Exercises for section 12.1

Ex 1. Domain: $A = \{0, 1, 2, 3, 4\}$, range: $\{2, 3, 4\}$, $f(2) = 4$, and $f(1) = 3$.

Ex 2. Domain: $A = \{a, b, c, d\}$, range: $\{2, 3, 4, 5\}$, $f(b) = 3$, and $f(d) = 5$.

Ex 3. Four different f functions:

1. $f = \{(a, 0), (b, 0)\}$
2. $f = \{(a, 0), (b, 1)\}$
3. $f = \{(a, 1), (b, 0)\}$
4. $f = \{(a, 1), (b, 1)\}$

Ex 4. Eight different f functions:

1. $f = \{(a, 0), (b, 0), (c, 0)\}$
2. $f = \{(a, 0), (b, 0), (c, 1)\}$
3. $f = \{(a, 0), (b, 1), (c, 0)\}$
4. $f = \{(a, 1), (b, 0), (c, 0)\}$
5. $f = \{(a, 1), (b, 1), (c, 0)\}$
6. $f = \{(a, 1), (b, 0), (c, 1)\}$
7. $f = \{(a, 0), (b, 1), (c, 1)\}$
8. $f = \{(a, 1), (b, 1), (c, 1)\}$

Ex 5. $R = \{(a, d), (b, d), (c, d), (d, d), (d, e)\}$. R is not a function because $(d, d), (d, e) \in R$.

Ex 6. Domain and codomain are both $\mathbb{Z}$. Note that $4x + 5 = 4(x + 1) + 1$, because domain is $\mathbb{Z}$, the range is $\{4(x + 1) + 1 : x\mathbb{Z}\} = \{..., -7, -3, 1, 5, 9, ...\}$. Finally, $f(10) = 45$.

Ex 7. Yes, f's domain is $\mathbb{Z}$, range is subset of the codomain, and for each $x \in \mathbb{Z}$, $f(x)$ is uniquely defined.

Ex 8. No, for f to be a function it must be the case that $f(x)$ is uniquely defined for all $x \in \mathbb{Z}$. Observe the counter-example: $f(0)$ is not defined.

Ex 9. No, for f to be a function it must be the case that $f(x)$ is uniquely defined for all $x \in \mathbb{Z}$. Observe the counter-example: $f(-1)$ is not defined.

Ex 10. Yes, f's domain and codmain are $\mathbb{R}$, and for each $x \in \mathbb{R}$, $f(x)$ is uniquely defined.

Ex 11. Yes. The domain is is the powerset of $\mathbb{Z}_5$. The range is $\{0, 1, 2, 3, 4, 5\}$.

Ex 12. Yes. The domain is $\mathbb{R}^2$, the codomain is $\mathbb{R}^3$, and the range is $\{(3y, 2x, x + y) : x, y \in \mathbb{R}\}$.