PeerGrader simulator - preliminary study

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December 6, 2021

1 Introduction

This report aims to propose a design for a PeerGrader simulator. The simulated system consists of a group of people grading each other homeworks. Specifically, taken a students population, every homework will be evaluated by a sample of the population. The objective of the simulation is to understand if such a grading system can provide a fair evaluation of a student, namely an evaluation that reflects the effective competence of a student.

2 Assumptions in the simulation model

2.1 Interval of evaluation

All the homeworks are evaluated with a real number in the range [0,1].

2.2 Quality of a student

Each student can be associated with a quality x_s, drawn from X_s, a random variable uniformly distributed in $[0, 1]$.

2.3 Quality of a homework

Each student tends to perform homeworks reflecting their quality. This means that the quality of the h-th homework of the s-th student Q_{hs} can be represented through a random variable (e.g. a truncated Gaussian), with mean x_s, and a given variance.

2.4 Aggregation reliability

The evaluations given by a group of students, once aggregated, provide a good estimator of a homework quality. This means that the evaluation given by the k-th student to the h-th homework of the s-th student $E_{hs}^{(k)}$ can be represented through a random variable (e.g. a truncated Gaussian), with mean Q_{hs}, and a given variance.

2.5 Grade of a homework

The grade (i.e. the estimated quality) of a homework obtained by aggregating[1] the K evaluations of the h-th homework of the s-th student:

$$\hat{Q}_{hs} = \frac{\sum_{k=1}^{K} E_{hs}^{(k)}}{K} \tag{1}$$

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[1]The aggregation shown is an average, but it is possible to propose different alternatives, such as the median of the evaluations.

3   Input parameters of the simulation model

- seed: the random seed of the simulator.

- S: the number of students to evaluate.

- K: the number of received evaluations for delivery.

- H: the number of homeworks each student has performed.

- σ_s: the standard deviation of the random variable representing the qualities of the homeworks.

- σ_e: the standard deviation of the random variable representing the evaluations of the homeworks.

4   Output parameters of the simulation model

The output is the accuracy of the model, which requires the definition of Average Relative Grading Error:

$$\epsilon = \frac{1}{S} \frac{|\frac{\sum_{h=1}^{H} Q_{hs}}{H} - \frac{\sum_{h=1}^{H} \hat{Q}_{hs}}{H}|}{\frac{\sum_{h=1}^{H} Q_{hs}}{H}} = \frac{1}{S} \frac{|\sum_{h=1}^{H} Q_{hs} - \sum_{h=1}^{H} \hat{Q}_{hs}|}{\sum_{h=1}^{H} Q_{hs}} \tag{2}$$

This is the error committed on the evaluation of a single student, normalized with respect to the mean of the student's homeworks. Thus, the accuracy is defined as follows:

$$acc = 1 - \epsilon, \tag{3}$$

resulting in a real number in the range]-inf ,1], assuming its maximum value when no error is committed.