COS 210 WORKSHEET 1

Scott Bebington
u21546216

Question 1

- Let n be a natural number. If n^2 is even, then n is even

$n \in N$, $n \neq 0$

n^2 is even $\rightarrow$ n is even

Proof

Assume n^2 is even and n is odd

$\rightarrow$ $n = 2k + 1$ for some k

$n^2 = (2k+1)^2$

$n^2 = 4k^2 + 4k + 1$

$n^2 = 2(2k^2 + 2k) + 1$

$\rightarrow 2(2k^2 + 2k)$ is even

$\rightarrow 2(2k^2 + 2k) + 1$ is odd

therefor n^2 is odd, this is a contradiction to the assumption that n^2 is even

This proves that if n is even, n^2 will be even.

Question 2

The number $\sqrt{2}$ is irrational

$\sqrt{2} \notin \mathbb{Q}$

Proof
$\sqrt{2} \in \mathbb{Q}$

$\rightarrow \sqrt{2} = \frac{m}{b}$
$\rightarrow 2 = m^2 / b^2$
$\rightarrow 2b^2 = m^2$

$2b^2$ must be an even number as $2k$ is an even number, this makes m^2 an even number aswell as it is = to $2b^2$.

because they are both even they share a common factor of 2.

because a rational number can be written as ~~m~~ $\frac{m}{b}$ with the greatest ~~comon~~ common divisor being 1, we proved that $\sqrt{2}$ is irrational.

Question 3

T: $\forall n \geq 1: 6^n - 1 \% 5 = 0$

Base Case:

$n=1: \quad 6^n - 1 = 6^1 - 1 = 5$

$\to 5/5 = 0 \to 5$ is divisible by 5

Inductive step

let $k = n$

$6^k - 1 = 5a$ for some value of a

expanding the left side
for $n = k+1$

$\to 6^{k+1} - 1$

$= \cancel{6}(6 \times 6^k - 1)$

$= (6 + 6^k) - 1$

$= (5 \times 6^k) + 6^k - 1$

$= 5(6^k) + (6^k - 1)$

$= 5(6^k) + 5a$ as per the inductive hypothesis

$= 5(6^k + a)$

as you can divide $5(6^k + a)$ ~~for 5~~ by 5 ~~not~~ without a remainder we have proved by induction that $6^k - 1$ is divisible for all $n >= 1$.