# Stat 509: Statistics for Engineers
## Homework Assignment 3

1. Solve one of the following three problems. HINT: You may need to use the geometric series.

   (a) You keep rolling a set of five dice until you get a set showing either exactly one six or no sixes. You win if there is exactly one six and you lose if there are no sixes. What is the probability that you win?

   (b) Ender and Harry are playing game in which they alternate making moves. When it is Ender's turn, the probability that he wins with his next move is 40%. When it is Harry's turn, the probability that he wins with his next move is $p$. If Ender is allowed to go first, what is the value of $p$ such that neither player is favored to win?

   (c) **Prove** that the $E(X) = \frac{1}{p}$ for a geometric distribution, where $X$ is the number of trials.

Solving Problem A.
Win = (1/6)*(5/6)^4
Lose = (5/6)^5

P(Winning) = (1/6) * (5/6)^4 + (5/6)^5 = 0.48

2. Calculate $E(X^3)$, where $X$ has the following PMF:

$$f(x) = \begin{cases} 0.5 & x=1 \\ 0.3 & x=2 \\ 0.2 & x=3 \end{cases}$$

E(X^3) = 1^3 * 0.5 + 2^3 * 0.3 + 3^3 * 0.2 = 8.3

3. You are designing a road system leading to a city. There are 9 lights on the road and the probability that a light is red is 0.3. Assume the probability that any particular light is red has nothing to do with any of the other lights being red.

   (a) What is the probability that 4 of the lights are red? Show by hand **and** provide the appropriate R code.

   (b) Consider that you are required to design another road leading to the city. This road has 8 lights. What must the probability of a red light be on this road so that neither road would be preferred over the other on the basis of average number of red lights?

   (c) Which road has more variability in the number of red lights?

P = 0.3

R = 4

X = 9

(9-1) * 0.3^4 * (1 – 0.3)^(9-4) = .17

(4-1)

Dbinom(4, 9, 0.3)

P = ? = 0.34

R = Assume 4

X = 8

Road 1 would have more variety in red lights since there are more lights that could be red.

4. A manufacturing facility produces items with a defect rate of 1/500. Assume that defective items are produced independently of one another. Items are produced in batches of 1000. A batch is rejected if it contains more than 5 defective items.

   (a) Assuming that the defect rate is constant, what is the probability that a randomly selected batch will be rejected? Show by hand (leaving in sigma notation is fine) **and** provide the appropriate R code.

(b) You examine a batch and notice that it contains 7 defective items, but you know the company you are selling this batch to is not very thorough as they will only examine 500 of the items and reject the batch if more than 5 defectives are found. What is the probability that the purchaser fails to reject the batch? Show by hand (leaving in sigma notation is fine) **and** provide the appropriate R code.

P = 1/500

R = 5

X = 1000

(1000-1) * (1/500)^5 * (1-(1/500))^(1000-5) = 0.036

(5-1 )

Dbinom(5,1000,1/500)

P = 7/1000

R = 5

X = 500

(500-1) * (7/1000)^5 * (1-(7/1000))^(500-5) = 0.13

(5-1 )

Dbinom(5,500,7/1000)

5. The number of earthquakes that occur per week in California follows a Poisson distribution with an average of of 1.5 earthquakes per week.

(a) What is the probability that no earthquakes occur during a randomly selected week? Show by hand **and** provide the appropriate R code.

(b) What is the probability that more than 1 earthquake occurs next week? Show by hand **and** provide the appropriate R code.

X(a) – Poisson(1.5)

P(X = 0)

(e^-1.5 * 1.5^0)/0! = 0.22

Dpois(0, 1.5)

X(b) – Poisson(1.5)

P(X = 1)

(e^-1.5 * 1.5^1)/1! = 0.33

Dpois(1, 1.5)