CSCE 580: Artificial Intelligence
Written Homework 4: Probability and Bayesian Networks
Due: 3/25/2024 at 11:58pm

You may either type your homework or upload a scanned copy of your written homework. However, your answers must be legible. **Answers that are not legible will be marked as incorrect.**

Turn in **PDF** (not Word or any other format) of your answers to Blackboard.

1 Probability - Inference by Enumeration (25 pts)

Consider the full joint distribution shown below for Boolean variables A, B, C. For each question below, write an arithmetic expression that evaluates to the correct answer. An arithmetic expression involves only +, -, *, /, parentheses, and numbers.

A	B	C	P(A, B, C)
t	t	t	0.05
t	t	f	0.10
t	f	t	0.17
t	f	f	0.18
f	t	t	0.05
f	t	f	0.10
f	f	t	0.24
f	f	f	0.11

1.1 (2 pts)

$P(B = t, C = t)$ = 0.05 + 0.05 = 0.10

1.2 (3 pts)

$P(A = f, C = t)$ 0.05 + 0.24 = 0.29

1.3 (5 pts)

$P(A = f)$ = 0.05 + 0.10 + 0.24 + 0.11 = 0.50

1.4 (5 pts)

$P(B = t)$ = 0.05 + 0.10 + 0.05 + 0.10 = 0.30

1.5 (5 pts)

$P(A = t | B = t)$ = (0.05 + 0.10) / (0.05 + 0.10 + 0.05 + 0.10) = 0.50

1.6 (5 pts)

$P(C = f | B = f)$ = (0.18 + 0.11) / (0.17 + 0.18 + 0.24 + 0.11) = 0.41

2 Probability (25 pts)

Your friend has a game night every week. For fun, he uses a fake 6-sided die on certain nights with probability $P(D = f)$, otherwise, he uses the real 6-sided die. In this question, $D = f$ denotes a fake die and $D = \neg f$ denotes a real die. $R \in \{1, 2, 3, 4, 5, 6\}$ denotes the result of a roll. For example $R = 1$ means that a 1 was rolled.

2.1 (5 pts)

Suppose you know the probability of rolling a 1, 2, 3, 4, and 5 given that the die is fake. In other words, you know $P(R = 1|D = f)$, $P(R = 2|D = f)$, $P(R = 3|D = f)$, $P(R = 4|D = f)$, and $P(R = 5|D = f)$. Given this information, how would you find $P(R = 6|D = f)$?

$P(R = 6|D = f) = 1 - (\mathrm{P(R = 1 | D = f) + P(R = 2 | D = f) + P(R = 3 | D = f) + P(R = 4 | D = f) + P(R = 5 | D = f)})$

2.2 (5 pts)

This night, you are the first to roll. What is the probability that you roll a 6 (i.e. $P(R = 6)$)? Use the product rule and marginalization to give your answer in terms of $P(R = 6|D = f)$, $P(R = 6|D = \neg f)$,$P(D = f)$, and $P(D = \neg f)$.

$P(R = 6) = \mathrm{P(R = 6 | D = f) * P(D = f) + P(R = 6 | D = -f) * P(D = -f)}$

2.3 (5 pts)

Say you rolled a 6. The next turn, your friend rolls a 1. Given that you rolled a 6 and your friend rolled a 1, what is the probability of the die being fake (i.e. $P(D = f|R_1 = 6, R_2 = 1)$)? Use Baye's Rule to give your answer in terms of $P(R_1 = 6, R_2 = 1|D = f)$, $P(D = f)$, and $P(R_1 = 6, R_2 = 1)$.

$P(D = f|R_1 = 6, R_2 = 1) = (\mathrm{P(R_1 = 6, R_2 = 1 | D = f) * P(D = f)) / P(R_1 = 6, R_2 = 1)}$

2.4 (5 pts)

Use the product rule and marginalization to put $P(R_1 = 6, R_2 = 1)$ in terms of $P(R_1 = 6, R_2 = 1|D = f)$, $P(R_1 = 6, R_2 = 1|D = \neg f)$, $P(D = f)$, and $P(D = \neg f)$.

$P(R_1 = 6, R_2 = 1) = \mathrm{P(R_1 = 6, R_2 = 1 | D = f) * P(D = f) + P(R_1 = 6, R_2 = 1 | D = -f) * P(D = -f)}$

2.5 (5 pts)

The probability of each roll is conditionally independent given D. Given this information, how can you rewrite $P(R_1 = 6, R_2 = 1|D = f)$ and $P(R_1 = 6, R_2 = 1|D = \neg f)$?

$P(R_1 = 6, R_2 = 1|D = f) = \mathrm{P(R_1 = 6 | D = f) * P(R_2 = 1 | D = f)}$

$P(R_1 = 6, R_2 = 1|D = \neg f) = \mathrm{P(R_1 = 6 | D = -f) * P(R_2 = 1 | D = -f)}$

3 Bayesian Networks - Representation and Independence (25 pts)

3.1 (9 pts)

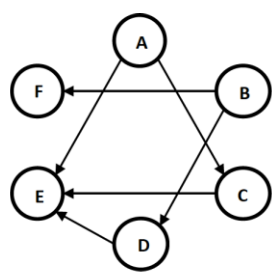
In the common effect structure, when we are given the effect, we cannot guarantee that the other two variables (the possible causes) are independent of one another. Suppose variables S and G are the possible causes and the common effect is variable P. Construct conditional probability tables such that S and G are not independent given P.

P	S=s1		S=s2	
P1	0.4	0.6	0.1	0.9
P2	0.8	0.2	0.3	0.7

P	G=g1		G=g2	
P1	0.7	0.3	0.5	0.5
P2	0.2	0.8	0.6	0.4

3.2

Consider the following Bayesian network:



3.2.1 (5 pts)

Write the factored joint probability distribution corresponding to the

Bayesian network above: P(A,B,C,D,E,F) = P(A) * P(B) * P(C|A) * P(D|B) * P(E|A,C,D) * P(F|B)

3.2.2 (5 pts)

Write the variables for the Markov blanket of the variable D above: B,E,F

3.2.3 (3 pts)

A and D are independent given E and F. True or False? False

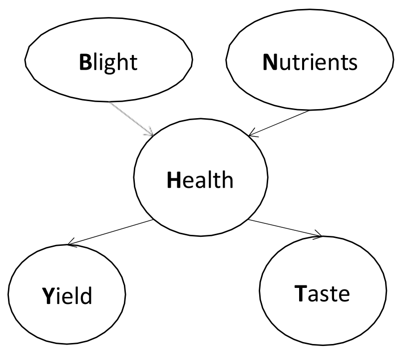
3.2.4 (3 pts)

B and E are independent given D. True or False? False

4 Bayesian Networks (15 pts)

Consider the following Bayesian network with the given conditional probability tables. For each question below, write an arithmetic expression that evaluates to the correct answer. An arithmetic expression involves only +, -, *, /, parentheses, and numbers. For each question, show the work that lead to your arithmetic expression.

P(B=+b)
0.1



P(N=+n)
0.95

B	N	P(H=+h|B,N)
+b	+n	0.5
+b	-n	0.05
-b	+n	0.99
-b	-n	0.6

H	P(Y=+y|H)
+h	0.7
-h	0.1

H	P(T=+t|H)
+h	0.9
-h	0.05

4.1 (5 pts)

Compute the probability that our crops suffer from blight, do not get enough nutrients, are not healthy, do not have high yield, and do not taste good. That is, compute $P(+b, -n, -h, -y, -t)$.

P(+b,-n,-h,-y,-t) = P(+b) * P(-n) * P(-h|+b,-n) * P(-y|-h) * P(-t|-h)

P(+b,-n,-h,-y,-t) = 0.1 * 0.05 * 0.05 * 0.9 * 0.95 = 0.00021375

4.2 (10 pts)

There was no blight and the crops had good nutrition. Given this information, compute the probability that you will have high yield and that the crops will taste good. That is, compute $P(+y, +t \mid -b, +n)$. Show your work.

P(+y,+t|-b,+n) = P(+y|+h) * P(+t|+h)

$P(+y|+h) = 0.7$

$P(+t|+h) = 0.9$

$P(+y,+t|-b,+n) = 0.7 * 0.9 = 0.63$