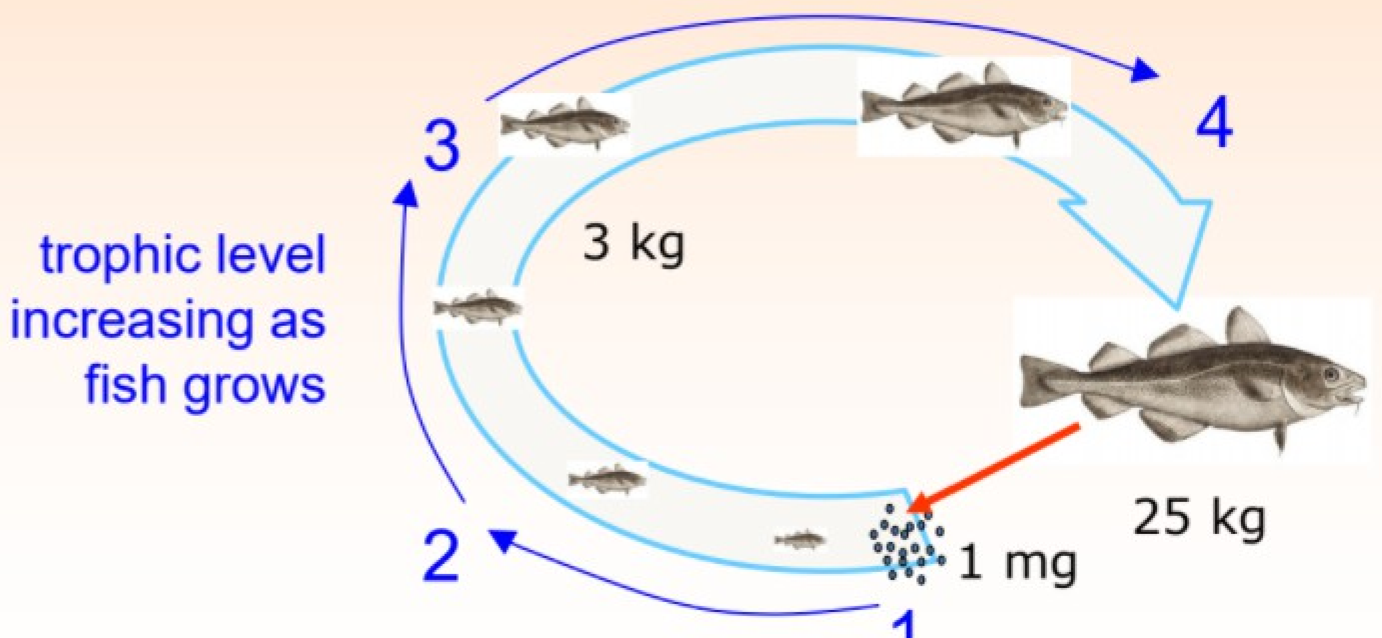
Coexistence In Size Spectrum Models

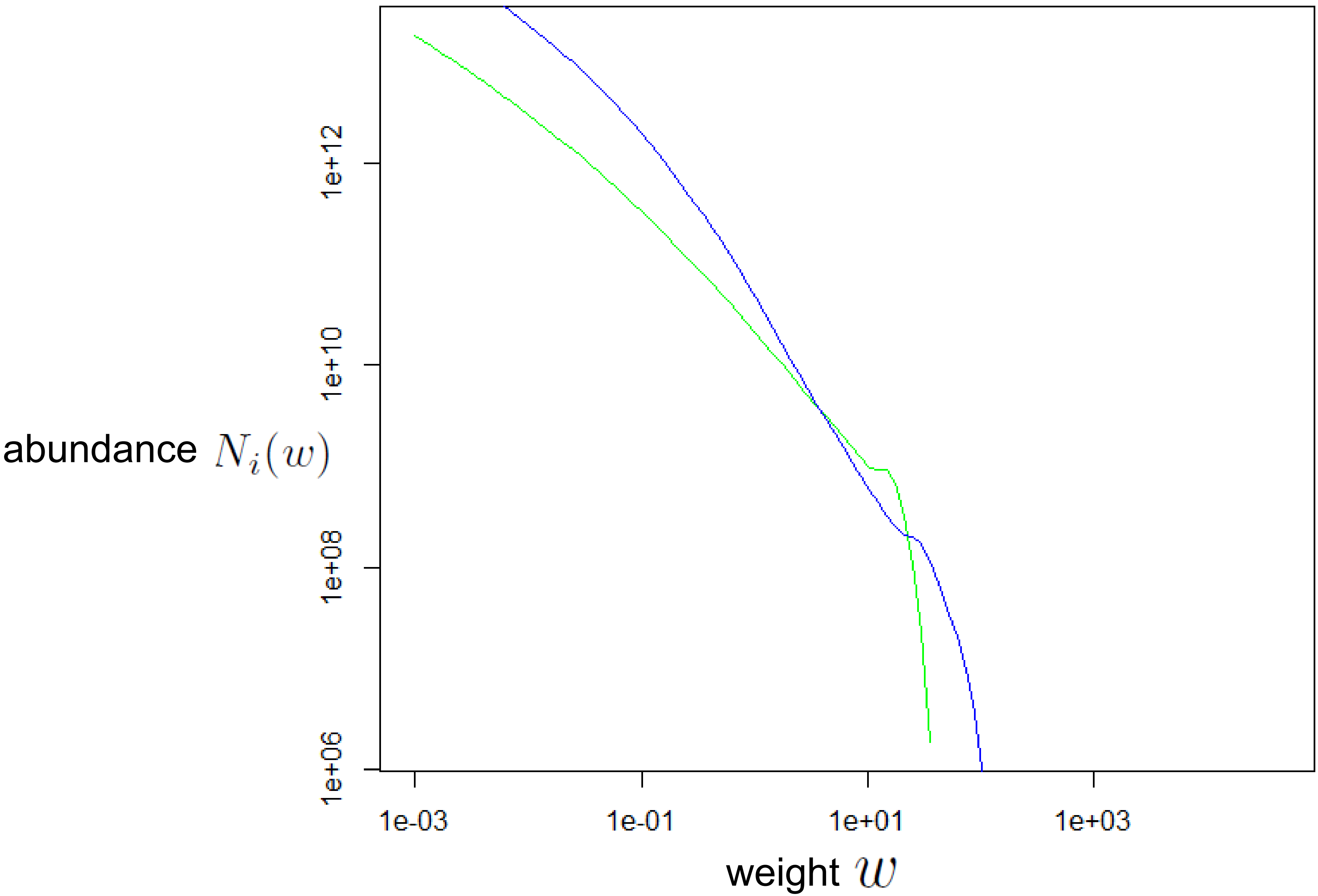
Gustav Delius

Richard Southwell

Richard Law



from Beyer 2010, Nagoya talk


1e+12
1e+10
1e+08
1e+06
1e-03
1e-01
1e+01
1e+03
abundance $N_i(w)$
weight w

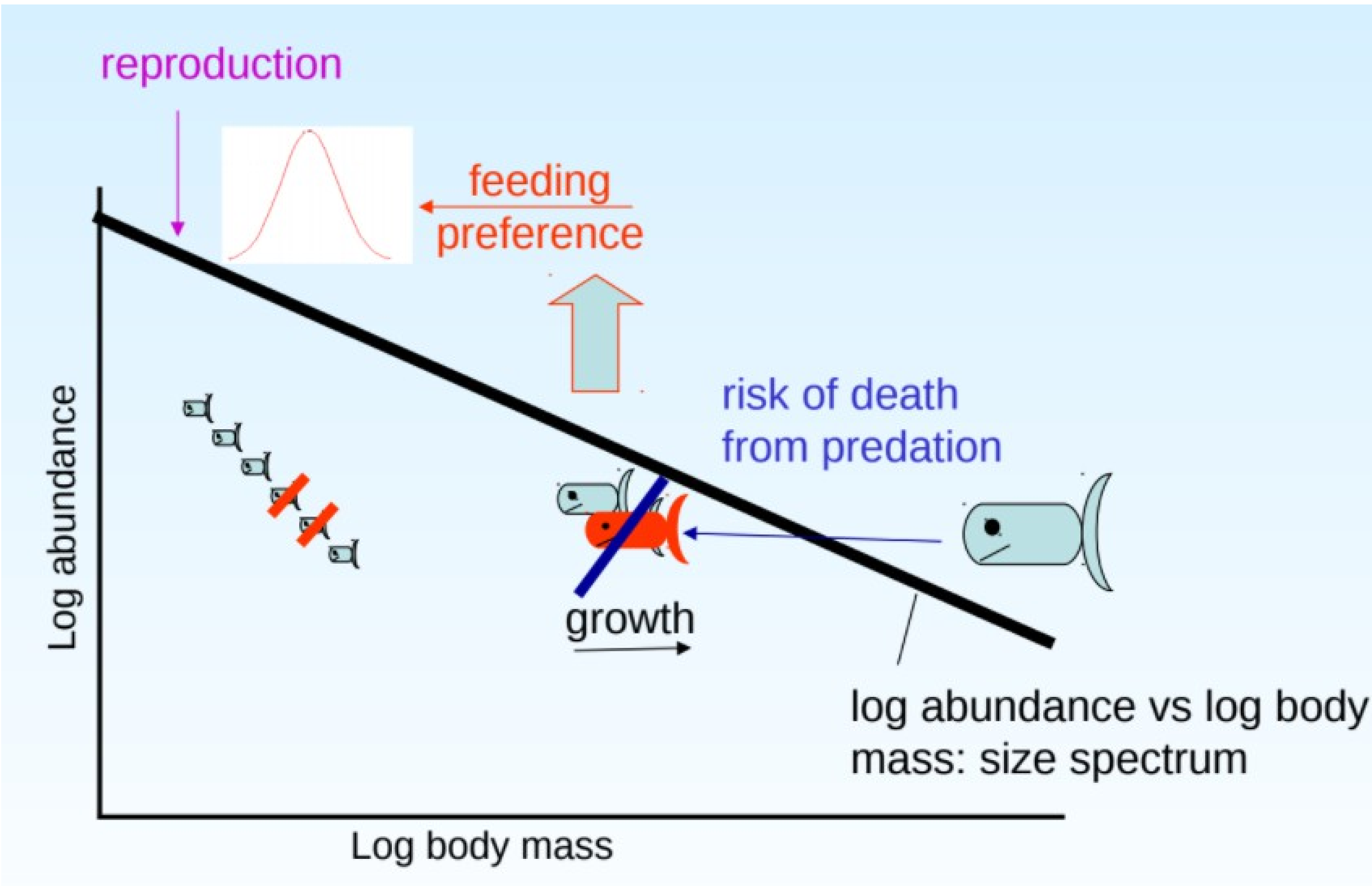
McKendrik-von Foerster equation

density of weight w individuals of species i

growth rate

$$\frac{\partial N_i(w)}{\partial t} = -\frac{\partial[N_i(w)g_i(w)]}{\partial w} - \mu_i(w)N_i(w)$$

death rate



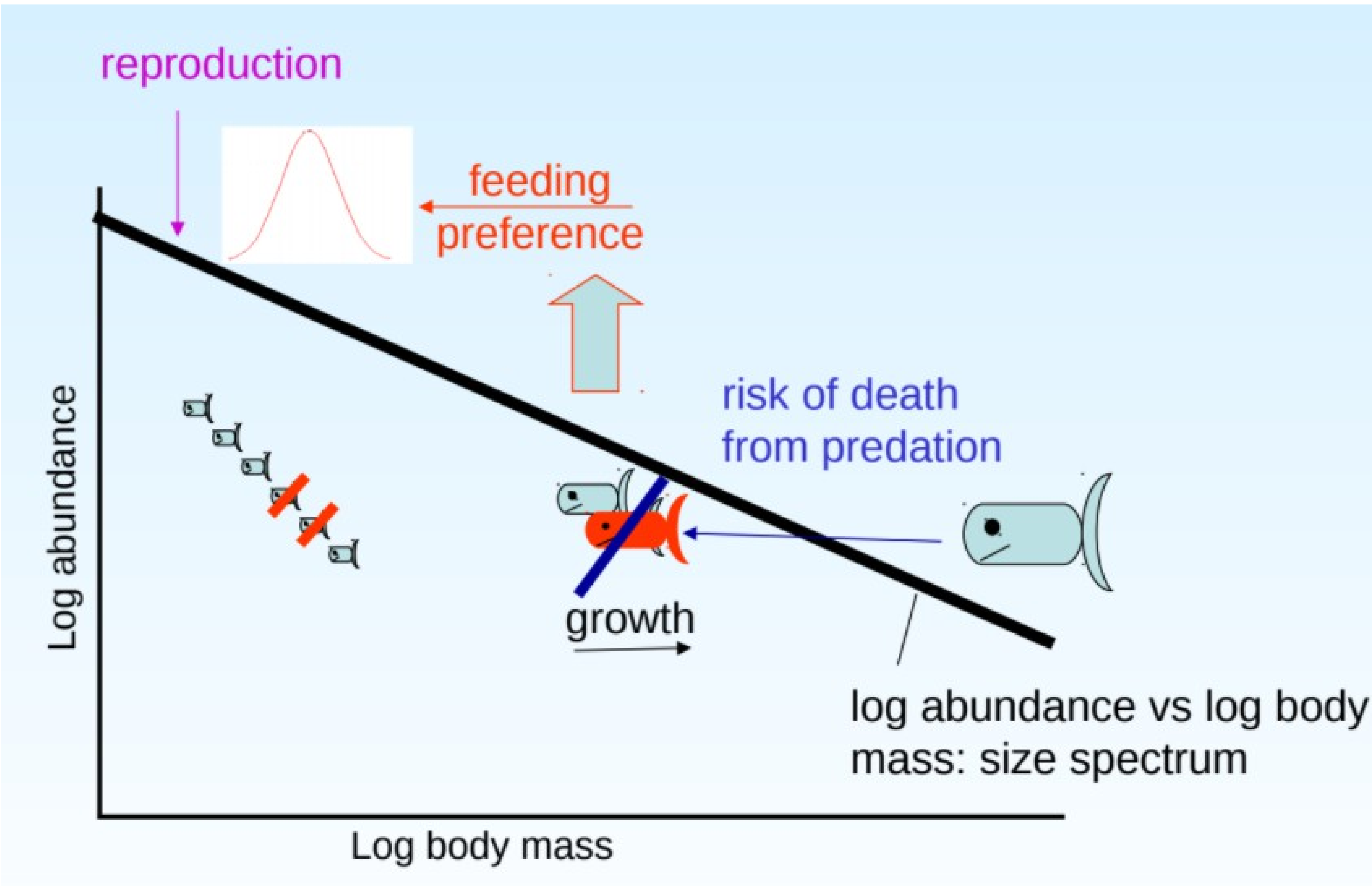
reproduction
feeding
preference
risk of death
from predation
growth
log abundance vs log body
mass: size spectrum
Log abundance
Log body mass

Number of recruits

Abundance at egg size

Growth rate of eggs

$$R_i = N_i(w_{0.i})g_i(w_{0.i})$$



Abundance

Energy

Reproduction efficiency

Fraction of energy used
for reproduction

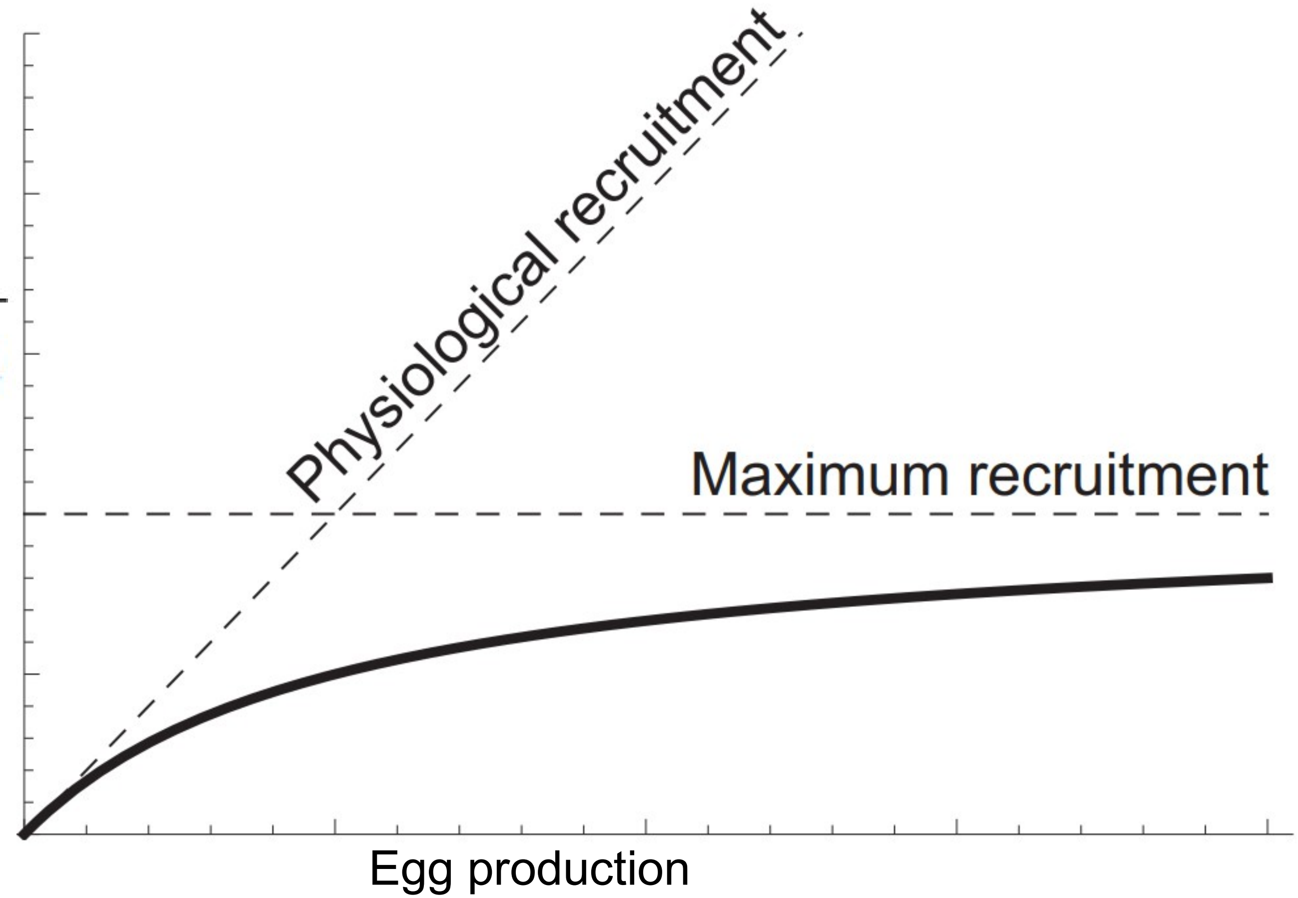
Egg production

$$R_{p.i} = \frac{\epsilon_i}{2w_{0.i}} \int_0^\infty N_i(w) E_{r.i}(w) \psi_i(w).dw$$

Egg size

Recruitment

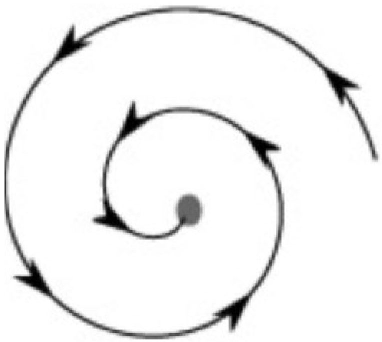
$$R_i = \frac{R_{p.i} R_{max.i}}{R_{p.i} + R_{max.i}}$$



$$R_{p.i} = \frac{\epsilon_i}{2w_{0.i}} \int_0^\infty N_i(w) E_{r.i}(w) \psi_i(w).dw$$

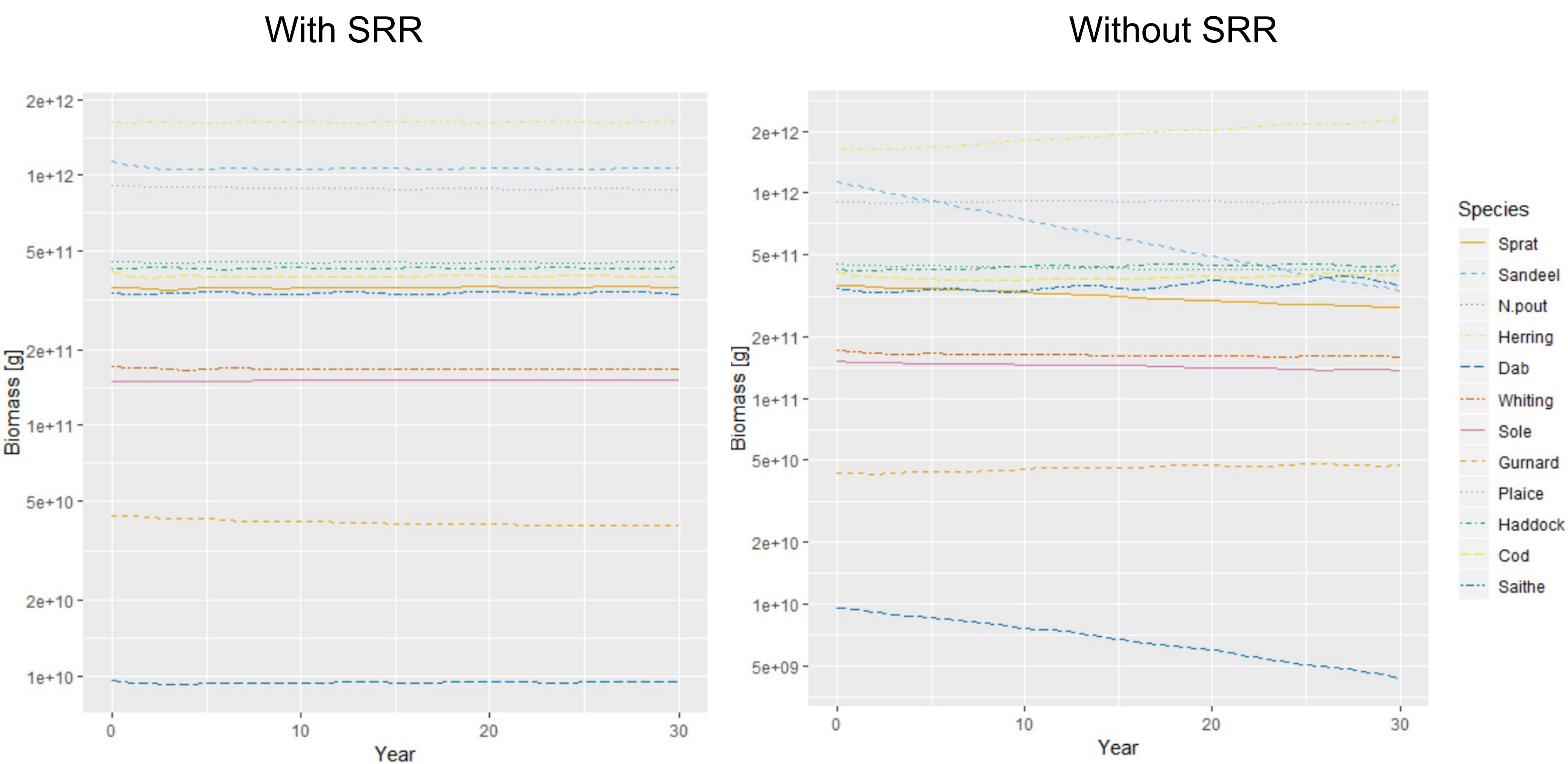
Hold recruitment fixed at $R_{f.i}$

Evolve system to steady state



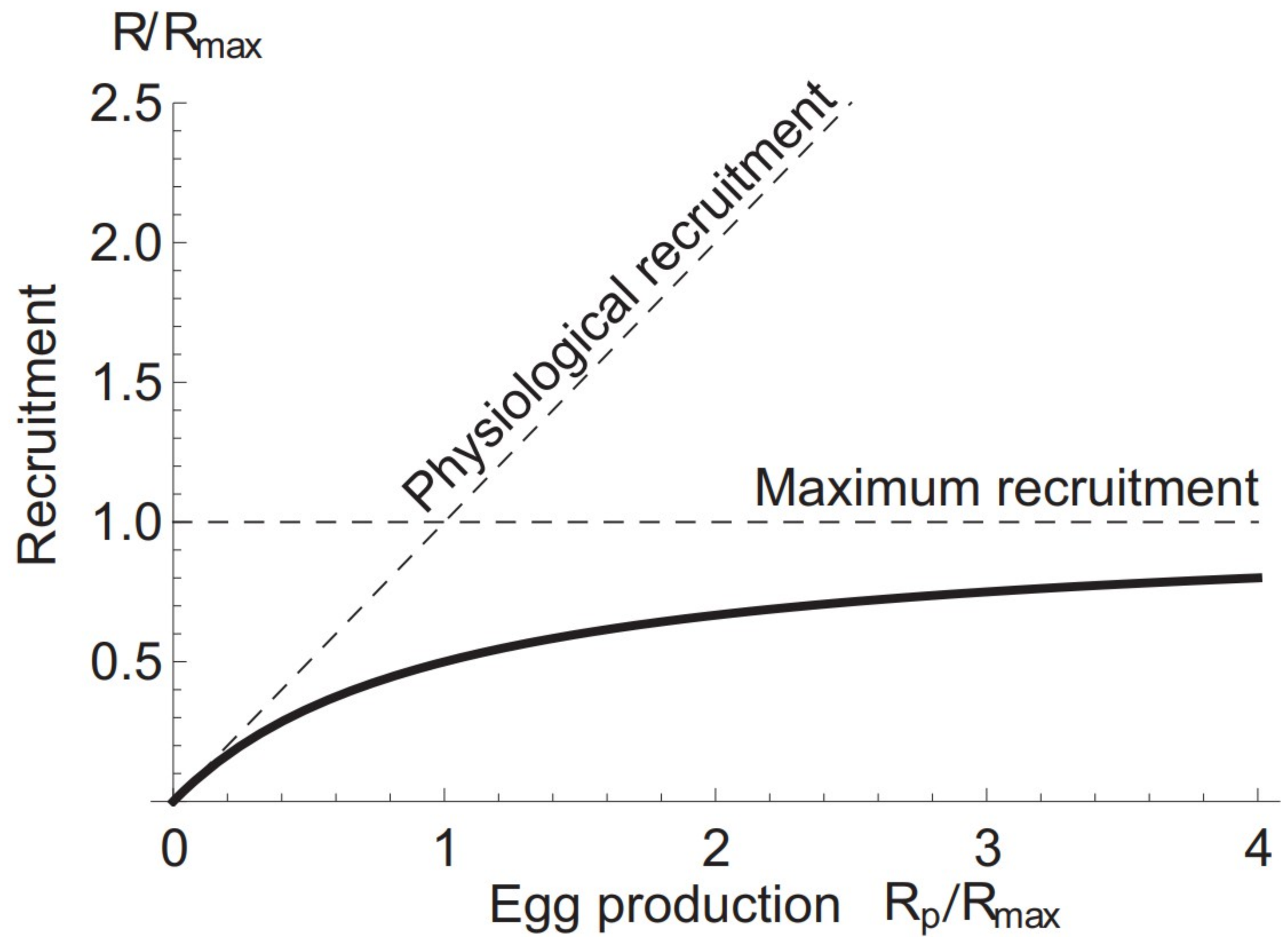
Choose reproduction efficiency ϵ_i
so

$$R_{f.i} = \frac{\epsilon_i}{2w_{0.i}} \int_0^\infty N_i(w) E_{r.i}(w) \psi_i(w).dw$$



With SRR
Without SRR
Biomass [g]
Year
2e+12
1e+12
5e+11
2e+11
1e+11
5e+10
2e+10
1e+10
5e+09
0
10
20
30
Species
Sprat
Sandeel
N.pout
Herring
Dab
Whiting
Sole
Gurnard
Plaice
Haddock
Cod
Saithe

$$R_{p.i} = \frac{\epsilon_i}{2w_{0.i}} \int_0^\infty N_i(w) E_{r.i}(w) \psi_i(w).dw$$



$$R_i = \frac{R_{p.i} R_{max.i}}{R_{p.i} + R_{max.i}}$$

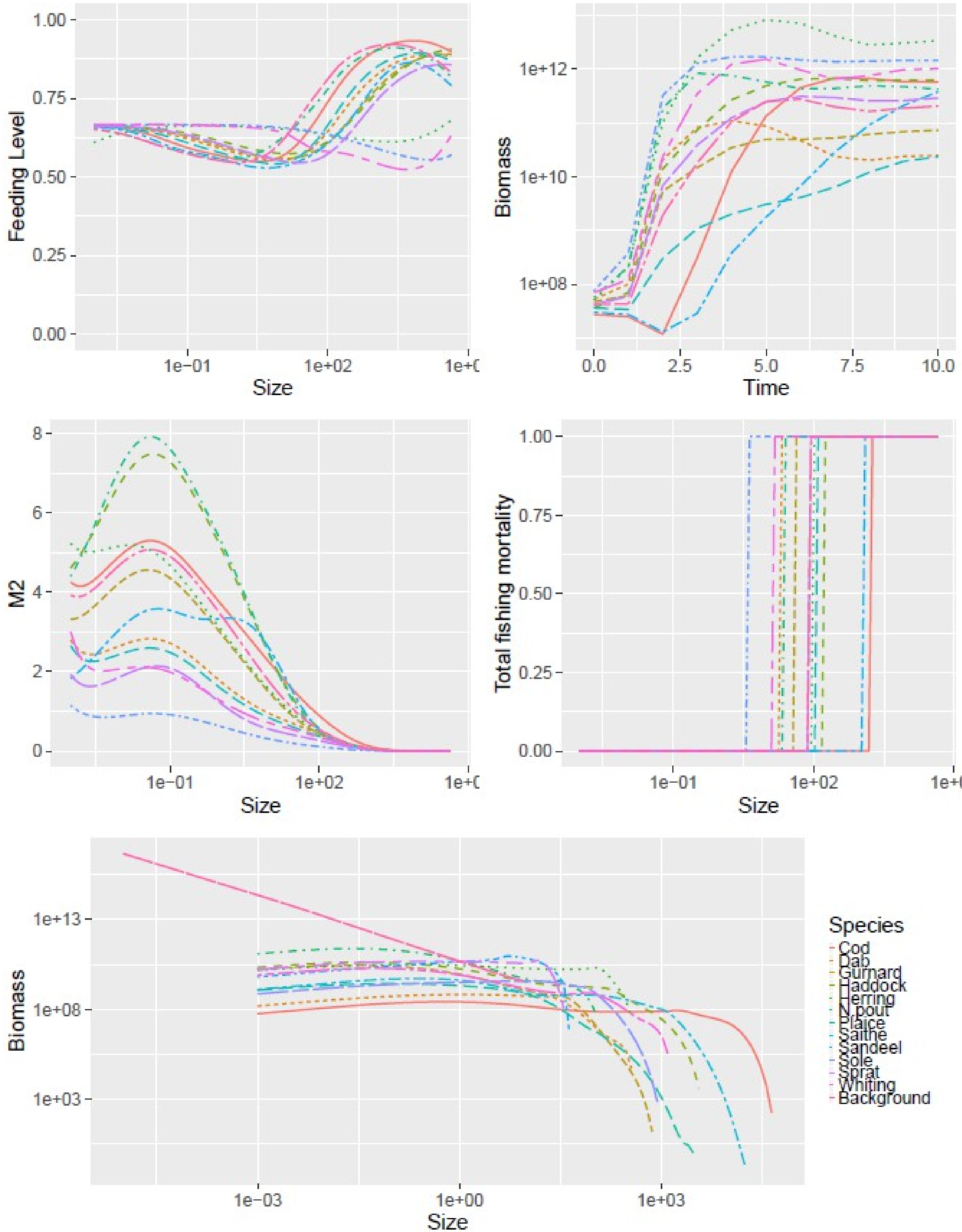
energy available for growth and reproduction

reproductive efficiency

$$g_i(w_{e.i})N_i(w_{e.i}) = \frac{\epsilon_i}{2w_{e.i}}\int_0^\infty N_i(w)E_{r.i}(w)\psi(w).dw$$

egg size for i th species

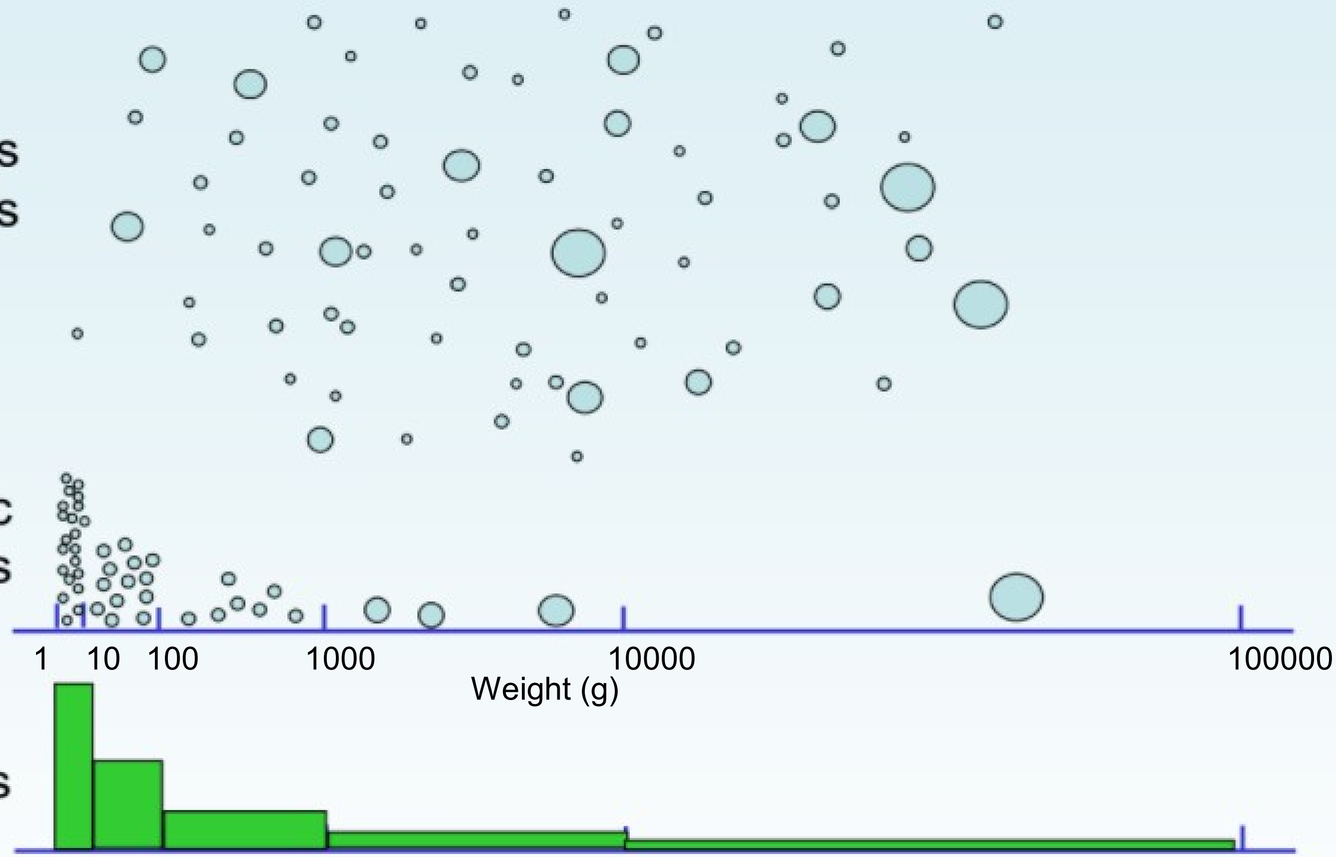
fraction of energy diverted into reproduction


Feeding Level
Size
Biomass
Time
M2
Size
Total fishing mortality
Size
Biomass
Size
Species
Cod
Dab
Gurnard
Haddock
Herring
N.pout
Plaice
Saithe
Sandeel
Sole
Sprat
Whiting
Background

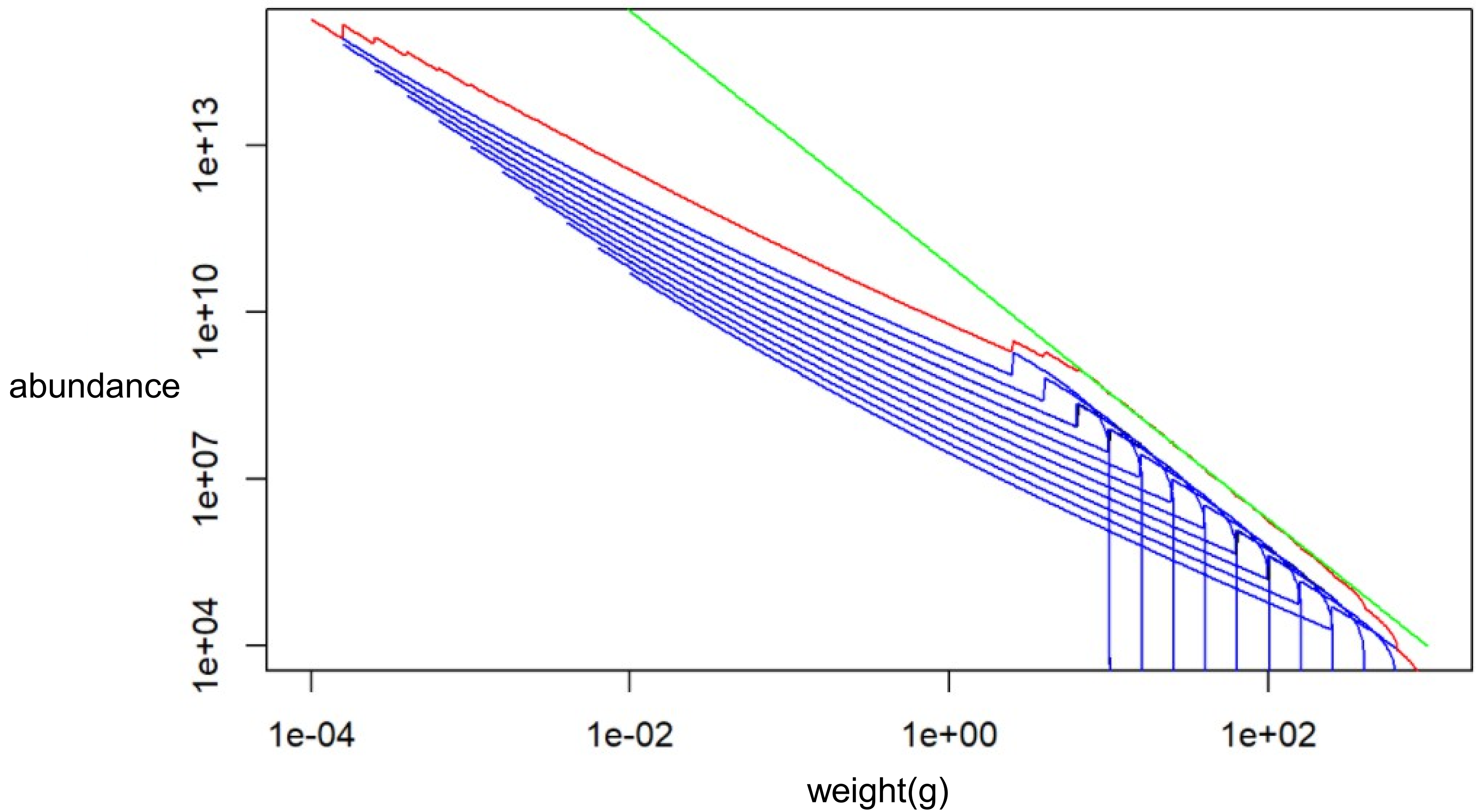
organisms as particles of different sizes

bin into logarithmic size intervals

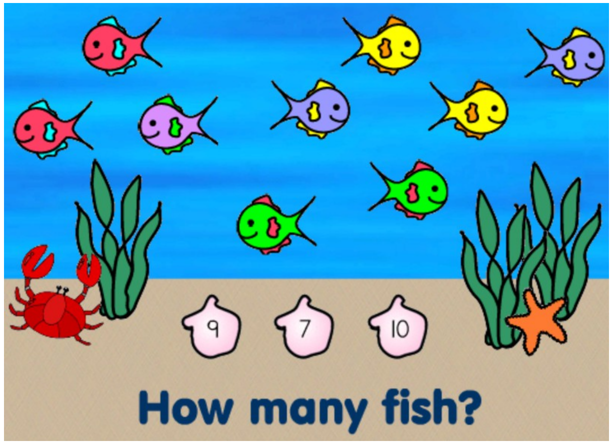
sum the biomass



biomass in log bins is approximately constant

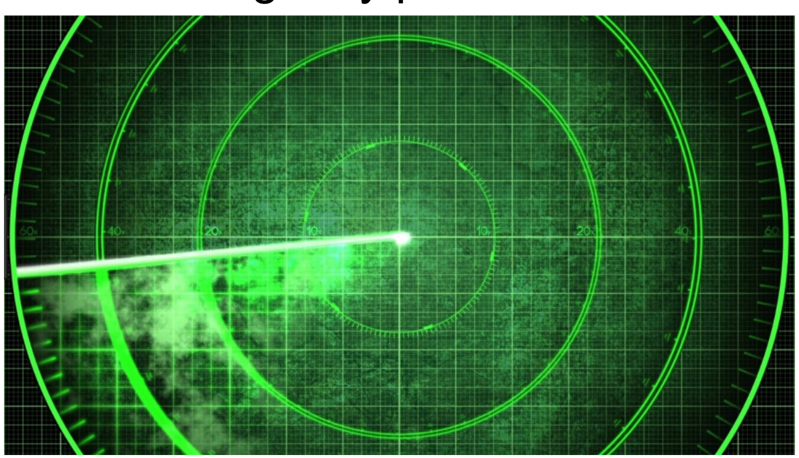


abundance
1e+04
1e+07
1e+10
1e+13
1e-04
1e-02
1e+00
1e+02
weight(g)



sum over prey

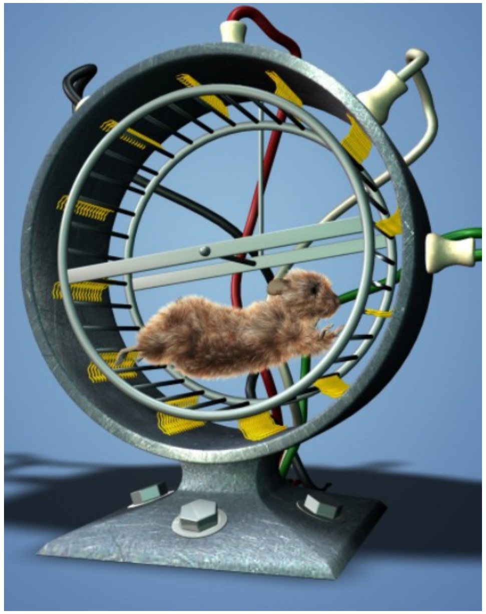
weight by preference



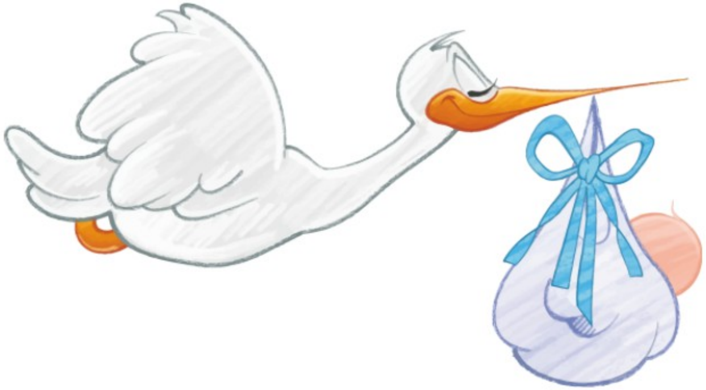
multiply by search rate of predator

energy encountered

limited eating rate

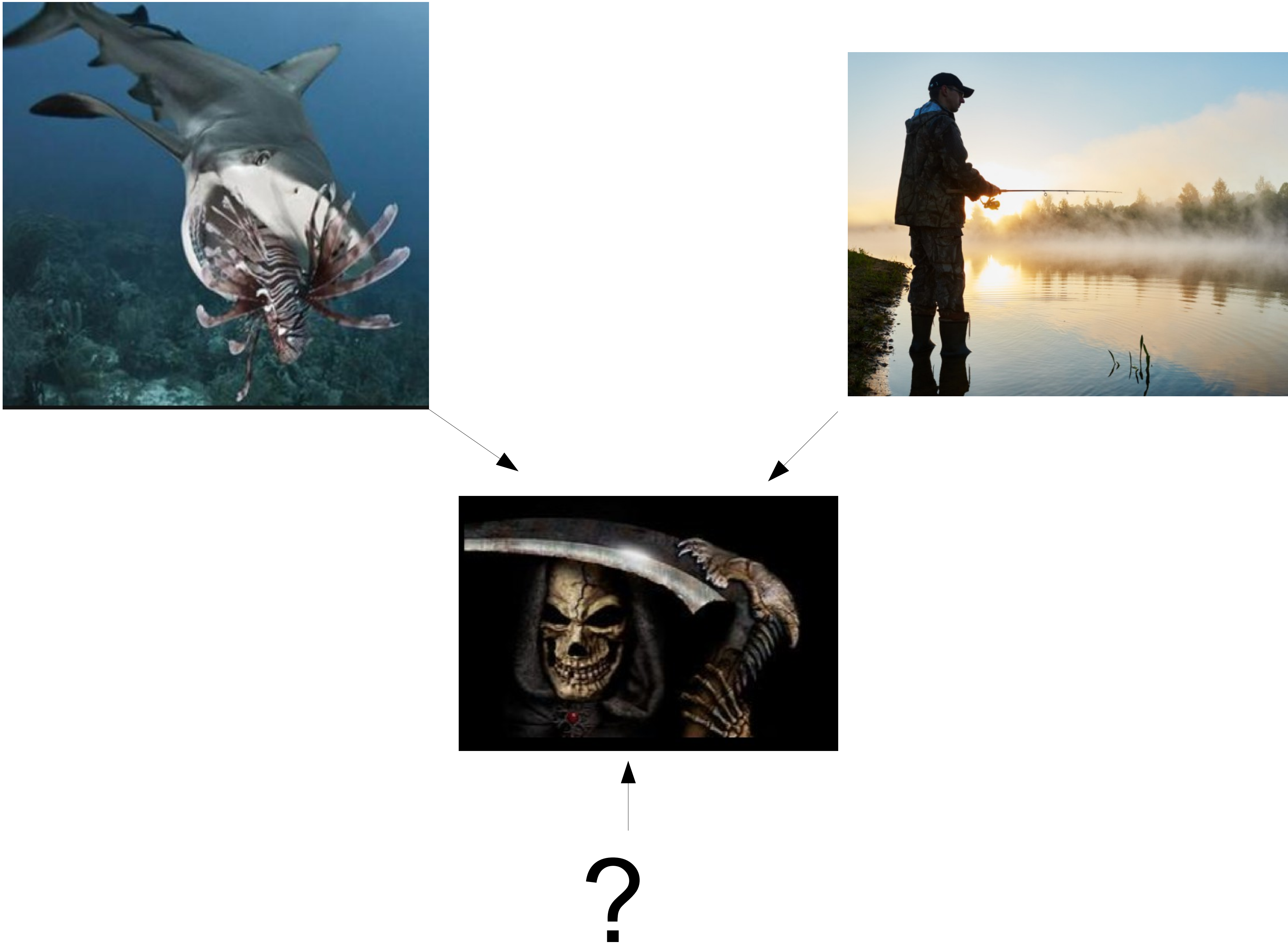


energy costs for movement
and metabolism



energy for reproduction

growth



preference level

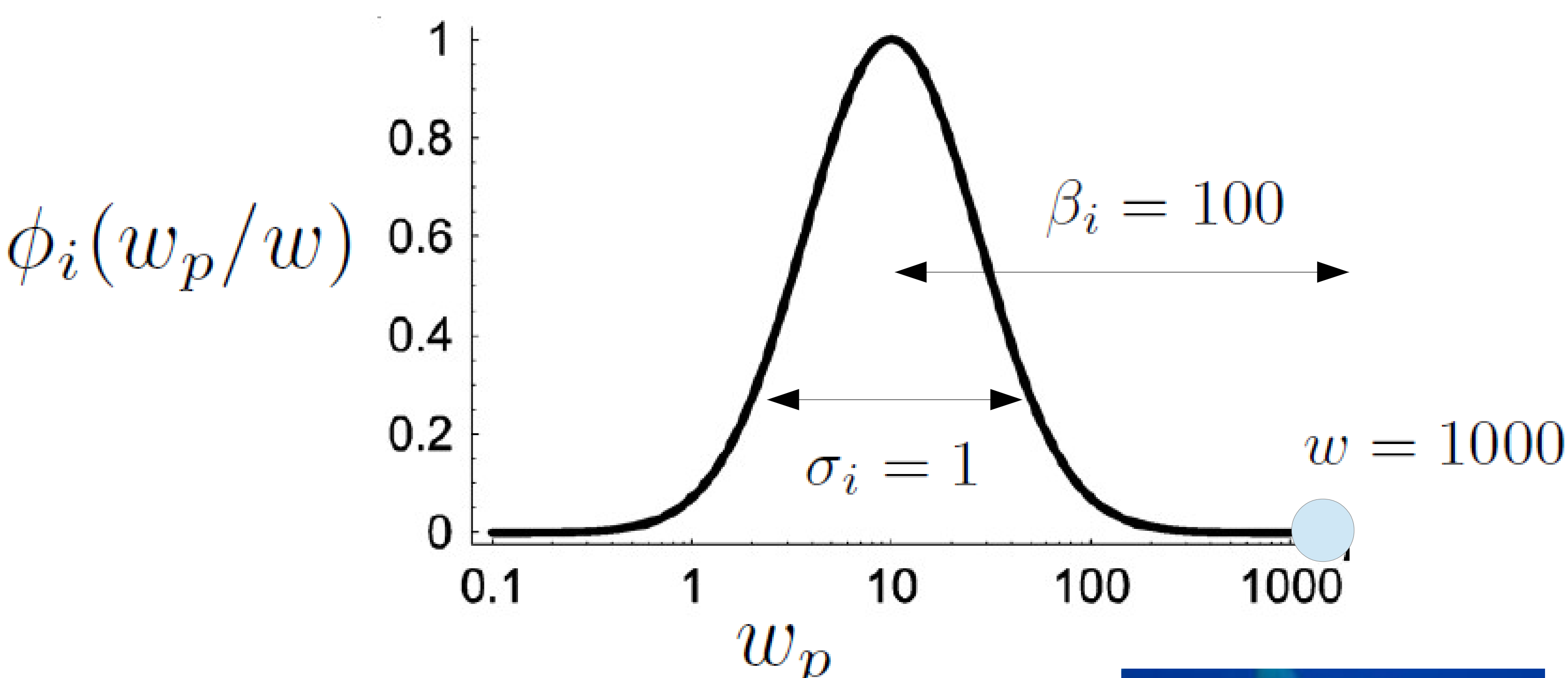
prey size

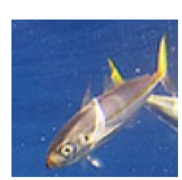
prefered predator-prey mass ratio

$$\phi_i(w_p/w) = \exp\left[\frac{-(\ln(w/(w_p\beta_i)))^2}{2\sigma_i^2}\right]$$

predator size

width of prey distribution





energy encountered

preference level of species i for species j

predator weight

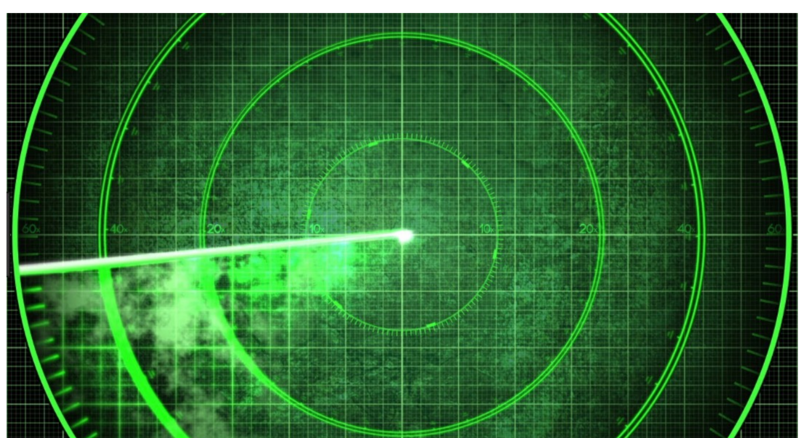
prey weight

$$E_{e.i}(w) = \gamma w^q \int_0^\infty \left(N_R(w_p) + \sum_j \theta_{ij} N_j(w_p) \right) \phi_i\left(\frac{w_p}{w}\right) w_p dw_p$$

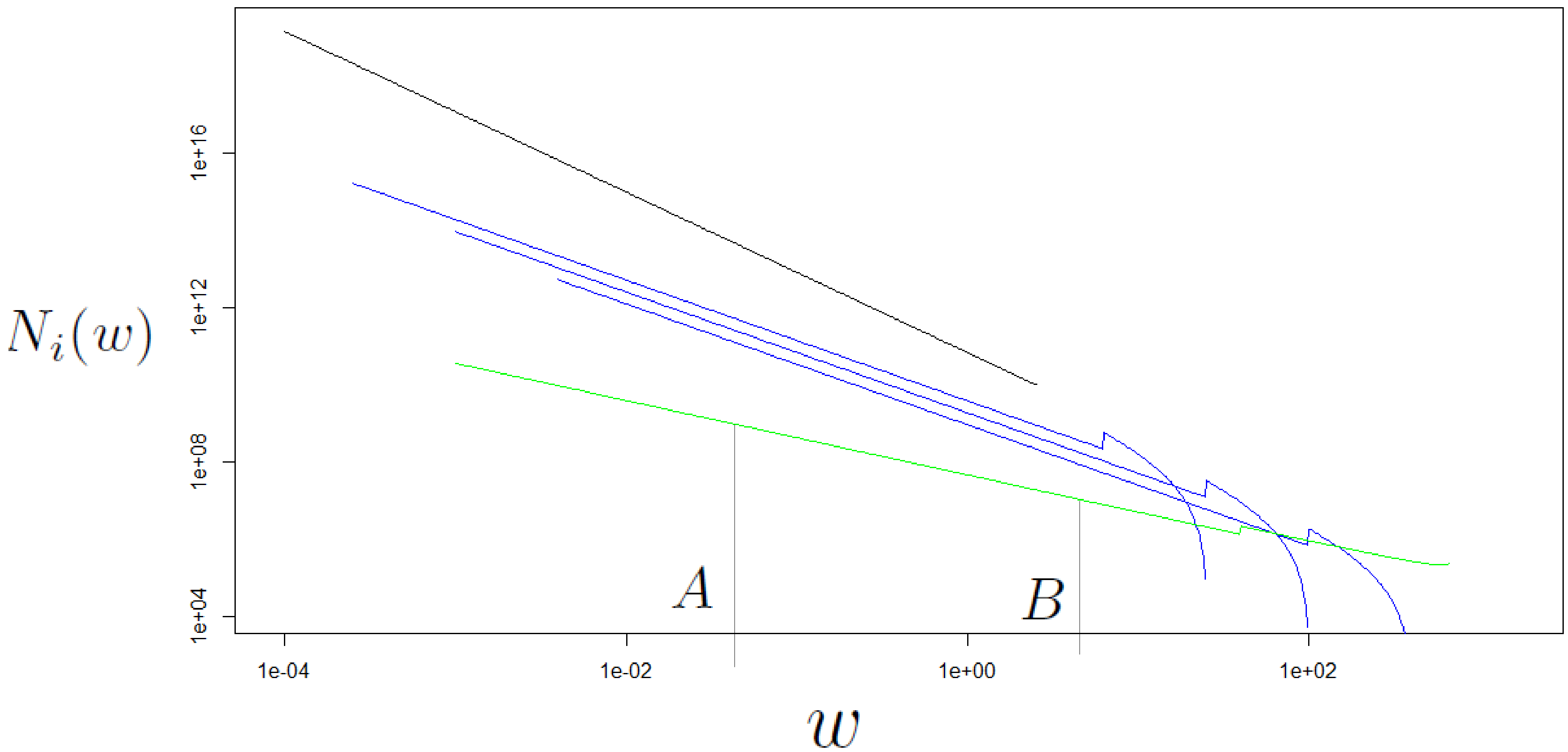
volumetric search rate

abundance of background resources at weight w_p

preference level of weight w predator for weight w_p prey



$N_i(w)$ =density of weight w individuals of species i



$\int_A^B N_i(w)dw$ = number of individuals with weight between A and B

Size Spectrum Modelling

Gustav Delius, Richard Southwell