Midterm Exam

Oct 28, 8:00 am - 12:00 pm, LW-836

Midterm Review

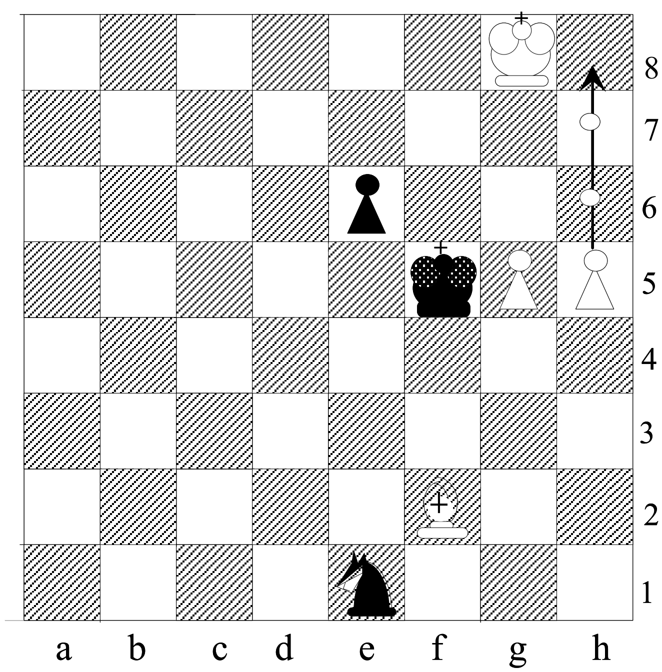
1. **Represent the system shown below as an ABG.**

2. **Generate Zone with the main trajectory a(h5)a(h6)a(h7)a(h8). Use grammars of Trajectories and Zones, show the generations with *<u>important</u>* values of the functions and sets including**

   DOCK, med_i, $lmed_i$, SUM, ST_k, MOVE, $next_i$,

   f, h_i^o, h_j, g, <u>v, w, TIME, and NEXTTIME</u>.

3. **Show your understanding of Translations. Show a translation table (analogous to table in handout No. 14) for the main variation (your choice) assuming that Black moves first. Include only the most important trajectories.**



Note. On the exam it is going to be a meaningful ABG (which may include non-chess-like pieces). It will be stated as a gaming problem so that generating a zone and moving in it would allow you to solve this ABG.

Class of Problems

Abstract Board Game

is the following eight-tuple

< X, P, R_p, {ON} , v, S_i, S_t, TR>

X = {x_i} is a finite set of ***points***;

P = {p_i} is a finite set of ***elements***; **P** = $P_1 \cup P_2$, $P_1 \cap P_2 = \emptyset$;

R_p(x, y) is a family of binary relations of ***reachability*** in X (**x** $\in$ X, **y** $\in$ X, **p** $\in$ P); y is ***reachable*** from x for p;

ON(p)=x is a partial function of ***placement*** of elements P into X;

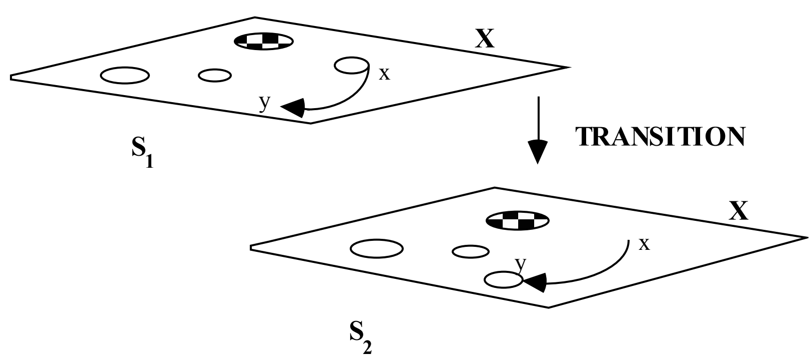
v > 0 is a real function, **v(p_i)** are the ***values*** of elements;

S_i is a set of ***initial*** states of the system, a certain set of formulas {**ON(p_i)** = **x_i**};

S_t is a set ***target*** states of the system (as **S_i**);

TR is a set of operators **TRANSITION(p, x, y)** for transition of the system from one state to another described as follows

precondition: ON(p) = x $\wedge$ R_p(x, y)
delete: ON(p) = x, ON(q) = y
add: ON(p) = y



1	Q_1	$S(u, v, w) \to A(u, v, w)$		*two*	ø
2_i	Q_2	$A(u, v, w) \to t(\, h_i^o(u), l_o+1)$ $A((0, 0, 0), g(h_i^o(u),w), zero)$	$\mathrm{TIME}(z) = DIST(z, h_i^o(u))$	3	ø
3	Q_3	$A(u, v, w) \to A(f\,(u, v), v, w)$	$\mathrm{NEXTTIME}(z) = init(u, \mathrm{NEXTTIME}(z))$	*four*	5
4_j	Q_4	$A(u, v, w) \to t(h_j(u), \mathrm{TIME}(y))$ $A(u, v, g(h_j(u), w))$	$\mathrm{NEXTTIME}(z) = ALPHA(z, h_j(u), \mathrm{TIME}(y) - l_j+1)$	3	3
5	Q_5	$A(u, v, w) \to A((0, 0, 0), w, zero)$	$\mathrm{TIME}(z) = \mathrm{NEXTTIME}(z)$	3	6
6	Q_6	$A(u, v, w) \to e$		ø	ø

$Q_1(u) = (\mathrm{ON}(p_o) = x) \wedge (\mathrm{MAP}_{x,p_o}(y) \le l \le l_o) \wedge (\exists q\, ((\mathrm{ON}(q) = y) \wedge (\mathrm{OPPOSE}(p_o, q))))$

$Q_2(u) = T$; $Q_3(u) = (x \ne n) \vee (y \ne n)$ $Q_5(w) = (w \ne zero)$; $Q_6 = T$;

$Q_4(u) = \exists p\, ((\mathrm{ON}(p) = x) \wedge (l > 0) \wedge (x \ne x_o) \wedge (x \ne y_o)) \wedge [(\neg \mathrm{OPPOSE}(p_o, p) \wedge (\mathrm{MAP}_{x,p}(y) = 1)) \vee (\mathrm{OPPOSE}(p_o, p) \wedge (\mathrm{MAP}_{x,p}(y) \le l\,)]$

$$init(u,\mathrm{r}) = \begin{cases} 2\mathrm{n}, & \text{if } u = (0,0,0), \\ \mathrm{r}, & \text{if } u \ne (0,0,0). \end{cases}$$

$$f(u,\ v) = \begin{cases} (\mathrm{x}+1, \mathrm{y}, l), & \text{if } ((\mathrm{x} \ne \mathrm{n}) \wedge (l > 0)) \vee ((\mathrm{y} = \mathrm{n}) \wedge (l \le 0)) \\ (1,\ \mathrm{y}+1,\ TIME(\mathrm{y}+1) \times v_{\mathrm{y}+1}), & \text{if } (\mathrm{x} = \mathrm{n}) \vee ((l \le 0) \wedge (\mathrm{y} \ne \mathrm{n})). \end{cases}$$

Let $t_o \in \mathbf{L_t}^{l_o}(S)$, $t_o = a(z_o)a(z_1)...a(z_m)$, $t_o \in t_{p_o}(z_o, z_m, m)$;

If $((z_m = y_o) \wedge (p = p_o) \wedge (\exists\, k\, (1 \le k \le m) \wedge (x = z_k))) \vee$
$(((z_m \ne y_o) \vee (p \ne p_o)) \wedge (\exists\, k\, (1 \le k \le m - 1) \wedge (x = z_k)))$
then $DIST(x, p_o, t_o) = k+1$ **else** $DIST(x, p_o, t_o) = 2n$

$$ALPHA(\mathrm{x}, \mathrm{p_o}, \mathrm{t_o}, \mathrm{k}) = \begin{cases} max\,(NEXTTIME(\mathrm{x}), \mathrm{k}), & \text{if}\,(DIST(\mathrm{x}, \mathrm{p_o}, \mathrm{t_o}) \ne 2\mathrm{n}) \wedge (NEXTTIME(\mathrm{x}) \ne 2\mathrm{n}); \\ \mathrm{k}, & \text{if } DIST(\mathrm{x}, \mathrm{p_o}, \mathrm{t_o}) \ne 2\mathrm{n}) \wedge (NEXTTIME(\mathrm{x}) = 2\mathrm{n}); \\ NEXTTIME(\mathrm{x}), & \text{if } DIST(\mathrm{x}, \mathrm{p_o}, \mathrm{t_o}) = 2\mathrm{n}). \end{cases}$$

$$\boldsymbol{g}_{\mathbf{r}}(\mathrm{p_o}, \mathrm{t_o}, w) = \begin{cases} 1, & \text{if } \boldsymbol{DIST}(\mathrm{r}, \mathrm{p_o}, \mathrm{t_o}) < 2n, \\ w_{\mathrm{r}}, & \text{if } \boldsymbol{DIST}(\mathrm{r}, \mathrm{p_o}, \mathrm{t_o}) = 2\mathrm{n}. \end{cases} \qquad \mathrm{TRACKS}_{p_o} = \{p_o\} \times (\bigcup_{1 \le k \le l} \mathrm{L}[\, \mathbf{G_t^{(2)}}(x, y, k, p_o)]$$

If $\mathrm{TRACKS}_{p_o} = e$
then $h_i^o(u) = e$

else $\mathrm{TRACKS}_{p_o} = \{(p_o, t_1), (p_o, t_2), \ldots, (p_o, t_b)\}, (b \le M)$ **and** $h_i^o(u) = \begin{cases} (p_o, t_i), & \text{if } i \le b, \\ (p_o, t_b), & \text{if } i > b. \end{cases}$

$\mathrm{TRACKS} = \bigcup_{\mathrm{ON}(p)=x} \mathrm{TRACKS}_p$, where TRACKS_p is the same as for h_i^o

If TRACKS = e
then $h_i(u) = e$

else $\mathrm{TRACKS} = \{(p_1, t_1), (p_1, t_2), \ldots, (p_m, t_m)\}$, $(m \le M)$ **and** $h_i(u) = \begin{cases} (p_i, t_i), & \text{if } i \le m, \\ (p_m, t_m), & \text{if } i > m. \end{cases}$

At the beginning: $u = (x_o, y_o, l_o)$, $w = zero$, $v = zero$, $x_o \in X$, $y_o \in X$, $l_o \in \mathbf{Z}_+$,
$p_o \in P$, and $\mathrm{TIME}(z) = 2n$, $\mathrm{NEXTTIME}(z) = 2n$ for all z from X.

Grammar $G_t^{(2)}$ of shortest and admissible trajectories.

L	Q	Kernel, π_k	π_n	F_T	F_F
1	Q_1	$S(x,y,l) \to A(x,y,l)$		*two*	ø
2_i	Q_2	$A(x,y,l) \to A(x,\ med_i(x,y,l),\ lmed_i(x,y,l))$ $A(med_i(x,y,l),\ y,\ l - lmed_i(x,y,l))$		*three*	*three*
3_j	Q_3	$A(x,y,l) \to a(x)A(next_j(x,l), y, f(l))$		*three*	4
4	Q_4	$A(x,y,l) \to a(y)$		*three*	5
5	Q_5	$A(x,y,l) \to e$		*three*	ø

$V_T = \{a\}$,

$V_N = \{S, A\}$,

V_{PR}

Pred $= \{Q_1, Q_2, Q_3, Q_4, Q_5\}$

$Q_1(x, y, l) = (MAP_{x,p}(y) \le l < 2MAP_{x,p}(y)) \wedge (l < 2n)$

$Q_2(x, y, l) = (MAP_{x,p}(y) \ne l)$

$Q_3(x, y, l) = (MAP_{x,p}(y) = l) \wedge (l \ge 1)$

$Q_4(y) = (y = y_o)$

$Q_5(y) = (y \ne y_o)$

Var $= \{x, y, l\}$;

Con $= \{x_o, y_o, l_o, p\}$;

Func $=$ *Fcon* $\cup$ *Fvar*;

Fcon $= \{f, next_1, \ldots, next_n,\ med_1, \ldots, med_n,$
$lmed_1, \ldots, lmed_n\}$ $(n = |X|)$,

$f(l) = l-1$, $D(f) = \mathbf{Z}_+ \setminus \{0\}$

functions $next_i$, med_i *and* $lmed_i$ are defined below.

Fvar $= \{x_o, y_o, l_o, p\}$

$E = \mathbf{Z}_+ \cup X \cup P$ is the subject domain;

Parm: $S \to Var$, $A \to Var$, $a \to \{x\}$;

$L = \{1, 4\} \cup two \cup three$, $two = \{2_1, 2_2, \ldots, 2_n\}$, $three = \{3_1, 3_2, \ldots, 3_n\}$

At the beginning of derivation: $x = x_o$, $y = y_o$, $l = l_o$, $x_o \in X$, $y_o \in X$, $l_o \in \mathbf{Z}_+$, $p \in P$.

Definition of functions *med*, *lmed*, *next*

med_i is defined as follows:

$D(med_i) = X \times X \times \mathbf{Z}_+ \times P$

DOCK = {v | v from X, $MAP_{x_o,p}(v) + MAP_{y_o,p}(v) = l$},

If

$DOCK_l(x) = \{v_1, v_2, ..., v_m\} \neq \emptyset$

then

$med_i(x, y, l) = v_i$ for $1 \leq i \leq m$ and

$med_i(x, y, l) = v_m$ for $m < i \leq n$,

otherwise

$med_i(x, y, l)=x$.

$lmed_i$ is defined as follows:

$D(med_i)= X \times X \times \mathbf{Z}_+ \times P$

$lmed_i(x, y, l) = MAP_{x,p}(med_i(x, y, l))$

$next_i$ is defined as follows:

$D(next_i)= X \times \mathbf{Z}_+ \times X^2 \times \mathbf{Z}_+ \times P$

SUM = {v | v from X, $MAP_{x_o,p}(v) + MAP_{y_o,p}(v)=l_o$},

$ST_k(x)$ = {v | v from X, $MAP_{x,p}(v) = k$},

$MOVE_l(x)$ is an intersection of the following sets:

$ST_1(x)$, $ST_{l_o-l+1}(x_o)$ and SUM.

If

$MOVE_l(x) = \{m_1, m_2, ..., m_r\} \neq \emptyset$

then

$next_i(x, l) = m_i$ for $i \leq r$ and

$next_i(x, l) = m_r$ for $r < i \leq n$,

otherwise

$next_i(x, l) = x$.