

# NASM

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## Abstract

NASM

Now, the “science of salience” proposed by Petitot and Smith (1997) illustrates the kind of formalized analysis made possible through the direct mathematization of phenomenological descriptions. Its aim is to account for the invariant descriptive structures of lived experience (what Husserl called “essences”) through formalization, providing a descriptive geometry of macroscopic phenomena, a “morphological eidetics” of the disclosure of objects in conscious experience (in Husserl’s words, the “constitution” of objects). Petitot employs differential geometry and morphodynamics to model phenomenal experience, and Smith uses formal structures from mereotopology (the theory of parts, wholes, and their boundaries) to a similar effect.

— Maxwell James Ramstead

Several authors have recently proposed “non-well-founded” or “non-anti-symmetric” mereologies. This terminology is relatively new, but at least some of the motivations behind their alternative systems derive from dilemmas that have inspired extensive commentary in the past. For example, is the clay out of which a sculpture is made a proper part of the statue? Most commentators seem to frame their intuitions in the parameters of part/whole hierarchies, and find some mechanism to reconcile the parts of their intuitions that don’t fit simple accounts of parthood. The more radical possibility, which is the starting point of this paper, is to re-engineer our conceptual of parthood from the ground up. In particular, we can drop the assumption that parthood is asymmetric or acyclic. Doing so results in a

Non-Asymmetric Mereology, or NAM. In NAM,  $x$  can be part of but different from  $y$  and *vice versa*.

As far as I can tell, most arguments for NAM derive from statue/clay-like cases where two things seem deeply, metaphysically intertwined by not identical. It is said that the statue (call it  $S$ ) is not identical to the lump of clay (call it  $C$ ) because  $C$  could be altered in ways  $S$  cannot — if a fire melts  $C$  down to a blob,  $C$  is still itself but  $S$  disappears. Ergo, an event can destroy  $S$  but not destroy  $C$ . Conversely, a small piece could fall out of  $C$ , leaving a hole then repaired with fresh clay, yielding a new lump  $C'$ . So  $S$  could end up being  $C'$  instead of  $C$ . But if we define  $C$  as just that exact lump of clay,  $C$  can’t be  $C'$  instead of  $C$ .<sup>1</sup> Ergo, as Aaron Cotnoir says, “it is natural to think that a lump of clay and a statue made from it have all the same proper parts [but not] everything true of the clay [is, arguably] also true of the statue” (p. 397). In sum, Cotnoir and others who have thought about NAM highlight cases where two non-identical things have all the same proper parts. Such examples then lead toward the possibility of parthood being cyclical in some sense (which I’ll try to pin down later).

I think statue/clay-like cases, while they raise important issues, are less than ideal as primes for NAM because they seem to invite numerous non-mereological

<sup>1</sup>Another issue: Michael Jubien imagines that the sculptor could just have built  $S$  from another lump of clay in his studio, call it  $D$ . So, modally,  $S$  could have been  $D$ , but  $C$  could not have been  $D$ . If we use possible world talk, we can posit a possible world where  $S$  is  $D$ , but we can’t imagine a possible world where  $C$  is  $D$ . This assumes that a hunk of matter just is the matter that it is; we’re not talking about essential or inessential parts or any object that may be *constituted* by some matter, like Tibbles the cat and his later-amputated tail. In other words, we’re assuming that the self-identity of matter, once we ignore any logical properties inhering in its form, is immune from counterfactuals. Matter can be arranged differently, but it can’t be different matter. This seems plausible but not self-evident, but in any case it may be a terminological rule in how philosophers often use the word “matter” — and by extension phrases like “lump of clay” — especially in discussions like statue/clay (non-)identity.

resolutions. For instance, we can say (drawing from Jubien again) that *C instantiates the property of being S*; *C* is *S*'s physical instantiation or realization. We have cognitive attitudes toward *C* that involve its physical form and nature; we also have cognitive attitudes toward *S* that thematize its aesthetic and social facets (qua artwork crafted for public appreciation). Our agreement that *C* designates *S*'s physical substrate — as described by terminology like *C is a material body instantiating the property of being S* — binds these two cognitive assemblies together, but not in a manner that readily propagates *C*-parthood to *S*-parthood or vice-versa. In other words, mereology itself is not a useful philosophical abstraction when too many divergent cognitive registers are involved. Or at least this is an escape hatch which brings us back to conventional mereology via the metaphilosophical claim that there are *other* analyses where the classical mereological models *do* fit our cognitive engagements. Statue/clay-like problems are not problems of mereology as a theory but problems of where the theory should be applied.

This won't be my final word on statues and clay, but I will pivot to other examples suggesting how NAM is more pervasive than people realize. The literature tends to treat NAM as an exotic, corner-case, enigmatic exception to partonomic common sense, as if it requires extra philosophical concentration to conceive of cases where parthood is really symmetric: *x* is part of *y* which is part of *x*. My goal is to argue the opposite: I think *asymmetric* mereology is really the special case. Of course, I understand that there's *some* notion of parthood which renders partonomic chains paradoxical. But I think in most common-sense cases where people think they are talking/reasoning about parthood, the kind of mereology they intuitively use has at last the potential to be NAM.

As my first exhibit, I'll mention several plausible and everyday-like sentences:

- ▼ (1) Mbappe was a big part of the 2018 World Cup.
- ▼ (2) Mbappe's footballing career is a big part of who Mbappe is.
- ▼ (3) The 2018 World Cup was a big part of Mbappe's footballing career.

Taken at face value, acquiescing to these sentences and to mereological transitivity, the 2018 World Cup is part of Mbappe, and Mbappe is part of the 2018 World Cup;

but of course Mbappe is not metaphysically identical to the 2018 World Cup. Granted, sometimes part-talk in natural language is metaphorical: we don't hear *my religion is part of me* as megalomaniacal, or *you are part of my life* as prima facie possessive. But if this metaphorical way of talking is *more* common than our commonsensical mereology, maybe we need to reconsider whether Classical mereology is really the base rather than derived notion.

Indeed, when we talk of *y* being part of *x* we rarely seem to talk as if *y* is *completely* part of *x*. Perhaps *total* parthood is a degenerate case. In fact, I could make an argument that classical mereology is paradoxical, like this: an intrinsic part of *y* is *being y*. If *y* is a proper part of *x*, by transitivity, *being y* is part of *x*. But if *x* is not *y*, it sounds absurd to say that *being y* is part of *x*. Now, I'm not actually raising this as an argument, because it can be countered: *being y* is not really "part" of *y* in the sense technically covered by mereology. Some of the more "philosophical" interpretations of parthood need to be quarantined from the mereological explanantia. Fair enough. But I think this exercise shows that preserving classical mereology requires filtering out a lot of notions of parthood based on a prior commitment to asymmetry, which becomes circular reasoning unless we have a good analysis that the apocryphal "notions of parthood" are in some definable sense atypical or deflationary.

Transitivity is problematic in many ways; Mbappe's left foot has the property *being a foot*, a property which is (arguably) part of the foot is not part of Mbappe. And Mbappe's foot is (arguably) not part of the team. Still, we'd like to preserve parthood-transitivity as much as possible. However, it seems that no matter how narrowly a part differs from the whole, it picks up some kernel of propositional difference that blocks transitivity without some elaborate patchwork: if we define parthood without a lot of conceptual detail *no y* not identical to *x*, however modestly, can ever be a proper part of *x* full stop.

That *y* is part of *x* does not seem to preclude *y* bearing predicative details that *x* lacks: for instance, if the camera zooms in on Mbappe it is not picturing the whole team, so although Mbappe is part of the team, Mbappe, but not the team, is covered by the predicate "being at the focus of the camera angle" at some moment. Of course, we already know that Mbappe is not *entirely* inside the team, so the prospect of Mbappe having

propositional attributes which the team lacks is not con-  
 certing. But a camera angle can show Mbappe’s *head*,  
 which we might think to say is “part of” Mbappe in a  
 stronger sense, or his *right side*, which seems even less  
 independent. Indeed any picture of Mbappe shows part  
 of him, making that part uniquely predicated as *what*  
*shown in this picture*. I cite these as further examples  
 of conceptual subtleties that can trip up strictly “sub-  
 suming” mereologies — where an axiomatic distinction  
 is made between *proper parthood* and *overlap*.

The rough progression to follow will migrate themat-  
 ically from cognition to science to computer science. I  
 will, in effect, first examine themes of parts and wholes  
 in cognitive — perceptual, conceptual, and enactive  
 — frames and schema, and in the overall phenomenol-  
 ogy of engaged world-experience. Phenomenologically,  
 parthood as such is preconditioned on an experience of  
 perceptual (and/or conceptual and enactive) *disconti-*  
*nuity*, which establishes the possibility of a part being  
 cognized independently of its whole. I will argue that  
 our inclination to recognize parts is influenced by con-  
 ceptual commitments: whether a putative part can be  
 recognized under a concept can determine the difference  
 between discontinuities passively perceived and the ac-  
 tive, explicit isolation of parts as distinct available foci  
 of attention.

This role of concepts then invites us to consider how  
 the part/whole organizations we experience correspond  
 to functional patterns in our environing spaces: what  
 are the physical and dynamic impulses producing mere-  
 ological patterns in physical reality? To what extent  
 are subjective experiences of parthood shaped by analo-  
 gous relations in the objects we perceive? I will consider  
 the dual options of treating mereology as a model of  
 parthood in our *concepts* — or cognition or language;  
 in general in the schema we mentally employ to find  
 order in our surroundings — or conversely as a formu-  
 lation representing parthood as a relation in nature and  
 in physical reality. The latter interpretation leads to  
 separate questions: what aspects of materiality are op-  
 erative in mereological instantiations, when we deem these  
 extramentally? Is a part compositionally enclosed in its  
 whole; or behaviorally dependent (but with some partial  
 autonomy) on a whole; or, more dynamically, is a part  
 some identifiably subspace of the state-space and dynamic  
 system of the whole? Qua scientific or “paleoscientific”  
 principle, how should parthood be understood?

Finally, having asserted a duality between cognitive  
 (phenomenological) and scientific (extramental) mereol-  
 ogy, I will discuss technological and computer-science  
 expressions of mereology which combine both mental  
 and extramental dimensions. Computationally curatd  
 and shared information systems are at once cognitive  
 and scientific phenomena because they represent human  
 judgments and engineering for human usage, but also  
 record humans’ attempts to systematically and, to the  
 degree warranted, non-subjectively record scientific facts  
 and data. How mereology functions and is modeled  
 in information spaces therefore unifies both “cognitive”  
 and “scientific” mereology, presenting new perspectives  
 on each.

## 1 Mereotopology and Phenomeno- logical Discontinuity

When we conceptualize something — or some class of  
 “things” — we inevitably interject cognitive and percep-  
 tual schema, mental summaries of various kind which  
 create cognitive routes away from the explicit presenta-  
 tion of things themselves in momentary consciousness.  
 Some sensory and perceptual details have (at least for  
 situations at hand) comparatively minor individual im-  
 portance even if they cumulatively synthesize into expe-  
 riential wholes which are important. These are details  
 we tend to forget, or not talk about, or notice only pre-  
 consciously. The detailing committed to memory, focal  
 attention, and language, tends to exhibit some recur-  
 rent schema and often to operate on a higher scale of  
 integration. While analyses of cognitive schema need  
 to integrate multiple perspectives, two facets of schema  
 deserve special attention: the mereological (concerned  
 with part/whole relations), and topological (concerned  
 with spatial and functional relations of contact, con-  
 nectedness, boundaries, and non-geometric properties  
 of objects’ shape, such as the presence or absence of  
 holes, the number of dimensions — contrast a point, line,  
 surface, solid — and relations like a surface covering a  
 solid, as in a table cloth, or a solid inside an open area  
 bounded by a surface, like an object in a bag).

Suppose I look at a Canadian flag. Insofar as I identify  
 the Maple Leaf pattern and the outer red stripes, I per-  
 ceptually divide it into distinct, visual pieces: I identify

discontinuities in the flag as an (otherwise) integrated union of material and color. On the other hand, if I observe its blowing to and fro in the wind, or physically manipulate it (folding it ritualistically, for example), I cease to attend specifically to the color patterns. The flag therefore lends itself to (at least) two different viewer comportments, either as a continuous material object or as a setting for visual patterns and icons, and (depending on which viewpoint we take) thereby exhibits two different “topologies”. This duality is indeed intrinsic to its semiotic functioning: a flag typically relies on visual forms for the construction of symbolic patterns, representing characteristic or celebrated aspects of a community’s culture and history. At the same time, a flag in its semiotic function must be a physical entity that can be perched over certain places and buildings, in plain air, with specific material properties. A reproduction of the visual appearance of a flag, for example in an encyclopedia or as a metonymic icon for a particular language, is not itself a flag (contrast this to the reasons why a picture of a stop sign in a driver’s instruction manual is not a stop sign, or conversely why a graphical reproduction of a copyrights or trademark symbol may indeed be a binding example of such a symbol). As *sign*, in short, a flag exists simultaneously as material cloth and visual tableau, and has different morphological properties depending on which (or both) of these guises dominate how it is disclosed to us on each occasion.

Our collectively surrounding environs, insofar as this forms the horizon and selection-space for our significations, embody multiple potential topologies, multiple parameters of continuity and discontinuity. Instead of a binary notion of (dis)continuity, we therefore need an image of *partial* discontinuity, in which separation in one parameter is joined with connectedness in another. The Canadian flag, for example, has regions separated by color but connected by material substrate. Such a combination is intrinsic to certain concepts: for example, a *footprint* is not a foreign object in some medium (like mud or sand), but involves a physical continuity crossed with a distinct shape and impression, typically outlined by a separation in one direction (down into the ground). Partial discontinuity has an intrinsic mereological aspect, insofar as its mixture of continuity and separateness implies both a distinct unit and a larger whole of which it is one part. Therefore mereotopology can be adopted as a study of topological properties in these cognitive and

semiotic contexts, where multiple topologies overlap.

Mereology in this kind of analysis can be more general than splitting a whole into crisp parts. Our experience of part/whole “articulation” can depend on the regularity or symmetries in some pattern, irregardless of whether we experience crisply delineated parts. For example, our experience of a chessboard depends on symmetries by which a color space is mapped onto a spatial extent, as much as on the visual discontinuity between black and white. Imagine an artistic chessboard on which the color lines are replaced with smooth gradient transitions — the underlying color space becoming a “loop” instead of a simple two-valued collection. Despite being topologically more complex, this color-loop is nonetheless a distinct subset of all possible color values, and the distribution of color values projected onto the checkerboard surface can exhibit the same translational symmetries of the two-valued, black/white board, creating the impression of similar mereological articulation even in the absence of identifiable distinct parts. Furthermore, the chessboard’s mereology has functional as well as purely geometric properties. Each square represents a distinct site where, in a game, at most one piece may be placed. It is certainly physically possible to violate this norm, but such an arrangement would not model a real game situation, or physically instantiate the abstract specifications of a chess match. The pattern of lighter and darker squares also helps to visually specify the rows, columns, and diagonals, so that players can clearly see possible moves for the different pieces.

As such, a board-game “square” is more one site in a functionally organized system than just a spatial shape; it may function just as well even if it is not actually clearly delineated as a shape (as in the hypothetical “artistic” board). By analogy, geospatial regions like islands or political units function as delineated wholes even if their precise spatial borders are impractical to specify with arbitrary granularity (considering how changing tides and water levels minutely alter the outline of islands, or how border conflicts can obscure the actual geospatial extent of nations).

For a mathematical model of the chess board or Canadian flag, we can consider “product spaces”, in these examples a simple, compact two-dimensional space crossed with a discrete color dimension (red/white or black/white). In the statistical classification of dimen-



sions as nominal, ordinal, interval, or ratio, these color dimensions would be nominal, so there is no concept of relative distance between colors. This changes in the “artistic board” example, but symmetries of the color/board product space preserve the mereological morphology. This suggests that a simple product-space model, such as one generated by an arbitrarily fine-grained mapping (one color value for each dimensionless point), does not fully capture the topological properties of the integrated space as a *functional* totality. In Peter Gärdenfors’s Conceptual Space theory, color is a canonical “geometrized” concept space, in which different (possibly overlapping) color-concepts can be associated with well-defined quantifiable regions in a “color space” suitable to several mathematical dimensionalizations (the RGB, CMYK, or “color wheel” of computer graphics). The coloration of a 2D surface is then a simple spatial-point to color-point map. Seen from a more functional perspective, however, color space is less crisply quantified; for example, a chess board functions pretty well even if fading and smudging corrupt the idealized black/white pattern. We have a tendency to “see” the pattern even if actual perceived colors merely approximate it, which suggests that our disposition to find such patterns is driven by organizational cues (like symmetries) as much as by crisp forms. So color may have several different functional roles. It may be purely “contrastive” — consider how many tournament chess boards replace black with green, to equal effect; or decorative, in which case precise hues become important; or also symbolic, like the colors in most flags, which often carry patriotic and/or geographic metaphors (blue for sky or ocean, brown for soil, etc.).<sup>2</sup>

The previous examples considered mereology in terms of color, a dimension which has functional as well as visual aspects, but similar analyses can involve parameters with more purely functional interpretations. Consider the contrast between a dumbbell fashioned from a single metal part and those separated into a central shaft and detached outer rings. While the topology of these shapes is different, they have comparable functional form: a good dumbbell must be easy to grasp but relatively hard to

lift, so it needs parameters of both bulk and graspability. In both types, a central shaft provides graspability while outer bulges provide heft; therefore these two parameters are distributed around the shape of the dumbbell, by analogy to the red/white of the Canadian flag. Relative to the “functional space” defined by these parameters — a two-valued graspable/bulk contrast — the central area is isolated by virtue of its graspability, analogous to the central Maple Leaf pattern isolated by color, even if in some dumbbells there is no comparable discontinuity in the physical extent considered alone. Functional parameters can introduce partonomies into an otherwise connected substratum, or, conversely, topologically unify disparate pieces.

Geospatial semantics provide good examples of both of these phenomena. For most of its extent, the border between the New York City boroughs of Brooklyn and Queens injects discontinuity into a terrestrial continuum. Conversely, a jogger whose itinerary crosses East Harlem and Randall’s Island is making a closed loop in the borough of Manhattan but not the island of Manhattan; also, the borough integrates several other islands and also the small Marble Hill neighborhood on the mainland, attached to the Bronx. The tendency of geospatial designations to either partition or bridge together geospatial extent, sometimes with no apparent relation to geographical features, is a canonical example of Barry Smith’s distinction of *fiat* and *bona fide* boundaries as constituting objects of reference (and therefore of domain-ontological record). The mere presence of a naming scheme induces “product spaces” where geospatial regions are crossed with names selected from an nominal axis, like M, B, Q, S, T for the New York boroughs (“T” for “The Bronx”).

Whether or not the outlines of named regions correspond with natural features, the mere presence of fiat boundaries alters the functional organization of geospatial extents seen as sites of human activity. A walk along Myrtle Avenue, for example, might cross between Brooklyn and Queens. This discontinuity may have no “natural” meaning, but could still be functionally significant; for example, if that path is actually a proposed route for a parade, or central strip of a business development zone, where regulations from both boroughs need to be accommodated. Fiat boundaries can also exercise causative influence on the natural land itself: the border between nations with different environmental laws might

<sup>2</sup>Flags of African nations often carry tones suggestive of the local landscapes (browns, greens, yellows), whereas flags of European nations feature colors which are more “abstract” (like red and white), or harder to match to geographic artifacts, except perhaps for blue, which in an admittedly rather speculative analysis might signify that the most concrete earthbound feature in those nations’ historical collective consciousness was the oceans — the routes to expansion, colonization, increasing national influence.

evolve into a line between lands with different ecological properties. Moreover, fiat boundaries tend to be drawn on functional considerations: fiat lines tend to be straight, presumably to facilitate surveying and resolving land-claims; alternatively, borders may group together speakers of a common language, or citizens who have voted to form a union. Geospatial regions, if they have political and legal significance, are not just spatial forms, but functional collections of people and places. So “fiat” boundaries are not really arbitrary. Evolutionary pressures act against truly random or arbitrary geospatial forms. For example, a border so misshapen that it causes inconvenience for citizens or government might end up being modified by popular demand. A dramatic example is the ill-conceived partition of India, which resulted in the topologically disjoint morphology of Pakistan, and finally a further war as East Pakistan seceded to become Bangladesh. As a more peaceful example, on the Brooklyn/Queens border, the Ridgewood neighborhood voted in 1950 to switch from being part of Brooklyn to part of Queens. There were several socioeconomic motives for this demand, but the areas’ architecture and urban space does seem more reminiscent of Queens, and, probably more important, the pedestrian and transportation routes seem more convenient heading toward Queens business and government districts than those of Brooklyn. The point of this example is that the notion of “fiat” boundary, while not entirely unmotivated, should not be understood too simplistically. Borders which seem arbitrary from a purely geometric or geographic perspective may make sense sociologically, or at least have their own social histories. The connection of geospatial regions to socially recognized names, and functional units (post codes, electoral districts, etc.), both encapsulates and helps to create a complex system in which human activity plays out against a geographic background.

A “topological” theory of functional organization should therefore explore how notions of contact, connection, continuity, and discontinuity are to be defined in functional as well as purely spatial terms. To say that two spaces are “in contact” can mean different things, and purely physical contact may be replaced with contact via travel, shipping, digital media, causal correlation, etc. A convey of cars can be modeled as a single unit, as if there were actually physical forces connecting them, even though the actual synergy between their movements depends instead on their collective de-

sire to stay together. Similarly, a group of hikers may adopt a spacing on a trail so that each hiker is visible to one behind, but the group covers a relatively large area (consider a search-and-rescue mission). Here “contact” most usefully means “visual contact”. Of course, different notions of contact can functionally interrelate, or fail to function properly in consort. Political union can forge contacts between disjoint regions, but (as in the Pakistan/Bangladesh example) such non-physical connectedness is not logistically equivalent to geospatial continuity. Or, suppose that I remove a key from a keychain, attach a small metal ring, and reattach that key to the new ring. During this exercise the keychain has undergone several topological modifications, but it has retained a certain cognitive unity as a functional object — even when disassembled into keys and rings, I can with some justice refer to the ensemble as “my keychain”. I can even carry around the disjoint lot in my pocket with the intention of completing it later, while still thinking of it in the singular. But the loose key is more likely to be lost in that scenario: there are (at least) two different topologies which are functionally relevant, one based primarily on direct physical morphology and one based on my intentions. Functional parameters merge disparate parts and introduce new topologies than yielded directly from three-dimensional geometry, usually without however rendering the original topology functionally irrelevant.

On the other hand, functionalities can also introduce new criteria of separation. A zipper is distinguished from a jacket, partly perhaps by a difference in color, but more importantly because it has dynamic affordances, patterns of movement, distinct from those of the jacket itself. Similarly, a black knob on a white stovetop stands out not only by color contrast, but by the actions it permits. The zipper and the knob introduce patterns of continuous but delimited change which give rise to new concepts — on or off, zipped or unzipped, or hitching the zipper just enough to keep the jacket from flying open, or zipping “all the way to the top” on a cold day. These dynamic patterns represent the emergence of geometric dimensions within an underlying topology; I can visually represent, and to some degree capture by concepts, the degree to which the zipper is zipped along an up/down axis, or the knob turned between “off” and “max”.<sup>3</sup>

<sup>3</sup>Perhaps it is not by accident that knobs and faucets, in North America, given our left-to-right orthography and our metaphor of “clockwise” as

A “functional mereology” does not only isolate parts by virtue of functional parameters (like the dumbbell’s graspable/bulk); it also reveals parthood isolated by the presence of functional dynamics concentrated within one area of a larger whole (the up/down of the zipper, or left/right of the knob, which have no correspondants across the general space of the jacket and stovetop). Moreover, these patterns tend to elicit schema which abstract from specific spatial and perceptual forms. The idea of the zipper or the knob as a zone of special dynamic affordances, relative to some larger whole — both the mereological fact of a parthood thus established, and the (at least partially) geometric fact of the smaller zone, when attended to, revealing a bounded but continuous dynamics — this pattern is instantiated by a given jacket or stovetop but is expected and replicated across almost all examples of these concept. The position of the zipper or the knob provides a specification of the jacket or stovetop’s “state”, and moreover a dynamics of how such state may be changed.

These examples demonstrate a correlation between *state* and *mereology*: a particular parameter for the state of a whole is often the state of one part, and a part can be isolated by virtue of its bearing this state, or being “in” one of a related set of possible states. The fact of some detail being part of a whole’s state therefore tends to suggest a mereological corollary: there may well be some part which embodies the state, and so the state is a state of the whole because of (or by causal or some other correlation with) being a state of the part. Moreover, the dynamics of the part-state reveal various degrees of autonomy relative to other parts or to the whole. If a stovetop has several burners, each knob can be on or off independent of each other. For the stovetop to be considered “on”, however, (e.g., to be a fire hazard if unattended), it is enough for any one knob to be on. This network of autonomy and correlation governs both the parthood and the functional organization of the whole: the knobs (and also the burners they control) are different parts because they are mutually autonomous, at least to some degree — they remain if nothing else physically connected, part of one single appliance — but they are all parts of one whole, whose state they can all influence.

According to Olav K. Wiegand,

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“forward in time”, that turning to the right tends to produce “on” or “more” (heat, light, volume).

The unfamiliarity of an object exhibits itself not by complete absence of anticipations that are induced by the type to which it belongs, but rather by the degree to which the typical latitude (*Spielraum*) of anticipations is determined. Hence, the *familiar* things of everyday life are exactly those things, perceived by virtue of their typicality, that possess a minimal amount of indeterminacy in their normal characteristics. Such familiar objects give rise to a certain “intimacy” and “automatism” in our dealings with them. However, the Phenomenologist views and analyzes these phenomena as a highly complicated structure of *explicit* and *implicit* expectations and memories.

A crucial point here, however, is that the anticipated perceptions, to which the present one points, are subject to two conditions: they must be in agreement and conformity both with one another and with the present perception.... The individual object is thus an *organized totality*, the members of which are subject to mutual *functional dependencies*. These dependencies can now be clarified as having two aspects: (a) each individual noema has a functional significance for the whole (one could say that it makes demands on the whole in order to fit into the whole) and (b) at the same time the complete noematic system determines the functional significance of each of its parts. Both of these make up what Gurwitsch calls the ‘principle of sense conformity’. Expressed noematically, the principle states that the complete noematic system must be constituted in such a way that it can adopt each perceptual noema as a part or element of itself. [5, p. 14]

The likely implication is that protention and retention are not based solely on perceptual continuity — imagine the continuous perceptual path accompanying the act of turning over an object held in one’s hand — but more significantly on our functionalized experience of objects as structured wholes.

What I have hoped to emphasize in the discussion so far, in any case, is that mereotopological structures within real-world phenomena sometimes take on functional patterns, which engender schema that abstract from explicit experiential engagement. These schema

then become rules of interacting with classes of objects, or metaphors which can be adoted in artifical-interactive settings, like the use of knobs or scrollbars in a Graphical User Interface, or scrolling though pages in an electronic document with a motion that simulates “flipping pages”. Philosophically, our study of these metaphors and these electronic environments involves crossing from a phenomenology of the experienced world, in which at some level of consciousness we register all sensory and perceptual details, to a phenomenology of special “operational” worlds — like the world of a computer application, or the World Wide Web — in which consciousness is directed toward norms and purposes of engagement, and less engrossed with fine-grained sensory detail. This is still phenomenology — in other words, we need to understand Phenomenology more broadly than an Analytic Philosophy of Mind sense preoccupied with qualia and “first-person experience”. Qualia, in all their experiential preciseness, are indeed intrinsic to the phenomenology of the perceptual life-world, but the *life-world* is also the world of all our actions and purposes, which often have schematic forms that abstract from sensory detail, and which in our modern age often engage us with constructed environments wherein consciousness adapts to hyletically imperfect conditions. Such ecosystems — watching television, videos online, using software — lack in full experiential “presence” as this is qualified in phenomenologies of Virtual Reality Environments. Yet as we become engrossed in these constructed “worlds”, the structures of mental intentionality adapt to noemata individuated less by sensory distinction and more by how artifacts in these worlds implement learnt or familiar functional patterns, so that these patterns permit (no less than “sense data”) the formation of attentional foci.

## 1.1 Individual Coherence and Internal Structuration

On this reasoning, a *cognitive* theory of mereology can originate with *perceptual* discontinuity but then generalize outward. A part is experienced as such not only because it is visually or sensorily distinguishable from its surroundings, but because it has some functional integrity, perhaps reinforced by conceptual specificity: the zipper in a jacket, the knob on a stovetop, the promotion rank on a chessboard, etc. A thorough “cogni-

tive mereology” should therefore both differentiate these perceptual/functional/conceptual foundations and also consider the structural accounts that unite them — there are aspects of parthood that are correspondingly envined in each partonomic “mode”. For example, whether we are considering the relation perceptually, functionally, or conceptually, a part is necessarily both continuous with and detached from the whole, in appropriate senses. A part, therefore, necessarily has some *individual coherence*, some features warranting its being experienced as noticeably distinct from the whole on some attentional premise.

Meanwhile, a whole must in turn have individual coherence, for it to be other than a trivial assortment of other wholes. The very idea of a *whole* implies a balance between unifying criteria and sufficient inner discontinuity, or *internal structuration*, to be appropriately cognized as a whole *with parts*. A putative whole with no coherent rationale can be deemed “trivial” insofar as a random assortment of physical objects, for example, is not itself a physical objects. On the other hand, an atomic or undifferentiated whole is trivial in a different sense, given that it becomes seemingly a partless a whole, a whole removed in its own part from mereological considerations.

In this latter kind of case a putative whole may have strong criteria of individuation, but its status as *whole* depends on a surrounding environment. We tend to experience squares on a chessboard, or stripes on a flag, or say a solid-color placemat, are *wholes* because they are *parts* of a larger totality, against which their individuation can be figured. So, cognitively, parthood and wholeness can be interdependent; there are cases where something’s cognitive status as *whole* relies on its simultaneous status as *part*. Here we do not experience compartments to whole-aspects or part-aspects as in tension; instead, they are mutually founding.

Jointly, then, *individual coherence* and *internal structuration* present a kind of two-valued criteriology modulating appraisals of wholeness and parthood. There are putative wholes that seem to slide toward triviality because there are few senses in which their integral unity is relevant. Consider the British Commonwealth, which nominally aggregates most former UK colonies but would rarely be classified as a geopolitical entity analogous to the United Nations or the European Union, much less a sovereign nation. These wholes effectively



have significant *internal structuration* but little *individual coherence*. Meanwhile, there are wholes which seem essentially insignificant — at least outside contexts where they are parts in turn — because they have *individual coherence* but little *internal structuration*.

The wholes which are important or meaningful, which tend to evolve toward distinct concepts and schematic conceptualization — entrenched in conceptual schema, e.g. via natural language lexicons — tend to lie at the intersection of high apportionings of both *internal structuration* and *individual coherence*. If we imagine these on a two-dimensional “scale”, say, an “IS/IC” scale, cognitively significant wholes would tend to be situated toward the upper-left of region.

An IS/IC scale is at best a suggestive idea, a philosophical trope; I don’t actually think we should try to “quantify” these dimensions. But as an image I think the notion of intersecting “IS” and “IC” can serve as an intuition not only for analyzing cognitive frames — our perceptual/enactive comportment in the moment — but also our conceptual systems that condition cognitive dispositions. Mereological models can have noteworthy structural significance in this domain as well.

## 1.2 Coherence and Concepts

In Formal Concept Analysis, a concept is defined by the combination of its instances and its indicators. Two distinct concepts can share the same extension: the set of black US presidents equals the set of Hawaiian-born presidents. Within a collection of objects and properties, a “formal concept” is a set with both objects and properties such that every object bears every property. In this formal setting, a concept is a statistical artifact, which may or may not correspond with concepts of thought and language (despite its well-defined extent, in the singular person of Barack Obama, *black Hawaiian president* probably does not express a concept with cognitive value beyond its constituent properties). Such analysis however suggests a more general truth, that concepts depend on both intension and extension; to be acquainted with a concept is to understand to some degree both a set of instances and the reasons or properties for why they belong.

Moreover, within real-world concepts — typically im-

precise and dynamically evolving, insofar as they are cognitive and communicative tools — intension and extension evolve in consort. Borderline cases need to be either excluded from or included into a concept’s reach, and this choice forces an evolution in intension. Suppose we start with a simple, provisional account for a familiar concept — a *house* is a place of residence. We can then consider places which we may or may not consider houses — an apartment, apartment building, hotel, cabin, tent, tree house, the White House. Based on how we classify these examples, our posited “house indicators” evolve; e.g., a house is a place of permanent, year-round residence for at most a few families, where habitation (as opposed to, say, government or commerce) is its primary purpose. Such thought experiments do not *define* concepts, but perhaps they retrace their history as linguistic and cognitive phenomena within a relevant community.

Although I will consider how conceptual attribution suggests patterns of *functional organization* within objects thereby identified, we can also consider the “function” of concepts as serving thought: what *roles* do typical concepts serve within mental life overall? Given the intension/extension co-evolution, we recognize at least two distinct roles; a concept both characterizes an *extension*, a set of instances, and also an *intension*, a set of properties or indicators which provide a rationale, and a suggestion of further characterization, for identifying an instance with a concept. To assert that something tokenizes a concept is not only to provide a very rough suggestion of what it is like, but to imply a strategy or “script” for providing more detail. Once I identify something as a *house*, I imply a kind of organization — a yard or some sort of surrounding property; an inside and outside; an external style and architecture; a set of several internal separated but interconnected rooms. I also imply (in contrast to a motor home) that the house is built in one fixed place and has a permanent location, perhaps a postal address, that visiting someone at their home means returning to the same location on each occasion. But implying this general schema also “initiates” a script for further specification; identifying something with a concept is more often the start of a conceptual and linguistic process, rather than a conclusion. I can learn the address of your new house, directions to get there, you can describe its various features, its yard and interior design and so forth.

Concepts play different roles, and these can be “activated” in turn by different situations, in particular by different linguistic and grammatic formations. Similar meanings can be achieved by subtle variation in which conceptual roles are suggested, providing a case study for how grammar connotes role. Consider Ronald Langacker’s comparison of sentences like

- ▼ (4) Three times, students asked an interesting question.
- ▼ (5) Three times, a student asked an interesting question.

In (4), the plural *students* reflects how a type of noteworthy situation occurred multiple times; whereas the singular in (5) reflects how, on each occasion, one student was involved. The “student” in (5) does not designate a particular person, but is rather a generic token of the concept *student* “conjured”, as Langacker says, to provide a kind of unspecified cognition of a conceptual tokening without implying reference to some specific token. Along these lines, consider

- ▼ (6) Rhinos are mammals.
- ▼ (7) A rhino is a mammal.
- ▼ (8) The rhino is a mammal.

While (6) refers via the set of all rhinos (each of them is a mammal), (7) second “conjures” a “generic” rhino, from an abstract “plane” — again using Langacker’s terminology — to make the point that any conceivable rhino is a mammal. That is, mammalness is a generic feature of the entire species, it is not a quality which depends on the nature of any given instance. In (8), the signification by contrast selects from the “plane” of all species types; so *rhino* here applies in the guise of naming a discrete element in categorizing thought. These examples illustrate roles of *selecting an extension*, *expressing intension* — in the specific sense of concept-intension; that is, envisioning a representative abstract case which explicates the conjunction of properties and indicators that are a concepts’ signature — and *naming a type*, referring into a space of *kinds* insofar as these are present in thought as discrete units.

Which of these roles is most clearly “activated”, given semantic and grammatic cues, then shapes the surrounding discourse:

- ▼ (9) The rhino is related to the horse.
- ▼ (10) Young rhinos are threatened by poachers.

- ▼ (11) Rhinos in that park are threatened by poachers.

Here (9) forefronts rhinos as a natural kind, invoking a familiar relation between species; (10), I would argue, invokes a more intensional sense of the concept insofar as it adds a further specifier to suggest a narrower concept (a young rhino has the properties associated with rhinos in general, and then further those associated with young animals, such as being exceptionally vulnerable and being cared for by parents); while (11) seems to construct a narrower concept by adding specifiers to *extent*, not *intent*. There is no suggestion that rhinos “in that park” have any further resemblance aside from their being there; as a result, (11) comes to designate some set of animals by invoking the *set* of rhinos and then adding semantics to focus on some select portion of that set. In these examples the concept *rhino* plays different semantic roles — guise of a species, a bundle of typical properties and indicators, and a set of jointly classified individually — corresponding to different cognitive roles.

This multiplicity of roles precludes simplistic theorizations of concepts as *just* instance-sets, or property-bundles, or taxonomic entries. Rather than a single (meta-) definition, a general theory of concepts should begin by classifying different ways that concepts are used. An initial distinction is that a concept can “profile” (another Langacker term) *either* an individual *or* a collection of individuals, including (but not necessarily) a set of all (actual or possible) instances. In the latter guise a concept (alone or in consort with other semantic elements) demarcates a collection from some more general or expansive collection. In the former case a concept *singles out* an individual; as I discussed earlier, this implies some mixture of continuity and discontinuity, insofar as the individual is (to some degree) posited as separable (in an act of cognition and/or perception) from its surroundings and also as internally connected or integrated. There is some “theory” of the *internal coherence* of the individual, of how it is appropriate in some context to consider it a discrete unit — acting as as singular whole, causally integrated, or in some other fashion disposed to function unitarily.

Certainly the “individual coherence” of a totality may be provisional. Some collections are bound together only by people’s desire to group them, e.g., the books in a library. Nevertheless, even this “external” connectedness is causally efficacious, with non-negligible effects of

cohering the integral whole more than a random baseline: books in a library are much more likely to remain spatially proximate than a random pair of books which happen to be in some one building at the same time. On the other hand, the degree to which parts *do* cohere into a whole is an essential conceptual detail, and, once a whole is characterized as an individual tokenizing a concept, part of that concept's role is to suggest the degree and nature of this coherence. For example, the United Nations is a more diffuse collection of political units than the United States. Nevertheless, there is a well-established semantics where the UN functions as a singular entity which can, say, pass a resolution. On the other hand, insofar as a totality has a strong degree of internal coherence, this may result from a complex causal or processual integration between many parts, or conversely by a relative lack of internal complexity or *internal structuration*. A torpedo and a school of fish may both follow a quantifiable trajectory through water, but the directedness of the former's motion depends on complex biosocial synchronization much different from the latter's straightforward physics.

For each conceptualized totality, then, there is a measure of Individual Coherence and of Internal Structuration — which can be quite independent from another (the “IS/IC” scale) — that specifies *in what sense* the whole is a (single, unitary) individual. The concept used to designate the whole, along with (in a language setting) surrounding grammar and context, suggests a particular account of the degree and schematic nature of this individuation.

Grammatical choices, like singular/plural and definite/indefinite article, play a role here:

- ▼ (12) A rhino is by the lake.
- ▼ (13) The rhino is by the lake.
- ▼ (14) Rhinos are by the lake.
- ▼ (15) The rhinos are by the lake.

These grammatical variations induce subtle cues to construct or indicate a “plane” from which individuation selects. In (12), the hearer's attention is newly directed to something which, the speaker suggests, either the hearer or both parties had not previously observed. In other words, (12) wants to shift the topic of conversation (even if only temporarily, so as to register the asserted fact as something collectively realized) and so functions

both to individuate the rhino, to propose it as a new object collectively recognized as something in their mutual surroundings, and to shift attention in its direction. By contrast, the rhino in (13) has a presumptive prior acknowledgement in the current discourse; the point of the definite article is to select the rhino from among the set of all entities which have in some fashion been talked about already. These different “planes of selection” carry over to the plural examples, but the designation there selects a group of animals, not one single individual. Nevertheless, (15) implies that the group has some sort of internal connectedness, perhaps moving roughly as a unit. By referring back, as does (12), to an earlier phase in the discourse, (15) implies that the group represents (more or less) “the same” rhinos as observed earlier, an effect of comparison which helps call attention to the rhinos in their totality. Without the definite article, (14) is less specific in asserting such a totality; it may, but need not, suggest that the rhinos by the lake are situated in some spatial or interacting formation so that the speakers should be disposed to consider them collectively. If the speaker wanted to nudge the emphasis more or less toward this grouping interpretation, she would have to select some further semantics: e.g., *A school of rhinos*, or conversely *Some rhinos*, are by the lake.

In these examples, the concept *rhino* does not, on its own, guide one interpretation or another (singular/plural; more integral or more scattered); it is rather a resource which is deployed in a linguistic and communicative setting along with further details, and the effect of conceptual resonance *in context* bears these further connotations. Whether a concept designates a group or an individual, it is the speaker (or a person using a concept as a tool in thought) who uses ideas activated by the concept so as to evoke patterns by which the selected individual or collection is internally connected and externally separated. Any conceptualization presents schema where continuities and discontinuities are mixed, but in different ways. Suppose a game warden says:

- ▼ (16) Those footprints were left by rhinos.
- ▼ (17) That trail of footprints was left by rhinos.
- ▼ (18) Those tracks were left by rhinos.

I argued above that the concept “footprint” internally suggests both continuity (of medium) and separation (of shape or contour). Analogously, a collection of footprints as in (16) has a presumptive internal separation

into discreet individuals, but enough spatial proximity to be plausibly grasped as a perceptual group, relative to a reasonably typical vantage point. The choice of words in (17) implies a further notion of directedness, and a somewhat different relation of the observer's spatial position vis-à-vis the observed. While the two sentences might describe the same situation, (17) carries an additional implication that the footprints suggest some line of direction which extends beyond the speakers' immediate line of sight, and can perhaps be followed. This directionality is also implied in (18), but in that case the internal continuity which supports this sense of direction is further emphasized. In this context "tracks" can mean not just footprints but also, say, bent grass or hairs or broken twigs, so that each particular artifact which cumulatively constitutes the "tracks" may have less precise distinctness than a particular footprint. Moreover, the word "tracks" tends overall to refer to physically continuous structures, like train tracks, whereas "trail" can be used in more metaphorical senses and with greater sense of internal pauses and separations (a detective *on the trail* of a fugitive; a scientist *on the trail* of a discovery).

So (16)-(18) show progressively less attention paid to internal constituents and more to the nature and direction of a whole, even if some scenarios tolerate all three sentences. Each usage would convey slightly different connotations to the scene at hand. The point of this comparison is that individuation through concepts depends on properly connoting the patterns of continuity and discontinuity which define individuals' coherence and separateness (as well as that of sets of individuals, when concepts are used to profile collections). A concept implies a schema both for individuals' internal structuration, and for its suitability as being conceptually and perceptually isolated from its surroundings. The function of concepts is, typically, to *set the stage* for further characterization of their tokens, first by individuating them as points of attention, and then by implying the presence of "scripts" or "schema" leading to more precise descriptions. Both of these roles present challenges for *formal* semantics, if this means defining rigorous or quasi-mathematical theories of how concepts select a *set* of instances, or of how each instance *instantiates* a concept. The act of linking a concept to an instance is a complex cognitive/semantic event, one which occurs against the backdrop of a mental and often dialogic context, and which should be theorized as one step in an evolving

process. A concept is an opening onto a more detailed (process of) characterization. To say that something is an instance of some concept is not a simple act of classification, then, analogous to declaring a variable in a programming language to be of some data type.

Of particular importance, I believe, is the notion that conceptualization (if we define this as concept-to-token identification) is only one stage in an extended process. This means that no one single concept provides a definitive account of an identified instance, even a provisional or imprecise account. Obviously concepts operate at some level of coarse-graining and can be combined together for greater precision. However, if we fail to theorize conceptualization as *process*, we can end up with a view of concepts as imprecise but, *at some degree of detail*, self-contained mental "pictures" which cover some range of possible cases. For each concept, there would correspond (on this account) a "rough" sketch of how a token of the concept is, insofar as it does bear the concept. The degree of (im)precision in these "sketches" would depend on how general or specific is the concept itself: scientific terms, for example, are more specific than everyday words. But each concept would connote a family of instances whose features are more clearly or more vaguely aligned with a set of indicators. Each house is vaguely aligned with a "sketch" of some locale with a yard, a front door, interior rooms, and so forth; each restaurant is aligned with a general notion of a place to sit at a table and eat prepared food, and so forth. The concept leaves particulars imprecise (what kind of food, what kind of yard) but presents them at a level of vagueness in which they match all examples. A concept, in other words, finds a suitable mix of vagueness and resemblance — it abstracts from particular details enough that all tokens can be considered to resemble each other.

While my analysis has likewise argued for *characterizing features*, where I differ from this kind of theory is in emphasizing conceptual imprecision as a matter of *process* and not of *vagueness*, or concepts as *scripts* rather than *sketches*. It is misleading, I believe, to think of a concept as *imprecise* in that it seeks to achieve a degree of nonspecificity which allows many somewhat mutually resembling things to be grouped together. This notion seems to both minimize the potential for using particular concepts to initiate more precise further characterization, and also to overstate the degree to which



concept-tokens resemble each other. As mental tools concepts are thereby weakened on two fronts: they are neither precise enough to truly describe their instances, nor general enough to group together these instance outside of some “resemblance” between them. I would argue, however, that real-world concepts both trend toward a degree of generality which is hard to account for in terms of token-resemblance, *and also* serve within cognitive and linguistic episodes in which general concepts provide a framework for precisely describing individuals, up to a degree of resolution suited for a given thought or dialog. The concept may not internally *imply* these details, but it implies a *framework* for accumulating them.

If we consider the concept of *fly*, for example, we note that it covers many different cases: the flight of birds, planes, comets, commercial passengers, leaves, kites, debris, shrapnel, types of aircraft (“that model flew during World War II”), carriers (“we fly Korean Air whenever possible”), their fleet (“that company flies the youngest planes of any discount carrier”). All of these somehow involve travel through air, so we might say that the role of “fly” is to invoke a highly stylized or fuzzy sketch of “movement above the ground”; in other words, to find the point of resemblance among all of these cases. The different senses of *fly* might then be considered *sub-concepts*, each with a more precise picture, which finds more detailed resemblances between its tokens. By this account, the concept “fly” is really a loose aggregate of more precise concepts, which manifest as different senses of the polysemous English word. However, such an analysis seems to too neatly group these various senses into a conceptual hierarchy, as if word-senses can assemble into a taxonomy akin to the classification of living species, each sense finding a degree of precision and inter-token resemblance relative to its hierarchical position. I would argue that different “senses” of words or concepts do not generally reveal such straightforward taxonomic patterns, at least unless (as with biological names for species, genera, etc.) they are deliberately constructed to do so. In normal usage new senses evolve gradually and can emerge ad-hoc for some specific language community (or “micro” community). For example, the sense of “fly” as in “Commercial Flight” — with its typical associations of buying a ticket, going to an airport, checking baggage, passing security, and so on — emerged only gradually from the more general sense of flying in an aircraft. Moreover, we can imagine specific situations

where new senses emerge — for example, executives of a multinational company who use the phrase “Flying to London” to mean a trip to specific offices, thereby mixing (in this specific context) the destination with the purpose of the trip.

The fecundity with which these senses arises suggests to me that we should not consider each “sense” as its own (sub)concept; instead, I would consider *fly* to be *one* concept, not a loose assortment of vaguely related concepts. This concept coverts many different cases, but the cases can be compared and characterized according to certain overall criteria, such as the *cause*, *reason*, *agent*, and *destination* of the flight. Flying *debris*, *leaves*, and *kites* all rely on the power of wind, but the former implies a relatively strong gust to dislodge solid objects naturally resting on the ground, whereas the latter implies a human agent who orients the kite to best gather the wind. Flying *birds* and *planes* are both self-powered, but the latter relies on human agency to provide fuel. The various senses of human flight are comparable based on the kind of craft used, the relation between passengers, pilots, and whoever owns the craft (contrast hobbyist who owns and flies planes, an air force pilot, a passenger service where flying is a commercial transaction). Each of these distinctions provide parameters for comparing different examples of flight. No parameters, except perhaps for the most general notion of airborne motion, apply to all cases and kinds of flights; but each case involves some mixture of some relevant parameters, and each parameter offers a ground of comparison. For example, only commercial flight involves a ticket, but once this parameter is “activated” we can compare the price of different flights.

It is hard to identify isolated “subconcepts” within general flight because different parameters overlap in different ways. Passenger flight has unique aspects of cost and value (and whatever legal and social considerations come along with commercial transactions), but it shares other parameters with other (but not all) senses: relatively specific times and destinations of travel, like the flight of birds but unlike that of leaves or debris; the distinction between the subject and the agent of flight, a *person* is flying but by virtue of being on an aircraft, so the subject/agent relation becomes a conceptual parameter, which is also present in some other contexts, like cargo. For an analogous example, we might think to subdivide the concept of “restaurant” into more specific

cases — after all, merely expressing the desire to visit a restaurant in general, or searching for one, can cover a wide spectrum much of which may not actually reflect the speaker’s/searcher’s interest. Someone may want a formal dining experience, a casual spot for family dinner, a healthy and inexpensive lunch, a quick meal, or a place to have a snack and go online. Certain lexemes cover some part of these more specific spectra — bistro, steak house, diner, coffee shop, cafeteria — yet none of these compete with “restaurant”’s dominance of the relevant semantic territory: one will often use the more general word even when one of the more specific ones apply. I believe this can be explained, in part, because the more general concept can be narrowed in several ways at once, thereby providing a more effective point of origin for a specified description; providing more “avenues” for elaboration. For example, a “Five Dishes and Soup” spot in a Chinatown might be considered a Chinese restaurant and a cafeteria, but there does not appear to be an established usage “Chinese Cafeteria”. Even if “cafeteria” might be plausibly defined as any restaurant without table service, it seems to connote further details about the kind and pricing of the food. Instead of specifiers offering a clear-cut segmentation of the “restaurant” spectrum, English speakers seem to prefer the more general usage and to provide further specification by an unfolding, dialogic, and open-ended process, relying on communicative cues to converge on an image of the “kind of place” some restaurant is, rather than trusting some narrower lexeme (like *cafeteria* or *diner*) to impart these cues via convention. Arguably, in some specific circumstances, a narrower lexeme has indeed achieved something like canonical status (*pizzeria*, *coffee bar*); so a speaker using those terms, amongst a particular language community, shows confidence that the allusions embedded in them convey a sufficiently precise picture of the “kind of place” being mentioned. But this specificity stands in semantic relation to the more general term, even if the latter is replaced on some occasions: the ubiquity of the word “restaurant” adds an extra dimension to a speaker’s choice on some occasion to instead say *pizzeria* or *coffee bar*. The concept “flight”, I would argue, has a similar semantic ubiquity; it encompasses a spectrum of cases so broadly that the occasional choice of a different word (“I jetted to London”; “The birds migrated south”) carries the semantics not only of the word used but the dominant one *not* used.

What this analysis suggests is that real-world language does not necessarily show a tendency to semantic specificity, to a preference for more specific usages and a relegation of more general terms to cases of deliberate imprecision or abstraction. If a language community had an instinctive drive toward semantic precision, then dominant but imprecise words like “restaurant” would tend to gradually be replaced in many cases by narrower concepts, and to be retained mostly when speakers deliberately intend to avoid any narrower connotations. However, this does not appear to be the way language in general evolves. I believe that, to an important degree, this fact can be explained by appeal to conceptual roles: the point of a concept is not to condense as precise a picture of its tokens into as concise a semantic unit as possible, but rather to initiate a further descriptive process as efficiently as possible.

### 1.3 Concepts’ Descriptive Spaces and Functional Schema

A concept-attribution, I have argued, does not primarily serve to suggest the nature of some token by aligning it with a vague schema, a fuzzy picture it “resembles”. Instead, saying that some instance bears some concept, like *restaurant*, initiates a process or “script” in which further characterizations are available. The attribution does not just imply a “bundle of properties” which can be enumerated, like in a relational database or “facts at a glance” — here is a restaurant; here is its address, kind of food, hours, coded numbers for price and quality, etc. The tabular form of a database relation can be considered one very specific kind of “script”, in which description proceeds just by listing a set of parameters and values. Typically, however, characterization is more unfolding and “branching”: does a restaurant have table service? If so, how polite/professional/knowledgable are the wait staff? Does it have a wine list? If so, how thoughtful/inexpensive/well-matched are its selections relative to the menu? And so forth.

Taking departure from Formal Concept Analysis, we can say that a single concept does not have a single “frame”; a single set of properties (or even parameters, property-dimensions which take different values for different instances, like the average price of a restaurant entrée) which collectively characterize all tokens. Instead, con-

cepts associate with *description spaces* in which different parameters are “packaged” to be activated collectively, or not — a table-service or wine-list package is appropriate to some, but not all, restaurants. In a characterizational process, these packages are sequentially activated and provide particular details, but the precise collection of details will vary from case to case. In other words, for a general concept like *restaurant*, there is no one set of properties or parameters which fully characterizes all tokens. There may be general parameters which apply everywhere (like a restaurant’s address or opening hours), but these tend to offer only the beginnings of a useful description.

Aside from the semantic subtlety of how a single concept gives rise to a spectrum of different “description packages”, we also need to honor the ontological complexity of how the properties attributed to concept-instances vary across these packages. Real world concept-tokens tend to be ontologically “complex”, which is to say that there is no single fashion in which they “exist”. A restaurant, for example, is a physical building and location, a collection of people (social institution), and a collection of dishes served. These various facets of a restaurant’s existence belong to different ontological “registers”. A restaurant which moves to a new location but keeps more or less the same staff, name, and menu might be considered “the same” restaurant. So too might a new operation which opens in the prior location — it is permissible to say “that restaurant is Chinese now”. We can say that a restaurant is *unionized*, *vegetarian*, and *remodeled*, asserting properties respectively of its staff, menu, and building, and by extension, metonymmically, of the restaurant itself — like calling a book with yellow cover (and white pages) — “yellow”.

Certainly the staff, building, and menu are “parts” of a restaurant. Yet “sum of parts” here is not simplistic set-theoretical union; it is a more complex functional interrelationship. A restaurant joins a set of people (staff) with a set of places and rooms (dining areas, kitchen areas, etc.) according to schema involving how the physical space enables the staff to carry out typical operations, and how the combination of physical and operational design (the procedures which employees understand and consent to enact) allow the restaurant to carry out its expected roles (taking orders, preparing food, bringing them to patrons). We expect the restaurant to be a “site” where certain kinds of things can happen, so that

certain kinds of activities are afforded to us, and it is the nature of these affordances (insofar as they have gustatory, interpersonal, economic, and spatial aspects) — to cross ontological “registers”.

## 1.4

### Mereology and Referential Vagueness

The ontological complexity of *objects* (like *restaurants*) then complicates semantics and reference (like *a restaurant* as a designated signified). Which “facet” of the restaurant is a referential target in different discursive contexts? These questions complicate attempts to map units of language to things “in the world”.

Granted, we should assess any “formal” semantics against a presumptive distinction of *semantics* and *pragmatics*, such that linguistic nuances which seem to violate referential rigidity are artifacts of actual *usage*, as separate from formal “meaning”. For example, the apparent ambiguity of *London* as either the larger city of the financial district, and *Manhattan* as either an island or a borough, is a product of how the respective language-communities reuse a single name for multiple, slightly different but overlapping purposes.

To be precise, the “City of London” is technically a small district in what people informally call “London”; and the *borough of Manhattan* includes Manhattan island but also several other islands and the small Marble Hill neighborhood on the mainland. If we accept a “model-theoretic” paradigm where formal semantics takes a collection of symbols (names, lexemes) and maps them to a collection of empirical things (like geospatial regions), we then ask, which regions are designated by the respective names?

Conventional usage in the two cities suggests that this question is not settled; New Yorkers for example will use “Manhattan” for either the borough or the island (or both) depending on context. There seems to be no nonarbitrary reason to name one or the other as the “real” (referent of) Manhattan.<sup>4</sup>

<sup>4</sup>Maps on the westbound   train show Roosevelt Island as the first stop in Manhattan (borough), but riders would not necessarily consider this stop (across the river from Midtown) to be “in” Manhattan, and Real Estate listings might be considered misleading if they classified Roosevelt Island addresses as in Manhattan.

Distinguishing *semantics* from *pragmatics*, we might argue that there is nonetheless an underlying semantic system here where the two locations actually are differentiated, so the name “Manhattan” actually has two senses. Speakers understand from context which sense is intended — this is the pragmatic layer — but *each* sense, if we can treat it as a distinct lexical unit, may still function in accord with (say) model-theoretic semantics. On this account, the name “Manhattan” reflects a pragmatic, communicational speech-gesture which singles out one or another formal-semantic meanings depending on context; underlying the surface pragmatics of everyday usage there is a formal semantic framework, where the two “Manhattan”s are distinguished.

The problem for this account is that most pragmatic usage does not make this distinction — and the reason is not because one sense or another can be inferred from context, but because it is not relevant. This can be called an example of *vagueness* rather than *ambiguity*; not that references are ambiguous between Manhattan-the-borough and the island, but rather they are vague about even positing the distinction. For example — to borrow an example due to Mark Heller — the assertion *Syracuse is north of Manhattan* holds despite fine-grained ambiguities in construing the designatum of “Manhattan” (or of Syracuse, for that matter). Heller does not mention the island/borough confusion, but points out that geospatial regions are in some sense “fuzzy” as sets of geospatial points — the precise borders of an island, for example, seem to depend on everchanging variations in water levels. Heller develops a theory according to which, considering this imprecision, the referents of proper names should be considered “conventional objects” which overlap to large extent with empirical objects or regions. For another example, he runs a Sculpture/Clay discussion: how a sculpture may begin as a block of clay, be completed and then exhibited as a work of art, and (let’s imagine) at some point destroyed; the block of clay is there all along, but only in the middle phase (when on display) do we consider it a statue. Therefore (for Heller) the statue is a “conventional object” distinct from the block but overlapping with it for much of its history.

In [3] Michael Jubien counters this analysis with a reference theory based on *properties*; specifically, corresponding to conventionalized concepts (like Manhattan or a sculpture) there is a property of *being* that thing (e.g., *being* Manhattan), and linguistic expressions (like

proper names) semantically designate those properties. “There is no contingent identity.... There are no conventional objects. There are just things and their properties” [3, p. 38]. Depending on contexts and speakers’ particular cognitive orientations, different specific entities can be conceived as instantiating the respective properties — adopting this notion to my examples, both the island and the borough can plausibly be supposed as *being* *Manhattan* for the sake of interpreting a particular speech-act. Better, since either island or borough may plausible be construed as *being* *Manhattan*, most uses need not commit to one interpretation or another. In other words, examples like the Syracuse/Manhattan directions would, on Jubien’s account, be interpreted roughly as asserting that there is some geospatial territory (its precise borders not necessarily characterized) with the property of *being* *Syracuse*, another with the property of *being* *Manhattan*, and the former is north of the latter.

Suppose someone actually asserts, or thinks to themselves about how, “Syracuse is north of Manhattan”. They may be driving down from Syracuse University to join a protest at Columbia University (to imagine one possible scenario). Neither of the two geospatial names are involved, in these thoughts or statements, in such a way that their precise geospatial extents are relevant. It is entirely possible that by “Syracuse” the speaker does not specifically mean that city per se but rather its immediate metro area — one could certainly describe a drive from Skaneateles to Columbia as “Syracuse to Manhattan”. The place names are understood “functionally”; that is, we conceive places not as precise sets of points on the ground, or on maps, but as functional ecosystems, *places* constituted conceptually by dynamic activities they afford: entering into and out of (which means, for *geospatial* regions, going to or leaving them by car, train, etc.), targeting a specific destination within them, associating specific such places with addresses, etc. In this functional sense the distinction between *Manhattan island* and *borough of Manhattan* is not often relevant, because many functional conceptualizations — planning a trip, anticipating taking the train to Penn Station and catching the right subway, and so forth — unfold mentally and operationally with no regard to the distinction. Depending on context, it is entirely possible that in someone’s mind the locale with the *property of being* *Syracuse* is roughly the metro area, and the locale



which instantiates the property *being Manhattan* is the general stretch of land extending outward from Penn Station or some other familiar landmark or neighborhood. Only in specific circumstances — like visiting a friend’s apartment on Roosevelt Island, or voting for the proper elected officials while living in Marble Hill — does the island/borough distinction become relevant. As a result, this distinction is only *semantically* relevant for the meaning of the lexeme “Manhattan” insofar as it is *functionally* relevant for the lifeworld activity relative to which the name is used.

Conventional “model-theoretic” or “truth-theoretic” semantics has to assume that, separate and apart from functional associations between word-uses and environments, there is a purely formal function definable to map specific lexemes to specific entities of a corresponding kind — place names to geospatial regions, personal names to human bodies, species names to sets of living things, etc. Such an assumption — that the world around us provides a “domain of reference” for a formal semantic, a “set” of “things” which can be singled out semantically or referentially — this perspective is questionable on ontological grounds, let alone invoking subtleties of language. Our macroscopic, processual lifeworld does not readily partition into a neat agglomeration of self-identical things, like molecules into atoms.

It is almost impossible to take any real-world concept or referent and formulate a precise definition (or “nomination”) of it as a wholly self-contained object or place. A restaurant is not just a building; a nation is not just a landmass; even a building is not just a mereological sum of architectural features. The “identity” of almost all concept-instances is dependent on how they function within a larger totality, whose activities include the people having occasion to speak about and conceive them through the structures of language.

In *Ontology, Modality, and the Fallacy of Reference* Jubien’s analysis centers on the apparent mereological ambiguity of referential semantics: how mereological composition seems functionally orthogonal to referring acts. Signifying designations of *Manhattan* seem unaffected by the fuzziness surrounding Manhattan’s borders (both because islands have imprecise extent and because there are several distinct but overlapping senses of Manhattan). Or, as Jubien analyzes in the context of modal logic, referring to *this statue* does not seem to be actually

referring to a *lump of clay*. As Cotnoir points out, modal contrasts (like *the clay could be flattened to a disk*) have inspired consideration of Statue/Clay pairs as “mutual proper parts”. Jubien’s theory, on the other hand, proceeds more on an underlying distinction between “the ‘is’ of predication rather than the ‘is’ of identity” (p. 34). It is not that the sculpture *is* the lump of clay; rather, the lump of clay is a physical mass bearing the property *being the statue*.

As Jubien puts it in *Possibility*:

While I believe that typical DR [Direct Reference] theorists would agree at the outset that the bearer is a physical object, I also believe they have not noticed the present sort of difficulty. A typical theorist ... would initially think that loading Daffy — a statue — into a proposition was nothing more nor less than loading a certain physical object into the proposition. But the present difficulty might lead such a theorist to think anew and seize upon the idea that although it’s a statue that gets loaded in, it is somehow not just a physical object. This might be thought to provide a way to avoid the great-divide problem of the object’s not being a statue essentially, since one would think that the statue — whatever exactly it is — is surely essentially a statue. Thus a DR theorist might seek a different way of understanding the proposition that Daffy might have been made of different matter, one that avoids the absurd conclusion that I claimed derails DR. But the options are not inviting. They would be views according to which a statue — the loaded entity — (1) is not a physical object at all, (2) is only partly physical, or (3) is a physical object of a kind somehow different from a simple mereological sum.... [2, p. 121].

The “present sort of difficulty” is the Sculpture/Clay “overlap” that inspires things like “conventional objects” or mutual-proper-parthood. Two years later Jubien would suggest his “property instantiation” model in [?], wherein the clay becomes an instantiator of *being the statue*, so the relevant relation is not parthood but property-substrate. Mereological sums are, on Jubien’s later theory, material aggregates that participate in conceptual or complex/functional organization as *substrata*

instantiating *properties*, which in turn are the actual referents of conceptual intentions [3, p. 12].

The consequence for conventional mereological analysis lies in how mereology is supposed to underlie semantics: in particular, the idea that meanings can be unpacked by mapping words to objects (or, say, geospatial regions) and higher-scale expressions to propositions. So *this sculpture is Greek* would assert a proposition whose intention can be recovered by substituting some object for “this sculpture”, and which should be deemed true if the corresponding worldly object did indeed originate in Greece. And *Syracuse is south of Manhattan* has concrete meaning by substituting geographical regions for the corresponding words. The underlying dynamic in both cases is that things and extents “in the world” are to be substituted for units “in language”, implying that linguistic complexes are composed of sites for eventual substitutions, laying out (at the sentence level) sentences’ “truth conditions”.

The mereological complexities here then arise from the indeterminacy of a mereological whole which can be the target of reference distinguished from other wholes. Since Manhattan is “fuzzy”, *which* whole should substitute for *Manhattan* when stating truth conditions for sentences about Manhattan? Which whole is *this statue* when the object may erode over time, losing its boundary precision?

I will return to these issues in a later section. At this point, however, I want to point out that these mereological controversies lie not within cognitive schemata but in the, perhaps it can be called, “cognitive/extramental” boundary or interface. The problem of “selecting” one whole from among mereologically similar candidates is not an analysis of the parts of our *conceptualization* of Manhattan or the statue, but rather trying to relate the conceptualized thing *as cognized* from the extramental object whose properties — at least on many theories — ground the accuracy and fallibility of cognitive attitudes.

## 2 Mereology and Externalism

One way to approach mereology as a philosophical topic is to define different mereological systems, including cases where these differences can be observed “axiomatically”:

the presence or absence of an anti-symmetry restriction on parthood, for example. To make this exercise worthwhile, it is then necessary to describe the philosophical or practical implications of the logical divergence: is the system with one logical form vs. another a more faithful model of thought, or a more useful directive for technology, or somehow better scientifically? After all, it’s not like the rules of mereology are written in the cosmic order; mereology is not an empirical science.

A related question is whether a given mereological theory is intended to represent how we, as humans, *think about* parthood, or to represent part/whole relations which have some causative or compositional role in nature. Given a partonomic assertion — that a leaf is part of a tree, say — we can read this as a description of conceptual attitudes: that most people (by virtue of perceptual gestalts or lexicosemantic pressures or sub-conscious internalization of others’ enactive-conceptual habits or some other means) instinctively see and comport to the leaf as part of the tree (and the tree as including and encompassing its leaves). We can also read this parthood as saying that *literally*, as a feature in how leaf and tree exists according to biological and physical laws, the latter encompasses the former.

Certainly these two senses are not completely independent. We probably would not entertain mereological attitudes without some pragmatic or physical sense that these attitudes are grounded in reality — that the part we ascribe to a whole does indeed behave as if under the constraints of parthood. On the other hand, insofar there are “physical” criteria of parthood, we presumably learn of them alongside other scientific aspects of phenomena, so that mereology becomes part of the overall package of our scientific model-building. At that point we may try to isolate partonomy as one important, recurring facet of scientific models and specify how, or the different ways in which, parthood is thematized within scientific explanations or proto-scientific intuition.

But still, the difference between partness as a matter of conceptual attitudes versus (in some sense) nomic given is consequential for how we read individual parthood assertions. Given  $x \triangleright y$ , do we see this as matching  $x$  and  $y$  to physical (or at least extramental) objects? Or does it mean that in our cognitive engagements with  $x$  we experience or believe  $x$  to include  $y$ ? If we read *the leaf is part of the tree* wholly extramentally, we have to

explain the referential logic of “the leaf” and “the tree”; i.e., what sort of entities these are such that they can be compactly signified. We could be fully realistic about (mentally) external things — let’s agree there really is a tree out there that many people see and therefore can be a topic of discussion. We still have to explain how there is a referential pattern which grounds use of notation like “the tree” as part of logically sensical assertions (including mereological ones). We presumably see the tree as a gestalt unifying perceptions of its trunk, branches, and leaves, but plugging this unadorned sense into the parthood assertion becomes circular, since the leaves then become *part of* the referential grounding of *the tree*, which empties the assertion of any content. Mereology would cease to be a philosophically relevant topic if its assertions were wholly on the order of, say, the number 11 being part of the set  $\{11, 22, 33\}$ .

Note that the problem of referential circularity does not necessarily arise in the same way when we think of mereology as a cognitive phenomenon. I don’t think it is trivial to the point of meaningless to ask if our *concept* of 11 is part of our *concept* of the set  $\{11, 22, 33\}$ . Philosophy is a conceptual activity, so it is simplest to read philosophical theories as models of other conceptual activities. Of course, however, usually our conceptual activity tries to stay oriented to extramental reality, so philosophy captures the structure of conceptualizations somehow interfaced with reality, and philosophy’s own concepts and notations and quantifications to some degree represents this dual appointment: sometimes we’re talking about cerebral artifacts and sometimes we’re talking about real things intended by (using *intend* phenomenologically) cognitive acts. In practice, it can be hard to disentangle the cerebral and (by discursive intent and cognitive intention) extramental artifacts as referential patterns, or quantification domains for logically-structured units that arise in the course of argumentation (like  $x \triangleright y$ ).

To clarify these points, consider for a moment the famous Putnam Twin Earth discussion. As the scenario is described, Twin Earth harbors an XYZ substance functionally identical (for all purposes relevant to Twin Earthers) but compositionally different from  $H_2O$ . We can then entertain questions about whether Twin Earthers’ water-concept — which apparently refers to a different Natural Kind — is the same as our water-concept.

This setup makes several assumptions of its own. First,

it assumes that we have a canonical water-concept, and that it (either essentially or incidentally) refers to a Natural Kind which is the substance  $H_2O$ . This is a simplifying assumption in multiple respects; one of which is that  $H_2O$  itself encompasses several different chemically distinguishable substances (if we consider various forms of heavy water). Second, even our everyday water-concept is divided across different contexts: we would probably call both ocean water and tap water in a bucket a *bucket of water*, but we probably wouldn’t call ocean water in a glass a *glass of water*. So our “water” is probably a fusion of several different concepts, the stuff in oceans and saline lakes plus the stuff in freshwater lakes and rivers (including the potable stuff that by aqualogical engineering is delivered to our taps).

The water in some saline lakes is actually much less “pure” than blood plasma in a hospital, yet we are not inclined to call plasma “water”. On the other hand, exceptionally pure water — distilled water — is not even a prototypical kind of water; that’s why it needs a special concept. So any trivial equation between *water* and  $H_2O$  is problematic.

Meanwhile, the Twin Earth discussion is also noncommittal about how XYZ is supposed to differ from  $H_2O$ . We can imagine the XYZ components as very similar to Hydrogen and Oxygen — for instance, imagine XYZ as a relabeling of DHO, or “semi-heavy” water with one Deuterium atom. I don’t think we should have trouble as accepting that XYZ is then just another kind of water (like heavy and semi-heavy water). Or perhaps Twin Earth has some new subatomic particle that can clink to Hydrogen and make it ‘X’/, like the extra neutron that makes Hydrogen into Deuterium. Perhaps, that is, XYZ is functionally similar to water because its constituent parts are similar to earthly Hydrogen and Oxygen. As with heavy water, there is already a precedent for expanding our earthly water-concept to accommodate more complex chemical models of the water molecule.

I think the Twin Earth discussion only really has philosophical weight if we assume that XYZ components are *significantly* different than Hydrogen and Oxygen. Of course, we also have to assume that XYZ behaves enough like earthly water that these differences have no practical effect for Twin Earthers. Among other things, we have to assume that they are technologically more primitive than we are. After all, among the functional characteristics of

water for us is that we can derive Hydrogen and Oxygen from it; assuming XYZ are not just special kinds of Hydrogen and Oxygen, presumably this is not true of XYZ. This could easily bias our assessment of Twin Earthers' water-concept, because *we* know that there are functionally salient differences between XYZ and  $H_2O$ . Perhaps we can't help but imagine that eventually Twin Earthers' knowledge may eventually reach the point where the differences become relevant to us, just as many years passed before humanity learned about Hydrogen and Oxygen; then of course the assumption that their water is (relative to their own needs) functionally identical to ours (is we factor out the level of our practical engagement with water that surpasses their technological capabilities) breaks down.

Intuitively, no doubt, we consider both functional and physical/material criteria when circumscribing the extent and intent of concepts. If I take coffee at a vegan friend's house and ask for milk, it is not impolite for her to bring almond milk. That is, we are prepared to subsume "almond milk" under the concept "milk" in some contexts. However, we are reluctant to draw concepts based solely on functional resemblance, ignoring obvious compositional differences such as those between milk and almond milk. Surely this is due in part because compositional differences, while they may be irrelevant in *some* contexts, are usually relevant in other contexts. For instance, milk and almond milk are nutritionally biologically different.

Our functional and "compositional" criteria for concepts are not usually in tension, because usually there is enough correlation between the two kinds of differences that, over a broad set of contexts, they tend to reinforce each other. That is, there are contexts where functional resemblance seems to warrant conceptual unification even pace apparent compositional differences. There are other contexts where functional *differences* might supercede compositional similitude: think *aluminum foil* or *copper wire* compared to blocks of aluminum or copper (or, concepts like *statue* and *pot* are different from each other and from *clay*). Of course, in these last examples, material form — shape and arrangement — contributes to functionally different behavior, so we can include physical morphology as properties as brute composition (consider, though, the special value accorded to statues and objects d'art of reputable creators, rooted in properties of provenance that are orthogonal to both physical consti-

tution and material form). But, in any case, we also have linguistic and situational faculties to construct contextually "local" maps of concepts — how *in this context* concepts subsume or fit together in particular ways — without confusing these local pictures for global schema. With sufficient integration of many contexts, our intellectual and linguistic dispositions tend to (collectively as a language- or social-community) converge on a mapping of concept's boundaries and inclusion/subsumption that reflects both functional and physical/material criteria with neither set of criteria dominating the other.

Putnam's Twin Earth experiment invites us to imagine scenarios where the approximate synergy between functional and compositional criteria breaks down. In a hypothetical case where functional resemblance persists despite (significant) compositional differences, and one a wide scale across contexts, are we prepared to find conceptual unity (siding with the functional resemblance) or conceptual bifurcation (siding with the compositional difference)? At one level, this is hard to thematize straight-on, because the very construction of the case-study seems to undermine its requisite presuppositions. As I said, I think the thought-experiment is most thought-inducing even we assume significant enough difference between XYZ and  $H_2O$  as compositional substrata of water (or *twater*); so with chemical knowledge akin to ours, XYZ *doesn't* behave like water. So there are meaningful contexts where functional resemblance *does* break down. We have to assume however that these contexts are not relevant on Twin Earth because Twin Earthers don't have, say, equipment to separate water into Hydrogen and Oxygen (or, analogously, XYZ into X, Y, and Z).

I think what began as a discussion about *concepts* ends up really being a discussion about *contexts*. There are of course local contexts where non-standard concept maps are drawn (like milk/almond-milk). As I have argued, we are competent in juggling local and "global" conceptual maps ("maps" in the sense of how conceptual "territory" is partitioned), exercising a mixture of linguistic and situational understanding; a partition in rational communities sharing language, norms, and the pragmatics of everyday life. Therefore we mark nonstandard conceptual maps to as *local* to given contexts, whereas we also have a sense of concepts as intellectually global resources, which adjust for local nuances in predetermined ways. Our concept "*water*", for example, is presumably a federation of narrower concepts (notably saltwater and



freshwater) which we unify for both physical and functional reasons: while not pure, the primary substance in both cases is  $H_2O$  (which is actually third concept *subsumed by* water), motivating the unification, plus they have functional similarities in many (albeit not all) contexts. So, by panning out from local contexts into a globally trant-contxt conceptualization that is the best compromise between global generality and local specificity, a canonical concept merges which unifies other concepts but also has some internal integrity (i.e., the identification of water with  $H_2O$  grounds our conceptual norms in established science). The panning from local to global contexts is then a key semantic detail in establishing “canonical” versions of concepts.

I think the real force of Twin Earth is that it introduces two different possibilities to “panning to global context”. Global becomes relative: do we mean to generalize Twin Earth contexts only to those which are likely to be efficacious on Twin Earth itself? That is, should we assume that there will never be a global context affecting Twin Earthers’ conceptualization of water that would establish a ground for contrasting this concept with (earthly)  $H_2O$ ? In that case, an “internalist” might say that twater is the same concept as water, applying the maxim that conceptual boundaries are drawn to reflect the interplay between function and composition as we pan from local to global contexts. No matter how “high” we pan out, on this argument, we will never encounter a situation where compositional difference triggers a potential functional difference that was lurking behind local functional resemblance — analogous to how the biological difference between milk and almond milk is bound to arise in many contexts, whether or not it is locally relevant. But water/twater differs from milk/almond milk because (according to the setup) there is *no* context we can encounter when we “pan out” from local to global which makes the water/twater difference consequential.

Conversely, we can read the same scenario differently and propose that “Global” cannot be read in such a restricted sense. The global context or context-synthesis available to Twin Earthers — the level of abstraction beyond their local contexts — is not the *real* global context, since *over and above that*, by stipulation, we provide an encompassing global context of which Twin Earthers’ global context is just a part. For *us*, the difference between XYZ and  $H_2O$  is functional, not just compositional: the thought experiment stipulates (or should do

so) that the two substances are compositionally different enough that functional differences would arise in contexts that depend on splitting water into its constituent parts (if X, Y, and Z are just slight variations on Hydrogen and Oxygen, or chemically transform to Hydrogen and Oxygen, I think the discussion becomes moot; on par with water/heavy water, which doesn’t involve any extraterrestrial stories). What makes Twin Earth (stipulationally) unique is that *its* global context is not global enough from *our* vantage point.

We can certainly debate whether Twin Earth’s context still deserves to be called “global”. If it does, I think we end up with an Internalist theory, since we’re saying that criteria of globalness should be measured against the cognitive resources of a rational community: if there’s no context where compositional difference is practically relevant, then we may as well use functional criteria alone to establish concept partitioning. Conversely, if we say Twin Earth’s context is *not* authentically global, I think we are led to an Externalist theory. If two conceptual cores refer to different physical kinds, and we can range over every possible context (without regard for how cognition is grounded in the practical machinery of knowledge acquisition), then we can certainly say that compositionally different kinds represent referentially or extensionally different concepts.

But, I would argue, this is really a debate about Externalism and Internalism vis-à-vis *contexts*, and specifically about *globality* of context. When we pan out from local contexts, do we reach “global” when we reach the horizon of conceptualization context that is empirically possible for the relevant cognitive community? Are latent functional differences beyond this horizon factors in concept-identity? Internalists basically say that this horizon *is* the global context, so “global” is an attribute relative to the epistemological possibilities of any cognitive community. If there is no epistemologically possible world where a stipulated compositional difference becomes functional — i.e. there is no possible world where the knowledge of the difference can be reached from a community’s present epistemological state — then we can say that in the “epistemologically possible horizon” there is no context which “functionalizes” the compositional difference. Then there is no point in saying that such a horizon is *not* global. But if we are allowed to imagine that any epistemological being whatsoever can “look down upon” that horizon and see beyond it — in

other words, that globalness is an external to any one mind (or any one community’s cognitive resources) — we end up being “contextual externalists”, assuming that the only real “global” context is the maximal context which is metaphysically possible, where everything knowable is factored in, such as the functional consequences of any compositional differences. In either case, any notion of competing externalist and internalist intuitions about *concepts* appear to piggyback on corresponding intuitions about (global horizons of) *contexts*.

Perhaps my line of reasoning here feels like I am presupposing the answer: the true externalist claim is that concepts which bear (even unbeknownst to those who have the concepts) referential relations to compositionally different kinds are different concepts *irrespective of* whether functional differences can (even *potentially*) follow. However, I question whether compositional difference can be completely free of functional differences in *all contexts*; function and composition are at least somewhat interdependent, so it’s hard to imagine the complete absence of contexts where this interdependence does not imply compositional difference that propagate to functional difference. One can be an Externalist in saying that concepts can be differentiated on the basis of compositional differences even outside the epistemological horizons of concept-holders; but an Internalist in the *rationale* for this Externalism — that with a sufficiently wide horizon of contexts (and sufficiently epistemologically endowed cognitive agents for whom these contexts are enviroing) compositional differences eventually become functionally noticeable.

In addition — more relevant to mereology — someone might react to my talking about how we *draw* conceptual boundaries; that we *choose* whether and in what circumstances to unify (e.g.) saltwater and freshwater into a canonical water-concept. This style of argument may seem to be vaguely internalist from the get-go, because if we define conceptual architecture as a mental exercise it’s hard to give due credence to concepts’ potential extramental reality. There are, of course, extramental factors influencing (e.g.) our water-concept. Heavy water isotopes could potentially complicate a simplistic mapping of *water* to  $H_2O$  but as a matter of empirical fact ordinary  $H_2O$  is by far the most common form on  $H_2O$  on our planet. Ocean water and freshwater have chemical differences, but they share the property of being predominantly  $H_2O$ . Moreover, they are uni-

fied by meteorological cycles and behavioral similarities that supercede their chemical differences — ocean water cycles to freshwater as rain, and rivers drain into the ocean. The individuation of the concept *water* depends on these geological and meteorological systems as much as by “water =  $H_2O$ ” chemistry; this is perhaps why plasma (which is more watery than salt water) has no conceptual status as water.

These facts don’t imply that we have no choice in which conceptual distinctions to recognize, or ignore; the space of *potential* concepts, as defined by whatever scientific regimes apply (chemistry, biology, geology, and so forth) may be finer-grained than our everyday concepts. On the other hand, we also have cases like a statue whose value changed depending on whether it is believed to be the work of a great artist; social constructs like artistic or accidental provenance (consider a baseball that, by coincidence of being hit for a historic home run, becomes a collector’s item) can introduce conceptual distinctions sometime more granular than warranted by science. But while human concept-mapping may be coarser or (occasionally) finer-grained, we cannot rationally entertain conceptual systems that would deviate too radically from what is scientifically warranted. Perhaps we could say that behind every practical concept-system there is a most scientifically transparent grounding, a system where spurious social distinctions (like artistic provenance) are bracketed, but material differences (even ones that may seem parenthetical to even most scientific contexts, like water/heavy water) are represented. This would then be the extramental background upon which are human concepts are established, and each human concept-system can be treated as modifying this background by unifying highly granular concepts into more general canonical concepts on practical terms, or occasionally superimposing scientifically-spurious criteria to split background-concepts into finer shadings. Arguably, it is a valid philosophical exercise to uncover the scientific grounding behind the culturally relative, socially ephemeral play of conceptual mergers and bifurcations above it.

We can develop a philosophically non-trivial theory of concept “mapping” considering only cognitive attitudes — for example, the balance between functional and scientific considerations that shapes a community’s choices (sometimes deliberate, sometimes emergent) about when to unify or to divide concepts. The fact that these choices are made against an extramental nomic order can be

seen as a relevant detail but not a defining theme of the analysis. On the other hand, parallel to that, we can consider a theory of concepts that considers our relative autonomy in *choosing* concept-boundaries to be a complicating detail, and the core analysis to engage the extramental constraints. Neither theory is better than the other on any purely speculative ground that I can think of — except that I think we need to formulate a notion of “extramental constraints” properly.

To the degree that our theory of concepts recognizes a domain of extramental fact — as in the Externalist implication that the chemical form of water/twater *could* affect whether they are the same concept — we need to pair the analysis of concepts with analysis of the relevant empirical conditions and how our scientific understanding should guide the concept theory. After all, the very possibility that water/twater’s chemical differences could make them *different concepts* — that our decision whether or not to *treat* them as the same concept does not *make* them the same concept — implies that there is at least one system of concepts whose boundaries are drawn independent of actual human conceptual activity. This is not a *prima facie* untenable paradigm, but it needs to be supported with a sufficient level of scientific backing. Which is fine, except that some analyses may seem to state simplistic referential norms as proxies for a realistic scientific analysis.

For example — leaving aside our relative autonomy and centering on empirical givens that extramentally shape the possibilities of our water concept(s) — the relevant background is not merely that water is  $H_2O$ . In other words, even if we consider (or narrow analysis to the strata where) concepts are extramental, we can’t just say that the concept *water* is  $H_2O$  irrespective of cognitive choice (or that we could create our own human concepts adding or subtracting details but we’re doing so on a foundational concept-system where water is  $H_2O$ ). Surely even in most purely scientific contexts water is not *pure*  $H_2O$ . Nor can we use some sort of fuzzy logic — substances close enough to pure  $H_2O$  below some threshold concentration of other compounds should be deemed water — because then plasma becomes water and some salt water becomes non-water. Nor can we say that we choose what to consider water by combining things like concentration levels with functional/behavioral criteria, because the whole point of the exercise is to analyze concepts at a layer outside intellectual *choice*.

What we *can* say is that water-criteria lie at the intersection or multiple sciences: geology, limnology, meteorology, chemistry, biology, not to mention hydrology itself. Water (unlike plasma, say) falls as rain, erodes cliffs, and flows down rivers. What makes the extramental dimension of the water-concept viable is the synergy between these different sciences, or rather the natural phenomena they study. This extramentality can be philosophically *lusive*, perhaps, because a robust theory of water’s extramental nature would seem to rely on expert testimony, and therefore to some degree on expert opinion, which seems to conflict with our effort to isolate the water-concept not that we can *choose* to reify, but that we are *compelled* to reify by empirical conditions. But at least the hypothetical scientists are testifying under the guise of reporting extramental reality to the best of our ability, trying to cancel the autonomy (and therefore potential arbitrariness at least vis-à-vis scientific criteria) of human conceptualizing as much as possible.

To the degree that this scientific process is possible, we have at least a picture of how an “extramental” theory of the water-concept could proceed. But this theory is possible because we do not consider the problem as a matter of *reference* — i.e., of determining what the signifier “water” refers to, yielding accounts like *water refers to  $H_2O$* . Instead, we are trying to isolate the factors that make *water* a conceptually coherent phenomenon involved with different sorts of natural processes: water as a meteorological substance (a factor in weather-patterns), as a geological substance, and so forth.

I use the water-concept as a case-study because I think there are analogous trajectories in mereology: here also we can develop theories which either foreground or background extramental empirica. For example, a nail is part of a toe, and presumably the end of a nail which can be snapped off is (beforehand) part of the nail. But what is the status of the mereological sum of the body and the snapped-off nail-part? Is this an actual whole? Presumably in a nontrivial mereological system not every combination of parts is necessarily a whole; otherwise there would be no room for a separate class of whole — the mereology would be just a set theory with some universe of members. So we can ask whether body-plus-snipped-nail-part is a nontrivial whole. Certainly there is a period of time when it was evidently non-trivial; does having been *in the past* non-trivial make the *current* whole non-trivial? It would seem on that theory

that very possible mereological sum involving myself and every snipped-nail-part or every cut-hair I've lost over the years then becomes non-trivial, which seems extravagant.

On the other hand, is that snipped sum *is* trivial, it once was not trivial, so snipping the nail made some mereological sum go from being a non-trivial whole to being a “mere” trivial sum (call it *Triv*). We then need to articulate what is going on with such a change: *Triv* is no longer physically connected, it no longer functions as a natural unit, and so forth. I juxtapose this case with the water example because I think (even though one analysis belongs to mereology and another to semantics and the theory of concepts) similar tropes come into play. We can debate whether it is mental or extramental that *Triv* is trivial; whether the quality of triviality is our *judging* that some sums are not worth our attention, or *choosing* not to conceptualize them as wholes; or whether we are compelled to these choices by empirical fact. We can debate whether beneath our culturally regulated mereological attitudes there is an extramental mereology guiding our cognitively selective mereology, just as empirical facts compel some maps to be more realistic than others. The intersection of scientific, cognitive, and philosophical criteria in comparing *Triv* to the nontrivial pre- and post-snip toe resonate with the relevant criteria in the water/twater distinction.

## 2.1 So, Mereology and Externalism

Suppose I clip a nail, and call the now-trivial sum *Triv* as before, and the non-trivial rest of myself *Me*. We want to explain why *Triv* is trivial and *Me* is not. We also want to explain why my friends think of me as *Me* and not as *Triv* (even if they see the nail-part that belongs now to *Triv* but not to *Me*).

Analogously, suppose I designate the conceptual sum *A* to be a substance which is either water or orange juice (suppose those are the two choices available on an airplane). Presumably *A* is a trivial conceptual sum, because there is no scientifically reasonable (or even functionally reasonable) conceptual unification of water and orange juice (this is different than saltwater-and-freshwater, or H<sub>2</sub>O and Deuterium Dioxide, or even milk and almond milk, or for that matter water and twater).

But why is water-plus-orange-juice *trivial*?

To the extent that we are reasoning in an Internalist spirit, this is obvious: there is no enactive situation where *A* would be usefully treated as one concept and not the logical-or of two distinct concepts. There is no compelling scientific case for *A*. There is no functional similarity between water and orange juice strong enough to make them interchangeable like (sometimes) milk and almond milk. In short, there is no criterion among the functional, situational, and scientific considerations we bring to bear on concept-maps that would make us *want* to draw *A* as an integral conceptual region.

However, if we are working in an Externalist mindset, we want to say that *A* is not a natural *kind*; that there is no scientific basis for unifying water and orange juice. Presumably we have no trouble making this assumption because there does not seem to be any scientific discipline, any nexus of sciences' topics, that would project onto *A* with any conceptual identity. Water is a non-trivial concept because it is a meteorological, biological, geological, limnological substance; *A* is not any of those things. *A* is trivial because there is no “ology” that makes *A* an “-ological” substance.

Analogously, *Triv* is trivial as a mereological sum by a similar account of why *A* is trivial as a conceptual sum. There is no “ology” that makes *Triv* an “-ological” whole. By contrast *Me* is a biological whole (and arguably psychological, sociological, and so forth). So the key factor in mereological triviality or holistic integrity would seem to be the presence or absence of an “ology” that renders the mereological sum into a concordant “-ological” whole.

I'll refer to this hypothesis as the *multiscientific* model of mereology. On this model, what distinguishes wholes from trivial mereological sums is that wholes can be reasonably judged as entities in the terms of one or (in the general case) more than one science. A whole is non-trivial if it is an “-ological” whole where we can plug in one or more sciences to get the relevant “-ologies”. I'll say that such a whole is *multiscientific*. So we can develop mereological theories rooted in cognition, and consider only whether we perceive some putative sum as a whole according to individual or collective cognition. In that sense any mereological expression quantifies over cognitive things: an  $x \triangleright y$  means that *x* and *y* are *my conception of* or *my cognition of* something; and in this



conception I perceive (feel, sense, whatever)  $y$  to be part of (included in, circumscribed by)  $x$ . If we want to get outside the intramental boundary of this way of talking, I think, our alternative to *cognitive* wholes is *multiscientific* wholes: this is the kind of mereological Externalism which I think is philosophically consistent.

This partition of the philosophical options — cognitive vs. multiscientific wholeness — may seem reasonable, but it imposes a discipline on philosophical discussions that can seem to complicate our intuitive discussions about mereology. For example, if I perceive a statue preserving its identity after losing a small piece, I don't think of the mereological integrity of the now slightly-smaller statue (but not the statue plus the broken-off piece) as merely a conceptual choice, or an artifact of cognitive framing: that the piece *looks* detached from the whole, but if the gestalt were a little perceptually different — say, the broken-off piece were resting on the statue — I could go back to thinking of the larger sum as the relevant whole. I feel that my cognitive-mereological attitudes are constrained by physical conditions and situations.

So we probably have a sense — generalizing from “my” impressions just noted — that, while mereology has an important cognitive dimension, a complete theory of mereology has to address how we experience mereology in the cognitive realm as constrained by some extramental mereological order. That would seem to take us, in the case of a sculpture/clay example, to look at the sculpture not just through the lens of its cognitive stature — not just as a cognitive (viz., cultural, aesthetic, intellectual) artifact — but as a material object whose nature is subject to extramental norms. It is qua material thing — qua lump of clay, say — that the sculpture carries out an extramental existence that make some mereological formulae concerning the statue more permissible than others, in a way we cannot fully command by the relative autonomy of our drawing conceptual boundaries. There is, in short, something extramental about the statue and therefore something subject to norms outside the play of cognitive frames. So we get discussions about the statue as lump-of-clay as well as archaeological artifact, or the lump as a proper part of the statue, or both lump and statue as proper parts of some social-physical hybrid object. However the argument runs, the “lump of clay” motif seems to serve as proxy for the extramental self-sufficiency of the statue.

Elsewhere we may have language about objects as collections of molecules or atoms, or “material extents”, or some other way to signify extra-mentality. That is, to discuss the extramental dimension of mereology we have to signify the extramental dimension of cognized *things*, and this seems to involve some language to the effect that “extramental” means *physical* according to a particular image of physicality: extension in space(time), composition of smaller physical parts, possessing geometric shape, and so forth. This general language of physicality — more a connected group of philosophical tropes perhaps than an explicit theory — is different in tenor than the formulation I have given here: namely, *multiscientific*. In its extramentality the sculpture is not (at least in the crucial details) a “lump of clay” or a collection of atoms or any other proxy designation of “physicalness”. Rather the sculpture is an archaeological object, perhaps also mineralogical, morphological, and so forth — its own “ologies” — and the mereological controversies about the sculpture's essence or nontriviality are really questions about what it means to be (say) an archaeological whole; a whole qua archaeological object.

I think questions about “multiscientific” stature or integralness sometimes get camouflaged by referential or quantificational issues. Next section I will clarify what I have in mind here and what I think the consequences.

## 2.2 Multiscientific Objects

A statue and the small piece that has fallen off of it is at best a “trivial” mereological sum because it is not a “meaningful” whole — it is not an archaeological object, say, while the statue now missing the piece *is* an archaeological whole. Similarly, *Me* after a nail-clip is a biological object, while *Triv* (*Me* plus the piece of nail) is not a biological whole.

Also, the (current) statue is additionally a mineralogical whole in the sense that clay mineralogy affects its properties (e.g., its fragility). The sum of the statue and the piece is probably not an analogous mineralogical whole (it is not a single clay object). Parallelwise, *Triv* is certainly not a *psychological* object as *Me* is; I don't feel sensations from the snapped-nail part, or see it as myself, and so forth.

A clear case for mereological integrity or nontriviality seems to be the convergence of multiple sciences: the sculpture is *archaeological* and *minerological*; *Me* is *biological* and *psychological*. The interesting or bonafide wholes here integrate properties of different sciences, or different ontological registers. So *Triv* may instantiate biological properties (like, even the snipped-nail-part is composed of cells) and the statue-plus-piece instantiates minerological properties (even the piece is composed of clay). But our attention — both literal perceptual attention and figurative philosophical attention — is drawn to the objects that tie between several ontological registers (which is presumably a precondition of objects lying in the circle of our life world and horizon of cognitive engagement).

Given the recurring interest in sculpture/clay examples, I'll focus on this case: the sculpture (unlike, say, the piece that has fallen off, alone or in combination with the larger whole) is actually an integral archaeological object (plus minerological, physical, aesthetic, and commercial). As such it instantiates different kinds of properties: archaeological (created during a particular historical era), minerological (composed of clay), physical (weight, dimensions such as maximum width, breadth, and height), aesthetic (artistic value and provenance), commercial (having a monetary value for insurance purposes; contributing to the traffic and revenue of the museum which houses it). Note that some of these aspects are interdependent: e.g., objects cannot be archaeological and minerological without being physical.

I will assume for the moment that archaeological, minerological, commercial objects, etc., exist: i.e., that it is unproblematic to take for granted that our world contains an arsenal of objects in numerous different ontological registers. This ontology gives rise to a kind of typed quantification: when asserting properties about an object, we can identify the register (or one of the registers) where the object belongs. In effect we say things like *there exists an archaeological object such that...* or *there exists an minerological object such that....* Let's say that for a register  $\mathcal{R}$  we can have  $\mathcal{R}$ -quantifiers that can iterate over things which have an  $\mathcal{R}$ -aspect. Thus an archaeological object is one that can be in the domain of an archaeological-quantifier.

My term “ $\mathcal{R}$ -quantifier” is at least distantly related to Olav K. Wiegand's “ $R$ -structured wholes” [4, p. 185]

(though the “ $R$ ” comes from different places). For Wiegand wholeness is figured relative to a “gestalt contexture” which primes identification of a whole as a coherent individual unifying its parts. Meanwhile, differences in “contexture” block the kind of unwelcome transitivity that suggests a violinist's arm might be part of an orchestra. Wiegand initially defines “ $R$ -structure” in formal terms but he soon thereafter, referencing Husserl, implies that contexture can be read as a *phenomenological* context, the cognitive/enactive considerations that dispose me to attend to a whole within its surroundings. Referring back to my discussion in Section 2 (including my reference to an earlier essay of Wiegand's), phenomenological context disposes us to treat some discontinuities as mereologically significant; wholes are “ $R$ -structured” insofar as their configurational integrity is evinced vis-à-vis a particular mode or domain of unification. Physical objects are unified in terms of spatial continuity and mechanical compactness in motion; engineered artifacts by functional unity; living things by biological autopoiesis; and so forth. When examined through concrete examples, I think the criteria of gestalt contexture naturally leads to the idea that “ $R$ -structured” wholes are wholes according to some ontological *register*.

Objects in multiple registers — each of which can be called a “facet” — are in the domain of different sorts of quantifiers, or of combined quantifiers. If  $\mathcal{R}$  and  $\mathcal{R}'$  are registers, then an  $\mathcal{R}/\mathcal{R}'$ -quantifier iterates over things that have a facet as  $\mathcal{R}$  and also a facet as  $\mathcal{R}'$ . Inter-register dependence means that some quantifiers are necessarily mixed: an archaeological-quantifier is necessarily actually an  $\mathcal{R}/\mathcal{R}'$ -quantifier where  $\mathcal{R}$  is archaeology and  $\mathcal{R}'$  is physicality.

I believe that a notion of  $\mathcal{R}$ -quantifiers is a natural correlate of what I am calling “multiscientific mereology”. Indeed, perhaps we should entrench the notion as “ $\mathcal{R}$ -quantifier mereology”. On this account, wholes intrinsically are wholes in one or multiple registers: there are biological wholes, archaeological wholes, and so forth. Any partness fact or assertion contains an embedded  $\mathcal{R}$ -quantifier: that a statue contains a horse, say, means that the statue *qua archaeological object* includes a depiction of a horse. Thus in any  $x \triangleright y$  (at least if  $x$  is not a trivial mereological sum, assuming our mereological system can even represent formulae involving trivial sums) there is an implicit  $\mathcal{R}$  from which  $x$  is drawn via an implicit  $\mathcal{R}$ -quantifier (and the  $\mathcal{R}$  may actually be a

mix of registers).

I'll say that an  $\mathcal{R}$ -quantifier mereology is one where *any* whole that can be part of  $x \triangleright y$  formulae has an implicit  $\mathcal{R}$ -quantifier, and therefore has an  $\mathcal{R}$ -facet for some  $\mathcal{R}$  (or a mixture of  $\mathcal{R}$ -facets). On a multidisciplinary account, most or all cognitively significant  $\mathcal{R}$ -quantifiers actually involve a mixture of facets. The universe of quantification is therefore partitioned into different regions because quantification itself is spread over different  $\mathcal{R}$ s (and the granularity increases because each mixture of  $\mathcal{R}$ s gives rise to a different sort of quantifier).

### 2.2.1 $\mathcal{R}$ -Quantification and Non-Antisymmetry

Adding  $\mathcal{R}$ -quantification casts a different perspective on discussions of NAM, such as Cotnoir's. He points out that statue/clay examples can potentially lead to symmetric parthood: "the clay is an improper part of the statue and the statue is an improper part of the clay" [?, p. 401] (I'll add, statue and clay need not be metaphysically identical). My account would express the analogous situation via "mixed quantifiers": the  $\mathcal{R}$ -quantifier which can select the statue is actually a mixture of an archaeological-quantifier and a minerological-quantifier. Thus by the very formulation of any mereological relation wherein the statue is the whole (the left-hand side of  $x \triangleright y$ ), the statue has archaeological and minerological facets, because it falls under the scope of a quantifier which *requires* those facets.

On the face of it, then, the relationship of facets to one another is not itself mereological (even allowing for *improper* parthood). How could we say that the clay-facet, for example, is a part of the sculpture-facet? That is, in a archaeological/minerological-quantifier, can the minerological facet be "part of" the archaeological facet? I think a reasonable  $\mathcal{R}$ -quantifier mereology could block that kind of formula because there is no apparent  $\mathcal{R}$ -quantifier that would come into play to express it. After all, the sculpture belongs to a domain of quantification which intrinsically mixes (at last) two facets; to formulate parthood *between* those facets would seem to require switching to a different domain of quantification.

It is reasonable, perhaps, that a mixed-domain quantification can be combined with quantification involving the integrated facets in isolation. So if I say that the sculpture is an archaeological-and-minerological object,

perhaps I should allow the sculpture's archaeological facet in isolation to be included in a archaeological-quantification domain which is not limited to minerological objects (not every sculpture, for instance — consider ice-sculptures — is executed in a minerological medium).

Certainly we need some semantics for migrating between different  $\mathcal{R}$ -quantification domains, because we do have reasonable conceptualizations like *this ice sculpture replicates that clay sculpture* or *this photograph shows that clay sculpture*. However, the "rules" for such "cross- $\mathcal{R}$ " domain-switching may be relatively restricted; we are not forced to theorize mereological systems where the component  $\mathcal{R}$ s in a mixed-register domain can be generically factored out to their own quantification-domains.

Moreover, this restriction captures why  $\mathcal{R}$ -quantification produces a more regulated mereological structure, because there are limited "parthood" relations between  $\mathcal{R}$ s in a mixture themselves. In an  $\mathcal{R}/\mathcal{R}'$ -quantification such as archaeological-minerological, it might be said that the specified domains (archaeological and minerological) are "parts" of the mixed  $\mathcal{R}$  which governs the quantification accessing the sculpture. In other words, the quantifier applicable to the sculpture seems like a "quantificational sum" of the archaeological and minerological domains. Analogously, the sculpture seems like a "conceptual sum" of aesthetic and physical properties. However, even though "summation" invites notions of membership and inclusion, we do not automatically proceed from conceptual sums to mereological sums: we can consider the conceptual sum of milk and almond milk without treating a glass of milk as a token of a milk/almond-milk disjunctive-or'ed substance, wherein milk-related-properties would be parts of hypothetical milk-or-almond-milk related properties. Analogously, even if we accept the language of an  $\mathcal{R}$ -quantifier as a "quantificational sum" of two finer domains, we can deny that this summation translates to *mereological* relations between facets in the finer and the mixed domains.

Suppose we *could* formulate that both clay and sculpture are parts of some enveloping whole which has both sculptural and clay-like aspects. With  $\mathcal{R}$ -quantification, this would require a distinct domain of quantification which I have not yet pondered — I'll call it  $\mathcal{X}$ . Of course, the actual domain for the sculpture combines (at least)

archaeological and minerological, but this mixture is not  $\mathcal{X}$ , because we want the various *facets* to be *part of* the  $\mathcal{X}$ -object in the internal vocabulary of a mereology. I have allowed that  $\mathcal{R}$ -mixtures are in some sense “quantificational sums” but blocked a concurrent mereological summation: we can’t necessarily take the *mereological sum* of an archaeological-facet and a minerological-facet even (or especially) if they are facets of “the same object” (a unit of iteration under one quantifier). Someone might find that restriction arbitrary.

In particular, someone might argue that domains of quantification should be arbitrarily generalizable. For example, archaeological, minerological, biological, and psychological objects are all among the things that exist, so we seem invited to quantify over “things that exist”, as a generalized domain abstracting from finer  $\mathcal{R}$ -domains. So quantifying over this general domain — say,  $\mathcal{X}$ -quantification — models a different semantic pattern than  $\mathcal{R}$ -quantification. Whereas  $\mathcal{R}$ -quantification reflects expressions like *there is a sculpture such that...*,  $\mathcal{X}$ -quantification corresponds to the more symbolic logical formulation like *there exists  $\underline{x}$  such that...*

It might seem as if  $\mathcal{X}$ -quantification can be logically converted to  $\mathcal{R}$ -quantification by, in effect, treating the  $\mathcal{R}$ -quantifier as a predicate under the scope of the (purported)  $\mathcal{X}$ -quantifier. So from *there is a sculpture that...* we can get something like *there exists an  $\underline{x}$  such that,  $\underline{x}$  is a sculpture and...* And also like *there exists an  $\underline{x}$  such that,  $\underline{x}$  is a molded expanse of clay and...* — in other words, we are logically lifting an  $\underline{x}$  from the sculpture/clay and transcribing sculpture-ness and clay-ness as predicates on this  $\underline{x}$ .

From that point we can see how mereological controversies arise, because now  $\underline{x}$  is logically neither sculpture nor clay but some kind of generic token of thing-ness of which sculpture-ness and clay-ness can be preicated. That is,  $\underline{x}$  is an  $\mathcal{X}$ -object, under the domain of a quantifier without any ontological structure or specificity. If any  $\mathcal{X}$ -object can be a whole, then we have mereological relations outside the regulations of  $\mathcal{R}$ -quantification. For instance, we have a stated theory of why *Triv* is trivial; because there is no  $\mathcal{R}$ -quantifier, for any  $\mathcal{R}$ , which covers *Triv* (or at last there is no multiscientific mixture of  $\mathcal{R}$ s that would render *Triv* a “multiscientific” whole). However, it seems reasonable to allow that *Triv* is an  $\mathcal{X}$ -object, because by definition  $\mathcal{X}$ -quantification

does not impose ontological structure on its domain of iteration.

I contend, then, that problems about trivial wholes are interrelated with problems about  $\mathcal{X}$ -quantification. If there is only  $\mathcal{R}$ -quantification, then there are only nontrivial wholes, because there are no quantifiers whose domain encompasses trivial mereological sums.

Whether or not this setup seems intuitive may depend on our beliefs about quantification. If one is already committed to the idea that  $\mathcal{X}$ -quantification is possible — or that philosophers at least have to give a good reason for excluding this generic quantification — then my reasoning could well be deemed circular. I am presupposing the mereological restrictions that should fall out of a theory, by restriction how quantification works; thus I avoid problematic objects by eliminating the quantifiers that could quantify over them, but this may seem like eliminating problems by fiat.

We can consider, then, arguments for and against unrestricted or  $\mathcal{X}$ -quantification. On the one hand, formal logic certainly accepts leaving quantification open-ended. It’s not clear what we should read into that philosophically — after all, logic is a study of abstract domains, rather than an empirical universe of statues and toes and such.<sup>5</sup> Another philosophical case for  $\mathcal{X}$ -quantification is that recognizing only  $\mathcal{R}$ -quantification *without abstracting to* an unrestricted “ $\mathcal{X}$ -domain” would seem to leave the  $\mathcal{R}$ -domains bearing a lot of weight. It is one thing to quantify over a set of objects bearing biological properties; it is another to establish a rule of quantification presupposing that there is some essential register in existence containing all and only biological objects.

We moderns presumably reject thoughts of an *elan vital* or any other metaphysical essence that suffuses biological wholes to make them a distinct order of reality. Even more so should we be skeptical of models implying a minerological *elan*, a psychological *elan*, and so forth. Rather we assume that there is an underlying physical

<sup>5</sup>Also, even in mathematics we have type-theoretic paradigms which are consistent with type-restricted quantification; no mathematical object is recognized which cannot be given a corresponding type (which excludes unrestricted set-formation). So quantification is limited to quantification over “fibers” induced by the value-to-type mapping. With a sufficiently expressive type theory, the category of types becomes effectively isomorphic to the collection of domains of quantification. In other words, universal quantification is not universally accepted even in logic and mathematics. Granted, though, type-restricted quantification is not the predominant logical construct.



reality — atoms and molecules and a small set of force fields — and that objects of physical composition take on biological or minerological properties, which *makes* them biological or minerological objects. But that sounds as if we envision a baseline of  $\mathcal{X}$ -quantification wherein biological and minerological domains, and so forth, are somehow “emergent” or higher-scale properties. Foreclosing  $\mathcal{X}$ -quantification may appear to leave us with inexplicable  $\mathcal{R}$ -domains, which have to be just accepted as metaphysical primitives, as if they are primordial “elans”. I’ll call this the “*elan*” problem.

I claim, though, that from the arguing-against-unrestricted-quantification side, the “*elan* problem” can be turned on its head. As stated, there is no *elan vital* which makes biological objects biological, or minerological objects mineral. Instead, at sufficiently small scales, all matter reveals itself as having the same minimal constituents (subatomic particles plus Standard Model forces, say). That is, we may believe we can avoid the *elan* problem by elevating a kind of subatomic-physics talk into a “prestige dialect”, an ontologically neutral way of talking about all physical things without mysteriously essentializing biological or any other higher-scale properties. In this vein, unrestricted quantification — at least with respect to physical reality — is essentially quantification over that subatomic domain.

But such an inscription of  $\mathcal{X}$ -quantification has its own blind-spots. The objects of the subatomic domain are not metaphysical essences outside the play of science; they are posits of a specific science, subatomic (and quantum) physics. Objects do not just “contain” quarks and electrons like bricks in a wall; the actual subatomic realm is apparently complexly structured by forces and quantum indeterminacy, where particles have some approximate and dispersive existence. When we knock on wood we’re not really encountering protons; there is a system of microphysical charges that creates the illusion of hardness.

So we can’t really capture the idea of unstructured materiality — a minimal unit of physical presence which lies conceptually outside any *elan*; which is too small to be biological, say — we can’t capture such an idea of an *ur*-particle by plugging in the latest scientific theories of the smallest constituents of the universe. The quantum model is subject to refinement, after all. And philosophers who talk of objects as “collections of molecules”,

say, are not really proposing that molecular chemistry is an operational part of their theory. Rather, this language tends to use talk of atoms and molecules — or, say, of material extension — as proxies for a picture of “ur-particles”, of an imagined smallest unit of physical nature.

These ur-particles are essentially a philosophical fiction; they don’t align with elementary particles recognized by physics, though we can plug various hypotheses (like string theory) to give them scientific detail. But it’s not like philosophers present ur-particles as an empirical claim and look for science to substantiate it. Rather, ur-particles are a way of talking: having (on metaphysical as much as empirical grounds) rejected the idea of *elans*, and also *accepted* the idea of physical things being (down to some tiny scale) made of smaller physical things, the conceptual merger of these ideas creates the notion of an “ur-particle”: something so small as to be indivisible and therefore unstructured, and something lacking any *elan* that would make it biological, minerological, or scientific in any sense of being an object *of* a science.

Larger objects then become figured as mereological sums of ur-particles, which in turn raises problems due to the dynamic nature of macroscopic objects (and their sheer differences in scale to “ur-particles”). Even leaving aside the point that ur-particles do not actually exist — they are conceptual proxies for a messier subatomic reality — we cannot simplistically equate biological objects (or objects of any other macroscopic register) with mereological sums of ur-particles. Through breathing, discharge of skin cells, the dynamics of our microbiome, and so forth, the boundary between our own molecules and the external world is (at sufficiently magnified scales) fluid and indeterminate. Indeed, consider any colored object: having color means that it absorbs some light from the environment, and emits heat; such perpetual exchange of photons prohibits any direct enumeration (even in principle) of the ur-particles that would “make up” an object.

Meanwhile, we still have to account for why (only) some mereological sums of ur-particles are non-trivial. Presumably sufficiently scattered sums are trivial, but we can’t assume any physically connected mass of ur-particles is non-trivial (even if we accept the approximation that solid objects are *really* solid). A glass atop a table forms a connected but, we’d say, physically trivial

sum. A metal chain has possible states where it is not connected (one of the rings not touching any others) but it is, even while in those states, non-trivial. A bucket of sand or glass of water have states where physical forces will destructure them (the water or sand spill out), but we may want to consider the glass-and-water or bucket-and-sand to be nontrivial sums. In short, the criterion of triviality — even just in the register of everyday objects — requires explicit consideration of physical forces promoting an object’s integrity.

So, while it may be appealing to adopt a discourse of “ur-particles” to encapsulate our atomistic and non-*elan* intuitions, this conceptual model does not really address concerns like the “*elan* problem” and it raises a host of new problems. Quantification over non-trivial mereological sums of ur-particles does not seem like a philosophical improvement over  $\mathcal{R}$ -quantification, even if this seems to lead us back to the question of how to think through  $\mathcal{R}$ -quantifiers without conjuring a realm of  $\mathcal{R}$ -elans.

Ultimately, I think the answer to this question lies not in philosophy alone but in the interface between philosophy and science(s). The question of what forces and autopoietic regulation allows  $\mathcal{R}$ -objects to sustain as non-trivial wholes is complex, and depends on laws and dynamics investigated by  $\mathcal{R}$ -sciences themselves. We do not have a succinct logical statement of what makes  $\mathcal{R}$ -quantified mereological wholes non-trivial. This does not mean we fall back on spooky *elans*; but rather that, for a given  $\mathcal{R}$ -quantifier, we can form (with sufficient scientific input) a *theory* of non-triviality in the  $\mathcal{R}$ -domain. Biologists, for example, do not rely on unexamined *elans* to demarcate the realm of living things; this is an scientific question, in light of astrobiological possibilities. So biology itself takes on the question of under which circumstances an object is a *biological* object. Similarly, psychologists can debate the proper demarcation of psychological objects (is a brain-dead patient still a psychological object, say); archaeologists the demarcation of archaeological objects, and so forth.

Of course, these scientists are not trying to distinguish trivial from non-trivial mereological sums, but sciences do include theories of their proper reach; that is, we can argue philosophically equipped with an inventory of theories drawn from multiple sciences, of what makes a biological object biological, etc. While a *prima facie*

claim that sums with no  $\mathcal{R}$ -quantifier just are trivial, a restriction that seems question-begging, seems at best philosophically incomplete, we can complete the picture by introducing constitutive theories developed by sciences to identify their ontic extent. For any  $\mathcal{R}$ -domain, we can assume that there is a scientific theory of  $\mathcal{R}$ -ness; of the conditions where objects are  $\mathcal{R}$ -objects. These theories may be provisional *a posteriori*, but they can complement an underlying mereological theory: a whole is non-trivial if it lies in the domain of an  $\mathcal{R}$ -quantifier, and we can philosophically defer to a suite of empirical theories delimiting  $\mathcal{R}$ -objects for different  $\mathcal{R}$ ’s; the corresponding “ $\mathcal{R}$ -theories” may lie outside philosophy proper, but they are not intellectually vacant *elans*.

### 2.2.2 $\mathcal{R}$ -Quantification and Cognitive Frames

My analysis of  $\mathcal{R}$ -quantification has been driven by a desire to theorize “Extramental” mereology. For those who intuit that mereological relations are really modeling how we cognitively (perceptually, conceptually, etc.) engage with objects, the metaphysical problems of triviality may never arise. Non-trivial wholes in *cognitive* mereology are non-trivial because we have innate perceptual tendencies and learned conceptualizations that dispose us to treat some wholes as integral. We can study different aspects of our whole- (or part-) forming dispositions: basic perceptual continuity, force-dynamic intuition, situational undersanding, and so forth. Indeed, a theory of mereological intuition will likely overlap with an outline of Cognitive Grammar. But in any case the cognitive criteria for non-triviality lie in our cognitive acts of synthesis and analysis, at the intersection of perceptual, situational, conceptual, and enactive criteria.

Quantification becomes an issue when we want to get beyond mereology within cognitive frames and define mereological relations as extramental features compelling or at least influencing those cognitive manifestations. Without the ambient cognitive discrimination which marks some cognized phenomena as wholes, we then have to introduce new criteria such that non-trivial mereological sums can be posited in reality so that they can be notated in formulae like  $x \triangleright y$ . We then face the question of how to properly restrict quantification for terms like the  $x$  in  $x \triangleright y$  so that only empirically/scientifically meaningful, naturally integral, non-trivial wholes are modeled by  $x \triangleright y$ -like formulae.

In short, I propose  $\mathcal{R}$ -quantification not as the starting point of a mereological theory but in conjunction with *cognitive* mereology: we want to pass beyond a theory wherein mereological relations are *only* cognitive without thereby ending up in vague ur-particle-like talk wherein everything extra-mental is a mereological sum of some hypothesized minimal quantum of physical reality. The problem with *unrestricted* quantification is that it seems to construct a simplistic register for the extra-mental: everything which is outside the mind is a sum of “stuff” and therefore its physical (i.e., extramental) reality is ontologically carried by all the stuff (e.g., ur-particles) composing it.

In effect, when passing beyond the cognitive register to the extramental, a mereological argument could reach for a convenient designation of physicality for “anything which is not mental”; e.g., ur-particles. Then ur-particles, or “being composed of ur-particles”, become conceptual proxy for extramentality. Since we then have all the ingredients to build ur-particle sums, it seems as if the very gesture of siting mereology outside cognition compels us toward unrestricted quantification. If the essence of being extramental is being composed of ur-particles, then as soon as our mereological universe ventures outside the mind it picks up ur-particles and therefore the prospect of mereological sums of ur-particles. Granted, we can introduce notions like *biological properties* to corral the interesting sums from the trivial ones, but this is a superstructure layered on to a mereological system which allows a lot of trivial “ $\mathcal{X}$ -quantified” wholes.

The problem in this intellectual trajectory is the ordinary intuition that extramentality automatically takes into a territory of ur-particles, as if we have to philosophically construct some minimal quantum of the extramental world. We don’t need to have this intuition. We can instead say that extramental objects are  $\mathcal{R}$ -quantified for various  $\mathcal{R}$ s: outside the mind are biological, minerological, archaeological objects, and so forth. These are extramental registers because our cognition does not *make* biological objects biological, for example, although there may be a synergy between our inclination to perceive objects framed in particular ways and their biological (etc.) status. Extramentality is not summarily dispatched by a fictional ur-particle, a conceptual proxy for “minimal unit of extramental existence”, but rather through a multiscientific patchwork of ontological registers which permit extramental quantification to be restricted. For

sake of discussion, I’ll call this multiscientific patchwork the  $\mathcal{R}$ -patchwork.

I’d like to argue that the  $\mathcal{R}$ -patchwork functions a lot like cognitive frames. The  $\mathcal{R}$ -patchwork contributes extramentally as cognitive frames contribute mentally: both project order onto existent/cognized phenomena; both select a realm of non-trivial wholes; both buttress a scaffolding of restricted quantification with interrelations and mixing between quantification domains. The structure of cognitive frames, at least from the vantage point of mereological analysis, seems architecturally resonant with the integrated structure of sciences, especially in their “patchwork” interconnections where sciences unify into our empirical picture of the world.

Perhaps this resonance is not surprising: science is shaped by cognition even if it concerns itself with extramental reality. Our cognitive patterns and propensities set the scientific agenda; the origins of biology, for instance, lie in our construal that living things behave in the world differently than inorganic matter. Scientific discipline guides us toward empirical models that can be tested apart from human beliefs and prejudices, but the structural outline of the sciences and their interrelationships still draws on the contrasts between different realms of existence as we cognize them.

In short, the extramental realm is not devoid of cognitive structure; it’s just that in our sciences of empirical reality we try to accept cognitive structure as a scaffolding to pass beyond, testing mental images (like scientific models) so they may remain as tools for formulating and communicating scientific theories but disproving models and impressions which are intellectually intuitive but end up being empirically misleading. Extramental reality is not essentially *non-cognitive*, which is not to say that our cognitive actions “create” extramental reality, but rather that our intellectual establishment of rational exercises which excavate the extramental from the envelop of cognition remains influenced by the structural order of the cognitive realm.

As such, bracketing that order — formulating a symbolic language wholly removed from cognitive structure — is not a prerequisite of extramentality. Quantifying outside the cognitive realm does not mean an unrestricted quantification with no intellectual structure, or within a quantification domain that never traces back at some level to cognitive attitudes. “Extramental” implies a

discipline correcting for cognitive prejudice, but not a radically non-cognitive world or a world (an ur-particle space with arbitrary composition, say) radically devoid of intellectual structure.



The upshot of my argument in this section was that the realm of mereological theory should be divided into *cognitive* and *multiscientific*. Criteria of mereological non-triviality are derived in the first case from *cognitive frames*, and in the second case from an “ $\mathcal{R}$ -patchwork”: a patchwork of scientific theories delimiting the extent of different scientific registers and the dynamics and autopoiesis sustaining the integrity of (in various domains) non-trivial wholes. Moreover, there is a structural resonance and interpenetration between cognitive frames and the “ $\mathcal{R}$ -patchwork”.

Whether from the perspective of cognitive frames or the “extramental” perspective of the  $\mathcal{R}$ -patchwork, mereological theories can be reasonably elaborated in parallel to theories of the cognitive/ontological status of wholes participating in  $x \triangleright y$  style parthood assertions. Any formalization of parthood relations can assume that all wholes have some integral unity that can be described on cognitive or scientific grounds (or really multiscientific, because any macroscopic object has properties at the intersection of different sciences, like physics and biology and/or chemistry).

Of course, we can investigate mereologies as abstract, axiomatic logical systems. I contend however that such essentially mathematical theories have only limited applicability to philosophy. I’ll expand on this hypothesis in the next section.

### 3 Mereology and Models

Last section I voiced skepticism that abstract logical models (in contrast to scientifically-inspired theories) concerning mereology are of great philosophical value. I’ll defend this opinion in this section by adopting (for sake of argument) a model-theoretic perspective. So assume that when we have a non-contradictory logical system we can analyze its collection of possible models, where each model is a set with some additional structure (stipulated by the logical axioms and whatever further detailing they entail).

What, then, would be *models* of mereological theories in this logical sense? Imagine a collection of “voxels” — three-dimensional indivisible cubes with two states (*empty* or *full*). Two (orthogonally) adjacent voxels can be said to form a “block”, and in general a set of connected voxels to form a “connected block”, say. The system of connected blocks then forms a classical mereological model where proper parthood is voxel set inclusion. Let’s call this a “Voxel Mereology”.

Some picture like this may feel like a natural bridge between logical-axiomatic mereological theories and philosophical analysis. The reason is that a Voxel Mereology undeniably instantiates Classical Mereology and also fits our intuitive picture of physical reality. In our folk physics, we experience objects as resisting force and sustaining shape as if they were composed of hard little bricks. And we can match this intuitive picture to actual science by imagining voxels sized to the small units of quantifying possible, e.g., the Planck scale. Science suggests that there is indeed a scale of reality finer than which there is no possible scientific measurement, information, or discrimination.

But, as I argued last section vis-à-vis “ur-particles”, such quantum analogies are of limited value. Space itself at the Planck scale does not fit the ambient Euclidean topology that voxels presuppose; nor is there a precise quantum distinction between “matter” and “emptiness”. So we can’t just assume our voxel picture plugs in to quantum physics. Instead, Voxel Mereology is at best a bridge — it encapsulates how we *experience* physical reality while also offering a rough conceptual figuration of physical principles, like the idea of a smallest measurable unit of space. Rather than as an analogy to the quantum realm, Voxel Mereology is perhaps most realistic as describing in approximate detail the Emergent Properties of ordinary macroscopic physical objects. Compact solids do tend to behave as if composed of hard “bricks” on a scale fine enough to mold to their convex geometry.

Having said that, *parthood* within macroscopic physical objects does not really work like little lego pieces being lifted apart. Usually parthood reflects some functional or manufactured assemblage; the knob on a drawer, the cap on a bottle; the twig on a branch, etc. Parts can of course break off with no apparent functional integrity: if the bottom of a table leg splinters away, or a small



tree branch is blown apart from the tree, the surface where the detachment occurs will presumably evince a jagged, apparently random edge. Setting the “voxel” scale small enough can still model the geometry of the parts, but that seems besides the point. In reality, patterns of breaks are not completely random; they reflect stress points and accumulated forces acting upon the larger object, and the geometry of the detached part reflects the structures of these forces. Like the patterns formed by water spilling over a surface, complexity and semi-randomness still has some mathematically tractable background.

In short, a “voxel” model has limited usefulness even as an approximate picture of solid wholes. While connected voxel-blocks are not unrealistic discrete approximations of three-dimensional shapes, voxels are less appropriate for representing part-whole relationships: a model of parthood (in the context of solid physical objects) is better organized around functional or mechanical constructs. There are parts induced by the accumulative processes which form wholes, like twigs on a branch. There are also parts deriving from stressors, fault-tolerance, and other material qualities of solid substance in the sense of materials science, where lines of force and/or vulnerability can dispose pieces of a whole to become semi-detached, giving them some definitional autonomy relative to the whole. But properly modeling these material tendencies presumably involves mathematical formations like vector fields, where the voxel discretization is at best conceptually tangential. Even if connected voxel-blocks can model induced parts as well as wholes, it is not clear what physical reality such a model would capture; it is more like a predefined conceptual scheme imposed on a mostly unrelated physical situation.

So “Voxel Mereology”, as a straightforward model of an axiomatic Classical Mereology, has at least dubious merit as a model of physical states of affairs. We could try to refine the model in a more scientifically rigorous way — in the direction of mereotopology, for example, allowing  $n$ -dimensional generalizations of voxels and/or infinite subdivision. So we could have a system whose elements have varying dimensions (perhaps to a maximum of four, representing temporal change) and smaller-dimensional objects can be part of larger-dimensional ones. Moreover, any objects of non-zero dimension can potentially have proper parts; there are no “atoms”. This picture may be more appropriate for topological or differential-geometric

representations of surfaces and manifolds as they would apply to science. To have a useful notion of parthood we may want to restrict partonomic assertions to parts which are not too “bizarre” — perhaps to connected open or closed manifolds, excluding “fractal” shapes of fractional dimension, or infinitely scattered subsets. By way of naming a juxtaposition with “Voxel Mereology”, I’ll call this theory “Manifold Mereology”.

Certainly topological spaces can be a foundation for Classical Mereology models. As with voxels, though, we need to consider how applicable these models are to empirical phenomena. The topology of manifolds may be a better (e.g., non-discrete) approximation of material objects’ nature; for instance solids’ outer boundary is reasonably seen as a 2-manifold immersed in 3-space. There still is not a perfect isomorphism between material parts and submanifolds, however. Granted, to a reasonable approximation every recognizable part of an object has a corresponding submanifold. For sake of discussion, suppose we equate all wholes and parts to their 2-manifold surface (and assume parts are three-dimensional submanifolds of three-dimensional wholes). Assume the surface of a part either overlaps with the surface of a whole or else is entirely inside the whole (that is, assume we are using a version of the Region Connected Calculus). The system of parts in relation to wholes is then an example of a system of submanifolds in relation to manifolds. However, the system of manifolds has some properties that may not be desired in a mereological theory. For instance, for a submanifold properly contained in a larger manifold, a sufficiently small deformation of the smaller manifold yields a new manifold also contained in the larger.

The mereological equivalent to this property is that if  $x \supset y$ , there is an infinite space of topological deformations of  $y$  yielding  $y$ ’s that, without some extra criteria, will also be in  $x$ . We can of course stipulate that only some submanifolds correspond to actual parts, but then the formalization of submanifolds is only one dimension in the description of parthood. Whereas Voxel Mereology was a flawed model of (solid, material) objects because its discrete architecture was at best a rather arbitrary structure superimposed on physical forms, Manifold Mereology has limits of applicability for essentially the opposite reason: mathematical continuity allows for infinitesimal modifications of geometric forms in ways that have no physical meaning.

Both the Voxel and the Manifold pictures are only indirectly representations of “solid matter”; they are actually models of geometric extension, figured either discretely or continuously. The “voxel” as a binary empty-or-filled spatial region is an idealization; but physical surfaces as 2-manifolds is an idealization also. Physical solids do not have a crisp boundary where their interior ends and surroundings space begins, such that exterior surfaces can be mathematically embodied in 2-manifolds. This may be a reasonable approximation for some solids, just as the voxel “little brick” picture is sometimes mostly appropriate. But neither are physically oriented theories of how matter actually extends in and relates to surrounding space. Instead, they are mathematical formulations of extension per se, as geometric primitive. This may yield valuably mathematics — consider differential geometry as a theory of physical processes on objects’ surface, or even tools like NURBS (Non-Uniform Rational B-Spline, a feature of Computer-Aided Design and Computer Graphics) for computationally modeling solid geometry. But mathematical formalizations always have at best an indirect relation to physical phenomena.

This may suggest an intrinsic limit to any axiomatic mereology invoked as part of a philosophical treatment of parts and wholes: any *model* of such a system has to be interpreted as a model (to some approximation) of physical wholes. But the most natural mereological models are likely to be mathematical systems, like discrete geometries or topologies — perhaps with certain mereologically-inspired restrictions (e.g. distinguishing connected submanifolds from submanifolds in general) — or perhaps algebraic models, like lattices. In any case we then need a further account of the relation between the mathematical idealization and physical reality. So the mereological system is removed at two steps from physical phenomena, which means the philosophical analysis has to provide at least two “bridge” theories alongside the formal statement of the mereological system, to explain why the proposed axiomatization is a philosophical subject-matter.

The tenor of this line of analysis is not restricted to mereology; I would make similar arguments about the theory of concepts, say, or “truth-theoretic” semantics. The *potential* problem in any use of logical-axiomatic systems in a philosophical context is that models of the system will always be fundamentally abstract. At best

they can serve as simplifying and/or explanatory idealizations, so long as *we* in an exercise of philosophical reasoning can connect the model to the phenomenal explananda. But doing so rigorously requires one (or maybe several) “bridge theories”, which means that however well-conceived a system may be as an artifact of pure logic, its philosophical value can be muted for want of an adequate bridge theory. To motivate a claim that this effect can indeed apply to mereology, I will present analyses of related situations, as I see it, in theories of concepts and semantics, harkening back to my earlier overview of Formal Concept Analysis and then Cognitive Semantics.

### 3.0.3 Concepts and lexemes are not model-theoretic models

Because Formal Concept Analysis can be rigorously defined axiomatically, it certainly has abstract models; but we need to explain how models of that theory actually relate to conceptualization as a phenomenon of human intellect. Formal Concept Analysis engages a structural morphology of concepts not very different from the building blocks of a mereology; intensional “features” and extensional “examples” jointly characterize concepts, which can then be merged together or split apart. For example, merging the feature-set of two concepts, and then selecting all examples which have all (or a sufficient number of) the combined features, represents an “addition” between concepts (a variation is to consider examples with *common* features of two concepts: so a potential milk-plus-almond-milk concept would have as examples any liquid exhibiting the features milk and almond milk have in common).

Formal Concept Analysis may provide an interesting systematization of discriminative or evolutionary factors shaping how humans intuit and modify concepts. There are other formal concept models as well, such as prototype-based models, or the idea of mapping concepts to regions in qualitative spaces, featured in Conceptual Space Theory. These various theories can be combined; for example, Formal Concept Analysis can be extended with prototype criteria such that features and examples are weighted in terms of how essential they are to the relevant concept (as features) or how characteristic they are (as examples). The most representative prototypes of a concept are (on such a model) ones that strongly

reflect the features most indicative of the concept. These criteria of weights and prototypes can be adjoined to the feature/example matrix of Formal Concept Analysis, yielding an axiomatic system which may model, to some approximation, human conceptual activity.

When we start to compare these formal models with actual conceptual patterns, however — for instance in Cognitive Semantics or Cognitive Grammar — the models start to seem unnaturally simplistic and superimposed. It's not that structural analysis of cognitive representations is impossible; but rather that each structure tends to have its own *sui generis* rationale, and trying to trace cognitive gestalts to an axiomatic foundation starts to feel, at best, reductionistic.

Analogously, a “truth-theoretic” model of linguistic meaning — mapping sentences to logical formulae or to truth conditions — have intuitive appeal but also many apparent counter-examples. A full discussion is beyond the scope of this paper, but I'll give some of my favorite examples demonstrating why truth-theoretic readings are, in my opinion, at best incomplete:

- ▼ (19) Everyone sang along to two songs.
- ▼ (20) Everyone performed two songs.
- ▼ (21) New Yorkers live in one of five boroughs.
- ▼ (22) New Yorkers often gripe about long commutes.
- ▼ (23) New Yorkers reliably vote Democratic.
- ▼ (24) Student after student came out against the tuition hikes.
- ▼ (25) A critical mass of students came out against the tuition hikes.
- ▼ (26) Students' anger about tuition hikes may have reached a tipping point.

If a truth-theoretic model is accurate, it should be possible to capture the approximate signifying content of each of these sentences by expressing them in logical form; or at least show a systematic translation of surface linguistic structure to truth-making conditions. I want to argue that every of these sentences fail the test.

Start with (19). The most natural interpretation to my ear — i.e., I imagine the kind of scenario where (19) is most likely to be expressed — involves some sort of concert which included two particularly popular songs, that most people in the audience could sing along with. On that reading, there were *exactly two songs* and the sentence does not commit to *literally* everyone

singing them. Conversely, (20) strikes me as talking about musicians onstage rather than the audience, and suggesting that each performer had a two-song set; i.e., each performed two *different* songs.

So, I hear a wide-scope reading for “two songs” in (19) but a narrow scope in (20). What I want to emphasize is *why* we would make that judgment: there is nothing in the sentence itself that points toward a wider or narrower scope. Instead this follows from a pragmatic construal of the verbs involved: to *perform* seems to bind the verb to its subject more tightly than *sing along*. The situational model most appropriate for *perform* leans toward narrow scope, while the situational model for *sing along* leans toward wide scope.

In (21), a reasonable interpretation is that, according to the sentence, *all* New Yorkers live in one of five boroughs. Indeed, the territory of New York is precisely partitioned into five boroughs. On the other hand, (22) does not appear to commit to referencing literally *all* New Yorkers; meanwhile it also appears to use the phrase “New Yorkers” differently than (21), to mean generically people who work in New York or live in New York's metro area. In the case of (23) we hear “New Yorkers” narrowly as in (21), because voting relates a person to their actual place of residence (not where they work or their metro area). However, the implicit quantifier in (23) is *most*, not *all*; it does not fit our conceptual picture of “voting” to imagine one party winning *all* votes. Again, these contrasts are not produced by sentence or phrasal form; they are instead driven by lexical peculiarities of words like *borough*, *commute*, and *vote*.

In the last three sentences, we can similarly hear an implied quantifier to the effect that *many* students are complaining or unhappy about some tuition increases. But each sentence adds a shading on the generic form like *many students complained*: in (24) the speaker tries to connote the pervasiveness of students' anger by giving it a temporal figure. She suggests how a temporal recurrence of some phenomenon reinforces our sense of its extent; not only is it asserted that many students are unhappy, but that this fact has come to hear awareness multiple times. The verbiage used to describe unhappiness, also — *come out against* — carries an extra spatial or narrative dimension than a plainer alternative like “complained”. To *come out against* implies a public, maybe even activist display of anger. In (25), a similar figuration of *many*

imposes an interpretive attitude on the sentence: to refer to *many* as a *critical mass* implies not just magnitude, but *enough* magnitude to effectuate something: (25) might be said of a case where student protests forced a school to cancel planned hikes. And (26) is similarly implying that some threshold may be crossed, without indicating (at least within the sentence) what the speaker thinks might happen then.

These readings do not dispute that each sentence in (19)-(26) have a logical substance that could be modeled as quantified assertions, using the appropriate quantifiers in each case (*every* performer/audience member; *all* New Yorkers; *most* New Yorkers; *many* students). Lexical variation (like “New Yorker” meaning both a resident of the city and the metro area) has to be accounted for, as does scope variation (like an *everyone ... two songs* case, where “everyone” as “space builder” introduces both a global and a local space such that scopes must be resolved for the rest of the sentence). I grant, however, that an underlying predicate logic can be enriched to represent these added scope and lexical details. So I believe that, at least among the sentences I’ve analyzed, we *can* construct a logical representation which captures the intended meaning of the sentences.

However, this by itself does not strike me as legitimating a truth-theoretic semantics. One reason is that the actual words chosen provide shades of meaning — narrative or figurative construals, interpretive connotations, visual imagery — which add communicative content that cannot be readily modeled within logical details themselves. In the “many students” examples, different formations taking the place of *many* present different interpretations or figurations of the described states of affairs. These added details, however, can still be said to have a logical structure: for instance the *critical mass* language implies the speaker thinks the sheer number of students complaining *caused* something to happen. As such, the interpretive or figurative implications of the word-choices can be seen as a compact or rhetorically effective way to denote an underlying logical structure. We can, even in these cases, find a propositional content; except, the language itself uses figurative or connotative devices to actually refer us toward that content.

My broader claim is that there is no effectively logical transformation *from* the linguistic content as presented *to* the constituent elements of the relevant, signified

propositional content. What *rules* are we following when we hear “sing along” as having wide scope and “perform” as having narrow scope? Or hear the implicit quantifier in (23) and (22) as *most*, but in (21) as *all*?

I am prepared to admit that a truth-theoretic *semantics* need not be a truth-theoretic *grammar*; that is, we can say that sentences have a propositional content that semantics should isolate, even if there is only an indirect mapping from the surface-level linguistic performance to the predicate structure which, in many cases, we could see as sentences’ *meaning*. Linguistic expressions do not, in general, structurally recapitulate the form of propositional assertions laid out mechanistically. Truth-theoretic models could still have semantic validity if we say that situational awareness and sensitivity to rhetorical nuance allows us to proceed, perhaps circuitously and interpretively, from language acts to predicate structures. The predicate logic of language is not so baldly apparent that we can program computers to understand language, but everything that seems not-quite-logical about language — its reliance on connotation and figuration — could, we might speculate, be cordoned off as an economy of *grammar*. We speak figuratively because it is grammatically more efficient — and, partly for this reason, rhetorically more persuasive — to designate propositional content indirectly, rather than speaking in surface forms mimicking predicate algebra.

However, my analysis suggests that the indirection of the expression-to-proposition path is *not* just grammatical. As I argued, our disposition to read logical formations as one way and not another — wide scope or narrow, *most* or *all* — depends in part on the semantics lexically embedded in words like (in these cases) *borough*, *commute*, *vote*, or phrases like *critical mass* and *tipping point*. Our belief that parsing a sentence includes mapping down to a propositional intention does not help us trace the semantic effects which different word-senses contribute to this process. In short, the cognitive structures which must receive language — the situational awareness and interpretive empathy that tunes us toward communicative intent — is not only manifest in how linguistic form translates to propositional signification. It is also embedded in the lexical specificity of word-senses themselves. Even when the semantics of *sentences* can be expressed propositionally, the semantics of their constituent words does not fit mechanically into the semantics of the whole.



Taking these analyses together, I am trying to describe cases where philosophers have turned to some logical, axiomatizable system to compose a model judged for its value as a philosophical sketch of some kind: a model of parthood as a precis of part/whole relations among physical phenomena; a feature/example statistic as a Formal Concept space; a prototyped and dimensionalized feature-space as a Conceptual Space theory; or a construal of semantics (abstracted from pragmatics and syntax) equating the meaning of sentences to their propositional content. In each of these cases we have to consider how a model for the logical *system* could also be a *philosophical* model; an explanatory tool or simplifying idealization that casts philosophical light on cognitive or physical phenomena.

Philosophical models do not need to be models of physical objects or systems directly; they could encapsulate or productively simplify how we perceive or conceptualize external reality. In that case we want to question the cognitive place for the *logical* model — the model-of-the-logical-system in the model-theoretic sense. For instance, is actual human conceptual process a model of Formal Concept Analysis in its guise as an axiomatic system whose formulae can be realized in axiomatic models? Do concepts as cognitive entities model those axiomatics?

Or if we look for extramental models, to what extent can physical objects and systems realize axiomatic structures? Certainly sciences present models (often useful and, on that basis, probably relatively accurate) of physical phenomena, and often do so via mathematical structures that are logical models of an axiomatic system. So the relevant scientifically studied phenomena become indirectly associated with the stipulated axiomatic. But as I have argued, the logic is two steps removed from the phenomena; at best the siting of logical structures in empirical reality — the idea that material systems realize structures that can be abstractly specified — depends on complex scientific investigation. It requires detailed analysis to show how a terrain of scientific givens — the populations of species in an ecosystem, say — play out as examples of a logically hypothesized (e.g. in this case Darwinian or genetic) theory.

Scientific models are therefore theoretical utilizations of axiomatic structures (usually via a mathematical intermediary), and provable consequences of the axiomatic

system can factor in to scientific reasoning. But scientific models are not “logical” models in the sense that science gives us sets that axiomatically realize logical systems by grounding quantification domains that allow positions in logical formulae to be inhabited. The logical model-theoretic picture — that axiomatic systems get instantiated in sets of entities into which logical symbols quantify — is at best only abstractly applicable to scientific models, which do not yield a physical reality composed of “sets” whose members can be unproblematically labeled with logical symbols.

To the degree that logical systems have even indirect axiomatic realization in philosophically salient contexts, then, they have to be “routed” either through cognition or through “science”. Either an axiomatic schema represents (perhaps with some deliberate simplification) how we cognize some region of phenomena, or else it models how the phenomena behave irrespective of our cognitive inclinations (of course, it can do both in mutual influence). But either we have to see axiomatic systems embedded in structures sutured in cognitive structure, or we have to see axiomatic systems embedded in scientific theories of phenomenal composition and behavior. In any case axiomatic models have to “latch on to” cognitive and/or scientific models to have a philosophical resonance.

This effectively metaphilosophical ideology, I think, includes mereology as an (important) special case. Introducing mereology via an axiomatic system is quite common, even in philosophy. But I’d argue that the philosophical value of an axiomatized mereological system is dependent on how and whether it can latch on to the requisite cognitive and/or scientific models to make its own, axiomatic model-building something more concrete than afforded by set-based model theory in pure logic.

This brings me to my final case for NAM: non-antisymmetric mereologies have more or better *concrete models* than Classical mereologies. I’ll call this the Model-Theoretic Argument: when we look at *concrete* models in cognitive or scientifically-described systems — not just imagined models like “Voxel Mereology” which are more about notating philosophical intuitions than philosophical exposés of thought or reality — non-antisymmetry, not antisymmetry, predominates.

## 3.1

# “Criteriological” and “Expository” Mereology

Usually when we hear part-like talk, we seem to instinctively look for overlap-style relations rather than subsuming inclusion of something smaller into something larger. The linguistic template *x is part of y* is usually interpreted in such a way that *y* actually has some existence or aspects *not* part of *x*, but the current speaker’s attention is focusing on contexts where *x* and *y* overlap. That’s why we don’t hear it as odd or metaphorical if something apparently larger is presented as a part rather than a whole:

- ▼ (27) Phenomenology is only part of Merleau-Ponty’s oeuvre.
- ▼ (28) Iraq and Afghanistan are only part of CENTCOMM’s responsibility.
- ▼ (29) Tomorrow’s chicken soup (with the leftover chicken in it) is my favorite part of the chicken.

If we’re doing formal semantics, we might want to say that the propositional form of these examples is a matter of overlap rather than proper parthood. But something like *Phenomenology overlapped with Merleau-Ponty’s oeuvre*, or vice-versa, sounds awkward in comparison. At the very least we have an apparent datum that semantic structures involving overlap often seem to invite surface articulations involving explicit parthood. From the angle of NAM set against Classical mereology, of course, overlap and parthood have a significant difference: overlap is symmetric while parthood is not.

I’ll give another example set on the lost continent of Atlantis, to be concrete in a hypothetical way. Suppose the continent’s largest newspaper, the Atlantis Times, creates a consortium of semi-autonomous local newspapers in smaller Atlantian towns. We can then consider the Consortium to be part of the Atlantis Times: perhaps its offices are in the Times’s headquarters, and the Times funds, administrates, and legally controls the Consortium. But we can also say that the Times is part of the Consortium, if the Consortium includes a portfolio of papers one of which is the Times itself.

For a final introductory example, consider a web portal which includes a collection of resources. Suppose one of those resources is another web portals which in turn includes a collection of resources — one of which is the original portal! Reading “includes” as “has as part”, this

is clearly a partonomic cycle. Indeed the whole “web” ideology is that resources link up in complex and dense (and often circular) ways.

I think these kinds of examples are both representative of “folk” mereology and structures where non-anti-symmetry is a more effective modeling assumption than anti-symmetry. In that case NAM is not the theory of a few ersatz cases but rather a general framework from which, if desired, classical mereology could be recovered as, so to speak, a proper part.

So here I will present my thoughts on what a “generalized” non-anti-symmetric mereology can look like and how it might be practically — i.e., technologically — applied. Then I’ll revisit philosophical terrain like the sculpture/clay conundrum.

## 3.1.1 Several Systems

In order to demonstrate a pragmatic, common-sensical NAM proposal, I will consider logical or conceptual relations which *seem* like mereological relations but which *also* seem like they involve a “cyclical” notion of parthood. To facilitate discussion I will use the symbol  $\triangleright$  to mean “has as proper part”, giving  $\triangleright$  different names in the course of debating which parthood-concepts are most appropriate. So parthood is modeled via  $\triangleright$  read left-to-right. I will pay special attention to cases where both (\*)  $x \triangleright y$  and  $y \triangleright x$ . I’ll assume by definition that  $\triangleright$  is “proper”, i.e. nothing is  $\triangleright$  itself, so in (\*) necessarily  $x$  is not identical to  $y$ . By this setup  $\triangleright$  is not transitive, since  $x \triangleright x$ , but call  $\triangleright^*$  the transitive closure of  $\triangleright$ . Then (\*) exemplifies a case where  $x \triangleright^* x$ .

Consider the Atlantis example, which posed that as the Consortium being an *administrative part* of the Atlantis Times, while the Times is a “constituant” part of the Consortium — if someone listed the papers in the consortium, this list would include the Times. Granted, these are not exactly the same notion of mereology, so we could treat this kind of case as an anomaly due to two different mereological relations which happen to conflict. Of course,  $y$  can be part of  $x$  on some criteria but not others: French Guyana is part of Europe politically but part of South America geographically.

However, there are millions of notions of parthood, and mereology is not very useful if we can only very rarely mix such notions together to form complex ideas.

Problems like the violinist’s arm — which is not part of the orchestra — suggest that transitivity between two *conceptually different* kinds of parthood needs be somehow restricted, but we should leave open the possibility that conceptually different mereologies can still find *some* propositional connections. This implies building in to mereological theories a sense of the conceptual structure of the parthood relations thereby theorized.

I’ll start by defining  $\triangleright$  as something like: “*has as proper part by all relevant criteria*”. In other words,  $x \triangleright y$  means *y is part of x by all criteria relevant to x*. This does not exclude the mere Ontological possibility that some part of  $y$  may be outside  $x$ ; but any such parts are of no importance for  $x$  — i.e., for any propositions of practical significance for people engaged with  $x$  insofar and during the time that they are engaged with  $x$ . On Atlantis, the Consortium is an “administrative” part of the Times, and let’s imagine that for all practical purposs, for everyone working for or otherwise engaged with the Times, the Consortium is just “part” of the times (no conceptual qualification needed).

But notice that  $\triangleright$  hereby conceptualized could easily be symmetric. We could have both  $x \triangleright y$  and  $y \triangleright x$ , if  $x$  is part of  $y$  by all criteria *relevant to y*. The fact that we are evaluating criteria relative to the whole allows  $\triangleright$  to be inverted, since the former part then *becomes* the whole and relevance-criteria are assessed on *its* terms. For employees of the Consortium — those who promote, deliver, index, or represent the papers, or whatever — the Times may be just one of several papers in the portfolio. By *their* criteria, the Times is part of the Consortium, not the other way around.

Someone looking from outside might prefer to say that neither institution is truly a proper part of the other. Their vantage point compels them to consider *both* parts’ criteria, and in this holistic sense there may be parts of the Consortium that are not practically speaking parts of the Times, and vice-versa. It might seem then that the “real” picture involves no proper parthood on either side — and therefore no NAM.

That analysis is not wrong *per se*, but it may pare down parthood relations unacceptably. After all, when are the occasions where we say that  $y$  is part of  $x$  *completely*, in every sense? An institute inside an acadmic department may be spatially and administratively part — its offices inside the building; its staff demed depart-

mental staff. But the institute might develop curricula, plan events, embrace intellectual paradigms, and form a social circle somewhat autonomous from and tangential to the department. A specialized imprint of a publishing house may prioritize disciplines different from the larger company. Usually parthood implies some level of autonomy, in real situations, because we usually don’t expend the conceptual and bureaucratic effort to keep track of some mere part if the part’s behavior or properties is fully predictable from the whole. A semi-autonomous part can still be a part — mereology can permit autonomy; indeed this is a defining characteristic of complex systems — but once we allow parts’ autonomy it is easy to soon realize aspects of those parts that make them no longer seem *completely* part of their wholes. We do jot have to abandon mereology completely if we argue that those residual parts are not *relevant* to the whole, and therefore do not interfere with propositional attitudes concerning the relationship on the whole’s terms.

This definition of mereology is arguably more robust, because it allows conceptually different mereological relations to be unified. The administrative nature of the Times including the Consortium can coexist with the “compositional” nature of the Consortium including the Times, as part of one mereological system. The key detail here is that we allow  $\triangleright$ -relations to exclude “irrelevant” parts — which of course means we introduce a criteria of relevance, which can “filter” the “Universe” of available parts. For any  $x$  whole, it can consider  $y$ s as its parts on the basis of filtering away irrelevant details. Therefore  $x \triangleright y$  does not force all  $y$ -parts to also be  $x$ -parts (like, say, *being y* qua metaphysical part of  $y$ ), but just that any stray  $z$  in  $y$  not be a *potentially relevant* part of  $x$ .

I’ll call the version of  $\triangleright$  just outlined “criteriological” because it depends on relevance-criteria localized to each whole. Note that  $\triangleright$  decays to “Classical” mereology if we stipulate that there is only one global set of relevance criteria across all wholes in the analysis. Thus classical mereology is a restricted form of this “criteriological” mereology.

As I have maintained, things are not in most common-sense perspectives *completely part* of other things. Usually the appearance of complete parthood is an illusion conjured by how parthood relations are disclosed. For example, surely Kyrian Mbappe is part of les Bleus. But

how do we know this? Presumably we see his name on a list of les Bleu's roster, or perhaps see him on the pitch with the squad. But the former case is not actually a warrant (with no further logic) for Mbappe being *part of les Bleus*; it is rather *Mbappe's name* being listed as *a member of les Bleus' roster*. We use the referential relation between his *name* and Mbappe himself to project to the idea of the player being part of the team.

But this referential indirection carries all the potential for criteriological criteria I have discussed. There is a sense of Mbappe being part of les Bleus relevant to les Bleus — their training, formation, tactics, marketing, popularity, etc. Obviously not every part of Mbappe is relevant to the team; he does not train with them every minute. Indeed, he has a whole other career at Paris St.-Germain. But aside from the cognitive complexity of tracing in what senses he is part of les Bleus, there is a nagging problem of defining what *Mbappe is part of les Bleus* actually *means*. Maybe this is simpler in well-defined contexts: surely he is part of the squad when he lines up in the starting eleven and the ball is kicked off. But surely also Mbappe being part of les Bleus is a more general phenomenon than just those moments on the pitch.

In real life, a concept like an athlete being part of a team can actually involve complex legal, financial, and procedural criteria, so it may require a detailed contract to state rather precisely what *being part of a team* actually means. However, supporters know the players on their teams without knowing the requisite contractual minutiae; in short, a fan's acquaintance with their team's history enables them to summon a list of current or past players on demand. Ask a fan who is part of the team, and they will rattle off a list of names. In this sense, they are conceiving a kind of parthood which we might call *enumerative*:  $x \triangleright y$  if we would (under ordinary circumstances) include  $y$  when enumerating a list of  $x$ 's parts.

Technically, though, an “enumerative” mereology is “indirect”: we use, say,  $y$ 's *name* on a *list* of  $x$ 's parts as proxy for  $y$  being part of  $x$ . Presumably  $y$ 's name is on the list because it *is* part of  $x$ , at least on some criteria. But as such criteriological relevance is built in to the list construction. To enumerate the parts of  $x$  we are not committed to those parts being wholly subsumed under  $x$ ; just that they are *members of  $x$*  in some salient

context. An  $x$  *taking* mereological relations is then a matter of  $x$  being conceptually figured as a collection, aggregate, or set. Of course, wholes can be conceived as multiples in different ways: all players in a sports franchise's history is one kind of plurality; the current roster is another; all the team's employees is a third.

I would argue that the most common kinds of mereologies in paractice are some variations on this theme: to conceive  $x$  as a *whole* means to conceive it as a *plurality*, which introduces the possibility of enumerating its members, which thereby become its *parts*. But rarely are part/whole relations thereby conceived where the whole “fully surrounds” the part, absorbing it so that mereological partiality vs. totality can be readily resolved, like a room being part of a house. Usually the members of  $x$  qua plurality bear instead some functional and integrative relation to  $x$  in some context.

In that sense “enumerative” mereology may seem to be incomplete, because the very act of conceiving a  $y$  in its functional role as a member of  $x$  seems to color how we are disposed to  $y$ ; we seem interested in  $y$  as member of  $x$  rather than on its own terms. In practice, however, conceptualizing many things — as least initially — as members of some multiple seems epistemologically unavoidable. Most people would never know of Kylian Mbappe *except* as a forward for les Bleus. Usually we are introduced to things via reference to a containing whole, and usually that whole is figured as a plurality, collection, or type (rather than as a physical or spatial part, say): we are introduced to a friend's cousin as a member of her family; we learn of a new young athlete when he is drafted by a team; we learn about our friend's dog by first being told his breed. Epistemologically the aggregate-whole gives us an *entré* which we can then follow up by learning about the part/member on its own terms.

Mereology in this kind of situation then models a kind of epistemological sequencing, tracing how our cognitive attention can migrate from whole to part. It is easy to see how parthood in this context can come out circular, because the pursuit of knowledge often circles back on itself. We learn about Mbappe because of our interest in les Bleus; then we learn about Mbappe's career, of which the 2018 World Cup was an important part; but then we circle back to les Bleus. Mbappe is part of an “epistemological” whole in the sense of our desiring to learn



a relatively complete picture of French international football. Mbappe is part of any encyclopediac treatment of les Bleus; likewise, les Bleus is part of any encyclopediac treatment of Mbappe. I might call this *encyclopediac* mereology. Encyclopedias, indeed, are almost essentially cyclical in how references link back and forth. But everyday language suggests that these information networks can be understood as a mereology; we can readily accept sentences like:

- ▼ (30) Husserl is part of that Encyclopedia’s article on Mathematical Foundations.
- ▼ (31) That Encyclopedia of Analytic Philosophy includes Husserl but not Brentano.
- ▼ (32) Iraq is a big part of Bush’s legacy.

Of course, Iraq is not literally part of Bush’s legacy or of CENTCOMM, as if these were geographic territories. But the *idea* of Iraq, or in some sense “responsibility for” Iraq, finds a place there conceptually.

Granted that only a hard-core idealist would equate the mereological relations of the *idea* of something with that thing’s own mereological relations. Surely the *idea* of Iraq is part of many things that Iraq itself is not part of. But in fact many practical applications of mereological theories depend on tracing mereological relations in a network of *ideas*, or at least something cerebral and/or computational rather than physical: Web Ontologies, Information Systems, and so forth. Consider the case of a web portal  $W$  whose resources include a portal  $W'$  which links back to  $W$ . In general, web portals *contain* resources in the sense that they link to or provide access to web resources — resources that are not necessarily “part of” the entry-point in the engineering sense of being on the same servers, or being URL subdomains. So *to be part of* in an Information Space  $I$  (I’ll use this as a generic word for database, portal, Information System, etc.) generally means that  $I$  provides access to, provides a kind of structured entryway to (e.g. a searchable front-end), or “exposes” some affiliated resource  $R$ . Here saying  $R$  is part of  $I$  also means that  $I$  *links to*  $R$ , which suggests  $R$  and  $I$  are peers rather than granularly mismatched.

In short, in structures that might be conceived as Information Spaces, the commonsense picture that mereology implies a difference in scale — parts are at least somewhat smaller than wholes — seems readily contradicted.

I will consider the consequences of this “cross-referencing” in the final section of this paper.

Taking a cue from the “linked data” nature of Information Spaces, the underlying model of parthood in this kind of theory might involve some form of epistemological linkage, where access to or information about parts is part of the epistemological interface afforded to their wholes. Mbappe as part of les Blues means, for instance, that an information-source profiling les Blues should link or provide entry to a comparable source profiling Mbappe. As a notion of parthood, this contains an extra layer of indirection, since we can distinguish a *link to*  $y$  from  $y$ : an encyclopedia entry on les Blues which Mbappe is a part of actually contains a *link* to Mbappe. And we get informed of Mbappe being part of les Blues’ roster by seeing his *name* on a list. Thus mereological relations often involve an intermediary name, link, or designation which stands in for an actually autonomous part in partonomic contexts.

To make this somewhat formal, we need two relations: first, a notation like  $\mathbf{p} \triangleright y$  which I’ll call (borrowing a term from Cognitive Grammar) “profiles”. The relation of a  $\mathbf{p}$  profiling a  $y$  could be read, according to context, as the conceptual tie between a web address, computer pointer, or other technological reference-artifact and its target; or the designatory relation between proper names and their objects; or a more cognitive form of reference. Then I’ll introduce a whole-to-profile relation  $\triangleright$  such that  $x \triangleright \mathbf{p}$  can read  $x$  *contains*  $\mathbf{p}$ , *which profiles something (other than  $x$ )*. Combining  $\triangleright$  and  $\triangleright$  yields a three-part double relation like  $x \triangleright \mathbf{p} \triangleright y$ , for  $x$  *contains*  $\mathbf{p}$ , *a profile of*  $y$ . Then finally a version of  $\triangleright$  can be defined as this three-part relation abstracting  $\mathbf{p}$ :  $x \triangleright y$  becomes  $x$  *contains a profile of*  $y$ .

This particular version of  $\triangleright$  may be useful because it is adaptable. It accommodates systems where profiles of  $y$  either are or aren’t *parts* of  $y$ , according to familiar mereological criteria. Depending on how that goes, this latest  $\triangleright$  could model “criteriological” mereology: suppose the profile of  $y$  in  $x$  is the only part or aspect of  $y$  which is relevant to  $x$ . Then  $\mathbf{p} \triangleright y$  acts as a relevance filter, the profile selecting salient parts of  $y$  and excluding residual parts from mereological consideration. Then  $x \triangleright \mathbf{p} \triangleright y$  can be read as  $x$  encompassing all parts of  $y$  when filtered by  $p$ . Either  $\mathbf{p}$  expressly operates to isolate  $x$ -relevant aspects, or  $x$ -relevance is derived from

a filtering according to more general criteria, like the properties of a restaurant being subdivided into culinary, operational, nutritional, and architectural dimensions — i.e., a restaurant has architecturally relevant parts, operationally relevant parts, etc.

I will do further analysis however of the alternative model where profiles are *not* in general parts of what they profile. Instead, a profile is like an epistemological or technological device that “exposes” or permits access to something else, like a pointer to a region of computer memory. I’ll call this model “Expository” mereology. The canonical idea of parthood is now that  $y$  is a *proper part* of  $x$  if and insofar as  $x$  “exposes” or provides an information link to  $y$ .

### 3.1.2 Normalizing Arbitrarily Granular Mereologies

One potential benefit of the Expository model is that it may be applicable to and/or reflective of how technology concretely implements mereological systems. I contend that any classical mereology can be directly mapped to an Expository mereology: take any classical parthood instance  $x \triangleright y$ . Designate a profile for  $y$  inside  $x$ , say,  $\mathbf{p}_x$ . Then reinterpret  $x \triangleright y$  to mean  $x \triangleright \mathbf{p} \triangleright y$  where  $\mathbf{p}$  is  $\mathbf{p}_x$ , along with a restriction that any  $y$  can have at most one profile, and has *exactly one* profile if it has a (classical) whole — actually this one profile is part of the whole *in lieu of*  $y$ .

So expository mereology includes Classical mereology as a special case, but it allows for generalizations which patch over conceptual objections that may be raised to Classical mereology in its unadulterated form. In practice the maxim that each  $y$  has only one profile may be too restrictive. Instead, multiple divergent wholes may overlap with  $y$  in different ways and contexts. For instance, Mbappe is part of les Blues, and also PSG, the Afro-French community, the Mbappe family, etc. This might be changing the subject somewhat: “overlap-systems” characterized by generally complex entities that overlap in different modes and contexts are a different area of philosophy than mereology. But in reality these two theories are intertwined: many conceptual phenomena can be approached both from a mereological perspective and an “overlap” perspective. The two kinds of theories may be viewed on a spectrum, with mereology emerging as we attend more to the filtering effects of  $x \triangleright y$  parthood, the

relation either witnessing or effectuating our disposition to ignore non- $x$  parts of  $y$ .

To put it differently, arguably any mereological system is an overlap-system which we are able to filter or simplify to reduce cases of practically inconsequential externality that would otherwise block proper parthood. If almost always  $y$  is *never* an *completely subsumed* part of  $x$  then (classical)  $x \triangleright y$  has to reference some kind of theory that  $y$ ’s non- $x$  parts are inconsequential. So “relevance” can be like a “knob” tuning in mereological *or* overlap theories depending on whether we are more or less sympathetic of filtering: mereologies emerge when we tolerate filtering non- $x$   $y$ -parts in  $x \triangleright y$  as practically appropriate, and overlap theories arise from mereologies when we realize that filtering skirts around legitimate Ontological, cognitive, conceptual, or natural-language/pragmatic concerns. I think that the system of operators  $\triangleright$ ,  $\triangleright$ , and  $\triangleright$  (when defined from the other two) models both mereological and overlap systems and accordingly can unify both kinds of theories.

Aside from this philosophical case, however, there is a practical benefit to the “Expository” definition of  $\triangleright$  which applies to Classical as well as non-antisymmetric mereologies. In a mereological system, profiles need not have their own parts, because any internal structure belongs not to the profile in itself but to the thing profiled. Accordingly, on Classical terms, we can have only one “wholes” layer and only one “parts” layer: there are wholes whose parts are profiles, and profiles refer to other wholes. This two-layer architecture takes the place of a classical system where partonomic nesting may be arbitrarily deep. Of course,  $\triangleright$ -chains can be arbitrarily long, but in Expository mereology — although  $\triangleright$  *conceptually* models parthood — the relations in  $x \triangleright y$  are not considered to be on intrinsically different scales. The  $\triangleright$  relation *is* across levels, but in  $x \triangleright y$  we go from a whole  $x$  to a *profile* and then back to another whole. This is not to rule out some scale of size within the order of wholes, but that detail is not intrinsic to the system.

I will generalize the two-level model a small bit, with the following rationale: on occasion the intuition that profiles point toward *one* target may be too restrictive. Suppose  $\mathbf{p}$  profiles a *set*,  $s$ . We could treat  $s$  as a whole and model its members via their own profiles in  $s$ , but then we have an extra layer of indirection that may serve no modeling purpose. For this reason I allow that there

may be a level higher-scale than wholes, which I'll call *frames*. This results in a three-level system: a higher level with frames, a lower level with profiles, and an intermediary level which contains most of the primary objects of investigation or conceptualization. Profiles can target multiple objects at this mid-level by targeting a frame rather than a single object.<sup>6</sup>

With this addendum, Expository mereology then becomes a three-level system. Arbitrarily complex scales of granularity can be modeled *within* the levels, particularly at the intermediate level, but the formal model can express a technological design where only those three levels need to be implemented as computational primitives.

A consequence of this design is that arbitrarily complex mereological systems can be encoded in hierarchy with only three levels, a process I'll call "normalization". This process is philosophically analogous to normalizing a hierarchical document database to a computationally more malleable graph database. Moreover, I will close this section by noting that the Expository mereology relation is quite naturally non-anti-symmetric: if  $x$  contains a profile of  $y$  there is no restriction against  $y$  containing a profile of  $x$ . With that in mind I'll refer to an Expository mereology using the general three-level model and its associated non-anti-symmetric  $\triangleright$  relation as an "N3" model or encoding.<sup>7</sup>

### 3.2 Concreteness and Computation

I would be remiss to draw a contrast between *abstract* and *concrete* models without mentioning the emergent field of "Ontology Engineering" and the role of mereology therein. Arguably, mereology in these contexts is neither abstract like logical-axiomatic models, nor concrete in

the sense of "routing" through cognitive or scientific models. Ontologies — as this term applies in computer science and Information Systems — are computational artifacts, effectively documents in a special-purpose language, enabling programmers and scientists to define and annotate data structures so as to describe logical patterns obtaining within data. Ontologies allow data to be correctly shared and reproduced, and also sometimes to be logically analyzed, identifying structures within a data set that can be extracted via logical rules. Embedding logical axioms, including those of mereology, in data modeling frameworks therefore becomes part of an overall data management technology. Logical analysis becomes a part of data analytics in general.

Ontologies are abstract prototypes; they play a determinative modeling role only in conjunction with actual data sets. Also, although computer programs can perform some "reasoning" on modeled data sets, the explicit correlation between Ontologies and empirical data depends on people consciously aligning data structures to Ontologies' types and relations. Data sets annotated via formal Ontologies therefore represent deliberate human attempts to relate conceptual and empirical structures — they are both cognitively and scientifically concrete.

Consider, for sake of demonstration, a derivation of the fact that an ingrown toenail should qualify as a malady of the foot. An Ontology might represent ingrown toenails afflicting toes, and also that toes are part of the foot, without identifying an ingrown toenail as a malady of the foot per se. Someone searching a database for foot afflictions, however, might well expect cases of ingrown toenails to be among the search results. Thus mereological rules can extend the semantic reach of an information system.

But that semantics depends first on conscious design of Ontologies to represent things like disease classifications and part/whole relations, and also on human classification activity. Someone has to specifically notate that a patient's condition exemplifies an ingrown toenail. The Information System is designed around human practice, according to different operational needs: entering a diagnostic code in a medical record, for example, and then querying a database to gather a set of results such as cases of diagnoses filtered by some condition (say, conditions that might be treated by a podiatrist). An Information System — its technology as well as its actual

<sup>6</sup>The significance of frames emerges in some semantic and technological contexts where we want to distinguish between relatively dispersed collections and complex wholes with some organizational coherence, such that we are inclined in many contexts to treat them as singular. That is, arbitrarily collating any collection into a whole may dilute a model's ability to distinguish between integrated wholes that often function as singles, from flat wholes that warrant encircling in a specific context but do not on most criteria cohere as individuals. So as not to bias "wholeness" toward either coherent or flat aggregates, I propose a "frame" level for "flat" wholes distinguished, as a system feature, from intermediate wholes with significant *individual coherence*.

<sup>7</sup>I don't propose this term outside the present writing because it conflicts with N3 in the Semantic Web — a notation for graph structures. N3 mereologies are actually a superset of N3-expressible data structures, where the second N3 is the Semantic Web term.

data — can evolve in response to operational preferences. In short, deductive reasoning like *ingrown toenail is a kind of foot malady* is desired not out of some abstract concern for logical completeness, but it fits operational needs to pair a data entry about a patient with an ingrown toenail matching a potential database query about podiatric diagnoses.

Mereology is, accordingly, part of an overall logical structure designed to facilitate the operational requirements of adding and filtering data from information spaces. This adds a further dimension to mereological analysis, because the mereology (along with other logical relations) becomes part of an engineered system, where we have prior anticipation of *how* we want the system to perform. We identify specific kinds of inferences which should be engineered into the system based on anticipated preferences of human users.

To clarify, we can recognize in an Information System at least two different classes of “behaviors”: *input* and *output*. Input behaviors are the system’s implementations, and the interactions it affords, for people entering new data (say a doctor entering a new ingrown-toenail diagnosis); output behaviors are implementations and affordances for getting information out of the system, such as by queries (perhaps, a list of patients with podiatric diagnoses). We can add *internal* behaviors involving internal processing to link inputs and outputs in (according to users’ expectation) correct ways. The system-outputs are products of the cumulative history of its inputs up to the point where output behavior is requisite. For example, the prior input of an ingrown-toenail diagnosis constitutes *relevant* input history for a query (demanding output behavior) for podiatric diagnoses.

So mereology and other logical relations play a role when it comes to identifying *input history* that is *relevant* for a certain output request. Logical reasoning expands the space of input history which will be considered relevant in a given output-context. This points to an interpretation of mereology in terms of *relevance*; properties or assertions involving a part (e.g., diagnoses) are relevant to corresponding whole.

Notionally, then, mereology serves as a kind of semantic enrichment of a system whose essential nature is a mapping from *input history* to *output behavior*. This core operational motive — *input history determines output behavior* — should be reiterated in the context of

the Semantic Web in particular, to help establish an underlying conceptual model to properly analyze (and implement) Semantic Web technology.

Conceptually, the Semantic Web is one example of a data-sharing platform; and the key ingredient of data sharing is the replication of information spaces, in whole or in part, between different places. Insofar as “input history determines output behavior” is the constituent feature of information spaces, replication means that the points where the data is shared will replicate analogous input-to-output patterns. Data sharing does not necessarily mean that all points process data identically; when data is exported between environments the new location may serve different operational purposes, so that analogous queries or interactions which receive output data may be evaluated differently at different points in a data-sharing network. But assuming the *receiver* of shared data does differ in its output structures from the *senders*, these differences should be systematically accounted for vis-à-vis how the respective systems are engineered.

Conventional Semantic Web data structures are organized around the principle of *labeled graphs*, where the building blocks of data models are so-called “triples”: *Subject/Predicate/Object* graph units where *Subject* and *Object* are graph-nodes and *Predicates* are labels applied to graph-edges, asserting that the *Object* bears to the *Subject* some identified relationship. Ontologies then, first and foremost, provide lists of identifier for both nodes and edge-labels: this allows nodes to be associated with a kind or type (a *person*, say, or a *diagnostic code*) while edges are labelled according to a controlled vocabulary (so edges represent concrete relations, for example, that a person has a diagnosis). Ontologies then superimpose logical axioms pertaining to restrictions or guarantees among relations and/or nodes: that a relation is transitive, say, or that a given relation is only reasonable for certain *Subject* and *Object* types (e.g., a diagnostic relation is only coherent as a relation between a person or patient and a diagnosis or diagnostic code).

Although a fair range of empirical semantics can be captured by individual *Subject/Predicate/Object* triples, in many cases a realistic semantics demands a model of how multiple relations aggregate to form meaningful units. For example, instead of a diagnosis being modeled



as a single relation between a patient and a diagnostic code, a system may stipulate a more detailed representation where a single “diagnosis” has several associated pieces of information, such as a diagnostic code, a date, a doctor’s name, and potential reference to laboratory or radiological findings that substantiate the diagnosis. This reflects how, in most computational settings, the basic units of evaluation are not single data points but internally structured data aggregates.

Thus far, no standard model has emerged to represent larger-scale data structures in the Semantic Web context. Intrinsically, the “semantics” of the Semantic Web depends on computer environments where data is shared and received. Since people generally do not consider data sets as a whole, any meaningfulness we can attribute to digital information depends on selecting smaller amounts of data, which in turn depends on the digital systems — user interface technology as well as data management software — through which people interact with information senses. So any discussion of the “semantics” of data, formally warehoused in information spaces, has to recognize the multiscale organization intrinsic to information spaces.

### 3.2.1 Mereology and Hypergraphs

The interaction between humans and information spaces is itself a technological artifact which has to be engineered and implemented. Such implementation in turn depends on implemented *procedures*, which classify, manipulate, and visually or textually present relevant data to human users. A major element in the semantics of information spaces is accordingly the semantics of procedures operating on this data — what data structures are valid operands for procedures’ operations; how procedures modify data structures or map inputs to outputs; how data manipulated procedurally must be internally structured, according to procedures’ computational assumptions and preconditions. The constituent elements of information spaces are internally structured data aggregates, more so than they are unstructured information atoms, because structured data is the kind of value which in the general case computational procedures operate upon.

For these reasons, part/whole aggregation and the rules for nesting data structures (directly or indirectly) should be essential topics for the Semantic Web (separate

and apart from mereological axioms built into web Ontologies). There are at least four different strategies that can be used to represent multiscale data in a graph or labeled graph context. These are not mutually exclusive but have different emphases; insofar as labeled graphs form the notional foundation of the Semantic Web they can carry over into that context as well. Graphs supporting multi-scale representations are generically called “hypergraphs”, although there are different hypergraph theories providing different spaces of models. For sake of illustration, I’ll use the hypothetical case of a diagnosis encompassing a diagnostic code, date, and the diagnosing doctor’s name, and refer to schematic diagrams in Fig1a-e.

One form of hypergraph is based on the idea of *hyperedges*, and further *directed* and *labeled hyperedges*, which generalize graph-edges to include more than two nodes. For instance, a record of a diagnosis even can involve a four-node edge where one node represents the patient, and the remaining three nodes represent a diagnostic code, date, and doctor’s name. Directed Hypergraphs group the nodes incident to a hyperedge into *source* and *target* nodes; in this example the patient’s node is the lone source and the remaining nodes are targets, indicating several pieces of information associated with a diagnosis (see Fig1a). The (potentially multi-node) source and target sets generalize the Semantic Web concept of *Subject* and *Object* for context where a given predicate-relation needs to be expressed with multiple subjects and/or objects.

A second model of hypergraphs can be defined by allowing each node to be (or internally contain) its own graph. This leads to nested levels of graphs, where a graph on one level can be incorporated into a graph on the next higher level by being nested into one of the latter graph’s nodes. Figure 1b shows how this could work in the running example: it shows the fact of a doctor giving a diagnosis on a given date as a miniature graph whose edges connect the doctor’s name to a “date” node and “diagnostic code” node, respectively. The mini graph is then placed as a package via a node inside the higher-level graph, where that node is joined to a node designating the diagnosed patient. Models may or may not allow nodes on one level to be linked with nodes on another level, or allow two nodes nested inside two different higher-level nodes to be linked to each other.

A third genre of hypergraph has similar aggregational features but does not distinguish between different levels. Instead, each graph has one level, but a collection of nodes and/or edges can be separately identified and associated with a separate node, or presented as some aggregate structure with special significance. Hypergraphs with this added structure function like conventional Semantic Web graphs upon which are superimposed added details. Figure 1c shows the extra layer of information via a dotted line around three graph edges. The underlying data in this case is a regular (labeled, directed) graph (not a hypergraph). A separate node is introduced representing a *diagnostic event*, serving essentially as a “placeholder” to which extra nodes (carrying factual details) can be attached. This is a common pattern sometimes called a “snowflake”: a *Subject* node (here the *patient*) is connected to an *Object* node (here a *diagnostic event* whose semantic role is to unify additional nodes, that provide the relevant actual details; the net effect is nodes branching off from a center, like a snowflake). The extra layer of information comes into play by unifying the three nodes and edges one step removed from the *patient*. These edges would be grouped together to indicate that their respective object-nodes form a logical unit (three components of a diagnosis: medical code, date, and doctor’s name). They could also indicate data requirements: an Ontology can stipulate that if a *diagnostic event* node is present in a graph, it must be accompanied by three additional edges providing pertinent info about that event.

Consider also the possible interrelationships between the first and third cases (hyperedges and frames). A consequence of hyperedges in *directed* hypergraphs is that each hyperedge can have distinct head- and tail-sets, which are called “hypernodes”. So a directed hyperedge connects two hypernodes. In a typical case, a hypernode packages together multiple interrelated values, like *name*, *address*, and *telephone number* or *date*, *doctor’s name*, and *diagnostic code*. Meanwhile, in a frame model, several edges may be grouped together to provide an interrelated package of information. Combining these two ideas, we can define a sort of *cross-reference graph* whose nodes are hypernodes and whose edges are defined by values within a hypernode that “point” to other hypernodes. A hypothetical case is outlined in Fig1d, with the assumption that elements in a hypernode are proxies for additional complex data structures. For example, diagnostic codes

can refer to a collection of known medical conditions each linked to additional data, such as symptoms and treatments. Similarly, a doctor’s name can link to a profile of that doctor in a database, and a *date* represented as a single value can be treated as a stand-in for an aggregate of *day*, *month*, and *year*. A set of hypernodes whose component values cross-reference to other hypernodes can be seen as defining a hypergraph whose edges are cross-references themselves; a computer representation of such a data space may not even need to recognize a separate model of “edges” apart from cross-reference values.

A final option for representing hypergraphs comes into effect if we recognize a *type system* applied to a graph; a classification of nodes according to an interrelated collection of types. One feature of most type systems is that for any type  $t$  there are corresponding types representing *lists* of  $t$ -values with different characteristics. Lists are often classified in terms of how they are modified. *Stacks* are lists where the last value added (the one which has been on the list for the shortest time) is the first value removed (think of a stack of trays in a cafeteria). *Queues* are lists where the value in the list for the *longest* time comes off the list first (think of people lining up for tickets). And *Dequeues* (double-ended queues) can have values added or removed from both ends. (There are more complex aggregate data structures as well, like associative arrays where values of one type are used as indices to retrieve values of a second type from a long list; consider how a phone book maps people’s names to telephone numbers). Collectively, these multi-valued types are often called “container” types.

One way to construct hypergraphs is to allow node-types to be container types as well as singled-value types. Therefore, a node may encapsulate a many-valued data structure. This is similar to the case depicted in Fig1b, where nodes could contain nested graphs; the difference here is that the nested data structures encapsulated in nodes are not necessarily graphs; they can also be different sorts of lists or tables. In Fig1e, this is shown by imagining a table that represents a patient’s diagnostic history. Each new diagnosis is added to the list, so the node can expand over time. The data structure “inside” the node then functions as a table, whose rows represent diagnoses made on a single occasion and whose columns are different facets of a diagnosis (date, code, and doctor’s name). The entire table is treated as one node,

linked to the *patient* node (indicating that the provided table is diagnostic history for that one patient).

These five different kinds of hypergraphs are to some degree interchangeable, in that algorithms can transform hypergraphs between these formats. They are still more or less optimal for different contexts. The fifth solution is similar to how container types work in conventional programming languages — and also to foreign key links in relational databases — so hypergraphs using that system can be easier to integrate with conventional (not necessarily Semantic Web) applications. On the other hand, the third solution is closer in spirit to the Semantic Web, adding an extra level of specification but preserving the underlying Semantic Web graph theory. Meanwhile, the nested-graph model is similar to how popular enterprise graph databases are designed; and the directed-hyperedge approach is closest to mathematical graph theory.

The diversity of related but not identical approaches points to an incomplete dimension to the Semantic Web; multiscale representations have not been standardized.<sup>8</sup> Not that a single form of multiscale structure should be preferred over others; there are competing priorities which come to the fore in different contexts. This actually reflects competing ideas about what the Semantic Web should try, first and foremost, to accomplish. The Semantic Web can be seen as a potentially global, open-ended data-sharing community; as a foundation for a future of AI-driven networking; as a kind of vast, decentralized graph database; or as an analytics ecosystem that allows logical analytic methods to be applied, alongside other (e.g., statistical) forms of data analysis, to traverse and filter information from the internet. Which priorities we emphasize dictate the kinds of technologies that have to be implemented, and the “semantics” of the Semantic Web can reflect the need to optimize representations and network interactions to benefit those prioritized technologies.

What this diversity of option reveals, however, is that the question of multiscale data representation is complex and multifaceted. Instead of just setting forth one system as a speculative enterprise, engineers have to identify different forms of multiscalar data — for example, I enumerated five versions of hypergraphs — and study

which flavor is best suited for different technological priorities and acceptable trade-offs. Implicitly, I believe that such a field of study represents a kind of computational investigation of mereology — not mereology tied to predefined logical axioms, but an organizing pattern for computational phenomena (e.g., hypergraphs) whose properties have to be codified and optimized.

This brings me to the final characterization of mereology I’ll propose here: Hypergraph Mereology. I think that analyzing hypergraphs in the computational and software engineering contexts presents a philosophically interesting picture of formalized mereology no less relevant — and perhaps more cognitively or scientifically appropriate — than a “logical” or “axiomatic” mereology.

Note that Hypergraph Mereology is not necessarily set forth axiomatically; we do not specifically define hypergraphs by stating their axiomatic conditions. We *could* proceed in this way, of course; hypergraphs are a meaningful mathematical subject. But often the formalization of hypergraph models is indirect, relying on software implementations rather than logical specification. In other words, much of the intellectual substance behind research into hypergraphs is built from practical implementations of hypergraph code libraries, allowing us to study and compare how these libraries are abstractly conceived and concretely implemented.

This demonstrates a phenomenon which I think Analytic Philosophy has not fully registered: the kind of formalizing efforts that in decades past may have been pursued via logical or logicomathematical axiomatics can increasingly be replaced by software and computer code. Testing philosophical intuitions in an intellectual environment oriented to concrete code development rather than logicomathematical abstraction has several implications: there is a kind of feedback and trial-and-error which can shape the intellectual process; the ecosystem of open-source code and software development tools has different institutional norms than communal deliberation over the validity of logical argumentation; and code development tangibly augments academic writing, in that philosophers (or writers from any other discipline) can pair their conventional publications with code repositories and data sets. This kind of research can lead philosophy in an empirical, experimental direction, but also simultaneously in a direction which accords to the

<sup>8</sup>There is an official “RDF Collections” model but this addresses only a small part of genuine multiscale modeling.

individual philosopher, I think, a renewed agency and autonomy. In this spirit we are not only sharing with our peers rational claims, but exhibiting practical technology created under the inspiration of philosophical arguments. Well-designed, open-source code projects have a certain inner empowerment for their contributors; in a world of (often imperfectly apportioned) intellectual capital, they act as fiat currency, neither curated nor endorsed by academic institutions in the conventional sense, and as such not diluted by the not-always-rational timelines of academic prestige.

It should be uncontroversial to say that “Analytic” philosophy is the prestige dialect of the modern philosophical community, especially in English. Partly this is no doubt topical — Analytic philosophers seem to entertain more “hard science” themes than the political or cultural concerns, or issues of subjective experience and personal identity, guiding “Continental” philosophy. But partly it also reflects the presupposition of analytic rigor accompanying analyses that can be grounded in, or at least shown to be inspired by, some kind of logical or mathematical framework.

With that said, some of the divergence between a logically-inspired Analytic paradigm and a more socio-cultural Continental thought is methodological, not just thematic. Logical models which discount the nuance and complexity of language and experience risk becoming reductionistic: limited explanatory value. Potentially, then, the emergence of a computational discipline for addressing philosophical structures can change the familiar Analytic/Continental narrative. Software models can be technically drawn without being either axiomatic nor reductionistic; they can engage a formalization in software implementation in lieu of specifications of logical structure; and they can present technological systems as suggestive approximations of human phenomena without committing to a reductive theory that cognitive processes — in situational understanding, linguistic meaning, or conceptualization, for example — are wholly characterized by the terms of the implemented software.<sup>9</sup>

<sup>9</sup>Type-theoretic analyses of natural-language semantics and syntax, for example, can represent linguistic artifacts according to formal/computational structures without implying that the human process of actually converting surface-level language to these derived representation happens mechanically or mathematically. Computational *models* of linguistic phenomena are different from Natural Language *Processing*, which makes the further metaphysical assumption that computers can “parse” language down to schematized representations analogously to people.

It is possible, then, to treat computational models as methodologically useful adjuncts to philosophical analysis while rejecting reductive analysis of cognition or language as transparently itself computational or logically tractable. In this way, computational models can bring a technical methodology to Continental thought analogous to the role of logical axiomatics on the Analytic side, but with different methodological commitments — and also in a different metatheoretical perspective. Whereas Analytic philosophers may look on logicomathematical techniques as a kind of rational kernel, a core ground of secure entailments on which to rest other claims, the Continental engagement with computational models can be more figured within the general plurality of Theory; the same pull toward hybridization that would wed phenomenology to analyses of race, class, and gender, for example, to elucidate the experiential as well as cultural political dimensions of social relations. Humanities “theory” acts as a kind of intellectual gravity, pulling in phenomenology, Critical Theory, Structuralism, linguistic and discourse analysis, and so forth; there is no *a priori* obstacle that would prevent the same force from reaching, say, formal and natural-language type theory and the analysis of multiscale information systems.

To my mind the most well-developed body of working merging phenomenology with computational methods is that of Jean Petitot; although I would add some at least provisional formal elaborations of Cognitive Grammar (if I may take this as a kind of applied phenomenology in the linguistic domain) by, for instance Kenneth Holmqvist and Matt Selway. I might add, implicitly, the larger tradition of type-theoretic semantics represented by the sometimes highly formal work of a Zhao-hui Luo, James Pustehovsky, or Bruno Mery; which in turn established technical links to linguistic models inspired directly by computer programming languages and computing paradigms, as represented by linguists such as Ash Asudeh, Gianluca Giorgolo, Oleg Kiselyov, and Chung-Chieh Shan. In short, we can develop an overarching model of linguistic representation which is anchored both in cognitive phenomenology and in computational procedures.

Alongside this linguistic line of development we can identify a parallel integration between phenomenology and theories of spatial comprehension, perceptual synthesis, and cognitive “visualistics”. One line of analysis, for example, considers the phenomenon of experiential



“presence”, motivating phenomenological treatment of computer imagery and Virtual Reality. Along this subjective assessment we can consider the mathematical infrastructure for creating realistic depictions of three-dimensional scenes and surfaces; and for applying lighting, textures, and space-filling effects (mimicking the impression of smoke or flowing water, say). This technology does not necessarily recreate the neurological processes by which we cognize our spatial environment — nor create perfectly lifelike graphics — but it presents a suite of technological formalisms that bear a simulative and structurally articulated relation to visual phenomenology. As such it provides a ground for complimentary but non-reductive analyses that can be set alongside, for example, theories of spatial synthesis as found in, say, Husserl or Merleau-Ponty.

These linguistic and spatial/visual examples have a common architecture, at least vis-à-vis phenomenology, in that they present computational structures that can be argued to encapsulate cognitive representations without eliminating the complexity, situationality, phenomenological immediacy, and interpersonal dimensions of consciousness. Computational models are suggestive analogies rather than axiomatic reconstructions of cognitive “logic”.

I think mereological theories can have an analogous intellectual architecture, albeit in a narrower topic area. Formal systems developed in the context of computer code — say, hypergraphs in one of the five models outlined above — provide a tangible but technological medium for articulating mereological concepts in a rigorous domain, while also allowing these models to be adjunct prototypes for theories of mereology as a cognitive phenomenon. Hypergraphs, in short, are to mereology what formal parse-representations and lexicons are to natural language, or computer-generated imagery is to visual experience.

## 4 Conclusion

The five variants of hypergraph representations I discussed in the last section all support some version of non-antisymmetry or whole-part-whole “cycles”. In these various models, nodes can encapsulate internally structured complexes of some kind — ordered lists of nodes

(as in hyperedges), nested graphs, groups of nodes and/or edges, or list- or table-like data structures. Insofar as complex aggregates are gathered into individual nodes, they can be referenced and/or linked from other contexts. This includes other nodes forming an edge which also includes a complex node, and also components within other complex nodes referencing a complex node as a kind of data value, for instance via a unique identifier selecting individual nodes from across every graph in some information system.

Hypergraph paradigms may or may not allow nodes in graphs or node-sets “nested” in complex nodes to link, via edges, to nodes on other “levels”. But even without such “explicit” links (manifest in actual graph edges) nodes or data values inside nodes can “point to” complex nodes via numeric codes or other unique identifiers. In general it may not be possible to technologically represent hypergraph structures without allowing some sort of “indirect” referencing of nodes; such indirection is a prerequisite of enumerating sets of nodes, iterating over nodes, or otherwise processing nodes otherwise than following graph-edges. Most analysis and computational examination of graphs involves larger-scale actions on graphs that are not restricted to “localized” navigation which incorporates only information immediately associated with nodes and edges (like types and labels) or the local “topology” around individual nodes. Creating lists of nodes, for example, involves structural associations among the nodes which lie outside the actual graph topology.<sup>10</sup> In effect, an option of designating (potentially complex, i.e., internally structured) nodes as their own values (or perhaps unique identifiers of nodes as values or data-points) is requisite for the computational expressibility of hypergraph instances.

Consequently, then, the possibility of cross-referencing between “levels” is an intrinsic feature of hypergraphs qua technological objects. A particular graph engine may restrict cross-referencing or at least formally represent a distinction among different levels, but this will be an extra structure superimposed on the core system. Intuitively, then, the “parts” of a complex node are not necessarily “smaller” than the outer nodes. Moreover, there is no limitation, without external restriction, that a complex node cannot contain a reference to *its own* identifier (in effect, to be a part of itself), or that by

<sup>10</sup>Topology in these contexts meaning network topology.

following chains of parthood we cannot cycle back with a reference to a prior whole. In other words, the mereology which organically emerges from hypergraph theory is intrinsically non-antisymmetric.

If this result seems counterintuitive, such an impression perhaps results from envisioning a mereology which is rooted in axiomatic formulation, and from there applied (at least as a simplifying or modeling device) to cognitive or empirical phenomena. It may be true that axiomatic mereologies are less interesting without restrictions on symmetry or cycles; otherwise parthood itself loses a lot of its *a priori* logical structure and becomes only a kind of vague association. The point of organizing mereological theory around hypergraphs, however, is to work against the background of concrete technology in lieu of axiomatic abstractions: hypergraph systems may have little in the way of *a priori* structure but they present a sophisticated technical structure when paired with concrete data; i.e., when instantiated.

It's also plausible that hypergraphs are a more convenient or legitimate representation of both scientific and cognitive phenomena than classical (axiomatic) mereology. Hypergraphs are, I think, certainly an improvement over (say) Semantic Web non-multiscale graphs for representing scientific data. I would argue that hypergraphs are better for representing cognitive phenomena as well, such as cognitive grammars and lexicons, or the prototypical forms of cognitive schema.

Ultimately, though, my main point is not to celebrate the expressive virtues of hypergraphs; instead I think the intellectual passage from mereology to hypergraphs can serve as a kind of parable. Rather than pursuing abstract axiomatic treatment for foundational philosophical concepts — here I have considered mereology, but I can see related discussions for perceptual synthesis, causal/constitutive emergence, supervenience, essences vs. accidents, theory-revision, graded/rough sets, prototypes and exemplars, indeed conceptualization in general — we may find more cognitive and metascientific traction by looking at structural systems that can embody the relevant philosophical or metaphysical topics in a computational, engineering-oriented milieu. Metaphilosophically, this may be analogized as a transition from a logico-mathematical paradigms to something more “structuralist”, with structures realized in engineered systems taking the intuition-forming role otherwise asso-

ciated with logical archetypes.

Our academic climate of open-source software and decentralized computational experimentation may provide the technological capabilities for a new, arguably belated “*Structuralist*” phase in Analytic philosophy. Structuralism in general — in linguistics, say, or sociology, anthropology, psychoanalysis, political economics — is associated with a technical but not logical/axiomatic theoretical architecture; a kind of *a posteriori* analyticity. Perhaps a Structuralist analytic philosophy will prove to overlap, in interesting ways, with Continental philosophy and, especially, with phenomenology.



More famous writers than myself, of course, have envisioned (or made their life's work) a synthesis of Analytic philosophy and phenomenology. At the least I would need to cite Petitot, David Woodruff Smith, and Barry Smith, with the latter's intellectual and logistical work on the Semantic Web showing how the foundational theory of the Semantic Web serves as a kind of applied and operationalized phenomenology. The scientists and linguists who have then identified limitations in and presented alternatives to the Semantic Web, like Peter Gärdenfors, are then joining a dialectic that phenomenologists can already see figured as like an “applied phenomenology” — the implicit dialog between Conceptual Space Theory and “Semantic Web Semantics” thereby linking Conceptual Spaces to phenomenology on an engineering basis, alongside phenomenological possibilities of Gärdenfors's theories in their philosophical roots.

The circle from Gärdenfors's to the (in his words) “mainly Californian” tradition embodied by Langacker and others, can then be extended to “a French semiotic tradition, exemplified by Desclès ... and Petitot-Cocorda ... which shares many features with the American (mainly Californian) group” [1, p.2]. In turn, Petitot's explicitly phenomenological foundation connects the cognitive *linguistic* perspective to the Analytic-phenomenology synthesis distilled for instance in Petitot's (et. al.) *Naturalizing Phenomenology* compilation from the year 2000. So at last one thematic circle ties phenomenology (especially in the project of its engagement with Analytic philosophy) with Cognitive Linguistics, and relates both in combination to the Semantic Web and related computational and “knowledge engineering”

concerns. This circle probably remains outside the dominant paradigms in both Continental and Analytic thought, but it has some internal coherence, and some unity of theoretical engagements spanning both philosophical and concrete (e.g., technological) subject-areas.

The circle between phenomenology and Cognitive Linguistics, and including computational and metascientific topics, deserves to be analyzed and reinforced. The only detail I would add is that the forum for such further unification may not in the future only be that of textualized philosophical argumentation. Perhaps this century's "Naturalizing Phenomenology" project will play out in software and code libraries — and in the intellectual spaces where these digital constructions are studied, debated, publicized, and refined — not just in the pages of books and journals.

## References

- 1 Peter Gärdenfors, *Some Tenets of Cognitive Semantics*.
- 2 Michael Jubien, *Possibility*.
- 3 Michael Jubien, *Ontology, Modality, and the Fallacy of Reference*. Cambridge University Press, 1993.
- 4 Olav K. Wiegand, *On Referring to Gestalts*.
- 5 Olav K. Wiegand, *The Phenomenological Semantics of Natural Language*.