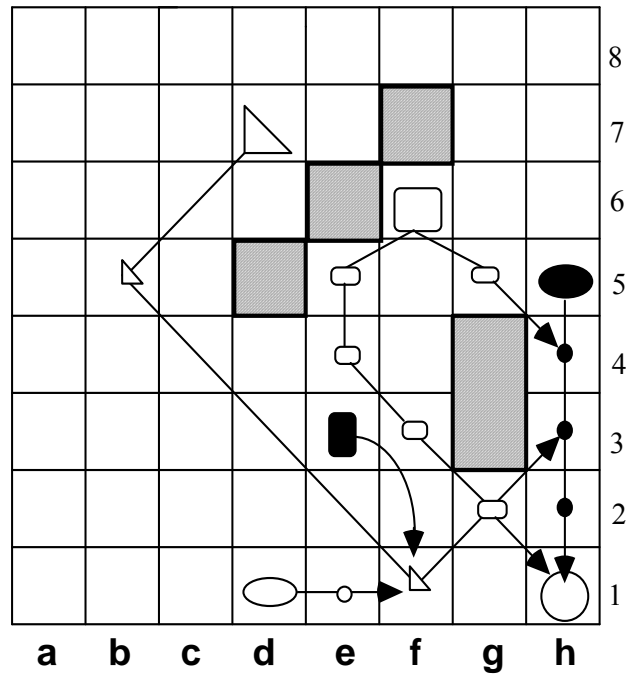


## Searches for Robot Control Model



### An *Abstract Board Game*

is the following eight-tuple

$$\langle X, P, R_p, \{ON\}, v, S_i, S_t, TR \rangle$$

$X = \{x_i\}$  is a finite set of *points*;

$P = \{p_i\}$  is a finite set of *elements*;  $P = P_1 \cup P_2, P_1 \cap P_2 = \emptyset$ ;

$R_p(x, y)$  is a family of binary relations of *reachability* in  $X$   
 $(x \in X, y \in X, p \in P)$ ;  $y$  is *reachable* from  $x$  for  $p$ ;

$ON(p) = x$  is a partial function of *placement* of elements  $P$  into  $X$ ;

$val > 0$  is a real function,  $val(p_i)$  are the *values* of elements;

$S_i$  is a set of *initial* states of the system,  
 a certain set of formulas  $\{ON(p_i) = x_i\}$ ;

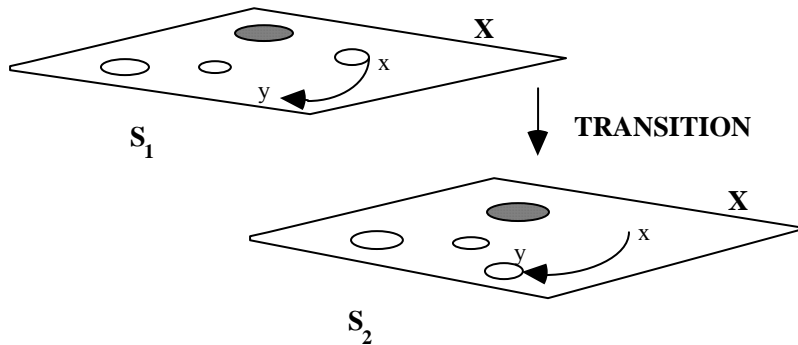
$S_t$  is a set *target* states of the system (as  $S_i$ );

$TR$  is a set of operators **TRANSITION**( $p, x, y$ ) for transition of the system from one state to another described as follows

**precondition:**  $(ON(p) = x) \wedge R_p(x, y)$

**delete:**  $ON(p) = x, ON(q) = y$

**add:**  $ON(p) = y$



**Family of Grammars of Reduced Searches  $G_{rs}$**

| $L$ | $Q$            | Kernel, $\pi_k$                             | $\pi_n$  | $F_T$ | $F_F$       |
|-----|----------------|---|--|-------|-------------|
| 1   | $Q_1$          | $S(i) \rightarrow A(i)$                     | $M(i) := m_s(STATE)$<br>formulas for $G_{rs}$  | 2     | $\emptyset$ |
| 2   | $Q_2^{G_{rs}}$ | $A(i) \rightarrow A(End)$<br>$\pi(End)A(i)$ | Parent(End) := i<br><b>If</b> $Child(i) \neq 0$<br><b>then</b><br>$Sibling(Child(i)) := End$<br><b>else</b><br>$Sibling(i) := 0$<br>$Child(i) := End$<br><b>Endif</b><br>End := End+1<br>d := d+1<br>SIGN := - SIGN<br><br>STATE := NEWSTATE<br>m(End) := $m_s(NEWSTATE)$<br>V(End) := <b>BIG_NUMBER</b> * SIGN<br><br>WHO(End) := <i>Element</i> (NEWMOVE)<br>FROM(End) := $X(NEWMOVE)$<br>TO(End) := $Y(NEWMOVE)$<br><br>formulas for $G_{rs}$ | 2     | 3           |
| 3   | $Q_3$          | $A(i) \rightarrow e$                        | <b>if</b> d $\neq 0$<br><b>then</b><br>SIGN := - SIGN<br>d := d - 1<br><b>endif</b><br>STATE := TRANSITION-1 (WHO(i), FROM(i), TO(i)) (STATE)<br><b>V</b> (Parent(i)) := <i>MINIMAX</i> (SIGN, <b>V</b> (Parent(i)), <i>LEAF</i> (V(i), m(i)))<br><br><b>V</b> (i) := <i>LEAF</i> (V(i), m(i))<br>formulas for $G_{rs}$  | 2     | $\emptyset$ |

**At the beginning:** i = 0, End = 1, d = 0; m(i), V(i), FROM(i), TO(i), Child(i), Sibling(i), Parent(i) are equal to 0; WHO(i)  $\in P \forall i \geq 0$ ; SIGN  $\in \{-1, 1\}$ , STATE  $\in SPACE$

$$V_T = \{\pi\} \quad V_N = \{A, S\} \quad Pred = \{Q_1, Q_2, Q_3\}$$

$$Q_1 = T$$

$$Q_2^{GrS} = (\exists p \exists x \exists y (((SIGN = 1) \wedge (p \in P_1)) \vee ((SIGN = -1) \wedge (p \in P_2))) \wedge \\ (ON(p) = x) \wedge R_p(x, y)) \wedge (d < d_{\max}) \wedge \neg CUT^{GrS}, \\ \text{where } CUT^{GrS} \text{ depends on the specific grammar } GrS$$

$$Q_3 = T$$

$$Fcon = \{MV^{GrS}, Element, X, Y, MINIMAX, LEAF, TRANSITION, \\ TRANSITION^{-1}, m_s\}$$

$MV^{GrS}$  depends on the grammar  $GrS$

$$Fvar = \{End, SIGN, m, V, WHO, FROM, TO, Child, Parent, Sibling, STATE\}$$

$$Parm: S \rightarrow \{i\}, A \rightarrow \{i\}, \pi \rightarrow \{i\}, L = \{1, 2, 3\}$$

**Functions are defined as follows:**

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$$D(m_s) = SPACE, m_s(STATE) = \sum_{p \in P_1} val(p) - \sum_{p \in P_2} val(p),$$

Functions  $val$ ,  $TRANSITION$ ,  $TRANSITION^{-1}$  are from the

$$\text{Definition of ABG; } BIG\_NUMBER = \sum_{p \in P} val(p) + 1$$


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$$D(LEAF) = \mathbf{Z}_+ \times \mathbf{Z}_+, \quad \text{If } a = BIG\_NUMBER \text{ then } LEAF(a, b) := b, \\ \text{else } LEAF(a, b) := a$$

$$D(MINIMAX) = \{-1, +1\} \times \mathbf{Z}_+ \times \mathbf{Z}_+$$

$$MINIMAX(SN, v_1, v_2) = \begin{cases} \max(v_1, v_2), & \text{if } SN = 1 \\ \min(v_1, v_2), & \text{if } SN = -1 \end{cases}$$


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$$MOVE = \{(p, x, y) \mid x \in X, y \in X, (\exists p (((SIGN = 1) \wedge (p \in P_1)) \vee \\ ((SIGN = -1) \wedge (p \in P_2))) \wedge (ON(p) = x) \wedge R_p(x, y)))\}$$

$$MOVE = \{(p_1, x_1, y_1), \dots, (p_k, x_k, y_k)\}$$

$$NEWMOVE = MV^{GrS}(d, End, SIGN, m, V, WHO, FROM, TO, Child, \\ Parent, Sibling, STATE, \dots)$$

$MV$  yields the ordinal of a triple from the list  $MOVE$

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$$Element(NEWMOVE) = p_{NEWMOVE},$$

$$X(NEWMOVE) = x_{NEWMOVE}$$

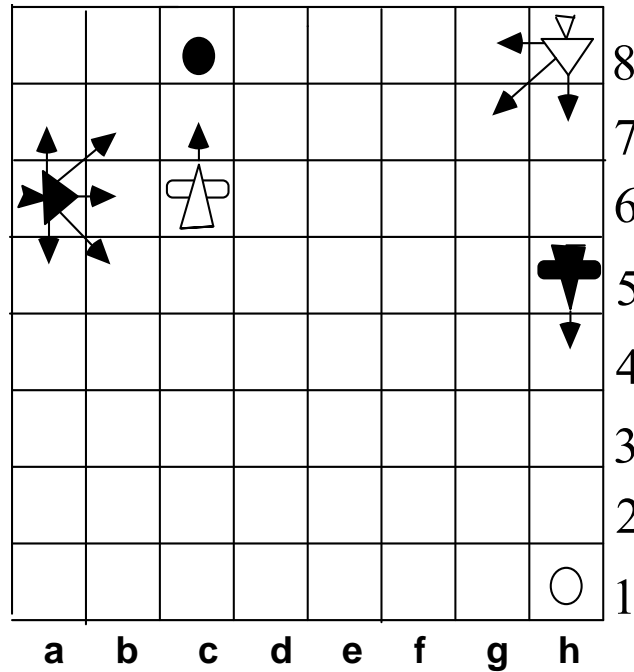
$$Y(NEWMOVE) = y_{NEWMOVE}$$

$$NEWSTATE = TRANSITION(Element(NEWMOVE), X(NEWMOVE), \\ Y(NEWMOVE))(STATE)$$


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# **Optimization problem for autonomous aerospace robotic vehicles with serial alternating motions**

## **2D/4A Problem**



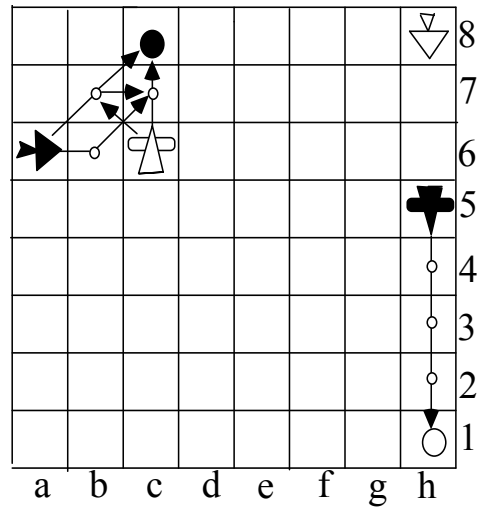
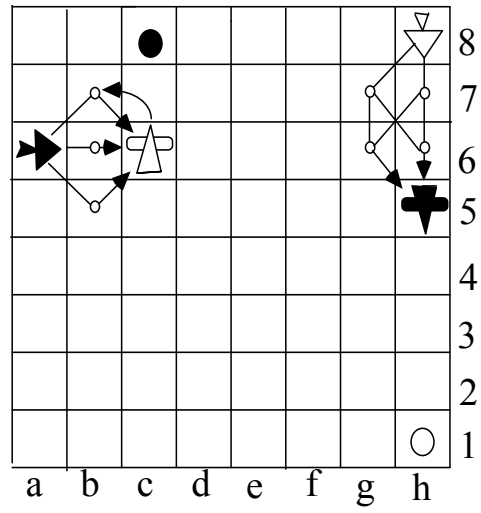
*Is there a strategy for the White to make a draw?*

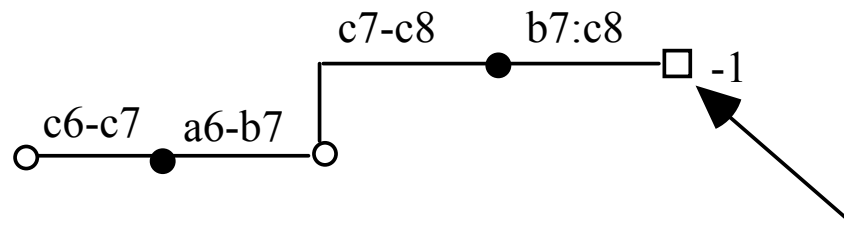
The specific question is as follows.

Is there an optimal strategy that provides one of the following?

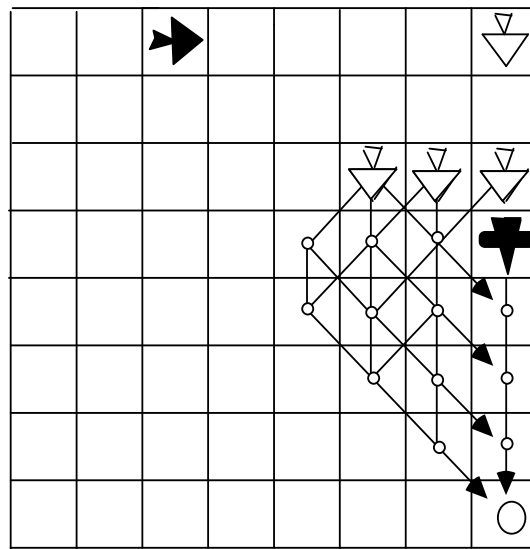
1. Both BOMBERS hit their targets on subsequent time increments and stay safe for at least one time increment.
2. Both BOMBERS are destroyed before they hit their targets or immediately after that.

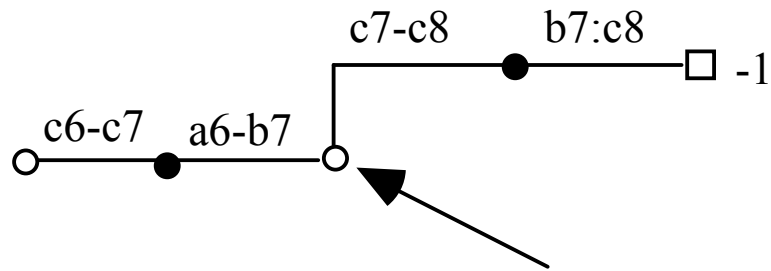
### Zones in the Start State of the Robot Control Model.



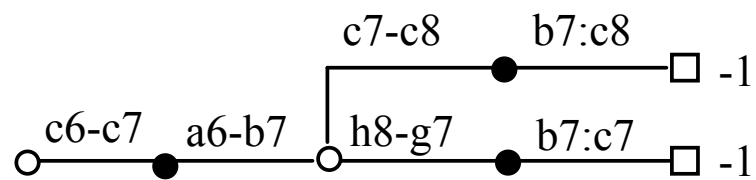
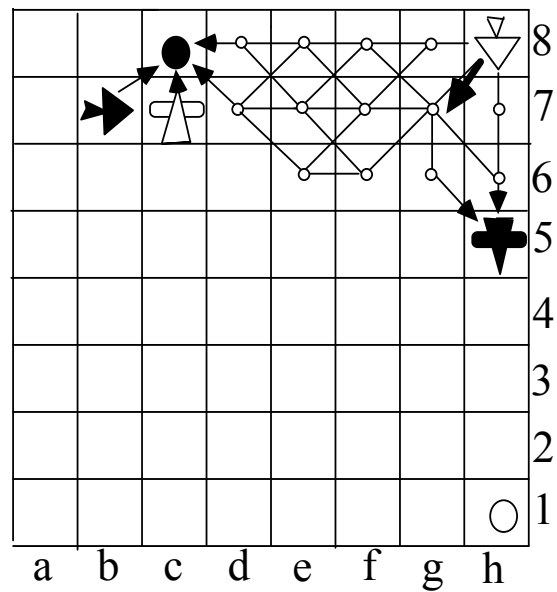


**State where the branch was terminated. Zone gateways.**

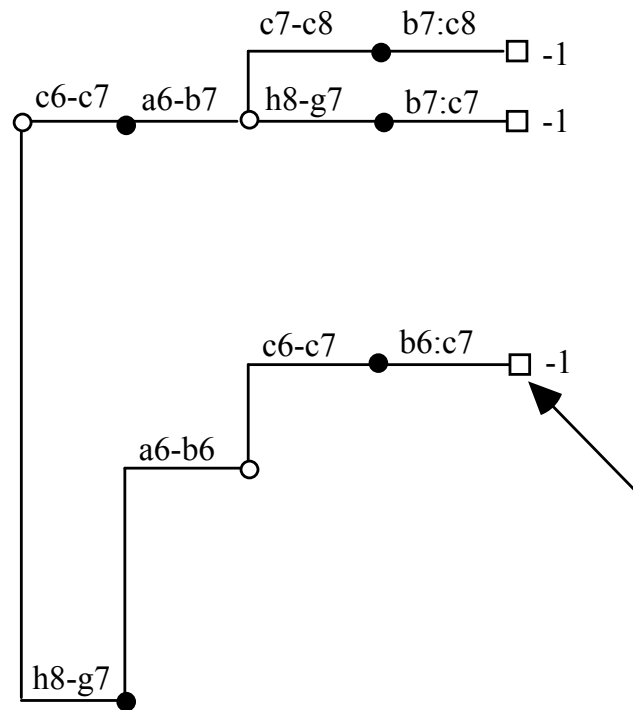




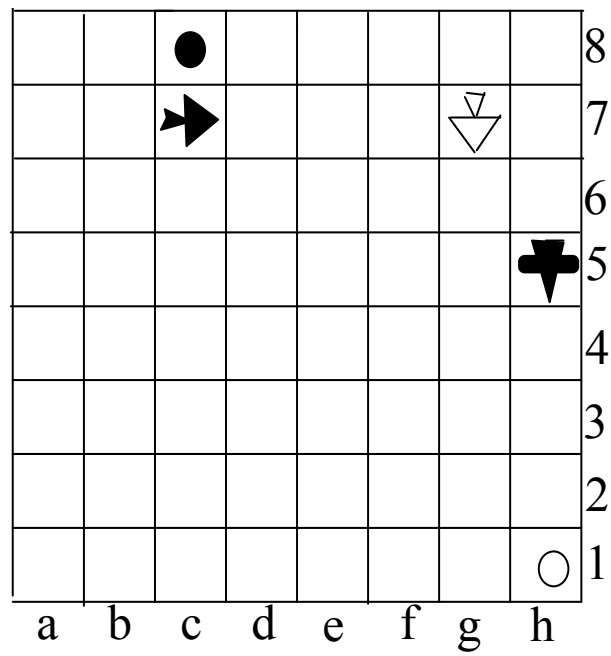
**State where the control Zone from h8 to c8  
was included into the search**





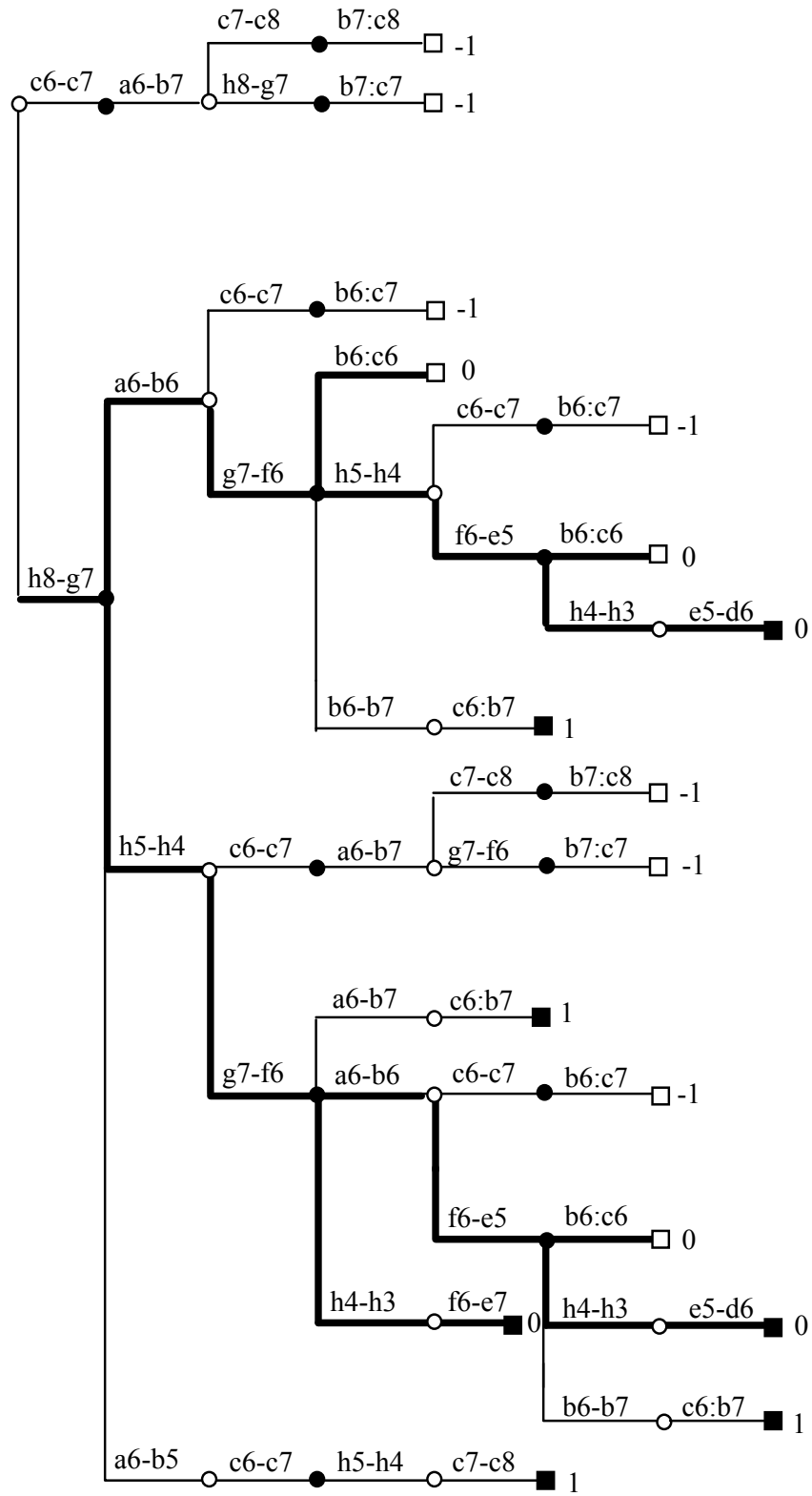


**State where the control Zone from g7 to c7 was detected**





# Search tree for the robotic vehicles with alternating serial motions



### Brute Force Search with limited depth

Add function  $NM(i)$  into **Fvar**. At the beginning of derivation  $NM(i) = 1$  for all  $i$  from  $\mathbf{Z}_+$ . Include new functional formula in the production 2 (section  $\pi_n$ ):

$$NM(i) := NM(i) + 1$$

Function  $MV$  in this case is as follows:

$$MV(r) = r, \quad r \text{ is from } \mathbf{Z}_+.$$

$$NEWMOVE = MV(NM(i)).$$

Predicate CUT:

$CUT = MV(i) > |MOVE|$ , where  $|MOVE|$  is the number of triples in  $MOVE$ .

### Best-Move Search without backtrack

Add function **Value**:  $P \times X \times X \rightarrow \mathbf{R}$  and function **DIRECTION**. The algorithm for computation of **Value** depends on the particular heuristic function.

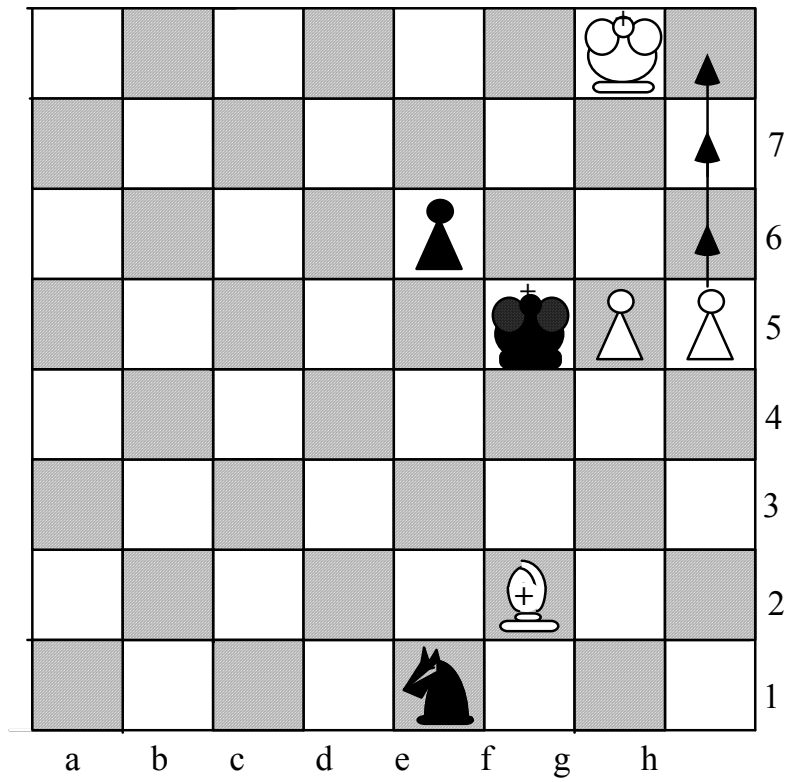
Function  $MV$  is defined as follows:

$MV(i, STATE)$  is the number of triple  $(p_i, x_i, y_i) \in MOVE$  for which **Value** $((p_i, x_i, y_i))$  is maximum.

Add formula **DIRECTION** := 1 into the section  $\pi_n$  of production 1 and

**DIRECTION** := -1 in production 3.

$$CUT = ((MOVE = \emptyset) \vee (DIRECTION = -1))$$

**Generate Search in this Zone**

**A Comparison of Searches  
for the same processing time**

