Constructing Steady States Of Multispecies Size Spectrum Models

Richard Southwell

February 1, 2018

We have a multispecies size spectrum model as described in the mizer vignette except

$$\psi_i(w) = \begin{cases} 0 & \text{if } w < w_{*i} \\ \left(\frac{w}{W_{\infty.i}}\right)^{1-n} & \text{otherwise} \end{cases} \tag{1}$$

we also suppose our parameters are such that
$n = p$
$\lambda = q + 2 - n$
$\mu_f(w) = k = 0$
$\theta_{ij} = 1$
Given that, here is my attempted recipe for constructing a steady state for some size spectrum setups:
Choose any value of $\mu_0 > 0$.
We explicitly model s species, which may have any different species specific parameters. For each species i let

$$f_{0i} = \frac{\gamma_i}{\frac{h_i \beta_i^{2-\lambda} \exp(-(\lambda-2)^2\sigma_i^2/2)}{\sqrt{2\pi}\kappa\sigma_i} + \gamma_i} \tag{2}$$

be the feeding level (as experienced by species i individuals feeding upon a community at abundance $\kappa w^{-\lambda}$) , and let $\hbar_i = \alpha_i h_i f_{0i} - k_{si}$.

Let $N_i(w)$ be a steady state of the MvF equation of species i with growth rate $\hbar_i w^n(1 - \psi_i(w))$ and death rate $\mu_0 w^{n-1}$. In particular

$$N_i(w) = \frac{1}{\hbar_i w^n}(w_{ei}/w)^{\mu_0/\hbar_i} \times \begin{cases} 1 & w < w_{*i} \\ \frac{\left(1-(w/W_{\infty_i})^{1-n}\right)^{(\mu_0/(\hbar_i(1-n)))-1}}{(1-(w_{*i}/W_{\infty i})^{1-n})^{\mu_0/(\hbar_i(1-n))}} & w > w_{*i} \end{cases} \tag{3}$$

Any multiple of $N_i(w)$ is also a steady state of the Mvf, and for our community construction, we can choose any abundance multipliers $A_i \geq 0$ and $A_i N_i(w)$ is also a steady state of the Mvf. We suppose that the abundance of background resource at weight w is $N_R(w) = \kappa w^{-\lambda} - \sum_{i=1}^{s} A_i N_i(w)$. In this resulting system, where species i has an abundance $A_i N_i(w)$ at weight w, and the background resource abundance is present at abundance $N_R(w)$, let $\mu_{pi}(w)$ denote the predation mortality level on species i from the explicitly modeled species $j \in 1,..,s$. Now if we choose a background mortality rate of $\mu_{bi}(w) = \mu_0 w^{n-1} - \mu_{p.i}(w)$ for the ith species, then its total mortality rate will be $\mu_i(w) = \mu_{p.i}(w) + \mu_{b.i}(w) = \mu_0 w^{n-1}$. Including the background resources and death term like this should result in the same growth and death terms as we assumed in construction of our abundance curves, so this should lead to a steady state of the Mvf. We also have to choose parameters e.g., the reproductive efficiencies ϵ_i so that the reproduction boundary condition

$$g_i(w_{e.i})N_i(w_{e.i}) = \frac{\epsilon_i}{2w_{e.i}} \int_0^\infty N_i(w)E_{r.i}(w)\psi(w).dw \tag{4}$$

is met for each species. This should result in a steady state of the size spectrum dynamics.

The mizer code [stable_community_gurnard_really.R] we did to include the gurnard corresponds to this type of setup where

$\mu_0 = (1 - f_{0B})\sqrt{2\pi}\kappa\gamma_B\sigma_B\beta_B^{n-1}\exp(\sigma_B^2(n-1)^2/2)$

in this case the the first $s-1$ species are the background species so $\beta_1 = .. = \beta_{s-1} = \beta_B \neq \beta_s$ etc, and the sth and final species is the gurnard with its own different parameters. Except this code has plankton dynamics, and we cut the plankton off at the first zero. We still need to test if we cut the cut the background mortality term off, will it still work.

More generally if we have two mizer steady states X and Y of different systems with different parameters if we can choose a subset x of species from X and a subset y of species from Y then we can sometimes transplant/swap populations x and y around in their ecosystems, (cutting their abundances our and swapping them round). The conditions required for the resulting two "post transplant" states to be steady states are:

(1) members of x expecience the same growth and death rate as a result of their contact with $X \setminus x$ before the tranplant, as they experience with the new surroundings (post-transplant) via their contact with $Y \setminus y$.

(2) members of $X \setminus x$ expecience the same growth and death rate as a result of their contact with x before the tranplant, as they experience with the new surroundings (post-transplant) via their contact with y.

(3) members of y expecience the same growth and death rate as a result of their contact with $Y \setminus y$ before the tranplant, as they experience with the new surroundings (post-transplant) via their contact with $X \setminus x$.

(4) members of $Y \setminus y$ expecience the same growth and death rate as a result of their contact with y before the tranplant, as they experience with the new surroundings (post-transplant) via their contact with x.