COS 210 Worksheet 1

- This worksheet consists of **3 questions** for a total of **10 marks**.
- Show your working for all calculations and reasoning.

Question 1 .... (2 marks)

Prove the following statement by **contradiction**:

Let n be a natural number. If n^2 is even, then n is even.

Question 2 .... (5 marks)

Prove the following statement by **contradiction**:

The number $\sqrt{2}$ is irrational.

Hint: Make use of the following definition in your proof:

A rational number can be written as a fraction $\dfrac{m}{b}$ where $m, b \in \mathbb{Z}$ and $\dfrac{m}{b}$ is non-reducible, i.e., the greatest common divisor of m and b is 1.

Question 3 .... (3 marks)

Prove by **induction** that for all natural numbers $n \geq 1$ the following holds:

$$6^n - 1 \text{ is divisible by } 5$$