COS 210

Worksheet 6

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Question 1

$L1 = \{va^{(n+1)} : v \in \{a, b\}^* , |v| = n, n \geq 0\}$

Step 1:

Assume that A is regular and therefor has a pumping length of $p >= 1$.

Step 2:

Consider $w = (va^{(p + 1)}) : v \in \{a, b\}^*) \in L1$

Step 3:

We have that $|w| = (p + 1)$, $p + 1 \geq p$. therefor w can be written as $w = xyz$ where

$y \neq \varepsilon$

$|xy| \leq p$

$xy^k \in L1$ for all $k \geq 0$

Step 4:

$|xy| \leq p$

xy = a's or b's or ε ($\{a,b\}^*$)

Step 5:

Since $|xy| \leq p$, we know that y consists of only a's or only b's, or it could be the empty string ε, because v can also be the empty string. Therefore, we can write $y = a^j$ or $y = b^j$ or $y = \varepsilon$, where $0 \leq j \leq p$.

Step 6:

Let's consider the pumped string xy^2z. We need to consider the following cases:

Case 1:

$y = \varepsilon$ In this case, xy^2z will be of the form $va^{(p + 1)}$ because pumping the empty string does not change the original string.

But since $|v| = n$, v must contain the same number of a's and b's as that of $a^{(n + 1)}$ where n must be a minimum of 1.

Case 2:

y consists of only a's or only b's.

Pumping y will increase the number of a's or b's in the string. Therefore, xy^2z will be of the form va^(n + k + 1) with k ≥ 0 since |v| = n, and not in the form va^(n +1)

Step 7:

Since we have arrived at a contradiction in all cases, our initial assumption that L1 is regular must be false. Therefore, L1 is not a regular language.

In conclusion, we have proven by contradiction using the Pumping Lemma that the language

L1 = {va^(n+1) : v ∈ {a, b}* , |v| = n, n ≥ 0} is not regular.

Question 2

Assume the language L2 = {0^n 1^m : n != m, n ≥ 0, m ≥ 0} is regular.

Then, by closure under complementation, the language L2' = must also be regular.

Then, by closure properties of regular languages, we know that the complement of L2 is also regular, L2' given by: L2' = {0^n 1^m : n = m, n ≥ 0, m ≥ 0}

Now, consider the intersection of L2' and the regular language A = {0^n 1^n : n ≥ 0}:

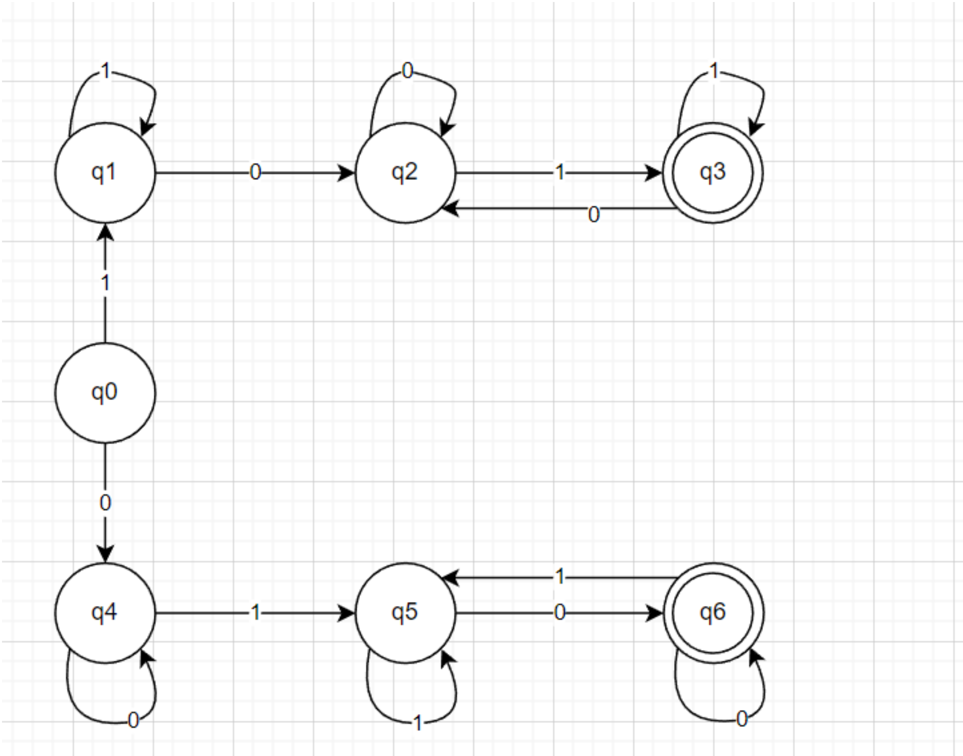
L = L2' ∩ A = {0^n1^n : n ≥ 0}

We know that A is not regular (given by proof in L12), but L is a subset of A, so if L were regular, it would contradict the fact that regular languages are closed under intersection.

Therefore, the assumption that L2 is regular must be false.

Question 3

In order to prove if a language is regular or not we need to see if a DFA can be constructed for the language, in this case it can be, given by the following: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