Exercises for Section 11.4

Ex 1.
Partitions on $A = \{a, b\}$

1. {{a, b}}
2. {{a}, {b}}

Ex 2.
Partitions on $A = \{a, b, c\}$

1. {{a, b, c}}
2. {{a}, {b, c}}
3. {{b}, {a, c}}
4. {{c}, {a, b}}
5. {{a}, {b}, {c}}

Ex 3.
The equivalence relation $\equiv \pmod 4$ partitions $\mathbb{Z}$ into
$\{[0], [1], [2], [3]\} = \{\{..., -4, 0, 4, ...\}, \{..., -3, 1, 5, ...\}, \{..., -2, 2, 6, ...\}, \{..., -1, 3, 7, ...\}, \}$.

Ex 4. Proposition: Suppose P is a partition of set A, R is a relation on A such that $(x, y) \in R$ if and only if $x, y \in X$ for some $X \in P$. Then R is an equivalence relation on A.

Proof.
First we show that R is reflexive. Suppose $x \in A$. Because P is a partition of A, we have that $x \in X$ for some $X \in P$. By definition of R, it follows that $(x, x) \in R$ and consequently R is reflexive.

Next we show that R is symmetric. Suppose $(x, y) \in R$. So $x, y \in X$ for some $X \in P$. From this it directly follows that $(y, x) \in R$, thus R is symmetric.

Finally we show that R is transitive. Suppose $(a, b), (b, c) \in R$. Then $a, b \in X_1$ and $b, c \in X_2$ for some $X_1, X_2 \in P$. Then the definition of P (two different subsets of P have no elements in common) combined with the fact that $b \in X_1$ and $b \in X_2$, implies that $X_1 = X_2$. So $a, b, c \in X_1$, implies that $(a, c) \in R$. Thus R is transitive.

$\square$

Ex 4. **Proposition:** Suppose P is a partition of set A, R is a relation on A such that $(x, y) \in R$ if and only if $x, y \in X$ for some $X \in P$. Then P is the set of equivalence classes of R.

Proof.

Skipped for now. There is a non-cogent proof here.

□

Ex 5. R is the equivalence relation $\equiv \pmod 2$.

Ex 6. R is the equivalence relation defined as: $(x, y) \in R$ if and only if $|x| = |y|$.