

Zones trajectories (<i>l</i> , τ)	TRANSITION(<i>p</i> , <i>x</i> ₀ , <i>x</i> ₁)					
	(<i>q</i> ₁ , 10, 11)	(<i>p</i> ₀ , 1, 2)	(<i>q</i> ₁ , 11, 12)	(<i>p</i> ₀ , 2, 3)	(<i>q</i> ₂ , 8, 9)	(<i>p</i> ₀ , 3, 4)
Z1, <i>a</i> (10) <i>a</i> (11) <i>a</i> (12) <i>a</i> (9) (3, 3)	(2,3)	(2, 2)	(1,2)	(1, 1)	Freeze	
Z2, <i>a</i> (10) <i>a</i> (13) <i>a</i> (14) <i>a</i> (9) (3, 3)	Freeze					
Z2 <i>a</i> (17) <i>a</i> (14) (1, 1)	Freeze					
Z1, Z2 <i>a</i> (15) <i>a</i> (16) <i>a</i> (9) (2, 3)	(2, 3)	(2, 2)	(2, 2)	(2, 1) Freeze		
Z1, Z2 <i>a</i> (18) <i>a</i> (9) (1, 3)	(1, 3)	(1, 2)	(1, 2)	(1, 1)	Freeze	
Z1, Z2 <i>a</i> (8) <i>a</i> (9) <i>a</i> (4) (2, 4)	(2, 4)	(2, 3)	(2, 3)	(2, 2)	(1, 2)	(1, 1)
Z1, Z2 <i>a</i> (6) <i>a</i> (7) <i>a</i> (4) (2, 4)	(2, 4)	(2, 3)	(2, 3)	(2, 2)	(2, 2)	(2, 1) Freeze

Time distribution function $timer_{\pi}$

Let $\pi_{M_0}(Z_1) = Z_2$ be a translation, with $Z_1 = t(p_0, t_0, \tau_0)t(p_1, t_1, \tau_1)...t(p_r, t_r, \tau_r)$, $Z_1 \in L_Z(S_1)$, $Z_2 \in L_Z(S_2)$. A mapping

$$timer_{\pi}: Con_{\Pi}(Z_1) \rightarrow \mathbf{Z},$$

where \mathbf{Z} is the set of all integer numbers, is constructed as follows. We consider three cases:

- (1) If $\Pi_{M_0}(t_0) = t_0'$, i.e., the main trajectory of Zone Z_1 is shortened, that is transformed into a substring with an excluded first symbol, then for all symbols $t(p_c, t_c, \tau) \in Con_{\Pi}(Z_1)$

$$timer_{\pi}(t(p_c, t_c, \tau)) = \tau - 1.$$

- (2) If $\Pi_{M_0}(t_k) = t_k'$, i.e., some other trajectory t_k of Zone Z_1 is shortened ($k \neq 0$), then we define $timer_{\pi}$ recursively.

(a) $timer_{\pi}(t(p_0, t_0, \tau_0)) = \tau_0$,
 $timer_{\pi}(t(p_i, t_i, \tau_i)) = \tau_i$ (if $C_{TA(Z_1)}(t_i, t_0) = T$)

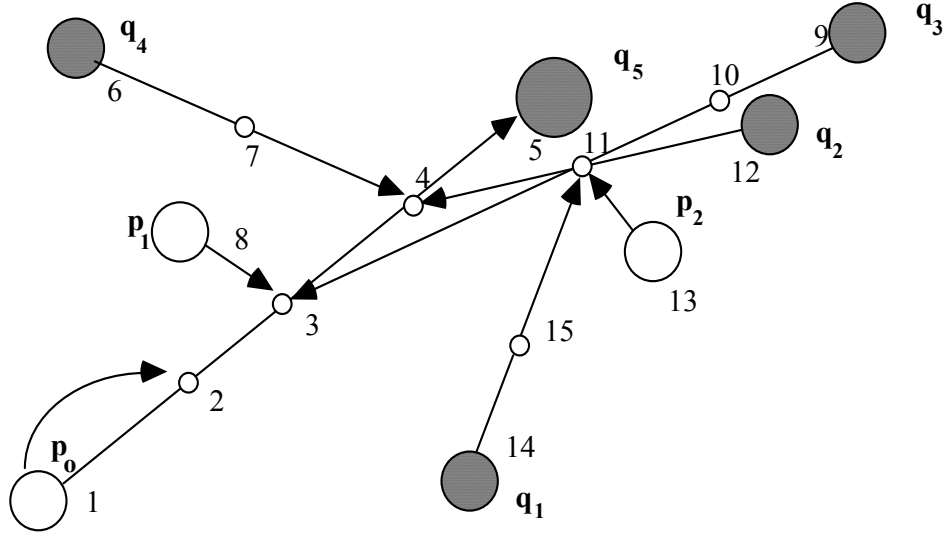
- (b) Let $t(p_c, t_c, \tau) \in Con_{\Pi}(Z_1)$,
denote $CA(t_c) = \{t_i \in Con_{\Pi}(Z_1) \mid C(t_c, t_i) = T\}$,
then $timer_{\pi}(t(p_c, t_c, \tau)) = \max \{TNEW(t_i)\}$, where

$$TNEW(t_i) = \begin{cases} timer_{\pi}(t(p_i, t_i, \tau_i)) - len(p_i, t_i) + 1, & \text{if } t_i \neq t_k, \\ (timer_{\pi}(t(p_i, t_i, \tau_i)) + 1) - len(p_i, t_i) + 1, & \text{if } t_i = t_k, \end{cases}$$

($len(p_i, t_i)$ is the length of t_i).

- (3) If $\Pi_{M_0}(t_m) = t_m$ for all $t_m \in TA(Z_1)$, then $timer_{\pi}(t(p_c, t_c, \tau)) = \tau$.

Interpretation of function $timer_{\pi}$



(1) $M_0 = \text{TRANSITION}(p_0, 1, 2)$

It means that function $timer_{\pi}$ for all the symbols of $A(Z_1)$ yields the value of $\tau - 1$, where τ is the value the third parameter of each symbol. For example,

$$timer_{\pi}(t(q_3, t_{q_3}, \tau)) = \tau - 1,$$

$$\text{where } t_{q_3} = a(9)a(10)a(11)a(3), \tau = 3.$$

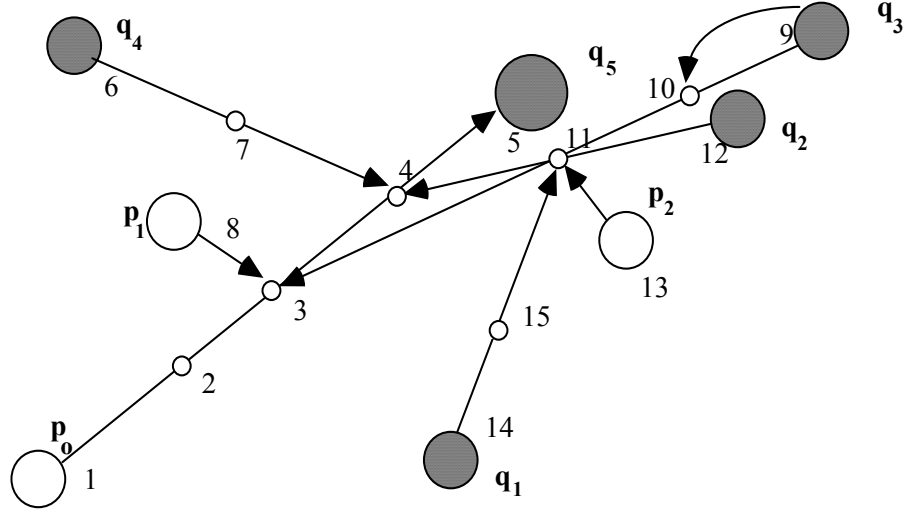
After $\text{TRANSITION}(p_0, 1, 2)$ time allocated to the motion along trajectory t_{q_3} is less than the length of this trajectory ($2 < 3$) and, thus, trajectory t_{q_3} should not be included into the translation $\pi_{M_0}(Z_1)$ of Zone Z_1 .

For the trajectory $t_{q_1} = a(14)a(15)a(11)$ connected with t_{q_3}

$$timer_{\pi}(t(q_1, t_{q_1}, 3)) = 2.$$

It means that its length does not exceed the time allocated for the motion and, consequently, t_{q_1} should be included into Z_2 . In spite of losing the C^+ connection with t_0 through t_{q_3} (which is not included), trajectory t_{q_1} keeps the C^+ connection with t_0 through t_{q_2} .

Interpretation of function timer_π (continued)



(2) $M_0 = \text{TRANSITION}(q_3, 9, 10)$

(a) For the main trajectory $\text{timer}_\pi(t(p_0, t_0, \tau)) = 5$,

for the 1st negation trajectories:

$$\text{timer}_\pi(t(p_1, t_{p_1}, \tau)) = 3, \text{timer}_\pi(t(q_2, t_{q_2}, \tau)) = 4,$$

$$\text{timer}_\pi(t(q_3, t_{q_3}, \tau)) = 3, \text{timer}_\pi(t(q_4, t_{q_4}, \tau)) = 4.$$

After transition M_0 elements q_2, q_3, q_4 still have enough time for interception of p_0 .

For the 2nd negation trajectories t_{q_1} and t_{p_2} we have case

(b) for both trajectories $\text{CA}(t_{q_1}) = \text{CA}(t_{p_2}) = \{t_{q_2}, t_{q_3}\}$. Then

$$\text{timer}_\pi(t(q_1, t_{q_1}, \tau)) = \max\{\text{TNEW}(t_{q_2}), \text{TNEW}(t_{q_3})\} = \max\{3, 2\} = 3,$$

where

$$\text{TNEW}(t_{q_2}) = \text{timer}_\pi(t(q_2, t_{q_2}, \tau)) - 2 + 1 = 4 - 2 + 1 = 3,$$

$$\text{TNEW}(t_{q_3}) = (\text{timer}_\pi(t(q_3, t_{q_3}, \tau)) + 1) - 3 + 1 = (3 + 1) - 3 + 1 = 2$$

Consequently, $\text{timer}_\pi(t(q_1, t_{q_1}, \tau)) = 3$. Thus because the length of t_{q_1} does not exceed the value of $\text{timer}_\pi(2 < 3)$ it should be included into the translation.

Theorem about Translations

THEOREM: *Let for a translation $\pi_{M_0}(Z_1) = Z_2$. Under certain constraints for every symbol $t(p, t_i, \tau) \in \text{Con}_{\Pi}(Z_1)$, (where $t_i \in t_p(x, y, l)$, $l > 1$)*

$$\pi_o(t(p, t_i, \tau)) = t(p, \Pi_{M_0}(t_i), \text{timer}_p(t(p, t_i, \tau)))$$

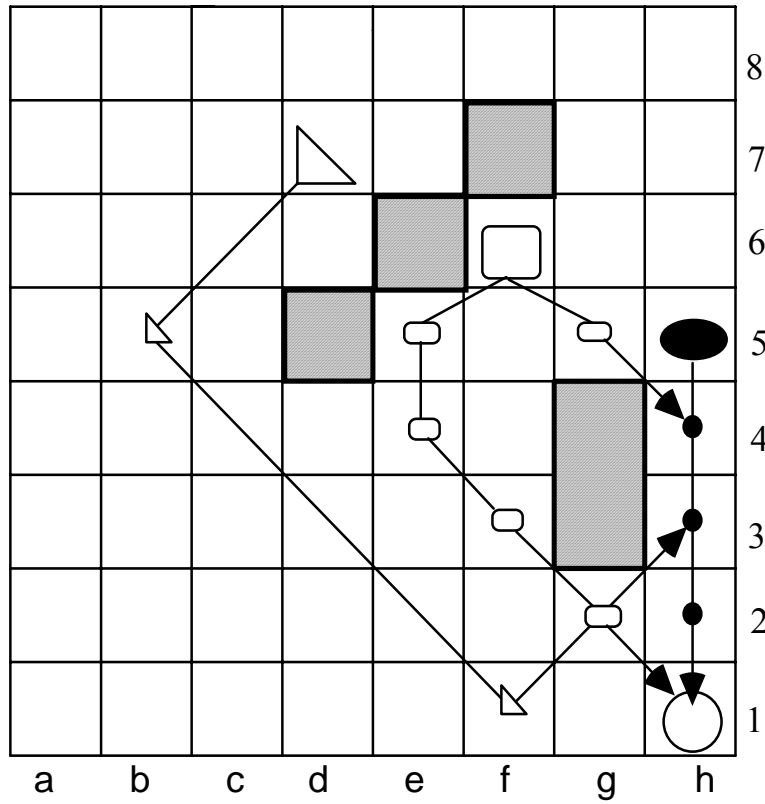
is a mapping onto $A(Z_2) \cap \Pi(\text{Con}_{\Pi}(Z_1))$, if and only if $l \leq \text{timer}_{\pi}(t(p, t_i, \tau))$.

Interpretation of the trajectory network language for the robot control model

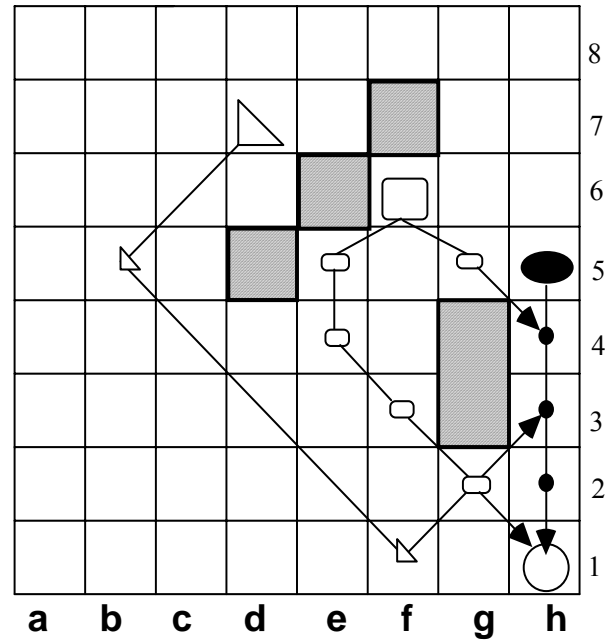
$$t(\text{BOMBER}, t_B, 5)t(\text{FIGHTER}, t_F, 5)t(\text{MISSILE}, t_M, 5)t(\text{MISSILE}, t_M^1, 3) \\ t(\text{FIGHTER}, t_F^1, 2),$$

where

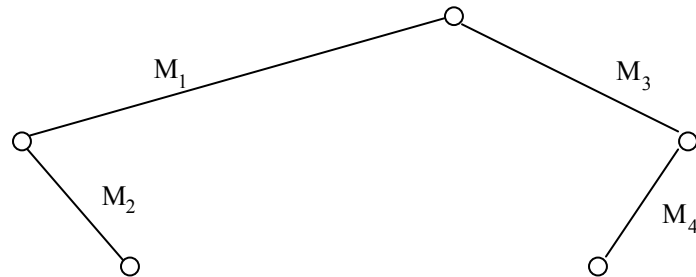
$$t_B = a(h5)a(h4)a(h3)a(h2)a(h1), \\ t_F = a(f6)a(e5)a(e4)a(f3)a(g2)a(h1), \\ t_M = a(d7)a(b5)a(f1)a(g2)a(h1), \\ t_M^1 = a(d7)a(b5)a(f1)a(h3), \\ t_F^1 = a(f6)a(g5)a(h4)$$



Translations for Robot Control Model



TRANSITIONS = $\{M_1, M_2, M_3, M_4\}$.



$M_1 = \text{TRANSITION}(\text{MISSILE}, d7, b5)$

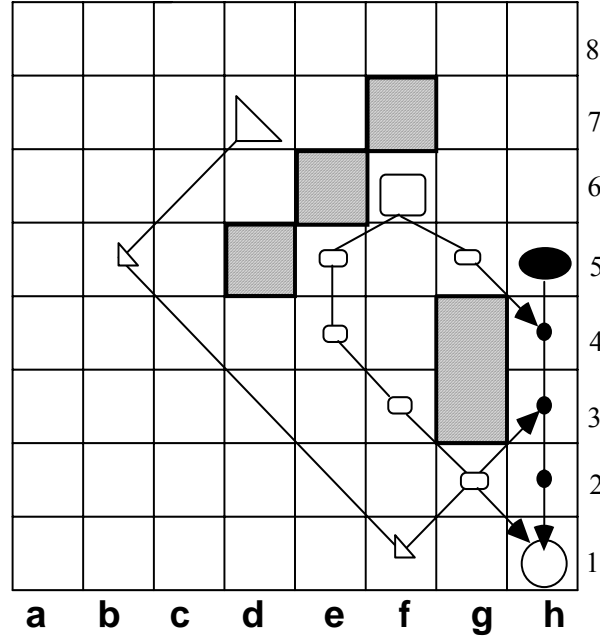
$M_2 = \text{TRANSITION}(\text{BOMBER}, h5, h4)$

$M_3 = \text{TRANSITION}(\text{FIGHTER}, f6, e5)$

$M_4 = \text{TRANSITION}(\text{BOMBER}, h5, h4)$

Translations

$M_1 = \text{TRANSITION}(\text{MISSILE}, d7, b5)$



$$\pi_o(t(\text{BOMBER}, t_B, 5)) = t(\text{BOMBER}, \Pi_{M_1}(t_B), \text{timer}_\pi(t(\text{BOMBER}, t_B, 5))) = t(\text{BOMBER}, t_B, 5)$$

$$\begin{aligned} \pi_o(t(\text{FIGHTER}, t_F, 5)) &= t(\text{FIGHTER}, t_F, 5), \\ \pi_o(t(\text{FIGHTER}, t_F^1, 2)) &= t(\text{FIGHTER}, t_F^1, 2), \\ \pi_o(t(\text{MISSILE}, t_M, 5)) &= t(\text{MISSILE}, \Pi_{M_1}(t_M), 5), \\ \pi_o(t(\text{MISSILE}, t_M^1, 3)) &= t(\text{MISSILE}, \Pi_{M_1}(t_M^1), 3), \end{aligned}$$

where $\Pi_{M_1}(t_M)$ and $\Pi_{M_1}(t_M^1)$ are shortened trajectories with excluded first symbol, i.e.,

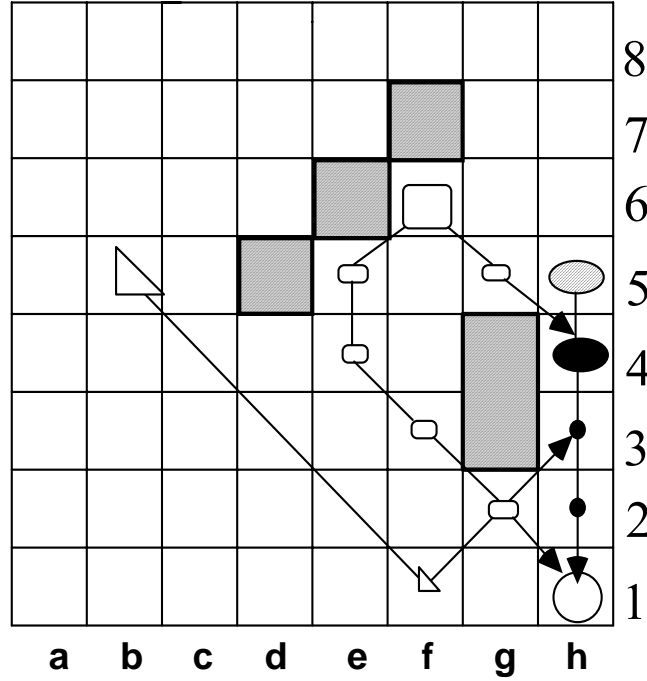
$$\Pi_{M_1}(t_M) = t_{M,s} = a(b5)a(f1)a(g2)a(h1),$$

$$\Pi_{M_1}(t_M^1) = t_{M,s}^1 = a(b5)a(f1)a(h3).$$

Lengths of all the 1-st negation trajectories of this Zone after the translation do not exceed values of timer_π , consequently, according to Theorem, all these trajectories should be included into the new Zone $Z_1 = \pi_{M_1}(Z_0)$.

Translations (continued)

$M_2 = \text{TRANSITION}(\text{BOMBER}, h5, h4).$



$$\pi_0(t(\text{BOMBER}, t_{B,s})) = t(\text{BOMBER}, \Pi_{M_2}(t_B), 4)$$

$$\pi_0(t(\text{MISSILE}, t_{M,s}, 5)) = t(\text{MISSILE}, t_{M,s}, 4),$$

$$\pi_0(t(\text{MISSILE}, t_{M,s}^1, 3)) = t(\text{MISSILE}, t_{M,s}^1, 2),$$

where $\Pi_{M_2}(t_B) = t_{B,s} = a(h4)a(h3)a(h2)a(h1)$ is a shortened trajectory.

For BOMBER and MISSILE the following inequalities hold

$$\text{len}(\text{BOMBER}, t_{B,s}) = 3 < 4,$$

$$\text{len}(\text{MISSILE}, t_{M,s}) = 3 < 4,$$

$$\text{len}(\text{MISSILE}, t_{M,s}^1) = 2 \leq 2.$$

According to Theorem it means that trajectories $t_{B,s}$, $t_{M,s}$, $t_{M,s}^1$ of BOMBER and MISSILE should be included into the new Zone $Z_2 = \pi_{M_2}(Z_1)$, i.e., MISSILE has enough time to intercept BOMBER at h3 or h1.

$$t(\text{FIGHTER}, t_F, \text{timer}_{\pi}(t(\text{FIGHTER}, t_F, 5))) = t(\text{FIGHTER}, t_F, 4),$$

$$t(\text{FIGHTER}, t_F^1, \text{timer}_{\pi}(t(\text{FIGHTER}, t_F^1, 2))) = t(\text{FIGHTER}, t_F^1, 1),$$

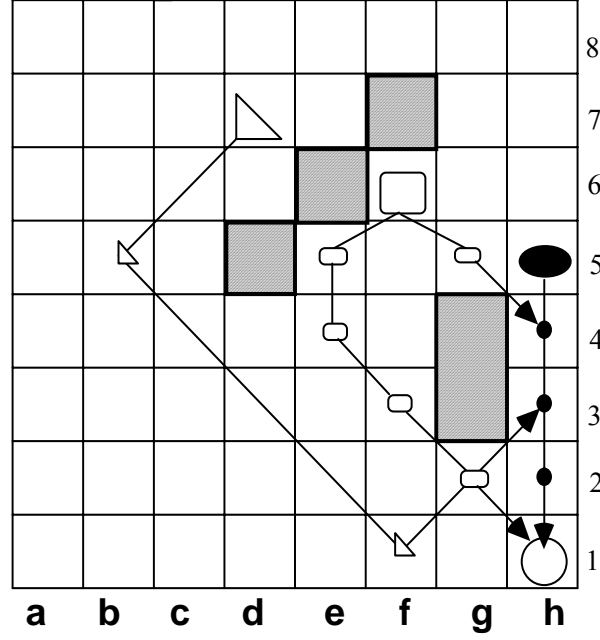
$$\text{len}(\text{FIGHTER}, t_F^1) = 2 > 1,$$

$$\text{len}(\text{FIGHTER}, t_F) = 5 > 4,$$

which means that trajectories t_F^1 , t_F of FIGHTER are not included into the new Zone Z_2 . Indeed, after transition M_2 FIGHTER does not have enough time for interception of BOMBER at h4 or at h1. In addition t_F^1 loses connection to $t_{B,s}$.

Translations (continued)

$M_3 = \text{TRANSITION}(\text{FIGHTER}, f6, e5)$



$$\pi_0(t(\text{BOMBER}, t_B, 5)) = t(\text{BOMBER}, t_B, 5)$$

For all the 1-st negation trajectories we obtain

$$\pi_0(t(\text{FIGHTER}, t_F, 5)) = t(\text{FIGHTER}, \Pi_{M_3}(t_F), 5),$$

$$\pi_0(t(\text{MISSILE}, t_M, 5)) = t(\text{MISSILE}, t_M, 5),$$

$$\pi_0(t(\text{MISSILE}, t_M^1, 3)) = t(\text{MISSILE}, t_M^1, 3),$$

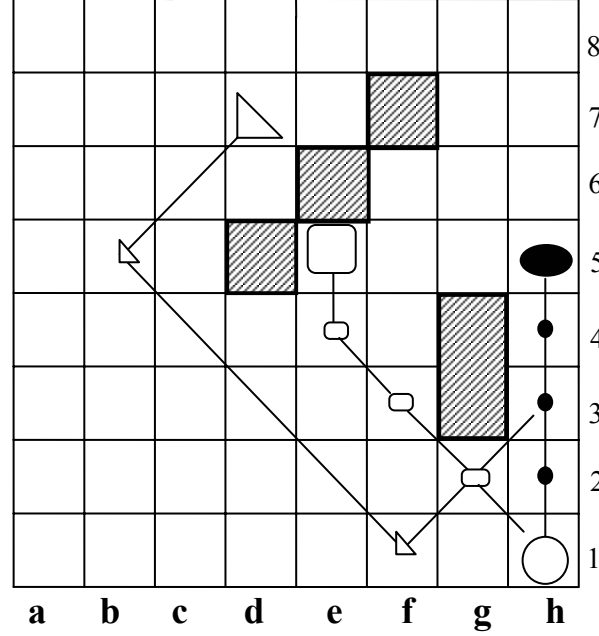
where $\Pi_{M_3}(t_F)$ is a shortened trajectory with excluded first symbol, i.e.,

$$\Pi_{M_3}(t_F) = t_{F,s} = a(e5)a(e4)a(f3)a(g2)a(h1).$$

Concerning $\Pi_{M_3}(t_F^1)$, we conclude that after transition M_3 t_F^1 loses the connection with the main trajectory t_B , $\Pi_{M_3}(t_F^1) = e$, hence $t_F^1 \notin \text{Con}_\Pi(Z_0)$. Lengths of the 1-st negation trajectories of this Zone, except for t_F^1 , after translation Π_{M_3} do not exceed values of **timer** π , consequently, according to the Theorem, all these trajectories should be included into the new Zone $Z_3 = \pi_{M_3}(Z_0)$. It means that both FIGHTER and MISSILE have enough time for interception.

Translations (continued)

$M_4 = \text{TRANSITION}(\text{BOMBER}, h5, h4)$



$$\pi_0(\mathbf{t}(\text{BOMBER}, t_B, 5)) = \mathbf{t}(\text{BOMBER}, \Pi_{M_4}(t_B), 4)$$

$$\pi_0(\mathbf{t}(\text{FIGHTER}, t_{F,s}, 5)) = \mathbf{t}(\text{FIGHTER}, t_{F,s}, 4),$$

$$\pi_0(\mathbf{t}(\text{MISSILE}, t_M, 5)) = \mathbf{t}(\text{MISSILE}, t_M, 4),$$

where $\Pi_{M_4}(t_B) = t_{B,s} = \mathbf{a}(h4)\mathbf{a}(h3)\mathbf{a}(h2)\mathbf{a}(h1)$ is a shortened trajectory. For BOMBER, FIGHTER and MISSILE the following inequalities hold

$$\text{len}(\text{BOMBER}, t_{B,s}) = 3 < 4,$$

$$\text{len}(\text{FIGHTER}, t_{F,s}) = 4 \leq 4,$$

$$\text{len}(\text{MISSILE}, t_M) = 4 \leq 4.$$

According to Theorem it means that trajectories $t_{B,s}$, $t_{F,s}$, t_M of BOMBER, FIGHTER and MISSILE should be included in the new Zone $Z_4 = \pi_{M_4}(Z_3)$, i.e., FIGHTER and MISSILE have enough time to intercept BOMBER at $h1$. But, considering trajectory t_M^1 of MISSILE, we have

$$\mathbf{t}(\text{MISSILE}, t_M^1, \text{timer} \pi(\mathbf{t}(\text{MISSILE}, t_M^1, 3))) = \mathbf{t}(\text{MISSILE}, t_M^1, 2),$$

$$\text{len}(\text{MISSILE}, t_M^1) = 3 > 2,$$

which means that this trajectory is not included in the new Zone Z_4 . Indeed, after transition M_4 MISSILE does not have enough time for interception of BOMBER at $h3$.

Translations for Robot Control Model

New Example

