Ex 1.

$$\begin{bmatrix} 1 & 5 & 7 \\ -2 & -7 & -5 \end{bmatrix} \implies \begin{bmatrix} 1 & 5 & 7 \\ 0 & 3 & 9 \end{bmatrix} \implies \begin{bmatrix} 1 & 5 & 7 \\ 0 & 1 & 3 \end{bmatrix} \implies \begin{bmatrix} 1 & 0 & -8 \\ 0 & 1 & 3 \end{bmatrix} \implies x_1 = -8, x_2 = 3$$

Ex 2.

$$\begin{bmatrix} 2 & 4 & -4 \\ 5 & 7 & 11 \end{bmatrix} \implies \begin{bmatrix} 1 & 2 & -2 \\ 5 & 7 & 11 \end{bmatrix} \implies \begin{bmatrix} 1 & 2 & -2 \\ 0 & -3 & 21 \end{bmatrix} \implies$$

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -7 \end{bmatrix} \implies \begin{bmatrix} 1 & 0 & -12 \\ 0 & 1 & -7 \end{bmatrix} \implies x_1 = 12, x_2 = -7$$

Ex 3.

$$\begin{bmatrix} 1 & 5 & 7 \\ 1 & -2 & -2 \end{bmatrix} \implies \begin{bmatrix} 1 & 5 & 7 \\ 0 & -7 & -9 \end{bmatrix} \implies \begin{bmatrix} 1 & 5 & 7 \\ 0 & 1 & 9/7 \end{bmatrix} \implies \begin{bmatrix} 1 & 0 & 4/7 \\ 0 & 1 & 9/7 \end{bmatrix} \implies x_1 = \frac{4}{7}, x_2 = \frac{9}{7}$$

Ex 4.

$$\begin{bmatrix} 1 & -5 & 1 \\ 3 & -7 & -5 \end{bmatrix} \implies \begin{bmatrix} 1 & -5 & 1 \\ 0 & 8 & -8 \end{bmatrix} \implies \begin{bmatrix} 1 & -5 & 1 \\ 0 & 1 & -1 \end{bmatrix} \implies$$

$$\begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -1 \end{bmatrix} \implies x_1 = -4, x_2 = -1$$

Ex 5. Add row-3 multiplied by 3 to row-2. Add row-3 multiplied by -5 to row-1.

Ex 6. Add row-3 multiplied by -3 to row-4. Multiply row-4 with $\dfrac{-1}{5}$.

Ex 7. Observe that row-3 has no solutions. So the system has no solutions.

Ex 8.

$$\begin{bmatrix} 1 & -4 & 9 & 0 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix} \implies \begin{bmatrix} 1 & -4 & 9 & 0 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \implies \begin{bmatrix} 1 & -4 & 9 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \implies$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \implies x_1 = x_2 = x_3 = 0$$

Ex 9.

$$\begin{bmatrix} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 2 & 4 \end{bmatrix} \implies \begin{bmatrix} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \implies \begin{bmatrix} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \implies$$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \implies \begin{bmatrix} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \implies x_1 = 4, x_2 = 8, x_3 = 5, x_4 = 2$$

Ex 10.

$$\begin{bmatrix} 1 & -2 & 0 & 3 & -2 \\ 0 & 1 & 0 & -4 & 7 \\ 0 & 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 1 & -3 \end{bmatrix} \implies \begin{bmatrix} 1 & -2 & 0 & 3 & -2 \\ 0 & 1 & 0 & 0 & -5 \\ 0 & 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 1 & -3 \end{bmatrix} \implies \begin{bmatrix} 1 & -2 & 0 & 0 & 7 \\ 0 & 1 & 0 & 0 & -5 \\ 0 & 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 1 & -3 \end{bmatrix} \implies$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 & -5 \\ 0 & 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 1 & -3 \end{bmatrix} \implies x_1 = -3, x_2 = -5, x_3 = 6, x_4 = -3$$

Ex 11.

$$\begin{bmatrix} 0 & 1 & 4 & -5 \\ 1 & 3 & 5 & -2 \\ 3 & 7 & 7 & 6 \end{bmatrix} \implies \begin{bmatrix} 0 & 1 & 4 & -5 \\ 1 & 3 & 5 & -2 \\ 0 & -2 & -8 & 12 \end{bmatrix} \implies \begin{bmatrix} 0 & 1 & 4 & -5 \\ 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -6 \end{bmatrix} \implies$$

$$\begin{bmatrix} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -5 \\ 0 & 1 & 4 & -6 \end{bmatrix} \implies \text{The bottom two rows lead to a contradiction. Hence there is no solution.}$$

Ex 12.

$$\begin{bmatrix} 1 & -3 & 4 & -4 \\ 3 & -7 & 7 & -8 \\ -4 & 6 & -1 & 7 \end{bmatrix} \implies \begin{bmatrix} 1 & -3 & 4 & -4 \\ 0 & 2 & -5 & 4 \\ -4 & 6 & -1 & 7 \end{bmatrix} \implies \begin{bmatrix} 1 & -3 & 4 & -4 \\ 0 & 2 & -5 & 4 \\ 0 & -6 & 15 & -9 \end{bmatrix} \implies$$

$$\begin{bmatrix} 1 & -3 & 4 & -4 \\ 0 & 2 & -5 & 4 \\ 0 & 2 & -5 & 3 \end{bmatrix} \implies \text{The bottom two rows lead to a contradiction. Hence there is no solution.}$$

Ex 13.

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{bmatrix} \implies \begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 1 & 5 & -2 \end{bmatrix} \implies \begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 0 & 5 & -5 \\ 0 & 1 & 5 & -2 \end{bmatrix} \implies \begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 5 & -5 \end{bmatrix} \implies$$

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & -1 \end{bmatrix} \implies \begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix} \implies \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix} \implies x_1 = 5, x_2 = 3, x_3 = -1$$

Ex 14.

$$\begin{bmatrix} 1 & -3 & 0 & 5 \\ -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0 \end{bmatrix} \implies \begin{bmatrix} 0 & -2 & 5 & 7 \\ -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0 \end{bmatrix} \implies \begin{bmatrix} 0 & 0 & 7 & 7 \\ -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0 \end{bmatrix} \implies \begin{bmatrix} 0 & 0 & 1 & 1 \\ -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0 \end{bmatrix} \implies \begin{bmatrix} -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \implies$$

$$\begin{bmatrix} -1 & 1 & 5 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \implies \begin{bmatrix} -1 & 1 & 0 & -3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \implies \begin{bmatrix} -1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \implies$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \implies x_1 = 2, x_2 = -1, x_3 = 1$$

Ex 15.

$$
\begin{bmatrix} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -2 & 3 & 2 & 1 \\ 3 & 0 & 0 & 7 & -5 \end{bmatrix} \implies
\begin{bmatrix} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -2 & 3 & 2 & 1 \\ 0 & 0 & -9 & 7 & -11 \end{bmatrix} \implies
\begin{bmatrix} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 3 & -4 & 7 \\ 0 & 0 & -9 & 7 & -11 \end{bmatrix} \implies
$$

$$
\begin{bmatrix} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 3 & -4 & 7 \\ 0 & 0 & 0 & -5 & 10 \end{bmatrix} \implies \text{System is consistent.}
$$

Ex 16.

$$
\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ -2 & 3 & 2 & 1 & 5 \end{bmatrix} \implies
\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 3 & 2 & -3 & -1 \end{bmatrix} \implies
\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 3 & 2 & -3 & -1 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \end{bmatrix} \implies
\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 3 & 2 & -3 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \end{bmatrix} \implies
$$

$$
\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 3 & 2 & -3 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \end{bmatrix} \implies
\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 3 & 2 & -3 & -1 \\ 0 & 0 & \frac{1}{3} & 1 & \frac{1}{3} \\ 0 & 0 & 1 & 3 & 1 \end{bmatrix} \implies
\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 3 & 2 & -3 & -1 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \implies
\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 3 & 0 & -9 & -3 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \implies
$$

$$
\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 0 & -3 & -1 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \implies \text{System is consistent.}
$$

Ex 17.

$$
\begin{bmatrix} 1 & -4 & 1 \\ 2 & -1 & -3 \\ -1 & -3 & 4 \end{bmatrix} \implies
\begin{bmatrix} 1 & -4 & 1 \\ 0 & -7 & 5 \\ -1 & -3 & 4 \end{bmatrix} \implies
\begin{bmatrix} 1 & -4 & 1 \\ -1 & -3 & 4 \\ 0 & -7 & 5 \end{bmatrix} \implies
\begin{bmatrix} 1 & -4 & 1 \\ 0 & -7 & 5 \\ 0 & -7 & 5 \end{bmatrix} \implies
\begin{bmatrix} 1 & -4 & 1 \\ 0 & -7 & 5 \\ 0 & 0 & 0 \end{bmatrix} \implies
$$

$$
\begin{bmatrix} 1 & 0 & -13/7 \\ 0 & -7 & 5 \\ 0 & 0 & 0 \end{bmatrix} \implies
\begin{bmatrix} 1 & 0 & -13/7 \\ 0 & 1 & -5/7 \\ 0 & 0 & 0 \end{bmatrix} \implies x_1 = \frac{-13}{7}, x_2 = \frac{-5}{7}
$$

Ex 18.

$$
\begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 1 & 3 & 0 & 0 \end{bmatrix} \implies
\begin{bmatrix} 0 & -1 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 1 & 3 & 0 & 0 \end{bmatrix} \implies
\begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & -1 & 1 & 4 \\ 0 & 1 & -1 & 1 \end{bmatrix} \implies
$$

$$
\begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & 0 & 5 \end{bmatrix} \implies \text{System is not consistent, hence there is no common point of intersection.}
$$

Ex 19.

$$
\begin{bmatrix} 1 & h & 4 \\ 3 & 6 & 8 \end{bmatrix} \implies
\begin{bmatrix} 1 & h & 4 \\ 0 & 6-3h & -4 \end{bmatrix} \implies
\begin{bmatrix} 1 & 6-2h & 0 \\ 0 & 6-3h & -4 \end{bmatrix} \implies \text{System is consistent when } h \neq 2.
$$

Ex 20.

$$
\begin{bmatrix} 1 & h & -3 \\ -2 & 4 & 6 \end{bmatrix} \implies
\begin{bmatrix} 1 & h & -3 \\ -1 & 2 & 3 \end{bmatrix} \implies
\begin{bmatrix} 1 & h & -3 \\ 0 & 2+h & 0 \end{bmatrix} \implies \text{System is consistent for all } h.
$$

Ex 21.

$$\begin{bmatrix} 1 & 3 & -2 \\ -4 & h & 8 \end{bmatrix} \implies \begin{bmatrix} 1 & 3 & -2 \\ -1 & h/4 & 2 \end{bmatrix} \implies \begin{bmatrix} 1 & 3 & -2 \\ 0 & h/4+3 & 0 \end{bmatrix} \implies \text{System is consistent for all } h.$$

Ex 22.

$$\begin{bmatrix} 2 & -3 & h \\ -6 & 9 & 5 \end{bmatrix} \implies \begin{bmatrix} 2 & -3 & h \\ 0 & 0 & 5+3h \end{bmatrix} \implies \text{System is consistent when } h = \frac{-5}{3}.$$

Ex 23.

a) True. b) False, rows are stated first. So a 5×6 matrix means 5 rows and 6 columns. c) False, the statement refers to a single solution. A solution set has one or more solutions. d) True, see middle of page 7.

Ex 24.

a) True. b) False. c) False. An inconsistent system has no solution. d) True

Ex 25.

$$\begin{bmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{bmatrix} \implies \begin{bmatrix} 1 & -4 & 7 & g \\ -2 & 5 & -9 & k \\ 0 & 3 & -5 & h \end{bmatrix} \implies \begin{bmatrix} 1 & -4 & 7 & g \\ 0 & -3 & 5 & k+2g \\ 0 & 3 & -5 & h \end{bmatrix} \implies \begin{bmatrix} 1 & -4 & 7 & g \\ 0 & -3 & 5 & k+2g \\ 0 & 0 & 0 & h+k+2g \end{bmatrix} \implies$$

System is consisten when $h + k + 2g = 0$

Ex 26.

$$\begin{bmatrix} -3 & -5 & 7 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \implies \begin{bmatrix} -3 & -5 & 7 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \implies \begin{bmatrix} -3 & -5 & 7 & 1 \\ 0 & 4 & 4 & 4 \\ 0 & 0 & 1 & 0 \end{bmatrix} \implies x_1 = -2, x_2 = 1, x_3 = 0$$

Ex 27.

$$\begin{bmatrix} 1 & 3 & f \\ c & d & g \end{bmatrix} \implies \begin{bmatrix} 1 & 3 & f \\ 0 & d-3c & g-fc \end{bmatrix} \implies d-3c \neq 0 \implies d \neq 3c$$

Ex 28.

$$\begin{bmatrix} a & b & f \\ c & d & g \end{bmatrix} \implies \begin{bmatrix} a & b & f \\ 0 & d-\dfrac{cb}{a} & g-\dfrac{fc}{a} \end{bmatrix} \implies d-\dfrac{cb}{a} \neq 0 \implies ad \neq cb$$

Ex 29 and 30. Skipped.

Ex 31.

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 5 & -2 & 8 \\ 4 & -1 & 3 & -6 \end{bmatrix} \leftrightarrow \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 5 & -2 & 8 \\ 0 & 7 & -1 & -6 \end{bmatrix}$$

Ex 32.

$$\begin{bmatrix} 1 & 2 & -5 & 0 \\ 0 & 1 & -3 & -2 \\ 0 & -3 & 9 & 5 \end{bmatrix} \leftrightarrow \begin{bmatrix} 1 & 2 & -5 & 0 \\ 0 & 1 & -3 & -2 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Ex 33.

$$4T_1 - T_2 - T_4 = 30$$
$$-T_1 + 4T_2 - T_3 = 60$$
$$-T_2 + 4T_3 - T_4 = 70$$
$$-T_1 - T_3 + 4T_4 = 40$$

Ex 34.

$$
\begin{bmatrix}
4 & -1 & 0 & -1 & 30 \\
-1 & 4 & -1 & 0 & 60 \\
0 & -1 & 4 & -1 & 70 \\
-1 & 0 & -1 & 4 & 40
\end{bmatrix}
\implies
\begin{bmatrix}
0 & 15 & -4 & -1 & 270 \\
-1 & 4 & -1 & 0 & 60 \\
0 & -1 & 4 & -1 & 70 \\
-1 & 0 & -1 & 4 & 40
\end{bmatrix}
\implies
\begin{bmatrix}
0 & 15 & -4 & -1 & 270 \\
1 & -4 & 1 & 0 & -60 \\
0 & -1 & 4 & -1 & 70 \\
-1 & 0 & -1 & 4 & 40
\end{bmatrix}
\implies
$$

$$
\begin{bmatrix}
1 & -4 & 1 & 0 & -60 \\
0 & 15 & -4 & -1 & 270 \\
0 & -1 & 4 & -1 & 70 \\
0 & -4 & 0 & 4 & -20
\end{bmatrix}
\implies
\begin{bmatrix}
1 & -4 & 1 & 0 & -60 \\
0 & 15 & -4 & -1 & 270 \\
0 & -1 & 4 & -1 & 70 \\
0 & 4 & 0 & -4 & 20
\end{bmatrix}
\implies
\begin{bmatrix}
1 & -4 & 1 & 0 & -60 \\
0 & 15 & -4 & -1 & 270 \\
0 & -1 & 4 & -1 & 70 \\
0 & 1 & 0 & -1 & 5
\end{bmatrix}
\implies
$$

$$
\begin{bmatrix}
1 & 0 & 1 & -4 & -40 \\
0 & 15 & -4 & -1 & 270 \\
0 & -1 & 4 & -1 & 70 \\
0 & 1 & 0 & -1 & 5
\end{bmatrix}
\implies
\begin{bmatrix}
1 & 0 & 1 & -4 & -40 \\
0 & 15 & -4 & -1 & 270 \\
0 & -1 & 4 & -1 & 70 \\
0 & 0 & 4 & -2 & 75
\end{bmatrix}
\implies
\begin{bmatrix}
1 & 0 & 1 & -4 & -40 \\
0 & 0 & 56 & -16 & 1320 \\
0 & -1 & 4 & -1 & 70 \\
0 & 0 & 4 & -2 & 75
\end{bmatrix}
\implies
$$

$$
\begin{bmatrix}
1 & 0 & 1 & -4 & -40 \\
0 & 0 & 7 & -2 & 165 \\
0 & -1 & 4 & -1 & 70 \\
0 & 0 & 4 & -2 & 75
\end{bmatrix}
\implies
\begin{bmatrix}
1 & 0 & 1 & -4 & -40 \\
0 & -1 & 4 & -1 & 70 \\
0 & 0 & 7 & -2 & 165 \\
0 & 0 & 4 & -2 & 75
\end{bmatrix}
\implies
\begin{bmatrix}
1 & 0 & 1 & -4 & -40 \\
0 & 1 & -4 & 1 & -70 \\
0 & 0 & 7 & -2 & 165 \\
0 & 0 & 4 & -2 & 75
\end{bmatrix}
\implies
\begin{bmatrix}
1 & 0 & 1 & -4 & -40 \\
0 & 1 & 0 & -1 & 5 \\
0 & 0 & 7 & -2 & 165 \\
0 & 0 & 4 & -2 & 75
\end{bmatrix}
\implies
$$

$$
\begin{bmatrix}
1 & 0 & 1 & -4 & -40 \\
0 & 1 & 0 & -1 & 5 \\
0 & 0 & 7 & -2 & 165 \\
0 & 0 & 0 & -6/7 & -135/7
\end{bmatrix}
\implies
\begin{bmatrix}
1 & 0 & 1 & -4 & -40 \\
0 & 1 & 0 & -1 & 5 \\
0 & 0 & 7 & -2 & 165 \\
0 & 0 & 0 & 1 & 22.5
\end{bmatrix}
\implies
\begin{bmatrix}
1 & 0 & 1 & -4 & -40 \\
0 & 1 & 0 & 0 & 27.5 \\
0 & 0 & 7 & -2 & 165 \\
0 & 0 & 0 & 1 & 22.5
\end{bmatrix}
\implies
\begin{bmatrix}
1 & 0 & 1 & -4 & -40 \\
0 & 1 & 0 & 0 & 27.5 \\
0 & 0 & 7 & 0 & 210 \\
0 & 0 & 0 & 1 & 22.5
\end{bmatrix}
\implies
$$

$$
\begin{bmatrix}
1 & 0 & 1 & -4 & -40 \\
0 & 1 & 0 & 0 & 27.5 \\
0 & 0 & 1 & 0 & 30 \\
0 & 0 & 0 & 1 & 22.5
\end{bmatrix}
\implies
\begin{bmatrix}
1 & 0 & 0 & -4 & -70 \\
0 & 1 & 0 & 0 & 27.5 \\
0 & 0 & 1 & 0 & 30 \\
0 & 0 & 0 & 1 & 22.5
\end{bmatrix}
\implies
\begin{bmatrix}
1 & 0 & 0 & 0 & 20 \\
0 & 1 & 0 & 0 & 27.5 \\
0 & 0 & 1 & 0 & 30 \\
0 & 0 & 0 & 1 & 22.5
\end{bmatrix}
\implies
T_1 = 20, T_2 =
$$

$27.5, T_3 = 30, T_4 = 22.5$