Gary Luo
Math 70
Hw 01
February 4, 2018

A1,A2,A3,A4,A5

A1

1. Find permutations p, q in S_3 such that $p \cdot q \neq q \cdot p$
2. Prove that if $n \geq 3$ then there exists $p, q \in S_n$ such that $p \cdot q \neq q \cdot p$
3. Let $a, b \in J_n$ with $a \neq b$. Prove that $(a, b) = (b, a)$
4. Prove that $(a, b)(a, b) = e$. Here e denotes the identity in S_n
5. Compute $(a, b, c) \cdot (a, b, c) \cdot (a, b, c)$.

1. Let $p = (1, 2, 3)$ and $q = (1, 3)$. Then, we see that $p \cdot q = (1, 2, 3)$ and $q \cdot p = (1, 2)$. Clearly, these are not the same.
2. We will prove this by induction. For our base case, we will look at part 1 and see that it holds true for $n = 3$. Next,

$\blacksquare$

A2

1. Let $i \in J_n$. Prove that the permutations $(i, i+3)$ may be written as

$$(i, i+1) \cdot (i+1, i+2) \cdot (i+2, i+3) \cdot (i+2, i+1) \cdot (i+1, i).$$

   Hint: Consider $x < i$, $i \leq x \leq i+3$ and $x > i+3$.

2. Prove that if c_i and c_2 are disjoint cycles then $c_1 \cdot c_2 = c_2 \cdot c_1$

■

A3 Prove that if $A = (a_{ij})$ is an $n \times n$ triangular matrix (either $a_{ij} = 0$ for all $i < j$ or $a_{ij} = 0$ for all $i > j$), then $det(A) = a_{11} \cdot a_{22} \cdots a_{nn}$.

■

A4

1. Let A, B, C be 2×2 matrices. Let X be the 4×4 matrix whose first two rows are $[A\ B]$ and its last two rows are $[0\ C]$, where 0 stands for the 2×2 matrix whose entries are the all equal to zero. Prove that $det(X) = det(A)det(C)$.

2. Generalize this result for $2n \times 2n$ matrices.

■

A5 For each $i, j \in J_n$ let $a_{ij}(x)$ be a differentiable functions of x. Let $A(x) = (a_{ij}(x))$. Prove that $(det(A(x)))' =_{i=1}^{n} det(B_i(x))$ where $B_i(x)$ is the matrix resulting from the replacing in $A(x)$ the i-th column by the column made up by the derivatives of the i-th column of $A(x)$.

■