

We can notice that:

$$\Delta(ADC) \sim \Delta(AED)$$

So:

$$\frac{b}{x} = \frac{d}{y} = \frac{x}{e}$$
 [1]

Also:

$$\Delta(ADB) \sim \Delta(DEB)$$

So:

$$\frac{x}{y} = \frac{a}{c} = \frac{c}{f}$$
 [2]

We know that:

$$a = e + f$$
 [3]

From first equation we get:

$$\frac{b}{d} = \frac{x}{y} \quad [4]$$

Which combined with [2] results in:

$$\frac{x}{y} = \frac{b}{d} = \frac{a}{c} = \frac{c}{f} \quad [5]$$

We extract e and f from [1] and [2] and add them together. We get:

$$a = e + f = \frac{x^2}{b} + \frac{c^2}{a} = \frac{x^2}{b} + \frac{cd}{b}$$
 [6]

Therefore:

$$x^2 = ab - cd$$