Multi 13.6 Notes

Omkar Tasgaonkar

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Contents

List of Figures

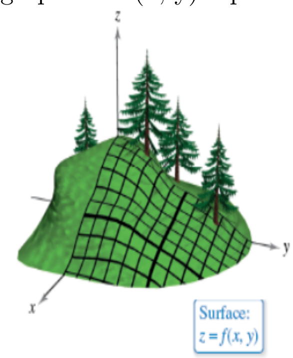
0.1 The Directional Derivative:

The directional derivative tells us the slope in a direction:

$$\frac{\partial f}{\partial x} = slope\ in\ x - dir \tag{1}$$

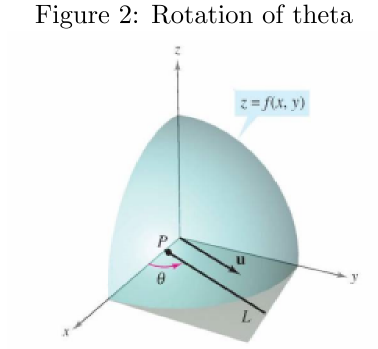
$$\frac{\partial f}{\partial y} = slope\ in\ y - dir \tag{2}$$

Figure 1: A graph z = f(x, y) represents the hill



We want to find the slope in a certain direction, a rotation of theta in cylindrical

Figure 2: Rotation of theta

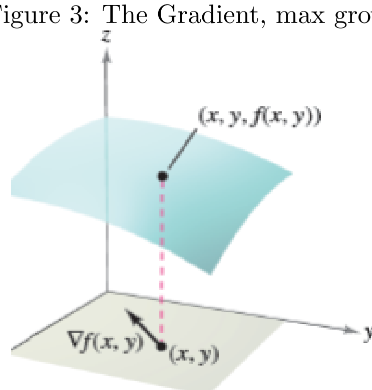


So our direction vector u Therefore our directional derivative is the dot product of two vectors, now which vectors?

0.2 The Gradient

The gradient tells us the biggest derivative vector at a certain point in the graph!

Figure 3: The Gradient, max growth



$$\nabla f(x,y) = \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} \tag{3}$$

So now, if we rotate this gradient by an angle theta, then we can say that the gradient is a subset of rotated gradients Or that ∇ f(x, y) ∈ G where G is the superset

For the final step, the directional derivative must be a scalar quantity as it is a slope, so:

$$Directional\ derivative = \nabla f(x,y) \cdot u \tag{4}$$

Whereby u is the direction vector for angle θ

0.2.1 Using ∇ f(x, y) to find Directional Derivative

Find the directional derivative of $f(x,y) = 3x^2 - 2y^2 at(-3/4, 0)$ in the direction of: $3/4\vec{i} + \vec{j}$

$$\nabla f(x,y) = 6x\vec{i} - 4y\vec{j} \tag{5}$$

$$u = 3/5\vec{i} + 4/5\vec{j} \tag{6}$$

$$u = 3/5\vec{i} + 4/5\vec{j} \tag{7}$$

$$\nabla f(-3/4, 0) \cdot u = -9/2\vec{i} \cdot (3/5\vec{i} + 4/5\vec{j}) \tag{8}$$

So the directional derivative for this question evaluates to: -27/10