# Markov Chains

**Background:**

Markov Chains are mathematical models used to describe systems that transition between different states based on probabilities. The feature that most defines Markov Chains is that the probability of transitioning to a future state depends on the current state, not on a sequence of past states. Markov Chains sequences are represented using transition matrices. In these matrices, each element represents the probability of moving from one state to another.

This project integrates mathematical concepts with real-world scenarios. These examples are a site-to-site transition system modeled as a Markov Chain, and a disease transmission model. Both problems utilize transition matricies to predict outcomes, demonstrating the practical applications of linear algebra in probabilistic systems.

**Solutions:**

Problem 1 uses the transition matrix of a Markov Chain. For part a, we need to determine the probability of an individual initially at site 3 transitioning to site 4 in 3 steps. In order to calculate this, we must use $P^3$. Then multiply that by the initial state, and you get the answer, which is 0.2882.

For part b, 100 individuals are evenly distributed across the five sites. We can represent this with a state vector of [20 ; 20 ; 20 ; 20 ; 20]. Using this and multiplying it by $P^4$, we get the answer, which is [14.8177 ; 21.5879 ; 19.4990 ; 28.9726 ; 15.1228].

For part c, we must find the steady-state vector, which is a long-term distribution of individuals across the sites. This will satisfy P'v = v. To solve this, we need to create a vector using P' and an identity matrix with a matrix of ones based on the rows of P. Then, we will create another vector using a matrix of zeroes based on the columns of P with 1. Finally, divide those two vectors and get [0.2000 ; 0.2000 ; 0.2000 ; 0.2000 ; 0.2000].

Problem 2 is based on how fast an influenza could spread. For part a, we must define a transition matrix based on students transitioning between two states, susceptible (S) and infected (I). Using the percentage provided, we can determine the following: P(S → S) = 85%, P(S → I )= 15%, P(I → S) = 0.35, P(I → I) = 0.65. With this, we can create the transition matrix, which is [0.85 0.35 ; 0.15 0.65].

For part b, for this question we see that 100/5000 students are infected. Thus, we can create an initial state vector: [4900 ; 100]. Then, we can use matrix multiplication to solve the equations. Multiplying $A^2$ and the vector, we find the number of infected students on the second day to be 1150. Multiplying $A^{10}$ and the vector, we find the number of infected students on the tenth day to be 4986e+03.

For part c, we need to determine the initial number of infected students if there are 1400 infected on day 3. Thus, we need to find $A^3$ * [$S_0$ ; $I_0$] = [$S_3$ ; 1400]. Using MatLab, we can do the following commands: syms $I_0$, $S_0$ = 5000 – $I_0$, then combine $S_0$ and $I_0$ into a vector, then multiply $A^3$ with that vector, then I typed eqn == day3symbol(2) == 1400, then the solution will be double(solve(eqn)). Using all of this, the solution for this questions is 700 initial infected students.

**Bibliography:**

“Markov Chains: Brilliant Math & Science Wiki.” *Brilliant*, brilliant.org/wiki/markov-chains/. Accessed 24 Nov. 2024.