CS310 Natural Language Processing
自然语言处理
Lecture 02 - Word Vectors

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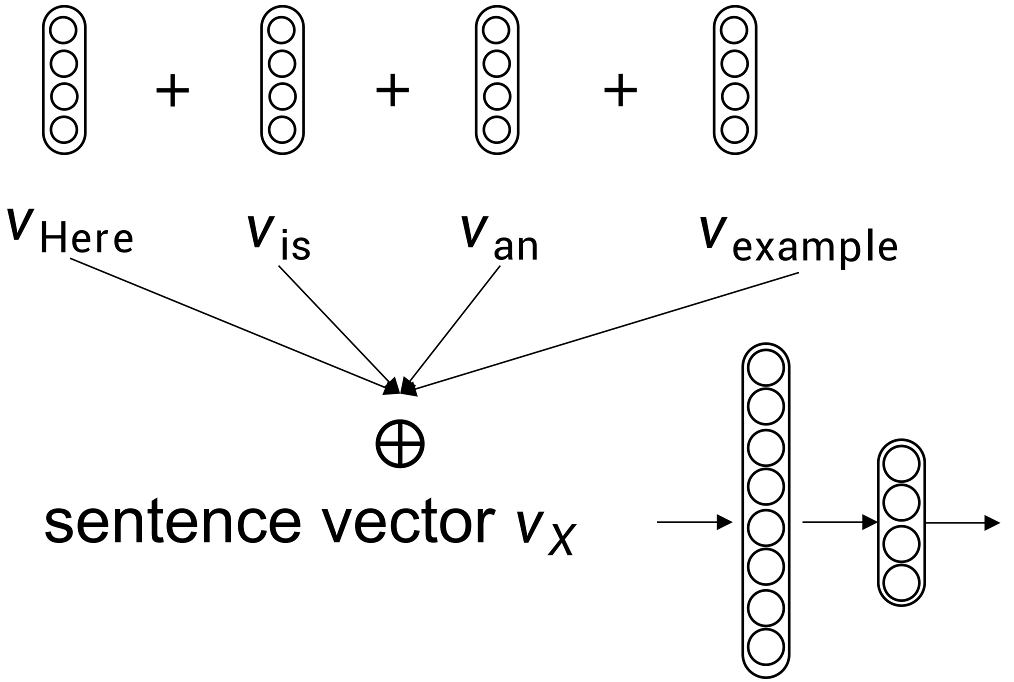
Content

- **Motivation**
- Documents and Counts-based Method
- Neural Network-based Method -- word2vec
- Evaluation and Applications

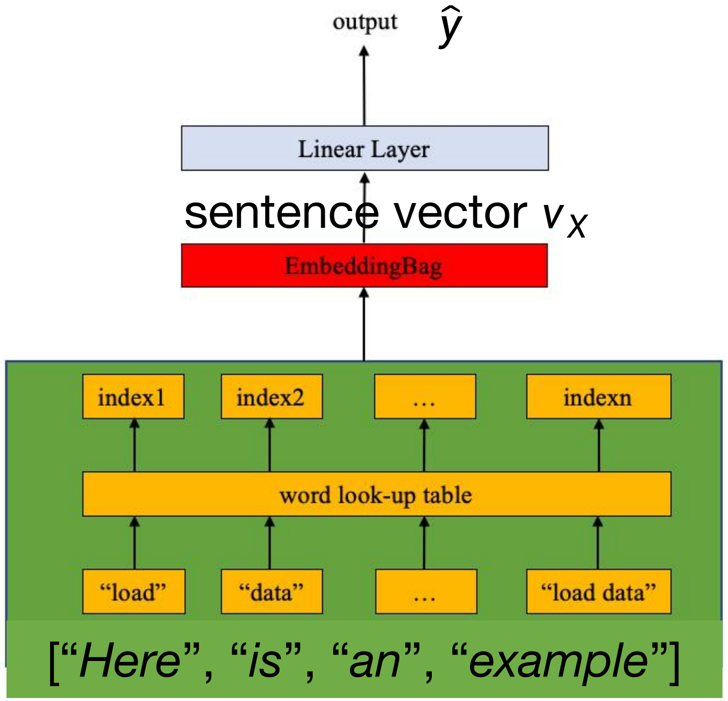
Recap: Bag-of-Words Neural Networks

Task: News text classification

X: [“*Here*”, “*is*”, “*an*”, “*example*”]



$$\hat{y} = [P(\text{World}|X), P(\text{Sports}|X), P(\text{Business}|X), P(\text{Sci/Tec}|X)]$$



Naïve method: one-hot vectors

- Words as discrete symbols ⇔ localist representations
- **One-hot** vectors

Vocabulary (10k) = [a, $about$, all, ⋮, zoo]

Apple [0 0 0 0 0 0 0 0 **1** 0 0 0 0 0 0 0 0 ... 0]

Orange [0 0 0 0 0 0 0 0 0 0 0 0 0 **1** 0 0 0 ... 0]

I would like some apple juice

I would like some orange ____

One-hot vector is not helpful

Distance between any pair of words is **constant:**

Euclidean distance = $\sqrt[2]{(1-0)^2 + (1-0)^2}$

Cosine distance = 0

Ideally ⇒ real-valued word vectors

	Man	Woman	King	Queen	Apple	Orange
Gender	-1	1	-0.98	0.97	0.00	-0.01
Royal	0.01	0.02	0.93	0.98	-0.01	0.00
Age	0.03	0.02	0.72	0.68	0.03	0.02
Food	0.00	0.00	0.01	0.02	0.95	0.97

main difference: gender

main difference: gender

$$e_{Man} = \begin{bmatrix} -1 \\ 0.01 \\ 0.03 \\ 0.0 \end{bmatrix} \quad e_{Woman} = \begin{bmatrix} 1 \\ 0.02 \\ 0.02 \\ 0.0 \end{bmatrix} \quad e_{Man} - e_{Woman} = \begin{bmatrix} -2 \\ -0.01 \\ 0.01 \\ 0.00 \end{bmatrix} \quad e_{King} - e_{Queen} = \begin{bmatrix} -1.95 \\ -0.05 \\ 0.04 \\ -0.01 \end{bmatrix}$$

With real-valued dense vectors, word similarity can be computed more accurately

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  - **LSA and TF-IDF**
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Documents and Word Counts

- **Goal**: Derive word vectors from a collection of documents
- without annotation -- unsupervised/self-supervised

- **Notations**:
- $\boldsymbol{x}$ is the collection of C documents
- $\boldsymbol{x_c}$ is the cth document in the corpus
- ℓ_c is the length of $\boldsymbol{x_c}$ (in # of tokens)
- N is the total number of tokens (words), $N = \sum_{c=1}^{C} \ell_c$

Adapted from: https://nasmith.github.io/NLP-winter23/assets/slides/vectors.pdf

Build Word-Document Matrix (term-document matrix)[1]

- Build matrix $\mathbf{A} \in \mathbb{R}^{V \times C}$, which contains the count of each word in each document
- **Example**:

x_1: 学 而 时 习 之

x_2: 学 而 不 思 则 罔

x_3: 思 而 不 学 则 殆

Entry $\mathbf{A}_{v,c} = \text{count}_{x_c}(v)$, count of word v in the cth document

	x_1	x_2	x_3
学	1	1	1
而	1	1	1
不	0	1	1
思	0	1	1
则	0	1	1
时	1	0	0
习	1	0	0
之	1	0	0
罔	0	1	0
殆	0	0	1

[1] https://en.wikipedia.org/wiki/Term-document_matrix

Q: how much surprise is in each word?

- What is the expected occurrence of word v in document c?
- Under a simple assumption, the chance of word v to occur at any position is $\frac{\text{count}_{\boldsymbol{x}}(v)}{N}$, (where $\text{count}_{\boldsymbol{x}}(v)$ is the count of v over all documents)
- So the expected occurrence of v in a document of length ℓ_c is $\frac{\text{count}_{\boldsymbol{x}}(v)}{N} \cdot \ell_c$
- Consider the **ratio** of *observed* count of v in document c, $\text{count}_{x_c}(v)$, to the expected count $\frac{\text{count}_{\boldsymbol{x}}(v)}{N} \cdot \ell_c$

Intuition of surprise in word

	x_1	x_2	x_3
学	1	1	1
而	1	1	1
不	0	1	1
思	0	1	1
则	0	1	1
时	1	0	0
习	1	0	0
之	1	0	0
罔	0	1	0
殆	0	0	1

$\text{count}_{\boldsymbol{x}}(\text{学}) = 1 + 1 + 1 = 3$

Expected count of 学 in x_1 is $\frac{\text{count}_{\boldsymbol{x}}(\text{学})}{N} \cdot \ell_1 = \frac{3}{17} \cdot 5 \approx 0.88$

The observed count of 学 in x_1 is $\text{count}_{\boldsymbol{x}_1}(\text{学}) = 1$

The **surprise** of seeing 学 in x_1 is:

$$\log \frac{\text{observed}}{\text{expected}} = \log \frac{\text{count}_{\boldsymbol{x}_1}(\text{学})}{\frac{\text{count}_{\boldsymbol{x}}(\text{学})}{N} \cdot \ell_1} \approx \log \frac{1}{0.88} \approx 0.125$$

Intuition of surprise in word

	x_1	x_2	x_3
学	1	1	1
而	1	1	1
不	0	1	1
思	0	1	1
则	0	1	1
时	1	0	0
习	1	0	0
之	1	0	0
罔	0	1	0
殆	0	0	1

$\text{count}_{\boldsymbol{x}}(\text{习}) = 1 + 0 + 0 = 1$

Expected count of 习 in x_1 is $\frac{\text{count}_{\boldsymbol{x}}(\text{习})}{N} \cdot \ell_1 = \frac{1}{17} \cdot 5 \approx 0.29$

The observed count of 习 in x_1 is $\text{count}_{\boldsymbol{x}_1}(\text{习}) = 1$

The **surprise** of seeing 习 in x_1 is:

$$\log \frac{\text{observed}}{\text{expected}} = \log \frac{\text{count}_{\boldsymbol{x}_1}(\text{习})}{\frac{\text{count}_{\boldsymbol{x}}(\text{习})}{N} \cdot \ell_1} \approx \log \frac{1}{0.29} \approx 1.223$$

> surprise of 学

Pointwise Mutual Information

- From matrix $\mathbf{A} \in \mathbb{R}^{V \times C}$, derive positive **pointwise mutual information**

$$[\mathbf{A}]_{v,c} = [\log \frac{\text{count}_{\boldsymbol{x}_c}(v)}{\frac{\text{count}_{\boldsymbol{x}}(v)}{N} \cdot \ell_c}]_+ = [\log \frac{N \cdot \text{count}_{\boldsymbol{x}_c}(v)}{\text{count}_{\boldsymbol{x}}(v) \cdot \ell_c}]_+ \qquad \text{where } [x]_+ = \max(0, x)$$

More examples:

$$[\mathbf{A}]_{学,2} = \log \frac{17 \cdot 1}{3 \cdot 6} \approx -0.057 \to 0$$

rounded to 0 because of max()

$$[\mathbf{A}]_{思,2} = \log \frac{17 \cdot 1}{2 \cdot 6} \approx 0.348$$

Adapted from: https://nasmith.github.io/NLP-winter23/assets/slides/vectors.pdf

Meaning of PMI

Random variable ***A*** and ***B***

$$\text{PMI}(a,b) = \log \frac{p(A=a, B=b)}{p(A=a)\cdot p(B=b)} = \log \frac{p(A=a \mid B=b)}{p(A=a)} = \log \frac{p(B=b \mid A=a)}{p(B=b)}$$

Mutual Information (MI):
The amount of information each r.v. offers about the other.
I.e., how much do we know about **B** by knowing about **A**

$$[\mathbf{A}]_{v,c} = [\log \frac{\text{count}_{\boldsymbol{x}_c}(v)}{\frac{\text{count}_{\boldsymbol{x}}(v)}{N}\cdot \ell_c}]_+ = [\log \frac{\frac{\text{count}_{\boldsymbol{x}_c}(v)}{\ell_c}}{\frac{\text{count}_{\boldsymbol{x}}(v)}{N}}]_+$$

Local probability

Global probability

Example:

$$\log \frac{\text{count}_{\boldsymbol{x}_1}(\text{习})}{\frac{\text{count}_{\boldsymbol{x}}(\text{习})}{N}\cdot \ell_1} \approx \log \frac{1}{0.29} \approx 1.223$$

is high, which means we learn a lot about the global meaning of "习" by reading x_1

How much do we know about the global meaning of *v* by knowing about its local meaning in document *c*

Slide credit: https://nasmith.github.io/NLP-winter23/assets/slides/vectors.pdf

Pointwise Mutual Information

$$PMI = [\mathbf{A}]_{v,c} = [\log \frac{\text{count}_{\boldsymbol{x}_c}(v)}{\frac{\text{count}_{\boldsymbol{x}}(v)}{N} \cdot \ell_c}]_+$$

- If a word v has nearly same frequency in every document, then its row $[\mathbf{A}]_{v,*}$ will be nearly all zeros
- If a word v only occurs in one document c, then its PMI will be large and positive
- Thus, PMI is sensitive to **rare** words; usually need to smooth the frequencies by filtering rare words

	x_1	x_2	x_3
学	1	1	1
而	1	1	1
不	0	1	1
思	0	1	1
则	0	1	1
时	1	0	0
习	1	0	0
之	1	0	0
罔	0	1	0
殆	0	0	1

Adapted from: https://nasmith.github.io/NLP-winter23/assets/slides/vectors.pdf

Reflection

- Can we directly use word-document matrix $\mathbf{A} \in \mathbb{R}^{V \times C}$ (or smoothed PMI $[\mathbf{A}]$) to represent word meanings?
- For example, can we use the row vectors as input features for a neural text classifier?
- What are the advantages/disadvantages?

Improvement: Latent Semantic Analysis

(Deerwester et al., 1990)

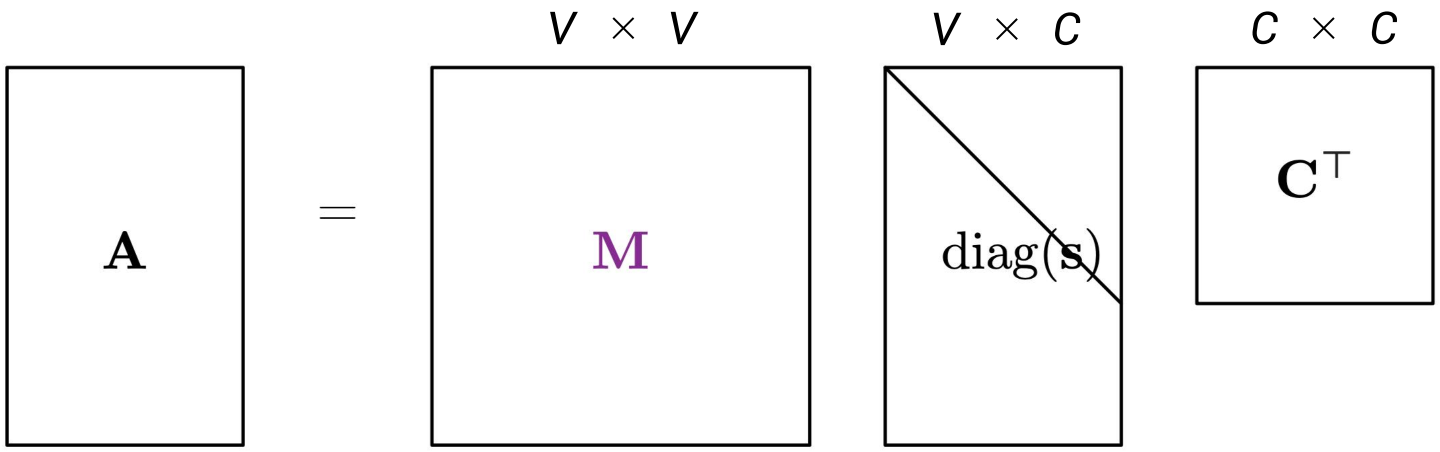
- LSA seeks to find a more compact (low rank) representation of document-word matrix **A**

$$\underset{V \times C}{\mathbf{A}} \approx \hat{\mathbf{A}} = \underset{V \times d}{\mathbf{M}} \times \underset{d \times d}{\mathrm{diag}(\mathbf{s})} \times \underset{d \times C}{\mathbf{C}^\top}$$

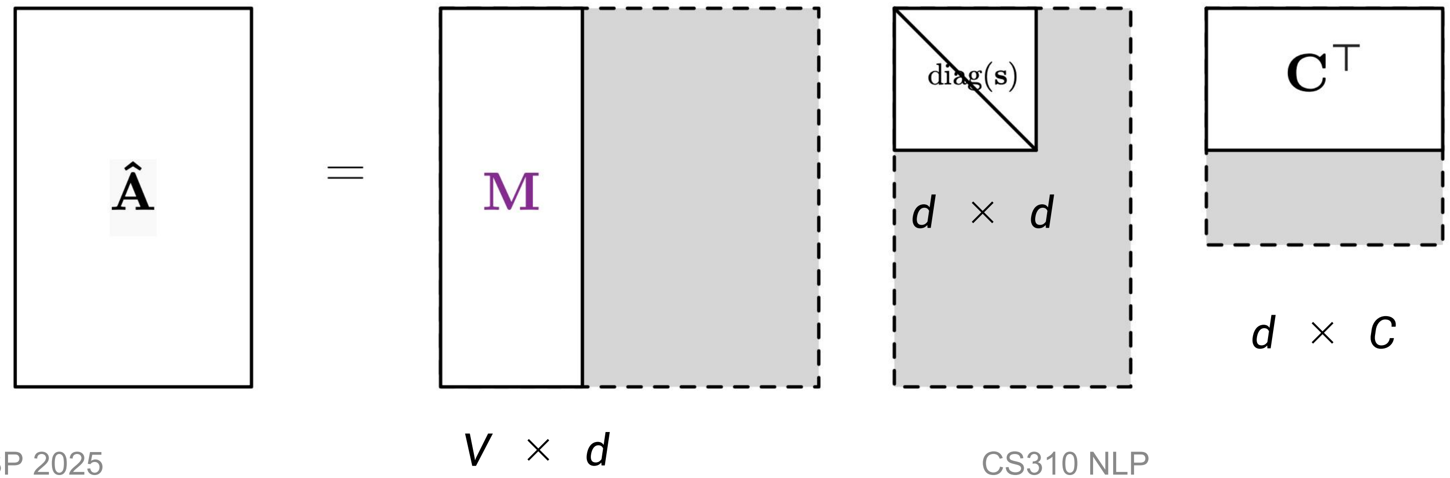
- Can be solved by applying singular value decomposition to **A**, and then truncating to *d* dimensions ($\hat{\mathbf{A}}$)
- **M** contains left singular vectors of **A**
- **C** contains right singular vectors of **A**
- **s** are singular values of **A**

SVD and Truncated SVD

SVD:



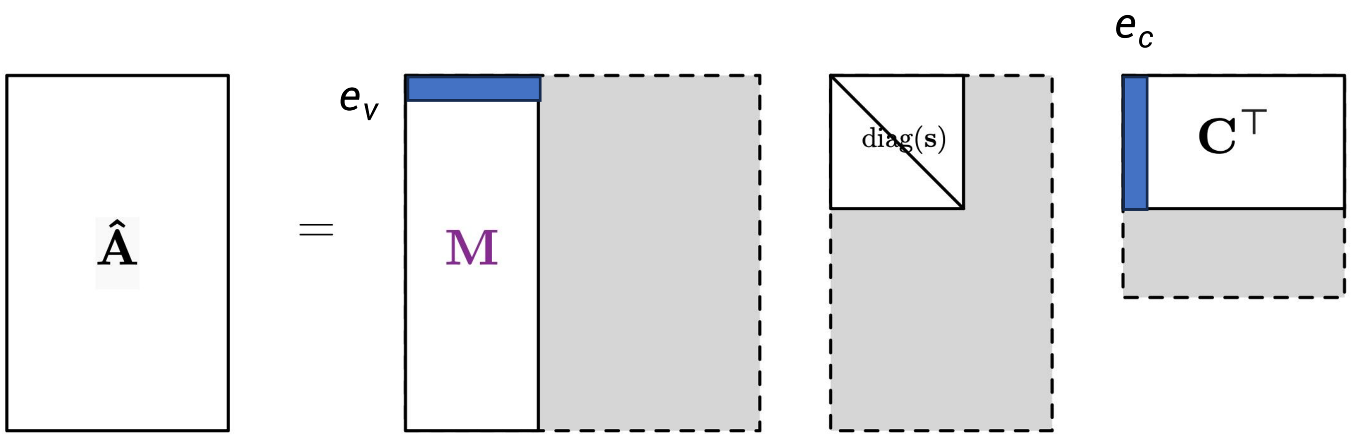
SVD truncated at *d* dimensions:



- **M** and **C** are unitary, i.e., $\mathbf{M}\mathbf{M}^\top = \mathbf{I}$ and $\mathbf{C}\mathbf{C}^\top = \mathbf{I}$
- $\mathrm{diag}(\mathbf{s})$ only has non-zero elements at diagonal
- **M** are eigenvectors of $\mathbf{A}\mathbf{A}^\top$
- **C** are eigenvectors of $\mathbf{A}^\top\mathbf{A}$
- $\mathbf{s}^2$ are eigenvalues
- Truncated: keeping only top *d* singular values in **s** corresponding *d* columns in **M** and **C**

Truncated SVD => word vectors

$$\mathbf{A} \approx \hat{\mathbf{A}} = \mathbf{M} \times \mathrm{diag}(\mathbf{s}) \times \mathbf{C}^{\top}$$

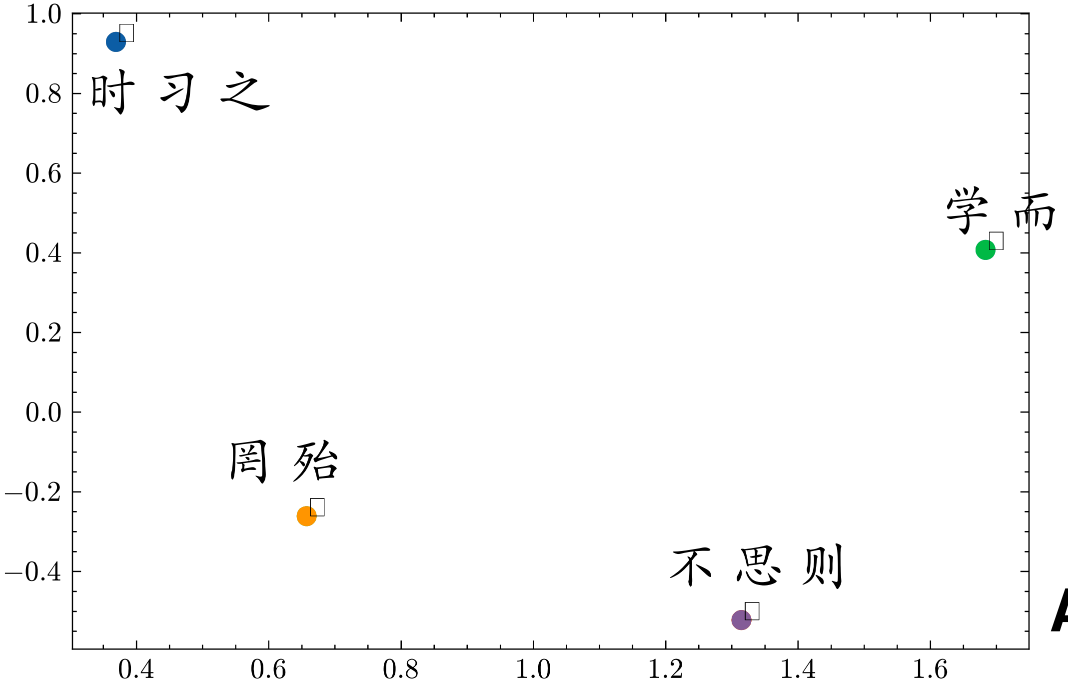


- *v*th row in **M** is the embedding vector for word *v*
- *c*th column in **C** is the embedding vector for document *c*

- **M** contains useful word vectors ("embeddings") of *d* dimensions
- **C** contains document vectors

LSA Example $d = 2$

- Word vectors **M** plotted
- Note that some words are in the same spot. Why?



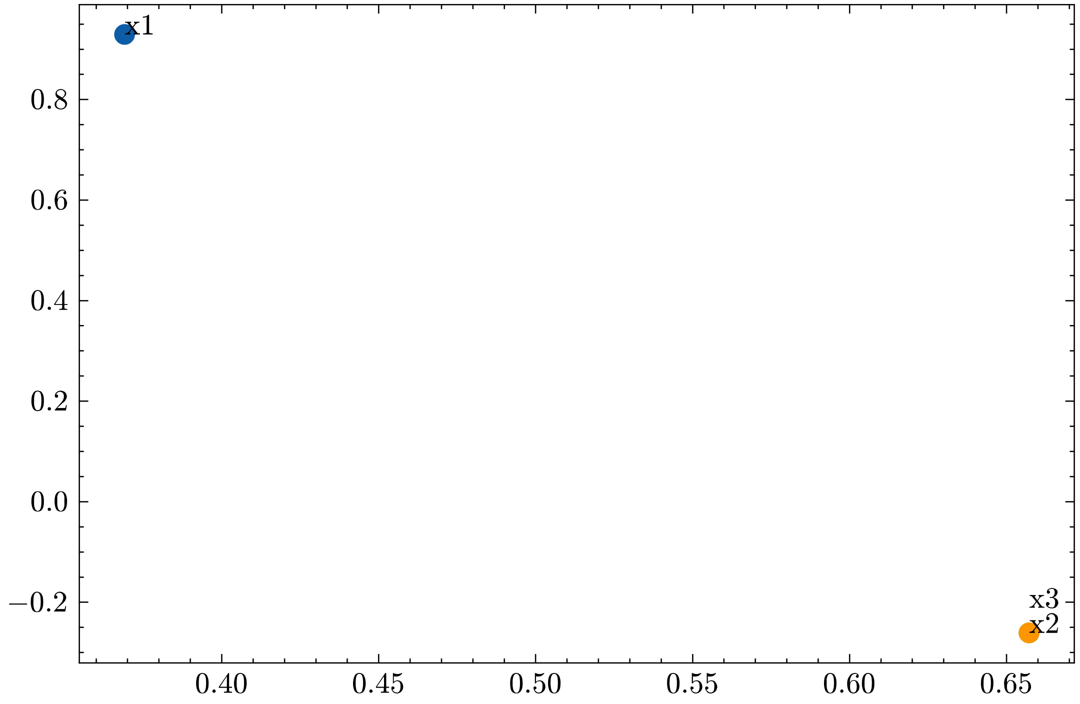
$\mathbf{A} \approx \hat{\mathbf{A}}$

$= \mathbf{M} \times \mathrm{diag}(\mathbf{s}) \times \mathbf{C}^\top$ $\mathbf{A} =$

	x_1	x_2	x_3
学	1	1	1
而	1	1	1
不	0	1	1
思	0	1	1
则	0	1	1
时	1	0	0
习	1	0	0
之	1	0	0
罔	0	1	0
殆	0	0	1

LSA Example $d = 2$

- Document vectors **C** plotted
- Note that documents x_2 and x_3 are in the same spot. Why?



A =

	x_1	x_2	x_3
学	1	1	1
而	1	1	1
不	0	1	1
思	0	1	1
则	0	1	1
时	1	0	0
习	1	0	0
之	1	0	0
罔	0	1	0
殆	0	0	1

LSA Summarized

- It creates a mapping of words and documents into the same low-dimensional space.
- Bag-of-words assumption (Salton et al., 1975):
  - A document is nothing more than the distribution of words it contains.
- Distributional hypothesis (Harris, 1954; J.R. Firth, 1957):
  - Words' meanings are nothing more than the distribution of *contexts* (here, documents) they occur in.
  - Words that occur in similar contexts have similar meanings.
- Word-document matrix **A** is sparse and noisy; LSA "fills in" the zeroes and tries to eliminate the noise.
- It finds the best rank-d approximation to **A**.

Adapted from: https://nasmith.github.io/NLP-winter23/assets/slides/vectors.pdf

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TF-IDF

- Background: Find the most relevant document among a collection of documents, using a query

tf: term frequency
idf: inverse document frequency

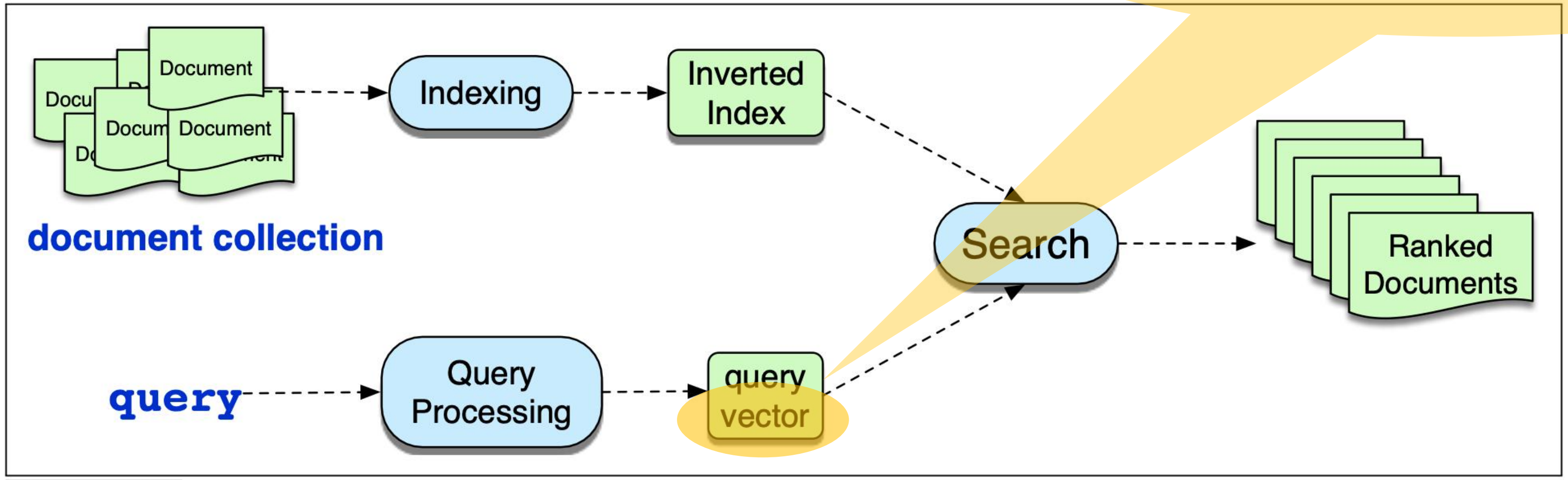


Figure 14.1 The architecture of an ad hoc IR system.

How to match a document a query?

- Compute a term weight for each document term
- **tf**: term frequency
- **idf**: inverse document frequency
- **tf-idf ≜ tf × idf** (product of the two)

term t; document d

$$\text{tf}_{t,d} = \begin{cases} 1 + \log_{10} \text{count}(t,d) & \text{if } \text{count}(t,d) > 0 \\ 0 & \text{otherwise} \end{cases}$$

- **tf**: words that occur more often in a document are likely to be informative about the document's content
- Use the $\log_{10}$ of word frequency count rather than raw count
- *Why? A word appearing 100 times doesn't make it 100 times more likely*

Tf-idf

$$\text{tf}_{t,d} = \begin{cases} 1 + \log_{10} \text{count}(t,d) & \text{if } \text{count}(t,d) > 0 \\ 0 & \text{otherwise} \end{cases}$$

term t; document d

term occurs 0 times in document: tf = 0
term occurs 1 times in document: tf = 1
term occurs 10 times in document: tf = 2, …

- **document frequency** df_t of a term t is the number of documents it occurs in
- Terms that occur in only **a few** documents are useful for discriminating those documents from the rest of the collection;
- terms that occur across the entire collection aren't as helpful (*the, a, an,* …)
- **inverse document frequency** or **idf** is defined as:

$$\text{idf}_t = \log_{10} \frac{N}{\text{df}_t}$$

N: total number of documents
The fewer documents in which t occurs, the higher idf_t

Inverse document frequency example

- Some idf values for some words in the corpus of Shakespeare plays

Word	df	idf
Romeo	1	1.57
salad	2	1.27
Falstaff	4	0.967
forest	12	0.489
battle	21	0.246
wit	34	0.037
fool	36	0.012
good	37	0
sweet	37	0

Extremely informative words that occur in only one play like *Romeo*

good or *sweet* tare completely non-discriminative since they occur in all 37 plays

Scoring with tf-idf

- We can score document d by the cosine of its vector $\vec{\boldsymbol{d}}$ with the query vector $\vec{\boldsymbol{q}}$:

$$\operatorname{score}(q,d) = \cos(\vec{\boldsymbol{q}},\vec{\boldsymbol{d}}) = \frac{\vec{\boldsymbol{q}}\cdot\vec{\boldsymbol{d}}}{|\vec{\boldsymbol{q}}|\cdot|\vec{\boldsymbol{d}}|}$$

- in which $\vec{\boldsymbol{q}}$ and $\vec{\boldsymbol{d}}$ are vectors of query length n, whose values are the **tf-idf** values (normalized):

$$\vec{\boldsymbol{q}} = \frac{[\operatorname{tfidf}(t_1,q),\ldots,\operatorname{tfidf}(t_n,q)]}{\sqrt{\sum_{t\in q}\operatorname{tfidf}^2(t,q)}}$$

$$\vec{\boldsymbol{d}} = \frac{[\operatorname{tfidf}(t_1,d),\ldots,\operatorname{tfidf}(t_n,d)]}{\sqrt{\sum_{t\in d}\operatorname{tfidf}^2(t,d)}}$$

$$\operatorname{score}(q,d) = \sum_{t_i\in q}\frac{\operatorname{tfidf}(t_i,q)}{\sqrt{\sum_{t\in q}\operatorname{tfidf}^2(t,q)}}\cdot\frac{\operatorname{tfidf}(t_i,d)}{\sqrt{\sum_{t\in q}\operatorname{tfidf}^2(t,q)}}$$

Tf-idf scoring example

- A collection of 4 nano documents

Query:	sweet love
Doc 1:	Sweet sweet nurse! Love?
Doc 2:	Sweet sorrow
Doc 3:	How sweet is love?
Doc 4:	Nurse!

Query vector $\vec{\boldsymbol{q}} = (0.383, 0.924)$

				Query		
word	**cnt**	**tf**	**df**	**idf**	**tf-idf**	**n'lized** = tf-idf/$\lvert q \rvert$
sweet	1	1	3	0.125	0.125	0.383
nurse	0	0	2	0.301	0	0
love	1	1	2	0.301	0.301	0.924
how	0	0	1	0.602	0	0
sorrow	0	0	1	0.602	0	0
is	0	0	1	0.602	0	0

$|q| = \sqrt{.125^2 + .301^2} = .326$

Tf-idf scoring example

Query vector $\vec{q} = (0.383, 0.924)$

			Document 1		
word	**cnt**	**tf**	**tf-idf**	**n'lized**	**× q**
sweet	2	1.301	0.163	0.357	**0.137**
nurse	1	1.000	0.301	0.661	0
love	1	1.000	0.301	0.661	**0.610**
how	0	0	0	0	0
sorrow	0	0	0	0	0
is	0	0	0	0	0

$|d_1| = \sqrt{.163^2 + .301^2 + .301^2} = .456$

			Document 2		
word	**cnt**	**tf**	**tf-idf**	**n'lized**	**×q**
sweet	1	1.000	0.125	0.203	**0.0779**
nurse	0	0	0	0	0
love	0	0	0	0	**0**
how	0	0	0	0	0
sorrow	1	1.000	0.602	0.979	0
is	0	0	0	0	0

$|d_2| = \sqrt{.125^2 + .602^2} = .615$

$\vec{d}_1 = (0.357, 0.661)$

$\text{score}(\vec{q}, \vec{d}_1) = \mathbf{0.747}$

$\vec{d}_2 = (0.203)$

$\text{score}(\vec{q}, \vec{d}_1) = \mathbf{0.0779}$

Therefore, d_1 is more relevant

Query: sweet love
Doc 1: Sweet sweet nurse! Love?
Doc 2: Sweet sorrow

Table of Content

Motivation: Distributional semantics

- Distributional semantics: A word's meaning is given by the words that frequently appear close-by
- "You shall know a word by the company it keeps" (J. R. Firth 1957: 11)
- When a word *w* appears in a text, its local **context** is the set of words that co-occur within a fixed-size window

…government debt problems turning into	***banking***	*crises as happened in 2009…*
…saying that Europe needs unified	***banking***	*regulation to replace the hodgepodge…*
…India has just given its	***banking***	*system a shot in the arm…*

The meaning of "banking" is represented by these context words

Example from: https://web.stanford.edu/class/archive/cs/cs224n/cs224n.1224/

In What Form of Representation?

- **Goal**: Obtain a dense vector for each word, so that word sense similarity can be computed via vector distance, such as dot product

$$e_{apple} = \begin{bmatrix} 0.00 \\ -0.01 \\ 0.03 \\ 0.95 \\ \dots \\ 0.21 \end{bmatrix} \quad e_{orange} = \begin{bmatrix} -0.01 \\ 0.00 \\ 0.02 \\ 0.97 \\ \dots \\ 0.22 \end{bmatrix}$$

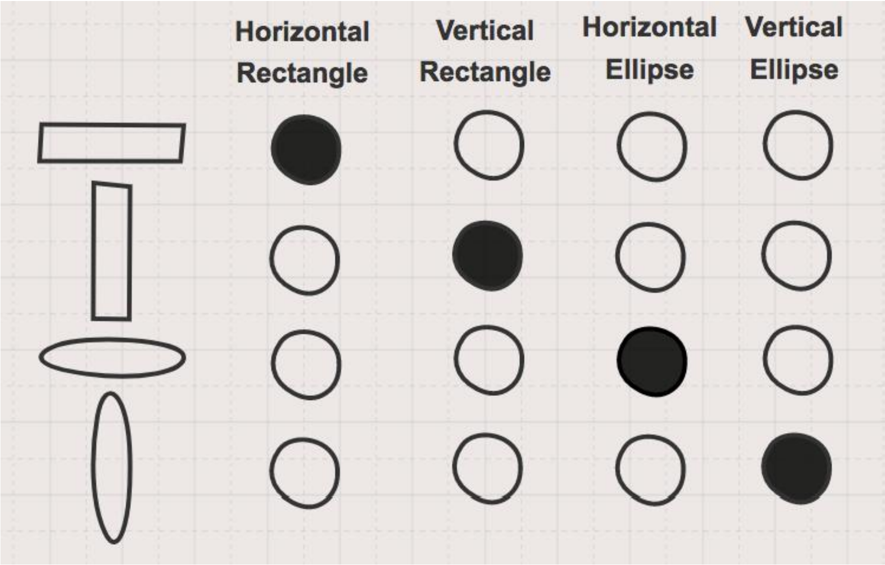
Common dimension size: 100-d,200-d,300-*d*, …

These dense word vectors are also called word **embeddings** (嵌入)

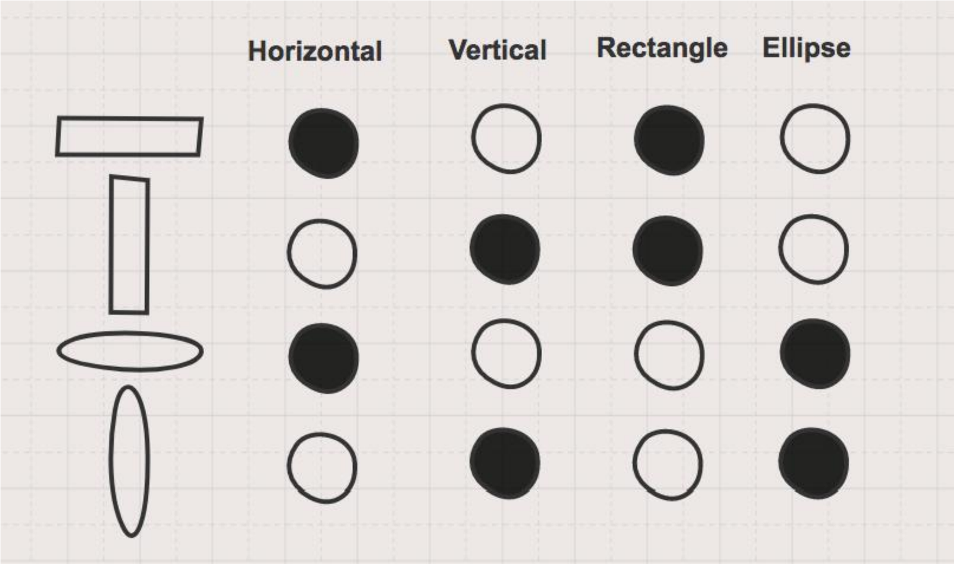
(which implies the idea of placing or mapping words into some continuous vector space)

Intuition: One-hot vs. Distributed repr.

One-hot representation



Distributed representation



The individual dimensions of a word embedding do not have concrete "meanings"

$$E_{Orange} = \begin{bmatrix} -0.01 \\ 0.00 \\ 0.02 \\ 0.97 \\ \cdots \\ 0.22 \end{bmatrix}$$

For instance, e_{orange} It does NOT mean

1st dimension -0.01 is for "animalness"

4th dimension 0.97 is for "fruitness"

They are only meaningful when compared to other words

Images source: https://www.oreilly.com/ideas/how-neural-networks-learn-distributed-representations

Question: How to obtain word embeddings?

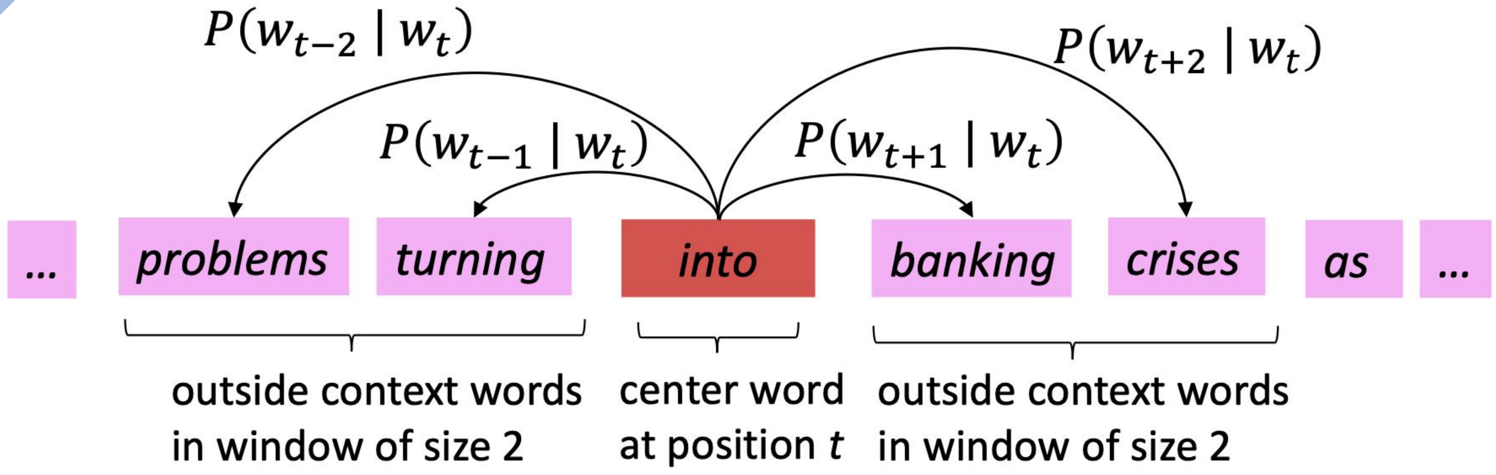
- An effective and efficient method: Word2vec (Mikolov et al. 2013 a&b)
- **Basic Idea**:
- Given a corpus as a list of words
- Go through each position t in the text, which has a center word c and context ("outside") words o
- Use the **similarity of word vectors** between c and o to **compute the probability** of o given c, i.e., conditional probability $P(o|c)$ (or vice versa)
- **Maximize this probability** by keep adjusting the word vectors

Two architectures of Word2vec

Word2vec
- **Skip-gram**: Maximize $P(o|c)$
- Continuous Bag-of-words (**CBOW**): Maximize $P(c|o)$

- "o" for outside (context) words
- "c" for center word

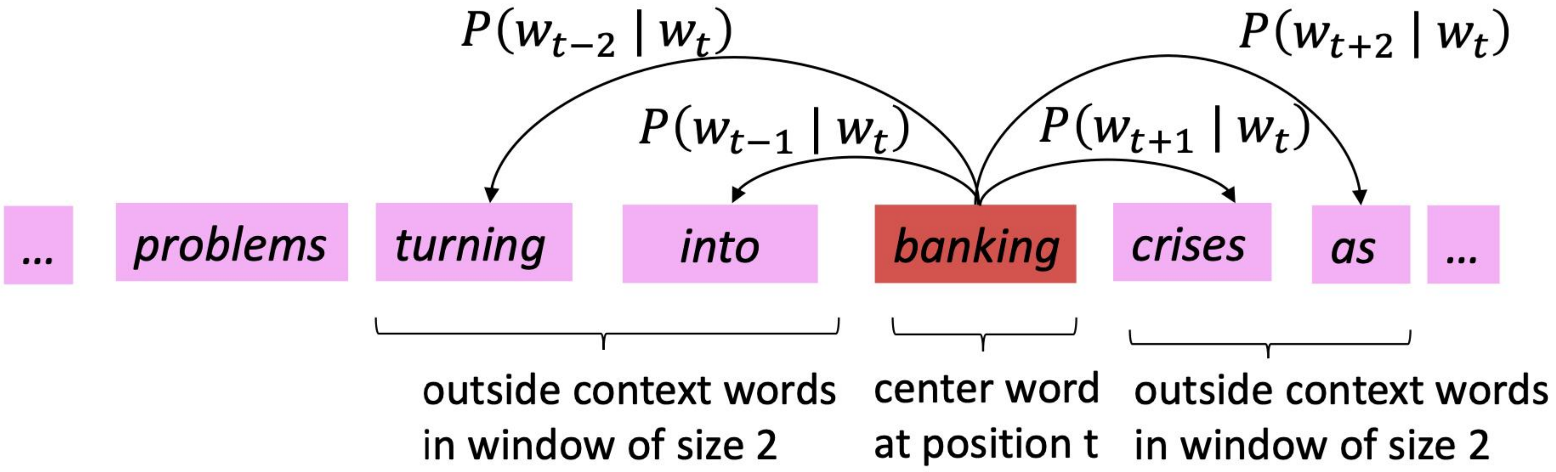
Compute probability $P(w_{t+j}|w_t)$, for $j \in \{-2, -1, 1, 2\}$ when window size is 2



Example from: https://web.stanford.edu/class/archive/cs/cs224n/cs224n.1224/

Use A Moving Window $t \leftarrow t + 1$

Skip-gram: Compute probability $P(w_{t+j}|w_t)$, for $j \in \{-2, -1, 1, 2\}$ when window size is 2



Example from: https://web.stanford.edu/class/archive/cs/cs224n/cs224n.1224/

Continuous Bag-of-Words (CBOW)

$P(w_t | w_{t-2}, w_{t-1}, w_{t+1}, w_{t+2})$

Aggregate all context words as if they a bag of words

I would like **some orange** **juice** **with some** white chocolate chips

2 outside context words | center word | 2 outside context words

Compute only one probability at position t:
$P(w_t | w_{t-2}, w_{t-1}, w_{t+1}, w_{t+2})$, for window size 2

Word2vec Objective Function (Skip-gram as example)

- Given a data set of T tokens, for each position $t = 1, \ldots, T$, we compute the conditional probability $P(w_{t+j} | w_t)$, for $j \in \{-m, \ldots, m\}$, with window size m
- Then the *likelihood* of data is:

$$\mathbb{L}(\theta) = \prod_{t=1}^{T} \prod_{\substack{-m \le j \le m \\ j \ne 0}} P(w_{t+j} | w_t; \theta)$$

θ denotes model parameters, that is, all the word **embeddings** to be learned!

- The objective function (cost/loss) is the negative log-likelihood

$$J(\theta) = -\frac{1}{T} \log \mathbb{L}(\theta) = -\frac{1}{T} \sum_{t=1}^{T} \sum_{\substack{-m \le j \le m \\ j \ne 0}} \log P(w_{t+j} | w_t; \theta)$$

Question: How to compute $P(w_{t+j}|w_t;\theta)$?

- **Solution**: Use *two* vectors per word w
- When w is a center word, its vector is v_w
- When w is a context (outside) word, its vector is u_w
- Then the conditional probability of context word o given center word c can be computed using **softmax** function:

$$P(o|c) = \frac{\exp\left(u_o^\top v_c\right)}{\sum_{w \in V} \exp\left(u_w^\top v_c\right)}$$

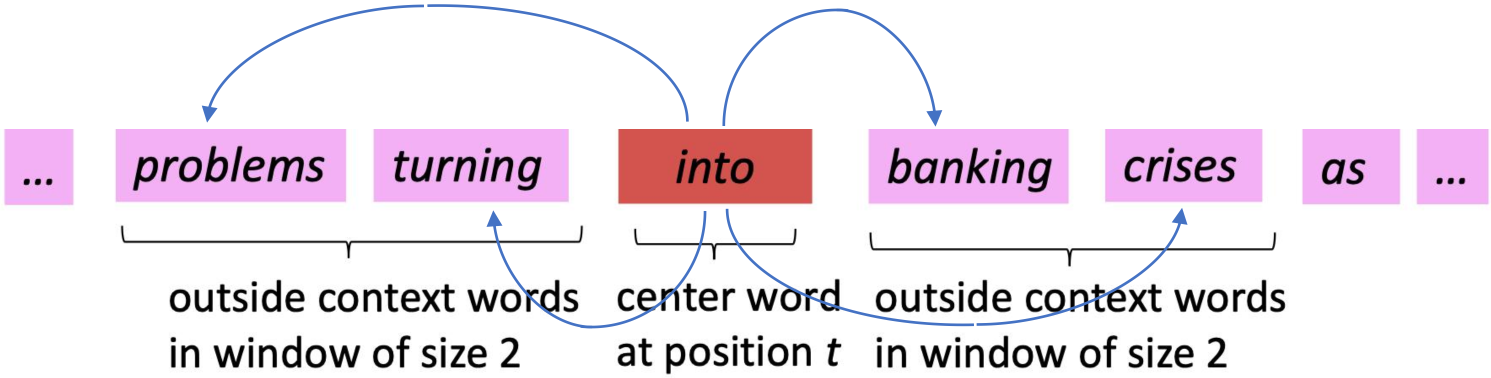
Dot product measures the similarity between o and c

Normalized over the entire vocabulary

Compute probabilities using softmax

$$P(problems|into) = \frac{\exp(u_{problems}^{\top} v_{into})}{\sum_{w \in V} \exp(u_w^{\top} v_{into})}$$

$$P(banking|into) = \frac{\exp(u_{banking}^{\top} v_{into})}{\sum_{w \in V} \exp(u_w^{\top} v_{into})}$$



$$P(turning|into) = \frac{\exp(u_{turning}^{\top} v_{into})}{\sum_{w \in V} \exp(u_w^{\top} v_{into})}$$

$$P(crises|into) = \frac{\exp(u_{crises}^{\top} v_{into})}{\sum_{w \in V} \exp(u_w^{\top} v_{into})}$$

Example from: https://web.stanford.edu/class/archive/cs/cs224n/cs224n.1224/

Number of Parameters

- Because *two* vectors are used per word w: v_w and u_w
- => Two parameter tables, or, embedding matrices

Usually we keep the target table **V** as the trained embeddings

Total # of params:
2 * [vocab-size] * [emb-size]

emb size

vocab size

j th row: v_j
target vector
for word w_j

j th row: u_j
context vector
for word w_j

emb size

vocab size

Table **V** contains all parameters for center vectors

Table **U** contains all parameters for context vectors

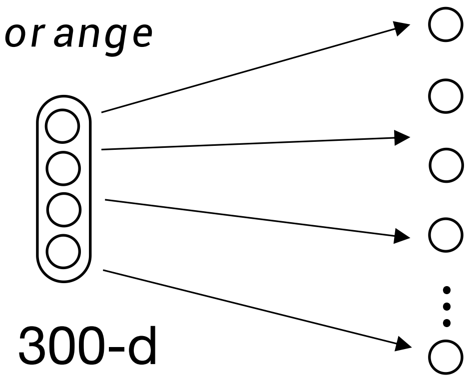
Problem with Softmax

center context

I	would	like	some	orange	juice	with	some	white	chocolate	chips

$$P(juice|orange) = \frac{\exp(u_{juice}^{\top} v_{orange})}{\sum_{w \in V} \exp(u_w^{\top} v_{orange})}$$

v_{orange}



300-d

$z_1 = u_a^{\top} v_{orange}$

$z_2 = u_{about}^{\top} v_{orange}$

$z_3 = u_{all}^{\top} v_{orange}$

$z_j = u_{juice}^{\top} v_{orange}$

$z_{10000} = u_{zoo}^{\top} v_{orange}$

For a vocabulary of 10,000 words

Needs 10,000 times of dot product to compute the denominator

To Overcome Softmax

Solutions

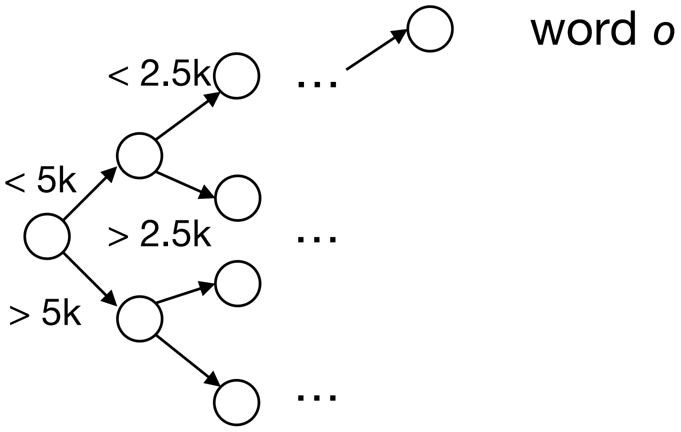
1. Hierarchical softmax
2. **Negative sampling**

Make binary predictions instead:

$$P\left(o < \frac{|V|}{2} \middle| c\right)$$

The probability of word o belongs to the 1st half of vocabulary

For vocabulary size $|V| = 10k$



Multiple steps of binary predictions until word o is found

Time complexity $O(\log(|V|))$

Then $P(o|c) = P(o < 5k|c) \cdot P(o < 2.5k|c) \cdot P(o < 1.25k|c)\ldots$

product of probabilities along the path

Reference: http://ruder.io/word-embeddings-softmax/

Solution 2: Negative sampling

window size = 2 | center | window size = 2

I *would* *like* *some* *orange* *juice* *with* *some* *white* *chocolate* *chips*

Goal: Given a center word, predict if a *randomly sampled* word is its context or not (within a fixed wi

Center	***Target Word***	***Label***
orange	*juice*	*1*
orange	*king*	*0*
orange	*the*	*0*
orange	*of*	*0*
orange	*book*	*0*

x *y*

Step 1: Pick a context word within the window

Positive sample

Step 2: Randomly pick *k* words from the entire vocabulary that do not appear in the window

Negative samples

Negative Sampling: Objective Function

- For token at position t, maximize the log-likelihood:

Word o is the positive sample

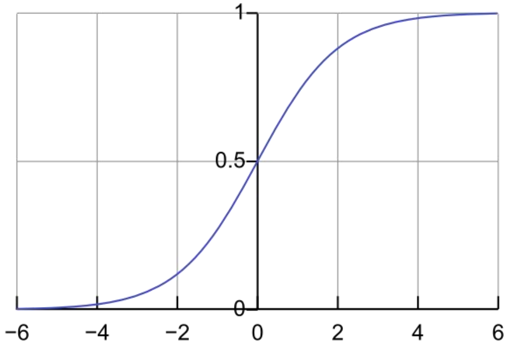
$$J_t(\theta) = \log \sigma(u_o^\top v_c) + \sum_{i=1}^{k} \mathbb{E}_{w_i \sim P(w)} [\log \sigma(-u_{w_i}^\top v_c)]$$

The k words w_i ($i = 1 \ldots k$) are the negative samples

- Sigmoid function $\sigma(u_o^\top v_c)$ outputs the probability of o in the context window

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

a monotone increasing function



Maximizing this term will push the dot product $u_o^\top v_c$ to **larger** values, i.e., making o and c closer in semantic space

Maximizing this term will push the dot product $u_{w_i}^\top v_c$ to **smaller** values, i.e., making w_i and c farther apart in semantic space

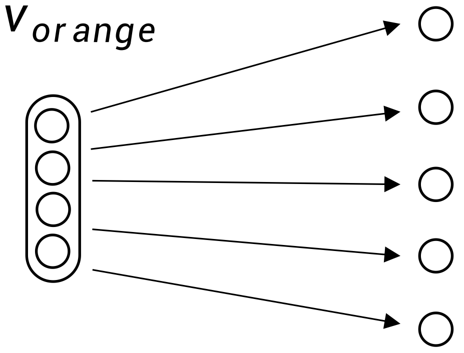
Negative sampling: Example

c	t	y
Center	Target	Context or not
orange	*juice*	1
orange	*king*	0
orange	*the*	0
orange	*of*	0
orange	*book*	0

Instead of using softmax: $P(t|c) = \dfrac{\exp(u_t \cdot v_c)}{\sum_{j=1...|V|} \exp(u_j \cdot v_c)} = \hat{y}_t$

Use logistic regression: $P(y = 1|c, t) = \sigma(u_t \cdot v_c)$

v_{orange}



$\hat{y}_1 = P(juice = 1) y_1 = 1$

$\hat{y}_2 = P(king = 1) y_2 = 0$

$\hat{y}_3 = P(the = 1) \quad y_3 = 0$

$\hat{y}_4 = P(of = 1) \quad y_4 = 0$

$\hat{y}_5 = P(book = 1) y_5 = 0$

k+1 times of logistic regression

For each center word, the k negative examples are different

$k = 5 \sim 20$ for small dataset

$k = 2 \sim 5$ for large dataset

Negative Sampling: More Details

- Maximize probability that *real* outside word appears;
- Minimize probability that *random* words appear around center word

- Sample from the distribution $P(w) = \frac{U(w)^{\frac{3}{4}}}{Z}$, the unigram frequency distribution $U(w)$ raised to the ¾ power (Z is normalization term)
- The power makes less frequent words be sampled more often
- $0.9^{3/4} \approx 0.924$ => a 2.7% increase in chance being sampled
- $0.1^{3/4} \approx 0.178$ => a 77.8% increase in chance being sampled

Content

- Motivation
- Documents and Counts-based Method
- Neural Network-based Method -- word2vec
- **Evaluation and Applications**

General Evaluation in NLP

- Intrinsic (内在的) vs. Extrinsic (外在的)
- Intrinsic:
  - Evaluation on a specific/intermediate subtask
  - Fast to compute
  - Not clear if really helpful unless correlation to real task is found
- Extrinsic:
  - Evaluation on a real task
  - Can take a long time to compute accuracy
  - Unclear if the subsystem is the problem or its interaction with other subsystems

Adapted from: https://web.stanford.edu/class/archive/cs/cs224n/cs224n.1224/

Evaluate Word Vectors (Embeddings)

- Intrinsic task: Word semantic similarity task

$d_1 = \cos(e_{book}, e_{library})$, cosine similarity

Word1	Word2	Human score	Cosine distance
book	library	7.46	d1
bank	money	8.12	d2
wood	forest	7.73	d3
professor	cucumber	0.31	d4
…	…	…	…

Spearman's correlation between the two columns are used to evaluate the quality of word embeddings

Evaluate Word Vectors (Embeddings)

- Intrinsic task: Word analogy task

Question: What is to "King" as "woman" to "man"?

$$e_{Man} - e_{Woman} \approx e_{King} - e_w \qquad w = ?$$

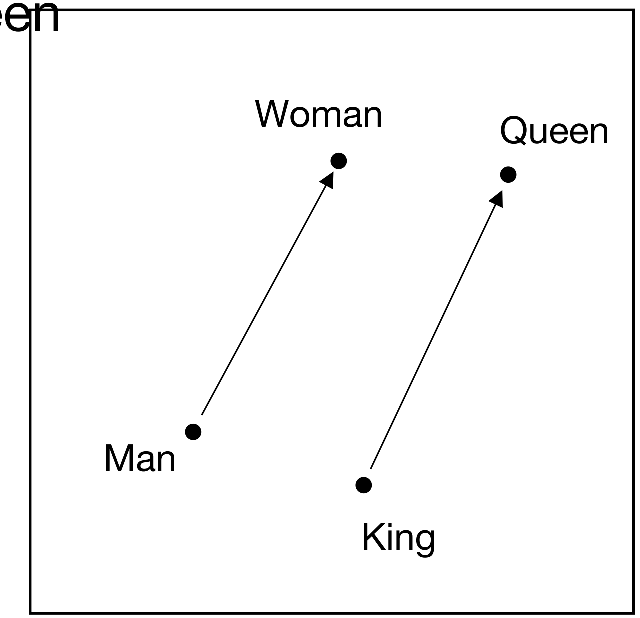
Find the word w so that:

$$\arg\max_w sim(e_w, e_{King} - e_{Man} + e_{Woman})$$

Here, $sim()$ is a similarity function, for example, cosine similarity

$$sim(u, v) = \frac{u^T v}{\|u\|\|v\|}$$

Finding the most similar vector e_w will hopefully pick up the word w = Queen



Word Analogy Task (as an interesting application)

Capital-common-countries:

Athens Greece Baghdad **Iraq**
Athens Greece Bangkok **Thailand**
Athens Greece Beijing **China**
Athens Greece Berlin **Germany**
…

Family:

boy girl brother **sister**
boy girl brothers **sisters**
boy girl dad **mom**
boy girl father **mother**
…

Comparative:

bad worse big bigger
bad worse bright brighter
bad worse cheap cheaper
bad worse cold colder
…

City-in-state:

Chicago Illinois Houston **Texas**
Chicago Illinois Philadelphia **Pennsylvani**
Chicago Illinois Phoenix **Arizona**
Chicago Illinois Dallas **Texas**
…

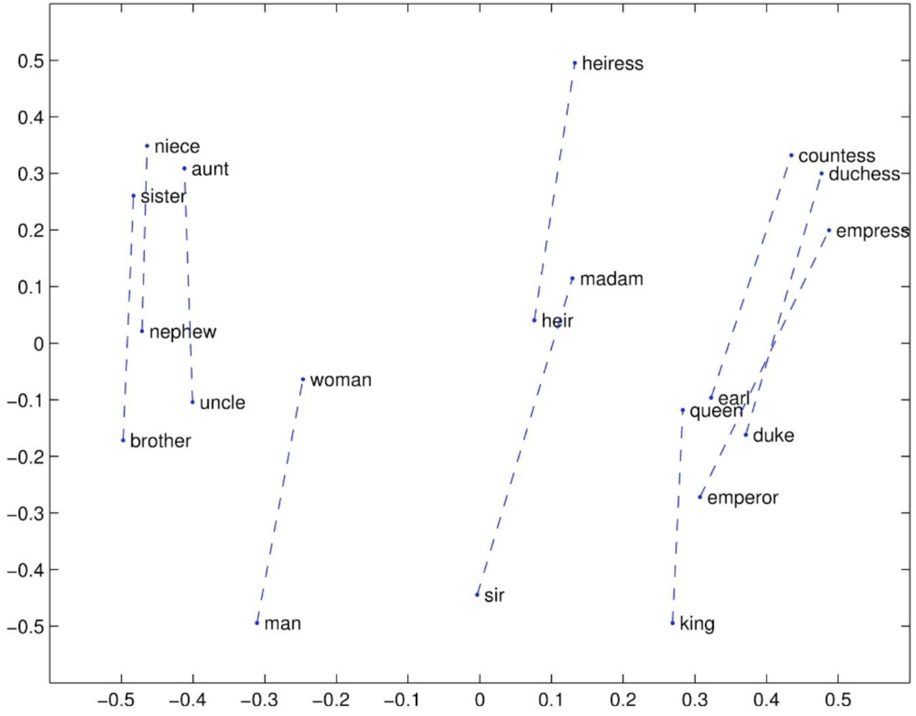


Figure from: Pennington et al. (2014). Glove: Global vectors for word representation.

70 - 80 % accuracy reported in Mikolov et al., 2013

Extension: GloVe

Pennington, J., Socher, R., & Manning, C. (2014). Glove: Global vectors for word representation.

A Different way from Word2vec

How the paper gets to this function step by step is deep and enlightening (reading recommended)

Cost function: $J = \sum_{i,j=1}^{V} f(X_{ij})(e'_i e_j + b_i + b'_j - \log(X_{ij}))^2$

Dot product of two embeddings

Frequency counts of word i and j co-occur (within a fixed window)

Basic idea: words that appear together more often (larger X_{ij}) should have closer meanings (larger dot product $e'_i e_j$)

Advantages: Fast training; scalable to huge corpra

Fun Application: Emoji2vec

Eisner, B., Rocktäschel, T., Augenstein, I., Bošnjak, M., & Riedel, S. (2016). emoji2vec: Learning emoji representations from their description. *arXiv preprint arXiv:1609.08359*.

To-Do

- Attend Lab 3
- Continue working on A1
- Read Chapter 9 - RNNs and LSTMs

References

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