



COS 210 Worksheet 4

- This worksheet consists of **5 questions** for a total of **12 marks**.

Question 1 (3 marks)

Prove that, if A is a regular language, then for all natural numbers $k \geq 0$ the language A^k is also regular, using induction. (The definition of A^k can be found in Lecture 5, Slide 10.)

(You may make use of the fact that regular languages are closed under concatenation.)

Question 2 (3 marks)

Prove by construction that, if A is a regular language, then A^+ is a regular language. Where A^+ is defined as

$$A^+ = \bigcup_{k=1}^{\infty} A^k$$

(A^+ is A^* without $k = 0$.)

Question 3 (2 marks)

For the alphabet $\Sigma = \{a, b\}$ we define the language A as

$$A = \{w : w \text{ contains exactly one } b \text{ and at least two } a\text{'s}\}.$$

Give the regular expression describing this language.

Question 4 (2 marks)

For the alphabet $\{a, b, c\}$ we define the language A as

$$A = \{w : w \text{ contains the substring } bac \text{ at least three times}\}.$$

Give the regular expression describing this language.

Question 5 (2 marks)

For the alphabet $\{a, b, c, d\}$ we define the language A as

$$A = \{w : \text{every even position is } a \text{ or } b, \text{ every odd position is } c \text{ or } d\}.$$

Give the regular expression describing this language.

(Take into account that a string w may have an even or an odd length.)