Algorithm 1

Require: A quantum algoritm A on n qubits initialized to $|0^n\rangle$ such that $0 \leq \nu(A) \leq 1$, integer t, real $\delta > 0$. A makes no measurement until the end of the algorithm and its final measurement is a measurement of the last $k \leq n$ qubits in the computational basis.

Ensure: An estimate of $\mathbb{E}[\nu(A)]$.

1: If necessary, modify A such that it makes no measurement until the end of the algorithm; operates on initial input state $|0^n\rangle$; and its final measurement is a measurement of the last $k \leq n$ of these qubits in the computational basis.

2: Let W be the unitary operator on $k+1$ qubits defined by

$$W|x\rangle|0\rangle = |x\rangle\left(\sqrt{1-\phi(x)}\,|0\rangle + \sqrt{\phi(x)}\,|1\rangle\right) \tag{1}$$

where each computational basis state $x \in \{0,1\}^k$ is associated with a real number $\phi(x) \in [0,1]$, such that $\phi(x)$ is the value output by A when measurement x is recieved.

3: Repeat the following step $O(\log(1/\delta)$ times and output the median of the results: Apply t iterations of amplitude estimation, setting $|\psi\rangle = (I \otimes W)(A \otimes I)|0^{n+1}\rangle$, $P = I \otimes |1\rangle\langle 1|$.