EE-223 Instrumentation & Measurement

SE EE Spring 2022

About the Course

- General Concepts
- Measurement of Electrical Quantities
- Instrumentation Transformers
- Electronic Instruments
- Transducers
- Measurement of Non Electrical Quantities

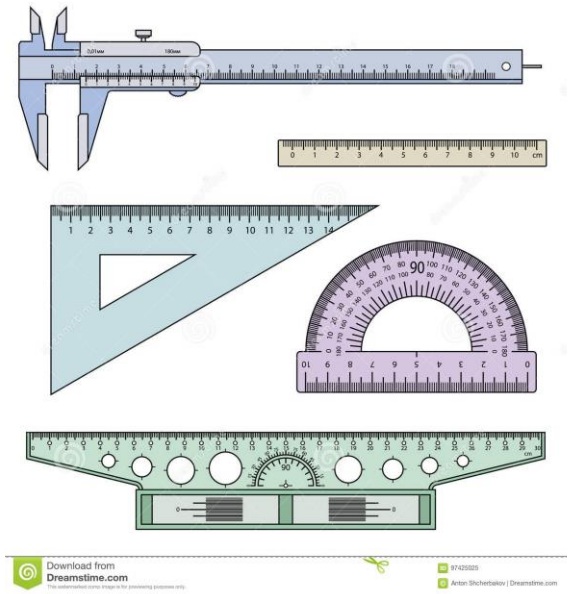
Reference Books

- Bell, David A. *Electronic instrumentation and measurements*. Englewood Cliffs, NJ: Regents/Prentice Hall,, 2011.
- Bakshi, Uday A., and Ajay V. Bakshi. *Electrical measurements and instrumentation*. Technical Publications, 2009.
- Kalsi, H. S. *Electronic Instrumentation*, *3e*. McGraw-Hill Education, 2010.

How to Reach the Instructor

- Write me an email, fezan@neduet.edu.pk

220/380V



Download from
Dreamstime.com

108
mg/dL

25 FT
1.The tape measure

What is Measurement

- Measurement is the process of converting physical parameter into a meaningful number.
- It enables us to describe a physical phenomena in quantitative manner.
- e.g. How fast a car is driving
  - How much electricity is consumed over a period of time
  - Amount of current flowing through a conductor
  - Blood pressure of a patient
  - Length of a road
  - TFLOPS

What is Measurement

- In engineering, measurement is the act or the result of a quantitative comparison between the quantity (whose magnitude is unknown) and a predefined standard.
- *Measurand : physical parameter to be measured*
- *Standard : a quantity of same kind chosen as a unit or basis for comparison of quantitative value to measure*
  - ** Standard must be accurately defined and commonly accepted*
  - ** Procedure and **instrument** used must be provable*
- Often in engineering a measurement is associated with a feedback to take regulating action for achieving desired objective (set point).

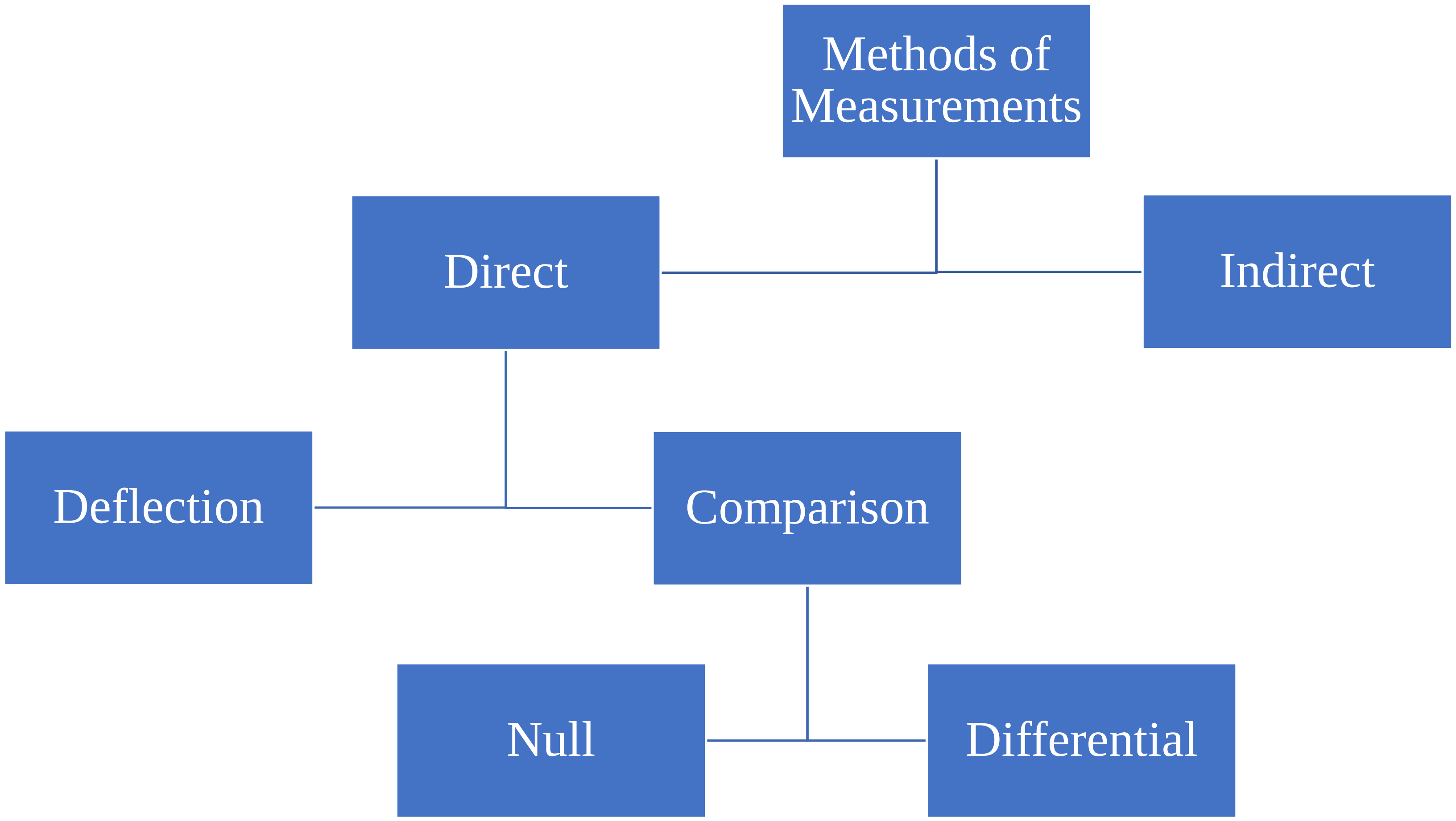
Methods of Measurement

- Methods of measurements can be classified as Direct and Indirect Measurements
- **DIRECT** Measurement : Simply put, this type of measurement is achieved directly from the measuring device and requires no further manipulation
- e.g. Voltage or current measurement, length measurement, temperature measurement etc

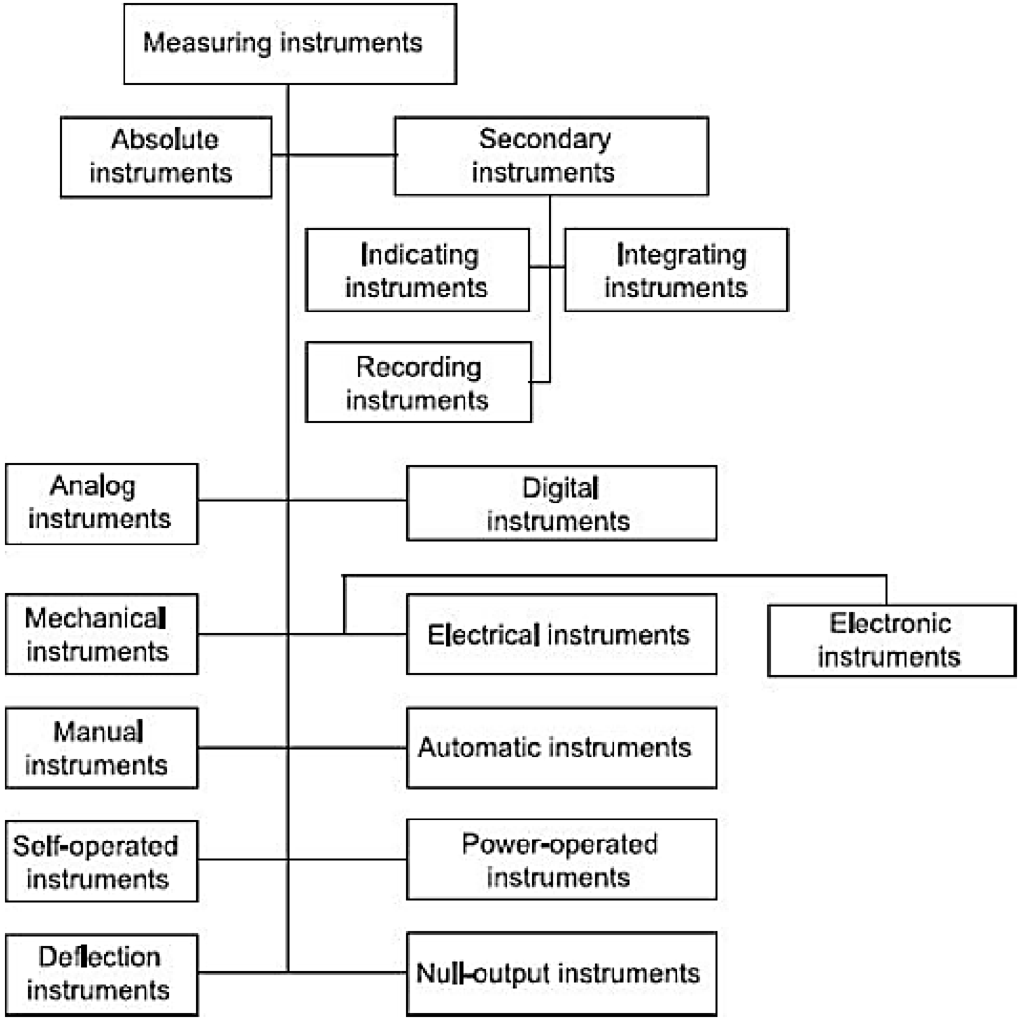
Methods of Measurement

- **<u>INDIRECT</u>** Measurement : Measurand is determined by measuring functionally related quantities which are then used to calculate the measurand
- e.g. Determining power by measuring voltage and current, power is determined as product of voltage and current
- Or determining volume by measuring length, width and height

Methods of Measurement



Classification of Instruments



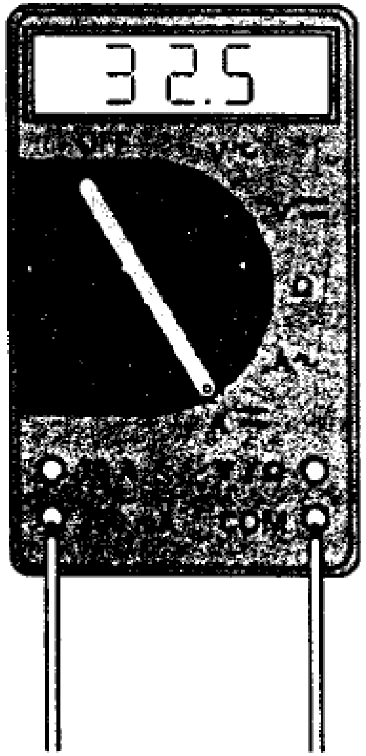
https://automationforum.co/classification-of-measuring-instruments

Measurement Errors

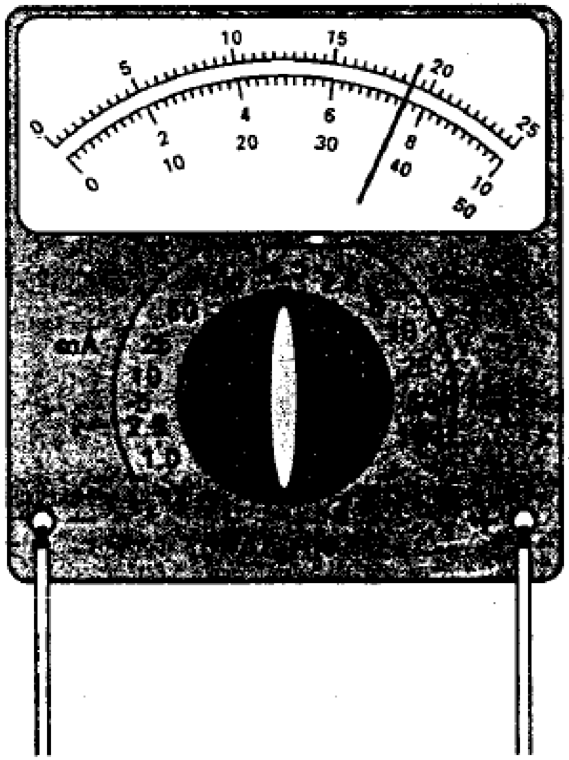
- Measurements may be prone to errors
- No equipment is perfectly accurate.
- Sometimes observer makes the error while taking measurements.

Gross Errors and Systematic Errors

- **Gross errors** are essentially human errors that are the result of carelessness or lack of experience. e.g. misreading of an instrument
- It can also occur due to incorrect adjustments of instruments
- (simply put, these errors are not due to the instrument, but the way it is used)



(a) Digital instrument
indicating 32.5 mA



(b) Analog instrument
indicating 0.76 V

- Sometimes meter is read correctly, but the reading is recorded incorrectly (either value or place in record table).
- These errors can be avoided through
  - Vigilant use of instrument
  - Putting instrument readings into appropriate equation or plotting a graph

- **Systematic errors** arise due to accuracy or correctness of the measuring instrument
- Instrument might not be calibrated
- Analog instrument might not be mechanically zeroed
- Sometime incorporating measuring device alters the system, hence affects the measured quantity, e.g. putting ammeter into a circuit changes its resistance
- If more than one instrument is involved, the errors due to instrument inaccuracy tend to accumulate.

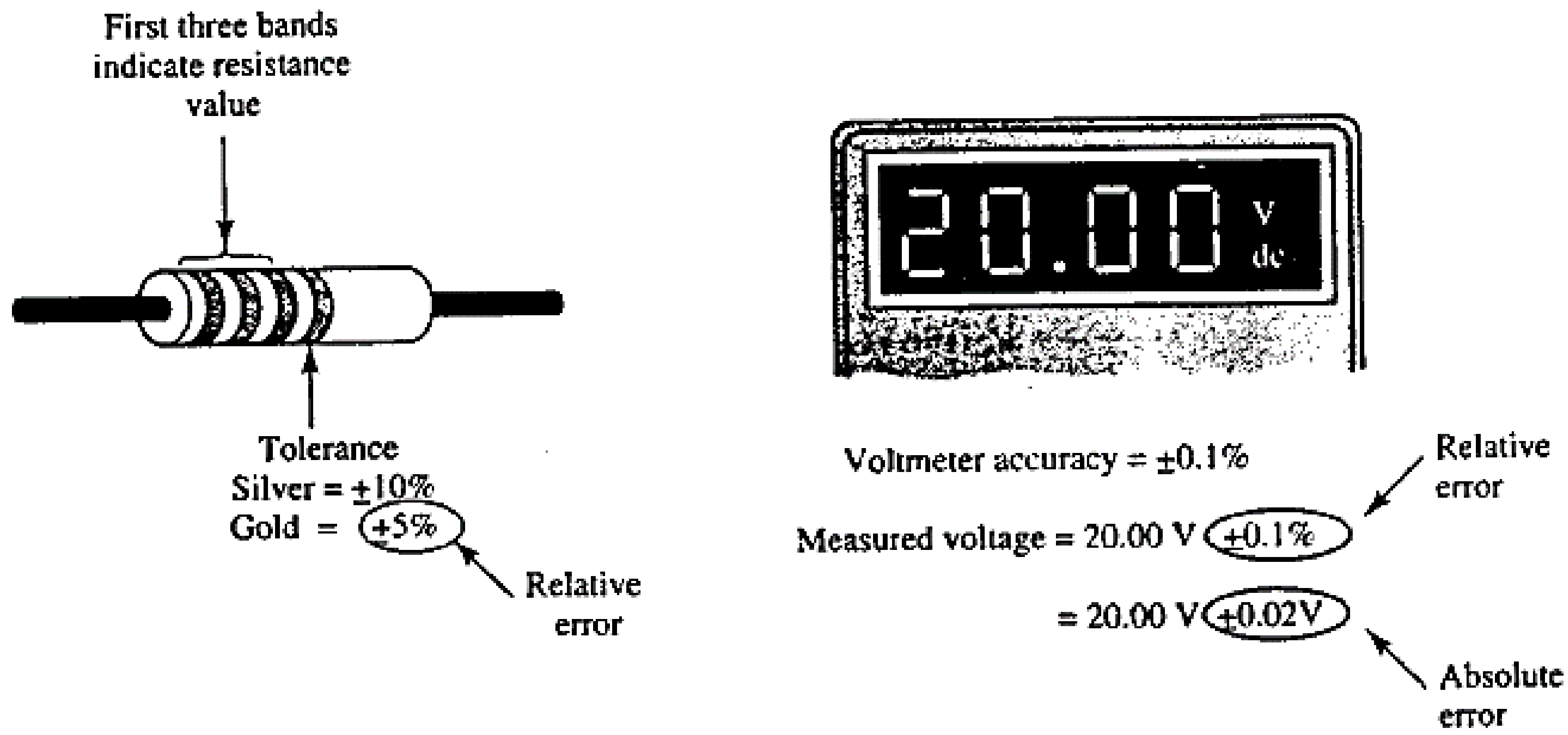
Absolute Errors and Relative Errors

If a resistor is known to have a resistance of 500 Ω with a possible error of ±50 Ω, the ±50 Ω is an *absolute error.* This is because 50 Ω is stated as an absolute quantity, *not* as a percentage of the 500 Ω resistance. When the error is expressed as a percentage or as a fraction of the total resistance, it becomes a *relative error.* Thus, the ±50 Ω is ±10%, relative to 500 Ω, or ±1/10 of 500 Ω. So the resistance can be specified as

$$R = 500\ \Omega \pm 10\%$$

Percentages are usually employed to express errors in resistances and other electrical quantities. The terms *accuracy* and *tolerance* are also used. A resistor with a possible error of ±10% is said to be accurate to ±10%, or to have a tolerance of ±10% [see Figure 2-2(a)]. *Tolerance* is the term normally used by component manufacturers. Suppose that a voltage is measured as 20.00 V using an instrument which is known to have a ±0.02 V

error. The measured voltage can be stated as 20.00 V ± 0.02 V. The 0.02 V is an absolute quantity, so it is an absolute error. But 0.02 V is also 0.1% relative to 20 V, so the measured quantity could be expressed as 20 V ± 0.1% [see Figure 2-2(b)], and now the error is stated as a relative error.



(a) Resistor tolerance is identified by a colored band

(b) Voltmeter accuracy defines the upper and lower limits of measured quantity

Figure 2-2 Percentage accuracy gives the relative error in a measured, or specified quantity. The absolute error can be determined by converting the percentage error into an absolute quantity.

Another method of expressing an error is to refer to it in *parts per million (ppm)* relative to the total quantity. For example, the temperature coefficient of a 1 MΩ resistor might be stated as 100 ppm/°C, which means 100 parts per million per degree Celsius. One millionth of 1 MΩ is 1 Ω; consequently, 100 ppm of 1 MΩ is 100 Ω. Therefore, a 1°C change in temperature may cause the 1 MΩ resistance to increase or decrease by 100 Ω (see Figure 2-3).

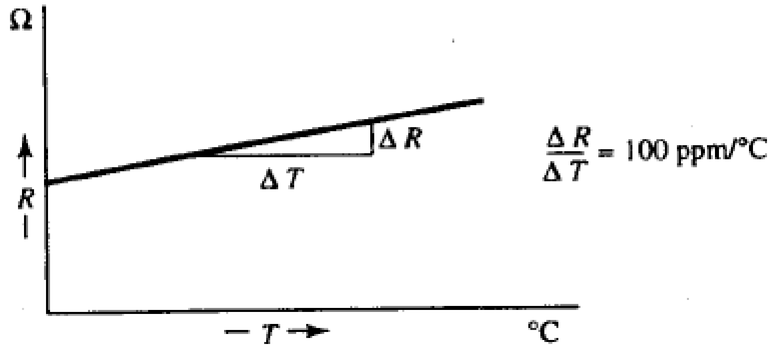


Figure 2-3 Instead of percentages, errors can be expressed in parts per million (ppm) relative to the total quantity. Resistance change with temperature increase is usually stated in ppm/°C.

A component manufacturer constructs certain resistances to be anywhere between 1.14 kΩ and 1.26 kΩ and classifies them to be 1.2 kΩ resistors. What tolerance should be stated? If the resistance values are specified at 25°C and the resistors have a temperature coefficient of +500 ppm/°C, calculate the maximum resistance that one of these components might have at 75°C.

Solution

$$\textit{Absolute error} = 1.26\ \text{k}\Omega - 1.2\ \text{k}\Omega = +0.06\ \text{k}\Omega$$

or

$$= 1.2\ \text{k}\Omega - 1.14\ \text{k}\Omega = -0.06\ \text{k}\Omega$$

$$= \pm 0.06\ \text{k}\Omega$$

$$\textit{Tolerance} = \frac{\pm 0.06\ \text{k}\Omega}{1.2\ \text{k}\Omega} \times 100\%$$

$$= \pm 5\%$$

Largest possible resistance at 25°C:

$$R = 1.2\ \text{k}\Omega + 0.06\ \text{k}\Omega$$

$$= 1.26\ \text{k}\Omega$$

Resistance change per °C:

$$500\ \text{ppm of}\ R = \frac{1.26\ \text{k}\Omega}{1\,000\,000} \times 500$$

$$= 0.63\ \Omega/^\circ\text{C}$$

Temperature increase:

$$\Delta T = 75^\circ\text{C} - 25^\circ\text{C}$$

$$= 50^\circ\text{C}$$

Total resistance increase:

$$\Delta R = 0.63\ \Omega/^\circ\text{C} \times 50^\circ\text{C}$$

$$= 31.5\ \Omega$$

Maximum resistance at 75°C:

$$R + \Delta R = 1.26\ \text{k}\Omega + 31.5\ \Omega$$

$$= 1.2915\ \text{k}\Omega$$

Accuracy, Precision, Resolution and Significant Figures

- Accuracy defines, how close the measured value is to the actual value.
- Precision defines, how consistently you can get that value. (specified with help of deviation)

- % of range
- % of reading

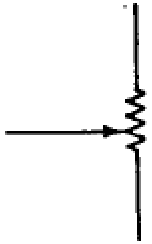
Resolution

- The smallest possible change which an instrument can produce, is the resolution.

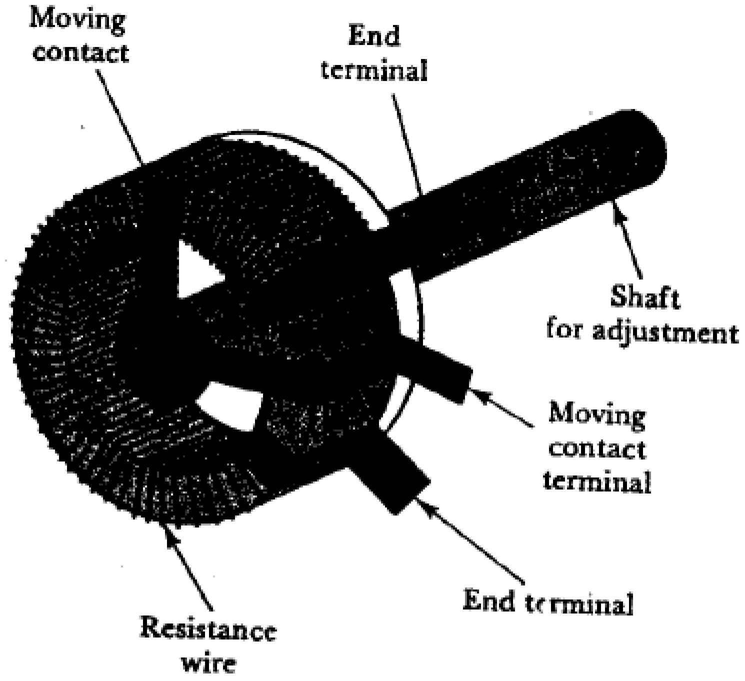
Consider the *potentiometer* illustrated in Figure 2-5. The circuit symbol in Figure 2-5(a) illustrates a resistor with two terminals and a contact that can be moved anywhere between the two. The potentiometer construction shown in Figure 2-5(b) reveals that the movable contact slides over a track on one side of a number of turns of resistance wire. The contact does not slide along the whole length of the wire but *jumps* from one point on one turn of the wire to a point on the next turn. Assume that the total potentiometer resistance is 100 Ω and that there are 1000 turns of wire. Each turn has a resistance of

$$\frac{100\ \Omega}{1000} = 0.1\ \Omega$$

When the contact moves from one turn to the next, the resistance from any end to the moving contact changes by 0.1 Ω. It can now be stated that the resistance from one end to the moving contact can be adjusted from 0 to 100 Ω with a *resolution* of 0.1 Ω, or a resolution of 1 in 1000. In the case of the potentiometer, the resolution defines how precisely the resistance may be set. It also defines how precisely the variable voltage from the potentiometer moving contact may be adjusted when a potential difference is applied across the potentiometer.

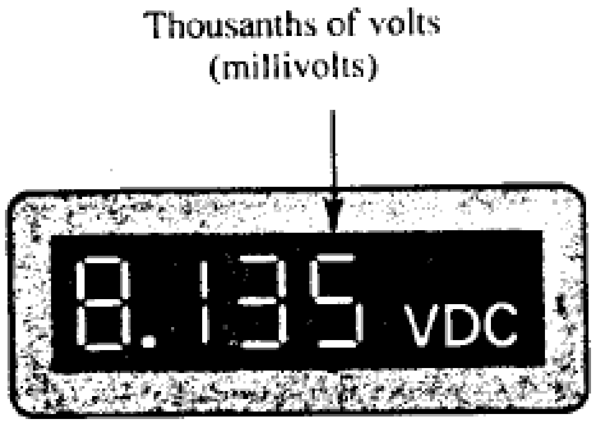


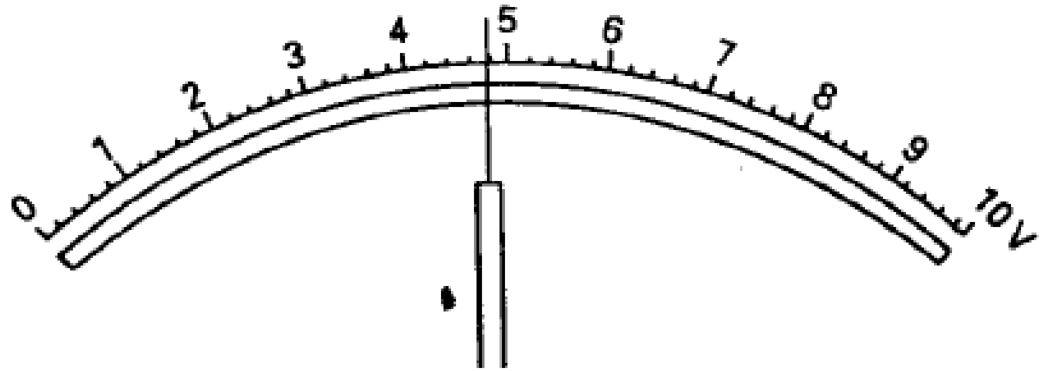
(a) Potentiometer circuit symbol



(b) Potentiometer construction

Figure 2-5 A potentiometer consists of a resistance wire wound around an insulating former. The movable contact slides from one turn to the next, changing the resistance (from one end to the moving contact) in steps. The potentiometer resolution depends on the number of steps.


Thousanths of volts
(millivolts)
8.135 VDC


0
1
2
3
4
5
6
7
8
9
10V

Significant Figures

The number of significant figures used in a measured quantity indicate the precision of measurement. For the 8.135 V measurement in Figure 2-4(a), the four significant figures

show that the measurement precision is 0.001 V, or 1 mV. If the measurement was made to a precision of 10 mV, the display would be 8.13 V or 8.14 V; that is, there would be only three significant figures.

In the case of a resistance value stated as 47.3 Ω, the actual value may not be exactly 47.3 Ω, but it is assumed to be closer to 47.3 Ω than it is to either 47.2 Ω or 47.4 Ω. The three significant figures show that measurement precision is 0.1 Ω. If the quantity was 47.3 kΩ, the measurement precision would be 0.1 kΩ, or 100 Ω. If 47.3 Ω is rewritten with two significant figures, it becomes 47 Ω, because 47.3 Ω is clearly closer to 47 Ω than it is to 48 Ω. If the quantity were written as 47.0 Ω, it would imply that the resistance is closer to 47 Ω than it is to 47.1 Ω, and in this case the zero (in 47.0 Ω) would be a significant figure.

Now consider the result of using an electronic calculator to determine a resistance value from digital measurement of voltage and current.

$$R = \frac{V}{I} = \frac{8.14\ \text{V}}{2.33\ \text{mA}} = 3.493\,562\,232\ \text{k}\Omega$$

Clearly, it does not make sense to have an answer containing 10 significant figures when each of the original quantities had only three significant figures. The only reasonable approach is to use the same number of significant figures in the answer as in the original quantities. So the calculation becomes

$$R = \frac{V}{I} = \frac{8.14\ \text{V}}{2.33\ \text{mA}} = 3.49\ \text{k}\Omega$$

As illustrated by the discussion above, the number of significant figures in a quantity defines the precision of the measuring instruments involved. No greater number of significant figures should be used in a calculation result than those in the original quantities. Where the quantities in a calculation have different precisions, the precision of the answer should not be greater than the least precise of the original quantities.

http://ccnmtl.columbia.edu/projects/mmt/frontiers/web/chapter_5/6665.html

Assignment

- Please prepare a list of about five to ten manufacturers of Digital Multimeters (and mention if you have used any of their product)
- Discover what are the differences among different models from the same manufacturer, please prepare a list of models and differences among them. Just Mention which one you will like to purchase for yourself and why
- Whenever you go the laboratory, please observe the manufacturer and features of available multimeters (please prepare a list), you may ask the technician to show you different kinds of devices they have
- Please go to the following webpage (or any other you find) and prepare a summary about accuracy of Digital Multimeters when it is specified as ±(0.5%+2) etc
- https://www.designworldonline.com/how-to-determine-digital-multimeter-accuracy/

Measurement Error Combinations

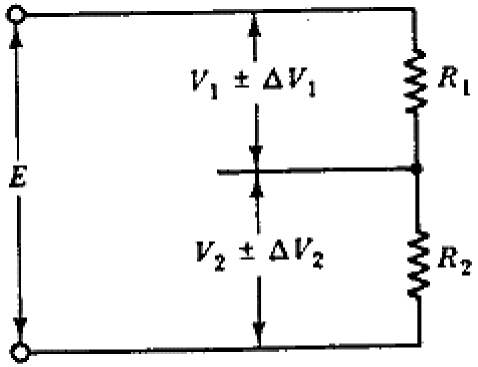
- When a quantity is calculated from measurements made on two (or more) instruments, it must be assumed that the errors due to instrument inaccuracy combine in the worst possible way. The resulting error is then larger than the error in any one instrument.

Sum of quantities

- Where a quantity is determined as the sum of two measurements, the total error is the sum of the absolute errors in each measurement.

$$E = (V_1 \pm \Delta V_1) + (V_2 \pm \Delta V_2)$$

$$\boxed{E = (V_1 + V_2) \pm (\Delta V_1 + \Delta V_2)}$$



$$\begin{aligned} E &= V_1 + V_2 \\ &= (V_1 \pm \Delta V_1) + (V_2 \pm \Delta V_2) \\ &= (V_1 + V_2) \pm (\Delta V_1 + V_2) \end{aligned}$$

(a) Error in sum of quantities equals sum of errors

Calculate the maximum percentage error in the sum of two voltage measurements when $V_1 = 100\ \text{V} \pm 1\%$ and $V_2 = 80\ \text{V} \pm 5\%$.

Solution

$$V_1 = 100\ \text{V} \pm 1\%$$
$$= 100\ \text{V} \pm 1\ \text{V}$$

$$V_2 = 80\ \text{V} \pm 5\%$$
$$= 80\ \text{V} \pm 4\ \text{V}$$

$$E = V_1 + V_2$$

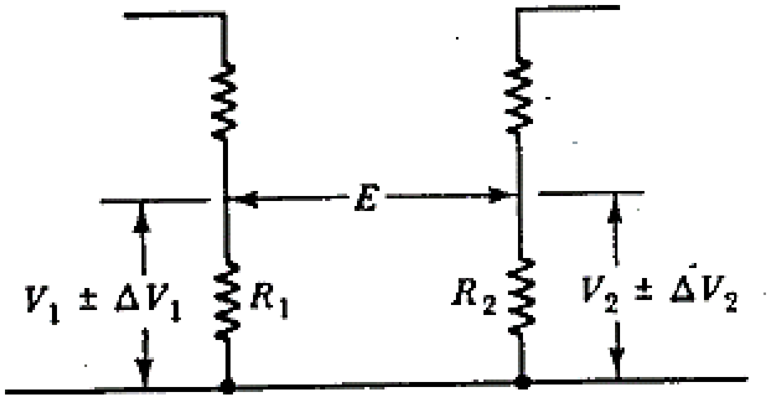
$$= (100\ \text{V} \pm 1\ \text{V}) + (80\ \text{V} \pm 4\ \text{V})$$
$$= 180\ \text{V} \pm (1\ \text{V} + 4\ \text{V})$$
$$= 180\ \text{V} \pm 5\ \text{V}$$
$$= 180\ \text{V} \pm 2.8\%$$

Please note, that the percentage error in the final quantity cannot be calculated directly from the percentage errors in the two measured quantities

When two or more measured quantities are summed to determine a final quantity, the absolute values of the errors must be summed to find the total possible error.

Difference of Quantities

- If a quantity is determined as the difference between two measured quantities (e.g, voltage), the ***errors are additive***



$$
\begin{aligned}
E &= V_1 - V_2 \\
&= (V_1 \pm \Delta V_1) - (V_2 \pm \Delta V_2) \\
&= (V_1 - V_2) \pm (\Delta V_1 + \Delta V_2)
\end{aligned}
$$

(b) Error in difference of quantities equals sum of error

$$
\begin{aligned}
E &= V_1 - V_2 \\
&= (V_1 \pm \Delta V_1) - (V_2 \pm \Delta V_2)
\end{aligned}
$$

$$\boxed{E = (V_1 - V_2) \pm (\Delta V_1 + \Delta V_2)}$$

Calculate the maximum percentage error in the difference of two measured voltages when $V_1 = 100\ \text{V} \pm 1\%$ and $V_2 = 80\ \text{V} \pm 5\%$.

Solution

$$\left.\begin{aligned} & V_1 = 100\ \text{V} \pm 1\ \text{V} \\ \text{and}\quad & V_2 = 80\ \text{V} \pm 4\ \text{V} \end{aligned}\right\}$$

The percentage error in the difference of two quantities can be very large. If the difference was smaller, the percentage error would be even larger.

$$\begin{aligned} E &= (100\ \text{V} \pm 1\ \text{V}) - (80\ \text{V} \pm 4\ \text{V}) \\ &= 20\ \text{V} \pm 5\ \text{V} \\ &= 20\ \text{V} \pm 25\% \end{aligned}$$

Product of Quantities

- When a calculated quantity is the product of two or more quantities, ***the percentage error is the sum of the percentage errors in each quantity.***

... each quantity [consider Figure 2-6(c)]:

$$P = EI$$

$$= (E \pm \Delta E)(I \pm \Delta I)$$

$$= EI \pm E\,\Delta I \pm I\,\Delta E \pm \Delta E\,\Delta I$$

Since $\Delta E\,\Delta I$ is very small,

$$P \simeq EI \pm (E\,\Delta I + I\,\Delta E)$$

$$\text{percentage error} = \frac{E\,\Delta I + I\,\Delta E}{EI} \times 100\%$$

$$= \left(\frac{E\,\Delta I}{EI} + \frac{I\,\Delta E}{EI}\right) \times 100\%$$

$$= \left(\frac{\Delta I}{I} + \frac{\Delta E}{E}\right) \times 100\%$$

$$\boxed{\%\ \text{error in } P = (\%\ \text{error in } I) + (\%\ \text{error in } E)}$$

Quotient of Quantities

- It is same as product of quantities

$$\boxed{\%\ error\ in\ E/I = (\%\ error\ in\ E) + (\%\ error\ in\ I)}$$

Quantity Raised to a Power

- When a quantity A is raised to a power B, the percentage error in A^B can be shown as

$$\boxed{\%\ error\ in\ A^B = B(\%\ error\ in\ A)}$$

For a current I with an accuracy of $\pm 3\%$, the error in I^2 is $2(\pm 3\%) = \pm 6\%$.

An 820 Ω resistance with an accuracy of ± 10% carries a current of 10 mA. The current was measured by an analog ammeter on a 25 mA range with an accuracy of ±2% of full scale. Calculate the power dissipated in the resistor, and determine the accuracy of the result.

Solution

$$P = I^2 R$$

$$P = (10\text{ mA})^2 \times 820\ \Omega$$

$$= 82\text{ mW}$$

$$\textit{error in } R = \pm 10\%$$

$$\textit{error in } I = \pm 2\% \text{ of } 25\text{ mA}$$

$$= \pm 0.5\text{ mA}$$

$$= \frac{\pm 0.5\text{ mA}}{10\text{ mA}} \times 100\%$$

$$= \pm 5\%$$

$$\%\ \textit{error in } I^2 = 2(\pm 5\%)$$

$$= \pm 10\%$$

$$\%\ \textit{error in } P = (\%\text{ error in } I^2) + (\%\text{ error in } R)$$

$$= \pm(10\% + 10\%)$$

$$= \pm 20\%$$

Summary

For $X = A \pm B$, $\quad$ $error\ in\ X = \pm\left[(error\ in\ A) + (error\ in\ B)\right]$

For $X = AB$, $\quad$ $\%\ error\ in\ X = \pm\left[(\%\ error\ in\ A) + (\%\ error\ in\ B)\right]$

For $X = A/B$, $\quad$ $\%\ error\ in\ X = \pm\left[(\%\ error\ in\ A) + (\%\ error\ in\ B)\right]$

For $X = A^B$, $\quad$ $\%\ error\ in\ X = \pm B(\%\ error\ in\ A)$

Statistical Analysis

- Arithmetic Mean Value

When a number of measurements of a quantity are made and the measurements are not all exactly equal, the best approximation to the actual value is found by calculating the average value, or *arithmetic mean*, of the results. For n measured values of $x_1, x_2, x_3, \ldots, x_n$, the arithmetic mean is

$$\boxed{\bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}} \tag{2-6}$$

Determining the arithmetic mean of several measurements is one method of minimizing the effects of *random errors*. Random errors are the result of chance or accidental occurrences. They may be human errors produced by fatigue, or they may be the result of such events as a surge in ac supply voltage, a brief draft upon equipment, or a variation in frequency.

When determining the mean value of a number of readings, it is sometimes found that one or two measurements differ from the mean by a much larger amount than any of the others. In this case, it is justifiable to reject these few readings as mistakes and to calculate the average value from the other measurements. This action should not be taken when more than a small number of readings differ greatly from the mean. Instead, the whole series of measurements should be repeated.

Deviation

The difference between any one measured value and the arithmetic mean of a series of measurements is termed the *deviation*. The deviations ($d_1, d_2, d_3, \ldots, d_n$) may be positive or negative, and the algebraic sum of the deviations is always zero. The *average deviation* may be calculated as the average of the *absolute* values of the deviations (neglecting plus and minus signs). If the measured quantity is assumed to be constant, the average deviation might be regarded as an indicator of the measurement precision (see Example 2-5).

$$\boxed{D = \frac{|d_1| + |d_2| + |d_3| + \cdots + |d_n|}{n}} \tag{2-7}$$

Example

The accuracy of five digital voltmeters are checked by using each of them to measure a standard 1.0000 V from a calibration instrument (see Section 12-3). The voltmeter readings are as follows: $V_1 = 1.001$ V, $V_2 = 1.002$, $V_3 = 0.999$, $V_4 = 0.998$, and $V_5 = 1.000$. Calculate the average measured voltage and the average deviation.

Solution

$$V_{av} = \frac{V_1 + V_2 + V_3 + V_4 + V_5}{5}$$

$$= \frac{1.001\text{ V} + 1.002\text{ V} + 0.999\text{ V} + 0.998\text{ V} + 1.000\text{ V}}{5}$$

$$= 1.000\text{ V}$$

$$d_1 = V_1 - V_{av} = 1.001\text{ V} - 1.000\text{ V}$$
$$= 0.001\text{ V}$$

$$d_2 = V_2 - V_{av} = 1.002\text{ V} - 1.000\text{ V}$$
$$= 0.002\text{ V}$$

$$d_3 = V_3 - V_{av} = 0.999\text{ V} - 1.000\text{ V}$$
$$= -0.001\text{ V}$$

$$d_4 = V_4 - V_{av} = 0.998\text{ V} - 1.000\text{ V}$$
$$= -0.002\text{ V}$$

$$d_5 = V_5 - V_{av} = 1.000\text{ V} - 1.000\text{ V}$$
$$= 0\text{ V}$$

$$D = \frac{|d_1| + |d_2| + |d_3| + |d_4| + |d_5|}{5}$$

$$= \frac{0.001\text{ V} + 0.002\text{ V} + 0.001\text{ V} + 0.002\text{ V} + 0}{5}$$

$$= 0.0012\text{ V}$$

Standard Deviation and Probable Error

As already discussed, measurement results can be analyzed by determining the arithmetic mean value of a number of measurements of the same quantity and by further determining the deviations and the average deviation. The *mean-squared value* of the deviations can also be calculated by first squaring each deviation value before determining the average. This gives a quantity known as the *variance*. Taking the square root of the variance produces the *root mean squared (rms)* value, also termed the *standard deviation* (σ).

$$\boxed{\sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \ldots + d_n^2}{n}}} \qquad (2\text{-}8)$$

For the case of a large number of measurements in which only random errors are present, it can be shown that the probable error in any one measurement is 0.6745 times the standard deviation:

$$\boxed{\text{probable error} = 0.6745\,\sigma} \qquad (2\text{-}9)$$

Example 2-6

Determine the standard deviation and the probable measurement error for the group of instruments referred to in Example 2-5.

Solution

Equation 2-8,

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \ldots + d_n^2}{n}}$$

$$= \sqrt{\frac{0.001^2 + 0.002^2 + 0.001^2 + 0.002^2 + 0^2}{5}}$$

$$= 0.0014\text{V}$$

Equation 2-9,

$$\textit{probable error} = 0.6745\,\sigma = 0.6745 \times 1.4\text{ mV}$$

$$= 0.94\text{ mV}$$

Summary so far

- We discussed what is measurement and why it is needed
- Classification of Measurements and Measuring Instruments
- Errors in Measurements
- Quantification of Errors
- Significant Figures
- Accuracy, Precision & Resolution
- Standard Deviation & Probable Error

Electromechanical Instruments

Introduction

- From Physics, we know that whenever a current carrying coil is placed inside a magnetic field, a force acts upon the coil.
- This phenomenon is used in electromechanical measuring devices
- Most common electromechanical instrument is **Permanent Magnet Moving Coil (PMMC)** instrument

The permanent-magnet moving-coil (PMMC) instrument consists basically of a light-weight coil of copper wire suspended in the field of a permanent magnet. Current in the wire causes the coil to produce a magnetic field that interacts with the field from the magnet, resulting in partial rotation of the coil. A pointer connected to the coil deflects over a calibrated scale, indicating the level of current flowing in the wire.

The PMMC instrument is essentially a low-level dc ammeter; however, with the use of parallel-connected resisors, it can be employed to measure a wide range of direct current levels. The instrument may also be made to function as a dc voltmeter by connecting appropriate-value resistors in series with the coil. Ac ammeters and voltmeters can be constructed by using rectifier circuits with PMMC instruments. Ohmmeters can be made from precision resistors, PMMC instruments, and batteries. Multirange meters are available that combine ammeter, voltmeter, and ohmmeter functions in one instrument.

and ohmmeter functions in one instrument.

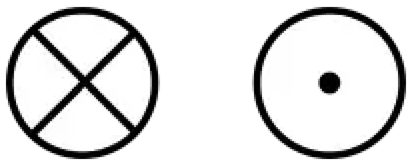
The electrodynamic instrument is similar to the PMMC instrument except that it uses stationary coils instead of a permanent mangnet. The most important application of the electrodynamic instrument is as a wattmeter.

Deflection Instruments Fundamentals

- A deflection instrument has a pointer, that moves over a calibrated scale to indicate a measured quantity.
- In order for this to occur, three forces act in this mechanism
  - Deflection Force
  - Controlling Force
  - Damping Force

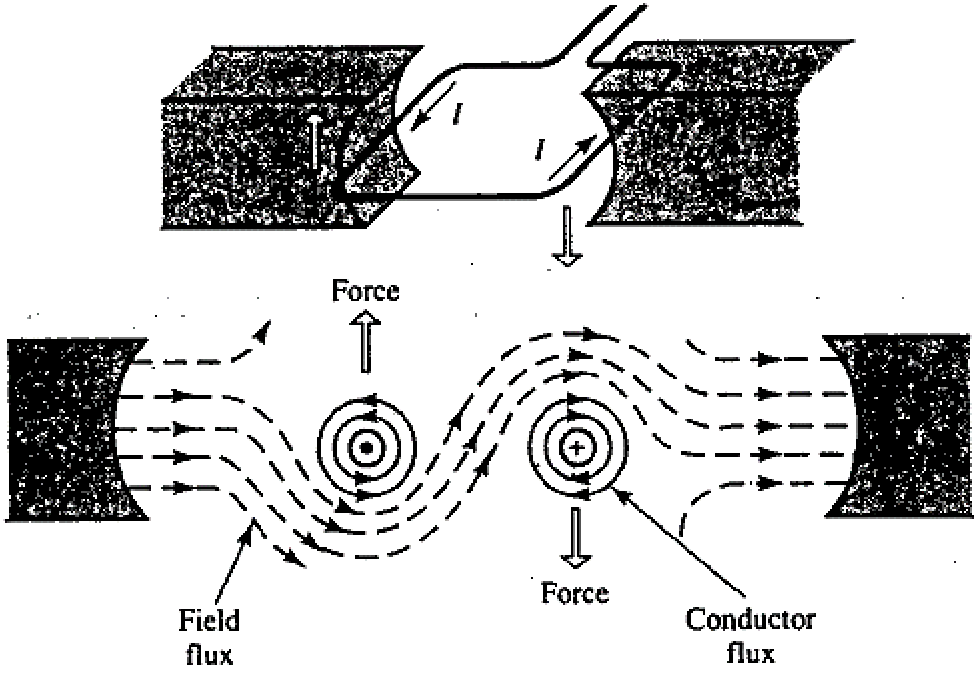
Deflection Force

The *deflecting force* causes the pointer to move from its zero position when a current flows. In the *permanent-magnet moving-coil* (PMMC) instrument the deflecting force is magnetic. When a current flows in a lightweight moving coil pivoted between the poles of a permanent magnet [Figure 3-1(a)], the current sets up a magnetic field that interacts with the field of the permanent magnet. A force is exerted on a current-carrying conductor situated in a magnetic field. Consequently, a force is exerted on the coil turns, as illustrated, causing the coil to rotate on its pivots. The pointer is fixed to the coil, so it moves over the scale as the coil rotates.



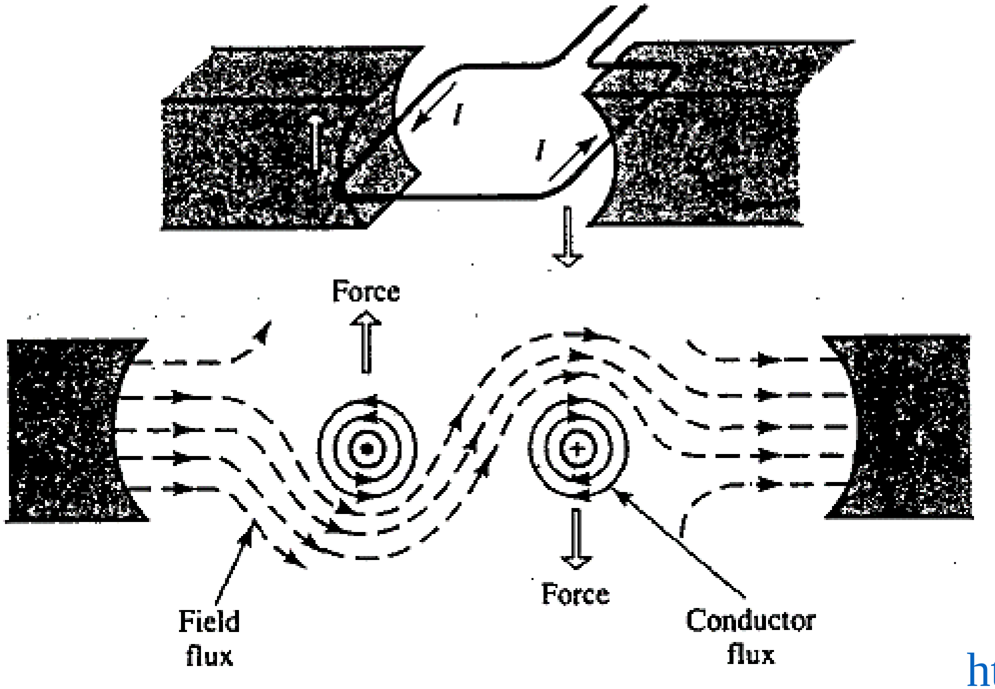
They represent the direction of current perpendicular to the two dimensional surface it has been represented upon (board, paper, screen etc.)

Perpendicular to the surface, there can be two possible directions a vector can take. It can either come out of the surface towards the observer and is denoted by the dot (.). Or it can go away from the observer into the surface, denoted by the cross (x).

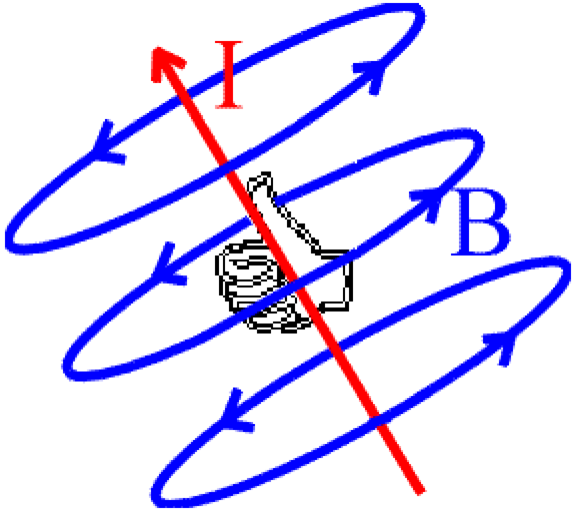


(a) The deflecting force in a PMMC instrument is provided by a current-carrying coil pivoted in a magnetic field.

The direction of the magnetic field due to moving charges depends on the right hand rule. For the case of a long straight wire carrying a current ***I***, the magnetic field lines wrap around the wire. **By pointing one's right thumb along the direction of the current, the direction of the magnetic field can by found by curving one's fingers around the wire.**



(a) The deflecting force in a PMMC instrument is provided by a current-carrying coil pivoted in a magnetic field.



https://web.pa.msu.edu/courses/1997spring/PHY232/lectures/amperesl aw/wire.html

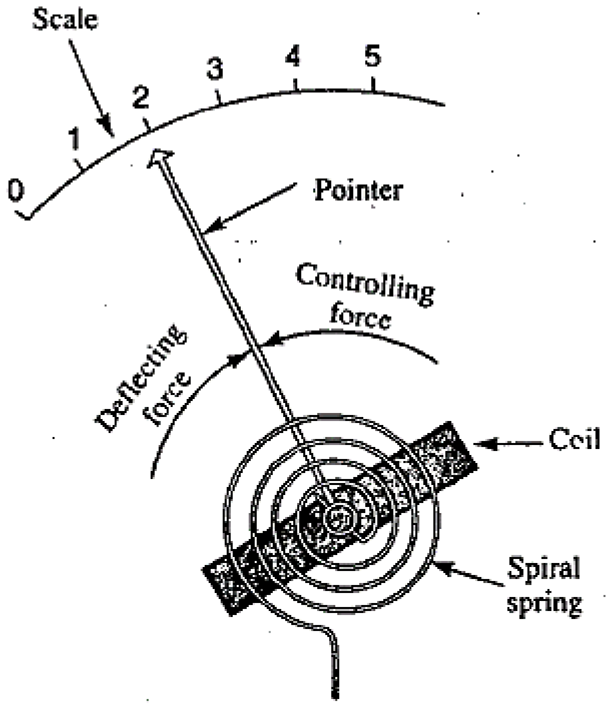
Bottom Line

- When current flows through a coil placed in a magnetic field, a force acts upon this coil (which moves the pointer), this force is called *Deflecting Force*
- *What would happen to an object, if it is subjected to a continuous force which is not opposed by any other force.*

Bonus Rading Material
https://www.khanacademy.org/test-prep/mcat/physical-processes/magnetism-mcat/a/using-the-right-hand-rule

Controlling Force

The controlling force in the PMMC instrument is provided by spiral springs [Figure 3-1(b)]. The springs retain the coil and poiner at their zero position when no current is flowing. When current flows, the springs "wind up" as the coil rotates, and the force they exert on the coil increases. The coil and pointer stop rotating when the controlling force becomes equal to the deflecting force. The spring material must be nonmagnetic to avoid any magnetic field influence on the controlling force. Since the springs are also used to make electrical connection to the coil, they must have a low resistance. Phosphor bronze is the material usually employed.



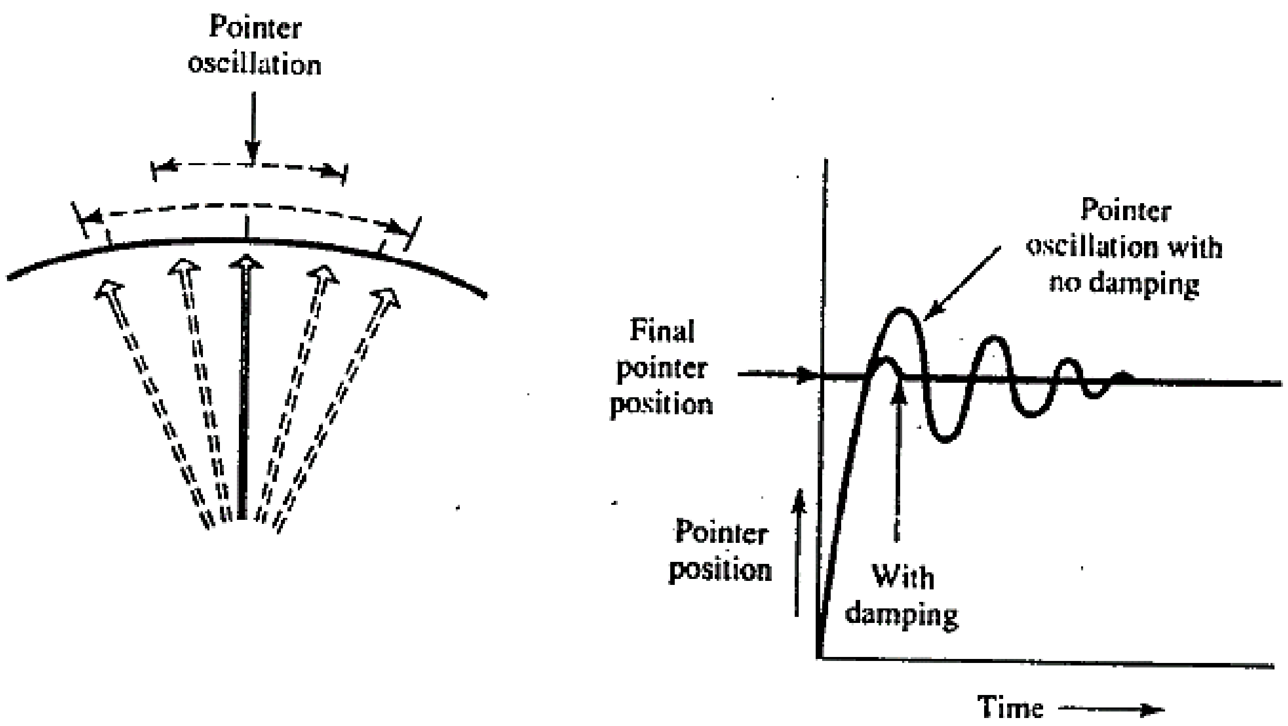
(b) The controlling force from the springs balances the deflecting force.

Damping Force

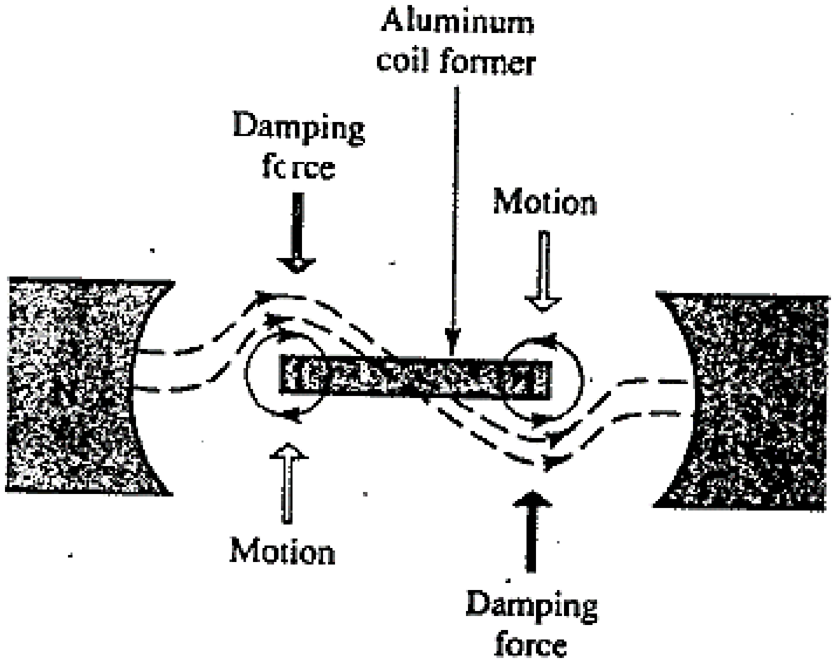
Damping is an influence within or upon an oscillatory system that has the effect of reducing, restricting or preventing its oscillations. In physical systems, **damping** is produced by processes that dissipate the energy stored in the oscillation. (Wikipedia)

... material usually employed.

As illustrated in Figure 3-2(a), the pointer and coil tend to oscillate for some time before settling down at their final position. A damping force is required to minimize (or damp out) the oscillations. The damping force must be present only when the coil is in motion; thus it must be generated by the rotation of the coil. In PMMC instruments, the damping force is normally provided by *eddy currents*. The coil former (or frame) is constructed of aluminum, a nonmagnetic conductor. Eddy currents induced in the coil former set up a magnetic flux that opposses the coil motion, thus damping the oscillations of the coil [see Figure 3-2(b)].



(a) Lack of damping causes the pointer to oscillate.



(b) The damping force in a PMMC instrument is provided by eddy currents induced in the aluminum coil former as it moves through the magnetic field.

Figure 3-2 A deflection instrument requires a damping force to stop the pointer oscillating about the indicated reading. The damping force is usually produced by eddy currents in a nonmagnetic coil former. These exist only when the coil is in motion.

- Idea of Taut-Band suspension
- Idea of Jewel-Bearing suspension

PMMC Construction

- PMMC is also called D' Arsonval Instrument
- So far we have discussed the conceptual implementation of PMMC, physical implementation is a little different
- Details of the construction of PMMC are shown in the figure

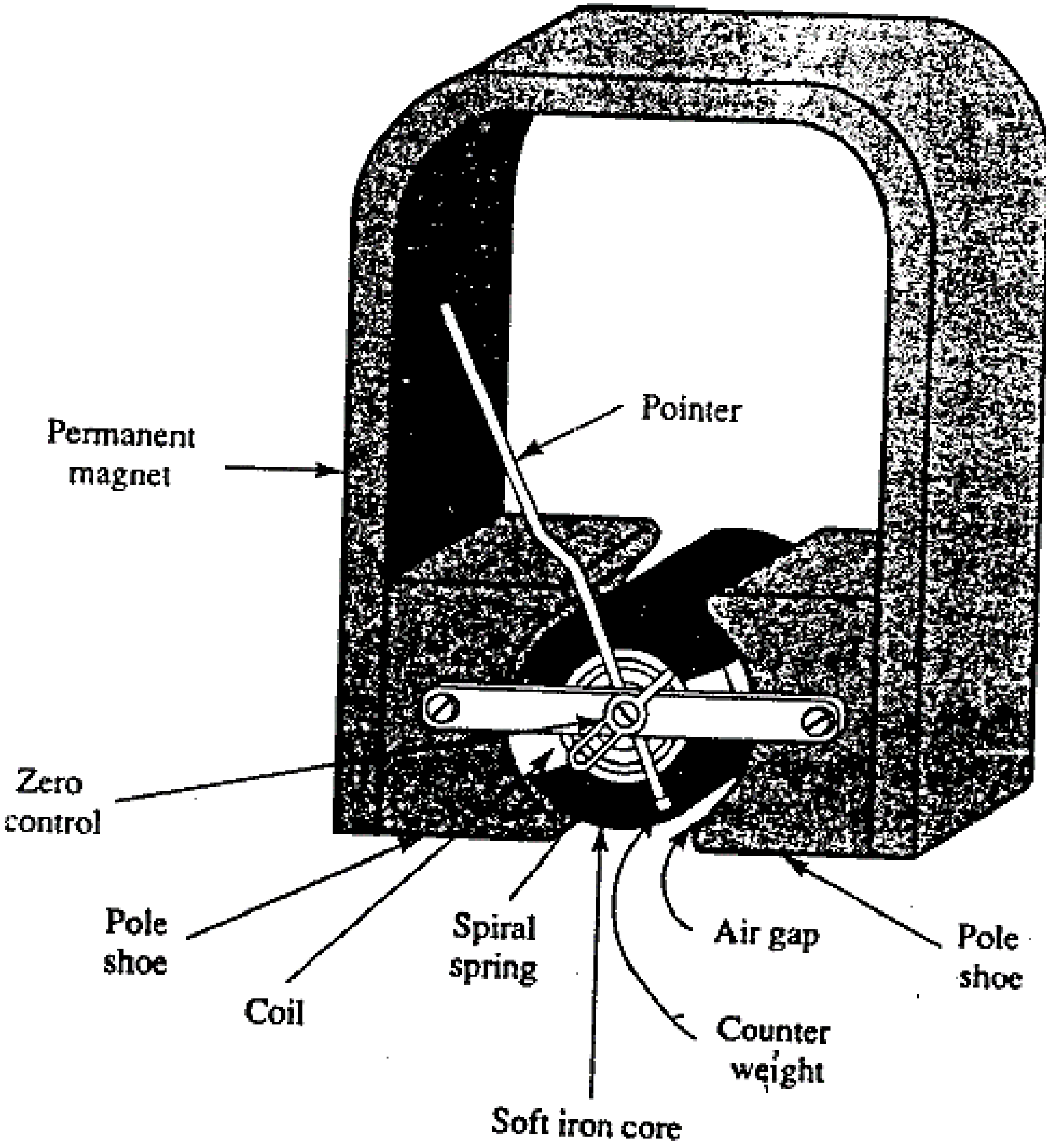


Figure 3-4 A typical PMMC instrument is constructed of a *horseshoe* magnet, soft-iron pole shoes, a soft-iron core, and a suspended coil that moves in the air gap between the core and the pole shoes.

Other Design

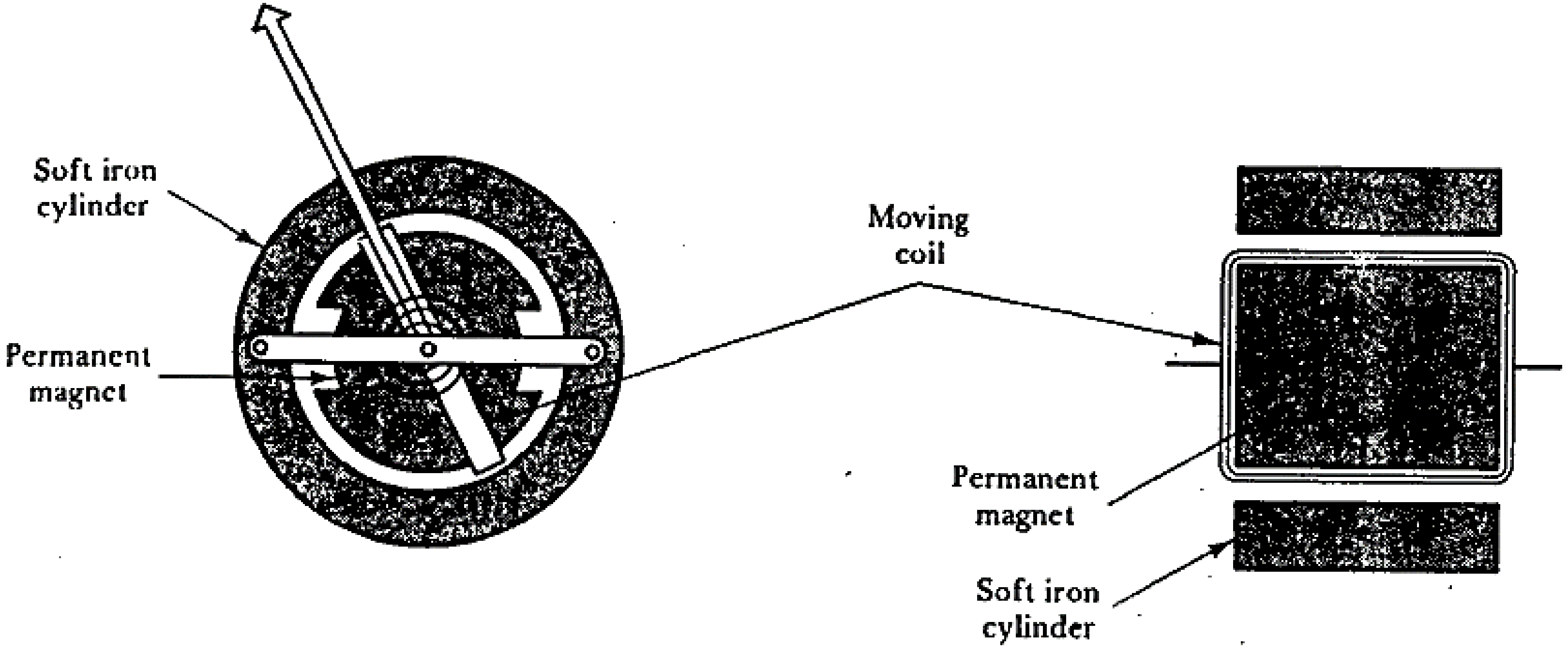


Figure 3-5 In a core-magnet PMMC instrument, the permanent magnet is located inside the moving coil, and the coil and magnet are positioned inside a soft-iron cylinder.

Torque Equation and Scale

When a current I flows through a one-turn coil situated in a magnetic field, a force F is exerted on each side of the coil [Figure 3-6(a)]:

$$F = BIl \quad \text{newtons}$$

where B is the magnetic flux density in tesla, I is the current in amperes, and l is the length of the coil in meters.

Since the force acts on each side of the coil, the total force for a coil of N turns is

$$F = 2BIlN \quad \text{newtons}$$

The force on each side acts at a radius r, producing a deflecting torque:

$$T_D = 2BIlNr \quad \text{newton meters (N} \cdot \text{m)}$$

$$= BlIN(2r)$$

$$\boxed{T_D = BlIND} \qquad (3\text{-}1)$$

where D is the coil diameter [Figure 3-6(b)].

The controlling torque exerted by the spiral springs is directly proportional to the deformation or "windup" of the springs. Thus, the controlling torque is proportional to the actual angle of deflection of the pointer:

$$T_C = K\theta$$

where K is a constant. For a given deflection, the controlling and deflecting torques are equal:

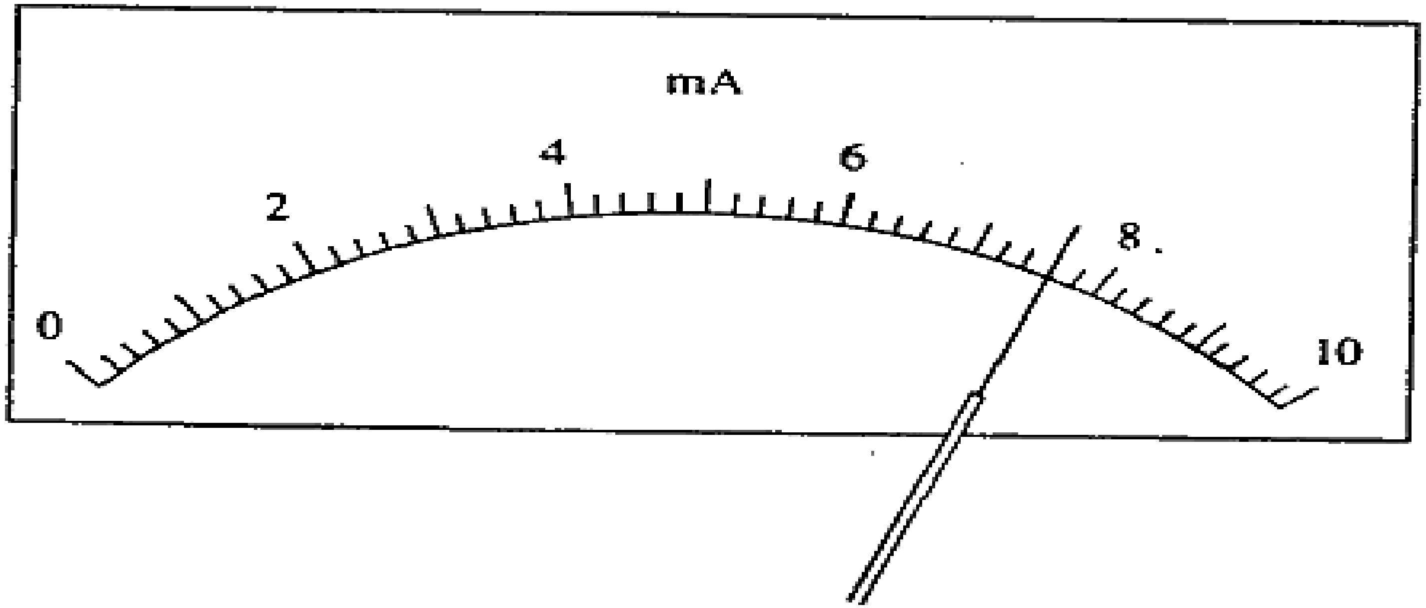
$$K\theta = BIIND$$

Since all quantities except θ and I are constant for any given instrument, the deflection angle is

$$\boxed{\theta = CI} \qquad (3\text{-}2)$$

where C is a constant.

Equation 3-2 shows that the pointer deflection is always proportional to the coil current. Consequently, the scale of the instrument is *linear*, or uniformly divided; that is, if 1 mA produces a 1 cm movement of the pointer from zero, 2 mA produces a 2 cm movement, and so on [see Figure 3-6(c)]. As will be explained the PMMC instrument can be used as a dc voltmeter, a dc ammeter, and an ohmmeter. When connected with rectifiers and transformers, it can also be employed to measure alternating voltage and current.



(c) Linear scale on a PMMC instrument

α

A PMMC instrument with a 100-turn coil has a magnetic flux density in its air gaps of $B = 0.2$ T. The coil dimensions are $D = 1$ cm and $l = 1.5$ cm. Calculate the torque on the coil for a current of 1 mA.

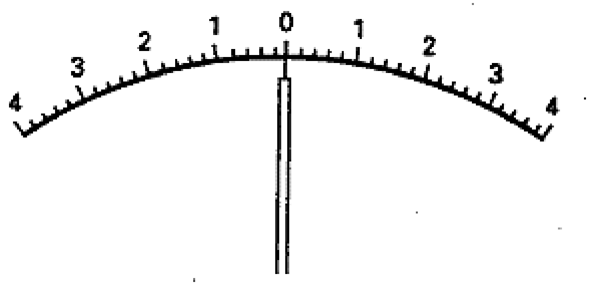
Solution

$$T_D = BlIND$$

$$= 0.2\ \text{T} \times 1.5 \times 10^{-2} \times 1\ \text{mA} \times 100 \times 1 \times 10^{-2}$$

$$= 3 \times 10^{-6}\ \text{N} \cdot \text{m}$$

Galvanometer

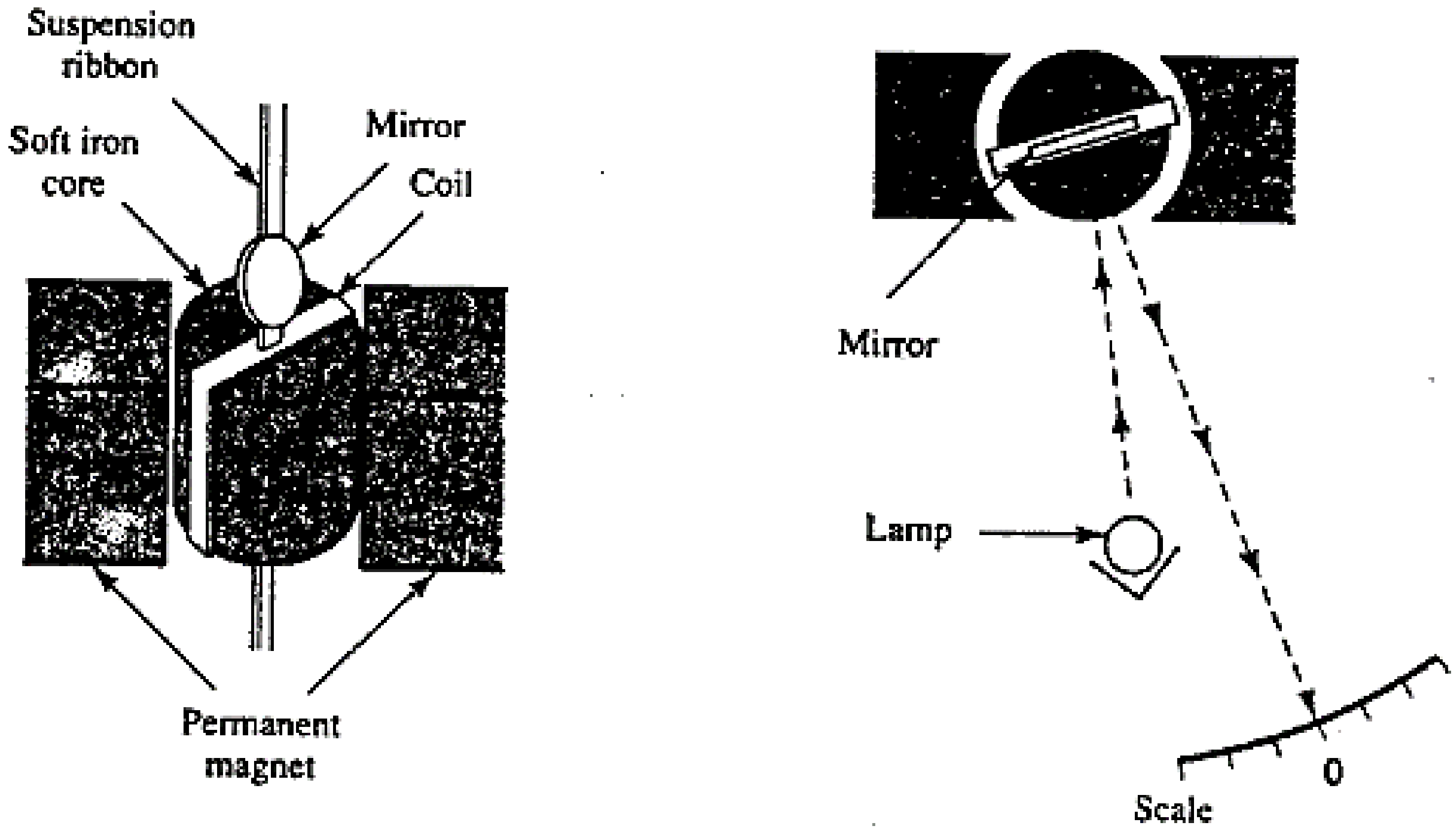


(a) Center-zero scale

- A galvanometer is essentially a PMMC instrument designed to be sensitive to extremely low current levels.
- The simplest galvanometer is a very sensitive instrument with the type of centre-zero scale illustrated in Figure.
- The deflection system is arranged so that the pointer can be deflected to either right or left of zero, depending on the direction of current through the moving coil.
- The scale may be calibrated in microamperes, or it may simply be a millimetre scale. In the latter case, the instrument current sensitivity (usually stated in μA/mm) is used to determine the current level that produces a measured deflection.

- The torque equation for a galvanometer is exactly as for PMMC.
- The deflecting torque is proportional to the number of coil turns, the coil dimensions, and the current flowing in the coil.
- The most sensitive moving-coil galvanometers use taut-band suspension, and the controlling torque is generated by the twist in the suspension ribbon. Eddy current damping may be provided, as in other PMMC instruments, by winding the coil on a nonmagnetic conducting coil former.
- Sometimes a nonconducting coil former is employed, and the damping currents are generated solely by the moving coil. In this case, the coil is shunted by a damping resistor which controls the level of eddy currents generated by the coil movements.
- Frequently, a critical damping resistance value is stated, which gives just sufficient damping to allow the pointer to settle down quickly with only a very small short-lived oscillation.

- With the moving-coil weight reduced to the lowest possible minimum for greatest sensitivity, the weight of the pointer can create a problem.
- This is solved in many instruments by mounting a small mirror on the moving coil instead of a pointer.
- The mirror reflects a beam of light on to a scale, as illustrated in Figure
- The light beam behaves as a very long weightless pointer which can be substantially deflected by a very small coil current.
- This makes light-beam galvanometers sensitive to much lower current levels than pointer instruments.
- Galvanometer *voltage sensitivity* is often expressed for a given value of critical damping resistance.
- This is usually stated in microvolts per millimeter.
- A megohm sensitivity is sometimes specified for galvanometers, and this is the value of resistance that must be connected in series with the instrument to restrict the deflection to one scale division when a potential difference of 1 V is applied across its terminals. Pointer galvanometers have current sensitivities ranging from 0.1 to 1 μA/mm.
- For light-beam instruments typical current sensitivities are 0.01 to 0.1 μA per scale division



(b) Basic deflection system of a galvanometer using a light beam

Figure 3-7 A galvanometer is simply an extremely sensitive PMMC instrument with a center-zero scale. For maximum sensitivity, the mass of the moving system is minimized by using a pointer that consists of a light beam reflected from a tiny mirror fastened to the coil.

- Galvanometers are often employed to detect zero current or voltage in a circuit rather than to measure the actual level of current of voltage.
- In this situation, the instrument is referred to as a null meter or null detector.
- A galvanometer used as a null meter must be protected from the excessive current flow that might occur when the voltage across the instrument terminals is not close to zero.
- Protection is provided by an adjustable resistance connected in shunt with the instrument
- When the shunt resistance is zero, all of the circuit current flows through the shunt. As the shunt resistance is increased above zero, an increasing amount of current flows through the galvanometer.
- Galvanometer applications have been largely taken over by electronic instruments that can measure extremely low levels of voltage and current.

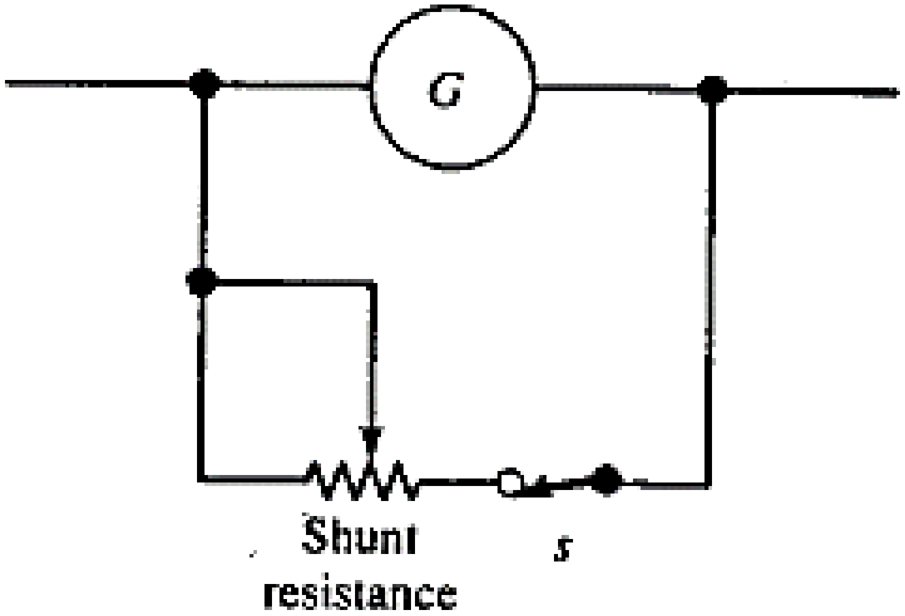


Figure 3-8 An adjustable shunt resistor is employed to protect the coil of a galvanometer from destructively excessive current levels. The shunt resistance is initially set to zero, and then gradually increased to divert current through the galvanometer.

Example 3-2

A galvanometer has a current sensitivity of 1 µA/mm and a critical damping resistance of 1 kΩ. Calculate (a) the voltage sensitivity and (b) the megohm sensitivity.

Solution

$$\textit{Voltage sensitivity} = 1\ \text{k}\Omega \times 1\ \mu\text{A/mm}$$

$$= 1\ \text{mV/mm}$$

For a voltage sensitivity of 1 V/mm,

$$\textit{megohm sensitivity} = \frac{1\ \text{V/mm}}{1\ \mu\text{A/mm}} = 1\ \text{M}\Omega$$

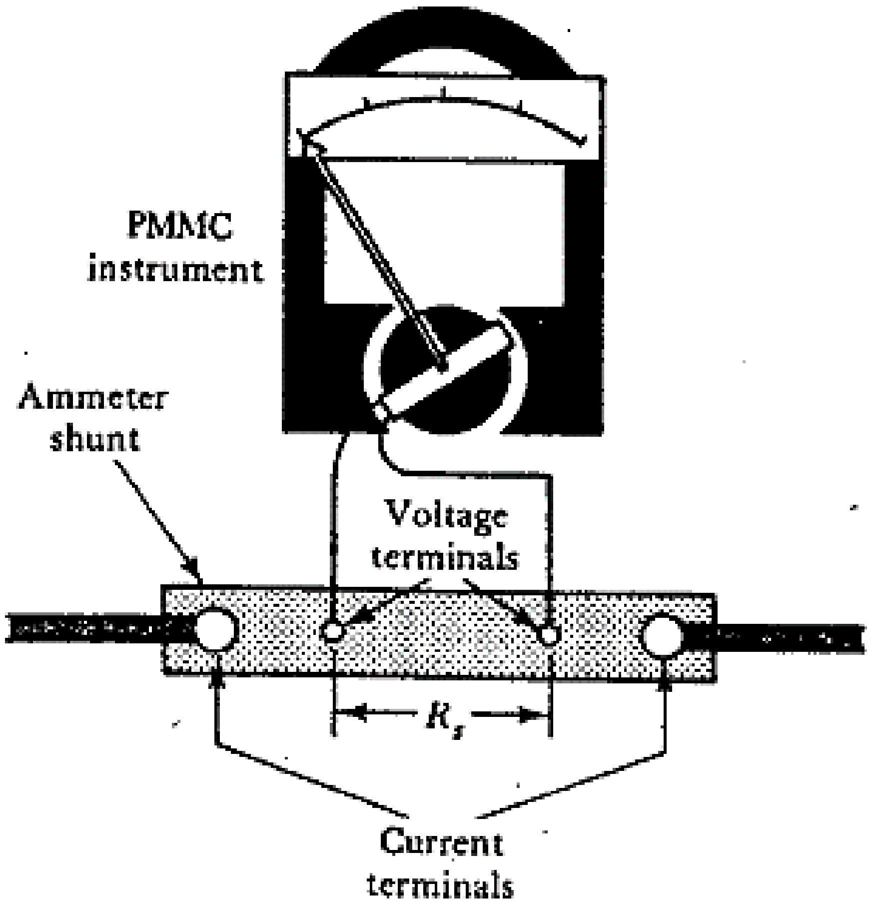
DC Ammeter

Ammeter Circuit

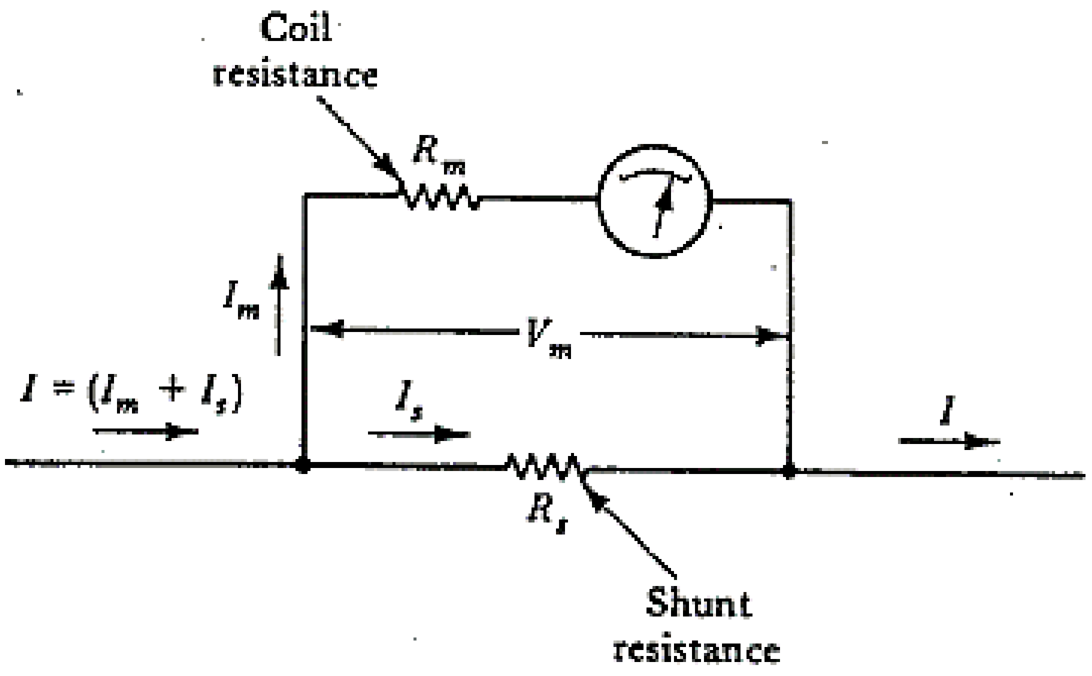
An ammeter is always connected in series with a circuit in which current is to be measured. To avoid affecting the current level in the circuit, the ammeter must have a resistance much lower than the circuit resistance. The PMMC instrument is an ammeter. Pointer deflection is directly proportional to the current flowing in the coil. However,

maximum pointer deflection is produced by a very small current, and the coil is usually wound of thin wire that would be quickly destroyed by large currents. For larger currents, the instrument must be modified so that most of the current to be measured is shunted around the coil of the meter. Only a small portion of the current passes through the moving coil. Figure 3-9 illustrates how this is arranged.

A *shunt*, or very low resistance, is connected in parallel with the instrument coil [Figure 3-9(a)]. The shunt is sometimes referred to as a *four-terminal resistor*, because it has two sets of terminals identified as *voltage terminals* and *current terminals*. This is to ensure that the resistance in parallel with the coil (R_s) is accurately defined and the contact resistance of the current terminals is removed from R_s. Contact resistance can vary with change in current level and thus introduce errors.



(a) Construction of dc ammeter



(b) Ammeter circuit

Figure 3-9 A direct-current ammeter consists of a PMMC instrument and a low-resistance shunt. The meter current is directly proportional to the shunt current, so that the meter scale can be calibrated to indicate the total ammeter current.

In the circuit diagram in Figure 3-9(b), R_m is the meter resistance (or coil circuit resistance) and R_s is the resistance of the shunt. Suppose that the meter resistance is exactly 99 Ω and the shunt resistance is 1 Ω. The shunt current (I_s) will be 99 times the meter current (I_m). In this situation, if the meter gives FSD for a coil current of 0.1 mA, the scale should be calibrated to read 100 × 0.1 mA or 10 mA at full scale. The relationship between shunt current and coil current is further investigated in Examples 3-3 and 3-4.

Example 3-3

An ammeter (as in Figure 3-9) has a PMMC instrument with a coil resistance of $R_m = 99\ \Omega$ and FSD current of 0.1 mA. Shunt resistance $R_s = 1\ \Omega$. Determine the total current passing through the ammeter at (a) FSD, (b) 0.5 FSD, and (c) 0.25 FSD.

Solution

(a) At FSD:

$$\text{meter voltage } V_m = I_m R_m \text{[see Figure 3-9(b)]}$$

$$= 0.1\ \text{mA} \times 99\ \Omega$$

$$= 9.9\ \text{mV}$$

and

$$I_s R_s = V_m$$

$$I_s = \frac{V_m}{R_s} = \frac{9.9\ \text{mV}}{1\ \Omega} = 9.9\ \text{mA}$$

$$\text{total current } I = I_s + I_m = 9.9\ \text{mA} + 0.1\ \text{mA}$$

$$= 10\ \text{mA}$$

(b) At 0.5 FSD:

$$I_m = 0.5 \times 0.1\ \text{mA} = 0.05\ \text{mA}$$

$$V_m = I_m R_m = 0.05\ \text{mA} \times 99\ \Omega = 4.95\ \text{mV}$$

$$I_s = \frac{V_m}{R_s} = \frac{4.95\ \text{mV}}{1\ \Omega} = 4.95\ \text{mA}$$

$$\text{total current } I = I_s + I_m = 4.95\ \text{mA} + 0.5\ \text{mA}$$

$$= 5\ \text{mA}$$

(c) At 0.25 *FSD:*

$$I_m = 0.25 \times 0.1 \text{ mA} = 0.025 \text{ mA}$$

$$V_m = I_m R_m = 0.025 \text{ mA} \times 99\ \Omega$$

$$= 2.475 \text{ mV}$$

$$I_s = \frac{V_m}{R_s} = \frac{2.475 \text{ mV}}{1\ \Omega} = 2.475 \text{ mA}$$

Ammeter Scale

The total ammeter current in Example 3-3 is 10 mA when the moving-coil instrument indicates FSD. Therefore, the meter scale can be calibrated for FSD to indicate 10 mA. When the pointer indicates 0.5 FSD and 0.25 FSD, the current levels are 5 mA and 2.5 mA, respectively. Thus, the ammeter scale may be calibrated to linearly represent all current levels from zero to 10 mA. Figure 3-10 shows a panel meter (for mounting on a control panel) that has a direct current scale calibrated linearly from 0 mA to 50 μA.

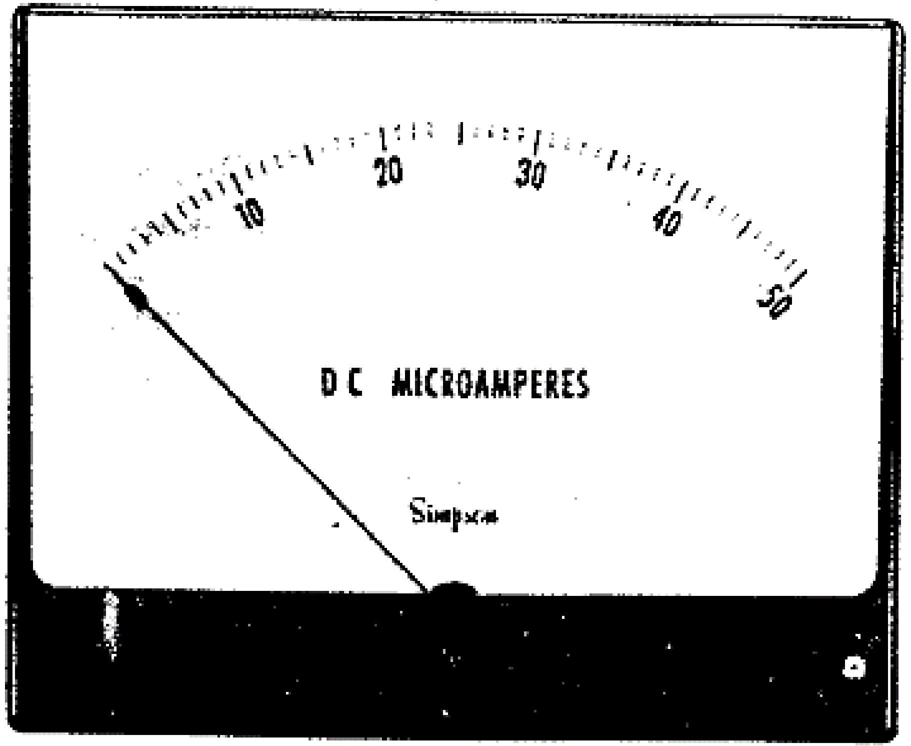


Figure 3-10 A dc ammeter made up of a PMMC instrument and a shunt has a linear current scale (Courtesy of bach-simpson limited.)

Shunt Resistance

Refer again to Example 3-3. If a shunt having a smaller resistance is used, the shunt current and the total meter current will be larger than the levels calculated. In fact, shunt resistance values can be determined to convert a PMMC instrument into an ammeter for measuring virtually any desired level of current. Example 3-4 demonstrates how shunt resistances are calculated.

Example 3-4

A PMMC instrument has FSD of 100 μA and a coil resistance of 1 kΩ. Calculate the required shunt resistance value to convert the instrument into an ammeter with (a) FSD = 100 mA and (b) FSD = 1 A.

Solution

(a) $FSD = 100\ \text{mA}$:

$$V_m = I_m R_m = 100\ \mu\text{A} \times 1\ \text{k}\Omega = 100\ \text{mV}$$

$$I = I_s + I_m$$

$$I_s = I - I_m = 100\ \text{mA} - 100\ \mu\text{A} = 99.9\ \text{m.}$$

$$R_s = \frac{V_m}{I_s} = \frac{100\ \text{mV}}{99.9\ \text{mA}} = 1.001\ \Omega$$

(b) $FSD = 1\ \text{A}$:

$$V_m = I_m R_m = 100\ \text{mV}$$

$$I_s = I - I_m = 1\ \text{A} - 100\ \mu\text{A} = 999.9\ \text{mA}$$

$$R_s = \frac{V_m}{I_s} = \frac{100\ \text{mV}}{999.9\ \text{mA}} = 0.10001\ \Omega$$

Swamping Resistance

The moving coil in a PMMC instrument is wound with thin copper wire, and its resistance can change significantly when its temperature changes. The heating effect of the coil current may be enough to produce a resistance change. Any such change in coil resistance will introduce an error in ammeter current measurements. To minimize the effect of coil resistance variation, a *swamping resistance* made of *manganin* or *constantan* is connected in series with the coil, as illustrated in Figure 3-11. Manganin and constantan have resistance temperature coefficients very close to zero. If the swamping resistance is nine times the coil resistance, a 1% change in coil resistance would result in a total (swamping plus coil) resistance change of 0.1%.

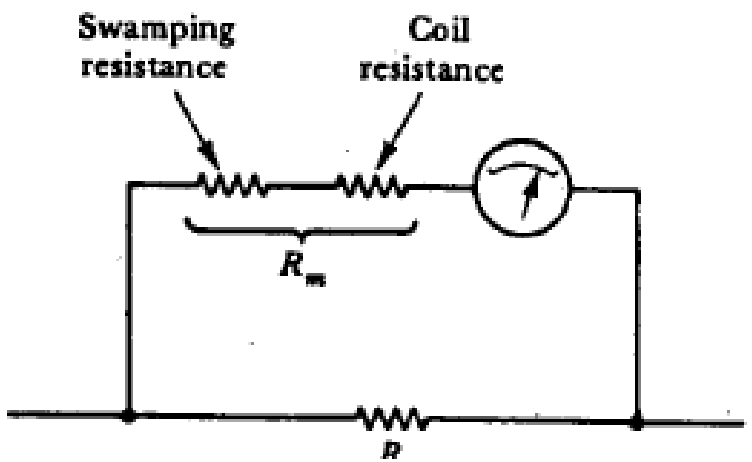
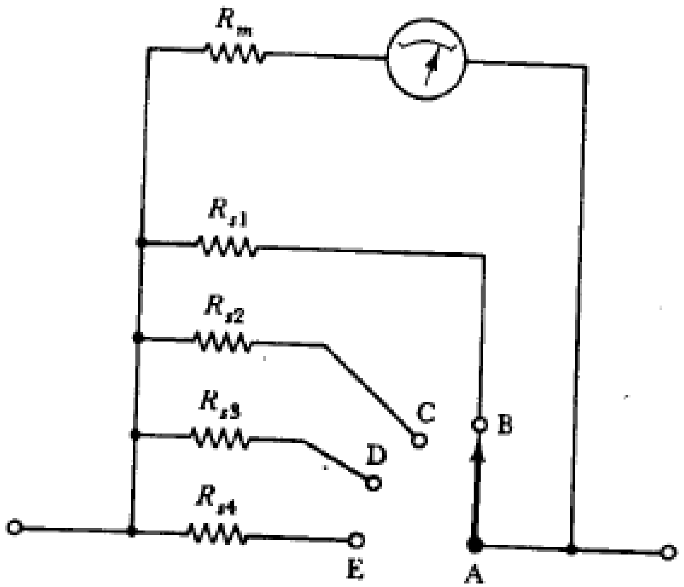


Figure 3-11 A swamping resistance made of a material with a near-zero temperature coefficient can be connected in series with the coil of a PMMC instrument to minimize temperature errors.

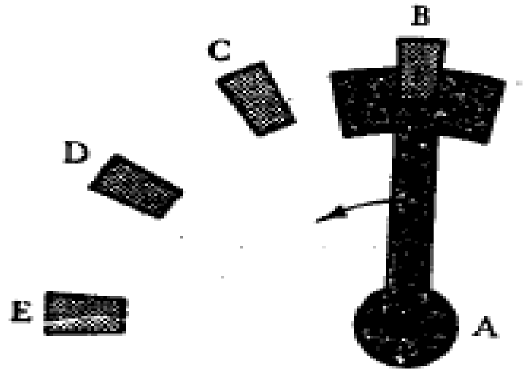
The ammeter shunt must also be made of manganin or constantan to avoid shunt resistance variations with temperature. As noted in Figure 3-11, the swamping resistance must be considered part of the meter resistance R_m when calculating shunt resistance values.

Multirange Ammeters

The circuit of a multirange ammeter is shown in Figure 3-12(a). As illustrated, a rotary switch is employed to select any one of several shunts having different resistance values. A *make-before-break switch* [Figure 3-12(b)] must be used so that the instrument is not left without a shunt in parallel with it even for a brief instant. If this occurred, the high resistance of the instrument would affect the current flowing in the circuit. More important, a current large enough to destroy the instrument might flow through its moving coil. When switching between shunts, the wide-ended moving contact of the make-before-break switch makes contact with the next terminal before it breaks contact with the previous terminal. Thus, during switching there are actually two shunts in parallel with the instrument.



(a) Multirange ammeter using switched shunts

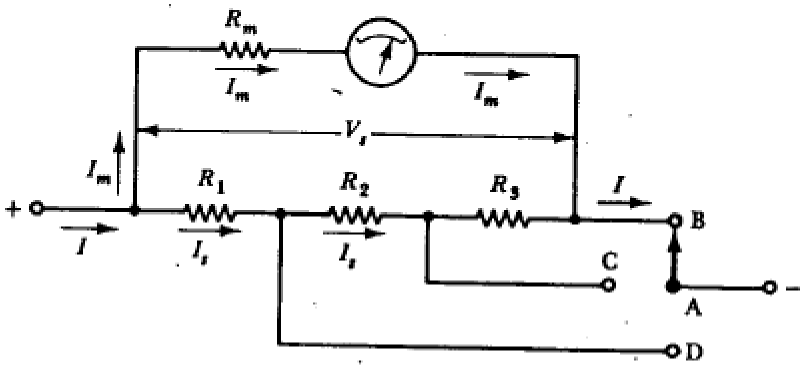


(b) Make-before-break switch

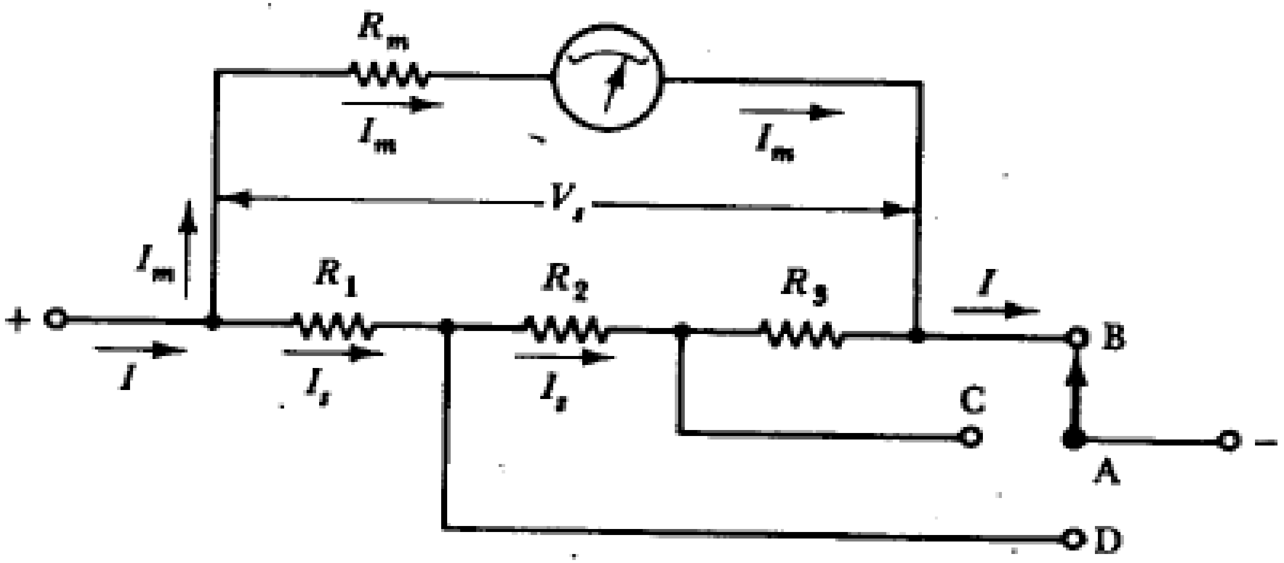
Figure 3-12 A multirange ammeter consists of a PMMC instrument, several shunts, and a switch that makes contact with the next shunt before losing contact with the previous one when range switching.

Ayrton Shunt

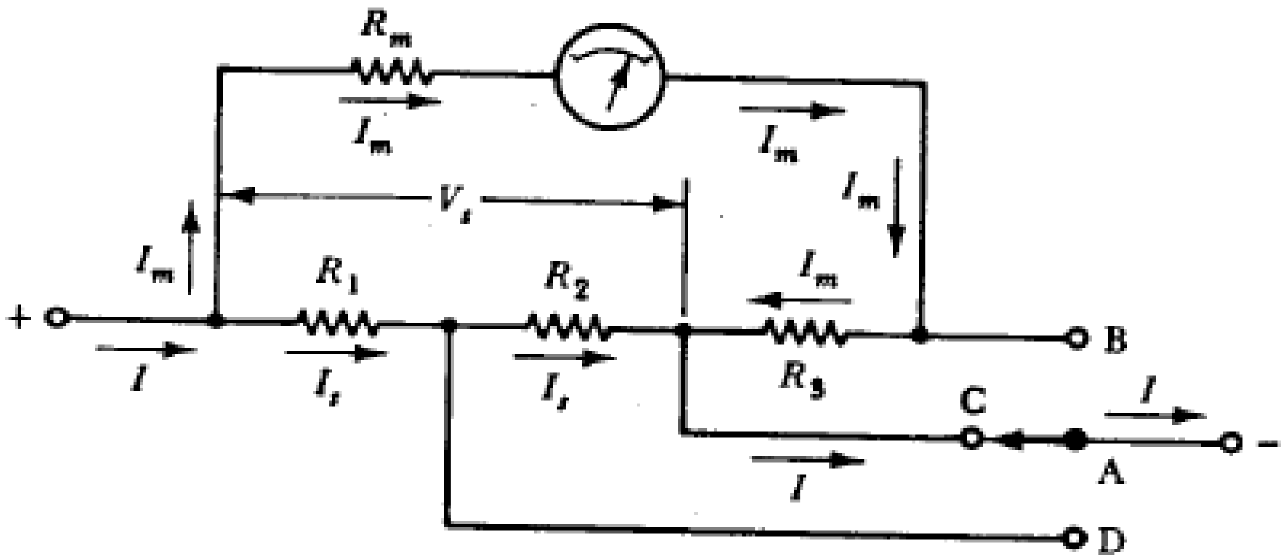
Figure 3-13 shows another method of protecting the deflection instrument of an ammeter from excessive current flow when switching between shunts. Resistors R_1, R_2, and R_3 constitute an *Ayrton shunt*. In Figure 3-13(a) the switch is at contact B, and the total resistance in parallel with the instrument is $R_1 + R_2 + R_3$. The meter circuit resistance remains R_m. When the switch is at contact C [Figure 3-13(b)], the resistance R_3 is in series with the meter, and $R_1 + R_2$ is in parallel with $R_m + R_3$. Similarly, with the switch at contact D, R_1 is in parallel with $R_m + R_2 + R_3$. Because the shunts are permanently connected, and the switch makes contact with the shunt junctions, the deflection instrument is never left without a parallel-connected shunt (or shunts). In Example 3-5 ammeter current ranges are calculated for each switch position on an Ayrton shunt.



(a) $(R_1 + R_2 + R_3)$ in parallel with R_m



(a) $(R_1 + R_2 + R_3)$ in parallel with R_m



(b) $(R_1 + R_2)$ in parallel with $(R_m + R_3)$

Example 3-5

A PMMC instrument has a three-resistor Ayrton shunt connected across it to make an ammeter, as in Figure 3-13. The resistance values are $R_1 = 0.05\ \Omega$, $R_2 = 0.45\ \Omega$, and $R_3 = 4.5\ \Omega$. The meter has $R_m = 1\ \text{k}\Omega$ and FSD = 50 μA. Calculate the three ranges of the ammeter.

Solution *Refer to Figure 3-13.*

Switch at contact B:

$$V_s = I_m R_m = 50\ \mu\text{A} \times 1\ \text{k}\Omega = 50\ \text{mV}$$

$$I_s = \frac{V_s}{R_1 + R_2 + R_3}$$

$$= \frac{50\ \text{mV}}{0.05\ \Omega + 0.45\ \Omega + 4.5\ \Omega} = 10\ \text{mA}$$

$$I = I_m + I_s = 50\ \mu\text{A} + 10\ \text{mA}$$

$$= 10.05\ \text{mA}$$

Ammeter range $\simeq$ 10 mA.

Switch at contact C:

$$V_s = I_m(R_m + R_3)$$

$$= 50\ \mu\text{A}(1\ \text{k}\Omega + 4.5\ \Omega)$$

$$\simeq 50\ \text{mV}$$

$$I_s = \frac{V_s}{R_1 + R_2}$$

$$= \frac{50\ \text{mV}}{0.05\ \Omega + 0.45\ \Omega}$$

$$= 100\ \text{mA}$$

$$I = 50\ \mu\text{A} + 100\ \text{mA}$$

$$= 100.05\ \text{mA}$$

Ammeter range $\simeq$ 1 A.

Switch at contact D:

$$V_s = I_m(R_m + R_3 + R_2)$$

$$= 50\ \mu A(1\ k\Omega + 4.5\ \Omega + 0.45\ \Omega)$$

$$\simeq 50\ mV$$

$$I_s = \frac{V_s}{R_1} = \frac{50\ mV}{0.05\ \Omega}$$

$$= 1\ A$$

$$I = 50\ \mu A + 1\ A$$

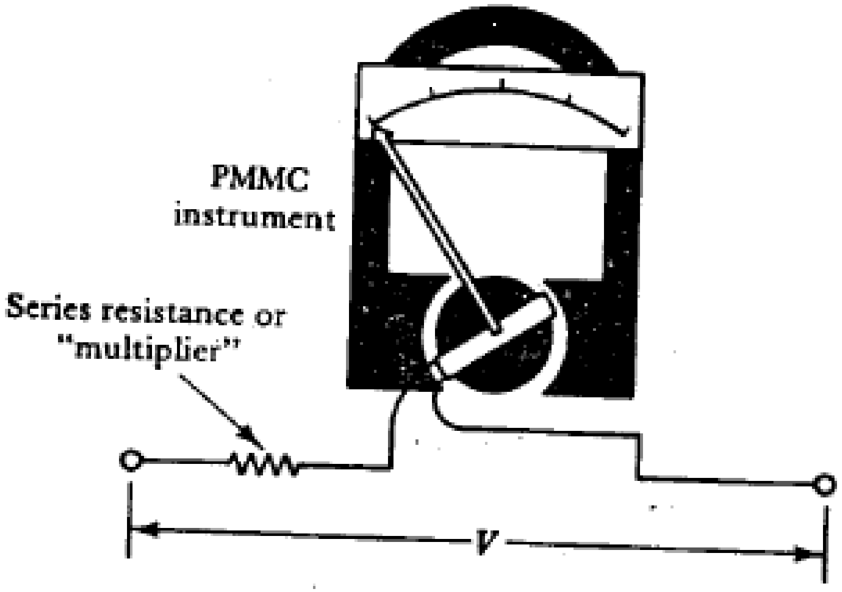
$$= 1.00005\ A$$

Ammeter range $\simeq$ 1 A.

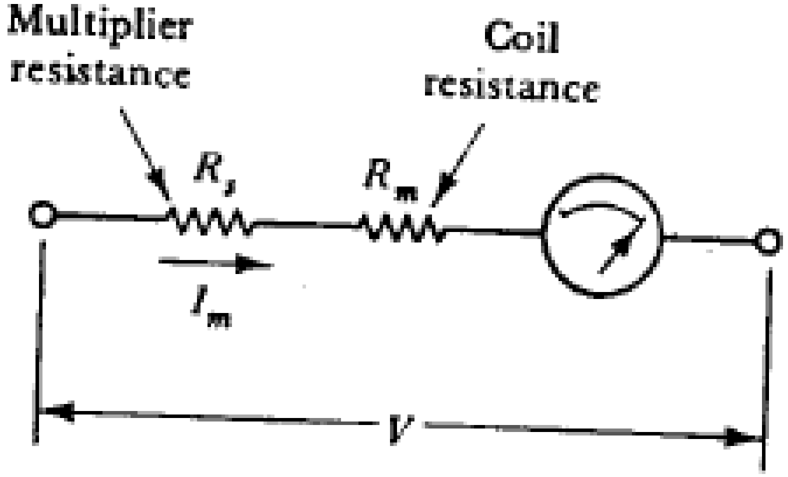
DC Voltmeter

Voltmeter Circuit

The deflection of a PMMC instrument is proportional to the current flowing through the moving coil. The coil current is directly proportional to the voltage across the coil. Therefore, the scale of the PMMC meter could be calibrated to indicate voltage. The coil resistance is normally quite small, and thus the coil voltage is also usually very small. Without any additional series resistance the PMMC instrument would only be able to measure very low voltage levels. The voltmeter range is easily increased by connecting a resistance in series with the instrument [see Figure 3-14(a)]. Because it increases the range of the voltmeter, the series resistance is termed a *multiplier resistance*. A multiplier resistance that is nine times the coil resistance will increase the voltmeter range by a factor of 10. Figure 3-14(b) shows that the total resistance of the voltmeter is (multiplier resistance) + (coil resistance).



(a) Construction of dc voltmeter



(b) Voltmeter circuit

Figure 3-14 A dc voltmeter is made up of a PMMC instrument and a series *multiplier* resistor. The meter current is directly proportional to the applied voltage, so that the meter scale can be calibrated to indicate the voltage.

Example 3-6

A PMMC instrument with FSD of 100 μA and a coil resistance of 1 kΩ is to be converted into a voltmeter. Determine the required multiplier resistance if the voltmeter is to measure 50 V at full scale (Figure 3-15). Also calculate the applied voltage when the instrument indicates 0.8, 0.5, and 0.2 of FSD.

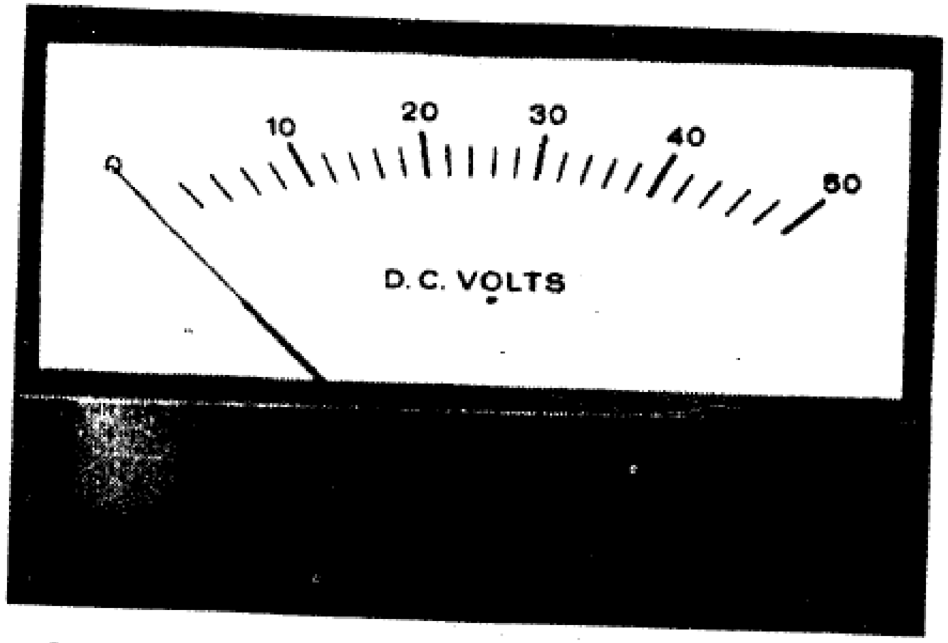


Figure 3-15 A dc voltmeter using a PMMC instrument has a linear voltage scale.

Solution

$$V = I_m(R_s + R_m)$$

$$R_s + R_m = \frac{V}{I_m}$$

and

$$R_s = \frac{V}{I_m} - R_m$$

For $V = 50\text{ V}$ *FSD,*

$$I_m = 100\ \mu\text{A}$$

$$R_s = \frac{50\text{ V}}{100\ \mu\text{A}} - 1\text{ k}\Omega$$

$$= 499\text{ k}\Omega$$

At 0.8 FSD:

$$I_m = 0.8 \times 100\ \mu\text{A}$$

$$= 80\ \mu\text{A}$$

$$V = I_m(R_s + R_m)$$

$$= 80\ \mu\text{A}(499\text{ k}\Omega + 1\text{ k}\Omega)$$

$$= 40\text{ V}$$

At 0.5 FSD:

$$I_m = 50\ \mu\text{A}$$

$$V = 50\ \mu\text{A}(499\text{ k}\Omega + 1\text{ k}\Omega)$$

$$= 25\text{ V}$$

At 0.2 FSD:

$$I_m = 20\ \mu\text{A}$$

$$V = 20\ \mu\text{A}(499\text{ k}\Omega + 1\text{ k}\Omega)$$

$$= 10\text{ V}$$

Example 3-6

A PMMC instrument with FSD of 100 μA and a coil resistance of 1 kΩ is to be converted into a voltmeter. Determine the required multiplier resistance if the voltmeter is to measure 50 V at full scale (Figure 3-15). Also calculate the applied voltage when the instrument indicates 0.8, 0.5, and 0.2 of FSD.

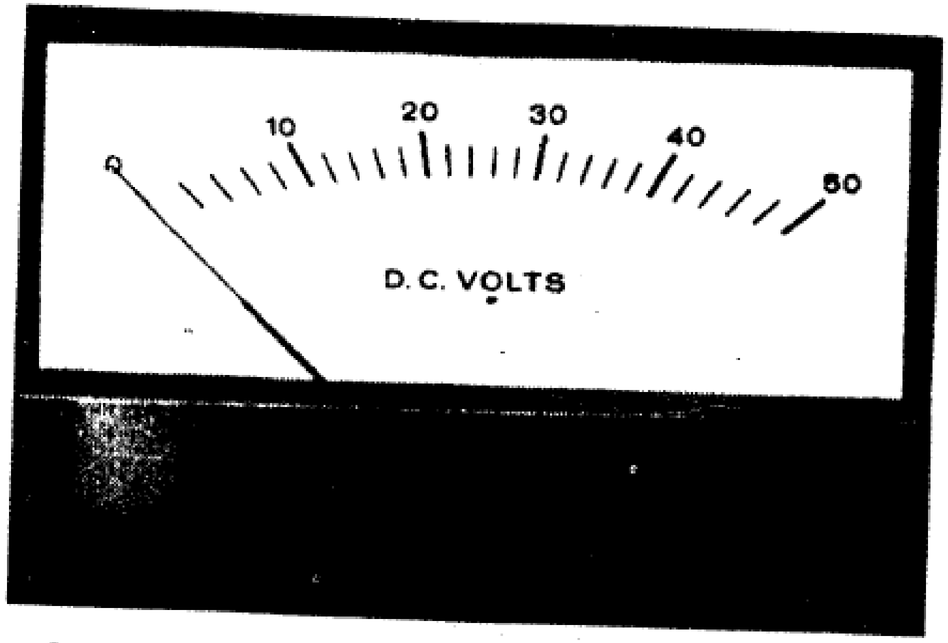


Figure 3-15 A dc voltmeter using a PMMC instrument has a linear voltage scale.

Solution

$$V = I_m(R_s + R_m)$$

$$R_s + R_m = \frac{V}{I_m}$$

and

$$R_s = \frac{V}{I_m} - R_m$$

For V = 50 V FSD,

$$I_m = 100\ \mu A$$

$$R_s = \frac{50\ V}{100\ \mu A} - 1\ k\Omega$$

$$= 499\ k\Omega$$

At 0.8 FSD:

$$I_m = 0.8 \times 100\ \mu A$$

$$= 80\ \mu A$$

$$V = I_m(R_s + R_m)$$

$$= 80\ \mu A(499\ k\Omega + 1\ k\Omega)$$

$$= 40\ V$$

At 0.5 FSD:

$$I_m = 50\ \mu A$$

$$V = 50\ \mu A(499\ k\Omega + 1\ k\Omega)$$

$$= 25\ V$$

At 0.2 FSD:

$$I_m = 20\ \mu A$$

$$V = 20\ \mu A(499\ k\Omega + 1\ k\Omega)$$

$$= 10\ V$$

Swamping Resistance

As in the case of the ammeter, the change in coil resistance (R_m) with temperature change can introduce errors in a PMMC voltmeter. However, the presence of the voltmeter multiplier resistor (R_s) tends to *swamp* coil resistance changes, except for low voltage ranges

where R_s is not very much larger than R_m. R_s will also be temperature sensitive to some degree (not as much as the copper wire coil), and in some cases it might be necessary to construct the multiplier resistor of manganin or constantan.

Voltmeter Sensitivity

The voltmeter designed in Example 3-6 has a total resistance of

$$R_V = R_s + R_m = 500\ \text{k}\Omega$$

Since the instrument measures 50 V at full scale, its *resistance per volt* is

$$\frac{500\ \text{k}\Omega}{50\ \text{V}} = 10\ \text{k}\Omega/\text{V}$$

This quantity is also termed the *sensitivity* of the voltmeter. The sensitivity of a voltmeter is always specified by the manufacturer, and it is frequently printed on the scale of the instrument. If the sensitivity is known, the total voltmeter resistance is easily calculated as (sensitivity × range). [It is important to note that the total resistance is *not* (sensitivity × meter reading).] If the full-scale meter current is known, the sensitivity can be determined as the reciprocal of full-scale current.

Ideally, a voltmeter should have an extremely high resistance. A voltmeter is always connected across, or in parallel with, the points in a circuit at which the voltage is to be measured. If its resistance is too low, it can alter the circuit voltage. This is known as *voltmeter loading effect*.

Multirange Voltmeter

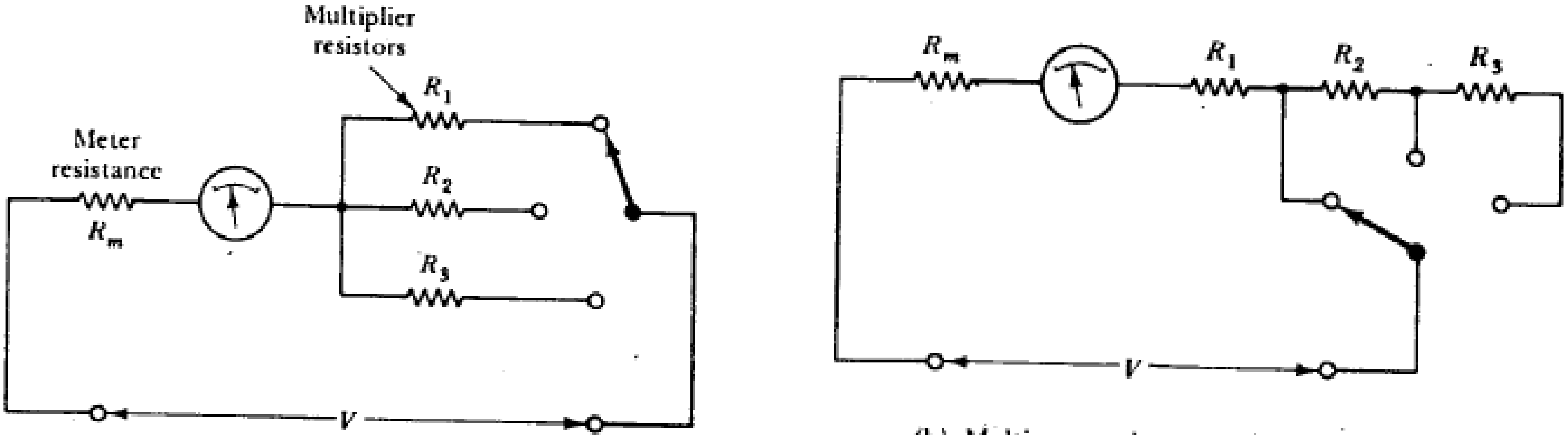
A multirange voltmeter consists of a deflection instrument, several multiplier resistors, and a rotary switch. Two possible circuits are illustrated in Figure 3-16. In Figure 3-16(a) only one of the three multiplier resistors is connected in series with the meter at any time. The range of this voltmeter is

$$V = I_m(R_m + R)$$

where R can be R_1, R_2, or R_3.

In Figure 3-16(b) the multiplier resistors are connected in series, and each junction is connected to one of the switch terminals. The range of this voltmeter can also be calculated from the equation $V = I_m(R_m + R)$, where R can now be R_1, $R_1 + R_2$, or $R_1 + R_2 + R_3$.

Of the two circuits, the one in Figure 3-16(b) is the least expensive to construct. This is because (as shown in Example 3-7) all of the multiplier resistors in Figure 3-16(a) must be special (nonstandard) values, while in Figure 3-16(b) only R_1 is a special resistor and all other multipliers are standard-value (precise) resistors.



(a) Multirange voltmeter using switched multiplier resistors

(b) Multirange voltmeter using series-connected multiplier resistors

Figure 3-16 A multirange voltmeter consists of a PMMC instrument, several multiplier resistors, and a switch for range selection. Individual, or series-connected resistors may be used.

Example 3-7

A PMMC instrument with FSD = 50 μA and R_m = 1700 Ω is to be employed as a voltmeter with ranges of 10 V, 50 V, and 100 V. Calculate the required values of multiplier resistors for the circuits of Figure 3-16(a) and (b).

Solution

Circuit as in Figure 3-16(a):

$$R_m + R_1 = \frac{V}{I_m}$$

$$R_1 = \frac{V}{I_m} - R_m$$

$$= \frac{10\text{ V}}{50\ \mu\text{A}} - 1700\ \Omega$$

$$= 198.3\text{ k}\Omega$$

$$R_2 = \frac{50\text{ V}}{50\ \mu\text{A}} - 1700\ \Omega$$

$$= 998.3\text{ k}\Omega$$

$$R_3 = \frac{100\text{ V}}{50\ \mu\text{A}} - 1700\ \Omega$$

$$= 1.9983\text{ M}\Omega$$

Circuit as in Figure 3-16(b):

$$R_m + R_1 = \frac{V_1}{I_m}$$

$$R_1 = \frac{V_1}{I_m} - R_m$$

$$= \frac{10\text{ V}}{50\ \mu\text{A}} - 1700\ \Omega$$

$$= 198.3\text{ k}\Omega$$

$$R_m + R_1 + R_2 = \frac{V_2}{I_m}$$

$$R_2 = \frac{V_2}{I_m} - R_1 - R_m$$

$$= \frac{50\text{ V}}{50\ \mu\text{A}} - 198.3\text{ k}\Omega - 1700\ \Omega$$

$$= 800\text{ k}\Omega$$

$$R_m + R_1 + R_2 + R_3 = \frac{V_3}{I_m}$$

$$R_3 = \frac{V_3}{I_m} - R_2 - R_1 - R_m$$

$$= \frac{100\text{ V}}{50\ \mu\text{A}} - 800\text{ k}\Omega - 198.3\text{ k}\Omega - 1700\ \Omega$$

$$= 1\text{ M}\Omega$$

Example 3-7

A PMMC instrument with FSD = 50 μA and R_m = 1700 Ω is to be employed as a voltmeter with ranges of 10 V, 50 V, and 100 V. Calculate the required values of multiplier resistors for the circuits of Figure 3-16(a) and (b).

Solution

Circuit as in Figure 3-16(a):

$$R_m + R_1 = \frac{V}{I_m}$$

$$R_1 = \frac{V}{I_m} - R_m$$

$$= \frac{10\ \text{V}}{50\ \mu\text{A}} - 1700\ \Omega$$

$$= 198.3\ \text{k}\Omega$$

$$R_2 = \frac{50\ \text{V}}{50\ \mu\text{A}} - 1700\ \Omega$$

$$= 998.3\ \text{k}\Omega$$

$$R_3 = \frac{100\ \text{V}}{50\ \mu\text{A}} - 1700\ \Omega$$

$$= 1.9983\ \text{M}\Omega$$

Circuit as in Figure 3-16(b):

$$R_m + R_1 = \frac{V_1}{I_m}$$

$$R_1 = \frac{V_1}{I_m} - R_m$$

$$= \frac{10\ \text{V}}{50\ \mu\text{A}} - 1700\ \Omega$$

$$= 198.3\ \text{k}\Omega$$

$$R_m + R_1 + R_2 = \frac{V_2}{I_m}$$

$$R_2 = \frac{V_2}{I_m} - R_1 - R_m$$

$$= \frac{50\ \text{V}}{50\ \mu\text{A}} - 198.3\ \text{k}\Omega - 1700\ \Omega$$

$$= 800\ \text{k}\Omega$$

$$R_m + R_1 + R_2 + R_3 = \frac{V_3}{I_m}$$

$$R_3 = \frac{V_3}{I_m} - R_2 - R_1 - R_m$$

$$= \frac{100\ \text{V}}{50\ \mu\text{A}} - 800\ \text{k}\Omega - 198.3\ \text{k}\Omega - 1700\ \Omega$$

$$= 1\ \text{M}\Omega$$

PMMC Instrument for AC Measurements

As discussed earlier, the PMMC instrument is *polarized*, that is, its terminals are identified as + and −, and it must be connected correctly for positive (on-scale) deflection to occur. When an alternating current with a very low frequency is passed through a PMMC instrument, the pointer tends to follow the instantaneous level of the ac. As the current grows postively, the pointer deflection increases to a maximum at the peak of the ac. Then as the instantaneous current level falls, the pointer deflection decreases toward zero. When the ac goes negative, the pointer is deflected (off-scale) to the left of zero. This kind of pointer movement can occur only with ac having a frequency of perhaps 0.1 Hz or lower. With the normal 60 Hz or higher supply frequencies, the damping mechanism of the instrument and the inertia of the meter movement prevent the pointer from following the changing instantaneous levels. Instead, the instrument pointer settles at the average value of the current flowing through the moving coil. The average value of purely sinusoidal ac is zero. Therefore, a PMMC instrument connected directly to measure 60 Hz ac indicates zero. It is important to note that although a PMMC instrument connected to an ac supply may be indicating zero, there can actually be a very large rms current flowing in its coils.

Full-Wave Rectifier Voltmeter

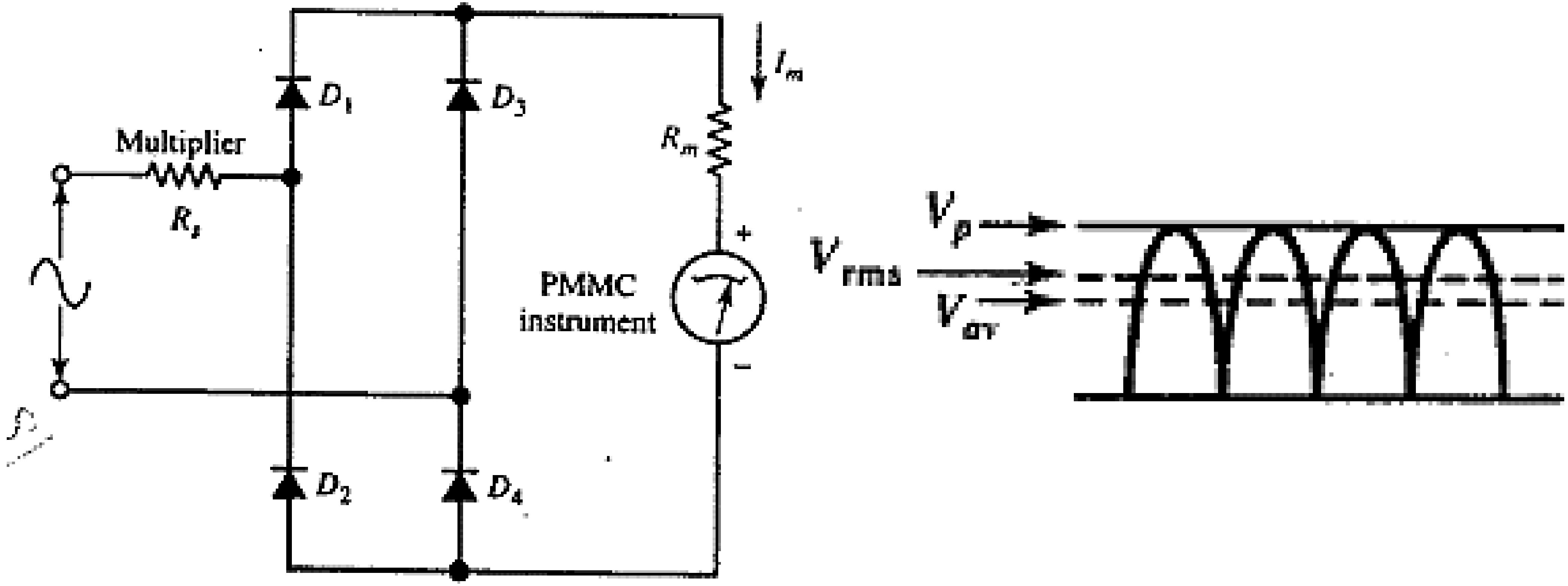


Figure 3-17 An ac voltmeter may be constructed of a PMMC instrument, a multiplier resistor, and a full-wave bridge rectifier. The instrument scale is correct only for pure sine waves.

SERIES OHMMETER

Basic Circuit

An *ohmmeter* (ohm-meter) is normally part of a *volt-ohm-milliammeter* (VOM), or *multi-function* meter. Ohmmeters do not usually exist as individual instruments. The simplest ohmmeter circuit consists of a voltage source connected in series with a pair of terminals, a standard resistance, and a low-current PMMC instrument. Such a circuit is shown in Figure 3-21(a). The resistance to be measured (R_x) is connected across terminals A and B.

The meter current indicated by the instrument in Figure 3-21(a) is (battery voltage)/(total series resistance):

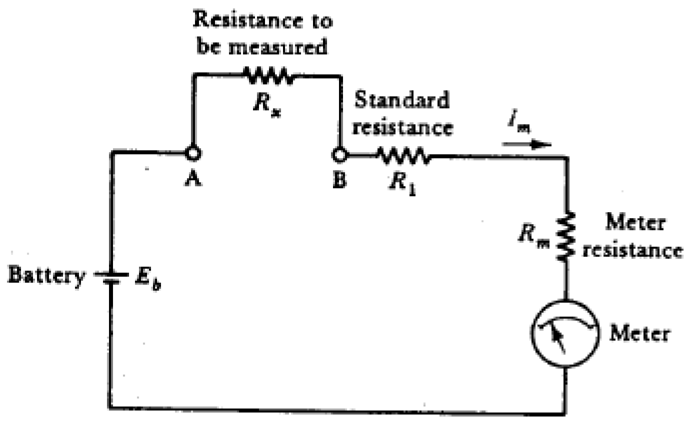
$$\boxed{I_m = \frac{E_b}{R_x + R_1 + R_m}} \qquad (3\text{-}3)$$

When the external resistance is zero (i.e., terminals A and B short-circuited), Equation 3-3 becomes

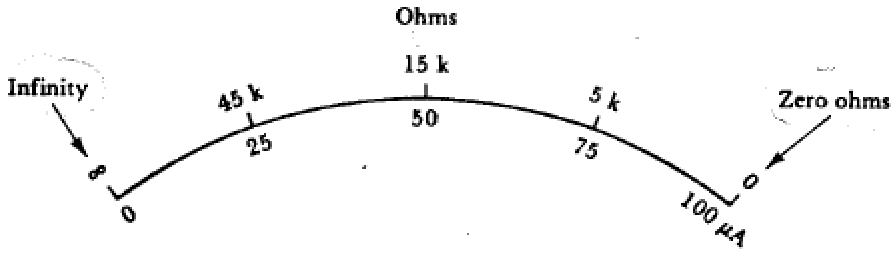
$$I_m = \frac{E_b}{R_1 + R_m}$$

If R_1 and R_m are selected (or if R_1 is adjusted) to give FSD when A and B are short-circuited, FSD is marked as *zero ohms*. Thus, for $R_x = 0$, the pointer indicates 0 Ω [see Figure 3-21(b)]. When terminals A and B are open-circuited, the effective value of resistance R_x is infinity. No meter current flows, and the pointer indicates zero current. This point (zero current) is marked as *infinity* (∞) on the resistance scale [Figure 3-21(b)].

If a resistance R_x with a value between zero and infinity is connected across terminals A and B, the meter current is greater than zero but less than FSD. The pointer position on the scale now depends on the relationship between R_x and $R_1 + R_m$. This is demonstrated by Example 3-14.



(a) Basic circuit of series ohmmeter



(b) Ohmmeter scale

Figure 3-21 Basic series ohmmeter circuit consisting of a PMMC instrument and a series-connected standard resistor (R_1). When the ohmmeter terminals are shorted ($R_X = 0$) meter full-scale deflection occurs. At half-scale deflection $R_X = R_1$, and at zero deflection the terminals are open-circuited.

The series ohmmeter in Figure 3-21(a) is made up of a 1.5 V battery, a 100 μA meter, and a resistance R_1 which makes $(R_1 + R_m) = 15\ \text{k}\Omega$.

(a) Determine the instrument indication when $R_x = 0$.

(b) Determine how the resistance scale should be marked at 0.5 FSD, 0.25 FSD, and 0.75 FSD.

Solution

(a) Equation 3-3,

$$I_m = \frac{E_b}{R_x + R_1 + R_m} = \frac{1.5\ \text{V}}{0 + 15\ \text{k}\Omega}$$

$$= 100\ \mu\text{A (FSD)}$$

(b) At 0.5 FSD:

$$I_m = \frac{100\ \mu\text{A}}{2} = 50\ \mu\text{A}$$

From Equation 3-3,

$$R_x + R_1 + R_m = \frac{E_b}{I}$$

$$R_x = \frac{E_b}{I_m} - (R_1 + R_m)$$

$$= \frac{1.5\ \text{V}}{50\ \mu\text{A}} - 15\ \text{k}\Omega$$

$$= 15\ \text{k}\Omega$$

At 0.25 FSD:

$$I_m = \frac{100\ \mu\text{A}}{4} = 25\ \mu\text{A}$$

$$R_x = \frac{1.5\ \text{V}}{25\ \mu\text{A}} - 15\ \text{k}\Omega$$

$$= 45\ \text{k}\Omega$$

At 0.75 FSD:

$$I_m = 0.75 \times 100\ \mu\text{A} = 75\ \mu\text{A}$$

$$R_x = \frac{1.5\ \text{V}}{75\ \mu\text{A}} - 15\ \text{k}\Omega$$

$$= 5\ \text{k}\Omega$$

The ohmmeter scale is now marked as shown in Figure 3-21(b).

The series ohmmeter in Figure 3-21(a) is made up of a 1.5 V battery, a 100 μA meter, and a resistance R_1 which makes $(R_1 + R_m) = 15\ k\Omega$.
(a) Determine the instrument indication when $R_x = 0$.
(b) Determine how the resistance scale should be marked at 0.5 FSD, 0.25 FSD, and 0.75 FSD.

Solution

(a) Equation 3-3,

$$I_m = \frac{E_b}{R_x + R_1 + R_m} = \frac{1.5\ V}{0 + 15\ k\Omega}$$

$$= 100\ \mu A\ (FSD)$$

(b) At 0.5 FSD:

$$I_m = \frac{100\ \mu A}{2} = 50\ \mu A$$

From Equation 3-3,

$$R_x + R_1 + R_m = \frac{E_b}{I}$$

$$R_x = \frac{E_b}{I_m} - (R_1 + R_m)$$

$$= \frac{1.5\ V}{50\ \mu A} - 15\ k\Omega$$

$$= 15\ k\Omega$$

At 0.25 FSD:

$$I_m = \frac{100\ \mu A}{4} = 25\ \mu A$$

$$R_x = \frac{1.5\ V}{25\ \mu A} - 15\ k\Omega$$

$$= 45\ k\Omega$$

At 0.75 FSD:

$$I_m = 0.75 \times 100\ \mu A = 75\ \mu A$$

$$R_x = \frac{1.5\ V}{75\ \mu A} - 15\ k\Omega$$

$$= 5\ k\Omega$$

The ohmmeter scale is now marked as shown in Figure 3-21(b).

Electrodynamic Instrument

- Instead of using Permanent Magnet, an Electrodynamic Instrument uses Electromagnets
- This makes it suitable for measuring AC quantities as well.

Construction & Operation

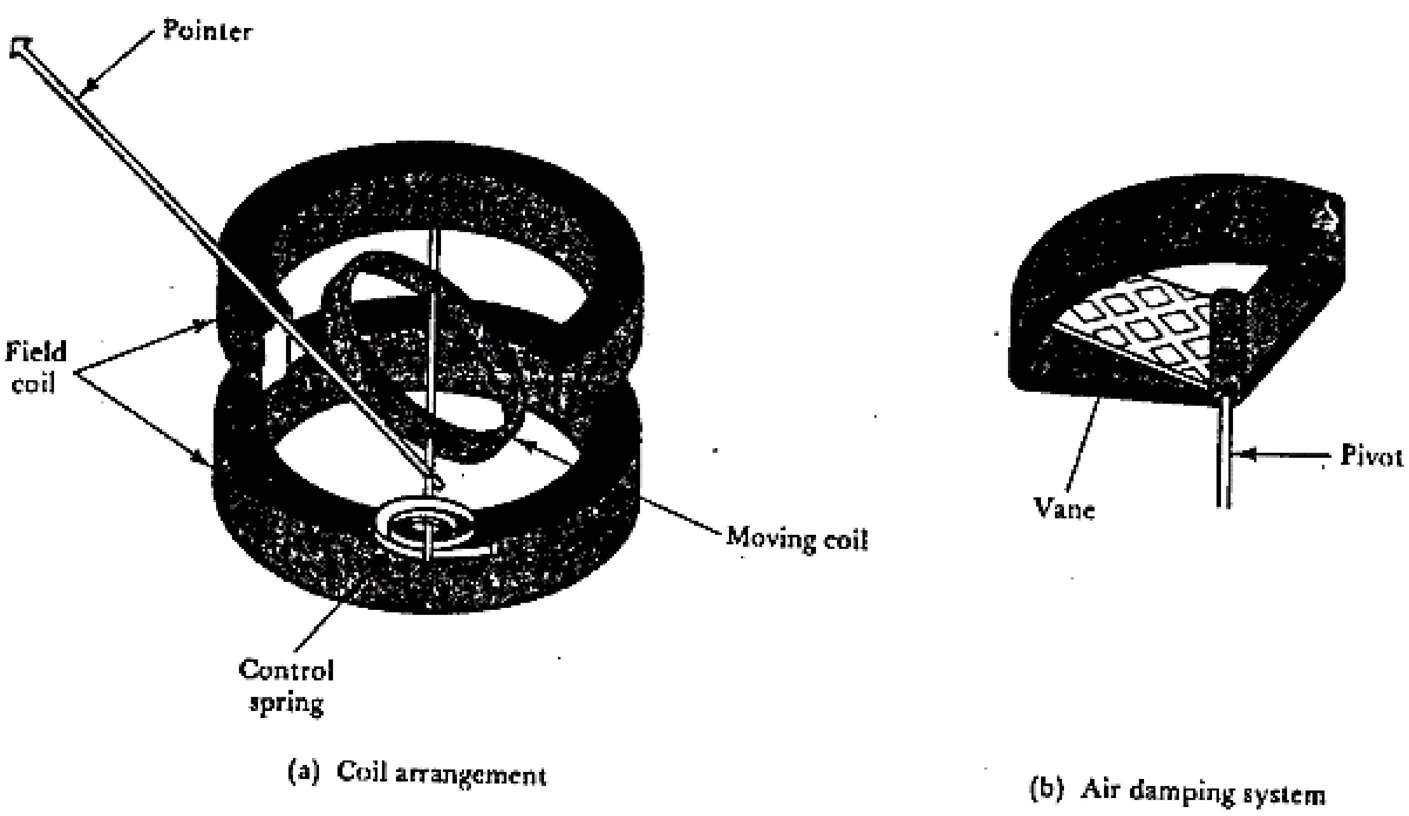


Figure 3-26 An electrodynamic instrument has a moving coil, as in a PMMC instrument, but the magnetic field is produced by two current-carrying field coils instead of a magnet. Damping is provided by an enclosed vane.

- The major difference between PMMC and Electrodynamic instrument is that two magnetic field coils are substituted in place of the permanent magnet.
- The magnetic field in which the moving coil is pivoted is generated by passing a current through the stationary field coils.
- When a current flows through the pivoted coil, the two fluxes interact (as in the PMMC instrument), causing the coil and pointer to be deflected.
- Spiral-springs provide controlling force and connecting leads to the pivoted coil.
- Zero adjustment and moving system balance are also as in the PMMC instrument.

Another major difference from the PMMC instrument is that the electrodynamic instrument usualy has air damping. A lightweight vane pushes air around in an enclosure when the pivoted coil is in motion [see Figure 3-26(b)]. This damps out all rapid movements and oscillations of the moving system. As will be explained, electrodynamic instruments can be used on ac. The alternating current would induce unwanted eddy currents in a metallic coil former. Therefore, the damping method employed in a PMMC instrument would not be suitable for an electrodynamic instrument.

Normally, there is no iron core in an electrodynamic instrument, so the flux path is entirely an air path. Consequently, the field flux is much smaller than in a PMMC instrument. To produce a strong enough deflecting torque, the moving-coil current must be much larger than the small currents required in a PMMC instrument.

As in the case of the PMMC instrument, the deflecting torque of an electrodynamic instrument is dependent on field flux, coil current, coil dimensions, and number of coil turns. However, the field flux is directly proportional to the current through the field coils, and the moving-coil flux is directly proportional to the current through the moving coil. Consequently, the deflecting torque is proportional to the product of the two currents:

$$T_D \propto I_{\text{field coil}} I_{\text{moving coil}}$$

When the same current flows through field coils and pivoted coil, the deflecting torque is proportional to the square of the current:

$$T_D \propto I^2$$

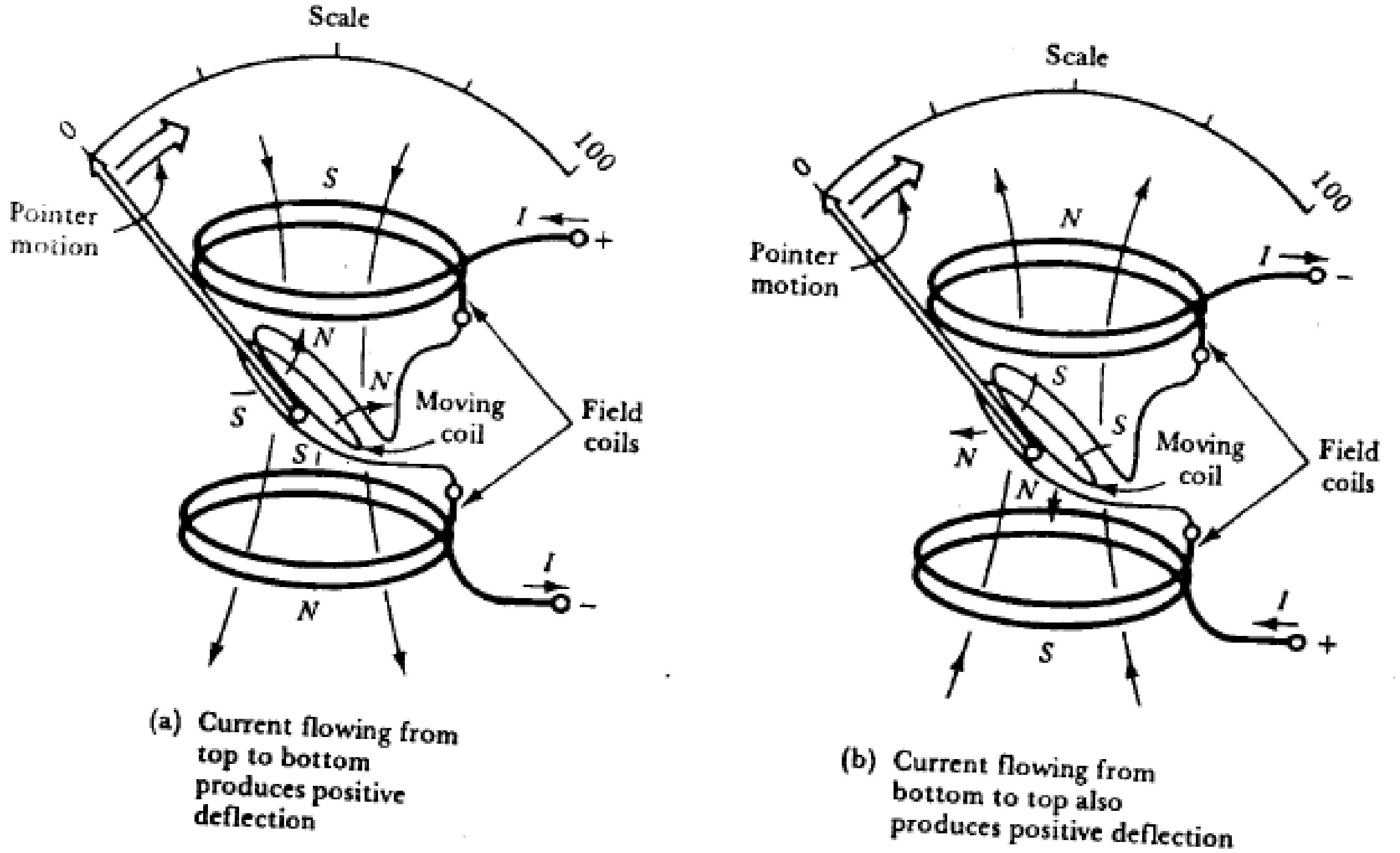
This gives the deflection angle as

$$\boxed{\theta = CI^2} \qquad (3\text{-}6)$$

where C is a constant. Because the deflection is proportional to I^2, the scale of the instrument is nonlinear: cramped at the low (left-hand) end and spaced out at the high end.

The major disadvantages of an electrodynamic instrument compared to a PMMC instrument are the lower sensitivity and the nonlinear scale. A major advantage of the electrodynamic instrument is that it is not polarized; that is, a positive deflection is obtained regardless of the direction of current in the coils. Thus the instrument can be used to measure ac or dc.

AC Operation



The simpler figure for coils and field can be referred from lectures

Consider Figure 3-27 in which the fixed and moving coils of an electrodynamic instrument are shown connected in series. In (a) the current direction is such that the flux of the held coils sets up S poles at the top. and N poles at the bottom. of each coil. The moving coil flux produces N pole at the right-hand side of the coil, and S pole at the left-hand side. The N pole of the moving coil is adjacent to the N pole of the upper field coil, and the S pole of the moving coil is adjacent to the S pole of the lower field coil.

Since like poles repel, the moving coil rotates in a clockwise direction, causing the poiner to move to the right from its zero position on the scale.

Now consider what occurs when the current through all three coils is reversed. Figure 3-27(b) shows that the reversed current causes the field coils to set up N poles at the top and S poles at the bottom of each coil. The moving-coil flux is also reversed so that it has an S pole at the right-hand side and an N pole at the left. Once again similar poles are adjacent, and repulsion produces clockwise rotation of the coil and pointer.

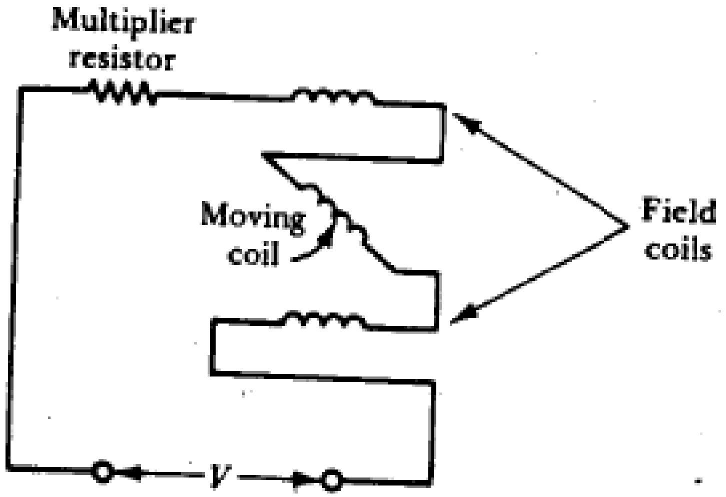
It is seen that the electrodynamic instrument has a positive deflection, regardless of the direction of current through the meter. Consequently, the terminals are *not* marked + and – (i.e., the instrument is *not* polarized).

Transfer Instrument

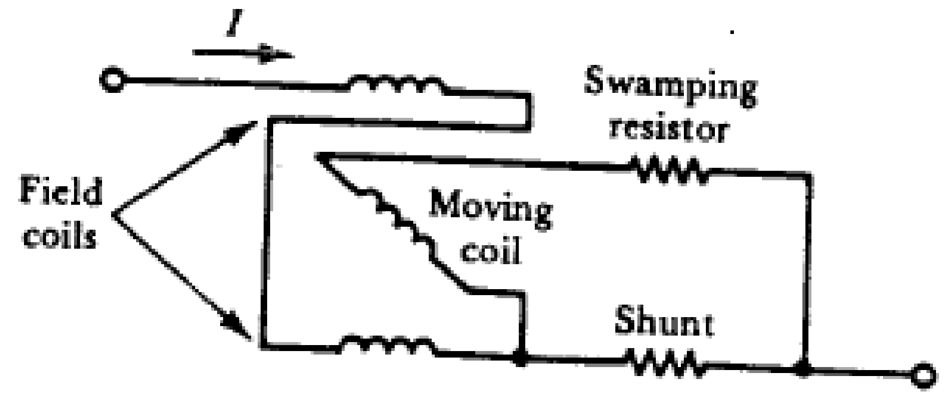
As already explained, the electrodynamic instrument deflection is proportional to I^2 (i.e., when the same current flows in the moving coil and field coils). When used on ac, the deflection settles down to a position proportional to the average value of I^2. Thus, the deflection is proportional to the *mean-squared value* of the current. Since the scale of the meter is calibrated to indicate I, rather than I^2, the meter indicates *root-mean-squared current*, or the rms value. The rms value has the same effect as a numerically equivalent dc value. Therefore, the scale of the instrument can be read as either dc or rms ac. This is the characteristic of a *transfer instrument*, which can be calibrated on dc and then used to measure ac. (Voltmeters are available that operate on an electrostatic principle. These are also ac/dc transfer instruments.)

Because the reactance of the coils increase rapidly with increasing frequency, electrodynamic instruments are useful only at low frequencies. Electrodynamic wattmeters, in particular, perform very satisfactorily at domestic and industrial power frequencies.

Electrodynamic Voltmeter & Ammeter



(a) Electrodynamic voltmeter



(b) Electrodynamic ammeter

Figure 3-28 For use as a voltmeter, an electrodynamic instrument has the field coils, moving coil, and multiplier resistor all connected in series. For use as an ammeter, the field coils are seriesed with the parallel-connected shunt and moving-coil circuit.

Voltmeter

Figure 3-28(a) shows the usual circuit arrangement for an electrodynamic voltmeter. Since a voltmeter must have a high resistance, all three coils are connected in series, and a multiplier resistor (made of manganin or constantan) is included. When the total resistance of the coils, and the required current for FSD are known, the multiplier resistance is calculated exactly as for dc voltmeters. The instrument scale can be read either as dc voltage or rms ac voltage.

Because electrodynamic instruments usually require at least 100 mA for FSD, an electrodynamic voltmeter has a much lower sensitivity than a PMMC voltmeter. At 100 mA FSD, the sensitivity is 1/100 mA = 10 Ω/V. For a 100 V instrument, this sensitivity gives a total resistance of only 1 kΩ. Therefore, an electrodynamic voltmeter is not suitable for measuring voltages in electronic circuits because of the loading effect.

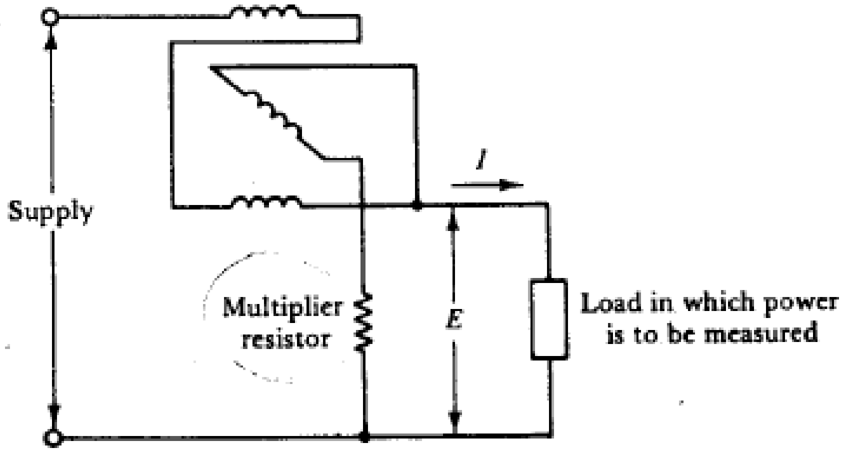
Ammeter

In an electrodynamic ammeter, the moving coil and its series-connected swamping resistance are connected in parallel with the ammeter shunt. This is illustrated in Figure 3-28(b). The two field coils should be connected in series with the parallel arrangement of shunt and moving coil, as shown.

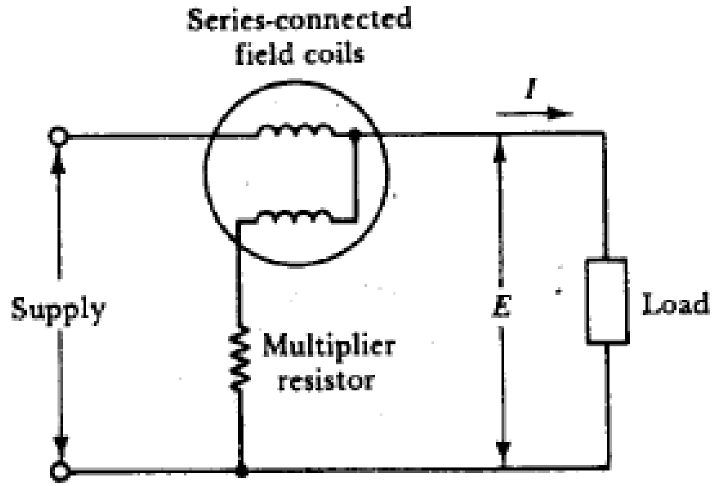
Because the field coils are always passing the actual current to be measured, resistance changes in the coils with temperature variations have no effect on the instrument performance. However, as in PMMC ammeters, the moving coil must have a manganin or constantan swamping resistance connected in series. Also, the shunt resistor must be made of manganin.

The scale of the electrodynamic ammeter can be read either as dc levels or rms ac values. Like the electrodynamic voltmeter, this instrument can be calibrated on dc and then used to measure either dc or ac.

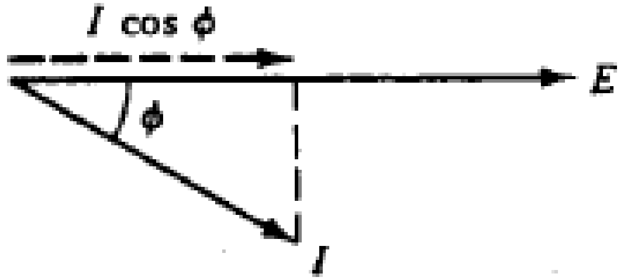
Electrodynamic Wattmeter



(a) Electrodynamic wattmeter circuit



(b) Another way to show the wattmeter circuit



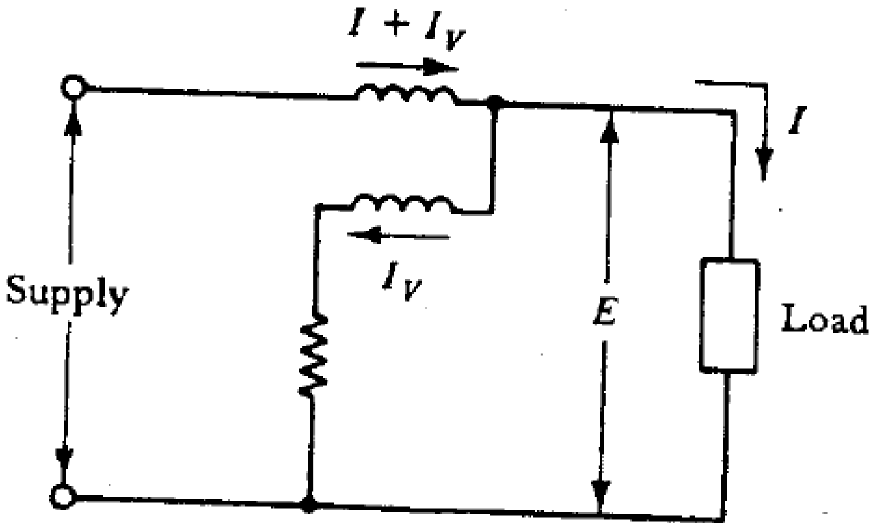
(c) Wattmeter measures $EI \cos \phi$

Figure 3-29 An electrodynamic wattmeter has the moving coil and multiplier resistor connected in parallel with the load, and the field coils in series with the load. Instrument deflection is proportional to $EI \cos \phi$.

For both dc and ac applications, the most important use of the electrodynamic instrument is as a wattmeter. The coil connections for power measurement are illustrated in Figure 3-29(a). The field coils are connected in series with the load in which power is to be measured. The moving coil and a multiplier resistor are connected in parallel with the load. Thus, the field coils carry the load current, and the moving-coil current is proportional to the load voltage. Since the instrument deflection is proportional to the product of the two currents, deflection = $C \times EI$, where C is a constant, or meter indication = EI watts. In Figure 3-29(b) the electrodynamic wattmeter is shown in a slightly less complicated form than in Figure 3-29(a). A single-coil symbol is used to represent the two series-connected field coils.

Suppose that the instrument is correctly connected and giving a positive deflection. If the supply voltage polarity were reversed, the fluxes would reverse in both the field coils and the moving coil. As already explained, the instrument would still have a positive deflection. In ac circuits where the supply polarity is reversing continuously, the electrodynamic wattmeter gives a positive indication proportional to $E_{rms}I_{rms}$. Like electrodynamic ammeters and voltmeters, the wattmeter can be calibrated on dc and then used to measure power in either dc or ac circuits.

An important source of error in the wattmeter is illustrated in Figure 3-30(a) and (b). Figure 3-30(a) shows that if the moving coil (or voltage coil) circuit is connected in parallel with the load, the field coils pass a current $(I + I_v)$, the sum of the load current and the moving-coil current. This results in the wattmeter indicating the load power (EI), plus a small additional quantity (EI_v). Where the load current is very much larger than I_v, this error may be negligible. In low-load-current situations, the error may be quite significant.



(a) Error due to moving-coil current

We didnt discuss source of error and compensation of wattmeter in lectures

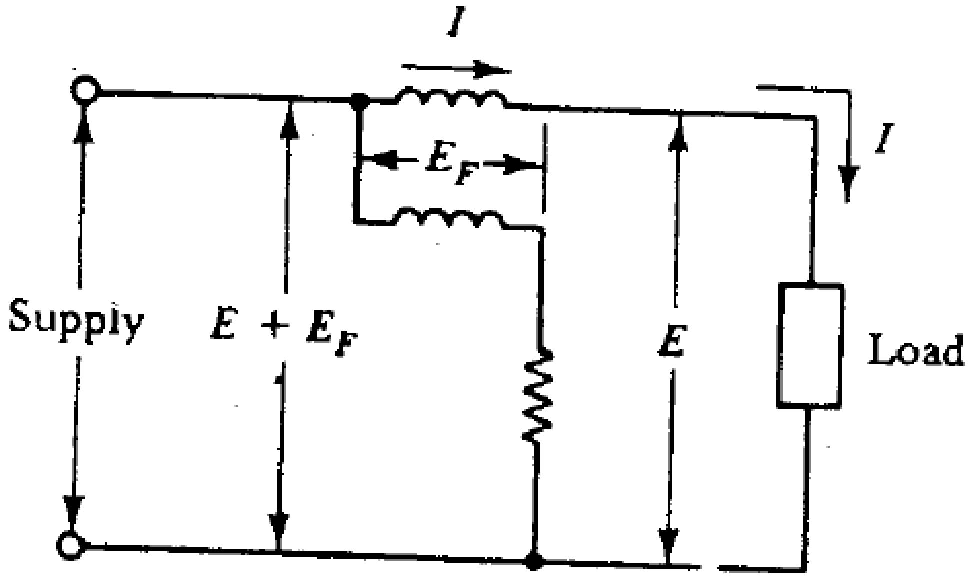
$$\text{Deflection} \propto E(I + I_V)$$

$$\propto EI + \underbrace{EI_V}_{\text{Error}}$$

... wattmeter connection in (c).

In Figure 3-30(b) the voltage coil is connected to the supply side of the field coils so that only the load current flows through the field coils. However, the voltage applied to the series-connected moving coil and multiplier is $E + E_F$ (the load voltage plus the voltage drop across the field coils). Now the wattmeter indicates load power (EI) plus an additional quantity ($E_F I$). In high-voltage circuits, where the load voltage is very much larger than the voltage drop across the field coils, the error may be insignificant. In low-voltage conditions, this error may be serious.

... illustrated in Figure 3-30(c) eliminates the errors de-

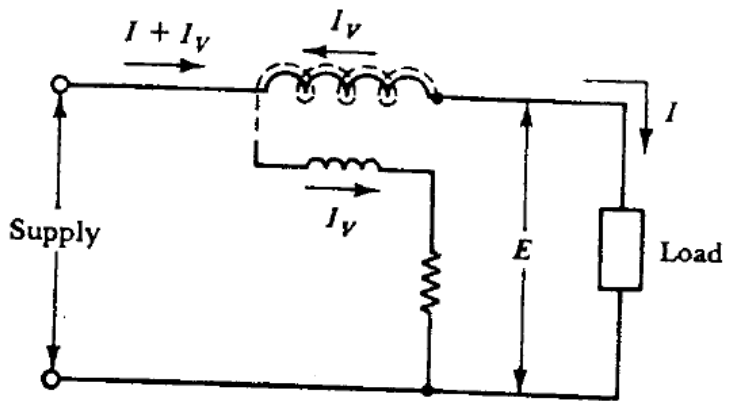


(b) Error due to field coils voltage drop

$$\text{Deflection} \propto (E + E_F)I$$

$$\propto EI + \underbrace{E_F I}_{\text{Error}}$$

In the compensated wattmeter, a compensating coil having the same number of turns as the current coil is wound on the current coil and is connected in series with the potential coil. The current in the compensating coil flows in a direction opposite to that of the current in the current coil, and therefore its effect on the wattmeter field is nullified.



(c) Compensated wattmeter using an additional coil wound alongside the field coils

$$\text{Deflection} \propto E(I + I_V - I_V) \propto EI$$

The *compensated wattmeter* illustrated in Figure 3-30(c) eliminates the errors described above. Since the field coils carry the load current, they must be wound of thick copper wire. In the compensated wattmeter, an additional thin conductor is wound right alongside every turn on the field coils. This additional coil, shown dashed in Figure 3-30(c), becomes part of the voltage coil circuit. The voltage coil circuit is seen to be connected directly across the load, so that the moving-coil current is always proportional to load voltage. The current through the field coils in $I + I_v$, so that a field coil flux is set up proportional to $I + I_v$. But the additional winding on the field coils carries the moving-coil current I_v, and this sets up a flux in opposition to the main flux of the field coils. The resulting flux in the field coils is proportional to $[(I + I_v) - I_v] \propto I$. Thus, the additional winding cancels the field flux due to I_v, and the wattmeter deflection is now directly proportional to EI.