Statistics for Engineers (MAT2001)- Lab Experiment-V: One Sample Z-test

1 Test for significance of single proportion

Suppose 60% of citizens voted in last election. 85 out of 148 people in a telephone survey said that they voted in current election. At 0.5 significance level, can we reject the null hypothesis that the proportion of voters in the population is above 60% this year?

- $H_0 : p = 0.6, H_1 : p > 0.6$

```
# Given data
P <- 0.60
x <- 85
n <- 148
alpha <- 0.05
p <- x / n
z_score <- (p - P) / sqrt((P* (1 - P)) / n)
z_critical <- qnorm(1 - alpha)
# Decision Rule: If Z-score > Z-critical,
# reject the null hypothesis
# if z_score < z_critical
if (z_score < z_critical) {conclusion="Accept null hypothesis"

} else {
  conclusion = "Reject the null hypothesis"
}
conclusion

## [1] "Accept null hypothesis"
```

Alternative Solution 1
Instead of using the critical value, we apply the pnorm function to compute the lower tail p-value of the test statistic. As it turns out to be greater than the .05 significance level, we do not reject the null hypothesis.

```r
pnorm(z_score)

## [1] 0.2618676
```

Suppose that 12% of apples harvested in an orchard last year was rotten. 30 out of 214 apples in a harvest sample this year turns out to be rotten. At .05 significance level, can we reject the null hypothesis that the proportion of rotten apples in harvest stays below 12% this year?

- $H_0 : p = 0.12, H_1 : p < 0.12$

```r
# Given data
P <- 0.12
x <- 30
n <- 214
alpha <- 0.05
p <- x / n
z_score <- (p - P) / sqrt((P * (1 - P)) / n)
z_critical <- qnorm( alpha)

# if z_score < z_critical
if (z_score < z_critical) {conclusion="Accept null hypothesis"

} else {
  conclusion = "Reject the null hypothesis"
}
conclusion

## [1] "Reject the null hypothesis"
```

Alternative Solution 2

```r
prop.test(30, 214, p = 0.12,alternative = "greater" )

##
##  1-sample proportions test with continuity correction
##
## data:  30 out of 214, null probability 0.12
## X-squared = 0.64573, df = 1, p-value = 0.2108
## alternative hypothesis: true p is greater than 0.12
## 95 percent confidence interval:
##  0.1035873 1.0000000
## sample estimates:
##         p
## 0.1401869
```

Suppose a coin toss turns up 12 heads out of 20 trials. At .05 significance level, can one reject the null hypothesis that the coin toss is fair?

- $H_0 : p = 0.5, H_1 : p \neq 0.5$

```
# Given data
P <- 0.5
x <- 12
n <- 20
alpha <- 0.05
p <- x / n
z_score <- (p - P) / sqrt((P * (1 - P)) / n)
z_critical <- qnorm(1 - alpha / 2)
## if z_score < z_critical
if (abs(z_score) < z_critical) {conclusion="Accept null hypothesis"

} else {
  conclusion = "Reject the null hypothesis"
}
conclusion

## [1] "Accept null hypothesis"
```

Test for significance of single mean

Suppose the manufacturer claims that the mean lifetime of a light bulb is more than 10,000 hours. In a sample of 30 light bulbs, it was found that they only last 9,900 hours on average. Assume the population standard deviation is 120 hours. At .05 significance level, can we reject the claim by the manufacturer?

- $H_0 : \mu = 10000, H_1 : \mu < 10000$

```
# Given data
mu <- 10000
x_bar <- 9900
sigma <- 120
n <- 30
alpha <- 0.05

# Calculate the Z-score
z_score <- (x_bar - mu) / (sigma / sqrt(n))
z_score
```

```
## [1] -4.564355

# Calculate the critical Z value for a left-tailed test at alpha = 0.05
z_critical <- qnorm(alpha)
z_critical

## [1] -1.644854

# Decision Rule: If Z-score < Z-critical, reject the null hypothesis
if (z_score < z_critical) {
  decision <- "Reject the null hypothesis"
} else {
  decision <- "Fail to reject the null hypothesis"
}

decision

## [1] "Reject the null hypothesis"
```

Suppose the food label on a cookie bag states that there is at most 2 grams of saturated fat in a single cookie. In a sample of 35 cookies, it is found that the mean amount of saturated fat per cookie is 2.1 grams. Assume that the population standard deviation is 0.25 grams. At .05 significance level, can we reject the claim on food label?

- $H_0 : \mu = 2, H_1 : \mu > 2$

```
# Given data
mu <- 2         # population mean (claimed by the label)
x_bar <- 2.1    # sample mean
sigma <- 0.25   # population standard deviation
n <- 35         # sample size
alpha <- 0.05   # significance level
z_score <- (x_bar - mu) / (sigma / sqrt(n))
z_score

## [1] 2.366432

z_critical <- qnorm(1 - alpha)
z_critical

## [1] 1.644854

# Decision Rule: If Z-score > Z-critical, reject the null hypothesis
if (z_score > z_critical) {
  decision <- "Reject the null hypothesis"
```

```r
} else {
  decision <- "Fail to reject the null hypothesis"
}
decision

## [1] "Reject the null hypothesis"
```

Suppose the mean weight of King Penguins found in an Antarctic colony last year was 15.4 kg. In a sample of 35 penguins same time this year in the same colony, the mean penguin weight is 14.6 kg. Assume the population standard deviation is 2.5 kg. At .05 significance level, can we reject the null hypothesis that the mean penguin weight does not differ from last year?

- $H_0 : \mu = 15.4, H_1 : \mu \neq 15.4$

```r
# Given data
mu <- 15.4
x_bar <- 14.6
sigma <- 2.5
n <- 35
alpha <- 0.05
z_score <- (x_bar - mu) / (sigma / sqrt(n))
z_score

## [1] -1.893146

z_critical <- qnorm(1 - alpha / 2)
z_critical

## [1] 1.959964

# Decision Rule: If |Z-score| > Z-critical, reject the null hypothesis
if (abs(z_score) > z_critical) {
  decision <- "Reject the null hypothesis"
} else {
  decision <- "Fail to reject the null hypothesis"
}
decision

## [1] "Fail to reject the null hypothesis"
```