Global Sharing, Local Meaning: Geospatial and Geo-Social Reasoning via Order-Sorted Mereotopoology

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Abstract

Mereotopology, the logical or axiomatic synthesis of mereological (part/whole) relations with topology, is a popular framework for developing theories of geospatial reasoning, Artificial Intelligence, Computer Vision, and related problems in cognitive science and computational simulations of cognitive activity. Mereotopology tries to concretely define topological notions like connectedness, interior, boundary, and overlap, considering how these relations are modeled by e.g. realistic spatial entities, rather than abstract point/set structures or dimensionless points. Here I explore how part/whole structures can be induced on (or are implicitly contained in) topological spaces (or their concrete approximators) in terms of maps into Order-Sorted Algebras. The idea of this representation is to highlight theoretical parallels and potential conceptual integration between Mereotopology and Conceptual Graph Semantics. I believe this fusion provides a model for expressing cognitive operations in realistic but formalized ways, and can particularly be used to study cognition as it relates to our geographic, social, and spatial surroundings. I will argue that Community Informatics, and the formal treatment of how people reason about their communal situations, provides a useful case-study for subtle cognitive structures which, while arguably not computationally intractable, are a challenge for formal reasoning and Ontology Engineering.

On connaît la célèbre affirmation de Claude Lévi-Strauss: "les sciences humaines seront structurales ou ne seront pas". Nous aimerions lui en adjoindre une autre: "les sciences humaines seront des sciences naturelles ou ne seront pas". Evidemment, sauf à en revenir à un réductionnisme dogmatique, une telle affirmation n'est soutenable que si l'on peut suffisamment généraliser le concept classique de "naturalité", le généraliser jusqu'à pouvoir y faire droit, comme à des phénomènes naturels, aux phénomènes d'organisation structurale.

—Jean Petitot, [46, p. 1]

The nature of any entity, I propose, divides into three aspects or facets, which we may call its form, appearance, and substrate. In an act of consciousness, accordingly, we must distinguish three fundamentally different aspects: its form or intentional structure, its appearance or subjective "feel", and its substrate or origin. In terms of this three-facet distinction, we can define the place of consciousness in the world.

—David Woodruff Smith, *Mind World* [53, p. 11]

Two friends in Toronto are discussing cafes in Burlington. Perhaps they mean Burlington, Vermont, but more likely they are referring to nearby Burlington, Ontario. One friend mentions an upcoming trip to London. Here confusion may arise, because while London, Ontario is closer, London, England, is a more prominent location, and a more popular destination for a trip.

We live in a universe of discourse, but the objects in this universe are not weighted equally. Different objects are more or less relevant to our typical situations, and so more or less likely to be involved in our thought and language. There are different criteria of proximity — geographic, topical, cultural, etc. Each provides a "metric" of semantic nearness; a partition of semantic "space" into overlapping, but partially individuated, conceptual frames. These frames pre-empt semantic ambiguity: given different meanings of a word, usually only one will fit comfortably into the most relevant, currently active, dialogic context. This applies to polysemes like "bank" (or like London, Ontario and England), but also to distinct but related meanings. In academia, for example, *Phenomenology* has two different (but not entirely unrelated) senses. Given a conversation between two philosophers, and a different dialog between two scientists, there will be implicit thematic frames which strongly suggest, to the conversants, which meaning of the term applies in which case. Similarly, consider the notion of Ontology, first in philosophy, and then in Computer Science and Knowledge Engineering: these are again distinct senses, but related, indeed more closely related than *phenomenology* in science and in philosophy.

Logical models of language and meaning need to resolve semantic ambiguities, while also respecting the different shades and varieties of polysemes and connotations. Sometimes a simple property — like being in Canada, or in England, respectively — can disambiguate proper names, like London. Nevertheless, even in these (superficially striaghtforward) cases, the relevant referential discriminator (like geographic location) may be only implicit in a discursive context, and therefore hard for language engines to recognize. Suppose someone from Toronto asks, over a social network, about good London hostels. Even if the forum is often used for Toronto-area recommendations, semantic cues take precedence over geographic ones: given

I am flying to London next week; anyone know a good hostel there?

words like "flying" and "hostel" connote a trip to England more than a trip to London, Ontario. Compare a post (still originating from Toronto):

I'm spending next Saturday in London; anyone know a good place for lunch?

— which implies a single-day trip. Unfortunately, even an automated processor which incorporates geospatial reasoning could misinterpret these semantic cues. As the second example demonstrates, semantic and geographic indications can work in consort: there is a semantic inference between "spending Saturday in London" and a *one day* trip; and a further geospatial inference that a one-day trip from Toronto probably involves London, Ontario, rather than England.

This example shows that semantic ambiguity cannot always be resolved, simply by providing a sufficiently detailed list of concepts and referents. In principle, the goal of computational Ontologies is to model a universe of discourse, where each individual bearer of a concept will be properly discriminated. Each token of the concept *city*, for example, can be uniquely "tagged" — given a universal and unambiguous "name" in lieu of an "overloaded" symbol, like *London*. Elements of discourse are crisply delineated at the level of each individual referent or concept-token, and then precise information spaces are built up in layers, through the levels of concept-types and conceptual relation. In the spirit of the "London" example, individual cities need unique "Uniform Identifiers"; then the concept *city* itself must be cleanly separated from related concepts like *town*, *village*, and *neighborhood*. Moreover, relations — like *mayor-of* — need to be schematized (e.g., a city has one mayor, who is a person), and their instances asserted. Given these multi-layered specifications and clarifications, reasoning engines can properly discriminate individual entities, and make correct inferences about them. Unfortunately, there are real-world nuances which may not be incorporated into the conceptual models, and which could engender improper or misleading inferences. In practice, conceptual tokens and relations are rarely "atomic": they have internal structure and fine-grained details, which can derail even those coarse-grained inferences which follow well-founded conceptual rules. Suppose we have a Sports database, which includes an "axiom" that two athletes are teammates, if their tenure for a team overlaps. The relation of being a player on a team seems simple enough, but it has its own nuances: while signed by a club, a player may be injured, or have international obligations, or out on loan. So a reasoning engine may infer two players as teammates, based on overlapping tenure, even if they never actually share the field.

Theories which influence the "Semantic Web" — like Description Logic [49, etc.] and Conceptual Graph Semantics (CGS) — model conceptual relations in graph or network models, which provide useful, flexible data representations, that can be expanded and merged. CGS provides a graph-oriented model of conceptual networks [54], [55], [21], whose subsequent theoretical developments have expanded from the graph-oriented paradigm to incorporate ideas from Modal Logic [8], [7], Category Theory [56], [9], Formal Concept Analysis [63], [41], etc. CGS and the Semantic Web can be contrasted with Relational Databases, for example, which can absorb new information only through an interface and records with a fixed, tabular structure. Conceptual graphs can expand and aggregate more freely, simply by adding new graph nodes and edges, as new facts are acquired, or new data sources come online. In these frameworks — partly to support this unrestricted, combinatorial network evolution — concept-tokens are treated as atomic semantic units (like nodes in a graph) and relations as atomic connections (like graph edges); "atomic" here meaning that internal structuration of concepts or relations is not posited or represented. This presumptive atomicity allows the interrelationships between concepts to be emphasized, and for new facts and data to be integrated in a free-form, combinatorial fashion. The problem with this paradigm is that fine-grained details may not be properly recognized: we can always build richer, more precise data models. For example, a sports database can distinguish between *playing for* and being *signed by* a team — which can guard against some incorrect inferences (like assuming two players were teammates based only on tenure). But it is impossible (and undesirable) to capture all conceptual subtleties in infinitely detailed information resources. Databases and data sources need precisely to develop simplified, rather coarse-grained approximations of real-world situations. Some degree of simplification is necessary if the generally mechanical and combinatorial structures of inferential and deductive reasoning are to be simulated.

Unfortunately, this very simplification, which allows network or graph based conceptual models to expand combinatorially, can make it difficult for them to automatically assimilate new information. This is particularly true for systems engineered to "learn" new concepts, based on access to language artifacts and real-world observations. It is one thing to represent relations among concept-types and tokens; but a different, and more subtle, kind of reasoning is needed to establish the proper conceptual and referential boundaries for new concepts and referents in the first place. Simplified data models can filter out precisely those nuances which allow human knowledge to perpetually evolve.

So there are two different challenges which confront any formal or comptuational system that operates on, or simulates, real-world concepts and conceptual understanding: first, properly modelling conceptual relations and networks, and, second, being flexible and nuanced enough to learn (and properly apply) new concepts and facts. We need to clarify the relationship (and sometimes the balance and trade-off) between these concerns. To this end, I think it is useful to recognize Conceptual Modelling as encompassing different aspects, which give rise to different problem-spaces. I would further like to propose a model of these aspects based on the "Multiple Aspect Theory" developed by the philosopher David Woodruff Smith in the book *Mind World*. This very general account applies to Phenomenology, Ontology, and cognition, and develops a philosophical ontology organized around a distinction of *form*, *appearance*, and *substrate*:

> 1. The form of an entity is how or what it is: its whatness or quiddity — the kinds, properties, relations that make it what it is.
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> 2. The appearance of an entity is how it is known or apprehended: how it looks if perceptible (its appearance in the everyday sense), but also how it is conceived if conceivable, how it is used if utilizable — and how it is experienced or "intended" as thus and so.
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> 3. The substrate of a thing is how it is founded or originated: how it comes to be, where it comes from, its history or genetic origin if temporal, its composition or material origin if material, its phylogenic origin if biological, its cultural origin if a cultural artifact — in short, its ecological origin in a wide sense, and ultimately its ontological origin in basic categories or modes of being.... The structure $<$ Form, Appearance, Substrate $>$ thus defines a special system of ontological categories. [T]he world includes attributes (of entities), minds (to which entities may appear), and contexts of foundation or origin (from or within which entities come to be). There may be possible worlds that lack such things, but our world has this much structure, and our ontology and phenomenology are accountable to this three-facet structure of the world. [53, p. 17-18]

This triad can be applied to Conceptual Modelling and the problems it engenders. The *form* of concepts is the structural organization of networks through which they acquire semantic content, as well as the internal structure of concepts seen as aggregate structures or "blends". The *appearance* of concepts is how they become known to and perceived by people, as perceptual qualities, scientific constructs, patterns in data, abstract ideas, etc., as well as how concept-tokens are represented by different technologies, such as internally in computer memory and visually in software. The *substrate* of concepts is the ontology of their instantiations, how they inhere in their bearers, what it means for a concept to be instantiated, and whether instantiation implies conceptual inter-dependence (as color depends on spatial material, for example). Conceptual Modelling therefore represents conceptual *form* through *relational* structures and concept networks; represents conceptual *substrata* by identifying entities in different domains (geospatial regions, physical objects, etc.) as tokens of concept-types; and considers the *appearance* of concept-types in terms of representational and sensory patterns through which concept-tokens become known to people and represented for people by technologies.

David Woodruff Smith develops his "multiple aspect" theory in a philosophical and speculative context; but I believe it can be applied to concrete, empircal examples. Consider the concept *epidemic*. The *relational* structures of a given instance of this concept include networks of information encompassing which disease pathogen is responsible for the epidemic, its treatments and genetic properties, the records of confirmed and suspected cases, mathematical models of its probable transmission between people in different situations, etc. Separate questions, which might be called *foundational* aspects, concern how we model the geospatial extent of the epidemic, and how we quantify different reports where clinical data confirming the details of (suspected) cases is, or is not (or not yet) available. These are *foundational* questions because they address how the concept-type *epidemic* is, for a given example, "founded" in empirical observations and situations. Finally, *representational* questions include how we can effectively structuralize medical and case data for transfer between different points (field workers, researchers, health officials), and for visually informative and interactive practical tools (like software in the field, and in medical offices).

Other examples are provided by geospatial terms and concepts. Consider the concept *nation*. Nations belong to complex relational networks involving their governments, treaties, histories, and so forth; indeed, most of the knowledge which we have of most nations comes from these relational predicates, rather than detailed knowledge of their geography or citizenry. On the other hand, each nation has a geographic and social extent. The proper boundaries of these extents can be ambiguous or contested: is Kashmir part of India, Pakistan, or both? How closely does an immigrant or expatriate need to follow political events in her first country — or to understand the government and history of her new one — to be considered an informed member of the respective communities? Is an undocumented Haitian, working underground sometimes in Crown Heights and sometimes in New Bordeaux, morally and conceptually a citizen of Haiti, New York, Montreal, Canada, the United States, or some combination of these? Is he morally part of the local community fabric in Montreal and Brooklyn but not (morally) the responsibility of the larger national governments? These illustrate (some of many) *foundational* ambiguities in how "nations", as concepts, are founded in social and geographic realities.

Moreover, as communities and govenments seek to establish productive online ecosystems for citizen participation, and a computational infrastructure for sharing government and community information, we encounter a space of *representational* challenges, where civic and information engineering overlap. These are examples of different Conceptual Aspects applying at the national level, but they have obvious correlates at the level of cities, local neighborhoods, and other geographic and social communities. Disputed territories (like Kashmir), or ambiguous classification (like the status of undocumented workers), exemplify problems of foundational ambiguity or *Foundational Vagueness*. These problems come to the fore when we consider reasoning engines which can potentially learn and assimilate new facts and concepts "on the fly". They are particularly significant in the context of "Community Informatics", because the kinds of concepts involved, as people reason and opine about their local communities, tend to be informal, colloquial, and subject to undirected, dynamic evolution.

In some formal settings, foundational ambiguities can be resolved by policy or fiat. Sometimes geospatial terms and entities are fixed by law, such as national borders, postal codes, electoral districts, etc., or changed only by official sanction: new nations are formed (like, recently, South Sudan); new electoral districts drawn (or "gerrymandered"), etc. With respect to these entities, the geospatial domain is comparable to bioinformatics, with an underlying universe of discourse that is generally static, and which only evolves in the presence of official declarations, like — taking biomedical examples — a new disease or disease-agent being recognized, a new drug patented, or a new medicine approved for clinical use. In these contexts, the primary challanges to successful data management are *relational*: the structural (in)compatibility between different data spaces which are to be extended, integrated, or to interoperate [6], [51], [18]. Different knowledge bases presume different fundamental concepts and relations, different ways of specifying further connections (which can then be entered into reasoning procedures), different kinds of logical inferential capabilities, and different representational paradigms or "algebras". These differences open the overall problem-space of Ontology Migration and Integration. General theories in this area, for example using mathematical Category Theory or the theory of "Institutions" [22], develop more abstract models in which distinct algebras or data representation systems can be expressed, and algorithms for their inter-translations potentially developed [37], [57], [25], [12][1].

The technical problems of Ontology Migration and Integration, as manifest in professional and institutional domains like bioinformatics, have received considerable research, funding, and institutional attention. But insofar as Conceptual Modelling seeks to simulate (and facilitate) a more community-oriented reasoning, the localized, informal discursive and contextual norms, implicit in

[1] Actually, this last, very interesting article is also thought-provoking with regard to what I would call *foundational* considerations.

community-based language and cognition, present a different suite of problems. Biomedical data tends to involve technical and formally defined concepts and observations. The “data” relevant to Community Informatics, by contrast, is more likely to come from peoples’ quotidian language and activity, and to reflect the familiar, colloquial patterns of presumed common background knowledge. Stories are told and retold, clever phrases repeated, and communities create new discursive elements and patterns by repetition. Semantically similar words and phrases acquire different connotations based on when they became popular, amongst whom. Stories engender new language uses: for example, “hiking the Appalacian trail” has become, in the United States, pseudonymous for an extra-marital affair, because of a well-known story involving a former South Carolina governer, and the subterfuge he used to surreptitiously visit his Argentine mistress. The concepts and idioms arising from narrative, event-based foundries are often adopted by a community gradually and unofficially, never being canonically defined or specified. Their pragmatic and foundational details emerge over time, from collective semantic and cognitive decisions. Whereas formal models of conceptual networks can often settle or ignore foundational issue by fiat — the ambiguities or imprecisions which accompany the extensions or reference of concepts may not be relevant to conceptual *relations* — these issues reappear when we attempt to model concepts in rapidly evolving, decentralized cognitive spaces, like community-oriented reasoning and discourse.

I will consider one very specific example of this “unofficial” semantic emergence, here in the context of geospatial reference. Because I will return to this example several times, let me set it up with a little detail. Suppose we take New York City’s Ⓝ train to the Eighth Avenue stop, in Sunset Park, Brooklyn. Once on street level we will immediately recognize one of New York’s “Chinatowns”; we notice a large Hong Kong Supermarket directly across from the subway, and many Chinese markets and restaurants, walking North along Eighth Avenue. Further West, in Borough Park, we would find a predominantly Hasidic and Orthodox neighborhood, with large Synagogues and Kosher markets and restaurants; to the East, toward the heart of the Sunset Park neighborhood (and the park itself), lies a largely Latina/o community. Given the pleasant Brownstone-fronted streets, the photogenic Sunsets over the Park, it is a testament to either generational change or Hollywood hyperbole that a major film’s theme song once riffed on these streets as the Valley of the Shadow of Death; those days are long gone, as they are in Flatbush, Prospect Heights, Bedford-Stuyesant, or West Harlem. But closer to Eighth avenue, while there are remnants of an older Eastern European community, the ambience has clearly been shaped and embraced, over the last one or two generations, by a Chinese and East Asian immigrant commnity. So, by reference to the “Flushing Chinatown” in Northeast Queens, and “Manhattan Chinatown” in Lower Manhattan, this “Sunset Park Chinatown” is “Brooklyn’s Chinatown”, in local parlance. The phrase *Sunset Park Chinatown* does not appear to be an “official” designator, separate from Sunset Park itself, and there is no fixed map or desciption of the area’s extent or boundaries. Indeed, as more businesses owned by Chinese immigrants appear on 7th and 9th avenues, the perceived borders of the district may change. One consequence of this imprecision is that, insofar as we construe the phrase "Sunset Park Chinatown" as referring to some geospatial entity (via a delimiting set of street corners, for example), we have no obvious procedure for specifying or determining which entity is in fact the proper referent. In point of fact, I will argue below that oversimplistic accounts of geospatial reference — where terms are considered "mapped" to (sets of) street-segments or corners or geospatial points or whatever — are problemmatic. But (even if we set this foundational question aside), with an unofficial term like *Sunset Park Chinatown*, the goal of fixing a referent in one of these fasions is unavoidably problemmatic, because no candidate region has referential warrant above (at least some of) its peers.

Let's consider this example in light of the framework of Conceptual Aspects I proposed earlier: the triad of *Relational*, *Foundational*, and *Representational* aspects is well illustrated by geospatial entities in particular, and more generally by the domain of geosocial reasoning and Community Informatics. In principle, we take Community Informatics as concerning both geographic and also virtual or online communities, interest-groups (such as dispersed communities of academic researchers), ethnic diasporas, etc. In practice, these dimensions tend to overlap. In "Sunset Park Chinatown", the association between this neighborhood (and its descriptive name) and Chinese businesses is *relational*, whereas the question of how we specify a region to serve as its geospatial substratum is *foundational*. More generally, communities provide different types of semantic contexts: geographical communities are associated with geospatial regions, but are further characterized by social, cultural, and political structures. Foundational structures of social communities involve collective (mutual) identification, as well as crtieria of historical or legal recognition. Foundational structures of geographic and geospatial regions involve their spatial boundaries and extents, as well as managing ambiguities such as territorial disputes. Whereas geosptial regions are theorized as mathematical spaces, in practice they can imprecise and "fuzzy": borders may not be perfectly surveyed, and they may be defined in terms of (intrinsically imprecise) natural features, like rivers or shorelines. When we then consider *social*-geographic communities, like nations, cities, and neighborhoods, these foundational structures intersect and overlap.

So the notion of a geographic community tends to blend together geospatial regions (or identifying geospatial landmarks) with social and cultural communal structures. With respect to virtual, diasporic, or language communities, etc., these social structures take prominence, and questions such as the mesh of social relations between community members, the shared criteria for communal recognition, etc., become foundational issues. These questions are also present in geographic communities. In the geographic case, though, issues of spatial proximity and the day-to-day realities of people living near each other are an integral part of this collective recognition. Geospatial surroundings provide a discursive context for locations, points of interests, news, events, etc.; people in a geographic community will discuss the new restaurant around the corner, the constuction work being planned, the local politicians and intrigues. Locals are assumed to be familiar with local people and places, so that their conversations can have a degree of intimacy or familiarity in contrast to conversations with outsiders. Similar presumptive familiarity is at work in "knowledge communities", for example, among people who share specialized interests and expertise. But whereas collective semantic knowledge, in these specialized cases, generally concerns abstract or technical concepts and terminology, the common domain of reference in the case of geographic communities is more day-to-day and informal. People absorb these concepts simply by living in and observing their surroundings, so they may seem simpler and more intuitive than formal domains whose concepts need to be carefully studied and mastered. Paradoxically, however, this very informatily which makes community-based reasoning natural and intuitive for humans, can make it intractable or at least very challenging for computers.

Exploring potential community-oriented applications may therefore serve two purposes: first, to develop practical tools, and, second, to provide a model for real-world, day-to-day human cognition and reasoning. Software applications can serve community development by effective (geo)spatial reasoning, but also through realistic 3d and Virtual Reality design, and by organizing and facilitating communication. For example, [48] describes tools used by European communities to route community members' emails to government officials based on their content. Community-oriented software can therefore serve citizens' communicating with one another and with elected leaders, expanding the reach and technological sophistication of the "civic sphere". Because automated semantic processing usually involves some expression of "semantic space" [20], [10], [31], [32], techniques developed for spatial analysis, assuming they can be generalized to high dimensions, can serve these more abstract discourse representations as well. With respect to more concerete notions of space, Community Informatics obviously calls for effective geospatial reasoning [14], [60], etc., but also for modelling in three-dimensions [17]; for example, creating 3d scenes by which proposed architecture and infrastructure can be represented to and commented upon by communities. Computer languages to create 3d scenes can be very powerful, but also very specialized; it is an open problem to generate 3d scene code from semantic representations of community structures, which are more high-level and natural-language based and abstract from technical details like surface triangulation and shaders [62], [61]. Imagine Conceptual Graphs representing ordinary-language descriptions: "A red brick pavillion at the Northwast entrance of the park on the corner of St. Clair Avenue and Avenue Road".

Of course, all of these concerns can overlap. Suppose an urban area intends to create or expand a subway line. Geospatial reasoning can quantify things like expected traffic based on population density in the surrounding neighborhoods, safety and engineering issues like the risk of track flooding, the underground relation of tracks to preexisting features like water lines and building foundations, etc. Three-dimensional modelling can prepare simulations of subway stations, or their street-level entrances, and internal connections to buildings. Semantic analysis can allow locals' comments on proposed routes and designs to be properly categorized, and rhetorically parsed (e.g., identifying which features or proposals are being approved or critiqued by a written evaluation). This kind of integrated platform requires a careful model of existing community infrastructure and discourse — existing buildings and transportation corridors need to be incorporated, but also local dialect and terminology. Street scenes should include place markers and information about local businesses, to promote them, and to model typical activities around and in the context of (here) subway stations, like where to shop or eat before or after catching a train. Popular expressions for local places and landmarks need to be acknowledged, so given an expression like "this station will provide family-friendly access to the park", the park in question will be correctly identified. Meanwhile, colloquial expressions of approval and disapproval need to be recognized and tagged accordingly.[2]

This kind of community-focussed reasoning and discourse presents a good case-study of human language of reasoning in our "natural" environment, at least among citizems of the urban First World. It is also a case-study of formal conceptual models and automated reasoning tools as they confront foundational ambiguity and imprecision, and to learn new concepts on the fly. Community Informatics also presents examples of foundational and relational aspects intersecting, because community-oriented concepts include domains where foundational vagueness can often be in effect (as with geospatial regions like Sunset Park Chinatown), but also with aggregates of facts and combinatorial concept networks, like businesses and other points of interest in a community. Locals' implicit knowledge includes both relational facts and spatial know-how (they can recommend restaurants serving a particular style of food, and give you directions to get there). Mereotopology [27], [50], [43] is often used to model geospatial Conceptual domains, whereas representing factual aggregates (my favorite local restaurant is a Thai restaurant in Woodside, on 39th avenue, popular with local Southeast Asian immigrants, with spicy Yum Talay, etc.), typically involves a framework like Description Logic or Conceptual Graph Semantics. Combining these aspects of community-oriented reasoning suggests a theoretical and practical interest in merging these network-like structures like `CGS` with more spatial and geospatial formalisms like Mereotopology.

Actually, I believe this is symptomatic of a deeper cognitive paradigm which permeates human reason and perception: it is quite natural for us to integrate spatial reasoning (about our surrounding community and physical space, as well as about objects, their spatial arrangements, and their surfaces) with conceptual

[2] In 2010's New York, say, "This neighborhood rocks", "This neighborhood is cool", and "This neighborhood is hot", respectively, are not references to geography or climate.

networks, expressing background knowledge and schematic represenions of immediate situations — practical activities we are engaged with, and conceptually meaningful relations between and amongst ourselves and the things around us. I introduced *foundational* and *relational* concept-aspects from a Phenomenological perspective, notably that of David Woodruff Smith; but I will also mention Jospeh Goguen's Unified Concept Theory [22][3], and his contrast between "symbolic" notions of Conceptual or Mental Spaces — which he associates with cognitive linguists such as George Lakoff and Gilles Fauconnier [15], [16], [39], [40] — and the more "Geometric" Concept Spaces of Peter Gargenförs [19], [59]. In community-related contexts, the Conceptual Modelling integration between spatial reasoning (for example as modelled via Mereotopology) and conceptual networks (such as represented by CGS) is particularly explicit.

There are also underappreciated formal connections between CGS and Mereotopology, which can be expressed by relating both systems to Order Sorted Algebra (OSA). The remainder of this paper will therefore sketch, rather informally, a brief theoretical integration involving these formal systems.[4] I will use terminology mixing OSA and CGS paradigms. Collections of Conceptual Graphs are often provided structures called *supports*, which establish "background knowledge" or fundamental concept-types, relations, and assumptions for all graphs involved [2], [11], [36]. These supports can be considered extensions of the OSA notion of *signatures*. In this context, a signature Σ is a collection of concept and relation types, which collectively represent *sorts* as in OSA, with the collection of sorts therefore fundamentally bifurcated into concept and relations. The elementary structures in CGS graphs are "double edges" or "triples", similar to Semantic Web "N-Triples", which include two concept-nodes linked by a relation-node. CGS certainly allows for relations of arity > 2, but I will assume binary relations only, for purposes of exposition. Given a set $\{\Gamma\}$ of graphs, an associated support S includes both sorts $\mathcal{S}$ for $\{\Gamma\}$ and provisional facts or assertions. In keeping with OSA, the sorts $\mathfrak{s} \in \mathcal{S}$ are assumed to form a partially ordered set: so each concept-type can be an extension or refinement of one or more other concept-types, and similarly for relation-types.

A graph Γ includes edges and two varieties of nodes — concept-nodes and relation-nodes — with the simplifying assumption that each relation-node has exactly two edges, each of which is a concept-node, and also that each edge links one concept-node with one relation-node. Let $\Gamma_{\mathbf{N}} = \Gamma_{\mathbf{C}} \cup \Gamma_{\mathbf{R}}$ be the total and the concept and relational node-sets, respectively. Given $n \in \Gamma_{\mathbf{N}}$, n has a sort $\mathfrak{s}(n) \in \mathcal{S}_{\Sigma}$, where $\mathcal{S}_{\Sigma}$ is the set of sorts in the signature. The map $n \mapsto \mathfrak{s}(n)$ provides semantic interpretations of Γ nodes. We can "strip away" this semantics to consider some underlying structure. Let $\dot{\gamma}$ be a transformation of Γ which eliminates all relation-nodes $r \in \Gamma_{\mathbf{R}}$, collapsing the double-edges around

[3]Particularly pp 10, 11, 17, 26, etc.

[4]The author may be contacted for longer theoretical papers or software implementations of some related concepts.

each r to a single edge between the spanned concept-nodes, and collapses the sort structure $\mathcal{S}$ to the single-sorted set Ⓢ. $\dot{\gamma}$ is therefore an ordinary undirected graph with Ⓢ as its vertex set.

Suppose Γ is a conceptual graph which represent a social network; perhaps information about people in a university or an organization. The support S for Γ may then include concepts such as student, professor, employee, or department. The stripped version $\dot{\gamma}$ ignores information about what title people have, and represents people as connected in any way, unifying different relations like being coworkers, classmates, students to teachers, etc. Even this minimal representation may provide interesting data, such as the connectedness of the social network. The available information becomes much more detailed when semantic interpretations are put back in, so that instead of a generic relation like x "knows" y we can indicate that x and y are employees and coworkers, etc.

So the inverse operation to "stripping" semantic data is "enriching" a simple graph $\dot{\gamma}$ by mapping its single-sorted vertex set Ⓢ into an order-sorted family $\mathcal{S}$; more precisely, mapping Ⓢ $\rightarrow \Gamma_{\mathbf{C}}$ and mapping $\dot{\gamma}$-edges into $\Gamma_{\mathbf{R}}$. Because concept and relation types in CGS are unrestricted posets and not, say, trees (in programming parlance, they allow "multiple inheritance"), the signatures for CGS types are consistent with OSA [42], [38], and simple graphs $\dot{\gamma}$ can be "enriched" to Conceptual Graphs by mapping their edges and vertices into an order-sorted algebraic structure. We can say that CGS is therefore in effect an Order Sorted Algebra "over" simple graphs, so that by analogy OSA in its canonical presentation represents Order Sorted Algebra over first-order logic. A collection of well-formed conceptual graphs (with respect to "syntactic" constraints, such as arities on relations) is analogous to a collection of well-formed first-order formulae. When semantic constraints are included (such as restricting relations to a list of concept-types which can appear as nodes around relation-nodes), there is a corresponding analogy between CGS graphs and OSA "term algebras".

Insofar as CGS graphs are OSA structures "over" single-sorted graphs, we can find a similar connection between OSA and Mereotopology. First, observe that CGS is a natural representation language for basic *mereology*, since the part-of relation can obviously be expressed as a collection of directed graph-edges. Insofar as mereological systems $\mathfrak{M}$ are to represent referential spaces for discursive domains, I think it is useful to consider a form of mereology that also includes a "coupling" relation $m \odot m'$, where m, m' are "mereons" in a mereological system. The semantic interpretation of $\odot$ can vary; this may involve spatial objects which are physically attached, nations which have entered into treaties, people who know each other, etc. The important point is that some collection of entities forms a discursive universe and that semantic expressions can refer to collections of those entities as well as to individuals. The semantics of these referring expressions is affected by how their referents, if they are collections, are organized as wholes. Consider the sentences:

(a) I invited the couple next door.
(b) I invited the first grade.
(c) I invited Abdul, Boris, and Chang.

Sentence (a) suggests a whole formed by a binary coupling amongst its parts; (b) suggests a whole formed by a membership criterion (being a child in some first-grade class), and (c) a provisional whole formed by listing its members.

In the context of semantics, mereological systems can model how a domain of discourse extends beyond a preliminary set **I** of individuals to include couplings, aggregates, and collections of individuals, insofar as these in turn are semantic referents. This theory can embrace "cognitive reference" involved in thinking about particular things, as well as actual linguistic reference via language artifacts or gestures (like pointing). Now, topology also helps us define domains of reference, for example with respect to spatial objects. Our immediate environment provides a collection of discrete objects which are available for mental or linguistic attention, and we recognize these individuals by perceiving their topological separation, on the basis of limited information provided by our visual (and also tactile and kinaesthetic) sensations. In general we see only the surfaces of objects, and vision requires that we create a mental model of surrouding space as occupied by different surfaces — manifolds which are sometimes adjacent, and sometimes physically attached. The mathematical reconstruction of these cognitive processes involves topology partly because the relations between distinct surfaces — separated, overlapping, or just touching — can be expressed topologically, but also because visual reconstruction of three-dimensional geometry is guided by qualities which are distributed across the surfaces, like coloration. These qualities can be modelled as vector fields over surfaces, and the global analysis of such fields depends on fundamental topological notions, like winding numbers and the Euler characteristic of a manifold.

Whereas mereology as modelling a discursive Domain of Reference is focussed on how individual referent-entities can be aggregated into wholes, topology within similar semantic theories plays more of a role of subdivision — dividing a visual field into different individuals, and then identifying how a single individual gives rise to multiple possible referents among component parts. We can refer to a teacup as a whole, or to its handle, its bottom, etc. These systems of referents within a single integrated whole involved mereological relations as well as topological ones — for example, the bottom and the handle of a teacup are mereological (or mereologically identified) individuals insofar as we consider the semantics of these words as largely *functional*; the meaning of "handle" involves the fact that we use it to grasp the teacup, and the "bottom" (in this context) is how we place the teacup on the table. But this functional organization results from its topology and geometry as a spatial, extended entity. So mereological relations with respect to function and organization can be manifest, or perceived, in geometric and topolgical structures, particularly as they are then cognitively reconstructed through visual (tactile, kinaesthetic) perception.

Of course, modelling "domains of reference" is only one aspect of Conceptual Modelling. But these examples show how mereology and topology may be connected. Mereological relations (for example, attachment or contact between surfaces) can yield topological predicates, like connectedness. One approach to Mereotopology is then to start with mereological systems (perhaps where mereological relations are modelled via conceptual graphs, given coupling $\odot$, as well as parthood, among the relation-types), and to define topological structures on top of them. The more popular approach however incorporates mereological notions into a topological context. For example, basic operators like closure, interior, and set-intersection, are sufficient to define at least one family of mereological relations. I will refer to sets in a topological space $\mathcal{T}$ as "topological mereons" or just mereons m. Relations such as m overlapping m' can be expressed as the topological interiors of m and m' overlapping; two mereons touching, but not overlapping, expressed by their boundaries overlapping but not their interior; two mereons being separated expressed as $m \cap m' = \emptyset$; and cases of $m \subset m'$ distinguished based on whether or not their respective boundaries touch.

There is a general philosophical or cognitive motivation behind Mereotopology, reflecting the idea that while "dimensionless points" may be mathematically precise vehicles for point-set topology, they do not obviously correspond to apparent cognitive (or real-world) individuals. We do not perceive truly zero-dimensional spaces, for example, but rather abstract the notion of a "point" from more (perceptually) immediate posits, like the intersection or endpoints of lines or a process of pereptually "zooming in" to an ever-finer spatial region. Of course the notion of "line" itself seems perceptually derived as, say, boundaries and intersections of regions. Perceptually, then, it is two-dimensional surface or three-dimensional volume which appears most fundamental. Technical analyses like [29], [30], [13], [58, Lemma 4.2], demonstrate that axioms, presupposing only relations (like contact and proximity) between regions, can recover topological theory on most spaces; this research effectively recreates, in a formal setting, the cognitive process of "deriving" points (or "pointhood"). Mereotopology in this sense can be called "pointless Topology", though again some construct demonstrably isomorphic to points can be defined, so points remain as analytic vehicles; they are technically constructed entities, rather than ontological primitives.

Most mathematical treatments of space and distance (whether topological or geometric) represent a form of spatial reasoning, but they emphasize the numerical encoding (or algebraic encoding, with respect to topological topics like homology or topological groups), of spatial locations or the distances between them. In other words, they presuppose an action upon spatial domains, such as measurement or $\mathbb{R}^n$ immersion, which transforms spatial relations into quantitative ones. Point-set topology abstracts the notion of metric nearness by positing open sets more generally, but clearly the intuitive foundations of this analysis are open intervals (disks, etc.) in metric spaces, and therefore of spatial proximity as numeric nearness, embodied by the real number line. If we accept dimensionless points, then the foundation of spatial measurements (distance, location, etc.) can be functions of the form $\mathcal{T} \longrightarrow \mathfrak{D}$, where $\mathfrak{D}$ is some observation domain. However, if we wish to (at least provisionally) set aside dimensionless points as ontologically primary, then we can instead consider a multidimensional space of possible observations $\mathfrak{O}$, for example using theories of "multidimensional mereotopology" [28]. If $\mathcal{F} = \mathcal{T} \times \mathfrak{F}$ is a "feature space" where a feature, like "color", reveals structure on $\mathcal{T}$, then an *observation* on $\mathcal{F}$ may measure (or approximately measure) a point, line, area, etc. We may have conditions for deriving or approximating a zero-dimensional observation $t \mapsto f \in \mathfrak{F}$, but consider this as derivative upon the general product $\mathcal{F}$, and perhaps upon a set of different kinds of observations, with different dimensions.

We can explore similar intuitions by noting that any process of visualizing points (or other constructions) on a topological space $\mathcal{T}$ requires visualizing $\mathcal{T}$ expanded with some discriminating criteria. For example, if I seek to visualize one point $p \in \mathcal{T}$, I may imagine $\mathcal{T}$ as a "white" region with one black-colored interior spot, representing p. We can also consider a demonstration in a textbook in lieu of a mental picture. Effectively, then, we cross $\mathcal{T}$ with some structure (a *symbol* p or a black-dot p or whatever) by which p and the rest of points — not p — are discriminated. A "point" in $\mathcal{T}$, at least *qua* symbolized or iconified entitiy, associates with a product space $\mathcal{T} \times \mathfrak{p}$, where $\mathfrak{p}$ represents a two-valued sort $\{p$, not-$p\}$. More generally, within real objects (rather than textbook diagrams, say), we visualize points, lines, etc., based on color patterns. We do not see isolated points because if a point lies in a monochromatic or smoothly varying patch, we tend to perceive the patch as a single unit; if a point lies on a boundary between differently-colored regions, we are disposed to perceive the regions as more primitive and their boundary as derived on their basis. In general, our perception of structures within a space $\mathcal{T}$ is defined by how $\mathcal{T}$ is crossed with some feature-granting principle, like coloration, and it is patterns or partitions in a compound space $\mathcal{T} \times \mathfrak{F}$, where $\mathfrak{F}$ represents a featural (e.g. color) space, which retroactively establish structures on $\mathcal{T}$ by a kind of "pullback".

From a mathematical point of view, the set of ordered pairs of the form (t, f), assuming $t \mapsto f$ via some function $\phi : \mathcal{T} \rightarrow \mathfrak{F}$, is a subset of $\mathcal{P} = \mathcal{T} \times \mathfrak{F}$ (stipulating, in particular, that each t maps to a unique f). This product space includes many other possible maps $\mathcal{T} \rightarrow \mathfrak{F}$, so it certainly does not uniquely identify any ϕ in particular. However, it is helpful to consider each given map ϕ in the context of this general space. Classically, a function is a special type of relation — it may be one-to-one or many-to-many, but not one-to-many. However, there are strategies and contexts where the space of functions or pseudo-functions can be expanded to a more general class of relations. Note that a one-to-many relation can be treated as a function whose codomain is a power set. Moreover, a relation can be described as a one-to-many deformation or approximation of a classical function. Suppose, say, that $\c{F}$ is a family of polynomial approxumations of a function ϕ, and we map points to the union of their images in each approximation $\c{\nu} \in \c{F}$. Or, in a model of cognitive process, suppose ϕ represents some perceptual pattern, like the coloration of a two-dimensional surface in three-space. In real life, a given empirical pattern will correspond to a range of different perceptual structures, since an object's coloration is viewed in different lighting conditions, for example. So a particular featural pattern ϕ is subject to empirical variation within a collection of related such maps.

If we entertain the idea that "dimensionless points" are arguably derivative rather than fundamental posits, then we can revisit the canonical definition of a product space, where *points* in a pair of spaces are matched up. A function $\mathcal{T} \rightarrow \mathfrak{F}$ defines a unique point f for each point p. If points themselves are derivative, however, then the ordered pairs $(p,\, f)$ are subject to the same derivative status as the p and f themselves. It is the patterning ϕ which retroactively defines structuration on $\mathcal{T}$; posits like points in $\mathcal{T}$, at least perceptually or as an abstract representation of perceptual processes, are derived from structures in the product space $\mathcal{P}$. It is only by associating $\mathcal{T}$ with a featural space of some kind (even an artificially constructed one, like a textbook diagram), that structures in $\mathcal{T}$ become expressible. So the product space $\mathcal{T} \times \mathfrak{F}$, in this paradigm, philosophically precedes structurations such as dimensionless $\mathcal{T}$ points. A "featuration" ϕ then may be intuitively pictured as derived from a general product space $\mathcal{P}$, for example as one coloration among all posible colorations of a surface, perceived at one possible moment in time, etc. — rather than compiled from points t mapped via $t \mapsto f \in \mathfrak{F}$. Regions in a compound space $\mathcal{F} \subset \mathcal{T} \times \mathfrak{F}$, like parts of a color pattern on a spatial surface, are more perceptually immediate than points in space (abstracted from color) or points of color (abstracted from spatial presentation) in isolation. Moreover, the topological properties of $\mathcal{F} = \{(t,\, \phi(t) \in\ \mathfrak{F})\}$ will be derived in part from the topology of $\mathcal{T} \times \mathfrak{F}$.

From a philosophical perspective, Mereotopology suggests that the most fundamental (get)spatial entities are *regions*, which have some spatial extent (and not points, say); but we can add that, given Phenomenological considerations, spatial extension is only observable in the context of featural distribution, like color. The truly fundamental entities by that argument are then regions in featural *product spaces*, like a product of spatial articulation or "spatiation" with coloration. A region in a product-space is subject to variation in multiple dimensions: for example, our eyes can follow around the surface of an object (spatial movement), where the perceived color may or may not change (perhaps simultaneous change in space and color); or we may watch the same spatial area as, say, the sun sets, causing observed color to change. The total set of possible observations of coloration on one surface is a subset of a general spatiation $\times$ coloration product space. We can analyze coloration as a "function" from spatial points to colors, but introducing function-like sets as product subspaces allows the notion of function to be generalized, optionally, to support ideas of measurement imprecision, or modelling changes in conditions of observation.

I will suggest the notation $\mathcal{F} \Subset \mathcal{P}$ and $\mathcal{F} = \mathcal{T} \otimes \mathfrak{F}$ to express that $\mathcal{F}$ is a space of ordered pairs where the second is unique given the first,[5] so derived from some function ϕ; however, so as to leave open (without mandating) the possibility that $\mathcal{F}$ is defined or isolated within $\mathcal{P}$ in some paradigm where *points*, in either or both component spaces, are not posited as fundamental entities. The canonical example would be a feature space of coloration on a surface. Points of color and spatial points are ontologically codependent, since color needs some spatial extension to become manifest, and spatial points need color contrast to provide designation or individuation. A given feature space $\mathcal{F}$ exists in a space of possible variation (for example involving different lighting conditions), and the possibilities of modifation upon $\mathcal{F}$ are defined by the global structure of the space $\mathcal{P} \Supset \mathcal{F}$. If Mereotopology is in one sense "point-free topology", then $\mathcal{T} \otimes \mathfrak{F}$ would be in the same sense a kind of "point-free map" from $\mathcal{T} \rightarrow \mathfrak{F}$.

In addition, we can make the further consideration that featural spaces $\mathfrak{F}$ belong to an ordered collection of sorts on which OSA structures may be defined. We can therefore consider properties (whether discrete, countable, continuous with some natural topology, algebraically enriched with certain operators, ordered or partially orderred, possessing boolean-algebraic or group structure or symmetries, etc.) of $\mathfrak{F}$, in the context of a larger sort collection $\mathcal{S}$. These properties will influence topological details and mereological patterns within $\mathcal{T} \times \mathfrak{F}$, and such details can moreover be contrasted amongst a collection of product spaces $\mathcal{T} \times s$, as s varies over a collection of sorts. A simple example will be discussed below, in terms of maps into different color-spaces, with different degrees of granularity. Moreover, given the supposition of OSA structure on (collections of) featural spaces $\mathfrak{F}$, we can develop frameworks for exploring "fuzzy" or multidimensional measurements, approximations, etc., in contexts such as cognitive-perceptual modelling, or simulated (geo)spatial reasoning. For example, consider signatures Σ, wherein one or more sorts s approximate or discretize an s'; or conversely derive continuous-valued variables from discrete s'-observations, via Simulation of Continuity. So, for the subsequent discussion, assume that we have feature spaces of the form $\mathcal{F} = \mathcal{T} \otimes \mathfrak{F}$ and moreover that $\mathfrak{F}$ belongs to an ordered collection of sorts consistent with an Order Sorted Algebra.

[5] Or as unique as can be feasibly measured in some context of observation; and we can allow t to be a "fuzzy point", or as near to a $\mathcal{T}$-atom, a part of $\mathcal{T}$ with no apparent internal structure, as observable in some context; similarly "points" in $\mathcal{T} \otimes \mathfrak{F}$ may be defined as atom-like $\mathcal{F}$ regions in the sense of having no *observable* internal structuration.

For practical purposes, the abstract topological notions of boundary, closure, and interior need to be modified to accommodate real-world data spaces — for example, if bitmap graphics are models or vehicles for merotopological theory and techniques, topological notions have to be adapted for discrete, pixelated contexts [47]. Whether in a more abstract or more applied context, it is worth paying closer attention to how mereological structures are actually defined within topological ones. Suppose a topological space $\mathcal{T}$ becomes a mereological system $\mathfrak{M}$ by an initial partition of the space, establishing a base layer of individuals which may then be combined. If the partition divides $\mathcal{T}$ into n pieces, $\{\mathcal{T}_i\}$, then there is a map $\mathfrak{m} : \mathcal{T} \rightarrow \{1, ..., n\}$ and each point $p \in \mathcal{T}$ becomes the ordered pair $\{p,\ \mathfrak{m}(p)\}$. However, $\mathcal{T}$ may exhibit apparent divisions even if individual parts cannot be crisply defined. A mereological system $\mathfrak{M}$ can be extended to a *weighted* mereology, where parthood (or coupling) relations are weighted based on how strongly two mereons are attached to each other, how much freedom they have relative to each other, etc. In this weighted context, even when parts are not crisply individuated, there may still be evident patterns or organizations of wholes which suggests that they contain component parts.

Consider a colored surface whose coloration includes gradient patterns with a repeated structure, like stripes. This is a part/whole organization even if there is no clear boundary between distinct parts. To investigate this organization we can represent coloration as a map from $\mathcal{T}$ into a color space $\mathcal{C}$. Note that a coloration corresponding to a set of n monochromatic patches is equivalent to a partition of $\mathcal{T}$ via a direct partition $\mathfrak{m} : \mathcal{T} \rightarrow \{1, ..., n\}$. Such a partition may also arise from an image segmentation algorithm, if $\mathcal{T}$ is for example a bitmap image. So the generic map $\mathfrak{m}$ can have different semantic interpretations. This can be seen as providing different sorts to act as domains $\mathcal{D}$ for $\mathfrak{m}$. If $\mathcal{D}_{\mathfrak{m}}$ is just $\mathbb{N}$, then $\mathfrak{m}$ represents a generic partition. If $\mathcal{D}_{\mathfrak{m}}$ is a color space, then the map may represent a surface divided into monochrome patches. Sending topological points $p \in \mathcal{T}$ to some sort-domain can be compared to mapping nodes and edges in a simple graph into an ordered collection $\mathcal{S}$ of sorts, yielding a Conceptual Graph. By analogy, mapping $\mathcal{T}$ into $\mathcal{S}$ yields (one kind of) Mereotopology.

If the image of $\mathfrak{m} : \mathcal{T} \rightarrow s$, for sort s, has finitely many discrete points, then $\mathfrak{m}$ induces a finite partition of $\mathcal{T}$. At least in theory, the resulting sets in $\mathcal{T}$ (preimages of distinct points in s) then provide a basis for Mereotopologies like RCC (Region Connection Calculus). If the points in s can be aggregated according to some (set of) criteria, the associated preimages can be aggregated accordingly, so relations involving overlap come into play. For example, an image segmentation may proceed by simplifying an image's coloration by selecting some set of color values, mapping nearby colors onto that set, and then segmenting the image as a collage of the resulting patches. These patches may then be grouped together on criteria like color similarity, contrast to surroudings, geometric connection, or some statistical matching of geometry or patterns to a corpus of standard figures (disks, spheres, etc.) or real-world object types (buildings, trees, tables, teacups). The according preimages then form an RCC-carrier system $\mathfrak{M}$, at least given a properly discretized topology for bitmap images [47, cf. §3].

The other case just mentioned — with a gradient stripes-pattern — does not have this crisp partition, but again we can consider a map $\mathfrak{m} : \mathcal{T} \rightarrow s$. The difference is that the image in s is now continuous, rather than discrete. A continuous image $\mathfrak{m}(\mathcal{T}) \subset s$ may still yield a partition of $\mathcal{T}$ if the (product-induced) topology on $\mathfrak{m} \boxtimes \{p\} =$ (or $\approx$) $\{(p, \mathfrak{m}(p))\}$ is not connected. Introduce the symbol $\mathfrak{m} \boxtimes \mathcal{T} := \mathcal{T} \otimes \mathfrak{m}(\mathcal{T})$, as a "box-product" $\Subset \mathcal{T} \times s$. Given $\mathcal{T}$ connected (or a connected subset of $\mathcal{T}$), a separation in the box-product $\mathfrak{m} \boxtimes \mathcal{T}$ results from discontinuities in $\mathfrak{m}$ itself; for example, if $\mathcal{T}$ is a colored image or surface, a border between two different color areas, perhaps suggesting one object lying in front of (and partially hiding) a different one. So a division of $\mathcal{T}$ can be defined by separating out regions where $\mathfrak{m}$ is continuous. Alternatively, it may be possible to reconstruct the feature space $\mathcal{F} = \{(p, \mathfrak{m}(p))\}$ by starting from some kernel image in s. Consider a gradient-stripes example. Suppose the coloration on a surface can be reconstructed by taking a loop in color space — representing the color gradient — which is then mapped to a colored circle in geometric space, then extended to a cylinder, by adding height dimension and "copying" the circle at each height-point, and then "unfolding" the cylinder onto the surface, like applying a paint roller which is coated with a striped paint color-pattern. If an original space $\mathcal{T}$ can be reconstructed from some kernel space k in a sort s (like a kernel color space), then this kernel provides a representation of $\mathcal{T}$'s mereological structure, even if the construction does not yield a crist partition of $\mathcal{T}$. If the kernel is a closed loop, like a circle in color-space, then we can expect the $\mathcal{T}$ mereology *not* to have crisp partitions, because (assuming $\mathcal{T}$ is reconstructed from k by geometric operations like extending, reflection, and copying), crisp borders in $\mathcal{T}$, or in a feature space $\mathcal{F} \Subset \mathcal{T} \times \mathfrak{m}(\mathcal{T})$, would suggest discontinuities or endpoints in k. If k instead is a closed loop, for example, we expect $\mathcal{T}$ to have mereological structure but not one whose pattern is identifiable in terms of crisp parts. Isolating a kernel k, and then reconstructing $\mathcal{T}$ via geometric operations, may be an alternative strategy for describing the $\mathcal{T}$ pattern.

Given $\mathfrak{m} : \mathcal{T} \rightarrow s$, the nature of sort s and of the image[6] $\mathfrak{m}(\mathcal{T}) \subseteq s$ determines what kind of mereology may be defined on $\mathcal{T}$ via $\mathfrak{m}$. Let $\dot{s} \subseteq s$ be this image $\mathfrak{m}(\mathcal{T})$. If $\dot{s}$ or s are discrete, then the mereology on $\mathcal{T}$ is a direct partition. If $\dot{s}$ and s are continuous, then a partition of $\mathcal{T}$ can potentially be induced by dividing $\mathcal{T} \otimes s = \mathfrak{m} \boxtimes \mathcal{T}$ into connected components. Even if a crisp partition is not possible, the internal structuration of $\mathcal{T}$ may be suggested by reconstructing this product from some kernel in s, as outlined in the last paragraph.

Moreover, a given sort s may belong to a hierarchical system S of sorts. For example, there are different sorts to represent color spaces. The simplest

[6]Or, technically, of the *carrier* for s, written A_s if $\{A\}$ is a collection of OSA sort-carriers.

such space is two-valued, e.g. the set {black, white}. Call this space $\mathcal{C}_0$. More general spaces include multi-valued named color sets (like the web colors); call an example of a such a space $\mathcal{C}_1$ (we can assume this includes black and white, so $\mathcal{C}_1 \supset \mathcal{C}_0$); or a continuous grayscale, say $\mathcal{C}_2$. Finer-grained represenions, with a full spectrum of colors, include RGB colors which can be modelled as a unit cube, where white = (1, 1, 1) and black = (0, 0, 0). Suppose $\mathcal{C}_3$ is this RGB space. These sorts then form a poset with $\mathcal{C}_0$ at the bottom, $\mathcal{C}_3$ on top, and the other two in between (and not compared to each other, so the poset is a simple diamond-shaped lattice). In practice it may be further necessary to consider finitary approximations of the theoretically continuous spaces $\mathcal{C}_2$ and $\mathcal{C}_3$.

In the case of an image segmentation or image analysis program, points in an initial space $\mathcal{T}$ (such as pixels) may also be associated with more complex data structures, beyond just their color-values. For example, it may be desirable to distinguish pixels on the border between regions of notably different color, so that a red pixel surrounded by other reddish pixels would be classified differently than a red pixel adject to several bluish ones. A pixel p may be associated with a data atructure considering its eight adject pixels; say, treating the pair of p and its neighbors as vectors in color space, measuring how radically the color pattern is changing across p in different directions. Here pixels are associated with a data structure which includes a color value and also these vectors, so including a color-sort (such as one of the $\mathcal{C}_i$) but extending it. The poset of sorts $\mathcal{C}_i$ is thereby extended by further sorts in which $\mathcal{C}_3$, say, is contained. The end goal of such an analysis may be to reconstruct three-dimensional geometry (thereby associating pixels with an extra numeric dimension, representing depth); or it may be to Semantically Annotate an image, segmenting it into parts labelled according to some semantic interpretation, like "beach" or "building". In the latter case, each pixel is then (along with a containing region) associated with one from a list of semantic predicates. An even more thorough analysis may combine three-dimensional reconstruction with semantic annotation, trying to recover the three-dimensional situation which produces a bitmap image, by creating a scene in a 3d declaration language, which includes structures like surface geometry, surface coloration, incandescent and directional lighting, and camera angles and resolution. Given a scene and a camera position, the original bitmap image can (in a perfect world) be recreated. In this case each pixel in the bitmap image is derived from a complex superposition of structures including surface coloration and geometry, lighting, etc. The data structures are therefore quite complex sorts which generalize the $\mathcal{C}_i$ color spaces mentioned above.

In this kind of analysis, pixels may be associated with a series of different and increasingly more complex data structures, a process which we can model by considering not a single sort s but a collection of sorts s_i and of maps $\mathfrak{m}_i(p) \in s_i$. This may include cases where $\mathfrak{m}_i$ is algorithmically generated from one or more preceding $\mathfrak{m}_j$ with $j < i$. So the image $\mathfrak{m}(\mathcal{T})$ may be generalized to a sequence $\mathfrak{m}_i(\mathcal{T})$ and therefore to a sequence of distinct mereological structures on $\mathcal{T}$, depending on the properties and the granularity of the different sorts s_i and the $\mathcal{T}$-images $\{\dot{s}_i\}$. Consider the case where one $\mathfrak{m}_i$ sends $\mathcal{T}$ to a simplified color-set, which may be derived from cluster-analysis of all the colors in $\dot{s}_i$ for this i, yielding a provisional $\mathcal{T}$-partition; this is similar to a Rough Set sorting on $\mathcal{T}$. A subsequent analysis may combine this information with the results of a different map $\mathfrak{m}_j$, which conducts a more complex analysis of points p, such as considering vectors between neighboring pixels. Combining these analyses may yield a third map, say $\mathfrak{m}_k$, and therefore three images $\dot{s}_i$, $\dot{s}_j$, and $\dot{s}_k$. If we then define mereologies on $\mathcal{T}$ based on these images — whether as a simple partition or something more complicated — we derive a collection of mereons m which belong to one of three systems $\mathfrak{M}_i$, etc., so label the mereons as m_i, m_j, or m_k. The entire set, call it $\mathfrak{M}_X = \bigcup\{m_x\}$, $x = i, j, k$, yields a compound system in which mereons m may overlap, contain each other, and so forth. Even if $\{m_i\}$ (in this example) is a simple partition, and so has no mereological structures such as overlap, or wholes other than $\mathcal{T}$ itself, the m_i could be part of or overlap mereons from the other systems, so the larger system $\mathfrak{M}_X$ will have a full range of mereological relations. Note also that these relations are influenced by the ordering among $\dot{s}_x$ $(x = i, j, k)$, insofar as (for example) images of mereons $m = m_i \Rightarrow \mathfrak{m}_i(m) \subset \dot{s}_i$, will also be subsets of $\dot{s}_j$ or $\dot{s}_k$ given some mapping $s_i \longrightarrow s_j$ or s_k. So if $\mathfrak{M}_X$ is a Mereotopology on $\mathcal{T}$, this demonstrates how mereotopological systems can be induced on $\mathcal{T}$ by mapping $\mathcal{T}$ into a collection $\mathcal{S}$ of sorts, whose internal relationships (a partial order on $\mathcal{S}$ as well as functors such as $s_i \longrightarrow s_j$ transforming or embeddeding elements betweens sorts in $\mathcal{S}$ — suitably enriched or simplified, depending on whether the target sort is more or less complex[7]) suggests $\mathcal{S}$ as an Order Sorted Algebra.

These examples hopefully draw an analogy between Mereotopology as, at least in one derivation, a kind of OSA "over" a topological space $\mathcal{T}$ — for example via a family $\mathfrak{m}_i$ of maps into sorts s_i yielding a family of mereological systems which can be unified into something like the $\mathfrak{M}_X$ above — and Conceptual Graph Semantics as similarly mapping from elements (nodes and edges) of simple graphs into some $\mathcal{S}$, to derive CGS graphs Γ. This is not the only possible connection between CGS and Mereotopology — as mentioned, mereological structures can themselves be modelled as Γ given a support including parthood and coupling relations, and, moreover, it is possible to define topological structures directly on a set $\{\Gamma\}$, deriving "topology over mereology" (rather than mereology over topology). One strategy for "topologizing" CGS structures is to consider pairs Γ_1, Γ_2 such that the "information" contained or asserted in Γ_1 is also contained or asserted in Γ_2. This can occur if Γ_2 contains Γ_1 directly as a subgraph, or contains a refinement Γ_1', where a Γ' is derived from Γ by replacing concept or relation nodes with subtypes (e.g. "I took an Amtrak" from "I took a train") or by replacing generic concept-tokens with specific ones ("I took the 8 a.m. Acela"). Suppose $\Gamma \approxeq \Gamma'$ if they are equivalent or if one is derived from the

[7]Note that OSA algebras may define "default values" δ_s which allow s to be mapped to more complex data structures in which the original s is present as a "field" or subinterface.

other by subtyping or specification; similarly consider a family ${}^{\Gamma}_{.} = \quad \approxeq \{\Gamma\}$.

If a set $\{\Gamma\}$ expresses a set of (natural language) sentences, then ${}^{\Gamma}_{.}$ would represent some common information between the sentences, if they are different surface representations of one underlying idea ("I visited Quebec" and "I took a trip to Quebec") or refinements ("I visited Montreal"). Given a space of Γs, we may want to group them together based on how they overlap in terms of information content. Note that some graphs can belong to different ${}^{\Gamma}_{.}$ classes ("I visited Montreal, then Toronto"). The OSA notion of "sort constructors" [23], [24, p. 6], has an analogue in ${}^{\Gamma}_{.}$ classes which provide minimal information needed to specify a concept-token for a given concept-type and circumstances; consider an airline, flight number, and date, to find a given flight. If Γ expresses information provided by a resource on the Semantic Web, then $\Gamma \in {}^{\Gamma}_{.<c>}$ means that Γ provides enough data to instantiate a programming object of type (or of a software class modelling concept type) c. These notions therefore suggest information *continuity*, such as the preservation of information across representation morphisms (modelled as maps $\mu : \Gamma \longrightarrow \Gamma'$ preserving a subgraph $\gamma \subset \Gamma \approxeq \mu(\gamma) \subset \Gamma'$), and also mereological structures among data aggregates; the combination of notions of continutity and mereology in this context suggest another possible area where Mereotopology can be developped and applied.

Consider the concept of a *keychain*, expressed in terms of functional synthesis, with one or more keys, gathered together by a metal ring, which is then fastened to some kind of strap, so that the whole can be picked or hung up. A graph will model a keychain if it has concept and relation types reflecting this organization, with conceps like "key" and "strap" and relations like *threaded through*. Such a graph then acts as a kind of sort constructor ${}^{\Gamma}_{.}$ for the keychain concept-type. Suppose Γ includes a subgraph $\ddot{\gamma}$ with this keychain-organization. Γ may be derived from visual perception, image analysis, etc.; an initial processing which recognizes spatial relations like *next-to* or *suspended-from* may then be refined, introducing more detailed relations like *threaded-through* and *attached to*. The singular objects of this structure — like keys, straps, and metal rings — may be individuated on topological grounds, but further aggregation depends on mereological and conceptual considerations. Topological notions, like connectedness, may then be extended to these unifications, by invoking notions like continuity across Conceptual Graphs. An example of where these extended notions could be relevant is perceiving sensible object histories: a simply-connected three-dimensional object, for example, has no reasonable trajectory in which it flies apart while moving through the air. Topological properties of empirical objects are also properties of their typical motions and movements. A keychain has multiple partially inter-connected objects which can move in some ways independently, so typical motion for a keychain (thrown through the air, say) is more complex. If we further consider temporal trajectories which are sensible given a keychain's functional organization — like removing a key from one chain and joining it to another — then we have a larger (but conceptually well-grounded) class of sensible histories for one or more keychain-tokens. These "sensible histories" act as a conceptual expansion of the primitive topological notion of connectedness, as it fits into reasoning about spatial dynamics.

I will mention one other example where Conceptual Graph and Mereotopological structures can be interwoven: the Semantic or Ontological modelling of User Interfaces, for Computer Applications. This provides an example of "representational" aspects of Conceptual Modelling, which I am otherwise neglecting to cover in this paper. Conceptual Application Modelling [65] [36] can be used both to simplify the development of visualization and interactive-visualization tools for displaying Ontologically and/or Semantically structured data, and also to facilitate the design of computer applications and User Interfaces in general. For example, an overall application can be modelled in terms of CGS structures which indicate the different relations between components (a particular visual control is linked to a memory object which is persisted in a database table, for example), and then individual controls and components can in turn be represented as providing or displaying objects and data values with their own Ontological specifications. A well-structured data model can allow visual controls to be automatically generated (or partially generated), so as to simplify and accelerate software development. In such a scenario, graph-oriented models of a User Interface are a good case-study in both applied Mereotopological and CGS modelling. Graphical User Interface (GUI) Regions can exhibit multiple RCC relations: two controls may border one another, one may be contained in another, and they can sometimes overlap — for example, specialized windows like find-and-replace dialogs can hover over other controls. GUI layout and design can be facilitated by specifying these relations: for example, toolbar buttons may be designed to always remain geometrically connected, forming a contiguous sequence even if the toolbar itself is repositioned by a user, in a GUI framework where toolbars may be detached, positioned as horizontal, or as vertical. Another example is rendering hovering dialogs or controls semi-transparently, when they overlap other controls and do not have keyboard focus. In addition to these region-connection relations, however, distinct controls and screen areas can be linked thematically or functionally. A GUI, say, may maintain two different lists of files, one for files included in some active project and another for recently viewed files. When a file changes state (opened, closed, revised without being saved, etc.), both of these representations should change accordingly (some applications, for example, indicate that a file may need saving by appending an asteriz to the displayed file name). These local relations are interwoven with spatial relations among controls, and are best modelled with graph-like structures, which are there combined with RCC-like models of GUI visual layout.

My discussion thus far has therefore covered several different kinds of visual spaces where both Mereotopology and CGS relations are relevant: bitmapped images; geospatial regions; physical objects with temporal histories; and, in the last example, application GUIs. I have used the example of image segmentation as a source of (hypothetical[8]) concrete examples, because this connects with real-world computational goals, like image tagging. Of all these visual spaces, hwoever, for a general theory of Mereotopology in a cognitive or pgenomenological context, surfaces of three-dimensional objects (perhaps modelled by a 3d computer language) are perhaps more intuitively precise. Here points on manifolds are sent into a feature space $\mathfrak{F}$, such as coloration, yielding a product $\mathcal{F}$. Consider $\mathcal{F}$ quantified as a vector field over the manifold. In addition to the mereological structures derived above in terms of box-product spaces $\mathfrak{m} \boxtimes \mathcal{T}$, vector-field analysis (such as locating critical points) can potentially explain some of the apparent mereological structure of manifolds [66, p. 86]. For example, an intuitive partition of a surface $\mathbf{S}$ with vector field $\mathbf{V}$ containing finitely many critical points, would assign one critical point of $\mathbf{V}$ to each part $\mathbf{p} \subset \mathbf{S}$. In the case of visual reasoning, we use both topological and mereological observations to develop general mental models of our surroundings: collections of predicative beliefs with real-world, quotidian concept and relation types ("the cup is on the table"; "the car keys are on that keychain"). These predicative networks include mereological relations (like coupling and parthood) among their relation-types, and both mereology and topology is involved in individuating objects to serve as concept-tokens. On the other hand, our immediate perceptions of parthood and continuity are influenced by cognitive appraisals of the semantic identity of different objects; their categorization into kinds like keychains and teacups. So general predicative beliefs are formed out of mereological structures which in turn derive from topological ones, but with the interaction between these layers serving as a kind of feedback loop, which can be modelled in terms of conceptual graph spaces. The most straightforward example of this feedback involves visual perception yielding cognitive appraisals of three-dimensional geometry, so a space of perceived three-dimensional surfaces, on which mereological operators then define complex wholes (like a keychain), in turn then subject to semantic and conceptual identification. An analogous computational process, which I alluded to earlier, involves reconstructing and then semantically annotating a 3d scene given two-dimensional bitmapped images.

This kind of example shows complex spaces where mereology, topology, and Conceptual Graphs intersect. While the cognitive operations involved are natural and almost automatic for people, they are very hard for computers; existing 3d reconstruction and annotation techniques are still very primitive. Analysis of geospatial information provides a different set of examples, which illustrate similar kinds of processes but in a more computationally tractable context. To return, then, to the earlier Sunset Park Chinatown example, an analysis which can semantically cluster geospatial locations (for example by identifying Chinese-themes businesses) provides both an estimation of the extent of Sunset Park Chinatown as geospatial entity, and also a collection of predicative assertions, such as particular restaurants serving a type of (here, Chinese) cuisine.

[8] Granting some poetic licence, in case "hypothetical concrete" sounds like a contradiction in terms — at least if we're not doing Phenomenology. Are we?

In general, a vistor to Sunset Park Chinatown will calibrate her sense of spatial location and direction with predicative observations. Whereas topological notions help model our sense of surrounding space and spatial locations, Conceptual Graphs provide models of predicative beliefs, like "this is a Sichuan restaurant" and "this is a Korean supermarket". Our overall conceptualization of spatial environs mixes these topological and predicative structures. This applies to perceiving objects in the immediate space around us, but also to an evolving mental picture of our local environment; which, at least if we do not live in a wilderness, will be characterized by built entities like roads, buildings, businesses, etc., as well as natural features.[9] The important concepts tokenized in these mental picture are then those of community-oriented data spaces: businesses, transportation routes, residences, markets, restaurants, and so forth.

How we calibrate these concept-tokens with our spatial modelling, knowing general direction, how to get between pairs of places which we frequent, etc., represents the intuitive and gradually acquired familiarity which people have of their neighborhoods, their local geographic communities. Conceptually Modelling this familiarity demands a proper synthesis and superposition of predicative and geospatial reasoning. Suppose we have a database of Brooklyn points of interest. Suppose also that we have a space where descriptors of locations have a semantic clustering and metric — identifying, for example, a conceptual proximity between Chinese restaurants and Korean supermarkets, or between Orthodox synagogues and kosher bakeries. We therefore have two different spaces identified with each place-of-interest datapoint: a geospatial expanse (say, $\mathcal{G}$) and a latent semantic space ($\mathbb{L}$). The semantic tagging yields a mapping and so a set ϑ within the product space $\vartheta \Subset \mathcal{P} = \mathcal{G} \times \mathbb{L}$, which we can consider in analogy to product spaces of coloration and surface points on three-dimensional objects. Since distances in $\mathcal{P} = \mathcal{G} \times \mathbb{L}$ reflect both geospatial and semantic nearness, mereological structures within ϑ can suggest phenomena like a "Chinatown" or an Orthodox neighborhood. If a geospatial partition is derived from some (e.g, clustering or self-organizing map) algorithm, real-world effects like the boundary between Sunset Park Chinatown and Borough Park may be simulated or "discovered". I mentioned earlier that entities such as geospatial regions (often) possess "Foundational Vagueness", because it can be hard to define precise criteria for what actual set of geospatial points comprise (the referent of) a geospatial designator. This example shows that algorithmic processes can provide one account of these foundational structures, for example of instantiators for referrring concepts or properties; like proper names.

[9]Indeed, the functional organization of our communities may be almost entirely a product of these man-made landmarks; natural features (like trees and rivers) may be present, but not as functionally integrated into communities' way of life, compared to their roles in traditional rural societies. We get water from a tap, not from rivers around us, and produce from stores, not from the neighborhood trees. So for a typical resident of first-world urban (and many rural) communities, mental models of our environment predominantly feature man-made places and their functional roles; where we eat and acquire food, how we get from place to place, etc.

Of course, defining a substratum for, say, Sunset Park Chinatown, in statistical terms based on a data space $\mathcal{D}$, means that the (precise) foundational entity depends on $\mathcal{D}$ and is subject to change along with $\mathcal{D}$. This may or may not be desirable. In the current example, since "Sunset Park Chinatown" is not an official designator, it is appropriate that the recognized expanse of the referent be dynamically evolving in a Conceptual Model, since it will be similarly evolving in real life. On the other hand, every change in $\mathcal{D}$, such as one new restaurant opening, can trigger algorithmic changes to the identified region, which may not mirror real-life changes as far as they are recognized by the community. Algorithms can of course be modified to mitigate these kinds of effects. We do not need to identify some region on a map, produced by an algorithm $\mathcal{A}$, as *the* referent of "Sunset Park Chinatown"; we can instead treat this region as a more foundationally complex ontological entity, perhaps as a set of overlapping regions each specific to a given set of observations, or as a space of possible substrata whose modal variation is driven by the space of possibilities in an observation-set. The semantics of these emergent referential structures, where referent boundaries are derived statistically from some observation set, are a good candidate for treatment in terms of Modal Logic. Moreover, since different geospatial regions are also topologically related, this suggests a kind of Modal Semantics based on topological models of Modal Logic, where topological spaces replace notions like "Possible Worlds" as providing interpretations for spaces of possible variation. [5], [33]; [34], [3] (for recent treatments.)

Geospaptial regions provide a Domain of Reference for geospatial referring expressions. Both topological and mereological properties influence what kinds of regions are best suited for these designatory roles. For example, topologically simpler regions are more likely to be named geospatial regions than topologically complex ones (such as ones which are disconneceted or "holely"). An example like Lesotho, a "hole" in a different region South Africa, is the exception rather than the norm. On the other hand, geospatial regions are often built up mereologically from topologically simpler parts. In New York City, the borough of Manhattan includes the island of Manhattan, several smaller islands, and the Kingsbridge neighborhood on the mainland. Locals generically refer to "Manhattan" without clarifying whether they mean the island or the borough. While there are occasions when this may be a source of ambiguity (knowing which elected official represents Kingsbridge addresses, for example), in most contexts distinguishing between the island and the borough has no conceptual bearing; on the other hand, when the distinction *does* matter in some discursive situation, this very fact is intrinsic to the semantics of that situation. There is perhaps a *philosophical* ambiguity so far as we want to identify some precise region to *be* Manhattan. For that matter, even the *island* is foundationally imprecise, since its (above-water) boundary changes with the tides. Referential expressions like "Manhattan" operate within a space of modal possibility, with different degrees of granularity and precision in different contexts. Instead of treating these designations as a direct map between discursive and (for example) geospatial entities, the semantics of reference needs to be theorized in this more complex setting. The space, in which appraisals of designated entities like *Manhattan* can be modelled, varies both mereologically and topologically.

If we turn to topology to provide a semantics for Modal Logic, perhaps in conjunction with a theory of referential semantics, then we need to consider mereological alongside of topological variation. Instead of treating the referent of "Manhattan island" as a fixed and arbitrarily fine-grained collection of points, we want to incorporate a semantic "fuzziness" where microscale spatial variations are incorporated into referential accounts. One way to proceed is to consider a space of modal possibilities in which are situated different fine-grained candidate regions for a referent of, say, Manhattan island. But given ambiguity between the island and the borough in this case, the modal space of "Manhattan-referents" reflects mereological as well as topological variation. This may provide examples for loosened criteria of topological continuity which apply to these modal referential spaces — and potentially to a derivation of Mereotopology in the "topology over mereology" paradigm. In the "Manhattan" case, the *borough* could be represented as a graph whose nodes are Manhattan island, other islands in the borough (Roosevelt Island, Governor's Island, etc.), and Kingsbridge, and graph edges modelling their mereological coupling into the whole designated by *Borough of Manhattan*. This kind of graph structure could provide a carrier for an expanded notion of topological continuity, for example by stipulating that given any topological space associated with these nodes individually (like a space of modal variation in fine-grained referential extent), an edge between the nodes (figuratively like a bridge between islands) "connects" these spaces, overriding the geospatial separation between the regions. For example, a loop representing a jogger's itinerary around East Harlem and Ward's Island will be *closed* in a box-product space indexed by borough names (since Ward's Island is in Manhattan borough), but not one indexed by island names.

Consider, for a moment, the parallels between this situation and the "keychain" example. I suggested that topological notions provide a basis on which a notion of "sensible histories" for keys and keychains can be defined — for example, constraints on a realistic computer animations of a keychain thrown through the air. In particular, a key should not "float through" the metal ring threaded through the key-holes. More precisely, the topological arrangement in which keys and the ring are linked to each other, via their respective holes, will typically be maintained. So any trajectory which preserves this topological structure may be a "sensible history", or at least this structure provides one criterion of sensibleness. On the other hand, there is one kind of sensible history in which a key is detached from a chain, by a very specific unthreading operation, and attached to a different chain. This is an additional class of sensible state-transitions for keys and keychains, which, since it is added on to a class defined in terms of topological arrangements, can be represented as an extension of the specific kind of topological connection usually manifest between the keys and a keychain-ring. We can model these state-transitions via Conceptual Graphs, so these graphs provide a basis for extending a base layer of topological relations. Consider the analogy, in the example just cited, of closed paths in the borough of Manhattan: there is a base layer of topological relations — here, being a closed path in a geospatial region. This base layer is then extended by treating distinct geospatial areas as linked in a graph structure, engendering an expanded notion of continuity and connectedeness, and thereby of closed paths.

This construction acts like an inversion of the use of semantic labels to partition a topological space. New York City has five boroughs, so there is in effect a 5-element discrete sort $\mathfrak{b} = \{$ Manhattan, Brooklyn, ... $\}$, and a product space between geospatial points in the city and this set. Let ϑ be the subset of this product space representing ordered pairs of points and the names of their containing borough. The regions represented by the borough names are internally not connected, but we can define a topology on ϑ so that different islands in one borough are joined, say via $\mathfrak{b}$ equality among pairs of the form $(p, b(p) \in \mathfrak{b})$. Here $\mathfrak{b}$ provides a criterion for unifying otherwise disconnected parts of a topological space, whereas in earlier examples a sort had provided criteria for the inverse mereological operation of separating an otherwise connected topological space. This kind of unification criterion would apply when topological spaces are used to model modal possibilities in interpreting referents of an expression like “Manhattan”, where either the island or the borough may be intended, or the distinction between them not known or deemed relevant, in a given context.

Both kinds of mereological operation may be involved in a case like Sunset Park, given the task of calculating a referent boundary from a set of observations. Semantic clustering can divide Sunset Park Chinatown from (say) Borough Park, but desire for topological and geometric simplicity may leave one or more clusters isolated — semantically related to the Chinatown “theme” but needing to be “gerrymandered” into the larger area. Algorithms may be trained to preserve the geometric isolation of these clusters, but to structurally link them when appraising referents — perhaps not gerrymandering a single connected region out of the data, but rather treating the referent as a graph connecting distinct geospatial clusters. *Sunset Park Chinatown* refers to a neighborhood, but also to a cultural phenomenon, one defined by thematics of social and economic development, ethnicity, immigration, entrepeneurship, etc. Observations which fit this theme are likely to be assimilated into a cognitive map of the neighborhood, even if they are recognized as on or beyond its periphery. This need not involve picturing the neighborhood’s boundary, as a line in “geospatial space”, meandering around to incorporate them; it is rather a cognitive change in focus, where the social thematics gain priority of attention over the geospatial picture-formation. We can model this conceptual duality by treating geospatial referents as data structures (or theoretically as *sorts*) with mereological unification operators as well as geospatial topology and geometry. Obviously a geospatial model or algebra can include set-theoretic union, but the kind of union at play here recognizes both the aggregation and the separation of regions as, jointly, conceptually (and semantically) meaningful: your mental picture of France probably does not include French Guyana, but sometimes it will (e.g., in the context of a World Cup roster). People spending time in Kingsbridge might mentally refer to their surroundings as the Bronx, even if technically they are in the borough of Manhattan; for practical purposes, like knowing which train to take, the latter classification may actually be more useful.

Developing accounts of ontological foundations, like defining a set of points to act as an instantiator for concepts like "Manhattan", is not just a philosophical exercise in matching proper names to real-world entities. The different senses of a name like Manhattan, and the different degrees of precision they imply — the island/borough distinction may or may not be relevant in a given context — has the potential to be a semantically meaningful distinction made or negotiated in the course of a dialogue. Consider apartment-seekers visiting addresses in Kingsbridge or on Roosevelt Island, and learning in the course of discussing those neighborhoods that they belong to the borough of Manhattan, despite not being on Manhattan island. The semantics of these dialogues will fundamentally depend on the referential scope of "Manhattan" differing across contexts. Similarly, imagine conversations about cultural life in the Borough Park / Sunset Park area, acknowledging the neighborhoods' evolving and perhaps recognizing the expanding boundaries of "Sunset Park Chinatown".

The semantics of these linguistic contexts — and, by extension, the cognitive maps which locals have of their neighborhoods, that such contexts exercise — literally put construals of referential scope and boundaries into play, topicalizing changeability in referential extent, or internalizing referential multiplicity. The case of Manhattan borough/island illustrates a partial polysemy, where distinct referential scopes coalesce around one term — not as a matter of foundational imprecision, like the literally unmeasurable (above-water and arbitrarily fine-grained) extension of an island; but as a deliberate superposition of partially distinct referential accounts, playing distinct cognitive roles. Conversely, the Sunset Park Chinatown case shows how referential imprecision can be internalized into discursive contexts and dialogic themes (like the neighborhoods' expanding borders). Such referential-semantic subtleties may transcend the expressive or inferential reach of Conceptual Modelling paradigms, even those with geospatial data structures or "algebras". Canonical relational models of predicative reasoning, which work with (at some nested depth) atomic concept and relation tokens, are ill-suited to the imprecision and dynamics of (geo)spatial reasoning. Mereotopology provides a theoretically richer account of (geo)spatial entities, but when reduced to a set of axioms (like the Region Connection Calculus), incorporated into relational networks, we lose its realism as a model for cognitive operations where foundational structures and foundational vagueness are involved: for example, developing an increasingly refined sense of geospatial boundaries by repeated observations. We may call these latter evolving, imprecise syntheses a kind of "Stochastic", in lieu of "Axiomatic", Mereotopology.

Insofar as "Stochastic" Mereotopology is not well-suited to direct axiomatization into relational networks, a richer synthesis of Mereotopology with predicative structures is needed. The canonical intersection between these aspects is that Mereotopology provides a domain of reference from which concepts in predicative networks can be instantiated. A given concept-token is then both a predicative entity (modelled as a node in a graph Γ) and also a Mereotopological region or "mereon" (a stochastically approximated spatial or geospatial entity; a collection of regions which collectively define a modal space of possible referents for a (geo)spatial term; etc.). Mereological structures are involved in both predicative networks (since part/whole and coupling are predicative relation-types) and in these foundational structures (since any substratum individuated as a concept-token needs some account of separation from its surroundings, and potentially also of synthesis from component parts). In general, because foundational and relational observations (and cognitive processes) mutually influence each other — cognitively suggesting a "feedback loop" — we need a theoretical framework in which both foundational and relational structure can be expressed and juxtaposed. If Mereotopolgy is used to model foundational structures, and CGS to model relational networks, then describing both of these paradigms in terms of Order Sorted Algebra (over topological spaces and graphs, respectively), can help set this integration on firm theoretical and expressive grounds.

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