

COS 210

Worksheet 4

Scott Bebington

U21546216

Question 1

Prove that, if A is a regular language, then for all natural numbers $k \geq 0$ the language A^k is also regular, using induction.

Base case: $k=0$

$A^0 = \epsilon$, ϵ is a regular language since it can be accepted by the starting state of function if the starting state is an accepting state. Therefore the statement A^k is regular for $k = 0$.

Inductive step: $k + 1$

Assume that the language A^k is regular for some natural number $k \geq 0$. We need to show that the language $A^{(k+1)}$ is also regular.

To expand:

$$A^{(k+1)} = A * A^k$$

Since regular languages are closed under concatenation, and A was given to be a regular language, and $A * A^k$ is a concatenation of A and A^k .

We can say that $A^{(k+1)}$ is also a regular language and that there exists a DFA such that the Language $M=(Q, \Sigma, \delta, q_0, F)$ accepts $A^{(k+1)}$.

Therefore, using induction, we have shown that if A is a regular language, then for all natural numbers $k \geq 0$, the language A^k is also regular,

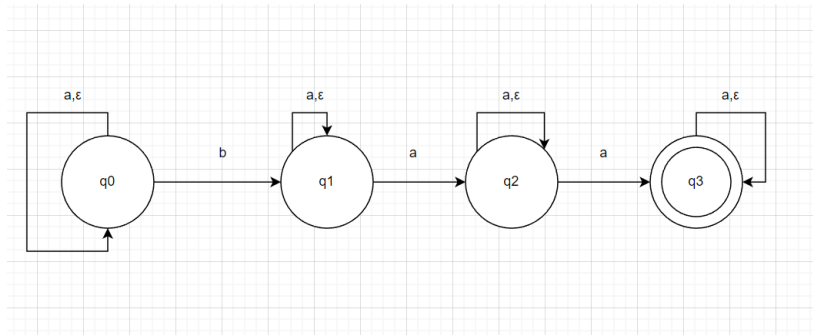
Question 2

For the alphabet $\Sigma = \{a, b\}$, let A be the language such that

$A = \{w : w \text{ contains exactly one } b \text{ and at least two } a\text{'s}\}$.

A^+ is the concatenation of A^k with $k > 0$

The NFA is as follows.



As you can see, no matter how many concatenations of A there are, the following state diagram holds, therefore A^+ is also a regular language.

Question 3

For the alphabet $\Sigma = \{a, b\}$ we define the language A as

$A = \{w : w \text{ contains exactly one } b \text{ and at least two } a\text{'s}\}.$

Give the regular expression describing this language.

$a^* (baa) a^* \cup a^* (aba) a^* \cup a^* (aab) a^*$

Question 4

For the alphabet $\{a, b, c\}$ we define the language A as

$A = \{w : w \text{ contains the substring } bac \text{ at least three times}\}.$

Give the regular expression describing this language.

$(a \cup b \cup c)^* bac (a \cup b \cup c)^* bac (a \cup b \cup c)^* bac (a \cup b \cup c)^*$

Question 5

For the alphabet $\{a, b, c, d\}$ we define the language A as

$A = \{w : \text{every even position is } a \text{ or } b, \text{ every odd position is } c \text{ or } d\}.$

Give the regular expression describing this language.

(Take into account that a string w may have an even or an odd length.)

$(c \cup d) \cup ((c \cup d) (a \cup b))^* \cup ((c \cup d) ((a \cup b) (c \cup d))^*)$

