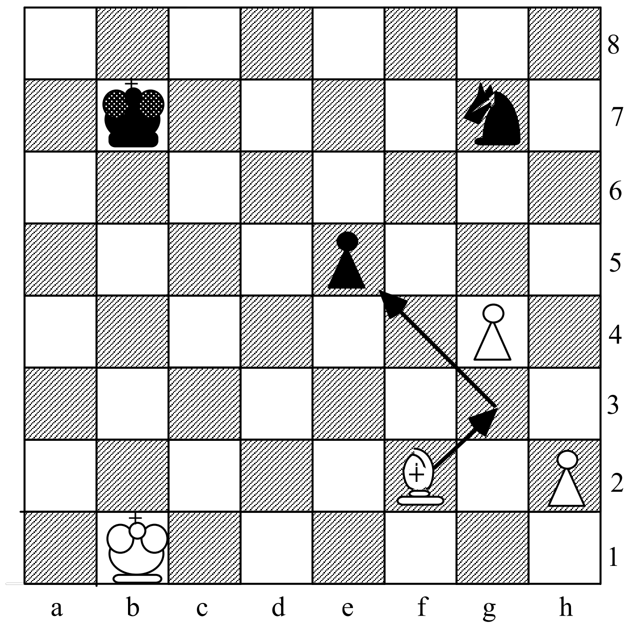
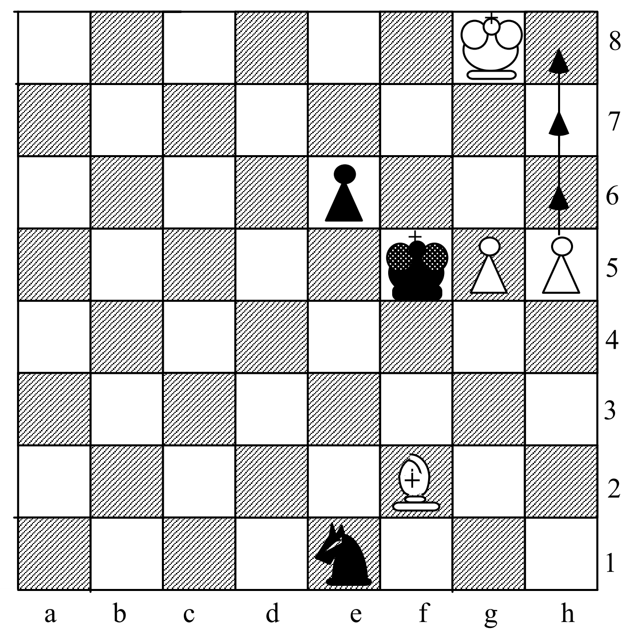
<u>Assignment 7. Due: 10/23/17</u>

15. **Generate motion in the Zone shown below.**
    Show three short variants (of 2-3 moves) employing different intercepting pieces. Show in details the translation of Zone after each move employing function *timer* considered in class.



See next page.

16. **Generate motion in the Zone shown below and represent the change of this Zone similarly to the table shown in the next lecture on page 1. You have to generate at least one variant with the final move such that the main element of the Zone (Pawn h5) is intercepted (and removed from the board) as a result of this move.**



Representation of *motion*: Translation of languages

Translation of Languages of Trajectories

Let the Complex System move from the state S_1 to the state S_2 by applying the operator T_o=TRANSITION(p, x_o,y_o). A ***Translation of Languages of Trajectories*** is a mapping

$$\prod_{T_o}: L_t^H(S_1) \rightarrow L_t^H(S_2),$$

of such a sort that trajectories of the form $\boldsymbol{a}(x)\boldsymbol{a}(y)\ldots\boldsymbol{a}(z)$ are transformed as follows:

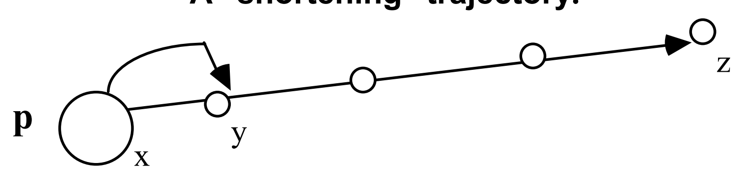
– are **"shortened"** by the exclusion of the first symbol $\boldsymbol{a}(x)$, if the transition T_o carries out along such a trajectory: $x = x_o$ & $y = y_o$. (If y = z, i.e., y is the ending point, the trajectory is transformed into the **empty trajectory *e***)

– are transformed into the **empty trajectory *e***, if
element p moves away from such a trajectory: $x = x_o$ & $y \neq x_1$,
or this element is withdrawn: $x = x_1$ and WFF ON(q) = x_1 comes into the Remove list of the transition T_o.

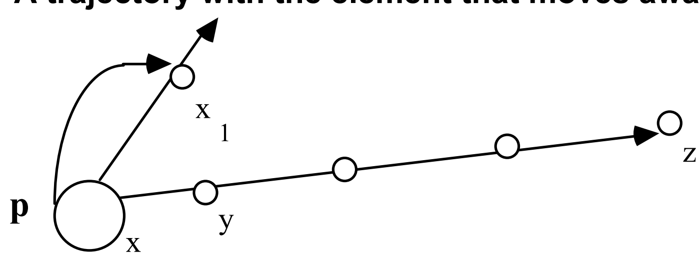
– are transformed **into itself** in all the other cases.

Obviously, mapping $\prod_{M_o}$ is not a mapping "onto" and has a non-empty kernel, i.e., a nonempty co-image of the empty trajectory ***e***.

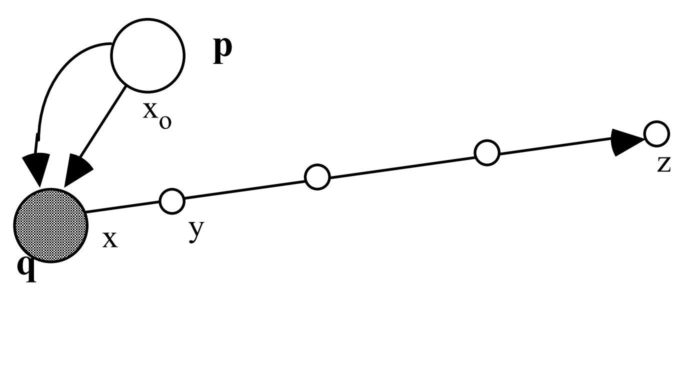
A "shortening" trajectory.



A trajectory with the element that moves away.



A trajectory whose element is withdrawn.

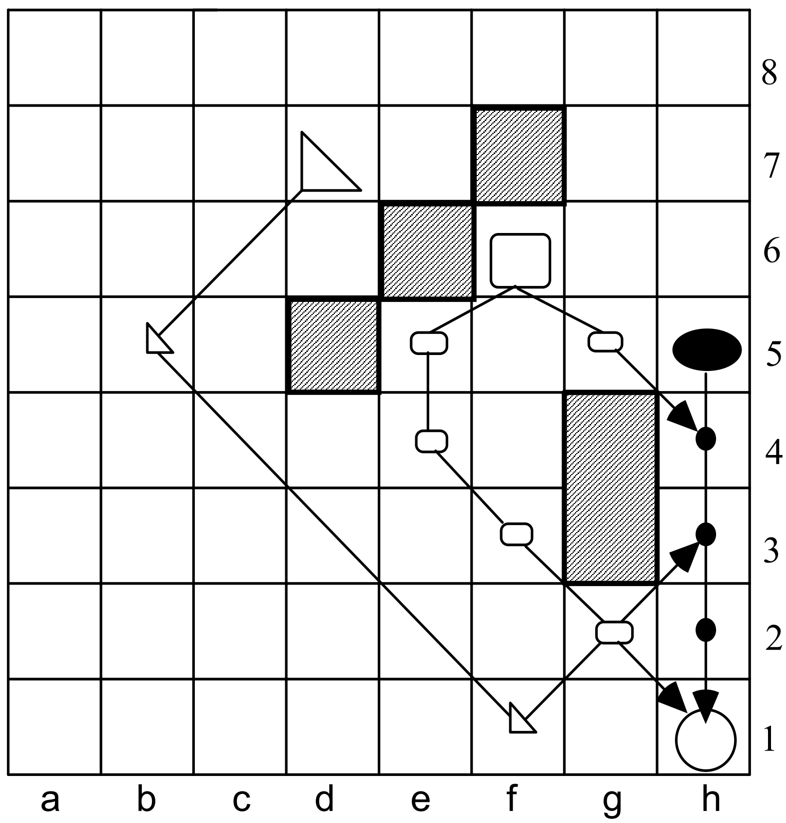


Interpretation of the trajectory network language for the robot control model

$$\boldsymbol{t}(\text{BOMBER}, t_B,5)\boldsymbol{t}(\text{FIGHTER}, t_F,5)\boldsymbol{t}(\text{MISSILE}, t_M,5)\boldsymbol{t}(\text{MISSILE}, t_M{}^1, 3)\ \boldsymbol{t}(\text{FIGHTER}, t_F{}^1, 2),$$

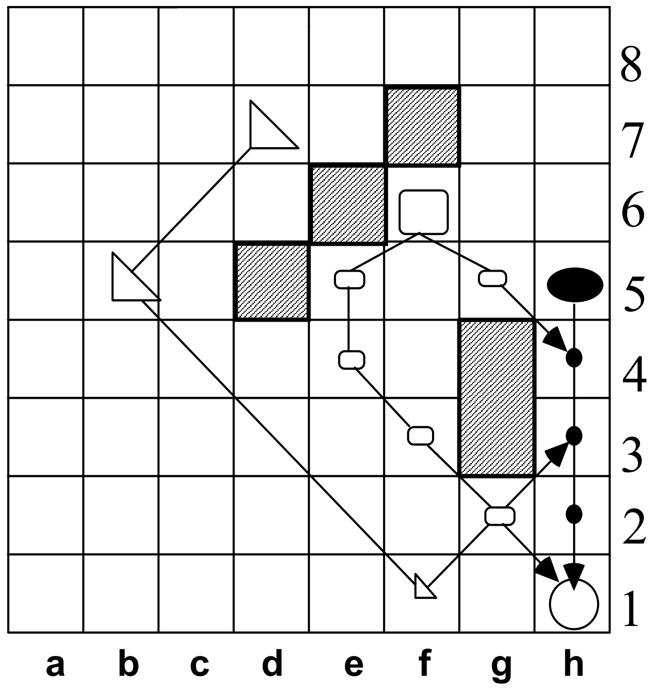
where

$$t_B=\boldsymbol{a}(h5)\boldsymbol{a}(h4)\boldsymbol{a}(h3)\boldsymbol{a}(h2)\boldsymbol{a}(h1),$$
$$t_F=\boldsymbol{a}(f6)\boldsymbol{a}(e5)\boldsymbol{a}(e4)\boldsymbol{a}(f3)\boldsymbol{a}(g2)\boldsymbol{a}(h1),$$
$$t_M=\boldsymbol{a}(d7)\boldsymbol{a}(b5)\boldsymbol{a}(f1)\boldsymbol{a}(g2)\boldsymbol{a}(h1),$$
$$t_M{}^1=\boldsymbol{a}(d7)\boldsymbol{a}(b5)\boldsymbol{a}(f1)\boldsymbol{a}(h3),$$
$$t_F{}^1=\boldsymbol{a}(f6)\boldsymbol{a}(g5)\boldsymbol{a}(h4)$$



Translations

M_1 = TRANSITION(MISSILE, d7, b5)



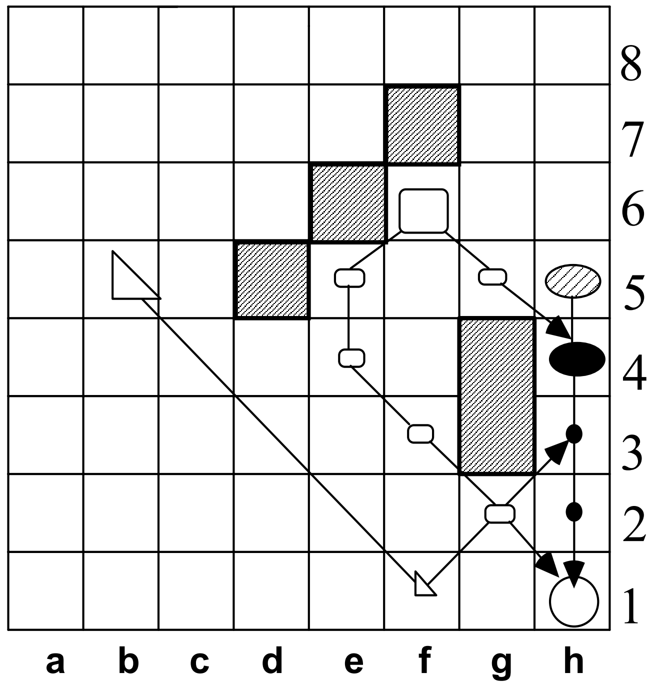
$\prod_{M_1}(t_M)$ and $\prod_{M_1}(t_M{}^1)$ are shortened trajectories with excluded first symbol, i.e.,

$$\prod_{M_1}(t_M) = t_{M,s} = \boldsymbol{a}(b5)\boldsymbol{a}(f1)\boldsymbol{a}(g2)\boldsymbol{a}(h1),$$

$$\prod_{M_1}(t_M{}^1) = t_{M,s}{}^1 = \boldsymbol{a}(b5)\boldsymbol{a}(f1)\boldsymbol{a}(h3).$$

All other trajectories are not changed.

M_2 = TRANSITION(BOMBER, h5, h4)



$\prod_{M_2}(t_B) = t_{B,s} = \boldsymbol{a}(h4)\boldsymbol{a}(h3)\boldsymbol{a}(h2)\boldsymbol{a}(h1)$ is a shortened trajectory.

Trajectories $t_{B,s}$, $t_{M,s}$, $t_{M,s}{}^1$ of BOMBER and MISSILE and trajectories $t_F{}^1$, t_F of FIGHTER are not changed

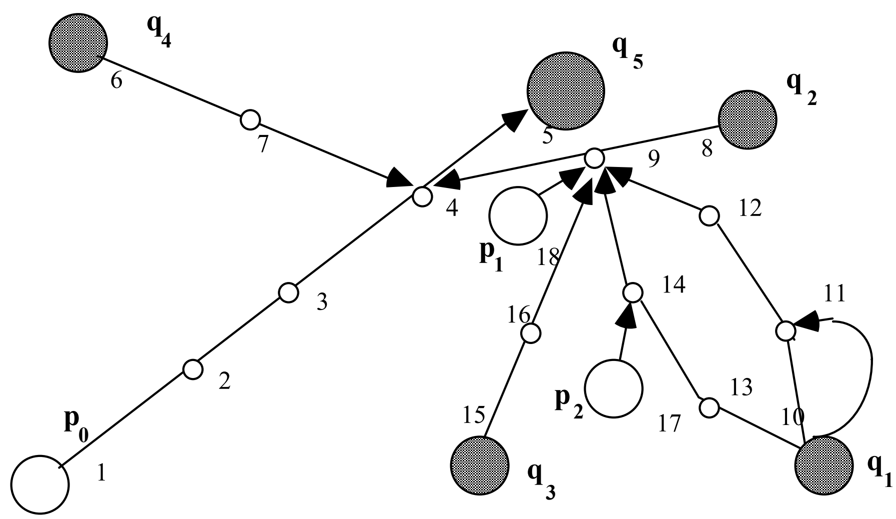
Translation of Languages of Zones

A ***Translation of Languages of Zones*** is the following mapping:

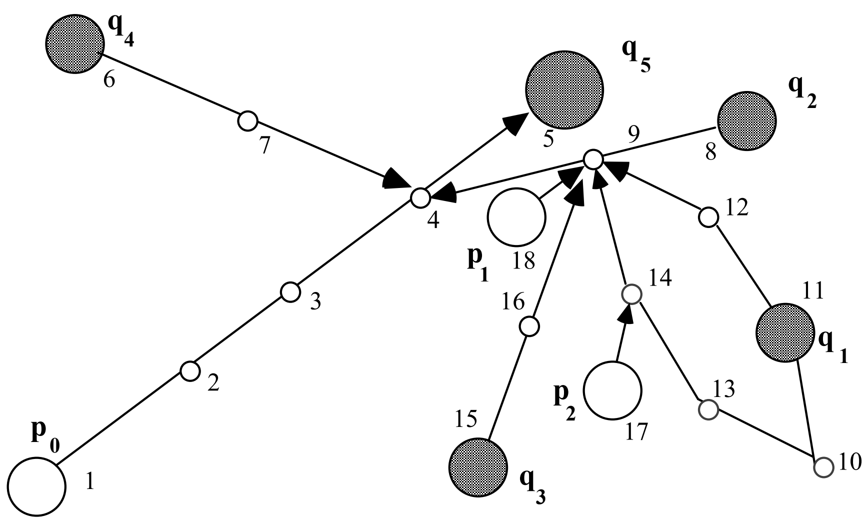
$$\boldsymbol{\pi}_{\mathbf{To}}: L_{\mathbf{Z}}(S_1) \rightarrow L_{\mathbf{Z}}(S_2),$$

where Zone Z_1 is translated into Zone Z_2, i.e., $\boldsymbol{\pi}_{\mathbf{To}}(Z_1) = Z_2$ if and only if the main trajectory t_o^1 of Zone Z_1 is translated into the main trajectory $t_o^{\mathbf{2}}$ of the Zone $Z_{\mathbf{2}}$ by the corresponding trajectory translation, $\prod_{\mathbf{To}}(t_o^1) = t_o^{\mathbf{2}}$.

State S_1

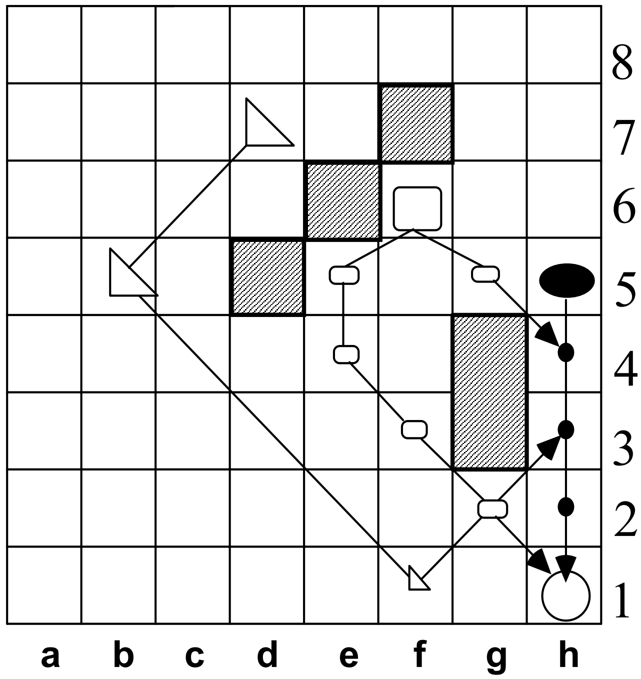


State S_2



Translation of Zones

M_1 = TRANSITION(MISSILE, d7, b5)



$\prod_{M_1}(t_M)$ and $\prod_{M_1}(t_M{}^1)$ are shortened trajectories with excluded first symbol, i.e.,

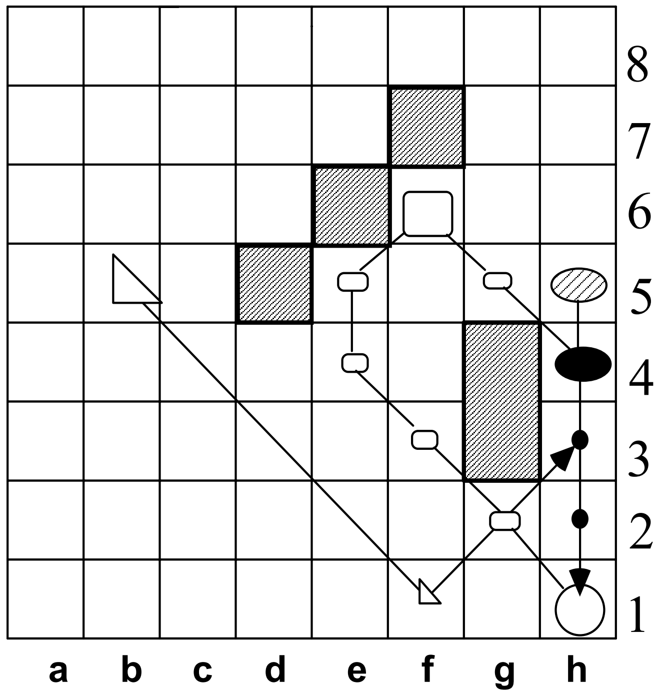
$$\prod_{M_1}(t_M) = t_{M,s} = \boldsymbol{a}(b5)\boldsymbol{a}(f1)\boldsymbol{a}(g2)\boldsymbol{a}(h1),$$

$$\prod_{M_1}(t_M{}^1) = t_{M,s}{}^1 = \boldsymbol{a}(b5)\boldsymbol{a}(f1)\boldsymbol{a}(h3).$$

All the other trajectories do not change

$$Z_1 = \pi_{M_1}(Z_o).$$

M_2 = TRANSITION(BOMBER, h5, h4).



$\prod_{M_2}(t_B) = t_{B,s} = \boldsymbol{a}(h4)\boldsymbol{a}(h3)\boldsymbol{a}(h2)\boldsymbol{a}(h1)$ is a shortened trajectory.

Trajectories $t_{B,s}$, $t_{M,s}$, $t_{M,s}{}^1$ of BOMBER and MISSILE should be included into the new Zone $Z_2 = \pi_{M_2}(Z_1)$., i.e., MISSILE has enough time to intercept BOMBER at h3 or h1.

Trajectories $t_F{}^1$, t_F of FIGHTER should not be included into the new Zone Z_2. Indeed, after transition M_2 FIGHTER does not have enough time for interception of BOMBER at h1 and it loses connection at h4.

A Structure of the Translation

An ***alphabet*** A(Z) ***of the string*** Z of the parameter language L is the set symbols of this language with given parameter values, where each of these symbols with parameters is included at least once in a string Z, and e (the empty symbol).

A ***trajectory alphabet*** TA(Z) of the Zone Z is the set of trajectories from $L_t^H(S)$ that correspond to the actual parameter values of the alphabet A(Z).

Let π_{Mo} be a translation of languages of Zones, with $\pi_{Mo}(Z_1) = Z_2$. Let also $\prod_{Mo}$ be a corresponding translation of languages of trajectories. ***Mapping of alphabets*** $\boldsymbol{\pi_o}$ of Zones Z_1 and Z_2 is the mapping $\boldsymbol{\pi_o}$: $A(Z_1) \rightarrow A(Z_2)$, which is constructed as follows. For all the symbols $\boldsymbol{t}(p, t_j, \tau_1)$ from $A(Z_1)$

$$\boldsymbol{\pi_o}(\boldsymbol{t}(p, t_j, \tau_1)) = \boldsymbol{t}(p, \textstyle\prod_{Mo}(t_j), \tau_2),$$

if there exists $\tau_2 \in \mathbf{Z}+$, $\tau_2 > 0$, such that $\boldsymbol{t}(p, \prod_{Mo}(t_j), \tau_2) \in A(Z_2) - \{e\}$;

$$\boldsymbol{\pi_o}(\boldsymbol{t}(p, t_j, \tau_1)) = e \text{ in the remaining cases.}$$

Denote by

$$Con_{\prod}(Z_1) = \{\boldsymbol{t}(p_i, t_i, \tau_i) \in A(Z_1) - \{e\} \mid C^+_{\prod_{M_o}(TA(Z_1))}(\textstyle\prod_{Mo}(t_i), \prod_{M_o}(t_o)) = T\}$$

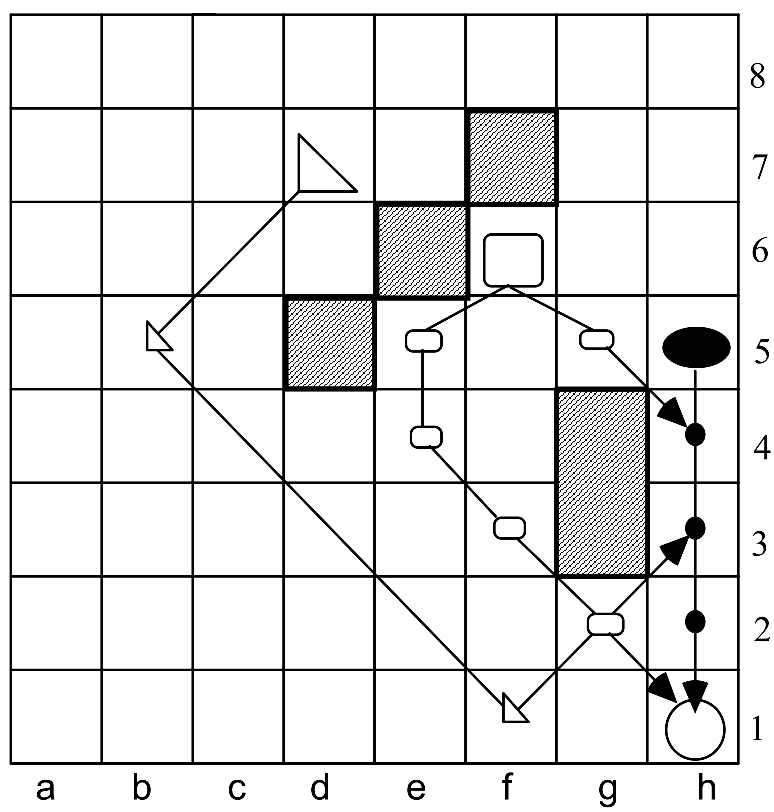
an ***invariant subnet*** of Zone Z_1 with respect to the translation π_{Mo}.

We need a time distribution function $\boldsymbol{timer_\pi}$. For every trajectory from the invariant subnet of Zone Z_1 function $\boldsymbol{timer_\pi}$ should yield a correct value of **"time"** (parameter τ) allocated to the image of this trajectory in the translation of Z_1. By comparing this value with the length of this image we should be able to conclude whether image of this trajectory is included in the translation of the Zone or not. Negative answer to this question means that the length of the trajectory image exceeds time allocated to the motion along it.

Interpretation of the trajectory network language for the robot control model

$\boldsymbol{t}$(BOMBER, t_B,5)$\boldsymbol{t}$(FIGHTER, t_F,5)$\boldsymbol{t}$(MISSILE, t_M,5)$\boldsymbol{t}$(MISSILE, $t_M{}^1$, 3)
$\boldsymbol{t}$(FIGHTER, $t_F{}^1$, 2),
where

t_B=$\boldsymbol{a}$(h5)$\boldsymbol{a}$(h4)$\boldsymbol{a}$(h3)$\boldsymbol{a}$(h2)$\boldsymbol{a}$(h1),
t_F=$\boldsymbol{a}$(f6)$\boldsymbol{a}$(e5)$\boldsymbol{a}$(e4)$\boldsymbol{a}$(f3)$\boldsymbol{a}$(g2)$\boldsymbol{a}$(h1),
t_M=$\boldsymbol{a}$(d7)$\boldsymbol{a}$(b5)$\boldsymbol{a}$(f1)$\boldsymbol{a}$(g2)$\boldsymbol{a}$(h1),
$t_M{}^1$=$\boldsymbol{a}$(d7)$\boldsymbol{a}$(b5)$\boldsymbol{a}$(f1)$\boldsymbol{a}$(h3),
$t_F{}^1$=$\boldsymbol{a}$(f6)$\boldsymbol{a}$(g5)$\boldsymbol{a}$(h4)



Time distribution function $\boldsymbol{timer_\pi}$

Let $\pi_{Mo}(Z_1) = Z_2$ be a translation, with $Z_1 = \boldsymbol{t}(p_o, t_o, \tau_o)\boldsymbol{t}(p_1, t_1, \tau_1)...\boldsymbol{t}(p_r, t_r, \tau_r)$, $Z_1 \in L_Z(S_1)$, $Z_2 \in L_Z(S_2)$. A mapping

$$\boldsymbol{timer_\pi}: \mathrm{Con}_\prod(Z_1) \to \mathbf{Z},$$

where $\mathbf{Z}$ is the set of all integer numbers, is constructed as follows. We consider three cases:

(1) If $\prod_{Mo}(t_o) = t_o'$, i.e., the main trajectory of Zone Z_1 is shortened, that is transformed into a substring with an excluded first symbol, then for all symbols $\boldsymbol{t}(p_c, t_c, \tau) \in \mathrm{Con}_\prod(Z_1)$

$$\boldsymbol{timer_\pi}(\boldsymbol{t}(p_c, t_c, \tau)) = \tau - 1.$$

(2) If $\prod_{Mo}(t_k) = t_k'$, i.e., some other trajectory t_k of Zone Z_1 is shortened ($k \neq 0$), then we define $\boldsymbol{timer_\pi}$ recursively.

(a) $\boldsymbol{timer_\pi}(\boldsymbol{t}(p_o, t_o, \tau_o)) = \tau_o$,
$\boldsymbol{timer_\pi}(\boldsymbol{t}(p_i, t_i, \tau_i)) = \tau_i$ (if $C_{TA(Z_1)}(t_i, t_o) = T$)

(b) Let $\boldsymbol{t}(p_c, t_c, \tau) \in \mathrm{Con}_\prod(Z_1)$,
denote $CA(t_c) = \{t_i \in \mathrm{Con}_\prod(Z_1) \mid C(t_c, t_i) = T\}$,
then $\boldsymbol{timer_\pi}(\boldsymbol{t}(p_c, t_c, \tau)) = \max\limits_{t_i \in CA(t_c)} \{TNEW(t_i)\}$, where

$$TNEW(t_i) = \begin{cases} \boldsymbol{timer}_\pi(\boldsymbol{t}(p_i, t_i, \tau_i)) - \boldsymbol{len}(p_i, t_i) + 1, & \text{if } t_i \neq t_k, \\ (\boldsymbol{timer}_\pi(\boldsymbol{t}(p_i, t_i, \tau_i)) + 1) - \boldsymbol{len}(p_i, t_i) + 1, & \text{if } t_i = t_k, \end{cases}$$

($\boldsymbol{len}(p_i, t_i)$ is the length of t_i).

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(3) If $\prod_{Mo}(t_m) = t_m$ for all $t_m \in TA(Z_1)$, then $\boldsymbol{timer_\pi}(\boldsymbol{t}(p_c, t_c, \tau)) = \tau$.