The Effect Of Discarding On Fish Size Spectra

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Introduction

In many places, regulations have encouraged fishermen to discard fish which have a length below a *minimum landing length.* The large amount of dead fish discarded in this way has led to a movement to ban discarding. On the surface of things, discarding seems to be wasteful, however it is important to be aware that it can lead to a significant re-injection of nutrients and energy into the the ecosystem, since dead fish act as food for fish and other organisms. We extend the size spectrum ecological modeling software package 'mizer' so that it can be used to model discarding. In this report we describe the modeling assumptions we made, how we included them in the mizer package, and what results we obtained from our models.

Governing Equations

The mizer modelling software we build upon uses the McKendrik-von Foerster equation to model the abundance of different species of fish of different weights. The underlying assumptions are detailed in the mizer vignette:

https://cran.r-project.org/web/packages/mizer/vignettes/mizer_vignette.pdf

and we shall only discuss the extra features we have added to this somewhat intricate mathematical modelling framework. Mizer can keep track of the abundance of plankton (referred to as background resource), and several species of fish of different sizes. In order to model the effect of discarding we added a new type of entity that we wish to keep track of, which we call 'dead fish'. We let $N_d(w)$ denote the abundance of dead fish of weight w. Here $N_d(w).dw$ denotes the number of dead fish (per unit volume) which a weight in $[w, w + dw]$. We assume that the abundance of dead fish obeys the partial differential equation:

$$\frac{\partial N_d(w)}{\partial t} = \frac{\partial (g_d N_d(w))}{\partial w} - \mu_d(w) N_d(w) + I(w), \tag{1}$$

where the sum is over the different species j, and:

- $g_d \geq 0$ is the *disintigaration rate* of dead fish, which we assume to be constant over time and weight.
- $\mu_d(w)$ is annihilation rate of dead fish given by the following equation, that takes a similar form to (3.12) from the mizer vignette, under the assumption that all species have equal preference for dead fish:

$$\mu_d(w) = \sum_j \int_0^\infty \phi_j(w/w_p)(1 - f_j(w_p))\gamma_j {w_p}^q N_j(w_p) dw_p. \tag{2}$$

- $I(w) = \sum_j N_j(w) D_j(w) \mu_{F.j}(w)$ is the influx of dead fish from fishing discards.
- $D_j(w) \in [0, 1]$ is the fraction of the caught fish of species j, and weight w which are discarded, we assume the it takes the form:

$$D_j(w) = \begin{cases} 1, & \text{if } w < \text{minimum landing weight} \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

- $\mu_{F.j}(w)$ is the fishing mortality rate of fish with species j, and weight w, we assume it takes the form

$$\mu_{F.j}(w) = (\text{effort}) \times (\text{catchability}) \times S_j(w). \tag{4}$$

Where $S_j(w)$ is the selectivity of our fishing gear for size w members of species j.

In this report we just consider the case where we have a single species, Whiting, and we suppose (effort)=(catchability)=1. We take the parameters for the Whiting and background resources from the *north sea model* encoded within mizer.

Downwind Difference Scheme

The partial differential equation governing the abundance of dead fish is like a reversal of the McKendrik-von Foerster equation underlying mizer. In mizer, an upwind difference scheme is used to solve the McKendrik-von Foerster equation, where the amount of new eggs produced corresponds to a left hand boundary condition. In a similar way we use a *downwind* scheme to solve the partial differential for the number of dead fish, using the right hand boundary condition that there are no dead fish of maximum size.

In order to set this scheme up we use evenly spaced discrete time steps so that $t = T \times \Delta t$ is the Tth time step. We also suppose that the size bins are spaced non-linearly so that the kth size bin is $w_k = w_0 \exp((k-1)(\Delta x))$, where $\Delta x > 0$ is a constant, and w_0 is the egg weight. The final bin is the maximum fish weight $W = w_K$. The non-linear spacing of the weight/size bins means that the kth step size $\Delta w_k = w_k - w_{k-1}$, varies with k.

We implement a downwind difference scheme in a way analogous to that discussed in Appendix G of *Food web framework for size-structured populations*,

https://arxiv.org/pdf/1004.4138.pdf

The scheme we use is described by the equation:

$$\frac{v_k^{T+1} - v_k^T}{\Delta t} = g_d \left(\frac{v_{k+1}^{T+1} - v_k^{T+1}}{\Delta w_{k+1}} \right) - \bar{\mu}_k^T v_k^{T+1} + I_k^T, \tag{5}$$

where

- v_k^T denotes the dead fish abundance $n_d(w_k)$, on the Tth time step.
- $\bar{\mu}$ denotes the predation based annihilation rate $\mu_d(w[k])$ on the Tth time step.
- I_k^T denotes the dead fish influx $I(w_k)$ on the Tth time step.

Letting

$$A_k = \frac{-g_d \Delta t}{\Delta w_{k+1}} \tag{6}$$

$$B_k = 1 + \frac{g_d \Delta t}{\Delta w_{k+1}} + \Delta t \bar{\mu}_k^T \tag{7}$$

$$C_k = v_k^T + \Delta t I_k^T \tag{8}$$

we can rearrange our difference equation into a form:

$$v_k^{T+1} = \frac{C_k - A_k v_{k+1}^{T+1}}{B_k}, \tag{9}$$

which we can use to determine the values v_k^{T+1} by working downwards through the k indexes, using our right hand boundary condition that $v_K^{T+1} = 0$, $\forall T$.

Parameter Settings

Our minimum discard weight is 250 grams (g). We assume that our selectivity function $S(w)$ has a sigmoidal form. We can rewrite this as a function $S^*(l) = S(w)$ of length l (in cm), where length and weight are related by the equation $w = al^b$, where for our species *whiting*, the North sea model gives parameter values $a = 0.006$, and $b = 3.080$. In terms of length, our selectivity function takes the form

$$S^*(l) = \frac{1}{1 + \exp(S_1 - S_2 l),}, \tag{10}$$

which smoothly varies from 0 to 1. Here the parameter values are $S_2 = \log(3)/(L_{50} - L_{25})$ and $S_1 = L_{50}S_2$, where in our case $L_{25} = 26cm$, is the length of fish that have a 25% chance of being selected for by the fishing gear, and $L_{50} = 29cm$, is the length of fish that have a 50% chance of being selected for by the fishing gear.

During the simulations we encountered problems because, although the selectivity $S(w)$ approaches zero as w approaches the egg size w_0, the abundance $N(w)$ of fish at such a weight becomes very large, and so the influx $I(w) = S(w)D(w)N(w) = \mu_F(w)D(w)N(w)$ (which has this form because effort and catchability are assumed to be 1), becomes unnaturally large when w is small. To avoid this effect we flatten out the left side of our sigmoid curve, and whenever $S(w) < 0.01$, we instead suppose that $S(w) = 0$.

The other parameter we need to select is the disintegration rate g_d. We were unsure how to select this parameter, and so we tried several different values.

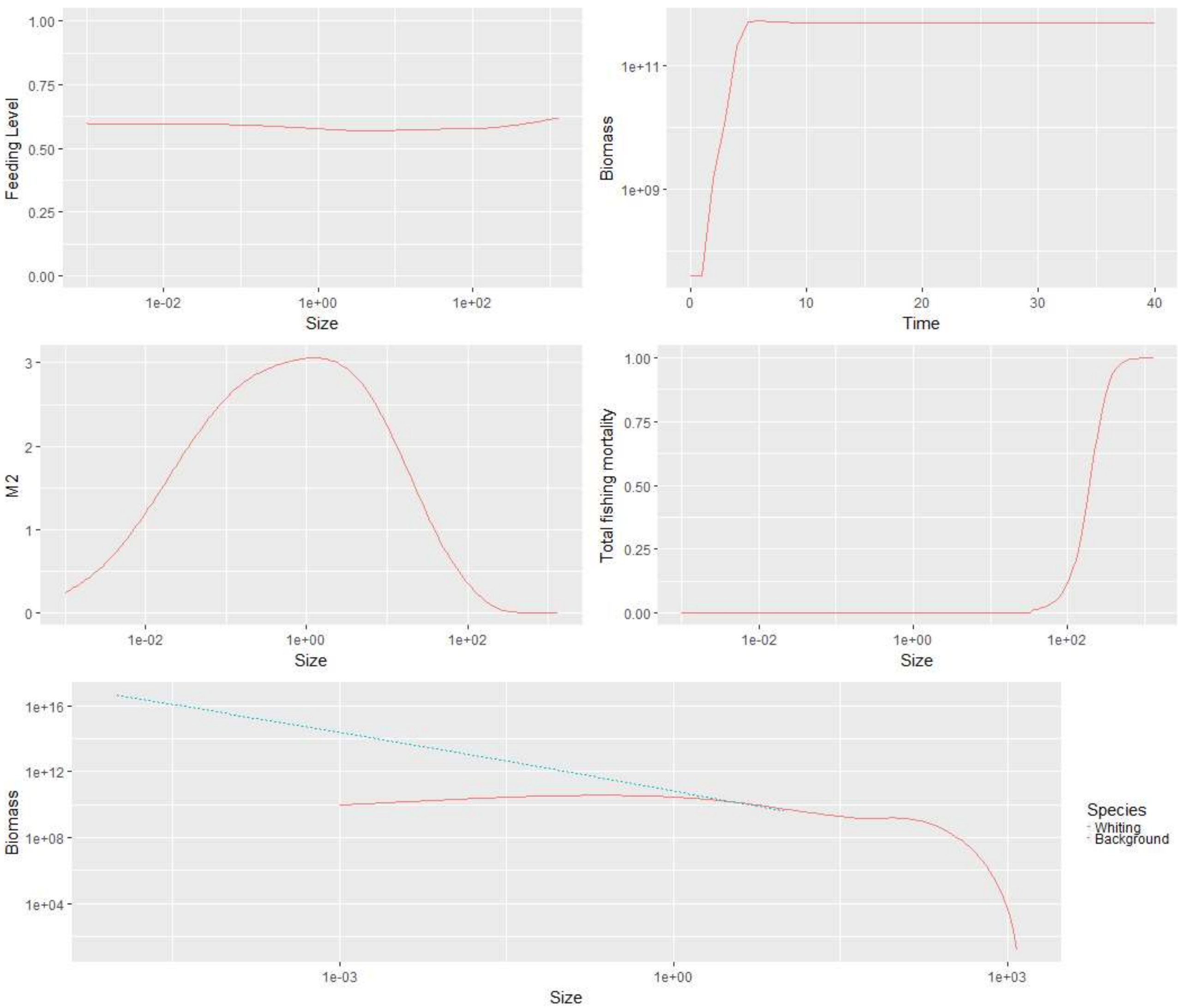
Simulation Results

The R code for this work is in the github repository

https://github.com/gustavdelius/mizer/tree/discard

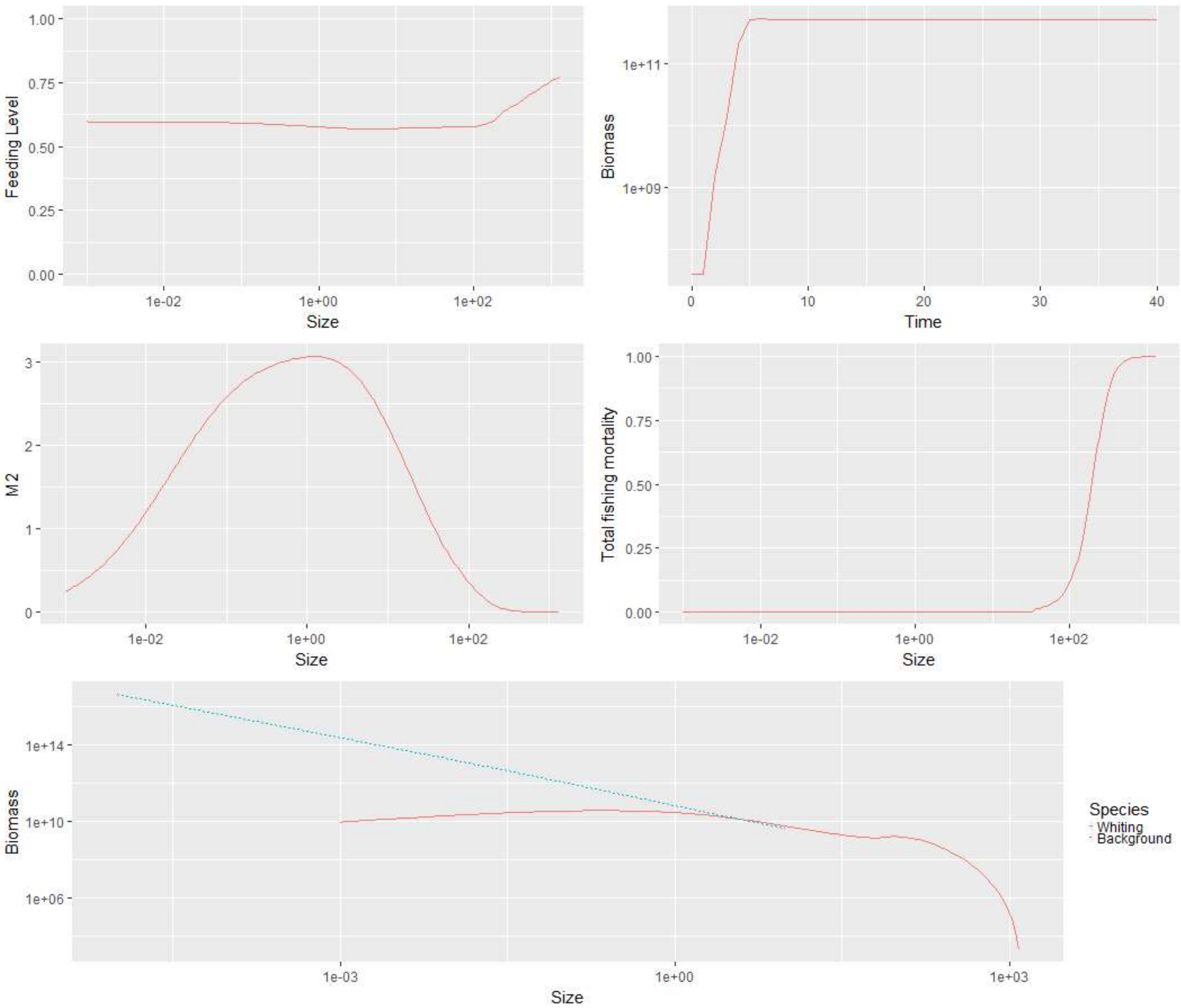
from here, it can be loaded by running the file BasicTestSingleWhitingMD.R

Mizer sets up the initial whiting spectrum, and underlying resource spectrum according to a power-law. Running the system without discarding for an amount of time corresponding to 40 years yields the basic results:



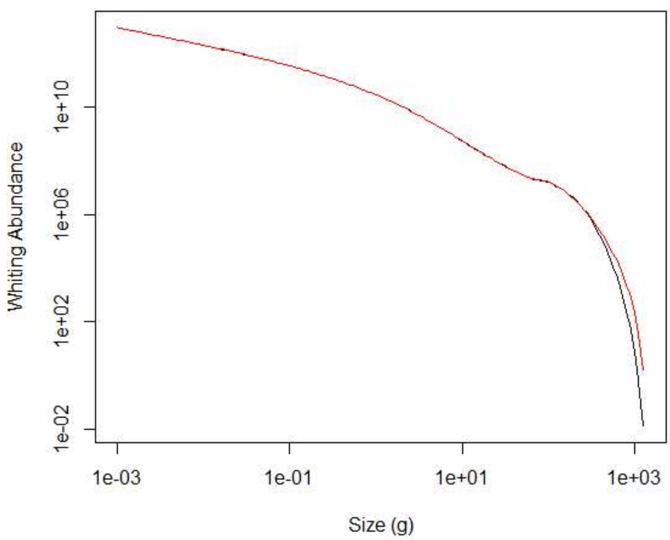
The system seems to be attracted to a fixed point where the biomass curve is mostly flat.

Now let us run the simulation again, with discarding, for now let us set the disintegration rate $g_d = 1$.

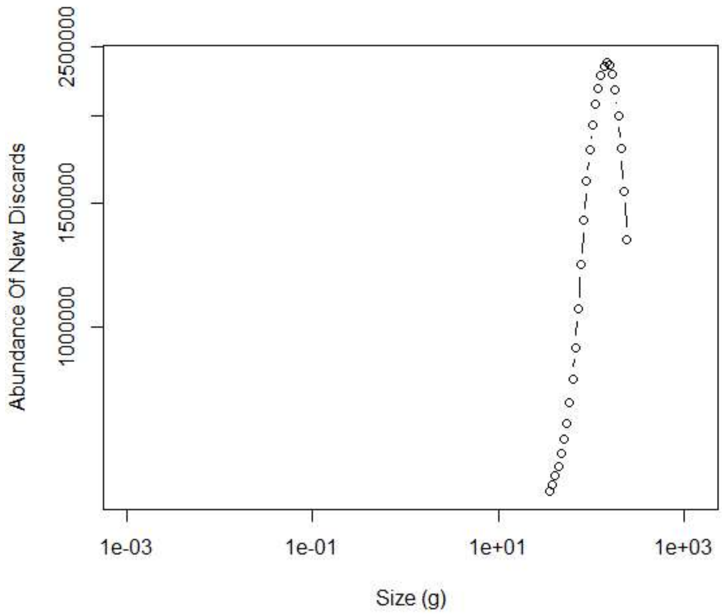


The results shown above (where fish caught under 250 g are discarded), look similar to those without discarding, however there are differences. The feeding level and biomass tend to be higher for larger fish when discarding is included.

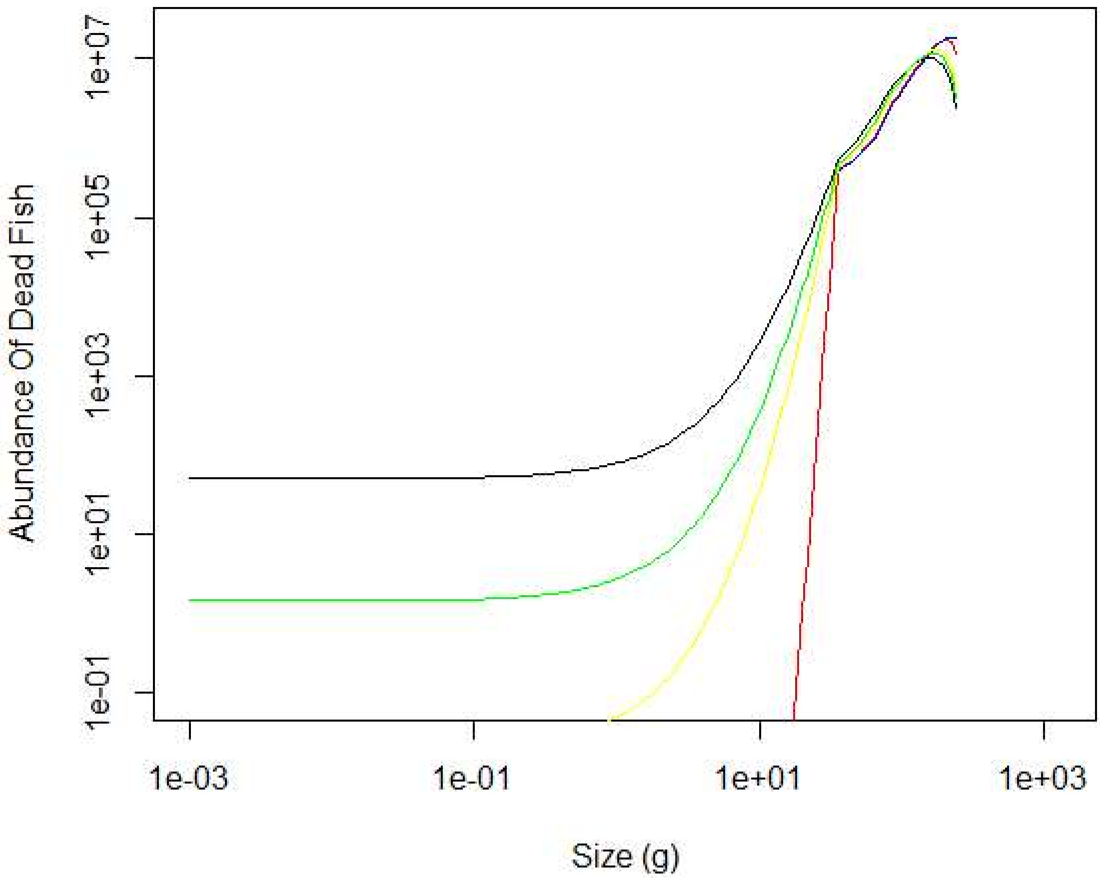
Comparing the final abundance of whiting in these two situations, we can see confirmation of this (here the red line is for the case where discarding is included):



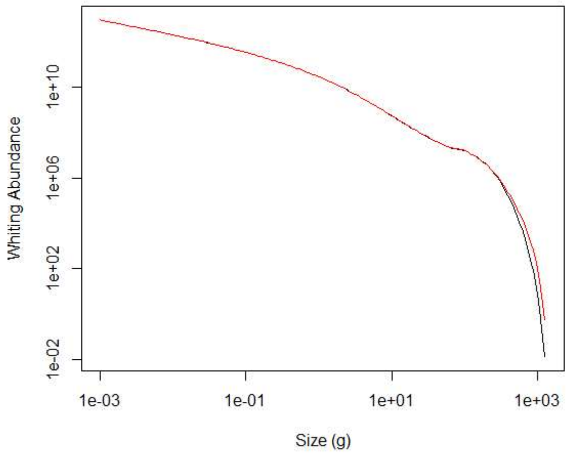
Let us look at more plots to increase our understanding of the dynamics of discarding. Below we show a plot on the influx I(w) of dead fish due to discarding. This appears to reach a steady state with time.



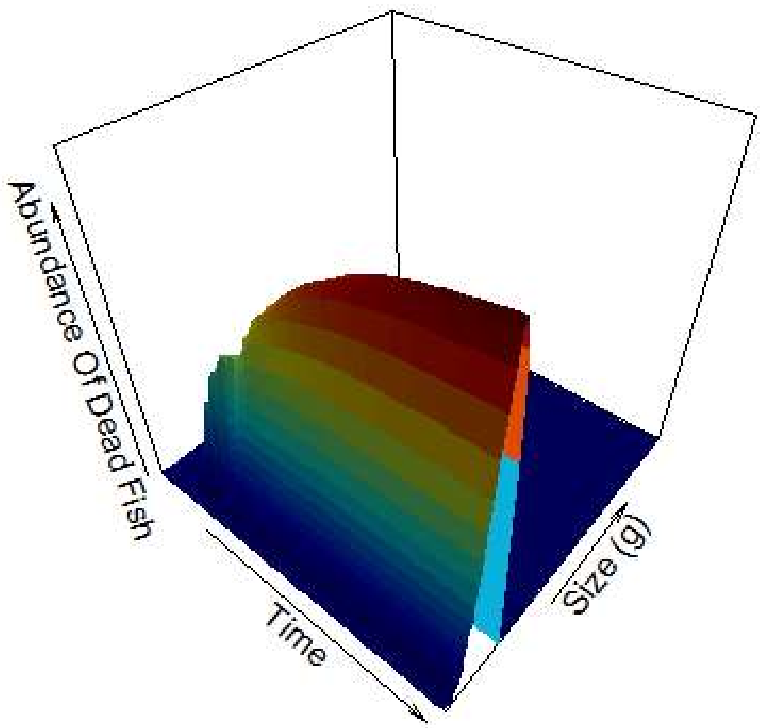
The drop to zero on the left happens because few fish below a given size are caught. The drop off on the right happens because no fish above 250 g are discarded. The final abundance of dead fish is plotted below:



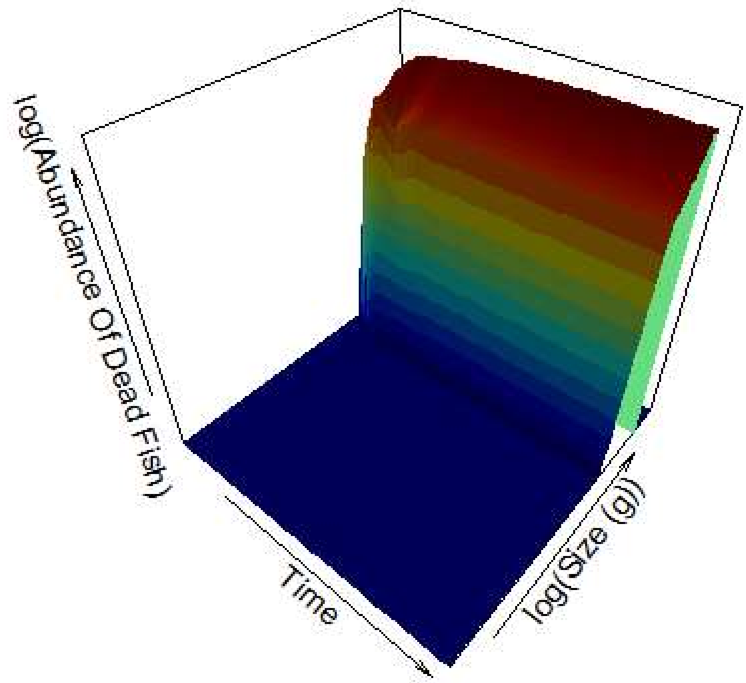
Here the blue, red, yellow, green and black curves show the final abundance of dead fish when the disintegration rate is 0, 1, 5, 7 and 10, respectively. It appears that there is a critical disintegration rate somewhere between 5 and 7, beyond which the amount of small dead fish build up significantly. It is worth bearing in mind that the disintegration can move masses of dead fish below the fish egg size, and such mass will no longer be able to influence our system. This above plot shows how the disintegration rate can have a significant effect upon the qualitative nature of our output, so it is worth checking how discarding effects abundance in the regime where the disintegration rate is high (equal to 10).
Below made another plot showing the difference that discarding makes to the final abundance of whiting in this regime where the disintegration rate is large (red is for the case with discarding):



Another important issue is how the abundance of dead fish changes over time. Below we provide a plot for this, for the case where the disintegration rate is 1.



Here all axes have linear scale/ This shows good evidence that the abundance of dead fish is reaching a stationary distribution.
The 3D plot for the case where the disintegration rate is higher looks similar to the above, because the linear scale is inappropriate for showing the build up of small dead fish. In order to see this comparison in 3D. In order to do this we first plot the same date as above (with disintegration rate 1), but we take the logs of the size and abundance:



Below we make a similar plot when the disintegration rate is 10: