Geometry of Zones

-----OLD DEFINITIONS----

A ***trajectory connection***
of the trajectories t_1 and t_2 is the relation $C(t_1,t_2)$. It holds, if the *ending link* of the trajectory t_1 coincides with an *intermediate link* of the trajectory t_2;

On the set **A** of trajectories it is defined:

$\mathbf{C_A}^{k}(\mathbf{t_1, t_2})$, a ***k-th degree of connection*** and

$\mathbf{C_A}^{+}(\mathbf{t_1,t_2})$, a ***transitive closure.***

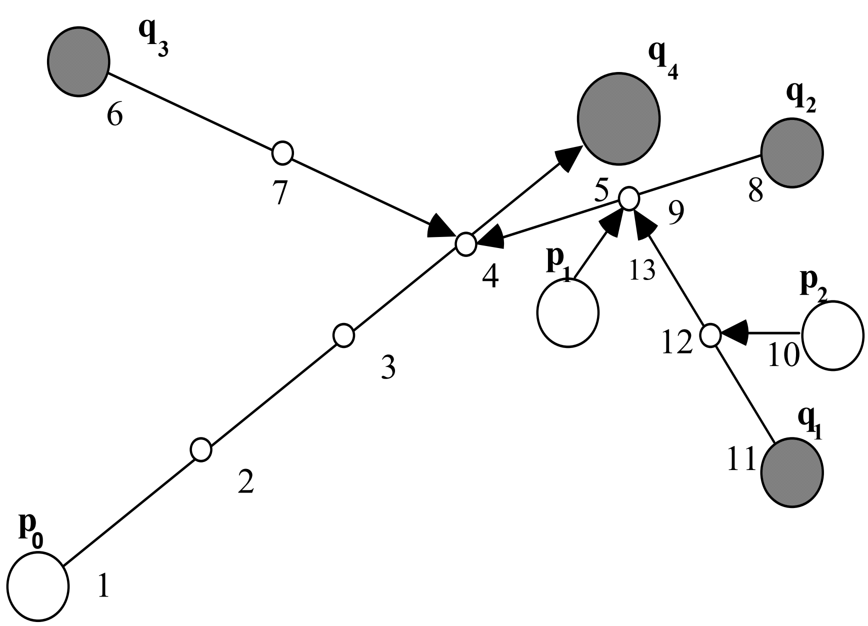
A ***trajectory network*** **W**
relative to trajectory t_o is a finite set of trajectories t_o, t_1,…, t_k from the language $L_t^H(S)$: for every trajectory t_i from W (i=1, 2,…, k) the relation $C_W^+(t_i, t_o)$ holds.

A ***family of trajectory network languages*** $\mathbf{L_C(S)}$
in a state S of the Complex System is the family of languages that contains strings of the form

$$\boldsymbol{t}(t_1,param)\boldsymbol{t}(t_2,param)\ldots\boldsymbol{t}(t_m,param),$$

where *param* in parentheses substitute for the other parameters of a particular language. All the symbols $t_1,t_2,\ldots,t_m$ correspond to trajectories which form a trajectory network **W** relative to t_1.

Network language interpretation.



Language of Zones

Definition
A language $\mathbf{L_Z(S)}$ generated by the grammar $\mathbf{G_Z}$ in a state S of a Complex System is called the ***Language of Zones***.

Grammar of Zones G_Z

L	Q	Kernel, π_k ($\forall z \in X$)	π_n ($\forall z \in X$)	F_T	F_F
1	Q_1	$S(u, v, w) \to A(u, v, w)$		*two*	$\varnothing$
2_i	Q_2	$A(u, v, w) \to t(h_i^o(u), l_o+1)$ $A((0, 0, 0), g(h_i^o(u),w), zero)$	$TIME(z) = DIST(z,h_i^o(u)$	3	$\varnothing$
3	Q_3	$A(u, v, w) \to A(f(u, v), v, w)$	$NEXTTIME(z) = init(u, NEXTTIME(z))$	*four*	5
4_j	Q_4	$A(u, v, w) \to t(h_j(u), TIME(y)))$ $A(u, v, g(h_j(u), w))$	$NEXTTIME(z) = ALPHA(z, h_j(u), TIME(y) - l+1)$	3	3
5	Q_5	$A(u, v, w) \to A((0, 0, 0), w, zero)$	$TIME(z) = NEXTTIME(z)$	3	6
6	Q_6	$A(u, v, w) \to \varepsilon$		$\varnothing$	$\varnothing$

$V_T = \{t\}$, $V_N = \{S, A\}$,

V_{PR}

Pred $=\{Q_1, Q_2, Q_3, Q_4, Q_5, Q_6\}$

$Q_1(u) = (\mathrm{ON}(p_o) = x) \wedge (\mathrm{MAP}_{x,p_o}(y) \le l \le l_o) \wedge$
$(\exists q\, ((\mathrm{ON}(q) = y) \wedge (\mathrm{OPPOSE}(p_o, q))))$

$Q_2(u) = T$

$Q_3(u) = (x \ne n) \vee (y \ne n)$

$Q_4(u) = (\exists p\, ((\mathrm{ON}(p) = x) \wedge (l > 0) \wedge (x \ne x_o) \wedge (x \ne y_o)) \wedge$
$((\neg \mathrm{OPPOSE}(p_o, p) \wedge (\mathrm{MAP}_{x,p}(y) = 1)) \vee$
$(\mathrm{OPPOSE}(p_o, p) \wedge (\mathrm{MAP}_{x,p}(y) \le l)))$

$Q_5(w) = (w \ne zero)$

$Q_6 = T$

Var $= \{x, y, l, \tau, \theta, v_1, v_2, \ldots, v_n, w_1, w_2, \ldots, w_n\}$; for the sake of brevity:
$u = (x, y, l)$, $v = (v_1, v_2, \ldots, v_n)$, $w = (w_1, w_2, \ldots, w_n)$, $zero = (0, 0, \ldots, 0)$

Con $= \{x_o, y_o, l_o, p_o\}$; *Func* = *Fcon* $\cup$ *Fvar*;

Fcon $= \{f_x, f_y, f_l, g_1, g_2, \ldots, g_n, h_1, h_2, \ldots, h_M,$
$h_1^o, h_2^o, \ldots, h_M^o, DIST, init, ALPHA\}$, $f = (f_x, f_y, f_l)$, $g = (g_{x1}, g_{x2}, \ldots, g_{xn})$,
$M = |\mathbf{L}_t^{l_o}(S)|$ is the number of trajectories in $\mathbf{L}_t^{l_o}(S)$

Fvar $= \{x_o, y_o, l_o, p_o, TIME, NEXTTIME\}$

$E = \mathbf{Z}_+ \cup X \cup P \cup \mathbf{L}_t^{l_o}(S)$ is the subject domain;

Parm: $S \varnothing Var$, $A \to \{u, v, w\}$, $t \to \{p, \tau, \theta\}$;

$L = \{1,3,5,6\} \cup two \cup four$, $two = \{2_1, 2_2, \ldots, 2_M\}$, $four = \{4_1, 4_2, \ldots, 4_M\}$

Definition of functions of the Grammar of Zones G_Z

$D(init) = X \times X \times \mathbf{Z}_+ \times \mathbf{Z}_+$

$$\boldsymbol{init}\,(u, r) = \begin{cases} 2n, & \text{if } u = (0,0,0), \\ r, & \text{if } u \neq (0,0,0). \end{cases}$$

$D(\boldsymbol{f}) = (X \times X \times \mathbf{Z}_+ \cup \{0, 0, 0\}) \cup \mathbf{Z}_+^n$

$$f(u,\ v) = \begin{cases} (x+1, y, l), & \text{if } ((x \neq n) \wedge (l > 0)) \vee ((y = n) \wedge (l \leq 0)) \\ (1,\ y+1,\ TIME(y+1) \times v_{y+1}), & \text{if } (x = n) \vee ((l \leq 0) \wedge (y \neq n)). \end{cases}$$

$D(DIST) = X \times P \times \mathbf{L_t}^{\boldsymbol{l_o}}(S)$.

Let $t_o \in L_t^{l_o}(S)$, $t_o = \boldsymbol{a}(z_o)\boldsymbol{a}(z_1)\ldots\boldsymbol{a}(z_m)$, $t_o \in t_{p_o}(z_o, z_m, m)$;

If $((z_m = y_o) \wedge (p = p_o) \wedge (\exists\, k\ (1 \leq k \leq m) \wedge (x = z_k))) \vee$

$(((z_m \neq y_o) \vee (p \neq p_o)) \wedge (\exists\, k\ (1 \leq k \leq m - 1) \wedge (x = z_k)))$

then $DIST(x, p_o, t_o) = k+1$

else $DIST(x, p_o, t_o) = 2n$

$D(ALPHA) = X \times P \times \mathbf{L_t}^{\boldsymbol{l_o}}(S) \times \mathbf{Z}_+$

$$ALPHA\ (x, p_o, t_o, k) = \begin{cases} max\ (NEXTTIME\ (x), k), & \text{if } (DIST\ (x, p_o, t_o) \neq 2n) \\ & \wedge\ (NEXTTIME\ (x) \neq 2n); \\ k, & \text{if } DIST\ (x, p_o, t_o) \neq 2n) \\ & \wedge\ (NEXTTIME\ (x) = 2n); \\ NEXTTIME\ (x), & \text{if } DIST\ (x, p_o, t_o) = 2n). \end{cases}$$

$D(g_r) = P \times \mathbf{L_t}^{\boldsymbol{l_o}}(S) \times \mathbf{Z}_+^n$, $r \in X$.

$$g_{\mathbf{r}}(p_o, t_o, w) = \begin{cases} 1, & \text{if } DIST(r, p_o, t_o) < 2n, \\ w_r, & \text{if } DIST(r, p_o, t_o) = 2n. \end{cases}$$

$D\,(h_i^o) = X \times X \times \mathbf{Z}_+$; Let $TRACKS_{p_o} = \{p_o\} \times (\bigcup_{1 \leq k \leq l} L[\mathbf{G_t}^{(2)}(x, y, k, p_o)]$

If $TRACKS_{p_o} = \varnothing$

then $h_i^o(u) = \varepsilon$

else $TRACKS_{p_o} = \{(p_o, t_1), (p_o, t_2), \ldots, (p_o, t_b)\}, (b \leq M)$ **and**

$$h_i^o(u) = \begin{cases} (p_o, t_i), & \textbf{if } i \leq b, \\ (p_o, t_b), & \textbf{if } i > b. \end{cases}$$

$D(h_i) = X \times X \times \mathbf{Z}_+$; Let $TRACKS_p = \{p\} \times (\bigcup_{1 \leq k \leq l} L[\mathbf{G_t}^{(2)}(x, y, k, p)]$

If $\mathrm{TRACKS}_p = \varnothing$

then $h_i(u) = \varepsilon$

else $\mathrm{TRACKS}_p = \{(p_1, t_1), (p_1, t_2), \ldots, (p_m, t_m)\}, \ (m \le M)$ **and**

$$h_i(u) = \begin{cases} (p_i, t_i), & \textbf{if } i \le m, \\ (p_m, t_m), & \textbf{if } i > m. \end{cases}$$ -

Trajectories t_i should not be embedded (as sub-trajectories) in the trajectories of the same negation.

At the beginning of generation:

$u = (x_o, y_o, l_o)$, $w = zero$, $v = zero$, $x_o \in X$, $y_o \in X$, $l_o \in \mathbf{Z}_+$, $p_o \in P$,

TIME (z) = 2n, *NEXTTIME* (z) = 2n for all z from X.

To study this language formally we need preliminary definitions.

Definition 1.
An ***alphabet*** **A(Z)** ***of the string*** **Z** of the parameter language **L** is the set symbols of this language with given parameter values, where each of the symbols with parameters is included at least once in a string **Z**, and ***e*** (empty symbol).

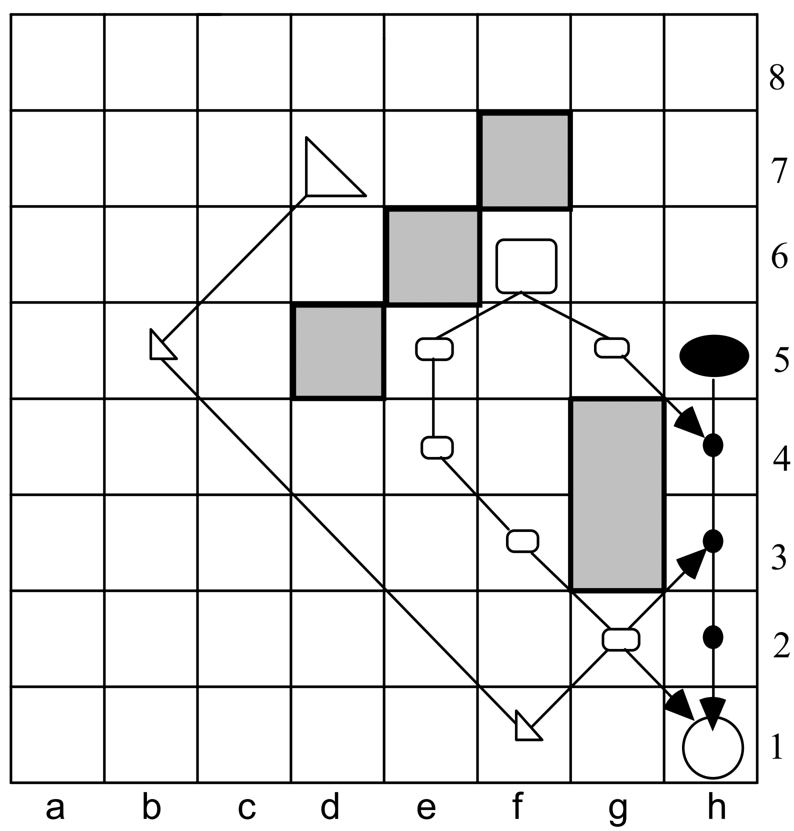
Definition 2.
A ***trajectory alphabet*** **TA(Z)** of the Zone **Z** is the set of trajectories from $\mathbf{L_t^H(S)}$ that correspond to the actual parameter values of the alphabet **A(Z)**.

Example of Zone

$$\boldsymbol{t}(\text{BOMBER}, t_B, 5)\boldsymbol{t}(\text{FIGHTER}, t_F, 5)\boldsymbol{t}(\text{MISSILE}, t_M, 5)\boldsymbol{t}(\text{MISSILE}, t_M^1, 3)$$
$$\boldsymbol{t}(\text{FIGHTER}, t_F^1, 2),$$

where

$$t_B=\boldsymbol{a}(h5)\boldsymbol{a}(h4)\boldsymbol{a}(h3)\boldsymbol{a}(h2)\boldsymbol{a}(h1),$$
$$t_F=\boldsymbol{a}(f6)\boldsymbol{a}(e5)\boldsymbol{a}(e4)\boldsymbol{a}(f3)\boldsymbol{a}(g2)\boldsymbol{a}(h1),$$
$$t_M=\boldsymbol{a}(d7)\boldsymbol{a}(b5)\boldsymbol{a}(f1)\boldsymbol{a}(g2)\boldsymbol{a}(h1),$$
$$t_M^1=\boldsymbol{a}(d7)\boldsymbol{a}(b5)\boldsymbol{a}(f1)\boldsymbol{a}(h3),$$
$$t_F^1=\boldsymbol{a}(f6)\boldsymbol{a}(g5)\boldsymbol{a}(h4)$$



Theorem
For any string Z from $L_Z(S)$, trajectories from TA(Z) form a trajectory network, i.e., $L_Z(S) \in L_C(S)$.

Proof
Let us consider a string $\mathbf{Z} = \boldsymbol{t}(\mathbf{p_o}, \mathbf{t_o}, \boldsymbol{\tau_o}) \ldots \boldsymbol{t}(\mathbf{p_k}, \mathbf{t_k}, \boldsymbol{\tau_k})$.

Grammar of Zones G_Z

Q		Kernel, π_k $(\forall z \in X)$	π_n $(\forall z \in X)$	F_T	F_F
1	Q_1	$S(u, v, w) \rightarrow A(u, v, w)$		*two*	$\varnothing$
2_i	Q_2	$A(u, v, w) \rightarrow t(h_i^o(u), l_o+1)$ $A((0, 0, 0), g(h_i^o(u),w), zero)$	$TIME(z) = DIST(z, h_i^o(u))$	3	$\varnothing$
3	Q_3	$A(u, v, w) \rightarrow A(f(u, v), v, w)$	$NEXTTIME(z) = init(u, NEXTTIME(z))$	*four*	5
4_j	Q_4	$A(u, v, w) \rightarrow t(h_j(u), TIME(y)))$ $A(u, v, g(h_j(u), w))$	$NEXTTIME(z) = ALPHA(z, h_j(u), TIME(y) - l+1)$	3	3
5	Q_5	$A(u, v, w) \rightarrow A((0, 0, 0), w, zero)$	$TIME(z) = NEXTTIME(z)$	3	6
6	Q_6	$A(u, v, w) \rightarrow \varepsilon$		$\varnothing$	$\varnothing$

$V_T = \{t\}$, $V_N = \{S, A\}$,
V_{PR}
$Pred = \{Q_1, Q_2, Q_3, Q_4, Q_5, Q_6\}$
$Q_1(u) = (ON(p_o) = x) \wedge (MAP_{x,p_o}(y) \leq l \leq l_o) \wedge (\exists q\ ((ON(q) = y) \wedge (OPPOSE(p_o, q))))$
$Q_2(u) = T$

Obviously under the condition that the predicate Q_1 is true, the symbol $\boldsymbol{t}(\mathbf{p_o}, \mathbf{t_o}, \boldsymbol{\tau_o})$ is attached to the string by applying the productions **1** and $\mathbf{2_i}$.

The following proof is by induction.

We assume that all the trajectories $\mathbf{TA(Z_m)}$ of the substring

$$\mathbf{Z_m} = t(\mathbf{p_o}, \mathbf{t_o}, \boldsymbol{\tau_o}) \ldots t(\mathbf{p_m}, \mathbf{t_m}, \boldsymbol{\tau_m})$$

form a trajectory network. Symbol $t(\mathbf{p_{m+1}}, \mathbf{t_{m+1}}, \boldsymbol{\tau_{m+1}})$ can be attached to a string only after applying the production $\mathbf{4_j}$.

L	Q	**Kernel,** π_k $(\forall z \in X)$	π_n $(\forall z \in X)$	F_T	F_F
1	Q_1	$S(u, v, w) \to A(u, v, w)$		*two*	$\varnothing$
2_i	Q_2	$A(u, v, w) \to t(h_i^o(u), l_o+1)$ $A((0, 0, 0), g(h_i^o(u),w), zero)$	$TIME(z) = DIST(z,h_i^o(u))$	3	$\varnothing$
3	Q_3	$A(u, v, w) \to A(f(u, v), v, w)$	$NEXTTIME(z) = init(u, NEXTTIME(z))$	*four*	5
4_j	Q_4	$A(u, v, w) \to t(h_j(u), TIME(y)))$ $A(u, v, g(h_j(u), w))$	$NEXTTIME(z) = ALPHA(z, h_j(u), TIME(y) - l+1)$	3	3
5	Q_5	$A(u, v, w) \to A((0, 0, 0), w, zero)$	$TIME(z) = NEXTTIME(z)$	3	6
6	Q_6	$A(u, v, w) \to \varepsilon$		$\varnothing$	$\varnothing$

$V_T = \{t\}$, $V_N = \{S, A\}$,

V_{PR}

$Pred = \{Q_1, Q_2, Q_3, Q_4, Q_5, Q_6\}$

$Q_1(u) = (ON(p_o) = x) \wedge (MAP_{x,p_o}(y) \le l \le l_o) \wedge$
$(\exists q\ ((ON(q) = y) \wedge (OPPOSE(p_o, q))))$

$Q_2(u) = T$

$Q_3(u) = (x \ne n) \vee (y \ne n)$

$Q_4(u) = (\exists p\ ((ON(p) = x) \wedge (l > 0) \wedge (x \ne x_o) \wedge (x \ne y_o)) \wedge$
$((\neg OPPOSE(p_o, p) \wedge (MAP_{x,p}(y) = 1)) \vee$
$(OPPOSE(p_o, p) \wedge (MAP_{x,p}(y) \le l)))$

$Q_5(w) = (w \ne zero)$ $\qquad Q_6 = T$

Among the parameters of the trajectory $\mathbf{t_{m+1}} \in \mathbf{t_p(x, y, \mathit{l})}$ we are interested in the value of **y**, the parameter value of the **last symbol** of the trajectory. One can pass to the production with the label $\mathbf{4_j}$ only after a successful application of a production with the label **3**, i.e., in F_T case. Here the $f(u, v)$ function changes the value of the parameter $u = (\mathbf{x}, \mathbf{y}, \mathit{l})$.

L	Q	Kernel, π_k $(\forall z \in X)$	π_n $(\forall z \in X)$	F_T	F_F
1	Q_1	$S(u, v, w) \to A(u, v, w)$		*two*	$\varnothing$
2_i	Q_2	$A(u, v, w) \to t(h_i^o(u), l_o+1)$ $A((0, 0, 0), g(h_i^o(u),w), zero)$	$TIME(z) = DIST(z,h_i^o(u))$	3	$\varnothing$
3	Q_3	$A(u, v, w) \to A(f(u, v), v, w)$	$NEXTTIME(z) = init(u, NEXTTIME(z))$	*four*	5
4_j	Q_4	$A(u, v, w) \to t(h_j(u), TIME(y)))$ $A(u, v, g(h_j(u), w))$	$NEXTTIME(z) = ALPHA(z, h_j(u), TIME(y) - l+1)$	3	3
5	Q_5	$A(u, v, w) \to A((0, 0, 0), w, zero)$	$TIME(z) = NEXTTIME(z)$	3	6
6	Q_6	$A(u, v, w) \to \varepsilon$		$\varnothing$	$\varnothing$

$V_T = \{t\}$, $V_N = \{S, A\}$,

V_{PR}

$Pred = \{Q_1, Q_2, Q_3, Q_4, Q_5, Q_6\}$

$Q_1(u) = (ON(p_o) = x) \wedge (MAP_{x,p_o}(y) \le l \le l_o) \wedge$
$(\exists q\ ((ON(q) = y) \wedge (OPPOSE(p_o, q))))$

$Q_2(u) = T$

$Q_3(u) = (x \ne n) \vee (y \ne n)$

$Q_4(u) = (\exists p\ ((ON(p) = x) \wedge (l > 0) \wedge (x \ne x_o) \wedge (x \ne y_o)) \wedge$
$((\neg OPPOSE(p_o, p) \wedge (MAP_{x,p}(y) = 1)) \vee$
$(OPPOSE(p_o, p) \wedge (MAP_{x,p}(y) \le l)))$

$Q_5(w) = (w \ne zero)$

$Q_6 = T$

$$f(u, v) = \begin{cases} (x+1, y, l), & \text{if } ((x \ne n) \wedge (l > 0)) \vee ((y = n) \wedge (l \le 0)) \\ (1,\ y+1,\ TIME(y+1) \times v_{y+1}), & \text{if } (x = n) \vee ((l \le 0) \wedge (y \ne n)). \end{cases}$$

The last change in the course of derivation of the value of v_y could occur only in a successful application of a production with the label **5** (if we already generated at least one 1st negation trajectory). Here, after applying the production, v_y was given the value of w_y. Consequently, $w_y \ne \mathbf{0}$.

Finally, such a change of the value of w_y for which it would become different from zero, could take place only in a successful application, earlier in the derivation, of one of the productions with the label $\mathbf{4}_j$.

L	Q	Kernel, π_k ($\forall z \in X$)	π_n ($\forall z \in X$)	F_T	F_F
1	Q_1	$S(u, v, w) \to A(u, v, w)$		*two*	$\varnothing$
2_i	Q_2	$A(u, v, w) \to t(h_i^o(u), l_o+1)$ $A((0, 0, 0), g(h_i^o(u), w), zero)$	$TIME(z) = DIST(z, h_i^o(u))$	3	$\varnothing$
3	Q_3	$A(u, v, w) \to A(f(u, v), v, w)$	$NEXTTIME(z) = init(u, NEXTTIME(z))$	*four*	5
4_j	Q_4	$A(u, v, w) \to t(h_j(u), TIME(y)))$ $A(u, v, g(h_j(u), w))$	$NEXTTIME(z) = ALPHA(z, h_j(u), TIME(y) - l+1)$	3	3
5	Q_5	$A(u, v, w) \to A((0, 0, 0), w, zero)$	$TIME(z) = NEXTTIME(z)$	3	6
6	Q_6	$A(u, v, w) \to \varepsilon$		$\varnothing$	$\varnothing$

$$\boldsymbol{g}_r(p_o, t_o, w) = \begin{cases} 1, & \text{if } \boldsymbol{DIST}(r, p_o, t_o) < 2n, \\ w_r, & \text{if } \boldsymbol{DIST}(r, p_o, t_o) = 2n. \end{cases}$$

Let $t_o \in L_t^{l_o}(S)$, $t_o = \boldsymbol{a}(z_o)\boldsymbol{a}(z_1)\ldots\boldsymbol{a}(z_m)$, $t_o \in t_{p_o}(z_o, z_m, m)$;

If $((z_m = y_o) \wedge (p = p_o) \wedge (\exists\, k\, (1 \le k \le m) \wedge (x = z_k))) \vee$

$(((z_m \ne y_o) \vee (p \ne p_o)) \wedge (\exists\, k\, (1 \le k \le m - 1) \wedge (x = z_k)))$

then $DIST(x, p_o, t_o) = k+1$

else $DIST(x, p_o, t_o) = 2n$

$D(\boldsymbol{h}_i) = X \times X \times \mathbf{Z}_+$; Denote $\text{TRACKS} = \bigcup_{ON(p)=x} \text{TRACKS}_p$, where TRACKS_p is the same as for $\boldsymbol{h}_i^o$

If $\text{TRACKS} = e$

then $\boldsymbol{h}_i(u) = e$

else $\text{TRACKS} = \{(p_1, t_1), (p_1, t_2), \ldots, (p_m, t_m)\}$, $(m \le M)$ **and** $\boldsymbol{h}_i(u) = \begin{cases} (p_i, t_i), & \text{if } i \le m, \\ (p_m, t_m), & \text{if } i > m. \end{cases}$

This means that at some stage of derivation symbol $\boldsymbol{t}(\mathbf{p_j}, \mathbf{t_j}, \boldsymbol{\tau}_j)$ was included in the string Z. At the same time, the parameter $\boldsymbol{w^o} = (w_1^o, \ldots, w_n^o)$ was changed under the action of the function $\mathbf{g}(\boldsymbol{h}_j(u), w^o)$ in such a way that $w_y = \mathbf{g}_y(\boldsymbol{h}_j(u), w^o)$.

But $w_y \neq 0$; consequently, $w_y = 1$, i.e., $DIST(y, p_j, t_j) < 2n$, and hence, **y** is included among the parameter values of the t_j trajectory. In addition, obviously, this trajectory is included among the trajectories $t_o, t_1, ..., t_m$, since symbol $t(p_j, t_j, \tau_j)$ was included in **Z** earlier in the course of derivation.

In accord with **<u>Definition of Trajectory Connection</u>**, $\exists\, t_i$ from the set $t_o, t_1, ..., t_m$ such that trajectory t_{m+1} is connected with trajectory t_i, i.e.,

$$C(t_{m+1}, t_i) = T \text{ holds, with } i \leq m.$$

By the assumption of induction

$$C^+_{TA(Z)}(t_i, t_o) = T$$

and we conclude that $C^+_{TA(Z)}(t_{m+1}, t_o) = T$ (because of the <u>transitivity</u> of C^+).

Thus all the trajectories $t_o, t_1, ..., t_{m+1}$ form a trajectory network.

<u>The theorem is proved</u>.