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Accretion Disc Models for Compact X-Ray Sources

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Summary. The accretion process is considered, in cases when the infalling matter possesses angular momentum and forms a disc spinning around a central compact mass. This situation occurs when gas falls onto a black hole or neutron star from a binary companion, and may be relevant to galactic X-ray sources. The spectrum of the radiation emitted by gas spiralling down into a black hole is calculated and compared with the data on Cygnus X-1. Rapid irregular variability is expected in this model. If the compact object is a neutron star, the gas dynamics near the star may be controlled by the stellar magnetic field. The main X-ray emission then comes not from the disc, but from regions on the stellar surface near the magnetic poles. An interpretation of Centaurus X-3 involving a rotating magnetized neutron star is proposed. It is emphasized that accretion discs can display a wide range of properties (depending on the accretion rate, viscosity, etc.). Tentative interpretation of some other phenomena are also proposed.

Key words: accretion – binary stars – X-ray sources

1. Introduction

A number of galactic X-ray sources are now known to be components of close binary systems. Their rapid ($\lesssim 1$ s) variability suggests that a compact object – either a neutron star or a black hole – is involved. Both these facts strongly support earlier suggestions (see, e.g. Shlovskii, 1967; Prendergast and Burbidge, 1968) that X-ray sources are an accretion phenomenon: capture of material from the binary companion could supply an abundant mass flux; and each gram of matter, falling into the deep gravitational well associated with a compact object, could yield $\sim 10^{20}$ erg of radiation.

Material transferred from the companion star is likely to have so much angular momentum that it cannot fall directly onto the compact object. The fairly extensive literature on spherically symmetric accretion (reviewed by, for example, Zeldovich and Novikov, 1971) is thus not directly relevant. The matter will, instead, form a differentially rotating disc – the circular velocity at any point being approximately Keplerian – composed of material which gradually spirals inward as viscosity transports its angular momentum outward. Prendergast and Burbidge (1968) have already considered some aspects of this kind of model. These authors estimated that as much as half the material transferred from the large star could fall onto the compact object, and discussed the spectrum of the radiation from the disc under certain assumptions. Schwartzman (1971b) has also speculated along the same lines as in the present discussion. A somewhat similar situation, but on a much larger scale, was discussed by Lynden-Bell (1969) and by Lynden-Bell and Rees (1971), with application to active galactic nuclei.

In the present paper we explore some further properties of accretion models for X-ray sources. For plausible parameters the energy released by the gradually infalling matter is radiated by bremsstrahlung. Because of the small size of the effective emitting region, the temperature $\gtrsim 10^7$ °K, and most of the energy is radiated as X-rays (see also footnote 2). We consider, in particular, the spectrum of the radiation from the disc, and the effects of a strong magnetic field attached to the compact central star (which may be rotating). Our results suggest that one may envisage six types of X-ray source, which fall into two main categories, depending on the nature of the compact object.

I. When the Central Object is a "Black Hole"

(a) If the mass flux lies within a certain range, the observed radiation comes almost solely from the disc. We show that in this case a power law X-ray spectrum may result.

(b) If the mass flux is *either* so *large* that radiation pressure prevents steady accretion, *or* so *small* that the gravitational energy released cannot be radiated efficiently, we expect some kind of flaring, with the black hole alternately accreting matter and expelling it from its environment.

II. When the Central Object is a Neutron Star

(a) If the mass flux is again too large or too small, the situation resembles I(b).

(b) If the star has a negligible magnetic field, the disc extends down to the surface. We then expect comparable amounts of X-ray power to come from the star and the disc. The spectrum would then be thermal, with perhaps an apparently non-thermal high energy "tail".

(c) If the star has a magnetic field and is spinning slowly, accretion takes place along field lines. The X-rays would then be emitted thermally from regions near the magnetic poles of the neutron star. Unless the field is axisymmetric, the received radiation would be pulsed, with a period equal to the star's rotation period.

(d) If the star rotates rapidly, or has a very strong magnetic field, accretion cannot take place, and energy would instead be supplied to the system from the stellar rotational energy. It would seem likely that, in this case, the rotation would give rise to some rapid but regular periodic effect.

The gas flow in binary systems is a complex subject which we do not even attempt to discuss here. However almost all the gravitational energy of the accreted material is liberated at distances from the compact object very small compared to the binary separation. We are therefore justified in ignoring the gravitational effect of the other star, and simply considering a circular disc surrounding a compact object. This we do in § 2. We then (§ 3) calculate the influence on the accretion process of a dipole-type field attached to the central star. In § 4 we tentatively apply our results to some particular observed sources.

2. Accretion Discs

The detailed properties of disc models depend on what assumptions are made about turbulence, viscosity, etc. (see, for example, Prendergast and Burbidge (1968) or Lynden-Bell (1969). We shall here, in general, follow the notation of the latter paper). For the sake of argument we shall take a specific – and, we hope, reasonable – model for the disc, at the same time allowing ourselves a few variable parameters.

The disc is held out by centrifugal force, and so the circular velocity, V_c, is Keplerian:

$$\text{i.e.}\quad V_c \approx 1.15\times 10^{10}\,(M/M_\odot)^{1/2}\,R_6^{-1/2}\,\mathrm{cm\,s^{-1}} \tag{1}$$

where R_6 is the radius of the orbit in units of 10^6 cm, and M is the mass of the central object. In all the situations we consider, the gravitational effects of the disc itself will be negligible. We also neglect relativistic effects. This is a good approximation when $R_6 \gg (M/M_\odot)$ and inclusion of the relativistic corrections would not significantly alter any of our conclusions.

The thickness of the disc is determined by the balance between the pressure in the disc (comprising turbulent and magnetic pressure, as well as ordinary gas and radiation pressure) and the relevant component of the gravitational pull of the central object. We shall take the disc semithickness b as

$$b = \frac{x}{20}\times R = 5\times 10^4\, x R_6\ \mathrm{cm}\,, \tag{2}$$

where x is a dimensionless parameter which we expect to be of order unity, and which may depend on the radius R. (Strictly speaking, b is the scale height of the disc, but throughout this paper we neglect any gradual dependence of density and velocity on distance from the plane of symmetry.)

The material in the disc will drift inward at a rate V_r which depends on how fast angular momentum can be transported outwards, i.e. on the viscosity. We assume

$$V_r = y\,\frac{V_c}{100} \tag{3}$$

where y is a parameter (which we again expect, following Lynden-Bell (1969), to be ~ 1) depending on the viscosity. In a steady state when the flux of accreted matter is F, the density $\varrho(R)$ is given by

$$F = 2\pi R\cdot 2b\cdot \varrho V_r\,. \tag{4}$$

Hence

$$\varrho = 1.4\times 10^{-4}\, y^{-1} x^{-1} (M/M_\odot)^{-1/2} F_{16} R_6^{-3/2}\ \mathrm{g\,cm^{-3}} \tag{5}$$

where F_{16} is the mass flux in the convenient units of $10^{16}\ \mathrm{gm\,s^{-1}} \approx 1.5\times 10^{-10}\, M_\odot\ \mathrm{year^{-1}}$.

To radiate the energy dissipated by viscosity, the disc must emit a power per unit area of

$$\begin{aligned} p(R) &= \frac{3FGM}{4\pi R^3} \\ &= 1.1\times 10^{23} F_{16} (M/M_\odot)\, R_6^{-3}\ \mathrm{erg\,cm^{-2}\,s^{-1}}\,. \end{aligned} \tag{6}$$

We note in passing that, if the central object is a neutron star of radius R_{ns}, and the disc extends inwards to its surface, the total luminosity of the disc is

$$L \approx 7\times 10^{35}\, F_{16}\,(M/M_\odot)\left(\frac{R_{ns}}{10^6\ \mathrm{cm}}\right)^{-1} \mathrm{erg\,s^{-1}}\,. \tag{7}$$

For a disc surrounding a Kerr black hole with the maximum allowable angular momentum to mass ratio, the material continues to spiral gradually inward until it reaches the most tightly bound stable orbit, whose binding energy is $0.42\,c^2$ per unit mass (Bardeen, 1970). We then have

$$L \approx 4\times 10^{36}\, F_{16}\ \mathrm{erg\,s^{-1}}\,. \tag{8}$$

If the disc radiated like a black body, the temperature would be

$$\begin{aligned} T_{bb}(R) &= \left(\frac{p(R)}{2\sigma}\right)^{1/4} \\ &= 7.3\times 10^6\, F_{16}^{1/4}\,(M/M_\odot)^{1/4}\, R_6^{-3/4}\ {}^\circ\mathrm{K} \end{aligned} \tag{9}$$

where σ is Stefan's constant.

This is, of course, a *lower limit* to the temperature: the disc would become substantially *hotter* than T_{bb} if *either*

(i) cooling processes were not efficient enough to radiate the power $p(R)$ at a temperature $\sim T_{bb}$;

or (ii) the main contribution to the opacity comes from *electron scattering*, in which case the surface brightness cannot attain the full black body intensity (Felten and Rees, 1972).

We consider first the inner part of the disc where $T_{bb} \gtrsim 10^4$ °K. Since the actual temperature cannot be less than T_{bb}, these regions must be almost completely ionized, the electron density n_e being comparable with the total particle density n.

For discs with temperatures high enough to be relevant to X-ray sources, the dominant emission mechanism is free-free radiation (bremsstrahlung [1]) which yields a cooling rate $\propto n_e^2 T^{1/2}$ per unit volume (if we neglect the T-dependence of the Gaunt factor). The temperature needed to give the required output by this process is

$$T_{ff}(R) \simeq 1.2 \times 10^{10} F_{16}^{-2} (M/M_\odot)^4 x^2 y^4 R_6^{-2} \,°\mathrm{K}. \quad (10)$$

If the effects of electron scattering ((ii) above) were negligible, the actual disc temperature would be

$$T \approx \max[T_{bb}, T_{ff}]. \quad (11)$$

Since T_{ff} decreases more sharply than T_{bb} with increasing R, (if x and y are taken to be constants) we expect the outer parts of the disc to have essentially the black body temperature given by (9); the inner parts, however may be unable to radiate the locally-generated power ($\propto R^{-3}$ per unit area) unless they are much hotter than T_{bb}. The critical radius at which $T_{bb} \approx T_{ff}$ – which is also the place where the disc has a free-free optical depth $\tau_{ff}(\nu)$ of order unity for photons near the black body peak ($h\nu \approx 3kT$) – is

$$\left(\frac{R_{ff}}{10^6\,\mathrm{cm}}\right) \approx 3.4 \times 10^2 F_{16}^{-9/5} (M/M_\odot)^3 x^{8/5} y^{16/5}. \quad (12)$$

We note that this is sensitive to the uncertain parameters x and y.

The optical depth of the disc to electron scattering is $2\tau_{es}$, where

$$\tau_{es} \approx 1.9 (M/M_\odot)^{-1/2} F_{16} R_6^{-1/2} y^{-1}. \quad (13)$$

If $\tau_{es} > 1$, *and* $\tau_{ff}(h\nu \approx 3kT) < \tau_{es}$, the peak power that can be radiated by a disc at temperature T is reduced by a factor $\mathfrak{R}^{-1/2}$ below the black body intensity, where $\mathfrak{R} = \tau_{es}/\tau_{ff}(\nu = 3kT/h)$.

As a corollary, the disc may have to be hotter than (11) would imply; and we then find that the disc temperature is

$$T = \max[T_{bb}, T_{\mathrm{mod}}, T_{ff}], \quad (14)$$

where T_{mod} ($> T_{bb}$ when $\mathfrak{R} > 1$ and electron scattering opacity is dominant) is

$$T_{\mathrm{mod}}(R) = 4.4 \times 10^7 F_{16}^{2/9} (M/M_\odot)^{5/9} y^{2/9} x^{2/9} R_6^{-1} \,°\mathrm{K}. \quad (15)$$

[1]) Detailed computations by, for example, Cox and Tucker (1969) show that free-free cooling dominates for $T \gtrsim 10^7$ °K in a collisionally ionized gas with normal "cosmic" abundances. In X-ray sources, photoionization by thermal photons raises the ionization level at a given temperature. This reduces the importance of cooling by line emission, and so free-free cooling may be dominant even below $\sim 10^7$ °K.

The change-over from T_{bb} to T_{mod} occurs at (point B in Fig. 1)

$$\left(\frac{R_B}{10^6\,\mathrm{cm}}\right) = 1.3 \times 10^3 F_{16}^{-1/9} (M/M_\odot)^{11/9} x^{8/9} y^{8/9} \quad (16)$$

and the change-over from T_{mod} to T_{ff} at (point C in Fig. 1)

$$\left(\frac{R_C}{10^6\,\mathrm{cm}}\right) = 2.7 \times 10^2 F_{16}^{-20/9} (M/M_\odot)^{31/9} x^{16/9} y^{34/9}. \quad (17)$$

Thus it is necessary to take electron scattering into account in calculating the radiation spectrum from the disc if $R_B > R_C$, or, approximately

$$1.7 \times F_{16} > (M/M_\odot)\, x^{2/5} y^{5/4}. \quad (18)$$

If the factor $\mathfrak{R}$ exceeds $m_e c^2/kT$, then the outgoing spectrum is modified by the cumulative effect of the frequency shifts (each of order $\delta\nu/\nu \approx (kT/m_e c^2)^{1/2}$) in successive scatterings. This process alters the value of T_{mod}, and also gives rise to an extra cooling mechanism, since electrons will lose energy by scattering the photons emitted as free-free radiation. But these effects introduce only logarithmic corrections into the foregoing equations, and we shall not consider them in any further detail here.

Some words of caution are necessary at this stage. The above discussion is only valid provided that the component of the star's gravitational field perpendicular to the disc is strong enough to balance the radiation pressure gradient in that direction, i.e. provided that

$$R_6 \gtrsim 0.6 F_{16} x^{-1}. \quad (19)$$

A consequence of this is, of course, that the luminosity/mass ratio of the source cannot exceed the usual Eddington limit. If F_{16} is very large, the disc would thicken until the configuration became almost spherical. The material would then fall radially inward, or be expelled, without yielding much energy in the form of radiation. Thermal gas pressure P_g would itself thicken the disc until x satisfied the inequality

$$(x/20) \lesssim V_c^{-1} (P_g/\varrho)^{1/2}.$$

This means that disc models are only consistent when

$$T \lesssim T_{\max} = 2.4 \times 10^9 (M/M_\odot) R_6^{-1} x^2 \,°\mathrm{K}. \quad (20)$$

If $T \approx T_{ff}$, and T_{ff} violates (20), then here again the disc expands into a spherical configuration. The fact that T_{ff}, like the right hand side of (20), also depends on x^2, means that the disc cannot stabilise itself against this instability merely by increasing x^2). T_{ff} satisfies (20)

[2]) Note that, in disc models, $\dfrac{kT(R)}{m_H}$ is always $\ll GM/R$. It is because of this that we can get X-rays rather than γ-rays, even from the immediate vicinity of a collapsed object, and thereby convert the rest-mass energy of infalling gas into X-rays with high efficiency. Typical galactic sources would require accretion rates of only $\sim 10^{-10} M_\odot$ per year.

when

$$R_6 \gtrsim 5 F_{16}^{-2} (M/M_\odot)^3 y^4 \,. \tag{21}$$

At radii where (21) is violated, the accreted matter falls inward without necessarily releasing much further energy until (if the central object is a neutron star rather than a black hole) it impacts on the surface.

It is clear that even the idealised disc models that we have considered can display several different kinds of behaviour, depending on the particular parameters. If x and y are assumed independent of R, then the temperature increases towards the centre. If electron scattering is *un*important, then T varies as $R^{-3/4}$ for $R > R_{ff}$ (Eq. (12)), each annulus radiating like a black body; for $R < R_{ff}$, T varies as R^{-2}, and the spectrum is "flat", with free-free absorption being important only for frequencies with $h\nu \ll kT_{ff}$. If electron scattering *is* important ($\Re > 1$) there is an intermediate range of radii over which $T \approx T_{\rm mod} \propto R^{-1}$. (If the electron scattering optical depth is very large, the emergent spectrum may be distorted towards a Wien law.) The dependence of T on R is illustrated in *Fig. 1*. The temperature T_{ff} rises as R decreases, and eventually the inequality (20) may be violated. This also is illustrated in the figure.

These results would be altered if x and y were R-dependent. Indeed, if this dependence were very strong even the qualitative features of Fig. 1 might change – for example T_{ff} might in some circumstances be an *increasing* function of R. (We suggest in § 4 that this may happen in Cygnus X-1.)

In the outermost parts of the disc, where T_{bb} falls below 10^4 °K and the power generation rate $p(R)$ is low, the gas will not necessarily be completely ionized. Instead, the ionization level will adjust so that the radiation losses – primarily line emission and recombination – balance $p(R)$. As is well known for the case of the interstellar medium, the electron temperature stays close to $\sim 10^4$ °K for a wide range of values of $p(R)$, with n_e/n adjusting itself somewhere between 0.01 and 1. We find $n_e/n \propto R^{-1/2} x^{1/2} y$.

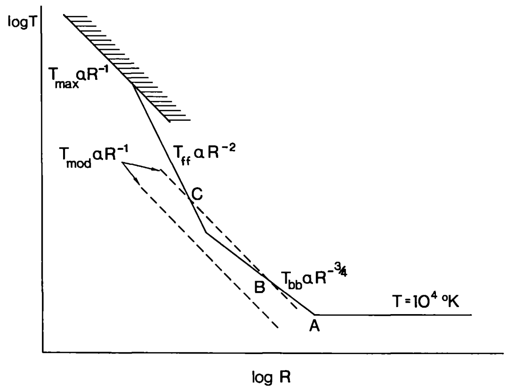

Fig. 1. Schematic representation of the dependence of T on R in models where x and y are taken as constant. In the outer parts (outside A) the ionization level adjusts itself to keep T constant $\simeq 10^4$ °K. Inside A, the matter is fully ionized and the disc is at the blackbody temperature T_{bb}. When the disc becomes optically thin, free-free emission becomes dominant and the temperature T_{ff} rises more steeply. This continues until $T = T_{\max}$ where gas pressure thickens the disc and the model is no longer applicable. The temperature $T_{\rm mod}$ due to modification of the spectra through electron scattering is shown as dotted lines. The two positions depend on the parameters x, y and demonstrate how it may or may not be relevant

The total emission spectrum $S(\nu)$ is calculated as an integral over the spectra emitted by the elemental rings that comprise the disc. Although the spectrum emitted by the material at a particular radius is thermal, the integrated spectrum from the whole disc may be of power-law form, thus mimicking a non-thermal spectrum. We shall also allow for a possible R-dependence of x and y, and take $x \propto R^\mu$, $y \propto R^\lambda$. (Note, however, that is no obvious reason why x and y should actually obey power laws. On the other hand we have shown that a power law dependence of T or R would arise more naturally.) The spectra are of the form $S(\nu) \propto \nu^{-\alpha}$, where the slope α is easily evaluated for the different cases

(a) Black Body Regime

$$\alpha = -\tfrac{1}{3}\,.$$

(b) Modified Black Body

$$\alpha = \frac{2 - \dfrac{\lambda}{3} - \dfrac{2\mu}{3}}{1 - \dfrac{2\lambda}{9} - \dfrac{2\mu}{9}} - 2\,.$$

(c) Free-free

$$\alpha = \frac{1-(4\lambda + 2\mu)}{2-(4\lambda+2\mu)}\,. \tag{23}$$

(d) Integrated Emission from the Partially Ionized outer Parts of the Disc

$$\alpha = -\tfrac{2}{3}\,.$$

The above formulae are only valid provided that the calculated spectrum falls off, at low frequencies, *less* rapidly than the spectrum contributed by the material

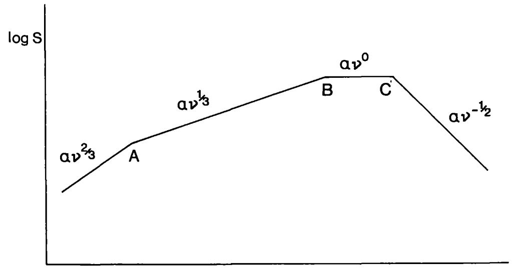

Fig. 2. A representation of the overall power spectrum from the disc in the case when x and y are constant, and when $T_{\rm mod}$ is relevant (see Fig. 1). The bends on the spectrum at A, B and C correspond to the points marked similarly in Fig. 1. The spectrum would cut off exponentially at $T_{\max}$

with a particular value of R. This has little effect on (a), (b) and (d), but in (c) must demand that the numerical value of $4\lambda+2\mu$ does *not* lie between 1 and 2.
Figure 2 shows the type of spectrum expected in the particular case when λ and μ are zero.

3. The Inner Edge of the Disc, and the Influence of a Stellar Magnetic Field

The inner edge of the disc is defined *either* by the smallest value of R for which (19) and (21) hold, *or* by the radius at which the influence of the central object becomes critical. If the latter is a black hole, then (provided (19) and (21) are satisfied), the disc will extend inwards to the position of the most tightly bound stable circular orbit. Once the inward-spiralling matter reaches this point, it can be swallowed by the black hole without releasing a significant further amount of energy. The total luminosity per gram may approach the theoretical limit given by (8) – note, however, that relativistic corrections modify the detailed spectrum of the inner parts of the disc, from which most of the energy comes.
If the central object is an *unmagnetized* neutron star, then (if (19) and (21) hold) the disc terminates at $R\approx R_{ns}$, where the matter grazes the surface of the star. The integrated luminosity L of the disc is given by (7). However the matter impinging on the star heats it, causing an amount of power comparable with L to be radiated thermally from the stellar surface. (This power is precisely equal to L if the star is not rotating, but is somewhat less if the star is spinning in the same sense as the disc.)
A neutron star – unlike an (uncharged) black hole – can possess a frozen – in magnetic field, which may be strong enough to affect the dynamics of the inflowing gas even at radii $R\gg R_{ns}$. We shall now attempt to estimate this effect. We have in mind particularly the case when the stellar field is non-axisymmetric (e.g. an oblique dipole field), since in this case the X-ray emission would be non-isotropic, giving rise to pulses with the rotation period of the star.
The matter in the disc will be highly ionized, so that once in contact with the stellar field it will only be able to move along field lines. Assuming the field to be dipolar, its magnitude is given by

$$B=10^{12}\,B_{*12}\left(\frac{R}{R_{ns}}\right)^{-3}\ \mathrm{G} \tag{24}$$

where B_{*12} is the field at the stellar surface, at the point on the same radius vector, in units of 10^{12} G (the typical field strength estimated for pulsars).
The star is taken to have a rotational period P seconds, and we define the corotation radius R_Ω to be the radius at which the keplerian and stellar angular velocities are equal,

$$\left(\frac{R_\Omega}{10^6\ \mathrm{cm}}\right)=1.5\times10^2\,P^{2/3}\,(M/M_\odot)^{1/3}\,. \tag{25}$$

For accretion to take place the inner edge of the disc must be within R_Ω. There the angular momentum of the matter is transferred to the star via the magnetic field, and accretion along field lines results. However we must first investigate whether the star can throw off any infalling matter *before* it penetrates within R_Ω.
Let us envisage a steady situation in which matter from the inner edge of the disc is being continuously ejected by the rotating magnetic field attached to the star. Consider a ring of matter, mass m and circular velocity V_c, being accelerated by the star. By Newton's law for the ring, we may write

$$d/dt(mV_c)=\frac{B_p B_\phi}{4\pi}\times A' \tag{26}$$

where B_ϕ and B_p are the azimuthal and poloidal parts of the magnetic field at the ring and A' is the area over which contact is made. From energy considerations we see that to throw matter out we need

$$d/dt(\tfrac{1}{2}mV_c^2)\gtrsim\frac{FGM}{R}\,, \tag{27}$$

i.e. the rate at which kinetic energy is being supplied by the star must exceed the rate at which potential energy is being lost in the disc.
Combining the above we obtain the inequality

$$F\lesssim\frac{B_\phi B_p V_c R A'}{4\pi GM}\,. \tag{28}$$

The size of the area A' depends on the magneto-dynamics at the interface and also on the orientation of the stellar field. However, as an order of magnitude estimate we may take

$$A'\approx 2\pi R\cdot 2b\,. \tag{29}$$

The poloidal field component B_p is part of the stellar field, so $B_p=B_{*12}\,10^{12}\left(\frac{R}{R_{ns}}\right)^{-3}$ G. The azimuthal component depends on the detailed magneto-dynamics at the interface; we shall leave the ratio B_ϕ/B_p as an unknown, hoping that it is of order unity.
Thus we find that (28) becomes

$$F_{16}\lesssim 5.5\times10^8\,xB_{*12}^2\left(\frac{B_\phi}{B_p}\right)(M/M_\odot)^{-1/2}\left(\frac{R_{ns}}{10^6\ \mathrm{cm}}\right)^6 R_6^{-7/2}. \tag{30}$$

The condition that accretion be able to take place is that the reverse inequality hold at R_Ω,
i.e.

$$F_{16}\gtrsim F_{\Omega 16}=13\,xB_{*12}^2\left(\frac{B_\phi}{B_p}\right)(M/M_\odot)^{-5/3}P^{-7/3}\left(\frac{R_{ns}}{10^6\ \mathrm{cm}}\right)^6 \tag{31}$$

An approximate upper limit to the incoming mass flux is given by the "Eddington limit", at which the pressure of outward flowing radiation on free electrons would limit accretion,

i.e. $F_{16} \lesssim F_{\mathrm{Edd}\,16} \approx 100(R_{ns}/10^6\ \mathrm{cm})$. (32)

The Eddington limit holds strictly only for a spherically symmetric situation but it is unlikely that it can be substantially exceeded in the geometry under consideration here.

By requiring that $F_\Omega \lesssim F_{\mathrm{Edd}}$ we obtain a lower limit on the rotational period P if accretion is to occur:

$$P \gtrsim 0.42\, B_{*12}^{6/7} \left(\frac{B_\phi}{B_P}\right)^{3/7} \left(\frac{F_{\mathrm{Edd}\,16}}{100}\right)^{-3/7} \cdot (M/M_\odot)^{-5/7} \left(\frac{R_{ns}}{10^6\ \mathrm{cm}}\right)^{15/7} x^{\frac{3}{7}}\ \mathrm{s}. \tag{33}$$

Schwartzman (1971a) has considered the inhibition of accretion by the particle flux from pulsar-like objects. By assuming a theoretical dependence of this flux on B and Ω, he obtains a condition on F involving these parameters. The process described here is quite distinct from that considered by Schwartzman.

For any disc model we may also define a magnetic radius R_M as being the radius down to which the gas can crush the stellar field. More precisely it is the radius at which the effective pressure in the disc is equal to the external magnetic pressure (for self consistency of the accretion model we require $R_M \lesssim R_\Omega$). To obtain an estimate of the pressure P' in the disc we equate the pressure gradient to the component of gravitation perpendicular to the disc,

$$P'/b \simeq \varrho \left(\frac{GM}{R^2}\right) \cdot \left(\frac{b}{R}\right). \tag{34}$$

By requiring the two pressures to be equal at R_M, we obtain

$$\left(\frac{R_M}{10^6\ \mathrm{cm}}\right) = 5.2 \times 10^2 B_{*12}^{4/7} F_{16}^{-2/7} (M/M_\odot)^{-1/7} \cdot x^{-2/7} y^{2/7} \left(\frac{R_{ns}}{10^6\ \mathrm{cm}}\right)^{12/7}. \tag{35}$$

The condition ($R_M \lesssim R_\Omega$) then yields a lower bound to the density of the disc, for a given value of x,

$$y^{-1} \gtrsim 76\, B_{*12}^2 F_{16}^{-1} (M/M_\odot)^{-5/3} P^{-7/3} x^{-1} \left(\frac{R_{ns}}{10^6\ \mathrm{cm}}\right)^6. \tag{36}$$

Change of the Stellar Period

We shall take the inner edge of the disc to be at $R_M \lesssim R_\Omega$. There the matter's angular momentum is transferred to the star, speeding the star up and thus tending to reduce R_Ω to R_M.

The flux of angular momentum at R_M is

$$h = F(GMR_M)^{1/2} = 1.2 \times 10^{32} F_{16} (M/M_\odot)^{1/2} \left(\frac{R_M}{10^6\ \mathrm{cm}}\right)^{1/2} \mathrm{g\ cm^{-2}\ s^{-2}}. \tag{37}$$

Taking the moment of inertia of the star to be

$$I = 2/5 M R_{ns}^2 = 8 \times 10^{44} (M/M_\odot) \left(\frac{R_{ns}}{10^6\ \mathrm{cm}}\right)^2 \mathrm{g\ cm^2}$$

we obtain

$$\dot{P}/P = -7.2 \times 10^{-7} F_{16} (M/M_\odot)^{-1/2} \cdot P \left(\frac{R_M}{10^6\ \mathrm{cm}}\right)^{1/2} \left(\frac{R_{ns}}{10^6\ \mathrm{cm}}\right)^{-2} \mathrm{year}^{-1}. \tag{38}$$

Note that the energy of the emitted radiation comes from the gravitational energy released by the infalling matter. It does *not* come from the rotational energy of the neutron star – indeed, (38) tells us that in all cases when accretion is allowed the matter falling in from the disc actually *speeds up* the stellar rotation.

Radiation from the Star

For $R_M/R_{ns} \gtrsim 3$ the flow near the star takes place along field lines the gas being funnelled down an almost undistorted dipole field. Thus most of the radiation will come from the stellar magnetic poles. The condition can be written.

$$B_{*12} \gtrsim 1.8 \times 10^{-4} F_{16}^{1/2} (M/M_\odot)^{1/4} \cdot \left(\frac{R_{ns}}{10^6\ \mathrm{cm}}\right)^{-5/4} x^{1/2} y^{-1/2}. \tag{39}$$

The area A on the star over which accretion takes place is given approximately by

$A \approx R_{ns}^3/R_M\ \mathrm{cm}^2$.

We shall take

$A = 10^{10} a^2\ \mathrm{cm}^2$

where we see that

$$a^2 = \left(\frac{R_{ns}}{10^6\ \mathrm{cm}}\right)^2 \frac{R_{ns}}{(R_M/100)}.$$

The infalling gas has plenty of time to radiate, and the accretion will be supersonic. In fact the funnelling effect modifies Bondi's (1952) condition to be $1 < \gamma < 7/5$ where γ is the usual ratio of specific heats.

The infall velocity is taken to be freefall, which yields the density in the column to be

$$\varrho = 6.2 \times 10^{-5} a^{-2} F_{16} (M/M_\odot)^{-1/2} R_6^{-5/2}\ \mathrm{g\ cm^{-3}}. \tag{40}$$

The kinetic energy of the gas will be dissipated when it strikes the stellar surface, the spectrum of radiation emitted being that of a hot thermal source with possibly a high energy tail. (See e.g. Zeldovich and Shakura (1969).) For a black body spectrum the temperature is given by

$$T = 1.3 \times 10^7 F_{16}^{1/4} (M/M_\odot)^{1/4} \cdot \left(\frac{R_M}{10^6\ \mathrm{cm}}\right)^{1/4} \left(\frac{R_{ns}}{10^6\ \mathrm{cm}}\right)^{-1} {}^\circ\mathrm{K}. \tag{41}$$

A large amount of both circular and linear polarization might be expected, even though the radiation is basically thermal, since it is produced in the presence of a large magnetic field. However the direction in which the radiation is emitted will be affected by the opacity of the infalling material. At these temperatures electron scattering provides the dominant opacity. The vertical optical depth from a point in the centre of the column is

$$\tau_v = 9.9\, a^{-2} F_{16} (M/M_{\odot})^{-1/2} R_6^{-3/2} \tag{42}$$

whereas the *horizontal* optical depth from the same point is

$$\tau_h = 1.24\, a^{-1} F_{16} (M/M_{\odot})^{-1/2} R_6^{-5/2}. \tag{43}$$

Thus the radiation will come out along a cone, and to an external observer would appear regularly pulsed.

4. Tentative Interpretations of Some Particular Sources

We may now attempt to relate the model to observations of some individual X-ray sources in the galaxy.

Cygnus X-1

Cygnus X-1 is a prime candidate for being a black hole accreting matter from a disc. It belongs to a binary system in which mass transfer seems to be taking place (Webster and Murdin, 1972; Bolton, 1972), and its mass has been estimated to lie well above the limiting mass for neutron stars or white dwarfs. Also, it has an apparently non-thermal X-ray spectrum, which is what we would expect for a disc whose temperature depends on radius.

The observed spectral index α varies between ~ 1.6 and ~ 4 in the 1–10 keV range (Schreier *et al.*, 1971), which exceeds the value arising from free-free emission in the case $\lambda = \mu = 0$. But Eq. (23) shows that a steeper slope may occur if λ and μ have suitable non-zero values. (For these values, however, T_{ff} increases with R. Also, the precise value of α is very sensitive to λ and μ, so the variable slope is not surprising if the disc is somewhat unsteady.) The intensity fluctuates on timescales $\lesssim 1$ s. There may be short "pulse trains" (though even this is controversial (Terrell, 1972)), but there is definitely no evidence for any single preferred period. The rotation rate at the inner edge of the disc is $\sim 10^{-3}\,(M/M_{\odot})$ s, and the observations suggest $M \simeq 4 M_{\odot}$. This model therefore allows variability on timescales as short as this. The infall time is ~ 100 times the rotation period, and this is perhaps the most likely timescale for variability. If the viscosity is primarily magnetic in origin, one could speculate that, even if F were constant, "flares" might sometimes burst out of the disc, giving rise to the rapid fluctuations. Should the amplitude or timescale of the X-ray variations depend on photon energies, this would tell us something about the form of $T(R)$.

Above 10 keV, the X-ray spectrum flattens. This could be due to a change in the density gradient (i.e. in λ or μ) at some radius, or to thermalization effects. Alternatively, these hard X-rays may be due to a different mechanism, as proposed by Jackson (1972), who has attempted to explain the "anti-eclipse" (Dolan, 1971; Bolton, 1972; Webster and Murdin, 1972) observed at $\gtrsim 20$ keV [3]).

Centaurus X-3

The most marked features of this source are the large amplitude pulsations, with a period of about 4.8 s. The period is steady apart from the 2.09 day modulation due to orbital motion around the inferred binary companion. The pulses certainly contain $\gtrsim 70\%$ of the X-ray energy, and may contribute as much as 99% (Schreier *et al.*, 1972).

We suggest that Centaurus X-3 is a neutron star with an oblique magnetic field and a 4.8 s rotation period, and that the X-ray pulses are due to accreted material funnelled down to the magnetic poles. The exact shape of the pulses would depend on the shape of the emission cone, the orientations of the magnetic and rotation axes, and the angle our line of sight makes with the plane of the disc. The pulse shape plotted by Schreier *et al.* (1972) gives some indications of an "interpulse", suggesting that both magnetic poles contribute to the observed pulsing.

The overall spectrum of Centaurus X-3 is thermal, as would be expected for our model (though substantial linear or circular polarization is quite possible); and it is clear from (41) that the observed temperature of $\sim 3 \times 10^7$ °K (Giacconi *et al.*, 1971) can be matched for a plausible choice of F_{16} and R_M. Equation (38) tells us that the accretion causes the stellar rotation rate to alter. The reported decrease of 1 part in ~ 4000 between January and May 1971 is of the right sign and order of magnitude. The mass of the compact object in Centaurus X-3 is not yet known. Our suggestion obviously requires that it should not exceed the limiting neutron star mass. (We note, in this connection, that a "pulsar" model for Centaurus X-3 – in which the kinetic energy of a spinning neutron star provides the power – is not tenable. The rotational energy of a neutron star with a period as long as ~ 5 s would only be able to power the X-ray emission for $\lesssim 100$ years so a substantial *increase* in the period would already have been detected.)

It is perhaps significant that both this source and the 1.25 s period pulsating X-ray source on Hercules, designated 2U1702 +35 in the UHURU catalogue (Giacconi *et al.*, 1972) are associated with *eclipsing* binary systems. For there to be noticeable pulsations, the mag-

[3]) Jackson's model attributes the hard X-rays to inverse Compton scattering of light from the companion B star in a region $\sim 10^{11}$ cm across. It would therefore not be tenable if the hard X-rays displayed rapid variability. An alternative interpretation of the "anti-eclipse" might then be that the binary orbit is eccentric, so that the mass transfer rate, and hence F, varies with the 6 day binary period.

netic axis may have to make a *large* angle with the rotation axis. If this is so, and if the half-angle of the emission cone is small (say $\lesssim 45°$), this may not be a coincidence.

Relevance of Model to Other Observations

The sources discussed above are the two whose binary character is most firmly established. We suspect, however, that our model has more general relevance, though applications to other specific sources are at the moment somewhat conjectural.

We have seen that even our present simple model can display a wide range of behaviour for different values of the few free parameters we have included. The X-ray emission could vary on any time-scale down to $\sim 10^{-3}$ s, either irregularly (black hole) or with a steady period (spinning magnetized neutron star). X-ray spectra of either power law or exponential form are obtainable. Violently flaring sources, such as the source in Crux observed by Lewin *et al.* (1971), may be discs with unsteady accretion rate F, which become unstable – because (19) or (21) is violated – for some values of F. It is possible that matter is alternately accreted and expelled.

Finally, we briefly indicate some reasons why searches for line emission from galactic X-ray sources may not prove fruitful. We have interpreted the X-ray pulses of Centaurus X-3 as thermal emission from "caps" around the magnetic poles of a neutron star. This radiation would have an essentially black body spectrum (but perhaps with the low energy photons attenuated by absorption), and any line features would be much less prominent than those from an optically thin plasma. In objects when the X-ray emissions comes primarily from the disc itself – as we suspect is the case in Cygnus X-1 – line emission may be comparable to free-free emission. However, the lines would be severely Doppler broadened so that $\delta v/v \simeq V_c/c$ (with V_c given by (1)). Since most of the energy is liberated near the inner edge of the disc, where V_c is large, we would generally expect line widths $\gtrsim 10\%$. The only exception to this would be cases when $F_{16} \gg 1$, in which case inequality (19) may be violated and the disc expanded by radiation pressure, out to values of R so large that $V_c/c \ll 1$. The emission lines, especially resonance lines may be further broadened by electron scattering (Angel, 1969; Felten *et al.*, 1972).

The source whose spectrum has been searched most thoroughly for evidence of line emission is Sco X-1. So far no narrow emission lines have been seen, the upper limits falling well below estimates based on simple models. Since the binary nature of Sco X-1 is still an open question, the relevance to this source of our present discussion is unclear. However the similarity of its hard X-ray/soft γ-ray spectrum to that of Cyg X-1 is worth bearing in mind (Haymes *et al.*, 1972). We wish to emphasize strongly that the "predicted" line strengths in Sco X-1 are based on a simple homogeneous model whose size ($\sim 10^9$ cm) and density ($n_e \sim 10^{16}$ cm^{-3}) are inferred by attributing the infrared cut-off in the spectrum to self-absorption by the hot X-ray-emitting plasma (see Neugebauer *et al.*, 1969). In disc models, the optical and infrared emission would come from the outer parts, and the X-rays from a more compact inner region. Indeed, it is interesting that a spectrum of the kind shown in Fig. 2 fits all the data on Sco X-1, including the infrared turnover, if the disc extends out to a radius of $\gtrsim 10^{11}$ cms. (The optical and infrared emission then comes predominantly from material at $\sim 10^4$ °K rather than $\sim 6 \times 10^7$ °K, and so that linear dimensions inferred from self-absorption ($\propto T^{-1/2}$) become $\sim 10^{11}$ cm rather than $\sim 10^9$ cm.) We would expect our simple discussion of the disc's dynamics to be applicable out to $\sim 10^{11}$ cms. Beyond that radius, the gravitational effects of the companion star would no longer be negligible.

Note added in proof

Because viscosity transports *energy* outwards, as well as angular momentum, the surface brightness $p(R)$ may exceed the rate of release of gravitational energy *at radius* R by a significant factor. It has been pointed out to us independently by D. Lynden-Bell and K. S. Thorne that, except at points near the inner and outer edges of the disc, this factor is 3, and we have included it in Eq. (6). The subsequent formulae are therefore somewhat inaccurate near the edges of the disc; also, our Eq. (7) is only approximate, since L depends on the precise inner boundary condition.

When a neutron star accretes matter, it contracts, and this leads to an additional energy release whose magnitude is comparable with (7). This energy, however, will be radiated from the whole stellar surface and not just from the magnetic polar caps. It therefore contributes either a steady background of lower-energy X-rays, or (if the star contracts in discrete jumps) could emerge as sporadic bursts.

We have recently received a preprint by N. I. Shakura and R. A. Sunyaev which discusses accretion discs around black holes. These authors make more specific assumptions about the viscosity than the present paper, but obtain results which are fully consistent with our own.

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