

TA Problems of the Week: Semester One

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0.1 Problem of the week (9/30 - 10/3): Integrals!

Given a function $f(x)$ at $x = 2\pi$, which outputs -1:

$$\text{If } f'(x) = -(1 - \sin^2(\pi/2 - x))^{1/2} \quad (1)$$

Find $f(x)$

0.1.1 Solution:

Assume domain restriction from 0 to $\pi/2$

$$\sin^2(\pi/2 - x) = \cos^2(x) \quad (2)$$

$$f'(x) = -(1 - \cos^2(x))^{1/2} \quad (3)$$

$$-(1 - \cos^2(x))^{1/2} = -(\sin^2(x))^{1/2} \quad (4)$$

$$-(\sin^2(x))^{1/2} = -\sin(x) \quad (5)$$

Now that we have simplified the derivative to a simple cos function, we can integrate it. Let's review the fundamental theorem of calculus first:

$$\int_a^b g(x) dx = f(b) - f(a)$$

$$\begin{aligned} & \int -\sin(x) dx \\ &= \cos(x) + C \end{aligned} \quad (6)$$

$$\cos(2\pi) + C = -1 \quad (7)$$

$$C = -2 \quad (8)$$

$$\text{So } f(x) = \cos(x) - 2 \quad (9)$$