# Guidance for Project 4 SLAM

## ESE650 University of Pennsylvania

## 1 Preparation

- 1. Read the assignment description and docs/config\_slam.pdf carefully. Try to understand (a) components of the given data, (b) how and where (frame) the data is collected (in order to understand the model), (c) how to use the given codes, and (d) which steps you need to follow to complete the project.
- 2. Try mapping from the first scan and plot the map
- 3. Try dead-reckoning and plot the robot trajectory
- 4. Try prediction only and plot the robot trajectories (100 for N = 100 particles)
- 5. Try the update step with only 3-4 particles and see if the weight update makes sense

## 2 Notations

- $x^{(i)}$  represents a vector or a scalar x in frame i. (g), (b), (h) mean global, body, and head frames respectively.
- $R(\theta)$  is a rotation matrix: For SO(2) (special orthonormal group),

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

For SO(3), rotations with respect to z, y, x-axis, respectively,

$$\begin{bmatrix} \cos(\theta_z) & -\sin(\theta_z) & 0 \\ \sin(\theta_z) & \cos(\theta_z) & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} \cos(\theta_y) & 0 & \sin(\theta_y) \\ 0 & 1 & 0 \\ -\sin(\theta_y) & 0 & \cos(\theta_y) \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_x) & -\sin(\theta_x) \\ 0 & \sin(\theta_x) & \cos(\theta_x) \end{bmatrix}$$

• Homogeneous transform matrix between Frame 0 and Frame 1,  $A_1^0$ , is defined as:

$$A_1^0 = \begin{bmatrix} R_1^0 & \mathbf{d}_1^0 \\ \mathbf{0}^T & 1 \end{bmatrix}$$

where  $R_1^0 \in SO(3)$  is a rotatio matrix of Frame 1 with respect to Frame 0,  $\mathbf{d}_1^0 \in \mathbb{R}^3$  is a vector of the origin of Frame 1 with respect to Frame 0, and  $\mathbf{0}^T = [0,0,0]$ . Therefore,  $A_1^0$  belongs to the *special Euclidean group* SE(3).

• Coordinate transformation of Frame n with respect to Frame 0,  $T_n^0$ :

$$T_n^0 = A_1^0 A_2^1 \cdots A_n^{n-1}$$

and

$$\mathbf{v}^{(0)} = T_n^0 \mathbf{v}^{(n)}$$

for a vector **v**.

• Smart Plus

$$x_{t+1} \oplus x_t \equiv \begin{bmatrix} p_t + R(\theta_t)p_{t+1} \\ \theta_t + \theta_{t+1} \end{bmatrix}$$
$$x_{t+1} \ominus x_t \equiv x_t^{-1} \oplus x_{t+1} = \begin{bmatrix} R^T(\theta_t)(p_{t+1} - p_t) \\ \theta_{t+1} - \theta_t \end{bmatrix}$$

## 3 Mapping

### Input

- 1. laser scan  $z_t$  (laser.scan)
- 2. transform from head to body frame:  $T_h^b$  (head and neck angles)
- 3. robot pose  $x_t$  determining transform from body to global frame:  $T_b^g$  (current best particle and could use laser.rpy but be careful with transform from IMU to body frame)
- 4. current log-odds map  $m_t$

## Output

1. Updated log-odds map  $m_{t+1}$ 

### Pseudo-code

- 1. Transform  $z_t = z_t^{(h)}$  via  $T_b^g * T_h^b$  to the global frame
- 2. Remove scan points that are too close, too far, or hit the ground
- 3. Use MapUtils/bresenham2D.py/getMapCellsFromRay to obtain the cell locations  $y_t^o$  that are occupied according to the laser and  $y_t^f$  that are free according to the laser
- 4. Increase the log-odds in  $m_t$  of the occupied cells  $y_t^o$  and decrease the odds on the free cells  $y_t^f$  to obtain  $m_{t+1}$

## 4 Dead-reckoning

## Input

- 1. Current robot pose  $p_t \in SE(2)$
- 2. Global frame odometry (laser.odom)  $o_t$  and  $o_{t+1}$

### Output

1. Updated robot pose  $p_{t+1} \in SE(2)$ 

### Pseudo-code

1.  $p_{t+1} = p_t \oplus (o_{t+1} \ominus o_t)$ 

## 5 Localization Prediction

## Input

- 1. Current particles:  $p_t^n \in SE(2), n = 1, ..., N$
- 2. Global frame odometry (laser.odom)  $o_t$  and  $o_{t+1}$

### Output

1. Updated particles:  $p_{t+1}^n \in SE(2), n = 1, ..., N$ 

## Pseudo-code

1.  $p_{t+1}^n = p_t^n \oplus (o_{t+1} \ominus o_t) \oplus w_t^n$ ,  $w_t^n \sim \mathcal{N}(0, W_{3\times 3})$ 

<sup>\*</sup> Note that some of notations may be different from notations in your reference materials.

## Localization Update

#### Algorithm 6.1

## Input

- 1. Current particle positions and weights:  $(p_t^n, a_t^n), n = 1, \dots, N$
- 2. Laser scan  $z_t$
- 3. Current map  $m_t$
- 4. Transform from head to body:  $T_h^b$  (head and neck angles)

## Output

1. Updated particle positions and weights:  $(p_{t+1}^n, a_{t+1}^n), n = 1, \dots, N$ 

## Pseudo-code

- 1. For each particle  $n = 1, \ldots, N$ :
  - Transform  $z_t$  via  $T_b^g * T_h^b$  to the global frame, where  $T_b^g$  is determined from  $p_t^n$  (and optionally laser.rpy but be careful with transform from IMU to robot center of mass)
  - Remove scan points that are too close, too far, or hit the ground

Find the cells  $y_t$  corresponding to the global-frame scan  $z_t$ . For speed you can just use the points that were hit by the laser instead of MapUtils/bresenham2D.py/getMapCellsFromRay

- Compute  $corr(m_t, y_t)$  using mapCorrelation in MapUtils.py. Call mapCorrelation with a grid of values (e.g., 9x9) around the current particle position to get a good correlation. See test\_MapUtils.py for an example variation in x and y. You can also consider adding variation in yaw for the particle.
- 2. Update the particle weights (see the below section)
- 3. If  $N_{eff} < N_{threshold}$ , re-sample the particles

#### 6.2Updating Weights

There are two ways to compute the weights: one easy but slightly incorrect and one easy but correct. We define the measurement likelihood as follows:

$$p_h(z_t|x,m) = \frac{exp(corr(z_t,m))}{\sum_z exp(corr(z,m))}$$

We are interested in the following in the particle filter update step

$$a_{t+1|t+1}^{(k)} = \eta_{t+1} a_{t+1|t}^{(k)} exp(corr(z_{t+1}, m))$$

where  $\eta_{t+1}$  is the normalization due to  $\sum_{z} exp(corr(z,m))$  and  $\sum_{j} a_{t+1|t}^{(j)} p_h(z_{t+1}|\mu_{t+1|t}^{(j)},m)$ .

- 1. The easy, slightly incorrect way:

  - a) say that  $p_h(z_t|x,m) \propto corr(z_t,m)$ b) update weights:  $a_{t+1|t+1}^{(k)} = a_{t+1|t}^{(k)} * corr(z_{t+1},m)$ c) normalize:  $a_{t+1|t+1}^{(k)} \leftarrow \frac{a_{t+1|t+1}^{(k)}}{\sum_j a_{t+1|t+1}^{(j)}}$
- 2. The easy, correct way
  - a) say that  $p_h(z_t|x,m) \propto exp(corr(z_t,m))$  and define  $w_{t|t}^{(k)} := log(a_{t|t}^{(k)})$

  - b) update weights:  $w_{t+1|t+1}^{(k)} = w_{t+1|t}^{(k)} + corr(z_{t+1}, m)$ c) normalize:  $w_{t+1|t+1}^{(k)} \leftarrow w_{t+1|t+1}^{(k)} \max_{j} w_{t+1|t+1}^{(j)} \log \sum_{i} exp(w_{t+1|t+1}^{(i)} \max_{j} w_{t+1|t+1}^{(j)})$

The function  $logsumexp(\mathbf{x},b) := b + \log \sum_{j} exp(x_{j}-b)$  is translation invariant in the second argument, so it can be computed robustly via:  $lse(\mathbf{x}, \max_{j} \mathbf{x}_{j}) = \max_{j} \mathbf{x}_{j} + \log \sum_{i} exp(x_{i} - \max_{j} \mathbf{x}_{j})$  as suggested above.

## 7 SLAM

Initialize  $p_0^n = (0, 0, 0), \ a_0^n = \frac{1}{N}, \ n = 1, \dots, N$ Input

- 1. Current particle positions and weights:  $(p_t^n, a_t^n), n = 1, \dots, N$
- 2. Laser scan  $z_t$  (laser.scan)
- 3. Current map  $m_t$
- 4. Transform from head to body:  $T_h^b$  (head and neck angles)
- 5. Global frame odometry  $o_t$  and  $o_{t+1}$  (laser.odom)

### Output

- 1. Updated particle positions and weights:  $(p_t^n, a_t^n), n = 1, \dots, N$
- 2. Updated log-odds map  $m_{t+1}$

### Pseudo-code

- 1. Find particle  $p_t^*$  with highest weight from  $(p_t^n, a_t^n), n = 1, \dots, N$
- 2.  $m_{t+1} \leftarrow Mapping(z_t, p_t^*, T_h^b, m_t)$
- 3.  $p_{t+1}^n \leftarrow LocalizationPrediction(p_t^n, o_t, o_{t+1})$
- $4. \ (p^n_{t+1}, a^n_{t+1}) \leftarrow Localization Update(p^n_{t+1}, a^n_t, z_t, m_{t+1}, T^b_h)$

## 8 Texture Mapping

## 8.1 Algorithm

### Input

- 1. rgb image  $I_t$  and depth image  $d_t$
- 2. transform from head to body:  $T_h^b$  (head and neck angles, note: the kinect is at different height from the laser)
- 3. robot pose  $x_t$  determining transform from body to global:  $T_b^g$  (current best particle and laser.rpy)
- 4. Current texture map:  $tm_t$

### Output

1. Updated texture map:  $tm_{t+1}$ 

## Pseudo-code

- 1. Transform  $I_t$  and  $d_t$  via  $T_b^g * T_h^b$
- 2. Find the ground plane in the transformed data via RANSAC or simple thresholding on the height
- 3. Color the cells in  $tm_t$  using the points from  $I_t$  and  $d_t$  that belong to the ground plane

## 8.2 Ground Detection

See docs/ground\_detection.pdf file.

- You need only 'fc' and 'cc' from IRcam\_Calib\_result.pkl and RGBcamera\_Calib\_result.pkl files.
- For 'fc' (focal length), the first element corresponds to  $f_u$  and the second element corresponds to  $f_v$  in the pdf file page 6.
- Find the parameters  $a_0, a_1, a_2, a_3$  as the page 10 instruction, choose an appropriate threshold value for  $\epsilon$  and detect ground points.
- As in the page 2, u corresponds to x-axis and columns and v corresponds to y-axis and rows. Note that you need to compute appropriate pixel positions using the principal point values before applying the equations in the document:
- Let (u', v') be a pixel position in a given RGB (or depth) image matrix and  $c = (c_x, c_y)$  be the RGB (or depth) principal point provided in the calibration pickle files. Then (u, v) used in the document is:  $u = u' c_x$  and  $v = v' c_y$ . For example, the correct (u, v) you will use for the pixel at (0, 0) in a depth image = (-258.42, -202.49).