Statistics for Engineers (MAT2001)- Lab Experiment-IV: Binomial distribution and Poisson distribution

1 BASICS IN PROBABILITY

If you want to pick five numbers at random from the set 1:50, then you can

```
sample(1:50,5)

## [1] 19 33  2 49 22
```

Sampling with replacement is suitable for modelling coin tosses or throws of a die.

```
sample(1:6,10,replace=TRUE)

##  [1] 3 5 2 3 4 3 3 1 6 1
```

```
sample(1:6,10,replace=FALSE)

## Error in sample.int(length(x), size, replace, prob):  cannot take
a sample larger than the population when 'replace = FALSE'
```

```
dice = as.vector(outer(1:6,1:6,paste))
print(dice)

##  [1] "1 1" "2 1" "3 1" "4 1" "5 1" "6 1" "1 2" "2 2" "3 2" "4 2" "5 2" "6 2"
## [13] "1 3" "2 3" "3 3" "4 3" "5 3" "6 3" "1 4" "2 4" "3 4" "4 4" "5 4" "6 4"
## [25] "1 5" "2 5" "3 5" "4 5" "5 5" "6 5" "1 6" "2 6" "3 6" "4 6" "5 6" "6 6"
```

Toss a coin

```
sample(c("H","T"),10,replace=TRUE)

##  [1] "T" "H" "T" "H" "H" "T" "H" "H" "H" "T"
```

Combination

```
choose(10,3)

## [1] 120
```

Permutation

```
# There is no separate permutation function in R
n <- 10
k <- 5
pnk <- factorial(n)/factorial(n-k)
pnk

## [1] 30240
```

Give all binomial coefficients for $10C_x$

```
choose(10,0:10)

##  [1]   1  10  45 120 210 252 210 120  45  10   1
```

Use a loop to print the first several rows of pasacal's triangle.

```
for (n in 0:10) print(choose(n, 0:n))

## [1] 1
## [1] 1 1
## [1] 1 2 1
## [1] 1 3 3 1
## [1] 1 4 6 4 1
## [1]  1  5 10 10  5  1
## [1]  1  6 15 20 15  6  1
## [1]  1  7 21 35 35 21  7  1
## [1]  1  8 28 56 70 56 28  8  1
##  [1]   1   9  36  84 126 126  84  36   9   1
##  [1]   1  10  45 120 210 252 210 120  45  10   1
```

2 Binomial Distribution

For a binomial(n,p) random variable X, the R functions involve the abbreviation "binom":

- dbinom(k,n,p), binomial(n,p) density at k: Pr(X = k)
- pbinom(k,n,p), binomial(n,p) CDF at k: $Pr(X \leq k)$

- qbinom(P,n,p), binomial(n,p) P-th quantile
- rbinom(N,n,p), N binomial(n,p) random variables

Problem:
Find the P(2) among ten dice by using binomial probability formula

```
choose(10,2)*(1/6)^2*(5/6)^8

## [1] 0.29071
```

Problem:
Find the Probability of getting two among ten dice

```
dbinom(2,size=10,prob=1/6)

## [1] 0.29071
```

Problem:
Find the Probability of getting less than or equal 3 among ten dice

```
bin_3 <- dbinom(0:3,size=10,prob=1/6)
bin_3

## [1] 0.1615056 0.3230112 0.2907100 0.1550454

P_3 <- sum(bin_3)
P_3

## [1] 0.9302722

pbinom(3,size=10,prob=1/6)

## [1] 0.9302722
```

Problem:
Find the table for BIN(n=10,P=1/6)

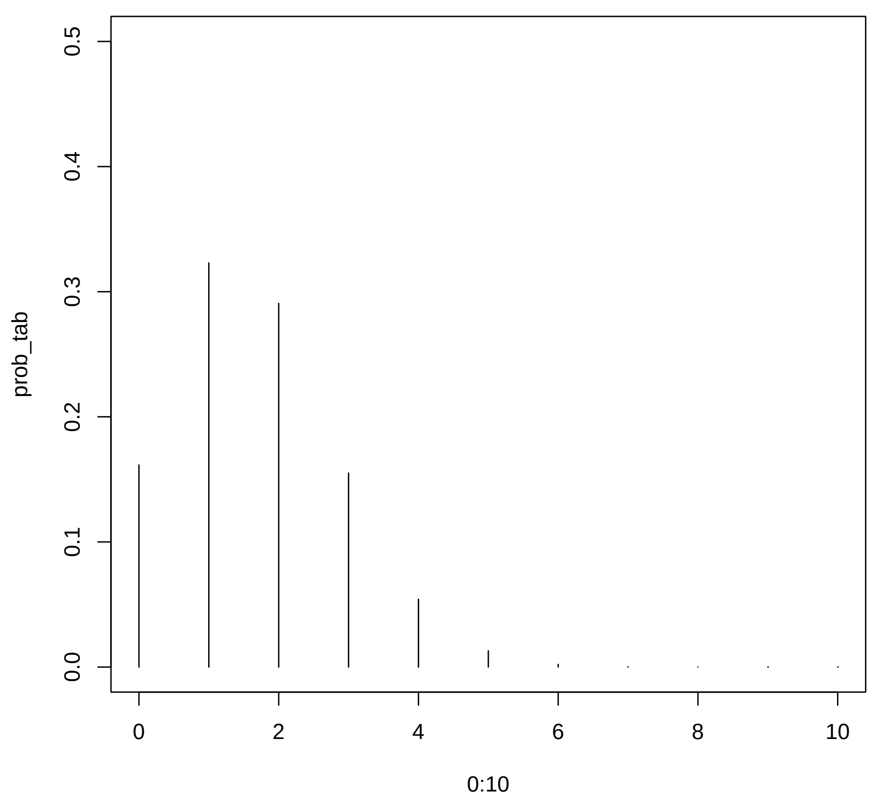
```
prob_tab <- dbinom(x = 0:10, size = 10, prob = 1/6)
data.frame(0:10, prob_tab)

##    X0.10     prob_tab
## 1      0 1.615056e-01
## 2      1 3.230112e-01
## 3      2 2.907100e-01
## 4      3 1.550454e-01
## 5      4 5.426588e-02
## 6      5 1.302381e-02
```

```
## 7       6 2.170635e-03
## 8       7 2.480726e-04
## 9       8 1.860544e-05
## 10      9 8.269086e-07
## 11     10 1.653817e-08
```

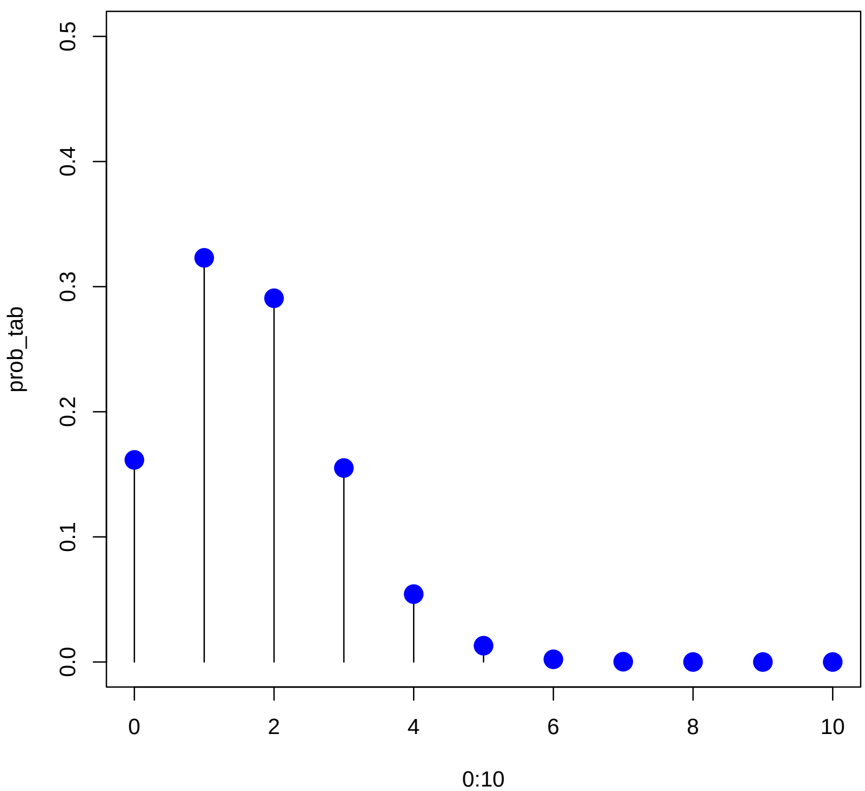
2.1 BINOMIAL PROBABILITY PLOTS

Problem:
Draw a Plot for the Binomial distribution Bin(n=10,p=1/6)

```
plot(0:10,prob_tab,type="h",xlim=c(0,10),ylim=c(0,0.5))
```



```r
plot(0:10,prob_tab,type="h",xlim=c(0,10),ylim=c(0,0.5))
points(0:10,prob_tab,pch=16,cex=2, col="blue")
```



Problem:
If 10% of the Screws produced by an automatic machine are defective, find the probability that out of 20 screws selected at random, there are
(i) Exactly 2 defective
(ii) At least 2 defectives
(iii) Between 1 and 3 defectives (inclusive)

```r
dbinom(2,20,10/100)

## [1] 0.2851798

1-dbinom(1,20,10/100)

## [1] 0.7298297
```

```
one_to_three=sum(dbinom(1:3,20,0.10))
one_to_three

## [1] 0.74547
```

Problem:
Show that Binomial distribution variance is less than mean with Binomial variable follows (7,1/4)

```
x <- dbinom(0:7,size = 7, prob = 1/4 )
x

## [1] 1.334839e-01 3.114624e-01 3.114624e-01 1.730347e-01 5.767822e-02
## [6] 1.153564e-02 1.281738e-03 6.103516e-05

Ex <- sum(x*1/4) #E(x)=np
Ex

## [1] 0.25

var <- sum((x-Ex)^2*x)
var

## [1] 0.008062817
```

Experiment

1. Plot Binomial distribution with n=50 and P=0.33
2. For a Binomial(7,1/4) random variable named X,
i. Compute the probability of two success
ii. Compute the Probablities for whole space
iii. Display those probabilities in a table
iv. Show the shape of this binomial Distribution
3. Suppose there are twelve multiple choice questions in an English class quiz. Each question has five possible answers, and only one of them is correct. Find the probability of having four or less correct answers if a student attempts to answer every question at random.

3 THE POISON DISTRIBUTION

Syntax:-

- dpois(x, lambda, log = FALSE)
- ppois(q, lambda, lower.tail = TRUE, log.p = FALSE)
- qpois(p, lambda, lower.tail = TRUE, log.p = FALSE)
- rpois(n, lambda)

Problem:

```
# a. P(x=5) with parameter 7
dpois(x=5,lambda=7)

## [1] 0.1277167
```

```
# b. #P(x=0)+P(x=1)+..........+P(x=5)
dpois(x=0:5,lambda=7)

## [1] 0.000911882 0.006383174 0.022341108 0.052129252 0.091226192 0.127716668
```

```
# c. > #P(x<=5)
sum(dpois(0:5,lambda=7))

## [1] 0.3007083

# or
ppois(q=4,lambda=7,lower.tail=T)

## [1] 0.1729916
```

```
ppois(q=12,lambda=7,lower.tail=F)

## [1] 0.02699977
```

Problem :
Check the relationship between mean and variance in Poisson distribution(4) with n=100

```
X.val=0:100
P.val=dpois(X.val,4)
EX=sum(X.val*P.val) #mean
EX
```

```
## [1] 4

sum((X.val-EX)^2*P.val)

## [1] 4
```

Problem:
Compute Probabilities and cumulative probabilities of the values between 0 and 10 for the parameter 2 in poisson distribution.

```
dpois(0:10,2) #

##  [1] 1.353353e-01 2.706706e-01 2.706706e-01 1.804470e-01 9.022352e-02
##  [6] 3.608941e-02 1.202980e-02 3.437087e-03 8.592716e-04 1.909493e-04
## [11] 3.818985e-05

# or
P=data.frame(0:10,dpois(0:10,2))
round (P,4)

##    X0.10 dpois.0.10..2.
## 1      0         0.1353
## 2      1         0.2707
## 3      2         0.2707
## 4      3         0.1804
## 5      4         0.0902
## 6      5         0.0361
## 7      6         0.0120
## 8      7         0.0034
## 9      8         0.0009
## 10     9         0.0002
## 11    10         0.0000
```

Problem:
Poisson distribution with parameter '2'
i)Calculate P(0),P(1),...,P(10) when lambda =2 and Make the output prettier
ii) Find $P(x \leq 6)$
iii) Sum all probabilities
iv)Find $P(Y > 6)$
v) Make a table of the first 11 Poisson probs and cumulative probs when mu=2 and make the output prettier
vi) Plot the probabilities Put some labels onthe axes and give the plot a title:

```
round(dpois(0:10, 2), 3)

##  [1] 0.135 0.271 0.271 0.180 0.090 0.036 0.012 0.003 0.001 0.000 0.000
```

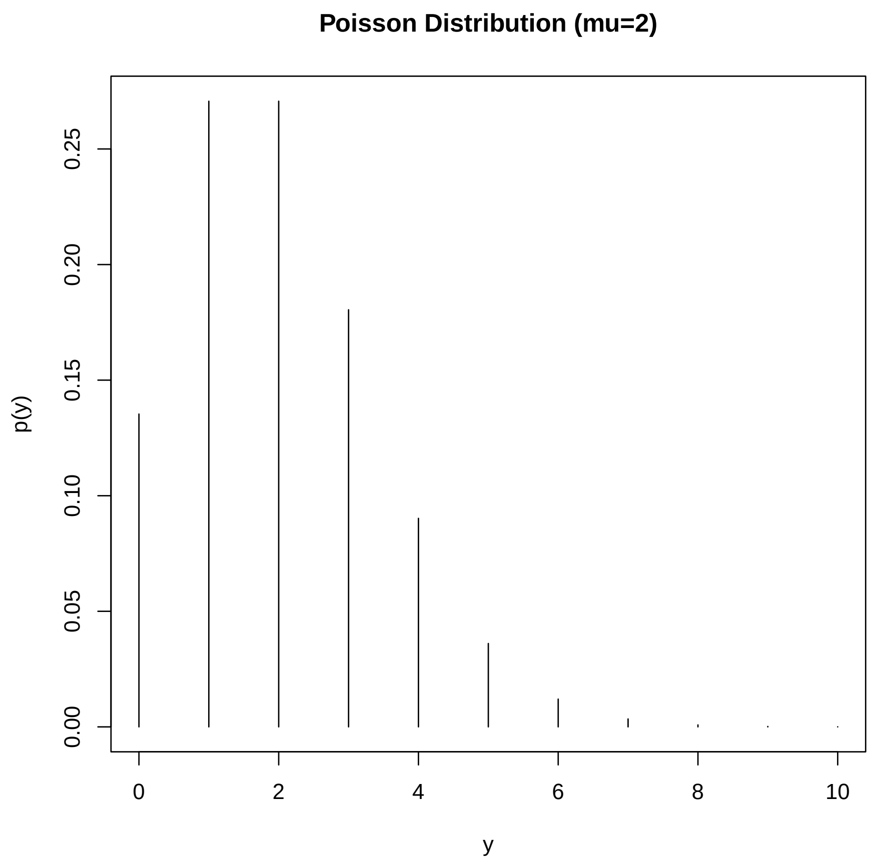
```r
ppois(6, 2)

## [1] 0.9954662
```

```r
sum(dpois(0:6, 2))

## [1] 0.9954662
```

```r
1 - ppois(6, 2)

## [1] 0.004533806
```

```r
round(cbind(0:10, dpois(0:10 ,2), ppois(0:10, 2)) ,3)

##       [,1]  [,2]  [,3]
##  [1,]    0 0.135 0.135
##  [2,]    1 0.271 0.406
##  [3,]    2 0.271 0.677
##  [4,]    3 0.180 0.857
##  [5,]    4 0.090 0.947
##  [6,]    5 0.036 0.983
##  [7,]    6 0.012 0.995
##  [8,]    7 0.003 0.999
##  [9,]    8 0.001 1.000
## [10,]    9 0.000 1.000
## [11,]   10 0.000 1.000
```

```r
plot(0:10,dpois(0:10,2),type="h",xlab="y",ylab="p(y)",
main="Poisson Distribution (mu=2)")
```



Experiment

The number of traffic accidents that occur on a particular stretch of road during a month follows a Poisson distribution with a mean of 7.6.
1. Find the probability that less than three accidents will occur next month on this stretch of road.
2. Find the probability of observing exactly three accidents on this stretch of road next month.
3. Find the probability that the next two months will both result in four accidents each occurring on this stretch of road.
4. Check the mean and variance of the poisson distribution
5. Plot the Poisson distribution and compare with binomial distribution