Properties of a Switching Algebra

P1 commutative:

(a)
$$a + b = b + a$$

(b)
$$ab = ba$$

P2 associative:

(a)
$$a + (b+c) = (a+b) + c$$

(b)
$$a(bc) = (ab)c$$

P3 identity:

(a)
$$a + 0 = a$$

(b)
$$a \cdot 1 = a$$

P4 null:

(a)
$$a+1=1$$

(b)
$$a \cdot 0 = 0$$

P5 complement:

(a)
$$a + a' = 1$$

(b)
$$a \cdot a' = 0$$

P6 idempotency:

(a)
$$a + a = a$$

(b)
$$a \cdot a = a$$

P7 involution: (a')' = a

P8 distributive:

(a)
$$a(b+c) = ab + ac$$

(b)
$$a + bc = (a + b)(a + c)$$

P9 adjacency:

(a)
$$ab + ab' = a$$

(b)
$$(a+b)(a+b') = a$$

P10 simplification:

(a)
$$a + a'b = a + b$$

(b)
$$a(a' + b) = ab$$

P11 DeMorgan's Theorem:

(a)
$$(a+b)' = a'b'$$

(b)
$$(ab)' = a' + b'$$

P12 absorption:

(a)
$$a + ab = a$$

(b)
$$a(a+b) = a$$

P13 consensus: $at_1 \not c a't_2 = t_1t_2$

(a)
$$at_1 + a't_2 + t_1t_2 = at_1 + a't_2$$

(b)
$$(a+t_1)(a'+t_2)(t_1+t_2) = (a+t_1)(a'+t_2)$$

P14 swap:

(a)
$$ab + a'c = (a+c)(a'+b)$$

(b)
$$(a+c)(a'+b) = ab + a'c$$