Cognitive Neighborhoods: A Phenomenology of Order Sorted Algebra over Conceptual Graphs and Mereotopology

November 21, 2016

Abstract

Joseph Goguen's Unified Concept Theory (UCT) explores the connections and integration between several general notions of *concept*, such as Formal Concept Analysis, Conceptual Graph Semantics, Conceptual Integration (aka Blending), Concept Spaces, and Description Logic. Goguen has also pioneered the use of Category Theory as a practical tool for Ontology Engineering, particularly Ontology Integration: unifying the structure and contents of distinct conceptual representation systems, encoded into data specifications for different software components which need to be integrated or to inter-communicate. Here I will distinguish semantic, logical, and representational aspects of concept-integration, and consider UCT extensions particularly concerning the former (semantic) integration domain, drawing on Modal Logic, Phenomenological Semantics, Mereotopology, and Michael Jubien's Property Theory. I will focus on community-specific complications of concept-integration: the fact that most real-life concepts are relative to specific linguistic, cognitive, and/or geographic communities, and that concepts which extend across communities can acquire different connotations in different contexts. I believe concepts need to be understood via their roles in our social and perceptual activity, from which their formal representation in computational domains is derived, and is therefore philosophically secondary. I will argue that mereotopology and modal topology provide rich theories for capturing conceptual nuance but also suggesting concrete computational application.

On connaît la célèbre affirmation de Claude Lévi-Strauss: "les sciences humaines seront structurales ou ne seront pas". Nous aimerions lui en adjoindre une autre: "les sciences humaines seront des sciences naturelles ou ne seront pas". Evidemment, sauf à en revenir à un réductionnisme dogmatique, une telle affirmation n'est soutenable que si l'on peut suffisamment généraliser le concept classique de "naturalité", le généraliser jusqu'à pouvoir y faire droit, comme à des phénomènes naturels, aux phénomènes d'organisation structurale.

—Jean Petitot, [33, p. 1]

The nature of any entity, I propose, divides into three aspects or facets, which we may call its form, appearance, and substrate. In an act of consciousness, accordingly, we must distinguish three fundamentally different aspects: its form or intentional structure, its appearance or subjective "feel", and its substrate or origin. In terms of this three-facet distinction, we can define the place of consciousness in the world.

—David Woodruff Smith, *Mind World* [41, p. 11]

Human cognition works with immediate perceptual data and with our background knowledge — things we have just seen and remember in our proximate environment, as well as knowledge-structures which orient given situations to pragmatic, worldly contexts. Perception deals with objects in the space around us, with their surfaces and appearances, qualitative and sensory properties, and the raw, immediate reality of our moment-to-moment consious states. Background knowledge is more schematic, relational, structured, and categorical: it groups concepts into relational networks, providing general cognitive "modules" for identifying things within their concept-types, tracking the characterstic properties of concept-tokens given their types, how different concepts relate together, and so forth. Between the raw givenness of perception, and the schematic logicality of the horizon of our knowledge, lies our situational awareness of the current moment, which extends beyond the actually perceived to the current place and activity where we are engaged, marshalling background-knowledge to identify and interact with the entities of immediate perception.

Although integrated together into productive, interactive episodes of practical reason, the cognitive structures of immedate perception and background knowledge or "mental schema" are different, or at least they engender different models and formulations. This difference carries over, I believe, into the schema with which different semantic theories represent the formal properties of semantic systems, such as semantic frames, conceptual networks, logical systems adopted as formal semantic models, etc. Since cognition unifies the perceptual foreground and the epistemological background into unfolding, world-disclosing judgment, these phenomenological syntheses can perhaps be formally represented, at least to some approximation, by a similar synthesis at the level of abstract semantic systems.

In particular, some version of Conceptual Graph Semantics (CGS) is a popular tool for modelling semantic networks, including those for representing natural language, cognitive frames (insofar as they are integrated in Concept Blending theories, for example), symbolic logic, and similarly well-structured semantic domains. CGS combines a graph-theoretic approach to semantic structure (with an emphasis on relational structures between concept-tokens, which are single or generic instances of concept-types), with a hierarcy or "partial order" of concept-types (and relation-types). This type-hierarchy corresponds to the hierarchy of types or "sorts" which are formally analyzed in Order Sorted Algebra (OSA), and provide semantics for object-oriented programming languages as well as formal representations of certain concept systems. We can define Conceptual Graphs as Order Sorted Algebras over directed and labelled hypergraphs, meaning that the "base structure" of these graphs is a collection of nodes or nested subgraphs, and that the semantic interpretation of graphs is represented by associating these nodes with concept and relation types.

While this graph formalism is useful for modelling many belief-systems and cognitive episodes, the perceptual experience of immediate consciousness — reasoning through qualitative sensory manifolds to create a mental image of our immediate surrondings — this aspect of cognitive reality is arguably too fine-grained, and too dependent on the perceptual integration of continuously varying sensory impressions, to be readily modeled with discrete graph structures, whose minimal units are concept-tokens, that is, atomic instances of concept-instantiation. Ultimately, as I will explore, a given concept-token may be more complex than these atomic instantiations, and may itself possess an internal structure, not one directly modelled by a contextualizing graph or network, but engendered by model properties of certain concept-types, by an inherent "fuzzy logic" in real-world concepts as they are born by real-world entities, and by the structure of "model spaces" over which possible concept-instantiators can be represented. These are not structures within conceptual networks, but modal or model-theoretic schema which qualify how conceptual networks are holistically related to worldly things. Within the narrower domain of modelling immediate perceptual judgment, both abstract and practical representations — including technologies such as three-dimensional computer graphs and Virtual Reality — tend to take on mathematical structures derived from topology or Differential Geometry. Mereotopology, which combines topology with mereological (part/whole) structures, is popular for modelling perceptual cognition, because it can capture how we use continuously varying (i.e., topologically structured) sensory "input space" to recognize a mental image of a surrounding environment comprised of discrete objects which aggregate into larger wholes.

By analogy with CGS as a graph-based vehicle for an OSA type hierarchy, mereotopology can be represented as OSA lattices defined over topological spaces. For example, a basic finite partition of a topological space $\mathcal{T}$ into distinct "parts", is obviously an indexing of $\mathcal{T}$ points based on which part they belong to, and therefore a map from a set of indices into $\mathcal{T}$. Such an index-set is therefore one example of a "sort" which can be mapped onto a topological space, or which can form the preimage of a mereotopological construction on $\mathcal{T}$. More complex mereotopological structures, such as partitions based on critical points of vector spaces, or level sets of a scalar field or "height function" over a manifold, can similarly be defined as the image from some type in an OSA hierarchy. Furthermore, given types $s \leqslant t$ in an OSA poset $\mathcal{O}$ (a partially ordered type hierarchy), with a mereotopology defined on a space $\mathcal{T}$ and an injection $f : \mathcal{O} \rightarrowtail \mathcal{T}$, the image of s under f will be contained in that of t, offerring a potential mereological structure. So this correlation between Mereotopology and Conceptual Graphs, as both representations of an ordered type lattice over otherwise simpler mathematical categories (ordinary graphs and topological spaces), can suggest a technical basis for integrating these two frameworks for semantic representations.

Here I will generally not work with mathematical details, but situate this discussion in a larger semantic and philosophical context. Our cognition modulates between the immediate, experiential, and qualitative, and the more abstract, systematic, and relational. Cognitive science and cognitive semantics seem to exhibit a similar split personality: there is a more formal, mathematical temperment and a more impressionistic, philosophical ambience which weave together through theories and subdisciplines, sometimes through individual documents. Jean Petitot and George Lakoff, in particular, are particularly skillful at harmonizing elements of mathematical and phenomenological discourse. In many ways, it is Petitot's approach to the project of "Naturalizing Phenomenology" (to cite the name of a compilation which he edited, along with notably Barry Smith; his article in that text is an excellent introduction to a Topological reading of Husserlian phenomenology), which provides the disciplinary framework through which I hope the current paper will be read.

The first half of this paper will discuss issues of modelling "dimensional structures" in semantics, both in the explicit semantics of some concept-types and then in the variations evidenced by "model spaces" for concept instantiation. I will try to sketch a formal apparatus for considering the internal structure of these spaces in light of the contextual form provided by relational networks (like Conceptual graphs) in which concepts are situated. With that schema at least sketched out, I will consider how this notion of continuously varying possibilities of concept-instantiation can apply to mereotopological spaces in which concepts are distributed over a ground space, or in which fusions of concepts are co-variant (like coloration and "spatiation" of two-dimensional surfaces and their visual representations). The technical models I sketch in the first part of the paper will focus on integrating CGS with Formal Concept Analysis (FCA); in the second part, I will consider similar unification between CGS and Concept Blending or Conceptual Integration (CI). In both cases I will consider how the relational structures mapped out by these integrated semantic systems corresponde to spaces of possible model-variation by appeal to geometric, topological, or modal-logic representations of model spaces associated with specific concept-types (or, formally, with sorts in an OSA poset). In returning to notions of modality and modal logic in various places, I will try to emphasize how the role for modal notions in the kind of semantic environment I am using here engenders a non-standard modal semantics in general, and indeed I will be critical of conventional "Possible World" interpretations of modality. The "Axiatropic" structures I propose on Conceptual Graphs, for example, can potentially I believe yield an alternative approach to defining modal semantic carriers; that is, models for modal logics and conceptual networks defined via their signatures. My treatment of modality is influenced by the work of Michael Jubien, which I will consider at different points here.

Finally, I will present a general partition of the "semantics of concepts" — the overall intersection between semantic and conceptual domains — which is influenced by the phenomenologist David Woodruff Smith, and, in particular, his Multiple Aspect Theory (which, as the quotation at the start of this paper suggests, is intended, though not exclusively, to model the "place" of consciousness in the world). David Woodruff Smith and Michael Jubien are lucid, engaging philosohers, and I have been frustrated by the scarcity of material connecting their ideas to widely-read researchers in current cognitive science and semantics (like George Lakoff, Gilles Fauconnier, Joseph Goguen, Mark Johnson, or Peter Gardenförs). I think there is clearly some overlap of philosophical intuitions: read Mark Johnson's *The Body in the Mind* alongside Woodruff Smith's *Mind World*, for example, or Woodruff Smith's collaboration with Barry Smith, on the one hand, against the latter's collaboration with Jean Petitot, on the other.

Humans live in a communal world, and it is the interaction between raw perceptual experience and the norms, obligations, and situations built for us by our communal cohabitants and predecessors, which drives our cognitive transition from the subjective world of sense-data to the systemantic world of propositional judgment and operational mental schema. At the same time, formal analyses of cognition and language nowadays can also be a service to our communities, because they provide computational tools for geographical and virtual communities to organize and retrieve information from their language and data resources. While my primary focus here is theoretical, I will on occasion mention possible practical applications. Even if I do not sketch concrete implementations in detail, the goal of building practical tools is very much part of the generally more speculative voice of the current paper.

1 Semantics and Dimensional Structures

An important question for contemporary semantics and computer science is how best to translate semantic structures into quantitative form — both for modelling their semantic content, and for efficient cataloguing and information retrieval involving language artifacts. Clearly some structures in natural language have associated dimensional forms, or implied meanings with numeric content or some form of numerical approximation. If I declare that a meal was expensive, for example, you can reasonably assume that there is some specific number corresponding to the cost of the meal, and probably also that I know this number. If I declare that a meal was long, we can also assume that there is some number corresponding to the duration of the meal in seconds, although most likely I do not know *that* number. If I call a meal "delicious", then there is no specific numeric measure involved, but presumably I could approximate such a measure by rating the meal in comparison to others (and professional restaurant critics create such metrics by using a system of, say, "five stars"). If I am imprecise about the length of the meal ("about an hour and a half"), this should less likely suggest some kind of evasiveness, compared with similar imprecision in telling you the cost of the meal. These expectations about empirical quantitative structures, known or unknown, are important semantic elements affecting the truth values and the dialogic norms of many linguistic expressions.

Within larger linguistic resources, such as written documents, similar quantification occurs at several stages and scales. For example, an article about train travel in France may be assigned some numerical measure so as to index its relation to relevant topics (trains, France) — in theory, by converting a raw count of its words into a statistical vector or "signature" measuring the document against recognized topics in some canon [17], [5]. Further quantitative structures are involved when trying to extract specific information from the document, such as the cost or duration of a train trip. Even when they are not obviously built in to concepts like "expensive", such dimensional structures are often implicit in semantic structures and individual expressions. For example, any name referring to a geospatial entity can presumably be associated with a location relative to some coordinate system, such as map indices or longitude and latitude. Given sufficient knowledge about a train ride, I expect be able to find its endpoints on a map. Furthermore, most geospatial names refer to regions with an interior, and with borders relative to other regions [38, p. 5]. Such borders, and their implicit spatial dimensions, are internal to the semantics of geospatial names. Moreover, the relation of a name to a geospatial structure is subject to change. When South Sudan became an independent nation, the semantics of the name "South Sudan" changed as well. One way to express this change is by noting how geospatial borders of the referent are now canonically fixed (compare with less precise geospatial designators, like "South Philly").

The dimensional structures which play a role in linguistic meanings do not always map onto numerical structure like integers or map coordinates. Objects whose extensions are defined mereologically — as collections of parts, rather than via geospatial boundaries — can still be associated with dimensional structures insofar as mereological systems possess a partial order which can emulate numerical dimensions [37, p. 7], to some approximation. I also believe that dimensional structures are latent to the Phenomenological concept of "intentionality", or to the cognitive tendency to discriminate more clearly or precisely the properties of objects which are experientially marked as foci of attention. Such cognitive or perceptual focussing implies a set of dimensions upon which a border can be defined between background and foreground, or a graduated transition articulated between perceptual focus and an ambient background.

Goguen's Unified Concept Theory (UCT) sketches a notion of *frames*, which supplement relational models between concepts with summaries of how certain concepts' instances reveal a spectrum or "space" of variation. Here I will suggest a way to codify the intuitive idea that this variation is an ontological property of certain concept-types. I propose to use the term *axiatrope* for semantic formations in natural or formal language which imply dimensional structures, or to representations of these internal dimensional patterns as "attached" to semantic formations. In this proposed terminology, *axiatropes* refer not specifically to measurable dimensions, but to larger relational pairings where they are linked with a qualitative context. I will sketch a theory of axiatropes within Conceptual Graph Semantics (CGS), which provides an elegant foundation for modelling quantitative / qualitative interconnections. Researchers have extended CGS with ideas from several other theories, including Modal Logic [5] and Formal Concept Analysis [30], [46]. The extensions I outline here are related to modal CGS, though they incorporate a more topological notion of modality [23], [3, p. 72].

Jean-Pierre Chevallet uses modal CGS as a theoretical and implementation tool for Information Retrieval, including an interpretation of document and query contents as logical propositions.[1] Joseph Goguen, via "Order Sorted Algebra" and "Unified Concept Theory", integrates qualitative and quantitative structure by distinguishing the structure of logical systems from the structures between their models. These modal and model-theoretic treatments can

[1]A document "implies" a query when "consider[ing] as true all the knowledge within the document, then the query is true" [5, p. 16]. If — to continue my earlier example — a query is for an inexpensive but scenic train route in France, then a document is relevant to this query if it describes one or more such routes. This is a more fine-grained notion of relevance than simply considering a document *about* French trains. This latter, "semantic" notion of relevance typifies the kind of document matching which is the goal of the Semantic Web, where internet-based resources can be queried with the kind of precision familiar to relational databases — for example a "WHERE" condition relative to a train trips' cost. In this example a document needs to be encoded not only with a numeric representation of topics like trains and France, but also with a collecton of numeric "axes" including, in this case, information about individual train trips.

themselves by unified by considering modal assertions as constraints on model-spaces. Similar intuitions, particularly in [15], perhaps motivate the analysis of mereotopological spaces as models for first-order algebras (including CGS deduction chains). In brief, and despite efforts like Goguen and Burstall's "Institution" theory, technical ontology seems to be clustered around two distinct theoretical poles, one based on discrete semantic structures which can be modelled (however tediously) by some first-order logic, and another which relies on some formulation of a continuous model-space, either by using modal operators in the logical system itself or by designating "cononical sortal interpretations" for some logical predicates. However, there is a smaller cluster of analyses which work to bridge this gap, though perhaps not specifically framing their research in these terms. I believe that the technical challenge of integrating more "qualitative" and more "quantitative" (or more *continuous*) semantics belies an underappreciated philosophical subtlety in the nature of concepts themselves.

These philosophical issues come to the fore when we consider how thoroughly numerical measures can replicate or indicate disursive meanings. Translating semantic form to quantitative measure is a central concern of Information Retrieval (IR) using artifacts of natural language. However, IR is not the same as *understanding* language. A search engine can use a travelog to get information about a train trip, without understand the more complex travel narrative that human readers may enjoy. On the other hand, IR systems will be more effeective, the more precisely real-world concepts are modelled. To what degree does quantitative analysis of concepts approximate this real-world understanding? This remains a controversial question in philosophy, cognitive science, and knowledge-engineering. I don't believe there is a simple answer: concept systems blend the quantitative *and* qualitative in complex ways.

Some concepts have straightforward numeric dimensions (like measures of time or money), but qualitative structures determine what substance or proportion is actually being measured. Other qualitative comparisons have measures of similarity or distance. In "What is a concept?" [13], Goguen describes how different theories of concepts can be integrated, including Formal Concept Analysis (FCA), Conceptual Integration (CI), Conceptual Graph Semantics, Peter Gardenförs' theory of Conceptual Spaces [11], and the "Description Logic" which provides a theoretical foundation for the Semantic Web. The theory of *institutions* allows the algebraic equivalence between different systems to be proven, or if needed engineered by logical extensions. Informally[2], an *institu-*

[2]The formal definition uses Category Theory, though its presentation is similar to classical symbolic logic. The main difference is that structures which are treated as mostly expository artifacts external to logical theory proper — particularly sets of logical variables and predicates — are here internalized into the theory. In particular, "sets of logical symbols" becomes a Category ($\mathbb{S}$ign) of "signatures", which are collections of symbols and their use-specifications (like relation arity) that can be mapped into each other (i.e., they are *morphisms* in $\mathbb{S}$ign). This theory lays the foudation for Category treatment of Ontology migration [**?**].

tion is a system for representing concepts and their interrelationships. Often systems can be inter-translated, which is practically important because different theoretical concept-systems lie behind concrete computational tools (like databases and published ontologies) which need to inter-operate.

Unfortunately (perhaps), insofar as there are (on the one hand) more "symbolic" or "logical" systems and (on the other) more "geometric" or "mereotopological" ones, migrating across this divide creates particular problems. A rough analogy is perhaps first-order and modal logic: modal operators are not directly expressible in first-order systems, but families of models for first-order theories can model modal theories, for example by defining each model as a "possible world". This analogy is not purely accidental: indeed, representations of domains with topological model-spaces (such as geospatial ontologies or geographical databases) have been implemented via Goguen/Burstall Institutions precisely by adopting modal signatures [**?**]. However, these representations still express only the axiomatic properties of modal operators, and need further specification to guarantee the proper modal semantics. In my opinion, these technical difficulties in integrating different genres of conceptual schema are really not unfortunate: it is a sign that a nascent "science of concepts" is on the right track. Human thought modulates and mixes concepts from symbolic or abstract realms (societal or legal norms, scientific ideas, etc.), and more continuous or spatial ones (perceptual qualities, geospatial regions).

Cognitive science has left behind a paradigm where all reasoning is foundationally deductive, the kind of "toy" semantics which infers from *John is a Bachelor*, to *John is unmarried*. Instead, cognition is "situated", "embodied", "phenomenological", etc. But in truth both of these paradigms (the more logical and the more situational) have a role to play. Goguen's Unified Concept paradigm, for example, works with theories whose originators — Eleanor Rosch, George Lakoff, Mark Johnson, Gilles Fauconnier, Mark Turner, J. F. Sowa, etc. — were sensitive to real-world conceptual nuance, critical of a narrowly mechanistic or symbol-processing view of cognition, but also interested in formal presentations and computational simulations of congitive processes[3]. Some conceptual activity — creating or appreciating works of art, scientific theories, political platforms, literature and its analysis, etc. — may be fundamentally beyond the reach of computational approximation. Other concept systems, like library card catalogs, have a structure simple enough that they even predate the computer age, while perhaps foreshadowing modern information science. Between these two extremes lies a vast range of concepts and artifacts which are crafted by people, reflecting the natural exercise of cognitive operations in different conceptual domains, but evincing formal structure potentially suited for computational analysis.

[3]Goguen and most of the scholars just mentioned — and also David Woodruff Smith and Michael Jubien — spent much of their careers in California, which may be a coincidence, or a matter of direct influence, or a less direct diffusion of a philosophical milieu.

I believe there has been a subtle philosophical shift from assuming that Information Technology can *replicate* human cognition, to merely *observing*, *simulating*, and *classifying* cognition to varying degrees. Embracing the nuances and experiential richness of human cognition no longer seems to researchers as a step *away from* science and formalization, but rather a challenge to design new generations of tools which are less constrained, less obtrusive, and better integrated into our natural cognitive habits. In this climate, researchers look beyond the theoretical simplifications accepted by earlier generations, where cognitive and conceptual structures were largely separated from the experiential, perceptual, and embodied reality where they operate. 20th-century disciplinary boundaries, like the distinction between a scientifically-minded "Analytic" philosophy and a humanistic "Continental" thought, increasingly seem anacronistic. In [13] Goguen appeals for more interdisciplinary research, reviewing analyses of concepts from social-scientific and linguistic perspectives as well as formal computational and Category Theoretic systems.[4] He and other cognitive and computer sciences (George Lakoff, Mark Johnson, Jean Petitot, Barry Smith) openly embrace analyses of human reality which emphasize, rather than problemmatize, the embodied and experiential nature of the human condition. Even if subtle conceptual and experiential reasoning cannot be replicated on computers, it can perhaps be simulated to some approximation.[5] At least in some domains, technology which is sensitive to human semantic and experiential nuance can help people search, archive, share, and analyze their linguistic and creative artifacts.

Information Technology provides specialized domains where people cognitively operate: for example, the practice of selecting topic-keywords for online articles, partly for the benefit of readers, but mostly for automated search tools. A document corpus may contain a list $\mathcal{L}$ of recognized topics and a map $d \to 2^{\mathcal{L}}$ assigning several topics to each document. This provides a simplified concept-system and a frame for cognitive operations, specifically designed to be computationally tractable. But superior tools do not constrain the cognitive operations to these pre-simplified structures. Unlike document keywords, for example, an *abstract* is targeted at human readers. Algorithms can analyze abstracts (or whole documents) for statistical profiles which match documents to topics probabilistically. In this case computer tools are not providing a simplified domain of cognitive action, but "observing" cognition in its "natural exercise". Previous generations' cognitive and philosophical theories may give the impression that *all* cognition is reducible to simple forms, compatible for

[4]'And making scattered references further afield: Nargajuna, Pablo Neruda, and Garcia Lorca, etc.

[5]For example, search engines already provide a much more sophisticated interface to document collactions than library-style catalogs, using little more than brute statistical force in measuring document-to-document and document-to-query similarity. Conteporary "Web 3.0", Artificial Intelligence, and Semantic Web tools point toward still more powerful software: search engines sensitive to rhetorical inter-textual relations and ontological annotations; image processors which convert digital images to 3D scenes; increasingly sophisticated 3D scene and VRE builders and rendering engines; 3D printers and physical modelling engines; etc.

example with control-flow or classifications within computer code.

Sometimes natural cognition does indeed reveal these kind of structures, at some nested layer; and, granted, people can be productive working in domains which are similarly restricted, such as programming or using computer software, or playing board games. Nevertheless, computer tools are more useful if they are *nonobtrusive*, able to make sense of linguistic and creative artifacts which are not deliberately designed for computer processing. These kinds of tools can only be developed by rejecting the idea that computer operations provide a natural model of human thought, but also seeking within natural human thought and language sufficient formal structure for computational analysis. These tools need not and probably cannot "understand" artifacts as we do — for example, classifying documents by a full linguistic parsing of the text, interpreting all syntactic, semantic, and technical or conceptual details within and between each sentence. Instead, conceptual structures have formal patterns whose "statistical signatures" can be algorithmically observed.

To honor this interdisciplinary spirit at the theoretic level, and these Information Engineering goals at the practical level, we need to include formal analysis of conceptual *systems* within broader analyses of the role concepts play in human personal, social, and perceptual reality. Concepts are the fundament units through which we perceive, experience, interact with, and modify our physical and social environments; to classify and reason about things in the world around us, but also to individuate objects from their physical and visual surroundings. This blend of perception and cognition occurs in social contexts; communal, interpersonal, and pragmatic spaces of day-to-day action and interaction, where our moment-to-moment surroundings are interpreted through linguistic, cultural, and situational norms. It also occurs in the context of our conscious, embodied selves, cognizant of our place as social beings in communities but also as privately experiencing the pleasures and fatigues of day-to-day sensory reality. This embodied, experiential, interpersonal "lifeworld" forms the background continuum from which formal conceptual structures emerge.

Understanding conceptual structures as *emergent systems* allows us to respect concepts' foundation within this subjective backdrop but also recognize formal organizations which conceptual network embody.[6] Conscious experience includes a perpetual affective realm, an unfolding of private sensations pleasant, uncomfortable, and neutral, but we almost always direct our attention past this realm into the public context of our day-to-day lives, and in that setting we operate and communicate through conceptual structures which can often be

[6] Consider how people engaged in a conversation will experience a broad range of emotional and experiential affects, conveyed through subtle accents in speech, "body language", or choice of words; but a significant part of the details of their conversation can still be captured by the formal semantics and syntax of their speech.

abstractly modeled. At the same time, concepts translate raw perceptual experience into mental models; they bridge affective consciousness to our purposive participation in a public world. Conceptual structures overlay this immediate affective reality, allowing us to perceive the world as an environment of individuated objects within physical and social situations, and to perceive ourselves as acting within these situations, turning our mental focus away from private affective sensation and toward this public, pragmatic world. Concepts therefore form the cognitive architecture of public reality, but also the transition within each person's consciousness from private experience to public interaction.

A mature theory of concepts needs to be interdisciplinary because concepts occupy these different registers: they belong to formally organized systems, private experiential cognition, and public situations and communities. Goguen's "Unified Concept Theory" is receptive to these different registers but concentrates on unifying different representations of conceptual *structures*. Here I propose an extension to this theory whose outline is inspired by David Woodruff Smith's "Multiple Aspect Theory". Woodruff Smith develops a philosophical ontology organized around a distinction of *form*, *appearance*, and *substrate*:

> 1. The form of an entity is how or what it is: its whatness or quiddity — the kinds, properties, relations that make it what it is.
>
> 2. The appearance of an entity is how it is known or apprehended: how it looks if perceptible (its appearance in the everyday sense), but also how it is conceived if conceivable, how it is used if utilizable — and how it is experienced or "intended" as thus and so.
>
> 3. The substrate of a thing is how it is founded or originated: how it comes to be, where it comes from, its history or genetic origin if temporal, its composition or material origin if material, its phylogenic origin if biological, its cultural origin if a cultural artifact — in short, its ecological origin in a wide sense, and ultimately its ontological origin in basic categories or modes of being.... The structure $<$ Form, Appearance, Substrate $>$ thus defines a special system of ontological categories. [T]he world includes attributes (of entities), minds (to which entities may appear), and contexts of foundation or origin (from or within which entities come to be). There may be possible worlds that lack such things, but our world has this much structure, and our ontology and phenomenology are accountable to this three-facet structure of the world. [41, p. 17-18]

In its own context, the term "Ontology" here refers to its more philosophical meaning, rather than to formal concept specifications considered by "Ontology Engineering". However, this language resonates with *technical* ontologies which model what may be called *foundational* aspects of conceptual relationships, such as the ontological dependence of concepts on one another or modal relations between them (consider [25, p. 25], citing examples of ontological dependence, such as colors on spatially extended material surfaces, and mereological dependence, the idea that certain parts are essential to the identity of a whole). Some of the more sophisticated published ontologies (particularly DOLCE: [**?**], which expresses details of human perceptual reasoning in the language of symbolic logic) appear to revisit (or rediscover) ontological analyses pioneered especially by Husserl, as early as the *Logical Investigatons* (1900), and especially the famous Fourth Investigation on Parts and Wholes, which introduced a theory of "dependent moments" (such as colors in relation to surfaces) and independent ones (such as detachable parts in mereological, or part-whole, complexes).

To understand concepts as *cognitive* as well as *technical* artifacts, we need to relate these foundational issues to concept-systems: to develop a *Concept Foundation Theory* to complement Ontology Engineering and its theorizing of Concept *Systems*. I propose adopting the above "Multiple Aspect" distinction (form / substrate / appearance) from *Mind World*, applied to the "metatheory" of concepts as follows: the *form* of concepts is the structural organization of networks through which they acquire semantic content, as well as the internal structure of concepts seen as aggregate structures or "blends". The *appearance* of concepts is how they become known to and perceived by people, as perceptual qualities, scientific constructs, abstract ideas, etc., as well as how concept-tokens are represented by different technologies, such as internally in computer memory and visually in software. The *substrate* of concepts is the ontology of their instantiations, how they inhere in their bearers, what it means for a concept to be instantiated, and whether instantiation implies conceptual inter-dependence (as color depends on spatial material, for example). Substrate analysis should also address how conceptual networks have coherent logic even *without* concept-instantiation. Consider fictional entities: how is it *true* that Sherlock Holmes was English, and that "Hercule Poirot was English" is false?

These are no longer merely abstract, philosophical questions. Attempts to systematize conceptual networks (the *form* aspect from this Multiple-Aspect distinction) are provisional at best without these deeper "foundational" analyses. Corresponding to a distinction of *form*, *substrate*, and *appearance* among concepts, we can distinguish multiple aspects of concept-systems: the *relational* structure between concepts, which is primarily the modelling goal of conventional ontology engineering; the *foundational* structure of concepts, which (I will argue) tend to be insufficiently theorized, except as indirectly manifest in relational patterns; and the *representational* structure of concepts, in particular technological concerns about how different conceptual domains can be represented as visual and data components for algorithms and software. I believe that only the first of these aspects (the relational) has been recognized as a core concern of technical ontology modelings. My point is not that the other concerns are ignored, but rather that they are addressed by adding logical structure or axioms to relational models, such as logical definitions of part/whole relations, or concept-types which are axiomatized as real-valued quantities or their computational approximations. These representations may be adequate for modeling *existing* concepts, but not for creating future engines which can learn *new* concepts in flexible, semt-automated ways.

The fundamental limitation of all technical ontologies is that they are intrinsically *static*; they are a snapshot of conceptual relations at a single point in time. Goguen's "Institution Theory" and the logical (or Category) theory of cross-ontology mapping allows ontologies to evolve and merge over time, or at least provides practical, computational tools for ontology migration. But these are still human-engineered artifacts, "samples" of domains at different moments, demanding an explicit updating, modification, and migration. Real-world conceptual evolution is fundamentally different: it unfolds "in real time", in the context of geographic and/or cognitive communities whose innate concept schemes evolve primarily by convention rather than by fiat. There is therefore an intrinsic connection between human *concept systems* and human *communities.* Only in the context of communities do concepts become systematic vehicles of thouht and communication, linking cognition and language.[7]

There is therefore a general philosophical and cognitive reason to emphasize the communal situation of concept-systems. If we envision technologies that "understand" concepts in flexible, evolutionary ways, then these will have to simulate our human receptivity to conceptual change in the context of (geographical, linguistic, and knowledge) communities. In our Internet Age the striaghtforward problem is semantic clashes, because localizing effects in how names and words are resolved can get lost.[8] Insofar as language and text from around the world is juxtaposed online, these local effects can be hard to recover, because they can project contexts presupposed by language-users rather than specifically mentioned. But while integrating local and global reference-frames is important, the more complex problem is to develop technologies which are sensitive to community-specific conceptual evolution. In this problem-space, the notion of community plays a role in theorizing interpersonal cognitive space and social and situational context; but in addition, the narrower domain of "Community Informatics", working with ontologies and semantics of local, virtual,

[7]I do not discount the possibility of "private" concepts, like a mathematician inventing new algebraic constructions to prove a theorem. But these must be symbolically represented or translated into other, pre-existing concepts, if the ideas they engender are to be communicated at all. Moreover, private concepts are representations of objects, structures, patterns, or other details within our environing world, and this world is shaped by collective thoughts and behavior, so private thoughts rest on a foundation of prior communal concepts.

[8]For example, the word "Chinatown" may refer to several different New York neighborhoods (in Sunset Park, Flushing, Manhattan, Elmhurst), but would not generally be confused with the Dundas/Spadina intersection in Toronto, because typically *either* New York *or* Toronto would be the expected frame of reference.

and knowledge communities — the discourse of geospatial, architectural, urban environment, social network, and related domains — provides useful case-studies of conceptual change. Geographically, for example, place names and boundaries are often in flux; political events cause new geospatial concepts to appear — like recently South Sudan; and old ones disappear, like the USSR. How can reasoning engines make sense of this dynamic process? This question will offer several examples, which I will use to motivate theoretical questions about ontological structures involved in concept-instantiation and concept relations.

So I am interested in "cognitive community" in a philosophical sense but will discuss *geographical* communities for some case-studies. Toward the end of this paper I will mention case-studies concerning other types of communities within the bounds of Community Informatics, like "virtual" and "knowledge" communities. My primary goal is to develop a model of conceptual spaces in which the distinction of *relational* and *foundational* aspects correspond roughlys to the distinction I suggeted earlier between *logical/symbolic* and *geometric/mereotopoological* concept-spaces. Goguen, for example, considers unifying numerous semantic theories, but perhaps the most significant integration (and the one he discusses in most depth, at least in [13]) relates the "Conceptual Space" or "Mental Spaces" model of Fauconnier with the "Conceptual Spaces" of Peter Gardenförs. Mental spaces in the former sense are symbolic networks, in which concepts are characterized by typically clusters of interrelations.[9] Gardenförs's notion of *Conceptual Spaces*, particularly in the book with that title, is more "geometric". It considers concepts organized in quasi-mathematical continua where "nearby" concepts have borders and spaces of internal variation.[10] Geospatial concepts also form "geometric" concept spaces; for example the neighborhoods of a city link up to cover the spatial extent of the city. Becuase geospatial terms embody both community-based semantic norms and semantic frames with complex spatial and topological models, they are interesting case-studies for the interaction of these semantic dimensions.

Consider dimensional structures as semantic phenomena, in the context of

[9]For example, a *house* typically has an owner, one or more residents, an address, etc. The distinct meaning of particular concepts is manifest in the specific set of other concepts which are canonically linked to them: a typical apartment, like a house, has an owner and resident, but lacks a unique street address; a business, like a house, has an owner and address, but not someone living there. These kinds of semantic networks create "frames" which individuate different concepts. *Blending* frames, via metaphor, compound phrases, or inventing new terms, leads to new concepts which mix the properties of pre-existing ones, by mixing the canonical relations which they individual take one.

[10]The canonical example of this kind of space is colors. Colors can be mathematically modelled as a geometric space in several different ways; every visible color will be recognized as falling within some named category (red, blue, etc.) which spans these spaces, with some hues being more or less ambiguous depending on how large the set of color-words we can use. An artist or designer may be familiar with a large set of color-terms and use them to identify color-samples, while other people may associate samples with the best match from the most familiar words, like red, blue, purple, etc.

collective or communal discursive frames and in terms of the form / substrate / appearance distinction I proposed earlier. Numeric dimensions can be used for topic extraction or semantic clustering, part of a statistical toolkit or Latent Semantic Analysis, designed for information retrieval but not generally for modelling conceptual structures directly. On the other hand, LSA techniques, or related schema like Semantic Indexing, represent concepts and texts for efficient future querying; they are therefore *representational* treatments of conceptual domains. These representations are "behind the scenes", probably not directly visible to technology consumers, but they are technologically connected to schema for visual or interactive presentation of texts and concepts. In other words, they belong to the "appearance" aspect of concepts and concept-types, at least as this aspect is extended beyond perceptual or scientific appearances within the sophisticated representational capacities of modern technology.

Leaving this aside, semantic dimensions intersect with both cognitive and communal perception and reasoning in different ways. For example, certain dimensional structures are internal to perceptual experience, at least insofar as it can be theoretically modelled: the directly perceptible shapes and forms of the objects in our immediate line of vision, for example. Insofar as cognition transfers these direct perceptions into relational mental schema, some of these precise structures are lost. When I simply contemplate interacting with some object, for example (anticipating, say, the arrival of a metro and thinking about how I have to study the subway map, look for a seat, etc.), its precise spatial form and dimension does not have the same direct presentation as when this object is before me (when the train arrives, say). On the other hand, there are other cases where dimensional structures are less apparent in direct perception but emerge insofar as we synchronize our perceptions and the mental pictures which emerge from them, with other peoples', both implicitly and explicitly. For example, the collective representation which residents of a neighborhood form of geographical features, which are disclosed in bits and pieces to each individual person, nevertheless coalesce into a relatively complete picture which cohabitants share (and help each other to develop). Most people have never walked aruond the perimeter of their city or neighborhood, but they still have a rough impression of its shape, how it should appear on maps, etc. In this case, dimensional structures are not present in direct perceptions (because each perception of a neighborhood, for example, is only one sample of a much larger whole), but these structures do emerge from a superposition of many such samples in an interpersonal context. Certain dimensions are internal to the *foundational structure* of perceived entities, such as the spatial form of three-dimensional objects, and engender dimensional structures internal to raw perception which then fall away, or become simplified, in the transition to cognitive schema. Other dimensional structures are not given in perception but emerge as perceptual episodes are combined, over time, into conceptual networks representing our belief systems and schema of our environment, incluing our social and geospatial surrondings.

Several geospatial case-studies will occupy me in the next section. I will try to motivate my proposed alignment between the David Woodruff Smith-inspired "Multiple Aspect" approach to concept ontology — the distinction and connection of concept *form* to *relational* concept networks; concept *substrata* to *foundational* concept ontology; and concept *appearance* to *representational* technologies. I will explore how different systems represent conceptual networks, particularly `CGS` and `FCA`, and then show how these representations can be extended to address more "foundational" issues. I will then briefly explore modal and mereotopological models for the resulting systems seen as formal logics.

2 Relational and Foundational Ontology

A new nation engenders a multi-faceted conceptual change: the semantics of the phrase "South Sudan" has fundamentally changed, for example, and new concepts have come into being alongside: the government, citizens, political leadership, currency, and military of this nascent state. Sometimes new geospatial concepts emerge more quietly and gradually: until recently most New Yorkers would have called the trendy Manhattan neighborhood "Nolita" (from "North of Little Italy"), just east of Soho, as simply part of Soho or Little Italy. Local residents and businesses felt that there was an (architectural, commercial) distinctness of their few square blocks and moreover that marking this distinctness, effectively inventing a new geospatial moniker, served local (business, realty) interests. Another New York City example: "Sunset Park Chinatown", a mile-long cluster of Asian restaurants and stores along Brooklyn's Eigth Avenue, would in earlier generations have been seen as just another street in Sunset Park. Although neighborhood names and boundaries are sometimes fixed by policy (postal codes, electoral districts, Zoning laws), the last two examples illustrate names being adopted by convention. Sometimes pre-existing names are maintained, but their perceived referents change: when Williamsburg, Brooklyn became trendy, adjoining Greenpoint — pleasant, historically Polish, but without the hipster vibe — magically shrunk, at least in terms of how local businesses described their locations. Concepts do not change arbitrarily: the "rebranding" of Nolita only worked because the blocks between Little Italy and Soho do indeed have an ambience different from their earlier-named neighbors. Concepts emerge within a distinct semantic niche, recognized and eventially embraced by a relevant community. Concepts therefore have *roles*, serving to individuate some object or idea from its surroundings, or to group together abstract or concrete entities which share some relation. Axiomatic representations of concepts often capture these roles only imperfectly or indirectly.

A geospatial ontology can certainly distinguish between nations and neighborhoods, and represent the extra conceptual structure (treaties, international relations) implied by the former. But each emerging concept has a unique pattern and history. The examples I just discussed are fundamentally different from each other: South Sudan is partly a geospatial concept, but mostly a political one; whereas Sunset Park Chinatown and Nolita emerged informally, individuated more by ambience than by social or political separation. These three concepts are each tokens of the concept-type *geospatial region*, so they share some foundational features.[11] But the difference between semantic networks where *nations* are situated, and *neighborhoods*, are just as significant. South Sudan is a political and cultural unit, a social group and category, a geospatial region, and these entities, while currently fused, are not *a priori* coextensive: the South Sudanese people can (and once did) exist without the state, as can the geographic area, and the people (via emmigration and cross-border migration) can spread beyond the geospatial region. On the other hand, because "Sunset Park Chinatown", for example, is not a formally recognized moniker, the evidence for its conceptual status and reference is purely indirect, which presents complexities of a different kind. These are differences in *foundational structure*, in the metaphysical patterns relating concepts to their (here geospatial) substrata. The concept *South Sudan* "inheres in" its geographical area in different ways than the two neighborhood-concepts. The latter two are also different from each other: the name *Nolita* has been more deliberately promoted and formally established; *Sunset Park Chinatown* is more informal, less exact, and to some measure a descriptive phrase rather than a proper name. These *differences in foundational structure* are central to the concepts' respective place in the world, and to any cognitive activity which properly involves them.

A geospatial ontology may (and should) properly distinguish neighborhoods from nations, and therefore, say, *South Sudan* from *South Philly*. "The President of South Sudan" makes sense; "the President of the South Bronx" does not, except in jest. But there is a subtler distinction between "South Bronx" or "Sunset Park Chinatown", on the one hand, from *Nolita* or *Montparnasse*: the latter are more precisely defined, for example by the Parisian Arrondissments, while the former are more informal, used by locals for orientation. A geospatial ontology probably would *not* represent this kind of distinction, though perhaps we can envision a distinction between, say, a neighborhood *district* (officially named and recognized) from a neighborhood *area* (an informal designation used by locals). Using this provisional terminology, there would be an expectation that *districts* (but not *areas*) are associated with further legal or political entities (like postal codes or electoral districts). We can cut the distinctions still finer: "South Bronx", say, is a somewhat informal designator which semantically marshals a geographic term, whereas "Sunset Park Chinatown" involves a cultural or "ethnic" reference. People visiting the Chinatown would presum-

[11] Their referential extent can be mapped via a single-dimensional boundary line, for example, and is therefore a geometric structure, and a topological one, separating an interior from an exterior. They have geospatial location relative to other regions both topologically (other neighborhoods or nations which they border) and geographically (relations of distance, and geometric and compass direction, with other places).

ably be looking around for markers like Chinese Restaurants, to check if they are in the right place. People looking for the South Bronx, at least on a map, would presumably look at the Bronx and then toward the south. With respect to their conversations, as they orient themselves, we might consider "South Bronx" say as *an informal neighborhood designation using geographic indicators* and "Sunset Park Chinatown" as *an informal neighborhood designation using ethno-cultural indicators*. Should a geospatial ontology include axioms for *that* distinction, or recognize these as distinct subconcepts of the subconcept "area" of the concept "neighborhood"? These details are of dubious merit in a geospatial ontology proper. They belong more to the *pragmatics* of these terms as proper names, than to the *semantics* of words like "nation" and "neighborhood". On the other hand, pragmatics and semantics overlap: a conceptual acquaintance with "South Bronx" must include some notion of its boundaries — *and also* knowledge of the imprecision of these boundaries, because this too can play a role in linguistic contexts where "South Bronx" appears.

The boundaries of South Bronx are imprecise because convention recognizes the general location of this area but not a rigid outline. The conventional popularity of a designator may spur more precise refinements — like a South Bronx police precinct — but this is not necessarily a rigidification of the earlier concept (unless it spurs a communal reinterpretation), but rather the emergence of a new concept (*South Bronx precinct*) which semantically and referentially overlaps with, but is not identical to, the prior one (*South Bronx*). "Sunset Park Chinatown" too is imprecise, but also "fluid": Chinese businesses are starting to emerge on 7th and 9th avenues, so the perceived extension of this concept is likely to expand. The rigidity, fluidity, or imprecision of a concept is intrinsic not only to its *pragmatics*, but also to the semantics of some discursive situations, like someone asking for directions. Since these situations will be among those from which reasoners (whether automated tools or human languate-users) extract uses of a concept, a failure to represent these pragamtic details means a failure to properly parse discursive artifacts: that is, a failure to properly map the static structures of a technical ontology onto the concrete semantics of particular conversations. We can formally introduce ever-finer conceptual distinctions: the contrast of *neighborhood* and *nation*, from which the humour in *the President of South Bronx* derives, and then further between imprecise designators like *South Bronx* and sharper ones like *Nolita*, fixing a "subclassification" of the neighborhood-concept into perhaps "areas" and "districts". But the attempt to pre-emptively capture fine-grained pragmatic contrasts through relational models of concept-systems is bound to both lag behind real-world semantics and to engender a combinatorial explosion of concepts and distinctions. The problem is that these pragmatic details, while partly reflecting conceptual classification and inter-relationships, is more significantly a reflection of distinct *foundational* patterns specific to each concept: ontological details of how specifically the concept inheres in its bearer or instantiators.

When we learn a new concept, we situate it in relational and classificatory networds, but we also internalize these foundational issues: the general category of its instantiation (geospatial or material, singular or plural, abtract or concrete, etc.), its precision or imprecision, the geometric, topological, or mereological (part/whole) properties of its extension, and so forth. We learn to recognize concept extension and criteria for separating that which falls within and outside the concept. This applies both to "sortal" concepts which have many instances (providing the scope of their extensions) and to geospatial concepts whose extensions are marked by spatial boundaries, or concepts like proper names and referential expressions (like "this book"). The latter cases give rise to their own extensional complications, which are often discussed in mereological terms: the referent of "this book", but not its referential semantics, perhaps changes if a page falls out; this can analyzed in terms of the mereological significance of a part in a whole (like a page in a book). I will discuss mereological concerns with respect to referent-individuation in greater depth below; my attention right here is on geospatial extensional complexity, but some of these ideas carry over to other cases as well. Users who learn a concept (or acquiesce to an emerging usage by repeating it) implicitly (sometimes explicitly) identify salient features of the concept's unique niche. These features (like Asian Markets in Sunset Park Chinatown, or the narrow Nolita alleys) provide both a rationale for concept individuation and a provisional map of their extensions, which in the case of place-names includes their geospatial boundaries. Automated tools, were they to emulate human reasoning, would need to replicate the process by which locals figure out the reference and rough extent of new geospatial names, identifying and mapping salient features (like Chinese Markets), and relating them to whatever semantic cues may be available (like Chinatowns typically having numerous Chinese Markets). Similarly, we were primed to accept the new conceptual role of the phrase "South Sudan" because there was a sequence of historical and political events, accessible to the general international community through news reports, events belonging to politics and treaties than to geography. Actual independence simply consolidated this evolution, but we were prepared the learn the new concept becuase it rests against a foundation of our prior knowledge.

Different foundational structures are part of concepts' individuality, and may be expressed by formal representations of conceptual relations, such as an axiom that every nation has a head of state. So relational models of concept-systems can subsume certain foundational issues, but only through deliberately engineering and/or definitions within an ontology. This reliance on human design "intervention" condemns ontologies to be rather artificial constructs which are disconnected from real-world, evolving conceptual plasticity. A truly sophisticated ontology engine would be able to *recognize* that "South Sudan" is (now) a nation, that "South Bronx" is (sort of) a neighborhood, that "Nolita" is (certainly) a neighborhood, that there are South Sudanese *citizens* but not "citizens of the South Bronx", except in jest. In other words, a mature ontological reasoner would construct relational inter-conceptual networks "on the fly" and also recognize the idiosyncracies of individual concepts' foundational properties. The reasoning involved in these operations can be modelled, but the models require a level of sophistication which exceeds the deductive structures programmed within axiomatic ontology tools.

To illustrate, consider a Brooklynite forming the habit of using the phrase "Sunset Park Chinatown". This is not the kind of concept one learns formally in a textbook; she probably first heard it casually mentioned, or overheard it, saw it in a local paper, etc. If she had been to the area and noticed the cluster of Chinese businesses, she perhaps connected the phrase and this prior observation; if not, the phrase may influence her initial impressions. In either case, the internal semantics of "Sunset Park Chinatown" should play a role in her expectations, even if the phrase becoming common parlance implies that it evolves from a definite description to a proper name, wherein internal semantics can be misleading (Little Italy is not very Italian anymore, Dartmouth is no longer on the Dart, there are few Germans in Philadelphia's Germantown or Ukrainians in Chicago's Ukrainian Village, etc.). But her sense of the area's extent, relative to the rest of Sunset Park and to adjacent neighborhoods, depends on observing places which fit the Chinatown "theme" and also where these become scarcer: Chinese markets, restaurants, signs, churches promoting Chinese-language services, etc. Gradually, perhaps subconsciously, walking through the area, she comes to recognize its center (along 8th avenue, to be concrete), its periphery, its adjacent but separate neighbors (observing perhaps the Spanish-language milieu along 5th avenue and the Hasidic stores and synagogues due west in Borough Park). Mentally behind the scenes, while her conscious attention is more on the architecture or where to have lunch, she peforms subtle operations to cluster salient features into semantic filters (Asian/Chinese, Latino/a, Hasidic/Orthodox) and then map these onto identification in her surroundings, which in an urban environment will prominently feature restaurants, shops, etc.

These semantic and geo-social spaces, overlayed together, provide an emerging mental map, which we can characterize in statistical-topological terms. A plausible cognitive model will identify regions where semantically clustered identifiers are borne by densely clustered individuals — a Chinese restaurant near a Hong Kong Supermarket near a Korean kimbap stand; a synagogue down the block from a Kosher bakery across the street from a tailor. The observation of these semantic and spatial proximities can be formally summarized: her mental map seeks to identify topologically simple regions in which semantically similar identifiers are associated with spatially proximate individuals — in this specific case, Borough Park, Sunset Park Chinatown, and the rest of Sunset Park align neatly together East to West. Her mental picture, in its emergence, may be simulatd by considering homology classes of topological spaces formed by connecting salient identifiers (Chinese restaurants, Kosher markets, etc.) with different distance thresholds — effectively a two-dimensional version of Persistent Homology Analysis on a point-cloud paritioned by semantic clustering.

Suppose a Formal Concept Analysis algorithm which considers, say, descriptions and locations of Brooklyn businesses, identifies a correlation between Chinese restaurants and markets and Sunset Park locations — that is, correlation between the feature of catering to people of Chinese origin and the feature of being located in Sunset Park. This correlation can then explain the semantics of a phrase like "Sunset Park Chinatown". On the other hand, the correlation itself does not specify the *referential* scope or spread of this phrase taken as a proper name — for example, which streets and blocks are actually inside Sunset Park Chinatown. This further detail may be approximated by plotting the geospatial points of Chinese-themed business and translating this "point cloud" into a contiguous region. Persistent Homology can translate numerical data into topological properties, yielding a simplified representation of the data; for example, disks of radius ϵ are drawn around each data point (such as "Chinese-themed businesses" in this example), and the manifold formed from the union of these disks will have topological properties (such as being connected or being simply connected) depending on the choice of ϵ. The homology algebra thereby derived from a data set, given different choices for ϵ, provides a very succinct representation of the "shape" of the data.[12] When defining a perimeter for Sunset Park Chinatown, it is not necessary to include every single Chinese restaurant or market in the neighborhood, but rather to outline the central cluster of these businesses with a topologically straightforward (perhaps simply connected) region.[13]

This kind of analysis, combining Formal Concept and Statistical Topological techniques, provides if nothing else a formal model of the kinds of cognitive operations which go on amongst linguistic communities as new descriptive and referential phrases are incorporated into a lexicon. So discourse-pragmatic reasoning can be simulated by algorithms; but this is still quite different from the decutive logic available to ontology engines. Using SQL (the Standard Query Language of relational databases) as a rough analog for this first-order logic, algorithms like Semantic Clustering and Persistent Homology are analogous to specialized libraries operating on database content; certinaly there are no *SQL queries* which calculate persistent homology classes! The kind of ontological reasoning involved in properlying identifying the rough outline of a neigborhood, for example, is different than the kind of reasoning which can be effectively

[12] A data set may be partitioned into different groups with their own topologies — in this case, to separate "Sunset Park Chinatown" from adjaceny areas, plotting Chinese-themed businesses alongside Latino/a-themed businesses (considering the significant Latino/a population of Sunset Park as a whole) and locations reflecting the Orthodox Jewish character of Borough Park. This could statistically recover the fact that the area in question has three rather self-contained, culturally distinct sub-communities.

[13] In more complex situations it may be also be useful to topologically suture the manifolds derived from criteria which are considered similar, or similar enough to serve as joint markers of some featural contiguity. For example, to demarcate the larger, more diverse Queens Chinatown in Flushing and Murray Hill, it may be necessary to overlay manifolds drawn from businesses "themed" relative to different Chinese ethnic groups (Fujian, Cantonese), as well as Korean, Taiwanese, etc.

modeled as deductive chains over conceptual graphs.

This hypothetical example implies that a large dataset is already at hand and then subject to a mathematical (statistical, topological) analysis. The idea is to show algorithms which might *simulate* human reasoning and allow automated tools to "learn" new concepts. On the other hand, the analogy is still inexact. The typical New Yorker, for example, would not assemble a database of Sunset Park locations and set out to consciously draw a map of the neighborhood. The proposed algorithms are representaitons of subconscious observations which people would make in the course of simply walking through or carrying out quotidian activities in the area. Someone who spends enough time there would presumably acquire a mental map similar to the one hypothetically constructed in this example; in other words, the Statistical-Topological algorithms condense into a single analysis a series of observations and assimilations of new information that, for a real person, could unfold over weeks or months. Even while this picture is still accruing, we should still recognize her as having some acquaintance (however incomplete) with the concept *Sunset Park Chinatown* — how to get there by subway, how to walk to the nearest station, etc. Alongside our analysis of the substrata-relationships which concetps entertain with (in these caes) geospatial bearers, we also need to consider epistemological questions concerning how relevant the details of these relations are to cognitive processes involving geospatial (for example) concepts. In other words, the geospatial extension of a concept like *Sunset Park Chinatown* is significant because it provides a space of ever-unfolding observations; each visit to the area adds a layer of detail to someone's existing mental map, until a well-informed picture the neighborhood's character and outline emerges. On the other hand, many of our beliefs and opinions about places are formed independent of any precise sense of their boundaries, even of their precise locations; and many of these propositional attitudes may be held with little if any properly spatial acquaintance with geospatial entities at all.

Indeed, most of our beliefs about places and geospatial regions are not founded in geographic or spatial figures at all, but rather reflect places' status as cultural, political, social, or economic entities. We might conclude that cities, nations, and neighborhoods are therefore not *really* geospatial entities after all, but this raises the problem of what they actually *are* then. The underlying problem here might be called *foundational vagueness*: we have a detailed collection of propositional attitudes vis-a-vis geospatial entities, but in most cases we have at best a vague sense of their actual geospatial identity: their locations, extents, and other geospatial details. The *facts* we know about them are not generally *geospatial* facts, but the *truthood* of these facts is presumably truthood *about* geospatial entities. They are non-geospatial facts which are *founded in* geospatial entities. There is, in other words, an ontological cross between the high-level concept-types where we may classify the *facts* and where we may classify the *entities* which bear these facts (or bear their "truthood"). Moreover, belief or acknowledgement of these facts can be entertained despite considerable vagueness in perceiving the geospatial entities *as* geospatial. Similar vagueness can apply to other domains than the geospatial: my computer is a physical objects, but most facts I know about my computer are not physical facts, etc. This complicates how we understand (collections of) geospatial or physical entities, etc. — entities in their foundational domains — as models of conceptual networks, analogous to models of logical systems. These model-theoretic issues will be discussed next.

3 Foundational Vagueness

Many people around the world may be vaguely away that South Sudan is now a nation but have little if any picture of its geography, cities, bordering countries, etc.; in short, little knowledge of South Sudan *qua* Geospatial Region. On the other hand, simply by placing it under the concept-type *nation* we form some beliefs about it, and acquire a framework for gathering more information: we can look up who or where are its capital city, head of state, how its flag looks, etc. The assertion $(\star)$ “Juba is the Capitol of South Sudan” relates two geospatial regions. Consider the standard logical interpretaton of $(\star)$: we express the concepts *Juba*, *South Sudan*, and *capitol of* with atomic symbols for convenience, yielding a formula like $(\check{\star})$ $c(j, s)$. Here c is a binary predicate; j and s are symbols for individuals (logical analogs for proper names). To recover the real-world fact which $(\check{\star})$ represents, these symbols are replaced by (here geospatial) entities to which they refer. So someone who believs $(\check{\star})$ believes that *a certain geospatial* entity j is the capital city of another such entity s. I believe $(\check{\star})$ because I saw it in a book. On this account, I therefore believe that *a certain geospatial entity* is a capital city. However, I frankly have no idea where Juba is, what its boundary looks like, have never seen in picture or in person any of its streets or buildings, etc. In other words, it is hard to say that I actually have some mental image of a geospatial entity Juba, and another such image of South Sudan, and assert that the thing represented by the former image bears that relation (capital-to-country) with the corresponding latter image.

I presume that I could acquire this information if I ever need to. Does this have any bearing on my belief that $(\check{\star})$? Should it affect the epistemological status we assign to this belief? Suppose a friend tells me that Padraig and Miyoku are engaged. Suppose that I have no idea who these people are; perhaps my friend thought mistakenly that I knew them. If I think he is being sincere, what exactly do I actually believe? That *Padraig and Miyoku are engaged*, full stop, without any sense of the referents of these names? That the sentence “Padraig and Miyoku are engaged” is true? That my friend knows someone named Padraig and someone else named Miyoku, and that they are engaged? For all I know, these people are actually characters in a television series. Or perhaps he is writing a novel and forget to tell me about it. I think we intuitively sense that I believe *something*, but struggle to articulate just what this is. For example, do I have an *actual belief* about two people, or only the *potential to have* a coherent such belief? Is my belief really only a "down payment" on a real belief, to put it figuratively; a framework for acquiring an actual belief? What if my friend were telling a long story and simply fabricated Padraig, Miyaku, and their engagement, to catch me not paying attention? How much information do I need for my putative belief or framework-for-a-belief to become a real belief?

Given this example, I believe the $(\breve{\star})$ case is similar. I know little more about Juba and South Sudan that I do about Padraig and Miyaku. Suppose my friend were indeed writing a novel set in a fictional nation. He might tell me: $(\star)$ "Abuj is the capitol of South Nadus". Perhaps I do not realize initially whethere these are real places, fictional, etc. As with Padraig and Miyaku, we intuit that some further information is needed for me to hold a real belief. In the case of $(\breve{\star})$, is it enough simply to know that South Sudan is a real country? A book asserting $(\star)$, in a context suggesting that South Sudan is a (real) nation, presumably is offering sufficient information: further geospatial details, like maps or pictures, are interesting but not necessary for a coherent belief that $(\breve{\star})$. Is the difference here simple the contextual implication that referents of $(\breve{\star})$ are real places? Does my framework-for-a-belief about Padraig and Miyaku become a real belief with nothing more than my asking, say, "Are those friends of yours?".

Evidently belief in a logical proposition p demands some minimal sense of how the names or symbols in p refer. Are they real people and places, fictional, completely made up,[14] etc.? But we seem prepared to accept beliefs as well-formed with little more than these quite minimal details. My knowledge of concepts like "capitol" and "nation", and my belief that $(\star)$, confirms that I could acquire a more-informed geospatial picture of Jubas and South Sudan (perhaps by studying a map). But we are not inclined to say that my belief that $(\star)$ is not a real belief, only a propositional attitude that can become a real belief upon this further investigation. Instead (so we intuit), the possibility of this further investigation, even in the absence of it actually occurring, consolidates the well-formedness of the belief.

Let us call this potential future clarification a *retroactive consolidation* of a belief b. If b is my belief that $(\star)$, then retroactive consolidation explains why b is belief that some specific entity, which is in fact a geospatial region (Juba), is related (is capitol-of) another geospatial entity (South Sudan), even though my entertaining b does not coincide with having real acquaintance with these entities *as* geospatial. Since I can defer to experts, who do know these geospatial

[14] Distinguishing *fictional* from *made up* in the sense that fictional persons and places in published or well-known works of literature, etc., have fixed properties. Conversations can refer to pre-existing fictional entities, or imagine completely new ones.

details in depth, my minimal sense of "Juba" as at least a real place is proxy for a detailed geospatial account. But who *does* know the full geospatial account of Juba and South Sudan? Where is this information available? A map is only a partial representation; and presumably the border between Sudan and South Sudan is not painted on the ground. Borders are inexact: they may be marked by a natural or man-made landmark, like a river or fence, but fences can blow away and river-banks ebb and flow. Even "experts" lack infinitely fine-grained accounts of geospatial regions, because they are simply not possible.

This vagueness presents nontrivial problems for symbolic logic as an expression of real-world semantics. We take ($\breve{\star}$) as representing a fact recovered by mapping j and s to geospatial regions. So my belief b is a belief about these regions, despite my vague sense of these regions *qua* geospatial, partly because I can get further information if needed. But there simply is not *any* non-vague representation of these regions. So we cannot simplistically use the possibility of future refinement as retroactively consolidating b, because these refinements will still yield beliefs whose referents are vague, needing further retroactive consolidation, etc. There are many people who have beliefs (including justified true beliefs) about South Sudan, but literally no-one who knows precisely what South Sudan *is*; in other words, which precise geo-spatial region *is* South Sudan, because this abitrarily fine-grained knowledge is not possible. By analogy, I may *believe* your assertion that Padraig and Miyaku are engaged, yet not *know* who these people are. We have several options: (1) these two cases are equivalent; this latter belief has similar epistemological standing as my belief that ($\star$). (2) Beliefs about South Sudan are equally well-formed to my belief about the engagement *after I have clarified* whether Padraig and Miyaku are real people, fictional characters, etc. (3) We know (or can find out) the *approximate* geospatial extent of regions like South Sudan, which allows beliefs like ($\star$) to be well-formed. This is in contrast to a fictional case like ($\ast$), in which we cannot form even an approximate geospatial account of "South Nadus" unless this happens to be mentioned in the fictional work where it appears.

Option (3) here allows the referential semantics of beliefs to be progressively refined, not to a set of *perfect* fine-grained accounts of referents, but at least to accounts which are as fine-grained as may be deemed necessary for a given circumstance (planning a trip to South Sudan, negotiating rights to underground resources near the Sudan/South Sudan border, etc.). So the prospect of this *progressive refinement* perhaps retroactively consolidates a b, even without some "terminal" refinement: the "consolidator" here is not any single refinement but rather the refinement process itself. But none of these options fully address the problem: if indeed *South Sudan* is a geospatial region, and *knowing what South Sudan is* means knowing its geospatial extent, then *knowing what South Sudan is* is not possible. Nevertheless people (apparently) know many facts *about* South Sudan. The follow-up question is whether it is indeed possible to know facts *about* something without knowing *what it is*. This is a nontrivial question, especially if we consider "knowledge" about fictional entities. We may know that Abuj is the capitol of South Nadus (in some novel), or that Sherlock Holmes lives in London. If b is belief that proposition p holds once its designators are replaced by real-world referents, then beliefs about "Abuj" or Holmes are not strictly well formed because these designators *can't be* thus replaced; and indeed beliefs about *Juba* and *South Sudan* also cannot be well-formed, because no-one actually knows what geospatial entity would stand in for these names. Moreover, the cognitive operation of forming (holding, validating) the fiction and non-fiction beliefs here are similar. I can read in a book that ($\star$), and I can read in a novel that Abuj is in South Nadus. In both cases I mobilize my familiarity with the general semantic frame of cities and nations. Unless I actually visit South Sudan, there is little in my actual mental states to distinguish my beliefs about this real nation from beliefs about fictional ones.

There are many entities about which we have coherent beliefs without direct perceptual acquaintance: there are places we have never visited, people we have never met, etc. We form these beliefs not from direct experience but from our experience with conceptual domains and our collective ability to use language for creating structures within these domains. We know that a nation has a capitol, a government, etc. This is why we so readily accept (and reason about) fictional narratives: they paint coherent pictures within semantic frames we already use in real life. A set of facts or assertions about some specific (real or fictional) entity can be called a *Symbolic frame*. We can formally represent symbolic frames using systems like Conceptual Graph Semantics, Ordeer Sorted Algebtra, etc., including combinations of these. A CGS representation would display frames via graphs Γ, whose nodes represent both concept-tokens and relations between them. A simple Γ may have nodes representing South Sudan, Juba, the relation capitol-of, etc.; the logical formula ($\breve{\star}$) $c(j,s)$ would be expressed as a "triple" or "doule-edge" $j \leftarrow c \rightarrow s$, this representing two graph-edges, with a "relation-node" c between them. Further information, like the colors of the South Sudan flag or the population of Juba, could be added as additional double-edges connected to the c and j nodes. As new data is added, the graph grows like a snowflake. This open-ended expandability, not being constrained by restrictions like database schema, lies behind the popularity of CGS or the related Semantic Web "Triplestore" format, because it allows information to accrue in a decentralized, collaborative way. Different aggregates of data can moreover be combined simply by joining them along common nodes.

This open-ended collaboration does present problems like the name-clashes I mentioned earlier; it can be difficult to distinguish when different nodes represent *the same* concept, or rather different concepts which, say, share the same name (like "Reds" the movie and the sports teams). It can also be hard to distinguish data intrinsic to some concept (like the Capitol of South Sudan) from data merely connected to it (like a traveler's review of South Sudan tours). These problems can be addressed in part by defining certain canonical representations or Symbolic frames which provide “core” information about certain concepts. Borrowing terminology from OSA, we can define a *constructor* for a concept-type as a graph which contains some prespecified set of core information (analogous to a “record” in relational databases). We can also define a *selector* as a process which identifies which double-edge or subgraph of a Γ provides a particular data point. As in OSA, construtors and selectors interoperate: given a constructor Γ for concept-type c, specific pieces of information are available within Γ via a set of corresponding selectors. For example, a graph providing information about nations might be considered a constructor, in some context, if it provides each nations’ capitol, flag colors, current head of state, etc.

When reasoning about concept systems, we typically base our beliefs about individual concept-tokens by these networks of information we have available, developing a set of background beliefs which precede or substitute for direct perceptual acquaintance. This is why we can so readily follow fictional narratives and why reasoning about fictional entities (like imaginary nations or empires) is so similar cognitively to reasoning about real-world places. We are prepared to accept fictional narratives so long as they are conceptually coherent: they do not present incompatible facts (no-one would accept a Sherlock Holmes revival where the detective lives and had always lived in Paris), nor use concepts in wildly unexpected ways (like declaring a place as a national capitol and then as an uninhabited paleolithic site). Without perceptual impressions to correlate with concept-tokens, the only “checks” on conceptual networks is that they aggregate concepts in sensible ways. Moreover, this same principle applies whether concept-networks are non-perceptual of necessity (because they’re fictional, say), or only contingently, for a particular person or context, because of physical distance or some other impediment to direct observation.

The fact that so much cognitive activity happens within these *nonperceptual*, structurally organized conceptual networks, means that when we *do* have perceptual or observational impressions, these merely supplement background knowledge already expreesed in Symbolic frames Γ.[15] By perceptually encountering entities which correspond to individual concept-tokens (Γ concept-nodes), we acquire a new mental relation with those entities which serve as models of Γ networks insofar as they express (sets of) logical propositions. That is, when a Concept Graph Γ represents a proposition (or a propositional attitude, like a belief), it expresses a fact (or putative fact) about the world “recovered” by replacing Γ concept-nodes with their worldly referents, just like symbols in first-order logic. But perceptual *acquaintance* with these referents is not equivalent to a thorough account of their nature as foundational (physical, geospatial, etc.) entities. Anyone who has visited (or perhaps even just seen pictures of) South Sudan has perceptual acqaintance with this geospatial region, but (as I have argued) no-one has (or *can* have) a complete geospatial *account* of this region.

[15] Using CGS as a representation vehicle for Symbolic frames in general.

If Γ represents a *true* proposition, what does this *truth* entail? Even when a community judges Γ uncontroverisally true they may not precisely *know* what these real-world entities are, in the sense that knowing *what South Sudan is* means knowing what precise geosptial region is South Sudan (a knowledge too fine-grained to be possible), or knowing *who Holmes is* means knowing *which person* is Holmes (which is impossible because there is no such person). Perhaps "knowing what something is" entails less a fine-grained foundational account but rather knowing some cluster of facts about it, perhaps enough to recognize it from among other entities of similar types. In other words, to know what x is we need to know that some Symbolic frame Γ expresses a set of facts about x, including some set of minimally necessary facts (for example I know who Padraig and Miyaku are, arguably, as soon as I know that they are your friends, and not fictional characters, etc.). We may call this epistemological perspective a kind of *Symbolic frame holism* in the sense that we do not "know" entities individually, but rather in the context of Γs through which we "know" multiple concept-tokens collectively. If we read a story about some international intrigue, we may not know whether the nations mentioned are real or fictional. When we learn that the story is true, then we recognize the conceptual network which the story presents (relating nations in terms of their treaties, wars, etc.) as grounded in the actual world; that is, we accept that there are concrete entities designated by Γ concept tokens. This does not mean that we have perfectly fine-grained foundational accounts of these entities, merely that we accept the collection of them as each instantiated in the actual world. If we accept this holistic account, then we have to revise our understanding of how logical propositions express facts. The truth of a p represented as Γ is not the truth of a fact recovered by substituting concrete entities for Γ concepts; instead, p is true insofar as the set of concept-nodes in Γ, and whatever other graphs may accompany it, are *collectively* instantiated by actual individuals who moreover bear the relationships asserted by Γ relation nodes.

One virtue of this (manner of) holistic epistemology is that it extends readily to the problem of facticity in fictional worlds. For example, it is *true* that Holmes was English. The classical approach to fictional truths is via Possible World Semantics: fictional narratives invent possible worlds, concept-tokens in stories refer to entities in these worlds, and the facts about them are truths in those worlds where they appear. So "Holmes" does not designate a person in the real world, but this name does refer to a person in some possible world. The problem with Possible Worlds is that they tend to be either metaphysically extravagent or semantically circular. Some authors have a very literal notion of possible worlds, including identifying them with real parallel universes which branch off from "our" universe due to quantum superposition. This is an ambitious synthsis of philosophers' Possible Worlds with the physicists' Multiverse concept, but it seems fundamentally incompatible with how philosophers or linguists actually use Possible World notions: for example, I find it highly unlikely that Sherlock Holmes lives in a possible world because there was some quantum fluctuation in the (real) past which caused our universe to split off so that, in one branch, there (really is) some person equivalent to Sherlock Holmes (and even if there were, how would Arthur Conan Doyle know him?). On the other hand, if we say that Holmes lives in a possible world because there is nothing self-contradictory about Conan Doyle's narrative — the existence of a person with the properties attributed to Holmes is *possible* — then Possible Worlds become circular. If "there is a possible world where x" means exactly "x is possible", then possible-world talk is just a fancy form of uncertainty and/or counterfactual talk.

Philosophers and logicians have explored various ways to rectify the notion of Possible Worls in light of these sorts of problems. One approach is to maintain the modal ideas of possibility and necessity, but to consider the diversity of formal modal logics and the semantic interpretations of the modal operators. While the classical sense of Possible Worlds provides one semantics for modal systems, quite a few others have evolved: there are epistemological, temporal, topological, computational, etc., modal logics with various axioms and model interpretations. Points in these models may be contexts for evaluating a proposition, belief-states of an agent, points in a topological space, intervals in a temporal continuum, etc. My discussion about "Symbolic frame holism" suggests a different treatment of possibility in terms of the internal consistency of conceptual networks: a given such network N represents a possible state of affairs (which we can define as a *possible world*). Some of the concepts in N may refer to real entities — for example, Holmes lives in London, the "real" London in the sense that the real English city has the property of *being the city "London" in the Holmes stories*.[16] N represents situations in the "actual" world if all of its concepts have real referents and moreover bear the relations present in N. A conceptual network can instead be designed to express counterfactual possibilities, including by referring only to real individuals but modifying the relations between them ("Al Gore could have been President"). Another solution to the problemmatic metaphysics of Possible Worlds is to treat Possible-World talk as only a semantic convenience, discursively reducible to some other theory — a detailed analysis of counterfactuals, notions of "fuzzy logic" or vagueness, Property Theory, etc. The property-theoretic option yields a very lucid treatment by the philosopher Michael Jubien, which I will disuss in the next section.

Here I have discussed the epistemology of (assertions concerning) fictional entites, but the modal and semantic questions raised by fictions have applications to real-world entities as well. Foundational Vagueness ensures that the referents of many real-world concept-tokens are vague or "non-crisp" individuals, imprecisely individuated relative to their surroundings and/or peers. An intrinsically coarse-grained concept may refer to many possible fine-grained individuals. Depending on our metaphysical account of vagueness or granularity, these may be distinct individuals which are modally connected within some

[16]See Jubien's defense of this claim.

kind of "possible referent-space", or "granular parts" of a single coarse-grained individual. In either case the granularity of the coarser space or individual is provisional and subject to refinement. Border disputes, or the challenges of surveying remote terrain, can make borders imprecise. For example, political or technological developments can yield a more fine-grained measurement or aggrement on the Sudan/South Sudan border than exists now. So any coarse-grained individual is associated with possible further refinements, measurements, and specifications. These modal possibilities do not relate single real-world individuals to their counterparts in *other possible worlds*, but represent modal variations ontologically internal to individuals within this one actual world.

Insofar as we reason about entities through conceptual networks, coarse-grained individuals acquire their specificity simply through the norms of a semantic framework, which stipulates that a given concept represents an integral, individuated referent. But when we consider the *grounding* of these networks in the actual world, when we mentally engage with these referents as entities in respective foundational domains (geospatial regions, physical objects, etc.) — by perception, observation, measurement, simulation, experimentation, etc.[17] — then we are no longer engaging with crisply singular individuals. Instead, we are operating in spaces of referential multiplicity, possible future refinements, and, in general, spaces of ontological possibility which demand modal treatment in lieu of conventional symbolic logic, at least if we want formal representations of our thought processes and semantics.

As I will now argue, Goguen's distinction of "symbolic" and "geometric" concept-spaces can be useful here, because they capture the distinct structure of reasoning via conceptual networks, where individuals tend to have semantic crispness, versus reasoning through the foundational imprecision of entities, as they are observed, perceived, measured, etc. Ultimately, our cognition and semantics fuses both of these modalities, because we reason through conceptual networks but orient this reasoning through perceptual and observational engagement (at least except for reasoning about fictional or purely abstract entities). So the integration of these different modalities of concept-spaces is a fundamental synthesis within human cognition, both in formal (e.g. scientific settings) and in moment-to-moment perception of the world. "Geometric" concept-spaces can provide a formal, perhaps mathematical representation of the modal variation which arises as coarse-grained concepts are engaged within spaces of fine-grained variation: hues in color-space, spatial regions embodying geospatial concepts, etc. Real-world colors and geography may not perfectly match these mathematical constructions, and most semantic domains do not even have these straightforward model-representations. Nonetheless, theories

[17]Distinguishing *observation* from *perception* because the former can be indirect; an apparatus may observe properties of something without providing a perceptible image. Consider observations of black holes via their gravitational effects. Also, experimentation can explore properties of some inaccessible object by using a proxy, both real and virtual.

such as Conceptual Blending, Prototypy Theory, and Latent Semantic Analysis show how concepts are modelled as integrations of distinct conceptual frames, points in a high-dimensional space of latent topics, distances from prototypical concept-type examples, and similar models which suggest a quantitative treatment. The mathematical representation may be an idealization of real-world semantics, but it can be both a vehicle for sharpening philosophical intuitions and also a useful tool for practical applications (as is well-illustrated by the use of LSA in search engines). Having argued in this section for a modal-logic representation of Foundational Vagueness, I will consider in the latter part of this paper how modal notions can facilitate the unification which Goguen initiates.

4 Conceptual Graphs and Formal Concept Analysis

Unified Concept Theory relies on the expressive power of Order Sorted Algebra to generalize (and construct mappings between) FCA, CGS, CI, and other semantic models. Goguen also discusses integration between these theories and Gardenförs-style "geometric" concept spaces, but this latter synthesis is more tenuous. Specifically, Goguen introduces the concept of "frames", which are essentially conceptual networks in which certain concept-types (or "sorts") have intended models which can be expressed in terms of mathematical stuctures and/or continuous spaces, such as colors. In these cases, concept extensions, and boundaries between them, can be given precise representations — for example, we can partition "color space" by associating color-samples with color words. There is, however, a fundamental difference between this latter integration, in contrast to the possible unification between (say) CGS and FCA. Most of the semantic models which Goguen considers are interrelated, and, while they differ in terms of their expressive power, within the specific semantic constructions where they overlap, their mutual differences tend to be mostly syntactic. For example, there are few technical differences between OSA and CGS. Order Sorted Algebra places greater emphasis on mathematical constructions like functions and equations, but these can certainly be modelled by Conceptual Graphs. Such graphs, in turn, can be directly mapped onto or initialized from practical software components, like Regular Expression match-objects or similar grammar or parsing products, Graphical User Interface layouts, or function call trees. Because CGS graphs can therefore translate between static data representation and active application logic, they are more intuitive for many semantic and computer applications. On the other hand, OSA can be used to study sophisticated kinds of type hierarchies as they relate to simpler logical systems (and in particular to algebraically model the kind of multiple-inheritance object systems used by a few programming languages, particularly C++). But while one or the other may be more natural for different contexts, there are clear analogies between the OSA and CGS framework. Ordered sorts in OSA are isomorphic to the CGS hierarchy of “concept types”; and OSA initial algebras are roughly analogous to the set of graphs Γ which can be formed from a given *support* (a set of concept and relation types[18]). Both are effective supersets for theories like Conceptual Blending or Formal Concept Analysis.

However, notions like Geometric concept-spaces are harder to integrate with CGS or OSA-compatible theories. As Goguen points out, these spaces tend to model a single OSA sort (or CGS concept-type). Just as each model for a logical system provides a collection of individuals, particular concept-tokens are born by entities which provide models for corresponding concept-types. So a “geometric” interpretation of an individual concept-type means that its *set of possible models* exhibits a certain mathemtical, geometric, and/or topological structure, such as the possibility of continuous variation[19] between different potential tokens. For example, bearers of the concept (-type) “red” exhibit a spectrum of distinct but similar color-hues, whose borderline cases overlap with peer concepts like “pink” or “purple”. If concept-types (or families of related concept-types, like color-terms) engender “geometric” concept-spaces, this is not a produt of relational networks through which their tokens are connected, but rather a property of their particular model-spaces. However, insofar as these concept-types *also* enter into relational networks, the geometric structure of these model-spaces becomes a substruture within the more complex model spaces[20] of the surrounding networks, Symbolic frames, Conceptual Graphs, etc., where they appear. Here I will explore the expansion of geometric model-spaces, from single concept-types outward into more complete conceptual networks, in more detail.

Note that CI, LSA, and Prototype theories also allow for “geometric” models concerning how concepts are “blended” or modeled as spaces of similarity or distance from prototypical examples. Concept-blends may integrate different component concepts in varying proportions, even if the core CI model does not discuss the possible quantitative modelling of blend-spaces in detail. Insofar as CI presents the integration of distinct *relational* concept-frames, it can be struturally incorporated into OSA or CGS frameworks; but when we consider quantitative variation within blend “proportions”, we can extend this account to continuous model-spaces, by analogy to how prototype theory extends the notion of type-models to include a (graded) spectrum of instantiators. Latent Semantic Spaces, meanwhile, allow linguistic artifacts to be located at points of relative nearness to latent concepts or topics, and Prototype theory recognizes concept-type model spaces (in the terminology I am using here) as organised around prototype examples. So a general theory of “geometric” concept-spaces

[18]Specifically, a *partially ordered* set, where the partial order $c \leqslant c'$ means that the c type extends c': all c's are c''s, but not vice-versa. Relation types are similarly ordered, though the notion of *arity* is a further restriction on the relation hierarchy lattice: two relations are comparable only if they take the same number of “arguments”.

[19]Consider the Phenomenological concept of “eidectic” variation.

[20]Now understood as sets of collections of individuals, as with models for logical theories.

can carry over into these related frameworks. Gardenförs, in particular, has emphasized how his notion of "concept-spaces" considers "similarity" in general, not only in directly mathematizable domains like color-terms. In this paper I am concentrating on the more straightforward domains, like colors or geospatial regions with their spectrum of different possible extensions, but I will assume that many ideas presented here can be generalized to more subtle model-spaces.

In the formal setting of Conceptual Graph Semantics — formats like Semantic Web "triples" or Conceptual Graph Interchange Format (CGIF) [42, p. 5] — concepts tend to be presented in terms of individual "concept nodes", or quasi-atomic semantic units which are labeled as instances of a concept-type.[21] But particular concept tokens are rarely atomic units of thought, even if they play atomic semantic roles. Conceptual graphs do not necessarily capture the internal structure of concepts, or the tendency of this structure to vary over time and across contexts. It is interesting to compare this situation to Formal Concept Analysis (FCA), which is another theoretical and computational technique for analyzing concepts and semantics. In its simplest form, FCA is a statistical tool which operates on data sets including objects which are identified as possessing attributes or *features*. A Formal Concept is a collection of objects and of features such that each object bears each feature. These may or may not correspond to ideas or semantic categories recognized ahead of time based on the objects and features in the data set. This can result in some Formal Concepts being calculated which are mere statistical anomalies [2, pp. 176, 187, etc.], but it also means that the set of concepts evolves based on the distribution of object/feature correspondance, rather than being fixed prior to analysis. One philosophical implication of FCA is its reminder that most concepts are defined neither only by some set of features (a concept's *intent*), nor by a set of examples (a concept's *extent*), but by a mixture of examples and features. For example, to define the concept *house* we might start with the most salient feature — say, "a house is a place of residence" — then modify this defintion to account for specific (counter-) examples: a house is a place of residence for one or at most a few families (unlike an apartment building); its primary purpose is residential (unlike the White House); it is freestanding (unlike an apartment or a motor home); it is suitable for permanent occupancy (unlike a tent or hotel room). The final definition relies both on a set of features and a set of candidate objects which (depending on whether or not they should indeed be classified as houses) suggest certain features as indicators (or counter-indicators) of "house-ness".

By allowing concepts to emerge statistically or even dialogically, rather than being defined by fiat, FCA (at least potentially) better captures the changeability

[21] The literature is inconsistent as to whether the word "concept" is used to name something general — a.k.a a *concept type* or *generic concept* — or conversely one *instance* of a concept — a.k.a a *concept node* or *specific concept*. To be consistent with Formal Concept Analysis, I will use the word "concept" in its generic fashion, and "concept node" or "object" to denote an instance of a concept.

of discursive contexts. On the other hand, the standard FCA algorithms rely on one single conceptual relation — the fact that some object bears some feature. This effectively inverts the situation with Conceptual Graph Sematics, which can model many different relations (many different edge types or "labels"), but is less suited for modeling the internal structure within concepts. Potentially, then, FCA and CGS naturally complement each other. There have been several papers merging these two theories, and several strategies for translating Conceptual Graphs into FCA matrices. Aside from technical options, there is a risk of terminological confusion: insofar as "concept nodes" play the role of FCA *objects*, they should not be confused with "Formal Concepts", which would be associated with *sets* of concept nodes. Another potential confusion is between graph systems whose relations are edge types or labels — for example, the basic unit of Semantic Web graphs are *triples* of two object-edges (or concept nodes) with a labeled edge between them, representing a specific relation — in contrast to systems like CGIF, where relations as well as concepts are *nodes*, and the basic "triple" is two concept nodes joined to one relation node. So long as graphs are restricted to binary relations, this difference is little more than terminological. CGS sometimes uses the word "arc" to denote edges incident to one concept node and one relation node. So long as all relation nodes are incident to precisely two arcs, we can use the word "edge" to name either a labeled edge (two nodes and a label) or a CGS triple (two concept nodes and a binary relation node). Here I will adopt the convention of using the term "double-edge" to refer to concept-relation-concept triples.[22]

CGIF also allows for what might be called "superbinary" relations — which connect three or more relata — although any superbinary relation $\widehat{R}$ can be reexpressed as one or more binary relations. We can define a single *application* relation $\mathcal{A}$ such that $\widehat{R}\vec{x}$ for n-tuple $\vec{x}$ transforms to $\vec{x}\mathcal{A}\widehat{R}$. Alternatively, if we allow some graph triples to relate two relation-nodes via a connecting "metarelation", a superbinary $\widehat{R}$ can be broken into a sequence of binary relations $\{\hat{R}_i\}$ for $1 \leq$ i $< |\widehat{R}|$ (the arity of $\widehat{R}$), so $\vec{x}\mathcal{A}\widehat{R}$ becomes $x_1\hat{R}_1x_2$, etc. The latter option enables conceptual graphs to be translated to Semantic Web style labeled graphs, with edge-labels replacing relation nodes, though with the added complication that different relation nodes (consequently edge-labels) should be explicitly marked or implicitly understood to be interrelated. Call an *unfolding* of a superbinary $\widehat{R}$ to be a subgraph with nodes of the form $\cdots\ x_i\hat{R}_ix_{i+1}\ \cdots$, and where if necessary each pair of relation nodes $\hat{R}_i$, $\hat{R}_{i+1}$ (in both cases i such that $i+1 \leq\ |\widehat{R}|$) is linked via an extra "metarelation" indicating that they form part of an unfolded sequence.[23] With the former type of translation (in terms of an application relation $\mathcal{A}$), any conceptual graph can be expresed as a

[22]Although CGS relation-nodes take the place of Semantic Web relation-edges, I think there is still a place for *labeled* edges, not for representing semantic relations as such but rather for structures which are part of the theoretical "metalanguage", such as multiple edges which collectively define relations of arity > 2 (discussed in the next paragraph).

[23]This can be done by adding edges between the relation-nodes or labelling the edges between successive x_i to clarify that they are grouped together.

single matrix (what we might call a "canonical matrix") whose rows are concept-node tuples and whose columns are relations. This is the strategy proposed by Mineau [30] for developing FCA methods on conceptual graphs. On the other hand, different FCA matrices can be formed from conceptual graphs for different (groups of) binary relations. Given a (possibly one-element) set $\widetilde{R} = \{R_i\}$ in such a graph there is a matrix whose non-null or non-zero row-and-column entries are the x, y such that $\exists i{:}xR_iy$ is an edge in the graph. Ultimately, to convert between conceptual graphs and FCA contexts it is necessary to extract, from the collection of relations either in themselves or in combination with concept nodes, a collection of *features*, which may be relations themselves (seen as features of tuples of objects); or concepts themselves (the concept *red* in most cases corresponds to a feature *red*, whatever precise relation or relation-node may connect it to a bearer of redness); or relations with one or more components fixed (as in, *being near Toronto* is a feature of Waterloo). I will write $o \hookrightarrow f$ to indicate that the featural representation of a graph yields an object/feature pair without specifying the actual relation such that oRf.

More general than individual features, families of features can be grouped together (including situations where features collectively partition "geometric" concept spaces), or when features are defined as projections from some relation (for example, for each city there is a feature "being near" that city). However they are formed, feature-sets, or *frames*, are the basic component of FCA methods. We assume that a conceptual graph can be mapped onto one or more feature-sets and therefore onto one or more frames, each giving rise to a matrix whose rows and/or columns are graph elements, individually or in tuples, either relations or concept-nodes. Potential frames include the "canonical matrix" mentioned earlier, as well as R-specific matrices for each relation R or sets $\widetilde{R}$ of "related" relations (as in, *having color red* and *being painted red* could potentially be different relations, but both implying *having the feature* red). Whatever the details, call a set of frames a "featural representation" of a conceptual graph, with each frame generating a matrix or "sheet" relating graph elements to features. We can therefore represent the collection of these sheets as a matrix bundle $\vec{\mathcal{M}}$ or a three-dimensional matrix, or 3-Matrix, encoding a featural representation in a manner suitable for FCA. I will use symbols $\vec{\mathcal{M}}$ and the term "3-Matrix" to apply to general matrix bundles regardless of whether all sheets $\mathcal{M} \in \vec{\mathcal{M}}$ have the same dimensions, since it is trivial to resize the frames as needed.[24] Call a 3-Matrix $\vec{\mathcal{M}}_\Gamma$ *complete* if represents a conceptual graph Γ in full detail, so that Γ can be reconstructed in its entirety from $\vec{\mathcal{M}}_\Gamma$. For example, if one sheet in $\vec{\mathcal{M}}_\Gamma$ is canonical for Γ then $\vec{\mathcal{M}}_\Gamma$ (even if restriced to just this one sheet) is complete. Alternatively, $\vec{\mathcal{M}}_\Gamma$ is complete if there is a distinct sheet $\mathcal{M}_R \in \vec{\mathcal{M}}_\Gamma$ for each relation-type R represented by relation nodes in Γ. Adopting language from Yiyu Yao and Yaohua Chen [50, p. 290], I will distinguish *property* concepts, which will tend to translate to features in FCA

[24] If we are prepared to allow empty or all-zero rows or columns.

matrices, from *object* concepts, which tend to translate to objects.

Existing literature on modal semantics relative to Conceptual Graphs [43, p. 40], [44, p. 15], [6] appears to focus on duplicating entire conceptual graphs across a set of possible worlds (or whatever concept replaces "possible worlds" in a given system of modal logic). By encoding graphs using 3-Matrices, it is possible to replace this distribution with a more concise distribution of 3-Matrices. Even when this change in perspective has little formal significance, it arguably better captures the intuition that the "counterparts" of a conceptual graph Γ in other "worlds" are not unrelated to Γ and may well share almost all of its structure. It may indeed be intuitive to say that all of these counterparts of Γ share all of its concept-nodes, and only differ as to the relations between them. Even if this seems too restrictive, it is obvious that two graphs Γ and Γ' are *similar* to the degree that they share the same structure. We can measure the similarity (and dissimilarity) between graphs, using an integer-valued metric, by saying that Γ and Γ' differ by one unit if either Γ or Γ' can be obtained from the other by adding and connecting one relation node, and then proceeding by induction. We can thereby define a notion of "accessibility" between graphs in terms of their dissimilarity falling below some threshold — so that two wholly unrelated graphs cannot be considered modal counterparts. Insofar as all counterparts of Γ will thereby share some structure, using 3-Matix representations can isolate that part of the structure which does vary. Here I will further assume that all counterparts of Γ share all of Γ's vertices, and so the set of these counterparts can be represented by a set Σ of 3-Matices such that given $\vec{\mathcal{M}}_i$, $\vec{\mathcal{M}}_j \in \Sigma$, the kth sheets $\vec{\mathcal{M}}_{i,k}$ and $\vec{\mathcal{M}}_{j,k}$ have the same dimensions.

In addition to the above notion of similarity, which might be called *structural* similarity, consider also *dimensional* similarity reflecting the meaning of some concepts in a graph: for example, the graphs expressing the phrases "62°" and "63°" have similar meanings, or certainly express similar facts, facts which are made true by similar states of affairs. These several notions of similarity and counterparts can be used to extend CGS and FCA, through 3-Matrix representations, via the introduction of modal operators. Aside from extending these forms of conceptual analysis, I believe that such a framework can be of interest to modal logicians in general, because CGS graphs become elegant models for modal languages. Collections of conceptual graphs, or single conceptual graphs which can take on a spectrum of precise meanings, are a concrete alternative to abstract "possible worlds", and structural or dimensional similarity provides a more concrete alternative to the notion of modal (inter-world) accessibility.

The approach to modal logic I am advocating here focuses on variations in the precise meaning expressed by or attributed to a conceptual graph, rather than variations over a set of distinct conceptual graphs. I will refer to different graphs as "variations" or "copies" of a given graph Γ, but this is largely a matter of terminological convenience. Because most concepts within a Γ will be in some measure coarse-grained, there is a variation of distinct real-world situations which could be considered as properly modeled by the graph, or as confirming the graph as expressing a true proposition. These more precise situations can potentially be rendered with their own graphs, so we can consider a family of graphs extending Γ. Say Γ models this sentence: "the temperature is warm"; Γ can be extended by adding a further relation to the concept node "the temperature" to express that the temperature is, say, "62°" or "63°". These further graphs will indeed be distinct from Γ, but they share Γ as a foundation or basis. They may be considered a family of *fine-grained elaborations* on Γ. Let Γ_ε be such an elaboration and write $\Gamma \preccurlyeq \Gamma_\varepsilon$. A set $\Gamma_{\mathcal{E}} = \{\Gamma_\varepsilon\}$, each $\succcurlyeq \Gamma$ for a given Γ can be introduced as a modal model, consistent with existing modal extensions to CGS which rely on sets of conceptual graphs. However, such extensions can be introduced not as separate graphs, but rather as "annotations" providing details specific to some part (such as one concept-node) of a graph. I will generally refer to a space of variation over a single Γ, but also to different graphs as variants or "copies" of Γ with the assumption that these are implicit elaborations that could be expressed also as annotations on Γ rather than as separate graphs, so describing them as separate from Γ is largely for convenience. As annotated elaborations on Γ they are also (structurally) similar to Γ.

Although CGS is a very expressive framework for modelling conceptual networks, it does not directly provide a mechanism for describing or placing constraints on model-spaces for particular concept-types. FCA allows these model spaces to be statistically sampled, from which the structure of these spaces as a whole can potentially be reconstructed, but this kind of empirical analysis may not substitute for the kind of theoretical investigation pursuted by researchers like Gardenförs or Gregor Strle[25]. From a philosophical perspective, I believe these weaknesses are symptomatic of larger flaws in the use of symbolic representation (which deliberately simplifies and formalizes language and thought) to capture the nuances of human cognition and discourse. I have already summarized how Foundational Vagueness challenges conventional applications of Symbolic Logic for analyzing thought and language. Similar problems of imprecision (or *flexibility*, to adopt a less perjorative term) affect both intentional and extensional properties of concepts (including such as may be modelled by property-sets and object-sets within Formal Concepts).

Some philosophers have argued that symbolic representation fails to capture how people rely on similarity and prototypical examples to define and map conceptual spaces (including Gardenförs, and notably George Lakoff and Mark Johnson [28, p. 20], [27, p. 7]). Others, from a phenomenological perspective, suggest that formalization abstracts away the role of background knowledge and

[25]Who extends Gardenförs' notion of conceptual-spaces with more precise Topological details, including techniques in Statistical Topology, such as persistent homology and Voronoi tesselations [45].

social context in shaping conceptual understanding (David Woodruff Smith [39], Dagfinn Føllesdall). Phenomenology, the tradition of Edmund Husserl, Maurice Merleau-Ponty, etc., also highlights the importance of intentionality, of the cognitive and experiential role of attention in marking a referential foreground as separate from a contextual background, which is crucial to understanding how humans speakers translate structures in their surrounding environment into linguistic expressions (as well as mental states in general). Scholars like Lakoff and Gardenförs suggest that symbolically representing concepts fails to capture the flexibility of real-world concepts to apply in a spectrum of different scenarios (the concept "red" is more general than one single color point, like 0xFF0000). In a similar vein, *objects* themselves possess imprecise boundaries, at least insofar as they are represented in thought and language. Researchers like Barry Smith and Jean Petitot have explored using topology (or non-set-theoretic variations, like Mereotopology) as a way of capturing the spatial perceptual dimensions of objects. Finally, Michael Jubien's book from 1993, *Ontology, Modality, and the Fallacy of Reference* [20] — unfairly neglected, I believe, in domains like Cognitive Semantics — presents provocative and persuasive critiques of certain influetial notions pertaining to modality and language.

Many of the challenges raised by the above-mentioned scholars concern how we distinguish objects from the features (attributes, properties, concepts, etc.) which they instantiate. Most formal systems take the object/feature distinction (not necessarily using this same terminology) as self-evident, and construe atomic propositions as the inhering of one feature in one object. However, the real-world relation between objects and features is more subtle. Here I will list several semantic phenomena or notions, labeled in capital letters, to identify object/feature relations that tend to undermine the rigid conceptual distinction between objects and features in the first place. My point with these labels is not to introduce new technical terms, but simply to mark down certain concerns for potential future discussion; to some degree I am following the rhetorical model here used by Lakoff and Johnson in [28]. With that said, consider the following:

- SINGLETON For each object (or at least many objects), there seems to exist a corresponding concept (feature, property, etc.): there is Paris the geospatial region, and also Paris the concept (and the property of being Parisian, being stereotypically Parisian, etc.). These might be called *singleton* concepts, properties, or features, identified by the fact they have (in some sense) at most one instance.

- PARTHOOD For each object there is also the mereological feature or property of being part of that object. In addition, an object may physically be a collection of its parts, but conceptually an object seems to involve more than its physical constituents: an automobile fully diassessembled so as to recycle its parts is not conceptually the same as that car when it was on the road.

- DEPICTION For each object there is also a property of being a representation or depiction of that object. One could plausibly point to a picture of the Boulevard Montparnasse and declare "that's Paris" — even if the photograph is really on a wall in Guelph and even though Montparnasse and Paris do not name the same geospatial regions. In a sense two quite different entities (a picture in Guelph and a geospatial collection of Arrondissements) both instantiate the property *being Paris*.

- SUBSTRATUM Objects can sometimes be identified with their physical parts, and sometimes not. Suppose a tree was planted 100 years ago, and one year ago someone cuts off a branch and fashions a sculpture. There is a sense that the sculpture is a piece of wood, that the piece of wood is 100 years old, and that the sculpture is one year old — but one cannot conclude that the scuplture is both one year and 100 years old. The scupture *is* a piece of wood in one sense, but in another sense the wood is the *substratum* which instantiates the property *being that statue*.

- PROTOTYPE Concepts are often defined not only intentionally (in terms of indicative features), but in terms of prototypical examples, objects with some special relation to the concept above and beyond their instantiating all of its features.

- MEASUREMENT Concepts often apply in a range of cases, and if it is desired to fix their proper range it is necessary to rely on more complex conceptual structures. For example, graphics software may wish to identify color words with some partition of the range of computer color values expressed as an RGB cube — so that "red", say, can be identified as a three-dimensional shape, with a list of endpoints that serve to mark the boundary between red and pink, purple, maroon, etc. Whereas "red", in its coarse-grained, flexible form seems to be a simple feature, attempting to fix it to a precise spectrum seems to involve a complex concept, which includes features involving the mathematical dimensionalization of a range of values and the specification of certain points within that range. This raises several concerns:

    - AXIATION To associate a concept with a range of values, it is necessary to recognize some axis or dimension from which values can be numerically measured (such as red, green, and blue within a color cube.
    - BOUNDARY It is then necessary to mark off some part of this dimensional structure as the boundary of the scope of a concept.
    - EXOTYPE This boundary may often be specified by marking certain specific boundary points or examples. These examples play a role complementary to prototype: they do not represent the *best* example of a concept, but rather the *worst* examples which nonetheless can still be subsumed by the concept, and therefore mark its furthest reaches.

- XENOTYPE On the other hand, some examples demarcate a concept by being cited as boundary cases which are *not* within the concept. In a sense then a "xenotype" is an exotype of the *negation* of a concept, or a counter-example which is nonetheless not far off: pink compared to red; an apartment compared to a house; Montreuil compared to Paris (being just beyond the city limits).
- PROTOVALUE Notions like prototype, exotype, and xenotype represent *examples* of typical or borderline cases where a concept applies. When a concept is specified in terms of some sort of MEASUREMENT, however, these notions can also be expressed not only as an *example* of the concept, but an example *value*; so in computer graphics we can call the color displayed by 0xFF0000 a prototypical red, but we can also call 0xFF0000 a *protovalue* for red. Similarly, define EXOVALUE and XENOVALUE.

◦ PERIMETER Similarly, objects which are generally referred to as singular entities (like place names) may need more complex specification when it comes to, say, identifying those streets or street intersections which mark the boundaries of a geographical neighborhood. Formal Concepts have extents (the list of objects which bear their features), but compound objects also have extents or extensions, such as the collection of their parts or the extents of the concepts *being part of o* for some object o, or special objects such as spatial points and lines, or geospatial locations, which mark extents or boundaries of a spatially (or geospatially) extended object. The PERIMETER of an extended object may beed to be specified or MEASURED akin to the BOUNDARY of a feature.

◦ EXOPART, XENOPART One way to describe the PERIMETER of an extended object is to identify certain parts — the streets which bound a neighborhood, for example — including parts just inside or just outside the perimeter, the analogues of EXOVALUES and XENOVALUES.

These kinds of subtle semantic phenomena often give rise to modal problems and theories, either because modality can be used to analyze them or because they can lead to modal enigmas if not properly theorized. A pair of examples: a wood sculpture could potentially have been executed in clay, but a piece of wood could not be a piece of clay (call this a SUBSTRATUM puzzle). Second, the lovely Queens neighborhood of Ridgewood was once part of Brooklyn, and presumably could have remained so (the story goes that locals felt their property values were depreciated because outsiders confused the area with Bushwick — not anticipating that Bushwick itself would become a trendy North Brooklyn destination). So Brooklyn could have included Ridgewood, but Brooklyn qua set of geospatial points presumably either does or does not include Ridgewood qua geospatial region (we could call this a SUBSTRATUM and PERIMETER puzzle). Using the (problematic) language of possible worlds, are the "counterparts" of Brooklyn and the sculpture in other possible worlds the geospatial region/ piece of wood, or the associated *concepts*? How do we modally disentangle objects-as-substrata from objects-as-concepts? In his [20], Jubien explores similar problems by distinguishing properties from the substrata which intantiate them: a geospatial region instantiates the *property* of *being Brooklyn*, and a piece of wood instantiates the property *being this statue.* These properties could have been instantiated by different geospatial regions (in the Brooklyn case) or by a piece of clay rather than wood (in the sculpture case).

Jubien's property theory is an elegant solution to these kinds of modal puzzles and more, but it depends on the nontrivial further notion of *singulary* or *singleton* properties. Singletons have their own nuances: the property *being Paris* may be singulary (since Paris is a proper name), but there is a *sense* in which the picture in Guelph also instantiates this property. The SINGLETON notion can, at least superficially, be modeled in terms of an *identification* relation, say $\mathfrak{I}$, such that $x\mathfrak{I}y$ iff $x = y$ (or better, if x *necessarily* equals y). Unfortunately, it may be difficult to define singleton properties without using modal notions, which means that modal notions cannot then be built on top of singletons (as in, say, the counterparts of an object in other worlds are the instantiators of the singleton property of being that object), on pain of circularity. Despite these remaining nuances, I think Jubien's work on properties offers many insights in the context of conceptual-graph semantics, concerning how properties relate both to objects and to concepts.

For the current discussion, I will use the term *property* for sets of features (and the term *collection* for sets of objects). There is a property *being a house*, identified by certain features, and the concept *house*, which combines this property (its intent) with the set of houses in some domain (its extent). In [20, pp. 111-115] and [21, pp. 68-76], Jubien argued that many apparent instances of the modal concept of necessity can be studied in terms of property relations — for example, that all horses are mammals because the property *being a mammal* is more general than the property *being a horse* (as opposed to the "Possible Worlds" formulation, that horses are necessarily mammals because they are mammals in all possible worlds). In the sense of FCA, properties can be included in other properties either intensionally or extensionally: in other words, it may be that one property (as a list of features) is a subset of another property (which contains all of those features and then others). It may also be that, relative to some data set, all objects which bear property P_1 (bearing all of its features) also bear P_2, even if P_1 and P_2 have distinct feature sets. Let $\mathcal{M}$ be an object/feature matrix, f_1 and f_2 be features, and say $f_1 \sqsubset f_2$ (relative to $\mathcal{M}$) iff every object bearing f_1 also bears f_2. The $\sqsubset$ relation can then be extended to sets of features: it may be that every object which bears every feature from some list (P_1) also bears every feature from some other list (P_2). If some of the features involved are distinct, does this demonstrate necessary relations between features or propeties, or is it a statistical anomaly? If we replace $\mathcal{M}$ with another matix in which the object/feature relations are distributed differently, would these inter-feature or inter-property relations be sustained?

5 From Properties to Modal Logic

As this example suggests, to incorporate notions of modality into the FCA/CGS synthesis, it is necessary to consider Conceptual Graphs as existing in a space of possible variations, in which the graph is potentially subject to variations such as the addition or subtraction of edges. As graphs are translated to object/feature 3-Matrices, this variation then generates, in lieu of one single 3-Matrix, a space of multiple possible "distributions" of features over objets. We can assume that all distributions share the same vertices (if necessary, those which would otherwise be exluded from a particular version of a graph can be added as isolated vertices, that is, intersecting no edges at all); therefore distributions vary in terms of which pairs of nodes are connected and which relation labels that connection. A second question is where these "distributions" come from: it may be that graphs are completed on the basis of certain real-world constraints which could vary, or from some calculation which can yield different results in different contexts. A conceptual graph may describe not only one single specific situation, but rather expresses a kind of compound concept which can be applied in different contexts. For example, different people's reactions to a meal may be captured with conceptual graphs and, since people may disagree over whether the meal should be called expensive, an edge linking the meal to the concept *expensive* may be present or absent in different reviews. The set of reactions therefore can form a *distribution space* over the graph where each single distribution includes some edges and, potentially, excludes others. I will call this kind of distributional variation *external* because it relies on various interpretations or applications of a concept graph. The graph of "an expensive meal" can vary as an interpretation of a given meal, and also be used as a generic, compound concept applied to different meals; when necessary, we can further distinguish *applicative* from *interpretive* variation.

On the other hand, variation also can result from the fact that a particular concept-node can be associated with a range of values in some measurement or dimensionalization: in a given context the concept "expensive", for example, can be specifically pegged to some range of values (in this case, prices). In this situation a distribution can be derived by plugging one value from such a range into an associated node and checking whether this results in the addition or subtraction of edges. More precisely, we can say that a conceptual graph may have a set $\Xi = \{\alpha_1 \ldots \alpha_n\}$ of "axial" nodes, which are nodes whose relation to a concept node could be satisfied by some range of values within some dimension or *axis*, taking an "axis" to be a labeled dimensional structure (for example, *prices* are integer values labelled according to some currency). A set of one or more such potential measurements or specifications can be called an *axis structure* on a node, and a set of points drawn from these axes called a *valuation* of an axial node. Each element of a distribution space is then a set of values $v_1, \ldots v_n$ applied to axial nodes α_k, $1 \leq k \leq n$, potentially affecting the object/feature distribution derived from edges which are thereby present or absent.

A concept-node becomes an axial node because of its relation to a property concept which suggests, or corresponds to a feature satisfied by, a range of values. Call these property concept-nodes *spectrum* nodes, and the triple relating an axial node to a spectrum node an *axial edge*. If the price of a given meal, depending on circumstances, can vary between \$50 and \$150, and if "expensive" in context is defined as at least \$100, then a meal/expensive edge will be present in some but not all distributions as the price of the meal varies — as will the meal having the feature *expensive* in the associated matrix, assuming expensive as a concept node is mapped to expensive as a feature. Call an "axial pair" the combination of an axial node over a distribution space, and a spectrum node corresponding to a range of values. An axial pair *holds* in a distribution if the axial node's valuation (which can be called an *axial measure* if there is no confusion as to which node and axes are involved) falls in the spectrum node's range; and the set of distributions where the pair holds can be called the *reach* of the pair. Call a pair *fragile* if it holds, but barely; the axial measure just barely fits within the spectrum range. If the pair is not a borderline case, but there is some leeway — in other words, if the pair holds in a given distribution and also in all "similar" distributions, where similar values apply to the axial measure and spectrum range — call the pair *robust.* Call a set of distributions a *distribution neighborhood* if they differ by sufficiently small variations in the axial measures and/or spectrum ranges of one or more nodes. More precisely, a distribution neighborhood includes a continuous range of values for axial and/or spectrum nodes depending on a suitable notion of "continuity" for the range of values and dimensionalization of the nodes involved.

Variation of an axial pair can be driven not only by spectrum-variation, but also by scope-variation on the axial node. As an example, suppose first of all that I shop online for a red scarf. Whether a web site describes a scarf as red may depend on the range of colors they have available; one site may offer a version in red and in maroon, whereas another may describe as "red" a shade similar to the former's "maroon". Assuming colors are dimensionalized as a 3-cube, the precise RGB points inside the region "red" will vary from context to context. This variance then provides a space of possible variations in the distribution of conceptual graphs including an edge like *this scarf is red.* For sake of argument, suppose this distribution is gathered from a collection of photographs of scarves; suppose each photograph is partitioned with one segment corresponding to a scarf; and suppose each scarf can be assigned a unique color by averaging the RGB values of each pixel in the segment. Each photograph provides a context in which, if the scarf is called red, this average will fall within some (potentially context-specific) red spectrum. Meanwhile, image segmentation is imprecise; different software components (or even different people), trying to mark off pixels *inside* the scarf, could plausibly draw the boundary differently. Each particular segmentation will yield a fine-grained object/feature pair, in which *this exact* pixel-set has color values which average to *this exact* RGB value. Such a triple is *not* the same as a triple like *this scarf is red*, because it is more fine-grained. One exact pixel-set is not the same as the object *this scarf*; it is rather one possible fine-grained representation of the precise pictorial extension of this scarf. Similarly, one exact RGB value is not the same as the feature *red*. I will call these fine-grained nodes *preobjects* and *prefeatures* to represent that they are specific to precise measurements. If in some distribution the proposition "this scarf is red" is true — meaning the object *this scarf* bears the feature *red* — then, albeit perhaps only in principle, there will be some preobject o' and some prefeature f', such that the "pretriple" $o' \hookrightarrow f'$ provides a fine-grained basis for the more general fact that $o \hookrightarrow f$.

If a conceptual graph semantically representing "this scarf is red" is used as a generic sentence, applied to a collection of distinct photographs, then the distribution space of a "scarf/red" axial edge varies over this range of cases, representing what I have called an "external" variation. However, even considering just one photograph, there may still be "internal" variation driven by different possible definitions of a range in color space for *red*, and for the precise pixel-set assigned to the scarf. Because image-segmentation is imperfect, different algorithms may include or exclude certain pixels from the scarf-segment, which could obviously vary the calculated color average. In this case a particular distribution will have one specific construal of the scope of the object, what we might call the "spread" of the axial node, as well as of the range of a spectrum node. If in some conceptual graph we have oRf for some object o, feature f, and relation R, then it may be that o and f are to some degree coarse-grained, in the sense that there are a spectrum of more fine-grained cases which are all subsumed under the coarse-grained oRf triple. A feature f may be associated with some quantitative measure (such as maps to points on a color cube, or even just to similarity measures to some prototypical case); and/or there may be some way to formally describe different options for the spread of an object (such as a set of pixels in a bitmap image). Either the object or the feature, or both, then presents a spectrum of possible measures or spreads; if this spectrum can be defined as a set of fine-grained values or constructs (a set of pixel-sets, or points in color space), we can say that there exists a *dimensionalization* of the (object-) spread or (feature-) spectrum. These dimensions can also be analyzed as approximations of more general, continuous or real-valued dimensions, which can be called "topologizations" of the spectra. As a theoretical tool, then, suppose that in a concept graph edge xRy both x and y are "sampled" from topological spaces $\mathcal{T}_x$ and $\mathcal{T}_y$. Depending on the dimensions involved, a distribution space $\mathcal{D}$ can be considered finitary approximations of an idealized case where the spread of the object x (here the scarf) and feature y (here red) vary continuously within topological spaces of (in this case) two-valued spatial locations and three-valued color spaces, respectively. If x and y are the only nodes subject to variation, then $\mathcal{D}$ is homeomorphic a subspace of the product topology $\mathcal{T}_x \times \mathcal{T}_y$, and sets of distributions $\{d\} \subset \mathcal{D}$ are open (or closed) according to this topology.

Of course, by introducing these further details (pixel sets, RGB colors) I am imagining more complex conceptual graphs than the simple two-node proposition that a scarf is red. Even restricting attention to one picture of a scarf, I am imagining a further connection which relates the scarf to a pixel set (via image segmentation) and red to a spectrum of color values. Introducing the pixel set and the spectrum as nodes in their own right, we can assume then further nodes marking the boundaries of the pixel-set and color region. Within this expanded, more complex conceptual network we then assume the existence of a measurement which associates the pixel set to an average color value (which may or may not be in the concept-spectrum). It is then assumed that whenever "the scarf is red" in a distribution, there is a fine-grained edge or "pretriple" such that the spread of the scarf is formally measured (say as a pixel set), then mapped into a color space, and that the result is an edge linking a fine-grained pixel set to a precise color value, within the red spectrum. Borrowing a term from measure theory, we can say that this hypothetical pretriple is an "axiation" of the original coarse-grained edge, because it permits the qualitative observation (this scarf is red) to be translated into a purely quantitative measurement.

The presence of such an axiation is not guaranteed for a given coarse-grained edge; it may be necessary to impose dimensional structures onto a conceptual graph, or to make an educated guess as to feature-spectra or object-spread. A web site which reviews restaurants may explicitly define a range for "expensive", but when a graph is extracted from social network posts it may be necessary to assign a spectrum for "expensive" based on further information about the network and the origin of a post. Moreover, in practice there may be only an approximate distinction between fine-grained and coarse-grained features. For example, the feature "red" certainly corresponds to a range of color-values. This very fact suggests that there is such a thing as an "atomic" color value, like a wavelength or an RGB cube point. But even a certain RGB color will appear differently on different computer screens. The distinction between features and "prefeatures" is less about the latters' *practical* association with precise numeric values under measurements, but instead about different semantics and conceptual roles. Most features are *conceptually* coarse-grained. Suppose **J** is a (cognitive or semantic) judgement (that is, representations of **J** can be considered either representations of a mental state involving believing something to be the case, or else as representations of the semantics of a language artifact). Suppose that f is a "coarse feature" (coarse-grained feature) involved in **J**, such as *red* in *this scarf is red.* It is conceptually and semantically internal to **J** and

f that slight variations in prefeatures associated with f will not significantly alter the semantics and cognitive status of $\mathbf{J}$. In other words, a small change in the situation which $\mathbf{J}$ describes — such as a picture of a red scarf appearing differently on my computer than on yours — should not trigger a change in our opinions of $\mathbf{J}$, or its semantic meaning. This is conceptually *internal* to f in the sense that when I use the word "red" to describe a scarf which we both see, but on different computers, I do not expect that the color you see is *exactly* the color I see. This allowance for variation, what I have called a $\mathbf{J}$ being "robust", is intuitively a modal operator in that it modifies a proposition; and with sufficient formalization, I have claimed, can be associated with technical modal operators, such as within topological semantics. But if it is conceptually *a piori* that a coarse feature can take on a range of values, it is equally *a piori* that we have some notion of a fine-grained "prefeature" which is one of the values over which the coarse feature can range. Even if the prefeature is itself somewhat coarse-grained in practice, it plays the *role* of a conceptual atom, arbitrarily fine-grained in meaning (or *eidectically*, to use the phenomenological term). The possibility of combining a fine-grained measurement with a coarse-grained feature is therefore *a priori* to many concepts, and this possibility can be formally modeled in conceptual graphs using axial pairs.

Let $\Gamma_{\mathbf{J}}$ model a judgment (cogitive or linguistically expressed). $\Gamma_{\mathbf{J}}$ then provides a context within which are situated each of its concept-nodes; I propose to call this a *syntagmatic* context or space, marshalling a linguistic term for the structural relations between units of meanings and the specific linguistic acts where they occur, particularly syntactic relations but without excluding relevant semantic details. Aside from this immediate context there is a larger pragmatic or *ontological* context which connects concepts to their cognitive horizons, the overall store of knowledge and its accepted layering into distinct pragmatic (social, spatial, scientific, etc.) domains, insofar as this is developed within a language or cognitive community. These domains can be considered in terms of the formal CGS notion of *supports*, but also in the Phenomenological terms of *regional ontologies*. Many concept-nodes are also situated in *eidetic* contexts, or spaces of variation, feature-ranges, and related concepts which share dimensional structures with a given concept and can subsume, be contained within, overlap, be adjacent to, or be detached from them, as in the case of red compared with colors like pink, purple, and blue. So in general concept-nodes are contextualized in three distinct kinds of "space": syntagmatic, ontological, and eidetic spaces, each of which can potentially be modeled with conceptual graphs of their own, which form horizons or background conditions for the structure of a "local" $\Gamma_{\mathbf{J}}$.

The capacity of certain concepts to range over a spectrum of values — and also of some objects, like geospatial neighborhoods, to have evolving spatial extents or "spread" — is intrinsic to their semantics. Therefore the notion of a "distribution space" of valuations, and consequently the possibility of using modal operators taking this space as their domain, is a natural extension to conceptual-graph semantics. It is possible to introduce modal notions into a CGS/FCA synthesis without this added notion of distributions — for example, Yiyu Yao has shown that FCA operators themselves have formal modal properties[49, p. 10]; the implication is that any given FCA frame is a model for modal logic. Given formal concepts $C1 \sqsubset C2$ then necessarily any object bearing $C1$ will also bear $C2$. This operator can then be extended to the case where $C1 \sqsubset\ C2$ everywhere in a distribution space. Other modal operators can be derived from the notion of "fragile" and "robust" axial pairs. If $\mathcal{D}$ is a distribution space over a conceptual graph Γ, with FCA 3-Matrix representation $\vec{\mathcal{M}}_\Gamma$, then call Γ_d the state of Γ at a particular distribution d — including whether possible edges hold and the values assigned to axial or spectrum nodes. Each Γ-node n then contains multiple "copies" n_d, and we can call n_d a "residence map" r between nodes and distributions. Define a "proposition" as the set of distributions where an axial pair holds. The set of propositions on a conceptual graph, together with the residence map r, form a system compatible with Kohei Kishida's topological or "sheaf" semantics for modal logics; a proposition being "robust" corresponding to Kishida's version of the necessity operator [23, p. ?]. I contend therefore that M3 representations of conceptual graphs provide models for sheaf semantics.[26] I also believe that such graphs provide models for the less abstract "topological semantics" explored by researchers like Peter Gardenförs, Barry Smith [], or Gregor Strle, whose models define what are (in Goguen's terminology) Geometric concept-spaces.

I have borrowed the notion of *annotations*, which are used in computer programming languages to provide information about data types and data values relevant to issues such as persisting values to a database. Call an *annotation* on a concept-node η a structure (which may be a conceptual graph in turn or just a data structure conforming to some data description language). A *dimensional annotation* includes a dimensional structure $\mathcal{S}$; let us further assume that such structures include both an "ideal" dimensional structure $\mathcal{T}$, which we can assume is a topological space, and a "practical" structure $\mathcal{P}$. The idea is that $\mathcal{T}$ serves as a hypothetical representation of the *semantics* of a concept, irregardless of practical limitations in creating or assessing its instances. In many contexts the sense of a concept includes both an "eidetic" dimension, where the concept can vary continuously over a range of possible exemplifications, as well as a practical dimension, where we recognize limitations in its production or measurement. Both of these should be considered part of the concept's meaning or "sense" (*Sinne*, borrowing a term from Husserl). Recognizing the spectrum of a concept's meaning as defined by two different structures — one ideal and semantically *a priori*, and one practical and informed by technological limitations — is one way to recognize a concept's eidetic aspect as including how it "appears" both in theory (as familiar with the concept *red* we have some internal vision of *how red appears* separate and apart from particular representational

[26] I emphasize Kishida's work here first because it is recent, but also because it gives a thorough formulation of (what amounts to) "trans-world identity".

media), as well as in practice. In computer graphics, for example, we recognize the concept "color" both in terms of the arbitrarily fine-grained spectrum of human perception (at least relative to our conscious capacities of discrimination), as well as the use of some limited color spectrum (such as recognizing only 2^{24} distinct color values). It may indeed be that there are only a finite set of colors which human vision can discriminate, but one can argue that the semantics of color include the notion that as perceptual objects, colors can vary continuously. This is an example of where the semantics of a concept can be captured by combining an ideal dimensional structure (such as a unit cube in $\mathbb{R}^3$) along with a restricted structure respecting how that concept applies within given technological and representational contexts. So a dimensional annotation includes a structure $\mathcal{S} = (\mathcal{T}, \mathcal{P})$, where the "practical" dimensional structure $\mathcal{P}$ may or may not be distinct from, and represents to some approximation, the ideal $\mathcal{T}$.[27] A *spectrum node* η in a graph Γ is then a concept-node annotated with dimensional structure $\mathcal{S}$ as well as a *specification* $(\mathcal{T}', \mathcal{P}')$ indicating a range of values covered by the concept.

Given a spectrum node η in Γ, an axial node α is a node related to η via a relation which implies that α bears a feature corresponding to η's concept type. An *axiation* of Γ is then a set $\mathfrak{A}$ of annotations and their corresponding spectrum nodes, and a map $\mathfrak{A} \mapsto \Xi$ (Γ's axial nodes), so that onto each $\alpha \in \Xi$ is mapped one or more dimensional structures of the form $\mathcal{S} = (\mathcal{T}, \mathcal{P}, \mathcal{T}', \mathcal{P}')$. Call this structure a *measurement framework* $\mathcal{S}_\alpha$ on α. A *hypothetical measurement* $\mathcal{S}_h$ is then a value $t \in \mathcal{T}$ for $\mathcal{S}_\alpha$, and a *practical measurement* is similarly a value p in the corresponding $\mathcal{P}$. A *complete* hypothetical (similarly, practical) measurement on Γ is a map from Γ into a tuple $\{t\}$ (similarly, $\{p\}$) where each t_a (p_a) belongs to $\mathcal{T}_\alpha$ ($\mathcal{P}_\alpha$) for some node α. A (hypothetical or practical) *elaboration*, written $\Gamma \preccurlyeq \Gamma_\varepsilon$, is a graph uniting Γ with a subgraph expressing the details of one or more measurements, and a *complete elaboration* incorporates measurements for all axial pairs in Γ. A measurement *confirms* (or *breaks*) an axial pair (α, η) if $t \in \mathcal{T}_\alpha{}' \subset \mathcal{T}_\alpha$ or not (similarly $p \in \mathcal{P}_\alpha{}' \subset \mathcal{P}_\alpha$). A (hypothetical or practical) *distribution space* $\mathcal{D}$ over Γ can then be defined as a set of (hypothetical or practical) complete measurements which assigns to each $\alpha \in \Xi_\Gamma$ one or more t (or p) as above, for each spectrum node attached to α. An axial pair *holds* (or *breaks*) in a distribution $d \in \mathcal{D}$ (by extension, in a subset $\{d\} \subset \mathcal{D}$) if the measurement corresponding to d (or to each $d \in \{d\}$) confirms (or breaks) the pair.

If a conceptual graph models a linguistic resource (which I'll call generically a "document"), then an axiation of the document represents a more fine-grained

[27]In terms of my earlier "multiple aspect" theory of conceptual domains, the practical dimension is part of the "representational" aspect of a concept, while the ideal dimension expresses the *foundational* structure of a concept's model-space, although (insofar as eidetic variation is also variation of the phenomenology of certain concepts relative to conscious perception), it also belongs to the phenomenological part of the "representation" aspect.

representation of the document's potential relevance than a simple topic classification. Returning to the train-travel example from the start of this paper, a document describing a train ride may be "axiated" in term of nodes representing the trip's cost, duration, and the degree to which it is "scenic". This information would have to extracted and codified with varying degrees of precision — unless the document is prepared in a context which offers a specific rating mechanism for this purpose, for example, there is no metric for a trip's "scenicness". Such imprecision, however, allows a given conceptual graph to map to different "points" in the compound space spanned by the set of its axial nodes. This variation is not undesired, because queries themselves are imprecise. Assuming that a relevant document is axiated according to triple dimensions of a trip's cost, duration, and scenicness, and a spectrum of values assigned to each parameter based on possible interpretations of the information given in the graph, then a query can be similarly axiated based on explicit or approximated input from a person executing the query — requesting, for example, a train trip whose cost, duration, and "scenicness" falls within a given range. The document and the query will therefore potentially overlap in some regions within this three-dimensional space. Call $\mathbf{R}_D$ and $\mathbf{R}_Q$ regions corresponding to the document and the query. We may have $\mathbf{R}_D \subset \mathbf{R}_Q$, or $\mathbf{R}_Q \subset \mathbf{R}_D$, or just $\mathbf{R}_D \cap \mathbf{R}_D \neq \emptyset$ (compare with [1, p. 7]). Although this does not substitute for a formal demonstration, I observe that these options correspond to Chevallet's modal operators applied to document/query relations: the axiation of the document plays the role of a modal possible world, and the axiation of the query plays the role of a proposition evaluated at a set of worlds [5, p. 14]. By formally representing these semantic "possible worlds" as dimensional structures induced by documents' axiatropes, I therefore claim that Chavallet's Modal Logic, developed in the context of Information Retrieval, can be related to the topological models explored by "sheaf semantics" theorists like Kohei Kishida and Steven Awodey.

Although this framework describes measurements combining spectrum nodes and axial nodes, we can similarly define measurements on the "spread" of an axial node, or a fine-grained representation of its precise extension (for example, spatial) as an object (for example, a geospatial region). As above, we can distinguish *hypothetical* and *practical* measurements. A hypothetical measurement is an abstract device to capture the semantics of a concept as a referring entity: for example, we assume that place names refer to geospatial regions whose spatial extension takes on whatever form may be recognized by a community of people who know and speak about that place. A practical measurement is instead constrained by the practical limitations of mapping geospatial entities. In the hypothetical sense we can conceive of geospatial regions as subsets of spatial maps which are rich topological spaces, perhaps equivalent to bounded regions of $\mathbb{R}^2$. Practically, these regions are defined with respect to more coarse-grained markers, such as listing roads or geographical features which form the border of a region. As another example, when objects are defined as segments of a bitmap image, practical measurements of their spread are offerred in terms of pixel sets, while hypothetically we can conceive of image segments as subsets of an idealized Euclidean space. When measurements of the "spread" of an axial node α can affect the measurements of axial pairs involving α, then the latter measurement, as part of a distribution space $\mathcal{D}$, can be replaced with a compound measurement involving the spread measurement as well as the axial measure. Measuring the variation of some feature across a space from within which an axial node's spread is defined, can also be a way of calculating a best approximation for that spread.

I have discussed several scenarios in which modal ideas can represent foundational vagueness, nuance, coarse-grainedness, etc., as these are intrinsic to the semantics of different concept-types. Simple examples are provided by relatively coarse-grained concepts like *expensive*, where we have an obvious axis structure (a dimensional structure or numeric measure, and a semantic identification of this quantity as a monetary amount, a measure of cost) — and also a clear (if not numerically precise) notion of how the concept represents one spectrum of possible values within this axiation, and peer concepts which share its axial terrain or one very similar, like concepts *cheap* or (less strightforwardly) *frugal*. A good semantics for concepts like "expensive" can use modal notions, though we do not need to consider a single meal, say, as spread across multiple worlds. Possibilia may come into play when we articulate how *an expensive meal* can have many possible actual costs, so perhaps there are different "possible worlds" where the meal costs different specific amounts. However, this appeal to possible worlds is only one way of elaborating the underlying semantic situations. The modal aspects in this scenario derive from the fact that *expensive* is an intrinsically coarse-grained concept, so *this meal is expensive* can asserts a spectrum of fine-grained propositions, though perhaps without intending precisely to single out any one, or to single out a fixed "set", or a fuzzy set, or any other abstract formulation. How coarse-grained assertions should be semantically modelled is an interesting question, but any mathemtical construction should be considered as an idealization or approximation of real cognitive operations.

This analysis of the concept *expensive* serves to introduce the larger topic of imprecision or coarse-grainedness as it affects concepts in different ways, including the ontological foundations of concept-tokens as individuated entities, the intensional and extensional spread of sortal concepts, and the synthesis of concepts in blending or instantiation relations. A more complete theory will need to consider these different manifestations of coarse-grainedness with respect not only to single dimensions with clear axial interpretations, but more subtle imprecisions in object individuation, discursive connotations, and so forth. Notice that FCA is mathematically based on idealized object/feature pairs, but real-world semantics will be more subtle: a real-world analog of a Formal Concept will have muliple indicative properties and multiple bearers, but some bearers will be more prototypical than others, and some indicators will be stronger than others. A Formal Concept semantically, then, covers a spectrum of different connotations, since the component "indicator" concepts may be exhibited to varying degrees. In a sense, then, a Formal Concept is a kind of Conceptual Blend of a set of component properties and/or object examples.[28]

At the same time, although we speak of an abstract *instantiator* for concepts or properties — some kind of bare matter or "stuff", in Jubien's words — real concepts are instantiated by tokens of other concepts. We do not typically have red "stuff", but a red book, a red car, a red scarf, etc. This notion of instantiation also then bears aspects of Concept Blending: there is one concept which fixes an individual (a book, say), blended with others expressing its attributes (maybe red, heavy, hardcover, leatherbound). Because real-world Formal Concepts or Concept Instantiators are generally coarse-grained, they can give rise to different possible refinements, so their semantics has an internal modal aspect. Meanwhile, the same coarse-grainedness which produces this modal dimension also contributes to the idea that both Formal Concepts and Concept Instantiations have aspects of Concept Blending, at least when we move from idealized schema to real-world semantics. This suggests a relation between CI and modality, which I will explore further in the second half of this paper.

6 Mereology and Mereotopology

The previous sections considered one integration within Unified Concept Theory, the synthesis of Conceptual Graph Semantics and Formal Concept Analysis. In the latter part of this paper I will consider integrating Conceptual Blending or CI schema as well. The "axiatropic" theory extending CGS focussed on model spaces of individual concept-types, and how these combined within relational structures between distinct concept-tokens. The formal motivation behind this provisional axiatropic theory is to represent, in a systematic way, how semantic intentions influence the model-spaces within which individual concept-types are instantiated. Introducing Concept Blends adds further structure to these model spaces, because they allow model-spaces to be "blended" together. In conventional CI, spaces are joined in combinatorial, discontinuous ways, but here I will explore "concept blends" like the merger of spatial articulation and qualitative patterns, like coloration, in which each component concept-type has a contuinous or "geometric" model-space. Mereotopology, in particular, emerges as an intuitive framework for models of these blended model-spaces.

[28]Consider an analysis of the concept *house*, say, as a blend of many actual houses. This does not appear to be a typical CI formulation, but it bears considering in light of FCA: concepts blend component other concepts, but perhaps they also blend specific examples, whether for a single individual, a local community, or a larger language community, so that prototypical houses (on TV, in magazines, realtor's ads, etc.) form part of a "blend" fixing the concept, alongside generic propositions like a typical house having a front door and front yard.

Mereotopological systems combine features of mereological algebras and topological spaces. They are a natural vehicle for semantic models because mereological systems, on the one hand, lie at the foundation of almost any collection of entities which provide a grounding for discursive domains, while topological spaces, on the other, provide useful representations of concepts which permit continuous variation. Mereotopology usually takes topological spaces as more primitive, and introduces mereology by considering partitions of these spaces, or topological relations such as connectedness or "contact" (intuitively, the contact between subspaces whose closures are connected but whose interiors are not). However, it is useful to consider mereological systems as fundamental insofar as they can articulate a "universe of discourse", or a domain of individuals which serve as concept tokens. Here I will assume that topological spaces are attached to elements of mereologcal systems, but also that these spaces have their own internal mereological properties, which allows for both perspectives (mereology based on topology and vice-versa). Because mereological structures can be expressed with Conceptual Graphs, which have their own topological analyses, it would be interesting to develop a topology-over-mereology theory on a CGS basis. In this context, however, I will focus on mereology as defining a referent-universe and as derived from topological structure, motivated by the idea that mereological structures can be induced on topological spaces by mappings from "sorts" in the sense of Order Sorted Algebra.

I assume that any semantic framework presumes a "referent-universe", a discursive context in which entities with some degree of individuation are available as tokens to concept types, and as referents of indexical expressions, insofar as designating a particular entitity is the most common way of identifying a concept-type as instantiated in a token. In terms of their *foundational ontology*, these referents may not be "crisp", or crisply individuated; however, to serve as identified concept-bearers, they need only have some provisional separation from their surroundings and their peers.

Entities within semantic and discursive contexts acquire a degree of individuality simply by being intended in meaning, and referred to in discourse, as a singular unit. Some of these unities are well-defined individuals which form obvious functional units in their interaction with other entities. Other unities are more diffuse and loosely integrated. For example, the United Nations is much less "coherent" as an individual political entity than the United States. Nonetheless, there are certain occasions and situations in which the United Nations operates as a discrete, functional whole. Conversely, there are many situations where particular US states act as isolated bodies. In general, mereological properties — the phenomena of being parts or wholes, and parts of wholes — these properties will vary across situations. Most things in mereological relations function as wholes on some occasions, and as parts on others.

The degree to which semantic units are also mereological wholes which tightly bind their component parts, in more circumstances than not, is part of the semantic meaning that we can attach to any usage which treats semantic units *as* units. The United Nations and the United States are two different types of political bodies partly because the principles of their syntheses into wholes are different. A semantic unit is semantically meaningfully not only *because* it is a whole, but also because of the distinct properties pertaining to *how* it is a whole. The nations in the UN are only loosely coupled to that body, but there is nonetheless some pressure on member nations to respect the decisions and initiatives of the UN as a whole. This balance between individual coherence — the degree to which parts cohere into a unity, and in which statements treating the whole as singular are coherent — and diffuseness, degrees of freedom among parts, forms part of the infrastructure of any entity which enters into a sphere of discourse and meaning. Any such entity must be associated with some balance between individual coherence and internal diffusion. The identification of this balance with regard to any semantic unit whatsoever is part of the fundamental conceptual infrastructure preconditioned for any semantic activity whatsoever.

Of course, some wholes are even more diffuse than the United Nations. The collection of nations which I have visited in my lifetime, is a set whose members (at least in any way relevant to the formation of the set) bear no relation to one another, in contrast even to the UN. A set like this is a "pure collection", with no principle of selection other than simply enumerating its members. Even here, though, there is some criterion which distinguishes the set from its complement. This criterion may have nothing to do with the relevant nations internally, so their comembership in the set is a purely external relation between them. Nevertheless, there is a criterion which marks a separation between the set and all nations not part of the set. In this sense the set forms a whole; figuratively, it has an inside and an outside. It functions as a logical unit in relation to other sets, such as its complement (the nations I have *not* visited) or other sets with which it may combine or intersect (like the nations *you* have visited). In this sense even pure collections are mereological wholes, because they can enter as single units into mereological relations such as overlap or containment.

In adopting a mereological rather than a set-theoretic perspective, the fundamental relations which form the basis for domains of semantic entities are not the relations of member to a set and of subsets to supersets. The distinction between part-of and member-of is logically secondary. The more properly mereological relations are relations of part/whole, but also relations of coupling or integration. When a part belongs to a whole, it can also be tightly bound to a whole or more loosely bound. Two parts of a whole can be coupled with each other while also both bound to the whole. I will use the word "coupling" to refer to degrees of attachment or connectedness between entities, and the word "bind" in cases where the coupling occurs between a part and a whole which contains it. The fundamental relations then are coupling and parthood.

I assume that these relations are not binary, but instead can apply to varying degrees, which we can consider to range from zero (no coupling or parthood whatsoever) to one (which we can assume to measure only coupling and parthood between an entity and itself). I will call any domain of entities which have these relations a "weighted mereological system", and I will call objects in such systems "mereons". All mereons are wholes to some degree, and most are also parts of other wholes.

A *pure collection*, such as the set of nations I have visited, is a whole whose parts have negligible coupling to the whole itself. Their membership in the collection does not imply any sort of functional interaction with the collection as a whole. As a technical artifact, I will treat "coupling" and "parthood" as fundamental notions with no further definition, though informally we can certainly consider the different ways in which things can be parts of and/or coupled to other things. Coupling relations generally imply that there exists some causal or functional influence between them: that changes in one thing tend, with some degree of probability, to cause changes in the other. Coupling relations may also be recognized between entities which share some property, particularly if this property is relatively "important" as a characteristic of the objects in question. Two objects which are similar according to some important criteria may not direectly influence each other, but may tend to behave similarly in similar circumstances. Later I will consider these semantic interpretations of coupling and parthood in more detail. For now, I will simply assume that a weighted mereological system ${}^{\mathcal{W}}\mathfrak{M}$ is a set of mereons m, such that between any two mereons m and m' there are two different weighted relations. If m is part of m' this will be notated as $m \lessdot m'$; the coupling relation will be written $m \odot m'$. For each relation I will assume that there is a real number α so that $m \lessdot_{\alpha} m'$ means that m is part of m' to degree α, and similarly $m \odot_{\alpha} m'$. When α is not written I will assume that the relation is asserted for some nonzero α or for some α above a negligible threshold, depending on context. I will use the single symbol $\mathfrak{M}$ to represent a weighted mereological system ${}^{\mathcal{W}}\mathfrak{M}$ (later I will further refine this contrast from traditional, non-weighted mereologies), since I will not have occasion here to talk about "binary" mereologies built around a single, unweighted part-whole relation.

A whole m has a collection of parts $m' \lessdot m$. Each of these parts in turn will in most cases have some coupling $m' \odot m$. How these two relations numerically vary determine the "kind" of whole represented by m, as suggested earlier. For example, m can be called a "pure collection" if all the coupling values α of $m \odot_{\alpha} m'$ given $m \lessdot m'$ are zero or negligible. At the other extreme, a mereon can be called an *aggregate* if the coupling weights $m' \odot_{\alpha} m$ are generally stronger than the couplings between pairs of mereons both in m. In other words, the parts inside an aggregate may have some functional interrelationships separate from their mutual attachment to the whole, but this relationship is less significant than their connections to the whole. A *pure aggregate* is an aggregate whose parts have zero or negligible coupling to each other except through their mutual coupling with the whole. Between pure aggregates and pure collections there are a range of situations corresponding to different degrees of integration between parts and wholes and between parts themselves. For example, we can describe an *associative network* as a mereon whose parts $m' \lessdot m$ have little coupling to m, but each part m' is relatively strongly coupled to at least one other part $m'' \lessdot m$. A mereon m becomes part of an associative network by establishing a coupling with another mereon in the network. A loose grouping of mereons which have relatively low coupling to the whole — in other words, relatively large freedom relative to the whole — but in which there is nonetheless some formal recognition or criterion of membership in the whole, can be called a *confederation*. So a confederation differs from an associative network in that, although members in a confederation have relative autonomy, there is some collective criterion or decision to include each mereon in the whole. For example, a social club is different from a group of friends in that there will typically be some formal process whereby a new person is admitted to a club subject to the approval of the club as a whole, not just a single member. In between these two cases, we can call an "associative conferation" a whole in which membership is collectively decided, but contingent on some sort of "invitation" or prior coupling with one (or a small group) of members already present.

One manifestation of the relative diffuseness of the United Nations, compared to nations like the United States, is that nations within the United Nations tend to have varying degrees of coupling with each other. In the current terminology, the United States would be an aggregate, and the United Nations would be a confederation. This is revealed by considering that nations are much less "coupled" to the UN than are states to the US. We can say that nations are *autonomous* relative to the UN, suggesting relatively low coupling despite parthood. A mereon m is "diffuse" if most of its parts are automous relative to m. Because nations in the UN also tend to couple with each other to varying degrees, we can say in addition that UN members are *pairwise autonomous* relative to the whole. This means that, notwithstanding their mutual membership in the whole, the degree to which two members of the UN possess further interrelationships can vary significantly. By comparison, states in the US do not generally display this kind of pairwise autonomy: states cannot enter into treaty relations with other states, for example, and in general the mutual relations and obligations between states, by virtue of their common membership in the union, significantly supercede whatever political or cultural ties that may tend to link some pairs of states more than others. This notion of pairwise autonomy (or the lack thereof) can then be scaled up to subnetworks and subcollections within a whole. For example, there are clusters of UN members who also have formal relationships in other political groupings: NATO, NAFTA, ASEAN, the EU and Euro Zone, etc. So in addition to individual and pairwise autonomy, we can say that the UN confederation exhibits *cluster autonomy*, meaning that their common membership in the UN does not inhibit groups of nations from forming other, more tightly bound unities. In some cases, these clusters act as individual parts of the UN in turn, in that they tend to function as unified blocks in the UN context, voting and advocating for similar policies. For example, UN dynamics are often characterized in terms of negotiations, cultural differences, or ideological clashes between "Western" and "Developing" nations.

This elaboration of different types of wholes, and also of potential numeric "signatures" in which the aggregative structure of different types of wholes can be statistically compared, perhaps serves as an intuitive picture of the conceptual structures underlying our language of parts and wholes. Although I will explore mereological relations as fundamental conceptual structures internal to language in general — and therefore logically prior to any particular domain within language, any kind of "talk" — nonetheless we should not ignore "mereological talk", the semantics of natural language concepts which pertain to parthood, aggregation and coupling, etc. Providing a quasi-mathematical account to back up semantic intuitions can be a productive exercise: the plausibility of the mathematical structures as a formal representation of a conceptual system said to undergird some domain of natural language, can serve as evidence that the proposed system is a successful model of the underlying conceptual network. However, I want to go beyond the use of formalization as simply a way to refine semantic intuitions, and explore mereological systems as mathematical objects in their own right. Since any system $\mathfrak{M}$ includes a collection of pairs with weights α for both coupling and parthood relations, the distribution of these values can be analyzed statistically, by analogy to the analysis of large-scale graph structures. By considering measures such as the average coupling or autonomy from among the set m' of mereons part of m, the range of pairwise couplings within this set, or structural analyses of coupling networks as graphs taking their vertices from sets $\{m'\} \lessdot m$, we can provide a more formal classification of different types of wholes based on these statistical signatures.

However, I want focus exploration here in a different direction. While wholes are constituted by a collection of parts, it is not necessarily the case that a whole is *nothing other* than a collection of parts. Aside from metaphysical questions of whether the whole is somehow "more" than its parts, there is a more concrete problem that for a given $\mathfrak{M}$ particular mereons m may have properties which are not wholly explained or exhibited by the relations between m and the $m's \lessdot m$. A mereological system may be designed not *only* to model mereological relations, but also to recognize properties of mereons themselves. Indeed, mereons are often individuated by virtue of properties which distinguish them from their surroundings, and/or from other mereons. It is not necessarily the case that these properties are wholly subsumed by the core mereological relations. In other words, it should not be assumed that all facts pertaining to how a mereon m is individuated relative to other mereons in an $\mathfrak{M}$ can be captured by listing the set of m's parts and non-parts, with their coupling and parthood weights. To capture how mereons are individuated in the first place, it will be useful to allow these extra features and properties to be expressed within mereological systems. Such features can then induce further structurations of mereons, and in some cases this structuration can cause parts of existing mereons to be identified as further merons in the $\mathfrak{M}$. However, the possibility exists that there may be patterns of structuration within mereons which can not readily be associated with particular parthood relations.

Consider the case of discrete objects in space. Certainly objects are sometimes joined to others to form wholes. Moreover, many objects are functional aggregates of parts which have some individuality or "individual coherence" of their own: a lid can be screwed onto a jar, and a chair contains legs, arms, a seat, perhaps a cushion, and so forth. On the other hand, these spatial parts may include variations in form or color which are harder to describe in terms of a fixed decomposition into parts. The full complexity and detail of a mereon can be called its *internal structuration*, and certainly how a mereon is composed of parts, how many parts and their degree of coupling to the whole, contributes to this internal structuration or "**IS**". We can assume, however, that there will be other structural features of m which contribute to its **IS** value — $m_{\mathbf{IS}}$ — apart from m's system of parts. In some cases these features will cause parts of m, other mereons, to be recognizably isolated, but with relatively ambiguous or imprecisely defined boundaries. For example, we can recognize the painted surface of an object as a collection of color patches, but these patches may overlap and merge so that we have no obvious criteria for their internal borders. These parts may still be recognized mereons in a $\mathfrak{M}$, but they will have some measure of indistinctness or fuzziness as individual unities. We can say that mereons in this case have comparatively less *individual coherence* or "**IC**". We tend to think of entities as either atomic simples — with obvious individual identity and little internal structure — or else as complexes whose internal structuration tends to diminish to some degree their coherence as individuals. But this account is imprecise: tightly unified aggregate wholes can have complex internal structure but still cohere as individual unities in may situations, and in other cases an apparent whole (like a color patch) can stand out only partially from its surroundings, even if its internal structure does not possess a clear internal division into component parts, so that it can act to some degree as an atomic unit — consider a color patch which is not purely monochrome, but which has relatively little internal color variation and a gently curved and somewhat indistinct outer boundary.

While there may be some correlation between **IS** and **IC** — greater internal structuration can tend to imply greater diffuseness among parts of a whole, so that the whole exhibits less individual coherence, whereas simple, quasi-atomic unities have clear demarcation as individuals by virtue of having little apparent inner structure — these are merely general trends. The **IS** and **IC** measures for a given mereon can vary independently, so that each mereon can take values within a two-dimensional range that we may visualize as a unit square, allowing each **IS** and **IC** measure to vary between zero and one. We can call this compond measure the **IS/IC** scale or **IS/IC** measure on a mereon. Like the degrees of coupling between parts and the whole as well as between each other, the **IS/IC** measure can be considered a measure of precisely how mereons are organized *as* wholes. I will set aside for now the question of whether **IS/IC** values for a collection of mereons can actually be quantified, and just assume that these values are defined on mereons m in a system $\mathfrak{M}$. So this measure $m_{\mathbf{IS/IC}}$ can be considered a fundamental property, defined on a all $m \in \mathfrak{M}$, alongside the parthood and coupling relations.

Thematically, the problem of relating mereological strucutre to internal structuration appears to be a crucial element in the historical relation of mereology, as a domain with potential formalization, to philosophical treatments of perception and complexity. Most cognitive objects — things perceived, propositions believed, meanings expressed in language, etc. — most of these, speaking generally, are compound unities whose meaning or signifance derives in part from the significance of these parts in turn and how they are unified into the whole. This compositional structure is perhaps best exhibited by statement in natural language, although similar comments apply to perceptual and cognitive domains more generally. There is a sense in which individual words are atomic units of meaning, and sentences are structural complexes formed by interrelating these units, giving graph-like structures which can be modelled as graphs whose nodes are words and whose edges are semantic and syntactic relations. This account needs to be qualified to handle things like polysemy and word-stems or compound words, like "waterfall". Even with these modifications, however, it seems that there are degrees of semantic nuance which are difficult or impossible to capture in a purely compositional framework. For example, different words can have similar meanings but different shades of emphasis. In a purely compositional paradigm, these differences would have to be captured by isolating one or more component concepts, some semantic atom or aggregate of atoms, which are present in one case and absent in the other, and/or vice/versa. At best a "compositional" semantics — the kind of overall philosophical orientation to meaning which we find in a work such as Wittgenstein's *Tractatus*, and in one fashion extended to a general theory of meaning and knowledge in Russell's Logical Atomism — presents a coarse-grained theory of meaning as a "molecular" synthesis of simpler parts. Some philosophers have contrasted this paradigm with a more interpretive or "hermeneutic" approach to meaning inspired, for example, by Husserl (cf. [**?**]). Husserl's distinction of dependent and independent "moments", for example, incorporates aspects of Logical Atomism but also cosniders situations where parts cannot be readily separated from their containing wholes, such as the two-dimensional surface of a three-dimensional objects.

It is easy to oversimplify philosophical trends when reviewing decades-long intellectual historylines in a few sentences, so I will not try to develop some precise picture of the differences and interrelationships between Phenomenological and Analytic approaches to meaning and semantics. I think it is more helpful to regard semantic reality as involving both compositional and connotational aspects (I'll use these terms, rather idiosyncratically, to express the kind of molecular paradigm evidenced by approaches such as Logical Atomism, and conversely the more Phenomenological intuitions which can be traced back to Husserl's *Logical Investigations*). Some analyses place more emphasis on the former, and others more on the latter. It is interesting to speculate on long-term paradigm trajectories which link Russell or Wittgenstein, during their Logical Atomism and picture-theory phases, forward in time to contemporary Description Logic and Conceptual Graph Semantics, and, on the other hand, to trace a trajectory through Husserlian texts such as the *Investigations* and *Thing and Space*, through Merleau-Ponty and a kind of Merleau-Ponty-inspired "dynamic phenomenology" represented by (for example) Jean Petitot and David Armstrong, forward to contemporary technological problems such as computer vision and Virtual Reality. Such a trace of intellectual history may not stand up to more detailed scrutiny, and it is easy to miscontrue the philosophical relations between distinct thinkers by contrasting specific works, without considering their place in a philosopher's total career: for example, neither the *Tractatus* nor Russel's *Philosophy of Logical Atomism* should be considered representative examples of their authors' philosophies, considering how these evolved. Nevertheless, I think these mentioned texts serve as good examples of a "compositional" approach to meaning which can be usefully contrasted with the stategies adopted in *Thing and Space* or Merleau-Ponty's *Phenomenology of Perception*.

Insofar as atomic or "molecular" compositional form cannot fully account for the meaning of semantic aggregates, there will be some degree of semantic nuance, some internal structuration, which is not wholly represented within the mereological partition of a whole. To what degree can this problem be modelled quantitatively, even if just as an intuition-sharpening exercise? Suppose we have a system $\mathfrak{M}$ with an $\mathbf{IS}/\mathbf{IC}$ measure $m_{\mathbf{IS}/\mathbf{IC}}$ for $m \in \mathfrak{M}$. We want to measure "how much" of $m_{\mathbf{IS}}$ is produced by m's parts and their interrelationships. This question is influenced by different factors: how distinct are these parts in contrast to the whole; that is, how much individual coherence do they have on their own? How complex are the interrelationships between these parts? Mereons of course can entertain many different kinds of mutual relationships. Wholes whose parts are interrelated in only a few ways arguably have less internal structuration than wholes whose parts exhibit many different relationship-types. Moreover, wholes whose parts tend to have many interrelationships amongst each other are more intricately structured than wholes whose typical parts have only a few relata. In other words, both the density and diversity of relations between mereons $m' \in m$ contribute to m's structuration. It seems plausible that these notions are statistically meaningful: we can count the number of relationship-types found among a system of parts (perhaps eliminating relationship-types whose tokens appear only in a few instances, below some threshold occurance criterion), and the average number of connections involving these parts (treating relationship-types as defining graphs), and each $m' \in m$ will have some individual coherence $m_{\mathbf{IC}}$ by stipulation, since $\mathbf{IC}$ is defined for all mereons in $\mathfrak{M}$. Each part will also have its own degree of internal structure which contributes to the $\mathbf{IS}$ measure of the whole. So we have a plausible quantitative framework for measuring the degree to which $m_{\mathbf{IS}}$ is a product of m's parts and their interrelationships. We therefore have a framework for measuring the degree to which $m_{\mathbf{IS}}$ is *not* caused by this compositional structure. In other words, we can potentially quantify a measure of holistic or *compound* structuration, say $\mathbf{CS}$, which considers the gap between a mereon's internal structuration $m_{\mathbf{IS}}$ and the composition of the structuration of its parts, considering in this composition not only their own $\mathbf{IS}$ but also their degrees of individual coherence and the patterns of their interconnections.

The presence of this $\mathbf{CS}$ — the fact that internal structuration of a whole is not wholly articulated by the mereological properties of the whole in relation to its parts — this problem fundamentally affects the degree to which mereological formalization can serve as a system modelling meaning and semantics in general. Just as mereology itself is proposed as a more philosphically intuitive system extending and incorporating set theory, and attempts to model language in terms of set theory and first-order logic, so mereology in turn needs to be further extended to handle problems such as the presence of "compound **IS**". To address these issues a mereological system $\mathfrak{M}$ needs some further elaboration of how internal structuration within mereons can be modelled apart from mereological relationships themselves. For example, among objects in space, details such as spatial form and color patterns can be captured through mathematical equations, and these provide a framework for defining structure on mereons in addition to the structure provide by parts m'. Suppose we can therefore define, for mereon m, an *equational system* which is a collection of numerical dimensions and equations defined on these dimensions which express details relevant to m' structure, composition, and form. These equations can model m structure more precisely than a coarse-grained specification of $m's$ identifiable component parts and their interrelationships in turn. I will call a weighted system ${}^{\mathcal{W}}\mathfrak{M}$ extended by these kinds of equations a "quasi-mereological" system ${}^{\mathcal{Q}}\mathfrak{M}$, one which for some $m \in {}^{\mathcal{Q}}\mathfrak{M}$ there exist systems of equations $\mathcal{Q}_m$ that express properties of m's structure. A ${}^{\mathcal{Q}}\mathfrak{M}$ system therefore represents the **IS** measure on m as a union between *compositional* and, let's say, "equational" **IS** — the latter being **IS** which is latent among the equations $\mathcal{Q}_m$ — allowing for the fact that compositional and equational structuration can overlap. I will from here on assume that all systems $\mathfrak{M}$ are also quasi-mereological in this sense and therefore continue to use the simpler symbol $\mathfrak{M}$ in lieu of ${}^{\mathcal{W}}\mathfrak{M}$ or ${}^{\mathcal{Q}}\mathfrak{M}$.

Problems such as compound structuration (**CS**), along with other complications such as the multiplicity of part-whole relation *types* in real-world situations, are one reason why mereology is suspect as a tool for modelling real-life meaning and semantics, as are formalisms like set theory and first-order logic. Systems which are precise enough to engender mathematical theory are usually not subtle and flexible enough to model human meaning and cognition, and vice-versa. This simplicity can be addressed in part by extending mereology in various ways — weighting part/whole relations, for example, or by supplementing part/whole relations with coupling relations and therefore with full-fledged graph structures, rather than lattices or tree structures engendered by nontransitive part/whole connetions. A more substantial extension to mereology, which introduces notions from Topology into a compound theory or Mereotopology, helps to capture the intuition that entities in mereological systems have internal structure which cannot be fully captured by part/whole relations, insofar as Topology generally connotes surfaces and continuously varying spaces in contrast to discrete aggregates of atomic simples. One approach to mereotopology is to define different types of part/whole relations either inspired by or technically incoporating topological notions — so for example being "deeply part of", an "internal part of", etc.[29] I suspect that these extensions still yield axiomatic systems without sufficient flexibility to serve as general-purpose foundations for studying meaning and semantics. Instead, I propose that we emphasize the problem of "compund **IC**" as

[29] In particular, axiom-based mereological systems often depend on notions like "contact" whose schematic formalization depends on the topological defininition of *boundary* as set difference between a closure and an interior. This is a useful but idealized representation of the contact-relation, and I will use it in several analyses below. In real life, however, situations where two identifiably distinct objects are "in contact" but not "overlapping", arise not from their touching only along a hypothetical *boundary*, which is dimensionally smaller than the objects themselves, but on some noncrisp internal structuration which demarcates a general boundary-region from the rest of the objects; or else as a situation of physical contact involving distribution of forces, like when one object rests on another and is held there by gravity.

the key intuitive problem with simpler mereological systems and the motivation for exploring topological extensions. In other words, the underlying rationale for turning from *mereology* to *mereotopology* is to derive an account of internal structuration which reflects but is not wholly dependent upon mereological composition.

If this is granted, then we can try to tackle this problem head-on rather than trying to capture it indirectly by introducing new part-of relations (like *internal part*) or hoping for a straightforward resolution by simply asserting that mereons are also elements of topological spaces. I believe that the motivation for mereotopological semantics emerges from an intuition that the combination of part/whole connections and topological continua is a fundamental feature of human perceptual reality, insofar as this forms a horizon and origin for semantics and discourse. This suggests that we may look to perceptual structures themselves as a framework from which can attempt to extract formal mereotopological schema.

7 Mereotopology and Perception

In some cases, it may be that mereological properties can be derived from topological ones and that the topological "information" available for a given mereon may have structures which reveal more fine-grained mereological details than can be directly observed from the topology of a particular space. Topological structure, in other words, may present information about features which do not appear to be either mereological or topological on face value. For example, consider the spatial structuration of simple objects like a dumbell, volleyball, teacup, and a gold ring. By a crude topological comparison the first two objects are equivalent, as are the last two; this kind of topology therefore does not reveal the important functional differences between them. It is not clear that these differences are better revealed by mereological considerations either, since for example the dumbell differs from the volleyball because it has somewhat distinct parts — the center of the dumbell where it is grasped being distinct from either end, reflecting a division which is not similarly apparent in the volleyball. However, these parts are not clearly delineated, so there is no immediate mereological analysis which would isolate the parts by splitting the dumbell into components, any more than there is a topological analysis which would divide the surface of the dumbell into three regions. To the degree that more sophisticated analyses are possible, they may reveal topological and mereological properties simultaneously. For example, we can imagine shrinking the center of the dumbell until it disappears, which has the effect of splitting the object into two pieces; there is no comparable alteration which can be performed on the volleyball. Sometimes structural properties of an object become topologically signifant as the object is deformed or simplified, like converting a peninsula into an island, or a concave region into a hole. The degree of alteration required for these alterations to induce topologically significant changes can provide some measure of the spatial form of an object, in other words of geometric properties which would not otherwise be topological. Similar ideas lie behind the theory of "persistent homology", which is used to calculate computational surfaces for objects which have been sampled (for example with lasers) into three-dimensional point-set clouds.

Moreover, obejcts whose spatial form varies continuously may have apparent features that predispose us to recognize certain parts, even if these are not cleanly individuated from the rest of the object. Consider a teacup with an incomplete handle and a flat bottom. We clearly interact with this object by grasping the handle and placing the bottom on the table, even if these features cannot be topologically isolated from the whole cup. Along these lines, the geometry of an object's spatial form will suggest features which predispose us to direct attention to certain areas as if they were parts with some degree of individuation. There may not be *topological* boundaries which clarify how these parts are individuated, but there may be topological analyses which can identify some of these patterns. For example, an object's spatial form determines how smoothly or complexly it bends around in space. This is a property of the surface as embedded in three-space, but it can also be identified if we imagine a surface as a fabric which is stretched or compressed depending on different areas of two-dimensional curvature. It is possible to reproduce shapes in computer graphics, with noteworthy accuracy, by drawing lines on a surface derived from tangent planes to the surface at different points, reflecting how and to what degree the surface curves from place to place. One common method is to consider spatial curvature as a vector field over a surface, and tangent planes as containing vectors whose magnitude measures curvatures in different directions. Sampling some of these vectors, by drawing them as projected onto the surface itself, creates the visual illusion of three-dimensional depth even in a monochrome coloration, like a sketch using a single pen and a single style of pen-stroke. The form of a surface embedded in space can therefore be reconstructed from curvature vectors on the surface itself, considered as a self-contained manifold. Moreover, the collection of tangent vectors can be treated as a topological space on its own, distinct from the topology of the original surface. This latter space may have topological features which are richer than features of the former space.

Spatial form alone is one obvious feature of an object; certainly there are other features which contribute to our perception of objects as wholes, such as coloration. Color patterns along the surface of an object can dispose us to recognize the surface as divided into distinct parts, or else to recognize some pattern of division but to have trouble identifying borders between parts. These features suggest structural patterns along a surface which are perceived alongside patterns in the surgace geometry itself. In some cases these additional features can similarly be represented in terms of vectors over the surface (at least insofar as features such as color have scales of featural variation, so for example we can measure the degree of change in color-value along different directions within a surface). If a feature which varies along a surface (like coloration) can be given a quantitative expression, then this variation may give rise to a vector space along the surface which can be analyzed using topological and/or differential methods. For example, a vector space can have critical points along a surface, such as maxima, minima, and saddle points ([**?**, p. 87]). While these point do not themselves divide surfaces into regions, they can form a nonarbitrary basis for identifying partitions on mereological grounds. For example, our perceptual impressions of mereological partitions of a surface may be modelled as a set of patches each centered on a critical point, with imprecise borders which generally move between and maintain a comfortable distance from these points. It may not be clear from geometric considerations alone how patches are to be identified around critical points, but insofar as certain partitions seem perceptually evident, critical points and the structure of the surrounding vector field can provide a quantitative explanation of where this instinctive partition comes from. Moreover, the topological structure of the vector space induced by features on a surface — including curvature of the surface itself — may have different topological details than the surface on its own, particular when subject to analytic variations, such as considering gradient lines or spaces between gradient lines. These subspaces may have topologically meaningful features (such as holes) which reveal structures in the variational pattern of features over the surface, patterns which are geometric but not topological in nature when considered only in terms of their spatial articulation on the surface.

Any attempt to model empirical and/or perceptual details topologically assumes a correspondance between topological notions (like points and sets) with empirical/perceptual locations and features — for example, zero-dimensional points in perceptual space, zero-dimensional color points, etc. This is clearly an idealization: even if we intuitively picture color space as subject to continuous variation (such as a three-dimensional RGB cube), we cannot perceive an infinite variety of color hues (or create such a variety on computer screens). On the other hand, it certainly seems as if we *perceive* coloration as varying continuously along the surface of objects. Even if the color does *not* appear to vary, this same color hue itself spreads out along the surface, and so there is variation in the points identified as colored by that hue. Even if the idea of a "point" in color space, or a zero-dimensional point in perceptual space, strikes us as an idealization, nonetheless we need to bear in mind that the purpose of some sort of quantitative or topological modeling of perceptual spaces is to capture patters of variation within perceptual contents, and this variation involves both variation in featural values or "hues" (like color hues, but perhaps we can borrow this word to stand for qualitative characters in other sensory dimensions, like pitch and tonality, flavor and scent, etc.), as well as in the spatial distibution of these hues along perceived surfaces (and sometimes within space itself, such as when we see a colored light creating an illusion of color filling space through a medium like fog or dust).

Insofar as we want to explore philosophical implications of topological models of perceptual contents, the crucial topological spaces which we can hypothesize as providing meaningful models are spaces whose continuities involve changes both in the distribution of features within a "manifold" space or the space of a surface, as well as changes within a qualitative space of featural "hues". Logically we may want to treat these compound "feature spaces" as product spaces of "manifold" and "featural" components — using "featural" to refer to spaces spanned by different featural "hues" and "feature space", by contrast, to refer to distributions of these hues within a geometric space. However, even if this logical separation can be a useful analytic device, it may not be the best representation of real-world phenomena of interest. For example, our perceptions of color hues and spatial forms are interrelated: we use color variation to perceive spatial form, but we also use spatial form to indicate coloration of objects, which we may believe to be different from the apparent coloration at a given moment, due to effects such as shadows and perspectival distortion. This is not a matter of cognitive judgment superceding perceptual ones (the kind of second-order bracketing of perceptual judgments which occur, say, when I realize that everything looks red because I am wearing red glasses). Instead, this qualifying of moment-to-moment appearances is part of perceptual judgment itself: I anticipate that as I view an object from different angles, walking or moving continuously from one vantage point to another, I will witness a subtle variation in apparent coloration based on differences in how the surrounding light illuminates the object relative to different positions when looking at it. Nevertheless these judgments, which affect how I perceive coloration, are based on assumptions about the spatial geometry of surfaces, which on the other hand I infer from color patterns. So judgments about coloration and surface geometry mutually influence one another. This lends some philosophical motivation to the idea that spaces of perceptual variation should be modelled as continuities of changes in both perceptual location and featural "hues" simultaneously.

Structural details about the organization of perceptual mereons — perceptual contents insofar as they are modelled through mereological systems — therefore may be clarified by considering structures in these compound feature spaces which may not be evident in geometric or featural variation when these are taken in isolation. Consider, for example, the surface of a Canadian flag. If we model this entity as a mereon with its own internal feature space, focusing here on color and spatial form, then assume for sake of argument that the flag has exactly two color values, red and white. Each point in the feature space therefore has a component which includes a point in a geometric space (say (x, y), treating the space as a rectangle)) and one of these two colors. Once again, I am "separating" the featural and manifold directions of variation for sake of analysis, without assuming that either of these two component spaces are meaningful in isolation from the other in their original setting; I assume immediately that modelling a perceptual situation like a mathematical system, with component numeric dimensions, is itself an idealization, and teasing apart some of these dimensions from others is still a further idealization. Nonetheless, this separation helps to provide a formal representation of the mereology of the original entity (the flag). Keeping "OSA" somewhat nontechnically in the background, we can identify the color points distributed over the flag as sampled from a *sort*, which in this case is a two-valued subsort of color space. In other words, we have a signature with sorts C (representing color space) and $s \leqslant C$ (a two-valued set {red, white}). A compound space is then a generated by a map $s \rightarrow$ the flag.

Within the compound feature space the two color points are disjoint, so the space as a whole is divided into two disjoint components, one set of points with the color fixed as red, and another set with the color fixed as white. The effect of this separation is akin to cutting the flag at the red/white borders and pulling the red portions forward, only the direction of this imaginary movement is not in physical space tangent to the flag, but in a more abstract "color space". In the feature space the red and white parts of the flag are therefore separate from each other, whereas when these are seen just as planar components of the flag (as components of the geometric base space without the compounding feature dimension), these components contact one another, sharing the same borders. Once separated out, we can concentrate on the structural properties of each component part. The red components are evidently three disjoint, simply connected regions. The single white component is topologically equivalent to an annulus (a one-dimensional analog to a torus), since the maple-leaf pattern of the interior "hole" is not topologically relevant. Topological details such as the presence of "holes" can be identified by how structures inside a topological space may be transformed or mapped into each other. The holes themselves are not part of the space — for example, if the universe were shaped like a giant torus, there would be no way for us to "see" a hole because there would be no space there for us to see anything; space would not exist outside the boundary of the universe (of course a torus in real life is not a self-contained universe but exists in a larger space, and relative to that space the hole can be seen). However, the presence of the hole within the space itself (here in two dimensions) is revealed by the fact that loops in the space which travel around the hole cannot be tranformed into those which do not do so, at least if we consider only continuous transformations wholly defined by paths inside the space.

The topological structure of a space, considered without reference to some larger space in which it is embedded, can be assessed by these considerations of mappings between structures of different dimension. For example, the fact that the red part of our flag-space has three disjoint components is expressed by the fact that there are effectively three ways in which two points may fail to be transformable into one another. If two points are in the same component, then there is a path within the space connecting them, and thereby mapping one point onto the other. If we imagine placing all points which *can* be mapped onto one another — which are "homologous" — into a bucket, then we end up with three different buckets, and two points from different buckets can *not* be mapped onto each other. The topological structure of the annulus is revealed by similar homology considerations but here concerning loops, rather than points. In this case there is only one component (the annulus is connected), so all points are homologous to each other. However, loops around the inner hole are not homologous to loops which fail to encircle the hole. Any time one loop fails to be homologous to another loop, this failure has one single explanation (one of the loops goes around the hole and one does not). This single explanation corresponds to the presence of one hole. If there were more holes, there would be more possible explanations (of why two loops are not homologous). Therefore homology between loops, which is an internal property of topological spaces, conveys information such as the presence of holes, even though holes are not part of the space (and if the space is a universe unto itself, do not actually "exist"). Homology in different dimensions therefore conveys structural information about a topology — homology of points counts how many components a two-dimensional manifold has, and homology of loops counts how many holes it has (more precisely *failures* of homology, situations where *simplexes* — constructive objects which in the zero-dimensional case are sets of points, and in the one-dimensional case are homeomorphic to loops — are *not* homologous).

Moreover, this kind of topological information is certainly also relevant to mereological structure, since the presence of holes or of disjoint parts obviously convey properties of how something is divided, or how some larger whole to which it belongs is divided. Of course, not all mereological structure can be "read off" from topological properties in this way. For example, parts may be unified into wholes according to systems of organization which have patterns or symmetries that cannot be directly read off from topological structure. However, some of these details can indeed be captured by topological analysis if we subject spaces to different kinds of analysis and variation. Consider a checkerboard pattern, and as above we can consider the feature space involving variation in both spatiation and color space. Once again we have just two colors, so we have two disjoint regions, one all of whose points share one color (say white) and the other with a different color point (say black). However, this checkerboard space has additional symmetries. In general, a symmetry on a space means that parts of the space can be mapped to other parts by symmetric transformations, transformations which collectively form a group. In other words, points in the space can be derived from points in a smaller "kernel" space by applying one from a group of possible transformations. In this checkerboard example (suppose we actually have a chessboard, and we can borrow the **a-g** and **1-8** notation from chess), all white-colored squares are mapped from one single square (say **a1**) by spatial translations, reflecting the spatial symmetries of the checkerboard. Specifically, each of these symmetries correspond to displacements in the x and y (horizontal and vertical) directions by even integral units, relative to the size of the edges of the square. If we set this size as unit one, then all points in the checkerboard can be derived from points in the unit square by equations of the form $(\star)$ $(a, b, x, y + k, C_k)$, with $0 < a, b < 1, x, y$ even integers, $k = 0$ or 1, and C_0, C_1 being two color values. The $k = 1$ case maps a black square (**a2**) to the other black squares; the color of each square is uniquely determined by whether its displacement from the **a1** in the two directions is two even or odd integers or one even and one odd, which is equivalent to mapping all squares from either **a1** or **a2** by even-number displacements. The set of points satisfying $(\star)$ provides an alternative representation of the points in the feature-space, one which highlights the partition of this space via its symmetries.

This alternative representation does not mean that the original space is topologically equivalent to $(\star)$ if $(\star)$ is considered as a topological space in its own right where the displacements x and y are extra dimensions. In the latter case, instead of these numbers mapping points in two dimensions to other points in the same plane, they would also "elevate" the points in two further dimensions, creating a subset of $\mathbb{R}^4$. Each square would then be disconnected from each other square. In the normal planar representation of a checkerboard, we certainly observe a partition into distinct squares, and these two squares are separate in the sense that their *interiors* do not overlap. On the other hand, some squares "contact" other squares in that their borders *do* overlap. There are different ways that things can contact each other: one being inside another, interior overlap, border-contact, or none at all. If we define these ideas in terms of topological criteria of interior and boundaries, then these ideas combine topological and mereological notions and become distinct "mereotopological" relations. Certainly the kinds of contact which parts have with each other is part of the mereological structure of a whole. This contact-information can be retrieved from the $(\star)$ representation, even if we treat $(\star)$ topologically as a subspace of $\mathbb{R}^4$, by modifying $(\star)$ so that boundaries of squares are fully represented (in $(\star)$ only their lower and left edges are represented). For this modification, allow a and b in $(\star)$ to be exactly one (rather than < 1). To recapture the planar topology (and the contacts between squares), we have to carefully define topological identification ("gluings") between some edges of squares; for example, the upper-left corner of **a1** coincides with the lower-right corner of **b2**, so the point (in $(\star)$ representation) $(1, 1, 0, 0, C_0)$ coincides with the points $(0, 0, 1, 1, C_0)$, and also $(1, 0, 1, 0, C_1)$ and $(0, 1, 0, 1, C_1)$. The latter cases show where topological identification is gluing points of different colors. Other points, along the edges but not the corners of squares, also glue points which otherwise would have different color values.

Given a set with an initial presentation p, "gluing" creates a new set where certain elements of the original set, which are distinct according to p, are treated in the new set as different labels for the same element. Many complex topological spaces, which would be difficult to visualize or analyze otherwise, are defined with such gluing operations. In the current example, the the effect of the indicated gluing scheme recaptures the connectivity of the original space, but it raises questions about how features around the borders of recognizable parts of a whole are to be understood. Does the border between **a1** and **a2** belong to **a1**, **a2**, both, or neither? Is it colored white, black, both, or neither? How we construct the feature-space which combines featural and spatial information is influenced by how we choose to represent these border-conditions. If we "glue" distinct points — points which may geometrically coincide but are separated by the featural dimensions — this seems to suggest that these points exhibit both featural variations, e.g., there are points on the checkerboard which are both black and white. This interpretation is the one which fits most comfortably with the mathematical framework of topological identifications. Other interpretations, perhaps more appealing from a philosophical point of view, could perhaps be given a mathematical expression as well — for example constructing a featural space $\mathcal{F}$ so that the property of being on the border between black and white is a different feature than being either black or white alone.[30] Under this interpretation, $\mathcal{F}$ as a collection of "hue points" can be assigned a topology such that any open neighborhood of feature-spaces using $\mathcal{F}$ (for example by combining $\mathcal{F}$ with a manifold) includes both black and white points. On the other hand, while the borders between apparent (i.e., clearly individuated) parts and their contact-relations are part of mereological structure, considering interiors of parts rather than their borders better focusses on the partition of a whole into a set of parts to begin with. In the current example, the space ($\star$), without the identifications, cleanly models this mereological division.

Other sorts of mereological breakdowns do not involve such cleanly articulated borders. Suppose a feature space **F** models a coloration with vertical stripes which blend into each other via a gradient between two colors. As part of the modelling paradigm we can assume that **F** has a featural component $\mathcal{F}$ and a geometrically axiated manifold component $\mathfrak{G}$. Each point **F** is associated with one $\mathcal{F}$-point and one $\mathfrak{G}$-point (so **F** is contained in the product-space $\mathcal{F} \times \mathfrak{G}$). Let's call a path in **F** *horizontal* if its restriction to $\mathfrak{G}$ is horizontal. The featural component of these paths will oscillate between two color points, forming closed loops if they are long enough. This repetition in the featural distribution is an obvious perceptual detail of the surface in question and influences how we tend to divide it into "stripes"; however, there are no obvious candidates for boundaries between distinct individual parts. Speaking informally, insofar as the stripes blend into each other we certainly seem to think that they are *there*, but we have no obvious criterion to say what they actually *are*, that is, what region is in fact a stripe. As with the checkerboard example, we can relabel points in **F** so as to highlight symmetries. **F** exhibits horizontal symmetry in integral increments and also continuous vertical symmetry. Along horizontal lines, the color pattern loops between two color values. So we can identify a kernel space of the form (a, C_a), choosing units so that $0 \leq a < 1$; if the color gradient is regular then we further stipulate that $C_a = C_{1-a}$. The full space is derived from this kernel by a vertical translation by some value $b < h$, where h is the height of the original planar

[30] In terms of "sorts", this implies that given a sort of color-concepts we construct a supersort whose additional elements are pairs (or more generally tuples) representing border regions between their respective colors. This operation is (loosely) similar to designing supersorts for programmatic situations like thrown exceptions, empty collections, etc. ([13, p. ?]])

figure relative to the selected units, followed by a horizontal translation x an integer. $\mathbf{F}$ then corresponds with a set of tuples ($\circledast$) (a, C_a, b, x), taking the above restrictions on a, b, and x. If we consider the parameter x as a factor in continuity between points in this space, then x induces a "jump" around points of the form $a = 0$, which is consistent with the idea that $\mathbf{F}$ has different "stripes". However, actually "cutting" $\mathbf{F}$ along these lines is rather arbitrary, and appears as an artifact of the mathematical expression of $\mathbf{F}$. Indeed, it makes more sense perceptually to consider the color-points between which the hues cycle as centers of stripes, rather than their boundaries.

In the checkerboard example, representing points in the feature space using a kernel space and symmetries also caused apparent discontinuities, but in that case these jumps had an intuitive perceptual meaning: they were boundaries between distinct squares. There are no similarly precise boundaries between the current stripes. To the degree that we wanted to recapture the connectivity of the original checkerboard-case, we had the choice before of just ignoring the displacement values and considering them irrelevant for tests of continuity, or else modifying the tuple representation ($\star$) to include borders and topologically identifying some of the resulting points. In the current "stripes" case, the discontinuities are manifest in the fact that horizontal lines in ($\circledast$) jump from (a, b, C_a, x) to $(0, b, C_a, x + 1)$ as a approaches and then crosses the line $a = 1$. To "suture" these jumps, we can modify ($\circledast$) so that a can be 1 (and mandating $C_1 = C_0$), then topologically identifying points of the form $(1, b, C_1, x)$ with $(0, b, C_0, x+1)$. Alternatively, we can ignore the x's and stipulate that the kernel space is not an interval [0, 1] but rather a circle, so that the region (a, b, C_a, x) for a given x is topologically a cylinder. On this interpretation, a line in $\mathbf{F}$ winds around the cynlinder n times, and $\mathbf{F}$ itself is like a cynlinder whose surface is laid down on a plane muliple times, the way that paint on a cynlindrical brush is applied by rolling the brush. Here we identify $(1, b, C_0, x)$ with $(0, b, C_0, x+1)$ but also with $(0, b, C_0, x)$, and in the kernel (a, C_a) we identify $(0, C_0)$ with $(1, C_1)$. The circularity of this space captures the fact that the original $\mathbf{F}$ has a repeating featural pattern, but also the fact that there is no obvious point at which to divide regions created by this pattern.

Notice that the fundamental organizing principle of the checkerboard case was a pair of two disjoint color-points: the ($\star$) tuples map these two color-points onto two unit squares, then displace these squares via symmetry to create a planar region. The foundation of the pattern is therefore a zero-dimensional set of just two points, which in homological terms would be called a "0-simplex". In the "stripes" case, the foundation of the featural pattern is a loop in color-space, or a path in color-space between two colors which becomes a closed loop in a restricted feature-space kernel when combined with a geometric circle, given that the space (a, C_a) under $(0, C_0)\#(1, C_1)$ is topologically a circle. Here the pattern is based on a one-simplex, so where the checkerboard pattern was so-to-speak zero-dimensional, based on a zero-simplex, we can say that the "stripes" pattern is one-dimensional. The dimensions here are not spatial dimensions in the sense of three-dimensional objects with two-dimensional surfaces, but are rather dimensions of topological spaces isolated from compound feature-spaces, spaces which include both featural and geometric variation, after these spaces are reconstructed so as to model their patterns as symmetric patterns derived from some kernel space and a group of symmetry-preserving transformations. In terms of "sorts", the checkerboard case defined a map $m : s \rightarrow \mathbf{F}$ from a sort s

such that the preimage of m was topologically a zero-simplex; that is, its model-space is a set of discrete (here two) points. In the stripes case, the equivalent preimage (model-space) is topologically a circle.

The idea that the "stripes" pattern is one-dimensional is not just a product of its geometric description — if the pattern were monochrome stipes of alternating color it would instead be zero-dimensional, like the checkerboard. Instead, the two-dimensional nature of the pattern is due to the smoothly changing color-gradient. On the other hand, the nature of the manifold aspect of these patterns also contributes to these measures of dimension: if the patterns did not have their symmetries, it may not be possible to construct kernel spaces within the feature-spaces which represent the pattern in isolation from the overall space. For example, if a surface were colored with monochrome patches in just two colors, but these patches intersected and varied with no apparent order, then it woold be impossible to describe the pattern without simply defining the borders of the color-regions geometrically — in other words, without stipulating these borders as curves on the manifold, or 1-simplexes, since they cannot be automatically generated by a symmetry group. The 1-simplexes in the "stripes" case combined a manifold dimension with a featural dimension ($a \mapsto C_a$), wheras those in this hypothetical case of random color-patches would require two manifold dimensions. This suggests that when assessing "complexity" of meroelogical structures we should consider featural and spatial variation together, because nuances in patterns can come from featural or geometric variation or the combination of the two. In addition, the clear boundaries of the squares in the checkerboard case can be demonstrated by the fact that its underlying pattern, if this analysis is correct, is "0-dimensoional"; the kind of pattern suggested by the "stripes" cases, where there is a clear sense of mereological division but not of part-boundaries, is more complex because its underlying pattern is "1-dimensional" in this technical sense. The crisp borders in the zero-dimensional case seem to be just a special case of more nuanced patterns in higher dimensions: the "shrinking" of dimensions down to zero has the effect of eliminating boundary ambiguity and reducing all parts of the surface to crisp individuals.

The one-simplexes referred to earlier in mapping $a \to C_a$, or defining paths around "random" patches, as well as tuple systems like ($\star$) and ($\circledast$), are examples of what I earlier suggested as "equational systems" $\mathcal{Q}_m$ on mereons m. If a mereological system models three-dimensional objects, then these types of equations help define structures on these objects, particularly (within the kind of examples discussed here) on their surfaces. Of course, mereological structures within the surface of three-dimensional objects is only one aspect of mereology in real-world, three-dimensional cases; the more common part/whole relation in these three-dimensional situations is the relation of physical objects to their component parts or larger aggregates (rather than structures on their surfaces). On the other hand, seeing objects from afar, we assess these three-dimensional connections by making judgments based on how surfaces appear to intersect. We see overlaps between *surfaces* of objects, and only rarely see objects overlapping each other in full three dimensions, like a handle partially embedded in a clear-glass mug. Our visual perception involves a collection of surface-structure components, which we provisionally group together in terms of which "patch" (not meaning a monochrome color patch, but some region of coloration and other featural variation with some degree of individuality as a two-dimensional space) belongs to which perceived object. We therefore need to account for surface mereology because we need to identify when structures are partitions of a single surface due to its coloration, as opposed to the visual juxtaposition of two different surfaces from two different objects, or the physical proximity of two surfaces of distinct objects in physical contact. A mereological system $\mathfrak{M}$ modelling these three-dimensional cases may or may not model surface patches, but even if two-dimensional regions are considered among the set of mereons, it is not clear how every single structural detail would be included in $\mathfrak{M}$. For example, how would the "stripes" case be handled? Equational systems like ($\star$) and ($\circledast$) demonstrate how the structuration of mereons can be described in greater precision even if structural features are not represented, or not represented in full detail, by the presence of other mereons in $\mathfrak{M}$ which are their parts.

In visual experience we perceive objects in terms of their visible surfaces, but we do not experience surfaces as depth-less fusions of pathes of coloration, like collages. Apparently this is due in part to "stereographic" projection, but not entirely, because even with one eye closed we experience depth in the visual contents seen with the other eye; it is not as if closing one eye magically turns our vision into a kaleidoscope of flat colors. Whatever mental processing occurs subconsciously, our visual experience contains implicit judgments about three-dimensional form. On the other hand, we do not experience three dimensional objects in all three dimensions fully; we experience surfaces with depth in the sense that they are receding from us at different rates, but still relative to a specific vantage point. The "manifolds" of visual experience are not two-dimensionally flat, but nor are they fully three-dimensionally detailed. They are not merely two dimensional projections of three-dimensional depth given a specific vantage-point, because for example a perfectly realistic photo of a scene, which exactly records this projectional data, still does not seem to us as identical to the scene itself. The manifolds of visual experience appear as a kind of experiential hybrid between two and three dimensions, which may be difficult to describe technically but which seems to be an undeniable aspeect of visual reality. Perhaps we cannot explain "what it is like" to have this kind of visual manifold somewhere between two and three dimensions, just as we cannot guess what it is like to have a bat's sonar (maybe it's a kind of auditory version of this visual 2-to-3 dimensional hybrid?). But the fact that this is how visual content appears to us seems like an undeniable aspect of perception.

The structure of this visual manifold is not a purely "subjective" structure (like imaginary scenes played out in my mind), but nor is it a purely "objective" structure, like the full three-dimensional geometry of a collection of objects. We can say that the "language" of visual perception is really three separate languages, with the conceptual and experiential details of the perception of surfaces being a "core" language which is extended in different ways. The core structures of visual perception are not *only* surfaces, because we sometimes have direct experience of features with three-dimensional depth (like fog or dust), but these occasions are exceptional. Most visual experience is the experience of surfaces, including sometimes the experience of some surfaces as transparent to some degree and therefore allowing vision of other surfaces behind them. So when considering the "language" of visual perception we can focus specifically on language of visually experienced manifolds, which are experienced as and believed to be syntheses of impressions of multiple object-surfaces. This core language can then give rise to two other "languages": on the one hand, I can focus attention on these visual details as specifically *personal* experiences, with their own subjective characteristics. This can involve my recognizing certain experiential details as *uniquely* subjective, somehow departing from what I guess to be "normal" perception in equivalent circumstances (for example if I acknowledge that my vision is distorted by sunglasses). It can also involve my focusing attention on subjective elements on visual experience rather than on judgments engendered by them. For example, I direct attention to the unique qualities of a certain shade of color (featured in a painting, for example), or the pleasure of contemplating an elaborate architectural pattern. Here I move from the core language of visual perception (mostly the perception of surfaces, though occasionally of perceiving features like coloration and lighting in three-dimensional regions), in a direction which foregrounds their subjective dimension. On the other hand, there is a different "language" which proceeds from visual perception to a cognitive model of three-dimensional objects, which we believe to exist and have properties independent of our subjective experience of them. We may not have full-fledged perceptual *presentations* of these cognitive models, but they influence our physical interactions with objects, our judments as to their nature and categories, our judgments about other people's beliefs about them, etc. For example, we may not fully "see" an object in three dimensions, but if I reach to grasp it, the mechanics of all the movements in my hand, arms, and torso when carrying out that act are subconsciously guided by a mental model of the objects' depth and geometry.

So the language of visual perception joins a more "subjective" language of visual experience as "private" — as containing distortions due to my own unique circumstances which need to be corrected for, but which also bring affective qualities to experience that can have private significance separate and apart from whatever information they convey — to a more "objective" language of three-dimensional objects in inpersonal and interpersonal physical space, with depth and geometry which I reconstruct (largely subconsciously) from surface-perceptions. My visual experience, in which the direct presentation takes the form of surface-manifolds, is accompanied in my conscious experience with beliefs about three-dimension form, beliefs with their own "language" and which are not sensorially present as immediate conscious contents, but are latent in all my physical interactions with and judgments about objects. So the core language, the language whose primary concerns are manifolds of visual experience which disclose objects' surface-form, this language is a kind of intersection or juncture between a more "subjective" and another more "objective" language. The subjective and objective are linked by a language and domain which is neither purely subjective, nor purely objcetive, but a combination of the two. We have to be careful not to misconstrue the nature of this combination and the connections of these three languages: it is not as if the subjective is a "copy" of the objective, that there is an object over there in physical space and another object in here in my mind. We cannot metaphysically separate the object-out-there and the object-as-experienced so as to say that one is real and the other some kind of ontological illusion — whether as an "Idealist" rejection of the object "in itself" or a reductionist problemmatization of talk about conscious experiences. This is not to say however that there is no distinction at all to be made between the object-out-there and the object-as-experienced, so the question is how best to phrase this distinction when exploring it philosophically. I will next consider this paradigmatic balance from a perspective based on Phenomenology.

8 From Perception to Situations

The "languages" of the subjective and objective are bridged, in phenomenology, through the discourse of "intentionality", by talking about conscious experience being "directed at" real objects. The object I experience is not a mental *copy* of the real object because I intentionally "target" the real object. I can indeed "bracket" this real object by philosophically refusing to simply presume that the object is there. But even if I agree to remain skeptical of the objects' reality as a philosophical maxim, I do not thereby abandon belief in the object if I remain true to my conscious experience. The belief that the object is actually *there* — not only seen but physically present, so that I will feel it if I touch it, others see it also, etc. — this belief is just as consciously real as the direct perceptual visual sensation of the object (or at last part of its surface), even if the belief itself, my awareness that I *have* this belief in the object's reality, is not *sensuously* present in the same way. This can be demonstrated through the obvious experiential difference between seeing objects and merely imagining them. The experience of objects believed to be real is an experience of objects situated in a surrounding space, a space in which I operate visually and kinaesthetically. The fact that I experience this space as a domain where I move and am myself situated expresses how this space is experienced as *objective*, but my perceptions of this space are through a conscious medium which includes some subjective details and distortions. Visual experience (and perceptual experience in general) becomes conscious not only as an aggregate of sense-data, but as sensed contents accompanied and intertwined with cognitive judgments, and part of the structure of these judgments is the disentangling of subjective affects and distortions from beliefs about the placement and geometry of objects in our environing physical space.

So we should consider the objective and subjective as two distinct aspects which are co-presented in any moment of consciousness, and the visual perception of surfaces is the most substantial point of contact and overlap between these aspects or "languages" — along with visualization of features in space as a whole, like fog, or perceptions through other senses like hearing and touch. For analyzing this intersection and cognitive integration of objective and subjective it therefore makes sense to focus on surface-visualization as a theoretical core, and then follow this analysis in the distinct subjective and objective directions, the directions of subjective affects and meta-judgments and of mental models of objects' geometry which are experienced to be objective even if necessarily imperfect (and we need them to be reasonably accurate). Other directions of analysis can attend to preception through other senses, and to visual experience apart from rea the perceptions of surfaces. For example, tactile sensations are sensations of surfaces through touch rather than vision, while visual experience of ambient lighting, including both the play of light and the effects of media which can distort light through three-dimensional extensions rather than surfaces — contrast the effects of a semi-transparent glass *surface* with a region of fog or dust that impedes light in similar ways, but over a three-dimensional area. Moreover, sensations like touch are localized on surfaces (like vision most of the time), whereas sensations like scent and hearing are experienced relative to ambient space, not through two-dimensional surfaces. Sound has a direction, like light, and we may sometimes experience sound coming from a specific source, but sound does does fill out a surface as does touch or vision. So vision and touch are fundamentally surface-oriented, whereas sound and odor are not; we might say that they are “ambient”, structured according to three-dimensional volume (however imperfectly experienced) rather than two-dimensional surface. Vision and touch have ambient dimensions also; there is both a visual and tactile sense of walking through a cold fog. Moreover, tactile/kinaesthetic sense of my body combines ambient and surface aspects; I experience some senations as localized in my body as a corporeal surface (such as sensations of muscular tightness of tension), but these sensations are distributed through an ambient space of corporal awareness which seems to occupy the space immediately around me. I experience my arms as extending out into space around me, for example, but I do not correlate surface-sensations on my arms with ambient spatial locations with the same precision as I do visual surfacs. For example, if I feel a fly moving along my hands I have some sense of this sensation occupying both a surface area and also a spatial location, but I instinctively *look* at my hand to properly fuse these presentations together.

From a core analysis of visual surface-perception, then, our cognitive or phenomenological investigations can proceed in different directions: from experience of surface to experience of ambient areas; from visual to multisensory perceptions; from surfaces as the mode through which apparently real ambient objects are disclosed to my consciousness to the more “subjective” affective and meta-epistemic aspects of experience, with their affective qualities and believed distortions. Here however I want to focus on the direction which leads from visual surface-perception to cognitive judments and mental models of objects in ambient space, with their geometries and placements relative to each other. In forming these mental models I hypothoesize spatial geometry and “colocation” (how objects are situared relative to each other) from the evidence latent in my direct visual surface-perception. Let me reiterate that as “mental” models these are not directly *visualized* models, the way that for example my imagining the map of North America provides me with some model of North America’s geography. These are more *latent* mental models, implicit in my physical movements and interactions with objects as well as my beliefs about other peoples’ beliefs insofar as we share the same spatial locales. Whereas visual or sensory perception operates on contents that have explicit presence in my consciousness, these latent mental models move beyond moment-to-moment perceptual awareness and repersent a mental “tracking” of entities in our environment. They remain perceptually present, as stored in recent memory or what Husserl calls “retention”, even if they cease to be perceptually explicit in a given moment. In this “horizon”, the relational structures between concepts feature more prominently than the precise details of their bearers in each moment. As a result, this transition from explicit perception to implicit background-beliefs corresponds to a transition between perceptual structures with intuitive mereotopological models, to conceptual networks consistent with CGS representations. I will explore this transition in greater detail by focusing on mental models of our immediate environment as a mereologically organized system.

As a mereological system, my experienced visual manifolds present a mosaic of visual features, which have apparent part/whole and aggregative structures. Some of these structures are due to featural details within discrete (real world) surfaces, and others are accidents of my line of vision and how objects are colocated. If I focus just on visual experience “as experienced”, with its particular structures of continuity and separation, then I have a sense of visual features which aggreage and separate from each other to varying degrees. This is a weighted mereological system, a system whose "contents" or *mereons* are subjective, experienced visual manifolds and their parts. These contents are not *purely* subjective, like imaginary worlds[31]. Perceived surface geometry *as perceived* is not a replica of actual object surfaces in three dimensions, but it is *derived from* this geometry and not subject to my own will or imaginations: I cannot control the shapes through which objects appear to me, which is why I can consider my visual experience as conveying objective information. The mereological structure of visual impressions as conscious experiences is not the same as the mereological structure of three-dimensional objects themselves, but the latter mereology can be recovered or inferred from the former through acts of judgment. We can debate the mereological structure of the "mereons" in the former system — in other words how exactly we classify their existing, since they are neither mere two-dimensional projections of real surface nor mere mental contents — but these ontological ambiguities should not distort how the former mereological system *as a system* is a representation of the latter one and a means for the mereology of real-world ambient objects to be disclosed to us experientially. From the former system we derive beliefs about how mereons in the latter system, the real-world objects and their surfaces, are connected, separated, and interrelated.

So in retracing this cognitive progression from the mereology of visual experience *as impressions* to the mereology of real objects *as presented in mental models* of our environing locales, we need to explore how the kinds of part-to-part and part-to-whole relationships in the former mereology serve as evidence for or representations of analogous mereological connections in the latter system. The play of visual angles can create the illusion of objects being connected which are not, so we have to account for this possibility through our estimations of visual depth. Leaving this aside, we still have several different possibilities for how the mereology of visual impressions is cognitively translated to the believed mereology of ambient objects. Mereological patterns can reflect surface patterns *within* one object, such as coloration on its surface (or occasional effects of partly seeing the interior of objects with semi-transparent surfaces). On the other hand, mereological patterns can reflect several varieties of contact between discrete or semi-individuated objects: one object can rest on another (provisionally connected by force of gravity); two objects can touch while both rest on a third; one can be attached to the other, etc. These are new genres of mereological connections which become relevent as our investigative attention turns to objects in space. Abstractly these are various sorts of part/whole and coupling relations, but we now have a whole inventory of such relations, suggesting the need for new formalizations to model them. In particular, CGS is a natural framework for modeling mereological systems because mereons can be treated as concept-nodes and their different forms of part/whole and coupling relations as different relation-types. A *Conceptual Graph Mereology* can according be developed with a support whose relationship-type set $\mathcal{R}$ includes multiple "parthood" relations $\lessdot_\rho \sqsubset \lessdot$ and $\odot_\gamma \sqsubset \odot$, where $\lessdot$ and $\odot$ are now more abstract relations in $\mathcal{R}$ which have these more concrete refinements.

[31] Though even imaginary scenes are perhaps not fully subjective either insofar as being imaginary "scenes", imaginary accounts of alternative physical realities, they are constrained to have some features in common with scenes experienced as real — for example, it is hard or impossible to directly imagine a *four-dimensional* reality, at least without some mental trick which merely embellishes what is truly a three-dimensional imaginary world, such as by visualizing hypothetical gradient lines of spacetime curvature.

In the first half of this paper I used CGS to model relational structures between concept-tokens, and discussed modal logic and topological notions to characterize model-spaces for concept-types. My argumentation here is similar: mereotopological structures, which can characterize the internal structuration of particular real-world objects and their surfaces, is defined as a topological synthesis of concept-types with continuously varying model-spaces, like coloration and spatiation. This corresponds to model-spaces of individual concept-tokens, though in lieu of modal variation across different interpretations or perspectives (or counterfactuals or "possible worlds"), this variation occurs in the blended space combining the two dimensions, that is, "feature space". This variation is internal to each concept-token. It is then incorporated into a relational structure between these tokens, and in the current context the relations under consideration are also mereological (including coupling relations). In a more general semantic context, mereological relations would still be important (as suggested earlier, notions of how entities in some domains combine or couple with each other, as the UN or US cases illustrate, are fundamental parts of their meanings, or their proper Symbolic frames); but these would be supplemented with other relation-types, including "situational" or situation-specific relations, which I will briefly mention below. The larger picture, then, is topological variation as *modal* or *featural* variation *within* concept-tokens, inducing one type of mereological structure, and then mereological relations *between* tokens. This correlates with a perceptual and cognitive shift from the individual perceived object to environing contexts and situations.

While tending to focus attention on attachments and aggregations between objects, a turn toward the mereology of three-dimensional ambient objects also has consequences for our picture of the structuration of individual objects. We do not only *see* objects, but interact with them, and their mereological form accordingly reflects different functional patterns or partitions as well as visual ones. Consider, for example, the shape of a dumbell, which we grasp around the narrower center while the thicker outer parts provide bulk. A dumbell has two functions — it should be relatively heavy, so as to provide resistance when we try to lift it, but it should be relatively easy to grasp. The outer bulges provide heft, while the inner shaft provides graspability. This suggests a mereological partition of the object, whose parts, though not crisply individuated, are separated through functional differences. It is true that a related partition is suggested by visual form, but the mereology of the visual shape is reinforced by our typical interaction with the obejct. This example shows how operational and functional features can be merged with spatiation and coloration as featural patterns. Earlier, I explored how spatial form and coloration operate in consort to create a more or less precise sense of parthood. Here we see a similar juxtaposition of featural distribution, but the features in question can be functional as well as sensory. For example, we can consider a mereology of the dumbell by defining features of "bulk" and "graspability" as distributed through the object. Points around the center are assigned greater values in the graspability dimension, and points around the thicker outer parts have greater "bulk" values. Since these two dimensions are roughly inversely proportional, we can envision points where bulk-lines and graspability-lines cross, and these points imply a partition of the object on functional grounds.

Of course, a similar functional organization can be seen in objects which are more clearly aggregates of discrete parts. Consider for example a different type of dumbell in which weights are attached to a central shaft. This object has a similar functional organization to the "one-piece" dumbell, but is more obviously an aggregate of three individual parts. These part have greater individual coherence, and their manner of physical connection to the whole is different, since it is possible to detach them; they have looser coupling to the whole both visually and functionally. Again we associate the parts with distinct functionality of bulk and graspability, but instead of these functions providing parameters of featural variation *within one* object, they are functional attributes attributed to crisply individual parts, the whole being an aggregate of parts synthesized precisely so as to implement a certain functional structure. The dumbell is both heavy and graspable because it combines parts which are heavy and graspable in turn, and in such a manner that the aggregate whole has both functions.

As our mental models of ambient objects extend beyond visual features such as spatiation and coloration, to include these kinds of functional features, we also go beyond immediate visual evidence to incorporate judgments concerning objects' types, concerning how I may interact with them, concerning anticipations of possible movements and interactions, etc. We are going beyond the realm of sensory evidence into a more situational cognition which depends on a prior inventory of beliefs and concepts, recognizing the specific shape of the object before me as "dumbell", that there are such thigs as dumbells in the world, that this is a *concept*. If I had no acquaintance with dumbells I could probably figure out what this concept involved first from the visual appearance, then tentatively lifting the object in question, like a collector examining an unidentified tchotske in an antique store. Indeed, perhaps a paleolithic fitness guru starting collecting stones with a dumbell-like shape to use as weights, and began to mentally refer to stones meeting this criteria as instances of a similar concept. By analogy, for us a "walking stick" may be something deliberately fashioned or else just a useful branch found in the woods. Insofar as a natural artifact is selected for a purpose, it conforms to a concept because only by virtue of meeting associated criteria does it call our attention; insofar as something is deliberately formed for a purpose, it is shaped according to these criteria. If a paleolithic exerciser accidentally discovers that a certain rock is good for weight-lifting, but then starts to collect similar rocks, she has effectively brought a new concept into being, since her stones do not just have vaguely similar shapes; they are similar *because they are all dumbells.* Concepts may be criteria for selecting objects using one particular dimension of similarity, or guides for constructing objects so that the end products meet these criteria (typically functional criteria) of similarity. We can say that someone who starts collecting dumbell-shaped stones for exercise, once having discovered a rock with that kind of shape which can serve that purpose, has *invented* the concept (or reinvented it); most people instead *learn* the concept; and someone who discovers a prefabricated dumbell, realizes that it was designed for some purpose and figures out what it was used for, has *discovered* the concept. Whether we invent, learn, or discover a concept, subsequently it serves as proxy for this initial exploratory process. With the concept *dumbell* in our arsenal, we are predisposed to recognize dumbell-like things as dumbell-tokens, which also predisposes our estimation of their three-dimensional geometry and mereology.

The "dumbell" concept in this example combines some notion of spatial form with a notion of functionality, though these are interrelated; it is because of the spatial form that an object can be both bulky and graspable, since a central narrow shaft provides a graspable area in an otherwise bulky object. For the type of dumbell which is one integrated whole object, but shaped so that there is an imprecise division of a central shaft from outer bulges, the functional differentiation between bulk and graspability provides a partial separation within an obviously singular whole. The superposition of these two functional criteria yields a mereological structure in which several parts are identifiable but with imprecise boundaries and limited individual coherence. Conversely, for the type of dumbell which is built from three freestanding pieces, these same functional features are joined to component parts prior to their synthesis into the whole. Functional criteria do not separate the whole into parts, but rather one part is selected because it meets one of these criteria, two other parts selected to meet the second criterion, and these three parts attached together. So functional criteria become criteria of synthesis rather than criteria of separation. Nevertheless, the end result in both cases are objects with similar functional organization and similar mereological structure, although with varying degrees of individual coherence for the parts, and also for the whole, insofar as in the latter case the whole can be "disassembled".

Mereological distinction between objects which are attached to other objects, which can be diassessmbled and reassembled, etc., are suggested but not wholly disclosed by the shape and position of their respective surfaces. Contact between surfaces can mean that objects are physically attached (with varying degrees of force) or merely touching, and it can also mean that the "objects" are imprecise parts of some larger whole more than they are individually coherent distinct mereons. We have to cognitively translate apparent surface geometry into this three-dimensional physical mereology, but in so doing we are guided not only by geometric considerations alone, but by conceptual and categorial considerations, as well as our understanding of physical relations. If two objects are in contact and if lifting one results in lifting the other, we reason that almost certainly they are materially attached (though occasionally, as in the case of static electricity, this attachment can be accidental and temporary). An object hanging from another object is probably attached to it, in contrast to an object *resting on* another. These physical considerations supplement our appraisal of apparent surface-relations. The degree to which objects or object-aggregates conform to apparent concepts like *dumbell* provides a much larger arsenal for interpreting visual forms. In order to propely recognize the geometry of objects, and their classification in terms of some relevant conceptual framework (including the general framework of all concepts we have available to apply to typical objects in our day-to-day "lifeworld"), we need to combine together evidence from visual manifold appearances, estimations of physical contact and connectedness, and conceptual classification.

If we consider the result of these subconscious deliberations to be canonical cognitive units — propositional molecules or "belief molecules" which have inner structure, but are basic units of synthesis into more complex thoughts and propositional attitudes, the kinds of thoughts which are minimally useful for real-life mental activity as it guides our actions in the world — the basic structure of these "molecules" involves cognitively individuating an ambient object on the basis of apparent visual surfaces, extending and completing these surfaces in a mental schema so we have some notion of how the object is closed off, via its surface, from its surroundings even if we do not actully see the objects from all sides. Moreover, these individuals are then represented as mereons in a system whose basic mereological structures are physical contact, attachment, connection, and aggregation, which (as I discussed) is a different kind of mereological system then the system of experienced visual manifolds. Finally, the cognitively grasped individuals are recognized as tokens of concept-types, albeit with such occasional gaps as the mysterious item in an antique shop (though here more generic concepts come into play, like "tool" or just "thing"). Often the complete form of these cognitive molecules has then a summarial object/predicate form — *"this" is a dumbell*, where the referent of "this" is an individuated object defined by a spatial geometry which, in a separate mental act, I infer from the appearance of its surface from some angle. The two different cognitive operations — forming a model of the geometry and the classifcation *as dumbell* — these operations are interconnected, just as forming an image of coloration and spatiation is interconnected, with apparent coloration influencing our sense of spatiation and vice-versa. For example, if I see the dumbell from one side but have reason to believe it is a dumbell, I will anticipate the presence of a central narrowing even if I do not see this. So classification and estimation of three-dimensional shape are interrelated, though they are logically separate because they involve two different kinds of cognitive activity, one forming a schematic sense of an object's geometry and another classifying an object as tokening some concept-type.

Insofar as the union of these operations is a cognitive unit of the form *this is (a)* $\underline{p}$, the resulting "belief molecule" has a familiar form, in the world of Conceptual Graph Semantics, of two concepts connected by a relation of general predication or property-instantiation. One concept may then be "this", this object, this perceived thing, etc., represented linguistically or mentally by a gestural reference like pointing, though we can imagine scenarios where the referent is semantically more like a proper name — suppose an antique dumbell is listed in the catalog for an auction, with an accomanying label like "Item 10-A". The other concept is the generic concept *dumbell*, or more properly the concept of being a token of this concept-type, being *a* dumbell. The double-edge connecting these concept-nodes via the relation of general predication would be a graph representation of statements like "this thing is a dumbell" or "Item 10-A is a dumbell". Assuming we use concept-graphs Γ as before to represent mereological relations between ambient objects (relations like being-attached-to, etc.), the existence of these more categorial double-edges like *this is a dumbell* represents a vast expansion of the expressive capabilities of Γ. The CGS "support" has been extended to include a large family of concepts representing kinds of three-dimensional objects, and a predicate relation to convey the notion that $\underline{x}$ *is a* $\underline{p}$. These Γ's then no longer model a mereological syatem $\mathfrak{M}$ on its own, but rather an immersion of $\mathfrak{M}$ systems in predicational and classificatory networks. This opens the possibility of seeing how relation-types deriving from the *mereological component* of the resulting CGS framework interact with relation-types deriving from the *predicative/classificatory component*. In other words, we have a system of graphs Γ sharing a support whose relation-types include two abstract relations of *being physically connected to*, quite generally, and of *being classified as*, or being recognized as, being an example of, etc. The former generic relation is refined as *being in contact with*, *being attached to*, *being inside*, etc.; in other words, this is the base relation for relation-nodes for those double-edges in graphs which derive from the meroelogical "parts" of Γ — we might say the edges or subgraphs involving these relations have meroelogical *valence*. Other Γ subgraphs involve relations which are derived from the abstract predicative relation of being an example of, being classified as, with different modifications: a painting is *believed to be* a Rembrandt; a number is *calculated to be* prime, a person *elected to be* President. Subgraphs involving these types of relations can be said to have "predicative valence". The fact that Γ's in the current context have a mixture of predicative and mereological *valence* indicates that they represent intersections of distinct general regions of cognitive activity.

As a preliminary to exploring these intersections further, consider first of all how typologies among concept-nodes and relation-nodes can influence our interpretation of conceptual graphs. In spirit with "Concept-Blend Semantics" ([27]), we can observe that most concepts are blends of other concepts: for examle, the concept "dumbell" is to some degree a blend of the concepts "heavy" and "graspable". Semantic and rhetorical context can imply variations on how the component concepts are blended, and allow a single concept to acquire subtly different meaning or connotations. For example, we may find it proper to define "terrorism" as "politically motivated violence", but our discussion of terrorist acts take on different shades of emphasis (such as moral emphasis) if we highlight the *violence* aspect or the *political motivation* aspect. As an example, it became something of an issue in the 2012 United States Presidential Campaign whether the phrase "Act of Terror" meant the same as "Terrorist Act". Someone whose intuitions are that these two expressions have different connotations might feel that the former phrase — perhaps more reminiscent of phrases like "terrifying act" or "acts causing terror", which have no connotation of political motivation at all — less clearly identifies the perpetrators as "terrorists", that is, as radicals who are formally involved with terrorist organizations and executing carefully planned attacks. The difference in connotation is perhaps that "acts of terror" need not be so deliberately planned, but for the purpose of this discussion we can say that this phrase connotes violence more strongly than political motivation, in contrast to "terrorist act". So the blend of "violence" and "political motivation" creates a matrix of possible connotations, and precise phraseology as well as context can influnce or "pull" connotations in one direction or another.

For another example, a "prodigy" is a very talented young person, but how we are to interpret this depends on context. A five-year old desribed as a prodigy is probably not being called a great pianist, in contrast to a 20-year old. Because "prodigy" suggests youth, it can also suggest someone with great talent but perhaps with limited experience, or (in the case of music, say), artistic expressiveness and sophistication. If we call some pianist a "maturing prodigy", we imply someone who is starting to marry talent with musical depth. If we describe someone instead as an "immature prodigy" we imply a talent being held back somewhat by personal issues. The concept "prodigy" is therefore rather pliable, and the component concepts which are blended in, like *youth* and *talent*, can contribute different connotations to the blend, providing dimensions or "degrees of freedom" for the concept's range. In a conceptual-graph situation, where concepts are linked to one another, the pliability of each concept in turn suggests a range of possible appraisals of a predicate structure or "double-edge". How we construe concept blends latent to these concepts disposes us to treat the compound predicate as accurate or not, or true or false.

Because concept blends can take on different connotations, their semantics can often acquire a modal dimension, since a propositional use of a blended concept may be subject to different evaluations depending on the connotations intended or understood.

I suggest that analyses of these variations is similar to cases like a meal being described as expensive, which I analyzed earler. In that example, the specific concept *expensive* needs to be assessed differently if a speaker (or a hearer) is a College President or a College Student, or if the meal is enjoyed in Paris or Part-au-Prince. Someone's inclination to accept an assertion like $(\star)$ "this meal is expensive" will depend both on their own private sense of what should be considered expensive in the current circumstances, as well as the actual cost of the meal. Since both of these are subject to variation, we can consider a Γ-triple expressing $(\star)$ as taking a range of possible fine-grained meanings as the related concepts take on a spectrum of possible precise details. If we can model the different possibilities as continuously varying, for some notion of continuity, then we can consider the concept-nodes as situated within topological spaces of conceptual possibility, representing for example the different range of prices which might bound the concept "expensive" from context to context. Adopting the Axiatropic analyses I described in the first part of this paper, suppose Γ takes a set $\mathfrak{T}$ of topological spaces $\{\mathcal{T}_i\}$, and let $\mathfrak{t}$ be a map associating some concept-nodes $c \in \Gamma$ with spaces $\mathfrak{t}(c) \in \mathfrak{T}$. If $\mathcal{T} = \mathfrak{t}(c)$, say that c is "$\mathcal{T}$-modal". If $\tau = c_1 \leftarrow r \rightarrow c_2$ is a triple in Γ, c_1 is $\mathcal{T}_1$-modal, and c_2 is $\mathcal{T}_2$-modal, say that τ is $\mathcal{T}^{\times}$-modal, where $\mathcal{T}^{\times}$ is the product topology $\mathcal{T}_1 \times \mathcal{T}_2$. This product reflects the possible variations of c_1 and c_2 as they take on different conceptual possibilities. For example, c_1 variation could be possible costs of a meal, and c_2 variation could be possible judgments as to what range of costs should be called expensive. The overlap between the c_1-spectrum (say, $\mathfrak{s}_1 \subset \mathcal{T}_1$) and c_2-spectrum ($\mathfrak{s}_2 \subset \mathcal{T}_2$) in $\mathcal{T}^{\times}$ represent the range of c_1, c_2 pairs where an assertion like $(\star)$ is judged true.

The nature of this overlap reflects different modal possibilities in assessing $(\star)$. Notice that in the $(\star)$ example $\mathfrak{s}_1$ and $\mathfrak{s}_2$ share a similar dimension (cost) and can be readily compared. If $\mathfrak{s}_1 \subseteq \mathfrak{s}_2$ then $(\star)$ is necessarily true; if $\mathfrak{s}_1 \cap \mathfrak{s}_2 \neq \emptyset$ then it is possibly true. So overlaps in product topologies associated with Γ-triples are a way to introduce modal notions into `CGS`. If a triple τ is $\mathcal{T}^{\times}$-modal then points in $\mathcal{T}^{\times}$ correspond to "possible worlds". Moreover, variations in c_1 and c_2 correspond to paths in their respective $\mathcal{T}$ spaces and, joined together, in $\mathcal{T}^{\times}$, so the possibility of variation linking differet fine-grained connotations of c_1 and c_2 provide a semantic interpretation for the modal relation of inter-world "accessibility". Conceptual graphs, at least when provided with topological $\mathfrak{T}$-extensions along these lines, provide conventient models for modal logic (and arguably provide an intuitively sensible of account inter-world accessibility, which tends to be a rather exotic aspect of the metaphysics of modal system), models which moreover seem grouned in real-world semantic phenomena.

The association $\mathfrak{t}(c)$ of Γ-nodes with topological spaces is straigthforward in the $(\star)$ case discussed above because the concept-tokens there have an obvious dimensional structure. However, I believe a similar model, perhaps allowing for more poetic licence when linking the formalisms with the real-life semantics, can also be applied to concept-tokens in terms of their latent semantic blends. In the case of a $\mathcal{T}^{\times}$-modal triple $\tau = c_1 \leftarrow r \rightarrow c_2$, we can associate τ's concept-nodes with specific dimensional structures (here related to cost and money). This provides an *axiation* for τ. Axiated Γ-triples have a straightforward numerical interpretation (such as the cost of a meal falling in some range), and can potentially be used to extract information from language artifacts modeled as conceptual graphs (consider Social Network posts about some restaurant).

Earlier I proposed "Axiatropic Semantics" as analysis of conceptual graphs with these kinds of axiated, topological extensions. Here I have revisited that theory to motivate an extension to *concept blends*, which we can explore with similar topological notions, even if the lack of precise dimensional structures perhaps renders any mathematical treatment mostly suggestive and demonstrative.

Insofar as a concept-token c represents a latent blend of inner concepts $\{\mathfrak{c}_i\}$, we can define $\tau(c)$ as a topological space representing different proportions of these latent $\mathfrak{c}_i$. This topological representation of concept-blends can be perhaps be motivated by the historical link between Concept-Blend Semantics and Latent Semantic Analysis (LSA), which associates words in a document or in document-corpora with a space of latent semantic categories, statistically derived from the distribution of words as they mutually occur in different contexts. Most often, co-occurance tables are built for words in documents within document corpora, and statistical techniques, such as "Scalar Value Decomposition", are used to reduce these quite large matrices into smaller representations, which has the effect of establishing latent semantic categories. These categories may or may not correspond to recognized concepts or topics, and the use of statistical analysis to expose semantic patterns is subject to mixed interpretations — some practictioners argue that these techniques reveal significant semantic connections, while others suggest that statistical tools at best demonstrate semantic correlations, whioch serve practical needs such as document-to-query matching but are of dubious merit when construed as unveiling semantic "deep structure". One interpretation of LSA, which (in a nod to "generative semantics") we can call the *generative* theory of LSA, holds that latent semantic regions, identified by statistical analysis, approximately correspond to latent concepts blended into surface concepts. For example, the status of terrorism as a conceptual blend of "violence" and "political motivation" can perhaps be revealed by plotting how documents about terrorism, within corpora subject to LSA techniques, would be clustered with violence and politics as topics.

Latent Semantic Analysis can potentially give concrete numeric form to various proportions available in a concept-blend, but even without such clear numeric interpretations I think it is intuitively plausible that the spectrum of connotations formed when different concepts are blended together, even if we cannot assign dimensions to "measure" these blends, can be modeled in terms of topological spaces. If this is reasonable, then Γ-triples whose concept-nodes are blends of latent concepts $\{\mathfrak{c}_i\}$ are subject to similar modal/topological interpretations as I suggested above for "axiated" triples. The effect of a conceptual relation r, formally modelled in Γ as a triple linking r to two concept-nodes, is to relate distinct conceptual blends, with (for the most part) different latent component concepts, via a relation-type. The result is a more elaborate blend potentially combining latent concepts from both concept-nodes; so for example the assertion "that pianist is a prodigy" blends concepts involved on the left side (youth, talent, etc.) to others latent on the right (musician, performer). Via these latent concepts we observe that conclusions like "this person is a talented musician" follow more or less automatically, which would not be similarly latent in either concept by itself (not all prodigies are musicians, and not all pianists are talented). Because triples τ in this context can join latent concepts in one concept-node to those latent in another, we can call these "cross-blend" triples: latent concepts on one side are crossed with those on the other. On the other hand, because latent concepts can be blended in different admixtures, these cross-blends take a spectrum of different connotations and can be applied or interpreted in a spectrum of contexts. Conceptual blending is not akin to straightforward logical connectives, as they may be in semantic models based on first-order logic ("A bachelor is an unmarried man"): the triples which combine different blends are not aggregates of atomic semantic units but conceptual "spaces" of possible connotations and variations. By analogy with a potential "Axiatropic" semantics, exploring triples linking concepts with well-defined dimensional structures, so that we have a precise notion of how a spectrum of interpretations for the two concepts can overlap with each other, we can perhaps develop a "Cross-Blend Semantics" which investigates similar overlap between latent concepts in double concept-blends. We can use a similar topological representation, even if the nature of variation in the latter case — in terms of how prominent are different latent concepts within an admixture — is less clearly suited to formalization than the former case, where there is some numeric dimension (like price or price-ranges) identified with and distinguishing concept-tokens.

The modal interpretation of cross-blend structures in conceptual graphs can be interpreted in light of Jubien's property-theoretic account of modality. Earlier, I discussed Jubien's analysis in the context of CGS overlapping or integrating with FCA. Here I want to consider Jubien's framework in the related integration between CGS and CI — in other words, in the use of Concept Graphs to express the structure of Conceptual Blends. Although this theory may have applications to CI in general, my primary focus in this case is to extend CI as a way of capturing perceptual qualities of objects, like the blend of coloration and spatiation, insofar as these contribute to objects' three-dimensional form and individuality being reconstructed from visual perceptions and kinaesthetic engagement with objects. This is not the primary application of the term "blend", but I think it opens the door for interesting analyses connecting CI with Mereotopology. Given the existing connections I have explored between CI and CGS, and from CGS to both Formal Concept Analysis and Mereotopology, I think this final theoretical link helps to yield a more detailed inter-theoretic network between these major semantic frameworks.

9 Conceptual Integration and Jubien Semantics

Jubien notes that some modal relations can be directly expressed in terms of property-relations: for example, the fact that all horses are mammals reflects the fact that the property *being a horse* is a refinement of the property *being a mammal*. At least in cases like these, there is no need for the (ontologically controversial) language of possible worlds, as in, horses are necessarily mammals because horses are mammals in all possible worlds. Jubien allows that talk of Possible Worlds is acceptable as a kind of informal commentary for purposes of demonstration, and when developed as a mathematical theory Modal Logic does not attempt to actually give an account of possible worlds, merely leaving this notion (and the related notion of inter-world accessibility, which I mentioned earlier) as primitive. There is no harm in using the terminology "possible world" to refer to formal entities in Modal Logic as an abstract system, rather than to metaphysical entities which provide *models* for Modal Logic; but if we do not have an ontology of possible worlds to provide these models, to provide a notion of what it means for some fact to be necessary or possible, then we need some other way to provide these models. Jubien's work provides one strategy; for example, the inter-property relation between horses and mammals can *be* a model for the modal necessity that all horses are mammals, rather than simply being evidence for this necessity. In other words, systems of property-relations can provide modal models themselves, whereas in most interpretations of modal logic such property-relations are merely derived from modal systems which then need some other framework to acquire models — for example by taking possible worlds as full-fledged ontological constructs, rather than undefined mathematical primitives or informal ways of talking.

Jubien's property-theoretic account of modality is mostly directed not at inter-property relations such as *being a horse* extending *being a mammal*, but rather toward ambiguities in the modal properties of the semantics of referring expressions, what we could call the "modal logic of individuals". Referring expressions designate particular objects, but we have an intuitive sense that the scope or extension of objects can change somewhat, without our believing that the truth-value of statements about these objects also changes. We have the intuition, for example, that a coconut which loses a hair is still the same coconut. Conventionally we would say that the hair is not an "essential part" of the coconut. Jubien, however, argues for "mereological essentialism" — the idea that all objects possess all of their parts essentially. In other words, the coconut $\bar{\mathfrak{C}}$ without the hair is a *different* object than the coconut $\mathfrak{C}$ with the hair; nonetheless, Jubien suggests that these two objects could be *the same coconut.* If the coconut loses a hair, then $\bar{\mathfrak{C}}$ (now a different object) still instantiates the property of ($\star$) *being this coconut.* (Technically Jubien considers objects to be four-dimensional, implying in my example that $\bar{\mathfrak{C}}$ and $\mathfrak{C}$ are two temporal-parts of one object; however insofar as the object *could lose* a hair then it *could be* a different object, and this other object would then instantiate *being this coconut*). The semantic force of the expression "this coconut" then is its relation to ($\star$). We can still consider the expression in question as referring to an object, in the specific sense that (if the expression has a referent at all) there is some single object which instantiates ($\star$); but except for being this specific instantiator the actual object, qua material thing, has no bearing on semantic meanings associated with ($\star$) — with statements where this property is connected with other properties. One benefit of Jubien's approach is that it minimizes the degree of semantic difference between referents to real things and referring expressions in fictional contexts and other contexts where by definition referring expressions "have no referents", or, more precisely, they semantically embody properties which have no instantiators.

In the current context I want to focus on the modal variations amongst objects which can serve as instantiators for properties nominated through referring expressions, what Jubien calls *singulary* properties. For Jubien, meaning is primarily a network of properties, some of these singulary properties representing individuals, which exist in a mesh of interconnections. Intuitively, Jubien's semantics is therefore similar to CGS, though I am not aware of any lines of influence between Jubien and CGS practitioners, such as J. F. Sowa. I propose to call "Jubien Semantics" the technique of modeling semantic structures in terms of graphs among properties, where edges represent not only inter-property relations (like *being a horse* extending *being a mammal*), but also co-instantiation relations which may involve singulary properties. For example, the phrase ($\mathcal{R}$) "this coconut is ripe" can be modeled as a coinstantiation network $\mathfrak{N}$, which verbally might be rendered as "the property *being this coconut* is coinstantiated with the property *being ripe*". These kinds of coinstantiations are the Jubien Semantic equivalents of CGS predicative triples. In the context of Jubien's thought, the important point (or one important point) concerning coinstantiation networks $\mathfrak{N}$ is that their semantic meaning or validity need not depend on singulary properties in $\mathfrak{N}$ actually having instantiators. For example, we might enunciate ($\mathcal{R}$) to explain the actions of an jungle explorer in a fairy tale. There are occasions when the realness, or not, of putative referents (like the Fountain of Youth, say) is indeed semantically relevant, and on those occasions there are properties like *being an object in the actual world* which are incorporated into networks $\mathfrak{N}$. In general, though, $\mathfrak{N}$'s carry over their structure into fictional contexts, and this accounts for how fictional objects can take on properties and how propositions concerning these objects can be true or false, even though representations of these objects do not have actual referents.

Possible World Semantics can address similar concerns by arguing that "fictional propositions" are to be evaluated in other possible worlds, so that the purpose of storytelling is to construct a family of possible worlds where, say, Sherlock Holmes exists and therefore statements about him can be true. Earlier I argued that this notion of possible worlds is either metaphysically extravagent or definitionally circular. In its stead I proposed a provisional "Symbolic frame holism", where collections of concept-tokens, which link into networks of conceptual relations, are instantiated in the actual world in their entirety (in non-fictional contexts), or else possess many tokens which are not thus instantiated (in fictional ones). In some cases (like Holmes's London) we identify actual-world referents for some concept-tokens even in a fictional setting where other concepts are non-referring. There may be non-fiction contexts where some tokens are also be non-referring in this sense (such as when true-to-life journalist reports have to alter some details to protect sources). In general, though, a conceptual network in which only some concept-tokens are intended to have real-world referents is one being used to create a fictional narrative. We interpret referring expression in this case not by assuming that they "quantify" over entities in some non-actual world, but rather that their semantic meaning depends wholly on their place in their conceptual network and is not affected by individual or collective acquaintance with their referents, because these do not exist. Given this absence of concrete referents, the sole "constraints" on these networks is their internal consistency.

Informally, we may say that the "referrent" of a name like Sherlock Holmes is a "structural place" in a conceptual network, a place defined by its relation to other concept tokens (friend of Dr. Watson, resident of London, etc.). However, it is better to say that a fictional name does not have a referent at all; that there are different kinds of referring expressions, some using ostensive or other gestural effects (like pointing at something), others using proper names, labels, or descriptions, and the semantics of these expressions will vary depending on language users' direct or indirect acquaintance, or no acquaintance, with the designated entities. In the latter case the referring expression serves to create or indicate a concept-token to be placed in the context of a larger conceptual network, thereby becoming a vehicle for representing the conceptual relations which connect with that token. For fictional properties, such concept-tokens have no properties except those attribted to them by their enclosing network. Even for non-fictions, though, whatever perceptual or epistemic acquaintance we have with concept-tokens is extended and completed by the conceptual networks where we identify them. The objects we see are not just the portions of them which we see, but insofar as they enter into conceptual relations with other objects, they extend beyond our direct acquaintance and are "completed", in terms of our cognitive familiarity with them, by our recognizing these networks as obtaining. Considering the rational organization of fictional narratives is therefore a suggestive analogy for how our perceptions of objects are transcended by their posited place in conceptual networks, which we reason about in terms of their internal consistency as well as their consistency with whatever perceptual details we do have available concerning their objects.

So for example, insofar as relations between Holmes and Watson are modeled as property networks, then assertions about these two characters are details of the structure of these networks and can be assessed by abstracting away questions of what are (or whether there are) instantiators of properties like *being Dr. Watson*. Identifying meaning with property-networks then renders Possible Worlds as ontological posits unnecessary. Jubien singulary properties play a role similar to "trans-world individuals" in Modal Logic, as can be seen by topological models of modal logic, like the Kishida spaces I have alluded to here, which consider possible worlds as "sheaves" in a bundle-space for which trans-world individuals are fibres: each such individual has a "residence map" which names the identity of that individual in each possible world where it exists. Like these residence maps, the image of singulary properties in each possible world (if we can use this talk for a moment) is at most one instantiator; however, here this is merely a special case of the image of any property at all, in some world, being a collection of individuals — singulary properties being different from other properties only in having at most one instantiator. Whereas properties may in other paradigms be defined as collections of individuals relative to possible worlds — each property in each possible world has a collection of representatives, thereby establishing properties as a derivative notion to objects in possible worlds, Jubien's paradigm inverts this approach: properties are logically fundamental, and individuals are properties which have the unique "meta-property" of having at most one instantiator, or, if we like, having exactly one instantiator at each world where they are instantiated at all. This captures the intuition that for each object there is a specific concept of being that specific object — and, more significantly, the intuition that most individuals function in prositional attitudes conceptually rather than through their particular object-features. For example, most people are acqainted with the *concept* Sherlock Holmes but have no idea how tall he (fictionally) was, his age or weight, etc. Since they bring these conceptual or intensional aspects of individuals to the fore, Jubien singulary properties act as a kind of theoretical channel between trans-world individuals in Modal Logic and object-concepts in CGS.

Singulary properties have (at most) one instantiator, but it is usually true that other slightly different objects could just as well be that instantiator, so the actual instantiator is modally connected with a series of possible alternatives. Once again, we grant for the sake of argument Jubien's "mereological essentialism", so that each object is uniquely determined by all of its parts, no matter how trivial they seem to be to the object's overall character. Another way of saying this is that objects in this theory are abitrarily fine-grained; we measure objects with sufficient precision that any variation, no matter how tiny the scale, yields a difference between two distinct objects. A singulary property like ($\star$) — *being this coconut* — may have one instantiator, but there is a space of many similar objects which nearly overlap this actual instantiator and could potentially replace it. While we do not need a notion of possible worlds — indeed, this scenario is intended as a replacement *for* possible worlds — we can adopt possible-world talk for ease of exposition, and say that in other possible worlds some of these other mereological sums are ($\star$)-instantiators in lieu of the actual bearer of that concept-token: i.e., the coconut $\mathfrak{C}$. If we imagine all of these possible-coconuts as we travel across these possible worlds varying one into another, we can consider the possible-world universe around $\mathfrak{C}$ as a topological space, with each point in this space associated with one world. Alternatively, we can say that the phrase "possible world" is simply a name for points in this topological space: there is a spectrum of possible variations in the precise mereology of the actual coconut and we use possible-world talk to name points in this space so as to emphasize its modal character.

Let's explore the thematic threads here. Jubien wants to replace possible-world talk with talk about property-relations, and in particular with a theory of singulary properties which capture the semantics of (typical) referring expressions. Different objects are connected by a relation according to which each could potentially be instantiators for a given singulary property. Modal topologists like Kohei Kishida want to replace possible-world talk with talk about points in topological spaces. Because Jubien considers objects to be distinct given arbitrary fine-grained differences, candidate instantiator-objects for some singulary property seem to exhibit a continuous spectrum of variations, a variation which therefore seems appropriate to model as a toopology space. On the other hand, Jubien discusses object-differences in terms of mereology, not topology (nor mereotopology, for that matter). Extending this thematic summary to CGS, conceptual graphs can model mereological variation by associating concept-nodes with parts and relation-nodes with parthood and coupling connections. For a suitable definition of continuity, we can certainly supply topological interpretations to these mereological conceptual graphs. We can describe a *mereological neighborhood* of a graph Γ as a set of alternative graphs Γ_k which differ from Γ by adding, subtracting, or reweighting triples whose relation-nodes possess mereological valence (i.e., they are some sort of parthood or coupling relation). A plausible account of mereological neighborhood systems $\mathcal{N}_\Gamma$ thereby yields a topology for the space of variations whose points are distinct graphs each modeling slightly different mereological structures. This is a kind of mereotopology defined on collections of distinct graphs, which contrasts with my earlier discussion in which I associated concept-nodes within *one single* graph with topological spaces; call the earlier case an *external* mereotopology and the current one an *internal* mereotopology (below I will show how these notions can be combined).

If a hair could have fallen off a coconut, this suggests that two different mereological sums (one with and one without the hair) share a mereological neighborhood. If we consider a set of all such neighborhoods around $\mathfrak{C}$ as a topological space $\mathcal{T}^{\mathfrak{C}}$, then the set of objects or mereological sums $\mathfrak{m} \in \mathcal{T}^{\mathfrak{C}}$ which continue to instantiate ($\star$) define a subspace, so that ($\star$) is modal in $\mathcal{T}^{\mathfrak{C}}$. We can shave all the hairs off $\mathfrak{C}$ and evidently it will still be the same coconut, which can be represented by a mereological graph $\mho$, figured a little simplistically by a flower-shaped graph with a central node representing the coconut shell, connected to one node representing the fruit (which might disappear soon), and connected to a bunch of nodes representing hairs. Shaving off a hair is then akin to eliminating the edge between a hair-node and the central node. If I shave off the hairs and then eat the coconut — eliminating the "fruit node" — we might agree that the scraps left over no longer collectively instantiate ($\star$). I am more likely to say: "that *was* a delicious coconut", than I am to point contentedly to the scatterred hairs and pieces of shell and say "this *is* a delicious coconut". Colloquially we might say that the fruit is an essential part of $\mathfrak{C}$ while the hairs are not, or that the hairs have relatively less coupling to $\mathfrak{C}$ as a mereological whole. We can also imagine different mereological structures among the same elements: I could absent-mindedly shave off the hairs and arrange them on the ground in a figure-eight. Considering the spectrum of possible relations between these parts, the couplings and spatial proximity which can vary incrementally from configuration to configuration, it is intuitive to imagine this space of mereological possibility as topological. Some subset of this space, say $\{\mho\} \subset \mathcal{T}^{\mathfrak{C}}$, represent possible substrata for ($\star$). We can then consider predicative relations involving ($\star$), such as *this coconut is ripe*. The property *being ripe*, in this context, presumably may be satisfied by a coconut with a range of different times on the tree. Knowing which precise $\mho$ actually instantiates ($\star$) is unnecessary, as is knowing the precise number of days that this $\mathfrak{C}$ has been ripening. It makes no semantic relevance whether $\mathfrak{C}$ has 1000 hairs and has been on the tree for 100 day, or 999 and 101, etc. In the actual world indeed *some* fine-grained object, with a precise number of hairs, will instantiate ($\star$), and will be ripe to a precise number of days. But the granularity of the instantiators of a predicative network, and of the precise details of properties being instantiated, are not specified by the semantic scope of the concepts in question. Instead, these concepts range over a spectrum of possibilties, and each fine-grained detail is one possibility among others. This modal variation can generally occur at all points in a graph, including with respect to object-concepts like "this coconut" as well as property-concepts like "being ripe".

I have discussed several kinds of parameters of modal variation. Some properties (being ripe, being expensive) have a simple dimensional correlate (days on a tree, cost of a meal). Others are "conceptual blends" whose latent components admit different admixtures and connotations. Object-concepts or singulary-properties can be associated with a spectrum of fine-grained mereological sums. We capture the intuition that $\mathfrak{C}$ could lose a hair, with the image of $\mathfrak{C}$ as a mereological sum or mereological structure (perhaps represented with a conceptual graph $\mho$) that can undergo some modification. We can also say that $\mathfrak{C}$ is only imprecisely individuated, so that there are multiple candidates $\{\mho\}$ which all have some modal possibility of *being* $\mathfrak{C}$. Given the distinction of object-concept-nodes and property-concept-nodes, we can distinguish object-modal-variation and property-modal-variation; for example, mereological imprecision in individuating referents for object-nodes demonstrates the former. The use of conceptual graphs $\mho$ to model these mereological possibilities within larger graphs Γ is an example of CGS "nested graphs", in which a separate graph is introduced as an inner structure within a single node. This is a different structure than a subgraph $\gamma \Subset \Gamma$. A nested graph $\eta \prec \Gamma$ is an internal structuration of a node $n \in \Gamma$ such that the graph as a whole enters into relations with other Γ-nodes. In this case, we can use nested graphs to represent the internal mereological structuration of individuals which are represented by object-nodes in graphs Γ.

Earlier, however, I discussed how the internal structuration of individuals within mereological systems is not wholly captured by their mereological breakdowns. This carries over also to their individuation from their surroundings. In a purely mereological context the individuation of an object separates parts within the object from those outside it; the structure amongst internal parts gives rise to internal structuration (**IS**), while the separation from external mereons gives rise to individual coherence (**IC**). Even when the distinction between inside and outside cannot be fully captured by mereological boundary-lines — some parts in, some putative parts out — internal structuration serves as a principle separating wholes from their surroundings, which typically will exemplify different internal patterns. So rather than modeling the individuation of mereons in terms of isolating their own parts, it is often more appropriate to characterize their **IS** in contrast to their surroundings — **IS** which often cannot be fully identified through mereological relations alone. The variability in these **IS** patterns gives rise to a form of object-modal-variance; in cases where this variance is more fine-grained than crisp mereological partitions allow, we need to model it more in terms of featural distribution and combination across an object, such as the interplay of coloration and spatiaion on an object's surface. Having discussed conceptual blending as a source of modal variation on the property-node side (of predicative triples, technically), we can see these effects of featural variation as related "blending" on the object-concept side. Features are rarely bases of object-individuation on their own: for example, it is difficult or impossible to distinguish the boundaries of objects via spatiation without coloration, as can be observed by replacing colors in a computer bitmap image with black or white points. Some object-details remain, but the sense of three-dimensional depth is lost. Conversely, image-compression can be achieved by averaging color-values of adjacent pixels (replaceing 3×3 or 5×5 pixel arrays with monochrome squares). The overall color depth of the image may not be reduced, but without fine-grained spatial lines of color variation the sense of depth, once again, is significantly curtailed. This effect is an accidental artifact of shrinking and then re-enlarging images, according to some resizing algorithms. Of course, similar effects were deliberately exploited, to rich aesthetic effect, by Cubist painters.

When we speak of "distribution" of features "across" or "across the surface" of objects, we implicitly suggest a blend of spatiation and featural variance — such as coloration — which we attribute to objects based on their manifold appearance, and which provides us a sense of their three-dimensional depth and geometry and their individuation. The fusion of coloration and spatiation, in particular, separates objects or object-parts from their surroundings. Obviously some objects are compound forms whose parts are thereby individuated, but where some other notion of functional organization or connectedness provides a sense of the whole. The cognitive acts of object-individuation can then be modeled in terms of mereological graphs $\mho$, but individual mereon-nodes in these graphs are individuated in turn by featural variation and contrast to surroundings. There is therefore a nested structure: nodes associated with patterns or featural spaces $\mathcal{F}$ are joined to mereological graphs $\mho$ which in turn are nested inside predicative graphs Γ, where again $\mho \prec \Gamma$ means that $\mho$ is treated as a *node* within Γ. A predicative triple $\rho \in \Gamma$ may contain a nested $\mho$ as its object-node, giving it the form $\mho \leftarrow r \rightarrow p$, where r is a predicative relation, p is a property-concept-node, and $\mho$ is a mereological graph nested $\prec \Gamma$ as an object-concept-node. As before, modal variation in ρ can be explored in both "ends": dimensional structures or concept-blend connotations on the right (p) side, and mereological and/or featural variation on the left side. I have already discussed topological interpretations of mereological variance, so, in order to focus on featural variation as a basis of modal structure, let's consider a triple ρ whose object-node o is a single individual identified by featural patterns. In other words, o does not have parts with particularly crisp **IC**, though its patterning might suggest a mereological partition suited to modeling as a graph $\mho$, which in that case can be nested $\prec \Gamma$. For now assume instead that o has structuration modeled as a feature space $\mathcal{F}$. Because $\mathcal{F}$ canonically fuses different parameters of featural patterning — almost certainly spatiation and some other feature, canonically coloration — we can analyze $\mathcal{F}$ as a concept-blend, though in contrast to the earlier examples the blend "pattern" is more complex. If points in $\mathcal{F}$ represent combinations of spatial points and color values, so patterns in $\mathcal{F}$ are color patterns or hue gradients tracing lines and regions in space, then $\mathcal{F}$ intertwines spatial form and color variation. Although ealirer I discussed the interpretation of $\mathcal{F}$ as a compound space or product topology, at this point I would like to point out the parellels with property-node concept blends such as the concept *prodigy* as a blend of youth and talent.

Whereas blends of the latter sort mix component concepts proportionally, yielding a unique connotation specific to a given circumstance or rhetorical suggestion, blends of the former sort are more intricate. The space of variation in $\mathcal{F}$ is not just a space of possibilities, but defines a patterning and structuration for an object even in one single state, perceived from one single viewpoint, etc. So whereas *dumbell*, for example, as a generic *concept* mixes bulk and graspability, a particualr *object* identified as a dumbell will have a corresponding blend of features, but the detail of this blending will vary from across perceptual circumstances. I take a provisional suggestion of an o's dumbell-ness to be its shape, but only in the context of general observations about its color and appearance. For example, a typical dumbell will look rather black or metallic: brightly-colored dumbell-like shapes might be perceived as some sort of decorative ornament, not an exercise tool; similar shapes with luminiscent bulges might be guessed as some sort of lighting; similar shapes in ceramic or wood might be judged to be some sort of kitchen implement. The belief that o is in fact a dumbell will not be confirmed until lifting it, synthesizing multiple features of its appearance, the materials from which it is made, how we can physically interact with it, etc. This synthesis of features is like a drawn-out, multi-faceted conceptual blending.

There is a generic blend of bulk and graspability within "dumbell" as a concept *type*. For the recognition of an object as a concept *token*, these and other constitutive and contextual concepts (found in a gym, made of metal) must be integrated into a pattern of perceptual synthesis. As a semantic unit the space of variation within *dumbell* qua concept-type may have connotative range, mixing component concepts to varying degrees; but as a perceptual synthsis the identification of an obejct as dumbell *token* — and the semantics of a referential expression like "this dumbell" — demands a more intricate conceptual synthesis articulated within a feature space $\mathcal{F}$. While modal variation within the concept *type* may be captured by points in a space of connotations, modal variation within the concept *token* is variation in the entire featural distribution — for example the systematic change in a dumbell shape's appearance from different angles — because any one particular featural synthesis involves many different featural points, canonically points of feature variation across an object's surface.

We are adept at multiple stages and layers of these perceptual and semantic syntheses: individuating objects by featural contrast, cognitively extending patterns around hidden sides to create a mental model of three-dimensional geometry; grouping parts into aggregate component objects; identifying these objects as instances of concept-types and linked with other objects and types into predicative networks; and embodying or retaining these networks in conscious beliefs, belief dispositions, propositional attitudes, linguistic expressions, etc. There is a still more general and holistic layer of situational understanding and identifcation which further guides our propensities to identify objects with concept-types: the association of contexts to typical objects, like a dumbell in a gym, as well as situational planning, anticipation, construction, debate, and so forth. We recognize situations that obtain, but also the practical activities which are involved with or can be carried out within them. But in addition to taking situations as given and treating them as contexts for cognitive acts, we also reason *about* situations; we construct them or call them into being through collective cognitive assent, like legal and political systems; we measure situations on moral or pragmatic grounds; we make personal and collective commitments to enter into and operate within situations according to specific maxims: go to the gym, eat healthy, protect the environment, etc. All of these normative, evaluative, moral, practical, and situational cognitive actions lie above the cognitive structures I have identified here.

Nevertheless, even within the narrow focus on object-individuation and feature-spaces from which we can generalize and contextualize at least up the scale of cognitive breadth to the layer of predicative networks and conceptual graphs, I would like to close with some comments on what appears to be uniquely human in this cognitive progression — or if not uniquely human, at least unique to conscious intelligence. Insofar as we can model cognitive operations schematically and even mathematically, we inevitably confront the question of whether these operations can be "mechanized", programmed, implemented on computers or by robots, etc., and in particular whether they depend on *conscious* minds. I will briefly offer some opinions on this matter.

10 Minds, Machines, and Manifolds

I believe that human intelligence benefits from a vivid capacity for imagination. Imagination allows for advance planning, in particularly vivid, precise ways. Because so much of our actions are carried out at least in part subconsciously, advance planning demands more than simply forming the intention to behave in certain ways. We must attempt to form a subconscious disposition for the behaviors we believe will be desirable, and for that end it is helpful to imagine future situations as vividly as possible. A striker practicing penalty-kicks must do more than commit certain rules to memory: take a breadth, shoulders down, imagine the ball as a low arrow parallel to the ground. It is one thing to remember these techniques on the practice pitch, and something else to do so under the stress and tension of the big match. This is why the footballer imagining a heroic goal in a Champions League is doing more than telling stories internally; imagining these scenarios is a way to commit to memory certain subconscious behaviors in the context of highly specific circumstances, ones which cannot be simulated in practice but can only be imagined.

On the other hand, it is equally important for us to be aware of what perceptual content is imaginary and what is not; therefore to experientially feel the presentational structure of perceptions which are *not* imaginary, and whose details and changes, as a result, are not products of our mental choice or subject to our will. I fail to see how this distinction between imaginary and nonvolitional contents, a distinction made without conscious deliberation and with full experiential immediacy, can be made without second-order awareness and self-consciousness. I am aware of myself having conscious experiences and of the fact that the details and changes of these experiences, for the most part, occur outside my mental control. I am therefore in position to consider nonvolitional perceptions as representation of external phenomena and bearers of information about an external world. This does not mean my perceptions are infallible, but rather that I may define *the external world* as *the world disclosed in my nonvolitional perceptions.*

A side-effect of how conscious experience lies beyond my control is the fact that perceptions carry affective force, which at the least serves as a marker of their nonvolitional valence but which can sometimes also carry distinct pleasure or displeasure. This affective intensity provides a further source of information — pain carries a particular urgency which magnifies the information that some situation is potentially dangerous, and pleasure serves as a general guide toward which situations I should seek out. But aside from this epistemic value, pain and pleasure emphasize the nonvolitional aspect of consciousness: I cannot mentally erase pain, and I cannot imaginatively generate pleasure comparable to actual pleasurable circumstances. We may sometime regret these limitations, but the inirascibility of conscious experience to our mental control is precisely what we need to cognitively distinguish reality from imagination.

I believe therefore that we have well-defined functional roles for conscious states as affective, presentational contents to a self-aware ego, for our personal sense of self and identity built through the perpetual inner narrative of our conscious experience and of our awareness of ourselves and the consciousness affected through them. However, I believe there is also well-defined functional roles for all the nuance and richness of qualitative consciousness. Nonconscious intelligences certainly can absorb signals and information from outside, and it is tempting to argue that the sole cognitive role for qualitative characteristics like *being red* is to provide a basis to distinguish objects and object-types. Certainly this discriminatory role is one function of qualia, but I am not sure as a result that there is no functional role for the precise qualitative character of red *as red*, as the distinct look of redness, rather than merely its contrast to blue and any other color. In particular, using color as typical example of qualia, colors are not discrete labels attached to objects, as if being red means being placed in a metaphysical red bucket, being blue places objects in a blue bucket, and so forth. The property of *being red* is not the property of having exactly one color, but rather a disposition to have a variety of color-pattens based on view angles, lighting, ambient luminoscity, etc. As perceptual detail, "red" is more a coloration than a color. We do not use colors only to disciminate objects from one another, the way that colors are arbitrary markers for game pieces or billiard balls. We need to perceive a rich space of color variation in order to identify three-dimensional geometry. We can clearly identify functional merit to perceiving a broad spectrum of color possibilities, suggesting a topological space with a clear (even if not infinitely fine-grained) notion of continuity.

The continuity of color variation means that distances in color space are fine-grained, allowing for a richer variety of visual information: we can use color to discriminate objects, and sustain this judgment across differences in viewpoint, lighting, ambient conditions, and so forth. So we can identify functional roles not only for a basic disciminatory color-perception, but perception of the broad spectrum of color-differences. Moreover, we have a subconscious tendency to associate colors with prototypical situations and conditions, as illustrated by the notion of "cool" and "warm" colors. A museum gallery with many wintry landscapes can physically feel a few degrees cooler than an adject gallery with fiery scenes in oranges and yellows. However modest, there is a correlation between colors like white, blue, and black, with objects and places that tend to be on the cold side, and colors like red, orange, and yellow, on the hotter side. There are plenty of exceptions of course, and even when these associations hold, like blue as a "cool" color, say, or yellow as "warm", perhaps they are nothing more than our remembering past correlations of color and situations. I am not convinced, however, that that equivalent associations would be made if the "mapping" between physical color-space and colors *as experienced* were rearranged in a manner that changed the character of color-*qualia* while leaving perception of color-space *distance* equally detailed. For example, in an "inverted color" universe, where blues looked like orange and greens like red, etc., would we simply invert our associations so that orange "feels cold" and blue "feels hot", and so forth? Would the gradations and boundaries of these associations have the same form as they do in reality? If not, then we have reason to identify functional roles not only for blue to look different from red, and for the whole spectrum of perceptual color differences, but also a functional role for blue to look precisely *like blue*; for all colors to have their specific qualitative character and appearance.

Proposing functional roles for these kinds of qualitative nuances can help resolve questions about how (and whether) talk about "qualitative feels", affective dimensions of perception, consciousness as subjective, first-person reality, etc., can be integrated into our philosophical and scientific discourse. Some mental and cognitive structures are suited to formal analysis — whether in terms of mathematics, algorithms, neuroscience, etc. If we believe that cognition is largely or exclusively the responsibility and result of the brain and nervous system, and that these are biological systems whose processes are ultimately biological and biochemical in nature, then we have grounds for hoping to extend biological science into the domain of mind and thought. At the same, time, if we believe that some human cognition can be simulated on computers, and has structural properties which can be modeled in in terms of computational concepts and formalisms, then we have grounds for also extending computer and information science into these same domains. Certainly the study of mind and consciousness also belongs to some degree to the domains of history, aesthetics, poetics, and human sciences generally; but we have good reason to believe that mind and consciousness is a proper domain for scientific investigation and that many, perhaps all, of the laws and properties of these domains are subject to natural explanation.

If we aspire to make the study of mind and consciousness into a domain within natural science, we face the obvious question of how talk *about* consciousness, insofar as one aspect of conscious reality is its privacy to each individual, its "first person" nature, and therefore the apparent reality that some of its immediate contents are not subject to "public observation" in the conventional sense. Ascribing functional roles to dimensions of conscious reality is one way to accommodate this situation, because functional roles can be defined in terms of posited functional organizations whose structures can be subject to public discussion, and whose schematic representations can be developed as publicly observable constructs, even if the entities (qualia, subjective affects, etc.) suggested as playing these roles are not themselves collectively observable. This suggests why functional descriptions of mind have become popular amongst Analytic Philosophers, in particular, who are concerned about the ontological status of the "first-personal". Functional roles, however, can be slippery constructs when asked to play this kind of metaphysical role as arbiters of ontological controversy. For example: how do we in some cases distinguish qualitative differences *between* functional roles, from different degrees of success or efficiency in performing *one single* functional role? Is an electric car functionally identical to a gas-powered car? Electric cars do not pollute the environment, but they have the drawback of needing to be recharged after a certain number of miles, and this process is less convenient than simply filling a gas tank. Are these functional differences, or differences in the efficacy by which the two types of cars perform the single role of allowing people to drive around? If charging stations were as common as gas stations, and cars could be recharged in only a few minutes, would this render a functional equivalence between two currently functionally different systems? Conversely, what if some filter were invented that could sanitize the emissions from gas cars? Can these empirical details be properly used to ontologically distinguish qualitative differences between functional roles, from mere differences in effectiveness between systems which exhibit the same functional abilities?

Even if nonconscious intelligent beings or Artificial Neural Networks can perform some of the same cognitive operations as human minds, we need to consider, in our comparison of functional operation, the efficacy and immediacy of human conscious judgment. I am not sure how to compare the energy consumption of human beings with that of comptuers, but the energy costs of modern server warehouses indicate that energy efficiency is at least relevant to comparisons of different sorts of intelligence. Aside from these issues of efficiency, biological minds and biological systems in general have avenues for evolutionary variance which are not available, at least in the same ways, for computing machines. This suggests that there are clear functional differences between biological and mechanical intelligent systems, even if these differences may not be captured in terms of the information-processing algorithms which they may respectively perform. If we contrast nonconscious biological minds with the cognitive abilities of humans and other advanced mammals, it seems clear that consciousness allows a richer, more forward-looking engagement with our environment.

If the goal of functional descriptions of cognitive processes is to identify some portion of what we commonly think of as the "mental", incorporating these cognitive structures into our scientific account of mind while defining them as scientific posits solely in terms of their functional characterization, then I think this program fails if it treis to be a kind of ontological elimination or simplification. I believe it is hard to find any aspect of the mental, including consciousness in its full nuance and first-person subjectivity, which is *not* functionally significant. On the other hand, if functional analysis suggests how consciousness can provide adaptative benefits to its biological bearers, in terms of mental flexibility and the imagination to reshape our environment, then we find at least the outline of an argument for consciousness having evolved from nonconscious biological intelligence — and therefore being ultimately explainabile in terms of physical laws, at least if all biological processes are.

Ultimately, I believe the distinctly first-person nature of conscious contents and experiences does indeed affect public discussion and argumentation, including scientific, mathematical, compuatation, or other formal models of conscious reality and of the cognitive structures involved in consciousness. A science of consciousness must account for the obliqueness of its crucial elements to the conventional norms of scientific discourse and observation. Often philosophers seem to interpret this very obliqueness as an indicator that something is wrong with the "first person" view of consciousness: either consciousness thereby presented is not a reasonable candidate for scientific theorizing, or else the presumption that consciousness is indeed scientifically tractable compels us to conclude that the first-personal paradigm is problemmatic. However, I do not think a first-personal or "subjective ontology" is *necessarily* inconsistent with a scientific approach to consciousness. If we simply accept that formal treatments of consciousness must respect *both* the norms of scientific communication *and* the intrinsic privation of subjective conscious contents, our priorities can then turn to developing a set of discursive and theoretical norms for talking about subjective reality in the context of a larger, scientifically-inclined spirit of investigation. Rather than taking the disjunction between scientific publicness and first-person ontology as a metaphysical breakdown which can cause the entire project of scientifically treating consciousness to break down, we should instead see it as a careful balance to be achieved through stylistic and methodological norms. Indeed, I would suggest that the history of Phenomenology from Husserl trough to "Analytic" or mathematically-inclined phenomenologists like David Woodruff Smith, Jean Petitot, Barry Smith, etc., reveals consistent scholarship and discourse approaching this balance.

In this paper I have emphasized a particular line of investigation, and a particular direction of cognitive conscious operation, focusing on perception of object-surfaces extended (potentially though not necessarily through syntheses of multiple viewpoints and view conditions) to a schematic image of three-dimensional depth and geometry, and then through aggregation and identification of objects in terms of organized wholes and conceptual categories, locating objects in predicative networks and associating them with concept-types in the context of situational models. This is only one aspect of the perpetual dynamic movement of consciousness, its focus migrating between fine-grained perceptual detail and holistic situational understanding. I have mentioned, but not attempted to systematically treat, multi-sensory perception of surfaces and spatiation which accompanies visual experience, or the experience of ambient space and our overall surrounding environment as a visual and perceptual manifold on its own, apart from the surfaces of objects located therein. These are certainly important subjects for a more complete theory of the interactions between consciousness and cognition. Nor have I explored the more holistic and broader-scale "situational understanding", the background conditions, normative principles, moral and pragmatic reasoning, etc., which conceptually frames all of this cognitive activity. Arguably, most of the truly "human" themes of human reason lie within this layer. However, I think that the progression from object-surfaces to object-geometry to situationally contextualized predicative networks is an ever-present and structurally rigorous progression within the space marked by these layers. This particular combination of cognitive processes can then be studied as a canonical structure in the overlap between cognition and consciousness, from which analyses of other topics, such as the multi-sensory synthesis, the role of kinaesthetic experience and embodied movement, perception of coloration and other features of full three-dimensional regions and ambient space, and the broad structures of situational awareness, can be added on.

I also think that the various stages from object-surfaces through geometry and mereology to the layer of predicative networks all demonstrate details in which the richness and self-awareness of our conscious experience play obvious functional roles. Whether in the nuances of coloration which reveal spatial form, the sensitivity to how similar featural structural will reveal a spectrum of distinct appearances in different perceptual conditions and angles, or in the flexibility of our situational understanding, it is hard to envision human selves living their lives through these cognitive formations, forming complex social and physical environments, without the full theater of first-person consciousness lieing behind the scenes.

As a final methodological comment, the analysis from object-surfaces up to the layer of predicative networks, as schematic representations of conscious or subconscious beliefs as well as semantic structures, highlights an interplay between two different overall genres of model. There is an apparent topological intuition behind mereotopological accounts of object structuration and featural variation, as well as behind themes of conceptual blending, "Conceptual Spaces" as theorized (in particular) by Peter Gardenförs, Latent Semantic Analysis, and "dynamic" models of perceptual synthesis. For some phenomenologists (notably Jean Petitot) this topological intuition is clearly identified. The methodological concerns here are perceptual and conceptual details subject to continuous variation (for some perhaps not infinitely fine-grained notion of continuity). Complementing these methodological emphases, and their general project of modeling these possibilities of variation within a schematic structure, there are semantic and conceptual theories whose intuitions are more graph-oriented: trying to capture the schematic and network shape of semantic or perceptual wholes through networks of concept-relations, property-coinstantiations, and rhetorical structures. Moreover, analyses of these networks often also highlight the surrounding conceptual structures, types, and "background beliefs" which provide context for explicit semantic networks. These backgrounds or "frames" are suggested by the "background conditions" in David Woodruff Smith and Dagfinn Føllesdall, Gilles Fauconnier's "mental spaces", Michael Jubien's treatment of modality (for example with respect to fictional instantiators, which reveals some similarities to Fauconnier)[32], and the CGS concept of "support".

Individual objects often lie at the intersection of these more graph-like and more topological semantic and cognitive patterns. Their internal, continuous variation often determines their specific individual character and individuation from peers and surroundings. Once individuated, however, this internal structuration becomes cognitively "tabled", at least in many circumstances, as our beliefs about objects in relational networks focuses on them as singular concept-tokens, rather than complex inter-

[32] Concepts in Fauconnier's mental spaces are similar to Jubien properties, and, in particular, object-concepts are similar to singulary properties.

nal spaces. Jubien's property-theoretic approach captures this dynamic well, because singulary properties like *being this Coconut* encapsulate a spectrum of fine-grained modal details; there are many precise objects which can instantiate these properties, but our cogintive priorities attend not to specific instantiator but to the conceptual role, a role which is captured by the singulary property, elevating this property, more than the precise object which bears it, into the space of a relational concept network. Indeed, I believe Jubien's theory is motivated by similar intuitions to those expressed in the phenomenological notion of *noemata*: our perceptual impressions of an intended object, which are provisional and continuously varying, nonetheless engender cognitive activity where perceived obejcts are figured as concepts, mentally "completed" beyond whatever perspectival sensory acquintance we have (or, in the case of ficta, no sense data whatsoever). The more topological nature of continuous qualitative variation can capture our phenomenological, experiential synthesis of perceptual objects, but the more graph-like struture of conceptual networks captures our subsequent cognitive operations as we situate objects in predicative and pragmatic contexts.

The cognitive transition from individual object-perception to broader situational reasoning, and practical action, corresponds to a transition from a conscious domain in which subjective, first-person affect is more prominent, albeit interwoven with predicate judgments, toward a domain of interpersonal, normative and collective practical reason. It is at this point that computer technology or Aritificial Intelligence, as practical tools, become relevant. There are some domains — particularly the engineering of Virtual Reality scenes and vehicles, and the reconstruction of three-dimensional geometry from two-dimensional binary images — where we have a practical and technical interest in stimulating and/or simulating fundamental perception: creating realist virtual scenes, and mimicking our visual faculties. Outside these specific problem-spaces, on the other hand, the goal of computational tools is to become part of that interpersonal, social landscape: helping people and communities to store and extract information from language resources, and providing a semantic window onto non-linguistic graphical or multi-media content. If we recognize three "aspects" of conceptual phenomena — relational, foundational, and representational, a distinction inspired by the form / substrate / appearance triad in David Woodruff Smith's *Mind World* — then these technological problem-domains roughly correspond to the representational aspect, while modal and perceptual/featural variation roughly correspond to the foundational aspect, and conceptual networks to the relational aspect.

Graph-structured concept networks are often explicitly defined or obtainable from digital content by direct transformation, using algorithms which may be complex but are by no means intractable, such as morphing object-representations between different languages and schema (XML, SQL, different programming languages, etc.). `CGS` can be a useful tool for modelling the shape-shifting which goes on as objects are moved from persisted memory (e.g., databases), to live memory, to on-screen visual controls or woven into text and documents. Automating this process, and allowing its general parameters to be expressed more simply by programmers, is a non-trivial problem. Even modern-day rich, expressive computer languages, like Java or Ruby, struggle to develop ORM (Object/Relational Mapping frameworks) and similar solutions which are both concise and intuitive. So it can be valuable to develop elegant algorithm architectures for managing data aggregates that, once we look past the technical details, are compatible to semantic concept-networks. Research in this araa can allow both geographical and local communities to operate more effecticaly *and democratically*: synergise data, but also allow group norms and priorities to be systematically expressed and discussed in equitable forums. But these tools can be even more effective if the less directly network-like, the more subtle and topological patterns of meaning and signifance are modelled, to some degree and approximation. Effective LSA tools, for example, can help online communities manage deliberative processes insofar as rhetorically similar or related texts can be clustered and compared. Good technology replaces the kind of influence confirmation bias where communities struggle to equitably share access to the "public exercise of reason". Indeeed, the technology of online publication, debate, and civic participation — serving geographic as well as virtual communities — leaves room for a still under-explored theoretical cross between Habermassian themes of the civic sphere and computational and semantic technologies.

Apart from their place in conceptual networks — "orthogonal" to these networks' graph structures, in a metaphorically mathematical sense — concept-tokens have spaces of internal variation, be it modal, perceptual, qualitative, connotational, etc. The conscious synthesis of different qualia and perspectives into the image of one object, is perhaps the most complex and immediate example of this structuration, but there are related semantic patterns which occur outside of each persons' conscious perception. I have discussed examples of geospatial tokens, conceptual blending, coarse-grained concepts like "expensive", and modal properties of coarse-grained assertions. In these cases the semantic intersection between graph-like conceptual networks and internal, "topological" variation operates within interpersonal reasoning and discourse directly, rather than being a bridge between the private perception and communal situational cognition.

For modeling cognition and consciousness, both these graph-oriented and these topological intuitions are appropriate. The continuous and the network-like structures of concepts and semantics are interleaved and mutually influencing. I have tried to convey this compound structure, this synthesis of graph-like structures with topological continuity, by a formal model of graphs whose nodes are embbeded in topological spaces. Whether this "Conceptual Graph Mereotopology" can be applied with enough rigor to warrant formal mathematical treatment, or should serve merely as a useful picture to sharpen intuitions with, may depend on whether formal algorithms based on CGS, LSA, FCA, and related computational semantic methods can be integrated with these hybrid graph-mereological models. I will continue to research computational applications in these directions.

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