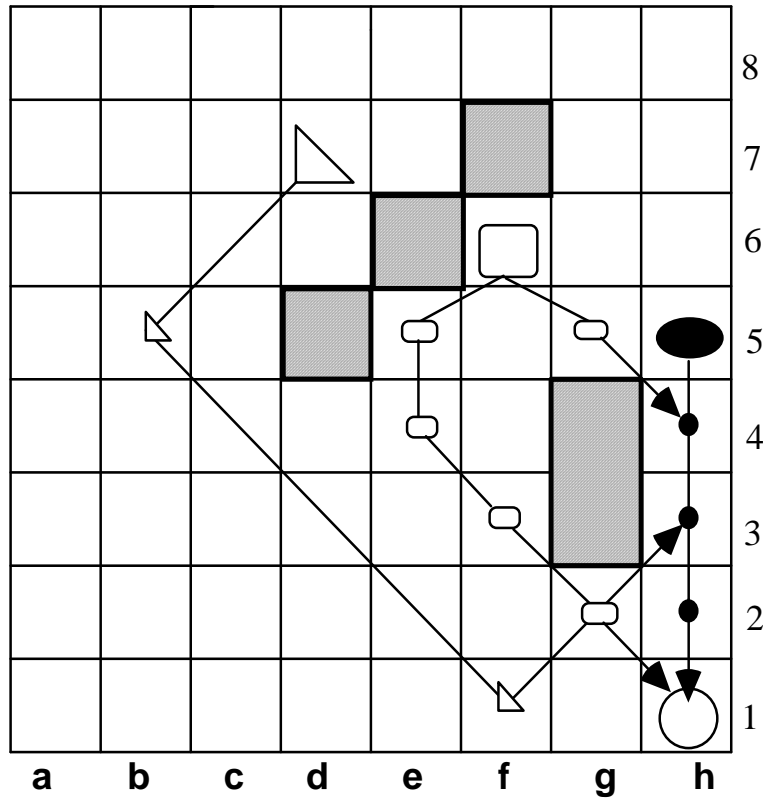


Interpretation of the Language of Zones for the robot control model

$t(\text{BOMBER}, t_B, 5)t(\text{FIGHTER}, t_F, 5)t(\text{MISSILE}, t_M, 5)t(\text{MISSILE}, t_M^1, 3)$
 $t(\text{FIGHTER}, t_F^1, 2),$

where

$t_B = a(h5)a(h4)a(h3)a(h2)a(h1),$
 $t_F = a(f6)a(e5)a(e4)a(f3)a(g2)a(h1),$
 $t_M = a(d7)a(b5)a(f1)a(g2)a(h1),$
 $t_M^1 = a(d7)a(b5)a(f1)a(h3),$
 $t_F^1 = a(f6)a(g5)a(h4)$



Grammar of Zones G_Z

L	Q	Kernel, π_k ($\forall z \in X$)	π_n ($\forall z \in X$)	F_T	F_F
1	Q_1	$S(u, v, w) \rightarrow A(u, v, w)$		<i>two</i>	\emptyset
2_i	Q_2	$A(u, v, w) \rightarrow t(h_i^0(u), l_0+1)$ $A((0, 0, 0), g(h_i^0(u), w), zero)$	$TIME(z) = DIST(z, h_i^0(u))$	3	\emptyset
3	Q_3	$A(u, v, w) \rightarrow A(f(u, v), v, w)$	$NEXTTIME(z) =$ $init(u, NEXTTIME(z))$	<i>four</i>	5
4_j	Q_4	$A(u, v, w) \rightarrow t(h_j(u), TIME(y))$ $A(u, v, g(h_j(u), w))$	$NEXTTIME(z) =$ $ALPHA(z, h_j(u), TIME(y) - l + 1)$	3	3
5	Q_5	$A(u, v, w) \rightarrow A((0, 0, 0), w, zero)$	$TIME(z) =$ $NEXTTIME(z)$	3	6
6	Q_6	$A(u, v, w) \rightarrow \varepsilon$		\emptyset	\emptyset

$V_T = \{t\}, V_N = \{S, A\},$

V_{PR}

$Pred = \{Q_1, Q_2, Q_3, Q_4, Q_5, Q_6\}$

$Q_1(u) = (ON(p_0) = x) \wedge (MAP_{x, p_0}(y) \leq l \leq l_0) \wedge$
 $(\exists q ((ON(q) = y) \wedge (OPPOSE(p_0, q))))$

$Q_2(u) = T$

$Q_3(u) = (x \neq n) \vee (y \neq n)$

$Q_4(u) = (\exists p ((ON(p) = x) \wedge (l > 0) \wedge (x \neq x_0) \wedge (x \neq y_0)) \wedge$
 $((\neg OPPOSE(p_0, p) \wedge (MAP_{x, p}(y) = 1)) \vee$
 $(OPPOSE(p_0, p) \wedge (MAP_{x, p}(y) \leq l)))$

$Q_5(w) = (w \neq zero)$

$Q_6 = T$

$Var = \{x, y, l, \tau, \theta, v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n\};$ for the sake of brevity:

$u = (x, y, l), v = (v_1, v_2, \dots, v_n), w = (w_1, w_2, \dots, w_n), zero = (0, 0, \dots, 0)$

$Con = \{x_0, y_0, l_0, p_0\}; Func = Fcon \cup Fvar;$

$Fcon = \{f_x, f_y, f_l, g_1, g_2, \dots, g_n, h_1, h_2, \dots, h_M,$

$h_1^0, h_2^0, \dots, h_M^0, DIST, init, ALPHA\}, f = (f_x, f_y, f_l), g = (g_{x1}, g_{x2}, \dots, g_{xn}),$

$M = |L_t^{l_0}(S)|$ is the number of trajectories in $L_t^{l_0}(S)$

$Fvar = \{x_0, y_0, l_0, p_0, TIME, NEXTTIME\}$

$E = Z_+ \cup X \cup P \cup L_t^{l_0}(S)$ is the subject domain;

Parm: $S \rightarrow Var, A \rightarrow \{u, v, w\}, t \rightarrow \{p, \tau, \theta\};$

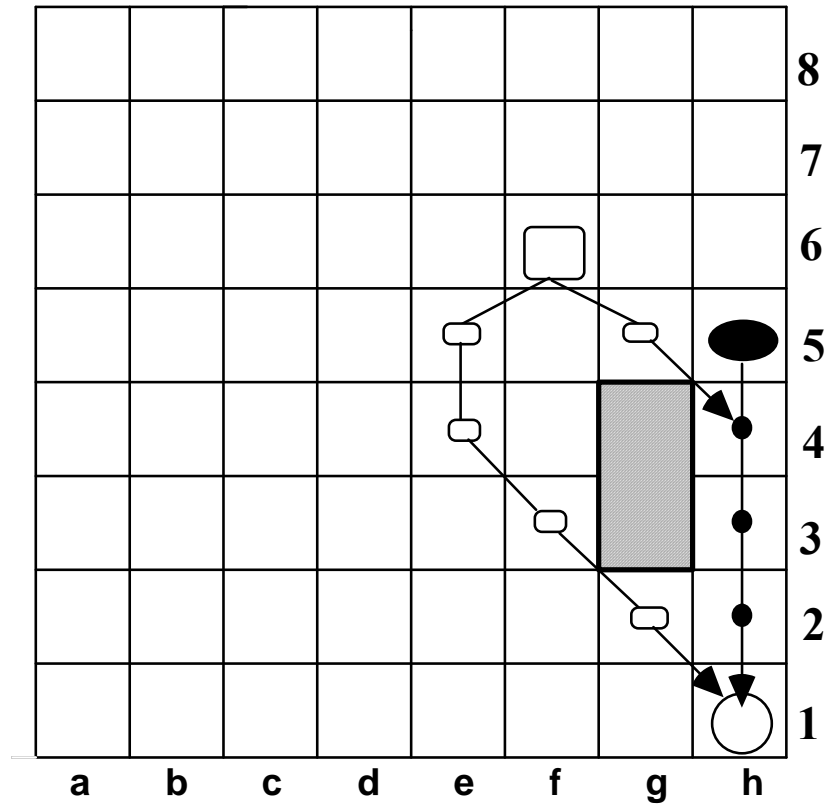
$L = \{1, 3, 5, 6\} \cup two \cup four, two = \{2_1, 2_2, \dots, 2_M\}, four = \{4_1, 4_2, \dots, 4_M\}$

Definition of functions of the Grammar of Zones Gz

$D(init) = X \times X \times \mathbf{Z}_+ \times \mathbf{Z}_+$ $init(u, r) = \begin{cases} 2n, & \text{if } u = (0, 0, 0), \\ r, & \text{if } u \neq (0, 0, 0). \end{cases}$
$D(f) = (X \times X \times \mathbf{Z}_+ \cup \{0, 0, 0\}) \times \mathbf{Z}_+^n$ $f(u, v) = \begin{cases} (x+1, y, l), & \text{if } ((x \neq n) \wedge (l > 0)) \vee ((y = n) \wedge (l \leq 0)) \\ (1, y+1, TIME(y+1) \times v_{y+1}), & \text{if } (x = n) \vee ((l \leq 0) \wedge (y \neq n)). \end{cases}$
$D(DIST) = X \times P \times \mathbf{L}_t^{I_0}(S).$ Let $t_0 \in \mathbf{L}_t^{I_0}(S)$, $t_0 = a(z_0)a(z_1)...a(z_m)$, $t_0 \in t_{p_0}(z_0, z_m, m)$; If $((z_m = y_0) \wedge (p = p_0) \wedge (\exists k (1 \leq k \leq m) \wedge (x = z_k))) \vee$ $((z_m \neq y_0) \vee (p \neq p_0)) \wedge (\exists k (1 \leq k \leq m - 1) \wedge (x = z_k))$ then $DIST(x, p_0, t_0) = k+1$ else $DIST(x, p_0, t_0) = 2n$
$D(ALPHA) = X \times P \times \mathbf{L}_t^{I_0}(S) \times \mathbf{Z}_+$ $ALPHA(x, p_0, t_0, k) = \begin{cases} max(NEXTTIME(x), k), & \text{if } (DIST(x, p_0, t_0) \neq 2n) \\ & \wedge (NEXTTIME(x) \neq 2n); \\ k, & \text{if } DIST(x, p_0, t_0) \neq 2n \\ & \wedge (NEXTTIME(x) = 2n); \\ NEXTTIME(x), & \text{if } DIST(x, p_0, t_0) = 2n. \end{cases}$
$D(g_r) = P \times \mathbf{L}_t^{I_0}(S) \times \mathbf{Z}_+^n$, $r \in X$. $gr(p_0, t_0, w) = \begin{cases} 1, & \text{if } DIST(r, p_0, t_0) < 2n, \\ w_r, & \text{if } DIST(r, p_0, t_0) = 2n. \end{cases}$
$D(h_i^0) = X \times X \times \mathbf{Z}_+;$ Let $TRACKS_{p_0} = \{p_0\} \times (\bigcup_{1 \leq k \leq l} L[G_t^{(2)}(x, y, k, p_0)])$ If $TRACKS_{p_0} = \emptyset$ then $h_i^0(u) = \varepsilon$ else $TRACKS_{p_0} = \{(p_0, t_1), (p_0, t_2), \dots, (p_0, t_b)\}, (b \leq M)$ and $h_i^0(u) = \begin{cases} (p_0, t_i), & \text{if } i \leq b, \\ (p_0, t_b), & \text{if } i > b. \end{cases}$
$D(h_i) = X \times X \times \mathbf{Z}_+;$ Let $TRACKS_p = \{p\} \times (\bigcup_{1 \leq k \leq l} L[G_t^{(2)}(x, y, k, p)])$ If $TRACKS_p = \emptyset$ then $h_i(u) = \varepsilon$ else $TRACKS_p = \{(p_1, t_1), (p_1, t_2), \dots, (p_m, t_m)\}, (m \leq M)$ and $h_i(u) = \begin{cases} (p_i, t_i), & \text{if } i \leq m, \\ (p_m, t_m), & \text{if } i > m. \end{cases}$
Trajectories should not be embedded (as sub-trajectories) in the trajectories of the same negation. At the beginning of generation: $u = (x_0, y_0, l_0)$, $w = zero$, $v = zero$, $x_0 \in X$, $y_0 \in X$, $l_0 \in \mathbf{Z}_+$, $p_0 \in P$, $TIME(z) = 2n$, $NEXTTIME(z) = 2n$ for all z from X .

Generation of the Language of Zones for robot control model (simplified version)

$t(\text{BOMBER}, t_B, 5)t(\text{FIGHTER}, t_F, 5)t(\text{FIGHTER}, t_F^1, 2)$



—STEP 1—

$u = (h5, h1, 4) = (38, 8, 4)$, $l = l_0 = 4$ because

$$Q_1(u) = (\text{ON}(\text{BOMBER}) = h5) \wedge (\text{MAP}_{h5, \text{BOMBER}}(h1) \leq 4 \leq 4) \wedge \\ ((\text{ON}(\text{Target}) = h1) \wedge (\text{OPPOSE}(\text{BOMBER}, \text{TARGET}))) = T.$$

$$S(u, \text{zero}, \text{zero}) \stackrel{1}{=} A(u, \text{zero}, \text{zero})$$

$$A(u, \text{zero}, \text{zero}) \stackrel{2}{=} t(h_i^0(u), 5) A((0, 0, 0), g(h_i^0(u), \text{zero}), \text{zero})$$

Computation of $h_i^0(u)$ ($l = 4$).

$$\text{TRACKS}_{\text{BOMBER}} = \{\text{BOMBER}\} \times \left(\bigcup_{1 \leq k \leq 4} L [G_t^{(2)}(h5, h1, k, \text{BOMBER})] \right).$$

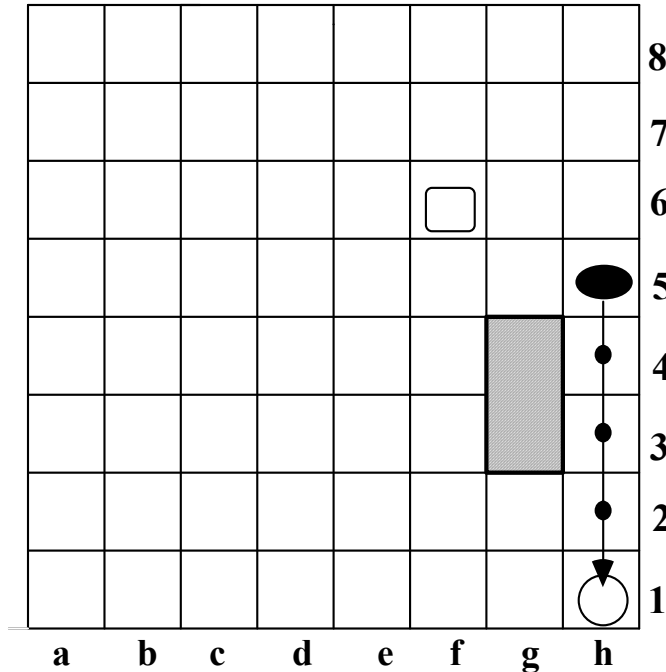
Only one such trajectory t_1 exists, and it is generated by this grammar:

$$t_B = a(h5)a(h4)a(h3)a(h2)a(h1).$$

Thus $\text{TRACKS} = \{(\text{BOMBER}, t_B)\}$, the number of trajectories $b = 1$ and

$$h_I^0(u) = (\text{BOMBER}, t_B).$$

$$t(\text{BOMBER}, t_B, 5).$$



—STEP 5—

Try $4j$ $Q_4(1, 8, 5) = F$

Try 3 with $l > 0$ and $x \neq 62$.

This means the beginning of a new loop which consists of multiple applications of production 3 after failures of attempts to apply one of productions $4j$.

$3 \Rightarrow t(\text{BOMBER}, t_B, 5) A((2, 8, 5), v, \text{zero})$

$3 \Rightarrow t(\text{BOMBER}, t_B, 5) A((3, 8, 5), v, \text{zero})$

.....

$3 \Rightarrow t(\text{BOMBER}, t_B, 5) A((44, 8, 5), v, \text{zero})$

With $u = (44, 8, 5)$ this loop will be terminated because

$Q_4(44, 8, 5) = (\text{ON}(\text{FIGHTER}) = 44) \wedge (5 > 0) \wedge$

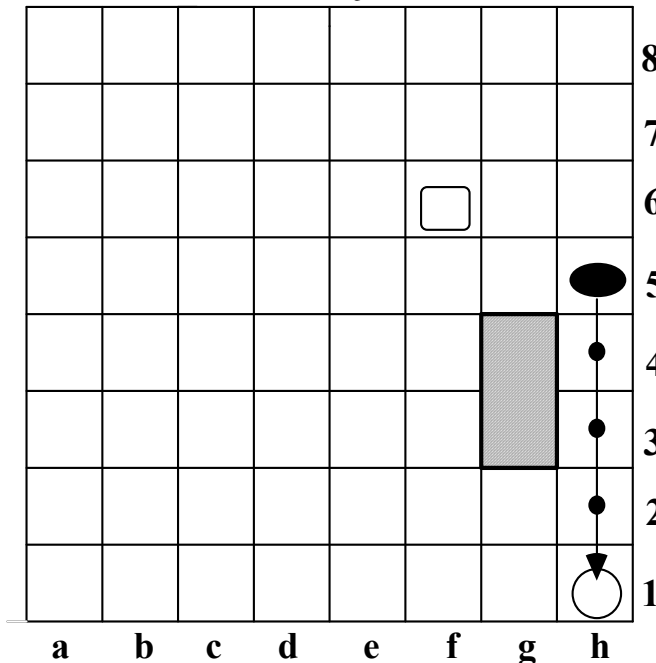
$(\chi(\text{BOMBER}, \text{FIGHTER}) = 0) \wedge (\text{MAP}_{f6, \text{FIGHTER}}(h1) = 5) = T,$

which means that productions $4j$ are applicable.

These productions will generate intercepting trajectories from f6 to h1.

$4j \Rightarrow t(\text{BOMBER}, t_B, 5) t(h_j(44, 8, 5), \text{TIME}(8))$

$A((44, 8, 5), v, g(h_j(44, 8, 5), \text{zero}))$



— STEP 5 (continued) —

$$4j=>t(\text{BOMBER}, t_B, 5)t(h_j(44, 8, 5), \text{TIME}(8))$$

$$A((44, 8, 5), v, g(h_j(44, 8, 5), \text{zero}))$$

Computation of $h_j(44, 8, 5)$

We have to generate all the shortest trajectories from point f6 to h1 for robot FIGHTER. The length of these trajectories should be less or equal 5.

$$\text{TRACKS}_{\text{FIGHTER}} = \{\text{FIGHTER}\} \times \bigcup_{1 \leq k \leq 5} L [G_t^{(2)}(f6, h1, k, \text{FIGHTER})]$$

$$\text{TRACKS} = \{(\text{FIGHTER}, t_1), (\text{FIGHTER}, t_2), (\text{FIGHTER}, t_3)\}, m = 3 \text{ and}$$

$$h_1(44, 8, 5) = (\text{FIGHTER}, t_1)$$

$$h_2(44, 8, 5) = (\text{FIGHTER}, t_2)$$

$$h_3(44, 8, 5) = (\text{FIGHTER}, t_3)$$

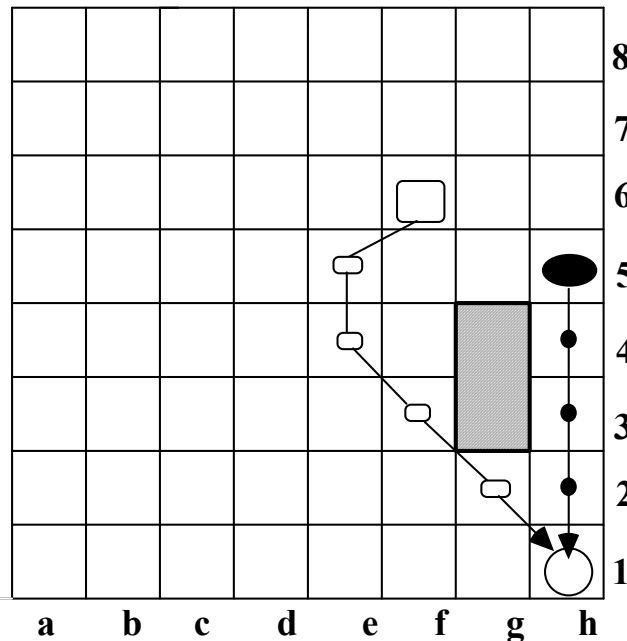
There are three such trajectories t_1 , t_2 and t_3 , and they are generated by the grammar $G_t^{(2)}$. (Of course, there is one more trajectory, $a(f6)a(g5)a(h4)a(h3)a(h2)a(h1)$, which partially coincides with the main trajectory of the Zone and thus should be rejected.)

$$t_F = t_1 = a(f6)a(e5)a(e4)a(f3)a(g2)a(h1).$$

Taking into account that $\text{TIME}(8) = 5$, we have

$$4_1=>t(\text{BOMBER}, t_B, 5)t((\text{FIGHTER}, t_F), 5)$$

$$A((44, 8, 5), v, g(\text{FIGHTER}, t_F, \text{zero}))$$



—STEP 6—

$$4_1 \Rightarrow t(\text{BOMBER}, t_B, 5) \cup ((\text{FIGHTER}, t_F), 5) \\ A((44, 8, 5), v, g(\text{FIGHTER}, t_F, \text{zero}))$$

Computation of $g(\text{FIGHTER}, t_F, \text{zero})$

For all $r \in X$ the r -th component of function g is as follows:

$$g_r(\text{FIGHTER}, t_F, \text{zero}) = \begin{cases} 1, & \text{if } \text{DIST}(r, \text{FIGHTER}, t_F) < 2n, \\ 0, & \text{if } \text{DIST}(r, \text{FIGHTER}, t_F) = 2n, \end{cases}$$

$\text{DIST}(x, \text{FIGHTER}, t_F) = k+1$, where k is the number of symbol of the trajectory t_F , whose parameter value equals x .

$$\text{DIST}(e5, \text{FIGHTER}, t_F) = 2$$

$$\text{DIST}(e4, \text{FIGHTER}, t_F) = 3$$

$$\text{DIST}(f3, \text{FIGHTER}, t_F) = 4$$

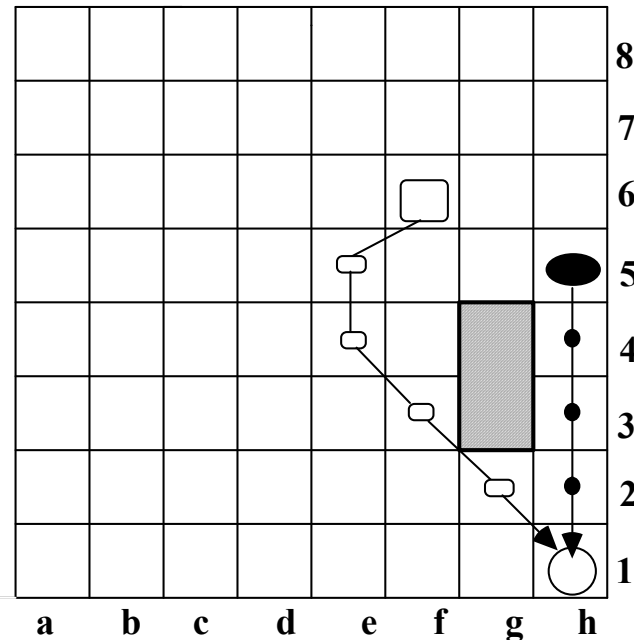
$$\text{DIST}(g2, \text{FIGHTER}, t_F) = 5$$

$$\text{DIST}(h1, \text{FIGHTER}, t_F) = 6$$

For the rest of x from X $\text{DIST}(x, \text{FIGHTER}, t_F) = 2 \times 62 = 124$.

Thus for $r \in \{e5, e4, f3, g2, h1\} = \{35, 28, 21, 15, 8\}$

$g_r(\text{FIGHTER}, t_F, \text{zero}) = 1$, for the rest of r $g_r = 0$.



—STEP 7—

Computation of NEXTTIME

$$\text{NEXTTIME}(z) = \text{ALPHA}(z, (\text{FIGHTER}, t_F), 5 - 5 + 1).$$

From previous steps $\text{NEXTTIME}(x) = 124$ for all x from X .

$$\text{ALPHA}(x, p_O, t_O, k) = \begin{cases} \max(\text{NEXTTIME}(x), k), & \text{if } (\text{DIST}(x, p_O, t_O) \neq 2n) \\ & \wedge (\text{NEXTTIME}(x) \neq 2n); \\ k, & \text{if } \text{DIST}(x, p_O, t_O) \neq 2n \\ & \wedge (\text{NEXTTIME}(x) = 2n); \\ \text{NEXTTIME}(x), & \text{if } \text{DIST}(x, p_O, t_O) = 2n. \end{cases}$$

For $x \in \{e5, e4, f3, g2, h1\}$ $\text{ALPHA}(x, \text{FIGHTER}, t_F, 1) = 1$,
while for other x $\text{ALPHA}(x, \text{FIGHTER}, t_F, 1) = 124$.

The same values should be assigned to $\text{NEXTTIME}(z)$.

A representation of values of w (left) and $\text{NEXTTIME}(z)$ (right) after generating trajectory $a(f6)a(e5)a(e4)a(f3)a(g2)a(h1)$.

				1			
				1			
					1		
						1	
							1

					1		
					1		
						1	
							1
							1

—STEP 8—

Try 3 with $u = (44, 8, 5)$, i.e., with $l > 0$ and $x \neq 62$.

New loop consists of multiple applications of production 3 after failures of attempts to apply one of productions 4j .

$3 \Rightarrow t(\text{BOMBER}, t_B, 5) t(\text{FIGHTER}, t_F, 5) A(45, 8, 5), v, w)$

$3 \Rightarrow t(\text{BOMBER}, t_B, 5) t((\text{FIGHTER}, t_F, 5) A(46, 8, 5), v, w)$

.....

$3 \Rightarrow t(\text{BOMBER}, t_B, 5) t(\text{FIGHTER}, t_F, 5) A(62, 8, 5), v, w).$

Computations of NEXTTIME(z) in production 3 will not change its values.

With $u = (62, 8, 5)$ this loop is terminated which means that no other starting points are found.

New loop begins. The grammar changes ending point of prospective trajectories:

$3 \Rightarrow t(\text{BOMBER}, t_B, 5) t(\text{FIGHTER}, t_F, 5) A(1, 9, 0), v, w)$

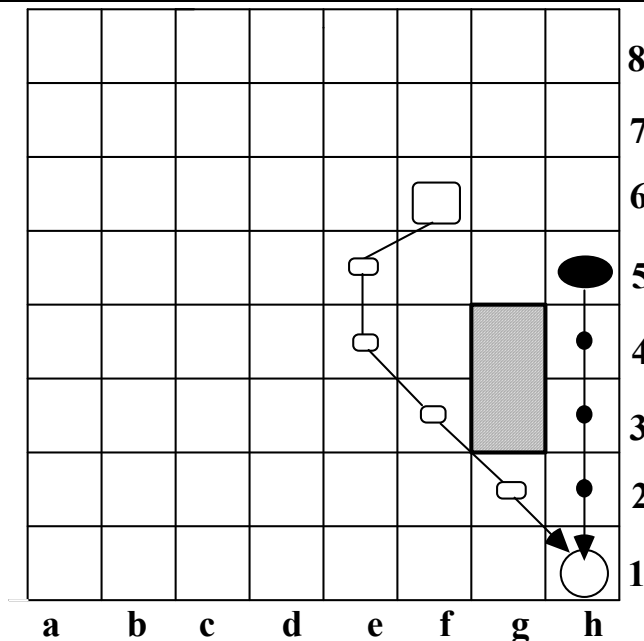
$3 \Rightarrow t(\text{BOMBER}, t_B, 5) t((\text{FIGHTER}, t_F, 5) A(1, 10, 0), v, w)$

.....

$3 \Rightarrow t(\text{BOMBER}, t_B, 5) t(\text{FIGHTER}, t_F, 5) A(1, 16, 4), v, w),$

With $u = (1, 15, 0)$ this loop is terminated;

$y+1$ corresponds to h2, $\text{TIME}(y+1) * v_{y+1} = \text{TIME}(h2) * 1 = 4.$



—STEP 9—

Point h2 is determined as the next ending point for generating trajectories of robots, which can intercept motion of the BOMBER. The following derivation steps would allow us to look for possible starting points of such trajectories. Obviously, nothing will be found, except

$$a(f6)a(g5)a(h4)a(h3)a(h2),$$

which will be rejected.

The same negative result will be achieved with the next ending point, h3. The only intercepting trajectory to be found and accepted is as follows:

$$t_F^1 = t_1 = a(f6)a(g5)a(h4)$$

$$4_1 \Rightarrow t(\text{BOMBER}, t_B, 5)t(\text{FIGHTER}, t_F, 5)t(\text{FIGHTER}, t_F^1, 2)$$

$$A((44,30,2), v, g(\text{FIGHTER}, t_F^1, w)),$$

$$\text{NEXTTIME}(z) = \text{ALPHA}(z, \text{FIGHTER}, t_F^1, 2-2+1).$$

A representation of values of w (left) and $\text{NEXTTIME}(z)$ (right) after generating trajectory $a(f6)a(g5)a(h4)$.

				1		1	
				1			1
					1		
						1	
							1

					1		1
					1		1
						1	
							1

Try 3 returning to it each time after unsuccessful attempt of applying production 4j. This loop will be terminated when $Q_3(u) = F$

$$Q_3(u) = (x \neq 62) \vee (y \neq 62) = F \text{ (in our case } x = 62 \text{ and } y = 62).$$

$$\text{Try 5: } Q_5(w) = (w \neq 0) = T.$$

$$5 \Rightarrow t(\text{BOMBER}, t_B, 5)t(\text{FIGHTER}, t_F, 5)t(\text{FIGHTER}, t_F^1, 2)$$

$$A((0, 0, 0), w, \text{zero})$$

$$\text{TIME}(z) = \text{NEXTTIME}(z)$$

This is the completion of generation of the 1-st negation trajectories

—STEP 10—

All the steps, 3 and 4j, which have been executed (or tried) for generating 1-st negation trajectories, will be repeated for generating 2-nd negation. No one such trajectory should be found.

The next return to production 5 will happen with $w = zero$ (nothing is found). This means that production 5 is not applicable, and we complete derivation by applying production 6:

$$6 \Rightarrow t(\text{BOMBER}, t_B, 5)t(\text{FIGHTER}, t_F, 5)t(\text{FIGHTER}, t_F^1, 2).$$

