PDE solution for non-interacting fish

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Our system is

$$\frac{\partial N(w,t)}{\partial t} = -\frac{\partial\left[g(w)N(w,t)\right]}{\partial t} - \mu(w)N(w,t), \forall w \in (w_e, w_*) \tag{1}$$

where

$$X(w_e, t) = KN_m \tag{2}$$

$$\frac{dN_m}{dt} = X(w_*, t) - \bar{\mu}N_m \tag{3}$$

Here $X(w,t) = N(w,t)g(w)$ and we consider the case where $\mu(w) = \mu_0 w^{n-1}$ and $g(w) = \hbar w^n$.

The system can be rewritten as the PDE

$$g(w)\frac{\partial X(w,t)}{\partial w} + \frac{\partial X(w,t)}{\partial t} = -\mu(w)X(w,t), \forall w \in (w_e, w_*) \tag{4}$$

with boundary conditions given by the delay ODE

$$\frac{\partial X(w_e,t)}{\partial t} = KX(w_*,t) - \bar{\mu}X(w_e,t). \tag{5}$$

The general solution to the PDE is

$$X(w,t) = F(L(w) - t)w^{-\frac{\mu_0}{\hbar}} \tag{6}$$

where F is an arbitrary function, and

$$L(w) = \frac{w^{1-n}}{\hbar(1-n)} \tag{7}$$

If $t < \frac{w^{1-n} - w_e^{1-n}}{\hbar(1-n)}$ then