Coexistence In Size Spectrum Models

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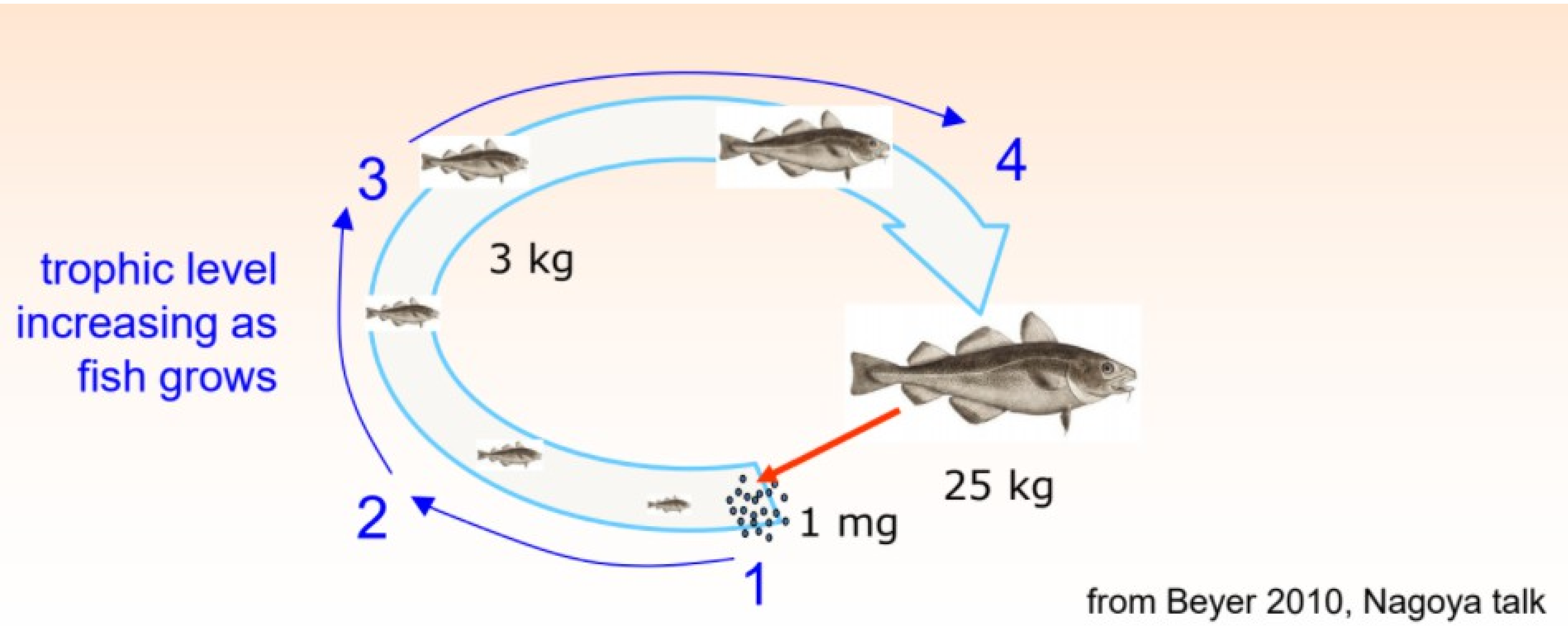
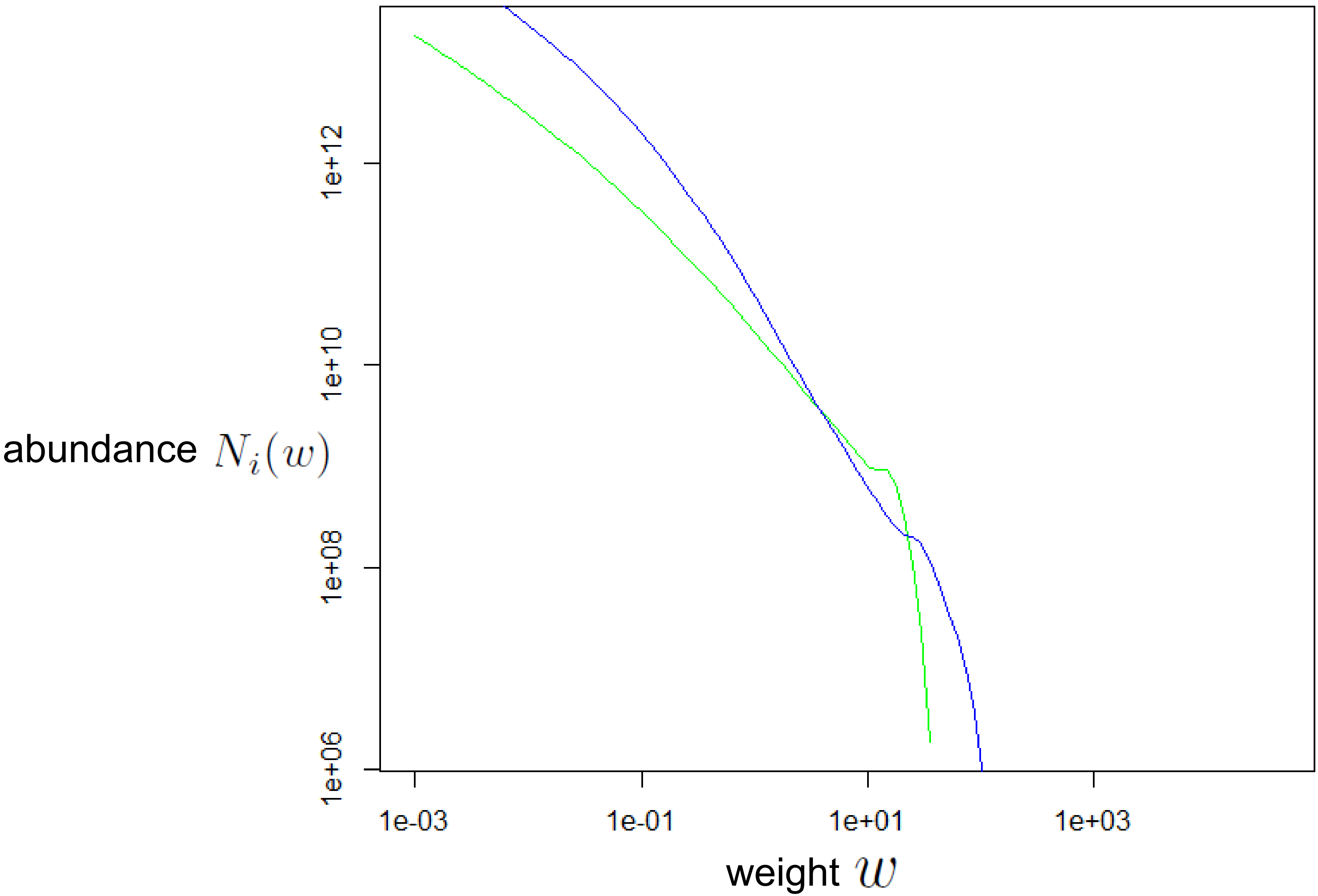


trophic level
increasing as
fish grows
3
4
3 kg
25 kg
2
1 mg
1
from Beyer 2010, Nagoya talk



abundance $N_i(w)$
1e+12
1e+10
1e+08
1e+06
1e-03
1e-01
1e+01
1e+03
weight w

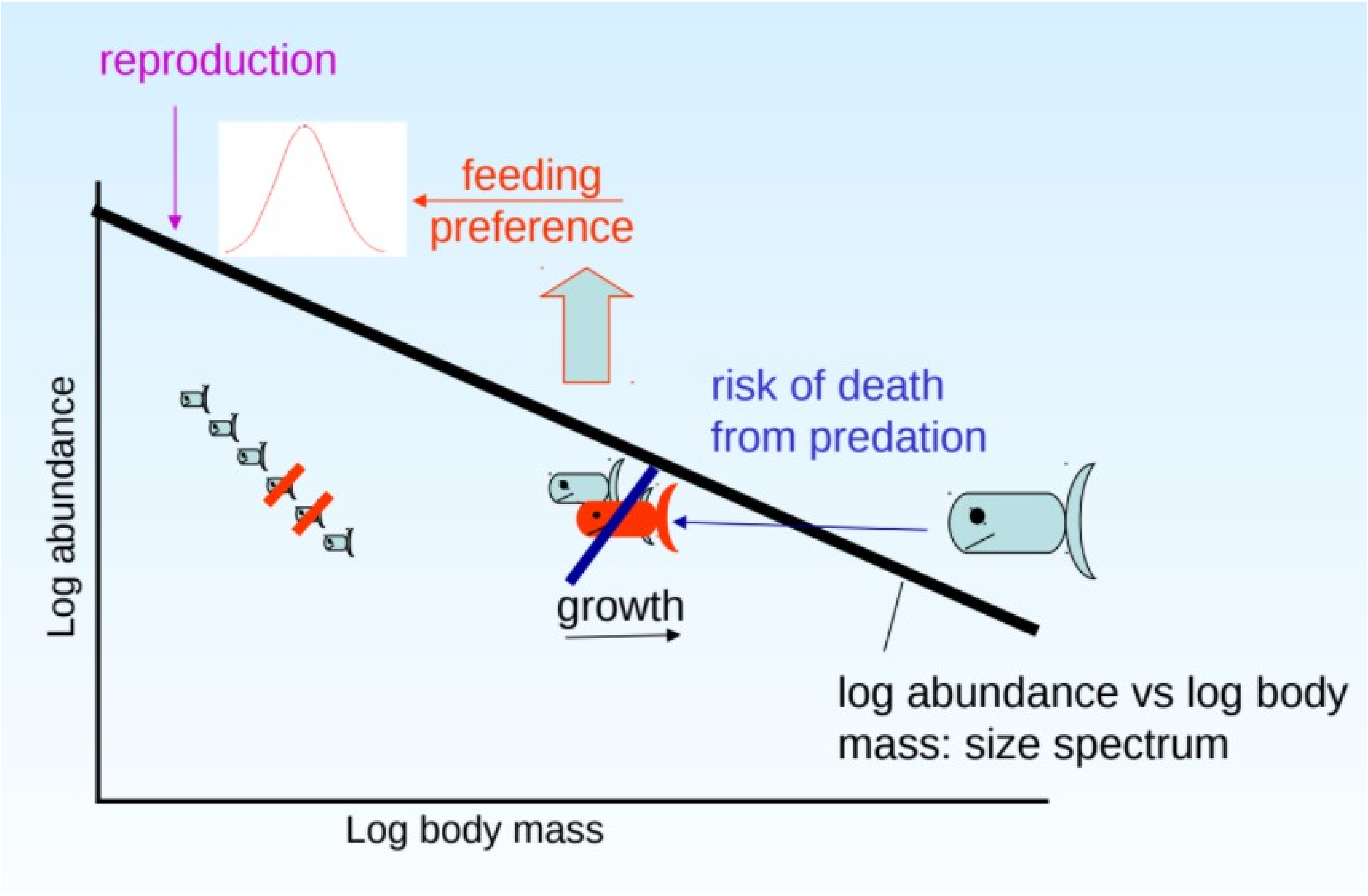
McKendrik-von Foerster equation

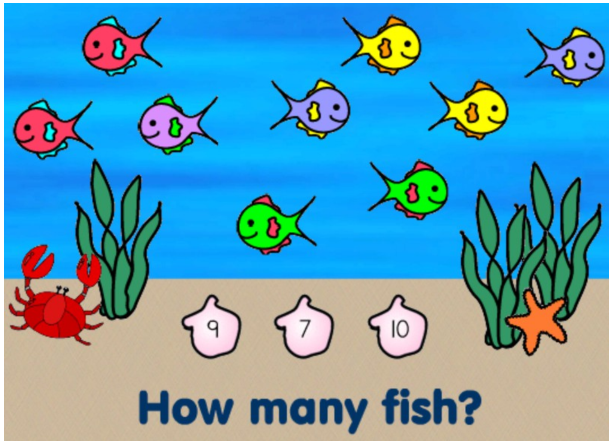
density of weight w individuals of species i

growth rate

$$\frac{\partial N_i(w)}{\partial t} = -\frac{\partial[N_i(w)g_i(w)]}{\partial w} - \mu_i(w)N_i(w)$$

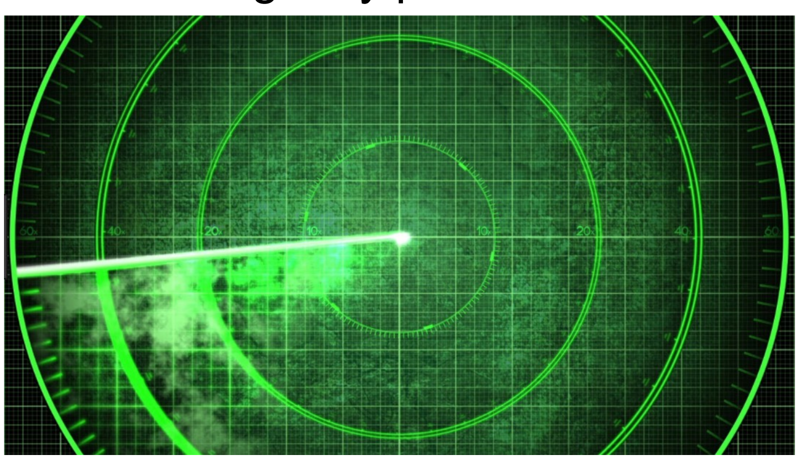
death rate


reproduction
feeding
preference
risk of death
from predation
Log abundance
growth
log abundance vs log body
mass: size spectrum
Log body mass



sum over prey

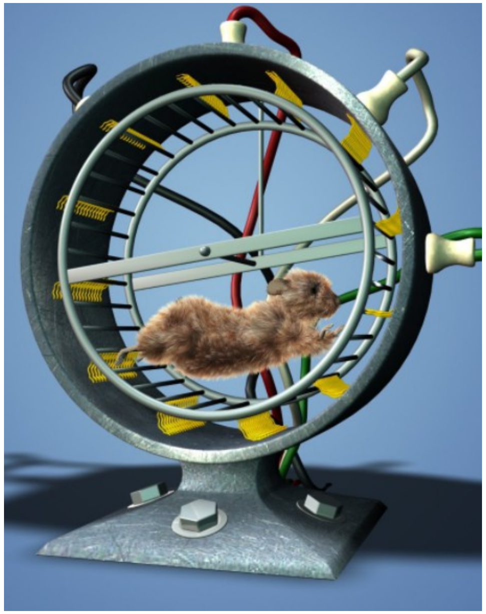
weight by preference



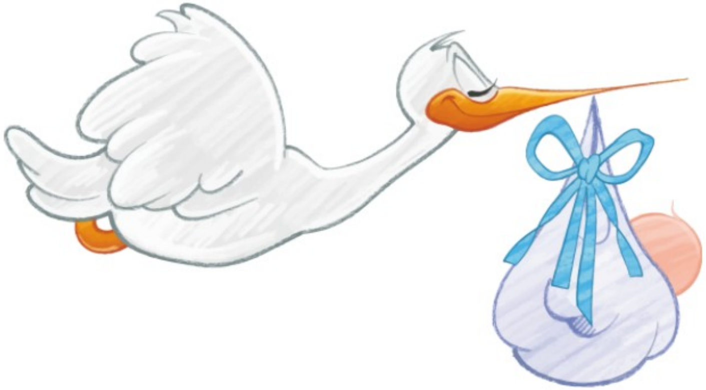
multiply by search rate of predator

energy encountered

limited eating rate

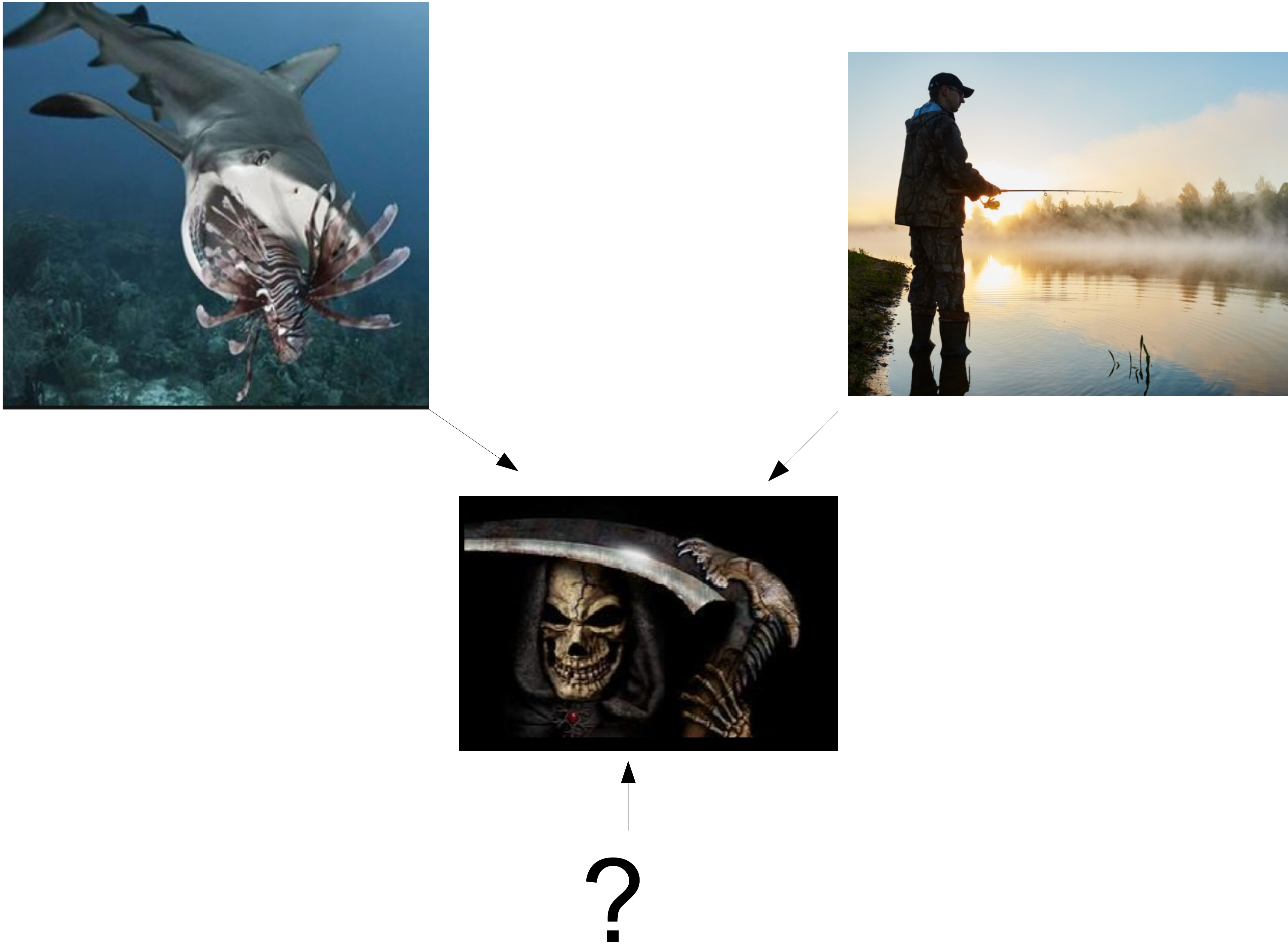


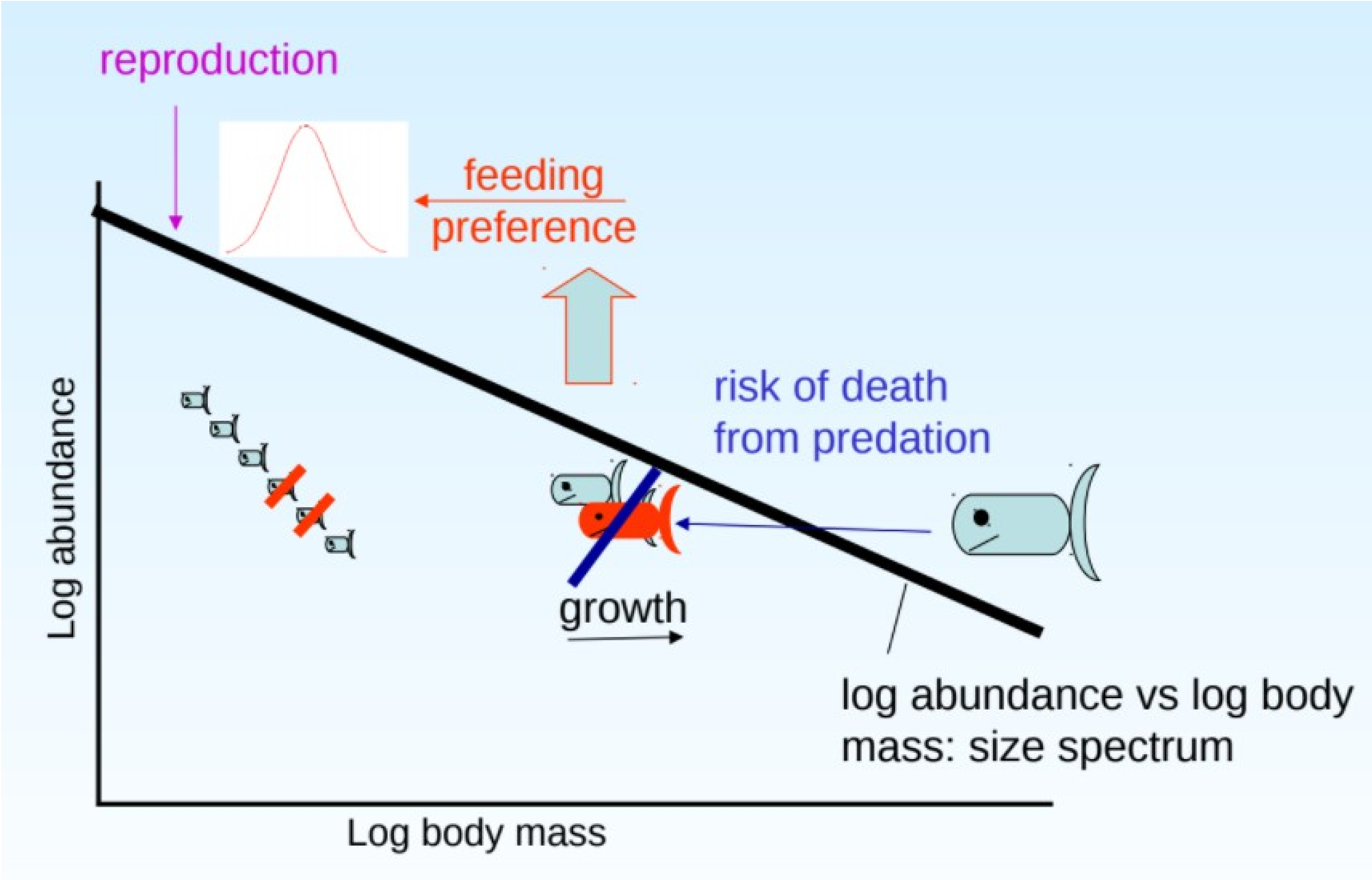
energy costs for movement and metabolism



energy for reproduction

growth



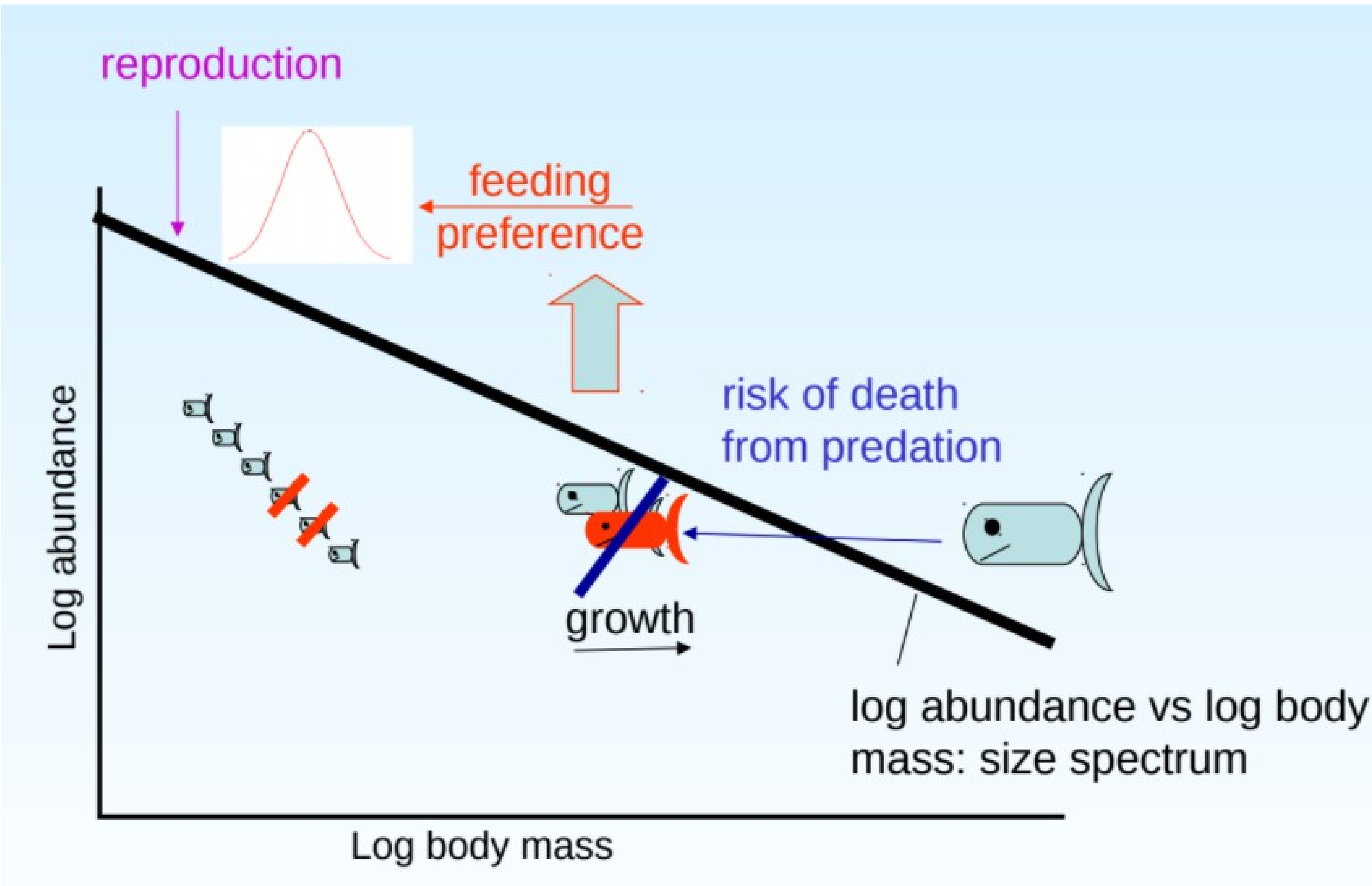

reproduction
feeding
preference
risk of death
from predation
Log abundance
growth
log abundance vs log body
mass: size spectrum
Log body mass

Abundance at egg size

Number of recruits

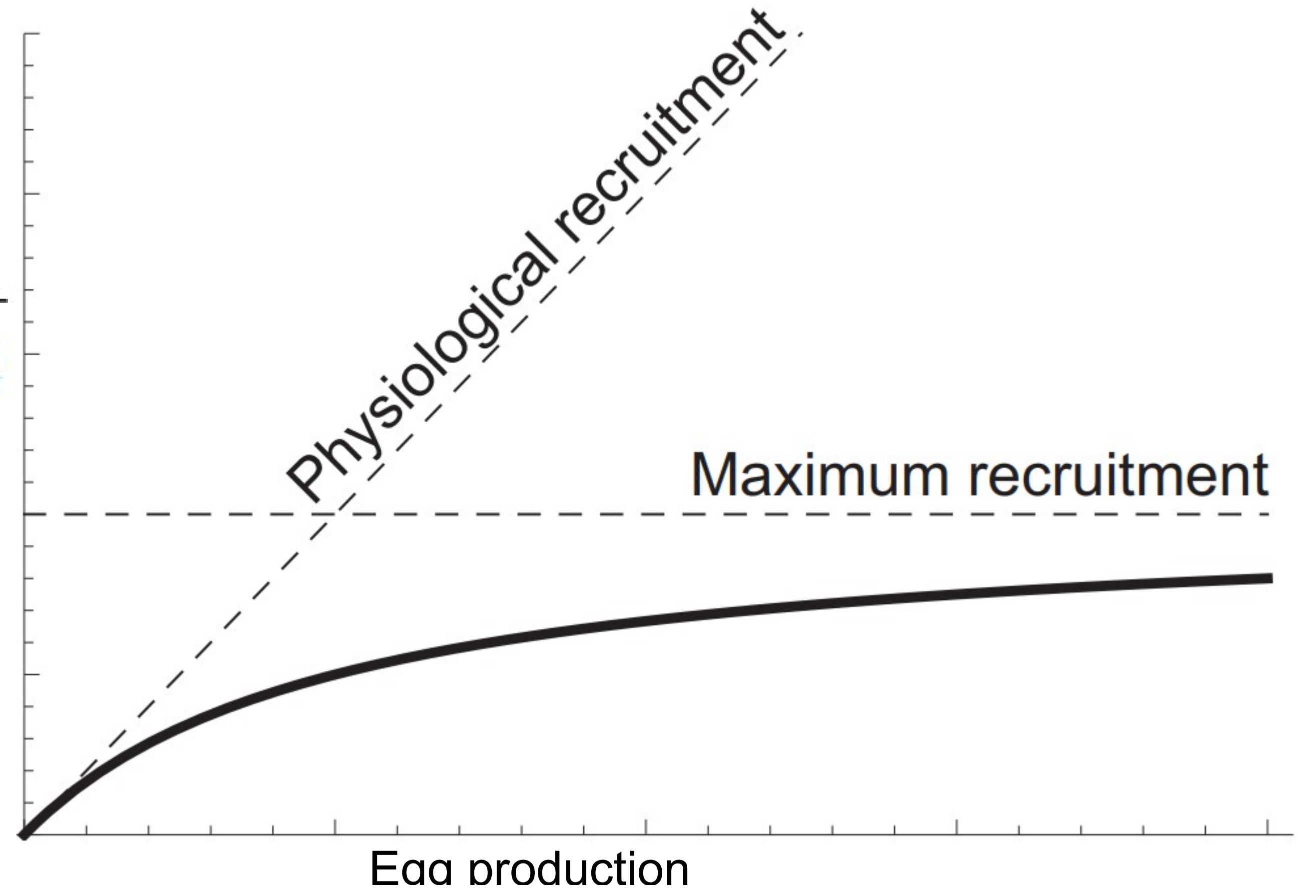
Growth rate of eggs

$$R_i = N_i(w_{0.i})g_i(w_{0.i})$$



Recruitment

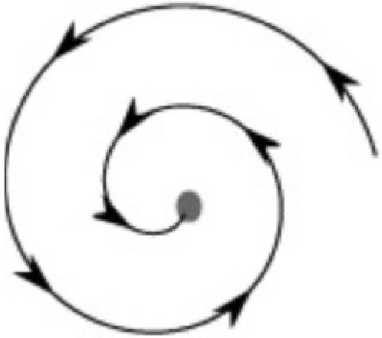
$$R_i = \frac{R_{p.i} R_{max.i}}{R_{p.i} + R_{max.i}}$$



$$R_{p.i} = \frac{\epsilon_i}{2w_{0.i}} \int_0^\infty N_i(w) E_{r.i}(w) \psi_i(w).dw$$

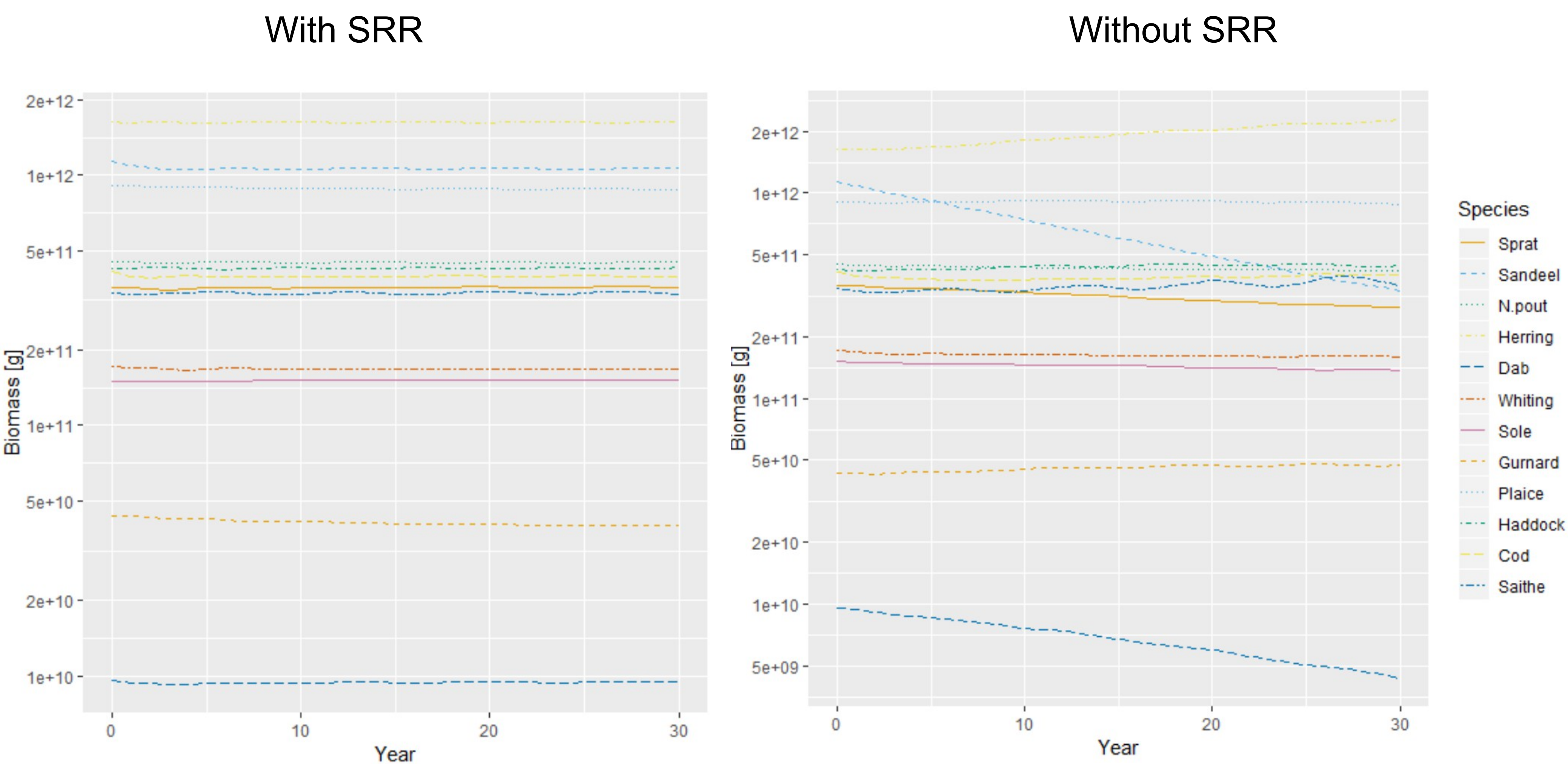
Hold recruitment fixed at $R_{f.i}$

Evolve system to steady state



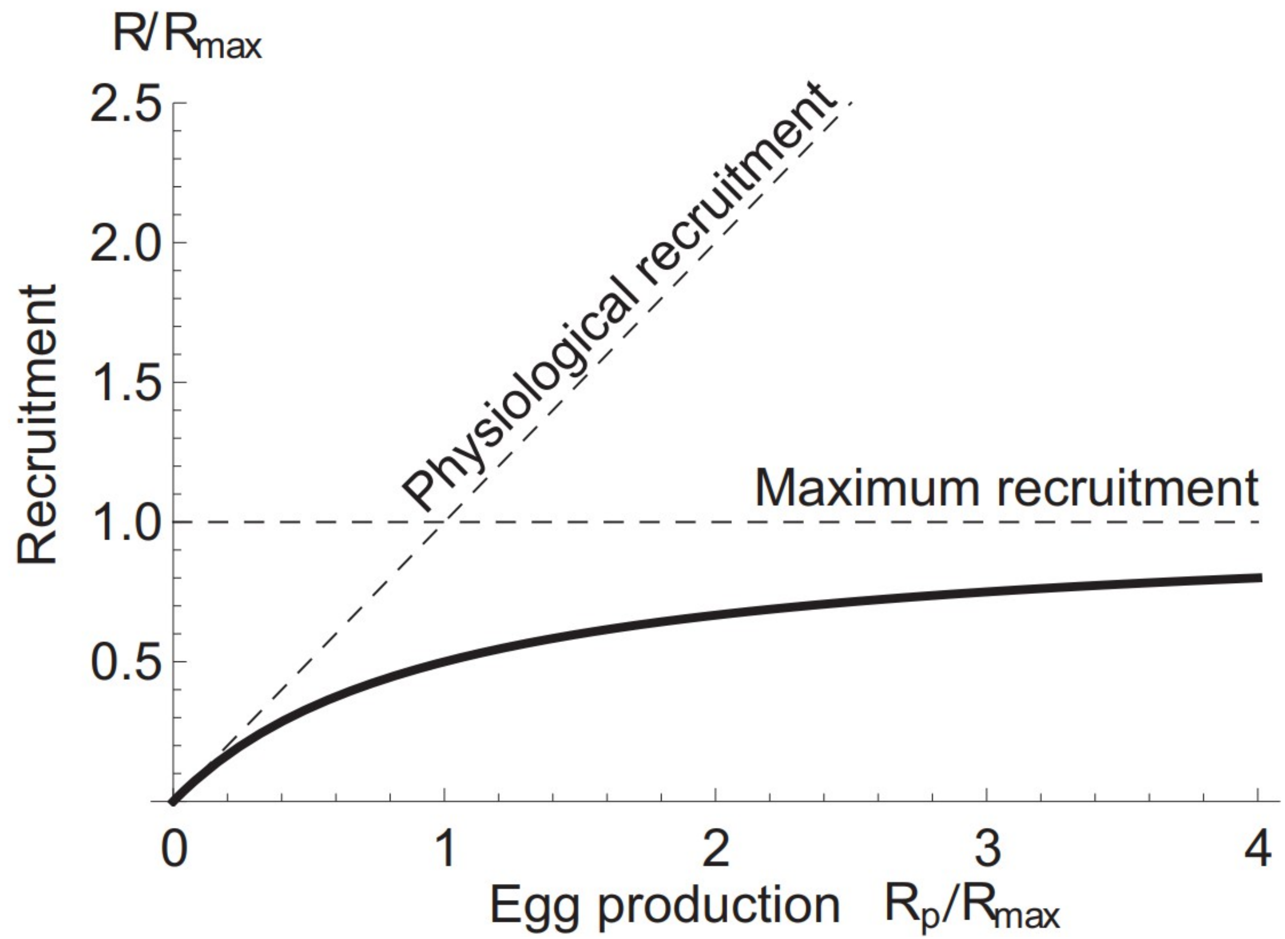
Choose reproduction efficiency ϵ_i so

$$R_{f.i} = \frac{\epsilon_i}{2w_{0.i}} \int_0^\infty N_i(w) E_{r.i}(w) \psi_i(w) . dw$$



With SRR
Without SRR
2e+12
1e+12
5e+11
2e+11
1e+11
5e+10
2e+10
1e+10
5e+09
Biomass [g]
0
10
20
30
Year
Species
Sprat
Sandeel
N.pout
Herring
Dab
Whiting
Sole
Gurnard
Plaice
Haddock
Cod
Saithe

$$R_{p.i} = \frac{\epsilon_i}{2w_{0.i}} \int_0^\infty N_i(w) E_{r.i}(w) \psi_i(w).dw$$



$$R_i = \frac{R_{p.i} R_{max.i}}{R_{p.i} + R_{max.i}}$$

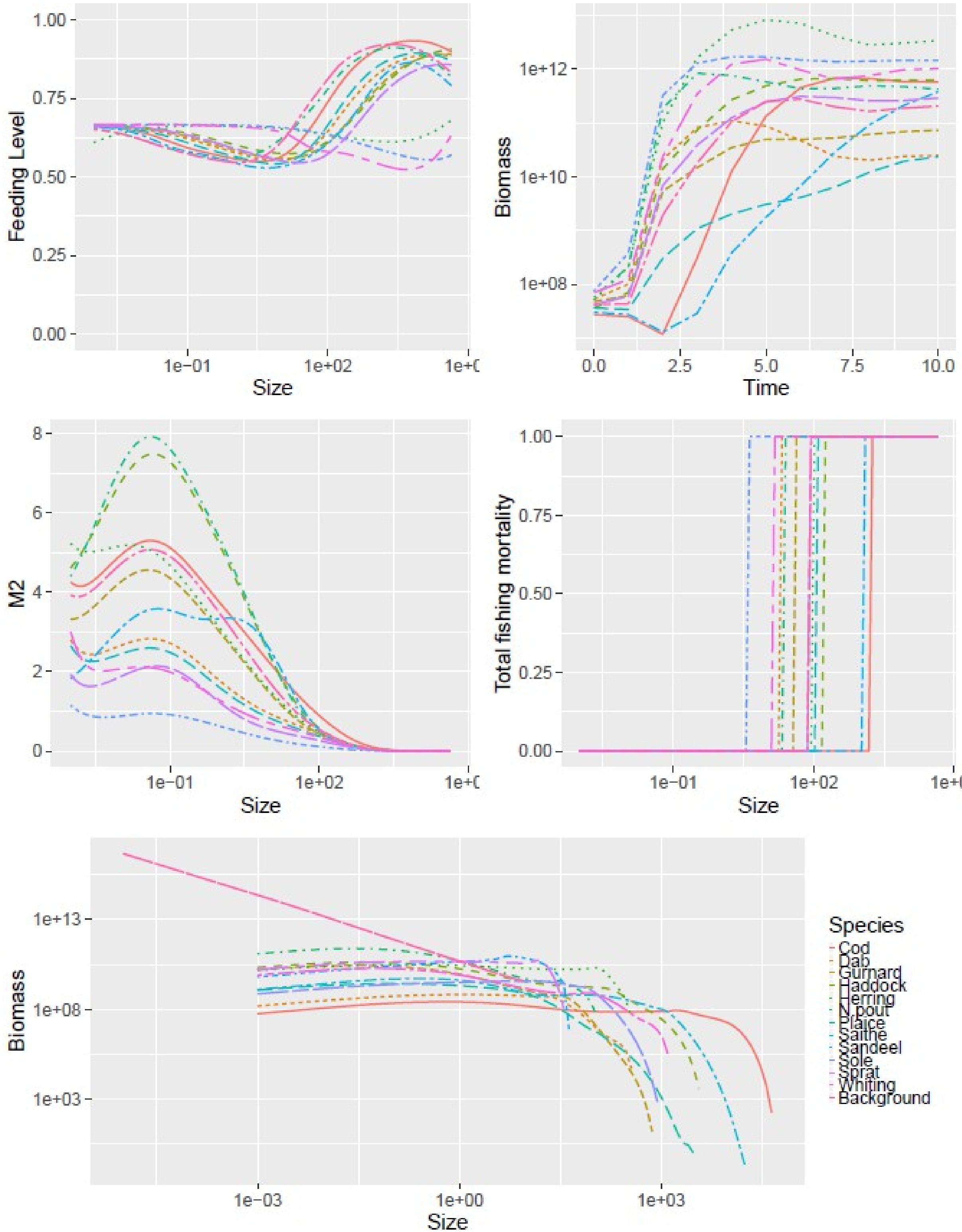
energy available for growth and reproduction

reproductive efficiency

$$g_i(w_{e.i})N_i(w_{e.i}) = \frac{\epsilon_i}{2w_{e.i}}\int_0^\infty N_i(w)E_{r.i}(w)\psi(w).dw$$

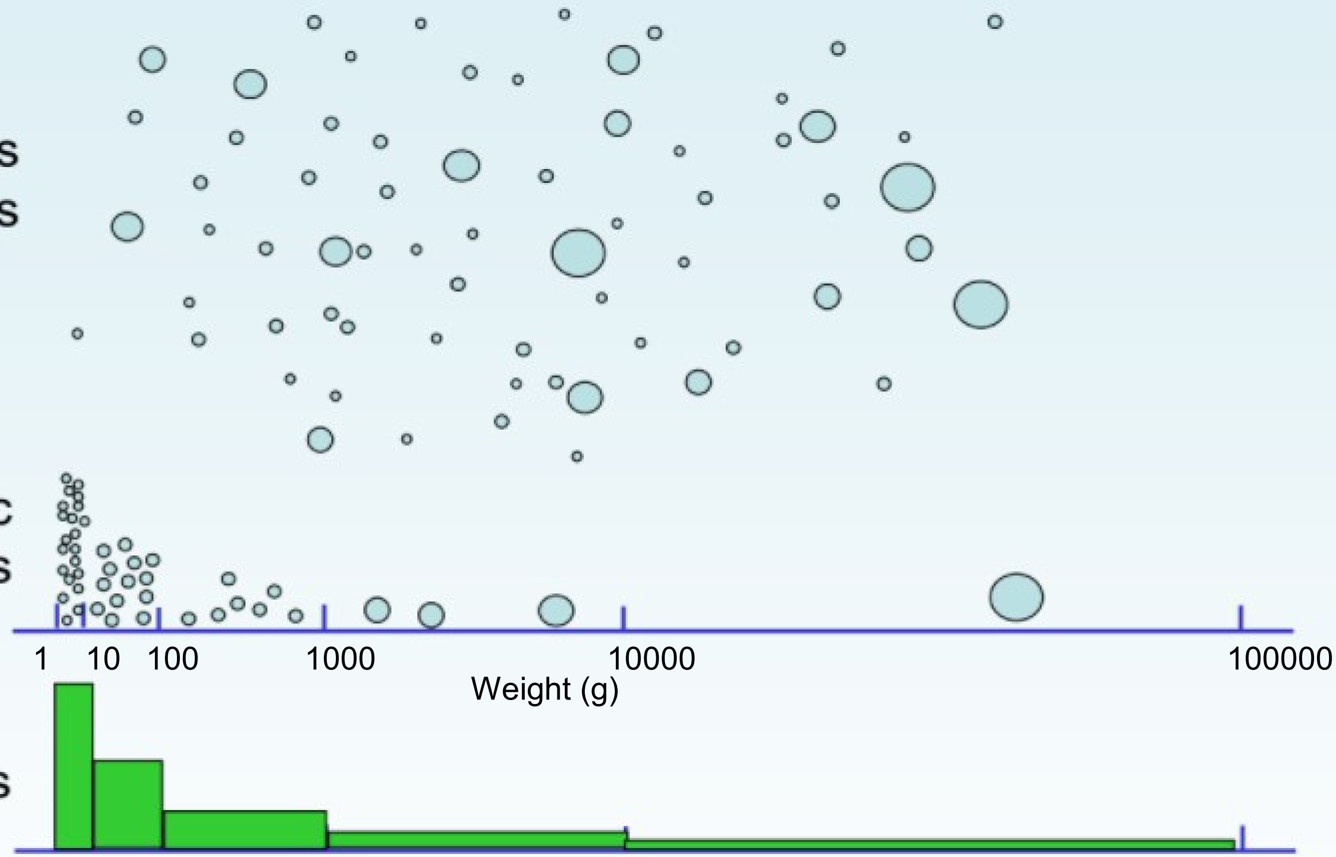
egg size for i th species

fraction of energy diverted into reproduction

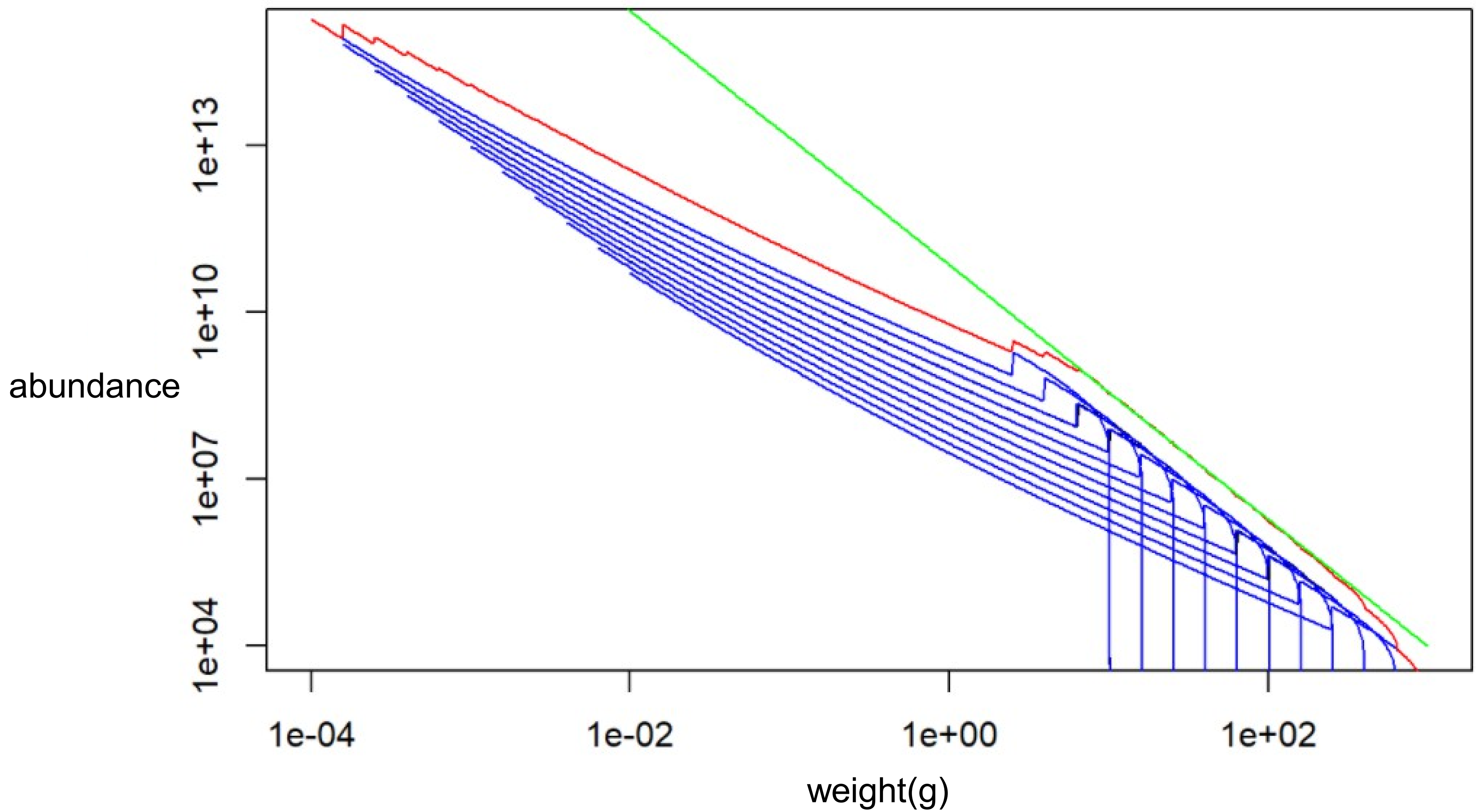


Feeding Level
Size
Biomass
Time
M2
Size
Total fishing mortality
Size
Biomass
Size
Species
Cod
Dab
Gurnard
Haddock
Herring
N.pout
Plaice
Saithe
Sandeel
Sole
Sprat
Whiting
Background

- organisms as particles of different sizes
- bin into logarithmic size intervals
- sum the biomass



- **biomass in log bins is approximately constant**


abundance
1e+13
1e+10
1e+07
1e+04
1e-04
1e-02
1e+00
1e+02
weight(g)

prefered predator-prey mass ratio

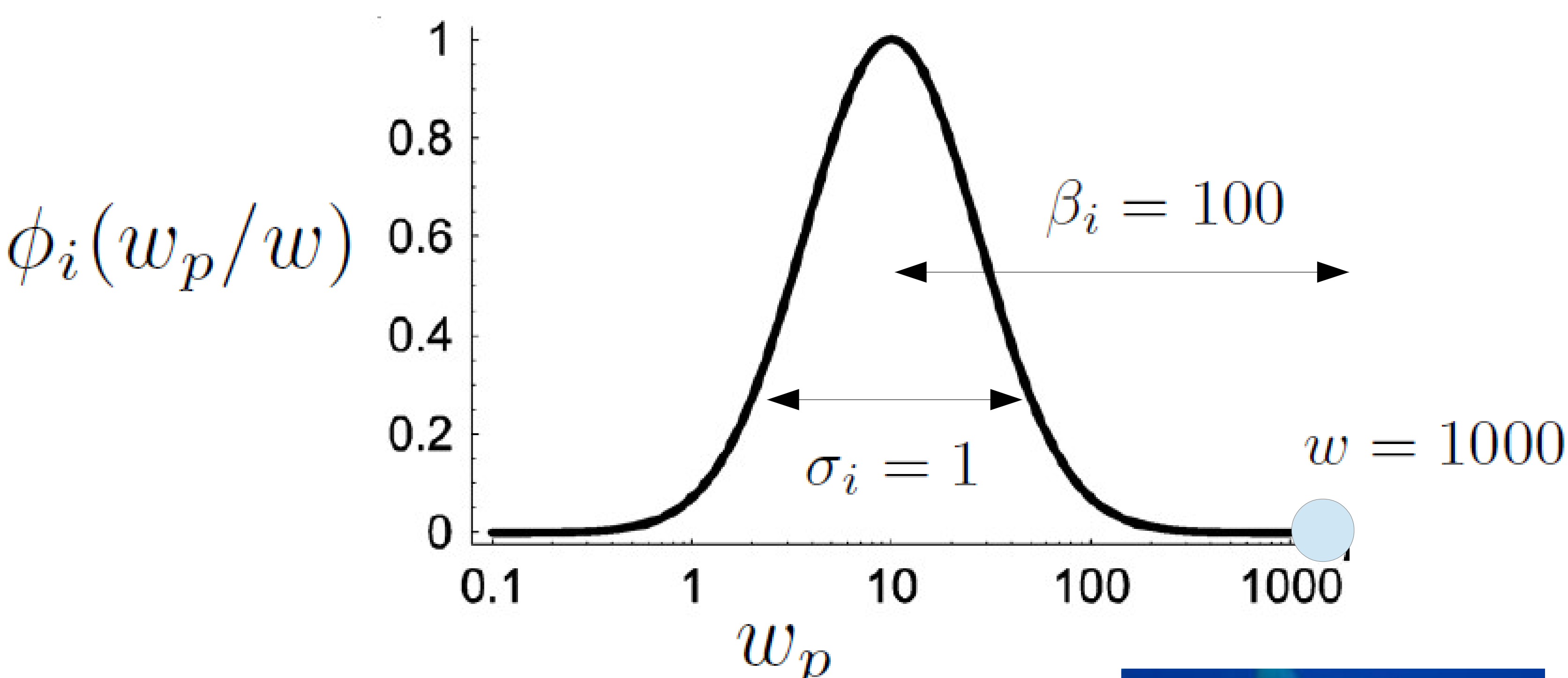
preference level

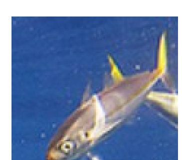
prey size

$$\phi_i(w_p/w) = \exp\left[\frac{-(\ln(w/(w_p\beta_i)))^2}{2\sigma_i^2}\right]$$

predator size

width of prey distribution





energy encountered

preference level of species i for species j

predator weight

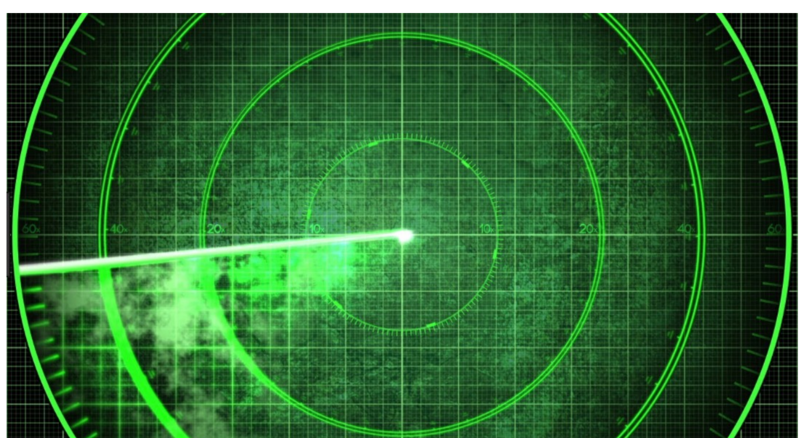
prey weight

$$E_{e.i}(w) = \underline{\gamma w^q} \int_0^\infty \left(N_R(w_p) + \sum_j \theta_{ij} N_j(w_p) \right) \phi_i\left(\frac{w_p}{w}\right) w_p \, dw_p$$

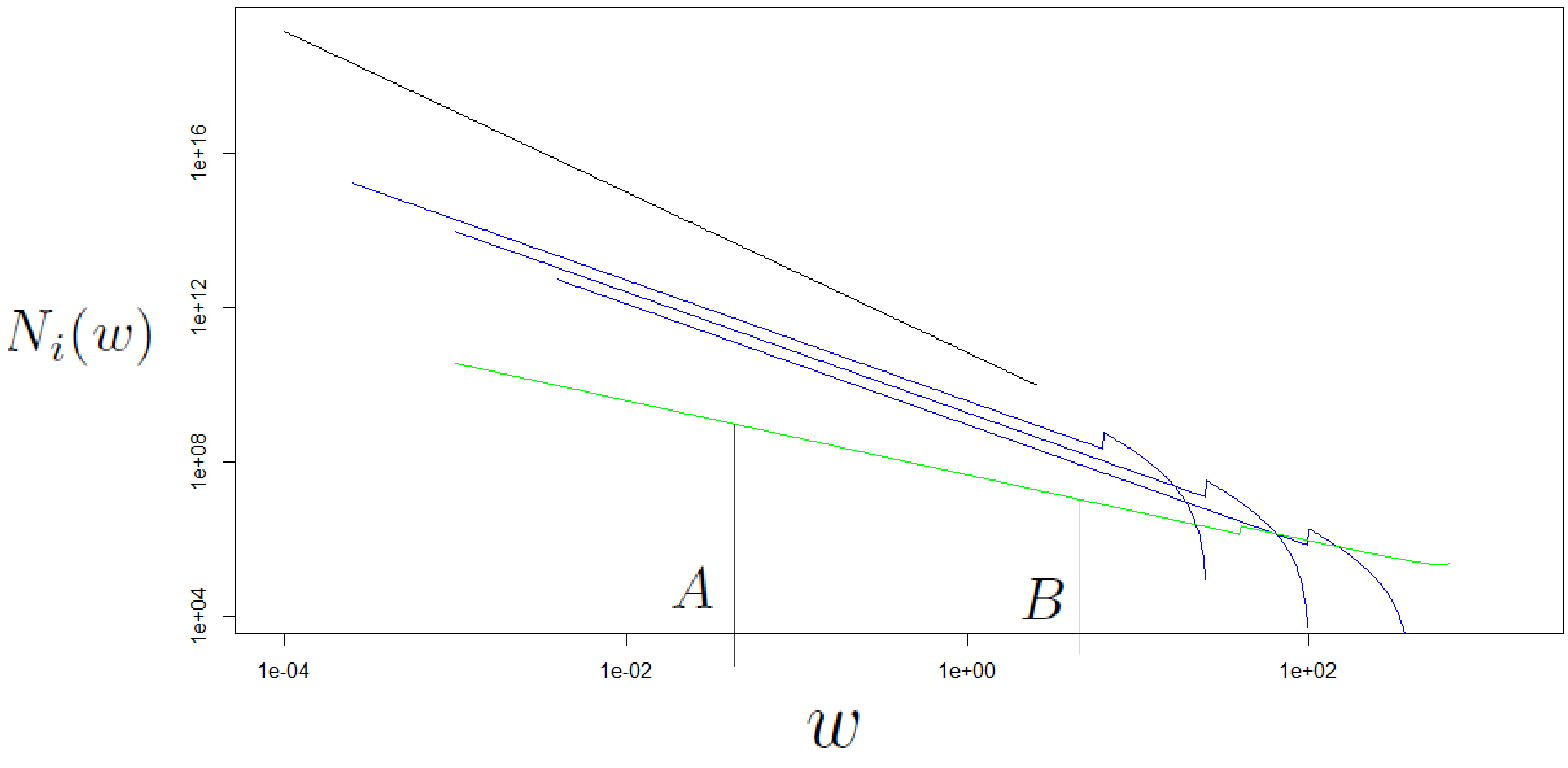
volumetric search rate

abundance of background resources at weight w_p

preference level of weight w predator for weight w_p prey



$N_i(w)$ =density of weight w individuals of species i



$\int_A^B N_i(w)dw$ = number of individuals with weight between A and B

Size Spectrum Modelling

Gustav Delius, Richard Southwell