Stat 509: Statistics for Engineers

Homework Assignment 1

1. Solve the following integral, where λ is a constant:

$$\int_0^{\infty} \lambda^2 x e^{-\lambda x} \, dx$$

$\int udv = uv - \int vdu$

Let u = x => du = dx
Dv = *λ^2e^- λx * dx => v = - λ/ λ^2 + 1 * e^- λx*

$\int_0^{\infty}$ *λ^2 * xe^- λx * dx = uv |* $_0^{\infty}$ *- f* $_0^{\infty}$ *vdu = [-(λx)/(λ^2 + 1) * e^- λx]* $_0^{\infty}$ *+ ∫* $_0^{\infty}$ *λ/(λ^2 + 1) * e^- λx*

Lim (*-(λx)/(λ^2 + 1) * e^- λx) = 0*
x->∞

Lim (*-(λx)/(λ^2 + 1) * e^- λx) = 0*
x -> 0

$\int_0^{\infty}$ *λ^2 * xe^- λx * dx =* $\int_0^{\infty}$ *λ/(λ^2 + 1) * e^- λx * dx = -1/(λ^2 + 1) * e^- λx |* $_0^{\infty}$

Lim (*-1/(λ^2 + 1) * e^- λx) = 0*
x->∞

lim (*-1/(λ^2 + 1) * e^- λx) = -1/(λ^2 + 1)*
x->0

2. Solve the following integral:

$$\int_0^t \frac{2x}{100} e^{-(x/10)^2}\,dx$$

Let u = x/10
du = 1/10 dx

$\int_0^{t/10}$ 2(10u)/100 * e^-u^2*1/10du

1/50 $\int_0^{t/10}$ e^-v dv

Let v = u^2, dv = 2u du

1/50 $\int_0^{(t/10)^2}$ e^-v dv

1/50 (-e^-v) $|_0^{(t/10)^2}$

1/50 (-e^-(u/10)^2) $|_0^{(t/10)^2}$

1/50 (-e^-(u/10)^2) $|_0^{t/10}$

1/50 (-e^-(x/100) + 1) $|_0^{t}$

1/50 (-(1 – e^-(t/100) + 1)

1/50 (e^-(t/100) – 1)