Stat 509: Statistics for Engineers

Homework Assignment 4

1. You have the following PDF: $f(x) = \frac{1}{9} \quad 1 \le x \le c$.

   (a) Find the value of c such that this is a valid PDF.

$\int_1^c 1/9\, dx = 1/9 * x |_1^c = c/9 - 1/9$

   (b) Calculate $\mathbf{E}(X)$.

$E(X) = \int_1^c x * 1/9\, dx = x^2/18 |_1^c = c^2/18 - 1/18$

   (c) Calculate $\mathbf{V}(X)$.

$V(X) = E(X^2) - [E(X)]^2$

$E(X^2) = \int_1^c x^2 * 1/9\, dx = x^3/27 |_1^c = c^3/27 - 1/27$

$[E(X)] = (c^2/18 - 1/18)^2$

$V(X) = (c^3/27 - 1/27) - (c^2/18 - 1/18)^2$

   (d) Calculate $F(x)$.

$\int_1^c 1/9\, dx = 1/9 * x |_1^c = c/9 - 1/9$

   (e) Calculate $P(2 < X < 5)$.

$\int_2^5 1/9\, dx = x/9 |_2^5 = 5/9 - 2/9 = 3/9$

$= 1/3$

   (f) Calculate $P(X = 4)$.

$\int_1^4 1/9\, dx = x/9 |_1^4 = 4/9 - 1/9 = 3/9$

$= 1/3$

   (g) Calculate the median of X.

$\int_1^{q*} 1/9\, dx = .5 \Rightarrow x/9 |_1^{q*} = .5 \Rightarrow q*/9 - 1/9 = .5 \Rightarrow q*/9 = .5 + 1/9 \Rightarrow q* = (.5 + 1/9) * 9$

2. Find the CDF of an exponential random variable with mean $\frac{1}{\Lambda}$.

$f(x) = \Lambda e^{-\Lambda x}$

$F(x) = \int_0^x f(t)\, dt$

$F(x) = \int_0^x 1/mean * e^{-t/mean}\, dt$

$F(x) = [-e^{-t/mean}]_0^x$

$F(x) = -e^{-x/mean} - (-e^0)$

$F(x) = 1 - e^{-x/mean}$

3. Let X be a random variable which represents the lifetime in years of a particular battery. We are given that X has an exponential distribution with rate $\lambda = 0.15$.

   (a) What is the expected value of X?

$1/\Lambda = 1/0.15$

   (b) What is the variance of X?

$1/\Lambda^2 = 1/0.15^2$

   (c) What is the CDF of X?

$f(x) = \Lambda e^{-\Lambda x}$

$f(x) = 0.15e^{-0.15x}$

   (d) What is the probability that the battery fails between the fifth and sixth year? Show by hand **and** provide the appropriate R code.

$\int_5^6 0.15e^{-0.15x}\, dx$

$-e^{-0.15x} |_5^6$

$-e^{-0.15(6)} + e^{-0.15(5)}$

Exp(6) – exp(5) = 255.02

(e) What is the probability that the battery is still working after three years? Show by hand **and** provide the appropriate R code.

Still Working = 0.85

$\int_0^3 0.85e^{-0.85x}\, dx = -e^{-0.85x} |_0^3 = -e^{-0.85(3)} + 1$

Exp(3) + 1 = 21.09

(f) What is the probability that the battery is still working after five years, given that the battery is still working after two years?

$\int_2^5 0.85e^{-0.85x}\, dx = -e^{-0.85x} |_2^5 = -e^{-0.85(5)} + e^{-0.85(2)}$

(g) You observe a battery who's lifetime is in the 99th percentile. How long has this battery lasted? Show by hand **and** provide the appropriate R code.

$\int_0^{q^*} 0.15e^{-0.15x}\, dx = .99$

$-e^{-0.15x} |_0^{q^*} = 0.99$

$-e^{-0.15q^*} + 1 = 0.99$

$0.01 = e^{-0.15q^*}$

Ln(0.15)/-0.01 = q*

Qexp(0.99,0.15) = 30.70

4. The number of earthquakes that occur per week in California follows a Poisson distribution with a mean of 1.5.

(a) What is the probability that an earthquake occurs within the first week? Show by hand **and** provide the appropriate R code.

$\int_0^{1.5} 1.5e^{-1.5y}\, dy = -e^{-1.5y} |_0^{1.5} = -e^{-1.5(1.5)} + 1$

Exp(1.5) = 4.48

(b) What is the expected amount of time until an earthquake occurs?

E(x) = 1/ʌ = 1/1.5

(c) What is the standard deviation of the amount of time until two earthquakes occur?

Sqrt(2ʌ)

(d) What is the probability that it takes more than a month to observe 2 earthquakes? Show by hand **and** provide the appropriate R code.

P(X > 2 x 4) = 1 – P(X <= 2 * 4)

Ppois(2 * 4, 1.5) = 0.99

(e) What is the probability that it takes more than a month to observe 4 earthquakes? Show by hand (you may simply leave it as an integral) **and** provide the appropriate R code.

K = 4 * weeks in a month => k = 4 * 4

Exp(1) = 2.72

(f) What is the median amount of time it takes for 5 earthquakes to occur? Show by hand (you may simply leave it as an integral, but be sure to explain how to find the median) **and** provide the appropriate R code.

ʌ * ln(2)

1.5 * log(2) = 1.04