial y} = -4y \tag{8}$$

$$\frac{\partial F}{\partial z} = 2z \tag{9}$$

$$-4(x-1) + 4(y+1) + 8(z-4) = 0 \tag{10}$$

0.3 Whats up this is test topic

0.3.1 Whats up this a test example

hello $2x^2 = 2x^2$ is the equation ∇