COS 284 TUTORIAL 2

SEMESTER TEST 1 RECAP

CYCLIC REDUNDANCY CHECK I

Determine the quotient and remainder

when 100111 is divided by 110

in modulo 2 arithmetic.

Provide your answers as bit strings.

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(dividend) (divisor)

`100111 ÷ 110 mod 2 =` (quotient)

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```
Modulo 2 subtraction:
0 - 0 = 0
0 - 1 = 1
1 - 0 = 1
1 - 1 = 0
```

(dividend) (divisor)

```
 100111 ÷ 110 mod 2 = 1      (quotient)
-110 mod 2
 ----
  10
```

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```
Modulo 2 subtraction:
0 - 0 = 0
0 - 1 = 1
1 - 0 = 1
1 - 1 = 0
```

(dividend)   (divisor)

```
 100111 ÷ 110 mod 2 = 1      (quotient)
-110 mod 2
 ----
  101
```

CYCLIC REDUNDANCY CHECK I

Determine the quotient and remainder

when 100111 is divided by 110

in modulo 2 arithmetic.

Provide your answers as bit strings.

```
(dividend)   (divisor)
 100111 ÷ 110 mod 2 = 11     (quotient)
-110  mod 2
 ----
  101
 -110 mod 2
  ----
   11
```

```
Modulo 2 subtraction:
0 - 0 = 0
0 - 1 = 1
1 - 0 = 1
1 - 1 = 0
```

CYCLIC REDUNDANCY CHECK I

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Provide your answers as bit strings.

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Modulo 2 subtraction:
0 - 0 = 0
0 - 1 = 1
1 - 0 = 1
1 - 1 = 0
```

(dividend) (divisor)

```
 100111 ÷ 110 mod 2 = 11    (quotient)
-110 mod 2
 ----
  101
 -110 mod 2
  ----
   111
```

CYCLIC REDUNDANCY CHECK I

Determine the quotient and remainder

when 100111 is divided by 110

in modulo 2 arithmetic.

Provide your answers as bit strings.

```
Modulo 2 subtraction:
0 - 0 = 0
0 - 1 = 1
1 - 0 = 1
1 - 1 = 0
```

```
(dividend)   (divisor)
 100111 ÷ 110 mod 2 = 111   (quotient)
-110 mod 2
 ----
  101
 -110 mod 2
  ----
   111
  -110 mod 2
   ----
      1
```

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Determine the quotient and remainder

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Provide your answers as bit strings.

```
Modulo 2 subtraction:
0 - 0 = 0
0 - 1 = 1
1 - 0 = 1
1 - 1 = 0
```

```
(dividend)   (divisor)
 100111 ÷ 110 mod 2 = 111   (quotient)
-110 mod 2
 ----
  101
 -110 mod 2
  ----
   111
  -110 mod 2
   ----
     11
```

CYCLIC REDUNDANCY CHECK I

Determine the quotient and remainder

when 100111 is divided by 110

in modulo 2 arithmetic.

Provide your answers as bit strings.

```
Modulo 2 subtraction:
0 - 0 = 0
0 - 1 = 1
1 - 0 = 1
1 - 1 = 0
```

```
(dividend)  (divisor)
 100111 ÷ 110 mod 2 = 1110 (quotient)
-110 mod 2
 ----
  101
 -110 mod 2
  ----
   111
  -110 mod 2
   ----
     11
    -00 mod 2
    ---
     11 (remainder)
```

CYCLIC REDUNDANCY CHECK II

Determine the quotient and remainder when the polynomial $x^5 + x^3 + x + 1$ is divided by 101 in modulo 2 arithmetic.

CYCLIC REDUNDANCY CHECK II

Determine the quotient and remainder when

the polynomial $x^5 + x^3 + x + 1$ is divided

by 101 in modulo 2 arithmetic.

Polynomial $x^5 + x^3 + x + 1$

$= 1 \cdot x^5 + 0 \cdot x^4 + 1 \cdot x^3 + 0 \cdot x^2 + 1 \cdot x^1 + 1 \cdot x^0$

represents bit string 101011

CYCLIC REDUNDANCY CHECK II

(dividend) (divisor)

`101011 ÷ 101 mod 2 =` (quotient)

Determine the quotient and remainder when the polynomial $x^5 + x^3 + x + 1$ is divided by 101 in modulo 2 arithmetic.

Polynomial $x^5 + x^3 + x + 1$

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represents bit string 101011

(dividend) (divisor)

```
 101011 ÷ 101 mod 2 = 1      (quotient)
-101 mod 2
----
   0
```

CYCLIC REDUNDANCY CHECK II

Determine the quotient and remainder when the polynomial $x^5 + x^3 + x + 1$ is divided by 101 in modulo 2 arithmetic.

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represents bit string 101011

(dividend) (divisor)

```
 101011 ÷ 101 mod 2 = 1      (quotient)
-101   mod 2
----
   00
```

CYCLIC REDUNDANCY CHECK II

Determine the quotient and remainder when the polynomial $x^5 + x^3 + x + 1$ is divided by 101 in modulo 2 arithmetic.

(dividend) (divisor)

```
 101011 ÷ 101 mod 2 = 10     (quotient)
-101 mod 2
 ----
   00
  -00 mod 2
```

Polynomial $x^5 + x^3 + x + 1$

$= 1 \cdot x^5 + 0 \cdot x^4 + 1 \cdot x^3 + 0 \cdot x^2 + 1 \cdot x^1 + 1 \cdot x^0$

represents bit string 101011

CYCLIC REDUNDANCY CHECK II

Determine the quotient and remainder when the polynomial $x^5 + x^3 + x + 1$ is divided by 101 in modulo 2 arithmetic.

(dividend) (divisor)

```
 101011 ÷ 101 mod 2 = 10     (quotient)
-101 mod 2
   00
  -00 mod 2
    01
```

Polynomial $x^5 + x^3 + x + 1$

$= 1 \cdot x^5 + 0 \cdot x^4 + 1 \cdot x^3 + 0 \cdot x^2 + 1 \cdot x^1 + 1 \cdot x^0$

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$= 1 \cdot x^5 + 0 \cdot x^4 + 1 \cdot x^3 + 0 \cdot x^2 + 1 \cdot x^1 + 1 \cdot x^0$

represents bit string 101011

(dividend) (divisor)

```
 101011 ÷ 101 mod 2 = 100   (quotient)
-101 mod 2
----
   00
  -00 mod 2
  ----
   01
   -00 mod 2
   ----
     1
```

CYCLIC REDUNDANCY CHECK II

Determine the quotient and remainder when the polynomial $x^5 + x^3 + x + 1$ is divided by 101 in modulo 2 arithmetic.

Polynomial $x^5 + x^3 + x + 1$

$= 1 \cdot x^5 + 0 \cdot x^4 + 1 \cdot x^3 + 0 \cdot x^2 + 1 \cdot x^1 + 1 \cdot x^0$

represents bit string 101011

(dividend) (divisor)

```
 101011 ÷ 101 mod 2 = 100   (quotient)
-101 mod 2
 ----
   00
  -00 mod 2
  ----
    01
   -00 mod 2
   ----
    11
```

CYCLIC REDUNDANCY CHECK II

Determine the quotient and remainder when the polynomial $x^5 + x^3 + x + 1$ is divided by 101 in modulo 2 arithmetic.

Polynomial $x^5 + x^3 + x + 1$

$= 1 \cdot x^5 + 0 \cdot x^4 + 1 \cdot x^3 + 0 \cdot x^2 + 1 \cdot x^1 + 1 \cdot x^0$

represents bit string 101011

(dividend) (divisor)

```
 101011 ÷ 101 mod 2 = 1000 (quotient)
-101 mod 2
 ----
   00
  -00 mod 2
  ---
    01
   -00 mod 2
   ---
     11
    -00 mod 2
    ---
     11 (remainder)
```

IEEE-754 STANDARD

Convert the number 6.71875 to binary using the IEEE-754 standard. Assume you only have 15 bits in total. Specifically, 8 bits for the exponent and 6 bits for the significand.

0														
sign	exponent								significand					

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- Decimal to binary: $(6.71875)_{10} = (110.10111)_2$

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0														

sign exponent significand

- Decimal to binary: $(6.71875)_{10} = (110.10111)_2$
- The format for the significand using IEEE standard is 1.xxxx
- Hence, $110.10111 = 1.1010111 \times 10^2$ (the 1. is implicit)

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Convert the number 6.71875 to binary using the IEEE-754 standard. Assume you only have 15 bits in total. Specifically, 8 bits for the exponent and 6 bits for the significand.

sign	exponent								significand					
0									1	0	1	0	1	1

- Decimal to binary: $(6.71875)_{10} = (110.10111)_2$
- The format for the significand using IEEE standard is 1.xxxx
- Hence, $110.10111 = 1.1010111 \times 10^2$ (the 1. is implicit)
- Since significand is only 6 bits, we lose some precision

IEEE-754 STANDARD

Convert the number 6.71875 to binary using the IEEE-754 standard. Assume you only have 15 bits in total. Specifically, 8 bits for the exponent and 6 bits for the significand.

0									1	0	1	0	1	1
sign				exponent							significand			

- 1.1010111×10^2
- The bias is $2^{k\text{-}1}\text{-}1$ where k is the length of the exponent, hence, $2^{8\text{-}1}\text{-}1 = 127$

IEEE-754 STANDARD

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0									1	0	1	0	1	1
sign				exponent							significand			

- 1.1010111×10^2
- The bias is $2^{k-1}-1$ where k is the length of the exponent, hence, $2^{8-1}-1 = 127$
- The exponent is the bias plus the true exponent, hence, $127+2 = 129$

IEEE-754 STANDARD

Convert the number 6.71875 to binary using the IEEE-754 standard. Assume you only have 15 bits in total. Specifically, 8 bits for the exponent and 6 bits for the significand.

0	1	0	0	0	0	0	0	1	1	0	1	0	1	1
sign				exponent							significand			

- 1.1010111×10^2
- The bias is $2^{k\text{-}1}\text{-}1$ where k is the length of the exponent, hence, $2^{8\text{-}1}\text{-}1 = 127$
- The exponent is the bias plus the true exponent, hence, $127+2 = 129$
- Decimal to binary: $(129)_{10} = (10000001)_2$

IEEE-754 STANDARD

Convert the number 6.71875 to binary using the IEEE-754 standard. Assume you only have 15 bits in total. Specifically, 8 bits for the exponent and 6 bits for the significand.

0	1	0	0	0	0	0	0	1	1	0	1	0	1	1
sign				exponent							significand			

- Conversion of result back to decimal:

$1.101011 \times 10^2 = (110.1011)_2 = (6.6875)_{10}$

HAMMING CODES I

In Hamming codes with data words of length 125, what is the minimum number of check/parity bits needed for correcting single-bit errors?

How long will the resulting code words be?

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How long will the resulting code words be?

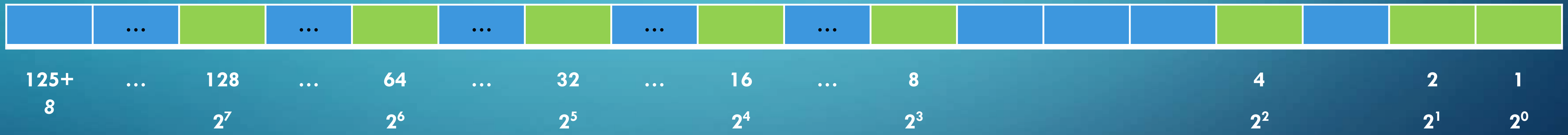
	…	?	…		…		…		…								
125+*r*	…	128?	…	64	…	32	…	16	…	8				4		2	1
		2^7?		2^6		2^5		2^4		2^3				2^2		2^1	2^0

Each bit whose position is a power of two is a parity bit.

HAMMING CODES I

In Hamming codes with data words of length 125, what is the minimum number *r* of parity bits needed for correcting single-bit errors?

How long will the resulting code words be?



Each bit whose position is a power of two is a parity bit.

$r = 8, \; 125 + 8 = 133$

HAMMING CODES I

In Hamming codes with data words of length $n = 125$, what is the minimum number r of parity bits needed for correcting single-bit errors?

How long will the resulting code words be?

	…		…		…		…		…								
125+8	**…**	**128**	**…**	**64**	**…**	**32**	**…**	**16**	**…**	**8**				**4**		**2**	**1**
		2^7		2^6		2^5		2^4		2^3				2^2		2^1	2^0

Each bit whose position is a power of two is a parity bit.

$r = 8,\ \ 125 + 8 = 133$

number of parity bits is the smallest number r that makes $(n + r + 1) \leq 2^r$ true

HAMMING CODES II

What is the hamming code word of the data word 10100101 using even parity?

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$n = 8$

smallest r for which $(8 + r + 1) \leq 2^r$ is true is 4

Hence, total number of bits is 12

HAMMING CODES II

What is the hamming code word of the data word 10100101 using even parity?

12	**11**	**10**	**9**	**8**	**7**	**6**	**5**	**4**	**3**	**2**	**1**

$n = 8$

smallest r for which $(8 + r + 1) \leq 2^r$ is true is 4

Hence, total number of bits is $8 + 4 = 12$

HAMMING CODES II

What is the hamming code word of the data word 10100101 using even parity?

12	11	10	9	8	7	6	5	4	3	2	1

$n = 8$

smallest r for which $(8 + r + 1) \leq 2^r$ is true is 4

Hence, total number of bits is 12

HAMMING CODES II

What is the hamming code word of the data word 10100101 using even parity?

1	0	1	0		0	1	0		1		
12	11	10	9	8	7	6	5	4	3	2	1

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What is the hamming code word of the data word 10100101 using even parity?

1	0	1	0		0	1	0		1		
12	11	10	9	8	7	6	5	4	3	2	1

Hamming algorithm:

Position 1: check 1 bit, skip 1 bit, …

set parity bit to 1 if number of checked bits that are 1 is odd

set parity bit to 0 otherwise

HAMMING CODES II

What is the hamming code word of the data word 10100101 using even parity?

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12	11	10	9	8	7	6	5	4	3	2	1

Hamming algorithm:

Position 2: check 2 bits, skip 2 bits, …

set parity bit to 1 if number of checked bits that are 1 is odd

set parity bit to 0 otherwise

HAMMING CODES II

What is the hamming code word of the data word 10100101 using even parity?

1	0	1	0		0	1	0		1	1	1
12	11	10	9	8	7	6	5	4	3	2	1

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Hamming algorithm:

Position 4: check 4 bits, skip 4 bits, …

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HAMMING CODES II

What is the hamming code word of the data word 10100101 using even parity?

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HAMMING CODES II

What is the hamming code word of the data word 10100101 using even parity?

1	0	1	0		0	1	0	0	1	1	1
12	11	10	9	8	7	6	5	4	3	2	1

Hamming algorithm:

Position 8: check 8 bits, skip 8 bits, …

set parity bit to 1 if number of checked bits that are 1 is odd

set parity bit to 0 otherwise

HAMMING CODES II

What is the hamming code word of the data word 10100101 using even parity?

1	0	1	0	0	0	1	0	0	1	1	1
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Hamming algorithm:

Position 8: check 8 bits, skip 8 bits, …

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BINARY ADDITION

Consider the following 4-bit additions of binary values in two's complement notation. Select all additions where an overflow occurs.

- 1101 + 1111
- 0010 + 0101
- 1001 + 0111
- 1001 + 1010

BINARY ADDITION

Consider the following 4-bit additions of binary values in two's complement notation. Select all additions where an overflow occurs.

Rule for detecting signed two’s complement overflow: When the “carry in” and the “carry out” of the sign bit differ, overflow has occurred. If the carry into the sign bit equals the carry out of the sign bit, no overflow has occurred

BINARY ADDITION

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- 1101 + 1111
- 0010 + 0101
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- 1001 + 1010

```
 1 1 1 1
   1101
+  1111
-------
   1100
```

carry in and carry out of sign bit the same, no overflow

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- 1101 + 1111
- 0010 + 0101
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```
 0000
  0010
+ 0101
------
  0111
```

carry in and carry out of sign bit the same, no overflow

BINARY ADDITION

Consider the following 4-bit additions of binary values in two's complement notation. Select all additions where an overflow occurs.

- 1101 + 1111
- 0010 + 0101
- 1001 + 0111
- 1001 + 1010

```
 1 1 1 1
   1001
+  0111
-------
   0000
```

carry in and carry out of sign bit the same, no overflow

BINARY ADDITION

Consider the following 4-bit additions of binary values in two's complement notation. Select all additions where an overflow occurs.

- 1101 + 1111
- 0010 + 0101
- 1001 + 0111
- 1001 + 1010

```
 1000
  1001
+ 1010
------
  0011
```

carry in and carry out of sign bit differ, overflow

DISK CAPACITY

A disk manufacturer sells a disk that it claims has a capacity of 90 gigabytes.

Compared to a 2 megabyte drive that the company used to manufacture, how many times will the 90 gigabyte drive be bigger than the 2 megabyte drive?

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- 90 GB = 90×1 billion bytes = 90×10^9 bytes

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- 2 MB = 2 × 1 million bytes = 2×10^6 bytes

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- 90 GB = 90 × 1 billion bytes = 90×10^9 bytes
- 2 MB = 2 × 1 million bytes = 2×10^6 bytes
- $\frac{90 \times 10^9}{2 \times 10^6} = \frac{90\,000\,000\,000}{2\,000\,000} = 45\,000$

DISK SIZE

You buy a 15 terabyte disk. Assuming that all the space on the disk is available to the operating system, what will the size of the disk be as reported by the operating system in gigabytes?

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  - 15 TB = 15×10^{12} bytes = 15 000 000 000 000 bytes
- Operating systems use binary interpretation of storage prefixes
  - 1 GB = 1×2^{30} bytes = 1 073 741 824 bytes

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- For hard disks the decimal interpretation of storage prefixes is used
  - 15 TB = 15×10^{12} bytes = 15 000 000 000 000 bytes
- Operating systems use binary interpretation of storage prefixes
  - 1 GB = 1×2^{30} bytes = 1 073 741 824 bytes
- $\frac{15 \times 10^{12}}{1 \times 2^{30}} = 13970$ (rounded to integer)