Token and Continuity: A Modal Mereotopological Theory of Reference and Intentionality

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Abstract

Logical and symbolic models of language and reference interpret natural language by analogy to formal theories, whose symbols and terms map onto those structures which are their possible models. Such maps, in these abstract contexts, are assumed to be straightforward one-to-one or one-to-many operations, that is, constants interpreted as a single entity, or predicates or relations (or functions *qua* many-to-one or one-to-one relations), interpreted as sets of (tuples of) model objects. In real life, however, the association between semantic elements and worldly things and classes is not nearly so clear-cut; classes and kinds have ambiguous cases, and single objects or regions have imprecise or "fuzzy" outlines. *Foundational vagueness*, which affects objects of discourse and perception, implies that perceptual synthesis, or contextual clarification, is needed to properly grasp the intended meaning of referring expressions. This suggests a theoretical intersection between referential semantics and phenomenological or cognitive accounts of perception and of (geo)spatial reasoning. Here I will explore theoretical opportunities to enrich this sythesis by drawing ideas from Mereotopology, Modal Logic, and Conceptual Graph Semantics.

Both Modal Logic and Mereotopology (sometimes together) have provided theories of semantics and of conceptual meaning. These theories do not appear to have been extensively applied to problems of linguistic and cognitive *reference*, although referential issues are often implicit in semantic investigations in general. Here I will try to bring these issues to the fore.

Before attacking reference directly, I will set up a theoretical framework which can clarify the contrast between referential and other aspects of semantics. This is partly to demarcate which are the most significant "referential" issues (and useful terminology), but also to introduce formal systems which can be applied both within referential semantics and elsewhere. In particular, I will refer to the systems or variants of Meretopology, Modal Logic, and Conceptual Graph Semantics. In this current paper, my goal is not to develop full-fledged implementations and practical tools, although I would like to keep these possible applications in mind. Semantic theory sheds light on languange and the mind, but it is also a crucial science and technology for the Information Age. Image annotation and segmentation, textual classification, Information Retrieval, Knowledge Engineering, 3d Reconstruction (based on two-dimensional images and/or three-dimensional point clounds), Computer Vision, Robotics, Virtual Reality, and the design and implementation of domain-specific as well as general-purpose comptuer programming languages, are all subjects in which formal semantics plays a major role. These domains seek to simulate or anticipate human reasoning and language-understanding, and, as a result, their formal models can be important tools for sharpening our intuitions about how concepts and meanings appear in the mind: not because formal systems can recreate human understanding *tout court*, but because their simplified representations, of particular cognitive structures in turn, can focus theory's lens on specific parts of the global structural and interpretive dynamics of human consciousness.

Not to belabor the obvious, but *semantics* concerns how entities or artifacts in language relate to things and classes in the world. How this works in context and in detail is not obvious at all. Predicates cannot just be defined in terms of their extensions, because different (unrelated) concepts can still be everywhere co-instantited: consider (as of 2013) the concepts *African American US President* and *Democratic 21st-Century US President*. The semantics of *referential* concepts, by analogy, is not just a mapping from names to single objects. There are cases where significant changes to a referent do not alter the referential semantics — for example, a newborn's name will still be hers in adulthood. In other cases, though, changes which are arguably less extreme than infancy-to-adulthood trigger substantial referential change: the referential term *USSR* is now vacuous, even though its most significant part, Russia, is still intact.

Speaking rather loosely, by *referential semantics* I mean semantic nuances concerning proper names, and other referring expressions which appear to single out at most one entity. Any analysis of this topic can then be extended to consider non-linguistic or extra-linguistic designative acts, like pointing, as well as cognitive reference or *intending* things which are topics of our thoughts. This can be contrasted with the semantics of predicates or concepts which are *not* assumed to have singular referents. With most *predicative* concepts, we relate some object to a class or attribute, or we compare two different objects. A minimal semantic aggregate, then, combines unary and binary elements: there is a unary map which relates semantic designators to things in the world, and a binary connection between these entities and each other, or to a class or attribute. Given *Clinton is a Centrist*, we need to establish who is referred to by "Clinton" (whether Bill or Hillary, for example), and then to establish the property or category meant by "Centrism". In this paper I will mostly discuss the unary or referential dimension, but it will prove helpful also to consider some terms and distinctions in the predicative or "relational" dimension.

1 Reference and Relation

So our semantic universe, I propose, can be divided into the *referential* and the *relational*. In this section I will further characterize the *relational*.

Following Conceptual Graph Semantics (CGS), let us divide the semantic universe into *concepts* and *relations*. Concepts are further divided into concept-*types*, which form a partially ordered set (some concepts are extensions or refinements of other concepts), and concept-*tokens*, which are instances of concept-types. Both concept types and tokens are "concepts", in the more informal sense: for example, Barack Obama is a token of concepts like *man* and *US President*, but there is also a property of *being Barack Obama*, or a concept *Barack Obama*. Most people around the world do not know Barack Obama, but they know *about* him; that is, they know him *conceptually* (his friends and colleagues also know him *personally*). Concept-tokens are further characterized as *generic* or *specific*: contrast "*An* African-American President", or even ($\star$) "*The first* African-American President‘" — as this latter phrase would have been heard before Barack Obama's ascendence — and then ($\star$) as understood since his election. With respect to relations, we similarly distinguish relation-types and relation-tokens, generic and specific relation-tokens, and we also distinguish binary and *superbinary* relations, which have *arity* two or > 2, respectively.

The subdivisions in the last paragraph are canonical in CGS. Semantic and conceptual structures (including mental models or belief systems as well as language artifacts) can then be modelled as graph structures, whose fundamental units are "compound edges" in which a relation token — so a *relation-node* — is associated with two or more *concept-nodes*. Many beliefs and assertions can be modelled using only binary relations, as collections of "double edges" which connect one relation-node to two concept-nodes (a *source* and a *target*). Conceptual Graphs using only binary relations, or translating superbinary relations into multiple distinct binaries, are similar to the graph structures used by the Semantic Web; the main difference being that CGS relation-nodes need to be mapped onto Semantic Web edge *labels*. CGS models use conceptual specifications called *supports*, which define provisional "rules" such as relation arity and a hierarchy (partial order) between concept types; these supports are analogous (if not always convertible) to Semantic Web "Ontologies". Heretofore I will use the symbol Γ to represent some given Conceptual Graph.

Within a Γ, the collection of compound edges around a concept-node represent knowledge available about the corresponding concept type and/or token. Suppose Γ conveys that Barack Obama is a Democrat, married to Michelle, 40 years old, etc.: this node-set includes (symbols for) Michelle Obama, the Democratic Party, *married-to*, *registered-as*, and *is-aged*. Imagine these edges and nodes arranged around Barack, in Γ, like a snowflake; we can exploit this image and call information available in a graph, about some concept, as a *(relational) snowflake*. This applies to generic as well as specific concept-nodes. For example, a US President is elected to a four-year term in November, inaugurated the following January, and must be a non-naturalized US citizen. These facts (which do not apply to any US president in particular) can be Γ-represented by a "snowflake" around a *generic* concept-node (US) *President*. Note that a *generic concept token* is not the same as a *concept type*: for example, all US Presidents must be US Citizens, which can be represented as part of that "snowflake", but can also be expressed through the type-hierarchy: the concept-type *US President* being contained in or extending the concept-type *US Citizen*.

Going beyond canonical CGS, I propose that relations can be generally divided into relations of *association* or *connection*, on the one hand, and relations of *similarity* or *comparison*, on the other. For a concise terminology, I will refer to the former kind of relations as *join* relations, and the latter kind as *link* relations. Both kinds can be binary or superbinary. Binary join relations include, say, *Barack* ***married-to*** *Michelle* and *Barack* ***registered-as*** *Democrat*. This second relation, however, is conceptually close to the superbinary relation that Obama is a member of the Democratic party, and therefore connected with the set of people who are likewise Democrats. I will argue that superbinary join and link relations sometimes overlap. A *binary* link relation expresses that two entities are linked by some criterion of similarity (or potentially dissimilarity): for example, Barack Obama and Hillary Clinton are *similar* in regards to their politics. Superbinary link relations group individuals into *classes* defined by some criterion of similarity; for example, describing Obama as a *Centrist Democrat*. Note that while the term *Democrat* has a referential meaning — in other words, there is a recognized distinct entity, the Democratic Party, which is usually designated by the term — the expression *Centrist Democrat* does not have a corresponding singulary interpretation. Instead, this phrase refers to a *set* of Democrats who share similar (centrist) beliefs. Such could potentially change if some group of Centrist Democrats tried to create an official or self-declared Caucus or organization, with aspirations to act as a unified body; in that case the term *Centrist Democrat* could become ambiguous, because it would be unclear whether the speaker intends to refer to members of this (perhaps semi-) formal body, or just to any Democrat with a generally Centrist platform.

This example invokes a distinction between generally descriptive (on the one hand) or designative expressions (on the other). If (in a political novel, say), there is a *Centrist Democratic Caucus*, then referring to a member as a *Centrist Democratic* implies her membership in that caucus, which supercedes her actual policies. In such a situation, it would be entirely meaningful to describe someone as ($\star$) "A *Centrist Democrat* whose not really a Centrist", just as (in real life) we have RINOs (Republicans In Name Only). If ($\star$) is instead taken as a descriptive phrase, then it becomes superficially paradoxical, which is not to say that a real-world token of the sentence would be dismissed as nonsense; it might be taken along the lines of suggesting that the politician in question likes to describe herself as centrist, but her voting record suggests otherwise. My point is that this more nuanced interpretation of ($\star$) would be foreclosed if *Centrist Democrat* were a recognized club, with some formal or informal membership criterion. The relation of a member to this club would then be a *join* relation, whereas describing a real-world Democrat as *centrist* instead suggests a *link* relation, comparing their politics to a slightly left-of-center norm. In principle, both of these effects can overlap, so a given semantic unit can simultaneously represent some individuals' membership in a collective and also its similarity to other such members, or similarity between its properties and the signature properties which demarcate the collective in question.

When a relation is asserted between an individual and a collective or aggregate, the degree of freedom or rigidity can vary from case to case. Some collections are loose confederations of parts (like the United Nations, for example), while others are self-contained wholes, whose parts have little conceptual stature on their own. How a particular part fits into the whole can be a property described or predicated of the part; alternatively, the general or typical nature of part/whole relations can be a characteristic of the whole. We can say, for example, the the United Nations is a loose and often ineffective organization (predicating properties of a whole, in terms of its general mereological organization); or that Puerto Rico is a semi-independent US territory (characterizing a part in terms of its relation to a whole, which is not necessarily asserted to be typical of other such parts). In these kinds of cases, mereological relations are invoked to ascribed properties to wholes and/or parts, that is, a whole or a part is the semantic "topic" of an assertion. In other cases, descriptive assertions can associate an individual with a collection of individuals, not in terms of part/whole structures but rather a collection defined largely by precisely those similarities being mentioned, as for example anything described as *red* is simultaneously described as belonging to the class of red things.

When individuals are described as connected to other (sets of) individuals, these groupings can be called *aggregates*, however loose or tight. Aggregates may be binary (like a married couple), and formed by one binary join relation, or superbinary (like the United Nations). In some cases, superbinary aggregates can be formed by binary relations, like a Family Tree whose core relations are those between spouses, and between parents and their children. Even relatively loose aggregates (like the UN) can be referred to, in some contexts, as singular bodies; we say that the UN *voted on* a resolution, or *is in session*. On the other hand, when groups are identified with no presumption of singularity in any context, but rather as sets defined by similarity by some criterion, we can refer to them as *collections*, *kinds*, *classes*, etc. There are still occasions when we refer to such collections in singular terms — we can say, for example, that all liquids will evaporate at high enough temperatures. These expressions, however, do not *refer to* the entire collection as a singular aggregate; instead, they are either abbreviated expressions for assertions concerning each of the collection's members in turn, or else they clarify or explicate the properties introduced as criteria defining the collection through similarity.

These rules establish basic semantics for mereological terms and relations, denoting aggregates and collections, part/whole and part-to-part relations, and multi-layered scales of description, as when we refer to a keychain and then to individual keys. These relations can be invoked to make assertions about individuals, whether "freestanding" or as parts within wholes, or wholes with respect to their parts. In this sense they provide predicative content, through which a semantic artifact asserts some property of some individual. Suppose we say (call this sentence $\mathbb{S}\mathbf{1}$) that "South Sudan has joined the United Nations". This sentence will be informative to us only if we have some basic, prior familiarity with South Sudan and the UN. It is true that the United Nations is constituted by the set of its members, so $\mathbb{S}\mathbf{1}$ is expressing a change in the compilation which defines *the UN* in the first place. Nevertheless, *the UN* is not only a list of nations; there are a number of properties borne by the UN separate and apart from its members, and *the UN* is a distinct token of concepts like *multi-national organization*. So, although $\mathbb{S}\mathbf{1}$ describes a constitutive change in the UN with respect to its basic delineation, nonetheless its semantics invokes *the UN* as a singular referent, and the primary sense of $\mathbb{S}\mathbf{1}$ is to provide new information about South Sudan. A variant sentense like "the UN is voting on admitting South Sudan" similarly is constructed to highlight new information about *the UN*, which is suggested as the latter sentence's *topic*, but again the semantics suggests prior definition of *the UN* to begin with.

Expressing an individual's membership in a collection or aggregate can provide further information about the individual (or collection or aggregate in turn), but these assertions rest upon a prior foundation of the individual and collective having been defined; even if the collective is constituted by its set of individual members. Once an entity is defined and entered into the space of discourse, further information accrues, the "snowflake graphs" figuratively form, and predicative constellations emerge which situate the entity in question in a network of conceptual relations. But these predications are only meaningful if the entity has been provisionally defined. The formalization of that original defintion are where referential semantics — and its complexities — come into play.

2 Conceptual and Foundational Definition

Many people may know that ($\mathbb{S}\mathbf{2}$): South Sudan is a new nation, in Northeast Africa, once part of Sudan. This information, along with the name *South* Sudan, is enough to suggest a minimal if broadly accurate picture of where South Sudan is on the map, and its status as a newly independent nation. We may consider ($\mathbb{S}$**2**) to be a basic but adequate defintion of South Sudan, even if we have not seen a map of or including South Sudan, we do not know any of its cities, etc. Of course, we assume that the nation does have relatively fixed borders, a capital, and so forth; we assume that *someone* knows this more detailed information, even if it is not part of the minimal definition. So a definition like ($\mathbb{S}$**2**) provides a basic conceptual account of its definandum, but we consider it resting on a prior, more *foundational* account, where South Sudan is defined as a geospatial region, as points on a map, as a political entity constituted by official international recognition and treaties and diplomatic ties, etc.

Another example, which illustrates a similar duality in our notion of *defintion*, was introduced by Mark Heller, who has had a long career teaching in Syracuse, New York. He discusses the fact ($\mathbb{S}$**3**) that Syracuse is north of Manhattan. People may certainly believe ($\mathbb{S}$**3**) who have never visited Syracuse or Manhattan or studied them closely on a map. They may be vaguely aware that Syracuse is a city in Upstate New York and that Manhattan is a borough in New York City. They therefore have an adequate conceptual definition of Syracuse and Manhattan, one which can establish these places as concept-tokens, from which the predicative relation ($\mathbb{S}$**3**) becomes meaningful. However, consider the *meaning* of ($\mathbb{S}$**3**), apart from the *information* it conveys to different people as that is further affected by their familiarity with the places involved. We presumably think that the *meaning* of ($\mathbb{S}$**3**) is expressed by taking *Syracuse* and *Manhattan* as names referring to geospatial entities (or some such notion); in other words, when we consider the fact which is *expressed* by ($\mathbb{S}$**3**), we are considering Syracuse and Manhattan not as minimally defined by vague familiarity, but as designated entities with specific and detailed definitions. We are prepared to accept informal, conceptual definitions as proxies for these more detailed accounts relative to our own degree of personal familiarity with the entities involved, but we assume their imprecision is a product of our *personal* limited knowledge; that we could, with further research, acquire a more precise definition, which for example provides a detailed picture of the geo-spatial entity which the name "Manhattan" designates. Insofar as any language artifact conveys information, it does so because those of its symbols which represent concept-tokens may be replaced by real-world entities, which serve as their referents. Linguistic meaning would be vacuous if there were not things in the world whose properties are disclosed through semantic formations.

To be sure, there are *de dicto* referents which *only* have conceptual definition, like "the First Woman US President". I may believe that Hillary Clinton will become president in a few years, and so use this expression with her specifically in mind; in most contexts, however, this expression (as of 2013) does not refer to a particular person. I can still opine, for example, that ($\star$) *the First Woman US President will be a Democrat*; but — assuming I do not hold ($\star$) simply, say, by projecting Hillary as a likely successor to Obama — the warrant for ($\star$) will be something like the Democratic party being more likely to nominate a woman; in other words, something based only on conceptual definitions. We furthermore need to distinguish the reasons why such an open-ended belief may be plausible, from the semantic *meaning* of the belief were the projected state of affairs to obtain. If and when ($\star$) becomes true, then *the First Woman President* will indeed become a *de re* as well as *de dicto* designator, and ($\star$) expresses a fact involving some worldly entity expressed as "the First Woman President". Relational structures involving generic, hypothetical, or even fictional concept-tokens describe *hypothetical* states of affairs, which are *meaningful* by analogy to similar or actual states of affairs whose topics are indeed worldly entities. I do not suppose that this argument resolves all issues with fictional or *de dicto* reference, but perhaps they at least justify the claim that the kinds of reference in which referring expressions are understood to stand for actual, real-world entities, are logically prior to these fictional, hypothetical, or *de re* varieties.

So this brings us to the main issue: to analyze referring expressions (call them REs) involving worldly entities. The simplistic (or desired) picture is that linguistic artifacts (and cognitive states) express relational states which describe worldly states of affairs, as their topical "symbols" are substituted by worldly entities. This formulation begs issues of mental counterparts to language "symbols": it seems reasonable enough to call the string of letters *Manhattan* a kind of compound symbol, but is my mental image or thought of Manhattan a symbol? — and, if so, which of the various presentations I have is *the* Manhattan-symbol, and which are further beliefs and experiences I have *about* Manhattan? Is my mental Manhattan-symbol a point on a map, or a larger geospatial outline on a more detailed map, or my memory of seeing Manhattan buildings and streets? Which of these presentations "iconifies" Manhattan for me, and which are *about* Manhattan insofar as Manhattan has been thus iconoified? Perhaps there is no particular mental content or picture which is my Manhattan-symbol, but rather a kind of inner awareness or acknowledgement that I am now thiking about (or perceiving or looking at) Manhattan; i.e., the Manhattan-symbol is really a *meta-thought*, a second-order thought or feel annotating the subsequent thoughts or accompanying presentations such that these are *about Manhattan*. I will not try to settle the issue here; instead, let us assume for now that we consider only *linguistic* reference, and delay until later a consideration of *cognitive* reference with these further Phenomenological possibilities.

This is not to argue, however, that cognitive issues are set aside even when we focus just on *linguistic* reference. I have read, for example, that ($\mathbb{S}\mathbf{4}$) Juba is the Capitol of South Sudan. However, I do not know where Juba is, or really anything about it, *except for* ($\mathbb{S}\mathbf{4}$). I do not have some prior definition for *Juba* on which ($\mathbb{S}\mathbf{4}$) then provides further predication. We can say that ($\mathbb{S}\mathbf{4}$) implicitly contains a definition (at least a conceptual one): ($\mathbb{S}\mathbf{5}$) *there is a city* called Juba, which is the Capitol of South Sudan. The state of my knowledge is better conveyed by ($\mathbb{S}$**5**) than by ($\mathbb{S}$**4**). If we read them as linguistic acts or statements, these two expressions have different semantics: ($\mathbb{S}$**4**) is constructed such that *Juba* is a referential term; its meaning is established by designating a particular entity, and ($\mathbb{S}$**4**) would be present (*qua* language act) in a context where prior familiarity with this referent is assumed. ($\mathbb{S}$**5**), by contrast (*qua* language act), sets up this referential structure internally; it is not phrased as if prior familiarity were dialogically assumed. I have emphasized "*qua* language act" because ($\mathbb{S}$**4**) and ($\mathbb{S}$**5**) could be used as metalanguage or analytic expressions apart from particular dialogic contexts, in which case the relationship between them may be understood differently. The semantics of ($\mathbb{S}$**4**) and ($\mathbb{S}$**5**) differ *in context*, but I find it an open question whether they have the same *factual* meaning; whether they express *the same fact*.

If I already know that there is a city called Juba — but not that it is the capitol of the new nation — then ($\mathbb{S}$**5**) conveys *for me* the same information as does ($\mathbb{S}$**4**); relative to *my particular* epistemic states, the two assertions have the same meaning. On the other hand, if I had never heard of Juba, then upon hearing ($\mathbb{S}$**4**) I would have to interpret it by inferring ($\mathbb{S}$**5**); in this case the meaning of ($\mathbb{S}$**4**) therefore "collapses" to ($\mathbb{S}$**5**). So relevant to a particular hearer's epistemic states, one or the other expression may "collapse to" the other. This may suggest that they share the same factual meaning, and that the semantic differences between them are differences in accounting for various epistemic states of potential conversants. On the other hand, the factual content of ($\mathbb{S}$**4**) seems to be recovered by substituting a specific entity (some geospatial region) for the sumbol *Juba* (and *South Sudan*, for that matter). On the other hand, the factual semantics of ($\mathbb{S}$**5**) does not seem to similarly depend on a referential substitution with a *specific* geospatial enity; only that such substitution is possible *in principle*; that there is some *unspecified* entity available for this substitution.

If someone, similarly, accepts ($\mathbb{S}$**3**) (about Syracuse and Manhattan), then they accept that a certain relational predicate between Syracuse and Manhattan obtains. We are then left to consider whether this belief is identical to belief about the specific fact which seems to be (factually-semantically) asserted by ($\mathbb{S}$**3**), namely that there are *some specific geospatial entities*, Syracuse and Manhttan, the former **north-of** the latter. Accepting this fact — holding it as a cognitive item in my belief-system — appears to require that I have a similar cognitive presentation entities "substituted in" for the aymbols *Manhattan* and *Syracuse*. On the other hand, even someone who has never visited or even seen these places on a map — in a position comparable to my own vis-a-vis Juba and South Sudan — and who accepts ($\mathbb{S}$**3**) as *true*, presumably has some nonvoid belief. My earlier comments implied one particular interpretation of this situation, in which the belief, where *Syracuse* and *Manhattan* are known only according to their *conceptual* definitions, is a kind of *proxy belief* for a belief related to the more detailed factual content, where there is *some specific* geospatial entities grasped as referents for *Syracuse* and *Manhattan*. It is possible to accue beliefs about entities given only conceptual defintions: collectively, for example, ($\mathbb{S}$**1**), ($\mathbb{S}$**2**), ($\mathbb{S}$**4**), etc., represent a collection of facts about South Sudan. Our familiarity with a space of predicative assertions concerning some concept-token implies knowledge of *conceptual relations* which this entity holds to other entities. This knowledge is meaningful even with only rather minimal, conceptual definitions of the entities involved. But when judging ourselves to have certain knowledge, we seem to construe ourselves as having knowledge of worldly *states of affiars*, and these states of affairs are truths about *specific entities*; such as geospatial entities, in these examples. We may not know what these entities are, but we seem to construe that the facts which we believe are facts *about these entities*; so those details which we actually posit, relative to how *we* define the relevant referents given our particular epistemic situations, are proxies for facts which "objectively" obtain *with respect to* worldly referents.

So there is a potential duality *within the factual interpretation* of assertions like ($\mathbb{S}$**3**) and ($\mathbb{S}$**4**) themselves: first, they can be taken to express *that some relational predicates* obtain, between Manhattan and Syracuse, and between Juba and South Sudan, respectively. On this *relational* interpretation, familiarity with these facts can be absorbed within an epistemic situation in which prior definition of the (here geospatial) entities involved are more or less detailed; including situations in which very little is known about them *as* geospatial regions, as such (in terms of their location, geospatial extent, etc.). On the other hand, the *factual content* can alternatively be taken as tokenizing a worldly state of affairs in which these entities *as geospatial* are implicated. Presumably, if ($\mathbb{S}$**3**) and ($\mathbb{S}$**4**) are *true*, they are true *about something.* Whether those geospatial things are represented in detail *as geospatial*, the fact that they do indeed exist, and are indeed designated with REs "Manhattan", etc., is presupposed by the very fact that ($\mathbb{S}$**3**) and ($\mathbb{S}$**4**) are taken as true. So the *relational* interpretation, which can somewhat bracket the "foundational" question of *what* — or *which (geospatial) entities*, — *are in fact Manhattan*, etc., nevertheless still rests on the possibility that such a foundational account can be provided.

On the face of it, it seems simple enough to define Manhattan, etc., as geospatial regions — (sets of) points or locations on maps or the surface of the Earth. I will now argue that this simplistic account is inadequate; if we want some Domain of Reference into which (geospatial) REs will refer, this domain must be defined much more carefully.

3 Domains of Reference

I have no particular reason to single out the Geospatial domain, but the Manhattan/Syracuse and Juba/South Sudan examples are useful, so I will stick with this theme. A straightforward account of referential semantics would suggest that *Manhattan*, say, "points" to a geospatial entity; linguistic assertions about "Manhattan" express facts insofar as the symbol is replaced by the appropriate geospatial region. Let's look a little closer. To begin with, there is a difference between the *borough* of Manhattan, and the *island*. This difference is not always relevant, but they do not name the same geospatial region; the Borough of Manhattan (say M_B) includes numerous islands in addition to the Island of Manhattan (say M_I), and also the Kingsbridge neighborhood on the mainland, attached to the Bronx. Does "Manhattan" in ($\mathbb{S}$**3**) refer to M_B or M_I, or both? Is ($\mathbb{S}$**3**) really two different assertions, one about M_B and the other about M_I? Even if we consider just M_I, how do we actually define what geospatial region M_I refers to? A boundary on a map? Which map? Points on the Earth? What about microscale changes in water levels, which cause the line between the island and the surrounding water to change?

I will review several possible tactics for addressing these kind of questions. These strategies have various literature and history, but I will not try to honor their provenance here; I just want to mention them as theoretical possibilities. I'm sure other approaches can be added to this list. But the following seem intuitive and substantial, so that if they fail to fully address the problems, this failure will be interesting. To whit, then:

Domain-Specific Ontologies. Manhattan, etc., are geospatial entities, which means that a foundational account of their referential status — of what specific geospatial entities they are — depends specifically on concepts and ontological posits within the geospatial domain. This domain includes notions such as geospatial locations and regions, their various mereological and mereotopological relations (being contained in, sharing a border with, etc.), and the specific ontological relations which geospatial entities hold to their referring designators. For example, there are particular ambiguities which are specific to this domain — national borders may be contested, so that parties disagree on where the border actually lies; or borders may be imperfectly surveyed, or defined relative to natural features subject to change, like the course of rivers. Comparable referential or mereological ambiguities may apply to other domains; for example, a proper name referring to a person refers to the same person (presumably) once she gets a haircut, so we need to account for the more and less hirsute individual as "the same" person. Instead of unifying all of these mereological subtleties with one global theory, they should be considered as domain-specific nuances, which are the formal responsibility of different domain-specific ontologies.

Property Theory. Michael Jubien's (1993) *Ontology, Modality, and the Fallacy of Reference* is a very lucid and (for me) persuasive account of mereological puzzles such as those I just alluded to. A very condensed overview of Jubien's theory, applied to these specific geospatial cases, would suggest an ontological distinction between whatever geospatial regions we think may be referred to by Manhattan (etc.), and the property of *being Manhattan* (etc.). I know that there is a city with the property *being Juba*, and (now) a nation with the property *being South Sudan*; the relation expressed by ($\mathbb{S}$**4**) connects these two *properties*. Similarly, Mark Heller used his Manhattan and Syracuse example — in light of foundational ambiguities with respect to these entities — to support his notion of "conventional objects". Both Heller and Jubien discuss examples related to statues: a statue is a lump of clay, but we can imagine that the same statue could perhaps have been executed instead in wood, or with a different lump of clay, etc.; but a lump of clay could not be a different lump of clay, and certainly could not be a block of wood. Because the Sudan/South Sudan treaty was contentious, we can imagine that some border towns could have ended up in Sudan rather than South Sudan; so South Sudan could have been a somewhat different geospatial region. But presumably a certain set of geospatial points either does or does not contain a particular subset. For these kinds of reasons, Heller suggests that the sculpture is a "conventional obejct", and the lump-of-clay a "physical obejct", and these two (different) objects coexist in one place for a while. Similarly, Manhattan is a *geospatial object* (a precise set of points, which might fluctuate with the tides, etc.) overlayed with a conventional object (a socially recognized communal territory, the most famous part of New York City). Jubien recasts these notions in property-theoretic terms. He rejects the idea of conventional objects distinct from physical ones; instead, he thematizes the theoretical role played by this distinction by distinguishing properties from their substrata. There is a property of being some particular statue, which is instantiated by a lump of clay, though it could instead have been instantiated by a different lump of clay, or even a block of wood. There is a property of *being South Sudan*, which is instantiated by some geospatial region, though it could have been instantiated by a different geospatial region (for example if the peace treaty had been negotiated differently).

Referential Modal Logic. There are multiple different regions which *could have been* such that they instantiante the property *being South Sudan*. Using "possible world" talk, we can say that there are possible worlds where they *are* South Sudan, though Jubien argues that Possible Worlds are metaphysically problemmatic, at least except as an eliminable but useful *façon á parler*. Jubien's property-theoretic account of modality offers an *alternative* to possible worlds. Not all interpretations of modal logic, however, need to invoke possible worlds, or traditional Kripke semantics or Kripke frames. Many different formal and mathematical systems provide models for modal logics, ranging from topological spaces and "bisimulations" to epistemic, temporal, game-theoretic, and mereotopological modalities. The idea that several different geospatial regions *could be* or *could have been* South Sudan, for example, may be modelled by considering each precise collection of geospatial points as a different geospatial entity, but associating the designator "South Sudan" with a modal operator which relates each of these as its possible extension. Such a "modal-referential" operator need not quantify only over sets of geospatial points; for example, it could also apply to geospatial *observations* or decisions, such as those made when surveying or negotiating the Sudan/South Sudan border.

Multi-Granular Objects. The precise *geospatial entities* just introduced as *possible extents* of, say, South Sudan, are arbitrarily fine-grained. Indeed, a geospatial ontology may theorize geospatial regions as infinitely fine-grained; every collection of distinct geospatial points is a distinct region, and the purpose of surveying, maps, GPS systems, and the like, is to create empirical measurements which approximate these fine-grained regions as precisely as possible. On the other hand, these measurements are always *just* approximations. If we take "Manhattan" or "South Sudan" as referring to arbitrarily fine-grained point-sets, then we also acknowlege that we will never have a thorough account of their actual identity. This assumes that no two regions are identical, no matter how fine-grained their points of difference, unless they are *perfectly* identical. However, we do not have to accept *a priori* the idea that granularity is metaphysically separated from identity; in other words, thar identity is never relative to a particular "scale of granularity", but is always to be assessed against a hypothetical "absolutely fine" granularity. It is possible, instead, to develop a framework in which identity is "granularity-relative", just as, for people who believe in "perduring" or *four dimensional* objects, identity is time-relative. An object persists in time, so that the object at t and at t' is the same object, even if its extent or properties are different. These differences are accommmodated by saying that the state or appearance of the object is relative to the "time slice" at or during which the relevant observation occurs. Observations, in other words, are temporally relative, and there is no assumption that they are expected (in general) to hold at other times, or that failure to hold would imply that the observed entity is *a different* entity. Similarly, we can say that observations are relative to a particular degree of precision, and more-fine grained observations may present a different picture of *the same* object. For example, when considering whether M_B and M_I are *the same* place, we can ask what degree of granularity applies to the desired criterion of "sameness". Similarly, when asking whether Manhattan in 1813 is *the same* as Manhattan in 2013, we need to specify whether the intended notion of *same* is or is not temporally relative.

Syracuse instantiates the property of ($\star$) *being North of Manhattan*, and Juba the property of ($\circledast$) *being a national capital*. Both of these properties have many instances; so Syracuse shares ($\star$) with many other places, and Juba shares ($\circledast$) with some other cities. But "Syracuse" and "Juba" are also proper names, so their subsrata also instantiate what Jubien calls *singulary* properties, which have at most one bearer — being the entity given those names. Jubien's approach to meaning and semantics is quite consistent with a relational or "semantic web" paradigm, because almost all semantic elements then become modelled as properties, which are set in relational networks. Some of this properties are singulary, and capture the semantics of REs and proper names. *Qua* referential term, for example, *Syracuse* encodes the property *being Syracuse*, and ($\mathbb{S}$**3**) encodes the property network which, stated inelegantly in words, would be something like "The bearer of the property *being Syracuse* also instantiates the property *being North of Manhattan*"; or "the property *being Syracuse* is co-instantiated with the property *being North of Manhattan*". These property "co-instantiation" networks, which in "Jubien semantics" model all or almost all natural language, are therefore quite similar to Conceptual Graphs, with singulary properties playing the role of (non-generic) concept-nodes, and non-singulary properties (or families of them) playing the role of relation-tokens.

Moreover, the sentences ($\mathbb{S}$**1**), ($\mathbb{S}$**2**), and ($\mathbb{S}$**4**), which I earlier suggested model a "predicative network" concerning South Sudan, also model a network of property relations or "coinstantiations", including, for example, that South Sudan instantiates the property of being a United Nations member; so that this latter property is coinstantiated with the property *being South Sudan*. By this account, the *semantics* of a referential phrase like "South Sudan" is not some specific, fine-grained geospatial (in this case) entity to which the phrase refers, but rather the *property of being* this entity, which allows for foundational ambiguity, and related ontological or epistemic imprecision, with respect to the fine-grained referent. Such imprecision does not necessarily have any bearing on the property-network and relational semantics. In other words, because the *semantically significant* element here is the *property of being* some specific entity, not the (precise, arbitrarily fine-grained) entity itself, the overall semantics are in many contexts independent from foundational or fine-grained ambiguities affecting referent-to-entity association. Similarly, *being Syracuse*, co-instantiated with ($\star$) as above, is a relation obtaining irregardless of the *being Manhattan* instantiator being the borough or the island. So call the distinction M_B and M_I *semantically external* to the property-relation expressed in ($\mathbb{S}$**3**). If Γ models ($\mathbb{S}$**3**), with nodes for Syracuse, Manhattan, and **north-of**, then the Manhattan-node represents the property *being Manhattan*, and within the implied granularity (or lack thereof) of the semantics, the distinction between M_B and M_I, as distinct possible instantiators of *being Manhattan*, have no semantic bearing. The property network holds with respect to the *property* of being Manhattan, and we do not need to specify whether we understand this property as being instantiated by the island, the borough, or both, or whatever other combination (modal, "fuzzy", etc.) we may envision as a *foundational* account.

As I have perhaps suggested by how I have blended the terminologies, I think all of the above tactics can play a role in developing a robust theory of foundational, referential semantics. We can accept, for example, that REs are primarily used to express singulary properties, and it is *as properties*, rather than as ontologically specific accounts of their *instantiators*, that these referential formations have semantic bearing. On the other hand, there is a significant semantic distinction between singulary properties which do indeed have real-world instantiators, as opposed to *de dicto* properties like *being the first Woman*

President, or fictional properties like *being Sherlock Holmes*. These latter kinds of properties may still be meaningful in predicative networks — consider "the property of *being Sherlock Holmes* implies the property of *being English*"; and "the property of *being the first Woman President* suggests the property of *being a Democrat*" — as property-theoretic construals of assertions such as Holmes being English, and the first Woman President being a Democrat.

The fact that we so readily accept fictional and hypothetical discourse suggests that most semantic weight is born by relational networks between concepts — networks whose structure does not depend on the concepts actually being instantiated. So an RE is not *meaningless* even if it does not have a referrent; it can still occupy a relational network "slot". The worldly referent, when it exists, nonetheless can play several roles: it can be mutually recognized by conversants to confirm that they have the same referring expression in mind (consider someone clarfying whether a use of the name "Clinton" refers to Bill or Hillary); gestures like pointing, which engage referential substrata directly, can be one way of clarifying these situations. Moreover, clarifying the sense in which, and "where", a presumed referrent-substratum exists, can suggest how a more fine-grained referential account may be developed, should this prove desirable. For example, if new natural resources were discovered on the Sudan/South Sudan border, it may be necessary to clarify the border's geometry more precisely, so as to identify which nation has rights to which of these resources.

Leaving aside these *epistemic* operations, accounts of referential substrata still play *ontological* roles, because the semantics of situations where referents are presumed to exist, in contrast to fictional or hypothetical cases, certainly invite different interpretations, even if they share similar relational structures. The Sherlock Holmes stories would be read differently, for example, if Holmes were a real person — perhaps an influential politican —, known to both author and readers and intended to be understood as such, with the novels inventing a "double life" for this person. For that matter, it is intrinsic to the "meaning" of the novels that *London* is the real London; we would read them differently if we expected Conan Doyle to intend "London" instead as a purely fictitious place which just happened to have that name. There is a subtle commentary which unmistakably alludes to a real, historicized London, and to English society.

There are, indeed, several different modalities in which REs can operate — as fictional references, references to fictional incorporations of real-world things, hypotheticals which may be realized (like the first woman president), hypotheticals which the speakers does not suggest will be realized, but is exploring the further hypotheticals which would plausibly arise if they were (like imagining the first *gay* US president); and references to real entities which are known to exist but are unspecified, as well as those which are known and located, so they can be pointed to and studied. Even if an *arbitrarily* fine-grained account of worldly referents, in this last mode, has no semantic bearing, the fact that the referent is *known* and is *posited* and available for study and investigation, is intrinsic to referential semantics in this mode. So the *singularity* of these referents, and the fact that *finer-grained* accounts can be developed, even if *arbitratily* fine-grained accounts are not practical, is presumed and semantically meaning. So a robust theory of referential semantics, within this mode — which arguably encomapsses the largest class of referring acts — needs to explicate this *singularity* and this *possibility of finer-grained elaboriation.* I have mentioned some approaches whcih can lead toward such a theory, but they need to be drawn together in a systematic way. I will briefly sketch one possible approach.

4 Singularity and Granularity

I think the most plausible referential theory, in the mode where the referent exists and is roughly located and identified (and not just existentially posited), supposes that there is *one* referent substratum-entity. There are many different fine-grained representaations of Manhattan, for example, but it seems implausible to argue that there are many different "Manhattans". The M_I and M_B distinction is an interesting variation, because it may be argued in that case that there are indeed two different senses of the word "Manhattan", but, even here, in a relational predicate like ($\mathbb{S}$**3**), the referential scope seems to unify both of these. But leaving this aside, and focussing just on Manhattan island — which may correspond to many different fine-grained point sets or geospatial regions or etc., considering say tidal variations — there is still (on a good theory, I believe) just one Manhattan, so either these fine-grained entities are all collectively this one Manhattan, or just one of them are, or they are all parts of the one Manhattan. The main question for (this part of) referential semantics is how the multiplicity of fine-grained candidate referents are merged into a single real-world referent, as instantiator of a singulary property.

There should be few problems with the buildings and streets of Manhattan being amongst its proper parts; any fine-grained ambiguities arise out by the rivers, where we consider rocks that are sometimes above-water and sometimes not. We could perhaps adopt some graded-set measure, and map geospatial points which are **part-of** Manattan to degree 1 — with no ambiguity or hesitation at all — and then assign measures to non-crisp parts on the periphery, perhaps granting some given rock a membership grade identical to the percentage of time where it is above water. Any actual quantity may be arbitrary; the point is simply to consider plausible accounts of Manhattan as a graded or "fuzzy" set of geospatial points. If we were interested in some fashion of "fuzzy Mereotopology" then we could consider the grade-1 Manhattan-points as the *interior* of the region Manhattan, and the points with grade in the open interval (0, 1) as its *boundary.* We should bear in mind that *points* (as zerodimensional entities) are problemmatic in (some forms of) Mereotopology, so the idea of a "grading", as a map like $\mathcal{T} \to [0,1]$ where $\mathcal{T}$ is a Mereotopological space, needs to be defined in a manner suitable for a "point-free" paradigm. We can define "point-free" maps in terms of more general product spaces, but I will not consider the formal possibilities at this point.

Insofar as "membership grade" of points in a region is ambiguous, then trying to resolve these ambiguities by a map $\mathcal{T} \to [0,1]$ may seem circular, because these grades may themselves be ambiguous. For example, shouldn't the boundary between the interior and the border of a region (if these are defined by where membership grades dip below 1) be imprecise, like the boundary between the region and its complement? The point is not, however, to assign one single membership function defining a region; instead, we assume that there are many different possible such functions, relative to different observations or membership criteria. We can therefore combine the modal and the granular strategies suggested in the previous section: a single coarse-grained entity can have many possible fine-grained elaborations. An (overly) simplistic *referential semantics for geospatial entities* would assign geospatial referents like *Manhattan* to *single*, arbitrarily fine-grained, sets of geospatial points. An attempt to represent foundational vagueness or "fuzziness" could replace this *crisp* set with a fuzzy or graded set, but a more compelling theory will consider a space which includes many different possible sets. Note that a graded set S is equivalent to a family of crisp sets $\{S_\alpha\}$ where $x \in S_\alpha$ iff $\ell(x) \geq \alpha$, writing $\ell(x)$ for the grade of x in the original S. So every family of graded sets is contained in a family of crisp sets, and we can consider each particular set as crisp without loss of generality.

Let $\mathcal{E}$ be a geospatial environment large enough to contain (any possible) extension for *Manhattan*, and consider the set of possible Manhattan-referents $\{M\}$ as a subset of the power-set $\wp(\mathcal{E})$. A set M is a plausible model of Manhattan's extent if $M \subset \mathcal{E}$; that is, if $M \in \wp(\mathcal{E})$. If we consider $\mathcal{E}$ as a topological space, then the power set $\wp(\mathcal{E})$ will have induced topological and metric properties; for example, two $\wp(\mathcal{E})$-points e and e' are proximate if they significantly overlap, as subsets $\subset \mathcal{E}$. For example, imagine the boundary of Manhattan island gradually contracting, as the water rises; this corresponds to a continuous path in $\wp(\mathcal{E})$. Not every point in $\wp(\mathcal{E})$ is a sensible model for Manhattan; we have to exclude sets with infinitely many disjoint parts, or overly complex constructions like space-filling curves. We can also exclude sets which topologically resemble an annulus: Manhattan has no "holes". The possible "Manhattans", then, are a subset say $\mathfrak{E} \subset \mathcal{E}$, essentially those e which are finite unions of simply connected parts that locally resemble a disk. If we intend to model Manhattan *island*, then only those e which have one component will be available. In general, $\wp(\mathcal{E})$ can be partitioned into those sets of sets which are *similarly situated* in $\wp(\mathcal{E})$, borrowing this notion from [58, p. 4]: i.e., there is an isomorphism $\mathcal{E} \mapsto \mathcal{E}$ which carries e to e'. In particular, on $\mathfrak{E}$, this sets up equivalence classes such that $e \equiv e'$ iff they have have same number of simply connected components.

I'll refer to this equivalence relation, restricted to $\mathfrak{E}$, with the symbol $\cong$.

This $\cong$ equivalence on $\mathfrak{E}$ has a potential concrete interpretation: we can say that $e \cong e'$ if they are *compatible* referential accounts. For example, imagine a catastrophic change in water levers which divides Manhattan into two islands; the RE *Manhattan* would clearly need to be reassessed in that situation. So an e representing one island, and an $e' \ncong e$ with two islands, are referentially incompatible because they could not be two different accounts of one referring term unless this term were subject to deliberate alteration (as opposed to fine-grained variation due to gradual changes in its target). Similarly, e-models of M_I and M_B are referentially incompatible because they depend upon a clarification of which sense applies to *Manhattan* in some usage. So $e \cong e'$ if they are different possible appraisals for the extent of *Manhattan* under one single usage. If we accept that each e models a possible (set of) observations or possible geospatial states (like water levels), then this $\cong$-equivalence also has a modal interpretation, as a kind of inter-world or "trans-state" accessibility. Observations may be performed under an initial set of intentions, such as an agreement to survey the boundary of Manhattan island, or to map the borough of Manhattan by surveying each of its islands. Different e sets are mutually accessible if they are possible results of such observations under the same initial assumptions.

Let's say a morphism $\mu : e \longrightarrow e'$ is $\cong$-compatible if $e \cong \mu(e)$. Perhaps e represents a computer simulation of the outline of M_I as oceans rise at the current pace, while $\mu(e)$ represents a scenario where carbon-reducing legislation is passed. The morphism μ in that scenario then measures the contribution of this legislation as one factor. We can also consider notions of equivalence relative to a scale of resolution. Consider a difference measure $\mathit{\Delta}$ on $\wp(\mathcal{E})$, or relatedly an operator $\blacktriangle_\mu$ on μ, such that $\mathit{\Delta}(e, e')$ is the greatest distance between pairs of points which are both in either e or e' but not their intersection; respectively for $\blacktriangle$ with e and $\mu(e)$. A map μ may then be "negligible" on a scale η if $\blacktriangle_\mu \leq \eta$. Two e may model "the same object" if their difference is negligible in this sense, for a context-relative η. For example, at sufficiently low resolution (say a map of New York State), even the difference between M_I and M_B may be unnoticeable.

Identifying two extents whose difference is only apparent at some fine-grained scale is analogous to identifying two different possible future versions of an object; when we refer to someone with a proper name, say, this referring term is assumed to hold for that person in the future, and therefore to refer to multiple possible combinations of a person with her set of attributes. For example, it may or may not turn out that referring to *Hillary Clinton*, as of 2017, is also referring to the first female US President. The expression "Hillary Clinton" will continue to refer across a wide spectrum of possible futures, so, in a temporal dimension, it unifies many different possibilities into a single referential term. Similarly, different fine-grained point-sets e may be non-identical, at high-resolution scales, just as different time-slices of an object may reveal different properties; "scale of resolution" therefore serves like a *dimension*, by analogy to time. An object with properties at one more coarse-grained scale may give rise to different observations at more high-resolution scales, just as an object at time t has different possible futures at subsequent times. The various finer-grained observations which are possible, relative to a given coarser-grained state, are unified by a modal possibility operator, just as, in temporal modal logic, different futures are modal variations related to a present time-point.

We can take the entities spanned by a "granularity" modal operator as possible point-sets, geospatial regions, observations, or observation-sets. Whatever our preferred paradigm, the multi-granular modal frame should resemble a Topological modal logic in which propositions (or observations or observation-sets) are associated with points in topological spaces (in lieu of, say, possible worlds). In conventional modal topologies, points in topological spaces are analogous to "worlds", and propositions correspond to sets of worlds at which the proposition holds. Depending on technical details of a particular theory (such as how inter-world "accessibility" is defined, and whether propositional variables are allowed to range over the worlds themselves), topological notions of closure, interior, complement, and boundary have various modal interpretations. For example, propositions whose corresponsing sets S are *closed* may model possible truths, whereas open S model necessary truths. Intuitively, points on the S boundaries are points where a proposition p is neither necessarily true nor necessarily untrue, with other "worlds", *accessible from* those points, showing both p and not-p. A similar picture emerges if we consider some geospatial points as *necessarily* within, say, Manhattan (like Columbus Circle), since there is no observation which will exclude these points; and others (like the top of a sometimes-submerged rock in the Hudson river) as *possibly* in Manhattan, considered part of Manhattan in some observations but not others.

In modal-topological models, the property of points being topologically connected in a set, generally corresponds to "worlds" being mutually accessible. In geospatial examples like Manhattan's extent, the proposed notion of *referential compatibility* suggests a concrete interpretation of this concept. But, in general, technical modal/topological systems may be hard to match with concrete intuitions, at least if we interpret modal logic in terms of "possible worlds". It is hard, for example, to correlate our idea of worlds being *similar* to one another, with topological proximity between points representing trans-world resemblance. In the case of $\mathfrak{E}$, nearby points are sets in $\mathcal{E}$ with minimal difference (and therefore significant overlap); so there is a visual analogy for the abstract model theory of topological spaces within modal logic. Another intuitive example is that modal logic provides a pictorial metaphor for the connection between Mereotopology and Boolean Algebra: the overlap between spatial regions corresponds to overlap between sets of "worlds", in modal-topological spaces, where Boolean **and** between propositions holds. In general, certain models of

Mereotopological logics — with operators like overlap and set-theoretic union — are also models of Boolean algebras with operators like $\wedge$ and $\vee$.

Predicative networks can also give rise to modal models with intuitive, visual summarizations, for example given any property which inheres relative to (or as a result of) some quantifiable parameter; like the property *hot* and temperature, or the property *red* and an RGB color cube. Imagine now that we taste a partly melted bowl of sherbert. There will be some border between the liquid part and the still solid part, and assume that the former part grows, and latter shrinks, as the bowl warms up. So different temperatures correspond to different conditions of evaluation at which the bowl's contents are to be interpreted as referent for "this sherbert". We can consider these possible referents to be sets v, namely different volumes, contained in the entire volume of the bowl, representing different sets where the sherbert is still solid. Suppose I want to judge whether the sherbert is *sweet*, and assume that we imagine the sugar level in the sherbert varying by the gram. So evaluation conditions of an assertion ($\star$) *this sherbert is sweet* will vary according to the grams of sugar, and according to the volume of still-solid sherbert. If there is no sugar, then I will judge ($\star$) to be false; if all the sherbert has melted, then ($\star$) is at best confusing, because all I really see is a bowl of liquid. We can plot the volume dimension and the sugar quantity as parameters in a compound space, and in some part of this space, ($\star$) will be judged coherent and true. Different boundaries of this truth-region will apply according to different evaluation conditions and to my own personal inclinations, such as how much sugar makes something taste sweet to me. So there is a set of possible truth-regions in this compound space, and the variation between these regions apparently exhibits a topological continuity. This is similar to the Manhattan case in that both modal and topological notions apply, and in such a manner that dimensions of variation have both a succinct quantitative representation as measures or locations, as well as an intuitive, concretely grounded empirical interpretation.

In particular, the two cases both reveal measurable or quantifiable parameters, such as spatial and geospatial extents and measurable quantities (like grams of sugar). These quantifiable parameters play different roles: the grams-of-sugar parameter, for example, may be called a *predicative* dimension, because points on (or portions of) this dimension influence whether the predicative relation that a substance *is sweet* will hold. On the other hand, parameters like the sets v or e — establishing the geospatial extent of *Manhattan* or the spatial extent of the (non-melted) sherbert — may be called *referential* parameters of variation, because they introduce a space of variation whereby different possible referents or fine-grained elaborations of referents are modelled. So in an assertion like ($\star$), the modal variation ranges over a *relational* or *predicative* dimension (sweetness) and also a *referential* or *foundational* dimension (the volume which is referent of "this sherbert"). There is a similar combination of different axes of variation with respect to ($\mathbb{S}$**3**): we can assume that both Syracuse and Manhattan rest in a space of possible fine-grained variation, so there are multiple fine-grained regions which can form pairs instantiating *Syracuse* and *Manhattan*, respectively. The difference in this case is that, ranging across all of these possible variations, there is never any doubt that Syracuse is North of Manhattan.

We can consider ($\star$) and ($\mathbb{S}\mathbf{3}$) to be (modelled in terms of) graph structures: say $\Gamma_\star$ and $\Gamma_{\mathbb{S}\mathbf{3}}$. So $\Gamma_\star$ will have concept-nodes for *this sherbert* and for the property of *being sweet*, with a relation-node such as **has-quality** (the sherbert has the quality of sweetness). Similarly, $\Gamma_{\mathbb{S}\mathbf{3}}$ has nodes for Syracuse, Manhattan, and **north-of**. The "dimensions of possible variation" within these graphs are further structures attached to the concept-nodes, representing a spectrum of situations where the relation asserted by a double-edge holds — or potentially fails to hold. We can understand this spectrum of possible evaluations, then, as an *embedding* of $\Gamma_\star$ and $\Gamma_{\mathbb{S}\mathbf{3}}$ in respective modal-topological spaces. The *topology* is induced by parameters of variation on the Γ nodes (spatial and geo-spatial extents, as well as varying sugar quantities, in these particular cases), and the *modality* is induced because there are topologically or geometrically identifiable areas within these spaces with a modal interpretation, as a boundary between where the double-edges hold and fail to hold (though note that the double-edge in $\Gamma_{\mathbb{S}\mathbf{3}}$ holds in all evaluations, as defined here).

$\Gamma_\star$ and $\Gamma_{\mathbb{S}\mathbf{3}}$ are very simple graphs, with only a single double-edge, but of course we can model more complex predicative networks, such as the several facts I have mentioned about South Sudan. Each node in a Γ could potentially be associated with some dimension or parameters of variation, which can potentially then influence whether relations asserted by its adjacent double-edges (are judged to) hold. Some of this variation reflects interpretations of predicates or relations or the conditions under which they hold; how much sugar renders something sweet, what temperature renders something hot or cold, etc. Other variation reflects the dependence of observations and evaluations on scales and parameters of time, place, observation conditions, granularity, etc.; for example, evidently South Sudan plans eventually to move their capital to an interior city, in place of Juba, which is near the border. A graph $\Gamma_{\mathbb{S}\mathbf{4}}$ for ($\mathbb{S}\mathbf{4}$) will then express a true proposition before that time, and a false one after. So the Juba-node is embedded in a temporal dimension such that *Juba is the capital of South Sudan* depends on whether *Juba* here refers to the temporal part of Juba before or after the change. Similarly, asserting that "Roosevelt Island is part of Manhattan" will be true if "Manhattan" refers to the borough, but false if it refers to the island; so variation in this context represents mereological variation in how the refeferent for *Manhattan* is assessed.

Modal variations due to temporal, granular, or mereological parameters affect the evaluated truth or falsehood of propositions, but they also represent variations in how the nodes in Γ graphs — those which designate specified, posited, concrete concept-tokens — are referentially accounted for. They represent how the predicative structures modelled by these graphs — or by relational networks in the beliefs, mental states, language artifacts, etc., which these graphs simulate — are *anchored* in concrete reality. If graphs express mental states or epistemic situations, then their *relational structures*, assuming that the nodes are fixed, represent shared beliefs. We share these epistemic dispositions despite having differing fine-grained assessments of nodal referents, based on our differing perceptual perspectives, history, and acquatinance with them. So graph relational structures express inter-subjective propositional attitudes, which transcend variations in our comportment to particular concept-tokens.

This argument does not entail that referring expressions (in general) have multiple targets. We should endorse Jubien's account of referential terms as semantic carriers of *singulary* properties, with (at most) one instantiator. However, we can develop an ontological construction that preserves their singularity across, and does not just foreclose or ignore, variations in perspective, conditions of observation, and descriptive scales of resolution. We can theorize referential targets as coarse-grained entities, situated in a space of fine-grained elaborations. The referent is ontologically the modal space itself where they vary, with an associated operator which unifies fine-grained possibilities into a single coarse-grained individual. This construction preserves the singularity of the referent, but it also preserves the possibility that different observation, and different people's conscious and cognitive comporment to the referents, will be nonidentical one to another. So Conceptual Graphs suggest a pictorial metaphor for social-phenomenological intersubjectivity, where individuals' perceptual histories are cognitively transcended into a joint representation of inter-personal factuality; this facticity here modelled by conceptual graphs' relational structure. The final section of this paper will explore this analogy further.

5 Conceptual Graphs and Phenomenology

Abdul, Boris, Chang, and Dikembe (I'll call them by their names' first letters) are planning a trip from Manhattan to Syracuse. A is looking at a New York Subway Map, where Manhattan is outlined in some detail; B looks at a New York State Map, where Manhattan is just a dot; C is sitting on the piers by the Brooklyn Brewery, looking over at Manhattan; and D sits in Columbus Circle looking around at the tall buidings. So their mental images of Manhattan, at the moment, are all quite different, but in thinking about a drive north to Syracuse, they are all believing (among other things) the proposition ($\mathbb{S}\mathbf{3}$). Of course, their impressions of Manhattan extend beyond what they are seeing right now. But we can imagine some time in the future — perhaps on the trip North, pausing for lunch in Ithaca — they consider ($\mathbb{S}\mathbf{3}$) and the thought of Manhattan brings to mind, for each of them respectively, the various presentations just described.

So these images tokenize *Manhattan* for them as their thoughts turn to ($\mathbb{S}$**3**).

At the same time, from each of these perspectives *Manhattan* is intended as beyond or outside of the particular image or icon which represents it, for each person: when seen from afar it is *over there*, but extends beyond the line of sight; when from *inside* Manhattan it is all around, but can still be conceptually referred as singular, as if we were pointing to it ("we were in Brooklyn earlier, but we are in Manhattan now"); the dot or outline on a map stands in for Manhattan, either as an arbitrary symbol or as a schematic representation. In each of these cases, in different ways, symbolic or perspectival representations refer beyond themselves to a Manhattan which "transcends" them.

Across the river, Chang is well aware that the island stretches beyond his visual line; he looks North and South, up and down, synthesizing a mental image encompassing a gradually wider scope. Chang knows that there is a *concept* Manhattan, and that each of his visual presentations represent one perspective on the larger entity which (uniquely) tokenizes this concept. He also knows that relational predicates, like ($\mathbb{S}$**3**), apply to the full Manhattan concept-bearer, not just whatever part he sees at any moment. His various Manhattan-presentations are overlayed, with the concept *being Manhattan* providing a *principle of synthesis*, its bearer belonging to a constellation of relational structures, from which his knowledge about Manhattan is sampled. The full entity *Manhattan* both transcends his particular perspective, but also acquires this individuality — this Ontological surpassing of any given perspectival impression — not only because it is a whole above and beyond parts seen, but also because, as an individual concept, it takes on these relational attributes viz-a-viz other concepts and their tokens. These relational structures ontologically found how Manhattan is in the world's facticity; in the way things are. Manhattan is both a multiplicity, a synthesis of many perspectival scenes and component parts (buildings, streets, etc.); and also a singularity, a single point in the worlds' empirical mesh, a single concept and concept-token. Conceptual relations "collapse" or "tokenize" the mulitiplicity of a concept-bearer into a cognitive singularity. Insofar as our thoughts turn to compound judgments, attention passes beyond the continuity and variation of the perceived as it sustains itself in perception, and relative to foreground attention this coninuous variation recedes to a mental point.

When perceiving any worldly entity I need to synthesize multiple perspectives into a conceptual image which unifies and transcends them, but I also need to *package* this unity into a single concept and concept-token, such that it may be situated into predicative networks (corresponding here to my cognitive states and the mesh of my propositional judgments). A stray red frisbee catches my attention flying over a grass lawn. I reach out to catch and return it. I become visually aware of the frisbee given the color contrast between its red and the green grass; but this perceptual detail is also an initial, preliminary predication, disposing me to believe a proposition like *this frisbee is red.* So the red plays a double role: first as one component of a perceptual process by which the frisbee is disclosed to me as a self-contained individual object, and second as a concept within a propositional network, analogous to a concept-node within a double-edge whose nodes include one for *this frisbee* and one for, say, **has-color**.

As I see the frisbee flying toward me, I very quickly estimate its three-dimensionsal geometry, based on my limited visual perspective but also presumably on my identifying it *as a frisbee.* So for a split second, let's assume, I have only the impression of a moving red object; given the context (in a park, say), and its general shape and appearance and how it moves, I quickly surmise that it is a frisbee, so I entertain the proposition modelled by a graph (say Γ_1) asserting, in effect, *this object is a frisbee.* Since my initial impression is of its redness, that act of judgment is simultaneously my acquiescing to a belief that (say Γ_2) *this frisbee is red.* Given that Γ_1 implies certain details with regards to the frisbee's shape and likely trajectory, I am able to predict how it will move and orient myself, my arm and hand, etc., to catch it. So perceptual details are marshalled into conceptual relational compound judgments, which then inform further perceptions, and so forth, with my cognitive attention oscillating between these perceptual and these predicative positings. Once I have the frisbee in hand, for example, I may look around to identify its owner.

As I perceive objects, I seek to consolidate them, in my cognitive schematics, within singular concepts and concept-tokens; the frisbee, for example, represents a specific concept *this frisbee* and also a token of the concept-type *frisbee.* Initially, at the brief time-point suggested above where I am about to first surmise its frisbee-ness, the concept within Γ_1 is *this object*; but my judgment confirms that whatever instantiates *being this object* is a frisbee, or also instantiates *being this frisbee*, so in subsequent judgments (like Γ_2) the referential concept becomes *this frisbee.* As I develop a mental model of my surroundings, I extrapolate visual perceptions "out" to objects' three-dimensional form. Apparent (mereo)topological relations, like contact and connectedness, are carried over, from the space of visual perception, into perceived ambient space. I identify the separation of surfaces of distinct objects, as well as where they touch or rest on each other. These topological and geometric intuitions, then, guide one aspect of our "mental model" of our immediate environment. But there is a separate aspect in which we situate objects, once their three-dimensional contours and separation are "packaged", within predicative networks. These networks can be modelled via Conceptual Graphs, by contrast (and also analogy) to how topology and manifold-geometry formally emulate structures of our visual and kinaesthetic engagement with objects, at a more immediate, perceptual level. Objects are present in predicative networks as nodes, their visual and geometric details encapsulated into their singular identity as concept-tokens.

These predicative or graph-structured networks also model our environing situations, but in different ways than immediate visual and kinaesthetic awareness; they represent the world more schematically, in terms of processes, plans, practical activities, social and inter-personal situations. For example, I recognize that a frisbee flying through the air was likely thrown by someone nearby, and it is polite for me to return it. This situational reasoning is encapsulated in a plan, which could be expressed in a Conceptual Graph like a Γ_3: catch the frisbee, look around for its owners, throw it back to them. My intentions (in the everyday, non-phenomenological sense) are just as vivid and self-aware as immediate conscious contents, like the color of the frisbee and its brisk contact with my hand; but this intention is an overview of a scenario, one which collapses spaces of fine-grained, perceptual variation into singular concepts. As subject of perceptual synthesis, the frisbee undergoes continuous apparent variation while it travels a trajectory toward me; its observed properties also may vary continuously, like its color, affected by ambient light or the sun's reflection or shadows. As these perceptual experience are absorbed into prdicative situational models, however, these spaces of variation condense onto atomic or quasi-atomic units of thought. The combinatory, constructive nature of propositional judgment — including its grammatical manifestation in language — depends on the possibility of treating component semantic and conceptual unit as quasi-atomic tokens, which can enter into various relational and syntactic permutations, their inner structure and perceptual imprecision bracketed and superceded.

When simulating the transition of visual experience to perceived, three-dimensional form — including in algorithmic settings such as 3d reconstruction — formal models emphasize the topological and geometric properties of objects, as these manifest in visual appearances and may be reconstructed accordingly. Part/whole relations also play a role in this 3d reformation, however, both in the internal structuration of single objects and in how they aggregate into complexes. A frisbee, for example, has an evident if not crisply demarcated division into a central disk (which may be decorated with stickers, logos, etc.), and the curved lip allowing it to be caught, thrown, and gracefully fly through the air. Imagine I recognize stickers in the center suggestive of a Brazilian flag, and then see someone walking toward me wearing a striped Fluminense jersey. I infer the jersey-wearer to be the frisbee's owner. This judgment attends specifically to the *center* of the frisbee, as a part whose separation has some contribution from geometric form, but also from the frisbee's functional organization.

Consider now a simple discursive situation, with a table, a shelf, a blue teacup, and a white teacup with a red handle. Presented with an expression like "place the teacup with the red handle on the shelf", I have to recognize mereological structures within the objects as bases for referential tokens, as well as the topological separation of these objects from one another. The semi-individuality of the red handle is partly a topological matter, but also a mereological detail, a product of the object's organization. Geometric considerations guide my perception of the objects' three-dimensional form, for example projecting their visible shape and coloration around to the invisible sides, exploiting for example the apparent rotational symmetry of the teacups, apart from their handles. But this three-dimensional perceiving has a topological dimension as well, identifying for example the teacup handles encircling a hole in the shape, but also recognizing their visible surface as statically enclosing a solid region. Surfaces can relate to 3d interiors in different ways — they may simply bound them as manifolds, but they can also open up to reveal an interior, like a backpack zipper, or peel away, like the skin of a grape, or be (semi) transparent and therefore visually disclosed some interior contents. These relations are predominantly topological, because they do not depend on geometric details.

Consider, then, the various roles which topology, geometry, mereology, and conceptual networks play in possible referrings and predications, using this simple situation as an example. One aspect concerns the reconstruction of 3d form, projecting visible coloration and spatial form or "spatiation" around to invisible sides. Structurally, the perceptual operations at this layer are largely topological and geometric. Another cognitive aspect identifies objects' functional organization and potential referential sub-objects, parts which may independently be referential targets: the teacups' handles, the level shelf-top. A third cognitive dimension represents general predicative relations: *this cup is blue*; *this one has a red handle*; *the white teacup is on the shelf*. Mereology is involved in each of these cognitive "layers", because mereological connectives are both predicative relations (*the handle is part of the teacup*), and also structures of objects' disclosure and apparent form. In general, mereology is mixed with geometry and topology in the more perceptual layer, and mixed with predicative judgment — perhaps modeled via `CGS` — at the more predicative layer. Bridging these two is a layer where objects and their functional organization are conceptually identified and cateogirized, where mereology is particularly important.

Within predicative networks, elements of thought are drawn in to combinatory structures, where they tend to be treated as mental atoms. Their internal structuration is not necessarily unrecognized, but it is not foregrounded. Semantic theories which model word-to-world associations in largely logical, symbolic terms, capture the combinatory form of these networks and judgments — but at the expense of overlooking fine-grained variation both within concepts and within substrata which bear them. These theories can be judged to overlook that red, for example, is not an atomic concept but blends continuously over into blue or pink, and also that red objects may be perceived only imperfectly from against their surroundings, with continuous perceptual variation leading to only imprecise perceptions of discrete objects. This *continuity of* (or within) concept- and discursive/referential spaces, is seen by some as a serious problem for older semantic models based on first-order logic. The fact that real-world concepts are not atomic units of meaning or cognition, but rather subject to continuous variation, has led to a counter-theory which rejects semantic paradigms that are too *symolic* or too dependent on logical formalisms. This counter-movement is perhaps manifestoed most passionately by George Lakoff and Mark Johnson, but it has evidently influenced a spectrum of scholars, many of whom are professionally associated with the US West Coast: Gilles Fauconnier, Mark Turner, Terry Regier, Michael Jubien, David Woodruff Smith, etc.[1]. A semantic theory based on continuous variation and degrees of similitude was also developed in depth with Peter Gardenförs' *Conceptual Spaces*, and was given subsequent elaboration, with particular emphasis on topology, by Gregor Strle.

Joseph Goguen, in his essay *What is a Concept*, contrasts and unifies many of these scholars' perspectives. He suggests that the "geometric" Concept Spaces of Gardenförs, the "Conceptual Blending" of Fauconnier and Turner, Fauconnier's "Mental Spaces", and Lakoff and Johnsons's emphasis on prototypes and metaphor in concept-formation, are all partial perspectives on conceptuality which need to be synthesized; he explores numerous other formal and philosophical perspectives as well. Here I will conclude by focussing on two dimensions of conceptuality which I believe demand synthesis: the "combinatorial" structure of predicative networks and judgment, and the perceptual continuous veriation of objects as we are most foundationally aware of them.

6 Predication and Continuity

Both the combinatorial paradigm, and the approach which instead highlights possibilities of continuous variation within concepts, are phenomenologically meaningful: they represent different layers of thetic awareness and different modalities of perceptual and cognitive attention. I believe that they can be unified with a combination of modal/topological or mereotopological formations at the perceptual level, and CGS at the predicative level. Intermediary between these we can theorize a level of functional and conceptual identification, where mereological operators are especially prevalent.

The sigficance of both mereology and topology, at several different points in this theoretical sketch of a cognitive/perceptual hierarchy, is one reason to consider how these two domains may be brought together within Mereotopology. However, there is a further, more technical significance. Note that coloration

[1]The presence of Jubien and Woodruff Smith on this list may seem surprising, because neither appears to have either associated themselves or their work with the others mentioned, or to be seen as sharing many intuitions or motivations with them. I think there is both textual and circumstantial evidence, however, that they should be considered as part of this milieu. I would note the following: Woodruff Smith, like the Mereotopologists, appears to have been influenced by Whitehead's Process Philosophy, and writes about connections between Whitehead and Husserl; Jubien's property-theoretic approach to modality has, I believe, many elements in common with Fauconnier "Mental Spaces", particular with regards to fictional reference; and both Woodruff Smith and Jubien spent much of their careers in California.

and spatial form or "spatiation" are ontologically codependent: we do not see color except as manifest in spatially extend objects, and we do not see spatial geometry except as conveyed by color modulation. This codependence should be considered alongside of the ontological relation between *points* and *regions* which inspired Mereotopology in the first place. The notion of *point*, as abstractly posited by geometry, by point-set topology, etc., appears cognitively dependent on the perceptually more primitive notion of (extended spatial) *region*. This logical priority may be reversed in abstract mathematics, but in terms of perception, we do not really "see" points, just ever-smaller regions, or intersections between lines.

But we do not ever see *regions*, either, except as their shape is disclosed in coloration or, with respect to tactile and kinaesthetic awareness, with respect to their solidity and feel. Perceptually most primitive, then, are *product spaces*, where spatiation and (especially) coloration are blended. Just as *points* are derivative on *regions*, so *colored points* are derivative on *colored regions*: neither the point without the extension, nor the color without the medium, are perceptually (and without abstracting cognition) given.

Although mereotopology studies "regions" in general, in concrete perceptual/cognitive situations a notion of topological *parthood* derives from perspective and detail: there is the part of an object seen and the part unseen, parts identified by functional organization, and parts marked by discontinuities in shape, feel, or coloration. Functional parts may be topologically identifiable (like a teacup handle), but not always: a teacup's *bottom*, for example (which rests on the table), or the frisbee's center (bearing a sticker), are referential sub-objects but are topologically contiguous with their super-objects. When I synthesize a mental picture by overlaying visual impressions from different angles, I combine together overlapping parts, identified by their presence to a view angle. So instead of an abstract notion of "parthood", we have a more phenomenologically grounded mereology which distinguishes various kinds of part/whole relations, based on their cognitive/perceptual roles.

All of these considerations pose several questions for Mereotopology. First, how can these different kinds of mereological relations (functional, perspectival, featural — derived from featural structures, like borders between color patches) be incorporated into formal Mereotopological systems? Second, how can we respect the "regions are primitive" paradigm in a context where regions themselves are dependent on a featuration, like color, which discloses them — in other words, where region-based *product spaces* are the truly fundamental entities? Third, how do we extend Mereotopology to a kind of Mereogeometry which studies shaped-based details, like convex regions or object symmetries, as contributing to our sense of objects' functional organization and three-dimensional form? Fourth, how do we model object rotation and perspectival synthesis from a Mereotopological perspective — contact algebras on $SO(3)$, on general matrix spaces, Quaternions, etc.? Fifth, insofar as objects are meretopologically synthesized and then conceptually and functionally characterized, and their mental representations or "tokens" introduced into predicative networks, how can we build graph or CGS structurs on top of mereotopological spaces? And, sixth, considering that almost all visual perception is perception of object *surfaces* — with rare exceptions, like a semitransparent decorative paperweight — how can we explore familiar geometric and topological properties of $\mathbb{R}^2$ surfaces in $\mathbb{R}^3$: region-based generalization of path homology, for example, and parameters of classification, like the fundamental group, orientedness, and boundedness?

While existing research has focussed on geospatial and "planar" mereotopology, there has been some extension in the direction of three-dimensional or "polyhedral" Mereotopology, four-dimensional or "perdurant" Mereotopology, and modal-logical or sorted-algebraic Mereotopology. Given considerations I have sketched here, I believe a theoretically and practically valuable application of Mereotopology to and within Phenomenology can benefit from exploring Mereotopology within some further spaces: physically realizable surface manifolds $\mathcal{M}$; product spaces crossing $\mathcal{M}$ with feature-granting measures or vectors, like coloration; Mereotopology over rotation spaces or (unit) quaternions; and over discrete graphs or Conceptual Graph Semantics.

The objects formally modelled by these analyses, I believe, have a good chance of being useful formalizations of objects of perception itself, or real-world, concrete objects as they are perceptually apprehended. These formal structures can therefore model the variability and synthesis which occurs as these objects are tokenized in predicative judgment. That is, they can model the modal domains and operators which unify multiple possible perspetival accounts into a single cognitive-referential structure, and by extention into a linguistic-referential latency. These spaces, in other words, provide a robust system foundation for a referential semantics of real-world, perceived, spatially extended objects; those everyday things which most of our language talks about.

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