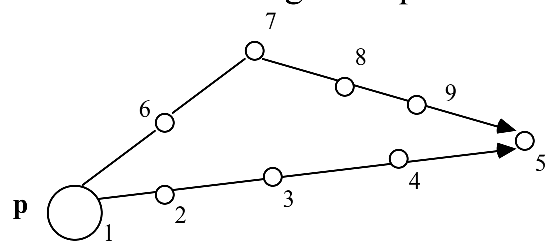
Admissible Trajectories

For example, a trajectory $a(1)a(2)a(3)a(4)a(5)$ is the shortest trajectory. Reasoning informally, an analogy can be set up: the shortest trajectory is an analogous to a straight line segment connecting two points in a plane. Let us consider an analogy to a **k-element segmented line** connecting these points.



DEFINITION

An ***admissible trajectory of degree k***

is the trajectory t_o which can be divided into k shortest trajectories; more precisely, there exists a subset $\{x_{i1}, x_{i2}, \ldots, x_{ik-1}\}$ of $\boldsymbol{P}(t_o)$, $i_1<i_2<\ldots<i_{k-1}$, $k \le l$, such that corresponding substrings $a(x_o)\ldots a(x_{i1})$, $a(x_{i1})\ldots a(x_{i2})$, …, $a(x_{ik-1})\ldots a(x_l)$ are the shortest trajectories.

Shortest and admissible trajectories of degree 2 play a special role in many problems. Obviously, every shortest trajectory is an admissible trajectory at the same time, but of course, inverse statement is not true. There exist admissible trajectories, e.g., of degree 2, which are not shortest. An example of such a trajectory $a(1)a(6)a(7)a(8)a(9)a(5)$ is shown in the figure above. As a rule, elements of the System should move along the shortest paths. In case of an obstacle, the element should move around this obstacle by tracing some intermediate point aside (e.g. point 7) and going to and from this point to the end along the shortest trajectories. Thus, in this case, an element should move along the admissible trajectory of degree 2.

Grammar $G_t^{(2)}$ of shortest and admissible trajectories.

L	*Q*	Kernel, π_k	π_n	F_T	F_F
1	Q_1	$S(x,y,l) \to A(x,y,l)$		*two*	ø
2_i	Q_2	$A(x,y,l) \to A(x, med_i(x,y,l), lmed_i(x,y,l))$ $A(med_i(x,y,l), y, l - lmed_i(x,y,l))$		*three*	*three*
3_j	Q_3	$A(x,y,l) \to a(x)A(next_j(x,l), y, f(l))$		*three*	4
4	Q_4	$A(x,y,l) \to a(y)$		*three*	5
5	Q_5	$A(x,y,l) \to e$		*three*	ø

$V_T = \{a\}$,
$V_N = \{S, A\}$,
V_{PR}

Pred $= \{Q_1, Q_2, Q_3, Q_4, Q_5\}$

$Q_1(x, y, l) = (MAP_{x,p}(y) \le l) \wedge (l < 2n)$

$Q_2(x, y, l) = (MAP_{x,p}(y) \ne l)$

$Q_3(x, y, l) = (MAP_{x,p}(y) = l) \wedge (l \ge 1)$

$Q_4(y) = (y = y_o)$

$Q_5(y) = (y \ne y_o)$

Var $= \{x, y, l\}$;
Con $= \{x_o, y_o, l_o, p\}$;

Func $=$ *Fcon* $\cup$ *Fvar*;

Fcon $= \{f, next_1, \ldots, next_n, med_1, \ldots, med_n, lmed_1, \ldots, lmed_n\}$ $(n = |X|)$, $f(l) = l - 1$, $D(f) = \mathbf{Z}_+$
functions $next_i$, med_i and $lmed_i$ defined below.

Fvar $= \{x_o, y_o, l_o, p\}$

E $= \mathbf{Z}_+ \cup X \cup P$ is the subject domain;

Parm: $S \to Var$, $A \to Var$, $a \to \{x\}$;
L $= \{1,4\} \cup two \cup three$, $two = \{2_1, 2_2, \ldots, 2_n\}$, $three = \{3_1, 3_2, \ldots, 3_n\}$

At the beginning of derivation:
$x = x_o$, $y = y_o$, $l = l_o$, $x_o \in X$, $y_o \in X$, $l_o \in \mathbf{Z}_+$, $p \in P$.

Definition of functions *med*, *lmed*, *next*

med$_i$ is defined as follows:

$D(med_i) = X \times X \times \mathbf{Z}_+ \times P$

DOCK = $\{v \mid v$ from X, $MAP_{x_o,p}(v) + MAP_{y_o,p}(v) = l\}$,

If

$DOCK_l(x) = \{v_1, v_2, ..., v_m\} \neq \varnothing$

then

$med_i(x, y, l) = v_i$ for $1 \le i \le m$ and

$med_i(x, y, l) = v_m$ for $m < i \le n$,

otherwise

$med_i(x, y, l) = x$.

lmed$_i$ is defined as follows:

$D(med_i) = X \times X \times \mathbf{Z}_+ \times P$

$lmed_i(x, y, l) = MAP_{x,p}(med_i(x, y, l))$

next$_i$ is defined as follows:

$D(next_i) = X \times \mathbf{Z}_+ \times X^2 \times \mathbf{Z}_+ \times P$

SUM = $\{v \mid v$ from X, $MAP_{x_o,p}(v) + MAP_{y_o,p}(v) = l_o\}$,

$ST_k(x) = \{v \mid v$ from X, $MAP_{x,p}(v) = k\}$,

$MOVE_l(x)$ is an intersection of the following sets:

$ST_1(x)$, $ST_{l_o - l + 1}(x_o)$ and SUM.

If

$MOVE_l(x) = \{m_1, m_2, ..., m_r\} \neq \varnothing$

then

$next_i(x, l) = m_i$ for $i \le r$ and

$next_i(x, l) = m_r$ for $r < i \le n$,

otherwise

$next_i(x, l) = x$.

THEOREM

All the admissible trajectories $t_p(x_o, y_o, l_o)$ of degree 2 from the point x_o to the point y_o of the length l_o for the element p on x_o can be generated by the grammar $G_t^{(2)}$.

Consider in detail application of such a grammar to generating trajectories in cases of visible and invisible obstacles. The difference between these two types of obstacles is as follows. Visible obstacles can be considered beforehand and represented as restricted areas. Invisible obstacles display themselves only during the motion of elements along the trajectories, and after being encountered, require new (usually longer) trajectories to be generated and examined.

We shall apply this grammar for generating trajectories for the robot from point h8 to point c6. The motion space for this robot is the square table of 8x8 with the restricted area. Let us consider the derivation of the shortest trajectory from h8 to point c6 for the robot K. Values of $MAP_{h8,K}$ are shown in figures. The restricted area shown in figure represents visible obstacles. Thus, the distance from h8 to c6 is equal to 5. Applying the grammar $G_t^{(2)}$ we have (symbol $\overset{l}{\longrightarrow}$ means application of the production with the label l):

$$\boldsymbol{S}(h8, c6, 5) \overset{1}{\longrightarrow} \boldsymbol{A}(h8, c6, 5) \overset{3_1}{\longrightarrow} \boldsymbol{a}(h8)\boldsymbol{A}(next_1(h8, 5), c6, 5)$$

Thus we have to compute MOVE (see definition of the function $next_i$ from the grammar $G_t^{(2)}$). First we have to determine the set of SUM, i.e., we need to know values of $MAP_{h8,K}$ and $MAP_{c6,K}$ on X. Adding these tables as matrices we compute

$$SUM = \{v \mid v \text{ from } X, MAP_{h8,K}(v)+MAP_{c6,K}(v)=5\}.$$

The next step is the computation of $ST_1(h8)=\{v \mid v \text{ from } X, MAP_{h8,K}(v)=1\}$. A result is the only point g8. In order to complete computation of the set $MOVE_5(h8)$ we have to determine the following intersection:

$$ST_1(h8),\ ST_{5-5+1}(h8) = ST_1(h8) \text{ and } SUM$$

Consequently, $MOVE_5(h8)=\{g8\}$; and $next_1(h8, 5)=g8$. Since the number of different values of *next* is equal to 1 we can not branch here. Let us proceed with the derivation.

$$\boldsymbol{a}(h8)\boldsymbol{A}(g8, f8, 4) \overset{2_1}{\longrightarrow} \boldsymbol{a}(h8)\boldsymbol{a}(g8)\boldsymbol{A}(next_1(g8, 4), c6, 3)$$

We have to compute $next_1(g8, 4)$. Obviously $next_1(g8, 4)=f8$.

$$\boldsymbol{a}(h8)\boldsymbol{a}(g8)\boldsymbol{A}(f8, c6, 3) \overset{2_1}{\longrightarrow} \boldsymbol{a}(h8)\boldsymbol{a}(g8)\boldsymbol{a}(f8)\boldsymbol{A}(next_1(f8, 3), c6, 2)$$

As on the preceding step we have to determine $MOVE_3(f8)$. To do that we have to compute

$$ST_1(f8)=\{v \mid v \text{ from } X, MAP_{f8,K}(v)=1\} \text{ and}$$

$$ST_{5-3+1}(h8)=ST_3(h8)=\{v \mid v \text{ from } X, MAP_{h8,K}(v)=3\}.$$

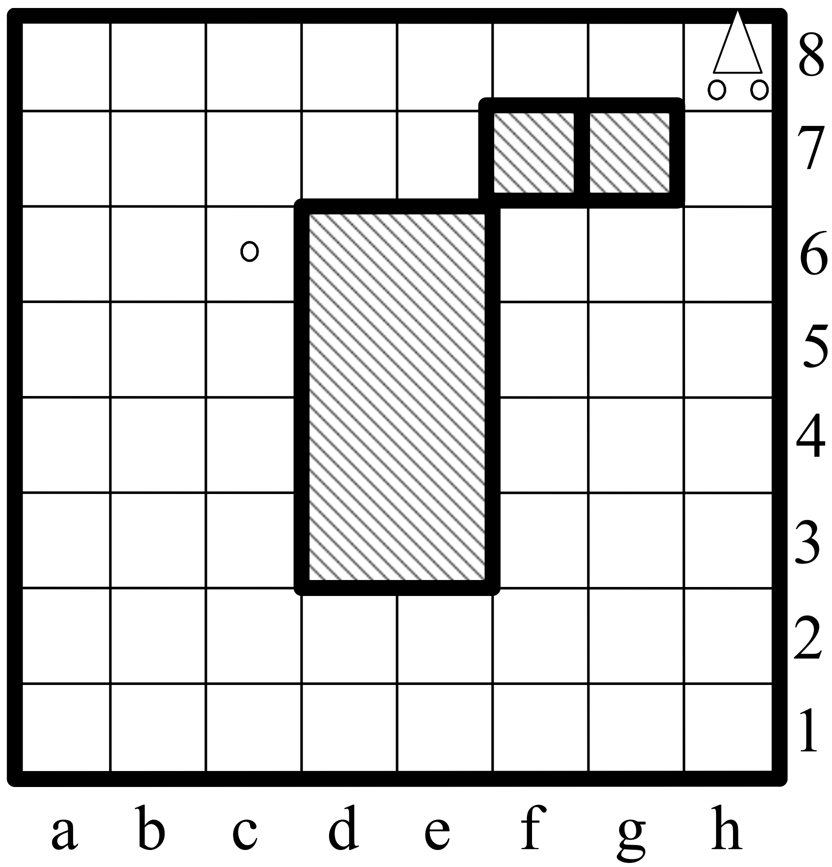
The set of SUM is the same on all steps of the derivation. Hence, $MOVE_3(f8)$ is the intersection of the sets shown in Figures, $MOVE_4(f8) = \{e7, e8\}$; and

$$next_1(f8, 3) = e7;\ next_2(f8, 3) = e8.$$

Thus, the number of different values of the function *next* is equal to 2 (r=2), so the number of continuations of derivation should be multiplied by 2; two shortest trajectories exist. This way, eventually, we will derive both of the shortest trajectories for the robot K from h8 to c6:

$$\boldsymbol{a}(h8)\boldsymbol{a}(g8)\boldsymbol{a}(f8)\boldsymbol{a}(e7)\boldsymbol{a}(d7)\boldsymbol{a}(c6) \text{ and } \boldsymbol{a}(h8)\boldsymbol{a}(g8)\boldsymbol{a}(f8)\boldsymbol{a}(e8)\boldsymbol{a}(d7)\boldsymbol{a}(c6).$$

Computation of shortest and admissible trajectories



Values of $MAP_{h8,K}$

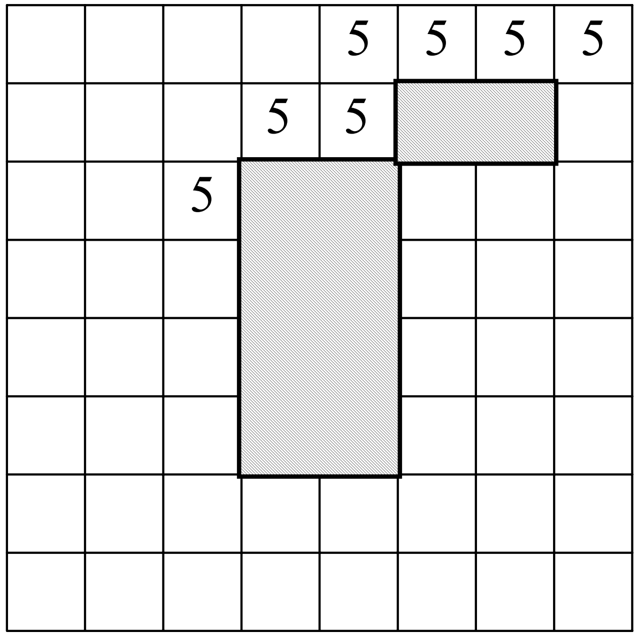
7	6	5	4	3	2	1	0
7	6	5	4	3			1
7	6	5			3	2	2
7	6	6			3	3	3
7	7	7			4	4	4
8	8	8			5	5	5
9	9	8	7	6	6	6	6
10	9	8	7	7	7	7	7

restricted area

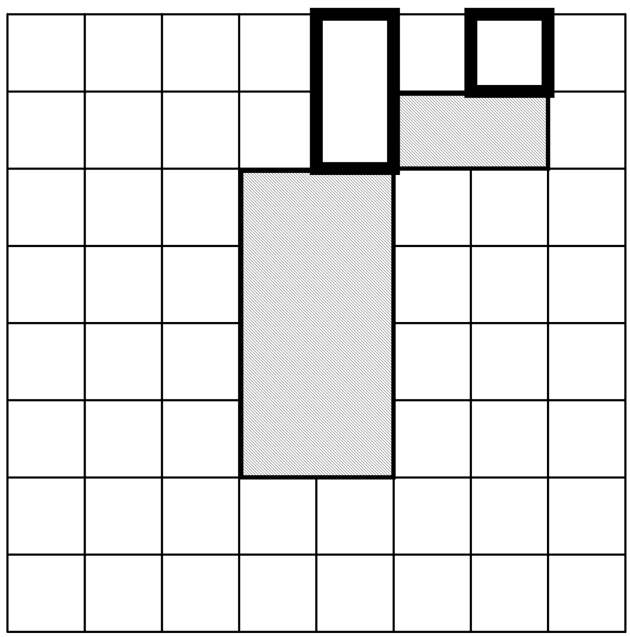
Values of $MAP_{c6,K}$

2	2	2	2	2	3	4	5
2	1	1	1	2			5
2	1	0			3	4	5
2	1	1			4	4	5
2	2	2			5	5	5
3	3	3			6	6	6
4	4	4	4	5	6	7	7
5	5	5	5	5	7	7	8

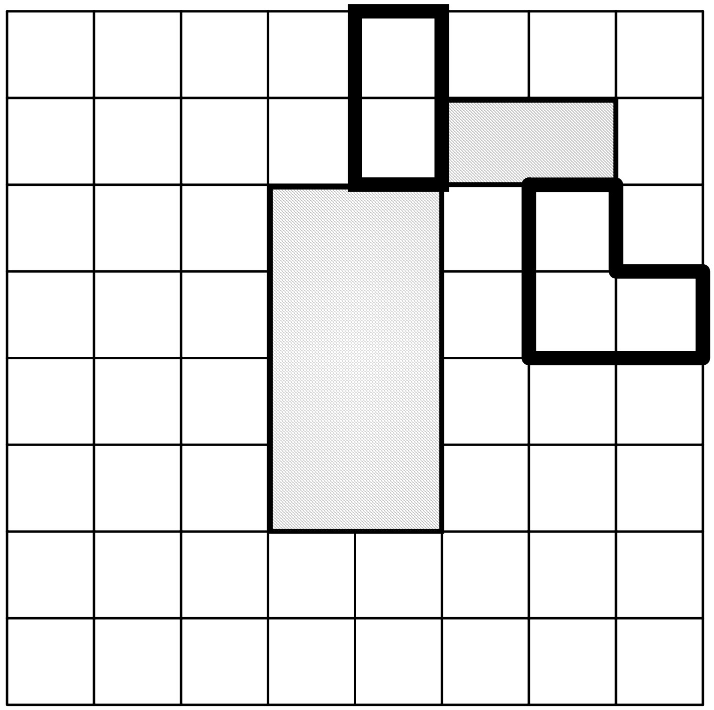
Points of X which belong to SUM.



The set of ST_1(f8)



The set of $ST_3(h8)$



Computation of admissible trajectories

Let us assume that after examining these shortest paths robot found out that both of them are not satisfactory because of invisible obstacles. So our robot is looking for different paths: they can be longer trajectories only. Applying grammar $G_t^{(2)}$ with $l = 6$ we have

$$\boldsymbol{S}(h8, c6, 6) \xrightarrow{1} \boldsymbol{A}(h8, c6, 6)$$

Then we try to apply one of the productions 2_i, check $\boldsymbol{Q_2} = (MAP_{h8,K}(c6)=5)$, i.e., $\boldsymbol{Q_2} \neq F$. Hence

$$\boldsymbol{A}(h8, c6, 6) \xrightarrow{2_1} \boldsymbol{A}(h8, med_i(h8, c6, 6), \ lmed_i(h8, c6, 6))$$
$$\boldsymbol{A}(med_i(h8, c6, 6), c6, 6\text{-}lmed_i(h8, c6, 6))$$

Thus we have to compute med_i and $lmed_i$ (the definitions of these functions are included into the definition of the grammar $G_t^{(2)}$). First we have to determine the set of DOCK, i.e., we have to know values of $MAP_{h8,K}$ and $MAP_{c6,K}$ on X. Adding these tables as matrices we compute

$$DOCK = \{v \mid v \text{ from } X, MAP_{h8,K}(v)+MAP_{c6,K}(v)=6\}.$$

This set represents the attaching points of admissible trajectories of degree 2. There are five attaching points: DOCK={f6, g6, c7, d8, h7}, and

$$med_1(h8, c6, 6)=c7, med_2(h8, c6, 6)=d8, med_3(h8, c6, 6)=h7,$$
$$med_2(h8, c6, 6)=f6, med_3(h8, c6, 6)=g6.$$

Thus, the number of different values of the function *med* is equal to 5 (m=5), so the number of continuations of derivation should be multiplied by 5; there exist five bundles of admissible trajectories coming through points f6, g6, c7, d8, and h7. Some of them may coincide with each other.

Let us continue the derivation of the trajectories coming through h7. According to definition of *lmed* and $MAP_{h8,K}$:

$$lmed_3(h8, c6, 6)= MAP_{h8,K}(med_3(h8, c6, 6))=1;$$
$$l - lmed_3(h8, c6, 6)=5.$$

It means that shortest trajectories forming admissible trajectories from h8 to c6 through h7 are as follows: the trajectories which come from h8 to h7 are of length 1, and from h7 to c6 — are of length 5. Proceeding with the derivation we have:

$$\boldsymbol{A}(h8, c6, 6) \xrightarrow{2_1} \boldsymbol{A}(h8, h7, \ 1)\boldsymbol{A}(h7, c6, 5),$$

i.e., each of nonterminals $\boldsymbol{A}$ corresponds to the bundle of shortest trajectories coming to and from the attaching point h7. Next we apply one of the productions 3_j:

$$\boldsymbol{A}(h8, h7, \ 1)\boldsymbol{A}(h7, c6, 5) \xrightarrow{3_j} \boldsymbol{a}(h8)\boldsymbol{A}(next_j(h8, 1), h7, 0)\boldsymbol{A}(h7, c6, 5)$$

Obviously, the number of different values of *next* is equal to 1 and $next_1(h8, 1) = h7$:

$$\boldsymbol{a}(h8)\boldsymbol{A} \ (h7, h7, 0)\boldsymbol{A}(h7, c6, 5).$$

Then we try to apply production 3_1 again to nonterminal $\boldsymbol{A}$ (h7, h7, 0), but fail because $\boldsymbol{Q_3}(h7, h7, 0) = F$ (l =1), and we go to the production 4. Due to y = h7, i.e. $y \neq y_o$ (y_o=c6),

$\boldsymbol{Q_4}$ = F, and we fail applying this production. So according to the set of $\boldsymbol{F_F}$ we go to the production 5:

$$\boldsymbol{a}(\text{h8})\boldsymbol{A}\ (\text{h7, h7, 0})\boldsymbol{A}(\text{h7, c6, 5}) \ {}^{5}\!\!\longrightarrow \boldsymbol{a}(\text{h8})\boldsymbol{A}(\text{h7, c6, 5})$$

Next we apply one of the productions 3_j:

$$\boldsymbol{a}(\text{h8})\boldsymbol{A}(\text{h7, c6, 5}) \ {}^{3_j}\!\!\longrightarrow \boldsymbol{a}(\text{h8})\boldsymbol{a}(\text{h7})\boldsymbol{A}(next_j(\text{h7, 5}), \text{c6, 4}).$$

Thus, one of the shortest components is done. This is the trajectory $\boldsymbol{a}$(h8)$\boldsymbol{a}$(h7).

Obviously, function $next_j$(h7, 5) yields the only value g6, and proceeding with derivation we have:

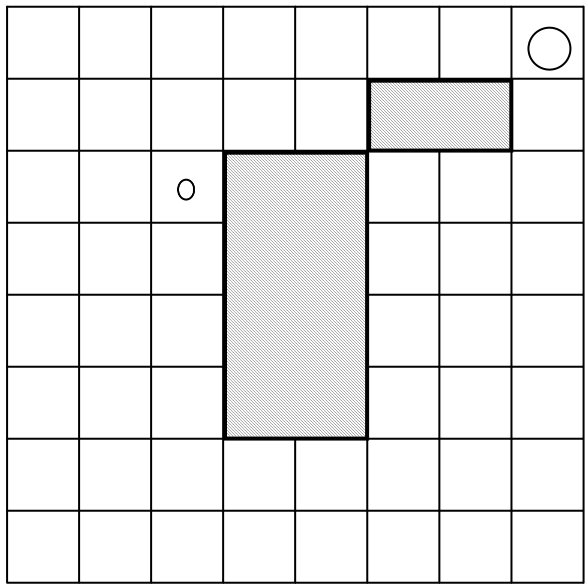
$$\boldsymbol{a}(\text{h8})\boldsymbol{a}(\text{h7})\boldsymbol{A}(next_1(\text{h7, 5}), \text{c6, 4}) = \boldsymbol{a}(\text{h8})\boldsymbol{a}(\text{h7})\boldsymbol{A}(\text{g6, c6, 4}).$$

At the conclusion of derivation we will get the admissible trajectory (Fig. 20):

$$\boldsymbol{a}(\text{h8})\boldsymbol{a}(\text{h7})\boldsymbol{a}(\text{g6})\boldsymbol{a}(\text{f6})\boldsymbol{a}(\text{e7})\boldsymbol{a}(\text{d7})\boldsymbol{a}(\text{c6}).$$

This trajectory is quite different of the trajectories generated earlier. It might be free of invisible obstacles. If it is not true, robot K should generate longer trajectories and try to move along them.

Computation of admissible trajectories of degree 2 of the length 6



COMPUTATION OF DOCK

Values of $MAP_{h8,K}$

7	6	5	4	3	2	1	0
7	6	5	4	3			1
7	6	5			3	2	2
7	6	6			3	3	3
7	7	7			4	4	4
8	8	8			5	5	5
9	9	8	7	6	6	6	6
10	9	8	7	7	7	7	7

Values of $MAP_{c6,K}$

2	2	2	2	2	3	4	5
2	1	1	1	2			5
2	1	0			3	4	5
2	1	1			4	4	5
2	2	2			5	5	5
3	3	3			6	6	6
4	4	4	4	5	6	7	7
5	5	5	5	5	7	7	8

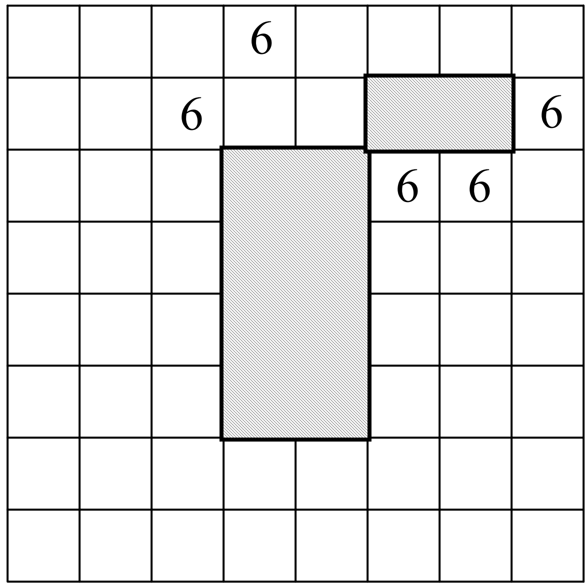
Values of $MAP_{h8,K}$

7	6	5	4	3	2	1	0
7	6	5	4	3			1
7	6	5			3	2	2
7	6	6			4	3	3
7	7	7			4	4	4
8	8	8			5	5	5
9	9	8	7	6	6	6	6
10	9	8	7	7	7	7	7

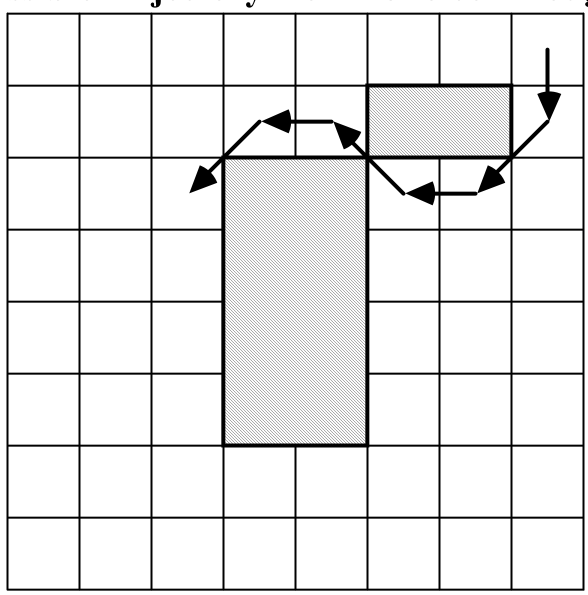
Values of $MAP_{c6,K}$

2	2	2	2	2	3	4	5
2	1	1	1	2			5
2	1	0			3	4	5
2	1	1			4	4	5
2	2	2			5	5	5
3	3	3			6	6	6
4	4	4	4	5	6	7	7
5	5	5	5	5	7	7	8

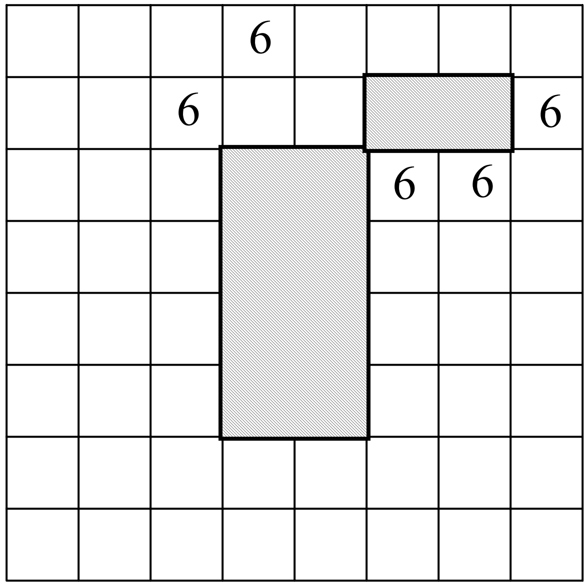
The set of DOCK



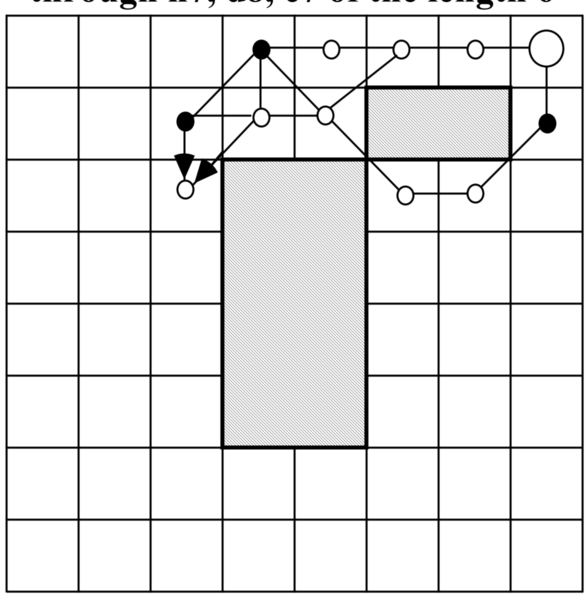
Admissible trajectory from h8 to c6 through h7

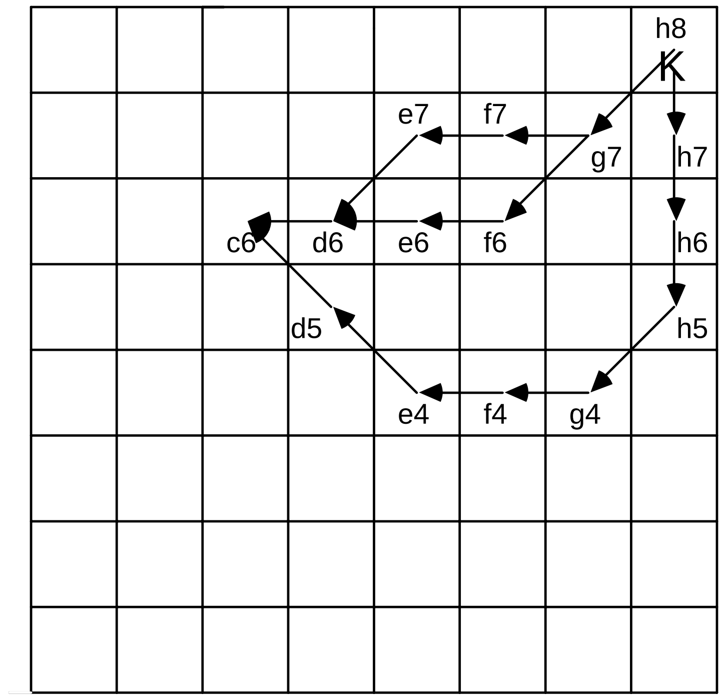


The set of DOCK



Admissible trajectories of degree 2 from h8 to c6 through h7, d8, c7 of the length 6





Shortest trajectories of the length **5** and
admissible trajectory of degree 2 of the length **8** (through g4).