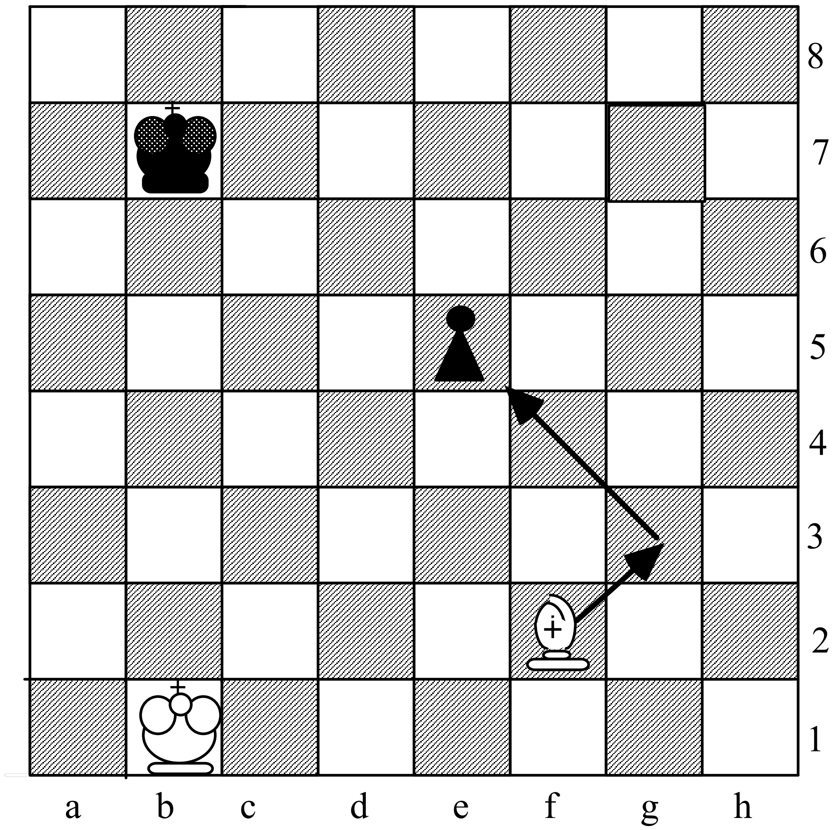
Assignment 4. Due Date: 09/18/17

10. Generate Zone with the main trajectory shown in the figure below. Use the grammar of Zones G_Z, show values of all the functions and sets.



Network Languages

A ***trajectory connection***
of the trajectories t_1 and t_2 is the relation $C(t_1,t_2)$. It holds, if the *ending link* of the trajectory t_1 coincides with an *intermediate link* of the trajectory t_2;

On the set **A** of trajectories it is defined:

$\mathbf{C_A}^{k}\mathbf{(t_1, t_2)}$, a ***k-th degree of connection*** and

$\mathbf{C_A^{+}(t_1,t_2)}$, a ***transitive closure.***

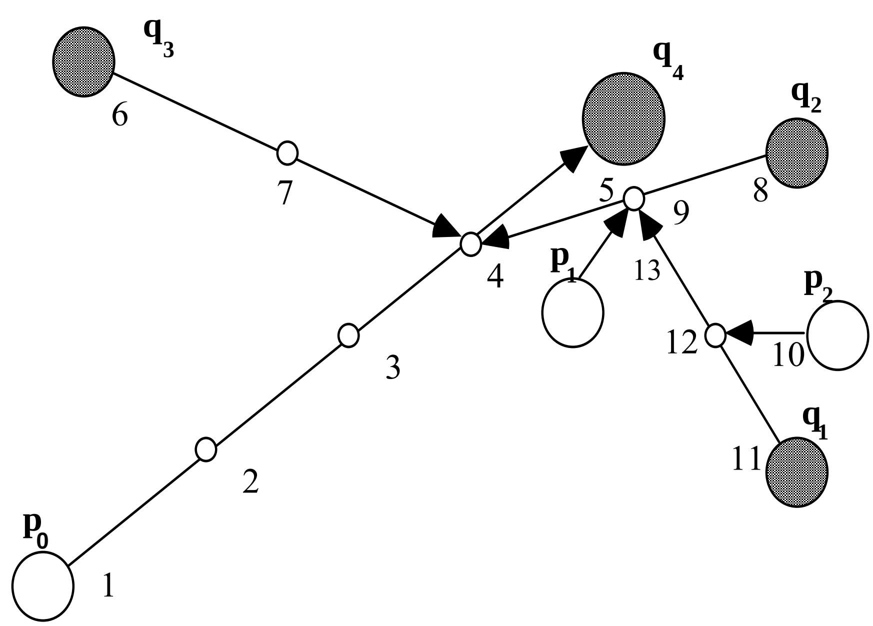
A ***trajectory network*** **W**
relative to trajectory t_o is a finite set of trajectories $t_o, t_1,\ldots, t_k$ from the language $L_t^H(S)$: for every trajectory t_i from W (i=1, 2,…, k) the relation $C_W^{+}(t_i, t_o)$ holds.

A ***family of trajectory network languages*** $\mathbf{L_C(S)}$
in a state S of the Complex System is the family of languages that contains strings of the form

$$\boldsymbol{t}(t_1,param)\boldsymbol{t}(t_2,param)\ldots\boldsymbol{t}(t_m,param),$$

where *param* in parentheses substitute for the other parameters of a particular language. All the symbols $t_1,t_2,\ldots,t_m$ correspond to trajectories which form a trajectory network **W** relative to t_1.

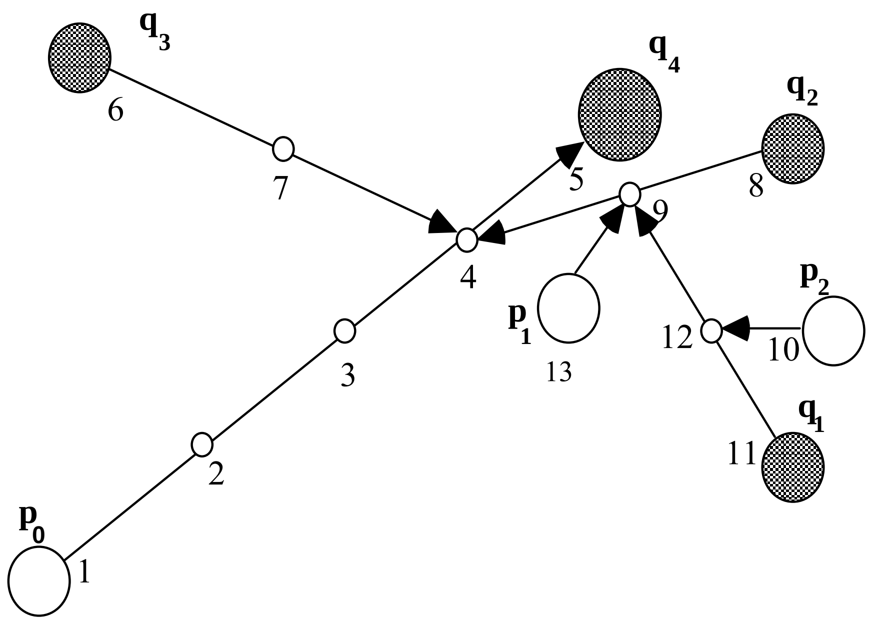
Network language interpretation.



Languages of Trajectory Networks

After defining the Language of Trajectories, we have tools for the breakdown of our System into subsystems. These subsystems should be various types of trajectory networks, i.e., some sets of interconnected trajectories with one singled out trajectory called the main trajectory. An example of such a network is shown in Figure.

Network language interpretation



The basic idea behind these networks is as follows. Element p_o should move along the main trajectory $\boldsymbol{a}(1)\boldsymbol{a}(2)\boldsymbol{a}(3)\boldsymbol{a}(4)\boldsymbol{a}(5)$ to reach the ending point 5 and remove the target q_4 (an opposite element). Naturally, the opposite elements should try to disturb those motions by controlling the intermediate points of the main trajectory. They should come closer to these points (to the point 4 in Fig. 2) and remove element p_o after its arrival (at point 4). For this purpose, elements q_3 or q_2 should move along the trajectories $\boldsymbol{a}(6)\boldsymbol{a}(7)\boldsymbol{a}(4)$ and $\boldsymbol{a}(8)\boldsymbol{a}(9)\boldsymbol{a}(4)$, respectively, and wait (if necessary) on the next to last point (7 or 9) for the arrival of element p_o at point 4. Similarly, element p_1 of the same side as p_o might try to disturb the motion of q_2 by controlling point 9 along the trajectory $\boldsymbol{a}(13)\boldsymbol{a}(9)$. It makes sense for the opposite side to include the trajectory $\boldsymbol{a}(11)\boldsymbol{a}(12)\boldsymbol{a}(9)$ of element q_1 to prevent this control.

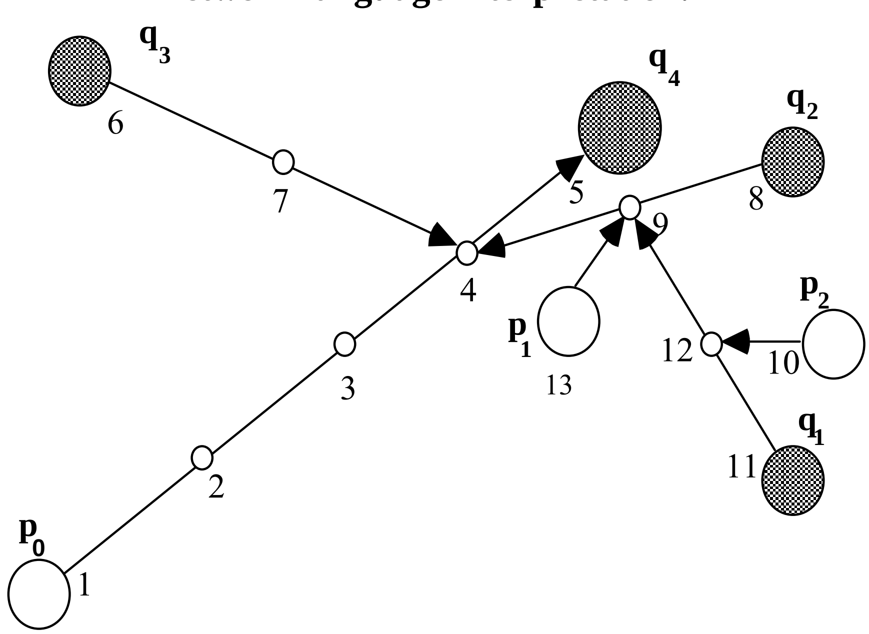
Similar networks are used for the breakdown of complex systems in different areas. Let us consider a formal linguistic formalization of such networks. The Language of Trajectories describes "one-dimensional" objects by joining symbols into a string employing reachability relation $R_p(x, y)$. To describe networks, i.e., “two (or multi)-dimensional" objects made up of trajectories, we use the relation of *trajectory connection.*

Definition
A *trajectory connection* of the trajectories t_1 and t_2 is the relation $C(t_1,t_2)$. It holds, if the ending link of the trajectory t_1 coincides with an intermediate link of the trajectory t_2; more precisely t_1 is connected with t_2, if among the parameter values $P(t_2) = \{y, y_1,\dots, y_l\}$ of trajectory t_2 there is a value $y_i = x_k$, where $t_1= a(x_o)a(x_1)\dots a(x_k)$. If t_1 belongs to a set of trajectories with the common end-point, than the entire set is said to be connected with the trajectory t_2.

For example, the trajectories ***a***(6)***a***(7)***a***(4) and ***a***(8)***a***(9)***a***(4) are connected with the main trajectory ***a***(1)***a***(2)***a***(3)***a***(4)***a***(5) via point 4. Trajectories ***a***(13)***a***(9) and ***a***(11)***a***(12)***a***(9) are connected with ***a***(8)***a***(9)***a***(4).

Definition
A set of trajectories $CA_B(t)$ from B, with which trajectory t is connected is called the *bundle of trajectories* for trajectory t relative to the set B of trajectories.

Network language interpretation.



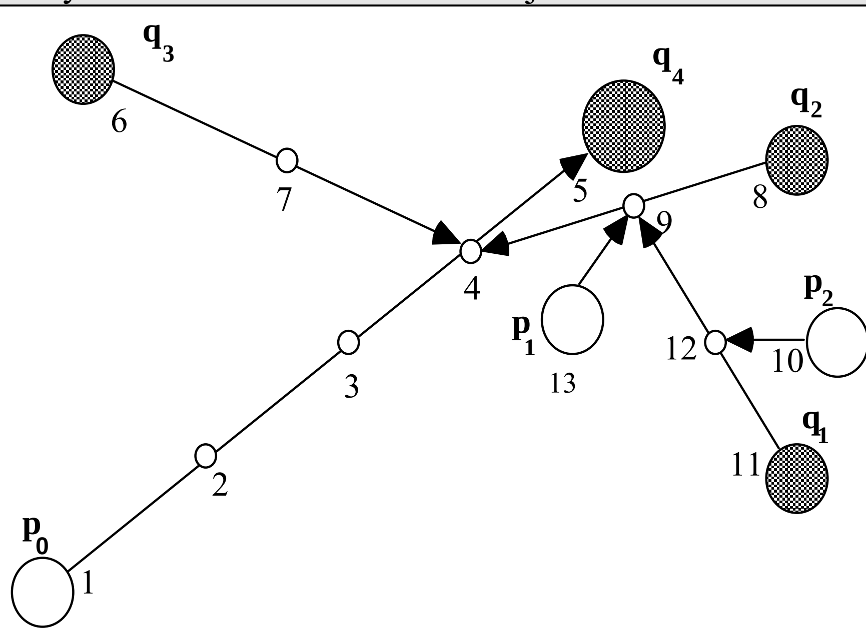
Definition: A *k-th degree of the relation C* on the set of trajectories A (denoted by C_A^k) is defined as usual by induction.
For k = 1 $C_A^k(t_1,t_2)$ coincides with $C(t_1,t_2)$ for t_1,t_2 from A.
For k > 1 $C_A^k(t_1,t_2)$ holds if and only if there exists a trajectory t_3 from A, such that $C(t_1,t_3)$ and $C_A^{k-1}(t_3,t_2)$ both hold.

Trajectory ***a***(11)***a***(12)***a***(9) shown in Figure is connected (degree 2) with trajectory ***a***(1)***a***(2)***a***(3)***a***(4)***a***(5), i.e., C^2(***a***(11)***a***(12)***a***(9), ***a***(1)***a***(2)***a***(3)***a***(4)***a***(5)) holds.

Definition: A *transitive closure of the relation* C on the set of trajectories A (denoted by C_A^+) is a relation, such that $C_A^+(t_1,t_2)$ holds for t_1 and t_2 from A, if and only if there exists i > 0 that $C_A^i(t_1,t_2)$ holds.

The trajectory ***a***(10)***a***(12) is in transitive closure to the trajectory ***a***(1)***a***(2)***a***(3)***a***(4)***a***(5) because C^3(***a***(10)***a***(12), ***a***(1)***a***(2)***a***(3)***a***(4)***a***(5)) holds by means of the chain of trajectories ***a***(11)***a***(12)***a***(9) and ***a***(8)***a***(9)***a***(4).

Definition: A *trajectory network* W relative to trajectory t_o is a finite set of trajectories $t_o,t_1,\ldots,t_k$ from the language $L_t^H(S)$ that possesses the following property: for every trajectory t_i from W (i = 1, 2,…,k) the relation $C_W^+(t_i,t_o)$ holds, i.e., each trajectory of the network W is connected with the trajectory t_o that was singled out by a subset of interconnected trajectories of this network.



Obviously, the trajectories shown in Figure form a trajectory network relative to the main trajectory ***a***(1)***a***(2)***a***(3)***a***(4)***a***(5). We are now ready to define network languages.

Definition

A *family of trajectory network languages* $L_C(S)$ in a state S of the Complex System is the family of languages that contains strings of the form

$$t(t_o, param)t(t_1, param)\ldots t(t_m, param),$$

where *param* in parentheses substitute for the other parameters of a particular language. All the symbols of the string t_o, t_1,…, t_m correspond to trajectories which form a trajectory network W relative to t_o.

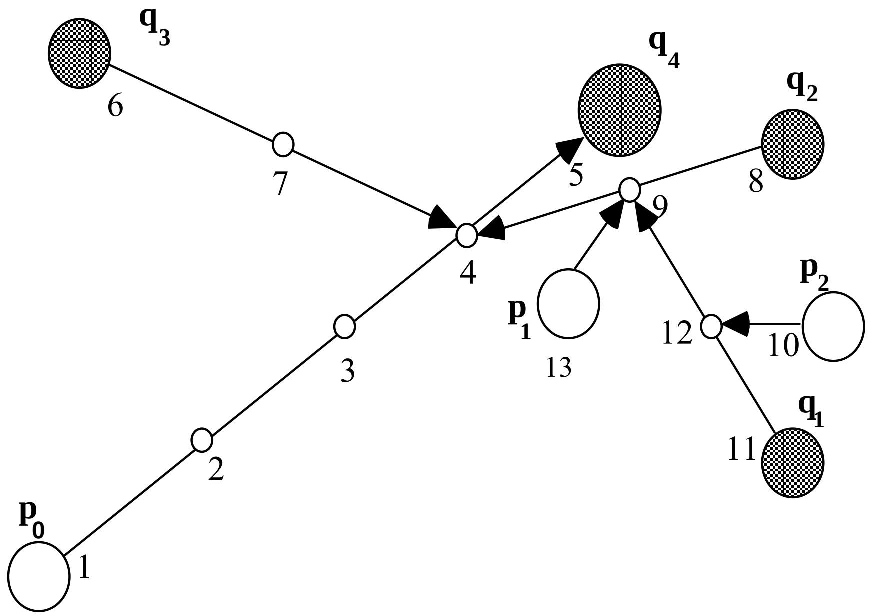
Different members of this family correspond to different types of trajectory network languages, which describe particular subsystems for solving search problems. One of such languages is a language, which describes specific networks called Zones. They play a main role in the model considered here. The formal definition of this language is essentially constructive and requires showing explicitly a method for generating this language, i.e., a certain formal grammar. This grammar will be discussed later. In order to make our points transparent, first of all, we define the Language of Zones informally.

A *Language of Zones* is a trajectory network language with strings of the form

$$Z = \boldsymbol{t}(p_o,t_o,\tau_o)\ \boldsymbol{t}(p_1,t_1,\tau_1)\ldots\boldsymbol{t}(p_k,t_k,\tau_k),$$

where $t_o,t_1,\ldots,t_k$ are the trajectories of elements $p_o,p_2,\ldots,p_k$ respectively; $\tau_o,\tau_1,\ldots,\tau_k$ are positive integer numbers (or 0) which "denote time allocated for the motion along the trajectories with respect to the mutual goal of this Zone: to remove the target element – for one side, and to protect it – for the opposite side. Trajectory $\boldsymbol{t}(p_o,t_o,\tau_o)$ is called the *main trajectory* of the Zone. The element q standing on the ending point of the main trajectory is called the *target*. The elements p_o and q belong to the opposing sides.

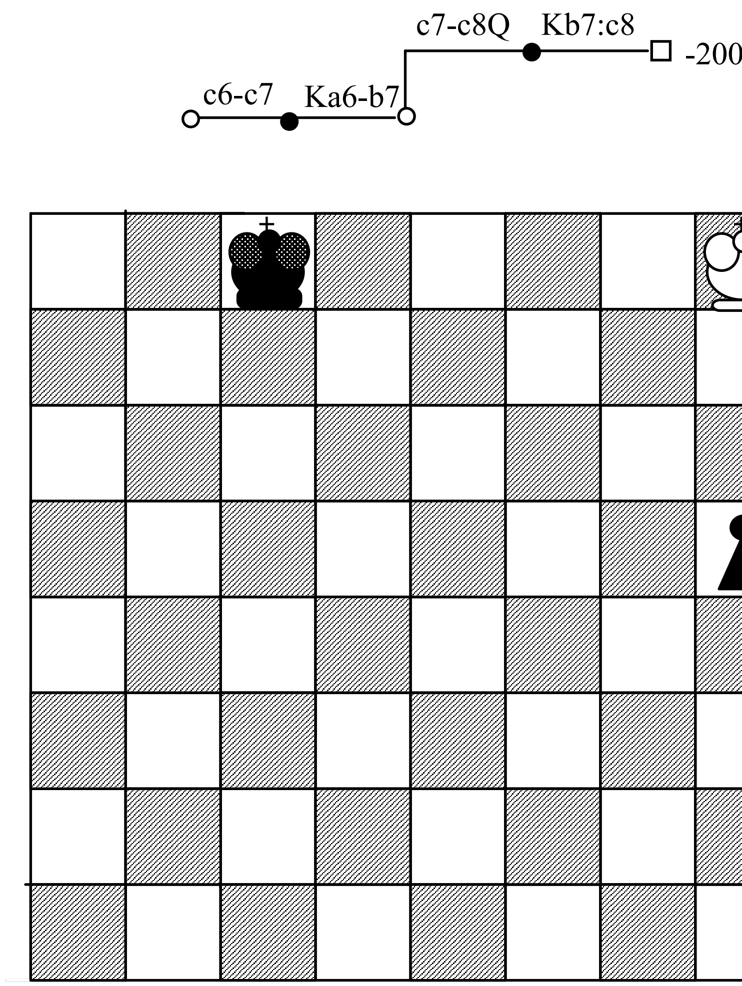
Network language interpretation.

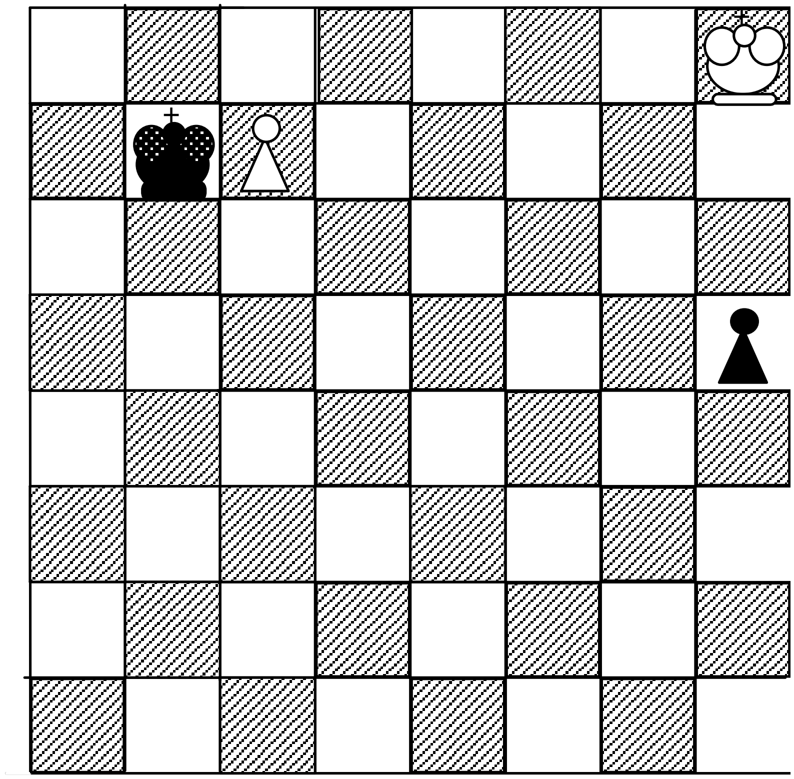


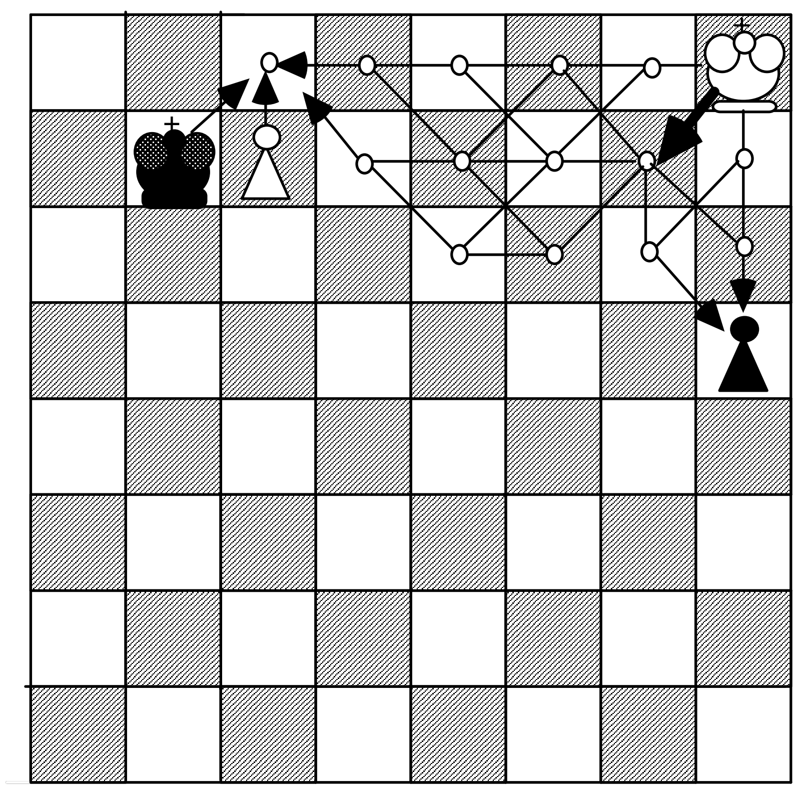
$$Z = \boldsymbol{t}(p_o, \boldsymbol{a}(1)\boldsymbol{a}(2)\boldsymbol{a}(3)\boldsymbol{a}(4)\boldsymbol{a}(5), 5)\boldsymbol{t}(q_3, \boldsymbol{a}(6)\boldsymbol{a}(7)\boldsymbol{a}(4), 4)\boldsymbol{t}(q_2, \boldsymbol{a}(8)\boldsymbol{a}(9)\boldsymbol{a}(4), 4)$$
$$\boldsymbol{t}(p_1, \boldsymbol{a}(13)\boldsymbol{a}(9), 3)\boldsymbol{t}(q_1, \boldsymbol{a}(11)\boldsymbol{a}(12)\boldsymbol{a}(9), 3)\ \boldsymbol{t}(p_2, \boldsymbol{a}(10)\boldsymbol{a}(12), 2)$$

Assume that the goal of the white side is to remove target q_4, while the goal of the black side is to protect it. According to these goals, element p_o begins motion to the target after Black starts to move their elements q_2 or q_3 to intercept element p_o. We will always assume that the protecting side (in this case Black side) moves first. Only those black trajectories are to be included into the Zone where the motion of the element makes sense, i. e., the *length of the trajectory is less than the amount of time (third parameter f) allocated to it.* For example, the motion along the trajectories $\boldsymbol{a}(6)\boldsymbol{a}(7)\boldsymbol{a}(4)$ and $\boldsymbol{a}(8)\boldsymbol{a}(9)\boldsymbol{a}(4)$ makes sense, because they are of length 2 and time allocated equals 4: each of the elements has 4 time intervals to reach point 4 to intercept element p_o assuming one would go along the main trajectory without move omission. According to definition of Zone the trajectories of white elements (except p_o) could only be of the length 1, e.g., $\boldsymbol{a}(13)\boldsymbol{a}(9)$ or $\boldsymbol{a}(10)\boldsymbol{a}(12)$. As far as element p_1 can intercept motion of the element q_2 at the point 9, Black include into the Zone the trajectory $\boldsymbol{a}(11)\boldsymbol{a}(12)\boldsymbol{a}(9)$ of the element q_1 which has enough time for motion to prevent this interception. The total amount of time allocated to the whole bundle of black trajectories connected (directly or indirectly) with the given point of main trajectory is determined by the number of that point. For example, for the point 4 it is equal to 4 time intervals.

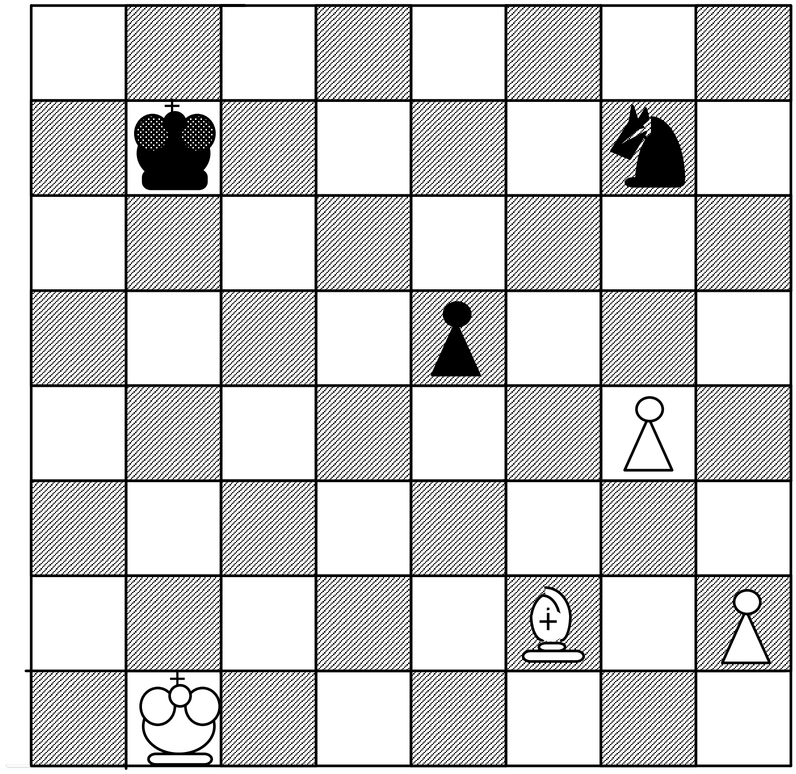
ZONES IN THE CHESS MODEL

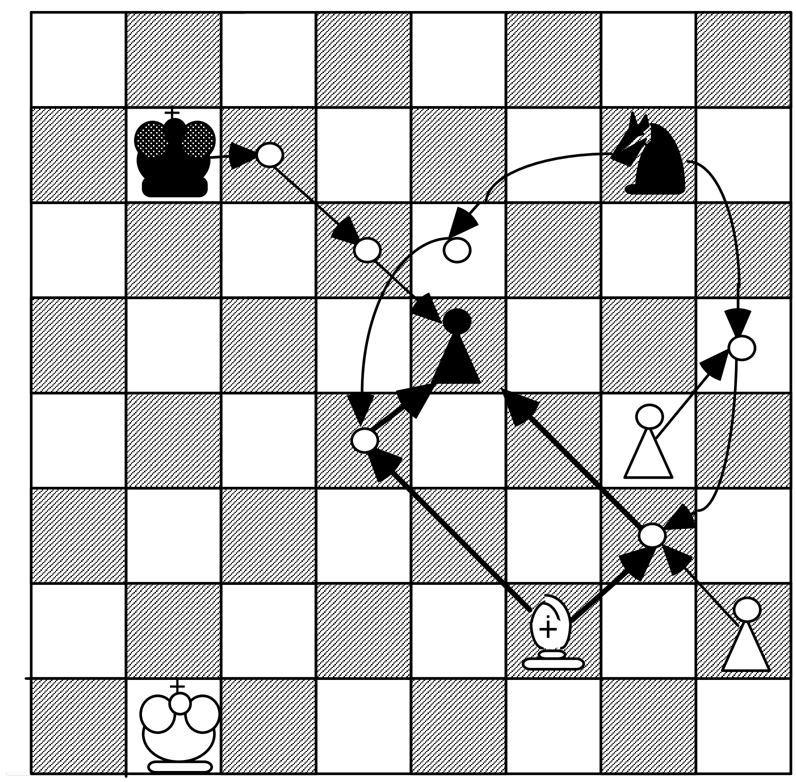




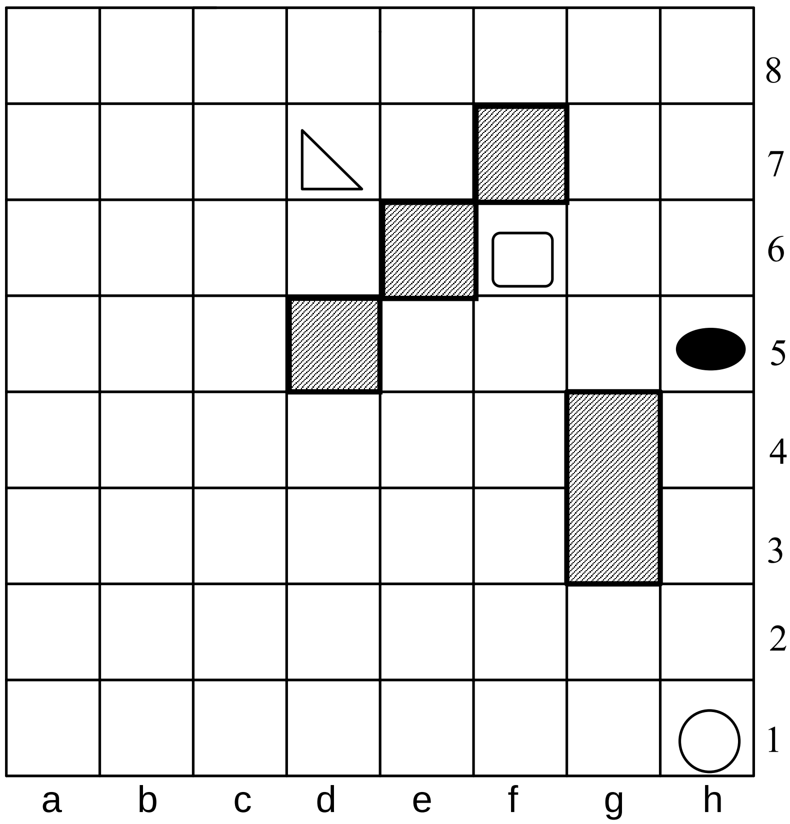


Interpretation of Language of Zones for the chess model





Interpretation of the trajectory network language for the robot control model

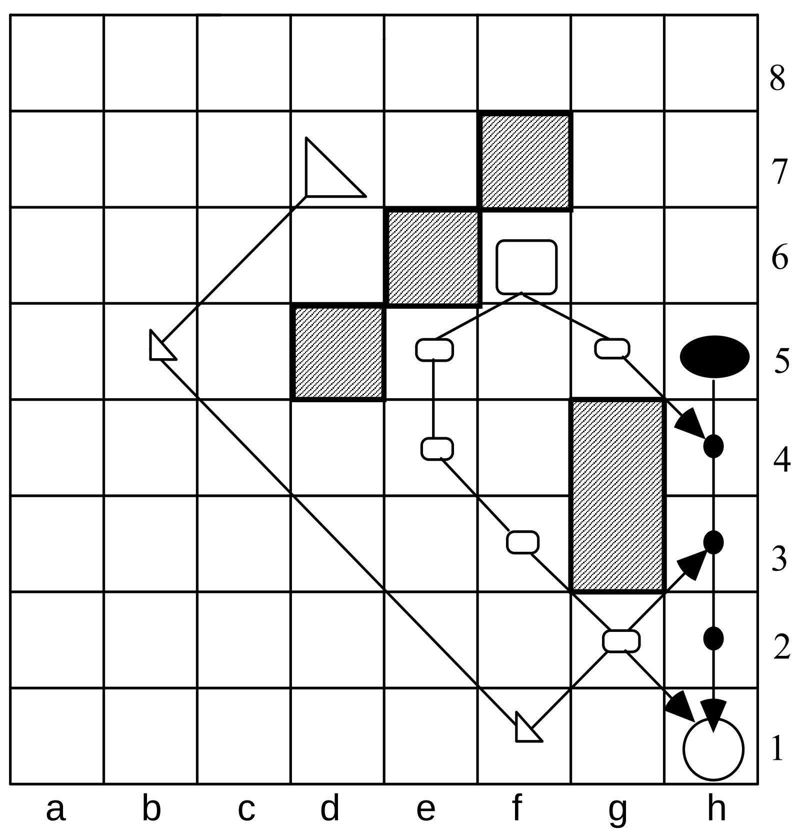


Interpretation of the trajectory network language for the robot control model

$$\boldsymbol{t}(\mathrm{BOMBER}, t_B, 5)\boldsymbol{t}(\mathrm{FIGHTER}, t_F, 5)\boldsymbol{t}(\mathrm{MISSILE}, t_M, 5)\boldsymbol{t}(\mathrm{MISSILE}, t_M{}^1, 3)$$
$$\boldsymbol{t}(\mathrm{FIGHTER}, t_F{}^1, 2),$$

where

$$t_B=\boldsymbol{a}(h5)\boldsymbol{a}(h4)\boldsymbol{a}(h3)\boldsymbol{a}(h2)\boldsymbol{a}(h1),$$
$$t_F=\boldsymbol{a}(f6)\boldsymbol{a}(e5)\boldsymbol{a}(e4)\boldsymbol{a}(f3)\boldsymbol{a}(g2)\boldsymbol{a}(h1),$$
$$t_M=\boldsymbol{a}(d7)\boldsymbol{a}(b5)\boldsymbol{a}(f1)\boldsymbol{a}(g2)\boldsymbol{a}(h1),$$
$$t_M{}^1=\boldsymbol{a}(d7)\boldsymbol{a}(b5)\boldsymbol{a}(f1)\boldsymbol{a}(h3),$$
$$t_F{}^1=\boldsymbol{a}(f6)\boldsymbol{a}(g5)\boldsymbol{a}(h4)$$



Zones for the Game of Chess.

The problem of programming the game of chess, is the most transparent example of a Linguistic Geometry application. This chess problem domain was the first application and experimental area for the formal linguistic approach. In this model of the Complex System **X** is represented by 64 squares of the chess board, i.e., n=64; $\mathbf{P_1}$ and $\mathbf{P_2}$ are the white and black pieces; $\mathbf{R_p(x, y)}$ are given by the rules of the game, permitting or forbidding a piece p to make a move from square x to square y; thus a point x is *reachable* from a point y for an element p, if a piece p can move from square x to square y according to the chess game rules; **ON(p)=x**, if a piece p stands on square x; **v(p)** is the value of piece p, e.g., P – 1, N – 3, B – 3, R – 5, Q – 9, K – 200; $\mathbf{S_i}$ is an arbitrary initial chess position for analysis, or the starting position of the game; $\mathbf{S_t}$ is the set of chess positions which can be obtained from all possible mating positions in two half moves by capturing the King (suppose, this capture is permitted). The sets of WFF $\{ON(p_j) = x_k\}$ correspond to the lists of pieces with their coordinates. **TRANSITION(p, x, y)** represents the move of the piece p from square x to square y; if a piece of the opposing color stands on y, a capture is made.

The chess problem does not completely meet the requirements of the definition of the Complex System. We have neglected such an important chess concept as a blockade: in the Complex System several elements (pieces of the same color) can stand on the same point (square). Besides that, we have neglected certain chess features, such as castling, capture en passant, Pawn promotion, etc. All these chess complications are not crucial for our model; at the implementation stage of the hierarchy of languages for this model (program PIONEER) all this was taken into account.

Let us consider an example of the Language of Zones for the chess model. We are going to present this language informally, listing Zones and trajectories, without explicit generating by the Grammar of Zones. An artificial chess position is shown in p. 11. Assuming that, the so-called horizon, H = 2 steps, in this range of lengths the only couple of attacking and attacked pieces are the Bishop on f2 and Pawn on e5, respectively. Thus, only such Zones can be generated. Trajectories ***a***(f2)***a***(g3)***a***(e5) and ***a***(f2)***a***(d4)***a***(e5) for the Bishop are the main trajectories of these Zones. They are shown by bold lines. All the other lines shown in p. 11 single out one Zone of the bundle of Zones generated by the grammar. The black side can intercept the Bishop employing one of the various intercepting trajectories, the 1-st negation trajectories. For example, the interception on square g3 can be accomplished by the black pieces located in the range of two steps from g3. (By definition of Zone it is generated in assumption that the protecting side is to move.) Thus one of the Knight's trajectories from g7 to g3, ***a***(g7)***a***(f5)***a***(g3) or ***a***(g7)***a***(h5)***a***(g3), should be included into the this Zone. Similarly either ***a***(g7)***a***(e6)***a***(d4), or ***a***(g7)***a***(f5)***a***(d4) can be included to intercept Bishop on d4. The last chance for interception is to approach the target, Pawn on e5, in 3 steps. It can be done by the King on b7 along one of two trajectories, ***a***(b7)***a***(c6)***a***(d5)***a***(e5) or ***a***(b7)***a***(c7)***a***(d6)***a***(e5). There are no other trajectories to prevent the attack. White side should include its own trajectories to support the attack, i.e., the motion of the Bishop along one of the main trajectories. By definition of Zone they are in the range of one step only. They are ***a***(h2)***a***(g3), ***a***(g4)***a***(h5) (if ***a***(g7)***a***(h5)***a***(g3) was included) or ***a***(g4)***a***(f5) (in case of ***a***(g7)***a***(f5)***a***(g3)).