

# NASM

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## Abstract

NASM

Several authors have recently proposed “non-well-founded” or “non-anti-symmetric” mereologies. This terminology is relatively new, but at least some of the motivations behind their alternative systems derive from dilemmas that have inspired extensive commentary in the past. For example, is the clay out of which a sculpture is made a proper part of the statue? Most commentators seem to frame their intuitions in the intuitive parameters of part/whole hierarchies, and find some mechanism to reconcile the parts of their intuitions that don’t fit simple accounts of parthood. The more radical possibility, which is the subject of this paper, is to re-engineer our conceptual of parthood from the ground up. In particular, we can drop the assumption that parthood is asymmetric or acyclic. Doing so results in a Non-Asymmetric Mereology, or NASM. In NASM,  $x$  can be part of but different from  $y$  and vice versa.

As far as I can tell, most arguments for NASM derive from statue/clay-like cases where two things seem deeply, metaphysically intertwined but not identical. It is said that the statue (call it  $S$ ) is not identical to the lump of clay (call it  $C$ ) because  $C$  could be altered in ways  $S$  cannot — if a fire melts  $C$  down to a blob,  $C$  is still itself but  $S$  disappears. Ergo, an event can destroy  $S$  but not destroy  $C$ . Conversely, a small piece could fall out of  $C$ , leaving a hole then repaired with fresh clay, yielding a new lump  $C'$ . So  $S$  could end up being  $C'$  instead of  $C$ . But if we define  $C$  as just that exact lump of clay,  $C$  can’t be  $C'$  instead of  $C$ .<sup>1</sup> Ergo, as Aaron Cotnoir says, “it is natural to think that a lump of clay and a

statue made from it have all the same proper parts [but not] everything true of the clay [is, arguably] also true of the statue” (p. 397). In sum, Cotnoir and others who have thought about NASM highlight cases where two non-identical things have all the same proper parts. Such examples then lead toward the possibility of parthood being cyclical in some sense (which I’ll try to pin down later).

I think statue/clay-like cases, while they do raise important issues, are less than ideal as primes for NASM because they seem to invite numerous non-mereological resolutions. For instance, we can say (drawing from Jubien again) that  $C$  *instantiates the property of being  $S$* ;  $C$  is  $S$ ’s physical instantiation or realization. We have cognitive attitudes toward  $C$  that involve its physical form and nature; we also have cognitive attitudes toward  $S$  that thematizes its aesthetic and social facets (qua artwork crafted for public appreciation). Our agreement that  $C$  designates  $S$ ’s physical substrate — as described by terminology like  *$C$  is a material body instantiating the property of being  $S$*  — binds these two cognitive assemblies together, but not in a manner that readily propagates  $C$ -parthood to  $S$ -parthood or vice-versa. In other words, mereology itself is not a useful philosophical abstraction when too many divergent cognitive registers are involved. Or at least this is an escape hatch which brings us back to conventional mereology via the metaphilosophical claim that there are *other* analyses where the classical mereological models *do* fit our cognitive engagements. Statue/clay-like problems are not problems of mereology

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matter just is the matter that it is; we’re not talking about essential or inessential parts or any object that may be *constituted* by some matter, like Tibbles the cat and his later-amputated tail. In other words, we’re assuming that the self-identity of matter, once we ignore any logical properties inhering in its form, is immune from counterfactuals. Matter can be arranged differently, but it can’t be different matter. This seems plausible but not self-evident, but in any case it may be a terminological rule in how philosophers often use the word “matter” — and by extension phrases like “lump of clay” — especially in discussions like statue/clay (non-)identity.

<sup>1</sup> Another issue: Michael Jubien imagines that the sculptor could just have built  $S$  from another lump of clay in his studio, call it  $D$ . So, modally,  $S$  *could have been*  $D$ , but  $C$  could not have been  $D$ . If we use possible world talk, we can posit a possible world where  $S$  is  $D$ , but we can’t imagine a possible world where  $C$  is  $D$ . This assumes that a hunk of

as a theory but problems of where the theory should be applied.

This won't be my final word on statues an clay, but I will pivot to other examples suggesting how NAM is more pervasive than people realize. The literature seems to treat NAM as an exotic, corner-case, enigmatic exception to partonomic common sense: it seems to require special philosophical concentration to conceive of cases where parthood is really symmetric:  $x$  is part of  $y$  which is part of  $x$ . My goal is to argue the opposite: I think *asymmetric* mereology is really the special case. Of course, I understand that there's *some* notion of parthood which renders partonomic chains paradoxical. But I think in most common-sense cases where people think they are talking/reasoning about parthood, the kind of mereology they intuitively use has at last the potential to be NAM.

As my first exhibit, I'll mention several plausible and everyday-like sentences:

- ▼ (1) Mbappe was a big part of the 2018 World Cup.
- ▼ (2) Mbappe's footballing career is a big part of who Mbappe is.
- ▼ (3) The 2018 World Cup was a big part of Mbappe's footballing career.

Taken at face value, acquiescing to these sentences and to mereological transitivity, the 2018 World Cup is part of Mbappe, and Mbappe is part of the 2018 World Cup; but of course Mbappe is not metaphysically identical to the 2018 World Cup. Granted, sometimes part-talk in natural language is metaphorical: we don't hear *my religion is part of me* as megalomaniacal, or *you are part of my life* as prima facie possessive. But if this seemingly metaphorical way of talking is *more* common than our seemingly commonsensical mereology, maybe we need to reconsider whether classical mereology is really the base rather than derived notion.

Indeed, when we talk of  $y$  being part of  $x$  we rarely seem to talk as if  $y$  is *completely* part of  $x$ . Perhaps *total* parthood is a degenerate case. In fact, I could make an argument that classical mereology is paradoxical, like this: an intrinsic part of  $y$  is *being y*. If  $y$  is a proper part of  $x$ , by transitivity, *being y* is part of  $x$ . But if  $x$  is not  $y$ , it sounds absurd to say that *being y* is part of  $x$ . Now, I'm not actually raising this as an argument, because it can be countered: *being y* is not really "part" of  $y$  in the sense technically covered by mereology. Some of the

more "philosophical" interpretations of parthood need to be quarantined from the mereological explanantia. Fair enough. But I think this exercise shows that preserving classical mereology requires filtering out a lot of notions of parthood based on a prior commitment to asymmetry, which becomes circular unless we have a good analysis that the apocryphal "notions of parthood" are in some definable sense atypical or deflationary.

Transitivity is problematic in many ways; Mbappe's left foot has the property *being a foot*, which if it is part of the foot is not part of Mbappe. And Mbappe's foot is (arguably) not part of the team. Still, we'd like to preserve parthood-transitivity as much as possible. However, it seems that no matter how narrowly a part differs from the whole, it picks up some kernel of propositional difference that blocks transitivity without some complex conceptual patchwork: if we define parthood without a lot of conceptual detail *no y* not identical to  $x$ , however modestly, can ever be a proper part of  $x$  full stop.

That  $y$  is part of  $x$  does not seem to preclude  $y$  bearing predicative details that  $x$  lacks: for instance, if the camera zooms in on Mbappe it is not picturing the whole team, so although Mbappe is part of the team, Mbappe, but not the team, is covered by the predicate "being at the focus of the camera angle" at some moment. Of course, we already know that Mbappe is not *entirely* inside the team, so the prospect of Mbappe having propositional attributes which the team lacks is not concerning. But a camera angle can show Mbappe's *head*, which we might think to say is "part of" Mbappe in a stronger sense, or his *right side*, which seems even less independent. Indeed any picture of Mbappe shows part of him, making that part uniquely predicated as *what shown in this picture*. I cite these as further examples of conceptual subtleties that can trip up strictly "subsuming" mereologies — where an axiomatic distinction is made between *proper parthood* and *overlap*.

Usually when we hear part-like talk, we seem to instinctively look for overlap-style relations rather than subsuming inclusion of something smaller into something larger. That's why we don't hear it as odd or metaphorical if something apparently larger is presented as a part rather than a whole:

- ▼ (4) Phenomenology is only part of Merleau-Ponty's oeuvre.
- ▼ (5) Iraq and Afghanistan are only part of CENTCOMM's responsibility.

- ▼ (6) Tomorrow’s chicken soup (with the leftover chicken in it) is my favorite part of the chicken.

If we’re doing formal semantics, we might want to say that the propositional form of these examples is a matter of overlap rather than proper parthood. But something like *Phenomenology overlapped with Merleau-Ponty’s oeuvre*, or vice-versa, sounds awkward in comparison. At the very least we have an apparent datum that semantic structures involving overlap often seem to invite surface articulations involving explicit parthood. From the angle of NAM set against Classical mereology, of course, overlap and parthood have a significant difference: overlap is symmetric while parthood is not.

I’ll give another example set on the lost continent of Atlantis, to be concrete in a hypothetical way. Suppose the continent’s largest newspaper, the Atlantis Times, creates a consortium of semi-autonomous local newspapers in smaller Atlantian towns. We can then consider the Consortium to be part of the Atlantis Times: perhaps its offices are in the Times’s headquarters, and the Times funds, administrates, and legally controls the Consortium. But we can also say that the Times is part of the Consortium, if the Consortium includes a portfolio of papers one of which is the Times itself.

For a final introductory example, consider a web portal which includes a collection of resources. Suppose one of those resources is another web portals which in turn includes a collection of resources — one of which is the original portal! Reading “includes” as “has as part”, this is clearly a partonomic cycle. Indeed the whole “web” ideology is that resources link up in complex and dense (and often circular) ways.

I think these kinds of examples are both representative of “folk” mereology and structures where non-anti-symmetry is a more effective modeling assumption than anti-symmetry. In that case NAM is not the theory of a few ersatz cases but rather a general framework from which, if desired, classical mereology could be recovered as, so to speak, a proper part.

So here I will present my thoughts on what a “generalized” non-anti-symmetric mereology can look like and how it might be practically — i.e., technologically — applied. Then I’ll revisit philosophical terrain like the sculpture/clay conundrum.

## 1 “Criteriological” and “Expository” Mereology

In order to demonstrate a pragmatic, common-sensical NAM proposal, I will consider logical or conceptual relations which *seem* like mereological relations but which *also* seem like they involve a “cyclical” notion of parthood. To facilitate discussion I will use the symbol  $\triangleright$  to mean “has as proper part”, giving  $\triangleright$  different names in the course of debating which parthood-concepts are most appropriate. So parthood is modeled via  $\triangleright$  read left-to-right. I will restrict attention to cases where both (\*)  $x \triangleright y$  and  $y \triangleright x$ . I’ll assume by definition that  $\triangleright$  is “proper”, i.e. nothing is  $\triangleright$  itself, so in (\*) necessarily  $x$  is not identical to  $y$ . By this setup  $\triangleright$  is not transitive, since  $x \text{ nhpp } x$ , but call *chpp* the transitive closure of  $\triangleright$ . Then (\*) exemplifies a case where  $x \text{ chpp } x$ .

Consider the Atlantis example. I posed that as in effect the Consortium is an *administrative part* of the Atlantis Times, but the Times is a “constituant” part of the Consortium: if list the papers in the consortium, this list would include the Times. Granted, these are not exactly the same notion of mereology, so we could treat this kind of case as an anomaly due to two different mereological relations which happen to conflict. Of course,  $y$  can be part of  $x$  on some criteria but not others: French Guyana is part of Europe politically but part of South America geographically.

However, there are millions of notions of parthood, and mereology is not very semantically useful if we can only very rarely mix such notions together to form complex ideas. Problems like the violinist’s arm — which is not part of the orchestra — suggest that transitivity between two *conceptually different* kinds of parthood needs be somehow restricted, but we should leave open the possibility that conceptually different mereologies can still find *some* propositional connections. This means trying to build in to the mereological theory a sense of the conceptual structure of the parthood relations thereby theorized.

I’ll start by defining  $\triangleright$  in conceptual terms as something like: “has as proper part by all relevant criteria”. In other words,  $x \triangleright y$  means *y is part of x by all criteria relevant to x*. This does not exclude the mere Ontological possibility that some part of  $y$  may be outside  $x$ ; but any such parts are of no importance for  $x$  — i.e.,

for any propositions of practical significance for people engaged with  $x$  insofar and during the time that they are engaged with  $x$ . On Atlantis, the Consortium is an “administrative” part of the Times, and let’s imagine that for all practical purposes, for everyone working for or otherwise engaged with the Times, the Consortium is just “part” of the times (no conceptual qualification needed).

But notice that  $\triangleright$  hereby conceptualized could easily be symmetric. We could have both  $x \triangleright y$  and  $y \triangleright x$ , if  $x$  is part of  $y$  by all criteria *relevant to y*. The fact that we are evaluating criteria relative to the whole allows  $\triangleright$  to be inverted, since the former part then *becomes* the whole and relevance-criteria are assessed on *its* terms. For employees of the Consortium — those who promote, deliver, index, or represent the papers, or whatever — the Times may be just one of several papers in the portfolio. By *their* criteria, the Times is part of the Consortium, not the other way around.

Someone looking from outside might prefer to say that neither institution is truly a proper part of the other. Their vantage point compels them to consider *both* parts’ criteria, and in this holistic sense there may be parts of the Consortium that are not practically speaking parts of the Times, and vice-versa. It might seem then that the “real” picture involves no proper parthood on either side — and therefore no NAM.

That analysis is not wrong *per se*, but it may pare down parthood relations unacceptably. After all, when are the occasions where we say that  $y$  is part of  $x$  *completely*, in every sense? An institute inside an academic department may be spatially and administratively part — its offices inside the building; its staff deemed departmental staff. But the institute might develop curricula, plan events, embrace intellectual paradigms, and form a social circle somewhat autonomous from and tangential to the department. A specialized imprint of a publishing house may prioritize disciplines different from the larger company. Usually parthood implies some level of autonomy, in real situations, because we usually don’t expend the conceptual and bureaucratic effort to keep track of some mere part if the part’s behavior or properties is fully predictable from the whole. A semi-autonomous part can still be a part — mereology can permit autonomy; indeed this is a defining characteristic of complex systems — but once we allow parts’ autonomy it is easy

to soon realize aspects of those parts that make them no longer seem *completely* part of their wholes. We do not have to abandon mereology completely if we argue that those residual parts are not *relevant* to the whole, and therefore do not interfere with propositional attitudes concerning the relationship of the whole’s terms.

This definition of mereology is arguably more robust, because it allows conceptually different mereological relations to be unified. The administrative nature of the Times including the Consortium can coexist with the “compositional” nature of the Consortium including the Times, as part of one mereological system. The key detail here is that we allow  $\triangleright$ -relations to exclude “irrelevant” parts — which of course means we introduce a criteria of relevance, which can “filter” the Universe. If any  $x$  is a whole, it can consider  $y$ s as its parts on the basis of filtering away irrelevant details. Therefore  $x \triangleright y$  does not force all  $y$ -parts to also be  $x$ -parts (like, say, *being y qua metaphysical part of y*), but just that any stray  $z$  not be a *potentially relevant* part of  $x$ .

I’ll call the version of  $\triangleright$  just outlined “criteriological” because it depends on relevance-criteria localized to each whole. Note that  $\triangleright$  decays to “Classical” mereology if we stipulate that there is only one global set of relevance criteria across all wholes in the analysis. Thus classical mereology is a restricted form of this “criteriological” mereology.

Next, I’ll generalize further. As I said, things are not usually *completely part* of other things. Usually the appearance of complete parthood is an illusion conjured by how parthood relations are disclosed. For example, surely Kyrian Mbappe is part of les Bleus. But how do we know this? Presumably we see his name on a list of les Bleus’ roster, or perhaps see him on the pitch with the squad. But the former case is not actually a warrant (with no further logic) for Mbappe being *part of* les Bleus; it is rather *Mbappe’s name* being listed as *a member of les Bleus’ roster*. We use the referential relation between his *name* and Mbappe himself to project to the idea of the player being part of the team.

But this referential indirection carries all the potential for criteriological criteria I have discussed. There is a sense of Mbappe being part of les Bleus relevant to les Bleus — their training, formation, tactics, marketing, popularity, etc. Obviously not every part of Mbappe is relevant to the team; he does not train with them



every minute. He has a whole other career at Paris St.-Germain. But aside from the cognitive complexity of tracing in what sense he is part of les Bleus, there is a nagging problem of defining what *Mbappe is part of les Bleus* actually means. Maybe this is simpler in well-defined contexts: surely he is part of the squad when he lines up in the starting eleven and the ball is kicked off. But surely also Mbappe being part of les Bleus is a more general phenomenon than just those moments on the pitch.

In real life, a concept like an athlete being part of a team can actually involve complex legal, financial, and procedural criteria, so it may require a detailed contract to state rather precisely what *being part of a team* actually means. However, supporters know the players on their teams without knowing the requisite contractual minutiae; in short, a fan's acquaintance with their team's history enables them to summon a list of current or past players on demand. Ask a fan who is part of the team, and they will rattle off a list of names. In this sense, they are conceiving a kind of parthood which we might call *enumerative*:  $x \triangleright y$  if we would (under ordinary circumstances) include  $y$  when enumerating a list of  $x$ 's parts.

Technically, though, an “enumerative” mereology is “indirect”: we use, say,  $y$ 's name on a list of  $x$ 's parts as proxy for  $y$  being part of  $x$ . Presumably  $y$ 's name is on the list because it *is* part of  $x$ , at least on some criteria. But as such criteriological relevance is built in to the list construction. To enumerate the parts of  $x$  we are not committed to those parts being wholly subsumed under  $x$ ; just that they are *members of*  $x$  in some salient context. An  $x$  taking mereological relations is then a matter of  $x$  being conceptually figured as a collection, aggregate, or set. Of course, wholes can be conceived as multiples in different ways: all players in a sports franchise's history is one kind of plurality; the current roster is another; all the team's employees is a third.

I would argue that the most common kinds of mereologies in practice are some variations on this theme: to conceive  $x$  as a *whole* means to conceive it as a *plurality*, which introduces the possibility of numerating its members, which thereby become its *parts*. But rarely are part/whole relations thereby conceived where the whole “surrounds” the part, absorbing it so that mereological partiality vs. totality can be readily resolved, like a room

being part of a house. Usually the members of  $x$  qua plurality bear instead some functional and integrative relation to  $x$  in some context.

In that sense “enumerative” mereology may seem to be incomplete, because the very act of conceiving a  $y$  in its functional role as a member of  $x$  seems to color how we are disposed to  $y$ ; we seem interested in  $y$  as member of  $x$  rather than on its own terms. In practice, however, conceptualizing many things — as least initially — as members of some multiple seems epistemologically unavoidable. Most people would never know of Kylian Mbappe *except* as a forward for les Bleus. Usually we are introduced to things via reference to a containing whole, and usually that whole is figured as a plurality, collection, or type (rather than as a physical or spatial part, say): we are introduced to a friend's cousin as a member of her family; we learn of a new young athlete when he is drafted by a team; we learn about our friend's dog by first being told his breed. Epistemologically the aggregate-whole gives us an *entré* which we can then follow up by learning about the part/member on its own terms.

Mereology in this kind of situation then models a kind of epistemological sequencing, tracing how our cognitive attention can migrate from whole to part. It is easy to see how parthood in this context can come out circular, because the pursuit of knowledge often circles back on itself. We learn about Mbappe because of our interest in les Bleus; then we learn about Mbappe's career, of which the 2018 World Cup was an important part; but then we circle back to les Bleus. Mbappe is part of an “epistemological” whole in the sense of our desiring to learn a relatively complete picture of French international football. Mbappe is part of any encyclopedic treatment of les Bleus; likewise, les Bleus is part of any encyclopedic treatment of Mbappe. I might call this *encyclopedic* mereology. Encyclopedias, indeed, are almost essentially cyclical in how references link back and forth. But everyday language suggests that these information networks can be understood as a mereology; we can readily accept sentences like:

- ▼ (7) Husserl is part of that Encyclopedia's article on Mathematical Foundations.
- ▼ (8) That Encyclopedia of Analytic Philosophy includes Husserl but not Brentano.
- ▼ (9) Iraq is a big part of Bush's legacy.

Of course, Iraq is not literally part of Bush’s legacy or of CENTCOM, as if these were geographic territories. But the *idea* of Iraq, or in some sense “responsibility for” Iraq, finds a place there conceptually.

Granted that only a hard-core idealist would equate the mereological relations of the *idea* of something with that thing’s own mereological relations. Surely the *idea* of Iraq is part of many things that Iraq itself is not part of. But in fact many practical applications of mereological theories depend on tracing mereological relations in a network of *ideas*, or at least something cerebral and/or computational rather than physical: Web Ontologies, Information Systems, and so forth. Consider the case of a web portal  $W$  whose resources include a portal  $W'$  which links back to  $W$ . In general, web portals *contain* resources in the sense that they link to or provide access to web resources — resources that are not necessarily “part of” the entry-point in the engineering sense of being on the same servers, or being URL subdomains. So *to be part of* in an Information Space  $I$  (I’ll use this as a generic word for database, portal, Information System, etc.) generally means that  $I$  provides access to, provides a kind of structured entryway to (e.g. a searchable front-end), or “exposes” some affiliated resource  $R$ . Here saying  $R$  is part of  $I$  also means that  $I$  *links to*  $R$ , which suggests  $R$  and  $I$  are peers rather than granularly mismatched.

In short, in structures that might be conceived as Information Spaces, the commonsense picture that mereology implies a difference in scale — parts are at least somewhat smaller than wholes — seems readily contradicted. It might be argued that this is an eccentric feature of mereology in “Information Spaces” which, as essentially cognitive domains (albeit somewhat depersonalized and mechanized technologically) don’t obey the usual laws of mereology. But Information Technology is at least where many philosophers are now trying to embed mereology (or mereotopology) as a technical artifact, so the kind of mereological relation germane to IT should be taken seriously as a candidate for mereology in general.

Taking a cue from the “linked data” nature of Information Spaces, the underlying model of parthood in this kind of theory might involve some kind of epistemological linkage, where access to or information about parts is part of the epistemological interface afforded to their wholes. Mbappe as part of les Blues means, for instance, that an information-source profiling les Blues

should link or provide entry to a comparable source profiling Mbappe. As a notion of parthood, this contains an extra layer of indirection, since we can distinguish a *link to*  $y$  from  $y$ : an encyclopedia entry on les Blues which Mbappe is a part of actually contains a *link* to Mbappe. And we get informed of Mbappe being part of les Blues’ roster by seeing his *name* on a list. Thus mereological relations often involve an intermediary name, link, or designation which stands in for an actually autonomous part in partonomic contexts.

To make this somewhat formal, we need two relations: first, a notation like  $p \triangleright y$  which I’ll call (borrowing a term from Cognitive Grammar) “profiles”. The relation of a  $p$  profiling a  $y$  could be read, according to context, as the conceptual tie between a web address, computer pointer, or other technological reference-artifact and its target; or the designatory relation between a proper names and their objects; or a more cognitive form of reference. Then I’ll introduce a whole-to-profile relation  $\triangleright$  such that  $x \triangleright p$  can read *x contains p, which profiles something (other than x)*. Combining  $\triangleright$  and  $\triangleright$  yields a three-part double relation like  $x \triangleright p \triangleright y$ , for *x contains p, a profile of y*. Then finally a version of  $\triangleright$  can be defined as this three-part relation abstracting  $p$ :  $x \triangleright y$  becomes *x contains a profile of y*.

This particular version of  $\triangleright$  may be useful because it is adaptable. It accommodates systems where profiles of  $y$  either are or aren’t *parts* of  $y$ , according to familiar mereological criteria. Depending on how that goes, this latest  $\triangleright$  could model “criteriological” mereology: suppose the profile of  $y$  in  $x$  is the only part or aspect of  $y$  which is relevant to  $x$ . Then  $p \triangleright y$  acts as a relevance filter, the profile selecting salient parts of  $y$  and excluding residual parts from mereological consideration. Then  $x \triangleright p \triangleright y$  can be read as *x encompassing all parts of y when filtered by p*. Either  $p$  expressly operates to isolate  $x$ -relevant aspects, or  $x$ -relevance is derived from a filtering according to more general criteria, like the properties of a restaurant being subdivided into culinary, operational, nutritional, and architectural dimensions — i.e., a restaurant has architecturally relevant parts, operationally relevant parts, etc.

I will do further analysis however of the alternative model where profiles are *not* in general parts of what they profile. Instead, a profile is like an epistemological or technological device that “exposes” or permits access

to something else, like a pointer to a region of computer memory. I'll call this model "Expository" mereology. The canonical idea of parthood is now that  $y$  is a *proper part* of  $x$  if and insofar as  $x$  "exposes" or provides an information link to  $y$ .

## 1.1

### Normalizing Arbitrarily Granular Mereologies

One potential benefit of the Expository model is that it may be applicable to and/or reflective of how technology concretely implements mereological systems. I contend that any classical mereology can be directly mapped to an Expository mereology: take any classical parthood instance  $x \triangleright y$ . Designate a profile for  $y$  inside  $x$ , say,  $\mathbf{p}_x$ . Then reinterpret  $x \triangleright y$  to mean  $x \triangleright \mathbf{p} \triangleright y$  where  $\mathbf{p}$  is  $\mathbf{p}_x$ , along with a restriction that any  $y$  can have at most one profile, and has *exactly one* profile if it has a (classical) whole — actually this one profile is part of the whole *in lieu of*  $y$ .

So expository mereology includes Classical mereology as a special case, but it allows for generalizations which patch over conceptual objections that may be raised to Classical mereology in its unadulterated form. In practice the maxim that each  $y$  has only one profile may be too restrictive. Instead, multiple divergent wholes may overlap with  $y$  in different ways and contexts. For instance, Mbappe is part of les Blues, and also PSG, the Afro-French community, the Mbappe family, etc. Granted, this may be changing the subject somewhat: "overlap-systems" characterized by generally complex entities that overlap in different modes and contexts are a different area of philosophy than mereology. But in reality these two theories are intertwined: many conceptual phenomena can be approached both from a mereological perspective and an "overlap" perspective. The two kinds of theories may be viewed on a spectrum, with mereology merging as we attend more to the filtering effects of  $x \triangleright y$  parthood, the relation either witnessing or effectuating our disposition to ignore non- $x$  parts of  $y$ .

To put it differently, arguably any mereological system is an overlap-system which we are able to filter or simplify to reduce cases of practically inconsequential externality that would otherwise block proper parthood. If almost always  $y$  is *never* an *completely subsumed* part of  $x$  then (classical)  $x \triangleright y$  has to reference some kind of theory that

$y$ 's non- $x$  parts are inconsequential. So "relevance" can be like a knob tuning in mereological or overlap theories depending on whether we are more or less sympathetic of filtering: mereologies emerge when we tolerate filtering non- $x$   $y$ -parts in  $x \triangleright y$  as practically appropriate, and overlap theories arise from mereologies when we realize that filtering skirts around legitimate Ontological, cognitive, conceptual, or natural-language/pragmatic concerns. I think that the system of operators  $\triangleright$ ,  $\triangleright$ , and  $\triangleright$  (when defined from the other two) models both mereological and overlap systems and accordingly can unify both kinds of theories.

Aside from this philosophical case, however, there is a practical benefit to the "Expository" definition of  $\triangleright$  which applies to Classical as well as non-antisymmetric mereologies. Note that according to Classical measures of parthood, Expository mereology only has parthood relations between wholes and *profiles*, and moreover we can assert with no loss of generality that [ro]files themselves do not *contain* (as opposed to "point to") other parts. I assume a semantics where profiles are not themselves organized data structures, but rather referential atoms leading to (arbitrarily complex) structures outside themselves. Proper reference in, say, natural language is not quite so simple — consider first and last names — but we certainly do seem to have a conceptual ability to coalesce cognitive quanta with almost no internal structuration, save for designating intellectually complex structures; and with this mechanism build up arbitrarily complex cognitive models. Analogously, computer software uses pointers to build up arbitrarily complex data structures without unworkable amounts of memory manipulation. Our ability to designate complex wholes with simpler icons — consider cities as dots on a map, or facial portraits as links to bibliographies — sure is a key enabler of complexity in our conceptual and semiotic systems.

In short, we lose no complexity when we envision "profiles" as intellectual quanta whose only signification is as a rational bridge to something else; i.e., in a mereological system, profiles need not have their own parts. Accordingly, on Classical terms, we have only one "wholes" layer and only one "parts" layer: there are wholes whose parts are profiles, and profiles refer to other wholes. This two-layer architecture takes the place of a classical system where partonomic nesting may be arbitrarily deep. Of course,  $\triangleright$ -chains can be arbitrarily long, but in Expository mereology — although  $\triangleright$  *conceptually* models

parthood — the relations  $x \triangleright y$  are not considered to be on intrinsically different scales. The  $\triangleright$  relation is across levels, but in  $x \triangleright y$  we go from a whole  $x$  to a *profile* and then back to another whole. This is not to rule out some scale of size within the order of wholes, but that detail is not intrinsic to the system.

I contend this kind of model with limited granular levels is a more accurate representation of how Information Systems actually work, considering the design of resource networks (like the World Wide Web) and software systems (with objects and pointers) or (relational) database architecture (with tables and primary keys). It is technologically simpler to have only two or three levels of organization and model complex structures via some kind of pointer or indirect (e.g., foreign-key) reference. Internally, technology that interacts directly with multi-level, hierarchical information — consider an XML database — transforms this structure into something more like an object-graph (consider an XML Document Object Model).

I will generalize the two-level model a small bit, with the following rationale: on occasion the intuition that profiles point toward *one* target may be too restrictive. Suppose  $p$  profiles a *set*,  $s$ . We could treat  $s$  as a whole and model its members via their own profiles in  $s$ , but then we have an extra layer of indirection that may serve no modeling purpose. For this reason I allow that there may be a level higher-scale than wholes, which I’ll call *frames*. This results in a three-level system: a higher level with frames, a lower level with profiles, and an intermediary level which contains most of the primary objects of investigation or conceptualization. Profiles can target multiple objects at this mid-level by targeting a frame rather than a single object. The significance of frames emerges in some semantic and technological contexts where we want to distinguish between relatively dispersed collections and complex wholes with some organizational coherence, such that we are inclined in many contexts to treat them as singular. That is, arbitrarily collating any collection into a whole may dilute a model’s ability to distinguish between integrated wholes that often function as singles, from fiat wholes that arrange encircling in a specific context but do not on most criteria cohere as individuals. So as not to bias “wholeness” toward either coherent or fiat aggregates, I propose a “frame” level for “fiat” wholes distinguished, as a system feature, from intermediary wholes with significant *individual*

*coherence*.

With this addendum, Expository mereology then becomes a three-level system. Arbitrarily complex scales of granularity can be modeled *within* the levels, particularly at the intermediate level, but the formal model can express a technological design where only those three levels need to be implemented as computational primitives.

A consequence of this design is that arbitrarily complex mereological systems can be encoded in hierarchy with only three levels, a process I’ll call “normalization”. This process is philosophically analogous to normalizing a hierarchical document database to a computationally more malleable graph database. Moreover, I will close this section by noting that the Expository mereology relation is quite naturally non-anti-symmetric: if  $x$  contains a profile of  $y$  there is no restriction against  $y$  containing a profile of  $x$ . With that in mind I’ll refer to an Expository mereology using the general three-level model and its associated non-anti-symmetric  $\triangleright$  relation as an “N3” model or encoding.<sup>2</sup>

## 2 N3 Mereology and Cognitive Frames

One way to approach mereology as a philosophical topic is to define different mereological systems, including cases where these differences can be observed “axiomatically”: the presence or absence of an anti-symmetry restriction on parthood, for example. To make this exercise worthwhile, it is then necessary to describe the philosophical or practical implications of the logical divergence: is the system with one logical form vs. another a more faithful model of thought, or a more useful directive for technology, or somehow better scientifically? After all, it’s not like the rules of mereology are written in the cosmic order; mereology is not an empirical science.

A related question is whether a given mereological theory is intended to represent how we, as humans, *think about* parthood, or to represent part/whole relations which have some causative or compositional role in nature. Given a partonomic assertion — that a leaf is

<sup>2</sup>I don’t propose this term outside the present writing because it conflicts with N3 in the Semantic Web — a notation for graph structures. N3 mereologies are actually a superset of N3-expressible data structures, where the second N3 is the Semantic Web term.



part of a tree, say — we can read this as a description of conceptual attitudes: that most people (by virtue of perceptual gestalts or lexicosemantic pressures or sub-conscious internalization of others’ enactive-conceptual habits or some other means) instinctively see and comport to the leaf as part of the tree (and the tree as including and encompassing its laves). We can also read this parthood as saying that *literally*, as a feature in how leaf and tree exists according to biological and physical laws, the latter encompasses the former.

Certainly these two senses are not completely independent. We probably would not entertain mereological attitudes without some pragmatic or physical sense that these attitudes are grounded in reality — that the part we ascribe to a whole does indeed behave as if under the constraints of parthood. On the other hand, insofar there are “physical” criteria of parthood, we presumably learn of them alongside other scientifizable aspects of phenomena, so that mereology becomes part of the overall package of our scientific model-building. At that point we may try to isolate parthood as one important, recurring facet of scientific models and specify how, or the different ways in which, parthood is thematized within scientific explanations or proto-scientific intuition.

But still, the difference between parthood as a matter of conceptual attitudes versus (in some sense) nomic given is consequential for how we read individual parthood assertions. Given  $x \triangleright y$ , do we see this as matching  $x$  and  $y$  to physical (or at least extramental) objects? Or does it mean that in our cognitive engagements with  $x$  we experience or believe  $x$  to include  $y$ ? If we read *the leaf is part of the tree* wholly extramentally, we have to explain the referential logic of “the leaf” and “the tree”; i.e., what sort of entities these are such that they can be compactly signified. We could be fully realistic about (mentally) external things — let’s agree there really is a tree out there that many people see and therefore can be a topic of discussion. We still have to explain how there is a referential pattern which grounds use of notation like “the tree” as part of logically sensical assertions (including mereological ones). We presumably see the tree as a gestalt unifying perceptions of its trunk, branches, and leaves, but plugging this unadorned sense into the parthood assertion becomes circular, since the leaves then become *part of* the referential grounding of *the tree*, which empties the assertion of any content. Mereology would cease to be a philosophically relevant

topic if its assertions were wholly on the order of, say, the number 11 being part of the set  $\{11, 22, 33\}$ .

Note that the problem of referential circularity does not necessarily arise in the same way when we think of mereology as a cognitive phenomenon. I don’t think it is trivial to the point of meaningless to ask if our *concept* of 11 is part of our *concept* of the set  $\{11, 22, 33\}$ . Philosophy is a conceptual activity, so it is simplest to read philosophical theories as models of other conceptual activities. Of course, however, usually our conceptual activity tries to stay oriented to extramental reality, so philosophy captures the structure of conceptualizations somehow interfaced with reality, and philosophy’s own concepts and notations and quantifications to some degree represents this dual appointment: sometimes we’re talking about cerebral artifacts and sometimes we’re talking about real things intended by (using *intend* phenomenologically) cognitive acts. In practice, it can be hard to disentangle the cerebral and (by discursive intent and cognitive intention) extramental artifacts as referential patterns, or quantification domains for logically-structured units that arise in the course of argumentation (like  $x \triangleright y$ ).

To clarify these points, consider for a moment the famous Putnam Twin Earth discussion. As the scenario is described, Twin Earth harbors an XYZ substance functionally identical (for all purposes relevant to Twin Earthers) but compositionally different from  $H_2O$ . We can then entertain questions about whether Twin Earthers’ water-concept — which apparently refers to a different Natural Kind — is the same as our water-concept.

This setup makes several assumptions of its own. First, it assumes that we have a canonical water-concept, and that it (either essentially or incidentally) refers to a Natural Kind which is the substance  $H_2O$ . This is a simplifying assumption in multiple respects; one of which is that  $H_2O$  itself encompasses several different chemically distinguishable substances (if we consider various forms of heavy water). Second, even our everyday water-concept is divided across different contexts: we would probably call both ocean water and tap water in a bucket a *bucket of water*, but we probably wouldn’t call ocean water in a glass a *glass of water*. So our “water” is probably a fusion of several different concepts, the stuff in oceans and saline lakes plus the stuff in freshwater lakes and rivers (including the potable stuff that by aqualogical engineering is delivered to our taps).

The water in some saline lakes is actually much less “pure” than blood plasma in a hospital, yet we are not inclined to call plasma “water”. On the other hand, exceptionally pure water — distilled water — is not even a prototypical kind of water; that’s why it needs a special concept. So any trivial equation between *water* and  $H_2O$  is problematic.

Meanwhile, the Twin Earth discussion is also noncommittal about how XYZ is supposed to differ from  $H_2O$ . We can imagine the XYZ components as very similar to Hydrogen and Oxygen — for instance, imagine XYZ as a relabeling of DHO, or “semi-heavy” water with one Deuterium atom. I don’t think we should have trouble as accepting that XYZ is then just another kind of water (like heavy and semi-heavy water). Or perhaps Twin Earth has some new subatomic particle that can click to Hydrogen and make it ‘X’, like the extra neutron that makes Hydrogen into Deuterium. Perhaps, that is, XYZ is functionally similar to water because its constituent parts are similar to earthly Hydrogen and Oxygen. As with heavy water, there is already a precedent for expanding our earthly water-concept to accommodate more complex chemical models of the water molecule.

I think the Twin Earth discussion only really has philosophical weight if we assume that XYZ components are *significantly* different than Hydrogen and Oxygen. Of course, we also have to assume that XYZ behaves enough like earthly water that these differences have no practical effect for Twin Earthers. Among other things, we have to assume that they are technologically more primitive than we are. After all, among the functional characteristics of water for us is that we can derive Hydrogen and Oxygen from it; assuming XYZ are not just special kinds of Hydrogen and Oxygen, presumably this is not true of XYZ. This could easily bias our assessment of Twin Earthers’ water-concept, because *we* know that there are functionally salient differences between XYZ and  $H_2O$ . Perhaps we can’t help but imagine that eventually Twin Earthers’ knowledge may eventually reach the point where the differences become relevant to us, just as many years passed before humanity learned about Hydrogen and Oxygen; then of course the assumption that their water is (relative to their own needs) functionally identical to ours (is we factor out the level of our practical engagement with water that surpasses their technological capabilities) breaks down.

Intuitively, no doubt, we consider both functional and physical/material criteria when circumscribing the extent and intent of concepts. If I take coffee at a vegan friend’s house and ask for milk, it is not impolite for her to bring almond milk. That is, we are prepared to subsume “almond milk” under the concept “milk” in some contexts. However, we are reluctant to draw concepts based solely on functional resemblance, ignoring obvious compositional differences such as those between milk and almond milk. Surely this is due in part because compositional differences, while they may be irrelevant in *some* contexts, are usually relevant in other contexts. For instance, milk and almond milk are nutritionally biologically different.

Our functional and “compositional” criteria for concepts are not usually in tension, because usually there is enough correlation between the two kinds of differences that, over a broad set of contexts, they tend to reinforce each other. That is, there are contexts where functional resemblance seems to warrant conceptual unification even pace apparent compositional differences. There are other contexts where functional *differences* might supercede compositional similitude: think *aluminum foil* or *copper wire* compared to blocks of aluminum or copper (or, concepts like *statue* and *pot* are different from each other and from *clay*). Of course, in these last examples, material form — shape and arrangement — contributes to functionally different behavior, so we can include physical morphology as properties as brute composition (consider, though, the special value accorded to statues and objects d’art of reputable creators, rooted in properties of provenance that are orthogonal to both physical constitution and material form). But, in any case, we also have linguistic and situational faculties to construct contextually “local” maps of concepts — how *in this context* concepts subsume or fit together in particular ways — without confusing these local pictures for global schema. With sufficient integration of many contexts, our intellectual and linguistic dispositions tend to (collectively as a language- or social-community) converge on a mapping of concept’s boundaries and inclusion/subsumption that reflects both functional and physical/material criteria with neither set of criteria dominating the other.

Putnam’s Twin Earth experiment invites us to imagine scenarios where the approximate synergy between functional and compositional criteria breaks down. In a hypothetical case where functional resemblance persists

despite (significant) compositional differences, and one a wide scale across contexts, are we prepared to find conceptual unity (siding with the functional resemblance) or conceptual bifurcation (siding with the compositional difference)? At one level, this is hard to thematize straight-on, because the very construction of the case-study seems to undermine its requisite presuppositions. As I said, I think the thought-experiment is most thought-inducing even we assume significant enough difference between XYZ and  $H_2O$  as compositional substrata of water (or twater); so with chemical knowledge akin to ours, XYZ *doesn't* behave like water. So there are meaningful contexts where functional resemblance *does* break down. We have to assume however that these contexts are not relevant on Twin Earth because Twin Earthers don't have, say, equipment to separate water into Hydrogen and Oxygen (or, analogously, XYZ into X, Y, and Z).

I think what began as a discussion about *concepts* ends up really being a discussion about *contexts*. There are of course local contexts where non-standard concept maps are drawn (like milk/almond-milk). As I have argued, we are competent in juggling local and “global” conceptual maps (“maps” in the sense of how conceptual “territory” is partitioned), exercising a mixture of linguistic and situational understanding; a partition in rational communities sharing language, norms, and the pragmatics of everyday life. Therefore we mark nonstandard conceptual maps to as *local* to given contexts, whereas we also have a sense of concepts as intellectually global resources, which adjust for local nuances in predetermined ways. Our concept “*water*”, for example, is presumably a federation of narrower concepts (notably saltwater and freshwater) which we unify for both physical and functional reasons: while not pure, the primary substance in both cases is  $H_2O$  (which is actually third concept *subsumed by* water), motivating the unification, plus they have functional similarities in many (albeit not all) contexts. So, by panning out from local contexts into a globally trant-contxt conceptualization that is the best compromise between global generality and local specificity, a canonical concept merges which unifies other concepts but also has some internal integrity (i.e., the identification of water with  $H_2O$  grounds our conceptual norms in established science). The panning from local to global contexts is then a key semantic detail in establishing “canonical” versions of concepts.

I think the real force of Twin Earth is that it introduces

two different possibilities to “panning to global context”. Global becomes relative: do we mean to generalize Twin Earth contexts only to those which are likely to be efficacious on Twin Earth itself? That is, should we assume that there will never be a global context affecting Twin Earthers’ conceptualization of water that would establish a ground for contrasting this concept with (earthly)  $H_2O$ ? In that case, an “internalist” might say that twater is the same concept as water, applying the maxim that conceptual boundaries are drawn to reflect the interplay between function and composition as we pan from local to global contexts. No matter how “high” we pan out, on this argument, we will never encounter a situation where compositional difference triggers a potential functional difference that was lurking behind local functional resemblance — analogous to how the biological difference between milk and almond milk is bound to arise in many contexts, whether or not it is locally relevant. But water/twater differs from milk/almond milk because (according to the setup) there is *no* context we can encounter when we “pan out” from local to global which makes the water/twater difference consequential.

Conversely, we can read the same scenario differently and propose that “Global” cannot be read in such a restricted sense. The global context or context-synthesis available to Twin Earthers — the level of abstraction beyond their local contexts — is not the *real* global context, since *over and above that*, by stipulation, *we* provide an encompassing global context of which Twin Earthers’ global context is just a part. For *us*, the difference between XYZ and  $H_2O$  is functional, not just compositional: the thought experiment stipulates (or should do so) that the two substances are compositionally different enough that functional differences would arise in contexts that depend on splitting water into its constituent parts (if X, Y, and Z are just slight variations on Hydrogen and Oxygen, or chemically transform to Hydrogen and Oxygen, I think the discussion becomes moot; on par with water/heavy water, which doesn’t involve any extraterrestrial stories). What makes Twin Earth (stipulationally) unique is that *its* global context is not global enough from *our* vantage point.

We can certainly debate whether Twin Earth’s context still deserves to be called “global”. If it does, I think we end up with an Internalist theory, since we’re saying that criteria of globalness should be measured against the cognitive resources of a rational community: if there’s

no context where compositional difference is practically relevant, then we may as well use functional criteria alone to establish concept partitioning. Conversely, if we say Twin Earth's context is *not* authentically global, I think we are led to an Externalist theory. If two conceptual cores refer to different physical kinds, and we can range over every possible context (without regard for how cognition is grounded in the practical machinery of knowledge acquisition), then we can certainly say that compositionally different kinds represent referentially or extensionally different concepts.

But, I would argue, this is really a debate about Externalism and Internalism vis-à-vis *contexts*, and specifically about *globality* of context. When we pan out from local contexts, do we reach “global” when we reach the horizon of conceptualization context that is empirically possible for the relevant cognitive community? Are latent functional differences beyond this horizon factors in concept-identity? Internalists basically say that this horizon *is* the global context, so “global” is an attribute relative to the epistemological possibilities of any cognitive community. If there is no epistemologically possible world where a stipulated compositional difference becomes functional — i.e. there is no possible world where the knowledge of the difference can be reached from a community's present epistemological state — then we can say that in the “epistemologically possible horizon” there is no context which “functionalizes” the compositional difference. Then there is no point in saying that such a horizon is *not* global. But if we are allowed to imagine that any epistemological being whatsoever can “look down upon” that horizon and see beyond it — in other words, that globalness is external to any one mind (or any one community's cognitive resources) — we end up being “contextual externalists”, assuming that the only real “global” context is the maximal context which is metaphysically possible, where everything knowable is factored in, such as the functional consequences of any compositional differences. In either case, any notion of competing externalist and internalist intuitions about *concepts* appear to piggyback on corresponding intuitions about (global horizons of) *contexts*.

Perhaps my line of reasoning here feels like I am presupposing the answer: the true externalist claim is that concepts which bear (even unbeknownst to those who have the concepts) referential relations to compositionally different kinds are different concepts *irrespective*

*of* whether functional differences can (even *potentially*) follow. However, I question whether compositional difference can be completely free of functional differences in *all contexts*; function and composition are at least somewhat interdependent, so it's hard to imagine the complete absence of contexts where this interdependence does not imply compositional difference that propagates to functional difference. One can be an Externalist in saying that concepts can be differentiated on the basis of compositional differences even outside the epistemological horizons of concept-holders; but an Internalist in the *rationale* for this Externalism — that with a sufficiently wide horizon of contexts (and sufficiently epistemologically endowed cognitive agents for whom these contexts are enviroining) compositional differences eventually become functionally noticeable.

In addition — more relevant to mereology — someone might react to my talking about how we *draw* conceptual boundaries; that we *choose* whether and in what circumstances to unify (e.g.) saltwater and freshwater into a canonical water-concept. This style of argument may seem to be vaguely internalist from the get-go, because if we define conceptual architecture as a mental exercise it's hard to give due credence to concepts' potential extramental reality. There are, of course, extramental factors influencing (e.g.) our water-concept. Heavy water isotopes could potentially complicate a simplistic mapping of *water* to  $H_2O$  but as a matter of empirical fact ordinary  $H_2O$  is by far the most common form on  $H_2O$  on our planet. Ocean water and freshwater have chemical differences, but they share the property of being predominantly  $H_2O$ . Moreover, they are unified by meteorological cycles and behavioral similarities that supersede their chemical differences — ocean water cycles to freshwater as rain, and rivers drain into the ocean. The individuation of the concept *water* depends on these geological and meteorological systems as much as by “water =  $H_2O$ ” chemistry; this is perhaps why plasma (which is more watery than salt water) has no conceptual status as water.

These facts don't imply that we have no choice in which conceptual distinctions to recognize, or ignore; the space of *potential* concepts, as defined by whatever scientific regimes apply (chemistry, biology, geology, and so forth) may be finer-grained than our everyday concepts. On the other hand, we also have cases like a statue whose value changes depending on whether it is believed to be



the work of a great artist; social constructs like artistic or accidental provenance (consider a baseball that, by coincidence of being hit for a historic home run, becomes a collector's item) can introduce conceptual distinctions sometime more granular than warranted by science. But while human concept-mapping may be coarser or (occasionally) finer-grained, we cannot rationally entertain conceptual systems that would deviate too radically from what is scientifically warranted. Perhaps we could say that behind every practical concept-system there is a most scientifically transparent grounding, a system where spurious social distinctions (like artistic provenance) are bracketed, but material differences (even ones that may seem parenthetical to even most scientific contexts, like water/heavy water) are represented. This would then be the extramental background upon which are human concepts are established, and each human concept-system can be treated as modifying this background by unifying highly granular concepts into more general canonical concepts on practical terms, or occasionally superimposing scientifically-spurious criteria to split background-concepts into finer shadings. Arguably, it is a valid philosophical exercise to uncover the scientific grounding behind the culturally relative, socially ephemeral play of conceptual mergers and bifurcations above it.